THREE ESSAYS ON MICROECONOMICS

Fee Evasion in Marketplaces, Buy-In Tournaments and Technology Theft in Joint Ventures

Inaugural-Dissertation
zur Erlangung des Grades
Doctor oeconomiae publicae (Dr. oec. publ.)
an der Ludwig-Maximilians-Universität München

2008

vorgelegt von

Tu-Lam Pham

Referent:	Prof. Sven Rady, PhD
Korreferentin:	Prof. Dr. Monika Schnitzer
Datum der mündlichen Prüfung:	30. Januar 2009
Promotionsabschlussberatung:	04. Februar 2009

DANKSAGUNG

Für eine erfolgreiche Promotion bedarf es guter Ideen, etwas Durchhaltevermögen, vor allem jedoch der Unterstützung all jener Personen, die mich während dieser Zeit und bereits davor mit ihrem Rat, ihrem Einsatz und ihrer Freundschaft begleitet und damit diese Arbeit ermöglicht haben. Bei diesen Personen möchte ich mich an dieser Stelle ganz herzlich bedanken.

Zunächst möchte ich meinem Erstbetreuer Prof. Sven Rady, PhD danken, den ich während meines Masterstudiums an der Universität Toulouse kennen gelernt habe. Während meiner Promotion hat er mir stets die Freiheit gelassen, auch unkonventionellen Fragestellungen nachzugehen und mich dabei stets mit seinem Rat unterstützt.

Weiterhin möchte ich mich bei meiner Zweitgutachterin Prof. Dr. Monika Schnitzer bedanken, die mir zu jedem Kapitel meiner Dissertation wertvolle Anregungen lieferte und mir die Möglichkeit gab, meine Arbeit regelmäßig im Rahmen ihres Lehrstuhlseminars vorzustellen und zu diskutieren.

Ebenfalls möchte ich mich bei Prof. Dr. Martin Kocher bedanken, der netterweise die Rolle des Drittgutachters übernommen hat.

Ich hatte das Glück, bereits zu Beginn meiner Promotion Basak Akbel, Jan Philipp Bender und Maria Lehner kennen zu lernen. Unsere Forschungsgruppe (manchmal eher Selbsthilfegruppe :-) hat mir viel Freude bereitet und mich inhaltlich sehr weiter gebracht. Weiterhin möchte ich Zouhair Belkoura, Benno Bühler und Joachim Klein für ihre wertvollen Ideen und Anregungen danken.

Ich möchte mich ebenfalls bei Manuela Beckstein für ihre Unterstützung sowie bei den Teilnehmern des Economic Theory Seminars und des Industrial Organization and Finance Seminars für ihre hilfreichen Kommentare und Anregungen bedanken.

Ich möchte auch ganz herzlich jenen Personen danken, die mich an meinen vorherigen akademischen Stationen an der Universität Mannheim, der Universität Toulouse und der Universität Cambridge unterstützt haben. Hervorheben möchte ich dabei Prof. Dr. Oliver Kirchkamp und PD Dr. Matthias Staat, die meine Begeisterung für die Mikro- und Industrieökonomik geweckt haben und sich stets für eine ausgezeichnete Lehre engagierten. Ebenfalls möchte ich Prof. Claude Crampes, Prof. Helmuth Cremer, Prof. Sanjeev Goyal, Prof. David Newbery, Prof. Dr. Martin Peitz, Prof. Jean-Charles Rochet und Aude Schloesing für ihre Unterstützung danken.

Für ihre großzügige finanzielle und ideelle Förderung möchte ich der Stiftung der Deutschen Wirtschaft, dem Förderverein Kurt Fordan für herausragende Begabungen e.V. sowie der Europäischen Union danken, die mir mein Studium in dieser Form erst ermöglich haben.

Ein großer Dank gebührt meinen Eltern, meiner Schwester und meiner gesamten Familie, die mich stets gefördert, bei meinen Entscheidungen unterstützt und mir dabei alle Freiheiten gelassen haben.

Weiterhin möchte ich meinen Freunden Lorenz, Lu, Matthias, Michael, Sebastian, Sebastian und Zouhair für ihre langjährige Freundschaft und Unterstützung sowie viele heitere Momente danken.

Ich möchte diese Arbeit Maria widmen und ihr für ihre große Unterstützung und die schönen letzten Jahre danken.

München, 08. Februar 2009 Tu-Lam Pham

Contents

List of Figures

Preface

Microeconomists study the economic behavior of households and firms, from individual decision problems to situations of strategic interaction. One might say that they spend a little too much "quality time" with the books of their good friends Hal (Varian), Jean (Tirole) and Andreu (Mas-Colell). The upside though is that after years of absorbing the theory and solving problem set after problem set, they are able to tackle and analyze new developments in the constantly-evolving business world and even make fairly educated guesses as to what optimal strategies and policies could involve.

In this thesis, we use microeconomic tools to apply and newly develop models of strategic interaction that aim to analyze and explain recent real-world phenomena. This dissertation consists of three self-contained essays on topics in industrial organization. Chapter 1 looks at the incentives of buyers and sellers to bypass a platform when they engage in a repeated business relationship. In chapter 2, we analyze buy-in tournaments, where the prize pool consists of the participants' entry fees, as opposed to traditional tournaments, where the prizes are usually provided by an external sponsor. In chapter 3, we analyze a multinational's incentives to enter a foreign market via a joint venture with a local firm, if it knows that the local firm might steal its technology and become a competitor not only in the foreign market, but also in the multinational's home market.

In Chapter 1, we examine the problem of fee evasion in platform-intermediated transactions, which have become increasingly important due to the rapid diffusion of the internet. In recent years, a large number of marketplaces have emerged where people can easily trade goods and services. Among them are global ecommerce giants like the auction platform eBay, but also more local institutions like dating clubs or platforms where one can hire babysitters. These marketplaces or platforms help buyers and sellers to find each other and facilitate the transactions between them. Without the platform, buyers and sellers would probably not even know of the other one's existence, so that the platform provides a valuable service in helping economic agents realize mutual gains of trade.

The platform's business model typically consists of charging a commission fee for providing this service. This fee may decrease the buyer's and the seller's gains of trade, which is why they might look for ways to use the platform's service but to avoid paying the commission fees. The incentives of buyers and sellers to avoid the commission fee threaten the platform's business model and therefore its very existence. This problem is particularly severe if buyers and sellers engage in a repeated business relationship that involves multiple transactions (like tutoring or babysitting), since buyers and sellers have great incentives to only use the platform in order to find each other, but to carry out all follow-up transactions without involving the platform. Hence, the platform enables both parties to carry out multiple transactions, but may only be able to charge a fee for the initial transaction.

Although there exists a large literature on optimal platform pricing in two-sided markets, no paper has yet studied the incentives of buyers and sellers to bypass the platform in order to save on the commission fee. The lack of research in this area is very surprising since this problem not only affects new internet marketplaces, but also more traditional intermediaries like real estate agents or job agencies. We contribute to the literature by establishing a model of fee evasion in repeated platform-intermediated transactions.

We consider a single platform that enables matches between a single seller and multiple buyers and charges a commission fee for this service. We distinguish between two groups of buyers: independent buyers, who are always willing to buy the seller's product and reputation buyers, whose purchasing decisions depend on the seller's reputation. The seller can propose to the independent buyers to make transactions outside the platform in order to avoid the commission fee. While this increases the seller's profit per transaction, it also sends a negative signal to the prospective reputation buyers who interpret a high number of official transactions over the platform as a positive signal about the seller's quality. Hence, from the seller's perspective, bypassing the platform and saving on the commission fee comes at the cost of fewer reputation buyers. The platform anticipates the seller's behavior and sets its commission fee accordingly.

Surprisingly, we find that in equilibrium the seller makes more official transactions if bypassing the platform is particularly profitable. This counter-intuitive result is due to the fact that private transactions hurt the platform in two ways: first, the platform does not earn a commission fee for the private transactions between the seller and the existing independent buyers and second, the lack of official transactions acts to decrease the number of new reputation buyers, which also decreases the platform's profits. Hence, it has a strong interest to set a low commission fee in order to make official transactions more attractive for the seller. This commission fee has to be lower the more attractive private transactions are. Interestingly, our model predicts that the platform will lower its fee to the point where the seller finds it profitable to declare a high share of follow-up transactions in order to increase his reputation and attract more reputation buyers. We further demonstrate that if the seller and the platform cooperate, the platform sets its fee to zero and the seller only makes official transactions, which maximizes the number of transaction as well as social welfare.

In chapter 2, we analyze buy-in tournaments, a new form of competition that has emerged in recent years. In a buy-in tournament, participants compete for a prize pool that consists of the sum of their entry fees. This is very different from traditional non-buy-in tournaments, where the prizes are usually provided by an external sponsor rather than by the players themselves. Buy-in tournaments are most prevalent in the world of the card game poker, where casinos and online poker rooms organize thousands of tournaments every day with buy-ins starting from as low as 10 cents up to $ 100,000 per participant. Other examples of buy-in tournaments include online business contests as well as chess and backgammon contests.

To date, the tournament literature has focused exclusively on non-buy-in tournaments, which have mainly been analyzed in the context of sports contests, political elections or promotion procedures in internal labor markets. However, there is no theoretical work on buy-in tournaments, which is surprising since they exhibit very interesting economic features that are widely applicable and that should make them a fascinating subject of study for economists in general and researchers in microeconomics and mechanism design in particular. We contribute to the literature by building a theoretical model of buy-in tournaments that studies the players' incentives to participate in tournaments with different buy-in levels.

In our model, players of different skill levels compete for a prize pool that is the sum of the players' entry fees. We first look at how the participation of a particular player affects the utility of all other players in a game with a given buy-in. A distinct feature of buy-in tournaments is that the effect is ambiguous: one the one hand, each player decreases the other players' chances of winning, on the other hand, his buy-in increases the prize pool. Second, we study the players' game selection problem when there are several tournaments available that differ in their buy-in level and difficulty. Finally, we investigate how a tournament organizer can achieve self-selection of the players according to their skill level by offering a menu of tournaments.

Our main results can be summarized as follows: first, the effect of a particular player on the other players' expected utilities does not depend on his absolute skill level, but on his skill level relative to the average of the field. More precisely, the effect is positive if his skill level is below the average of the field and negative, if his skill level is above the average. This ambiguous effect is in contrast to traditional tournaments, where additional players always decrease the other players' utilities, as they decrease everyone else's chances of winning, while the prize pool remains the same.

Second, we find that tournaments with higher buy-in levels and higher potential profits are also associated with more skillful competitors. Hence, players face a trade-off between their chances of winning and the expected prize money. We show that under certain conditions, players with different skill levels also choose different optimal buy-in levels, thereby truthfully revealing their skill level.

Finally, we show that a tournament organizer can achieve self-selection of the different types by offering a menu of tournaments and selecting a correct ratio between the buy-ins.

In Chapter 3, we analyze the problem of technology theft in international joint ventures. In recent years, world markets have increasingly been flooded by counterfeited products, which represents a major concern for the companies that have invested great resources in creating innovative technologies and unique intellectual property, only to see product pirates reap part of the benefits. At the origin of the technology theft is often a multinational's decision to form a joint venture with a local firm in order to enter a developing market (e.g. China). Host country governments often require such joint ventures with the intention that the local firm will learn as much as possible about the multinational's technology and management skills. Once the local firm has acquired the multinational's know-how, it is in prime position to break up the joint venture and compete with the multinational. So by agreeing to the joint venture, the multinational creates a potential competitor, not only in the host country, but also in the multinational's home markets.

Foreign market entry has been extensively studied in the international trade literature. However, the papers focus almost exclusively on the foreign market's characteristics in determining the multinational's entry incentives. Our approach is different, as we are interested in the implications that the multinational's entry decision has on its home market. We contribute to the literature by establishing a model of foreign market entry, technology theft and competition, where we focus on the competitive pressure in the multinational's home market as the main determinant for the entry decision.

In our model, we consider a multinational that can enter a foreign market through a joint venture with a local firm. The multinational is aware of the risk that the local company may copy its technology and become a competitor not only in the foreign market, but also in the home market. We first study the joint venture relationship between the two firms. In particular, we are interested in whether the multinational wants to implement cooperation by the local company by offering a one-time payment. We then analyze the multinational's incentives to enter the foreign market if it knows that this will create a new competitor.

We find that if the local company enters the multinational's home market, the negative impact on the multinational's profits is lower the higher the initial level of domestic competitive pressure. We further show that the multinational's incentives to protect its home market by implementing cooperation depend on the level of domestic competition in a non-monotonic way: the incentives are highest when the multinational is a monopolist, lowest for low levels of domestic competitive pressure and increasing but negative as competition increases.

We are particularly interested in the way that domestic competitive pressure affects the multinational's incentives to enter the foreign market. First, we consider the case where the multinational's entry motive is market expansion. We show that its entry incentives are strictly increasing in the number of domestic firms. Second, we analyze the case in which the multinational's motivation for entry is cost reduction. The relationship between the multinational's entry incentives and the number of domestic firms depends on the margins in the home market and the degree of cost reduction. Our main findings are that if margins are low, the entry incentives are strictly increasing and concave in the number of domestic firms. If margins are high, the entry incentives initially increase and then decrease in the number of firms.

Our work also makes an interesting contribution to the literature on innovation and market structure. In most of the literature, the costs of innovation include expenses for patents, licences and/or research and development. These costs are usually fixed and independent of the market structure. In our model, entering the foreign market creates a new competitor in the home market, which alters the market structure and increases competitive pressure. Hence, the cost of pursuing the cost-reducing innovation is not a monetary expense, but the emergence of a new competitor. The cost of entry is the negative impact of the new competitor and depends on the market structure at home.

Chapter 1

Fee Evasion and Seller-Reputation in Marketplaces

1.1 Introduction

In this Chapter, we analyze fee evasion in platform-intermediated transactions, which have become increasingly important due to the rapid diffusion of the internet. Intermediaries help sellers and buyers to find each other and facilitate the transactions between both parties. Auctions sites like eBay come to mind, as well as marketplaces for babysitters, piano teachers and other freelance workers, but also brick-and-mortar institutions like real estate agents (we will refer to all these marketplaces and intermediaries as platforms). Without the platform, buyers and sellers would probably not even know of the other one's existence, so the platform provides a valuable service in helping economic agents realize mutual gains of trade. The platform's business model consists of charging a commission fee for providing this service. This fee may decrease the buyer's and the seller's gains of trade. Thus, they look for ways to use the service of the platform but to avoid paying the commission fees. The incentives of buyers and sellers to avoid the commission fee threaten the platform's business model and therefore its very existence.

At eBay, the world's largest online marketplace, the commission fee is between six and fifteen percent of the final sales price. The sellers who sell the most via eBay, so-called powersellers, pay many hundreds of thousands of dollars per year in commission fees alone. Naturally, the sellers try to find ways to bypass eBay in order to save on the commission fee, two of which are redirection and price-shifting. Redirection is the common practice by a seller to post a link on its eBay site that directs the prospective buyer to the seller's website outside of eBay where no commission fees apply. So the seller uses eBay's popularity to acquire customers, but tries to deal with them outside of eBay. Another way for the seller to save on the commission fee is by shifting prices: for the product itself, he quotes a price that is much lower than the normal price, but charges very high shipping costs, so that the final price reflects the real value of the product. The seller does this because eBay only charges its fees on the sales price, but not on shipping costs, so price-shifting allows the seller to reduce his fees. While these practices are a problem for eBay, they do not threaten eBay's business model, since most buyers dislike to pay high shipping costs (even if the final price is the same) and prefer to buy at eBay rather than at the seller's own website, especially if the buyer does not know the seller (buyers can punish seller's with negative ratings at eBay, something they cannot do at the seller's own shop). In general, fee evasion is not that big of a problem on platforms like eBay, where buyers and sellers usually only interact once, and therefore depend more on the platform's service.

Fee evasion is a much more severe problem if buyers and sellers engage in a repeated relationship that involves multiple transactions. Buyers and sellers have great incentives to only use the platform in order to find each other and to bypass the platform in all future follow-up transactions in order to save on the commission fee which is then shared between the two parties. Hence, the platform enables both parties to carry out multiple transactions, but may only be able to charge a fee for the initial one. This conflict of interest between buyers and sellers on the one side and the platform on the other side arises in many markets. Think of parents who make private arrangements for future transactions with their children's music teachers, tutors and babysitters after the parents have found them through an agency or platform; landlords of summer residences who want to let their houses to the same tenant every year without involving the agency that matched landlord and tenant in the first place; advertising companies who would like to book the same model for several campaigns without having to pay the model agency every time. In this study, we are interested in such repeated platform-intermediated relationships. Our analysis applies most to platforms for freelance workers like getacoder.com (market for programmers), myhammer.de (market for craftsmen) or care.com (marketplace for babysitters, petsitters etc.).

Obviously, the platform wants all follow-up transactions to be carried out over the official sales channel rather than privately. The challenge for the platform is to incentivize the seller to officially declare his follow-up transactions with the platform so that it can charge commission fees. One way for the platform to achieve this is by making the transaction history of the seller public information which allows the seller to build a positive reputation on the platform that will help him attract new customers. This creates a trade-off for the seller: on the one hand, private transactions allow him to save on part of the commission fee, on the other hand, official transactions help him attract new customers. Examples of such reputation-building schemes can be found at internet marketplaces like eBay, Amazon or Yahoo!

Although there exists a large literature on optimal platform pricing in two-sided markets, no paper has yet studied the incentives of buyers and sellers to bypass the platform in order to save on the commission fee. The lack of research in this area is very surprising since this problem not only affects new internet marketplaces, but also more traditional intermediaries like real estate agents or job agencies. We contribute to the literature by establishing a model of fee evasion in repeated platform-intermediated transactions. .

We consider a single platform that enables matches between a single seller and multiple buyers and charges a commission fee for this service. We distinguish between two groups of buyers: independent buyers, who are always willing to buy the seller's product and reputation buyers, who base their purchasing decisions on the seller's reputation. The seller can propose to the independent buyers to make transactions outside of the platform in order to save on the commission fee, which buyer and seller then split. While this increases the seller's profit per transaction, it also sends a negative signal to the prospective reputation buyers, who interpret a high number of official transactions as a positive signal about the seller's quality. This creates an interesting trade-off for the seller: if he declares a small number of repeat transactions, this increases his profits from the transactions with the independent buyers, but decreases his reputation and hence the number of reputation buyers. The seller's incentives to bypass the platform depend on the platform's commission fee and the general attractivity of the private transactions. The platform anticipates the seller's behavior and sets its optimal commission fee accordingly. The platform faces the following tradeoff: while a high commission fee increases the profit per transaction, it also reduces the number of official repeat transactions, which in turn decreases the number of reputation buyers. The higher the commission fee, the lower the number of transactions that will be carried out over the platform.

Our results show that the platform's optimal fee and the seller's optimal share of official follow-up transactions depend on two main parameters: the first parameter is the number of independent buyers who do not base their purchasing decisions on the seller's reputation. The higher the number of independent buyers, the less does the seller depend on attracting reputation buyers. The second parameter is the seller's ability to capture a high share of the saved commission fee when making a private transaction with a buyer. The higher these two parameters, the more attractive are private transactions for the seller.

Surprisingly, we find that in equilibrium the seller makes more official transactions if bypassing the platform is particularly attractive. This counter-intuitive result is due to the fact that private transactions hurt the platform in two ways: first, the platform does not earn a commission fee for the private transactions between the seller and the existing independent buyers and second, the lack of official transactions acts to decrease the number of new reputation buyers, which also decreases the platform's profits. Since the platform anticipates the seller's behavior, it has great incentives to set a low commission fee in order to make official transactions more attractive for the seller. This commission fee has to be lower the more attractive private transactions are. We show that the platform lowers its fee to the point where the seller finds it profitable to declare a high share of follow-up transactions in order to increase his reputation and attract more reputation buyers.

We also look at the case in which platform and seller behave cooperatively in order to maximize the total number of transactions. We find that this increases the joint profits of the platform because the platform will find it optimal to set its commission fee to zero and as a result, the seller will officially declare all of his follow-up transactions, which attracts the most reputation buyers and maximizes the number of customers. The effect on consumer surplus is ambiguous, since there are some buyers who benefit from the cooperation between platform and seller and other buyers who prefer platform and seller to act independently from each other. We show that social welfare is unambiguously higher in the cooperative setting.

The remainder of this chapter is structured as follows. After reviewing the related literature in Section 1.2, we set up a two-period model in Section 1.3 that illustrates the conflict of interest between platform and seller when buyers and sellers engage in a repeated relationship. In Section 1.4, we derive the platform's optimal commission fee and the seller's optimal share of official follow-up transactions. In Section 1.5, we then study the effect of relevant parameters on the platform's and the seller's equilibrium choices. In Section 1.6, we consider the case in which platform and seller behave cooperatively. Section 1.7 concludes. All proofs are relegated to the Appendix.

1.2 Related Literature

In recent years, platforms have received great interest in the economic literature. The literature on two-sided markets has focused on platform intermediaries, emphasizing the indirect network effects which arise between the two sides of the market when the latter have to affiliate with the platforms in order to be able to transact with one another. Examples for such two-sided markets include marketplaces like eBay where buyers and sellers meet; dating clubs, where women and men hope to find a match; video game systems, where consumers play the software companies' games; TV channels, where consumers watch advertising; and many more. The most important contributions are by papers by Caillaud and Jullien (2003), Rochet and Tirole (2003) and (2006) and Armstrong (2006). Virtually all of these papers focus on the pricing structure chosen by two-sided platforms in order to internalize (partially) network externalities. The main finding is that in order to overcome the "chicken and egg" problem, the platform should subsidize the members of the side of the market, that attract the most members of the other side of the market. This explains why women usually pay no membership fees for dating clubs while men do and why watching TV is (mostly) free for consumers, but airing advertising is costly for firms.

Another related strand of literature concerns the effect of seller reputation on prices. Theoretical models have typically generated a positive relationship between seller reputation and the price (Klein and Leffler (1981), Shapiro (1983), Allen (1984), Houser and Wooders (2000)) in large part because the seller's reputation is a proxy for quality characteristics that are unobserved prior to the transaction. Important contributions have also been made by Tadelis (1999, 2002, 2003, 2008). There also exist recent empirical studies that analyze extensive data collected from internet-based auction websites, including the website's own index of the seller's reputation, to estimate the impact of reputation on the price of the seller's product. Melnik and Alm (2002) and Houser and Wooders (2006) both find that bidders pay an economically and statistically significant premium to sellers with better reputation.

1.3 The Model

Consider a single seller S selling a product for a fixed price $p = 1$. We assume that the seller has zero costs. There are several buyers B who each have a valuation of v for the seller's product. Each buyer only buys one unit and his utility from a transaction is given by $U = v - 1$. Transactions between both parties may be intermediated by a platform P which charges a commission fee $0 < \alpha < 1$ for each transaction. We assume that the fee is being paid by the seller[1]. The platform has zero cost. We further assume that buyer and sellers only need the platform's services for their first transaction but are able to carry out all further transactions without the platform. We distinguish between two groups of buyers: independent buyers and reputation buyers. We assume that there are x independent buyers who are always willing to purchase the seller's

[1] This is the case at eBay, care.com and many other platforms.

product. Reputation buyers however are somewhat more cautious and base their purchasing decisions on the seller's reputation.

Consider a two period setting, with $t = 1, 2$. In $t = 1$, the seller uses the platform in order to acquire x new independent buyers. The profit of the seller is given by $(1 - \alpha) x$ and the platform's commission revenue is αx. We assume that all independent buyers are also willing to buy in $t = 2$. Now that the seller and the independent buyers know each other, they could carry out their follow-up transaction in $t = 2$ privately and without involving the platform, thereby saving on the commission fee. If both parties agree to this, they split the saved commission fee in a way that makes them both better off compared to an official transaction over the platform. Let $0 < \beta < 1$ be the share of the commission fee that the seller can capture in private transactions. We also refer to this as the seller's bargaining power. Hence, in a private transaction, the seller receives a price $p' = 1 - \alpha (1 - \beta) > 1 - \alpha$ and the buyer's utility is given by $U' = v - (1 - \alpha (1 - \beta)) > v - 1$. We assume that if the seller proposes a private transaction, the buyer will always accept. We denote the number of official follow-up transactions in $t = 2$ by x^O and the number of private follow-up transactions by x^P. Since we assume that all independent buyers also buy in $t = 2$, we have $x^O + x^P = x$. We define $0 \leq \gamma \equiv \frac{x^O}{x} \leq 1$ as the share of follow-up transactions that the seller officially declares.

We now turn to the reputation buyers, who enter the market in $t = 2$. The platform makes the number of official repeat purchases $x^O = \gamma x$ public information. We assume that reputation buyers interpret a high number x^O as a positive signal about the seller's quality, since a customer is more likely to do a repeat purchase with a high quality seller. We further assume that the number of reputation buyers is increasing and concave in the number of official follow-up transactions. This is in line with the empirical observation that a seller has diminishing marginal benefits of reputation. On eBay, for example, the first 50 positive user comments greatly increase the seller's reputation and have a positive effect on his profits, but another 1000 positive ratings are relatively less valuable. With respect to the number of reputation buyers, we choose the simple function $x^R = f(x^O) = \sqrt{\gamma x}$ that captures the idea of diminishing returns of reputation.

The timing is as follows: at the start of $t = 1$, the platform chooses a commission fee α. In $t = 1$, the platform helps the seller to acquire x independent buyers. At the start of $t = 2$, the seller decides on the share of follow-up transactions that he will carry out over the platform. This share also determines the number of reputation buyers. The game is solved by backwards induction: the platform anticipates the seller's incentives to make private transactions and sets its optimal fee accordingly.

1.4 Equilibrium in the Platform-Seller Relationship

In this section, we study the equilibrium in the platform-seller relationship. First, we derive the seller's reaction function, which specifies his optimal share of official follow-up transactions given any commission fee of the platform. We then calculate the equilibrium values of the platform's commission fee and the seller's optimal share of official follow-up transactions.

1.4.1 The Seller's Decision Problem

Over the two periods, the seller may serve up to four different groups of buyers: in $t = 1$, he serves the x independent buyers; in $t = 2$, he serves the x^R reputation buyers, the x^O official repeat buyers and the x^P private repeat buyers. The transactions with the first three groups are all carried out over the platform, while the transactions with the last group are unofficial. Hence, the seller's total profit is given by

$$\pi_S = (1-\alpha)\left(x + x^O + x^R\right) + (1 - (1-\beta)\,\alpha)x^P. \tag{1.1}$$

The existence of the reputation buyers creates an interesting trade-off: while private transactions are more profitable, they also limit the seller's ability to attract reputation buyers. The seller's profit crucially depends on the platform's commission fee α. The seller's reaction function specifies his profit-maximizing share of private follow-up transactions given any commission fee α.

Proposition 1.1 *The seller's reaction function is given by*

$$\arg\max_{\gamma} \pi_S = \gamma^*(\alpha) = \frac{1}{x}(\frac{1-\alpha}{2\alpha\beta})^2 \tag{1.2}$$

with

$$\frac{d\gamma^*(\alpha)}{d\alpha} < 0, \frac{d\gamma^*(\alpha)}{d\beta} < 0 \text{ and } \frac{d\gamma^*(\alpha)}{dx} < 0. \tag{1.3}$$

The optimal share γ^* depends on the platform's commission fee α, the seller's bargaining power β and the number of independent buyers x. The higher α, the less attractive official transactions. The higher β, the more profitable private transactions. Finally, the higher x, the less the seller depends on attracting reputation buyers. All three effects decrease the seller's incentives to officially report follow-up transactions.

1.4.2 The Platform's Decision Problem

The platform earns the commission fee α on each official transaction. Hence, its profit function is given by

$$\pi_P = \alpha\left(x + x^O + x^R\right) = \alpha(x + \gamma x + \sqrt{\gamma x}). \tag{1.4}$$

Using the reaction function $\gamma^*(\alpha) = \frac{1}{x}(\frac{1-\alpha}{2\alpha\beta})^2$ yields

$$\pi_P = \alpha(x + (\frac{1-\alpha}{2\alpha\beta})^2 + (\frac{1-\alpha}{2\alpha\beta})). \tag{1.5}$$

The platform faces the following tradeoff: while a high commission fee increases the profit for every transaction, it also gives the seller more incentives to make private transactions. This has two negative effects: the number of official follow-up transactions declines, which also decreases the number of reputation buyers. The platform takes these effects into account and chooses the profit-maximizing commission fee.

Proposition 1.2 *In equilibrium, the platform's profit-maximizing commission fee is given by*

$$\alpha^* = \sqrt{\frac{1}{4x\beta^2 - 2\beta + 1}}. \tag{1.6}$$

The seller takes this equilibrium commission fee as given and chooses his profit-maximizing share γ^* according to his reaction function.

Proposition 1.3 *In equilibrium, the seller's optimal share of official follow-up transactions is given by*

$$\gamma^* = \frac{1}{x}\left(\frac{1 - \sqrt{\frac{1}{4x\beta^2 - 2\beta + 1}}}{2\beta\sqrt{\frac{1}{4x\beta^2 - 2\beta + 1}}}\right)^2. \tag{1.7}$$

In equilibrium, the platform's and the seller's optimal actions are determined by the seller's bargaining power β and the number of independent buyers x. In the next section, we analyze how these parameters affect α^* and γ^*.

1.5 Comparative Statics

The seller's bargaining power and the number of independent buyers x determine the optimal choice of the seller (platform) directly and indirectly through the effect on the optimal choice of the platform (seller).

Interestingly, the direct effect and the indirect effect work in different directions and as a result, the overall effects are ambiguous. We will show that if private follow-up transactions are very attractive, the seller finds it optimal to officially declare a high share of follow-up transactions. This counterintuitive result arises because the more attractive private follow-up transactions are, the more the platform has to lower its fees in order to make official transactions more attractive. This indirect effect outweighs the direct effect so that given the low commission fee, the seller finds it profitable to officially declare a high share of follow-up transactions in order to attract more reputation buyers.

1.5.1 The Determinants of the Equilibrium Commission Fee

First, we analyze how the platform's optimal fee α^* depends on the seller's bargaining power β. The seller's bargaining power determines the share of the seller's official follow-up transactions γ^* which in turn determines the platform's equilibrium fee.

Proposition 1.4 *The platform's equilibrium commission fee is decreasing in the seller's bargaining power, that is,*

$$\frac{d\alpha^*}{d\beta} < 0. \tag{1.8}$$

This result is intuitive. The seller's incentives to make a private transaction are increasing in his ability to negotiate a favorable deal. In order to give the seller an incentive to make official transactions, the platform has to make this option more attractive by lowering its commission fee. Hence, the higher the seller's bargaining power, the lower the platform's optimal fee. The positive effect of a low commission fee for the platform is that a higher number of official repeat purchases also increases the number of reputation buyers, which benefits the platform.

Let us now look at the effect of the number of independent buyers x on the platform's optimal fee. The number of independent buyers determines the seller's share of official follow-up transactions which in turn determines the platform's equilibrium fee.

Proposition 1.5 *The platform's equilibrium commission fee is decreasing in the number of independent buyers, that is,*

$$\frac{d\alpha^*}{dx} < 0. \tag{1.9}$$

If the number of independent buyers is high, the seller depends less on reputation building which tends to decrease his share of official transactions. Hence, the higher the number of independent buyers, the lower will be the commission fee set by the platform, in order to give the seller an incentive to use the official sales channel.

1.5.2 The Determinants of the Equilibrium Share of Follow-Up Transactions

Next, we analyze how the seller's bargaining power β determines his optimal share of official follow-up transactions γ^*.

Proposition 1.6 *The optimal share of official follow-up transactions is increasing in the seller's bargaining power, that is,*

$$\frac{d\gamma^*}{d\beta} > 0. \tag{1.10}$$

Interestingly we find that the seller's bargaining power in private transactions positively affects the share of official transactions. This result is not surprising, since one would suspect that the opposite was the case. There are two effects in play. The direct effect is that private transactions are more attractive if the seller can capture a large part of the saved commission fee. This effect tends to reduce the share of official transactions. However, there is also a countervailing indirect effect. The platform takes the seller's incentives into account when setting its commission fee. Hence, if the seller's incentives to bypass the platform are high, the platform must react by lowering its commission fee in order to make official transactions more attractive. We find that the indirect effect outweighs the direct effect, so that in equilibrium, a seller with a high negotiation skill will be charged such a low commission fee that he finds it more profitable to officially declare a high share of follow-up transactions and thereby attract a higher number of reputation buyers.

Finally, we analyze how the optimal share of official transactions γ^* depends on the number of independent buyers x who are interested in the seller's product regardless of his reputation.

Proposition 1.7 *The optimal share of official follow-up transactions is increasing in the number of independent buyers, that is,*

$$\frac{d\gamma^*}{dx} > 0. \tag{1.11}$$

Again, we find a surprising effect. One would think that a high number of independent buyers would result in a low share of official transactions, since the seller depends less on reputation buyers. But similarly to our previous result, the platform has to react by lowering its fee to an extent that the seller finds it profitable to declare a higher share of transactions.

We find that the seller finds it optimal to report more transactions the higher his bargaining power and the higher the number of independent buyers, even though their direct effect should be negative. This is due to the fact that these two parameters have have an even stronger effect on the platform. The two parameters act to decrease both the number of official transactions and, as a result, the number of reputation buyers. Hence, the platform is forced to lower its fee by a significant amount, which makes official transactions more attractive for the seller. The more attractive private transactions, the more will the platform reduce its fees, which results in a high share of official transactions.

1.6 Cooperation between Platform and Seller

So far, we have analyzed the situation in which the platform and the seller are separate agents, each with their own profit-maximizing objectives. We call this the non-cooperative case. Next, we consider the case when the platform and the seller cooperate and act to maximize joint profits. We call this the cooperative case. The goal of this section is to compare producers' and consumers' surplus as well as total welfare in both cases.

1.6.1 Platform and Seller Perspective

If the seller and the platform cooperate, the objective of the firms is to maximize joint profits by maximizing the number of transactions on the platform.

Proposition 1.8 *If the platform and the seller behave cooperatively, the platform charges no commission fee, that is, $\alpha^* = 0$ and the seller officially declares all his follow-up transactions, that is, $\gamma^* = 1$. Hence joint profits are given by*

$$\pi_C = 2x + \sqrt{x} \tag{1.12}$$

and are higher than the sum of profits in the non-cooperative case,

$$\pi_P + \pi_S = x + \gamma x + \sqrt{\gamma x} + (1 - \alpha + \beta\alpha)(1 - \gamma)x. \tag{1.13}$$

In the non-cooperative case, the platform charges a positive commission fee which induces the seller to carry out a certain share of his follow-up transactions privately. This, however, makes the seller less attractive for reputation buyers and there are fewer transactions in $t = 2$ from which both platform and seller could benefit. It is clearly optimal for both parties to act in a way that attracts as many buyers as possible. The joint profit can be split among the firms so that both are better off than in the non-cooperative case.

1.6.2 Buyer perspective

We now examine how the cooperation between platform and seller affects the buyers' utility. Interestingly, from the buyers' perspective, it is not clear whether the cooperative or the non-cooperative case is more beneficial because there are several groups of buyers with different objectives. We have to distinguish between the x independent buyers in $t = 1$, and the x^O official repeat buyers, the x^P private repeat buyers and the x^R reputation buyers in $t = 2$. We will look at each of these groups in turn.

For the x independent buyers and the x^O official repeat buyers in $t = 2$, it makes no difference whether platform and seller cooperate or not, as they have to pay the full price $p = 1$ in both cases.

The analysis is more interesting for the private repeat buyers x^P. The repeat buyers favor the non-cooperative case, since the seller then has an incentive to bypass the platform by proposing a favorable deal with a lower price to the buyer.

Finally, there are the reputation buyers x^R. Reputation buyers favor the cooperative setting, in which both platform and seller work together to maximize the number of official follow-up transactions in order to signal a high degree of quality and trustworthiness, which gives rise to a higher number of reputation buyers, who are able to enjoy a customer surplus $v - 1$.

In the cooperative case, the platform's commission fee is equal to zero and the seller officially declares all his follow-up transactions, which maximizes the number of reputation buyers $x^R = x$. In the cooperative case, total consumer surplus is given by

$$CS_C = (v - 1) \left(2x + \sqrt{x}\right). \tag{1.14}$$

In the non-cooperative case, the platform charges a positive commission fee, which gives the seller an incentive to make private follow-up transactions. This benefits the x^P private repeat buyers who will be able to buy at a lower price. However, the smaller share of official follow-up transactions results in a smaller number of reputation buyers. In the non-cooperative case, total consumer surplus is given by

$$CS_{NC} = (v-1)\left(x + \gamma x + \sqrt{\gamma x}\right) + (1 - \alpha(1-\beta))(1-\gamma)x. \quad (1.15)$$

Proposition 1.9 *The consumers' surplus in the cooperative case is given by*

$$CS_C = (v-1)\left(2x + \sqrt{x}\right). \quad (1.16)$$

and the consumers' surplus in the non-cooperative case is given by

$$CS_{NC} = (v-1)\left(x + \gamma x + \sqrt{\gamma x}\right) + (1 - \alpha + \beta\alpha)(1-\gamma)x. \quad (1.17)$$

We have $CS_C > CS_{NC}$ for a high commission fee α, for a high number of independent buyers x, for a low valuation of the product v and for a low seller bargaining power β.

The result is ambiguous because of the conflicting interests of reputation buyers and those repeat buyers who want to benefit from the cheaper private follow-up transactions.

First, if the commission fee α is small, the increase in consumer surplus from having more reputation buyers would outweigh the benefits from the cheaper private follow-up transactions. In this case, a cooperative setting would be more favorable for consumers.

Second, if the number of independent buyers x is small, the increase in consumer surplus from having more reputation buyers would outweigh the benefits from the cheaper private follow-up transactions. Also, the number of private follow-up transactions is smaller the smaller x. In this case, a cooperative setting would be more favorable for consumers.

Third, if the buyers' valuation v for the product is high, the increase in consumer surplus from having more reputation buyers would outweigh the benefits from the cheaper private follow-up transactions. In this case, a cooperative setting would be more favorable for consumers.

Finally, if the seller is able to capture a large share β of the commission in private follow-up transactions, the latter become less attractive to $t = 1$ consumers and the increase in consumer surplus from having more reputation buyers would outweigh the benefits from the cheaper private follow-up transactions. In this case, a cooperative setting would be more favorable for consumers.

1.6.3 Social welfare

Total welfare is given by the sum of the firms' profits and the consumer surplus. As seen above, the firms' profits are always higher in the cooperative case than in the non-cooperative case. However, whether consumers' surplus is higher in the cooperative case is ambiguous. What does this mean overall for social welfare?

Proposition 1.10 *In the cooperative case, social welfare is given by*

$$SWF_C = v(2x + \sqrt{x}) \tag{1.18}$$

and is unambiguously higher than in the non-cooperative case.

We have seen that the platform and the seller prefer the cooperative case. Even though it is ambiguous whether consumer surplus is higher in the cooperative case, it is clear that social welfare is higher in the cooperative case. The reason why some buyers might prefer the non-cooperative case is because they can save on some part of the commission fee. From a social point of view however, the consumers' savings in commission fees come at the expense of the selling side, so these two effects even out. From a social point of view, it is always more desirable to maximize the number of transactions, which is why the cooperative case results in higher social welfare.

1.7 Conclusion

We have set up a two period model that analyzes the relationship between a platform and a seller where the latter engages in a repeated business relationship with a group of buyers. In particular, we have looked at how the attractiveness of bypassing the platform affects the seller's choice of the share of transactions he officially declares and how this determines the platform's optimal commission fee.

We find that the higher the seller's incentives to bypass the platform, the more transactions he will officially declare. This effect arises because the platform knows about the seller's incentives to bypass the official channel and lowers his commission fee to the point where it becomes profitable for the seller to make fewer private transactions and to benefit from the reputation benefits associated with having many official customers.

We also look at the case in which platform and seller behave cooperatively in order to maximize the total number of transactions. We find that this increases the joint profits of the platform and the seller and even though the effect on consumer surplus is ambiguous, it can be shown that the social welfare is higher in such a cooperative setting compared to the case in which the seller and the platform do not maximize joint profits.

In this study, we have assumed that the number of reputation buyers is increasing and concave in the number of official transactions. While the concavity of the function is certainly reasonable and in line with empirical observations, in a next step, we would like build a micro-foundation in order to endogenize the number of reputation buyers. One approach could be to assume that these buyers are risk averse and uncertain about the seller's quality. They could interpret the share of official follow-up transactions as the probability that the seller's products are of high quality or that the seller is honest. Hence, making a transaction with the unknown seller is a lottery for the buyer. One could derive the buyer's willingness to pay from his expected utility when making a transaction. We think that this could be a promising approach in order better explain the purchasing decisions of the reputation.

It would further be interesting to test our results empirically. Our model predicts that in markets where the seller can easily bypass the platform, commission fees should be lower, which gives the seller an incentive to make more official follow-up transactions. In a next step, one could collect data on commission fees of different marketplaces for goods and services use our model in order to try to explain differences in commission fees.

1.8 Appendix

Proof of Proposition 1.1:

The seller's profit function is given by

$$\begin{aligned} \pi_S &= (1-\alpha)\left(x + x^O + x^R\right) + (1-\alpha(1-\beta))x^P & (1.19)\\ &= (1-\alpha)\left(x + \gamma x + \sqrt{\gamma x}\right) + (1-\alpha(1-\beta))(1-\gamma)x. & (1.20) \end{aligned}$$

The first order condition with respect to γ is given by

$$\frac{d\pi_S}{d\gamma} = -\frac{1}{2\gamma}\sqrt{x\gamma}\,(\alpha + 2\alpha\beta\sqrt{x\gamma} - 1) \stackrel{!}{=} 0. \quad (1.21)$$

We obtain the seller's reaction function by solving the first-order condition for γ:

$$\gamma^*(\alpha) = \frac{1}{x}\left(\frac{(1-\alpha)}{2\alpha\beta}\right)^2. \quad (1.22)$$

Differentiating $\gamma^*(\alpha)$ with respect to α gives

$$\frac{d\gamma^*(\alpha)}{d\alpha} = \frac{\alpha - 1}{2x\alpha^3\beta^2} < 0. \quad (1.23)$$

Differentiating $\gamma^*(\alpha)$ with respect to x gives

$$\frac{d\gamma^*(\alpha)}{dx} = -\frac{(\alpha-1)^2}{4x^2\alpha^2\beta^2} < 0. \quad (1.24)$$

Differentiating $\gamma^*(\alpha)$ with respect to β gives

$$\frac{d\gamma^*(\alpha)}{d\beta} = -\frac{(\alpha-1)^2}{2x\alpha^2\beta^3} < 0. \quad (1.25)$$

Proof of Proposition 1.2

The platform's profit function is given by

$$\pi_P = \alpha(x + x^O + x^R) = \alpha\left(x + \gamma^*(\alpha)x + \sqrt{\gamma^*(\alpha)x}\right). \tag{1.26}$$

Inserting the seller's reaction function $\gamma^*(\alpha) = \frac{1}{x}(\frac{1-\alpha}{2\alpha\beta})^2$ yields

$$\pi_P = \alpha(x + (\frac{1-\alpha}{2\alpha\beta})^2 + (\frac{1-\alpha}{2\alpha\beta})). \tag{1.27}$$

The first order condition of π_P with respect to α is given by

$$\frac{d\pi_P}{d\alpha} = \frac{\left(4x\alpha^2\beta^2 - 2\alpha^2\beta + \alpha^2 - 1\right)}{4\alpha^2\beta^2} \overset{!}{=} 0. \tag{1.28}$$

Solving for α yields

$$\alpha^* = \sqrt{\frac{1}{4x\beta^2 - 2\beta + 1}}. \tag{1.29}$$

Note that $\alpha^* < 1$ is equivalent to $x > \frac{1}{2\beta}$. We will henceforth refer to this as *condition (1).*

Proof of Proposition 1.3:

By inserting $\alpha^* = \sqrt{\frac{1}{4x\beta^2-2\beta+1}}.$ into the seller's reaction function $\gamma^*(\alpha)$, we obtain the equilibrium share of official follow-up transactions:

$$\gamma^* = \frac{1}{x} * (\frac{(1 - \sqrt{\frac{1}{4x\beta^2-2\beta+1}}.)}{2\beta\sqrt{\frac{1}{4x\beta^2-2\beta+1}}.})^2. \tag{1.30}$$

Proof of Proposition 1.4:

Differentiating the platform's optimal fee α^* with respect to the seller's bargaining power β gives

$$\frac{d\alpha^*}{d\beta} = \frac{1-4x}{\left(4x\beta^2-2\beta+1\right)^2\sqrt{\frac{1}{4x\beta^2-2\beta+1}}}. \tag{1.31}$$

Note that $\frac{d\alpha^*}{d\beta} < 0$ is equivalent to $x > \frac{1}{4\beta}$. From *condition (1)*, we know that $x > \frac{1}{2\beta}$ must hold. It follows that $x > \frac{1}{4\beta}$ must also hold and hence, $\frac{d\alpha^*}{d\beta} < 0$.

Proof of Proposition 1.5:

Differentiating the platform's optimal fee α^* with respect to x yields

$$\frac{d\alpha^*}{dx} = -\frac{2\beta^2}{\left(4x\beta^2-2\beta+1\right)^2\sqrt{\frac{1}{4x\beta^2-2\beta+1}}} < 0. \tag{1.32}$$

Proof of Proposition 1.6:

Differentiating the seller's optimal share

$$\gamma^* = \frac{1}{x} * \left(\frac{(1-\sqrt{\frac{1}{4x\beta^2-2\beta+1}})}{2\beta\sqrt{\frac{1}{4x\beta^2-2\beta+1}}}\right)^2 \tag{1.33}$$

with respect to β gives

$$\frac{d\gamma^*}{d\beta} = \frac{1}{2x\beta^3}\left((2-3\beta+4x\beta^2)\sqrt{\frac{1}{4x\beta^2-2\beta+1}}-(2-\beta)\right). \tag{1.34}$$

First, note that $2-3\beta+4x\beta^2 > 0$ is true for $x > \frac{(3\beta-2)}{4\beta^2}$. Remember that due to *condition (1)*, it must hold that $x > \frac{1}{2\beta}$. Note that $\frac{1}{2\beta} > \frac{(3\beta-2)}{4\beta^2}$ is equivalent to $\beta < 2$ which is true since $\beta < 1$. Hence, $x > \frac{(3\beta-2)}{4\beta^2}$ also holds and $2-3\beta+4x\beta^2 > 0$.

Second, note that $(2-3\beta+4x\beta^2)\sqrt{\frac{1}{4x\beta^2-2\beta+1}} - (2-\beta) > 0$ is equivalent to $2\beta^3\,(4x-1)\,(2x\beta-1) > 0$. Due to *condition (1)*, $2x\beta-1 > 0$ holds. Note further that $4x - 1 > 0$ for $x > \frac{1}{4}$. Also, $\frac{1}{2\beta} > \frac{1}{4}$ is equivalent to $\beta < 2$ which clearly holds. Hence, we have shown that $\frac{d\gamma^*}{d\beta} > 0$.

Proof of Proposition 1.7:

Differentiating the seller's optimal share

$$\gamma^* = \frac{1}{x} * (\frac{1 - \sqrt{\frac{1}{4x\beta^2 - 2\beta + 1}}}{2\beta\sqrt{\frac{1}{4x\beta^2 - 2\beta + 1}}})^2 \tag{1.35}$$

with respect to x gives

$$\frac{d\gamma^*}{dx} = \frac{1}{2x^2\beta^2}\left((2x\beta^2 + 1 - 2\beta)\sqrt{\frac{1}{4x\beta^2 - 2\beta + 1}} - (1 - \beta)\right). \tag{1.36}$$

First, note that $2x\beta^2 + 1 - 2\beta > 0$ is true for $x > \frac{(2\beta - 1)}{2\beta^2}$. Remember that due to *condition (1)*, it must hold that $x > \frac{1}{2\beta}$. Note that $\frac{1}{2\beta} > \frac{(2\beta-1)}{2\beta^2}$ is equivalent to $\beta < 1$ which clearly holds. Hence $x > \frac{(2\beta-1)}{2\beta^2}$ also holds and $2x\beta^2 + 1 - 2\beta > 0$.

Second, note that $(2 - 3\beta + 4x\beta^2)\sqrt{\frac{1}{4x\beta^2 - 2\beta + 1}} - (2 - \beta) > 0$ is equivalent to $\beta^2\,(2x\beta - 1)\,(1 + 2x\beta - 2\beta) > 0$. Due to *condition (1)*, $2x\beta - 1 > 0$ must hold. Note further that $1 + 2x\beta - 2\beta > 0$ for $x > \frac{(2\beta-1)}{2\beta}$. Furthermore, $\frac{1}{2\beta} > \frac{(2\beta-1)}{2\beta}$ is equivalent to $\beta < 1$ which clearly holds. Hence, we have shown that $\frac{d\gamma^*}{dx} > 0$.

Proof of Proposition 1.8:

The platform's profits are given by

$$\pi_P = \alpha(x + \gamma x + \sqrt{\gamma x}). \tag{1.37}$$

The seller's profits are given by

$$\pi_S = (1 - \alpha)(x + \gamma x + \sqrt{\gamma x}) + (1 - \alpha(1 - \beta))(1 - \gamma)x. \tag{1.38}$$

The sum of profits is given by

$$\pi_P + \pi_S = x + \gamma x + \sqrt{\gamma x} + (1-\alpha(1-\beta))(1-\gamma)x. \tag{1.39}$$

The derivative of the sum of profits with respect to α is given by

$$\frac{d\left(\pi_P + \pi_S\right)}{d\alpha} = -x\left(1-\beta\right)\left(1-\gamma\right) < 0. \tag{1.40}$$

Since the joint profits decrease in α, the platform will set $\alpha = 0$.

The derivative of the sum of profits with respect to γ is given by

$$\frac{d\left(\pi_P + \pi_S\right)}{d\gamma} = \frac{1}{2\gamma}\left(\sqrt{x\gamma} + 2x\alpha\gamma\left(1-\beta\right)\right) > 0. \tag{1.41}$$

Since the joint profits increase in γ, the seller will set $\gamma = 1$

With $\alpha = 0$ and $\gamma = 1$, total joint profit in the cooperative case is given by

$$\pi_C = 2x + \sqrt{x}. \tag{1.42}$$

We now show that joint profits in the cooperative case π_C are larger than the sum of profits in the non-cooperative case $\pi_P + \pi_S$. Note that $\pi_C > \pi_P + \pi_S$ is equivalent to

$$(1-\sqrt{\gamma})\sqrt{x} + (1-\beta)\alpha(1-\gamma)x > 0 \tag{1.43}$$

which clearly holds.

Proof of Proposition 1.9:

In the non-cooperative case the consumer surplus of the independent buyers is given by

$$(v-1)\left(x+\gamma x\right) + (v-(1-\alpha+\beta\alpha))(1-\gamma)x \tag{1.44}$$

and the consumer surplus of the reputation buyers is given by

$$(v-1)\sqrt{\gamma x}. \tag{1.45}$$

Hence total consumer surplus in the non-cooperative case is given by

$$CS_{NC} = (v-1)(2x + \sqrt{\gamma x}) + \alpha(1-\beta)(1-\gamma)x. \tag{1.46}$$

In the cooperative case, the consumer surplus of the independent buyers is given by

$$2(v-1)x \tag{1.47}$$

and the consumer surplus of the reputation buyers is given by

$$(v-1)\sqrt{x}. \tag{1.48}$$

Hence, total consumer surplus in the cooperative case is given by

$$CS_C = (v-1)\left(2x + \sqrt{x}\right) \tag{1.49}$$

Note that

$$CS_C > CS_{NC} \text{ for } \frac{(v-1)}{(1-\beta)\alpha\sqrt{x}} \frac{1-\sqrt{\gamma}}{1-\gamma} > 1. \tag{1.50}$$

The left hand side of the inequality increases in v, γ, β and decreases in α and x.

Proof of Proposition 1.10:

Welfare in the cooperative case is given by

$$WF_C = CS_C + \pi_C = v(2x + \sqrt{x}). \tag{1.51}$$

Welfare in the non-cooperative case is given by

$$WF_{NC} = CS_{NC} + \pi_P + \pi_S = vx + (v-1)x + v\sqrt{\gamma x} + x. \tag{1.52}$$

Note that $WF_C > WF_{NC}$ since $\gamma < 1$.

Chapter 2

A Model of Buy-In Tournaments

2.1 Introduction

In recent years, a new form of tournaments has emerged, where players need to pay a "buy-in" in order to participate and where the prize pool is the sum of these buy-ins. This is very different from traditional tournaments, where there is no buy-in and the prize pool is normally provided by external sources rather than by the players themselves.

Buy-in tournaments are most prevalent in the world of the card game poker, where casinos and online poker rooms organize thousands of tournaments every day with buy-ins starting from as low as 10 cents up to $ 100,000 per participant. At the 2006 World Series of Poker Main Event in Las Vegas (poker's most prestigious tournament), almost 9000 players each paid $ 10,000 to enter the tournament. The total prize pool was more than $ 80,000,000 of which the eventual winner took home $12 million. Another example of buy-in tournaments are fantasy sports, a very popular pastime in the U.S. Websites like Yahoo! allow sports fans to manage fictional sports teams that compete against each other in different fantasy leagues. Each participant must pay a fee in order to enter the competition and at the end of the season, the organizing body distributes the sum of the participants' entry fees among the top-ranked players. Even in very traditional games such as chess or backgammon, competitions are increasingly organized as buy-in tournaments.

In traditional non-buy-in tournaments, participants compete for a fixed prize that is not a direct function of the number of participants. The best known examples of such tournaments are sports contests such as soccer, tennis or golf championships, but also tender procedures, beauty contests, political elections and promotion procedures in the labor market. In all these tournaments, the prizes to be won are provided by some external source and not by the participants themselves. In the case of sports contests, the money comes from sponsors, TV stations and ticket sales. In the job market example, the prize is provided by the employer. In traditional tournaments, even if there is no buy-in, participants may still have to incur some cost if they want to participate such as acquiring the necessary skills, qualifying for the tournament or the cost of application. However, these costs have no effect on the prize pool. As a consequence, additional participants tend to have negative external effects on all other participants, as the newcomers only decrease everyone else's chances of winning the tournament while the prize remains constant. Hence, in traditional tournaments with a fixed prize, tournament participants would like to have as few other competitors as possible.

The situation is different in buy-in tournaments, where each participant's entry fee is directed towards the prize pool, which benefits all other players. This raises some interesting questions: what is the overall impact of an additional participant on all other participants if the additional player increases the prize pool but also decreases everyone else's chances of winning? How does the skill level of the additional player influence the utility of all other players? Under which circumstances does a player find it profitable to participate in a given tournament? What determines the players' optimal game selection when they can choose between several tournaments with different buy-in levels? Do players have an incentive to reveal their type by self-selecting into the corresponding games? How can the tournament organizer achieve self-selection?

To analyze these questions, we establish the first theoretical model of buy-in tournaments, where the prize pool is a direct function of the number of players, and each player's probability of winning the tournament is a function of the number of players at the different skill levels. To our knowledge, no such model already exists. We first look at the participation externalities that all players exert on each other. We then look at the players' game selection problem, when they can choose to participate in one of several tournaments that differ in their buy-in levels and the strength of the competition Finally, we analyze how a tournament organizer can use different buy-in levels to sort the players into different ability groups.

Our main results can be summarized as follows: first, we find that in a buy-in tournament, the participation externalities of one player on all other players are ambiguous. On the one hand, more players make it more difficult for any individual to win the tournament. On the other hand, their additional entry fees increase the prize pool. Interestingly, the overall effect and whether a particular player increases or decreases the other players' expected utility of participating in the tournament does not depend on his absolute skill level, but on his skill level relative to the average of the field. More precisely, we show that an additional player increases everyone else's expected utility if his skill level is below the average of the field and decreases everyone else's expected utility if his skill level is above the average of the field.

Second, when we look at the players' game selection problem, we find that tournaments with higher buy-in levels and higher potential profits are also associated with more skillful competitors. Hence, when players have to choose between different buy-in levels, they face a trade-off between their chances of winning the tournament and the expected prize money. We show that if the expected average skill level and the non-monetary utility from participating in the tournament do not increase too rapidly in the buy-in levels, players with different skill levels also have different optimal buy-in levels, thereby truthfully revealing their skill level.

Finally, we show that a tournament organizer can achieve self-selection by the different types by organizing multiple tournaments and selecting a correct ratio between the buy-ins.

We want to emphasize that this chapter is not about poker but about the theory of buy-in tournaments. Poker is just the field where this form of tournament is very prevalent, but in principle, all the traditional tournaments could also be organized as buy-in tournaments. Therefore, it is important to analyze their distinct features and the underlying principles. As a next step, it would be interesting to compare both kinds of tournaments with respect to criteria such as efficiency or incentive provision and determine which tournament form is most appropriate for which real world application. This, however, is beyond the scope of this chapter.

The remainder of the chapter is structured as follows. Section 2 reviews the related literature. In Section 2 we set up a model of buy-in tournaments. In Section 3, we analyze how participation of players of different skill affects the expected of all other players. In Section 4, we analyze the players' game selection problem. In Section 5, we look at the tournament organizer's ability to achieve self-selection. Section 6 concludes. All proofs are relegated to the Appendix.

2.2 Related Literature

To date, the tournament literature has exclusively focused on non buy-in tournaments. Rank-order tournaments are often presented as incentive devices which are useful in practice since rewards are tied only to ordinal comparisons. Originating in sports, these tournaments have been widely applied to align incentives in a principal-agent setting. The main application which the theory has found relates to an internal labor market. This literature focuses mainly on the provision of effort incentives. In their seminal work, Lazear and Rosen (1981) show that rank-order tournaments can, under certain conditions, function as optimal labor contracts yielding the first-best level of effort in an environment characterized by moral hazard. Other important contributions include Green and Stokey (1983), Nalebuff and Stiglitz (1983) and Rosen (1986).

Furthermore, the literature also focuses on the selection properties of tournaments if the ability of the participants is private information and the tournament sponsor wants to identify their true ability. Bhattacharya and Guasch (1988) use a reward-penalty structure to demonstrate how rank-order tournaments can be used to achieve that works self-select into ability groups. Clark and Riis (2001) use a bonus structure in order to achieve self-selection of the participants.

More recently, there has been an increased interest in optimal tournament structures and optimal prize allocations (Moldovanu and Sela, 2001 and 2006).

There has been some empirical literature related to buy-in tournaments, but none of the papers models the theoretical features that are distinct to buy-in tournaments. Goldreich and Pomorski (2007) study bargaining at the end of high-stakes online poker tournaments where the prize pool is a direct function of the participants' entry fees and in which participants often negotiate a division of the prize money rather than risk playing until the end. The participants have an incentive to negotiate such deals since even in tournaments with over 7,000 players, the disparity in prize money between first and say tenth place is generally very large. Goldreich and Pomorski analyze data from 1246 online poker tournaments with an average prize pool of more than \$ 80,000 and the largest tournaments have prize pools in excess of \$1 million. They find that the likelihood of a successful deal increases in the stakes but that overall, risk-reducing deals are not completed as often as one would expect given the top-heavy prize distribution[2]. Lee (2004) studies risk-taking behavior of professional poker players. He analyzes data from tournaments of the World Poker Tour over the span of two years. He finds that the degree of risk-taking when facing a particular decision in a tournament depends on the player's relative position at that time and the structure of the prize pool, which is usually very convex. Moreover, he shows that the players' incentives for risk taking are significantly more responsive to expected losses than gains.

To our knowledge, there is no theoretical work on buy-in tournaments, which is surprising since they exhibit very interesting economic features that are widely applicable and that should make them a fascinating subject of study for economists in general and researchers in microeconomics and mechanism design in particular. We contribute to the literature by building a theoretical model of buy-in tournaments that studies the players' incentives to participate in tournaments with different buy-in levels.

[2]At a recent online tournament, over 7,300 players created a prize pool of over \$ 3.6million. The winner received more than \$ 450,000 while the tenth-place finisher received less than \$ 20,000

2.3 The Model

Consider a one-shot winner-takes-all[3] tournament. To participate, players must pay a buy-in B, which is set aside entirely for the prize pool. Participation is not restricted. The players are risk-neutral and differ in their skill level s_i, with $i = 1, .., N$ and $s_1 = 1 < s_2 < ... < s_N$. The number of players with skill level s_i that enter the tournament is n_i. The total number of participants is $n_T = \sum_{i=1}^{N} n_i$ and the prize pool, that is, the first prize is given by $P = n_T B$.

The winning probability of a player of particular skill level s_j is given by $p_j = f(s_j, s_{i \neq j}, n_i, n_{i \neq j})$ and is a function of his own skill level s_j, the other players' skill levels $s_{i \neq j}$ and the number of players n_i at each skill level s_i. Skill levels are common knowledge among the players[4]. We assume that the probability of winning the tournament is a linear function of the skill level. If there are two players of skill level s_k and s_l with $s_k < s_l$, then the player of skill level s_l is s_l/s_k times as likely to win as the player of skill level s_k. Let $p_1 = p_{\min}$ be the winning probability of the player with the lowest skill level s_1. Then all winning probabilities can be expressed as multiples of the minimum winning probability $p_{\min}$ and are given by $p_i = s_i p_{\min}$. Clearly, we have $\sum_{i=1}^{N} n_i p_i = 1$, that is, the sum of the winning probabilities of all players must be equal to one.

In addition to the prospect of winning the first prize, players may also enter the tournament because they derive some non-monetary utility[5] $v(B)$ from participating that is independent from the outcome of the tournament. This additional utility may include non-monetary benefits like the joy and thrill of competition or the learning experience that players gain from participating. These non-monetary

[3] Most of the literature has focused on winner-takes-all contests. See Moldovanu and Vela (2006).

[4] This assumption is widely used in the literature.

[5] A couple of years ago, billionaire Andy Beal famously challenged the best professional poker players in the world to play for millions of dollars. He became a very capable player and was even winning initially, before losing several million dollars. Billionaire Guy Laliberté, founder of successful Cirque du Soleil, has been a big loser in high-stakes poker games since 2006. Even economist Steven Levitt admits to play poker in situations where he will be a loser in expectation. Goldreich and Pomorski (2008) also talk about the existence of non-monetary utility.

benefits may even be the main factor why some players decide to play in the tournament. We assume that the non-monetary utility is increasing in the buy-in level B, $v'(B) > 0$. The higher the stakes, the higher the non-monetary utility[6].

Consider two different types of players, professional players and amateur players. What distinguishes the two types is not their skill level: amateur players may be of lower, equal or even higher skill than professional players. The two types differ in their utility of participating in the tournament. More specifically, we assume that professional players are only interested in the potential monetary benefits of entering the tournament and have $v(B) = 0$. Amateur players also care about the potential winnings and, in addition, derive some non-monetary benefits $v(B) > 0$ from playing.

We assume that all players have rational expectations about the other players' skill levels and are able to correctly anticipate in which tournament a player of a given skill level wants to participate. These expectations lead to players' participation decisions that in equilibrium are consistent with the expectations. Hence, players have correct expectations about the expected average skill level at each buy-in level and decide to participate in the tournament that maximizes their expected utility.

2.4 Expected Utility and Participation Externalities

In this section, we will derive the players' expected utility from playing in the tournament and how their entry decisions affect all other players. A player of skill level s_j has a probability $p_j = f(s_j, s_{i \neq j}, n_i)$ of winning the tournament. This probability is a function of his own skill level, and the number and skill levels of his competitors.

[6] Goldreich and Pomorski (2008) also make this observation.

Proposition 2.1 *A player of skill level s_j has a probability*

$$p_j = s_j / \sum_{i=1}^{N} n_i s_i \tag{2.1}$$

of winning the tournament, with

$$\frac{dp_j}{ds_j} > 0, \frac{dp_j}{ds_i} < 0 \text{ for } i \neq j \text{ and } \frac{dp_j}{dn_i} < 0 \text{ for } i = 1, ..., N. \tag{2.2}$$

The probability of winning increases in the own skill level and decreases in the skill level of all other players and in the number of players at each skill level.

Next, we are interested in the expected utility from participating in a tournament. We define

$$\overline{s} = \sum_{i=1}^{N} s_i n_i / \sum_{i=1}^{N} n_i \tag{2.3}$$

as the average skill level of the field.

Proposition 2.2 *The expected utility of participating in a tournament for a player of skill level s_i is given by*

$$EU_i = (\frac{s_i}{\overline{s}} - 1)B + v(B) \tag{2.4}$$

with

$$\frac{dEU_i}{ds_i} > 0, \frac{dEU_i}{d\overline{s}} < 0. \tag{2.5}$$

The expected utility increases in the own skill level and decreases in the average skill level of the field. We find that the expected profit crucially depends on the relationship of the own skill level s_i relative to the average skill level of the field.

We now look at the impact of an additional player on all other players. Each additional player has a positive externality on the other players because his buy-in increases the prize pool. The negative effect of course is that each player also reduces the chances of all other players to win the tournament. Because of these opposing externalities, the overall effect of one player on the expected utility of all other players is not obvious. We will look at this overall effect next by differentiating the expected profit EU_i with respect to the number n_j of players of skill s_j.

Proposition 2.3 *The overall external effect of one tournament participant on the expected utility of the other participants does not depend on his absolute skill level, but rather on his skill level relative to the average skill level of the field. The overall external effect is given by*

$$\frac{dEU_i}{dn_j} = s_i(\overline{s} - s_j)/(\sum_{i=1}^{N} n_i s_i)^2 B \tag{2.6}$$

with:

$$\frac{dEU_i}{dn_j} > 0 \text{ for } s_j < \overline{s} \text{ and } \frac{dEU_i}{dn_j} < 0 \text{ for } s_j > \overline{s}. \tag{2.7}$$

In particular,

$$\frac{dEU_i}{dn_1} > 0 \text{ and } \frac{dEU_i}{dn_N} < 0. \tag{2.8}$$

As we have seen in Proposition 2.2, the expected utility decreases in the average skill level of the field. Since all players of above average skill level act to increase the average skill level, they must have a negative effect on the expected utility of the other players. The argument for players of below average skill is analogous. Proposition 2.3 states that expected utility increases with the arrival of additional players as long as their skill level s_j is below the average skill level $\overline{s}$. We know that $s_j < \overline{s}$ is always true for $s_j = s_1 = s_{\min}$, and therefore the expected utility of players of all skill levels increases with the arrival of new competitors of the lowest skill level. We also know that $s_j > \overline{s}$ is always true for $s_j = s_N = s_{\max}$ and therefore expected profits of players of all skill levels decrease with the arrival of new competitors of the highest skill level.

However, it is not clear how the arrival of new competitors of skill level $s_1 < s_j < s_N$ affects the expected utility of the other players, as they could either increase or decrease the average quality of the field, which would increase or decrease expected utility. Hence the effect of additional players does not depend on their absolute skill level but rather on their skill level relative to the average skill of the current field. An additional player with a particular absolute skill level s_j may be below or above the average skill level of the current field, depending on the composition of the player pool at that moment. Hence, from the point of view of any given player, it does not matter that much if the newcomer is of higher or lower skill level than himself, he only cares about how the newcomer's skill level relates to the average of the field. This has important implications. First, it is possible that players welcome the arrival of more and even better players, which sounds counter-intuitive at first. However, as long as they do not increase the average skill level of the field, their additional buy-ins more than compensate for the increased competition they represent to the other players. Second, it is possible that very skilled players are not happy about the arrival of inferior players, even though this increases the prize pool. If the additional player is above average skill, his additional buy-in does not compensate the other players for the decrease in their winning probabilities. Hence, the additional player has a negative overall effect even on the very best players who have the biggest skill advantage and therefore the highest chance of winning the tournament. The participation externalities are illustrated in the figure below.

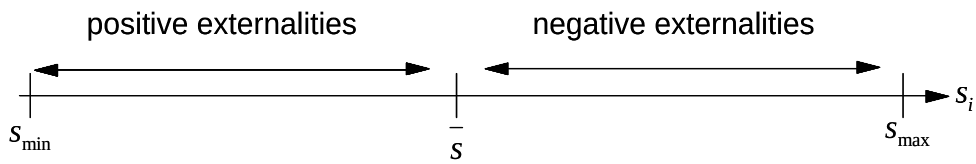

Figure 2.1: Participation Externalities

2.5 Game Selection

We next turn to the case where players can choose to participate in different tournaments with different buy-in levels. We assume that each player can only play one tournament at a time. We look at the trade-offs involved in the players' game selection. This gives rise to interesting questions: is it optimal to play in a game where one enjoys the greatest skill-edge over the competition? Do players of different skill levels have different optimal buy-in levels? Will players of comparable skill compete in the same tournaments? We look at the players' optimal decisions, the participation patterns in equilibrium and illustrate this with an example.

In the model setup, we have distinguished between two types of players, professional players and amateur players, the difference between the two being that the former only care about the monetary benefits associated with playing in a tournament whereas the latter derive some non-monetary utility from participating. Since the two types have different objective functions, they will also behave differently when it comes to game selection. We now examine the differences in the game selection of professionals and amateurs.

2.5.1 The Professional's Decision

Suppose that there are several tournaments with different buy-in levels B. The expected average skill levels at each buy-in level are denoted by $\overline{s}^e(B)$. We have assumed that all players have rational expectations about the expected average skill level at each buy-in level. The professional's expected utility of participating is given by

$$EU_{P_i} = (\frac{s_i}{\overline{s}^e(B)} - 1)B \tag{2.9}$$

and depends on the buy-in and his own skill relative to the expected average skill at a given buy-in level. The player chooses the buy-in level that maximizes his expected utility.

Proposition 2.4 *The derivative of the expected utility function with respect to the buy-in is given by*

$$\frac{dEU_{P_i}}{dB} = s_i(\frac{\overline{s}^e(B) - \overline{s}^{e\prime}(B)B}{(\overline{s}^e(B))^2}) - 1. \tag{2.10}$$

This implies the following for the professional's game selection decision:

1) He will always play in the highest buy-in games if the expected average skill is decreasing in the buy-in level, that is, $\overline{s}^{e\prime}(B) < 0$.
2) He will always play in the lowest buy-in games if the expected average skill is increasing and linear or convex in the buy-in level, that is $\overline{s}^{e\prime}(B) > 0$ *and* $\overline{s}^{e\prime\prime}(B) \geq 0$.
3) There can only be an interior solution if the expected average skill is increasing and concave in the buy-in level, that is, $\overline{s}^{e\prime}(B) > 0$ *and* $\overline{s}^{e\prime\prime}(B) < 0$.

First, consider the case in which the average skill level decreases in the buy-in level, that is $\overline{s}^{e\prime}(B) < 0$ and the expected utility strictly increases in the buy-in level, that is, $\frac{dEU_{P_i}}{dB} > 0$. Here, players would choose to play in the highest possible game. Playing in the high stakes game would be a free lunch: higher potential profits *and* worse competition. However, it seems natural that the average skill increases in the stakes, that is $\overline{s}^{e\prime}(B) > 0$.

Next, consider $\overline{s}^{e\prime}(B) > 0$ and $\overline{s}^{e}(B) - \overline{s}^{e\prime}(B)B \leq 0$. Here, the relationship between the average skill level and the buy-in level is positive and linear/convex, that is $\overline{s}^{e\prime}(B) > 0$ and $\overline{s}^{e\prime\prime}(B) \geq 0$. As buy-ins increase, the competition becomes over-proportionately tougher, meaning that the higher potential winnings in the higher stakes game do not compensate for the drastic reduction in skill edge. We have $\frac{dEU_{P_i}}{dB} < 0$ and the expected profit decreases in the buy-in. Hence, players would elect to play at the lowest buy-in games available.

Finally, consider $\overline{s}^{e\prime}(B) > 0$ and $\overline{s}^{e}(B) - \overline{s}^{e\prime}(B)B > 0$. Here, the average skill level is increasing and concave in the buy-in level, that is, $\overline{s}^{e\prime}(B) > 0$ and $\overline{s}^{e\prime\prime}(B) < 0$. In the higher buy-in games, the average skill level of the competition still increases, but less rapidly than the buy-in levels, meaning that the higher potential winnings in the higher stakes game compensate for the reduction in skill edge. Only under these conditions, $\frac{dEU_{P_i}}{dB} \overset{!}{=} 0$ is possible and there can be an interior solution.

We are interested in situations with interior solutions, because in reality we can observe that players choose games at all levels, not only at the highest or lowest ones available[7]. For the remainder of the chapter, we only consider situations

[7] At PokerStars, the world's largest online poker room, there are up to 150,000 players

where the expected average skill is increasing and concave in the buy-in level. From now on, we assume $\overline{s}^e(B) = B^c$ with $c < 1$. Differentiating the expected profit $EU_{P_i} = \left(\frac{s_i}{B^c} - 1\right) B$ with respect to B yields the optimal buy-in level $B^*_{P_i}$ for a player of skill s_i.

Proposition 2.5 *If the expected average skill is given by $\overline{s}^e(B) = B^c$ with $c < 1$, the optimal buy-in level of a professional is given by*

$$B^*_{P_i} = \left((1-c)s_i\right)^{1/c}. \tag{2.11}$$

The optimal buy-in level increases in the own skill level, that is

$$\frac{dB^*_{P_i}}{ds_i} > 0. \tag{2.12}$$

Our result shows that the optimal buy-in level increases in the skill level of the player. This is in line with empirical observations: players with a higher skill level tend to participate in higher buy-in tournaments, because the higher prize money compensates for the tougher competition. Players with a lower skill level tend to participate in lower buy-in tournaments, since the weak competition compensates for the smaller prize pools.

2.5.2 The Amateur's Decision

We will now analyze the game selection problem of an amateur, who derives some non-monetary utility $v(B) > 0$ from playing. His expected utility of participating is given by

$$EU_{A_i} = \left(\frac{s_i}{\overline{s}^e(B)} - 1\right)B + v(B). \tag{2.13}$$

We are still only interested in the case in which $\overline{s}^e(B) - \overline{s}^{e\prime}(B)B > 0$ holds and the expected average skill level is increasing and concave in the buy-in level. The player chooses the buy-in level that maximizes his expected utility.

playing simultaneously at buy-in levels ranging from \$ 0.1 to \$ 100,000. Most of the players are playing the lower-stakes games.

Proposition 2.6 *The derivative of the expected utility function with respect to the buy-in is given by*

$$\frac{dEU_{A_i}}{dB} = s_i(\frac{\overline{s}^e(B) - \overline{s}^{e\prime}(B)B}{(\overline{s}^e(B))^2}) + v'(B) - 1. \tag{2.14}$$

This implies the following for the amateur's game selection decision:
1) He will always play in the highest games available if $v'(B) > 1$.
2) An interior solution is only possible for $v'(B) < 1$.

The intuition is as follows. If $v'(B) > 1$, the increase in non-monetary utility would always outweigh the higher costs and possible expected monetary losses in the more expensive, more difficult game. As a result, amateurs would play in infinitely high games So there is only a trade-off in game selection if $v'(B) < 1$.

We now determine the optimal buy-in level. Again, we assume $\overline{s}^e(B) = B^c$, with $c < 1$. We further assume a linear relationship between the non-monetary utility v and B which is given by $v(B) = \alpha B$, $\alpha < 1$. Differentiating the expected profit $EU_{A_i} = \left(\frac{s_i}{B^c} - (1-\alpha)\right) B$ with respect to B yields the optimal buy-in level $B^*_{A_i}$ for a player of skill s_i.

Proposition 2.7 *For an amateur player, the optimal buy-in level is given by*

$$B^*_{A_i} = \left(\frac{(1-c)\, s_i}{(1-\alpha)}\right)^{1/c}. \tag{2.15}$$

The optimal buy-in level increases in the own skill level, that is,

$$\frac{dB^*_{A_i}}{ds_i} > 0 \tag{2.16}$$

and in the factor of non-monetary utility, that is,

$$\frac{dB^*_{A_i}}{d\alpha} > 0. \tag{2.17}$$

2.5.3 An illustrative example

Consider a situation with 5 amateur players and 5 professional players. There are five skill levels $s_i = i$ with $i = 1, 2, 3, 4, 5$. Let P_i and A_i be a professional (amateur) of skill level s_i. There is one amateur and one professional player at each skill level, that is, $n_{P_i} = n_{A_i} = 1$. Suppose that there are two tournaments T_L, T_H with different buy-ins $B_L = 2$ and $B_H = 5$. Each player can choose one of three actions $a \in \{0, L, H\}$ with

$a = 0$: the player does not play in any tournament

$a = L$: the player plays in T_L

$a = H$: the player plays in T_H.

Let a^e_{Pi} (a^e_{Ai}) be the expected participation decision of a professional (amateur) player of skill level i, with $a^e_i \in \{0, L, H\}$ and let a^*_{Pi}, a^*_{Ai} be the optimal actions in equilibrium given the expected actions of all other players.

Suppose that the expected participation decisions of all players are given by:

$$a^e_{P_1} = a^e_{P_2} = 0, a^e_{P_3} = a^e_{P_4} = L, a^e_{P_5} = H \tag{2.18}$$

$$a^e_{A_1} = L, a^e_{A_2} = a^e_{A_3} = a^e_{A_4} = a^e_{A_5} = H. \tag{2.19}$$

We assume that $v(B) = 0.7B$. The expected utility of an amateur player is then given by

$$EU_{A_i} = s_i \left(\sum_{i=1}^{N} n_i / \sum_{i=1}^{N} n_i s_i \right) / B - (1 - \alpha) B = \left(\frac{s_i}{\overline{s}^e(B)} - 0.3 \right) B \tag{2.20}$$

and the expected utility of a professional player is given by

$$EU_{P_i} = \left(\frac{s_i}{\overline{s}^e(B)} - 1 \right) B. \tag{2.21}$$

Given the expected actions of the other players, each player chooses the game that maximizes his expected utility.

Proposition 2.8 *Given the set of expectations, all players correctly anticipate the equilibrium average skill levels of* $\overline{s}_L^e = 2.87$ *and* $\overline{s}_H^e = 3.8$*. Players* P_1 *and* P_2 *choose not to play in any tournament,* A_1*,* P_3 *and* P_4 *choose to play in the low buy-in tournament and* A_2*,* A_3*,* A_4*,* A_5 *and* P_5 *choose to play in the high buy-in tournament. The players' equilibrium choices are consistent with the expected actions.*

The players correctly anticipate the equilibrium average skill levels at each buy-in. Given these expectations, no player has an incentive to deviate from the expected actions in equilibrium. If A_1 does not play in T_L, P_3 would no longer find it profitable to play in this game and leave. If A_2 were to play in T_L, this game would become so attractive that P_5 would elect to play in the smaller stakes game. However these would again cause other players to switch games are therefore no stable equilibria.

2.5.4 Participation pattern in equilibrium

In this section, we discuss the players' participation patterns in equilibrium and analyze the role of the so-called "losing amateurs" and the role of non-monetary benefits.

In equilibrium, some of the potential players choose to participate in a tournament while others do not find it profitable to do so. None of the participating players has an incentive to leave his current game. At any given buy-in level, the participants consist of professional players and amateur players. All the participating professionals are of above average skill. Otherwise, their expected utility would be negative and they would not participate. Among the amateur players participating in the tournament there are some players of above and some players of below average skill. We will call those who are of above average skill "winning amateurs" since their skill edge allows them to make a monetary profit from playing in addition to the non-monetary benefits of playing. We will call those who are of below average skill "losing amateurs". They incur monetary losses from playing but still find it worthwhile to participate because of the non-monetary benefits.

Let us look at the role of the losing amateurs. As we know, professionals will only play if they have a skill edge, so there must be some participants in the game who are of below average skill and therefore expected losers in this game, at least monetary-wise. The presence of these losing amateurs ensures that a certain number of professionals finds it worthwhile playing. Without the losing amateurs, the games would be unprofitable for all but the very best professional players.

The participation decision of the amateur players crucially depends on the non-monetary benefits $v(B)$. These benefits may induce amateurs to participate in a tournament even when they are of below average skill and therefore expected losers monetary-wise. If $v(B)$ is very high, even amateurs of very low skill find it worthwhile to play which decreases the average skill level and makes the games more attractive for amateurs and professionals alike. If $v(B)$ is very small, only the highly-skilled amateurs and, as a result, only the very best professionals will find it worthwhile playing, while all other players of inferior skill will choose not to participate.

2.6 Tournament Design

In this section, we look at the tournaments from the perspective of the tournament organizer. We assume that the organizer's goal is to sort players into different tournaments according to their skill levels. While skill levels are common knowledge among the players, the tournament organizer cannot distinguish between the different types[8]. We look at the case in which the tournament organizer offers two tournaments with different buy-in levels and show that he can achieve self-selection of the players by setting the correct ratio of buy-in levels.

As we have argued before, in each game the presence of some losing amateurs is crucial. If an organizer offers two tournaments with different buy-in levels and wants to attract the desired number of players, he must ensure that the low buy-in tournament is just attractive enough for some low-skilled amateur to be

[8]This assumption is widely used in the literature. See Clark and Riis (2001).

the losing player in this game. However, it must not be too attractive, because otherwise some amateurs of higher skill who are the crucial losing amateurs in the higher buy-in game could decide to play the low buy-in game instead. The migration of these amateurs from the high stakes game to the low stakes game could lead to the collapse of the high stakes game, as the absence of the crucial losing players could make the game too tough even for the professional of highest skill. This, in turn, could also lead to the collapse of the low-stakes game, as the players of the highest skill level would move down to the low stakes game which would give the weaker players in the low-stakes game incentives to play in the now-deserted high-stakes game, which would again attract the strong players to play in the high-stakes game which would drive out the weak players and so on. There would be no stable equilibrium in which the players of different skill levels have no incentive to leave their current game.

We have argued before that if an organizer wants different tournaments with different buy-in levels to be sustainable, it is of major importance to give the "losing amateurs" the right incentives to play in the game where they are "needed". Suppose that there are two different tournaments T_k, $k = H, L$, with $B_H > B_L$ and the expected average skill levels $\overline{s}_H^e > \overline{s}_L^e$. Let us assume that the organizer has identified the players of skill levels s_{j-1} and s_j, with $s_j > s_{j-1}$, to be the crucial losing amateurs and that he wants the players of skill level s_j to play the role of losing amateur in tournament T_H and the players of skill level s_{j-1} to play the role of losing amateur in tournament T_L. Our analysis focuses on the crucial losing amateurs. In order to achieve self-selection of the types, the buy-ins B_H, B_L must be set such that the following participation and incentive constraints are fulfilled.

$$\begin{aligned} EU_{A_j}(T_H) &\geq 0 && \left(PC_{A_j}\right) && (2.22)\\ EU_{A_{j-1}}(T_L) &\geq 0 && \left(PC_{A_{j-1}}\right) && (2.23)\\ EU_{A_j}(T_H) &> EU_j^A(T_L) && \left(IC_{A_j}\right) && (2.24)\\ EU_{A_{j-1}}(T_L) &> EU_{j-1}^A(T_H) && \left(IC_{A_{j-1}}\right) && (2.25) \end{aligned}$$

By setting different buy-in levels, the tournament organizer determines the players' incentives to participate in a particular tournament and can achieve self-selection by choosing the correct ratio of buy-in levels $\frac{B_H}{B_L}$.

Proposition 2.9 *The tournament organizer can achieve self-selection by setting a ratio*

$$\frac{B_H}{B_L} > \frac{\overline{s}^e_H}{\overline{s}^e_L}\frac{s_j - (1-\alpha)\overline{s}^e_L}{s_j - (1-\alpha)\overline{s}^e_H} \text{ for } s_{j-1} - (1-\alpha)\overline{s}^e_H < 0 \tag{2.26}$$

and a ratio

$$\frac{\overline{s}^e_H}{\overline{s}^e_L}\frac{s_j - (1-\alpha)\overline{s}^e_L}{s_j - (1-\alpha)\overline{s}^e_H} < \frac{B_H}{B_L} < \frac{\overline{s}^e_H}{\overline{s}^e_L}\frac{s_{j-1} - (1-\alpha)\overline{s}^e_L}{s_{j-1} - (1-\alpha)\overline{s}^e_H} \text{ for } s_{j-1} - (1-\alpha)\overline{s}^e_H > 0. \tag{2.27}$$

Given these buy-in levels, the players with skill level s_{j-1} find it optimal to play in the low-stakes game. By moving to the high-stakes game, they would enjoy more non-monetary utility, but this would be more than offset by the monetary losses due to the higher buy-ins and the tougher competition in the high stakes game. The players with skill level s_j enjoy such a high level of non-monetary utility at the high stakes game that they find it optimal to stay there even though moving down would increase their monetary benefits. If the ratio between the buy-in levels is set correctly, the players have no incentive to leave their current games and therefore the games are stable.

If the ratio were chosen incorrectly, the tournaments would no longer be stable, as players would then have incentives to leave their current game for another or not to play at all. Too small a ratio could entice amateur players of skill level s_{j-1} to move to the high stakes game, since, if the spread in buy-in is small, the higher monetary losses in the high stakes game would be offset by the increase in non-monetary utility. This could cause the low buy-in game to collapse. Too big a ratio could entice amateur players of skill level s_j to move to the low stakes game, since their monetary losses in the high-stakes game would be too high. The migration of these players could make the high-stakes game collapse.

This shows that the organizer can achieve self-selection of the types, but needs to be careful in setting the correct ratio between the different buy-in levels. Players with higher skill level will play in higher buy-in tournaments and truthfully reveal their ability.

Another illustrative example

Again, consider our example from Section 3.5.3. The equilibrium average skill levels were given by $\overline{s}^e(B=2) = 2.87$ and $\overline{s}^e(B=5) = 3.8$. Players A_1, P_3 and P_4 elect to play in tournament T_L, players A_2, A_3, A_4 A_5 and P_5 play in T_L and players P_1 and P_2 do not play at all. Now suppose that the tournament organizer has yet to determine the buy-ins for the two tournaments but wants players to self-select in the same way as when $B_L = 2$ and $B_H = 5$. What is the ratio between the two buy-in levels that achieves self-selection in that manner?

As we have argued before, the key agents are the losing amateurs in each game. As we have shown in Section 3.5.3, players will self-select as described above when A_1 plays in the low buy-in tournament and A_2 plays in the high buy-in tournament. The organizer must now set the buy-in levels in way that ensures that A_1 and A_2 play in the games they are supposed to. First, look at the expression $s_{j-1} - (1-\alpha)\overline{s}^e_H$ from Proposition 2.9. Inserting $\alpha = 0.7$, $s_{j-1} = 1$ and $\overline{s}^e_H = 3.8$ yields $1 - 0.3 \cdot 3.8 < 0$, hence self-selection is achieved for $\frac{B_H}{B_L} > \frac{\overline{s}^e_H}{\overline{s}^e_L}\frac{s_j-(1-\alpha)\overline{s}^e_L}{s_j-(1-\alpha)\overline{s}^e_H}$. Inserting $\alpha = 0.7$, $s_j = 2$, $\overline{s}^e_L = 2.87$ and $\overline{s}^e_H = 3.8$ yields $\frac{B_H}{B_L} > 1.99$.

Hence, if the buy-in for tournament T_H is at least 1.99 as large as the buy-in for tournament T_L,this ensures that A_1 plays in T_L and that A_2 plays in T_H. The presence of the losing amateurs in these games ensures that the other players self-select in the desired manner. Hence, for $\frac{B_H}{B_L} > 1.99$, we have a_{P_1}, a_{P_3}, $a_{P_4} = L$, a_{A_2}, a_{A_3}, a_{A_4} $a_{A_5}, a_{P_5} = H$ and $\overline{s}^e_L = 2.87$ and $\overline{s}^e_H = 3.8$.

2.7 Conclusion

We have built a model of buy-in tournaments and analyzed some of their main properties. In particular, we have focused on participation externalities, the players' game selection in equilibrium and the selection properties of buy-in tournaments. We find that whether a particular player increases or decreases the other players' expected utility does not depend on his absolute skill level, but on his skill level relative to the average of the field. In particular, he will increase the other players' expected utility if he is of below average skill and decreases the other players' expected utility if he is of above average skill. Moreover, we show that players have an incentive to truthfully reveal their skill level by choosing optimal buy-in levels that are increasing in the players' skill levels if the tournament organizer chooses a correct ratio between the buy-in levels and some conditions about the relationships between the expected average skill level, the non-monetary utility and the buy-in levels hold.

2.8 Appendix

Proof of Proposition 2.1:

The winning probabilities of all players add up to one, that is

$$\sum_{i=1}^{N} n_i p_i = 1. \tag{2.28}$$

All winning probabilities can be expressed as multiples of the winning probability $p_{\min}$ of the player of lowest skill level s_1, that is $p_i = s_i p_{\min}$. Inserting $p_i = s_i p_{\min}$ into $\sum_{i=1}^{N} n_i p_i = 1$ gives

$$p_{\min} = 1/\sum_{i=1}^{N} n_i s_i \tag{2.29}$$

and

$$p_i = s_i/\sum_{i=1}^{N} n_i s_i. \tag{2.30}$$

Differentiating the winning probability p_j of a particular player of skill s_j with respect to his own skill level s_j gives

$$\frac{dp_j}{ds_j} = \frac{\sum_{i=1}^{N} n_i s_i - n_j}{(\sum_{i=1}^{N} n_i s_i)^2}. \tag{2.31}$$

As $\sum_{i=1}^{N} n_i s_i - n_j > 0$, it follows that

$$\frac{dp_j}{ds_j} > 0. \tag{2.32}$$

Differentiating p_j with respect to all other skill levels $s_{i \neq j}$ gives

$$\frac{dp_j}{ds_i} = \frac{-\sum_{i=1}^{N} n_i}{(\sum_{i=1}^{N} n_i s_i)^2} < 0 \text{ for } i \neq j. \tag{2.33}$$

Differentiating p_j with respect to the number of participants at skill level n_i yields

$$\frac{dp_j}{dn_i} = \frac{-\sum_{i=1}^{N} s_i}{(\sum_{i=1}^{N} n_i s_i)^2} < 0 \text{ for } n = 1, ..., N. \tag{2.34}$$

Proof of Proposition 2.2:

When entering a given tournament, a participant will win the first prize $P = n_T B$ with probability p_i. Using the expression for the winning probability from Proposition 2.1, deducting the buy-in B and taking into account the non-monetary benefits $v(B)$, the expected utility from playing in the tournament is given by $EU_i = p_i P + v(B) - B$. Using $p_i = s_i / \sum_{i=1}^{N} n_i s_i$, $P = B \sum_{i=1}^{N} n_i$ and $s = \sum_{i=1}^{N} n_i s_i / \sum_{i=1}^{N} n_i$ gives

$$EU_i = \left(\frac{s_i}{\overline{s}} - 1\right) B + v(B). \tag{2.35}$$

Remember that $v(B) = 0$ for professional players. A professional player only plays when his expected utility is positive. Note that $EU_i^A > 0$ is equivalent to $s_i > \overline{s}$, which must always hold for a professional to participate.

Differentiating EU_i with respect to the own skill level s_i gives

$$\frac{dEU_i}{ds_i} = \frac{B}{\overline{s}} > 0. \tag{2.36}$$

Differentiating EU_i with respect to the average skill level $\overline{s}$ gives

$$\frac{dEU_i}{d\overline{s}} = -\frac{s_i B}{(\overline{s})^2} < 0. \tag{2.37}$$

Proof of Proposition 2.3:

Expected utility is given by $EU_i = (\frac{s_i}{\overline{s}} - 1)B + v(B)$. Differentiating with respect to the number of players n_j of skill level s_j gives

$$\frac{dEU_i}{dn_j} = -\frac{s_i B}{\overline{s}^2}\frac{d\overline{s}}{dn_j}. \tag{2.38}$$

Note that $\frac{d\overline{s}}{ds_j} = (s_j - \overline{s}) \,/ \sum_{i=1}^{N} n_i$. Hence

$$\frac{dEU_i}{dn_j} = -\frac{s_i B}{\overline{s}^2}\,(s_j - \overline{s}) \,/ \sum_{i=1}^{N} n_i. \tag{2.39}$$

It follows that

$$\frac{dEU_i}{dn_j} > 0 \text{ for } s_j < \overline{s} \text{ and } \frac{dEU_i}{dn_j} < 0 \text{ for } s_j > \overline{s}. \tag{2.40}$$

Note further that

$$\frac{dEU_i}{dn_1} > 0 \tag{2.41}$$

since $s_1 < \overline{s}$ and

$$\frac{dEU_i}{dn_N} < 0 \tag{2.42}$$

since $s_N > \overline{s}$.

Proof of Proposition 2.4:

Differentiating the expected utility of a professional player $EU_i^P = (\frac{s_i}{\overline{s}^e(B)} - 1)B$ with respect to the buy-in level B gives

$$\frac{dEU_i^P}{dB} = s_i(\frac{\overline{s}^e(B) - \overline{s}^{e\prime}(B)B}{(\overline{s}^e(B))^2}) - 1. \tag{2.43}$$

The sign of the derivative is determined by $\overline{s}^e(B) - \overline{s}^{e\prime}(B)B$. We consider two cases:

Case (1): $\overline{s}^{e\prime}(B) < 0$

Note that $\frac{dEU_{P_i}}{dB} > 0$ is equivalent to

$$s_i\overline{s}^e(B) - \overline{s}^e(B)^2 - s_i\overline{s}^{e\prime}(B)B > 0. \tag{2.44}$$

We know that $s_i > \overline{s}^e(B)$, since professionals only play in games where they have an edge, because otherwise, their expected utility from playing would be negative. Hence, $s_i\overline{s}^e(B) - \overline{s}^e(B)^2 > 0$. Since $\overline{s}^{e\prime}(B) < 0$, it follows that

$$\frac{dEU_{P_i}}{dB} > 0 \tag{2.45}$$

and the player will always choose the game with the highest buy-in level.

Case (2): $\overline{s}^{e\prime}(B) > 0$

Note that $\frac{dEU_{P_i}}{dB}$ can be expressed as

$$\frac{dEU_{P_i}}{dB} = \frac{s_iB}{(\overline{s}^e(B))^2}(\frac{\overline{s}^e(B)}{B} - \overline{s}^{e\prime}(B)) - 1 \tag{2.46}$$

.Note further that the sign of $\frac{dEU_i^P}{dB}$ crucially depends on the term $\frac{\overline{s}^e(B)}{B} - \overline{s}^{e\prime}(B)$.

(a) If $\frac{\overline{s}^e(B)}{B} = \overline{s}^{e\prime}(B)$, $\overline{s}^e$ is linear in B and hence

$$\overline{s}^{e\prime\prime}(B) = 0. \tag{2.47}$$

(b) If $\frac{\overline{s}^e(B)}{B} < \overline{s}^{e\prime}(B)$, $\overline{s}^e(B)$ is increasing and convex in B and hence

$$\overline{s}^{e\prime\prime}(B) > 0. \tag{2.48}$$

(c) If $\frac{\overline{s}^e(B)}{B} > \overline{s}^{e\prime}(B)$, $\overline{s}^e(B)$ is increasing and concave in B and hence

$$\overline{s}^{e\prime\prime}(B) < 0. \tag{2.49}$$

Clearly, if $\overline{s}^{e\prime\prime}(B) \geq 0$, then $\frac{dEU_{P_i}}{dB} < 0$ and the player will always choose the game with the lowest buy-in level.

If $\overline{s}^{e\prime\prime}(B) < 0$, the sign of $\frac{dEU_{P_i}}{dB}$ is ambiguous. Hence, an interior solution at $\frac{dEU_{P_i}}{dB} = 0$ can only exist for $\overline{s}^{e\prime}(B) > 0$ and $\overline{s}^{e\prime\prime}(B) < 0$.

Further, note that

(1) $\frac{dEU_i^P}{dB}_{B=0} > 0$ is equivalent to $s_i > \overline{s}^e(B)$ which always holds for professional players.

(2) $\frac{dEU_i^P}{dB} < 0$ is equivalent to $\frac{\overline{s}^{e\prime}(B)}{\overline{s}^e(B)/B} + \frac{\overline{s}^e(B)}{s_i} > 1$. Hence, $\frac{dEU_i^P}{dB}_{B\to\infty} < 0$.

Proof of Proposition 2.5:

Differentiating the expected profit $EU_{P_i} = \left(\frac{s_i}{B^c} - 1\right) B$ with respect to B gives

$$\frac{dEU_{P_i}}{dB} = (1-c)s_i B^{-c} - 1 \overset{!}{=} 0. \tag{2.50}$$

Solving for B gives the optimal buy-in level

$$B^*_{P_i} = ((1-c)s_i)^{1/c}. \tag{2.51}$$

Differentiating $B^*_{P_i}$ with respect to s_i yields

$$\frac{dB^*_{P_i}}{ds_i} = \frac{1}{cs_i}\left(s_i\left(1-c\right)\right)^{\frac{1}{c}} > 0. \tag{2.52}$$

Proof of Proposition 2.6:

Differentiating the expected profit

$$EU_{A_i} = \left(\frac{s_i}{\overline{s}^e(B)} - 1\right) B + v(B) \tag{2.53}$$

with respect to B yields the optimal buy-in level B_i^* for an amateur player of skill

s_i. The first order condition is given by

$$\frac{dEU_{A_i}}{dB} = s_i(\frac{\overline{s}^e(B) - \overline{s}^{e\prime}(B)B}{(\overline{s}^e(B))^2}) + v'(B) - 1.$$

Since we are only interested in cases in which $\overline{s}^e(B)$ is increasing and concave in B, that is $\overline{s}^e(B) - \overline{s}^{e\prime}(B)B > 0$, we have $\frac{dEU_{A_i}}{dB} > 0$ for $v'(B) > 1$. Hence for $v'(B) > 1$, amateur players will always play in the highest games. Note that an interior solution at $\frac{dEU_{A_i}}{dB} = 0$ only exists for $v'(B) < 1$ and is given by arg max $EU_{A_i}(B)$.

Proof of Proposition 2.7:

Again, we consider the case in which $\overline{s}^e(B) = B^c$. We further assume a linear relationship between v and B and set $v(B) = \alpha B$ with $0 < \alpha < 1$ in order to ensure an interior solution. Solving $\frac{dEU_{A_i}}{dB} = 0$ for B yields the optimal buy-in level

$$B^*_{A_i} = \left(\frac{(1-c)\, s_i}{1-\alpha}\right)^{1/c} \tag{2.54}$$

Note that

$$\frac{dB^*_{A_i}}{ds_i} > 0 \tag{2.55}$$

and

$$\frac{dB^*_{A_i}}{d\alpha} > 0 \tag{2.56}$$

clearly hold.

Proof of Proposition 2.8:

We consider the following expected participation decisions:

$$a^e_{P_1} = a^e_{p2} = 0, a^e_{P_3} = a^e_{P_4} = L, a^e_{P_5} = H \tag{2.57}$$

$$a^e_{A_1} = L, a^e_{A_2} = a^e_{A_3} = a^e_{A_4} = a^e_{A_5} = H. \tag{2.58}$$

We assume that $v(B) = 0.7B$. The expected utility of an amateur player is then given by

$$EU_{A_i} = s_i \left(\sum_{i=1}^{N} n_i / \sum_{i=1}^{N} n_i s_i \right) /B - B + v(B) = \left(\frac{s_i}{\overline{s}^e(B)} - 0.3 \right) B$$

and the expected utility of a professional player is given by

$$EU_{P_i} = \left(\frac{s_i}{\overline{s}^e(B)} - 1 \right) B.$$

Now we check if, given these expectations, any player has an incentive to deviate from his expected behavior a_i^e. A player's optimal decision is denoted by a_i^*.

The expected utilities of the professional players are given by:

$EU_{A_1}(a_{A_1} = L, a^e_{A_2} = a^e_{A_3} = a^e_{A_4} = a^e_{A_5} = H, a^e_{P_1} = a^e_{p2} = 0, a^e_{P_3} = a^e_{P_4} = L, a^e_{P_5} = H) = \left(\frac{1 \cdot 3}{1+3+4} - 0.3\right) 2 = 0.15$

$EU_{A_1}(a_{A_1} = H, a^e_{A_2} = a^e_{A_3} = a^e_{A_4} = a^e_{A_5} = H, a^e_{P_1} = a^e_{p2} = 0, a^e_{P_3} = a^e_{P_4} = L, a^e_{P_5} = H) = \left(\frac{1 \cdot 6}{1+2+3+4+2 \cdot 5} - 0.3\right) 5 = 0$

Hence, $a^*_{A_1} = a^e_{A_1} = H$.

$EU_{A_2}(a_{A_2} = L, a^e_{A_1} = L, a^e_{A_3} = a^e_{A_4} = a^e_{A_5} = H, a^e_{P_1} = a^e_{p2} = 0, a^e_{P_3} = a^e_{P_4} = T_L, a^e_{P_5} = H) = \left(\frac{2 \cdot 4}{1+2+3+4} - 0.3\right) 2 = 1$

$EU_{A_2}(a_{A_2} = H, a^e_{A_1} = L, a^e_{A_3} = a^e_{A_4} = a^e_{A_5} = H, a^e_{P_1} = a^e_{p2} = 0, a^e_{P_3} = a^e_{P_4} = T_L, a^e_{P_5} = H) = \left(\frac{2 \cdot 5}{2+3+4+2 \cdot 5} - 0.3\right) 5 = 1.13$

Hence, $a^*_{A2} = a^*_{A_2} = H$.

$EU_{A_3}(a_{A_3} = L, a^e_{A_1} = L, a^e_{A_2} = a^e_{A_4} = a^e_{A_5} = H, a^e_{P_1} = a^e_{p2} = 0, a^e_{P_3} = a^e_{P_4} = L, a^e_{P_5} = H) = 1.58$

$EU_{A_3}(a_{A_3} = H, a^e_{A_1} = L, a^e_{A_2} = a^e_{A_4} = a^e_{A_5} = H, a^e_{P_1} = a^e_{p2} = 0, a^e_{P_3} = a^e_{P_4} = L, a^e_{P_5} = H) = 2.45$

Hence, $a^*_{A_3} = a^e_{A_3} = H$.

$EU_{A_4}(a_{A_4} = L, a^e_{A_1} = L, a^e_{A_2} = a^e_{A_3} = a^e_{A_5} = H, a^e_{P_1} = a^e_{p2} = 0, a^e_{P_3} = a^e_{P_4} = L, a^e_{P_5} = H) = 2.01$

$EU_{A_4}(a_{A_4} = H, a^e_{A_1} = L, a^e_{A_2} = a^e_{A_3} = a^e_{A_5} = H, a^e_{P_1} = a^e_{p2} = 0, a^e_{P_3} = a^e_{P_4} = L, a^e_{P_5} = H) = 3.76$

Hence, $a^*_{A_4} = a^e_{A_4} = H$.

$EU_{A_5}(a_{A_5} = L, a^e_{A_1} = L, a^e_{A_2} = a^e_{A_3} = a^e_{A_4} = H, a^e_{P_1} = a^e_{p2} = 0, a^e_{P_3} = a^e_{P_4} = L, a^e_{P_5} = H) = 2.48$

$EU_{A_5}(a_{A_5} = H, a^e_{A_1} = L, a^e_{A_2} = a^e_{A_3} = a^e_{A_4} = H, a^e_{P_1} = a^e_{p2} = 0, a^e_{P_3} = a^e_{P_4} = L, a^e_{P_5} = H) = 5.08$

Hence, $a^*_{A_5} = a^e_{A_5} = H$.

The expected utilities of the professional players are given by:

$EU_{P_1}(a_{P_1} = 0, a^e_{p_2} = 0, a^e_{P_3} = a^e_{P_4} = L, a^e_{P_5} = H, a^e_{A_1} = L, a^e_{A_2} = a^e_{A_3} = a^e_{A_4} = a^e_{A_5} = H) = 0.$

$EU_{P_1}(a_{P_1} = L, a^e_{p_2} = 0, a^e_{P_3} = a^e_{P_4} = L, a^e_{P_5} = H, a^e_{A_1} = L, a^e_{A_2} = a^e_{A_3} = a^e_{A_4} = a^e_{A_5} = H) = \left(\frac{1\cdot 4}{2*1+3+4} - 1\right) 2 = -\frac{10}{9}.$

$EU_{P_1}(a_{P_1} = H, a^e_{p_2} = 0, a^e_{P_3} = a^e_{P_4} = L, a^e_{P_5} = H, a^e_{A_1} = L, a^e_{A_2} = a^e_{A_3} = a^e_{A_4} = a^e_{A_5} = H) = \left(\frac{1\cdot 6}{1+2+3+4+2\cdot 5} - 1\right) 5 = -3.5.$

Hence, $a^*_{P_1} = a^e_{P_1} = 0$.

$EU_{P_2}(a_{p2} = 0, a^e_{P_1} = 0, a^e_{P_3} = a^e_{P_4} = L, a^e_{P_5} = H, a^e_{A_1} = L, a^e_{A_2} = a^e_{A_3} = a^e_{A_4} = a^e_{A_5} = H) = 0.$

$EU_{P_2}(a_{p2} = L, a^e_{P_1} = 0, a^e_{P_3} = a^e_{P_4} = L, a^e_{P_5} = H, a^e_{A_1} = L, a^e_{A_2} = a^e_{A_3} = a^e_{A_4} = a^e_{A_5} = H) = -0.4.$

$EU_{P_2}(a_{P_2} = H, a^e_{P_1} = 0, a^e_{P_3} = a^e_{P_4} = L, a^e_{P_5} = H, a^e_{A_1} = L, a^e_{A_2} = a^e_{A_3} = a^e_{A_4} = a^e_{A_5} = H) = -2.14.$

Hence, $a^*_{P_2} = a^e_{p_2} = 0$.

$EU_{P_3}(a_{P_3} = 0, a^e_{P_1} = 0, a^e_{p_2} = 0, a^e_{P_4} = L, a^e_{P_5} = H, a^e_{A_1} = L, a^e_{A_2} = a^e_{A_3} = a^e_{A_4} = a^e_{A_5} = H) = 0.$

$EU_{P_3}(a_{P_3} = L, a^e_{P_1} = 0, a^e_{p_2} = 0, a^e_{P_4} = L, a^e_{P_5} = H, a^e_{A_1} = L, a^e_{A_2} = a^e_{A_3} = a^e_{A_4} = a^e_{A_5} = H) = 0.25$

$EU_{P_3}(a_{P_3} = H, a^e_{P_1} = 0, a^e_{p2} = 0, a^e_{P_4} = L, a^e_{P_5} = H, a^e_{A_1} = L, a^e_{A_2} = a^e_{A_3} = a^e_{A4} = a^e_{A_5} = H) = -0.91.$

Hence, $a^*_{P_3} = a^e_{P_3} = L.$

$EU_{P_4}(a_{P_4} = L, a^e_{P_1} = 0, a^e_{p2} = 0, a^e_{P_3} = L, a^e_{P_5} = H, a^e_{A_1} = L, a^e_{A_2} = a^e_{A3} = a^e_{A4} = a^e_{A_5} = H) = 1.$

$EU_{P_4}(a_{P_3} = H, a^e_{P_1} = 0, a^e_{p2} = 0, a^e_{P_4} = L, a^e_{P_5} = H, a^e_{A_1} = L, a^e_{A_2} = a^e_{A3} = a^e_{A4} = a^e_{A_5} = H) = 0.22.$

Hence, $a^*_{P_4} = a^e_{P_4} = L.$

$EU_{P_5}(a_{P_5} = L, a^e_{P_1} = 0, a^e_{p2} = 0, a^e_{P_3} = L, a^e_{P_4} = L, a^e_{A_1} = L, a^e_{A_2} = a^e_{A3} = a^e_{A4} = a^e_{A_5} = H) = 1.08.$

$EU_{P_5}(a_{P_5} = H, a^e_{P_1} = 0, a^e_{p2} = 0, a^e_{P_3} = L, a^e_{P_4} = L, a^e_{A_1} = L, a^e_{A_2} = a^e_{A3} = a^e_{A4} = a^e_{A_5} = H) = 1.58.$

Hence, $a^*_{P_5} = a^e_{P_5} = H.$

Players correctly anticipate average skill level of $\overline{s}^e(B = 2) = 2.67$ and $\overline{s}^e(B = 5) = 3.8$ and select their games accordingly, in a way that is consistent with their expectations.

Proof of Proposition 2.9:

The tournament organizer wants to choose B_H, B_L in a way that gives an amateur of skill level s_j the right incentives to choose the high buy-in game T_H and that makes it rational for an amateur of skill level s_{j-1} to choose the low buy-in game T_L. Hence, participation and incentive constraints for both types must be fulfilled.

The participation constraints of the player of skill level s_j and skill level s_{j-1} are given by

$$EU^A_j(T_H) = \frac{s_j}{\overline{s}^e_H} B_H - B_H + \alpha B_H \geq 0$$

and

$$EU^A_{j-1}(T_L) = \frac{s_{j-1}}{\overline{s}^e_L} B_L - B_L + \alpha B_L \geq 0.$$

The incentive constraints of the player of skill level s_j and skill level s_{j-1} are given by

$$EU_j^A(T_H) \geq EU_j^A(T_L), \tag{2.59}$$

that is,

$$\frac{s_j}{\overline{s}_H^e} B_H - B_H + \alpha B_H \geq \frac{s_j}{\overline{s}_L^e} B_L - B_L + \alpha B_L \tag{2.60}$$

and

$$EU_{j-1}^A(T_L) \geq EU_{j-1}^A(T_H), \tag{2.61}$$

that is,

$$\frac{s_{j-1}}{\overline{s}_L^e} B_L - B_L + \alpha B_L \geq \frac{s_{j-1}}{\overline{s}_H^e} B_H - B_H + \alpha B_H. \tag{2.62}$$

From $EU_j^A(T_H) \geq EU_j^A(T_L)$, it follows that

$$\frac{B_H}{\overline{s}_H^e}(s_j - (1-\alpha)\overline{s}_H^e) > \frac{B_L}{\overline{s}_L^e}(s_j - (1-\alpha)\overline{s}_L^e). \tag{2.63}$$

Note that due to $EU_j^A(T_H) \geq 0$, it must be the case that $s_j \geq (1-\alpha)\overline{s}_H^e$. As a result, since $\overline{s}_H^e > \overline{s}_L^e$, we have

$$s_j - (1-\alpha)\overline{s}_L^e > s_j - (1-\alpha)\overline{s}_H^e \tag{2.64}$$

Hence, $EU_j^A(T_H) \geq EU_j^A(T_L)$ can be rewritten as

$$\frac{B_H}{B_L} > \frac{\overline{s}_H^e}{\overline{s}_L^e} \frac{s_j - (1-\alpha)\overline{s}_L^e}{s_j - (1-\alpha)\overline{s}_H^e}. \tag{2.65}$$

From $EU_{j-1}^A(T_L) \geq EU_{j-1}^A(T_H)$, it follows that

$$\frac{B_L}{\overline{s}_L^e}(s_{j-1} - (1-\alpha)\overline{s}_L^e) > \frac{B_H}{\overline{s}_H^e}(s_{j-1} - (1-\alpha)\overline{s}_H^e). \tag{2.66}$$

Note that due to $EU_{j-1}^A(T_L) \geq 0$, it must hold that $s_{j-1} \geq (1-\alpha)\overline{s}_L^e$. However, the sign of $s_{j-1} - (1-\alpha)\overline{s}_H^e$ is ambiguous.

Case (1) $s_{j-1} > (1-\alpha)\overline{s}_H^e$.

In this case,

$$EU_{j-1}^A(T_L) \geq EU_{j-1}^A(T_H) \tag{2.67}$$

is equivalent to

$$\frac{B_H}{B_L} < \frac{\overline{s}_H^e}{\overline{s}_L^e}\frac{s_{j-1} - (1-\alpha)\overline{s}_L^e}{s_{j-1} - (1-\alpha)\overline{s}_H^e}. \tag{2.68}$$

Incentive compatibility for both types is achieved for

$$\frac{\overline{s}_H^e}{\overline{s}_L^e}\frac{s_j - (1-\alpha)\overline{s}_L^e}{s_j - (1-\alpha)\overline{s}_H^e} < \frac{B_H}{B_L} < \frac{\overline{s}_H^e}{\overline{s}_L^e}\frac{s_{j-1} - (1-\alpha)\overline{s}_L^e}{s_{j-1} - (1-\alpha)\overline{s}_H^e}.$$

Note that

$$\frac{\overline{s}_H^e}{\overline{s}_L^e}\frac{s_{j-1} - (1-\alpha)\overline{s}_L^e}{s_{j-1} - (1-\alpha)\overline{s}_H^e} > \frac{\overline{s}_H^e}{\overline{s}_L^e}\frac{s_j - (1-\alpha)\overline{s}_L^e}{s_j - (1-\alpha)\overline{s}_H^e} \tag{2.69}$$

is equivalent to

$$(1-\alpha)\left(s_j - s_{j-1}\right)\left(\overline{s}_H^e - \overline{s}_L^e\right) > 0, \tag{2.70}$$

which clearly holds.

Case (2) $s_{j-1} < (1-\alpha)\overline{s}_H^e$.

In this case,

$$EU_{j-1}^A(T_L) \geq EU_{j-1}^A(T_H) \tag{2.71}$$

is equivalent to

$$\frac{B_H}{B_L} > \frac{\overline{s}_H^e}{\overline{s}_L^e}\frac{s_{j-1} - (1-\alpha)\overline{s}_L^e}{s_{j-1} - (1-\alpha)\overline{s}_H^e}, \tag{2.72}$$

which is clearly fulfilled since the right hand side is negative. Hence only

$$EU_j^A(T_H) \geq EU_j^A(T_L) \tag{2.73}$$

is binding and the organizer can achieve self-selection for

$$\frac{B_H}{B_L} > \frac{\overline{s}_H^e}{\overline{s}_L^e}\frac{s_j - (1-\alpha)\overline{s}_L^e}{s_j - (1-\alpha)\overline{s}_H^e}.$$

Chapter 3

Technology Theft in Joint Ventures

3.1 Introduction

In recent years, world markets have increasingly been flooded by counterfeited products. Their widespread availability represents a major concern for the companies that have invested great resources in creating innovative technologies and unique intellectual property, only to see product pirates reap part of the benefits. While simple consumer and media goods have traditionally been subject to product piracy, today even highly sophisticated technological products are being copied and sold internationally. Name any major brand and chances are that there is a counterfeit version of it somewhere. According to the World Customs Organization, counterfeiting accounts for 5 to 7 percent of global merchandise trade, equivalent to lost sales of as much as US$ 512 billion in 2004. China is widely considered to be the main culprit in the international business of counterfeiting and piracy, as counterfeited products account for a large share of the nation's GDP.

At the origin of the technology theft is often a multinational's decision to enter a country with little intellectual property rights enforcement. The multinational's entry makes it much easier for the counterfeiting firms to get hold of prototypes or blueprints of the products that they want to copy. In order to enter a developing market, multinationals often need to form a joint venture with a local company. This may be a voluntary decision, because multinationals often lack local capabilities such as personnel, contacts, language skills, market knowledge or distribution networks. However, the decision to form a joint venture could also be a forced upon the multinational by the host country government. This occurs in industries that are of great importance to the local government (national security, transport, utilities) and where it does not want to cede control to foreign companies. Another reason why local governments may insist on joint ventures is that they want to force a know-how transfer from the multinational to the local partner. Hence, joint ventures can be used as means of acquiring the latest technology and management skills. Once the local firm has acquired the multinational's know-how, it is in prime position to break up the joint venture and compete with the multinational. Thus, if the multinational engages in the joint venture, it potentially contributes to the emergence of a new competitor.

The loss in sales caused by pirated goods is not limited to the multinational's sales in the country where the copies originate from. Multinationals increasingly face competition from counterfeited products in their home markets: almost exact copies of the multinational's products can be found in the open market, at trade shows or in product catalogues. Hence, multinationals need to take into account the potential damage of pirated products not only in the foreign market but especially in their home markets.

A natural question to ask is why multinationals keep on being active in markets like China if that means exposing their technology to potential theft. Some multinational executives point at the sheer size and the rapid growth of the Chinese market, while others want to take advantage of the cheap labor costs and other producer-friendly conditions in order to significantly reduce their production cost and gain a competitive advantage. We will look at both market expansion and cost reduction as possible motives for multinational entry. We are particularly interested in how the level of domestic competition affects the multinational's incentives to enter a foreign market.

Our work is related to different strands of literature. International trade theory examines the motives and the optimal modes for multinational entry. Another focus of study is the effect of spillovers on the firms' incentives to enter cooperative agreements and on the host country in general. Contract theory studies incentives problems within joint ventures. Our work is also related to the industrial organization literature on innovation and market structure. We contribute to the literature by establishing a model of foreign market entry, technology theft and competition Our main idea is that foreign market may create a new competitor which increases competitive pressure in the multinational's home market.

In our model, we consider a multinational that can enter a foreign market through a joint venture with a local firm. The multinational is aware of the risk that the local company may copy its technology and become a competitor not only in the foreign market, but also in the home market. We first study the joint venture relationship between the two firms. In particular, we are interested in the question whether the multinational wants to implement cooperation by the local by offering a one-time payment. We then look at the multinational's incentives to enter the foreign market, if it knows that this will create a new competitor in both the foreign and its home market. We consider two motives for entry: market expansion and cost reduction. We are especially interested in how domestic competitive pressure affects the multinational's entry decision.

We first analyze the joint venture relationship between the multinational and the local company. We look at the local company's incentive problem, as to whether it should remain the multinational's partner or become a competitor. We find that the multinational can implement cooperative behavior by the local company by paying a sufficiently high share of the joint venture profits in the foreign market if these are relatively large compared to the potential profits in the home market. The latter are decreasing in the number of domestic firms. We show that the fewer firms there are in the home market, the more attractive it is for the local company to enter and the more costly it becomes for the multinational to implement cooperation. However, the multinational also has a higher incentive to implement cooperation and protect its home markets if there are only few competitors and profits are high. We show that for very low levels of competition, the multinational's incentives to protect the home market exceed the local company's entry incentives and the multinational always finds it profitable to implement cooperation if it is a monopolist the home market. For higher levels of competition, the local company's entry incentives exceed the multinational's incentives to protect the home market. However, we show that overall, the multinational is more likely to implement cooperation if the number of domestic firms high.

We then look at the multinational's entry decision if it knows that entry will inevitably create a new competitor. We find that the multinational's entry incentives are very different depending on whether its entry motive is market expansion or cost reduction. If the multinational's motive for entry is market expansion, it is more likely to enter the foreign market the higher domestic competition, since in this case, the negative impact of the local company becoming a competitor would be smaller.

In the cost reduction case, we find that both the costs and benefits of entering the foreign market are decreasing in the number of domestic firms. The overall effect depends on the margins in the home market and on the degree of cost reduction. We show that if margins in the home market are low, the multinational's entry incentives are increasing and concave in the number of domestic firms. However if margins are high, the multinational's entry incentives are highest for intermediate levels of domestic competition and decreasing as competition increases. Our results suggest that in the case of market expansion and in the case of cost reduction when home market margins are low, multinationals should find foreign market entry most attractive when domestic competition is high. In the case of cost reduction when home market margins are high, multinational's should find foreign market entry most attractive for intermediate levels of domestic competition.

Our work also makes an interesting contribution to the literature on innovation and market structure. Our work also contributes an interesting aspect to the literature of innovation and market structure. In most of the literature, the costs of innovation include expenses for patents, licences and/or research and development. These costs are usually fixed and independent of the market structure. In our model, entering the foreign market creates a new competitor in the home market, which alters the market structure and increases competitive pressure. Hence, the cost of pursuing the cost-reducing innovation is not a monetary expense, but the emergence of a new competitor. The cost of entry is the negative impact of the new competitor and depends on the market structure at home.

3.2 Related Literature

Our work is related to different strands of literature ranging from international trade theory to the economics of innovation and market structure.

There exists a strand of literature that analyzes spillovers in joint ventures. Blomström and Sjöholm (1999) argue that the interaction between multinationals and local firms facilitates spillovers and that this might reduce the multinationals' incentives to cooperate. Müller and Schnitzer (2006) analyze a multinational's incentives to engage in a joint venture if this gives rise to technology spillovers. They show that spillovers do not necessarily have a negative effect on the incentives to transfer technology and that a joint venture may not always be in the host country government's best interest, even if it could potentially benefit from the spillovers.

Since we focus on domestic competitive pressure as the main determinant of the multinational's entry decision, our work is related to the large literature on innovation and market structure. Various papers have analyzed the link between a firm's incentives to innovate and the intensity of competition. There are different measures for the intensity of competition. According to Arrow (1962), the main determinant is the number of firms in a given industry and competition is more intense the higher the number of firms. Aghion, Harris and Vickers (1995) emphasize the mode of competition, rather than the number of firms, They argue that Cournot competition is less intense than Bertrand competition, since Cournot competition normally leads to lower output and higher prices than Bertrand competition. Boone (2000) defines a number of axioms that a good measure of market competition must satisfy. He lists the switch from Cournot to Bertrand competition and a reduction in travel costs on a Hotelling Beach as well-known examples of such parameterizations of competition.

The literature mainly focuses on cost-reducing innovation. Delbono and Denicolo (1990) show that, under the assumption of a homogenous product, the incentive to introduce cost-reducing innovation is greater for a Bertrand competitor than for a Cournot competitor. Bonnano and Haworth (1998) use a model of horizontal differentiation à la Mussa and Rosen (1978) and show that the increase in profits associated with any given cost reduction is higher in the case of Cournot competition than in the case of Bertrand competition, no matter how small the degree of differentiation. They find that there are cost-reducing innovations that would be pursued under Cournot competition but not under Bertrand Competition. Bester and Petrakis (1993) also consider the case of differentiated products and obtain a mixed result: if the degree of differentiation is "large", the incentive to introduce a cost-reducing innovation is higher for the Cournot competitor, while if the degree of differentiation is "small", then the incentive is higher for a Cournot competitor.

Another related strand of literature concerns horizontal and vertical foreign direct investment (FDI). The literature on horizontal FDI addresses the multinational's market expansion motives. The multinational must decide whether it wants to serve the foreign market with exports from the home market which involves transport costs or whether it wants to set up a production facility in the foreign market, which involves a fixed cost. Important contributions have been made by Brainard (1997) or Helpman et al. (2004). The literature on vertical FDI addresses the multinational's cost-reducing motives. Vertical FDI takes place when the multinational fragments the production process internationally, locating each stage of the production where it can be done at least cost. Grossmann and Helpman (2002, 2003) have recently made important contributions in this field.

3.3 The Model

Consider a multinational enterprise that is currently selling a single good in its home market A and is considering entering a foreign market B. We assume that in market A, there are n symmetric firms producing the good at constant marginal cost c. There are no fixed costs. Consumer demand in market A is given by $p^A(q^A) = a^A - q^A$. The n firms compete in quantities à la Cournot.

We assume that only the multinational enterprise has the option of entering market B. We consider two motives for entry, market expansion and cost reduction. In the market expansion case, the multinational can expand its market reach and sell its product in both markets A and B. Consumer demand in market B is given by $p^B(q^B) = a^B - q^B$. In the cost reduction case, the multinational can lower its marginal cost from c to c' with $c' < c$ and gain a competitive advantage in its home market. We analyze both cases separately, so the multinational's motivation is either market expansion *or* cost reduction, but never both at the same time.

In our model, the only mode of entry into market B is via a joint venture with a local company. We assume that there is only one potential joint venture partner and that if both firms cooperate, they are the only supplier of the good in market B and share the joint profit. Let β $(1 - \beta)$ be the share of the joint profit that the local firm (multinational) receives, with $0 \leq \beta \leq 1$. In case the firms do not cooperate, they engage in Cournot competition.

There are two periods $t = 1, 2$. In $t = 1$ the multinational can either enter market B through a joint venture with the local company or stay in its home market A. We assume that the multinational can only enter market B in $t = 1$. In $t = 2$, the multinational continues to operate in all the markets in which it was active in $t = 1$. We assume that if the multinational decides to enter market B, the local company is always willing to cooperate with the multinational in $t = 1$ and receives a share $0 < \beta_1 < 1$ of the joint profits.

At the start of $t = 2$, the local company has to decide whether it wants to remain the multinational's partner or become a competitor instead. We assume that if the multinational and the local company cooperate in $t = 1$, the local company has access to all production and business processes of the multinational, so that in $t = 2$ it is able to produce and sell a perfect substitute of the multinational's product in both markets A and B. We assume that if the local company decides to compete with the multinational in $t = 2$, it can instantly enter both markets without entry or transport costs. In market A, the emergence of the local company as a new competitor would increase the number of firms from n to $n + 1$. In market B, there would be a duopoly between the multinational and the local company.

At the start of $t = 2$, the multinational can offer the local company a share $0 \leq \beta_2 \leq 1$ of the joint venture profits to be made in market B if both cooperate in $t = 2$. This payment is conditional on the local company not becoming a competitor in market A or B. We assume that this is verifiable and that no other side payments are possible.

3.4 Market Expansion

In this section, we look at the case in which the multinational's only motive for entry is market expansion. We first look at the different profits that both firms make in market A and B depending on whether they cooperate or not. Then we analyze the incentives of the joint venture partners to extend their cooperation in $t = 2$. Finally, we look at the multinational's incentives to enter market B if it knows that the local company will become a competitor in $t = 2$.

3.4.1 Profits in Market A and Market B

The Home Market A

In $t = 1$, there are initially n firms in market A. If the local company enters market A in $t = 2$, the total number of firms increases to $n + 1$. In both cases, all firms engage in Cournot competition.

Proposition 3.1 *If there are n firms in market A, the profit of each firm is given by*

$$\pi_n^A = \left(\frac{a^A - c}{n+1}\right)^2. \tag{3.1}$$

If the local firm enters market A in $t = 2$ there are $n + 1$ firms and each firm's profit is given by

$$\pi_{n+1}^A = \left(\frac{a^A - c}{n+2}\right)^2, \tag{3.2}$$

with

$$\pi_n^A > \pi_{n+1}^A. \tag{3.3}$$

The profits are decreasing in n.

This is a well-known result for a Cournot game with n symmetric players. The profits in market A decrease in the number of competing firms n. The entry of the local company in $t = 2$ decreases the multinational's profit from π_n^A to π_{n+1}^A.

The Foreign Market B

If the multinational enters market B via a joint venture with the local company and both cooperate in $t = 1$, they are the only supplier of the good and can act as a monopoly. If the local company breaks up the joint venture in $t = 2$, the two firms compete à la Cournot.

Proposition 3.2 *If the multinational and the local company cooperate in market B, their joint monopoly profit is given by*

$$\pi_M^B = \left(\frac{a^B - c}{2} \right)^2 . \tag{3.4}$$

If the multinational and the local company engage in Cournot competition in $t = 2$, *the profit of each firm is given by*

$$\pi_{Duop}^B = \left(\frac{a^B - c}{3} \right)^2 . \tag{3.5}$$

These are also standard monopoly and Cournot results.

3.4.2 The Joint Venture Relationship

In this section we analyze the joint venture relationship between the multinational and the local company. First, we look at the local company's incentives to steal the multinational's technology and to become a competitor in both markets. Second, we look at the impact of the local company's entry on the multinational's profits. Third, we look at the multinational's incentives to implement cooperation by making a payment to the local company so that it does not become a competitor.

The Local Company's Entry Incentives

In the first period, there is no conflict of interest between the multinational and the local company, since the local company is not yet able to copy the multinational's product. The multinational and the local company cooperate in market B and share the monopoly profit π_M^B.

At the start of $t = 2$, the local company has to decide whether it wants to remain in the partnership with the multinational or become a competitor. If the local company competes with the multinational in market A, the local company can make a profit of π_{n+1}^A as opposed to zero profits if it chooses not to enter the market.

Lemma 3.1 *The local company's entry incentive is given by*

$$\pi_{n+1}^A = \left(\frac{a^A - c}{n+2}\right)^2 \tag{3.6}$$

and is decreasing in n.

The more competitive market A, the lower the potential profits for the local company and the lower its incentives to break up the joint venture and enter market A.

The Multinational's Cost of Entry

If the multinational's decision to enter market B results in the local company entering market A in $t = 2$, the multinational's profit in market A decreases from π_n^A to π_{n+1}^A. This decrease in profits can be interpreted as the multinational's cost of entering the foreign market B and depends on n.

Proposition 3.3 *The multinational's cost of entering the foreign market is given by the decrease in profits in its home market*

$$\pi_n^A - \pi_{n+1}^A = \left(\frac{a^A - c}{n+1}\right)^2 - \left(\frac{a^A - c}{n+2}\right)^2. \tag{3.7}$$

This cost is decreasing in n.

We will refer to this as the cost of entry or the competitive threat faced by the multinational. The negative impact of the local company entering market A on the multinational's profit is smaller the higher the initial number of domestic firms n. If initially the number of domestic firms is high, the multinational only makes small profits even before the entry of the local firm, so having one more firm in the market does not greatly affect profits. If initially there are very few competitors, the multinational's profits are high. Hence, an additional competitor would have a greater impact on the competitive situation and on the multinational's profits. This implies that firms with very strong positions in their home markets have the most to lose from entering a new market and, in the process, creating a new competitor. The cost of entry is illustrated in figure 3.1:

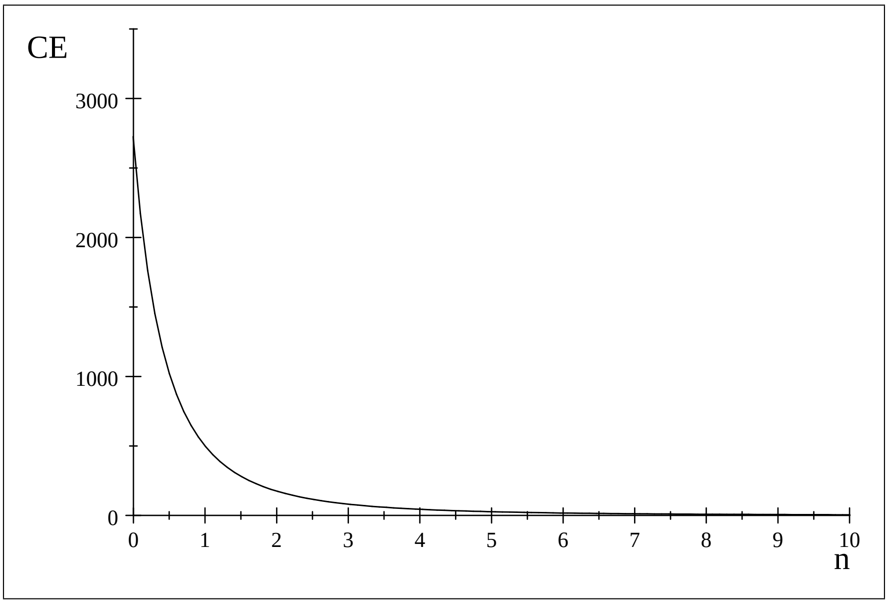

Figure 3.1: Cost of Entry

In our model, the competitive pressure in market A is characterized by the number of firms n. One could also interpret n as a proxy for the maturity of the product. First, consider the case where a low n characterizes a rather new product, for which there are only few substitutes (and few competitors). Hence, the multinational's market position is strong and profit margins are high. If another firm were to enter this market, this would have a large negative impact on the multinational's position, so the cost of entering the foreign market is higher for low values of n. Second, if a high value of n is indicative of a more mature product, for which there are many substitutes, the multinational's position is less strong and profit margins are lower. Entering the foreign market and creating a new competitor would therefore have less of a negative impact on the multinational's profits. This implies that the multinational should rather enter foreign markets with more mature products that only yield small profit margins in the home market and stay away with cutting-edge technologies, for which it can charge a high premium in its home market.

Implementing Cooperation

If the local company becomes a competitor in $t = 2$, it would earn π^A_{n+1} in market A and π^B_{Duop} in market B. We assume that the multinational can offer to pay the local company an amount $\beta_2 \pi^B_M$ with $0 \leq \beta_2 \leq 1$ under the condition that the local company remains the multinational's partner and does not enter market A. We assume that the local company accepts the multinational's offer if it is indifferent between cooperating and becoming a competitor.

Proposition 3.4 *The local company will cooperate if it receives at least a share*

$$\widehat{\beta}_2 = \frac{\pi^B_{Duop} + \pi^A_{n+1}}{\pi^B_M} \tag{3.8}$$

of the monopoly profit in market B in $t = 2$. *The necessary share* $\widehat{\beta}_2$ *is higher the higher* π^B_{Duop}, *the higher* π^A_{n+1} *and the lower* π^B_M. *The share* $\widehat{\beta}_2$ *is increasing in* n.

The local company knows that if both firms compete in market B, the total profit to be made in a competitive market would be smaller than if both cooperate.

This is due to the efficiency of monopoly: the monopoly profit is always greater than the sum of oligopoly profits. The local company will cooperate if what it can earn from the joint venture in market B is greater than the sum of the duopoly profits in market B and the profits that it would earn in market A. The share $\widehat{\beta}_2$ that is necessary to implement cooperation is higher the more attractive it is for the local company to break up the joint venture and become a competitor. This is the case if the duopoly profits π^B_{Duop} are relatively high compared to the joint venture profits π^B_M, and if the number of domestic firms n is low, since this results in higher profits π^A_{n+1}.

We now turn to the multinational. It can implement cooperation by paying $\widehat{\beta}_2 \pi^B_M = \pi^B_{Duop} + \pi^A_{n+1}$. In this case, the multinational and the local company continue to have a monopoly in market B, which gives them a joint profit of π^B_M. Moreover, the local company would not enter market A and the multinational's profits in market A would remain at π^A_n. If the firms cooperate in $t = 2$, the multinational's period profit is given by $\pi^B_M + \pi^A_n - \left(\pi^B_{Duop} + \pi^A_{n+1}\right)$. If both firms compete in $t = 2$, the multinational's period profit is given by $\pi^B_{Duop} + \pi^A_{n+1}$. The multinational wants to implement cooperation if the payment to the local company is small relative to what it would lose if the local company were to become a competitor.

Lemma 3.2 *The multinational finds it profitable to implement cooperation by paying* $\pi^B_{Duop} + \pi^A_{n+1}$ *to the local company if*

$$\pi^B_M - 2\pi^B_{Duop} + \left(\pi^A_n - 2\pi^A_{n+1}\right) \geq 0. \tag{3.9}$$

Implementation is more attractive the higher the monopoly profits π^B_M *and the lower the duopoly profits* π^B_{Duop}.

Given a price of $\pi^B_{Duop} + \pi^A_{n+1}$, the multinational finds it worthwhile to implement cooperation and to protect its home market if the inequality in Lemma 3.2 is satisfied. Cooperation is clearly more attractive if the joint venture profit π^B_M is high compared to the duopoly profit π^B_{Duop}.

The multinational's incentives to implement cooperation also depend on the potential profits in market A, which in turn are influenced by the number of domestic firms n.

Proposition 3.5 *With respect to market A, the multinational's incentives to implement cooperation are given by*

$$\pi_n^A - 2\pi_{n+1}^A = \left(\frac{a^A - c}{n+1}\right)^2 - 2\left(\frac{a^A - c}{n+2}\right)^2 \tag{3.10}$$

and depend on the number of domestic firms n in a non-monotonic way. The multinational always implements cooperation if it is a monopolist in market A. The multinational is least likely to implement cooperation when there are $n = 3$ firms in market A. However as n increases, implementing cooperation becomes relatively more attractive.

The effect of the number of domestic firms n on the multinational's implementation incentives is not straightforward, since there are two opposing effects. First, consider the local company's entry incentives. The higher the initial n, the smaller π_{n+1}^A that the local company can earn by entering market A and the less attractive it is to break up the joint venture. This makes implementing cooperation cheaper and thus more attractive for the multinational. Second, consider the competitive threat $\pi_n^A - \pi_{n+1}^A$ which is the decrease in the multinational's profits due to the entry of the local company. The higher the initial number of firms n, the lower the multinational's profits in market A and the smaller the negative impact of an additional competitor. For high values of n, the multinational has less incentives to protect its home market by implementing cooperation.

As n increases, implementing cooperation becomes less costly, but at the same time less important for the multinational. So what is the overall effect? For $n = 1$ the multinational initially is a monopolist in market A and the arrival of the local company in $t = 2$ creates a duopoly. The term $\left(\pi_n^A - 2\pi_{n+1}^A\right)$ becomes $\left(\pi_M^A - 2\pi_{Duop}^A\right)$ which as we know is positive due to the efficiency of monopoly. Hence the inequality in Proposition 2.5 is fulfilled and a monopolist would always agree to pay $\widehat{\beta}_2\pi_M^B$ to the local company if this prevents the local company from entering market A. For $n \geq 2$ the local company's entry incentive π^A_{n+1} is greater than the competitive threat $\pi^A_n - \pi^A_{n+1}$ faced by the multinational, which makes implementing cooperation more costly. However, as n increases, the difference between π^A_{n+1} and $\pi^A_n - \pi^A_{n+1}$ decreases which makes implementing cooperation less costly again. This is depicted in figure 3.2

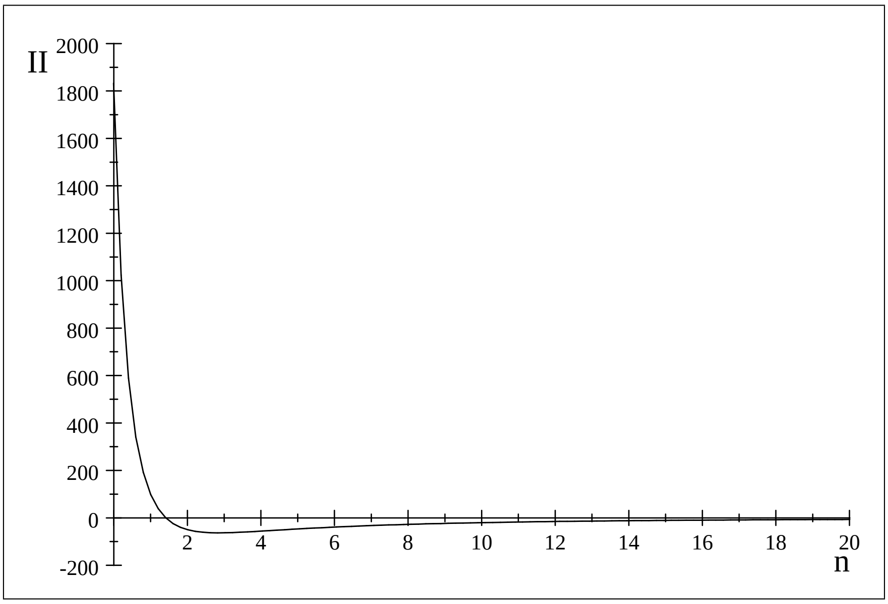

Figure 3.2: Implementation Incentives

In this section we have shown that the willingness of the multinational to protect its home market by implementing cooperation in $t = 2$ critically depends on the number of domestic firms n. In the monopoly case, the multinational will always implement cooperation while in other cases, this may not be optimal since the local company's entry incentives exceed the multinational's incentives to protect the home market, which makes implementation relatively costly. We are mostly interested in cases in which the local company breaks up the joint venture in $t = 2$ and becomes a competitor of the multinational in both markets. For the remainder of this chapter, we assume that if the multinational enters market B and cooperates with the local company in $t = 1$, the local company will always compete with the multinational in $t = 2$.

3.4.3 The Entry Decision

In this section, we investigate the multinational's entry decision if it knows that entering market B in $t = 1$ will create a new competitor in $t = 2$. If the multinational enters the foreign market in order to get access to new customers, its benefits are given by $(1 - \beta_1)\pi_M^B + \pi_{Duop}^B$. The cost of entering market B is the decrease in profits $\pi_n^A - \pi_{n+1}^A$ in market A in $t = 2$.

Proposition 3.6 *In the market expansion case, the multinational enters market B if*

$$(1 - \beta_1)\pi_M^B + \pi_{Duop}^B \geq \left(\pi_n^A - \pi_{n+1}^A\right) \tag{3.11}$$

which is equivalent to

$$(1 - \beta_1)\left(\frac{a^B - c}{2}\right)^2 + \left(\frac{a^B - c}{3}\right)^2 \geq \left(\frac{a^A - c}{n+1}\right)^2 - \left(\frac{a^A - c}{n+2}\right)^2. \tag{3.12}$$

Entry is more profitable the higher the profits in the foreign market and the higher the number of domestic firms n.

The multinational has to weigh the benefits and costs of entry against each other. In the market expansion case, the benefits of entry only depend on the attractiveness of the foreign market and are independent from the number of domestic firms n. The cost of entry $\pi_n^A - \pi_{n+1}^A$ however depends on the number of domestic competitors. The higher domestic competition already is, the smaller the damage caused by an additional competitor. Hence, entry is more profitable if the number of domestic firms n is large.

3.5 Cost reduction

We now turn to the case in which the multinational enters market B in $t = 1$ in order to reduce its marginal cost from c to c'. We no longer consider the multinational's market expansion motives, so the potential profits in market B

are irrelevant for our analysis. We know from our previous results that if the multinational does not reduce cost, its per period profit in market A is given by

$$\pi_n^A(c,c) = \frac{\left(a^A - c\right)^2}{(n+1)^2}. \tag{3.13}$$

We assume that if the multinational enters market B, it is immediately able to produce at the lower cost level c', while in $t=1$, all competitors in market A still produce at marginal cost c. As a result, there is asymmetric Cournot competition. We denote the multinational's profit in $t=1$ by $\pi_n^A(c',c)$.

Proposition 3.7 *If the multinational enters market B in $t=1$ and is able to produce at marginal cost $c' < c$, there is asymmetric Cournot competition in market A and the multinational's profit in $t=1$ is given by*

$$\pi_n^A(c',c) = \frac{\left(a^A - c + n\left(c - c'\right)\right)^2}{(n+1)^2} > \pi_n^A(c,c). \tag{3.14}$$

The cost reduction increases the multinational's profit. Naturally, the multinational's profits increase in the cost level c of the other competitors and decrease in the own cost level c'.

In $t=2$, the local company enters market A and there is asymmetric Cournot competition between $n+1$ firms. The multinational and the local company produce at marginal cost c', while the $n-1$ other firms produce at marginal cost c. We denote the multinational's profit by $\pi_{n+1}^A(c',c',c)$.

Proposition 3.8 *If the local company enters market A in $t=2$, there is asymmetric Cournot competition between the $n+1$ firms and the multinational's profit is given by*

$$\pi_{n+1}^A(c',c',c) = \frac{\left(a^A - c + n\left(c - c'\right)\right)^2}{(n+2)^2} \tag{3.15}$$

with $\pi_{n+1}^A(c',c',c) > \pi_n^A(c,c)$ for

$$c' < \widehat{c} = \frac{\left(1 - \frac{(n+2)}{(n+1)}\right)(a-c) + cn}{n}. \tag{3.16}$$

Entering market B allows the multinational to produce at a lower marginal cost $c < c'$, but creates a new competitor in $t = 2$. For very high levels of cost reduction, that is $c' < \widehat{c}$, the cost reduction benefits in $t = 2$ alone already outweigh the decrease in profits due to the increase in competition and $\pi^A_{n+1}(c', c', c) > \pi^A_n(c, c)$ holds. From Proposition 3.7, we know that $\pi^A_n(c', c) > \pi^A_n(c, c)$ always holds in $t = 1$. Hence, for $c' < \widehat{c}$, the multinational would increase profits in both periods by entering the foreign market in $t = 1$.

We now look at the multinational's entry decision. If the multinational chooses not to enter market B, its profit in each period is given by

$$\pi^A_n(c, c) = (\frac{a - c}{n + 1})^2. \tag{3.17}$$

If it chooses to enter market B , its profit in $t = 1$ is given by

$$\pi^A_n(c', c) = \frac{\left(a^A - c + n\,(c - c')\right)^2}{(n + 1)^2} \tag{3.18}$$

and its profit in $t = 2$ is given by

$$\pi^A_{n+1}(c', c', c) = \frac{\left(a^A - c + n\,(c - c')\right)^2}{(n + 2)^2}. \tag{3.19}$$

Lemma 3.3 *The multinational will enter market B if*

$$EI = \frac{(a - c + n\,(c - c'))^2}{(n + 1)^2} + \frac{(a - c + n\,(c - c'))^2}{(n + 2)^2} - 2(\frac{a - c}{n + 1})^2 > 0.$$

The multinational will only enter market B if its cost reduction benefits in $t = 1$ are greater than what it could potentially lose due to the additional competitor in $t = 2$. The multinational's incentives to enter market B are a function of the number of domestic firms n. The shape of the function depends on the parameters a, c and c'. The difference $a - c$ characterizes the initial margins in market A while the difference $c - c'$ characterizes the degree of cost reduction.

We are interested in the relationship between the multinational's entry incentives and the number of domestic firms n. Since the derivative of the incentive function is highly dependent on the parameters, we have made numerous simulations for different parameter values in order to get a good idea of the possible shape of the function. The corresponding graphs are included in the Appendix. In this section, we present three representative cases. In the three graphs below, the value for c is set to $c = 40$. We vary the parameter a and distinguish between low margins for $a - c = 20$, intermediate margins for $a - c = 40$ and high margins for $a - c = 60$. Within each of these three regimes, we further distinguish between a low degree of cost reduction for $c' = 0.75c$, an intermediate degree of cost reduction for $c' = 0.5c$ and a high degree of cost reduction for $c' = 0.25c$. Each of the following three graphs depicts the entry incentives in a market with a given margin $a - c$ and three different levels of cost reduction $c - c'$.

We first look at the scenario with small margins, with $a - c = 20$. The dotted line represents a high, the broken line an intermediate degree and the solid line a low degree of cost reduction.

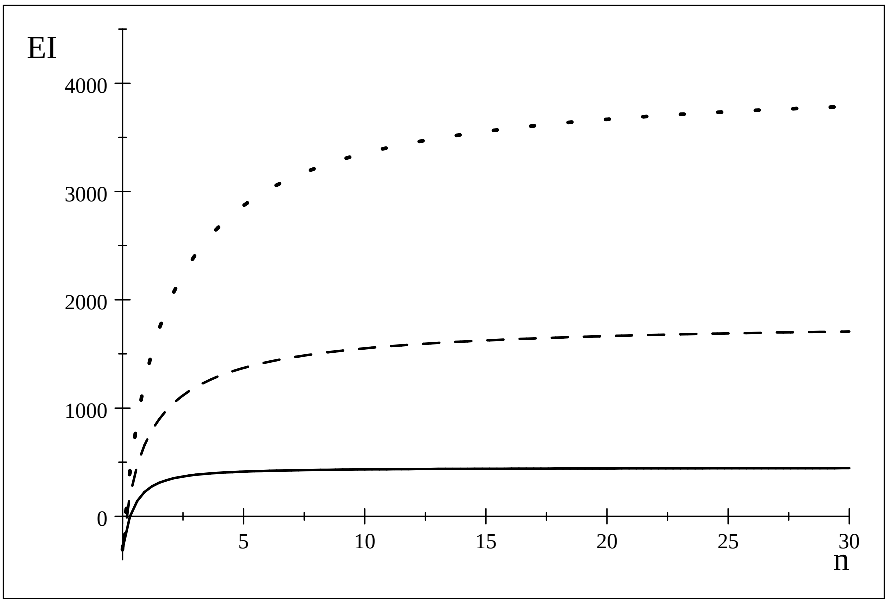

Figure 3.3: Entry Incentives (low margins)

Proposition 3.9 *If the home market is characterized by low margins, the incentives to pursue cost reduction by entering market B are increasing and concave* *in the number of domestic firms n for all degrees of cost reduction The entry incentives are higher the higher the degree of cost reduction.*

For most values of n, the entry incentives are positive. Hence, the benefits of cost reduction outweigh the cost of having an additional competitor in market A. The higher the number of domestic firms, the higher the multinational's incentives to enter market B in order to reduce cost. For a given n, entry incentives are higher the higher the degree of cost reduction. The marginal entry incentives however are decreasing in n, which is illustrated by the concave shape. The intuition here is as follows. On the one hand, if n is high, the multinational has a small market share in market A and hence, the cost reduction only applies to a small quantity which makes the cost reduction less valuable and entering market B less attractive. On the other hand, if n is high the multinational's profits in market A are small and the negative impact of the local company's entry is also small, which makes entering market B more attractive. As n increases, the cost reduction benefits still outweigh the cost of entry, but become relatively smaller which is why the marginal incentives decrease.

The intersection points of the incentive functions with the $n-$axis determine the threshold values where entering market B becomes profitable. We can see that the threshold values are lower for higher degrees of cost reduction. If the threshold value is low, the multinational is keen on entering market B even though market A is very profitable and thus, the negative impact of a new competitor would be high. If the degree of cost reduction is high, the multinational finds entering market B profitable even for low values of n. If the degree of cost reduction is low, the multinational is more hesitant and only enters market B if market A is less profitable, that is, for higher values of n. The graphs for even lower margins look similar, only that the entry incentives are higher for any given n and that the threshold values are smaller. The intuition here is that cost reduction is even more attractive when initial margins are small and the multinational finds entering market B profitable even for very small values of n, because due to the low margins, home market profits are small even when there is little competition, and hence, any damage caused by an additional competitor would be relatively small.

We next look at the scenario with intermediate margins, $a - c = 40$. Again, the dotted line represents a high, the broken line an intermediate degree and the solid line a low degree of cost reduction.

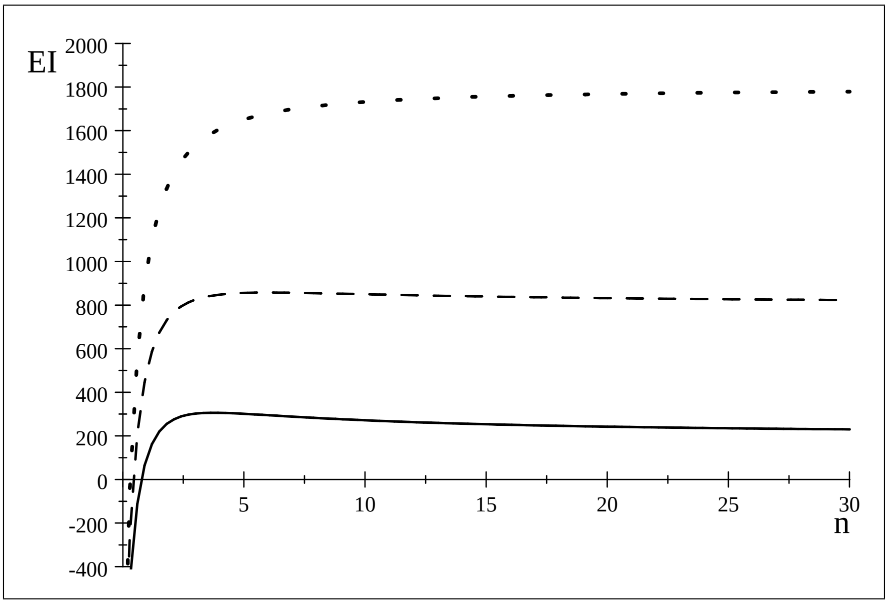

Figure 3.4: Entry Incentives (intermediate margins)

Proposition 3.10 *If the home market is characterized by intermediate margins, the incentives to pursue cost reduction by entering market B are increasing and concave in the number of domestic firms n if the degree of cost reduction is high. For intermediate and low degrees of cost reduction, the incentives initially increase in n and then decrease as n increases. The entry incentives are higher the higher the degree of cost reduction.*

If the degree of cost reduction is high, the entry incentives are increasing and concave in n, similar to the case when margins are low. For intermediate and low levels of cost reduction, the entry incentives initially increase for low values of n and then decrease as n increases. The intuition is as follows. If n is small, the multinational has a large market share and hence the cost reduction is very valuable and outweighs the cost of entry which is also high if n is low. As n increases, both the benefits and the cost of entry decrease. If the level of cost reduction is only intermediate or low, at some point, the cost reduction benefits decrease more rapidly and become less significant than the cost of entry, which is why entry becomes less attractive. This explains the part of the function where the slope is negative.

Last, we look at the scenario with high margins, $a - c = 60$. The dotted line represents a high, the broken line an intermediate degree and the solid line a low degree of cost reduction.

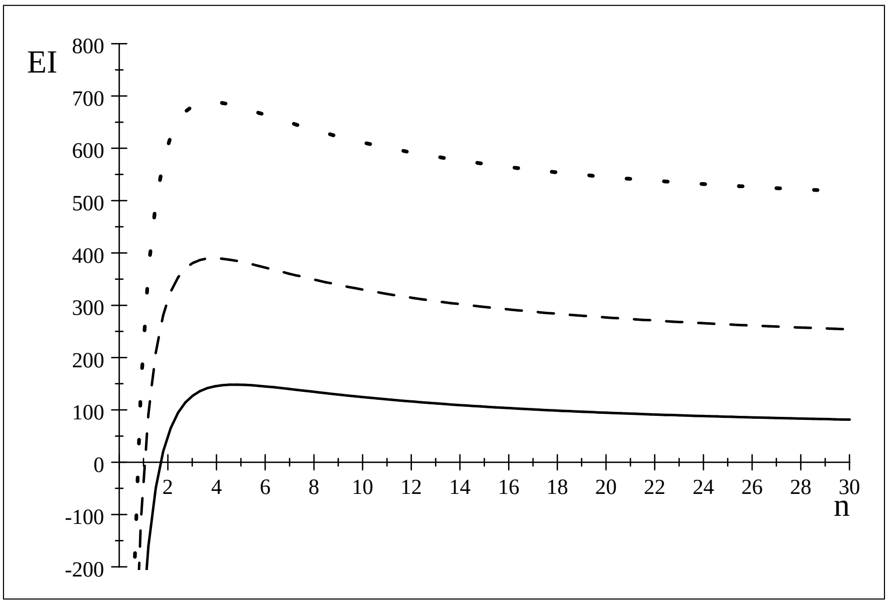

Figure 3.5: Entry Incentives (high margins)

Proposition 3.11 *If the home market is characterized by high margins, the incentives to pursue cost reduction by entering market B initially increase in n and then decrease as n increases for all degrees of cost reduction. The entry incentives are higher the higher the degree of cost reduction.*

For all degrees of cost reduction, the entry incentives initially increase in n and then decrease an n increases, similar to the cases with intermediate margins and intermediate and low levels of cost reduction. The intuition is the same: at some point, the cost reduction benefits decrease more rapidly than the cost of entry. This is even the case for a high level of cost reduction if margins are high. This is due to the fact that the high margins increase the cost of entry, so that at some point, the costs become relatively more important than the benefits of entry.

If we compare the three scenarios with the different margins, we can make interesting observations with respect to the multinational's entry incentives.

Proposition 3.12 *The multinational's incentives to pursue cost reduction by entering market B are higher if market A is characterized by low margins. The threshold values for entry are higher the higher the margins in market A.*

If margins in market A are low, the negative impact of an additional competitor is small compared to the benefits of cost reduction. Hence, entering market B is more attractive if margins in market A are low. The threshold values for entry are higher if the margins are high. This is due to the fact if the home market is more profitable, the cost of an additional competitor is higher, so the multinational is only willing to create this new competitor by entering market B if the number of domestic firms is high and the profits relatively low. If margins are low, the multinational is much less protective of its home market and is willing to create a new competitor by entering market B even if competition in market A is low.

3.6 Conclusion

We have studied a multinational's incentives to enter a foreign market via a joint venture with a local company that might copy the multinational's product and become a competitor in both the foreign and the multinational's home market. First, we show that the cost of entering the foreign market and creating a new competitor decreases in the number of domestic firms, as the negative impact of an additional competitor is smaller the higher the initial competitive pressure. Our second finding is that the multinational's willingness to pay to protect its home market depends on the number of domestic firms in the domestic market in a non-monotonic way. If the multinational is a monopolist, it always wants to implement cooperation. For a small number of firms, the incentives to implement cooperation are smallest and increase in the number of firms.

We are particularly interested in the way that domestic competitive pressure affects multinational's incentives to enter the foreign market. First, we considered the case where the multinational's entry motive is market expansion. We show that its entry incentives are strictly increasing in the number of domestic firms. Second, we analyzed the case in which the multinational's motivation for entry is cost reduction. The relationship between the multinational's entry incentives depends on the margins in market A and the degree of cost reduction. Our main findings are that if margins are low, the entry incentives are strictly increasing and concave in the number of domestic firms. If margins are high, the entry incentives initially increase and then decrease in the number of firms.

Our work also makes an interesting contribution to the literature on innovation and market structure. In most of the literature, the costs of innovation include expenses for patents, licenses and/or research and development. These costs are usually fixed and independent of the market structure. In our model, entering the foreign market creates a new competitor in the home market, which alters the market structure and increases competitive pressure. Hence, the cost of pursuing the cost-reducing innovation is not a monetary expense, but the emergence of a new competitor. The cost of entry is the negative impact of the new competitor and depends on the market structure at home. This chapter focuses on Cournot competition. If the number of firms prior to the local company's entry is small, then the impact of an additional competitor is large and the cost of innovation is high. If the initial number of domestic firms is already high then the impact of an additional competitor is small and the cost of innovation is low.

The fact that the cost of innovation depends on market structure has important implications for the firm's decision as to whether it should pursue innovation or not. If the innovative activity creates a new market and the benefits are independent of the home market structure, then pursuing innovation is more attractive the higher the domestic competitive pressure since this is equivalent to low costs of innovation. If the benefits of innovation decrease in the domestic competitive pressure as in the Cournot example, we have countervailing effects:

on the one hand, the benefits of innovation decrease in the number of firms, which makes innovation less attractive. On the other hand, the cost of innovation also decreases in the number of firms which makes innovation more attractive. The overall effect depends on home market demand, the initial cost level and the degree of cost reduction. We find that the overall incentives to pursue innovation are increasing and concave in the number of firms when the initial cost level is close to home market demand and thus margins are small. When margins are high, that is the initial cost level is small compared to home market demand, then the incentives to pursue innovation are increasing for small numbers of domestic firms and decreasing for large numbers of domestic firms. The empirical implications of these results are that a firm whose home market is characterized by low margins should pursue innovation more strongly the higher domestic competition. The implication is different for firms whose home market is characterized by high margins. In this case, the firm has strong incentives to pursue innovation when the number of firms is small and these incentives decrease when the number of firms is high.

3.7 Appendix

Proof of Proposition 3.1:

In $t = 1$, there are n symmetric firms in market A that compete in quantities á la Cournot. The profit function of the multinational is given by

$$\begin{aligned}\pi^A_{MNC} &= p^A(q^A)q^A_{MNC} - cq^A_{MNC} &(3.20)\\ &= (a^A - (q^A_{MNC} + \sum_{i=1}^{n} q^A_i))q^A_{MNC} - cq^A_{MNC}, i \neq MNC, &(3.21)\end{aligned}$$

the first order condition is given by

$$a^A - 2q^A_{MNC} - \sum_{i=1}^{n} q^A_i - c \stackrel{!}{=} 0, i \neq MNC \tag{3.22}$$

and the multinational's reaction function is given by

$$q^A_{MNC}(q^A_i) = \frac{a^A - \sum_{i=1}^{n} q^A_i - c}{2}, i = 1, ..., n, i \neq MNC \tag{3.23}$$

In the symmetric Cournot equilibrium, the equilibrium values are given by

$$\begin{aligned}q^{A*}_{MNC} &= \frac{a^A - c}{(n+1)}, &(3.24)\\ Q^{A*} &= nq^{A*}_{MNC} = \frac{n(a^A - c)}{(n+1)}, &(3.25)\\ p^{A*} &= \frac{a^A + nc}{n+1} &(3.26)\\ \pi^{A*}_{MNC} &= \pi^A_n = (\frac{a^A - c}{n+1})^2. &(3.27)\end{aligned}$$

Analogously, we derive the multinational's profit when there are $n+1$ symmetric firms in the market. The profit is then given by

$$\pi^A_{n+1} = (\frac{a^A - c}{n+2})^2 \tag{3.28}$$

Proof of Proposition 3.2:

The profit function of the monopolist in market B is given by

$$\pi^B(q^B) = (a^B - q^B)q^B - cq^B. \tag{3.29}$$

Maximizing the profit function with respect to quantity gives the monopoly profit

$$\pi_M^B = \left(\frac{a^B - c}{2}\right)^2. \tag{3.30}$$

In a duopoly, the profit function of each firm is given by

$$\pi_{MNC}^B(q_{MNC}^B, q_{LC}^B) = (a^B - q_{MNC}^B - q_{LC}^B)q_{MNC}^B - cq_{MNC}^B. \tag{3.31}$$

Solving this Cournot game gives the equilibrium profits

$$\pi_{MNC}^{B*} = \pi_{LC}^{B*} = \pi_{Duop}^B = \left(\frac{a^B - c}{3}\right)^2. \tag{3.32}$$

Proof of Lemma 3.1:

The local company's entry incentive is given by

$$\pi_{n+1}^A = (\frac{a^A - c}{n+2})^2. \tag{3.33}$$

Differentiating π_{n+1}^A with respect to n gives

$$\frac{d\pi_{n+1}^A}{dn} = -\frac{2\left(a^A - c\right)^2}{(n+2)^3} < 0. \tag{3.34}$$

Proof of Proposition 3.3:

The cost of entry is given by

$$\pi_n^A - \pi_{n+1}^A = (\frac{a^A - c}{n+1})^2 - (\frac{a^A - c}{n+2})^2. \tag{3.35}$$

Differentiating $\pi_n^A - \pi_{n+1}^A$ with respect to n gives

$$\frac{d\left(\pi_n^A - \pi_{n+1}^A\right)}{dn} = -2\left(a^A - c\right)^2 \frac{3n^2 + 9n + 7}{\left(n^2 + 3n + 2\right)^3} < 0. \tag{3.36}$$

Proof of Proposition 3.4:

For the local company to cooperate, the following incentive constraint must be fulfilled:

$$\beta_2 \pi_M^B \geq \pi_{Duop}^B + \pi_{n+1}^A. \tag{3.37}$$

Solving for β_2 yields

$$\widehat{\beta}_2 = \frac{\pi_{Duop}^B + \pi_{n+1}^A}{\pi_M^B} = \frac{\left(\frac{a^B - c}{3}\right)^2 + \left(\frac{a^A - c}{n+2}\right)^2}{\left(\frac{a^B - c}{2}\right)^2}. \tag{3.38}$$

For $\widehat{\beta}_2 < 1$ it must hold that $\frac{\pi_{Duop}^B + \pi_{n+1}^A}{\pi_M^B} < 1$ or $\pi_{n+1}^A < \pi_M^B - \pi_{Duop}^B$. This means that what the local company can gain in the home markets must be smaller than the difference between the joint venture profits and the duopoly profits in B. So the local company would only cooperate if the profit opportunities in the home markets are relatively small compared to the profit opportunities in B. $\widehat{\beta}_2$ is clearly bigger the bigger π_{Duop}^B, π_{n+1}^A and the lower π_M^B

Differentiating $\widehat{\beta}_2$ with respect to n yields

$$\frac{d\widehat{\beta}_2}{dn} = -\frac{8\left(a^A - c\right)^2}{\left(a^B - c\right)^2 (n+2)^3} \tag{3.39}$$

which is clearly negative.

Proof of Lemma 3.2:

The multinational wants to implement cooperation if

$$(1 - \widehat{\beta}_2)\pi_M^B + \pi_n^A \geq \pi_{Duop}^B + \pi_{n+1}^A. \tag{3.40}$$

After inserting $\widehat{\beta}_2 = \frac{\pi^B_{Duop} + \pi^A_{n+1}}{\pi^B_M}$, this is equivalent to

$$\pi^B_M - 2\pi^B_{Duop} + \pi^A_n - 2\pi^A_{n+1} \geq 0. \tag{3.41}$$

The multinationals incentives to implement cooperation are clearly higher the higher π^B_M and the lower π^B_{Duop}.

Proof of proposition 3.5:

We are interested in the difference between the competitive threat $\pi^A_n - \pi^A_{n+1}$ and the entry incentive π^A_{n+1}, which is equivalent to

$$\pi^A_n - 2\pi^A_{n+1} = \left(\frac{a-c}{n+1}\right)^2 - 2\left(\frac{a-c}{n+2}\right)^2. \tag{3.42}$$

For illustrative purposes, we provide the numerical values for $\pi^A_n - 2\pi^A_{n+1}$ for $n = 1, .., 7$.

$$\pi^A_1 - 2\pi^A_2 = \frac{1}{36}(a-c)^2 = 0.0278\,(a-c)^2 > 0 \tag{3.43}$$

$$\pi^A_2 - 2\pi^A_3 = -\frac{1}{72}(a-c)^2 = -0.0138\,(a-c)^2 < 0 \tag{3.44}$$

$$\pi^A_3 - 2\pi^A_4 = -\frac{7}{400}(a-c)^2 = -0.0175\,(a-c)^2 < 0 \tag{3.45}$$

$$\pi^A_4 - 2\pi^A_5 = -\frac{7}{450}(a-c)^2 = -0.0156\,(a-c)^2 < 0 \tag{3.46}$$

$$\pi^A_5 - 2\pi^A_6 = -\frac{23}{1764}(a-c)^2 = -0.0130\,(a-c)^2 < 0 \tag{3.47}$$

$$\pi^A_6 - 2\pi^A_7 = -\frac{17}{1568}(a-c)^2 = -0.0108\,(a-c)^2 < 0 \tag{3.48}$$

We can see that

$$\pi^A_n - 2\pi^A_{n+1} > 0 \tag{3.49}$$

only holds for $n = 1$. For $n \geq 2$, we have

$$\pi^A_n - 2\pi^A_{n+1} < 0 \tag{3.50}$$

and

$$\lim_{n\to\infty}\left(\pi^A_n - 2\pi^A_{n+1}\right) = 0. \tag{3.51}$$

Proof of proposition 3.6:

The multinational will enter market B if

$$(1-\beta_1)\left(\frac{a^B-c}{2}\right)^2+\left(\frac{a^B-c}{3}\right)^2 \geq \left(\frac{a^A-c}{n+1}\right)^2-\left(\frac{a^A-c}{n+2}\right)^2. \quad (3.52)$$

We know from Proposition 3.3 that $\left(\frac{a^A-c}{n+1}\right)^2-\left(\frac{a^A-c}{n+2}\right)^2$ is decreasing in n. Hence, entry is more attractive the higher n.

Proof of proposition 3.7:

In t=1, there are n firms. The multinational's produces at marginal cost c' while the $n-1$ other firms produce at marginal cost c. The multinational's profit function is given by

$$\pi^A_{MNC}=p^A(q^A)q^A_{MNC}-cq^A_{MNC}=(a^A-(q^A_{MNC}+(n-1)q^A_j))q^A_{MNC}-c'q^A_{MNC}. \quad (3.53)$$

The multinational's reaction function is given by

$$q^A_{MNC}=\frac{a^A-(n-1)q^A_j-c'}{2}. \quad (3.54)$$

The profit function of firm i, $i=1,...,n$, $i\neq MNC$ is given by

$$\pi^A_i=p^A(q^A)q^A_i-cq^A_i=(a^A-(q^A_i+q^A_{MNC}+(n-2)q^A_k))q^A_i-cq^A_i. \quad (3.55)$$

The first order condition is given by

$$\frac{d\pi^A_j}{dq^A_i}=a^A-2q^A_i-q^A_{MNC}-(n-2)q_k-c\overset{!}{=}0. \quad (3.56)$$

Using $q^A_i=q^A_k$ yields

$$q^A_i=\frac{a^A-q^A_{MNC}-c}{n}. \quad (3.57)$$

Inserting $q^A_{MNC}=\frac{a^A-(n-1)q^A_i-c'}{2}$ yields

$$q^A_i=\frac{\left(a^A-2c+c'\right)}{(n+1)}. \quad (3.58)$$

Hence,

$$q^A_{MNC} = \frac{\left(a^A - c + n\left(c - c'\right)\right)}{n+1}, \quad (3.59)$$

$$q^A_{ges} = \frac{\left(c - c' + n\left(a^A - c\right)\right)}{n+1}, \quad (3.60)$$

$$p^A(q_{ges}) = \frac{\left(a^A - c + c' + cn\right)}{n+1}, \quad (3.61)$$

$$\pi_{MNC} = \pi^A_{n+1}(c', c) = \frac{\left(a^A - c + n\left(c - c'\right)\right)^2}{(n+1)^2} \quad (3.62)$$

$$\pi_j = \frac{\left(a^A - 2c + c'\right)^2}{(n+1)^2} \quad (3.63)$$

Proof of proposition 3.8:

In t=2, there are n+1 firms in market A and the multinational and the local firm produce at c' while the $n-1$ other firms produce at c.

The multinational's profit function is given by

$$\pi^A_{MNC} = p(q)q^A_{MNC} - c'q^A_{MNC} = (a^A - (q^A_{MNC} + q^A_{LC} + (n-1)q^A_i))q^A_{MNC} - c'q^A_{MNC} \quad (3.64)$$

and the multinational's reaction function is given by

$$q^A_{MNC} = \frac{a^A - q^A_{LC} - (n-1)q^A_i - c'}{2}. \quad (3.65)$$

Analogously, the local company's reaction function is given by

$$q^A_{LC} = \frac{a^A - q^A_{MNC} - (n-1)q^A_i - c'}{2}. \quad (3.66)$$

Using $q^A_{MNC} = q^A_{LC}$ yields the reaction functions

$$q^A_{MNC} = q^A_{LC} = \frac{a^A - (n-1)q^A_i - c'}{3}. \quad (3.67)$$

The profit function of a representative domestic firm producing at cost level $c < c'$

is given by

$$\pi_j = p(q)q_i^A - cq_i^A = (a^A - (q_i^A + q_{MNC}^A + q_{LC}^A + (n-2)q_k))q_i^A - cq_i^A \tag{3.68}$$

and its reaction function is given by

$$q_i^A = \frac{a^A - q_{MNC}^A - q_{LC}^A - c}{n}. \tag{3.69}$$

Inserting the reaction functions and solving for the quantities yields

Inserting these reaction functions into $q_i^A = \frac{a^A - q_{MNC}^A - q_{LC}^A - c}{n}$ yields

$$q_i^{A*} = \frac{\left(a^A - 3c + 2c'\right)}{n+2} \tag{3.70}$$

$$q_{MNC}^{A*} = q_{LC}^{A*} = \frac{\left(a^A - c + n\left(c - c'\right)\right)}{n+2}, \tag{3.71}$$

$$q^* = \frac{\left(a^A + c - 2c' + n\left(a - c\right)\right)}{n+2}, \tag{3.72}$$

$$p^* = \frac{\left(a^A - c + 2c' + cn\right)}{n+2}, \tag{3.73}$$

$$\pi_{MNC}^{A*} = \pi_{LC}^{A*} = \pi_{n+1}^A(c', c', c) = \frac{\left(a^A - c + n\left(c - c'\right)\right)^2}{\left(n+2\right)^2} \tag{3.74}$$

$$\pi_j^* = \frac{\left(a^A - 3c + 2c'\right)^2}{\left(n+2\right)^2} \tag{3.75}$$

Note that $\pi_{n+1}^A(c', c', c) > \pi_n^A(c, c)$ is equivalent to

$$\frac{\left(a - c + n\left(c - c'\right)\right)^2}{\left(n+2\right)^2} > (\frac{a-c}{n+1})^2 \tag{3.76}$$

which is equivalent to

$$c' < \frac{\left(1 - \frac{(n+2)}{(n+1)}\right)(a-c) + cn}{n}. \tag{3.77}$$

Proof of Lemma 3.3:

The multinational will enter market A if

$$\pi_n^A(c', c) + \pi_{n+1}^A(c', c', c) > 2\pi_n^A(c, c) \tag{3.78}$$

which is equivalent to

$$\frac{\left(a^A - c + n\,(c - c')\right)^2}{(n+2)^2} + \frac{\left(a^A - c + n\,(c - c')\right)^2}{(n+1)^2} - 2\frac{\left(a^A - c\right)^2}{(n+1)^2} > 0. \tag{3.79}$$

Illustrations of the simulations for Propositions 3.9-3.11:

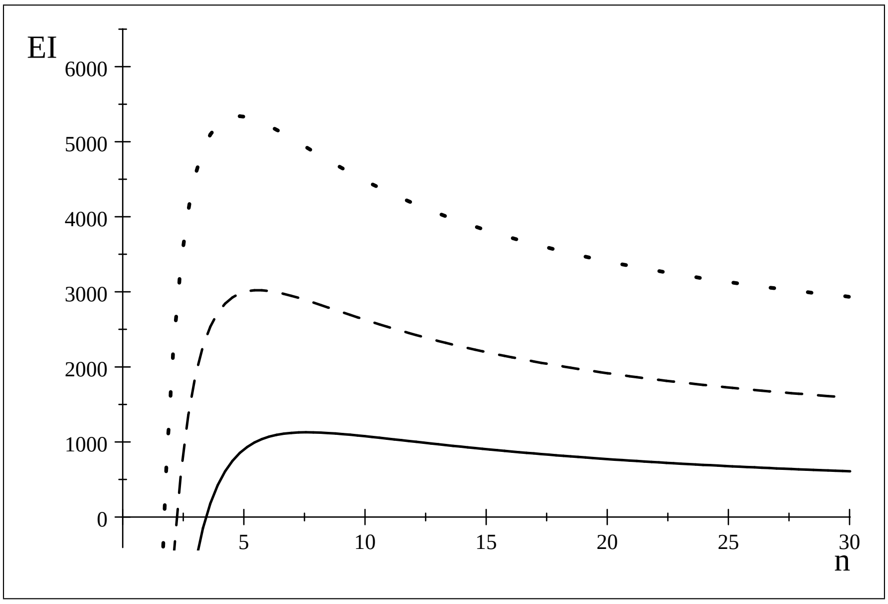

Figure 3.6: Simulation 1: $a = 400$, $c = 40$

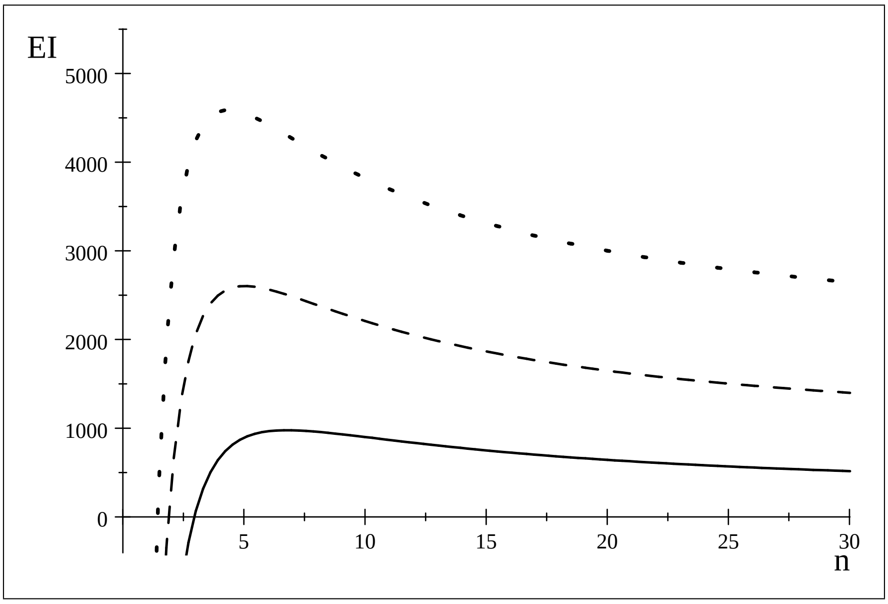

Figure 3.7: Simulation 2: $a = 320$, $c = 40$

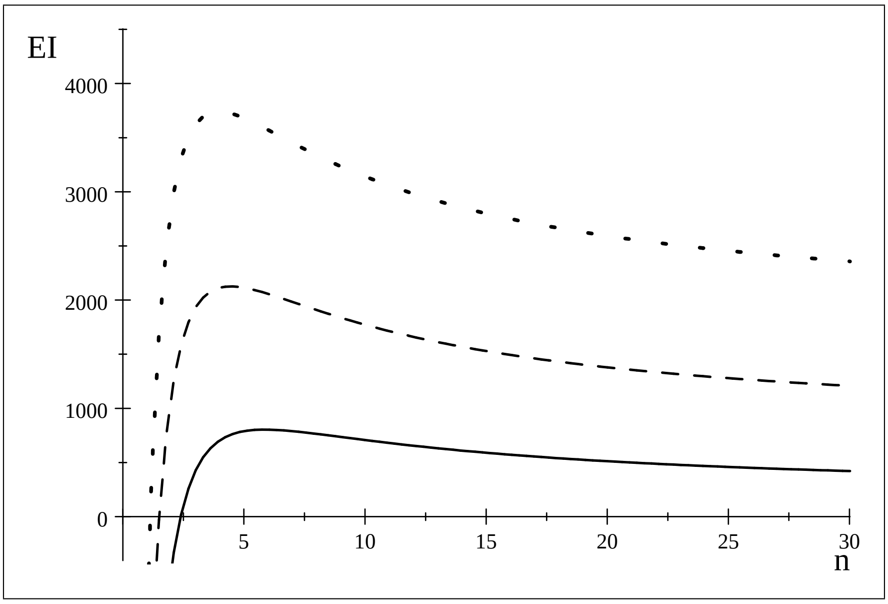

Figure 3.8: Simulation 3: $a = 240$, $c = 40$

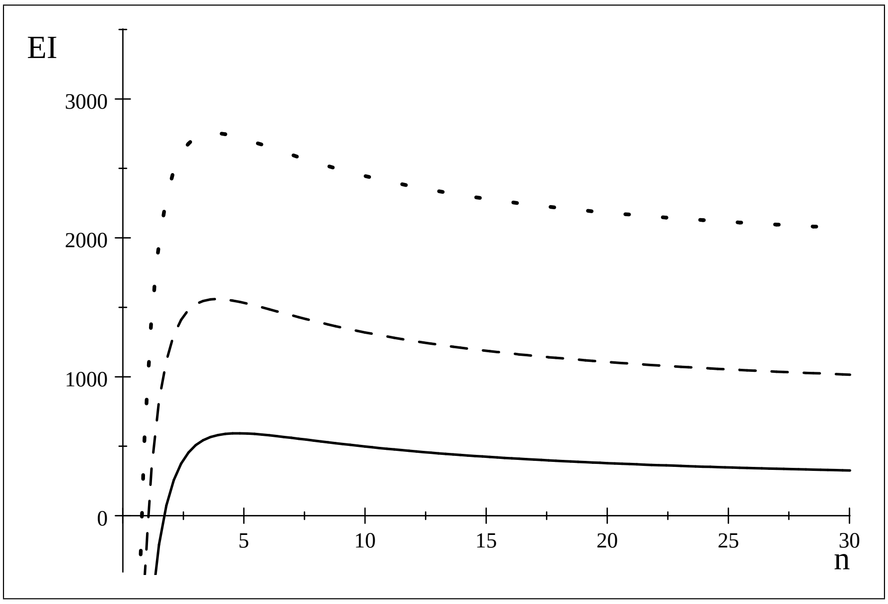

Figure 3.9: Simulation 4: $a = 160$, $c = 40$

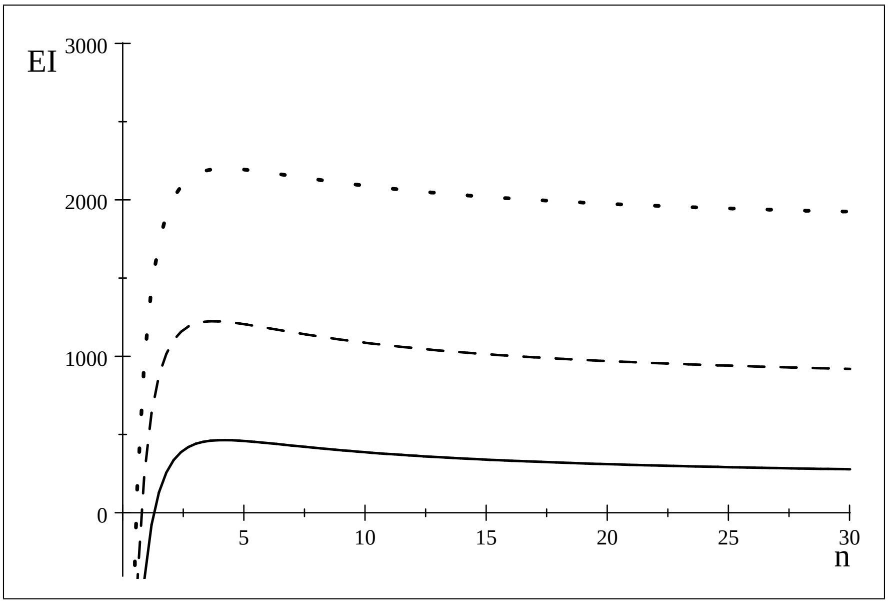

Figure 3.10: Simulation 5: $a = 120$, $c = 40$

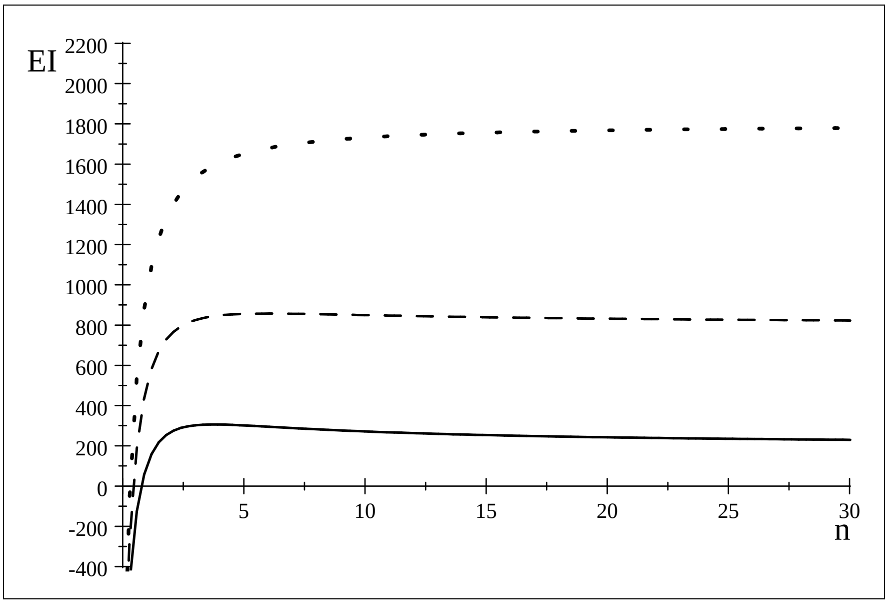

Figure 3.11: Simulation 6: $a = 80$, $c = 40$

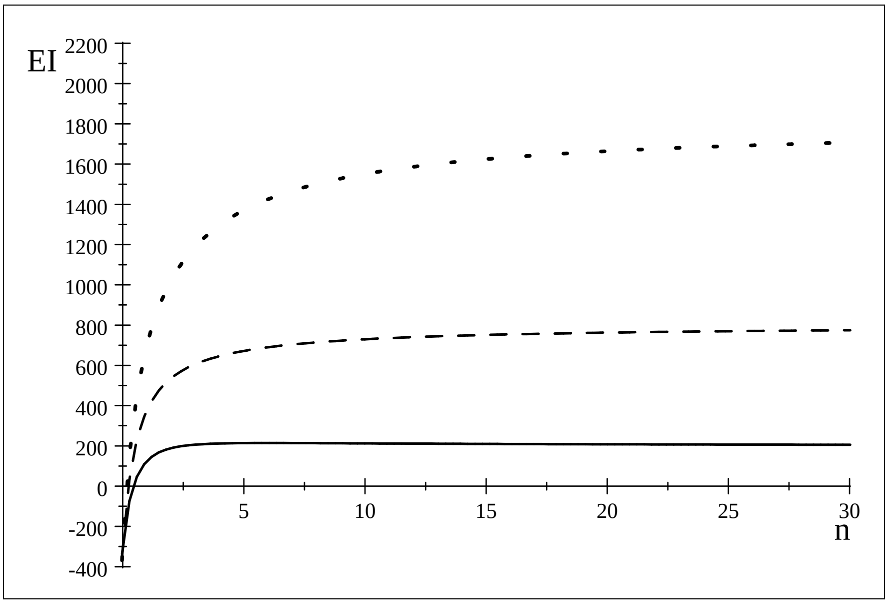

Figure 3.12: Simulation 7: $a = 60$, $c = 40$

Bibliography

Aghion, P., C. Harris and J. Vickers (1995), "Competition and Growth with Step-by-Step Technological Progress", mimeo, Nuffield College.

Aitken, B.J. and A.E. Harrison (1999), "Do Domestic Firms Benefit from Foreign Direct Investment?" Evidence from Venezuela", American Economic Review, 89, 605-618.

Armstrong, M. (2006), "Competition in Two-Sided Markets", RAND Journal of Economics, Vol. 37, 668-691.

Arrow, K. (1962), "Economic Welfare and The Allocation of Resources for Invention", R. Nelson, ed., The Rate and Direction of Inventive Activity: Economic and Social Factors. Princeton University Press.

Bar-Isaac, H. and S. Tadelis (2008), "Seller Reputation", Foundations and Trends in Microeconomics, forthcoming.

Bester, H. and E. Petrakis (1993), "The Incentives for Cost Reduction in a Differentiated Industry", International Journal of Industrial Organization, Vol. 11, 519-34.

Bhattacharya, S. and J.L. Guasch (1988), "Heterogeneity, Tournaments and Hierarchies", Journal of Political Economy, Vol. 96, 867-881.

Blomström, M. and A. Kokko (1998), "Multinational Corporations and Spillovers", Journal of Economic Surveys, 12, 247-277.

Blomström, M. and F. Sjöholm (1999), "Technology Transfer and Spillovers: Does local Participation with Multinationals matter?", European Economic Review, 43, 915-923.

Blomström, M. and M. Zejan (1991), "Why do Multinational Firms Seek out Joint Ventures?", Journal of International Development, 3, 53-63.

Bonanno, G. and B. Haworth (1998), "Intensity of Competition and the Choice between Product and Process Innovation," International Journal of Industrial Organization, Vol. 16, 495-510.

Brainard, S.L. (1997), "An empirical assessment of the Proximity-Concentration Tradeoff between Multinational Sales and Trade", American Economic Review, 87, 520-544.

Boone, J. (2000), "Competitive Pressure: The Effects on Investments in Product and Process Innovation", RAND Journal of Economics, Vol. 31, 549-569.

Boone, J. (2001), "Intensity of Competition and the Incentive to Innovate", International Journal of Industrial Organization, Vol. 19, 705-726.

Clark, D.J. and C. Riis (2001), "Rank-Order Tournaments and Selection", Journal of Economics, Vol. 73, 167-191.

Coe, D.T. and E. Helpman (1995), "International R&D Spillovers", European Economic Review, 39, 859-887.

Craig, M. (2005),"The Professor, the Banker and the Suicide King", Grand Central Publishing.

Delbono, F. and V. Denicolo (1991), "Incentives to Innovate in a Cournot Oligopoly" The Quarterly Journal of Economics, MIT Press, Vol. 106(3), 951-61.

Gattai, V. and C. Molteni (2007), " Dissipation of Knowledge and the Boundaries of the Multinational Enterprise", Review of World Economics, 143, 1-26.

Goldreich, D. and L. Pomorski (2008), "When is Bargaining Successful? Negotiated Division of Tournament Prizes", CEPR Discussion Paper 6556.

Groh, C., B. Moldovanu, A. Sela and U. Sunde, "Optimal Seedings in Elimination Tournaments", forthcoming in Economic Theory.

Grossman, G.M. and E. Helpman (2002), "Integration vs. Outsourcing in Industry Equilibrium", Quarterly Journal of Economics, 1, 317-327.

Grossman, G.M. and E. Helpman (2003), "Outsourcing vs. FDI in Industry Equilibrium", Journal of the European Economic Association, 117, 85-120.

Helpman, E., Melitz, M. and S. Yeaple (2004) "Export versus FDI with Heterogeneous Firms", American Economic Review 94(1), 300-316.

Horstmann, I. and J.R. Markusen (1996), Exploring New Markets: Direct Investment, Contractual Relations and the Multinational Enterprise", International Economic Review, 37, 1-19.

Keller, W. (2004), "International Technology Diffusion", Journal of Economic Literature, Vol. 42, 752-782.

Kokko, A. (1994), "Technology, Market Characteristics and Spillovers", Journal of Development Economics, 43, 279-293.

Lazear, E.P. and S Rosen (1981), "Rank-Order Tournaments as Optimum Labor Contracts", Journal of Political Economy, Vol. 89, 841-864.

Lee, J. (2004), "Prize and Risk-Taking Strategy in Tournaments: Evidence from Professional Poker Players", IZA Discussion Papers 1345.

Moldovanu, B. and A. Sela (2001), "The Optimal Allocation of Prizes in Contests", American Economic Review, Vol. 91, 542-558.

Moldovanu, B. and A. Sela (2006), "Contest Architecture", Journal of Economic Theory, Vol. 126, 70-97.

Mugele, C. and M. Schnitzer (2008), "Organization of Multinational Activities and Ownership Structure", International Journal of Industrial Organization, forthcoming.

Müller, T. and M. Schnitzer (2006), "Technology Transfer and Spillovers in International Joint Ventures", Journal of International Economics, 68, 456-468.

Mussa, M. and S. Rosen (1978), "Monopoly and Product Quality" Journal of Economic Theory 18, 301–317.

Rochet, J.C. and J. Tirole (2003), "Platform Competition in Two-Sided Markets", Journal of the European Economic Association, Vol. 1, 990-1029.

Rochet, J.C. and J. Tirole (2004), "Two-Sided Markets: An Overview", mimeo, IDEI University of Toulouse.

Rochet, J.C. and J. Tirole (2006), "Two-Sided Markets: A Progress Report", RAND Journal of Economics, Vol. 35, 645-667.

Tadelis, S. (1999), "What's in a Name? Reputation as a Tradeable Asset", American Economic Review, Vol. 89, 548-563.

Tadelis, S. (2002), "The Market for Reputations as an Incentive Mechanism", Journal of Political Economy, Vol. 92, 854-882.

Vives, X. (2004), "Innovation and Competitive Pressure" CEPR Discussion Papers 4369.

Eidesstattliche Versicherung

Ich versichere hiermit eidesstattlich, dass ich die vorliegende Arbeit selbständig und ohne fremde Hilfe verfasst habe. Die aus fremden Quellen direkt oder indirekt übernommenen Gedanken sowie mir gegebene Anregungen sind als solche kenntlich gemacht.

Die Arbeit wurde bisher keiner anderen Prüfungsbehörde vorgelegt und auch noch nicht veröffentlicht.

Datum: 25.09.2008

Unterschrift:

Curriculum Vitae

10/2004 - 09/2008	Research and teaching assistant Chair of Prof. Sven Rady, PhD Ph.D. program in Economics Munich Graduate School of Economics Ludwig-Maximilians-University, Munich, Germany
09/2006 - 02/2007	Visiting Ph.D. student in Economics Cambridge University, Cambridge, United Kingdom
09/2004	MSc in Economics Toulouse University, France
09/2003 - 09/2004	Studies in Economics Toulouse University, France
10/2000 - 08/2003	Studies in Economics Mannheim University, Mannheim, Germany
07/2000	European Baccalaureate European School Karlsruhe, Karlsruhe, Germany
01/04/1982	born in Karlsruhe, Germany