Essays in Sustainable Development

Inaugural-Dissertation

zur Erlangung des Grades Doctor oeconomiae publicae (Dr.oec.publ.)

an der Ludwig-Maximilians-Universität München

2006

vorgelegt von

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Promotionsabschlussberatung:	07 Februar 2007

Acknowledgment

To my supervisor Professor Peter Egger I express my deepest gratitude for all the wonderful support, professional as well as human, he gave me and because he left me the freedom in continuing to do research on a topic which I am extremely interested in.

If these four difficult years can be crowned with the present dissertation, that is also because Professor Sven Rady, from the beginning to the end, has always encouraged and helped me and he has given me precious teachings which I will never forget. For all of that I thank him.

I want to thank Professor Marco Runkel because he was one of the first persons who stimulated me to persevere in my work and found always time to listen to my research ideas. It is also especially thanks to him that I had such an enrichment and wonderful experience as that of being a teaching assistant at the chair of Professor Bernd Huber.

For another unforgettable experience I thank Professor Sjak Smulders who made my three months stay at the Economic Department of Tilburg University possible. I extremely benefited from his comments and the stimulating environment.

For all his engagement I am very grateful to Professor Gerhard Illing.

The mathematical part of chapter 4 is based upon derivations in work by Professor Georg Schlüchtermann. I gratefully acknowledge his assistance with the exposition and his kind help in preparing the draft.

For the precious help with all the administrative questions along these years I am thankful to Ingeborg Buchmayr and to Dirk Roesing with all the technical ones.

Alessia dell'Acqua, Rossella Bargiacchi, Michela Coppola, Frauke Eckermann, Federico Flora, Anita Hofmann, Friederike Koehler, Tobias Kronenberg, Karolina Leib, Corrado di Maria, Nadine Riedel, Don Rinaldo, Tiziana Rucli, Marco Sahm, Marco Salcoacci, Chiara di Taranto, I want to thank for the help and all the positive energy they gave me during this Munich experience.

To my wonderful family, my mum, my dad - who unfortunately left us from this world last year - and my brother: thank you for all.

To my beloved husband my special thanks. You have always encouraged, motivated, supported and helped me. You are a special and wonderful person.

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Part I

Concepts

Chapter 1

Introduction

1.1 Purpose and Structure

The present work is the result of my interest in two themes: poverty and environment. Scarcity, which is in fact the essential economic problem in all the history of economic thought, is inevitably related to the finiteness of the natural environment. And it is not a coincidence that "eco"-nomics and "eco"-logy have the same Greek root "oikos", meaning household. After World War II, economic growth is considered as the ultimate remedy for poverty, but it is only since the 1990s that the notions of natural resources and environmental economics systematically enter the theory of economic growth to model sustainable development, which recognizes the value of the environmental services for the growth process.

The dissertation is organized in two parts. The first one comprises the present chapter 1 "Introduction" and chapter 2 "Economy and environment joined together" and provides an overview of key notions for a better understanding of the second part, which presents new theoretical applications. Explicitly the second part analyzes different aspects of the sustainability problem that are not already captured by the existing growth literature: in chapter 3, the non-constancy of the regenerative capacity of the environment; in chapter 4, the connection between use and depletion of renewable resources to address the problem of biodiversity loss and in chapter 5 the role of nature as a knowledge reservoir.

The motivation for chaper 3 "Role of the regenerative capacity of nature in the sustainability debate: a Schumpeterian endogenous growth model" is the study of the waste sink and life support services. In the construction of a model of economic growth, the analysis of these environmental services requires the insertion of the regenerative capacity of nature in the regeneration function of the environment, which is captured by an environmental quality indicator reflecting all ecosystems and their interactions. But the regenerative capacity of nature is different from just the regenerative capacity of biological populations because it includes also the regenerative ability of particular types of renewable resources, namely water, soil, atmosphere, to maintain the quality necessary for human life (assimilation of pollutants). Therefore it is not constant, as assumed in the model of Aghion and Howitt (1998), but it depends on the impact of pollution on the environment.

The recently growing interest in the problem of biodiversity loss is the motivation for chapter 4 "Biodiversity loss and stochastic technological processes: a sustainable growth analysis". In contrast to the greenhouse effect which is well studied in the economic growth literature, to address the problem of biodiversity loss we need a new methodological approach. Namely to combine the ideas of the standard environmental quality literature of economic growth, which investigates pollution awareness, with the "corn-eating" framework, used in the analysis of optimal use of renewable resources. So, it is possible to investigate the joint effect of harvesting and induced pollution degradation on renewable resources. In addition to that, the model extends the lesson coming from the previous chapter about the need of developing different types of technologies, introducing all three possible types of environmentally friendly technologies: techniques that affect the productivity of harvested resources, techniques that reduce pollution damages, and techniques that reduce the production of pollution itself.

The motivation for chapter 5 "Nature as a knowledge reservoir: a non scale endogenous growth model with relaxation of knife edge assumptions" is the recognition of the positive role of nature as knowledge reservoir in the advancement of scientific research. In an economic model this implies inserting an environmental indicator variable, which will be called natural knowledge, not only into the production function of the final good but also into the production function of the standard technological sector. This model specification, in addition to giving a new explanatory variable for the growth process, eliminates the presence of scale effects and the recourse to knife edge assumptions about the returns to scale in the produced factors of production. Here, as well as in the previous two chapters, the final goal of the analysis is to conjecture whether the model predicts sustainable growth, and under which assumptions.

Each chapter of the second part is therefore a self-contained paper which can be read independently of the others, although the chosen sequence is not casual. It represents an evolution not only in the results of the models (from ones without sustainable development to ones with sustainable development) but also in the focus of the environmental analysis (from the particular role of the regenerative capacity of nature in the regeneration function, to the general one of nature as basis for scientific advancement).

1.2 The sustainability issue

It was during the 1970s that the new concept of economic sustainability entered the international political agenda. At that time politicians and researchers recognized that the environment plays an important role for the maintenance of economic growth. Nevertheless note that this consciousness was already present in classical economics two centuries before. For all classical economists the central question of research was what determined national wealth and its growth (Perman et al. (2003)), and natural resources were important explanatory variables, as well for Thomas Malthus in his "Essay on the Principle of Population" (1798), as for David Ricardo in his "Principles of Political Economy and Taxation" (1817).

In the 1970s the connection between natural environment and economic growth returned to the center of attention for many reasons, most important the energy crisis, environmental catastrophes and discouraging scientific publications, which produced a lot of debate.[1] That new sensibility for the relationship between nature and the economic world flowed into the United Nations Conference on the Human Environment in 1972 which was the first of a long series of international conferences about the role of the natural environment in the economic development. The successive decades, in fact, are characterized by an increasing awareness of the role of the natural environment and therefore for its preservation, which is testified by all international conventions, programmes, conferences, publications that followed, some of them listed in the appendix to this introduction.[2]

But it was only in 1987 that the concept of sustainable development was formalized. In that year the final report of the World Commission on Environment and Development "Our Common Future" (WCED (1987)) was published.[3] It states that "environment and development are not separate challenges: they are inexorably linked" and "attempts to maintain social and ecological stability through old approaches to development and environmental protection will increase instability." Therefore the new concept of sustainable development was presented, which is development that "seeks to meet the needs and aspirations of the present without compromising the ability to meet those of the future." In the report this new concept is absolutely not associated with reduction of the economic activities, instead: "Far from requiring the cessation of economic growth, it recognizes that the problems of poverty and underdevelopment cannot be solved unless we have a new era of growth in which developing countries play a large role and reap large benefits." And further "The medium term prospects for industrial countries are growth of 3-4 per cent, the minimum that international financial institutions consider necessary if these countries are going to play a part in expanding the world economy. Such growth rates could be environmentally sustainable if industrialized nations can continue the re-

[1] The Costs of Economic Growth, Mishan (1967); The Limits to Growth, Meadows et al. (1972).

[2] For a detailed historical reconstruction see UNEP (2002).

[3] Both the Commission and the report are also called Brundtland after the chairperson of the commission, Gro Harlem Brundtland. She was both Minister of the Environment and Prime Minister of Norway.

cent shifts in the content of their growth towards less material- and energy-intensive activities and the improvement of their efficiency in using materials and energy."

Then after this cornerstone report, the United Nation Conference on Environment and Development in 1992 followed. Among other goals it produced the document "Agenda 21" (UN (1993)), which is an action programme for the realization of sustainable development and the creation of a permanent UN agency called Commission on Sustainable Development. Ten years later in 2002 the World Summit on Sustainable Development followed. It states again in the "Report of the World Summit on Sustainable Development" (UN (2002)), the necessity to implement the programs for sustainable development asserting that:"Thirty years ago, in Stockholm, we agreed on the urgent need to respond to the problem of environmental deterioration. Ten years ago, at the United Nations Conference on Environment and Development, held in Rio de Janeiro, we agreed that the protection of the environment and social and economic development are fundamental to sustainable development, based on the Rio Principles. To achieve such development, we adopted the global programme entitled Agenda 21 and the Rio Declaration on Environment and Development, to which we reaffirm our commitment. The Rio Conference was a significant milestone that set a new agenda for sustainable development."

But, if the Brundtland report is commonly recognized to have put the concept of sustainable development on the international scene, in the literature there are many different definitions of sustainability as recalled by Pezzey (1997). In the standard view the term sustained growth is used to indicate increases in consumption, while sustained development refers to increases in utility. In this dissertation the terms growth and development are used interchangeably, as long as the utility function considers only consumption. To the extent that also the second goal of an increase in the environmental indicator is met, we speak of sustainable growth/development.

1.3 Environmental facts

For the pursuit of reliable information about the state of the environment and its evolution along the years, the United Nation Environment Programme (UNEP) started the Global Environmental Outlook (GEO) project in 1995. The first report, GEO-1 was published in 1997, the second in 2000 and the third and so far last, GEO-3 in 2002.[4] Since 2003, due to an increased request of updated information, also an annual report was prepared, the last one is the GEO Year Book 2006. There are eight macro indicators under observation: atmosphere, disasters caused by natural hazards, forests, biodiversity, coastal and marine areas, freshwater, urban areas, global environmental governance, (UNEP (2002)).

The macro indicator "atmosphere" comprises several sub-indicators: the energy use per unit of GDP which is decreasing and indicates therefore an improvement in the energy use, the renewable energy supply index which presents a small increase only for the wind energy, the total carbon dioxide emissions which are increasing especially for the Asia and Pacific regions, the mountain glacier mass balance which is steadily decreasing indicating an accelerating global warming, the consumption of chlorofluoro- and hydrochlorofluoro carbons and methyl bromide, substances which are responsible for the stratospheric ozone depletion. Thanks to the Vienna Convention and the Montreal Protocol their use is decreasing.

Deforestation is continuing at a high rate for the need of the agricultural sector, the surface of protected areas to maintain biological diversity is staying constant and the Red List Index for birds, which indicates the extinction's risk of species, is steadily worsening. The levels of Biological Oxygen Demand in freshwater, which indicates water contamination, is increasing in Africa and Latin America and the Caribbean. But as a signal for environmental commitment the number of ratifications of the major multilateral environmental agreements (indicated with MEA in the appendix to this introduction) is increasing.

[4] The new GEO-4 is planned to be published between March and August 2007.

1.4 Appendix 1

1972 UNESCO Convention Concerning the Protection of the World Cultural and Natural Heritage (World Heritage), MEA

1973 Convention on International Trade in Endangered Species of Wild Fauna and Flora (CITES), MEA

1975 Great Barrier Reef Marine Park declared in Australia

1977 United Nations Conference on Desertification, Nairobi, Kenya

1979 First World Climate Conference, Geneva, Switzerland;
Convention on the Conservation of Migratory Species of Wild Animals (CMS), MEA

1980 World Climate Programme established;
"World Conservation Strategy" launched by IUCN, UNEP and WWF;
Beginning of the International Decade for Drinking Water and Sanitation

1982 United Nations Convention on the Law of the Sea (UNCLOS), MEA;
United Nations General Assembly adopts the World Charter for Nature

1984 World Industry Conference on Environmental Management

1985 Vienna Convention for the Protection of the Ozone Layer (Ozone), MEA;
International Conference on the Assessment of the Role of Carbon Dioxide and other Greenhouse Gases, Villach, Austria

1986 International Whaling Commission imposes a moratorium on commercial whaling

1987 Montreal Protocol on Substances that deplete the Ozone Layer adopted;
"Our Common Future" published

1989 Basel Convention on the Transboundary Movements of Hazardous Wastes and their Disposals (Basel), MEA;
Inter-governmental Panel on Climate Change established

1990 Second World Climate Conference, Geneva, Switzerland;
Global Climate Observing System (GCOS) created

1991 Global Environment Facility established to finance conventions

1992 UN Conference on Environment and Development (the Earth Summit), Rio de Janeiro, Brazil;
Convention on Biological Diversity (CBD), MEA;
UN Framework Convention on Climate Change

1993 Chemical Weapons Convention

1994 UN Convention to Combat Desertification (UNCCD), MEA;
International Conference on Population and Development, Cairo, Egypt;
Global Conference on the Sustainable Development of Small Island Developing States, Bridgetown, Barbados

1995 World Summit for Social Development, Copenhagen, Denmark;
World Business Council for Sustainable Development created

1996 ISO 14 000 created for environmental management systems in industry;
Comprehensive Nuclear Test Ban Treaty

1997 Kyoto Protocol adopted (Kyoto), MEA;
Rio + 5 Summit reviews implementation of "Agenda 21"

1998 Rotterdam Convention on the Prior Informed Consent Procedure for Certain Hazardous Chemicals and Pesticides in International Trade (PIC), MEA

1999 Launch of "Global Compact" on labour standards, human rights and environmental protection

2000 Millenium Summit, New York, United States;
World Water Forum, The Hague

2001 Stockholm Convention on Persistent Organic Pollutants (POPs), MEA

2002 World Summit on Sustainable Development, Johannesburg, South Africa

Chapter 2

Economy and environment joined together

2.1 What an ideal model should encompass

As indicated in the previous chapter, to study the sustainability issue the natural environment has to be incorporated into the functional specifications of an optimal growth model. An almost complete and standard model could be constructed with

- a production function of the final good (Y) which is affected by the labor force (L), the human-made capital stock, natural capital and pollution. Respectively human-made capital embraces physical capital (K), human capital (H) and technological capital (A); natural capital describes the flows of renewable (R_R) and non-renewable (R_{NR}) resources, and the stocks of renewable (S_R) and non-renewable (S_{NR}) resources; pollution in form of stock (P) and flow (F),
- a growth function respectively for K, H and A,
- a growth function for L,
- a growth function for S_R and S_{NR},

- a growth function for P,
- a social welfare function (U) which depends on consumption (C) and the state of the environment (S_R, S_{NR}, F P).

The introduction of the flows of natural resources into the production function of the final good corresponds to the standard interpretation of nature as a resource base of raw materials for the agricultural and industrial sectors. Instead, the incorporation of the state of the environment into the production function together with its incorporation into the growth functions of the stock of resources describes the life support feature of nature (see Bovenberg and Smulders (1995), Gradus and Smulders (1996)). This feature guarantees that the conditions for human life are maintained on earth as the quantity of UV-B radiation on earth, air quality, or temperature.[1] Then the waste sink service is captured by the assimilative capacity of nature in the growth function of the stock of pollution and the amenity base service by the introduction of the state of the environment into the social welfare function. The importance of this service can be best understood if one imagines its absence.[2]

From the above analysis, it is evident that such a complex model, which takes into consideration all economic and environmental aspects, cannot be solved to produce analytical solutions. So, the strategy of the theoretical literature is to focus on a particular aspect of the more general model.[3] For example the model of Aghion and Howitt (1998), which is presented in the next chapter, focuses on the environmental degradation of nature caused by the flow of pollution. Using the same variable labels as above, in their model, the production function of the final good is described only by K, A and F, where the flow of pollution in the production function is attributed

[1] Some empirical studies find a correlation between average temperature and income, demonstrating that warmer countries have a worse economic performance than cooler ones; Gallup et al. (1999), Horowitz (2001).

[2] Which would also have big consequences for a portion of the tourist industry.

[3] The other strategy is that of the Computable General Equilibrium Model literature, see Boehringer (2003), which computes the solution using real data and estimated functional forms, e.g. the MIT Integrated Global System model (Reilly et al. (2005)).

a positive role (opposite to the negative one that it would have in respect to the life support service). Their interpretation about the flow of pollution is that if it is possible to pollute more, then it is also possible to produce more. The stocks of resources S_R, S_{NR} are combined to form the maximal environmental quality and the loss in that maximal quality, called E, is introduced into the utility function. So, considering that the population is constant, the model is reduced to only three state variables making it possible to find an analytical solution.

2.2 The basic model

The first model which describes an interaction between the economic and the ecological systems is that of Dasgupta and Heal (1974). The production function of output Y, which is increasing, strictly concave, twice differentiable and homogenous of degree 1, is described by

$$Y_t = Y(K_t, R_{NR,t}).$$

Capital accumulation follows the rule

$$\dot{K}_t = Y(K_t, R_{NR,t}) - C_t$$

where C_t represents consumption. The change in the resource stock is described by

$$\dot{S}_{NR,t} = -R_{NR,t}$$

and the utilitarian social welfare function reads

$$W = \int_{t=0}^{t=\infty} U(C_t)e^{-\rho t}dt$$

where $U_C > 0, U_{CC} < 0$. The corresponding current-value Hamiltonian is

$$H = U(C_t) + \lambda_{1,t}[Y(K_t, R_{NR,t}) - C_t] - \lambda_{2,t}R_{NR,t},$$

and the necessary condition with respect to the control variables C_t and $R_{NR,t}$ and with respect to the state variables K_t and $S_{NR,t}$ are

$$U_{C,t} = \lambda_{1,t} \tag{2.1}$$

$$\lambda_{1,t} Y_{R_{NR,t}} = \lambda_{2,t} \tag{2.2}$$

$$\dot{\lambda_{2,t}} = \rho \lambda_{2,t} \tag{2.3}$$

$$\dot{\lambda_{1,t}} = \rho \lambda_{1,t} - Y_{K_t} \lambda_{1,t}. \tag{2.4}$$

Equations (2.1) and (2.2) describe the static efficiency conditions. The former states that the marginal net benefit of one unit of output either used for consumption or for increases in the capital stock must be equal. The latter condition implies that the marginal value of the resource stock must be equal to the value of the marginal product of the resource.

The other two equations instead represent the dynamic efficiency conditions. Equation (2.3) is known as the Hotelling rule and assures that the growth rate of the shadow price of the resources is equal to the utility discount rate. The same happens for the other asset of this economy in equation (2.4). It guarantees in fact that capital appreciation (the growth rate of the shadow price of capital) plus marginal productivity of capital is equal to the discount rate.

Another way to see that would be to differentiate equation (2.2) with respect to time and substituting the value with equation (2.3) and (2.4). This operation leads to

$$Y_{K_t} = \frac{\dot{Y}_{R_{NR,t}}}{Y_{R_{NR,t}}}$$

which states a no-arbitrage condition of equality among rates of return.

Then differentiating equation (2.1) with respect to time and inserting equation (2.4), the growth rate of consumption is found:

$$g_{C_t} = \frac{1}{\eta}(Y_{R_{NR,t}} - \rho)$$

where η is the elasticity of marginal utility, $-\frac{U_{CC,t}C_t}{U_{C,t}}$. With η being positive, whether consumption is growing, decreasing or stays constant, depends on the difference between the marginal productivity of capital and the rate of time preference. But with capital accumulation the marginal productivity of capital decreases, so Dasgupta and Heal (1974) demonstrated that permanent growth is possible only if the elasticity of substitution σ between the exhaustible resource and capital is greater than 1 and the asymptotic marginal productivity of capital is greater than the rate of time preference.

Whether and to what extent substitution between natural and human-made capital is possible, has long been debated between economists (Solow (1986)) and is difficult to imagine for the amenity base and life support services. In the context of the present model it means that we are able to bequeath to future generations substitutes for exhaustible resources, so e.g. it should not be important how much of one specific resource we leave to future generations, but whether we leave them the ability to satisfy the need that we satisfy today with that resource (Perman et al. (2003)).

2.3 Hotelling rule for renewable resources

Renewable resources, and nature as an environmental indicator in general, have the capacity to regenerate. Biological populations such as animals or forests reproduce themselves, and natural resources such as water, air, soil are reproduced by biochemical and biophysical processes. This means that the functional specification for the change in the resource stock should be substituted with

$$\dot{S}_{R,t} = -R_{R,t} + G(S_{R,t})$$

where the basic functional form $G_t = G(S_{R,t})$ states that the resources' regeneration positively depends on the stock.

The current-value Hamiltonian is now slightly different,

$$H = U(C_t) + \lambda_{1,t}[Y(K_t, R_{R,t}) - C_t] - \lambda_{2,t}[R_{R,t} - G(S_{R,t})],$$

and the necessary conditions are

$$U_{C,t} = \lambda_{1,t} \tag{2.5}$$

$$\lambda_{1,t} Y_{R_{R,t}} = \lambda_{2,t} \tag{2.6}$$

$$\dot{\lambda}_{2,t} = \rho\lambda_{2,t} - G_{S_{R,t}}\lambda_{2,t} \tag{2.7}$$

$$\dot{\lambda}_{1,t} = \rho\lambda_{1,t} - Y_{K_t}\lambda_{1,t}. \tag{2.8}$$

Equation (2.7) is different from the previous model only because the Hotelling rule now is

$$\frac{\dot{\lambda}_{2,t}}{\lambda_{2,t}} = \rho - G_{S_{R,t}}.$$

Differently from before, the growth rate of the shadow value of resources is smaller because the ability of resources to regenerate themselves decreases their physical scarcity.

Chapter 4 presents a complex functional form for the change of the resource stock in order to investigate the problem of biodiversity loss. Even though the structure of that model is different from the present deterministic one, focusing the attention only on the environmental part, we can still appreciate how the Hotelling rule will be modified. The first consideration, following chapter 3, is that $G_{S_{R,t}}$ is not a constant but a function, $G_{S_{R,t}} = G_S(P_t, S_{R,t})$, which is negatively affected by the stock of pollution P_t, so that

$$\frac{\dot{\lambda}_{2,t}}{\lambda_{2,t}} = \rho - G_S(P_t, S_{R,t}).$$

The second step is to treat jointly the negative effects on the resources coming from the harvesting ($R_{R,t}$) and the environmental degradation, let's call it D_t, which is a function of the flow of pollution F_t, P_t and $S_{R,t}$. Therefore the growth rate of the shadow value is increased again:

$$\frac{\dot{\lambda}_{2,t}}{\lambda_{2,t}} = \rho - G_S(P_t, S_{R,t}) + D_{S_{R,t}}.$$

Finally, the positive role of different types of environmentally friendly technologies (T) is considered. It makes the reduction of the physical scarcity of the resources possible, affecting positively the regenerative capacity $G_{S_{R,t}}$ and negatively the environmental degradation rate $D_{S_{R,t}}$. Thus the new Hotelling rule is

$$\frac{\dot{\lambda}_{2,t}}{\lambda_{2,t}} = \rho - G_S(P_t, S_{R,t}, T_{G,t}) + D_S(S_{R,t}, T_{D,t})$$

where T_G are technologies targeted to reduce pollution damages and T_D to reduce the production of pollution. This happens because the regeneration function for the resource stock would correspond in this deterministic framework to

$$\dot{S}_{R,t} = -R_t(R_{R,t}, T_{R,t}) + G_t(S_{R,t}, P_t, T_{G,t}) - D_t(F_t, S_{R,t}, P_t, T_{D,t})$$

where T_R are technologies targeted to increase the productivity of harvested resources.

Part II

Applications

Chapter 3

Role of the regenerative capacity of nature in the sustainability debate: a Schumpeterian endogenous growth model

3.1 Introduction

Since the publication of "The Brundtland Report" in 1987, the growth literature has been using a new concept: sustainable development.[1] About since then, a new framework for studying growth has been developed. If the neoclassical growth literature of the '70s focused predominantly on the optimal use of non-renewable resources[2] in response to the "oil crises", new growth models of the '90s devote increasingly attention to environmental quality problems as a consequence of pollution-induced global changes (greenhouse effect, biodiversity loss).

The United Nations Conference on Environment and Development held in Rio de Janeiro in 1992 recognized the pressing environment and development problems of the world and, through adoption of Agenda 21, produced a strategy for sustainable development in the 21st century. After a decade known as the rhetoric decade, the World Summit for Sustainable Development, held in Johannesburg in August 2002, made clear that urgent action is necessary. The main result of the summit was that "one of the three pillars of sustainable development - the environment - is seriously damaged because of the distortions placed on it by the actions of human population. The collapse of the environmental pillar is a serious possibility if action is not taken as a matter of urgency to address human impacts, which have left: increased pollutants in the atmosphere, vast areas of land resources degraded, depleted and degraded forests, biodiversity under threat, reduction of the fresh water resources, depleted marine resources" (UNEP (2002)).

One of the best-established endogenous growth models is the Schumpeterian approach of Aghion and Howitt (1998), henceforth referred to as A&H. They overcome the shortcoming of the Stokey (1998) setup in reaching the combination of two goals which count for the most pragmatic definition of sustainable development (see Brock and Taylor (2004b)): "... a balanced growth path with the joint result of increasing environmental quality and ongoing growth in income per capita", the

[1] The broad concept of sustainable development was discussed also before but that report put it on the international agenda. For a comprehensive survey of definitions see Pezzey (1992).

[2] Even though there are exceptions see Forster (1980).

so-called sustained growth. But their model ignores the lasting effects of cumulative pollutants by modeling the regenerative capacity of the environment as a constant. I will demonstrate that in the Schumpeterian set-up sustainable development, as defined above, cannot be reached if, in line with insights from ecological and biological sciences, the regenerative capacity of the environment depends on the lasting effects that pollutants have on the environment. It is argued that a more sophisticated theoretical framework, incorporating different types of innovation, is needed.

I will proceed as follows: in section 2 the theoretical background for a regeneration function of the environment with a non-constant regenerative capacity will be presented; in section 3 the economic part of the model will be briefly introduced. In section 4 the new concept about the regenerative capacity of the environment will be introduced in the model. In section 5 the main idea will be explicitly modeled and section 6 will comment on the results and conclude.

3.2 The ecological part of a growth model: the regeneration function

Two of the four services[3] that the environment provides are the waste sink and the life support. The former means the capacity to disperse pollutants. The latter subsumes services like regulation of the hydrological cycle (material cycles of water and phosphorus), regulation of the gaseous composition of the atmosphere (material cycle of carbon), generation and conservation of soils (material cycle of nitrogen). In the growth literature, these two services can be associated with the regenerative capacity of the environment in the regeneration function of the environment. The environment or nature is captured by an environmental quality indicator reflecting all biosystems and their interactions. So, the regenerative capacity of the environment is different from just the regenerative capacity of one biological population because it includes also the ability of particular types of renewable resources, namely

[3]The other two are the resource base and the amenity base (see Perman et al. (2003)).

water, soil, atmosphere, to maintain the quality necessary for human life (assimilation of pollutants). The regenerative capacity of the environment is a combination of biophysical and biochemical processes. Therefore, it also depends on living organisms (bacteria, fungi, algae, plants, animals, in one word: the biodiversity). As Dasgupta and Maeler (1994) put it, "... the speed of regeneration depends, among other things, upon: the current state of the resource, the rate at which pollutants are deposited, the nature of the pollutants."

Therefore it is worthy to stress that we are not going to transfer a specific logistic function for representing the natural growth rate of nature. This would be correct if we wished to investigate one specific population of renewable resources (as the well studied populations of fish). We do not know which is the function for nature as a whole but we want to capture the fact that a linear function is not appropriate. In fact, the regenerative capacity of the environment (which, transferred to a logistic function of one specific population would correspond to the intrinsic growth rate of the resource) is not independent of changes that will happen in the environment such as the direct and indirect negative effects of different types of pollutants.

The most serious environmental problem our societies face is the increasing prevalence of cumulative pollutants (that is, pollutants that cannot be absorbed by nature in historical but only in geological times). This increase is not only due to a higher presence of strictly cumulative pollutants (e.g. organochlorine pesticides) but also due to the change of noncumulative pollutants into cumulative ones (e.g. CO_2). The change is caused by too high a rate of emission in comparison to the normal rate of regeneration. If that condition persists, the normal rate of regeneration will also start to decline and eventually go to zero (causing extinction of biological populations and quality loss for other types of renewable resources).

3.3 The Schumpeterian growth approach

The model developed by Aghion and Howitt is not a pure endogenous-growth model with technological progress, but it combines two alternative visions of the growth process:

1. capital accumulation as in Solow (1956);
2. innovation as in Aghion and Howitt (1992).

That is the reason why in the optimization problem there is one law of motion for the tangible capital K,

$$\dot{K}(t) = Y(t) - c(t),$$

and another one for the intellectual capital B

$$\dot{B}(t) = \eta \sigma n(t) B(t).$$

B is nothing else than the quality of the technology incorporated in the intermediate goods, needed for the production of the final output. It is called a Schumpeterian approach because the research activities produce technological innovation not in the sense of an increase of the number of intermediate goods as in Romer (1990), but in the sense of an increase of their quality. So, vertical instead of horizontal innovation produces obsolescence of the old technologies or the so-called Schumpeterian "creative destruction" feature. As Aghion and Howitt (1998) write, "σ indicates the rate at which the flow of innovations pushes out the economy's technological frontier; η is a positive parameter of the research technology indicating the Poisson arrival rate of innovation to a single research worker; n is the number of researchers".

The aggregate production function of the final good then is

$$Y(t) = K(t)^{\alpha} \left(B(t)(1 - n(t))\right)^{1-\alpha}$$

where $\alpha \in (0, 1)$.

3.4 The Schumpeterian approach to the environmental quality with endogenous regenerative capacity of the environment

As Aghion and Howitt did, I use the specifications proposed by Stokey (1998) for 1) the final good production function with environmental awareness, 2) the flow of pollution production function, and 3) the environmental disutility in the utility function.

In contrast, a new regeneration function of the environment is introduced. Formally, this implies that the final good production function with environmental awareness is

$$Y(t) = K(t)^{\alpha} \left(B(t)(1 - n(t))\right)^{1-\alpha} z(t)$$

where $0 < \alpha < 1$ and $0 \leqslant z \leqslant 1$ is a measure of the dirtiness of the existing technologies or of the emission standard of the existing techniques. "The cost of using a cleaner technique is that less output can be obtained per unit of input" (Aghion and Howitt (1998)).

The production function for the flow of pollution is

$$P(t) = Y(t)(z(t))^{\gamma}$$

where $\gamma > 0$.

The loss of environmental quality E is inserted into the utility function of the representative agent. This captures the amenity base services of the environment. The instantaneous utility function has an additive isoelastic form:

$$u(c, E) = \frac{(c)^{(1-\varepsilon)} - 1}{1 - \varepsilon} - \frac{(-E)^{(1+\omega)} - 1}{1 + \omega},$$

with $\varepsilon, \omega > 0$. c denotes the consumption and E is defined as the difference between the actual environmental quality and the maximal environmental quality, which could only be reached if all production would cease indefinitely.

E is also subject to an ecological threshold of the form $E_{\min} \leq E(t) \leq 0$ because the authors want to take into account that a lower limit exists below which environmental quality cannot fall without starting an irreversible deterioration process.[4]

The new regeneration function of the environment is

$$\dot{E}(t) = -P(t) - \theta(t)E(t)$$

where $\theta(t) = 1 + aE(t)$. Here, the regenerative capacity of the environment, θ, is not constant any more.[5] Since θ depends on E, the establishment of an ecological threshold $E^{\min} \leq E(t) \leq 0\ \forall t$ implies that the regenerative capacity of the environment, i.e. $\theta(t)$, faces a threshold as well.[6] So, E is increased by the flow of pollution P and reduced by the regenerative function which now is not any more linear, $\theta E(t)$, but quadratic, $(1 + aE(t))E(t)$.

The social planner's problem is described (where for concision we drop the time index with all the variables) by:

$$\max_{c,z,n} \int_0^\infty e^{-\rho t} u(c, E)\, dt$$

subject to

$$\begin{aligned}
\dot{K} &= Y - c = K^\alpha (B(1-n))^{1-\alpha} z - c \\
\dot{B} &= \eta \sigma n B \\
\dot{E} &= -P - \theta E = -K^\alpha (B(1-n))^{1-\alpha} z^{1+\gamma} - (1 + aE)E
\end{aligned}$$

and the initial conditions for K, B, E, the ecological threshold $E^{\min} \leq E_0 \leq 0$, the requirement that $K(t)$ and $B(t) \geq 0$, and the transversality conditions for K, B, E.

[4]By assumption, see page 165 of Aghion and Howitt (1998); in any case, this threshold is not relevant for the optimal balanced steady state.

[5]In A&H, θ represents the maximal potential rate of regeneration. This interpretation is no longer suitable here.

[6]Note that $E(t)$ is a measure of cumulative pollutants as well as of the current state of resources (actual environmental quality). Hence, $\theta(t)E(t)$ controls for the loss of environmental quality associated with cumulative and non-cumulative pollutants.

The new current-value Hamiltonian is

$$\begin{aligned} H &= u(c,E) + \lambda \left[K^{\alpha} \left(B \left(1-n \right) \right)^{1-n} z - c \right] + \mu\sigma\eta n B \\ & -\zeta \left[K^{\alpha} \left(B \left(1-n \right) \right)^{1-\alpha} z^{\gamma+1} + (1+aE)E \right] \end{aligned}$$

and the new first order conditions for E and ζ are[7]

$$\frac{\partial H}{\partial E} = -\dot{\zeta} + \rho\zeta \Rightarrow \dot{\zeta} = \rho\zeta - (-E)^{\omega} + \zeta \left(1 + 2aE \right),$$

$$\frac{\partial H}{\partial \zeta} = \dot{E} \Rightarrow \dot{E} = -K^{\alpha} B^{1-\alpha} (1-n)^{1-\alpha} z^{\gamma+1} - \left(1 + aE \right) E.$$

This leads to a completely different result than A&H. In the original model of A&H a balanced steady state[8] with sustainable development is possible even though under four[9] really special assumptions, because sustained development ($g_K, g_c, g_y > 0$)[10] is combined with environmental improvement ($g_E < 0$).

Proposition 1 *In this model along the balanced steady state, there is no sustainable development defined as joint achievement of sustained development and environmental improvement. Hence, an improvement of the environmental quality ($g_E < 0$) and, at the same time, sustained development ($g_K, g_c, g_y > 0$) cannot be achieved. But rather, there is a constant environmental quality ($g_E = 0$) and non-sustained development ($g_K, g_c, g_y = 0$).*

Proof: See Appendix 1 to this chapter.

[7] We only present the most relevant first order conditions, here. The other ones are presented in Appendix 1 to this chapter.

[8] That is where all variables growth at a constant rate.

[9] First assumption: $\varepsilon - 1 > 0$, second assumption: $\eta\sigma - \rho > 0$, third assumption: $(\varepsilon-1)(\eta\sigma-\rho) < \theta \left[\varepsilon(1+\omega) + \frac{\varepsilon+\omega}{(1-\alpha)\gamma} \right]$ and fourth assumption: $E^{\min} \leq E_0 \leq 0$.

[10] g means growth rates.

3.5 Endogenous regenerative capacity of the environment and the stock pollution function

In the previous section the regenerative capacity of the environment θ was negatively affected by the loss of environmental quality the society experiences, E. It is a very general and useful environmental indicator, but for capturing explicitly the special negative role of cumulative pollutants on the regenerative capacity, we have to introduce a new stock variable: the pollution stock S for the pollutants that accumulate, formally defined by $\dot{S} = \phi PS$. ϕ is the fraction of the flow of pollutants that accumulates.

The regenerative capacity of the environment $\theta(t)$, is now equal to $bS(t)^{-\delta}$, where b is a positive constant, $S(t)$ is the stock of pollution and $\delta > 0$, which captures the sensitivity of the potential maximal regeneration rate b to the negative effect of the accumulation of pollutants.

The new social planner's problem is

$$\max_{c,z,n} \int_0^\infty e^{-\rho t} u\left(c, E\right) dt$$

subject to

$$\begin{aligned}
\dot{K} &= Y - c = K^\alpha (B(1-n))^{1-\alpha} z - c \\
\dot{B} &= \eta\sigma n B \\
\dot{E} &= -P - \theta E = -K^\alpha (B(1-n))^{1-\alpha} z^{1+\gamma} - bS^{-\delta} E \\
\dot{S} &= \phi P S
\end{aligned}$$

and the initial conditions for K, B, E, S, the ecological threshold $E^{\min} \leq E_0 \leq 0$, the requirement that $K(t), B(t)$ and $S(t) \geq 0$, and the transversality conditions for K, B, E, S.

The new current-value Hamiltonian is

$$\begin{aligned}
H &= u(c, E) + \lambda \left[K^\alpha \left(B\left(1-n\right)\right)^{1-n} z - c\right] + \mu\sigma\eta n B \\
&\quad -\zeta \left[K^\alpha \left(B\left(1-n\right)\right)^{1-\alpha} z^{\gamma+1} - bS^{-\delta} E\right] + \xi\phi P S
\end{aligned}$$

and the derivation of all first order conditions is affected by the model specification.

Proposition 2 *Also in this new model specification, along the balanced steady state there is no sustainable development. Again, an improvement of the environmental quality ($g_E < 0$) and, at the same time, sustained development ($g_K, g_c, g_y > 0$) cannot be achieved. But rather, there is a constant environmental quality ($g_E = 0$) and non-sustained development ($g_K, g_c, g_y = 0$).*

Proof: See Appendix 2 to this chapter.

So, even though the regeneration function has a more precise formulation than before (in the sense that the regenerative capacity is explicitly negatively affected by the indirect effect of cumulative pollutants through the stock of them) and a more general one (in the sense that $\delta \neq 0$), the results do not change.

3.6 Conclusion: what the new environmental specifications tell us

In both cases, even though the model specifications are different, the theoretical motivations for a non-constant regenerative capacity are the same, and this is the reason why the same results appear. The intuition for them is the following. The intellectual capital B does not grow enough to offset the decline in dirtiness z. In the model of A&H, z has to fall to compensate the environmental costs of just the flow of pollution. In my models, z needs to fall even faster. The reason is that not only the direct environmental costs associated with the flow of pollution but also the indirect ones, generated by the accumulation of pollutants, have to be compensated to avoid increased environmental damage.

Whereas non-cumulative pollutants can be controlled by acting on the emission flow, cumulative pollutants need to be controlled through a management of the environmental stock. This suggests that, in order to avoid the accumulation of environmental damage and to restore environmental quality, incentives and innovations that are suited to manage the stock of damage should be targeted.

3.7 Appendix 1

3.7.1 Optimal control problem

The problem exhibits three state variables, $K(t)$, $B(t)$ and $E(t)$, and three control variables, $c(t)$, $n(t)$ and $z(t)$. Formally, it can be written as

$$\max_{c,n,z} \int_0^\infty u(c,E)e^{-\rho t}dt$$

subject to

$$\begin{aligned}
\dot{K} &= Y - c = K^\alpha (B(1-n))^{1-\alpha} z - c \\
\dot{B} &= \sigma \eta n B \\
\dot{E} &= -K^\alpha (B(1-n))^{1-\alpha} z^{\gamma+1} - (1+aE)E \\
K_0, B_0 &\geq 0 \\
E_0 &\in (E^{\min}, 0) \\
\lim_{t\to\infty} e^{-\rho t}\lambda K &= 0 \\
\lim_{t\to\infty} e^{-\rho t}\mu B &= 0 \\
\lim_{t\to\infty} e^{-\rho t}\zeta E &= 0 \\
E(t) &\in [E^{\min}, 0] \ \forall t.
\end{aligned}$$

The current-value Hamiltonian is

$$H = u(c,E) + \lambda \left[K^\alpha \left(B\left(1-n\right)\right)^{1-\alpha} z - c\right] + \mu\sigma\eta n B - \zeta \left[K^\alpha \left(B\left(1-n\right)\right)^{1-\alpha} z^{\gamma+1} + (1+aE)E\right].$$

The necessary conditions are:

Static part:

$$\frac{\partial H}{\partial c} = 0 \;\Rightarrow\; \frac{\partial u(c,E)}{\partial c} - \lambda = 0$$

$$\Rightarrow\; \lambda = c^{-\varepsilon} \tag{3.1}$$

$$\frac{\partial H}{\partial z} = 0 \;\Rightarrow\; \lambda - \zeta\,(\gamma+1)\,z^{\gamma} = 0$$

$$\Rightarrow\; \zeta = \frac{\lambda}{(1+\gamma)\,z^{\gamma}} \tag{3.2}$$

$$\frac{\partial H}{\partial n} = 0 \;\Rightarrow\; \mu\eta\sigma B = (1-\alpha)\,\frac{\gamma}{1+\gamma}\frac{\lambda Y}{1-n} \tag{3.3}$$

Dynamic part:

$$\frac{\partial H}{\partial K} = -\dot{\lambda} + \rho\lambda \;\Rightarrow\; \frac{\dot{\lambda}}{\lambda} = \rho - \alpha\frac{Y}{K}\left(1 - \frac{\zeta}{\lambda}z^{\gamma}\right) \tag{3.4}$$

$$\frac{\partial H}{\partial B} = -\dot{\mu} + \rho\mu \;\Rightarrow\; \dot{\mu} = \rho\mu - \mu\eta n\sigma - (1-\alpha)\,\lambda\frac{Y}{B}\frac{\gamma}{1+\gamma} \tag{3.5}$$

$$\frac{\partial H}{\partial E} = -\dot{\zeta} + \rho\zeta \;\Rightarrow\; \dot{\zeta} = \rho\zeta - (-E)^{\omega} + \zeta(1+2aE). \tag{3.6}$$

A&H instead have

$$\frac{\partial H}{\partial E} = -\dot{\zeta} + \rho\zeta \Rightarrow \dot{\zeta} = \rho\zeta - (-E)^{\omega} + \zeta\theta.$$

The three resource constraints are

$$\frac{\partial H}{\partial \lambda} = \dot{K} \;\Rightarrow\; \dot{K} = K^{\alpha}B^{1-\alpha}(1-n)^{1-\alpha}z - c \tag{3.7}$$

$$\frac{\partial H}{\partial \mu} = \dot{B} \;\Rightarrow\; \dot{B} = \sigma\eta n B \tag{3.8}$$

$$\frac{\partial H}{\partial \zeta} = \dot{E} \;\Rightarrow\; \dot{E} = -K^{\alpha}(B(1-n))^{1-\alpha}z^{\gamma+1} - (1+aE)E. \tag{3.9}$$

A&H instead have

$$\frac{\partial H}{\partial \zeta} = \dot{E} \Rightarrow \dot{E} = -K^{\alpha}(B(1-n))^{1-\alpha}z^{\gamma+1} - \theta E.$$

3.7.2 Derivation of growth rates in a balanced steady state

From

$$g_K \equiv \frac{\dot{K}}{K} = \frac{Y}{K} - \frac{c}{K}$$

a balanced steady state $\dot{g}_K = 0$ is found either when

$$g_K = g_Y = g_c, \tag{3.10}$$

as in the endogenous growth literature is assumed, or when

$$g_K = 0, \;\; g_Y = g_c. \tag{3.11}$$

From (3.1), we obtain

$$g_\lambda = -\varepsilon g_c. \tag{3.12}$$

From (3.6) the growth rate of ζ is

$$g_\zeta = \rho - \frac{(-E)^\omega}{\zeta} + (1 + 2aE)$$

and therefore in balanced steady state the growth rate of g_ζ must satisfy

$$-\omega g_E + g_\zeta = 0$$

and

$$g_E = 0. \tag{3.13}$$

Thus also

$$g_\zeta = 0. \tag{3.14}$$

From

$$g_B \equiv \frac{\dot{B}}{B} = \eta\sigma n$$

and $\dot{g}_B = 0$, we get

$$g_n = 0. \tag{3.15}$$

From (3.5) and $\dot{g}_\mu = 0$, we obtain

$$g_Y - g_B + g_\lambda - g_\mu = 0.$$

With (3.12) and knowing that $g_Y = g_c$, we obtain

$$g_c (1 - \varepsilon) - g_B = g_\mu. \tag{3.16}$$

From (3.9) and $\dot{g}_E = 0$, we find

$$\frac{Yz^{\gamma}}{E}\left(-g_Y - \gamma g_z + g_E\right) - aEg_E = 0.$$

Therefore, recalling that $g_Y = g_c$ and that $g_E = 0$, we have

$$\gamma g_z = -g_c.$$

From (3.4) and $\dot{g}_\lambda = 0$ and using (3.12) and (3.14), we obtain

$$\gamma g_z = -\varepsilon g_c$$

and, after inserting the previous equation,

$$-g_c + \varepsilon g_c = 0.$$

Thus we obtain

$$g_c = 0 \tag{3.17}$$

and

$$g_Y = 0. \tag{3.18}$$

Substituting back in γg_z we get

$$g_z = 0 \tag{3.19}$$

and in (3.12)

$$g_\lambda = 0. \tag{3.20}$$

From the production function we must have $g_Y = \alpha g_K + (1 - \alpha)\, g_B + g_z$. Knowing that $g_Y = 0$, $g_z = 0$ and $g_K = 0$ (either because of (3.10) or (3.11)), we obtain

$$g_B = 0. \tag{3.21}$$

Therefore from (3.16), we find also that

$$g_\mu = 0. \tag{3.22}$$

3.8 Appendix 2

3.8.1 Optimal control problem

The problem exhibits four state variables, $K(t)$, $B(t)$, $E(t)$ and $S(t)$, and three control variables $c(t)$, $n(t)$ and $z(t)$. Formally, it can be written as

$$\max_{c,n,z} \int_0^\infty u(c,E)e^{-\rho t}dt$$

subject to

$$\begin{aligned}
\dot{K} &= Y - c = K^\alpha(B(1-n))^{1-\alpha}z - c \\
\dot{B} &= \sigma\eta nB \\
\dot{E} &= -P + bS^{-\delta}E \\
\dot{S} &= \phi PS \\
K_0, B_0, S_0 &\geq 0 \\
E_0 &\in (E^{\min}, 0) \\
\lim_{t\to\infty} e^{-\rho t}\lambda K &= 0 \\
\lim_{t\to\infty} e^{-\rho t}\mu B &= 0 \\
\lim_{t\to\infty} e^{-\rho t}\zeta E &= 0 \\
\lim_{t\to\infty} e^{-\rho t}\xi S &= 0 \\
E(t) &\in [E^{\min}, 0]\ \forall t.
\end{aligned}$$

The current-value Hamiltonian is

$$H = u(c,E) + \lambda\left[K^\alpha\left(B\left(1-n\right)\right)^{1-\alpha}z - c\right] + \mu\sigma\eta nB - \zeta\left[P - bS^{-\delta}E\right] + \xi\phi PS.$$

We recall that $P = Yz^\gamma = K^\alpha\left(B\left(1-n\right)\right)^{1-\alpha}z^{1+\gamma}$. The necessary conditions are:

Static part:

$$\frac{\partial H}{\partial c} = 0 \;\Rightarrow\; \frac{\partial u(c,E)}{\partial c} - \lambda = 0$$

$$\Rightarrow\; \lambda = c^{-\varepsilon} \tag{3.23}$$

$$\frac{\partial H}{\partial z} = 0 \;\Rightarrow\; \lambda\frac{Y}{z} - \zeta(1+\gamma)\frac{P}{z} + \xi\phi(1+\gamma)\frac{P}{z}S = 0$$

$$\Rightarrow\; \zeta = \frac{\lambda Y}{(1+\gamma)P} + \xi\phi S \tag{3.24}$$

$$\frac{\partial H}{\partial n} = 0 \;\Rightarrow\; -\lambda(1-\alpha)\frac{Y}{(1-n)} + \mu\sigma\eta B + \zeta(1-\alpha)\frac{P}{(1-n)} - \xi\phi(1-\alpha)\frac{P}{(1-n)}S = 0$$

$$\Rightarrow\; \mu\eta\sigma B = \lambda(1-\alpha)\frac{Y}{(1-n)} - \zeta(1-\alpha)\frac{P}{(1-n)} + \xi\phi(1-\alpha)\frac{P}{(1-n)}S = 0$$

$$\Rightarrow\; \mu\eta\sigma B = (1-\alpha)\,\frac{\gamma}{1+\gamma}\frac{\lambda Y}{1-n} \tag{3.25}$$

Dynamic part:

$$\frac{\partial H}{\partial K} = -\dot{\lambda} + \rho\lambda \;\Rightarrow\; \frac{\dot{\lambda}}{\lambda} = \rho - \alpha\frac{Y}{K}\left(1 - \frac{\zeta}{\lambda}z^{\gamma} + \frac{\xi}{\lambda}z^{\gamma}\phi S\right) \tag{3.26}$$

$$\frac{\partial H}{\partial B} = -\dot{\mu} + \rho\mu \;\Rightarrow\; \dot{\mu} = \rho\mu - (1-\alpha)\lambda\frac{Y}{B} - \mu\sigma\eta n + \zeta\frac{P}{B}(1-\alpha) - \xi\phi\frac{P}{B}(1-\alpha)$$

$$\Rightarrow\; \dot{\mu} = \rho\mu - \mu\eta n\sigma - (1-\alpha)\,\lambda\frac{Y}{B}\frac{\gamma}{1+\gamma} \tag{3.27}$$

$$\frac{\partial H}{\partial E} = -\dot{\zeta} + \rho\zeta \;\Rightarrow\; \dot{\zeta} = \rho\zeta - (-E)^{\omega} - \zeta b S^{-\delta} \tag{3.28}$$

$$\frac{\partial H}{\partial S} = -\dot{\xi} + \rho\xi \;\Rightarrow\; \dot{\xi} = \rho\xi - \xi\phi P + \zeta b\delta S^{-\delta-1}E. \tag{3.29}$$

The three resource constraints are

$$\frac{\partial H}{\partial \lambda} = \dot{K} \;\Rightarrow\; \dot{K} = K^{\alpha}B^{1-\alpha}(1-n)^{1-\alpha}z - c \tag{3.30}$$

$$\frac{\partial H}{\partial \mu} = \dot{B} \;\Rightarrow\; \dot{B} = \sigma\eta n B \tag{3.31}$$

$$\frac{\partial H}{\partial \zeta} = \dot{E} \;\Rightarrow\; \dot{E} = -P + bS^{-\delta}E \tag{3.32}$$

$$\frac{\partial H}{\partial \xi} = \dot{S} \;\Rightarrow\; \dot{S} = \phi P S. \tag{3.33}$$

3.8.2 Derivation of growth rates in a balanced steady state

From

$$g_K \equiv \frac{\dot{K}}{K} = \frac{Y}{K} - \frac{c}{K}$$

a balanced steady state $\dot{g_K} = 0$ is found either when

$$g_K = g_Y = g_c, \tag{3.34}$$

as in the endogenous growth literature is assumed, or when

$$g_K = 0, \; g_Y = g_c. \tag{3.35}$$

From (3.23), we obtain

$$g_\lambda = -\varepsilon g_c. \tag{3.36}$$

From (3.28) the growth rate of ζ is

$$g_\zeta = \rho - \frac{(-E)^\omega}{\zeta} - bS^{-\delta}$$

and therefore in balanced steady state the growth rate of g_ζ must satisfy

$$-\omega g_E + g_\zeta = 0 \tag{3.37}$$

and

$$g_S = 0. \tag{3.38}$$

From

$$g_B \equiv \frac{\dot{B}}{B} = \eta\sigma n$$

and $\dot{g_B} = 0$, we get

$$g_n = 0. \tag{3.39}$$

From (3.27) and $\dot{g_\mu} = 0$, we obtain

$$g_Y - g_B + g_\lambda - g_\mu = 0.$$

With (3.36) and knowing that $g_Y = g_c$, we obtain

$$g_c \left(1 - \varepsilon\right) - g_B = g_\mu. \tag{3.40}$$

From (3.32) and $\dot{g_E} = 0$, we find

$$-\frac{Yz^\gamma}{E}\left(g_Y + \gamma g_z - g_E\right) - \delta b S^{-\delta} g_S = 0.$$

Therefore, recalling that $g_S = 0$, we have

$$g_E = g_Y + \gamma g_z. \tag{3.41}$$

From (3.33) and $\dot{g}_S = 0$, we find

$$g_Y + \gamma g_z = 0. \tag{3.42}$$

Thus from (3.41), we obtain

$$g_E = 0, \tag{3.43}$$

and from (3.37)

$$g_\zeta = 0. \tag{3.44}$$

Equation (3.26) is the growth rate of λ and therefore in balanced steady state the growth rate of g_λ must satisfy

$$g_\zeta - g_\lambda + g_Y + \gamma g_z - g_K = 0 \tag{3.45}$$

and

$$g_\xi - g_\lambda + g_Y + \gamma g_z - g_K + g_S = 0. \tag{3.46}$$

Knowing that $g_\zeta = 0$, $g_Y + \gamma g_z = 0$ and $g_\lambda = -\varepsilon g_c$, from (3.45), we obtain

$$\varepsilon g_c - g_K = 0.$$

Thus either because of (3.34) or (3.35), we get

$$g_c = 0, \tag{3.47}$$

and

$$g_Y = 0. \tag{3.48}$$

From (3.46) and knowing that $g_S = 0$, $g_Y + \gamma g_z = 0$ and $\varepsilon g_c - g_K = 0$, we obtain

$$g_\xi = 0. \tag{3.49}$$

From (3.47) and (3.36), we obtain

$$g_\lambda = 0. \tag{3.50}$$

From (3.48) and (3.42), we obtain

$$g_z = 0. \tag{3.51}$$

From the production function we must have $g_Y = \alpha g_K + (1-\alpha)\, g_B + g_z$. Knowing that $g_Y = 0$, $g_z = 0$ and $g_K = 0$ (either because of (3.34) or (3.35)), we obtain

$$g_B = 0. \tag{3.52}$$

Therefore from (3.40), we find also that

$$g_\mu = 0. \tag{3.53}$$

Chapter 4

Biodiversity loss and stochastic technological processes: a sustainable growth analysis

4.1 Introduction

The two most critical environmental challenges that our society faces nowadays are human-induced global ones: the greenhouse effect and the biodiversity loss. We develop a stochastic endogenous growth model to investigate the biodiversity loss challenge for the purpose of sustainability. In fact we investigate the conditions for an optimal growth path to be sustainable.

There are four motivations for developing such a model. First, the problem of biodiversity loss has not received attention from the growth literature yet (in contrast to the greenhouse effect). Second, the analysis of the biodiversity loss requires an approach that is fundamentally different from that one used for assessing the greenhouse effect. Third, an optimal growth model is the best way to study sustainability, since we can use such a model to derive conditions under which sustained growth can go hand in hand with environmental improvement. Finally, the chosen stochastic approach is very effective to incorporate different types of technological progress and to find an analytical solution. In a deterministic version we should add a state variable for each different type of technological progress. This would increase the complexity of the model and, as already discussed in Chapter 2, the chance to find a general mathematical solution, see Pezzey (1992).

The importance of biodiversity loss as an indicator of environmental sustainability has only recently come to the limelight of research. For instance, the current observed rate of extinction per century just for birds and mammals is 100-1000 times the "natural" background rate, based on fossil records, see Townsend et al. (2003). Furthermore, the tropical moist forest clearance and burning due to land conversion (one of the major causes of biodiversity loss) will increase over the next 50 years by 30 percent and 20 percent, respectively, see Tilman et al. (2001).

Why do we need a different approach to account for biodiversity loss? This can be seen immediately from recalling that the two major reasons for biodiversity loss (at the level of species, communities, and ecosystems) are overexploitation of renewable resources (which in the model corresponds to the variable R) and habitat disruption.

The latter can be directly caused by the flow of pollution (Γ), or indirectly by the stock of pollution (P). The consideration of biodiversity loss requires viewing renewable resources from a broader perspective (including biological populations and water, soil and atmosphere). Hence, it is insufficient to analyze either the optimal use of renewable resources or the lasting effects of pollution problems resulting from that use. In fact both aspects are simultaneously relevant for biodiversity loss. This is inherently different from the greenhouse effect whose cause is the cumulation of pollutants rather than the exhaustion of non-renewable resources per se.

There is a notable disconnection between the "70s growth models" being interested in the optimal use of non-renewable resources (Dasgupta and Heal (1974), Stiglitz (1974), Solow (1974a)) and the present pollution-induced awareness intrinsic in the "90s growth models" on environmental quality (Bovenberg and Smulders (1995), Stokey (1998), Aghion and Howitt (1998), Brock and Taylor (2004a)). Brock and Taylor (2004b) provide a critical discussion of this 'unbundling' of interests in environmental issues. Obviously, the case of renewable resources calls for a simultaneous treatment of "optimal use" and "quality degradation" issues.

This chapter combines the ideas of the standard environmental quality literature of economic growth (which investigates pollution awareness) with that of the "corn-eating" framework (used in the analysis of optimal use of renewable resources, see Pezzey (1992)). Thereby, three standard shortcomings will be overcome: (i) we investigate the joint effect of harvesting and induced pollution degradation on renewable resources; (ii) in contrast to previous research, the recruitment curve will not treat the environment as invariant (Townsend et al. (2003)) or, put differently, the regenerative capacity (Clark (1990)) will not be exogenous and fixed but "conditional on the particular environment circumstances that happen to prevail, and [it will] change if any of those circumstances change" (Perman et al. (2003));[1] (iii) the regeneration function will be affected by the scientific and technological advancements, unlike in previous research. We introduce three possible types of environ-

[1] In the previous chapter we have demonstrated the importantce of the non-constancy of the regenerative capacity.

mentally friendly technologies: techniques that affect the productivity of harvested resources (e.g., to avoid clear-felling in the forest harvest), techniques that reduce pollution damages (e.g., to restore water quality), and techniques that reduce the production of pollution (e.g., to increase the efficiency of the practices used to prevent soil degradation). The technologies are generated by a non-stationary Poisson process whose arrival rate depends on both time and R&D investments (Lafforgue (2004)). Only in steady state, this process will be stationary. If these R&D investments produced an infinite number of innovations, the three lasting effects on renewable resources would potentially disappear.

This chapter is organized in five sections. We present the model in the following one. In section 3 we examine the new Hotelling rule, taking into account the negative effects of pollutants and the positive ones of new innovations. In section 4 analytical optimal solutions are discussed. Then, in section 5, we study the optimal trajectories of extraction and consumption and we determine the necessary conditions for the optimal growth paths to be sustainable. A summary and the concluding remarks are given in section 6.

4.2 Structure of the model

Assume an economy that produces a homogeneous final good Y_t, using labor (L) and renewable resources according to

$$Y_t = F(R_t, L_t) = R_t^{\theta} L_t^{1-\theta} \tag{4.1}$$

where R_t is the harvested resources (i.e., the flow) at period t. The population is constant over time and equal to 1 and each individual can supply up to one unit of labor per unit of time, i.e. $0 \leq L_t \leq 1$. Furthermore, there are constant returns to scale, $\theta \in (0, 1)$.

The stock of resources is modeled as a stochastic process. It evolves over time according to the following stochastic equation

$$dS_t = (\mu P_t^{-\zeta} S_t^{\kappa} dt + \sigma_1 \mu P_t^{-\zeta} S_t^{\kappa} dq_{1,t}) - (R_t dt - \sigma_2 R_t dq_{2,t}) - (\Gamma P_t^{\xi} S_t^{\rho} dt - \sigma_3 \Gamma P_t^{\xi} S_t^{\rho} dq_{3,t}) \tag{4.2}$$

where $\mu, \kappa, \zeta, \rho, \xi \geq 0$ and $\sigma_1, \sigma_2, \sigma_3 \geq 0$ are parameters. The parameters $\sigma_i (i = 1, 2, 3)$ denote the expected rate of growth in the availability of the resources due to technological innovations. The components on the right-hand side of (4.2) can be interpreted as follows. $\mu P_t^{-\zeta} = \Xi_t$ is the regenerative capacity of resources that is endogenously determined in equilibrium. Then, $\mu P_t^{-\zeta} S_t^{\kappa} dt$ is a modified regeneration function of resources. Note that this differs from the one used in existing sustainable growth models, Aghion and Howitt (1998), where the regenerative capacity is constant. Here, this rate inter alia depends on the stock of pollution of the economy (P). There is a direct negative effect on the change in resource stocks from the harvesting (R) and from the flow of pollution (Γ), which depending on the type of pollutants is reinforced by the level of pollution and resources. However, there is a threefold role to play for technological innovations. Among those, there are two ways of how technological innovations affect the impact of pollution on the availability of resources. First, σ_1 is the rate at which the indirect negative effect of pollution on resources is reduced. Second, σ_3 reflects the rate at which the direct negative effect of pollution on resources is reduced. Note that σ_1 and σ_3 are due to innovations affecting resource abatement. In contrast, σ_2 is due to innovations in the efficiency of resources usage. Therefore, there is a direct positive effect on dS_t from the latter.

Note that the law of motion for the renewable resources takes into account all the considerations given in the introduction. 1) The regenerative capacity of resources Ξ_t is not exogenously given but negatively influenced by the lasting effects of pollutants, especially, of cumulative ones. 2) The regeneration function is reduced not only by either harvesting $R_t dt$ or environmental degradation induced by the emission of pollutants $\Gamma P_t^{\xi} S_t^{\rho} dt$, but by both of them simultaneously. 3) These three lasting effects on the regeneration function can be reduced by three possible types of environmental friendly innovations: $\sigma_1 \mu P_t^{-\zeta} S_t^{\kappa} dq_{1,t}$, $\sigma_2 R_t dq_{2,t}$, and $\sigma_3 \Gamma P_t^{\xi} S_t^{\rho} dq_{3,t}$. 4) The variations in the random cumulated number of innovations ($dq_{1,t}$,$dq_{2,t}$, $dq_{3,t}$) follow a non stationary Poisson process[2] with arrival rates $\lambda_i(N_t)(i = 1, 2, 3)$, where N_t denotes the fraction of labor devoted to R&D as in Lafforgue (2004):

$$\mathbb{P}(q_{i,t} - q_{i,s} = k) = \frac{(\int_s^t \lambda_i(N_u)du)^k}{k!} e^{-\int_s^t \lambda_i(N_u)du} \tag{4.3}$$

for $0 \leq s \leq t$, and $q_{1,t} - \int_0^t \lambda_1(N_s)ds, q_{2,t} - \int_0^t \lambda_2(N_s)ds, q_{3,t} - \int_0^t \lambda_3(N_s)ds$ are independent martingales ($E[q_{i,T}|q_{i,t}] = q_{i,t}$ for $T \geq t$). The intensity function $\lambda_i(.)$ is assumed to be increasing and concave in N_t. In the time intervals between the innovation jumps of the technological sector, the resource evolves deterministically.

Labor can be devoted to either production or R&D activities. Therefore, the following constraints must hold:

$$N_t \geq 0,$$

$$L_t \geq 0,$$

$$1 - L_t - N_t \geq 0.$$

We consider pollutants as an inevitable consequence of human activity. Following the argument of the law of thermodynamics and the considerations of Common (1995) about the environmental impact, in the very long run, we model the evolution of the stock of pollution as:

$$\dot{P} = \Gamma P.$$

Note that this specification is consistent with an inverted U-shape of the environmental Kuznets curve. However, it also covers the case where no such inverted U-shaped environmental Kuznets curve prevails.[3] What is important here is that

[2]A Poisson process (q_t) is a time dependent family of identically and independent distributed (iid) random variables with integer values $q_0 = 0$. The increments $q_t - q_s$ and $q_v - q_u$ are stochastically independent and stationary.

[3]Although the empirical finding of an inverted U-shaped environmental Kuznets curve is uncontroversial for some particular forms of pollution (see Grossman and Krueger (1995)), recent papers cast doubt on an application of the concept to pollution in general (see Bertinelli and Strobl (2005), Stern (2004)).

the emission of pollutants does not converge to zero but to a maybe even small level $\Gamma > 0$ as income grows.

Equation (4.3) tells us that the current probability of a new successful innovation increases with the effort devoted to R&D activities. As long as no such effort is undertaken, the probability of success is zero ($\lambda_i(0) = 0$). In each point of time either a new innovation is developed ($dq_{i,t} = 1$) with probability $\lambda_i(N_t)dt$, or is not ($dq_{i,t} = 0$) with probability $[1 - \lambda_i(N_t)dt]$. In this last case equation (4.2) is reduced to its deterministic components and becomes

$$dS_t = \mu P_t^{-\zeta} S_t^{\kappa} dt - R_t dt - \Gamma P_t^{\xi} S_t^{\rho} dt$$

where the technological progress does not mitigate the negative effects on the regenerative function. Assume that the consequences of successful innovations are instantaneous. Then, each time a new innovation occurs, q_i is instantaneously increased by one unit and $dt = 0$. Thus, the availability of resources instantaneously grows but in a discontinuous manner since the stock trajectory jumps upward at each new success of the technological process. The size of such a jump is given by

$$\begin{aligned} \Delta_1 S_t &= \sigma_1 \mu P_t^{-\zeta} S_t^{\kappa}, \\ \Delta_2 S_t &= \sigma_2 R_t, \\ \Delta_3 S_t &= \sigma_3 \Gamma P_t^{\xi} S_t^{\rho}. \end{aligned}$$

The discrete changes in the availability of resources are assumed to be proportional to the size of the lasting effects, to maintain the notion that the R&D activity is proportional to the severity of the lasting effects. Each of the three possible types of innovations happens independently of each other. On average, the positive effects of innovations may balance the lasting pressures of harvesting and pollutants.

The instantaneous utility function of the infinitely lived representative agent is characterized by

$$u(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma};\ \gamma > 0;\ \gamma \neq 1$$

where C_t is the consumption quantity of the final good at date t and $\frac{1}{\gamma}$ is the elasticity of intertemporal substitution of consumption.

4.3 The new Hotelling Rule

The program (Ω) of the social planner is to maximize the expected present value of the utility

$$V(S) = \max_{R_t, N_t \geq 0} E \int_0^\infty u(C_t) e^{-\delta t} dt$$

subject to

$$C_t = F(R_t, 1 - N_t)$$

$$dS_t = (\mu P_t^{-\zeta} S_t^\kappa dt + \sigma_1 \mu P_t^{-\zeta} S_t^\kappa dq_{1,t}) - (R_t dt - \sigma_2 R_t dq_{2,t}) - (\Gamma P_t^\xi S_t^\rho dt - \sigma_3 \Gamma P_t^\xi S_t^\rho dq_{3,t})$$

$$R_t, N_t, 1 - N_t, S_t \geq 0 \quad \forall t \geq 0$$

where future utility flows are discounted at rate $\delta > 0$ and one control variable is redundant through $L_t = 1 - N_t$. Using the dynamic programming technique (Merton (1990)) and the results of Sennewald and Waelde (2005) and Sennewald (2005) we find the Hamilton-Jacobi-Bellman equation associated with the value function of (Ω), $V(S_t)$.

$$\delta V(S_t) = \max_{R_t, N_t \geq 0} \{u(C_t) + \frac{1}{dt} E dV(S_t)\}. \quad (4.4)$$

If we expand the stochastic differential $dV(S_t)$, equation (4.4) becomes

$$\begin{aligned} \delta V(S) &= \max_{R_t, N_t \geq 0} \{u(C_t) + V'(S)[\mu P_t^{-\zeta} S^\kappa - R_t - \Gamma P_t^\xi S^\rho] + \\ &\quad + \lambda_1(N_t) \Delta_1 V(\hat{S}) + \lambda_2(N_t) \Delta_2 V(\hat{S}) + \lambda_3(N_t) \Delta_3 V(\hat{S})\} \end{aligned} \quad (4.5)$$

where $\Delta_1 V(\hat{S})$, $\Delta_2 V(\hat{S})$, and $\Delta_3 V(\hat{S})$ are the respective instantaneous increases in social welfare due to the development of a new environmental friendly innovation, reducing the permanently negative effects of pollutants on the regenerative capacity μ, the harvesting pressure, and the temporary emission intensity:

$$\begin{aligned} \Delta_1 V(\hat{S}) &= \underbrace{V(S + \sigma_1 \mu P_t^{-\zeta} S_t^\kappa)}_{V_1(\hat{S})} - V(S) \\ \Delta_2 V(\hat{S}) &= \underbrace{V(S + \sigma_2 R_t)}_{V_2(\hat{S})} - V(S) \\ \Delta_3 V(\hat{S}) &= \underbrace{V(S + \sigma_3 \Gamma P_t^\xi S^\rho)}_{V_3(\hat{S})} - V(S). \end{aligned}$$

The first order conditions for (Ω) are

$$\frac{\partial u}{\partial C}(C_t)\frac{\partial F}{\partial R}(R_t, L_t) = V'(S) + \lambda_2(N_t)\sigma_2 V'(S + \sigma_2 R_t), \tag{4.6}$$

$$\frac{\partial u}{\partial C}(C_t)\frac{\partial F}{\partial L}(R_t, L_t) = \lambda_1'(N_t)\Delta_1 V(\hat{S}) + \lambda_2'(N_t)\Delta_2 V(\hat{S}) + \lambda_3'(N_t)\Delta_3 V(\hat{S}). \tag{4.7}$$

As usual equation (4.6) indicates that along any optimal path the marginal benefit of using (harvesting) the resource in terms of instantaneous utility must be equal to the resource rent $V'(S)$, corrected for the second type of innovations – in the efficiency of resources usage (σ_2) – that are not proportional to the stock of resources. Equation (4.7) assures that along any optimal path the marginal cost of the R&D activity in terms of instantaneous utility must equal the expected utility gain due to the development of a new type of innovation, being the marginal probability of success in the development of a new innovation $\lambda_i'(N_t)$, and the associated instantaneous increment of social welfare, $\Delta_i V(\hat{S})$. If we replace the $u(.)$ and $F(.)$ functions by their analytical analogues and divide equation (4.6) by equation (4.7), then

$$\frac{\theta(1-N_t)}{(1-\theta)R_t} = \frac{V'(S) + \lambda_2(N_t)\sigma_2 V'(S_t + \sigma_2 R_t)}{\lambda_1'(N_t)\Delta_1 V(\hat{S}_t) + \lambda_2'(N_t)\Delta_2 V(\hat{S}_t) + \lambda_3'(N_t)\Delta_3 V(\hat{S}_t)}. \tag{4.8}$$

In models of optimal use of renewable resources the standard result is that the resource rent (or shadow price of the resource) grows at the difference between the interest rate and the regeneration rate of the resource. This result is known as the Hotelling Rule extended to renewable resources and describes the permanent intertemporal trade-off resource users are faced with. But in our model, with the lasting effects of pollutants and the R&D activities, the standard Hotelling Rule needs to be modified and takes a new form.

Proposition 3 *If the stock of the renewable resources* (S_t) *satisfies (4.2), then the resource rent in average grows as follows:*

$$\begin{aligned} \frac{\frac{1}{dt}E(dV'(S_t))}{V'(S_t)} &= \delta - (\mu P_t^{-\zeta}\kappa S^{\kappa-1} - \Gamma P_t^{\xi}\rho S_t^{\rho-1}) \qquad (4.9) \\ &\quad -[\lambda_1(N_t)\sigma_1\mu P_t^{-\zeta}\kappa S_t^{\kappa-1}\frac{V_1'(\hat{S}_t)}{V'(S_t)} + \lambda_3(N_t)\sigma_3\Gamma P_t^{\xi}\rho S_t^{\rho-1}\frac{V_3'(\hat{S}_t)}{V'(S_t)}] \end{aligned}$$

where $V_1'(\hat{S}_t) = V'(S_t + \sigma_1 \mu P_t^{-\zeta} S_t^{\kappa})$ *is the value of the resource rent following an innovation for the mitigation of the lasting effects of pollutants on the regenerative capacity and* $V_3'(\hat{S}_t) = V'(S_t + \sigma_3 \Gamma P_t^{\xi} S_t^{\rho})$ *is the value of the resource rent following an innovation for reducing the production of pollutants.*

Proof: See Appendix 1 to this chapter.

In this model, the rate of growth of the resource rent is not simply equal to the social discount rate net of the regeneration rate of the resource. But rather, the latter has to be corrected for two crucial factors of influence: (i) the effect on the growth rate through technological progress (σ_1 and σ_3), and (ii) the sum of the indirect effects on the regenerative capacity μ of the stock of pollution and the direct one on the stock of resources of the flow. The corrections connected with the direct and the indirect effects of pollution increase the growth rate due to the fact that lasting effects of pollutants increase the physical scarcity of the resource. The effect of technology as such reduces the growth rate because at the time a new innovation is developed, the physical scarcity of resources is instantaneously reduced. As a result, the adverse effects of pollutants on the regeneration rate may slow down the use of the resource, but uncertainty on the return of innovations may speed up the harvesting.

Equation (4.9) is also important to verify the transversality condition of (Ω). A sufficient condition for the transversality condition to hold is

$$\frac{1}{dt} E[\frac{d(e^{-\delta t} V'(S))}{V'(S)}] < 0.$$

From equation (4.9) it becomes

$$\begin{aligned} \frac{1}{dt} E[\frac{d(e^{-\delta t} V'(S_t))}{V'(S_t)}] &= e^{-\delta t}[-\delta + \frac{\frac{1}{dt} E(dV'(S_t))}{V'(S_t)}] \\ &= -e^{-\delta t}[\lambda_1(N_t)\sigma_1 \mu P_t^{-\zeta} \kappa S_t^{\kappa-1} \frac{V_1'(\hat{S}_t)}{V'(S_t)} \\ &\quad +\lambda_3(N_t)\sigma_3 \Gamma P_t^{\xi} \rho S_t^{\rho-1} \frac{V_3'(\hat{S}_t)}{V'(S_t)} \\ &\quad +\mu\kappa P_t^{-\zeta} S^{\kappa-1} - \rho \Gamma P_t^{\xi} S_t^{\rho-1}] < 0. \end{aligned}$$

Hence, the transversality condition holds if the expression in the parenthesis on the right hand side is positive.

Note that $\mu\kappa P_t^{-\zeta} S^{\kappa-1} - \rho\ \Gamma P_t^{\xi} S_t^{\rho-1} > 0$ is a sufficient condition for this. However, even if the effect on the regeneration rate and the direct effect of pollutants on S is negative, $\mu\kappa P_t^{-\zeta} S^{\kappa-1} - \rho\Gamma P_t^{\xi} S_t^{\rho-1} < 0$, the transversality condition holds as long as it is smaller in absolute value than the positive technological effect, $\lambda_1(N_t)\sigma_1\mu P_t^{-\zeta}\kappa S_t^{\kappa-1}\frac{V_1'(\hat{S}_t)}{V'(S_t)} + \lambda_3(N_t)\sigma_3\Gamma P_t^{\xi}\rho S_t^{\rho-1}\frac{V_3'(\hat{S}_t)}{V'(S_t)}$.

4.4 The optimal paths

For finding an analytical solution of the optimal policy functions of harvesting and R&D effort, we set $\lambda_i(N_t) = \lambda_i N_t$, with $\lambda_i \in [0,1]$. In fact only a linear functional form for the intensity allows us to solve (Ω) analytically. So, an increase in the R&D effort leads to a higher probability of technological innovation, but leaving the marginal probability unchanged. For analytical simplicity we also restrict ourself to the case where $\sigma_2 = \sigma_3 = 0$ because the main result of the positive effect of newly developed innovations will not be affected and no additional qualitative insight will be gained.

Proposition 4 *The optimal paths of harvesting, R&D effort, and consumption are unique and regular. They are*

$$1 - N_t^* = L_t^* = \frac{(1-\theta)(1-\gamma)m_t}{\lambda\eta_t} \tag{4.10}$$

$$R_t^* = m_t S_t \tag{4.11}$$

$$C^*(S_t) = m_t[\frac{(1-\gamma)(1-\theta)}{\lambda\eta_t}]^{1-\theta} S_t^{\theta} \tag{4.12}$$

where

$$\begin{aligned}
\eta_t &= (1 + \sigma_1\mu P_t^{-\zeta} S_t^{\kappa-1})^{\theta(1-\gamma)} - 1 \\
m_t &= \frac{1}{\gamma}[\delta - \lambda\eta_t - \theta(1-\gamma)(\mu P_t^{-\zeta} S_t^{\kappa-1} - \Gamma P_t^{\xi} S_t^{\rho-1})].
\end{aligned}$$

Proof: See Appendix 2 to this chapter.

Since η_t and μ_t are time dependent, the optimal allocation of labor in the production sector and in the R&D sector are not constant. But since we have to meet the constraints $0 \leq N^*, L^* \leq 1$, we have to characterize a feasible set of parameters that guarantees the existence of an interior optimal solution.

Proposition 5 *An interior optimal solution exists if and only if*

$$\delta_t \in \{\underline{\delta}_t, \overline{\delta}_t\}$$

where, for $\mu P_t^{-\zeta} S_t^{\kappa-1} - \Gamma P_t^{\xi} S_t^{\rho-1} > 0$,*the lower bound is*

$$\underline{\delta}_t = \begin{cases} \theta(1-\gamma)(\mu P_t^{-\zeta} S_t^{\kappa-1} - \Gamma P_t^{\xi} S_t^{\rho-1}) + \lambda\eta_t & if \gamma < 1 \\ 0 & if \gamma > 1 \end{cases}$$

and the upper bound is for $\gamma < 1$

$$\overline{\delta}_t = \theta(1-\gamma)(\mu P_t^{-\zeta} S_t^{\kappa-1} - \Gamma P_t^{\xi} S_t^{\rho-1}) + \lambda(\eta_t + \frac{\eta_t \gamma}{(1-\gamma)(1-\theta)})$$

and for $\gamma > 1$

$$\overline{\delta}_t = \begin{cases} 0 & if \lambda \leq \Omega \\ \theta(1-\gamma)(\mu P_t^{-\zeta} S_t^{\kappa-1} - \Gamma P_t^{\xi} S_t^{\rho-1}) + \lambda(\eta_t + \frac{\eta_t \gamma}{(1-\gamma)(1-\theta)}) & if \lambda > \Omega \end{cases}$$

where

$$\Omega_t = \frac{-\theta(1-\gamma)(\mu P_t^{-\zeta} S_t^{\kappa-1} - \Gamma P_t^{\xi} S_t^{\rho-1})}{\frac{[1-\theta(1-\gamma)]}{(1-\gamma)(1-\theta)}\eta_t}.$$

See figures 4.1a) and 4.1b).

For $\mu P_t^{-\zeta} S_t^{\kappa-1} - \Gamma P_t^{\xi} S_t^{\rho-1} = 0$

$$\underline{\delta}_t = \begin{cases} \lambda\eta_t & if \gamma < 1 \\ 0 & if \gamma > 1 \end{cases}$$

and both for $\gamma < 1$ *and* $\gamma > 1$

$$\overline{\delta}_t = \frac{[1-\theta(1-\gamma)]}{(1-\gamma)(1-\theta)}\eta_t \lambda.$$

See figures 4.1c) and 4.1d).

For $\mu P_t^{-\zeta} S_t^{\kappa-1} - \Gamma P_t^{\xi} S_t^{\rho-1} < 0$ *and* $\gamma < 1$

$$\underline{\delta}_t = \begin{cases} 0 & if \lambda \leq \Upsilon \\ \theta(1-\gamma)(\mu P_t^{-\zeta} S_t^{\kappa-1} - \Gamma P_t^{\xi} S_t^{\rho-1}) + \lambda \eta_t & if \lambda > \Upsilon \end{cases}$$

$$\overline{\delta}_t = \begin{cases} 0 & if \lambda \leq \Phi \\ \theta(1-\gamma)(\mu P_t^{-\zeta} S_t^{\kappa-1} - \Gamma P_t^{\xi} S_t^{\rho-1}) + \lambda(\eta_t + \frac{\eta_t \gamma}{(1-\gamma)(1-\theta)}) & if \lambda > \Phi \end{cases}$$

where

$$\Upsilon_t = \frac{-\theta(1-\gamma)(\mu P_t^{-\zeta} S_t^{\kappa-1} - \Gamma P_t^{\xi} S_t^{\rho-1})}{\eta_t}$$

and

$$\Phi_t = \frac{-\theta(1-\gamma)(\mu P_t^{-\zeta} S_t^{\kappa-1} - \Gamma P_t^{\xi} S_t^{\rho-1})}{\frac{[1-\theta(1-\gamma)]}{(1-\gamma)(1-\theta)}\eta_t}.$$

See figure 4.1e).

For $\gamma > 1$

$$\underline{\delta}_t = \begin{cases} \theta(1-\gamma)(\mu P_t^{-\zeta} S_t^{\kappa-1} - \Gamma P_t^{\xi} S_t^{\rho-1}) + \lambda \eta_t & if \lambda \leq \Psi \\ 0 & if \lambda > \Psi \end{cases}$$

and

$$\overline{\delta}_t = \theta(1-\gamma)(\mu P_t^{-\zeta} S_t^{\kappa-1} - \Gamma P_t^{\xi} S_t^{\rho-1}) + \lambda(\eta_t + \frac{\eta_t \gamma}{(1-\gamma)(1-\theta)})$$

where

$$\Psi_t = \frac{-\theta(1-\gamma)(\mu P_t^{-\zeta} S_t^{\kappa-1} - \Gamma P_t^{\xi} S_t^{\rho-1})}{\eta_t}.$$

See figure 4.1f).

Proof: See Appendix 3 to this chapter.

The lower and upper constraints delimitate our feasible set and are time dependent. But since we are interested in balanced steady state solutions, we use now the definition given to the evolution of pollutants $g_P = \Gamma = \dot{P}/P$ and assume that also $g_S = \dot{S}/S$ is a constant. If and only if $-\zeta g_P = (1-\kappa) g_S$ and $\xi g_P = (1-\rho) g_S$ hold, we have that $P_t^{-\zeta} S_t^{\kappa-1} = constant$ and $P_t^{\xi} S_t^{\rho-1} = constant$, and therefore we find a balanced steady state solution for the optimal paths where both the feasible set for the parameters and the optimal path are no more time

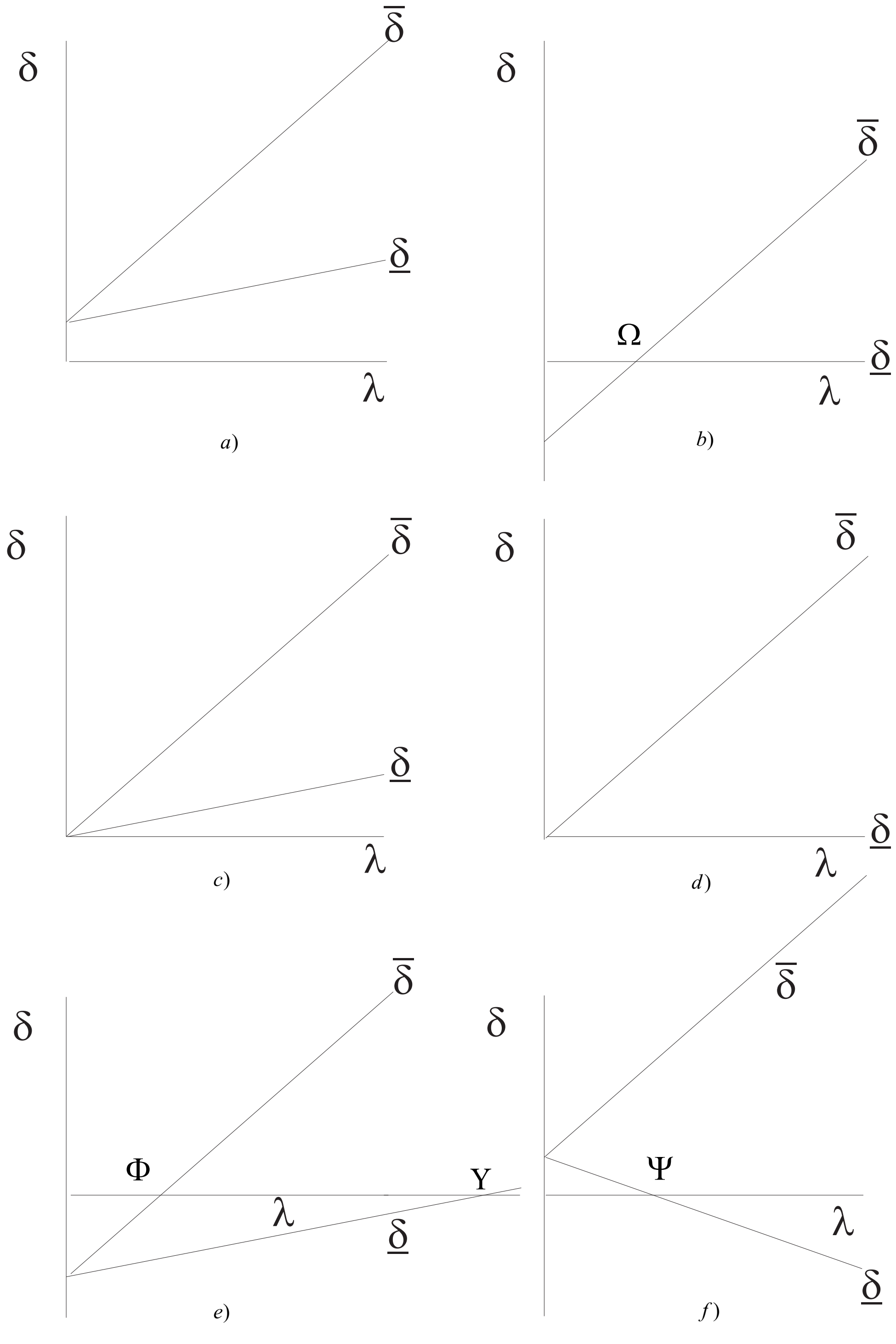

Figure 4.1: Lower and upper constraints for $\gamma <, > 1$ and $\mu P_t^{-\zeta} S_t^{\kappa-1} - \Gamma P_t^{\xi} S_t^{\rho-1} >, =, < 0$

dependent. Hence $\mu P_t^{-\zeta} S_t^{\kappa-1} - \Gamma P_t^{\xi} S_t^{\rho-1} = \mu P^{-\zeta} S^{\kappa-1} - \Gamma P^{\xi} S^{\rho-1}$ and when, for e.g., $\mu P^{-\zeta} S^{\kappa-1} - \Gamma P^{\xi} S^{\rho-1} > 0$ and $\gamma < 1$ we can rewrite:

$$\underline{\delta} = \theta(1-\gamma)(\mu P^{-\zeta} S^{\kappa-1} - \Gamma P^{\xi} S^{\rho-1}) + \lambda\eta$$

and

$$\overline{\delta} = \theta(1-\gamma)(\mu P^{-\zeta} S^{\kappa-1} - \Gamma P^{\xi} S^{\rho-1}) + \lambda(\eta + \frac{\eta\gamma}{(1-\gamma)(1-\theta)}).$$

In this case $\underline{\delta}$ and $\overline{\delta}$ are linear in λ with slope η and $\eta + \frac{\gamma\eta}{(1-\gamma)(1-\theta)}$, respectively and have a common intercept $(1-\gamma)\theta(\mu P^{-\zeta} S^{\kappa-1} - \Gamma P^{\xi} S^{\rho-1})$. They span a cone in which all (λ, δ) -pairs are associated with feasible equilibria given the constraints. This is shown in figure 4.1a).

The following Table 1, similar to Lafforgue (2004), summarizes the qualitative effects of parameters on the optimal paths.

	δ	λ	σ_1	S_t	θ
N^*	$-$	$+$	$+$	/	$\gamma < 1 : +$ $\gamma > 1 : -$
L^*	$+$	$-$	$-$	/	$\gamma < 1 : -$ $\gamma > 1 : +$
R^*	$+$	$\gamma < 1 : -$ $\gamma > 1 : +$	$\gamma < 1 : -$ $\gamma > 1 : +$	$+$	$\gamma < 1 : -$ $\gamma > 1 : +$
C^*	$+$	$\gamma < 1 : -$ $\gamma > 1 : \pm$	$\gamma < 1 : -$ $\gamma > 1 \pm$	$+$	$\gamma < 1 : -$ $\gamma > 1 : +$

Let us concentrate on the social discount rate (δ). An increase in δ causes the (labor and resources) input usage to increase, the R&D effort to diminish, and the consumption to increase. An increase in the marginal probability of success increases the effort N^* in R&D and consequently decreases the productive labor input L^*; it also increases current extraction R^* if and only if society favors its present consumption capacity, i.e. $\gamma > 1$. In that case, it raises consumption C^* if the positive effect on extraction more than compensates the negative effect on labor.

4.5 The optimal paths analysis

We first characterize the exact optimal paths and then find their asymptotic behavior. In a balanced steady state, according to solutions (4.10) and (4.11), our stochastic differential equation (4.2), if we recall from Appendix 2 to this chapter that $P^{-\zeta}S^{\kappa-1} = A$ and $P^{\xi}S^{\rho-1} = B$, is

$$dS_t = \mu AS_t dt - mS_t dt - \Gamma BS_t dt + \sigma_1 \mu AS_t dq_t \tag{4.13}$$

where we recall

$$m = \frac{1}{\gamma}(\delta - \lambda\eta - (1-\gamma)\theta(\mu A - \Gamma B)).$$

As long as there is no innovation (no jump) $dq_t = 0$, then equation (4.13) has the solution

$$S(t, 0) = S_0 e^{(\mu A - m - \Gamma B)t}.$$

When an innovation is developed $dq_t = 1$ and the availability of resources is instantaneously increased by σ_1 percent. Then the resources follow the optimal trajectory

$$S^*(t, q_t) = (1 + \sigma_1 \mu A)^{q_t} S(t, 0) = (1 + \sigma_1 \mu A)^{q_t} S_0 e^{(\mu A - m - \Gamma B)t}, \tag{4.14}$$

the optimal harvesting trajectory is

$$R^*(t, q_t) = mS^*(t, q_t) = m(1 + \sigma_1 \mu A)^{q_t} S_0 e^{(\mu A - m - \Gamma B)t}, \tag{4.15}$$

and the one for consumption reads

$$C^*(t, q_t) = m^{\theta}(1 + \sigma_1 \mu A)^{\theta q_t} S_0^{\theta} e^{\theta(\mu A - m - \Gamma B)t}(1 - N^*)^{1-\theta}. \tag{4.16}$$

Since the exact trajectories (4.14),(4.15) and (4.16) are piecewise discontinuous (they jump upwards at the instant an innovation is developed) and their asymptotic behavior is undetermined (q_t tends to infinity in probability over an infinite time horizon and $S_0 e^{(\mu A - m - \Gamma B)t}$ could decline to zero if $\mu A < m + \Gamma B$), we compute the smoothed trajectories. Hence, we consider the paths of the expected value of S_t^*, R_t^* and C_t^*.

Integrating (4.13) and computing the expected value, we get

$$\overline{S_t} = S_0 \exp((\mu A - m - \Gamma B + \lambda N^* \sigma_1 \mu A)t)$$

and therefore the solution for (4.15) (the expected optimal extraction rate path) is

$$\overline{R_t} = m\overline{S_t} \tag{4.17}$$

and in a similar way the solution for (4.16) (the expected optimal consumption path) is

$$\overline{C_t} = (\mu S_0)^\theta (1 - N^*)^{1-\theta} \exp(\{\theta(\mu A - m - \Gamma B) + \lambda N^*[(1 + \sigma_1\mu A)^\theta - 1]\}t). \tag{4.18}$$

The corresponding growth rates are constant over time. For R and S they are

$$\begin{aligned} g_{\overline{S}} &= g_{\overline{R}} = \\ &= \mu A - m - \Gamma B + \lambda N^* \sigma_1 \mu A = \\ &= \mu A - \Gamma B - \frac{1}{\gamma}(\delta - \lambda\eta - (1-\gamma)\theta(\mu A - \Gamma B))(1 + \frac{(1-\gamma)(1-\theta)}{\eta}\sigma_1\mu A) + \lambda\sigma_1\mu A \end{aligned} \tag{4.19}$$

and for C the growth rate is

$$\begin{aligned} g_{\overline{C}} &= \\ &= \theta\mu A - \theta\Gamma B - \theta m + \lambda N^*(1+\sigma_1\mu A)^\theta - \lambda N^* = \\ &= \theta(\mu A - \Gamma B) + \lambda[(1+\sigma_1\mu A)^\theta - 1] + \\ &\quad -\frac{1}{\gamma}(\delta - \lambda\eta - (1-\gamma)\theta(\mu A - \Gamma B))(\theta + \frac{(1-\gamma)(1-\theta)}{\eta}[(1+\sigma_1\mu A)^\theta - 1]. \end{aligned} \tag{4.20}$$

We are interested in the signs of those rates. In particular, the aim of this chapter is to determine the necessary conditions for sustainable growth, i.e., positive consumption growth and increasing resources over time. Therefore we find the two functions that guarantee $g_{\overline{S}} > 0$ and $g_{\overline{C}} > 0$. Using (4.19)

$$(\delta - \lambda\eta - (1-\gamma)\theta(\mu A - \Gamma B))[1 + \frac{(1-\gamma)(1-\theta)}{\eta}\sigma_1\mu A] < \gamma(\mu A - \Gamma B) + \gamma\lambda\sigma_1\mu A$$

if and only if

$$\delta < \delta_S = (\mu A - \Gamma B)((1-\gamma)\theta + \frac{\gamma}{Q}) + \lambda(\eta + \frac{\gamma\sigma_1\mu A}{Q}) \tag{4.21}$$

where $Q = 1 + \frac{(1-\gamma)(1-\theta)}{\eta}\sigma_1\mu A$. Using (4.20)

$$(\delta - \lambda\eta - (1-\gamma)\theta(\mu A - \Gamma B))[1 + \frac{(1-\gamma)(1-\theta)}{\eta}[(1+\sigma_1\mu A)^\theta - 1]] < \theta\gamma(\mu A - \Gamma B) + \gamma\lambda[(1+\sigma_1\mu A)^\theta - 1]$$

if and only if

$$\delta < \delta_C = \theta(\mu A - \Gamma B)((1-\gamma) + \frac{\gamma}{Z}) + \lambda(\eta + \frac{\gamma[(1+\sigma_1\mu A)^\theta - 1]}{Z}) \tag{4.22}$$

where $Z = \theta + \frac{(1-\gamma)(1-\theta)}{\eta}[(1+\sigma_1\mu A)^\theta - 1]$.

Now, recall that the original constraints $\underline{\delta}(\lambda)$ and $\overline{\delta}(\lambda)$ assure a feasible set of parameters. Together with the new non-negativity constraints $\delta_S(\lambda)$ and $\delta_C(\lambda)$ given in (4.21) and (4.22), we can study the sustainability issue in the δ and λ space. For the sake of better readability, let us give this system of four equations again:

$$\begin{aligned}
\underline{\delta}(\lambda) &= (\mu A - \Gamma B)\theta(1-\gamma) + \lambda\eta \\
\overline{\delta}(\lambda) &= (\mu A - \Gamma B)\theta(1-\gamma) + \lambda(\eta + \frac{\eta\gamma}{(1-\gamma)(1-\theta)}) \\
\delta_S(\lambda) &= (\mu A - \Gamma B)(\theta(1-\gamma) + \frac{\gamma}{Q}) + \lambda(\eta + \frac{\gamma\sigma_1\mu A}{Q}) \\
\delta_C(\lambda) &= \theta(\mu A - \Gamma B)((1-\gamma) + \frac{\gamma}{Z}) + \lambda(\eta + \frac{\gamma[(1+\sigma_1\mu A)^\theta - 1]}{Z}).
\end{aligned}$$

When $\gamma < 1$ and $\mu A - \Gamma B > 1$ (see figure 4.2a)), $\underline{\delta}(\lambda)$ and $\overline{\delta}(\lambda)$ have the same positive intercept at $\lambda = 0$, namely $\theta(1-\gamma)(\mu A - \Gamma B)$. Since $Q, Z > 0$ and $\gamma > 0$ the intercept of $\delta_S(\lambda)$ and $\delta_C(\lambda)$ is higher than that of $\underline{\delta}(\lambda), \overline{\delta}(\lambda)$. The slope of $\overline{\delta}(\lambda)$ is bigger than that of $\underline{\delta}(\lambda)$. Also, the slope of both $\delta_S(\lambda)$ and $\delta_C(\lambda)$ are bigger than that of $\underline{\delta}(\lambda)$ since $\frac{\gamma\sigma_1\mu A}{Q} > 0$ and $\frac{\gamma[(1+\sigma_1\mu A)^\theta - 1]}{Z} > 0$. They exhibit a bigger intercept than $\underline{\delta}(\lambda)$ so that they cannot intersect with $\underline{\delta}(\lambda)$. Comparing the slopes of $\delta_S(\lambda)$ and $\overline{\delta}(\lambda)$,

$$\begin{aligned}
\frac{\gamma\eta}{(1-\gamma)(1-\theta)} &\geq \frac{\gamma\sigma_1\mu A}{Q} \Leftrightarrow \\
\frac{\eta}{(1-\gamma)(1-\theta)} + \sigma_1\mu A &\geq \sigma_1\mu A.
\end{aligned} \tag{4.23}$$

The last condition always holds with strict inequality, since $\eta(1-\gamma) > 0$. Thus, the slope of $\overline{\delta}(\lambda)$ is strictly bigger than that of $\delta_S(\lambda)$. Knowing that the intercept of $\delta_S(\lambda)$ is bigger than that of $\overline{\delta}(\lambda)$, then the two functions have an intersection point.

Comparing $\delta_C(\lambda)$ and $\overline{\delta}(\lambda)$,

$$\begin{aligned}
\frac{\gamma\eta}{(1-\gamma)(1-\theta)} &\geq \frac{\gamma[(1+\sigma_1\mu A)^\theta - 1]}{Z} \Leftrightarrow \\
\frac{\eta}{(1-\gamma)(1-\theta)} + (1+\sigma_1\mu A)^\theta - 1 &\geq (1+\sigma_1\mu A)^\theta - 1.
\end{aligned} \tag{4.24}$$

The last condition holds with strict inequality for the same reasons as above. Again the slope of $\overline{\delta}(\lambda)$ is strictly bigger than that of $\delta_C(\lambda)$, and because also the intercept of $\delta_C(\lambda)$ is bigger than that of $\overline{\delta}(\lambda)$ a new intersection point exists.

Then, comparing the slopes of $\delta_S(\lambda)$ and $\delta_C(\lambda)$,

$$\begin{aligned} \frac{\gamma\sigma_1\mu A}{Q} &\geq \frac{\gamma[(1+\sigma_1\mu A)^\theta - 1]}{Z} \Leftrightarrow \\ \theta\sigma_1\mu A &\geq [(1+\sigma_1\mu A)^\theta - 1]. \end{aligned} \tag{4.25}$$

The last condition holds with strict inequality throughout, since the auxiliary function θx and $[(1+x)^\theta - 1]$ start both in 0 but the first one has a bigger derivative, thus lies always above the second one. The slope of $\delta_S(\lambda)$ is strictly bigger than that of $\delta_C(\lambda)$ and together with the fact that $\delta_C(\lambda)$ has a bigger intercept than that of $\delta_S(\lambda)$ a last intersection point must be found.

Therefore the intersection point I_1 from $\delta_C(\lambda) = \delta_S(\lambda)$ is

$$(\mu A - \Gamma B)(\theta Q - Z) = \lambda(Z\sigma_1\mu A - Q[(1+\sigma_1\mu A)^\theta - 1])$$

hence,

$$\lambda_1^* = \frac{(\mu A - \Gamma B)(\theta Q - Z)}{\theta\sigma_1\mu A - [(1+\sigma_1\mu A)^\theta - 1]} = \frac{(\mu A - \Gamma B)(1-\gamma)(1-\theta)}{\eta}. \tag{4.26}$$

The intersection point I_2 from $\overline{\delta}(\lambda) = \delta_S(\lambda)$ is

$$\frac{\mu A - \Gamma B}{Q} = \lambda(\frac{\eta}{(1-\gamma)(1-\theta)} - \frac{\sigma_1\mu A)}{Q})$$

hence,

$$\lambda_2^* = \frac{(\mu A - \Gamma B)(1-\gamma)(1-\theta)}{\eta Q - (1-\gamma)(1-\theta)\sigma_1\mu A} = \frac{(\mu A - \Gamma B)(1-\gamma)(1-\theta)}{\eta}. \tag{4.27}$$

And finally the intersection point I_3 from $\overline{\delta}(\lambda) = \delta_C(\lambda)$ is

$$\frac{(\mu A - \Gamma B)\theta}{Z} = \lambda(\frac{\eta}{(1-\gamma)(1-\theta)} - \frac{[(1+\sigma_1\mu A)^\theta - 1]}{Z})$$

hence,

$$\lambda_3^* = \frac{(\mu A - \Gamma B)\theta(1-\gamma)(1-\theta)}{\eta Z - (1-\gamma)(1-\theta)[(1+\sigma_1\mu A)^\theta - 1]} = \frac{(\mu A - \Gamma B)(1-\gamma)(1-\theta)}{\eta}. \tag{4.28}$$

We see that $\lambda_1^* = \lambda_2^* = \lambda_3^*$ and hence I_1, I_2, and I_3 are identical and the three linear constraints $\overline{\delta}(\lambda)$, $\delta_S(\lambda)$, $\delta_C(\lambda)$ intersects in the same point I^*. This implies that positive consumption growth is always consistent with growth of the resources, whenever we are within the feasible parameter range constrained by $\underline{\delta}(\lambda)$ and $\overline{\delta}(\lambda)$ until I^* and by $\underline{\delta}(\lambda)$ and $\delta_C(\lambda)$ after I^*.

After the intersection point I^* we can therefore distinguish three areas:

$\delta_\lambda \in [\underline{\delta}, \delta_C]$	$\delta_\lambda \in (\delta_C, \delta_S]$	$\delta_\lambda \in (\delta_S, \overline{\delta}]$
C increasing	C decreasing	C decreasing
S increasing	S increasing	S decreasing

We have these three areas also when $\mu A - \Gamma B = 0$, but with the difference that they start already in the origin, all the intercepts being zero; see figure 4.2c).

When $\mu A - \Gamma B < 0$ (see figure 4.2e)), all the intercepts are negative and those of $\delta_S(\lambda)$ and $\delta_C(\lambda)$ smaller than those of $\underline{\delta}(\lambda)$ and $\overline{\delta}(\lambda)$ (which are identical). This means that the intersection points now happen with the line $\underline{\delta}(\lambda)$, the slopes of $\delta_S(\lambda)$ and $\delta_C(\lambda)$ being bigger than that of $\underline{\delta}(\lambda)$.

The intersection point I_1^* between $\delta_S(\lambda)$ and $\underline{\delta}(\lambda)$ is

$$\lambda_1^* = -\frac{(\mu A - \Gamma B)}{\sigma_1 \mu A}$$

and the intersection point I_2^* between $\delta_C(\lambda)$ and $\underline{\delta}(\lambda)$ corresponds to

$$\lambda_2^* = -\frac{\theta(\mu A - \Gamma B)}{[(1 + \sigma_1 \mu A]^\theta - 1]}$$

which is the key point for the beginning of the sustainable growth area.

Finally for $\gamma > 1$, all the previous results for the three cases apply if we assume respectively for $\delta_S(\lambda)$ that $\theta(1-\gamma) + \frac{\gamma}{Q} > 0$ and $\eta + \frac{\gamma\sigma_1\mu A}{Q} > 0$ and for $\delta_C(\lambda)$ that $(1-\gamma) + \frac{\gamma}{Z} > 0$ and $\eta + \frac{\gamma[(1+\sigma_1\mu A)^\theta - 1]}{Z} > 0$.

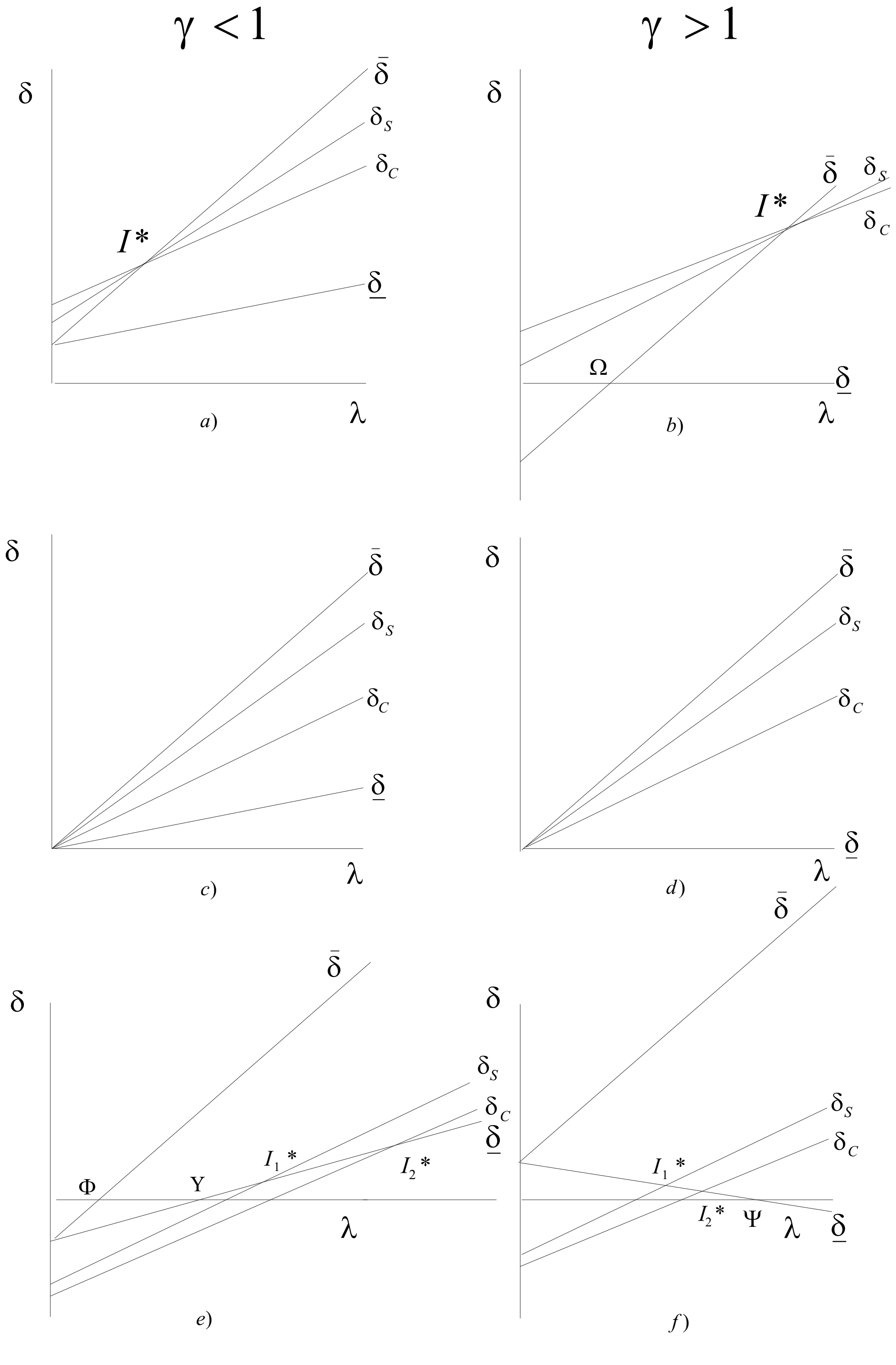

Figure 4.2: Sustainability areas for $\gamma <,> 1$ and $\mu P_t^{-\zeta} S_t^{\kappa-1} - \Gamma P_t^{\xi} S_t^{\rho-1} >,=,< 0$

4.6 Conclusion

Thanks to all the three new considerations in the regeneration function (connection between harvesting and quality degradation of renewable resources; non-constancy of the regenerative capacity; technological processes that directly affect the availability of resources), we obtain a new Hotelling rule. In fact, in this model, the rate of growth of the resource rent is not simply equal to the social discount rate net of the regeneration rate of the resource. But rather, it has to be corrected for two crucial factors of influence: (i) the effect on the growth rate through technological progress, and (ii) the sum of the indirect effects of pollutants on the regeneration rate and the direct one on the stock of resources. The correction connected with the direct and indirect effects of pollutants increases the growth rate of the resource rent due to the fact that lasting effects of pollutants increase the physical scarcity of the resource. The effect of technology as such reduces the growth rate of the resource rent because at the time a new innovation is developed, the physical scarcity of resources is instantaneously reduced. As a result, the adverse effects of pollutants on the regeneration function may slow down the use of the resource, but uncertainty on the return of innovations may speed up the harvesting.

We have also determined the necessary conditions for sustainable growth i.e., positive consumption growth and positive resources growth over time. For that reason as in Lafforgue (2004), we have firstly found an analytical solution of the optimal policy functions of harvesting, R&D effort and consumption; secondly characterized the smoothed optimal paths (the exacted ones being only piecewise continuous and asymptotically undetermined); thirdly computed the growth rates. As final result we have that, if the marginal probability of innovations is high enough compared with the degree of impatience of society, the expected positive effect of R&D activities overrides expected negative effects of harvesting and environmental degradation (caused by the direct and indirect impacts of pollutants) so that the smoothed trajectory of renewable resources and consumption increases over time.

4.7 Appendix 1

We start with the description of the eight possible states according to the three independent Poisson process $(q_{1,t}), (q_{2,t}), (q_{3,t})$. For this we consider a given time t and a later time $t + dt$.

Event in "dt"	State	Probability
Only jump of $(q_{1,t})$	I	$\lambda_1(N_t)dt(1-\lambda_2(N_t))dt(1-\lambda_3(N_t))dt$
Only jump of $(q_{2,t})$	II	$\lambda_2(N_t)dt(1-\lambda_1(N_t))dt(1-\lambda_3(N_t))dt$
Only jump of $(q_{3,t})$	III	$\lambda_3(N_t)dt(1-\lambda_1(N_t))dt(1-\lambda_2(N_t))dt$
Only jump of $(q_{1,t})$ and $(q_{2,t})$	IV	$\lambda_1(N_t)dt\lambda_2(N_t)dt(1-\lambda_3(N_t))dt$
Only jump of $(q_{1,t})$ and $(q_{3,t})$	V	$\lambda_1(N_t)dt\lambda_3(N_t)dt(1-\lambda_2(N_t))dt$
Only jump of $(q_{2,t})$ and $(q_{3,t})$	VI	$\lambda_2(N_t)dt\lambda_3(N_t)dt(1-\lambda_1(N_t))dt$
Jump of $(q_{1,t})$, $(q_{2,t})$, $(q_{3,t})$	VII	$\lambda_1(N_t)dt\lambda_2(N_t)dt\lambda_3(N_t)dt$
No jump at all	VIII	$(1-\lambda_1(N_t))dt(1-\lambda_2(N_t))dt(1-\lambda_3(N_t))dt$

For the derivative $V'(S)$ at time $t + dt$ we have

$$V'(S)|_{t+dt} = V'(S)|_t + V''(S)dS_t.$$

This gives according to the above table the following representation of $V'(S)|_{t+dt}$ depending of the state

$$V'(S)|_{t+dt} = \begin{cases} V'(S)|_t + \mu P_t^{-\zeta} S_t^{\kappa} V''(S)dt - R_t V''(S)dt - & \\ \Gamma P_t^{\xi} S_t^{\rho} V''(S)dt & \text{state VIII} \\ V'_{1,2,3} = V'(S_t + \sigma_1 \mu P_t^{-\zeta} S_t^{\kappa} + \sigma_3 \Gamma P_t^{\xi} S_t^{\rho} + \sigma_2 R_t) & \text{state VII} \\ V'_{2,3} = V'(S_t + \sigma_3 \Gamma P_t^{\xi} S_t^{\rho} + \sigma_2 R_t) & \text{state VI} \\ V'_{1,3} = V'(S_t + \sigma_1 \mu P_t^{-\zeta} S_t^{\kappa} + \sigma_3 \Gamma P_t^{\xi} S_t^{\rho}) & \text{state V} \\ V'_{1,2} = V'(S_t + \sigma_1 \mu P_t^{-\zeta} S_t^{\kappa} + \sigma_2 R_t) & \text{state IV} \\ V'_3(\hat{S}) = V'(S_t + \sigma_3 \Gamma P_t^{\xi} S_t^{\rho}) & \text{state III} \\ V'_2(\hat{S}) = V'(S_t + \sigma_2 R_t) & \text{state II} \\ V'_1(\hat{S}) = V'(S_t + \sigma_1 \mu P_t^{-\zeta} S_t^{\kappa}) & \text{state I} \end{cases}$$

This implies

$$\frac{V'(S)|_{t+dt}-V'(S)|_t}{dt}=\begin{cases}[\mu P_t^{-\zeta}S_t^{\kappa}-R_t-\ \Gamma P_t^{\xi}S_t^{\rho}]V''(S) & \\ =M_{V''(S)} & \text{state VIII}\\ \frac{1}{dt}(V'_{1,2,3}-V'(S)|_t) & \text{state VII}\\ \frac{1}{dt}(V'_{2,3}-V'(S)|_t) & \text{state VI}\\ \frac{1}{dt}(V'_{1,3}-V'(S)|_t) & \text{state V}\\ \frac{1}{dt}(V'_{1,2}-V'(S)|_t) & \text{state IV}\\ \frac{1}{dt}(V'_3(\hat{S})-V'(S)|_t) & \text{state III}\\ \frac{1}{dt}(V'_2(\hat{S})-V'(S)|_t) & \text{state II}\\ \frac{1}{dt}(V'_1(\hat{S})-V'(S)|_t) & \text{state I}\end{cases}$$

We apply the expectation

$$\begin{aligned}E\quad & (\frac{V'(S)|_{t+dt}-V'(S)|_t}{dt})=([1-(\lambda_1(N_t)+\lambda_2(N_t)+\lambda_3(N_t))dt]+ \qquad (4.29)\\ + \quad & [\lambda_1(N_t)\lambda_2(N_t)+\lambda_2(N_t)\lambda_3(N_t)+\lambda_1(N_t)\lambda_3(N_t)](dt)^2+\lambda_1(N_t)\lambda_2(N_t)\lambda_3(N_t)(dt)^3)M_{V''(S)}\\ + \quad & \lambda_1(N_t)\lambda_2(N_t)\lambda_3(N_t)(dt)^2(V'_{1,2,3}-V'(S)|_t)\\ + \quad & \lambda_2(N_t)\lambda_3(N_t)dt(1-\lambda_1(N_t)dt)(V'_{2,3}-V'(S)|_t)\\ + \quad & \lambda_1(N_t)\lambda_3(N_t)dt(1-\lambda_2(N_t)dt)(V'_{1,3}-V'(S)|_t)\\ + \quad & \lambda_1(N_t)\lambda_2(N_t)dt(1-\lambda_3(N_t)dt)(V'_{1,2}-V'(S)|_t)\\ + \quad & \lambda_3(N_t)(1-\lambda_1(N_t)dt)(1-\lambda_2(N_t)dt)(V'_3(\hat{S})-V(S)|_t)\\ + \quad & \lambda_2(N_t)(1-\lambda_1(N_t)dt)(1-\lambda_3(N_t)dt)(V'_2(\hat{S})-V(S)|_t)\\ + \quad & \lambda_1(N_t)(1-\lambda_2(N_t)dt)(1-\lambda_3(N_t)dt)(V'_1(\hat{S})-V(S)|_t)\end{aligned}$$

We partially differentiate (4.5) with respect to S

$$\begin{aligned}\delta V'(S) &= V''(S)(\mu P_t^{-\zeta}S_t^{\kappa}-R_t-\Gamma P_t^{\xi}S_t^{\rho})+V'(S)(\kappa\mu P_t^{-\zeta}S_t^{\kappa-1}-\rho\Gamma P_t^{\xi}S_t^{\rho-1})\\ &+ \lambda_1(N_t)\Delta_1 V'(\hat{S})+\lambda_1(N_t)\sigma_1\mu\kappa P_t^{-\zeta}S_t^{\kappa-1}V'_1(\hat{S})\\ &+ \lambda_2(N_t)\Delta_2 V'(\hat{S})\\ &+ \lambda_3(N_t)\Delta_3 V'(\hat{S})+\lambda_3(N_t)\sigma_3\rho\Gamma P_t^{\xi}S_t^{\rho-1}V'_3(\hat{S}).\end{aligned}$$

After rearranging terms, we derive

$$\begin{aligned} M_{V''(S)} + \sum_{i=1}^{3} \lambda_i(N_t)\Delta_i V'(\widehat{S}) &= \delta V'(S) - \lambda_1(N_t)\sigma_1\mu\kappa P_t^{-\zeta} S_t^{\kappa-1} V_1'(\hat{S}) \\ &- \lambda_3(N_t)\sigma_3\rho\Gamma P_t^{\xi} S_t^{\rho-1} V_3'(\hat{S}) - V'(S)(\kappa\mu P_t^{-\zeta} S_t^{\kappa-1} - \rho\Gamma P_t^{\xi} S_t^{\rho-1}). \end{aligned} \tag{4.30}$$

We now divide (4.29) by "$V'(S)$"and let dt tend to zero so that (4.29) becomes

$$\frac{\frac{1}{dt}E(dV'(S_t))}{V'(S_t)} = \frac{M_{V''(S)} + \sum_{i=1}^{3} \lambda_i(N_t)\Delta_i V'(\hat{S})}{V'(S_t)}.$$

With (4.30) we conclude the proof:

$$\begin{aligned} \frac{\frac{1}{dt}E(dV'(S_t))}{V'(S_t)} = \delta &- [\lambda_1(N_t)\sigma_1\mu\kappa P_t^{-\zeta} S_t^{\kappa-1} \frac{V_1'(\hat{S}_t)}{V'(S_t)} \\ &+ \lambda_3(N_t)\sigma_3\rho\Gamma P_t^{\xi} S_t^{\rho-1} \frac{V_3'(\hat{S}_t)}{V'(S_t)} \\ &+ (\mu\kappa P_t^{-\zeta} S_t^{\kappa-1} - \rho\Gamma P_t^{\xi} S_t^{\rho-1})]. \end{aligned}$$

4.8 Appendix 2

We follow the standard technique and use the first order conditions in (4.6) and (4.7) to compute the optimal solution for the extraction rate and labour input rate:

$$\begin{aligned} \frac{\partial u}{\partial C}(C_t) &= C_t^{-\gamma} \\ \frac{\partial F}{\partial R}(R_t, L_t) &= \theta R_t^{\theta-1} L_t^{1-\theta} \\ \frac{\partial F}{\partial L}(R_t, L_t) &= (1-\theta) R_t^{\theta} L_t^{-\theta}. \end{aligned}$$

Using $C_t = F(R_t, L_t)$ and $\sigma_2 = \sigma_3 = 0$, the conditions (4.6) and (4.7) read

$$\frac{\partial u}{\partial C}\frac{\partial F}{\partial R} = \theta R_t^{\theta(1-\gamma)-1} L_t^{(1-\gamma)(1-\theta)} = V'(S) \tag{4.31}$$

and

$$\frac{\partial u}{\partial C}\frac{\partial F}{\partial L} = (1-\theta) R_t^{\theta(1-\gamma)} L_t^{-\gamma-(1-\gamma)\theta} = \lambda\Delta_1 V(\hat{S}). \tag{4.32}$$

Dividing (4.32) by (4.31) and omitting the time index reveals

$$R = \frac{\theta}{1-\theta}\frac{\lambda\Delta_1 V(\hat{S})}{V'(S)} L. \tag{4.33}$$

Inserting back into (4.32) gives

$$(1-\theta)\big(\frac{\theta}{1-\theta}\big)^{\theta(1-\gamma)}\frac{(\lambda\Delta_1 V(\widehat{S}))^{\theta(1-\gamma)}}{V'(S)^{\theta(1-\gamma)}}\cdot L^{\theta(1-\gamma)}\cdot L^{-\gamma-(1-\gamma)\theta}=\lambda\Delta_1 V(\hat{S}).$$

After rearrangements we get

$$L^{\gamma}=(\frac{1-\theta}{\lambda\Delta_1 V(\hat{S})})^{1-\theta(1-\gamma)}(\frac{\theta}{V'(S)})^{\theta(1-\gamma)}$$

and finally

$$L=(\frac{1-\theta}{\lambda\Delta_1 V(\hat{S})})^{\frac{1-\theta(1-\gamma)}{\gamma}}(\frac{\theta}{V'(S)})^{\frac{\theta(1-\gamma)}{\gamma}}. \tag{4.34}$$

We insert (4.34) into (4.33):

$$R=(\frac{1-\theta}{\lambda\Delta_1 V(\hat{S})})^{\frac{(1-\theta)(1-\gamma)}{\gamma}}(\frac{\theta}{V'(S)})^{\frac{1-(1-\theta)(1-\gamma)}{\gamma}}. \tag{4.35}$$

We now insert this into the HJB equation (4.5) and, using the abbreviations $G_1 = \mu P_t^{-\zeta}S_t^{\kappa}$ and $G_3 = \Gamma P_t^{\xi}S_t^{\rho}$, we get

$$\delta V(S)=\frac{R^{\theta(1-\gamma)}L^{(1-\theta)(1-\gamma)}}{1-\gamma}+V'(S)G_1-V'(S)R-V'(S)G_3+\lambda(1-L)\Delta_1 V(\hat{S}).$$

With (4.34) and (4.35) we realize after rearrangements that

$$\begin{aligned}\delta V(S) &= (\frac{\gamma}{1-\gamma})[\frac{1-\theta}{\lambda\Delta_1 V(\hat{S})}]^{\frac{(1-\theta)(1-\gamma)}{\gamma}}[\frac{\theta}{V'(S)}]^{\frac{\theta(1-\gamma)}{\gamma}} \\ &+ \lambda\Delta_1 V(\hat{S})+\frac{G_1}{S}SV'(S)-\frac{G_3}{S}SV'(S).\end{aligned} \tag{4.36}$$

Thus,

$$\begin{aligned}\delta V(S) &= (\frac{\gamma}{1-\gamma})[\frac{1-\theta}{\lambda\Delta_1 V(\hat{S})}]^{\frac{(1-\theta)(1-\gamma)}{\gamma}}[\frac{\theta}{V'(S)}]^{\frac{\theta(1-\gamma)}{\gamma}} \\ &+ \lambda\Delta_1 V(\hat{S})+\mu P_t^{-\zeta}S_t^{\kappa-1}[SV'(S)]-\Gamma P_t^{\xi}S_t^{\rho-1}[SV'(S)].\end{aligned} \tag{4.37}$$

Now assume that for all $t \geq 0$

$$P_t^{-\zeta}S_t^{\kappa} = A_t S_t \tag{4.38}$$

$$P_t^{\xi}S_t^{\rho} = B_t S_t, \tag{4.39}$$

with A_t, B_t continuous. Condition (4.38) guarantees that the HJB equation (4.5) defines a necessary condition for an optimal path. For the sufficient condition no additional assumption is required (see Sennewald (2005)). Then equation (4.37) turns into

$$\begin{aligned} \delta V(S) &= (\frac{\gamma}{1-\gamma})[\frac{1-\theta}{\lambda\Delta_1 V(\hat{S})}]^{\frac{(1-\theta)(1-\gamma)}{\gamma}}[\frac{\theta}{V'(S)}]^{\frac{\theta(1-\gamma)}{\gamma}} \\ &+ \lambda\Delta_1 V(S) + \mu A_t[SV'(\hat{S})] - \Gamma B_t[SV'(S)]. \end{aligned} \tag{4.40}$$

Equation (4.40) shows an ordinary differential equation for V. To solve it we use the approach

$$V(S) = \Psi S^{\theta(1-\gamma)} \tag{4.41}$$

where $\Psi \in \mathbb{R}$ is unknown, and needs to be determined below. We compute the derivative and $\Delta_1 V(\hat{S})$:

$$V'(S) = \theta(1-\gamma)\Psi S^{\theta(1-\gamma)-1} \tag{4.42}$$

$$\Delta_1 V(\hat{S}) = \Psi[(1+\sigma_1\mu A_t)^{\theta(1-\gamma)} - 1]S^{\theta(1-\gamma)}. \tag{4.43}$$

Insertion of (4.41), (4.42) and (4.43) into (4.40) gives

$$\begin{aligned} \delta\Psi S^{\theta(1-\gamma)} &= (\frac{\gamma}{1-\gamma})(\frac{1-\theta}{\lambda})^{\frac{(1-\theta)(1-\gamma)}{\gamma}}[\Psi[(1+\sigma_1\mu A_t)^{\theta(1-\gamma)} - 1]]^{-\frac{(1-\theta)(1-\gamma)}{\gamma}} \\ &\cdot S^{-\frac{(1-\theta)(1-\gamma)}{\gamma}\theta(1-\gamma)}\theta^{\frac{\theta(1-\gamma)}{\gamma}}(\theta(1-\gamma)\Psi)^{-\frac{\theta(1-\gamma)}{\gamma}}S^{\frac{\theta(1-\gamma)}{\gamma}(1-\theta(1-\gamma)} \\ &+ \lambda\Psi[(1+\sigma_1\mu A_t)^{\theta(1-\gamma)} - 1]S^{\theta(1-\gamma)} \\ &+ \mu A_t\theta(1-\gamma)\Psi[S\cdot S^{\theta(1-\gamma)-1}] - \Gamma B_t\theta(1-\gamma)\Psi[S\cdot S^{\theta(1-\gamma)-1}]. \end{aligned}$$

Collecting terms that involve S, we obtain in a first step

$$\begin{aligned} \delta\Psi S^{\theta(1-\gamma)} &= (\frac{1-\theta}{\lambda})^{\frac{(1-\theta)(1-\gamma)}{\gamma}}[\Psi[(1+\sigma_1\mu A_t)^{\theta(1-\gamma)} - 1]]^{-\frac{(1-\theta)(1-\gamma)}{\gamma}} \\ &\cdot (\frac{\gamma}{1-\gamma})\theta^{\frac{\theta(1-\gamma)}{\gamma}}(\theta(1-\gamma)\Psi)^{-\frac{\theta(1-\gamma)}{\gamma}}S^{\theta(1-\gamma)} \\ &+ \lambda\Psi[(1+\sigma_1\mu A_t)^{\theta(1-\gamma)} - 1]S^{\theta(1-\gamma)} \\ &+ \mu A_t\theta(1-\gamma)\Psi S^{\theta(1-\gamma)} - \Gamma B_t\theta(1-\gamma)\Psi S^{\theta(1-\gamma)}. \end{aligned} \tag{4.44}$$

In (4.44) each term contains $S^{\theta(1-\gamma)}$ by which we divide to derive

$$\begin{aligned}
\delta\Psi &= (\frac{1-\theta}{\lambda})^{\frac{(1-\theta)(1-\gamma)}{\gamma}}[(1+\sigma_1\mu A_t)^{\theta(1-\gamma)}-1]^{-\frac{(1-\theta)(1-\gamma)}{\gamma}} \\
&\cdot (\frac{\gamma}{1-\gamma})\theta^{\frac{\theta(1-\gamma)}{\gamma}}(\theta(1-\gamma))^{-\frac{\theta(1-\gamma)}{\gamma}}\Psi^{-\frac{\theta(1-\gamma)}{\gamma}}\Psi^{-\frac{(1-\theta)(1-\gamma)}{\gamma}} \\
&+ \lambda[(1+\sigma_1\mu A_t)^{\theta(1-\gamma)}-1]\Psi \\
&+ \mu A_t\theta(1-\gamma)\Psi - \Gamma B_t\theta(1-\gamma)\Psi.
\end{aligned} \tag{4.45}$$

Now note that $\Psi^{-\frac{\theta(1-\gamma)}{\gamma}-\frac{(1-\theta)(1-\gamma)}{\gamma}} = \Psi\cdot\Psi^{-\frac{1}{\gamma}}$. Hence, each term in (4.45) contains Ψ. We divide again and use the abbreviation $x=\sigma_1\mu A_t$, $y=\mu A_t$ and $z=\Gamma B_t$ to derive

$$\begin{aligned}
&\delta-\lambda[(1+x)^{\theta(1-\gamma)}-1]-(y-z)\theta(1-\gamma) = \\
&\Psi^{-\frac{1}{\gamma}}(\frac{\gamma}{1-\gamma})(\frac{1-\theta}{\lambda})^{\frac{(1-\theta)(1-\gamma)}{\gamma}}[(1+x)^{\theta(1-\gamma)}-1]^{-\frac{(1-\theta)(1-\gamma)}{\gamma}}\theta^{\frac{\theta(1-\gamma)}{\gamma}}(\theta(1-\gamma))^{-\frac{\theta(1-\gamma)}{\gamma}}.
\end{aligned} \tag{4.46}$$

This implies

$$\Psi = [\frac{\gamma(1-\gamma)^{-\frac{\gamma+\theta(1-\gamma)}{\gamma}}[\frac{1-\theta}{\lambda[(1+x)^{\theta(1-\gamma)}-1]}]^{\frac{(1-\theta)(1-\gamma)}{\gamma}}}{\delta-\lambda[(1+x)^{\theta(1-\gamma)}-1]-(y-z)\theta(1-\gamma)}]^{\gamma}. \tag{4.47}$$

Now we use the expression $V(S)=\Psi S^{\theta(1-\gamma)}$ and insert (4.47) into the expressions for L and R given in (4.34) and (4.35), respectively:

$$\begin{aligned}
R &= (\frac{1-\theta}{\lambda\Psi[(1+x)^{\theta(1-\gamma)}-1]S^{\theta(1-\gamma)}})^{\frac{(1-\theta)(1-\gamma)}{\gamma}}(\frac{\theta}{\theta(1-\gamma)\Psi S^{\theta(1-\gamma)-1}})^{\frac{1-(1-\theta)(1-\gamma)}{\gamma}} = \\
&= (\frac{1}{1-\gamma})^{\frac{1}{\gamma}}(\frac{(1-\theta)(1-\gamma)}{\lambda[(1+x)^{\theta(1-\gamma)}-1]})^{\frac{(1-\theta)(1-\gamma)}{\gamma}}\Psi^{-\frac{1}{\gamma}}S.
\end{aligned} \tag{4.48}$$

Collecting terms, this can be rewritten as

$$R = \frac{\delta-\lambda[(1+x)^{\theta(1-\gamma)}-1]-(y-z)\theta(1-\gamma)}{\gamma}S. \tag{4.49}$$

And for L we compute

$$\begin{aligned}
L &= (\frac{1-\theta}{\lambda\Psi[(1+x)^{\theta(1-\gamma)}-1]S^{\theta(1-\gamma)}})^{\frac{1-\theta(1-\gamma)}{\gamma}}(\frac{\theta}{\theta(1-\gamma)\Psi S^{\theta(1-\gamma)-1}})^{\frac{\theta(1-\gamma)}{\gamma}} = \\
&= (\frac{1-\theta}{\lambda[(1+x)^{\theta(1-\gamma)}-1]})^{\frac{1}{\gamma}}(\frac{\lambda[(1+x)^{\theta(1-\gamma)}-1]}{(1-\gamma)(1-\theta)})^{\frac{\theta(1-\gamma)}{\gamma}}\Psi^{-\frac{1}{\gamma}}.
\end{aligned} \tag{4.50}$$

Collecting terms we derive

$$L = \frac{(1-\theta)(1-\gamma)}{\lambda[(1+x)^{\theta(1-\gamma)}-1]\gamma}(\delta-\lambda[(1+x)^{\theta(1-\gamma)}-1]-(y-z)\theta(1-\gamma)). \tag{4.51}$$

Recalling the definition of x and y and setting

$$\begin{aligned}
\eta_t &= [(1+x)^{\theta(1-\gamma)} - 1] = [(1+\sigma_1 \mu P^{-\zeta} S_t^{\kappa-1})^{\theta(1-\gamma)} - 1] \\
m_t &= \frac{\delta - \lambda[(1+x)^{\theta(1-\gamma)} - 1] - (y-z)\theta(1-\gamma)}{\gamma} \\
&= \frac{\delta - \lambda[(1+\sigma_1 \mu P^{-\zeta} S_t^{\kappa-1})^{\theta(1-\gamma)} - 1] - \theta(1-\gamma)[\mu P^{-\zeta} S_t^{\kappa-1} - \Gamma P^{\xi} S_t^{\rho-1}]}{\gamma}
\end{aligned}$$

we complete the proof.

4.9 Appendix 3

We verify the constraints $0 \leq L_t, N_t \leq 1$. From equation (4.10) we recall that

$$1 - N_t^* = L_t^* = \frac{(1-\theta)(1-\gamma)m_t}{\lambda \eta_t}$$

where

$$\begin{aligned}
\eta_t &= (1+\sigma_1 \mu P_t^{-\zeta} S_t^{\kappa-1})^{\theta(1-\gamma)} - 1 \\
m_t &= \frac{1}{\gamma}[\delta - \lambda \eta_t - \theta(1-\gamma)(\mu P_t^{-\zeta} S_t^{\kappa-1} - \Gamma P_t^{\xi} S_t^{\rho-1})].
\end{aligned}$$

Because $\mu P_t^{-\zeta} S_t^{\kappa-1} - \Gamma P_t^{\xi} S_t^{\rho-1}$ can be greater, equal or smaller than 0 and γ greater or smaller than 1, six cases follow.

Case 1: $\mu P_t^{-\zeta} S_t^{\kappa-1} - \Gamma P_t^{\xi} S_t^{\rho-1} > 0$ and $\gamma < 1$. When $\gamma < 1$ then $\eta_t > 0$. Hence according to equation (4.10)

$$L_t^* \geq 0 \Leftrightarrow m_t \geq 0.$$

The lower bound on δ becomes

$$\underline{\delta}_t = \theta(1-\gamma)(\mu P_t^{-\zeta} S_t^{\kappa-1} - \Gamma P_t^{\xi} S_t^{\rho-1}) + \lambda \eta_t.$$

According to (4.10) again, we have that

$$L_t^* \leq 1 \Leftrightarrow 1 - \frac{(1-\gamma)(1-\theta)m_t}{\lambda \eta_t} \geq 0.$$

This condition is equivalent to

$$\lambda \eta_t - (1-\gamma)(1-\theta)\frac{1}{\gamma}(\delta - \lambda \eta_t - \theta(1-\gamma)(\mu P_t^{-\zeta} S_t^{\kappa-1} - \Gamma P_t^{\xi} S_t^{\rho-1})) \geq 0$$

and the upper bound on δ becomes

$$\overline{\delta}_t = \theta(1-\gamma)(\mu P_t^{-\zeta} S_t^{\kappa-1} - \Gamma P_t^{\xi} S_t^{\rho-1}) + \lambda(\eta_t + \frac{\eta_t \gamma}{(1-\gamma)(1-\theta)})$$

Case 2: $\mu P_t^{-\zeta} S_t^{\kappa-1} - \Gamma P_t^{\xi} S_t^{\rho-1} > 0$ as before, but $\gamma > 1$. When $\gamma > 1$ then now $\eta_t < 0$. According to (4.10)

$$L_t^* \geq 0 \Leftrightarrow m_t \geq 0.$$

The lower bound on δ becomes now $\underline{\delta}_t = 0$, as $\underline{\delta}_t = \theta(1-\gamma)(\mu P_t^{-\zeta} S_t^{\kappa-1} - \Gamma P_t^{\xi} S_t^{\rho-1}) + \lambda\eta_t$ is negative. To find the new upper bound on δ again,

$$L_t^* \leq 1 \Leftrightarrow 1 - \frac{(1-\gamma)(1-\theta)m_t}{\lambda\eta_t} \geq 0.$$

This condition is equivalent to

$$\lambda\eta_t - (1-\gamma)(1-\theta)\frac{1}{\gamma}(\delta - \lambda\eta_t - \theta(1-\gamma)(\mu P_t^{-\zeta} S_t^{\kappa-1} - \Gamma P_t^{\xi} S_t^{\rho-1})) \geq 0$$

and the upper bound on δ becomes

$$\overline{\delta}_t = \begin{cases} 0 & if \lambda \leq \Omega \\ \theta(1-\gamma)(\mu P_t^{-\zeta} S_t^{\kappa-1} - \Gamma P_t^{\xi} S_t^{\rho-1}) + \lambda(\eta_t + \frac{\eta_t\gamma}{(1-\gamma)(1-\theta)}) & if \lambda > \Omega \end{cases}$$

where

$$\Omega = \frac{-\theta(1-\gamma)(\mu P_t^{-\zeta} S_t^{\kappa-1} - \Gamma P_t^{\xi} S_t^{\rho-1})}{\frac{[1-\theta(1-\gamma)]}{(1-\gamma)(1-\theta)}\eta_t},$$

which is the intersection point between the $\delta_t = 0$ line and the $\delta_t = \theta(1-\gamma)(\mu P_t^{-\zeta} S_t^{\kappa-1} - \Gamma P_t^{\xi} S_t^{\rho-1}) + \lambda(\eta_t + \frac{\eta_t\gamma}{(1-\gamma)(1-\theta)})$ line.

When $\mu P_t^{-\zeta} S_t^{\kappa-1} - \Gamma P_t^{\xi} S_t^{\rho-1} = 0$ we have similar results to Lafforgue.

Case 3: $\mu P_t^{-\zeta} S_t^{\kappa-1} - \Gamma P_t^{\xi} S_t^{\rho-1} = 0$ and $\gamma < 1$. When $\gamma < 1$ then $\eta_t > 0$. Hence according to equation (4.10)

$$L_t^* \geq 0 \Leftrightarrow m_t \geq 0.$$

The lower bound on δ becomes

$$\underline{\delta}_t = \lambda\eta_t.$$

According to (4.10) again, we have that

$$L_t^* \leq 1 \Leftrightarrow 1 - \frac{(1-\gamma)(1-\theta)m_t}{\lambda\eta_t} \geq 0.$$

This condition is equivalent to

$$\lambda\eta_t - (1-\gamma)(1-\theta)\frac{1}{\gamma}(\delta - \lambda\eta_t) \geq 0$$

and the upper bound on δ becomes

$$\overline{\delta}_t = \frac{[1-\theta(1-\gamma)]}{(1-\gamma)(1-\theta)}\eta_t\lambda.$$

Case 4: $\mu P_t^{-\zeta} S_t^{\kappa-1} - \Gamma P_t^{\xi} S_t^{\rho-1} = 0$ and $\gamma > 1$. When $\gamma > 1$ then $\eta_t < 0$. Hence according to equation (4.10)

$$L_t^* \geq 0 \Leftrightarrow m_t \geq 0.$$

The lower bound on δ becomes $\underline{\delta}_t = 0$, as $\underline{\delta}_t = \lambda\eta_t$ is negative. According to (4.10), we have that

$$L_t^* \leq 1 \Leftrightarrow 1 - \frac{(1-\gamma)(1-\theta)m_t}{\lambda\eta_t} \geq 0.$$

This condition is equivalent to again

$$\lambda\eta_t - (1-\gamma)(1-\theta)\frac{1}{\gamma}(\delta - \lambda\eta_t) \geq 0$$

and the upper bound on δ becomes

$$\overline{\delta}_t = \frac{[1-\theta(1-\gamma)]}{(1-\gamma)(1-\theta)}\eta_t\lambda.$$

In the same way we proceed for the last two cases.

Case 5: $\mu P_t^{-\zeta} S_t^{\kappa-1} - \Gamma P_t^{\xi} S_t^{\rho-1} < 0$ and $\gamma < 1$. When $\gamma < 1$ then $\eta_t > 0$. Hence according to equation (4.10)

$$L_t^* \geq 0 \Leftrightarrow m_t \geq 0.$$

The lower bound on δ becomes

$$\underline{\delta}_t = \begin{cases} 0 & if \lambda \leq \Upsilon \\ \theta(1-\gamma)(\mu P_t^{-\zeta} S_t^{\kappa-1} - \Gamma P_t^{\xi} S_t^{\rho-1}) + \lambda\eta_t & if \lambda > \Upsilon \end{cases}$$

where

$$\Upsilon_t = \frac{-\theta(1-\gamma)(\mu P_t^{-\zeta} S_t^{\kappa-1} - \Gamma P_t^{\xi} S_t^{\rho-1})}{\eta_t},$$

which is the intersection point between the $\delta_t = 0$ line and the $\delta_t = \theta(1-\gamma)(\mu P_t^{-\zeta} S_t^{\kappa-1} - \Gamma P_t^{\xi} S_t^{\rho-1}) + \lambda\eta_t$ line. According to (4.10) again, we have that

$$L_t^* \leq 1 \Leftrightarrow 1 - \frac{(1-\gamma)(1-\theta)m_t}{\lambda\eta_t} \geq 0.$$

This condition is equivalent to

$$\lambda\eta_t - (1-\gamma)(1-\theta)\frac{1}{\gamma}(\delta - \lambda\eta_t - \theta(1-\gamma)(\mu P_t^{-\zeta} S_t^{\kappa-1} - \Gamma P_t^{\xi} S_t^{\rho-1})) \geq 0$$

and the upper bound on δ becomes

$$\overline{\delta}_t = \begin{cases} 0 \quad if \lambda \leq \Phi \\ \theta(1-\gamma)(\mu P_t^{-\zeta} S_t^{\kappa-1} - \Gamma P_t^{\xi} S_t^{\rho-1}) + \lambda(\eta_t + \frac{\eta_t\gamma}{(1-\gamma)(1-\theta)}) \quad if \lambda > \Phi \end{cases}$$

where

$$\Phi_t = \frac{-\theta(1-\gamma)(\mu P_t^{-\zeta} S_t^{\kappa-1} - \Gamma P_t^{\xi} S_t^{\rho-1})}{\frac{[1-\theta(1-\gamma)]}{(1-\gamma)(1-\theta)}\eta_t},$$

which is the intersection point between the $\delta_t = 0$ line and the $\delta_t = \theta(1-\gamma)(\mu P_t^{-\zeta} S_t^{\kappa-1} - \Gamma P_t^{\xi} S_t^{\rho-1}) + \lambda(\eta_t + \frac{\eta_t\gamma}{(1-\gamma)(1-\theta)})$ line.

Case 6: $\mu P_t^{-\zeta} S_t^{\kappa-1} - \Gamma P_t^{\xi} S_t^{\rho-1} < 0$ and $\gamma > 1$. When $\gamma > 1$ then $\eta_t < 0$. Hence according to equation (4.10)

$$L_t^* \geq 0 \Leftrightarrow m_t \geq 0.$$

The lower bound on δ becomes

$$\underline{\delta}_t = \begin{cases} \theta(1-\gamma)(\mu P_t^{-\zeta} S_t^{\kappa-1} - \Gamma P_t^{\xi} S_t^{\rho-1}) + \lambda\eta_t \quad if \lambda \leq \Psi \\ 0 \quad if \lambda > \Psi \end{cases}$$

where

$$\Psi_t = \frac{-\theta(1-\gamma)(\mu P_t^{-\zeta} S_t^{\kappa-1} - \Gamma P_t^{\xi} S_t^{\rho-1})}{\eta_t},$$

which is the intersection point between the $\delta_t = \theta(1-\gamma)(\mu P_t^{-\zeta} S_t^{\kappa-1} - \Gamma P_t^{\xi} S_t^{\rho-1}) + \lambda\eta_t$ line and the $\delta_t = 0$ line. To find the new upper bound on δ again,

$$L_t^* \leq 1 \Leftrightarrow 1 - \frac{(1-\gamma)(1-\theta)m_t}{\lambda\eta_t} \geq 0.$$

This condition is equivalent to

$$\lambda\eta_t - (1-\gamma)(1-\theta)\frac{1}{\gamma}(\delta - \lambda\eta_t - \theta(1-\gamma)(\mu P_t^{-\zeta} S_t^{\kappa-1} - \Gamma P_t^{\xi} S_t^{\rho-1})) \geq 0$$

and the upper bound on δ becomes

$$\overline{\delta}_t = \theta(1-\gamma)(\mu P_t^{-\zeta} S_t^{\kappa-1} - \Gamma P_t^{\xi} S_t^{\rho-1}) + \lambda(\eta_t + \frac{\eta_t\gamma}{(1-\gamma)(1-\theta)}).$$

Chapter 5

Nature as a knowledge reservoir: a non-scale endogenous growth model with relaxation of knife edge assumptions

5.1 Introduction

Since the period after World War II economic growth is normally seen as the only solution for poverty. As Perman et al. (2003) p. 16 writes: "Economic growth increases the size of the cake. With enough of it, it may be possible to give everybody at least a decent slice, without having to reduce the size of the larger slices." Because of this fundamental consideration a flourishing growth literature has developed over the decades to investigate two crucial questions: what determines worldwide growth (in order to raise it) and why are there cross-country income differences (in order to close the gap in standards of living between poor and rich countries). But the exercises trying to answer those questions have been done without taking natural systems into consideration which indeed are the ultimate foundation of the worldwide economic system. Starting with the milestone works arising from the Symposium on the Economics of Exhaustible Resources organized by The Review of Economic Studies in 1974 (see Dasgupta and Heal (1974), Solow (1974a), Stiglitz (1974)), a new literature was born, namely natural resource economics emerged out of the neoclassical growth economics. The further consideration that pollution (and therefore the ability of the nature "to act as a sink for human wastes", see Brock and Taylor (2004b)) can also be a drag on growth, lead to the sustainable development literature of the '90s. Sustainability reflects the aim of reducing poverty without damaging the environment in a way that negatively affects future economic improvement.

So, since the last decade we have experienced two parallel discipline advancements, one improving the growth literature in itself without any consideration of the role of nature, and the other improving the sustainability literature which tries to incorporate the limitations arising from either natural resource scarcity or nature's limited capacity to absorb pollution in the best available growth model, in order to check the consistency of growth prediction. Significant examples are Verdier (1993) as an enrichment of Romer (1990), Stokey (1998) of Rebelo (1991), Aghion and Howitt (1998) of Aghion and Howitt (1992), Brock and Taylor (2004a) of Solow

(1956) and di Maria and Valente (2006) of Acemoglu (2003).

Along this line of research, nature even though being recognized as producing important services,[1] is constrained, ultimately, to be a limiting factor[2] to growth, either because natural resources are exhaustible or because the environmental quality is strongly negatively affected by pollution.[3] Another interesting point is that with this interpretation of nature as a limiting factor no insight can be gained to improve the answers the growth literature gives to its two above mentioned fundamental questions.[4] But if the attention is concentrated on the inestimable role that nature plays for the advancement of the sciences, e.g. as a knowledge reservoir, then this new interpretation can help in climbing up the quality ladder for growth models and thus contributes to explaining the sources of the growth process. Along with attributing a new role to nature in the context of economic growth, we take into consideration that as Romer (2006) writes: "The principal conclusion of the Solow model is that the accumulation of physical capital cannot account for either the vast growth over time in output per person or the vast geographic differences in output per person." We therefore exclude physical capital from our analysis and endogenise

[1]The four big categories of environmental services are: resource base that enters directly into the production function of output, waste sink which enters into the environmental quality function, amenity base service entering the utility function and the life support that can enter either in the production function directly or in the regenerative capacity function.

[2]Along the direct negative effect of nature on growth, there is also a possible indirect one, through environmental policy. Whether stringent environmental policies have a negative direct input effect or a win-win outcome (the Porter hypothesis) is long debated. Ricci (2004) surveys the related literature. It should be noted that if a positive effect is found, it relies either on knife edge assumptions, that should be avoided, or on an indirect effect through the standard explanatory variables for growth.

[3]The empirical literature is also debating the existence or not of the so called "curse of natural resources" when nature is seen as supplier of raw materials; see Gylfason (2001), Gylfason (2004), Sachs and A.M.Warner (1995), Bretschger (2006), Brunnschweiler (2006). Instead, the paper of Bloom and Sachs (1998) stresses the role of a better understanding of how climate and natural ecology work for development policies.

[4]This conceptually means that the sustainable development literature is a follower of the growth literature.

the Solow effectiveness of labor (A), interpreting it as technology. In addition, the introduction of the positive role of nature in the production function of technologies eliminates the low empirical success of the scale endogenous growth literature and the necessity of knife edge assumptions about the returns to scale to the produced factors in the production function of technologies.

In the following section the debate about scale effects and non-robustness (need of knife edge assumptions) will be briefly summarized; in section 3 the role of nature as a knowledge reservoir will be illustrated. Section 4 presents the basic model with natural knowledge and its dynamic implications for economic growth. In section 5 a more detailed version of the model is introduced in order to investigate in section 6 the role that the technological sector can play in the presence of environmental constraints. Section 7 concludes.

5.2 The scale effects and knife edge debate

The endogenous growth literature is motivated by the desire to explain what in the Solow model is exogenous and the driving force for sustained growth, namely the technological progress. The standard endogenous growth literature, also referred to as first-generation R&D-based growth models, is based upon the knife edge assumption of constant returns to scale in the produced factors of production.[5] In addition to that, these models imply scale effects, because the scale of the economy (L), or the fraction of the resources it gives to the R&D sector ($\frac{L_R}{L}$), influences the long-run growth rate. In fact, both the horizontal innovation approach of Romer (1990),[6] where the manufacturing sector and the innovative sector are described by

$$Y = K^{\alpha}(AL_Y)^{(1-\alpha)}$$

$$\dot{A} = \lambda A L_R, \quad \lambda > 0,$$

[5]For a detailed survey and discussion see Groth (2004) and Jones (1999).

[6]The production function of the final good is the result of static efficiency for $Y = \left(\sum_{i=1}^{A} x_i^{\alpha}\right) L_Y^{(1-\alpha)}, 0 < \alpha < 1$, where x_i, the input of intermediate good, is equal to $x = \frac{K}{A}$ for all i.

and the vertical innovation approach of Grossman and Helpman (1991) and Aghion and Howitt (1992),[7] with

$$Y = K^{\alpha}(AQL_Y)^{(1-\alpha)}$$

$$\dot{Q} = \lambda Q L_R, \quad \lambda > 0$$

lead to the steady state result

$$g_y = \frac{\dot{y}}{y} = \lambda s_R L, \quad where \quad s_R = \frac{L_R}{L}.$$

Therefore policy can affect the long-run growth rate by influencing s_R, which is the fraction of labor devoted to the innovative sector.

But Jones's critique (Jones (1995a) and Jones (1995b)) claims that the assumed scale effects are contradicted by empirical evidence. He proposes an alternative with decreasing returns to scale

$$\dot{A} = \lambda A^{\varphi} L_R, \quad \varphi < 1$$

$$L_R = s_R L, \quad \frac{\dot{L}}{L} = n \geq 0.$$

This produces that in steady state

$$g_y = \frac{n}{1-\varphi}$$

and thus the scale effects are cleared out.

The response to Jones was the second generation R&D-based models, Dinopoulos and Thompson (1998), Howitt (1999), Peretto (1998), Young (1998). These models, connecting the horizontal and the vertical innovation approach, managed to get rid only of the scale effect arising from the scale of the economy (L) but not of that deriving from the fraction of the resources devoted to the different R&D sectors (s_Q). In fact, given the production function

$$Y = K^{\alpha}(AQL_Y)^{1-\alpha}$$

[7] Also here the production function of the final good is the result of static efficiency for $Y = \left(\sum_{i=1}^{Q} x_i^{\alpha}\right) L_Y^{(1-\alpha)}, 0 < \alpha < 1$, where $x_i = x = \frac{\frac{K}{A}}{Q}$ and A, the number of different intermediate goods, is fixed and Q is the quality attached to the latest version of intermediate good.

and

$$\dot{Q} = \lambda Q \frac{s_Q s_R L}{A}, \quad \lambda > 0$$
$$\dot{A} = \mu(1 - s_Q) s_R L, \quad \lambda > 0$$

where s_Q is the fraction of the researchers working in the vertical innovation sector, the steady state is

$$g_y = (1 + \frac{\lambda s_Q}{\mu(1 - s_Q)})n.$$

In addition to that, see Jones (1999) and Li (2002), they are based upon knife edge assumptions on the spillovers within and between types of innovatios: zero among horizontal innovations, one among vertical innovations, and zero across horizontal and vertical innovation.

5.3 Nature as a knowledge reservoir

The natural environment, or commonly said nature, is a thermodynamically closed system and is composed by the earth and the atmosphere. With the rest of the universe it has only an exchange of energy (no matter exchange) and the way in which it absorbs or reflects that energy influences the functioning of the climate system. The earth and the atmosphere are a complex interaction of different types of ecosystems (Olson (1994) singles out 94 ecosystem classes) where biological populations coact with the abiotic environment in which they are set and where a vast amount of biological, chemical and physical processes take place. How terrestrial and aquatic communities are distributed around the globe, depends on topographical and geological factors, on the soil and water chemical characteristics, on the solar radiation and the ocean currents. All the scientific knowledge we possess nowadays is the result of a challenging process along the centuries to understand and, in the end, control nature in favor of our precarious human condition. The observation and the study of nature and its phenomena are at the origin of physics, astronomy, geometry, mathematics in the antique world and later in the Renaissance, when "the scientific method" was developed, of sciences (in the same meaning as nowadays[8]) as chemistry, biology, medicine, topography, cartography, geography and geology. All these types of scientific knowledge backgrounds were the prerequisite for the big land discoveries of the XV/XVI centuries, for the industrial revolution and the later improvements in human health conditions.

If we only consider living organisms in their three realms of animals, plants and microorganisms, we know nowadays that just only the described species are respectively: 52.000 for animal vertebrates, 1.272.000 for animal invertebrates, 270.000 for plants, 4.000 for bacteria, 80.000 for algae and protozoa and 72.000 for fungi; yielding a total of 1 750 000 described species. But the real number of species is estimated to be 14.000.000 (UNEP-WCMC (2000)). The knowledge about this overwhelming biological diversity thanks to the science of ecology, can help us in saying today that "without the appropriate level of diversity, natural ecosystems cannot adjust to natural variations in the environment" (Heal (2004)). Nature is an infinite basis of possible information which raises opportunities to directly or indirectly (through inventions) increase our utility. The difficulty is to find this information through a more or less slow process of scientific advancement. We could not have invented the steam engine without knowing the physical law of thermodynamics or the plane without the desire to copy birds and knowledge about aerodynamic laws. Every new discovery is an improvement of the scientific knowledge and therefore in the end our ability to survive better on the earth and develop new inventions.

Knowledge about microorganisms like bacteria, fungi, viruses, yeasts, which in the common imagination are seen more like a threat to our life, are of fundamental importance for the human utility in every-day life: in classical microbial processes (in the food industry for the production of cheeses, wines, beers, bakery products and preservatives; in the chemical industry for that of ethanol, acetone, butanol between

[8]Indeed the oral transmission of the Odyssey can be seen as a first encyclopedic work of geography and cartography but not in the meaning that is given today at that sciences; also knowledge of anatomy and medicine were available even before the period of the Classical Greece but they were mixed with religion and philosophy.

many others); in the pharmaceutical industry for the production of antibiotics, vaccines, active ingredients and therapeutical approaches; in new microbial processes (in the chemical industry for the production of enzymes, amino acids, nucleotide or steroids); in the utilization and conversion of crude oil, natural gas and cellulose; in the growing sectors of gene and biotechnology; in the treatment of wastes (see Dixon (1996), Schlegel (1992)). The relevance of studying nature can also be seen in the growing sector of biomechanics[9] which has lead to the development and production of nanostructures and in computer sciences to robotics and cybernetics.[10]

5.4 Natural knowledge as a prerequisite for sustained growth

5.4.1 Model structure

Having in mind the economic implications of the environment illustrated in the previous section and the economic evidence about the factor capital (K) mentioned in the introduction, the model structure is straightforward. It is described by four variables, namely output (Y), labor (L), knowledge (A) in the standard interpretation of technology (see Romer (1990), Grossman and Helpman (1991), Aghion and Howitt (1992)), and knowledge (D) in the interpretation of basic scientific research which arises by the study and the understanding of nature. In the analysis to follow A will be called technology and D natural knowledge. There are, therefore, three sectors: the final good sector where the output is produced; the standard R&D sector which is mostly private and characterized by the strength of developing technologies which have a clear target for their utilization in the production of goods; the

[9]Well known examples of drawing from nature in engineering are Leonardo da Vinci's flying machines and ships.

[10]The sector of biomechanics, or also said bionics from the connection of biology and electronics, is nowadays one of the most promising sectors, especially if we consider that the overlap between biology and technology in terms of mechanisms used is only 10% approximately.

scientific research sector which is mostly non-private and fundamentally motivated by the intrinsic human aspiration of enlightenment and improvement of the human condition. Normally in this sector, gained knowledge that afterwards is found to be relevant for the development of a specific market good (either directly in the production function or indirectly through the development of a new invention) is only a side product rather than the target of research,[11] such as the discovery of the first antibiotic[12] or the invention of new materials/tissue.[13]

The production function of the final good is

$$Y(t) = D(t)^{\alpha}[A(t)(1 - a_A - a_D)L(t)]^{1-\alpha}, \tag{5.1}$$

that for technologies is

$$\dot{A}(t) = D(t)^{\beta}(a_A L(t))^{\gamma} A(t)^{\theta}, \tag{5.2}$$

and finally the production function of natural knowledge is

$$\dot{D}(t) = (a_D L(t))^{\chi} D(t)^{\varepsilon}. \tag{5.3}$$

Population growth is exogenous and follows the standard differential equation $\dot{L}(t) = nL(t)$, with n exhibiting a positive value. Fraction a_A of the labor force is used for the invention of technologies; fraction a_D for the discovery of natural knowledge; and $1 - a_A - a_D$ is used for the production of the final good.[14] The three production functions have a standard Cobb-Douglas specification and follow the standard literature, see e.g.Romer (2006). The production function for the final

[11]Indeed in all natural sciences, as Pasteur said, the relationship between basic and applied research is very close: "Il n'y a pas des sciences appliquees... Mais il y a des applications de la science", see Schlegel (1992).

[12]Penicillin, which is a substance produced by the mould Penicillium notatum, was the first antibiotic discovered by Alexander Fleming in 1928 by chance on a nutrient agar which was thrown away after the study of another bacterium.

[13]As to the promising expectations arising from the study of the echinoderm sea cucumber, see Thurmond and Trotter (1996) and Jangoux et al. (2002).

[14]So, in a very elementary interpretation, we could also rename the variables A as inventions and D as discoveries.

good presents constant returns to labor and to natural knowledge for a given technology, with $0 < \alpha < 1$. Together with the introduction of the technological process as Harrod-neutral, the model therefore exhibits constant returns to scale in the production function of the final good for both factors of production, namely technology and natural knowledge. Thus, on net, whether this economy has increasing, decreasing or constant returns to scale to the produced factors depends on the returns to scale it has in the production function of knowledge, equation (5.2), and so on $(\beta + \theta) \gtreqless 1$. Note that in this model, we will see that the type of returns to scale to the produced factors is no more a key determinant for the existence of a balanced growth path, thus there is no need for a knife edge assumption like $\beta + \theta = 1$ or decreasing returns, $\beta + \theta < 1$.

There is no specific assumption with respect to the type of returns to scale to natural knowledge and labor in the production function of technology, and therefore, $\beta \geq 0$ and $\gamma \geq 0$. The same applies for the production function of natural knowledge where the returns to scale to labor could be decreasing, constant or increasing, $\chi \geq 0$. There are good reasons for all three possibilities therefore, we do not impose a specific formulation.

Finally, the parameters θ and ε, which represent the contribution of existing inventions to the success of the standard R&D sector and the contribution of existing discoveries to the advancement of scientific research, are also not subject to any assumption leaving them to be positive or negative. In fact, the contribution could be positive if we believe that inventions or discoveries in the past make future improvements easier; or it could also be negative if we assume that the bigger the stock of improvements, the more difficult to add new ones.

5.4.2 Dynamics of technology and natural knowledge

For simplicity we omit time indices. The growth rates of the two endogenous stock variables, A and D, become

$$g_A = D^{\beta} a_L^{\gamma} L^{\gamma} A^{\theta-1}, \tag{5.4}$$

$$g_D = a_D^{\chi} L^{\chi} D^{\varepsilon-1}. \tag{5.5}$$

Finding the balanced steady state implies

$$\dot{g_A} = 0 \;\Rightarrow\; \beta g_D + \gamma g_L + (\theta-1) g_A = 0$$

$$\Rightarrow\; g_D = -\frac{\gamma}{\beta} g_L + \frac{1-\theta}{\beta} g_A \tag{5.6}$$

$$\dot{g_D} = 0 \;\Rightarrow\; \chi g_L + (\varepsilon-1) g_D = 0$$

$$\Rightarrow\; g_D = \frac{\chi}{1-\varepsilon} g_L \tag{5.7}$$

Proposition 6 *Independent of the initial values of g_a and g_D, the economy always converges to its balanced steady state (g_A^*, g_D^*) where A and D grow steadily. The existence of this balanced steady state is independent of the returns to scale in the production function of technology.*

Proof: The values of g_A and g_D at the balanced steady state can be found if we recall that in equilibrium both $\dot{g_D} = 0$ and $\dot{g_A} = 0$. Thus $g_A^*$and g_D^* must satisfy

$$\beta g_D^* + \gamma g_L + (\theta-1) g_A^* = 0 \tag{5.8}$$

and

$$\chi g_L + (\varepsilon-1) g_D^* = 0, \tag{5.9}$$

which by substituting $g_D^* = \frac{\chi}{1-\varepsilon} g_L$ from equation (5.9) in equation (5.8) leads to

$$g_A^* = \frac{\frac{\chi}{1-\varepsilon} + \frac{\gamma}{\beta}}{\frac{1-\theta}{\beta}} g_L. \tag{5.10}$$

As depicted in figure 5.1 the phase diagram represents the locus of points where g_D and g_A are constant for $\theta < 1$ and $\varepsilon < 1$. Equation (5.6) corresponds to the set of points where g_A is constant. Above this locus, g_A is rising and, correspondingly, the

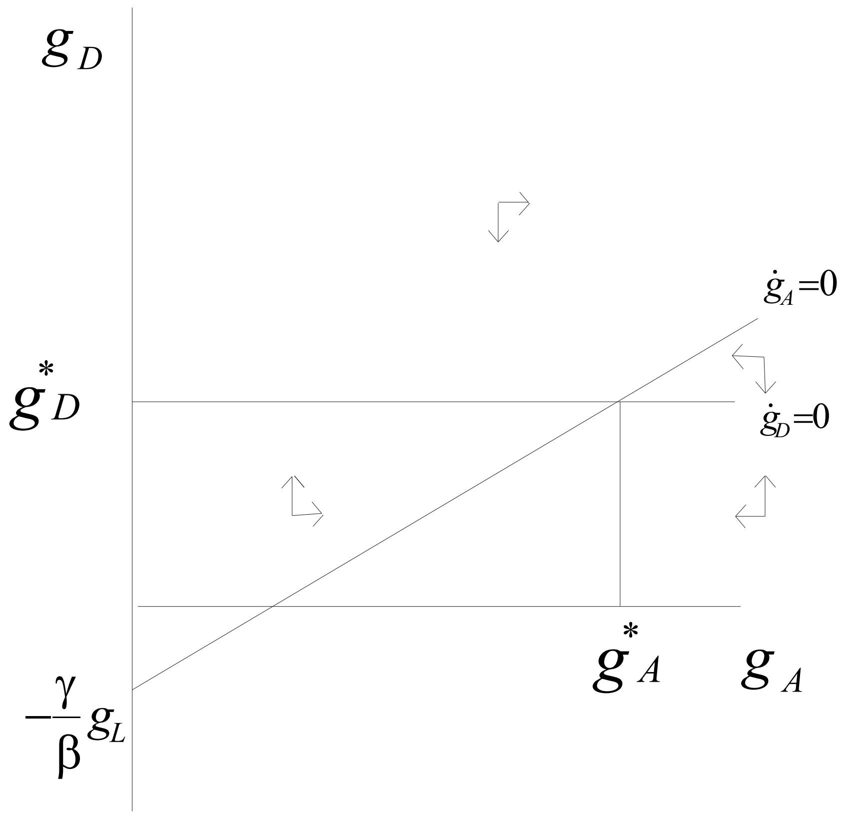

Figure 5.1: Sustainable Growth Equilibria

arrows point east; below the locus, it is falling and therefore the arrows point west. Similarly, equation (5.7) corresponds to the set of points where g_D is constant. Above the locus, g_D is falling and the arrows point south; below the locus, it is rising and the arrows point north. Thus, the two schedules divide the space into four regions. The arrows point southwest in the first quadrant, southeast in the second, northeast in the third, and northwest in the fourth. The balanced steady state is (g_A^*, g_D^*) and this point is stable. For any values of g_A and g_D, the dynamics of the system takes it back to the balanced steady state.■

The model does not imply scale effects because the long-run growth rates are not permanently influenced by changes in the resources devoted either (as in the early new growth literature) to the R&D sector (a_A) or the scientific research sector (a_D). At the same time the existence of the equilibrium is independent of the type of returns to scale in the produced factors of production in the production function of technology: increasing if $\beta+\theta > 1$, constant if $\beta+\theta = 1$, decreasing if $\beta+\theta < 1$. This overcomes both the knife edge assumption of the endogenous growth literature where $\beta+\theta = 1$ and the assumption of Jones's critique (Jones (1995a)) that $\beta+\theta < 1$. This is because the driving force of the economy is now the production function of natural knowledge where the only limitation is the human thinking capacity.

5.5 The threat from what gets lost: pollution damages on nature as a knowledge reservoir

The production function for the final good is again

$$Y = D^{\alpha}[A(1 - a_A - a_D)L]^{1-\alpha}$$

and the production function for technology is maintained as before

$$\dot{A} = D^{\beta}(a_A L)^{\gamma} A^{\theta}.$$

The production function of natural knowledge as potential for the maximal scientific improvement, differently from the basic model, additionally captures the realistic feature that something of the nature is destroyed due to pollution damages as result of the human activity. Therefore precious sources of information get lost, reducing the basis of scientific knowledge. These damages are modeled as an inevitable consequence of the output production. Thus the new function is

$$\dot{D} = (a_D L)^{\chi} D^{\varepsilon} - d\{D^{\alpha}[A(1 - a_A - a_D)L]^{1-\alpha}\}. \tag{5.11}$$

For finding an equilibrium the dynamics of the model must be studied.[15] The new growth rate for g_D is

$$g_D = a_D^{\chi} L^{\chi} D^{\varepsilon-1} - dD^{\alpha-1} A^{1-\alpha}[(1 - a_A - a_D)L]^{1-\alpha} \tag{5.12}$$

and therefore balanced steady state implies that the growth rate of g_D must satisfy

$$\chi g_L + (\varepsilon - 1)g_D = 0 \tag{5.13}$$

and

$$(\alpha - 1)g_D + (1 - \alpha)g_A + (1 + \alpha)g_L = 0, \tag{5.14}$$

hence

$$g_D = \frac{\chi}{1 - \varepsilon} g_L \tag{5.15}$$

$$g_D = g_A + g_L. \tag{5.16}$$

Equations (5.15) and (5.16) are indicating the constraints that must be satisfied for having $\dot{g_D} = 0$. This happens only in one point (g'_A, g'_D) which is the intersection point of the two equations if $\frac{\chi}{1-\varepsilon} > 1$ (even though it does not mean any restriction on the type of returns to scale on labor (χ) in equation (5.3), recalling that from the basic model $\varepsilon < 1$). Thus

$$g'_D = \frac{\chi}{1 - \varepsilon} g_L$$

and

$$g'_A = (\frac{\chi}{1 - \varepsilon} - 1)g_L.$$

[15]As we ate interested in the existence of equilibrium at this stage, we will not investigate issues of convergence.

The locus of points where g_D is constant is, as before, the straight line with intercept $-\frac{\gamma}{\beta}g_L$ and slope $\frac{1-\theta}{\beta}$:

$$\dot{g_A} = 0 \Rightarrow g_D = -\frac{\gamma}{\beta}g_L + \frac{1-\theta}{\beta}g_A. \tag{5.17}$$

Summing all the information together the second proposition follows.

Proposition 7 *This economy possesses an equilibrium ($\dot{g_D} = 0$ and $\dot{g_A} = 0$) with $(g_A^*, g_D^*) = (g_A', g_D')$ where A and D grow steadily but if and only if $\beta + \theta < 1$.*

Proof: If g_A' and g_D' are substituted into equation (5.17), it follows

$$\frac{\chi}{1-\varepsilon}g_L = -\frac{\gamma}{\beta}g_L + \frac{1-\theta}{\beta}(\frac{\chi}{1-\varepsilon} - 1)g_L$$

where it must be that

$$\frac{1-\theta}{\beta} = \frac{\left(\frac{\chi}{1-\varepsilon} + \frac{\gamma}{\beta}\right)}{\left(\frac{\chi}{1-\varepsilon} - 1\right)} > 1 \tag{5.18}$$

and therefore $\beta + \theta < 1$. See figure 5.2a).■

This economy can perform sustainable growth without scale effects, but it needs decreasing returns to scale in the two produced factors of production, namely A and D, in the production function of technologies. This brings the model back to run on the same assumption as Jones (1995b), yet not to the early new growth literature with $\beta+\theta = 1$. Now the main relevant force of the economy is the constraint arising from the loss of useful information $d\{D^\alpha[A(1 - a_A - a_D)L]^{1-\alpha}$ from equation (5.11). But because the production function of the final good has constant returns to scale in A and D, in the end, whether that constraint is too strong or not for having sustaiable growth depends on the returns to scale of A and D in the production function of technologies. Therefore only with decreasing returns the limiting effect is not too severe for the growth process.

5.6 How technological progress influences a knife edge assumption

If the consideration that technologies could play an important role in this economy is taken into account, then the production function for natural knowledge becomes

$$\dot{D} = (a_D L)^{\chi} D^{\varepsilon} - d\{D^{\alpha}[A(1 - a_A - a_D)L]^{1-\alpha}\}A^{-\lambda} \tag{5.19}$$

where $\lambda > 0$. In fact, technological progress can mitigate the environmental impact of the production activities on nature and therefore relax the constraint on D.

Again the dynamics of the new model are studied. The growth rate of natural knowledge is

$$g_D = ba_D^{\chi} L^{\chi} D^{\varepsilon-1} - dD^{\alpha-1} A^{1-\alpha-\lambda}[(1 - a_A - a_D)L]^{1-\alpha} \tag{5.20}$$

and therefore balanced steady state implies that the growth rate of g_D must satisfy

$$\chi g_L + (\varepsilon - 1)g_D = 0 \tag{5.21}$$

and

$$(\alpha - 1)g_D + (1 - \alpha - \lambda)g_A + (1 + \alpha)g_L = 0, \tag{5.22}$$

hence

$$g_D = \frac{\chi}{1 - \varepsilon} g_L \tag{5.23}$$

$$g_D = \frac{1 - \alpha - \lambda}{1 - \alpha} g_A + g_L. \tag{5.24}$$

Again, these two straight lines are the conditions that must be jointly respected in order to find the only point which guarantees that $\dot{g_D} = 0$. The new intersection point is (g''_A, g''_D) with

$$g''_D = \frac{\chi}{1 - \varepsilon} g_L$$

as before and

$$g''_A = \frac{(\frac{\chi}{1-\varepsilon} - 1)}{\frac{1-\alpha-\lambda}{1-\alpha}} g_L$$

which is positive either when $\chi > 1-\varepsilon$ and $\lambda < 1-\alpha$ or $\chi < 1-\varepsilon$ and $\lambda > 1-\alpha$ or $\chi = 1-\varepsilon$ and $\lambda = 1-\alpha$.

When $\chi > 1-\varepsilon$, as in the previous section, and if we recall that

$$\dot{g_A} = 0 \Rightarrow g_D = -\frac{\gamma}{\beta} g_L + \frac{1-\theta}{\beta} g_A, \tag{5.25}$$

then

Proposition 8 *The economy possesses an equilibrium with* $(g_A^*, g_D^*) = (g_A'', g_D'')$ *where A and D grow steadily without any assumption on the returns to scale in the produced factors of production in the production function of technologies, namely the sum* $\beta + \theta$.

Proof: If g_A'' and g_D'' are substituted into equation (5.25) then

$$\frac{\chi}{1-\varepsilon} g_L = -\frac{\gamma}{\beta} g_L + \frac{1-\theta}{\beta} \frac{\left(\frac{\chi}{1-\varepsilon} - 1\right)}{\frac{1-\alpha-\lambda}{1-\alpha}} g_L \tag{5.26}$$

which implies that

$$\frac{1-\theta}{\beta} = \frac{\left(\frac{\chi}{1-\varepsilon} + \frac{\gamma}{\beta}\right)}{\left(\frac{\chi}{1-\varepsilon} - 1\right)} \frac{1-\alpha-\lambda}{1-\alpha}. \tag{5.27}$$

From the proof to Proposition 7 the first part of the right hand side of the equation is greater than 1 and the second part is smaller than 1, thus the sum $\beta + \theta$ is free to be $>, <$ than 1. See figure 5.2b).

The same conclusion applies for the cases $\chi < 1-\varepsilon$, $\lambda > 1-\alpha$ and $\chi = 1-\varepsilon$, $\lambda = 1-\alpha$, but the last case leads to an interesting constellation. If $\dot{g_D} = 0$ was derived for $\chi = 1-\varepsilon$, $\lambda = 1-\alpha$, then the two straight lines corresponding to equations (5.23) and (5.24) collapse to the same one $g_D = g_L$. So, similarly to Section 5.4, this straight line will be the locus of points where g_D is constant but with intercept 1 instead of $\frac{\chi}{1-\varepsilon}$. In this case, as in Section 5.4, $\beta + \theta$ can be smaller, equal or bigger than one.■

In this version of the model sustainable growth can go hand in hand with both increasing, decreasing and constant returns to scale in the produced factors of production in the production function of technologies, because the constraint which drives this economy is $d\{D^{\alpha}A^{1-\alpha-\lambda}[(1-a_A-a_D)L]^{1-\alpha}\}$ from equation (5.19). Differently from the previous section where no mitigation arising from the technological progress was considered, here, in the environmental constraint, the returns to scale on D and A are always decreasing, independent of whether λ is $>, =, < 1-\alpha$. This guarantees that the environmental constraint is not strong enough to affect sustainable growth predictions.

5.7 Conclusion

A new model structure is developed where nature is given a positive interpretation as a knowledge reservoir which is a maximal source for scientific improvement. Three different versions of the model, which do not predict scale effects, are presented to investigate how the role of the returns to scale in the produced factors of production changes. Starting with no constraints at all on the production function of natural knowledge, we move to their inclusion, ending with the recognition of the positive role that the technological progress has in the mitigation of the environmental threat.

It is demonstrated that only in the case with the environmental constraint and without technological mitigation, a specific assumption about the returns to scale in the produced factor of production is needed in order to guarantee sustainable growth. This is $\theta + \beta < 1$, which also Jones (1995) found necessary to eliminate scale effects. This assumption returns to be non-binding when technological mitigation is introduced, increasing therefore the generality of the model in predicting sustainability.

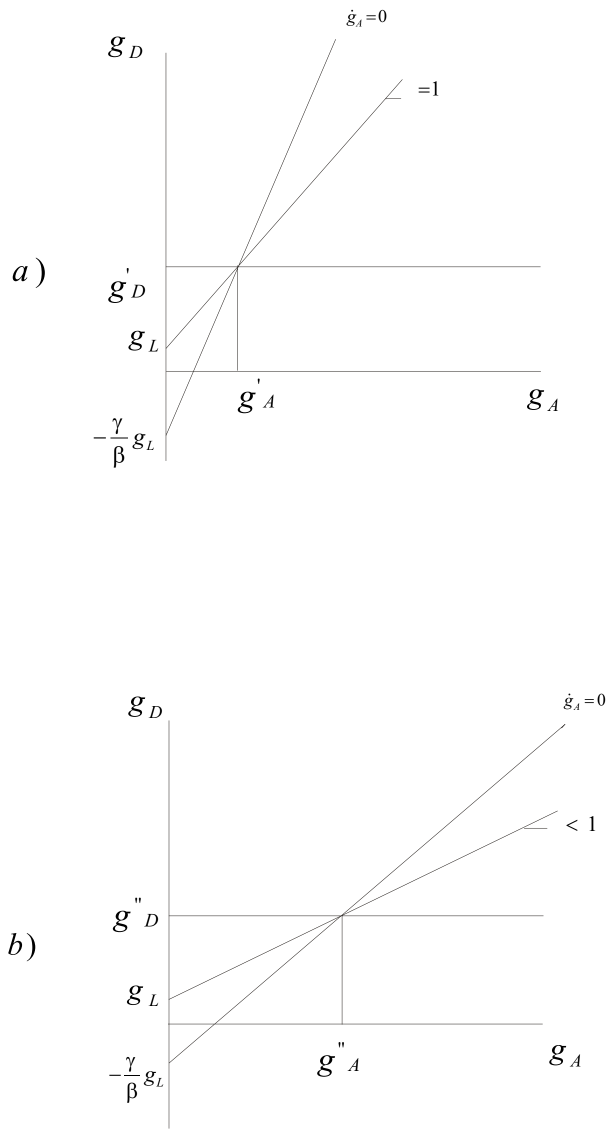

Figure 5.2: Sustainable Growth Equilibria

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