NEW FORMS OF COMPETITION - HOW MARKETS WORK IN THE INFORMATION SOCIETY

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Für meine Eltern, Jürgen und Anny,
und für meine Geschwister, Manfred und Birgit.

Und ganz besonders für Julia.

Contents

Chapter 1

Introduction

"The Information Society is a society in which the creation, distribution and manipulation of information is becoming a significant economic and cultural activity. The knowledge economy is its economic counterpart whereby wealth is created through the economic exploitation of knowledge."[1]

Wikipedia, the free internet encyclopedia, which is by itself a product of our Information Society, provides this definition. The existence of Wikipedia, a costless product, created by individuals, and meanwhile a serious substitute to encyclopedias like Britannica, is a good example how the Information Society is changing markets and competition.

In this study we analyze how markets work in the Information Society. In particular we concentrate on three important markets: the software market, the broadcasting market and "technology markets" where intellectual property rights can be traded.[2] All these markets are characterized by modes of competition that are rather unorthodox and beyond simple Cournot or Bertrand models. Therefore, extended models are needed to gain insights about competition in the Information Society.

Before the onset of game theory, the existing price theory addressed effectively only situations of perfect competition and pure monopoly. However, in most cases,

[1] http://en.wikipedia.org/wiki/Information_society

[2] "Technology markets consist of the intellectual property that is licensed (the "licensed technology") and its close substitutes–that is, the technologies or goods that are close enough substitutes significantly to constrain the exercise of market power with respect to the intellectual property that is licensed." (FTC (1995))

and in particular in the Information Society, markets are characterized by imperfect competition. As Karl-Göran Mäler said in his presentation speech for the Nobel Prize 1994 that was awarded to John Forbes Nash, Reinhard Selten and John Charles Harsanyi, "Many economists and social scientists subsequently tried to analyze the outcome in other specific forms of strategic interaction. However, prior to the birth of game theory, there was no toolbox that gave scholars access to a general but rigorous method of analyzing different forms of strategic interaction." We open this toolbox and use its concepts of non-cooperative and cooperative game theory to analyze how markets work in the Information Society. In the following we introduce the three different markets and point out our contribution to their understanding.

The software market is highly concentrated. In particular Microsoft has a very strong position due to its operating system Windows. Nevertheless, individuals and firms invest in the development of Open Source Software like Linux. Approximately 50 % of the development is done by commercial firms.[3] Interestingly, Linux evolves as the only serious alternative to Windows. From an industrial organization perspective, commercial investment in a non-excludable public good like Open Source Software is on a first glance puzzling. A firm cannot directly profit from the enhanced public good because it cannot sell the improvement. However, firms are able to sell more complementary proprietary products at a potential higher price by improving the non-excludable public good. Obviously, such an incentive to invest in Open Source Software is beyond the treatment of simple competition models.

In chapter 2 we set up a model to analyze a market environment where firms can produce a private good and can invest in its complementary non-excludable public good like Open Source Software. However, firms can only sell the private good. By studying the incentive to invest in such a non-excludable public good, we ask (1) how market entry and (2) how a government investment in the public good affects the firms' output levels and profits. Surprisingly, we find that cases exist where incumbents benefit from market entry. Moreover, we show the counter-intuitive result that a government investment in the public good can increase the private investment in the public good. Hence, we provide conditions under which a crowding in instead of a crowding out occurs.

The second market we address is the broadcasting market. This market is gaining more and more importance in our Information Society. This is clearly evident

[3] International Institute of Infonomics (2002)

from the data. In Germany, the average time people watch television has increased from 144 minutes in 1988 to 211 minutes in 2005. Furthermore, expenditures for television advertising have risen from 880 million Euro in 1988 to 8.1 billion Euro in 2005.[4]

In this study we consider competition between free-to-air broadcasting channels. Their business idea is to match potential consumers and advertisers. Hence, the channels air their programs to attract viewers in order to sell the attention of these viewers to the advertisers. Competition in such a two-sided market is quite different from usual markets because platforms have to take into account the link between the two markets. The existing literature on such two-sided markets only deals with participation externalities: A change in a channel's advertising level changes its own and the competitors' number of viewers. So far, the literature has neglected pecuniary externalities between broadcasting channels. Pecuniary externalites change the equilibrium advertising price on all channels if a channel changes its advertising level. This externality plays an important role in the real world, and we show that it changes the theoretical predictions of the existing models.

In Chapter 3 we build a model that includes both externalities. In our setup differentiated platforms compete in advertising. They offer consumers a service free of charge, such as a TV program, which is financed by advertising. We show that advertising can exhibit the property of a strategic substitute or complement. This is in contrast to the existing literature. Surprisingly, cases exist in which platforms benefit from market entry. Moreover, we show that perfect competition is not always desirable from a welfare point of view.

Knowledge and its protection is gaining more and more importance in the Information Society. Often, firms rely on patents to protect their innovations. In the US, the number of patents granted per year almost tripled from 70.000 to 190.000 between 1970 and 2004.[5] Hence, patents and the outcomes of patent litigation play an important role in many markets.

In this study we consider the mode of competition in technology markets where patents are involved. For many years economists just assumed that issued patents are valid for sure. Hence, in former models no uncertainty existed whether the

[4]Source: www.agf.de/daten

[5]Leonard and Stiroh (2005)

patent is infringed by another party or not. Recently, research has focused on the probabilistic nature of patents. As Shapiro (2003) states, patents give the patentholder not the right to exclude a potential infringer, but the right *to try* to exclude the potential infringer by a lawsuit. Empirically, roughly half of all lawsuits are lost by the patentholder. This highlights the probabilistic nature of patents.[6] In most cases firms do not rely on a court decision to resolve a patent dispute, but settle their conflict out of court. For example, Lanjouw and Schankerman (2002) find that 95% of all patent lawsuits are settled prior to a court judgment. These patent settlements often include licensing contracts between the firms. For the antitrust authorities the design of such contracts is highly suspicious, particularly if the parties are (potentially) horizontal competitors in the relevant market. Firms can easily use these contracts to fix prices or to split up markets in order to soften competition between them. Hence, antitrust limits to patent settlements are urgently needed.

In Chapter 4 we look at such antitrust limits to patent settlements given that patents are probabilistic. So far, the literature addressed probabilistic patents only in a static environment. In contrast, we allow for market entry. Hence, we consider the impact of different antitrust limits to patent settlements on market entry incentives. We show that unconstrained settlements are not preferable at all from a welfare point of view. Furthermore, even constrained settlements, as proposed by Shapiro (2003), can decrease social welfare and harm the patentholder. Surprisingly, we find that a patentholder may prefer very restrictive antitrust limits.

[6] Estimations of the probability that the patentee wins the lawsuit are between 35% and 70%. See Lanjouw and Schankerman (1998) for a survey of the corresponding empirical literature.

Chapter 2

Private Provision of a Complementary Public Good

2.1 Introduction

An increasing number of firms, like IBM and Hewlett-Packard or Suse and Red Hat, have begun to invest in Open Source Software. Open Source Software, such as Linux, is typically under the General Public License. This license implies that the software, including any improvement, has to be provided for free. Hence, an Open Source Software can be seen as a non-excludable public good. Therefore, firms are not able to sell the Open Source Software or their improvements. This raises the question why companies invest in such a public good.

Lerner and Tirole (2000) argue that firms expect to benefit from some market segment the demand of which is boosted by the improvement of a complementary Open Source Software. Even though the companies cannot directly capture the value of an open source program's improvement, they can profit indirectly through selling more complementary proprietary goods at a potentially higher price.

This incentive to invest in a non-excludable public good does not only arise in the case of Open Source Software. For example, a similar argument could be made in the case of advertising that increases the demand of the advertising firm and at the same time the demand of its competitors. Friedman (1983) calls this "cooperative advertising". Another example are lobby-activities of a firm that have a positive effect on the whole industry.

In this chapter we study the incentive of firms to invest in such a non-excludable public good. In particular we address the following two questions:

(1) What is the effect of a higher public good investment by the government on the firms' output levels and profits?

(2) How does market entry affect the private incentive to invest in the public good? Furthermore, how does market entry influence the incumbents' profits?

We contribute to answer the questions (1) and (2) by analyzing a model with Cournot competition. Firms can produce a private good, and they can invest in a non-excludable public good in order to enhance its quality. However, they can only sell the private good. The private good and the public good are complements for the consumers. An increase in the quality of the public good increases their willingness to pay for the private good.

The first question is particularly interesting because of the ongoing discussion whether or not the government should support Open Source Software and if so, how.[1] A concern might be that higher government investment in the public good decreases firms' investments as it is known from the public good literature.[2] Hence, one usually expects a crowding out. Interestingly, such a crowding out does not have to take place with a complementary public good. We show that it might occur that the firms' investments increase if the government increases its investment in the public good. Hence, a crowding in can occur. Thus, it is not obvious whether the government investment in the public good is a strategic substitute or complement to the firms' public good investment.

The second question is shortly addressed by Lerner and Tirole (2000). They argue that the usual free-rider problem might appear because firms are not able to capture all the benefits of their investments. Therefore, one might argue that the free-rider problem gets worse with an increasing number of firms. Hence, a firm's investment in the public good decreases with market entry. As we will show in our model, it might occur that the opposite happens and thus each firm invests more. Furthermore, we show that market entry of an additional firm has a positive

[1] See e.g. Hahn (2002), Evans and Reddy (2003) or Schmidt and Schnitzer (2002).

[2] e.g Bergstrom et al. (1986)

externality (through the entrant's investment in the public good) and a negative externality (through the entrant's production of the private good) on the incumbents. We find that for certain cost and demand functions each firm reduces its private good production and its public good investment when market entry occurs. In this case incumbents suffer a decrease in profits and dislike market entry. Surprisingly, for certain cost and demand constellations it is also possible that every firm expands its investment in the public good, combined with a higher private good output, when market entry occurs. In this case market entry increases the incumbents' profits. However, a social planer unambiguously prefers market entry.

This chapter is related to the public good literature that is concerned with the private provision of a non-excludable public good. In standard models of private provision of a public good, households can buy the private good, they can contribute to the public good, and they face a budget constraint.[3] A household receives utility directly from the consumption of both goods and has to decide how to allocate his budget between the two goods. This setup differs from our model. We consider agents that have no direct benefit from the public good. The firms produce the public good solely due to the complementarity. Furthermore, firms face no budget constraint.

A second strand of literature our chapter is related to is the literature on Multimarket Oligopoly. Bulow et al. (1985) analyze the spillovers of a change in one market environment on the related markets. In contrast to their model, in our model the markets are not related via the production technology, but via the demand function. Bulow et al. (1985) address this issue, but do not formalize it. They mention that firms have to take care of cross-effects in making marginal revenue calculations. Moreover, firms have to consider the strategic effects of their actions in one market on the competitors' actions in a second market. In our model we extend this setup to the case of a non-excludable public good. This is in contrast to the model of Bulow et al. (1985) where only private goods are considered.

Becker and Murphy (1993) analyze a model in which advertisement and an advertised good enter the utility function of the households. Advertisement has the property that it rises the willingness to pay for the advertised good. Hence, it is complementary to the advertised good from an economic point of view. Neverthe-

[3]See e.g. Bergstrom et al. (1986) for a general approach and e.g Bitzer and Schröder (2002) or Johnson (2002) for an application to Open Source Software.

less, a firm's advertisement is only complementary to its own private good in their setup. By contrast, in our model it is also complementary to their competitors' private good.

Friedman (1983) considers, as a special case, cooperative advertising in a dynamic setup. He models advertising like capital in such a way that a firm can use advertising to build up a "goodwill stock". The firms have to decide how much to spend on advertising and how to allocate these spendings over time. By using "a symmetric quadratic model", he shows that an increasing number of active firms leads to a steady-state in which conventional competitive effects dominate. Hence, prices converge to marginal costs. In contrast, we look at a static game to concentrate explicitly on the externalities between the firms. As we will see, this leads to different results.

We will proceed as follows. In the next section we set up the model. In Section 3 we look at the properties of the market equilibrium. In Section 4 we analyze the consequences of a government investment in the public good. In Section 5 we consider the effects of market entry. The final section concludes.

2.2 The Model

We assume that firms are engaged in a one-period Cournot competition. They decide simultaneously about their private good production and their investment in the public good, taking as given the competitors' production and investment. We denote by x_i ($x_i \in [0; \overline{x}_i]$) the firm i's ($i \in \{1, ..., N\}$) production of the homogenous private good. By y_i ($y_i \in [0; \overline{y}_i]$) we denote the firm i's investment in the homogenous non-excludable public good. Such an investment increases the quality of the public good linearly $Y = \sum_{i=1}^{N} y_i$.[4] The private good and the public good are complements for the consumers. Their willingness to pay for the private good is increasing in the quality of the public good and hence in the firms' public good investments. For an illustration consider a computer server (=the private good) and an Open Source Software (=the public good). The performance of the server depends crucially on the ability of the server operating system to use the power of the hardware. If the quality of the operating system increases, then the consumers' willingness to pay for

[4] Throughout this chapter we speak about quality of the public good. In some cases, like advertising, one can interpret this quality as a measure of quantity.

the server increases due to the better performance. This yields the following private good demand function

$$p = p(x_i, y_i, X_{-i}, Y_{-i}), \tag{2.1}$$

$$\frac{\partial p}{\partial x_i} < 0, \ \forall i \in \{1, 2, ..., N\}, \tag{2.2}$$

$$\frac{\partial p}{\partial y_i} > 0, \ \forall i \in \{1, 2, ..., N\}. \tag{2.3}$$

We denote by $X_{-i} = \sum_{j=1;j\neq i}^{N} x_j$ the other firms' production of the private good. By $Y_{-i} = \sum_{j=1;j\neq i}^{N} y_j$ we denote the other firms' investment in the public good.

Furthermore, we have the following revenue function of firm i

$$R_i(x_i, y_i, X_{-i}, Y_{-i}) = x_i p(x_i, y_i, X_{-i}, Y_{-i}) \tag{2.4}$$

and the following profit function

$$\pi_i = R_i(x_i, y_i, X_{-i}, Y_{-i}) - K^x(x_i) - K_i^y(y_i). \tag{2.5}$$

We denote by $K^x(x_i)$ the private good cost function. For simplicity, we assume that this cost function is the same for all firms. We assume that firm i has to incur the costs $K_i^y(y_i)$ for investing y_i in the public good. Hence, we call this function "public good cost function". Furthermore, we assume that the public good cost function can be different for different firms.

In order to have a well defined maximization problem, we assume that the profit function π_i is continuous and strictly concave in x_i and y_i. The following conditions, satisfied for all (x_i, y_i), yield a Hessian matrix that is everywhere negative definite. This a sufficient condition for a strictly concave profit function

$$\text{Condition 1: } \frac{\partial^2 \pi_i}{\partial x_i^2} < 0; \tag{2.6}$$

$$\text{Condition 2: } \frac{\partial^2 \pi_i}{\partial y_i^2} < 0; \tag{2.7}$$

$$\text{Condition 3: } \frac{\partial^2 \pi_i}{\partial x_i^2}\frac{\partial^2 \pi_i}{\partial y_i^2} > [\frac{\partial^2 \pi_i}{\partial x_i \partial y_i}]^2. \tag{2.8}$$

In order to satisfy Condition 1, we assume that the private good cost function is convex $\frac{\partial^2 K^x(x_i)}{\partial x_i^2} \geq 0$ and that the revenue function is strictly concave $\frac{\partial^2 R_i}{\partial x_i^2} < 0$. For example, a concave demand function $\frac{\partial^2 p}{\partial x_i^2} \leq 0$ is sufficient for a strictly concave revenue function.

In order to satisfy Condition 2, we assume a concave revenue function and a convex public good cost function, with at least one strict, because subtracting a convex (strictly convex) function from a strictly concave (concave) function yields a strictly concave function.

$$\frac{\partial^2 R}{\partial y^2} \leq 0 \text{ and } \frac{\partial^2 K_i^y(y_i)}{\partial y_i^2} \geq 0, \text{ at least one strict.} \tag{2.9}$$

For example, a concave demand function $\frac{\partial^2 p}{\partial y_i^2} \leq 0$ yields a strictly concave profit function if the public good cost function is strictly convex.

In order to satisfy Condition 3, we have to assume that

$$[\frac{\partial^2 R_i}{\partial x_i^2} - \frac{\partial^2 K^x}{\partial x_i^2}][\frac{\partial^2 R_i}{\partial y_i^2} - \frac{\partial^2 K_i^y}{\partial y_i^2}] > [\frac{\partial^2 R_i}{\partial x_i \partial y_i}]^2, \tag{2.10}$$

$$\Leftrightarrow [2\frac{\partial p}{\partial x_i} + x_i\frac{\partial^2 p}{\partial x_i^2} - \frac{\partial^2 K^x}{\partial x_i^2}][x_i\frac{\partial^2 p}{\partial y_i^2} - \frac{\partial^2 K_i^y}{\partial y_i^2}] > [x_i\frac{\partial^2 p}{\partial x_i \partial y_i} + \frac{\partial p}{\partial y_i}]^2 \, . \tag{2.11}$$

The stationary point does not have to be a maximum without this technical assumption.

2.3 Market Equilibrium

In this section we solve the game and determine the three different kinds of market equilibria. We show that a unique equilibrium can exist if the marginal costs of the public good are increasing. In contrast, a multiplicity of equilibria may arise if the marginal costs of the public good are constant.

We start with the case of increasing marginal costs of the public good.

Proposition 2.1 *Suppose that the public good's marginal costs are increasing. There exists a unique Nash Equilibrium where the firms produce* $x^* = (x_1^*, x_2^*, ..., x_N^*)$ *of the private good and invest* $y^* = (y_1^*, y_2^*, ..., y_N^*)$ *in the public good if*

$$(N-1)|x_i \frac{\partial^2 p}{\partial x_i \partial x_j} + \frac{\partial p}{\partial x_j}| + N|x_i \frac{\partial^2 p}{\partial x_i \partial y_j} + \frac{\partial p}{\partial y_j}| + (\frac{\partial^2 p}{\partial x_i \partial x_i} x_i + 2\frac{\partial p}{\partial x_i}) < \frac{\partial^2 K^x(x_i)}{\partial x_i \partial x_i}, \forall i \tag{2.12}$$

and

$$(N-1)|x_i \frac{\partial^2 p}{\partial y_i \partial x_j}| + |x_i \frac{\partial^2 p}{\partial y_i \partial x_i} + \frac{\partial p}{\partial x_i}| + (2-N)x_i \frac{\partial^2 p}{\partial y_i \partial y_j} < \frac{\partial^2 K_i^y(y_i)}{\partial y_i \partial y_i}, \forall i \tag{2.13}$$

Proof. See Appendix. ∎

In order to see the intuition behind Proposition 2.1, we write down firm i's profit function

$$\pi_i = x_i p\left(x_i, X_{-i}, y_i, Y_{-i}\right) - K^x(x_i) - K_i^y(y_i). \tag{2.14}$$

Profit maximization yields the following first order conditions

$$\frac{\partial \pi_i}{\partial x_i} = p + x_i \frac{\partial p}{\partial x_i} - \frac{\partial K^x(x_i)}{\partial x_i} = 0, \tag{2.15}$$

$$\frac{\partial \pi_i}{\partial y_i} = x_i \frac{\partial p}{\partial y_i} - \frac{\partial K_i^y(y_i)}{\partial y_i} = 0. \tag{2.16}$$

Equation (2.15) displays the standard optimality condition. It says that the marginal costs of the private good have to be equal to the marginal revenue of the private good. Equation (2.16) shows that the investment in the public good has only an indirect effect on the profit. If the public good's quality increases, then the consumers' willingness to pay increases. This rises the price of the private good. Therefore, the revenue of firm i increases.

Proposition 2.1 states that there is a unique Nash Equilibrium x^* and y^*. From the Equations (2.15) and (2.16) we see that the production of the private good can reinforce the incentive to invest in the public good and vice versa. In order to illustrate this, consider an increase in the private good production of firm i. This increases the incentive to invest in the public good. A higher public good investment increases the price of the private good. Compared to the situation with a lower private good production, this price increase works on more private good units. Hence, for a firm it gets more attractive to invest in the public good if its private good production increases.[5]

At the same time, a higher public good investment also increases the incentives to produce the private good. The public good investment leads to a higher price of the private good because it increases the consumers' willingness to pay. Therefore, selling an additional unit gets more attractive because a firm earns a higher price out of the additional unit sold.[6] But once again, this higher output level can yield a higher incentive to invest in the public good. Hence, the incentives to produce the private good and to invest in the public good can reinforce each other.

Conditions 1-3 ensure that this described process converges to a unique maximum for each firm if one takes the production and investment of the other firms as given. In order to determine the Nash-Equilibrium, we additionally have to take into account that the investment of a firm in the public good increases the incentive to produce the private good for all other firms. This may increase the other firms' incentive to invest in the public good and so on.

The two technical assumptions of Proposition 2.1 guarantee a unique Nash-Equilibrium (x^*, y^*) by ensuring a contraction mapping of the best response functions. Hence, the equilibrium (x^*, y^*) is globally stable and therefore unique.

[5]Of course, this is only true if the effect of the cross derivative $\frac{\partial^2 p}{\partial x_i \partial y_i}$ does not work in the opposite direction and dominates.

[6]Of course, as above, this is only true if the effect from the cross derivative $\frac{\partial^2 p}{\partial x_i \partial y_i}$ does not work in the opposite direction and dominates.

In the two next propositions we consider constant marginal costs of the public good. We show that a multiplicity of equilibria or an asymmetric equilibrium arises.

Proposition 2.2 *Suppose that the public good's marginal costs are constant and equal for all firms. There are an infinite number of equilibria. In all equilibria* x_i^* *is the same and the firms' investment in the public good always sums up to a certain level* Y^* *if the following condition is fulfilled*

$$(N-1)|x_i\frac{\partial^2 p}{\partial x_i \partial x_j}+\frac{\partial p}{\partial x_j}|+N|x_i\frac{\partial^2 p}{\partial x_i \partial y_j}+\frac{\partial p}{\partial y_j}|+(\frac{\partial^2 p}{\partial x_i \partial x_i}x_i+2\frac{\partial p}{\partial x_i})<\frac{\partial^2 K^x(x_i)}{\partial x_i \partial x_i}. \tag{2.17}$$

Proof. See Appendix. ■

In contrast to Proposition 2.1, Proposition 2.2 states that all public good investment vectors $y=(y_1,...,y_N)$ that sum up to $Y^*=\sum_{i=1}^N y_i$ are Nash-Equilibria. In the previous case the individual investment level y_i^* was determined by the increasing marginal costs. If these are constant, then a coordination problem between the firms occurs. For an illustration suppose two firms that produce the same quantities of the private good. Suppose that firm 1 assumes that firm 2 invests nothing in the public good. Then firm 1 should invest until its marginal revenue of the public good is equal to its marginal costs. Given this investment of firm 1, firm 2 should invest nothing because its marginal revenue and its marginal costs are the same as for firm 1. Generally, if firm 1 is in an optimum given to the investment of firm 2, then firm 2 is also in an optimum given to the investment of firm 1. This fact yields the infinite number of possible equilibria.[7]

In the next proposition we show the properties of an asymmetric equilibrium. For simplicity, we only address the case of two firms $\{i,-i\}$. A generalization to N firms is straightforward.

Proposition 2.3 *Suppose that firm i's constant marginal costs for investing in the public good are lower than the constant marginal costs of firm* $-i$

$$\frac{\partial K_i^y}{\partial y_i}<\frac{\partial K_{-i}^y}{\partial y_{-i}}. \tag{2.18}$$

[7] The technical assumption in Proposition 2.2 ensures that the best response functions with respect to the private good are a contraction mapping.

In the unique Nash Equilibrium both firms produce the same quantity of the private good ($x_i^* = x_{-i}^*$) and only firm i invests in the public good ($y_i^* = Y^*; y_{-i}^* = 0$) if the following condition is fulfilled

$$(N-1)|x_i \frac{\partial^2 p}{\partial x_i \partial x_j} + \frac{\partial p}{\partial x_j}| + N|x_i \frac{\partial^2 p}{\partial x_i \partial y_j} + \frac{\partial p}{\partial y_j}| + (\frac{\partial^2 p}{\partial x_i \partial x_i} x_i + 2\frac{\partial p}{\partial x_i}) < \frac{\partial^2 K^x(x_i)}{\partial x_i \partial x_i} \tag{2.19}$$

Proof. See Appendix. ■

In the following we first check whether the described equilibrium is indeed an equilibrium. Afterwards, we show that this equilibrium is unique.

Suppose that the firms choose ($x_i^* = x_{-i}^*, y_i^* = Y^*, y_{-i}^* = 0$). With respect to the private good we know that the marginal costs of i and $-i$ are equal at the point $x_i^* = x_{-i}^*$. What can we say about the marginal revenues? With respect to the marginal revenue of the private good we know that it does not matter whether firm i or $-i$ invests in the public good. Only the resulting quality of the public good $Y = y_i + y_{-i}$ is important. Therefore, the marginal revenue with respect to the private good is the same for both firms. Thus, both firms produce the same quantity of the private good.

Furthermore, suppose that firm i's optimal response to $y_{-i}^* = 0$ is $y_i^* = Y^*$ given $x_i^* = x_{-i}^*$. This an equilibrium if firm $-i$ has no incentive to deviate from $y_{-i}^* = 0$. We know that the optimal response of firm i implies that

$$x_i \frac{\partial p}{\partial y_i} |_{x_i^* = x_{-i}^*, y_i = Y_i^* y_{-i}^* = 0} = \frac{\partial K_i^y}{\partial y_i}. \tag{2.20}$$

Furthermore, we know that the marginal revenue with respect to the public good is the same for both firms with $x_i^* = x_{-i}^*$. Hence, if firm $-i$ has higher constant marginal costs than firm i, then it directly follows that

$$x_{-i} \frac{\partial p}{\partial y_{-i}} |_{x_i^* = x_{-i}^*, y_i = Y_i^* y_{-i}^* = 0} < \frac{\partial K_{-i}^y}{\partial y_{-i}}. \tag{2.21}$$

Therefore, it is indeed optimal for firm $-i$ to choose $y_{-i} = 0$. Hence, $x_i^* = x_{-i}^*, y_i = Y^*, y_{-i} = 0$ is an equilibrium.

Uniqueness follows from the iterated deletion of strictly dominated strategies.[8] Assume that firm i invests nothing ($y_i = 0$) and denote the best response of firm $-i$ as $\widetilde{y}_{-i} \mid_{y_i=0}$. This is the maximal value of y_{-i} that makes sense for firm $-i$ because if $y_i > 0$, then it follows $\widetilde{y}_{-i} \mid_{y_i>0} < \widetilde{y}_{-i} \mid_{y_i=0}$. Hence, all $y_{-i} > \widetilde{y}_{-i} \mid_{y_i=0}$ are strictly dominated by $y^a_{-i} = \widetilde{y}_{-i} \mid_{y_i=0}$. Given this, the smallest y_i that firm i should choose is $\widetilde{y}_i \mid_{y^a_{-i}}$. This is the best response to y^a_{-i}. Hence, all $y_i < \widetilde{y}_i \mid_{y^a_{-i}}$ are strictly dominated by $y^a_i = \widetilde{y}_i \mid_{y^a_{-i}}$. Given this, firm $-i$ should maximal invest $\widetilde{y}_{-i} \mid_{y^a_i} < y^a_{-i}$. Hence, all $y_{-i} > \widetilde{y}_{-i} \mid_{y^a_i}$ are strictly dominated by $\widetilde{y}_{-i} \mid_{y^a_i} = y^b_{-i}$. It is obvious that this process converges to the unique equilibrium where $y_i = Y^*$ and $y_{-i} = 0$.

2.4 Government Intervention

Let us now consider the effects of government intervention. Assume that the government starts to invest or changes its investment in the public good. This is not a hypothetical assumption as one can see from the direct US Government support for Linux. For example, the US government decided to finance a research project at a university to improve Linux (Evans and Reddy (2003)). In the following we assume that the government can invest directly in the public good as the firms. By y_G we denote the investment of the government in the public good.

If the government increases its investment in the public good, one usually expects a crowding out in such a way that the firms decrease their investment in the public good. At least, this is what the standard public good literature states in the context of households.[9] In these models households have a certain budget. They have to decide how to allocate their budget between a private and a public good. In an interior equilibrium each household splits up his budget in such a way that his marginal utility of the private good is equal to his marginal utility of the public good. Suppose that the government increases its public good investment. Usually this decreases the marginal utility of the public good. Therefore, households shift money from the investment in the public good to the consumption of the private good in such a way that the marginal utilities are again equal. Thus, a crowding out occurs.

In this section we show that such a crowding out does not have to occur in

[8] Uniqueness of $x^*_i = x^*_{-i}$ follows from the technical condition in the Proposition 2.3. This yields that the best response functions with respect to the private good are a contracting mapping.

[9] see e.g. Bergstrom et.al. (1985)

our setup. Instead, a crowding in is possible.[10] Thus, a higher government investment in the public good can yield a higher investment of the firms in the public good.

In order to understand how the government investment influences the incentive of a firm to invest in the public good, we have to distinguish between direct and indirect effects. Therefore, we take the total derivative of the first order conditions:

Total derivative of $\frac{\partial \pi_i}{\partial x_i}$

$$[2 * \frac{\partial p}{\partial x_i} + x_i \frac{\partial^2 p}{\partial x_i^2} - \frac{\partial^2 K(x_i)}{\partial x_i^2}]dx_i + \overbrace{[\frac{\partial p}{\partial y_i} + x_i \frac{\partial^2 p}{\partial x_i \partial y_i}]dy_i}^{\text{indirect effect}}$$
$$\overbrace{+[\frac{\partial p}{\partial X_{-i}} + x_i \frac{\partial^2 p}{\partial x_i \partial X_{-i}}]dX_{-i} + [\frac{\partial p}{\partial Y_{-i}} + x_i \frac{\partial^2 p}{\partial x_i \partial Y_{-i}}]dY_{-i}}^{\text{direct effects}} = 0. \quad (2.22)$$

Total derivative of $\frac{\partial \pi_i}{\partial y_i}$

$$\overbrace{[\frac{\partial p}{\partial y_i} + x_i \frac{\partial^2 p}{\partial x_i \partial y_i}]dx_i}^{\text{indirect effect}} + [x_i \frac{\partial^2 p}{\partial y_i^2} - \frac{\partial^2 K_i(y_i)}{\partial y_i^2}]dy_i$$
$$\overbrace{+[x_i \frac{\partial^2 p}{\partial y_i \partial X_{-i}}]dX_{-i} + [x_i \frac{\partial^2 p}{\partial y_i \partial Y_{-i}}]dY_{-i}}^{\text{direct effects}} = 0, \quad (2.23)$$

with

$$dX_{-i} = \sum_{j=1, i \neq j}^{N} dx_j \text{ and } dY_{-i} = \sum_{j=1, i \neq j}^{N} dy_j + dy_G. \quad (2.24)$$

Equation (2.22) and (2.23) illustrate the decomposition into direct and indirect effects.

1. The direct effect influences x_i (y_i) by a change in X_{-i} and Y_{-i} without depending on a change in the corresponding complement y_i (x_i).

[10] Because one can consider several different cases, we restrict our attention to only two examples which highlight the possible outcomes.

2. The indirect effect influences the optimal level of x_i (y_i) by a change in the corresponding complement y_i (x_i).

For an illustration, suppose that IBM, a producer of hardware like computer servers, faces an exogenous increase in the quality of the complementary public good Linux. For example, IBM learns that the government finances a research project at a university in order to improve Linux. We denote this by $dy_G > 0$. How does this exogenous shock influences IBM's incentives to produce the private good and to invest in the public good? On the one hand, the government investment in the public good increases the consumers' valuation of IBM's hardware. This gives IBM an incentive to increase its production of the hardware (= a direct effect of dY_{-i} on dx_i). On the other hand, the additional public good investment may decrease IBM's incentive to invest in the public good because keeping the old investment level leads to an inefficient high quality of the public good from IBM's point of view (= a direct effect of dY_{-i} on dy_i). Furthermore, indirect effects appear. If IBM produces more servers, this changes its incentive to invest in the public good Linux (=indirect effect of dx_i on dy_i). Additionally, if IBM changes its investment in Linux, the quality of Linux changes. This has an effect on IBM's incentive to produce the private good (= indirect effect of dy_i on dx_i).

As we show in the following proposition, these direct and indirect effects can yield a crowding in.

Proposition 2.4 *Suppose that the government increases its public good investment $dy_G > 0$. Each firm increases its private good production and its public good investment if*

- *the marginal revenue of the public good is constant ($\frac{\partial^2 p}{\partial y_i^2} = 0$);*
- *the marginal production costs of the public good are increasing ($\frac{\partial^2 K_i^y}{\partial y_i^2} > 0$);*
- *the cross derivative of the price is non-negative ($\frac{\partial^2 p}{\partial y_i \partial x_i} \geq 0$).*

Proof. See Appendix. ■

The firms are in an equilibrium before the government changes its investment. In this equilibrium the marginal costs of the public good are equal to the marginal revenue of the public good. If the government increases its investment in the public good, then the marginal revenue with respect to the public good does not change ($\frac{\partial^2 p}{\partial y_i^2} = 0$). Therefore, the firms have no direct incentive to adjust their investment in the public good. Nevertheless, if the quality of the public good increases, then the firms have an incentive to increase their production of the private good (=direct effect) because the private good's price increases. This higher private good output has a feedback-effect (= indirect effect) on the incentives to invest in the public good. A firm's marginal revenue of the public good increases if the firm produces more of the private good. Finally, the investment of the government yields a higher public good investment of the firms. Hence, no crowding out, but a crowding in occurs. Furthermore, the total supply of the private good increases.

As a second case we consider the opposite to a crowding in. We show that a total crowding out might occur. Hence, firms reduce their investment in the public good in such a way that the quality of the public good remains constant. In addition, this yields no change in the private good production.

Proposition 2.5 *Suppose that the government increases its public good investment dy_G. Each firm does not change its private good production and the public good investment of the firms gets adjusted in such a way that a total crowding out occurs $\sum_{i=1}^N dy_i = -dy_G$ if*

- *the marginal revenue of the public good is decreasing ($\frac{\partial^2 p}{\partial y_i^2} < 0$);*
- *the marginal production costs of the public good are constant ($\frac{\partial^2 K_i^y}{\partial y_i^2} = 0$) and*
- *the change in the government investment is not higher then the total investment of the firms $\sum_{i=1}^N y_i^* > dy_G$.*

Proof. From Proposition 2.2 and 2.3 we know that the investment in the public good has always to sum up to Y^* with constant marginal costs. The last condition ensures that the firms' new optimal investment level remains non-negative.

■

Given constant marginal costs and a decreasing marginal revenue of the public good, there exists an intersection where marginal revenue is equal to marginal costs. This point determines the individually optimal investment level in the public good. If the government increases its investment in the public good, then the marginal costs are higher than the marginal revenue because the marginal revenue decreases. Therefore, the firms have an incentive to decrease their investment in the public good. They should decrease their investment until the marginal costs are again equal to marginal revenue. This is achieved by a total crowding out. Finally, the quality of the public good does not change. This leads to no adjustment of the private good production. From a welfare point of view, the consumers do not benefit from the government intervention. Only the firms are better off. They invest less in the public good. Nevertheless, they get the same price for their proprietary product as before. In this case the investment of the government only increases the profits of the firms. The government investment has neither an impact on the firms' production of the private good nor on the quality of the public good.

2.5 Market Entry

Let us now turn to the effects of market entry. In the last section we have assumed that the firms face a given exogenous shock with respect to the quality of the public good. Given market entry, the dimension of this shock is determined endogenously. Furthermore, market entry changes the total supply of the private good. Therefore, we use a linear-quadratic framework to solve this game analytically. Hence, we use the following inverse demand function

$$p = A - bX + cY. \tag{2.25}$$

In this function b and c represent weight factors. They determine the impact of the private good and the impact of the public good on the market price. One can easily derive such a demand function from the following maximization problem of a representative consumer

$$\max_{x,z} U = (A + cy)x - \frac{1}{2}bx^2 + z \quad \text{s.t.} \quad z + xp = m \ . \tag{2.26}$$

Furthermore, we consider N identical firms that have the same quadratic cost functions

$$K^x(x_i) = dx_i^2, \tag{2.27}$$

$$K^y(y_i) = fy_i^2. \tag{2.28}$$

The parameters d and f represent the weight of the cost functions in the profit function.

In the following we introduce Lemma 2.1 to ensure an interior solution of this game.

Lemma 2.1 *There exists a unique Nash Equilibrium* (x^*, y^*) *where each firm chooses the same* x_i^* *and* y_i^* *if the following conditions are fulfilled:*

Case 1: If $f \geq \frac{c^2}{2b}$, *then it has to be true that* $N < \frac{3b+2d-\frac{c^2}{f}}{b-\frac{c^2}{2f}}$.

Case 2: If $f < \frac{c^2}{2b}$, *then it has to be true that* $N < \frac{b+2d}{-b+\frac{1}{2f}}$.

Proof. See Appendix. ■

The conditions in Lemma 2.1 are always easily to fulfill because one can find a sufficiently high d for each value of N. Intuitively, Lemma 2.1 ensures that the firms' best reply functions are a contraction mapping. This is the case if the costs of the private good have enough weight, so d is high enough.

To find the optimal values $(x_i^*; y_i^*)$, we write down the profit function of a firm i. We denote by x_j and y_j the production of firm $j \in \{1, ..., N\} \setminus i$

$$\pi_i = x_i(A - bx_i - (N-1)bx_j + cy_i + (N-1)cy_j) - dx_i^2 - fy_i^2. \tag{2.29}$$

The first-order conditions are

$$\frac{\partial \pi_i}{\partial x_i} = A - 2bx_i - (N-1)bx_j + cy_i + (N-1)cy_j - 2dx_i = 0, \tag{2.30}$$

$$\frac{\partial \pi_i}{\partial y_i} = cx_i - 2fy_i = 0. \tag{2.31}$$

Solving (2.31) for y_i^* yields

$$y_i^* = \frac{c}{2f}x_i. \tag{2.32}$$

Equation (2.32) shows that y_i^* depends on the firm's own production of the private good, on the weight of the public good cost function f, and on the impact of the public good on the price. Therefore, the optimal level of y_i^* is independent of the production of the other firms and changes only if x_i^* changes. This follows from the fact that the cross derivative of the price is zero. We summarize this observation in Lemma 2.2.

Lemma 2.2 *A firm i's optimal public good investment y_i^* depends only on its private good production*

$$y_i^* = \frac{c}{2f} x_i.$$

Intuitively, a firm's public good investment increases the private good's price. The effect of y_i on p is constant and equal to c. Therefore, the marginal revenue of y_i is cx_i. The marginal costs are $2fy_i$. The marginal revenue has to be equal to the marginal costs in the optimum. We see that the relationship between x_i and y_i is linear and the constant slope depends on the weight of the public good's production cost f and on the impact of y_i on p.

In a next step we determine firm i's optimal private good supply. Solving (2.30) for x_i^* and using (2.32) and symmetry yields the optimal private good production of firm i.

$$x_i^* = \frac{A}{b(1+N) - \frac{c^2}{2f}N + 2d}. \tag{2.33}$$

After deriving the firms' optimal production and investment levels, we consider how these change due to market entry. Suppose that an additional firm enters the market that has access to the same technology as the incumbents. Hence, it can invest in the public good and it can produce the private good. The new firm's production of the private good and its investment in the public good leads to two effects. On the one hand, the price decreases because competition in the private good market gets tougher. This decreases the incentives to produce x_i. On the other hand, the entrant's investment in the public good increases the consumers' valuation of the private good. This yields an incentive to increase the production of the private good. Hence, it is not obvious whether an incumbent increases or decreases its production of the private good.

Proposition 2.6 *Suppose that the number of competing firms increases.*

- *If the weight of the public good's production costs is relatively high ($f > \frac{c^2}{2b}$), then each incumbent reduces its production x_i^* and investment y_i^*.*
- *If the weight of the public good's production costs is relatively low ($f < \frac{c^2}{2b}$), then each incumbent increases its production x_i^* and investment y_i^*.*

If $f = \frac{c^2}{2b}$, then each incumbent does not change its production x_i^* and investment y_i^*.

Proof.

To prove Proposition 2.6 we take the first derivative of x_i^* with respect to N.

$$\frac{\partial x_i^*}{\partial N} = -\frac{A}{\left(b\,(1+N) - \frac{1}{2}\frac{c^2}{f}N + 2d\right)^2}\left(b - \frac{1}{2}\frac{c^2}{f}\right)$$

We see that $\frac{\partial x_i^*}{\partial N}$ is negative if $f > \frac{c^2}{2b}$, is zero if $\frac{c^2}{2b}$, and is positive if $f < \frac{c^2}{2b}$. Furthermore, from Lemma 2.2 follows that y_i^* changes in the same direction as x_i^*. ■

If the weight of the public good cost function is small ($f < \frac{c^2}{2b}$) then, in equilibrium, the entrant invests so much in the public good that it makes up for its negative pecuniary externality. This leads to an increase in the price. The increase in the price yields a higher incentive to produce the private good. It directly follows that the firms increase their public good investment, too (Lemma 2.2). If the weight of the public good cost function is high ($f > \frac{c^2}{2b}$), then the opposite is true.

Let us now consider the total supply of the private good X^* and the quality of the public good Y^*. By taking the derivative with respect to N, we see that X^* and $Y*$ are increasing in N.

$$\frac{\partial X^*}{\partial N} = \frac{A * [b + 2d]}{[b(1+N) - \frac{1}{2}\frac{c^2}{f}N + 2d]^2} > 0 \tag{2.34}$$

$$\Rightarrow \frac{\partial Y^*}{\partial N} > 0 \text{ (Lemma 2.2)} \tag{2.35}$$

We see that the entrant's production is always high enough to make up for a possible decline in the incumbents' production and investment. Hence, if the number of firms increases, then the total production of the private good and the quality of the public good increases. However, even if the total supply of the private good always increases in N, the market price does not have to decrease. Hence, counter-intuitively, market entry can increase the price of the private good.

Proposition 2.7 *Suppose that the number of competing firms increases.*

- *If the weight of the public good's production costs is relatively high ($f > \frac{c^2}{2b}$), then the price p decreases.*
- *If the weight of the public good's production costs is relatively low ($f < \frac{c^2}{2b}$), then the price p increases.*

If $f = \frac{c^2}{2b}$, then the price does not change.

Proof. See Appendix. ■

Intuitively, we know that an increase in the private good production decreases the price and that a higher investment level in the public good increases the price. Hence, it depends on the dimension of the two effects whether the price increases or decreases. If the weight of the public good cost function is relatively low, then the firms invest so much in the public good that the price increases. If weight of the public good cost function is relatively high, then the opposite is true.

Let us now consider the firms' profits. We know that in a "normal" Cournot-Game, i.e. without a public good, incumbents dislike market entry because the entrant has a negative pecuniary externality. In contrast, in our setup the entrant has also a positive pecuniary externality by investing in the public good. Hence, it is not obvious whether the negative or positive effect dominates.

Proposition 2.8 *Suppose that the number of competing firms increases.*

- *If the weight of the public good's production costs is relatively high ($f > \frac{c^2}{2b}$), then the profits of the incumbents decrease.*
- *If the weight of the public good's production costs is relatively low ($f < \frac{c^2}{2b}$), then the profits of the incumbents increase.*

If $f = \frac{c^2}{2b}$, then the profits of the incumbents do not change.

Proof. See Appendix. ∎

We know from Proposition 2.6 that market entry reduces each firm's production and investment if the weight of the public good cost function is high ($f > \frac{c^2}{2b}$). Furthermore, the price decreases. Hence, it is obvious that the profits of the incumbents decrease. This is in line with the usual effect of tougher competition. If $f < \frac{c^2}{2b}$, we get the surprising result that the incumbents prefer more competition. This is due to the fact that the entrant does not only produce the private good, but also invests in the public good. Hence, the entrant has a positive and negative externality on the incumbents. If the weight of the public good cost function is small, the positive external effect dominates the negative one.

Usually higher market prices and higher profits are indices for a lower consumer surplus and a lower social welfare. Hence, one might think that welfare reacts ambiguously to market entry due to the fact that the price and the profits can increase with market entry. Nevertheless, we show that market entry unambiguously increases social welfare.

Proposition 2.9 *If the number of competing firms increases, then the social welfare increases.*

Proof. See Appendix. ∎

If we consider the consumer surplus, we see that it increases in N. This is due to two effects. Firstly, the quality of the public good increases if market entry occurs. This leads to a higher private good valuation, which has a positive effect on the consumer surplus. Secondly, market entry leads to tougher competition in the proprietary sector. This increases the supply and has again a positive effect on the consumer surplus. Thus, even if the market entry leads to higher prices, we get a higher consumer surplus. Proposition 2.8 shows that market entry increases the profits of the firms if the public good's production costs have a low weight. Hence, social welfare increases. If the firms' profits decrease with market entry, the gain in the consumer surplus dominates. Therefore, market entry is always welfare enhancing.

2.6 Conclusion

In this chapter we have studied the incentive to invest in a non-excludable public good that is complementary to a private good. We have shown that an increase of the government investment in the public good leads to ambiguous results. Firms may decrease their investment and a crowding out may occur. In this case the government investment is a strategic substitute to the investment of the firms. However, it is also possible that it is a strategic complement. Hence, firms may invest more in the public good if the government increases its public good investment. This leads to the following policy implication: If a government thinks about supporting Open Source Software by directly investing, then the concern that a crowding out occurs can be without any reason. Exactly the opposite can be true. The government investment can induce the firms to increase their investment in the public good and to increase their private good supply. Furthermore, we have considered market entry. We have shown that an entrant has positive and negative pecuniary externalities on the incumbents. Therefore, market entry can increase the profits of the incumbents.

Our analysis sheds new light on the incentives to license a proprietary product to horizontal competitors. Usually it is argued that licensing gives incumbents a commitment device to higher quality or, in network industries, to a bigger network by inducing competition. This results in an overall higher demand and offsets the loss in market power.[11] Another argument arises from our paper. A firm can use licensing to induce market entry. The incumbent anticipates that the entrant does not only produce the private good, but also invests in public goods like e.g. cooperative advertising. This public good investment can make up for the tougher competition in the proprietary sector. Hence, it may increase the incumbent's profit.

In this context, we would highlight the advertising campaign of Apple in the year 1981 as an example of such a "warm welcome" of competition. Apple Computer had responded to the entry of IBM in the PC Market with full-page newspaper advertisements, *"Welcome IBM. Seriously."*(See Figure 2.1). In these advertisements Apple claims that there will be a huge market as soon as the people understand the value of a PC. It seems that Apple thought that IBM will help them to convince people, *"We look forward to responsible competition in the massive effort to distribute this American technology to the world"*. This is in line with our argumentation if one interprets the meaning of "responsible competition" as helping to convince people,

[11] See e.g. Shepard (1987) and Economides (1997).

and not as selling only proprietary products. Hence, IBM takes part in the private provision of the non-excludable public good "knowledge" or "awareness".

Figure 2.1: Advertisement of Apple

2.7 Appendix

Proof of Proposition 2.1:

We proceed in two steps. Firstly, we prove existence. Secondly, we show uniqueness.

1. Existence:

Given $x_i \in [0; \overline{x}_i]$ and $y_i \in [0; \overline{y}_i]$, it follows that the strategy spaces are nonempty compact convex subsets of R^2. Furthermore, by assumption, the profit function is continuous. Given a strictly concave profit function, it follows directly that the profit functions satisfy the quasi-concavity criteria. It follows that there exists a Nash equilibrium in pure strategies (Debreu (1952)).

2. Uniqueness

To show uniqueness we apply the contraction mapping approach. Due to Bertsekas (1999) it is sufficient to show that the Hessian of the profit functions fulfills the "diagonal dominance" condition.

$$H = \begin{vmatrix} \frac{\partial^2\pi_1}{\partial x_1\partial x_1} & \frac{\partial^2\pi_1}{\partial x_1\partial y_1} & \frac{\partial^2\pi_1}{\partial x_1\partial x_2} & \frac{\partial^2\pi_1}{\partial x_1\partial y_2} & \cdots & \frac{\partial^2\pi_1}{\partial x_1\partial x_n} & \frac{\partial^2\pi_1}{\partial x_1\partial y_n} \\ \frac{\partial^2\pi_1}{\partial y_1\partial x_1} & \frac{\partial^2\pi_1}{\partial y_1\partial y_1} & \frac{\partial^2\pi_1}{\partial y_1\partial x_2} & \frac{\partial^2\pi_1}{\partial y_1\partial y_2} & \cdots & \frac{\partial^2\pi_1}{\partial y_1\partial x_n} & \frac{\partial^2\pi_1}{\partial y_1\partial y_n} \\ \frac{\partial^2\pi_2}{\partial x_2\partial x_1} & \frac{\partial^2\pi_2}{\partial x_2\partial y_1} & \frac{\partial^2\pi_2}{\partial x_2\partial x_2} & \frac{\partial^2\pi_2}{\partial x_2\partial y_2} & \cdots & \frac{\partial^2\pi_2}{\partial x_2\partial x_n} & \frac{\partial^2\pi_2}{\partial x_2\partial y_n} \\ \frac{\partial^2\pi_2}{\partial y_2\partial x_1} & \frac{\partial^2\pi_2}{\partial y_2\partial y_1} & \frac{\partial^2\pi_2}{\partial y_2\partial x_2} & \frac{\partial^2\pi_2}{\partial y_2\partial y_2} & \cdots & \frac{\partial^2\pi_2}{\partial y_2\partial x_n} & \frac{\partial^2\pi_2}{\partial y_2\partial y_n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \frac{\partial^2\pi_n}{\partial x_n\partial x_1} & \frac{\partial^2\pi_n}{\partial x_n\partial y_1} & \frac{\partial^2\pi_n}{\partial x_n\partial x_2} & \frac{\partial^2\pi_n}{\partial x_n\partial y_2} & \cdots & \frac{\partial^2\pi_n}{\partial x_n\partial x_n} & \frac{\partial^2\pi_n}{\partial x_n\partial y_n} \\ \frac{\partial^2\pi_n}{\partial y_n\partial x_1} & \frac{\partial^2\pi_n}{\partial y_n\partial y_1} & \frac{\partial^2\pi_n}{\partial y_n\partial x_2} & \frac{\partial^2\pi_n}{\partial y_n\partial y_2} & \cdots & \frac{\partial^2\pi_n}{\partial y_n\partial x_n} & \frac{\partial^2\pi_n}{\partial y_n\partial y_n} \end{vmatrix}$$

Therefore, the diagonal of the Hessian dominates the off-diagonal entries if

$$\sum_{j=1, i\neq j}^{N} |\frac{\partial^2\pi_i}{\partial x_i\partial x_j}| + \sum_{j=1}^{N} |\frac{\partial^2\pi_i}{\partial x_i\partial y_j}| < |\frac{\partial^2\pi_i}{\partial x_i\partial x_i}|, \forall i$$

and if

$$\sum_{j=1}^{N} |\frac{\partial^2\pi_i}{\partial y_i\partial x_j}| + \sum_{j=1, i\neq j}^{N} |\frac{\partial^2\pi_i}{\partial y_i\partial y_j}| < |\frac{\partial^2\pi_i}{\partial y_i\partial y_i}|, \forall i$$

Calculating the derivatives leads to

$$\sum_{j=1,i\neq j}^{N}|x_i\frac{\partial^2 p}{\partial x_i \partial x_j}+\frac{\partial p}{\partial x_j}|+\sum_{j=1}^{N}|x_i\frac{\partial^2 p}{\partial x_i \partial y_j}+\frac{\partial p}{\partial y_j}|<|\frac{\partial^2 p}{\partial x_i \partial x_i}x_i+2\frac{\partial p}{\partial x_i}-\frac{\partial^2 K^x(x_i)}{\partial x_i \partial x_i}|,\forall i$$

$$(N-1)|x_i\frac{\partial^2 p}{\partial x_i \partial x_j}+\frac{\partial p}{\partial x_j}|+N|x_i\frac{\partial^2 p}{\partial x_i \partial y_j}+\frac{\partial p}{\partial y_j}|<-(\frac{\partial^2 p}{\partial x_i \partial x_i}x_i+2*\frac{\partial p}{\partial x_i})+\frac{\partial^2 K^x(x_i)}{\partial x_i \partial x_i},\forall i$$

$$(N-1)|x_i\frac{\partial^2 p}{\partial x_i \partial x_j}+\frac{\partial p}{\partial x_j}|+N|x_i\frac{\partial^2 p}{\partial x_i \partial y_j}+\frac{\partial p}{\partial y_j}|+(\frac{\partial^2 p}{\partial x_i \partial x_i}x_i+2\frac{\partial p}{\partial x_i})<\frac{\partial^2 K^x(x_i)}{\partial x_i \partial x_i},\forall i$$

Calculating the derivatives leads to

$$\sum_{j=1,i\neq j}^{N}|x_i\frac{\partial^2 p}{\partial y_i \partial x_j}|+|x_i\frac{\partial^2 p}{\partial y_i \partial x_i}+\frac{\partial p}{\partial x_i}|+\sum_{j=1,i\neq j}^{N}|x_i\frac{\partial^2 p}{\partial y_i \partial y_i}|<|x_i\frac{\partial^2 p}{\partial y_i \partial y_i}-\frac{\partial^2 K_i^y(y_i)}{\partial y_i \partial y_i}|,\forall i$$

$$(N-1)|x_i\frac{\partial^2 p}{\partial y_i \partial x_j}|+|x_i\frac{\partial^2 p}{\partial y_i \partial x_i}+\frac{\partial p}{\partial x_i}|+(N-1)|x_i\frac{\partial^2 p}{\partial y_i \partial y_j}|<|x_i\frac{\partial^2 p}{\partial y_i \partial y_i}-\frac{\partial^2 K_i^y(y_i)}{\partial y_i \partial y_i}|,\forall i$$

$$(N-1)|x_i\frac{\partial^2 p}{\partial y_i \partial x_j}|+|x_i\frac{\partial^2 p}{\partial y_i \partial x_i}+\frac{\partial p}{\partial x_i}|+(1-N)x_i\frac{\partial^2 p}{\partial y_i \partial y_j}<-(x_i\frac{\partial^2 p}{\partial y_i \partial y_i})+\frac{\partial^2 K_i^y(y_i)}{\partial y_i \partial y_i},\forall i$$

$$(N-1)|x_i\frac{\partial^2 p}{\partial y_i \partial x_j}|+|x_i\frac{\partial^2 p}{\partial y_i \partial x_i}+\frac{\partial p}{\partial x_i}|+(2-N)x_i\frac{\partial^2 p}{\partial y_i \partial y_j}<\frac{\partial^2 K_i^y(y_i)}{\partial y_i \partial y_i},\forall i$$

■

Proof of Proposition 2.2

We proceed in two steps. Firstly, we prove existence. Secondly, we show a multiplicity of equilibria.

1. Existence

See first part of the proof of Proposition 2.1.

2. Multiplicity

We proceed in 4 steps:

Step 1:
Due to the first technical assumption in the Proposition 2.2, we have a contraction mapping of the best reply functions with respect to the private good. This leads to a unique Nash Equilibrium $x^* = (x_i^*, x_2^*, ..., x_N^*)$.

Step 2:
By the proof of existence, we know that there exists at least one Nash Equilibrium in pure strategies. Therefore, we can assume that

$$s^* = ((x_1^*, y_1^*), (x_2^*, y_2^*), ..., (x_j^*, y_j^*), (x_k^*, y_k^*), ..., (x_N^*, y_N^*))$$

is a Nash Equilibrium in pure strategies.

Step 3:
We define $y_j' = y_j + \mu$ and $y_k' = y_k - \mu$ with $\mu \neq 0$.

s^* implies that the FOCs of every firm $i \in \{1, ..., N\}$ has to be fulfilled at the values of s^*:

$$\frac{\partial \pi_i}{\partial x_i} |_{s^*} = \frac{\partial R_i}{\partial x_i} - \frac{\partial K^x}{\partial x_i} = 0$$

$$\frac{\partial \pi_i}{\partial y_i} |_{s^*} = \frac{\partial R_i}{\partial y_i} - \frac{\partial K_j^y}{\partial y_i} = 0$$

If this is the case, then these first order conditions are also fulfilled with the values $s' = ((x_1^*, y_1^*), (x_2^*, y_2^*), ..., (x_j^*, y_j'), (x_k^*, y_k'), ..., (x_N^*, y_N^*))$.

This is the case because $Y = \sum_{i=1}^N y_i$ does not change. The marginal costs of y_i are always constant and the same for all firms. Furthermore, the firms' marginal revenues do not change.

$$x_i \frac{\partial p}{\partial y_i} |_Y = const.$$

This leads to the conclusion that all first order conditions are fulfilled at s'.

Step 4:
Now one can go back to Step 3 and repeat the procedure with s' instead of s^*.

By repeating Step 3 and 4 it is obvious that there exist an infinite number of equilibria. ■

Proof of Proposition 2.3

Firstly, we prove that the described equilibrium is indeed a Nash Equilibrium. Secondly, we show that this is the unique Nash Equilibrium by ruling out all other possible equilibria.

1. Is $(x_i^* = x_{-i}^*, y_i = Y^*, y_{-i}^* = 0)$ an equilibrium?

The FOC of the firms with respect to the public good is

$$\frac{\partial \pi_j}{\partial y_j} = x_j^* \frac{\partial p}{\partial y_j} - \frac{\partial K_j^y}{\partial y_j} = 0, j \in \{i, -i\}$$

Given $x_i^* = x_{-i}^*$, it follows that $x_i^* \frac{\partial p}{\partial y_i} = x_{-i}^* \frac{\partial p}{\partial y_{-i}}$. Hence, if

$$x_i^* \frac{\partial p}{\partial y_i} \mid_{y_i = Y^*} = \frac{\partial K_i^y}{\partial y_i},$$

then

$$x_{-i}^* \frac{\partial p}{\partial y_{-i}} \mid_{y_i = Y^*} < \frac{\partial K_{-i}^y}{\partial y_{-i}}.$$

Therefore, $(x_i^* = x_{-i}^*, y_i = Y^*, y_{-i}^* = 0)$ is an equilibrium.

Next, we rule out all other possible equilibria to show uniqueness. Firstly, we show that y_i and y_{-i} are perfect strategic substitutes. The total derivative of

$$\frac{\partial \pi_i}{\partial y_i} = x_i \frac{\partial p}{\partial y_i} - \frac{\partial K_i^y}{\partial y_i} = 0$$

is

$$[x_i \frac{\partial^2 p}{\partial y_i^2} - \frac{\partial^2 K_i^y}{\partial y_i^2}] dy_i + [x_i \frac{\partial^2 p}{\partial y_i \partial y_{-i}} - \frac{\partial^2 K_i^y}{\partial y_i \partial y_{-i}}] dy_{-i} = 0.$$

Due to $\frac{\partial^2 K_i^y}{\partial y_i^2} = \frac{\partial^2 K_i^y}{\partial y_i \partial y_{-i}} = 0$ it is true that

$$dy_i = -dy_{-i}.$$

Secondly, we solve the game by iterated deletion of strictly dominated strategies. We denote by $y_i^* \mid_{y_{-i}=0}$ the optimal response of firm i to $y_{-i} = 0$. Because of $\frac{\partial K_i^y}{\partial y_i} < \frac{\partial K_{-i}^y}{\partial y_{-i}}$, it follows that $y_{-i}^* \mid_{y_i=0} < y_i^* \mid_{y_{-i}=0}$.
Hence, firm i knows that firm $-i$ never chooses a higher y_{-i} than $y_{-i}^* \mid_{y_i=0} = y_{-i}^{max}$.
Given this, firm i should at least choose $y_i^{min} = y_i^* \mid_{y_{-i}=0} - y_{-i}^{max}$.
Hence, firm $-i$ knows that firm i never invests less than y_i^{min}.
Given this, firm $-i$ should never choose a higher y_{-i} than $y_{-i}^{max'} = y_{-i}^* \mid_{y_i=y_i^{min}}$.
Hence, firm i knows that firm $-i$ never chooses a higher y_{-i} than $y_{-i}^{max'}$.
Given this, the smallest y_i that firm i should choose is $y_i^{min'} = y_i^* \mid_{y_{-i}=0} - y_{-i}^{max'}$.
Continuing, one sees that in the limit y_i^{min} converges to $y_i^*|_{y_{-i}=0}$ and y_{-i}^{max} converges to 0. Hence, $\{(x_1^*, y_1^*|_{y_2=0}), (x_2^*, y_2^* = 0)\}$ with $y_1^*|_{y_2=0} > y_2^*|_{y_1=0}$ is the unique Nash Equilibrium.

■

Proof of Proposition 2.4

For the proof we proceed in two steps. Firstly, we show that $\frac{\partial x_i}{\partial y_G} > 0$. Secondly, we prove that $\frac{\partial y_i}{\partial y_G} > 0$.

1. Using the implicit function theorem combined with symmetry yields

$$\frac{\partial x_i}{\partial y_G} = \frac{\partial x_1}{\partial y_G} = \frac{|D_x|}{|D|}$$

with

$$|D_x| =$$

$$\begin{vmatrix}
\partial f^1/\partial y_G & \partial f^1/\partial y_1 & \partial f^1/\partial x_2 & \partial f^1/\partial y_2 & \dots & \dots & \partial f^1/\partial x_{N-1} & \partial f^1/\partial y_{N-1} & \partial f^1/\partial x_N & \partial f^1/\partial y_N \\
\partial g^1/\partial y_G & \partial g^1/\partial y_1 & \partial g^1/\partial x_2 & \partial g^1/\partial y_2 & \dots & \dots & \partial g^1/\partial x_{N-1} & \partial g^1/\partial y_{N-1} & \partial g^1/\partial x_N & \partial g^1/\partial y_N \\
\partial f^2/\partial y_G & \partial f^2/\partial y_1 & \partial f^2/\partial x_2 & \partial f^2/\partial y_2 & \dots & \dots & \partial f^2/\partial x_{N-1} & \partial f^2/\partial y_{N-1} & \partial f^2/\partial x_N & \partial f^2/\partial y_N \\
\partial g^2/\partial y_G & \partial g^2/\partial y_1 & \partial g^2/\partial x_2 & \partial g^2/\partial y_2 & \dots & \dots & \partial g^2/\partial x_{N-1} & \partial f^4/\partial y_{N-1} & \partial g^2/\partial x_N & \partial g^2/\partial y_N \\
. & . & . & . & . & . & . & . & . & . \\
\partial f^N/\partial y_G & \partial f^N/\partial y_1 & \partial f^N/\partial x_2 & \partial f^N/\partial y_2 & \dots & \dots & \partial f^N/\partial x_{N-1} & \partial f^N/\partial y_{N-1} & \partial f^N/\partial x_N & \partial f^N/\partial y_N \\
\partial g^N/\partial y_G & \partial g^N/\partial y_1 & \partial g^N/\partial x_2 & \partial g^N/\partial y_2 & \dots & \dots & \partial g^N/\partial x_{N-1} & \partial g^N/\partial y_{N-1} & \partial g^N/\partial x_N & \partial g^N/\partial y_N
\end{vmatrix}$$

and

$$|D| =$$

$$\begin{vmatrix}
\partial f^1/\partial x_1 & \partial f^1/\partial y_1 & \partial f^1/\partial x_2 & \partial f^1/\partial y_2 & \dots & \dots & \partial f^1/\partial x_{N-1} & \partial f^1/\partial y_{N-1} & \partial f^1/\partial x_N & \partial f^1/\partial y_N \\
\partial g^1/\partial x_1 & \partial g^1/\partial y_1 & \partial g^1/\partial x_2 & \partial g^1/\partial y_2 & \dots & \dots & \partial g^1/\partial x_{N-1} & \partial g^1/\partial y_{N-1} & \partial g^1/\partial x_N & \partial g^1/\partial y_N \\
\partial f^2/\partial x_1 & \partial f^2/\partial y_1 & \partial f^2/\partial x_2 & \partial f^2/\partial y_2 & \dots & \dots & \partial f^2/\partial x_{N-1} & \partial f^2/\partial y_{N-1} & \partial f^2/\partial x_N & \partial f^2/\partial y_N \\
\partial g^2/\partial x_1 & \partial g^2/\partial y_1 & \partial g^2/\partial x_2 & \partial g^2/\partial y_2 & \dots & \dots & \partial g^2/\partial x_{N-1} & \partial f^4/\partial y_{N-1} & \partial g^2/\partial x_N & \partial g^2/\partial y_N \\
. & . & . & . & . & . & . & . & . & . \\
\partial f^N/\partial x_1 & \partial f^N/\partial y_1 & \partial f^N/\partial x_2 & \partial f^N/\partial y_2 & \dots & \dots & \partial f^N/\partial x_{N-1} & \partial f^N/\partial y_{N-1} & \partial f^N/\partial x_N & \partial f^N/\partial y_N \\
\partial g^N/\partial x_1 & \partial g^N/\partial y_1 & \partial g^N/\partial x_2 & \partial g^N/\partial y_2 & \dots & \dots & \partial g^N/\partial x_{N-1} & \partial g^N/\partial y_{N-1} & \partial g^N/\partial x_N & \partial g^N/\partial y_N
\end{vmatrix}$$

where $f^i = \frac{\partial \pi_i}{\partial x_i}$ for $i \in \{1, 2, 3, ..., N\}$ and $g^i = \frac{\partial \pi_i}{\partial y_i}$ for $i \in \{1, 2, 3, ..., N\}$.

For the proof that

$$\frac{\partial x_i}{\partial y_G} = \frac{|D_x|}{|D|} > 0$$

we show that the numerator and denominator are positive.

1.1 $|D_x| > 0$

Using the conditions from Proposition 2.4 we can substitute:

$$-\frac{\partial f^i}{\partial y_G} = -a \ \forall i \in \{1, 2, 3, ..., N\}$$

$$\frac{\partial f^i}{\partial y_w} = a \ \forall i \in \{1, 2, 3, ..., N\}, \forall w \in \{1, 2, 3, ..., N\}$$

$$\frac{\partial f^i}{\partial x_z} = c \ \forall i \in \{1, 2, 3, ..., N\}, \forall z \in \{1, 2, 3, ..., N\} \setminus i$$

$$\frac{\partial f^i}{\partial x_i} = e \ \forall i \in \{1, 2, 3, ..., N\}$$

$$\frac{\partial g^i}{\partial y_G} = 0 \ \forall i \in \{1, 2, 3..., N\}$$

$$\frac{\partial g^i}{\partial y_i} = b \ \forall z \in \{1, 2, 3, ..., N\}$$

$$\frac{\partial g^i}{\partial x_z} = d \; \forall i \in \{1, 2, 3, ..., N\}, \forall z \in \{1, 2, 3, ..., N\}$$

$$\begin{vmatrix}
-a & a & c & a & c & a & ... & ... & c & a & c & a \\
0 & b & d & 0 & d & 0 & ... & ... & d & 0 & d & 0 \\
-a & a & e & a & c & a & ... & ... & c & a & c & a \\
0 & 0 & a & b & d & 0 & ... & ... & d & 0 & d & 0 \\
-a & a & c & a & e & a & ... & ... & c & a & c & a \\
0 & 0 & d & 0 & a & b & ... & ... & d & 0 & d & 0 \\
. & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . \\
-a & a & c & a & c & a & ... & ... & e & a & c & a \\
0 & 0 & d & 0 & d & 0 & ... & ... & a & b & d & 0 \\
-a & a & c & a & c & a & ... & ... & c & a & e & a \\
0 & 0 & d & 0 & d & 0 & ... & ... & d & 0 & a & b
\end{vmatrix}$$

By subtracting line 1 from all lines z with $z \in \{3, 5, 7, ..., N-1\}$ we derive

$$\begin{vmatrix}
-a & a & c & a & c & a & ... & ... & c & a & c & a \\
0 & b & d & 0 & d & 0 & ... & ... & d & 0 & d & 0 \\
0 & 0 & e-c & 0 & 0 & 0 & ... & ... & 0 & 0 & 0 & 0 \\
0 & 0 & a & b & d & 0 & ... & ... & d & 0 & d & 0 \\
0 & 0 & 0 & 0 & e-c & 0 & ... & ... & 0 & 0 & 0 & 0 \\
0 & 0 & d & 0 & a & b & ... & ... & d & 0 & d & 0 \\
. & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . \\
0 & 0 & 0 & 0 & 0 & 0 & ... & ... & e-c & 0 & 0 & 0 \\
0 & 0 & d & 0 & d & 0 & ... & ... & a & b & d & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & ... & ... & 0 & 0 & e-c & 0 \\
0 & 0 & d & 0 & d & 0 & ... & ... & d & 0 & a & b
\end{vmatrix}$$

One sees that there exists a line z ($z \in \{3, 5, 7, ...N\}$) where only the element in the z row is not zero. Therefore, we use theses lines to derive a upper triangle matrix:

$$
\begin{vmatrix}
-a & a & c & a & c & a & ... & ... & c & a & c & a \\
0 & b & 0 & 0 & d & 0 & ... & ... & d & 0 & d & 0 \\
0 & 0 & e-c & 0 & 0 & 0 & ... & ... & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & b & d & 0 & ... & ... & d & 0 & d & 0 \\
0 & 0 & 0 & 0 & e-c & 0 & ... & ... & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & b & ... & ... & d & 0 & d & 0 \\
. & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . \\
0 & 0 & 0 & 0 & 0 & 0 & ... & ... & e-c & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & ... & ... & 0 & b & d & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & ... & ... & 0 & 0 & e-c & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & ... & ... & 0 & 0 & 0 & b
\end{vmatrix}
$$

Hence, the Determinant is

$$|D_x| = -ab^N(e-c)^{N-1}$$

Substituting back gives

if N even:

$$|D_x| = -\overbrace{[\frac{\partial p}{\partial y_i} + x_i \frac{\partial^2 p}{\partial y_i \partial x_i}]}^{+}\overbrace{[x_i \frac{\partial^2 p}{\partial y_i^2} - \frac{\partial^2 K_i(y_i)}{\partial y_i^2}]^N}^{+}\overbrace{[\frac{\partial p}{\partial x_i} - \frac{\partial K_i(x_i)}{\partial x_i^2}]^{N-1}}^{-} > 0$$

if N uneven:

$$|D_x| = -\overbrace{[\frac{\partial p}{\partial y} + x_i \frac{\partial^2 p}{\partial y_i \partial x_i}]}^{+}\overbrace{[x_i \frac{\partial^2 p}{\partial y_i^2} - \frac{\partial^2 K_i(y_i)}{\partial y_i^2}]^N}^{-}\overbrace{[\frac{\partial p}{\partial x} - \frac{\partial K_i(x_i)}{\partial x_i^2}]^{N-1}}^{+} > 0$$

Therefore if follows $|D_x| > 0 \; \forall \; N$.

1.2 $|D| > 0$

$$|D| =$$

$$
\begin{vmatrix}
\partial f^1/\partial x_1 & \partial f^1/\partial y_1 & \partial f^1/\partial x_2 & \partial f^1/\partial y_2 & ... & ... & \partial f^1/\partial x_{N-1} & \partial f^1/\partial y_{N-1} & \partial f^1/\partial x_N & \partial f^1/\partial y_N \\
\partial f^2/\partial x_1 & \partial f^2/\partial y_1 & \partial f^2/\partial x_2 & \partial f^2/\partial y_2 & ... & ... & \partial f^2/\partial x_{N-1} & \partial f^2/\partial y_{N-1} & \partial f^2/\partial x_N & \partial f^2/\partial y_N \\
\partial f^3/\partial x_1 & \partial f^3/\partial y_1 & \partial f^3/\partial x_2 & \partial f^3/\partial y_2 & ... & ... & \partial f^3/\partial x_{N-1} & \partial f^3/\partial y_{N-1} & \partial f^3/\partial x_N & \partial f^3/\partial y_N \\
\partial f^4/\partial x_1 & \partial f^4/\partial y_1 & \partial f^4/\partial x_2 & \partial f^4/\partial y_2 & ... & ... & \partial f^4/\partial x_{N-1} & \partial f^4/\partial y_{N-1} & \partial f^4/\partial x_N & \partial f^4/\partial y_N \\
. & . & . & . & . & . & . & . & . & . \\
\partial f^N/\partial x_1 & \partial f^N/\partial y_1 & \partial f^N/\partial x_2 & \partial f^N/\partial y_2 & ... & ... & \partial f^N/\partial x_{N-1} & \partial f^N/\partial y_{N-1} & \partial f^N/\partial x_N & \partial f^N/\partial y_N
\end{vmatrix}
$$

We know that this matrix is diagonally dominant. Furthermore, if a matrix is diagonal dominant and its diagonal elements are positive, then the matrix is positive definite. On the diagonal, there are the second-order conditions that are all negative. Through multiplying every line by (-1) one gets positive diagonal elements and through the odd number of lines a scalar of $+1$. This gives a positive definite matrix. Therefore, the determinant has to be positive.

2.

Total derivative of $\frac{\partial \pi_i}{\partial y_i}$:

$$\overbrace{[\frac{\partial p}{\partial y_i} + x_i \frac{\partial^2 p}{\partial x_i \partial y_i}]dx_i}^{\text{indirect effect}} + [x_i \frac{\partial^2 p}{\partial y_i^2} - \frac{\partial^2 K_i(y_i)}{\partial y_i^2}]dy_i$$
$$\overbrace{+[x_i \frac{\partial^2 p}{\partial y_i \partial X_{-i}}]dX_{-i} + [x_i \frac{\partial^2 p}{\partial y_i \partial Y_{-i}}]dY_{-i}}^{\text{direct effects}} = 0$$

One sees that the direct effect of dY_{-i} is zero. The direct effect of dX_{-i} is non-negative. The indirect effect of dx_i is positive. Therefore, y_i has to increase to ensure that $\frac{\partial \pi_i}{\partial y_i}$ remains zero.

■

Proof of Lemma 2.1

The existence of a Nash-Equilibrium in pure strategies follows immediately from Proposition 2.1. For the uniqueness we apply the contraction mapping principle. Beforehand, we reduce the strategy space form R^2 to R because y_i is directly determined through x_i. To see this we write down the FOC with respect to the public good

$$\frac{\partial \pi_i}{\partial y_i} = cx_i - 2fy_i = 0.$$

Rewriting leads to

$$y_i = \frac{c}{2f} x_i.$$

Plugging back into the profit function of firm i yields

$$\pi_i = x_i(A - bx_i - b\sum_{j=1;i\neq j}^{N} x_j + c\frac{c}{2f}x_i + c\sum_{j=1;i\neq j}^{N}\frac{c}{2f}x_j) - dx_i^2 - f(\frac{c}{2f}x_i)^2.$$

The first order conditions of the N firms with respect to the private good are a contraction mapping if

$$\sum_{j=1;i\neq j}^{N} \mid \frac{\partial^2\pi_i}{\partial x_i \partial x_j} \mid < \mid \frac{\partial^2 \pi_i}{\partial x_i^2} \mid \;\; \forall i \in \{1, ..., N\},$$

$$(N-1) \mid -b + \frac{c^2}{2f} \mid < \mid -2b - 2d + \frac{c^2}{2f} \mid .$$

If $f \geq \frac{c^2}{2b}$, then $-b + \frac{c^2}{2f} \leq 0$. Therefore:

$$(1-N)(-b + \frac{c^2}{2f}) < 2b + 2d - \frac{c^2}{2f}$$

$$N < \frac{3b + 2d - \frac{c^2}{f}}{b - \frac{c^2}{2f}}$$

If $f < \frac{c^2}{2b}$, then $-b + \frac{c^2}{2f} > 0$. Therefore:

$$(N-1)(-b + \frac{c^2}{2f}) < 2 + 2d - \frac{1}{2f}$$

$$N < \frac{b + 2d}{\frac{c^2}{2f} - b}$$

■

Proof of Proposition 2.7

For the proof we use the fact that $y_i = \frac{c}{2f}x_i$ (2.32) and rewrite the demand function (2.25) as follows:

$$p = A - bX + cY = A - bX(N) + c\frac{c}{2f}X(N) = A + X(N)(-b + \frac{c^2}{2f})$$

$$p = A + N * \frac{A}{b(1+N) - \frac{1}{2}\frac{c^2}{f}N + 2d}(-b + \frac{c^2}{2f})$$

Now we can take the first derivative of p with respect to N yields

$$\frac{\partial p}{\partial N} = (\,-b + \frac{c^2}{2f}) * \frac{A(b+2d)}{[b(1+N) - \frac{1}{2}\frac{c^2}{f}N + 2d]^2}.$$

Therefore:

- $\frac{\partial p}{\partial N} < 0$ if $f > \frac{c^2}{2b}$
- $\frac{\partial p}{\partial N} = 0$ if $f = \frac{c^2}{2b} \rightarrow \; p = A$
- $\frac{\partial p}{\partial N} > 0$ if $f < \frac{c^2}{2b}$

■

Proof of Proposition 2.8

$$\pi_i = p * x_i - dx^2 - f[\frac{1}{2f}x_i]^2 = p * x_i - x_i^2(d + \frac{1}{4f})$$

$$\pi_i = \frac{A^2(b + d - \frac{c^2}{4f})}{(b\,(1+N) - \frac{c^2}{2f}N + 2d)^2}$$

$$\frac{\partial \pi_i}{\partial N} = \frac{A^2(b - \frac{c^2}{2f})(-2b - 2d + \frac{c^2}{2f})}{\Big(b\,(1+N) - \frac{c^2}{2f}N + 2d\Big)^3}$$

The last term $(-2b - 2d + \frac{c^2}{2f})$ is always negative due to the second order conditions. Therefore, we have to look at $b - \frac{c^2}{2f}$ and at the denominator.

Case 1: $f = \frac{c^2}{2b}$
Then $b - \frac{c^2}{2f} = 0$ and therefore $\frac{\partial \pi}{\partial N} = 0$.

Case 2: $f > \frac{c^2}{2b}$
Then $b - \frac{c^2}{2f} > 0$ and the sign of $\frac{\partial \pi}{\partial N}$ depends on N due to $b\,(1+N) - \frac{c^2}{2f}N + 2d$. This term is always positive

$$b\,(1+N) - \frac{c^2}{2f}N + 2d = b + N(b - \frac{c^2}{2f}) + 2d.$$

Therefore, $\frac{\partial \pi}{\partial N} < 0$ for all $N > 0$.

Case 3: $f < \frac{c^2}{2b}$
Then $b - \frac{c^2}{2f} < 0$ and the sign of $\frac{\partial \pi}{\partial N}$ depends on N due to the term $b\,(1+N) - \frac{c^2}{2f}N + 2d$.
This term is zero if

$$N = \frac{2d+b}{\frac{c^2}{2f} - b} > 0.$$

The slope of $b\,(1+N) - \frac{c^2}{2f}N + 2d$ with respect to N is

$$\frac{\partial(b\,(1+N) - \frac{c^2}{2f}N + 2d)}{\partial N} = b - \frac{c^2}{2f} < 0.$$

Therefore, $\frac{\partial \pi}{\partial N} > 0$ in the relevant area where $N < \frac{2d+b}{\frac{c^2}{2f}-b}$ (Lemma 2.1).

■

Proof of Proposition 2.9

Firstly, we calculate the consumer surplus

$$CS = (A + cY - p)X0.5 = 0.5bX^2.$$

The total surplus is the sum of the firms' profits and the consumers' surplus

$$TS = N * \pi_i + CS,$$

$$\frac{\partial TS}{\partial N} = \frac{\partial(N * \pi_i)}{\partial N} + \frac{\partial CS}{\partial N},$$

$$\frac{dTS}{dN} = A^2 \frac{bN(d + \frac{c^2}{4f}) + (b + d - \frac{c^2}{4f})(b + .5\frac{c^2}{f}N + 2d)}{(b\,(1+N) - .5\frac{c^2}{f}N + 2d)^3} > 0.$$

■

Chapter 3

Two-Sided Markets with Pecuniary and Participation Externalities

3.1 Introduction

A series of recent papers have looked at competition and regulation in the television and radio broadcasting market by using "two-sided market" models. In these setups platforms, which are broadcasting channels, match viewers and advertisers. Viewers dislike advertisements and competition has the special feature that each platform has a "participation externality" on its competitors. If a platform changes its advertising level, then it influences its own and the competitors' number of viewers. This is an externality that works via the viewer market.

In the real world, however, platforms do not only experience participation externalities, but also pecuniary externalities. In contrast to participation externalities, which work via the viewer market, pecuniary externalities work via the advertising market. Hence, we will take into account that a change in a channel's advertising level does not only affect the distribution of the viewers between the channels, but it also changes the broadcasters' total supply in the advertising market. When the broadcasters' total supply changes, then the market price for advertising has to adjust in order to clear the market. This price adjustment changes the revenue of all platforms and yields a pecuniary externality. So far, this pecuniary externality has been neglected in the existing literature. In this chapter we show that pecuniary externalities are important because they influence the theoretical predictions substantially.

In the following we illustrate the existence of pecuniary externalities with two examples. In Germany, free available public broadcasting channels are competing with free available private broadcasting channels. The public broadcasting channels are financed through fees and are not allowed to offer advertising after 8 pm. The private channels are allowed to offer advertising after 8 pm. The public broadcasting channels are currently running a deficit. Hence, there is a debate in Germany: Should one increase the fee or should one allow the public broadcasting channels to offer advertising after 8 pm in order to balance their budget? The position of the private channels is: Do not allow them to offer advertisements after 8 pm. Instead, increase the fee or force them to reduce their expenditures! [1] This statement is surprising. Given our existing knowledge of competition in such a two-sided market, advertising after 8 pm decreases the attractiveness of the public channels. The resulting participation externality should be in the interest of the private channels. Therefore, private channels should like the idea of advertising on public channels. However, the neglected aspect here is that private channels fear that the additional time for commercials decreases the price for advertisements. Due to this pecuniary externality, they dislike the idea of advertisements on public channels.

The other example is taken from the US television market. In the 70s the Department of Justice (DOJ) alleged that the "Code of Conduct" of the National Association of Broadcasters (NAB), which regulated the competition between the broadcasting channels, violated antitrust laws.[2] In particular the "Code of Conduct" included

(i) a limitation of the advertising time on each channel to 9.5 minutes per hour in prime time and sixteen minutes per hour at all the other times

(ii) a limitation of the number of commercials per hour

(iii) a limitation of the number of advertised products in one hour.

The DOJ argued that these rules had the purpose and effect of manipulating the supply of commercial television time with the result that the price for advertisements had been raised. This would violate the Section 1 of the Sherman Act. As a result of

[1] Press release "Verband Privater Rundfunkt und Telekommunikation (VPRT), 26.9.2003"

[2] Campell (1999)

the allegation and subsequent legal judicial proceedings, the NAB voluntarily abandoned its "Code of Conduct" in the year 1983. From an economic point of view, these advertising ceilings cannot be explained without pecuniary externalities. In the existing models, like Anderson and Coate (2005), collusion between the channels results in an agreement that every channel has to offer a minimum level of advertising because the equilibrium advertising level is below the the collusive advertising level. Hence, collusion would not lead to a "Code of Conduct" that determines a maximum advertising level.

In order to analyze a two-sided market with both externalities, we build a model where platforms are symmetrically located on a Salop circle. They simultaneously decide how much time they offer to advertisers on their channels. The consumers are uniformly distributed on the circle, dislike advertisements, and have to choose exactly one platform. Advertisers want to advertise their products. Therefore, they have a certain willingness to pay for "viewer-time" units. In our model the crucial ingredient is that the advertisers' aggregated demand function for viewer-time units is decreasing. We show that one can derive such a property from two kinds of microfoundations. One way is to assume that advertisers have a convex cost function for producing the advertised good. A second way is to assume word-of-mouth advertising.[3]

In this chapter we show that advertising can either have the property of a strategic substitute or of a strategic complement. This is in contrast to the existing literature like Anderson and Coate (2005). By using linear demand functions, we are able to solve the model analytically. We show that advertising exhibits the property of a strategic complement (substitute) in the market equilibrium if the differentiation between the platforms is low (high). In addition, we show that market entry can lead to more or less advertising on each channel. This is also in contrast to the existing literature. For example Choi (2003) shows that market entry unambiguously decreases the advertising level. Furthermore, we get the surprising result that market entry can increase the incumbents' profits. This is the case if the equilibrium advertising level is above the per viewer revenue maximizing advertising level. Due to the pecuniary externalities, market entry can shift the equilibrium advertising level in the direction of the per viewer revenue maximizing advertising level. This increases the profits per viewer. If this increase of the profit is higher than the decrease of the profit through the loss of viewers, then the incumbents are better

[3] In the Appendix we consider a third microfoundation, namely switching viewers.

off. Moreover, we show that entry has ambiguous effects from a welfare point of view. A social planer has to consider the following trade-off. On the one hand, more advertising increases the surplus of the advertisers. On the other hand, more advertising decreases viewers' utility. We show that perfect competition can drive the equilibrium advertising level below the socially optimal advertising level. Therefore, perfect competition may not be desirable even if the sum of viewers' transportation costs decrease with market entry.

This chapter is related to the literature on two-sided markets. The basic literature describes the interaction between two groups that are mediated by a platform (see e.g. Armstrong (2005), Cauillad and Jullien (2003), Nocke et.al. (2004), Rochet and Tirole (2003, 2004)). The media market, and particularly the broadcasting market, is a subclass of such a two-sided market because it has a special feature. One side, namely advertisers, likes the interaction with the second side, namely viewers. At the same time viewers dislike the interaction with advertisers.[4]

Several papers have addressed this peculiarity of the broadcasting market. Usually the advertising market is modelled in such a way that advertisers have no bargaining power and take the decisions (advertising prices or quantities) of the channels as given[5] and viewers dislike advertising.[6] Papers that consider price competition are Reisinger (2004), Nilssen and Sorgard (2001) and Kind et.al. (2005). Given the structure of the US TV advertising market, we argue that the assumption of quantity competition seems to better fit reality. Papers that consider quantity competition are e.g. Anderson and Coate (2005), Anderson (2005), Gabszewicz et.al. (2004), Crampes et.al. (2004), Choi (2003), Peitz and Valetti (2004), and Kohlschein (2004). All papers that consider quantity competition have a similar microfoundation of advertising, which we describe in Section 3.3. Hereafter, we refer to these papers as the existing literature. Anderson and Coate (2005) prove that there could be too much or too little advertising compared to the social optimum. Anderson (2005) considers how advertising ceilings influence the quality decisions of the broadcasting channels. Choi (2003) looks at the endogenous number of broadcasting channels. Kohlschein (2004) considers competition between public and private channels. Gabszewicz et.al.

[4]By contrast, there is often the assumption that viewers like advertising in models concerned with the press industry (see e.g. Häckner and Nyberg (2000)).

[5]One exception is the work of Gal-Or and Dukes (2003, 2006) where platforms and advertisers bargain.

[6]One paper that differs is Cunningham and Alexander (2004). They use a completely different modelling approach and incorporate competition in the advertising market, but do not explicitly consider competition for the viewers.

(2004), like Peitz and Valetti (2004), consider the location decision of the channels. Crampes et.al. (2004) allow for (dis)economies of scale of advertising. By including pecuniary externalities between the platforms, we extend the framework that is used in the existing literature.

We will proceed as follows. In the next section we set up the model. In Section 3 we provide different microfoundations for pecuniary externalities. In Section 4 we solve the model and derive the symmetric market equilibrium. In Section 5 we determine whether advertising exhibits the property of a strategic substitute or complement. In Section 6 we consider the effects of market entry on platforms' profits and on welfare. Section 7 concludes.

3.2 The Model

There are three kinds of agents in our model: viewers, broadcasting channels and advertisers.

Viewers of a broadcasting channel, who are potential consumers of the advertisers, can watch one channel at a certain point in time. In the two-sided market framework this means that they singlehome. The viewers are heterogeneous with respect to their preferences of watching a certain channel. Hence, we assume that they are uniformly distributed on a Salop circle. We denote by x the location of a viewer on this circle. The mass of consumers is normalized to 1. This leads to the following utility of a viewer that watches channel i and is located at 0:

$$U(x) = B - w_i - tx. \tag{3.1}$$

We denote by w_i the advertising level of channel i. Due to the fact that advertising is annoying, w_i enters the utility function with a negative sign. Furthermore, we denote by tx the viewer's disutility from not watching exactly his preferred programm. B is a constant which ensures that the utility is always positive. We normalize the outside option to $\underline{U} = 0$ and hence the market is always covered. Furthermore, we assume that the N broadcasting channels are located equidistantly on the Salop circle. Therefore, the marginal consumer between channel i and j for

$w_j - \frac{t}{N} \leq w_i \leq w_j + \frac{t}{N}$ is

$$B - w_i - tx = B - w_j - t(\frac{1}{N} - x). \tag{3.2}$$

The marginal consumer between channel i and l for $w_l - \frac{t}{N} \leq w_i \leq w_l + \frac{t}{N}$ is

$$B - w_i - tx = B - w_l - t(\frac{1}{N} - x). \tag{3.3}$$

Hence, channel i faces the following demand function

$$D_i(w_i, w_j, w_l) = 2x = \frac{1}{N} + \frac{w_j - w_i}{2t} + \frac{w_l - w_i}{2t} \tag{3.4}$$

for $w_l - \frac{t}{N} \leq w_i \leq w_l + \frac{t}{N}$ and $w_j - \frac{t}{N} \leq w_i \leq w_j + \frac{t}{N}$.

Using symmetry between channels j and l yields

$$D_i = \frac{1}{N} + \frac{w_l - w_i}{t} \quad \text{for } w_l - \frac{t}{N} \leq w_i \leq w_l + \frac{t}{N}. \tag{3.5}$$

Broadcasting channels try to attract viewers in order to sell time of these viewers to the advertisers. Therefore, the profit function of a channel i is

$$\pi_i = D_i w_i p \ . \tag{3.6}$$

By p we denote the "viewer-time" unit price. If one multiplies the number of viewers of a channel ($= D_i$) by the advertising time ($= w_i$), then this yields the channel i's supply of viewer-time units. This supply multiplied by the price per viewer-time unit yields the channel i's profit. We assume zero marginal costs for serving a viewer. This is due to the public good nature of broadcasting. Furthermore, we abstract from fixed costs. The channels choose w_i in order to maximize their profits, taking as given the other channels advertising levels.

The advertisers advertise in order to inform viewers about their products. Therefore, they have a willingness to pay for viewer-time units in order to reach consumers. We assume that viewer-time units are a homogenous good, which we justify with our microfoundation. Furthermore, we assume that the advertisers' inverse demand function for viewer-time units is decreasing in the total supply of viewer-time units

$$p = p(w_1 D_1, w_2 D_2, ..., w_N D_N), \tag{3.7}$$

$$\frac{\partial p}{\partial w_i D_i} < 0 \ , \forall i \in \{1, 2, ..., N\}. \tag{3.8}$$

We are able to solve the model analytically with the following linear demand function, which we will use throughout the chapter.

$$p = A - \sum_{k=1}^{N} w_k D_k. \tag{3.9}$$

To model the television market, we choose the following time structure. Firstly, the channels choose simultaneously their advertising levels w_i.[7] Secondly, the viewers decide which channel to watch. Thirdly, a price p, which is determined by a walrasian auctioneer, clears the market in such a way that the supply of viewer-time units is equal to the advertisers' demand for viewer-time units.

We think that this is the appropriate time structure to model the television market. Goettler (1998) describes how the television advertising market works in the US. The broadcasting channels present their programm schedule in the so called "Upfront Market", which is in May for the upcoming season in September. 70% to 80% of the advertising time is sold during this upfront market at a market clearing price.[8] The rest is sold in the "scatter market" some weeks before the advertising slot is aired or is used to promote the channel's own movies and shows.

3.3 Microfoundation of the Advertisers' Demand Function

The main difference to the existing literature is our inverse demand function of the advertisers. In the existing literature it is assumed that the price of a viewer-time unit on channel i is only decreasing in w_i, but independent of D_i, w_j and

[7] Consider e.g. that a channel decides to produce, or to buy a sitcom, that is 25 minutes long. If the channel starts this sitcom at 8 pm and starts the next show on the channel at 8.30 pm, then the channel commits to 5 minutes advertising.

[8] To quote from one report about the upfront market: "If supply exceeds demand in the network Upfront, then let the suppliers battle among themselves to the advantage of advertisers." Source: Jack Myers Report, April 5, 2004 (www.jackmyers.com)

$D_j, \ \forall j \in \{1,2,...,N\}/i$

$$p_i = p_i(w_i), \tag{3.10}$$

$$\frac{\partial p_i}{\partial w_i} < 0, \frac{\partial p_i}{\partial D_i} = 0, \tag{3.11}$$

$$\frac{\partial p_i}{\partial D_j} = 0, \frac{\partial p_i}{\partial w_j} = 0 \ \forall j \in \{1,2,...,N\}/i. \tag{3.12}$$

Hence, no pecuniary externalities appear because a change in w_i does not influence the price p_j on a channel j. In the following we explain the microfoundation of the existing literature. Afterwards we show how one derives pecuniary externalities due to simple modifications of the existing microfoundation.[9] Therefore, we think that having no pecuniary externalities is an artefact of the particular microfoundation that is used in the existing literature. Hence, in our opinion pecuniary externalities should be the expected case.

Following Bagwell (2003), we can distinguish between three views of advertising: the persuasive view, the informative view, and the complementary view. As the existing literature, we concentrate on the informative view. In the existing models an advertiser g produces one product with constant marginal costs. This product has no substitute. Viewers are unaware of this product and advertising has the function of informing a viewer about the existence of this product. By seeing a commercial of an advertiser g, a viewer learns about the existence of this product. Each viewer has, by assumption, the same valuation v_g for one unit of a product of firm g. A viewer's valuation is zero for all further units of a product. This is common knowledge. Obviously, an advertiser asks a price v_g for his product. Hence, a viewer buys only once one unit of a product after seeing the corresponding commercial. Given this setup, each advertiser wants to reach each viewer exactly once. Every additional contact is useless because the consumer is already informed about the product and never buys a second unit of this product. Furthermore, it is assumed that the consumers' valuation v_g is different for each kind of product ($v_g \in [\underline{v}; \overline{v}]$). This assumption leads to a decreasing aggregated inverse demand function for advertising on one channel without generating pecuniary externalities between the channels. For example, suppose a price of 10 for one viewer-time unit on a certain channel. Then only the

[9] In the following we only consider a microfoundation with a convex cost function or with word-of-mouth advertising. A third possibility are switching viewers. However, given switching viewers, we are not able to derive that viewer-time units are a homogenous good. We can only justify pecuniary externalities. Therefore, we consider the case of switching viewers in the Appendix.

advertisers that have a product $v_g \geq 10$ advertise on this channel i. If another channel increases his advertising level, then this does not influence the willingness to pay of the advertisers to advertise on channel i because the revenue from advertising remains constant. Thus, 10 remains the equilibrium price on channel i and no pecuniary externalities appear in this existing setup.

In order to derive microfoundations for the pecuniary externalities, we modify this framework. We assume that all advertisers are homogenous in such a way that each advertiser has one kind of product for that no substitute exists. Furthermore, a consumer has the valuation v for exactly one unit of each product. As in the existing literature, the willingness to pay is zero for further units of a product. Viewers are unaware of the existence of an advertiser's product. Advertising has the function of informing the viewers about its existence. In contrast to the framework above, we do not assume that a viewer learns for sure that the product exists after he has seen the product's commercial. In fact, we take into account that viewers might be inattentive. Therefore, we assume that a viewer gains knowledge with a certain probability z after seeing an advertisement. This probability increases in the length of the commercial. Given the assumption of homogenous advertisers, we can concentrate on one representative advertiser g.

In order to derive the advertiser g's demand function for viewer-time units, we write down his profit function

$$\pi_g = v \sum_{k=1}^{N} z(w_{gk}) D_k - C(\sum_{k=1}^{N} z(w_{gk}) D_k) - \sum_{k=1}^{N} p_k w_{gk} D_k \ . \tag{3.13}$$

By w_{gi} we denote the advertiser g's advertising level on channel i. The advertising level w_{gi} determines the probability $z = z(w_{gi})$ with which a consumer gets aware of the advertiser's product if he watches channel i. If a viewer becomes aware of the product, we know that his product valuation is v. Obviously, the optimal price that an advertiser g should ask for his product is v. Hence, the first term displays the revenue of selling the advertised good, the second the costs of producing the advertised good, and the third the costs of advertising.

Microfoundation 1: Convex Cost Function for Producing the Advertised Good

Suppose that the probability z increases linearly in w_{gi}

$$z(w_{gi}) = \frac{w_{gi}}{A}. \tag{3.14}$$

A is a parameter that normalizes the probability $z(w_{gi}) \in [0;1]$. Hence, the advertiser g sells $\sum_{k=1}^{N} \frac{w_{gk}}{A} D_k$ units at a price of v.

Furthermore, suppose that the advertiser has a strictly convex cost function for producing the advertised good[10]

$$C'(\sum_{k=1}^{N} \frac{w_{gk}}{A} D_k) \geq 0; \quad C''(\sum_{k=1}^{N} \frac{w_{gk}}{A} D_k) > 0. \tag{3.15}$$

This yields the following profit function

$$\pi_g = v \sum_{k=1}^{N} \frac{w_{gk}}{A} D_k - C(\sum_{k=1}^{N} \frac{w_{gk}}{A} D_k) - \sum_{k=1}^{N} p_k w_{gk} D_k \ . \tag{3.16}$$

The advertiser maximizes his profit by his choice of $w_{gi}D_i$. Therefore, maximizing the advertiser g's profit function with respect to $w_{gi}D_i$ leads to his willingness to pay for the last viewer-time unit on channel i[11]

$$p_i = \frac{v}{A} - C' \ . \tag{3.17}$$

We see that viewer-time units are a homogenous good because C' does not depend on the particular channel i. Therefore, the prices for viewer-time units have to be the same on all channels

$$p_i = p \ \forall i \in \{1, 2, ..., N\}. \tag{3.18}$$

[10] Armstrong (2004) mentions this idea in the context why the advertisers' payoffs need not to be constant. Cunningham and Alexander (2004) also use a convex cost funtion in their setup.

[11] The SOC is globally satisfied.

Furthermore, the inverse demand function $p = \frac{v}{A} - C'$ is decreasing in $w_{gi}D_i$

$$\frac{\partial p}{\partial w_{gi}D_i} = -C'' < 0 \ , \forall i \in \{1, 2, ..., N\} \ . \tag{3.19}$$

Summing up, we see that the broadcasting channels have pecuniary externalities given increasing marginal costs for producing the advertised good. The intuition for the pecuniary externality is obvious. If an advertiser g increases his advertising level on a channel, then the advertising firm g sells more products. Hence, the firm g's willingness to pay for the last viewer-time unit decreases on all channels because his marginal costs are higher compared to the situation before.

In order to derive the linear demand function

$$p = A - \sum_{k=1}^{N} w_k D_k, \tag{3.20}$$

suppose that a consumer has the valuation $v = A^2$. Furthermore, we assume that the cost function for producing the advertised good is

$$C(\sum_{k=1}^{N} z_g(w_{gk}) D_k) = \frac{A^2}{2} (\sum_{k=1}^{N} \frac{w_{gk}}{A} D_k)^2 \ . \tag{3.21}$$

Plugging into the profit function yields

$$\pi_g = A^2 \sum_{k=1}^{N} \frac{w_{gk}}{A} D_k - \frac{A^2}{2} (\sum_{k=1}^{N} \frac{w_{gk}}{A} D_k)^2 - \sum_{k=1}^{N} p w_{gk} D_k \ . \tag{3.22}$$

Maximizing with respect to $w_{gi}D_i$ and solving for p leads to

$$p = A - \sum_{k=1}^{N} w_{gk} D_k \ . \tag{3.23}$$

Summing up over all M advertisers yields the aggregated inverse demand function

$$p = A - \sum_{k=1}^{N} w_k D_k \ . \tag{3.24}$$

Microfoundation 2: Word-of-Mouth Advertising

Another way to give a microfoundation is word-of-mouth advertising. In the following we assume that the marginal costs for producing the advertised good are constant. We normalize these costs to zero without loss of generality.

The idea behind the word-of-mouth advertising setup is that a viewer can learn about the existence of a certain product in two different ways. On the one hand, he can become aware of the product through advertising. On the other hand, he can become aware of it by recognizing that another person has bought this product. In order to capture this idea, we extend the model by introducing two periods. In the first period the viewers are exposed to the advertising spots of the different firms. Depending on the length of advertisement w_{gi}, they buy the product of advertiser g with probability $z(w_{gi}) = \frac{w_{gi}}{A}$. In the second period we assume that always pairs of viewers meet each other. In particular each viewer meets one other viewer. A viewer sees whether the other viewer has bought a product of firm g or not. Suppose that a viewer has not become aware of the product in the first period and meets a viewer in the second period who has bought the product in the first period. Given this, the non-buyer of the first period becomes aware of the product and also buys it. Such a constellation, where a non-buyer meets a buyer, is the only possibility how sales occur in the second period. Formalizing this idea leads to the following profit function of an advertiser g

$$\pi_g = v[\sum_{k=1}^{N} D_k \frac{w_{gi}}{A} + \sum_{k=1}^{N} D_k (1 - \frac{w_{gk}}{A}) \sum_{j=i}^{N} D_j \frac{w_{gj}}{A}] - \sum_{k=1}^{N} p_k w_{gk} D_k \ . \tag{3.25}$$

Maximizing with respect to $w_{gk} D_k$ leads to

$$p_k = v(\overbrace{\frac{1}{A}}^{\text{Effect 1}} + \overbrace{(-\frac{1}{A} \sum_{k=1}^{N} D_k \frac{w_{gk}}{A})}^{\text{Effect 2}} + \overbrace{\frac{1}{A} \sum_{k=1}^{N} D_k (1 - \frac{w_{gk}}{A})}^{\text{Effect 3}}) \ . \tag{3.26}$$

We see that if channel k increases his supply of viewer-time units, then three effects appear:

Effect 1: The probability increases that a viewer of this channel buys the product in the first period.

Effect 2: The probability decreases that a viewer of this channel buys the product in the second period 2. This is due to the fact that it is less likely that a viewer of channel k does not buy in the first period.

Effect 3: The probability increases that a non-buyer meets a buyer in the second period.

The sum of the three effects multiplied by the price v yields the marginal revenue of advertising on channel k. This is the advertiser's willingness to pay for the last viewer-time unit on channel k. We see that viewer-time units are a homogenous good because the willingness to pay for the last viewer-time unit is the same on all channels

$$p = p_i \ \forall i \in \{1, ..., N\}. \tag{3.27}$$

Furthermore, simplifying 3.26 yields

$$p = 2\frac{v}{A}(1 - \sum_{i=1}^{N} D_i \frac{w_{gi}}{A}) \ . \tag{3.28}$$

We see that the inverse demand function is decreasing in $w_{gi}D_i$

$$\frac{\partial p}{\partial w_{gi} D_i} = -2\frac{v}{A^2} < 0 \ \forall i \in \{1, 2, ..., N\} \ . \tag{3.29}$$

In particular there are two reasons why the willingness to pay decreases if the supply increases:

Reason 1: It is less attractive to increase the probability that a viewer buys in the first period because the probability that he meets a buyer in the second period increases ($= -\frac{1}{A^2}$).

Reason 2: It is less attractive to increase the probability that a viewer buys the product in the first period because the probability that this buyer meets a non-buyer in the second period decreases ($= -\frac{1}{A^2}$).

As the microfoundation with a convex cost function, this mircrofoundation yields the linear demand function with which we work later on. Suppose that the consumers' valuation is

$$v = \frac{A^2}{2} \ . \tag{3.30}$$

This yields the following profit of advertiser g

$$\pi_g = \frac{A^2}{2}[\sum_{k=1}^{N} D_k \frac{w_{gk}}{A} + \sum_{k=1}^{N} D_k(1 - \frac{w_{gk}}{A}) \sum_{j=1}^{N} D_j \frac{w_{gj}}{A}] - \sum_{k=1}^{N} p w_{gk} D_k \ . \tag{3.31}$$

Maximizing with respect to $w_{gk}D_k$ leads to

$$p = A - \sum_{k=1}^{N} w_{gk} D_k \ . \tag{3.32}$$

Summing up over all M advertisers yields the linear inverse demand function

$$p = A - \sum_{k=1}^{N} w_k D_k \ . \tag{3.33}$$

3.4 Competition in Advertising Levels

In this section we look at the profit maximization problem of the channels and we solve the model for the symmetric market equilibrium. The broadcasting channel i has the following maximization problem

$$\max_{w_i} \pi_i = D_i w_i p, \tag{3.34}$$

with

$$D_i = \frac{1}{N} + \frac{w_j - w_i}{2t} + \frac{w_l - w_i}{2t} \quad \text{for } w_j - \frac{t}{N} \leq w_i \leq w_j + \frac{t}{N} \tag{3.35}$$

and

$$p = A - \sum_{k=1}^{N} w_k D_k. \tag{3.36}$$

The first order condition of channel i can be written as

$$\frac{\partial \pi_i}{\partial w_i} = \frac{\partial D_i}{\partial w_i} p w_i + D_i \frac{\partial (p w_i)}{\partial w_i} = 0 \ . \tag{3.37}$$

Hence, the optimal advertising level solves the trade-off between losing the revenue from a viewer and increasing the revenue per viewer.

Another possibility to express the first order condition of channel i is

$$\frac{\partial \pi_i}{\partial w_i} = \overbrace{D_i p}^{\text{Scale Up Effect}} + \overbrace{\frac{\partial D_i}{\partial w_i} w_i p}^{\text{Quantity Effect}} +$$

$$\overbrace{D_i w_i \frac{\partial p}{\partial w_i} + D_i w_i \frac{\partial p}{\partial D_i}\frac{\partial D_i}{\partial w_i} + D_i w_i \left(\frac{\partial p}{\partial D_j}\frac{\partial D_j}{\partial w_i} + \frac{\partial p}{\partial D_l}\frac{\partial D_l}{\partial w_i}\right)}^{\text{Price Effect}} = 0. \tag{3.38}$$

To understand this FOC, we consider a "One-Sided Market" maximization problem. Usually the marginal revenue, $R = pq$, exhibits in a "One-Sided Market", depending on the kind of strategic variable, a simple trade-off:

- Suppose that the quantity q is the strategic variable, so $R(q) = p(q)q$. If a firm increases q, then it sells one more unit (=Scale Up Effect), but every unit at a smaller price (=Price Effect)

$$\frac{\partial R}{\partial q} = \overbrace{p}^{\text{Scale Up Effect}} + \overbrace{\frac{\partial p}{\partial q} q}^{\text{Price Effect}} \ . \tag{3.39}$$

- Suppose that the price p is the strategic variable, so $R(p) = pq(p)$. If a firm increases p, then it sells all units at a marginal higher price (=Scale Up Effect), but only a smaller number of units (=Quantity Effect)

$$\frac{\partial R}{\partial p} = \overbrace{q}^{\text{Scale Up Effect}} + \overbrace{\frac{\partial q}{\partial p} p}^{\text{Quantity Effect}} \ . \tag{3.40}$$

In our two-sided-market setup we have the combination of a price and a quantity competition. The advertising level is a kind of price for the viewers. For the advertisers it is, multiplied by the number of viewers, the supply. This is reflected in the the first order condition. There we have, beside the "Scale Up Effect", the "Price Effect" as well as the "Quantity Effect". For the "Price Effect" we have to take into account that if a channel i increases its advertising level, then it directly increases the supply of viewer-time units (=direct effect). But furthermore, it changes the distribution of the viewers on the channels. This changes the supply of viewer-time units, which has a further impact on the price (=indirect effect).

$$\frac{\partial p}{\partial w_i} = \overbrace{\frac{\partial p}{\partial w_i}}^{\text{direct effect}} + \overbrace{\frac{\partial p}{\partial D_i}\frac{\partial D_i}{\partial w_i} + \frac{\partial p}{\partial D_j}\frac{\partial D_j}{\partial w_i} + \frac{\partial p}{\partial D_l}\frac{\partial D_l}{\partial w_i}}^{\text{indirect effect}} \tag{3.41}$$

with l and j as the neighbors of i.

In order to derive the symmetric market equilibrium, we look at the corresponding first order condition

$$\frac{\partial \pi_i}{\partial w_i} |_{w_1^*=w_2^*=\ldots=w_N^*} = \frac{1}{N}p^* - \frac{1}{t}w_i^* p^* + \frac{1}{N}w_i^*\frac{\partial p^*}{\partial w_i^*} = 0. \tag{3.42}$$

Using the specified demand functions, we can solve for the symmetric equilibrium advertising level w_i^*.

Proposition 3.1 *Assume that the viewers' demand function is (with $-i$ as the symmetric neighbors of i)*

$$D_i = \frac{1}{N} + \frac{w_{-i} - w_i}{t} \quad \textit{for} \quad w_{-i} - \frac{t}{N} \leq w_i \leq w_{-i} + \frac{t}{N} \tag{3.43}$$

and that the advertisers' inverse demand function is

$$p = A - \sum_{k=1}^{N} D_k w_k. \tag{3.44}$$

The advertising level

$$w_i^* = \frac{(Nt + t + N^2A) - \sqrt{(Nt + t + N^2A)^2 - 4N^3tA}}{2N^2} \tag{3.45}$$

is the unique symmetric Nash-Equilibrium.

Proof. See Appendix. ∎

3.5 Has Advertising the Property of a Strategic Substitute or Complement?

In the existing literature advertising has the property of a strategic complement. In these setups advertising is a kind of price for the consumers and no pecuniary externality appears. Hence, as in price competition, each channel decreases his advertising level if another channel decreases his advertising level. In the following we show that it is not obvious that advertising has the property of a strategic complement given pecuniary and participation externalities.

For simplicity we consider a duopoly ($N = 2$). We denote the two channels by $\{i; -i\}$. Whether we have strategic substitutes or complements depends on the sign of

$$\frac{\partial^2 \pi_i}{\partial w_i \partial w_{-i}} = \frac{\partial p}{\partial w_{-i}} \frac{\partial (D_i w_i)}{\partial w_i} + \frac{\partial D_i}{\partial w_{-i}} \frac{\partial (p w_i)}{\partial w_i} + w_i [\frac{\partial^2 D_i}{\partial w_i \partial w_{-i}} p + D_i \frac{\partial^2 p}{\partial w_{-i} \partial w_i}] \lesseqgtr 0. \tag{3.46}$$

We see that channel $-i$ influences the price function p and the demand function D_i. This has four consequences:

- it changes the price p $(= \frac{\partial p}{\partial w_{-i}})$
- it changes the number of viewers of channel i $(=\frac{\partial D_i}{\partial w_{-i}})$
- it changes the impact of firm i on the price p $(=\frac{\partial^2 p}{\partial w_i \partial w_{-i}})$
- it changes the impact of firm i on the number of viewers D_i $(= \frac{\partial^2 D_i}{\partial w_i \partial w_{-i}})$

The change in the price p influences the incentive to change the quantity of viewer-time units $(\frac{\partial (D_i w_i)}{\partial w_i})$. If the price decreases, then it gets less desirable to have more viewer-time units. The change in the number of viewers D_i changes the incentive to change the revenue per viewer $\frac{\partial (p w_i)}{\partial w_i}$. Given more viewers, it gets more preferable to have a high revenue per viewer. Furthermore, the changes of the impact of the advertising level on the audience size and on the price have an additional effect on the incentives to change the advertising level. The number of viewers that one loses with a higher advertising level changes. Moreover, the impact of an increase in the advertising level on the price changes.

With our specified viewers' demand function, we have that $\frac{\partial D_i}{\partial w_{-i}} = \frac{1}{t}$ given $w_{-i} - \frac{t}{2} \le w_i \le w_{-i} + \frac{t}{2}$ and $\frac{\partial^2 D_i}{\partial w_i \partial w_{-i}} = 0$ because

$$D_i = \begin{cases} 1 & \text{if} \quad w_i < w_{-i} - \frac{w_{-i}}{t} \\ \frac{1}{2} + \frac{w_{-i} - w_i}{t} & \text{if} \quad w_{-i} - \frac{t}{2} \le w_i \le w_{-i} + \frac{t}{2} \\ 0 & \text{if} \quad w_{-i} + \frac{t}{2} < w_i \end{cases} . \tag{3.47}$$

Furthermore, given the specified advertisers' demand function $p = A - \sum_{k=1}^{N} D_k w_k$, the cross derivative is $\frac{\partial^2 p}{\partial w_i \partial w_{-i}} = -\frac{2}{t}$ for $w_{-i} - \frac{t}{2} \le w_i \le w_{-i} + \frac{t}{2}$. We cannot make any statement about the sign of $\frac{\partial p}{\partial w_i}$ and $\frac{\partial p}{\partial w_{-i}}$ without knowing the actual values of w_i and w_{-i} because

$$\frac{\partial p}{\partial w_i} = \begin{cases} -1 & \text{if} \quad w_i < w_{-i} - \frac{t}{2} \\ -D_i + \frac{w_i}{t} - \frac{w_{-i}}{t} \le 0 & \text{if} \quad w_{-i} - \frac{t}{2} \le w_i \le w_{-i} + \frac{t}{4} \\ -D_i + \frac{w_i}{t} - \frac{w_{-i}}{t} > 0 & \text{if} \quad w_{-i} + \frac{1}{4}t < w_i \le w_{-i} + \frac{t}{2} \\ 0 & \text{if} \quad w_{-i} + \frac{t}{2} < w_i \end{cases} . \tag{3.48}$$

Hence, it is possible that the price per viewer-time unit increases if a channel extends his advertising level. To illustrate this point, suppose two channels. Assume that channel 1 has a higher advertising level than channel 2 in such a way that channel 1 has only "one" remaining viewer. If channel 1 increases his advertising level even further, then this last viewer switches to channel 2. Thus, the higher advertising level of channel 1 decreases the aggregated supply of viewer-time units and increases the market price p.

Let us now determine the reaction function of channel i: $w_i^* = w_i^*(w_{-i})$. In order to do this, we have to distinguish between three cases:

Case 1: $w_{-i} > w_{-i}^H = \frac{A}{2} + \frac{t}{2}$

Suppose that $w_{-i} > \frac{A}{2} + \frac{t}{2}$. We can calculate that $w_i = \frac{A}{2}$ maximizes channel i's revenue per viewer. Furthermore, we have that

$$D_i(w_i = \frac{A}{2}; w_{-i} \ge w_{-i}^H) = 1 \ .$$

Hence, $w_i = \frac{A}{2}$ has to be channel i's best response. If channel i chooses $w_i = \frac{A}{2}$, then he all viewers watch his program and he has the highest possible revenue per viewer.

Case 2: $w^H_{-i} \geq w_{-i} \geq w^C_{-i} = \frac{7}{4}t + \frac{1}{2}A - \frac{1}{4}\sqrt{(25t^2 + 4At + 4A^2)}$

In this second case w_{-i} is not high enough to allow channel i to choose $w_i = \frac{A}{2}$ and still to have $D_i = 1$. Given this w_{-i}, the profit of channel i decreases in w_i ($\frac{\partial \pi_i}{\partial w_i} < 0$). A higher advertising level leads to a lower number of viewers and the change in the revenue per viewer cannot make up for this loss. Hence, reducing the advertising level increases the profit as long as $D_i < 1$. Therefore, we derive a corner solution $w_i^*(w_{-i}) = w_{-i} - \frac{t}{2}$ where $D_i = 1$.

Case 3: $w_{-i} < w^C_{-i} = \frac{7}{4}t + \frac{1}{2}A - \frac{1}{4}\sqrt{(25t^2 + 4At + 4A^2)}$

In Case 3 we have an interior solution. The reaction function can be calculated and is

$$\begin{aligned} w_i^{Case3} = {} & (1 + \frac{1}{48}t^2 + \frac{5}{24}w_{-i}t - \frac{1}{6}At + \frac{1}{16}w^2_{-i})[\frac{3}{64}tw^2_{-i} - \frac{1}{16}w_{-i}At - \frac{1}{64}w^3_{-i} + \\ & \frac{1}{576}[1440t^4w_{-i}A + 1296Atw^4_{-i} - 432A^2t^2w^2_{-i} + 2376w^3_{-i}t^2A \\ & -5760w_{-i}t^3A^2 + 7632t^3Aw^2_{-i} - 3540t^3w^3_{-i} - 3t^6 - 90t^5w_{-i} - 927t^4w^2_{-i} - 576t^4A^2 - \\ & 2052t^2w^4_{-i} - 1296w^5_{-i}t + 1536A^3t^3 + 72t^5A]^{\frac{1}{2}}]^{\frac{1}{3}} + \frac{1}{4}t + \frac{3}{4}w_{-i} \end{aligned}$$

We summarize these findings in the following proposition.

Proposition 3.2 *In a duopoly with*

$$D_i = \begin{cases} 1 & \text{if} \quad w_i < w_{-i} + \frac{t}{2} \\ \frac{1}{2} + \frac{w_{-i} - w_i}{t} & \text{if} \quad w_{-i} - \frac{t}{2} \leq w_i \leq w_{-i} + \frac{t}{2} \\ 0 & \text{if} \quad w_{-i} + \frac{t}{2} < w_i \end{cases} \tag{3.49}$$

and

$$p = A - D_i w_i - D_{-i} w_{-i}, \tag{3.50}$$

the best response function $w_i^*(w_{-i})$ of platform i is

$$w_i^* = \begin{cases} \frac{A}{2} & \text{if} \quad w_{-i}^H = \frac{A}{2} + \frac{t}{2} < w_{-i} \\ w_{-i} - \frac{t}{2} & \text{if} \quad w_{-i}^H \leq w_{-i} \leq w_{-i}^C \\ w_i^{Case3}{:=}\textit{interior solution} & \text{if} \quad w_{-i} < w_{-i}^C = \frac{7}{4} + \frac{A}{2} - \frac{1}{4}\sqrt{25t^2 + 4At + 4A^2} \end{cases} \tag{3.51}$$

Proof. See Appendix. ■

In the first case it is obvious that advertising is neither a strategic substitute nor a strategic complement. In the second case we have, due to the corner solution, a perfect strategic complement. Whether advertising exhibits the property of a strategic complement or substitute in the case of an interior solution depends on the actual values of w_i and w_{-i}. We have plotted two numerical examples to illustrate this (see Figure 3.1).

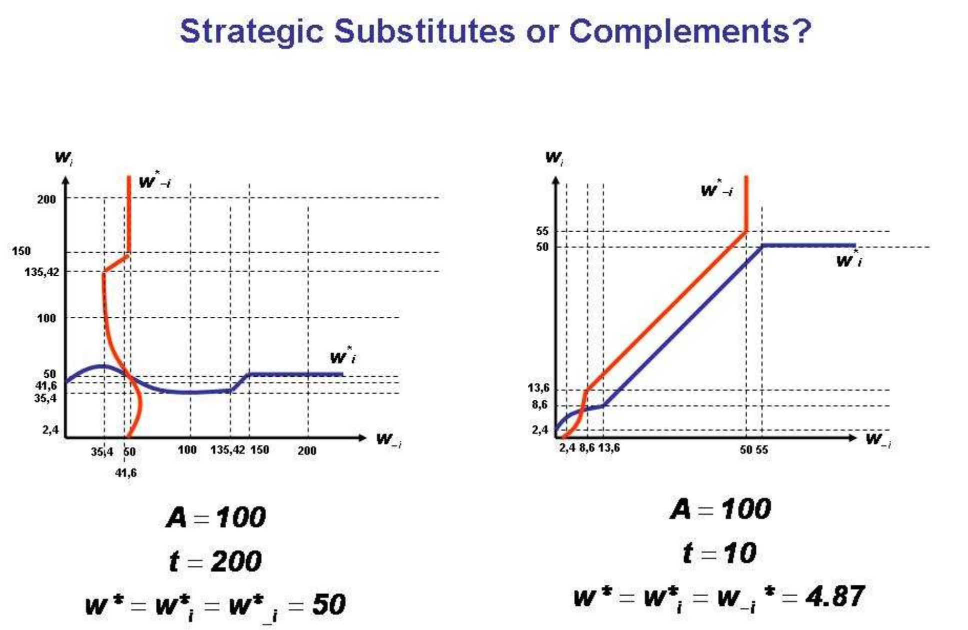

Figure 3.1: Strategic Substitute or Complement

Given a high differentiation parameter ($t = 200$), we see that advertising can be either a strategic substitute or complement. In the corresponding equilibrium,

$w_i^* = 50$, it is a strategic substitute. In contrast, advertising exhibits the property of a strategic complement given a low differentiation ($t = 10$). However, we are able to derive analytical results whether advertising has the property of a strategic substitute or complement in a symmetric market equilibrium.

Proposition 3.3 *In the symmetric Nash-Equilibrium of a duopoly advertising has the property of a*

- *strategic substitute if differentiation between platforms is relatively high* ($t > \frac{4}{7}(2\sqrt{2} - 1)A$)
- *strategic complement if the differentiation between platforms is relatively low* ($t < \frac{4}{7}(2\sqrt{2} - 1)A$)

Proof. See Appendix. ■

If one considers the advertising market, then one expects that advertising has the property of a strategic substitute because channels compete in quantities. In contrast, if one considers the viewer market, then one expects the property of a strategic complement because advertising is a kind of price for the viewers. The intuition of the proposition is the following. Due to the fact that we consider a symmetric situation, no effects that result from asymmetric advertising levels appear. Furthermore, suppose that the differentiation is high. In this case a change in the advertising level has no big impact on the viewer side and the effects of the advertising market dominate. Advertising has on the advertising market the property of a strategic substitute. Therefore, we have exactly this property if the differentiation is high. If the differentiation is low, then the opposite is true.

3.6 Market Entry

In this section we consider market entry. Given the introduction of Digital Television, broadcasting frequencies are no longer a scare resource. Governments use this to award additional licenses. This yields tougher competition between the broadcasting channels. Choi (2003) shows, by using the Anderson and Coate (2005) framework, that market entry leads to a lower advertising level and that it decreases the profits of the channels. In the following we show that this does not have to be true if we take into account pecuniary and participation externalities.

In order to do this, we start by looking at the role of the differentiation parameter t and the demand parameter A. If the differentiation parameter t increases, then the viewers' demand function gets less elastic. Furthermore, in the symmetric market equilibrium, the size of the price change $\frac{\partial p}{\partial w_i}$ is independent of t and always $-D_i = -\frac{1}{N}$. Hence, an increase in t leads to a higher equilibrium advertising level due to the less elastic demand function.

Additionally, if the parameter A increases, then the equilibrium advertising level rises because the advertisers' willingness to pay increases. This has two effects. On the one hand, it is more attractive to increase the advertising level because the revenue per advertising unit $(= D_i p)$ increases. On the other hand, it is less attractive to increase the advertising level because the revenue per viewer increases $(= w_i p)$ and more advertising leads to a smaller audience size. Therefore, the two effects work in the opposite direction. One can summarize both effects in the term $p\frac{\partial(D_i w_i)}{\partial w_i}$. In a symmetric equilibrium we have $\frac{dp}{dw_i} < 0$. Therefore, it has to be that $p\frac{\partial(D_i w_i)}{\partial w_i} > 0$ in order to fulfill the first order condition. If the price p increases, then the value of $\frac{\partial(D_i w_i)}{\partial w_i}$ has to decrease in order to satisfy the first order condition. We see that the second derivative is negative ($\frac{\partial^2(D_i w_i)}{\partial w_i^2} < 0$). Therefore, due to the higher price p, the revenue per viewer has to decrease, which the channels achieve by a higher advertising level.

We summarize these findings in the following lemma.

Lemma 3.1 *If the differentiation between the firms increases, then the equilibrium advertising level increases*

$$\frac{\partial w_i^*}{\partial t} > 0.$$

If the advertisers' demand function shifts out, then the equilibrium advertising level increases

$$\frac{\partial w_i^*}{\partial A} > 0.$$

Proof. See Appendix. ■

In the following we use Lemma 3.1 to show that market entry can either lead to a higher level of advertising or a lower level of advertising.

Proposition 3.4

Market entry yields

- *a higher level of advertising* ($\frac{\partial w_i^*}{\partial N} > 0$) *if the differentiation between the channels is relatively high* ($t > \frac{2N^2 A}{2+N}$);
- *a lower level of advertising* ($\frac{\partial w_i^*}{\partial N} < 0$) *if the differentiation between the channels is relatively low* ($t < \frac{2N^2 A}{2+N}$).

Proof. See Appendix. ∎

To understand the intuition behind this Proposition 3.4, let us consider the first order condition. In a symmetric equilibrium we have

$$\frac{\partial \pi_i}{\partial w_i} \mid_{w_1^* = \ldots = w_N^*} = -\frac{1}{t} p w_i^* + \frac{1}{N} \frac{\partial (w_i^* p)}{\partial w_i} = 0. \tag{3.52}$$

Using the implicit function theorem yields

$$\frac{dw^*(N)}{dN} = -\frac{-\frac{1}{N^2} \frac{\partial (w_i^* p)}{\partial w_i} + \frac{1}{N} \frac{\partial \frac{\partial (p w_i^*)}{\partial w_i}}{\partial N}}{[SOC]}. \tag{3.53}$$

Therefore, we see that $\frac{dw^*(N)}{dN} > 0$ if

$$-\frac{1}{N^2} \frac{\partial (w_i^* p)}{\partial w_i} + \frac{1}{N} \frac{\partial \frac{\partial (p w_i^*)}{\partial w_i}}{\partial N} > 0. \tag{3.54}$$

The crucial point is the change of the marginal revenue per viewer multiplied by the number of viewers ($D_i^* \frac{\partial (w_i^* p)}{\partial w_i}$). If this term increases, then firms increase their advertising levels. Otherwise, they decrease their advertising levels. Note that only the mentioned term is relevant because market entry does not affect the negative effect of increasing advertising, namely losing viewers $\frac{\partial (\frac{\partial D_i}{\partial w_i} w_i^* p)}{\partial N} = 0$.

After market entry each channel has a lower number of viewers in the equilibrium. This has two effects on the incentives to offer advertising. We can distinguish between them by taking the derivative of $D_i^* \frac{\partial(w_i^* p)}{\partial w_i}$ with respect to N.

On the one hand, an increase in the advertising level, which yields a higher revenue per viewer, has a smaller positive effect on the profit because it works on a smaller number of viewers. This is reflected in the term

$$\frac{\partial D_i^*}{\partial N}\frac{\partial(w_i^* p)}{\partial w_i} = -\frac{1}{N^2}\frac{\partial(w_i^* p)}{\partial w_i} < 0 \tag{3.55}$$

and gives an incentive to decrease the advertising level.

On the other hand, the smaller audience size decreases the impact of channel i on the price because a change in the advertising level leads to smaller change in the total supply of viewer-time units. This is reflected in the term

$$D_i^* \frac{\partial(\frac{\partial(w_i^* p)}{\partial w_i})}{\partial N} = \frac{1}{N} w_i^* \frac{1}{N^2} > 0 \tag{3.56}$$

and gives an incentive to increase the advertising level.

Therefore, we see that a smaller audience size has two counteracting effects. The total effect depends on the size of the different effects. The dimension of the first effect decreases in the advertising level,[12] and the dimension of the second effect increases in the advertising level.[13] Hence, if the advertising level w_i^* is relatively high, then the total effect is positive and market entry leads to an even higher advertising level. We have already seen that a higher t leads to a higher w_i^* (see Lemma 3.1). Therefore, if the differentiation is relatively high, then the equilibrium value of the advertising level is above the threshold value. Hence, the advertising level increases if market entry occurs. One may wonder why t has to be higher if A increases ($t^{crit} = \frac{2N^2 A}{2+N}$) because w_i^* increases if A increases. But a higher A has two effects. A higher A leads to a higher w_i^* but it increases the equilibrium value of p^*, which puts more weight on the first effect (see Equation 3.55).

To summarize, we see that the effect of market entry on the advertising level is ambiguous. Given a high level of differentiation, the advertising level increases, and

12 $\frac{dp}{dw_i}\Big|_{w_1^*=\ldots=w_N^*} = -\frac{1}{N}$; $\frac{\partial(\frac{\partial p}{\partial w_i} w_i^* + p(w_i^*))}{\partial w_i} = \frac{\partial(-\frac{1}{N} w_i^* + (A - w_i^*))}{\partial w_i} < 0$

13 $\frac{\partial(-\frac{1}{N} w_i^* + p)}{\partial N} = \frac{1}{N^2} w_i^*$; $\frac{\partial(\frac{1}{N^2} w_i^*)}{\partial w_i} > 0$

given a low level of differentiation, it decreases.

In a next step we consider how market entry affects the profits of the broadcasting channels. There exists an advertising level $\widehat{w} = \frac{A}{2}$ that maximizes the revenue per viewer

$$\frac{\partial[(A - w_i)w_i]}{\partial w_i} = A - 2w_i = 0; \tag{3.57}$$

$$\widehat{w}_i = \frac{1}{2}A. \tag{3.58}$$

If the broadcasting channels colluded, then they would choose this advertising level. In the following we have to distinguish between four situations.

Situation 1. $\widehat{w}_i < w_i^*$ and $\frac{\partial w_i^*}{\partial N} > 0 \rightarrow \frac{\partial \pi}{\partial N} < 0$

In this situation the equilibrium advertising level is higher than the advertising level that maximizes the profits per viewer. Furthermore, the differentiation parameter t is above the threshold level $t > \frac{2N^2 A}{2+N}$. Hence, market entry increases the equilibrium advertising (see Proposition 3.4). It is obvious that the profits of the channels decrease. Firstly, given market entry, a channel has less viewers in equilibrium because $\frac{1}{N}$ decreases in N. Secondly, the equilibrium advertising level increases, which yields a smaller revenue per viewer.

Situation 2. $\widehat{w}_i > w_i^*$ and $\frac{\partial w_i^*}{\partial N} < 0 \rightarrow \frac{\partial \pi}{\partial N} < 0$

In this second situation the equilibrium advertising level is lower than the advertising level that maximizes the revenue per viewer. Furthermore, the differentiation parameter t is smaller than the threshold level. Hence, given market entry, the equilibrium advertising decreases. Therefore, it is obvious that the profit of an incumbent decrease with market entry.

Situation 3. $\widehat{w}_i < w_i^*$ and $\frac{\partial w_i^*}{\partial N} < 0 \rightarrow \frac{\partial \pi}{\partial N} \lesseqgtr 0$

In this third situation the equilibrium advertising level is higher than the level that maximizes the revenue per viewer. Furthermore, given market entry, the equilibrium level of advertising decreases. Hence, the revenue per viewer increases, which has a positive effect on the profit of an incumbent. At the same time the incumbent loses viewers, which has a negative impact on the profit. To summarize, there are two counteracting effects on an incumbent's profit. Whether the profit decreases or increases depends on the size of the different effects.

$$\pi_i = (A - w_i^*) w_i^* \frac{1}{N} \tag{3.59}$$

$$\frac{\partial \pi_i}{\partial N} = -\frac{1}{N^2}(A - w_i^*) w_i^* + \frac{1}{N}\frac{\partial w_i^*}{\partial N}(A - 2w_i^*) \lesseqgtr 0 \text{ with } \widehat{w}_i < w_i^* \text{ and } \frac{\partial w_i^*}{\partial N} < 0 \tag{3.60}$$

Let us illustrate this case with a numerical example. We assume that $A = 10$ and $t = 300$. The first derivative of a channel's profit function with respect to N is zero at the values $N_1 \approx 25.597$ and $N_2 \approx 40.838$. Therefore, we have two extrema. Checking the second order condition yields

$\frac{\partial^2 \pi_i}{\partial N^2} = 0.002224 > 0$ with $N_1 = 25.597$;

$\frac{\partial^2 \pi_i}{\partial N^2} = -0.0010858 < 0$ with $N_2 = 40.838$.

Hence, the profit of a channel is increasing in N for $N \in]N_1; N_2[$. Figure 3.2 shows a channel's advertising level and a channel's profit for the numerical example. We see that the profit starts to decrease in N, then it increases in N, and if $N > 41$, then it decreases again.

Situation 4. $\widehat{w}_i > w_i^*$ and $\frac{\partial w_i^*}{\partial N} > 0$

Theoretically, this situation may be possible. It could be that the equilibrium advertising level is too low and that market entry leads to a higher equilibrium advertising level. Nevertheless, we can exclude this case with our chosen functions. Given that the differentiation parameter t is above the threshold level, the advertising level is always higher than $\widehat{w}_i$ with $N > 2$.

To show this we calculate the smallest possible w_i^* given $t \geq \frac{2N^2 A}{2+N}$ and compare it to $\hat{w}$

$$w_i^*(t = \frac{2N^2 A}{2+N}) = \frac{NA}{2+N} > \frac{A}{2} \; \forall N > 2. \tag{3.61}$$

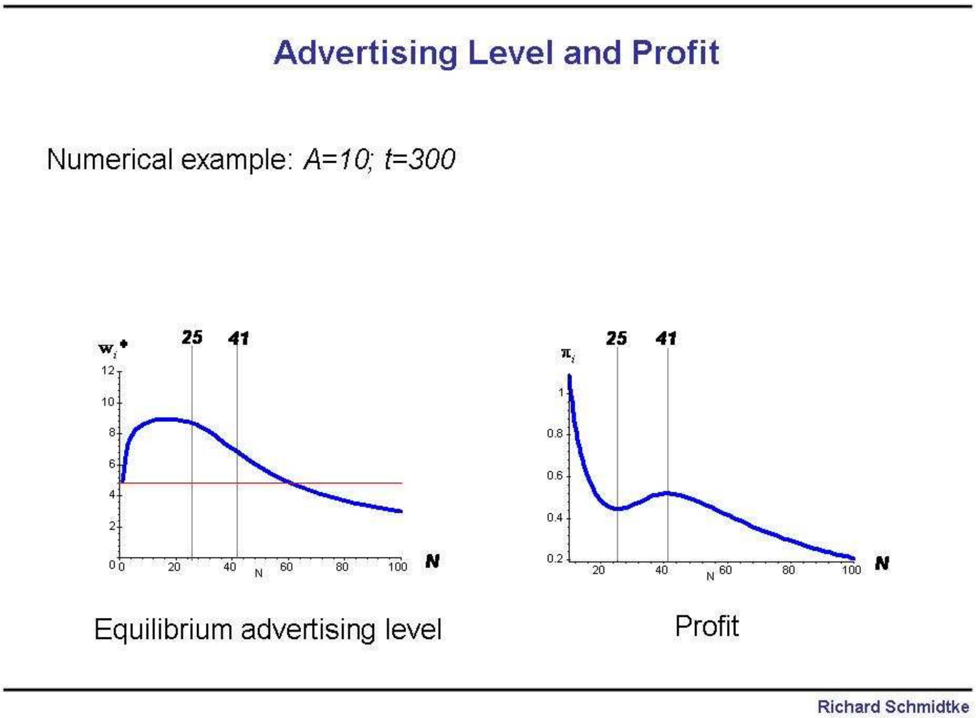

Figure 3.2: Advertising level and profit

We summarize our findings in the following proposition.

Proposition 3.5 *Market entry decreases the profits of the incumbents*

- *if the differentiation is relatively high($t > \frac{2N^2A}{2+N}$). The equilibrium advertising level is above the revenue maximizing advertising level and market entry increases the equilibrium advertising level.*
- *if the differentiation is relatively low ($t < \frac{N^2A}{2N-2}$). The equilibrium advertising level is below the revenue maximizing advertising level and market entry decreases the equilibrium advertising level.*

If the differentiation parameter has a medium value ($\frac{2N^2A}{2+N} > t > \frac{N^2A}{2N-2}$), then the equilibrium advertising level is above the revenue maximizing advertising level. Furthermore, market entry leads to a lower equilibrium advertising level. In this case it might occur that the incumbents' profits increase with market entry.

Proof.

We know that $\widehat{w}_i = \frac{A}{2}$. Looking for t s.t. $w_i^* = \frac{A}{2}$ yields

$$t = \frac{N^2 A}{2N - 2} \tag{3.62}$$

For the rest see above. ■

The surprising finding that market entry can lead to higher profits comes from the two externalities that are incorporated in our model. Given the pecuniary externality, a platform does not internalize that it decreases the other platforms' profits if he increases the supply of viewer-time units. This goes in the direction of a too high advertising level compared to the collusive advertising level. Hence, the platforms would like to commit to a lower advertising level. Therefore, individually rational behavior leads to a too high advertising level compared to collectively rational behavior.

On the other hand, we have the participation externality. Given this externality, a platform does not internalize that it increases the profits of the competing platforms if he chooses a higher advertising level. This goes in the direction of a too low advertising level. Hence, the platforms would like to commit to a higher advertising level. Therefore, individually rational behavior leads to a too low advertising level compared to collectively rational behavior.

To illustrate this point, consider the example of two channels and the parameter values $A = 100$ and $t = 200$. Although we have competition, the equilibrium advertising level $w_i^* = 50$ equals the collusive advertising level $\hat{w}_i = 50$. We see that the "suboptimal" behavior of the firms on one side of the market, from a collusive point of view, corrects the "suboptimal" behavior on the other side of the market.[14]

[14] Reisinger (2004) shows a related effect of the interaction between the two markets. In his paper he considers a situation where two platforms compete in prices and the advertisers total demand is fixed. Suppose that the differentiation parameter is very high. Hence, viewers never switch between the platforms. Competition between the channels yields an advertising price of zero. In such a situation Reisinger (2004) shows that an exogenous decrease of the differentiation parameter increases the profits. The intuition is that viewers begin to switch. Therefore, the channels have to take care of their advertising levels. This leads to higher prices for advertising, which yields a higher profit per channel.

In the following we consider the impact of market entry on social welfare. A social planer, who wants to determine the optimal advertising level $\widetilde{w}$, has to solve the following trade-off. On the one hand, a higher advertising level increases the surplus of the advertisers. On the other hand, a higher advertising level decreases the surplus of the viewers. One can easily determine the optimal advertising level that solves this trade-off. The willingness to pay of the advertisers (p) should be equal to the marginal change in the viewers' utility ($= 1$).

$$1 = A - \widetilde{w} \tag{3.63}$$

$$\widetilde{w} = A - 1 \tag{3.64}$$

One can compute the necessary relationship between t, A and N such that the equilibrium advertising level is equal to the social optimal advertising level $\widetilde{w}_i$

$$\widetilde{w}_i = \frac{Nt + t + N^2A - \sqrt{(Nt + t + N^2A)^2 - 4N^3tA}}{2N^2} = A - 1; \tag{3.65}$$

$$t = N^2 \frac{A-1}{N+1-A}. \tag{3.66}$$

Concerning the effects of market entry on the social welfare, we have to take into account the reduction of the consumers' transportation costs. Hence, the social welfare function is

$$WF = (A - w)w + \frac{1}{2}w^2 - \frac{t}{4N} - w. \tag{3.67}$$

In order to consider whether market entry leads to a higher or lower welfare, we have again to distinguish between four situations:

Situation 1: $w_i^* > \widetilde{w}_i$ and $\frac{\partial w_i^*}{\partial N} < 0 \rightarrow \frac{\partial WF}{\partial N} > 0$

In this first case the differentiation parameter t is between $\frac{2N^2A}{2+N} > t > N^2\frac{A-1}{N+1-A}$. Hence, the advertising level is too high from a welfare point of view. Market entry leads to a lower advertising level because the transportation costs are below the threshold level. In this case market entry unambiguously increases welfare. The advertising level moves in the right direction and the consumers' transportation costs decrease. Given that the differentiation parameter t can be in the mentioned area, the parameter A has to be $A > 1 + \frac{N}{2}$.[15]

Situation 2: $w^* < \widetilde{w}$ and $\frac{\partial w^*}{\partial N} > 0 \rightarrow \frac{\partial WF}{\partial N} > 0$

As in case 1, in this second case market entry increases the social welfare. If $A > 1+\frac{N}{2}$ then t can be between $N^2 \frac{A-1}{N+1-A} > t > \frac{2N^2A}{2+N}$. Hence, the advertising level is below the social optimal advertising level. Market entry increases the advertising level. Furthermore, the viewers' transportation costs decrease if market entry occurs.

Situation 3: $w^* > \widetilde{w}$ and $\frac{\partial w^*}{\partial N} > 0 \rightarrow \frac{\partial WF}{\partial N} \lesseqgtr 0$

If t is very high ($t > max\{N^2 \frac{A-1}{N+1-A}, \frac{2N^2A}{2+N}\}$), then the equilibrium advertising level is above the social optimal level. Market entry yields two counteracting effects. On the one hand, the consumers' transportation costs are reduced. On the other hand, the advertising level moves in the "wrong" direction. Nevertheless, numerical examples let us expect that the positive effect on social welfare always dominates.

Situation 4: $w^* < \widetilde{w}$ and $\frac{\partial w^*}{\partial N} < 0 \rightarrow \frac{\partial WF}{\partial N} \lesseqgtr 0$

If the differentiation parameter t is very low ($t < min\{N^2 \frac{A-1}{N+1-A}, \frac{2N^2A}{2+N}\}$), then the advertising level is above the social optimal level. Market entry decreases the advertising level even further. Even if the consumers' transportation costs are reduced, it can happen that the social welfare decreases with market entry. In particular this is true when N goes to infinity and A above the critical value $\frac{5}{4}$. Given a high N and further market entry, the decrease of the transportation costs is negligibly and the negative effect of a lower advertising level dominates. Therefore, we conclude that perfect competition has not to be desirable from a welfare point of view.

Proposition 3.6 *The equilibrium advertising level can be above or below the social optimal advertising level. Therefore, market entry has two welfare effects. On the one hand, it changes the equilibrium advertising level. This can increase or decrease social welfare. On the other hand, it decreases the transportation costs of the view-*

[15] $\frac{2N^2A}{2+N} > N^2 \frac{A-1}{N+1-A} \rightarrow A > 1 + \frac{N}{2}$

ers, which unambiguously increases welfare. In particular if $A > \frac{5}{4}$, then perfect competition does not maximize social welfare.

Proof. See Appendix. ∎

3.7 Conclusion

In this chapter we have analyzed a two-sided market model where broadcasting channels offer a costless program to viewers. The channels make profits by selling viewer-time units to advertisers. In contrast to the existing literature, we have combined the "participation externality" (more advertisements on platform i decrease its own audience size and increase the other platforms' audience sizes) and the "pecuniary externality" (more advertisements on platform i changes the advertisers' willingness to pay on all platforms). We have given two examples: the "Code of Conduct" of the National Association of Broadcasters in the US and the discussion about advertising on public broadcasting channels in Germany. Both examples illustrate the existence of pecuniary externalities. We have provided two microfoundations for pecuniary externalities, namely a convex cost function and word-of-mouth advertising. We have shown that advertising can have the property of a strategic substitute or complement. Furthermore, market entry can increase or decrease the equilibrium advertising level. Both is in contrast to the existing literature. Moreover, we have shown that market entry can make incumbents better off and that perfect competition does not have to be desirable from a welfare point of view.

Given the introduction of Digital Television, our model leads to the policy implication that governments should be careful with additional broadcasting licenses. In particular they should take into account that the equilibrium advertising level can move in the wrong direction and that this can decrease welfare. Furthermore, our model shows that advertising ceilings, which restrict the time that a channel can offer to advertisers, can be used as a collusive device between the broadcasting channels in order to increase the price for advertising. This function of advertising ceilings does not appear in the existing literature. Therefore, we conclude that a government should be careful with such advertising ceilings, particularly if the channels ask for such ceilings in order to protect viewer against "too much" advertising.

Natural extensions of this model would be to consider the program quality choices of the broadcasters and to consider the introduction of subscription fees. In particular the last point seems quite interesting. The existing literature, like Peitz and Valetti (2004), only considers subscription fees in the Anderson and Coate (2005) framework, so without pecuniary externalities. It would be interesting to analyze subscription fees under perfect competition in a model that includes pecuniary externalities. Without pecuniary externalities, channels set monopoly advertising levels and compensate the viewers by a corresponding lower subscription price. Hence, an inefficiency arises due to the monopoly advertising level, which is persistent even under perfect competition. Given pecuniary externalities, perfect competition would drive the advertising level to the social optimal level. Therefore, with two instruments, advertising prices and subscription prices, and two externalities, pecuniary and participation externalities, more competition would always increase social welfare. Hence, perfect competition would be unambiguously desirable from a welfare point of view.

3.8 Appendix

Switching Viewers

We consider "switching viewers" in order to derive pecuniary externalities. As in Anderson and Coate (2005), we consider two periods (say day 1 and day 2) where the channels broadcast their programs and viewers watch these programs. Assume furthermore that the channels have to commit to their advertising levels for the two periods in advance. Additionally, we make the assumption that the viewers reallocate after the day 1 on the Salop circle at random. A possible explanation is that the kind of differentiation of the channels is different. For an illustration assume that channel A broadcasts always US movies and channel B broadcasts always French movies. If a viewer decides to watch the US movie, then this has not to imply that this viewer always watches this channel. Perhaps he has chosen the US movie because it was a black and white movie and next time the French movie is black and white. Or he preferred the US movie because it was a love story and next time the French channel has the better love story to offer.

Let us look at a representative advertiser g. If he advertises w_{gi1} on channel i in the first period, then the viewers of this channel buy his product with a probability of $z = z(w_{gi1})$. We assume that the probability function z is concave in the advertising level

$$z' > 0, z'' < 0 \text{ with } z(0) = 0, z(\infty) = 1 \ .$$

This assumption ensures that the second order condition of the advertisers are globally satisfied. We stick to the assumption that each consumer is only interested in one unit of a certain product. This implies that all viewers who have bought a product in the first period will for sure not buy such a product in the second period. Hence, only a non-buyer of the first period can get informed about the product of advertiser g in the second period. In this second period the probability z depends again on the length of the advertising spot.

This leads to the following expected profit function for advertiser g:

$$\pi_g = v[\sum_{j=1}^{N} D_{j1}(z(w_{gj1})+(1-z(w_{gj1}))\sum_{i=1}^{N} D_{i2}z(w_{gi2})]-\sum_{j=1}^{N} p_{j1}w_{gj1}D_{j1}-\sum_{i=1}^{N} p_{i2}w_{gi2}D_{i2} \ .$$

Maximizing with respect to the optimal advertising level on channel k in both periods leads to

$$p_{k1} = v[1 - \sum_{i=1}^{N} D_{i2} z(w_{gi2})] z'(w_{gk1})$$

$$p_{k2} = v[1 - \sum_{j=1}^{N} D_{j1} z(w_{gj1})] z'(w_{gk2})$$

For simplicity let us assume, as Anderson and Coate (2005), that every channel offers the same time for commercials in both periods.[16] This implies that given w_i, a channel i has the audience size D_i in both periods. In particular the channel k gets the same per viewer-time unit time price p_k in both periods. This price p_k is

$$p_k = p_{k1} = p_{k2} = v[1 - \sum_{j=1}^{N} D_j z(w_{gj})] z'(w_{gk}).$$

If another channel l changes its advertising level in such a way that firm g advertises more on this channel, then it distorts the price p_k

$$\frac{\partial p_k}{\partial w_g l} = -v D_l z'(w_{gl}) z'(w_{gk}) < 0 \ .$$

Thus, the broadcasting channels have pecuniary externalities. The intuition for this is straightforward: if an advertiser advertises more on channel l in both periods, then this leads to a decrease in the effectiveness of advertising on channel k. The reason is simply that the probability to get a non-buyer gets smaller (through the higher advertising level on channel l) and therefore the net payoff from advertising decreases.

Proof of Proposition 3.1

Symmetry and the knowledge $D_i = \frac{1}{N}$ yields:

$$\frac{\partial \pi_i}{\partial w_i} = -\frac{1}{t} w_i [A - N w_i \frac{1}{N}] + \frac{1}{N}[A - N w_i \frac{1}{N}] - \frac{1}{N^2} w_i = 0$$

[16]Given the assumption, the channels have pecuniary externalities on each other in both periods. If we allow for different advertising levels in the two periods, then we derive intertemporal externalities in the sense that the advertising level of channel l in one period influences the willingness to pay of the advertiser in the other period. Such a dynamic analysis is beyond the scope of this paper. But we expect that such a dynamic model yields qualitatively the same results.

$$\frac{\partial \pi_i}{\partial w_i} = -\frac{1}{t} w_i A + \frac{1}{t} w_i^2 + \frac{1}{N} A - \frac{1}{N} w_i - \frac{1}{N^2} w_i = 0$$

$$\frac{\partial \pi_A}{\partial w_A} = -N^2 w_i A + N^2 w_i^2 + NtA - Ntw_i - tw_i = 0$$

$$\frac{\partial \pi_A}{\partial w_A} = N^2 w_i^2 + [-Nt - t - N^2 A] w_i + NtA = 0$$

$$w_{i_{1,2}} = \frac{(Nt + t + N^2 A) \pm \sqrt{(Nt + t + N^2 A)^2 - 4N^3 tA}}{2N^2}$$

Checking the second-order condition:

$$\frac{\partial^2 \pi_i}{\partial w_i^2} = \frac{\partial^2 D_i}{\partial w_i^2} w_i p + 2\frac{\partial D_i}{\partial w_i} p + 2\frac{\partial D_i}{\partial w_i} w_i \frac{dp}{dw_i} + 2D\frac{dp}{dw_i} + 2D_i w_i \frac{d^2 p}{dw_i^2} < 0.$$

Evaluating at a symmetric equilibrium yields:

$$-\frac{1}{t} p^* + 2\frac{1}{t} w_i^* \frac{1}{N} - \frac{1}{N^2} < 0$$

$$p > 2w_i^* \frac{1}{N} - \frac{t}{N^2}$$

$$A - w_i^* > 2w_i^* \frac{1}{N} - \frac{t}{N^2}$$

$$w_i < \frac{t + AN^2}{N^2 + 2N}$$

Now plugging in

$$w_i^* = \frac{(Nt + t + N^2 A) - \sqrt{(Nt + t + N^2 A)^2 - 4N^3 tA}}{2N^2}$$

This gives

$$\frac{(Nt + t + N^2 A) - \sqrt{(Nt + t + N^2 A)^2 - 4N^3 tA}}{2N^2} < \frac{t + AN^2}{N^2 + 2N}$$

$$\frac{(Nt + t + N^2 A) - \sqrt{(Nt + t + N^2 A)^2 - 4N^3 tA}}{2N^2} - \frac{t + AN^2}{N^2 + 2N} < 0$$

$$N^2 t + Nt + 2t - N^3 A + 2N^2 A - (N + 2)\sqrt{(Nt + t + N^2 A)^2 - 4N^3 tA} < 0$$

Case 1: If $N^2 t + Nt + 2t - N^3 A + 2N^2 A \leq 0$, then SOC is fulfilled.

Case 2: If $N^2 t + Nt + 2t - N^3 A + 2N^2 A > 0$, then we have to show that

$$N^2 t + Nt + 2t - N^3 A + 2N^2 A < (N+2)\sqrt{(N^2 t^2 + 2Nt^2 - 2N^3 tA + t^2 + 2tN^2 A + N^4 A^2)}$$

$$(N^2t+Nt+2t-N^3A+2N^2A)^2 < (N+2)^2\left(N^2t^2+2Nt^2-2N^3tA+t^2+2tN^2A+N^4A^2\right)$$

$$-8N^2t^2-8Nt^2-8N^5A^2-4N^3t^2+8N^4tA<0$$

$$2Nt^2+2t^2+2N^4A^2+N^2t^2-2N^3tA>0$$

We know that the term $2Nt^2+2t^2+2N^4A^2+N^2t^2-2N^3tA$ is always positive or negative because the Determinant with respect to t and A is negative. We see immediately that the term is positive for $N=1$ $(2t^2+2t^2+2A^2+t^2-2tA = 4t^2+A^2+(A-t)^2>0)$. Furthermore, we can show that the term $2Nt^2+2t^2+2N^4A^2+N^2t^2-2N^3tA$ is increasing in N. Therefore it is positive for all $N>1$.

$$\frac{\partial(2Nt^2+2t^2+2N^4A^2+N^2t^2-2N^3tA)}{\partial N} = 2t^2+8N^3A^2+2Nt^2-6tN^2A>0$$

$$8N^3A^2+2Nt^2-6tN^2A>0$$

$$4N^3A^2+Nt^2-3tN^2A>0$$

$$4N^2A^2+t^2-3tNA>0$$

$$(2NA-t)^2+NAt>0$$

Now showing that

$$w=\frac{(Nt+t+N^2A)+\sqrt{(Nt+t+N^2A)^2-4N^3tA}}{2N^2}$$

is a minimum.

Plugging into

$$w_i^* < \frac{t+AN^2}{N^2+2N}$$

yields

$$N^2t+Nt+2t-N^3A+2N^2A+(N+2)\sqrt{(Nt+t+N^2A)^2-4N^3tA}>0$$

Case 1: If $N^2t+Nt+2t-N^3A+2N^2A>0$, then SOC is not fulfilled.

Case 2: If $N^2t+Nt+2t-N^3A+2N^2A\leq 0$, then we have to show

$$(N^2t+Nt+2t-N^3A+2N^2A)^2<(N+2)^2((Nt+t+N^2A)^2-4N^3tA).$$

We know that this is true. See proof of maximum. ∎

Proof of Proposition 3.2

For Case 2:

$$\frac{\partial \pi_i}{dw_i}\mid_{w_i=w_{-i}-\frac{t}{2}}=\frac{1}{2t}\left(-7w_{-i}t+3t^2+2w_{-i}^2+3At-2Aw_{-i}\right)=0$$

$$\rightarrow w_{-i}=\frac{7}{4}t+\frac{1}{2}A\pm\frac{1}{4}\sqrt{(25t^2+4At+4A^2)}$$

Hence, if player i plays $w_i=w_{-i}-\frac{t}{2}$, then the FOC has two nulls. We can exclude the point $w_{-i}=\frac{7}{4}t+\frac{1}{2}A+\frac{1}{4}\sqrt{(25t^2+4At+4A^2)}$ because

$$w_{-i}=\frac{7}{4}t+\frac{1}{2}A+\frac{1}{4}\sqrt{(25t^2+4At+4A^2)}>w_{-i}^H=\frac{A}{2}-\frac{t}{2}$$

and therefore it falls in the range of Case 1.
Now we check if $w_i=\frac{7}{4}t+\frac{1}{2}A-\frac{1}{4}\sqrt{(25t^2+4At+4A^2)}-\frac{t}{2}$ is indeed a maximum given $w_{-i}=\frac{7}{4}t+\frac{1}{2}A-\frac{1}{4}\sqrt{(25t^2+4At+4A^2)}$. Checking the SOC at the value $w_i=\frac{7}{4}t+\frac{1}{2}A-\frac{1}{4}\sqrt{(25t^2+4At+4A^2)}-\frac{t}{2}$ and $w_{-i}=\frac{7}{4}t+\frac{1}{2}A-\frac{1}{4}\sqrt{(25t^2+4At+4A^2)}$ yields

$$\frac{\partial^2\pi_i}{\partial w_i^2}=\frac{1}{4t}\left(6A+23t-7\sqrt{(25t^2+4At+4A^2)}\right).$$

To show $\frac{1}{4t}\left(6A+23t-7\sqrt{(25t^2+4At+4A^2)}\right)<0$. Simplifying yields

$$-160A^2+80At-696t^2<-(\frac{1}{2}A)^2+At-t^2=-(\frac{1}{2}A-t)^2<0$$

Now more generally: Given the nulls at $w_{-i}=\frac{7}{4}t+\frac{1}{2}A\pm\frac{1}{4}\sqrt{(25t^2+4At+4A^2)}$ for $w_i=w_{-i}-\frac{t}{2}$ and the fact that the SOC is negative at the point $w_i=w_{-i}-\frac{t}{2}$ with $w_{-i}=\frac{7}{4}t+\frac{1}{2}A-\frac{1}{4}\sqrt{(25t^2+4At+4A^2)}$ we have that the FOC is negative for all $w_i=w_{-i}-\frac{t}{2}$ with

$$w_{-i}\in]\frac{7}{4}t+\frac{1}{2}A-\frac{1}{4}\sqrt{(25t^2+4At+4A^2)};\frac{7}{4}t+\frac{1}{2}A+\frac{1}{4}\sqrt{(25t^2+4At+4A^2)}[$$

. Therefore, it would payoff to decrease w_i. But we are at a kink of the profit function. Hence, $w_i=w_{-i}-\frac{t}{2}$ is indeed a maximum. Further decreasing w_i does not change the audience size (it is $D_i=1$), but reduces the revenue per viewer.
Next, we show that

$$\frac{7}{4}t+\frac{1}{2}A-\frac{1}{4}\sqrt{(25t^2+4At+4A^2)}<\frac{A}{2}+\frac{t}{2}$$

$$\frac{7}{4}t+\frac{A}{2}-\frac{1}{4}\sqrt{25t^2+4At+4A^2}-(\frac{A}{2}+\frac{t}{2})<0$$

$$\frac{5}{4}t - \frac{1}{4}\sqrt{(25t^2 + 4At + 4A^2)} < 0$$
$$\frac{5}{4}t < \frac{1}{4}\sqrt{(25t^2 + 4At + 4A^2)}$$

This is true for all $t > 0$ and $A > 0$.

For Case 3:

We have to show that the value w_i^{Case3} is indeed an optimum. Therefore, we show that for all values of w_i s.t. $w_{-i} - \frac{t}{2} \leq w_i \leq w_{-i} + \frac{t}{2}$ the second order condition is fulfilled given that $w_{-i} < \frac{7}{4}t + \frac{1}{2}A - \frac{1}{4}\sqrt{(25t^2 + 4At + 4A^2)}$.

$$\frac{\partial^2 \pi_i}{\partial w_i^2} = -\frac{1}{2}\frac{4At - 12w_i t + 24w_i^2 - 36w_i w_{-i} + 4w_{-i}t + 12w_{-i}^2 + t^2}{t^2}$$

Given $w_i = w_i + x$ with $-\frac{t}{2} \leq x \leq \frac{t}{2}$, $\frac{\partial^2 \pi_i}{\partial w_i^2} < 0$ if

$$4At - 8w_{-i}t - 12tx + 12w_{-i}x + 24x^2 + t^2 > 0$$

This term is decreasing in w_i due to $-\frac{t}{2} \leq x \leq \frac{t}{2}$. The term is zero if

$$w_{-i}^{zero} = \frac{4At - 12tx + 24x^2 + t^2}{8t - 12x}$$

Hence, it is sufficient to show that $w_{-i}^{zero} > w_{-i}^{C}$

$$w_{-i}^{zero} > \frac{4At - 12t * 0 + 24 * 0^2 + t^2}{8t - 12(-\frac{t}{2})} > w_{-i}^{C}$$

$$\frac{2}{7}A + \frac{1}{14}t > \frac{7}{4}t + \frac{1}{2}A - \sqrt{25t^2 + 4At + 4A^2}$$
$$\frac{3}{14}A + \frac{47}{28}t < \sqrt{25t^2 + 4At + 4A^2}$$
$$\frac{9}{196}A^2 + \frac{141}{196}At + \frac{2209}{784}t^2 - (25t^2 + 4At + 4A^2) = -\frac{775}{196}A^2 - \frac{643}{196}At - \frac{17391}{784}t^2 < 0$$

■

Proof of Proposition 3.3

With $w_A = w_B = w_i$ it follows:

$$\frac{\partial^2 \pi}{\partial w_A \partial w_B} = \frac{A - w_i}{t} - \frac{1}{4} - \frac{w_i}{t}$$

Strategic substitutes:

$$w_i^* = \frac{3}{8}t + \frac{1}{2}A - \frac{1}{8}\sqrt{9t^2 - 8At + 16A^2} \; i \in \{A, B\}$$

$$\frac{\partial^2 \pi}{\partial w_A \partial w_B} = -1 + \frac{1}{4t}\sqrt{(9t^2 - 8At + 16A^2)} < 0$$

$$\frac{1}{4t}\sqrt{(9t^2 - 8At + 16A^2)} < 1$$

$$\frac{1}{16t^2}\left(9t^2 - 8At + 16A^2\right) < 1$$

$$9t^2 - 8At + 16A^2 < 16t^2$$

$$-7t^2 - 8At + 16A^2 < 0$$

$$\frac{d(-7t^2 - 8At + 16A^2)}{dt} = -14t - 8A < 0$$

$$-7t^2 - 8At + 16A^2 = 0$$

$$t_1 = -\frac{4}{7}A - \frac{8}{7}\sqrt{2}A < 0$$

$$t_2 = -\frac{4}{7}A + \frac{8}{7}\sqrt{2}A > 0$$

Therefore given A, the function is positive for all $t \in]t_1; t_2[$. Hence, we have strategic substitutes if

$$t > -\frac{4}{7}A + \frac{8}{7}\sqrt{2}A = \frac{4}{7}A(2\sqrt{2} - 1)$$

Otherwise we have strategic complements. ■

Proof of Lemma 3.1

We show $\frac{\partial w_i^*}{\partial t} > 0$ by using the implicit function theorem.

$$d\frac{\partial \pi_i}{\partial w_i}\,|_{w_1^* = \ldots = w_N^*} = \frac{\partial \frac{\partial \pi_i}{\partial w_i}}{\partial t}dt + \frac{\partial^2 \pi_i}{\partial w_i^2}dw_i = 0$$

$$\frac{dw_i^*}{dt} = -\frac{p w_i^* \frac{\partial D_i}{\partial w_i \partial t}}{[SOC]}$$

$$\frac{dw_i^*}{dt} = -\frac{pw_i^* \frac{1}{t^2}}{[SOC]} > 0$$

We show $\frac{\partial w_i^*}{\partial A} > 0$ by using the implicit function theorem.

$$d\frac{\partial \pi_i}{\partial w_i} \mid_{w_1^*=...=w_N^*} = \frac{\partial \frac{\partial \pi_i}{\partial w_i}}{\partial A} d + A\frac{\partial^2 \pi_i}{\partial w_i^2} dw_i = 0$$

$$\frac{dw_i^*}{dA} = -\frac{pw_i^* \frac{\partial D_i}{\partial w_i \partial A}}{[SOC]}$$

$$\frac{dw_i^*}{dA} = -\frac{-\frac{1}{t}\frac{\partial p}{\partial A} w_i^* + \frac{1}{N}\frac{\partial p}{\partial A}}{[SOC]}$$

$$\frac{dw_i^*}{dA} = -\frac{\frac{\partial p}{\partial A}\frac{\partial (D_i w_i)}{\partial w_i}}{[SOC]} > 0$$

because

$$\frac{\partial \pi_i}{\partial w_i} \mid_{w_1^*=...=w_N^*} = p\overbrace{\frac{\partial (D_i w_i)}{\partial w_i}}^{+} + \overbrace{\frac{\partial p}{\partial w_i} D_i w_i}^{-} = 0$$

■

Proof of Proposition 3.4

The first order condition at a symmetric equilibrium $w_1^* = ... = w_N^*$ is

$$\frac{\partial \pi_i}{\partial w_i} = -\frac{1}{t} pw_i^* + \frac{1}{N}\frac{\partial (w_i^* p)}{\partial w_i} = 0$$

By totally differentiating this first order condition we get

$$[SOC]dw_i + [-\frac{1}{t}\frac{\partial (w_i^* p)}{\partial N} - \frac{1}{N^2}\frac{\partial (w_i^* p)}{\partial w_i} + \frac{1}{N}\frac{\partial \frac{\partial (pw_i^*)}{\partial w_i}}{\partial N}]dN = 0$$

$$[SOC]dw_i + [-\frac{1}{N^2}\frac{\partial (w_i^* p)}{\partial w_i} + \frac{1}{N}\frac{\partial \frac{\partial (pw_i^*)}{\partial w_i}}{\partial N}]dN = 0$$

$$\frac{\partial w_i^*(N)}{\partial N} = -\frac{-\frac{1}{N^2}\frac{\partial (w_i^* p)}{\partial w_i} + \frac{1}{N}\frac{\partial \frac{\partial (pw_i^*)}{\partial w_i}}{\partial N}}{[SOC]}$$

Therefore, $\frac{\partial w_i^*(N)}{\partial N} > 0$ if

$$-\frac{1}{N^2}\frac{\partial (w_i^* p)}{\partial w_i} + \frac{1}{N}\frac{\partial \frac{\partial (pw_i^*)}{\partial w_i}}{\partial N} > 0$$

$$\frac{\partial \frac{\partial (p w_i^*)}{\partial w_i}}{\partial N} > \frac{1}{N}\frac{\partial (w_i^* p)}{\partial w_i}$$
$$\frac{\partial (p - \frac{1}{N} w_i^*)}{\partial N} > \frac{1}{N}(p - \frac{1}{N} w_i^*)$$
$$\frac{1}{N^2} w_i^* > \frac{1}{N}(p - \frac{1}{N} w_i^*)$$
$$\frac{2}{N} w_i^* > p$$
$$\frac{2}{N} w_i^* > (A - w^*)$$
$$w_i^* > \frac{AN}{2+N}$$

Plugging in w_i^*, one sees that this is the case if $t > \frac{2N^2 A}{2+N}$

■

Proof of Proposition 3.6

It remains to show:

$$\lim_{N\to\infty} (\frac{\partial WF}{\partial N} = \frac{\partial w_i^*}{\partial N}(A - w_i^* - 1) + \frac{t}{4N^2} \leq 0)$$

Using the Implicit Function Theorem we know that

$$\frac{\partial w_i^*}{\partial N} = -\frac{\frac{1}{N^2}(\frac{N+2}{N} w_i^* - A)}{-\frac{A}{t} + \frac{N+2}{Nt} w_i^* - \frac{1}{N^2}}$$

This leads to

$$\frac{\partial WF}{\partial N} = -\frac{\frac{1}{N^2}(\frac{N+2}{N} w_i^* - A)}{-\frac{A}{t} + \frac{N+2}{Nt} w_i^* - \frac{1}{N^2}}(A - w_i^* - 1) + \frac{t}{4N^2}$$

$$\lim_{N\to\infty} \frac{1}{N^2}\{\frac{-(\frac{N+2}{N} w_i^* - A)}{-\frac{A}{t} + \frac{N+2}{Nt} w_i^* - \frac{1}{N^2}}(A - w_i^* - 1) - \frac{t}{4}\} = 0$$

due to

$$\lim_{N\to\infty} w_i^* = \frac{(\frac{t}{N} + \frac{t}{N^2} + A) - \sqrt{\frac{(Nt+t+N^2A)^2 - 4N^3 tA}{N^2}}}{2} = 0$$

Now to show that we approach zero from below with $A > \frac{5}{4}$

$$\lim_{N\to\infty} \{\frac{-(\frac{N+2}{N} w_i^* - A)}{-\frac{A}{t} + \frac{N+2}{Nt} w_i^* - \frac{1}{N^2}}(A - w_i^* - 1) - \frac{t}{4}\} < 0$$

$$\frac{A}{-\frac{A}{t}}(A-1)+\frac{t}{4}<0$$

$$A>\frac{5}{4}$$

■

Chapter 4

Patent Settlements and Market Entry*

4.1 Introduction

Licensing of intellectual property rights is a two-edged sword. On the one hand, it can be procompetitive and welfare enhancing. Firms are able to allocate property rights efficiently and this increases social welfare. Furthermore, licensing can be an efficient way to settle patent disputes out of court because the licensor and the licensee can avoid uncertainty and litigation costs. Moreover, courts are strongly favoring settlements because these conserve public administrative and judical resources.[1]

On the other hand, some licensing arrangements may raise antitrust concerns by reducing competition. This is widely recognized and, for example, the FTC's Guidelines for Intellectual Property Rights state:

"Antitrust concerns may arise when a licensing arrangement harms competition among entities that would have been actual or likely potential competitors in a relevant market in the absence of a license (entities in a "horizontal relationship"). A restraint in a licensing arrangement may harm such competition, for example, if it

*This chapter is based on joint work with Jing-Yuan Chiou, University of Toulouse.

[1] O'Roureke and Brodley (2003), Shapiro (2003).

facilitates market division or price fixing."[2]

In this chapter we discuss the needed antitrust limits to patent settlements. In particular we analyze the implications of these antitrust limits with endogenous market entry, which has not been considered in the literature so far. The basic idea of the model is that a potential entrant, who has to incur fixed costs for entering the market, anticipates the payoff from a potential settlement for his entry decision. This payoff depends on the kind of settlement the parties are allowed to agree upon and hence on the antitrust limits to such patent settlements. From a social point of view, this raises the question what is the appropriate antitrust limit to patent settlements? On the one hand, the parties should be able to enjoy high enough profits by settling their dispute in order to prevent inefficient court decisions. This goes in the direction of low antitrust limits. On the other hand, the limit should not be too low because this may lead to inefficient market entry without increasing consumer surplus in the appropriate way.

In order to analyze different antitrust limits with endogenous market entry, we build a model with a patentholder already active in the market and an entrant who has to decide whether to enter the market or not. The entrant has to incur fixed costs for entering the market. If the entrant enters, then he infringes the patent of the patentholder with a certain probability. We assume that the parties can solve such a patent dispute through a court decision or by settling it out of court. The possible profits from a settlement depend on the different kinds of antitrust limits, which we will consider.

We find that, from a social point of view, having no limits to patent settlements is not desirable at all because firms use such settlements to split up monopoly profits and entry costs occur additionally. Shapiro (2003) proposes a limit that leaves consumers equally well off and increases the profits of the firms compared to a lawsuit. We show that even such an antitrust limit may generate the wrong incentives to enter the market and hence may decrease social welfare. Interestingly, we find that a very restrictive antitrust limit can be optimal from a social point of view. Furthermore, such a restrictive limit can maximize the profits of the patentholder. Hence, we show that maximizing R&D incentives and having restrictive antitrust limits can go hand in hand and do not have to work against each other.

[2] FTC (2002)

Several authors stress the tension between intellectual property rights and antitrust.[3] Maurer and Scotchmer (2006) suggest three unifying principles for acceptable terms on licensing. Firstly, "Profit Neutrality" implies that the possible reward of a patent should not depend on the pantentee's ability to work the patent himself. Secondly, "Derived Reward" holds that the patentholder's profit should be earned from the social value created by the invention. Thirdly, "Minimalism" holds that licensing contracts should not contain more restrictions than are necessary to achieve neutrality. However, if it is not obvious whether a party infringes a valid patent or not, a problem arises with the principal "derived reward". In this case it is not obvious what the appropriate reward should be.

To illustrate this point, consider the following example. Suppose that firm A produces one good and that firm B produces one good. Both goods are different, but they have some similar or related parts. These two goods are substitutes for the consumers. We assume that no other substitute exists. Both firms compete against each other. Now suppose that firm A receives a patent that protects a certain feature of his product. It is not clear to both parties whether firm B infringes A's patent. Firstly, it is not obvious that A's patent is indeed valid because of possible prior art that is yet unknown to them. Secondly, it is not obvious that A's patent covers B's product. For simplicity, let us assume that both parties agree that with probability α the patent is indeed valid and B's product infringes A's patent.

Suppose that the firms decide to merge in order to solve the patent dispute or that they write a settlement contract that replicates the merger outcome.[4] This merger, or the corresponding settlement, will certainly not be accepted by the antitrust authorities if no patent is involved. This raises the question: Should such a merger or settlement contract be possible only because there is an "uncertain" patent infringement involved?

Applying the mentioned principle "derived reward" raises the question concerning the right patent's reward that the society should grant. The answer to this question is not obvious. Given that B is indeed infringing A's valid patent, monopoly profits are justifiable and a merger or the corresponding settlement would be perfectly fine. In contrast, given B does not infringe or given A's patent is not valid,

[3]among others: Choi (2005), Farell and Shapiro (2005), Hovenkamp, Janis and Lemley (2003), Lemely and Shapiro (2005), Maurer and Scotchmer (2006), O'Rourke and Brodley (2003), Shapiro (2003)

[4]Shapiro (1985),(2003) discusses several possible contracts. See also Maurer and Scotchmer(2006).

these actions should be illegal. Hence, one possibility would be to let a court decide whether A has indeed a valid patent and B is indeed infringing A's patent to clarify the situation.

But such a court decision is completely inefficient from a welfare point of view. Firms have to pay lawyers and some of their internal resources are bounded because managers have to take care of the lawsuit. Furthermore, the society has to afford a big enough court system in order to handle all these cases. In addition, the welfare function is usually concave in the price. Hence, the society is "risk-averse" (Shapiro (2003), Gilbert and Shapiro (1990)). Therefore, welfare is higher under the certain price $\widehat{p} = \alpha p^m + (1-\alpha)p^c$ than under the price p^m with probability α and the price p^c with probability $1-\alpha$. A court decision is a kind of lottery because either the patentholder wins and there is a monopoly price p^m, or the entrant wins and there is a duopoly price p^c. Hence, a settlement that leads to a price that is between the monopoly and duopoly price can be preferable to a court decision by avoiding uncertainty.

Meurer (1989) considers, as we do, settlements between a patentee and a competitor under different antitrust limits. Nevertheless, his work differs from ours in two respects. Firstly, he assumes that the competitor is already in the market, or equivalently that the entry costs are zero, whereas we look explicitly at the entry decision in the presence of entry costs. Secondly, he gives the patentee the whole bargaining power. Therefore, the competitor gets his outside option. In contrast, we assume a symmetric Nash-bargaining, which leaves room for strategic actions of the potential entrant.

Shapiro (2003) introduces a new idea concerning antitrust limits to patent settlements. He sets up the rule that "the proposed settlement generates at least as much surplus for consumers as they would have enjoyed had the settlement not been reached and the dispute instead been resolved through litigation".[5] Shapiro (2003) argues that this rule fully respects the patentholder's rights because he gets at least the same payoff as without a settlement. Furthermore, his rule would lead to efficient settlements and therefore would increase social welfare. Shapiro (2003) states that the observed enforcement actions of the DOJ and FTC are consistent with his rule. For example, in the case of Schering-Plough, the FTC compared the amount of competition that occurred under the considered settlement to the amount

[5] Shapiro (2003)

of competition that was likely to occur in the absence of such a settlement. In this particular case the FTC concluded that the settlement unreasonably restrained commerce.[6] However, Shapiro (2003) does not consider market entry. By extending his framework, we evaluate the Shapiro policy, beside other antitrust limits, under endogenous market entry.

Choi (2005) considers the Shapiro policy with cross-licensing. Instead of considering market entry, he asks whether the parties obtain the right incentives to litigate under the Shapiro policy. Considering cross-licensing, he shows that the parties may have no incentive to sue each other, which leaves the society worse off. Therefore, the Shapiro policy generates the wrong litigation incentives for firms that have entered the market. We do not consider cases of cross-licensing and so avoid problems related to litigation incentives in order to fully concentrate on the problems concerning market entry.

Waterson (1990) considers the entry and location decision of a firm in the presence of a patent. In contrast to our work, he does not consider patent settlements and hence does not deal with antitrust limits to licensing contracts.

This chapter proceeds as follows. In the next section we set up the model. In Section 3 we assume that the entrant has to incur the full entry costs before a court decides whether the entrant is infringing a valid patent or not. In Section 4 we consider the case where the entrant can resolve this uncertainty before he incurs the full entry costs. In Section 5 we allow the parties to avoid sinking the entry costs by settling out of court. Section 6 links our findings to the actual debate about the patent system and Section 7 concludes.

4.2 Model

We assume that there are two firms: the patentholder P and a potential entrant E. The patentholder is already operating in the market. The potential entrant E has to incur an entry cost f to enter. P can sue E for patent infringement, or equivalently, E can challenge P's patent. We denote by α the probability that the court finds that E infringes P's valid patent.

[6] Schering-Plough I, 2003, WL 22989651

We denote by π^m monopoly profits in the market. If E does not enter the market or if a court rules in the patentee's favor, P is a monopolist and his profit is π^m. In this case consumer surplus is denoted by CS^m and social welfare is denoted by $W^m = CS^m + \pi^m$.

By π^c we denote each firm's profit with unconstrained competition between them. If E enters and the court finds that he does not infringe a valid patent of P, then each firm's profit is π^c. Social welfare is in this case $W^c = CS^c + 2\pi^c - f$. We assume that $\pi^m \geq 2\pi^c \geq 0$, $CS^c \geq CS^m$ and $CS^c + 2\pi^c > CS^m + \pi^m$.

Firms can avoid court rulings by settling their patent infringement dispute. We assume, as Shapiro (2003) and Choi (2005), zero litigation costs without loss of generality. Introducing litigation costs would only change the threshold values of entry costs under which settlements or lawsuits occur, which leads to no new insights. Beside avoiding litigation costs, settlements have two additional features on which we would like to concentrate.

Firstly, a settlement avoids uncertainty. Absence of a settlement, a court decision solves a patent dispute. That court decision has two possible outcomes. Either the patentholder wins and we have a monopoly, or the entrant wins and we have a duopoly. With a concave welfare function, which we assume throughout the chapter, the society is "risk-avers".[7] Instead of having such a kind of lottery by a court decision, a settlement leads to a certain price and hence generates an efficiency gain.

Secondly, the parties can use settlements to prevent competition that would have arisen had the patentholder lost the lawsuit. Hence, even if the patent is very weak or worthless, the parties can enjoy extraordinary profits by using settlement contracts to sustain collusion. One can consider different possible designs of such settlements like including price fixing, quantity restrictions, per-unit royalties, or territory agreements. The parties can split up these collusive profits by appropriate side-payments.

We look at three different antitrust limits to patent settlements, which we introduce in the following. As the most restrictive policy, we consider that firms are not allowed to use a licensing contract to collude. Such an antitrust approach would de-

[7]For example a falling demand function and a weakly convex cost function lead to a concave welfare function.

clare contracts as illegal that include, for example, price fixing, quantity restrictions, per-unit royalties, or territory agreements. Only fixed side payments are allowed. Meurer (1989) calls this "lump-sum policy". Under such a lump-sum policy, the firm's profit is π^c before the fixed side payment.

The "laissez-faire policy" is the opposite to a lump-sum policy.[8] All licensing contracts are legal under such an antitrust limit. Given such a policy, the firms are able to realize the highest possible joint profit π^m. Afterwards they can use fixed side payments to split up π^m in the desired way.

Shapiro (2003) proposes a constrained settlement. He sets up a "Shaprio policy" to strike a balance between the lump-sum policy and the laissez- faire policy. This Shapiro policy ensures that the consumers are always equally well off compared to a lawsuit. In particular the consumer surplus after the settlement CS^s has to be equal to the expected consumer surplus of a court decision

$$CS^s = \alpha CS^m + (1-\alpha)CS^c. \tag{4.1}$$

Therefore, the firms are allowed to collude on a price p^s in such a way that $CS(p^s) = CS^s$. We denote by Π^s the corresponding joint profits with p^s. Due to the concave welfare function, a settlement realizes an efficiency gain because it avoids uncertainty. We denote by λ this efficiency gain. In particular we can define λ in the following way

$$\Pi^s - \lambda = \alpha\pi^m + (1-\alpha)2\pi^c, \tag{4.2}$$

because the firms are allowed to appropriate λ under the Shapiro policy. This is the case because the consumers only have to be equal off.

Bargaining Scenarios

We assume Nash-Bargaining with equal bargaining power. For the patentholder's bargaining payoff and for the entrant's bargaining payoff, it is important whether the bargaining takes place before or after market entry. The entrant prefers an earlier bargaining because this allows him to include his entry costs in the bargaining. In contrast, the patentholder prefers a bargaining after the entrant has sunk his entry

[8] Meurer (1989)

costs because this increases his payoff compared to an early bargaining. Therefore, the entrant tries to achieve an early bargaining and the patentholder tries to have a late bargaining. This raises the following question: When does the entrant has a credible threat to go to court? As soon as the entrant has this credible threat, the patentholder can no longer deny a bargaining.

According to the actual law situation in the US, the validity of a patent may be challenged only by an "alleged infringer as an affirmative defense or counterclaim to an infringement action brought by the patentee, or by a declaratory judgment plaintiff, who must show (1) an explicit threat or other action by the pantentee which creates a reasonable apprehension on the part of the declaratory judgment plaintiff that it will face an infringement suit, and (2) present activity by the declaratory judgement plaintiff which could constitute infringement, or concrete steps taken by the declaratory judgment plaintiff with the intend to conduct such activity".[9] Given such a legal situation, the patentholder can force the entrant to wait with the bargaining until the entrant has fully sunk his entry costs. Hence, in Scenario 1 we consider a game where the entrant has to incur the full entry costs before a potential lawsuit or settlement takes place.

In contrast, in Europe everybody can challenge a patent without any restrictions. Hence, an entrant does not need not to incur the full entry costs in order to challenge the patent. Therefore, in Scenario 2 we consider a game where the entrant can challenge the patent whenever he wants. Nevertheless, the entrant has to incur the entry costs after a settlement.

In Scenario 3 we assume that the parties can agree on a settlement in such a way that the potential entrant stays off the market and receives a compensation payment from the patentholder. Therefore, in this scenario the parties can avoid the entry costs by a settlement. Such settlements, which include so called reverse payments, have been quite often seen in the Pharmaceutical Industry.[10]

[9] Teva Pharma. USA, Inc. v. Pfizer, Inc., 395 F.3d 1324, 1330 (Fed. Cir. 2005)
[10] FTC (2002)

4.3 Scenario 1: Bargaining after Entry Costs are sunk

In Scenario 1 we assume that the bargaining or lawsuit takes place after the complete entry costs have been sunk. The exact timing of the game is the following. Firstly, E decides whether to enter the market or not. If he enters, then he has to incur the entry costs f. Secondly, if E has entered, then the two firms E and P bargain for a settlement or go to court. If E has not entered, then P remains monopolist. Figure 4.1 shows the structure of the game.

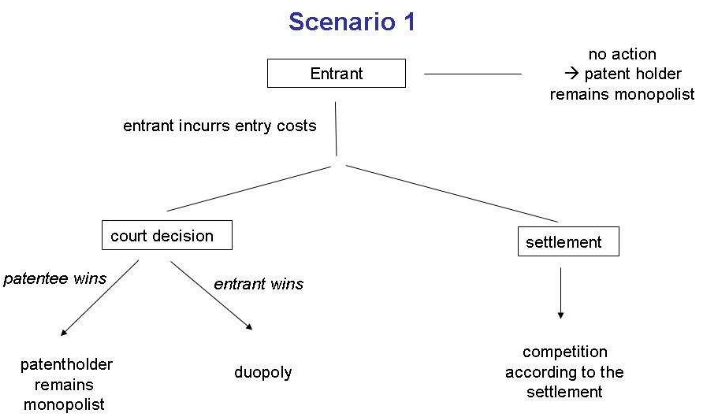

Figure 4.1: Scenario 1

In the following we determine the equilibrium under the different antitrust limits.

Lump-Sum Policy (LS)

Given that only fixed side payments are allowed, there is no scope for a bargaining solution. The firms can only realize the profits $2\pi^c \leq \alpha\pi^m + (1-\alpha)2\pi^c$ by a settlement, which is less than the sum of the outside options. Hence, the parties see each other before court. The entrant anticipates that he has to complete the legal process to survive in the market after incurring f. Market entry yields the following expected profit for the entrant

$$\pi^E = \alpha \cdot 0 + (1-\alpha)\pi^c - f. \tag{4.3}$$

Therefore, entry occurs if $f \leq (1-\alpha)\pi^c$.

Finding 4.1: *Assume that settlements have to obey the lump-sum policy and that a potential lawsuit or bargaining takes place after market entry. If*

- $f \leq (1-\alpha)\pi^c$, *then the entrant sinks* f *and a lawsuit occurs*;
- $f > (1-\alpha)\pi^c$, *then the entrant stays off the market.*

Laissez-Faire Policy (LF)

The parties can split up monopoly profits π^m under a laissez-faire policy. Therefore, there is always scope for a bargaining solution

$$\pi^m - \alpha\pi^m - (1-\alpha)2\pi^c \geq 0. \tag{4.4}$$

The corresponding threat points are

$$\begin{aligned} T_P &= \alpha\pi^m + (1-\alpha)\pi^c, \quad \text{for } P, \text{ and} \\ T_E &= (1-\alpha)\pi^c, \quad \text{for } E. \end{aligned} \tag{4.5}$$

The entrant anticipates his payoff from a settlement. The corresponding entrant's payoff if he enters is

$$\pi^s_E = (1-\alpha)\pi^c + \frac{1}{2}[\pi^m - \alpha\pi^m - (1-\alpha)2\pi^c] - f = \frac{1}{2}(1-\alpha)\pi^m - f. \tag{4.6}$$

Hence, entry occurs if f is smaller than $\frac{1}{2}(1-\alpha)\pi^m$.

Finding 4.2: *Assume that settlements have to obey the laissez-faire policy and that a potential lawsuit or bargaining takes place after market entry. If*

- $f \leq \frac{1}{2}(1-\alpha)\pi^m$, *then the entrant sinks* f *and a settlement occurs*;
- $f > \frac{1}{2}(1-\alpha)\pi^m$, *then the entrant stays off the market.*

Shapiro Policy (SP)

Suppose that the antitrust authority approves every settlement that obeys the Shapiro policy. Given the Shapiro policy, the firms have to respect the expected consumer surplus from a lawsuit. Hence, the cooperative value of the bargaining is $\Pi^s = \alpha\pi^m + (1-\alpha)2\pi^c + \lambda$. We see that there is always scope for a bargaining solution

$$\alpha\pi^m + (1-\alpha)2\pi^c + \lambda - (\alpha\pi^m + (1-\alpha)2\pi^c) = \lambda \geq 0. \tag{4.7}$$

This yields the following entrant's profit from market entry

$$\pi_E^s = (1-\alpha)\pi^c + \frac{1}{2}[\Pi^s - \alpha\pi^m - (1-\alpha)2\pi^c] - f = \frac{1}{2}\lambda + \pi^c(1-\alpha) - f. \tag{4.8}$$

Finding 4.3: *Assume that settlements have to obey the Shapiro policy and that a potential lawsuit or bargaining takes place after market entry. If*

- $f \leq \frac{1}{2}\lambda + \pi^c(1-\alpha)$, *then the entrant sinks f and a settlement occurs*;
- $f > \frac{1}{2}\lambda + \pi^c(1-\alpha)$, *then the entrant stays off the market f.*

Discussion

It is obvious that a laissez-faire policy generates the highest level of market entry and an entrant prefers this antitrust limit to the other two. From the entrant's point of the view, the lump-sum policy is the worst case. Intuitively, the laissez-faire policy leads to the highest bargaining surplus and thus yields the highest profit for the entrant. In contrast, the lump-sum policy minimizes the bargaining surplus and therefore yields the lowest profit for the entrant.

A ranking of the different antitrust limits is not obvious for the patentholder. For some values of f it depends on the antitrust limit whether the entrant enters the market or not. Hence, the patentholder's ranking depends on the actual value of f. We summarize this in the following proposition.

Proposition 4.1 *Assume that a potential lawsuit or bargaining takes place after market entry. The patentholder*

- *prefers LF to SP to LS if*
$$f < \pi^c(1-\alpha);$$
- *prefers LS to LF to SP if*
$$\pi^c(1-\alpha) \leq f < \frac{1}{2}\lambda + \pi^c(1-\alpha);$$
- *prefers LS and SP to LF and is indifferent between LS and SP if*
$$\frac{1}{2}\lambda + \pi^c(1-\alpha) \leq f < \frac{1}{2}(1-\alpha)\pi^m;$$
- *is indifferent between LS, SP and LF if*
$$\frac{1}{2}(1-\alpha)\pi^m \leq f.$$

Proposition 4.1 states that even an antitrust limit that leads to an ex post Pareto improvement, as the Shapiro policy compared to the lump-sum policy, can decrease the patentholder's profit and increase the entrant's profit ex ante. Intuitively, a less restrictive limit increases the entrant's profit. Hence, market entry occurs for a higher level of f. This market entry implies that the patentholder has to pass some profits from his patent to the entrant, which leaves the patentee worse off. Therefore, a patentholder prefers a restrictive limit if this limit effectively prevents market entry. Otherwise, he prefers the less restrictive limit to maximize the bargaining surplus.

We have seen that low antitrust limits may trigger additional market entry. From a social point of view, it is questionable if this market entry is welfare enhancing.

Proposition 4.2 *Assume that a potential lawsuit or settlement takes place after market entry. A laissez-faire policy never increases the social welfare compared to a lump-sum policy or to the Shapiro policy. Hence, a laissez-faire policy is weakly dominated from a social point of view.*
In contrast, in comparison to the lump-sum policy, the Shapiro policy

- *increases welfare if the entry costs are low* $f \leq \pi^c(1-\alpha)$*;*
- *increases or decreases welfare if the entry costs are medium* $\pi^c(1-\alpha) < f < \frac{1}{2}\lambda + \pi^c(1-\alpha)$*;*

- *does not change welfare if the entry costs are high* $f \geq \frac{1}{2}\lambda + \pi^c(1-\alpha)$.

Proof. See Appendix. ■

Given a laissez-faire policy, the consumer surplus is always CS^m and total profits are $\pi^m - f$ after entry. Furthermore, this regime triggers additional market entry. Due to the fact that neither the consumer surplus nor the the total profits change, these entry costs are wasted resources from a social point of view. Therefore, the social welfare cannot be higher under a laissez-faire policy compared to the other two policies. In the following we compare the Shaprio policy to the lump-sum policy.

If the entry costs f are low, applying the Shapiro policy does not change the market entry decision of the potential market entrant compared to the lump-sum policy. He always enters with low entry costs. If the entrant enters the market anyway, then the Shapiro policy yields a Pareto improvement. Therefore, it is welfare enhancing.

If f is high enough, then the entrant neither enters under the Shapiro policy nor under the lump-sum policy. Therefore, social welfare is the same under both limits.

The welfare analysis is not obvious in the case of medium entry costs. The Shapiro policy leads to more market entry than the lump-sum policy. Therefore, the Shapiro policy can increase consumer surplus by inducing additional entry. On the other hand, this market entry decreases joint profits and entry costs occur. In sum, total welfare may decrease if the entry costs are corresponding high. For an illustration, consider the extreme case where f approaches the upper bound $\frac{1}{2}\lambda + \pi^c(1-\alpha)$. Social welfare is in this case

$$\begin{aligned} W^{LS} &= CS^m + \pi^m, \quad \text{with a lump-sum policy, and} \\ W^{SP} &= CS^s + \Pi^s - f, \quad \text{with the Shapiro policy.} \end{aligned}$$

Simple calculations show that

$$W^s - W^{ns} = (1-\alpha)(CS^c - CS^m) + (\Pi^s - \pi^m) - f. \tag{4.9}$$

If f approaches the upper bound $f = \frac{1}{2}\lambda + \pi^c(1-\alpha)$, then social welfare is lower when the settlement is allowed if

$$(1-\alpha)(CS^c - CS^m) + (\Pi^s - \pi^m) < \frac{1}{2}[\Pi^s - \alpha\pi^m] \tag{4.10}$$

$$\Leftrightarrow \quad (1-\alpha)(CS^c - CS^m) < \frac{1}{2}(\pi^m - \Pi^s) + \frac{1-\alpha}{2}\pi^m \tag{4.11}$$

$$\Leftrightarrow \quad (1-\alpha)(CS^c - CS^m) < (1-\alpha)(\pi^m - \pi^c) - \frac{1}{2}\lambda. \tag{4.12}$$

This will be more likely the case if the demand is more inelastic. Competition only moderately increases consumer surplus and hence the difference $CS^c - CS^m$ is not large. For example, this is satisfied for all $\alpha < 1$ with linear demand $P = 1 - bQ$ and Cournot competition (with zero marginal cost for both firms).[11]

4.4 Scenario 2: Bargaining before Sinking the full Entry Costs

In this second scenario we assume that the entrant can challenge the patent whenever he wants. Therefore, he can even litigate before the full entry costs have been sunk. This is in contrast to Scenario 1. Furthermore, we assume that the entrant can sink parts of the entry costs before the bargaining starts. This can change his bargaining position as we will see later on. Figure 4.2 shows the structure of the game.

Obviously, it makes only sense to go to court for the entrant if his remaining entry costs are smaller than the duopoly profit π^c. Otherwise, even if he succeeds in the lawsuit, he will stay off the market. Hence, the patentholder can deny any bargaining as long as the entrants remaining entry costs are higher than π^c because the entrant's threat point, to go to court and to enter the market if he succeeds, is not credible.

As in Scenario 1, we start by considering the case of a lump-sum policy.

Lump-Sum Policy (LS)

There is no scope for a bargaining solution under a lump-sum policy. Even if the entrant has a credible threat point, the cooperative value of a settlement is lower than the sum of the outside options. It follows that a lawsuit occurs if $f \leq \pi^c$.

[11] see Appendix.

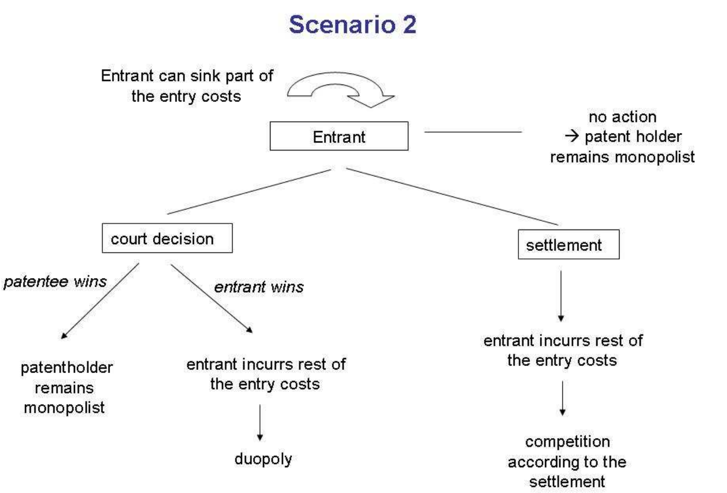

Figure 4.2: Scenario 2

Finding 4.4: *Assume that settlements have to obey the lump-sum policy and that a potential lawsuit or bargaining can take place before the full entry costs have been sunk. If*

- $f \leq \pi^c$, *then a lawsuit occurs*;
- $f > \pi^c$, *then the entrant stays off the market.*

Laissez-Faire Policy (LF)

Given a laissez-faire policy, there can be scope for a bargaining solution. Suppose that $f \leq \pi^c$. This implies that the entrant has a credible threat to sue and thus a credible threat point in the bargaining. Due to the fact that bargaining takes place before market entry, the parties incorporate f in their bargaining. Thus, they split up $\pi^m - f$ in the bargaining stage. This is less than in the corresponding case of Scenario 1 where the bargaining profit was π^m. In particular now it is possible that $\pi^m - f$ is smaller than the sum of the outside options

$$\pi^m - f < (\alpha\pi^m + (1-\alpha)\pi^c) - (1-\alpha)(\pi^c - f); \tag{4.13}$$

$$\Leftrightarrow \pi^m - 2\pi^c < \frac{\alpha}{1-\alpha} f. \tag{4.14}$$

In this case bargaining breaks down and the parties meet each other before court.

The remaining question is if the potential entrant can achieve a settlement with $f > \pi^c$. On a first glance it seems that the entrant has no credible threat to sue and to enter the market if he succeeds before court. Hence, the incumbent can simply deny any bargaining because he knows that the entrant will never enter the market anyway. However, the entrant can strategically sink some parts of the fixed costs in order to get "tough". After sinking parts of the entry costs, to sue and to enter the market if he succeds, can be a credible threat. Hence, after the entrant has sunk a certain part of the entry costs, the incumbent can no longer deny any bargaining.

This raises the question what is the optimal amount of entry costs w that the entrant should sink in order get tough for the bargaining.

Lemma 4.1 *The optimal w that the entrant should sink in order to get tough is $w = f - \pi^c$.*

Proof. See Appendix. ■

Intuitively, increasing w by one unit decreases E's profit exactly by one unit. At the same time, it increases his outside option by $(1-\alpha)$ units and the bargaining surplus by α. Due to the fact that the entrant gets $\frac{1}{2}$ of the bargaining surplus, sinking w makes only sense to get tough. Obviously, the entrant stays off the market and does not sink f if f is too high. In this case it is possible that the payoff from a settlement is not high enough to make up for the market entry costs. Given this, the following equilibrium occurs.

Finding 4.5: *Assume that settlements have to obey the laissez-faire policy and that a potential lawsuit or bargaining can take place before the full entry costs have been sunk.*
If $f \leq \pi^c$, there occurs

- *no settlement but a lawsuit if $\pi^m - 2\pi^c < \frac{\alpha}{1-\alpha} f$;*
- *a settlement if $\pi^m - 2\pi^c \geq \frac{\alpha}{1-\alpha} f$.*

If $\pi^c < f \leq \frac{1}{2}[(1-\alpha)\pi^m + \alpha\pi^c)]$, *there occurs*

- *neither a settlement nor a lawsuit if* $(1-\alpha)\pi^m < \pi^c$;
- *a settlement if* $(1-\alpha)\pi^m \geq \pi^c$.

If $\frac{1}{2}[(1-\alpha)\pi^m + \alpha\pi^c] < f$, *there occurs neither a settlement nor a lawsuit.*

Proof. See Appendix. ■

Shapiro Policy (SP)

Given the Shapiro policy, the firms can split up $\Pi^s - f$ by settling out of court. This is less compared to a laissez-faire policy $\Pi^s - f \leq \pi^m - f$. Thus, the region of f decreases where entry occurs. If $f > \pi^c$, the entrant can have an incentive to sink $w = f - \pi^c$ in order to get tough, as long as his entry costs are not too high. As the intuition is the same as above, we directly state the resulting equilibrium.

Finding 4.6: *Assume that settlements have to obey the Shapiro policy and that a potential lawsuit or bargaining can take place before the full entry costs have been sunk.*
If $f \leq \pi^c$, *there occurs*

- *no settlement but a lawsuit if* $\lambda < \alpha f$;
- *a settlement if* $\lambda \geq \alpha f$.

If $\pi^c < f \leq \pi^c(1 - \frac{1}{2}\alpha) + \frac{1}{2}\lambda$, *there occurs*

- *neither a settlement nor a lawsuit if* $\lambda < \alpha\pi^c$;
- *a settlement if* $\lambda \geq \alpha\pi^c$.

If $\pi^c(1 - \frac{1}{2}\alpha) + \frac{1}{2}\lambda < f$, *there occurs neither a settlement nor a lawsuit.*

Proof. See Appendix. ■

Discussion

We see that the entrant's ranking of the different policies is the same as before. He prefers less restrictive limits to more restrictive limits. Thus, the laissez-faire policy is better than the Shapiro policy, and the Shapiro policy is better than the lump-sum policy.

In contrast, the patentholder trades off the following effects. On the one hand, less restrictive limits imply higher joint profits and therefore higher payoffs from a settlement. On the other hand, higher payoffs may trigger additional market entry, which leaves the patentholder worse off. We summarize the different possible rankings for the patentholder in the following proposition.

Proposition 4.3 *Assume that a potential lawsuit or settlement can take place before the full entry cost have been sunk. The patentholder*

1. *is indifferent between SP, LS and LF if*

   a) there is always no entry: $(1-\alpha)\pi^m < \pi^c \wedge \pi^c < f < \frac{1}{2}[(1-\alpha)\pi^m + \alpha\pi^c]$;

   b) there is always no entry: $\frac{1}{2}[(1-\alpha)\pi^m + \alpha\pi^c] < f$;

   c) there is always a lawsuit: $f < \pi^c \wedge (1-\alpha)(\pi^m - 2\pi^c) < \alpha f$.

2. *prefers LS and SP to LF and is indifferent between LS and SP if there is only a settlement under LF*

   a) $\pi^c(1-\frac{1}{2}\alpha) + \frac{1}{2}\lambda \leq f < \frac{1}{2}[(1-\alpha)\pi^m + \alpha\pi^c] \wedge (1-\alpha)\pi^m \geq \pi^c$; *or*

   b) $\pi^c \leq f < \pi^c(1-\frac{1}{2}\alpha) + \frac{1}{2}\lambda \wedge \lambda < \alpha\pi^c < (1-\alpha)(\pi^m - 2\pi^c)$.

3. *prefers LS to LF to SP if there is no entry under LS and there are settlements under LF and SP*

   $\pi^c < f < \frac{1}{2}\lambda + \pi^c(1-\frac{1}{2}\alpha) \wedge \lambda > \alpha\pi^c$.

4. *prefers LF to SP to LS if there are always settlements*

   $f < \pi^c \wedge \lambda > \alpha f$.

5. *prefers LF to SP and LS and is indifferent between SP and LS if there is a lawsuit under SP and LS and a settlement under LF*

   $f < \pi^c \wedge \lambda < \alpha f < (1-\alpha)(\pi^m - 2\pi^c)$.

Proof. See Appendix. ■

Intuitively, there is never market entry in (1a) and (1b) because either the entry costs are too high or because no scope exists for a bargaining solution. There is always market entry in (1c) without scope for a bargaining solution. Therefore, a lawsuit occurs. In all these cases antitrust limits play no role. In (2) only the laissez-faire policy leads to a settlement. The other two policies yield neither a settlement nor a lawsuit. The Shapiro policy and the lump-sum policy prevent entry because either the entry costs are too high or because no scope exists for a bargaining solution. Therefore, the patentholder prefers the two more restrictive policies because they protect him from passing profits to the entrant. In (3) only the lump-sum policy prevents market entry. The Shapiro policy and the laissez-faire policy lead to market entry. Given market entry under both policies, the patentholder prefers the laissez-faire policy as the less restrictive regime compared to the Shapiro policy. In (4) no antitrust limit prevents market entry and all policies yield a settlement. Thus, the patentholder prefers less restrictive limits to more restrictive limits. In (5) only a laissez-faire policy leads to a settlement and the other two limits lead to a lawsuit. Thus, the patentholder prefers the laissez-faire policy and is indifferent between the other two policies.

Again, from a social point of view, we derive that the laissez-faire policy is weakly dominated by the Shapiro policy and the lump-sum policy. Intuitively, market entry does not change joint profits or consumer surplus under a laissez-faire policy. Thus, only the additional entry costs occur, which decreases social welfare. The comparison between the Shapiro policy and a lump-sum policy leads to the same results as in the scenario before. The Shapiro policy increases social welfare for low entry costs, the comparison is ambiguous for medium entry costs and social welfare is the same for high entry costs. Only the threshold levels of f change because a court decision is possible before entry occurs.

Proposition 4.4 *Assume that a potential lawsuit or settlement can take place before the full entry cost have been sunk. From a social point of view, the laissez-faire policy is weakly dominated by the lump-sum policy and the Shapiro policy. Comparing the Shapiro policy to the lump-sum policy shows that*

- *the welfare is higher under the Shaprio policy for low entry costs* $f \leq \pi^c$*;*

- *the welfare comparison is ambiguous for medium entry costs* $\pi^c < f \leq \frac{1}{2}\lambda + (1-\alpha)\pi^c$*;*
- *the welfare is the same under both antitrust limits for high entry costs* $f > \frac{1}{2}\lambda + (1-\alpha)\pi^c$.

Proof. See Appendix. ∎

In the following we describe the intuition for the different welfare outcomes if one compares the Shapiro policy to the lump-sum policy. We start with medium entry costs and then consider low and high entry costs.

For medium entry costs, the intuition is the same as in Scenario 1. On the one hand, market entry occurs for higher levels of entry costs under the Shapiro policy than under the lump-sum policy. This is due to the efficiency gain that the firms can appropriate under the Shapiro policy. This additional entry increases the consumer surplus. On the other hand, the firms have to incur the entry costs. This trade-off can go in either direction.

Given low entry costs, one could have the impression that the lump-sum policy does not have to be better than the Shapiro policy. On the one hand, firms can realize the efficiency gain by a settlement under the Shapiro policy. On the other hand, they have to pay the entry costs for sure. Hence, it might be that the entry costs outweigh the efficiency gains. But if this had been the case, then the firms would rather choose to have a trial instead of a settlement. Intuitively, the consumer surplus has to be the same from a settlement as from a court decision. The firms decide whether to settle or not with respect to their joint profits. This coincides with welfare maximization.

Given high entry costs, no entry occurs under both policies. Thus, social welfare is the same under both antitrust limits.

4.5 Scenario 3: Bargaining Without Having to Incur the Entry Costs

Let us now assume that it is possible to write settlement contracts in such a way that the entrant does not have to enter the market. Hence, he can avoid incurring f. A possible contract would be that the entrant stays off the market and receives a compensation payment from the patentholder, a so called "reverse payment". Such kinds of settlements have been seen quite often in the Pharmaceutical Industry.[12] The FTC set up a study about such settlements because these are obviously suspicious from an antitrust perspective (FTC (2002)). We are interested in the consequences of these settlements on firms' profits and on social welfare.

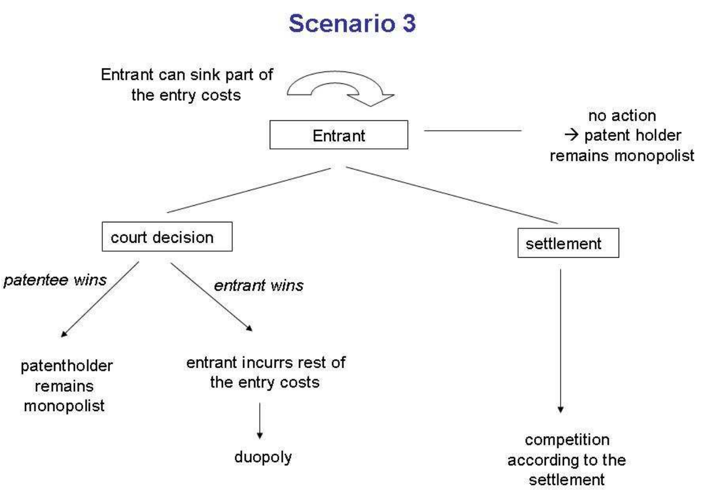

Figure 4.3: Scenario 3

As in Scenario 2, we assume that the entrant can sink parts of his entry costs, which can be infinitesimal, before a bargaining occurs. If the incumbent still denies a bargaining after the entrant has sunk parts of the entry cost, then the entrant has again the option to sink parts of entry costs. Only when the entrant decides not to sink a further part of the entry costs and no court decision or settlement occurs, then the patentholder remains monopolist. Figure 4.3 shows the game structure.

[12]FTC (2002)

Lump-Sum Policy

A lump-sum policy constrains the cooperative value of a settlement to $2\pi^c$. In contrast to Scenario 1 and 2, entry costs can be avoided by a settlement. Thus, if the fixed costs are high enough $f \geq \frac{\alpha}{1-\alpha}(\pi^m - 2\pi^c)$, there is scope for a bargaining solution

$$2\pi^c - [\alpha\pi^m + (1-\alpha)\pi^c - (1-\alpha)(\pi^c - f)] \geq 0; \tag{4.15}$$

$$\Leftrightarrow f \geq \frac{\alpha}{1-\alpha}(\pi^m - 2\pi^c). \tag{4.16}$$

As in Scenario 1 and 2, the question remains if the entrant can achieve a settlement with $f > \pi^c$. On a first glance it seems as if the entrant's outside option is not to sue and to stay off the market. Hence, there might be no reason for the patentholder to settle with the potential entrant because the entrant has no credible threat to sue. But again, the potential entrant can incur parts of the entry costs in order to get tough. After incurring parts of the entry costs ex ante, to sue and to enter the market if he succeeds, can get optimal ex post.

But in Scenario 3, where the firms can settle without incurring the entry costs, the entrant's upfront sinking of parts of the fixed costs is a wasting of resources. A rational patentholder anticipates the possible strategic actions of the entrant. Thus, both firms can do better if they avoid the sinking of parts of the entry costs. Instead, they should include these costs in the bargaining. This leads to the following market equilibrium.

Finding 4.7: *Assume that settlements have to obey the lump-sum policy and that a potential lawsuit or settlement can take place before the full entry costs have been sunk. Furthermore, the remaining entry costs do not have to be incurred with a settlement.*

1. *There is a lawsuit if* $f \leq \pi^c \wedge f < \frac{\alpha}{1-\alpha}(\pi^m - 2\pi^c)$.
2. *There is neither a lawsuit nor a settlement if*

   a) $\pi^c < f \leq 2\pi^c + \alpha(\pi^c - \pi^m) \wedge \pi^c < \frac{\alpha}{1-\alpha}(\pi^m - 2\pi^c)$, *or*

   b) $f > 2\pi^c + \alpha(\pi^c - \pi^m)$.

3. *There is a settlement if*
   a) $f < \pi^c \wedge f \geq \frac{\alpha}{1-\alpha}(\pi^m - 2\pi^c)$, *or*
   b) $\pi^c \leq f \leq 2\pi^c + \alpha(\pi^c - \pi^m) \wedge \pi^c \geq \frac{\alpha}{1-\alpha}(\pi^m - 2\pi^c)$.

Proof. See Appendix. ■

In the first case the entry costs are so small that no scope exists for a bargaining solution. Hence, the entrant challenges the patent. In the first part of the second case no scope exists for a bargaining solution, too. Hence, the entrant anticipates that he gets no settlement even if he sinks parts of his entry costs. Therefore, he stays off the market. The entry costs are so high in (2b) that the entrant stays off the market. In contrast, in case (3) the parties settle because there is scope for a bargaining solution.

Laissez-Faire Policy

Given a laissez-faire policy, there is always scope for a bargaining solution and a lawsuit never occurs in equilibrium. The joint profits from a settlement have the highest possible value π^m because the parties can avoid the entry costs f.

As in the cases before, the potential entrant can achieve a settlement with $f > \pi^c$ by sinking parts of the entry costs. A rational patentholder anticipates this possible strategic action of the entrant. Thus, both firms can do better by including these costs in the bargaining. Only if the entry costs are too high, then a settlement does not pay off for the potential entrant.

Finding 4.8: *Assume that settlements have to obey the laissez-faire policy and that a potential lawsuit or settlement can take place before the full entry costs have been sunk. Furthermore, the remaining entry costs do not have to be incurred with a settlement. If* $f \leq (1-\alpha)(\pi^m - \pi^c) + \pi^c$, *then a settlement occurs. If* $f > (1-\alpha)(\pi^m - \pi^c) + \pi^c$, *then the patentholder remains monopolist.*

Proof. See Appendix. ■

Shapiro Policy (SP)

The analysis is the same as above under the Shapiro policy. Only the cooperative value of a settlement decreases from π^m to $a\pi^m + (1-\alpha)2\pi^c + \lambda$. Thus, the threshold level of f where entry occurs decreases because the entrant's profit from entering decreases.

Finding 4.9: *Assume that settlements have to obey the Shapiro policy and that a potential lawsuit or settlement can take place before the full entry costs have been sunk. Furthermore, the remaining entry costs do not have to be incurred with a settlement. If $f \leq \lambda + \pi^c(2-\alpha)$, then a settlement occurs. If $f > \lambda + \pi^c(2-\alpha)$, then the patentholder remains monopolist.*

Proof. See Appendix. ∎

Discussion

Once again we see that the entrant prefers a less restrictive policy to a more restrictive policy. In contrast, the ranking depends on the actual level of f for the patentholder. If a certain regime is able to prevent market entry, then he prefers this one. Otherwise, he prefers a less restrictive regime in order to increase his payoff from the settlement.

Proposition 4.5 *Assume that a potential lawsuit or settlement can take place before the full entry costs have been sunk. Furthermore, the remaining entry costs do not have to be incurred by the entrant with a settlement. The patentholder*

1. *prefers LF to SP to LS if*
   a) $f < \pi^c$, *or*
   b) $\pi^c \leq f < 2\pi^c + \alpha(\pi^c - \pi^m) \wedge f \geq \frac{\alpha}{1-\alpha}(\pi^m - 2\pi^c)$;
2. *prefers LS to LF to SP if*
   a) $\pi^c \leq f < 2\pi^c + \alpha(\pi^c - \pi^m) \wedge f < \frac{\alpha}{1-\alpha}(\pi^m - 2\pi^c)$, *or*
   b) $2\pi^c + \alpha(\pi^c - \pi^m) \leq f < \lambda + \pi^c(2-\alpha)$;

3. *is indifferent between LS and SP but prefers both antitrust limits to LF if* $\lambda + \pi^c(2 - \alpha \leq f < (1-\alpha)(\pi^m - \pi^c) + \pi^c$;

4. *is indifferent between LS, SP and LF if* $(1-\alpha)(\pi^m - \pi^c) + \pi^c \leq f$.

Proof. See Appendix. ■

Intuitively, in case (1) the entry costs are in such a way that no antitrust limit prevents a settlement or a lawsuit. Given this, the patentholder would like to have a less restrictive limit. Therefore, his ranking is laissez-faire to Shapiro to a lump-sum policy. If the entry costs are in such a way that a lump-sum policy prevents a settlement or lawsuit (2), then the patentholder's profit is maximal and equal to π^m under a lump-sum policy. Thus, he prefers a lump-sum policy to the two other policies. Comparing the remaining two policies, we see that a settlement occurs under both. Thus, he prefers the less restrictive one, namely the laissez-faire approach. In case (3) the lump-sum and the Shapiro policy prevent a settlement or a lawsuit. Thus, his profit is π^m under both policies. Furthermore, a settlement occurs under a laissez-faire policy. Therefore, the profits of the patentholder are lower under a laissez-faire policy compared to the other two limits. In case (4) the entry costs are so high that neither a settlement nor a lawsuit occurs under all policies. Hence, the patentholder is indifferent between the three antitrust limits because his profit is always π^m.

With regard to social welfare, a laissez-faire policy is weakly dominated by the two other policies. Interestingly, the Shapiro policy is weakly better than the lump-sum policy in nearly all cases. Only if a lump-sum policy yields a settlement, the lump-sum policy is better than the Shapiro policy.

Proposition 4.6 *Assume that settlements have to obey the lump-sum policy and that a potential lawsuit or settlement can take place before the full entry costs have been sunk. Furthermore, the remaining entry costs do not have to be incurred by the entrant with a settlement. From a social point of view, the laissez-faire policy is weakly dominated by the lump-sum policy and by the Shapiro policy. The Shapiro policy weakly dominates the lump-sum policy in nearly all cases. Only if* $\pi^c < f \leq 2\pi^c + \alpha(\pi^c - \pi^m) \wedge \pi^c \geq \frac{\alpha}{1-\alpha}(\pi^m - 2\pi^c)$ *or* $f \leq \pi^c \wedge f \geq \frac{\alpha}{1-\alpha}(\pi^m - 2\pi^c)$, *then the lump-sum policy is better than the Shapiro policy.*

Proof. See Appendix. ■

The resulting market price is always p^m under a laissez-faire policy. Entry costs do not have to be incurred. Hence, they are irrelevant from a social point of view. Therefore, every antitrust limit which leads, at least in some cases, to a lower price than p^m is better. This implies that the laissez-faire policy is weakly dominated by the other two antitrust limits.

Comparing the Shapiro policy with the lump-sum policy shows that the Shapiro policy is always better if the lump-sum policy leads to a lawsuit. The consumer surplus is the same under both policies. However under the Shaprio policy the parties realize the efficiency gain λ and avoid the fixed costs. Only if the lump-sum policy leads to a settlement $f \leq 2\pi^c + \alpha(\pi^c - \pi^m) \wedge \pi^c \geq \frac{\alpha}{1-\alpha}(\pi^m - 2\pi^c)$ or $f < \pi^c \wedge f \geq \frac{\alpha}{1-\alpha}(\pi^m - 2\pi^c)$, then the lump-sum policy is better because it yields a lower price compared to the Shapiro policy.

4.6 Post-Grant Challenge and Antitrust Limits to Settlements

In this section we link our findings to the actual debate about the patent system and the antitrust limits to patent settlements.

There is an ongoing discussion about how to organize a post-grant challenge and review process. Hall and Harhoff (2004) consider different design choices. In particular they argue that there should be no restrictions on the type of opponent who is allowed to challenge the patent. Thus, a challenger does not have to show that he is involved in an infringement suit or that the patentee is threatening him to sue for infringement. Shapiro (2004) even proposes that third parties should have the right to initiate a post-grant challenge any time during the lifetime of the patent. These proposals are different to the actual US legislation regarding patent litigation where a patent can be challenged only by an alleged or threatened infringer. Hall and Harhoff (2004) argue that leaving out such restrictions enables public-interest groups and non-governmental organizations to participate. Hence, it may give access to more information regarding prior art.

We would like to point out that a system that allows everyone to challenge a patent has another important impact. In our view, it is not only about enabling public-interest groups or non-governmental organizations to challenge, but it is more about enabling potential entrants. These firms usually have the best knowledge about prior art because it deals with their core business. Under the current US law, a competitor can only litigate the validity of a patent when he is involved in an infringement suit or when he can show that the patentee is actually threatening him to sue. Given such regulations, the patentee can force the entrant to sink his entry costs before he agrees to bargain over a license because the entrant has no threat point before sinking f. As we have shown, this introduces a kind of hold-up problem and it weakens the entrant's bargaining position. Consequently, this decreases the entrant's payoff and leads to less market entry.[13] In contrast, if it is possible to challenge a patent without having to be involved in an infringement suit or without having to show a concrete threat, then the entrant can bargain over a licensing contract before incurring the entry costs. Hence, the entrant can include his entry costs in the bargaining, which increases his profit. In the end, such a post-grant challenge system where everyone can challenge a patent expands the range of entry costs under which market entry occurs. Therefore, the number of lawsuits and the number of licensing contracts should increase. But as we have shown, such a system can backfire from a social point of view. It may generate too high incentives for market entry and it may decrease social welfare due to the entry costs (see Scenario 2). This is particularly true if the antitrust authority uses a laissez-faire approach to settlements.

Another issue we would like to stress is that it is not obvious that low antitrust limits to settlements are always in the interest of a patentee and thus increase R&D incentives. In contrast, the patentholder can gain from more restrictive antitrust limits that avoid additional market entry. These findings are in contrast to Shapiro (2003). Without considering market entry, he shows that his rule fully respects the patentholder's rights and never decreases his payoff. We show that this is no longer true with endogenous market entry. We conclude that keeping the incentives for R&D high and having restrictive antitrust limits do not necessarily work against each other, but possibly they go hand in hand.

Regarding antitrust limits to patent settlements, we have shown that a laissez-faire policy is definitely not desirable from a social point of view. Interestingly, in the recent past courts have approved settlements that heavily restrain competition regardless of the validity (or the probability of the validity) of the patent, as long

[13] Compare Scenario 1 to 2.

as it is not total obvious that the patent is invalid.[14] To cite from the District Court in In re Ciprofloxacin Hydrochloride Antitrust Litigation (2005): "In sum, it is inappropriate for an antitrust court, in determining the reasonableness of a patent settlement agreement, to conduct an after-the-fact inquiry into the validity of the underlying patent. Such an inquiry would undermine any certainty for patent litigants seeking to settle their disputes. (...) To summarize, it would be inappropriate to engage in an after-the-fact analysis of the patent's likely validity. Nor is it appropriate to discount the exclusionary power of the patent by any probability that the patent would haven been found invalid." In our opinion, this approach goes completely in the wrong direction of a laissez-faire policy.

But even an ex-post Pareto improvement, as the Shapiro policy, is not always optimal. At least, such a rule is definitely better than the laissez-faire policy. Taking all together, we have to conclude that to the best of our knowledge, there is no easy general rule applicable to all cases. Antitrust authorities and the courts should therefore rely on the "rule of reason" and treat a given settlement term differently in different circumstances.

The Federal Trade Commission challenged several settlements that included so called reverse payments, which are very common in the Pharmaceutical Industry. These reverse payments usually go from the patentholder to the licensor. In exchange, the licensor agrees not to enter the market for a certain period of time. Several authors stress the anti-competitive impact of such settlements.[15] In contrast, we would like to point out that these contracts avoid the fixed costs of entry (Scenario 3). Without reverse payments, the parties must rely on price fixing or territorial agreements, which may lead to the same consumer welfare, but only make the firms worse off and hence decrease social welfare.

[14] Examples include the District Court approach in In re Ciprofloxacin Hydrochloride Antitrust Litigation (E.D.N.Y. 2005), the Eleventh Circuit's approach in Valley Drug Co. v. Geneva Pharma., Inc. 344 F.3d 1294 (11th Cir. 2003), the Sixth Circuit's approach in In re Cardizem CD Antitrust Litig., 332 F.3d 896 (6th Cir. 2003), and the District Court's approach in In re Tamoxifen Citrate Antitrust Litig., 277 F. Supp. 2d 121 (E.D.N.Y. 2003)

[15] For example Farrell and Shapiro (2005) or Bulow (2004).

4.7 Conclusion

In this chapter we have studied the implications of antitrust limits to patent settlements on market entry incentives. We have considered three different antitrust limits: laissez-faire policy, lump-sum policy and the Shapiro policy. Counter-intuitively, a patentholder can prefer more restrictive antitrust limits. Furthermore, the patentholder's profits and the social welfare can decrease under the Shapiro policy compared to a lump-sum policy. This is in contrast to the findings of Shapiro (2003), who does not consider market entry.

In this context we discussed several different scenarios when the bargaining or a lawsuit takes place. A patentholder prefers bargaining or lawsuits after the entrant has sunk his entry costs. The opposite is true for the entrant. An early bargaining allows the entrant to incorporate his fixed costs in the bargaining. This increases the range of entry costs under which market entry occurs. Hence, a strengthening of the post-grant challenge system does not only increase the participation of public or private interest parties. Furthermore, potential competitors get more incentives to participate in the market, which decreases the patentee's profits.

Reflecting on our results, they crucially depend on the inability of the patentholder to commit not to license his product. This raises the questions if the patentholder has any opportunities to do this? For the patentee one possibility might be to give an exclusive license to the first entrant in order to avoid additional market entry. Further potential entrants anticipate that they would have to go through a trial for sure if they entered the market. Therefore, it might be better for them to stay off the market. To consider such an argument, one has to extend our model to at least two entrants. This introduces another important issue, namely the free-rider problem. An entrant who has successfully challenged a patent provides a public good. In our opinion, it leaves room for further research to build a model with several entrants and an appropriate timing in order to analyze such a new motivation for exclusive licensing.

4.8 Appendix

Proof of Proposition 4.1

Lump-Sum Policy

$$\pi_P^{LS} = \begin{cases} \alpha\pi^m + (1-\alpha)\pi^c & \text{if } f < (1-\alpha)\pi^c \\ \pi^m & \text{if } f \geq (1-\alpha)\pi^c \end{cases}$$

Laissez-Faire Policy

$$\pi_P^{LF} = \begin{cases} \frac{1}{2}(1+\alpha)\pi^m & \text{if } f < \frac{1}{2}(1-\alpha)\pi^m \\ \pi^m & \text{if } f \geq \frac{1}{2}(1-\alpha)\pi^m \end{cases}$$

Shapiro-Policy

$$\pi_P^{SP} = \begin{cases} (\alpha\pi^m + (1-\alpha)\pi^c) + \frac{1}{2}\lambda & \text{if } f < \frac{1}{2}\lambda + (1-\alpha)\pi^c \\ \pi^m & \text{if } f \geq \frac{1}{2}\lambda + (1-\alpha)\pi^c \end{cases}$$

because if $f < \frac{1}{2}\lambda + (1-\alpha)\pi^c$ then

$$\pi_P = (\alpha\pi^m + (1-\alpha)\pi^c) + \frac{1}{2}[\Pi^s - (\alpha\pi^m + (1-\alpha)\pi^c)) - (1-\alpha)(\pi^c)]$$

$$\pi_P = (\alpha\pi^m + (1-\alpha)\pi^c) + \frac{1}{2}\lambda.$$

Comparison

If $f < \pi^c(1-\alpha)$ then $\pi_P^{LF} > \pi_P^{SP} > \pi_P^{LS}$.

If $\pi^c(1-\alpha) \leq f < \frac{1}{2}\lambda + \pi^c(1-\alpha)$ then $\pi_P^{LS} > \pi_P^{LF} > \pi_p^{SP}$.

If $\frac{1}{2}\lambda + \pi^c(1-\alpha) \leq f < \frac{1}{2}(1-\alpha)\pi^m$ then $\pi_P^{LS} = \pi_P^{SP} > \pi_P^{LF}$.

If $\frac{1}{2}(1-\alpha)\pi^m \leq f$ then $\pi_P^{LS} = \pi_P^{LF} = \pi_P^{SP}$.

■

Proof of Proposition 4.2

1. Lump-Sum Policy:

$$W^{LS} = \begin{cases} \alpha(CS^m + \pi^m) + (1-\alpha)(CS^c + 2\pi^c) - f & \text{if } f < (1-\alpha)\pi^c \\ CS^m + \pi^m & \text{if } f \geq (1-\alpha)\pi^c \end{cases}$$

2. Laissez-Faire Policy:

$$W^{LF} = \begin{cases} CS^m + \pi^m - f & \text{if } f < \frac{1}{2}(1-\alpha)\pi^m \\ CS^m + \pi^m & \text{if } f \geq \frac{1}{2}(1-\alpha)\pi^m \end{cases}$$

3. Shapiro Policy:

$$W^{SP} = \begin{cases} \alpha(CS^m + \pi^m) + (1-\alpha)(CS^c + 2\pi^c) + \lambda - f & \text{if } f < \frac{1}{2}\lambda + (1-\alpha)\pi^c \\ CS^m + \pi^m & \text{if } f \geq \frac{1}{2}\lambda + (1-\alpha)\pi^c \end{cases}$$

Part 1: A laisser-faire policy never increases the social welfare compared to a lump-sum policy or to the Shapiro policy.

Because of

$$CS^m + \pi^m < \alpha(CS^m + \pi^m) + (1-\alpha)(CS^c + 2\pi^c)$$

$$CS^m + \pi^m < CS^c + 2\pi^c$$

and

$$\frac{1}{2}(1-\alpha)\pi^m \geq \frac{1}{2}\lambda + (1-\alpha)\pi^c \geq (1-\alpha)\pi^c$$

it directly follows that

$$W^{LF} \leq W^{SP} \wedge W^{LF} \leq W^{SP} \; \forall f.$$

Part 2: Shapiro policy in comparison to the lump-sum policy

1. To show: $W^{SP} \geq W^{LS}$ if $f < \pi^c(1-\alpha)$

$$\begin{aligned} W^{SP} = \alpha(CS^m + \pi^m) + (1-\alpha)(CS^c + 2\pi^c) + \lambda - f \geq \\ \alpha(CS^m + \pi^m) + (1-\alpha)(CS^c + 2\pi^c) - f = W^{LS} \end{aligned}$$

$$\lambda \geq 0$$

2. To show: $W^{SP} \lesseqgtr W^{LS}$ if $\pi^c(1-\alpha) \leq f \leq \frac{1}{2}\lambda + \pi^c(1-\alpha)$

We know that $(1-\alpha)[\pi^m - 2\pi^c] \geq \lambda \geq 0$ because of

$$\pi^m \geq \Pi^s = \alpha\pi^m + 2(1-\alpha)\pi^c + \lambda.$$

Example: $W^{SP} \lesseqgtr W^{LS}$ with $f = (1-\alpha)\pi^c$ and $\lambda = (1-\alpha)[\pi^m - 2\pi^c]$

$$W^{SP} = \alpha(CS^m + \pi^m) + (1-\alpha)(CS^c + 2\pi^c) + \lambda - f \lesseqgtr CS^m + \pi^m = W^{LS}$$

$$\alpha(CS^m) + (1-\alpha)(CS^c) - (1-\alpha)\pi^c \lesseqgtr CS^m$$

$$CS^c - CS^m \lesseqgtr \pi^c$$

This is true because without more restrictions $CS^c - CS^m$ can be smaller or bigger than π^c.

3. To show: $W^{SP} = W^{LS}$ if $f > \frac{1}{2}\lambda + \pi^c(1-\alpha)$

With $f > \frac{1}{2}\lambda + \pi^c(1-\alpha)$ we directly have $W^{SP} = W^{LS} = CS^m + \pi^m$.

■

Shapiro Policy vs. Lump-Sum Policy with Cournot Competition

Demand function: $p = 1 - bx$

Monopoly: $\pi^m = \frac{1}{4b}$; $CS^m = \frac{1}{2}$; $p = \frac{1}{2}$; $x = \frac{1}{2b}$

Duopoly: $\pi^c = \frac{1}{9b}$; $CS^c = \frac{2}{9b}$; $p = \frac{1}{3}$ $x_1 = x_2 = \frac{1}{3b}$

Price p_s with a settlement:

$$CS(p_s) = \alpha CS^m + \alpha CS^c$$

$$\frac{1}{2}(1-p_s)(\frac{1-p_s}{b}) = a\frac{1}{8b} + (1-a)\frac{2}{9b}$$

$$p_s = 1 - \frac{1}{6}\sqrt{(-7\alpha + 16)}$$

Resulting joint profits Π^s:

$$\Pi^S = (1 - \frac{1}{6}\sqrt{(-7\alpha + 16)}) * \frac{1 - (1 - \frac{1}{6}\sqrt{(-7\alpha + 16)})}{b} = -\frac{1}{36}\left(-6 + \sqrt{(-7\alpha + 16)}\right)\frac{\sqrt{(-7\alpha + 16)}}{b}$$

This yields the following λ

$$\lambda = \frac{1}{6}\frac{\sqrt{(-7a + 16)} + a - 4}{b}.$$

Calculating the critical λ where lump-sum is better than Shapiro:

$$\alpha(\frac{1}{8b} + \frac{1}{4b}) + (1 - \alpha)(\frac{2}{9b} + \frac{2}{9b}) + \frac{1}{6}\frac{\sqrt{(-7\alpha + 16)} + \alpha - 4}{b} - f < \frac{1}{8b} + \frac{1}{4b}$$

Hence with $f > f_{crit} = -\frac{1}{72}\frac{-7a+43-12\sqrt{(-7a+16)}}{b}$ lump-sum policy is better.

To show $f_{min} = (1 - \alpha)\pi^c > f_{crit}$.

$$\frac{1}{9b}(1 - \alpha) > -\frac{1}{72}\frac{-7\alpha + 43 - 12\sqrt{(-7\alpha + 16)}}{b}$$

$$-\frac{1}{24}\frac{-17 + 5a + 4\sqrt{(-7a + 16)}}{b} > 0 \forall\alpha \in [0; 1]$$

■

Proof of Lemma 4.1

What is the optimal amount w he should sink in the first place?

Entrant's payoff with settlement after entry:

$$\pi_E^s = (1-\alpha)(\pi^c-(f-w))+\frac{1}{2}[\pi^m-(f-w)-[\alpha\pi^m+(1-\alpha)\pi^c]-(1-\alpha)(\pi^c-(f-w))]-w$$

$$\pi_E^s = \frac{1}{2}[\pi^m - (f - w) - [\alpha\pi^m + (1 - \alpha)\pi^c] + (1 - \alpha)(\pi^c - (f - w))] - w$$

$$\frac{\partial\pi_E^s}{\partial w} = \frac{1}{2}[1 + (1 - \alpha)] - 1 < 0$$

Hence, we have a corner solution. The entrant should sink only the amount to get a non-negative threat point:

$$\pi^c = f - w.$$

$\blacksquare$

Proof of Finding 4.5

To show: Upper bound of f

$$\pi^e = (1-\alpha)(\pi^c - f') + \frac{1}{2}[\pi^m - f' - \alpha\pi^m - (1-\alpha)\pi^c - (1-\alpha)(\pi^c - f')] - w \geq 0.$$

By f' we denote the remaining entry costs which get incorporated in the bargaining.

We know that $f' = \pi^c$ (Lemma 4.1). Hence,

$$\pi^e = \frac{1}{2}[\pi^m - \pi^c - \alpha\pi^m - (1-\alpha)\pi^c] - (f - \pi^c) \geq 0;$$

$$\pi^e = \frac{1}{2}[(1-\alpha)\pi^m + \alpha\pi^c] \geq f.$$

Hence, the entrant enters if $f \leq \frac{1}{2}[(1-\alpha)\pi^m + \alpha\pi^c]$.

To show: If $\pi^c < f \leq \frac{1}{2}[(1-\alpha)\pi^m + \alpha\pi^c$, then there is a settlement if $\pi^m - 2\pi^c < \frac{\alpha}{1-\alpha}\pi^c$.

We know that the entrant has to sink $f - \pi^c$ for the bargaining with $\pi^c < f \leq \frac{1}{2}[(1-\alpha)\pi^m + \alpha\pi^c$. Given π^c as the remaining entry costs, there is a settlement if

$$\pi^m - \pi^c - (\alpha\pi^m + (1-\alpha)\pi^c) - (1-\alpha)(\pi^c - \pi^c) \leq 0$$

$$\Leftrightarrow (1-\alpha)\pi^m \geq \pi^c$$

$$\Leftrightarrow \pi^m - 2\pi^c \geq \frac{\alpha}{1-\alpha}\pi^c$$

$\blacksquare$

Proof of Finding 4.6

Part 1: $f \leq \pi^c$

There is no scope for bargaining if

$$\Pi^s - f < \alpha\pi^m + (1-\alpha)2\pi^c + (1-\alpha)(-f).$$

We defined λ through

$$\lambda = \Pi^s - \alpha\pi^m - (1-\alpha)2\pi^c.$$

This leads to

$$\lambda - f < -(1-\alpha)f;$$

$$\lambda < \alpha f.$$

Part 2: $\pi^c < f \leq \pi^c(1-\frac{1}{2}) + \frac{1}{2}\lambda$

Using Lemma 4.1, we know that $f - w = \pi^c = f'$. There is no scope for bargaining after sinking w if

$$\Pi^s - \pi^c < \alpha\pi^m + (1-\alpha)2\pi^c + (1-\alpha)(-\pi^c).$$

We defined λ through

$$\lambda = \Pi^s - \alpha\pi^m - (1-\alpha)2\pi^c.$$

This leads to

$$\lambda - \pi^c < -(1-\alpha)\pi^c;$$

$$\lambda < \alpha\pi^c.$$

With $\lambda \geq \alpha\pi^c$ a settlement leads to the following entrant's profit:

$$\pi_E^s = \frac{1}{2}[\Pi^s - (f-w) - [\alpha\pi^m + (1-\alpha)\pi^c] + (1-\alpha)(\pi^c - (f-w)] - (f - \pi^c)$$

$$\pi_E^s = \frac{1}{2}[\Pi^s - \pi^c - (\alpha\pi^m + (1-\alpha)\pi^c)] - (f - \pi^c)$$

$$\pi_E^s = \frac{1}{2}[\lambda + (1-\alpha)\pi^c - \pi^c] - f + \pi^c$$

$$\pi_E^s = \frac{1}{2}[\lambda - \alpha\pi^c] - f + \pi^c$$

If this expression is negative then it does not pay off to achieve a settlement for the entrant. Hence, the critical value of f is

$$f = \frac{1}{2}\lambda + \pi^c[1 - \frac{1}{2}\alpha].$$

■

Proof of Proposition 4.3

1. is indifferent between SP, LS, LF if

   a) there is always no entry: $(1-\alpha)\pi^m < \pi^c$ and $\pi^c < f < \frac{1}{2}[(1-\alpha)\pi^m + \alpha\pi^c]$

   - from Finding 4.4 no entry under LS because $f > \pi^c$
   - from Finding 4.5 no entry under LF because$(1-\alpha)\pi^m < \pi^c$ and $\pi^c < f < \frac{1}{2}[(1-\alpha)\pi^m + \alpha\pi^c]$
   - from Finding 4.6 no entry under SP because $f > \pi^c$ and $\lambda < \alpha\pi^c$. This follows from $(1-\alpha)\pi^m < \pi^c$. We know that

     $$\Pi^s - \lambda \leq \pi^m$$

     $$\alpha\pi^m + (1-\alpha)2\pi^c + \lambda \leq \pi^m$$

     $$\lambda^{max} = (1-\alpha)(\pi^m - 2\pi^c)$$

     Combining with $\lambda < \alpha\pi^c$ yields

     $$(1-\alpha)(\pi^m - 2\pi^c) < \alpha\pi^c$$

     $$(1-\alpha)\pi^m < \alpha\pi^c + (1-\alpha)2\pi^c$$

     $$(1-\alpha)\pi^m < (1-\alpha)\pi^c + \pi^c$$

     This is always true with $(1-\alpha)\pi^m < \pi^c$.

   With no entry anyway, the profits are always π^m.

   b) there is always no entry: $\frac{1}{2}[(1-\alpha)\pi^m + \alpha\pi^c] < f$

   This comes directly from Finding 4.4-4.6. With no entry anyway the profits are always π^m.

   c) there is always a lawsuit: $f < \pi^c$ and $(1-\alpha)(\pi^m - 2\pi^c) < \alpha f$

- from Finding 4.4 and $f < \pi^c$ it directly follows that there is a lawsuit under LS.
- from Finding 4.5 and $(1-\alpha)(\pi^m - 2\pi^c) < \alpha f$ it follows directly that is a lawsuit under LF.
- $(1-\alpha)(\pi^m - 2\pi^c) < \alpha f$ implies that $\lambda < \alpha f$ because we know that $\lambda^{max} = (1-\alpha)(\pi^m - 2\pi^c)$. Then from Finding 4.6 it is straight forward that a lawsuit occurs under SP.

If there is always a lawsuit then the profit is under each policy $\alpha\pi^m + (1-\alpha)\pi^c$.

2. prefers LS and SP to LF and is indifferent between LS and SP if there is only a settlement under LF

a) $\pi^c(1 - \frac{1}{2}\alpha) + \frac{1}{2}\lambda \leq f < \frac{1}{2}[(1-\alpha)\pi^m + \alpha\pi^c]$ and $(1-\alpha)\pi^m \geq \pi^c$

- from Finding 4.4 and $f > \pi^c$ it directly follows that there is no entry under LS
- from Finding 4.6 and $\pi^c(1 - \frac{1}{2}\alpha) + \frac{1}{2}\lambda \leq f$ it directly follows that there is no entry under SP
- from Finding 4.5 and $\pi^c(1 - \frac{1}{2}\alpha) + \frac{1}{2}\lambda \leq f < \frac{1}{2}[(1-\alpha)\pi^m + \alpha\pi^c]$ and $(1-\alpha)\pi^m \geq \pi^c$ it follows there is no settlement under LF.

Hence, profits under LF and SP are π^m. These are always higher than the profits from the settlement under LF because the joint profits are smaller than the monopoly profits:

$$f - w = \pi^c$$

$$\Rightarrow \pi^m - (f - w) \leq \pi^m$$

b) $\pi^c \leq f < \pi^c(1 - \frac{1}{2}\alpha) + \frac{1}{2}\lambda$ and $\lambda < \alpha\pi^c < (1-\alpha)(\pi^m - 2\pi^c)$

- from Finding 4.4 and $\pi^c < f$ it directly follows that there is no entry under LS
- from Finding 4.6 and $\lambda < \alpha\pi^c$ it directly follows that there is no entry under SP
- from Finding 4.5, $\alpha\pi^c < (1-\alpha)(\pi^m - 2\pi^c)$ and $\pi^c \leq f < \pi^c(1-\frac{1}{2}\alpha) + \frac{1}{2}\lambda$ it follows there is a settlement under LF.

Hence, profits under LF and SP are π^m. These are always higher than the profits with LF under the settlement because the joint profits are smaller than the monopoly profits. See above.

3. prefers LS to LF to SP if

   $\pi^c < f < \frac{1}{2}\lambda + \pi^c(1 - \frac{1}{2}\alpha)$ and $\lambda > \alpha\pi^c$ or

   - from Finding 4.4 and $\pi^c < f$ it directly follows that there is no entry under LS
   - from Finding 4.6, $\pi^c < f < \frac{1}{2}\lambda + \pi^c(1 - \frac{1}{2}\alpha)$ and $\lambda > \alpha\pi^c$ it directly follows that there is a settlement under SP
   - from Finding 4.5 $\pi^c < f < \frac{1}{2}\lambda + \pi^c(1 - \frac{1}{2}\alpha)$ and $\lambda > \alpha\pi^c$ it directly follows that there is a settlement under LF

   The patentholder has the highest profits π^m under LS. Under LF he has higher profits than under SP because in the bargaining the firms have the same outside options but the joint profit $\pi^m - \pi^c$ is higher under LF than under SP $\Pi^s - \pi^c$.

4. prefers LF to SP and SP to LS if

   $f < \pi^c$ and $\lambda > \alpha f$

   From Findings 4.4-4.6 it directly follows that under LS there is a lawsuit and under SP and LF there is a settlement.

   SP and LF is better because the patentholder gets in the bargaining the outside option plus a part of the bargaining surplus. This is higher than under LS where he gets the payoff from a lawsuit which is equal to the outside option.

   LF yields higher profits for the patentholder than the SP because under both policies the outside option is the same but the bargaining surplus is higher under LF.

5. prefers LF to SP and LS and is indifferent between SP and LS if $f < \pi^c$ and $\lambda < \alpha f < (1 - \alpha)(\pi^m - 2\pi^c)$

   - from Finding 4.4, $f < \pi^c$ it directly follows that there will be a lawsuit under LS
   - from Finding 4.6, $f < \pi^c$ and $\lambda < \alpha f$ it directly follows that there will be a lawsuit under SP

- from Finding 4.5, $f < \pi^c$ and $\alpha f < (1-\alpha)(\pi^m - 2\pi^c)$ it follows that there is a settlement under LF

Profits under LS and SP are therefore $\alpha\pi^m + (1-\alpha)\pi^c$. This less than under LF because the profits under LS and SP are the outside option in the bargaining under LF.

■

Proof of Proposition 4.4

Welfare with laissez-faire policy :

$$W^{LF} = \begin{cases} CS^m + \pi^m - f & \text{if } f < \pi^c \wedge \pi^m - 2\pi^c \geq \frac{1}{1-\alpha} f \\ \alpha(CS^m + \pi^m) + (1-\alpha)(CS^c + 2\pi^c - f) & \text{if } f < \pi^c \wedge \pi^m - 2\pi^c < \frac{1}{1-\alpha} f \\ CS^m + \pi^m & \text{if } \pi^c \leq f < \frac{1}{2}[(1-\alpha)\pi^m + \alpha\pi^c)] \wedge 1 - \alpha\pi^m < \pi^c \\ CS^m + \pi^m - f & \text{if } \pi^c \leq f < \frac{1}{2}[(1-\alpha)\pi^m + \alpha\pi^c)] \wedge 1 - \alpha\pi^m \geq \pi^c \\ CS^m + \pi^m & \text{if } \frac{1}{2}[(1-\alpha)\pi^m + \alpha\pi^c] \leq f \end{cases}$$

Welfare with lump-sum policy :

$$W^{LS} = \begin{cases} \alpha(CS^m + \pi^m) + (1-\alpha)(CS^c + 2\pi^c - f) & \text{if } f < \pi^c \\ CS^m + \pi^m & \text{if } f \geq \pi^c \end{cases}$$

Welfare with Shapiro policy:

$$W^{SP} = \begin{cases} \alpha(CS^m + \pi^m) + (1-\alpha)(CS^c + 2\pi^c - f) & \text{if } f < \pi^c \wedge \lambda < \alpha f \\ \alpha(CS^m + \pi^m) + (1-\alpha)(CS^c + 2\pi^c) + \lambda - f & \text{if } f < \pi^c \wedge \lambda \geq \alpha f \\ CS^m + \pi^m & \text{if } \pi^c \leq f < \pi^c(1 - \frac{1}{2}\alpha) + \frac{1}{2}\lambda \wedge \lambda < \alpha\pi^c \\ \alpha(CS^m + \pi^m) + (1-\alpha)(CS^c + 2\pi^c) + \lambda - f & \text{if } \pi^c \leq f < \pi^c(1 - \frac{1}{2}\alpha) + \frac{1}{2}\lambda \wedge \lambda \geq \alpha\pi^c \\ CS^m + \pi^m & \text{if } \pi^c(1 - \frac{1}{2}\alpha) + \frac{1}{2}\lambda \leq f \end{cases}$$

1. By comparison one sees that $W^{LF} \leq W^{SP} \wedge W^{LF} \leq W^{LS} \forall f$.

2. With $f < \pi^c$ market entry occurs with lump-sum policy and yields

$$W^{LS} = \alpha(CS^m + \pi^m) + (1-\alpha)(CS^c + 2\pi^c - f).$$

With $f < \pi^c$ the Shapiro policy leads to

$$W^{SP} = \begin{cases} \alpha(CS^m + \pi^m) + (1-\alpha)(CS^c + 2\pi^c - f) & \text{if } f < \pi^c \wedge \lambda < \alpha f \\ \alpha(CS^m + \pi^m) + (1-\alpha)(CS^c + 2\pi^c) + \lambda - f & \text{if } f < \pi^c \wedge \lambda \geq \alpha f \end{cases}$$

To show:

$$\alpha(CS^m + \pi^m) + (1-\alpha)(CS^c + 2\pi^c - f) \leq \alpha(CS^m + \pi^m) + (1-\alpha)(CS^c + 2\pi^c) + \lambda - f$$

$$-(1-\alpha)f \leq \lambda - f$$

This true with $\lambda \geq \alpha f$.

2. With $f \geq \pi^c$ no market entry occurs under lump-sum policy, thus

$$W^{LS} = CS^m + \pi^m.$$

With $f \geq \pi^c$ the Shapiro policy yields

$$W^{SP} = \begin{cases} CS^m + \pi^m & \text{if } \pi^c \leq f < \pi^c(1-\frac{1}{2}\alpha) + \frac{1}{2}\lambda \wedge \lambda < \alpha\pi^c \\ \alpha(CS^m + \pi^m) + (1-\alpha)(CS^c + 2\pi^c) + \lambda - f & \text{if } \pi^c \leq f < \pi^c(1-\frac{1}{2}\alpha) + \frac{1}{2}\lambda \wedge \lambda \geq \alpha\pi^c \end{cases}$$

To show:

$$\alpha(CS^m + \pi^m) + (1-\alpha)(CS^c + 2\pi^c) + \lambda - f \lesseqgtr CS^m + \pi^m$$

See Part 2 of the proof of Proposition 4.2.

■

Proof of Finding 4.7

1. *There is a lawsuit if $f \leq \pi^c \wedge f < \frac{\alpha}{1-\alpha}(\pi^m - 2\pi^c)$.*

   We know the critical value s.t. the bargaining surplus is positive: $f \geq \frac{\alpha}{1-\alpha}(\pi^m - 2\pi^c)$. Furthermore, suing and entering makes only sense if $f \leq \pi^c$.

2. *There is neither a lawsuit nor a settlement if*

   a) $\pi^c < f \leq 2\pi^c + \alpha(\pi^c - \pi^m) \wedge \pi^c < \frac{\alpha}{1-\alpha}(\pi^m - 2\pi^c)$, or

   b) $f > 2\pi^c + \alpha(\pi^c - \pi^m)$.

   Calculation of the value of f where the entrant gets a settlement with a positive payoff. By w we denote the fixed costs he sinks to get a settlement

$$\pi_E = \frac{1}{2}[2\pi^c - (\alpha\pi^m + (1-\alpha)\pi^c) + (1-\alpha)(\pi^c - (f-w))] - w$$

$$\frac{\partial \pi_E}{\partial w} = \frac{1}{2}(1-\alpha) - 1 < 0$$

Thus, the optimal w would be $w = f - \pi^c$.

Anticipating this, the entrant's profit from a settlement with $f > \pi^c$ is

$$\pi_E^s = \frac{1}{2}[2\pi^c - (\alpha\pi^m + (1-\alpha)\pi^c)] - \frac{1}{2}w$$
$$\pi_E^s = \frac{1}{2}[\pi^c - \alpha\pi^m + \alpha\pi^c] - \frac{1}{2}w$$

This leads to the critical value of f:

$$f = w + \pi^c = 2\pi^c + \alpha(\pi^c - \pi^m)$$

Combined with $f - w = \pi^c \geq \frac{\alpha}{1-\alpha}(\pi^m - 2\pi^c)$ this ensures a settlement and proves Part 2.

Part 3:

3. *There is a settlement if*

   a) $f < \pi^c \wedge f \geq \frac{\alpha}{1-\alpha}(\pi^m - 2\pi^c)$*, or*

   b) $\pi^c \leq f \leq 2\pi^c + \alpha(\pi^c - \pi^m) \wedge \pi^c \geq f$

   This directly follows from Part 2.

■

Proof of Finding 4.8

1. Firstly, we derive the optimal $w = f - \pi^c$ for $f > \pi^c$

$$\pi_E = \frac{1}{2}[\pi^m - (\alpha\pi^m + (1-\alpha)\pi^c) + (1-\alpha)(\pi^c - (f - w))] - w$$
$$\frac{\partial \pi_E}{\partial w} = \frac{1}{2}(1-\alpha) - 1 < 0$$

2. Secondly, incorporating w in the bargaining gives the critical value of f:

$$\pi_E^S = \frac{1}{2}(1-\alpha)(\pi^m - \pi^c) - \frac{1}{2}w \geq 0$$
$$(1-\alpha)(\pi^m - \pi^c) + \pi^c \geq f_{max}$$

■

Proof of Finding 4.9

Entrant's payoff with $f < \pi^c$:

$$\pi_E^s = (1-\alpha)(\pi^c - f) + \frac{1}{2}[\alpha\pi^m + (1-\alpha)2\pi^c + \lambda - (\alpha\pi^m + (1-\alpha)\pi^c) - (1-\alpha)(\pi^c - f)]$$

$$\pi_E^s = \frac{1}{2}[\alpha\pi^m + (1-\alpha)2\pi^c + \lambda - (\alpha\pi^m + (1-\alpha)\pi^c) + (1-\alpha)(\pi^c - f)]$$

$$\pi_E^s = \frac{1}{2}[(1-\alpha)\pi^c + \lambda + (1-\alpha)(\pi^c - f)] > 0$$

Entrant's payoff with $f = \pi^c$:

$$\pi_E^s = \frac{1}{2}[(1-\alpha)\pi^c + \lambda] > 0$$

Optimal w that the entrant should sink ex ante with $f > \pi^c$:

$$\pi_E = \frac{1}{2}[\alpha\pi^m + (1-\alpha)2\pi^c + \lambda - (\alpha\pi^m + (1-\alpha)\pi^c) + (1-\alpha)(\pi^c - (f - w)] - w$$

$$\frac{\partial \pi^E}{\partial w} = \frac{1}{2}(1-\alpha) - w < 0$$

Hence, the optimal w is $f - \pi^c$. The resulting payoff is:

$$\pi_E^s = \frac{1}{2}[(1-\alpha)\pi^c + \lambda] - \frac{1}{2}w \geq 0$$

$$(1-\alpha)\pi^c + \lambda \geq w$$

$$(1-\alpha)\pi^c + \lambda + \pi^c \geq w + \pi^c = f$$

$$(2-\alpha)\pi^c + \lambda \geq f$$

■

Proof of Proposition 4.5

Part 1a:

$f < \pi^c \wedge f < \frac{\alpha}{1-\alpha}(\pi^m - 2\pi^c)$

With LS → lawsuit: $\pi_P = \alpha\pi^m + (1-\alpha)\pi^c$.

With SP $\rightarrow$ settlement: $\pi_P^S = \frac{1}{2}[\alpha\pi^m + (1-\alpha)2\pi^c + \lambda + \alpha\pi^m + (1-\alpha)\pi^c - (1-\alpha)(\pi^c - f)]$.

With LF $\rightarrow$ settlement:$\pi_P^S = \frac{1}{2}[\pi^m + \alpha\pi^m + (1-\alpha)\pi^c - (1-\alpha)(\pi^c - f)]$.

$\Rightarrow$ LF>SP>LS.

$f < \pi^c \wedge f \geq \frac{\alpha}{1-\alpha}(\pi^m - 2\pi^c)$

With LS $\rightarrow$ settlement: $\pi_P = \frac{1}{2}[2\pi^c + \alpha\pi^m + (1-\alpha)\pi^c - (1-\alpha)(\pi^c - f)]$.

With SP $\rightarrow$ settlement: $\pi_P^S = \frac{1}{2}[\alpha\pi^m + (1-\alpha)2\pi^c + \lambda + \alpha\pi^m + (1-\alpha)\pi^c - (1-\alpha)(\pi^c - f)]$.

With LF $\rightarrow$ settlement:$\pi_P^S = \frac{1}{2}[\pi^m + \alpha\pi^m + (1-\alpha)\pi^c - (1-\alpha)(\pi^c - f)]$.

$\Rightarrow$ LF>SP>LS.

Part 1b:

$\pi^c \leq f < 2\pi^c + \alpha(\pi^c - \pi^m) \wedge \pi^c \leq \frac{\alpha}{1-\alpha}(\pi^m - 2\pi^c)$

With LS $\rightarrow$ settlement: $\pi_P = \frac{1}{2}[2\pi^c + \alpha\pi^m + (1-\alpha)\pi^c - (1-\alpha)(\pi^c - f)]$.

With SP $\rightarrow$ settlement: $\pi_P^S = \frac{1}{2}[\alpha\pi^m + (1-\alpha)2\pi^c + \lambda + \alpha\pi^m + (1-\alpha)\pi^c - (1-\alpha)(\pi^c - f)]$.

With LF $\rightarrow$ settlement:$\pi_P^S = \frac{1}{2}[\pi^m + \alpha\pi^m + (1-\alpha)\pi^c - (1-\alpha)(\pi^c - f)]$.

$\Rightarrow$ LF>SP>LS.

Part 2a:

$\pi^c \leq f < 2\pi^c + \alpha(\pi^c - \pi^m) \wedge \pi^c > \frac{\alpha}{1-\alpha}(\pi^m - 2\pi^c)$

With LS $\rightarrow$ no settlement/ lawsuit: $\pi_P = \pi^m$.

With SP $\rightarrow$ settlement: $\pi_P^S = \frac{1}{2}[\alpha\pi^m + (1-\alpha)2\pi^c + \lambda + \alpha\pi^m + (1-\alpha)\pi^c - (1-\alpha)(\pi^c - f)]$.

With LF $\rightarrow$ settlement:$\pi_P^S = \frac{1}{2}[\pi^m + \alpha\pi^m + (1-\alpha)\pi^c - (1-\alpha)(\pi^c - f)]$.

$\Rightarrow$ LS>LF>SP.

Part 2b:

$2\pi^c + \alpha(\pi^c - \pi^m) \leq f < \lambda + \pi^c(2-\alpha)$

With LS $\rightarrow$ no settlement/lawsuit: $\pi_P = \pi^m$.

With SP $\rightarrow$ settlement: $\pi_P^S = \frac{1}{2}[\alpha\pi^m + (1-\alpha)2\pi^c + \lambda + \alpha\pi^m + (1-\alpha)\pi^c - (1-\alpha)(\pi^c - f)]$.

With LF $\rightarrow$ settlement:$\pi_P^S = \frac{1}{2}[\pi^m + \alpha\pi^m + (1-\alpha)\pi^c - (1-\alpha)(\pi^c - f)]$.

$\Rightarrow$ LS>LF>SP.

Part 3:

$\lambda + \pi^c(2-\alpha) < f < (1-\alpha)(\pi^m - \pi^c) + \pi^c$

With LS $\rightarrow$ no settlement/lawsuit: $\pi_P = \pi^m$.

With SP $\rightarrow$ no settlement/lawsuit:$\pi_P = \pi^m$.

With LF $\rightarrow$ settlement:$\pi_P^S = \frac{1}{2}[\pi^m + \alpha\pi^m + (1-\alpha)\pi^c - (1-\alpha)(\pi^c - f)]$.

$\Rightarrow$ LS=SP>LF.

Part 4:

$(1-\alpha)(\pi^m - \pi^c) + \pi^c \leq f$

With LS $\rightarrow$ no settlement/lawsuit: $\pi_P = \pi^m$.

With SP $\rightarrow$ no settlement/lawsuit:$\pi_P = \pi^m$.

With LF $\rightarrow$ no settlement/lawsuit:$\pi_P = \pi^m$.

$\Rightarrow$ LS=SP=LF.

■

Proof of Proposition 4.6

Welfare under LS:

$$WF^{LS} = \begin{cases} \alpha(\pi^m + CS^m) + (1-\alpha)(2\pi^c + CS^c - f) & \text{if } f < \pi^c \wedge f < \frac{\alpha}{1-\alpha}(\pi^m - 2\pi^c) \\ 2\pi^c + CS^c & \text{if } f < \pi^c \wedge f \geq \frac{\alpha}{1-\alpha}(\pi^m - 2\pi^c) \\ 2\pi^c + CS^c & \text{if } \pi^c \leq f < 2\pi^c + \alpha(\pi^c - \pi^m) \wedge \pi^c \geq \frac{\alpha}{1-\alpha}(\pi^m - 2\pi^c) \\ \pi^m + CS^m & \text{if } \pi^c \leq f < 2\pi^c + \alpha(\pi^c - \pi^m) \wedge \pi^c < \frac{\alpha}{1-\alpha}(\pi^m - 2\pi^c) \\ \pi^m + CS^m & \text{if } f > 2\pi^c + \alpha(\pi^c - \pi^m) \end{cases}$$

Welfare under LF:

$$WF^{LF} = \pi^m + CS^m$$

Welfare under SP:

$$WF^{SP} = \begin{cases} \alpha(\pi^m + CS^m) + (1-\alpha)(2\pi^c + CS^c) + \lambda & \text{if } f \leq \lambda + \pi^c(2-\alpha) \\ \pi^m + CS^m & \text{if } f > \lambda + \pi^c(2-\alpha) \end{cases}$$

Comparison:

1. $W^{LF} \leq WF^{SP}$ and $W^{LF} \leq W^{LS} \forall f$.

2.

If $f < \pi^c \wedge f \geq \frac{\alpha}{1-\alpha}(\pi^m - 2\pi^c)$ then $W^{LS} \geq W^{SP}$.

If $\pi^c \leq f < 2\pi^c + \alpha(\pi^c - \pi^m) \wedge \pi^c \geq \frac{\alpha}{1-\alpha}(\pi^m - 2\pi^c)$ then $W^{LS} \geq W^{SP}$.

Otherwise $W^{LS} < W^{SP}$.

■

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