Essays in Applied Game Theory: Competition and Cooperation

Yixuan Shi

Munich, 2026

Essays in Applied Game Theory: Competition and Cooperation

Inaugural-Dissertation
zur Erlangung des Grades
Doctor oeconomiae publicae (Dr. oec. publ.)
an der Ludwig-Maximilians-Universität München

2026

vorgelegt von
Yixuan Shi

Referent: Prof. Dr. Dr. h.c. Kai A. Konrad
Korreferent: Prof. Dr. Matthias Lang

Promotionsabschlussberatung: 28 January 2026

MÜNDLICHE PRÜFUNG: 13. JANUAR 2026

BERICHTERSTATTENDE: PROF. DR. DR. h.c. KAI A. KONRAD
PROF. DR. MATTHIAS LANG
PROF. RABAH AMIR, PH.D.

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Acknowledgements

It has been a long journey to reach this point. Along the way there were moments of doubt, but I'm deeply grateful to be crossing the finish line with this thesis. This would not have been possible without the support of many people. First and foremost, I would like to thank my first supervisor Kai Konrad for giving me the opportunity to pursue a PhD, for his constructive advice whenever I needed it, and for his constant encouragement. Second, I would like to thank my second supervisor Matthias Lang for never hesitating to discuss my research ideas no matter how preliminary they are, and for the detailed comments on my projects from which I benefited greatly. I would also like to thank Rabah Amir for his insightful discussions during his visits to the institute and for kindly agreeing to be my third examiner. I am also very thankful to my co-authors, Stefano Barbieri and Sven Simon, for being such wonderful collaborators on my third chapter. I have learned a lot from working with them and I am especially grateful for their generosity in adjusting their schedules to accommodate my thesis deadline.

I am glad to meet all my former and current colleagues at the Max Planck Institute for Tax Law and Public Finance, who made working there a much more enjoyable experience. In particular, I would like to thank Marco Serena for his patient discussions with me when I first started working on theory and his generous feedback; Sven Simon for his kind inductions when I joined the institute and the countless times he has helped me since; Raisa Sherif for her lively and comforting conversations whenever I felt uncertain; Georg Thunecke for organising the internal seminar and for the insightful comments and suggestions we received; Anwesha Banerjee for her kind guidance and encouragement; Afiq bin Oslan for being both a wonderful conference travel companion and a much-needed calm presence; and Behanz Fard for joining us and bringing more lively discussions to the group.

I'm also grateful to my fellow PhD colleagues both at the institute and at the LMU. I will fondly remember our conversations over lunch, dinner and coffee, about research, the PhD and life. In particular, I would like to thank Keshav Choudhary for generously hosting many get-togethers, Pascal Nieder for his nudges to try out new drinks, and Carmen Sainz Villalba for her company when we started the PhD around the similar times. I'm also thankful to Michael Landschützer, Jonas Send, Raphaela Hennings, Biljana Meiske, Jinju Rhee, Yifan Mao and Leyi Tu.

I would also like to thank Natalie Struwe for her generous help with experimental methods when I started working on them; our wonderful student assistants for their support with many tasks; Sandra Sundt-Johannesen for her invaluable assistance with administrative matters; Hans Müller and Andreas Kraus for their help with all IT-related issues; and the General Administration for ensuring such a smooth work experience.

Last but not least, I would like to thank my mum, my grandparents and my aunts for supporting my education and giving me the opportunity to see a bigger world, as well as my cousins for their warm welcome whenever I return home. I'm deeply grateful to my partner, Thomas Furber, who even after witnessing my many frustrations, has continued to support me, encourage me and give me confidence. I'm also thankful to my friends, whom I have met at different stages of my life, for their companionship along the way.

Introduction

Many social and economic situations require a group of people to decide whether to cooperate in order to achieve outcomes that benefit everyone, even when doing so involves personal costs, such as collective action to address climate change or the private provision of public goods. Other situations, in contrast, involve people expending resources and efforts to compete with each other, with rewards going only to the winners, as in sports, electoral competition or R&D races. Game theory provides a powerful framework to study such strategic interactions of both cooperation and competition. It allows us to examine how agents' preferences and costs of provision influence the timing and success of public goods provision, and how total effort depends on the prize allocation rules and information available in the contests. These insights also raise broader questions of institutional design: how we can foster and sustain cooperation, or structure contests to maximise or minimise aggregate efforts.

This dissertation applies game theory to study cooperation and competition in three distinct but related settings. More specifically, all three chapters highlight situations involving both elements of cooperation and competition. The first chapter examines a dynamic public goods game in which agents must decide whether to provide a public good that benefits all. While everyone prefers the public good to be provided, each agent prefers that someone else bears the cost of provision. As a result, cooperation is necessary for provision, yet a 'war-of-attrition' emerges over who should provide first. The second and third chapters study team contests in which a group of agents compete collectively against another group. Only the winning team secures a prize, which creates incentives for members to cooperate within their own team while competing against the opposing team. The aim is to understand how agents' preferences and institutional structures shape the balance between cooperative and competitive elements of the game and how this, ultimately, affects public goods provision and contest efforts.

Chapter 1 studies cooperation through the framework of the 'Volunteer's Dilemma', a game in which a public good is provided as long as at least one player provides it. Although every individual in the group finds it beneficial to provide the public good by herself, each prefers that someone else bears the costs. Such a trade-off arises, for example, when only one friend is needed to plan a trip for the group to enjoy, or when a single country is sufficient to coordinate international talks that benefit all participants. While much of the existing literature focuses on static settings or assumes exponential discounting, this study examines the timing of volunteering when agents are hyperbolic discounters in a dynamic volunteer's dilemma.

Specifically, we study a dynamic volunteering dilemma game in which two players repeatedly decide whether to volunteer or wait, provided that no volunteering has occurred in previous periods. Agents may have time-inconsistent preferences, modelled using $\beta - \delta$ preferences introduced by Phelps and Pollak (1968). We fully characterise the stationary Markov Strotz-Pollak equilibria. When the cost of volunteering is sufficiently small or agents' present-bias parameters are sufficiently close, there always exists an equilibrium in which both players randomise. This equilibrium features stochastic delay in provision and the delay is exacerbated if one or both agents become more present-biased. Although all players prefer to be 'cooperative' and volunteer if no one else volunteers, the 'competitive' tension between potential volunteers generates inefficient delay. However, when agents differ significantly in their present-bias parameters, this difference can serve as a 'natural coordination device': the unique stationary equilibrium predicts that only the less severe procrastinator volunteers, potentially leading to an even quicker provision of the public good compared with the case of two exponential discounters.

Chapter 2 investigates political candidates' incentives in electoral competition under mixed electoral systems using a contest-theoretical model. In mixed electoral systems, candidates can win parliamentary seats either by securing the most votes in local districts or through party lists. Candidates' successes in district elections often hinge on personal charisma, whereas the total number of party seats depends on the overall party vote share. With limited campaign budgets, candidates face a trade-off between allocating resources towards their individual or team battles: more individualised campaigning boosts individual appeal and attract voters at local districts, while more party-centric campaigning increases the party's seat share.

In the benchmark model, two parties, each with two candidates, compete for at most three parliamentary seats. At the district level, candidates face one-on-one contests, with the winners securing a seat in the parliament. The final seat is awarded to the winning party at the party-level contest and then distributed internally, either through a closed or an open list. Each candidate is endowed with a campaign budget, which must be allocated between their district and party battles. In making their decisions, candidates weigh not only the intensity of competition against their district opponents but also the extent to which they wish to cooperate with their fellow party members. Under a closed-list system, where party-level prizes are distributed according to a predetermined ranking, lower-ranked candidates concentrate their efforts on their individual battles, while higher-ranked candidates balance resources in both battles. Their exact allocation also depends on the rank of their district-level opponents. In contrast, in an open-list system, where party-level prizes are tied to candidates' vote shares, all candidates are incentivised to contribute to party-centric campaigning.

Chapter 3 analyses a team contest in which team members must not only cooperate in their contest efforts, but also coordinate their 'directions'. Coordination often requires communication, yet in some cases opponents can gain access to this internal communication, a phenomenon often referred to as "Sign-Stealing" in sports. While the term originates from sports, the underlying idea is relevant in many competitive contexts, such as espionage in joint military actions, industrial espionage in business rivalries and information leakage of campaign strategies in political contests. This chapter studies the impact of sign-stealing on teams' competition behaviour, both theoretically and experimentally.

In our team contest model, two players (team A) compete against a single opponent (team B), with both players' contest efforts and binary direction choices affecting their team's winning probability. A mismatch in directions within team A results in an immediate loss, while successfully coordinating on a direction different from the opponent yields a directional advantage. To coordinate, team A members may send messages, but these communications may be observed by the opposing team. The equilibrium analysis shows that when communication is possible, team A members always send messages, adhere to them and achieve perfect coordination with no sign-stealing. Even under the threat of sign-stealing, team A continues to send messages and coordinate perfectly if the directional advantage that team B gains from matching team A's direction is sufficiently small. Sign-stealing has no effect on contest efforts when neither team holds a prior advantage. However, when team A has a prior advantage, sign-stealing levels the playing field and increases the efforts of both teams. In contrast, when team B has an advantage, sign-stealing further exacerbates this imbalance, leading both teams to exert less effort.

We test these theoretical predictions in a laboratory experiment under the assumption that neither team holds a prior advantage and the directional advantage is small. In line with the theory, sign-stealing by team B does not significantly influence contest efforts of either team. In contrast to the theoretical prediction, however, it does affects communication decisions: team A members are more reluctant to send messages about their intended action. This leads to lower coordination rates within team A as compared to the regime without sign-stealing. Nevertheless, once a message is sent, both team A and team B members follow the indicated action almost perfectly with and without sign-stealing.

Chapter 1

Dynamic Volunteer's Dilemma with Procrastinators

This chapter is based on a published article.[1]

1.1 Introduction

We consider the situation in which a public good is provided as long as at least one player provides it. Even though every individual in the group finds it beneficial to provide the public good by herself, she would prefer it if someone else supplied the public good and shouldered the cost of providing instead. This social dilemma is often referred to as the 'volunteer's dilemma'. Many social situations resemble the volunteer's dilemma game. Suppose some friends are planning a trip together on a future weekend, and they need one volunteer to conduct research on the potential destinations, plan and coordinate the trip and book the tickets. Suppose they know each other well enough and are aware that everyone in the group finds the benefits from a pleasant trip greater than the cost of being the trip planner. The question is then, who will volunteer: everyone in the group wants the trip to be organised but each group member prefers someone else to plan it. This question also appears in the international arena: which country sets up an environmental research centre when a group of countries share the same forest or river; which country adopts a new technology first when multiple countries will benefit from the implementation; which country coordinates the talks when both France and the UK want to make progress in tackling a surge in migrants across the Channel. The above examples are special in one aspect: the benefits of the volunteering action are relatively delayed compared to the costs – the trip can be weeks or months away from the point of planning.

Rather than a one-time choice, many decisions are made over a long period and agents' time preferences can affect the timing of the volunteering. Previous studies on the dynamic volunteering dilemma mostly assume exponential discounting. However, many experimen-

[1] See Shi (2025).

tal and empirical studies suggest that agents can be present-biased and time-inconsistent[2], and such preferences are used to explain behaviours such as procrastination, smoking and unhealthy eating. A procrastinator may delay the volunteering action because of its immediate costs and delayed benefits, but will she change her mind if other potential volunteers are also procrastinators?

In this study, we aim to answer the following research questions: firstly, comparing with only time-consistent agents, how will the equilibrium outcome of the dynamic volunteering dilemma game change if agents are procrastinators and the benefits occur later than the costs? Secondly, what does the equilibrium outcome suggest for the timing of volunteering – will there be more or less delay in volunteering compared to the case with only exponential discounters? Thirdly, what is the identity of the volunteer – will the more severe or the less severe procrastinator be more likely to volunteer?

We consider a dynamic volunteering dilemma with complete information: there are an infinite number of time periods and two agents with potentially different time preferences, and each agent can volunteer or wait out each time period given there were no volunteering actions in the past. The players make no further choices once at least one of the agents volunteers. The benefits of volunteering are delayed compared to the costs. Agents can have time-inconsistent preferences, captured by $\beta - \delta$ preferences introduced by Phelps and Pollak (1968). We focus on sophisticated agents in this study and provide brief discussions of cases with naive agents. We fully characterise the stationary Markov Strotz-Pollak equilibria. When the costs of volunteering are sufficiently small or the agents' present-bias parameters are sufficiently close, there always exists a stationary equilibrium in which both players randomise, suggesting an overall delay in volunteering. The delay is exacerbated if one or both agents become more present-biased. Moreover, the severe procrastinator has a higher probability of volunteering. When the agents' present-bias parameters differ sufficiently, the unique stationary equilibrium predicts that only the slight procrastinator will volunteer. This may imply a quicker provision compared with the case of two exponential discounters. Introducing present bias into the model allows us to identify a unique stationary equilibrium when the two agents differ sufficiently in their degree of present bias. This is not possible when we have only exponential discounters, even when they differ significantly in their discount factors.

This study is related to two main strings of literature: the volunteer's dilemma and time-inconsistent preferences. For the static version of the volunteer's dilemma game, Diekmann (1985) studied a symmetric N-player volunteering dilemma game. The game has N asymmetric strong equilibria in which one of the players volunteers and one symmetric mixed strategy equilibrium. The author also noted that social norms or coordination devices may be required to achieve one of those asymmetric equilibria. Diekmann (1993) extended the

[2] See Frederick et al. (2002), DellaVigna and Paserman (2005), Paserman (2008), Fang and Silverman (2009) and Augenblick et al. (2015).

model with asymmetric agents. The mixed strategy equilibrium yields a counter-intuitive result: the player's probability of volunteering increases with costs. Weesie and Franzen (1998) extended the model such that the costs of providing are shared equally among the volunteers. This assumption changed the magnitude of the probability but not the comparative statics.

Volunteer's dilemma game has also been extended to dynamic settings. Bliss and Nalebuff (1984) considered a game with incomplete information about other players' costs, where equilibrium involves a delay in provision, which serves as a signalling mechanism for volunteering costs. Our study examines a model of complete information. Weesie (1993) considered a game with complete information and declining benefits of volunteering: with symmetric players, there exists an equilibrium where players randomise the timing of their provision; with asymmetric players, the unique equilibrium involves the agent in the most favourable position (in terms of benefit to cost ratio) volunteering. Bilodeau and Slivinski (1996) reached a similar conclusion under a finite-horizon assumption. Standard models of the dynamic volunteer's dilemma with complete information are generally non-stationary, as players' benefits may change over time or players have a final period in which to act. Such non-stationarity is also crucial in determining the unique equilibrium under asymmetry. In contrast, our model preserves stationarity and demonstrates that a unique stationary equilibrium can still emerge when agents are present-biased and the benefits of volunteering arrive after the costs. The dynamic volunteer's dilemma is structurally similar to war-of-attrition games, in which players decide the timing to concede or exit the market, with each preferring that the other concedes first[3]. In addition, the existing studies either neglect discounting or assume time-consistent preferences[4]. Our study contributes to this literature by examining the role of present-biased preferences in the dynamic volunteer's dilemma.

The second string of literature relates to time-inconsistent preferences. In individual decision-making, these preferences have been used to explain behaviours such as procrastination, addiction, smoking and under-saving. For example, O'Donoghue and Rabin (1999a) showed that present-biased individuals tend to procrastinate over tasks with immediate costs and delayed benefits[5]. A growing literature also studies the implications of time-inconsistent preferences in non-cooperative games. Prior studies have explored their effects in settings such as infinitely repeated games[6], bargaining games, team production, policy implementations and public goods provision.

[3]For games related to war-of-attrition, see also Maynard Smith (1974), Hendricks and Wilson (1985), Ghemawat and Nalebuff (1985), Fudenberg and Tirole (1986), Hendricks et al. (1988) and Shapira and Eshel (2001).

[4]For more extensions or experimental studies of the static and dynamic volunteering dilemma, see Weesie (1994), Bilodeau et al. (2004), Otsubo and Rapoport (2008), Goeree et al. (2017), Barbieri et al. (2020), Campos-Mercade (2021) and Leo and Pate (2025). There are also some evolutionary studies such as He et al. (2014) and Konrad and Morath (2021).

[5]See also Strotz (1955), Phelps and Pollak (1968), Peleg and Yaari (1973), Goldman (1980), Laibson (1997), O'Donoghue and Rabin (2001) and O'Donoghue and Rabin (2008); studies on contracting with time-inconsistent agents include O'Donoghue and Rabin (1999b), DellaVigna and Malmendier (2004), Eliaz and Spiegler (2006), Gilpatric (2008), Heidhues and Kőszegi (2009), Akin (2012) and Yılmaz (2013).

[6]See Chade et al. (2008) and Obara and Park (2017).

Akin (2007) studied an alternating-offers bargaining game with potentially naive agents who can learn over time. The study found that bargaining delays arise as naive agents gradually recognise their own time-inconsistency[7]. Brocas and Carrillo (2001) examined scenarios in which agents with time-inconsistent preferences compete or cooperate on a joint project. They found that task complementarity exacerbates procrastination among symmetric agents, a result similar to ours. Our study further considers agents with different extent of present bias[8]. Furusawa and Lai (2011) modelled policy implementation as a war-of-attrition between two competing political parties; unlike in the dynamic volunteering dilemma, both parties cannot make decisions simultaneously. Altınok and Yılmaz (2018) analysed a dynamic voluntary contribution game with quasi-hyperbolic discounting, showing that time-inconsistent sophisticated agents complete the project earlier than time-consistent agents. Altınok and Yılmaz (2018) allow agents to make partial contributions while we consider a binary public good and focus on the problem of who provides the public good when every agent finds it beneficial to provide on their own. Gans and Landry (2022) studied a dynamic threshold public good game with naive present-biased agents, who upon discovering their own present bias, also update their beliefs regarding the other player's present bias and higher-order beliefs. They showed that the public good is always provided if players have limited reassessment and differs with respect to whether one player reassesses their partner's present bias more deeply than their own. As the authors acknowledge, the game requires each player to contribute at least once, and the subgame in $t = 1$ resembles a static volunteer's dilemma. When adapted to a dynamic volunteer's dilemma, the present bias no longer plays a role. In contrast, our study examines a dynamic volunteer's dilemma where present bias directly affects the public goods provision. This effect arises because the public good is provided once the threshold is reached and benefits of provision always arrive later than the costs[9]. Our study complements the literature by studying the implications of time-inconsistent preferences in a game addressing coordination issues.

This paper is organised as follows: Section 1.2 presents the model. Section 1.3 expounds the stationary Markov Strotz-Pollak equilibria and studies the implications. Section 1.4 discusses some special cases, the case of naive agents and multiple volunteers. Section 1.5 concludes. The Appendices will show the detailed calculations of our propositions and results.

1.2 Formal Model

We consider a dynamic volunteering dilemma game in a complete information environment with two players with possibly different time preferences; more specifically, different extents

[7] For other bargaining games with time-inconsistent agents, see Lu (2016) and Schweighofer-Kodritsch (2018).

[8] For other studies on team production, see Battaglini et al. (2005), Weinschenk (2016), Fahn and Hakenes (2019), Cerrone (2021) and Weinschenk (2021).

[9] The authors also have other papers with a similar setting, see Gans and Landry (2019).

of present bias measured by the value of β[10]. In the first period, two players are asked simultaneously to volunteer or not: if only one of them volunteers, the volunteer pays a strictly positive cost of c and both players receive a one-off benefit of 1, s periods after the volunteering act, where we assume s is a strictly positive integer; if both players volunteer at the same time, then they share the cost equally (each pays $\frac{1}{2}c$). Alternatively, it is equivalent to assume that there is a randomisation device to decide who is the volunteer that will pay the cost. Note that as long as at least one of the players volunteers, there are no more choices for players to make in the future. If no one volunteers in the first period, the game continues to the second period and repeats as in the first period, and so on.

1.2.1 Players, actions, histories and strategies

There are two players $i \in \{A, B\}$, who make decisions in an infinite series of time periods, $t = 1, 2, 3, ...,$. Players cannot bind their future choices. The actions of players at different time periods are denoted as a_t^i. The action space of players at different time periods is contingent on whether there have been any volunteering actions in the past. If there have not been any volunteering actions in the past, then player i can take one of two possible actions in period t. The actions are $a_t^i \in \{1, 0\}$. The action $a_t^i = 1$ means that player i takes a volunteering action at time t. The action $a_t^i = 0$ means that player i does not take such an action at time t. Other than the word 'volunteering', we might also use the phrase 'provides the public good' to describe the same action. If there were volunteering actions in the past, we assume the actions of both players are just $a_t^i = 0$. The sequence of these actions generates a history of actions. The history at the beginning of period t is denoted by h_t and is a series

$$h_t = (\mathbf{a}_{t-1}^A, \mathbf{a}_{t-1}^B) = ((a_1^A, ..., a_{t-1}^A), (a_1^B, ..., a_{t-1}^B))$$

The set of all possible histories in period t is denoted by $\mathbf{H}_t$. A series of actions for which $t \to \infty$ is called a trajectory and is denoted by h; the set $\mathbf{H}$ is the set of all possible trajectories. At each period t, we define a state variable $j(t) \in \{0, 1\}$. $j(t) = 1$ implies that there have already been volunteering actions in the past. $j(t) = 0$ implies that there have been no volunteering actions up to period t. Note that there can only be one such history at each t satisfying $j(t) = 0$, that is $h_t = ((0, ..., 0), (0, ..., 0))$.

Since agents are allowed to randomise their actions, we define a behavioural Markov strategy σ_i for player $i \in \{A, B\}$ as a sequence of mappings: $\sigma_i = \{\sigma_{i,t}\}_{t=0}^{\infty}$, $\sigma_{i,t} : \{0, 1\} \to \Sigma_{\{0,1\}}$. Given players make choices only when no one has volunteered in the past, we focus on their strategies when $j(t) = 0$. When $j(t) = 0$, the strategy at t is denoted as $\sigma_{i,t}(0) = (1 - P_{i,t}, P_{i,t})$ and $P_{i,t} \in [0, 1]$, where $P_{i,t}$ is the probability that player i volunteers at t if no one volunteered in the past. A behavioural Markov strategy is completely mixed if $P_{i,t} \in (0, 1)$ for all t, meaning the player randomises between volunteering and waiting with strictly positive probabilities.

[10]The case of two symmetric players can also be included as a special case.

Since this game is stationary, we further partition all finite histories by the same state variable, $j \in \{0,1\}$. We can focus on (stationary) behavioural Markov strategies $\sigma_i^s : \{0,1\} \to \Sigma_{\{0,1\}}$. This means that across all t, the probability of volunteering remains constant $\sigma_i^s(0) = (1-P_i, P_i)$ and $P_i \in [0,1]$. A (stationary) completely mixed behavioural Markov strategy implies agents always volunteer with probability $P_i \in (0,1)$. Other stationary strategies include that agents always wait ($P_i = 0$) or volunteer ($P_i = 1$).

1.2.2 Payoffs

We first define the instantaneous payoff function of player i at time t. The players will only receive the benefit of 1 in one period, which is at the period of the volunteering action plus s. Therefore, the instantaneous payoff of agents not only depends on their current actions, but also their actions exactly s periods earlier. We assume agents are risk-neutral and the instantaneous payoff is defined as follows:

$$v_t^i(\mathbf{a}_t^A, \mathbf{a}_t^B) = \begin{cases} -c, & \text{if } a_t^i = 1 \text{ and } a_t^{-i} = 0 \\ -\frac{1}{2}c, & \text{if } a_t^i = 1 \text{ and } a_t^{-i} = 1 \\ 1, & \text{if } t \geq s \text{ and } \exists j \in \{A,B\} \text{ such that } a_{t-s}^j = 1 \\ 0, & \text{otherwise} \end{cases} \tag{1.1}$$

where a_t^{-i} denotes the action of the player who is not player i. If agent i volunteers at period t but the other player does not, then she has to pay the cost of volunteering herself immediately, thus her current payoff at time t is $-c$. If the other agent also volunteers, then they share the costs equally. If the public good is provided s periods before time t, then both agents receive the benefit of 1 at time t. For all other scenarios, the instantaneous payoff is 0.

Given the instantaneous payoff functions, we define the discounted payoff functions for player i at time period t given a trajectory h:

$$u_t^i(h) = v_t^i(\mathbf{a}_t^A, \mathbf{a}_t^B) + \beta_i \sum_{\tau=t+1}^{\infty} \delta^{\tau-t} v_\tau^i(\mathbf{a}_\tau^A, \mathbf{a}_\tau^B) \tag{1.2}$$

Note the future payoffs are discounted using the quasi-hyperbolic discounting function, at t: the agent discounts the instantaneous payoffs at $t+1$ by the discount factor $\beta_i\delta$ and the agent discounts the instantaneous payoffs at $t+2$ by the discount factor $\beta_i\delta^2$. Therefore, the parameter β_i is applied once to capture the agents' present bias, $0 \leq \beta_i \leq 1$. When $\beta_i = 0$, the players discount the future completely; when $0 < \beta_i < 1$, the players are quasi-hyperbolic discounters; when $\beta_i = 1$, the players are exponential discounters. When $\beta_i = 0$, the public good gives no benefit to the agents so that the agents will not want to volunteer even if nobody else does; this violates the definition of a volunteer's dilemma and we will not consider this case. We also allow β_i to differ between the two agents; the symmetric case in which two agents have the same β parameter is also included as a special case. Every behavioural Markov strategy profile $\sigma = (\sigma_A, \sigma_B)$ generates a probability distribution over all possible history trajectories. The expected discounted payoff for player i for any t, given that $j(t) = 0$, is calculated as $E^i_{\sigma|j(t)=0}\left(u^i_t(h|j(t) = 0)\right)$.

1.2.3 Equilibrium

In this paper, we adopt the Markov Strotz-Pollak equilibrium concept. Similar to Markov perfect equilibrium, which is a subgame perfect equilibrium where all players use Markov strategies (Maskin and Tirole, 2001), a Markov Strotz-Pollak equilibrium is a Strotz-Pollak equilibrium where all players use Markov strategies. A strategy profile σ is a Strotz-Pollak equilibrium if and only if there are no profitable one-shot deviations for any player after any history. This means that, after any history, no player can find a strictly better strategy given that their future selves and the other player will adhere to the equilibrium strategies following any deviations. The intuition is that sophisticated agents are aware of their future self-control problems. They choose the best possible strategy considering what their future selves will actually behave.[11]

Note that the Strotz-Pollak equilibrium differs from the subgame perfect Nash equilibrium (SPNE). While the SPNE requires robustness to all possible deviations, the Strotz-Pollak equilibrium checks only for one-shot deviations. The two concepts coincide because the one-shot deviation principle applies with time-consistent agents. However, the one-shot deviation principle no longer applies with time-inconsistent preferences and the SPNE is in general a subset of the Strotz-Pollak equilibrium (Chade et al., 2008). This implies that there may exist strategy profiles where players cannot identify any profitable one-shot deviations (satisfying Strotz-Pollak equilibrium) but fails the stricter SPNE requirements. For example, a player's current self might identify improvements in their payoffs from altering their strategies in some future periods, violating SPNE. Importantly, this does not mean they will actually change their strategies when the future period arrives, as these deviations are only deemed profitable by their current self and not their future selves due to time-inconsistent

[11]Note that this equilibrium concept only applies to sophisticated agents. If we consider naïve agents, they falsely perceive themselves as time-consistent. These agents fail to anticipate their future self-control problems and choose strategies based on what they think their future selves would do, rather than what their future selves will actually do. In such case, a different equilibrium concept is required, which we will discuss in Section 1.4. For more discussions of the equilibrium concept, please also see Phelps and Pollak (1968), Peleg and Yaari (1973), Goldman (1980), Chade et al. (2008), Obara and Park (2017) and Schweighofer-Kodritsch (2018).

preferences[12]. Given the stationary structure of the game, we focus on stationary Markov Strotz-Pollak equilibrium, where players play stationary behavioural Markov strategies.

1.2.4 Assumptions

We assume:

1. All players find it worthwhile to provide the public good themselves, that is, $0 < c < \min\{\beta_A \delta^s, \beta_B \delta^s\}$. In words, for both players, the delayed benefit is strictly larger than the costs of volunteering.

2. When both players volunteer, we assume there is cost-sharing: both players share the costs of volunteering equally. This assumption matches some situations better than others, for example, saving a drowning victim or calling the police when an accident happens does not normally accommodate cost-sharing. However, cost-sharing is more plausible when considering planning a trip with friends or organising a conference. For most of the scenarios with delayed benefits we can think of, cost-sharing seems to be a more reasonable assumption than without, the model is also ready to extend to the case without cost-sharing. See Weesie and Franzen (1998) for more discussion.

1.3 Main Results

1.3.1 One Player Case

To help us understand the underlying mechanism, we start by analysing the case where there is only one potential volunteer who is potentially present-biased ($0 < \beta_i \leq 1$). The player needs to decide whether to volunteer or not in each period.

When the player is time-consistent ($\beta = 1$), the player will always choose to provide immediately. To see this, the player's payoff from providing now is $\delta^s - c$; her expected payoff from waiting is largest if she volunteers next period, this gives her the payoff $\delta^{s+1} - \delta c = \delta(\delta^s - c) < \delta^s - c$. Therefore, no matter what the agent's future selves do, she always finds it more profitable to provide now rather than wait. Thus, the only strategy profile that survives one-shot deviations is that the agent always volunteers. The public good will be

[12] **Illustrative example:** consider a public goods provision game where two players contribute to a threshold public good over three periods. In each period, they can contribute either 10 or 50 units. The public good is provided if the total contribution meets or exceeds 60 units, and the benefit $b = 150$ is realised one period after. Assume the cost of provision is 1 per unit, and both players have discounting factors $\delta = \frac{1}{2}$ and $\beta = \frac{1}{3}$. A strategy profile where both players contribute 10 units each period has no profitable one-shot deviations and thus constitutes a Strotz-Pollak equilibrium. However, it fails SPNE: at period 1, the players find it optimal to contribute 50 units in period 2 if the public good has not been provided ($-10 - \beta\delta 10 - \beta\delta^2 10 + \beta\delta^3 b < -10 - \beta\delta 50 + \beta\delta^2 b$), the proposed strategy thus fails to be a sincere NE in the root subgame. Nevertheless, when period 2 actually arrives, the players prefer to contribute 10 units ($-10 - \beta\delta 10 + \beta\delta^2 b > -50 + \beta\delta b$). This discrepancy arises due to time-inconsistent preferences, where different periods' selves disagree with one another.

provided immediately. The intuition is that when the agent realises it is beneficial to volunteer, she prefers to provide it sooner rather than later because that future payoff will be discounted. When the player is present-biased, $\beta < 1$, the player's payoff from providing now is $\beta\delta^s - c$. As for the expected payoff from waiting, it is largest if she volunteers in the next period, which gives her an expected payoff of $\beta\delta^{s+1} - \beta\delta c$. This comparison is not immediately clear and depends on the exact parameter values. More specifically, if the cost of volunteering c is sufficiently small (less than or equal to $\frac{\beta\delta^s(1-\delta)}{1-\beta\delta}$), then the agent still prefers volunteering and the public good is provided immediately. However, if $c > \frac{\beta\delta^s(1-\delta)}{1-\beta\delta}$, the agent now prefers to delay volunteering to the next period. The agent wants the public good to be provided but she also prefers to delay the provision. When she is sophisticated and aware of her tendency to procrastinate, this allows the existence of a stationary completely mixed behavioural Markov strategy in which the agent volunteers with a probability strictly between 0 and 1 every period. This is also the only stationary strategy that survives one-shot deviations. Unlike an exponential discounter, this strategy features stochastic delay in provision. Also, the probability of volunteering is calculated as $\frac{(1-\delta)(\beta\delta^s-c)}{c\delta(1-\beta)}$ and it decreases as the agent becomes more present-biased.

To summarise, when there is only one time-consistent potential volunteer, she always volunteers immediately no matter the value of the discount factor δ. When the potential volunteer is present-biased, there may be stochastic delay in volunteering. What will happen if there exists another potential volunteer?

1.3.2 Equilibrium Results

In this section, we discuss the main equilibrium results when there are two potential volunteers with potentially different levels of present bias. In particular, we fully characterise the stationary Markov Strotz-Pollak equilibria. There are three different cases to consider. We assume without loss of generality $\beta_A \geq \beta_B$ – player A is less present-biased than player B.

Proposition 1*: If the costs of volunteering are sufficiently small* $\left(c \leq \frac{\beta_B\delta^s(1-\delta)}{1-\beta_B\delta}\right)$ [13]*, there exist three stationary Markov Strotz-Pollak equilibria: (a) player A always waits and player B always volunteers; (b) player A always volunteers and player B always waits; (c) a completely mixed behavioural Markov strategy in which both players randomise. The probabilities are calculated from the indifference conditions for both agents:*

$$(1-P_A)(1-P_B) = \frac{1}{\delta} + \frac{\beta_A\delta^{s+1} - \beta_A\delta^s - P_A c\beta_A\delta(1-\frac{1}{2}P_B)}{\frac{1-\frac{1}{2}P_B}{1-P_B}\delta c} \tag{1.3}$$

$$(1-P_A)(1-P_B) = \frac{1}{\delta} + \frac{\beta_B\delta^{s+1} - \beta_B\delta^s - P_B c\beta_B\delta(1-\frac{1}{2}P_A)}{\frac{1-\frac{1}{2}P_A}{1-P_A}\delta c} \tag{1.4}$$

We prove that the three strategy profiles above are indeed Strotz-Pollak equilibria by es-

[13] Note that this condition is always satisfied if both players are exponential discounters.

tablishing that there are no profitable one-shot deviations for any player after any non-volunteering history. Equilibria (a) and (b) are straightforward to understand, when one of the players chooses to always wait, while the other player prefers to volunteer than not. Furthermore, when the costs of volunteering are sufficiently small, as we explained in the one-player case, the player prefers to volunteer now than later. If one of the agents chooses to always volunteer, then the other agent will always find it more profitable to wait and free ride on the volunteer. We can further rule out other possible stationary strategy profiles involving one player playing pure strategies. The detailed proof is in Appendix 1A.

Given that each player wants the public good to be provided but also prefers the other agent to provide it, there also exists a stationary equilibrium (c) in which both players randomise between volunteering and waiting. In such an equilibrium, given the other player's and her future selves' strategies, the players must be indifferent between volunteering and waiting. Equations (1.3) and (1.4) represent the indifference conditions for player A and player B, respectively, since we consider stationary strategies, the indifference conditions are the same for all time periods.

If no one has volunteered previously, given player B always volunteers with probability P_B and player A volunteers with P_A in the future, we calculate player A's expected payoff of volunteering:

$$EU^A(volunteer) = (1-P_B)(\beta_A\delta^s - c) + P_B\left(\beta_A\delta^s - \frac{1}{2}c\right) = \beta_A\delta^s - \left(1 - \frac{1}{2}P_B\right)c \quad (1.5)$$

The first part refers to the probability that the other player does not volunteer, in which case player A shoulders the cost of volunteering by herself. The second part refers to the possibility that the other player volunteers as well so players share the costs evenly. As for the expected utility from waiting, it is calculated as follows:

$$\begin{aligned} EU^A(wait) = P_B\beta_A\delta^s + (1-P_B)\Big[P_AP_B\left(\beta_A\delta^{s+1} - \frac{1}{2}\beta_A\delta c\right) \\ + (1-P_A)P_B\beta_A\delta^{s+1} + P_A(1-P_B)\left(\beta_A\delta^{s+1} - \beta_A\delta c\right) + (1-P_A)(1-P_B)... \quad (1.6) \end{aligned}$$

This is an infinite sequence, which simplifies to:

$$= P_B\beta_A\delta^s + (1-P_B)\frac{(1-(1-P_A)(1-P_B))\beta_A\delta^{s+1} - P_A\beta_A\delta\left(1-\frac{1}{2}P_B\right)c}{1-(1-P_A)(1-P_B)\delta} \quad (1.7)$$

When player A chooses to wait, there is a probability P_B that player B volunteers, in which case player A enjoys the delayed benefit without incurring any cost. However, with probability $1 - P_B$, no one volunteers in the current period, and the game continues to the next period. In the next period, four possible outcomes arise: both agents volunteer with probability P_AP_B; only agent A volunteers with probability $P_A(1-P_B)$; only agent B volunteers with probability $(1-P_A)P_B$; no one volunteers with probability $(1-P_A)(1-P_B)$, leading to another continuation of the game, discounted by a further δ. This will form an infinite sequence and the sum can be simplified as in equation (1.7). For the completely mixed behavioural Markov strategy profile to be an equilibrium, player A must be indifferent between volunteering and waiting. Therefore, $EU^A(volunteer) = EU^A(wait)$; the indifference condition for player A is therefore:

$$\beta_A\delta^s - \left(1-\frac{1}{2}P_B\right)c = P_B\beta_A\delta^s + (1-P_B)\frac{(1-(1-P_A)(1-P_B))\beta_A\delta^{s+1} - P_A\beta_A\delta\left(1-\frac{1}{2}P_B\right)c}{1-(1-P_A)(1-P_B)\delta} \tag{1.8}$$

Rearranging and dividing both sides by $(1-P_B)$ gives:

$$\beta_A\delta^s - \frac{1-\frac{1}{2}P_B}{1-P_B}c = \frac{(1-(1-P_A)(1-P_B))\beta_A\delta^{s+1} - P_A\beta_A\delta\left(1-\frac{1}{2}P_B\right)c}{1-(1-P_A)(1-P_B)\delta} \tag{1.9}$$

Simplifying further gives:

$$(1-P_A)(1-P_B) = \frac{\beta_A\delta^{s+1} - \beta_A\delta^s - P_Ac\beta_A\delta\left(1-\frac{1}{2}P_B\right) + \frac{1-\frac{1}{2}P_B}{1-P_B}c}{\frac{1-\frac{1}{2}P_B}{1-P_B}c\delta} \tag{1.10}$$

For player B, the indifference condition will be exactly symmetric:

$$(1-P_A)(1-P_B) = \frac{\beta_B\delta^{s+1} - \beta_B\delta^s - P_Bc\beta_B\delta\left(1-\frac{1}{2}P_A\right) + \frac{1-\frac{1}{2}P_A}{1-P_A}c}{\frac{1-\frac{1}{2}P_A}{1-P_A}c\delta} \tag{1.11}$$

We can further prove that there exists one and only one pair of values (P_A, P_B) satisfying both indifference conditions, implying that there can only exist one such stationary equilibrium with completely mixed behavioural Markov strategies. For the complete proof, see Appendix 1A.

Suppose two countries share the same river and they plan to set up a research centre to gain better insights into improving the river's water quality. One of the two countries needs to host the research centre. Suppose both governments are present-biased but the costs of hosting are not too high, for example, the research agenda of the new centre does not require experimental equipment or field trips, so the hosting government only needs to coordinate researchers, find office spaces and hire administration staff, etc. Equilibria (a) and (b) imply that one of the countries will set up the research centre immediately. However, there is another possibility that both countries will wait in the hope of the other country volunteering to host first, implying an overall delay in setting up the research centre.

Proposition 2*: If the costs of volunteering are not sufficiently small* $\left(c > \frac{\beta_B\delta^s(1-\delta)}{1-\beta_B\delta}\right)$ *and* β_A *and* β_B *are sufficiently close*[14]*, then there always exist three stationary Markov Strotz-Pollak equi-*

[14]The exact condition is $\beta_A < \beta_B\frac{(\delta^s-c)(c\delta(1-\beta_B)-\frac{1}{2}(\beta_B\delta^s-c)(1-\delta))}{(\delta^s-\beta_B\delta^s)(c\delta(1-\beta_B)-(\beta_B\delta^s-c)(1-\delta))}$. Since the part in the fraction is larger than 1, the condition implies that the two present-bias parameters β_A and β_B need to be sufficiently close.

libria: (a) Player B randomises and player A always waits; (b) Player B always waits and player A always volunteers or randomises (depending on the exact parameter values); (c) the equilibrium in which both players play completely mixed Markov behavioural strategies as in Proposition 1.

The main differences between propositions 1 and 2 are mainly from equilibria (a) and (b). More specifically, in Proposition 1, when one of the players always waits, the other player will always volunteer. However, in case 2, the costs of volunteering are no longer sufficiently small for player B; if player A waits, player B randomises rather than always volunteers. The intuition is as follows: present-biased agents tend to procrastinate over tasks with immediate costs and delayed benefits; they would prefer to volunteer in the next period rather than now. However, they will not delay forever since the sophistication effect kicks in; this means that the agent realises their future selves may not volunteer. This is in contrary to the naive agents who falsely believe their future selves will always volunteer. This explains how player B starts to randomise, even when she is the only potential volunteer. Similarly, if player B always waits, then player A may volunteer immediately if her cost of volunteering is sufficiently small; otherwise, she also randomises. Moreover, if the two players are sufficiently similar in their tendencies to procrastinate; the agents still find free-riding on the other agent attractive even if it comes with some delay, the equilibrium in which both players randomise also exists and is unique.

To prove there exists a unique equilibrium in which both players randomise, we must show there exists a unique pair of (P_A, P_B) satisfying the two indifference conditions (1.3) and (1.4). Illustrating these two conditions in P_A and P_B space (Figure 1.1), this means that the two curves representing the two conditions will intersect and intersect only once.

The blue curve illustrates player A's indifference condition for a specific value of β_A. When player A always waits in the future ($P_A = 0$), player B must volunteer with probability P_C each period to make player A feel indifferent between volunteering and waiting. When player B always waits ($P_B = 0$), player A feels indifferent when her future selves volunteer with probability P_D each period. We can further show that blue curve is downward-sloping. The green and red curves illustrate player B's indifference condition, let us focus on the green curve for now. When player B always waits in the future ($P_B = 0$), player A must volunteer with probability P_E each period to make player B feel indifferent. When player A always waits ($P_A = 0$), player B feels indifferent when her future selves volunteer with probability P_F each period. We can further show: (1) P_D must be greater than P_E; (2) at the point of intersection between player A and player B's indifferences curves, player B's indifference curve must be steeper. We can now see that the two indifference curves intersect exactly once if and only if P_F is above P_C, and this is our Case 2 (green curve). P_F is always above P_C when β_A is sufficiently close to β_B $\left(\beta_A < \beta_B \frac{(\delta^s - c)(c\delta(1-\beta_B) - \frac{1}{2}(\beta_B\delta^s - c)(1-\delta))}{(\delta^s - \beta_B\delta^s)(c\delta(1-\beta_B) - (\beta_B\delta^s - c)(1-\delta))}\right)$. For a more detailed proof, see Appendix 1B.

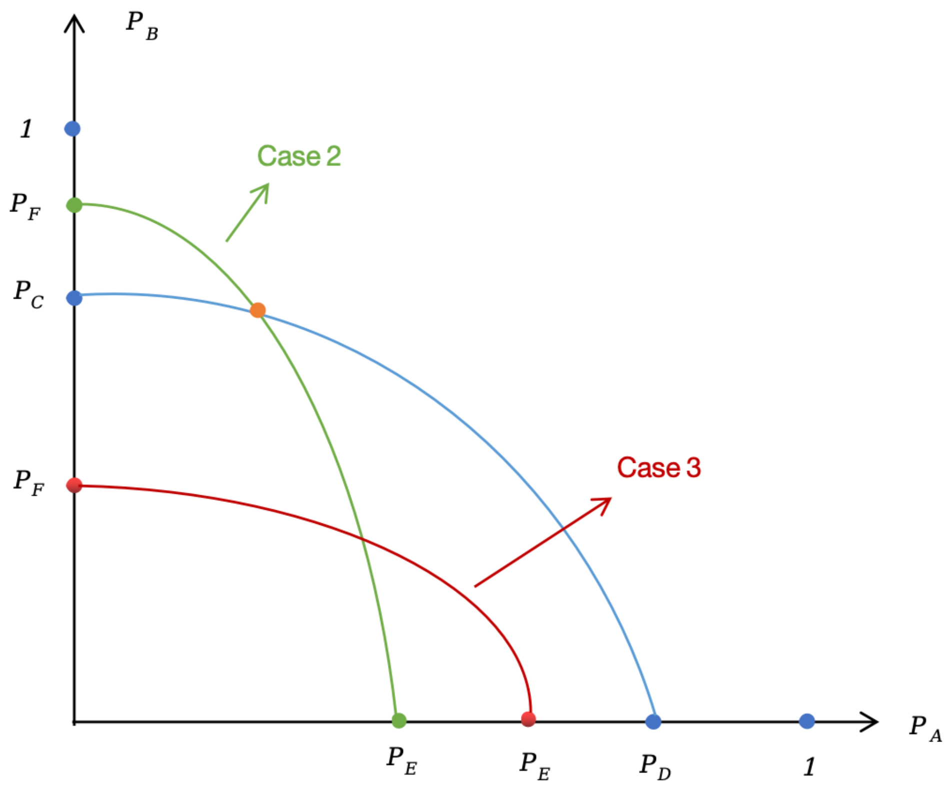


Figure 1.1: Indifference Conditions in P_A–P_B space

We explain using a similar example with two countries volunteering to set up a new research centre. Suppose now the agenda of the research centre requires many experimental projects and local field trips, meaning the costs of volunteering are no longer small for the two governments. If the two governments are similar procrastinators, i.e. they are similarly slow to action when deciding whether to implement long-run projects, then the equilibrium results suggest that there may be a stochastic delay in setting up the research centre, from governments' incentives to free-ride on the other government and their own future administration.

From Figure 1.1, we can further deduce that when P_F is below P_C, there exists no intersections between the two curves and therefore no equilibrium in which both players randomise (Case 3). P_F will always be below P_C if β_A is not sufficiently close to β_B, as opposed to Case 2. We will now explain this case in Proposition 3.

Proposition 3: *If $c > \frac{\beta_B \delta^s (1-\delta)}{1-\beta_B \delta}$ and β_A and β_B differ sufficiently*[15]*, there exists a unique sta-*

[15]The exact condition is $\beta_A > \beta_B \frac{(\delta^s - c)(c\delta(1-\beta_B) - \frac{1}{2}(\beta_B \delta^s - c)(1-\delta))}{(\delta^s - \beta_B \delta^s)(c\delta(1-\beta_B) - (\beta_B \delta^s - c)(1-\delta))}$. Note here that we left the case where $\beta_A = \beta_B \frac{(\delta^s - c)(c\delta(1-\beta_B) - \frac{1}{2}(\beta_B \delta^s - c)(1-\delta))}{(\delta^s - \beta_B \delta^s)(c\delta(1-\beta_B) - (\beta_B \delta^s - c)(1-\delta))}$, the equilibrium in which player A always waits coincides with the equilibrium in which both players randomise. So we have two equilibria in this special case. However, this is a case with specific parameter values and we will omit this case in the analysis.

tionary Markov Strotz-Pollak equilibrium in which player A always volunteers or randomises, while player B always waits.

In this case, the two players are significantly different procrastinators: they differ a lot in their present-bias parameters. There exists a unique stationary equilibrium in which only the less present-biased agent volunteers and the more present-biased agent always waits. Moving from case 2 to case 3, the strategy profile in which the less present-biased always waits can no longer be sustained as an equilibrium. The intuition is as follows: when the more present-biased agent is a very severe procrastinator, she has a very low probability of volunteering even when the other player always waits, resulting in a severe delay in volunteering. Thus, rather than waiting for the severe procrastinator to volunteer with so much delay, the less present-biased agent would rather volunteer herself. When the option to free-ride on the more present-biased agent is no longer attractive, the equilibrium in which both players randomise also disappears. This is also the case where we really see the differences in equilibrium when we introduce present-bias into the model. The proof is in Appendix 1C.

In the example of setting up a research centre, suppose the two countries are now very different procrastinators. One government is delaying the implementation of long-run projects much more than the other country. The less present-biased government realises that if she does not volunteer, the other country will delay setting up the research centre to such an extent that it prefers to set up the research centre itself. When the less present-biased government stops waiting for the other government to volunteer, this may suggest a relatively quick set-up of the new research centre.

Consider a setting with two potential volunteers who both discount the future exponentially but have different discount factors δ. Regardless of how much their discount factors differ, if one player always waits, the other will always choose to volunteer immediately, no matter how low their discount factor is. This implies that the option to free ride remains appealing for both agents and the equilibrium where both players randomise will always exist. As a result, when both players are exponential discounters, there always exist three stationary equilibria, even if their discount factors differ significantly. However, when we introduce present-bias into the model, we can identify a unique stationary equilibrium when agents have sufficiently different present-bias parameters. This occurs because present-biased agents tend to procrastinate volunteering even when the other agent always waits.

Table 1.1 summarises the three cases discussed. Notably, when the cost of volunteering is no longer small, the more present-biased agents may begin to randomise. If she delays volunteering for too long, the less present-biased agent may no longer find it beneficial to wait and chooses to volunteer immediately instead. This is also the reason why equilibria 2 and 3 disappear.

<table>
<tr><th colspan="2"></th><th>Case 1</th><th>Case 2</th><th>Case 3</th></tr>
<tr><td>Conditions</td><td></td><td>Low costs $c \leq \frac{\beta_B \delta^*(1-\delta)}{1-\beta_B \delta}$</td><td colspan="2">High costs $c > \frac{\beta_B \delta^*(1-\delta)}{1-\beta_B \delta}$</td></tr>
<tr><td></td><td></td><td></td><td>Close β^*</td><td>Distant β^*</td></tr>
<tr><td rowspan="3">Equilibria</td><td>(1)</td><td>One player volunteers, the other waits</td><td colspan="2">More present-biased waits, less present-biased volunteers/randomises</td></tr>
<tr><td>(2)</td><td>One player volunteers, the other waits</td><td>Less present-biased waits, more present-biased randomises</td><td></td></tr>
<tr><td>(3)</td><td colspan="2">Both players randomise</td><td></td></tr>
<tr><td>Notes</td><td colspan="4">Case 1 $\rightarrow$ 2: More present-biased player randomises when the other waits.
Case 2 $\rightarrow$ 3: More present-biased player delays so much that the other player loses the incentive to free-ride.</td></tr>
<tr><td colspan="5">* β is close when:
$$\beta_A < \beta_B \left(\frac{(\delta^s - c)(c\delta(1-\beta_B) - \frac{1}{2}(\beta_B \delta^s - c)(1-\delta))}{(\delta^s - \beta_B \delta^s)(c\delta(1-\beta_B) - (\beta_B \delta^s - c)(1-\delta))} \right)$$
and distant otherwise.</td></tr>
</table>

Table 1.1: Summary table of all three cases

1.3.3 Comparative Statics

In this section, we mainly focus on studying the equilibrium with both players playing completely mixed behavioural strategies. Thus, we focus on the cases where this equilibrium exists. We will discuss equilibrium selection in Section 1.3.4.

Proposition 4: *In the stationary equilibrium with completely mixed behavioural Markov strategies, when one player becomes more present-biased, their own probability of volunteering increases and the other player's probability of volunteering decreases.*

$$\frac{dP_i}{d\beta_i} \leq 0$$

$$\frac{dP_j}{d\beta_i} \geq 0$$

where $i, j \in \{A, B\}$, $i \neq j$.

When one of the players becomes more or less present-biased, the players' probabilities of volunteering change and those probabilities still need to satisfy the players' indifference conditions. Therefore, if we take the total derivatives of the left-hand side and right-hand side of both indifference conditions with respect to β, they need to be equal. We can then derive how the players' probabilities of volunteering change with one player's present-bias parameter. See Appendix 1D for more details.

This result implies that if player A becomes marginally more present-biased, in the equilibrium in which both players randomise, player A's probability of volunteering increases while player B's probability of volunteering decreases. This also implies that when players are different procrastinators, the more severe procrastinator has a higher probability of volunteering. To see this, suppose we start from the situation in which both agents have the same present-bias parameter β_A, in which case both agents will have the same probability of volunteering owing to symmetry. Suppose now player B becomes more present-biased and so $\beta_A > \beta_B$, then we know from the derivative results that player A has a lower probability of providing while player B has a higher probability. Thus, it must be the case that player B has a higher probability of volunteering than player A.

We refer to the slight procrastinator as the 'strong' player and we refer to the severe procrastinator as the 'weak' player. If one player becomes less present-biased, we say the player becomes 'stronger'. The above results are rather counter-intuitive in the sense that the 'strong' player has a lower probability of providing.[16] To explain the intuition better, we write the derivatives of both players' indifference conditions with respect to player A's present bias β_A as follows:

$$\frac{dP_B}{d\beta_A} = \frac{\partial P_B}{\partial \beta_A}\Big|_{\bar{P_A}} + \frac{\partial P_B}{\partial P_A}\Big|_{\bar{\beta_A}} \frac{dP_A}{d\beta_A} \tag{1.12}$$

$$\frac{dP_A}{d\beta_A} = \frac{dP_A}{dP_B}\Big|_{\bar{\beta_A}} \frac{dP_B}{d\beta_A} \tag{1.13}$$

Equation (1.12) describes how P_B needs to change if β_A changes to keep player A indifferent. Similarly, equation (1.13) explains how P_A needs to change if β_A changes marginally to keep player B's indifference condition satisfied. The partial derivative only enters equation (1.12) because that β_A only directly appears in player A's indifference condition. Suppose player A becomes more present-biased, in other words, 'weaker', and she prefers waiting to volunteering now given her future selves' and the other player's strategies. Player B exploits this advantage and lowers her own conditional probability of volunteering, which will make player A indifferent between providing and waiting. This can be interpreted using the partial derivative in equation (1.12). When player B's future selves have a lower probability of providing, player B no longer feels indifferent[17] – she would prefer to provide now rather than wait – player A will increase her own probability to make player B indifferent. This can also be interpreted using equation (1.13). This will further make player A prefer to wait rather than feel indifferent and player B will need to adjust her conditional probabilities downwards to make player A indifferent. The probabilities keep adjusting like this until we find a pair of conditional probabilities that satisfy both indifference conditions. To summarise, the player that is more present-biased, the 'weak' player will always have a higher conditional probability of volunteering than the 'strong' player. Therefore, the 'strong' player enjoys a higher utility than the 'weak' player. We can also say that the 'strong' player uses this advantage and exploits the 'weak' player. Moreover, as one player becomes more present-biased, or 'weaker', this can be exploited by the other player and will be reflected in both players' conditional probabilities.

[16] This also shares similarity with the results in Diekmann (1993), in which the player with a lower cost of volunteering has a lower probability of volunteering.

[17] If player B is an exponential discounter, she still feels indifferent between providing and waiting even after changes in her future selves' strategies. This will be explained more in Section 1.4.1 and the Appendices.

Proposition 5: *The expected time period for the provision of the public good, $E(T)$, increases when one or both players become more present-biased.*

The expected time period for the provision of the public good is calculated as:

$$E(T) = \sum_{t=1}^{\infty} t((1 - P_A)(1 - P_B))^{t-1}(1 - (1 - P_A)(1 - P_B)) \tag{1.14}$$

Using the geometric series formula, this simplifies as follows:

$$\begin{aligned} E(T) &= (1 - (1 - P_A)(1 - P_B)) \sum_{t=1}^{\infty} t((1 - P_A)(1 - P_B))^{t-1} \\ &= \frac{1 - (1 - P_A)(1 - P_B)}{(1 - (1 - P_A)(1 - P_B))^2} \\ &= \frac{1}{1 - (1 - P_A)(1 - P_B)} \end{aligned} \tag{1.15}$$

The expected time period for the provision of the public good $E(T)$ increases with the value of $(1 - P_A)(1 - P_B)$. The term $(1 - P_A)(1 - P_B)$ represents the probability that neither player volunteers in a given period. When this probability increases, the expected delay before the public good is provided also increases. To understand how present bias affects $E(T)$, we can take the derivative of $(1 - P_A)(1 - P_B)$ with respect to one of the player's present-bias parameters, e.g. β_A. The derivative is negative, it implies that the probability $(1 - P_A)(1 - P_B)$ increases as player A becomes more present-biased. Note that this comparison is only valid when the equilibrium in which both players play completely mixed behavioural Markov strategies still exists after the change in parameters β_A and/or β_B. We can further infer that when both players become more present-biased, the probability $(1 - P_A)(1 - P_B)$ increases. The overall delay in the provision implied by the randomised strategies is lowest when both agents are exponential discounters; when one or both agents become more present-biased, there is a longer delay before the public good is provided. More detailed proof is in Appendix 1E.

We illustrate the expected timing of volunteering across all cases using a simulation (Figure 1.2). The parameter values are fixed as follows: $\delta = 0.9$, $s = 1$, $c = 0.2$ and $\beta_A = 0.8$. We vary the value of β_B and analyse how changes in β_B affect the expected timing of volunteering $E(T)$. The simulation shows that as player B becomes more present-biased (decreasing β_B), the dynamics move through different cases: (1) When $\beta_B = 0.8$, this is Case 1 and the equilibrium with completely mixed strategy suggests the public good takes around 2.67 periods to be provided; (2) As β_B decreases to 0.7, 0.6 and 0.5, β_B remains sufficiently close to β_A, this is Case 2 and the provision takes longer as player B becomes more present-biased; (3) When β_B decreases further and becomes too far apart from β_A, this is Case 3. The only stationary equilibrium suggests that the public good is provided immediately.

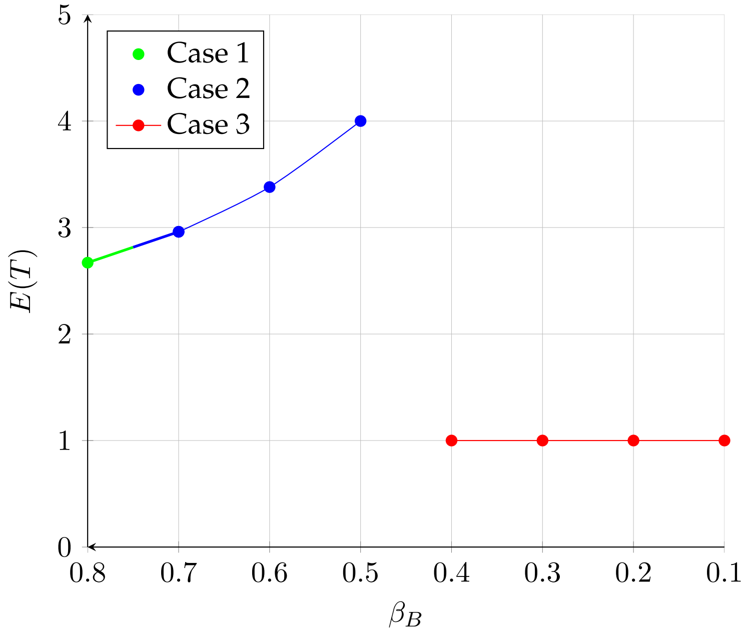


Figure 1.2: The expected time period for the provision of the public good

1.3.4 Equilibrium Selection

With the costs of volunteering sufficiently small or the present-bias parameters sufficiently close, there exist three stationary equilibria. Various theoretical and experimental studies have discussed the equilibrium selection using the one-shot game. With limited coordination devices, selecting the strategy profiles in which both players randomise is commonly supported with symmetric players.[18] With asymmetric players' differing costs of volunteering, Diekmann (1993) and Przepiorka and Diekmann (2018) argued towards selecting the equilibrium in which only the 'strong' player volunteers[19]; results from experimental studies also support the argument. However, this prediction is not perfect: the strong player does not volunteer with a probability of 1, and the prediction weakens if the heterogeneity decreases. He et al. (2014) studied an evolutionary volunteer's dilemma with asymmetric players. They showed that the mixed strategy equilibrium is not evolutionarily stable but contradicted the idea that the good can only be provided by the 'strong' player.

For the dynamic version of the game, previous theoretical studies pin down the unique equilibrium using assumptions like declining benefits and/or finite horizon. Bilodeau et al. (2004) conducted an experiment with the prediction of a unique subgame perfect equilibrium, but the experimental results do not confirm this. Apart from this study, Otsubo and Rapoport (2008) and Campos-Mercade (2021) experimentally tested a dynamic volunteer-

[18] See Diekmann (1985), Diekmann (1993) and Campos-Mercade (2021). Diekmann (1993) argued that theory underestimates the probability of volunteering in the symmetric equilibrium.

[19] See also Przepiorka and Diekmann (2013) and Healy and Pate (2018).

ing dilemma with symmetric players and declining benefits.[20] By borrowing some insights from earlier studies, we would like to argue the following: when there are limited coordination opportunities, with symmetric players, the symmetric and stationary equilibrium in which both players randomise is more likely to be chosen; with asymmetric players, there is not enough experimental evidence on the dynamic volunteering dilemma similar to our game setting for us to draw any strong arguments towards selecting any of the stationary equilibria. We do not want to overlook the possibility that the equilibrium in which both players randomise may occur. Moreover, the delay in volunteering can indeed be observed in some real-life scenarios, such as delays in travel planning.

1.3.5 Welfare Analysis

The welfare analysis of hyperbolic discounters is complicated since the time preference is not time-consistent; thus there could be a strategy that benefits oneself at time t but makes the self at other times worse off. We adopt the welfare criterion used in O'Donoghue and Rabin (1999a) and evaluate welfare from a 'long-run perspective'. Suppose there is a period before the first period agents make the decisions; the 'long-run self' at that period will not make any choices but will evaluate the agent's long-run utilities. Note that the 'long-run self' will discount future utilities exponentially using the long-run discount factor δ rather than hyperbolically. For example, if player A provides at period 1 and player B waits, player A's long-run utility is $U_A^L = \delta^s - c$; while player B's long-run utility is $U_B^L = \delta^s$. We adopt the 'utilitarian' social welfare in which welfare is calculated as the total 'long-term' utilities summed across both agents, $W = U_L^A + U_L^B$. We further argue that social welfare is decreasing in the probability $(1 - P_A)(1 - P_B)$.

Proposition 6*: If there is a stationary strategy profile σ such that the probability $(1 - P_A)(1 - P_B)$ is smaller compared to another stationary strategy profile $\hat{\sigma}$, then the strategy profile σ has a higher social welfare than strategy profile $\hat{\sigma}$. This holds for all stationary strategies.*

To derive this result, we first write out the social welfare function of the 'long-term selves' given the probabilities of volunteering P_A and P_B; then we take the derivative of the welfare function with respect to the probability $(1 - P_A)(1 - P_B)$. The sign of the derivative is negative. For more details, see Appendix 1F.

This result implies that the welfare of all possible stationary strategies can be indirectly assessed by the overall delay in volunteering. The sooner the public good is expected to be provided, the higher the social welfare. Note that we do not address the issue of wasting resources when multiple players volunteer in the classic volunteer's dilemma game since we assumed cost-sharing. We would rather focus on the issue of delay or no provision of

[20] There are also studies on the repeated volunteering dilemma, but they have different game structures to ours, they repeat the game even if the public good is provided, while there are no more choices to make in this study if someone volunteers. See Diekmann and Przepiorka (2015), Przepiorka et al. (2021) and Nunner et al. (2022).

the public good rather than on having too many volunteers.

1.4 Discussions

1.4.1 Special Cases

In this section, we will present some special examples of this model including the symmetric player case in which we have two exponential discounters or two hyperbolic discounters with the same present-bias parameter; and the case of one exponential discounter and one hyperbolic discounter. Starting with the case of symmetric agents, as we argue in Section 1.3.3, the symmetric equilibrium - where both players randomise - is more likely to occur when there are no coordination opportunities. In this scenario, the probabilities of volunteering are identical for both players. The indifference condition to calculate the equilibrium probability P is as follows:

$$(1-P)^2 = \frac{1}{\delta} + \frac{\beta\delta^{s+1} - \beta\delta^s - Pc\beta\delta(1-\frac{1}{2}P)}{\frac{1-\frac{1}{2}P}{1-P}\delta c} \tag{1.16}$$

This probability P decreases as both agents become more present-biased. Intuitively, as shown in the comparative statics with asymmetric players: the probability of one player volunteering increases when she becomes more present-biased and the probability decreases when the other player becomes more present-biased. In the symmetric case, when both players become more present-biased, the second effect outweighs the first. Consequently, the equilibrium probabilities P for both players decrease.

We now consider the case where one player is an exponential discounter and the other is a hyperbolic discounter, focusing primarily on situations where the cost of volunteering is not small. When the hyperbolic discounter is not extremely present-biased, an equilibrium in which both players randomise always exists. In this equilibrium, the probability of the hyperbolic discounter volunteering remains constant, even as she becomes marginally more or less present-biased, while the probability of the exponential discounter volunteering changes as the hyperbolic discounter becomes more or less present-biased. This result stems from the fact that the exponential discounter's indifference hinges on the other player volunteering with a specific probability and is independent of her future selves' volunteering probabilities. Moreover, the hyperbolic discounter always has a higher probability of volunteering than the exponential discounter. However, when the hyperbolic discounter is extremely present-biased, the unique stationary equilibrium implies that the exponential discounter volunteers immediately.

1.4.2 Naive Agents

In previous sections, we focused on sophisticated agents - those who are present-biased but fully aware of their future self-control problems. However, agents can also be naive or partially naive about their present bias. Naive agents believe they will behave like a time-consistent agents in the future, while partially naive agents recognise their future present bias but underestimate its degree. To model this, we define $\hat{\beta}_i$ as the perceived extent of present bias: $\hat{\beta}_i = 1$ for naive agents, and $1 > \hat{\beta}_i > \beta_i$ for partially naive agents. For naive and partially naive agents, the Strotz-Pollak equilibrium cannot be used because (partially) naive agents make decisions based on perceived future strategies rather than actual future behaviours. To address this, we adopt the concept of a perception perfect outcome as in Haan and Hauck (2023)[21]. In this framework, in each time period t, players observe each other's current time preferences. Given a player's perceptions of their own and their opponent's future present biases, we construct a "perceived game" for each player. Within this game, we can derive strategy profiles in which there are no profitable one-shot deviations for both players. This is called the 'equilibrium as perceived by A' or 'equilibrium as perceived by B' at t. We can derive such 'equilibrium' for both players at every t. The perception perfect outcome aggregates these 'equilibrium' across all t, ensuring that strategies at t are part of the 'equilibrium' at t. As in Haan and Hauck (2023), we assume both intra-player perception naivety and inter-player perception naivety to address the complexity of higher-order beliefs. Intra-player perception naivety assumes all players' future selves have the same perception as their current self. Inter-player perception naivety imposes players assume that the other player shares their perceptions.

In this dynamic volunteer's dilemma game with potentially naive agents, we assume the following: Both player A and B are potentially present-biased, with $\beta_A \geq \beta_B$. Both players are (partially) naive about their own future present biases: $\hat{\beta}_A = \hat{\beta}_B = \hat{\beta} > \beta_A \geq \beta_B$. Moreover, both players are (partially) naive about the other player's biases in a symmetric way: player A believes that player B's future present-bias to be $\hat{\beta}$, and player B believes player A has future present-bias $\hat{\beta}$. Given this symmetry, an 'equilibrium perceived by A' is also an 'equilibrium perceived by B' at any t. Therefore, it is sufficient to analyse the 'equilibrium perceived by A'. Moreover, since the game structure is stationary, we focus on the 'equilibrium perceived by A' that is the same across all t.

We now establish the following strategy profile constitutes an 'equilibrium perceived by A'. It suggests that, in this perceived game of A's, there are no profitable one-shot deviations by either player A or B. This strategy profile consists of the following:

1. In all future periods $s > t$, both players volunteer with probability P_s, calculated as:

$$(1 - P_s)^2 = \frac{1}{\delta} + \frac{\hat{\beta}\delta^{s+1} - \hat{\beta}\delta^s - P_s c\hat{\beta}\delta(1 - \frac{1}{2}P_s)}{\frac{1-\frac{1}{2}P_s}{1-P_s}\delta c} \tag{1.17}$$

2. (a) In the current period t, if $\left(\frac{1-\frac{1}{2}P_s}{1-P_s}\right)\left(\frac{\beta_A}{\hat{\beta}}\right) \geq \left(\frac{1-\frac{1}{2}P_s}{1-P_s}\right)\left(\frac{\beta_B}{\hat{\beta}}\right) > 1$, player A volun-

[21] For other solution concepts with time-inconsistent agents, see also Sarafidis (2006).

teers with probability P_A and player B volunteers with probability P_B[22]. P_A and P_B satisfy the following indifference conditions:

$$\beta_A \delta^s - \frac{1 - \frac{1}{2}P_B}{1 - P_B}c = \frac{(1 - (1 - P_s)^2)\beta_A \delta^{s+1} - P_s \beta_A \delta c(1 - \frac{1}{2}P_s)}{1 - (1 - P_s)^2 \delta} \tag{1.18}$$

$$\beta_B \delta^s - \frac{1 - \frac{1}{2}P_A}{1 - P_A}c = \frac{(1 - (1 - P_s)^2)\beta_B \delta^{s+1} - P_s \beta_B \delta c(1 - \frac{1}{2}P_s)}{1 - (1 - P_s)^2 \delta} \tag{1.19}$$

(b) In the current period t, if $\left(\frac{1-\frac{1}{2}P_s}{1-P_s}\right)\left(\frac{\beta_A}{\hat{\beta}}\right) \geq 1 \geq \left(\frac{1-\frac{1}{2}P_s}{1-P_s}\right)\left(\frac{\beta_B}{\hat{\beta}}\right)$, player B waits and player A volunteers.

(c) In the current period t, if $1 > \left(\frac{1-\frac{1}{2}P_s}{1-P_s}\right)\left(\frac{\beta_A}{\hat{\beta}}\right) \geq \left(\frac{1-\frac{1}{2}P_s}{1-P_s}\right)\left(\frac{\beta_B}{\hat{\beta}}\right)$, both players wait.

The detailed proof is in Appendix 1G. In Case (a), the perception perfect outcome is that player A always volunteers with probability P_A and player B always volunteers with probability P_B. In Case (b), the perception perfect outcome is that player A always volunteers and player B always waits, the public good is provided immediately. In Case (c), the perception perfect outcome is that both players always wait, leading to indefinite delays in public good provision. The underlying intuition is as follows: when players perceive both themselves and the other as highly time-consistent, they believe they will volunteer with high probability in the future. This expectation provides a strong justification for waiting. They fail to recognise their actual present bias and keep waiting indefinitely, ultimately leading to a permanent delay in provision. As the perceived present-bias parameter $\hat{\beta}$ decreases, for some values of $\hat{\beta}$, the more severe procrastinator player B remains 'optimistic' about the future and continue to wait; while player A, recognising the delay, prefers to volunteer immediately. This results in an immediate provision. Finally, when both players have a relatively precise perception of their and the other's present-bias, they both volunteer with positive probabilities. To summarise, highly naive players may delay volunteering indefinitely, players near sophistication may randomise their actions, while intermediate naivety may facilitate immediate provision of the public good.

1.4.3 Multiple Volunteers

What happens if there are multiple potential volunteers? We begin by analysing the symmetric case with $N > 2$ players, where all players share the same present-bias parameter β. In this setting, we derive the unique stationary and symmetric equilibrium in which each player volunteers with probability P, given no one has volunteered in the past. The indifference condition is:

$$(1 - P)^N = \frac{\beta \delta^{s+1} - \beta \delta^s + c(A - \beta \delta \frac{1}{N})}{c \delta A - \beta \delta \frac{c}{N}} \tag{1.20}$$

[22]Under this case, there also exist two other 'equilibria' in which one player waits and the other volunteers in the current period.

where

$$A := \sum_{n=0}^{N-1} \binom{N-1}{n} \left(\frac{P}{1-P}\right)^n \left(\frac{1}{n+1}\right)$$

We can further prove that there always exists a unique solution for P. That means, there exists a unique symmetric and stationary equilibrium in which all players randomise. Moreover, we prove the probability P decreases when all players become more present-biased. The detailed proof is in Appendix 1H.

We also consider the case where the N potential volunteers are divided into two groups: m players have present-bias parameter β_A and the remaining $N-m$ players have present-bias parameters β_B. We analyse group-symmetric equilibria, where all players in the same group adopt the same strategy. Suppose that in the equilibrium, each player with β_A volunteers with probability P_A and all players with β_B volunteers with probability P_B. For such an equilibrium to hold, P_A and P_B must satisfy the following indifference conditions:

$$\beta_A \delta^s (1-\delta) = cB(1 - \delta(1-P_A)^{m-1}(1-P_B)^{N-m}(1-(1-\beta_A)P_A)) \tag{1.21}$$

where

$$B := \sum_{n=0}^{m-1} \sum_{s=0}^{N-m} \binom{m-1}{n} \left(\frac{P_A}{1-P_A}\right)^n \binom{N-m}{s} \left(\frac{P_B}{1-P_B}\right)^s \left(\frac{1}{1+n+s}\right)$$

and

$$\beta_B \delta^s (1-\delta) = cD(1 - \delta(1-P_A)^{m}(1-P_B)^{N-m-1}(1-(1-\beta_B)P_B)) \tag{1.22}$$

where

$$D := \sum_{n=0}^{m} \sum_{s=0}^{N-m-1} \binom{m}{n} \left(\frac{P_A}{1-P_A}\right)^n \binom{N-m-1}{s} \left(\frac{P_B}{1-P_B}\right)^s \left(\frac{1}{1+n+s}\right)$$

If such an equilibrium exists, it must mean that the equilibria in which one group always waits and the other group volunteers with a certain probability also exist. However, we cannot characterise analytically the exact conditions for such equilibria to exist. We conjecture that their existence depends not only on the values of β_A and β_B, but also on the relative group sizes N and m.

1.5 Conclusion

This paper examines a dynamic volunteer's dilemma game in which two sophisticated, potentially present-biased players decide whether to volunteer or wait in each period. While volunteering is beneficial for both players, they may prefer waiting for the other player, or their future selves to volunteer instead. When the cost of volunteering is sufficiently small or the players have sufficiently close present-bias, an equilibrium in which both players randomise between volunteering and waiting always exists, leading to stochastic delays in volunteering. When one or both players become more present-biased, it increases the expected delay in the provision of the public good. When the two potential volunteers differ significantly in their present-bias, the unique stationary equilibrium suggests that only the less present-biased player volunteers, potentially leading to quicker provision than in the case of two time-consistent agents. Interestingly, strong naivety about present-bias can lead to indefinite delays, while a moderate degree of naivety may facilitate immediate volunteering.

When coordination opportunities are limited, both free-riding motives and present bias contribute to delays in public good provision. From the perspective of 'long-term selves', reducing delay improves social welfare. Previous studies emphasised the role of coordination mechanisms, such as pre-communication, to counteract free-riding. However, even with coordination, the delay is still possible because of the tendency of hyperbolic discounters to procrastinate. Thus, setting deadlines or allowing agents to pre-commit can counter such tendencies and reduce delay. Notably, the presence of present-biased individuals does not always lead to a more inefficient outcome. When potential volunteers exhibit sufficiently heterogenous present-bias, this heterogeneity can help identify the volunteer and may achieve a more efficient outcome, even without explicit coordination.

Future research may explore several directions: understanding the effect of learning from naive agents on the outcome of the game; incorporating incomplete information about agents' time preferences to better reflect some real-world scenarios; conducting experimental studies to inform equilibrium selection; and applying time-inconsistent preferences to other non-cooperative games to uncover new welfare implications.

Appendices

Appendix 1A. Proof of Proposition 1

Firstly, we prove that equilibria (a) and (b) in Proposition 1 are indeed stationary Strotz-Pollak equilibria by checking there are no profitable one-shot deviations. Given player B always waits and player A's future selves always volunteer, the expected payoff from volunteering is $\beta_A\delta^s - c$, while the payoff from waiting is $\beta_A\delta^{s+1} - \beta_A\delta c$. Since $c \leq \frac{\beta_A\delta^s(1-\delta)}{1-\beta_A\delta}$, the payoff from volunteering is always larger than the payoff from waiting, so there are no profitable one-shot deviations for player A. As for player B, given player A always volunteers and player B's future selves always wait, if player B waits, she receives payoff $\beta_B\delta^s$; if she volunteers, then her payoff is $\beta_B\delta^s - \frac{1}{2}c$. The payoff for waiting is always larger than volunteering, so there are no profitable one-shot deviations for player B. Similarly, we can also show that the strategy profile in which player A always waits and player B always volunteers is an equilibrium.

Secondly, we rule out all other possible strategy profiles involving one player playing stationary non-mixed behavioural strategies being an equilibrium. Strategy profiles in which player A always volunteers and player B volunteers with any strictly positive probability P_B can never be equilibria. To see this, by waiting, player B receives payoff $\beta_B\delta^s$; if she volunteers with probability P_B, her payoff is $\beta_B\delta^s - \frac{1}{2}P_B c$, which is always smaller than the payoff from waiting if P_B is strictly positive. This means that player B will find it profitable to deviate to waiting. Similarly, we can also rule out the profile in which player B always volunteers and player A volunteers with any strictly positive probability P_A. Moreover, the strategy profile in which player A always waits and player B always volunteers with any probability P_B strictly less than 1 can never be an equilibrium. By sticking to the original strategy, player B receives the payoff $P_B(\beta_B\delta^s - c) + (1-P_B)\frac{P_B}{1-(1-P_B)\delta}(\beta_B\delta^{s+1} - \beta_B\delta c)$. Since $c \leq \frac{\beta_A\delta^s(1-\delta)}{1-\beta_A\delta}$, by deviating to volunteering and earning payoff $\beta_B\delta^s - c$, player B is always better off. Thus, we find a profitable one-shot deviation for player B and this profile cannot be an equilibrium. Similarly, we can also rule out the profile in which player B always waits and player A volunteers with any probability P_A strictly less than 1.

For equilibrium (c) in which both players randomise, we show that when the cost of volunteering is sufficiently small $c \leq \frac{\beta_B\delta^s(1-\delta)}{1-\beta_B\delta}$, there exists a unique pair of values (P_A, P_B) that satisfies both indifference conditions. Thus, there can only exist one such stationary equilibrium in which both players randomise. To prove this, we draw the two indifference conditions on the same graph, with P_A on the x-axis and P_B on the y-axis. (Note that we want to check whether we can find P_A and P_B strictly in between 0 and 1, so we only want to consider the part of the graph with P_A and P_B in between 0 and 1).

Firstly, we consider player A's indifference condition. Note that when $\beta_A = 1$, the indifference condition reduces to $\left(1 - \frac{1}{2}P_B\right)(1-P_B)c\delta + \delta^s(1-\delta)(1-P_B) = \left(1 - \frac{1}{2}P_B\right)c$. The equation only contains P_B, we further show that as long as $c < \delta^s$, there always exists a unique solution for P_B strictly between 0 and 1. This implies that when player A is an exponential discounter, her indifference condition can be drawn as a horizontal line with P_B being the solution of the above equation. When $\beta_A < 1$, we draw the indifference condition for player A as follows. When $P_A = 0$, the equation becomes $(1-P_B)\left(1-\frac{1}{2}P_B\right)c\delta + (1-P_B)\beta_A\delta^s(1-\delta) = \left(1-\frac{1}{2}P_B\right)c$. Given $c < \beta_A\delta^s$, there exists a unique solution for P_B strictly between 0 and 1; we denote this value as P_C. (To obtain this result, draw the left-hand side and right-hand side of the equation separately, when $P_B = 0$, the LHS equals $c\delta + \beta_A\delta^s(1-\delta)$ and the RHS is equal to c, and the LHS value is strictly greater than the RHS because $c < \beta_A\delta^s$; when $P_B = 1$, LHS equals 0, RHS is equal to $\frac{1}{2}c > 0$. Both sides are monotonically decreasing in P_B and the LHS is always steeper than RHS. Therefore, there can only be a unique solution for P_B between 0 and 1). Moreover, for $P_B = 0$, $P_A = \frac{(1-\delta)(\beta_A\delta^s - c)}{c\delta(1-\beta_A)}$, denote this value as P_D. If $c \leq \frac{\beta_A\delta^s(1-\delta)}{1-\beta_A\delta}$, $P_D \geq 1$. We further show that for $P_A = 1$, there always exists a unique P_B between 0 and 1, and $P_B = 0$ when $c = \frac{\beta_A\delta^s(1-\delta)}{1-\beta_A\delta}$. Moreover, for any P_A between 0 and 1, we can find a unique P_B that is between 0 and 1 that satisfies player A's indifference condition. Lastly, we take the derivative of P_B with respect to P_A using the first indifference condition. The derivative is calculated as:

$$\frac{dP_B}{dP_A} = -\frac{\left(1-\frac{1}{2}P_B\right)c\delta(1-\beta_A)}{\frac{1}{2}(1-P_A)c\delta + \frac{1}{2}c\left(\frac{1}{(1-P_B)^2}\right) + \frac{1}{2}P_A\beta_A c\delta} \tag{1.23}$$

this derivative is strictly negative for P_A and P_B between 0 and 1. Therefore, we know that the graph for the first indifference condition is such that P_B is monotonically decreasing in P_A.

Secondly, we draw the indifference condition for player B using similar procedures as above. Note that when $\beta_B = 1$, the graph of the indifference condition will be a vertical line with P_A equal to the unique solution between 0 and 1 to $\left(1-\frac{1}{2}P_A\right)(1-P_A)c\delta + (1-P_A)\delta^s(1-\delta) = \left(1-\frac{1}{2}P_A\right)c$. For $\beta_B < 1$, when $P_B = 0$, the condition becomes $(1-P_A)\left(1-\frac{1}{2}P_A\right)c\delta + (1-P_A)\beta_B\delta^s(1-\delta) = \left(1-\frac{1}{2}P_A\right)c$. Given $c < \beta_B\delta^s$, there always exists a unique solution for P_A between 0 and 1. We denote this value as P_E. Moreover, for $P_A = 0$, $P_B = \frac{(1-\delta)(\beta_B\delta^s - c)}{c\delta(1-\beta_B)}$, denote this value as P_F. Similar to above, when the cost is sufficiently small, $P_F \geq 1$. Moreover, we take the derivative of P_B with respect to P_A using the second condition. The derivative is calculated as follows:

$$\frac{dP_B}{dP_A} = -\frac{\frac{1}{2}(1-P_B)c\delta + \frac{1}{2}c\left(\frac{1}{(1-P_A)^2}\right) + \frac{1}{2}P_B\beta_B c\delta}{\left(1-\frac{1}{2}P_A\right)c\delta(1-\beta_B)} \tag{1.24}$$

For P_A and P_B between 0 and 1, the sign of this derivative is negative, therefore, P_B is monotonically decreasing in P_A.

Thirdly, we check whether the graphs of the two indifference conditions intersect and if they only intersect once (strictly between 0 and 1). When both agents are exponential discounters, the first indifference condition is just a horizontal line and the second indifference condition is a vertical line. Since $0 < P_C < 1$ and $0 < P_E < 1$, the two lines will intersect and only intersect once at (P_E, P_C) (Figure 1.3). If only one of the agents is an exponential discounter, the graph for the first or second indifference condition will be a horizontal or vertical line; while the graph for the other indifference condition is a downward-sloping curve. When the cost of volunteering is small enough, we know that $P_F \geq 1 > P_C > 0$ and $P_D \geq 1 > P_E > 0$, the two curves will intersect and intersect only once, see Figures 1.4 and 1.5. When both agents are hyperbolic discounters, both curves are downward-sloping and will intersect at least once.

Now we show that the two curves will only intersect once. It is useful to argue first that at any intersection between the two indifference conditions, it must be the case that the curve for the second indifference condition will be steeper than the curve for the first indifference condition. To show this, recall the derivatives of P_B with respect to P_A for the two indifference conditions. We compare the numerators from the two derivatives first: the numerator for the derivative of the first indifference condition is $-\left(1-\frac{1}{2}P_B\right)c\delta(1-\beta_A)$, and the numerator for the second indifference condition is $-\left(\frac{1}{2}(1-P_B)c\delta+\frac{1}{2}c\left(\frac{1}{(1-P_A)^2}\right)+\frac{1}{2}P_B\beta_B c\delta\right)$. The latter can be rewritten as $-\left(\left(1-\frac{1}{2}P_B\right)c\delta(1-\beta_B)+c\left(\frac{1}{2}\delta+\frac{1}{2}\frac{1}{(1-P_A)^2}\right)-c\delta(1-\beta_B)\right)$, since $\beta_A \geq \beta_B$ and $c\left(\frac{1}{2}\delta+\frac{1}{2}\frac{1}{(1-P_A)^2}\right)-c\delta(1-\beta_B) > 0$. The numerator for the derivative of the first condition is negative and greater than the numerator in the second condition. The denominator for the first condition is $\frac{1}{2}(1-P_A)c\delta+\frac{1}{2}c\left(\frac{1}{(1-P_B)^2}\right)+\frac{1}{2}P_A\beta_A c\delta$ while the denominator for the second condition is $\left(1-\frac{1}{2}P_A\right)c\delta(1-\beta_B)$. Rewrite the first expression as $c\left(\frac{1}{2}\delta+\frac{1}{2}\frac{1}{(1-P_B)^2}\right)+\frac{1}{2}P_A\beta_A c\delta-\frac{1}{2}P_A c\delta$, and rewrite the second expression as $c\delta(1-\beta_B)+\frac{1}{2}P_A\beta_B c\delta-\frac{1}{2}P_A c\delta$. We know that $\frac{1}{2}\delta+\frac{1}{2}\frac{1}{(1-P_B)^2} > \delta(1-\beta_B)$ and $\beta_A \geq \beta_B$, hence we can conclude that the denominator of the $\frac{dP_B}{dP_A}$ of the first indifference condition is greater than that in the second condition. Overall, we can say that at any intersection point, the graph of the second indifference condition must be steeper than the graph for the first condition. Therefore, we can only find one intersection between the two curves, so there exists a unique solution for (P_A, P_B) strictly between 0 and 1 that satisfies both indifference conditions (Figure 1.6).

Appendix 1B. Proof of Proposition 2

We first prove that the strategy profile in which player A always waits and player B volunteers with probability $P_B = \frac{(\beta_B\delta^s-c)(1-\delta)}{c\delta(1-\beta_B)}$ is indeed an equilibrium. Under the assumption $c > \frac{\beta_B\delta^s(1-\delta)}{1-\beta_B\delta}$, $\frac{(\beta_B\delta^s-c)(1-\delta)}{c\delta(1-\beta_B)}$ is strictly between 0 and 1. Given player A always waits and player B's future selves volunteer with probability P_B, if player B volunteers now, her payoff is $\beta_B\delta^s - c$; by waiting, her payoff is $P_B(\beta_B\delta^s-c)+(1-P_B)\frac{P_B}{1-(1-P_B)\delta}(\beta_B\delta^{s+1}-\beta_B\delta c)$. By substituting the value $P_B = \frac{(\beta_B\delta^s-c)(1-\delta)}{c\delta(1-\beta_B)}$ in the expression, we find that her payoff of volunteering is equal to waiting. Thus, we do not find any profitable one-shot deviation for player

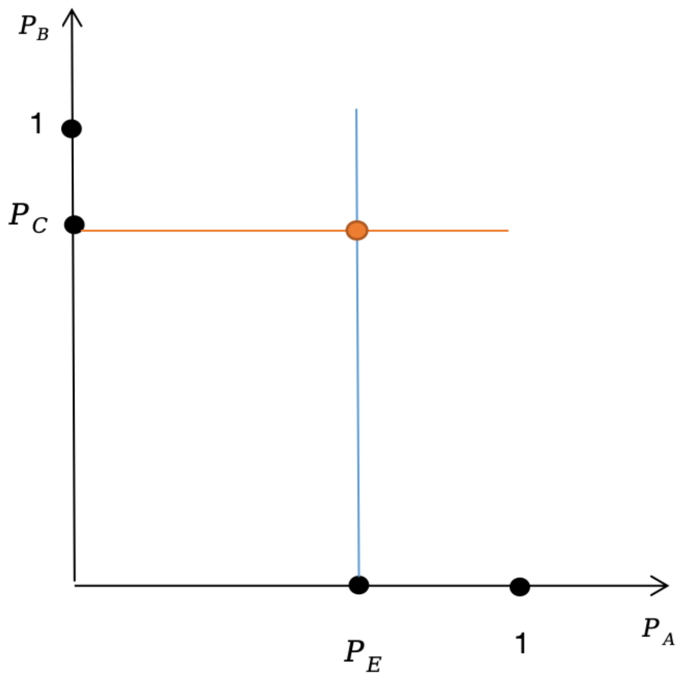


Figure 1.3: Indifference Conditions when both agents are exponential discounters

B. We can further rule out the strategy profile in which player A always waits and player B volunteers with any probability other than $P_B = \frac{(\beta_B\delta^s - c)(1-\delta)}{c\delta(1-\beta_B)}$ being an equilibrium: if $P_B > \frac{(\beta_B\delta^s - c)(1-\delta)}{c\delta(1-\beta_B)}$, then there is a profitable one-shot deviation for player B to deviate to waiting; if $P_B < \frac{(\beta_B\delta^s - c)(1-\delta)}{c\delta(1-\beta_B)}$, player B finds it profitable to deviate to volunteering. Moreover, we also need to prove that there are no profitable one-shot deviations for player A. For player A, by waiting, her payoff is calculated as $\frac{P_B\beta_A\delta^s}{1-(1-P_B)\delta}$; if she volunteers, her payoff is $\beta_A\delta^s - c(1-\frac{1}{2}P_B)$. The payoff of waiting is larger than volunteering if and only if

$$c\left(1-\frac{1}{2}P_B\right)\left(\frac{1}{1-P_B}-\delta\right) \geq \beta_A\delta^s(1-\delta)$$

After we substitute $P_B = \frac{(\beta_B\delta^s - c)(1-\delta)}{c\delta(1-\beta_B)}$ into the LHS and RHS, the LHS is calculated as:

$$\frac{\left(c\delta(1-\beta_B) - \frac{1}{2}\left(\beta_B\delta^s - c\right)(1-\delta)\right)\left(\beta_B(1-\delta)\left(\delta^s - c\right)\right)}{(1-\beta_B)^2 c\delta - (1-\beta_B)(1-\delta)\left(\beta_B\delta^s - c\right)} \tag{1.25}$$

while the RHS is

$$\frac{\left(c\delta(1-\beta_B) - \left(\beta_B\delta^s - c\right)(1-\delta)\right)\left(\beta_A(1-\delta)(1-\beta_B)\delta^s\right)}{(1-\beta_B)^2 c\delta - (1-\beta_B)(1-\delta)\left(\beta_B\delta^s - c\right)} \tag{1.26}$$

Another assumption we have in Proposition 2 is such that β_A and β_B are sufficiently close, in particular:

$$\beta_A < \beta_B \frac{(\delta^s - c)(c\delta(1-\beta_B) - \frac{1}{2}(\beta_B\delta^s - c)(1-\delta))}{(\delta^s - \beta_B\delta^s)(c\delta(1-\beta_B) - (\beta_B\delta^s - c)(1-\delta))} \tag{1.27}$$

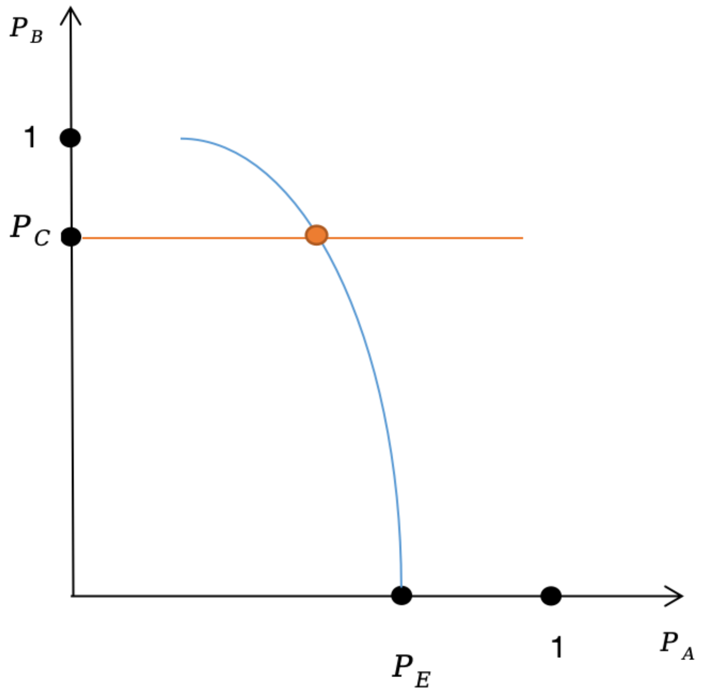


Figure 1.4: Indifference Conditions when player A is an exponential discounter and the cost of volunteering is sufficiently small

This condition implies that the LHS is indeed larger than the RHS, thus, there are no profitable one-shot deviations for player A. The strategy profile in which player A always waits and player B volunteers with probability $P_B = \frac{(\beta_B\delta^s - c)(1-\delta)}{c\delta(1-\beta_B)}$ is indeed an equilibrium.

We then prove that if $c > \frac{\beta_A\delta^s(1-\delta)}{1-\beta_A\delta}$, the strategy profile in which player B always waits and player A volunteers with probability $P_A = \frac{(\beta_A\delta^s - c)(1-\delta)}{c\delta(1-\beta_A)}$ is indeed an equilibrium. The proof is very similar to the last paragraph. There are no profitable one-shot deviations for player A because player A is indifferent between volunteering and waiting. The strategy profile in which player B always waits and player A volunteers with any probability other than $P_A = \frac{(\beta_A\delta^s - c)(1-\delta)}{c\delta(1-\beta_A)}$ cannot be an equilibrium since player A may find it profitable to deviate to volunteering or waiting. We further show that player B has no profitable one-shot deviations. Player B's payoff from waiting is $\frac{P_A\beta_B\delta^s}{1-(1-P_A)\delta}$; if she volunteers, her payoff is $\beta_B\delta^s - c(1-\frac{1}{2}P_A)$. The payoff of waiting is larger than volunteering if and only if

$$c\left(1 - \frac{1}{2}P_A\right)\left(\frac{1}{1-P_A} - \delta\right) \geq \beta_B\delta^s(1-\delta)$$

Under the assumption $\beta_A \geq \beta_B$, $\beta_A\delta^s(1-\delta) \geq \beta_B\delta^s(1-\delta)$, it is also sufficient to show that

$$c\left(1 - \frac{1}{2}P_A\right)\left(\frac{1}{1-P_A} - \delta\right) \geq \beta_A\delta^s(1-\delta)$$

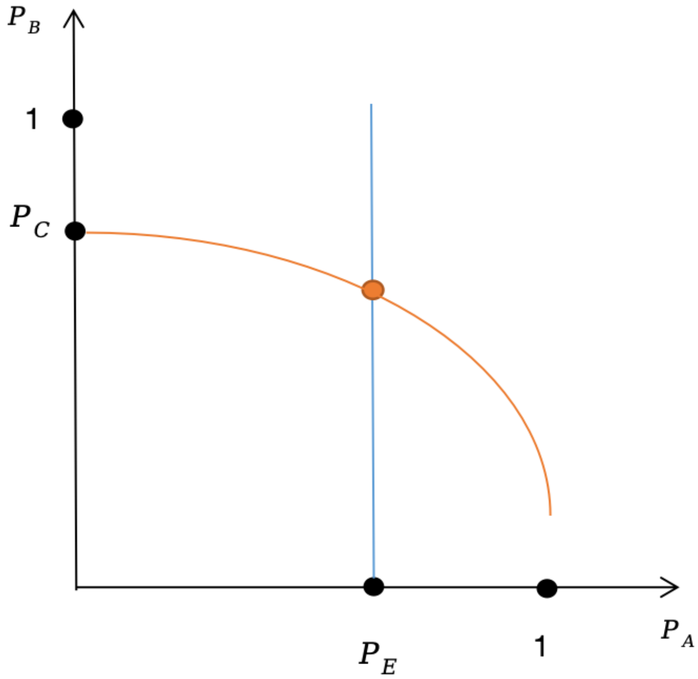


Figure 1.5: Indifference Conditions when player B is an exponential discounter and the cost of volunteering is sufficiently small

By substituting $P_A = \frac{(\beta_A\delta^s - c)(1-\delta)}{c\delta(1-\beta_A)}$ into the LHS and RHS, the LHS is calculated as:

$$\frac{\left(c\delta(1-\beta_A) - \frac{1}{2}\left(\beta_A\delta^s - c\right)(1-\delta)\right)\left(\beta_A(1-\delta)\left(\delta^s - c\right)\right)}{(1-\beta_A)^2 c\delta - (1-\beta_A)(1-\delta)\left(\beta_A\delta^s - c\right)} \tag{1.28}$$

while the RHS is

$$\frac{\left(c\delta(1-\beta_A) - \left(\beta_A\delta^s - c\right)(1-\delta)\right)\left(\beta_A(1-\delta)(1-\beta_A)\delta^s\right)}{(1-\beta_A)^2 c\delta - (1-\beta_A)(1-\delta)\left(\beta_A\delta^s - c\right)} \tag{1.29}$$

Since we assumed that $c > \frac{\beta_A\delta^s(1-\delta)}{1-\beta_A\delta}$, the denominator is then positive and $c\delta(1-\beta_A) - \frac{1}{2}\left(\beta_A\delta^s - c\right)(1-\delta) > c\delta(1-\beta_A) - \left(\beta_A\delta^s - c\right)(1-\delta) > 0$. Moreover, $\beta_A(1-\delta)\left(\delta^s - c\right) > \beta_A(1-\delta)(1-\beta_A)\delta^s$ because that $c < \beta_A\delta^s$. Therefore the LHS is indeed larger than the RHS. Hence, there are no profitable one-shot deviations for player B. Furthermore, if $c \leq \frac{\beta_A\delta^s(1-\delta)}{1-\beta_A\delta}$, the strategy profile in which player B always waits and player A always volunteers is an equilibrium. There exists no other equilibrium in which player B always waits following similar arguments as in Appendix 1A. Finally, we argue that in the equilibrium, there cannot be any equilibrium in which one player always volunteers, while the other player volunteers with a strictly positive probability.

The last family of stationary strategy profiles left to check is such that both players randomise. Similar to the proof of Proposition 1, we need to check whether there exist values of P_A and P_B that satisfy both players' indifference conditions. Again, we try to draw both players' indifference conditions in $P_A - P_B$ space. Let's start with player A's indifference

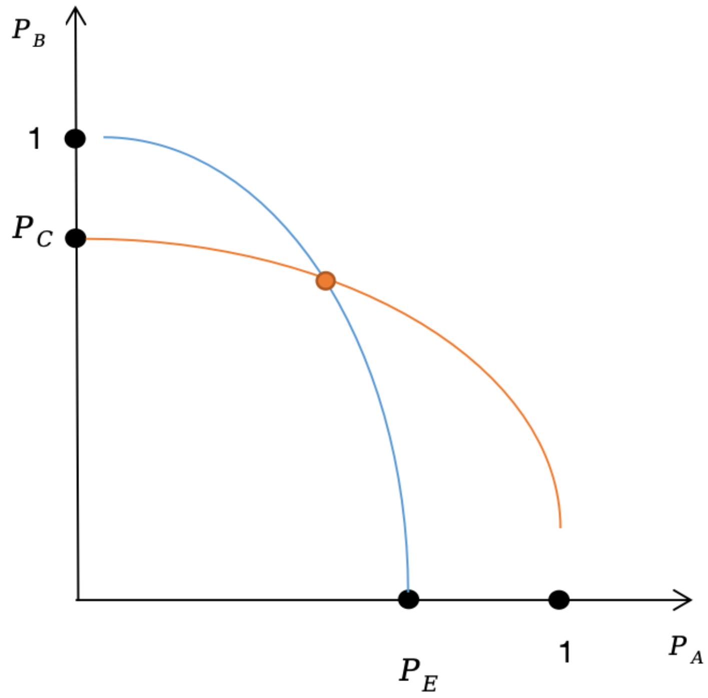


Figure 1.6: Indifference Conditions when both players are hyperbolic discounters and the cost of volunteering is sufficiently small

condition. When player A is an exponential discounter, the graph for A's indifference condition is again a horizontal line with $P_B = P_C$. When player A is a hyperbolic discounter and $c \leq \frac{\beta_A \delta^s (1-\delta)}{1-\beta_A \delta}$, the graph is a downward-sloping curve. If $c > \frac{\beta_A \delta^s (1-\delta)}{1-\beta_A \delta}$, then $0 < P_D < 1$. Moreover, for every P_A between 0 and P_D, we can find a unique P_B between 0 and 1; for other values of P_A, we cannot find a P_B between 0 and 1. We also know from the last section that P_B is monotonically decreasing in P_A, we can then draw player A's indifference condition. Similarly, we can also draw player B's indifference condition. When $c > \frac{\beta_B \delta^s (1-\delta)}{1-\beta_B \delta}$, $0 < P_F < 1$.

We also want to check if the two curves intersect and intersect only once. We can show that $P_D > P_E$. If $c \leq \frac{\beta_A \delta^s (1-\delta)}{1-\beta_A \delta}$, it must be the case that $P_D \geq 1$ and $P_E < P_D$. If $c > \frac{\beta_A \delta^s (1-\delta)}{1-\beta_A \delta}$, $0 < P_D < 1$. The equation to calculate P_E is $(1 - P_A)\left(1 - \frac{1}{2}P_A\right) c\delta + (1 - P_A)\beta_B \delta^s (1 - \delta) = \left(1 - \frac{1}{2}P_A\right) c$, we rewrite the equation as $\beta_B \delta^s (1-\delta) = \left(1 - \frac{1}{2}P_A\right) c \left(\frac{1}{1-P_A} - \delta\right)$. If we substitute $P_A = P_D$ in the RHS and show that $\beta_B \delta^s (1-\delta) < \left(1 - \frac{1}{2}P_D\right) c \left(\frac{1}{1-P_D} - \delta\right)$, we can prove that P_D must be larger than P_E because that RHS is increasing in P_A. Since $\beta_A \geq \beta_B$, it is sufficient to show $\beta_A \delta^s (1-\delta) < \left(1 - \frac{1}{2}P_D\right) c \left(\frac{1}{1-P_D} - \delta\right)$. We already showed this in the earlier part of the proof. Thus, when $\beta_A \geq \beta_B$, P_D will always be larger than P_E. Next, we want to compare the values of P_C and P_F. Remember that the equation to derive P_C is $(1 - P_B)\left(1 - \frac{1}{2}P_B\right) c\delta + (1 - P_B)\beta_A \delta^s (1-\delta) = \left(1 - \frac{1}{2}P_B\right) c$, we can rewrite this as $\beta_A \delta^s (1-\delta) = \left(1 - \frac{1}{2}P_B\right) c \left(\frac{1}{1-P_B} - \delta\right)$. Substituting $P_B = P_F = \frac{(1-\delta)(\beta_B \delta^s - c)}{c\delta(1-\beta_B)}$ on the right-hand side. If we can find that $\beta_A \delta^s (1-\delta) < \left(1 - \frac{1}{2}P_F\right) c \left(\frac{1}{1-P_F} - \delta\right)$ and we know that the RHS is increasing in P_B, we can conclude that $P_C < P_F$. From the assumption that β_A and β_B are sufficiently close:

$$\beta_A < \beta_B \frac{(\delta^s - c)(c\delta(1-\beta_B) - \frac{1}{2}(\beta_B\delta^s - c)(1-\delta))}{(\delta^s - \beta_B\delta^s)(c\delta(1-\beta_B) - (\beta_B\delta^s - c)(1-\delta))} \tag{1.30}$$

we can conclude that P_C must be smaller than P_F using the same procedure as previously.

Lastly, recall that the slope of the graph of the second condition must be steeper than the first condition at all intersections of the two curves for P_A and P_B between 0 and 1. If $P_C < P_F$ and $P_D > P_E$, then the two curves intersect and can only intersect once. Therefore, there can only exist one such equilibrium in which both players randomise (Figures 1.7 and 1.8).

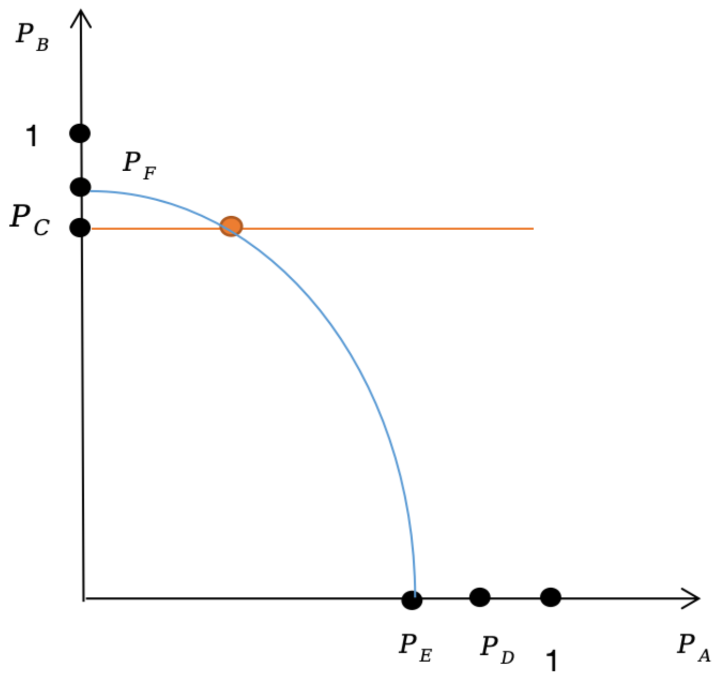


Figure 1.7: Indifference Conditions when player A is an exponential discounter and present-bias parameters are sufficiently close

Appendix 1C. Proof of Proposition 3

Following the proof of Proposition 2, the proof of Proposition 3 is straightforward. The proof that the strategy profile in which player B always waits and player A always volunteers/randomises is indeed an equilibrium is the same as in Appendix 1B. We mainly focus on arguing that the strategy profiles in which player B randomises or both players randomise can no longer be sustained as equilibria. Firstly, for the strategy profile in which player B randomises and player A waits, player A has a profitable one-shot deviation to deviate to volunteering. To see this, if player A waits, her payoff is $\frac{P_B\beta_A\delta^s}{1-(1-P_B)\delta}$; if she volunteers, her payoff is $\beta_A\delta^s - c(1-\frac{1}{2}P_B)$. The payoff of waiting is larger than volunteering if and only

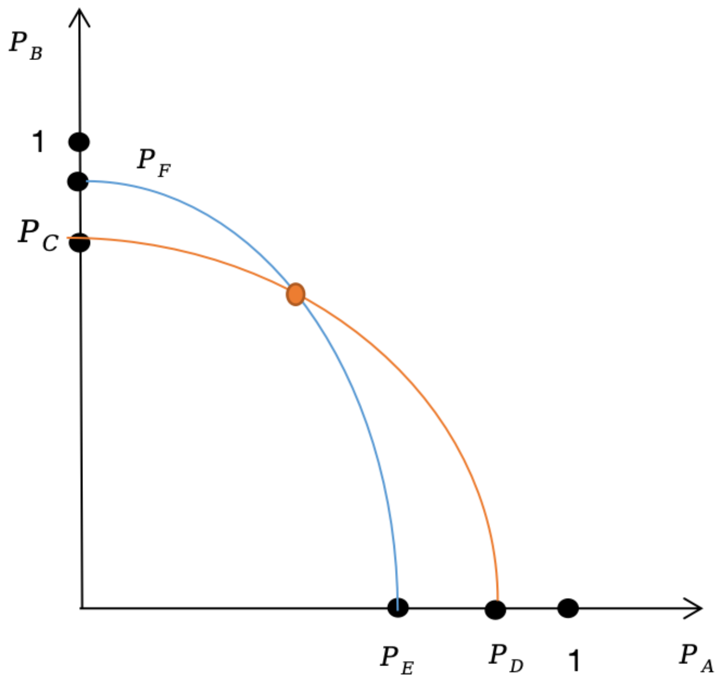


Figure 1.8: Indifference Conditions when both players are hyperbolic discounters and present-bias parameters are sufficiently close

if

$$c\left(1-\frac{1}{2}P_B\right)\left(\frac{1}{1-P_B}-\delta\right) \geq \beta_A\delta^s(1-\delta)$$

After we substitute $P_B = \frac{(\beta_B\delta^s - c)(1-\delta)}{c\delta(1-\beta_B)}$ into the LHS and RHS, the LHS is calculated as:

$$\frac{\left(c\delta(1-\beta_B)-\frac{1}{2}\left(\beta_B\delta^s-c\right)(1-\delta)\right)\left(\beta_B(1-\delta)\left(\delta^s-c\right)\right)}{(1-\beta_B)^2c\delta-(1-\beta_B)(1-\delta)\left(\beta_B\delta^s-c\right)} \tag{1.31}$$

while the RHS is

$$\frac{\left(c\delta(1-\beta_B)-\left(\beta_B\delta^s-c\right)(1-\delta)\right)\left(\beta_A(1-\delta)(1-\beta_B)\delta^s\right)}{(1-\beta_B)^2c\delta-(1-\beta_B)(1-\delta)\left(\beta_B\delta^s-c\right)} \tag{1.32}$$

The assumption we have in Proposition 3 is such that β_A and β_B are no longer sufficiently close, in particular:

$$\beta_A > \beta_B\frac{(\delta^s-c)(c\delta(1-\beta_B)-\frac{1}{2}(\beta_B\delta^s-c)(1-\delta))}{(\delta^s-\beta_B\delta^s)(c\delta(1-\beta_B)-(\beta_B\delta^s-c)(1-\delta))} \tag{1.33}$$

This implies that the payoff from volunteering is always larger than from waiting, therefore player A finds it profitable to deviate to volunteering. As for the strategy profile in which both players randomise, P_D is still larger than P_E. However, since the present-bias parameters of players A and B are no longer close, P_F is now smaller than P_C. Note that at any point of intersection between the two indifference conditions, the second indifference condition must be steeper. This implies that there cannot be any intersection between the two curves. Therefore, there cannot exist any equilibrium in which both players randomise.

Appendix 1D. Proof of Proposition 4

Suppose player A becomes more or less present-biased, then the players' probabilities of volunteering change and those probabilities still need to satisfy the players' indifference conditions. Alternatively, changes in probabilities change both the LHS and RHS of both players' indifference conditions, but the change needs to be equal to still keep the indifference conditions satisfied. Thus, we first take the total derivative of the RHS of player A's indifference condition with respect to β_A:

$$\left(\delta^{s+1} - \delta^s - P_A c\delta\left(1 - \frac{1}{2}P_B\right) - c\beta_A\delta\left(1 - \frac{1}{2}P_B\right)\frac{dP_A}{d\beta_A} + \frac{1}{2}P_A c\beta_A\delta\frac{dP_B}{d\beta_A}\right)\frac{1 - P_B}{\left(1 - \frac{1}{2}P_B\right)\delta c}$$

$$+\left(\beta_A\delta^{s+1} - \beta_A\delta^s - P_A c\beta_A\delta\left(1 - \frac{1}{2}P_B\right)\right)\frac{-\left(1 - \frac{1}{2}P_B\right)\delta c\frac{dP_B}{d\beta_A} + \frac{1}{2}\delta c(1 - P_B)\frac{dP_B}{d\beta_A}}{\left(\left(1 - \frac{1}{2}P_B\right)\delta c\right)^2} \quad (1.34)$$

Expanding terms:

$$\left(\delta^{s+1} - \delta^s\right)\frac{1 - P_B}{\left(1 - \frac{1}{2}P_B\right)\delta c} - \frac{P_A c\delta\left(1 - \frac{1}{2}P_B\right)(1 - P_B)}{\left(1 - \frac{1}{2}P_B\right)\delta c} - \frac{c\beta_A\delta\left(1 - \frac{1}{2}P_B\right)(1 - P_B)}{\left(1 - \frac{1}{2}P_B\right)\delta c}\frac{dP_A}{d\beta_A}$$

$$+\frac{\frac{1}{2}P_A c\beta_A\delta(1 - P_B)}{\left(1 - \frac{1}{2}P_B\right)\delta c}\frac{dP_B}{d\beta_A} - \left(\beta_A\delta^s(\delta - 1)\right)\frac{\frac{1}{2}c\delta}{\left(\left(1 - \frac{1}{2}P_B\right)\delta c\right)^2}\frac{dP_B}{d\beta_A} + \frac{\frac{1}{2}P_A c\beta_A\delta\left(1 - \frac{1}{2}P_B\right)c\delta}{\left(\left(1 - \frac{1}{2}P_B\right)\delta c\right)^2}\frac{dP_B}{d\beta_A}$$

Simplifying further gives:

$$\left(\delta^{s+1} - \delta^s\right)\frac{1 - P_B}{\left(1 - \frac{1}{2}P_B\right)\delta c} - P_A(1 - P_B) - \beta_1(1 - P_B)\frac{dP_A}{d\beta_A} + \frac{\frac{1}{2}P_A\beta_A(1 - P_B)}{1 - \frac{1}{2}P_B}\frac{dP_B}{d\beta_A}$$

$$-\left(\beta_A\delta^{s+1} - \beta_A\delta^s\right)\frac{\frac{1}{2}c\delta}{\left(\left(1 - \frac{1}{2}P_B\right)\delta c\right)^2}\frac{dP_B}{d\beta_A} + \frac{\frac{1}{2}P_A\beta_A}{1 - \frac{1}{2}P_B}\frac{dP_B}{d\beta_A} \quad (1.35)$$

Collecting terms:

$$\left(\delta^s(\delta - 1)\right)\frac{1 - P_B}{\left(1 - \frac{1}{2}P_B\right)\delta c} - P_A(1 - P_B) - \beta_A(1 - P_B)\frac{dP_A}{d\beta_A} - \frac{\frac{1}{2}\beta_A\left(\delta^{s+1} - \delta^s\right)\delta c}{\left(\left(1 - \frac{1}{2}P_B\right)\delta c\right)^2}\frac{dP_B}{d\beta_A} + P_A\beta_A\frac{dP_B}{d\beta_A} \quad (1.36)$$

Taking the total derivative of the RHS of player B's indifference condition with respect to β_A:

$$\left(-c\beta_B\delta\left(1 - \frac{1}{2}P_A\right)\frac{dP_B}{d\beta_A} + \frac{1}{2}P_B c\beta_B\delta\frac{dP_A}{d\beta_A}\right)\frac{1 - P_A}{\left(1 - \frac{1}{2}P_A\right)c\delta}$$

$$+\left(\beta_B\delta^{s+1} - \beta_B\delta^s - P_B c\beta_B\delta\left(1 - \frac{1}{2}P_A\right)\right)\frac{-\frac{dP_A}{d\beta_A}\left(1 - \frac{1}{2}P_A\right)c\delta + \frac{1}{2}\frac{dP_A}{d\beta_A}c\delta(1 - P_A)}{\left(\left(1 - \frac{1}{2}P_A\right)c\delta\right)^2} \quad (1.37)$$

Expanding the terms gives:

$$-\frac{c\beta_B\delta\left(1-\frac{1}{2}P_A\right)(1-P_A)}{\left(1-\frac{1}{2}P_A\right)c\delta}\frac{dP_B}{d\beta_A}+\frac{\frac{1}{2}P_Bc\beta_B\delta(1-P_A)}{\left(1-\frac{1}{2}P_A\right)c\delta}\frac{dP_A}{d\beta_A}$$

$$-\left(\beta_B\delta^{s+1}-\beta_B\delta^s\right)\frac{\frac{1}{2}c\delta}{\left(\left(1-\frac{1}{2}P_A\right)c\delta\right)^2}\frac{dP_A}{d\beta_A}-\frac{\frac{1}{2}c\delta(1-P_A)P_Bc\beta_B\delta\left(1-\frac{1}{2}P_A\right)}{\left(\left(1-\frac{1}{2}P_A\right)c\delta\right)^2}\frac{dP_A}{d\beta_A}$$

Simplifying the terms gives:

$$-\beta_B(1-P_A)\frac{dP_B}{d\beta_A}+\frac{\frac{1}{2}P_B\beta_B(1-P_A)}{1-\frac{1}{2}P_A}\frac{dP_A}{d\beta_A}$$

$$-\left(\beta_B\delta^{s+1}-\beta_B\delta^s\right)\frac{\frac{1}{2}c\delta}{\left(\left(1-\frac{1}{2}P_A\right)c\delta\right)^2}\frac{dP_A}{d\beta_A}-\frac{\frac{1}{2}P_B\beta_B}{1-\frac{1}{2}P_A}\frac{dP_A}{d\beta_A} \tag{1.38}$$

Collecting terms gives:

$$-(1-P_A)\beta_B\frac{dP_B}{d\beta_A}+\left(P_B\beta_B+\frac{\frac{1}{2}\beta_B\delta^s(1-\delta)\delta c}{\left(1-\frac{1}{2}P_A\right)\delta c)^2}\right)\frac{dP_A}{d\beta_A} \tag{1.39}$$

Since the two indifference conditions have the same LHS, we can equalise the total derivative of the RHS of the two indifference conditions with respect to β_A. Therefore, we have the expression (1.36) that is equal to (1.39). Rearranging terms gives:

$$\left(-(1-P_A)\beta_B-P_A\beta_A-\frac{\frac{1}{2}\beta_A\delta^s(1-\delta)\delta c}{\left(\left(1-\frac{1}{2}P_B\right)\delta c\right)^2}\right)\frac{dP_B}{d\beta_A}$$

$$=\left(\delta^{s+1}-\delta^s\right)\frac{1-P_B}{\left(1-\frac{1}{2}P_B\right)\delta c}+\left(-\beta_A(1-P_B)-P_B\beta_B-\frac{\frac{1}{2}\beta_B\delta^s(1-\delta)\delta c}{\left(\left(1-\frac{1}{2}P_A\right)\delta c\right)^2}\right)\frac{dP_A}{d\beta_A} \tag{1.40}$$

This allows us to write the expression as follows:

$$\frac{dP_B}{d\beta_A}=A+B\frac{dP_A}{d\beta_A} \tag{1.41}$$

$$A:=\frac{-\delta^s(1-\delta)\frac{1-P_B}{\left(1-\frac{1}{2}P_B\right)\delta c}-P_A(1-P_B)}{-(1-P_A)\beta_B-P_A\beta_A-\frac{\frac{1}{2}\beta_A\delta^s(1-\delta)\delta c}{\left(\left(1-\frac{1}{2}P_B\right)\delta c\right)^2}}>0$$

$$B:=\frac{-\beta_A(1-P_B)-P_B\beta_B-\frac{\frac{1}{2}\beta_B\delta^s(1-\delta)\delta c}{\left(\left(1-\frac{1}{2}P_A\right)\delta c\right)^2}}{-(1-P_A)\beta_B-P_A\beta_A-\frac{\frac{1}{2}\beta_A\delta^s(1-\delta)\delta c}{\left(\left(1-\frac{1}{2}P_B\right)\delta c\right)^2}}>0$$

Then we substitute this expression back into the total derivatives of the indifference conditions, for example using the indifference condition for player B:

$$-(1-P_B)\frac{dP_A}{d\beta_A}-(1-P_A)\frac{dP_B}{d\beta_A} = -(1-P_A)\beta_B\frac{dP_B}{d\beta_A}+\left(P_B\beta_B + \frac{\frac{1}{2}\beta_B\delta^s(1-\delta)\delta c}{\left(\left(1-\frac{1}{2}P_A\right)\delta c\right)^2}\right)\frac{dP_A}{d\beta_A} \tag{1.42}$$

Rearranging gives:

$$\frac{dP_A}{d\beta_A}\left(-(1-P_B)-\left(P_B\beta_B + \frac{\frac{1}{2}\beta_B\delta^s(1-\delta)\delta c}{\left(\left(1-\frac{1}{2}P_A\right)\delta c\right)^2}\right)-B(1-P_A)(1-\beta_B)\right) = A(1-P_A)(1-\beta_B) \tag{1.43}$$

Thus,

$$\frac{dP_A}{d\beta_A} = \frac{A(1-P_A)(1-\beta_B)}{\left(-(1-P_B)-\left(P_B\beta_B + \frac{\frac{1}{2}\beta_B\delta^s(1-\delta)\delta c}{\left(\left(1-\frac{1}{2}P_A\right)\delta c\right)^2}\right)-B(1-P_A)(1-\beta_B)\right)} \tag{1.44}$$

Note the denominator is negative while the numerator is positive, so the derivative of P_A with respect to β_A is negative. This means that the probability of a particular agent volunteering decreases as she gets marginally less present-biased.[23]

As for $\frac{dP_B}{d\beta_A}$, using the relationship with $\frac{dP_A}{d\beta_A}$:

$$\frac{dP_B}{d\beta_A} = A + B\frac{dP_A}{d\beta_A}$$

$$= A + \frac{BA(1-P_A)(1-\beta_B)}{\left(-(1-P_B)-\left(P_B\beta_B + \frac{\frac{1}{2}\beta_B\delta^s(1-\delta)\delta c}{\left(\left(1-\frac{1}{2}P_A\right)\delta c\right)^2}\right)-B(1-P_A)(1-\beta_B)\right)}$$

$$= A\left[1 - \frac{-B(1-P_A)(1-\beta_B)}{\left(-(1-P_B)-\left(P_B\beta_B + \frac{\frac{1}{2}\beta_B\delta^s(1-\delta)\delta c}{\left(\left(1-\frac{1}{2}P_A\right)\delta c\right)^2}\right)-B(1-P_A)(1-\beta_B)\right)}\right] \tag{1.45}$$

As A and the part in the square bracket are all greater than 0, the derivative of P_B with respect to β_A is positive, meaning that players increase their probabilities of volunteering if the other player becomes less present-biased. Note that for the change of β_B, the result would be exactly symmetric, such that $\frac{dP_B}{d\beta_B} \leq 0$ and $\frac{dP_A}{d\beta_B} \geq 0$.

Appendix 1E. Proof of Proposition 5

We calculate the derivative of $(1-P_A)(1-P_B)$ with respect to β_A, the total derivative is calculated as:

$$-(1-P_B)\frac{dP_A}{d\beta_A} - (1-P_A)\frac{dP_B}{d\beta_A} \tag{1.46}$$

[23] Notice that if $\beta_B = 1$, the derivative of P_A with respect to β_A is 0. Thus, if player B is an exponential discounter, the probability P_A will not be affected by changes in her own present bias.

We then substitute the values of the derivatives from equations (1.44) and (1.45) in the expression and find that:

$$-(1-P_B)\frac{dP_A}{d\beta_A} - (1-P_A)\frac{dP_B}{d\beta_A}$$

$$= -(1-P_B)\frac{A(1-P_A)(1-\beta_B)}{\left(-(1-P_B) - \left(P_B\beta_B + \frac{\frac{1}{2}\beta_B\delta^s(1-\delta)\delta c}{((1-\frac{1}{2}P_A)\delta c)^2}\right) - B(1-P_A)(1-\beta_B)\right)}$$

$$-(1-P_A)A\left[1 - \frac{-B(1-P_A)(1-\beta_B)}{\left(-(1-P_B) - \left(P_B\beta_B + \frac{\frac{1}{2}\beta_B\delta^s(1-\delta)\delta c}{((1-\frac{1}{2}P_A)\delta c)^2}\right) - B(1-P_A)(1-\beta_B)\right)}\right]$$

$$= \frac{A(1-P_A)(1-P_B)\beta_B + (1-P_A)A\left(\left(P_B\beta_B + \frac{\frac{1}{2}\beta_B\delta^s(1-\delta)\delta c}{((1-\frac{1}{2}P_A)\delta c)^2}\right)\right))}{\left(-(1-P_B) - \left(P_B\beta_B + \frac{\frac{1}{2}\beta_B\delta^s(1-\delta)\delta c}{((1-\frac{1}{2}P_A)\delta c)^2}\right) - B(1-P_A)(1-\beta_B)\right)} \tag{1.47}$$

The sign of the numerator of expression (1.47) is positive and the sign of the denominator is negative. The overall sign is therefore negative. This means that the probability $(1-P_A)(1-P_B)$ increases if player A becomes more present-biased in the equilibrium in which both players randomise. Similarly, this derivative also applies to the change in β_B. This result also implies that if both agents become more present-biased, the probability $(1-P_A)(1-P_B)$ increases.

Appendix 1F. Proof of Proposition 6

The utilities of agents are evaluated by the 'long-term' self criterion as defined in Section 1.3.4 and the social welfare is calculated as the sum of the two agents' long-run utilities. Given a stationary behavioural strategy profile σ in which player A volunteers with probability P_A, and player B volunteers with probability P_B, the 'long-term' utility of player A is calculated as:

$$U_L^A = \frac{(1-(1-P_A)(1-P_B))\delta^s - P_A\left(1-\frac{1}{2}P_B\right)c}{1-(1-P_A)(1-P_B)\delta} \tag{1.48}$$

Similarly, the 'long-term' utility of player B is calculated as follows:

$$U_L^B = \frac{(1-(1-P_A)(1-P_B))\delta^s - P_B\left(1-\frac{1}{2}P_A\right)c}{1-(1-P_A)(1-P_B)\delta} \tag{1.49}$$

Therefore, the social welfare is calculated as the sum of the two utilities:

$$W = U_L^A + U_L^B = \frac{2\delta^s\left(1-(1-P_A)(1-P_B)\right) - \left(1-(1-P_A)(1-P_B)\right)c}{1-(1-P_A)(1-P_B)\delta} \tag{1.50}$$

Taking the derivative of the social welfare function with respect to $(1-P_A)(1-P_B)$:

$$\frac{dW}{d((1-P_A)(1-P_B))} = \frac{(-2\delta^s + c)\,(1-(1-P_A)(1-P_B)\delta)}{(1-(1-P_A)(1-P_B)\delta)^2} + \frac{\delta\,(2\delta^s(1-(1-P_A)(1-P_B)) - (1-(1-P_A)(1-P_B))\,c)}{(1-(1-P_A)(1-P_B)\delta)^2} \tag{1.51}$$

$$= \frac{-2\delta^s + 2\delta^{s+1}(1-P_A)(1-P_B) + c - c\delta(1-P_A)(1-P_B)}{(1-(1-P_A)(1-P_B)\delta)^2} + \frac{2\delta^{s+1} - 2\delta^{s+1}(1-P_A)(1-P_B) - c\delta + (1-P_A)(1-P_B)c\delta}{(1-(1-P_A)(1-P_B)\delta)^2} \tag{1.52}$$

$$= \frac{-2\delta^s(1-\delta) + c(1-\delta)}{(1-(1-P_A)(1-P_B)\delta)^2} \tag{1.53}$$

$$= \frac{(1-\delta)\,(c-2\delta^s)}{(1-(1-P_A)(1-P_B)\delta)^2} \tag{1.54}$$

Since the cost of volunteering is less than the delayed benefit, $c < 2\delta^s$, the sign of the derivative is negative. This means that as the probability $(1-P_A)(1-P_B)$ increases, social welfare decreases. The results also imply a delay in volunteering leads to lower social welfare. Hence, if there exists another stationary strategy profile with lower delay in volunteering, then it also has a higher social welfare.

Appendix 1G. Naive Agents

Since all players perceive both their future selves and the other players' future selves to have a present-bias parameter $\hat{\beta}$, players' strategic reasoning for periods $s > t$ is based on these perceived present-bias parameters. Equation (1.17) guarantees that there are no profitable one-shot deviations for any players in any future periods $s > t$. In equilibrium, agents are indifferent between volunteering and waiting given their own future strategies and those of the other player. Moreover, from previous results, we know that there exists a unique solution for P_s between 0 and 1. In the current period t, given that future selves of both players volunteer with probability P_s, a player's desire to volunteer is highest when the other player waits. Suppose player B waits in the current period, player A's payoff from volunteering is: $\beta_A\delta^s - c$. Player A's utility from waiting is:

$$\frac{(1-(1-P_s)^2)\beta_A\delta^{s+1} - P_s\beta_A\delta c(1-\frac{1}{2}P_s)}{1-(1-P_s)^2\delta}$$

Player A prefers to wait if her utility from waiting is larger than volunteering:

$$-c + c\delta(1-P_s)^2 < \beta_A\delta^{s+1} - \beta_A\delta^s - P_s\beta_A\delta c(1-\frac{1}{2}P_s) \tag{1.55}$$

Using equation (1.17), we can rewrite the RHS as:

$$c\left((1-P_s)^2-1\right) < c\left(\frac{\beta_A}{\hat{\beta}}\right)\left((1-P_s)\left(1-\frac{1}{2}P_s\right)\delta - \frac{1-\frac{1}{2}P_s}{1-P_s}\right)$$

$$c\left((1-P_s)^2-1\right) < c\left((1-P_s)^2-1\right)\left(\frac{\beta_A}{\hat{\beta}}\right)\left(\frac{1-\frac{1}{2}P_s}{1-P_s}\right)$$

$$1 > \left(\frac{\beta_A}{\hat{\beta}}\right)\left(\frac{1-\frac{1}{2}P_s}{1-P_s}\right) \tag{1.56}$$

Similarly, if player A waits, player B prefers to wait if:

$$1 > \left(\frac{\beta_B}{\hat{\beta}}\right)\left(\frac{1-\frac{1}{2}P_s}{1-P_s}\right) \tag{1.57}$$

Therefore, when $1 > \left(\frac{\beta_A}{\hat{\beta}}\right)\left(\frac{1-\frac{1}{2}P_s}{1-P_s}\right) \geq \left(\frac{\beta_B}{\hat{\beta}}\right)\left(\frac{1-\frac{1}{2}P_s}{1-P_s}\right)$, it can only be an 'equilibrium' when both players wait in the current period. This is the Case (c).

Following a similar argument, when $\left(\frac{\beta_A}{\hat{\beta}}\right)\left(\frac{1-\frac{1}{2}P_s}{1-P_s}\right) > 1 > \left(\frac{\beta_B}{\hat{\beta}}\right)\left(\frac{1-\frac{1}{2}P_s}{1-P_s}\right)$, player B prefers to wait regardless of player A's decision. If player B waits, player A prefers to volunteer. This is Case (2). When $\left(\frac{\beta_A}{\hat{\beta}}\right)\left(\frac{1-\frac{1}{2}P_s}{1-P_s}\right) \geq \left(\frac{\beta_B}{\hat{\beta}}\right)\left(\frac{1-\frac{1}{2}P_s}{1-P_s}\right) > 1$, if one player waits, the other player always prefers to volunteer. That implies there also exists one equilibrium in which both players randomise. The indifference conditions are characterised by equations (1.18) and (1.19).

Appendix 1H. Multiple Volunteers

We derive the indifference condition for a symmetric equilibrium in which N potential volunteers, each with present-bias parameter β, volunteer with probability P if no one has volunteered previously. The player's expected payoff of volunteering is calculated as

$$(1-P)^{N-1}(\beta\delta^s - c) + \sum_{n=1}^{N-1}\binom{N-1}{n}P^n(1-P)^{N-1-n}\left(\beta\delta^s - \frac{c}{n+1}\right) \tag{1.58}$$

$$= \beta\delta^s - c\sum_{n=0}^{N-1}\binom{N-1}{n}P^n(1-P)^{N-1-n}\left(\frac{1}{n+1}\right)$$

$$= \beta\delta^s - c(1-P)^{N-1}\sum_{n=0}^{N-1}\binom{N-1}{n}\left(\frac{P}{1-P}\right)^n\left(\frac{1}{n+1}\right) \tag{1.59}$$

The first term represents the probability that no other players volunteer, in which case the player bears the full cost. The second term accounts for cost-sharing, where at least one other player volunteers, sharing the cost among them.

The player's expected payoff of waiting is calculated as:

$$(1-(1-P)^{N-1})\beta\delta^s + (1-P)^{N-1}\frac{\sum_{n=1}^{N}\binom{N}{n}P^n(1-P)^{N-n}\left(\beta\delta^{s+1}-\beta\delta\frac{n}{N}\frac{c}{n}\right)}{1-\delta(1-P)^N} \tag{1.60}$$

$$= (1-(1-P)^{N-1})\beta\delta^s + (1-P)^{N-1}\frac{\sum_{n=1}^{N}\binom{N}{n}P^n(1-P)^{N-n}\beta\delta\left(\delta^s-\frac{c}{N}\right)}{1-\delta(1-P)^N}$$

$$= (1-(1-P)^{N-1})\beta\delta^s + (1-P)^{N-1}\frac{(1-(1-P)^N)(\beta\delta^{s+1}-\beta\delta\frac{c}{N})}{1-\delta(1-P)^N} \tag{1.61}$$

When one waits, there is probability $1-(1-P)^{N-1}$ that at least one other player volunteers, the player can receive the benefit without paying the cost. Otherwise, the game continues to the next period, where the process repeats. More specifically, if at least one player volunteers in the next period, the player receives the delayed benefit. We need to consider the number of volunteers and the probability the player is one of the volunteers to account for cost-sharing.

The indifference condition requires that the expected payoff from volunteering equal to the expected payoff from waiting.

$$\beta\delta^s - c(1-P)^{N-1}\sum_{n=0}^{N-1}\binom{N-1}{n}\left(\frac{P}{1-P}\right)^n\left(\frac{1}{n+1}\right)$$

$$= (1-(1-P)^{N-1})\beta\delta^s + (1-P)^{N-1}\frac{(1-(1-P)^N)(\beta\delta^{s+1}-\beta\delta\frac{c}{N})}{1-\delta(1-P)^N} \tag{1.62}$$

We define

$$A := \sum_{n=0}^{N-1}\binom{N-1}{n}\left(\frac{P}{1-P}\right)^n\left(\frac{1}{n+1}\right)$$

The indifference condition simplifies to:

$$(1-P)^N = \frac{\beta\delta^{s+1}-\beta\delta^s+c(A-\beta\delta\frac{1}{N})}{c\delta A-\beta\delta\frac{c}{N}} \tag{1.63}$$

We now prove that there exists a unique solution for P. For equation (1.63), when $P=0$, the value of LHS is 1; when $P=1$, the value of LHS is 0. We also know the LHS is monotonically decreasing in P. When $P=0$, the value of the RHS is $\frac{c-\frac{\beta\delta c}{N}-\beta\delta^s(1-\delta)}{c\delta-\frac{\beta\delta c}{N}}$. We can also show that $c-\frac{\beta\delta c}{N}-\beta\delta^s(1-\delta) < c\delta-\frac{\beta\delta c}{N}$ because $c<\beta\delta^s$. Thus, the value of RHS when $P=0$ must be smaller than 1. When P approaches 1, the RHS approaches $\frac{1}{\delta}$. Moreover, the RHS is increasing in P. This implies that LHS and RHS intersect and intersect only once.

We further show that the probability P increases with parameter β. To prove this, we take the derivative of both sides of equation (1.63) with respect to β.

$$-N(1-P)^{N-1}\frac{dP}{d\beta} = \frac{(\delta^{s+1}-\delta^{s}+c\frac{dA}{dP}\frac{dP}{d\beta}-\frac{c\delta}{N})(c\delta A-\frac{\beta\delta c}{N})}{(c\delta A-\frac{\beta\delta c}{N})^2}$$
$$-\frac{(\beta\delta^{s+1}-\beta\delta^{s}+cA-\frac{c\beta\delta}{N})(c\delta\frac{dA}{dP}\frac{dP}{d\beta}-\frac{c\delta}{N})}{(c\delta A-\frac{\beta\delta c}{N})^2}$$

This can be rewritten as:

$$\frac{dP}{d\beta}\left(c\frac{dA}{dP}(c\delta A-\frac{\beta\delta c}{N})-c\delta\frac{dA}{dP}(\beta\delta^{s+1}-\beta\delta^{s}+cA-\frac{c\beta\delta}{N})+N(1-P)^{N-1}\left(c\delta A-\frac{\beta\delta c}{N}\right)^2\right)$$
$$=-\left(\delta^{s+1}-\delta^{s}-\frac{c\delta}{N}\right)\left(c\delta A-\frac{\beta\delta c}{N}\right)-\left(\beta\delta^{s+1}-\beta\delta^{s}+cA-\frac{c\beta\delta}{N}\right)\frac{c\delta}{N}$$

We can then calculate $\frac{dP}{d\beta}$ as:

$$\frac{dP}{d\beta}=\frac{(1-\delta)c\delta A(\delta^{s}-\frac{c}{N})}{c\delta\frac{dA}{dP}\left((\beta\delta^{s}-\frac{\beta c}{N})(1-\delta)\right)+N(1-P)^{N-1}\left(c\delta A-\frac{\beta\delta c}{N}\right)^2} \tag{1.64}$$

Since $c<\beta\delta^{s}<\delta^{s}$, the derivative of P with respect to β is positive. This implies that the more present-biased players are, the lower the probability of volunteering is.

We now derive the indifference conditions for the group-symmetric equilibrium, where m agents with β_A volunteer with probability P_A and $N-m$ agents with β_B volunteer with probability P_B. For players with β_A, her utility from volunteering is:

$$\beta_A\delta^{s}-\sum_{n=0}^{m-1}\sum_{s=0}^{N-m}\binom{m-1}{n}P_A^{n}(1-P_A)^{m-1-n}\binom{N-m}{s}P_B^{s}(1-P_B)^{N-m-s}\left(\frac{c}{1+n+s}\right) \tag{1.65}$$

$$\begin{aligned}
&=\beta_A\delta^{s}-c(1-P_A)^{m-1}(1-P_B)^{N-m}\\
&\quad\times\sum_{n=0}^{m-1}\sum_{s=0}^{N-m}\binom{m-1}{n}\left(\frac{P_A}{1-P_A}\right)^{n}\binom{N-m}{s}\left(\frac{P_B}{1-P_B}\right)^{s}\left(\frac{1}{1+n+s}\right).\\
&=\beta_A\delta^{s}-c(1-P_A)^{m-1}(1-P_B)^{N-m}B
\end{aligned}$$

we define

$$B:=\sum_{n=0}^{m-1}\sum_{s=0}^{N-m}\binom{m-1}{n}\left(\frac{P_A}{1-P_A}\right)^{n}\binom{N-m}{s}\left(\frac{P_B}{1-P_B}\right)^{s}\left(\frac{1}{1+n+s}\right)$$

Again we need to consider all the possibilities other players with β_A and/or β_B also volunteer. The number of players volunteering matters since it concerns cost-sharing. The utility from waiting is:

$$\begin{aligned}&(1-(1-P_A)^{m-1}(1-P_B)^{N-m})\beta_A\delta^s \\ &\quad+(1-P_A)^{m-1}(1-P_B)^{N-m}\left[\frac{(1-(1-P_A)^m(1-P_B)^{N-m})\beta_A\delta^{s+1}}{1-(1-P_A)^m(1-P_B)^{N-m}\delta}\right. \\ &\quad\left.-\frac{P_A\beta_A\delta c(1-P_A)^{m-1}(1-P_B)^{N-m}B}{1-(1-P_A)^m(1-P_B)^{N-m}\delta}\right]\end{aligned} \tag{1.66}$$

When a player waits and at least one other player volunteers, she receives the benefits without incurring any costs. If no one volunteers, the game advances to the next period. In subsequent periods, as long as at least one player volunteers, she receives the delayed benefit. The player incurs costs only when she volunteers, and the amount depends on how many other players also volunteer.

Equating the utility from volunteering and waiting gives:

$$\beta_A\delta^s(1-\delta)=cB(1-\delta(1-P_A)^{m-1}(1-P_B)^{N-m}(1-(1-\beta_A)P_A))$$

Similarly, we can calculate the indifference condition for players in the group with β_B:

$$\beta_B\delta^s(1-\delta)=cD(1-\delta(1-P_A)^m(1-P_B)^{N-m-1}(1-(1-\beta_B)P_B))$$

with

$$D:=\sum_{n=0}^{m}\sum_{s=0}^{N-m-1}\binom{m}{n}\left(\frac{P_A}{1-P_A}\right)^n\binom{N-m-1}{s}\left(\frac{P_B}{1-P_B}\right)^s\left(\frac{1}{1+n+s}\right)$$

Chapter 2

Campaign Contests in Mixed Electoral Systems

2.1 Introduction

Political candidates' campaign strategies vary widely across countries, shaped significantly by the electoral systems in place as argued in Carey and Shugart (1995). In countries like the UK, which adopt a First-Past-The-Post (FPTP) system, voters elect individual candidates in local districts, and the candidate with the most votes wins a seat. This system encourages candidates to adopt highly personalised and locally-focused campaign strategies. For example, UK politician Martin Bell won a parliamentary seat by cultivating his 'man in the white suit' persona, which symbolises integrity and a stand against political sleaze; while independent candidate Richard Taylor won his district by focusing on a single hyper-local issue: opposing the downgrading of Kidderminster Hospital[1].

In contrast, countries like Sweden and Spain use Proportional Representation (PR) system in which voters cast a single vote for a party and the seats are allocated based on parties' vote share. The party seats are then distributed within each party according to a pre-determined party list (closed list) or preference voting (open list[2]). This system incentivises a more party-centric campaigning and emphasises promoting the party's broader policy platform. For example, Spain's Socialist Workers' Party (PSOE) emphasises European integration under the slogan *"Más Europa" ("More Europe")*, while Sweden's Social Democrats focuses on unity and employment[3].

[1] Martin Bell and 'man in the white suit': Barnett (2004); Richard Taylor and his win as independent candidate on a single local issue: `http://news.bbc.co.uk/news/vote2001/hi/english/newsid_1376000/1376722.stm` (BBC News, 2001)

[2] With an open list, voters vote for candidates, these votes are aggregated at the party level to determine the overall party seat share, and the distribution of seats within the party is determined by how many votes each candidate receives.

[3] Spain's PSOE and *"Más Europa"*: `https://www.socialistas-parlamentoeuropeo.eu/que-defendemos/mas-europa-eu/` (Socialistas en el Parlamento Europeo, 2025) and Sweden's Social Democratic Party's website: `https://www.socialdemokraterna.se/` (Sveriges socialdemokratiska arbetareparti, 2025)

In addition to First-Past-The-Post and Proportional Representation, many countries, including Japan and Germany, have adopted a third type of electoral system known as the mixed electoral system[4]. This system in general combines elements of both FPTP and PR, allowing voters to cast two votes: one for a local candidate and another for a party. Candidates who receive the most votes in their districts secure a seat directly, while additional seats are distributed based on party vote shares[5]. In many countries, candidates may also run under both tiers simultaneously (dual candidacy), so that a candidate who loses their district race might still enter parliament via the party list.

In single-vote systems such as FPTP and PR, candidates typically focus on either personalised or party-centered campaigning as previous studies suggest. However, the dual vote structure in mixed systems introduces a unique strategic trade-off: more personalised campaigning may enhance a candidate's individual appeal, while more party-centric campaigning can increase the party's seat share. The question is thus: How will candidates allocate resources between personalised and party-centric campaigning to maximise their chances of winning a seat?

Allocating resources to personalised rather than party-centric campaigning can be interpreted in different ways – investing in local gatherings rather than party rallies, prioritising policy development at the local policy level over the national level, or emphasising personal stories rather than party slogans on election posters. Candidates' focus between personal/local and party/national strategies has important implications for electoral institution designers. Excessive personalisation in campaigns is argued to have limited informational value[6], whereas a balanced campaigning that integrates both local and national priorities may strengthen policy platforms at both levels[7]. Additionally, parties can influence candidates' campaign choices through mechanisms like party list placements and campaign budget allocations, which can help party leaders achieve strategic objectives such as enhancing party presence or securing parliamentary majority. To address these questions, this study presents a foundational step towards understanding candidates' incentives in a given electoral framework. Here, we study how various design elements (open versus closed list) and party organisational structures (list placements and budget distribution) shape candidates' strategic resource allocations.

To address the question above, we develop a contest-theoretical model of political compe-

[4]There are also other countries adopting a mixed electoral system, such as New Zealand, South Korea, Thailand, Scotland (UK) and Mexico, etc.

[5]There are multiple implementations of mixed electoral systems, which vary across countries. Parallel voting are used in countries such as Japan and Thailand. Countries like South Korea and Germany employ mixed-member proportional representation (MMP). Countries such as Italy allow only a single vote despite being classified as mixed systems. These implementations may differ in some institutional details.

[6]A news article about the over-personalised campaigning in Japan: `https://www.npr.org/2024/07/05/nx-s1-5030098/toky-wacky-election-campaigning-tests-tokyos-patience` (NPR, 2024)

[7]Advantages of both local and national policy campaigning are discussed in Fox (2018).

tition to explore how candidates allocate resources between personalised and party-centric campaigning within a specific mixed electoral system known as parallel voting[8]. In this study, we investigate the following specific questions: (1) In parallel voting with a closed list, how do candidates allocate resources between personalised and party-centric campaigning? (2) How does this allocation depend on candidates' ranks on the party list and their pairings in district-level contests? (3) How might resource allocation change in a mixed system with an open list? Additionally, we also analyse whether these equilibrium allocations align with the optimal strategies from the party's perspective. Furthermore, we consider extensions to the model, such as the effects of asymmetric campaign budgets, positive spillover effects, and factors like safe districts and local attachments.

In the benchmark model, two parties, each with two candidates, compete for at most three parliamentary seats. Candidates compete one versus one in the district-level battles, where winners secure a parliamentary seat. The winning party at the party-level battle distributes the last seat in the parliament within its party according to a closed or an open list. Under a closed list, the winning party distributes the seat according to a predetermined candidate ranking. With an open list, candidates who contribute more towards the party are more likely to be awarded the seat. Each candidate has a campaign budget, which they can allocate between their district and party battles. The resources they allocate affect their winning probabilities in their district battle and their party's success in the party battle.

Our model reveals several key insights. With a closed list, lower-ranked candidates pick their battles: they only allocate their resources towards their district-level battle. The higher-ranked candidates split their resources into both battles and the exact allocation depends on the pairings at the district battle. With an open list, both candidates contribute to the party battle. We also consider the optimal candidates' allocations from the parties' perspective. When party leaders aim to achieve a majority in the parliament, candidates' equilibrium allocations in a closed list are sub-optimal from the parties' perspective, but optimal with an open list. In terms of extensions to the model, when one battle has positive spillover effects on the other, or candidates attach special meanings towards winning through one particular battle, candidates spend more resources towards that battle than in the benchmark model. Furthermore, in-party budget asymmetry can affect which candidates contribute to the party battle. When the low-ranked candidates have a sufficiently large budget, they may become the sole contributors to the party despite their rank.

This study is related to the literature on candidates' incentives or campaigning strategies across different electoral systems. Carey and Shugart (1995) discussed how election formulas can affect candidates' incentives to campaign on a personal or the party reputation.

[8]In parallel voting, local candidates compete directly at local districts and the candidates win most votes get a seat in the parliament. The distribution of the rest of the seats in the parliament depends on the proportion of votes parties receive. The party then distribute those seats according to the party list. Although we focus mainly on parallel voting in this study, we conjecture some main results may also apply to other forms of mixed system like mixed-member proportional representation (MMP).

Important determinants include the degree to which candidates are elected on individual votes, party control over ballot rankings and whether voters cast a single party vote. Crutzen et al. (2020) used team contests to model candidates' incentives in Proportional Representation (PR) system. They find that with a closed list, the candidates in the middle of the party list have the strongest incentives to exert effort. In addition, a closed list can yield a higher team output than an open list when the effort complementarity is low and the cost of effort is not too convex[9]. In PR, the high-ranked candidates may have very little incentives to exert effort since they are "safe" on the party list and the middle-ranked candidates will fight hard for the party since they are in marginal list positions. Our study shows that in a mixed system, high-ranked candidates are strongly incentivised to contribute to party efforts because the lower-ranked candidates have an alternative battle to fight. Moreover, some studies also address the role of parties in influencing candidates' efforts, such as candidates selection in PR/FPTP and deciding candidates' party list positions in PR, see Galasso and Nannicini (2015), Cox et al. (2021), Buisseret et al. (2022), Crutzen and Sahuguet (2023) and Fiva et al. (2024)[10]. There is also a body of research specifically on candidates' incentives in mixed electoral systems. Trumm (2018) used Welsh data to show that the dual candidates - those competing in both the district election and the party list - tend to have more intense, as well as more complex campaigning than those candidates running solely on party list or districts. In addition, Zittel and Gschwend (2008) used a candidate survey from Germany to argue that individualised campaigning is driven by electoral incentives [11]. While the literature on mixed electoral systems is largely empirical, our study contributes by developing a contest-theoretical model that formalises candidates' incentives in a mixed electoral system.

This study also contributes to the literature on contest theory. Candidates expend resources or exert efforts to campaign or improve policy platforms, influencing how many votes they or their party may receive, and ultimately their probabilities of winning a seat in the parliament. For comprehensive surveys of contest theory, see Konrad (2009) and Corchón and Serena (2018). More specifically, the party-level contest relates to the work on team contests, where the efforts from team members are aggregated and determines the contest results. The winning team can distribute the prize within the team according to specified rules, such as egalitarian or proportional rule. Examples of such studies include Nitzan (1991), Lee (1995), Esteban and Ray (2001), Baik (2008) and Nitzan and Ueda (2011). The structure of district battles in our model resembles a team contest with pairwise battles, in which candidates from opposing parties compete one-on-one. Studies on team contests with pairwise battles include Fu et al. (2015) and Konishi et al. (2022). In these studies, the team prize is normally determined by the overall results of the pairwise battles. In contrast, our model treats the

[9]For more studies on candidates' incentives in different electoral systems, see Myerson (1993), Caillaud and Tirole (2002), Crutzen et al. (2009), Castanheira et al. (2010), Hangartner et al. (2019), Gulzar et al. (2022) and Buisseret and Prato (2022).

[10]For similar studies, see also Crutzen and Sahuguet (2018) and Crutzen et al. (2024).

[11]For other studies on mixed electoral systems, see also Massicotte and Blais (1999), Moser and Scheiner (2004), Zittel (2015), Papp and Zorigt (2018), Demirkaya et al. (2022) and Bochsler et al. (2024).

party-level contest as separate from these pairwise battles. Additionally, our study relates to multi-battle contests, as candidates need to decide how to allocate resources between district and party battles. When we model the campaign budget as a "use-it-or-lose-it" budget, this is also related to the Colonel Blotto game. Studies on Colonel Blotto games with Tullock contest success functions include Friedman (1958), Robson (2005), Osorio (2013), Duffy and Matros (2015), Klumpp et al. (2019), Morales and Thraves (2021), Li and Zheng (2022) and Anbarci et al. (2023)[12]. In the existing literature, multi-battle contests typically consist of multiple individual or team contests. By contrast, our study introduces a novel dimension by allowing a choice between individual and team contests, with the allocation of the team prize contingent on results from the individual contests. Finally, when there are multiple seats to be distributed within and/or between parties, this is related to contests with multiple prizes such as in Clark and Riis (1996) and Crutzen et al. (2020).

The remainder of the article is organised as follows. Section 2.2 presents the benchmark model. Section 2.3 illustrates the main results from the model. Section 2.4 discusses the robustness of the model when there are more candidates and prizes. Section 2.5 explores extensions to the benchmark model. Section 2.6 concludes. Detailed proofs and calculations are provided in the Appendices.

2.2 The Benchmark Model

We consider the following contest: there are 2 parties and each party has 2 candidates. They compete for at most 3 seats in the parliament and each candidate can win at most 1 seat. Throughout, we use the terms "prize" and "seat" interchangeably.

2.2.1 Battlefields and Prize Allocations

Each player competes in two battlefields: a district battle and the party battle. In the district battles, each candidate from one party faces a candidate from the opposing party in a one-on-one contest. Thus, there are 2 district battles in total, with each candidate participating in only 1. In the party battle, the two candidates from each party compete collectively against the two candidates from the other party.

The prizes are allocated in two stages. In the first stage, the winners of the two district battles each secure a prize. In the second stage, the winning party from the party battle earns the right to distribute the final prize within its party according to a closed or an open list. More specifically, if only one candidate within the winning party has not won a prize in the district battle, that candidate receives the final prize. If both candidates in the winning party have not won a prize, the highest-ranked candidate wins the prize with a closed list, while

[12]More studies on Colonel Blotto games include Borel (1953), Laslier and Picard (2002), Roberson (2006), Kvasov (2007) and Kovenock and Roberson (2021). More studies on multi-battle contests (not necessarily with a fixed budget) include Snyder (1989), Strumpf (2002), Konrad and Kovenock (2009) and Send (2020).

with an open list, the probability of winning depends on each candidate's resources allocated to the party battle. If both candidates in the winning party have already won prizes, we assume the final prize remains unclaimed, and only two prizes are allocated in total[13].

We illustrate the prize allocation process using an example. Imagine the following scenario: Party A has two candidates, $A1$ and $A2$, and Party B has two candidates, $B1$ and $B2$. There are two districts, candidates $A1$ and $B1$ compete for the seat in district 1, while candidates $A2$ and $B2$ compete for the seat in district 2. Here, $A1$ wins more votes than $B1$ in district 1, thus $A1$ secures a seat in the parliament; $B2$ wins more votes than $A2$ in district 2, thus $B2$ also secures a seat. To allocate the final seat, Party A receives more party-level votes than Party B, giving Party A the right to distribute the final seat. Since $A1$ has already won a seat in the district-level contest, $A2$ is awarded the final seat in the parliament. In this example, $A1$, $A2$ and $B2$ claim the three parliamentary seats.

2.2.2 Formal Model

Players, Strategies and Payoffs

In the benchmark model, there are 2 parties: $j \in \{A, B\}$. Each party has 2 candidates: $i \in \{1, 2\}$. Thus, there are 4 candidates in total: $A1$, $A2$, $B1$ and $B2$. Each candidate has a fixed budget of 1, which they allocate between two contests: their district battle (e_D^{ji}) and the party battle (e_P^{ji}). Candidates simultaneously decide how much of their budget to allocate to each battle. The resources they allocate need to be non-negative: $e_D^{ji} \geq 0 \;\; \forall j, i$ and $e_P^{ji} \geq 0 \;\; \forall j, i$. Additionally, their total allocations across both battles is bounded by their budget constraint, $e_D^{ji} + e_P^{ji} \leq 1 \;\; \forall j, i$. Importantly, the budget is a 'use-it-or-lose-it' budget: any resources not allocated generate no payoff. Thus, the setup corresponds to a Colonel Blotto game, where candidates strategically distribute their fixed resources across multiple battlefields. We assume (for now) that candidates only care about winning a seat in the parliament: they receive a payoff of 1 if they win a seat and 0 if they do not.

Contest Success Function (CSF)

Candidates' resource allocations to the district and party battles affect their probabilities of winning their district and party battles. We model both contests using the Tullock Lottery CSF. For the district battles, the probability candidate ji winning her district battle (P_D^{ji}) is calculated as:

$$P_D^{ji} = \begin{cases} \frac{1}{2}, & \text{if } e_D^{ji} = e_D^{-ji} = 0 \\ \frac{e_D^{ji}}{e_D^{ji} + e_D^{-ji}}, & \text{otherwise} \end{cases} \tag{2.1}$$

We denote the candidate ji's opponent at the district level as $-ji$, this could be any candidate from the opposing party. Thus, the probability of ji winning her district battle increases with resources she allocates to her district battle and decreases with her opponent's alloca-

[13] Alternatively, we assume that the winning party has some backup candidates to take the seat.

tion.

For the party battle, each party's resource allocation is the sum of the resources allocated by its two candidates. Thus, the total allocation for party j is $e_P^j = e_P^{j1} + e_P^{j2}$. The probability of party j winning the party battle (P_P^j) is calculated as:

$$P_P^j = \begin{cases} \frac{1}{2}, & \text{if } e_P^j = e_P^{-j} = 0 \\ \frac{e_P^j}{e_P^j + e_P^{-j}}, & \text{otherwise} \end{cases} \tag{2.2}$$

Here, the probability of party j winning the party battle increases with the resources allocated to the party battle by its party members and decreases with the resources allocated by the opposition party members.

Equilibrium

We will solve the model using Nash Equilibrium (NE). A strategy profile of allocations, denoted by $\mathbf{E} = (e_D^{ji}, e_P^{ji}) \mid j \in \{A, B\}, i \in \{1, 2\}$, is a NE if and only if each candidate chooses a strategy that maximises their payoff given the strategies chosen by all other players.

2.3 Main Results

2.3.1 A Closed List

We begin by examining the closed list setup, where each party has a predetermined, ordered list of candidates. The party distributes the party-level prize to its members based on this fixed order. In this context, we interpret the candidate index $i \in \{1, 2\}$ as indicating each candidate's rank within the party. Thus, candidate $A1$ is the rank-1 candidate in Party A, and $A2$ is the rank-2 candidate in Party A; the same applies for candidates in Party B.

In the district battle, candidates from the two parties compete in one-on-one contests. There are two possible pairing types: (1) symmetric pairing, where rank-1 candidates face each other ($A1$ vs. $B1$) and rank-2 candidates face each other ($A2$ vs. $B2$); and (2) highest-versus-lowest pairing, where the highest-ranked candidate from one party competes against the lowest-ranked candidate from the other party ($A1$ vs. $B2$ and $A2$ vs. $B1$). We begin our analysis with the symmetric pairing.

To find the NE, each candidate aims to maximise her payoff given the allocation strategies of the other players. From candidate $A1$'s perspective, given the allocation decisions of $A2$, $B1$ and $B2$, $A1$ seeks to maximise her probability of winning a prize. The objective function can be written as:[14]

[14]To write the objective function as follows, we temporarily ignore the cases where $e_D^{A1} = e_D^{B1} = 0$, $e_D^{A2} = e_D^{B2} = 0$ and/or $e_P^{A1} = e_P^{B1} = e_P^{A2} = e_P^{B2} = 0$. We will show in Appendix 2A why these values cannot be part of an equilibrium.

$$\max_{e_D^{A1}, e_P^{A1}} \frac{e_D^{A1}}{e_D^{A1}+e_D^{B1}} + \frac{e_D^{B1}}{e_D^{A1}+e_D^{B1}} \cdot \frac{e_P^{A1}+e_P^{A2}}{e_P^{A1}+e_P^{A2}+e_P^{B1}+e_P^{B2}} \tag{2.3}$$

$$s.t. \quad e_D^{A1}+e_P^{A1} \leq 1$$

$$e_D^{A1} \geq 0 \quad e_P^{A1} \geq 0$$

From $A1$'s perspective, if she wins her district battle, she wins a prize with certainty. If she loses her district battle, she can still win a prize if her party wins the party battle. As the highest-ranked candidate in her party, $A1$ will receive the party-level prize as long as her party wins.

Similarly, we can formulate the maximisation problem of candidate $A2$ as follows:

$$\max_{e_D^{A2}, e_P^{A2}} \frac{e_D^{A2}}{e_D^{A2}+e_D^{B2}} + \frac{e_D^{B2}}{e_D^{A2}+e_D^{B2}} \frac{e_P^{A1}+e_P^{A2}}{e_P^{A1}+e_P^{A2}+e_P^{B1}+e_P^{B2}} \frac{e_D^{A1}}{e_D^{A1}+e_D^{B1}} \tag{2.4}$$

$$s.t. \quad e_D^{A2}+e_P^{A2} \leq 1$$

$$e_D^{A2} \geq 0 \quad e_P^{A2} \geq 0$$

From $A2$'s perspective, if she wins the district battle, she wins a prize with certainty. If she loses, she can still win a prize if Party A wins at the party battle. However, since $A2$ is the rank-2 candidate in Party A, she can claim the prize only if $A1$ has already won a prize in the district battle.

The maximisation problems of candidates $B1$ and $B2$ are symmetric to those of $A1$ and $A2$. Allocating more resources to the district battle increases a candidate's probability of winning at the district level, thereby increasing the chance of securing a prize directly. However, this reduces the probability of winning a prize through the party battle, as each candidate can only win one prize. On the other hand, allocating more resources to the party battle raises the probability of the party winning the party-level prize, enhancing the candidate's chance of winning a prize through this route. Given the budget constraint, candidates face a trade-off between the returns from allocating resources to the district and party battles. Since the rank-1 and rank-2 candidates between the two parties face symmetric problems, we look for between-party symmetric equilibria in which $e_D^{A1} = e_D^{B1} = e_D^1$, $e_P^{A1} = e_P^{B1} = e_P^1$, $e_D^{A2} = e_D^{B2} = e_D^2$ and $e_P^{A2} = e_P^{B2} = e_P^2$.

Proposition 1: *In a closed-list model with symmetric pairings in the district battles, the unique party-symmetric Nash Equilibrium implies: (1) Rank-2 candidates pick their battles - they allocate all resources to the district level; (2) Rank-1 candidates divide their resources equally between the district and party battles.*

To characterise the equilibrium, we derive the Kuhn–Tucker conditions for all candidates'

(e_D^{ji}, e_P^{ji})	Party A	Party B
Rank-1 Candidate	$(\frac{1}{2}, \frac{1}{2})$	$(\frac{1}{2}, \frac{1}{2})$
Rank-2 Candidate	$(1, 0)$	$(1, 0)$

Table 2.1: Nash Equilibrium in a closed list with symmetric pairing

maximisation problems. Since each objective function is concave for $0 \leq e_D^i \leq 1$ and $0 \leq e_P^i \leq 1$ (the Hessian matrices of the objective functions are negative semi-definite) and the set defined by the budget constraint is convex, any solution that satisfies the KT conditions constitutes a global constrained maximum. An allocation profile is therefore an equilibrium if and only if it jointly satisfies the KT conditions of both rank-1 and rank-2 candidates. Restricting attention to symmetric equilibria simplifies the analysis, and the full proof is presented in Appendix 2A.[15]

With a closed list, if the rank-1 candidates do not win their district battles, they can still win a prize as long as their party wins the party battle. For the rank-2 candidates, however, winning the party prize is only possible if their rank-1 teammate has already secured a prize in the district battle. Consequently, rank-2 candidates may have a much lower incentive to allocate resources to the party battle. Recognising this, rank-1 candidates anticipate limited contributions from rank-2 candidates and are therefore incentivised to allocate some resources to the party battle themselves.

Using the same method, we next solve for the NE in a closed list with highest-versus-lowest pairing. In this setup, the rank-1 candidates compete against the rank-2 candidates in the district battles.

Proposition 2: *In a closed-list model with highest-versus-lowest pairings, the unique party-symmetric NE implies: (1) Rank-2 candidates pick their battles, allocating all resources to the district level; (2) Rank-1 candidates allocate $\frac{2}{3}$ to the party battle, a higher amount than with symmetric pairing.*

The proof follows a similar procedure as Proposition 1, see Appendix 2B for details. With highest-versus-lowest pairings in the district battles, the rank-2 candidates continue to focus solely on the district battles, as in symmetric pairings. However, the rank-1 candidates allocate more resources to the party battle than with symmetric pairing. The intuition is that rank-1 candidates now compete against rank-2 candidates who are more "aggressive" in district battles. When opponents already allocate significant resources to the district battle,

[15]In the proof, we include KT conditions for all three constraints for each player. One could alternatively argue that candidates always exhaust their budget and substitute the equality constraint directly. However, this only holds when the district opponent exerts positive effort. To cover all cases consistently, we adopt the full KT approach.

(e_D^{ji}, e_P^{ji})	Party A	Party B
Rank-1 Candidate	$(\frac{1}{3}, \frac{2}{3})$	$(\frac{1}{3}, \frac{2}{3})$
Rank-2 Candidate	$(1, 0)$	$(1, 0)$

Table 2.2: Nash Equilibrium in a closed list with highest-versus-lowest pairing

adding an extra unit of resources may yield very little return. Consequently, rank-1 candidates may shift more resources to the party battle, where the marginal impact on their probability of winning a prize is greater.

2.3.2 An Open List

We now discuss the main results with an open list. In an open list system, voters can vote directly for individual candidates on the party ballot. The total number of party votes is still pooled to determine the party's seat share, as in a closed list. However, within each party, seats are distributed based on the votes each candidate receives on the party ballot. This means that voters may not only consider the party's policy platform, but also be influenced by candidates who make a strong impression while campaigning on behalf of the party. We model the open list by assuming that the party prize is distributed within the party according to a "merit rule": candidates who allocate more resources to party-centric campaigning are more likely to win the party prize. Specifically, if both members of party j lose their district battles and party j wins the party battle, the probability that candidate $j1$ wins the party prize (P_i^j) is calculated as:

$$P_1^j = \begin{cases} \frac{1}{2}, & \text{if } e_P^{j1} = e_P^{j2} = 0 \\ \frac{e_P^{j1}}{e_P^{j1} + e_P^{j2}}, & \text{otherwise} \end{cases} \tag{2.5}$$

We now formulate the maximisation problem of candidate $A1$ as follows:

$$\max_{e_D^{A1}, e_P^{A1}} \frac{e_D^{A1}}{e_D^{A1} + e_D^{B1}} + \frac{e_D^{B1}}{e_D^{A1} + e_D^{B1}} \frac{e_P^{A1} + e_P^{A2}}{e_P^{A1} + e_P^{A2} + e_P^{B1} + e_P^{B2}} \left(\frac{e_D^{A2}}{e_D^{A2} + e_D^{B2}} + \frac{e_D^{B2}}{e_D^{A2} + e_D^{B2}} \frac{e_P^{A1}}{e_P^{A1} + e_P^{A2}} \right) \tag{2.6}$$

$$s.t. \quad e_D^{A1} + e_P^{A1} \leq 1$$

$$e_D^{A1} \geq 0 \quad e_P^{A1} \geq 0$$

From $A1$'s perspective, if she wins the district battle, she secures a prize with certainty. If she does not win her district battle, she can still win a prize if Party A wins the party battle. More specifically, there are two scenarios in which she can win a prize: (1) the other party member, $A2$, has already won a prize, making $A1$ the only remaining candidate eligible for the party prize; (2) both party members have lost their district battles, and $A1$ receives more votes on the party ballot than $A2$, thereby winning the party prize. These probabilities are reflected in the objective function above. The maximisation problems for the other candidates are symmetric to that of $A1$. Since all 4 candidates face symmetric problems, we look for a symmetric equilibrium where $e_D^{A1} = e_D^{A2} = e_D^{B1} = e_D^{B2} = e_D$ and $e_P^{A1} = e_P^{A2} = e_P^{B1} = e_P^{B2} = e_P$.

Proposition 3: *In an open list model, the unique symmetric Nash Equilibrium implies that all candidates allocate $\frac{1}{3}$ to the party battle and $\frac{2}{3}$ to their district battle. Compared to a closed list, rank-1 candidates decrease their allocations to the party battle, while rank-2 candidates increase their allocations to the party battle.*

The proof follows a similar procedure as previously, refer to Appendix 2C for the detailed proof. With an open list, both candidates within a party are incentivised to allocate resources to the party battle. Allocating resources to the party battle not only supports the party's chances of winning but also improves each candidate's probability of winning the party prize. In a closed list, rank-2 candidates have minimal chances of winning the party prize, while an open list enables them to compete for it. As a result, rank-2 candidates in an open list allocate more to the party battle than they would in a closed list. Conversely, rank-1 candidates lose their ranking advantages and their party members start to contribute to the party battle in an open list, making them allocate less to the party battle compared to the case of a closed list.

2.3.3 Parties' Choices

After determining the equilibrium allocations under both closed and open lists, a natural question arises: are these equilibrium allocations optimal from the perspective of the party leaders? Alternatively, if party leaders could choose the allocation decisions for their candidates, would they choose any differently? This question is valuable not only as a benchmark but also for understanding scenarios in countries where parties exercise significant control over candidates' decisions. The answer depends on parties' objectives. If a party's goal is to maximise total allocations towards the party battle, the solution is straightforward: allocate all resources to the party battle. Alternatively, if the party seeks to maximise its chance of achieving a "majority" in parliament - when both of its candidates are elected - the strategy becomes less obvious.

We formulate the problem as follows. Suppose there are 2 party leaders. They simultaneously choose allocations for their party's two candidates while respecting each candidate's budget constraint. For example, Party A's leader allocates candidate $A1$'s budget between the district and party battles, and similarly for $A2$. Party A's leader aims to maximise the probability that both Party A candidates are elected to parliament. This includes cases where both candidates win their district battles or where one candidate wins the district battle and party A wins the party battle. Note that for party leaders aiming to achieve majority, a closed or an open list will not matter for the maximisation problem. The maximisation problem for

Party A's leader with symmetric pairings[16], is therefore formulated as follows:

$$\max_{e_D^{A1}, e_P^{A1}, e_D^{A2}, e_P^{A2}} \frac{e_D^{A1}}{e_D^{A1}+e_D^{B1}} \frac{e_D^{A2}}{e_D^{A2}+e_D^{B2}} + \frac{e_D^{B1}}{e_D^{A1}+e_D^{B1}} \frac{e_D^{A2}}{e_D^{A2}+e_D^{B2}} \frac{e_P^{A1}+e_P^{A2}}{e_P^{A1}+e_P^{A2}+e_P^{B1}+e_P^{B2}}$$

$$+\frac{e_D^{A1}}{e_D^{A1}+e_D^{B1}} \frac{e_D^{B2}}{e_D^{A2}+e_D^{B2}} \frac{e_P^{A1}+e_P^{A2}}{e_P^{A1}+e_P^{A2}+e_P^{B1}+e_P^{B2}} \tag{2.7}$$

$$s.t. \quad e_D^{A1}+e_P^{A1} \leq 1 \quad e_D^{A2}+e_P^{A2} \leq 1$$

$$e_D^{A1} \geq 0 \quad e_P^{A1} \geq 0 \quad e_D^{A2} \geq 0 \quad e_P^{A2} \geq 0$$

The maximisation problem for Party B is symmetric to that of Party A. Therefore, we seek a symmetric equilibrium.

Proposition 4: *In both closed and open lists, when party leaders control candidates' allocations, they allocate $\frac{2}{3}$ to each candidate's district battle and $\frac{1}{3}$ to each candidate's party battle. In a closed list, candidates' equilibrium allocations are suboptimal from the party's perspective, whereas in an open list, they are optimal.*

To determine the equilibrium allocations chosen by the parties, we solve the maximisation problems for both Party A and Party B. The detailed proof is provided in Appendix 2D.

Recall that in a closed list, lower-ranked candidates, lacking a top position for the party prize, allocate all resources to their district battles while the higher-ranked candidates split their resources across both district and party battles. However, from the party's perspective, the higher-ranked candidates are allocating too little in the district battle while the lower-ranked candidates are allocating too much. The misalignment arises because individual candidates maximise only their own winning probabilities, whereas the party would prefer their own candidates to maximise the joint probability that both candidates secure prizes.

The key intuition is that each candidate's allocation creates externalities for their party partner. The candidates do not account for these externalities when maximising their own winning probabilities. More specifically, allocating more resources to the party battle generates positive externalities on the other party member as it increases the party's probability of winning the party prize. At the same time, when rank-1 candidates shift resources from their districts to the party battle, they lower their own chance of winning at the district battle, which in turn lowers the probability that their rank-2 partners can claim the party prize, creating a negative externality for the rank-2 candidates. If the candidates were to account for these externalities as maximising their joint winning probabilities, rank-2 candidates would allocate more to the party battle, while rank-1 candidates would allocate less. In an open list, for both candidates, the positive externalities from allocating to the district battle counterbalance those from allocating to the party battle, resulting in equilibrium allocations

[16]The set up of the problems is very similar in the case of highest-versus-lowest pairings.

that are optimal from the party's perspective.

2.4 Robustness

A natural question to ask is whether the general patterns identified also hold with a larger number of candidates and prizes. Will lower-ranked candidates still focus heavily on the district battles? Will higher-ranked candidates' allocations be influenced by their district opponents? How might allocations shift with an open list? To capture the trade-off between the district and party battles, the model must ensure that the number of prizes is greater than the number of candidates per party (or number of districts) but less than the total number of candidates. When increasing candidates and prizes, three considerations arise:

- **Distribution of Multiple Prizes among Parties:** With more prizes, there may be more than 1 party prize to distribute among parties. We model the distribution of party seats between parties using the binomial Tullock function (Crutzen et al., 2020). Here, prizes are awarded to party j using independent draws from a Bernoulli distribution with probability $p_j = \frac{e_P^j}{e_P^j + e_P^{-j}}$ determined by the party-level allocations. Thus, the probability that party j wins m out of k seats is calculated as

$$P_j(m) = C_m^k (p_j)^m (1 - p_j)^{k-m}.$$

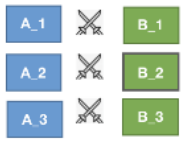
- **Distribution of Multiple Prizes within Parties:** In an open list, the parties may also need to distribute multiple prizes among their party members. Following Clark and Riis (1996), we model this prize allocation process as a sequential contest: the first prize is determined by all candidates' party contributions using a Tullock Lottery CSF. The winning candidate is then removed, and the next prize winner is determined using the remaining candidates' allocations, continuing until all prizes are allocated.

- **Increased Pairings in District Battles:** In a closed list, as the number of candidates grows, the possible pairings in district battles increase substantially. To manage this complexity, we focus on a restricted set of tractable pairings or those likely to yield new insights.

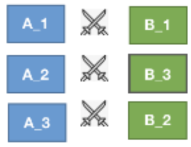
The analysis begins with the case of three candidates in each party.

2.4.1 Three candidates in each party

With 3 candidates in each party, there are 3 district-level battles in total, and the total number of seats can be either 4 or 5. For a closed list, there are 6 different pairings at district battles. The equilibrium results largely mirror those found in the benchmark model. The increased number of possible pairings allows for a richer comparative analysis with respect to different district pairings. More specifically, we can investigate how a candidate's equilibrium allocation changes when her opponent remains the same across two pairings, while the district opponents of the other party members differ. To explore this, we analyse the case with 5 seats in total and consider two distinct pairings shown in Figure 2.1. By observing the equilibrium allocations, we derive Proposition 5.



Candidates	(D, P)
Rank 1	$(0.4, 0.6)$
Rank 2	$(0.8, 0.2)$
Rank 3	$(1, 0)$



Candidates	(D, P)
Rank 1	$(0.43, 0.57)$
Rank 2	$(0.71, 0.29)$
Rank 3	$(1, 0)$

Figure 2.1: Nash Equilibrium with three candidates in each party and five seats in total

Proposition 5: *In a closed list model, the allocation decisions of higher-ranked candidates are influenced not only by their own district opponents but also by the district opponents of the other party members.*

We consider the following two pairings: (1) a symmetric pairing where candidates of the same rank compete against each other in the district battles, and (2) a pairing where rank-1 candidates continue to compete against rank-1 candidates, but rank-2 candidates face rank-3 candidates. In this comparison, the district opponents of the rank-1 candidates remain the same, while the opponents of the other party members differ. This setup allows us to investigate whether and how rank-1 candidates adjust their allocations between the two pairings.

Across the two pairings, the rank-1 candidates' opponents remain unchanged, while the opponents of the other party members vary. More specifically, in the second pairing, the rank-2 candidates face lower-ranked opponents. With limited chances of winning the party-level prize, the rank-3 candidates are extremely 'aggressive' in their district battles. Consequently, rank-2 candidates might find it advantageous to allocate more resources to the party battle. The rank-1 candidates, in turn, are inclined to free-ride on this increased contribution from rank-2 candidates and reallocate more resources towards their district battles. In the open list model, all candidates allocate positive resources to both battles. Detailed results are provided in Appendix 2E.

2.4.2 More candidates in each party

In this section, we extend the analysis to scenarios with a greater number of candidates and prizes. As the number of candidates per party increases, the number of possible pairings grows substantially. Thus, we focus on two specific pairings: symmetric pairing and highest-versus-lowest pairing. The objective is to assess if the patterns observed in the benchmark model hold with a large number of candidates and prizes. Our findings indicate that an allocation profile in which higher-ranked candidates contribute to the party battle while lower-ranked candidates allocate all resources towards their district battles remains an equilibrium across any number of candidates and prizes. When there is only one party-level prize, analytical solutions are feasible; however, for multiple party-level prizes, numerical solutions are required. Additionally, the tendency for the highest-ranked candidates to allocate more to the party battle in the highest-versus-lowest pairing also appears to persist for any number of candidates and prizes. Overall, this analysis provides evidence that the general patterns from the benchmark model are robust with more candidates and prizes. With an open list, since all candidates are symmetric, we can derive analytical solutions for any number of candidates and prizes. For a more detailed analysis, see Appendix 2F.

2.5 Extensions

2.5.1 In-party Budget Asymmetry

In the benchmark model, we assume that all candidates have the same budget of 1. However, some candidates may possess a larger budget than others. In this extension, we explore in-party budget asymmetry, where each party still has a total budget of 2, but the rank-1 and rank-2 candidates may have different budgets. More specifically, we allow the rank-1 candidates to have either a larger or smaller budget than the rank-2 candidates. In this extended model, rank-1 candidates have a budget of $1+c$, while rank-2 candidates have a budget of $1-c$, $-1 < c < 1$. When $c > 0$, the higher-ranked candidates have a larger budget than the lower-ranked candidates. When $c < 0$, the higher-ranked candidates have a smaller budget than the lower-ranked candidates.

Proposition 6: *In a closed-list model with symmetric pairing and in-party budget asymmetry, the unique symmetric NE implies that: (1) when higher-ranked candidates do not have a significantly smaller budget than lower-ranked candidates, higher-ranked candidates remain the sole contributors to the party battle; (2) when lower-ranked candidates have a sufficiently larger budget, lower-ranked candidates may become the sole contributors to the party.*

We analyse three cases under symmetric pairing:

1. **Higher-ranked candidates as sole contributors** ($c > -\frac{1}{5}$): When the budget of higher-ranked candidates is above or not too much below 1, the rank-1 candidates remain the sole contributors to the party. In this case, $e_D^1 = e_P^1 = \frac{1+c}{2}$, $e_D^2 = 1-c$, and $e_P^2 = 0$. The total party allocation is greater than $\frac{2}{5}$.

2. **Both candidates contribute** ($-\frac{3}{5} \le c \le -\frac{1}{5}$): In this range, both candidates contribute to the party. More specifically, $e_D^1 = \frac{2}{5}$ and $e_D^2 = \frac{6}{5}$, with a total party allocation consistently at $\frac{2}{5}$.

3. **Lower-ranked candidates as sole contributors** ($c < -\frac{3}{5}$): When the lower-ranked

candidates have a sufficiently larger budget, they become the sole contributors to the party, with $e_P^2 = \frac{1-c}{4}$ and $e_P^1 = 0$. The total party allocation again exceeds $\frac{2}{5}$.

To summarise, the total allocation to the party is the lowest when rank-2 candidates receive only a moderate increase in budget. Intermediate negative values of c reduce rank-1 candidates' ability to contribute to the party battle, though their contribution remains positive. This compels rank-2 candidates to allocate a substantial proportion of their budget toward the district battle.

In contrast, under highest-versus-lowest pairing, the results shift significantly: when $c \geq -\frac{1}{3}$, rank-1 candidates remain the sole contributors to the party battle. For $c < -\frac{1}{3}$, rank-1 candidates not only stay as the sole contributors to the party, but also allocate all their resources towards the party. When lower-ranked candidates gain additional budget, they use this increase as a leverage in the district battle, pressuring rank-1 candidates to invest even more heavily in the party, ensuring rank-2 candidates can secure a seat for sure. See Appendix 2G for the detailed proof.

2.5.2 Safe Districts

In the benchmark model, all districts are considered fair, meaning that the winning probabilities of candidates depend solely on the resources they allocate to the district battle. However, some districts may be "safer" for certain candidates, such as those who have been running in the district for an extended period. This image could grant those candidates certain advantages, or the district battle may be inherently biased in their favour. We model this using a "biased" Tullock Lottery CSF. If candidate ji has an advantage in her district battle, her winning probability is calculated as:

$$\frac{\gamma e_D^{ji}}{\gamma e_D^{ji} + e_D^{-ji}}$$

where $\gamma > 1$ represents the degree of bias. As a result, campaigning in these safe districts becomes more productive for the favoured candidates due to their local popularity or extensive knowledge of the area. We consider the highest-versus-lowest pairings in order to derive a closed-form solution.

Proposition 7: *In a closed-list model with highest-versus-lowest pairing, the unique symmetric NE implies the following: (1) when the district battles are biased in favour of rank-1 candidates, rank-1 candidates allocate more resources in the district battle than in the benchmark model; (2) when the district battles are biased in favour of rank-2 candidates, rank-1 candidates focus exclusively on their party battle if $\gamma > 2$.*

When rank-1 candidates are running for their safe districts, they face a trade-off: on one hand, they are motivated to allocate resources to the party, as they are more likely to receive the party prize. On the other hand, their campaign is significantly more effective in the district battle. Thus, the exact allocation depends on the value of γ. More specifically, we find that $e_D^1 = \frac{2\gamma-1}{3\gamma}$ and $e_P^1 = \frac{\gamma+1}{3\gamma}$. As γ increases, candidates allocate more resources toward the district battle. Given that the effectiveness of local campaigning is greater than in the benchmark model, we observe that $e_D^1 = \frac{2\gamma-1}{3\gamma} > \frac{1}{3}$; thus, rank-1 candidates allocate more to the district battle than in the benchmark model. For rank-2 candidates, although they may have some incentives to adjust resources in response to more "aggressive" district campaigning by rank-1 candidates, the limitations of being ranked second still dominate, so their allocation remains unchanged from the benchmark model.

When rank-2 candidates conduct more productive campaigning in their districts, this adds further pressure on their rank-1 opponents in the district battle. Consequently, rank-1 candidates may increase their allocations to the party battle and, if γ is sufficiently large, may even focus exclusively on the party battle. For a more detailed proof, see Appendix 2H.

2.5.3 Positive Spillover Effects

A charismatic personalised campaign may create positive impressions not only for the candidate but also for the candidate's party. Similarly, a successful party campaign can attract more attention to its candidates. Thus, spending resources on one battle can produce positive spillover effects on the other. We extend the model as follows: if e_D^{ji} is spent at the district battle, it contributes to the party battle as if an additional αe_D^{ji} is spent there. Similarly, if the spillover works the other way around, if e_P^{ji} is allocated to the party battle, it boosts the district battle as if a further βe_P^{ji} is spent in the district battle. We assume $0 < \alpha < 1$ and $0 < \beta < 1$.

Proposition 8: *When allocating resources in one battle generates a positive spillover effect on the other battle, candidates allocate more resources to the initial battle than they would in the benchmark model.*

In a closed-list model with symmetric pairing, when resources allocated to personalised campaigning have a positive spillover effect on the party battle, candidates allocate more to the district battle. More specifically, when $0 < \alpha \leq \frac{1}{3}$, we derive $e_D^1 = \frac{1+\alpha}{2(1-\alpha)}$ and $e_D^2 = 1$. In this case, $e_D^1 = \frac{1+\alpha}{2(1-\alpha)} > \frac{1}{2}$, so rank-1 candidates allocate more to the district battle than in the benchmark case. When $\frac{1}{3} < \alpha < 1$, the positive spillover effect is strong enough that $e_D^1 = e_D^2 = 1$, and both rank-1 and rank-2 candidates allocate all their resources to the district battle. A similar effect occurs when considering spillover from the party battle to the district battle. The same intuition applies with highest-versus-lowest pairings and an open list. For a more detailed proof, refer to Appendix 2I.

2.5.4 Local District Attachments

In the benchmark model, candidates are assumed to only care about winning a seat, regardless of which battles secures their victory. However, some candidates may have strong attachments to their local districts and thus place greater value on winning through district battles. We extend the model to capture this by assuming that candidates who win their district battles receive an enhanced payoff of $V > 1$.

Proposition 9: *If candidates place greater value on winning through the district battle, they allocate more resources to the district battle compared to the benchmark model.*

In a closed list model with symmetric pairings, when $V > 1$, rank-2 candidates allocate all of their resources to the district battle, as in the benchmark model. For rank-1 candidates, we find that $e_D^1 = \frac{2V-1}{2V}$ and $e_P^1 = \frac{1}{2V}$. The value of e_D^1 increases with V, and since $V > 1$, we have $e_D^1 = \frac{2V-1}{2V} > \frac{1}{2}$. Thus, compared to the benchmark model where rank-1 candidates allocate $\frac{1}{2}$ to the district battle, rank-1 candidates with local attachments allocate more to their district battles. The same intuition applies when considering highest-versus-lowest pairings and an open list. For a more detailed proof, see Appendix 2J.

2.6 Discussions and Conclusion

In this study, we explore how dual candidates in a mixed electoral system trade off between allocating resources to personalised campaigning to win their district battles and party-centric campaigning aimed at securing more seats for their party. Using a contest theoretical model, we find that candidates' resource allocations are influenced by whether they operate under a closed or an open list, as well as by their ranks on the party list and the specific district-level pairings they encounter. Under a closed list, our findings indicate that higher-ranked candidates are more incentivised to allocate resources to the party battle, given their favourable positions to claim party-level prizes. In contrast, lower-ranked candidates focus primarily on their district battles. In addition, candidates' allocations are shaped by the ranks of their opponents in the district battles, as well as the ranks of the other party members' opponents. For parties aiming to maximise representation, higher-ranked candidates over-invest in the party battle while lower-ranked candidates under-invest. Candidates' equilibrium allocations are, therefore, suboptimal under a closed list.

This model has practical implications for both parties and policymakers. Parties could strategically construct their party lists, taking into account the differing district-party trade-offs associated with different rank positions. For institution designers, altering election formulas or campaign funding rules could shape parties' and candidates' incentives, potentially influencing the types of campaigning that candidates prioritise.

Future extensions of this model could explore scenarios where parties have asymmetric budgets, distinguishing between large and small political parties. Moreover, it would be valuable to consider how parties might construct their party lists or assign campaign budgets to different candidates, especially when candidates vary in terms of popularity or ability. Future research could also investigate alternative implementations of mixed electoral systems beyond the closed and open lists, examining factors such as the proportion of seats designated for district versus party representation, multi-member versus single-member districts, or the sequencing of seat allocation.

Appendices

Appendix 2A. Proof of Proposition 1

We begin by deriving the Kuhn–Tucker (KT) conditions for each candidate's maximisation problem. For candidate $A1$, the Lagrangian is

$$L(e_D^{A1}, e_P^{A1}, \lambda^{A1}, \mu_D^{A1}, \mu_P^{A1}) =$$

$$\frac{e_D^{A1}}{e_D^{A1}+e_D^{B1}} + \frac{e_D^{B1}}{e_D^{A1}+e_D^{B1}} \frac{e_P^{A1}+e_P^{A2}}{e_P^{A1}+e_P^{A2}+e_P^{B1}+e_P^{B2}} - \lambda^{A1}(e_D^{A1}+e_P^{A1}-1) + \mu_D^{A1}e_D^{A1} + \mu_P^{A1}e_P^{A1} \quad (2.8)$$

From this, the KT conditions are:

$$\frac{e_D^{B1}}{(e_D^{A1}+e_D^{B1})^2} - \frac{e_D^{B1}}{(e_D^{A1}+e_D^{B1})^2} \frac{e_P^{A1}+e_P^{A2}}{e_P^{A1}+e_P^{A2}+e_P^{B1}+e_P^{B2}} - \lambda^{A1} + \mu_D^{A1} = 0 \quad (2.9)$$

$$\frac{e_D^{B1}}{e_D^{A1}+e_D^{B1}} \frac{e_P^{B1}+e_P^{B2}}{(e_P^{A1}+e_P^{A2}+e_P^{B1}+e_P^{B2})^2} - \lambda^{A1} + \mu_P^{A1} = 0 \quad (2.10)$$

$$e_D^{A1}+e_P^{A1} \leq 1, \qquad \lambda^{A1} \geq 0, \qquad (e_D^{A1}+e_P^{A1}-1)\lambda^{A1} = 0 \quad (2.11)$$

$$e_D^{A1} \geq 0, \qquad \mu_D^{A1} \geq 0, \qquad \mu_D^{A1}e_D^{A1} = 0 \quad (2.12)$$

$$e_P^{A1} \geq 0, \qquad \mu_P^{A1} \geq 0, \qquad \mu_P^{A1}e_P^{A1} = 0 \quad (2.13)$$

Furthermore, we check the Hessian matrix of the objective function:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

where

$$a = -\frac{2e_D^{B1}}{(e_D^{A1}+e_D^{B1})^3} \frac{e_P^{B1}+e_P^{B2}}{e_P^{A1}+e_P^{A2}+e_P^{B1}+e_P^{B2}}$$

$$b = c = -\frac{e_D^{B1}}{(e_D^{A1}+e_D^{B1})^2} \frac{e_P^{B1}+e_P^{B2}}{(e_P^{A1}+e_P^{A2}+e_P^{B1}+e_P^{B2})^2}$$

$$d = -\frac{e_D^{B1}}{e_D^{A1}+e_D^{B1}} \frac{2(e_P^{B1}+e_P^{B2})}{(e_P^{A1}+e_P^{A2}+e_P^{B1}+e_P^{B2})^3}$$

The determinant of the matrix, calculated as

$$ad - bc = \frac{3(e_D^{B1})^2}{(e_D^{A1}+e_D^{B1})^4} \frac{(e_P^{B1}+e_P^{B2})^2}{(e_P^{A1}+e_P^{A2}+e_P^{B1}+e_P^{B2})^4}$$

For $e_D^i \geq 0$ and $e_P^i \geq 0$, $a \leq 0$, $d \leq 0$ and $ad - bc \geq 0$, the Hessian of the objective function is negative semi-definite over the feasible region, confirming that the objective function is concave in this region. Since the set defined by the constraint $e_D^{A1} + e_P^{A1} \leq 1$ is convex, any point satisfying KT conditions is a global constrained maximum.

We repeat the same steps for candidate $A2$, setting up the Lagrangian:

$$L(e_D^{A2}, e_P^{A2}, \lambda^{A2}, \mu_D^{A2}, \mu_P^{A2}) =$$

$$\frac{e_D^{A2}}{e_D^{A2}+e_D^{B2}} + \frac{e_D^{B2}}{e_D^{A2}+e_D^{B2}} \frac{e_P^{A1}+e_P^{A2}}{e_P^{A1}+e_P^{A2}+e_P^{B1}+e_P^{B2}} \frac{e_D^{A1}}{e_D^{A1}+e_D^{B1}} - \lambda^{A2}(e_D^{A2}+e_P^{A2}-1) + \mu_D^{A2} e_D^{A2} + \mu_P^{A2} e_P^{A2} \tag{2.14}$$

The KT conditions are:

$$\frac{e_D^{B2}}{(e_D^{A2}+e_D^{B2})^2} - \frac{e_D^{B2}}{(e_D^{A2}+e_D^{B2})^2} \frac{e_P^{A1}+e_P^{A2}}{e_P^{A1}+e_P^{A2}+e_P^{B1}+e_P^{B2}} \frac{e_D^{A1}}{e_D^{A1}+e_D^{B1}} - \lambda^{A2} + \mu_D^{A2} = 0 \tag{2.15}$$

$$\frac{e_D^{B2}}{e_D^{A2}+e_D^{B2}} \frac{e_P^{B1}+e_P^{B2}}{(e_P^{A1}+e_P^{A2}+e_P^{B1}+e_P^{B2})^2} \frac{e_D^{A1}}{e_D^{A1}+e_D^{B1}} - \lambda^{A2} + \mu_P^{A2} = 0 \tag{2.16}$$

$$e_D^{A2}+e_P^{A2} \leq 1, \qquad \lambda^{A2} \geq 0, \qquad (e_D^{A2}+e_P^{A2}-1)\lambda^{A2} = 0 \tag{2.17}$$

$$e_D^{A2} \geq 0, \qquad \mu_D^{A2} \geq 0, \qquad \mu_D^{A2} e_D^{A2} = 0 \tag{2.18}$$

$$e_P^{A2} \geq 0, \qquad \mu_P^{A2} \geq 0, \qquad \mu_P^{A2} e_P^{A2} = 0 \tag{2.19}$$

Furthermore, we check the Hessian matrix of the objective function:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

where

$$a = -\frac{2e_D^{B2}}{(e_D^{A2}+e_D^{B2})^3}\left(1 - \frac{e_P^{A1}+e_P^{A2}}{e_P^{A1}+e_P^{A2}+e_P^{B1}+e_P^{B2}} \frac{e_D^{A1}}{e_D^{A1}+e_D^{B1}}\right)$$

$$b = c = -\frac{e_D^{B2}}{(e_D^{A2}+e_D^{B2})^2} \frac{e_D^{A1}}{e_D^{A1}+e_D^{B1}} \frac{e_P^{B1}+e_P^{B2}}{(e_P^{A1}+e_P^{A2}+e_P^{B1}+e_P^{B2})^2}$$

$$d = -\frac{e_D^{B2}}{e_D^{A2}+e_D^{B2}} \frac{e_D^{A1}}{e_D^{A1}+e_D^{B1}} \frac{2(e_P^{B1}+e_P^{B2})}{(e_P^{A1}+e_P^{A2}+e_P^{B1}+e_P^{B2})^3}$$

The determinant of the matrix, calculated as

$$ad - bc = \frac{3(e_D^{B2})^2(e_D^{A1})^2(e_P^{B1}+e_P^{B2})^2 + 4(e_D^{B2})^2 e_D^{A1} e_D^{B1}(e_P^{B1}+e_P^{B2})(e_P^{A1}+e_P^{A2}+e_P^{B1}+e_P^{B2})}{(e_D^{A2}+e_D^{B2})^4(e_D^{A1}+e_D^{B1})^2(e_P^{A1}+e_P^{A2}+e_P^{B1}+e_P^{B2})^4}$$

For $e_D^i \geq 0$ and $e_P^i \geq 0$, $a \leq 0$, $d \leq 0$ and $ad - bc \geq 0$, the Hessian of the objective function is negative semi-definite over the feasible region, confirming that the objective function is concave in this region. Since the set defined by the constraint $e_D^{A1} + e_P^{A1} \leq 1$ is convex, any point satisfying KT conditions is a global constrained maximum. An analogous derivation holds for candidates in party B. Combining the conditions for all four candidates yields the full set of KT constraints characterising an equilibrium. Imposing party symmetry then allows us to simplify these conditions to the following form:

$$\frac{1}{8e_D^1} - \lambda^1 + \mu_D^1 = 0 \tag{2.20}$$

$$\frac{1}{8(e_P^1 + e_P^2)} - \lambda^1 + \mu_P^1 = 0 \tag{2.21}$$

$$\frac{3}{16e_D^2} - \lambda^2 + \mu_D^2 = 0 \tag{2.22}$$

$$\frac{1}{16(e_P^1 + e_P^2)} - \lambda^2 + \mu_P^2 = 0 \tag{2.23}$$

$$\lambda^1 \geq 0, \quad e_D^1 + e_P^1 \leq 1, \quad \lambda^1(e_D^1 + e_P^1 - 1) = 0 \tag{2.24}$$

$$\lambda^2 \geq 0, \quad e_D^2 + e_P^2 \leq 1, \quad \lambda^2(e_D^2 + e_P^2 - 1) = 0 \tag{2.25}$$

$$e_D^1 \geq 0, \quad \mu_D^1 \geq 0, \quad \mu_D^1 e_D^1 = 0 \tag{2.26}$$

$$e_P^1 \geq 0, \quad \mu_P^1 \geq 0, \quad \mu_P^1 e_P^1 = 0 \tag{2.27}$$

$$e_D^2 \geq 0, \quad \mu_D^2 \geq 0, \quad \mu_D^2 e_D^2 = 0 \tag{2.28}$$

$$e_P^2 \geq 0, \quad \mu_P^2 \geq 0, \quad \mu_P^2 e_P^2 = 0 \tag{2.29}$$

The function form restricts cases where $e_D^1 = 0$, $e_D^2 = 0$ or $e_P^1 = e_P^2 = 0$. Thus, we have $\lambda^1 > 0$, $\lambda^2 > 0$ and $\mu_D^1 = \mu_D^2 = 0$. We need to examine a total of 3 possible cases: (i) $\mu_P^1 = \mu_P^2 = 0$ (ii) $\mu_P^1 = 0$ and $e_P^2 = 0$ (iii) $\mu_P^2 = 0$ and $e_P^1 = 0$. Let's first consider Case (i). Under this scenario, the KT conditions simplify to:

$$\frac{1}{8e_D^1} - \lambda^1 = 0 \tag{2.30}$$

$$\frac{1}{8(e_P^1 + e_P^2)} - \lambda^1 = 0 \tag{2.31}$$

$$\frac{3}{16e_D^2} - \lambda^2 = 0 \tag{2.32}$$

$$\frac{1}{16(e_P^1 + e_P^2)} - \lambda^2 = 0 \tag{2.33}$$

$$e_D^1 + e_P^1 = 1, \quad e_D^2 + e_P^2 = 1 \tag{2.34}$$

$$e_D^1 > 0, \quad e_P^1 \geq 0, \quad e_D^2 > 0, \quad e_P^2 \geq 0 \tag{2.35}$$

The system of equations solves $e_D^1 = \frac{2}{5}$, $e_D^2 = \frac{6}{5}$, $e_P^1 = \frac{3}{5}$ and $e_P^2 = -\frac{1}{5} < 0$. This result contradicts the condition $e_P^2 \geq 0$; no solutions therefore satisfy Case (i).

Next, we consider Case (ii). Under this scenario, the KT conditions simplify to:

$$\frac{1}{8e_D^1} - \lambda^1 = 0 \tag{2.36}$$

$$\frac{1}{8e_P^1} - \lambda^1 = 0 \tag{2.37}$$

$$\frac{3}{16e_D^2} - \lambda^2 = 0 \tag{2.38}$$

$$\frac{1}{16e_P^1} - \lambda^2 + \mu_P^2 = 0 \tag{2.39}$$

$$e_D^1 + e_P^1 = 1, \quad e_D^2 = 1 \tag{2.40}$$

$$e_D^1 > 0, \quad e_P^1 \geq 0, \quad \mu_P^2 \geq 0 \tag{2.41}$$

Equations (2.36) and (2.37) imply that $e_D^1 = e_P^1 = \frac{1}{2}$. Solving (2.38) gives $\lambda^2 = \frac{3}{16}$. Substituting into (2.39), we find $\mu_P^2 = -\frac{1}{8} + \frac{3}{16} = \frac{1}{16} > 0$. Therefore, all conditions are satisfied, and the strategy profile $(e_D^1 = e_P^1 = \frac{1}{2}, e_D^2 = 1, e_P^2 = 0)$ is indeed a Nash equilibrium. Finally, we examine Case (iii), under this scenario, the following KT conditions must hold:

$$\frac{1}{8e_D^1} - \lambda^1 = 0 \tag{2.42}$$

$$\frac{1}{8e_P^2} - \lambda^1 + \mu_P^1 = 0 \tag{2.43}$$

$$\frac{3}{16e_D^2} - \lambda^2 = 0 \tag{2.44}$$

$$\frac{1}{16e_P^2} - \lambda^2 = 0 \tag{2.45}$$

$$e_D^1 = 1, \quad e_D^2 + e_P^2 = 1 \tag{2.46}$$

$$\mu_P^1 \geq 0, \quad e_D^2 > 0, \quad e_P^2 \geq 0 \tag{2.47}$$

Solving (2.42) gives $\lambda^1 = \frac{1}{8}$. Further, solving (2.44) and (2.45) gives $e_P^2 = \frac{1}{4}$. This results in $\mu_P^1 = \frac{1}{8} - \frac{1}{2} < 0$, which contradicts the condition $\mu_P^1 \geq 0$. Thus, this cannot constitute an equilibrium. The last step to confirm the strategy profile $(e_D^1 = e_P^1 = \frac{1}{2}, e_D^2 = 1, e_P^2 = 0)$ is the unique party-symmetric equilibrium is to check whether $e_D^1 = 0$, $e_D^2 = 0$ or $e_P^1 = e_P^2 = 0$ could be part of an equilibrium. If $e_D^1 = 0$ or $e_D^2 = 0$, candidates would have an incentive to allocate slightly more to the district battle to win a prize with probability 1. Similarly, if $e_P^1 = e_P^2 = 0$, the rank-1 candidates would be incentivised to allocate slightly more to the party battle to guarantee a prize. Thus, none of these cases can be part of an equilibrium. We conclude that the strategy profile $(e_D^1 = e_P^1 = \frac{1}{2}, e_D^2 = 1, e_P^2 = 0)$ is indeed the unique party-symmetric equilibrium.

Appendix 2B. Proof of Proposition 2

We follow a similar procedure as in Proposition 1, first applying symmetry to simplify the KT conditions:

$$\frac{e_D^2}{2(e_D^1 + e_D^2)^2} - \lambda^1 + \mu_D^1 = 0 \tag{2.48}$$

$$\frac{e_D^2}{e_D^1+e_D^2}\frac{1}{4(e_P^1+e_P^2)}-\lambda^1+\mu_P^1=0 \tag{2.49}$$

$$\frac{3e_D^1}{4(e_D^1+e_D^2)^2}-\lambda^2+\mu_D^2=0 \tag{2.50}$$

$$\frac{e_D^1}{e_D^1+e_D^2}\frac{1}{8(e_P^1+e_P^2)}-\lambda^2+\mu_P^2=0 \tag{2.51}$$

$$\lambda^1\geq 0,\quad e_D^1+e_P^1\leq 1,\quad \lambda^1(e_D^1+e_P^1-1)=0 \tag{2.52}$$

$$\lambda^2\geq 0,\quad e_D^2+e_P^2\leq 1,\quad \lambda^2(e_D^2+e_P^2-1)=0 \tag{2.53}$$

$$\mu_D^1\geq 0,\quad e_D^1\geq 0,\quad e_D^1\mu_D^1=0 \tag{2.54}$$

$$\mu_P^1\geq 0,\quad e_P^1\geq 0,\quad e_P^1\mu_P^1=0 \tag{2.55}$$

$$\mu_D^2\geq 0,\quad e_D^2\geq 0,\quad e_D^2\mu_D^2=0 \tag{2.56}$$

$$\mu_P^2\geq 0,\quad e_P^2\geq 0,\quad e_P^2\mu_P^2=0 \tag{2.57}$$

The allocation profile constitutes an equilibrium if it satisfies all KT conditions above. We need to consider the following cases:

- **Case (i):** $\mu_D^1=0$, $\mu_P^1=0$, $\mu_D^2=0$ and $\mu_P^2=0$, this implies $\lambda^1>0$ and $\lambda^2>0$.
- **Case (ii):** $e_D^1=0$, $\mu_P^1=0$, $\mu_D^2=0$, and $\mu_P^2=0$, this implies $\lambda^1>0$ and $\lambda^2=0$.
- **Case (iii):** $\mu_D^1=0$, $e_P^1=0$, $\mu_D^2=0$, and $\mu_P^2=0$, this implies $\lambda^1>0$ and $\lambda^2>0$.
- **Case (iv):** $\mu_D^1=0$, $\mu_P^1=0$, $e_D^2=0$, and $\mu_P^2=0$, this implies $\lambda^1=0$ and $\lambda^2>0$.
- **Case (v):** $\mu_D^1=0$, $\mu_P^1=0$, $\mu_D^2=0$, and $e_P^2=0$, this implies $\lambda^1>0$ and $\lambda^2>0$.
- **Case (vi):** $e_D^1=0$, $\mu_P^1=0$, $\mu_D^2=0$, and $e_P^2=0$, this implies $\lambda^1>0$ and $\lambda^2=0$.
- **Case (vii):** $\mu_D^1=0$, $e_P^1=0$, $e_D^2=0$, and $\mu_P^2=0$, this implies $\lambda^1=0$ and $\lambda^2>0$.

We will examine each of these cases individually. For Case (i), solving equations (2.48) and (2.49) gives $\frac{1}{e_D^1+e_D^2}=\frac{1}{2(e_P^1+e_P^2)}$, while equations (2.50) and (2.51) give $\frac{3}{e_D^1+e_D^2}=\frac{1}{2(e_P^1+e_P^2)}$, resulting in a clear contradiction. For Case (ii), we solve for $\lambda^1=\frac{1}{4(1+e_P^2)}$, which leads to $\mu_D^1=\lambda^1-\frac{1}{2e_D^2}<0$, contradicting the condition $\mu_D^1\geq 0$. In Case (iii), solving equations (2.50) and (2.51) gives $e_D^2=\frac{5}{7}$. However, the condition $\mu_P^1>0$ requires $e_D^2<\frac{1}{3}$, leading to a contradiction. In Case (iv), we first solve for $\lambda^2=\frac{1}{8(1+e_P^1)}$, then calculate $\mu_D^2=\lambda^2-\frac{3}{4e_D^1}<0$, contradicting $\mu_D^2\geq 0$. In Case (v), solving equations (2.48) and (2.49) gives $e_D^1=\frac{1}{3}$, and we calculate $\lambda^2=\frac{9}{64}$. Checking further, we find $\mu_P^2=\lambda^2-\frac{e_D^1}{e_D^1+e_D^2}\frac{1}{8e_P^1}>0$. Thus, all KT conditions are satisfied, and the allocation profile $(e_D^1=\frac{1}{3},e_P^1=\frac{2}{3},e_D^2=1,e_P^2=0)$ is indeed a NE. For Case (vi), solving for $\lambda^1=\frac{1}{4}$ yields $\mu_D^1=\lambda^1-\frac{1}{2e_D^2}<0$, contradicting $\mu_D^1\geq 0$. Lastly, in Case (vii), calculating $\lambda^2=\frac{1}{8}$ gives $\mu_D^2=\lambda^2-\frac{3}{4e_D^1}<0$, which contradicts $\mu_D^2\geq 0$.

The last step to confirm the strategy profile $(e_D^1 = \frac{1}{3}, e_P^1 = \frac{2}{3}, e_D^2 = 1, e_P^2 = 0)$ is the unique party-symmetric equilibrium is to check whether $e_D^1 = e_D^2 = 0$ or $e_P^1 = e_P^2 = 0$ can be supported as an equilibrium. Note that if $e_D^1 = e_D^2 = 0$, candidates would always have an incentive to allocate slightly more to the district battle to secure a prize with probability 1. Similarly, if $e_P^1 = e_P^2 = 0$, the rank-1 candidates would be incentivised to allocate slightly more to the party battle to ensure a prize. Thus, these cases cannot constitute an equilibrium. We conclude that the strategy profile $(e_D^1 = \frac{1}{3}, e_P^1 = \frac{2}{3}, e_D^2 = 1, e_P^2 = 0)$ is indeed the unique party-symmetric equilibrium.

Appendix 2C. Proof of Proposition 3

To solve $A1$'s maximisation problem, we set up the Lagrangian expression as follows:

$$
\begin{aligned}
&L(e_D^{A1}, e_P^{A1}, \lambda^{A1}, \mu_D^{A1}, \mu_P^{A1}) = \\
&\frac{e_D^{A1}}{e_D^{A1} + e_D^{B1}} + \frac{e_D^{B1}}{e_D^{A1} + e_D^{B1}} \frac{e_P^{A1} + e_P^{A2}}{e_P^{A1} + e_P^{A2} + e_P^{B1} + e_P^{B2}} \left(\frac{e_D^{A2}}{e_D^{A2} + e_D^{B2}} + \frac{e_D^{B2}}{e_D^{A2} + e_D^{B2}} \frac{e_P^{A1}}{e_P^{A1} + e_P^{A2}} \right) \\
&-\lambda^{A1}(e_D^{A1} + e_P^{A1} - 1) + \mu_D^{A1} e_D^{A1} + \mu_P^{A1} e_P^{A1}
\end{aligned} \tag{2.58}
$$

Derive all KT conditions:

$$
\begin{aligned}
&\frac{e_D^{B1}}{(e_D^{A1} + e_D^{B1})^2} - \frac{e_D^{B1}}{(e_D^{A1} + e_D^{B1})^2} \frac{e_P^{A1} + e_P^{A2}}{e_P^{A1} + e_P^{A2} + e_P^{B1} + e_P^{B2}} \left(\frac{e_D^{A2}}{e_D^{A2} + e_D^{B2}} + \frac{e_D^{B2}}{e_D^{A2} + e_D^{B2}} \frac{e_P^{A1}}{e_P^{A1} + e_P^{A2}} \right) \\
&-\lambda^{A1} + \mu_D^{A1} = 0
\end{aligned} \tag{2.59}
$$

$$
\begin{aligned}
&\frac{e_D^{B1}}{e_D^{A1} + e_D^{B1}} \frac{e_P^{B1} + e_P^{B2}}{(e_P^{A1} + e_P^{A2} + e_P^{B1} + e_P^{B2})^2} \left(\frac{e_D^{A2}}{e_D^{A2} + e_D^{B2}} + \frac{e_D^{B2}}{e_D^{A2} + e_D^{B2}} \frac{e_P^{A1}}{e_P^{A1} + e_P^{A2}} \right) \\
&+ \frac{e_D^{B1}}{e_D^{A1} + e_D^{B1}} \frac{e_P^{A1} + e_P^{A2}}{e_P^{A1} + e_P^{A2} + e_P^{B1} + e_P^{B2}} \frac{e_D^{B2}}{e_D^{A2} + e_D^{B2}} \frac{e_P^{A2}}{(e_P^{A1} + e_P^{A2})^2} - \lambda^{A1} + \mu_P^{A1} = 0
\end{aligned} \tag{2.60}
$$

$$
e_D^{A1} + e_P^{A1} \leq 1, \quad \lambda^{A1} \geq 0, \quad (e_D^{A1} + e_P^{A1} - 1)\lambda^{A1} = 0 \tag{2.61}
$$

$$
e_D^{A1} \geq 0, \quad \mu_D^{A1} \geq 0, \quad \mu_D^{A1} e_D^{A1} = 0 \tag{2.62}
$$

$$
e_P^{A1} \geq 0, \quad \mu_P^{A1} \geq 0, \quad \mu_P^{A1} e_P^{A1} = 0 \tag{2.63}
$$

For $e_D^i \geq 0$ and $e_P^i \geq 0$, the Hessian of the objective function is negative semi-definite over the feasible region, confirming that the objective function is concave in this region. Since the set defined by the constraint $e_D^{A1} + e_P^{A1} \leq 1$ is convex, any point satisfying KT conditions is a global constrained maximum.

For an allocation profile to constitute a NE, all candidates must maximise their payoffs given the other candidates' strategies. That requires that the Kuhn-Tucker conditions for all candidates must be satisfied simultaneously. Therefore, a symmetric NE strategy profile must meet the following KT conditions:

$$\frac{5}{32e_D} - \lambda + \mu_D = 0 \tag{2.64}$$

$$\frac{1}{16e_P} - \lambda + \mu_P = 0 \tag{2.65}$$

$$\lambda \geq 0, \quad e_D + e_P \leq 1, \quad \lambda(e_D + e_P - 1) = 0 \tag{2.66}$$

$$e_D \geq 0, \quad \mu_D \geq 0, \quad \mu_D e_D = 0 \tag{2.67}$$

$$e_P \geq 0, \quad \mu_P \geq 0, \quad \mu_P e_P = 0 \tag{2.68}$$

Note that, given symmetry, the objective function excludes cases where $e_D = 0$ or $e_P = 0$. Solving for simultaneous equations gives $e_D = \frac{2}{3}$ and $e_P = \frac{1}{3}$. Therefore, the strategy profile $(e_D = \frac{2}{3}, e_P = \frac{1}{3})$ is indeed an equilibrium. To confirm this is the unique symmetric NE, we also need to check if $e_D = 0$ or $e_P = 0$ could be sustained as an equilibrium. In these cases, candidates would be incentivised to allocate slightly more to the district battle (for $e_D = 0$) or to the party battle (for $e_P = 0$), as this will increase their probability of winning to 1.

Appendix 2D. Proof of Proposition 4

To solve Party A's maximisation problem with symmetric pairings, we write the Lagrangian expression as follows:

$$\begin{aligned}
L(e_D^{A1}, e_P^{A1}, e_D^{A2}, e_P^{A2}, \lambda^{A1}, \lambda^{A2}, \mu_D^{A1}, \mu_P^{A1}, \mu_D^{A2}, \mu_P^{A2}) &= \frac{e_D^{A1}}{e_D^{A1} + e_D^{B1}} \frac{e_D^{A2}}{e_D^{A2} + e_D^{B2}} \\
&+ \frac{e_D^{B1}}{e_D^{A1} + e_D^{B1}} \frac{e_D^{A2}}{e_D^{A2} + e_D^{B2}} \frac{e_P^{A1} + e_P^{A2}}{e_P^{A1} + e_P^{A2} + e_P^{B1} + e_P^{B2}} \\
&+ \frac{e_D^{A1}}{e_D^{A1} + e_D^{B1}} \frac{e_D^{B2}}{e_D^{A2} + e_D^{B2}} \frac{e_P^{A1} + e_P^{A2}}{e_P^{A1} + e_P^{A2} + e_P^{B1} + e_P^{B2}} \\
&+ \lambda^{A1}(e_D^{A1} + e_P^{A1} - 1) + \lambda^{A2}(e_D^{A2} + e_P^{A2} - 1) + \mu_D^{A1} e_D^{A1} + \mu_P^{A1} e_P^{A1} + \mu_D^{A2} e_D^{A2} + \mu_P^{A2} e_P^{A2}
\end{aligned} \tag{2.69}$$

All KT conditions are calculated as:

$$\begin{aligned}
&\frac{e_D^{B1}}{(e_D^{A1} + e_D^{B1})^2} \frac{e_D^{A2}}{e_D^{A2} + e_D^{B2}} - \frac{e_D^{B1}}{(e_D^{A1} + e_D^{B1})^2} \frac{e_D^{A2}}{e_D^{A2} + e_D^{B2}} \frac{e_P^{A1} + e_P^{A2}}{e_P^{A1} + e_P^{A2} + e_P^{B1} + e_P^{B2}} \\
&+ \frac{e_D^{B1}}{(e_D^{A1} + e_D^{B1})^2} \frac{e_D^{B2}}{e_D^{A2} + e_D^{B2}} \frac{e_P^{A1} + e_P^{A2}}{e_P^{A1} + e_P^{A2} + e_P^{B1} + e_P^{B2}} - \lambda^{A1} + \mu_D^{A1} = 0
\end{aligned}$$

$$\begin{aligned}
&\frac{e_D^{B1}}{e_D^{A1} + e_D^{B1}} \frac{e_D^{A2}}{e_D^{A2} + e_D^{B2}} \frac{e_P^{B1} + e_P^{B2}}{(e_P^{A1} + e_P^{A2} + e_P^{B1} + e_P^{B2})^2} \\
&+ \frac{e_D^{A1}}{e_D^{A1} + e_D^{B1}} \frac{e_D^{B2}}{e_D^{A2} + e_D^{B2}} \frac{e_P^{B1} + e_P^{B2}}{(e_P^{A1} + e_P^{A2} + e_P^{B1} + e_P^{B2})^2} - \lambda^{A1} + \mu_P^{A1} = 0
\end{aligned}$$

$$\frac{e_D^{A1}}{e_D^{A1}+e_D^{B1}}\frac{e_D^{B2}}{(e_D^{A2}+e_D^{B2})^2}+\frac{e_D^{B1}}{e_D^{A1}+e_D^{B1}}\frac{e_D^{B2}}{(e_D^{A2}+e_D^{B2})^2}\frac{e_P^{A1}+e_P^{A2}}{e_P^{A1}+e_P^{A2}+e_P^{B1}+e_P^{B2}}$$
$$-\frac{e_D^{A1}}{e_D^{A1}+e_D^{B1}}\frac{e_D^{B2}}{(e_D^{A2}+e_D^{B2})^2}\frac{e_P^{A1}+e_P^{A2}}{e_P^{A1}+e_P^{A2}+e_P^{B1}+e_P^{B2}}-\lambda^{A2}+\mu_D^{A2}=0$$
$$\frac{e_D^{B1}}{e_D^{A1}+e_D^{B1}}\frac{e_D^{A2}}{e_D^{A2}+e_D^{B2}}\frac{e_P^{B1}+e_P^{B2}}{(e_P^{A1}+e_P^{A2}+e_P^{B1}+e_P^{B2})^2}$$
$$+\frac{e_D^{A1}}{e_D^{A1}+e_D^{B1}}\frac{e_D^{B2}}{e_D^{A2}+e_D^{B2}}\frac{e_P^{B1}+e_P^{B2}}{(e_P^{A1}+e_P^{A2}+e_P^{B1}+e_P^{B2})^2}-\lambda^{A2}+\mu_P^{A2}=0$$
$$\lambda^{A1}\geq 0,\quad e_D^{A1}+e_P^{A1}\leq 1,\quad \lambda^{A1}(e_D^{A1}+e_P^{A1}-1)=0$$
$$\lambda^{A2}\geq 0,\quad e_D^{A2}+e_P^{A2}\leq 1,\quad \lambda^{A2}(e_D^{A2}+e_P^{A2}-1)=0$$
$$e_D^{A1}\geq 0,\quad \mu_D^{A1}\geq 0,\quad \mu_D^{A1}e_D^{A1}=0$$
$$e_P^{A1}\geq 0,\quad \mu_P^{A1}\geq 0,\quad \mu_P^{A1}e_P^{A1}=0$$
$$e_D^{A2}\geq 0,\quad \mu_D^{A2}\geq 0,\quad \mu_D^{A2}e_D^{A2}=0$$
$$e_P^{A2}\geq 0,\quad \mu_P^{A2}\geq 0,\quad \mu_P^{A2}e_P^{A2}=0$$

Applying symmetry $\left(e_D^{A1}=e_D^{B1}=e_D^1, e_P^{A1}=e_P^{B1}=e_P^1, e_D^{A2}=e_D^{B2}=e_D^2, e_P^{A2}=e_P^{B2}=e_P^2\right)$ gives the following KT conditions:

$$\frac{1}{8e_D^1}-\lambda^1+\mu_D^1=0 \tag{2.70}$$
$$\frac{1}{8(e_P^1+e_P^2)}-\lambda^1+\mu_P^1=0 \tag{2.71}$$
$$\frac{1}{8e_D^2}-\lambda^2+\mu_D^2=0 \tag{2.72}$$
$$\frac{1}{8(e_P^1+e_P^2)}-\lambda^2+\mu_P^2=0 \tag{2.73}$$
$$\lambda^1\geq 0\quad e_D^1+e_P^1\leq 1\quad \lambda^1(e_D^1+e_P^1-1)=0 \tag{2.74}$$
$$\lambda^2\geq 0\quad e_D^2+e_P^2\leq 1\quad \lambda^2(e_D^2+e_P^2-1)=0 \tag{2.75}$$
$$e_D^1\geq 0\quad \mu_D^1\geq 0\quad \mu_D^1e_D^1=0 \tag{2.76}$$
$$e_P^1\geq 0\quad \mu_P^1\geq 0\quad \mu_P^1e_P^1=0 \tag{2.77}$$
$$e_D^2\geq 0\quad \mu_D^2\geq 0\quad \mu_D^2e_D^2=0 \tag{2.78}$$
$$e_P^2\geq 0\quad \mu_P^2\geq 0\quad \mu_P^2e_P^2=0 \tag{2.79}$$

When $\mu_D^1=\mu_P^1=\mu_D^2=\mu_P^2=0$, $\lambda>0$. The equations solve for $e_D^1=e_D^2=\frac{2}{3}$ and $e_P^1=e_P^2=\frac{1}{3}$. Thus, the strategy profile $(e_D^1=e_D^2=\frac{2}{3}, e_P^1=e_P^2=\frac{1}{3})$ satisfy all KT conditions. We further verify through the reduced two-dimensional problem that these are indeed the maximum points for each party, thus the profile is indeed a NE. We can further confirm that this is the only symmetric NE.[17] We repeat the same analysis for the highest-versus-lowest pairing. After applying symmetry, the KT conditions are as follows:

$$\frac{(e_D^2)^2 + e_D^1 e_D^2}{2(e_D^1 + e_D^2)^3} - \lambda^1 + \mu_D^1 = 0 \tag{2.80}$$

$$\frac{(e_D^2)^2 + (e_D^1)^2}{4(e_P^1 + e_P^2)(e_D^1 + e_D^2)^2} - \lambda^1 + \mu_P^1 = 0 \tag{2.81}$$

$$\frac{(e_D^1)^2 + e_D^1 e_D^2}{2(e_D^1 + e_D^2)^3} - \lambda^2 + \mu_D^2 = 0 \tag{2.82}$$

$$\frac{(e_D^2)^2 + (e_D^1)^2}{4(e_P^1 + e_P^2)(e_D^1 + e_D^2)^2} - \lambda^2 + \mu_P^2 = 0 \tag{2.83}$$

$$\lambda^1 \geq 0 \quad e_D^1 + e_P^1 \leq 1 \quad \lambda^1(e_D^1 + e_P^1 - 1) = 0 \tag{2.84}$$

$$\lambda^2 \geq 0 \quad e_D^2 + e_P^2 \leq 1 \quad \lambda^2(e_D^2 + e_P^2 - 1) = 0 \tag{2.85}$$

$$e_D^1 \geq 0 \quad \mu_D^1 \geq 0 \quad \mu_D^1 e_D^1 = 0 \tag{2.86}$$

$$e_P^1 \geq 0 \quad \mu_P^1 \geq 0 \quad \mu_P^1 e_P^1 = 0 \tag{2.87}$$

$$e_D^2 \geq 0 \quad \mu_D^2 \geq 0 \quad \mu_D^2 e_D^2 = 0 \tag{2.88}$$

$$e_P^2 \geq 0 \quad \mu_P^2 \geq 0 \quad \mu_P^2 e_P^2 = 0 \tag{2.89}$$

When $\mu_D^1 = \mu_P^1 = \mu_D^2 = \mu_P^2 = 0$, $\lambda > 0$. The equations solve for $e_D^1 = e_D^2$, we can further solve for $6(e_D^1)^3 = 12(e_D^1)^2(1 - e_D^1)$, thus $e_D^1 = 2(1 - e_D^1)$. This results in $e_D^1 = e_D^2 = \frac{2}{3}$ and $e_P^1 = e_P^2 = \frac{1}{3}$. Thus, the strategy profile $(e_D^1 = e_D^2 = \frac{2}{3}, e_P^1 = e_P^2 = \frac{1}{3})$ satisfy all KT conditions. We further verify through the reduced two-dimensional problem that these are indeed the maximum points for each party, thus the profile is indeed a NE. We can further confirm this is the only symmetric NE.

Appendix 2E. Proof of Proposition 5

In this section, we determine the equilibrium allocations for two pairings in the case of three candidates per party and a total of five seats. We begin with the symmetric pairing, where candidates of the same rank compete against each other in district battles. The objective function for candidate $A1$ is formulated as follows:

$$\max_{e_D^{A1}, e_P^{A1}} \left\{ \frac{e_D^{A1}}{e_D^{A1} + e_D^{B1}} + \frac{e_D^{B1}}{e_D^{A1} + e_D^{B1}} \left[\left(\frac{e_P^{A1} + e_P^{A2} + e_P^{A3}}{e_P^{A1} + e_P^{A2} + e_P^{A3} + e_P^{B1} + e_P^{B2} + e_P^{B3}} \right)^2 + \right. \right. \tag{2.90}$$

$$\left. \left. 2 \left(\frac{e_P^{A1} + e_P^{A2} + e_P^{A3}}{e_P^{A1} + e_P^{A2} + e_P^{A3} + e_P^{B1} + e_P^{B2} + e_P^{B3}} \right) \left(\frac{e_P^{B1} + e_P^{B2} + e_P^{B3}}{e_P^{A1} + e_P^{A2} + e_P^{A3} + e_P^{B1} + e_P^{B2} + e_P^{B3}} \right) \right] \right\}$$

$$s.t. \quad e_D^{A1} + e_P^{A1} \leq 1$$

[17] If we solve for the equivalent formulation where each candidate maximises the joint winning probability of themselves and their party partners, we can also solve for the same unique symmetric NE.

$$e_D^{A1} \geq 0 \quad e_P^{A1} \geq 0$$

When there are 5 prizes in total, there are 2 party-level prizes. Thus, the party-level resource allocations determine the probabilities that a party wins either 1 or 2 party-level prizes. As the top-ranked candidate, $A1$ can win the party prize as long as party A secures at least 1 party-level prize. The above calculation accounts for scenarios in which Party A wins both prizes or one of the two party-level prizes. By formulating the Lagrangian, we can derive the corresponding KT conditions as follows:

$$\begin{aligned} \frac{e_D^{B1}}{(e_D^{A1}+e_D^{B1})^2} - \frac{e_D^{B1}}{(e_D^{A1}+e_D^{B1})^2} \Bigg(\bigg(& \frac{e_P^{A1}+e_P^{A2}+e_P^{A3}}{e_P^{A1}+e_P^{A2}+e_P^{A3}+e_P^{B1}+e_P^{B2}+e_P^{B3}} \bigg)^2 \\ & + 2\left(\frac{e_P^{A1}+e_P^{A2}+e_P^{A3}}{e_P^{A1}+e_P^{A2}+e_P^{A3}+e_P^{B1}+e_P^{B2}+e_P^{B3}} \right) \\ & \times \left(\frac{e_P^{B1}+e_P^{B2}+e_P^{B3}}{e_P^{A1}+e_P^{A2}+e_P^{A3}+e_P^{B1}+e_P^{B2}+e_P^{B3}} \right) \Bigg) \\ & - \lambda^{A1} + \mu_D^{A1} = 0 \end{aligned} \tag{2.91}$$

$$\begin{aligned} \frac{e_D^{B1}}{e_D^{A1}+e_D^{B1}} \Bigg(& 2\frac{e_P^{A1}+e_P^{A2}+e_P^{A3}}{e_P^{A1}+e_P^{A2}+e_P^{A3}+e_P^{B1}+e_P^{B2}+e_P^{B3}} \cdot \frac{e_P^{B1}+e_P^{B2}+e_P^{B3}}{\left(e_P^{A1}+e_P^{A2}+e_P^{A3}+e_P^{B1}+e_P^{B2}+e_P^{B3}\right)^2} \\ & + 2\frac{e_P^{B1}+e_P^{B2}+e_P^{B3}}{\left(e_P^{A1}+e_P^{A2}+e_P^{A3}+e_P^{B1}+e_P^{B2}+e_P^{B3}\right)^2} \cdot \frac{e_P^{B1}+e_P^{B2}+e_P^{B3}}{e_P^{A1}+e_P^{A2}+e_P^{A3}+e_P^{B1}+e_P^{B2}+e_P^{B3}} \\ & - 2\frac{e_P^{A1}+e_P^{A2}+e_P^{A3}}{e_P^{A1}+e_P^{A2}+e_P^{A3}+e_P^{B1}+e_P^{B2}+e_P^{B3}} \cdot \frac{e_P^{B1}+e_P^{B2}+e_P^{B3}}{\left(e_P^{A1}+e_P^{A2}+e_P^{A3}+e_P^{B1}+e_P^{B2}+e_P^{B3}\right)^2} \\ & \Bigg) - \lambda^{A1} + \mu_P^{A1} = 0 \end{aligned} \tag{2.92}$$

$$e_D^{A1} + e_P^{A1} \leq 1, \quad \lambda^{A1} \geq 0, \quad (e_D^{A1} + e_P^{A1} - 1)\lambda^{A1} = 0 \tag{2.93}$$

$$e_D^{A1} \geq 0, \quad \mu_D^{A1} \geq 0, \quad \mu_D^{A1} e_D^{A1} = 0 \tag{2.94}$$

$$e_P^{A1} \geq 0, \quad \mu_P^{A1} \geq 0, \quad \mu_P^{A1} e_P^{A1} = 0 \tag{2.95}$$

Similarly, we can formulate the objective functions for the rank-2 candidate $A2$:

$$\begin{aligned} \max_{e_D^{A2}, e_P^{A2}} \Bigg\{ \frac{e_D^{A2}}{e_D^{A2}+e_D^{B2}} + \frac{e_D^{B2}}{e_D^{A2}+e_D^{B2}} \Bigg[& \left(\frac{e_P^{A1}+e_P^{A2}+e_P^{A3}}{e_P^{A1}+e_P^{A2}+e_P^{A3}+e_P^{B1}+e_P^{B2}+e_P^{B3}} \right)^2 \\ + 2\left(\frac{e_P^{A1}+e_P^{A2}+e_P^{A3}}{e_P^{A1}+e_P^{A2}+e_P^{A3}+e_P^{B1}+e_P^{B2}+e_P^{B3}} \right) & \left(\frac{e_P^{B1}+e_P^{B2}+e_P^{B3}}{e_P^{A1}+e_P^{A2}+e_P^{A3}+e_P^{B1}+e_P^{B2}+e_P^{B3}} \right) \\ \times \frac{e_D^{A1}}{e_D^{A1}+e_D^{B1}} \Bigg] \Bigg\} & \end{aligned} \tag{2.96}$$

$$s.t. \quad e_D^{A2} + e_P^{A2} \leq 1$$

$$e_D^{A2} \geq 0 \quad e_P^{A2} \geq 0$$

If $A2$ loses her district battle and if Party A wins both party-level seats, she is guaranteed a seat, as she is the top two candidates on the party list. However, if Party A wins only 1 seat, $A2$ can secure a seat only if $A1$ has already won her district battle. We proceed by formulating the Lagrangian expression and deriving the KT conditions:

$$\begin{aligned}\frac{e_D^{B2}}{(e_D^{A2}+e_D^{B2})^2} - \frac{e_D^{B2}}{(e_D^{A2}+e_D^{B2})^2} &\Bigg(\left(\frac{e_P^{A1}+e_P^{A2}+e_P^{A3}}{e_P^{A1}+e_P^{A2}+e_P^{A3}+e_P^{B1}+e_P^{B2}+e_P^{B3}}\right)^2 \\ &+ 2\left(\frac{e_P^{A1}+e_P^{A2}+e_P^{A3}}{e_P^{A1}+e_P^{A2}+e_P^{A3}+e_P^{B1}+e_P^{B2}+e_P^{B3}}\right) \\ &\times \left(\frac{e_P^{B1}+e_P^{B2}+e_P^{B3}}{e_P^{A1}+e_P^{A2}+e_P^{A3}+e_P^{B1}+e_P^{B2}+e_P^{B3}}\right) \\ &\times \frac{e_D^{A1}}{e_D^{A1}+e_D^{B1}}\Bigg) - \lambda^{A2} + \mu_D^{A2} = 0\end{aligned} \tag{2.97}$$

$$\begin{aligned}\frac{e_D^{B2}}{e_D^{A2}+e_D^{B2}} \Bigg(&2\frac{e_P^{A1}+e_P^{A2}+e_P^{A3}}{e_P^{A1}+e_P^{A2}+e_P^{A3}+e_P^{B1}+e_P^{B2}+e_P^{B3}} \cdot \frac{e_P^{B1}+e_P^{B2}+e_P^{B3}}{(e_P^{A1}+e_P^{A2}+e_P^{A3}+e_P^{B1}+e_P^{B2}+e_P^{B3})^2} \\ &+ 2\frac{e_P^{B1}+e_P^{B2}+e_P^{B3}}{(e_P^{A1}+e_P^{A2}+e_P^{A3}+e_P^{B1}+e_P^{B2}+e_P^{B3})^2} \cdot \frac{e_P^{B1}+e_P^{B2}+e_P^{B3}}{e_P^{A1}+e_P^{A2}+e_P^{A3}+e_P^{B1}+e_P^{B2}+e_P^{B3}} \\ &\times \frac{e_D^{A1}}{e_D^{A1}+e_D^{B1}} - 2\frac{e_P^{A1}+e_P^{A2}+e_P^{A3}}{e_P^{A1}+e_P^{A2}+e_P^{A3}+e_P^{B1}+e_P^{B2}+e_P^{B3}} \\ &\cdot \frac{e_P^{B1}+e_P^{B2}+e_P^{B3}}{(e_P^{A1}+e_P^{A2}+e_P^{A3}+e_P^{B1}+e_P^{B2}+e_P^{B3})^2} \cdot \frac{e_D^{A1}}{e_D^{A1}+e_D^{B1}}\Bigg) \\ &- \lambda^{A2} + \mu_P^{A2} = 0\end{aligned} \tag{2.98}$$

$$e_D^{A2} + e_P^{A2} \leq 1, \quad \lambda^{A2} \geq 0, \quad (e_D^{A2} + e_P^{A2} - 1)\lambda^{A2} = 0 \tag{2.99}$$

$$e_D^{A2} \geq 0, \quad \mu_D^{A2} \geq 0, \quad \mu_D^{A2} e_D^{A2} = 0 \tag{2.100}$$

$$e_P^{A2} \geq 0, \quad \mu_P^{A2} \geq 0, \quad \mu_P^{A2} e_P^{A2} = 0 \tag{2.101}$$

Similarly, we can apply the same process for the rank-3 candidate by first formulating candidate $A3$'s objective function:

$$\begin{aligned}\max_{e_D^{A3},e_P^{A3}} \Bigg\{ & \frac{e_D^{A3}}{e_D^{A3}+e_D^{B3}} + \frac{e_D^{B3}}{e_D^{A3}+e_D^{B3}} \Bigg[\left(\frac{e_P^{A1}+e_P^{A2}+e_P^{A3}}{e_P^{A1}+e_P^{A2}+e_P^{A3}+e_P^{B1}+e_P^{B2}+e_P^{B3}}\right)^2 \\ & \times \left(\frac{e_D^{A1}}{e_D^{A1}+e_D^{B1}}\cdot\frac{e_D^{A2}}{e_D^{A2}+e_D^{B2}} + \frac{e_D^{A1}}{e_D^{A1}+e_D^{B1}}\cdot\frac{e_D^{B2}}{e_D^{A2}+e_D^{B2}} + \frac{e_D^{B1}}{e_D^{A1}+e_D^{B1}}\cdot\frac{e_D^{A2}}{e_D^{A2}+e_D^{B2}}\right) \\ & +\ 2\left(\frac{e_P^{A1}+e_P^{A2}+e_P^{A3}}{e_P^{A1}+e_P^{A2}+e_P^{A3}+e_P^{B1}+e_P^{B2}+e_P^{B3}}\right) \\ & \times \left(\frac{e_P^{B1}+e_P^{B2}+e_P^{B3}}{e_P^{A1}+e_P^{A2}+e_P^{A3}+e_P^{B1}+e_P^{B2}+e_P^{B3}}\right)\cdot\frac{e_D^{A1}}{e_D^{A1}+e_D^{B1}}\cdot\frac{e_D^{A2}}{e_D^{A2}+e_D^{B2}}\Bigg]\Bigg\}\end{aligned} \tag{2.102}$$

$$s.t. \quad e_D^{A3}+e_P^{A3} \le 1$$

$$e_D^{A3} \ge 0 \quad e_P^{A3} \ge 0$$

Since $A3$ is the lowest-ranked candidate on party A's list, if she loses her district battle and party A wins both seats, she will win a seat as long as at least one of the higher-ranked candidates ($A1$ or $A2$) has won their district battle. However, if party A secures only one seat, $A3$ can claim it only if both $A1$ and $A2$ have already won their district battles. We then proceed to calculate the KT conditions:

$$\begin{aligned}\frac{e_D^{B3}}{(e_D^{A3}+e_D^{B3})^2} - \frac{e_D^{B3}}{(e_D^{A3}+e_D^{B3})^2}\Bigg(& \left(\frac{e_P^{A1}+e_P^{A2}+e_P^{A3}}{e_P^{A1}+e_P^{A2}+e_P^{A3}+e_P^{B1}+e_P^{B2}+e_P^{B3}}\right)^2 \\ & \times\left(\frac{e_D^{A1}e_D^{A2}+e_D^{A1}e_D^{B2}+e_D^{B1}e_D^{A2}}{(e_D^{A1}+e_D^{B1})(e_D^{A2}+e_D^{B2})}\right) \\ & +2\frac{e_P^{A1}+e_P^{A2}+e_P^{A3}}{e_P^{A1}+e_P^{A2}+e_P^{A3}+e_P^{B1}+e_P^{B2}+e_P^{B3}} \\ & \times\frac{e_P^{B1}+e_P^{B2}+e_P^{B3}}{e_P^{A1}+e_P^{A2}+e_P^{A3}+e_P^{B1}+e_P^{B2}+e_P^{B3}}\frac{e_D^{A1}e_D^{A2}}{(e_D^{A1}+e_D^{B1})(e_D^{A2}+e_D^{B2})}\Bigg) \\ & -\lambda^{A3}+\mu_D^{A3}=0\end{aligned} \tag{2.103}$$

$$\frac{e_D^{B3}}{e_D^{A3}+e_D^{B3}}\Bigg(2\frac{e_P^{A1}+e_P^{A2}+e_P^{A3}}{e_P^{A1}+e_P^{A2}+e_P^{A3}+e_P^{B1}+e_P^{B2}+e_P^{B3}}\frac{e_P^{B1}+e_P^{B2}+e_P^{B3}}{(e_P^{A1}+e_P^{A2}+e_P^{A3}+e_P^{B1}+e_P^{B2}+e_P^{B3})^2}$$
$$\times\left(\frac{e_D^{A1}e_D^{A2}}{(e_D^{A1}+e_D^{B1})(e_D^{A2}+e_D^{B2})}+\frac{e_D^{A1}e_D^{B2}}{(e_D^{A1}+e_D^{B1})(e_D^{A2}+e_D^{B2})}+\frac{e_D^{B1}e_D^{A2}}{(e_D^{A1}+e_D^{B1})(e_D^{A2}+e_D^{B2})}\right)$$
$$+2\frac{e_P^{B1}+e_P^{B2}+e_P^{B3}}{(e_P^{A1}+e_P^{A2}+e_P^{A3}+e_P^{B1}+e_P^{B2}+e_P^{B3})^2}\frac{e_P^{B1}+e_P^{B2}+e_P^{B3}}{e_P^{A1}+e_P^{A2}+e_P^{A3}+e_P^{B1}+e_P^{B2}+e_P^{B3}}$$
$$\times\frac{e_D^{A1}e_D^{A2}}{(e_D^{A1}+e_D^{B1})(e_D^{A2}+e_D^{B2})}-2\frac{e_P^{A1}+e_P^{A2}+e_P^{A3}}{e_P^{A1}+e_P^{A2}+e_P^{A3}+e_P^{B1}+e_P^{B2}+e_P^{B3}}$$
$$\times\frac{e_P^{B1}+e_P^{B2}+e_P^{B3}}{(e_P^{A1}+e_P^{A2}+e_P^{A3}+e_P^{B1}+e_P^{B2}+e_P^{B3})^2}\frac{e_D^{A1}e_D^{A2}}{(e_D^{A1}+e_D^{B1})(e_D^{A2}+e_D^{B2})}\Bigg)-\lambda^{A3}+\mu_P^{A3}=0 \tag{2.104}$$

$$e_D^{A3}+e_D^{A3}\leq 1,\quad \lambda^{A3}\geq 0,\quad (e_D^{A3}+e_P^{A3}-1)\lambda^{A3}=0 \tag{2.105}$$

$$e_D^{A3}\geq 0,\quad \mu_D^{A3}\geq 0,\quad \mu_D^{A3}e_D^{A3}=0 \tag{2.106}$$

$$e_P^{A3}\geq 0,\quad \mu_P^{A3}\geq 0,\quad \mu_P^{A3}e_P^{A3}=0 \tag{2.107}$$

Using between-party symmetry, we can characterise the necessary conditions a symmetric NE must satisfy:

$$\frac{1}{16e_D^1}-\lambda^1+\mu_D^1=0 \tag{2.108}$$

$$\frac{1}{8(e_P^1+e_P^2+e_P^3)}-\lambda^1+\mu_P^1=0 \tag{2.109}$$

$$e_D^1+e_P^1\leq 1,\quad \lambda^1\geq 0,\quad (e_D^1+e_P^1-1)\lambda^1=0 \tag{2.110}$$

$$\mu_D^1\geq 0,\quad e_D^1\geq 0,\quad \mu_D^1e_D^1=0 \tag{2.111}$$

$$\mu_P^1\geq 0,\quad e_P^1\geq 0,\quad \mu_P^1e_P^1=0 \tag{2.112}$$

$$\frac{1}{8e_D^2}-\lambda^2+\mu_D^2=0 \tag{2.113}$$

$$\frac{1}{8(e_P^1+e_P^2+e_P^3)}-\lambda^2+\mu_P^2=0 \tag{2.114}$$

$$e_D^2+e_P^2\leq 1,\quad \lambda^2\geq 0,\quad (e_D^2+e_P^2-1)\lambda^2=0 \tag{2.115}$$

$$\mu_D^2\geq 0,\quad e_D^2\geq 0,\quad \mu_D^2e_D^2=0 \tag{2.116}$$

$$\mu_P^2\geq 0,\quad e_P^2\geq 0,\quad \mu_P^2e_P^2=0 \tag{2.117}$$

$$\frac{11}{64e_D^3}-\lambda^3+\mu_D^3=0 \tag{2.118}$$

$$\frac{3}{32(e_P^1+e_P^2+e_P^3)}-\lambda^3+\mu_P^3=0 \tag{2.119}$$

$$e_D^3 + e_P^3 \leq 1, \quad \lambda^3 \geq 0, \quad (e_D^3 + e_P^3 - 1)\lambda^3 = 0 \tag{2.120}$$

$$\mu_D^3 \geq 0, \quad e_D^3 \geq 0, \quad \mu_D^3 e_D^3 = 0 \tag{2.121}$$

$$\mu_P^3 \geq 0, \quad e_P^3 \geq 0, \quad \mu_P^3 e_P^3 = 0 \tag{2.122}$$

Firstly, since e_D^1, e_D^2, and e_D^3 must all be greater than zero, all budget constraints are binding. We examine several potential equilibrium cases:

Case (1): $\mu_D^1 = 0$, $\mu_D^2 = 0$, $\mu_D^3 = 0$, $\mu_P^1 = 0$, $\mu_P^2 = 0$, and $e_P^3 = 0$, leading to $e_D^1 = \frac{2}{5}$, $e_D^2 = \frac{4}{5}$, $e_D^3 = 1$, $e_P^1 = \frac{3}{5}$, $e_P^2 = \frac{1}{5}$, $e_P^3 = 0$; with $\lambda^1 = \lambda^2 = \frac{5}{32}$, $\lambda^3 = \frac{11}{64}$, and $\mu_P^3 = \frac{7}{128}$.
Case (2): $\mu_D^1 = 0$, $\mu_D^2 = 0$, $\mu_D^3 = 0$, $\mu_P^1 = 0$, $\mu_P^2 = 0$, and $\mu_P^3 = 0$, leading to $e_D^1 = \frac{9}{26}$, $e_D^2 = \frac{18}{26}$, and $e_D^3 = \frac{33}{26} > 1$, which is a contradiction.
Case (3): $\mu_D^1 = 0$, $\mu_D^2 = 0$, $\mu_D^3 = 0$, $e_P^1 = 0$, $\mu_P^2 = 0$, and $\mu_P^3 = 0$, resulting in $e_D^2 = \frac{12}{23}$, $e_D^3 = \frac{22}{23}$, $\lambda^1 = \frac{1}{16}$, and $\mu_P^1 < 0$, which is a contradiction.
Case (4): $\mu_D^1 = 0$, $\mu_D^2 = 0$, $\mu_D^3 = 0$, $\mu_P^1 = 0$, $e_P^2 = 0$, and $\mu_P^3 = 0$, yielding $e_D^1 = \frac{3}{10}$, $e_D^3 = \frac{11}{10} > 1$, which is a contradiction.
Case (5): $\mu_D^1 = 0$, $\mu_D^2 = 0$, $\mu_D^3 = 0$, $e_P^1 = 0$, $e_P^2 = 0$, and $\mu_P^3 = 0$, leading to $e_D^3 = \frac{11}{17}$, $e_P^3 = \frac{6}{17}$, $\lambda^1 = \frac{1}{16}$, $\lambda^2 = \frac{1}{8}$, and $\mu_P^1 < 0$, which is a contradiction.
Case (6): $\mu_D^1 = 0$, $\mu_D^2 = 0$, $\mu_D^3 = 0$, $e_P^1 = 0$, $\mu_P^2 = 0$, and $e_P^3 = 0$, implying $e_D^2 = e_P^2 = \frac{1}{2}$, $\lambda^1 = \frac{1}{16}$, and $\mu_P^1 < 0$, which is a contradiction.
Case (7): $\mu_D^1 = 0$, $\mu_D^2 = 0$, $\mu_D^3 = 0$, $\mu_P^1 = 0$, $e_P^2 = 0$, and $e_P^3 = 0$, resulting in $e_D^1 = \frac{1}{3}$, $e_P^1 = \frac{2}{3}$, $\lambda^2 = \frac{1}{8}$, and $\mu_P^2 < 0$, which is also a contradiction.

Overall, only the first case satisfies all KT conditions. Since we cannot establish concavity of the objective functions over the entire feasible region, this alone does not guarantee that the proposed profile is a Nash equilibrium. To verify optimality, given others' allocations in the proposed profile, we note that each agent exhausts their budget in equilibrium, which reduces the problem to a one-dimensional optimisation. In this reduced form, the objective function is strictly increasing below the equilibrium allocation and strictly decreasing above it, ensuring that the proposed allocation is the unique team-symmetric equilibrium.

The next step is to determine the symmetric NE for the alternative pairing, in which the rank-2 candidates compete against the rank-3 candidates. By applying between-party symmetry, we can characterise necessary conditions required for a symmetric NE:

$$\frac{1}{16e_D^1} - \lambda^1 + \mu_D^1 = 0 \tag{2.123}$$

$$\frac{1}{8(e_P^1 + e_P^2 + e_P^3)} - \lambda^1 + \mu_P^1 = 0 \tag{2.124}$$

$$e_D^1 + e_P^1 \leq 1, \quad \lambda^1 \geq 0, \quad (e_D^1 + e_P^1 - 1)\lambda^1 = 0 \tag{2.125}$$

$$\mu_D^1 \geq 0, \quad e_D^1 \geq 0, \quad \mu_D^1 e_D^1 = 0 \tag{2.126}$$

$$\mu_P^1 \geq 0, \quad e_P^1 \geq 0, \quad \mu_P^1 e_P^1 = 0 \tag{2.127}$$

$$\frac{e_D^3}{2(e_D^2 + e_D^3)^2} - \lambda^2 + \mu_D^2 = 0 \tag{2.128}$$

$$\frac{e_D^3}{e_D^2 + e_D^3} \frac{1}{4(e_P^1 + e_P^2 + e_P^3)} - \lambda^2 + \mu_P^2 = 0 \tag{2.129}$$

$$e_D^2 + e_P^2 \leq 1, \quad \lambda^2 \geq 0, \quad (e_D^2 + e_P^2 - 1)\lambda^2 = 0 \tag{2.130}$$

$$\mu_D^2 \geq 0, \quad e_D^2 \geq 0, \quad \mu_D^2 e_D^2 = 0 \tag{2.131}$$

$$\mu_P^2 \geq 0, \quad e_P^2 \geq 0, \quad \mu_P^2 e_P^2 = 0 \tag{2.132}$$

$$\frac{e_D^2}{(e_D^2 + e_D^3)^2} \left(\frac{7}{8} - \frac{3}{8} \frac{e_D^2}{e_D^2 + e_D^3} \right) - \lambda^3 + \mu_D^3 = 0 \tag{2.133}$$

$$\frac{e_D^2(2e_D^2 + e_D^3)}{2(e_D^2 + e_D^3)^2} \frac{1}{4(e_P^1 + e_P^2 + e_P^3)} - \lambda^3 + \mu_P^3 = 0 \tag{2.134}$$

$$e_D^3 + e_P^3 \leq 1, \quad \lambda^3 \geq 0, \quad (e_D^3 + e_P^3 - 1)\lambda^3 = 0 \tag{2.135}$$

$$\mu_D^3 \geq 0, \quad e_D^3 \geq 0, \quad \mu_D^3 e_D^3 = 0 \tag{2.136}$$

$$\mu_P^3 \geq 0, \quad e_P^3 \geq 0, \quad \mu_P^3 e_P^3 = 0 \tag{2.137}$$

We examine several potential cases:

Case (1): $\mu_D^1 = 0$, $\mu_D^2 = 0$, $\mu_D^3 = 0$, $\mu_P^1 = 0$, $\mu_P^2 = 0$, and $e_P^3 = 0$, leading to $e_D^1 = \frac{3}{7}$, $e_D^2 = \frac{5}{7}$, $e_D^3 = 1$, $e_P^1 = \frac{4}{7}$, $e_P^2 = \frac{2}{7}$, $e_P^3 = 0$, with $\lambda^3 = \frac{805}{4608}$ and $\mu_P^3 > 0$. This configuration holds as a potential equilibrium.
Case (2): $\mu_D^1 = 0$, $\mu_D^2 = 0$, $\mu_D^3 = 0$, $\mu_P^1 = 0$, $\mu_P^2 = 0$, and $\mu_P^3 = 0$, leading to $e_D^1 = \frac{3}{7}$, $e_D^2 + e_D^3 = \frac{12}{7}$, and $e_P^1 + e_P^2 + e_P^3 = \frac{6}{7}$. However, no values of $e_D^2 > 0$ and $e_D^3 > 0$ satisfy these conditions simultaneously, so this case does not yield a solution.
Case (3): $\mu_D^1 = 0$, $\mu_D^2 = 0$, $\mu_D^3 = 0$, $e_P^1 = 0$, $\mu_P^2 = 0$, and $\mu_P^3 = 0$, leading to $e_D^2 = \frac{12}{23}$, $e_D^3 = \frac{22}{23}$, $\lambda^1 = \frac{1}{16}$, and $\mu_P^1 < 0$, which is a contradiction.
Case (4): $\mu_D^1 = 0$, $\mu_D^2 = 0$, $\mu_D^3 = 0$, $\mu_P^1 = 0$, $e_P^2 = 0$, and $\mu_P^3 = 0$, leading to $e_D^1 = \frac{3}{10}$, $e_D^3 = \frac{11}{10} > 1$, which is a contradiction.
Case (5): $\mu_D^1 = 0$, $\mu_D^2 = 0$, $\mu_D^3 = 0$, $e_P^1 = 0$, $e_P^2 = 0$, and $\mu_P^3 = 0$, resulting in $e_D^3 = \frac{11}{17}$, $e_P^3 = \frac{6}{17}$, $\lambda^1 = \frac{1}{16}$, $\lambda^2 = \frac{1}{8}$, and $\mu_P^1 < 0$, which is a contradiction.
Case (6): $\mu_D^1 = 0$, $\mu_D^2 = 0$, $\mu_D^3 = 0$, $e_P^1 = 0$, $\mu_P^2 = 0$, and $e_P^3 = 0$, implying $e_D^2 = e_P^2 = \frac{1}{2}$, $\lambda^1 = \frac{1}{16}$, and $\mu_P^1 < 0$, which is a contradiction.
Case (7): $\mu_D^1 = 0$, $\mu_D^2 = 0$, $\mu_D^3 = 0$, $\mu_P^1 = 0$, $e_P^2 = 0$, and $e_P^3 = 0$, resulting in $e_D^1 = \frac{1}{3}$, $e_P^1 = \frac{2}{3}$, $\lambda^2 = \frac{1}{8}$, and $\mu_P^2 < 0$, which is also a contradiction.
Case (8): $\mu_D^1 = 0$, $e_D^2 = 0$, $\mu_D^3 = 0$, $\mu_P^1 = 0$, $\mu_P^2 = 0$, and $\mu_P^3 = 0$. This configuration requires that $\lambda^3 = 0$. Further calculations reveal that no positive values of e_P^1 and e_P^3 satisfy the remaining constraints, so this case does not yield a solution.
Case (9): $\mu_D^1 = 0$, $\mu_D^2 = 0$, $e_D^3 = 0$, $\mu_P^1 = 0$, $\mu_P^2 = 0$, and $\mu_P^3 = 0$, also does not yield a solution upon further examination of constraints for positive values.

After examining all cases, only Case (1) satisfies all KT conditions. We further confirm that this is indeed a NE by examining the reduced one-dimensional optimisation problem as previously.

In an open list, we characterise necessary conditions required for a symmetric NE:

$$\frac{23}{192e_D} - \lambda + \mu_D = 0 \tag{2.138}$$

$$\frac{53}{576e_P} - \lambda + \mu_P = 0 \tag{2.139}$$

$$e_D + e_P \leq 1, \quad \lambda \geq 0, \quad (e_D + e_P - 1)\lambda = 0 \tag{2.140}$$

$$\mu_D \geq 0, \quad e_D \geq 0, \quad \mu_D e_D = 0 \tag{2.141}$$

$$\mu_P \geq 0, \quad e_P \geq 0, \quad \mu_P e_P = 0 \tag{2.142}$$

Solving for the systems of equations give $e_D = \frac{69}{122}$ and $e_P = \frac{53}{122}$.

Appendix 2F. More candidates and prizes

We will start with the closed list model, examining symmetric pairing and the scenario with one party-level prize. Assume there are N candidates per party, indexed by rank i. The symmetric equilibrium must satisfy all of the equations below:

$$\frac{1}{4e_D^i}\left(1 - \left(\frac{1}{2}\right)^i\right) - \lambda^i + \mu_D^i = 0, \quad \forall i \in \{1, \ldots, N\} \tag{2.143}$$

$$\left(\frac{1}{2}\right)^i \frac{1}{4(e_P^1 + e_P^2 + \cdots + e_P^N)} - \lambda^i + \mu_P^i = 0, \quad \forall i \in \{1, \ldots, N\} \tag{2.144}$$

$$e_D^i + e_P^i \leq 1, \quad \lambda^i \geq 0, \quad \lambda^i(e_D^i + e_P^i - 1) = 0, \quad \forall i \in \{1, \ldots, N\} \tag{2.145}$$

$$e_D^i \geq 0, \quad \mu_D^i \geq 0, \quad e_D^i \mu_D^i = 0, \quad \forall i \in \{1, \ldots, N\} \tag{2.146}$$

$$e_P^i \geq 0, \quad \mu_P^i \geq 0, \quad e_P^i \mu_P^i = 0, \quad \forall i \in \{1, \ldots, N\} \tag{2.147}$$

We want to prove that an allocation profile, in which only the highest-ranked candidate contributes positively to the party battle while all other candidates allocate all their resources to district battles, is a NE. Specifically, we check the following cases: $e_D^1 > 0$, $e_D^i = 1 \quad \forall i \in \{2, ..., N\}$, $e_P^1 > 0$, $e_P^i = 0 \quad \forall i \in \{2, ..., N\}$, $\mu_D^1 = 0$, $\mu_D^i = 0 \quad \forall i \in \{2, ..., N\}$, $\mu_P^i = 0$ and $\mu_P^i > 0 \quad \forall i \in \{2, ..., N\}$. Solving these conditions gives $e_D^1 = e_P^1 = \frac{1}{2}$. Next, we need to check if $\mu_P^i > 0 \quad \forall i \in \{2, ..., N\}$ holds. We calculate as follows: $\mu_P^i = \frac{1}{4}\left(1 - \left(\frac{1}{2}\right)^i\right) - \left(\frac{1}{2}\right)^i \left(\frac{1}{2}\right) \quad \forall i \in \{2, ..., N\}$. The first part of this expression is increasing in i, while the second part is decreasing in i. This means that the overall expression for μ_P^i is increasing in i. Thus, if we can show that $\mu_P^2 \geq 0$, that also implies $\mu_P^i \geq 0$ for all $i > 2$. We then calculate $\mu_P^2 = \lambda^2 - \frac{1}{8} = \frac{3}{16} - \frac{1}{8} > 0$. We further check that agents have no profitable deviations given the proposed allocation using the reduced one-dimensional optimisation problem. This confirms that the allocation profile in which the rank-1 candidates splits resources evenly between both battles, while all other candidates allocate 1 to their district battles, is indeed a NE.

We will use a similar procedure with the highest-versus-lowest pairing. In this case, the symmetric equilibrium must satisfy the following equations:

$$\frac{1}{2}\frac{e_D^N}{(e_D^1 + e_D^N)^2} - \lambda^1 + \mu_D^1 = 0 \tag{2.148}$$

$$\frac{e_D^N}{e_D^1 + e_D^N}\frac{1}{4(e_P^1 + e_P^2 + \cdots + e_P^N)} - \lambda^1 + \mu_P^1 = 0 \tag{2.149}$$

$$\frac{1}{4e_D^i}\left(1 - \left(\frac{1}{2}\right)^{i-1}\frac{e_D^1}{e_D^1 + e_D^N}\right) - \lambda^i + \mu_D^i = 0, \quad \forall i \in \{2, \ldots, N-1\} \tag{2.150}$$

$$\frac{1}{4(e_P^1 + e_P^2 + \cdots + e_P^N)}\frac{e_D^1}{e_D^1 + e_D^N}\left(\frac{1}{2}\right)^{i-1} - \lambda^i + \mu_P^i = 0, \quad \forall i \in \{2, \ldots, N-1\} \tag{2.151}$$

$$\frac{e_D^1}{(e_D^1 + e_D^N)^2}\left(1 - \left(\frac{1}{2}\right)^{N-1}\frac{e_D^1}{e_D^1 + e_D^N}\right) - \lambda^N + \mu_D^N = 0 \tag{2.152}$$

$$\frac{1}{4(e_P^1 + e_P^2 + \cdots + e_P^N)}\left(\frac{e_D^1}{e_D^1 + e_D^N}\right)^2\left(\frac{1}{2}\right)^{N-2} - \lambda^N + \mu_P^N = 0 \tag{2.153}$$

To prove that the allocation profile where only the highest-ranked candidate contributes positively to the party battle, while all other candidates allocate all resources to their district battles, is a NE, we consider the following cases: $e_D^1 > 0$, $e_D^i = 1 \quad \forall i \in \{2, ..., N\}$, $e_P^1 > 0$, $e_P^i = 0 \quad \forall i \in \{2, ..., N\}$, $\mu_D^1 = 0$, $\mu_D^i = 0 \quad \forall i \in \{2, ..., N\}$, $\mu_P^i = 0$ and $\mu_P^i > 0 \quad \forall i \in \{2, ..., N\}$. Solving these conditions yields $e_D^1 = \frac{1}{3}$ and $e_P^1 = \frac{2}{3}$. Note that e_P^1 is larger than in the symmetric pairing case. Next, we verify that $\mu_P^i > 0 \quad \forall i \in \{2, ..., N\}$ holds. We calculate $\mu_P^i = \frac{1}{4}\left(1 - \left(\frac{1}{2}\right)^{i-1}\frac{1}{4}\right) - \left(\frac{1}{2}\right)^{i-1}\left(\frac{1}{4}\right)\left(\frac{3}{8}\right) \quad \forall i \in \{2, ..., N-1\}$. Here, the first part of the expression is increasing in i and the second part is decreasing in i, making the whole expression for μ_P^i increasing in i. If we can show that $\mu_P^2 \geq 0$, it follows that $\mu_P^i \geq 0$ for all $i > 2$ and $i < N$. We calculate $\mu_P^2 = \lambda^2 - \frac{3}{64} = \frac{7}{32} - \frac{3}{64} > 0$. Additionally, we need to calculate $\mu_P^N = \lambda^N - \left(\frac{3}{8}\right)\left(\frac{1}{4}\right)^2\left(\frac{1}{2}\right)^{N-2} = \frac{3}{16} - \left(\frac{3}{16}\right)\left(\frac{1}{4}\right)\left(\frac{1}{2}\right)^{N-1} - \left(\frac{3}{8}\right)\left(\frac{1}{4}\right)^2\left(\frac{1}{2}\right)^{N-2}$. This simplifies to $\frac{3}{16} - \frac{3}{16}\left(\frac{1}{2}\right)^N > 0$. We further check that agents have no profitable deviations given the proposed allocation using the reduced one-dimensional optimisation problem. Thus, we conclude that the allocation profile in which the rank-1 candidates contribute $\frac{2}{3}$ to the party battle and all other candidates allocate 0 to their party battles is indeed a NE.

To confirm a certain allocation profile is NE with multiple party-level prizes, we begin by establishing the conditions that must be met for any number of party-level prizes. Using

Mathematica, we then calculate and verify the equilibrium allocation profile. For symmetric pairing, the symmetric equilibrium must satisfy all of the equations below (for all i):

$$\frac{1}{4e_D^i}\left(1-\left(\frac{1}{2}\right)^K\left(\sum_{j=i}^{K}\binom{K}{j}+\left(\frac{1}{2}\right)^{i-1}\sum_{j=1}^{\min\{i-1,K\}}\binom{K}{j}\sum_{m=i-j}^{i-1}\binom{i-1}{m}\right)\right)-\lambda^i+\mu_D^i=0 \tag{2.154}$$

$$\begin{aligned}&\left(\frac{1}{2}\right)^K\frac{1}{4(e_P^1+\cdots+e_P^N)}\left(\sum_{j=i}^{K}\binom{K}{j}(2j-K)\right.\\&\left.+\left(\frac{1}{2}\right)^{i-1}\sum_{j=1}^{\min\{i-1,K\}}\binom{K}{j}(2j-K)\sum_{m=i-j}^{i-1}\binom{i-1}{m}\right)-\lambda^i+\mu_P^i=0\end{aligned} \tag{2.155}$$

$$e_D^i+e_P^i\leq 1,\quad \lambda^i\geq 0,\quad \lambda^i(e_D^i+e_P^i-1)=0,\quad \forall i\in\{1,\ldots,N\} \tag{2.156}$$

$$e_D^i\geq 0,\quad \mu_D^i\geq 0,\quad \mu_D^i e_D^i=0,\quad \forall i\in\{1,\ldots,N\} \tag{2.157}$$

$$e_P^i\geq 0,\quad \mu_P^i\geq 0,\quad \mu_P^i e_P^i=0,\quad \forall i\in\{1,\ldots,N\} \tag{2.158}$$

We aim to find a potential NE that satisfies all of the conditions above for any N and K. For $K>1$, obtaining analytical solutions may be difficult, so we employ tools like Mathematica to compute numerical solutions. The procedures to find the potential NE is as follows: (1) Assume that only the rank-1 candidates allocate positive resources towards the party battle. Calculate the optimal allocations for rank-1 candidates, then verify if μ_P^i is positive for all other candidates. If not, proceed to the next step. (2) Assume that only the rank-1 and rank-2 candidates allocate positive resources toward the party battle. Calculate the optimal allocations for both rank-1 and rank-2 candidates, then verify if μ_P^i is positive for all remaining candidates. If not, we will continue to search. This iterative approach continues until we find a profile that satisfies all conditions. (3) After finding this solution, we further verify that we indeed find the maximum point using the reduced one-dimensional optimisation problem, confirming the profile is indeed a NE. The table below presents the NE identified through this process for the case with 6 candidates in each party and a total of 2, 3, 4 and 5 party-level prizes.

Symmetric NE with N=6 — Symmetric Pairing						
(D,P)	Rank-1	Rank-2	Rank-3	Rank-4	Rank-5	Rank-6
K=2	$\left(\frac{2}{5},\frac{3}{5}\right)$	$\left(\frac{4}{5},\frac{1}{5}\right)$	$(1,0)$	$(1,0)$	$(1,0)$	$(1,0)$
K=3	$\left(\frac{9}{25},\frac{16}{25}\right)$	$\left(\frac{3}{5},\frac{2}{5}\right)$	$\left(\frac{24}{25},\frac{1}{25}\right)$	$(1,0)$	$(1,0)$	$(1,0)$
K=4	$\left(\frac{10}{29},\frac{19}{29}\right)$	$\left(\frac{15}{29},\frac{14}{29}\right)$	$\left(\frac{22}{29},\frac{7}{29}\right)$	$(1,0)$	$(1,0)$	$(1,0)$
K=5	$\left(\frac{420}{1259},\frac{839}{1259}\right)$	$\left(\frac{588}{1259},\frac{671}{1259}\right)$	$\left(\frac{812}{1259},\frac{447}{1259}\right)$	$\left(\frac{1116}{1259},\frac{143}{1259}\right)$	$(1,0)$	$(1,0)$

Table 2.3: Symmetric Nash Equilibrium with symmetric pairing (N=6)

We can apply the same procedure for the case of highest-versus-lowest pairing, where the rank-1 candidates compete against rank-N candidates in district battles. The conditions characterising the equilibrium are as follows for the rank-1 candidates:

$$\frac{e_D^N}{(e_D^1+e_D^N)^2}\left(1-\left(\frac{1}{2}\right)^K\left(\sum_{j=i}^{K}\binom{K}{j}\right)\right)-\lambda^1+\mu_D^1=0 \tag{2.159}$$

$$\left(\frac{e_D^N}{e_D^1+e_D^N}\right)\left(\frac{1}{2}\right)^{K-1}\frac{1}{4(e_P^1+...+e_P^N)}\left(\sum_{j=i}^{K}\binom{K}{j}(2j-K)\right)-\lambda^1+\mu_P^1=0 \tag{2.160}$$

For Rank-2 to rank-(N-1) candidates:

$$\begin{gathered}\frac{1}{4e_D^i}\left(1-\left(\frac{1}{2}\right)^K\left(\sum_{j=i}^{K}\binom{K}{j}+\left(\frac{1}{2}\right)^{i-2}\left(\frac{1}{e_D^1+e_D^N}\right)\right.\right.\\ \left.\left.\sum_{j=1}^{\min\{i-1,K\}}\binom{K}{j}\left(\sum_{m=i-1-j}^{i-2}e_D^1\binom{i-2}{m}+\sum_{m=i-j}^{i-2}e_D^N\binom{i-2}{m}\right)\right)\right)-\lambda^i+\mu_D^i=0\end{gathered} \tag{2.161}$$

$$\begin{gathered}\left(\frac{1}{2}\right)^K\frac{1}{4(e_P^1+...+e_P^N)}\left(\sum_{j=i}^{K}\binom{K}{j}(2j-K)+\left(\frac{1}{2}\right)^{i-2}\left(\frac{1}{e_D^1+e_D^N}\right)\right.\\ \left.\sum_{j=1}^{\min\{i-1,K\}}\binom{K}{j}(2j-K)\left(\sum_{m=i-1-j}^{i-2}e_D^1\binom{i-2}{m}+\sum_{m=i-j}^{i-2}e_D^N\binom{i-2}{m}\right)\right)-\lambda^i+\mu_P^i=0\end{gathered} \tag{2.162}$$

For the rank-N candidates:

$$\begin{gathered}\frac{e_D^1}{(e_D^1+e_D^N)^2}\left(1-\left(\frac{1}{2}\right)^K\left(\left(\frac{1}{2}\right)^{N-2}\left(\frac{1}{e_D^1+e_D^N}\right)\right.\right.\\ \left.\left.\sum_{j=1}^{\min\{N-1,K\}}\binom{K}{j}\left(\sum_{m=N-1-j}^{N-2}e_D^1\binom{N-2}{m}+\sum_{m=N-j}^{N-2}e_D^N\binom{N-2}{m}\right)\right)\right)-\lambda^N+\mu_D^N=0\end{gathered} \tag{2.163}$$

$$\begin{gathered}\left(\frac{1}{2}\right)^{K-1}\left(\frac{e_D^1}{e_D^1+e_D^N}\right)\frac{1}{4(e_P^1+...+e_P^N)}\left(\left(\frac{1}{2}\right)^{N-2}\left(\frac{1}{e_D^1+e_D^N}\right)\right.\\ \left.\sum_{j=1}^{\min\{N-1,K\}}\binom{K}{j}(2j-K)\left(\sum_{m=N-1-j}^{N-2}e_D^1\binom{N-2}{m}+\sum_{m=N-j}^{N-2}e_D^N\binom{N-2}{m}\right)\right)-\lambda^N+\mu_P^N=0\end{gathered} \tag{2.164}$$

We use Mathematica to search for the NE, summarised in the table below:

Symmetric NE with N=6 — Highest-versus-Lowest Pairing						
(D,P)	Rank-1	Rank-2	Rank-3	Rank-4	Rank-5	Rank-6
K=2	$(0,1)$	$(1,0)$	$(1,0)$	$(1,0)$	$(1,0)$	$(1,0)$
K=3	$(0,1)$	$\left(\frac{4}{5},\frac{1}{5}\right)$	$(1,0)$	$(1,0)$	$(1,0)$	$(1,0)$
K=4	$(0,1)$	$\left(\frac{3}{5},\frac{2}{5}\right)$	$\left(\frac{24}{25},\frac{1}{25}\right)$	$(1,0)$	$(1,0)$	$(1,0)$
K=5	$(0,1)$	$\left(\frac{15}{29},\frac{14}{29}\right)$	$\left(\frac{66}{87},\frac{21}{87}\right)$	$(1,0)$	$(1,0)$	$(1,0)$

Table 2.4: Symmetric Nash Equilibrium with highest-versus-lowest pairing (N=6)

Comparing the two examples, we observe that: (1) Higher-ranked candidates tend to allocate a larger share, sometimes even all of their resources, towards the party battle, while lower-ranked candidates allocate less towards the party battle, focusing primarily on their district battles. (2) The resource allocation of higher-ranked candidates is influenced by the pairings at the district battle.

In an open list, we derive all conditions the symmetric equilibrium must satisfy:

$$\frac{1}{4e_D}\left(1-\left(\frac{1}{2}\right)^{K+N-1}\sum_{j=1}^{K}\binom{K}{j}\left(\sum_{m=N-j}^{N-1}\binom{N-1}{m}+\sum_{m=0}^{N-j-1}\binom{N-1}{m}\frac{j}{N-m}\right)\right)-\lambda+\mu_D=0 \tag{2.165}$$

$$\begin{aligned}&\left(\frac{1}{2}\right)^{K+N}\left(\frac{1}{e_P}\right)\sum_{j=1}^{K}\binom{K}{j}\left(\frac{2j-K}{2N}\left(\sum_{m=N-j}^{N-1}\binom{N-1}{m}\right.\right.\\&\left.+\sum_{m=0}^{N-j-1}\binom{N-1}{m}\frac{j}{N-m}\right)+\sum_{m=0}^{N-j-1}\binom{N-1}{m}\left(\frac{j(N-m-1)}{(N-m)^2}\right.\\&\left.\left.-\frac{1}{N-m}\sum_{s=1}^{j-1}\frac{j-s}{N-m-s}\right)\right)-\lambda+\mu_P=0\end{aligned} \tag{2.166}$$

$$e_D+e_P\leq 1,\quad \lambda\geq 0,\quad (e_D+e_P-1)\lambda=0 \tag{2.167}$$

$$\mu_D\geq 0,\quad e_D\geq 0,\quad \mu_De_D=0 \tag{2.168}$$

$$\mu_P\geq 0,\quad e_P\geq 0,\quad \mu_Pe_P=0 \tag{2.169}$$

Using equations above, we can solve for the closed-form solutions for the equilibrium e_D and e_P.

Appendix 2G. Proof of Proposition 6

The system of KT conditions characterising the symmetric NE is as follows:

$$\frac{1}{8e_D^1}-\lambda^1=0 \tag{2.170}$$

$$\frac{1}{8(e_P^1+e_P^2)} - \lambda^1 + \mu_P^1 = 0 \tag{2.171}$$

$$\frac{3}{16e_D^2} - \lambda^2 = 0 \tag{2.172}$$

$$\frac{1}{16(e_P^1+e_P^2)} - \lambda^2 + \mu_P^2 = 0 \tag{2.173}$$

$$e_D^1 + e_P^1 \leq 1+c, \quad \lambda^1 \geq 0, \quad \lambda^1(e_D^1 + e_P^1 - (1+c)) = 0 \tag{2.174}$$

$$e_D^2 + e_P^2 \leq 1-c, \quad \lambda^2 \geq 0, \quad \lambda^2(e_D^2 + e_P^2 - (1-c)) = 0 \tag{2.175}$$

$$e_D^1 \geq 0, \quad \mu_D^1 \geq 0, \quad e_D^1\mu_D^1 = 0 \tag{2.176}$$

$$e_P^1 \geq 0, \quad \mu_P^1 \geq 0, \quad e_P^1\mu_P^1 = 0 \tag{2.177}$$

$$e_D^2 \geq 0, \quad \mu_D^2 \geq 0, \quad e_D^2\mu_D^2 = 0 \tag{2.178}$$

$$\mu_P^2 \geq 0, \quad e_P^2 \geq 0, \quad e_P^2\mu_P^2 = 0 \tag{2.179}$$

We begin by examining the case where $\mu_D^1 = 0, \mu_P^1 = 0, \mu_D^2 = 0, e_P^2 = 0$, this implies $e_D^2 = 1-c$. Solving for the remaining terms, we find that $e_D^1 = e_P^1 = \frac{1+c}{2}$ and $\lambda^2 = \frac{3}{16(1-c)}$. We then calculate $\mu_P^2 = \frac{5c+1}{16(1-c)(1+c)}$. Thus, $\mu_P^2 \geq 0$ if and only if $c \geq -\frac{1}{5}$. This implies that, as long as the rank-1 candidates do not have a significantly smaller budget than rank-2 candidates, they will continue to be the sole contributors to the party. Next, we consider the case where $\mu_D^1 = 0, \mu_P^1 = 0, \mu_D^2 = 0, \mu_P^2 = 0$, solving for the equations give $e_D^1 = \frac{2}{5}$, $e_P^1 = \frac{3}{5} + c$, $e_D^2 = \frac{6}{5}$ and $e_P^2 = -\frac{1}{5} - c$. These allocations remain within budget only if $-\frac{3}{5} < c < -\frac{1}{5}$. This implies that if the rank-2 candidates have a moderately higher budget than the rank-1 candidates, both candidates will allocate positively to the party. Finally, we examine the scenario where $\mu_D^1 = 0, e_P^1 = 0, \mu_D^2 = 0, \mu_P^2 = 0$, this implies $e_D^1 = 1+c$. We then solve for $e_D^2 = \frac{3(1-c)}{4}$ and $e_P^2 = \frac{1-c}{4}$ and $\lambda^1 = \frac{1}{8(1+c)}$. We find that $\mu_P^1 = \frac{-3-5c}{8(1-c)(1+c)}$. Thus, $\mu_P^1 \geq 0$ if and only if $c \leq -\frac{3}{5}$. This implies that the lower-ranked candidates can become the sole contributors of the party if they have a sufficiently larger budget than the higher-ranked candidates. We can confirm as in the benchmark case that the objective functions are concave in the feasible region and any point satisfying KT conditions is a global constrained maximum.

Similarly, we explore the case with the highest-versus-lowest pairing, the system of KT conditions is as follows:

$$\frac{e_D^2}{2(e_D^1+e_D^2)^2} - \lambda^1 + \mu_D^1 = 0 \tag{2.180}$$

$$\frac{e_D^2}{e_D^1+e_D^2}\frac{1}{4(e_P^1+e_P^2)} - \lambda^1 + \mu_P^1 = 0 \tag{2.181}$$

$$\frac{3e_D^1}{4(e_D^1+e_D^2)^2} - \lambda^2 + \mu_D^2 = 0 \tag{2.182}$$

$$\frac{e_D^1}{e_D^1+e_D^2}\frac{1}{8(e_P^1+e_P^2)} - \lambda^2 + \mu_P^2 = 0 \tag{2.183}$$

$$e_D^1 + e_P^1 \leq 1 + c, \quad \lambda^1 \geq 0, \quad \lambda^1(e_D^1 + e_P^1 - (1+c)) = 0 \tag{2.184}$$

$$e_D^2 + e_P^2 \leq 1 - c, \quad \lambda^2 \geq 0, \quad \lambda^2(e_D^2 + e_P^2 - (1-c)) = 0 \tag{2.185}$$

$$e_D^1 \geq 0, \quad \mu_D^1 \geq 0, \quad e_D^1 \mu_D^1 = 0 \tag{2.186}$$

$$e_P^1 \geq 0, \quad \mu_P^1 \geq 0, \quad e_P^1 \mu_P^1 = 0 \tag{2.187}$$

$$e_D^2 \geq 0, \quad \mu_D^2 \geq 0, \quad e_D^2 \mu_D^2 = 0 \tag{2.188}$$

$$\mu_P^2 \geq 0, \quad e_P^2 \geq 0, \quad e_P^2 \mu_P^2 = 0 \tag{2.189}$$

We first consider the case where $\mu_D^1 = 0, \mu_P^1 = 0, \mu_D^2 = 0, e_P^2 = 0$, this implies $e_D^2 = 1 - c$. Solving for the remaining terms, we find that $e_D^1 = \frac{3c+1}{3}$, $e_P^1 = \frac{2}{3}$ and $\lambda^2 = \frac{9(1+3c)}{64}$. We then calculate $\mu_P^2 = \frac{6(1+3c)}{64}$. Thus, $\mu_P^2 \geq 0$ if and only if $c \geq -\frac{1}{3}$. This implies that, as long as the rank-1 candidates do not have a budget that is significantly lower than that of the rank-2 candidates, they will remain the sole contributors to the party. Next, we examine the case where $e_D^1 = 0, \mu_P^1 = 0, \mu_D^2 = 0, e_P^2 = 0$, this implies $e_P^1 = 1 + c$ and $\lambda^2 = 0$. We can then solve for $\lambda^1 = \frac{1}{4(1+c)}$ and $\mu_D^1 = \frac{e_D^2 - 2(1+c)}{4e_D^2(1+c)}$. $\mu_D^1 \geq 0$ if and only if there exists values of e_D^2 that are greater than $2(1+c)$, which is possible only if $2(1+c) \leq 1 - c$, or equivalently, when $c \leq -\frac{1}{3}$. This implies that, if rank-2 candidates have a sufficiently larger budget than rank-1 candidates, they can use this additional budget to compel rank-1 candidates to focus entirely on the party battle. In this case, the rank-2 candidates will win their district battle for sure.

Appendix 2H. Proof of Proposition 7

With the rank-1 candidates having a more productive district campaigning, the system of KT conditions is written as follows:

$$\frac{\gamma e_D^2}{2(\gamma e_D^1 + e_D^2)^2} - \lambda^1 + \mu_D^1 = 0 \tag{2.190}$$

$$\frac{e_D^2}{4(\gamma e_D^1 + e_D^2)(e_P^1 + e_P^2)} - \lambda^1 + \mu_P^1 = 0 \tag{2.191}$$

$$\frac{(\gamma e_D^1)^2 + 2\gamma e_D^1 e_D^2}{2(e_D^2 + \gamma e_D^1)^3} - \lambda^2 + \mu_D^2 = 0 \tag{2.192}$$

$$\frac{\gamma (e_D^1)^2}{4(\gamma e_D^1 + e_D^2)^2(e_P^1 + e_P^2)} - \lambda^2 + \mu_P^2 = 0 \tag{2.193}$$

$$e_D^1 + e_P^1 \leq 1, \quad \lambda^1 \geq 0, \quad \lambda^1(e_D^1 + e_P^1 - 1) = 0 \tag{2.194}$$

$$e_D^2 + e_P^2 \leq 1, \quad \lambda^2 \geq 0, \quad \lambda^2(e_D^2 + e_P^2 - 1) = 0 \tag{2.195}$$

$$e_D^1 \geq 0, \quad \mu_D^1 \geq 0, \quad e_D^1 \mu_D^1 = 0 \tag{2.196}$$

$$e_P^1 \geq 0, \quad \mu_P^1 \geq 0, \quad e_P^1 \mu_P^1 = 0 \tag{2.197}$$

$$e_D^2 \geq 0, \quad \mu_D^2 \geq 0, \quad e_D^2 \mu_D^2 = 0 \tag{2.198}$$

$$\mu_P^2 \geq 0, \quad e_P^2 \geq 0, \quad e_P^2 \mu_P^2 = 0 \tag{2.199}$$

We consider the case where $\mu_D^1 = 0, \mu_P^1 = 0, \mu_D^2 = 0, e_P^2 = 0$, this implies that $e_D^2 = 1$. We then solve for $e_D^1 = \frac{2\gamma-1}{3\gamma}$ and $e_P^1 = \frac{\gamma+1}{3\gamma}$. Next, we calculate $\lambda^2 = \frac{(\gamma e_D^1)(\gamma e_D^1 + 2\gamma e_D^1)}{2(1+\gamma e_D^1)^3}$ and $\mu_P^2 = \lambda^2 - \frac{3\gamma^2 (e_D^1)^2}{4(\gamma+1)(\gamma e_D^1+1)^2}$. This reduces to $\frac{4(1+\gamma)(\gamma e_D^1)^2}{4(\gamma+1)(\gamma e_D^1+1)^2} > 0$. We confirm this is indeed a NE by examining the reduced one-dimensional optimisation problem and we also rule out other possible NE, so we confirm this is the unique symmetric equilibrium. We can further show that $e_D^1 = \frac{2\gamma-1}{3\gamma} > \frac{1}{3}$.

When the rank-2 candidates have a more productive district campaigning, the system of KT conditions is written as follows:

$$\frac{\gamma e_D^2}{2(e_D^1 + \gamma e_D^2)^2} - \lambda^1 + \mu_D^1 = 0 \tag{2.200}$$

$$\frac{\gamma e_D^2}{4(e_D^1 + \gamma e_D^2)(e_P^1 + e_P^2)} - \lambda^1 + \mu_P^1 = 0 \tag{2.201}$$

$$\frac{\gamma e_D^1 (2\gamma e_D^2 + e_D^1)}{2(\gamma e_D^2 + e_D^1)^3} - \lambda^2 + \mu_D^2 = 0 \tag{2.202}$$

$$\frac{(e_D^1)^2}{4(e_D^1 + \gamma e_D^2)^2 (e_P^1 + e_P^2)} - \lambda^2 + \mu_P^2 = 0 \tag{2.203}$$

$$e_D^1 + e_P^1 \leq 1, \quad \lambda^1 \geq 0, \quad \lambda^1 (e_D^1 + e_P^1 - 1) = 0 \tag{2.204}$$

$$e_D^2 + e_P^2 \leq 1, \quad \lambda^2 \geq 0, \quad \lambda^2 (e_D^2 + e_P^2 - 1) = 0 \tag{2.205}$$

$$e_D^1 \geq 0, \quad \mu_D^1 \geq 0, \quad e_D^1 \mu_D^1 = 0 \tag{2.206}$$

$$e_P^1 \geq 0, \quad \mu_P^1 \geq 0, \quad e_P^1 \mu_P^1 = 0 \tag{2.207}$$

$$e_D^2 \geq 0, \quad \mu_D^2 \geq 0, \quad e_D^2 \mu_D^2 = 0 \tag{2.208}$$

$$\mu_P^2 \geq 0, \quad e_P^2 \geq 0, \quad e_P^2 \mu_P^2 = 0 \tag{2.209}$$

We start by examining the case where $\mu_D^1 = 0, \mu_P^1 = 0, \mu_D^2 = 0, e_P^2 = 0$, this implies that $e_D^2 = 1$. We then solve for $e_D^1 = \frac{2-\gamma}{3}$ and $e_P^1 = \frac{\gamma+1}{3}$. Next, we can calculate $\lambda^2 = \frac{(\gamma e_D^1)(2\gamma + e_D^1)}{2(\gamma + e_D^1)^3}$ and $\mu_P^2 = \lambda^2 - \frac{(e_D^1)^2}{4(\gamma + e_D^1)^2 e_P^1} > 0$. For $e_D^1 \geq 0$, we must have $\gamma \leq 2$. This implies that, when γ is not too large, the rank-1 candidates allocate less to the district battle than in the benchmark model, but still allocate a positive amount. Next, we consider the case where $e_D^1 = 0, \mu_P^1 = 0, \mu_D^2 = 0, e_P^2 = 0$, this implies that $e_P^1 = 1$ and $\lambda^2 = 0$. Solving for $\lambda^1 = \frac{1}{4}$ and $\mu_D^1 = \lambda^1 - \frac{1}{2\gamma e_D^2}$, we find that $\mu_D^1 \geq 0$ if and only if $e_D^2 \geq \frac{2}{\gamma}$. Such values are achievable as long as $\gamma > 2$. This analysis shows that, when γ is sufficiently large, the rank-1 candidates allocate resources solely to the party battle, so that rank-2 candidates win their district battles and a seat for sure.

Appendix 2I. Proof of Proposition 8

With the positive spillover effects from the district battle to the party battle, the KT conditions are written as follows with symmetric pairing:

$$\frac{1}{8e_D^1} + \frac{\alpha}{8(e_P^1 + e_P^2 + \alpha e_D^1 + \alpha e_D^2)} - \lambda^1 = 0 \tag{2.210}$$

$$\frac{1}{8(e_P^1 + e_P^2 + \alpha e_D^1 + \alpha e_D^2)} - \lambda^1 + \mu_P^1 = 0 \tag{2.211}$$

$$\frac{3}{16e_D^2} + \frac{\alpha}{8(e_P^1 + e_P^2 + \alpha e_D^1 + \alpha e_D^2)} - \lambda^2 = 0 \tag{2.212}$$

$$\frac{1}{16(e_P^1 + e_P^2 + \alpha e_D^1 + \alpha e_D^2)} - \lambda^2 + \mu_P^2 = 0 \tag{2.213}$$

$$e_D^1 + e_P^1 \leq 1, \quad \lambda^1 \geq 0, \quad \lambda^1(e_D^1 + e_P^1 - 1) = 0 \tag{2.214}$$

$$e_D^2 + e_P^2 \leq 1, \quad \lambda^2 \geq 0, \quad \lambda^2(e_D^2 + e_P^2 - 1) = 0 \tag{2.215}$$

$$e_D^1 \geq 0, \quad \mu_D^1 \geq 0, \quad e_D^1\mu_D^1 = 0 \tag{2.216}$$

$$e_P^1 \geq 0, \quad \mu_P^1 \geq 0, \quad e_P^1\mu_P^1 = 0 \tag{2.217}$$

$$e_D^2 \geq 0, \quad \mu_D^2 \geq 0, \quad e_D^2\mu_D^2 = 0 \tag{2.218}$$

$$\mu_P^2 \geq 0, \quad e_P^2 \geq 0, \quad e_P^2\mu_P^2 = 0 \tag{2.219}$$

We first examine the case where $\mu_P^1 = 0$ and $e_P^2 = 0$, which implies that $e_D^2 = 1$. We then solve for $e_D^1 = \frac{1+\alpha}{2(1-\alpha)}$ and $e_P^1 = \frac{1-3\alpha}{2(1-\alpha)}$. Here, $e_D^1 \geq 1$ if and only if $\alpha \leq \frac{1}{3}$. We further show that $\mu_P^2 = \frac{\frac{7\alpha+1}{2}}{8(1+\alpha)} > 0$. Next, we consider the case where $e_P^1 = 0$ and $e_P^2 = 0$. We solve for $\lambda^1 = \frac{3}{16}$ and $\lambda^2 = \frac{7}{32}$. Next, we solve for $\mu_P^1 = \frac{3\alpha-1}{16\alpha}$, this is greater than 0 if $\alpha > \frac{1}{3}$. We also find $\mu_P^2 = \frac{7\alpha-1}{32\alpha}$, which is positive if $\alpha > \frac{1}{7}$. We confirm these profiles are indeed NE by examining the reduced one-dimensional optimisation problem. When $\alpha < \frac{1}{3}$, rank-1 candidates increase their allocations to the district battle compared to the benchmark model, while still allocating positively to the party battle. However, when α is sufficiently large, both candidates allocate all their resources to the district battle.

Similarly, if spending resources on the party battle generates positive spillover effects on the district battle, the system of KT conditions can be written as:

$$\frac{1}{8(e_D^1 + \beta e_P^1)} - \lambda^1 + \mu_D^1 = 0 \tag{2.220}$$

$$\frac{\beta(e_P^1 + e_P^2) + e_D^1 + \beta e_P^1}{8(e_P^1 + e_P^2)(e_D^1 + \beta e_P^1)} - \lambda^1 + \mu_P^1 = 0 \tag{2.221}$$

$$\frac{3}{16(e_D^2 + \beta e_P^2)} - \lambda^2 + \mu_D^2 = 0 \tag{2.222}$$

$$\frac{3\beta(e_P^1 + e_P^2) + e_D^2 + \beta e_P^2}{16(e_P^1 + e_P^2)(e_D^2 + \beta e_P^2)} - \lambda^2 + \mu_P^2 = 0 \tag{2.223}$$

$$e_D^1 + e_P^1 \leq 1, \quad \lambda^1 \geq 0, \quad \lambda^1(e_D^1 + e_P^1 - 1) = 0 \tag{2.224}$$

$$e_D^2 + e_P^2 \leq 1, \quad \lambda^2 \geq 0, \quad \lambda^2(e_D^2 + e_P^2 - 1) = 0 \tag{2.225}$$

$$e_D^1 \geq 0, \quad \mu_D^1 \geq 0, \quad e_D^1 \mu_D^1 = 0 \tag{2.226}$$

$$e_P^1 \geq 0, \quad \mu_P^1 \geq 0, \quad e_P^1 \mu_P^1 = 0 \tag{2.227}$$

$$e_D^2 \geq 0, \quad \mu_D^2 \geq 0, \quad e_D^2 \mu_D^2 = 0 \tag{2.228}$$

$$\mu_P^2 \geq 0, \quad e_P^2 \geq 0, \quad e_P^2 \mu_P^2 = 0 \tag{2.229}$$

We first consider the case where $\mu_D^1 = 0$, $\mu_D^2 = 0$, $\mu_P^1 = 0$ and $e_P^2 = 0$, this implies that $e_D^2 = 1$. We then solve for $e_D^1 = \frac{1-2\beta}{2(1-\beta)}$ and $e_P^1 = \frac{1}{2(1-\beta)}$. Here, $e_P^1 \leq 1$ if and only if $\beta \leq \frac{1}{2}$. We also calculate $\lambda^2 = \frac{3}{16}$ and confirm that $\mu_P^2 > 0$. Next, we examine the case where $e_D^1 = 0$, $\mu_D^2 = 0$, $\mu_P^1 = 0$ and $e_P^2 = 0$. We then calculate $\lambda^1 = \frac{1}{4}$ and find that $\mu_D^1 = \frac{2\beta-1}{8\beta}$, which is positive if $\beta > \frac{1}{2}$. In addition, we find $\lambda^2 = \frac{3}{16}$ and $\mu_P^2 = \frac{2-3\beta}{16}$, which is positive if $\beta < \frac{2}{3}$. We then consider the case where $e_D^1 = 0$, $\mu_D^2 = 0$, $\mu_P^1 = 0$ and $\mu_P^2 = 0$, this implies that $e_P^1 = 1$. We then calculate $e_P^2 = \frac{3\beta-2}{4-4\beta}$ and $e_D^2 = \frac{6-7\beta}{4-4\beta}$. These values lie between 0 and 1 if $\frac{2}{3} \leq \beta \leq \frac{6}{7}$. We can also confirm that $\mu_D^1 > 0$ as long as $\beta > \frac{2}{5}$. Finally, we check the case where $e_D^1 = 0$, $e_D^2 = 0$, $\mu_P^1 = 0$ and $\mu_P^2 = 0$. Here, $\mu_D^1 > 0$ as long as $\beta > \frac{2}{3}$ and $\mu_D^2 > 0$ if $\beta > \frac{6}{7}$.

Appendix 2J. Proof of Proposition 9

When the candidates attach some special meanings towards winning their district battles, the system of KT conditions is written as follows with symmetric pairings:

$$\frac{v}{4e_D^1} + \frac{1}{8e_D^1} - \lambda^1 = 0 \tag{2.230}$$

$$\frac{1}{8(e_P^1 + e_P^2)} - \lambda^1 + \mu_P^1 = 0 \tag{2.231}$$

$$\frac{v}{4e_D^2} + \frac{1}{16e_D^2} - \lambda^2 = 0 \tag{2.232}$$

$$\frac{1}{16(e_P^1 + e_P^2)} - \lambda^2 + \mu_P^2 = 0 \tag{2.233}$$

$$e_D^1 + e_P^1 \leq 1, \quad \lambda^1 \geq 0, \quad \lambda^1(e_D^1 + e_P^1 - 1) = 0 \tag{2.234}$$

$$e_D^2 + e_P^2 \leq 1, \quad \lambda^2 \geq 0, \quad \lambda^2(e_D^2 + e_P^2 - 1) = 0 \tag{2.235}$$

$$e_D^1 \geq 0, \quad \mu_D^1 \geq 0, \quad e_D^1 \mu_D^1 = 0 \tag{2.236}$$

$$e_P^1 \geq 0, \quad \mu_P^1 \geq 0, \quad e_P^1 \mu_P^1 = 0 \tag{2.237}$$

$$e_D^2 \geq 0, \quad \mu_D^2 \geq 0, \quad e_D^2 \mu_D^2 = 0 \tag{2.238}$$

$$\mu_P^2 \geq 0, \quad e_P^2 \geq 0, \quad e_P^2 \mu_P^2 = 0 \tag{2.239}$$

We first check the case where $\mu_P^1 = 0$ and $e_P^2 = 0$, this implies that $e_D^2 = 1$. Then we solve for $e_D^1 = \frac{2v-1}{2v}$ and $e_P^1 = \frac{1}{2v}$. Next, we calculate $\lambda^2 = \frac{4v-1}{16}$ and $\mu_P^2 = \lambda^2 - \frac{v}{8} = \frac{2v-1}{16} > 0$. We confirm this is indeed a NE by examining the reduced one-dimensional optimisation problem and we also rule out other possible NE, so we confirm this is the unique symmetric equilibrium. When $v > 1$, rank-1 candidates allocate more resources to the district battle than in the benchmark model.

Chapter 3

Contests under Sign-Stealing

This chapter is based on joint work with Stefano Barbieri and Sven A. Simon.

3.1 Introduction

In 2017, Major League Baseball penalised the Houston Astros for using a camera in center field to record the opposing catcher's hand signals and relay the decoded instructions to their own batters in real time[1]. In team sports, communication is crucial for coordination but must often be coded to avoid revealing strategies to the opposition, see, e.g., Eccles and Tran (2012) and Ishak (2017). By decoding the hand signals, the Astros learned the type of pitch about to be thrown, an advantage that can potentially alter game outcomes. This practice, known as *Sign-Stealing*, occurs when an opponent observes the communication intended for internal coordination. While the term originates from sports, its underlying meaning appears in many competitive settings, including spying activities in joint military operations, industrial espionage in business rivalries and information leakage of campaign strategies in political contests. Even unilateral military actions may require internal coordination across multiple government departments, each responsible for distinct policy domains. This coordination is typically conducted through encrypted communication channels, which are nonetheless susceptible to leaks.[2]

Importantly, although the act of sign-stealing typically occurs during the actual baseball game or military operation itself, baseball players and states often make upfront investments and preparations. In such settings, sign-stealing has the potential to change both the intensity and the nature of competition, particularly in how teams invest in preparation and use internal signals for coordination. This paper combines theoretical analysis with an experi-

[1] See Longman (2020) for a detailed description.

[2] A prominent example is the so-called "Signalgate" incident, in which former US National Security Adviser Michael Waltz initiated a group chat to coordinate the "Rough Rider" operation. The operation aimed to counter Houthi forces and ensure the security of international shipping lanes. Participants included the U.S. Secretaries of Defense, State, and Treasury, among others, as well as an unintended journalist. Although the leak did not appear to compromise the military execution of the operation, its political and diplomatic consequences were significant (See Greenwood (2025) and Hutzler and Stoddart (2025) for a detailed description).

ment to examine the impact of sign-stealing on teams' competitive behaviour. Specifically, we address three questions: (i) how the possibility of sign-stealing affects teams' upfront investments; (ii) whether its effects differ when one team has a prior advantage (or disadvantage); and (iii) how the threat of sign-stealing shapes teams' use of internal signals for coordination.

In the theoretical model, we consider a contest between two teams: one consisting of two players (team A) and the other of a single player (team B). All members of the winning team receive a prize. The contest unfolds in three stages. In stage 1, players exert upfront contest efforts. A team has a prior advantage if its effort aggregation is more 'productive' than that of the other team. In stage 2, the members of team A may communicate to coordinate their actions in the next stage. In stage 3, all players choose a binary "direction". If the two members of team A choose different directions, they automatically lose the contest. If they choose the same direction, their joint choice interacts with team B's choice to determine which team gains a directional advantage. We analyse the equilibrium under three regimes, which differ only in stage 2: (i) *No Coordination*: communication within team A is not allowed; (ii) *Coordination without Sign-stealing*: the members in team A may send private messages to coordinate that are unobservable to team B; (iii) *Coordination with Sign-stealing*: the opposing team "steals" this message, making the message observable to team B.

The equilibrium analysis shows that when communication is possible and sign-stealing is absent, team A always sends messages and coordinates perfectly. If the directional advantage that team B gains from matching team A's direction is sufficiently small, even the threat of sign-stealing does not deter communication. Thus, with a small directional advantage, team A achieves perfect coordination, while sign-stealing ensures team B always secures the directional advantage. The effect of this additional directional advantage team B gains from sign-stealing on teams' contest efforts depends on which team holds a prior advantage. When neither team holds a prior advantage, sign-stealing has no effect on effort. When team A has a prior advantage, sign-stealing levels the playing field and increases the efforts of both teams. In contrast, when team B has a prior advantage, sign-stealing further exacerbates this imbalance, leading both teams to exert less effort.

We test our theoretical predictions in a laboratory experiment. The design of our treatments closely follows the three regimes *No Coordination*, *Coordination without Sign-stealing* and *Coordination with Sign-stealing*. Consistent with the theory, allowing for communication within team A increases effort of both team A members and team B. Moreover, sign-stealing by team B does not significantly affect contest efforts of either team. However, sign-stealing affects communication choices: team A members refrain from sending messages indicating their intended action more often. This behavioural effect results in lower coordination rates within team A as compared to the regime without sign-stealing. Nevertheless, once a message is sent, both team A and team B members follow the indicated action almost perfectly. Our results also provide causal evidence for the dual role of communication. First, communication increases coordination rates within team A, in particular when compared to sessions of *No Coordination* in which team A fails to achieve implicit coordination. Second, communication enables team A to conceal its action choice from team B in the absence of sign-stealing. Specifically, team B fails to match team A's action at a rate above chance in *Coordination without Sign-stealing*, whereas in *No Coordination* sessions where team A achieves implicit coordination, team B is significantly more successful in matching their actions.

This paper contributes to the literature on espionage and spying in contests. Most existing studies model spying as the acquisition of information about opponents' abilities or private valuations. These studies examine how such information acquisition affects their efforts and expected payoffs, and in some cases, the optimal level of spying. Baik and Shogren (1995) considered a setting where players spy to reduce uncertainty about relative abilities and found that effort is lower when spying is unobservable than when it is observable. In Chen (2025), players acquire private information about an opponent's value by selecting a signal of varying accuracy before the contest[3]. Other studies view spying as a means of securing an additional advantage in the contest. For example, Stupak (2023) studied a dynamic R&D race in which espionage can confer an advantage. They found that industrial espionage has an ambiguous effect on overall investment. Send (2023) modelled idea-stealing in innovation contests by allowing players to copy each others' efforts[4]. Our study models espionage as the leaking of internal signals used for within-team coordination, which can confer an advantage to the spying team. This approach enables us to study not only the impact of espionage on contest effort, but also how its threat shapes the communication strategies of the targeted team.

Beyond contests, several studies examine the effects of espionage in broader classes of games where players can spy on others' actions or strategies. Solan and Yariv (2004) studied extensive-form and normal-form games in which players can buy a noisy signal about their opponents' future actions. They found that the existence of espionage equilibria depends on the difference between the Stackelberg and Subgame Perfect Equilibrium (SPE) payoffs. Barrachina et al. (2014) investigated industrial espionage in an entry game. Lee (2023) studied a two-stage hide-and-seek game in which players may forgo immediate payoffs to manipulate an opponent's suspicion. They found, through an experiment, a sizeable proportion of spying players do not pretend to be ignorant[5]. Our study also examines espionage on opponents' actions, but within a contest framework that features distinct strategic trade-offs: the spied-upon team can use internal communication to improve in-team coordination, but

[3]This is also related to the literature on information disclosure in contests, such as Fu et al. (2011), Kovenock et al. (2015), Zhang and Zhou (2016), Wu and Zheng (2017) and Serena (2022), which typically adopt the contest designer's perspective.

[4]Since sign-stealing effectively grants a team an advantage, this work is also related to studies on favouritism in contests, including Kirkegaard (2012), Franke (2012), Franke et al. (2013), Lee (2013), Drugov and Ryvkin (2017) and Franke et al. (2018).

[5]Related studies include Matsui (1989), Whitney and Gaisford (1999) and Billand et al. (2010).

doing so carries the risk of revealing its intended actions to the opposing team. In this respect, the regime *Coordination with Sign-stealing* parallels cheap-talk models with multiple audiences, as in Farrell and Gibbons (1989) and Battaglini and Makarov (2014). Whereas this literature examines how to "persuade" multiple receivers whose actions are independent, our study examines information "leakage", where each receiver's payoff may depend on the action of others[6].

Sign-stealing has also received considerable attention in the sports literature, particularly in baseball. Stitzel et al. (2021) investigated the impact of the prominent 2017 sign-stealing scandal in baseball, using player performance data to argue that sign-stealing significantly affected outcomes and contributed to the Houston Astros' increased win total. The value and ethics of sign-stealing have also been heavily debated[7] (Barna, 2019; Longman, 2020; Champion, 2021). Our contribution is to develop a formal theoretical framework to examine how sign-stealing influences competitive behaviour. Interestingly, not all forms of sign stealing are prohibited in baseball; Barna (2019) noted that sign stealing aided by "mechanical means" is banned because "aided sign stealing also gives the home-field team an unfair advantage, as they can use their stadium and technology to steal signs while the away team cannot." If the home team holds a prior advantage, our theoretical result that sign stealing can reduce effort aligns with the rationale behind this rule[8].

The remainder of the article is organised as follows. Section 3.2 introduces the theoretical framework and derives the main predictions. Section 3.3 describes the experiment design and procedures. Section 3.4 reports the main experimental results. Section 3.5 provides additional exploratory analysis. Section 3.6 concludes. Detailed theoretical derivations and the experiment instructions and example screenshots are presented in Appendices.

3.2 Theoretical Framework

3.2.1 Formal Model

We consider a contest between two teams A and B. Team A consists of two players A_1 and A_2, while team B consists of the single player B_1. The game unfolds in three stages:

- **Stage 1 (Effort Choices):** All players simultaneously choose their initial contest efforts (or investments).
- **Stage 2 (Pre-play Communication):** Without observing efforts of the other players, players in team A engage in pre-play communication: player A_1 may send a message to player A_2, indicating her intended "direction" (U or D) in the next stage.

[6] For related studies, see Goltsman and Pavlov (2011), Sobel (2013), Drugov et al. (2017), Grandjean et al. (2017) and Fehrler and Hahn (2023).

[7] Other baseball-related studies include Stephenson (2008), Patterson (2011) and Howard (2017).

[8] Home advantage in professional team sports is discussed in Pollard and Pollard (2005).

- **Stage 3 (Direction Choices):** All players simultaneously choose their directions.

We analyse three different regimes: *No Coordination*, *Coordination without Sign-stealing* and *Coordination with Sign-stealing*. These three cases differ only in stage 2.

Actions, Histories and Strategies

Stage 1: Each player simultaneously chooses their initial contest efforts. Let $g_i \in \mathbb{R}_{\geq 0}$ denote the effort choice of player i for $i \in \{A_1, A_2, B_1\}$. The total contest effort of team A is the sum of individual efforts, scaled by a factor $\gamma > 0$: $g_A = \gamma(g_{A_1} + g_{A_2})$. Since team B consists of a single player, its total effort is simply $g_B = g_{B_1}$. The parameter γ captures a team's intrinsic strength or prior advantage. When $\gamma > 1$, team A has a prior advantage; when $\gamma < 1$, team B holds the advantage. We restrict attention to pure strategies in stage 1. Effort costs are linear, with a per-unit cost of 1, and are incurred regardless of the contest outcome.

Stage 2: Players in team A may engage in pre-play communications to coordinate their directions. More specifically, player A_1 sends a costless message $m \in M$ to A_2. The set of possible messages M depends on the regime we consider:

- *No Coordination:* Messaging is not allowed beyond an empty message. $M = \{\emptyset\}$.
- *Coordination without Sign-stealing* and *Coordination with Sign-stealing*: Player A_1 can either send an empty message or communicate her intended direction (direction U or D). The message set is $M = \{\emptyset, u, d\}$[9].

Effort choices in stage 1 are private information: each player only observes their own contest efforts. Accordingly, the history available to player i at the beginning of Stage 2 is $h_2^i = g_i$. The set of all possible stage-2 histories for player i is denoted by $\mathbf{H}_2^i$. Player A_1 may randomise over which messages to send. We define a strategy of A_1 in stage 2 as a mapping:

$$\sigma_2^{A_1} : \mathbf{H}_2^{A_1} \to \Delta(M), \qquad \sigma_2^{A_1}(g_{A_1}) = \big(p_\emptyset(g_{A_1}), p_u(g_{A_1}), p_d(g_{A_1})\big).$$

The triple $\sigma_2^{A_1}(g_{A_1})$ denotes the probability distribution over possible messages conditional on own effort g_{A_1}.

Stage 3: Each player simultaneously chooses a direction $a_i \in \{U, D\}$ for $i \in \{A_1, A_2, B_1\}$. While the message sent by A_1 is always observable to player A_2, whether player B_1 can observe the message depends on the specific coordination regime:

- *Coordination without Sign-stealing*: The message is private and only observable to A_2. Player B_1 knows that A_1 can send a message and knows M, but she cannot observe the content of the message.

[9] Implicitly here, we focus on the cases in which there exists a common language among players, and the messages carry a literal meaning all players understand in the same way, which is to communicate A_1's intended direction in the next stage.

- *Coordination with Sign-stealing*: The message is public and observable to both A_2 and B_1. This can be interpreted as communication leakage, where player B_1 gains access to team A's internal communication[10].

Accordingly, at the start of stage 3, the history of each player consists of the stage 2 history h_2^i and the observed message m, if applicable. Specifically:

- *No Coordination:* only messaging an empty message is allowed, $h_3^i = (g_i, \emptyset)\ \forall i$.
- *Coordination without Sign-stealing:* the message is only observable to A_2. Thus, $h_3^i = (g_i, m)$ for $i \in \{A_1, A_2\}$ and $h_3^{B_1} = g_{B_1}$.
- *Coordination with Sign-stealing:* the message is observable to both A_2 and B_1. Thus, $h_3^i = (g_i, m)$ for $i \in \{A_1, A_2, B_1\}$.

The set of all possible stage-3 histories for player i is denoted as $\mathbf{H}_3^i$. Players may randomise over their choices, we define a direction strategy for player i in stage 3 as a mapping: $\sigma_3^i : \mathbf{H}_3^i \to \Delta(\{U, D\})$, where $\sigma_3^i(h_3^i) = (p_U^i(h_3^i), p_D^i(h_3^i))$ represents the probability distribution over directions.

Payoffs

At the end of stage 3, the contest outcome is determined and prizes are awarded. If team A wins, each of its members receives a prize of value 1. If team B wins, player B_1 receives a prize of value 1. The players always need to pay their effort costs regardless of the contest outcome. Winning probabilities depend on both teams' contest efforts and their stage 3 directions. There are three possible cases:

1. **Miscoordination within the team** A: If A_1 and A_2 choose different directions, team A automatically loses the contest and team B wins, regardless of effort choices.
2. **Team** A **coordinates on a direction different from** B_1: For example, both A_1 and A_2 choose direction U while player B_1 chooses D. Team A gains the direction advantage, represented by the parameter $\beta > 1$.
3. **Team** A **coordinates on the same direction as** B_1: For example, all players choose direction U. Team B gains the direction advantage, represented by the parameter $\beta > 1$.

If g_A and g_B are not both zero, the probabilities of each team winning are modelled using a Tullock Lottery Contest Success Function (CSF) and summarised in the two tables below, where P_A refers to team A's winning probability and P_B refers to team B's winning probability:

[10]Equivalently, player A_1 sends the same message to both A_2 and B_1. While A_1 can choose the content of the message, she cannot control its recipient. In either case, players in team A are fully aware that their communication is visible to player B_1.

When B_1 chooses U:

(P_A, P_B)	U	D
U	$\left(\frac{g_A}{g_A+\beta g_B}, \frac{\beta g_B}{g_A+\beta g_B}\right)$	$(0,1)$
D	$(0,1)$	$\left(\frac{\beta g_A}{\beta g_A+g_B}, \frac{g_B}{\beta g_A+g_B}\right)$

Table 3.1: Winning probabilities when B_1 chooses U

When B_1 chooses D:

(P_A, P_B)	U	D
U	$\left(\frac{\beta g_A}{\beta g_A+g_B}, \frac{g_B}{\beta g_A+g_B}\right)$	$(0,1)$
D	$(0,1)$	$\left(\frac{g_A}{g_A+\beta g_B}, \frac{\beta g_B}{g_A+\beta g_B}\right)$

Table 3.2: Winning probabilities when B_1 chooses D

If $g_A = g_B = 0$, we assume that, if team A coordinates on a direction, each team wins with probability $\frac{1}{2}$, and if team A miscoordinates then team B wins.

Equilibrium

In all regimes, we adopt the solution concept of a Perfect Bayesian Equilibrium (PBE) (Fudenberg and Tirole, 1991; Mas-Colell et al., 1995). We say that a strategy profile s is part of a PBE if, at each stage, given other players' strategies and own beliefs about others' actions, each player maximises her expected payoff. Completing the description of a PBE, players' beliefs about others' efforts must be consistent with the equilibrium strategies on the equilibrium path[11].

To implement the optimality requirement for strategies, it is helpful to introduce notation describing expected payoffs calculated at the different stages of the game. At stage 1, given any strategy profile s, the expected probability of winning for player i is denoted as $EP_i(s)$. The stage 1 expected payoff for player i is given by:

$$U_1^i(s) = EP_i(s) - g_i$$

[11]That is, on the equilibrium path, players believe that others have chosen their equilibrium efforts. Since we restrict effort choices to pure strategies, players' beliefs about others' efforts are required to be degenerate (concentrated on a single effort level) on the equilibrium path.

At stage 2, for any history h_2^i and the continuation strategy profile $s(h_2^i)$, the stage 2 expected payoff for player i is:

$$U_2^i(s(h_2^i)) = EP_i(s(h_2^i))$$

At stage 3, for any history h_3^i and the continuation strategy profile $s(h_3^i)$, the stage 3 expected payoff for player i is:

$$U_3^i(s(h_3^i)) = EP_i(s(h_3^i))$$

3.2.2 Equilibrium Results

No Coordination

In the *No Coordination* regime, player A_1 is only allowed to send an empty message. A strategy profile consists of: (i) a pure effort choice g_i^* for each player $i \in \{A_1, A_2, B_1\}$, and (ii) a direction strategy σ_3^i that assigns a probability distribution over directions conditional on the player's own effort level. That is, for each g_i, the strategy specifies $\sigma_3^i(g_i) = (p_U^i(g_i), p_D^i(g_i))$.

A strategy profile and a belief system constitute a PBE if the following conditions are satisfied:

NC1 Given the strategies of the other players, each player i chooses an effort level g_i^* that maximises their expected utility in stage 1, $U_1^i(\cdot)$.

NC2 For every possible effort level, and given beliefs about others' effort choices and the resulting direction strategies, each player chooses a direction distribution that maximises their expected payoff in stage 3, $U_3^i(\cdot)$.

NC3 Players' beliefs about others' efforts are consistent with the equilibrium strategies on the equilibrium path[12].

Stage 3: Even restricting attention to this stage, there is a great multiplicity of possible PBE direction profiles. Nonetheless, we can characterise possible equilibrium outcomes in a compact way. To do so, we first identify all possible equilibrium direction choices on the equilibrium path, given belief consistency NC3. One can verify that all possible direction profiles are summarised in Table 3.3.

[12] Since effort choices are pure and privately observed, it follows that beliefs must assign probability one to the equilibrium effort of other players on the equilibrium path.

	$(p_U^{A_1}, p_D^{A_1})$	$(p_U^{A_2}, p_D^{A_2})$	$(p_U^{B_1}, p_D^{B_1})$
(1)	$(1, 0)$	$(1, 0)$	$(1, 0)$
(2)	$(0, 1)$	$(0, 1)$	$(0, 1)$
(3)	$(\frac{1}{2}, \frac{1}{2})$	$(\frac{1}{2}, \frac{1}{2})$	$(\frac{1}{2}, \frac{1}{2})$
(4)	$(q, 1-q)$	$(q, 1-q)$	$(1, 0)$
(5)	$(m, 1-m)$	$(m, 1-m)$	$(0, 1)$

Table 3.3: Possible direction choices on the equilibrium path (No Coordination)

Here, q and m depend on the (anticipated) equilibrium effort choices g_A^* and g_B^* as follows:

$$q = \frac{\frac{\beta g_A^*}{\beta g_A^* + g_B^*}}{\frac{\beta g_A^*}{\beta g_A^* + g_B^*} + \frac{g_A^*}{g_A^* + \beta g_B^*}} \;;\; m = \frac{\frac{g_A^*}{g_A^* + \beta g_B^*}}{\frac{\beta g_A^*}{\beta g_A^* + g_B^*} + \frac{g_A^*}{g_A^* + \beta g_B^*}}$$

Intuitively, if A_1 plays direction U on the equilibrium path as in row (1) of Table 3.3, A_2 also plays U to avoid direction mismatch, and B_1 also chooses U to gain a direction advantage. The same reasoning applies to the profile where all players choose D as in row (2). If both A_1 and A_2 randomise between the two directions with probability $\frac{1}{2}$ on the equilibrium path, player B_1 is indifferent between choosing U or D; team A players also feel indifferent between the two directions if the others randomise uniformly. Thus, all players randomising with probability $\frac{1}{2}$ is also a possible direction profile on the equilibrium path. If B_1 chooses U, then A_1 can only be indifferent between the two directions if A_2 chooses U with probability $q > \frac{1}{2}$ (row (4)). Similar reasoning applies to row (5) with probability $m < \frac{1}{2}$.

To illustrate the multiplicity of possible PBE strategies, we use Table 3.3 to construct possible full equilibrium direction profiles, specifying direction choices for all possible own effort levels. We further specify off-path beliefs to support each profile. From profile (1)/(2), we can construct a possible equilibrium profile in which all players choose direction U/D for every possible effort level; and any off-path beliefs about others' efforts can support this profile. With profile (3), we construct a profile in which all players always randomise between directions U and D with probability $\frac{1}{2}$, again supported by any off-path beliefs.

Profile (3) can also generate other possible direction profiles. For example, players may randomise uniformly only when they chose their equilibrium efforts, and otherwise play U deterministically. This profile is supported if each player always assigns probability one to others choosing their equilibrium efforts. Many similar direction profiles can be constructed. Profile (4) leads to a more nuanced direction profile. That is, team A members randomise between U and D with probabilities q and $1-q$ when $g_{A_1} = g_{A_1}^*$; they choose D if $g_{A_1} < g_{A_1}^*$ and U if $g_{A_1} > g_{A_1}^*$, while player B_1 always chooses U. This profile is supported by the belief system that players always believe all other players have chosen their equilibrium efforts.[13] However, none of these more nuanced strategy profiles changes the equilibrium outcomes with respect to Table 3.3.

Stage 1: Given the possible direction profiles we constructed, the equilibrium effort choices must form mutual best responses to the others' strategies. First, consider the case in which all players always randomise between U and D with probability $\frac{1}{2}$, corresponding to row (3) of Table 3.3. In this case, the probability that team A miscoordinates (i.e., chooses different directions) is $\frac{1}{2}$, leading to an immediate loss. With probability $\frac{1}{4}$, team A coordinates on the same direction as B_1, in which case team B gains a direction advantage. With the remaining probability $\frac{1}{4}$, team A coordinates on the opposite direction from B_1, obtaining a direction advantage. Each player chooses her effort to maximise her expected payoff given the effort choices of others. Specifically, player A_1 solves:

$$\max_{g_{A_1}} \frac{1}{4}\left(\frac{\gamma(g_{A_1}+g_{A_2})}{\gamma(g_{A_1}+g_{A_2})+\beta g_B}\right)+\frac{1}{4}\left(\frac{\beta\gamma(g_{A_1}+g_{A_2})}{\beta\gamma(g_{A_1}+g_{A_2})+g_B}\right)+\frac{1}{2}*0-g_{A_1}$$

The first-order condition (FOC) is $(1/4)\mu_A g_B\,\gamma+(1/4)\mu_B g_B\,\gamma=1$, where

$$\mu_A=\frac{\beta}{(\beta g_A+g_B)^2} \text{ and } \mu_B=\frac{\beta}{(g_A+\beta g_B)^2};$$

note that $\mu_A g_B$ ($\mu_B g_B$) is the marginal effect of g_A on team A's winning probability, illustrated in Tables 3.1 and 3.2, when team A has a directional advantage (disadvantage), while γ is marginal effect of g_{A1} on g_A and 1 is the marginal cost of g_{A1}. The FOC is the same for player A_2. Symmetrically, the FOC for player B_1 is $(1/4)\mu_A g_A+(1/4)\mu_B g_A=1$, where $\mu_A g_A$ ($\mu_B g_A$) is the marginal effect of g_B on team B's winning probability when team A has a directional advantage (disadvantage).

These two FOCs imply $g_A=\gamma g_B$, and, letting $\epsilon_A=\frac{\beta\gamma}{(\beta\gamma+1)^2}$ and $\epsilon_B=\frac{\beta\gamma}{(\gamma+\beta)^2}$, we obtain:

$$g_A^{*(3)}=\frac{\gamma}{4}(\epsilon_A+\epsilon_B),\ g_B^{*(3)}=\frac{1}{4}(\epsilon_A+\epsilon_B). \tag{3.1}$$

In the equilibrium, team A's total effort is $g_A^{*(3)}=\gamma(g_{A_1}^{*(3)}+g_{A_2}^{*(3)})$, where $g_A^{*(3)}$ is in (3.1) and any combination of $g_{A_1}^{*(3)}$ and $g_{A_2}^{*(3)}$ satisfying the previous equality is valid. Player B_1 exerts $g_B^{*(3)}$ in (3.1). All players always randomise uniformly between U and D regardless of their effort levels. On the equilibrium path, players' beliefs assign probability one to others' equilibrium effort levels; off-path beliefs are unrestricted[14]. When $\gamma=1$, neither team has a prior advantage and equilibrium requires equal total efforts: $g_A^{*(3)}=g_B^{*(3)}=\frac{\beta}{2(1+\beta)^2}$. When

[13] A closely related profile works as follows: players in team A randomise with probability m only when $g_{A_1}=g_{A_1}^*$; they choose direction U if $g_{A_1}<g_{A_1}^*$ and D if $g_{A_1}>g_{A_1}^*$; player B_1 always chooses D.

[14] Note that there exist other equilibria that produce the same equilibrium outcome. For example, player A_1 may always choose U when $g_{A_1}\neq g_{A_1}^{*(3)}$ and randomise uniformly only when $g_{A_1}=g_{A_1}^{*(3)}$. This is optimal under the belief system that the other players always believe A_1 has chosen her equilibrium effort. We can construct many such variations and all such variants yield the same equilibrium outcome.

$\gamma > 1$, $g_A^{*(3)} > g_B^{*(3)}$ and when $\gamma < 1$, $g_A^{*(3)} < g_B^{*(3)}$.

Now consider an alternative profile in which all players always choose the same direction - either U or D - regardless of their own effort levels, as in rows (1) and (2) of Table 3.3. Under this profile, miscoordination never occurs, and team B always holds a direction advantage. Solving in the same manner as earlier in this section, we obtain equilibrium efforts

$$g_A^{*(1)} = g_A^{*(2)} = \gamma\epsilon_B,\ g_B^{*(1)} = g_B^{*(2)} = \epsilon_B. \tag{3.2}$$

In the equilibrium, team A's total effort is $g_A^{*(r)} = \gamma(g_{A_1}^{*(r)} + g_{A_2}^{*(r)})$ for $r \in \{1, 2\}$, where $g_A^{*(1)}$ and $g_A^{*(2)}$ are in (3.2) and any combination of $g_{A_1}^{*(r)}$ and $g_{A_2}^{*(r)}$ satisfying the previous equality is valid. Player B_1's effort is $g_B^{*(r)}$ in (3.2), for $r \in \{1, 2\}$. All players always choose U or D regardless of their effort level. As before, on the equilibrium path, beliefs assign probability one to others' equilibrium effort levels; off-path beliefs are unrestricted.

Interpretation: The characterisation in (3.2) helps us interpret ϵ_B ($\gamma\epsilon_B$): it is the equilibrium total effort in team B (A) in a contest in which team B always holds the directional advantage. In a similar manner the following strategy profile (which is **not** an equilibrium of our game) helps us interpret ϵ_A. Suppose in Stage 3 players in team A choose U and player B_1 chooses D, while in Stage 1 effort choices are chosen optimally given this assumed behaviour. In this case, team A would always hold the directional advantage. Standard calculations then show how the effort in team B is ϵ_A and the total effort in team A is $\gamma\epsilon_A$. All in all, we can interpret the equilibrium efforts in (3.1) as a linear combination of the equilibrium of a contest in which team A has a directional advantage with probability $1/4$, a contest in which team B has a directional advantage with probability $1/4$, and a contest that is decided in favour of B regardless of efforts with probability $1/2$.

Finally, consider the more nuanced profile associated with rows (4) and (5) of Table 3.3, in which team A members mix with probabilities q or m when $g_i = g_i^*$ but switch to deterministic directions otherwise. We argue that such profiles cannot be supported in equilibrium, because players in team A have an incentive to deviate to a different effort level and select a deterministic direction. The details of this argument are provided in Appendix 3A.

We have identified three possible equilibrium outcomes: two in which members of team A perfectly coordinate on either U or D, and one in which members of team A randomise with equal probability on the directions. We argue that perfect coordination on any pure direction seems implausible without communication, salience cues or repeated play. Experimental evidence supporting this argument can be found in Isoni et al. (2019)[15]. We therefore view the equilibrium in which all players always randomise between U and D with probability

[15]This paper presents experimental evidence on pure coordination games with symmetric payoffs and minimal cues as benchmark cases, the evidence shows that coordination rates are close to chance level. Other experimental studies on different types of coordination games include: Crawford and Haller (1990), Mehta et al. (1994a), Mehta et al. (1994b), Crawford (1995) and Bardsley et al. (2010).

$\frac{1}{2}$ and efforts are $\{g^{*(3)}_{A_1}, g^{*(3)}_{A_2}, g^{*(3)}_{B_1}\}$ as the most plausible prediction for our experimental environment in which interactions between players should be limited.

Result 1: *In the regime of No Coordination, the more plausible equilibrium predicts that team A's aggregate effort is* $\frac{\beta\gamma^2((\beta\gamma+1)^2+(\gamma+\beta)^2)}{4(\beta\gamma+1)^2(\gamma+\beta)^2}$ *and team B's effort is* $\frac{\beta\gamma((\beta\gamma+1)^2+(\gamma+\beta)^2)}{4(\beta\gamma+1)^2(\gamma+\beta)^2}$. *All players always randomise between U and D with probability* $\frac{1}{2}$*, resulting in a coordination rate within team A of* $\frac{1}{2}$.

Coordination without Sign-stealing

In the *Coordination without Sign-stealing* regime, player A_1 can send meaningful and private messages to coordinate with her teammate. A strategy profile consists of: (i) a pure effort choice g_i^*; (ii) a messaging strategy for A_1, mapping her effort level to a probability distribution over messages in $M = \{\emptyset, u, d\}$; and (iii) a direction strategy that assigns a probability distribution over directions conditional on each player's information at the start of stage 3. More specifically, A_1 and A_2 condition their direction choices on their own respective efforts and the message, while B_1 conditions her direction choice only on her own effort.

A strategy profile and a belief system constitute a PBE if the following conditions hold:

CNS1 Given the strategies of the other players, each player i chooses an effort level g_i^* that maximises their expected utility in stage 1, $U_1^i(\cdot)$.

CNS2 Given others' strategies and beliefs about other players' effort choices, A_1 chooses a messaging strategy in stage 2 that maximises $U_2^i(\cdot)$.

CNS3 Given others' strategies and beliefs about others' effort choices, players A_1 and A_2 choose direction profiles in stage 3 that maximise $U_3^i(\cdot)$ for $i \in \{A_1, A_2\}$.

CNS4 Given others' direction strategies, beliefs about others' effort choices and about the message choice of A_1, B_1 chooses her direction profile in stage 3 to maximise $U_3^{B_1}(\cdot)$.

CNS5 Players form beliefs about others' effort choices that are consistent with equilibrium strategies on the equilibrium path. B_1's belief about A_1's message choices must also be consistent with equilibrium strategies on the equilibrium path.

We focus on the case in which there exists a common language among players, so players understand the literal meaning of the messages. Message $\emptyset$ means *"No message"*, message u means *"I intend to choose direction U"* and message d means *"I intend to choose direction D"*. Recall that communication is private in this regime: player B_1 does not observe the content of the message.

Stage 2 & 3: Suppose players chose their equilibrium efforts. In Appendix 3B, we systematically categorise all possible direction and message profiles satisfying conditions $CNS2$–$CNS4$, given belief consistency[16]. For players A_1 and A_2, we must specify their directions following each of the three possible messages. Player B_1, who does not observe the message content, cannot condition her strategies on messages directly. Since this message is cheap-talk, its literal meaning does not need to be believed literally. This generates a large set of possible equilibrium strategy profiles.

We refine the set of equilibrium profiles using the arguments in Farrell and Rabin (1996). Specifically, if the literal meaning of a message is considered credible by its receivers, it becomes common knowledge that the message will be believed literally. A message is credible if it is both *self-signalling* and *self-committing*. A message is self-signalling if A_1 wants her message to be believed literally when she intends to choose the direction stated in the message. A message is self-committing if A_1 wants to follow the stated direction when A_2 believes the message literally.

We argue that both messages u and d are credible. We first check if message u is self-signalling. Suppose A_1 intends to play U in stage 3. If A_2 always believes her messages literally, then A_1 strictly prefers sending u over d. Sending u leads A_2 to choose U, giving team A a positive probability of winning. Sending d instead would induce A_2 to choose D, causing an immediate loss for team A. Thus, if A_1 intends to play U, she has no incentives to misrepresent her intention, making u a self-signalling message. Message u is also self-committing: If A_1 sends u and A_2 believes it, A_2 will choose U, and A_1 will want to match and choose U, validating her message. The same reasoning applies to message d. Therefore, u and d satisfy both self-signalling and self-committing conditions and are credible, making it common knowledge that they will be believed literally. Under this refinement, if A_1 sends u or d, the only possible equilibrium direction profile is for A_1 and A_2 to match the direction indicated in the message. After applying this refinement, three potential profiles remain (Table 3.4)[17]. Note that B_1 only mixes here; otherwise A_1 gains a direction advantage by sending a message indicating the opposite direction.

	$(p_U^A, p_D^A)\vert\emptyset$	$(p_U^A, p_D^A)\vert u$	$(p_U^A, p_D^A)\vert d$	$(p_U^{B_1}, p_D^{B_1})$	$(p_\emptyset^{A_1}, p_u^{A_1}, p_d^{A_1})$
(1)	$(1, 0)$	$(1, 0)$	$(0, 1)$	$(\frac{1}{2}, \frac{1}{2})$	$(p_\emptyset^{A_1} + p_u^{A_1} = \frac{1}{2}, \frac{1}{2})$
(2)	$(0, 1)$	$(1, 0)$	$(0, 1)$	$(\frac{1}{2}, \frac{1}{2})$	$(p_\emptyset^{A_1} + p_d^{A_1} = \frac{1}{2}, \frac{1}{2})$
(3)	$(\frac{1}{2}, \frac{1}{2})$	$(1, 0)$	$(0, 1)$	$(\frac{1}{2}, \frac{1}{2})$	$(0, \frac{1}{2}, \frac{1}{2})$

Table 3.4: Stage 2&3 profiles after refinement (Coordination without Sign-stealing)

Using the profile in row (3), we can construct a full message and direction profile as follows: Upon receiving messages u or d, players in team A always choose the direction indicated in

[16] To summarise the possible profiles systematically, we temporarily restrict the belief system so that players continue to believe others have chosen their equilibrium efforts, even after receiving a message off the equilibrium path (i.e., a message sent with zero probability).

[17] Since two team A members choose the same profile, they are combined into one column.

the message. Upon receiving an empty message, players in team A always randomise with probability $\frac{1}{2}$. Player B_1 always randomise uniformly between U and D. Player A_1 always randomise between sending messages u and d with probability $\frac{1}{2}$. This is supported by any off-path beliefs regarding others' efforts, and B_1's beliefs about A_1's messaging choice are always correct[18]. Similarly, we can construct full message and direction profiles for profiles in rows (1) and (2).

Stage 1: For all equilibrium messaging and direction profiles we described above, miscoordination within team A does not occur. With probability $\frac{1}{2}$, team A coordinates on a direction different from B_1's choice, gaining a direction advantage; with probability $\frac{1}{2}$, they coordinate on the same direction, resulting in a disadvantage. Given these probabilities, A_1 chooses g_{A_1} to maximise her expected payoff:

$$\max_{g_{A_1}} \frac{1}{2}\left(\frac{\gamma(g_{A_1}+g_{A_2})}{\gamma(g_{A_1}+g_{A_2})+\beta g_B}\right)+\frac{1}{2}\left(\frac{\beta\gamma(g_{A_1}+g_{A_2})}{\beta\gamma(g_{A_1}+g_{A_2})+g_B}\right)-g_{A_1}$$

Using the same first-order approach as in Section 3.2.2, the equilibrium team efforts are:

$$\tilde{g}_A^* = \frac{\gamma}{2}(\epsilon_A+\epsilon_B),\ \tilde{g}_B^* = \frac{1}{2}(\epsilon_A+\epsilon_B). \tag{3.3}$$

In the equilibrium, total team A effort is $\tilde{g}_A^* = \gamma(\tilde{g}_{A_1}^* + \tilde{g}_{A_2}^*)$ as in (3.3), any combination of $(\tilde{g}_{A_1}^*, \tilde{g}_{A_2}^*)$ satisfying the previous equality is valid. Player B_1 exerts effort $\tilde{g}_B^*$ as in (3.3). Upon receiving messages u or d, A_1 and A_2 always choose the direction indicated in the message for all efforts. Player B_1 always chooses each direction with probability $\frac{1}{2}$. Note here that regardless of team A members' direction profiles upon receiving an empty message (as in Table 3.4), the resulting equilibrium contest efforts and coordination success are the same, though the implied messaging behaviour differs. To pin down messaging behaviour more precisely, we argue as follows: an empty message conveys no meaningful information for coordinating on a pure direction. Consequently, the profile in which team A members always randomise with probability $\frac{1}{2}$ upon receiving an empty message (row (3) in Table 3.4) is the most plausible selection. Therefore, player A_1 always randomises between sending messages u and d with probability $\frac{1}{2}$. In terms of the belief system, players believe others have chosen their equilibrium efforts on the equilibrium path. Off-path beliefs about efforts may be arbitrary, and B_1's beliefs about A_1's messaging choice are always correct[19].

The equilibrium efforts in (3.3) can be interpreted in the same manner as in *No Coordination*

[18] There also exist possible alternative profiles where A_1 and A_2 randomise between U and D with probability $\frac{1}{2}$ upon receiving empty messages only for certain effort levels (e.g. equilibrium efforts) and play a pure direction otherwise. These direction profiles are optimal if players believe others exert efforts leading to randomisation. Similarly, there may be profiles where B_1 or A_1 randomise only under certain effort levels. However, all such variants yield the same equilibrium outcome as row (3) in Table 3.4.

[19] Other equilibria exist with slightly different off-path strategies. For example, A_1 chooses U upon receiving an empty message when $g_{A_1} \neq \tilde{g}_{A_1}^*$, but randomises when $g_{A_1} = \tilde{g}_{A_1}^*$. This is optimal if others believe A_1 always chose her equilibrium effort. Similarly, A_1 sends u or d for off-path effort levels, while randomising only at $\tilde{g}_{A_1}^*$. All such variants are equivalent in terms of the equilibrium outcomes.

as a linear combination of the equilibrium of a contest in which team A has a directional advantage with probability $1/2$ and a contest in which team B has a directional advantage with probability $1/2$. When $\gamma = 1$, no team has a prior advantage and $\tilde{g}_A^* = \tilde{g}_B^* = \frac{\beta}{(1+\beta)^2}$. When $\gamma > (<)1$, the equilibrium effort of team A is larger (smaller) than that of team B.

Result 2: *In the Coordination without Sign-stealing regime, the equilibrium predicts that team A's aggregate effort is $\frac{\beta\gamma^2((\beta\gamma+1)^2+(\gamma+\beta)^2)}{2(\beta\gamma+1)^2(\gamma+\beta)^2}$ and team B's effort is $\frac{\beta\gamma((\beta\gamma+1)^2+(\gamma+\beta)^2)}{2(\beta\gamma+1)^2(\gamma+\beta)^2}$. Contest efforts are higher than in the No Coordination regime.*

Result 3: *In the Coordination without Sign-stealing regime, the message sender in team A always sends non-empty messages and team A coordinates perfectly on directions. Each team has a $\frac{1}{2}$ probability of gaining a direction advantage.*

The intuition for the higher contest efforts relative to Result 1 for the *No Coordination* regime is that eliminating miscoordination within team A raises the 'effective prize' of the contest for all parties, thereby incentivising higher investment in the contest.

Coordination with Sign-stealing

In the *Coordination with Sign-stealing* regime, team B gains access to team A's internal communication, so the message is also observable to B_1. A strategy profile consists of: (i) a pure effort choice g_i^* for each player $i \in \{A_1, A_2, B_1\}$; (ii) a messaging strategy for player A_1, mapping her effort to a message in $M = \{\emptyset, u, d\}$, allowing for randomisation; (iii) a direction strategy for each player that assigns a probability distribution over directions conditional on the message and their own effort levels. The key difference from the case without sign-stealing is that B_1 can now condition her direction choice on the observed message.

A strategy profile and belief system constitute a PBE if the following conditions hold:

CS1 Given the strategies of the other players, each player i chooses an effort level that maximises their expected utility in stage 1, $U_1^i(\cdot)$.

CS2 Given the others' strategies and beliefs about others' efforts, A_1 chooses a messaging strategy in stage 2 that maximises $U_2^{A_1}(\cdot)$.

CS3 Given others' strategies and beliefs about others' efforts, each player i chooses direction profiles that maximise their expected utility in stage 3, $U_3^i(\cdot)$.

CS4 Players form beliefs about others' effort choices that are consistent with the equilibrium strategies on the equilibrium path.

Stage 3: Suppose players have chosen their equilibrium effort levels in stage 1. We first identify all possible equilibrium direction profiles, specifying how each player responds to every possible message. Initially, we impose no restrictions on whether messages are believed literally. This results in a large set of possible profiles, which we characterise systematically in

Appendix 3C[20].

As before, we assume that if a message's literal meaning is deemed credible by its receivers, it becomes common knowledge that it will be believed literally. The key difference under sign-stealing is that B_1 also observes the message, so her beliefs become relevant. We apply the same credibility conditions as in *Coordination without Sign-stealing*: a message is credible if it is both *self-signalling* and *self-committing*. We argue that both messages u and d remain credible under sign-stealing. To illustrate, consider message u. It is self-signalling: If A_1 truly intends to play U, and if both A_2 and B_1 believe her message, A_1 prefers sending the truthful message u to sending d. Misrepresenting her choice causes miscoordination with A_2 and this loss is larger then the gain from "fooling" B_1. Specifically, if A_1 sends u and both receivers believe it, both A_2 and B_1 select U, giving A_1 a positive probability of winning despite the directional disadvantage. Sending d instead would lead both to choose D, causing team A to lose. The message u is also self-committing: If A_1 sends message u and both receivers believe it literally, both A_2 and B_1 choose U. In this case, it is optimal for A_1 to follow through her message and choose U. The same reasoning applies to d. Thus, both messages u and d are credible and will be believed literally. Under this refinement, whenever A_1 sends messages u or d, all players choose directions indicated in the message. This reduces the set of direction profiles that generate equilibrium outcomes to those in Table 3.5.

	$(p_U^A, p_D^A)\|\emptyset$	$(p_U^A, p_D^A)\|u$	$(p_U^A, p_D^A)\|d$	$(p_U^{B_1}, p_D^{B_1})\|\emptyset$	$(p_U^{B_1}, p_D^{B_1})\|u$	$(p_U^{B_1}, p_D^{B_1})\|d$
(1)	$(1,0)$	$(1,0)$	$(0,1)$	$(1,0)$	$(1,0)$	$(0,1)$
(2)	$(0,1)$	$(1,0)$	$(0,1)$	$(0,1)$	$(1,0)$	$(0,1)$
(3)	$(\frac{1}{2},\frac{1}{2})$	$(1,0)$	$(0,1)$	$(\frac{1}{2},\frac{1}{2})$	$(1,0)$	$(0,1)$
(4)	$(q,1-q)$	$(1,0)$	$(0,1)$	$(1,0)$	$(1,0)$	$(0,1)$
(5)	$(m,1-m)$	$(1,0)$	$(0,1)$	$(0,1)$	$(1,0)$	$(0,1)$

Table 3.5: Direction profiles after refinement (Coordination with Sign-stealing)

Note that upon receiving an empty message, the five possible direction profiles are identical as in the *No Coordination* regime. As in the previous sections, there is a great multiplicity of full PBE direction profiles that "implement" the equilibrium outcomes in Table 3.5. Take row (3): Upon receiving an empty message, all players always randomise with probability $\frac{1}{2}$; Upon receiving messages u or d, all players always choose the direction indicated in the message. Any off-path beliefs support this profile[21]. Similarly for other rows in Table 3.5.

[20] As in the *Coordination without Sign-stealing* regime, we assume temporarily that players continue to believe others have chosen their equilibrium efforts even when receiving off-equilibrium messages (i.e., messages sent with probability zero).

[21] We can construct alternative profiles upon receiving an empty message using rows (4) and (5). For example, players in team A choose U with probability q conditional on their equilibrium efforts; they choose U when effort is greater than the equilibrium effort; and they choose D when effort is smaller than the equilibrium effort. Player B_1 always chooses direction U. This profile can be supported by the belief that all players always believe the others have chosen their equilibrium efforts. We can construct a similar profile for row (5) in Table 3.5.

Stage 2: Given how players react to each message, player A_1 aims to choose a messaging strategy that maximises her probability of winning given any own efforts. If A_1 sends a non-empty message (u or d), there is no miscoordination within team A and team B always gains a direction advantage. A_1's probability of winning (for any g_{A_1} and beliefs about others' efforts) is given by:

$$\frac{g_A}{g_A + \beta g_B}$$

If she sends an empty message, her expected winning probability depends on players' strategies upon receiving an empty message. Suppose all players always randomise between U and D with probability $\frac{1}{2}$, her expected winning probability is:

$$\frac{1}{4}\frac{g_A}{g_A + \beta g_B} + \frac{1}{4}\frac{\beta g_A}{\beta g_A + g_B}$$

This means that A_1 prefers to send a non-empty message if

$$\frac{g_A}{g_A + \beta g_B} \geq \frac{1}{4}\frac{g_A}{g_A + \beta g_B} + \frac{1}{4}\frac{\beta g_A}{\beta g_A + g_B} \iff 2\beta g_A + (3 - \beta^2) g_B \geq 0$$

The validity of this condition depends on the values of β, g_A and g_B. Notably, if β is not too large ($\beta \leq \sqrt{3}$), the inequality holds for any values of g_A and g_B. Intuitively, if the direction advantage team A can gain from mismatching the direction with team B is not so large, A_1 avoids the risk of miscoordination within team A by sending non-empty messages. However, if the direction advantage is sufficiently large, it may become worthwhile for A_1 to increase the chance of gaining the direction advantage, even with the risk of miscoordination within its team. We restrict attention to sufficiently small β, ensuring A_1 always prefers to send non-empty messages. A_1 remains indifferent between sending u or d. As a result, randomising between sending u and d with any probabilities is optimal in stage 2.

We also consider cases where players in team A randomise with probability q or m upon receiving an empty message. We arrive at a similar conclusion: A_1 always prefers to send non-empty messages when β is sufficiently small. We include the details in Appendix 3D.

Stage 1: For all possible direction profiles we discussed in Table 3.5 and the corresponding messaging choices, team A members always coordinate on their directions and B_1 always matches team A's direction, thus team A is always disadvantaged in the contest. Players' effort choices must be mutual best responses to each other. Player A_1's objective function is:

$$\max_{g_{A_1}} \frac{\gamma(g_{A_1} + g_{A_2})}{\gamma(g_{A_1} + g_{A_2}) + \beta g_B} - g_{A_1};$$

the familiar first-order approach leads to the following equilibrium efforts:

$$\hat{g}_A^* = \gamma \epsilon_B, \ \hat{g}_B^* = \epsilon_B. \tag{3.4}$$

In the equilibrium, total team A effort is $\hat{g}_A^* = \gamma(\hat{g}_{A_1}^* + \hat{g}_{A_2}^*)$ as in (3.4), any combination of $\hat{g}_{A_1}^*$ and $\hat{g}_{A_2}^*$ satisfying the previous equality is valid. Player B_1 exerts effort $\hat{g}_B^*$ as in (3.4). Upon receiving message u, all players choose U, regardless of their own efforts. Upon receiving message d, all players always choose D.

Upon observing an empty message $\emptyset$, various direction profiles can be part of an equilibrium. For example: all players always choose pure directions U or D; all players always randomise between U and D with probability $\frac{1}{2}$; players condition their direction on their own effort levels, mixing with probability $\frac{1}{2}$ at equilibrium effort levels and playing pure directions otherwise; team A members randomise with probabilities q or m, while B_1 plays either U or D. All these strategies result in the same equilibrium contest efforts and coordination success, though the implied messaging behaviour differs. To pin down the messaging behaviour more precisely, we argue as follows: an empty message conveys no meaningful information for coordinating on a pure direction. Consequently, the profiles in which team A members always choose a pure direction upon receiving an empty message (rows (1) and (2) in Table 3.5) are not so plausible. The remaining profiles suggest that A_1 always sends non-empty messages but is indifferent between which messages to send. In all cases, players believe others have chosen their equilibrium efforts on the equilibrium path. Any off-path beliefs support the profile in which all players always randomise between U and D with probability $\frac{1}{2}$, while the other examples require beliefs in which players always believe others have chosen their equilibrium efforts.

Result 4: *In the Coordination with Sign-stealing regime, the equilibrium predicts that team A's total effort is* $\frac{\beta\gamma^2}{(\gamma+\beta)^2}$ *and team B's effort is* $\frac{\beta\gamma}{(\gamma+\beta)^2}$*. Compared to the case without sign-stealing, teams' efforts are the same when* $\gamma = 1$*, higher when* $\gamma > 1$ *and lower when* $\gamma < 1$.

Result 5: *In the Coordination with Sign-stealing regime, the sender in team A always sends non-empty messages and team A members coordinate on their directions perfectly. Moreover,* B_1 *always matches the direction indicated in the message.*

To understand the comparison in Result 4, recall that, in the equilibrium of the no-sign-stealing regime, each team receives the directional advantage β with probability $1/2$. The resulting sum of teams' total equilibrium efforts is then $\left(\frac{1}{2}\epsilon_A + \frac{1}{2}\epsilon_B\right)(1+\gamma)$, as we see from (3.3). In contrast, in the sign-stealing regime team B always gets the directional advantage and total equilibrium efforts are $(\epsilon_B)(1+\gamma)$, as we see from (3.4). Thus, with probability $1/2$ the directional advantage switches from A to B when switching from no sign-stealing to sign-stealing. Exploiting the linearity of equilibrium efforts in the probability of each team getting a directional advantage we see from (3.3) and (3.4), the comparison of equilibrium efforts then boils down to the difference between ϵ_A and ϵ_B: are teams' total efforts larger in a contest in which the directional advantage goes to A or B? The answer depends on the value of γ.

If $\gamma = 1$, then no team has a prior advantage and the team switch in directional advantage perfectly offsets: $\epsilon_A = \epsilon_B$, as the name of the (directionally) advantaged team does not matter if no team has a prior advantage. When $\gamma > 1$, team A has a prior advantage, so the direction advantage that B gains with sign-stealing balances the playing field and both teams' efforts increase with respect to the no sign-stealing regime. When $\gamma < 1$, team A has a prior disadvantage and sign-stealing further exacerbates this imbalance, resulting in lower effort levels from both teams.

3.3 Experiment Design and Hypotheses

3.3.1 Experimental Design

Overview

Our laboratory experiment mirrors the team contest framework of the theoretical model. Two teams compete for a prize, making both upfront effort choices and subsequent action choices, and both choices affect their probability of winning. We used a between-subject design with three treatments corresponding to the three regimes over multiple rounds. We refer to the *No Coordination* treatment as *NC*, *Coordination without Sign-stealing* treatment as *CNS* and *Coordination with Sign-stealing* treatment as *CS*. The experiment pursued three objectives: first, to empirically test our main theoretical predictions; second, to allow for empirical equilibrium selection across regimes; and third, to identify potential behavioural effects of sign-stealing that go beyond our model.

Structure of the experiment

The experiment consisted of three parts presented in a fixed order. Part 1 was the main decision task: each subject took part in the team contest for exactly one regime/treatment over 30 rounds. Player roles (Team A member or Team B member) were randomly assigned before the first round and remained fixed throughout the 30 rounds. In each round, subjects were randomly re-matched into groups of three within matching groups of 12 by stranger matching. Four rounds were randomly selected for payment. Part 2 consisted of five incentivised post-tests measuring the joy of winning, impulsive decision-making, risk attitudes, ambiguity aversion and loss aversion[22]. Part 3 was a non-incentivised questionnaire including demographic questions and self-reported risk attitudes.

Before the experiment started, subjects received the instructions for part 1 in hard copy, which were also read out aloud via audiotape. Subsequently, they answered a set of com-

[22] For the joy of winning, subjects form groups of three and play a standard Tullock lottery contest with a prize value of 0 (Sheremeta, 2010). Impulsive decision-making was elicited by the CRT (Frederick, 2005). Risk attitudes were elicited using the method in Holt and Laury (2002). Ambiguity aversion was measured via an urn task (Ellsberg, 1961; Halevy, 2007). Loss aversion was elicited by asking subjects whether to accept or reject mixed-gamble lotteries involving both gains and losses (Kahneman and Tversky, 1979; Tom et al., 2007).

prehension questions and completed two practice rounds in their randomly assigned role. During the main decision task in part 1, we also elicited subjects' beliefs about other group members' decisions every 5 rounds. Upon completion of part 1, subjects were asked about their understanding of the experiment and their motivations for their decisions before proceeding to part 2.

Procedure of the main experimental task

In each round of the main decision task, a team of two subjects (Team A) competes against a team of one subject (Team B). Each member of the winning team earns a prize worth 400 points (£4), while members of the losing team earn nothing. A round involves three stages in the *No Coordination* treatment and four stages in the *Coordination without Sign-stealing* and *Coordination with Sign-stealing* treatments:

- **Investment Stage – All treatments**: Each subject receives a round endowment of 250 points (£2.50). Subjects decide how many of the endowment points to invest into team Competition Points and how many to keep themselves. All investments must be paid and all points kept themselves will be paid out to them if the round is selected for payment.

- **Communication Stage – CNS and CS only**: Subjects are allowed to send a message about their intended action in the Action Stage. They can choose one of three preformatted messages: *No message*, *I intend to choose action U* and *I intend to choose action D*. Both team A members make the message choice, but only one team A member is randomly selected as the Sender in each round. Only the message chosen by the Sender is sent, while the other team A member automatically becomes the Receiver.

- **Action Stage – All treatments**: Each subject chooses one of two actions: Action U or Action D. Subjects' action choices determine (i) whether team A qualifies for the Competition Stage and (ii) whether team B gets an advantage or a disadvantage if team A qualifies. Team A qualifies only if both members in team A choose the the same action; otherwise team A is disqualified and team B wins automatically. If team A qualifies and both teams choose the same action, team B gets an advantage: team B's total Competition Points are its invested endowment points multiplied by 1.5. If team A chooses a different action from team B, team B gets a disadvantage: team B's total Competition Points are its invested endowment points multiplied by 0.67.

- **Competition Stage – All treatments**: This stage determines the winning team via a Tullock contest if team A qualifies in the Action Stage[23]. Each team's winning probability is calculated as its Competition Points divided by the total Competition Points of both teams. A fortune wheel, with coloured segments proportional to teams' winning probabilities (red for team A and blue for team B), is then spun to determine the winner.

[23]Subjects are first informed whether team A qualifies and, if so, whether team B gets an advantage or a disadvantage.

Across all treatments, subjects receive full feedback after each round on all group members' payoffs, investment choices, action choices and the message sent if applicable.

Key treatment characteristics

The three treatments differ in the following key characteristics:

- *NC*: Team A members are not allowed to send messages to coordinate their actions. Teams move directly from the Investment Stage to the Action Stage.
- *CNS*: Team A members are allowed to send a message to coordinate their actions in the Communication Stage. This message is private and only visible to the Receiver in team A. Team B sees only a waiting screen and subsequently makes its action choice.
- *CS*: Team A members are allowed to send a message to coordinate their actions in the Communication Stage. This message is public and visible to both the Receiver in team A and the opposing team B member. Importantly, sign-stealing is framed neutrally and subjects are simply informed that the message is public to both teams.

Implementation

The experiment was conducted in the CeDEx Laboratory at the University of Nottingham, UK. The experiment consisted of 10 sessions held in June 2025, and was divided across treatments as follows: 4 sessions/7 matching groups for CS, 3 sessions/6 matching groups for CNS and 3 sessions/6 matching groups for NC. In 9 out of 10 sessions, 24 participants took part; in the remaining session, 12 participants participated. In total, 228 participants were involved in the main experiment. Each session lasted around 60-90 minutes, and participants earned on average £24. Additionally, we conducted a pilot with 12 participants for technical pre-check prior to the main experiment. The experiment was computerised and implemented using the software oTree (Chen et al., 2016). The ethical approval of the experiment is obtained from the German Association for Experimental Economic Research. The experiment was pre-registered at AsPredicted.org.

3.3.2 Main Hypotheses

Our hypotheses directly follow from the theoretical framework in section 3.2. In order to keep the experimental design intuitive and simple, we chose the parameters $\gamma = 1$ and $\beta = 1.5$. This implies that neither team has a prior advantage and we abstract from introducing this dimension to the experiment. Second, it implies that the sender in team A should always prefer to send non-empty messages, both with and without sign-stealing. Third, the parameters allow for a straightforward computation of the (dis)advantage of matching actions between teams. We formulate the following main hypotheses based on our theoretical results:

***Hypothesis 1 – Effort Choices**: For both teams, total team effort is...*
(i) lower in NC than in CNS;
(ii) lower in NC than in CS;
(iii) not different in CNS and CS.

***Hypothesis 2 – Communication**: The probability that a team A member sends a non-empty message is equivalent in CNS and CS.*

***Hypothesis 3 – Action Matches**: The probability of miscoordination within team A is...*
(i) lower in CNS and CS than in NC;
(ii) not different in CNS and CS.

3.4 Experimental Results

Our analysis unfolds in two steps. In this section, we systematically test the pre-registered hypotheses and model predictions concerning effort choices, communication behaviour, and coordination successes at the treatment level. The following section 3.5 provides additional insights into behaviour in the 'No Coordination' treatment and is of exploratory nature.

For consistency, all reported test-statistics and p-values are based on random-effects GLS panel regressions for effort choices, and on random-effects linear probability panel regressions for communication and coordination success (binary variable). The independent variables are treatment dummies, with the treatment 'No Coordination' serving as the reference group. Standard-errors are clustered at the matching-group level ($N = 12$). The results are robust to alternative specifications[24].

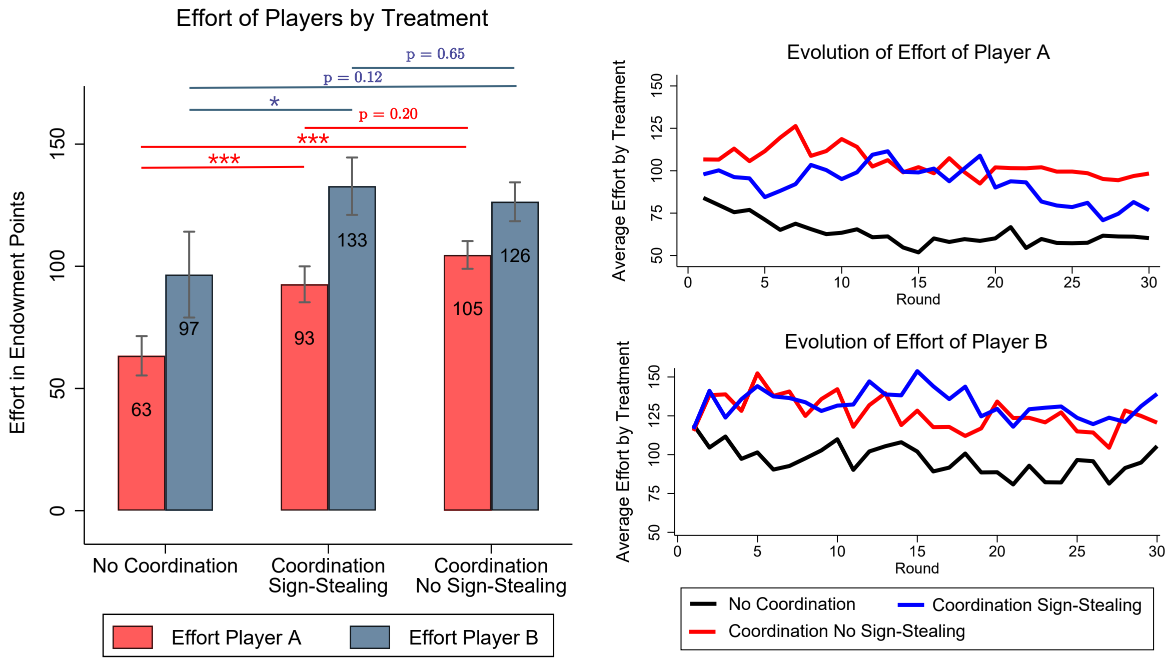
3.4.1 Effort choice by players in stage 1

Aggregate behavior

We start our analysis by examining players' investment decisions in the first stage. Figure 3.1 (left) displays average effort choices by player type and treatment. For team A, total investment is twice the average effort of player A.

In the baseline treatment *'NC'*, where communication is not allowed, player As (red bars) invest on average 63 endowment points in the team competition ($N = 1440$). This corresponds to 25% of their round endowment and 16% of the prize value. In treatment *'CNS'*, where communication is possible, they invest 105 endowment points ($N = 1680$). Hence, the possibility to coordinate their actions leads to a highly significant increase of player As'

[24] These include panel probit specifications for binary variables, non-parametric tests, and an ultra-conservative approach that averages outcome variables by player type at the matching group level. The ultra-conservative approach yields higher p-values due to the small number of independent observations ($N = 19$), but most effects remain significant at the $p < 0.1$ level.



Note: The left panel shows the average effort invested over the 30 rounds by player type and treatment. Red (Blue) bars refer to player A (player B). The upper (lower) right panel shows the evolution of player As' (player Bs') effort over rounds by treatment. Error bars indicate mean +/- SE (as estimated by a random effects GLS panel regression with standard errors clustered at the matching-group level, see specification (1a)/(1b) in Table 3.6). Brackets indicate significant treatment differences, $^{*}p < 0.1$. and $^{***}p < 0.01$.

Figure 3.1: Effort by player type and treatment.

effort by 67% ($p < 0.001$). This effect of communication persists when the communication is observable by player B. In treatment *'CS'*, player As invest on average 93 endowment points ($N = 1440$), which also represents a substantial and highly significant increase of 48% relative to *'NC'* ($p = 0.007$). We find no evidence that sign-stealing by player B has a large deterrent effect: the difference in average player As' effort between *'CNS'* and *'CS'* is 12 endowment points. This corresponds to a relative decrease of 11% from *'CNS'* to *'CS'*, which does not reach statistical significance ($p = 0.196$). Hence, in line with our model prediction, the ability to communicate intended action choices is decisive for player A's effort choice, while sign-stealing by player B plays at most a secondary role.

Result 1*: In line with Hypotheses 1,*
(i) Player As exert more effort in CNS than in NC;
(ii) Player As exert more effort in CS than in NC;
(iii) There is no significant difference in player As' effort between CNS and CS.

Treatment effects on effort choices of player Bs (blue bars) mirror those of player As. In the baseline treatment *'NC'* ($N = 720$), player Bs invest on average 97 endowment points, corresponding to 39% of their round endowment and 24% of the prize value. When player As can coordinate their actions in *'CNS'*, player Bs' effort increases to 126 endowment points

($N = 720$), a 30% increase relative to *'NC'* ($p = 0.122$)[25]. In *'CS'*, player Bs invest 133 endowment points ($N = 840$). Thus, the possibility for player Bs to observe team A's message significantly increases player Bs' effort relative to *'NC'* ($p = 0.087$), but has no additional effect compared to *'CNS'* ($p = 0.654$). Consistent with the model's prediction, the ability of player As to coordinate their actions significantly increases player B' effort choices, while sign-stealing by player B has no additional impact on its effort.

Result 2*: In line with Hypotheses 1,*
(i) Player Bs exert more effort in CNS than in NC;
(ii) Player Bs exert more effort in CS than in NC;
(iii) There is no significant difference in player Bs' effort between CNS and CS.

Figure 3.1 (right) displays the evolution of effort over time for player As (upper panel) and Player Bs (lower panel). Overall, investment decisions of both player types are remarkably stable across rounds. For player As, treatment difference emerge immediately: they invest more in *'CNS'* and *'CS'* than in *'NC'* from round 1 onwards. While effort levels in *'CNS'* and *'CS'* remain broadly stable in early rounds, player As' effort in *'NC'* declines from 84 points in round 1 to about 60 points from round 10 onward, suggesting slower convergence than other treatments. Toward the end of the experiment, player As invest roughly 20 points less in *'CS'* than in *'CNS'*. For player Bs, treatment differences become persistent from round 5 onward.[26] Their investment decisions in *'CNS'* and *'CS'* show little variation and are almost congruent over time, while effort in *'NC'* declines from 120 points in round 1 to around 100 points from round 5 onward. Hence, the effort trajectories of both player types move largely in parallel across treatments and over rounds. Taken together, these dynamics suggest only limited learning effects, as behaviour in early rounds does not differ qualitatively from behaviour in later rounds.

Result 3*: For both player As and player Bs, effort decisions are remarkably stable over time.*

Multivariate analysis

We corroborate our findings with a multivariate analysis. Table 3.6 reports results of a random effects GLS panel regression of the investment decisions (measured in endowment points) for player As (specifications (1a)–(5a)) and player Bs (specifications (1b)–(5b)) as the dependent variable. In all specifications, the treatment *'NC'* serves as the reference group, and standard errors are clustered at the matching-group level. The two baseline specifications (1a) and (1b) replicate the treatment effects presented in Section 3.4.1. Most importantly, none of the alternative specifications (2a/b)–(5a/b) alter the treatment effects.

Alternative specifications (2a) and (2b) add a dummy for female and non-binary gender

[25] Statistical power to detect treatment effects is lower for player Bs' effort choices as the team structure implies twice as many observations for player As.

[26] In the first round, effort choices are similar across treatments, but differences emerge as early as round 2.

Table 3.6: Impact of Treatments on Effort Choice

	Player A					Player B				
	(1a)	(2a)	(3a)	(4a)	(5a)	(1b)	(2b)	(3b)	(4b)	(5b)
Treatment CNS	41.26***	43.00***	43.00***	38.00***	44.50***	29.81*	28.62*	28.62*	32.96**	30.47
	(8.41)	(8.71)	(8.71)	(8.33)	(10.29)	(16.74)	(16.77)	(16.77)	(14.66)	(19.95)
Treatment CS	29.25***	30.51***	30.51***	28.04***	26.59**	36.18**	35.33**	35.33**	46.52**	38.43*
	(8.11)	(8.27)	(8.27)	(10.18)	(11.46)	(16.13)	(16.15)	(16.15)	(19.39)	(21.50)
Non-Male		-5.57	-5.57	-2.06	-5.70		-14.34	-14.34	-22.06*	-15.11
		(7.08)	(7.08)	(5.07)	(5.06)		(13.72)	(13.72)	(12.89)	(11.87)
Learning			18.21***					7.87		
			(3.44)					(5.06)		
Joy of Winning				34.94***					68.57**	
				(8.92)					(31.27)	
Prev. Opponent Effort					0.08***					0.10***
					(0.02)					(0.01)
Prev. Team Member Effort					0.12***					
					(0.03)					
Constant	63.38***	65.70***	63.28***	47.67***	48.61***	96.60***	106.16***	105.11***	90.96***	88.51***
	(5.95)	(6.65)	(6.66)	(9.73)	(8.26)	(11.84)	(14.96)	(14.97)	(19.98)	(18.24)
Observations	4560	4560	4560	4560	4408	2280	2280	2280	2280	2204
Diff CNS - CS: $p - Value$	0.139	0.125	0.125	0.331	0.027	0.693	0.677	0.677	0.374	0.527
Further Post-Tests	No	No	No	Yes	No	No	No	No	Yes	No
Further Lagged Behaviour	No	No	No	No	Yes	No	No	No	No	Yes
Between-R^2	0.1466	0.1502	0.1502	0.2259	0.1935	0.0707	0.0845	0.0845	0.2210	0.1324

Note: The table presents results of a random effects panel GLS specification with players' effort as the dependent variable. Specifications (1a)–(5a) refer to player As' effort, specifications (1b)–(5b) refer to player Bs' effort. The reference group in all specifications is the 'No Coordination' Treatment. 'Non-Male' is a dummy for female and non-binary subjects, 'Learning' is defined as $1/Round$, 'Joy of Winning' is a subject's effort choice in a contest with a prize of 0 (Sheremeta, 2010), 'Prev. Opponent Effort' and 'Prev. Team Member Effort' refer to effort of the opponent team and effort of the team member encountered in the previous round. 'Further Post-Tests' include the Cognitive Reflection Test (Frederick, 2005) and attitudes towards risk (Holt and Laury, 2002), ambiguity (Ellsberg, 1961; Halevy, 2007) and losses (Kahneman and Tversky, 1979; Tom et al., 2007). 'Further Lagged Behavior' includes a dummy for winning in the previous round and interaction terms for lagged opponent effort/team member effort and treatment dummies. Robust SEs are clustered at the matching-group level and reported in brackets. $^{*}\ p < .10$, $^{**}\ p < .05$, $^{***}\ p < .01$

identity. We employ this gender dummy throughout our alternative specifications. Male participants tend to invest more across both player types, but this effect is not statistically significant (except in specification (4b)). Specifications (3a) and (3b) control for learning and convergence effects. The variable 'Learning' is defined as 1 divided by the round number, capturing stronger learning and convergence effects in early rounds.[27] For player As, the coefficient is positive and highly significant ($p = 0.012$). It implies that player As invest, on average, 17.6 endowment points more in the first round than in the last round. For player Bs, no relevant learning effect is detected ($p = 0.479$).

Specifications (4a) and (4b) control for behavioural measures elicited in the post-tests. We include subjects' attitude towards risk (Holt and Laury, 2002), losses (Kahneman and Tversky, 1979; Tom et al., 2007) and ambiguity (Ellsberg, 1961; Halevy, 2007) in small stake environments, impulsive decision-making (Frederick, 2005) and the joy of winning (Sheremeta, 2010). Among these, only the joy of winning is significantly associated with higher efforts ($p < 0.001$ and $p = 0.028$): A player A/player B who allocates her entire post-test endowment to a zero-prize contest, on average, invests 35/69 endowment points more in the main experiment than one who invests nothing in the zero-price-contest. No significant relationships emerge for impulsivity or attitudes toward risk, ambiguity or losses.

Finally, specifications (5a) and (5b) control for individual-specific experience in the previous round. The specifications include lagged winning, lagged effort of the opponent team and lagged effort of the team member (for player As only), and the interaction of the two latter variables with treatment dummies to account for treatment-specific effort levels.[28] Effort of the opponent team in the previous round predicts higher effort in the current round ($p < 0.001$ for both player types). The estimated effect size is similar across player types and treatments: each additional point invested by the previous opponent team increases current effort by 0.08-0.10 points. In addition, player As respond positively to their teammate's previous effort, investing 0.12 additional points for each extra point invested by their prior teammate ($p < 0.001$)[29]. By contrast, winning the prize in the previous round has only a marginal effect for player As (estimate is 2.39 points, $p = 0.106$), but significantly increases player Bs' effort (estimate is 8.17 points, $p < 0.001$).

Result 4: *Treatment differences in effort choices are robust after controlling for procedural and socio-economic control variables, behavioural traits and experience in the previous rounds.*

[27] Alternative measures for learning that do not impose a functional form, such as a dummy variable for early rounds, lead to qualitatively similar results.

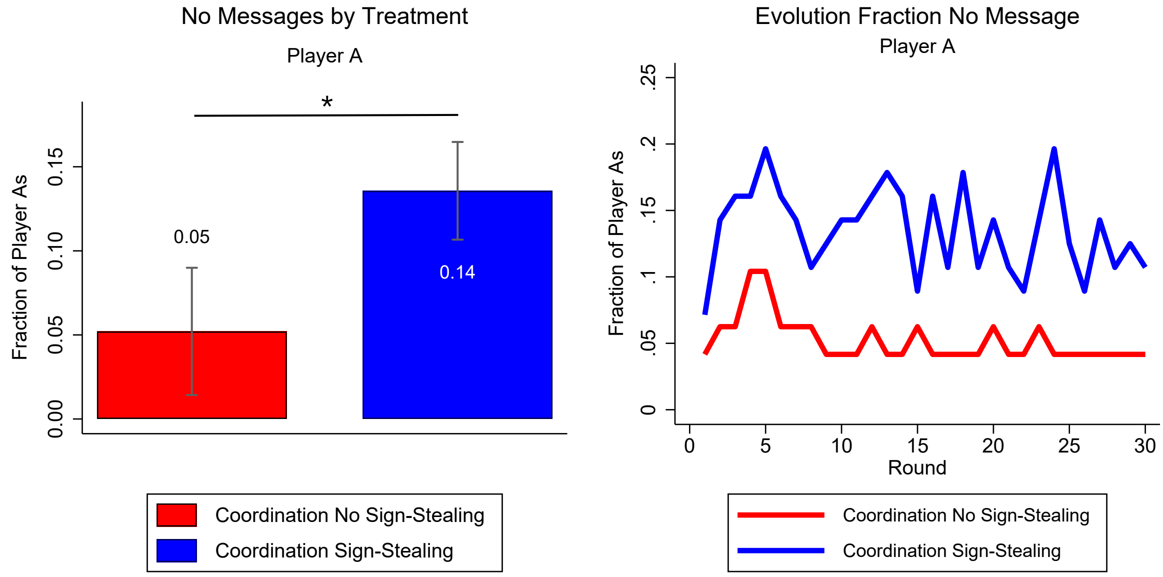
[28] The number of observations in specifications (5a) and (5b) is lower as there are no lagged values for the first round, which is consequently excluded. It is important to note that lagged values reflect individual-specific rather than team-specific history as players are re-matched each round.

[29] For both players, we find no evidence for heterogenous treatment effects of lagged opponent effort and lagged team partner effort (player A).

3.4.2 Communication choices in stage 2

This section examines communication behaviour in the two treatments *'CNS'* and *'CS'* that allowed player As to send a message on their intended action to their teammate. The message was private in *'CNS'* but observable to player B in *'CS'*.

Fraction of no messages



Note: The left panel shows the average fraction of player As sending "No message" in *'CNS'* and *'CS'* over the 30 rounds. The right panel shows the evolution of this fraction over time. Error bars indicate mean +/- SE (as estimated by a random effects linear probability panel regression with clustering of standard errors at the matching-group level). Brackets indicate significant treatment differences, $^{*}p < 0.1$.

Figure 3.2: Fraction of "No message" by treatment.

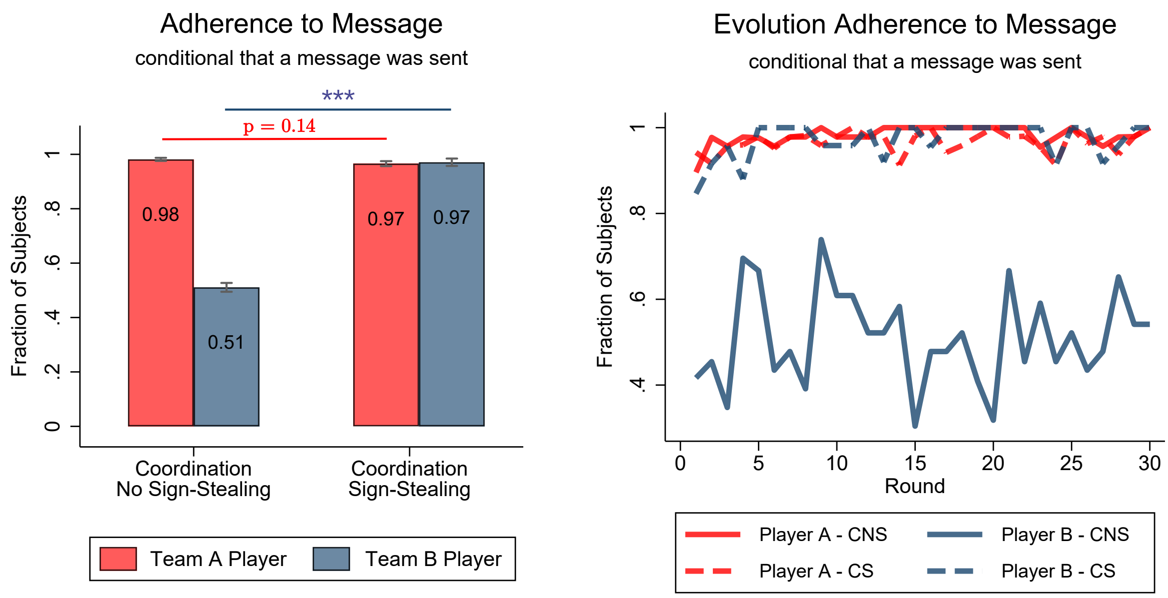
In the Communication Stage, player As had the choice between sending a message about their intended action (action U or action D) or abstaining from sending a message. Figure 3.2 (left) reports the aggregate fraction of player As who send "No message" in *'CNS'* and *'CS'*. In *'CNS'*, behaviour closely aligns with the theoretical prediction: only 5% of player As refrain from sending a message. This fraction is not statistically different from zero ($p = 0.169$). In contrast, under sign-stealing (*'CS'*), the fraction of "No message" increases to 14%. This is a significant increase of 9 percentage points relative to *'CNS'* ($p = 0.080$) and significantly above 0 ($p < 0.001$). Hence, sign-stealing has a behavioural effect and discourages some player As from using the communication channel. We reject Hypothesis 2.

Figure 3.2 (right) shows the evolution of the fraction of "No message" over the 30 rounds. In both treatments, player As' behaviour is stable over time. In *'CNS'*, the fraction of "No message" remains consistently low. With sign-stealing, "No message" is sent persistently more often, suggesting that players' reluctance to communicate is not confined to early rounds.

Nevertheless, the majority of player As send a message with their intended action even under sign-stealing.

Result 5: *Team A members send "No message" significantly more often in CS than in CNS. Hypothesis 2 is rejected.*

Adherence to observed messages



Note: The left panel shows the average fraction of players following the message sent by the sender in team A (adherence) over the 30 rounds, by player type and treatment. Red (Blue) bars refer to player A (player B). The right panel shows the evolution of adherence for player As' (red) and player Bs' (blue) in *'CNS'* (solid lines) and *'CS'* (dashed lines). Error bars indicate mean +/- SE (as estimated by a random effects linear probability panel regression with clustering of standard errors at the matching-group level). Brackets indicate significant treatment differences, ***$p < 0.01$.

Figure 3.3: Adherence to observed messages by player type and treatment.

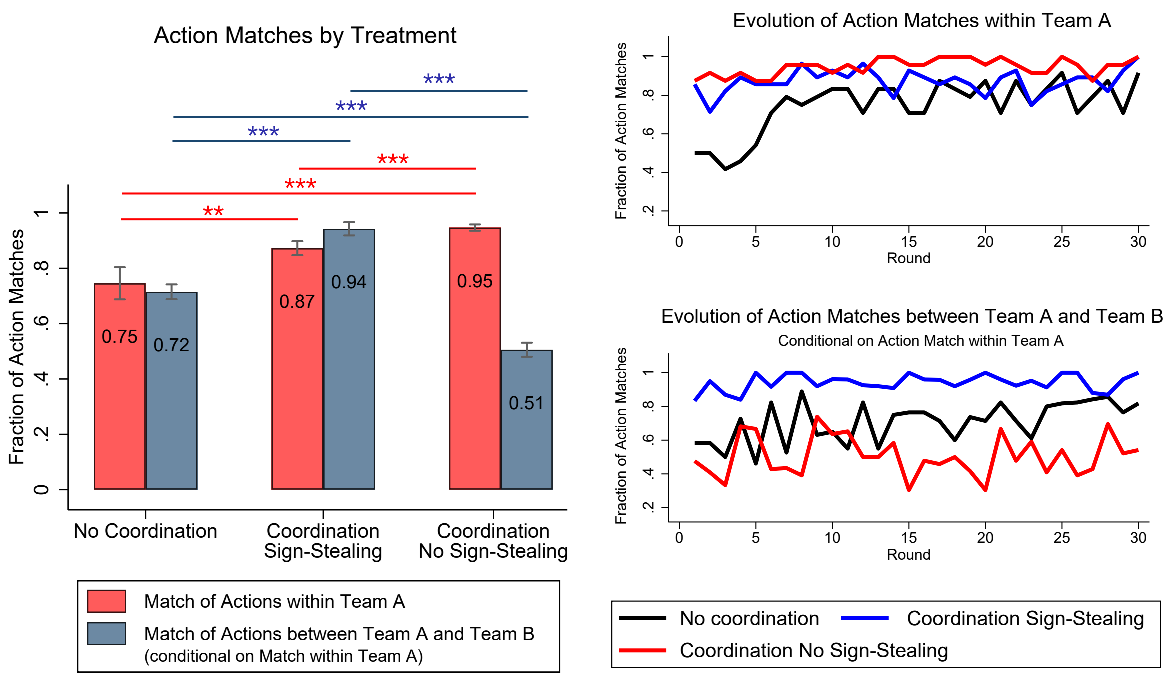
Figure 3.3 (left) reports adherence to messages, defined as choosing the same action as indicated in the message, conditional on a message being sent. Player As (red bars) adhere to the indicated actions almost perfectly, with adherence rates of 0.98 in *'CNS'* and 0.97 in *'CS'*, in line with the theoretical refinement. The difference between both treatments is small and not significant ($p = 0.144$). Therefore, sign-stealing has no effect on whether team A members adhere to the messages. Player Bs (blue bars) do not observe the message of player As in *'CNS'*, and adherence to the unobserved message is no better than chance. This indicates that communication allows team As to hide their action choice in the absence of sign-stealing[30]. In *'CS'*, however, the adherence rate of player Bs rises to 97%, a highly significant increase relative to *'CNS'* ($p < 0.001$).

[30]The pattern is consistent with player As randomising over indication of action U and D, and player Bs randomising over their actual action choice.

Figure 3.3 (right) further shows that adherence to *observed* messages is high and stable across rounds for both player types. Moreover, player Bs do not improve in guessing the content of the *unobserved* messages in 'CNS' over time.

Result 6: *Upon observing a message that indicates an action, almost all players As and Bs choose the indicated action.*

3.4.3 Action choices in stage 3



Note: The left panel shows the average fraction of action matches over the 30 rounds by team and treatment. Red bars refer to the fraction of action matches within team A, blue bars refers to action matches between team A and team B, conditional on team A matching internally. The right panel shows the evolution of internal action matches of team A (upper) and for action matches between team A and B (lower). Error bars indicate mean +/- SE (as estimated by a team level random effects linear probability panel regression with clustering of standard errors at the matching-group level). Brackets indicate significant treatment differences, $^{**}p < 0.05$ and $^{***}p < 0.01$.

Figure 3.4: Action matches by team and treatment.

Finally, Figure 3.4 (left) reports action matches across the three treatments. For team A, an action match occurs when both members choose the same action in the Action Stage. For team B, an action match is defined as player Bs choosing the same action as team A, conditional on team A members matching their actions. We refer to such action matches as a 'coordination success'. Unlike effort and communication choices, action matches are measured at the team rather than the individual level.

For team As (red bars), the fraction of action matches is lowest in 'NC', where no communication is possible. The average matching rate is 75%, which is well above random chance

($p < 0.001$)[31]. In line with *Hypothesis 3(i)*, allowing communication between team A members significantly increases their coordination success. The fraction of action matches rises to 0.95 in *'CNS'* ($p < 0.001$) and to 0.87 in *'CS'* ($p = 0.045$), corresponding to increases of 27% and 16% relative to *'NC'*. This is evidence for the first role of communication for team A: enabling coordination within the team. In contrast, we reject *Hypothesis 3(ii)*: sign-stealing by team B is not neutral to team A's coordination success. It reduces action matches within team As by about 8 percentage points ($p < 0.007$), which is driven by the higher frequency of "No message" in *'CS'*.

Result 7: *Communication between team A members increases their coordination success. This effect is hampered when team B observes team A's communication.*

For team B (blue bars), the fraction of action matches in *'NC'* is comparable to team A: the matching rate averages 0.72, which is significantly above the matching rate under random choice ($p < 0.001$). In contrast, team B's matching rate is significantly smaller at 0.51 in *'CNS'* ($p < 0.001$). This corresponds to a 30% reduction in team B's coordination success. This is evidence for the second role of communication for team A: successful hiding of action choices from team B in the absence of sign-stealing. Finally, player Bs almost perfectly match the action of team A with sign-stealing in *'CS'*. The matching rate of 0.94 is significantly higher than in *'NC'* ($p < 0.001$) and in *'CNS'* ($p < 0.001$). Thus, communication within team A reduces coordination success of team B in the absence of sign-stealing, but increases coordination success in the presence of sign-stealing.

Result 8: *Communication of team A has an asymmetric effect on team B: it reduces its matching rate without sign-stealing, but increases it when team B observes team A's message.*

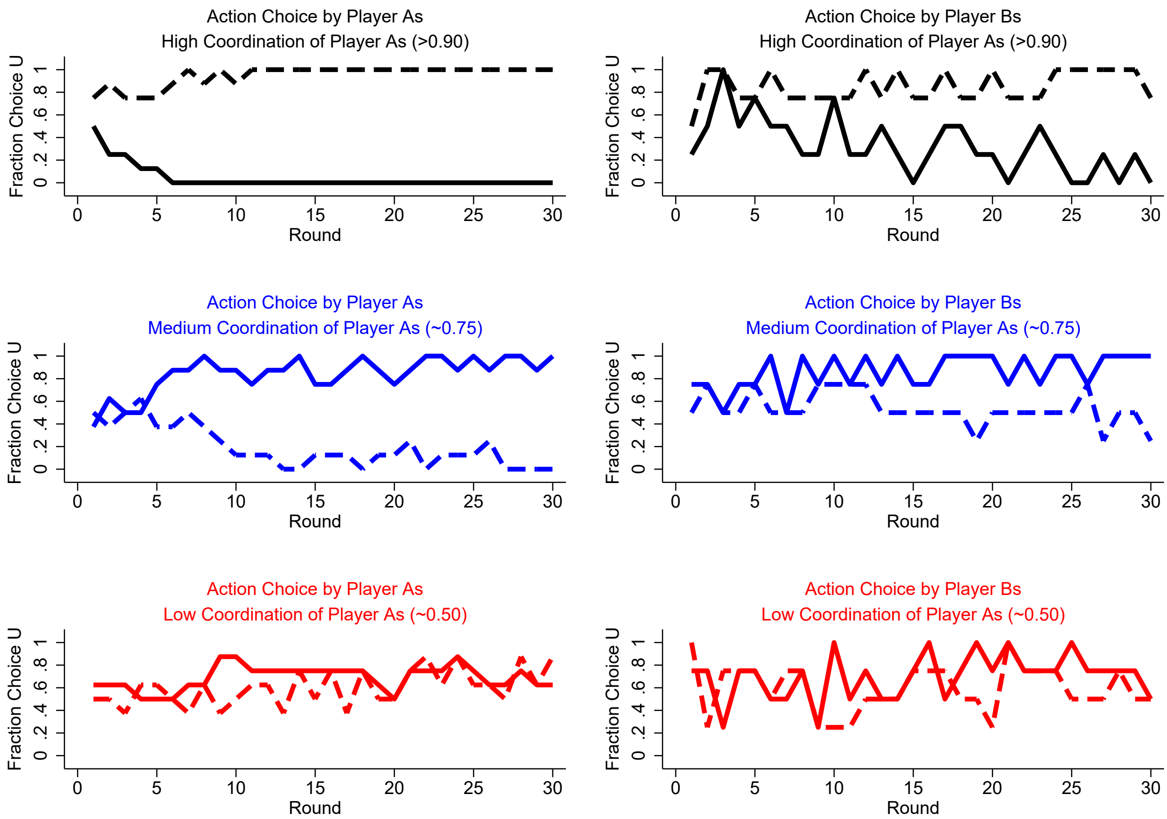
For the evolution over rounds, Figure 3.4 (upper right) shows that action matching rates are stable within team A in *'CNS'* and *'CS'*. However, there is a break in action matches in *'NC'*. The matching rate is at roughly 0.5 in early rounds, but rises to 0.8 from round 5 onwards. For team B, matching rates are consistently low in *'CNS'* and consistently high in *'CS'* over time (Figure 3.4 (lower right)). Similar to team A, team Bs' matching rates in *'NC'* increases across rounds, but at a slower pace as compared to team A.

3.5 Exploratory Results

3.5.1 Coordination in No-Coordination

This section provides a deeper analysis of the behaviour in the *No Coordination* treatment. As outlined in section 3.2.2, multiple equilibria are feasible in this treatment. Our hypotheses built on the equilibrium in which all players randomise between action U and D with

[31] Regarding the detailed analysis of why this is the case, see section 3.5.1.



Note: Panels show the evolution of action choices in the 'No Coordination' treatment by player type and by the coordination level of team A. The left (right) panels show action choices of player As (player Bs). The upper, middle and lower rows correspond to sessions with high, intermediate and low coordination levels within team A. Each panel comprises two out of the six matching groups in 'No Coordination.' The solid and dashed line each represent one of the two matching groups of each division.

Figure 3.5: Evolution of action choices in 'No Coordination' by player type.

probability $\frac{1}{2}$[32]. To identify which equilibria materialised in the experiment, we categorise each of the six matching groups of the *No Coordination* treatment into three divisions based on team A's average coordination success.

Figure 3.5 presents the evolution of action choices within the three divisions. The left (right) figures refer to the fraction of player As (player Bs) choosing action U of the respective division[33]. Each of the divisions *Low Coordination*, *Medium Coordination* and *High Coordination* consists of two matching groups. Solid and dashed lines each identify one matching group, respectively. In the *High Coordination* division (upper figures in black), team As have an average coordination success rate above 0.9. This is achieved by implicit coordination: player As choose either consistently action U or consistently action D, and this tendency spreads fast in the matching group. By round 5, convergence of actions is already high and it is

[32] We considered this equilibrium most plausible when communication is not allowed.
[33] The fraction of players choosing action D is equal to 1 minus the fraction choosing action U.

perfect from round 10 onwards[34]. Most importantly, player As in *High Coordination* do not deviate from their implicit coordination once convergence is achieved, even when team B matches their action. Hence, we find evidence for the two pure strategy equilibria described in section 3.2.2. Player Bs in *High Coordination* gradually recognise the implicit coordination and match team As' action more often in later rounds, though in a more delayed and less stable manner. This results in an average matching rate of 0.77. In the *Medium Coordination* division (middle figures in blue), behaviour of player As and player Bs follows a similar trend, but implicit coordination emerges more slowly and with greater volatility. Perfect coordination is not observed in this division. The average fraction of action matches is 0.76 within team A and 0.72 between team A and team B.

Finally, closest to our theoretical prediction, we find evidence for the mixed strategy equilibrium. In the division *Low Coordination* (lower figures in red), player As do not achieve an implicit coordination of actions. The fraction of choices of action U fluctuates around 0.64 and no convergence of actions is achieved within matching groups over time. This leads to an average matching rate of 0.57 within team A. As a consequence, team Bs cannot recognise an implicit pattern, which results in a matching rate between team A and team B of 0.62 in *Low Coordination*. Hence, observed behaviour in *Low Coordination* is close to the predicted matching rate of 0.5 in the mixed strategy equilibrium[35].

Additional Result*: We find evidence for both the pure strategy equilibrium with implicit coordination and for the mixed strategy equilibrium in the No Coordination treatment.*

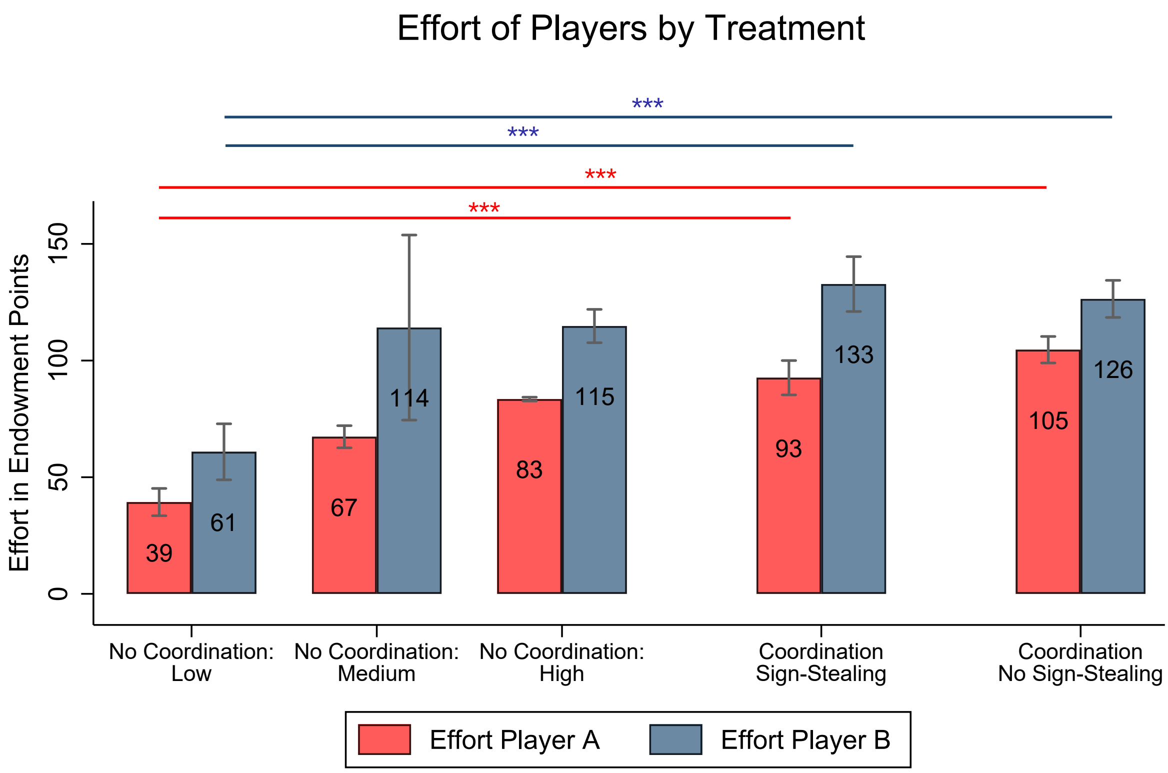
3.5.2 Effort choice for different coordination levels in No Coordination

The disaggregation of the *No Coordination* treatment allows for a more fine-grained test of our model predictions. Specifically, effort choices should be lowest in the mixed strategy equilibrium of the division *Low Coordination*. In contrast, the pure strategy equilibrium in *High Coordination* should lead to similar effort choices as in the treatment *Coordination Sign-stealing*.

The left side of Figure 3.6 reports players' efforts across the three divisions of the *No Coordination* treatment. In the *Low Coordination* division, average efforts are low for both player As and Bs. Player As invest on average 39 endowment points, which is equivalent to 16% of their initial endowment. A similar finding holds for Player Bs who invest on average 61 endowment points, or 24% of their endowment. In line with our model prediction, effort levels increase markedly in the *Medium Coordination* and *High Coordination* divisions. In *High Coordination*, player As invest 83 endowment points, more than doubling their efforts relative to Low Coordination ($p < 0.001$). Player Bs likewise increase their efforts to 115

[34] Notably, this is not driven by a sunspot in which player As choose the same actions by chance and then stick to the action in the following rounds.

[35] We find evidence that some subjects randomise between action U and action D, whereas others consistently stick to an action without achieving convergence within the matching group.



Note: The figure shows the average effort invested over 30 rounds by player type and treatment. Red (Blue) bars represent player A (player B). Effort in the 'No Coordination' treatment is disaggregated by matching-groups with low, intermediate and high coordination rates of player As (see section 3.5.1). Error bars indicate mean +/- SE (as estimated by a random effects GLS panel regression with clustering of standard errors at the matching group level). Brackets indicate significant treatment differences, ***$p < 0.01$.

Figure 3.6: Effort by player type and treatment for subdivisions of 'No Coordination.'

endowment points, corresponding to an increase by 89% ($p < 0.001$).

Additional Result: *Both player A and player B effort choices are significantly higher in the division corresponding to the pure strategy equilibrium than in the mixed strategy equilibrium of No Coordination.*

Since the *Low Coordination* division of *NC* is closest to the mixed strategy equilibrium benchmark of *Hypotheses 1(i) and 1(ii)*, we compare players' efforts in this division to those in the *CNS* and *CS* treatments. Effects are qualitatively in line with the aggregate results from section 3.4.1 but quantitatively stronger. Player As exert 136% more effort in *CS* and 166% more effort in *CNS* relative to the *Low Coordination* division ($p < 0.001$, respectively). Similarly, player Bs invest 118% more in *CS* and 108% more in *CNS* than in the *Low Coordination* division ($p < 0.001$, respectively). These comparisons confirm that communication substantially increases players' efforts relative to cases where communication is not possible and implicit coordination fails to emerge.

Additional Result: *In line with Hypothesis 1(i) and 1(ii), both player types exert significantly more effort in CNS and CS than NC-Low Coordination.*

Finally, we compare perfect implicit coordination in the *High Coordination* division of *NC* to the *CS* treatment. Both divisions share two key features: (i) team A is able to coordinate on matching actions, and (ii) team B is able to match team A's action. Consequently, there should be no differences in effort choices between the two divisions for both player types. Our data is in line with this prediction: player As invest 83.5 endowment points in the *High Coordination* division and 92.6 endowment points the *CS* treatment. The difference of 9 endowment points is small and not significant ($p = 0.215$). For player Bs, the difference in effort between two divisions is 18 endowment points and also fails to reach statistical significance (114.8 points vs. 126.4 points, $p = 0.191$).

Additional Result: *The effort of both player types in NC-High Coordination with implicit coordination is not different from the effort in CS.*

3.6 Conclusion

Sign-stealing occurs when an opponent gains access to a team's internal communication. In this paper, we examine how sign-stealing affects teams' competition behaviour, both in terms of their upfront efforts and their use of internal signs. Using contest-theoretical modelling, we analyse three regimes: *No Coordination*, *Coordination without Sign-stealing* and *Coordination with Sign-stealing*. When communication is allowed, our theoretical analysis predicts that players always send non-empty messages to coordinate in the absence of sign-stealing. With sign-stealing – when the opposing team also observes the message – players send non-empty messages if the direction advantage is sufficiently small. Under the assumption of a small advantage, players always use messages to coordinate and coordination is perfect. Moreover, teams exert more efforts when communication is allowed. Comparing the cases with and without sign-stealing, whether teams exert more or less efforts depends on which team holds a prior advantage. When no team has a prior advantage, efforts are the same in both regimes. When the team whose signal is being "stolen" holds an advantage, sign-stealing balances the playing field and increases efforts for both teams. Conversely, when the "stealing" team holds a prior advantage, sign-stealing exacerbates the imbalance and reduce both teams' efforts.

We test the main theoretical predictions in a laboratory experiment, fixing parameters so that players always prefer to send non-empty messages and contest efforts should not change with sign-stealing. We find that communication increases both teams' efforts and coordination rates, in-line with the hypotheses. As predicted, teams' contest efforts do not differ with and without sign-stealing. However, with the threat of sign-stealing, subjects send empty messages more often than in the case without sign-stealing, resulting in lower coordination rates.

In our model, sign-stealing succeeds with certainty. Future work could explore settings in which stealing succeeds only with some probability, whether known or unknown to players. Another extension would be to relax the assumption that stealing is exogenous and instead study the optimal level of stealing as a strategic choice, both theoretically and experimentally, incorporating the behavioural aspects of endogenous sign-stealing.

Appendices

Appendix 3A. The more nuanced profiles involving q and m

Here, we explain in details why equilibria cannot exist in which players in team A randomise with probability q or m. Suppose B_1 always plays U as in row (4) in Table 3.3. In the full direction profile constructed previously, players in team A always believe others have chosen their equilibrium efforts. Consider A_1, if she exerts her equilibrium effort, she is indifferent between directions U and D. However, if A_1 chooses an effort different from her equilibrium effort, this indifference is broken. The payoff from choosing direction U is calculated as:

$$q\frac{\gamma(g_{A_1}+g_{A_2})}{\gamma(g_{A_1}+g_{A_2})+\beta g_B}$$

Taking the derivative with respect to g_{A_1} gives:

$$q\frac{\beta\gamma g_B}{(g_A+\beta g_B)^2}$$

Hence, starting from equilibrium efforts, marginally increasing g_{A_1} increases the payoff from U by $q\frac{\beta\gamma g_B^*}{(g_A^*+\beta g_B^*)^2}$. The payoff from choosing D is:

$$(1-q)\frac{\beta\gamma(g_{A_1}+g_{A_2})}{\beta\gamma(g_{A_1}+g_{A_2})+g_B}$$

Taking the derivative with respect to g_{A_1} gives:

$$(1-q)\frac{\beta\gamma g_B}{(\beta g_A+g_B)^2}$$

Thus, starting from equilibrium efforts, marginally increasing g_{A_1} increases the payoff from D by $(1-q)\frac{\beta\gamma g_B^*}{(\beta g_A^*+g_B^*)^2}$. We know that at equilibrium $q\frac{g_A^*}{g_A^*+\beta g_B^*}=(1-q)\frac{\beta g_A^*}{\beta g_A^*+g_B^*}$. This further implies:

$$q\frac{\beta\gamma g_B^*}{(g_A^*+\beta g_B^*)^2}>(1-q)\frac{\beta\gamma g_B^*}{(\beta g_A^*+g_B^*)^2} \tag{3.5}$$

Hence, if A_1 exerts more than her equilibrium effort, she strictly prefers U; if she exerts less than her equilibrium effort, she strictly prefers D. At stage 1, her payoff from exerting a greater effort is therefore:

$$q\frac{\gamma(g_{A_1}+g_{A_2})}{\gamma(g_{A_1}+g_{A_2})+\beta g_B}-g_{A_1}$$

Starting from the equilibrium efforts, increasing g_{A_1} marginally increases A_1's payoff by

$$q\frac{\beta\gamma g_B^*}{(g_A^*+\beta g_B^*)^2}-1$$

Similarly, marginally decreasing g_{A_1} increases A_1's payoff by

$$1-(1-q)\frac{\beta\gamma g_B^*}{(\beta g_A^*+g_B^*)^2}$$

For there to be no profitable deviations for A_1, both expressions $q\frac{\beta\gamma g_B^*}{(g_A^*+\beta g_B^*)^2}-1$ and $1-(1-q)\frac{\beta\gamma g_B^*}{(\beta g_A^*+g_B^*)^2}$ must be negative. That requires, simultaneously,

$$q\frac{\beta\gamma g_B^*}{(g_A^*+\beta g_B^*)^2}<1;\ (1-q)\frac{\beta\gamma g_B^*}{(\beta g_A^*+g_B^*)^2}>1$$

However, as we proved previously in (3.5), these inequalities cannot both hold. This contradiction implies that A_1 always has a profitable deviation to increase or decrease her effort. Thus, profiles involving q or m cannot be sustained as equilibria. By contrast, the strategy in which players randomise with probability $\frac{1}{2}$ is unaffected by this argument: when others randomise uniformly, players remain indifferent between the two directions for all effort levels. With q or m, players in team A are only indifferent at a specific effort level.

Appendix 3B. Direction and message profiles (Coordination without Sign-stealing)

In this section, we characterise possible equilibrium message and direction profiles systematically. Since in equilibrium $p_U^{A_1}$ must be equal to $p_U^{A_2}$, we merge them into one column.

<table>
<tr><th>$(p_U^A, p_D^A)|\emptyset$</th><th>$(p_U^A, p_D^A)|u$</th><th>$(p_U^A, p_D^A)|d$</th><th>$(p_U^{B_1}, p_D^{B_1})$</th><th>$(p_\emptyset^{A_1}, p_u^{A_1}, p_d^{A_1})$</th></tr>
<tr><td>$(1,0)$</td><td>$(1,0)$</td><td>$(1,0)$</td><td>$(1,0)$</td><td>Any</td></tr>
<tr><td colspan="5">Notes: One class of equilibria has players in team A completely ignoring the message, choosing their directions independently of it.</td></tr>
<tr><td>$(1,0)$</td><td>$(1,0)$</td><td>$(0,1)$</td><td>$(\frac{1}{2},\frac{1}{2})$</td><td>$(p_\emptyset^{A_1} + p_u^{A_1} = \frac{1}{2}, \frac{1}{2})$</td></tr>
<tr><td>$(1,0)$</td><td>$(0,1)$</td><td>$(1,0)$</td><td>$(\frac{1}{2},\frac{1}{2})$</td><td>$(p_\emptyset^{A_1} + p_d^{A_1} = \frac{1}{2}, \frac{1}{2})$</td></tr>
<tr><td>$(1,0)$</td><td>$(0,1)$</td><td>$(0,1)$</td><td>$(\frac{1}{2},\frac{1}{2})$</td><td>$(\frac{1}{2}, p_u^{A_1} + p_d^{A_1} = \frac{1}{2})$</td></tr>
<tr><td colspan="5">Notes: Another class of equilibria has players in team A responding to messages with different directions while B_1 randomises between U and D with probability $\frac{1}{2}$. Player A_1 randomises over messages leading to U or D.</td></tr>
<tr><td>$(q, 1-q)$</td><td>$(1,0)$</td><td>$(1,0)$</td><td>$(1,0)$</td><td>$(0, p_u^{A_1} + p_d^{A_1} = 1)$</td></tr>
<tr><td>$(q, 1-q)$</td><td>$(q, 1-q)$</td><td>$(1,0)$</td><td>$(1,0)$</td><td>$(p_\emptyset^{A_1} = p_u^{A_1} = 0, 1)$</td></tr>
<tr><td colspan="5">Notes: A further class of equilibria has team A mixing after some messages and playing deterministic directions after others. B_1 always matches the deterministic direction. A_1 sends only messages that induce deterministic play.</td></tr>
<tr><td>$(\frac{1}{2},\frac{1}{2})$</td><td>$(1,0)$</td><td>$(0,1)$</td><td>$(\frac{1}{2},\frac{1}{2})$</td><td>$(0, \frac{1}{2}, \frac{1}{2})$</td></tr>
<tr><td>$(\frac{1}{2},\frac{1}{2})$</td><td>$(0,1)$</td><td>$(1,0)$</td><td>$(\frac{1}{2},\frac{1}{2})$</td><td>$(0, \frac{1}{2}, \frac{1}{2})$</td></tr>
<tr><td colspan="5">Notes: Finally, equilibria exist when team A randomises after one type of message and chooses deterministic but opposing directions after others. B_1 randomises with probability $\frac{1}{2}$, and A_1 randomises over messages leading to deterministic directions.</td></tr>
</table>

Table 3.7: Direction and message profiles (Coordination without Sign-stealing)

Appendix 3C. Direction profiles (Coordination with Sign-stealing)

$(p_U^A, p_D^A)\vert\emptyset$	$(p_U^A, p_D^A)\vert u$	$(p_U^A, p_D^A)\vert d$	$(p_U^{B_1}, p_D^{B_1})\vert\emptyset$	$(p_U^{B_1}, p_D^{B_1})\vert u$	$(p_U^{B_1}, p_D^{B})\vert d$
$(1,0)$	$(1,0)$	$(1,0)$	$(1,0)$	$(1,0)$	$(1,0)$
$(1,0)$	$(1,0)$	$(0,1)$	$(1,0)$	$(1,0)$	$(0,1)$
$(0,1)$	$(1,0)$	$(0,1)$	$(0,1)$	$(1,0)$	$(0,1)$

Notes: Pure direction profiles exist where players in team A choose one direction for some or all messages and the opposite direction for others, while B_1 always matches team A's direction.

$(q,1-q)$	$(1,0)$	$(0,1)$	$(1,0)$	$(1,0)$	$(0,1)$
$(m,1-m)$	$(1,0)$	$(0,1)$	$(0,1)$	$(1,0)$	$(0,1)$

Notes: Similar profiles exist where players in team A randomise upon receiving one type of messages and B_1 chooses direction U or D depending on q or m. Upon receiving other types of messages, players in team A select a deterministic direction. Player B_1 matches the same direction as team A.

$(\frac{1}{2},\frac{1}{2})$	$(1,0)$	$(0,1)$	$(\frac{1}{2},\frac{1}{2})$	$(1,0)$	$(0,1)$

Notes: Similar profiles exist where players in team A randomise after one message type and B_1 responds with randomisation. Upon receiving other types of messages, players in team A select a deterministic direction and B_1 matches the same direction.

$(q,1-q)$	$(q,1-q)$	$(0,1)$	$(1,0)$	$(1,0)$	$(0,1)$
$(\frac{1}{2},\frac{1}{2})$	$(q,1-q)$	$(0,1)$	$(\frac{1}{2},\frac{1}{2})$	$(1,0)$	$(0,1)$
$(\frac{1}{2},\frac{1}{2})$	$(\frac{1}{2},\frac{1}{2})$	$(0,1)$	$(\frac{1}{2},\frac{1}{2})$	$(\frac{1}{2},\frac{1}{2})$	$(0,1)$
$(q,1-q)$	$(q,1-q)$	$(q,1-q)$	$(1,0)$	$(1,0)$	$(1,0)$
$(\frac{1}{2},\frac{1}{2})$	$(q,1-q)$	$(q,1-q)$	$(\frac{1}{2},\frac{1}{2})$	$(1,0)$	$(1,0)$
$(\frac{1}{2},\frac{1}{2})$	$(\frac{1}{2},\frac{1}{2})$	$(q,1-q)$	$(\frac{1}{2},\frac{1}{2})$	$(\frac{1}{2},\frac{1}{2})$	$(1,0)$
$(\frac{1}{2},\frac{1}{2})$	$(\frac{1}{2},\frac{1}{2})$	$(\frac{1}{2},\frac{1}{2})$	$(\frac{1}{2},\frac{1}{2})$	$(\frac{1}{2},\frac{1}{2})$	$(\frac{1}{2},\frac{1}{2})$

Notes: Similar profiles exist in which players in team A randomise upon receiving more than one type of messages and B_1 randomises or chooses pure directions. Upon receiving other types of messages, players in team A choose a deterministic direction and B_1 matches the same direction.

Table 3.8: Direction profiles (Coordination with Sign-stealing)

Appendix 3D. Messaging choices in Coordination with Sign-stealing

We now consider the following directions profile upon receiving an empty message. Players in team A choose U with probability q when they choose their equilibrium efforts; they choose U when they exert more than their equilibrium efforts, and D when they exert less than the equilibrium efforts. B_1 always chooses U. The belief system is such that players always believe others have chosen their equilibrium efforts. Our aim is to examine whether A_1 prefers to send non-empty messages both when she exerts her equilibrium effort and when she chooses other effort levels. Consider first the case where A_1 chooses her equilibrium effort. If she sends a non-empty message (u or d), her winning probability is:

$$\frac{g_A^*}{g_A^* + \beta g_B^*}$$

If instead she sends an empty message, her winning probability is:

$$q\frac{g_A^*}{g_A^* + \beta g_B^*}$$

Since $q < 1$, A_1 strictly prefers sending non-empty messages. Now consider the case where A_1 exerts more than the equilibrium effort, her winning probability from sending u or d is $\frac{g_A}{g_A+\beta g_B^*}$, whereas sending an empty message (A_1 chooses U while A_2 randomises with q) yields $q\frac{g_A}{g_A+\beta g_B^*}$. Since $q < 1$, A_1 strictly prefers to send non-empty messages. The final case arises when A_1 exerts less than the equilibrium effort. If she sends an empty message (A_1 chooses D while A_2 randomises with q), her winning probability is:

$$(1-q)\frac{\beta g_A}{\beta g_A + g_B^*}$$

We want to compare this probability with $\frac{g_A}{g_A+\beta g_B^*}$. Since the value of q depends on the equilibrium efforts, this comparison depends on the values of g_A^*, g_B^*, g_A and β. Since $q > \frac{1}{2}$, $(1-q)\frac{\beta g_A}{\beta g_A+g_B^*} < \frac{1}{2}\frac{\beta g_A}{\beta g_A+g_B^*}$. One can verify that when $\beta < \sqrt{2}$, $\frac{g_A}{g_A+\beta g_B^*} > \frac{1}{2}\frac{\beta g_A}{\beta g_A+g_B^*}$ holds for any values of g_A^*, g_B^* and g_A, and A_1 always prefers to send non-empty messages.

As the last step, we check the results under the parameterisation used in the experiment ($\gamma = 1, \beta = 1.5$). The parameter values pin down the equilibrium efforts $\hat{g}_A^*$ and $\hat{g}_B^*$ and $q = \frac{3}{5}$. One can verify:

$$\frac{g_A}{g_A + 1.5 * g_B^*} > \frac{2}{5}\frac{1.5 * g_A}{1.5 * g_A + g_B^*}$$

holds for any values of g_A and g_B^*. This implies, under these parameter values, A_1 always prefers to send non-empty messages. This confirms that direction profiles involving q or m upon receiving an empty message do not generate a profitable deviations in the equilibrium considered under *Coordination with Sign-stealing*. The same reasoning applies to direction profile involving probability m.

Appendix 3E. Experiment Instructions

This section presents the instructions for all three treatments of the experiment. The instructions for the *No Coordination* treatment serve as the baseline. Whenever differences arise across treatments, they are highlighted explicitly. These differences occur primarily in the Communication Stage.

Overview of the Study:
The study consists of one main part (the experiment) and several shorter subsequent parts (the scenarios). In each part, you can earn money based on your decisions. All parts are independent, meaning that your decisions in one part will not affect your earnings in another part. The currency in this experiment is **Points**, where **100 Points = 1 pound**. Your earnings will be converted to pounds at the end of the study.

Experiment Instructions (Main Part of the Study):
Overview:
Two teams compete for a prize. Everyone on the winning team gets 400 points. Everyone on the losing team gets 0 points. You can affect your teams' chance of winning in two ways:

1. By investing into **Competition Points**

2. By choosing an *action (U or D)*

Investing in competition points gives your team a better chance to win the prize in the Competition Stage. The action choices decide if Team A qualifies for the Competition Stage and if Team B has an advantage or disadvantage. The experiment consists of 30 rounds. You will make the same type of decisions in each round. All rounds are separate from each other. This means that your decisions in one round do not affect other rounds. Sometimes, we will ask you what you think the other participants might choose. At the end of the study, **4 out of 30 rounds** will be randomly selected and paid out.

Team Composition:
At the start of the experiment, you will be randomly placed in the role of either a Team A member or a Team B member. **You will keep this role throughout the experiment**. In each round, you will be randomly grouped with two other participants to form a group of three:

1. Team A has TWO members

2. Team B has ONE member

Groups typically change every round. This means you will likely **have different group members** in each new round.

Timeline in each round:
Each round has three stages:

Investment Stage: Influences your chances of winning in the Competition Stage.
Action Stage: Decides whether Team A qualifies and whether Team B gains an advantage.
Competition Stage: Decides which team wins the competition (if Team A qualifies).

In treatments CNS and CS (replacing the section above):

Timeline in each round:
Each round has four stages:

Investment Stage: Influences your chances of winning in the Competition Stage.
Communication Stage: Allows Team A members to coordinate on their actions.
Action Stage: Decides whether Team A qualifies and whether Team B gains an advantage.
Competition Stage: Decides which team wins the competition (if Team A qualifies).

Investment Stage:
In this Stage, all participants decide how many points from their endowment of each round to invest into **Competition Points**. The more you invest, the better your team's chances of winning - if it makes it to the Competition Stage.

How it works?
At the start of each round, each participant gets an endowment of 250 points in total.

- Each Team A member gets **250 points**
- The Team B member gets **250 points**

You choose how many points from your endowment to convert into Competition Points.

- Endowment points you invest **have to be paid** to the lab and you can't get them back.
- Endowment points you do not invest are **yours** and will be paid out to you.

How Competition Points are calculated:

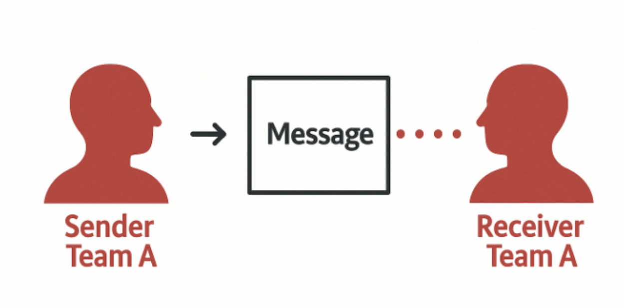
- **For Team A:**
  The conversion rate is always 1 to 1:
  1 endowment point = 1 Competition Point
  Both Team A members' Competition Points are summed up. This gives Team A's total Competition Points.

- **For Team B:**
  The conversion rate depends on whether Team B has an advantage or disadvantage:
  Advantage: 1 endowment point = 1.5 Competition Points
  Disadvantage: 1 endowment point = 0.67 Competition Points
  Team B's total Competition Points are the endowment points invested multiplied by 1.5 (advantage) or 0.67 (disadvantage).

Important:

- **No one will know** if Team B has an advantage or disadvantage when you invest - **this is only decided in the Action Stage**.
- You will only find out how much everyone invested at the end of the round.

In treatments CNS (adding the section):
Communication Stage:
In this stage, the two members of Team A can try to coordinate their action choice for the next stage. For Team A, coordination of actions is important to qualify for the Competition Stage.



One Team A member is randomly chosen as **the Sender**, and the other Team A member as **the Receiver**. The Sender decides which message to send about the planned action in the Action Stage. The Sender can choose one of these three messages:

- **"No message"**
- **"I intend to choose action U"**
- **"I intend to choose action D"**

Important details about the message:

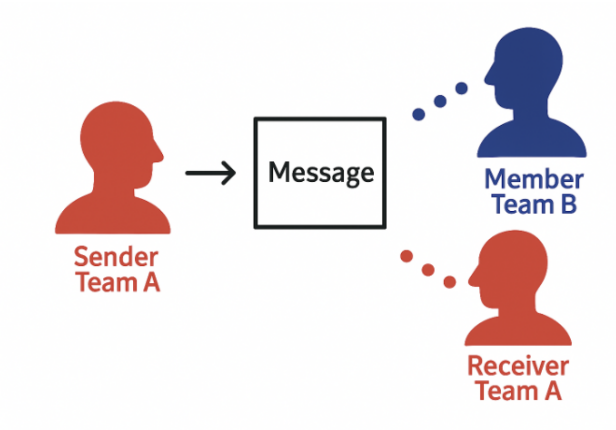
- The message is **private** — only the Receiver in Team A will see it. This means the Team B member cannot see the message.
- The message is **non-binding** — this means everyone can still choose any action in the next stage, no matter what the message says.

If you are the Team B member, you don't make any decision here and you will only see a waiting screen.

In treatments CS (adding the section):

Communication Stage:

In this stage, the two members of Team A can try to coordinate their action choice for the next stage. For Team A, coordination of actions is important to qualify for the Competition Stage. However, this may allow Team B to gain an advantage by observing Team A's communication and matching their action choice.



One Team A member is randomly chosen as **the Sender**, and the other Team A member as **the Receiver**. The Sender decides which message to send about the planned action in the Action Stage. The Sender can choose one of these three messages:

- **"No message"**
- **"I intend to choose action U"**
- **"I intend to choose action D"**

Important details about the message:

- The message is **public** — both the Receiver and the Team B member will see it.
- The message is **non-binding** — this means everyone can still choose any action in the next stage, no matter what the message says.

If you are the Team B member, you don't make any decision here. You just see the message sent by the Sender.

Action Stage:

In this stage, all participants choose one of two actions: **U or D**.

- If you are in Team A, the action choice decides **whether you move on to the Competition Stage or get disqualified**.

- If you are in Team B, the action choice decides **whether you get an advantage or a disadvantage** in the Competition Stage.

How it works:

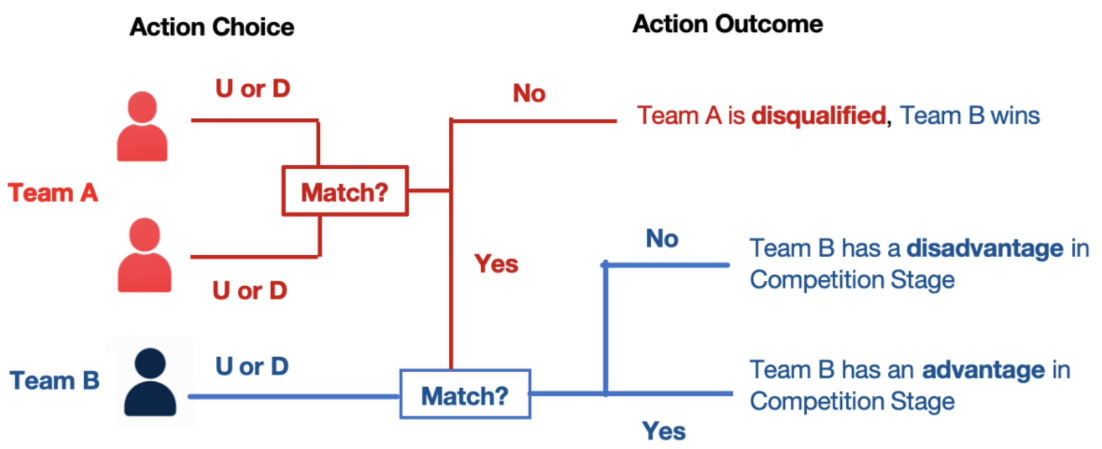
- Everyone chooses their action (U or D) at the same time. This means that you won't know what the others actually choose before you decide.
- After all actions are chosen, the computer checks:
  Whether both Team A members chose the same action
  Whether Team A and Team B chose the same action

If you are in Team A:

- If you and your team member choose **different actions**, your team is **disqualified**. You only get points you have kept yourself, and Team B wins the prize.
- If you and your team member choose the **same action**, your team qualifies for the Competition Stage, where the winner is decided.

If you are in Team B:

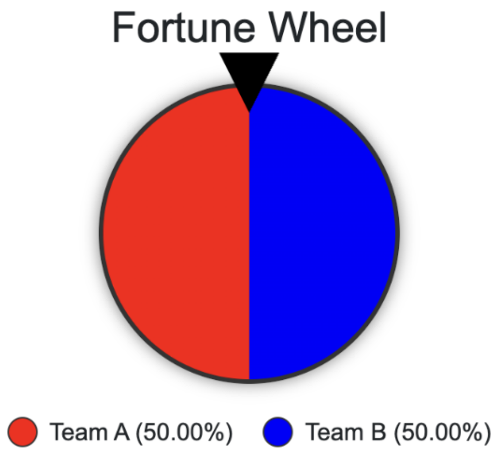
- You always move on to the Competition Stage, unless Team A is disqualified and you win the prize automatically.
- Your **advantage** or **disadvantage** depends on whether you match Team A's action:
  If you choose a **different action** from Team A, you get a disadvantage.
  ⇒ Your Competition Points = your invested endowment points x **0.67**
  If you choose the **same action** as Team A, you get an advantage.
  ⇒ Your Competition Points = your invested endowment points x **1.5**



Competition Stage:
This stage only happens if Team A qualifies. A wheel of fortune decides which team wins.
How the wheel of fortune works:

- The wheel has two colored sections: red for Team A and blue for Team B.
- The size of each section shows each team's chance of winning. The more Competition Points a team has relative to the opposing team, the bigger its section.
- There is a pointer at the top of the wheel of fortune. The wheel will start spinning and stop at a random position.
- If the wheel stops in the red section, Team A wins. If the wheel stops in the blue section, Team B wins.

You can see an example of the wheel of fortune here:



Winning probabilities:
Your team's chance of winning depends on how many Competition Points your team has relative to the opposing team.

$$Probability\ that\ your\ team\ wins$$

$$= \frac{Your\ team's\ Competition\ Points}{Your\ team's\ Competition\ Points + Opposing\ team's\ Competition\ Points}$$

In case both teams invest 0, each team has a 50% chance of winning.

What this means for you:

- If you (or your team member) invest more into Competition Points, your team's chance of winning **goes up**.
- If the other team invests more into Competition Points, your team's chance of winning **goes down**.
- If your team has invested very little, each extra point you invest makes **a big difference**.

- But if your team has already invested a lot, each extra point you invest only has **a small effect**.

Your payoff in each round:

- If your team wins the team competition:
  Payoff = 400 Points (Prize) + Points you kept for yourself in the Investment Stage
- If your team does not qualify or loses the team competition:
  Payoff = Points you kept for yourself in the Investment Stage

At the end of each round, you are informed about:

- All participants' action and investment choices
- The winning team
- All participants' payoffs

In treatments CNS and CS (replacing the section above):
At the end of each round, you are informed about:

- All participants' action and investment choices
- The content of the message sent by the Sender in Team A
- The winning team
- All participants' payoffs

Payment for the Experiment:

- At the end of the study, 4 out of the 30 rounds will be randomly selected for payment.
- The rounds will be randomly drawn by the computer and announced on your screen.
- You will only receive the payment from those randomly chosen rounds.

Appendix 3F. Experiment Screenshots

In this section, we present screenshots from the main decision tasks.

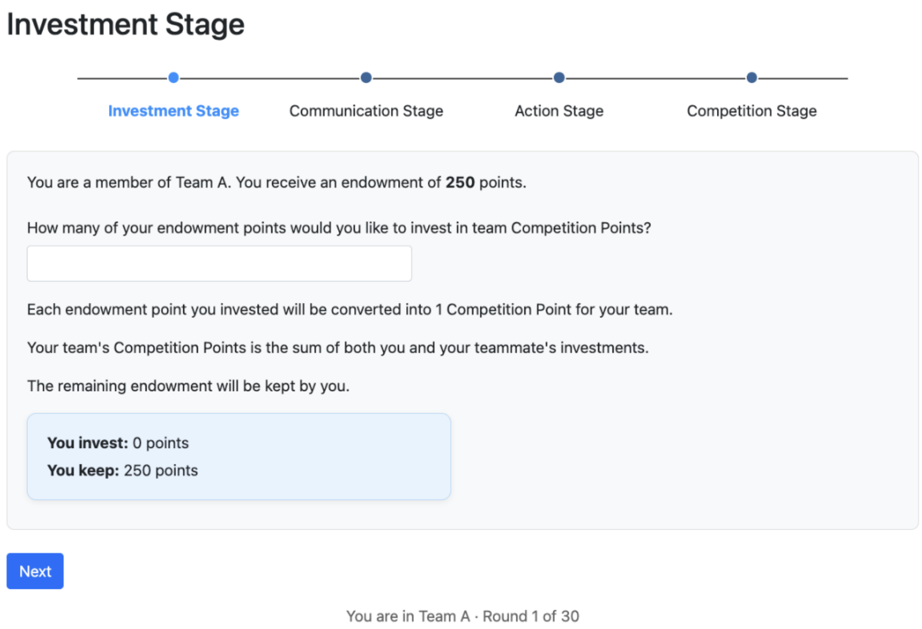


Figure 3.7: Experiment Screenshots - Investment Stage

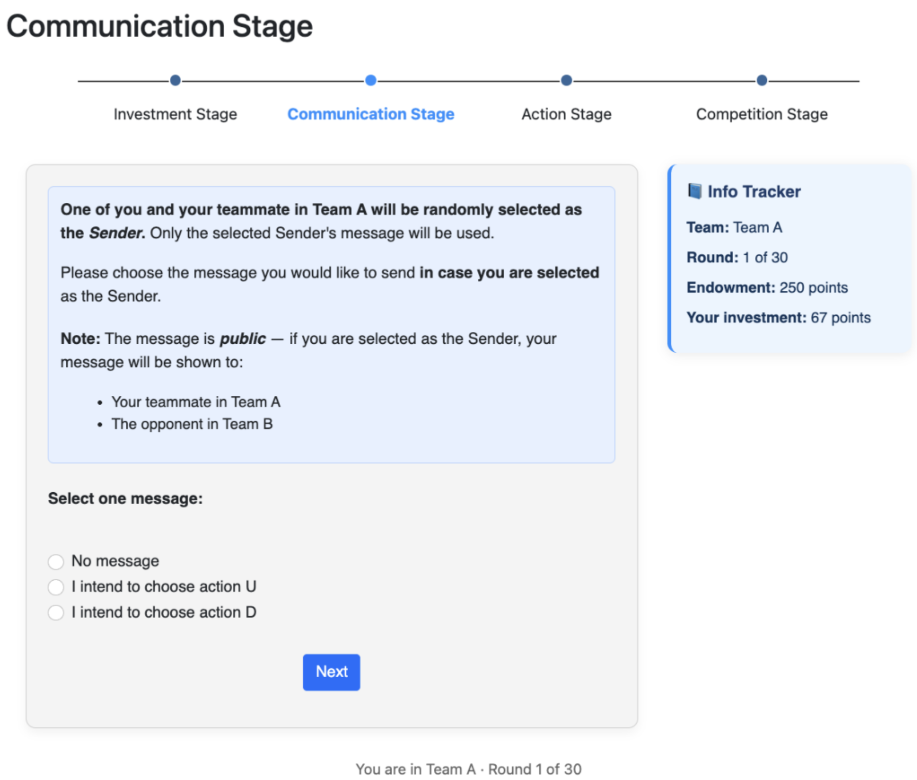


Figure 3.8: Experiment Screenshots - Communication Stage (CS)

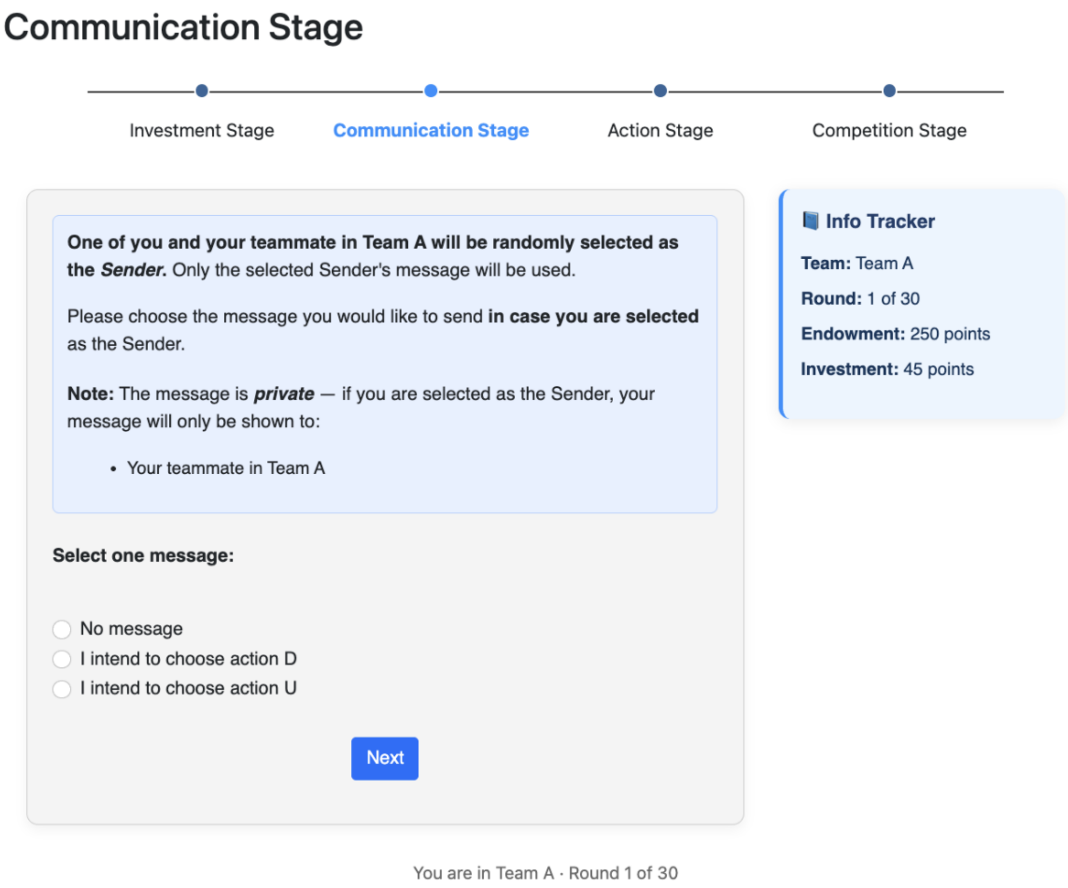


Figure 3.9: Experiment Screenshots - Communication Stage (CNS)

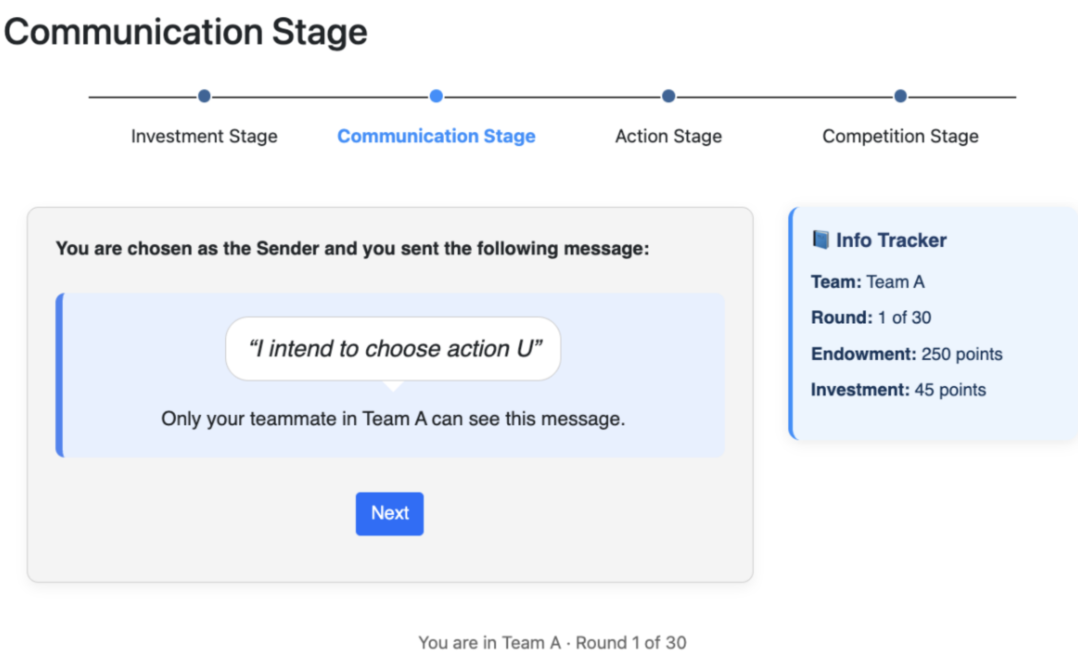


Figure 3.10: Experiment Screenshots - Sender (CNS)

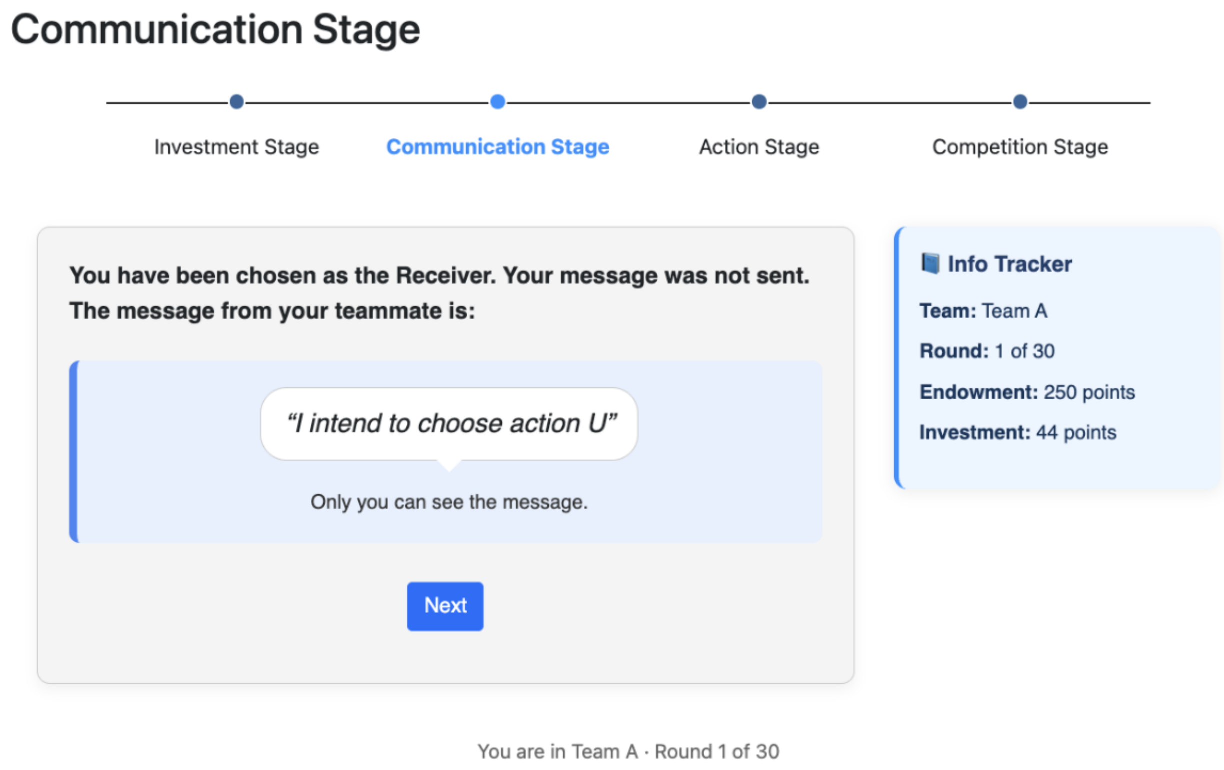


Figure 3.11: Experiment Screenshots - Receiver (CNS)

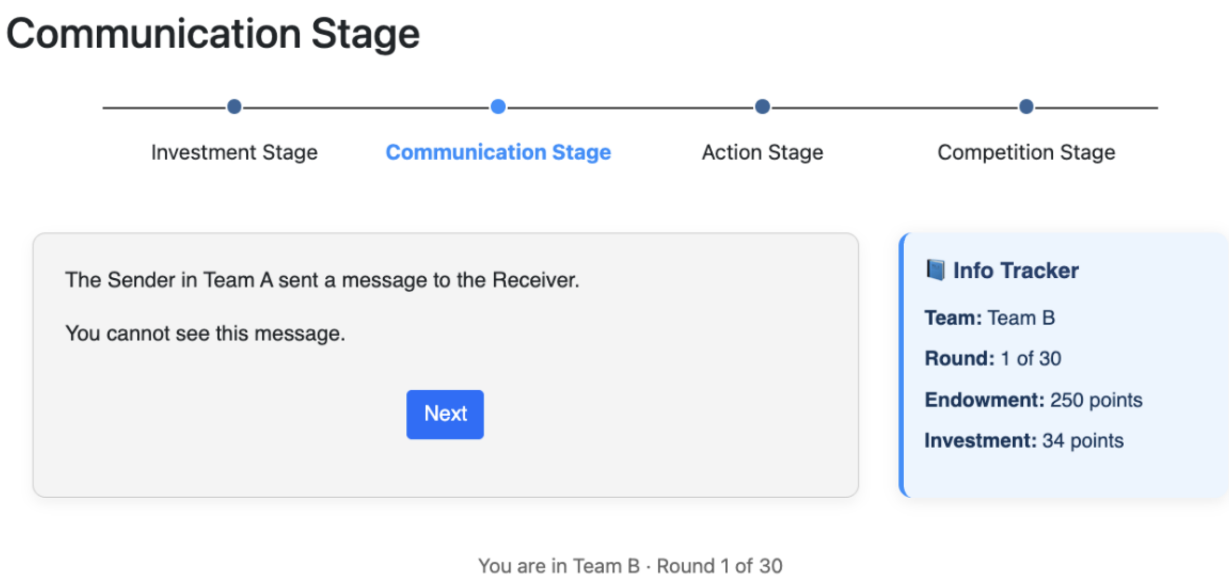


Figure 3.12: Experiment Screenshots - Team B Receive (CNS)

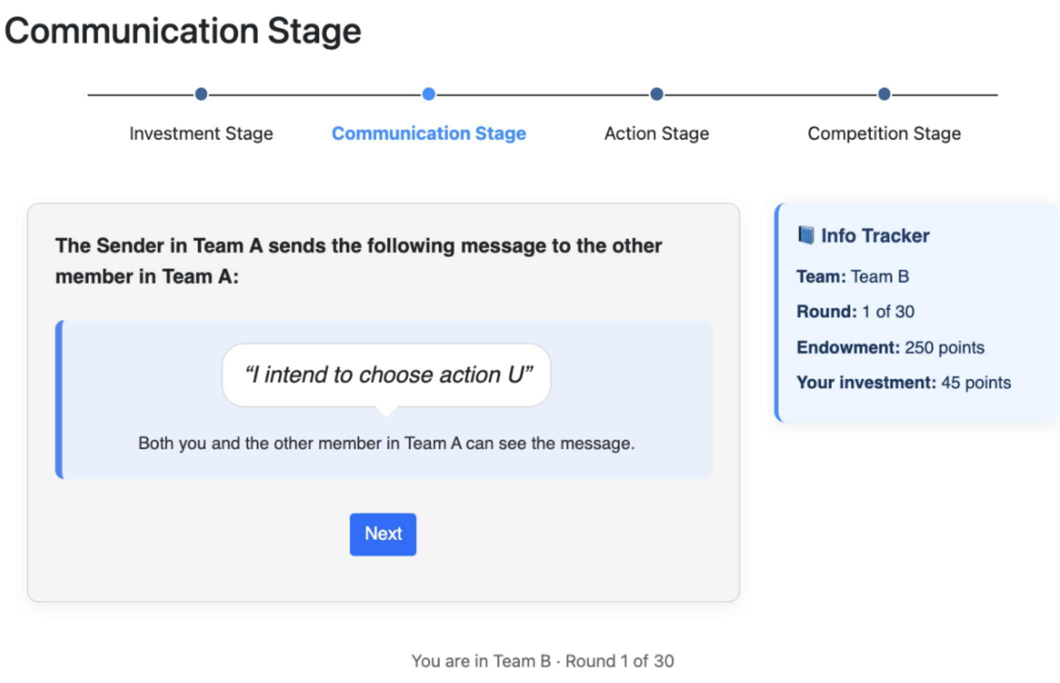


Figure 3.13: Experiment Screenshots - Team B Receive (CS)

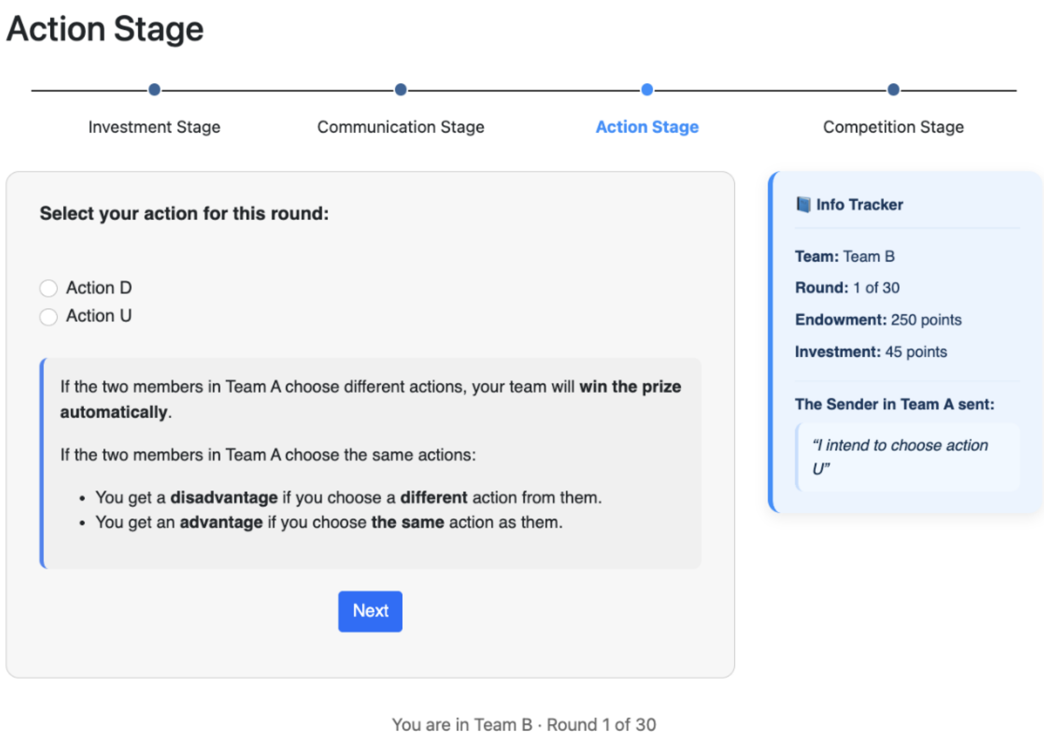


Figure 3.14: Experiment Screenshots - Action Stage

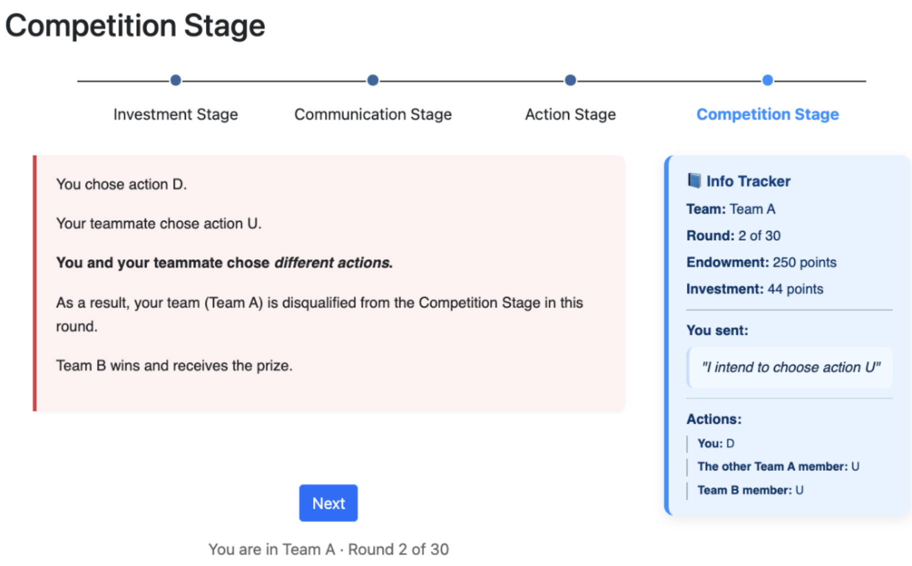


Figure 3.15: Experiment Screenshots - Competition Stage (when A is disqualified)

Figure 3.16: Experiment Screenshots - Competition Stage (when A qualifies)

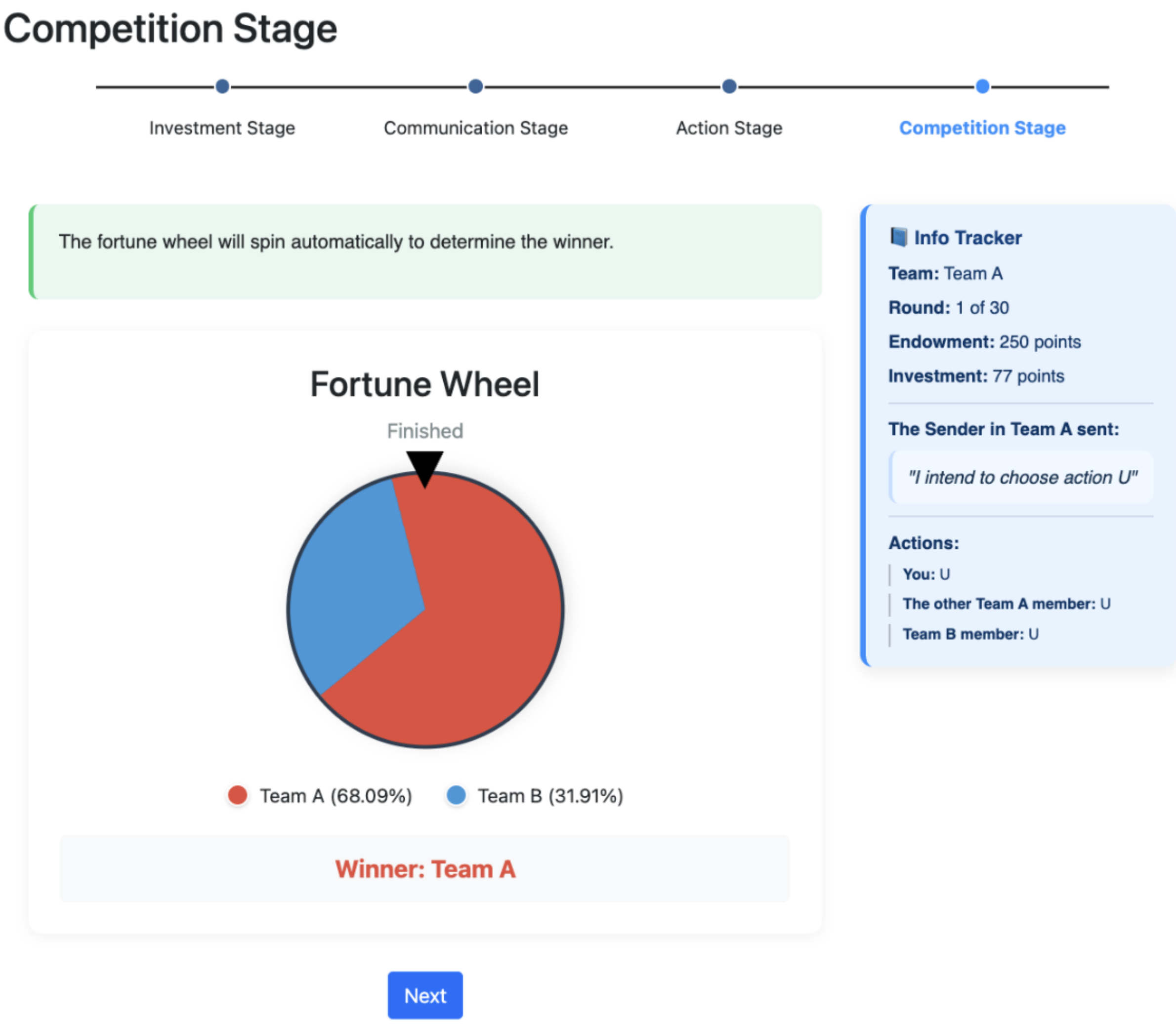


Figure 3.17: Experiment Screenshots - Fortune Wheel

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