Effective Learning Mechanisms of External Representations in Quantum Technology

Dissertation zum Erwerb des Doctor of Philosophy (Ph.D.)

am Munich Center of the Learning Sciences

der Ludwig-Maximilians-Universität

München

vorgelegt von

Linda Qerimi-Osmani

aus Erding

München, 2025

First Supervisor / Erstgutachter: Prof. Dr. Jochen Kuhn, LMU München

Second Supervisor / Zweitgutachter: Prof. Dr. Harald Weinfurter, LMU München

Weitere Mitglieder der Prüfungskommission:

Prof. Dr. Frank Fischer, LMU München

PD Dr. Sarah Malone, Universität des Saarlandes

Datum der mündlichen Prüfung: 24.10.2025

Acknowledgement

Science does not work in isolation; it requires action and interaction with others. Various people have supported me in this work, for which I am very thankful.

I would like to express my deepest gratitude to those who have supported and accompanied me from day one at my PhD. My heartfelt thanks go to Jochen Kuhn. Without his valuable educational input and multifaceted advice on research and strategic thinking, this work would not have been possible. I would also like to thank my second supervisor, Harald Weinfurter. We have been working together for many years on the communication and experimental implementation of quantum technologies for learners. I have always found it exciting to learn from your perspective on quantum physics. Your expertise consistently helped me to explore different approaches and realize that teaching quantum physics is far more meaningful for learners than just delivering a curriculum topic. Furthermore, I would like to thank my third supervisor, Frank Fischer. You made me see learning sciences and educational psychology in a new light. This has accompanied me throughout the entire process, and I am very grateful for that. I would also like to thank Sarah Malone for our excellent collaboration. We often understood each other without needing many words. I would also like to thank Stefan Küchemann. He helped me with every minor or major hurdle and supported me. I have learnt so much from you, thank you!

My thanks also go to Silke Stähler-Schöpf the Head of the PhotonLab, Tatjana Wilk the General Manager of the cluster of excellence MCQST and the Munich Quantum Valley. They enriched this work by offering new perspectives and engaging in practical interactions.

Finally, I would like to thank my colleagues, fellow PhD researchers from the MCSL and family members — without them, this dissertation would not have been possible. I am especially grateful to my husband, Lori, his grateful mother, and my daughter, Klea, who endured many demanding times with me and continually motivated me. In addition to my sister Elisa, who was consistently informed and well-versed in my research process, even though she didn't know exactly what I was doing.

Last, I would like to thank my parents for always encouraging me to follow my path and for giving me the courage to believe in myself. Thank you so much!

Executive Summary

Research into how learners visualize and learn quantum phenomena has been conducted in various ways for several decades (Küblbeck & Müller, 2002; Lichtfeldt, 1992; Müller & Wiesner, 1999; Wiesner, 1996). With the emergence of potential applications such as quantum computers, quantum cryptography and quantum sensor technology, interest in conveying quantum physics in an accessible, audience-appropriate way has grown markedly, corresponding competence frameworks for professional requirements in quantum technologies have been developed (European Commission et al., 2025; Greinert et al., 2022), and many representations have been developed or refined (e.g., Bley et al., 2024; Donhauser et al., 2024; Dür & Heusler, 2012, 2014; Huber & Glaser, 2024; Johnston et al., 2019; Yeung, 2020). Learners without a strong mathematical background—whether in schools or in professional settings (e.g., Kelly et al., 2024; Piña et al., 2025)—need approaches that make the field's central element, the qubit, tangible. As an instructional strategy, the so-called "spin first approach" is recommended (Dür & Heusler, 2012; Sadaghiani, 2016; Sadaghiani & Munteanu, 2015); it introduces a two-state system early on, thereby enabling an early representation of the qubit.

This dissertation investigates which aspects of visual qubit representations differ in terms of effectiveness in learning quantum physics, without completely ignoring the underlying mathematics. To that end, it introduces a category system grounded in representation research, physics education, and quantum science, which was evaluated by experts using four exemplary visual qubit representations (Bloch sphere, Quantum Bead (Huber & Glaser, 2024), Pie-chart Model (Qake) (Donhauser et al., 2024; Yeung, 2020) and the circle notation (Bley et al., 2024; Johnston et al., 2019)).

Key objective was to find out how the features of visual qubit representations differ in terms of effectiveness in learning quantum physics concepts. First from an expert perspective (1) then from learners' perspective (2, 3). This led to research focusing on multiple external representations, asking: (2) Do informational redundant qubit representations influence cognitive load and learning behavior? It also led to research in the context of direct application, asking: (3) Are the features identified by experts also beneficial for students' learning?

Experts highlighted, in particular, the features for visualizing phase and amplitude, the combination of different representations, and the avoidance of learning difficulties or

misconceptions. They also agreed that no single representation could meet all requirements equally well—making a repertoire of multiple representations essential.

To verify these evaluations and to examine the use of multiple representations in more detail, two additional studies were conducted. One focused on variations of informational redundant representations in the context of the Mach–Zehnder interferometer with single photons and compared four groups: (1) Text only (control), (2) Text + formula, (3) Text + Bloch sphere, (4) Text + formula + Bloch sphere.

No significant differences emerged in learning outcome or cognitive load. However, eye-tracking observations showed that groups working with the Bloch sphere exhibited a significant increase in transitions between text and representation.

The other study was carried out to verify the experts' evaluations at the learner level. Conceptual understanding, cognitive load, and application-oriented tasks on *phase, amplitude, quantum state, superposition,* and *quantum measurement* for each representation (Bloch sphere and Quantum Bead) were examined. The results showed that students completed the application-oriented tasks significantly more efficiently when using the Bloch sphere, even though no group differences appeared in conceptual understanding or cognitive load. These findings partially confirm the expert ratings and demonstrate how the category system can guide the use of other representations that share characteristics with the four examples investigated.

The results indicate that our category system with representations can be applied in various settings—for instance, to experimental setups or practice-oriented scenarios in quantum technology. While neither study revealed group differences in conceptual understanding or cognitive load, the process data from eye tracking and timing measurements uncovered subtle distinctions in how learners interacted with the representations.

These findings partially validate the category system: it is useful both for selecting suitable representations and for guiding the design of new ones.

Research into learning with representations is far from complete, yet the feature structure presented here offers a solid starting point for future work—whether on different variations of multiple external representations or on specific concepts such as entanglement. Overall, this dissertation provides an insight into the broad, complex landscape of representations in quantum physics.

Deutsche Zusammenfassung

Die Art und Weise, wie sich Lernende Quantenphänomene vorstellen und erlernen, wird seit mehreren Jahrzehnten auf unterschiedliche Weise erforscht (Küblbeck & Müller, 2002; Lichtfeldt, 1992; Müller & Wiesner, 1999; Wiesner, 1996). Mit dem Aufkommen von möglichen Anwendungen wie Quantencomputern, Quantenkryptographie oder Quantensensorik ist das Interesse an einer verständlichen, adressatengerechten Vermittlung der Quantenphysik deutlich gestiegen entsprechende Kompetenzrahmen für berufliche Anforderungen in den Quantentechnologien wurden entwickelt (European Commission et al., 2025; Greinert et al., 2022) und zahlreiche Repräsentationen ausgearbeitet oder verfeinert (Bley et al., 2024; Donhauser et al., 2024; Dür & Heusler, 2012, 2014; Huber & Glaser, 2024; Johnston et al., 2019; Yeung, 2020). Lernende ohne ausgeprägten mathematischen Hintergrund – sei es in der Schule oder im Berufsleben (z. B. Kelly et al., 2024; Piña et al., 2025) – benötigen Zugänge, die das zentrale Element dieser Technologien, das Qubit, anschaulich und verständlich machen. Als didaktische Strategie empfiehlt sich der sogenannte spin first-Ansatz (Dür & Heusler, 2012; Sadaghiani, 2016; Sadaghiani & Munteanu, 2015); es behandelt früh ein Zwei-Zustands-System und ermöglicht so eine frühe Einführung von Qubit Repräsentationen.

In dieser Dissertation wird untersucht, welche Aspekte visueller Qubit-Repräsentationen sich hinsichtlich ihrer Effektivität beim Erlernen der Quantenphysik unterscheiden, ohne die zugrunde liegende Mathematik vollständig auszublenden. Dazu wird ein Kategoriensystem vorgestellt, das auf Erkenntnissen der Repräsentationsforschung, der Quantenphysikdidaktik und der Quantenwissenschaften basiert. Experten bewerteten dieses Kategoriensystem anhand von vier exemplarischen Repräsentationen: Blochkugel, Quantum Bead (Huber & Glaser, 2024), Kuchenmodell (Qake) (Donhauser et al., 2024; Yeung, 2020) und Circle Notation (Bley et al., 2024; Johnston et al., 2019).

Ein zentrales Ziel war es, herauszufinden, wie sich die Merkmale visueller Qubit-Darstellungen im Hinblick auf ihre Wirksamkeit beim Erlernen von Konzepten der Quantenphysik unterscheiden. Zunächst aus der Sicht von Experten (1), dann aus der Perspektive der Lernenden (2, 3). Dies hat zu weiterer Forschung mit multiplen externen Repräsentationen geführt, mit der Frage: (1) Beeinflussen informationsredundante Repräsentationen die kognitive Belastung und das Lernverhalten? Und zur Forschung im Kontext der direkten Anwendung, mit der Frage: (2) Sind

die Merkmale, die von Experten bewertet wurden, auch für SchülerInnen förderlich für das Lernen?

Die Experten betonten insbesondere die Merkmale von Darstellung für Phase und Amplitude, die Kombination verschiedener (mehrerer) Repräsentationen sowie die Vermeidung von Lernschwierigkeiten durch Fehlvorstellungen. Zugleich betonen sie, dass keine einzelne Darstellung alle Anforderungen gleichermaßen erfüllt – weshalb ein Repertoire multipler Repräsentationen unverzichtbar sei.

Zur Überprüfung dieser Einschätzungen und zur genaueren Analyse des Einsatzes mehrerer Darstellungen wurden zwei weitere Studien durchgeführt. In einer Studie zu mehrfach informationsredundanten Qubit-Repräsentationen im Kontext des Mach-Zehnder-Interferometers mit Einzelphotonen wurden vier Gruppen verglichen: (1) Nur Text (Kontrollgruppe), (2) Text + Formel, (3) Text + Blochkugel, (4) Text + Formel + Blochkugel.

Es zeigten sich weder beim Lernzuwachs noch bei der kognitiven Belastung signifikante Unterschiede. Eye-Tracking-Daten zeigen jedoch, dass Gruppen mit der Blochkugel signifikant mehr Übergänge (Transitionen), also einen verstärkten Wechsel der Augenbewegungen zwischen den Repräsentationen auslöst.

Eine weitere Studie prüfte die Experteneinschätzungen direkt auf Lernenden Ebene in dem anwendungsbezogenen Kontext von Quantencomputing. Untersucht wurden das konzeptuelle Verständnis, die kognitive Belastung sowie anwendungsorientierte Aufgaben mit den jeweiligen Repräsentationen zu *Phase, Amplitude, Quantenzustand, Superposition* und *Quantenmessung*. Hier lösten die SchülerInnen die Aufgaben mit der Blochkugel signifikant effizienter, obwohl sich wiederum keine Gruppenunterschiede im konzeptuellen Verständnis oder in der kognitiven Belastung zeigten. Diese Ergebnisse verifizieren die Expertenratings teilweise und zeigen, wie das Kategoriensystem den Einsatz anderer, ähnlich gelagerter Repräsentationen leiten kann.

Die Befunde verdeutlichen, dass das Kategoriensystem mit Repräsentationen flexibel einsetzbar ist – etwa in experimentellen Aufbauten oder praxisnahen Szenarien der Quantentechnologie. Obwohl in beiden Studien keine Unterschiede im konzeptuellen Verständnis oder in der kognitiven Belastung gefunden wurden, legten Prozessdaten aus Eye-Tracking und Zeitmessung Unterschiede im Umgang mit den Repräsentationen offen.

Damit wird das Kategoriensystem teilweise validiert: Es unterstützt sowohl die Auswahl geeigneter Repräsentationen als auch die Entwicklung neuer Repräsentationen.

Die Forschung zum Lernen mit Repräsentationen ist keineswegs abgeschlossen; doch bietet die hier vorgestellte Merkmalsstruktur einen soliden Ausgangspunkt für weitere Arbeiten – etwa zum unterschiedlichen Einsatz mit *Multiple External Repräsentationen* oder zu spezifischen Konzepten wie der Verschränkung. Insgesamt liefert diese Dissertation einen ersten Einblick in das breite und komplexe Feld der Repräsentationen in der Quantenphysik.

Table of Contents

1. General Introduction

1.1 Aims of the Dissertation

As quantum technologies continue to develop, it can expect to see not only changes in science but also significant social changes (de Wolf, 2017). Quantum technologies are interdisciplinary, connecting physics, computer science, and mathematics (e.g., de Wolf, 2017). In addition, the promotion of a deeper conceptual understanding of quantum physics has become an important curricular goal Europe-wide and beyond (European Commission et al., 2025; Greinert et al., 2022; Krijtenburg-Lewerissa et al., 2017; Stadermann & van den Berg, 2019; KMK 2020), spanning different educational contexts—from schools and universities to adult and industrial education.

However, learning quantum physics poses particular challenges because it involves abstract, non-intuitive concepts (Corsiglia et al., 2023) that differ significantly from classical thinking (Marshman & Singh, 2017). As Stadermann (2019, p. 1) noticed:

> “In contrast to most classical physics topics, we cannot find a consistent visualization for quantum phenomena.”

In addition, Bouchée et al. (2022) point out that the abstract mathematical formalism of quantum mechanics often obscures the underlying concepts for students in an early learning stage. Learning with visual-graphical representations in quantum physics is versatile and can create a variety of learning opportunities. In order to provide students with more suitable access to quantum physics, and in particular to quantum technologies, it is necessary to understand how learning processes unfold when representations are used—and what impact these representations have on learning.

From a learning science perspective, studying how students conceptualize quantum concepts—such as superposition, measurement and entanglement—when working with (multiple) representations provides valuable insights into their underlying cognitive processes. The attempt to make quantum physics and quantum technologies understandable with representations or visualizations tools e.g., *Quantum Composer* (Küchemann et al., 2023; Weidner et al., 2021) or *The Quantum Mechanics Visualisation Project QuVis* (n.d.) and the development and design of existing or new (qubit) representations show the great and ever-increasing interest in it (Bley et al., 2024; Donhauser et al., 2024; Huber & Glaser, 2024; Just, 2020; Küchemann et al., 2023; Weidner et al., 2021). There are already initial approaches to investigating the behavior of educators and their use of multiple external representations, such as the online survey by Rexigel et al. (2025). There is some research on how learners deal with representations or learn in a simulated learning environment (e.g, Bley et al., 2025; Kohnle et al., 2014, 2020; Küchemann et al., 2023; Marshman et al., 2024). One promising approach to support learning in this area is the use of multimedia with visual-graphical representations or augmented reality (AR) environments (Coban et al., 2025), dynamic visualization tools (Kohnle et al., 2020), or in simple online learning material. However, there is still no insight into the mechanisms/aspects of representations which are responsible for outcomes.

Given the growing importance of quantum education, more research needs to be done on how learners acquire quantum concepts through representations and how to support them in doing so.

The aim of this dissertation is to identify features of representations and systematically examining their effectiveness for learners. Across several empirical studies, it explores how visual features, representational formats, and prior knowledge influence conceptual understanding in quantum physics from the perspectives of both learners and instructors (experts).

By integrating insights from physics education research, cognitive psychology, representation theory, and domain-specific aspects relevant to quantum technologies, this work contributes both theoretical understanding and practical implications for the design of effective learning environments with representations in quantum physics.

1.2 Learning with Representations

Lemke (1998) showed that content is used together with representations—such as text, diagrams, tables, photos, and equations—in scientific papers to foster the construction of meaning via multiple external representations.

The term ‘representation’ has many meanings and different categorizations. Lemke (1998) includes mathematical (-operational) (e.g. formula), visual–graphical (e.g. diagrams, graphs), visual–gestural (e.g. gestures), verbal–semantic (e.g. text). Other authors such as Bertin (1983), Kosslyn (1989) or Schnotz (2001) have proposed different ways of categorizing representations. However, these category systems are not able to determine the effectiveness of learning and its appropriate use. Building on our earlier work (Qerimi et al., 2025), a categorization system (see Table 1) was developed and applied, which is presented in more detail in Study 1 (see Section 2). This system draws on insights from representation research, physics education and misconceptions in quantum physics, as well as domain-specific content in quantum science and technology.

A conceptual foundation for the overarching categories is provided by Ainsworth’s Design, Function and Task (DeFT) framework, which outlines how multiple external representations can be used effectively to support learning (Ainsworth, 2006). Ainsworth (2006, 2008) emphasizes the importance of effective learning with multiple external representations involving at least two representations. A key finding from the meta-analysis by Rexigel, Kuhn et al., (2024) indicates that the benefits of multiple external representations are not limited to well-established combinations of two representation types. Rather, positive effects are also expected when combining three or even more representations (Rexigel, Kuhn, et al., 2024).

The category system (in Table 1) differs from the DeFT framework but the four overarching classifications—Design, Function, Task, and Cross-Concept—are inspired by Ainsworth’s (2006) DeFT framework with more detail added for representations themselves and allow differentiation criteria between representations. They can be described as follows: ***Design*** includes features related to the visual appearance and structure of the representation. ***Function*** refers to features that describe how a representation interacts with learners or with other representations and the role it plays in the learning process. The ***Task*** cluster encompasses features that are directly associated with fundamental applications of quantum technologies. Finally, ***Cross-Concept*** includes features that go beyond individual tasks and address broader conceptual connections across representations. All 16 categories are fully described in Qerimi et al. (2025) (see Section 2).

Table 1

Refined Category system of visual representations adapted from Qerimi et al. (2025)

Categories
Design
1. Salience
2. Dimension
3. Understanding difficulties
4. Color
Function
5. Actions/Steps
6. Interaction with
7. Contiguity
8. Overlaps/Redundancy
9. Complementary
10. Predictability
Tasks
11. Phase
12. Amplitude
13. Concepts
14. Quantum Technology
Cross-Concept
15. Generability
16. Effort in explanations

Note: The four overarching classifications: Design refers to visual and structural aspects of a representation; Function describes its role in learning and interaction; Task includes features tied to quantum applications; and Cross-Concept captures broader conceptual links across representations.

Visual–graphical representations are typically not presented to learners in isolation, but rather in combination with other forms of external representations such as verbal texts or mathematical formulas. A key challenge for learners is to comprehend each representation, extract the essential information, and integrate these elements to form coherent mental models or schemas (Schnotz, 2005).

A description of how such representations in multimedia environments affect learners' cognitive processing is provided by Mayer's Cognitive Theory of Multimedia Learning (CTML) (Mayer,

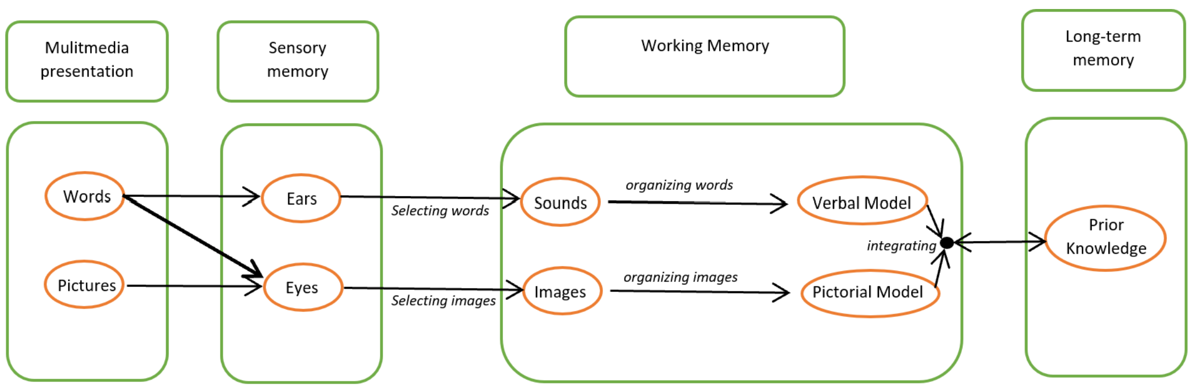
2021) and Schnotz's Integrated Model of Text and Picture Comprehension (ITPC) (Schnotz, 2005, 2014).

1.2.1 Cognitive Processes

CTML assumes that humans possess two separate channels for processing visual/pictorial and auditory/verbal information (Mayer, 2021; Mayer & Fiorella, 2014). Effective learning occurs when learners can *select* relevant information, *organize* it into coherent structures within each modality, and *integrate* it with prior knowledge and across modalities (Mayer, 2021; Schnotz, 2005, see Figure 1).

Figure 1

Cognitive processes in multimedia learning adapted from Mayer (2014, 2021)



Note. First, the sensory register must be accessible via a multimedia environment in the form of visual and/or verbal representations via the eyes and/or ears. The representations then enter the working memory via selection processes. According to the active process by CTML proposed by Mayer (2021) learning involves the construction of internal representations and their integration with prior knowledge stored in long-term memory to form a coherent mental model (Schnotz, 2005, 2014).

The CTML is based on the three assumptions about *channel duality*, *limited capacity* and *active processing* (Mayer, 2021)

- *Dual-channel* processing is receiving and processing information through different modalities; Mayer focuses specifically on visual and verbal input, following Paivio (1990) (Mayer, 2014, 2021; Paivio, 1990, Chap. 4). Learning with multiple representations involves the cognitive processing of information presented through multiple modes, such as visual and verbal (Mayer, 2021; Moreno & Mayer, 2007; Paivio, 1990; Seufert, 2003). This enables learners to integrate complementary information and make more efficient use of their working memory (Mayer, 2021). *Mode* describes the form of a representation, such as verbal (e.g. spoken or written text) or visual (non-verbal) (e.g. images, diagrams or animations) (Mayer, 2021; Moreno & Mayer, 2007; Paivio, 1990). Modality refers to the sensory perception (e.g. seeing or hearing).

- *Limited capacity* concerns how much information the working memory can process at a time (Mayer, 2021; Sweller, 1988; Sweller et al., 2019). In order to support these processes and enable knowledge building, the limited cognitive resources in working memory should be utilized optimally (Mayer, 2021; Sweller et al., 2019). Sweller's cognitive load theory (CLT) offers in-depth perspective in this context. He distinguishes between different types of cognitive load: intrinsic cognitive load (ICL), which is caused by the inherent complexity of the learning content; extraneous cognitive load (ECL), which can be caused by poor implementation in the instructional design; and germane cognitive load (GCL), which supports the processing, organization and integration of learning material (Sweller, 2010; Sweller et al., 2019). Optimal teaching materials should be designed to minimize extraneous cognitive load, optimize intrinsic load and promote germane load (Sweller et al., 2019). A useful guideline for the number of information units that learners can process at the same time comes from Miller (1956). He estimated the capacity of working memory to be approximately 7 ± 2 elements. More recent work, such as that of Mayer (2014), suggests a slightly lower range, that learners can typically process about 5 to 7 new units of information at the same time. Above all, processes should be promote not only increase the extraneous process by supplementing it with additional representations, but also enable essential and generative processes to promote selection or organization and integrative learning processes (Mayer & Fiorella, 2014).
- *Active processing* refers to the learner's effort to select relevant information and to process it through organizational and structural processes (Mayer, 2021). The goal of this integration is to connect new information with prior knowledge stored in long-term memory.

An additional theoretical perspective is provided by Schnotz's Integrated Model of Text and Picture Comprehension (ITPC) (Schnotz, 2005, 2014), which emphasizes that meaningful learning results from the construction of coherent mental models that integrate propositional (verbal) and pictorial (visual) representations (Mayer, 2021; Schnotz, 2005). The model posits that the capacity to internalize, i.e. to comprehend, is contingent upon learners' ability to semantically align verbal and visual information into a unified mental representation (Schnotz, 2005).

Furthermore, to what learners can experience and acquire through external representations, mental models also play a critical role—both those that learners bring with them from prior experiences (background knowledge) and those that can be intentionally supported through external representations (Dutke, 1994; Hettmannsperger, 2014).

Cognitive and mental processes specific in quantum physics: Ubben and Bitzenbauer, (2022) investigated the structure of perception through models in quantum physics, initially using single photons as a specific example and subsequently exploring the broader applicability of mental models to other domains (Bitzenbauer & Ubben, 2025). In this context, the concepts for mental models in quantum physics, *fidelity of function* and *fidelity of Gestalt* are central (Bitzenbauer & Ubben, 2025; Ubben & Bitzenbauer, 2022; Ubben & Heusler, 2021).
Originally developed by Ubben and Heusler (2021) in the context of atomic models, these concepts were further extended by Ubben and Bitzenbauer (2022, 2025). The dimension *fidelity of Gestalt* describes the mental model in which it is assumed to be like an exact visual representation of a phenomenon or quantum object (Ubben & Heusler, 2021). For example, when using wave representation to explain interference, the dimension *fidelity of function* describes the mental model in which it is assumed to be the underlying abstract functionality of a phenomenon or quantum object (Ubben & Heusler, 2021). The goal is to guide learners from a primarily *Gestalt*-oriented model toward one with a more abstract *functional fidelity*, in order to support the development of a coherent understanding of quantum physics (Bitzenbauer & Ubben, 2025).

Although it has been suggested that visual–graphical representations are beneficial, their perceived simplicity can pose a risk of misunderstanding. Learners sometimes fail to recognize that they are not direct representations of reality, but rather scientific models (Garcia Garcia & Cox, 2010). Garcia Garcia and Cox (2010) investigated how learners perceive and interpret visual graphics, with a particular focus on the misconception of "Graph as pictures"—the tendency to misinterpret abstract representations as physical images. The results showed that graphics can facilitate understanding but often lead to misinterpretations if their abstract nature is not clearly communicated. The use of external representations and individual adaptation to the learners' prior knowledge can play an important role (Dutke, 1994; Hettmannsperger, 2014; Schnotz, 2014) in preventing misconceptions like "Graph as pictures" (Garcia Garcia & Cox, 2010) and to promote the potential of a more functional understanding of abstract phenomena in quantum physics (Bitzenbauer & Ubben, 2025).

Traditionally, quantum physics has been characterized by symbolic–formal representations such as texts, equations and mathematical operators. This has historically led to various theoretical approaches. For example, Heisenberg (1925) developed matrix mechanics, while Schrödinger (1926) introduced wave mechanics. Soon afterwards, (Dirac, 1939) formulated an abstract representation using bra–ket notation, which has since become standard in research and teaching quantum physics and technology. But the abstract mathematical formalism of quantum physics can obscure the conceptual meaning behind symbolic expressions, making it difficult for learners in an early stadium to develop a deep understanding (Bouchée et al., 2022). In this context, visual–graphical representations can play a crucial role by serving as intuitive bridges to abstract mathematical structures. Alongside this formalism, visual–graphical representations have gained increasing relevance for teaching and learning (e.g,. Bley et al., 2024; Donhauser et al., 2024; Huber & Glaser, 2024; Just, 2020; Küchemann et al., 2023; Weidner et al., 2021).

A well-known example is the Bloch sphere, which is frequently employed in higher education to visualize two-state quantum systems. Recently, new representations have been also developed to visualize quantum states in Hilbert space, such as the Circle Notation (Bley et al., 2024; Johnston et al., 2019) or the Quantum Bead (Huber & Glaser, 2024).

In order to better understand the use of visual–graphic representations, as well as the effect that their representational characteristics have on learning processes, it is important to consider process-oriented evidence rather than just outcome-oriented measures (Huang et al., 2009; Schewior & Lindner, 2024). Measures such as reaction time, task efficiency, and particularly gaze paths from eye trackers, can reveal cognitive strategies of which learners are often unaware. These findings help to reveal how representations are processed and interpreted by learners in real time.

1.2.2 Measuring learning processes

Multimedia learning and multimedia testing partly overlap, as learners are initially confronted with a multimedia environment in both contexts (Schewior & Lindner, 2024). However, a fundamental difference lies in their primary focus: while multimedia *learning* aims to support the understanding and processing of content—as described in the previous section—multimedia *testing* refers to assessment situations in which the test question is presented simultaneously with a multimedia element such as a visual representation (Schewior & Lindner, 2024).

To examine the extent to which the implemented representations support learning, various methods could be used in empirical educational research to capture both cognitive changes, such as knowledge acquisition or conceptual understanding, and the performative aspects of learning. These include pre-/posttest designs with standardized achievement tests (e.g., Bitzenbauer et al., 2024), response time measurements (e.g., Huang et al., 2009; Schewior & Lindner, 2024), as well as process-oriented approaches such as eye tracking (Becker et al., 2022; Holmqvist & Andersson, 2017; Klein et al., 2021), the analysis of learning behavior, and the assessment of cognitive load (e.g., Klepsch et al., 2017; Sweller, 1988, 2011).

a) Learning gain

Learning gain measures the difference in students performance between two time points (McGrath et al., 2015). These two stages could come before and after an introduction, learning unit or teaching lesson. Learners can be tested in many ways, for example content-specifically, skills-specifically or via competencies (McGrath et al., 2015; Vermunt et al., 2018). To reliably assess learning of certain concepts and learning gain, validated test items are useful. Validation ensures that each item accurately measures the specific concept targeted by the instructional intervention (e.g., test items by Bitzenbauer et al., 2024; Waitzmann, 2023). As a standard practice, the difference between the knowledge after the intervention and the knowledge prior, which are evaluated through specific learning results such as test scores or progress in conceptual understanding, defines the learning gain. However, since learning is a process, essential aspects of this process cannot always be directly captured by collecting results (Huang et al., 2009).
In order to gain a comprehensive impression of how learners deal with representations, process-related data such as eye movements (e.g. using eye tracking), reaction times or cognitive load could therefore also be considered to become a refine perspective of the learning process with representations.

b) Eye tracking

To record eye movements, eye-tracking instruments are required. These can be stationary (e.g. mounted on a screen) or mobile devices (e.g., glasses). Furthermore, suitable software (such as Tobii Pro Studio) is required to record and analyze the gaze data. For a targeted analysis, areas of interest (AOIs) must first be defined. These mark specific regions in the visual material, such as different representations that are spatially separated. Various eye-tracking metrics can then be analyzed—for example, the number of transitions between AOIs or the fixation duration within a specific AOI (Holmqvist & Andersson, 2017).

Establishing a link between the data collected through eye tracking and learning performance is proving to be a complex challenge (Alemdag & Cagiltay, 2018; Coskun & Cagiltay, 2022; . Mayer, 2010). Drawing on Coskun and Cagiltay, 2022, possible *selection*, *organization* and *integration* processes could be clearly classified by Mayer (2014) from eye movement metrics, e.g. time to first fixation in selection or number of transitions in integration (Alemdag & Cagiltay, 2018). Eye tracking has proven reliable for tracking the learning process, but, there are also inconsistencies in the interpretation of eye-tracking data. The data should therefore be used in conjunction with other tests, such as cognitive load tests or concept tests, in order to make reliable and transparent statements (e.g., Van Gog & Jarodzka, 2013).

c) Time reaction

Hou & Zhang (2006) demonstrated that visual information processing is highly dependent on viewing time: the longer a visual stimulus is observed, the more detail can be perceived. They found a clear relationship between reaction time and the spatial resolution of visual attention. Schewior and Lindner (2024) also emphasize reaction time as an important indicator of cognitive processes in multimedia learning and testing environments. CLT considers that an increase in element interactivity can also lead to an increased load on working memory (Sweller et al., 1998, 2019). This has been shown to lead to an increase in processing time and may indicate either deeper cognitive engagement, increased effort or comprehension difficulties (Sweller, 2010; Sweller et al., 1998). Therefore, processing time should always be interpreted in relation to task accuracy (Lindner et al., 2021; Schewior & Lindner, 2024), cognitive load (Hou & Zhang, 2006) or eye-tracking data.

d) Cognitive Load

In addition to time reaction, cognitive load can provide valuable insights into learners' information processing (Hou & Zhang, 2006; Sweller, 1988, 2010; Sweller et al., 2019). Participants answer on a scale of 1 to 7 how difficult or demanding they found a particular task or learning material to be (e.g., Klepsch et al., 2017 or Thees et al., 2020). In addition to the numerical Likert scale, there are also other scales, such as sliding regulator or color-differentiated smileys (Ouwehand et al., 2021). The various items on the scale make it possible to draw conclusions about the underlying mental demands. As described in the previous section, different types of cognitive load can be distinguished. According to Sweller et al., (2011, 2019), various methods exist for measuring cognitive load, including performance measures, secondary tasks, physiological indicators, and subjective rating scales. The latter—such as the Cognitive Load Test developed by Klepsch et al., (2017)—assess cognitive load retrospectively and can be influenced by learners' self-concept and self-assessment (Klepsch et al., 2017; Sweller et al., 2011; Thees et al., 2020). Although the method is not entirely free from subjective bias (Sweller et al., 2011), studies have shown that the instrument provides a reliable way of assessing cognitive load (Klepsch et al., 2017; Krieglstein et al., 2022).

All considered, it becomes clear that, in order to get a comprehensive picture of learning with representations, suitable methods, formats and instruments are needed. Learning with representations not only promotes cognitive processes in content-related competences, but using representations also activates so-called representational competences and further promotes them (Rau, 2017).

1.2.3 Representational Competence

Learning environments in physics—especially in quantum physics—often involve a variety of representations such as symbolic, verbal (e.g., text-based), or visual–graphical representations (Lemke, 1998). Learners are often faced with the challenge of linking these representations and integrating them conceptually (diSessa & Sherin, 2000; Rau, 2017). At the same time, working with multiple representations offers educational opportunities (Ainsworth, 2006).

In her 2017 review, Rau provides a comprehensive overview of representational competencies—that is, the specific knowledge and skills that learners need in order to work effectively with (multiple) representations, as well as the learning processes through which these competencies are developed (Rau, 2017). One of these competencies is described as *connectional understanding*, the ability to relate multiple visual representations to each other, to identify relevant similarities between them, and to understand the conventions for using different types of representations (Rau, 2017).

In contrast, meta-representational competencies describe a more global competences by interacting with representations. According to diSessa (2000, p. 386) meta-representational competencies include

> "…the ability to select, produce and productively use representations, but also the ability to criticize and modify representations and even to design completely new representations."

Meta-representational competencies can encourage learners to develop an understanding of the roles that representations play and their potential limitations. This can lead to greater transparency and reflection on possible misconceptions.

The development of representational competencies—particularly connectional understanding or meta-representational competence—requires tailored instructional support (Rau, 2017). These competencies can be fostered through a combination of social mediated sense-making process, nonverbal inductive learning, and reflective engagement with multiple representations, depending on the type of competence being addressed (Rau, 2017).

1.3 Representations in Quantum Physics

The theoretical aspects and cognitive processes discussed in the previous sections provide a foundation for understanding learning with representations. However, quantum physics presents particular challenges: its content is very abstract, dominated by mathematical formalism, and often unintuitive for learners (Corsiglia et al., 2023; Marshman & Singh, 2017). In this context, representations are not merely educational tools but serve as mediators between mathematical formalism and conceptual understanding. A wide range of representations is available. It is more difficult to find consistent representations in quantum physics than in other domains of physics (Stadermann, 2019). It is important that representations in quantum physics remain connectable and transferable to mathematics, as they play an important role in the deeper understanding of quantum physics and quantum technologies.

The spin-first approach introduces a two-state system such as the behavior of photon or electron spin earlier than the position-first approach (see Table 2) (Sadaghiani, 2016; Sadaghiani & Munteanu, 2015). From an educational perspective, the spin-first approach is particularly valuable as it enables the use of two-state system representations—which can be described by a qubit representations—from the outset, helping to make modern concepts from quantum information and computing more accessible (Dür & Heusler, 2012, 2014). This allows for learners to describe quantum states, quantum measurement, probabilities in an early stage without first being introduced to the Schrödinger equation (Sadaghiani, 2016; Sadaghiani & Munteanu, 2015). The Schrödinger equation is important, of course, and can be connected in a later stage depending on the prior knowledge of the target group.

In contrast, the position-first approach follows a more historically motivated progression (e.g., double-slit experiment, wave function and Schrödinger equation) and introduces the two-state system later (Sadaghiani, 2016; Sadaghiani & Munteanu, 2015). In the position-first approach, wave–particle duality is further discussed via the double-slit experiment and allows for a deepening of this conflict, which learners often find unintuitive to accept.

Both instructional approaches allow for an introduction to quantum physics and quantum technology. Which approach is more accessible depends on the target group and the objectives of the lesson.

In the context of designing instructions that introduce quantum technologies and quantum computing, the spin-first approach offers a focused entry point through the representation of two-state systems. Every two-state system can be represented by a qubit representation. A qubit representation can depict the state of a quantum object, e.g. the behavior of a photon after passing through a beam splitter.

Table 2

Overview: spin-first compared to position-first, modified from Sadaghiani (2016)

Spin-First	Position-First
Birth of modern physics, blackbody radiation	
Photoelectric effect	
Structure of the atom, Thomson & Rutherford models	
Wave–particle duality, de Broglie waves	
Polarization, two-state system (spin as context), probability	Double slit and electron scattering
Dirac notation, postulate (for students: basic principles), matrix notation, quantum state	Wave function, properties, Schrödinger equation, probability, expectation value
Schrödinger equation, expectation value	Infinite and finite potential pot
…	…

1.3.1 Qubit Representations

According to Benjamin Schuhmacher (1995), qubits are the fundamental units of quantum information. Qubit representations describe quantum states of two-level systems, represented in the basis $|0\rangle$ and $|1\rangle$. The qubit representation can contextualize the content to be conveyed by referring to possible applications of quantum technologies (Dür & Heusler, 2012), which additionally can promotes learners' motivation in quantum physics (Müller, 2006).

Both in scientific literature and in educational practice, various visual, symbolic, and formal representations have been established, each bringing its own potentials and challenges for supporting student learning (Hennig et al., 2024; Hu, Li, Mong, et al., 2024; Hu, Li, & Singh, 2024; Wawro et al., 2020).

As Lautesse et al., 2015 point out, the simultaneous use of classical models such as wave and particle—for example, in the context of the double-slit experiment—can lead to confusion, as it does not provide a coherent overall model. Moreover, there is a risk of reinforcing or even creating misconceptions (Lautesse et al., 2015). This makes the selection and use of appropriate representations a particularly challenging educational task.

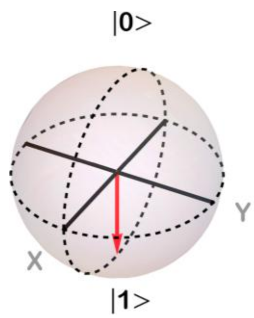
Prior instructional experiences from classical physics may inadvertently hinder further learning, as learners often rely on familiar but incompatible models (Krijtenburg-Lewerissa et al., 2017;

Majidy, 2024; Merzel et al., 2024; Müller & Wiesner, 1999; Singh & Marshman, 2015). Singh and Marshman (2015) provide empirical evidence of students struggling, for example, in conceptualizing photon polarization states as orthogonal vectors in a two-dimensional Hilbert space (Singh & Marshman, 2015). Instead, learners frequently interpreted these phenomena through the lens of classical optics—focusing on physical components like wave plates or polarization rotators. Their classical association of polarization with light waves that, after passing through a polarizer are filtered in one orientation, or waveplate that the orientation rotated accordingly, hinders recognition of the abstract vector nature of polarization as a quantum state (Singh & Marshman, 2015). Notably, such difficulties did not occur when students dealt with electronic spin-based systems that they did not know from classical physics lessons (Singh & Marshman, 2015). The authors mentioned are not the only ones to have investigated how learners deal with misconceptions or incompletely formed quantum concepts, and to what extent these can impede their understanding (see Bouchée et al., 2022; Brang et al., 2024; Fischler & Lichtfeldt, 1992; Krijtenburg-Lewerissa et al., 2017; Majidy, 2024; Özcan, 2011; Wiesner, 1996). The background of the core concepts from the "Wesenzüge der Quantenphysik" from Küblbeck and Müller (2002), also known as "reasoning tools for quantum physics" (Küblbeck & Müller, 2002; Müller & Greinert, 2022; Müller et al., 2021), which include key concepts relevant to understanding modern quantum technologies (Merzel et al., 2024), such as entanglement, are central concepts of this work. Potential misconceptions could arise from the design aspects of visual–graphical qubit representations and have been identified based on empirical findings, which was the theoretical base of Study 1 (Qerimi et al., 2025). As an aspect of *learning difficulties*, it is included as an own category in the category system (see Table 1).

These insights emphasize the crucial role of representations: The selection and design of visual representations e.g. the Bloch sphere (Figure 2) in quantum physics lessons must be guided not only by their explanatory power, but also by their potential to accidentally reinforce misconceptions. To support conceptual learning while minimizing the risk of reinforcing classical misconceptions, it is useful to analyze and design representations that are cognitively accessible, mathematically consistent, and visually meaningful.

Figure 2

Bloch sphere

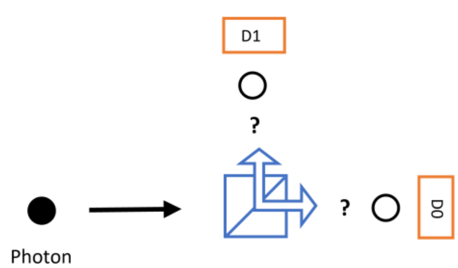


Note. Demonstration of a visual-graphical qubit representation. The Bloch sphere maps any pure state to a point on the surface of a unit sphere, using spherical coordinates to express quantum states geometrically. It offers a way to represent quantum phenomena such as superposition, phase differences, by a vector rotation. The Bloch sphere can be used to visualize product states, but not entangled states. Bell states are exceptions.

Moreover it is important to note that a (real) experimental setup also constitutes a form of representation (Lemke, 1998.; Schnotz, 2014). As Kozma and Russell (1997) already recognized, scientific experiments can significantly support the learning of scientific concepts. Particularly interesting is the fact that two-state systems can be realized in a simple and authentic way—for example, by discussing the qubit state of a photon after a beam splitter (see Figure 3). The bit values 1 and 0 are assigned to the paths—in this case, reflection (1) and transmission (0)—so that, after the measurement, one classical bit of information is obtained by the measurement signal.

Figure 3

Superposition state after the beam splitter



Note: A single photon can be detected at either detector 0 (D0) or detector 1 (D1). As long as no measurement is made, the photon's state can be described as a superposition of the two paths—in other words, it can be represented as a simple two-state quantum system.

And the qubit state can be also represented mathematically as follows:

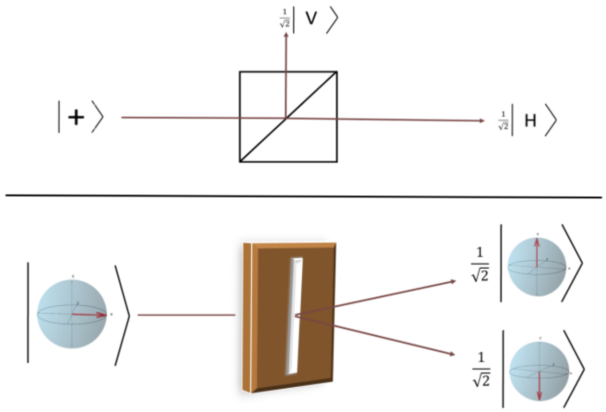
$$|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \quad (1)$$

After passing through the beam splitter, the photon is in a superposition state of the two possibilities: “transmitted photon” $|0\rangle$ and “reflected photon” $|1\rangle$. For an ideal 50:50 beam splitter, the probability of detecting the photon in either output is ½. Such processes are also investigated, for example, in the context of quantum random number generators (e.g. Fürst, 2011).

A combination of an experimental setup with qubit representations has already been realized by Dür and Heusler (2012). Using visual–graphical representations such as the Bloch sphere, the quantum state can be illustrated in an intuitive way (see Figure 4). To illustrate the measuring process, Dür and Heusler suggest to imagine the Bloch sphere passing by “slitting” in a certain spatial direction. The usual measuring axis is taken to be in the *Z* direction, so the measurement ensures that the state vector must be oriented in either the positive or negative *Z* direction by passing through the slit. The probability of obtaining a corresponding measurement result is determined by the angle between the slit and the state vector (Dür and Heusler, 2012). For example, if the state vector is close to the positive *Z*-axis, the experiment is more likely to measure state $|0\rangle$. In Figure 4, the slit represents the possible outcomes of the experiment, as previously mentioned in the context of the beam splitter. It serves merely as an illustrative aid for the measurement process (Dür & Heusler, 2012).

Figure 4

Adapted from Dür and Heusler (2012)



Note. The figure establishes a link between the experiment and the Bloch sphere model, providing a description of the quantum state after a beam splitter and depicting the possible measurement outcomes. The polarization of photons at polarizing beam splitters is used to describe the photon's quantum state. The slit illustrates that, after measurement, one of the possible outputs is always realised and no intermediate state exists. Figure originally created for "Exploring the mechanisms of qubit representations and introducing a new category system for visual representations: results from expert ratings" by L. Qerimi, S. Malone, E. Rexigel, S. Mehlhase, J. Kuhn, & S. Küchemann (2025), *EPJ Quantum Technology, 12*(1), 45. Licensed under CC BY 4.0 (http://creativecommons.org/licenses/by/4.0/).

Research has also demonstrated that the Mach–Zehnder interferometer (MZI) with single photons can be used as an example to reduce comprehension difficulties and to illuminate students' understanding of wave–particle duality, the probabilistic nature of quantum measurement, and the principles of quantum physics within a tangible experimental setting (Marshman & Singh, 2017). Other studies on the use of the Bloch sphere show that it serves as an effective cognitive aid for learning about qubits (Hu, Li, Mong, et al., 2024; Hu, Li, & Singh, 2024).

In quantum technological contexts, employing qubit representations in combination with experimental setups like the MZI with single photons has been demonstrated to facilitate comprehension of fundamental concepts including quantum computing (e.g., Ekert, 2010). Quantum gates, including the NOT gate, can be demonstrated using the MZI with single photons (Ekert, 2010). While further gates, including the Controlled NOT (CNOT), can also be demonstrated via extensions to multi-qubit systems, these are not described in detail here. In addition, various quantum technology approaches can be realized through visual-graphical qubit representations, such as using the Bloch sphere to describe the behavior of single photons in the MZI.

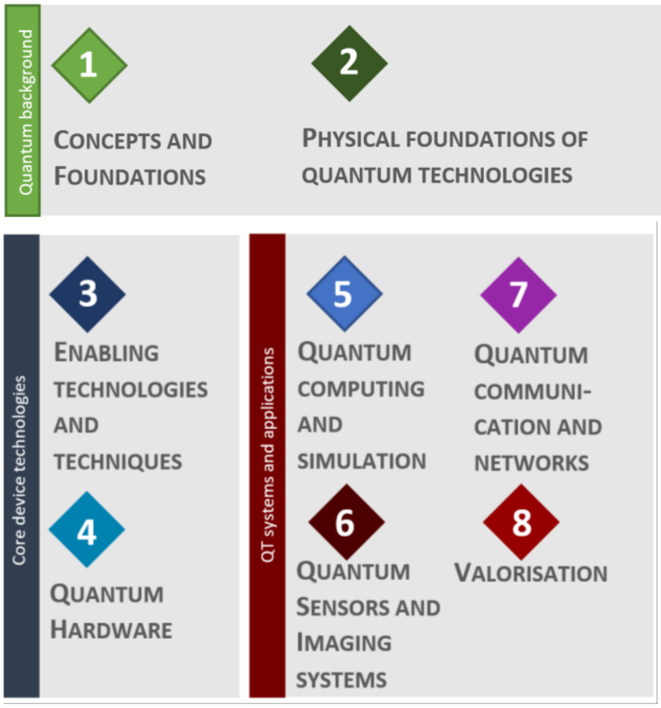
This highlights the potential of combining multiple external representations: it allows fundamental principles and applications of quantum technologies to be conveyed in an authentic learning environment, making quantum physics and quantum technologies more tangible and accessible for learners.

1.3.2 Relevance of Quantum Technologies in Education

The use of representations in teaching materials, lessons or other learning opportunities us based on instructional or curriculum guidelines for teaching or lessons. These vary from country to country but, in 2021, the European Competence Framework for Quantum Technologies (CFQT) was developed, a structured system for recording and describing the competences and skills required in the field of quantum technology. This provides an overview of the content relevant for learning in quantum technologies from a more global perspective (see Figure 5) (European Commission et al., 2025; Greinert et al., 2022). The current version (April 2025) even shows that, according to the language-oriented skill levels (A1–C2), under A2 literacy ‘knowledge of basic quantum concepts and the underlying representations’ (European Commission et al., 2025; Greinert & Müller, 2025, p. 15), representations of two-level-systems play a relevant role both in the initial phase (level 1) and at an advanced level (level 5) (see Figure 5). The Competence Framework is more strongly orientated towards professional training, but can be used in the early stages (1 and 2) for learners with low prior knowledge, like students.

Figure 5

Competence Framework for Quantum Technologies adapted from Greinert and Müller (European Commission et al., 2025; Greinert et al., 2022)



Note. Content Map of the Competence Framework for Quantum Technologies: Across domain levels 1 to 8, the competence framework addresses simple fundamentals of quantum physics and quantum technology (levels 1 and 2) to core device or implementation for realization such as neutral atoms in an optical lattice (levels 3 and 4) and in-depth study of quantum technology systems and more concrete applications (levels 5 to 8).

In addition to the orientation toward frameworks or curricular structures for the sensible use and utilization of representations, approaches that allow learners to be reached in teaching sequences also play a role in teaching quantum technology for learners.

The importance of qubit representations and the conceptual understanding of quantum physics can be effectively demonstrated using the spin-first approach (Dür & Heusler, 2012, 2014; Sadaghiani, 2016; Sadaghiani & Munteanu, 2015). Studies have already shown that the understanding qubit processes represented on the Bloch sphere may support learning (Hu, Li, Mong, et al., 2024; Hu, Li, & Singh, 2024). Other studies show that visual–graphical representations are helpful in problem solving (Bley et al., 2025; Kohnle et al., 2014; Küchemann et al., 2023).

Despite this, there has been a lack of systematic research into which aspects/features in visual representations effectively support learning processes in quantum physics and quantum technology.

To address this gap, **Study 1** developed a feature-based category system. The system was evaluated by experts of four exemplary visual–graphical qubit representations.

The insights from **Study 1** informed the design of **Study 2**, which investigated how learners engage with multiple informational-redundant representations and how these influence understanding.

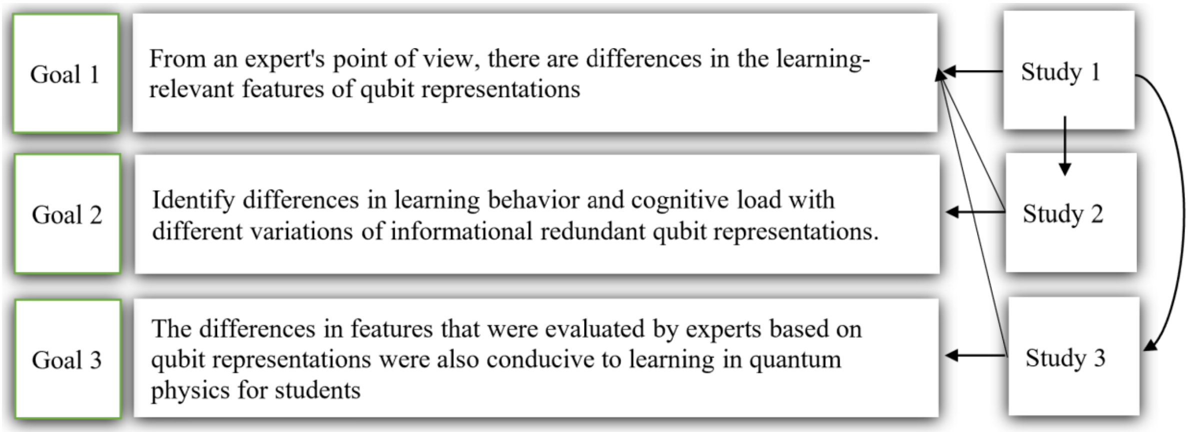
Finally, **Study 3** applied the expert-based evaluations from **Study 1** to learner data in order to assess how expert-identified features translate into actual learning outcomes.

1.4 General Research Questions and Overview of the Cumulative Dissertation

This cumulative dissertation addresses the question of how visual–graphical representations can support the learning of fundamental concepts in quantum physics and quantum technologies. The focus lies on understanding the features that make certain representations more effective for learning than others, from both theoretical and empirical perspectives. Its goals are to investigate how learners engage with different external representations—especially of qubit representations in the context of quantum technologies—and how features/aspects of representations affect the cognitive processes and influence learning (see Figure 6).

Figure 6

Goals of the studies



To address these goals, three empirical studies were conducted that build upon each other methodologically and conceptually:

- Study 1 explores theoretical mechanisms and introduces a category system with expert rating of qubit representations.

- Study 2 investigates how the integration of informationally redundant qubit representations (e.g., Dirac notation and Bloch sphere) affects learning processes, using eye-tracking to analyze visual attention and integration behavior.

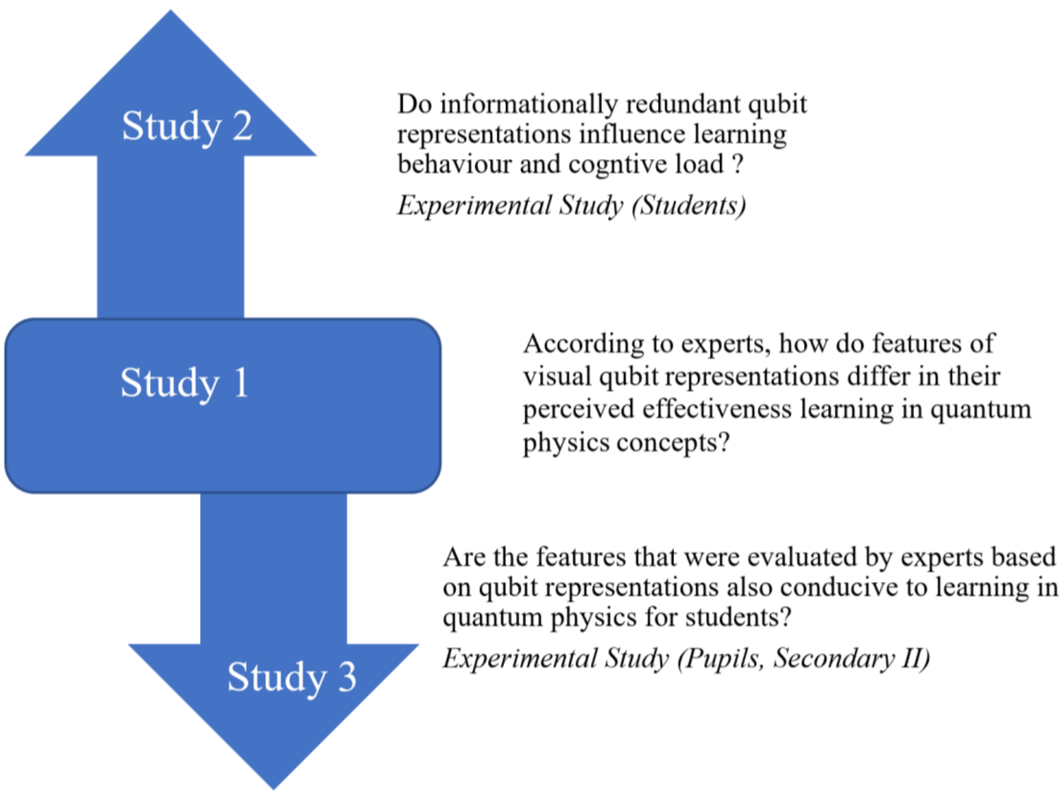
- Study 3 compares qubit representations in terms of their effects on learning gain, task performance in accuracy and time, and cognitive load among upper secondary students.

Together, these studies offer a multi-perspective insight into learning with representations in quantum physics and contribute to a deeper theoretical and empirical understanding of how instructional materials and learning environments can be designed to support meaningful learning in complex scientific domains.

The structure of this dissertation (see Figure 7) highlights the interplay between quantum-specific expertise, didactics, and learning sciences. Through this interdisciplinary perspective, a comprehensive understanding can be developed of how quantum physics can be effectively taught and learned using representations. This integrative perspective also shapes the methodological approach: the starting point is a categorization system evaluated by experts, which serves as the theoretical and empirical foundation for the selection and assessment of representations in the subsequent studies (by students and high school students).

Figure 7

Structure of the dissertation and main research questions



Note. Structure of the context, the associated research questions and the applied methodological strategies and the main target group in the study.

1.4.1 Outline of Study 1

Learning quantum physics is particularly challenging, as the subject is characterized by abstract concepts, strong formalism, and frequent conflicts with classical intuitions (Corsiglia et al., 2023). Visual–graphical representations—such as the Bloch sphere—have already been shown to support learning about qubits by providing conceptual access to otherwise abstract content (Hu, Li, & Singh, 2024). Their potential as a bridge between mathematical formalism and conceptual understanding makes them especially attractive for educational purposes. Bouchée et al. (2022) even emphasize that abstract mathematical formalism can obscure the underlying meaning of quantum physics concepts for students in the beginning.

However, in quantum physics, it remains difficult to identify representations that are both scientifically accurate and accessible to learners, particularly without requiring extensive prior knowledge in mathematics (Stadermann, 2019). The risk of triggering or reinforcing misconceptions is high (Krijtenburg-Lewerissa et al., 2017; Lautesse et al., 2015; Müller & Wiesner, 1999) if representations are not carefully designed and introduced.

Despite the growing use and development of visual–graphical representations (Bley et al., 2024, 2025; Coban et al., 2025; Hu, Li, Mong, et al., 2024; Huber & Glaser, 2024; Johnston et al., 2019; Just, 2020; Kohnle et al., 2014; Küchemann et al., 2023), a systematic understanding of which representational features support learning, and how these features are realized across different representations, is still lacking. To address this gap, the study developed a category system comprising 16 categories that describe features that promote learning. These categories were selected at the intersection of representation research, quantum education, and aspects of quantum science and technologies. A top-down process was followed to evaluate four exemplary qubit representations using this category system by experts in quantum physics and quantum technologies and, after that, to investigate the learners' perspective. Twenty-one experts from ten institutions across Germany, Italy, Switzerland, and the USA participated in the online rating via Google Forms. The rating was conducted using a five-point Likert scale (1–5) (Likert, 1932), supplemented with the option "I don't know". Qualitative data were also collected from the experts' perspective via free-text questions to discern which criteria are important for differentiating representations and to identify concepts missing from the rating.

To ensure a well-informed and comparable evaluation, so-called **cheat sheets** were developed for each qubit representation (see supplementary material in Section 2. Study 1, Qerimi et al., 2025). These were uniformly designed and included all relevant information needed for the rating. All experts had teaching experience in quantum physics—two professors/junior professors, nine postdoctoral researchers, and ten PhD students (only those in their second year or above were eligible). Their primary research focus was theoretical (*n* = 7), experimental (*n* = 5), educational (*n* = 7), or interdisciplinary (*n* = 2) in quantum technology. On average, they had been engaged in quantum-physics research for 5.1 years (SD = 1.9 years).

The aim of this study was to systematically analyze differentiation criteria, key features and the educational potential of qubit representations. The investigation focused not only on their suitability for conveying fundamental quantum physics concepts on an expert perspective, but also the risk to promote misconceptions.

The study addressed the following research questions:

- RQ1: According to experts, which differences exist between the learning-relevant features of four selected visual-graphic qubit representations?
- RQ2: According to experts, what factors should be considered when creating new qubit representations to promote learning?

No hypotheses were formulated, as the study adopted an exploratory approach.

Beyond this, the study sought to derive design principles for future representations that support effective and sustainable learning in the context of quantum physics and quantum technologies. In doing so, it contributes to the overarching research question of this dissertation: According to experts, how do features of visual qubit representations differ in their perceived effectiveness learning in quantum physics concepts?

1.4.2 Outline of Study 2

The effective use of multiple external representations (MERs) has already been emphasized in Ainsworth’s work, particularly through her Design, Functions, Tasks (DeFT) framework (Ainsworth, 2006). In this framework and in her earlier work, Ainsworth outlines different functions that MERs can serve in learning—such as promoting deeper understanding, constraining interpretation, or fulfilling complementary roles (Ainsworth, 1999, 2006). The latter refers to the idea that representations may convey redundant information but, due to their different modalities, they can activate complementary cognitive processes and thus enhance learning (Ainsworth, 2006).

At the same time, Mayer's Cognitive Theory of Multimedia Learning (CTML) highlights the redundancy principle, which describes how redundant information can unnecessarily burden cognitive resources, which may hinder learning (Mayer & Fiorella, 2014).

However, Ott et al. (2018) found out that students performed better on mathematical problem-solving tasks when they had access to multiple, even redundant, representations. Additionally, experts in Study 1 pointed out that learning in quantum physics particularly benefits from the combination and alternation of multiple representations (Qerimi et al., 2025).

Still, it remains unclear under which conditions redundant information actually supports more effective formation of conceptual understanding in quantum physics. To address this issue, the study used a 2×2 between-subjects factorial design, comprising four groups: CG, IG1, IG2 and IG3.

- Control Group (CG): Text + static illustration
- Intervention Group 1 (IG1): CG + Dirac notation (symbolic)
- Intervention Group 2 (IG2): CG + Bloch sphere (graphical)
- Intervention Group 3 (IG3): CG + Dirac notation + Bloch sphere (symbolic + graphical)

A total number of 113 STEM students were randomly assigned to one of these groups. All participants worked with a multimedia learning environment on the Mach–Zehnder interferometer (MZI). The experimental manipulation was as follows:
To evaluate learning outcomes and cognitive processing, participants completed pre- and posttests of content knowledge (Waitzmann, 2023; Waitzmann et al., 2024). Cognitive load was assessed using validated questionnaires that measured extraneous, intrinsic and germane load (Klepsch et al., 2017). During the learning phase, eye movements were tracked and recorded while the learning unit was being completed. Finally, spatial abilities were measured using a mental rotation test (RCube Vis test, Fehringer, 2020).

Study 2 investigates whether and how the learning of quantum physical properties is improved when informationally redundant external representations—specifically Dirac notation and the Bloch sphere—are used to complement an existing multimedia learning setting.

The study addressed the following research questions:

- RQ1: Does adding an information-redundant symbolic-mathematical or graphical geometric representation to a multimedia learning unit enhance learning (in terms of content knowledge and cognitive load) of quantum properties?
- RQ2: Does the integration of both informationally redundant representations additionally promote learning?
- RQ3: Are advantages in learning with information-redundant representations correlated with visual integration processes across representations or, rather, the selection of one preferred representation?

Furthermore, the study examined how learners interact with redundant representations and to what extent both the number and the type of representations influence conceptual understanding and cognitive load. In doing so, it contributes to addressing the overarching research question of this dissertation: Do informationally redundant qubit representations influence cognitive load and learning behavior?

1.4.3 Outline of Study 3

Learning quantum physics remains challenging due to the abstractness of its concepts and the difficulty of bridging the gap between formalism and conceptual understanding. Representation research shows that combining symbolic mathematical elements (e.g. equations) with visual–graphical representations promotes conceptual understanding more effectively than representations that merely depict phenomena (e.g., Ainsworth, 1999, 2006; Mayer, 2021). Although the previous expert rating in Study 1 identified relevant features of visual–graphical qubit representations (Qerimi et al., 2025), the impact of these representations on actual student learning remains unclear. To keep the study as simple as possible, two representations (Quantum Bead and Bloch sphere) were compared directly. The focus was on those features that showed significant differences between the Quantum Bead and Bloch sphere. These included, in particular, the salience in visualizing phase and amplitude, as well as the conveying key concepts such as superposition.

In context of school students in particular, the aim was to understand how design such as salience and the depiction of application-specific features such as visualization of superposition, demonstrating quantum measurement, phase or amplitude affect learners' task performance, learning gain, cognitive load and retention.

To address this gap, this study employed a mixed factorial design with two representations (Quantum Beads vs. Bloch sphere) as the between-subjects factor, and test occasion (pre-, post-, and follow-up tests) as the within-subjects factor (time). The study involved a total N = 149 high school students (secondary II). Concept understanding was assessed through pre- and posttests (Bitzenbauer et al., 2024; Hu, Li, & Singh, 2024; Waitzmann, 2023; Waitzmann et al., 2024), while task performance was measured by process time (in milliseconds) and accuracy. Cognitive load was measured using validated scales for intrinsic, extraneous, and germane load (Klepsch et al., 2017), and participants provided confidence ratings to indicate the certainty of their answers. To investigate mid-term retention, a follow-up test (posttest 2) was carried out 1–2 weeks later with the same items in the pre- and posttest.

This study aimed to address this gap by comparing two representations that were previously rated by experts—the Bloch sphere and the Quantum Beads—with the goal of linking expert evaluations to student learning outcomes and process-based indicators such as task efficiency and cognitive resource use. Specifically, the study evaluated how the two visual–graphical qubit representations affect learners' understanding of quantum properties, cognitive load, and retention. In particular, it sought to empirically validate whether the expert assessments from Study 1 align with actual learning behavior.

The study addressed the following research questions:

- **RQ1:** To what extent do different visual–graphical representations (Quantum Bead vs. Bloch sphere) foster learning quantum concepts differently?

    H1.1: Participants who learn with the Bloch sphere achieve a higher learning outcome than those who learn with the Quantum Bead representation. **H1.2:** Participants using the Bloch sphere perform more efficiently on application-oriented quantum tasks in *phase gate, amplitude, quantum state, superposition,* and *quantum measurement* than those using the Quantum Bead.

- **RQ2:** How do different visual–graphical representations (Bloch sphere vs. Quantum Bead) affect the use of cognitive resources in the learning of quantum concepts?

  H2: Participants who learn with the Bloch sphere show a more effective use of cognitive resources than those who learn with the Quantum Bead.

- **RQ3:** How does the use of different visual–graphical representations (Quantum Bead and Bloch sphere) influence medium-term retention of fundamental quantum concepts?

  H3: Learners who used the Bloch sphere demonstrate higher medium-term retention of basic quantum concepts compared to those who used the Quantum Bead.

This study contributes to the overarching research question of the dissertation: Are the features that were evaluated by experts based on qubit representations also conducive to learning in quantum physics for students?

2. Study 1: Exploring the mechanisms of qubit representations and introducing a new category system for visual representations: Results from expert ratings

Contribution:

Qerimi, Küchemann, Kuhn, Malone designed the study, Qerimi and Küchemann developed the questionnaires and collected the data, Qerimi and Küchemann analyzed the data, Qerimi wrote the first draft of the manuscript. All authors reviewed and edited the manuscript. Küchemann supervised the study. All authors have read and agreed to the submitted version of the manuscript.

Reference: Qerimi, L., Malone, S., Rexigel, E., Mehlhase, S., Kuhn, J., & Küchemann, S. (2025). Exploring the mechanisms of qubit representations and introducing a new category system for visual representations: results from expert ratings. *EPJ Quantum Technology*, *12*(1), 45.

Qerimi et al. *EPJ Quantum Technology* (2025) 12:45
https://doi.org/10.1140/epjqt/s40507-025-00346-1
EPJ.org
EPJ Quantum Technology
a SpringerOpen Journal

RESEARCH **Open Access**

Exploring the mechanisms of qubit representations and introducing a new category system for visual representations: results from expert ratings

Linda Qerimi[1,2,3*], Sarah Malone[4], Eva Rexigel[5], Sascha Mehlhase[2,3], Jochen Kuhn[1] and Stefan Küchemann[1]

*Correspondence: linda.qerimi@physik.uni-muenchen.de
[1]Chair of Physics Education, Faculty of Physics, Ludwig-Maximilians-Universität München (LMU), Geschwister-Scholl-Platz 1, 80539 Munich, Germany
[2]Max Planck Institute of Quantum Optics, 85748 Garching, Germany
Full list of author information is available at the end of the article

Abstract

In quantum physics (QP) education, the use of representations such as diagrams and visual aids that connect to mathematical concepts is crucial. Research in representation theory indicates that combining symbolic-mathematical elements (e.g., formulae) with visual-graphical representations enhances conceptual understanding more effectively than representations that merely depict phenomena. However, common representations vary widely, and existing category systems do not adequately distinguish between them in QP. To address this, we developed a new set of differentiation criteria based on insights from representation research, QP education, and specific aspects of the quantum sciences. We created a comprehensive category system for evaluating visual QP representations for educational use, grounded in Ainsworths (2006) DeFT Framework.

Twenty-one experts from four countries evaluated this category system using four qubit representations: the Bloch sphere, Circle Notation, Quantum Bead, and the pie chart (Qake) model. This evaluation enabled us to assess the discriminative power of our criteria and to gain expert-based insights into the perceived effectiveness of each representation in supporting the learning of QP concepts. It evaluated how well each representation conveyed quantum concepts such as quantum state, measurement, superposition, entanglement, and quantum technologies (X-, Z-, and H-gates) across 16 criteria.

The results showed significant differences in the effectiveness of these representations, particularly in conveying key concepts like superposition and measurement from an expert perspective. Additionally, expert ratings indicated notable variations in the potential of each representation to induce misconceptions, linked to differences in shape, measurement behaviour, and requirements for understanding entanglement. We also discuss considerations for developing new representations and suggest directions for future empirical studies.

Keywords: Quantum technologies; Representations; Qubit; Quantum education; Expert rating; Category system

1 Introduction

QP plays a central role in the development of emerging quantum technologies (QT), which include applications such as quantum computing and quantum communication. The growing importance of these technologies highlights the need to teach QP effectively in order to prepare students for scientific and technological challenges. However, teaching quantum concepts is a major educational challenge, as these concepts are often abstract and unintuitive (e.g. [1]).

Diagrams, models, and visual aids can play a valuable role in QP education by helping students engage with and better understand mathematical concepts. Visual-graphical representations, in particular, are powerful tools for conveying complex quantum phenomena, as they help bridge the gap between abstract mathematical formalism and intuitive understanding. Research in representation theory indicates that combining symbolic-mathematical elements (e.g. formulae) with visual-graphical representations enhances conceptual understanding more effectively than representations that merely depict phenomena [2].

However, common visual-graphical representations differ in their properties, and existing category systems often do not cover these differences adequately [3–6]. Against this background, a new set of differentiation criteria was developed, drawing on recent findings from representation research, quantum education, and specific aspects of quantum sciences and technologies (see categories in Sect. 3.3). Ainsworth's (2006) Design, Functions, and Tasks Framework was used as a conceptual basis to refine and create a comprehensive category system for evaluating visual-graphical qubit representations [7].

Among these categories, for example the category of 'Salience' exemplifies how visual design can guide learners' attention. Salience refers to the perceptual prominence of specific features that stand out and automatically capture attention. According to Itti & Koch (2001), salience-driven attention is primarily guided by stimulus properties, such as color, contrast, and motion, rather than by an individual's goals or expectations [8]. This means that certain elements in a scene naturally attract attention due to their visual distinctiveness, regardless of the observer's. While salience determines which stimuli are noticed first, its influence on cognitive processing and learning is complex. Reynolds & Anderson (1982) found that directing attention to specific textual information—by posing targeted questions—led to longer reading times for relevant segments and increased reaction times in a secondary task [9]. These results suggest that attention allocation can influence the depth of information processing, as readers who spent more time on question-relevant text sections also tended zu better post-test performance.

While Ainsworth (2006) highlights the importance of designing multiple external representations (MERs) to support accessibility and clarity in learners' understanding of complex scientific concepts [7], Schnotz and Bannert (2003) focus on how the structure and design of external representations influence cognitive processing [10]. Ainsworth (2006) frames design within her DeFT framework, which emphasizes the functions of MERs. In contrast, Schnotz and Bannert (2003) analyse design from a cognitive processing perspective, arguing that representations fundamentally differ in how they convey information [10]. The distinction made by these authors between descriptive representations (e.g. text, equations) that rely on symbolic encoding and depictive representations (e.g. diagrams, images) that share structural similarities with what they represent is also of interest [10]. Their research highlights that the effectiveness of multiple representations depends not only on their function but also on their structural alignment with cognitive processing demands [10]. While well-designed combinations of text and image can enhance learning, inappropriately structured representations may interfere with the construction of mental models. Both perspectives emphasise the importance of a thoughtful representation design. Ainsworth (2006) underlines the necessity of aligning the design of representations with their pedagogical function and the learning task [7], whereas Schnotz and Bannert (2003) stress the importance of ensuring that representations are cognitively compatible to support mental model construction [10].

The objective of this study is to provide educators with valuable insights into the characteristics of representations that are perceived as conducive to learning, and to identify relevant factors for designing new representations. The study relies on expert evaluations to derive key design principles that may contribute to effective representations. Furthermore, it addresses general challenges in developing effective representations that facilitate a solid understanding of QP (see Sect. 2.4).

The structure of the paper is as follows: Sect. 2 provides an overview of the theoretical background, Sect. 3 outlines the methodology, Sect. 4 presents the results, and Sects. 5 and 6 discuss the implications and future research directions.

2 Theoretical background

2.1 Quantum education

Quantum science and technology doesn't change only our fundamentally view of the world, but can also influence people's living conditions through its influence on society and technology. In light of the growing importance of QT, the teaching of QP plays a central role in various stages of education and professional training (school, university, and industry). Research has already been carried out in various directions, for example in course structure [11–13], representational [14–16] or instructional [17–19] and many other. This should only show a small number of the research directions that make a contribution. At the same time, it is important to ensure that QP concepts are delivered in a sustainable and targeted manner, enabling individuals to actively contribute to the advancement of these technologies at different stages. In order to provide a basic education in this field, suitable concepts with supporting representations [20], which are needed to get more and deeper knowledge.

Introducing the fundamentals of QT, in particular the qubit, not only leads to new approaches in the teaching of QP but also opens up the possibility of an application-oriented teaching methodology [18, 21, 22]. In addition to the different approaches that can be used to introduce learners to QT, a recently updated competence framework addresses the understanding of QT at different levels and in different relevant sectors [11, 12]. It can be used as a guide for matching representations to the cognitive and content-related needs of different learning levels, and as a starting point for developing appropriate teaching materials.

The spin-first approach appears to be a suitable method for teaching the fundamentals of QP and, with its focus on qubits as two-level systems, also for teaching QT [18, 22, 23]. This approach introduces the spin-half context with the qubit at an early stage, providing a basic understanding of QP and, in particular, QT. In this context, the spin-1/2 system serves as an ideal introduction, as it allows learners to work with a fundamental two-state quantum system that can be easily contextualized in different physical implementations.

Common examples include the spin states of an electron (spin-up/spin-down) or the polarization states of a photon (horizontal/vertical). By introducing spin at an early stage, this approach establishes a clear conceptual link between the abstract concepts of QP and possible applications in QT. The *reasoning tools* for QP, also called *the basic rules of quantum physics*, from the German 'Wesenzüge der Quantenphysik' [24–26], include fundamental concepts that describe the behavior of quantum systems.

A quantum state characterizes a physical system and encodes the probabilities of measurement results. In a two-state system, such as a qubit, the state is defined by two basis states, $|0\rangle$ and $|1\rangle$ [27]. The principle of superposition allows the system to exist in a combination of these states, leading to characteristic quantum effects such as interference.

These *reasoning tools* [24–26] can also be addressed with the spin-first approach:

- Quantum measurement
- Complementarity
- Indeterminism and statistical predictability
- Interference of single quantum objects

Current research has highlighted a set of *fundamental concepts* that educators consider essential for teaching QP and QT. These include superposition, quantum measurement, quantization, the Heisenberg principle, entanglement, statistical nature, wave-particle duality, non-locality, decoherence, and complementarity [28].

Within this framework, certain concepts emerge as particularly central to understanding quantum technologies. In particular, superposition and entanglement are frequently emphasized as *key concepts*, as they underpin quantum computing, quantum communication, and other QT applications. Their role in enabling quantum parallelism and secure information transfer highlights their significance beyond fundamental physics. Sadaghiani et al. found that students demonstrated a higher understanding of QP concepts when they followed the spin-first approach compared with a position-first approach [23].

To convey the complex content of QP in an intuitive and memorable way, it is essential to create an appropriate learning environment that encompasses not only an effective teaching approach but also the use of suitable representations. The selection of representations in the context of QP is a challenging endeavour. Stadermann elucidates the complexities in identifying useful representations in QP [29]:

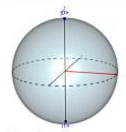
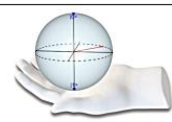
> *'In contrast to most classical physics topics, we cannot find a consistent visualisation for quantum phenomena. QP offers students new views on physical reality, which conflict with earlier learnt classical concepts such as the nature of particles, locality, and determinism. Scientists still discuss how – and if at all – QP should be interpreted.'*

Therefore, it is essential to identify valuable elements of visual representations within the context to generate hypotheses regarding their learning effectiveness and, potentially, to generate new representations.

2.2 Qubit representations

The term 'representation' can be interpreted in many different ways. There is a diversity of perspectives, definitions, and categorisations of representations in the context of education. In this work, we are orientated towards Lemke [5], who analysed the construction and conveyance of meanings through signs and symbols, both in verbal and non-verbal languages. He found that in scientific papers, content such as text, graphs, tables, photos,

Table 1 Example for representation in different variations

Description	Example
Visual-graphical (e.g. diagrams, graphs, images)	 Bloch sphere
Mathematical (–operational) (e.g. formulae)	$$\lvert\Psi\rangle = a_1\,\lvert 0\rangle + a_2\,\lvert 1\rangle \quad (1)$$ Remark: $a_{1,2} \in \mathbb{C}$, $\sum_{i=1}^{2} \lvert a_i\rvert^2 = 1$.
Visual-gestural (e.g. physical or gestural movements)	 Interaction between the teacher and the student through physical and gestural movement
Verbal-semantic (e.g. text)	'Superposition can be realised with the aid of a beam splitter.'
Tangible (e.g. 3D printed Bloch sphere in the hand)	 Hand-sized 3D print of a Bloch sphere

and equations are used together because meaning is constructed through a multimodal process. This meaning is now taken up for learning. To determine the conditions in which this multimodality can be employed in the context of learning QP and QT, we categorise possible representations. Examples of different types of representations that could be used and combined in multimodal quantum instruction are shown in Table 1.

Previous research has already attempted to implement various representations in combination with the spin-first approach [30]. The study included symbolic representations such as bra-ket notation and matrix representations, graphical visualisations in the form of histograms, colour-coded probability distributions and three-dimensional representations, as well as dynamic approaches such as time-dependent animations and simulations of experiments. The results show that this combination of multiple representations promoted conceptual understanding and the ability to translate between different forms of representation [30]. However, challenges remain, especially in dealing with complex-valued states and projections in Hilbert spaces [30]. A common and often used visual-graphical representation of two-level systems is the Bloch sphere, which has several variants in its implementation [18, 22, 31]. However, a variety of other representations are also available, for example, the "arrow" formalism by Richard Feynman [32–35] and the Circle Notation [15, 36, 37]. In recent years, many other visualisations of qubits have been developed and refined, for example, the Quantum Bead by Steffen Glaser's group [38, 39], and the pie chart model [40]: Qubit cake model (Qake, paper in progress) [41]. For the visualisation of quantum phenomena, not only have different representations been used and analysed, but also media such as the quantum Composer [42–44] or the Quantum

Mechanics Visualisation Project (QuVis) [45, 46]. The field of visual-graphical quantum representation is undergoing constant development and refinement. There is a significant interest in making QP accessible to an interdisciplinary audience, particularly with regard to the future development of QT [13].

Teaching QP and QT to beginners requires a structured approach to mathematical concepts. While mathematical formalism is essential for a deeper understanding, its abstract nature can pose challenges for learners with little or no prior knowledge. According to Bouchee et al. (2021), the abstract mathematical formalism can obscure for students the meaning of the associated concepts of QP [47]. Visual-graphical representations can serve as an intuitive bridge to mathematical concepts—for example, the Bloch sphere helps learners understand quantum superposition and rotations in Hilbert space and later maintain a connection to the underlying linear algebra. Rather than immediately introducing formal mathematical structures, visual representations provide an entry point by illustrating abstract principles in a concrete way. They help learners develop an intuition before moving on to the corresponding algebraic formulations, such as state vectors, operators and unitary transformations, and can provide the link to them [12].

Learning often involves (multiple) representational resources [7], including verbal-semantic formats (e.g., written text), mathematical-operational (e.g., equations), and visual representations [5]. Visual representations can be further categorised into visual-graphical representations (e.g. Bloch spheres, quantum circuits or vector diagrams) and visual-gestural representations, which involve interactive elements such as hand gestures by interacting with each other [5]. According to Lemke's (1998) categorisation, also physical representations such as those used in chemistry (e.g. molecular models) also belong to graphical representations [5]. They are part of the broader concept of representation, which is categorised differently by different authors [4–6, 48]. Schnotz (2005), for example, distinguishes between descriptive and depictive (pictorial) representations. Descriptive representations use symbols or signs that have no direct similarity to their reference object, while pictorial representations have a structural similarity to the depicted object through a spatial arrangement (e.g. molecular models) [48]. Models and representation are not the same thing. Models can be understood as representations, among other things, but often fulfil other characteristics in their model concept. According to Kircher et al. (2015) characteristics of models are clarity, simplicity, transparency, familiarity, productivity, importance of models [49]. As the mathematical formalism of QP is challenging to understand and apply, visual representations of qubits can facilitate understanding by making abstract concepts more tangible.

Therefore, teachers need to carefully evaluate their properties and determine their suitability for conveying key quantum concepts. This includes checking that a representation accurately reflects the underlying mathematical principles. At the same time, learners should be encouraged to actively connect different representations, such as linking quantum state vectors to their Bloch sphere representation. Such integration supports mathematical understanding and reinforces structural relationships within QP. Making these connections requires identifying relevant similarities between different representations and understanding the conventions that guide their combined use. This process is referred to as connectional understanding [2]

As previously mentioned, qubit representations within the spin-first approach allow for the visualization of fundamental principles of QP using a two-state system. A qubit is a physical system that can exist in two basis states and is described by their superposition. Well-designed visual-graphical representations can help to illustrate concepts such as superposition and measurement in a more accessible way by providing an intuitive entry point before formal mathematical descriptions are introduced. This facilitates the teaching of key QP concepts without immediately introducing the full mathematical formalism. The focus on visual-graphical representations of qubits is a deliberate didactic strategy, aiming to enhance the accessibility and understanding of fundamental concepts in QP.

Although it has been suggested that visual-graphical qubit representations are beneficial due to their accessibility and close relationship with mathematics, their perceived simplicity can pose a risk that they will be misunderstood. Learners sometimes fail to recognise that they are not direct representations of reality, but scientific models. A prominent class of such difficulties is the "graph-as-picture" misconception, in which, for example, a learner may misinterpret a line graph as a picture of a mountain [50]. The study investigated how people perceive and interpret visual graphs, with a particular focus on the 'graph-as-picture' misconception - the tendency to misinterpret abstract representations as physical images [50]. Experiments were conducted with subjects who interpreted various visual graphs, supplemented by questionnaires and qualitative interviews to analyse the influence of design elements such as colours, shapes and layouts. The results show that visual graphs can facilitate understanding, but often lead to misinterpretation if their abstract nature is not clearly communicated. Particularly relevant for visual graphical qubit representations is that while intuitive representations can improve accessibility, they also carry the risk of learners misinterpreting them as physical structures. Therefore, their risk of supporting the development of misconceptions (e.g. the Bloch sphere describes the behaviour of a photon, then the photon is associated with the shape of the Bloch sphere as a small sphere) should be considered when using or creating visual-graphical representations. In addition to the misinterpretations caused by visual-graphic representations, meta-representation competences (MRC) can play a central role in avoiding these [51]. MRC enable a reflective approach to representations and promote an understanding of their use and limitations [51]. According to diSessa, MRC is crucial for not only utilising representations, but also for critically questioning them and meaningfully integrating them into the learning process [51]. He defines MRC as follows:

> *MRC includes the ability to select, produce and productively use representations but also the abilities to critique and modify representations and even to design completely new representations (diSessa und Sherin, 2000, p. 386).*[51]

With regard to the use of qubit representations, difficulties and misconceptions in QP have been analysed extensively by various authors [1, 52–56]. Based on our literature research, the design and presentation of visual-graphical representations play a crucial role in concept acquisition and the avoidance of misconceptions. Additionally, the manner in which a representation is presented can influence learning outcomes and potentially lead to difficulties [57, 58]. The following are selected misconceptions identified in the literature regarding the concepts of quantum state, quantum measurement, superposition, entanglement and general perception (spin as rotation) that may be encountered when using visual-graphical representations:

- **Quantum state:** Several studies have shown that understanding quantum states involves several conceptual challenges. A common problem is the regression to classical thinking, especially with regard to the non-determinism of quantum mechanics [56]. Students have difficulty distinguishing between pure superposition and mixed states, and tend to overlay quantum mechanical concepts with deterministic ideas. In addition, many students believe that a time-dependent wave function automatically implies a time-dependent probability of a particle. Another misunderstanding concerns the state after a measurement: some students incorrectly assume that the wave function remains the same after a measurement, or that it returns to its initial state over time [56].

- **Measurement:** Understanding difficulties with measurement and expected value have been identified [1, 52]. An important insight related to three-dimensional representations comes from Singh et al. [1]. They show that students often incorrectly assume that the states labelled x, y and z are spatially orthogonal and independent of each other, based on their experience in classical physics, where these axes are conventional labels for orthogonal vector components. However, if it is not explicitly explained that the eigenstates of a quantumsystem (e.g. spin components) are vectors in abstract Hilbert space - and not in the three-dimensional physical space in which, for example, a magnetic field propagates - this misconception may persist and lead to learning difficulties [1]. The role of representation is as a means of predicting measurement results based on probabilities, rather than determining them directly. Bouchée et al. (2021) point out that measurement is a general problem in learning QP, which is related to the situational relevance of (linguistic) representations [47]. For example, he mentions that learners who are confronted with the uncertainty principle draw incorrect conclusions from the experimental physics representation (e.g., measurements are accompanied by errors related to the measuring device), thus hindering their process of meaning formation [47].
- **'Spin as rotation':** In a study on mental models, it was found that the majority of participants held the misconception of 'spin as a rotation of particles around their own axis' [54, p. 1374].
- **Superposition:** Many students struggle to understand that photons can exist in two states (e.g., horizontal or vertical polarisation). The majority of students encountered difficulty in accepting that the polarisation states of a photon can be employed as the basis for a two-state system [1]. This issue was frequently observed in students who exhibited a pronounced inclination towards their established understanding of polarisation within the context of classical optics [1]. A common misconception is that a photon can have an infinite number of polarisation states and therefore cannot be reduced to two base states [1]. Students are strongly orientated towards polarisers, which in classical physics can be rotated at will, thereby determining the state of the light [1]. This idea means that they do not understand polarisation as a quantum mechanical two-state system.

Interestingly, this resistance does not occur with spin-1/2 states, although these are physically isomorphic to photon polarisation [1]. The reason is probably that polarisation is usually taught in a classical context, while students only learn about spin in quantum mechanics. As a result, they lack the connection between the two concepts, which makes it difficult to accept polarisation as a two-state system [1].

These difficulties also affect the understanding of superposition. If students do not accept polarisation as a quantum mechanical two-state system, it is difficult for them to understand that a photon can be in a superposition of these states.

- **Entanglement:** To our knowledge, there are few studies that address specific misconceptions about entanglement held by learners. Brang et al. (2024) are one of them, who explored the perspectives of physics teachers and students on quantum entanglement, quantum teleportation, and their applications. The study highlighted several learner conceptions, including challenges in differentiating quantum entanglement from superposition, misconceptions regarding the role of measurement in entangled states, and a limited understanding of practical applications such as quantum communication and computing [55]. For one, they categorised some of the participants' responses as 'hidden variable explanations.' As they assumed that the measurement results were predetermined by local hidden variables, reminiscent of the EPR (Einstein, Podolsky, Rosen) perspective [55, 59]. A classical, deterministic understanding is implicit in this view [55]. Other microconceptions were described in the text as 'direct influence or action at a distance.' In that section, learners assumed that one entangled particle actively influences or transmits information to its partner particle, and even thought that manipulating one particle changes the other in a controllable way [55]. Furthermore, the misconception that measurements on entangled particles always show perfect (anti)correlations, regardless of the measurement basis, was identified. To investigate common misconceptions in QP and their teaching challenges, Majidy et al. (2024) also conducted a review and interviewed QP instructors [20]. Their study provides an overview of misconceptions in higher education physics, discusses their sources and remediation strategies. Interviews have shown that many students wrongly assume that a measurement on an entangled particle causes an immediate physical change in the partner particle [20]. Equally widespread, according to the lecturers, is the misconception that entanglement enables information to be transmitted at faster-than-light speeds, although quantum mechanics does not permit such communication [20]. Another key misconception from the lecturers' point of view is the distinction between correlation and causality, as students often do not understand that measurements on entangled particles only show statistical correlations, but that there is no direct influence between them [20]. Popular scientific presentations, insufficiently precise school material and unsuitable classical analogies were identified as the main causes of these misconceptions [20]. To improve understanding, the researchers recommend the targeted use of representations and simulations to make the non-local but non-causal nature of entanglement comprehensible [20].

These findings underscore the necessity for the development of effective representations to address these conceptual gaps and to facilitate a more profound comprehension of abstract quantum phenomena, such as entanglement.

Based on our literature research, the way visual-graphical representations are designed plays a crucial role in concept acquisition and the avoidance of misconceptions. Additionally, the manner in which a representation is presented can influence learning outcomes and potentially lead to difficulties [57, 58].

2.3 Describing visual-graphical qubit representations

As in other areas of physics, educators in QP are confronted with the challenge of comparing and selecting appropriate representations. With this study, we want to help categorise representations and describe them in a consistent manner to evaluate their strengths, weaknesses, and special features. Previous categorisations of representations, such as those proposed by Lemke [5], Schnotz [3], Kosslyn [6] and Bertin [4], differentiate visual-graphical representations but do not support drawing conclusions about their effectiveness or appropriate use. However, if educators could gain a deeper understanding of the aspects of qubit representations that promote the acquisition of content knowledge, they could assign qubit representations to different levels of learners' prior knowledge and develop more effective, targeted approaches.

In this work, we present a refined categorisation system for representation and QP visualisation research. We selected Ainsworth's Design, Functions, and Tasks (DeFT) [7] as the conceptual framework and extended it with relevant aspects of QP representations, including their respective potential risks of inducing misconceptions in learners. Independent of the learning content, the DeFT Framework provides an overview of how multiple external representations (MER) can be used effectively to support students' learning [7]. Ainsworth outlined relevant aspects of design, functions, and tasks when learning with MER. The design aspect of the DeFT framework concerns the structure of multi-representational learning environments and how they influence learner interaction. Specifically, it addresses how accessibility, comprehensibility, active engagement, and the integration of multiple representations impact the effectiveness of learning. According to Ainsworth (2006), key design considerations include the number of representations, the distribution of information, the format of representations (e.g., text, diagrams, or tables), the sequence in which they are introduced, and the ease of translating between them. These factors determine how learners process information and interact with different representations. The "functions" refer to the roles that can be played by multiple representations in supporting learning: providing complementarity, constraining interpretation, and constructing deeper understanding. These functions influence how learners process and integrate information. The 'tasks' refer to the cognitive demands that learners must manage to work effectively with these representations. In this aspect, we have differentiated ourselves from Ainsworth (2006) and have focused on content-related processes and operations that can be shown with the representations to solve tasks [7]. Together, these aspects provide an understanding of how incorporating MER into educational settings can influence learning processes and outcomes.

For our purpose we redefined DeFT [7] as a theoretical framework to derive useful dimensions for categorising visual representations (see Table 2).

Under **design**, categories were included that allow statements about the shape and visual impression of a representation.

Under **function**, categories were included that primarily characterise the representations according to their interaction with the learners or other representations.

Under **task**, categories have been chosen that primarily take place in a basic QT application or task.

Finally **cross-concepts**, include aspects that do not fit in the categories above (the categories are described in more detail in Sect. 3.3).

Table 2 Refined categorisation of visual representations

Category	Classification
1. Salience 2. Dimension 3. Understanding difficulties 4. Colour	Design
5. Actions/Steps 6. Interaction with mathematics 7. Contiguity 8. Overlap/redundancy 9. Complementarity 10. Predictability	Functions
11. Phase visualisation 12. Amplitude visualisation 13. Concepts 14. Quantum technology	Tasks/applications
15. Generability 16. Effort in explanation	Cross-concepts

2.4 Research questions

Our goal is to refine a categorisation of representations that allows us to make decisions about the selection and design of visual representations for appropriate, effective, and sustainable learning of QP and QT content. We use expert ratings to profile and cluster representations and obtain answers to the research questions:

RQ1: According to experts, which differences exist between the learning-relevant features of four selected visual-graphic qubit representations?

RQ2: According to experts, what factors should be considered when creating new qubit representations to promote learning?

3 Methods

To answer the research questions, we conducted online sessions in which experts were asked to rate four visual-graphical qubit representations across 16 categories using cheat sheets developed for the study. Rather than directly asking experts to define relevant aspects, we inferred their importance from expert evaluations of existing representations. This approach allowed us to identify which features are perceived as most relevant for learning QP and QT. The Bloch sphere, the Cirle Noation [15, 36], the Quantum Bead, a further development of the Spindrops representation [38, 39] (second paper in progress) and the pie-chart model: Qubit Cake Model (Qake) [41] were the four representations evaluated by the experts (Fig. 1). The Quantum Bead is not substantially different from the Spin Drops representation (or DROPS for short [38, 60]) for single qubits. The difference lies in the fact that the Quantum Bead is also suitable for visualising two or more qubits, including entanglement [39], as requested in the expert rating. Therefore, we refer here to the Quantum Bead [39].

3.1 Cheat sheets

The structure and layout of the cheat sheets with the relevant terminology were identical for each of the four representations (see supplementary material 1). Brief explanations were given on qubits (in general), quantum states (two-state system), the representation itself, and how the respective representations visualise the following: quantum measurement, superposition, entanglement and quantum gates (X-Gate, Z-Gate and H-Gate). The

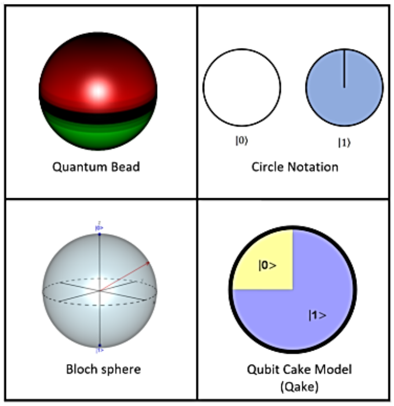


Figure 1 The Quantum Bead and Bloch sphere are three-dimensional representations, whereas the Qubit Cake Model (Qake) and Circle Notation are two-dimensional representations. The Circle Notation uses the third dimension for multi-qubit systems. Own illustration based on Quantum Bead [39, 60], Circle Notation [36], the Bloch sphere [19] and Qake Model [40, 41]

experts were advised first to look at the cheat sheets to familiarise themselves with all the qubit representations and then complete the rating sheet. The cheat sheets could optionally remain open. Experts were asked to assess each of the four representations within the different categories via Google Forms (https://docs.google.com/forms).

3.2 Sample

For the present study, we selected and invited experts whose current field of research is related to QT: theoretical (N = 7), experimental (N = 5), educational (N = 7) and across all interfaces (N = 2). A total of twenty-one experts from ten locations across four countries (Germany, Italy, Switzerland, and the USA) were involved. Only QT experts with teaching experience in QP were included (two professors or junior professors, nine postdoctoral researchers, and ten PhD students). PhD students were eligible to participate only if they were in their second year or above. The mean number of years spent engaged in research in QP was 5.1 years (standard deviation ±1.9 years). One participant with over 30 years of experience was considered an outlier and so was not included in this average. However, the ratings of this expert was considered for the analysis in this paper in the same way as the other data. The participants demonstrated expertise in various areas, including mathematics, quantum optics, quantum field theory, quantum computing, and quantum education. All experts were contacted personally or by email.

3.3 Categories

The categories for evaluation were chosen based on Ainsworth's DeFT framework and research on (mis-)conceptions in quantum education and quantum sciences.

1. *Salience*: This describes how clearly a concept is perceived through a representation. The salience of a stimulus can depend on its intensity, novelty, ecological validity, movement, and interactivity [61]. The signalling principle states that learning materials are more effective when they contain cues or elements that

draw learners' attention to the relevant content or information or highlight the organisation of the content [62].

2. *Dimension*: Ainsworth mentions dimensionality as a relevant design factor [7]. Although visual-graphical representations are always two-dimensional, they can differ in whether and how three-dimensional information is visualised [7]. The capacity for spatial abilities, including mental rotation, differs between learners of different genders [63–65]. The visuospatial experience provided by representations depends on whether learners can adequately perform any necessary explicit operations (e.g. mental rotation). A study showed that spatial ability is the decisive factor for learning success, with male participants performing better than female participants regardless of the type of multimedia resources [64]. Three-dimensional representations may require a higher level of spatial imagination than two-dimensional representations, that not all learners can achieve in the same way [64].
3. *Understanding difficulties*: This assesses whether the representations could lead to students misunderstanding the underlying concepts. Topics should be considered which may lead to possible understanding difficulties due to their function or external features such as the design of the representation. Here we have limited ourselves to certain concepts such as simple structures, quantum measurement, superposition and entanglement by known literature. A more detailed background on misunderstandings that can arise from visual-graphical qubit representations is provided in Sect. 2.2 and how we implemented there in the study is shown in Sect. 3.4.
4. *Colour:* Another component of representations is the *colour* coding of concepts/information. The point here is not that colour can be used to visually draw attention, but that information is encoded with the colour, which can increase the extrinsic cognitive load and have a detrimental effect on learning [66]. The additional information processing through colour requires increased (extrinsic) cognitive load and has a negative effect on learning. For example, both the colour and the mixture of colours (in the case of a function of the representation) contain information about the underlying content.
5. *Actions/steps*: This category describes the cognitive steps required to extract relevant information from the representation of a concept or to perform a specific operation with the representation. It is based on Sweller's element interactivity [66]. Learning content or representations that require numerous cognitive steps place a significant load on working memory, which is inherently limited in its capacity to process information effectively [67]. According to Miller, the human working memory can typically manage only a limited number of elements, often cited as 7 ± 2 at any given time [68]. Therefore, representations should be as comprehensive as possible while remaining simple to avoid cognitive overload and instead provide support to learners. It is important to distinguish between extrinsic cognitive load, which arises from the way information is presented, and intrinsic cognitive load, which depends on the inherent complexity of the material relative to the learner's prior knowledge. Certain representations might require many cognitive steps for learners with low prior knowledge, leading to overload, yet be entirely suitable for learners with higher levels of expertise [69].

6. *Mathematics:* For QP, the interaction between mathematics and visual-graphical representation is an important aspect in the choice of representation. The integration between mathematical and visual-graphical representations promotes conceptual understanding more than one representation that visualises certain phenomena [2, 7]. Erwin Schrödinger captured the mathematical meaning of QP succinctly when he said *"... then the mathematical apparatus of the new theory can give us a well-defined probability distribution for every variable..."* [70] emphasising how fundamental a connectable representation is. To introduce mathematics at the right level or to enable later levels of learning, it is important to provide access to mathematics with visual-graphical representations.
7. *Contiguity*: This is based on the principle of contiguity described by van Gog [62], who found that learners achieve greater learning success when text is placed next to the graphic [71]. The combination of both proved to be conducive to learning, which is why this category is included to consider the direct integration of a visual-graphical qubit representation with a mathematical formula or some additional text for a more detailed description. This could also ease the transition to the introduction of mathematical representations.
8. *Overlap/redudancy:* There is a paucity of data concerning the impact of redundant combinations of representations on learning outcomes. The combination of images and written text supports the acquisition of knowledge more effectively than the simultaneous presentation of the same information in the form of images, written text, and speech [71]. However, a study has demonstrated that combining multiple representations, especially text with formulae, improves problem-solving performance in mathematics, even if they contain redundant information [72]. Furthermore, the use of multiple visual-graphic representations can be beneficial for learning [7]. This includes the use of redundant graphical representations. A categorisation of representations in connection with one or more redundant representations enhances the use of MER and increases the potential to improve learning [7]. The challenge here is to identify compatible representations and use them in a targeted manner.
9. *Complementarity*: This describes the use of two visual-graphical representations that complement each other. The principle of complementarity implies that the representations differ either in the information they present or the related cognitive processes [7]. For example, two visual-graphical representations may depict the information of a superposition state while triggering different cognitive processes, thus promoting learning through stronger integration [2]. Another possibility is that one representation may depict the superposition well but not the entanglement, whereas the other representation may visualise the entanglement but not the superposition. Multiple representations can, therefore, be used to promote learning if they complement each other in their representation of concepts. As in *overlap/redundancy*, the categorisation of representations in relation to one or more other complementary representations also has the potential to enhance the effectiveness of MER and improve learning [7].
10. *Predictability*: For representations to be used effectively by learners, they should enable predictions to be made. Thus, statements should be made about possible measurement results, taking into account the properties or rules of the

representation [73, 74]. Particularly important here are the tasks/applications categories, which capture how certain representations can be used to demonstrate specific applications or tasks of QT. These categories are specific to QT and the use of the qubit in the spin-first approach.

11. *Phase (or phase change)*: This is a particularly important property of a quantum state. Global phases and relative phases were considered separately. The global phase refers to a phase-related transformation applied to all states of a quantum system. It has no direct influence on the observable phenomena of the system. Relative phase refers to the phase relationship between different states within a quantum system [75].
12. *Amplitude*: To analyses a two states system in detail, the term of 'amplitude' is required in addition to 'phase' [75, 76]. To interpret and understand these terms as clearly as possible, it is important to visualise them.
13. *Concepts:* The concepts (A) quantum measurement, (B) superposition, (C) entanglement, (D) probabilistics were considered to assess which concepts are adequately conveyed by the representations. The concepts were selected based on relevance of the spin-first process and the reasoning tools for QP, as well as the competence framework [11, 23, 24, 26].
14. *Quantum technologies*: This category focuses on direct application and captures which quantum gates are adequately conveyed by the representation of experimental components. This concerns common gates: the H-gate, the X-gate, and the Z-gate [38].
15. *Generability*: This refers to how readily a representation can be reproduced by learners. The difficulty or complexity required to create the representation should be assessed. For example, in a school context, we would consider how difficult/complex it is for learners to draw a representation in their exercise books when they are shown it on a blackboard [7].
16. *Effort in explanation*: This aims to categorise the effort required to explain a representation. For example, there are some representations that can illustrate many concepts, but the effort or complexity of explaining all these concepts may be so great that these representations are ineffective.

3.4 Rating

The structure of the ratings shown in Fig. 2 indicates the weightings assigned to the different categories. To rate the *salience, dimension and actions/steps* and *concepts*, four items (for each QP concept) per representation are included. For *understanding difficulties*, we include the items in statements A, B, C, and D. For *redundancy and complementarity*, the combinations A–F are taken into account only once. The total number of items is calculated by adding the number of items on the sub-path of the characteristic of external representation, here QP concepts, so *redundancy and complementarity* are not taken into account as they are cross-representation items (32 items for one representation and 128 items for four). The number of all cross-representation items (redundancy and complementary) is shown here by 12 (6 + 6). The total number of items is calculated by multiplying the 32 items by 4 (for the representations) and adding 12 cross-representation items (into account once).

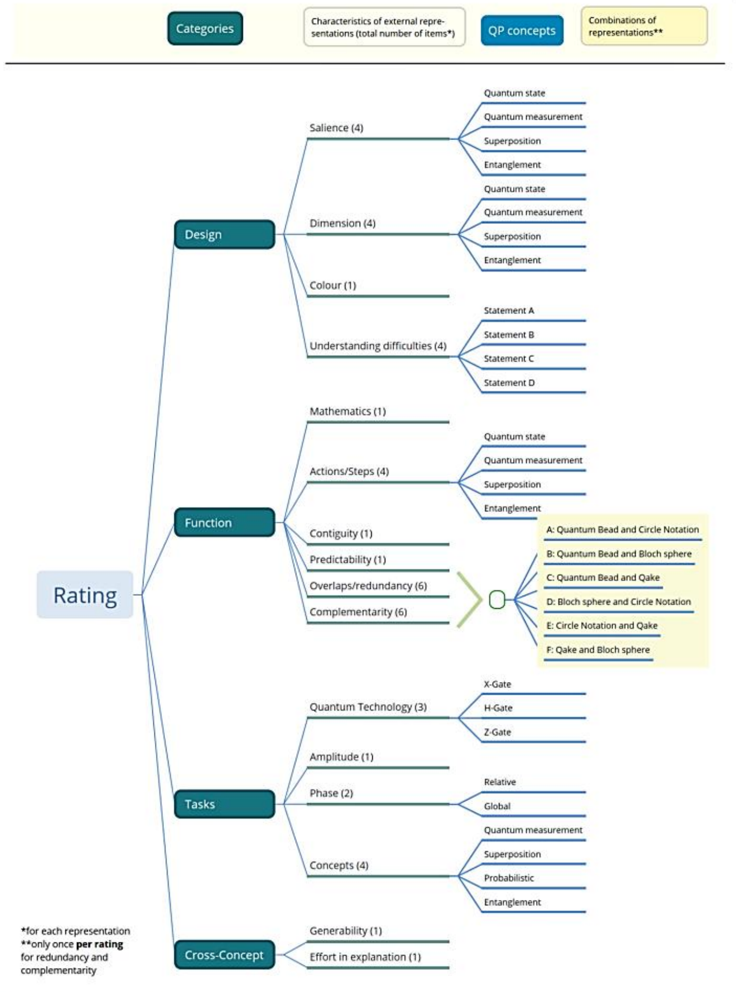


Figure 2 Overview of the rating structure used in the expert evaluation. The diagram illustrates how different categories were weighted across qubit representations, For a detailed explanation of the rating process, see Sect. 3.4

On a Likert scale from 1 to 5 ('strongly disagree' to 'strongly agree' or the reverse), the experts were asked to assess the four qubit representations based on 16 characteristics. In addition to these options, the raters could select 'I can't judge, I do not know' for each item. *Salience, Dimension and Action/Steps*, were rated separately for the concepts of:

- quantum state
- quantum measurement
- superposition and probabilistic

- entanglement

We also measured how many cognitive actions/steps were required to extract information about the above mentioned relevant concepts from each representation or to perform a specific operation with the representation. The procedure for counting cognitive steps was introduced through two examples in showing steps by "extraction of the slope of a linear graph" and example 2: "divergence of a vector field", so that it becomes clear how 'steps' can be recorded during graphical readout.

The example **'extraction of the slope of a linear graph'** shows how the cognitive steps were exemplified to the experts:

1. Search for the relevant x-position.
2. Find the corresponding points on the graph.
3. Read off the associated y-value.
4. Identify another point on the graph for which the difference in y and x can be easily determined (e.g. a point of intersection with grid lines).
5. Find the relevant x-position of the second point.
6. Read off the corresponding y-value of the second point.
7. Determine the difference in y and x and calculate the gradient.

In this case, a total number of seven steps were required.

To systematically analyze the expert data and ensure comparability across different scales, we transformed the original number of steps evaluations into the usual rating scale from 1 to 5 in increments of 0.5. This transformation allows for a structured comparison of expert assessments across all examined representations and categories. The rating 5 was used for 'one to two steps' and 4.5 for 'three to four steps', with this pattern continuing down to a rating of 2 for '13 to 14 steps'. A rating of 1 was used if 15 or more steps were required. This transformation ensures a consistent interpretation of the expert evaluations and facilitates statistical analysis by mapping categorical assessment onto a continuous scale.

A lower number of steps means a lower number of cognitive processes, based on Sweller's [66, 77] concept of 'element interactivity'. This term is used to describe the intrinsic cognitive load associated with the processing of different pieces of information or elements/aspects [66]. It encompasses the manner in which these elements interact with each other and how they are processed by the brain in order to be understood. A higher level of element interactivity is indicative of a greater (intrinsic) cognitive load, as it necessitates establishing more connections between the elements [66].

In *understanding difficulties*, the experts were asked to rate four possible statements (A–D) that might be made from the perspective of a students who may be unfamiliar with the respective representations. Based on works by a range of authors about mental models and misconceptions in QP [1, 52–54, 78, 79], the following statements were chosen:

A: 'Based on the representation, I imagine the quantum object as a small sphere.'
B: 'After a beamsplitter, the qubit will be split in two directions.'
C: 'The information of the qubit was already known *before* the actual measurement and was confirmed with the representation (deterministic behaviour).'

D: 'An entangled state is only possible with two or more qubits.'

Statement D is correct and was included to capture difficulties in the transition to multi-qubit systems, with regard to the concept of entanglement. In our work, we have limited ourselves to the fundamental requirements for understanding entanglement: going from a one-qubit to a two-qubit system. The focus was on clarifying the visual-graphical representations, which show the necessity of at least two qubits for entanglement.

For *complementarity* and *overlaps/redundancy*, following groups were formed to evaluate the representations in relation to each other:

QB/CN: Quantum Bead and Circle Notation
QB/B: Quantum Bead and Bloch sphere
QB/Qake: Quantum Bead and Qake
B/CN: Bloch sphere and Circle Notation
CN/Qake: Circle Notation and Qake
Qake/B: Qake and Bloch sphere

The participants were asked to provide a separate rating for the visualisation of global and relative phases. For *quantum technologies*, the rating was divided into the following topics: X-gate, Z-gate, and Hadamard gate (H-gate). The experts gave their ratings for each gate. An attempt was made to limit the rating to simple variants of operations that could be experimentally transferred to optical elements [80–82].

The 16 categories are taken into account, with four items to refine them in four concepts for *salience, dimension, actions/steps* and also four items in *concepts* to specify them, which (in general) appropriately convey the concepts. Four items were also included for *understanding difficulties* because of the four 'statements' describing interpretations or misinterpretations of concepts in QP. With *phase* (2 items) and *quantum technologies* (3 items), there are 32 items. Participants were asked about these for all four representations, which brings us to 128 items. Finally, the items were used in combinations due to *overlaps/redundancy and complementarity*; here there are 12 items, bringing us in total to 140.

In addition, the experts were asked to indicate which category they thought was important to discriminate the representations and which concepts, if any, were missing. These were presented as free-text questions.

The responses to the free-text questions were collected in tabular form and examined for recurring concepts and key terms. Concepts and key terms that recurred in the data were identified and grouped into thematic categories. Particular attention was paid to specific text passages to generate a meaningful structure, oriented on the principles in Chap. 8: qualitative analysis and interpretation (Patton, 2002, p. 452ff.) [83]. Similar responses were grouped together based on content-related keywords, while unique statements that did not align with other responses were recorded separately and included in Sect. 4.2.5. This approach allowed for an overview of common themes while also preserving individual expert perspectives.

3.5 Statistical analysis

We calculated the mean and median values across design, functions, tasks, and cross-concepts for each representation. Then, we narrowed our focus to the 16 categories and analysed the differences between the four representations. The coefficient of variation was calculated for each rating item. This represents the standard deviation in relation to the mean value and is dimensionless. If the coefficient of variation is less than or equal to 0.5, we can say that at least 50% of the experts provided ratings close to the mean value [84, 85]. For each of the 140 rating items within the representations, we checked whether there was at least 50% agreement. If an item rating of a representation fell below this value, this item would not be considered further for statistical analysis, because this indicates that there was substantial disagreement among the raters.

Next, the Levene test was used to test the homogeneity of the variances between the representations. The Friedman test was then performed to determine whether there were significant differences in the mean rating values between the representations. For each rater, the ratings for each representation were converted into ranks, with the lowest rating ranked first. For equal ratings, the ranks were assigned as the average of the positions of the tied ratings. The ranks were summed for each representation to obtain the rank sum for each condition. A significant Friedman test result indicates that there are differences between the representations [86].

Raters evaluated four different representations. Each representation was rated by the same raters in different conditions, such as quantum measurement, superposition and probability. The ratings were made on a scale from 1 to 5, and since the same raters judged each representation in each condition, the ratings were related (dependent).

Finally, the post hoc Wilcoxon signed rank test with Bonferroni correction was used to identify specific differences between the representations [86]. This methodological approach enabled a precise and differentiated analysis of the data, providing deeper insights into the variance and significance within the representations studied. In order to identify the impact of the significant values among the representations, Cohen's effect size *d* was calculated [87].

Each representation was rated by the same rater in different conditions. Ratings were made on a scale of 1 to 5, and because the same raters rated each representation in each condition, the ratings were related (dependent). Given the nature of our data, we carefully selected non-parametric methods to ensure meaningful analysis. The coefficient of variation was used to assess the agreement between raters, the Friedman test was chosen as the most appropriate approach to detect significant differences in dependent samples, and the median was used for the ordinal nature of the ratings and the mean for clarity, to challenges reposed by a relative but expert sample.

We also calculated the correlations between the overarching categories, design, functions, tasks, and cross-concepts, with Spearman's correlation coefficient and used this to refine our analysis of the categories. All statistical analyses were performed using R (version 4.4.0, R Core Team, 2024). The R code used for the analysis is available on request.

4 Results

4.1 Variations in expert ratings

We calculated the coefficient of variation to determine the level of agreement between raters for each rating item. Most coefficients of variation (Appendix B) were at or below 0.5, which is usually accepted as the threshold for adequate internal agreement [84, 85], with the exception of those for the items in the categories *actions/steps*, and *complementarity*, which were above 0.5. In these cases, no statement could be made due to disagreement among the experts.

A similar disagreement arose in the rating of entanglement within the category *concepts*, resulting in its exclusion from the *concepts* category. The ratings for global and relative phase visualisation were made separately for the category *phase*. Due to the disagreement in the assessment of the global phase, this was also not taken into account.

For *understanding difficulties* in item statement B, generability for the Qake representation, and *effort in explanation* for the Quantum Bead representation, the level of agreement was 49% agreement. Further consideration of the category understanding difficulties was limited to statements A, C, and D, with statement B excluded due to discrepancies between the experts. The same procedure was applied to colour for representation, generability for the Qake representation, and effort in explanation representation Quantum Bead. If two or more representations within a category had a coefficient of variation greater than or equal to 0.5, they were excluded completely.

4.2 Considered categories after agreement

Table 3 and 4 show the means for all categories and the significance within them. The mean values were inverted for specific categories (*understanding difficulties, colour and effort in explanation*) to provide an overview of which categories were rated higher and therefore have a positive impact on learning. For answering RQ1 and RQ2, details are provided in Sects. 4.2.1 - 4.2.4.

Table 3 Mean value of expert ratings for each category

Category	Quantum Bead	Circle Notation	Bloch sphere	Qake	p-value
			Design		
Salience	3.33 ± 1.03	2.88 ± 0.93	2.79 ± 0.90	3.11 ± 0.88	***
Dimension	3.40 ± 1.10	3.97 ± 1.10	3.66 ± 1.19	3.99 ± 1.07	***
Understanding difficulties[1]	3.20 ± 1.32	4.24 ± 0.90	3.61 ± 1.14	3.82 ± 1.09	***
Colour[2]	–	4.08 ± 1.12	4.90 ± 0.31	1.29 ± 0.46	***
			Functions		
Actions/steps	–	–	–	–	–
Mathematics	3.18 ± 1.29	3.94 ± 1.00	4.16 ± 0.60	3.71 ± 0.92	n.s.
Contiguity	3.41 ± 1.33	3.74 ± 1.10	3.16 ± 1.30	3.50 ± 0.99	n.s.
Predictability	3.82 ± 0.73	4.32 ± 0.58	4.05 ± 0.71	4.11 ± 0.74	*
			Tasks/applications		
Concepts	3.38 ± 1.16	4.02 ± 1.07	3.41 ± 1.16	4.19 ± 1.13	***
Quantum measurement	3.48 ± 1.08	4.00 ± 1.18	3.66 ± 1.06	4.19 ± 1.12	**
Superposition	3.24 ± 1.26	4.10 ± 1.00	3.62 ± 1.12	4.24 ± 1.16	**
Probabilistics	3.43 ± 1.16	3.95 ± 1.07	3.33 ± 1.15	4.14 ± 1.20	*
Quantum technologies	3.88 ± 1.05	3.87 ± 1.29	4.17 ± 0.96	3.92 ± 1.27	n.s.
Phase (relative)	3.12 ± 1.36	4.53 ± 0.61	4.56 ± 0.76	4.21 ± 0.85	***
Amplitude	3.30 ± 1.03	4.50 ± 0.51	3.85 ± 0.88	4.60 ± 0.50	***
			Cross-concepts		
Generability	2.10 ± 1.22	3.29 ± 1.06	2.52 ± 0.75	–	***
Effort in explanation[3]	–	2.81 ± 1.03	2.10 ± 0.89	3.15 ± 1.14	**

The p-value was calculated using the Friedman test to determine the difference between the representations. ***$p < 0.001$, **$p < 0.01$, and *$p < 0.05$. n.s. = not significant

[1] 5 means less prone to difficulties in learning. 1 means very prone to difficulties in learning

[2] A high mean value means less need for representations to visualise concepts with colour. This category is based on the need for colour representation.

[3] A high mean value means less 'effort in explanation' of the representation.

Table 4 Mean values of function categories

Category	Function						
	QB&CN	B&QB	Qake&QB	B&CN	CN&Qake	Qake&CN	p-Value
Redundancy	3.05 ± 1.00	3.45 ± 0.89	3.15 ± 1.09	3.50 ± 1.32	3.95 ± 1.05	3.20 ± 1.28	**
Complementarity	–	–	–	–	–	–	–

Combinations of representations: QB&CN = Quantum Bead and Circle Notation, B&QB = Bloch sphere and Quantum Bead, Qake&QB = Qake and Quantum Bead, B&CN = Bloch sphere and Circle Notation, CN&Qake = Circle Notation and Qake, Qake&CN = Qake and Circle Notation

4.2.1 Results from design categories

Based on the data, we observe that there are significant differences between the representations in the ratings for *salience* ($\chi^2(3) = 21.706$, $p < 0.001$). The rating for the Quantum Bead was significantly higher than for the Bloch sphere ($d = 0.36$) and the Circle Notation ($d = 0.38$). In addition, there was a significant difference between Qake and the Circle Notation with $d = 0.31$, and between Qake and the Bloch sphere ($d = 0.32$). The Quantum Bead (mean: 3.33 ± 1.03) was rated the most salient, followed by Qake (mean: 3.11 ± 0.88).

Furthermore, there are significant differences in the category *dimension* ($\chi^2(3) = 17.810$, $p < 0.001$), particularly between the Quantum Bead and the Circle Notation ($d = 0.39$) representations and between Qake and Quantum Bead ($d = 0.38$). The experts rated the Circle Notation and Qake as more adequate regarding their spatial dimensionality. There is also a significant positive correlation $r = 0.41$ between *dimension* and *salience* (95% confidence interval of 0.31 to 0.50).

We found significant differences between the representations in the generation of misconceptions respectively *understanding difficulties* ($\chi^2(3) = 37.090$, $p < 0.001$). According to the expert ratings, the Circle Notation tends to cause fewer difficulties than the Quantum Bead, ($p < 0.001$, $d = 0.50$), the Bloch sphere ($p < 0.034$, $d = 0.38$), and the Qake representation ($p < 0.042$), $d = 0.36$). The experts rated the Bloch sphere and the Quantum Bead as more likely to lead to understanding difficulties than the Circle Notation and Qake.

We also found significant differences between the representations in the category *colour* ($\chi^2(2) = 38.297$, $p < 0.001$). Significant differences were found between the Bloch sphere and Qake ($p < 0.001$, $d = 0.90$) and between the Circle Notation and the Bloch sphere ($p < 0.001$, $d = 0.69$). There was also a significant difference between Qake and the Circle Notation ($p < 0.001$, $d = 0.86$). These differences all have a high effect size. The Bloch sphere was rated high as colour independent respectively no relevant in colour (mean: 4.90 ± 0.31) while the Qake model was rated highly colour dependent to visualise the concepts (mean: 1.29 ± 0.46).

4.2.2 Results from function categories

The results indicate non-significant differences between the representations in terms of *mathematics* and *contiguity*. Despite the similar rating regarding interaction with *mathematics*, the Circle Notation and Qake were rated significantly higher in regarding whether concepts ($\chi^2(3) = 37.258$, $p < 0.0001$) such as quantum measurement, superposition, and probabilistics were appropriately visualised.

The data indicate a significant difference in *predictability* ($\chi^2(3) = 10.451$, $p = 0.015$). Moreover, there is a significant positive correlation $r = 0.47$ between the ratings for *predictability* and *mathematics* (95% confidence interval of 0.27 to 0.63).

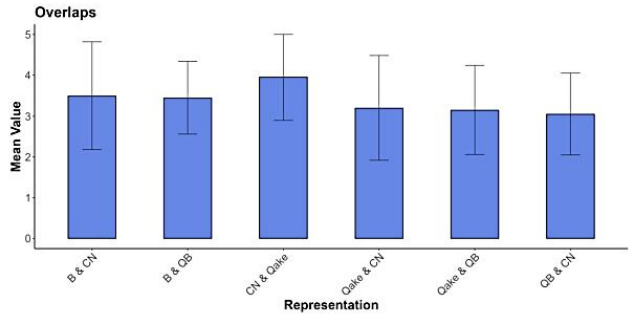


Figure 3 Experts' ratings of whether representations have the same information content of concepts. Combinations of representations: QB&CN = Quantum Bead and Circle Notation, B&QB = Bloch sphere and Quantum Bead, Qake&QB = Qake and Quantum Bead, B&CN = Bloch sphere and Circle Notation, CN&Qake = Circle Notation and Qake, Qake&CN = Qake and Circle Notation

The experts rating also analysed how representations are used in combination with each other *overlap/redundancy* (Fig. 3), i.e., how many complementary or redundant information they contain. This follows prior research on MER, which suggests that redundancy and complementarity can impact learning differently [7]. The category *complementary* was not analysed further due to disagreement among the experts. However, there is a significant difference between particular groups in *overlap/redundancy* ($\chi^2(5) = 16.126$, $p < 0.01$). Between groups QB/CN and CN/Qake ($p < 0.05$) a significant difference (with an effect size of $d = 0.64$) could be determined, as well as between groups QB/B and Qake/B ($p = 0.05$, and $d = 0.19$). Between all other pairs, no significant difference could be identified. Representations with the same dimensions were assigned the highest rating values for *overlap/redundancy* (QB/B: 3.45 ± 0.89; CN/Qake: 3.95 ± 1.05).

4.2.3 Results from task categories

As already mentioned, there was a significant difference between the expert ratings of the suitability of the representations for visualising *concepts* ($\chi^2(3) = 37.258$, $p < 0.001$). This concerns the concepts of quantum measurement, superposition, and probabilistic. This difference was found to exist mainly between the Circle Notation and the Bloch sphere ($d = 0.43$), the Quantum Bead and the Circle Notation ($d = 0.42$), Qake and the Bloch sphere ($d = 0.48$), and Qake and the Quantum Bead ($d = 0.51$). Circle Notation (mean: 4.02 ± 1.07) and Qake (mean: 3.88 ± 1.05) were rated higher on average.

The results indicate significant differences in the representations regarding the visualisation of *phase (relative)* ($\chi^2(3) = 15.528$, $p < 0.001$). Post hoc tests showed that these differences were mainly between the Quantum Bead and Bloch sphere representations ($p < 0.01$, $d = 0.64$), and also between the Quantum Bead and Circle Notation ($p < 0.01$, $d = 0.60$). A high effect size can be assigned to this. The Circle Notation and Qake were rated higher on average.

There was also a significant difference in the visualisation of the *amplitude* ($\chi^2(3) = 29.008$, $p < 0.001$). The data showed a difference between the Quantum Bead and Circle Notation representations ($p < 0.01$, $d = 0.67$) and also a significant difference between the Quantum Bead and Qake ($p < 0.01$, $d = 0.76$). Significant differences were also found in the comparisons between the Qake representation and the Bloch sphere ($d = 0.66$). Qake (mean: 4.60 ± 0.52) and the Circle Notation (mean: 4.50 ± 0.51) were rated higher on average. A high effect size can be assigned to this.

Moreover, ratings for visualisation of the *amplitude* correlate positively with those for the *predictability* category $r = 0.49$ (95% confidence interval of 0.30 to 0.64).

There are no significant differences between the representations concerning the category *quantum technology*. This is likely due to the fact that all representations of a qubit were visualised, thus allowing us to demonstrate the elementary operation of a quantum computer (the X-, Z-, and H-gates).

4.2.4 *Results from cross-concept categories*

The data indicated significant differences between the representations in terms of effort in explanation ($\chi^2(2) = 8.9333$, $p < 0.01$) specifically between the Circle Notation and the Bloch sphere ($d = 0.55$, $p = 0.034$) and between Qake and the Bloch sphere ($d = 0.56$, $p = 0.032$).

The expert assessments differed significantly across representations for the category *generability* according to the Friedman test results ($\chi^2(2) = 8.4595$, $p < 0.01$). However, the subsequent pairwise comparisons using the Wilcoxon signed-rank test with Bonferroni correction showed no significant differences between the individual representations ($p > 0.05$). This suggests that, although there were overall differences in the ratings, none of the specific pairwise differences were statistically significant after the correction for multiple comparisons was applied.

The coefficients of variation for *generability* the Qake representation and *effort in the explanation* for the Quantum Bead exceeded 0.5, indicating significant disagreement among the experts. Due to the lack of consensus reflected in the data, these representations were excluded from consideration in these categories.

In the following, we present an overview of the results and indicate how they can play a role in the development of new qubit representations. Fig. 4 visualises how the four representations differ in their profiles across categories. The categories *overlap/redundancy* and *complementarity* are not included as they do not refer to individual representations but to combinations of them.

It can be seen that representations were rather similar regarding the categories *contiguity, mathematics, and quantum technologies*. However, the experts perceived considerable differences between the representations in the following categories: *phase, amplitude, concepts*, and *understanding difficulties*. Moderate differences were observed for *salience, dimension*, and *predictability*. Table 5 shows the effect sizes of the differences within the representations. These can be helpful when deciding on the choice and creation of new representations, in which a categorisation can be made beforehand according to similarities and average, strong differences in the aspects of representations. It is essential to plan how these are used in their tasks/applications. The mean value was inverted for *understanding difficulties* to provide an overview of which categories were rated higher and thus have a positive impact on learning. *Colour* is intended to show how relevant the colour component is for understanding the concepts and represents a limitation of the representation, which is why it is inverted.

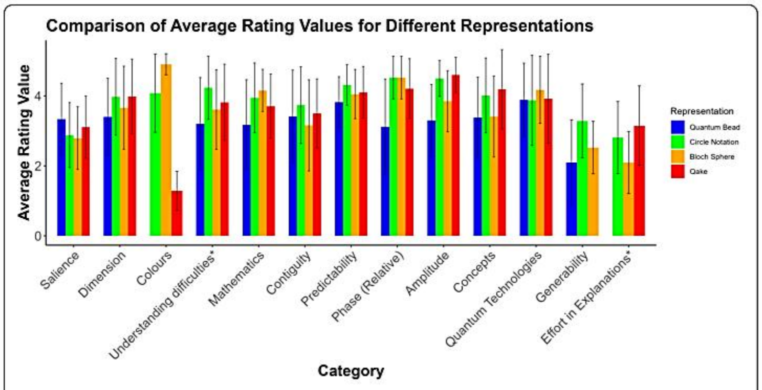


Figure 4 Overview of the differences in means between the representations in each category. Categories marked with * are inverted. The mean values were inverted for *understanding difficulties*, that is, high values in these items mean that experts associate the representation with a lower tendency to cause understanding difficulties, because we want to give an overview in this figure of which categories were rated higher and are, therefore, positive for learning. *Effort in explanations* is also inverted: high values in these cases mean that experts perceive less effort in presenting information with the representation. The categories and representations that were not analysed further due to a lack of agreement are as follows: *actions/steps and complementarity*, *colour* for the Quantum Bead, *generability* for the Cake, and *effort in explanations* for the Quantum Bead.

Table 5 Overview of effect size for each category

Category	Effect sizes
Moderate to high effect	
Amplitude	$0.66 \leq d \leq 0.76$
Phase *relative	$0.60 \leq d \leq 0.64$
Understanding difficulties	$0.40 \leq d \leq 0.52$
Overlap/redundancy	$d = 0.64$
Moderate effect	
Concepts	$0.42 \leq d \leq 0.51$
Dimension	$d \leq 0.39$
Salience	$0.31 \leq d \leq 0.38$
No difference	
Mathematics	
Contiguity	
Predictability	
Quantum technologies	

Only effect sizes for which the experts were in agreement for all representations are shown. Thus, *colour, generability and effort in explanations* are not listed.

4.2.5 Results of the free-text questions

The experts were also asked to indicate which **category** they thought was important to differentiate the representations and which concepts, if any, were missing.

An expert noted that the *salience* category is difficult to assess, as it is hard to fulfil the conditions for salience via the rating.

The *actions/steps* category is also difficult to quantify. This is also reflected in the wide spread of rater responses in this category, which is still meaningful for categorising the representations, but should be recorded differently. One possibility would be to record the individual steps qualitatively via what is spoken or written and to quantify them via defined sections in "steps".

Another expert, felt that the *effort in explanation* category is the most important from an educational perspective. This proves to be a good categorisation of the representation for effective and target group-oriented use.

Some experts suggested including additional categories for correctness, enjoyment in learning, the possible construction of real models for visualisation, and interactive software or videos.

The following additional **concepts** were suggested: mixed states, partial traces, errors, types of entanglement, entanglement entropy, systems with more than two qubits, C-X (controlled-X) gates for two qubits in all representations, as this is essential for quantum computing, and visualisation of multiple qubits and algorithms.

5 Discussion

The expert rating process had two goals: to identify features of qubit representations that may support learning and to identify factors that need to be considered when developing new qubit representations.

5.1 Interpretation of findings

The data show that, according to the experts, of the four compared representations, the Circle Notation and Qake are especially well suited for visualising concepts such as phase or amplitude. They provide clarification of the theoretical relationships between phase and amplitude, which play a central role in the precise description of a quantum state.

In general, the aim in learning environments—whether in traditional instructional materials, multimedia-based instruction, or technology-enhanced approaches such as AR/VR—is to reduce difficulties that can arise from misconceptions in visual-graphical representations. More attention needs to be paid to shape, dimension, and preconceptions to avoid misunderstandings [1, 64, 79]. The exclusion of one statement (B) has shown that it is difficult to judge on the basis of statements whether visual-graphical representations can cause or reinforce misconceptions or difficulties.

The remaining statements allow us to identify the differences in perception between the raters regarding the potential for triggering difficulties or false concepts in learners, with a medium to high effect size, which is why we think it is important to consider difficulties with regard to misconceptions.

According to the ratings, the qubit representations differ in *salience*, which may play a key role in learning. The correlation of the ratings in the salience category with those in the *dimension* category suggests that more *salient* representations like the Quantum Bead also have more appropriate dimensionality (2D or 3D) for understanding the concept. It is possible that this correlation is due to other factors, or that the variety of representations employed is insufficient to fully recognise this effect. Particularly highly salient representations show a strong colour component (e.g. the Quantum Bead). Brightly coloured representations such as the Quantum Bead are beneficial for learning if they are used to attract attention to relevant components [62].

Moreover, the expert rating results indicate that the dimensionality of a representation could play a role in *understanding difficulties*. In consideration of the theory and the findings [63–65], it has already been shown that spatial dimensionality can lead to difficulties in understanding. Overall, however, the expert ratings only refer to a small, selected number of statements regarding difficulties or misconceptions, which could potentially be caused by learners being overwhelmed by or misinterpreting the representations' features.

There are no significant differences between the representations regarding the *function* categories for qubit representations, except for *predictability* and *overlap/redundancy*. This is likely because most categories were considered elementary for qubit representations or because the selected representations may be similar in these respects.

The expert rating has shown that the individual qubit representations have different feature profiles, which is reflected in the different strengths and weaknesses of the representations. None of the representations could be found to be clearly superior or inferior in all categories. In other contexts, the use of several representations, also known as multiple representations, has proven to be useful (see Sect. 2.3). The following quotes from experts in the free-text responses illustrate this:

> *'[...] But there should never be the 'one representation,' since more visual models like the qubit cakes will always be easier to digest at first, while models like the bloch-sphere help understand more complex topics.' (Expert no. 16)*

> *'The Circle Notation and the quantum cake model purely represent the mathematical tool of expressing a qubit (or several qubits). Hence they are mostly helpful to students who are struggling with the basic math. The other two representations (Quantum Bead and Bloch sphere) on the other hand are somewhat more advanced, because they try to represent the fact that a qubit can be visualized in 3-dimensional space. [...]' (Expert no. 3)*

This emphasises how important it is to know features/aspects of qubit representations in order to use them in a targeted and learning-promoting way.

5.2 Further research

An attempt has been made to draw conclusions for learners from the experts' assessments and current learning research, but further studies are needed to verify the results and obtain a more detailed perspective. Table 6 provides an overview of expert opinions on the individual features of the representations using the mean values. This helps to raise questions for follow-up studies, such as which of the features that differ between the represen-

Table 6 Scaling of the different categories in relation to representations

Categories	Quantum Bead	Circle Notation	Bloch sphere	Qake
Salience	high	low	low	middle
Dimension	middle	high	middle	high
Understanding difficulties[1]	middle	high	middle	middle
Colour[2]	no information	middle	high	low
Predictability	middle	high	middle	middle
Concepts	middle	high	middle	high
Phase (relative)	middle	high	high	middle
Amplitude	low	high	middle	high
Generability	middle	middle	middle	no information
Effort in explanations[3]	no information	high	low	high

When the internal agreement among the raters was insufficient—applying only to this representation—we classified it as 'no information'

[1]'High' means less prone to difficulties; 'low' means very prone to difficulties

[2]'High' means that colour are not necessary to visualise concepts in QP, which is positive for representation.

[3]'High' means less effort is needed to explain how a representation presents information; 'low" means more effort is needed.

tations according to the experts' ratings are relevant to learning. The scaling from low to high was chosen based on whether there were significant differences in the data from the tests reported in this study and the corresponding orientation by mean value. If there were no significant differences between the representations, the same category was chosen for them. In future studies that investigate the influence of single features of representations on learning, as many aspects as possible should be controlled to study the effect of a particular aspect. Maintaining this balance is a challenge. The gains in learning provided by representations could be measured. In any case, a follow-up study is required to capture the effect of the representations on learners' achievements. It is, therefore, necessary to investigate whether the identified features of qubit representations, which differ from expert opinion, can be transferred to the perspective of the learners and examined for their potential to facilitate learning. For instance, it could be investigated whether certain representations are more likely to cause difficulties than others. It would also be interesting to investigate the relationship between predictability and amplitude visualisation in more detail. Are there effects on learning when the amplitude is visualised more strongly in the representations?

Draft of a study: In Table 6, the differences and similarities between the representations are shown. One possible study would be to first analyse two similar representations that nevertheless have substantial differences in a certain category, such as the Quantum Bead and the Bloch sphere in the salience category. The challenge, however, is to reduce or avoid compensation effects or effects that are stronger due to other categories than in the other representation (in this case, phase and amplitude for the Bloch sphere). Both representations could be delivered via a learning unit for a specific concept, for example, for superposition. Eye tracking could be used to analyse the learners' gaze behaviour and determine the duration of fixation. Pre- and post-tests could be used to compare the learning gain with the representations and thus determine how high the learning gain was when using the representations. Many other studies are possible.

As mentioned, MER can be used in different ways. When there are overlaps or redundancies between representations, these can be useful for learning concepts in QP and QT. In favour of the use of multiple representations: overlaps in representations of the same spatial dimensionality were rated higher on average by the experts. Further research is needed to investigate the learning potential of the use of multiple representations and, in particular, which combinations are conducive to learning. We know from MER research that the use of multiple representations can fulfil various functions that can promote learning with representations [7]. They can be analysed for different functions. Specifically, there are three key functions that MERs can perform (even simultaneously) to support the learning process:

1. supporting complementary processes
2. enabling representations to constrain each other
3. developing of deeper understanding

As part of a study, it could be examined to what extent and with combinations the use of MER for qubit representations makes sense. The combinations from the rating can be used for this purpose. The study could contrast learning using a combination of representations with learning using individual representations.

5.3 Limitations

The expert ratings provide us with a categorisation based on people with a wealth of experience, but there are also some limitations, such as the small sample size and the specific selection of representations. The experts had a limited view of the representations as they only had access to the contents of the cheat sheet and, where applicable, material from the references. It is therefore possible that the strength of the representations was limited.

Additionally, the relatively high number of rating items (140) may have led to fatigue effects among the raters, potentially influencing their responses towards the later items. However, the structured nature of the rating aimed to minimise this effect, and raters had the option to pause and resume their assessment at any time, ensuring that they could complete the ratings without time pressure.

Furthermore, the categories that were not further analysed due to a lack of agreement are not necessarily unsuitable, including *complementarity* and *actions/steps*. With a larger sample, it could be valuable to analyse these further. Similarly, the use of tangible tools for visualizing representations was not included in the ratings. Due of the study as an online survey and the highly variable and, in some cases, limited development status of tangible tools across all representations, this aspect could not be assessed.

Moreover, while expert perspectives provide valuable insights into the structure and conceptual affordances of representations, they do not necessarily reflect the intuitive understanding and learning processes of novice learner. Experts tend to evaluate representations based on their completeness, efficiency, and formal correctness, which may differ from the cognitive accessibility that would be most beneficial for learners. This distinction should be kept in mind when interpreting the findings.

Due to the lack of consensus among the raters, items in certain categories or even entire item sets, and thus the entire category, were excluded, so that the expert ratings only offer a limited view of the features of visual-graphical qubit representations. The concepts were exclusively focused on the consideration of single-qubit cases. Multi-qubit systems were not directly analysed.

6 Conclusion

The expert ratings provide key insights into the features of qubit representations that facilitate learning in QP and highlight important considerations for developing new, more effective representations in QP education. Choosing appropriate representations is challenging. Leveraging the category system based on the DeFT framework could help make visual representations more effective and optimise their use in different educational contexts.

6.1 Key insights from expert rating

- **Phase and Amplitude:** The expert ratings in this study suggest that representations like the Circle Notation or Qake could be more effective in teaching fundamental QP concepts, than the other two representations analysed in this study. The Circle Notation was also rated highly in terms of phase and amplitude and the experts attached particular importance to representations that clearly visualise these aspects. In addition, representations that explicitly visualise the relative phase with an arrow or a line, such as the Circle Notation and the Bloch sphere, received higher ratings.
- **Colour and Salience:** Although colour can enhance salience for specific purposes, it is not deemed essential for conveying the core concepts.

- **Combination of representations:** Since no single representation can effectively convey all concepts (for example, the Bloch sphere does not visualise entanglement), multiple representations should be combined for a more comprehensive understanding.
- **Understanding difficulties:** Misconceptions can arise from different sources, and careful consideration of these factors is necessary when developing educational materials.

Key factors in developing new qubit representations include minimising difficulties, particularly through clear visualisation of the (relative) phase and amplitude. Our findings indicate that the Circle Notation and the Qake model are particularly effective in visualising quantum concepts and are considered easier for learners to understand. To prevent misunderstandings, especially with less intuitive representations like the Quantum Bead and the Bloch sphere, a more detailed introduction or explanation may be required. This suggests that the Circle Notation or a pie chart model, like Qake, may be more suitable or efficient for teaching the fundamental concepts of QP and QT (see Fig. 4 and Table 5).

Finally, representations such as the Bloch sphere cannot be used for all concepts (e.g., entanglement), so they must be used in combination with other qubit representations when teaching QT. According to experts, they can certainly be used for educational purposes and may even be capable of supporting a transition to the use of MER to fill this "gap" [2, 7]. This enables a comprehensive representation of quantum concepts, particularly those that are fundamental to QT.

These findings have important implications for the development of future teaching materials and could significantly enhance the teaching of QP. Future research should focus on refining these representations for different educational levels and contexts, exploring their effectiveness across diverse learning environments, and integrating additional visual elements to further aid comprehension.

6.2 Challenges and open questions

While this study focuses on expert perspectives, an important open question remains: How do learners perceive and learn with different representations? Future research should involve learners in empirical studies so that it is possible to investigate how representations support or hinder learning processes in terms of the category system.

Moreover, this study is not without limitations, including its relatively small sample size, limited range of representations, and exclusive focus on single-qubit cases. Thus, although the findings are promising, further research is needed to confirm the results across broader settings and with more varied representations. Future studies should also investigate multi-qubit systems to provide a more comprehensive understanding of effective representations in QP education.

By leveraging the expert-rated category system based on the DeFT Framework, educators and researchers can develop more effective visual representations that align with learners' needs and support deeper conceptual understanding in quantum physics education.

Appendix A: Medians

After a significant Friedman test (see column p-value), comparing the medians between conditions can help to identify patterns in the ratings and clarify the direction of differ-

Table 7 Median expert rating for each category

Category	Quantum Bead	Circle Notation	Bloch sphere	Qake	p-value
			Design		
Salience	3.0	3.0	3.0	3.0	***
Dimension	4.0	4.0	4.0	4.0	***
Understand difficulties[1]	4.0	3.0	4.0	3.0	***
colour	–	4.0	5.0	1.0	***
			Function		
Actions/steps	–	–	–	–	–
Mathematics	3.0	4.0	4.0	4.0	n.s.
Contiguity	4.0	4.0	3.0	4.0	n.s.
Predictability	4.0	4.0	4.0	4.0	*
			Tasks/applications		
Concepts	3.0	4.0	3.0	5.0	***
Quantum measurement	3.0	4.0	4.0	5.0	**
Superposition	3.0	4.0	4.0	5.0	**
Probabilistics	4.0	4.0	3.0	5.0	*
Quantum technologies	4.0	4.0	4.0	4.0	n.s.
Phase[2]	4.0	5.0	5.0	4.0	***
Amplitude	3.5	4.5	4.0	5.0	***
			Cross-concepts		
Generability	2.0	3.0	2.0	–	***
Effort in explanation[3]	–	3.0	2.0	3.0	**

The p-value was calculated using the Friedman test to determine the difference between the representations. ***$p < 0.001$, **$p < 0.01$ and *$p < 0.05$. n.s. = not significant
[1]Inverted: 5 means very prone to difficulties, 1 means less prone to difficulties
[2]Relative Phase
[3]Inverted

Table 8 Medians of function categories

Category	Function						
	QB&CN	B&QB	Qake&QB	B&CN	CN&Qake	Qake&CN	p-Value
Redundancy	3.0	3.5	3.0	4.0	4.0	3.0	**
Complementarity	–	–	–	–	–	–	–

Combinations of representations: QB&CN = Quantum Bead and Circle Notation, B&QB = Bloch sphere and Quantum Bead, Qake&QB = Qake and Quantum Bead, B&CN = Bloch sphere and Circle Notation, CN&Qake = Circle Notation and Qake, Qake&CN = Qake and Circle Notation

ences. Medians are used to indicate central tendencies. Table 7 and Table 8 provide information about the median rating levels.

Appendix B: Coefficient of variation

The coefficient of variation provides a measure of agreement among expert ratings by expressing the standard deviation relative to the mean (see Table 9 and 10). All items with a value of 0.5 or higher were excluded from the analysis, as they exceed the threshold value, indicating high variability among responses. These excluded items are highlighted in grey in the table for better visibility. The exclusion criterion ensures that only items with sufficiently low response variability remain in the analysis, enhancing the robustness of the comparisons between the different representations. This appendix provides an overview of the coefficient of variation values used to assess rating consistency across categories.

Table 9 Coefficient of variation for the total items of the categories

Item	Quantum Bead	Circle Notation	Bloch sphere	Qake
Salience_QS	0.27	0.36	0.27	0.25
Salience_QM	0.32	0.35	0.30	0.29
Salience_Sp	0.29	0.32	0.31	0.27
Salience_E	0.37	0.27	0.40	0.31
Dimension_QS	0.29	0.29	0.21	0.21
Dimension_QM	0.21	0.23	0.27	0.25
Dimension_Sp	0.27	0.19	0.27	0.20
Dimension_E	0.50	0.38	0.50	0.35
Actions_steps_QS	0.52	0.39	0.34	0.38
Actions_steps_QM	0.53	0.70	0.63	0.48
Actions_steps_Sp	0.56	0.52	0.53	0.48
Actions_steps_E	0.59	0.55	0.66	0.57
Understanding_difficulties_A	0.19	0.45	0.35	0.49
Understanding_difficulties_B	0.51	0.47	0.41	0.51
Understanding_difficulties_C	0.42	0.44	0.42	0.43
Understanding_difficulties_D	0.28	0.29	0.29	0.39
Colour	0.55	0.28	0.06	0.47
Mathematics	0.40	0.25	0.14	0.25
Contiguity	0.39	0.29	0.41	0.28
Predictability	0.19	0.13	0.17	0.18
Phase_Gl	0.64	0.36	0.81	0.32
Phase_Rl	0.43	0.13	0.13	0.21
Amplitude	0.31	0.11	0.23	0.11
Concept_QM	0.31	0.30	0.29	0.27
Concept_Sp	0.39	0.24	0.31	0.27
Concept_E	0.56	0.51	0.44	0.45
Concept_prob	0.34	0.27	0.35	0.29
QT_H-Gate	0.27	0.36	0.26	0.36
QT_X-Gate	0.23	0.31	0.18	0.26
QT_Z-Gate	0.29	0.33	0.24	0.34
Generability	0.32	0.39	0.22	0.51
Effort_in_explanation	0.51	0.37	0.42	0.36

Any item with a coefficient of variation of 0.5 or higher was excluded from further analysis.

Table 10 Coefficient of variation for complementarity and overlap/redundancy in combination of representations. We name the items according to the requested combinations

Item	QB&CN	B&QB	Qake&QB	B&CN	CN&Qake	Qake&CN
Redundancy	0.33	0.26	0.35	0.38	0.27	0.40
Complementarity	0.46	0.51	0.48	0.53	0.48	0.50

Combinations of representations: QB&CN = Quantum Bead and Circle Notation, B&QB = Bloch sphere and Quantum Bead, Qake&QB = Qake and Quantum Bead, B&CN = Bloch sphere and Circle Notation, CN&Qake = Circle Notation and Qake, Qake&CN = Qake and Circle Notation

Appendix C: Mean rating value (graphs)

The following section presents the mean rating values across the evaluated categories. To facilitate comparison, the bar charts have been structured as grouped bar charts, displaying multiple categories for each representation side by side.

Figure 5 shows the mean values for the design-related categories, including salience, dimension, and understanding difficulties. Figure 6 presents the mean values for function-related categories, including mathematics, contiguity and predictability, and Fig. 7 provides an overview of the ratings for application-related categories. These visualizations support the analysis of differences in expert ratings across the evaluated representations.

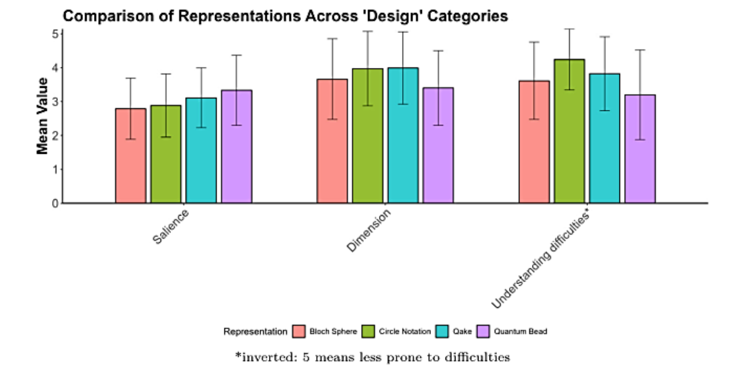


Figure 5 Mean values for categories under design categories

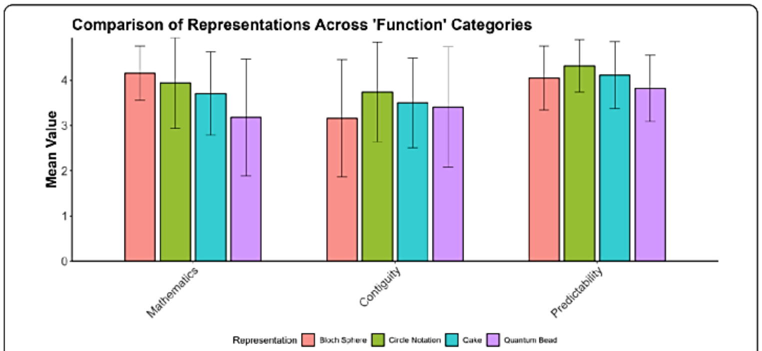


Figure 6 Mean values for categories under function categories

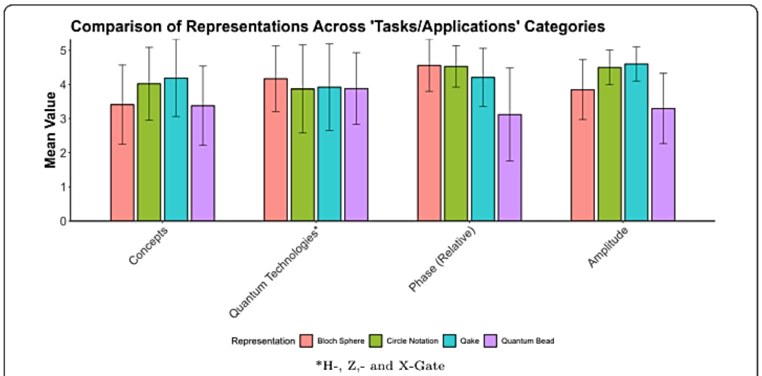


Figure 7 Mean values for categories under tasks/applications categories

It should be noted that the mean values presented in these figures provide an overview of the ratings but do not necessarily indicate statistically significant differences in all categories. For a detailed statistical analysis, incorporating significance testing, please refer to Table 3.

Abbreviations
DeFT, Design, Functions and Tasks; MER, Multiple external representations; QP, Quantum physics; QT, Quantum technology.

Supplementary information

Supplementary information accompanies this paper at https://doi.org/10.1140/epjqt/s40507-025-00346-1.

Additional file 1. The cheatsheets were developed for the rating so that all experts are familiar with the qubit representations to be rated and can see the visualisation of the concepts (PDF 3.4 MB)

Acknowledgements
We would like to thank Tatjana Wilk, General Manager of the Munich Center for Quantum Science and Technology (MCQST), and Silke Stähler-Schöpf, Head of the student laboratory PhotonLab at the Max Planck Institute of Quantum Optics (MPQ), for their valuable contributions to this article. We are also grateful to all participants who contributed to this study.

Author contributions
L.Q., St.K., J.K., S.Ma. designed the study, L.Q., St.K., developed the questionnaires and collected the data, L.Q., St.K., analysed the data, L.Q. wrote the first draft of the manuscript, L.Q., St.K., J.K., S.Ma., E.R., S.M. reviewed and edited the manuscript. St.K. supervised the study. All authors have read and agreed to the submitted version of the manuscript.

Funding information
Open Access funding enabled and organized by Projekt DEAL. This research is part of the Munich Quantum Valley (MQV), which is supported by the Bavarian State Government.

Data Availability
No datasets were generated or analysed during the current study.

Declarations

Ethics approval and consent to participate
Ethical review and approval was not required for the study involving human subjects in accordance with local legislation and institutional requirements. The study involved data collection using an online questionnaire. Participation was voluntary and anonymous. Participants had the option of giving their name voluntarily. Participants were informed that their answers would be kept confidential and would only be used for research purposes. Data collected included the location of each participant's research institution, their field of research, and their years of teaching experience. By voluntarily completing the online questionnaire, participants confirmed their participation and consented to the use of their anonymised data for research purposes and publication.

Consent for publication
Not applicable.

Competing interests
The authors declare no competing interests.

Author details
[1]Chair of Physics Education, Faculty of Physics, Ludwig-Maximilians-Universität München (LMU), Geschwister-Scholl-Platz 1, 80539 Munich, Germany. [2]Max Planck Institute of Quantum Optics, 85748 Garching, Germany. [3]Munich Quantum Valley (MQV), 80807 Munich, Germany. [4]Saarland University, Department of Education, 66123 Saarbrücken, Germany. [5]University of Kaiserslautern-Landau, Department of Physics, 67663 Kaiserslautern, Germany.

Received: 21 September 2024 Accepted: 25 March 2025 Published online: 08 April 2025

References

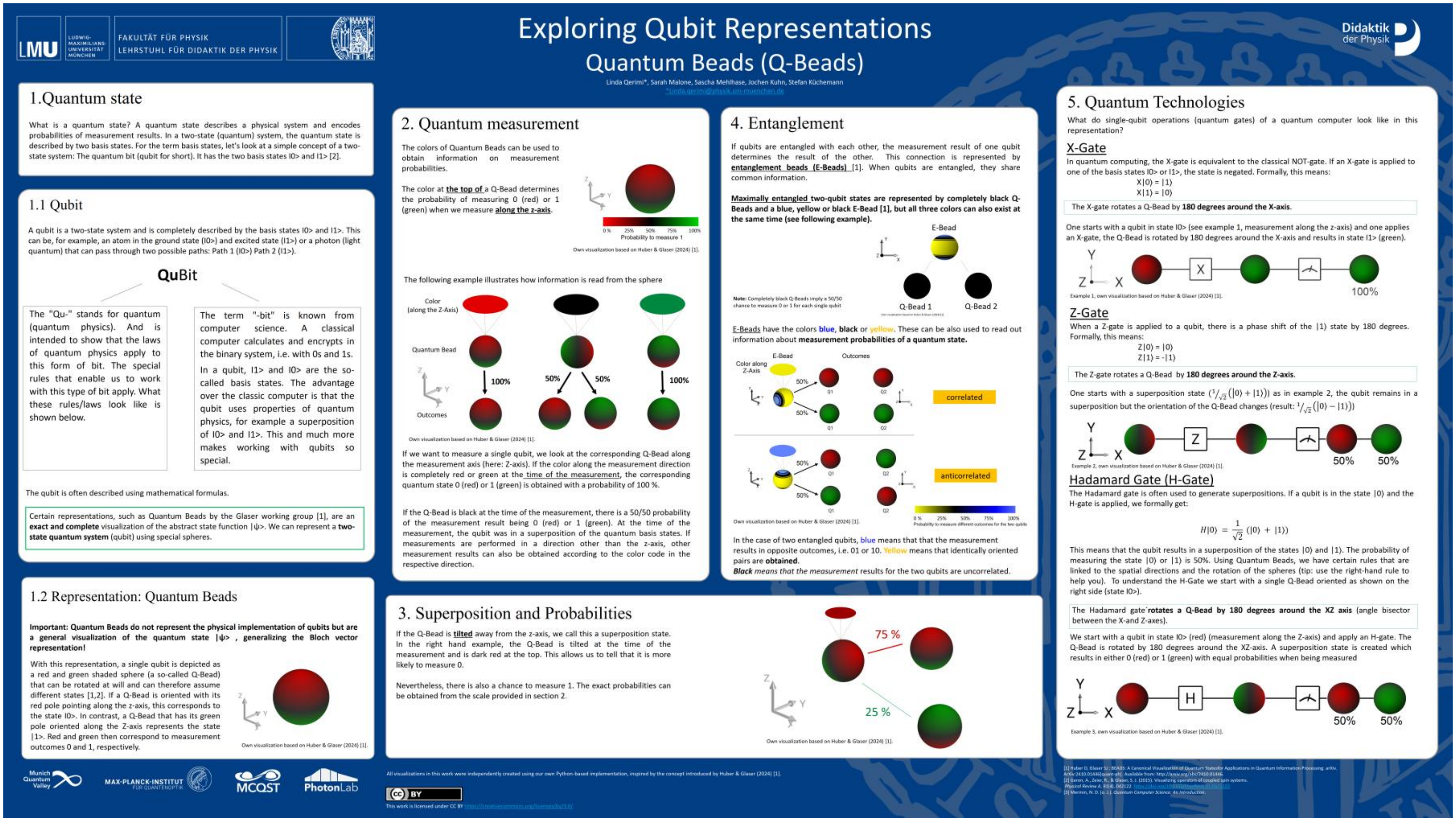
1. Singh C, Marshman E. Review of student difficulties in upper-level quantum mechanics. Phys Rev ST Phys Educ Res. 2015;11:020117. https://doi.org/10.1103/PhysRevSTPER.11.020117.
2. Rau MA. Conditions for the effectiveness of multiple visual representations in enhancing STEM learning. Educ Psychol Rev. 2017;29:717–61. https://doi.org/10.1007/s10648-016-9365-3.
3. Schnotz W. Kognitive Prozesse bei der sprach- und bildgestützten Konstruktion mentaler Modelle. In: Sichelschmidt L, Strohner H, editors. Sprache, Sinn und Situation: Festschrift für Gert Rickheit zum 60. Geburtstag. Wiesbaden: Deutscher Universitätsverlag; 2001. p. 43–57.

4. Bertin J. Semiology of graphics. Madison: University of Wisconsin Press; 1983.
5. Lemke J. Multiplying meaning. In: Reading science: critical and functional perspectives on discourses of science; 1998. p. 87.
6. Kosslyn SM. Understanding charts and graphs. Appl Cogn Psychol. 1989;3:185–225. https://doi.org/10.1002/acp.2350030302.
7. Ainsworth S. DeFT: a conceptual framework for considering learning with multiple representations. Learn Instr. 2006;16:183–98. https://doi.org/10.1016/j.learninstruc.2006.03.001.
8. Itti L, Koch C. Computational modelling of visual attention. Nat Rev Neurosci. 2001;2:194–203. https://doi.org/10.1038/35058500.
9. Reynolds RE, Anderson RC. Influence of questions on the allocation of attention during reading. J Educ Psychol. 1982;74(5):623–32. Place: US Publisher: American Psychological Association. https://doi.org/10.1037/0022-0663.74.5.623.
10. Schnotz W, Bannert M. Construction and interference in learning from multiple representation. Learn Instr. 2003;13:141–56. https://doi.org/10.1016/S0959-4752(02)00017-8.
11. Greinert F, Müller R, Bitzenbauer P, Ubben MS, Weber KA. Future quantum workforce: competences, requirements, and forecasts. Phys Rev Phys Educ Res 2023;19(1):010137. https://doi.org/10.1103/PhysRevPhysEducRes.19.010137.
12. Directorate-General for Communications Networks, Content and Technology (European Commission), Greinert F, Müller R. European competence framework for quantum technologies (CFQT): reference framework for planning, mapping and comparing QT related educational activities, personal qualification and job requirements. Publications Office of the European Union. 2024. Available from https://data.europa.eu/doi/10.2759/389764.
13. Goorney S, Bley J, Heusler S, Sherson J. The Quantum Curriculum Transformation Framework for the Development of Quantum Information. Science and Technology Education. arXiv.org. Preprint.
14. Marshman E, Maries A, Singh C. Using multiple representations to improve student understanding of quantum states. Phys Rev Phys Educ Res. 2024;20(2):020152.
15. Bley J, Rexigel E, Arias A, Longen N, Krupp L, ea Maximilian K-E. Visualizing entanglement in multiqubit systems. Phys Rev Res. 2024;6(2):023077. https://doi.org/10.1103/PhysRevResearch.6.023077.
16. Rexigel E, Bley J, Arias A, Qerimi L, Küchemann S, Kuhn J, et al. Investigating the use of multiple representations in university courses on quantum technologies. EPJ Quantum Technol. 2025;12(1):22.
17. Hu P, Li Y, Singh C. Investigating and improving student understanding of the basics of quantum computing. Phys Rev Phys Educ Res. 2024;20(2):020108.
18. Dür W, Heusler S. Was man vom einzelnen Qubit über Quantenphysik lernen kann. PhyDid A - Phys Didakt Sch Hochsch. 2012;1(11):01.
19. Dür W, Heusler S. Visualization of the invisible: the qubit as key to quantum physics. Phys Teach. 2014;52(8):489–92. https://doi.org/10.1119/1.4897588.
20. Majidy S. Addressing misconceptions in university physics: a review and experiences from quantum physics educators. 2024. arXiv preprint. arXiv:2405.20923.
21. Passante G. Energy measurement resources in spins-first and position-first quantum mechanics. In: 2016 physics education research conference proceedings. Sacramento: American Association of Physics Teachers; 2016. p. 236–9.
22. Dür W, Heusler S. Was man von zwei Qubits über Quantenphysik lernen kann: Verschränkung und Quantenkorrelationen. PhyDid A - Phys Didakt Sch Hochsch. 2014;1(13):11–34.
23. Sadaghiani HR, Munteanu J. Spin first instructional approach to teaching quantum mechanics in sophomore level modern physics courses. In: 2015 physics education research conference proceedings. College Park: American Association of Physics Teachers; 2015. p. 287–90.
24. Küblbeck J, Müller R. Die Wesenszüge der Quantenphysik: Modelle, Bilder, Experimente. Aulis-Verlag Deubner. 2002.
25. Müller R, Mishina O. Milq—quantum physics in secondary school. In: Jarosievitz B, Sükösd C, editors. Teaching-learning contemporary physics: from research to practice. Cham: Springer; 2021. p. 33–45. https://doi.org/10.1007/978-3-030-78720-2_3.
26. Müller R, Greinert F. Quantum technologies. Berlin: De Gruyter Oldenbourg; 2024. https://doi.org/10.1515/9783110717457.
27. Mermin ND. Quantum computer science: an introduction. Cambridge: Cambridge University Press; 2007.
28. Merzel A, Bitzenbauer P, Krijtenburg-Lewerissa K, Stadermann K, Andreotti E, Anttila D, et al. The core of secondary level quantum education: a multi-stakeholder perspective. EPJ Quantum Technol. 2024;11:27. https://doi.org/10.1140/epjqt/s40507-024-00237-x.
29. Stadermann K, van den Berg E. Analysis of secondary school quantum physics curricula of 15 different countries: different perspectives on a challenging topic. Phys Rev Phys Educ Res. 2019;15(1):010130. https://doi.org/10.1103/PhysRevPhysEducRes.15.010130.
30. Manogue C, Gire E, McIntyre D, Tate J, Rebello NS, Engelhardt PV, et al. Representations for a spins-first approach to quantum mechanics. In: AIP conference proceedings. Omaha: AIP Publishing; 2012. p. 55–8.
31. Woitzik AJ. Quanteninformationsverarbeitung in der gymnasialen Oberstufe. 2020. arXiv:2011.01029. Preprint.
32. Feynman RPQE. The strange theory of light and matter. Princeton: Princeton University Press; 2006.
33. Bader F. Optik und quantenphysik nach feynmans QED. Phys Sch. 1994;32(7):8.
34. Küblbeck J. Modellbildung in der Physik. Landesinstitut für Schulentwicklung.
35. Küblbeck J. Quantenphysik. In: Kircher E, Girwidz R, Häußler P, editors. Physikdidaktik: theorie und praxis. Berlin: Springer; 2015. p. 479–501.
36. Johnston ER, Harrigan N, Gimeno-Segovia M. Programming quantum computers: essential algorithms and code samples. Sebastopol: O'Reilly Media, Inc.; 2019.
37. Just B. Quantencomputing kompakt: Spukhafte Fernwirkung und Teleportation endlich verständlich. Berlin: Springer; 2020. Available from https://link.springer.com/10.1007/978-3-662-61889-9.
38. Garon A, Zeier R, Glaser SJ. Visualizing operators of coupled spin systems. Phys Rev A. 2015;91(4):042122. https://doi.org/10.1103/PhysRevA.91.042122.
39. Huber D, Glaser SJ. BEADS: a Canonical Visualization of Quantum States for Applications in Quantum Information Processing. arXiv. Preprint. Available from http://arxiv.org/abs/2410.01446.

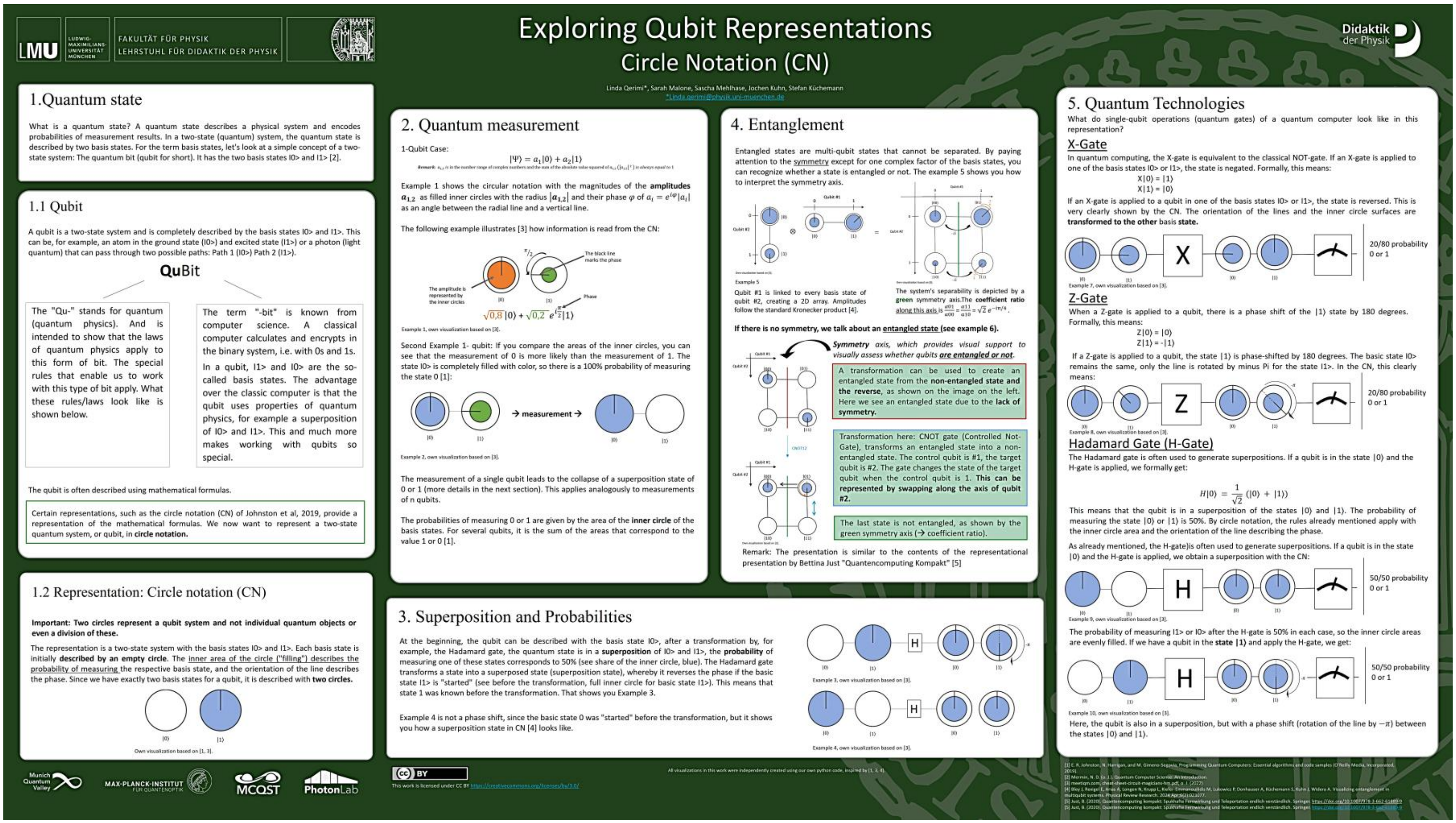
40. Yeung K. Quantum computing & some physics: the quantum computing comics notebook. Verlag nicht ermittelbar. 2020. Available from. https://books.google.de/books?id=3LEVzgEACAAJ.
41. Donhauser A, Kuhn J, et al. Qubit Cake Model. 2025.
42. Weidner CA, Ahmed SZ, Jensen JH, Sherson JF, Lewandowski H. Investigating student use of a flexible tool for simulating and visualizing quantum mechanics. In: 2020 physics education research conference proceedings. 2020.
43. Zaman Ahmed S, Jensen JHM, Weidner CA, Sørensen JJ, Mudrich M, Sherson JF. Quantum composer: a programmable quantum visualization and simulation tool for education and research. Am J Phys. 2021;89(3):307–16.
44. Küchemann S, Ubben M, Dzsotjan D, Mukhametov S, Weidner CA, Qerimi L, et al. The impact of an interactive visualization and simulation tool on learning quantum physics: results of an eye-tracking study. arXiv.org. Preprint.
45. Kohnle A, Ainsworth SE, Passante G. Sketching to support visual learning with interactive tutorials. Phys Rev Phys Educ Res. 2020;16(2):020139. https://doi.org/10.1103/PhysRevPhysEducRes.16.020139.
46. Kohnle A, Baily C, Ruby S. Investigating the influence of visualization on student understanding of quantum superposition. In: 2014 physics education research conference proceedings. Minneapolis: American Association of Physics Teachers; 2015. p. 139–42.
47. Bouchée T, de Putter-Smits L, Thurlings M, Pepin B. Towards a better understanding of conceptual difficulties in introductory quantum physics courses. Stud Sci Educ. 2022;58(2):183–202.
48. Schnotz W, Rasch T. Enabling, facilitating, and inhibiting effects of animations in multimedia learning: why reduction of cognitive load can have negative results on learning. Educ Technol Res Dev. 2005;53(3):47–58.
49. Kircher E. Modellbegriff und Modellbildung in der Physikdidaktik. In: Physikdidaktik: Theorie und Praxis; 2015. p. 783–807.
50. Garcia Garcia G, Cox R. "Graph-as-picture" misconceptions in young students. In: Goel AK, Jamnik M, Narayanan NH, editors. Diagrammatic representation and inference. Berlin: Springer; 2010. p. 310–2.
51. diSessa AA, Sherin BL. Meta-representation: an introduction. J Math Behav. 2000;19(4):385–98. https://doi.org/10.1016/S0732-3123(01)00051-7.
52. Wiesner H. Verständnisse von Leistungskursschülern über Quantenphysik. Phys Sch. 1996;34:95.
53. Fischler H, Lichtfeldt M. Modern physics and students' conceptions. Int J Sci Educ. 1992;14(2):181–90. https://doi.org/10.1080/0950069920140206.
54. Özcan O. What are the students' mental models about the "spin" and "photon" concepts in modern physics? Proc, Soc Behav Sci. 2011;15:1372–5. https://doi.org/10.1016/j.sbspro.2011.03.295.
55. Brang M, Franke H, Greinert F, Ubben MS, Hennig F, Bitzenbauer P. Spooky action at a distance? A two-phase study into learners' views of quantum entanglement. EPJ Quantum Technol. 2024;11(1):33. https://doi.org/10.1140/epjqt/s40507-024-00244-y.
56. Krijtenburg-Lewerissa K, Pol HJ, Brinkman A, van Joolingen WR. Insights into teaching quantum mechanics in secondary and lower undergraduate education. Phys Rev Phys Educ Res. 2017;13(1):010109.
57. Henriksen EK, Angell C, Vistnes AI, Bungum B. What is light? Students' reflections on the wave-particle duality of light and the nature of physics. Sci Educ. 2018;27:81–111.
58. Ubben MS, Bitzenbauer P. Two cognitive dimensions of Students' mental models in science: fidelity of gestalt and functional fidelity. Educ Sci. 2022;12(3):163. https://doi.org/10.3390/educsci12030163.
59. Einstein A, Podolsky B, Rosen N. EinsteinPodolsky. Phys Rev. 1935;47:777.
60. Leiner D, Zeier R, Glaser SJ. Wigner tomography of multispin quantum states. Phys Rev A. 2017;96(6):063413. https://doi.org/10.1103/PhysRevA.96.063413.
61. Hewstone M, Stroebe W. An introduction to social psychology. Hoboken: Wiley; 2021.
62. van Gog T. The signaling (or cueing) principle in multimedia learning. 2nd ed. In: Mayer RE, editor. The Cambridge handbook of multimedia learning. Cambridge handbooks in psychology. Cambridge: Cambridge University Press; 2014. p. 263–78.
63. Castro-Alonso JC, Jansen P. Sex differences in visuospatial processing. In: Castro-Alonso JC, editor. Visuospatial processing for education in health and natural sciences. Cham: Springer; 2019. p. 81–110.
64. Heo M, Toomey N. Learning with multimedia: the effects of gender, type of multimedia learning resources, and spatial ability. Comput Educ. 2020;146:103747. https://doi.org/10.1016/j.compedu.2019.103747.
65. Saha S, Halder S. He or she: does gender affect various modes of instructional visual design? Texas State University. Center for Diversity and Gender Studies.
66. Sweller J. Cognitive load theory, learning difficulty, and instructional design. Learn Instr. 1994;4(4):295–312. https://doi.org/10.1016/0959-4752(94)90003-5.
67. Baddeley A. Working memory. Science. 1992;255(5044):556–9.
68. Miller GA. The magical number seven, plus or minus two: some limits on our capacity for processing information. Psychol Rev. 1956;63(2):81–97.
69. Sweller J, van Merriënboer JJ, Paas F. Cognitive architecture and instructional design: 20 years later. Educ Psychol Rev. 2019;31(2):261–92. https://doi.org/10.1007/s10648-019-09465-5.
70. Schrödinger E. Die gegenwärtige Situation in der Quantenmechanik. Naturwissenschaften. 1935;23(50):844–9.
71. Mayer RE, Fiorella L. Principles for reducing extraneous processing in multimedia learning: coherence, signaling, redundancy, spatial contiguity, and temporal contiguity principles. In: Mayer RE, editor. The Cambridge handbook of multimedia learning. Cambridge: Cambridge University Press; 2014. p. 279–315.
72. Ott N, Brünken R, Vogel M, Malone S. Multiple symbolic representations: the combination of formula and text supports problem solving in the mathematical field of propositional logic. Learn Instr. 2018;58:88–105. https://doi.org/10.1016/j.learninstruc.2018.04.010.
73. Krüger D, Kauertz A. Upmeier zu Belzen A. Modelle und das Modellieren in den Naturwissenschaften. In: Krüger D, Parchmann I, Schecker H, editors. Theorien in der naturwissenschaftsdidaktischen Forschung. Berlin: Springer; 2018. p. 141–57.
74. Treagust DF, Duit R, Fischer HE, editors. Multiple representations in physics education. vol. 10. Cham: Springer; 2017.
75. Nielsen MA, Chuang IL. Quantum computation and quantum information: 10th anniversary edition. Cambridge: Cambridge University Press; 2010.
76. Feynman RP, Hibbs AR, Styer DF. Quantum mechanics and path integrals. Courier Corporation; 2010.

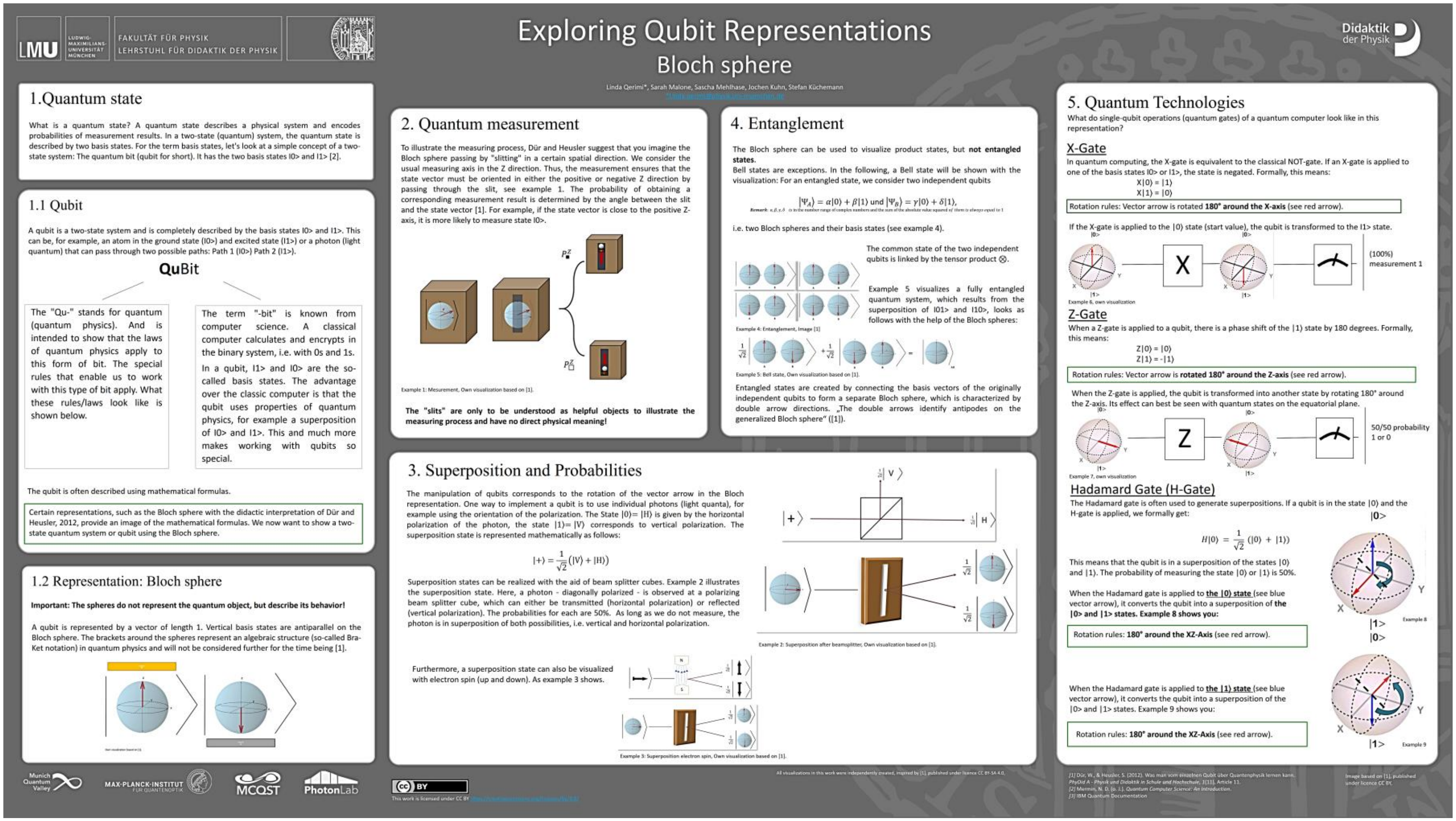
77. Sweller J. Cognitive load during problem solving: effects on learning. Cogn Sci. 1988;12(2):257–85.
78. Marshman E, Singh C. Investigating and improving student understanding of quantum mechanics in the context of single photon interference. Phys Rev Phys Educ Res. 2017;13(1):010117. https://doi.org/10.1103/PhysRevPhysEducRes.13.010117.
79. Ubben MS, Heusler S. Gestalt and functionality as independent dimensions of mental models in science. Res Sci Educ. 2021;51(5):1349–63. https://doi.org/10.1007/s11165-019-09892-y.
80. Ekert A. From Interferometers to Quantum Computers. Retrieved at Supplementary material. 2010; p. 1–11.
81. Scholz M, Aichele T, Ramelow S, Benson O. Deutsch-Jozsa algorithm using triggered single photons from a single quantum dot. Phys Rev Lett. 2006;96(18):180501. https://doi.org/10.1103/PhysRevLett.96.180501.
82. O'Brien JL, Pryde GJ, White AG, Ralph TC, Branning D. Demonstration of an all-optical quantum controlled-NOT gate. Nature 2003;426(6964):264–7. https://doi.org/10.1038/nature02054.
83. Patton MQ. Qualitative research & evaluation methods. Sage; 2002.
84. Zinn J, Zalokowski A, Hunter L. Identifying indicators of laboratory management performance: a multiple constituency approach. Health Care Manage Rev. 2001;26(1):40.
85. von der Gracht HA. Consensus measurement in delphi studies: review and implications for future quality assurance. Technol Forecast Soc Change. 2012;79(8):1525–36. https://doi.org/10.1016/j.techfore.2012.04.013.
86. Field A, Field Z, Miles J. Discovering statistics using R. SAGE Publications; 2012.
87. Cohen J. Statistical power analysis for the behavioral sciences. 2nd ed. Hillsdale: L. Erlbaum Associates; 1988.

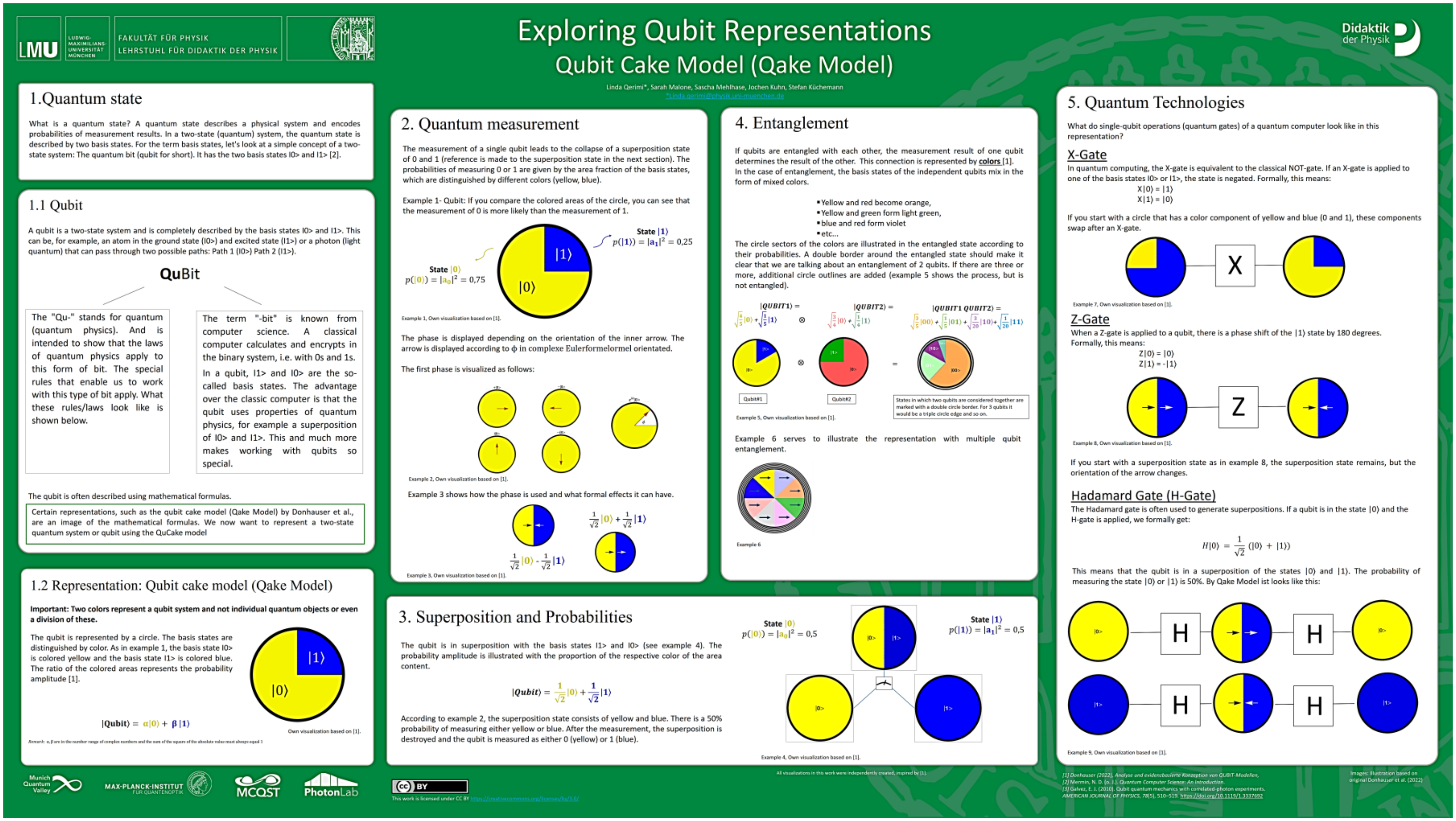
Publisher's note



LMU
LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN
FAKULTÄT FÜR PHYSIK
LEHRSTUHL FÜR DIDAKTIK DER PHYSIK
Exploring Qubit Representations
Quantum Beads (Q-Beads)
Linda Qerimi*, Sarah Malone, Sascha Mehlhase, Jochen Kuhn, Stefan Küchemann
Didaktik der Physik
1.Quantum state
What is a quantum state? A quantum state describes a physical system and encodes probabilities of measurement results. In a two-state (quantum) system, the quantum state is described by two basis states. For the term basis states, let's look at a simple concept of a two-state system: The quantum bit (qubit for short). It has the two basis states I0> and I1> [2].
1.1 Qubit
A qubit is a two-state system and is completely described by the basis states I0> and I1>. This can be, for example, an atom in the ground state (I0>) and excited state (I1>) or a photon (light quantum) that can pass through two possible paths: Path 1 (I0>) Path 2 (I1>).
QuBit
The "Qu-" stands for quantum (quantum physics). And is intended to show that the laws of quantum physics apply to this form of bit. The special rules that enable us to work with this type of bit apply. What these rules/laws look like is shown below.
The term "-bit" is known from computer science. A classical computer calculates and encrypts in the binary system, i.e. with 0s and 1s. In a qubit, I1> and I0> are the so-called basis states. The advantage over the classic computer is that the qubit uses properties of quantum physics, for example a superposition of I0> and I1>. This and much more makes working with qubits so special.
The qubit is often described using mathematical formulas.
Certain representations, such as Quantum Beads by the Glaser working group [1], are an exact and complete visualization of the abstract state function |ψ>. We can represent a two-state quantum system (qubit) using special spheres.
1.2 Representation: Quantum Beads
Important: Quantum Beads do not represent the physical implementation of qubits but are a general visualization of the quantum state |ψ> , generalizing the Bloch vector representation!
With this representation, a single qubit is depicted as a red and green shaded sphere (a so-called Q-Bead) that can be rotated at will and can therefore assume different states [1,2]. If a Q-Bead is oriented with its red pole pointing along the z-axis, this corresponds to the state I0>. In contrast, a Q-Bead that has its green pole oriented along the Z-axis represents the state |1>. Red and green then correspond to measurement outcomes 0 and 1, respectively.
Own visualization based on Huber & Glaser (2024) [1].
2. Quantum measurement
The colors of Quantum Beads can be used to obtain information on measurement probabilities.
The color at the top of a Q-Bead determines the probability of measuring 0 (red) or 1 (green) when we measure along the z-axis.
0 % 25% 50% 75% 100%
Probability to measure 1
Own visualization based on Huber & Glaser (2024) [1].
The following example illustrates how information is read from the sphere
Color (along the Z-Axis)
Quantum Bead
100%
50%
50%
100%
Outcomes
Own visualization based on Huber & Glaser (2024) [1].
If we want to measure a single qubit, we look at the corresponding Q-Bead along the measurement axis (here: Z-axis). If the color along the measurement direction is completely red or green at the time of the measurement, the corresponding quantum state 0 (red) or 1 (green) is obtained with a probability of 100 %.
If the Q-Bead is black at the time of the measurement, there is a 50/50 probability of the measurement result being 0 (red) or 1 (green). At the time of the measurement, the qubit was in a superposition of the quantum basis states. If measurements are performed in a direction other than the z-axis, other measurement results can also be obtained according to the color code in the respective direction.
4. Entanglement
If qubits are entangled with each other, the measurement result of one qubit determines the result of the other. This connection is represented by entanglement beads (E-Beads) [1]. When qubits are entangled, they share common information.
Maximally entangled two-qubit states are represented by completely black Q-Beads and a blue, yellow or black E-Bead [1], but all three colors can also exist at the same time (see following example).
E-Bead
Q-Bead 1
Q-Bead 2
Note: Completely black Q-Beads imply a 50/50 chance to measure 0 or 1 for each single qubit
E-Beads have the colors blue, black or yellow. These can be also used to read out information about measurement probabilities of a quantum state.
Color along Z-Axis
E-Bead
Outcomes
50%
50%
Q1
Q2
correlated
anticorrelated
Own visualization based on Huber & Glaser (2024) [1].
0 % 25% 50% 75% 100%
Probability to measure different outcomes for the two qubits
In the case of two entangled qubits, blue means that that the measurement results in opposite outcomes, i.e. 01 or 10. Yellow means that identically oriented pairs are obtained.
Black means that the measurement results for the two qubits are uncorrelated.
3. Superposition and Probabilities
If the Q-Bead is tilted away from the z-axis, we call this a superposition state. In the right hand example, the Q-Bead is tilted at the time of the measurement and is dark red at the top. This allows us to tell that it is more likely to measure 0.
Nevertheless, there is also a chance to measure 1. The exact probabilities can be obtained from the scale provided in section 2.
75 %
25 %
Own visualization based on Huber & Glaser (2024) [1].
5. Quantum Technologies
What do single-qubit operations (quantum gates) of a quantum computer look like in this representation?
X-Gate
In quantum computing, the X-gate is equivalent to the classical NOT-gate. If an X-gate is applied to one of the basis states I0> or I1>, the state is negated. Formally, this means:
X|0) = |1)
X|1) = |0)
The X-gate rotates a Q-Bead by 180 degrees around the X-axis.
One starts with a qubit in state I0> (see example 1, measurement along the z-axis) and one applies an X-gate, the Q-Bead is rotated by 180 degrees around the X-axis and results in state I1> (green).
Y
Z
X
100%
Example 1, own visualization based on Huber & Glaser (2024) [1].
Z-Gate
When a Z-gate is applied to a qubit, there is a phase shift of the |1) state by 180 degrees. Formally, this means:
Z|0) = |0)
Z|1) = -|1)
The Z-gate rotates a Q-Bead by 180 degrees around the Z-axis.
One starts with a superposition state $(^1/_{\sqrt{2}}(|0)+|1)))$ as in example 2, the qubit remains in a superposition but the orientation of the Q-Bead changes (result: $^1/_{\sqrt{2}}(|0)-|1))$)
50%
50%
Example 2, own visualization based on Huber & Glaser (2024) [1].
Hadamard Gate (H-Gate)
The Hadamard gate is often used to generate superpositions. If a qubit is in the state |0) and the H-gate is applied, we formally get:
$H|0) = \frac{1}{\sqrt{2}}(|0) + |1))$
This means that the qubit results in a superposition of the states |0) and |1). The probability of measuring the state |0) or |1) is 50%. Using Quantum Beads, we have certain rules that are linked to the spatial directions and the rotation of the spheres (tip: use the right-hand rule to help you). To understand the H-Gate we start with a single Q-Bead oriented as shown on the right side (state I0>).
The Hadamard gate rotates a Q-Bead by 180 degrees around the XZ axis (angle bisector between the X-and Z-axes).
We start with a qubit in state I0> (red) (measurement along the Z-axis) and apply an H-gate. The Q-Bead is rotated by 180 degrees around the XZ-axis. A superposition state is created which results in either 0 (red) or 1 (green) with equal probabilities when being measured
H
50%
50%
Example 3, own visualization based on Huber & Glaser (2024) [1].
Munich Quantum Valley
MAX-PLANCK-INSTITUT FÜR QUANTENOPTIK
MCQST
PhotonLab
All visualizations in this work were independently created using our own Python-based implementation, inspired by the concept introduced by Huber & Glaser (2024) [1].
cc BY
This work is licensed under CC BY


Exploring Qubit Representations
Circle Notation (CN)
Linda Qerimi*, Sarah Malone, Sascha Mehlhase, Jochen Kuhn, Stefan Küchemann
*Linda.qerimi@physik.uni-muenchen.de
LMU
LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN
FAKULTÄT FÜR PHYSIK
LEHRSTUHL FÜR DIDAKTIK DER PHYSIK
Didaktik der Physik
1.Quantum state
What is a quantum state? A quantum state describes a physical system and encodes probabilities of measurement results. In a two-state (quantum) system, the quantum state is described by two basis states. For the term basis states, let's look at a simple concept of a two-state system: The quantum bit (qubit for short). It has the two basis states I0> and I1> [2].
1.1 Qubit
A qubit is a two-state system and is completely described by the basis states I0> and I1>. This can be, for example, an atom in the ground state (I0>) and excited state (I1>) or a photon (light quantum) that can pass through two possible paths: Path 1 (I0>) Path 2 (I1>).
QuBit
The "Qu-" stands for quantum (quantum physics). And is intended to show that the laws of quantum physics apply to this form of bit. The special rules that enable us to work with this type of bit apply. What these rules/laws look like is shown below.
The term "-bit" is known from computer science. A classical computer calculates and encrypts in the binary system, i.e. with 0s and 1s. In a qubit, I1> and I0> are the so-called basis states. The advantage over the classic computer is that the qubit uses properties of quantum physics, for example a superposition of I0> and I1>. This and much more makes working with qubits so special.
The qubit is often described using mathematical formulas.
Certain representations, such as the circle notation (CN) of Johnston et al, 2019, provide a representation of the mathematical formulas. We now want to represent a two-state quantum system, or qubit, in circle notation.
1.2 Representation: Circle notation (CN)
Important: Two circles represent a qubit system and not individual quantum objects or even a division of these.
The representation is a two-state system with the basis states I0> and I1>. Each basis state is initially described by an empty circle. The inner area of the circle ("filling") describes the probability of measuring the respective basis state, and the orientation of the line describes the phase. Since we have exactly two basis states for a qubit, it is described with two circles.
Own visualization based on [1, 3].
2. Quantum measurement
1-Qubit Case:
$|\Psi\rangle = a_1|0\rangle + a_2|1\rangle$
Example 1 shows the circular notation with the magnitudes of the amplitudes $a_{1,2}$ as filled inner circles with the radius $|a_{1,2}|$ and their phase φ of $a_i = e^{i\varphi}|a_i|$ as an angle between the radial line and a vertical line.
The following example illustrates [3] how information is read from the CN:
The amplitude is represented by the inner circles
The black line marks the phase
Phase
$\sqrt{0{,}8}\,|0\rangle + \sqrt{0{,}2}\,e^{i\frac{\pi}{2}}|1\rangle$
Example 1, own visualization based on [3].
Second Example 1- qubit: If you compare the areas of the inner circles, you can see that the measurement of 0 is more likely than the measurement of 1. The state I0> is completely filled with color, so there is a 100% probability of measuring the state 0 [1]:
→ measurement →
Example 2, own visualization based on [3].
The measurement of a single qubit leads to the collapse of a superposition state of 0 or 1 (more details in the next section). This applies analogously to measurements of n qubits.
The probabilities of measuring 0 or 1 are given by the area of the inner circle of the basis states. For several qubits, it is the sum of the areas that correspond to the value 1 or 0 [1].
3. Superposition and Probabilities
At the beginning, the qubit can be described with the basis state I0>, after a transformation by, for example, the Hadamard gate, the quantum state is in a superposition of I0> and I1>, the probability of measuring one of these states corresponds to 50% (see share of the inner circle, blue). The Hadamard gate transforms a state into a superposed state (superposition state), whereby it reverses the phase if the basic state I1> is "started" (see before the transformation, full inner circle for basic state I1>). This means that state 1 was known before the transformation. That shows you Example 3.
Example 4 is not a phase shift, since the basic state 0 was "started" before the transformation, but it shows you how a superposition state in CN [4] looks like.
Example 3, own visualization based on [3].
Example 4, own visualization based on [3].
4. Entanglement
Entangled states are multi-qubit states that cannot be separated. By paying attention to the symmetry except for one complex factor of the basis states, you can recognize whether a state is entangled or not. The example 5 shows you how to interpret the symmetry axis.
Example 5
Qubit #1 is linked to every basis state of qubit #2, creating a 2D array. Amplitudes follow the standard Kronecker product [4].
The system's separability is depicted by a green symmetry axis.The coefficient ratio along this axis is $\frac{a01}{a00} = \frac{a11}{a10} = \sqrt{2}\,e^{-i\pi/4}$.
If there is no symmetry, we talk about an entangled state (see example 6).
Symmetry axis, which provides visual support to visually assess whether qubits are entangled or not.
A transformation can be used to create an entangled state from the non-entangled state and the reverse, as shown on the image on the left. Here we see an entangled state due to the lack of symmetry.
Transformation here: CNOT gate (Controlled Not-Gate), transforms an entangled state into a non-entangled state. The control qubit is #1, the target qubit is #2. The gate changes the state of the target qubit when the control qubit is 1. This can be represented by swapping along the axis of qubit #2.
The last state is not entangled, as shown by the green symmetry axis (→ coefficient ratio).
Remark: The presentation is similar to the contents of the representational presentation by Bettina Just "Quantencomputing Kompakt" [5]
5. Quantum Technologies
What do single-qubit operations (quantum gates) of a quantum computer look like in this representation?
X-Gate
In quantum computing, the X-gate is equivalent to the classical NOT-gate. If an X-gate is applied to one of the basis states I0> or I1>, the state is negated. Formally, this means:
X|0⟩ = |1⟩
X|1⟩ = |0⟩
If an X-gate is applied to a qubit in one of the basis states I0> or I1>, the state is reversed. This is very clearly shown by the CN. The orientation of the lines and the inner circle surfaces are transformed to the other basis state.
20/80 probability
0 or 1
Example 7, own visualization based on [3].
Z-Gate
When a Z-gate is applied to a qubit, there is a phase shift of the |1⟩ state by 180 degrees. Formally, this means:
Z|0⟩ = |0⟩
Z|1⟩ = -|1⟩
If a Z-gate is applied to a qubit, the state |1⟩ is phase-shifted by 180 degrees. The basic state I0> remains the same, only the line is rotated by minus Pi for the state I1>. In the CN, this clearly means:
20/80 probability
0 or 1
Example 8, own visualization based on [3].
Hadamard Gate (H-Gate)
The Hadamard gate is often used to generate superpositions. If a qubit is in the state |0⟩ and the H-gate is applied, we formally get:
$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$
This means that the qubit is in a superposition of the states |0⟩ and |1⟩. The probability of measuring the state |0⟩ or |1⟩ is 50%. By circle notation, the rules already mentioned apply with the inner circle area and the orientation of the line describing the phase.
As already mentioned, the H-gate)is often used to generate superpositions. If a qubit is in the state |0⟩ and the H-gate is applied, we obtain a superposition with the CN:
50/50 probability
0 or 1
Example 9, own visualization based on [3].
The probability of measuring I1> or I0> after the H-gate is 50% in each case, so the inner circle areas are evenly filled. If we have a qubit in the state |1⟩ and apply the H-gate, we get:
50/50 probability
0 or 1
Example 10, own visualization based on [3].
Here, the qubit is also in a superposition, but with a phase shift (rotation of the line by $-\pi$) between the states |0⟩ and |1⟩.
Munich Quantum Valley
MAX-PLANCK-INSTITUT FÜR QUANTENOPTIK
MCQST
PhotonLab
This work is licensed under CC BY https://creativecommons.org/licenses/by/3.0/
All visualizations in this work were independently created using our own python code, inspired by [1, 3, 4].


LMU
LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN
FAKULTÄT FÜR PHYSIK
LEHRSTUHL FÜR DIDAKTIK DER PHYSIK
Exploring Qubit Representations
Bloch sphere
Linda Qerimi*, Sarah Malone, Sascha Mehlhase, Jochen Kuhn, Stefan Küchemann
Didaktik der Physik
1.Quantum state
What is a quantum state? A quantum state describes a physical system and encodes probabilities of measurement results. In a two-state (quantum) system, the quantum state is described by two basis states. For the term basis states, let's look at a simple concept of a two-state system: The quantum bit (qubit for short). It has the two basis states I0> and I1> [2].
1.1 Qubit
A qubit is a two-state system and is completely described by the basis states I0> and I1>. This can be, for example, an atom in the ground state (I0>) and excited state (I1>) or a photon (light quantum) that can pass through two possible paths: Path 1 (I0>) Path 2 (I1>).
QuBit
The "Qu-" stands for quantum (quantum physics). And is intended to show that the laws of quantum physics apply to this form of bit. The special rules that enable us to work with this type of bit apply. What these rules/laws look like is shown below.
The term "-bit" is known from computer science. A classical computer calculates and encrypts in the binary system, i.e. with 0s and 1s. In a qubit, I1> and I0> are the so-called basis states. The advantage over the classic computer is that the qubit uses properties of quantum physics, for example a superposition of I0> and I1>. This and much more makes working with qubits so special.
The qubit is often described using mathematical formulas.
Certain representations, such as the Bloch sphere with the didactic interpretation of Dür and Heusler, 2012, provide an image of the mathematical formulas. We now want to show a two-state quantum system or qubit using the Bloch sphere.
1.2 Representation: Bloch sphere
Important: The spheres do not represent the quantum object, but describe its behavior!
A qubit is represented by a vector of length 1. Vertical basis states are antiparallel on the Bloch sphere. The brackets around the spheres represent an algebraic structure (so-called Bra-Ket notation) in quantum physics and will not be considered further for the time being [1].
2. Quantum measurement
To illustrate the measuring process, Dür and Heusler suggest that you imagine the Bloch sphere passing by "slitting" in a certain spatial direction. We consider the usual measuring axis in the Z direction. Thus, the measurement ensures that the state vector must be oriented in either the positive or negative Z direction by passing through the slit, see example 1. The probability of obtaining a corresponding measurement result is determined by the angle between the slit and the state vector [1]. For example, if the state vector is close to the positive Z-axis, it is more likely to measure state I0>.
Example 1: Mesurement, Own visualization based on [1].
The "slits" are only to be understood as helpful objects to illustrate the measuring process and have no direct physical meaning!
4. Entanglement
The Bloch sphere can be used to visualize product states, but not entangled states.
Bell states are exceptions. In the following, a Bell state will be shown with the visualization: For an entangled state, we consider two independent qubits
$|\Psi_A\rangle = \alpha|0\rangle + \beta|1\rangle$ und $|\Psi_B\rangle = \gamma|0\rangle + \delta|1\rangle$,
i.e. two Bloch spheres and their basis states (see example 4).
The common state of the two independent qubits is linked by the tensor product ⊗.
Example 5 visualizes a fully entangled quantum system, which results from the superposition of I01> and I10>, looks as follows with the help of the Bloch spheres:
Example 4: Entanglement, Image [1]
Example 5: Bell state, Own visualization based on [1].
Entangled states are created by connecting the basis vectors of the originally independent qubits to form a separate Bloch sphere, which is characterized by double arrow directions. „The double arrows identify antipodes on the generalized Bloch sphere" ([1]).
3. Superposition and Probabilities
The manipulation of qubits corresponds to the rotation of the vector arrow in the Bloch representation. One way to implement a qubit is to use individual photons (light quanta), for example using the orientation of the polarization. The State $|0\rangle = |H\rangle$ is given by the horizontal polarization of the photon, the state $|1\rangle = |V\rangle$ corresponds to vertical polarization. The superposition state is represented mathematically as follows:
$|+\rangle = \frac{1}{\sqrt{2}}(|V\rangle + |H\rangle)$
Superposition states can be realized with the aid of beam splitter cubes. Example 2 illustrates the superposition state. Here, a photon - diagonally polarized - is observed at a polarizing beam splitter cube, which can either be transmitted (horizontal polarization) or reflected (vertical polarization). The probabilities for each are 50%. As long as we do not measure, the photon is in superposition of both possibilities, i.e. vertical and horizontal polarization.
Example 2: Superposition after beamsplitter, Own visualization based on [1].
Furthermore, a superposition state can also be visualized with electron spin (up and down). As example 3 shows.
Example 3: Superposition electron spin, Own visualization based on [1].
5. Quantum Technologies
What do single-qubit operations (quantum gates) of a quantum computer look like in this representation?
X-Gate
In quantum computing, the X-gate is equivalent to the classical NOT-gate. If an X-gate is applied to one of the basis states I0> or I1>, the state is negated. Formally, this means:
X|0⟩ = |1⟩
X|1⟩ = |0⟩
Rotation rules: Vector arrow is rotated 180° around the X-axis (see red arrow).
If the X-gate is applied to the |0⟩ state (start value), the qubit is transformed to the I1> state.
X
(100%) measurement 1
Example 6, own visualization
Z-Gate
When a Z-gate is applied to a qubit, there is a phase shift of the |1⟩ state by 180 degrees. Formally, this means:
Z|0⟩ = |0⟩
Z|1⟩ = -|1⟩
Rotation rules: Vector arrow is rotated 180° around the Z-axis (see red arrow).
When the Z-gate is applied, the qubit is transformed into another state by rotating 180° around the Z-axis. Its effect can best be seen with quantum states on the equatorial plane.
Z
50/50 probability 1 or 0
Example 7, own visualization
Hadamard Gate (H-Gate)
The Hadamard gate is often used to generate superpositions. If a qubit is in the state |0⟩ and the H-gate is applied, we formally get:
$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$
This means that the qubit is in a superposition of the states |0⟩ and |1⟩. The probability of measuring the state |0⟩ or |1⟩ is 50%.
When the Hadamard gate is applied to the |0⟩ state (see blue vector arrow), it converts the qubit into a superposition of the |0> and |1> states. Example 8 shows you:
Rotation rules: 180° around the XZ-Axis (see red arrow).
When the Hadamard gate is applied to the |1⟩ state (see blue vector arrow), it converts the qubit into a superposition of the |0> and |1> states. Example 9 shows you:
Rotation rules: 180° around the XZ-Axis (see red arrow).
Example 8
Example 9
Munich Quantum Valley
MAX-PLANCK-INSTITUT FÜR QUANTENOPTIK
MCQST
PhotonLab
This work is licensed under CC BY
All visualizations in this work were independently created, inspired by [1], published under licence CC BY-SA 4.0.
[1] Dür, W., & Heusler, S. (2012). Was man vom einzelnen Qubit über Quantenphysik lernen kann. PhyDid A - Physik und Didaktik in Schule und Hochschule, 1(11), Article 11.
[2] Mermin, N. D. (o. J.). Quantum Computer Science: An Introduction.
[3] IBM Quantum Documentation
Image based on [1], published under licence CC BY.



LMU
LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN
FAKULTÄT FÜR PHYSIK
LEHRSTUHL FÜR DIDAKTIK DER PHYSIK
Exploring Qubit Representations
Qubit Cake Model (Qake Model)
Linda Qerimi*, Sarah Malone, Sascha Mehlhase, Jochen Kuhn, Stefan Küchemann
Didaktik der Physik
1.Quantum state
What is a quantum state? A quantum state describes a physical system and encodes probabilities of measurement results. In a two-state (quantum) system, the quantum state is described by two basis states. For the term basis states, let's look at a simple concept of a two-state system: The quantum bit (qubit for short). It has the two basis states I0> and I1> [2].
1.1 Qubit
A qubit is a two-state system and is completely described by the basis states I0> and I1>. This can be, for example, an atom in the ground state (I0>) and excited state (I1>) or a photon (light quantum) that can pass through two possible paths: Path 1 (I0>) Path 2 (I1>).
QuBit
The "Qu-" stands for quantum (quantum physics). And is intended to show that the laws of quantum physics apply to this form of bit. The special rules that enable us to work with this type of bit apply. What these rules/laws look like is shown below.
The term "-bit" is known from computer science. A classical computer calculates and encrypts in the binary system, i.e. with 0s and 1s. In a qubit, I1> and I0> are the so-called basis states. The advantage over the classic computer is that the qubit uses properties of quantum physics, for example a superposition of I0> and I1>. This and much more makes working with qubits so special.
The qubit is often described using mathematical formulas.
Certain representations, such as the qubit cake model (Qake Model) by Donhauser et al., are an image of the mathematical formulas. We now want to represent a two-state quantum system or qubit using the QuCake model
1.2 Representation: Qubit cake model (Qake Model)
Important: Two colors represent a qubit system and not individual quantum objects or even a division of these.
The qubit is represented by a circle. The basis states are distinguished by color. As in example 1, the basis state I0> is colored yellow and the basis state I1> is colored blue. The ratio of the colored areas represents the probability amplitude [1].
|Qubit⟩ = α|0⟩ + β|1⟩
Own visualization based on [1].
2. Quantum measurement
The measurement of a single qubit leads to the collapse of a superposition state of 0 and 1 (reference is made to the superposition state in the next section). The probabilities of measuring 0 or 1 are given by the area fraction of the basis states, which are distinguished by different colors (yellow, blue).
Example 1- Qubit: If you compare the colored areas of the circle, you can see that the measurement of 0 is more likely than the measurement of 1.
State |0⟩
p(|0⟩) = |a₀|² = 0,75
State |1⟩
p(|1⟩) = |a₁|² = 0,25
Example 1, Own visualization based on [1].
The phase is displayed depending on the orientation of the inner arrow. The arrow is displayed according to ϕ in complexe Eulerformelormel orientated.
The first phase is visualized as follows:
Example 2, Own visualization based on [1].
Example 3 shows how the phase is used and what formal effects it can have.
Example 3, Own visualization based on [1].
3. Superposition and Probabilities
The qubit is in superposition with the basis states I1> and I0> (see example 4). The probability amplitude is illustrated with the proportion of the respective color of the area content.
According to example 2, the superposition state consists of yellow and blue. There is a 50% probability of measuring either yellow or blue. After the measurement, the superposition is destroyed and the qubit is measured as either 0 (yellow) or 1 (blue).
State |0⟩
p(|0⟩) = |a₀|² = 0,5
State |1⟩
p(|1⟩) = |a₁|² = 0,5
Example 4, Own visualization based on [1].
4. Entanglement
If qubits are entangled with each other, the measurement result of one qubit determines the result of the other. This connection is represented by colors [1].
In the case of entanglement, the basis states of the independent qubits mix in the form of mixed colors.
•Yellow and red become orange,
•Yellow and green form light green,
•blue and red form violet
•etc...
The circle sectors of the colors are illustrated in the entangled state according to their probabilities. A double border around the entangled state should make it clear that we are talking about an entanglement of 2 qubits. If there are three or more, additional circle outlines are added (example 5 shows the process, but is not entangled).
|QUBIT1⟩ =
|QUBIT2⟩ =
|QUBIT1 QUBIT2⟩ =
Qubit#1
Qubit#2
States in which two qubits are considered together are marked with a double circle border. For 3 qubits it would be a triple circle edge and so on.
Example 5, Own visualization based on [1].
Example 6 serves to illustrate the representation with multiple qubit entanglement.
Example 6
5. Quantum Technologies
What do single-qubit operations (quantum gates) of a quantum computer look like in this representation?
X-Gate
In quantum computing, the X-gate is equivalent to the classical NOT-gate. If an X-gate is applied to one of the basis states I0> or I1>, the state is negated. Formally, this means:
X|0⟩ = |1⟩
X|1⟩ = |0⟩
If you start with a circle that has a color component of yellow and blue (0 and 1), these components swap after an X-gate.
X
Example 7, Own visualization based on [1].
Z-Gate
When a Z-gate is applied to a qubit, there is a phase shift of the |1⟩ state by 180 degrees. Formally, this means:
Z|0⟩ = |0⟩
Z|1⟩ = -|1⟩
Z
Example 8, Own visualization based on [1].
If you start with a superposition state as in example 8, the superposition state remains, but the orientation of the arrow changes.
Hadamard Gate (H-Gate)
The Hadamard gate is often used to generate superpositions. If a qubit is in the state |0⟩ and the H-gate is applied, we formally get:
H|0⟩ = 1/√2 (|0⟩ + |1⟩)
This means that the qubit is in a superposition of the states |0⟩ and |1⟩. The probability of measuring the state |0⟩ or |1⟩ is 50%. By Qake Model ist looks like this:
H
Example 9, Own visualization based on [1].
All visualizations in this work were independently created, inspired by [1].
This work is licensed under CC BY
Images: Illustration based on original Donhauser et al. (2022)
Munich Quantum Valley
MAX-PLANCK-INSTITUT FÜR QUANTENOPTIK
MCQST
PhotonLab

3. Study 2: Learning quantum properties with informationally redundant external representations: An eye-tracking study

Eva Rexigel[+] * *Linda Qerimi*[+] * *Jonas Bley* * *Stefan Küchemann* * *Sarah Malone* * *Jochen Kuhn*

[+]Authors contributed equally to this work

Contribution:

Rexigel designed the study, Qerimi and Rexigel developed the questionnaires and collected the data, Rexigel carried out mainly the analyses within the manuscript, with the support of Qerimi., Rexigel and Qerimi interpreted the data, Qerimi wrote mainly the first draft of the manuscript, with the support of Rexigel. All authors reviewed and edited the manuscript. Kuhn supervised the study. All authors have read and agreed to the submitted version of the manuscript.

Reference: Rexigel, E., Qerimi, L., Bley, J., Malone, S., Küchemann, S., & Kuhn, J. (2025). Learning quantum properties with informationally redundant external representations: An eye-tracking study. *arXiv preprint arXiv:2501.07389*.

Learning quantum properties with informationally redundant external representations: An eye-tracking study

Eva Rexigel*
Department of Physics and State Research Center OPTIMAS, University of Kaiserslautern-Landau, Erwin-Schroedinger-Str. 46, 67663 Kaiserslautern, Germany

Linda Qerimi*
Chair of Physics Education, Faculty of Physics, Ludwig-Maximilians-Universität München, Geschwister-Scholl-Platz 1, 80539 Munich, Germany, and Munich Quantum Valley (MQV), Max Planck Institute of Quantum Optics (MPQ), Germany

Jonas Bley
Department of Physics and State Research Center OPTIMAS, University of Kaiserslautern - Landau, Erwin-Schroedinger-Str. 46, 67663 Kaiserslautern, Germany

Sarah Malone
Department of Education, Saarland University, Campus A4 2, 66123 Saarbrücken, Germany

Stefan Küchemann and Jochen Kuhn
Chair of Physics Education, Faculty of Physics, Ludwig-Maximilians-Universität München, Geschwister-Scholl-Platz 1, 80539 Munich, Germany

Recent research indicates that the use of Multiple External Representations (MERs) has the potential to support learning, especially in complex scientific areas, such as quantum physics. In particular, the provision of informationally redundant external representations can have advantageous effects on learning outcomes. This is of special relevance for quantum education, where various external representations are available and their effective use is recognised as crucial to student learning. However, research on the effects of informationally redundant external representations in quantum learning is limited. The present study aims to contribute to the development of effective learning materials by investigating the effects of informationally redundant external representations on students' learning of quantum physics. Using a between-subjects design, 113 students were randomly assigned to one of four conditions. The control group learnt with a traditional multimedia learning unit on the behaviour of a single photon in a Mach-Zehnder interferometer. The three intervention groups received redundant essential information in the Dirac formalism, the Bloch sphere, or both. The use of eye tracking enabled insight into the learning process depending on the external representations provided. While the results indicate no effect of the study condition on learning outcomes (content knowledge and cognitive load), the analysis of visual behaviour reveals decreased learning efficiency with the addition of the Bloch sphere to the multimedia learning unit. The results are discussed based on current insight in learning with MERs. The study emphasises the need for careful instructional design to balance the associated cognitive load when learning with informationally redundant external representations.

INTRODUCTION

Background and Motivation

Science education, particularly in the domain of physics, is characterised by the effective use of external representations. These may include textual descriptions, equations and formulas, diagrams, and graphs, or educators' explanations. This is especially the case for complex physics concepts, such as those encountered in the context of quantum physics, a field characterised by its abstract principles and counterintuitive phenomena. In such cases, external representations play a crucial role in the communication and education of the subject [1]. It has been shown that quantum education based on classical analogies often leads to conceptual difficulties [2]. Consequently, the judicious use of external representations in quantum physics education is essential to prevent misconceptions and facilitate a more profound understanding of quantum phenomena. Across a range of science, technology, engineering and mathematics (Science, Technology, Engineering, & Mathematics (STEM)) disciplines, the use of multiple external representations (MERs) has been evidenced as an effective tool for fostering student learning (for an overview, see [3, 4]). This is particularly the case in contexts characterised by high complexity [5]. Consequently, it may also prove to be a valuable method for assisting students in the effective acquisition of quantum concepts.

Understanding key concepts in quantum physics has become increasingly important in recent decades as the relevance of quantum technologies has grown [6]. With its rapidly developing pillars of quantum communication, quantum computation, quantum simulation, and quantum sensing, a particular focus is placed on two-state systems, commonly referred to as qubits. The introduction of a qubit and its quantum properties has been demonstrated to be an effective method to introduce the fundamental principles of quantum physics and quantum technologies [7, 8]. The behaviour of a single photon in a Mach-Zehnder interferometer (Mach-Zehnder Interferometer (MZI)) is a common experimental approach that provides a valuable foundation to understand fundamental quantum concepts [9]. Studies show that the MZI is a helpful tool to reduce comprehension difficulties and improve students' understanding of wave-particle duality and the probabilistic nature of quantum measurement, as it demonstrates the principles of quantum mechanics in a tangible experimental setting [9]. Despite the introduction of a variety of teaching strategies in quantum physics in recent years [10], quantum physics concepts continue to present a considerable challenge to learners across different levels of education and academic backgrounds [11, 12]. Indeed, previous research indicates that developing a comprehensive understanding of quantum physics requires a substantial shift in perspective, diverging from classical concepts, which often leads to misconceptions [1].

The use of MERs with shared information enables different representations of the same essential information. It is important to emphasize that informational redundancy in this context refers specifically to the essential information aligned with the learning objective. For example, the understanding that the general state of a qubit is a superposition of two basis states can be conveyed either through Dirac notation or via a graphical representation such as the Bloch sphere. Both external representations hold the essential information that the general state of a qubit is a superposition of two basis states, yet they differ in the additional information they convey and the manner in which it is presented. These differences provide learners with complementary perspectives on the concept and can support learning by engaging distinct cognitive processes [13].

The aim of this study is to investigate the potential of MERs, particularly those that are informationally redundant, to facilitate the learning of fundamental quantum properties, illustrated by the single-photon behaviour in a MZI.

Learning with MERs

The acquisition of scientific knowledge is contingent on the use of suitable external representations. These serve as the foundations for effective communication, allowing us to convey information in a multitude of formats tailored to the specific requirements of the situation. It is generally accepted that there is a distinction to be made between symbolic representations, encompassing text, equation, and formula, and graphical representations, which include, for example, diagram and graph [14, 15]. In contrast to symbolic external representations, which are based on symbols that bear no direct resemblance to the referent, graphical external representations are based on icons that share structural characteristics with the referent, such as similarity in shape or form [15].

Current research indicates that the use of MERs has the potential to facilitate learning in different STEM contexts, in contrast to the use of a single external representation citeAinsworth.2021. In this context, a notable focus has been on the advantages of learning with text and pictures, known as multimedia learning, compared to learning through text alone [16]. The beneficial effect of combining text and pictures, as opposed to text alone, is commonly referred to as the multimedia principle [17]. According to cognitive theories such as the Cognitive Theory of Multimedia Learning (CTML) and the Integrated Theory of Text and Picture Comprehension (ITPC), the multimedia principle can be explained by a more efficient use of cognitive resources due to the dual structure of sensory memory and working memory, which allows for parallel processing of symbolic and graphical structures [15, 18]. In line with the CTML, the cognitive process of learning with MERs consists of three fundamental stages, including selection, organisation, and integration processes [18]. First, learners must select relevant information encoded in the external representations provided. Second, they need to organise the relevant information into mental structures. Third, learners must use these mental structures to build a comprehensive mental model by combining them with existing knowledge retrieved from long-term memory. The benefits of MERs have also been identified for various combinations of symbolic and graphical representations [19]. In particular, recent research has shown that the advantages of MERs are not limited to heterogeneous combinations of symbolic and graphical external representations. In fact, they can also be detected at a similar level in homogeneous combinations of multiple symbolic external representations [20, 21]. The ITPC, developed by Schnotz and Bannert, complements this perspective by highlighting the importance of semantic coherence between representations [22]. According to the ITPC, graphical representations only have the potential to facilitate learning only if they are semantically aligned with the accompanying symbolic representation(s) and do not contain any contradictory information [23, 24].

In addition to the cognitive theories of multimedia learning, the Design, Functions, and Tasks (DeFT) framework defines three main functions that MERs can fulfil to support learning [3, 13]. Regardless of the specific types of external representations combined, MERs can facilitate learning by complementing each other, constraining each other, or constructing a deeper understanding [13]. In doing so, external representations can complement each other, either through information or through cognitive processes induced by the different representation of information. They can constrain cognitive processing by focusing attention on relevant aspects. Finally, they can construct deeper understanding by allowing learners to integrate information from different sources of information [13].

The DeFT framework provides explanations for the learning effectiveness of various combinations of external representations, particularly for learning with informationally redundant representations. Providing MERs with shared information has the potential to support learners by inducing different cognitive processes and thus providing different access to the essential information [13]. In their recent meta-analysis, [19] found that the provision of additional informationally redundant external representations has the potential to help students use cognitive resources more efficiently without providing additional essential information. As a possible explanation for the beneficial effects of a higher number of MERs with shared information, the authors suggest that additional informationally redundant external representations increase the options for choosing the most appropriate external representation [19]. However, in order to benefit from multiple sources of the same information, learners need representational competence [25, 26]. According to [25] representational competence covers three areas of expertise. First, conceptual competencies are needed, including visual understanding of each external representation and connectional understanding of how the representations relate to each other. Second, learners need perceptual competencies to be able to apply visual and connectional understanding fluently. The third area of competence is given by meta-representational competencies, including the ability to choose an appropriate external representation based on the learning setting and personal characteristics [25].

Despite the potential advantages of MERs with shared information, previous research has also revealed instances where the provision of multiple informationally redundant representations hinders learning. According to the redundancy principle in its traditional form, learning with pictures and spoken text is more beneficial to learning than the additional presentation of printed text [27]. Based on the most prominent version of the Cognitive Load Theory (CLT), cognitive load when learning can be categorised in extraneous cognitive load (Extraneous Cognitive Load (ECL)), intrinsic cognitive load (Intrinsic Cognitive Load (ICL)), and germane cognitive load (Germane Cognitive Load (GCL)). Extraneous cognitive load is the result of the learner's interaction with elements introduced by the instructional design and should be reduced when learning with MERs to support learning [28]. In contrast, ICL is the result of the learner's interaction with those elements that are intrinsic to the task and must be processed in parallel. Finally, GCL is determined by the amount of cognitive resources allocated to ICL rather than ECL [28]. Various approaches exist for measuring cognitive load. However, in the context of multimedia learning, subjective rating scales are most commonly used [29]. Although such scales are influenced by retrospective self-assessment and individual self-concept [30], several instruments have been developed and validated in recent years to provide reliable instruments for assessing ECL, ICL and GCL separately in diverse educational settings (for an overview, see [29]).

The CLT provides an explanatory approach for the redundancy principle. Each external representation provided to learners constitutes an additional source of information that needs to be processed and coordinated, resulting in an increase in ECL [31]. In line with this, avoiding informationally redundant external representations frees cognitive resources for learning [31]. However, it has been shown that the learner characteristics play an important role in the effectiveness of redundant external representations [32]. While the presentation of redundant information in different forms may be valuable for novices in providing different accesses to the relevant information, this advantage may diminish with increasing expertise. This is because the additional external presentation does not add value, but only increases ECL. This constitutes the expertise-reversal principle [32, 33].

Thus, previous research both supports advantageous effects of learning with multiple informationally redundant external representations [13, 19] and disadvantageous effects [31].

The relevance of Dirac notation and the Bloch sphere in Quantum Education

Especially in the field of quantum technology education the Bloch sphere and Dirac notation have been identified as external representations with high relevance [4]. In educational contexts, conceptual advantages of the Dirac notation were recently discussed [34], with the results suggesting that the use of the Dirac notation facilitates the sensemaking of mathematics (probability rule, superpositions, orthogonality) and physics (connection to phenomena such as polarisation, measurements, and wave functions) and therefore acts as a bridge between mathematical structures and physical phenomena. The use of the Dirac notation has been shown to facilitate the understanding of intricate concepts in quantum mechanics [35, 36]. In particular, the Dirac notation provides a concise representation of eigenvalues and eigenstates, establishing a strong connection between mathematical and physical concepts.

While symbolic representations are often used in quantum education to formally explain quantum phenomena, graphical representations, such as the Bloch sphere, provide a vivid way to visualise and facilitate the understanding of quantum states [e.g., 37]. However, previous research has also revealed some difficulties in learning with the Bloch sphere. For example, students were found to have learning difficulties in constructing Bloch sphere states, understanding relative and global phases, and describing measurements when learning with the Bloch sphere [37]. Every dynamic of a quantum state can be interpreted in the Bloch sphere as a rotation of the state vector. When dynamic content is presented statically, learners have to perform cognitively demanding mental transformations that are closely related to spatial visualisation skills [38]. While learners with high spatial competencies are capable of executing such processes mentally, learners with low spatial competencies benefit more from external animations [38]. Consequently, learners with higher spatial competences, in particular those with superior mental rotation skills, may benefit more from the Bloch sphere than those with less developed mental rotation ability. Tests such as the RCube-Vis test [39] provide a differentiated measure of individual differences in mental rotation ability, while minimising the influence of other visual processing factors. In the RCube-Vis test, two static representations of a Rubik's Cube are presented simultaneously, one in a rotated position and the other solved. The participant has to decide whether the presented cubes can be transformed into each other. Similar to the rotation of the Bloch vector within the Bloch sphere, the individual layers of the cube must be mentally rotated.

Interaction of visual and cognitive processes in learning with MERs

The use of eye-tracking technology has proven to be a valuable tool in gaining insight into cognitive processing when learning with MERs [40, 41]. For example, Klein et al. (2020) found that eye tracking provides valuable insight into the cognitive processes involved in graph comprehension, revealing different visual attention patterns when students solve kinematics problems depending on their response accuracy and confidence [42]. According to the systematic review by Hahn and Klein (2022), the analysis of gaze transitions also provides valuable insight into how learners integrate different sources of information, revealing differences in cognitive processing and problem-solving strategies [43]. For instance, the number of transitions, defined as gaze shifts between defined areas of interest, such as different external representations, is a commonly used measure of learners' integration processes [40]. Current research suggests that the frequency of transitions reflects the degree of cognitive interplay between text and visualisations, with more transitions indicating active efforts to connect both sources [e.g., 44]. Canham and Hegarty (2010) showed that learners with higher prior knowledge focus their transitions on task-relevant features [45], while those with less knowledge may allocate their attention inefficiently. Similarly, Hannus and Hyönä (1999) found that high-achieving students made more targeted transitions between text and illustrations in science textbooks than low-achieving students [46], highlighting the importance of deliberate gaze shifts for effective comprehension. In addition, transitions can be influenced by design features. Visually salient or cued elements tend to attract attention and promote smoother transitions between different components of the material [47].

Research Questions

Learning quantum physics is particularly challenging due to its abstract and counterintuitive nature. Current research suggests that the use of MERs with shared information may be an effective way of supporting learning through a more efficient use of cognitive resources compared to learning with a single one [19]. However, it is not clear whether integration processes are responsible for this advantage or the fact that learners have the opportunity to choose the most appropriate external representation as opposed to learning with an individual representation. For example, [20] showed that the number of transitions between heterogeneous combinations of text and picture was higher than the number of transitions between homogeneous symbolic combinations of text and equation. This could suggest that in the case of heterogeneous combinations of symbolic and graphical representations, integration processes are more likely to provide advantages of MERs and, in the case of homogeneous combinations, the possibility of choosing an appropriate one. In light of the previous considerations, we investigate three research questions:

RQ1: Does adding an information-redundant symbolic-mathematical or graphical geometric representation to a multimedia learning unit enhance learning (content knowledge and cognitive load) of quantum properties?

RQ2: Does the integration of both informationally redundant representations additionally promote learning?

RQ3: Are advantages in learning with information-redundant representations correlated with visual integration processes across representations or rather the selection of one preferred representation?

This study was preregistered on the Open Science Framework (OSF) to ensure transparency and rigour [48].

METHODS

Participants

A total of 113 students from three German universities (RPTU Kaiserslautern-Landau, Ludwig-Maximilians-Universität München, and Saarland University) participated in the study. Participants were selected from a variety of fields related to STEM and randomly assigned to one of four groups: the control group ($N = 28$), the intervention group IG1 ($N = 28$), the intervention group IG2 ($N = 28$) or the intervention group IG3 ($N = 29$). A detailed overview of the number of participants in each group according to the field of study can be found in Table I. Three participants did not specify their field of study. In total, 71 men and 40 women were involved in the study. Two participants declined to specify their gender.

Discipline	CG	IG1	IG2	IG3
Physics	18	17	15	16
Mathematics	1	1	0	2
Biology	2	1	4	3
Biophysics	1	0	0	1
Business	1	0	0	0
Chemistry	0	1	1	0
Education	0	6	4	7
Engineering	3	1	1	0
Pharmacy	1	1	0	0
NA	1	0	2	0

Table I: Number of participants in each of the four study conditions (CG, IG1, IG2, IG3) depending on the stated field of study.

Study Design and Procedure

The study employed a between-subjects design with a 2×2 factorial structure. Each participant was randomly assigned to one of four study conditions. All participants were individually presented with the same multimedia learning unit, which consisted of complementary text and image elements that provided non-redundant information. This baseline unit was identical for all groups. The participants' visual behaviour was recorded using a Tobii Pro Nano eye tracker during the learning unit. A nine-point calibration was performed immediately prior to the start of the learning unit to ensure data accuracy.

Two factors were manipulated:

1. The presence or absence of an additional graphic-geometric representation (Bloch sphere) that provided redundant information to the text (factor 1: graphic-geometric representation present vs. absent).
2. The presence or absence of an additional symbolic-mathematical representation (equation) that was also informationally redundant to the text (factor 2: symbolic-mathematical representation present vs. absent).

This design resulted in four experimental groups:

- A control group (CG) that received only the baseline multimedia unit without any additional redundant representations,
- Intervention Group 1 (IG1), which received the baseline unit plus a graphic-geometric representation,
- Intervention Group 2 (IG2), which received the baseline unit plus a symbolic-mathematical representation, and
- Intervention Group 3 (IG3), which received the baseline unit plus both the additional graphic-geometrical and the symbolic-mathematical representation.

The entire study was conducted through digital means on a computer. The study procedure is described in Figure 1. In the following paragraphs, we will elucidate the individual stages and materials used in more detail.

Materials

Participants were first given an overview of the basic principles of physics as they relate to light, including a description of the properties of photons. In this regard, the authors designed and recorded a video for use in this study. The participants were permitted to pause, rewind, and fast-forward the video as often as they desired. The video itself did not make any reference to the Dirac formalism or the Bloch sphere. Similar to the first introductory video, the participants were presented with another pre-recorded video outlining the components of the MZI. This introduction encompassed the identification of each component and a description of its function within the interferometer.

After a general introduction to the subject, each participant was introduced to the external representations specific to their respective group. A brief introductory video was prepared for the Dirac formalism and the Bloch sphere, respectively, in which the method for describing a photon state with the respective external representation was outlined. As in the general introduction, participants were allowed to pause, rewind, and fast-forward as often as they wanted. Participants were instructed to

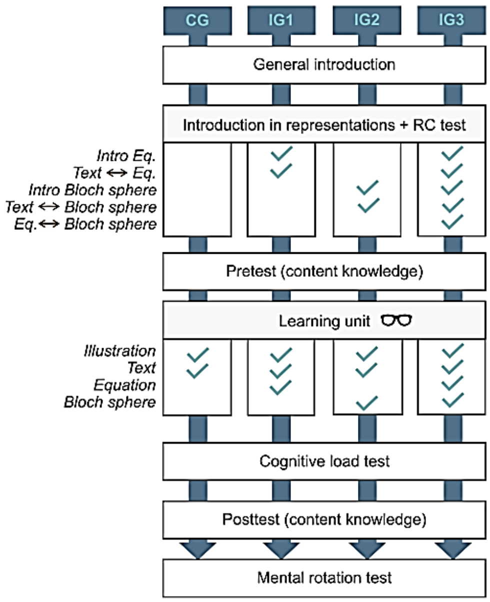


Figure 1: Illustration of the individual stages of the study four the four study conditions CG, IG1, IG2, and IG3. *Note:* Eq., Equation; RC, representational competence.

A photon is in the following state:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|O\rangle + |U\rangle\right)$$

Select the answer that describes the same state:

- The state of the photon is the basis state *up*.
- The state of the photon is an equal superposition of the basis states *up* and *down* with a relative phase of π.
- The state of the photon is the basis state *down*.
- The state of the photon is an equal superposition of the basis states *up* and *down* with a relative phase of 0.

Figure 2: Example item of the representational competence test for the translation between text and equation. Analogous items were used for the translation between text and Bloch sphere and equation and Bloch sphere.

move on to the test phase at their own discretion, ideally after feeling confident in their understanding of the external representation. In the representational competence test, the students were presented with a specific photon state represented in a given external representation and were asked to select the corresponding state in a sample of four presented in another external representation. Depending on the condition assigned, participants worked on different versions of the representational competence test (see Figure 1). The control group was not subjected to this phase of the study. Participants in IG1 completed the test for translations between equation and text and IG2 for translations between Bloch sphere and text. Participants in IG3 were asked for both translations between equation and text and Bloch sphere and text, as they were introduced to both additional external representations. In addition, IG3 was tasked with translating directly between equation and Bloch sphere. For each set of external representations, participants had to solve four equivalent tasks which differed only in the specific state present. An example task for translating between equation and text is provided in Figure 2.

As a third stage of the study, the prior content knowledge of the participants was evaluated. A total of five content-related multiple-choice items were selected from a questionnaire developed by [49] to assess students' use of quantum reasoning. The questionnaire was designed and validated to assess students' understanding of the core ideas of Probability, Superposition, and Interference (PSI) and has been developed specifically for high school and early undergraduate students (e.g., physics students in their first to third semester). Modifications were made to the items to align them with the formulations used in the study. The items and options were presented in a randomised sequence. In addition to solving the items, the students were asked to indicate their level of confidence in answering each item on a six-point Likert scale, ranging from "very unsure" to "very sure."

The learning unit comprised three consecutive stages, corresponding to the scenarios of a photon striking a beam splitter, the addition of a second beam splitter, and the measurement following the second beam splitter. For each stage, participants received a one-page study sheet tailored to their specific study group, with external representations adapted accordingly (see Figure 3). For each stage of the learning unit, participants were asked to answer two to three questions about the content presented in the corresponding material. The students were allowed to switch between the study material and the questions as often as they needed to complete the task. Across the conditions, the participants spent comparable time on the learning unit, with 12.03 minutes ($SD = 5.07$) in CG, 12.08 minutes ($SD = 4.49$) in IG1, 12.89 minutes ($SD = 5.10$) in IG2, and 13.01 minutes ($SD = 5.33$) in IG3.

The learning unit was presented on a 22-inch computer screen with a resolution of 1920×1080 pixels. To capture the visual attention of students during learning, their eye movements were recorded with a stationary Tobii Pro Nano eye tracker. Different types of eye movement (fixations and saccades) were identified using the Identification by Velocity Treshold (I-VT) algorithm with thresholds of $8500°/s^2$ for acceleration and $30°/s$ for velocity. A nine-point calibration was performed before the learning unit for each participant to ensure the accuracy of the detected data. If necessary, the calibration was repeated until it was deemed suitable.

The cognitive load of the participants was evaluated after the completion of the learning unit. For this purpose, the instrument developed by Klepsch et al. [50] was chosen, as it provides a validated measure of the different types of cognitive load (ICL, ECL, GCL) in a scale from 1 to 7 (disagree or fully agree). The test instrument used is developed and validated in the same language as that of the study participants (German), which was beneficial in its application. It consists of eight items designed to assess cognitive load during the learning unit on the basis of statements. Moreover, compared to other instruments, such as the one proposed by Leppink et al. [51], the items were found to align best with the learning context of our study. Subsequently, the content knowledge test based on [52] was conducted as a post-test. The test was identical to the one administered as a pretest, with the exception of a randomised order of items and answer options. The capacity for mental rotation was evaluated through the administration of the RCube-Vis test, as proposed by [39].

Data Analysis

Concerning RQ 1, we analysed the performance and cognitive load of the intervention groups IG1 and IG2 compared to the control group CG. The performance of each participant was measured in terms of the proportion of correctly solved items, both before and after the learning unit. The cognitive load was calculated on the basis of subjective ratings according to the dimensions of ECL, ICL, and GCL. To investigate possible differences in performance and cognitive load between the study conditions, we performed a multiple linear regression for each outcome measure, including the pretest accuracy and condition as independent variables.

In order to address RQ2, we also included IG3, receiving both additional external representations, in the respective multiple linear regressions for performance and cognitive load measures. To establish a linear relationship between each outcome and the condition variable, we transformed the four conditions (CG, IG1, IG2, and IG3) into dummy variables in ascending order according to their average scores on the respective outcome measure. As representational competence and mental rotation ability were considered potential influencing factors a priori, we subsequently analysed both variables to determine correlations with participants' performance and cognitive load. To this end, representational competence was defined as the proportion of correct responses on the representational competence test. The mean log-time for correct responses on the mental rotation test was used as a measure of students' mental rotation speed. In line with previous works [53, 54] participants with less than 70% correct answers were excluded from the analysis. In doing so, we ensure a reasonable level of accuracy to derive valid information about spatial ability from the time measure. Scatterplots were created to illustrate the relationship between the variables and each of the outcome measures. In order to enhance the robustness of the subsequent statistical analyses, the multiple linear regressions were extended to include the respective variable where feasible.

Third, to answer RQ 3, we performed an analysis of the visual behaviour exhibited by the students within the learning unit. In line with comparable studies in the research field, the areas of interest (AOIs) were designated for each external representation included in the learning unit, depending on the condition [40]. In the maximum case of condition IG_3, each slide of the learning unit comprised four pairs of AOIs, associated with the illustration, the text, the equation, and the Bloch sphere (see 3). We considered transitions between two AOIs of different external representations [40], while transitions between the both AOIs for one representation type were omitted. The raw data was detected using the software Tobii Pro Lab, and Python was employed for the identification of transitions, defined as shifts of fixations from one AOI to another. For this purpose, only fixations within the predefined AOIs were taken into account. The total number of transitions made by the students within the learning unit was analysed using a one-way analysis Analysis of Variance (ANOVA) with the condition (CG, IG1, IG2, IG3) as independent variable. Moreover, to gain further insight into the distribution of transitions contingent on the specific external representations incorporated into the material in the intervention groups, we conducted an unpaired-sample t-test to compare the relative number of transitions from and to the equation for IG1 and from and to the Bloch sphere for IG2. Similarly, we conducted a paired-samples t-test to compare the relative number of transitions for the two additional external representations in IG3. All statistical analyses were performed with RStudio, version 2023.06.0. Unless otherwise stated, the prerequisites for the respective statistical procedure were verified and found to be satisfied.

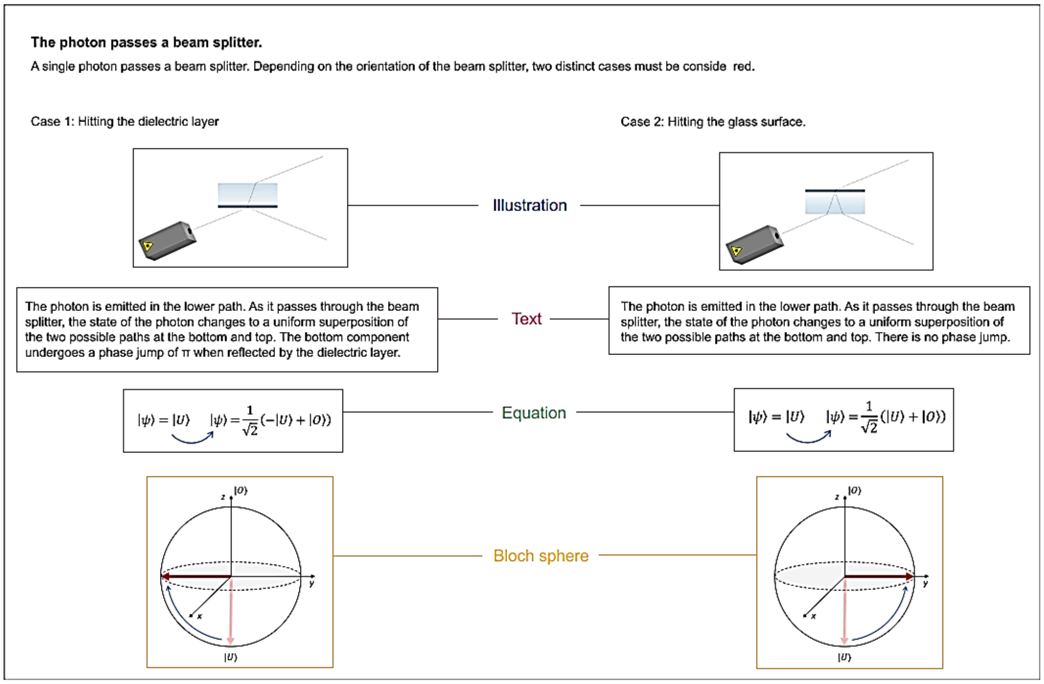


Figure 3: Study material for the initial stage of the learning unit, as presented to group IG3. In about half of the cases, the placement of the equation and the Bloch sphere was reversed. Depending on the study group, the Dirac formalism and/or the Bloch sphere were omitted. The areas of interest selected for the Eye Tracking (ET) analysis are highlighted in colour.
Note. The text was translated into English for publication, but the study used a German version.

RESULTS

Learning Effectiveness

An overview of the descriptive results for the pretest accuracy, posttest accuracy, and the cognitive load, in terms of ECL, ICL, and GCL, is presented in Figure 4 for each of the four conditions involved in the study. To identify potential differences in student learning across the four conditions, we performed a multiple linear regression analysis for each outcome measure, including the condition and the pretest accuracy as independent variables. The results indicated an overall effect for the accuracy post ($F(4, 108) = 14.2$, $p < .001^{***}$, $R^2 = 0.345$, $R^2_{adj} = 0.320$) and the ICL ($F(4, 108) = 4.525$, $p < .001^{***}$, $R^2 = 0.144$, $R^2_{adj} = 0.112$). In contrast, no significant overall effect could be identified for the ECL ($F(4, 108) = 1.165$, $p = .33$, $R^2 = 0.041$, $R^2_{adj} = 0.006$) and the GCL ($F(4, 108) = 1.547$, $p = .19$, $R^2 = 0.054$, $R^2_{adj} = 0.019$). The results for each independent variable in the statistically significant outcomes of the accuracy post and ICL visualized in Figure 4 are presented in Table II.

In order to increase the robustness of the previous analyses, we analysed the effect of participants' representational competence in the external representations relevant for the respective intervention group, as well as their mental rotation ability for participants learning with the Bloch sphere. The findings revealed that the participants demonstrated notably strong performance in the representational competence test. Based on the 49 data sets available for IG2 and IG3 ($M = 0.911$, $SD = 0.167$), it was observed that 71.43% of the participants attained the maximum score, indicating a high level of proficiency in the external representations provided. Due to the ceiling effect, the data proved to be unsuitable for identifying potential correlations. Furthermore, we conducted scatterplots to illustrate the relationship between the mental rotation ability of participants in IG2 and IG3, learning with the Bloch sphere, and each of the outcome measures

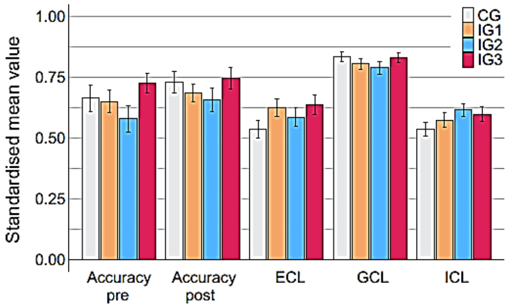


Figure 4: Standardized mean values for the accuracy pre, the accuracy post, ECL, GCL, and ICL for the study conditions CG, IG1 and IG2 (28 participants each) and IG3 (29 participants). The error bars represent one standard error.

	β	SE	t	p
Accuracy post				
Intercept (IG2)	1.798	0.273	6.598	$< .001^{***}$
CG	0.137	0.257	0.532	.60
IG1	-0.041	0.256	-0.159	.87
IG3	0.064	0.258	0.248	.80
Accuracy pre	0.514	0.071	7.284	$< .001^{***}$
ICL				
Intercept (CG)	4.601	0.299	15.365	$< .001^{***}$
IG1	0.250	0.258	0.966	.34
IG2	0.444	0.260	1.706	.09
IG3	0.516	0.257	2.010	$.05^{*}$
Accuracy pre	-0.256	0.071	-3,587	$< .001^{***}$

Table II: Individual results for the coefficients of the conditions (CG, IG1, IG2, and IG3) and the pretest accuracy (accuracy pre) of the multiple linear regression for the outcome measures of accuracy post, as well as the ICL. $^{*}p < .05$, $^{***}p < .001$

.

(see Figure 5).

To analyse possible correlations between mental rotation ability and learning outcomes when learning with the Bloch sphere, we performed an extended multiple linear regression for each outcome measure, i.e. accuracy post, ICL, ECL and GCL, based on the data of the 44 participants included in the analysis. In doing so, we included the learners' average log time for correct answers in the R-Cube-Vis test as an additional independent variable to the pretest accuracy. The analysis yielded a significant overall effect for the accuracy post ($F(2, 41) = 18.01$, $p < .001^{***}$, $R^2 = 0.468$, $R^2_{adj} = 0.442$) and ICL ($F(2, 41) = 3.356$, $p = .04^{*}$, $R^2 = 0.141$, $R^2_{adj} = 0.099$). However, no significant correlation was identified between mental rotation ability scores and the precision of either of the two outcome measures accuracy post ($\beta = 0.081$, $SE = 0.087$, $t = 0.924$, $p = .36$) and ICL ($\beta = -0.083$, $SE = 0.069$, $t = -1.207$, $p = .23$) (see Table III). Similarly to the basis regression, the

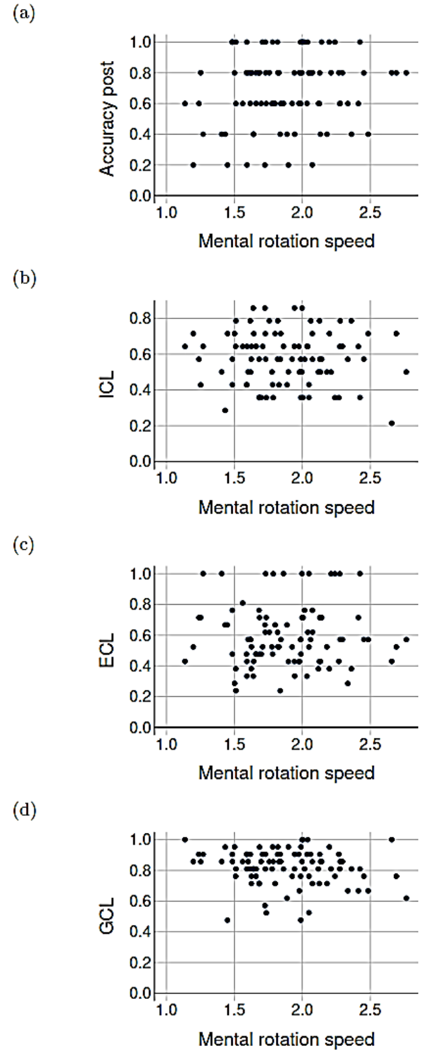


Figure 5: Scatterplots of the mental rotation ability and (a) the accuracy post (b) the ICL (c) the ECL, and (d) the GCL based on the data from 44 participants from IG2 and IG3.

overall effect for the outcomes of ECL and GCL could not be determined to be statistically significant (ECL: $F(2,41) = 0.411,\ p = .67,\ R^2 = 0.020, R^2_{adj} = -0.028$, GCL: $F(2,41) = 2.516,\ p = .09,\ R^2 = 0.109,\ R^2_{adj} = 0.066$).

	β	*SE*	t	p
Accuracy post				
Intercept	0.137	0.176	0.778	.44
Accuracy pre	0.617	0.105	5.902	$< .001^{***}$
MR speed	0.081	0.087	0.924	.36
ICL				
Intercept	0.874	0.139	6.302	$< .001^{***}$
Accuracy pre	−0.186	0.082	−2.259	$.029^{*}$
MR speed	−0.083	0.069	−1.207	.23

Table III: Summary of regression coefficients of the multiple linear regression for Accuracy pre and MR speed (mental rotation speed) for the outcome measures Accuracy post and ICL, for which significant overall effects were found. No significant effects of mental rotation speed were observed. $^{*}p < .05$, $^{***}p < .001$

Visual Behaviour

The descriptive results for the total number of transitions k_{tot} are presented in Table IV. The one-way ANOVA with the condition (CG, IG1, IG2, IG3) as the independent variable and the total number of transitions as the dependent variable yielded a significant overall effect $F(3,94) = 8.802,\ p < .001^{***}$. The results of the subsequent pairwise t-tests with Bonferroni correction are presented in Table V, including the t-value t, degree of freedom df, the Bonferroni corrected p-value p_{adj} and the effect size Cohen's d for each t-test.

Condition	N	k_{tot}	*SD*
CG	24	60.58	41.38
IG1	23	79.22	39.52
IG2	22	107.59	58.07
IG3	25	122.72	44.48

Table IV: Descriptive data of the number of participants N, the mean total number of transitions k_{tot} and the standard deviation *SD* for each of the four conditions.

We calculated the relative number of transitions from and to the equation $k_{rel,eq}$ for IG1 and the Bloch sphere $k_{rel,b}$ for IG2. Moreover, we calculated the relative number of transitions for the two additional external representations in IG3. The results are presented in Table VI. Since IG3 was presented with the equation and the Bloch sphere at the same time, transitions between the additional external representations are included in both $k_{rel,eq}$ and $k_{rel,b}$. Therefore, for IG3, the sum of $k_{rel,eq}$ and $k_{rel,b}$ is not necessarily equal to the total number of transitions made by the participants. The unpaired t-test for the intervention groups IG1 and IG2 did not yield statistically significant differences for $k_{rel,eq}$ and $k_{rel,b}$ ($t(44) = 1.74,\ p = .09$). Furthermore, the corresponding paired sample t-test for the intervention group IG3 did not reveal significant differences ($t(24) = -0.81,\ p = .42$).

Conditions	t	*df*	p_{adj}	d
IG1 vs. CG	1.58	45	.73	0.46
IG2 vs. CG	3.18	44	$.02^{*}$	0.94
IG3 vs. CG	5.06	47	$< .001^{***}$	1.45
IG2 vs. IG1	1.92	43	.37	0.57
IG3 vs. IG1	3.57	46	$.01^{**}$	1.03
IG3 vs. IG2	1.01	45	1.00	0.30

Table V: Results of the pairwise t-tests for the total number of transitions k_{tot} between the four conditions CG, IG1, IG2, and IG3 with Bonferroni corrected p-values p_{adj} and effect size Cohen's d. $^{**}p < .01$, $^{***}p < .001$.

Condition	N	$k_{rel,eq}$	$k_{rel,b}$	*SD*
IG1	23	0.54	–	0.17
IG2	22	–	0.46	0.15
IG3	25	0.48	0.52	0.14

Table VI: Overview over the number of participants N, the relative number of transitions from and to the equation $k_{rel,eq}$ and the Bloch sphere $k_{rel,b}$, and the standard deviation *SD* for each of the intervention groups.

DISCUSSION

The objective of this study was to investigate the effects of extending a multimedia learning unit with additional symbolic external representation, specifically equations expressed in the Dirac formalism, or a graphical representation, namely the Bloch sphere, on students learning of quantum properties. In particular, both additional external representations are redundant in terms of the relevant information content, given the multimedia basis of the text and illustration.

Learning Effectiveness

In regard to RQ1, no significant effects on students' content knowledge could be detected when learning with the additional symbolic external representation or when provided with the additional graphical external representation, in comparison to the basis multimedia unit. Contrary to previous results and assumptions [19], the provision of more informational redundant external representations was not associated with better learning outcomes. Similarly, students enrolled in IG1, who received additional instruction through equations, and students enrolled in IG2, who received additional instruction through the Bloch sphere, exhibited comparable cognitive load (as indicated by ICL, ECL and GCL) to that observed in the CG, who were provided with the fundamental multimedia setting alone. Consequently, with regard to RQ1, providing students with an additional symbolic or graphical external representation did not result in discernible improvements in content knowledge or cognitive load. Therefore, the findings of this study do not support the conclusions of previous research in other contexts [e.g., 20], proposing a possible advantage of learning with MERs with shared essential information. According to the DeFT framework, MERs with shared essential information have the potential to improve learning outcomes by prompting different cognitive processes or providing the opportunity to choose the external representation most appropriate for learning [3], especially in settings of more than two external representations [19]. Despite the fact that the vast majority of the participants demonstrated a high level of proficiency in using the external representations provided, as evidenced by their notable achievements in the representational competence test, the findings suggest that the learners in this study did not realise the potential benefits of learning with multiple informational redundant external representations. However, the incorporation of additional redundant representations did neither result in a decline in students' learning outcomes, as measured by content knowledge and cognitive load. Consequently, the analysed outcomes do not provide clear support for the preceding research that indicated positive effects of the incorporation of redundant external representations in students' learning [e.g, 3, 19], or negative effects [31]. One explanation for the absence of observed effects might be found in the measurement tools used to assess content knowledge and cognitive load. While both instruments employed in this study are validated, it is possible that they lack sufficient sensitivity to detect differences in the given context. An alternative explanation may be found in a more intricate interaction of student characteristics and the effect of redundant external representations on learning quantum properties.

In order to account for possible influencing factors of the findings, especially for students learning with the additional Bloch sphere, data were collected about students' representational competence and mental rotation ability. The participants obtained commendable results in the representational competence test, suggesting a high level of proficiency in the use of the respective external representations that were presented. Consequently, it is not reasonable to assume that any potential benefits of the additional redundant representation would be outweighed by inadequate representational competencies. Considering the potential impact of mental rotation ability on the efficacy of learning using the Bloch sphere, the absence of a significant correlation between mental rotation ability and either performance or cognitive load indicates that, if such effects exist, they are overshadowed by the influence of prior knowledge on the learning outcome. A possible explanation for the lack of effects of mental rotation ability on learning with the Bloch sphere might also be that the learning tasks used in our study did not require continuous or complex spatial transformations. It is possible that the learners have relied more on conceptual or symbolic strategies. In addition, didactic support is provided by the clear display of the directions of vector rotation. This may have reduced the need for high mental rotation ability, thus diminishing the predictive power of individual differences in spatial ability in this context. Nevertheless, given the limited number of participants, particularly with regard to their mental rotation ability ($k = 45$), and the consequent limited statistical power, it is possible that some statistically significant results may have been missed.

With regard to RQ2, we also investigated the potential impact of incorporating both informationally redundant external representations into the multimedia learning unit (IG3). As in the intervention groups IG1 and IG2, who received one of the two additional external representations, the presentation of the equation and the Bloch sphere did not result in an improved knowledge of the content. However, students who learnt with the maximum combination of four external representations demonstrated an increased ICL. Following the CLT [55] and the CTML [18] the results imply that the addition of MERs with informational redundancy leads to enhanced element interactivity and, correspondingly, enhanced essential processing. According to Mayer's definition, learning with both additional external representations is associated with greater cognitive processing in order to represent the essential information in working memory [18]. As IG3 did not result in an enhancement of content knowledge, the findings indicate that the provision of supplementary external representations induced students to perceive the learning content as more complex and challenging, with no evident advantages in content knowledge.

In consideration of RQ1 and RQ2, the provision of an additional informationally redundant symbolic or/and graphical external representation was not associated with advanced learning outcomes.

Visual Behaviour and Learning Effectiveness

The analysis of the learning outcome in relation to the presence of additional informationally redundant external representations indicated that there was no discernible impact on students' content knowledge when learning with a multimedia learning unit. However, the analysis of the cognitive load of the students when learning indicated that the participants in IG3, who received the maximum set of four external representations, experienced a higher level of ICL than the participants in CG, who learnt in the basic multimedia setting with two complementary external representations. This suggests that, although there were no differences in final content knowledge, the additional external representations may have prompted the use of different learning strategies. To gain insight into the learning processes employed according to the study condition, we conducted an analysis of the visual behaviour exhibited by students during the learning process. In line with previous research, we analysed the total and relative number of transitions between external representations as an indicator of attempted integration processes [41].

A higher number of transitions between external representations can be related to students' learning outcome in different ways. Research has indicated that an increased number of transitions is associated with better understanding and transfer performance when learning with MERs [e.g., 56, 57]. In other contexts, frequent transitions between external representations can also be indicative of processing difficulties and have a detrimental effect on learning success [40, 41]. Consequently, a high number of transitions may reflect successful integration processes or processing difficulties [41]. It is therefore essential to consider both the instructional design of external representations, individual learner characteristics and the learning outcome when interpreting transition frequency as a proxy for learning effectiveness.

The statistical analysis indicates that students demonstrated a higher total number of transitions between the external representations presented when the Bloch sphere was provided as an additional graphical external representation in the learning material. This was observed not only in IG2, who learnt only with the additional Bloch sphere, but also in IG3, who learnt both with the additional equation in the Dirac notation and the Bloch sphere. Given that an additional external representation, even if it does not provide any new information content, represents a further processing source, it is reasonable to expect an increase in integrations with more representations. However, the results indicate that the enhancement in transitions is only related to the presentation of the additional graphical external representation, not the symbolic one. Although the basic multimedia unit comprised a symbolic external representation (text) and a graphical one (illustration), the essential information about the quantum state in different phases when passing the MZI is conveyed by the text. Moreover, the text constitutes the informationally redundant reference representation. Therefore, redundant information is still presented in the homogeneous combination of text and equation for IG1. In contrast, the incorporation of the Bloch sphere results in the presentation of redundant information in the heterogeneous combination of text and Bloch sphere for IG2. Consequently, the increase in attempted integration behaviour exhibited by participants learning with the Bloch sphere is consistent with the findings of previous research [e.g., 20]. Here, a higher number of transitions was observed in heterogeneous combinations of MERs compared to homogeneous combinations comprising only symbolic external representations [20].

In line with the previous considerations, an increase in the number of transitions was not only observed when comparing CG, who received the basic multimedia setting, with IG2 or IG3, who received either the additional Bloch sphere (IG2) or additional equations using the Dirac formalism and the Bloch sphere (IG3). An increase in transitions was also detected when IG1, which received additional equations, was compared to IG3, where the Bloch sphere was added to the IG1 setting. While in IG1 the essential information regarding the basis state itself is provided by a homogeneous combination of text and equation, redundantly, the additional Bloch sphere in IG3 results in a presentation of redundant information across the heterogeneous combination of text, equation and Bloch sphere. Once more, the presentation of a heterogeneous combination of redundant representations, in this case given by text, equation and Bloch sphere, is associated with an increase in attempted integration processes, in line with previous research [20]. It can thus be concluded that in the present study the Bloch sphere plays a central role in the learning process, encouraging learners to proactively seek to connect information from different sources by facilitating the presentation of redundant information in heterogeneous external representations.

Interestingly, these increased transitions were not limited to the Bloch sphere itself with the other external representations presented, as indicated by the subsequent analysis of transitions to and from the additional external representation. When comparing IG1, receiving additional equations and IG2, receiving the additional Bloch sphere, similar relative numbers of transitions were found for each of the additional external representations. Similar findings were observed when the relative number of transitions from and to the equation and the Bloch sphere in group IG3 was considered. As a result, the provision of the Bloch sphere appears to encourage an increased level of attempted integration that encompasses all of the learning material. This could indicate an attempt to establish connections between the various external representations with the aim of developing a more comprehensive understanding. Although the integration of diverse external representations can be advantageous [25], the additional cognitive effort required did not result in improved learning outcomes. Consequently, the approach was not efficient in the context of this study. We found ceiling effects in the representational competence test, conducted previous to the learning unit. This suggests that learners were well-versed in handling the external representations used in the study. Consequently, the observed increase in transitions is unlikely to result from insufficient representational competence.

Another possible explanation for the observed cognitive processing differences might lie in the design of the graphical external representation itself. The Bloch sphere is not only based on icons, the fundamental unit of any graphical external representation [15]. It also incorporates symbolic elements to signify the fundamental states and the labelling of the axes. Thus, it combines properties of both graphical and symbolic representations, which are partly also found in the other external representations provided. In particular, the Bloch sphere encompasses the presentation of the two basis states in Dirac notation, as also included in the equation. Consequently, the additional equation may be regarded as a logical reference point, as it unifies the symbolic representation of the basis states in terms of the Dirac notation. It can thus be concluded that the promotion of unused cognitive processing may be attributed to the particular characteristics of the Bloch sphere, rather than being a phenomenon inherent to graphical external representations.

Limitations and Future Research

There are some limitations in our study that may serve as a starting point for further research. In the current study, the incorporation of a redundant graphical external representation, the Bloch sphere, was found to be associated with less efficient learning processes. Despite the lack of detected benefits in terms of content knowledge and cognitive load, it is possible that the test methods employed have failed to identify potential benefits of the Bloch sphere. For instance, it is conceivable that more profound integration processes may have led to the formation of more robust and connectible schemata, which were not detected by the outcome assessments used. Nevertheless, the eye-tracking analysis conducted proved to be highly sensitive, uncovering differences that a simple multiple-choice post-test would not have been able to detect. It may be advantageous for further research to focus on outcome measurements that are more sensitive, and to extend the scope of immediate performance assessments. For example, conceptual knowledge could be measured through open-ended explanations or concept-mapping tasks to assess a deeper understanding of the underlying principles. Transfer effects might be evaluated by examining how well learners apply acquired knowledge to new problems or different contexts. Additionally, follow-up tests, such as delayed assessments, could provide insight into the long-term retention and solidity of learning effects.

The ET analysis conducted may provide a foundation for subsequent fine-grained analyses of students' cognitive processes when learning with MERs in quantum education. The present study provides initial insights into the different visual processing of the Dirac formalism and the Bloch sphere in the given context. Further studies could focus on which elements of the external representations are relevant for the respective visual processing. For instance, subsequent studies could investigate which parts of the text precede or follow the transition to or from the equation and Bloch sphere. This approach may facilitate a more precise understanding of the relevant elements of the representations involved in the learning process.

To gain further insight into the generalisability of the findings, more research is required on different combinations of informationally redundant external representations. In particular, future studies could explore additional graphical external representations commonly used in quantum physics, such as Feynman diagrams [58] or recent external representations such as the Circle notation [59, 60]. Investigating these alternatives could help determine whether the observed facilitation of integration behaviour is specific to the Bloch sphere or reflects a more general phenomenon of heterogeneous MERs with shared information. At this point, it is unclear whether the different learning strategies associated with the additional symbolic and graphical external representation are a generalisable phenomenon across different types of external representations or whether they are triggered by individual characteristics of the Dirac formalism and the Bloch sphere. Future research should include different symbolic and graphical external representations to investigate whether the findings can be replicated.

Another limitation of our study is that most of the participants had a STEM background and were already accustomed to mathematical formulas as external representations in their studies, which may have influenced their perception and processing of these external representations. We did not detect an increased cognitive load associated with their use, which might be explained by the fact that STEM students are already familiar with this type of external representation from their studies. This familiarity could have mitigated the cognitive demands typically associated with the processing of complex symbolic external representations.

Furthermore, investigating the effects of Meta-Representational Competencies (MRC) when learning with informationally redundant external representations could be a valuable addition to future research. It could provide deeper insight into how learners choose and use external representations effectively. As diSessa (2004) states,

> *"MRC includes the ability to select, produce, and use external representations productively, as well as the ability to critique, modify, and even design entirely new representations." [61]*

Addressing MRC in future studies would allow a more nuanced understanding of the strategies associated with learning with redundant external representations and related learning outcomes.

CONCLUSION

This study provides initial insight into the role of redundant external representations in learning fundamental quantum concepts in the context of the MZI. It is among the first investigations into the use of MERs in this domain, particularly with regard to their effects on learning and cognitive processing. Consequently, the findings cannot yet be directly translated into concrete recommendations for teaching. However, one key observation is that adding one or more informationally redundant external representations to multimedia learning materials in the field of quantum properties does not necessarily lead to significant learning gains or losses.

Nevertheless, the inclusion of graphical-geometric external representations, such as the Bloch sphere, appears to encourage learners to attempt integration between different external representations. This is reflected in an increase in transition behaviour, which, in turn, results in higher intrinsic cognitive load (ICL). These findings align with prior research on MERs, which suggests that graphical external representations may facilitate cognitive integration, even if this does not directly translate into measurable learning benefits [15, 20].

Although this study does not yet allow definitive conclusions regarding practical applications, it demonstrates that the choice of external representations significantly influences how learners interact with the material. Further targeted research in quantum physics education with MERs is therefore warranted.

Practical Implication

The findings provide preliminary insights into how redundant external representations influence learning processes in complex domains such as quantum physics. Although no differences in learning outcomes were detected depending on the number and type of informationally redundant MERs included, differences in cognitive processing suggest that the design of instructional materials should carefully consider the role of additional external representations. In particular, the inefficient visual behaviour observed when learning with the Bloch sphere suggests that additional scaffolding or targeted cues may be necessary to help learners effectively integrate such external representations.

Key aspects to consider for the design of instructional material, especially in the context of quantum physics:

1. **Strategic integration of redundant external representations:** The use of additional external representations should be approached deliberately, balancing their potential to promote visual integration with their impact on cognitive load [3, 62].

2. **Developing representational competence:** Learning materials should not only support the understanding of individual external representations, but also help learners develop the ability to transition between different formats. Graphical external representations, such as the Bloch sphere, may foster these transitions. While this might not directly enhance content learning, it could contribute to representational fluency by facilitating students ability to connect MERs efficiently [25].

3. **Supporting learners in handling complex external representations**: The benefits of complex graphical external representations, such as the Bloch sphere, may only be fully realised if the learners receive adequate support. Scaffolding approaches, including guided instructions or structured tasks, could be beneficial in helping students navigate and integrate these external representations effectively [16].

We would like to thank Prof. Dr. Jürgen Eschner and Prof. Dr. Christoph Becher from the Saarland University for their support in the data collection. E.R. and J.B. acknowledge the support by the German Federal Ministry of Education and Research (BMBF) through the QuanTUK project. Furthermore, E.R. acknowledges the support by the European Union via the Interreg Oberrhein programme in the project Quantum Valley Oberrhein (UpQuantVal). This work was supported by LMUexcellent, funded by the Federal Ministry of Education and Research (BMBF) and the Free State of Bavaria under the Excellence Strategy of the Federal Government and the Länder.

* These authors contributed equally to this work.

[1] C. Singh and E. Marshman, Phys. Rev. Special Top Phys. Educ. Res. 11, 020117 (2015).

[2] T. Bouchée, L. de Putter-Smits, M. Thurlings, and B. Pepin, Studies in Science Education **58**, 183 (2022).
[3] S. Ainsworth, Learn Instr. **16**, 183 (2006).
[4] E. Rexigel, J. Bley, A. Arias, L. Qerimi, S. Küchemann, J. Kuhn, and A. Widera, EPJ Quantum Technology **12**, 22 (2025).
[5] S. Ainsworth, in *Visualization: Theory and Practice in Science Education*, Models and Modeling in Science Education, edited by J. K. Gilbert, M. Reiner, and M. Nakhleh (Springer Netherlands, s.l., 2008) pp. 191–208.
[6] A. Merzel, P. Bitzenbauer, K. Krijtenburg-Lewerissa, K. Stadermann, E. Andreotti, D. Anttila, M. Bondani, M. L. M. Chiofalo, S. Faletič, R. Frans, S. Goorney, F. Greinert, L. Jurčić, Z. Koupilová, M. Malgieri, R. Müller, P. Onorato, G. Pospiech, M. Ubben, A. Woitzik, and H. Pol, EPJ Quantum Technology **11**, 1 (2024).
[7] H. R. Sadaghiani and J. Munteanu, in *2015 Physics Education Research Conference Proceedings* (American Association of Physics Teachers, College Park, MD, 2015) pp. 287–290.
[8] W. Dür and S. Heusler, PhyDid A - Phys Didakt Sch Hochsch. **1**, 01 (2012), number: 11.
[9] E. Marshman and C. Singh, Phys. Rev. Phys. Educ. Res. **13**, 010117 (2017).
[10] A. Donhauser, P. Bitzenbauer, L. Qerimi, S. Heusler, S. Küchemann, J. Kuhn, and M. S. Ubben, Phys. Rev. Phys. Educ. Res. **20**, 020601 (2024).
[11] P. Bitzenbauer, J. M. Veith, B. Girnat, and J.-P. Meyn, Physics **4**, 1180 (2022).
[12] F. Hennig, K. Tóth, J. Veith, and P. Bitzenbauer, Physical Review Physics Education Research **20**, 020147 (2024), publisher: American Physical Society.
[13] S. Ainsworth, in *The Cambridge handbook of multimedia learning*, Cambridge Handbooks in Psychology, edited by R. E. Mayer and L. Fiorella (Cambridge University Press, New York, NY, 2021) pp. 158–170.
[14] W. Schnotz, in *Sprache, Sinn und Situation: Festschrift für Gert Rickheit zum 60. Geburtstag*, edited by L. Sichelschmidt and H. Strohner (Deutscher Universitätsverlag, Wiesbaden, 2001) pp. 43–57.
[15] W. Schnotz, in *The Cambridge Handbook of Multimedia Learning*, Cambridge Handbooks in Psychology, edited by R. E. Mayer and L. Fiorella (Cambridge University Press, Cambridge, 2021) pp. 82–99.
[16] R. E. Mayer and L. Fiorella, in *The Cambridge handbook of multimedia learning*, Cambridge Handbooks in Psychology, edited by R. E. Mayer and L. Fiorella (Cambridge University Press, New York, NY, 2021) pp. 3–16.
[17] R. E. Mayer, in *The Cambridge Handbook of Multimedia Learning*, Cambridge Handbooks in Psychology, edited by R. E. Mayer and L. Fiorella (Cambridge University Press, Cambridge, 2021) pp. 145–157.
[18] R. E. Mayer, in *The Cambridge Handbook of Multimedia Learning*, Cambridge Handbooks in Psychology, edited by R. E. Mayer and L. Fiorella (Cambridge University Press, Cambridge, 2021) pp. 57–72.
[19] E. Rexigel, J. Kuhn, S. Becker, and S. Malone, Educational Psychology Review **36**, 124 (2024).
[20] N. Ott, R. Brünken, M. Vogel, and S. Malone, Learn Instr. **58**, 88 (2018).
[21] S. Malone, K. Altmeyer, M. Vogel, and R. Brünken, Journal of Computer Assisted Learning **36**, 781 (2020).
[22] W. Schnotz and M. Bannert, Learning and instruction **13**, 141 (2003).
[23] W. Schnotz, The Cambridge handbook of multimedia learning **49**, 69 (2005).
[24] W. Schnotz, Sprache, Sinn und Situation: Festschrift für Gert Rickheit zum 60. Geburtstag , 43 (2001).
[25] M. A. Rau, Educ Psychol Rev **29**, 717 (2017).
[26] K. L. Daniel, C. J. Bucklin, E. Austin Leone, and J. Idema, in *Towards a Framework for Representational Competence in Science Education*, edited by K. L. Daniel (Springer International Publishing, Cham, 2018) pp. 3–11.
[27] L. Fiorella and R. E. Mayer, in *The Cambridge Handbook of Multimedia Learning*, Cambridge Handbooks in Psychology, edited by L. Fiorella and R. E. Mayer (Cambridge University Press, Cambridge, 2021) pp. 185–198.
[28] F. Paas and J. Sweller, in *The Cambridge handbook of multimedia learning*, Cambridge Handbooks in Psychology, edited by R. E. Mayer and L. Fiorella (Cambridge University Press, New York, NY, 2021) pp. 73–81.
[29] D. Mutlu-Bayraktar, V. Cosgun, and T. Altan, Computers & Education **141**, 103618 (2019).
[30] J. Sweller, P. Ayres, and S. Kalyuga, Measuring cognitive load, in *Cognitive Load Theory* (Springer New York, New York, NY, 2011) pp. 71–85.
[31] S. Kalyuga and J. Sweller, in *The Cambridge Handbook of Multimedia Learning*, Cambridge Handbooks in Psychology, edited by R. E. Mayer and L. Fiorella (Cambridge University Press, Cambridge, 2021) pp. 212–220.
[32] S. Kalyuga, in *The Cambridge handbook of multimedia learning*, Cambridge Handbooks in Psychology, edited by R. E. Mayer and L. Fiorella (Cambridge University Press, New York, NY, 2021) pp. 171–182.
[33] S. Kalyuga and A. Renkl, Instructional Science **38**, 209 (2010).
[34] A. Merzel, E. Y. Weissman, N. Katz, and I. Galili, Phys. Rev. Phys. Educ. Res. **20**, 020134 (2024).
[35] M. Wawro, K. Watson, and W. Christensen, Physical Review Physics Education Research **16**, 020112 (2020), publisher: American Physical Society.
[36] A. Merzel, E. Y. Weissman, N. Katz, and I. Galili, Physical Review Physics Education Research **20**, 020134 (2024), publisher: American Physical Society.
[37] P. Hu, Y. Li, R. S. K. Mong, and C. Singh, European Journal of Physics **45**, 025705 (2024), publisher: IOP Publishing.
[38] T. N. Höffler and D. Leutner, Computers in human behavior **27**, 209 (2011).
[39] B. C. O. F. Fehringer, Open Psychology **2**, 138 (2020).
[40] E. Alemdag and K. Cagiltay, Computers & Education **125**, 413 (2018).
[41] A. Coskun and K. Cagiltay, Journal of Computer Assisted Learning **38**, 581 (2022).
[42] P. Klein, A. Lichtenberger, S. Küchemann, S. Becker, M. Kekule, J. Viiri, C. Baadte, A. Vaterlaus, and J. Kuhn, European Journal of Physics **41**, 025701 (2020).
[43] L. Hahn and P. Klein, Physical Review Physics Education Research **18**, 013102 (2022).
[44] F. Schmidt-Weigand, A. Kohnert, and U. Glowalla, Learning and instruction **20**, 100 (2010).
[45] T. Rasch and W. Schnotz, Learning and Instruction **19**, 411 (2009).
[46] M. Hannus and J. Hyönä, Contemporary educational psychology **24**, 95 (1999).

[47] J. Hyönä, Learning and Instruction **20**, 172 (2010), eye tracking as a tool to study and enhance multimedia learning.
[48] E. Rexigel, L. Qerimi, S. Malone, J. Bley, S. Küchemann, and J. Kuhn, Learning Quantum Properties with Informationally Redundant Representations (2024).
[49] M. Waitzmann, R. Scholz, and S. Wessnigk, Physical Review Physics Education Research **20**, 010122 (2024).
[50] M. Klepsch, F. Schmitz, and T. Seufert, Frontiers in Psychology **8**, 10.3389/fpsyg.2017.01997 (2017), publisher: Frontiers.
[51] J. Leppink, F. Paas, C. P. M. van der Vleuten, T. van Gog, and J. J. G. van Merriënboer, Behavior research methods **45**, 1058 (2013).
[52] M. Waitzmann, *Wirkung eines quantenoptischen Realexperiments auf das physikalische Reasoning*, Ph.D. thesis, Hannover : Institutionelles Repositorium der Leibniz Universität Hannover (2023).
[53] P. Jansen, A. Schmelter, C. Quaiser-Pohl, S. Neuburger, and M. Heil, Cognitive Development **28**, 51 (2013).
[54] B. C. Fehringer, European Journal of Psychological Assessment (2021).
[55] J. Sweller, J. J. G. van Merriënboer, and F. Paas, Educ Psychol Rev **31**, 261 (2019).
[56] M. Kragten, W. Admiraal, and G. Rijlaarsdam, International Journal of Science Education **37**, 1915 (2015).
[57] P. A. O'Keefe, S. M. Letourneau, B. D. Homer, R. N. Schwartz, and J. L. Plass, Computers in Human Behavior **35**, 234 (2014).
[58] R. P. Feynman, *QED: the strange theory of light and matter* (Princeton, NJ: Princeton University Press, 2006).
[59] E. R. Johnston, N. Harrigan, and M. Gimeno-Segovia, *Programming quantum computers: essential algorithms and code samples* (Sebastopol, CA: O'Reilly Media, Inc., 2019).
[60] J. Bley, E. Rexigel, A. Arias, N. Longen, L. Krupp, M. Kiefer-Emmanouilidis, P. Lukowicz, A. Donhauser, S. Küchemann, J. Kuhn, and A. Widera, Phys. Rev. Res. **6**, 023077 (2024).
[61] A. A. diSessa and B. L. Sherin, The Journal of Mathematical Behavior **19**, 385 (2000).
[62] W. Schnotz, U. Ludewig, M. Ullrich, H. Horz, N. McElvany, and J. Baumert, Journal of Educational Psychology **106**, 974 (2014).

LIST OF ACRONYMS

STEM Science, Technology, Engineering, & Mathematics

DeFT Design, Functions, and Tasks

MERs Multiple External Representations

ITPC Integrated Theory of Text and Picture Comprehension

CTML Cognitive Theory of Multimedia Learning

CLT Cognitive Load Theory

ANOVA Analysis of Variance

ECL Extraneous Cognitive Load

ICL Intrinsic Cognitive Load

GCL Germane Cognitive Load

ET Eye Tracking

MRC Meta-Representational Competencies

MZI Mach-Zehnder Interferometer

4. Study 3: Comparing visual qubit representations in quantum education: The Bloch sphere enhances task efficiency

*Linda Qerimi * Sarah Malone * Eva Rexigel * Jochen Kuhn * Stefan Küchemann*

Contribution:
Qerimi, Küchemann, Kuhn, Malone designed the study, Qerimi developed the questionnaires and collected the data, Qerimi analysed the data, Qerimi wrote the first draft of the manuscript. All authors reviewed and edited the manuscript. Küchemann supervised the study. All authors have read and agreed to the submitted version of the manuscript.

Reference:
Qerimi, L., Malone, S., Rexigel, E., Kuhn, J., & Küchemann, S. (2025). Comparing visual qubit representations in quantum education: The Bloch sphere enhances task efficiency [Preprint]. ***arXiv*****. https://arxiv.org/abs/2507.21721**

Comparing Visual Qubit Representations in Quantum Education: The Bloch sphere Enhances Task Efficiency

Linda Qerimi*
Chair of Physics Education, Faculty of Physics, Ludwig-Maximilians-Universität München, Geschwister-Scholl-Platz 1, 80539 Munich, Germany, and Munich Quantum Valley (MQV), Max Planck Institute of Quantum Optics (MPQ), Germany

Sarah Malone
Department of Education, Saarland University, Campus A4 2, 66123 Saarbrücken, Germany

Eva Rexigel
Department of Physics and State Research Center OPTIMAS, University of Kaiserslautern-Landau, Erwin-Schroedinger-Str. 46, 67663 Kaiserslautern, Germany

Jochen Kuhn and Stefan Küchemann
Chair of Physics Education, Faculty of Physics, Ludwig-Maximilians-Universität München, Geschwister-Scholl-Platz 1, 80539 Munich, Germany

(Dated: July 30, 2025)

Visual-graphical qubit representations offer a means to introduce abstract quantum concepts – such as quantum state, superposition, or measurement – in an accessible manner, particularly for learners with low prior knowledge. Building on a previous expert rating of the mechanisms of qubit representations, this study compared two representations – the Bloch sphere and the Quantum Bead – in terms of learning outcomes, task performance, cognitive load, and mid-term retention. The study was conducted with N=149 secondary school students. The study assessed conceptual understanding via pre- and post-tests, application-oriented task performance by measuring accuracy per time, and cognitive load via intrinsic, extrinsic and germane dimensions. A follow-up test after 1-2 weeks assessed medium-term retention. Results showed no significant effect of representation type on post-test learning outcomes. However, process data revealed that learners using the Bloch sphere completed application-oriented tasks significantly more efficiently. The cognitive load was similar in both groups. Mid-term retention of quantum concepts was stable across groups, and early learning performance emerged as the strongest predictor of mid-term retention.

In conclusion, these findings emphasize that instructional impact is not solely determined by outcome measures, but also by how representations influence cognitive processing, task integration, and learners' interaction with complex content.

I. INTRODUCTION

A. Motivation und Backgroud

Learning in Quantum physics (QP) has become increasingly relevant, not least because of the growing importance of emerging Quantum technologies (QT) such as quantum computing, quantum cryptography, and quantum sensing. The physical conditions under which these technologies can be realised, for example, through ion traps, superconducting or neutral atoms, and are an active subject of fundamental research(e.g. [1]). Key concepts such as quantum measurement, superposition, probability, and entanglement are essential across all implementations. The interest in teaching these concepts now spans a broad spectrum of learners from school to industry with varying levels of prior knowledge and diverse learning goals [2, 3]. The use of representations in QP enables learners to gain access to an otherwise highly abstract and formalised world. Particularly in a school context, where mathematical skills are often not yet fully developed, suitable representations can help to make fundamental concepts of QP understandable. Various forms of representation are available in QP, ranging from symbolic-mathematical representations (such as Dirac notation or Schrödinger formalism) to visual-graphical representations such as the Bloch sphere. The latter offer the possibility of making central concepts such as superposition, measurement or quantum state accessible by visual means, independently of an intensively formal derivation [4, 5]. The review by Donhauser et al. (2024) analyses teaching and learning elements, innovative tools and process-oriented studies from the period between 2018 and early 2024 [6]. The results show that more than half of the articles examined are based on the qubit idea methodology or are preparing for it [6]. This demonstrates the upcoming relevance of qubits as teaching strategies [6]. In particular, external representations of two-level quantum systems – referred to as qubits – combined with the 'spin-first' approach, are well suited for introducing the elementary structure of QP in a clear and accessible manner and for embedding it into contextualised learning environments [5, 7, 8]. Visual representations can establish a connection between phenomena and concepts, thereby supporting conceptual development. In QP, it is more difficult to find consistent visualisations than for classical physics lessons [9], so the selection and design of suitable representations is of particular importance. In our previous study, we identified 16 features of visual-graphical qubit representations that are generally capable of supporting students during learning with visual representations [10]. These features were rated by experts on a scale of 1 to 5 for specific qubit representations. The results show that experts anticipate differences in the support of specific learning processes and suggest that certain representations activate different mechanisms in the learning process [10]. The expert ratings provide insights into how qubit representations are evaluated with regard to features that support learning. However, it remains unclear how students actually perceive these representations and benefit from them. In the present study, two representations were selected based on their expert ratings to investigate whether differences in learning outcomes emerge and to what extent these align with the expert evaluations. To address this aspect, we focus on the following research questions (RQs) and research hypotheses (RHs).

B. Research Questions and Hypotheses

The study examines whether visual representations support students' conceptual understanding of QP, and whether the Bloch sphere and quantum beads representations differ in their instructional effectiveness, particularly in the context of QT.

RQ1: To what extent do different visual-graphical representations (Quantum Bead vs. Bloch sphere) foster learning quantum concepts differently?
H1.1: Participants who learn with the Bloch sphere achieve a higher learning outcome than those who learn with the Quantum Bead representation.
H1.2: Participants using the Bloch sphere perform more efficient on application-oriented quantum tasks in *phase gate, amplitude, quantum state, superposition, quantum measurement*, than those using the Quantum Bead.

RQ2: How do different visual-graphical representations (Bloch sphere vs. Quantum Bead) affect the use of cognitive resources in the learning of quantum concepts?
H2: Participants who learn with the Bloch sphere show a more effective use of cognitive resources than those who learn with the Quantum Bead.

RQ3: How does the use of different visual-graphic representations (Quantum Bead and Bloch sphere) influence medium-term retention of fundamental quantum concepts?
H3: Learners who learned the Bloch sphere will demonstrate higher medium-term retention of basic quantum concepts compared to those who use the Quantum Bead.

II. THEORY

A. Theoretical Foundations of Representational Learning

Learning environments in physics – especially in QP – often involve a variety of representations, such as symbolic, verbal (e.g. text-based) or visual- graphical representations [11]. Learners are often faced with the challenge of linking these representations and integrating them conceptually. At the same time, working with multiple representations offers educational opportunities. A key finding of the meta-analysis by Rexigel et al. (2025) suggests that the advantages of Multiple external representations (MERs) are not limited to established combinations of two forms of representation [12]. Positive effects are also evident when three or more representations are combined [12].

A key task for learners is to understand the individual representations and to extract and connect essential information from them in order to form coherent mental representations (schemata) [13, 14]. The cognitive processes involved in this integration are described, for example, in the Cognitive Theory of Multimedia Learning (Cognitive Theory of Multimedia Learning (CTML) [15]. The CTML is based on fundamental assumptions such as the limited capacity of working memory and posits processes such as *selecting*, *organizing*, and *integrating* (SOI) information as central to meaningful learning [16]. To support these processes and facilitate efficient learning, instructional design should aim to optimize the use of limited working memory resources, thereby promoting effective learning outcomes.

These considerations set the stage for examining the cognitive mechanisms that underlie learning with (multiple) external representations.

Cognitive Theory of Multimedia Learning

The multimedia principle states that learners understand content better when it is presented both verbally and visually than when it is purely text-based [16]. Learning QP with multiple representation offers the opportunity to utilise this advantage and support students in the learning process [15].

The CTML is based on the following three assumptions: channel duality, limited capacity and active processing [16]. The term 'dual channels' refers to different modalities, such as visual and verbal perception [16].

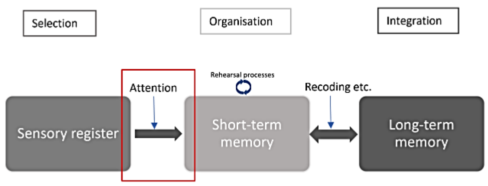


Figure 1: Based on the multi-store model of memory, cognitive processing involves the temporary storage and manipulation of information in working memory. Through processes such as rehearsal, elaboration, and integration, information is transferred to mid-term memory and linked to prior knowledge. Effective learning requires meaningful organization and retrieval of both existing and newly acquired information.

Limited capacity refers to the limited number of information in working memory that can be processed simultaneously [16–18]. Active processing involves selecting relevant information so that it can be further processed through organisational and structural processes [16]. The goal of this integration is to connect new information with existing knowledge.

Mayer (2014) sees multimedia design as a possible approach to support learners in mental modelling [16].

According to Mayer (2014), two implications for multimedia design can be derived from this assumption: (1) The material presented should have a coherent structure and (2) the message should provide learners with guidance on how the structure can be constructed [15].

To support this goal, multimedia design can be used strategically, in combination with knowledge and findings of representational mechanisms, to shape the design of learning environments and promote the development of a functional mental model [10, 15, 19].

Cognitive Load

Any new information presented to a learner puts a strain on working memory, which refers to the short-term memory system and has a limited capacity to process and organise information (see Figure 1) [15, 17, 18]. Ideally, learning processes in working memory allow not only for processing but also for initial integration with prior knowledge. According to Sweller (1998), cognitive load can be categorised into three types [18]:

- intrinsic cognitive load, which arises from the inherent complexity and difficulty of the learning material,
- extraneous cognitive load, which is caused by irrelevant or poorly designed information from outside the content, and
- germane cognitive load, which refers to the proportion of cognitive resources that directly support learning and the construction of mental structures.

The interplay between new information about representation and concepts in QP can increase and strain processing capacity, thereby becoming a barrier to learning [17]. But Sweller (1988, 1998) also demonstrates, that highlighted elements in instructional materials can facilitate processing and thereby enhance learning [18, 20]. However, visual-graphical representations have the potential to reduce cognitive load if they are accessible and stimulate learners' prior knowledge and different processing channels [15, 21]. The Cognitive Load Theory (CLT) emphasizes that learning is most effective when instructional design takes into account the limitations of working memory[18]. Selecting appropriate representations plays a crucial role in optimizing cognitive load – not necessarily by reducing it, but by aligning it with the task and learners' prior knowledge.

Process-Based Indicators of Learning

Learning success is often assessed based on learning outcomes such as test results or progress in conceptual understanding. However, learning is a process and important aspects of this process cannot always be captured directly by measuring outcomes (e.g. [22]).

Time: To gain a deeper insight into how learners deal with representations and tasks, it is essential to consider process-related indicators such as reaction time, subjective confidence or even visual attention (e.g. through eye tracking). Hou and Zhang (2006) show that visual information processing is dependent on processing time [23]. The longer a visual stimulus is viewed, the greater the resolvable depth of detail, particularly for complex or finely structured content. The authors present a model to quantify the information capacity of attention [23]. In this model, Hou and Zhang propose that visual attention dynamically adjusts its resolution over time and that this relationship can be described quantitatively. Their findings demonstrate a clear relationship between reaction time and the spatial resolution of attention [23]. As emphasized by Schewior and Lindner (2024), response time serves as an important indicator of cognitive processes in multimedia testing formats [24]. In the context of multimedia learning and testing [24, 25], representational pictures (RPs) is a term that describes images that depict conceptually relevant content and aim to support understanding by visually representing the underlying subject matter [24], in our interpretation: visual-graphical representations.

The review by Schewior & Linder (2024) shows that, performance-based studies on the effects of learning with visual-graphical representations often show mixed results, while process-based indicators such as time on task can provide deeper insights into the cognitive impact of visual-graphical representations. Several studies found no significant changes in time on task when visual-graphical representations were included compared to text-only materials (e.g., [24–27]), while others reported either shorter ([28, 29]) or longer task durations [24, 30]. However, eye-tracking data reveal that learners spend less time processing textual elements and instead focus more on visual components when visual-graphical representations are present [24, 25, 31]. This shift in attention suggests that visual-graphical representations may not change overall task time but do influence how time is allocated during problem solving. Furthermore, visual-graphical representations appear to facilitate the construction of mental representations in early problem phases and aid in updating them during later stages [24, 25]. These findings highlight that even when time-on-task effects are ambiguous, visual-graphical representations can have measurable effects on learners' cognitive processes.

From the perspective of Cognitive Load Theory [32], tasks with high element interactivity require more working memory resources due to increased intrinsic cognitive load. According to Sweller et al. (1998, 2019), the interactivity of elements in a task determines the intrinsic cognitive load, as multiple interacting elements must be processed simultaneously and integrated meaningfully [17, 32]. This suggests that tasks with high element interactivity are likely to require greater working memory resources, which may be reflected in increased processing time. Depending on the context of the task, longer processing times may be associated with increased cognitive effort, deeper engagement, or conversely with uncertainty and difficulties in understanding. As such, processing time should be interpreted with caution and always in relation to task performance and cognitive load.

Cognitive Load: In addition to the processing time, the cognitive load can provide information on the processing of the information. According to Sweller et al. (2011), various methods exist to measure cognitive load, including performance measures, secondary tasks, physiological indicators, and subjective rating scales [33]. The latter – such as the Cognitive Load Test developed by Klepsch et al. (2017) – assess cognitive load retrospectively and may be influenced by learners' self-assessment and self-concept [33, 34].

Self-assessment: In addition, learners' self-assessment (e.g. confidence in their answers) can serve as a metacognitive indicator of meaningful learning processes [24, 35]. Concepts that are processed with a higher level of subjective confidence are more likely to be stored in mid-term memory and can be retrieved. A comprehensive understanding of learning therefore requires not only outcome-oriented data but also process-oriented data, such as processing time or mental effort during the learning phase.

B. Mechanisms/Aspects of qubit representations

In order to enhance comprehension of the manner in which visual-graphical qubit representations can facilitate learning in QP, a comprehensive category system was developed [10]. This system was employed in the structuring and evaluation of pivotal representational features in our previous research. The category system, which is grounded in the Design, Functions, and Tasks (DeFT) framework (Ainsworth, 2006) and has been extended by findings from quantum education, was applied in an expert rating [19]. In this study, four representations (Bloch sphere, Quantum Bead [36], Circle Notation [37, 38], and a pi chart model: Qake [39]) were evaluated by experts across 16 features based on their perceived potential to facilitate learning. According to the ratings, representations differ in how well they addressed central quantum concepts such as quantum state, superposition, and measurement [10]. In particular, the experts evaluated whether and how well the representations depict phase and amplitude – two essential components of understanding QP. The Bloch sphere and Circle Notation were rated as more suitable for visualizing relative phase than the Quantum Bead and Qake representations. Another category assessed by the experts was visual salience, which is the extent to which visual elements draw attention. Salience refers to how strongly a stimulus stands out in a particular context and attracts attention [40]. In educational contexts, this characteristic is crucial, as salient stimuli can effectively direct learners' attention and support cognitive processing[41–43]. According to Cowan (1999), salient elements play a central role in the control of attention within working memory [44].

In this context, the Quantum Bead emerged as a particularly noteworthy element, garnering high ratings from experts who recognised its 3D design and utilisation of colour as key characteristics contributing to its prominence.

Overall, the expert evaluations make clear that different representations have different strengths and limitations, depending on the learning goal, the conceptual focus, and the visual and cognitive accessibility. It is evident that these insights provide a valuable basis for the selection or design of representations that are not only visually engaging but also educational effective. In order to facilitate a more profound comprehension of the manner in which the insights of the experts can be applied by learners, the following study was conducted.

Based on differences identified through expert evaluations, specifically in *Phase, Amplitude, Salience* and *Concepts* on the representations: the Bloch sphere and the Quantum Bead, the study was conducted. To examine whether and how the expert-identified differences between the representations translate into differences in student learning, we used application-oriented tasks involving each representation. These tasks were analyzed with regard to processing time. We also measured content knowledge [45–47] and cognitive load [34] to learn more about the effects of different qubit representations on students' learning and the mid-term effects.

III. METHOD

A. Piloting

A pilot study was carried out with N=8 participants (N=3 in group Bloch sphere and N=5 in group Quantum Bead) to ensure that the learning material and test items were neither too difficult or too easy. The necessity to modify the material and items was rendered obsolete by the students' feedback, which indicated that the material was comprehensible and the items were neither excessively challenging nor unduly simplistic. The methodology employed in the pilot study was replicated for the main study, as no adverse effects were observed in either case.

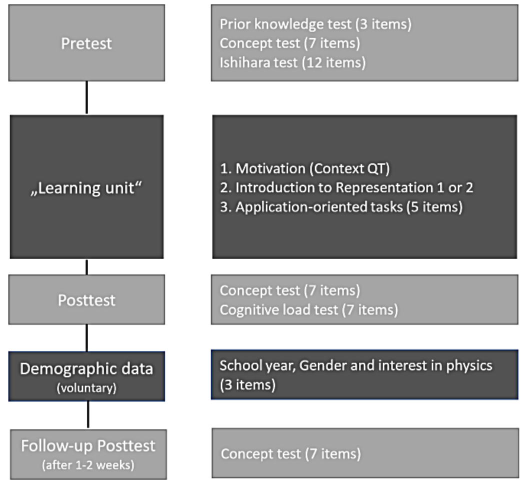


Figure 2: Schematic overview of the process: After pseudonymised registration and a colour perception screening (Ishihara test), the participants completed a prior knowledge test and a conceptual pretest. This was followed by a learning unit that included an introduction to the respective representation and application-oriented tasks. The participants then completed a posttest and an assessment of cognitive load (based on Klepsch et al., 2017) [34]. A follow-up (Posttest$_2$) test was conducted 1–2 weeks later to assess medium-term retention performance.

B. Participants

The sample in the main study comprised $N = 149$ upper secondary school students in Bavaria. The participants were distributed across class level 11 ($n = 46$), level 12 ($n = 32$) and level 13 ($n = 52$) (between the ages of 15 and 18). For $n = 20$ participants, no information on the class level was available. Participants were primarily recruited through teachers who had scheduled visits to the student lab (PhotonLab) at the Max Planck Institute of Quantum Optics. These teachers were contacted in advance and asked whether they would be willing to participate in the study with their classes. Upon agreement, the entire class took part, provided that students presented a signed consent form from their legal guardians.

C. Study design and process

The study followed a structured sequence (see Figure 2).

The study was designed as a mixed design with a between-subjects factor (representation: *Quantum Bead* or *Bloch sphere*) and a within-subjects factor (testing time: pretest, posttest, follow-up-test (Posttest$_2$)).

First, all participants completed a pseudonymised survey. A colour perception test (Ishihara test) was conducted [48] to rule out possible difficulties in interpreting the colour-coded Quantum Bead representations. If participants have three or more errors in the test, participants in the group of quantum beads were excluded [49]. Following this, prior knowledge was assessed using three items related to waves, electromagnetic radiation, and beam splitters (items were used[46]. In the pretest phase, students responded to conceptual questions on QP, which were later repeated in the posttest to evaluate learning gains and the follow-up test (items were used [45–47]. A learning unit followed, consisting of a motivational introduction and an explanation of the assigned visual representation (Bloch sphere or Quantum Bead). After that, application-related tasks were carried out with the corresponding representations of the concepts, quantum state, superposition, quantum measurement, quantum measurement/quantum state, phase and amplitude. The time taken to complete the tasks was measured in milliseconds [ms]. After the learning phase, the posttest was conducted. To assess cognitive load during the learning process, items based on Klepsch et al. (2017) were administered [34]. Finally, a follow-up test (Posttest$_2$) was carried out 1–2 weeks later to examine the retention of knowledge over time.

D. Selection of representations and Materials of the Learning unit

To ensure a fair comparison, two three-dimensional representations were used that differed primarily in the specific features under investigation. The *Quantum Beads* use colour-coded structures to visualise quantum concepts, while the *Bloch sphere* uses the classic spherical representation with vector arrows. Concrete: The Bloch sphere represents the state of a single qubit as a vector on the surface of a unit sphere, offering a geometric interpretation of superposition and phase through spatial rotation. In contrast, Quantum Beads visualize quantum states using a red-green color gradient mapped onto a sphere: red indicates a high probability of measuring —0⟩, green corresponds to —1⟩ – each relative to the conventional Z-axis measurement. Intermediate colors, such as black, represent a 50/50 superposition. As the quantum state evolves, the sphere rotates, and the visible color at the top changes accordingly (see Figure 3).

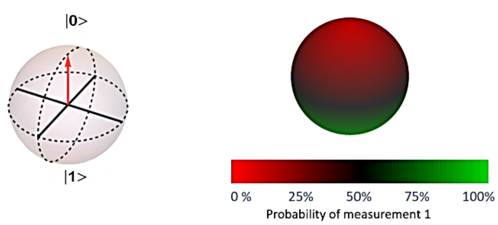


Figure 3: The Bloch sphere is shown on the left, while the Quantum Beads and their bar chart are on the right. The latter serves as a guide to the meaning of the colour changes. Images are self-created. Quantum Bead adapted from [36]

The learning materials were delivered digitally via iPads and were identical in content, structure and language, differing only in the way quantum information was represented. The unit was embedded in SoSci Survey (https://www.soscisurvey.de/en/index). Participants in group 1 received a version with the Quantum Bead, while group 2 worked with the Bloch sphere. Both groups start with the same context about quantum computing and the relevance of qubits, central concepts of QP, including quantum states, superposition, quantum measurement, phase (in form of a phase gate) and amplitude, followed by the introduction of the relevant representations (see Figure 4). The explanations were kept simple and adapted to the prior knowledge of the secondary school students.

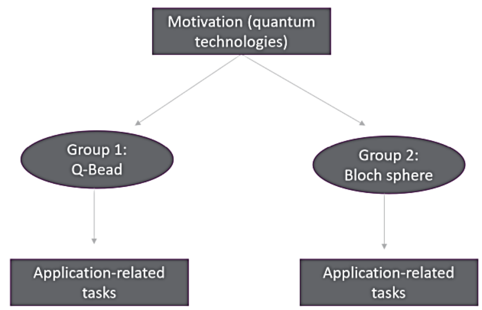


Figure 4: Structure of the learning unit and the application-oriented tasks.

After the introduction, the participants worked on five application-oriented tasks directly related to the respective representation. The tasks could only be solved once the essential information had been extracted from the representation. The tasks were designed as multiple-choice questions and targeted specific aspects of the representations (e.g., spatial orientation, probability distribution, rotations, phase and amplitude). For each task, the processing time in milliseconds [ms] was recorded to allow for process-oriented analysis. An example task requiring participants to determine the probability of measuring $|1\rangle$ using the *Bloch sphere* is shown in Figure 5.

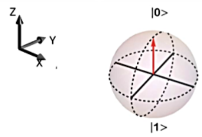
The material for the learning unit, including subsequent application-oriented tasks (example in figure 5, could be made available on request (Note:The material is in german language.

E. Statistical Methods

Various statistical methods were used to answer the research questions and test the hypotheses. First, an Analysis of variance (ANOVA) was planned to examine the influence of the different forms of presentation (Quantum Bead vs. block sphere) on learning success, while at the same time controlling for the effects of covariates such as prior knowledge or pre-test performance. However, due to the violation of the homogeneity of the regression slopes, a classic ANCOVA was not used and a multiple linear regression model was used instead. This allowed the predictors and their effects on the post-test score to be modelled separately.

To evaluate the application-oriented tasks, we consid-

Task 1: Quantum measurement/-state



What is the probability of measuring $|1\rangle$?

A: 100 %

B: 50 %

C: 30 %

D: 0 %

Figure 5: Task 1: Quantum measurement/ -state for the Bloch sphere group.The same task was set for Quantum Bead.

ered three variables: the number of correctly solved tasks (task accuracy), the total time spent on all tasks in seconds (task time), and the ratio of correct solutions to time (efficiency). Since the data did not meet the assumptions for an ANOVA—particularly due to a lack of normal distribution—we used Mann–Whitney U tests to compare the groups.

A multivariate analysis of variance (MANOVA) was conducted to the effects of representation type on cognitive load. Three dependent variables were considered simultaneously: *intrinsic cognitive load (ICL)*, *extraneous cognitive load (ECL)* and *germane cognitive load (GCL)*. This procedure is particularly suitable for investigating multivariate effects when it is suspected that the dependent variables interact with each other, as is the case with the cognitive load dimensions. In the event of significant main effects, subsequent post-hoc tests were carried out, whereby the Bonferroni correction was applied to control for the alpha error in multiple comparisons.

The effect sizes of the results were given in Cohen's d to better classify their practical relevance. This measure allows the size of an effect to be estimated independently of the sample size. In case of violations of the normal distribution assumption, in particular with smaller samples or skewed distributions, Wilcoxon tests were used as a non-parametric alternative to the t-test in order to be able to make robust statements about differences between two measurement points or groups.

To evaluate whether group differences emerged in medium-term retention, an Analysis of Covariance (ANCOVA) was conducted. In this model, the dependent variable was the test performance in the follow-up posttest (Posttest$_2$). The independent variable was the representation group (Quantum Bead vs. Bloch sphere), and the covariate was the performance in the immediate posttest (Posttest$_1$), which served to control for individual differences in initial learning successes.

To analyse the development of learning over time, not only between Posttest$_1$ and Posttest$_2$, but across the entire learning process, a mixed ANOVA with repeated measures was conducted. The within-subjects factor was time (Pretest, Posttest$_1$, Posttest$_2$), and the between-subjects factor was the representation group (Quantum Bead vs. Bloch sphere). In addition, a separate mixed ANOVA with repeated measures was performed for response confidence across the same three time points, in order to investigate changes in learners' perceived confidence over the course of the study.

All statistical analyses were performed in R (version 4.4.0; R Core Team, 2024), and the corresponding analysis scripts are available upon request.

IV. RESULTS

A. Sample considered

The final sample consisted of $N = 146$ students. Due to suspected red-green color vision deficiency, three participants were excluded from the analysis [49]. Of the remaining participants, 85 identified as male, 46 as female, 1 as diverse, and 14 did not provide gender information. Group allocation was balanced: 75 students were randomly assigned to the Quantum Bead group (23 female, 46 male, 1 diverse, 5 no gender specified), and 71 to the Bloch sphere group (23 female, 39 male, 0 diverse, 9 no gender specified).

B. RQ1 – Learning Outcome Based on Representation Type

Pre-Post Comparison: Wilcoxon signed-rank tests revealed significant learning gains in both groups from pretest to posttest in Bloch sphere ($V = 238$, $p < .001$) and in the Quantum Bead ($V = 370$, $p < .01$) group, indicating that students improved regardless of representation.

To investigate whether learners' outcomes differ as a function of the visual representation used (Quantum Bead vs. Bloch sphere), we first explored pre-post differences and then conducted a multiple linear regression analysis controlling for covariates.

Multiple Linear Regression (MLR). To test the effect of the representation while controlling for prior knowledge and pretest performance, we conducted a multiple regression analysis with the posttest score as dependent variable. The model included the predictor variables *group*, *pretest*, and *prior knowledge*. The regression model was statistically significant overall, $F(3, 142) =$

29.21, $p < .001$, explaining $R^2 = .38$ of the variance in posttest scores (adjusted $R^2 = .37$).

$$Y_i = 0.42 + 0.08 \cdot X_{1i} + 0.55 \cdot X_{2i} + 0.30 \cdot X_{3i} + \varepsilon \quad (1)$$

Y_i = Posttest score; X_{1i} = Group (0 = Bloch, 1 = Q-Bead); X_{2i} = Pretest score; X_{3i} = Prior knowledge; ε = residual error.

As shown in Table I, pretest performance ($p < .001$) and prior knowledge ($p = .008$) were significant predictors of posttest outcomes. The representation group, however, had no significant effect ($p = .703$). Thus, participants with higher pretest and prior knowledge scores tended to perform better on the posttest, regardless of the representation used.

Table I: Multiple Linear Regression Coefficients Predicting Posttest Score

Predictor	Estimate	Std. Error	t	p
(Intercept)	0.420	0.526	0.80	.426
Group (X_1)*	0.083	0.217	0.38	.703
Pretest Score (X_2)	0.547	0.068	8.00	***
Prior Knowledge (X_3)	0.300	0.112	2.68	**

* Dummy coding was introduced to correctly account for the group variable (categorical variable) in the regression equation.
Significance levels: *** $p < .001$, ** $p < .01$, * $p < .05$

Summary: Overall, no direct effect of representation on posttest performance was observed. Learning gains occurred in both groups, and individual differences (pretest, prior knowledge) had stronger predictive value than the type of representation.

C. RQ1 - Task Performance

To evaluate the application-oriented performance when learning with the visual-graphical representations, participants solved five applications-oriented tasks addressing key quantum concepts: quantum state, superposition, quantum measurement, as well as phase gate and amplitude with the representations (see Figure 2). Five tasks were set for this. The fourth, the phase gate task was excluded. The task did not differentiate between groups, indicating that it may not have been sensitive enough to capture group-specific effects of the representation. It was excluded for further analysis.

For the remaining tasks, the following indicators were calculated for each participant:

- **Task accuracy:** Number of correctly solved tasks (0–4)
- **Task time:** Total time spent on all tasks (transferred in seconds [s])
- **Efficiency:** Correct solutions divided by time

Task accuracy and Task time: Descriptive results showed that participants in the Quantum Bead group solved on average [2.99] tasks correctly (SD = [1.09]), compared to [3.27] in the Bloch sphere group (SD = [1.00]). The average task time was [11.40] s in the Bloch sphere group (SD = [3.95]), and [18.20] s in the Quantum Bead group (SD = [7.10]). No significant difference was found between the two representations for response accuracy ($W = 2906.5$, $p = .090$). However, a highly significant difference was found for the response time ($W = 882$, $p < .001$, $r = .56$), with the group with the Bloch sphere completing the tasks significantly faster. This is also clearly shown in Figure 6.

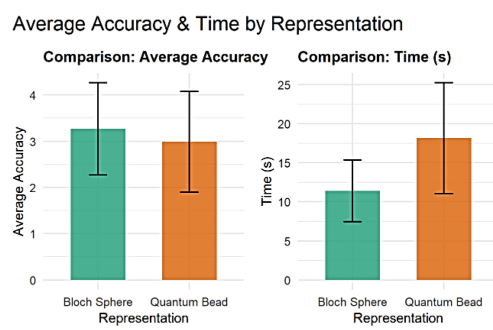


Figure 6: Mean response time (seconds) and the standard deviation (SD) after representation. The Bloch sphere group showed significantly shorter processing times.

Efficiency: The efficiency (correct answers per second) was slightly higher in the Bloch sphere group (*M = [0.323], SD = [0.162]*) than in the Quantum Bead group (*M = [0.187], SD = [0.099]*). The Mann–Whitney U test revealed a significant difference in efficiency in task performance between the groups (W = 3642, $p < .001$), with a medium effect size according to Cohen (r = .48).

Summary: Task accuracy and task time were also analyzed separately. While no significant difference was found between the two groups in terms of accuracy, a highly significant difference was observed in task time: participants in the Bloch sphere group completed the tasks substantially faster. Consequently, learning processes with the Bloch sphere representation were more efficient—measured as correct answers per second.

D. RQ2: Cognitive Load differences between groups

To investigate whether different visual-graphical representations lead to a more effective use of cognitive resources, a multivariate analysis of variance (MANOVA)

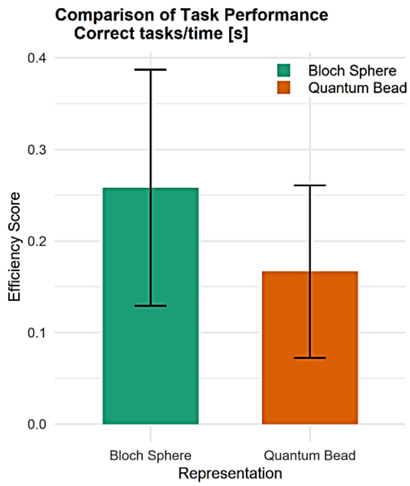


Figure 7: Mean efficency score and SD: The tasks were answered much more efficiently by pupils as soon as they were presented with the Bloch sphere. The efficiency score is significantly higher in the Bloch sphere group.

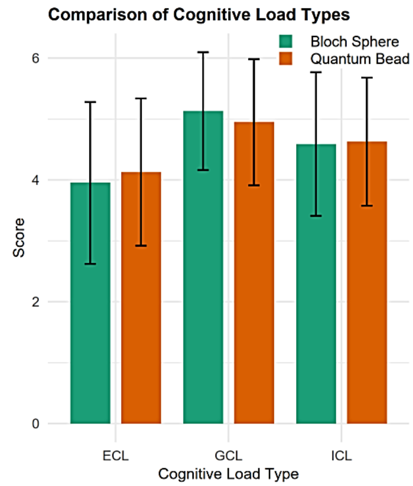


Figure 8: Mean cognitive load score and SD. There were no significant differences between the groups.

was conducted. The independent variable was study group (Quantum Bead vs. Bloch sphere), and the dependent variables were three types of cognitive load: intrinsic cognitive load (ICL), extraneous cognitive load (ECL), and germane cognitive load (GCL).

Descriptive statistics indicated only slight differences between the groups. For ICL, the Bloch sphere group had a mean of $M = 4.59$ ($SD = 1.18$), while the Quantum Bead group had a mean of $M = 4.63$ ($SD = 1.04$). Regarding ECL, participants in the Bloch sphere group reported $M = 3.95$ ($SD = 1.33$), and the Quantum Bead group $M = 4.17$ ($SD = 1.25$). In terms of GCL, the Bloch sphere group scored slightly higher ($M = 5.13$, $SD = 0.97$) than the Quantum Bead group ($M = 4.94$, $SD = 1.03$).

The results of the MANOVA revealed no statistically significant overall effect of group on the combined cognitive load measures, $Pillai's\ Trace = 0.0186$, $F(3, 148) = 0.90$, $p = .440$.

Summary:

- Hence, no significant group differences were found for any type of cognitive load.
- These findings suggest that the use of different visual-graphical representations did not lead to significantly different cognitive load experiences between the groups.

E. Results for RQ3: Medium-Term Retention of Quantum Concepts

To examine potential differences in the retention of information, participants were asked to complete the same posttest items again approximately 1–2 weeks later. Using their pseudonymised codes, responses could be matched to the initial data set. As not all participants completed the follow-up test, the sample size was accordingly reduced.

To assess medium-term retention, an ANCOVA was conducted using the follow-up posttest score (Posttest_2) as the dependent variable. The independent variable was the representation group (Quantum Bead vs. Bloch sphere), with the immediate posttest score (Posttest_1) as covariate.

Sample: The analysis included $N = 98$ students (Group 1: $n = 53$, Group 2: $n = 45$). Group 1 included 15 females, 32 males, 1 diverse, and 5 participants without gender information. Group 2 consisted of 17 females, 20 males, and 8 without gender information.

The covariate Posttest_1 significantly predicted the follow-up posttest score (Posttest_2) ($F(1, 97) = 18.82$, $p < .001$), indicating that higher scores in the immediate posttest were associated with better mid-term performance. However, there was no significant group effect on follow-up performance when controlling for prior performance ($F(1, 97) = 0.03$, $p = .863$). The means were nearly identical between groups (Quantum Bead: $M = 3.44$, Bloch sphere: $M = 3.49$).

Table II: ANCOVA results for predicting mid-term retention (Posttest_2)

Predictor	Sum Sq	Mean Sq	F	p
Posttest_1	43.34	43.34	18.82	***
Group	0.07	0.07	0.03	n.s.
Residuals	223.43	2.30		

Note. *** $p < .001$; n.s. = not significant. The covariate Posttest_1 significantly predicted Posttest_2 performance. No group differences were observed after controlling for Posttest_1.

Mixed ANOVA with repeated measure: Development of learning process

To examine the development of conceptual understanding over time, a mixed ANOVA with repeated measure was conducted with **time** as a within-subjects factor *(Pretest → Posttest*$_1$ *→ Posttest*$_2$*)*.

The analysis yielded a significant main effect of time, $F(2, 188) = 4.23$, $p = .016$, $\eta^2_G = .013$, indicating a learning effect. No significant group difference was found, $F(1, 94) = 0.82$, $p = .818$, and the interaction time × group was also non-significant, $F(2, 188) = 0.10$, $p = .902$. Post-hoc analyses revealed significant differences between the pretest and both posttests ($p < .001$), but no difference between Posttest$_1$ and Posttest$_2$. The learning gains remained stable over time following the learning unit.

Mixed ANOVA with repeated measure: Development of Confidence

We also analyzed response security over time using a mixed ANOVA with repeated measures. It showed a significant main effect of the test time, $F(2, 186) = 39.36$, $p < .001, \eta = .29$, indicating a significant increase in response certainty over time. Figure 9 shows a descriptive difference in response confidence between the groups after the learning unit; however, this difference is not statistically significant ($p = .066$). Post hoc analyses revealed significant differences between pretest and both posttests ($p < .001$), but no difference between posttest and posttest 2 see Figure 9. Confidence gains remained stable over time after the learning unit.

Summary

- **ANCOVA:** After controlling for Posttest_1, no significant group difference in Posttest_2 ($p = .863$); Posttest_1 was a strong predictor of retention.
- **Learning process:** Significant improvement over time ($p = .016$); no difference between groups ($p = .821$).

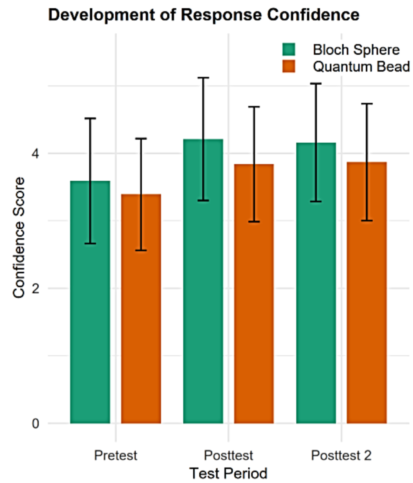


Figure 9: Response confidence over time and the SD. Confidence increased significantly, with no significant group difference after the learning unit.

- **Confidence:** Increased significantly after instruction ($p < .001$); remained stable over time; no group × time interaction.

V. DISCUSSION

A. Learning Outcome and Task performance

A comparison of pre- and posttest scores revealed learning gains overall, but no significant differences between the two instructional groups. As both groups received identical textual explanations—differing only in the type of visual representation introduced—this suggests that while the inclusion of a representation may have supported learning in general, the specific representational features under investigation contributed only marginally to differences in conceptual learning outcomes.

However, process-based data from the application-oriented tasks on amplitude, superposition, quantum measurement, and quantum state revealed clear differences in processing time and efficiency. This indicates that learning with visual-graphical representations can indeed vary – especially in terms of how learners process and apply information [17, 24, 25].

The results of our study support the assumptions made by experts in the prior expert rating: the Bloch sphere appears to be more effective for application-oriented *tasks* involving representations. This is also reflected in the task results — learners in this group were significantly more efficient (task/time) in answering questions related to quantum state, amplitude, superposition, and quantum measurement.

The results also suggest that the Bloch sphere enabled more efficient task processing, while the higher salience attributed to the Quantum Bead – according to the expert ratings – did not produce a compensatory effect. Based on the qualitative data from the expert rating (free text responses), it was already noted that the category of salience poses a challenge in the rating process, as the necessary conditions for salience are difficult to achieve in a static rating context [10]. This issue is also reflected in the present study, which found that visual salience alone did not result better. Another possible explanation is that specific aspects such as phase difference or amplitude are more easily localized via the vector arrow in the Bloch sphere and helps to highlighted the relevant aspect better [17, 50]. This may allow learners to extract relevant information more quickly and efficiently compared to the Quantum Bead representation, where such features are less visually explicit. The clearer directional cues in the Bloch sphere may have helped to guide learners' attention more effectively toward conceptually relevant elements.

The effectiveness of the Bloch sphere as a cognitive support medium for learning about quantum computing has already been highlighted in previous studies [47]. The more efficient processing of application-oriented tasks with the Bloch sphere may be attributed to the clearer and more structured visualization of key information–especially phase. The distinct direction vector and visible coordinate axes allow learners to extract relevant information more directly, for example when estimating measurement probabilities along the Z-axis. In such cases, a simple projection of the state vector onto the Z-axis is sufficient to infer the likely measurement outcome.

In contrast, while the Quantum Beads were perceived as visually salient, their use of color gradients and state-dependent rotation appears to spatially delocalize relevant information. For instance, learners must interpret the color at the top of the bead—ranging from red to green—to estimate probabilities, which may be less immediate and require additional cognitive steps. This may increase cognitive load and lead to longer task processing times, as learners must first interpret and integrate the visual information.

On the other hand, studies suggest that increased attention can be associated with more intensive information processing (e.g. [15, 44, 51]). Interestingly, however, the longer processing times in the Quantum Bead group did not lead to significant differences in pre-to-posttest learning gains. This suggests that while Quantum Beads may influence attention and processing time, they do not necessarily result in improved learning outcomes. One possible explanation is that the increased salience required learners to invest more cognitive resources, but without yielding additional learning benefits.

According to Cognitive Load Theory, the split-attention effect is particularly relevant in the context of text–image combinations [18]. This effect states that spatially separating related visual information sources – such as text and image – can increase cognitive load and thereby impair learning performance [18]. In this case, the students in the Quantum Bead group had to interpret both representations (bar and sphere) in order to arrive at a clear solution (see figure 3). The reduced efficiency observed in the application-oriented task performance may thus be attributed to the split-attention effect. To mitigate this effect, the representations were placed in close spatial proximity. However, no differences in cognitive load were found.

These findings suggest that the instructional impact of visual representations should not only be assessed by learning outcomes, but also by their role in supporting cognitive processing and structuring task performance.

B. Cognitive Load differences between groups

This study investigated whether different visual-graphical representations (Bloch sphere and Quantum Bead) facilitate different effective use of cognitive resources when learning QP concepts. Based on expert rating and cognitive load theory, we expected differences in cognitive load in particular [10, 18, 20]. However, the MANOVA showed no significant group differences in intrinsic, extraneous or germane cognitive load. Although the Bloch sphere group reported slightly higher germane load and slightly lower extraneous load than the Quantum Bead group, these differences were not statistically significant. Therefore, the hypothesis was not supported by the data.

Although the efficiency results suggest that participants in the Bloch Sphere group processed information more quickly, this did not result in a significantly lower self-reported cognitive load. According to Sweller et al. (2019), differences in cognitive load should become apparent when information is easier to locate or process [17]. The absence of such differences in our data may be due to the limitations of the subjective rating method employed. Retrospective self-assessments, such as the one employed here, are sensitive to individual interpretation and prior expectations [33]. In future studies, it may be advisable to use more objective measures of cognitive load, such as dual-task paradigms or eye tracking, or to use rating scales that are more closely tailored to the specific demands of the learning material. Alternatively, a within-subjects design, in which each participant works with multiple representations and compares their perceived cognitive load across different conditions, could offer greater sensitivity to subtle differences in cognitive demands. An other possible explanation is that both representations were presented in a clearly structured and well-supported learning environment, which is likely to have reduced extraneous load in both groups. In addition, both representations were unfamiliar to the learners, which may have led to similar levels of cognitive processing in both conditions. In contrast to the expectations from the expert rating, the visual salience of the Quantum Bead alone did not lead to an efficient use of cognitive resources or enhanced processing—possibly because the learners lacked prior experience with this type of representation [10, 44]. These findings suggest that the impact of a representation depends not only on its design, but also on how well it relates to learners' prior knowledge and how it is embedded in the instructional context. While previous studies (e.g., [44]; [52]) have highlighted the role of salience in directing attention, this effect did not translate into measurable differences in cognitive load in our study.

C. Medium-Term Retention of learning

The analysis of medium-term retention (RQ3) revealed no significant differences between the representation groups after controlling for immediate post-test performance. This finding indicates that, while the representations varied in terms of processing efficiency during the tasks, there were no significant differences in the retention of the concepts over time. The strongest predictor of follow-up (Posttest$_2$) performance was the immediate posttest result, suggesting that early learning success played a crucial role in mid-term retention. These results are consistent with long-established findings by Hattie, who showed that prior knowledge is a strong indicator of overall learning outcomes [53].

The repeated-measures ANOVA showed that learning gains were maintained after the learning unit but did not increase further, pointing to a stabilisation effect rather than continued learning. Both groups improved over time and followed a similar learning trajectory, with no evidence that one group retained information more effectively than the other. With regard to learners' response confidence, a significant increase was observed over time, which remained stable after the instructional phase. Learners thus became more confident following the intervention, and this confidence was sustained in the follow-up. Although the Bloch sphere group showed slightly higher confidence values, the difference was not statistically significant ($F(1, 93) = 3.45$, $p = .063$), and both groups developed similarly.

As noted in the review by Schewior&Lindner (2024), the present findings similarly indicate that response confidence increased through the use of visual-graphical representations—regardless of group—and remained stable over time [24].

VI. LIMITATIONS

The conclusions of the study are primarily be drawn based on process-related (time reaction) data regarding the processing time of the two representations (Bloch sphere vs. Quantum Beads). More detailed insights into the processing mechanisms – such as fixation duration, gaze patterns, or visual attention distribution – would have required the use of eye-tracking data. Using specific gaze data, it would have been possible to recognise how often and for how long the learners looked at the presentation until they answered the task.

It should also be noted that the majority of participants came from a school student group that had already shown an interest in QP prior to the study. Recruitment took place mainly in the context of scheduled visits to the student lab at the Max Planck Institute of Quantum Optics, which may have led to a certain degree of prior content exposure or motivation.

Another limitation concerns the short time (1-2 weeks) interval between the two posttests. It is possible that participants remembered their previous answers and responded accordingly, which may have influenced the measured stability of learning outcomes.

Furthermore, it is worth mentioning that working with the Q-Beads may have posed additional demands on learners, as they needed to integrate two representational elements (color bars and the Quantum Bead). Alternatively, some learners may have already internalised which color corresponds to which state—without this prior knowledge being explicitly controlled for in the study.

The evaluation of salience was based on an expert rating and did not reflect the subjective perception of the learners themselves. It therefore remains unclear whether visually highlighted elements were actually perceived and processed more strongly during learning.

VII. FUTURE RESEARCH

Given that expert ratings revealed both strengths and limitations across all representations, future studies could build on this work by systematically comparing additional representational features [10]. For example, it would be valuable to examine which specific quantum concepts or instructional goals benefit most from the use of Quantum Beads in classroom settings. Furthermore, future research could explore learning scenarios in which multiple qubit representations are provided simultaneously, to investigate potential advantages of representational flexibility. To examine the impact of design features such as salience or visual complexity more directly, methodologies like eye tracking could be employed to gain deeper insight into learners' visual attention and processing strategies.

VIII. CONCLUSION

This study investigated the influence of two visual-graphical representations—the Bloch sphere and Quantum Beads—on students' learning of QP concepts in the context of quantum computing. Overall, no significant differences in learning outcomes were observed between the two groups in the pre-post comparison. However, clear differences emerged in process-related measures during task completion: participants in the Bloch sphere group completed application-oriented tasks significantly faster and more efficiently. This suggests that the Bloch sphere may better support cognitive processing, particularly in terms of quickly identifying and applying relevant information [17]. The more efficient processing of application-oriented tasks when using the Bloch sphere may be due to the fact that the distinct vector arrow and the visible coordinate axes within the sphere [17] – an advantage also confirmed by expert ratings in [10]. In contrast, the higher visual salience of the Quantum Beads did not lead to improved learning outcomes or efficiency, but was associated with longer processing times—possibly due to increased cognitive demands when interpreting the representation. Rather, it can be assumed that the use of color gradients and sphere rotation delocalizes relevant information, thereby increasing cognitive load and processing time for learners. No significant group differences were found in terms of cognitive load. This may be explained by the limited validity of retrospective self-reports [33], as well as the structured learning environment, which likely reduced extraneous load for all participants. Similarly, no significant differences were found between the representations in terms of medium-term retention. Both groups maintained their learning gains over time. Overall, the findings suggest that not all visually salient representations are equally effective for learning. Although Quantum Beads were more visually prominent, the Bloch sphere enabled more efficient task performance. Future research should therefore take a more differentiated look at the alignment between representational features and specific learning goals, and employ more objective methods—such as eye tracking—to assess cognitive processing more precisely.

Appendixes

Task Performance for each Task

The following figures show the mean efficiency scores and standard errors (SE) for each task performance item, separated by group. We see a clear difference between the groups in terms of quantum state and quantum measurement, which is also evident in amplitude and superposition. It appears that the students using the Bloch sphere are more efficient in all of the tasks mentioned. For the *phase* task, no significant difference was found between the representations.

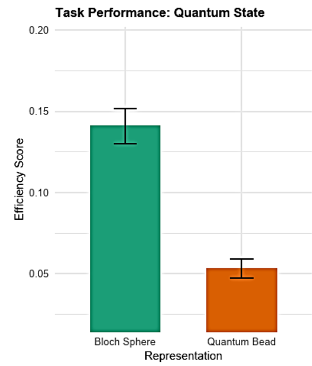


Figure 10: Quantum State

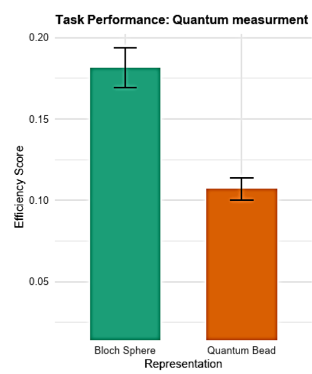


Figure 11: Quantum Measurement

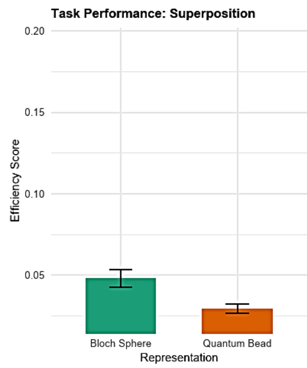


Figure 12: Superposition

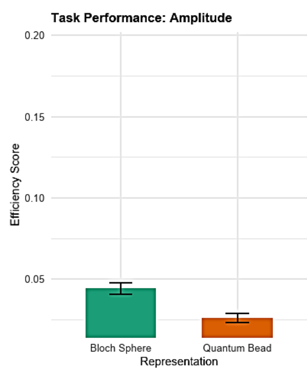


Figure 13: Amplitude

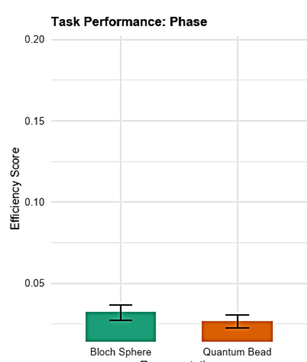


Figure 14: Phase

ACKNOWLEDGMENTS

We would like to thank Silke Stähler-Schöpf, head of the PhotonLab student laboratory at the Max Planck Institute of Quantum Optics (MPQ), for her valuable contributions to this article. We would also like to thank all participants who contributed to this study.

* linda.qerimi@physik.uni-muenchen.de

[1] Munich Quantum Valley, Research areas (2025), accessed May 6, 2025.
[2] F. Greinert, R. Müller, P. Bitzenbauer, M. S. Ubben, and K.-A. Weber, Phys Rev Phys Educ Res **19**, 010137 (2023).
[3] A. Merzel, P. Bitzenbauer, K. Krijtenburg-Lewerissa, K. Stadermann, E. Andreotti, D. Anttila, M. Bondani, M. L. M. Chiofalo, S. Faletič, R. Frans, S. Goorney, F. Greinert, L. Jurčić, Z. Koupilová, M. Malgieri, R. Müller, P. Onorato, G. Pospiech, M. Ubben, A. Woitzik, and H. Pol, EPJ Quantum Technology **11**, 1 (2024).
[4] W. Dür and S. Heusler, PhyDid A - Phys Didakt Sch Hochsch **1**, 01 (2012).
[5] W. Dür and S. Heusler, Phys Teach **52**, 489 (2014).
[6] A. Donhauser, P. Bitzenbauer, L. Qerimi, S. Heusler, S. Küchemann, J. Kuhn, and M. S. Ubben, Phys. Rev. Phys. Educ. Res. **20**, 020601 (2024).
[7] H. R. Sadaghiani and J. Munteanu, in *2015 Physics Education Research Conference Proceedings* (American Association of Physics Teachers, College Park, MD, 2015) pp. 287–290.
[8] R. Müller and O. Mishina, in *Teaching-Learning Contemporary Physics: From Research to Practice*, edited by B. Jarosievitz and C. Sükösd (Springer International Publishing, Cham, 2021) pp. 33–45.
[9] K. Stadermann and E. van den Berg, Phys Rev Phys Educ Res **15**, 010130 (2019).
[10] L. Qerimi, S. Malone, E. Rexigel, S. Mehlhase, J. Kuhn, and S. Küchemann, EPJ Quantum Technology **12**, 45 (2025).
[11] J. Lemke, in *Reading science: Critical and functional perspectives on discourses of science*, edited by J. Martin and R. Veel (Routledge, London, 1998) p. 87.
[12] E. Rexigel, J. Kuhn, S. Becker, and S. Malone, Educational Psychology Review **36**, 124 (2024).
[13] W. Schnotz and M. Bannert, Learn Instr **13**, 141 (2003).
[14] W. Schnotz, in *Sprache, Sinn und Situation: Festschrift für Gert Rickheit zum 60. Geburtstag*, edited by L. Sichelschmidt and H. Strohner (Deutscher Universitätsverlag, Wiesbaden, 2001) pp. 43–57.
[15] R. E. Mayer and L. Fiorella, in *The Cambridge Handbook of Multimedia Learning*, edited by R. E. Mayer (Cambridge University Press, Cambridge, 2014) pp. 279–315.
[16] R. E. Mayer, *The Cambridge Handbook of Multimedia Learning*, edited by R. E. Mayer and L. Fiorella, Cambridge Handbooks in Psychology (Cambridge University Press, Cambridge, 2021) pp. 57–72.
[17] J. Sweller, J. J. van Merriënboer, and F. Paas, Educ Psychol Rev **31**, 261 (2019).

[18] J. Sweller, Learn Instr **4**, 295 (1994).
[19] S. Ainsworth, Learn Instr **16**, 183 (2006).
[20] J. Sweller, Cogn Sci **12**, 257 (1988).
[21] A. Baddeley, Science **255**, 556 (1992).
[22] W. Huang, P. Eades, and S.-H. Hong, Information Visualization **8**, 139 (2009).
[23] X. Hou and L. Zhang, in *International Conference on Neural Information Processing* (Springer, 2006) pp. 127–136.
[24] L. Schewior and M. A. Lindner, Educational Psychology Review **36**, 49 (2024).
[25] M. A. Lindner, A. Eitel, B. Strobel, and O. Köller, Learning and instruction **47**, 91 (2017).
[26] T. Ehrhart and M. A. Lindner, Contemporary Educational Psychology **73**, 102151 (2023).
[27] R. Zheng and A. Cook, British Journal of Educational Technology **43**, 233 (2012).
[28] M. A. Lindner, O. Lüdtke, S. Grund, and O. Köller, Contemporary Educational Psychology **51**, 482 (2017).
[29] S. Sass, J. Wittwer, M. Senkbeil, and O. Köller, Applied Cognitive Psychology **26**, 70 (2012).
[30] I. E. Berends and E. C. Van Lieshout, Learning and Instruction **19**, 345 (2009).
[31] M. Ögren, M. Nyström, and H. Jarodzka, Instructional Science **45**, 263 (2017).
[32] J. Sweller, J. J. Van Merrienboer, and F. G. Paas, Educational psychology review **10**, 251 (1998).
[33] J. Sweller, P. Ayres, and S. Kalyuga, Measuring cognitive load, in *Cognitive Load Theory* (Springer New York, New York, NY, 2011) pp. 71–85.
[34] M. Klepsch, F. Schmitz, and T. Seufert, Frontiers in Psychology **8**, 10.3389/fpsyg.2017.01997 (2017), publisher: Frontiers.
[35] M. A. Lindner, A. Eitel, J. Barenthien, and O. Köller, Learning and Instruction **71**, 101100 (2021).
[36] D. Huber and S. J. Glaser, BEADS: A canonical visualization of quantum states for applications in quantum information processing (2024), preprint.
[37] E. R. Johnston, N. Harrigan, and M. Gimeno-Segovia, *Programming Quantum Computers: Essential Algorithms and Code Samples* (O'Reilly Media, Inc., Sebastopol, CA, 2019).
[38] J. Bley, E. Rexigel, A. Arias, N. Longen, L. Krupp, and e. a. Kiefer-Emmanouilidis, Maximilian, Phys Rev Res **6**, 023077 (2024).
[39] A. Donhauser, J. Kuhn, and et al. (2025).
[40] E. T. Higgins, Social psychology: Handbook of basic principles , 133 (1996).
[41] T. Parr and K. J. Friston, Current opinion in psychology **29**, 1 (2019).
[42] D. M. Rumbaugh, J. E. King, M. J. Beran, D. A. Washburn, and K. L. Gould, International Journal of Primatology **28**, 973 (2007).
[43] L. Itti and C. Koch, Nat Rev Neurosci **2**, 194 (2001).
[44] N. Cowan, Models of working memory: Mechanisms of active maintenance and executive control **20**, 1013 (1999).
[45] P. Bitzenbauer, S. Faletič, M. Michelini, K. Tóth, and G. Pospiech, Physical Review Physics Education Research **20**, 020136 (2024), publisher: American Physical Society.
[46] M. Waitzmann, *Wirkung eines quantenoptischen Realexperiments auf das physikalische Reasoning*, DoctoralThesis, Hannover : Institutionelles Repositorium der Leibniz Universität Hannover (2023).
[47] P. Hu, Y. Li, and C. Singh, Physical Review Physics Education Research **20**, 020108 (2024).
[48] `https://www.de.colorlitelens.com/farbenblindheit-test.html#TEST`.
[49] J. Birch, Ophthalmic and Physiological Optics **17**, 403 (1997).
[50] T. van Gog, in *The Cambridge Handbook of Multimedia Learning*, Cambridge Handbooks in Psychology, edited by R. E. Mayer (Cambridge University Press, Cambridge, 2014) 2nd ed., pp. 263–278.
[51] R. E. Reynolds and R. C. Anderson, Journal of Educational Psychology **74**, 623 (1982), place: US Publisher: American Psychological Association.
[52] R. E. Mayer, in *Psychology of learning and motivation*, Vol. 41 (Elsevier, 2002) pp. 85–139.
[53] J. Hattie and G. C. Yates, *Visible learning and the science of how we learn* (Routledge, 2013).

List of acronyms

ANOVA: Analysis of variance

CLT: Cognitive Load Theory

CTML: Cognitive Theory of Multimedia Learning

DeFT: Design, Functions, and Tasks

MERs: Multiple external representations

QP: Quantum physics

QT: Quantum technologies

SD: standard deviation

5. General Discussion

The aim of this dissertation is to identify and categorize the aspects/features of qubit representation with regard to their suitability for learning, and to analyze, quantify and verify their individual characteristics in order to optimize their targeted use. For this purpose, a category system was developed and evaluated by experts (Study 1). The analysis sought to determine how features of visual-graphical qubit representations differ in their perceived effectiveness learning in quantum physics concepts. The experts' evaluation and the category system then formed the basis for Study 2 and Study 3. These further studies investigated the impact of different features from the learners' perspective. This included investigating whether information-redundant combinations of representations influence learning and cognitive behavior (Study 2), and whether application-oriented features (e.g. phase, amplitude, or superposition) of different qubit representations, which experts had rated high or low for learning, are also transferable to students learning quantum physics. In Section 5.2, the theoretical implications and transferability of the category system are discussed in the following. Practical recommendations are then derived in Section 5.3. Finally, the limitations of the work (in Section 5.4), open questions for future research in Section 5.5, and the central conclusions are presented in Section 6.

5.1 Summary of the central Results

Study 1 (Qerimi et al., 2025):

This study aimed to determine, from an expert perspective, which features of visual qubit representations are relevant differentiated criteria for effective learning in quantum physics and quantum technologies. Existing categorizations of representations (e.g., Bertin, 1983; Kosslyn, 1989; Lemke,1998; Schnotz, 2001) are not extensively specified in learning so the objective was to demonstrate that visual representations can be systematically categorized with regard to their features that offer potential to support learning. To categorize the qubit representations, insights from representational research, physics education, and key concepts necessary for understanding quantum technologies were considered to determine a set of aspects and features.

The aspects and features were transferred into a category system, informed theoretically by Ainsworth's (2006) DeFT framework. This includes overarching classifications such as **design**, **function**, **task**, and **cross-concepts**—the latter referring to overarching aspects and transferability.

To empirically ground the category system, 21 experts from the fields of physics education, and areas of quantum technology science research participated in the online rating.

The qubit representations were evaluated with regard to their effectiveness in conveying the following core concepts:

- Quantum State
- Superposition
- Quantum measurement
- Entanglement
- Quantum technology applications, particularly through basic gates (Hadamard gate, X-gate, Z-gate)

In addition to the structured rating, experts were given the opportunity to provide free-text feedback. They could indicate which categories they considered important for distinguishing between the representations, and which concepts they felt had not been sufficiently considered.

When low agreement occurred among the experts (i.e. when less than 50% of the ratings were in the range around the mean value), certain categories and representations were excluded from further analyses (von der Gracht, 2012; Zinn et al., 2001). This specifically applied to the ratings to the concept of entanglement.

The results show that, within the *design* categories, salience stood out in particular: The Quantum Bead representation was rated by experts as significantly more salient than all other qubit representations. Specifically, the Quantum Bead was rated as the most visually prominent representation in terms of salience with a moderate effect size between the other qubit representations ($0.31 \leq d \leq 0.39$). However, several experts noted in their free-text responses that they found this category difficult to define or evaluate.

In the learning difficulties categories, the experts rated Circle Notation as less prone to causing learning difficulties than the Quantum Bead, the Bloch sphere, or the Qake model with a moderate to high effect size between the representations ($0.40 \leq d \leq 0.52$). In the color category, the Bloch sphere was considered more flexible, as it remains interpretable even without color highlighting. In contrast, the Qake model relies heavily on color coding, which is used to convey essential information about the quantum state (see cheat sheets for more explanation of the single qubit representations). This can limit the usability of the Qake model and other representations that rely on their color component, as they always require the color overlay to use the representation.

Among the *function* categories, only minor differences in predictability were observed. Overall, the representations were rated as highly similar in their functional characteristics as qubit representations.

In contrast, the *task* category revealed a key aspect: the appropriate depiction of the superposition**,** quantum measurement, and probability concepts for learners. As already mentioned, the entanglement has been excluded. According to the experts, the representations differed in their visualizations of the concepts.

For the concepts (quantum measurement, superposition and probabilistic) significant differences emerged particularly between Circle Notation and the Qake model, as well as between the Bloch sphere and the Quantum Bead. Significant differences were also found regarding the visualization of phase and amplitude. The Bloch sphere received the highest rating in the visualization of (relative) phase with a high effect size between the representations ($0.60 \leq d \leq 0.64$). In the visualization of amplitude, the Qake pie-chart model received the highest rating, followed by Circle Notation and the Bloch sphere with a high effect size between the representations ($0.66 \leq d \leq 0.67$). The Bloch sphere was rated higher than Quantum Beads for the visualization of both phase and amplitude.

In the cross-concept categories (generability and effort in explanations), only one notable difference appeared: the Bloch sphere was rated as requiring more explanatory effort than Circle Notation and the Qake model. The Quantum Bead was excluded from the effort in explanations category due to a lack of consensus among the experts. No differences were found in the post hoc test between the representations in the generability category, so we could not identify which qubit representations are easier to generate or create on e.g. a blackboard or in an exercise book than others.

A central theme in the expert free-text response was the idea that conceptual understanding in quantum physics cannot be achieved through a single representation alone. Instead, using multiple external representations (MERs) was emphasized as essential for effective learning.

This perspective is reflected in the following free-text comment from an expert:

> “*But there should never be the ‘one representation’, since more visual models like the qubit cakes will always be easier to digest at first, while models like the Bloch-sphere help understand more complex topics.”* (Qerimi et al., 2025, p. 26)

Overall, the expert rating makes clear that different representations have different strengths and limitations. For example, the Quantum Bead is more salient, Circle Notation is considered to rate lower in learning difficulties with regard to misconceptions, the Bloch sphere is considered to have a significantly higher visualization of the phase, and the Qake model is considered to offer a higher visualization of the amplitude. The findings serve as an important foundation for developing representations that combine visual clarity with educational value.

In order to facilitate a deeper understanding of how learners can apply the insights provided by the experts, Studies 2 and 3 were conducted. These studies build upon the theoretical foundation of the category system as well as the expert rating data and free-text feedback, which highlighted the relevance of multiple representations in learning quantum physics.

Study 2 (Rexigel & Qerimi et al., 2025):

Using multiple representations can effectively support learning (Ainsworth, 2006, 2008). One advantage is that combining multiple representation has the potential to compensate for the limitations of a single representation mode (Mayer, 2021; Schnotz, 2001): if only one representation is used, limitations may arise due to its modality or its inability to clearly convey certain concepts. According to Mayer’s multimedia principle, these limitations can be overcome by combining complementary representations (Mayer, 2021). This allows each representation to contribute its specific strengths while compensating for the limitations of the others, which is essential for effective learning (Ainsworth, 2006).

Although Ainsworth’s DeFT framework advocates multiple external representations (MERs) for deeper understanding, and Mayer’s CTML warns that redundant information can overload cognitive resources, mixed findings (e.g., Ott et al., 2018) and the expert rating in Study 1 leave open the conditions under which informational redundancy truly benefits conceptual learning in quantum physics.

To investigate whether informationally redundant qubit representations influence cognitive load and learning behavior, a 2×2 between-subjects study was conducted with 113 STEM students learning about the behavior of single photons in a Mach–Zehnder interferometer. The study involved a control group (text and illustration) with three intervention groups that received additional Dirac notation, Bloch-sphere visuals, or both. Cognitive load, eye movements, and learning outcomes were collected to determine whether the provision of MERs with informational redundancy improves understanding of quantum-physical properties.

Study 2 showed that, among all four conditions, no statistically significant differences emerged in content knowledge gains from pre- to posttest. Thus, adding Dirac notation, Bloch-sphere visuals, or their combination did not enhance conceptual learning relative to the text + illustration control. No group differences appeared for the cognitive load. ICL, GCL and ECL were in a similar range in all groups. A high ICL was shown in students who learned with the maximum combination of four external representations. In light of the CLT (Sweller, 1998, 2019), this suggests that the addition of multiple external representations with informational redundancy leads to increased element interactivity and correspondingly increased essential processing. But there was no significant difference between the groups in ICL und IG3, showing no significant improvement or deterioration in students' content knowledge.

In addition, learners who learned with the Bloch sphere (IG2, IG3) made significantly more cross-representational transitions than those without it, which indicates integration (Alemdag & Cagiltay, 2018). This corresponds with data from Gegenfurtner et al. (2011) who shows that, in cases of redundant representations, transition behavior decreases when experts focus on one representation, whereas it increases significantly among novices. Since the study involved learners in the early stages of learning quantum physics, it is reasonable to assume that they were novices in this field. Dirac notation alone (IG1) did not elicit a comparable effect. Despite this heightened attempt at integration, the groups with the Bloch sphere (IG2 and IG3) showed no corresponding boost in the learning outcome. Participants displayed generally high representational competence scores, and mental rotation ability did not predict learning performance. The increased number of transitions in IG2 and IG3 could indicate that integration in representational competence was promoted, creating translation between representations or identification of relevant similarities between the representations, i.e. conceptual understanding (in the sense of Rau, 2017). This possibility was not investigated in the study.

The study demonstrates, for the first time, the results of adding informational redundant external qubit representations to instruction on fundamental quantum concepts. No significant learning gains resulted, yet graphical–geometric visuals such as the Bloch sphere could prompt integrative viewing behavior, underscoring that the choice of representations shapes learner interaction and that further research on multiple external representations in quantum physics education is warranted.

Study 3 (Qerimi et al., 2025):

Study 1 both sorted the aspects/features underlying this work into a category system and, through expert ratings, identified differences among the various visual–graphical qubit representations. Study 3 builds directly on these findings to develop insights from secondary school students' perspective.

Using a between-subjects design, the study measured students' conceptual understanding with pre- and posttests after a learning unit. The learning unit provided a motivational introduction to quantum computing using qubit representations and drew partly on the spin first approach, specifically, the section introducing a two-state system. N = 149 students were involved.

Study 3 investigates whether visual qubit representations support students' conceptual understanding in quantum physics and whether two types of qubit representations (the Bloch sphere and Quantum Beads)—especially in the context of quantum technologies—differ in their instructional effectiveness.

To investigate whether the expert-identified differences among the representations translate into differences in students' learning, the study administered application-oriented tasks that required each student to work with the same representation that they had studied during the learning unit. The representation was displayed, and a question was posed, to be answered exclusively by reading the information contained in that representation. The tasks were analyzed for response time and accuracy. The quotient of accuracy and time was determined for the efficiency of the application-oriented tasks. In study 3, conceptual understanding and cognitive load were also measured, then a follow-up posttest was administered 1–2 weeks later to investigated mid-term retention. Taken together, these measures provide a more comprehensive view of how different qubit representations influenced students' learning.

The central results of study 3 show that conceptual understanding did not differ significantly between the experimental groups: both achieved comparable learning gain. But, overall, both groups achieved significant learning gains (Bloch sphere ($V = 238$, $p < .001$); Quantum Bead ($V = 370$, $p < .01$)). A reason could be that, in both groups, the spin-first instruction combined with a visual–graphical qubit representation was implemented. This is reflected in the theoretical approach predicted by Dür and Heusler (2012): a qubit representation combined with the spin-first approach is a promising teaching method for students with limited prior knowledge of quantum physics. However, it should be noted that no control group was included, so these results require further validation.

Process data from the application-oriented tasks, however, revealed that students who worked with the Bloch sphere were markedly more efficient: they responded items on quantum state, quantum measurement, superposition, and amplitude significantly faster without losing accuracy. In the task for phase, there was no difference between the two groups. The accuracy remained similarly high in both groups, yet the Bloch sphere group required substantially less time, resulting in a higher efficiency index (accuracy divided by response time).

Despite this time advantage—contrary to expectations based on Sweller's cognitive-load theory—no significant difference emerged in perceived cognitive load, indicating that the Bloch sphere did not demonstrably differ in its demand on cognitive resources (Sweller et al., 2019). This may also be due to the test instrument; a questionnaire was chosen here that retrospectively (after the learning unit) records the perceived cognitive load (Sweller, 2011). In addition, the evaluation depends on the participants' self-assessment, which can deviate from the actual value when self-perception is difficult. Finally, the mid-term retention test showed that learning remained stable in both groups, with no rapid decline in performance.

To summarize, the results show: They corroborate expert ratings, that task-specific features of the Bloch sphere—most notably the explicit visualization of amplitude, and the conceptual depiction of superposition, quantum measurement and probabilistic behavior—are critical for efficient learning, even though this representation is generally rated as less visually salient.

5.2 Theoretical Implications

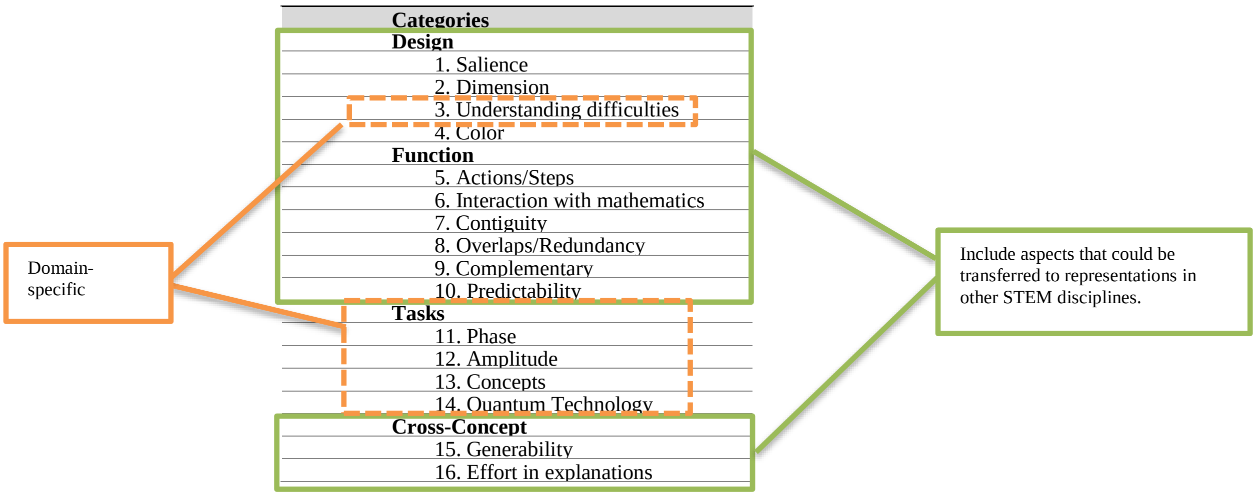
The category system developed draws on three intersecting domains—representation research, quantum education, and quantum science and technology—and can be theoretically implemented in various ways. Most of the features of the category system—design, function, and cross-concepts—capture general representational features that are equally relevant to other STEM areas on (multiple) external representations in biology (e.g., Tsui & Treagust, 2013) or chemistry (e.g., Gilbert & Treagust, 2009). Only the overarching tasks classification is explicitly linked to quantum physics and technology contexts and specifies how a given representation is applied within that domain (see Figure 8).

The expert ratings from Study 1 could be partially verified by Studies 2 and 3. In Study 1, differences between representations were identified from an expert's perspective, illustrating that these features can vary in intensity. In Study 2 and Study 3, the learners' perspective was investigated. In Study 2, the number of transitions increased in the eye-tracking data when visual–graphical representations (the Bloch sphere) were used with other representations, indicating integration attempts. Study 3 investigated specific features in the *tasks* classification from the learners' perspective. Process data were used to verify some of the features evaluated by the experts.

The combination of the category system and these evaluations provides a solid basis for selecting or designing representations in practice in the context of quantum physics. In Study 2, it is assumed that learning with redundant representations can ultimately be used in different context. Ott et al. (2018) used redundant representations in problem solving in mathematics and demonstrated their positive effect, and showed that this can be transferred to other STEM subjects. In the following, the features of the categorization system that theoretically allow transfer to other areas are discussed.

Figure 8

Overview of the transferability of the category system from Table 1



Design features such as salience, spatial dimensionality and the use of color play crucial roles in learning with visual–graphic representations, irrespective of the quantum physics context. Only the "understanding difficulties" category is domain-specific in misconceptions but also includes more general issues like the graphs-as-pictures problem (Garcia Garcia & Cox, 2010).

In the category system, salience describes how much a representation catches the learner's eye and directs their attention (Higgins, 1996). Considering Mayer's signaling principle and the studies from Alpizar et al. (2020) meta-analysis shows that targeted highlighting focuses attention and thus improves learning. For example, studies using static diagrams and spoken explanations have shown that signaling effectively directs learners' attention, even outside the specific quantum context (Jamet, 2014). Study 1 also showed that experts rated representations with the characteristics of the Quantum Bead as significantly more salient, including the strong color contrast from red to green and the round shape. However, the results of study 3 show that these characteristics of the Quantum Bead are not especially relevant to the effectiveness of learning.

Similarly, the spatial dimensions of a representation can be transferred to the category system. Ainsworth (2006) explicitly refers to dimensionality as a key design parameter. Studies show that female learners in particular have greater problems with three-dimensional representations than male learners (Castro-Alonso & Jansen, 2019; Heo & Toomey, 2020; Saha & Halder, 2016). Heo & Toomey (2020) also shows that visual–spatial abilities are a significant predictor of learning success and that male test subjects achieve higher performance regardless of the multimedia representation used. Three-dimensional representations seem, therefore, to require a higher degree of spatial reasoning than two-dimensional ones, which is not equally available to all learners (Heo & Toomey, 2020). Chemistry, for example, uses 3D models to explain bonds and reaction mechanisms at the molecular level (e.g. ball-and-stick or space-filling models), while biology illustrates structure–function relationships over scales from macromolecules to entire cells (e.g. ribbon models for proteins). Vijapurkar et al. (2014) found that pupils had difficulty switching from a 2D cell representation to a 3D one. Dimensionality is, therefore, a critical aspect that should be systematically considered for learning when selecting and designing representations.

Another way in which representations can convey information is through color coding. For this reason, color was included as a feature. However, the additional information processing required by colors increases cognitive load and has a negative effect on learning (Sweller, 1994). For example, both the color and the color mixture (in the case of a representation function) contain information about the underlying content. For example, the ratings from Study 1 show that the Qake model strongly needs to be visualized in color and is, therefore, restrictive, while the Bloch sphere is much more flexible as it can clearly visualize the concepts even without color. In a cell representation in biology, where two cells are connected to each other, the cell model changes specifically to a certain color depending on the connection. This also applies to other connections. This arrangement of color coding is actually very rare in representations, but it is not specific to quantum mechanical situations, but is rather a property that can occur in representations in general.

A similar pattern emerges when the function aspects of the representations were examined. The actions/steps illustrate the cognitive operations needed to visualize domain-specific concepts; they reflect the element interactivity described by Sweller (2010). Element interactivity refers to the portion of intrinsic cognitive load that arises when multiple pieces of information—or elements—must be processed together (Sweller, 2010). The more tightly these elements have to be connected, the higher the element interactivity becomes, because working memory must establish and maintain a greater number of relationships among them (Sweller, 2010).

In this context, the number of actions/steps indicates how many cognitive steps must be performed simultaneously to use a representation to convey a concept, and is therefore applicable to any type and domain of representation.

Additional features such as interaction with mathematics and predictability were included because mathematics plays a central, interdisciplinary role in STEM disciplines, and predictive power is an inherent aspect of modeling, as described in detail by Kircher (2015).

The continuity feature aligns with Mayer's contiguity principle in multimedia learning: content and representations should be positioned to maintain spatial proximity (Mayer, 2021). This principle applies to any domain-specific instructional unit that employs representations. In addition, Noetel et al. (2022) demonstrates the advantage of contiguity in both spatial and temporal terms, referring to the meta-analysis by Ginns (2006), which suggests that this can lead to significant learning gains, particularly when learning materials are complex. It can also be helpful for various complex materials by using external representations in STEM, for example by combining verbal and visual–graphical representations that are spatially close to describe the individual components of the visual representation (e.g., in biology, the cell nucleus and its components are explained through a combination of visual representation and written text.) In addition, the two supplementary categories—Redundancy/Overlap and Complementarity—are derived from Ainsworth's functional aspects for MERs (Ainsworth, 2006); they are designed to capture the general use of multiple representations in STEM contexts, independent of whether quantum physics is involved. Against the context of Mayer's redundancy principle (Mayer & Fiorella, 2014) and the findings from Study 2, it is not yet possible to draw a definitive statement as to whether multiple redundant representations are fundamentally conducive or hindering to learning; at the same time, however, the data do not support argument against a positive effect. The increased number of transitions in the eye-tracking data of groups IG 2 and IG 3 provides initial indications of integration processes (Kragten et al., 2015; O'Keefe et al., 2014). The transferability of these results to other STEM subjects is possible and has already been demonstrated in mathematics by Ott et al. (2018).

In addition, Ainsworth remark on the generatability of representations and their practical feasibility (Ainsworth, 2006, 2008). Generability refers to how easily a representation can be reproduced on the media available in a classroom—whether paper, an iPad, or a whiteboard. The explanatory effort required is another cross-disciplinary criterion, as it strongly influences which representation is ultimately selected, depending on time, targets and resources. Consequently, both generatability and explanatory effort are embedded as core elements within the overarching cross-concepts category of our framework.

All of the categories what were identified can be applied to other disciplines and STEM domains in which representations play a central role, informing both the selection and use of suitable representations. Although the expert rating data were collected specifically for qubit representations, they nevertheless underscore the relevance of these categories; their partially confirmation in Study 2 and Study 3.

The overarching “task” classification contains concrete application points that are invariably domain-specific and thus particularly important. Representations must ultimately be effective within their subject context. As Krey & Schwanewedel (2018, chap. 10, p. 159) note, “without subject-specific representations, scientific ideas and thoughts can only be processed, formulated and communicated to a limited extent”. Consequently, each field should determine which overarching concepts are most relevant to its own domain.

5.3 Practical Implications

The category system we developed is designed to help instructors make informed decisions about which visual–graphical representations to use, based on their instructional goals and their learners’ prior knowledge. Each representation has distinct strengths and limitations. The expert ratings from Study 1 were partially confirmed by Study 2 and Study 3, so the combination of the category system and the expert ratings offers an opportunity for selecting appropriate representations.

Findings from Study 1 suggest that experts perceive the Quantum Bead as more salient than the other qubit representations. Prior research shows that salient objects can guide attention (Parr & Friston, 2019) in learning (Cowan, 1999; Itti & Koch, 2001; Rumbaugh et al., 2007). Quantum Beads have potential to spark learners’ interest in the complex phenomena of quantum physics, drawing their attention to relevant concepts. They are, therefore, an appropriate choice when the primary teaching objective is to stimulate curiosity and initial interest. However, this salience advantage did not translate into greater processing efficiency in Study 3.

Self-determination theory (Deci & Ryan, 2008; Ryan & Deci, 2000) provides clues for building on the initial interest in sustainable motivation and enabling a more intensive engagement with quantum physics. This includes the learner’s need for autonomy, competence and social relatedness (Ryan & Deci, 2000). Meeting these needs requires varied, time-intensive learning opportunities in which students can (a) work independently with the representation, (b) experience mastery of their operations, and (c) present their findings to the class.

For a more efficient teaching unit and learning material, suggest the results of Study 3 that the Bloch sphere is particularly effective in conveying concepts such as *quantum states*, *quantum measurement*, *superposition* and *amplitudes*. Although the Quantum Bead was given a higher rating in the salience category, no compensating effect could be identified. Furthermore, it became clear that certain aspects such as phase difference or amplitude are easier to locate using the vector arrow on the Bloch sphere. This could be one of the reasons why the experts in Study 1 rated the Bloch sphere higher than the Quantum Beads in these aspects. The Bloch sphere helps to highlight aspects relevant to learning and enables learners to extract relevant information more quickly and efficiently than the Quantum Beads representation. The clearer directional cues in the Bloch sphere may also have helped to focus learners' attention more effectively on conceptually relevant elements. The study of Hu, Li, Mong, et al. (2024) had previously shown that the Bloch sphere was helpful for learning quantum computing.

Gegenfurtner et al. (2011) shows that experts focus on one explicit representation when there are several redundant representations, whereas beginners (novices) switch frequently between the redundant representations. These transition processes could indicate integration processes (Alemdag & Cagiltay, 2018), but they could also indicate confusion on the part of the learner, especially if they are at an early stage of learning quantum physics. From the results of Study 2 it can be deduced that, when redundant multiple external representations are employed, instructors could explicitly guide students, with low (novice level) prior knowledge, on when and why to switch between different representations of quantum systems, thereby avoiding unnecessary, non-productive transitions. Building on the findings from Study 2 regarding redundant representations, the learning unit in study 3 was designed so that text and supplementary illustrations were purposefully integrated with the visual qubit representation. Partial redundancy was employed: essential information was intentionally overlapped to illuminate the connections among representations, while each medium also contributed unique additional content. The significant learning gains in both groups in study 3 could indicate the effectiveness of this partially redundant design and indicate that it offers a promising model for introductory instruction in quantum physics and quantum technologies. On the one hand, it can be recommended to introduce or guide novices to different informational redundant representations during lessons in order to promote integration processes and to learn more effectively or, on the other hand, to try to choose partially informational-redundant representations in order to enable students to promote integration processes more independently.

The results of study 3 suggests that designing a learning unit based on the spin-first approach could beneficial—a point already advocated by Dür and Heusler (2012) and further supported by the learning gains observed in both groups of Study 3. To provide students with an accessible entry point into quantum technologies, the curriculum needs to be adapted to spin-first content sequencing (Sadaghiani, 2016; Sadaghiani & Munteanu, 2015). This makes it possible to introduce qubit representations early on (Dür and Heusler 2012). The spin-first approach also offers opportunity to introduce core principles such as quantum measurement, probabilistic reasoning, and complementarity (Müller & Mishina, 2021), which are now already part of many german curriculum (KMK, 2021), while simultaneously placing them in a meaningful and more relevant technology context. Palmgren et al., (2022) shows that spin-first curriculum reforms have already occurred in some countries, such as Finland.

5.4 Limitations

The development of the category system, including expert ratings, provides a theoretical and empirical basis for selecting and evaluating representations in future work. Nevertheless, it cannot be ruled out that other relevant categories exist that have not yet been considered due to the current state of knowledge or future technological developments. For example, the category system developed here only partially respects the interactivity of representations and does not take embodiment aspects into account. Dzsotjan et al. (2021) demonstrates how embodiment and augmented reality have been combined to aid understanding of graphs, in a process described as 'Walk the Graph'. The learner's bodily steps and variations in speed are used to visually project the slope of the graph in AR (Dzsotjan et al., 2021). These aspects may become much more important for learning with representations in the near future.

In addition, only certain of the identified categories could be investigated on learners in studies 2 and 3. In addition, the category of learning difficulties—that can be caused or exacerbated by misconceptions which, in turn, are triggered by visual–graphical representations—have not been studied for their effects on the learners.

In addition, the expert rating was developed by consulting only a relatively small group of experts with varying degrees of expertise in quantum physics and quantum technologies; nevertheless, this assessment serves as the basis for the subsequent studies.

Study 3 confirms the category system in principle, but only considered two representations (the Bloch sphere and Quantum Bead). Unlike in the expert rating, no differences were found for the ‘phase’ task in Study 3. This could be due to the complexity of the test task (phase gate) or to the fact that both representations may have made the phase concept difficult for learners to access. In general, it is hardly possible to examine features in complete isolation; compensatory effects between different properties of a representation can never be completely ruled out in empirical studies.

In study 2, the investigation of redundant representations in the context of the Mach–Zehnder interferometer (MZI) with single photons may have been unclear to the students. The instructions included an illustration of the MZI together with text, formulas and/or Bloch spheres. However, the relevant phenomena in the interference arm can be explained clearly using an explicit wave diagram—a representation that was missing from the learning material of the study, but which has proven to be helpful in other studies (Marshman & Singh, 2017). This limitation suggests that the complexity of the experimental setup, combined with the multiple redundant representations, resulted in overly demanding instructions and suboptimal use of representations.

The process-related analyses in Study 3 were primarily based on processing times, but it cannot be directly determined whether this was due to the processing of the respective representations. Supplementary eye-tracking data could reveal whether and in which area of the visual–graphical representation (e.g., on the Bloch vector) the learners’ attention was actually focused (Holmqvist & Andersson, 2017), thereby further refining the interpretation of the efficiency findings.

The primary aim of Study 3 was not, however, to systematically evaluate the spin-first approach. The learning gain observed in both groups in Study 3 may therefore also be due to factors that were not captured, such as the structure of the learning environment, the multimedia design or motivational influences. No reliable statements can be made about these aspects at present; they require further research.

5.5 Directions for Future Research

One goal pursued by the implementation of external representations is to make learning content more efficient and sustainable (Ainsworth, 2006). The use of external representations that target functional thinking (Ubben & Bitzenbauer, 2022; Ubben & Heusler, 2021) enables the development of coherent mental models in quantum physics. These representations can facilitate learning and improve learners' connectivity. Large language models (LLMs) can be used to promote functional thinking via external representations in a targeted and individualized way. Kasneci et al. (2023) highlights the potential of LLMs for instruction, particularly in the realm of personalized learning. LLMs can analyze students' texts, respond to their answers, and deliver tailor-made feedback precisely matched to the learners' needs (Kasneci et al., 2023). Building on the features developed in the category system, it could be possible to design an interconnected platform that uses AI-driven feedback to meet learners exactly where they are in their (mis)conceptions about quantum systems or quantum objects. Platforms that integrate large language models such as GPT-4o already exist, for example LEAP (Steinert et al., 2024). They allow teachers to formulate tasks in advance and assign them to the LLM so that learners receive formative feedback (Steinert et al., 2024). The category system can be used to classify the inputs and highlight features that are suitable and relevant for the learner, enabling the LLM to initiate conceptual changes step by step (depending on prior knowledge) and offering the potential to promote functional thinking. Such a system could make potential misunderstandings transparent while simultaneously outlining ways to avoid them during the learning process (see Duit, 2020 for handling misconceptions). For example, from a strongly Gestalt-based mental model "photon is a particle/sphere" to a more functional thinking "representations such as the Bloch sphere describe the behavior of a photon and not the photon itself" (Ubben & Bitzenbauer, 2022; Ubben & Heusler, 2021).

Moreover, this approach would also allow representations to be introduced gradually or switched easily, for example, starting with the two-dimensional Bloch sphere, then moving to the three-dimensional version, and finally linking to the underlying mathematics. In this way, the content can be systematically aligned with students' prior knowledge and learning progress without overloading their cognitive resources (Sweller et al., 2019).

Although the use of such AI technologies in the context of quantum physics has so far received little research attention, it holds considerable promise for developing effective and adaptive learning environments.

The introduction of personalized feedback systems creates the opportunity to meet learners where they currently are with their ideas about visualization. Personalized feedback can build functional thinking and stimulate a change of concept if ideas deviate from physically correct direction. Feedback can be provided in different ways—in person by teachers or in interaction with learning partners—but technologies with LLMs offer the potential to respond to learners individually and, indeed, severally at the same time when there is a large audience.

Ainsworth (2006) has already demonstrated that employing—or switching between—multiple representations can effectively support learning. However, even after Study 2 it remains unclear under which conditions and through which mechanisms the successful use of multiple external representations (MERs) is promoted. Study 1 merely suggested that representations of the same dimensionality tend to be perceived as redundant. The category system from Study 1 also explicitly lists the category “contiguity”. In this case, it is intended to describe the spatial contiguity that was not optimal in terms of the positioning of the information-redundant qubit representations used in Study 2. Further studies could investigate whether spatial contiguity exerts an influence on learning with information-redundant representations, or could investigate how different visual–graphical representations of the same dimensionality (2D/3D) can be used most effectively for learners. Using more than two representations appears to offer additional learning potential (Rexigel, Kuhn, et al., 2024). To identify the key mechanisms at play, further research is needed that systematically examines both complementary and redundant combinations of representations.

Study 3 provides process-based data showing that participants worked significantly more efficiently with the Bloch-sphere representation. Eye-tracking could yield more precise insights into how learners extract information from the representations, for example by analyzing fixation durations (individual focus points) and transition processes (shifts between text and visualization) (e.g. Hahn & Klein, 2023). Such gaze data would reveal where learners’ attention (Klein et al., 2020) is directed and clarify whether the Quantum Beads, rated by experts in Study 1 as particularly salient, indeed possess this quality.

This dissertation points to various directions for further research: On the one hand, it concerns supporting learners through formative feedback in order to guide personalized learning with regard to (mis)conceptions through conceptual change to further mental models more in functional thinking. On the other hand, it lends itself to more intensive investigation of the use of multiple external representations and the identification of mechanisms conducive to learning for a change of representation. Finally, for further studies using visual–graphical representations, in addition to time recording in Study 3, parallel recording using eye-tracking data would be useful to enable more concrete connections to be made and to learn more about the representation strategies of learners who solved the tasks more successfully and quickly.

6. Conclusion

This dissertation developed a category system that structures the use of representations in quantum physics and quantum technologies in a way that promotes learning. The expert rating in Study 1 showed that specific features—such as the visualization of amplitude and (relative) phase—as well as more global factors such as learning difficulties and the targeted use of redundant/overlapping representations played a strong role in distinguishing them from each other among the four representations rated (medium to high effect size). Subsequent studies with students provided additional insights into the effect of redundant representations and domain-specific requirements for learning quantum physics and quantum technologies.

In Study 2, no group differences were found in either learning gains or cognitive load. However, the eye-tracking data indicate initial integration processes: with the inclusion of a visual-graphical representation—specifically the Bloch sphere—the number of transitions between the graphical and the other representations increased significantly. Study 3 confirmed that students worked more efficiently on application-oriented tasks with the Bloch sphere; several features of this representation highlighted by experts could thus be partially verified. The Quantum Bead, which experts rated highly for salience, did not, however, yield any measurable advantage in the study. No differences in learning outcome or cognitive load were found here between groups.

Nevertheless, the results illustrate that process-related data do provide deep insights into learning with representations and confirm several assumptions of the category system and expert evaluations.

However, the view is limited, as the expert rating drew only on a small sample of experts (21). The process-related eye-tracking data from Study 2 show that the increased transitions may not have been effective enough in the groups with the Bloch sphere, as there were no learning differences. The implementation of the learning unit from study 3 with the spin-first approach proved successful for both groups, but there was no control group to refer to specifically. This shows that further research in this area is worthwhile.

Further research is needed particularly with regard to the learning difficulties category, which is crucial for the didactic preparation of content, and on how multiple representations can be used in a targeted manner, taking into account the characteristics described in the category system. In the context of quantum physics education, the purposeful use of multiple representations in learning environments can enhance learning, for example by combining visual–graphical representations during experimentation with the simultaneous display of mathematical formulas to illustrate basis states and their superposition.

Well-designed multimedia environments using appropriate (multiple) representations hold great promise for leveraging the unique affordances of learning quantum physics and effectively supporting student learning. The integration of different forms of representation opens up the possibility of designing learning environments in quantum physics that lower the entry barrier and could support conceptual understanding (Ainsworth, 2006; Mayer, 2021; Schnotz, 2005; Sweller et al., 2019).

7. References

Ainsworth, S. (1999). The functions of multiple representations. *Computers & Education, 33*(2), 131–152. https://doi.org/10.1016/S0360-1315(99)00029-9

Ainsworth, S. (2006). DeFT: A conceptual framework for considering learning with multiple representations. *Learning and Instruction, 16*(3), 183–198. https://doi.org/10.1016/j.learninstruc.2006.03.001

Ainsworth, S. (2008). The educational value of multiple representations when learning complex scientific concepts. In J. K. Gilbert, M. Reiner, & M. Nakhleh (Eds.), *Visualization: Theory and practice in science education* (pp. 191–208). Springer Netherlands. https://doi.org/10.1007/978-1-4020-5267-5_9

Alemdag, E., & Cagiltay, K. (2018). A systematic review of eye tracking research on multimedia learning. *Computers & Education*, *125*, 413–428. https://doi.org/10.1016/j.compedu.2018.06.023

Alpizar, D., Adesope, O. O., & Wong, R. M. (2020). A meta-analysis of signaling principle in multimedia learning environments. *Educational Technology Research and Development*, *68*(5), 2095–2119. https://doi.org/10.1007/s11423-020-09748-7

Becker, S., Küchemann, S., Klein, P., Lichtenberger, A., & Kuhn, J. (2022). Gaze patterns enhance response prediction: More than correct or incorrect. *Physical Review Physics Education Research*, *18*(2), 020107. https://doi.org/10.1103/PhysRevPhysEducRes.18.020107

Bertin, J. (1983). *Semiology of graphics*. University of Wisconsin Press.

Bitzenbauer, P. (2020). *Quantenoptik an Schulen: Studie im mixed-methods-Design zur Evaluation des Erlanger Unterrichtskonzepts zur Quantenoptik* (Vol. 303). Logos Verlag Berlin.

Bitzenbauer, P., Faletič, S., Michelini, M., Tóth, K., & Pospiech, G. (2024). Design and evaluation of a questionnaire to assess learners' understanding of quantum measurement in different two-state contexts: The context matters. *Physical Review Physics Education Research, 20*(2), 020136. https://doi.org/10.1103/PhysRevPhysEducRes.20.020136

Bitzenbauer, P., & Ubben, M. S. (2025). *The structure of learners' perceptions of models (not only) in quantum physics: Spotlight on fidelity of gestalt and functional fidelity*. *EPJ Quantum Technology, 12*(1), 16. https://doi.org/10.1140/epjqt/s40507-025-00316-7

Bley, J., Rexigel, E., Arias, A., Krupp, L., Longen, N., Lukowicz, P., Küchemann, S., Kuhn, J., Kiefer-Emmanouilidis, M., & Widera, A. (2025, May 13). *Visualization enhances problem solving in multi-qubit systems: An eye-tracking study* [Preprint]. arXiv. https://arxiv.org/abs/2505.21508

Bley, J., Rexigel, E., Arias, A., Longen, N., Krupp, L., Kiefer-Emmanouilidis, M., Lukowicz, P., Donhauser, A., Küchemann, S., Kuhn, J., & Widera, A. (2024). Visualizing entanglement in multiqubit systems. *Physical Review Research, 6*(2), 023077. https://doi.org/10.1103/PhysRevResearch.6.023077

Bouchée, T., de Putter-Smits, L., Thurlings, M., & Pepin, B. (2022). Towards a better understanding of conceptual difficulties in introductory quantum physics courses. *Studies in Science Education, 58*(2), 183–202.

Brang, M., Franke, H., Greinert, F., Ubben, M. S., Hennig, F., & Bitzenbauer, P. (2024). Spooky action at a distance? A two-phase study into learners' views of quantum entanglement. *EPJ Quantum Technology, 11*(1), Article 1. https://doi.org/10.1140/epjqt/s40507-024-00244-y

Castro-Alonso, J. C., & Jansen, P. (2019). Sex differences in visuospatial processing. In J. C. Castro-Alonso (Hrsg.), *In: Visuospatial processing for education in health and natural sciences* (S. 81–110). Springer International Publishing.

Coban, A., Dzsotjan, D., Küchemann, S., Durst, J., Kuhn, J., & Hoyer, C. (2025). AI support meets AR visualization for Alice and Bob: Personalized learning based on individual ChatGPT feedback in an AR quantum cryptography experiment for physics lab courses. *EPJ Quantum Technology*, *12*(1), 15. https://doi.org/10.1140/epjqt/s40507-025-00310-z

Corsiglia, G., Pollock, S., & Passante, G. (2023). Intuition in quantum mechanics: Student perspectives and expectations. *Physical Review Physics Education Research*, *19*(1), 010109. https://doi.org/10.1103/PhysRevPhysEducRes.19.010109

Coskun, A., & Cagiltay, K. (2022). A systematic review of eye-tracking-based research on animated multimedia learning. *Journal of Computer Assisted Learning*, *38*(2), 581–598. https://doi.org/10.1111/jcal.12629

Cowan, N. (1999). An Embedded-Processes Model of Working Memory. In A. Miyake & P. Shah (Hrsg.), *Models of working memory: mechanisms of active maintenance and executive control* (S. 62–101). Cambridge University Press. https://doi.org/10.1017/CBO9781139174909.006

de Wolf, R. (2017). The potential impact of quantum computers on society. *Ethics and Information Technology*, *19*(4), 271–276. https://doi.org/10.1007/s10676-017-9439-z

Deci, E. L., & Ryan, R. M. (2008). Self-determination theory: A macrotheory of human motivation, development, and health. *Canadian Psychology / Psychologie canadienne*, *49*(3), 182–185. https://doi.org/10.1037/a0012801

Dirac, P. A. M. (1939). A new notation for quantum mechanics. *Mathematical proceedings of the Cambridge philosophical society*, *35*, Article 3.

diSessa, A. A., & Sherin, B. L. (2000). Meta-representation: An introduction. *The Journal of Mathematical Behavior*, *19*(4), 385–398. https://doi.org/10.1016/S0732-3123(01)00051-7

Donhauser, A., Kuhn, J., & others. (2024). *Qubit cake model* [Unpublished manuscript].

Duit, R. (2020). Alltagsvorstellungen und Physik lernen. In E. Kircher, R. Girwidz, & H. E. Fischer (Hrsg.), *Physikdidaktik | Grundlagen* (S. 337–366). Springer. https://doi.org/10.1007/978-3-662-59490-2_9

Dür, W., & Heusler, S. (2012). Was man vom einzelnen Qubit über Quantenphysik lernen kann. *PhyDid A – Physik und Didaktik in Schule und Hochschule*, *11*(1), 11.

Dür, W., & Heusler, S. (2014). Was man von zwei Qubits über Quantenphysik lernen kann: Verschränkung und Quantenkorrelationen *PhyDid A – Physik und Didaktik in Schule und Hochschule*, *1*(13), 13.

Dutke, S. (1994). *Mentale Modelle: Konstrukte des Wissens und Verstehens. Kognitionspsychologische Grundlagen für die Software-Ergonomie*. Verlag für Angewandte Psychologie.

Dzsotjan, D., Ludwig-Petsch, K., Mukhametov, S., Ishimaru, S., Kuechemann, S., & Kuhn, J. (2021). The predictive power of eye-tracking data in an interactive AR learning environment. In *Adjunct proceedings of the 2021 ACM International Joint Conference on Pervasive and Ubiquitous Computing and the 2021 ACM International Symposium on Wearable Computers* (pp. 467–471). https://doi.org/10.1145/3460418.3479358

Ekert, A. (2010). *From interferometers to quantum computers* [Supplementary notes]. International Centre for Theoretical Sciences. https://www.icts.res.in/sites/default/files/1265634590Ekert_QI_Sup_notes.pdf

European Commission, Directorate-General for Communications Networks, Content and Technology, & Quantum Flagship. (2025). *European competence framework for quantum technologies (CFQT): Reference framework for planning, mapping and comparing QT-related educational activities, personal qualification and job requirements* (F. Greinert, R. Müller, S. R. Goorney, R. Laurenza, J. Sherson, & M. S. Ubben, Eds.). Publications Office of the European Union. https://doi.org/10.2759/8917117

Fehringer, B. C. O. F. (2020). Spatial thinking from a different view: Disentangling top-down and bottom-up processes using eye tracking. *Open Psychology*, *2*(1), 138–212. https://doi.org/10.1515/psych-2020-0105

Fischler, H., & Lichtfeldt, M. (1992). Modern physics and students' conceptions. *International Journal of Science Education*, *14*(2), 181–190. https://doi.org/10.1080/0950069920140206

Fürst, M. (2011). *Quantenoptisches Verfahren zur Erzeugung zufälliger Bitfolgen* [Doctoral dissertation, Ludwig-Maximilians-Universität München]. LMU München.

Garcia Garcia, G., & Cox, R. (2010). "Graph-as-Picture" misconceptions in young students. In A. K. Goel, M. Jamnik, & N. H. Narayanan (Hrsg.), *Diagrammatic representation and inference* (S. 310–312). Springer. https://doi.org/10.1007/978-3-642-14600-8_38

Gegenfurtner, A., Lehtinen, E., & Säljö, R. (2011). Expertise differences in the comprehension of visualizations: A meta-analysis of eye-tracking research in professional domains. *Educational Psychology Review*, *23*(4), 523–552. https://doi.org/10.1007/s10648-011-9174-7

Gilbert, J. K., & Treagust, D. F. (2009). Introduction: Macro, submicro and symbolic representations and the relationship between them: Key models in chemical education. In J. K. Gilbert & D. F. Treagust (Eds.), *Multiple representations in chemical education* (pp. 1–8). Springer Netherlands. https://doi.org/10.1007/978-1-4020-8872-8_1

Ginns, P. (2006). Integrating information: A meta-analysis of the spatial contiguity and temporal contiguity effects. *Learning and Instruction*, *16*(6), 511–525. https://doi.org/10.1016/j.learninstruc.2006.10.001

Greinert, F., & Müller, R. (2022). *Qualification profiles for quantum technologies*. https://zenodo.org/records/6834686

Hahn, L., & Klein, P. (2023). Analysis of eye movements to study drawing in the context of vector fields. *Frontiers in Education*, *8*. https://www.frontiersin.org/articles/10.3389/feduc.2023.1162281

Heisenberg, W. (1925). Über quantentheoretische Umdeutung kinematischer und mechanischer Beziehungen. *Zeitschrift für Physik*, *33*(1), 879–893. https://doi.org/10.1007/BF01328377

Hennig, F., Tóth, K., Veith, J., & Bitzenbauer, P. (2024). Introducing quantum physics concepts and Dirac notation at the secondary school level: Insights into student reasoning from an acceptance survey. *Physical Review Physics Education Research*, *20*(2), 020147. https://doi.org/10.1103/PhysRevPhysEducRes.20.020147

Heo, M., & Toomey, N. (2020). Learning with multimedia: The effects of gender, type of multimedia learning resources, and spatial ability. *Computers & Education*, *146*, 103747. https://doi.org/10.1016/j.compedu.2019.103747

Hettmannsperger, R. (2015). *Lernen mit multiplen Repräsentationen aus Experimenten: Ein Beitrag zum Verstehen physikalischer Konzepte*. Springer Fachmedien. https://doi.org/10.1007/978-3-658-07436-4

Higgins, E. T. (1996). Activation: Accessibility, and salience. *Social psychology: Handbook of basic principles*, 133–168.

Holmqvist, K., & Andersson, R. (2017). *Eye tracking: A comprehensive guide to methods, paradigms, and measures*. Oxford University Press.

Hou, X., & Zhang, L. (2006). A time-dependent model of information capacity of visual attention. In I. King, J. Wang, L.-W. Chan, & D. Wang (Eds.), *Neural information processing* (Vol. 4233, pp. 127–136). Springer. https://doi.org/10.1007/11893028_15

Hu, P., Li, Y., Mong, R. S. K., & Singh, C. (2024). Student understanding of the Bloch sphere. *European Journal of Physics*, *45*(2), 025705. https://doi.org/10.1088/1361-6404/ad2393

Hu, P., Li, Y., & Singh, C. (2024). Investigating and improving student understanding of the basics of quantum computing. *Physical Review Physics Education Research*, *20*(2), 020108. https://doi.org/10.1103/PhysRevPhysEducRes.20.020108

Huang, W., Eades, P., & Hong, S.-H. (2009). Measuring effectiveness of graph visualizations: A cognitive load perspective. *Information Visualization*, *8*(3), 139–152. https://doi.org/10.1057/ivs.2009.10

Huber, D., & Glaser, S. J. (2024). BEADS: A canonical visualization of quantum states for applications in quantum information processing (arXiv:2410.01446) [Preprint]. *arXiv*. https://doi.org/10.48550/arXiv.2410.01446

Itti, L., & Koch, C. (2001). Computational modelling of visual attention. *Nature Reviews Neuroscience*, *2*(3), 194–203. https://doi.org/10.1038/35058500

Jamet, E. (2014). An eye-tracking study of cueing effects in multimedia learning. *Computers in Human Behavior*, *32*, 47–53. https://doi.org/10.1016/j.chb.2013.11.013

Johnston, E. R., Harrigan, N., & Gimeno-Segovia, M. (2019). *Programming quantum computers: Essential algorithms and code samples*. Sebastopol, CA: O'Reilly Media, Inc.

Just, B. (2020). *Quantencomputing kompakt: Spukhafte Fernwirkung und Teleportation endlich verständlich*. Springer. https://doi.org/10.1007/978-3-662-61889-9

Kasneci, E., Sessler, K., Küchemann, S., Bannert, M., Dementieva, D., Fischer, F., Gasser, U., Groh, G., Günnemann, S., Hüllermeier, E., Krusche, S., Kutyniok, G., Michaeli, T., Nerdel, C., Pfeffer, J., Poquet, O., Sailer, M., Schmidt, A., Seidel, T., … Kasneci, G. (2023). ChatGPT for good? On opportunities and challenges of large language models for education. *Learning and Individual Differences*, *103*, 102274. https://doi.org/10.1016/j.lindif.2023.102274

Kelly, A. M., Darienzo, M., Wei, T.-C., & Schneble, D. (2024). Quantum information science and technology professional learning for secondary science, technology, engineering, and mathematics teachers. *Physical Review Physics Education Research*, *20*(2), 020154. https://doi.org/10.1103/PhysRevPhysEducRes.20.020154

Kircher, E. (2015). Modellbegriff und Modellbildung in der Physikdidaktik. In E. Kircher, R. Girwidz, & P. Häußler (Hrsg.), *Physikdidaktik: Theorie und Praxis* (S. 783–807). Springer. https://doi.org/10.1007/978-3-642-41745-0_27

Klein, P., Küchemann, S., Susac, A., Karabulut, A., Bubic, A., Planinic, M., Palmovic, M., & Kuhn, J. (2021). Students' understanding of diagrams in different contexts: Comparison of eye movements between physicists and non-physicists using eye-tracking. In I. Devetak & S. A. Glažar (Eds.), *Applying bio-measurements methodologies in science education research* (pp. 243–260). Springer International Publishing. https://doi.org/10.1007/978-3-030-71535-9_12

Klein, P., Lichtenberger, A., Küchemann, S., Becker, S., Kekule, M., Viiri, J., Baadte, C., Vaterlaus, A., & Kuhn, J. (2020). Visual attention while solving the test of understanding graphs in kinematics: An eye-tracking analysis. *European Journal of Physics*, *41*(2), 025701. https://doi.org/10.1088/1361-6404/ab5f51

Klepsch, M., Schmitz, F., & Seufert, T. (2017). Development and Validation of Two Instruments Measuring Intrinsic, Extraneous, and Germane Cognitive Load. *Frontiers in Psychology*, *8*. https://doi.org/10.3389/fpsyg.2017.01997

Kohnle, A., Ainsworth, S. E., & Passante, G. (2020). Sketching to support visual learning with interactive tutorials. *Physical Review Physics Education Research*, *16*(2), Article 020139. https://doi.org/10.1103/PhysRevPhysEducRes.16.020139

Kohnle, A., Baily, C., & Ruby, S. (2015). Investigating the influence of visualization on student understanding of quantum superposition. *2014 Physics Education Research Conference Proceedings*, 139–142. https://doi.org/10.1119/perc.2014.pr.031

Kosslyn, S. M. (1989). Understanding charts and graphs. *Applied Cognitive Psychology*, *3*(3), 185–225. https://doi.org/10.1002/acp.2350030302

Kozma, R. B., & Russell, J. (1997). Multimedia and understanding: Expert and novice responses to different representations of chemical phenomena. *Journal of Research in Science Teaching*, *34*(9), 949–968. https://doi.org/10.1002/(SICI)1098-2736(199711)34:9<949::AID-TEA7>3.0.CO;2-U

Kragten, M., Admiraal, W., & Rijlaarsdam, G. (2015). Students' learning activities while studying biological process diagrams. *International Journal of Science Education*, *37*(12), 1915–1937. https://doi.org/10.1080/09500693.2015.1057775

Krey, O., & Schwanewedel, J. (2018). Lernen mit externen Repräsentationen. In D. Krüger, I. Parchmann, & H. Schecker (Hrsg.), *Theorien in der naturwissenschaftsdidaktischen Forschung* (S. 159–175). Springer. https://doi.org/10.1007/978-3-662-56320-5_10

Krieglstein, F., Beege, M., Rey, G. D., Ginns, P., Krell, M., & Schneider, S. (2022). A systematic meta-analysis of the reliability and validity of subjective cognitive load questionnaires in experimental multimedia learning research. *Educational Psychology Review*, *34*(4), 2485–2541. https://doi.org/10.1007/s10648-022-09683-4

Krijtenburg-Lewerissa, K., Pol, H. J., Brinkman, A., & van Joolingen, W. R. (2017). Insights into teaching quantum mechanics in secondary and lower undergraduate education. *Physical Review Physics Education Research*, *13*(1), 010109. https://doi.org/10.1103/PhysRevPhysEducRes.13.010109

Küblbeck, J., & Müller, R. (2002). *Die Wesenszüge der Quantenphysik: Modelle, Bilder, Experimente*. Aulis-Verlag Deubner.

Küchemann, S., Ubben, M., Dzsotjan, D., Mukhametov, S., Weidner, C. A., Qerimi, L., Kuhn, J., Heusler, S., & Sherson, J. F. (2023). *The impact of an interactive visualization and simulation tool on learning quantum physics: Results of an eye-tracking study* (ArXiv:2302.06286). https://doi.org/10.48550/arXiv.2302.06286

Lautesse, P., Vila Valls, A., Ferlin, F., Héraud, J.-L., & Chabot, H. (2015). Teaching quantum physics in upper secondary school in France. *Science & Education*, *24*(7), 937–955. https://doi.org/10.1007/s11191-015-9755-9

Lemke, J. (1998). Multiplying meaning. In J. R. Martin & R. Veel (Eds.), *Reading science: Critical and functional perspectives on discourses of science* (pp. 87–113). Routledge.

Lemke, J. L. (1998). Teaching all the languages of science: Words, symbols, images, and actions. In *Proceedings of the Conference on Science Education in Barcelona* (pp. 483–492).

Lichtfeldt, M. (1992). *Schülervorstellungen in der Quantenphysik und ihre möglichen Veränderungen durch Unterricht*. Westarp-Wiss.

Likert, R. (1932). A technique for the measurement of attitudes. *Archives of Psychology*.

Lindner, M. A., Eitel, A., Barenthien, J., & Köller, O. (2021). An integrative study on learning and testing with multimedia: Effects on students' performance and metacognition. *Learning and Instruction*, *71*, 101100. https://doi.org/10.1016/j.learninstruc.2018.01.002

Majidy, S. (2024). *Addressing misconceptions in university physics: A review and experiences from quantum physics educators* (arXiv:2405.20923). arXiv. https://doi.org/10.48550/arXiv.2405.20923

Marshman, E., Maries, A., & Singh, C. (2024). Using multiple representations to improve student understanding of quantum states. *Physical Review Physics Education Research*, *20*(2), 020152. https://doi.org/10.1103/PhysRevPhysEducRes.20.020152

Marshman, E., & Singh, C. (2017). Investigating and improving student understanding of quantum mechanics in the context of single photon interference. *Phys. Rev. Phys. Educ. Res.*, *13*(1), 010117. https://doi.org/10.1103/PhysRevPhysEducRes.13.010117

Mayer, R. E. (2010). Unique contributions of eye-tracking research to the study of learning with graphics. *Learning and Instruction, 20*(2), 167–171. https://doi.org/10.1016/j.learninstruc.2009.02.012

Mayer, R. E. (2014). *The Cambridge Handbook of Multimedia Learning*. Cambridge University Press.

Mayer, R. E. (2021). Cognitive theory of multimedia learning. In R. E. Mayer & L. Fiorella (Hrsg.), *The Cambridge handbook of multimedia learning* (S. 57–72). Cambridge University Press. https://doi.org/10.1017/9781108894333.008

Mayer, R. E., & Fiorella, L. (2014). Principles for reducing extraneous processing in multimedia learning: Coherence, signaling, redundancy, spatial contiguity, and temporal contiguity principles. In R. E. E. Mayer (Hrsg.), *The Cambridge handbook of multimedia learning* (S. 279–315). Cambridge University Press.

McGrath, C. H., Guerin, B., Harte, E., Frearson, M., & Manville, C. (2015). Learning gain in higher education. *Santa Monica, CA: RAND Corporation*.

Merzel, A., Bitzenbauer, P., Krijtenburg-Lewerissa, K., Stadermann, K., Andreotti, E., Anttila, D., Bondani, M., Chiofalo, M. L. (Marilù), Faletič, S., Frans, R., Goorney, S., Greinert, F., Jurčić, L., Koupilová, Z., Malgieri, M., Müller, R., Onorato, P., Pospiech, G., Ubben, M., … Pol, H. (2024). The core of secondary level quantum education: A multi-stakeholder perspective. *EPJ Quantum Technology, 11*(1), 1–28. https://doi.org/10.1140/epjqt/s40507-024-00237-x

Miller, G. A. (1956). The magical number seven, plus or minus two: Some limits on our capacity for processing information. *Psychological Review, 63*(2), 81–97. https://doi.org/10.1037/h0043158

Moreno, R., & Mayer, R. (2007). Interactive Multimodal Learning Environments. *Educational Psychology Review*, *19*(3), 309–326. https://doi.org/10.1007/s10648-007-9047-2

Müller, R. (2006). Kontextorientierung und alltagsbezug. In H. F. Mikelskis (Hrsg.), *Physikdidaktik* (S. 102–118). Cornelsen.

Müller, R., & Greinert, F. (2024). *Quantum technologies*. De Gruyter Oldenbourg. https://doi.org/doi:10.1515/9783110717457
Müller, R., & Mishina, O. (2021). Milq—Quantum physics in secondary school. In B. Jarosievitz & C. Sükösd (Hrsg.), *Teaching-learning contemporary physics: From research to practice* (S. 33–45). Springer International Publishing. https://doi.org/10.1007/978-3-030-78720-2_3
Müller, R., & Wiesner, H. (1999). Students' conceptions of quantum physics. *Papers presented at the annual meeting national association for research in science teaching march, 1999*, 20.
Noetel, M., Griffith, S., Delaney, O., Harris, N. R., Sanders, T., Parker, P., del Pozo Cruz, B., & Lonsdale, C. (2022). Multimedia design for learning: An overview of reviews with meta-meta-analysis. *Review of educational research*, *92*(3), 413–454. https://doi.org/10.3102/00346543211052329
O'Brien, J. L., Pryde, G. J., White, A. G., Ralph, T. C., & Branning, D. (2003). Demonstration of an all-optical quantum controlled-NOT gate. *Nature, 426*(6964), 264–267. https://doi.org/10.1038/nature02054
O'Keefe, P. A., Letourneau, S. M., Homer, B. D., Schwartz, R. N., & Plass, J. L. (2014). Learning from multiple representations: An examination of fixation patterns in a science simulation. *Computers in Human Behavior*, *35*, 234–242. https://doi.org/10.1016/j.chb.2014.02.040
Ott, N., Brünken, R., Vogel, M., & Malone, S. (2018). Multiple symbolic representations: The combination of formula and text supports problem solving in the mathematical field of propositional logic. *Learning and Instruction, 58*, 88–105. https://doi.org/10.1016/j.learninstruc.2018.04.010
Ouwehand, K., van der Kroef, A., Wong, J., & Paas, F. (2021). Measuring cognitive load: Are there more valid alternatives to Likert rating scales? *Frontiers in Education, 6*, Article 702616. https://doi.org/10.3389/feduc.2021.702616
Özcan, O. (2011). What are the students' mental models about the "spin" and "photon" concepts in modern physics? *Procedia - Social and Behavioral Sciences, 15*, 1372–1375. https://doi.org/10.1016/j.sbspro.2011.03.295
Paivio, A. (1990). *Mental Representations: A Dual Coding Approach*. Oxford University Press.
Palmgren, E., Tuominen, K., & Kontro, I. (2022). Self-efficacy and conceptual knowledge in quantum mechanics during teaching reforms and the COVID-19 pandemic. *Physical Review Physics Education Research*, *18*(2), 020122. https://doi.org/10.1103/PhysRevPhysEducRes.18.020122
Parr, T., & Friston, K. J. (2019). Attention or salience? *Current Opinion in Psychology*, *29*, 1–5. https://doi.org/10.1016/j.copsyc.2018.10.006
Piña, A. R., El-Adawy, S., Verostek, M., Boyle, B. T., Cacheiro, M., Lawler, M., Pradeep, N., Watts, E., West, C. G., Lewandowski, H. J., & Zwickl, B. M. (2025, April 18). *Landscape of Quantum Information Science and Engineering Education: From Physics Foundations to Interdisciplinary Frontiers*. arXiv.Org. https://arxiv.org/abs/2504.13719v1
Qerimi, L., Malone, S., Rexigel, E., Mehlhase, S., Kuhn, J., & Küchemann, S. (2025). Exploring the mechanisms of qubit representations and introducing a new category system for visual representations: Results from expert ratings. *EPJ Quantum Technology*, *12*(1), Article 1. https://doi.org/10.1140/epjqt/s40507-025-00346-1

QuVis. (n.d.). *QuVis: Quantum Mechanics Visualization Project.* Retrieved June 20, 2024, from https://www.st-andrews.ac.uk/physics/quvis/

Rau, M. A. (2017). Conditions for the effectiveness of multiple visual representations in enhancing STEM learning. *Educational Psychology Review*, *29*(4), 717–761. https://doi.org/10.1007/s10648-016-9365-3

Rexigel, E., Bley, J., Arias, A., Qerimi, L., Küchemann, S., Kuhn, J., & Widera, A. (2025). Investigating the use of multiple representations in university courses on quantum technologies. *EPJ Quantum Technology*, *12*(1), Article 22. https://doi.org/10.1140/epjqt/s40507-025-00327-4

Rexigel, E., Kuhn, J., Becker, S., & Malone, S. (2024). The More the Better? A Systematic Review and Meta-Analysis of the Benefits of More than Two External Representations in STEM Education. *Educational Psychology Review*, *36*(4), 124. https://doi.org/10.1007/s10648-024-09958-y

Rumbaugh, D. M., King, J. E., Beran, M. J., Washburn, D. A., & Gould, K. L. (2007). A salience theory of learning and behavior: With perspectives on neurobiology and cognition. *International Journal of Primatology*, *28*(5), 973–996. https://doi.org/10.1007/s10764-007-9179-8

Ryan, R. M., & Deci, E. L. (2000). Intrinsic and extrinsic motivations: Classic definitions and new directions. *Contemporary Educational Psychology*, *25*(1), 54–67. https://doi.org/10.1006/ceps.1999.1020

Sadaghiani, H. R. (2016). Spin first vs. Position first instructional approaches to teaching introductory quantum mechanics. *2016 Physics Education Research Conference Proceedings*, 292–295. https://doi.org/10.1119/perc.2016.pr.068

Sadaghiani, H. R., & Munteanu, J. (2015). Spin first instructional approach to teaching quantum mechanics in sophomore level modern physics courses. *2015 Physics Education Research Conference Proceedings*, 287–290. https://doi.org/10.1119/perc.2015.pr.067

Saha, S., & Halder, S. (2016). *He or she: Does gender affect various modes of instructional visual design?* Texas State University, Center for Diversity and Gender Studies.

Schewior, L., & Lindner, M. A. (2024). Revisiting picture functions in multimedia testing: A systematic narrative review and taxonomy extension. *Educational Psychology Review*, *36*(2), Article 49. https://doi.org/10.1007/s10648-024-09883-0

Schnotz, W. (2001). Kognitive Prozesse bei der sprach- und bildgestützten Konstruktion mentaler Modelle. In L. Sichelschmidt & H. Strohner (Hrsg.), *Sprache, Sinn und Situation: Festschrift für Gert Rickheit zum 60. Geburtstag* (S. 43–57). Deutscher Universitätsverlag. https://doi.org/10.1007/978-3-663-08962-9_3

Schnotz, W. (2005). An integrated model of text and picture comprehension. *The Cambridge handbook of multimedia learning*, *49*(2005), 69.

Schnotz, W. (2014). An integrated model of text and picture comprehension. In R. E. Mayer (Ed.), *The Cambridge handbook of multimedia learning* (2nd ed., pp. 72–103). Cambridge University Press.

Schrödinger, E. (1926). Quantisierung als Eigenwertproblem (Zweite Mitteilung). *Annalen der Physik*, *384*(4), 361–376. https://doi.org/10.1002/andp.19263840404

Schumacher, B. (1995). Quantum coding. *Physical Review A*, *51*(4), 2738–2747. https://doi.org/10.1103/PhysRevA.51.2738

Seufert, T. (2003). Supporting coherence formation in learning from multiple representations. *Learning and Instruction*, *13*(2), 227–237. https://doi.org/10.1016/S0959-4752(02)00022-1

Singh, C., & Marshman, E. (2015). Review of student difficulties in upper-level quantum mechanics. *Physical Review Special Topics - Physics Education Research, 11*(2), Article 020117. https://doi.org/10.1103/PhysRevSTPER.11.020117

Stadermann, K., & van den Berg, E. (2019). Analysis of secondary school quantum physics curricula of 15 different countries: Different perspectives on a challenging topic. *Phys. Rev. Phys. Educ. Res.*, *15*(1), 010130. https://doi.org/10.1103/PhysRevPhysEducRes.15.010130

Steinert, S., Avila, K. E., Ruzika, S., Kuhn, J., & Küchemann, S. (2024). Harnessing large language models to develop research-based learning assistants for formative feedback. *Smart Learning Environments*, *11*(1), 62. https://doi.org/10.1186/s40561-024-00354-1

Sweller, J. (1988). Cognitive load during problem solving: Effects on learning. *Cognitive Science*, *12*(2), 257–285. https://doi.org/10.1207/s15516709cog1202_4

Sweller, J. (1994). Cognitive load theory, learning difficulty, and instructional design. *Learning and Instruction*, *4*(4), 295–312. https://doi.org/10.1016/0959-4752(94)90003-5

Sweller, J. (2010). Element interactivity and intrinsic, extraneous, and germane cognitive load. *Educational Psychology Review, 22*(2), 123–138. https://doi.org/10.1007/s10648-010-9128-5

Sweller, J., Ayres, P., & Kalyuga, S. (2011). Measuring cognitive load. In *Cognitive load theory* (S. 71–85). Springer New York. https://doi.org/10.1007/978-1-4419-8126-4_6

Sweller, J., van Merriënboer, J. J. G., & Paas, F. (2019). Cognitive architecture and instructional design: 20 years later. *Educational Psychology Review, 31*(2), 261–292. https://doi.org/10.1007/s10648-019-09465-5

Sweller, J., van Merrienboer, J. J. G., & Paas, F. G. W. C. (1998). Cognitive Architecture and Instructional Design. *Educational Psychology Review*, *10*(3), 251–296. https://doi.org/10.1023/A:1022193728205

Thees, M., Kapp, S., Strzys, M. P., Beil, F., Lukowicz, P., & Kuhn, J. (2020). Effects of augmented reality on learning and cognitive load in university physics laboratory courses. *Computers in Human Behavior*, *108*, 106316. https://doi.org/10.1016/j.chb.2020.106316

Tsui, C.-Y., & Treagust, D. F. (2013). Introduction to multiple representations: Their importance in biology and biological education. In D. F. Treagust & C.-Y. Tsui (Eds.), *Multiple representations in biological education* (pp. 3–18). Springer Netherlands. https://doi.org/10.1007/978-94-007-4192-8_1

Ubben, M. S., & Bitzenbauer, P. (2022). Two cognitive dimensions of students' mental models in science: Fidelity of gestalt and functional fidelity. *Education Sciences*, *12*(3), 163. https://doi.org/10.3390/educsci12030163

Ubben, M. S., & Heusler, S. (2021). Gestalt and functionality as independent dimensions of mental models in science. *Research in Science Education, 51*(5), 1349–1363. https://doi.org/10.1007/s11165-019-09892-y

Van Gog, T., & Jarodzka, H. (2013). Eye tracking as a tool to study and enhance cognitive and metacognitive processes in computer-based learning environments. In R. Azevedo & V. Aleven (Eds.), *International handbook of metacognition and learning technologies* (pp. 143–156). Springer. https://doi.org/10.1007/978-1-4419-5546-3_10

Vermunt, J. D., Ilie, Sonia, & and Vignoles, A. (2018). Building the foundations for measuring learning gain in higher education: A conceptual framework and measurement instrument. *Higher Education Pedagogies*, *3*(1), 266–301. https://doi.org/10.1080/23752696.2018.1484672

Vijapurkar, J., Kawalkar, A., & Nambiar, P. (2014). What do cells really look like? An inquiry into students' difficulties in visualising a 3-D biological cell and lessons for pedagogy. *Research in Science Education, 44*(2), 307–333. https://doi.org/10.1007/s11165-013-9379-5

von der Gracht, H. A. (2012). Consensus measurement in Delphi studies: Review and implications for future quality assurance. *Technological Forecasting and Social Change, 79*(8), 1525–1536. https://doi.org/10.1016/j.techfore.2012.04.013

Waitzmann, M. (2023). *Wirkung eines quantenoptischen Realexperiments auf das physikalische Reasoning* [Doctoral thesis, Hannover : Institutionelles Repositorium der Leibniz Universität Hannover]. https://doi.org/10.15488/14065

Waitzmann, M., Scholz, R., & Wessnigk, S. (2024). Testing quantum reasoning: Developing, validating, and application of a questionnaire. *Physical Review Physics Education Research*, *20*(1), 010122. https://doi.org/10.1103/PhysRevPhysEducRes.20.010122

Wawro, M., Watson, K., & Christensen, W. (2020). Students' metarepresentational competence with matrix notation and Dirac notation in quantum mechanics. *Physical Review Physics Education Research*, *16*(2), 020112. https://doi.org/10.1103/PhysRevPhysEducRes.16.020112

Weidner, C. A., Ahmed, S. Z., Jensen, J. H., & Sherson, J. F. (2021). *Publications using Quatomic software*.

Wiesner, H. (1996). Verständnisse von Leistungskursschülern über Quantenphysik. *Physik in der Schule, 34*(3), 4.

Yeung, K. (2020). *Quantum computing & some physics: The quantum computing comics notebook.* Verlag nicht ermittelbar. https://books.google.de/books?id=3LEVzgEACAAJ https://doi.org/10.1097/00004010-200101000-00006

Zinn, J., Zalokowski, A., & Hunter, L. (2001). Identifying indicators of laboratory management performance: A multiple constituency approach. *Health Care Management Review, 26*(1), 40–49. https://doi.org/10.1097/00004010-200101000-00006