Strategic Interactions on Party and Digital Hybrid Platforms

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Für mein ‘Zuhause’.

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Preface

"Models are never true;
but there is truth in models. [...]
We can understand the world only by simplifying it."
Dani Rodrik

This dissertation contains three essays from seemingly unrelated fields of economics: political economy and industrial organization. To provide a deep understanding of how competition is organized in these distinct settings, I apply fundamental concepts of non-cooperative game theory to both fields. I set up stylized models containing the essential competitive dynamics of the issues under investigation to derive answers to complex real-world questions. Throughout, I search for Nash equilibria (and their refinements) to explain puzzling observations and, ultimately, derive far-reaching conclusions from them.

A further feature shared by all essays is their direct or indirect reference to politics. The first chapter describes the electoral competition between two ideological parties. It provides an explanation for why parties are offering polarized platforms - a question which, ironically, becomes non-trivial when considered from an economist's perspective. The other chapters investigate the business model of hybrid platforms that provide a marketplace where they act as direct competitors alongside hosted third-party retailers. These essays are particularly relevant from a policy perspective as they yield insightful implications for regulators.

The third parallel has already been implicated by the title and by what has been set out so far. All essays refer to 'platforms' to describe very different objects. Do not let this confuse you.

Chapter 1: This chapter outlines a voting model with two ideologically opposed parties that compete by offering a multidimensional platform. A reduced form of electoral competition is modeled where parties compete by choosing a cumulative distribution function over ideological positions. Furthermore, it is assumed that voters act 'boundedly rational' as they rely on a heuristic that lets them evaluate each party's platform based on randomly sampled ideologies. This framework enables me to explain electoral competition and demonstrate that the dispersion of voter ideologies and the parties' own preferences determine in a complementary way to what extent party platforms are polarized in equilibrium. If the ideological preferences of the electorate are relatively more dispersed across political dimensions than parties are ideologically polarized, the parties' ideological bliss points directly determine equilibrium platform polarization. Otherwise, the dispersion of voter ideologies is crucial. For a given level of voter ideology dispersion, equilibrium polarization depends on the freedom parties have to 'obfuscate' the electorate by randomizing over ideological positions in their platforms.

Chapter 2: In this chapter, which is joint work with Felix Montag, we analyze how hybrid platforms that operate as a marketplace and a retailer leverage their access to third-party demand data to gain a competitive advantage over independent retailers. We set up a theoretical model to explain how platforms learn about demand by hosting third-party retailers. Subsequently, it is shown how policy interventions that protect third-party competitors affect consumer surplus, profits and overall welfare. The platform hosts a specialized producer to learn about the demand to possibly inform the launch of a private label product at a later stage. If the platform launches its own product version, it has an incentive to foreclose its marketplace to the competing specialist. We show that allowing a platform to learn about demand from third-party sellers and eventually launch a competing product can affect consumer surplus in either direction, depending on the competitive environment. Finally, we elaborate on how policy interventions such as break-ups, line of business restrictions and mandatory access regulation can benefit consumers. From this analysis, it becomes evident that a regulator faces a trade-off between maximizing consumer surplus or overall welfare, where the latter is positively correlated with the profit of a specialized retailer. The trade-off becomes less severe if there exists an additional fringe seller.

Chapter 3: The third chapter further investigates the business model of hybrid platforms. Data-enabled learning makes shopping on digital platforms more convenient and allows dominant hybrid platforms to obtain superior knowledge about expected on-platform demand. These circumstances create a competitive advantage if a platform informs its launching decision with demand data from third-party sales and simultaneously offers a form of insurance to third-party retailers through an exclusive distribution agreement. By entering an exclusive contract, a specialized retailer is guaranteed to continue operations as a monopolistic supplier. Therefore, a retailer is no longer threatened to create a future competitor after revealing on-platform demand through being hosted. By offering an exclusive contract, the platform can also capture an additional rent, even in conditions where it could never profitably launch an in-house produced product. Furthermore, entering an exclusive distribution agreement constitutes a pareto-efficient solution to an adverse selection problem that occurs if the specialized retailer is otherwise (in the absence of an exclusive distribution offer) not hosted. Based on these results, this chapter discusses the impact of two contrary policies that both resolve the information asymmetry. A policy that raises the specialist's information level to that of the platform outperforms a policy that erases the platform's information advantage by restricting its access to (consumer) data in most conditions. It becomes evident, however, that in comparison to the analyzed policies, an unregulated market is already quite competitive. As a consequence, regulators face a trade-off between maximizing consumer surplus or overall welfare. Specialized retailers can also be expected to lobby for different policies than what is beneficial for consumers.

In summary, this dissertation offers new insights into the driving forces behind the determinants and effects of competition in different settings. The mechanisms uncovered in Chapter 1 may hopefully contribute to a better understanding of electoral competition, allowing observers of the political process to draw more accurate conclusions. Chapters 2 and 3 serve two interdependent purposes. They provide an in-depth description of key aspects of the business model of hybrid platforms that might inform market-relevant decisions of affiliated stakeholders. Likewise, the results are relevant when designing economic policies that help to solve current and future regulatory challenges for the benefit of society.

Chapter 1

What Determines Platform Polarization? A Theory Based on Heuristic Voting and Ideological Parties

1.1 Introduction

What determines the political polarization of party platforms? This question became increasingly popular again since party polarization in the US increased over the last decades and now appears to be at an all-time high (McCarty, Poole, and Rosenthal, 2016). Abramowitz and Saunders (2008) and Fiorina and Abrams (2008), however, find conflicting arguments if the ideological polarization of the electorate has likewise increased. Given this lack of clear evidence, the observed polarization appears even more surprising.

I want to shed light on this puzzling observation by modeling a reduced form of multidimensional electoral competition and boundedly rational voters.[1] This paper answers the following research questions: How does the polarization of average party platforms depend on the dispersion of voter ideologies across different issues and the parties' own ideological preferences? What mechanisms drive the relationship? What is the distribution

[1]It should be mentioned here that the model in this paper is based on an idea that can be found in a significantly simplified version in Schader (2018). Schader (2018) essentially describes a limiting scenario of the model presented here, where $b = 0$ and $X_i = 1$, and answers the question of whether these conditions lead to polarized platforms. Results from Schader (2018) also partially correspond to the analysis in Appendix A.2 when platform polarization is not driven by the parties' ideological preferences.

of ideologies that shapes a party's platform in equilibrium? When is a polarized two-party system robust towards the entry of a third party?

To ultimately justify the assumed voting behavior, I leverage two stylized facts. First, it is documented on various occasions that heuristics to simplify complex decision problems determine the outcome and nature of electoral competition.[2] With the so-called 'Take-Best' heuristic (Gigerenzer and Goldstein, 1996), for instance, the electorate evaluates a party's platform based on very few (or a single) political issues.[3] Second, random events impact the importance of distinct issues, which lets a party's position on a specific issue become more or less relevant during an electoral campaign.[4]

Based on these stylized facts, I set up a model where two ideological parties choose a platform that spans a continuum of political issues (henceforth: dimensions). The platform itself can be ranked on the left-right spectrum. Instead of modeling an explicit mapping from dimensions to ideologies,[5] I leverage a reduced form approach of electoral competition where parties compete by effectively choosing a distribution over ideological positions. Therefore, the average ideology of the equilibrium distribution determines the degree to which platforms are polarized. On the side of the electorate, I exploit the assumption that voters act boundedly rational in the following way: Voters evaluate the parties' platforms based on a triplet of randomly sampled ideologies from each party's distribution function and their own distribution of ideological preferences.

Equilibrium platform polarization is determined in a complementary way by the parties' and the electorate's ideological preferences. If, relative to the parties' ideological preferences, the dispersion of voter ideologies is large, the former crucially determines platform

[2]Examples include: Canes-Wrone, Brady, and Cogan (2002), Ansolabehere and Jones (2010), Jones (2011), Jessee (2012) and Shor and Rogowski (2018).

[3]Issue voting is well documented: Madson (2021), Lau, Kleinberg, and Ditonto (2018), Hanretty, Lauderdale, and Vivyan (2020), Flavin and Law (2022), Rice, Schaffner, and Barney (2021), König and Waldvogel (2021) and Leeper and Robison (2020).

[4]Examples include severe natural disasters (e.g. Masiero and Santarossa (2021)), weather and climate conditions (e.g. Gasper and Reeves (2011), Ramos and Sanz (2020) and Liao and Junco (2022)), public health crisis (e.g. Fernandez-Navia, Polo-Muro, and Tercero-Lucas (2021), Beall, Hofer, and Schaller (2016)) economic shocks (e.g. Wolfers et al. (2002)) or terrorist attacks (e.g. Montalvo (2011), Peri, Rees, and Smith (2020)). The outlined story is also valid if alternatively assuming that the decisive issue is determined as good as random from the parties' perspective, for instance, through media coverage (McCombs and Shaw (1972) and Dearing, Rogers, and Rogers (1996)).

[5]It can be shown that there does not always exist an equilibrium in pure strategies in an explicit model.

polarization. Otherwise, the ideological preferences of the electorate are decisive. I find that party platforms never converge to the ideology position that decides the election in expectation, even in conditions where the uncertainty about voter preferences vanishes. The underlying reason is that, due to the applied heuristic, parties may not be able to predict the political ideologies from their own and their opponent's distribution function that determine the outcome of the election. Hence, they can leverage the possibility to obfuscate the electorate by randomizing across ideologies to offer polarized platforms. Likewise, if the dispersion of voter ideologies is sufficiently large, polarized platforms occur even in conditions where parties' play degenerate distributions such that there exists no uncertainty about which dimension is drawn by the voter. In conclusion, all three sources of uncertainty about the triplet of sampled ideologies that informs the voting decision determine to what extent platforms are polarized in equilibrium, also independently of each other.

In this model, each party is ideological in the sense that it has preferences over the average platform position of the winning party. The parties' ideological bliss points are opposed on the left-right spectrum. As they compete by playing a cumulative distribution function (henceforth: cdf) over ideological positions, each *cdf* indicates the share of political dimensions in a party's platform that is associated with a (weakly) more 'leftish' ideology than a given ideology that enters the *cdf* as an argument.

Voter ideologies are uniformly distributed around the center of the ideology spectrum, which is known to both parties. The dispersion of voter ideologies is measured by the length of the support of the uniform distribution. Due to the applied heuristic, both parties do not understand on what basis the electorate analyzes their platforms. From their perspective, nature randomly draws three sample points from the electorate's ideology distribution and from both parties' *cdfs* that decide the election.

When choosing a *cdf*, each party faces a trade-off between including ideologies closer to the center of the ideological spectrum to maximize the probability of winning the election and more extreme ideologies to decrease the distance between its average platform position and its preferred bliss-point conditional on winning the election.

Finally, I allow a purely office-motivated third party to enter the electoral competition to test the stability of the outlined two-party system.

I find that parties split up the ideological space between each other. A party whose ideology bliss-point is on the left does not include ideological positions from the right side of the spectrum in its distribution and vice versa. The parties' ideological bliss points and the dispersion of voter ideologies determine the polarization of average platform positions in a complementary way. If the former is decisive, equilibrium polarization and party preferences are positively correlated. Otherwise, the equilibrium polarization depends on how strongly platforms can leverage their ability to 'obfuscate' the electorate by playing a non-degenerate ideology distribution that randomizes across different ideologies. For sufficiently dispersed voter ideologies, parties are not restricted when choosing their platforms. Consequentially, platform polarization and the dispersion of voter ideologies are positively correlated. Finally, the results are robust towards opportunistic entry of a third party if the costs associated with running for office are sufficiently high.

The paper makes five contributions. First, it adds to the discussion of how voter- and party ideology preferences determine platform polarization. Unlike existing literature that focuses on just one of the two factors,[6] this paper unveils a complementary relationship of both determinants to inform equilibrium platform polarization. Furthermore, a mechanism is outlined that explains the correlation of voter ideologies and platform polarization. The polarization that occurs in equilibrium depends on how much freedom parties have to randomize across ideologies in their platforms for a given degree of voter ideology dispersion.

Second, the proposed voting heuristic is complementary to a recently evolving branch of literature that endogenizes how (rational) voters evaluate multidimensional party platforms to explain platform divergence. Matějka and Tabellini (2021) model rationally inattentive voters that optimally allocate scarce attention to selected political dimensions. Platform divergence occurs as ideologically more extreme voters are more attentive to proposed ideologies. In Yuksel (2022) voters differ in how they attribute subjective importance to different policy dimension. The more fractionalized societies prioritize different political dimensions, the higher equilibrium platform polarization is. The heuristic-based approach

[6] Examples include: Bernhardt, Duggan, and Squintani (2009), Callander and Carbajal (2022) and Fauli-Oller, Ok, and Ortuno-Ortin (2003).

in this paper inhibits characteristics that correspond to limiting scenarios in both outlined papers: Voters are equally (in)attentive and the degree of societal fractionalization is minimized as the election outcome is determined by a single draw from the electorate's ideology distribution. Nevertheless, substantial platform polarization occurs as the applied heuristic induces uncertainty about the relevant consideration set of the electorate.

The third contribution follows from the above-outlined argument. Even in conditions where policy preferences are perfectly known, platform polarization occurs as parties can 'obfuscate' the electorate, while anticipating that their platforms are not evaluated in their entirety. Therefore, the paper bridges the literature from models that assume informational frictions on the individual level to models that explain equilibrium platform divergence by assuming an exogenous uncertainty about voter preferences.[7]

Fourth, the paper contributes to the (small) literature that leverages a reduced form approach where players' utilize cumulative distribution functions as strategies by substantially extending the framework. Myerson (1993) first applied this concept in a context where purely office-motivated parties compete by proposing a distribution over transfer payments to the electorate.[8] In contrast, this paper applies the framework to a multidimensional policy space where all players exhibit ideological preferences. Therefore, each party's strategy constitutes a negative externality for the opponent and ideological preferences are *not* monotonously increasing or decreasing in any direction. On a purely methodological level, Spiegler (2006) is more closely related to this paper than Myerson (1993).[9] He uses a related approach to describe oligopoly competition between firms that offer a multidimensional product where distinct prices are attached to each dimension. The focus is on dynamics that are induced by a change in the number of firms, which is different for this paper. Furthermore, the last two points raised to differentiate the model from Myerson (1993) also distinguish the model from Spiegler (2006).[10]

[7]McCarty, Rodden, Shor, Tausanovitch, and Warshaw (2019) provide empirical justification of the latter approach. Examples include: Hansson and Stuart (1984), Wittman (1983), Wittman (1990), Roemer (1994) and Calvert (1985)). Both approaches require ideological parties to explain platform divergence.

[8]Lizzeri and Persico (2001) use a similar approach as introduced in Myerson (1993) to explain the underprovision of public goods.

[9]Like in Spiegler (2006), the players' utility functions in this paper are also quadratic in the chosen *cdf*.

[10]This becomes particularly evident when comparing the equilibrium in Spiegler (2006) and Myerson (1993) for $N = 2$ to the subsequently outlined equilibria.

Finally, this paper pins down an equilibrium where parties compete in a policy space that spans multiple dimensions. Unlike in conventional models,[11] the reduced form approach mutes the incentive to 'leapfrog' the opponent on specific policy issues.[12]

The remainder of the paper is structured as follows: I continue with introducing the formal model setup in Section 1.2. I start Section 1.3 by displaying some intermediary results that crucially inform the equilibrium solution process of the subsequently outlined equilibria. Next, I discuss the underlying patterns. Finally, I test the model's robustness towards the entry of a third party.

1.2 Model Setup

There are two expected-payoff maximizing and ideological parties $i \in \{L, R\}$ and a representative voter. Each party chooses a cumulative distribution function *cdf* $G_i(p)$ over a set of feasible ideological positions p that are taken from the $[-1, 1]$ domain. The interval can be interpreted as a left-to-right spectrum of ideological positions.[13] Let T_i denote the support of G_i and E_i the average ideology of G_i. I restrict the analysis to scenarios where $G_i(p)$ admits a density $g_i(p) \in [0, \infty)$ over $(inf(T_i), sup(T_i))$.[14] Thus, mass points (henceforth: atoms) are allowed to be placed on the infimum and supremum of each party's support.

The voter's ideological preferences x across political dimensions are also taken from the one-dimensional left-to-right spectrum.[15] They follow the uniform distribution $F(x)$ over the interval $[-b, b]$ where $b \in (0, 1)$ measures the dispersion of ideologies.[16]

The voter relies on a heuristic that lets him evaluate each parties' platform along a random sample of two ideologies $\{p_L, p_R\}$ from G_L and G_R and a sample point $\{x_m\}$ from its own ideology distribution F.[17] The vector $\Gamma = (x_m, p_L, p_R)$ fully determines the outcome of

[11] Examples include: Plott (1967) Hinich, Ledyard, and Ordeshook (1973) and McKelvey (1976).

[12] This finding is also based on the assumption that parties care about average platform ideologies, which simplifies their decision problem.

[13] The constitution of a country gives an intuitive explanation why the interval is bounded.

[14] This assumption is a precautionary measure. The subsequent proofs should also hold in more general scenarios where $G_i(p)$ is not restricted.

[15] Policy preferences tend to be aligned on the left-to-right scale (Poole and Rosenthal (2000)).

[16] Assuming that voter ideologies are uniformly distributed allows me to analytically solve the model.

[17] This resembles the $S(1)$ procedure outlined in Osborne and Rubinstein (1998).

the election: The voter's choice correspondence selects the party i that is associated with position p_i that is closer to x_m:

$$C(\Gamma) = \underset{i \in \{L,R\}}{\operatorname{argmin}} \{|x_m - p_L|, |x_m - p_R|\} \tag{1.1}$$

In case of a tie, the voter tosses a fair coin. It is evident from the choice correspondence that preferences over p_i do not monotonously increase or decrease in a certain direction.

The distribution of voter ideologies is common knowledge. Each party also take the voter's decision rule into account when calculating its expected profit from a given strategy profile. Before outlining the parties' payoff functions, I want to provide some helpful intuition as this stylized model setup is open to a number of interpretations.

Each party's strategy shall be interpreted as a reduced form representation of a party platform that specifies ideological positions for a continuum of political dimensions.[18] Thus, $G_i(p)$ indicates the share of dimensions where party i chooses an ideology that is at least as 'leftish' as a given ideology p.

Evaluating multidimensional party platforms is a complex task. Therefore, the voter leverages a heuristic that lets him evaluate one dimension at random and vote for the party that offers the position that is closest to his preferred ideology in that dimension. Alternatively, one could assume that, after parties choose their platforms, a random shock draws the voter's attention to a specific dimension. Based on this dimension, the voter makes his voting decision by comparing his preferred ideology to these sampled from the two rivaling parties. With both interpretations of the choice procedure, the ideologies that are drawn from the parties' distribution functions enter the voter's consideration set and $F(x)$ describes the distribution of the voter's bliss points across all possible consideration sets.

It needs to be mentioned that in this model, the uncertainty about the representatives voter's decisive ideology, which is captured by the dispersion of voter ideologies b, may determine the degree to which party platforms are polarized, but is *not* crucial for platform divergence

[18]To grasp the intuition behind the multidimensionality of the parties' platforms, consider each platform covering diverse topics like 'spending on education' or 'income taxation'. Besides addressing budgeting issues, parties also state their position on topics like '*LGBTQ+* Rights' or 'Abortion'. Even candidate characteristics like gender, age or marriage status could contribute to a party's platform.

to occur at the extensive margin.[19]

It is the combined uncertainty about each individual element of the voter's choice correspondence that creates an informational friction on the individual level and drives platform divergence. As long as both parties play a non-degenerate distribution across different ideologies, they are not able to predict the ideology that is drawn from their *own and the opponent's* distribution by the voter. In this sense, $\{p_L, p_R\}$ can be a random draw in this reduced form model.[20] Likewise, the uncertainty about the relevant voter ideology itself can also (co-)determine the polarization of platforms in equilibrium. This, however, is familiar from more conventional models.

In terms of the above interpretation of the voter behavior, it is crucial that both parties cannot predict the *dimension* (and the ideology associated with it) that is crucial during an upcoming election. Otherwise, the well known median-voter dynamics occur and parties fiercely compete in the decisive dimension and are not restricted elsewhere.

Parties are ideological in the sense that they have opposed ideological bliss points $X_i, \ \forall\, i \in \{L, R\}$ concerning the average ideological position of the winning party. I assume that $0 < -X_L = X_R \leq min\{2b, 1\}$. Although the parties' ideological preferences are exogenous, it appears reasonable to not model them completely independent of voter ideologies. Furthermore, with $|X_i| < 2b$, I can limit the number of subcases that need to be analyzed in the subsequent analysis to a minimum without restricting the model dynamics.[21] Each party's ex-post payoff π_i is defined as follows:

$$\pi_i = \begin{cases} \pi_i^i & = -\sqrt{(X_i - E_i(p))^2} \quad \text{if party i wins} \\ \pi_i^{j \neq i} & = -\sqrt{\left(X_i - E_{j \neq i}(p)\right)^2} \quad \text{if party } j \neq i \text{ wins} \end{cases} \tag{1.2}$$

[19] Appendix A.2 shows that the model dynamics hold for $b = 0$, where there exists no uncertainty about the *voter ideology* that is decisive in the upcoming election.

[20] This is the fundamental difference to conventional models that explain platform divergence with uncertainty about median-voter preferences.

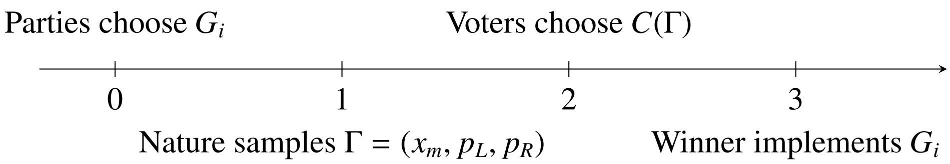
[21] As the voter is assumed to be representative for the electorate, allowing preferred party ideologies to be relatively more dispersed than voter ideologies also appears reasonable, especially in conditions where the dispersion of the voter's ideologies is small. Notice that if $b \geq 0.5$, the restriction is not binding as party ideologies are in any case bounded by the $[-1, 1]$ ideology interval.

It becomes evident from the above outlined ex-post payoff function that each parties *cdf* G_i constitutes a (negative) externality for the opponent. Furthermore, also the parties' preferences over the winning party's average platform position do not monotonously increase or decrease in a certain direction.

The intuition of the payoff functions directly follows from the above-outlined model interpretation. In contrast to the electorate, parties are familiar with the election process and understand that each ideology specified in their programs is equally important. Additionally assuming that they are ideological and that platforms are binding, their utility depends on the distance of the winning party's average platform position to their bliss point. Alternatively, one could argue that parties understand that political issues, which are decisive during the electoral campaign, might be less relevant ex-post as new random topics arise. Then, their payoff depends on a independent draw of the winning party's ideology distribution G_i. With the latter interpretation, X_i defines a bliss point that is constant across political dimensions. For the sake of expositional clarity, I adhere to the former interpretation throughout.[22]

The following timeline again sums up the timing of the game from the parties' perspective.

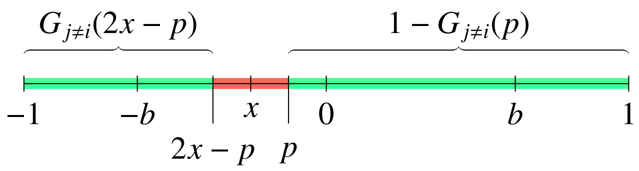
Figure 1.1: Timing of the Electoral Competition



Let me now continue with deriving the parties' expected utility function EU_i step-by-step. Conditional on nature drawing $x_m = x$ and $p_i = p$, party i wins the election for $x < p$ if nature either draws $p_{j \neq i} < 2x - p$ or $p_{j \neq i} > p$, which happens with probability $G_{j \neq i}(2x - p) + 1 - G_{j \neq i}(p)$. Abstracting from a tie, the following graphic illustrates the probability that party i wins the election:

[22] These intuitive explanations also implicitly demonstrate the fundamental difference to consumer search where consumers end up paying the sampled price (e.g. Burdett and Coles (1997)).

Figure 1.2: Conditional Probability of Winning



A reversed logic applies for $x > p$.

More generally, $H_i(p)$, which defines party $i's$ ex-ante probability of winning the election conditional on nature drawing $p_i = p$ from G_i, is defined as:[23]

$$\begin{aligned} H_i(p) = & \int_{-b}^{p} \Big[\Big(1 - G_{j\neq i}(p)\Big) + G_{j\neq i}(2x-p)\Big] f(x)dx \\ & + \int_{p}^{b} \Big[\Big(1 - G_{j\neq i}(2x-p)\Big) + G_{j\neq i}(p)\Big] f(x)dx \end{aligned} \tag{1.3}$$

Thus, H_i mutually depends on the opponent's behavior and on the distribution of voter ideologies. Party $i's$ probability of winning conditional on the voter sampling $p_i = p$ can be interpreted as a weighted average of voter ideologies. The weight attached to each ideology is the probability that party i wins the election conditional on the respective voter ideology being drawn from $F(x)$, given the opponent's strategy.

If both parties play a cdf $G_i(p)$, the expected probability of being elected is found by integrating over $H_i(p)$:

$$EH_i = \int_{-1}^{1} H_i(p)\, dG_i(p) = 1 - EH_{j\neq i} \tag{1.4}$$

Vice versa, Equation 1.4 characterizes the expected probability of losing from the oppo-

[23] As subsequently shown, scenarios where both parties tie on a position $p \neq 0$ do not occur in equilibrium and are therefore not considered here. For the special case where $p = 0$, the outlined ex-ante probability of winning the election conditional on nature drawing $p_i = 0$ implicitly accounts for the possibility that parties tie with positive probability as shown in Appendix A.1. Furthermore, x is assumed to be continuously distributed. Thus, a scenario where the election is tied because a specific voter ideology is sampled and both parties assign an atom to positions with identical distance from that voter ideology does not occur with positive probability.

nent's point of view as $EH_i = 1 - EH_{j \neq i}$ by definition.

A party chooses its distribution function to maximize the expected utility that depends on the probability of winning, its own and the opponent's average platform position and its own ideological preferences. Given Equations 1.2 and 1.4 from above, party $i's$ expected utility is equal to:

$$EU_i = EH_i * \pi_i^i + (1 - EH_i) * \pi_i^{j \neq i} \tag{1.5}$$

Although the parties' strategies in this model are probability distributions, they are not 'mixed strategies' with $[-1, 1]$ being the corresponding set of 'pure' strategies. As in Myerson (1993), Lizzeri and Persico (2001) and Spiegler (2006) the probability distributions in this model are genuinely pure strategies. Technically, the contrasting difference becomes clear from Equation 1.5 as each parties' expected utility is quadratic in G_i, not linear. Therefore, parties have an incentive to randomize over ideological positions in their platforms to 'obfuscate' the electorate and shall not be expected to be indifferent between G_i and the individual elements of T_i.

Link to alternative model setups

Analogous to Spiegler (2006), G_i can be interpreted as a reduced form representation of a multidimensional party platform where no pure strategy equilibrium in an elaborate model with an explicit mapping from dimensions to ideologies may exist. Likewise, the reduced form equilibria subsequently outlined are linked to mixed strategy equilibria in the elaborate model in the following way: Suppose a party's strategy is an explicit mapping from political dimensions to ideological positions. Then, $G_i(p)$ again describes the share of dimensions where the chosen ideology is at least as 'leftish' as p. If each party i chooses a mixed strategy that uniformly randomizes over all pure strategies that induce a given *cdf* G_i, the elaborate formalism boils down to this model. In both model setups, parties are aware of the share of dimensions in which the other party chooses a given ideology, without being able to map certain dimensions to certain ideologies. The ideology that is decisive from the opponent's distribution appears to be a random draw. As in Spiegler (2006), the set of pure-strategy equilibria in this model is isomorphic to a subclass of mixed-strategy equilibria in the elaborate model where parties uniformly randomize over all pure strategies that induce a given *cdf* G_i.

The outlined model also fits an alternative interpretation that coincides with the *proportional* vote system analyzed in, e.g., Lizzeri and Persico (2001) and Austen-Smith and J. Banks (1988) where the expected vote share (co-)determines the expected payoff.[24] One needs to assume that there exists a continuum of voters whose preferences for ideologies are distributed according to $F(\cdot)$. Each voter evaluates the parties' platforms along one political dimension, but parties cannot predict the sample points a voter with a specific ideology draws from their own and the opponent's distribution function. The outlined model describes the electoral competition if each party's expected vote share (co-)determines the expected payoff.

1.3 Solution Concept and Strategic Considerations

I start this section by establishing crucial intermediary results to derive all subsequent propositions. They give some intuition about the underlying strategic incentives and define the shape of any best replying distribution function G_i.

Let me first define the concept of mirror-inversion in the following way: If G_i is such that an atom equal to $\alpha_i \in (0, 1]$ is assigned to some $p \in [-1, 1]$ or $\exists\ p : g_i(p) \in (0, \infty)$, mirror-inverting party $i's$ strategy requires that $G_{j\neq i}$ assigns $\alpha_{j\neq i} = \alpha_i$ to $-p$ or $g_{j\neq i}(-p) = g_i(p)$. Furthermore, $g_{j\neq i}(-p) = 0,\ \forall\ p : g_i(p) = 0$.

Lemma 1.1. *For G_i to be eligible for a best-reply:*

1. *The points $(p, H_i(p))$, $\forall\ p \in T_i$ must lie on a straight line.*
2. *No $(p, H_i(p))$ where $p \in [inf(T_i), sup(T_i)] \setminus T_i$ (such that $g_i(p) = 0$) may lie above the line connecting the points $(inf(T_i), H_i(inf(T_i)))$ and $(sup(T_i), H_i(sup(T_i)))$.*

[24] Such a model setup could be motivated by assuming that how well parties are able to implement their policies depends on the support they receive in parliament and therefore on their vote share.

3. *Given three points p_1, p_2, p_3 where $p_1 \in T_i$ and $\exists\ n \in \{2, 3\} : p_n \notin T_i$ and $p_2 < p_3$, the point $(p_1, H_i(p_1))$ may not lie below the straight line running through $(p_2, H_i(p_2))$ and $(p_3, H_i(p_3))$ if $p_2 < p_1 < p_3$.*

Proof: See Appendix A.1

Lemma 1.1 constitutes a necessary equilibrium condition that pins down the shape of a party's distribution function.[25] It requires that, in equilibrium, $H_i(p)$ is linear over its support T_i. Thus, in equilibrium, all points $(p, H_i(p)),\ \forall\ p \in T_i$ lie on straight line.

The key takeaway from Lemma 1.1 directly follows: Given that for a given G_i all points $(p, H_i(p)),\ \forall\ p \in T_i$ lie on straight line, party i is indifferent between playing G_i and an alternative strategy where it reallocates some weight within T_i in a mean-preserving way.[26] This, however, may not be confused with the indifference condition that is required when solving for a mixed strategy equilibrium as the ideologies that are included in T_i are not payoff equivalent, given the opponent's strategy.

Furthermore, for this reasoning to apply, it is a straightforward prerequisite that T_i is such that it allows for a mean-preserving shift of weight.[27]

Corollary 1.1. *Lemma 1.1 implies that if G_i is a non-degenerate distribution that qualifies as a best-reply, any point $(p', H_L(p')),\ \forall\ p' \notin T_i$ may not lie above the line connecting the points $(p, H_L(p)),\ \forall\ p \in T_i$.*

Proof: Directly follows from Lemma 1.1.

The second main insight from Lemma 1.1 is that it implies a form of quasi-concavity restriction on $H_i(p)$ over the entire domain of ideological positions: Relative to each position's absolute distance from party $i's$ ideology bliss-point, there may not exist an ideological position outside T_i that yields a disproportionately higher probability of winning the election than the ideologies included in T_i.

[25] Lemma 1.1 is a generalized version of Myerson (1993) and Spiegler (2006) that accounts for scenarios where $H_i(p)$ is not monotone in p.

[26] This equals the 'indifference principle' outlined in Spiegler (2006).

[27] T_i does not allow for a mean-preserving shift of weight if G_i is a degenerate distribution or a simple *cdf* that places an atom on $inf(T_i)$ and $sup(T_i)$ and $g_i(p) = 0,\ \forall\ p \in [inf(T_i), sup(T_i)]$.

Notice that $H_i(p)$ is defined in terms of the distribution $G_{j \neq i}$ that is played by the opponent. Thus, in equilibrium each party chooses its strategy such that, for a given distribution of voter ideologies, there exists no 'obvious' room for improvement. This means that party i cannot achieve a higher payoff by reallocating some weight in a mean-preserving way to achieve a higher expected probability of winning the election.

Lemma 1.2. *A strategy G_i that generates $|E_i(p)| > |X_i|$ does not survive the iterative elimination of strictly dominated strategies. As a consequence, $|E_i(p)| \leq |X_i|$ and $EU_L + EU_R = X_L - X_R$ in equilibrium.*

Proof: See Appendix A.1

Suppose that $|E_i(p)| > |X_i|$. Then, after leveraging Lemma 1.1 to identify dominated strategies of the opponent, there exists an alternative distribution function with an expected value closer to X_i that also yields an expected probability of winning at least as big as G_i. Such a strategy strictly payoff dominates any strategy where $|E_i(p)| > |X_i|$. In conclusion, each party plays, in equilibrium, a distribution over ideologies such that the resulting average platform positions is (weakly) closer to the center of the ideology distribution than each party's preferred bliss point.

Lemma 1.2, thus, outlines a constraint for the first moment of each party's G_i that must be satisfied in equilibrium. The second takeaway is more of a technical nature: Given $EH_i = 1 - EH_{j \neq i}$ and Equation 1.5 from Section 1.2, the game adds up to a constant sum in expected utilities in equilibrium.

Lemma 1.3. *A given pair of strategies $(G_i, G_{j \neq i})$ can only constitute an equilibrium if mirror-inverting G_i is a best reply for party $j \neq i$.*

Proof: See Appendix A.1

Taking Lemma 1.2 as given, Lemma 1.3 leverages an imitation argument: As the game is a symmetric, two-player, (expected) constant sum game, G_i can only be a potential equilibrium candidate if mirror-inverting G_i is no dominated strategy for the opponent.[28]

[28]Fey (2012) find that a symmetric two-player zero sum games always have a symmetric equilibrium if an equilibrium exists.

Otherwise, there exists a better reply for the opponent that guarantees a payoff that is in expectation strictly greater than half of the constant sum. It follows from an analogous imitation argument for the given optimal reaction of the opponent that the initially chosen strategy G_i cannot be optimal.

Lemma 1.4. *Given $G_{j \neq i}$ of the opponent such that the conditions on H_i specified in Lemma 1.1 hold, a non-degenerate distribution G_i is a best reply if and only if:*

> *Any unilateral shift of weight of player i from G_i to a best-replying simple distribution function G_i^* that places an atom of size α_i on $sup(T_i)$ and an atom of size $(1 - \alpha_i)$ on $inf(T_i)$ yields $|E_i^*(p)| = |E_i(p)| \leq |X_i|$ (where $sgn(E_i(p)) = sgn(X_i)$ if $E_i(p) \neq 0$).*[29]

Proof: See Appendix A.1

On its own, Lemma 1.1 is a necessary but not a sufficient equilibrium condition. It rules out that there exists a profitable mean-preserving deviation for any of the two parties. Yet, there additionally may not exist a profitable *non*-mean-preserving deviation in equilibrium. Taking Lemma 1.1 and Corollary 1.1 as given, Lemma 1.4 pins down this second optimality condition for each distribution function.

The intuition of Lemma 1.4 is as follows: Suppose that Lemma 1.1 holds for a given $G_{j \neq i}$. Then, any function G_i^* that reallocates some weight in a mean-preserving way within T_i must also be a best reply since, by Lemma 1.1, such a function is associated with the same expected probability of winning and must, therefore, also generate an identical expected payoff such that party i is indifferent between G_i^* and G_i. Vice versa, if there exists a better reply than G_i, it generates a distinct $E_i^{'}(p) \neq E_i(p)$ and consequently is associated with a higher expected probability of winning the election at the cost of an average platform position further away from party $i's$ average bliss point or the way around. Testing this requirement with a simple *cdf* G_i^*, as outlined in Lemma 1.4, is the most straightforward way.

[29] Lemma 1.4 is an adapted version of the indifference principle in Spiegler (2006). Furthermore, if G_i itself is a simple distribution function as described, the outlined logic applies directly.

Furthermore, notice that for a given $G_{j\neq i}$ where Lemma 1.1 holds for part i, the best replying strategy of party i to $G_{j\neq i}$ is found by a shift of weight within T_i by the following argument: It directly follows from Lemma 1.1 and Corollary 1.1 that any reply that randomizes over an ideological position $p' \notin T_i$ cannot be strictly better as all points $(p', H_L(p'))$ may not lie above the straight line connecting the points $(p, H_L(p)),\ \forall\ p \in T_i$.

In conclusion, if no such better reply as described in Lemma 1.4 exists, G_i itself is a best reply to $G_{j\neq i}$ as it perfectly balances a parties' incentive to win the election and its incentive to implement a party platform closer to its preferred ideology bliss point in case of winning the election.

Proposition 1.1. *In any equilibrium, $p \leq 0,\ \forall\ p \in T_L$ and $p \geq 0,\ \forall\ p \in T_R$.*

Proof: See Appendix A.1

Entering the ideology space of the opponent's side of the political spectrum either creates a direct incentive for the opponent to 'leapfrog' the entrant or is a dominated strategy in the first place. Proposition 1.1, therefore, fundamentally determines how the parties' distribution functions are shaped in equilibrium. Each party's support exposes some form of ideological proximity to the parties' exogenous bliss points since parties endogenously divide the ideological space between each other. As a consequence, party L exclusively includes ideological positions from the 'left' side of the political spectrum in its support and vice versa for party R. In equilibrium, both parties may only share ideologies in their platforms that equal the expected voter ideology at $x = 0$. With Proposition 1.1, $H_i(p)$ from Equation 1.3 can be updated to:

$$H_L(p) = F\left(\frac{sup(T_R) + p}{2}\right) - \int_{\frac{inf(T_R)+p}{2}}^{\frac{sup(T_R)+p}{2}} G_R\,(2x - p)\, f(x)dx \tag{1.6}$$

$$H_R(p) = 1 - F\left(\frac{sup(T_L) + p}{2}\right) + \int_{\frac{inf(T_L)+p}{2}}^{\frac{sup(T_L)+p}{2}} G_L\,(2x - p)\, f(x)dx \tag{1.7}$$

Corollary 1.2. *A given strategy profile* $(G_i(p), G_{j\neq i}(p))$ *that is associated with average platform ideologies* $(E_i(p), E_{j\neq i}(p))$ *constitutes an equilibrium if:*

- *Proposition 1.1 and Lemma 1.1 are mutually satisfied for* $(G_i(p), G_{j\neq i}(p))$.

...and in case G_i^**, as outlined in Lemma 1.4, is well defined, if additionally:*[30]

- $|E_i(p)| = |E_i^*(p)|$ *if* $|E_i^*(p)| \leq |X_i|$.
- $|E_i(p)| = |X_i|$ *if* $|E_i^*(p)| > |X_i|$.

...if G_i^**, as outlined in Lemma 1.4, is not well defined, Lemmas 1.2 and 1.3 need to hold.*[31]

In any case: $|E_i(p)| = |E_{j\neq i}(p)| \leq |X_i|$.

Proof: See Appendix A.1

Proposition 1.1 restricts the strategy space where an equilibrium can occur. Lemma 1.1 specifies an essential requirement that mutually needs to hold for a given pair of strategies to qualify as an equilibrium candidate.

If $G_i(p)$ is a non-degenerate distribution such that the simple *cdf* G_i^* is well defined, Lemma 1.4 needs to additionally hold for the given pair of strategies to constitute mutual best replies if Lemma 1.2 is not binding. Otherwise, the average platform position directly follows from Lemma 1.2. In any case, Lemma 1.3 ensures that, in equilibrium, $|E_i(p)| = |E_{j\neq i}(p)|$ by the following argument: As mirror-inverting G_i must be an optimal strategy for party $j \neq i$ any alternatively played 'asymmetric' strategy must be associated with the same average platform position as the mirror-inverted strategy of party i since otherwise Lemma 1.4 is certainly violated. If Lemma 1.2 is binding, $|E_i(p)| = |E_{j\neq i}(p)|$ directly follows as the parties' bliss points are assumed to be symmetrically distributed around the center of the ideology spectrum.

If $G_i(p)$ is a degenerate distribution such that G_i^* is not well-defined, $|E_i(p)| = |E_{j\neq i}(p)| \leq |X_i|$ directly follows from combining Lemmas 1.3 and 1.2.

[30]This is the case for any non-degenerate G_i.
[31]This is the case for any degenerate G_i.

1.4 Equilibrium Analysis

The following introductory example where $b = 0$ and $|X_L| = 1$ demonstrates that parties do not fully converge to the expected voter ideology and gives an intuitive explanation for why this is the case.[32] Considering Proposition 1.1, there can only exist an equilibrium where both parties fully converge to the mean if they play a degenerate distribution that places all weight on $p = 0$. Assuming that party R plays such a distribution, all points $(p, H_L(p))$ where $p \in (-1, 0)$ lie below the line connecting the points $(-1, H_L(-1))$ and $(0, H_L(0))$ since $H_L(p) = 0,\ \forall\ p < 0$. In this case, Lemma 1.1 can only be satisfied if party L best replies with a simple *cdf* that places an atom on $p = -1$ and another atom on $p = 0$. A straightforward expected utility optimization shows that party L places equal weight on both ideologies.

How strong party $L's$ incentive to win the election is, depends on the average platform position of party R. Given the centrist platform position of its opponent, the negative externality that is attached to losing the election is relatively small. Therefore, party L randomizes over ideological positions in a way that it chooses an ideological position with the intention to win the election in only half of the political dimensions. While being aware that the voter does not evaluate each party's platform in its entirety, party L leverages this knowledge and chooses an ideology to achieve an average platform position that is closer to its preferred bliss-point in the remaining half of ideological dimensions to, thereby, profit in case of winning the election.

The outlined reasoning already suggests that, in equilibrium, parties do not fully converge to $E_i(p) = 0$ in average platform positions. In Appendix A.2, I outline the formal equilibrium for $b = 0$ and $|X_i| \in (0, 1]$ to prove this claim.[33]

The introductory example constitutes a limiting scenario for two reasons. First, with $b = 0$, decisive voter ideologies are identical across all political dimensions. Thus, there exists no uncertainty about which voter ideology decides the election. Second, party R plays a de-

[32] The example relaxes the assumption that $|X_L| < 2b$ for illustrative purposes. Lemmas 1.1-1.4 and Proposition 1.1 do not depend on this assumption.

[33] $|X_i| \in (0, 1]$ does not align with the assumption that $|X_i| < min\{2b, 1\}$ for $b = 0$, which is a necessary assumption to limit the number of subcases in the following analysis.

generate distribution with all weight placed on $p = 0$. Thus, there also exists no uncertainty about which ideological position from the opponent's distribution is going to decide the election. Nevertheless, party L can leverage its own ability to generate uncertainty about the election outcome by randomizing across ideologies to ultimately propose a polarized platform that allows for an expected utility strictly greater than that of the opponent.

Proposition 1.2. *Given that* $|X_i| \geq 1/3$*, any best-replying* G_i *chooses* $|E_i(p)| \geq \frac{1}{3}$.

Proof: See Appendix A.1

Proposition 1.2 outlines a lower bound on the polarization in average party platforms that can potentially occur in equilibrium as long as $|X_i| \geq 1/3$ such that Lemma 1.2 is not directly binding. It demonstrates that the equilibrium polarization is substantial.

The underlying reason is that parties know that a platform is not evaluated in its entirety. Hence, it is the applied voting heuristic that *always* allows parties to leverage their ability to 'obfuscate' the voter by randomizing over ideologies. This rationale generates polarized platforms, even for low levels of voter ideology dispersion.

For more general settings than outlined in the introductory example where $b \neq 0$, decisive voter ideologies are continuously distributed across dimensions such that there exists additional uncertainty on the parties' side about which voter ideology is decisive during an electoral campaign. Depending on b, there are three subcases to consider. The following propositions outline the distinct equilibria uniquely defined in terms of $|E_i(p)|$ and T_i for each subcase.

Proposition 1.3. *If* $b \in \left(0, \frac{1}{4}\right)$ *(and* $|X_i| < 2b$*), there exists an equilibrium where both parties play a distribution function* G_i *that places an atom of size* $\alpha_i = 1 - |X_i|$ *on* $p = 0$ *and an atom of size* $1 - \alpha_i = |X_i|$ *on* $p = -1$ *(party L) /* $p = 1$ *(party R) such that* $E_i(p) = X_i$*. In case* $b \in \left(\frac{1}{6}, \frac{1}{4}\right)$*, the equilibrium is unique.*

Proof: See Appendix A.1

Suppose the dispersion of the voter ideologies across dimensions is small. In that case, there exists an equilibrium where the polarization of average platform positions is determined by the distance between both parties' exogenous ideology bliss points as they can implement their preferred average platform positions in equilibrium.

With $|X_i| < 2b$, both parties place a relatively big atom on the expected voter ideology such that the overall polarization of platform ideologies is still low in absolute terms. Therefore, parties have no incentive to reallocate some weight from centrist to intermediate positions as such positions, conditional on being drawn, are associated with an expected probability of winning, which is, in proportion to their distance to the expected voter positions, lower than that of centrist positions. As a consequence, allocating the (relatively small) remaining weight to extreme positions more effectively decreases the distance of each party's average platform position to its preferred ideology bliss point than randomizing across intermediate ideologies.

Figure 1.3: Conditional Probability of Winning for Party L with $X_R = 0.2$

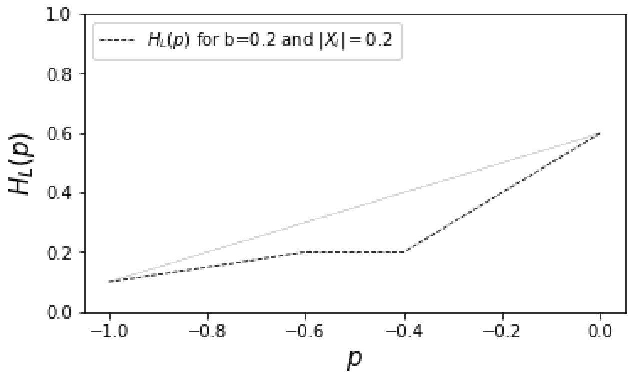


Figure 1.3 depicts $H_L(p)$, given party $R's$ equilibrium distribution for $X_R = 0.2$ (and $b = 0.2$). The phenomenon that *'no compromises'* occur in equilibrium is clearly observable. Party L has an incentive to only include extreme and centrist positions in its platform as all ideological position $p \in (0, 1)$ lie below the line connecting $(-1, H_L(-1))$ and $(0, H_L(0))$.

In the spirit of the intuition from the introductory example, the atom on $p = 0$ in each party's equilibrium distribution as outlined in Proposition 1.3 addresses the incentive to win the election. The atom on the most extreme position addresses each party's incentive to increase the payoff conditional on winning the election.

Finally, as the point $(X_i, H_i(X_i))$ lies below the line connecting $(inf(T_i), H_i(inf(T_i)))$ and $(sup(T_i), H_i(sup(T_i)))$, playing a degenerate distribution that allocates all weight to $p = E_i(p) = X_i$ is a strictly dominated strategy for both parties. In conclusion, they have a strict incentive to randomize across ideological positions in equilibrium. This rationale is also evident from Figure 1.3.

Proposition 1.4. *If* $b \in \left[\frac{1}{4}, \frac{1}{3}\right]$[34] *and* $|X_i| \geq 1 - 2b$*, in any equilibrium both parties play a distribution function* G_i *where an atom of size* $1 - \alpha_i = \frac{2-6b}{2-4b}$ *is placed on* $p = -1$ *(party L) / on* $p = 1$ *(party R) and where the remaining weight is used to randomize over* $[2b - 1, 0]$ *(party L) and* $[0, 1 - 2b]$ *(party R) in a way such that* $\int_0^{1-2b} p \; d\,G_R(p) = -\int_{2b-1}^{0} p \; d\,G_L(p) = 4b - 1$*, which consequentially yields* $|E_i(p)| = 1 - 2b$*.*

Otherwise, if $|X_i| < 1 - 2b$*, there exists a unique equilibrium where both parties play a distribution function* G_i *that places an atom of size* $\alpha_i = 1 - |X_i|$ *on* $p = 0$ *and an atom of size* $1 - \alpha_i = |X_i|$ *on* $p = -1$ *(party L) /* $p = 1$ *(party R) such that* $E_i(p) = X_i$*.*[35]

Proof: See Appendix A.1

For a slightly bigger dispersion of voter ideologies where $b \in [1/4, 1/3]$, both parties are again able to implement their ideology bliss points as average platform position as long as the distance between the parties' bliss points is sufficiently low. If $|X_i| \leq 1 - 2b$, an equilibrium similar in style to Propositions 1.3 occurs. The intuition also follows.

If $|X_i| > 1 - 2b$, however, the polarization in preferred party ideologies becomes excessive such that parties are no longer able to implement their preferred ideology as average platform position.

The underlying reason is that with increasing levels of b, intermediate ideologies, which are located in between centrist and extreme positions, become increasingly attractive for both parties: $\frac{\partial H_L(p)}{\partial b} > 0, \;\forall\; p \in [2b - 1, 0)$. With $b \in (1/4, 1/3]$ the distribution functions outlined in Proposition 1.3 can no longer constitute an equilibrium as Lemma 1.1 is violated. The points $(p, H_L(p)), \;\forall\; p \in [2b - 1, 0)$ would lie above the line connecting $(-1, H_L(1))$ and $(0, H_L(0))$.[36]

To better understand the comparative statics, consider the following illustrative example

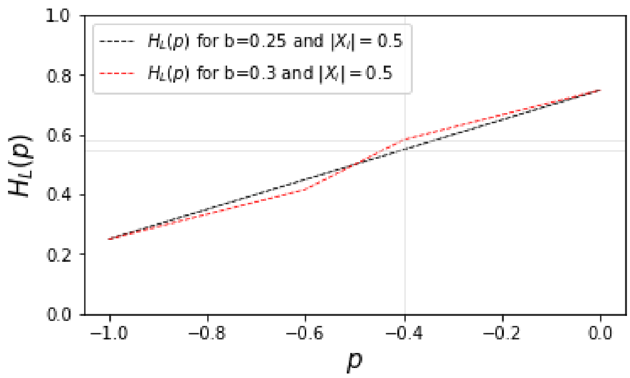
[34]The equilibria discussed in Propositions 1.3 & 1.4 converge for $b = \frac{1}{4}$. The same is true for Propositions 1.5 & 1.4 for $b = \frac{1}{3}$.

[35]For $|X_i| = 1 - 2b$, both equilibria mutually coexist.

[36]An argument that is similar in spirit holds for party R. In the limit where $b = 1/4$ the distributions outlined in Propositions 1.4 and 1.3 converge.

where $X_R = -X_L = 0.5$ and $b = 0.25$ in scenario (i) and $b = 0.3$ in scenario (ii). Furthermore, suppose that both parties play the equilibrium strategy outlined in Proposition 1.3. Thus, party R places an atom of size $1/2$ on ideologies $p = 1$ and $p = 0$. Figure 1.3 depicts $H_L(p)$ for both scenarios (i) (black) and (ii) (red), given party $R's$ equilibrium distribution for $X_R = 0.5$.

Figure 1.4: Violation of Lemma 1.1 for $b = 0.3$

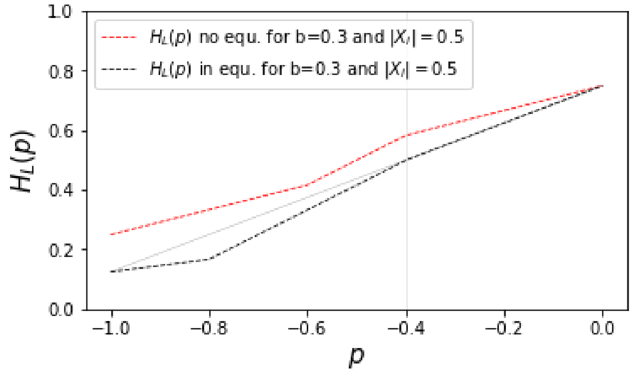


It is observable from Figure 1.4 that in scenario (i), the points $(p, H_L(p))$, $\forall\ p \in (0, 1)$ lie on the line connecting $(-1, H_L(-1))$ and $(0, H_L(0))$. If party L mirror inverts the opponent's strategy, an equilibrium occurs that satisfies the conditions outlined in Proposition 1.4. However, in scenario (ii), this is not the case. Compared to scenario (i), any position $p \in [-0.4, 0)$ is associated with a higher probability of winning the election conditional on being sampled by the electorate in scenario (ii). For instance, $p = -0.4$ wins the election with probability $11/20$ in scenario (i), whereas the probability of winning increases to $7/12$ in scenario (ii) for party L.

To make use of the analogy introduced earlier, there now occurs a *'willingness for a compromise'* as, in equilibrium, both parties allocate some weight on positions in $[2b - 1, 0)$ (party L) / $[0, 1 - 2b)$ (party R). To avoid that the opponent has an incentive to 'leapfrog' one's own chosen distribution, the increased attractiveness of intermediate positions is balanced by reallocating relatively more weight from extreme positions to achieve an equilibrium. Comparing Figure 1.5 below to Figure 1.4 from above, one observes the effect of this reallocation of weight to intermediate ideologies.

Figure 1.5 depicts the conditional probability of winning the election for party L (black), given party R plays an equilibrium *cdf* that satisfies the requirements outlined in Proposi-

Figure 1.5: Conditional Probability of Winning for Party L with $b = 0.3$



tion 1.4 for $b = 0.3$ (and $|X_i| = 0.5$). Additionally, scenario (ii) (red) from Figure 1.4 is included as a reference point.

The equilibrium *cdf* of party R allocates more weight to intermediary ideologies and yields a lower average platform position than the non-equilibrium *cdf* played in scenario (ii) from above. Therefore, the attractiveness of intermediary ideologies relatively decreases such that party L has no incentive to 'leapfrog' the strategy of the opponent. This implies that Lemma 1.1 is not violated if mirror inverting the opponent's strategy, which is also evident from Figure 1.5 as all points $(p, H_L(p))$, $\forall\ p \in \{\{-1\}, [-0.4, 0]\}$ lie on a straight line. Furthermore, given the opponent's average platform position, mirror-inverting G_R indeed constitutes an equilibrium as party $L's$ incentives to win the election and to increase its payoff conditional on winning are perfectly balanced.[37]

All distribution functions that are played in equilibrium are necessarily of a non-degenerate shape. Yet, the incentive to 'obfuscate' the electorate is not strict. It is evident from Figure 1.5 that $E_i(p) \in T_i$ such that a unilateral switch to a degenerate distribution that allocates all weight to $p = E_i(p)$ is not payoff decreasing for either party by Lemma 1.1.[38] Therefore, the occurring obfuscation should be understood as a means to establish an equilibrium rather than as parties' being strictly incentivized to randomize.

In the limit, where $b = \frac{1}{3}$, the 'reservoir' of extreme positions that are suitable for realloca-

[37]However, there exist additional asymmetric equilibria, where party L also randomizes over $\{\{-1\}, [-0.4, 0]\}$ in way that that generates an identical average platform position $E_L(p) = 2b - 1$ as outlined in Proposition 1.4.

[38]For $b = 0.3$, it follows from Proposition 1.4 that $E_L(p) = -0.4$ in equilibrium. It is evident from Figure 1.5 that $(-0.4, H_L(-0.4))$ lies on the straight line connecting $(-1, H_L(-1))$ and $(0, H_L(0))$.

tion is completely exhausted. There exists a (unique) equilibrium where both players play a degenerate distribution function which allocates all weight to $|p| = \frac{1}{3}$.

Proposition 1.5. *If* $b \in \left(\frac{1}{3}, 1\right]$[39] *and* $|X_i| \geq b$, *in any equilibrium both parties play a distribution function* G_i *where* $|E_i(p)| = b$ *and* $T_L \subseteq [-1, min\{2b-1, 0\}]$ *and* $T_R \subseteq [max\{0, 1-2b\}, 1]$.[40]

Otherwise, if $b \geq |X_i| \geq 1-2b$, *in any equilibrium both parties play a distribution function* G_i *where* $|E_i(p)| = |X_i|$ *and* $T_L \subseteq [-1, min\{2b-1, 0\}]$ *and* $T_R \subseteq [max\{0, 1-2b\}, 1]$.

Otherwise, if $b \geq 1-2b > |X_i|$,[41] *there exists a unique equilibrium where both parties play a distribution function* G_i *that places an atom of size* $\alpha_i = 1 - |X_i|$ *on* $p = 0$ *and an atom of size* $1 - \alpha_i = |X_i|$ *on* $p = -1$ *(party L) / p* $= 1$ *(party R) such that* $|E_i(p)| = |X_i|$.[42]

Proof: See Appendix A.1

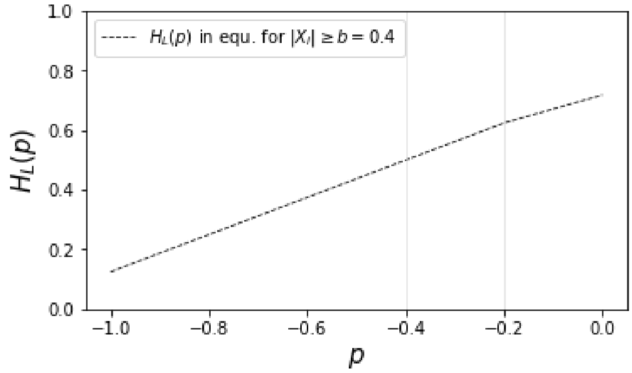
Let me first consider the case for intermediate levels of voter dispersion where $b \in \left[\frac{1}{3}, \frac{1}{2}\right)$. If, additionally, $|X_i| > b$, there occurs a phenomenon associated with an *'empty center'*. In any scenario where centrist positions were included in a party's support, a similar incentive would exist to switch to relatively more attractive intermediate positions as outlined when discussing the results from Proposition 1.4. If $b > \frac{1}{3}$, however, there cannot exist a strategy profile that balances each party's trade-off to ultimately reach a stable outcome. As argued after Proposition 1.4, the reservoir of extreme positions that can be reallocated to, again, decrease the relative attractiveness of intermediate positions is already exhausted at $b = \frac{1}{3}$.

In equilibrium, both parties, therefore, come to terms with only including ideologies in the support of their distribution strategy that are sufficiently extreme. Figure 1.6 depicts the conditional probability of winning the election for party L, given party R plays an equilibrium *cdf* (black) that satisfies the requirements outlined in Proposition 1.5 for $b = 0.4$. It is assumed that Lemma 1.2 is not violated in this case.

It is evident from Figure 1.6 that party L has no incentive to randomize over ideological positions close to the center of the ideology distribution, given that party R does

[39]The equilibria discussed in Propositions 1.4 & 1.5 converge for $b = \frac{1}{3}$.
[40]Notice that $1 - 2b \leq 0 \leq 2b - 1, \ \forall \ b \geq 1/2$.
[41]Notice that this can never be true for $b \geq 1/2$.
[42]If $|X_i| = 1 - 2b$, this and the equilibrium outlined for $b \geq |X_i| \geq 1 - 2b$ coexist.

Figure 1.6: Conditional Probability of Winning for Party L with $b = 0.4$



also not include the analogous positions on the right side of the ideology distribution in its platform: The points $(p, H_L(p))$, $\forall\ p \in (-0.2, 0]$ lie below the line connecting the points $(p, H_L(p))$, $\forall\ p \in [-1, -0.2]$. Additionally, it becomes evident that the points $(p, H_L(p))$, $\forall\ p \in [-1, -0.2]$ lie on a straight line such that Lemma 1.1 is satisfied.

Not including centrist positions in their platforms guarantees that the voter has certain core ideologies. This means that for a given pair of equilibrium distribution functions, one of the two parties certainly wins and the other one certainly loses the election if such a core ideology is drawn from $F(x)$.[43]

Notice that there, again, exists no strict incentive to randomize. Even more so, there now even exist equilibria where at least one party plays a degenerate distribution with all weight placed on $p = b$ (party R) / $p = -b$ (party L). This also becomes evident from Figure 1.4. As the point $(-0.4, H_L(-0.4))$ lies on the straight line connecting $(-1, H_L(-1))$ and $(-0.2, H_L(-0.2))$, there exists an equilibrium where party L replies with a degenerate distribution where all weight is placed on $p = b = -0.4$. With such a degenerate distribution yielding $E_L(p) = -0.4$, all equilibrium requirements outlined in Proposition 1.4 are satisfied.

The underlying reason is that the linearity of $H_i(p)$ does no longer depend on the opponent's strategy but only on the distribution of voter ideologies.[44] Therefore, it is no longer an equilibrium requirement that parties randomize across ideologies as the linearity of

[43]Such an equilibrium cannot occur in scenarios outlined in Proposition 1.4 as the resulting average platform position would be excessively extreme such that there always exists an incentive to reallocate some weight to positions closer to the expected voter position.

[44]Check the proof of Proposition 1.5 for the detailed argument.

$H_i(p)$ is trivially satisfied, given voter ideologies are uniformly distributed. Consequentially, parties' are no longer restricted in choosing their *cdf*. They balance the incentive to win the election and to increase the payoff conditional on winning solely by optimally choosing an average platform position for a given distribution of voter ideologies.

The rationale does not change if considering $b \geq \frac{1}{2}$ and $|X_i| > b$. Unlike outlined above for $b < \frac{1}{2}$, Proposition 1.1 ensures that there now exist core ideologies of the voter, independent of whether any of the two parties places some weight on centrist positions.[45] As a consequence, parties are completely free to choose any distribution that implements $|E_i(p)| = b$ in equilibrium as long as it does not include ideological positions from the opponent's side of the ideology distribution. Likewise, there exists no strict incentive to obfuscate the electorate by randomizing over ideologies and, again, even degenerate distributions can constitute an equilibrium.

Finally, for all $b \in \left[\frac{1}{3}, 1\right]$, both parties are able to implement their ideology bliss points as average platform position as long as $|X_i| \leq b$. If additionally $|X_i| \geq 1 - 2b$, which certainly is the case with $b \geq 1/2$, they are not restricted in choosing their equilibrium distribution as long as $E_i(p) = |X_i|$ and $sup(T_L) \leq min\{2b-1, 0\}$ and $inf(T_R) \geq max\{0, 1-2b\}$. However, if $|X_i| < 1-2b$, which is only possible with $b < 1/2$, an equilibrium similar to Propositions 1.3 occurs.

1.5 Discussion of Results and Mechanisms

In this section, I discuss the crucial results from Section 1.3 in more detail. Furthermore, I want to clarify what mechanisms drive the polarization of the parties' average platform ideologies. To unveil the fundamental dynamics, I show how $|E_i(p)|$ depends on the two crucial model parameters X_i and b. All subsequently outlined Corollaries follow from Proposition 1.3-1.5 and therefore require no further proof.

Corollary 1.3. *Equilibrium existence is guaranteed and* $|E_i(p)| \neq 0$ *in equilibrium as long as* $|X_i| > 0$.

[45] Notice that with $b \geq \frac{1}{2} \to 2b - 1 \geq 0$.

Given Proposition 1.1, an equilibrium where $|E_i(p)| = 0$ could only occur if both parties play a non-degenerate distribution with all weight on $p = 0$. However, all equilibria outlined previously are characterized by a certain degree of polarization in average platform ideologies. The basic intuition explained in the introductory example in Section 1.3 is preserved for all $|X_i| > 0$.[46] The assumed voting heuristic always ensures that both parties can offer polarized platforms in equilibrium. All outlined equilibrium distribution functions, therefore, balance each party's incentive to win the election and maximize the payoff conditional on doing so, while taking the opponent's strategy as given. Nevertheless, an equilibrium where $|E_i(p)| = 1$ can only occur if the dispersion of voter ideologies across ideologies and the parties' ideology bliss-points are maximum extreme.

Any comment on whether voters benefit from the occurring polarization in average platform positions should be formulated with caution. The observed voting behavior of boundedly rational voters is not suited to draw general conclusions about the underlying preferences. Having said this, a 'natural comparison' would be to benchmark the results against a model with a fully rational electorate that evaluates each party's platform in its entirety. To allow for a straightforward comparison, let me assume that such an electorate is only interested in the average platform position of each party across all political dimensions and that all dimensions are equally weighted. In this case, the game boils down to a one-dimensional decision problem with perfect information. It is a well-known result that in such a case, both parties fully converge to the center of the ideology distribution and offer a platform such that $|E_i(p)| = 0$, independent of the parties' ideological preferences.

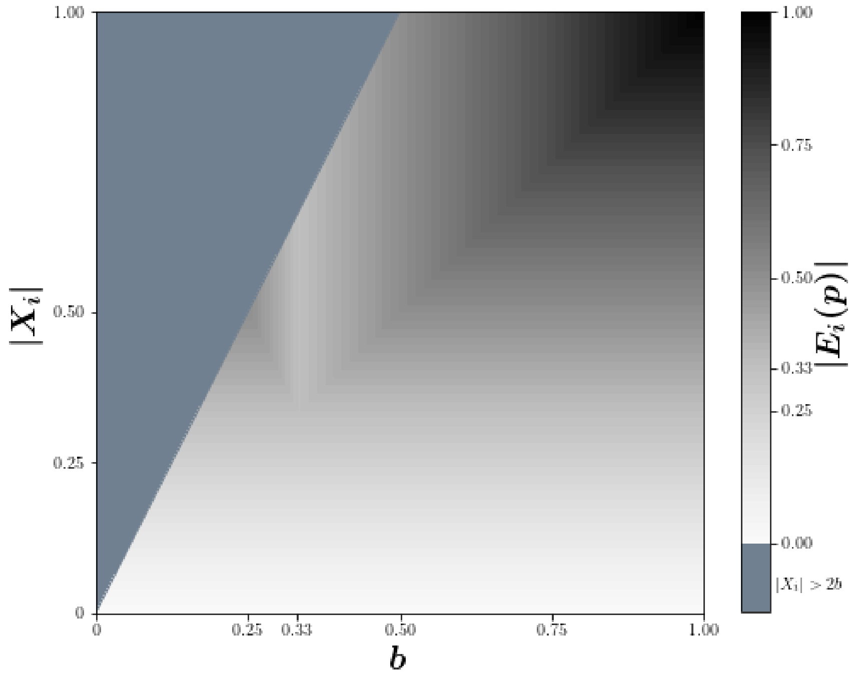
Benchmarked against this counterfactual, voters face, in expectation, the *identical* average platform position within this model. Since the game is a constant sum game, parties are not able to truly leverage their possibility to 'obfuscate' the electorate as in both scenarios $EU_i = -|X_i|, \ \forall \ i \in \{L, R\}$ in equilibrium. If assuming that the costs of analyzing a party platform increase in the number of dimensions evaluated, applying the discussed heuristic appears cost-efficient and might benefit the electorate if it can be considered risk neutral. Otherwise, if the electorate exhibits some risk aversion, it depends on the specific costs

[46]Even in the no uncertainty benchmark scenario where $b = 0$ outlined in Appendix A.2, parties do not fully converge.

associated with analyzing a party platform in its entirety, whether voters are better off with the applied heuristic.

Figure 1.7 depicts how the polarization in average platform ideologies, which is measured by $|E_i(p)|$, depends on the exogenous polarization in preferred party ideologies, which is measured by $|X_i|$, and the dispersion of median voter ideologies across political dimensions, which is measured by b.

Figure 1.7: Platform Polarization Depending on $|X_i|$ and b



Corollary 1.4. *$|X_i|$ and b determine $|E_i(p)|$ in a perfectly complementary way. Which parameter is decisive, depends on their relative size in the following way*

- *If $|X_i|$ is relatively small, it solely determines $|E_i(p)|$.*
- *If $|X_i|$ is relatively big, $|E_i(p)|$ is solely determined by b.*

It becomes evident from Figure 1.7, that $|X_i|$ and b perfectly co-determine $|E_i(p)|$. If the exogenous polarization in the ideological preferences of both parties is sufficiently small, it perfectly determines the equilibrium platform polarization. In these cases, the dispersion of median voter ideologies would allow both parties to choose an average platform position that is even more extreme than they would prefer. However, as argued in Lemma 1.2, this cannot be an optimal strategy. Thus, both parties choose their ideological bliss point as average platform position in equilibrium.

The dispersion of voter ideologies measured by b is the factor that solely determines the degree to which equilibrium platforms are polarized if $|X_i|$ is 'relatively big'. In this case, implementing the ideological bliss-point would incentivize the opponent to 'leapfrog' one's own strategy by choosing a distribution associated with an average platform position closer to the center. Hence, mirror-inverting the chosen distribution is a dominated strategy for the opponent in conditions where both parties' ideological bliss-points are excessively extreme relative to the dispersion of median voter ideologies. Consequentially, the dispersion of median voter ideologies solely determines the polarization of average platform positions in equilibrium.

How close to the center both parties' ideological bliss-points need to be located to be considered 'relatively small' also follows from Lemma 1.2. However, it requires knowledge about the counterfactual equilibrium that would be realized if Lemma 1.2 was not binding and average platforms were, consequently, not determined by the parties ideologies. Therefore, the cutoff crucially depends on how the dispersion of voter ideologies determines the polarization in average (counterfactual) equilibrium platform positions.

Corollary 1.5. *If b determines $|E_i(p)|$, there exists a non-monotonous relationship between b and $|E_i(p)|$.*

a) If $b \in (1/4, 1/3)$, the correlation is negative and crucially driven by the freedom parties have in randomizing across ideologies for a given level of b.

b) If $b \geq 1/3$, the correlation is positive and crucially driven by the dispersion of voter ideologies.

To better understand this insight let Table 1.1 briefly summarize the fundamental results from Section 1.4, conditional on b crucially determining $|E_i(p)|$. Notice that it follows from Proposition 1.3 that b never determines $|E_i(p)|$ if $b \leq 1/4$. Thus, in Table 1.1: $b \in (1/4, 1/3)$ in scenario (a) and $b \geq 1/3$ in scenario (b).

Table 1.1: Summary of Propositions 1.4 and 1.5

Scenarios	$Corr(b, E_i(p))$	Randomize in Equ.	Exist Core Ideologies	Crucial Mechanism
(a)	−	Necessary	No	$Corr(b, Var_i(p))$
(b)	+	Not necessary	Yes	b

Depicted results are conditional on $|E_i(p)| > |X_i|$ where $E_i(p)$ denotes the average platform position in equilibrium and $Var_i(p)$ describes the variance of an equilibrium function G_i that follows a given shape.

It follows from Proposition 1.4 that it is a necessary equilibrium requirement that both parties randomize across ideological positions in equilibrium if $b \in (\frac{1}{4}, \frac{1}{3})$ as long as b crucially determines the average platform position. However, it also became evident from the discussion in Section 1.4 that parties need to shift relatively more weight closer to intermediate ideologies to disincentivize the opponent from 'leapfrogging' the chosen distribution. In the limit, where $b = 1/3$, all weight is placed on $p = 1/3$ in equilibrium. Consequently, the variance of an equilibrium distribution of a given shape decreases with increasing levels of b.

Imagine, for instance, a simple *cdf* G_R that places an atom of size $1 - \alpha_R = \frac{2-6b}{2-4b}$ on $p = 1$ an atom of the remaining size on $p = 4b - 1$, which constitutes an equilibrium function as outlined in Proposition 1.4. If b increases, the variance associated with such an equilibrium distribution decreases.

This correlation between the dispersion of voter ideologies and the variance that is associated with an equilibrium distribution of party i is the mechanism that ultimately drives the correlation of the average equilibrium platform position and b. The intuition is the following: The less freedom each party has to randomize across ideologies for a given level of b, the more certain it is which ideology is ultimately drawn from its own distribution to decide the election. An identical reasoning also applies for the opponent's strategy. It is a common pattern of all results in this model that an overall increasing uncertainty about the sampled triplet $\{x_m, p_L, p_R\}$ governs the polarization of average party platforms in equilibrium. The uncertainty concerning each of the sample points included in the triplet is determined by the corresponding *cdfs* $F(x), G_i$ and $G_{j\neq i}$. For $b \in (1/4, 1/3)$, the predictability of two (G_i and $G_{j\neq i}$) of the three outlined sources of uncertainty increases with increasing levels of b. As the aggregated impact numerically outweighs an increased uncertainty about the third decisive uncertainty factor (the sampled voter ideology), the overall polarization of average platform positions must also decrease in equilibrium with higher levels of voter ideology dispersion.

The reasoning drastically changes for $b \geq 1/3$. Here, it is no longer an equilibrium requirement that parties randomize across ideological positions in their platforms. The parties' trade-off is entirely determined by the dispersion of voter ideologies, measured by b. There even exist equilibria in fully degenerate distributions. Thus, in the spirit of the just outlined intuition, the critical uncertainty within this model is solely generated by $F(x)$. As increasing levels of voter ideology dispersion also increase the uncertainty about x_m, the average platform position realized in equilibrium becomes intuitively more extreme with higher levels of b.

The observation that the opponent's ideology distribution no longer governs the parties' trade-off also becomes evident as core ideologies exist where one of the two parties certainly loses. The other one certainly wins, conditional on such a core ideology being sampled. Hence, for the given distribution function of each party, the chances of winning the election become less dependent on which ideology is being sampled from their distribution, with increasing voter ideology dispersion. It follows intuitively that each party can choose an average platform closer to its ideology bliss-point if b increases.

Corollary 1.6. *If $|X_i|$ crucially determines $|E_i(p)|$, there exists a monotonous and positive relationship between $|X_i|$ and $|E_i(p)|$.*

In contrast to conditions where the average platform position is determined by b, the intuition is straightforward if $|X_i|$ is the decisive factor: If an equilibrium requires that parties implement their ideology bliss point as average platform position, the latter becomes larger if the former increases.

Corollary 1.7. *If Lemma 1.2 is not binding such that, in equilibrium, $|E_i(p)|$ $\forall$ $i \in \{L, R\}$ is* *solely determined by b, $|X_L| = |X_R|$ is no necessary assumption for an equilibrium to exist as long as min$\{|X_L|, |X_R|\} > |E_i(p)|$.*

As long as Lemma 1.2 is not binding for any of the two parties, the average platform position in equilibrium is solely determined by the dispersion of voter ideologies b. Hence, each party's ideology bliss point $|X_i|$ does not impact a party's trade-off when choosing an ideology distribution, as argued in Corollary 1.4. Consequentially, the assumption that the parties' bliss-points are opposed around the expected median voter position is not crucial for an equilibrium to exist as all equilibria outlined in Propositions 1.3 - 1.5 hold analogously. However, it cannot be ruled out that additional equilibria exist where the chosen distribution functions also expose asymmetric first moments.

1.6 Third Party Entry

Let me assume in the following that the electoral competition between two incumbent parties L and R would result in one of the equilibria outlined in Propositions 1.3 - 1.5. In addition, there now exists a third party M that is interested in entering the electoral competition and that is not politically motivated and only cares about maximizing the expected probability of winning the election (EH_M) against both incumbent parties L and R.

Let me further assume that both parties are unaware of a third party M when designing their platforms. In addition, M can observe the distribution functions of both opponents before entering the electoral competition. As for the two incumbents, a chosen distribution is binding for M. After all parties choose their ideology distribution G_i, the representative voter randomly draws a quartet of sample points $\{x_m, p_L, p_R, p_M\}$ along which he evaluates the parties' platforms as familiar from Section 1.2. Consequentially, M can also not predict which ideologies will become relevant in the upcoming electoral campaign.[47]

The open question is if the entrant is willing to enter the electoral competition or not. Let

[47]Notice that such a setting describes a benchmark-scenario, where M winning the electoral competition is most likely without shutting down the role of nature randomly drawing ideologies.

me assume that the entrant maximizes its expected payoff from entering the election and that M incurs some costs from running an electoral campaign. These costs determine the minimum expected probability of winning $\underline{EH_M}$ for which the third-party entrant is just willing to enter the electoral competition. Without further specifying the costs of running an electoral campaign, I assume that M enters the political competition if its expected probability of winning $EH_M > \underline{EH_M}$.

Proposition 1.6. *If $\underline{EH_M} > \frac{7}{12}$, M is never willing to enter the electoral competition.*[48]

Proof: See Appendix A.1

Notice that $\underline{EH_M} > \frac{7}{12}$ poses an upper bound for the entry cutoff of party M. There exist levels of b where a lower cutoff is sufficient for M not to enter. For instance, if $|X_i| = b = 1$, M is not willing to enter the electoral competition if $EH_M > \frac{1}{2}$ is required for running a costly electoral campaign.

Within this model, the assumed voting heuristic creates uncertainty from different sources. For any possible equilibrium outlined in Propositions 1.3 - 1.5 at least one of the elements in $\{x_m, p_L, p_R, p_M\}$ is random. As a consequence, there 'certainly occurs uncertainty' in this model, which makes the outlined two-party system stable against opportunistic third-party entry, given sufficiently high costs of running for office.

1.7 Concluding Remarks

The emergence of polarized platforms appears intuitively appealing to the general public. However, it is much more difficult to argue why this observable phenomenon is expected theoretically. One reason for the unclear relationship may be that many different influencing factors can have an effect.

The presented results contribute to a better understanding of this phenomenon by disentangling the impact party and voter ideologies have on the polarization of party platforms. Both factors have a complementary impact when assuming that voters rely on a voting

[48]This result also holds in the no-uncertainty benchmark scenario where $b = 0$ that is outlined in Appendix A.2.

heuristic, and which of the two factors is decisive depends on their relative significance. Linking back to the introduction, the rising polarization of party platforms in the US can be explained by an increased polarization of the parties' ideologies, even if the dispersion of voter ideologies within the electorate remained vastly unchanged. However, it is a prerequisite for this explanation that voter ideologies were already quite dispersed at the start of the observation period.

The voting heuristic leveraged in this model creates three sources of uncertainty from each party's perspective: What ideology does the voter sample from the party's own and the opponent's distribution and what ideology of the voter's distribution is decisive? Since each party has power over one of the three dimensions, the model can explain polarized platforms in the absence of exogenous uncertainty about voter preferences. Likewise, the two-party model is robust against the entry of a third party if running for office is sufficiently costly. Even purely office-motivated candidates cannot win the election with certainty, although parties offer polarized platforms. This result might intuitively explain why the incumbent parties in the US have an incentive to keep the costs high that are associated with running an election campaign.

The model, furthermore, proposes mechanisms that govern the impact of each decisive explanatory factor on the equilibrium polarization. If party ideologies are decisive, they intuitively have a positive and monotonous impact on the equilibrium platform polarization. If voter ideologies are decisive, however, the occurring polarization is governed by the freedom each party has to randomize across ideological positions in their platforms.

The model results can be generalized in multiple directions. I want to sketch two briefly:[49] First, the findings also apply to more general settings where parties know that voters sample a specific set of ideologies C^* with certainty and one additional random ideology outside this set. If the game consists of two proper subgames, this model describes how parties compete in the dimensions outside C^*.

Second, the outlined analysis can also be generalized in a direction where the electorate randomly draws $K > 1$ sample points from both parties' distribution functions if assuming

[49] For a more detailed discussion check Spiegler (2006).

it has preferences over the unweighted average of the drawn ideologies. The equilibrium distribution functions then describe the distribution of average ideologies of distinct bundles with K elements. However, this approach generates convoluted distributions where it is generally impossible to back out the underlying distribution of ideologies across single issues.

As a final thought, the results show that the structure of party platforms follows complex patterns, which can be interpreted as a subtle ex-post verification of the exploited heuristic as a self-fulfilling prophecy: Assuming that party platforms are not analyzed in their entirety lets the structure of the parties' platforms become involved, which makes them indeed hard to analyze.

Chapter 2

Learning by Hosting: What Platforms Gain from Third-Party Data

2.1 Introduction

In many industries, platforms have become ubiquituous and unavoidable for market participants. Most recently, a large variety of competitive concerns around digital platform business models were raised.[1] One of the most prominent concerns is about Amazon's two-sided nature that leverages the access to third-party seller data obtained via its marketplace to gain a competitive advantage as a retailer.[2] A recent example that went viral is Amazon's copy of Peak Design's best-selling 'Everyday Sling' bag (Statt, 2021).[3] However, several questions remain unanswered. How exactly does a hybrid platform leverage its two-sided business model to gain a competitive advantage over third-party retailers? Why do third-party retailers offer their data to the platform if this could later lead the platform to drive the retailer out of the market? What are the trade-offs for a regulator when

[1]Numerous reports for policymakers across multiple jurisdictions were recently written on digital platforms, including the "Digital platforms inquiry" by the Australian Competition & Consumer Commission, the "Competition policy for the digital era" report written for the European Commission or the "Investigation of competition in digital markets" by the U.S. House Judiciary Subcommittee on Antitrust. The antitrust concerns of demand data usage by platforms have also been discussed by legal scholars (e.g. Khan (2016)).

[2]This practice led to investigations by the European Commission and led the recommendation of breaking-up Amazon by the U.S. House Judiciary Subcommittee on Antitrust (2020).

[3]For an in-depth investigation of the practice check, for instance, Mattioli (2020).

designing a market intervention?

We answer these questions by proposing a mechanism where hybrid platforms learn about uncertain demand for specialized products by hosting third-party specialist producers. We set up a two-period model that features a specialist producer and a hybrid platform on the supply side. On the demand side, there are some consumers that would buy the specialized product off-platform from the specialist or on-platform from any vendor. There are other consumers that already visit the platform to make other purchases and would never visit the specialist off-platform. Some of these have demand for the specialized product.

From a regulatory perspective, a platform's two-sided nature can lead to market failure if it causes the specialist to refrain from offering its product via the platform. In conditions where this is not the case, however, regulators face a fundamental trade-off between maximizing consumer surplus or overall welfare. Also, specialist retailers are expected to lobby for different policies than consumers, who may benefit from the platform's ability to launch a rivaling product.

Demand of platform-only consumers is revealed to the specialist and the platform only if the specialized product is offered on-platform. Through hosting, the platform can learn about demand from its existing customer base for the specialized product without investing the fixed costs of developing its own version. The specialist can tap into new markets by being hosted. However, if demand for the specialized product by existing platform customers turns out to be high, the platform launches its own version of the product and can foreclose the platform to the specialist. Thus, the specialist risks to create a future competitor for its existing customer base by selling via the platform. To generalize the findings, we also consider a setting where there exists a competitive fringe of sellers that can offer the specialized product at a lower quality. In light of these dynamics, we compare how a line of business restriction, a structural separation and a mandatory access regulation impact consumer surplus, profits and overall welfare.

We find that although hosting is profitable for the specialist and the platform ex-ante, it can be unprofitable for the specialist ex-post. This is the case, if observed demand from on-platform consumers is such that the platform is willing to launch the product itself, but also willing to compete for additional off-platform consumers at the expense of foregoing profits from on-platform consumers. The underlying dynamic remains in place after the introduction of an additional fringe seller. All consumers again benefit if the platform is incentivized to compete for off-platform consumers. If this is not the case, on-platform consumers can be harmed by the platform's ability to foreclose its marketplace. Thus, the above listed policy interventions potentially reduce consumer surplus, if, and only if, the platform would have otherwise aggressively competed for off-platform consumers. In all alternative scenarios, consumers weakly benefit from limiting the platform's foreclosure ability. An additional fringe seller amplifies the positive impact of regulation for on-platform consumers.

If the specialist is hosted, it pays the platform an exogenous per-unit transfer. Consumers that can buy from either seller incur fixed shopping costs when either visiting the platform or the specialist. Loyal platform consumers, however, never buy off-platform.[4] When buying on-platform, they do not incur shopping costs. We further extend the model by introducing a fringe seller that offers the specialized product at a basic quality, lower than the product sold by the specialist or the platform.

The platform launches its own version of the product in the second period and subsequently forecloses its platform to third-party retailers after having learned through previously hosting the specialist that precise on-platform demand is sufficient to recover the fixed costs and the foregone transfer payments. In some cases, the platform does not only try to sell the product to on-platform customers but also to the specialist's off-platform customers. This leads to an increase in consumer surplus. The possibility of the platform entering the product market in the second period affects the specialist's incentive to be hosted in the first period. An additional fringe seller exerts pressure on the specialist to get hosted with certainty. Concerning the platform's incentives, the fringe seller does not impact the decision to launch a product. In contrast, the willingness to compete on the off-platform

[4]An example for this would be a specialist that has a brick-and-mortar store or an online presence in an ecosystem which requires a separate registration. In many cases, off-platform customers of the specialist may already be customers of Amazon and so the additional shopping costs of visiting the platform are small. In contrast, loyal Amazon customers may not be willing to visit the specialist's brick-and-mortar store or setting up an account in a new ecosystem.

market is lower.

Finally, we use our model to study how three different policy interventions would affect market participants. We begin by describing how a line of business restriction and a structural separation, where the retail and platform business are broken-up into two separate companies, affect the equilibrium outcome. The third policy measure that we consider is a mandatory access regulation. Here, we allow the platform to launch and operate as a platform and a retailer, however, it is not allowed to foreclose its platform to third-party retailers. We find that whether these measures increase or decrease consumer surplus, firm profits and total welfare depend on the market specificities and the timing of the intervention.

Our paper makes three main contributions. First, we formalize under what circumstances the platform's access to demand data from third-party retailers distorts competition.[5]

Second, we explore how a hybrid platform's ability to distort competition impacts the incentives for third-party retailers to get hosted on the platform.[6] We can show under what circumstances a fully rational third-party retailer wants to sell via a platform ex-ante while regretting its decision ex-post.

Third, we provide a theoretical framework which allows studying how different, novel policy measures discussed by competition enforcers affect consumers, third-party retailers and platforms. We outline the conditions where regulators must trade off between maximizing consumer surplus or overall welfare but also identify circumstances under which market interventions simultaneously increase consumer surplus and the profits of third-party retailers.[7]

[5]There exists related work that also exploits demand uncertainty and the possibility of market entry by a hybrid platform to explain the pricing incentives of third-party retailers. Jiang, Jerath, and Srinivasan (2011) assume that third-party sellers know the on-platform demand for their products for sure whereas the hybrid platform does not. Such asymmetric information gives third-party sellers an incentive to keep its on-platform demand artificially low. Lam and X. Liu (2021) show that third-party sellers are more incentivized to set higher prices the more detailed demand data the platform is able to obtain.

[6]Farrell and Katz (2000) provide a somewhat related analysis of complementary input products of a system good.

[7]E.g. Hagiu, Teh, and Wright (2022), Etro (2021) and Kang and Muir (2021) argue that regulating a monopoly platform ultimately harms consumers under almost all circumstances. However, Anderson and Bedre-Defolie (2021) find that platforms operating in a hybrid mode harm consumers through their ability to set prices and third-party seller fees strategically.

Concurrent with this paper, Madsen and Vellodi (2021) and Hervas-Drane and Shelegia (2022) investigate how hybrid platforms leverage third-party (demand) data. Compared to their approaches, we do not focus on niche products that are exclusively sold via a dominant hybrid platform.[8] We explicitly model an off-platform distribution channel that gives independent retailers the possibility to abstain from being hosted on the platform.[9] In distinction to Madsen and Vellodi (2021), we model a specific mechanism that allows the platform to distort competition through its ability to foreclose the platform to exploit a loyal customer base.[10] Furthermore, our analysis considers launching already existing products, which is only costly for the platform and does not focus on the incentive to innovate for independent sellers. Hervas-Drane and Shelegia (2022) explicitly model a multiproduct platform. They propose a mechanism that does not resolve uncertainty about product demand, which is deterministic in their model. Their mechanism lets the platform become aware of the existence of certain products. The launching decision is ultimately determined by a capacity constraint.

Our work is complementary to a branch of literature that models alternative mechanisms to analyze what determine competition between third-party retailers and hybrid platforms. Where Hagiu and Spulber (2013) argue that platforms provide first-party content if it is complementary to offerings from third-parties, we show that this can also be the case for rivaling products. We follow Hagiu, Teh, et al. (2022) and Etro (2021) by assuming that the platform can introduce an inferior version of the specialist's product. In contrast to Hagiu, Teh, et al. (2022), we assume that there exist consumers that exclusively buy on the platform. Furthermore, our work is related to the literature that determines the differences between a platform- and selected alternative business modes.[11] More broadly, this paper

[8]Based on the intuition from Kang and Muir (2021), highly innovative niche producers could self-select into contract schemes that protect them from imitation. Amazon's *Exclusive Brands* program, for instance, targets highly successful niche product providers to exclusively distribute their products via Amazon in exchange for protection against market entry from Amazon's own private labels and competing third-party sellers.

[9]Here we follow Edelman and Wright (2015), Ronayne and Taylor (2022), Shen and Wright (2019), Wang and Wright (2020) and Hagiu, Teh, et al. (2022) who also model such an off-platform channel to highlight alternative mechanisms.

[10]This is in line with the intuition in Gutiérrez (2021), Padilla, Perkins, and Piccolo (2022) and Corniere and Taylor (2019) and the empirical evidence from N. Chen and Tsai (2019), Cure, Hunold, Kesler, Laitenberger, and Larrieu (2022), Zhu and Q. Liu (2018) and Wen and Zhu (2019)

[11]E.g. Hagiu and Wright (2015) or Hagiu, Jullien, and Wright (2020)

also relates to the literature on multisided platforms[12] and the recently increasing literature on the collection of consumer data.[13]

The paper is structured as follows: We continue with introducing Amazon's business model to ultimately inform some of our model's assumptions outlined in the subsequent model setup in Section 2.3. Next, we determine the players' equilibrium strategies under a monopolistic and a competitive market structure. Finally, we discuss the effect of different market interventions.

2.2 Empirical Observations: Amazon

To motivate our theoretical assumptions, we begin by pointing out several empirical observations about Amazon's business model.

Observation 1. *Third-party sellers play an important role on Amazon's marketplace.*

After steady growth over the past decade, Amazon's overall sales jumped to $519 bn in 2020.[14] Furthermore, the contribution of third-party sellers grew from 34% to 62% over the last decade.[15]

Observation 2. *Amazon has loyal customers who predominantly buy on-platform.*

Subscriptions of Amazon's Prime service rose to $140 mn in the US during the past decade. In 2020, nearly 80% of all sales on Amazon's US website came from Prime customers.[16] Amazon Prime customers are also more valuable to Amazon. In the US, they spent on average an amount close to $1400 in 2018, whereas non-Prime members spent on average $600 on Amazon's marketplace (CIRP, 2018).[17] 79% of Prime members name free shipping as the primary reason for their membership.[18]

[12]E.g. Caillaud and Jullien (2003), Armstrong (2006), Eisenmann, Parker, and Alstyne (2006) and Karle, Peitz, and Reisinger (2020)

[13]E.g. Acemoglu, Makhdoumi, Malekian, and Ozdaglar (forthcoming) and Bergemann, Bonatti, and Gan (2020)

[14]Amazon's e-commerce market share was estimated at 50% in 2020 ("Statista" 2021)

[15]See also Figure B.2 in Appendix B.3.

[16]See also Figure B.3 in Appendix B.3.

[17]In 2018, the average U.S. adult spent $2,500 on e-commerce.

[18]Check the "It's All About Free Shipping" (2018) report for details.

Observation 3. *Amazon learns about demand by hosting third-party retailers.*

Before launching a product, a hybrid platform can leverage its access to detailed data on transactions via its marketplace to identify the most profitable product categories. Zhu and Q. Liu (2018) and Jiang et al. (2011) show descriptive evidence that Amazon is launching own, in-house produced products in categories with high demand for third-party products.[19]

Observation 4. *Amazon forecloses its platform through steering and self-preferencing.*

N. Chen and Tsai (2019) show that Amazon places products it sells as a retailer more prominently in the 'Frequently bought together' section than identical products from hosted third-party retailers. Zhu and Q. Liu (2018) find that third-party sellers that directly compete with Amazon reduce their product offerings. For instance, descriptive evidence from the paper shredder market shows that third-party sellers drastically reduced their offerings after Amazon's entry.[20] There is also evidence suggesting that Amazon delisted rival third-party sellers on short notice (e.g., Bezos, 2020).

Observation 5. *Amazon charges standardized seller fees on a per transaction basis.*

Depending on the product category, Amazon charges a standardized revenue share for each third-party seller transaction facilitated via the platform. While product categories are defined quite broadly, transfer fees are fixed within each category. For most categories, fees lie between 8% and 15%.[21] If a shipment resulting from a third-party transaction is fulfilled by Amazon (FBA), Amazon charges an additional per-transaction-fee depending on the package's size. In 2020 more than 90% of third-party sellers used Amazon's FBA services (Kaziukenas, 2020). As Amazon's launching decision is determined on the product level, category-specific transfers that drive Amazon's decision to introduce new products can be assumed exogenous.

[19] Seller-specific data was used in the past to aid Amazon's private label business (Bezos, 2020).

[20] See Figure B.4 in Appendix B.3.

[21] For more detailed information on Amazon seller fees, visit https://sell.amazon.com/pricing.html

2.3 Model Setup

We begin by describing a simple two-period model. Future period payoffs are discounted by a factor δ expressing time preferences or uncertainty about the future.

On the supply side, there is a specialist S, a fringe seller f and a platform M.[22] S and f produce a product A. The cost of production is normalized to zero for both. They sell A via their own off-platform distribution channel and set off-platform prices p^o_R and p^o_f. If the specialist or the fringe seller is hosted to additionally sell A via the platform, they charge distinct on-platform prices p^m_R and p^m_f.

The platform is an established platform for sellers in other product categories from which we abstract in this model. Product A can be made available on the platform not only through hosting the specialist and/or the fringe seller as third-party retailers, but the platform could also launch its own version of product A, sell this as a retailer and become a hybrid platform (i.e., a direct retailer and a platform for third-party retailers).[23] When hosting a third-party retailer on its platform, M charges a transfer payment τ for each transaction facilitated via the platform. We assume τ to be exogenous and constant over time. If M launches its version of product A, it incurs a fixed and sunk cost F. In addition, we assume that M can only launch its own version of A in the second period after having hosted the specialist to learn about on-platform demand in the previous period.[24] Furthermore, we assume that the platform can foreclose its distribution channel for third-party retailers. If the platform launches its own version of product A, it sells it at p^m_M. Finally, we also normalize the platform's marginal cost of production to zero such that there is *no marginal cost advantage* for any of the players.

On the demand side, there exists a unit mass of atomistic consumers. There are three different types of consumers:

[22]The fringe seller is only relevant when analyzing the competitive market structure in section 2.5.

[23]Even if M chooses not to host S or f and launch its own version of A, it remains a platform for other products that we do not consider more closely here.

[24]This assumption can be made an endogenous decision by the specialist if assuming sufficiently high fixed costs F such that launching product A is ex-ante not profitable for the platform. Please refer to the discussion in Appendix B.2 for further details. Imposing this assumption streamlines the analysis, especially in the second part of the paper.

A fraction ϕ of consumers are of *Type 1*. These consumers can buy from either seller off- or on-platform. Whenever *Type 1* consumers visit the specialist, the fringe seller or the platform, they incur shopping costs s. They buy A from the specialist (or the fringe seller) as long as the platform does not launch its own version of product A. However, if the platform also launches product A, it can compete over these consumers. Therefore, they are the so-called 'contested consumers'.

A fraction $(1-\phi)\xi$ of consumers are of *Type 2A*. They are existing customers of the platform and do not incur any shopping costs when buying product A via the platform, independent of whether the product is offered by the platform itself or by the specialist through hosting.[25] Their shopping costs for buying any product off-platform are assumed to be prohibitively high such that they only purchase product A if offered on the platform. Therefore, they are the so-called 'loyal consumers'.

Both consumer types have an inelastic unit demand for good A. They value the version of good A produced by M at $u_A + \Delta_M$, the version of S at $u_A + \Delta_R$ and the version of f at u_A. We assume here that $0 \leq \Delta_M < \Delta_R$. Hence, if the platform decides to introduce good A, it does so at a lower quality than the specialist and at a (weakly) higher quality than the fringe seller.[26] The remaining fraction $(1-\phi)(1-\xi)$ of consumers are of *Type 2B*. These are also existing customers of the platform (i.e., they buy other goods on the platform), but they do not have any demand for product A. Table 2.1 summarizes the characteristics of the distinct consumer types.

Table 2.1: Characteristics of Consumer Types

Consumer Type	Valuation for A	Population share	Shopping costs on-platform	Shopping costs off-platform
Type 1	$u_A + \left(\Delta_j\right)^*$	ϕ	s	s
Type 2A	$u_A + \left(\Delta_j\right)^*$	$(1-\phi)\xi$	0	∞
Type 2B	–	$(1-\phi)(1-\xi)$	0	∞

"*": produced by $j \in \{R, M\}$

[25] We assume that *Type 2A* consumers already purchase other products via the platform and so do not incur any additional shopping costs from visiting the platform to buy product A.

[26] Shopova (2021) shows that a hybrid platform prefers to launch a lower quality product than a specialized retailer.

The share ϕ of *Type 1* consumers is deterministic, fixed over time and known to all players. In contrast, ξ that determines the demand from *Type 2A* consumers is unknown to the specialist, the fringe seller and the platform and only becomes common knowledge for all players if the specialist or the fringe seller is hosted. ξ is drawn from a continuous distribution $G(\xi)$ with support over the closed interval $[0, 1]$. Once revealed, ξ is constant over time.

The timing of the game is as follows: In the first period, the specialist and the fringe seller start by simultaneously deciding whether they want to be hosted on the platform for a given transfer τ. If this is the case, the platform itself accepts or denies the hosting inquiries, taking τ as given. On- and off-platform prices by all players are chosen simultaneously. After prices are set, consumers decide if, where and from which seller to buy. If product A is sold on the platform, stochastic demand ξ is revealed. The timing in $t = 2$ is the same with one exception. If ξ was revealed in the first period, it is already known when entering the second period and the platform starts first with determining whether it wants to launch its own version of product A.

Throughout, we solve for subgame perfect Nash equilibria. We apply the following tie-breaking rules: First, whenever consumers are indifferent between purchasing A from different suppliers, they choose the highest quality product. Second, if there exist several equilibria which are payoff equivalent for M, we focus on the one which is most desirable from a consumer perspective, which means that M launches A and R (and f) is hosted in case of indifference.[27] Henceforth, we rule out equilibria that rely on playing weakly dominated pricing strategies.

Lastly, we assume that $\tau < u_A - s$ to streamline the analysis (or vice versa $s < u_A - \tau$). This ensures that shopping costs are not prohibitive such that *Type 1* consumers may buy on-platform if competitive prices are charged.

[27] This is similar in spirit to Dinerstein, Einav, Levin, and Sundaresan (2018), who argue that if long-run revenues of the platform largely depend on attracting customers (rather than sellers), the actions taken to maximize long-run profits are highly correlated with the actions taken to maximize short-run consumer surplus. The assumption is particularly important when selecting between equilibria where just one or several third-party sellers are hosted on $M's$ platform.

2.4 Monopolistic Market Structure

This section determines the conditions where M is willing to become a hybrid platform by launching A. We begin with analyzing the case without a fringe seller, where R is a monopolist if M does not launch its own version of product A.

It follows from a straightforward argument that in all second-period subgames that are eventually reached, the platform never wants to host the specialist and simultaneously launch its own version of product A. Vice versa, the platform does want to host the specialist if not launching A. If the platform launches product A on its own, hosting the specialist cannot be optimal since it (R) can leverage its quality advantage when competing in prices. If, however, the platform does not want to offer A itself, hosting the specialist generates transfer payments and additionally offers an opportunity to learn about product demand. As a direct consequence, the platform never prefers to operate in a dual mode where M competes alongside a hosted third-party retailer in the same product category on its platform.[28]

In addition, the specialist certainly wants to be hosted in the second period, which also appears intuitive. Since the game ends afterward by assumption, the specialist does not risk creating a future competitor by revealing precise on-platform demand if being hosted in the second period.

To find the subgame perfect Nash equilibrium of the game, we solve it backward.

2.4.1 Hosting Subgame

Let us begin with analyzing the second-period subgame where the specialist was hosted in the previous stage such that a specific level of stochastic demand ξ is realized at the beginning of the second period.[29] Then, it crucially depends on the platform's launching

[28]This does not rule out that M may well be a platform for some products and a retailer for others. However, such a multiproduct case is not modeled here.

[29]Notice that stochastic demand ξ could also be observed if M launches A itself in the first period. However, we rule this out by assumption as it does not coincide with the spirit of the learning mechanism we want to analyze in this project. In Appendix B.2, we show that with sufficiently high fixed costs F, M also never launches A as long as ξ is unknown since launching is not profitable in expectation.

and pricing decisions, which outcome is realized in the second period. There may exist three distinct equilibrium outcomes in the second period. As shown in the subsequent Lemmas, it depends on a specific realization of the demand parameter ξ, which of them is realized.

Pooling Equilibrium outcome: The platform does not launch A. The specialist is hosted and charges $p_R^o = u_A + \Delta_R - s$ off-platform and $p_R^m = u_A + \Delta_R$ on-platform. *Type 1* and *Type 2A* consumers both buy from R on- and off-platform.

Separating Equilibrium outcome: The platform launches A and charges $p_M^m = u_A + \Delta_M$; The specialist is not hosted and charges $p_R^o = u_A + \Delta_R - s$. *Type 1* consumers purchase A directly from the specialist. *Type 2A* consumers purchase A from the platform.

Mixed Equilibrium outcome: The platform launches A. The specialist is not hosted and there exists a mixed strategy equilibrium where:

- p_M^m is distributed according to the $c.d.f.$ F_M with support $T_M = \left[\frac{u_A+\Delta_M}{\frac{\phi}{(1-\phi)\xi}+1}, u_A + \Delta_M - s\right) \cup \{u_A + \Delta_M\}$, where...

$$F_M(p_M^m) = \begin{cases} 1 - \frac{1}{p_M^m+\Delta_R-\Delta_M}\left(\frac{u_A+\Delta_M}{\frac{\phi}{(1-\phi)\xi}+1} + \Delta_R - \Delta_M\right) & \text{if} \quad \frac{u_A + \Delta_M}{\frac{\phi}{(1-\phi)\xi} + 1} \leq p_M^m < u_A + \Delta_M - s \\ 1 - \frac{1}{u_A-s+\Delta_R}\left(\frac{u_A+\Delta_M}{\frac{\phi}{(1-\phi)\xi}+1} + \Delta_R - \Delta_M\right) & \text{if} \quad u_A + \Delta_M - s \leq p_M^m < u_A + \Delta_M \\ 1 & \text{if} \quad u_A + \Delta_M \leq p_M^m \end{cases}$$

- p_R^o is distributed according to the $c.d.f.$ F_R with support $T_R = \left[\Delta_R - \Delta_M + \frac{u_A+\Delta_M}{\frac{\phi}{(1-\phi)\xi}+1}, \Delta_R + u_A - s\right]$, where...

$$F_R(p_R^o) = \begin{cases} 1 - \frac{(1-\phi)\xi}{\phi}\left(\frac{u_A+\Delta_M}{p_R^o-\Delta_R+\Delta_M} - 1\right) & \text{if} \quad \frac{u_A + \Delta_M}{\frac{\phi}{(1-\phi)\xi} + 1} + \Delta_R - \Delta_M \leq p_R^o < u_A + \Delta_R - s \\ 1 & \text{if} \quad u_A + \Delta_R - s \leq p_R^o \end{cases}$$

Depending on the prices drawn from $F_M(p^m_M)$ and $F_R(p^o_R)$, *Type 1* consumers can purchase A from M or R. *Type 2A* consumers purchase A from M.

From the pricing strategies played in each equilibrium outcome listed above, we find that the *Mixed Equ.* outcome is the most desirable one from a consumer perspective. From the specialist's perspective, the *Pooling Equ.*, where M does not launch A, is the most profitable. Lemmas 2.1 - 2.3 specify which of the above-specified equilibria is realized in $t = 2$ and how this affects the payoff of all players in the market.

Lemma 2.1. *If R is hosted in $t = 1$, the* ***Pooling Equ.*** *outcome is the unique equilibrium outcome if, and only if:*

$$\xi < \frac{F}{(1-\phi)(u_A + \Delta_M - \tau)} \equiv C^{Monop.}_{Launch} \tag{2.1}$$

In this case, profits of R are $\pi_R = \phi(u_A + \Delta_R - s) + (1-\phi)\xi(u_A + \Delta_R - \tau)$, profits of M are $\pi_M = \tau(1-\phi)\xi$ and consumer surplus is $CS = 0$ in $t = 2$.

Proof: See Appendix B.1

Thus, the *Pooling Equ.* outcome occurs if the demand from *Type 2A* consumers is relatively small, i.e. if stochastic demand ξ lies below the launching cutoff. Then, on-platform demand for product A is insufficient for the platform to recover its fixed costs F from launching A and the foregone transfer payment it receives from hosting the specialist. Hence, the specialist is the sole supplier of product A and can extract the entire rent from all consumers.

Lemma 2.2. *If R is hosted in $t = 1$, the* ***Separating Equ.*** *outcome is the unique equilibrium outcome in $t = 2$ if, and only if:*

$$\xi > max\{C^{Monop.}_{Competition}, C^{Monop.}_{Launch}\} \quad where \quad C^{Monop.}_{Competition} \equiv \frac{\phi(u_A + \Delta_M - s)}{(1-\phi)s} \tag{2.2}$$

In this case, profits of R are $\pi_R = \phi(u_A + \Delta_R - s)$, profits of M are $\pi_M = (1-\phi)\xi(u_A + \Delta_M) - F$ and consumer surplus is $CS = 0$ in $t = 2$.

Proof: See Appendix B.1

By the reversed reasoning of Lemma 2.1, launching A is now profitable for the platform at relatively high realizations of stochastic demand above the launching-cutoff. If stochastic demand ξ also lies above the competition-cutoff, on-platform demand from just *Type 2A* consumers is large enough to disincentivize the platform from lowering its price by the shopping costs s to also trying to attract *Type 1* consumers. If both conditions hold, M forecloses its platform to the specialist and extracts the entire rent from *Type 2A* consumers that are loyal to the platform by charging monopoly prices. The specialist also continues to charge monopoly prices from *Type 1* consumers on the off-platform market. As a consequence, markets become segmented and consumers do not benefit from the platform acting as an additional supplier of product A. However, compared to a situation where the *Pooling Equ.* outcome is realized overall welfare decreases as M can only offer a product of inferior quality and incurs additional fixed costs F.

Lemma 2.3. *If R is hosted in $t = 1$, the **Mixed Equ.** outcome is the unique equilibrium outcome in $t = 2$ if, and only if:*

$$C^{Monop.}_{Launch} < \xi < C^{Monop.}_{Comp.} \tag{2.3}$$

In this case, (expected) profits of R are $E(\pi_R) = \phi\left(\frac{u_A+\Delta_M}{\frac{\phi}{(1-\phi)\xi}+1} + \Delta_R - \Delta_M\right)$, (expected) profits of M are $E(\pi_M) = (1-\phi)\xi(u_A + \Delta_M) - F$ and expected consumer surplus of Type 1 and Type 2A consumers is strictly greater than zero in $t = 2$.

Proof: See Appendix B.1

For intermediate realizations of stochastic demand, i.e., in between the launching- and the competition-cutoff, it is still profitable for the platform to launch product A on its own. However, at such intermediate levels of stochastic demand, the platform is incentivized to lower its price for product A by the shopping costs s as additional demand from *Type 1* consumers outweighs foregone profits from selling A at a price below the willingness to pay to loyal *Type 2A* consumers. Therefore, the specialist and the platform compete over the contested *Type 1* consumers.

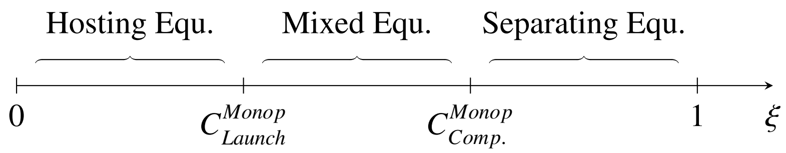
As both players' equilibrium strategies place an atom on the monopoly prices, perfect market segmentation, where the platform extracts monopoly rents from *Type 2A* consumers and the specialist from *Type 1* consumers, can still occur in the *Mixed Equ.* outcome. Nevertheless, both consumer types profit from the increased competition as they are, in expectation, strictly better off than under any alternative equilibrium outcome. In particular, this is true for *Type 2A* because the platform cannot price discriminate between different types of consumers. The increase in expected consumer surplus is borne entirely by the specialist. The platform's expected profit is the same as in the alternatively reached *Separating Equ.* outcome where the platform charges monopoly prices from loyal *Type 2A* consumers. Because of this outside option, the platform never fully commits to contest *Type 1* consumers, which still allows the specialist to capture some rent that is strictly greater than its quality advantage.

Notice also that the transition from the *Mixed Equ.* outcome to the *Separating Equ.* outcome is smooth. Thus, in the limit, where stochastic demand ξ approaches the competition-cutoff from below, the pricing strategies in the *Mixed Equ.* outcome and the *Separating Equ.* outcome converge. This shows that the market segmentation and likewise the (expected) profit of each supplier increases, the larger or the more valuable the loyal customer base of *Type 2A* consumers becomes for the platform.

Finally, (expected) overall welfare is the lowest in the *Mixed Equ.* outcome since the platform additionally sells its weakly inferior version of product *A* to *Type 1* consumers with positive probability.

To close this section, Figure 2.1 once more sums up how the distinct second-period outcomes depend on a specific realization of the demand parameter ξ in the *Hosting Subgame*.

Figure 2.1: Outcomes Hosting Subgame



2.4.2 No Hosting Subgame

If the specialist is not hosted in the first period, an alternative subgame is reached in the second period. There exists a unique outcome:

Lemma 2.4. *If the specialist is not hosted in $t = 1$, the* ***Pooling Equ.*** *outcome is the unique equilibrium outcome in $t = 2$.*[30]

Proof: See Appendix B.1

If the specialist is not hosted in the previous period, ξ is not revealed. Then, the platform does not launch A by assumption. Consequently, it follows from our initial reasoning in this section that the platform and the specialist mutually benefit if the specialist is hosted.

2.4.3 Equilibrium analysis

We now analyze the dynamic equilibrium of the baseline model. In the previous section, we derived the equilibria in the second period for all possible actions in the first period. Given that we assume that the platform does not launch A without knowing stochastic demand ξ, the platform certainly wants to host the specialist in the first period. The open question is: Will the specialist be willing to get hosted in the first period or not?

We now use our knowledge about the realization of equilibrium outcomes and profits in each second period subgame to inform the specialist's hosting decision in the first period. Although the specialist cannot observe ξ, it is forward-looking and forms an expectation about the realization of ξ that ultimately determines its decision. Depending on the distribution of ξ and a given set of parameters, each equilibrium outcome previously specified in the *Hosting* subgame is expected to occur with a well-specified probability in the second period. In the following exposition, $Pr(Mixed\ Equ.)$ denotes the probability that the *Mixed Equ.* outcome occurs and $Pr(Pooling\ Equ.)$ denotes the probability that the *Pooling Equ.* outcome occurs where the platform does not launch A.

[30]See Lemma 2.1 for details on the Pooling Equilibrium outcome.

Proposition 2.1. *There is a unique equilibrium outcome in which the specialist is hosted in $t = 1$ if, and only if:*

$$\Delta_R > \frac{\delta\phi \boldsymbol{Pr(Mixed\ Equ.)}\left(\frac{\phi(u_A+\Delta_M)}{\phi+(1-\phi)E(\xi|Mixed\ Equ.)} - s\right)}{(1-\phi)\,E(\xi)\left(1-\delta\left(1-\boldsymbol{Pr(Pooling\ Equ.)}\frac{E(\xi|Pooling\ Equ.)}{E(\xi)}\right)\right)} - u_A + \tau \tag{2.4}$$

Then, depending on a given vector of parameters, one of the equilibrium outcomes outlined in Lemmas (2.1)-(2.3) is realized in $t = 2$.

Otherwise, the specialist is not hosted in $t = 1$ and the Pooling Equ. outcome is realized in $t = 2$.[31]

Proof: See Appendix B.1

Proposition 2.1 follows from the specialist trading off the expected discounted gains and potential losses from being hosted in the first period against the expected outcome if only being hosted in the second period. We see that a higher probability of the platform not launching A increases the incentive of the specialist to get hosted in the first period as the *Pooling Equ.* outcome, which otherwise only occurs in the second period, is more beneficial for the specialist. The same is true if the threat of ending up in the competitive *Mixed Equ.* outcome in the second period decreases. Δ_R directly impacts the immediate gains from getting hosted. However, the odds of ending up in the *Mixed Equ.* outcome in the second period remain unchanged as the specialist's quality parameter Δ_R does not impact the platform's pricing and launching decision. The same is true for δ, which determines how harmful potential second-period losses are for the specialist when deciding whether to be hosted in the first period.

If the specialist is willing to be hosted in the first period, consumers strictly gain in expectation since only if the specialist is hosted in $t = 1$, the competitive *Mixed Equ.* outcome can occur in the subsequent stage. Overall welfare also increases as the demand of *Type 2A* consumer gets served in $t = 1$. However, the positive impact on overall welfare is counteracted by giving the platform the possibility to introduce its inferior version of A in $t = 2$ after the specialist is hosted.

[31] If Condition 2.4 holds with equality both equilibrium outcomes co-exist.

In addition, this result explains why even sophisticated specialized retailers are hosted on the platform, although they might create a future competitor that can infiltrate the off-platform market after learning about demand through hosting.

Corollary 2.1. *A forward looking specialist might regret being hosted in the first period ex-post, if there occurs a realization of stochastic demand ξ such that the Mixed Equ. outcome is realized in $t = 2$.*

Proof: Follows from Proposition 2.1

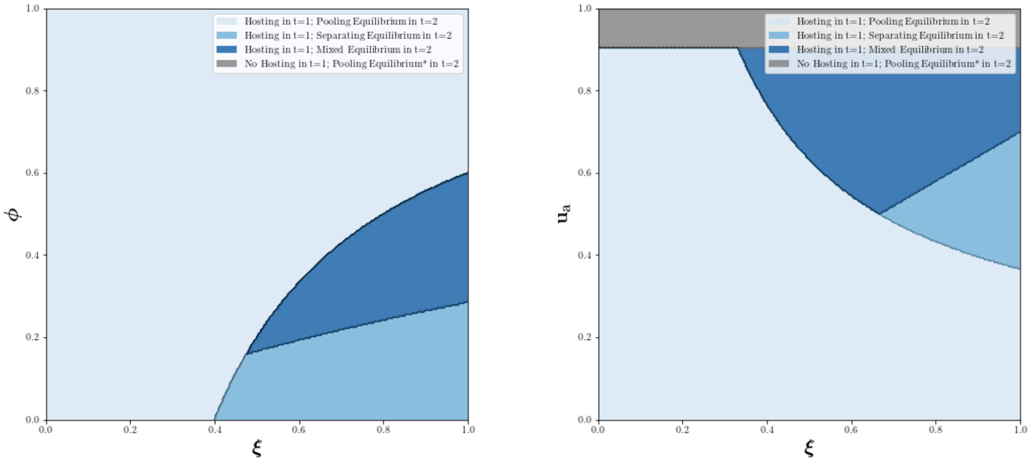
With Proposition 2.1, however, there can also occur scenarios where the specialist finds it ex-ante optimal to be hosted in the first period but regrets its decision ex-post in the second period if ending up with the *Mixed Equ.* outcome. This is the case if a forward looking specialist would not have been hosted if knowing with certainty that the *Mixed Equ.* outcome occurs in the second period.

Figure 2.2 displays parameter constellations where a situation as described in Corollary 2.1 can occur. It displays how the dynamic equilibrium outcome depends on the realized stochastic demand parameter ξ and the share of *Type 1* customers ϕ (Panel (a)) or the customers' base valuation u_A (Panel (b)).

From both panels in Figure 2.2, we can conclude that there exist parameter vectors where the specialist is hosted in the first period and then there occurs a realization of stochastic ξ such that the *Mixed Equ.* outcome is realized in the second period. Furthermore, Panel (a) shows that for the chosen parameter values, the specialist always wants to be hosted in $t = 1$, independent of the size of the off-platform market that is characterized by ϕ. Panel (b) demonstrates that this might not be true for high levels of consumer valuation denoted by u_A. Although a high consumer valuation for good A directly scales up immediate profits from being hosted in the first period, increasing levels of u_A also increase the overall likelihood that the platform launches A and that the *Mixed Equ.* outcome is realized in the second period. We see that for high levels of consumer valuation, the latter effect dominates the immediate effect, such that the specialist is unwilling to be hosted in the first period.[32]

[32]Appendix B.2 provides a more detailed discussion of how different model parameters impact the real-

Figure 2.2: Numerical Simulation



(a) Equilibrium depending on ξ and ϕ

(b) Equilibrium depending on ξ and u_A

The following parameter values are chosen for this numerical example: $s = 0.2$, $F = 0.2$, $\delta = 1$, $\Delta_R = 0.3$, $\Delta_M = 0.1$, $\tau = 0.2$, $u_A = 0.6$ (Panel (a)) and $\phi = 0.25$ (Panel(b)). ξ is drawn from the Kumaraswamy distribution $G(\xi) = 1 - (1 - \xi^{\alpha})^{\beta}$ where $\alpha = 1.25$ and $\beta = 5$.

2.5 Competitive Market Structure

In the previous section, the specialist and the platform can set monopoly prices off- and on-platform except for parameter constellations that induce the *Mixed Equ.* outcome in $t = 2$. To analyze the determinants of the platform's launching and pricing decision under an alternative market structure, we now introduce an additional third-party fringe seller f to our model. The fringe seller also produces and sells A and has identical strategy options as the specialist. Likewise, no costs of producing A accrue. Consumers value the version of A produced by the fringe seller at u_A.

With the introduction of a fringe seller, the off-platform market is now competitive. Again, the platform is willing to accept any hosting inquiry as long as it does not launch product A on its own. The existence of the fringe seller, however, exerts pressure on the specialist to get hosted in the first period.

Lemma 2.5. *With an additional fringe seller, R is hosted with certainty in $t = 1$.*

ization of second-period and dynamic outcomes.

Proof: See Appendix B.1

The fringe seller is not threatened by additional competition from the platform for *Type 1* consumers in the second period as its profit from selling on the off-platform market equals zero in any case due to its inferior quality product. Hence, if the specialist is not hosted in the first period, it is a best reply for the fringe seller to get hosted in $t = 1$ as it is thereby able to charge monopoly prices from *Type 2A* consumers. Therefore, stochastic demand ξ is revealed in any case, independent of the specialist's hosting decision. Thus, by also getting hosted in the first period, the specialist can at least generate some profits from additional sales to on-platform consumers in the first period.

With Lemma 2.5, stochastic demand is certainly revealed in the first period. Hence, only a single subgame exists in the subsequent stage, similar to the *Hosting Subgame* under the monopolistic market structure. As in the previous section, there again exist three distinct equilibrium outcomes.[33]

Lemma 2.6. *With an additional fringe seller M launches A in* $t = 2$ *if, and only if:*

$$\xi > \frac{F}{(1-\phi)(u_A + \Delta_M - \tau)} \equiv C^{Fringe}_{Launch} = C^{Monop.}_{Launch} \tag{2.5}$$

Otherwise, the Pooling Equ.(f) is the unique equilibrium outcome in $t = 2$.

Proof: See Appendix B.1

The platform's incentive to host a third-party retailer remains unaffected by the introduction of a fringe seller as the platform can still subsequently foreclose its marketplace to extract monopoly rents from *Type 2A* consumers. Again, launching is not profitable if demand from loyal *Type 2A* customers is insufficient to recover the fixed costs from launching *A* and foregone transfer payments from hosting a third-party supplier.

Lemma 2.7. *With the existence of a fringe seller the Mixed Equ.(f) outcome is the unique*

[33]The specific outcomes are included in the proofs of the subsequent Lemmas in Appendix B.1.

equilibrium outcome in $t = 2$ *if, and only if:*

$$C^{Fringe}_{Launch} < \xi < \frac{\Delta_M \phi}{u_A(1-\phi)} \equiv C^{Fringe}_{Competition} < C^{Monop.}_{Competition} \tag{2.6}$$

Otherwise, if $\xi > max\{C^{Fringe}_{Competition}, C^{Fringe}_{Launch}\}$*, the Separating Equ.(f) outcome is the unique equilibrium outcome in* $t = 2$*.*

Proof: See Appendix B.1

As the fringe seller causes the off-platform market to become more competitive, the platform's incentive to contest *Type 1* consumers conditional on launching *A* reduces compared to a monopolistic market structure. Therefore, the cutoff above which markets are perfectly segmented decreases.

Most interestingly, the consumer valuation u_A enters $C^{Fringe}_{Competition}$ in the exact opposite way it enters $C^{Monop.}_{Competition}$ as it is no longer scaling up profits on the off-platform market. Therefore, increasing levels of consumer valuation now have an overall negative effect on the platform's incentive to compete over *Type 1* consumers as they only let forgone profits from selling to *Type 2A* consumers at a reduced price loom larger. Nevertheless, situations still can occur where *M* is incentivized to contest *Type 1* consumers at intermediate realizations of stochastic demand ξ.

Proposition 2.2. *There exists a unique equilibrium outcome where the specialist and the fringe seller are hosted with certainty on M′s platform and the Pooling Equ.(f) outcome is realized in* $t = 1$*.*

Then, depending on the specific realization of ξ *in* $t = 1$*, one of the three distinct equilibrium outcomes outlined in Lemmas 2.6 - 2.7 is realized in* $t = 2$*.*

Proof: See Appendix B.1

Proposition 2.2 summarizes the results from this section. The rationale is the same as with a monopolistic market structure. The only difference in the first period is that the specialist (and the fringe seller) are hosted with certainty and precise on-platform demand from loyal *Type 2A* is revealed. Furthermore, the specialist can no longer extract monopoly rents on both markets.

It directly follows from the Lemmas outlined above how the realization of equilibrium outcomes in the second period depends on a specific realization of the demand parameter ξ. When discussing the impact of a fringe seller, it is straightforward to benchmark the results from this section against these under a monopolistic market structure known from Lemmas 2.1 - 2.3. We thereby implicitly assume that the *Hosting Subgame* is realized without a fringe seller. Eventually, we distinguish between the following four mutually exclusive counterfactual scenarios outlined in Table 2.2.[34]

Table 2.2: Counterfactual Scenarios: Fringe Seller

		Equilibrium Outcome:	
Scenario		Without f	With f
(i):	$\xi < C^{Fringe/Monop.}_{Launch}$	Pool. Equ.	Pool. Equ. (f)
(ii):	$\xi \in \left(C^{Fringe/Monop.}_{Launch}, C^{Fringe}_{Competition}\right)$	Mixed Equ.	Mixed Equ. (f)
(iii):	$\xi \in \left(C^{Fringe}_{Competition}, C^{Monop.}_{Competition}\right)$	Mixed Equ.	Separ. Equ. (f)
(iv):	$\xi > C^{Monop.}_{Competition}$	Separ. Equ.	Separ. Equ. (f)

The following Corollary summarizes the impact of introducing a fringe seller. Where applicable, the comparison is made based on expected prices faced by all players in equilibrium. All necessary outcomes follow from (the proofs) of Lemmas 2.1 - 2.3 and Lemmas 2.6 and 2.7.

Corollary 2.2. *Compared to the Hosting Subgame under a monopolistic market structure and given the counterfactual scenarios (i)-(iv) from Table 2.2, an additional fringe seller impacts the model in* $t = 2$ *as outlined in Table 2.3.*

Discussion: See Appendix B.1

As M can still foreclose its platform to third-party sellers by assumption, the fringe seller does not impact the platform's profit, independent of which counterfactual scenario applies. The most interesting counterfactual scenarios to analyze in more detail are sce-

[34]The scenarios are not mutually exhaustive since we do not consider scenarios where the inequalities outlined in Lemmas 2.1 - 2.3 and Lemmas 2.6 - 2.7 hold with equality. In such case, several equilibria would co-exist, which does not allow for a straightforward comparison.

Table 2.3: Impact Fringe Seller

	Impact on:				
Scenario	$E(W)$	$E(CS_1)$	$E(CS_{2A})$	$E(\pi_R)$	$E(\pi_M)$
(i)	·	↑	↑	↓	·
(ii)	↑	↑	↓	·	·
(iii)	↑	↑	↓	↓	·
(iv)	·	↑	·	↓	·

"·" = unchanged; "↑" = increasing; "↓" = decreasing

narios (ii) and (iii). Let us start with the former: As the platform cannot price discriminate between off- and on-platform consumers, extracting a relatively higher profit from on-platform consumers becomes more attractive since the platform's ability to set off-platform prices is restricted through the existence of a fringe seller. Consequently, the platform applies a less competitive pricing strategy to contest *Type 1* consumers, which is to the disadvantage of loyal *Type 2A* consumers. However, because the specialist is more likely to serve *Type 1* consumers, it is not harmed by the introduction of a fringe seller if the *Mixed Equ.* outcome would otherwise occur in an unregulated market. For the same reason, (expected) overall welfare increases as relatively more consumers buy a higher quality product. Although the platform is less incentivized to contest *Type 1* consumers, the off-platform market is still more competitive after introducing a fringe seller, which benefits *Type 1* consumers.

If scenario (iii) is the relevant counterfactual, the specialist and the platform directly compete for *Type 1* consumers under a monopolistic market structure, but markets are segmented if an additional fringe seller exists. *Type 2A* consumers are worse off as the platform can now extract on-platform monopoly profits. However, since the additional fringe seller makes the off-platform market competitive, *Type 1* consumers again benefit even in the absence of direct competition between the specialist and the platform. In contrast, the specialist is now worse off as for such high levels of demand ξ the platform's mixed pricing strategy in a counterfactual monopolistic market would be sufficiently relaxed to allow the specialist a higher (expected) profit than if directly competing against the fringe seller.

Finally, (expected) overall welfare increases for the same reason as in scenario (ii).

The impact of a fringe seller in scenarios (i) and (iv) is straightforward to analyze. The additional fringe seller makes all markets competitive in which it can operate. This directly harms the specialist. However, since the fringe seller is not competitive, overall welfare depends only on how the specialist and the platform divide up the two distinct markets between each other. As the introduction of the fringe seller does not affect how markets are segmented in scenarios (i) and (iv), no effect on (expected) overall welfare is observed.

2.6 Policy Interventions

So far, we have seen how the combination of being a platform, learning from third-party data and being able to launch a product as a retailer can give hybrid platforms a competitive advantage. In the following, we discuss the implications of different policy interventions.

In the case of Amazon and other dominant hybrid platforms, often discussed policy measures include structural remedies like an ex-post separation or an ex-ante line of business restriction (see for example U.S. House Judiciary Subcommittee on Antitrust, 2020). Structural separation means that if the hybrid platform is already active as a platform and a retailer, it is broken up into two separate entities. A line of business restriction establishes that the platform is not allowed to become a retailer on its own digital marketplace. An alternative and novel approach to structural remedies is to impose mandatory access for third-party sellers.

In light of distinct outcome variables, we first benchmark the different policies against each other and an additional 'laissez-faire' outcome obtained in an unregulated market. As the policies are designed to restrict the platform's competitive advantage, they all have a (weakly) negative impact on the profit of the platform. Therefore, we only outline the impact on overall welfare, the specialist's profit and consumer surplus in the subsequent analysis.

A line of business restriction is imposed ex-ante, i.e. before the platform can launch its version of product A. A structural separation is discussed ex-post, i.e. after the platform has launched product A. We assume that the former retail arm of the platform does not have an independent off-platform distribution channel. By definition, the ex-post separation can only have an impact in the second period. In contrast, the line of business restriction and the mandatory access regulation already have an impact in the first period as they are implemented ex-ante.

The remainder of this section is structured as follows: We first discuss how the (expected) profit of the specialist and (expected) overall welfare are affected by the distinct policies before analyzing the impact on (expected) consumer surplus. Up to this point, we analyze the impact of the distinct policies under a monopolistic market structure. Therefore, we conclude by briefly discussing how the previously outlined arguments differ if there exists an additional fringe seller.

We denote the different policies with the following abbreviations: 'SEP' for the structural separation, 'LOB' for the lines of business restriction, 'MAN' for the mandatory access regulation and 'LAF' for the laissez-faire outcome.

2.6.1 Impact on Welfare and Specialist

Impact on Hosting Incentive

As the platform can, by assumption, only launch product A in the second period, each policy's impact in the first period is solely driven by the specialist's incentive to be hosted.

Proposition 2.3. *A line of business restriction and a mandatory access regulation have a (weakly) positive impact on the specialist's incentive* (I) *to get hosted in* $t = 1$*:*

$$I^{LOB} \geq I^{MAN} \geq I^{SEP} = I^{LAF}$$

Proof: See Appendix B.1

A structural separation does not affect the specialist's rationale in the first period as it is imposed ex-post by definition.[35] Both alternative policies, however, either directly restrict

[35]If forward-looking players anticipate ex-ante that a structural separation is imposed ex-post, it would

or indirectly reduce the platform's incentive to launch product *A* in the second period such that the specialist is not (or less) threatened to create a future competitor by revealing on-platform demand. As a consequence, its incentive to be hosted in the first period increases.

Corollary 2.3. *If a policy solves an adverse selection problem that otherwise occurs in an unregulated market, its implementation is pareto-efficient.*

Proof: Directly follows from Proposition 2.1 and Lemmas 2.1 -2.3.

It follows from a straightforward argument that the specialist is better of if a policy makes being hosted, in expectation, profitable. The same is true for the platform as it can generate additional transfer payments in the first period and might be able to launch product *A* itself in the second period.

Whenever a policy ensures that the specialist is hosted in the first period if it would not be hosted in an unregulated market, (expected) overall welfare increases in the first period since demand from *Type 2A* consumers is served. This also outweighs a potentially negative impact on overall welfare that might occur in the second period if the platform launches *A* because the platform only launches the product itself if demand from *Type 2A* is significant, which also makes solving the adverse selection problem in the first period relatively more important.

A line of business restriction ensures that the platform never launches product *A* in the second period. Therefore, it is the most desirable policy from the specialist's and an overall welfare perspective in conditions where the specialist is otherwise not hosted.

With a mandatory access regulation, the specialist's increased incentive to be hosted in the first period can also positively impact consumer surplus in the second period if it initiates a realization of ξ where the platform launches *A* and additionally has an incentive to compete with the specialist. Otherwise, consumers are not impacted as the suppliers of product *A* are able to extract the entire consumer surplus from the market, independent of whether the specialist is hosted in the first period.

have a similar impact on the specialist's incentive to get hosted in $t = 1$ as a line of business restriction.

Impact Second Stage

In the analysis of the second period, we abstract from the beneficial impact a policy might have in the first period. As when analyzing the impact of an additional fringe seller in the previous section, we assume that the *Hosting Subgame* is as well realized with a laissez-faire policy.

In the second period, equilibria similar in spirit to those outlined in Section 2.4 can occur on the equilibrium path with all policies.[36] There is one exception: With a structural separation, a highly competitive *Hosting Equ.(SEP)* outcome can occur where the platform launches A and the specialist is hosted to directly compete on- and off-platform.[37]

The discussion of the policies is made conditional on a specific realization of the demand parameter ξ. We distinguish between four mutually exclusive scenarios summarized in Table 2.4.

Table 2.4: Counterfactual Scenarios for Distinct Policies

Scenario		Equilibrium Outcome: SEP	LOB	MAN	LAF
(i):	$\xi < C^{Monop.}_{Launch}$	Pool.	Pool.	Pool.	Pool.
(ii):	$\xi \in \left(C^{Monop.}_{Launch}, C^{SEP}_{Comp.}\right)$[38]	Mix.(SEP)[39]	Pool.	Mix./Pool.	Mix.
(iii):	$\xi \in \left(C^{SEP}_{Comp.}, C^{Monop.}_{Comp.}\right)$	Separ./Host.[40]	Pool.	Mix./Pool.	Mix.
(iv):	$\xi > C^{Monop.}_{Competition}$	Separ./Host.	Pool.	Separ./Pool.	Separ.

The key takeaway from Table 2.4 is that a structural separation decreases the platform's

[36]Check Appendix B.1 for further details.

[37]A similarly shaped equilibrium outcome can occur with a mandatory access regulation. However, it is never reached on the equilibrium path.

[38]Check equilibirum outcome 'SEP' for $C^{SEP}_{Comp.}$ in Appendix B.1. For the subsequent analysis it is only important that $C^{SEP}_{Competition} < C^{Monop.}_{Comp.}$

[39]Check the equilibirum outcome 'SEP' for the Mixed Equ.(SEP) in Appendix B.1. For the subsequent analysis it is only important that it is more competitive than the Mixed Equ..

[40]Strictly speaking, there can occur the Seprarating Equ.(SEP) outcome with the structural separation policy. However, the applied pricing strategies are the same as in the Separating Equ.. The Hosting Equ.(SEP) outcome is more competitive than both. However, it depends on the given set of parameters whether it is also more competitive than the Mixed Equ.. Check the equilibirum outcome 'SEP' for the Separating Equ.(SEP) and the Hosting Equ.(SEP) in Appendix B.1.

willingness to contest *Type 1* consumers at the extensive margin. The reason is that the former retail arm also has to make transfer payments to the platform when selling to *Type 1* consumers. Therefore, attracting additional *Type 1* consumers becomes less profitable compared to charging monopoly prices from loyal *Type 2A* consumers.

It is known from the analysis in Section 2.3 that *M* eventually launches *A* if the revealed on-platform demand from loyal *Type 2A* consumers is sufficiently large. Therefore, none of the policies has an impact on the platform's launching decision if ξ lies below the launching cutoff in scenario (i).[41] If a policy allows the specialist to access the platform freely, it depends on the specialist's incentive to be hosted, which equilibrium outcome is realized.

The following Proposition ranks the equilibrium outcomes of the distinct policies for scenarios (ii)-(iv) according to their impact on (expected) overall welfare and the specialist's profit and demonstrates how the ranking relates to the conditions that determine the specialist's willingness to be hosted.

Proposition 2.4. *Compared to the Hosting Subgame in a monopolistic market, the distinct policies are ranked according to their impact on (expected) overall welfare and the specialist's profits in* $t = 2$ *as outlined in Table 2.5, given the counterfactual scenarios (ii)-(iv) from Table 2.4.*

Proof: See Appendix B.1

Expected overall welfare and the specialist's profit are positively correlated as the specialist can serve all consumers with a product of superior quality and no fixed costs from launching occur as long as it is the sole provider of product *A*. Hence, the *Pooling Equ.* outcome, where the specialist is a monopolistic supplier of product *A*, is the first best outcome from an overall welfare *and* the specialist's perspective. Table 2.5 shows that the line of business restriction is the (weakly) most desirable policy under all circumstances as it ensures that the *Pooling Equ.* outcome is realized with certainty. Therefore, only a binding mandatory access regulation can match the impact of a line of business restriction as the specialist deters the platform from launching *A* by credibly threatening to get hosted subsequently. The *Pooling Equ.* outcome is not achievable with the structural separation

[41] To anticipate the next section: The policies also have no impact on consumer surplus In scenario (i).

by definition. Furthermore, fixed costs accrue, which is not desirable from a welfare point of view.

Table 2.5: Ranking *w.r.t.* Welfare and Profits of Specialist

Scenario	Impact on: $E(W)$ and $E(\pi_R)$	Conditions
(ii:)	$LOB > SEP > MAN = LAF$	(1)
	$LOB = MAN > SEP > LAF$	(2)
(iii:)	$LOB > SEP > MAN = LAF$	(1) ∧ (5) / (1) ∧ (6)
	$LOB = MAN > SEP > LAF$	(2) ∧ (5) / (2) ∧ (6)
(iv:)	$LOB > SEP = MAN = LAF$	(3) ∧ (5)
	$LOB > SEP > MAN = LAF$	(3) ∧ (6)
	$LOB = MAN > SEP > LAF$	(4)[42]

$''(1)'' \equiv \pi_R^{Host\,(MAN)} < \pi_R^{MixedEqu.}$[43]; $''(2)'' \equiv \pi_R^{Host\,(MAN)} > \pi_R^{MixedEqu.}$;
$''(3)'' \equiv \pi_R^{Host\,(MAN)} < \pi_R^{Separ.Equ}$; $''(4)'' \equiv \pi_R^{Host\,(MAN)} > \pi_R^{Separ.Equ}$;
$''(5)'' \equiv \pi_R^{Host\,(SEP)} < \pi_R^{Separ.\,(SEP)}$; $''(6)'' \equiv \pi_R^{Host\,(SEP)} > \pi_R^{Separ.\,(SEP)}$

Given that scenarios (ii)-(iv) are the relevant counterfactuals, the *Pooling Equ.* outcome never occurs in an unregulated market, which explains why the laissez-faire policy is found towards the lower end of the ranking.

With a mandatory access regulation, it is not necessarily the case that the specialist is hosted in the second period. As the platform makes its launching decision before the specialist makes its hosting decision, it might be an implausible threat of the specialist to get hosted subsequently if the specialist risks cannibalizing its own off-platform profit by fiercely competing with M for loyal on-platform customers. Hence, whenever this is the case, the mandatory access regulation is not binding and therefore equivalent to a laissez-faire policy as the platform still launches A and the specialist is not hosted, although it could freely access the platform.

With a structural separation, the platform owner still has the ability to grant access to its

[42] Notice that with (4): $\pi_R^{Host\,(MAN)} > \pi_R^{Separ.Equ}$, which implies that condition (6) must also be satisfied. Check the proof of Proposition 2.4 to understand this argument.

[43] Check the 'MAN' equilibrium outcome for the Hosting Equ.(MAN) in Appendix B.1.

marketplace selectively. The platform owner is willing to foreclose rivaling suppliers if the hosted supplier is subsequently incentivized to attract additional *Type 1* consumers to the platform that would otherwise purchase product A on the off-platform market. This is the case in scenario (ii), where the *Mixed Equ.(SEP)* outcome occurs. Compared to the *Mixed Equ.* in an unregulated market, the competition over contested *Type 1* consumers becomes less severe at the internal margin because the incurred transfer payments let the retail arm's off-platform sales become less attractive. As a consequence, the specialist can sell its superior quality product to more consumers and it is able to capture a relatively higher (expected) profit with the structural separation.

The *Hosting Equ.* outcome, where the specialist and the platform offer their product on the platform's digital marketplace, is the second best outcome from an overall welfare perspective since it also enables all consumers to certainly buy the superior quality product. Therefore, a structural separation outperforms a mandatory access regulation whenever the latter is not binding. However, opposed to the *Pooling Equ.* outcome that is realized with a line of business restriction, fixed costs from launching A accrue.

The key takeaway from this section is that a policy restricting the platform in its ability to launch product A is desirable from the specialist's and an overall welfare point of view. This observation is driven by the assumption that the specialist offers a superior product and by economies of scale, which occur as the specialist does not incur additional fixed costs from launching.

2.6.2 Impact on Consumer Surplus

Extrapolating on the results from the previous section, the specialist can always extract the entire consumer surplus from the market in the first period as a monopolistic supplier of product A. Hence, opposed to the results from the previous section, an increased incentive of the specialist to be hosted in the first period does *not* impact consumer surplus in the first period.

In the second period, we discuss the policies' impact conditional on the *Hosting Subgame* being realized in an unregulated market. Therefore, Table 2.4 from the previous section, again, describes the relevant counterfactual scenarios for a given realization of the demand parameter ξ.

Proposition 2.5. *Compared to the Hosting Subgame in a monopolistic market, the distinct policies are ranked according to their impact on consumer surplus in $t = 2$ as outlined in Table 2.6, given the counterfactual scenarios (ii)-(iv) from Table 2.4. .*

Proof: See Appendix B.1

In contrast to the welfare and profit analysis, it is of secondary importance for the consumer, whether the *Pooling Equ.* occurs or an alternative equilibrium in which the on- and off-platform markets are perfectly segmented. In both cases, all potential suppliers of product A can extract the entire consumer surplus from the market. Therefore, each policy only benefits consumers if it induces the platform to launch product A in the second period *and* if it creates an (additional) incentive for both suppliers to compete for revenue shares on (any of) the two distinct markets.

Table 2.6: Ranking *w.r.t.* Consumer Surplus

Scenario	Impact on: $E(CS_1)$ and $E(CS_{2A})$	Conditions
(ii:)	$LAF = MAN > SEP > LOB$	(1)
	$LAF > SEP > MAN = LOB$	(2)
(iii:)	$LAF = MAN > SEP = LOB$	(1) ∧ (5)
	$LAF = MAN > SEP > LOB$	(1) ∧ (6)
	$LAF > MAN = SEP = LOB$	(2) ∧ (5)
	$LAF > SEP > MAN = LOB$	(2) ∧ (6)
(iv:)	$LAF = SEP = MAN = LOB$	(3) ∧ (5)
	$SEP > LAF = MAN = LOB$	(4)[44] / (3) ∧ (6)

$"(1)" \equiv \pi_R^{Host\,(MAN)} < \pi_R^{MixedEqu.45}$; $"(2)" \equiv \pi_R^{Host\,(MAN)} > \pi_R^{MixedEqu.}$;
$"(3)" \equiv \pi_R^{Host\,(MAN)} < \pi_R^{Separ.Equ}$; $"(4)" \equiv \pi_R^{Host\,(MAN)} > \pi_R^{Separ.Equ}$;
$"(5)" \equiv \pi_R^{Host\,(SEP)} < \pi_R^{Separ.\,(SEP)}$; $"(6)" \equiv \pi_R^{Host\,(SEP)} > \pi_R^{Separ.\,(SEP)}$

[44] Notice that with (4): $\pi_R^{Host\,(MAN)} > \pi_R^{Separ.Equ}$, which implies that condition (6) must also be satisfied. Check the proof of Proposition 2.4 to understand this argument.

[45] Check the 'MAN' Equilibirum outcome for the Hosting Equ. (MAN) in Appendix B.1.

With a line of business restriction, the specialist is guaranteed to keep its monopoly position on both markets, which is the worst possible outcome from a consumer perspective. The same is true for a mandatory access regulation whenever it is binding as it induces the *Pooling Equ.* outcome in such conditions.

In conditions where the *Mixed Equ.* outcome is realized in an unregulated market, we find that the laissez-faire policy and a non-binding mandatory access regulation top the ranking from a consumer perspective. This result stresses that the unregulated market is already quite competitive whenever the platform has an incentive to contest *Type 1* consumers.

With the *Mixed Equ.(SEP)* outcome, which is realized with the structural separation in scenario (ii), the platform's former retail arm is incentivized to set an (expected) price strictly below the monopoly price. Hence, consumers prefer a structural separation over a mandatory access regulation if the latter is binding. However, the specialist and the platform do not compete as fiercely over *Type 1* consumers as in the *Mixed Equ.* as selling off-platform is less attractive for the platform's retail than in an unregulated market since the former retail arm also has to make transfer payments to the platform.

A structural separation outperforms a laissez-faire policy (and all other policies) in circumstances where the *Separating Equ.* is realized in an unregulated market if the *Hosting Equ.(SEP)* outcome is realized in scenario (iv), where the specialist only captures the quality differential $\Delta_R - \Delta_M$ as a markup on both markets. Otherwise, markets are also perfectly segmented with a structural separation.

The key takeaway from this section is that in most conditions, a social planner faces a trade-off between maximizing consumer surplus or overall welfare and the specialist's profits. In light of the latter two outcomes, it is optimal that the specialist operates as a monopolistic supplier, which is clearly not beneficial for consumers.

2.6.3 Regulation: Competitive Market Structure

In contrast to a monopoly market, the off-platform market is always competitive if there exists an additional fringe seller. Furthermore, the policies have no additional impact on the specialist's incentive to be hosted in the first period as the specialist is hosted with certainty in a competitively structured market by Lemma 2.5, which continues to hold.

Corollary 2.4. *With a line of business restriction or a mandatory access regulation, the surplus of Type 2A consumers is weakly greater than* $u_A - \tau$.

Proof: See Appendix B.1

With a line of business restriction, the *Pooling Equ.(f)* is realized with certainty. With a mandatory access regulation, the platform does not have an incentive to launch product *A* if the specialist is subsequently hosted. In this case, the *Pooling Equ.(f)* is again realized. Otherwise, at least the fringe seller is hosted with certainty, which also puts an upper bound on the prices that occur on-platform. Therefore, both policies create conditions that allow the on-platform market to become as competitive as the off-platform market, which benefits *Type 2A* consumers. *Type 1* consumers, however, are negatively impacted by the policies whenever they prevent the highly competitive *Mixed Equ.(f)* outcome from occurring.[46]

With a structural separation, the platform owner still is in a position to selectively grant specific suppliers access to its marketplace. As in a monopoly market, it is incentivized to do so if it creates an incentive for the hosted supplier to attract additional off-platform customers to its marketplace. This might be the case in conditions where the *Mixed Equ.(f)* is realized in an unregulated market. Whenever the platform owner has no incentive to restrict access to its platform, the impact of the structural separation on consumer surplus is equal to that of a line of business restriction or a mandatory access regulation.

Compared to a monopolistic market structure, the platform's trade-off between maximizing (expected) consumer surplus and overall welfare is less severe if an additional fringe seller exists. The reason is that the *Pooling Equ.(f)* outcome is still the first-best outcome from an overall welfare perspective. However, with an additional fringe seller, it also is relatively more beneficial from a consumer perspective, especially if the share of loyal *Type 2A* consumers is large.

[46] It might depend on the specific parameter vector if, relative to the *Mixed Equ.(f)* outcome, *Type 2A* consumers are better or worse off with a guaranteed consumer surplus equal to $u_A - \tau$.

2.7 Concluding Remarks

The emergence of hybrid platforms creates severe distortions in the competition between independent retailers and the operator of a hybrid platform. The platform can leverage its ability to observe third-party demand data to opportunistically time its market entry decision and launch a private label product if on-platform demand shows to be sufficient. Conditional on entering the market, the platform can subsequently leverage its control over the marketplace and the loyalty of its customer base.

A specialized retailer's decision to be hosted in a monopoly market is determined by its incentive to expand its customer base. The potential to generate additional profits is traded off against the risk of creating a future competitor by revealing on-platform demand. We outline conditions where a sophisticated, forward-looking specialized retailer finds it ex-ante optimal to be hosted but might regret the decision ex-post. The underlying rationale remains vastly unchanged if there exists an additional fringe seller. The platform's ability to launch an in-house product is beneficial for consumers if it is incentivized to subsequently expand its operations beyond the platform market to contest the specialized retailer's off-platform customer base.

To investigate how different regulatory measures impact the market outcome, we model policies designed to decrease or directly restrict the platform's incentive to launch its own product, which positively impacts the specialist's profits, its incentive to be hosted and overall welfare.

In conditions where markets fail as demand from loyal platform consumers is not served, policies that increase the specialist's incentive to be hosted in the first period are (weakly) pareto-optimal.

While concerns that relate to a hybrid platform's competitive advantage are valid, the platform's ability to launch a private label product can be beneficial from a consumer point of view in conditions where a specialized retailer is harmed. This finding is relevant as it suggests that a specialized retailer might lobby for very different policies than what is beneficial from a consumer perspective.

A key message from our results is that in conditions where an adverse selection problem does not occur in the first period, a social planner faces a trade-off between maximizing consumer surplus or overall welfare. The latter is additionally positively correlated with the profits of third-party sellers. In light of these findings, not intervening in the platform market can be interpreted as prioritizing consumer surplus. However, this conclusion has to be drawn with caution as there also exist conditions where the platform's ability to launch a private label product opportunistically is exclusively to the disadvantage of third-party retailers without benefiting consumers.

Our results demonstrate that market interventions need to be designed very carefully. Their contrary impact on different market stakeholders needs to be addressed and communicated in a differentiated way. However, the directions in which the policies impact the distinct outcome variables are more aligned in competitive markets. If there exists a competitive fringe, the platform can leverage its dominant position to create an on-platform monopoly that would otherwise not exist, which harms independent retailers *and* loyal customers likewise.

Finally, we want to point out two promising avenues for future research. First, a deviation from unit demand would add an interesting perspective for a social planner as this could create an incentive for third-party sellers to overprice their products to keep on-platform demand artificially low. Extrapolating on our results, we suppose that this might impact the positive correlation of overall welfare and the specialist's profits. Second, it is intriguing to ask how our results differ if the platform has a possibility to price discriminate between different consumer types. This extension is of particular interest in a setting with informed and uninformed consumers or in a setting where the platform can sell its product via a separate off-platform channel.

Chapter 3

Exclusive Selling: How Data-Driven Hybrid Platforms Leverage Asymmetric Information

3.1 Introduction

Numerous regulation authorities recently raised concerns about different business practices of dominant hybrid platforms that revolve around their two-sided nature.[1] These platforms act as an intermediary that facilitates interactions between consumers and third-party sellers, while simultaneously operating as a competitor on their own marketplaces. Also, their ability to collect and combine data from different third-party sources to gain an information advantage is an issue that is constantly up for debate (e.g., Furman, Coyle, Fletcher, McAuley, and Marsden, 2019 Crémer, Montjoye, and Schweitzer, 2019 and Morton, Bouvier, Ezrachi, Jullien, Katz, Kimmelman, Melamed, and Morgenstern, 2019). The addressed concerns interact if a platform can leverage its superior information about consumer preferences to amplify its (already existing) competitive advantage over independent retailers resulting from its hybrid business model (e.g., Parker, Petropoulos, and Van Al-

[1]Recent reports that raise such concerns include the "Digital platforms inquiry" by the Australian Competition & Consumer Commission, the "Competition policy for the digital era" report written for the European Commission or the "Investigation of competition in digital markets" by the U.S. House Judiciary Subcommittee on Antitrust.

styne, 2021 and Martens, Parker, Petropoulos, and Van Alstyne, 2021). This is the case when the platform can offer insurance in the form of an exclusive distribution agreement to outside sellers guaranteeing that the platform will not act as a competitor, a practice that gained momentum over the past years.

In Amazon's 'Exclusive Brands' program, for instance, third-party sellers commit to abandon alternative off-platform distribution channels to sell exclusively via Amazon's marketplace. Among other perks, they receive active support from Amazon in developing their brand and increased brand protection in exchange.[2] For Amazon, such exclusive contracts are an opportunity to extend its customer base even further and a convenient way to outsource the risk of manufacturing own products. The observation that a large share of Amazon's recent private label launches did not resonate well with consumers (Marketplacepulse, 2019) might additionally explain why Amazon currently appears to force a shift in operations – away from private labels and towards exclusive brands.[3]

While taking it as given that the private label strategy of a hybrid platform is informed by observing third-party data, this paper asks the following three research questions: How does an information asymmetry determine the market outcome? Related to that: What are the conditions where exclusive contracts occur? How do different policies that resolve the information asymmetry impact the market outcome?

The paper builds on a two-period model from Schader and Montag (2022) that contains a monopolistic, specialized retailer and a hybrid platform as potential suppliers of a given product. There exist consumers that either buy via the platform or directly from the specialist. In contrast, there also exist loyal platform consumers that buy exclusively via the platform. Shopping on-platform is considered more convenient for both consumer types. The precise demand from loyal platform consumers is uncertain and is only revealed after the specialist is hosted.

[2] For more details, check, for instance, a recent summary by Pattern: https://pattern.com/blog/what-is-amazon-exclusives/

[3] After introducing the Amazon exclusive brand program in 2015, the number of exclusive brands has multiplied quickly. 2018 was the first year that Amazon's exclusive brand outnumbered Amazon's launches of private labels. In 2019 Amazon hosted 434 exclusive brands, which was more than double the number of private label brands (Milnes, 2019).

Ultimately, the platform's decision to launch its own product version depends on the demand from loyal on-platform consumers. Therefore, the specialist risks creating a future competitor by being hosted. Through hosting, the platform can learn if the specialist's product sells successfully on-platform. As a consequence, the platform can launch its own product version after observing high on-platform demand. However, the platform can also offer an exclusive contract by which it guarantees the specialist not to enter the market with a rivaling product. In exchange, the specialist commits to abandoning its off-platform distribution channel to sell exclusively via the platform.

Having the possibility to enter an exclusive contract prevents market failure if it is otherwise not optimal for a third-party retailer to get hosted in the first period. There can, however, still occur scenarios where the platform and the specialist do not enter an exclusive contract and the specialist is hosted subsequently. If the platform launches its own product in the second period, the specialist might regret its initial hosting decision ex-post. From a consumer's point of view, an exclusive contract manifests the monopoly position of the specialist on- and off-platform.

In this two-period model, the platform has ex-ante a more precise expectation about on-platform demand than the third-party retailer. Depending on the platform's type, certain demand states can be realized. A strong-type platform knows ex-ante that on-platform demand will either be high or low with a certain probability. A weak-type platform knows that the low demand level occurs with the same probability as for the strong type platform. Otherwise, an intermediate demand level occurs. Put differently, both platform types take into account that the introduction of a new product could fail with a given probability. The platform types differ in the demand state that is realized if the product introduction is no failure, but a success.[4]

Hence, there are two sources of uncertainty in this model. First, at the extensive margin: Is the product going to fail on-platform? Second, at the intensive margin: How successful is the product going to be conditional on not being a failure? The first source of uncertainty

[4] To clarify, 'success' implies that the product does not fail. It does not necessarily imply that the high demand level is realized. Thus, in case of facing a weak-type platform, introducing the product is also considered successful if the intermediate demand level is realized.

is shared by both players on the supply side. It ultimately drives the platform's incentive to learn about demand through hosting the specialist, which also motivates the hybrid structure of the platform's business model within the chosen setup. The second source of uncertainty characterizes the critical information asymmetry as a platform's type is private information, where only the distribution of types is known to the specialist.

Launching a private label product in the second period is only profitable if the high demand state is realized. Hence, if knowing with certainty that the platform is of the weak type, the specialist does not fear to create a future competitor when getting hosted in the first period. Even if facing a strong-type platform, it can be optimal to get hosted. However, if the platform subsequently launches a private label product, the specialist is incentivized to abandon the platform. Otherwise, it would exert additional pressure on its own off-platform profit.

In the second part of the paper, I discuss the implications of three different policy interventions. First, I model banning exclusive contracts. As a consequence, there can occur scenarios where the product is not offered on-platform. Otherwise, consumers benefit and the specialist is harmed if the policy induces the platform to launch a rivaling product in the second period.

The other two discussed policies are contrary approaches to address the information asymmetry that stems from the platform's ability to access data from interactions with on-platform consumers, which are not explicitly modeled here.[5] The first policy is to strengthen data protection rights such that the platform can no longer access data from on-platform activities of consumers since the data is stored externally or not stored at all.[6] Such an 'ex-situ' policy resolves the information asymmetry by eliminating the information advantage of the platform. This also erases the convenience benefit associated with on-platform shopping as it is assumed to be driven through data enabled learning.[7] There-

[5]A platform could gain superior knowledge about demand if successfully introducing a product by observing sales in complementary product categories or by observing unanswered search queries or otherwise communicated preferences of potential customers.

[6]This approach has been followed in most of the policy measures adopted by various national and supranational regulation authorities, e.g., the European Union's "General Data Protection Regulation" or the "California Consumer Protection Act".

[7]For a detailed discussion of data-enabled learning see for instance Hagiu and Wright (2020) or Schäfer and Sapi (2020).

fore, overall welfare and the profit from selling via the platform directly decreases. The policy has a negative impact on consumer surplus whenever it pushes the specialist and the platform into entering an exclusive contract.

In contrast, an 'in-situ' policy does not restrict the platform from storing and evaluating consumer data. Instead, it grants the specialized retailer the right to access the data such that the platform's private information is revealed.[8] The idea behind this approach is that aggregating and systematically analyzing data generates value on both sides of the market and that analyzing the data is most valuable in the context in which the data is generated.[9] The impact on overall welfare and the specialist's profit is positive. Consumers are, however, (weakly) worse off with the policy as there can no longer occur scenarios where the specialist is 'accidentally' hosted and the platform launches its own product in the second period.

The paper makes four contributions. First, it introduces an information asymmetry to a setting where the platform learns (about on-platform demand) through hosting the specialist that is familiar from Madsen and Vellodi (2021), Hervas-Drane and Shelegia (2022) and Schader and Montag (2022). By assuming that the platform has access to superior information, this paper bridges the literature to Hagiu and Wright (2020) and Corniere and Taylor (2019) who outline how an information asymmetry grounded on data-enabled learning evolves and determines a platform's competitiveness.

Second, the paper finds that it is in the interest of the specialized retailer to abandon the platform distribution channel if the platform launches a rivaling product. This finding is contrary to Padilla et al. (2022) and Corniere and Taylor (2019) and adds an alternative perspective to the empirical findings of, e.g., Cure et al. (2022) and Zhu and Q. Liu (2018) who argue that active foreclosure causes specialized retailers to abandon the platform. The difference occurs as this paper explicitly models an off-platform distribution channel. Therefore, increasing competition on the platform's marketplace restricts the specialist's ability to set prices off-platform.

[8] For instance, free access rights are part of the "Digital Markets Act" that was recently enacted by the European Union.

[9] See, for instance, Brynjolfsson, Dick, and Smith (2010) and Manyika, Chui, Brown, Bughin, Dobbs, Roxburgh, Hung Byers, et al. (2011) or Martens et al. (2021).

Third, the paper also proposes an explicit framework that compares the impact of an in-situ regulation in the spirit of Martens et al. (2021) to a more conventional ex-situ policy that limits the platform's ability to access consumer data.

Finally, the paper outlines an application that bridges the literature between signaling games with cheap talk (e.g., Crawford and Sobel, 1982) and more traditional signaling models where the chosen signal directly impacts all players' payoffs (e.g., Spence, 1978). Offering an exclusive contract is not associated with any direct costs and therefore open to imitation. However, the signal to offer an exclusive contract impacts the strategy space available to the responded and thereby also indirectly both players' payoffs.

More broadly, this paper is complementary to a branch of recent literature that models alternative mechanisms that determine the market entry decision of hybrid platforms (e.g. Anderson and Bedre-Defolie, 2021, Zennyo, 2021, Etro, 2021, Shopova, 2021 and Hagiu, Teh, et al., 2022). The paper also refers to models that more fundamentally address under what circumstances hybrid platforms emerge (e.g. Hagiu and Spulber, 2013, Hagiu and Wright, 2015 and Hagiu, Jullien, et al., 2020 and Corniere and Taylor, 2019). Finally, this paper also relates to the recently increasing literature on the collection of consumer data (e.g., Brynjolfsson, Collis, and Eggers, 2019, Choi, Jeon, and Kim, 2019, Acemoglu et al., forthcoming and Bergemann et al., 2020)

The remainder of the paper is structured as follows: I start with introducing the model setup. Subsequently, I analyze the strategic interaction in the second period of the game. Next, I characterize the equilibrium outcomes and how they depend on the distinct parameters. Finally, I discuss the effect of different market interventions.

3.2 Model Setup

I model a two-period game where future period payoffs are discounted by a factor δ. As the information asymmetry is related to each player's expectation about demand, I discuss the type structure in more detail *after* introducing the demand side.

Supply side: On the supply side, there are a specialist retailer R and a (potentially hybrid) platform M. R is a monopolistic supplier of a newly introduced product A. The specialist's costs of production are normalized to zero. R sells A via its own outside distribution channel such that R is able to set the off-platform price p_R directly. If R is hosted to additionally sell A via the platform, it charges a distinct on-platform price p_R^{on}. Furthermore, the specialist pays M a transfer $\tau_H \geq 0$ for each unit sold via the platform if being hosted.

M is an established and dominant platform for sellers in other product categories that are not modeled here. Product A can be made available on the platform through hosting the specialist as a third-party retailer. M can launch a copy of product A in the second period, sell this as a retailer and become a hybrid platform. To do so, M needs to observe the product characteristics of product A in the first period. Thus, launching A is only possible in the second period for M. If launching A, M incurs a fixed costs equal to F and cannot directly foreclose its platform for the specialized retailer. Subsequently, it sets an on-platform price p_M^{on}. $M's$ marginal costs of production are also normalized to zero such that no supplier of product A has an advantage in marginal cost.

Alternatively, the platform can also offer an 'exclusive' contract before the specialist decides whether to be hosted. If the specialist accepts such a contract, it commits to abandon its off-platform distribution channel and to exclusively sell product A via the platform at a per-unit transfer $\tau_E > 0$. In turn, the specialist is protected from competition as the platform guarantees not to launch a rivaling product in the future (if offered in $t = 1$) or in the current period (if offered in $t = 2$).

It is helpful to discuss the role of the transfers τ_E and τ_H, which are assumed to be exogenous and constant over time.[10] Endogenizing transfers would enable the platform to always extract the maximum possible rent from the market, so launching would never occur within this model. However, on Amazon's marketplace and other dominant platforms, transfers are set on the product category level. They are standardized such that a platform cannot discriminate within a product category or between different third-party suppliers.[11] Therefore, transfers can be assumed exogenous on the product level where market entry occurs. Furthermore, a platform's transfer scheme might also depend on its position in the

[10] A rationalizability constraint is imposed on τ_E that is explained in detail at the end of this section.
[11] For more details, check: https://sell.amazon.com/pricing.

market, which is not explicitly modeled here and might also be independent of its decision of whether to launch a specific product.

Demand side: There is a unit mass of atomistic consumers of different types. To streamline the analysis, I deploy the simplest possible demand structure: Conditional on having positive demand, all consumer types have an inelastic unit demand for good A. Let me assume that buying product A via the platform is more convenient for all consumers than buying A via the off-platform market. Hence, whenever product A is purchased via the platform, both consumer types value the good at $u_A > max\{\tau_H, \tau_E\}$.[12] Otherwise, if bought via the off-platform market, additional shopping costs equal to $b \geq 0$ occur. I implicitly assume here that the platform's ability to collect data from different stakeholders allows for data-enabled learning, which creates a superior on-platform shopping experience for consumers (Hagiu and Wright, 2020). There eventually exist two relevant and distinct types of consumers:

A fraction ϕ of consumers are of *Type 1* that can buy product A from either seller. They are the so-called 'contested' consumers. In case of indifference, *Type 1* consumers buy product A off-platform. In contrast, there also exist 'loyal' platform consumers that only become aware of product A if offered via the platform and therefore never buy off-platform.[13] An (uncertain) fraction equal to $(1-\phi)\xi$ of these are *Type 2A* consumers with demand for product A. The assumption that *Type 2A* consumers never buy product A off-platform ensures that both suppliers can generate profits strictly greater than zero in scenarios where markets become (partially) segmented. The remaining fraction $(1-\phi)(1-\xi)$ are *Type 2B* consumers. They buy other products on the platform that are not explicitly modeled here but have no demand for product A.

The share of *Type 1* consumers is exogenously given, fixed over time and known to all players. Furthermore, I assume that $\phi \in (\frac{1}{4}, 1)$ such that specialized retailer can also reach

[12] This ensures that the transfers are not prohibitive.

[13] This assumption is made to streamline the analysis. It could be made endogenous if alternatively assuming prohibitively high shopping costs when buying off-platform. Considering, for instance, that 'Amazon Prime members' experience exclusive benefits when buying on-platform, their opportunity cost (of buying off-platform) should be higher than those of *Type 1* consumers: https://www.amazon.com/primeinsider/about.

a customer group of significant size if not selling via the platform.[14] The specific share of *Type 2A* consumers, however, is uncertain as ξ is a random draw from a binomial distribution discussed in detail in the next paragraph when introducing the information asymmetry and the distinct platform types. However, as soon as A is sold via the platform for the first time, ξ is drawn. Once ξ becomes common knowledge, it stays constant over time.

Information asymmetry: On-platform demand, which crucially depends on the demand parameter ξ, is only observed if good A is sold via the platform. There exist three distinct demand states $\xi \in \{\xi_l = 0, \xi_m = C_E^+, \xi_h = 1\}$. An auxiliary assumption explained in more detail in the last paragraph of this section ensures $\xi_l < \xi_m < \xi_h$. So, demand can either be low, intermediate or high. The constant C_E, which defines intermediate demand levels, is a cutoff for possible realizations of ξ, above which the platform can credibly demonstrate to the specialist that it is more profitable to launch its own version of product A than to host the specialist in the second period.[15] The cutoff is ex- ante known to all players. The '+' indicates that ξ_m is slightly above, but arbitrarily close to C_E.[16]

The characterization of platform types is based on their knowledge about which of the three distinct on-platform demand levels can be realized. There exist two platform types: A strong- and weak-type platform $\theta \in \{\theta_s, \theta_w\} = \Theta$.

For a 'strong' type (s) $\xi \in X_s = \{\xi_l = 0, \xi_h = 1\}$. Thus, the strong type *knows* that product A is either a best selling product that all on-platform consumers want to buy or introducing the product fails as *Type 2A* consumers have no demand for the product. A strong-type platform can, however, rule out that an intermediate demand level occurs.

For a 'weak' type (w) $\xi \in X_w = \{\xi_l = 0, \xi_m = C_E^+\}$. Thus, the weak-type platform can rule out that product A is a best selling product. Either the product fails, or an intermediate demand level occurs.

Irrespective of which platform type is realized, $\xi = \xi_l$ occurs with probability $Pr(\xi =$

[14] $\phi > \frac{1}{4}$ also ensures that the specialist finds it optimal to enter an exclusive in $t = 2$ if it can thereby prevent the platform from entering the market with a rivaling product for a given parameter vector. Check the proof of Lemma 3.2 for details.

[15] For further details, check the proofs of Lemma 3.1.

[16] Alternatively, one could assume that the platform launches A in case of indifference to break ties. Furthermore, notice that C_E is independent of τ_E, which is relevant for auxiliary assumption *A2* introduced below as it depends on ξ_m.

$\xi_l| \theta) = 1 - \alpha, \forall \theta \in \Theta$ where $\alpha \in (0, 1)$. Vice versa, $Pr(\xi \neq \xi_l) = \alpha, \forall \theta \in \Theta$. The probability that the success-case scenario occurs (α) is independent of the platform type and known to the specialist.

Thus, the characterization of platform types is based on the demand state that is realized in the best-case scenario. The platform's type is private information. The specialist's ex-ante prior that Nature (N) selects a type $\theta \in \Theta$ is the following: With probability $\mu(\theta_s) = \mu_s$, nature draws a strong-type platform and with probability $\mu(\theta_w) = 1 - \mu_s$ a weak-type platform, where $\mu_s \in (0, 1)$.

The type structure reflects the critical information asymmetry between a specialized retailer and a hybrid platform. Both suppliers take into account that the introduction of a new product fails for unforeseeable reasons. This is the case if $\xi = \xi_l$ occurs. The failure probability is a parameter that is determined through observing failed product launches in the past, which are not modeled explicitly.[17] Thus, the parameter is fed by ex-post data that is publicly available and therefore common knowledge. However, the assessment of on-platform demand in case the product does not fail requires information that allows for a forward-looking statement. By observing detailed demand data in complementary product categories or unanswered consumer search queries, which are both not modeled explicitly, the platform can leverage its access to superior information to get a precise estimate of on-platform demand that is associated with successfully introducing the product. In contrast, the specialist is uncertain about the demand state that is realized in the best-case scenario.

As the platform has an information advantage over the specialist, its initial action of offering (ω_o) or not offering (ω_{no}) an exclusive contract, can be interpreted as a signal $\Psi : \Theta \rightarrow \Omega = \{\omega_o, \omega_{no}\}$. Given a platform's type and a specific signaling function $\Psi(\theta)$, the specialist updates its belief about $M's$ type according to Bayes rule: $\mu(\theta|\Psi) = q \in [0, 1], \forall \theta \in \Theta$. Whenever $\Psi(\theta_w) = \Psi(\theta_s) = \omega \in \Omega$, the signal is not informative.[18] In this case, I assume that the specialist holds an off-the-equilibrium-path belief that nature selects a type $\theta \in \Theta$

[17] A recent survey outlines that Amazon's failure rate of newly introduced private label products is relatively high (Marketplacepulse, 2019).

[18] It should be mentioned here that the signaling cost do not directly depend on the platform's type. However, the platform's opportunity cost of offering such an exclusive contract depend on its type as it determines a platform's (expected) outside option.

when observing $\omega' \neq \omega$ according to: $\sigma(\theta_s) = \sigma_s$ and $\sigma(\theta_w) = 1 - \sigma_s$, where $\sigma_s \in [0, 1]$.

Timing and payoffs: The timing of the game, again, summarizes all strategies of the individual players. Figure C.1 in Appendix C.2 depicts the decision problem in the first period. In $t = 1$:

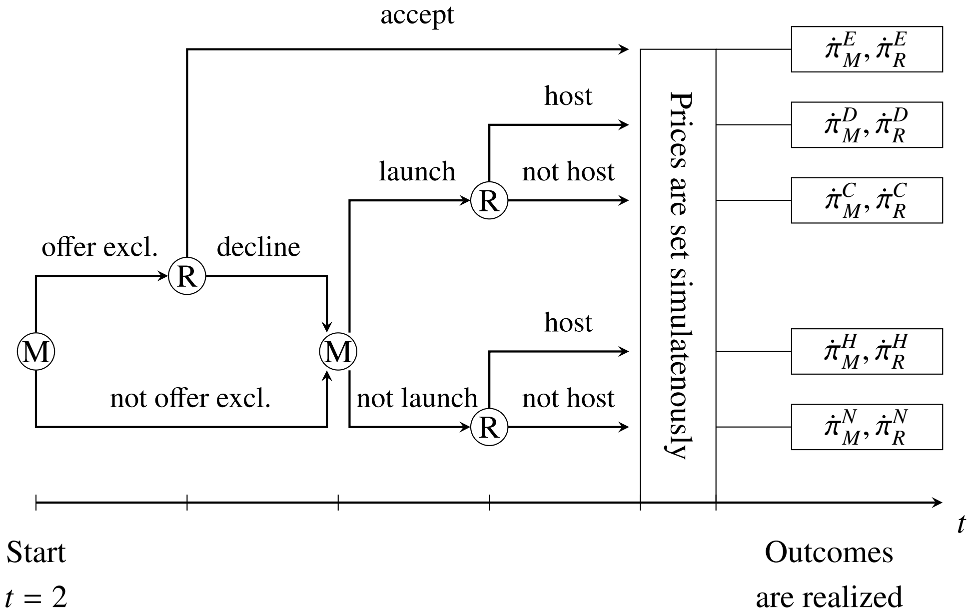
1. N selects $\theta \in \Theta$ where $\mu(\theta_s) = 1 - \mu(\theta_w) = \mu_s \in (0, 1)$.
2. M observes its type θ and chooses a signal $\omega \in \Omega$.
3. R updates it belief about $M's$ type according to Bayes rule: $\mu(\theta_s|\Psi) = 1 - \mu(\theta_w|\Psi) = q$. Off-the-equilibrium-path beliefs are defined by $\sigma(\theta_i) = 1 - \sigma(\theta_{j \neq i}) \in [0, 1]$.
4. If $\Psi(\theta) = \omega_o$, R chooses an action $a \in \{\text{accept, not accept}\}$.
5. If no exclusive contract was entered, R chooses an action $\beta \in \{\text{host, not host}\}$.
6. Where applicable, M and R set prices p_M^{on}, p_R^{on} and p_R simultaneously.
7. Consumers decide if, where and from which seller to buy.

The timing in $t = 2$ is similar, with few exceptions. The platform's type, which is drawn in the previous period, is constant over time. The same is true for ξ if revealed in $t = 1$. If the specialist accepts an exclusive contract in $t = 1$, R and M commit to also fulfill it in $t = 2$ by definition. Otherwise, M can additionally decide whether to launch product A before the specialist decides whether to be hosted. Figure 3.1 summarizes the timing in $t = 2$ if no exclusive contract is entered in $t = 1$.

The platform's and the specialist's payoffs depend on the vector of exogenously given parameters $\Gamma = (F, \phi, u_A, \tau_H, \tau_E, b, \alpha)$, the specialist's type θ and the specialist's (updated) belief $q = \mu(\theta_s|\Psi)$. Furthermore, the expected profits depend on the subgames that are reached in the first and second period. The exact payoffs become clear throughout the subsequent analysis.[19] For the sake of expositional clarity, I only sketch the second stage

[19]Corollary 3.1 in Section 3.4 formally outlines the expected payoffs of the platform and the specialist in the first stage after having derived the relevant first and second stage payoffs in the subsequent analysis.

Figure 3.1: Timing: Second Period without Exclusive Contract in $t = 1$



ex-post payoffs depicted in Figure 3.1 that are reached on the equilibrium path. The subsequent analysis shows that there exist three distinct outcomes that can be reached in the second stage. An exclusive contract is offered and accepted in the *Exclusive Outcome (E)*. In the *Competition Outcome (C)* the platform launches A and the specialist is not hosted. In the *Hosting Outcome (H)* the specialist is hosted and the platform does not launch A. Given a specific realization of $\xi \in \{\xi_l, \xi_m, \xi_h\}$ and the vector of exogenously given parameters Γ, the respective ex-post payoffs are as follows:

$$\dot{\pi}_R^E = (\phi + (1-\phi)\xi)(p_R^{on} - \tau_E) \qquad \dot{\pi}_M^E = (\phi + (1-\phi)\xi)\tau_E$$

$$\dot{\pi}_R^H = \phi p_R + (1-\phi)\xi(p_R^{on} - \tau_H) \qquad \dot{\pi}_M^H = (1-\phi)\xi\tau_H$$

$$\dot{\pi}_R^C = \phi p_R \mathbb{1}[p_R \leq p_M^{on} - b] \qquad \dot{\pi}_M^C = ((1-\phi)\xi)p_M^{on} + \phi p_M^{on}\mathbb{1}[p_R > p_M^{on} - b] - F$$

Solution concept and auxiliary assumptions: Throughout, I solve for the (subgame) perfect Bayesian equilibrium. Off-the-equilibrium-path beliefs are required to satisfy the divine criterion (J. S. Banks and Sobel, 1987). However, there still exist conditions where implausible equilibria may occur. Henceforth, I rule out equilibria in pricing subgames that rely on firms playing weakly dominated strategies. In case of indifference, I assume the specialist is hosted on the platform to break ties.[20]

Furthermore, I impose a 'conditional rationalizability' (CR) criterion that restricts the signaling function to only include signals for each platform type that are weakly dominant signals, independent of the signal that is specified for the other platform type.[21]

Finally, if conditions on specific or multiple parameters hold with equality, several equilibrium outcomes of the game may co-exist. Therefore, I limit the following analysis to scenarios where parameter conditions that determine the equilibrium outcome hold with strict inequality.[22]

I introduce three auxiliary assumptions to focus on the most interesting scenarios where the platform is incentivized to host the specialist to learn about on-platform demand and where entering an exclusive contract is not a dominant strategy for a strong-type platform and the specialist.

A1: $F \in ((1-\phi)\alpha(u_A - \tau_H), (1-\phi)(u_A - \tau_H))$

The upper bound on the fixed costs ensures that the platform can credibly demonstrate towards the specialist to launch product A if $\xi \in \{\xi_m, \xi_h\}$ is realized. Technically, it ensures that the cutoff $C_E < 1$, which is introduced in Lemma 3.1.[23] Otherwise, M launching A would never be an equilibrium outcome.

The lower bound on the fixed costs ensures that launching is, in expectation, not profitable for a strong-type platform, which also implies that the same is true for a weak-type platform.[24] The lower bound requires that it is an optimal strategy of the specialist to be hosted if the platform does not launch A. Consequentially, the platform launches product A in the

[20]This is similar in spirit to Dinerstein et al. (2018), who argue that if long-run revenues of the platform largely depend on attracting customers (rather than sellers), the actions taken to maximize long-run profits are highly correlated with the actions taken to maximize short-run consumer surplus.

[21]Check the *'Final Remark'* in *Step 6* of the proof of Proposition 3.3 to understand why equilibria that rely on weakly dominated signals being played appeal implausible. Ruling out implausible equilibria is particularly relevant when defining counterfactual market outcomes to conduct the policy analysis in Section 3.6.

[22]Focusing on conditions where a unique equilibrium outcome exists is without loss of generality and substantially streamlines the analysis.

[23]For further details check the proof of Lemma 3.1 where C_E is derived.

[24]For further details on the lower bound check the proof of Lemma 3.4.

second period if and only if it *learns* that on-platform demand is sufficiently high from hosting the specialist in the previous period.

A2: $\tau_E \in \left(\frac{F\tau_H}{F+\phi(u_A-\tau_H)}, min\left\{(1-\phi)u_A - F, \frac{\phi(1+\delta)b+x(1-\phi)\alpha[u_A+\delta\tau_H]}{(1+\delta)[\phi+x(1-\phi)\alpha)]}\right\}\right)$

where $x = \mathbb{1}[\delta < \Delta_{no}] + (\bar{\mu}_{no} + (1-\bar{\mu}_{no})\xi_m)\mathbb{1}[\delta > \Delta_{no}]$

The lower bound on τ_E ensures that a weak-type platform prefers to enter an exclusive contract instead of hosting the specialist in the second period. Technically, it ensures that the cutoffs $C_E < C_C$, which are introduced in Lemmas 3.1 and 3.2.[25]

The upper bound on τ_E serves two purposes. First, it ensures that although there exists the possibility to enter an exclusive contract, a strong-type platform is still willing to launch A in $t = 2$ if $\xi = \xi_h$ is realized. Thus, if $\tau_E < (1-\phi)u_A - F$, $C_C < 1$.[26] In combination with the lower bound of *A2* and *A1* it is therefore ensured that $\xi_l = 0 < \xi_m = C_E^+ < \xi_h = 1$, which is what was implicitly assumed earlier when discussing the distinct demand states.

Second, the assumption that the exclusive transfer does not exceed a certain threshold also satisfy a rationalizabilty concern: If facing a strong-type platform, the upper bound on τ_E ensures that the specialist rather accepts an exclusive contract instead of not being hosted in $t = 1$, which would be the worst possible outcome from the platform's perspective.[27] $\bar{\mu}_{no}$ and Δ_{no} are endogenous cutoffs that are outlined in more detail below in Lemma 3.6. They are independent of τ_E and ex-ante known to all players. Given that $\xi_m = C_E^+ < 1$ by *A1*, it follows that $x \in (\xi_m, \xi_h = 1)$ as $\bar{\mu}_{no} < 1$.

A3: $0 \leq b < \frac{\tau_H}{F+\phi(u_A-\tau_H)}[F - (1-\phi)\alpha(u_A - \tau_H)]$

Assumption *A3* ensures that it is an optimal strategy for the specialist to be hosted in the second period if M does not launch A, given τ_E satisfies *A2* as outlined above. To find the lower bound of *A1*, I implicitly assume that *A3* holds. It also directly follows from *A3* that $b < \tau_H$. Hence, the convenience benefit that consumers experience when shopping on-platform is assumed to be lower than the the per-unit transfer if the specialist is hosted.

[25]For further details, check the proofs of Lemmas 3.1 and 3.2 where the cutoffs are derived.

[26]For further details, check the proof of Lemma 3.2 where the cutoff is derived.

[27]The upper bound follows from *A1* and from *Step 1* of the proof of Lemma 3.7 in Section C.1.

3.3 Second Period

Let me start by analyzing the second period. Given the specialist's and the platform's actions in the first stage, three distinct subgames in the second period exist that differ in the information and strategies available to both players. The proofs of all pricing outcomes and equilibria are included in the proofs of the following Propositions and Lemmas.

3.3.1 Hosting Subgame

This subgame is reached if R is hosted in the first period. In this case, the stochastic demand parameter ξ is common knowledge such that there is no uncertainty about the demand state and the platform's type when entering the second stage. Furthermore, the entire set of strategies is available to both players as no exclusive contract was signed in the previous stage. The second-period outcome is derived from backward induction. Let me, therefore, start with outlining how the specialist optimally reacts if no exclusive contract is signed in $t = 2$ and if M decides to launch product A. If the specialist is willing to get hosted in such conditions, any equilibrium that potentially occurs is associated with the following *Direct Competition Outcome (D)*:

Outcome 1. ***Direct Competition Outcome (D)****: M does not offer an exclusive contract. M launches A and plays a mixed pricing strategy* $F_{M,D}\left(p_M^{on}\right)$*. R is hosted, but is not competitive on-platform, given* $F_{M,D}\left(p_M^{on}\right)$*.*[28] *R plays a mixed pricing strategy* $F_{S,D}\left(p_R\right)$ *on the off-platform market. Type 1 consumers can buy from R off-platform or from M on-platform. Type 2A consumers buy from M on-platform.* $E(\pi_R^D) = \phi\left(\frac{\tau_H}{\frac{\phi}{(1-\phi)\xi}+1} - b\right)$*,* $E(\pi_M^D) = (1-\phi)\xi\tau_H - F$*.* $E(CS_1^D), E(CS_{2A}^D) > 0$*.*

The specialist's presence on the platform exerts direct pressure on M's pricing strategy because the per-unit transfer τ_H, which denotes the platform's effective marginal cost on the platform, constitutes an upper bound on the platform's pricing strategy. For on-platform prices exceeding τ_H, both suppliers would be incentivized to mutually undercut the opponent's price. In the Direct Competition Outcome, M and R effectively compete for *Type 1*

[28] The strategies and their proofs are included in the proof of Proposition 3.1.

consumers that buy good A directly from the specialist via the off-platform market or from M via the platform. There does not exist an equilibrium in pure strategies as any such strategy profile would, again, give one of the two players an incentive to slightly undercut the opponent to sell to all *Type 1* consumers. However, the familiar Bertrand dynamics also do not show in full display since extracting the maximum possible profit from just *Type 2A* consumers is a form of 'safe haven' for the platform that directly determines both players' equilibrium pricing strategies. Since the (expected) price on the off- and on-platform market is smaller than the price a monopolist would charge, (expected) consumer surplus is greater than zero for both consumer types.

Otherwise, if the specialist is not hosted after the platform launches product A, any equilibrium outcome is associated with the following *Cross Competition Outcome (C)*:

Outcome 2. ***Cross Competition Outcome (C)****: M does not offer an exclusive contract. M launches A and plays a mixed pricing strategy* $F_{M,C}\left(p_M^{on}\right)$*. R is not hosted and plays a mixed pricing strategy* $F_{R,C}\left(p_R\right)$ *on the off-platform market. Type 1 can buy from R off-platform or from M on-platform. Type 2A consumers buy from M on-platform.* $E(\pi_R^C) = \phi\left(\frac{u_A}{\frac{\phi}{(1-\phi)\xi}+1} - b\right)$, $E(\pi_M^C) = (1-\phi)\xi u_A - F$. $E(CS_1^D) > E(CS_1^C) > 0, E(CS_{2A}^D) > E(CS_{2A}^C) > 0$

Likewise as with the *Direct Competition Outcome*, the specialist and the platform compete over *Type 1* consumers. However, since the specialist is not hosted, the platform's outside option improves since it is, in principle, able to charge monopoly prices from just *Type 2A* consumers. Therefore, the platform's expected equilibrium profit must be higher than in the *Direct Competition Outcome*. Also, the specialist is better off since the platform's incentive to compete over *Type 1* consumers is relatively smaller. As a result, both consumer types again receive an expected consumer surplus strictly greater than zero, but they are in expectation worse off than in the *Direct Competition Outcome*.

Proposition 3.1. *If M launches product A, the specialist abandons the platform such that the Cross Competition Outcome is subsequently realized in* $t = 2$.

Proof: See Appendix C.1

Proposition 3.1 demonstrates that the platform does not need to actively foreclose its marketplace after launching product A as it is in the specialist's interest to abandon the on-platform distribution channel. The hosting decision of the specialist directly determines how much profit the platform can extract from loyal *Type 2A* consumers. This profit constitutes the platform's opportunity cost of contesting *Type 1* consumers. Thus, the higher these are the less competitive the off-platform market becomes. Consequently, the specialist strategically abandons the on-platform distribution channel to protect its off-platform profit. Hence, the specialist cannot credibly threaten to get hosted to deter the platform from launching A. Therefore, the platform can base its launching decision on the expectation that it can subsequently act as a monopolistic supplier of product A on its marketplace. To sum up, if M launches A, the *Cross Competition Outcome* is the unique equilibrium outcome of the game.

If the platform does not launch product A and if no exclusive contract is entered in the second period, the specialist is willing to be hosted by a straightforward argument: The specialist is not concerned about M becoming a competitor in the current or a future period since the game terminates after the second stage by assumption. Therefore, the specialist and the platform are better off if the specialist is hosted in the second stage. There exists a unique equilibrium outcome:

Outcome 3. ***Hosting Outcome (H)****: R rejects an exclusive contract if offered. M does not launch A. R is hosted and charges* $p_R = u_A - b$ *and* $p_R^{on} = u_A$ *off- and on-platform. Type 1 consumers buy from R off-platform and Type 2A consumers buy from R on-platform. R pays* τ_H *for each unit sold via the platform.* $\pi_R^H = \phi(u_A - b) + (1-\phi)\xi(u_A - \tau_H)$, $\pi_M^H = (1-\phi)\xi\tau_H$. $CS_1 = CS_{2A} = 0$

In the *Hosting Outcome* the specialist serves both markets as a monopolistic supplier of good A. Given the assumed unit demand, it can extract the entire surplus from both consumer types off- and on-platform. The platform receives a per-unit transfer τ_H for each unit sold via the platform. Although the specialist needs to reimburse *Type 1* consumers for their incurred shopping costs, serving them via the off-platform market is still optimal.

Finally, if an exclusive contract is entered in $t = 2$, M cannot launch A and R needs to abandon its off-platform distribution channel. There exists a unique equilibrium outcome:

Outcome 4. ***Exclusive Outcome (E)****: M offers an exclusive contract that is accepted by R. R sells exclusively via the platform and sets* $p_R^{on} = u_A$ *and pays* τ_E *for each unit sold via the platform. Type 1 and Type 2A consumers buy from R via the platform.* $\pi_R^E = (\phi + (1-\phi)\xi)(u_A - \tau_E)$, $\pi_M^E = (\phi + (1-\phi)\xi)\tau_E$ *and* $CS_1 = CS_{2A} = 0$.

After entering an exclusive contract in $t = 2$, M is not allowed to launch product A and R is hosted with certainty to serve *Type 1* and *Type 2A* consumers via the platform. It pays τ_E for each unit sold via the platform. As the specialist is the monopolistic supplier of product A, it can again extract the entire consumer surplus from the market. Compared to the *Hosting Outcome*, the specialist is worse off and the platform is better off with an exclusive contract.

It depends on a specific realization of the demand parameter ξ, which of the mutually exclusive outcomes is realized on the equilibrium path.

Lemma 3.1. *If R is hosted in* $t = 1$*, the* ***Hosting Outcome*** *is the unique equilibrium outcome in* $t = 2$ *if:*

$$\xi < \frac{F}{(1-\phi)(u_A - \tau_H)} \equiv C_E \tag{3.1}$$

This is only possible if the demand state ξ_l *is realized.*

Proof: See Appendix C.1

The *Hosting Outcome* occurs if demand from loyal *Type 2A* consumers, which determines the platform's profit if launching product A, is insufficient to compensate the platform for foregone transfer payments and incurred fixed costs from launching. This is the case if stochastic demand ξ lies below the cutoff C_E. Therefore, threatening the specialist to launch product A if it rejects an exclusive contract is not a credible platform strategy. As being hosted is the specialist's preferred strategy, independent of whether the platform offers an exclusive contract, the specialized retailer is hosted and becomes the monopolistic supplier of product A on- and off-platform. Given the set of demand states as defined in Section 3.2, the *Hosting Outcome* occurs independent of the platform's type if introducing product A on-platform failed in the first period such that the demand state ξ_l is realized.[29]

Lemma 3.2. *If R is hosted in $t = 1$, the* ***Exclusive Outcome*** *is the unique equilibrium outcome in $t = 2$ if:*

$$C_E < \xi < \frac{F + \phi\tau_E}{(1-\phi)(u_A - \tau_E)} \equiv C_C \tag{3.2}$$

This is only possible if the demand state ξ_m is realized.

Proof: See Appendix C.1

The *Exclusive Outcome* occurs for intermediate levels of demand from *Type 2A* consumers. In this case, demand from *Type 2A* consumers is sufficiently high such that launching product A is profitable for the platform and therefore a credible threat if the specialist rejects an exclusive offer. Since the specialist also prefers the *Exclusive Outcome* over the *Cross Competition Outcome* that is alternatively reached in the outlined conditions, an exclusive contract is entered. By assumptions *A1* and *A2*, $0 < C_E < C_C < 1$ such that the stochastic demand ξ only lies between the two cutoffs if $\xi = \xi_m = C_E^+$ is realized. Hence, the *Exclusive Outcome* occurs in the second period if the platform is of the weak type and the introduction of product A in the first period was successful.

Lemma 3.3. *If R is hosted in $t = 1$, the* ***Cross Competition Outcome*** *is the unique equilibrium outcome in $t = 2$ if:*

$$C_C < \xi \tag{3.3}$$

This is only possible if the demand state ξ_h is realized.

Proof: See Appendix C.1

By reversing the reasoning from the previous two Lemmas, the *Cross Competition Outcome* is the unique outcome of the game if $C_C < \xi$. In this case, demand from loyal *Type*

[29] It is to mention here, that the specialist is in fact indifferent between being hosted and not being hosted if $\xi = \xi_l = 0$ is realized. The specified tie-breaking rule ensures that the *Hosting Outcome* occurs. Admittedly, the assumption that $\xi_l = 0$ is unfortunate to grasp the full intuition behind the *Hosting Outcome* but drastically streamlines the subsequent analysis. However, the reasoning outlined in Lemma 3.1 would also hold more generally for any $\xi \in (0, C_E)$, where the specialist's incentive to be hosted is strict.

2A consumers is sufficiently high such that launching product *A* is the most profitable strategy for the platform. Consequentially, it is a dominant strategy of the platform not to offer an exclusive contract since otherwise, the specialist would accept such an offer to avoid ending up in the *Cross Competition Outcome*, which is the most unfortunate outcome from the specialist's perspective. Since $C_C < \xi$ is only possible for $\xi = \xi_h$, the platform only launches product *A* in the second period if it is of the strong type and if the introduction of product *A* in the first period turned out to be successful.

To sum up, the characterization of the different platform types is based on the distinct second stage outcomes that occur in the hosting subgame if the low demand state is *not* realized: If product *A* was successfully introduced in the first period, an exclusive contract is entered with a weak-type platform and a strong-type platform launches product *A*.

3.3.2 Exclusive Subgame

If the specialist and the platform enter an exclusive contract in the first stage, the demand parameter ξ is known at the beginning of the second stage. However, both players are restricted in their available strategies by the definition of an exclusive contract. Hence, the *Exclusive Outcome* occurs with certainty in the second period.

3.3.3 No Hosting Subgame

Demand is still uncertain when entering the second period if *R* is not hosted in the first period. In this case, all possible equilibria are associated with the following outcome.

Lemma 3.4. *If the specialist is not hosted in $t = 1$, the Hosting Outcome occurs with certainty in $t = 2$.*

Proof: See Appendix C.1

Launching product *A* is not profitable for the platform as long as a stochastic demand ξ is unknown. As a consequence, the platform cannot credibly threaten to launch product *A* if no exclusive contract is entered. As a result, the already outlined *Hosting Outcome* occurs with certainty in the second period.

3.4 First Period

This section sketches the optimal strategies in the first period and outlines the game's solution. There are two open questions concerning the optimal decisions in the first period: First, does the platform offer an exclusive contract? Second, what is the specialists optimal strategy? If the specialist finds it not optimal to be hosted in the first period, product A is not sold via the platform such that ξ is not revealed in the first period.

3.4.1 First Stage Outcomes

If the specialist does not accept an exclusive contract and is not hosted in the first period, a unique equilibrium occurs that is associated with the following outcome:

Lemma 3.5. *If the specialist does not accept an exclusive contract and is not hosted, the* ***No Hosting Outcome*** *(N) is the unique equilibrium outcome where: R sets* $p_R^o = u_A - b$ *and Type 1 consumers buy product A on the off-platform market. Type 2A consumers do not buy product A.* $\pi_R^N = \phi(u_A - b)$, $\pi_M^N = 0$ *and* $CS_1 = (CS_{2A} =)\ 0$.

Proof: See Appendix C.1

Since the platform cannot launch product A in the first period by assumption, *Type 2A* consumers are not served and the platform generates no profit if the specialist decides to stay away from the platform and only operate on the off-platform market.

If this is not the case, however, the already known *Exclusive Outcome* occurs in the first period if an exclusive contract is offered and accepted. Otherwise, if no exclusive contract is entered in $t = 1$ and the specialist is hosted, the familiar *Hosting Outcome* occurs.

With the reasoning from this and previous sections, all payoffs are known that are potentially realized in the first and the second stage. The following Corollary outlines the expected payoffs and how they depend on the decision made in the first stage, where either the *Exclusive (E)-*, the the *No hosting (N)-* or the *Hosting Outcome (H)* is realized.[30]

Corollary 3.1. *Given a type $\theta \in \Theta$, a signal $\omega \in \Omega$, an updated belief $\mu(\theta_s|\Psi) = q$ and the payoffs as specified in Lemmas 3.1-3.5, the (expected) payoffs of the specialist and the platform depicted in Figure C.1 are as follows:*[31]

$$\begin{aligned}
E(\Pi_R^E|q) =&(1+\delta)\left[(1-\alpha)\pi_R^E(\xi_l)+\alpha(q\pi_R^E(\xi_h)+(1-q)\pi_R^E(\xi_m))\right]\\
E(\Pi_R^N|q) =&\pi_R^N+\delta\left[(1-\alpha)\pi_R^H(\xi_l)+\alpha(q\pi_R^H(\xi_h)+(1-q)\pi_R^H(\xi_m))\right]\\
E(\Pi_R^H|q) =&(1+\delta)(1-\alpha)\pi_R^H(\xi_l)+q\alpha\left[\pi_R^H(\xi_h)+\delta E(\pi_R^C(\xi_h))\right]\\
&+(1-q)\alpha\left[\pi_R^H(\xi_m)+\delta\pi_R^E(\xi_m)\right]
\end{aligned}$$

$$E(\Pi_M^E|\theta) = \quad (1+\delta)\alpha * \begin{cases}\pi_M^E(\xi_m) & if \quad \theta=\theta_w\\ \pi_M^E(\xi_h) & otherwise\end{cases}$$

$$E(\Pi_M^N|\theta) = \quad \delta\alpha * \begin{cases}\pi_M^H(\xi_m) & if \quad \theta=\theta_w\\ \pi_M^H(\xi_h) & otherwise\end{cases}$$

$$E(\Pi_M^H|\theta) = \quad \alpha * \begin{cases}\pi_M^H(\xi_m))+\delta\pi_M^E(\xi_m) & if \quad \theta=\theta_w\\ \pi_M^H(\xi_h))+\delta E(\pi_M^C(\xi_h)) & otherwise\end{cases}$$

3.4.2 Strategic Considerations

In the analysis of the hosting subgame in the second stage in Section 3.3, entering an exclusive contract ex-post, i.e. after demand is realized, is a means of avoiding competition that is leveraged if it is mutually benficial for both players on the supply side. This is the case if the product was successfully introduced in the first period and the platform is of the weak type. However, ex-ante, i.e while specific demand is still unknown at the beginning of the first period, entering an exclusive contract additionally is a means of hedging against an unfortunate realization of demand (for the platform) or to insure against the risk of competing against a strong-type platform (for the specialist).

Ex-ante, the specialist is particularly concerned about ending up in the *Cross Competition*

[30]Figure C.1 in Appendix C.2 refers to Corollary 3.1.

[31]Notice that with $\xi_l = 0$, $\pi_R^H(\xi_h) = \pi_R^N$ and $\pi_M^H(\xi_l) = \pi_M^N(\xi_l) = 0$.

Outcome in the second period. If no exclusive contract is offered, its only possibility to avoid this unfortunate outcome is not to get hosted in the first period.

Lemma 3.6. *There exists a cutoff value $\bar{\mu}_{no} \in (0,1)$ where the specialist is rather not hosted if not being offered an exclusive contract if $\mu(\theta_s|\Psi) \geq \bar{\mu}_{no}$ if:*

$$\delta > \frac{(1-\phi)(u_A - \tau_H)}{\phi^2 u_A + (1-\phi)(u_A - \tau_H)} \equiv \Delta_{no} \tag{3.4}$$

Proof: See Appendix C.1

If knowing with certainty that the platform is of the weak type, the specialist is certainly better off if being hosted in $t = 1$ since it is not threatened by M entering the market with a rivaling product in the second period.

Otherwise, if the specialist knows with certainty that the platform is of the strong type, concerns about expected future losses outweigh immediate gains from generating additional on-platform sales by getting hosted in the first period when Condition 3.4 is satisfied. In this case, there exists a threshold $\bar{\mu}_{no} \in (0,1)$ concerning the specialist's (updated) belief about the platform's type, above which the specialist is not willing to be hosted.

If, however, δ lies below the cutoff outlined in Condition 3.4, the specialist is hosted, even if certainly facing a strong-type platform. In this case, immediate gains from being hosted are always more important than potential future losses.

If the specialist is offered an exclusive contract, it has an additional alternative to circumvent the threat of M entering the market as a competitor in the second period.

Lemma 3.7. *There exists a cutoff value $\bar{\mu}_o \in (0, min\{\bar{\mu}_{no}, 1\})$ where the specialist is willing to accept an exclusive contract if $\mu(\theta_s|\Psi) \geq \bar{\mu}_o$ if:*

$$\delta > \frac{\tau_E\left[(1-\phi) + \frac{\phi}{\alpha}\right] - (1-\phi)\tau_H - \frac{\phi}{\alpha}b}{\phi^2 u_A + (1-\phi)u_A + \frac{\phi}{\alpha}b - \tau_E\left[(1-\phi) + \frac{\phi}{\alpha}\right]} \equiv \Delta_o < \Delta_{no} \tag{3.5}$$

Proof: See Appendix C.1

In any scenario where the specialist considers not being hosted, the specialist is better off if being able to enter an exclusive contract. Even more so, there also exist additional conditions where the specialist also chooses entering an exclusive contract over being hosted as compared to Lemma 3.6, the specialist's outside option of entering an exclusive contract is strictly better. As the specialist is more willing to enter an exclusive contract than not to be hosted if no such contract is offered, $\Delta_o < \Delta_{no}$ and $\bar{\mu}_o < \bar{\mu}_{no}$.

It becomes evident from a comparison of the cutoffs Δ_o and Δ_{no} that the consumer valuation for good A has a positive impact on the likelihood that the specialist prefers entering an exclusive contract over being hosted. Such a contract allows the specialist to extract the entire rent from the market in both periods. If not being hosted is the specialist's outside option, however, increasing levels of consumer valuation might positively impact the chances that the specialist is hosted. Foregoing the possibility of capturing the consumer valuation of loyal platform customers in the first period lets immediate gains from being hosted become relatively more attractive the higher u_A is.

Corollary 3.2. *Independent of the vector of parameters, not being hosted in $t = 1$ is strictly dominated for the specialist if (and only if) the platform offers an exclusive contract.*

Proof: Follows from Lemmas 3.6 and 3.7

It follows from Lemma 3.6 that the specialist prefers being hosted over not being hosted for any (updated) belief $\mu(\theta_s|\Psi) \in [0, \bar{\mu}_{no})$. Thus, if no exclusive contract is offered, the specialist is only not hosted for $\mu(\theta_s|\Psi) > \bar{\mu}_{no}$. It additionally follows from Lemma 3.7 that the specialist prefers accepting an exclusive contract over not being hosted for any (updated) belief $\mu(\theta_s|\Psi) \in [\bar{\mu}_{no}, 1]$ and that it prefers accepting an exclusive contract over being hosted for any (updated) belief $\mu(\theta_s|\Psi) \in [\bar{\mu}_o, 1]$. Since $\bar{\mu}_o < \bar{\mu}_{no}$, Corollary 3.2 directly follows. In summary, not being hosted is a never-best reply for the specialist if an exclusive contract is offered.

Lemma 3.8. *If $\theta = \theta_w$, offering an exclusive contract is a weakly dominant strategy for the platform.*

Proof: See Appendix C.1

Given the reasoning in Corollary 3.2, the worst-case outcome if offering an exclusive contract is that the specialist rejects the offer and is hosted in the first period. However, if not offering an exclusive contract, it actually is the best-case outcome for a weak-type platform that the specialist is hosted in the first period. As it follows from Lemma 3.2 that a weak-type platform will never launch product A in $t = 2$, even if product A is successfully introduced in the first period, it is always weakly better off if offering an exclusive contract in $t = 1$.

The second takeaway of Lemma 3.8 is more of a technical nature. Having identified a weakly dominant signaling strategy for the weak-type platform enables the selection of a unique equilibrium outcome in the equilibrium analysis below in Section 3.5.[32]

Lemma 3.9. *If* $\theta = \theta_s$, $E(\Pi_M^E|\theta_s) < E(\Pi_M^H|\theta_s)$ *if:*

$$\alpha > \underline{\alpha} \equiv \frac{(1+\delta)\phi\tau_E}{\delta[(1-\phi)(u_A-\tau_E)-F]-(1-\phi)(\tau_E-\tau_H)} \tag{3.6}$$

Otherwise, $E(\Pi_M^E|\theta_s) > E(\Pi_M^H|\theta_s)$.

Proof: See Appendix C.1

As opposed to a weak-type platform, a strong-type platform might find offering an exclusive contract in $t = 1$ not optimal if the specialist is otherwise hosted. This is the case if the chances of successfully introducing the product in the first period are sufficiently high such that the platform can generate a higher (expected) profit if having the possibilty to launch product A in the second period than if entering an exclusive contract in the first period. Having the possibility to launch A in the second period becomes relatively more attractive the lower the fixed costs F and the exclusive transfer τ_E are and the higher (lower) the excess net profits (losses) in the first and second period compared to not offering an exclusive contract are, which are positively impacted by u_A and τ_H in the first period.

If, however, the chances of successfully introducing the product are insufficient, entering an exclusive contract in the first period also is a way to hedge against an unfortunate realization of a demand state for a strong-type platform.

The outlined rationale is based on assuming that hosting the specialist in the first period is

[32]For further details, check the proof of Proposition 3.3 in Appendix C.1.

the counterfactual for a strong-type platform. If this is not the case, however, not offering an exclusive contract cannot be optimal for a strong-type platform.[33] As a consequence, there does not necessarily exist a weakly dominant signaling strategy for the specialist if $\alpha > \underline{\alpha}$ as there might exist a belief about the platform's type for which the specialist is not hosted if no exclusive contract is offered.

Proposition 3.2. *With the possibility of entering an exclusive contract, there cannot exist an equilibrium where the specialist is not hosted with positive probability in the first period.*

Proof: See Appendix C.1

It follows from the discussion in this section and from Corollary 3.1 that not offering an exclusive contract cannot be optimal for the platform if the specialist is not hosted in the first period, independent of the platform type.

It is additionally known from Corollary 3.2 that the specialist prefers to enter an exclusive contract if not being hosted is its outside option. Thus, if the platform infers that the specialist responds with not being hosted if no exclusive contract is offered, it is incentivized to certainly offer an exclusive contract to avoid this outcome.

In conclusion, entering an exclusive contract is a (weakly) pareto-efficient solution in conditions where markets would otherwise fail if loyal platform consumers are not served in the first period because there exists no possibility to enter an exclusive contract.[34]

3.5 Equilibrium Analysis

The equilibrium outcome of the game depends on the given vector of parameters. Five mutually exclusive scenarios depicted in Table 3.1 need to be considered.

[33] For details, check *Step 1* of the proof of Proposition 3.2.

[34] A more detailed discussion of how the possibility to enter an exclusive contract impacts the market outcome follows in Section 3.6.

Table 3.1: Counterfactual Scenarios and Conditions

Scenario	Condition(s)
(i):	$\mu_s < \bar{\mu}_o$
(ii):	$\{\delta \in (\Delta_o, \Delta_{no}) \ \wedge \ \mu_s > \bar{\mu}_o \ \wedge \ E(\Pi_M^E\|\theta_s) < E(\Pi_M^H\|\theta_s)\}$
(iii):	$\{\delta \in (\Delta_o, \Delta_{no}) \ \wedge \ \mu_s > \bar{\mu}_o \ \wedge \ E(\Pi_M^E\|\theta_s) > E(\Pi_M^H\|\theta_s)\}$
(iv):	$\{\delta > \Delta_{no} \ \wedge \ \mu_s \in (\bar{\mu}_o, \bar{\mu}_{no})\}$
(v):	$\mu_s > \bar{\mu}_{no}$

Proposition 3.3 outlines the parameter spaces where both players on the supply side are willing to enter an exclusive contract and the parameter spaces where this is not the case.

Proposition 3.3. *For scenarios (i) and (ii) as outlined in Table 3.1:*

- *The Hosting Outcome occurs with certainty in $t = 1$.*
- *Depending on a specific realization of ξ, one of the outcomes specified in Lemmas 3.1 -3.3 is realized in $t = 2$.*

Otherwise, for scenarios (iii)-(v) as outlined in Table 3.1:

- *There exists an equilibrium where the Exclusive Outcome occurs with certainty in both periods.*
- *For (v), the Exclusive Outcome is the unique outcome in both periods if the divine criterion applies.*
- *For (iii) and (iv), the Exclusive Outcome is the unique outcome, given a 'conditional rationalizability' criterion for each platform type's signaling strategy.*

Proof: See Appendix C.1

Proposition 3.3 shows that it is primarily the specialist's expectation about the platform's type and its concern about future period payoffs that are decisive for structuring the market in the first period.

In scenario (v), the specialist is willing to accept an exclusive contract in $t = 1$ if the ex-ante threat of the platform becoming a future competitor that cuts deeply into the specialist's off-platform customer base is sufficiently big. As a result, no updated belief exists where the specialist is willing to get hosted if no exclusive contract is offered. This reasoning makes offering such a contract an optimal strategy for the platform, independent of its type.

In scenario (iv), a one step more sophisticated but otherwise identical reasoning applies. The specialist infers that the platform must be of the strong type if not being offered an exclusive contract. Conditional on facing a strong type platform, the specialist is not willing to be hosted if no exclusive contract is offered. Consequently, the platform is urged to offer an exclusive contract, independent of its type.

Likewise, it becomes evident that the specialist's incentives also determine conditions for which an exclusive contract is never entered. In scenario (i), the ex-ante probability of facing a weak-type platform is relatively high such that the specialist is sufficiently confident that it is unlikely to create a future competitor through being hosted. As a consequence, the specialist rejects an exclusive contract if offered and is subsequently hosted.

Complementing these findings, it shows that the platform's incentive to enter an exclusive contract only impacts the market outcome if the specialist is willing to accept such a contract and simultaneously cannot make a credible case to not get hosted if no exclusive contract is offered. In such conditions, the specialist has no 'means available' to exert pressure on the platform in any direction. This is the case in scenarios (ii) and (iii).

As known from Lemma 3.8, the incentives of a weak type-platform are clear: It certainly wants to enter an exclusive contract in the first period, which is known to the specialist. Hence, how the specialist updates its belief crucially depends on the signal a strong-type platform sends. A strong-type platform does not offer an exclusive if the chances of successfully introducing the product are sufficiently high in scenario (ii). Thus, the specialist can infer that it certainly faces a weak-type platform if an exclusive contract is offered to subsequently reject the offer. In scenario (iii), however, the incentives of the distinct platform types are perfectly aligned as it also is in the interest of a strong-type platform to enter an exclusive contract. Being offered an exclusive contract is, therefore, not informative for the specialist such that it ultimately accepts the offer.

Corollary 3.3. *Compared to the market outcome as outlined in Proposition 3.3, a setting where, independent of its type, the platform always offers an exclusive contract and the specialized retailer self selects into such a contract can generate an identical market outcome in scenarios (i) and (iii)-(v) from Table 3.1.*

Proof: Follows from the proof of Proposition 3.3

Corollary 3.3 directly follows from the proof of Proposition 3.3. It shows that there exists a pooling equilibrium where both platform types offer an exclusive contract in scenarios (i) and (iii)-(v).

Such an opt-in policy where third-party sellers always have the possibility to apply for an exclusive dealership agreement might be more cost-efficient for large platforms that are trying to attract multiple sellers. In combination with Corollary 3.3, this might explain why hybrid platforms like Amazon primarily apply such an opt-in policy rather than explicitly offer an exclusive contract to individual third-party sellers.

Also, in scenario (ii), an opt-in policy might yield the same outcome as outlined in Proposition 3.3 if the platform can reject third-party inquiries for exclusive contracts. Although the specialist could infer that it faces a strong-type platform if the exclusive inquiry is rejected, the specialist is still willing to get hosted in scenario (ii).

3.6 Policy Interventions

Up to now, I showed how the combination of having an information advantage over third-party retailers and being a hybrid platform that learns from third-party data to inform its launching decision determines the outcome of the game. In the following, I discuss the implications of different policy interventions that ban particular business practices or reduce the information asymmetry between the platform and the specialist. In particular, I discuss banning exclusive contracts, an ex-situ regulation where both potential suppliers share the same *uninformed* prior about expected demand and an in-situ regulation where both potential suppliers share the same *informed* prior about expected demand.

I evaluate the policies' impact by benchmarking the induced market outcome against an unregulated market outcome. Thus, the relevant counterfactual depends on a specific set of parameters (scenarios (i)-(v)) as outlined in Proposition 3.3 and Table 3.1.

3.6.1 Banning Exclusive Contracts

Let me begin by analyzing the impact of banning exclusive contracts. Implementing such a policy is straightforward: Offering an exclusive contract is removed from the platform's strategies. Thus, after nature selects a platform type, the specialist directly decides whether it wants to get hosted on the platform or not in the first period. The specialist bases its decision on its initial belief about the platform type, which equals μ_s. Likewise, the platform directly decides whether it wants to launch product A at the beginning of the second period.

Proposition 3.4. *Given scenarios (i)-(v) as outlined in Table 3.1, banning exclusive contracts has the following effects:*

	Impact on:					
Scenario(s)	$E(W)$	$E(CS_1)$	$E(CS_2)$	$E(\Pi_R)$	$E(\Pi_M\|\theta_w)$	$E(\Pi_M\|\theta_s)$
(i)/(ii)	$\downarrow$ / $\cdot$	$\uparrow$ / $\cdot$	$\uparrow$ / $\cdot$	$\downarrow$ / $\cdot$	$\downarrow$	$\cdot$
(iii)/(iv)	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$ / $\uparrow$
(v)	$\downarrow$	$\cdot$	$\cdot$	$\downarrow$	$\downarrow$	$\downarrow$

" $\cdot$ " = unchanged; " $\uparrow$ " = increasing; " $\downarrow$ " = decreasing; " $\downarrow$ / $\uparrow$ " = ambiguous impact

Proof: See Appendix C.1

By a straightforward argument, the policy has no impact on the market outcome in the first period in scenarios (i) and (ii), where no exclusive contract is entered in an unregulated market. However, conditional on the platform being of the weak type, the policy has an impact in the second stage if the demand parameter $\xi = \xi_m$ is realized, as it follows from the reasoning in Lemma 3.2 that a weak-type platform is now incentivized to enter the market with a private label product, which benefits consumers. Even more so, the weak-type platform applies a more aggressive pricing strategy to attract *Type 1* consumers than a strong-type platform since its relatively smaller loyal customer base reduces the opportunity cost of pricing below the monopoly price. Furthermore, overall welfare decreases as the weak-type platform now incurs fixed costs from launching *A* and fewer consumers can conveniently buy via the platform.

In scenarios (iii) and (iv), the policy is already binding in the first period. As the specialist does not obtain a signal, being hosted is optimal for the specialist, given that its expectation about the platform being of the strong type is equal to μ_s. Consumers are not impacted in the first period since the specialist can still extract the entire surplus from the market if being hosted. In the second period, however, the policy enables the platform to enter the market as a rivaling supplier, which benefits both consumer types. Opposed to that, overall welfare reduces for the same reason as before. Furthermore, the policy is to the detriment of the specialist and a weak-type platform as both would prefer to enter an exclusive contract in the first period. A strong-type platform, however, might be better off if it also prefers that the specialist is hosted without the policy.

In scenario (v), the specialist is not hosted if no exclusive contract is offered. In that case, the *Hosting Outcome* occurs in the second period and the *No Hosting Outcome* in the first period, which is the worst possible outcome from the specialist's and the platform's perspective. Furthermore, the impact on overall welfare is strictly negative since loyal platform customers are not served in the first period. However, consumer surplus remains unchanged with the policy as the specialist can extract the entire rent from markets in which it is active, independent of whether the policy applies.

To summarize, the two key takeaways from this section are: First, it follows from the analysis of scenario (v) that exclusive contracts are a (weakly) pareto-efficient solution to solve an adverse selection problem that occurs if the specialist is otherwise not hosted in the first period. Second, the platform and the specialist benefit from having the possibility to enter an exclusive contract in almost all conditions. This finding is likewise driven through leveraging exclusive contracts to collect monopoly profits and from occurring economies of scale and synergies that stem from savings on fixed costs and from the convenience benefit consumers experience if buying on-platform. The latter two arguments are also the underlying reason why exclusive contracts have a positive effect on overall welfare.

3.6.2 Ex-situ Regulation

Let me continue with analyzing the impact of an ex-situ policy. Implementing such a policy resolves the information asymmetry by restricting the platform's access to consumer data, which entails two effects. First, the convenience benefit associated with buying product A via the platform vanishes as it is (assumed to be) grounded on data-enabled learning. Hence, consumers experience shopping costs equal to b, independent of where they purchase product A. Second, the specialist and the platform are assumed to share the same *uninformed* expectation about on-platform demand. Thus, offering an exclusive contract is no longer understood as a signal that indicates the platform's type. Both players on the supply side maximize their expected utility based on their (shared) expectation μ_s.

Independent of which of the scenarios (i)-(v) realizes in an unregulated market, an ex-situ policy has a direct negative impact on:

- Expected overall welfare.
- The expected on-platform profit of each potential supplier of product A.

Both effects occur since all consumers that buy via the platform need to be reimbursed for their additional shopping costs equal to b. Consequently, launching product A becomes relatively less attractive for the platform. To allow for a straightforward comparison to an unregulated market, let me replace assumptions *A1 - A3* with the following two auxilliary assumptions:

A1': $F \in ((1-\phi)\alpha(u_A - b - \tau_H), (1-\phi)(u_A - b - \tau_H))$

A2': $\tau_E \in \left(max\left\{ \frac{\tau_H(1-\phi)\xi_m}{\phi(1-\phi)\xi_m}, \frac{(1-\phi)\alpha\tau_H}{\phi+(1-\phi)\alpha} \right\}, min\left\{ (1-\phi)(u_A - b) - F, \frac{x(1-\phi)\alpha[u_A - b + \delta\tau_H]}{(1+\delta)[\phi + x(1-\phi)\alpha)} \right\} \right)$

where $x = \mathbb{1}[\delta < \Delta_{no}^{'}] + (\bar{\mu}_{no}^{'} + (1-\bar{\mu}_{no}^{'})\xi_m)\mathbb{1}[\delta > \Delta_{no}^{'}]$

Both assumptions ensure that, in the second period, launching is still profitable for a strong-type platform if $\xi = \xi_h$ is realized, but unprofitable in expectation. Furthermore, a weak-type platform rather enters an exclusive contract if $\xi = \xi_m$. In addition, the auxiliary assumptions guarantee that entering an exclusive contract in the first period is a better alternative for the platform than not hosting the specialist, independent of its type, and simultaneously ensure that it is optimal for the specialist to be hosted in the second period if the platform does not launch A. Finally, the specialist prefers entering an exclusive contract instead of not being hosted.

Proposition 3.5. *Given scenarios (i)-(v) as outlined in Table 3.1, introducing an ex-situ policy has the following effects:*

		Impact on:					
Scenario(s)	Exclusive	$E(W)$	$E(CS_1)$	$E(CS_2)$	$E(\Pi_R)$	$E(\Pi_M \vert \theta_w)$	$E(\Pi_M \vert \theta_s)$
(i)	(no)	↓	↓	↓	↑/↓	↓	↓
(ii)	no	↓	↓	↓	↑/↓	↓	↓
	yes	↑/↓	↓	↓	↑/↓	↑	↓
(iii)-(v)	no	↓	↑	↑	↑/↓	↓	↑/↓
	yes	↓	·	·	↓	·	·

" ·" = unchanged; " ↑" = increasing; " ↓" = decreasing; " ↓ / ↑" = ambiguous impact

Proof: See Appendix C.1

The specialist is again hosted with certainty after the policy is introduced if scenario (i) is the counterfactual. Conditional on launching A in the second period, the platform now also needs to reimburse *Type 1* consumers for their incurred shopping costs when selling via the platform. Therefore, the platform's incentive to contest these consumers is relatively lower. Consequentially, the platform and the specialist apply less competitive pricing strategies than in an unregulated market, which ultimately harms both consumer types and attenuates the direct negative impact on the specialist's profit.

Given that scenario (ii) is the relevant counterfactual, a similar reasoning applies if the specialist is again hosted in the first period. However, it is now also possible that an exclusive contract is entered as the relatively higher uncertainty about expected demand reduces the platform's willingness to launch product A, which disadvantages consumers. The policy's direct negative effect on expected welfare is, however, counteracted by the fact that no fixed costs from launching A occur in the second period. As the specialist would have preferred to enter an exclusive contract in an unregulated market, it might profit from the policy despite its direct negative impact on the on-platform profit.

A weak-type platform is certainly worse off with the policy if the specialist is hosted in the first period for scenarios (i) and (ii) as it can no longer credibly threaten to launch A in the second period if $\xi = \xi_m$ and consequentially, no exclusive contract is entered. In case an exclusive contract is entered in scenario (ii), however, it is certainly better off as this is its preferred outcome.

Given that scenarios (iii)-(v) apply, the market structure with an ex-situ policy depends on the specific vector of parameters. Other than the direct impact outlined initially, an ex-situ policy does not additionally impact the market outcome if it results in an exclusive contract being entered. If, however, the parameters are such that the specialist is hosted, the strong-type platform can now launch its own product version in the second period, which profits both consumer types but additionally decreases overall welfare due to the incurred fixed costs. Considering the reasoning from the discussion on banning exclusive contracts, not entering such a contract is to the specialist's and the platform's disadvantage in almost all conditions.

The key takeaway from this section is that an ex-situ policy harms most market participants and reduces overall welfare in most conditions. This finding is only partially due to the assumption that consumers no longer experience a convenience benefit when purchasing via the platform. The underlying reasoning also applies for small levels of b as the policy increases the uncertainty associated with the market outcome. This limits the potential to generate economies of scale that stem from savings on fixed costs.

3.6.3 In-situ Regulation

Let me continue with analyzing the impact of an in-situ regulation. With such a policy, the specialist is guaranteed the same access to consumer data as the platform. Therefore, the specialist perfectly observes the platform's type ex-ante, i.e. before deciding whether to accept an exclusive contract if offered, or to be hosted in the first period. The policy's impact depends on the specific platform type, which is only observed ex-post in an unregulated market, i.e. after a demand state is realized.

Proposition 3.6. *Given $\theta = \theta_w$ and given scenarios (i)-(v) as outlined in Table 3.1, introducing an in-situ policy has the following effects:*

	Impact on:				
Scenario(s)	$E(W)$	$E(CS_1\|\theta_w)$	$E(CS_2\|\theta_w)$	$E(\Pi_R\|\theta_w)$	$E(\Pi_M\|\theta_w)$
(i)/(ii)	·	·	·	·	·
(iii)-(v)	↓	·	·	↑	↓

" ·" = unchanged; " ↑" = increasing; " ↓" = decreasing; " ↓ / ↑" = ambiguous impact

Proof: See Appendix C.1

Conditional on the platform being of the weak type, the specialist is not threatened by creating a future competitor through hosting. It, therefore, is an optimal strategy to be hosted with certainty in the first period. Thus, compared to scenarios (i) and (ii), the market outcome is not impacted by the regulation. If benchmarked against an unregulated market in scenarios (iii)-(v), the policy advantages the specialist as it certainly prefers to be hosted in the first period instead of entering an exclusive contract if facing a weak-type platform. Vice versa, a weak-type platform is disadvantaged. Consumers are not impacted as a weak-type platform never launches A, independent of the policy. Overall welfare decreases as *Type 1* consumers now buy off-platform where shopping costs equal to b occur.

Proposition 3.7. *Given $\theta = \theta_s$ and given scenarios (i)-(v) as outlined in Table 3.1, introducing an in-situ policy has the following effects:*

	Impact on:				
Scenario(s)	$E(W)$	$E(CS_1\|\theta_s)$	$E(CS_2\|\theta_s)$	$E(\Pi_R\|\theta_s)$	$E(\Pi_M\|\theta_s)$
(i | $\delta < \Delta_o$)	·	·	·	·	·
(i | $\delta \in (\Delta_o, \Delta_{no})$)*	· / ↑	· / ↓	· / ↓	· / ↑	· / ↑
(i | $\delta > \Delta_{no}$)	↑	↓	↓	↑	↓ / ↑
(ii)-(v)	·	·	·	·	·

" ·" = unchanged; " ↑" = increasing; " ↓" = decreasing; " ↓ / ↑" = ambiguous impact;
*" * " = without excl. / with excl.*

Proof: See Appendix C.1

If the specialist observes that the platform is of the strong type, the specialist is threatened by the platform becoming a future competitor.

For $\delta < \Delta_o$, the specialist nevertheless strictly prefers to be hosted. As this can only be the case if scenario (i) is the relevant counterfactual, the policy has no impact here.

For $\delta \in (\Delta_o, \Delta_{no})$, which is only possible in scenarios (i)-(iii), the specialist prefers to enter an exclusive contract but is hosted if not offered. Therefore, the platform's incentive to offer such a contract is crucial. Thus, the policy has no impact on the outcome of the game in scenarios (ii) and (iii) where this is also the case in an unregulated market. The same is true if scenario (i) is the relevant counterfactual and the platform's incentives are again such that the specialist is hosted. Otherwise, an exclusive contract is entered in the first period, which benefits the specialist and the platform. Likewise, overall welfare increases as no fixed costs from launching accrue. The opposite, however, applies for consumers by an already familiar reasoning.

For $\delta > \Delta_{no}$, which is only possible in scenarios (i), (iv) and (v), the specialist is again willing to enter an exclusive contract in the first period but is not hosted if no such contract is offered. As the latter is never optimal for a strong-type platform, it certainly offers an exclusive contract in the first period. Thus, compared to scenarios (iv) and (v), the policy has no impact on the market outcome. However, if scenario (i) is the relevant counterfactual, an exclusive contract is entered in the first period. The effect of the policy then is identical as outlined before, with one exception: If the platform's and the specialist's incentives to enter an exclusive contract are not aligned, the platform is now worse off.

There are two key takeaways from this section. First, an in-situ policy increases the information level in the market, which is to the advantage of both players on the supply side whenever their incentives align. Therefore, exclusive contracts can also be leveraged in a setting where no information asymmetry exists to avoid competition when this is mutually beneficial for both potential suppliers.

Second, when benchmarked against an ex-situ policy, one finds that the in-situ policy is preferable from the specialist's perspective and also, in most conditions, from an overall welfare perspective. Moreover, even a high-type platform can be better off in some conditions. However, both policies are, in expectation, outperformed by an unregulated market in situations where the specialist is hosted in the first period if a social planner is particularly concerned about consumer surplus. Otherwise, the ex-situ policy has the potential to outperform both. However, it needs to be ensured that no exclusive contract is entered after implementing the policy. Furthermore, its relative performance deteriorates as b increases.

3.7 Concluding Remarks

Accessing data from multiple stakeholders creates an information asymmetry by giving the platform a clear outlook on the demand from loyal platform consumers if introducing a new product on-platform does not fail for unforeseeable reasons. This information advantage over third-party retailers can be exploited by a hybrid platform that competes on its product platform and that informs its launching decision with third-party demand data through offering an exclusive contract.

In the outlined setting, exclusive contracts constitute a possibility to hedge against unfortunate realizations of uncertain demand states. The platform is concerned that introducing the product might fail such that it is not able to recover its fixed costs. Thus, from the platform's perspective, exclusive contracts are a convenient way to outsource the risk of launching private label products. The specialist is concerned that it might create a future competitor by revealing on-platform demand if being hosted. Thus, an exclusive contract is understood as a way to insure against this eventuality from the specialist's perspective.

From a regulatory perspective, the possibility to insure against worst-case scenarios is key when exclusive contracts solve an adverse selection problem that occurs if the specialized retailer is otherwise not willing to be hosted.

If this is not the case, however, a regulator faces a severe trade-off. On the one side, exclusive contracts allow the specialist to capture a higher profit in most conditions and allow both players on the supply side to exploit synergies and economies of scale from savings on fixed costs and leveraging a more convenient consumer platform, which is beneficial for overall welfare. On the other side, entering an exclusive contract manifests the monopoly position of the specialized retailer if the platform would otherwise have an incentive to launch a rivaling product version. Therefore, such contracts also serve as a self-enforced constraint to escape the dynamics of abrasive competition by limiting each player's strategy space, which clearly harms consumers. This argument might also serve as an alternative explanation for why this business model has become increasingly popular in recent years.

The trade-off becomes even more fundamental for national regulation authorities if third-party retailers operate locally but the hybrid-platform is headquartered in a different jurisdiction as this increases the relative importance of third-party seller profits.

The outlined results also demonstrate that third-party retailers can be expected to propose very different policies than what is beneficial for consumers. Policies that aim at resolving the information asymmetry are beneficial for specialized retailers if they allow them to target more precisely when to enter an exclusive contract, or if they increase the platform's uncertainty about the market outcome such that it is more likely willing to offer an exclusive contract in conditions where this is also beneficial for third-party retailers. For consumers, the exact opposite is true: The analyzed policies are not suited to increase consumer surplus whenever they cause a strong-type platform and the specialist to enter an exclusive contract.

Let me finish with a final thought on how the results could differ if there exists a competitive fringe. An additional fringe seller could make a protected on-platform monopoly position more valuable for specialized retailers. Furthermore, a fringe seller pressures the specialized retailer to get hosted in any case, which makes the adverse selection problem obsolete (Schader and Montag, 2022). Therefore, the specialist's incentive to enter an exclusive contract and the conditions where this harms consumers should be expected to increase.

Appendices

Appendix A

Appendix to Chapter 1

A.1 Proofs

Throughout, I assume that $inf(T_i), sup(T_i) \in T_i, \ \forall \ i \in \{L, R\}$, which is without loss of generality.

A *simple cdf* G_i^* of party $i \in \{L, R\}$ places an atom of size α_i on $sup(T_L)$ (party L) / $inf(T_R)$ (party R) and an atom of size $(1 - \alpha_i)$ on $inf(T_L)$ (party L) / $sup(T_R)$ (party R).

Unless noted differently, I apply the following notation:

- $F(x) = \frac{x+b}{2b} \rightarrow \frac{\partial F(x)}{\partial x} = f(x) = \frac{1}{2b} = k$
- $\bar{G}_i(\cdot)$ is the antiderivative of $G_i(\cdot)$ and $g_i(\cdot)$ is the *pdf* of $G_i(\cdot)$
- $\frac{\partial H_i(p)}{\partial p} = H_i^{'}(p)$ and $\frac{\partial^2 H_i(p)}{\partial^2 p} = H_i^{''}(p)$

Proof of Lemma 1.1

Suppose that for $i \in \{L, R\}$ and for a given distribution function G_i, $p_1, p_2, p_3 \in T_i$. Let $p_2 < p_3$ and let α_q denote the mass placed on p_q where $q \in \{1, 2, 3\}$.

If $(p_1, H_i(p_1))$ lies below the line connecting $(p_2, H_i(p_2))$ and $(p_3, H_i(p_3))$, party i can profitably deviate to a strategy G_i^* that differs from G_i only in the mass it assigns to p_1, p_2, p_3, such that $\alpha_q^* = \alpha_q + \epsilon_q$ where $\sum_{q=1}^{3} \epsilon_q p_q = 0$:

- If $p_2 < p_1 < p_3$, party i can profitably deviate from G_i: By shifting some weight in a mean-preserving way such that $\epsilon_2, \epsilon_3 > 0$ and $\epsilon_1 < 0$, party i can achieve a higher EH_i without changing $E_i(p)$.

- If $p_1 < p_2 < p_3$, party i can deviate by a similar reasoning if setting $\epsilon_2 > 0$ and $\epsilon_1, \epsilon_3 < 0$.

- If $p_2 < p_3 < p_1$, party i can deviate by a similar reasoning if setting $\epsilon_3 > 0$ and $\epsilon_1, \epsilon_2 < 0$.

Any situation where $(p_1, H_i(p_1))$ lies above the line running through $(p_2, H_i(p_2))$ and $(p_3, H_i(p_3))$ cannot be an equilibrium. Following the above intuition, party i can achieve a higher EH_i without changing $E_i(p)$ by setting $\epsilon_2, \epsilon_3 < 0$ and $\epsilon_1 > 0$. Notice that it does not matter for this argument whether $p_1 \in T_i$ or not.

From the above reasoning it follows that all $(p, H_i(p))$, $\forall\, p \in T_i$ must lie on a straight line and no $p \in [inf(T_i), sup(T_i)] \setminus T_i$ may lie above this line.

Suppose $p_1 \in T_i$ and $\exists\ n \in \{2, 3\} : p_n \notin T_i$ such that $(p_1, H_i(p_1))$ lies below the line connecting $(p_2, H_i(p_2))$ and $(p_3, H_i(p_3))$ for $p_2 < p_1 < p_3$. Then, party i can, again, profitably deviate by shifting some weight in a mean-preserving way from p_1 to p_2 and p_3 to increase EH_i.

Otherwise, if $\neg[p_2 < p_1 < p_3]$, there may exist no profitable deviation for party i if $(p_1, H_i(p_1))$ lies below the line running through $(p_2, H_i(p_2))$ and $(p_3, H_i(p_3))$ since party i faces a trade-off between increasing (decreasing) EH_i and increasing (decreasing) the distance to its average preferred platform position.

Extending the proof to the case where G_i does not place an atom on $p_q \in T_i$, $\forall\, q \in \{1, 2, 3\}$ is straightforward: Assume that G_i assigns positive probability to the neighborhood p_q. Then, the above deviations can be reproduced, only now the weight is shifted from all points in an arbitrarily small neighborhood of p_q.

□

Proof of Lemma 1.2

Step 1: Any strategy where $sup(T_L) \in [sup(T_R), 1]$ for $sup(T_R) > 0$ is a strictly dominated strategy for party L:

If G_R is continuous around $sup(T_R)$, it directly follows from Equation 1.3 in Section 1.2 that $H_L'(p) \leq 0, \forall\, p \in [sup(T_R), 1]$ and $H_L(sup(T_R) - \epsilon) > H_L(sup(T_L)) > 0$ such that party L strictly increases EH_L if shifting all weight placed on $sup(T_L)$ (or its neighborhood if continuous around $sup(T_L)$) to $sup(T_R) - \epsilon$ where ϵ is arbitrarily small. This constitutes a profitable deviation as thereby π_L^L additionally strictly increases.

If G_R places an atom of size $\alpha_R \in (0, 1]$ on $sup(T_R)$, it follows from Equation 1.3 that $H_L'(p) \leq 0, \forall\, p \in (sup(T_R), 1]$. Conditional on the voter sampling $p_R = sup(T_R)$, $H_L(sup(T_R)\,|\,p_R = sup(T_R)) = 1/2$, given the tie-breaking rule specified in Section 1.2. However, since $sup(T_R) > 0$ by assumption, it follows from Equation 1.3 and the definition of $F(x)$ that $H_L(sup(T_R) - \epsilon\,|\,p_R = sup(T_R)) > 1/2$ where ϵ is arbitrarily small. Thus, the above-outlined argument applies, too.

Step 2: A scenario where $E_L(p) < X_L$ and $EH_L = 0$ cannot constitute an equilibrium:

Suppose $E_L(p) < X_L$ and $EH_L = 0$ constitutes an equilibrium. It follows from Equation 1.3 that this directly implies $H_L(inf(T_R) + \epsilon) > 0$ where ϵ is arbitrarily small.

Furthermore, if G_L is a non-degenerate distribution such that $inf(T_L) < sup(T_L)$, $EH_L = 0$ implies $H_L(inf(T_L)) = H_L(sup(T_L)) = 0$ such that Lemma 1.1 is violated: The point $(sup(T_L), H_L(sup(T_L)))$ lies below the line connecting $(inf(T_L), H_L(inf(T_L)))$ and $(inf(T_R) + \epsilon, H_L(inf(T_R) + \epsilon))$.

Otherwise, if G_L is a degenerate distribution that places all weight on an ideology $p < X_L$, $EH_L = 0$ implies $H_L(p) = 0$. Then, the Intermediate Value Theorem ensures that $\exists\, \gamma \in (0, 1]$ such that it constitutes a profitable deviation for party L to shift a fraction γ from $p < X_L$ to $inf(T_R) + \epsilon$ to thereby strictly increase EH_L and π_L^L.

Step 3: Suppose, in equilibrium, $E_L(p) < X_L$, which implies that $inf(T_L) < X_L < 0$ by the definition of X_L.

As, by symmetry, a similar argument as outlined in *Step 1* applies for party R, party $R's$ best-reply is such that $\nexists\ p : p \in [-1, inf(T_L)],\ and\ p \in T_R$. This implies that $inf(T_L) < inf(T_R)$.

It follows from a similar argument as outlined in *Step 1* that for $inf(T_L) < inf(T_R)$ party L can profitably shift the weight placed on $inf(T_L)$ (or its neighborhood if G_L is continuous around $inf(T_L)$) to $p = inf(T_L) + \epsilon$ where epsilon is arbitrarily small. Thereby, EH_L (weakly) increases and π_L^L strictly increases as $inf(T_L) < X_L$, which cannot constitute an equilibrium.

It follows that this is a strictly profitable deviation such that the initially described setting cannot constitute an equilibrium. The only exception is a scenario where $EH_L = 0$ before and after the shift. However, in such case *Step 2* is contradicted, which cannot constitute an equilibrium either.

By symmetry, a similar argument applies for party R.

□

Proof of Lemma 1.3

Given that $EH_L = 1 - EH_R$ and given Lemma 1.2, it directly follows from Equation 1.5 in Section 1.2 that, in equilibrium, the game is a constant sum game where:

$$\begin{aligned} EU_L + EU_R =& EH_L(X_L - E_L(p) - X_R + E_L(p)) \\ &+ (1 - EH_L)(X_L - E_R(p) - X_R + E_R(p)) = X_L - X_R \end{aligned}$$

Assume now that mirror-inverting G_i is no best reply for party $j \neq i$. Then, there must exist an alternative strategy for party $j \neq i$ which yields $EU_{j\neq i} > \frac{X_L - X_R}{2} > EU_i$.

In this case, party i can profitably deviate from its original strategy as reverse-mirroring party $j's$ best reply must yield $EU_{j\neq i} = \frac{X_L - X_R}{2} = EU_i$ and, therefore, a higher expected payoff for party i.

□

Corollary A.1.1. *If $E_i(p) \neq 0$, a strategy $G_i(p)$ with $sgn\,(E_i(p)) \neq sgn\,(X_i)$ does not survive the iterative elimination of dominated strategies.*

Proof: If $sgn\,(E_i(p)) \neq sgn\,(X_i)$ and $E_i(p) \neq 0$, there exist two mutually exclusive and exhaustive scenarios:

For $sgn\,(E_i(p)) = sgn\left(E_{j\neq i}(p)\right)$, Lemma 1.3 is contradicted as $EU_i < \frac{X_L - X_R}{2} < EU_{j\neq i}$ with certainty. Otherwise, if $sgn\,(E_i(p)) \neq sgn\left(E_{j\neq i}(p)\right)$ such that $sgn\left(E_{j\neq i}(p)\right) = sgn\,(X_i)$, either party i is strictly better off if $1:1$ copying $G_{j\neq i}$ or party $j \neq i$ is strictly better off if $1:1$ copying G_i or both as $EH_i + EH_{j\neq i} = 1$ by definition.[1]

□

Proof of Lemma 1.4

It is taken as given that for G_i to be a best reply $sgn(E_i\,(p)) = sgn(X_i)$ if $E_i\,(p) \neq 0$, which follows from Corollary A.1.1, and $|E_i\,(p)| \leq |X_i|$, which follows from Lemma 1.2.

If a best replying simple distribution G_i^* that places an atom of size α_i on $sup(T_i)$ and an atom of size $(1-\alpha_i)$ on $inf(T_i)$ yields $E_i^*\,(p) \neq E_i\,(p)$, it directly follows that G_i^* is a better reply than G_i such that G_i itself cannot be a best-reply to $G_{j\neq i}$ by the reversed reasoning.

If, however, a best replying simple distribution G_i^* yields $E_i^*\,(p) = E_i\,(p)$, G_i itself must also be a best-reply to $G_{j\neq i}$ as, by Lemma 1.1, $H_i(p)$ is linear on T_i and, therefore, party i is indifferent for any mean-preserving shift within T_i.

Considering also Corollary 1.1, the points $(p', H_i(p'))$, $\forall\ p' \notin T_i$ do not lie above the line connecting the points $(p, H_i(p))$, $\forall\ p \in T_i$. Thus, G_i is indeed a best-reply by a similar reasoning as in Lemma 1.1 if G_i^* yields an interior solution where $\alpha_i \in (0, 1)$ for $E_i^*\,(p)$. Notice that this certainly is the case if $E_i^*\,(p) = E_i\,(p)$ as $E_i(p) \in (inf(T_i), sup(T_i))$ by definition of a non-degenerate distribution.

□

[1]The intuition is that at least one party must have a strict incentive to copy the opponent as it cannot be the case that both simultaneously win the election with probability zero where no strict incentive would exist.

Proof of Proposition 1.1

Corollary A.1.1 ensures that $inf(T_L) \leq 0$ and $sup(T_R) \geq 0$. Suppose that $inf(T_R) < 0$ for the remainder of this proof. Then, Lemma 1.3 requires that $sup(T_L) = -inf(T_R) > 0$ is a best reply. The proof leverages Lemma 1.1 to show that there cannot exist an equilibrium where this is the case.

Step 1: By *Step 1* of the proof of Lemma 1.2, any G_L with $sup(T_L) > sup(T_R)$ is a dominated strategy for party L. If replacing $sup(T_R)$ by b, the identical argument as outlined in *Step 1* of the proof of Lemma 1.2 applies. Thus, it follows that, in equilibrium, $sup(T_L) < min\{b, sup(T_R)\}$ and, by symmetry, $inf(T_R) > max\{-b, inf(T_L)\}$.[2]

Furthermore, it follows from a similar argument as outlined in *Step 1* of the proof of Lemma 1.2 that it can never be a best reply for party R to choose $inf(T_R) = p < 0$ if G_L places an atom on $p = inf(T_R)$ and vice versa for party L.

Disclaimer: The remainder of this proof requires a deep understanding of $H_L(p)$ and $H_R(p)$. Check, therefore, the subsequent Corollary A.1.2 where the result from Step 1 is taken as given.

Step 2: Given $inf(T_R) < 0$, $inf(T_L) = -1$, there can only exist an equilibrium if G_L places no weight on $(inf(T_L), inf(T_R))$ and an atom on $p = -1$:

- If $sup(T_R) - inf(T_R) < 4b$ and $inf(T_R) \leq 2b - sup(T_R)$, no weight is placed on any $p \in (-1, inf(T_R))$ since $0 < H_L'(p_1) < H_L'(p_2)$ where $-1 < p_1 < -2b - inf(T_R) < p_2 < inf(T_R)$ such that all $(p, H_L(p))$ for $p \in (-1, inf(T_R))$ certainly lie below the line connecting $(-1, H_L(-1))$ and $(inf(T_R), H_L(inf(T_R)))$. Considering also *Step 1*, there can only exist an equilibrium if G_L places an atom of size $1 - \alpha_L$ on $p = -1$.

- If $sup(T_R) - inf(T_R) \geq 4b$ or if $sup(T_R) - inf(T_R) < 4b$ and $inf(T_R) > 2b - sup(T_R)$, $H_L'(p_1), H_L''(p_1) \geq 0$ and $H_L'(p_2) \geq 0, H_L''(p_2) \leq 0$ where $-1 < p_1 < 2b - sup(T_R) < p_2 < inf(T_R)$. By a similar reasoning as outlined above, $\nexists\ p_1 \in T_i,\ \forall\ p_1 \in (-1, 2b - sup(T_R))$.

[2] The strict inequalities might be weak inequalities if b is selected by *max* or *min* operator, which is without loss of generality.

It can still be the case that all $(p_2, H_L(p_2))$ lie below the line connecting $(-1, H_L(-1))$ and $(inf(T_R), H_L(inf(T_R)))$. Then, any strategy G_L places an atom of size $1-\alpha_L$ on $p=-1$ in equilibrium by a similar reasoning as above.

Otherwise, there cannot exist an equilibrium by the following argument: Suppose that there $\exists\ p^*, p^{**} : p^* \in T_L,\ and\ 2b - sup(T_R) \leq p^* < inf(T_R) < 0 < p^{**} \leq sup(T_L)$. Then, it follows from Corollary A.1.2 that $H_L^{'}(p^*) > H_L^{'}(p^{**})$ for all p^*, p^{**} as just defined.

In this case, Lemma 1.1 is certainly violated as Lemma 1.1 requires that $H_L^{'}(p) = const.,\ \forall\ p \in T_L$. Thus, in the just outlined scenario $\exists\ p^{***} \in [inf(T_R), sup(T_L))$ where $(p^{***}, H_L(p^{***}))$ lies above the line connecting $(p^*, H_L(p^*))$ and $(sup(T_L), H_L(sup(T_L)))$ for any p^* as just defined.

It follows from the above outlined argument that there can only exist an equilibrium where G_L places an atom of size $1-\alpha_L$ on $p=-1$ and no weight is placed on $(-1, inf(T_R))$.

Step 3: If $b \geq \frac{1}{2}$, any strategy with $sup(T_L) > 0$ is a dominated strategy for party L:

- $b \geq \frac{1}{2}$ implies $sup(T_R) - inf(T_R) \leq 4b$ and $inf(T_R) < 2b - sup(T_R) = 2b - 1$ since $sup(T_R) = 1$ by *Steps 1 & 2*.

- Given $p_1, p_2, p_3 : p_1 < inf(T_R) < p_2 < 0 < p_3$, it follows from Corollary A.1.2 that $H_L^{'}(p_1) > H_L^{'}(p_3)$ and $H_L^{'}(p_2) \geq H_L^{'}(p_3)$. Hence, $(0, H_L(0))$ certainly lies above the line connecting $(inf(T_L), H_L(inf(T_L)))$ and $(sup(T_L), H_L(sup(T_L)))$, which contradicts Lemma 1.1 and, therefore, is a dominated strategy.

Step 4: If $b < \frac{1}{2}$ and given *Step 2*, any strategy with $sup(T_L) > 0$ is a dominated strategy for party L:

- Given $sup(T_L) > 0$, a similar reasoning as in *Step 2* applies such that G_R places an atom of size $(1-\alpha_R)$ on $sup(T_R) = 1$ and may not attribute any weight to $(sup(T_L), 1)$ in any potential equilibrium. It follows that, in equilibrium, $2b - sup(T_R) = 2b - 1 < 0$.

- In addition, $2b - p \geq sup(T_L),\ \forall\ p \in (2b - sup(T_R), sup(T_L))$ since $sup(T_L) \in (0, b]$ by *Step 1*. Consequentially, it follows from Corollary A.1.2 and the observation that G_R

places no weight on $(sup(T_L), 1)$ that, in equilibrium, $G_R(2b - p) = G_R(sup(T_L)),\ \forall\ p \in (2b - sup(T_R), sup(T_L))$.

- Then, independent of $sup(T_R) - inf(T_R) \lesseqgtr 4b$, $H_L^{'}(p)\big|_{p=sup(T_L)} < 0$ as long as $sup(T_L) > 0$. Thus, a strategy where $sup(T_L) > 0$ is a dominated strategy since the point $(0, H_L(0))$ certainly lies above the line connecting $(inf(T_L), H_L(inf(T_L))$ and $(sup(T_L), H_L(sup(T_L))$.

□

The following Corollary A.1.2 serves as an aid to better understand all subsequent proofs.

In any potential equilibrium, the probability of getting elected conditional on nature sampling $p_L = p$ for party L depends on $p \in [-1, min\{sup(T_R), b\})$ where $b > 0$, $sup(T_R) > 0$ and $inf(T_R) \geq -b$.[3] For illustrative purposes, I assume in the following Corollary A.1.2 that G_R admits a density $g_R(\cdot) > 0,\ \forall\ p \in (inf(T_R), sup(T_R))$ such that the antiderivative $\bar{G_R}(\cdot)$ of G_R is, in general, well defined and situations where G_R places an atom on $sup(T_R)$ or $inf(T_R)$ can be neglected as:

$$\lim_{p \to sup(T_R)^+} \bar{G_R}(p) = \lim_{p \to sup(T_R)^-} \bar{G_R}(p)$$

A similar argument holds if an atom is placed on $inf(T_R)$ and, likewise, for G_L.

Corollary A.1.2 is leveraged when deriving Proposition 1.1. However, already anticipating that Proposition 1.1 applies, I do not account for the possibility that the election ties with probability $Pr(p_R = p) > 0$ if voters sample identical positions from both parties' platforms in the exposition of $H_L(p)$.[4] For the special case where voters sample $p_L = p_R = 0$, which is still possible with Proposition 1.1, $Pr(p_R = p = 0) > 0$. However, the outlined

[3] Notice that any G_L with $sup(T_L) \geq min\{sup(T_R), b\}$ is a dominated strategy by a similar argument as in *Step 1* of the proof of Proposition 1.1. Likewise, $inf(T_R) \geq max\{inf(T_L), -b\}$. Furthermore, $sup(T_R) \geq 0$ and $inf(T_L) \leq 0$ by Corollary A.1.1.

[4] When deriving Proposition 1.1, *Step 1* rules out that situation constitutes an equilibrium.

Expression implicitly accounts for this possibility. This becomes evident when applying a similar argument for party L as outlined in *Step 1* of the proof of Proposition 1.6. For an arbitrarily small ϵ:

$$H_L(p=0) = P_R(p_R=0)\frac{1}{2} + (1-P_R(p_R=0))\frac{H_L(0-\epsilon) - P_R(p_R=0)F(0)}{1-P_R(p_R=0)} = H_L(0-\epsilon)$$

Given $F(p=0) = 1/2$ and the specified tie-breaking rule, the $P_R(p_R=0)$ terms cancel each other out.

Corollary A.1.2. *$H_L(p)$, the probability of winning the election conditional on nature sampling $p_L = p$, follows from Equation 1.3 in Section 1.2:*

If $-2b - inf(T_R) < 2b - sup(T_R)$ or $sup(T_R) - inf(T_R) < 4b$:

$$H_L(p) = \begin{cases} 0 & \text{if } p \leq -2b - sup(T_R) \\ F\left(\frac{sup(T_R)+p}{2}\right) - \frac{k}{2} * \left[\bar{G}_R(sup(T_R)) - \bar{G}_R(-2b-p)\right] & \text{if } -2b - sup(T_R) < p \leq -2b - inf(T_R) \\ F\left(\frac{sup(T_R)+p}{2}\right) + G_R(p)(1-2F(p)) - \frac{k}{2} * \left[\bar{G}_R(sup(T_R)) - \bar{G}_R(inf(T_R)) - 2\bar{G}_R(p)\right] & \text{if } -2b - inf(T_R) < p < 2b - sup(T_R) \\ 1 + G_R(p)(1-2F(p)) - \frac{k}{2} * \left[\bar{G}_R(2b-p) - \bar{G}_R(inf(T_R)) - 2\bar{G}_R(p)\right] & \text{if } 2b - sup(T_R) \leq p < 2b - inf(T_R) \\ 1 + G_R(p)(1-2F(p)) + 2\bar{G}_R(p) & \text{if } 2b - inf(T_R) \leq p \end{cases}$$

$$H_L'(p) = \begin{cases} 0 & \text{if } p \leq -2b - sup(T_R) \\ \frac{k}{2}\left[1 - G_R(-2b-p)\right] & \text{if } -2b - sup(T_R) < p \leq -2b - inf(T_R) \\ \frac{k}{2} - k * G_R(p) + g_R(p)(1-2F(p)) & \text{if } -2b - inf(T_R) < p < 2b - sup(T_R) \\ \frac{k}{2} * G_R(2b-p) - k * G_R(p) + g_R(p)(1-2F(p)) & \text{if } 2b - sup(T_R) \leq p < 2b - inf(T_R) \\ -k * G_R(p) + g_R(p)(1-2F(p)) & \text{if } 2b - inf(T_R) \leq p \end{cases}$$

If $-2b - inf(T_R) > 2b - sup(T_R)$ *or* $sup(T_R) - inf(T_R) > 4b$*:*

$$H_L(p) = \begin{cases} 0 & if\ p \leq -2b - sup(T_R) \\ F\left(\frac{sup(T_R)+p}{2}\right) - \frac{k}{2} * \left[\bar{G_R}\left(sup(T_R)\right) - \bar{G_R}\left(-2b - p\right)\right] & if \begin{array}{l} -2b - sup(T_R) \leq p \\ <2b - sup(T_R) \end{array} \\ 1 - \frac{k}{2}\left[\bar{G_R}\left(2b - p\right) - \bar{G_R}\left(-2b - p\right)\right] & if \begin{array}{l} 2b - sup(T_R) \leq p \\ < -2b - inf(T_R) \end{array} \\ 1 + G_R\left(p\right)\left(1 - 2F(p)\right) - \frac{k}{2} * \left[\bar{G_R}\left(2b - p\right) - \bar{G_R}\left(inf(T_R)\right) - 2\bar{G_R}\left(p\right)\right] & if \begin{array}{l} -2b - inf(T_R) \leq p \\ <2b - inf(T_R) \end{array} \\ 1 + G_R\left(p\right)\left(1 - 2F(p)\right) + 2\bar{G_R}\left(p\right) & if\ 2b - inf(T_R) \leq p \end{cases}$$

$$H_L^{'}(p) = \begin{cases} 0 & if\ p \leq -2b - sup(T_R) \\ \frac{k}{2}\left[1 - G_R\left(-2b - p\right)\right] & if \begin{array}{l} -2b - sup(T_R) \leq p \\ <2b - sup(T_R) \end{array} \\ \frac{k}{2}\left[G_R\left(2b - p\right) - G_R\left(-2b - p\right)\right] & if \begin{array}{l} 2b - sup(T_R) \leq p \\ < -2b - inf(T_R) \end{array} \\ \frac{k}{2} * G_R\left(2b - p\right) - k * G_R\left(p\right) + g_R\left(p\right)\left(1 - 2F(p)\right) & if \begin{array}{l} -2b - inf(T_R) \leq p \\ <2b - inf(T_R) \end{array} \\ -k * G_R\left(p\right) + g_R\left(p\right)\left(1 - 2F(p)\right) & if\ 2b - inf(T_R) \leq p \end{cases}$$

□

Given a non-degenerate function G_i that is a potential equilibrium candidate, let me define the expected utility of a simple *cdf* played by party L that places an atom of size α_L on $sup(T_L)$ and an atom of size $1 - \alpha_L$ on $inf(T_L) \leq sup(T_L)$. Given $|X_i| > 0$ and given the opponent's strategy G_R that yields a well defined $E_R(p) = \mu_R$ and given the reasoning from Lemmas 1.2 and Corollary A.1.1, the expected utility of such a best replying simple *cdf* for party L is:

$$\begin{aligned} EU_L = \alpha_L\Big[&H_L(sup(T_L))[X_L - (\alpha_L sup(T_L) + (1-\alpha_L)inf(T_L))] \\ &+ (1 - H_L(sup(T_L)))(X_L - \mu_R)\Big] \\ +(1-\alpha_L)\Big[&H_L(inf(T_L))[X_L - (\alpha_L sup(T_L) + (1-\alpha_L)inf(T_L))] \\ &+ (1 - H_L(inf(T_L)))(X_L - \mu_R))\Big] \end{aligned}$$

After simplifying, this expression becomes:

$$\begin{aligned} EU_L = X_L - \mu_R + &[\alpha_L H_L(sup(T_L)) + (1-\alpha_L)H_L(inf(T_L))] \\ &* (\mu_R - X_L - (1-\alpha_L)inf(T_L) - \alpha_L sup(T_L) + X_L) \end{aligned} \tag{A.1}$$

After differentiating Equation A.1 with respect to α_L, one finds that:

$$\begin{aligned} \frac{\partial EU_L}{\partial \alpha_L} = &[H_L(sup(T_L)) - H_L(inf(T_L))](\mu_R + 2\alpha_L(inf(T_L) - sup(T_L)) - inf(T_L)) \\ &+ (inf(T_L) - sup(T_L))H_L(inf(T_L)) \end{aligned} \tag{A.2}$$

After equating Equation A.2 with zero, one finds that EU_L has an extreme value at:

$$\alpha_L^{opt.} = \frac{\mu_R - inf(T_L)}{2(sup(T_L) - inf(T_L)} - \frac{H_L(inf(T_L))}{2(H_L(sup(T_L)) - H_L(inf(T_L)))} \tag{A.3}$$

By symmetry, $\alpha_R^{opt.}$ can be derived in a similar way for party R.

Corollary A.1.3. *$\alpha_L^{opt.}$ as defined in Equation A.3 is a unique solution for a best replying simple cdf G_i^* as defined in Lemma 1.4. An analogous unique solution $\alpha_R^{opt.}$ exists for party R.*

Proof: After twice differentiating Equation A.1 with respect to α_L and given

$inf(T_L) \leq 0$ by Lemma A.1.1, one finds that:

$$\frac{\partial^2 EU_L}{\partial^2 \alpha_L} = 2[H_L(sup(T_L)) - H_L(inf(T_L))](inf(T_L) - sup(T_L)) < 0$$

As $inf(T_L) < sup(T_L)$ by definition of a non-degenerate *cdf* and $H_L(sup(T_L)) - H_L(inf(T_L)) > 0$, which immediately follows from Proposition 1.1 and Equation A.1 and a similar argument as outlined in *Step 2* of the proof of Proposition 1.2, $\alpha_L^{opt.}$ constitutes a global maximum that does not violate Lemma 1.2 as long as the corresponding $E_L^{opt.} \geq X_L$.

By symmetry, $\alpha_R^{opt.}$ is defined analogously.

□

Corollary A.1.4. *In case Lemma 1.1 is satisfied and Lemma 1.2 is not obviously violated, but $\alpha_L^{opt.}$ from Equation A.3 yields an average program position where $E_L^{opt.}(p) < X_L$ for a given $inf(T_L)$ and $sup(T_L) \geq X_L$, party L should optimally switch to an alternative simple cdf G_L^* that places an atom of size $\alpha_L^* = \frac{X_L - inf(T_L)}{sup(T_L) - inf(T_L)} > \alpha_L^{opt.}$ on $sup(T_L) \geq X_L$ and an atom of size $1 - \alpha_L^*$ on $inf(T_L) \leq X_L$, which implies a corner solution where $E_L^*(p) = X_L$. Consequentially, $E_L^*(p) = X_L = E_L(p)$ in equilibrium, where $E_L(p)$ describes the average position of the underlying strategy G_i. Vice versa for party R.*

Proof: Notice first that with $sup(T_L) < X_L$ Lemma 1.2 is obviously violated. Otherwise, $E_L^{opt.}(p) < X_L$ can only be reached if (i) $\alpha_L^{opt.} \in [0, 1)$ *and* (ii) $inf(T_L) < X_L$ since otherwise $E_L^{opt.}(p) \geq X_L$.

Given $E_L^{opt.}(p) < X_L$, it follows from Lemma 1.2 that such a scenario cannot constitute an equilibrium. Furthermore, it follows from the proof of Corollary A.1.3 that $\frac{\partial EU_L(\alpha_L')}{\partial \alpha_L'} < 0 < \frac{\partial EU_L(\alpha_L'')}{\partial \alpha_L''}$, $\forall\ \alpha_L' > \alpha_L^{opt.} > \alpha_L''$.

Thus, EU_L is decreasing in α_L' for all $\alpha_L' > \alpha_L^{opt.}$ such that a corner solution where $\alpha_L^* = \frac{X_L - inf(T_L)}{sup(T_L) - inf(T_L)} \leq 1$ on $sup(T_L) \geq X_L$ and an atom of size $1 - \alpha_L^*$ on $inf(T_L) \leq 0$ is required for any simple *cdf* that places an atom on $sup(T_L) \geq X_L$ and an atom on $inf(T_L) < X_L$ to be a best reply.

Given the best-replying simple *cdf* as just outlined, it follows from a similar reasoning as in the proof of Lemma 1.4 that the underlying strategy G_L must also yield $E_L(p) = X_L$ to be an equilibrium candidate.

By symmetry, a similar argument applies for party R.

□

Proof of Corollary 1.2

Lemma 1.1 needs to mutually hold in equilibrium by the definition of a Nash Equilibrium. Notice that Lemma 1.1 is trivially satisfied if G_i is a degenerate distribution with all weight placed on p when there exists a non-vertical line running through the point $(p, H_L(p))$ such that the points $(p', H_L(p'))$, $\forall\ p' \neq p$ do not lie above this line.

Likewise, $sgn(E_i\,(p)) = sgn(X_i)$ need to mutually hold by Corollary A.1.1 as long as $E_i(p) \neq 0$, independent of G_i being degenerate or non-degenerate. If G_i is a non-degenerate distribution such that G_i^* as outlined in Lemma 1.4 is well defined and $|E_i^*(p)| \leq |X_i|$, it follows from Lemma 1.4 and Corollary A.1.3 that in equilibrium $E_i(p) = E_i^*(p)$. Otherwise, if $|E_i^*(p)| > |X_i|$, it follows from Lemma 1.2 that G_i^* cannot be a best-reply. Thus, Corollary A.1.4 applies such that $|E_i(p)| = |X_i|$ in equilibrium.

Considering that, by symmetry, similar arguments apply for the opposing party and that Corollaries A.1.3 and A.1.4 yield a unique solution for $\alpha_i^{opt.}$, it directly follows from Lemma 1.3 that $|E_i(p)| = |E_{j\neq i}(p)|$.

If $G_i(p)$ is a degenerate distribution such that G_i^* is not well-defined, $|E_i(p)| = |E_{j\neq i}(p)| \leq |X_i|$ directly follows from combining Lemma 1.2 and Lemma 1.3.

□

Proof of Proposition 1.2

Throughout the subsequent argument, I assume that Lemma 1.2 is not binding.

Given a best replying simple *cdf* that places an atom of size $\alpha_L^{opt.}$ on $p = 0$ and an atom of size $1 - \alpha_L^{opt.}$ on $p = -1$, it follows from Corollary A.1.3 and Equation A.3 that $\alpha_L^{opt.}$, which is optimally chosen by party L, is maximized and, therefore, $E_L^{max.}(p)$ is maximized for $H_L(-1) = 0$.

Assuming such adverse conditions, it follows from Corollary 1.2 and Lemma 1.3 that there cannot exist an equilibrium in these, or less adverse, conditions if $E_L^{max.}(p) < -\mu_R$. Given Corollary A.1.3:

$$E_L^{max.}(p) = -\left(1 - \frac{1+\mu_R}{2}\right) < -\mu_R \ \forall \ \mu_R < \frac{1}{3} \tag{A.4}$$

It is taken as a prerequisite from Lemma 1.1 and Corollary 1.1 that, in equilibrium, the points $(-1, H_L((-1))$ and $(0, H_L(0))$ may not lie above the line connecting $(inf(T_L), H_L((inf(T_L)))$ and $(sup(T_L), H_L(sup(T_L)))$. Notice, however, that for the above-outlined argument it does not matter whether $(-1, H_L((-1))$ and $(0, H_L(0))$ lie on or below this line. If one or both points lie below the line, it follows from Lemma 1.1 that the optimally chosen strategy achieves $E_L(p) < E_L^{max.}(p)$ without reducing the expected probability of winning compared to a simple *cdf* which places an atom on $p = 0$ and on $p = -1$.

By symmetry, a similar argument holds for party R. Thus, the result presented in Equation A.4 represents a lower bound on $|E_i(p)|$ in any equilibrium where Lemma 1.2 is not binding.

□

From here onwards, I take Proposition 1.1 as given when proofing the subsequent propositions. For instance, a statement like $inf(T_R) < 2b$ actually implies $0 \leq inf(T_R) < 2b$. Furthermore, $G_R(p), \bar{G}_R(p), g_R(p) = 0, \ \forall \ p < 0$.

If argued from the perspective of a specific party in the subsequent proofs, the reversed argument holds analogously from the opponent's perspective, given the assumed symmetry of the game. Statements on $H_R(p)$ can be understood after reversing Corollary A.1.2 to find $H_R(p)$. The distinct steps within each proof iteratively eliminate scenarios that cannot constitute an equilibrium.

Proof of Proposition 1.3

The proof relies on $b \in \left(0, \frac{1}{4}\right)$. In *Steps 1-6*, I consider the case where $b \in \left[\frac{1}{6}, \frac{1}{4}\right)$. The reasoning in the subsequent steps assumes that $b \in \left(0, \frac{1}{6}\right)$.

Step 1: There can only exist an equilibrium if $inf(T_R) \leq 2b$:

Suppose that $inf(T_R) > 2b$. Then, $H_L(p) = 1, \ \forall \ p \in [2b - inf(T_R), inf(T_R))$. Thus, party L wins with certainty when playing $p = 2b - inf(T_R) + \epsilon$ where ϵ is arbitrarily small. Therefore, any best-reply of L must yield at least $EU_L = X_L - 2b + inf(T_R) > X_L$, which results in a contradiction of Lemma 1.3 since EU_L is certainly greater than if mirroring-inverting G_R. By a similar argument, $sup(T_L) \geq -2b$.

Step 2: There can only exist an equilibrium if $sup(T_R) \geq 4b$.

Suppose that $sup(T_R) \in [inf(T_R), 4b)$. Then, $H_L^{''}(p) \geq 0, \ \forall \ p \in (-1, 2b - sup(T_R))$ and $H_L^{'}(p)\big|_{p=-1} > H_L^{'}(p)\big|_{p=2b-sup(T_R)}$ where $2b - sup(T_R) > -sup(T_R)$. Thus, the point $(-sup(T_R), H_L(-sup(T_R)))$ certainly lies below the line connecting the points $(-1, H_L(-1))$ and $(2b - sup(T_R), H_L(2b - sup(T_R)))$. Then, by Lemma 1.3, there cannot exist an equilibrium where $sup(T_R) \in [inf(T_R), 4b]$ since mirror-inverting such a strategy is a strictly dominated for party L as Lemma 1.1 is violated.

Step 3: In equilibrium, $\nexists \ p : p \in T_R, \ and \ p \in (1 - 4b, 1 - 2b)$:

Suppose that G_R is such that $\exists \ p : p \in T_R, \ and \ p \in (1 - 4b, 1 - 2b)$. Then, G_L may not place any weight on $(-1, 2b - sup(T_R))$ since $H_L^{''}(p) \geq 0, \ \forall \ p \in [-1, 2b - sup(T_R)]$ and $H_L^{'}(p)\big|_{p=-1} < H_L^{'}(p)\big|_{p=2b-sup(T_R)}$ such that Lemma 1.1 is violated otherwise. As $2b -$

$sup(T_R) > -4b$, party L must place an atom of size $(1-\alpha_L)$ on $p = -1$ by *Step 2* and, considering also *Step 1*, choose $sup(T_L) \geq -2b$ in any equilibrium.

Suppose first that $sup(T_L) > 4b - 1$ such that $sup(T_L) - inf(T_L) > 4b$. Then, $H_R^{'}(p_1) \leq H_R^{'}(p_2)$, $\forall\ 1-4b \leq p_1 < 2b - sup(T_L) < p_2 \leq 1-2b$. Furthermore, $H_R^{'}(p)\big|_{p=1-4b} < H_R^{'}(p)\big|_{p=1-2b}$ since G_L does not attribute any weight to $(-1, 2b-1)$, but some weight to $[4b-1, 0]$. Then, Lemma 1.1 is violated for any G_R if $\exists\ p : p \in T_R,\ and\ p \in (1-4b, 1-2b)$ since any such $(p, H_R(p))$ certainly lies below the line connecting $(1-4b, H_R(1-4b))$ and $(1-2b, H_R(1-2b))$, which cannot constitute an equilibrium by a similar argument as outlined in Lemma 1.1 and Corollary 1.1.

Suppose now that $sup(T_L) \in [-2b, 4b-1]$ such that $sup(T_L) - inf(T_L) \leq 4b$ and suppose further that G_L also attributes some weight to $[2b-1, 4b-1)$. Then, by a similar reasoning as above, G_R places an atom of size $1-\alpha_R$ on $sup(T_R) = 1$ and no weight on $(1-2b, 1)$. For $inf(T_R) \geq 1-4b$, there can exist scenarios where Lemma 1.1 holds for both parties. However, they cannot constitute an equilibrium (for $b \in (0, 1/4)$). I proof this claim in *Remark I* of *Step 5* of the proof of Proposition 1.4. If $inf(T_R) < 1-4b$ it follows from reversing the above reasoning for party R that attributing some weight to $(2b-1, 4b-1)$ is dominated for party L such that mirror-inverting G_R is again no best reply for L.

To conclude, there cannot exist an equilibrium where $\exists\ p : p \in T_R,\ and\ p \in (1-4b, 1-2b)$ since otherwise one of Lemmas 1.1, 1.3 or 1.4 is certainly violated.

Step 4: Given that $sup(T_R) \geq 4b$ and $inf(T_R) \leq 4b-1$, $sup(T_R) - inf(T_R) > 4b$, in equilibrium, as long as $inf(T_R) \neq 0$. This implies $inf(T_R) < 1-4b$ with certainty:

Suppose $sup(T_R) - inf(T_R) \leq 4b$ and $0 < inf(T_R) < 1-4b$, which implies $sup(T_R) < 1$. Then, by a similar argument as outlined in *Step 2*, any G_L that plays $p = -sup(T_R)$ is a dominated strategy as $(-sup(T_R), H_L(-sup(T_R))$ lies below the line connecting $(-1, H_L(-1)$ and $(2b - sup(T_R), H_L(2b - sup(T_R))$ such that Lemma 1.1 is contradicted. Thus, there cannot exist an equilibrium as Lemma 1.3 does not hold. If $inf(T_R) = 1-4b$ and $sup(T_R) - inf(T_R) < 4b$, a similar argument applies

However, for $inf(T_R) = 1-4b$ and $sup(T_R) - inf(T_R) = 4b$, which implies $sup(T_R) = 1$, G_R must, in equilibrium, place an atom of size α_R on $inf(T_R) = 1-4b$, given *Step 3*:

- If $\alpha_R > \frac{1}{2}$ party L best replies with a simple *cdf* which places an atom of size α_L on $sup(T_L) = 0$ and an atom of size $(1 - \alpha_L)$ on $inf(T_L) = -1$ since it follows from Corollary A.1.2 that all $H_L^{''}(p) \geq 0,\ \forall\ p \in (-1, 0)$ and $H_L^{'}(p)\big|_{p=-1} < H_L^{'}(p)\big|_{p=0}$ such that otherwise, Lemma 1.1 is contradicted. By Lemma 1.3, such a situation cannot constitute an equilibrium.

- If $\alpha_R \leq \frac{1}{2}$, it follows from Corollary A.1.3 that if party L also chooses $sup(T_L) = 4b - 1$, the best-replying α_L is minimized in case $H_L(-1)$ is relatively big. Ceteris paribus, $H_L(-1)$ is maximized if G_R places another atom of size $(1-\alpha_R)$ on $sup(T_R) = 1$. Assuming such adverse conditions, it follows from Corollary A.1.3 that $\alpha_L^{opt} = \frac{1-4b\alpha_R}{8b} + \frac{1}{8b} - (1 - \alpha_R)\frac{1}{2} > \frac{1}{2},\ \forall\ b < \frac{1}{4}$ such that there again cannot exist an equilibrium as Lemma 1.3 is certainly violated, given that $\alpha_R \leq \frac{1}{2}$.

Step 5: In equilibrium, $\nexists\ p : p \in (-1, 0),\ and\ p \in T_L$.

It follows from *Step 4* that $sup(T_R) - inf(T_R) \geq 4b$ and $inf(T_R) > 4b - 1$ in any potential equilibrium. Given also the reasoning in *Step 3*, G_R does not place any weight on the interval $(1 - 4b, 1 - 2b)$. Furthermore, $4b - 1 > -2b - inf(T_R),\ \forall\ b \in (1/6, 1/4)$ and any $inf(T_R) \leq 2b$ as required by *Step 1*.

Then, it follows from Corollary A.1.2 that, in equilibrium, $H_L^{'}(p) = \frac{k}{2}\alpha_R^{[0,1-4b]},\ \forall\ p \in [4b - 1, 0]$ where $\alpha_R^{[0,1-4b]}$ denotes the weight G_R places on the interval $[0, 1 - 4b]$. Furthermore, $H_L^{'}(p) = \frac{k}{2}\left(1 - \alpha_R^{[0,1-4b]}\right),\ \forall\ p \in [-1, 2b - 1]$.

Consequentially, in equilibrium, $(0, H_L(0))$ must lie on the same line as $(p, H_L(p))\,,\ \forall\ p \in T_L \setminus 0$ by the following argument: Since $H_L^{''}(p) = 0,\ \forall\ p \in [4b - 1, 0]$, either Lemma 1.1 is violated since $(0, H_L(0))$ lies above this line or $sup(T_L) = 4b - 1$ in any alternative scenario if all $(p, H_L(p))\,,\ \forall\ p \in (4b - 1, 0]$ lie below the line connecting $(4b - 1, H_L(4b - 1))$ and $(inf(T_L), H_L(inf(T_L)))$. Notice, however, that $sup(T_L) = 4b - 1$ cannot constitute an equilibrium, given *Step 4*.

From the last argument and Corollary A.1.2 it also follows that $\alpha_R^{[0,1-4b]} < 1/2$ cannot constitute an equilibrium as otherwise $(2b - 1, H_L(2b - 1)$ certainly lies above the line connecting $(inf(T_L), H_L(inf(T_L))$ and any point $(p, H_L(p))\,, \forall p \in [4b-1, 0]$, which violates

Lemma 1.1.

Furthermore, if the points $(p, H_L(p))$, $\forall\ p \in (-1, 2b-1]$ lie below the line connecting $(-1, H_L(-1))$ and $(0, H_L(0))$, no $p \in (-1, 2b-1]$ can be part of T_L. In this case, also all $(p, H_L(p))$, $\forall\ p \in [4b-1, 0)$ lie below the line connecting $(-1, H_L(-1))$ and $(0, H_L(0))$ since it follows from Corollary A.1.2 that $H_L^{''}(p) = 0,\ \forall\ p \in [4b-1, 0]$ and, with $\alpha_R^{[0,1-4b]} > 1/2$ $H_L^{'}(p_1) < H_L^{'}(p_2)$ where $1 \leq p_1 < 4b-1 < p_2 \leq 0$.

Thus, if I can show that $(0, H_L(0))$ lies above the line that connects all $(p, H_L(p))$, $\forall\ p \in (-1, 2b-1]$, there cannot exist an equilibrium where any $p \in (-1, 0)$ is also part of T_L. The proof continues by showing that this is the case.

Notice that it follows from the above-outlined argument that the straight line connecting all these points $(p, H_L(p))$, $\forall\ p \in (-1, 2b-1]$ has a slope (Δ) equal to:

$$\Delta = \frac{k}{2}\left(1 - \alpha_R^{[0,1-4b]}\right) \tag{A.5}$$

Let me now leverage a property of any measurable *cdf*: **Tonelli's Theorem**

Given G_R yields $E_R(p) = \mu_R$, μ_R can be defined in terms of $G_R(p)$ only. Given $inf(T_R) \geq 0$ and $\lim_{p \to inf(T_R)^-} G_R(p) = 0$ and $\lim_{p \to sup(T_R)^+} G_R(p) = 1$, μ_R is defined as:

$$\mu_R = \int_0^{sup(T_R)} 1 - G_R(p)dp = sup(T_R) - \int_{inf(T_R)}^{sup(T_R)} G_R(p)dp = sup(T_R) - \bar{G}_R(sup(T_R)) \tag{A.6}$$

Notice further that $\mu_R^{[0,1-2b]}$, which describes the conditional average of G_R on the $[0, 1-2b]$ interval, is equal to $\mu_R^{[0,1-4b]}$,which also describes the conditional average of G_R on the $[0, 1-4b]$ interval as, in equilibrium, G_R does not attribute any weight to $(1-4b, 1-2b]$ by *Step 3*. Likewise, $\alpha_R^{[0,1-4b]}$ describes the entire weight attributed to $[0, 1-2b]$ by G_R. After, again, leveraging Tonelli's Theorem, one finds:

$$\mu_R^{[0,1-4b]} = \mu_R^{[0,1-2b]} = \int_0^{1-2b} 1 - \frac{G_R(p)}{\alpha_R^{[0,1-4b]}}dp = 1 - 2b - \frac{\bar{G}_R(1-2b)}{\alpha_R^{[0,1-4b]}} \tag{A.7}$$

After rearranging (A.6) and (A.7), one finds:

$$\bar{G_R}(sup(T_R)) = sup(T_R) - \mu_R \ \ and \ \ \bar{G_R}(1-2b) = \alpha_R^{[0,1-4b]}(1-2b-\mu_R^{[0,1-4b]}) \tag{A.8}$$

$H_L(-1)$ follows from Corollary A.1.2:

$$H_L(-1) = F\left(\frac{sup(T_R)-1}{2}\right) - \frac{k}{2}[\bar{G_R}(sup(T_R)) - \bar{G_R}(1-2b)] \tag{A.9}$$

After substituting $\bar{G_R}(sup(T_R))$ and $\bar{G_R}(1-2b)$ (from A.8) into (A.9):

$$H_L(-1) = \frac{k}{2}(sup(T_R)-1) + \frac{1}{2} - \frac{k}{2}\left[sup(T_R) - \mu_R - \alpha_R^{[0,1-4b]}(1-2b-\mu_R^{[0,1-4b]})\right] \tag{A.10}$$

Given also the slope Δ (from A.5), I can back out the function $m(p)$ that describes the entire straight line that connects all $(p, H_L(p))$, $\forall \ p \in [-1, 2b - sup(T_R)]$. $m(p)$ is given by:

$$m(p) = \frac{k}{2}\left(1-\alpha_R^{[0,1-4b]}\right) * p + \frac{1}{2} - \frac{k}{2}\left(\alpha_R^{[0,1-4b]}(2b+\mu_R^{[0,1-4b]}) - \mu_R\right) \tag{A.11}$$

$H_L(0)$ also follows from Corollary A.1.2:

$$H_L(0) = 1 - \frac{k}{2}\left(\alpha_R^{[0,1-4b]}(2b - \mu_R^{[0,1-2b]})\right) \tag{A.12}$$

Given $m(p)$ from Equation A.11 and $H_L(0)$ from Equation A.12, $H_L(0)$ lies above the line $m(p)$ if:

$$1 - \frac{k}{2}\left(\alpha_R^{[0,1-4b]}(2b - \mu_R^{[0,1-4b]})\right) > \frac{1}{2} - \frac{k}{2}\left(\alpha_R^{[0,1-4b]}(2b+\mu_R^{[0,1-4b]}) - \mu_R\right) \tag{A.13}$$

With $\frac{k}{2} = \frac{1}{4b}$ it follows after rearranging that Inequality A.13 holds if:

$$\mu_R < 2b + 2\alpha_R^{[0,1-4b]}\mu_R^{[0,1-2b]} \tag{A.14}$$

Step 6: Notice that $\mu_R < 2b$ as, by assumption, $|X_i| < 2b$ such that $\mu_R \geq 2b$ would violate Lemma 1.2. Therefore, it immediately follows from Inequality (A.14) and the reasoning in *Step 5* that in any potential equilibrium all $(p, H_L(p))$, $\forall\ p \in (0,1)$ lie below the line connecting $(-1, H_L(-1))$ and $(0, H_L(0))$.

Thus, in equilibrium, G_R necessarily needs to be a simple *cdf* that places an atom of size α_R on $p = 0$ and an atom size $1 - \alpha_R$ on $p = 1$.

If G_R is such a simple *cdf*, it follows from Corollary A.1.3 that $\alpha_L^{opt.} = 1/2$, which violates Lemma 1.2 as $E_L^{opt.} = -(1/2) < -2b < X_L,\ \forall\ b \in [1/6, 1/4)$ (and also $\forall\ b \in (0, 1/6)$).

Consequentially, Corollary A.1.4 applies such that only a 'corner solution' where $|E_i(p)| = |X_i| < 2b$ can mutually satisfies Lemmas 1.4 and 1.2.

By symmetry, analogous arguments apply for party R. As a consequence, only a scenario where both parties play a simple *cdf* where G_L places an atom of size $(1 - \alpha_L) = -X_L$ on $p = -1$ and an atom of size $\alpha_L = X_L + 1$ on $p = 0$ and vice versa for G_R can constitute an equilibrium. Uniqueness immediately follows from the outlined arguments.

Step 7: Let me now consider the case where $b \in (0, \frac{1}{6})$. By Proposition 1.2, there cannot exist an equilibrium that is not a corner solution as defined in Corollary A.1.4 if $\mu_R < 1/3$. By assumption, however, $1/3 > 2b > |X_i|\ \forall\ b \in (0, 1/6)$ such that Lemma 1.2 is violated for any $|E_i(p)| \geq 1/3$.

Then, the same strategy profile as outlined in *Step 6* again constitutes an equilibrium if $b \in (0, 1/6)$ as the arguments in *Step 5* and *Step 6* hold analogously for $b \in (0, 1/6)$.

However, the question whether there exist alternative equilibria remains open. If such alternative equilibria exist, Proposition 1.2 requires that these also constitute a 'corner solution' where $|E_i(p)| = |X_i|$, given the just outlined reasoning.

□

Proof of Proposition 1.4

The proof relies on $b \in \left[\frac{1}{4}, \frac{1}{3}\right]$ such that $sup(T_i) - inf(T_i) < 4b$ in any potential equilibrium by Proposition 1.1. To streamline exposition, the following arguments consider $b \in \left(\frac{1}{4}, \frac{1}{3}\right)$. The characteristic equilibrium *cdf* converges to the equilibrium outlined in Proposition 1.3

in the limiting scenario where $b = 1/4$.

Notice that *Step 1* of the proof of Proposition 1.3 also holds in this scenario such that $inf(T_R) \leq 2b$ and $sup(T_L) \geq -2b$.

The reasoning in the subsequent *Steps 1-5* implicitly assumes that, in equilibrium, $|E_i(p)| \leq |X_i|$ such that Lemma 1.2 is not violated.

Step 1: In equilibrium, $sup(T_R) \geq 2b$:

Suppose that $0 < sup(T_R) < 2b$[5], which implies $2b - sup(T_R) > 0 \ \forall \ sup(T_R) < 2b$. It follows from Corollary A.1.2 that $H_L''(p) \geq 0, \ \forall \ p \in (-1, 0)$ where ϵ is arbitrarily small and $H_L'(p)\big|_{p=-1} < H_L'(p)\big|_{p=0}$ such that $(-sup(T_R), H_L(-sup(T_R))$ certainly lies below the line connecting $(0, H_L(0))$ and $(-1, H_L(-1))$, which constitutes a contradiction of Lemma 1.3.

Step 2: In equilibrium, G_R assigns an atom of size $(1 - \alpha_R)$ to $sup(T_R) = 1$ and no weight to $(1 - 2b, 1)$:

First, let me show that $sup(T_R) = 1$. Suppose that $sup(T_R) \in (2b, 1)$. Given $inf(T_R) \geq 0$ by Proposition 1.1, it follows from Corollary A.1.2 that $H_L''(p) \geq 0, \ \forall \ p \in (-1, 2b - sup(T_R))$ and $H_L'(p)\big|_{p=-1} < H_L'(p)\big|_{p=2b-sup(T_R)}$. In this case, any G_L that assigns some weight to $p \in (-1, 2b - sup(T_R))$ is a dominated strategy for party L. As a consequence, Lemma 1.3 is violated as $2b - sup(T_R) > -sup(T_R)$, which implies, that there can only exist an equilibrium if $sup(T_R) = 1$.

Given that this is the case, it follows from the exact same reasoning that G_R may not place any weight on any $p \in (2b, 1)$. As a consequence, there can only exist an equilibrium if G_R assigns an atom of size $(1 - \alpha_R)$ to $sup(T_R) = 1$ and no weight to $(1 - 2b, 1)$.

Notice that by Lemma 1.3 similar arguments must hold analogously for party L such that $inf(T_L) = -1$ and no weight is placed on $(-1, 2b - 1)$.

Taking this as given and additionally considering Proposition 1.1, G_R can place the remaining weight equal to α_R on some $p : 0 \leq p \leq 1 - 2b$ or randomize over (a subset of) $[0, 1 - 2b]]$.

[5] Notice that $sup(T_R) \leq 0$ would certainly violate Proposition 1.2.

Step 3: In equilibrium, the points $(p, H_L(p)),\ \forall\ p \in [2b-1, 0]$ lie on a straight line:

Given $-2b \leq 2b-1,\ \forall\ b \in [1/4, 1/3)$, it follows from Corollary A.1.2 that $H_L^{'}(p) = \frac{k}{2}\alpha_R,\ \forall\ p \in [2b-1, 0]$. Thus, all $(p, H_L(p)),\ \forall\ p \in [2b-1, 0]$ certainly lie on a straight line with a slope equal to:

$$\Delta_1 = \frac{k}{2}\alpha_R \tag{A.15}$$

Step 4: In equilibrium, $\alpha_R = \frac{2b}{1-\mu_R^{[0,1-2b]}}$:

Let me define $\mu_L^{[2b-1,0]} = \int\limits_{2b-1}^{0} p\ dG_L(p)$. $\mu_R^{[0,1-2b]}$ is defined analogously. Furthermore, let $E_i(p) = \mu_R$.

As in *Step 5* of the proof of Proposition 1.3, it follows after leveraging Tonelli's Theorem from Corollary A.1.2 that:

$$H_L(-1) = \frac{k}{2}\left(sup(T_R) - 1\right) + \frac{1}{2} - \frac{k}{2}\left[sup(T_R) - \mu_R - \alpha_R(1 - 2b - \mu_R^{[0,1-2b]})\right] \tag{A.16}$$

$$H_L(2b-1) = 1 - \frac{k}{2}\left[\bar{G}_R(sup(T_R))\right] = 1 - \frac{k}{2}\left[1 - \mu_R\right] \tag{A.17}$$

Thus, with $sup(T_R) = 1$ from *Step 2* one finds that the slope (Δ_2) of the line connecting the points $(-1, H_L(-1))$ and $(2b-1, H_L(2b-1))$ is given by:

$$\Delta_2 = \frac{H_L(2b-1) - H_L(-1)}{2b} = \frac{\frac{1}{2} - \frac{k}{2}\left[\alpha_R(1 - 2b - \mu_R^{[0,1-2b]})\right]}{2b} \tag{A.18}$$

Considering also the results from *Step 3*, it follows from a straightforward argument that $(2b-1, H_L(2b-1))$ lies above the line connecting $(-1, H_L(-1))$ and $(sup(T_L), H_L(sup(T_L)))$ if:

$$\Delta_1 < \Delta_2 \rightarrow \alpha_R < \frac{2b}{1 - \mu_R^{[0,1-2b]}} \tag{A.19}$$

Thus, whenever Condition A.19 is satisfied, there can only exist an equilibrium if $sup(T_L) \leq 2b - 1$ as otherwise Lemma 1.1 is violated. Notice that by Lemma 1.3 a similar argument must hold for party R. A straightforward optimization, which is outlined in detail in the proof of Proposition 1.4, shows that there cannot exist an equilibrium in such case for $b \in [1/4, 1/3)$.

It follows from the reversed reasoning that all $(p, H_L(p)),\ \forall\ p \in (-1, 0)$ lie below the line connecting $(-1, H_L(-1))$ and $(0, H_L(0))$ if $\Delta_1 > \Delta_2$. In this case, $sup(T_L) = 0$ in any equilibrium since otherwise Lemma 1.1 is violated.

It follows from Lemma 1.3 that the same must be the case for party R such that G_R is a simple *cdf* that places an atom of size α_R on $inf(T_R) = 0$ and an atom of size $(1 - \alpha_R)$ on $sup(T_R) = 1$ and no weight in between.

It follows from Corollary A.1.3 that, given G_R as just outlined, a best-replying simple *cdf* of L places an atom of size $\frac{1}{2}$ on $inf(T_L) = -1$ and an atom of size $\frac{1}{2}$ on $sup(T_L) = 0$. However, $\frac{1}{2} < 2b = \frac{2b}{1+\mu_L^{[2b-1,0]}}$ with $\mu_L^{[2b-1,0]} = 0$. A similar argument holds for party R such that the initial assumption of $\alpha_R > \frac{2b}{1-\mu_R^{[0,1-2b]}}$ is contradicted.

It also immediately follows from the above-outlined arguments that there can only exist an equilibrium if $\alpha_R = \frac{2b}{1-\mu_R^{[0,1-2b]}}$ such that all points $(p, H_L(p)),\ \forall\ p \in [2b-1, 0)$ lie on the same straight line that connects $(-1, H_L(-1))$ and $(0, H_L(0))$ and the points $(p, H_L(p)),\ \forall\ p \in [-1, 2b-1)$ lie below this line. A similar argument holds for G_L.

Step 5: In equilibrium, $E_R(p) = 1 - 2b = -E_L(p)$:

Given the results from the previous steps and give $k = 1/2b$, it follows with $sup(T_R) = 1$ and $\mu_R = (1 - \alpha_R) + \alpha_R \mu_R^{[0,1-2b]}$ that $H_L(-1)$ outlined in Equation A.16 simplifies to:

$$\begin{aligned} H_L(-1) &= \frac{1}{2} - \frac{k}{2}\left[1 - (1-\alpha_R) - \alpha_R\mu_R^{[0,1-2b]} - \alpha_R(1 - 2b - \mu_R^{[0,1-2b]})\right] \\ &= \frac{1}{2}(1-\alpha_R) \end{aligned} \tag{A.20}$$

Furthermore, it follows from Corollary A.1.2 that $H_L(0)$ is given by:

$$H_L(0) = 1 - \frac{k}{2}\bar{G}_R(2b) \tag{A.21}$$

Since it is known from *Step 2* that, in equilibrium, G_R does not assign any weight to

$(1-2b, 2b]$ and randomizes with a weight equal to α_R over $[0, 1-2b]$ Equation A.21 can be rewritten as:

$$H_L(0) = 1 - \frac{k}{2}\bar{G}_R(2b) = 1 - \frac{k}{2}\left[\bar{G}_R(1-2b) + \alpha_R(4b-1)\right] \tag{A.22}$$

With $\bar{G}_R(1-2b)$ and α_R being known from the previous step, Equations A.20 and A.22 become:

$$H_L(-1) = \frac{1}{2}\left(1 - \frac{2b}{1-\mu_R^{[0,1-2b]}}\right) \tag{A.23}$$

$$\begin{aligned} H_L(0) = 1 - \frac{k}{2}\bar{G}_R(2b) &= 1 - \frac{k}{2}\left[\alpha_R(1-2b-\mu_R^{[0,1-2b]}) + \alpha_R(4b-1)\right] \\ &= 1 - \frac{1}{2}\frac{2b-\mu_R^{[0,1-2b]}}{1-\mu_R^{[0,1-2b]}} \end{aligned} \tag{A.24}$$

With (A.23) and (A.24), it follows from Corollary A.1.3 that, given G_R, a best replying simple *cdf* G_L^* that places an atom of size $\alpha_L^{opt.}$ on $p = 0$ and an atom of size $1-\alpha_L^{opt.}$ on $p = -1$ optimally chooses:

$$\alpha_L^{opt.} = \frac{\mu_R+1}{2} - \frac{\frac{1}{2}\left(1-\frac{2b}{1-\mu_R^{[0,1-2b]}}\right)}{1+\frac{\mu_R^{[0,1-2b]}}{1-\mu_R^{[0,1-2b]}}} = \frac{\mu_R+1}{2} - \frac{1-\mu_R^{[0,1-2b]}-2b}{2} \tag{A.25}$$

Given $\alpha_L^{opt.}$, it follows from Lemma 1.4 that $E_L(p) = \mu_L = -(1-\alpha_L^{opt.})$. Lemma 1.3 consequentially requires: $-\mu_L = (1-\alpha_L^{opt.}) = \mu_R$. Therefore:

$$\mu_R \overset{!}{=} \frac{2-2b-\mu_R^{[0,1-2b]}}{3} \tag{A.26}$$

Also, given α_R as defined in previous step, the average program position must, by definition, be equal to:

$$\mu_R = \frac{1-2b-\mu_R^{[0,1-2b]}}{1-\mu_R^{[0,1-2b]}} + \frac{2b\mu_R^{[0,1-2b]}}{1-\mu_R^{[0,1-2b]}} \tag{A.27}$$

After equating the right-hand-sides of Equations A.26 and A.27, one finds that they mutually hold for:

$$\mu_R^{[0,1-2b]} = 4b - 1 \tag{A.28}$$

It, therefore, follows immediately from the previous step that, in equilibrium, $\alpha_R = \frac{2b}{2-4b}$. Given Lemma 1.3, Equation A.28 and $\alpha_R = \frac{2b}{2-4b}$:

$$\begin{aligned} -E_L(p) = E_R(p) = \mu_R = (1-\alpha_R) + \alpha_R \mu_R^{[0,1-2b]} &= \frac{2-6b}{2-4b} + \frac{2b(4b-1)}{2-4b} \\ \frac{2-8b+8b^2}{2-4b} = \frac{(1-2b)(2-4b)}{2-4b} &= 1-2b \end{aligned} \tag{A.29}$$

To conclude, any strategy G_R with $\mu_R^{[0,1-2b]} = 4b-1$, $\alpha_R = \frac{2b}{2-4b}$, $sup(T_R) = 1$, where $\nexists\ p : p \in (1-2b, 1)$ *and* $p \in T_R$ and that yields $\mu_R = \frac{2b(4b-1)}{2-4b} + \frac{2-6b}{2-4b} = 1-2b$ is an equilibrium strategy.

By symmetry, analogous arguments hold for G_L

Remark I: I still owe the argument that the above-outlined strategies do not constitute an equilibrium in *Step 3* of the proof of Proposition 1.3 if $sup(T_L) \in [-2b, 4b-1]$ and $inf(T_R)[1-4b, 2b]$. If $\frac{1}{6} \leq b < \frac{1}{4}$, $\mu_R = 4b - 1 < 0$ (and vice versa for party L) such that the outlined strategies would violate Proposition 1.1.

Remark II: This is a forward looking statement concerning the argument presented in *Step 3* of the proof of Proposition 1.5. To understand why the above-outlined strategies do not constitute an equilibrium if $\frac{1}{3} < b \leq \frac{1}{2}$ notice that it would require $\alpha_R = \frac{2b}{2-4b} > 1$, which is not feasible with $\alpha_R \in [0, 1]$.

Step 6: If, however, $|X_i| < 1-2b$, any pair of strategies as described in the previous step violates Lemma 1.2. Corollary A.1.4 requires that there now exists a 'corner solution'. Notice, however, that *Steps 1 & 2* still apply. By an identical reasoning as in *Steps 3 & 4*, a corner solution can only be achieved for a pair of strategies that place an atom of size $1-\alpha_i = |X_i|$ on $inf(T_L) = -1$ (party L) and on $sup(T_R) = 1$ (party R) and an atom of size $\alpha_i = 1-|X_i|$ on $p = 0$. With $1-|X_i| > 2b$, $\forall\ b \in \left(\frac{1}{4}, \frac{1}{3}\right)$ and $|X_i| < 1-2b$, the condition $\Delta_1 > \Delta_2$ stated in *Step 4*, which is necessary for Lemma 1.1 to hold, is satisfied since $1 - |X_i| > 2b$.

In the limit, where $|X_i| = 1 - 2b$, the equilibria outlined in *Steps 5 & 6* mutually co-exist as all $(p, H_L(p))$, $\forall\ p \in [-1, 0]$ lie on a straight line if G_R places an atom of size $2b$ on $p = 0$ and another atom of size $1 - 2b$ on $p = 1$. A similar argument applies for party L.

In case $b = \frac{1}{4}$ the equilibria outlined here and in Proposition1.3 converge.

□

Proof of Proposition 1.5

The proof relies on $b \in \left(\frac{1}{3}, \frac{1}{2}\right)$ in *Steps 1-7*. The proof relies on $b \in \left[\frac{1}{2}, 1\right]$ in *Steps 8-10*. In both cases, it is ensured that $sup(T_i) - inf(T_i) < 4b$ in any potential equilibrium by Proposition 1.1.

In general, it is assumed that, in equilibrium, $|E_i(p)| \leq |X_i|$ such that Lemma 1.2 is not violated. In *Steps 7 & 10*, this assumption is relaxed.

Step 1: The reasoning in this first step is very general. Let me abstract for the following argument form Proposition 1.1. If $0 \leq inf(T_R) < 1 - 2b$, it follows from Corollary A.1.2 that party $L's$ best-reply is characterized by:

- $2b - 1 \leq min\{2b - sup(T_R), inf(T_R) - \epsilon\} \leq sup(T_L)$ where ϵ is arbitrarily small:

  Suppose $sup(T_L) < min\{2b - sup(T_R), inf(T_R) - \epsilon\}$. Then, party L is always better off shifting the weight from $sup(T_L)$ in a mean-preserving way to -1 and some $p^* \geq min\{2b - sup(T_R), inf(T_R) - \epsilon\}$ since it follows from Corollary A.1.2 that $H_L^{'}(p)$, $H_L^{''}(p) \geq 0$ where $p < min\{2b - sup(T_R), inf(T_R) - \epsilon\}$ and $H_L^{'}(p)\big|_{p=-1} < H_L^{'}(p)\big|_{p=min\{2b-sup(T_R), inf(T_R)-\epsilon\}}$ such that otherwise Lemma 1.1 is violated.

  $2b-1 \leq min\{2b-sup(T_R), inf(T_R)-\epsilon\}$ follows for all $b \in (1/3, 1/2)$ as $sup(T_R) \leq 1$ and $inf(T_R) \geq 0$.

- By a similar argument, $\nexists\ p \in (-1, min\{2b - sup(T_R), inf(T_R) - \epsilon\})$ that is also part of T_L. As a consequence, G_L places an atom of size $1 - \alpha_L$ on $inf(T_L) = -1$ if G_L places any weight on $[-1, min\{2b - sup(T_R), inf(T_R) - \epsilon\})$.

Step 2: If $0 < inf(T_R) < 1-2b$, Lemma 1.3 requires that $0 > sup(T_L) = -inf(T_R) > 2b - 1$ is a best reply. However, there cannot exist an equilibrium in such case.

If $inf(T_R) > 0$, $sup(T_R) \geq 2b$ is necessary for an equilibrium to exist since otherwise, $sup(T_L) > 0$ by *Step 1*, which contradicts Proposition 1.1. By a similar argument, $inf(T_L) \leq 2b$.

Given the (reversed) reasoning of *Step 1*, there can only exist an equilibrium where parties L and R place an atom of size $1 - \alpha_L$ on $p = -1$ / $1 - \alpha_R$ on $p = 1$. Furthermore, G_L does not assign any weight to $(-1, 2b - 1)$ and G_R to $(1 - 2b, 1)$.

Given such a pair of strategies, it follows from Corollary A.1.2 that $H_L^{'}(p_1) \geq \frac{k}{2}(1 - \alpha_R), H_L^{'}(p_2) = \frac{k}{2}$ and $H_L^{'}(p_3) = \frac{k}{2}\alpha_R, \ \forall \ -1 \leq p_1 < -2b - inf(T_R) < p_2 < 2b - 1 < p_3 \leq 0$.

It follows from a similar argument as outlined in Proposition 1.4 that a situation where $0 > sup(T_L) > 2b - 1$ can only constitute an equilibrium if all points $(p, H_L(p)), \ \forall \ p \in \{[2b - 1, 0] \cup \{-1\}\}$ lie on a straight line since otherwise either $(2b - 1, H_L(2b - 1))$ or $(0, H_L(0))$ lies above this line such that Lemma 1.1 is violated. A similar argument must hold for party R.

It, however, follows from of the proof of Proposition 1.4 that this requirement can never be satisfied with $b \in (1/3, 1/2)$ as stated in *Remark II* outlined in *Step 5*.

Step 3: There cannot exist an equilibrium if $inf(T_R) = 0$ and $sup(T_R) < 2b$:

Given the reasoning in *Steps 1 and 2* and Proposition 1.1, $inf(T_R) = 0$ is the only viable equilibrium candidate for a scenario where $inf(T_R) < 1 - 2b$ and $sup(T_R) < 2b$. Notice that it also follows immediately from Proposition 1.2 that $0 < sup(T_R) < 2b$.

Again by the initial reasoning in *Step 1*, $inf(T_L) = -1$ and G_L places no weight on $(-1, 0)$. Thus, G_L places an atom of size α_L on $p = 0$ and an atom of size $1 - \alpha_L$ on $p = -1$ such that Lemma 1.3 is certainly violated for $inf(T_R) = 0$ and $0 < sup(T_R) < 2b$.

Step 4: In equilibrium, $1 - 2b \leq inf(T_R) < 2b$:

Given the reasoning in *Steps 1-4* and Lemma 1.3, there remains one last equilibrium candidate for $inf(T_R) < 1 - 2b$, which needs to be ruled out:

1. $inf(T_R) = 0$ and G_R places an atom of size α_R on $inf(T_R) = 0$ and an atom of size $(1 - \alpha_R)$ on $sup(T_R) = 1$

2. $sup(T_L) = 0$ and G_L places an atom of size α_L on $sup(T_L) = 0$ and an atom of size $(1 - \alpha_L)$ on $inf(T_L) = -1$

By a similar argument as outlined in *Step 4* of the proof of Proposition 1.4, such a strategy profile can only constitute an equilibrium if $\alpha_R \geq 2b \geq 2/3, \ \forall \ b \in [1/3, 1/2)$ since otherwise the point $(2b-1, H_L(2b-1))$ certainly lies above the line connecting $(-1, H_L(-1))$ and $(0, H_L(0))$, which would contradict Lemma 1.1.

However, it, again, follows from Corollary A.1.3 that, given G_R as just outlined, a best-replying simple *cdf* of L places an atom of size $\frac{1}{2}$ on $inf(T_L) = -1$ and an atom of size $\frac{1}{2}$ on $sup(T_L) = 0$ and vice versa for party R, given the symmetry. Notice that $\frac{1}{2} < 2b, \ \forall \ b \in [1/3, 1/2)$.

Thus, by a similar argument as in *Step 4* of the proof of Proposition 1.4, the outlined strategy profile cannot constitute an equilibrium as Lemma 1.4 is, again, violated.

To summarize *Steps 1-4*,in any equilibrium, $1 - 2b \leq inf(T_R) < 2b$ and vice versa for $sup(T_L)$.

Step 5: Given *Steps 1-4*, the linearity of $H_i(p)$ does not depend on $G_{j\neq i}$:

As a direct consequence of *Step 4*, $-2b - inf(T_R) \leq p \leq 2b - sup(T_R), \ \forall \ p \in [-1, 2b - 1]$. Thus, it follows from Corollary A.1.2 that the linearity of $H_L(p)$ required for Lemma 1.1 to hold does not depend on G_R since $\bar{G}_R\left(sup(T_R)\right) - \bar{G}_R\left(inf(T_R)\right)$ is constant such that $H_L(p)$ is linear for all $p \in [-1, 2b - 1]$ as $F(\cdot)$ is a linear function by definition.

Step 6: In equilibrium, $E_R(p) = b$:

Lemma 1.4 is satisfied if G_L yields the same $E_L(p) = \mu_L = E_L^*$ as a best replying simple *cdf* G_L^*, given the opponent's strategy generates $E_R(p) = \mu_R$.

I continue by assuming that G_L^* places an atom of size α_L on $2b-1$ and an atom of size $(1-\alpha_L)$ on -1, which is without loss of generality, given the result of *Step 5*.

Let me again leverage Tonelli's Theorem:[6]

$$\frac{H_L(-1)}{2\left(H_L(2b-1)-H_L(-1)\right)} = \frac{\mu_R - 1 + 2b}{4b} \tag{A.30}$$

After substituting this expression into Corollary A.1.3 the optimal $\alpha_L^{opt.}$ is equal to:

$$\alpha_L^{opt.} = \frac{\mu_R + 1}{4b} - \frac{\mu_R - 1 + 2b}{4b} = \frac{2-2b}{4b} \tag{A.31}$$

It follows from Lemma 1.4 that $E_L^*(p) = \alpha_L^{opt.}(2b-1) - (1-\alpha_L^{opt.}) = E_L(p)$ needs to be satisfied. Given $\alpha^{opt.}$ as just outlined, it therefore follows that: $E_L^*(p) = \alpha_L^{opt.}(2b-1)-(1-\alpha_L^{opt.}) = E_L(p) = -b$, which is independent of μ_R. A similar argument applies for party R such that, in equilibrium, $E_R(p) = b$.

Since $-b \in [-1, 2b-1],\ \forall\ b \in \left[\frac{1}{3}, \frac{1}{2}\right)$, it immediately follows from *Steps 1-4* that any strategy profile (G_L, G_R) constitutes an equilibrium if and only if $-E_L(p) = E_R(p) = b$ and G_L, G_R randomize over the interval $[-1, 2b-1]$ (party L) / $[1-2b, 1]$ (party R) or a subinterval thereof. Notice that also an equilibrium in degenerate distributions is possible where all weight is placed on $p = -b$ (party L) / $p = b$ (party R).

Notice that the above-outlined equilibrium and the equilibrium outlined in Proposition 1.4 converge for $b = \frac{1}{3}$.

Step 7: By Corollary A.1.4, only a corner solution where $|E_i(p)| = |X_i|$ mutually satisfies Lemma 1.4 and 1.2 if $|X_i| < b$. Yet, *Steps 1-5* from above continue to hold.

If $|X_i| \in [1-2b, b)$, only a pair of strategies satisfying (i) $|E_i(p)| = |X_i|$ and (ii) $-sup(T_L), inf(T_R) \geq 1-2b$ constitutes an equilibrium: (i) guarantees a corner solution and with (ii) Lemma 1.1 again implicitly holds with a similar argument as outlined in *Step 5*.

[6] The relevant information to derive this result follows from Equations A.16 and A.17 in *Step 4* of the proof of Proposition 1.4.

If $|X_i| < 1-2b$, there does not exist a strategy combination with $-sup(T_L), inf(T_R) \geq 1-2b$ where $|E_i(p)| = |X_i|$. By the identical reasoning as applied in *Step 4*, there can only exist an equilibrium with a pair of strategies which place an atom of size $1 - |X_i|$ on $p = 0$ and an atom of size $|X_i|$ on $sup(T_R) = 1$ (party R) / $inf(T_R) = -1$ (party L). The outlined pait of strategies (i) yield a corner solution and (ii) Lemma 1.1 is also satisfied, which folllows from reversing the reasoning outlined in *Step 4* since $1 - |X_i| > 2b$ if $|X_i| < 1 - 2b$. The uniqueness of the equilibrium in this scenario follows from a similar argument as in Proposition 1.3.

The proof for $b \in [1/3, 1/2)$ is now complete. The following steps consider $b \geq 1/2$.

Step 8: Given Proposition 1.1, Lemma 1.1 holds independent of the opponent's strategy by a similar argument as applied in *Step 5*.

Step 9: Given the opponent's strategy yielding a well defined $E_R(P) = \mu_R$, Lemma 1.4 is satisfied for any distribution function G_L with a well defined support T_L, which yields the same $E_L(p)$ as a best replying simple *cdf* G_L^*. Without further restrictions on G_R and T_L, I continue with assuming that G_L^* places an atom of α_L on 0 and an atom of size $(1-\alpha_L)$ on -1, which is without loss of generality, given the result in *Step 8*.

From here onward the proof follows the same logic as in *Steps 6 and 7* of the proof of Proposition 1.5. After conducting similar steps one finds that:

$$\alpha_L^{opt.} = \frac{\mu_R + 1}{2} - \frac{\mu_R - 1 + 2b}{2} = 1 - b$$

Thus, any distribution G_L with $E_L(p) = (1 - \alpha_L^{opt.}) = -b$ satisfies Lemma 1.4. By a similar reasoning, any distribution G_R with $E_L(p) = b$ also satisfies Lemma 1.4.

Given *Step 8*, it immediately follows that a pair of strategies constitutes an equilibrium if, and only if, $-E_L(p) = E_R(p) = b$ and the support of each parties' distribution function is such that Lemma 1.1 is not violated.

Step 10: By Corollary A.1.4, only a corner solution where $|E_i(p)| = |X_i|$ mutually satisfies Lemma 1.4 and 1.2 if $|X_i| < b$.

Given *Step 8*, only a pair of strategies satisfying $|E_L(p)| = |E_R(p)| = |X_i|$ guarantee a corner solution and satisfy Lemma 1.1.

□

Proof of Proposition 1.6

The proof first outlines the scenario where entering the electoral competition is most likely for M and then defines the cutoff at which the entrant is just indifferent between entering the electoral competition and not doing so.

Step 1: Let me start by defining $H_R(p|p_L \neq p)$ if G_L places an atom of size $P_L(p)$ on $p \leq 0$:[7]

$$\begin{aligned} H_R(p|p_L \neq p) = &\int_{-b}^{p} \left[\frac{1 - G_L(p)}{1 - P_L(p)} + \frac{G_L(2x - p)}{1 - P_L(p)} \right] f(x)dx \\ &+ \int_{p}^{b} \left[\frac{1 - G_L(2x - p)}{1 - P_L(p)} + \frac{G_L(p) - P_L(p)}{1 - P_L(p)} \right] f(x)dx \\ = &\frac{H_R(p + \epsilon) - P_L(p)(1 - F(p))}{1 - P_L(p)} \end{aligned} \tag{A.32}$$

Where ϵ is arbitrarily small and $G_L(p)$ places no atom on $p + \epsilon$. $G_L(p)$ is the probability that $p_L \leq p$ is drawn from G_L. Analogously, $H_L(p|p_R \neq p)$ where G_R places an atom of size $P_R(p)$ on $p \geq 0$ is defined as:[8]

$$H_L(p|p_R \neq p) = \frac{H_L(p - \epsilon) - P_R(p)F(p)}{1 - P_R(p)} \tag{A.33}$$

Leveraging these results, the probability of winning for party M when playing a position

[7] In words: This is the conditional probability of winning when playing an ideology p conditional on the voter sampling $p_L \neq p$ from G_L.

[8] The necessary argument when deriving Lemma A.1.2 follows from here.

$p \in [sup(T_L), inf(T_R)]$ is defined as:

$$\begin{aligned} H_M(p) =&(1 - P_L(p)) * (1 - P_R(p)) * [H_R(p|p_L \neq p) - (1 - H_L(p|p_R \neq p)] \\ &+ P_L(p) * P_R(p)\frac{1}{3} + (1 - P_L(p)) * P_R(p)\frac{1}{2}[H_R(p|p_L \neq p)] \\ &+ P_L(p) * (1 - P_R(p))\frac{1}{2}[H_L(p|p_R \neq p)] \end{aligned} \tag{A.34}$$

Step 2: For all $p \in T_M$, $H_M(p)$ must be maximized since otherwise party M could shift some weight to positions which yield a higher probability of winning an thereby achieve a higher EH_M.

Step 3: From Step 1 it becomes evident that $H_M(p)$ is increasing in both $H_L(p)$ and $H_R(p)$. For $p < 0$, $H_L(p)$ is strictly increasing in p for all $p \in [-1, sup(T_L))$ by Lemma 1.1. $H_R(p)$ is also strictly increasing in p for all $p \in [-1, sup(T_L))$ by a similar argument as in *Step 1* of the proof of Proposition 1.1. Thus, any $p : p \in T_M,\ and\ p \in [-1, sup(T_L))$ would contradict *Step 2*. By a similar argument there can also not exist any $p : p \in T_M,\ and\ p \in (inf(T_R), 1]$.

Step 4: For $b \in \left(0, \frac{1}{4}\right)$, $EH_M < \frac{7}{12}$.

It follows from Proposition 1.3 that both parties can implement their preferred average platform position in equilibrium. Nevertheless, I want to sketch that also here $EH_M < \frac{7}{12},\ \forall\ b \in (0, \frac{1}{4})$.

It follows from Proposition 1.3 that G_L and G_R necessarily place an atom of size $\alpha_i = \alpha > 1/2$ on $p = 0$ and no weight on $(-1, 0)$ (G_L) or $(0, 1)$ (G_R). In this case, it follows from *Step 2* that $\nexists\ p : p \in T_M,\ and\ p \neq 0$. It, furthermore, follows from Equations A.32 and A.32 that $H_R(p = 0|p_L \neq p = 0) = H_L(p = 0|p_R \neq p = 0) = 1$. Combining these arguments it follows from Equation A.35 that:

$$\frac{\partial H_M(p = 0)}{\partial \alpha}\Big|_{\alpha > 1/2} < 0 \tag{A.35}$$

Thus, $H_M(p)$ is maximized in the limit where $b = \frac{1}{4}$ such that the smallest possible $\alpha_i = \frac{1}{2}$ is placed on $inf(T_R) = sup(T_L) = 0$. It follows that, in the limit, $EH_M|_{b=1/4} = H_M(p =$

$0)|_{b=1/4} = \frac{7}{12}$. Hence, it follows immediately that $EH_M < \frac{7}{12},\ \forall\ b \in \left(0, \frac{1}{4}\right)$.

Step 5: For $b \in \left[\frac{1}{4}, \frac{1}{3}\right)$, $EH_M \leq \frac{7}{12}$.

Given *Step 3*, M and L can only tie if $p = sup(T_L) \in T_M$ and if G_L places an atom on $p = sup(T_L)$. A similar argument applies for G_R. For this to be optimal, $\frac{\partial H_M(p)}{\partial P_L(p)}|_{p=sup(T_L)}$ needs to be weakly greater than 0. If both parties place an atom on $sup(T_L) = inf(T_R) = 0$, M necessarily ties with both parties by *Step 3*.

The following expression denotes for $b \in (1/4, 1/3)$ how $H_M(sup(T_L))$ changes if L marginally shifts $sup(T_L) < 0$ slightly closer to 0 and rearranges G_L in a way that $P_L(p)-\eta_1$ is shifted to $p^* = sup(T_L) + \eta_2$ and $\eta_1 = \frac{P_L(p)(\eta_2)}{p^*-2b+1}$ is shifted to $2b - 1 < sup(T_L) < 0$ such that the rearranged distribution function G_L still constitutes an equilibrium. Furthermore, notice that $P_R(sup(T_L)) = 0$ with $sup(T_L) < 0$.

With Equations A.32 and A.32 it follows from Equation A.35:

$$\begin{aligned}\frac{\partial H_M(sup(T_L))}{\partial sup(T_L)} &= \frac{\partial P_L(sup(T_L))}{\partial sup(T_L)}(F(sup(T_L)) - \frac{1}{2}H_L(sup(T_L)))\\ &+ \frac{\partial H_L(sup(T_L))}{\partial sup(T_L)}(1 - \frac{1}{2}P_L(sup(T_L))) + \frac{\partial H_R(sup(T_L) + \epsilon)}{\partial sup(T_L)} + f(sup(T_L))P_L(p)\end{aligned}$$

For an arbitrarily small η_2, the difference in $P_L(sup(T_L)) - P_L(p^*)$ is negligible such that $\frac{\partial P_L(sup(T_L))}{\partial sup(T_L)} \approx 0$. Furthermore, $\frac{\partial H_R(sup(T_L)+\epsilon)}{\partial sup(T_L)} - \frac{\partial H_L(sup(T_L))}{\partial sup(T_L)} \approx 0$ by a similar argument. Given $f(sup(T_L)) \geq \frac{1}{2},\ \forall\ b \leq 1$ and $\frac{\partial H_L(sup(T_L))}{\partial sup(T_L)} \leq 1$,[9] for all equilibria outlined in Proposition 1.4 and Lemma 1.1 it follows that:

$$\frac{\partial H_M(sup(T_L))}{\partial sup(T_L)} = f(sup(T_L))P_L(p) - \frac{\partial H_L(sup(T_L))}{\partial sup(T_L)}\frac{1}{2}P_L(sup(T_L)) > 0$$

In conclusion, H_M and EH_M increase if i) the maximum possible $P_L^{max}(sup(T_L))$ is placed on ii) $sup(T_L)$ closest to zero. Notice that for an arbitrarily small ϵ it follows from Equation

[9] If the latter would not hold, Lemma 1.1 would be violated which cannot be the case in equilibrium. This follows from $H_L(p) \leq 1$ by definition and $H_L(-1) > 0$ in equilibrium.

A.33:

$$H_L(p=0) \approx P_R(p_R=0)\frac{1}{2} + (1-P_R(p_R=0))\frac{H_L(0-\epsilon) - P_R(p_R=0)F(0)}{1-P_R(p_R=0)}$$
$$= H_L(0-\epsilon)$$

Thus, given $F(p = 0) = 1/2$ and the specified tie-breaking rule, the $P_R(p_R = 0)$ terms cancel each other out. A similar argument holds for Equation A.32 such that condition ii) must also be satisfied in the limit where $sup(T_L) = 0$.

Given an equilibrium as outlined in Proposition 1.4, $P_L^{max}(sup(T_L))$ is achieved if a fraction $1 - \frac{4b-1}{1-2b}$ of α_R is placed on $sup(T_L) = 0$ and the remaining fraction $\frac{4b-1}{1-2b}$ is placed on $p = 2b - 1$. As $P_L^{max}(sup(T_L) = 0)$ is decreasing in b, one finds that $H_M(p) = \frac{7}{12}$ is maximum in the limiting scenario where $b = \frac{1}{4}$.

Step 6: For $b \in \left[\frac{1}{3}, \frac{1}{2}\right]$, $EH_M \leq \frac{7}{12}$.

For $sup(T_L) < 0$ and $\frac{\partial H_M(p)}{\partial P_L(p)}|_{p=sup(T_L)} < 0$, M rather slightly leapfrogs $sup(T_L)$ than ties with L at $sup(T_L) < 0$. Whenever such a scenario occurs, $H_M(p) = H_R(p) - (1 - H_L(p)) < 0.5,\ \forall\ p \in (sup(T_L), 0]$. [10]

Assume now that tying at $p = sup(T_L) < 0$ is indeed optimal for M. It immediately follows that $H_M(p)$ is maximized if the maximum possible $P_L(p)$ is placed on $p = sup(T_L)$ by G_L. For this to be the case, G_L needs to place an atom of size $1 - P_L(sup(T_L)) = \frac{b+sup(T_L)}{1+sup(T_L)}$ on $p = -1$ and an atom of size $P_L(p) = \frac{1-b}{1+sup(T_L)}$ on $p = sup(T_L)$ to still achieve $E_L(p) = b$ as required by Proposition 1.5.

From a similar reasoning as in *Step 5*, it follows that H_M, and consequently EH_M, is maximized if i) $P_L^{max}(sup(T_L) = 2b - 1)$ is placed on ii) the $sup(T_L)$ that is closest or equal to zero. ii) is satisfied in the limit where $b = \frac{1}{2}$ and the corresponding $P_L^{max}(sup(T_L) = 2b - 1) = \frac{1}{2}$. In this case, $EH_M = H_M(0) = \frac{7}{12}$. It follows that for $b \in \left[\frac{1}{3}, \frac{1}{2}\right)$, $EH_M < \frac{7}{12}$

Step 7: For $b \in \left(\frac{1}{2}, 1\right]$, $EH_M < \frac{7}{12}$.

[10] Check Corollary A.1.2 to understand this argument.

The reasoning is identical to *Step 6*. $H_M(p)$, and therefore EH_M, is maximized if both parties place the maximum possible weight on $sup(T_L) = inf(T_R) = 0$. Given an equilibrium as outlined in Proposition 1.4, this is achieved if a fraction $1 - b$ is placed on $sup(T_L) = inf(T_R) = 0$ and the remaining fraction b is placed on $p = -1$ (party L) and $p = 1$ (party R). As $P_L^{max}(sup(T_L))$ are decreasing in b, one finds that $H_M(p)$ is maximized for $b = \frac{1}{2}$. In such case, $EH_M = H_M(0) = \frac{7}{12}$. It then immediately follows that $EH_M < \frac{7}{12}$ for $b \in (\frac{1}{2}, 1]$.

Step 8: From *Steps 5-7* it follows that $EH_M \leq \frac{7}{12}$ for all $b \in \{0, [\frac{1}{4}, 1]\}$, whenever both parties are not able to implement their preferred average platform ideology in equilibrium.

□

A.2 No Uncertainty Benchmark

Proposition 3.1. *If $b = 0$ and $|X_i| \geq \frac{1}{3}$ there exists a unique and symmetric equilibrium where both parties choose a distribution function that places an atom of size $\frac{1}{2}$ on $p = 0$ and uniformly randomizes with the remaining weight over $\left[-1, -\frac{1}{3}\right]$ (party L) and $\left[\frac{1}{3}, 1\right]$ (party R) such that $|E_i(p)| = \frac{1}{3}$.*

Otherwise, if $|X_i| < \frac{1}{3}$, there exists a unique and symmetric equilibrium where both parties choose a distribution function that places an atom of size $\frac{1-2|X_i|}{1-|X_i|} > \frac{1}{2}$ on $p = 0$ and uniformly randomizes with the remaining weight over $[-1, 2|X_i| - 1]$ (party L) and $[1 - 2|X_i|, 1]$ (party R) where $1 - 2|X_i| > \frac{1}{3}$ such that $|E_i(p)| = |X_i|$.

Proof:

Notice that Lemmas 1.1 - 1.4 and Proposition 1.1 and 1.2 continue to hold. Likewise, Corollary A.1.1 applies. With Proposition 1.1 and $b = 0$, the probability of winning the election for either party conditional on nature sampling $p_L = p$ / $p_R = p$ is given by:[11]

$$H_L(p) = 1 - G_R(-p) + \frac{1}{2}Pr(p_R = -p) \ // \ H_R(p) = G_L(-p) + \frac{1}{2}Pr(p_L = -p) \quad \text{(A.36)}$$

The reasoning in the subsequent Steps 1-7 implicitly assumes that, in equilibrium, $|E_i(p)| \leq |X_i|$ such that Lemma 1.2 is not violated. If argued of the perspective of party L, the reversed argument for party R immediately follows by the symmetry of the game.

Step 1: In equilibrium, G_L does not place an atom on any $p' \in (-1, 0)$.

Suppose that this is not the case. Then, $H_R(p)$ discontinuously jumps downwards at $p = -p'$. Thus, there exists a sufficiently small ϵ such that party R does not place any weight on $[-p', -p' + \epsilon]$ since party R is always better off if shifting this weight to a position slightly smaller than $-p'$. However, if party R does not place any weight on $[-p', -p' + \epsilon]$, placing an atom on p' is not optimal for party L since it could increase π_L^L without decreasing EH_L by shifting this atom to $p' - \epsilon$.

Step 2: In equilibrium, G_L does not place an atom on $p = -1$.

Otherwise, $H_R(p' = 1 - \epsilon) > H_R(p = 1)$ where ϵ is arbitrarily small. Thus, by placing an equally sized atom on p', no weight on $p = 1$ and elsewhere reverse-mirroring the strategy of party L, R achieves $EU_R > EU_L$ such that Lemma 1.3 is certainly violated if G_L places an atom on $p = -1$.

Step 3: In equilibrium, $sup(T_R) = 1$ and $sup(T_R) \in T_R$.

Suppose that $sup(T_R) = 1 - \Delta$ where $\Delta \in (0, sup(T_R)]$ and that $|inf(T_L)| \leq |sup(T_R)|$. Notice that G_R must be continuous around $sup(T_R)$ by *Step 1.* Then, only a strategy where G_L places an atom on $p = -1$ can be a best reply for party L since $H_L(p) = 0, \ \forall \ p \in [-1, -1 + \Delta]$. Otherwise, Lemma 1.1 is obviously violated: By shifting all the weight in $(-1, -1 + \Delta]$ to $p = -1$ party L can increase π_L^L without decreasing EH_L. Thus, $(-sup(T_R), H_L(-sup(T_R)))$ lies below the line connection $(-1, H_L(-1))$

[11] Although Lemma Proposition 1.1 still applies, the election is tied with certainty whenever voters sample two positions with identical distances from $b = 0$ since voter ideologies are not continuously distributed around zero. Therefore, $Pr(p_R = -p)$ is included in Equation A.36 and an equi-tie-breaking rule applies.

and $(-inf(T_L), H_L(-inf(T_L)))$. Such a best reply violates *Step 2* and Lemma 1.3 and cannot be part of an equilibrium. Thus, there can be an equilibrium if, and only if, $sup(T_R) = -inf(T_L) = 1$. By a similar argument, $sup(T_R) \in T_R$ and $inf(T_L) \in T_L$.

Step 4: If G_L attributes some weight to $p < 0$ (or $g_L(p) = t > 0$), G_R must also attribute some weight to $-p$ (or $g_L(-p) = t$) in equilibrium.

By *Steps 1 - 3* and Proposition 1.1, G_R must be continuous over $[p_1, 1]$ and G_L must be continuous over $[-1, p_2]$ where $|p_1|, |p_2| \in (0, 1)$. Let me now assume that $1 > |p_1| > |p_2|$ such that G_L attributes some weight to $(-p_1, p_2]$ where G_R does not attribute some weight to $[-p_2, p_1)$. In such case, $H_L(p) = c \in (0, 1]$, $\forall\ p \in [-p_1, p_2]$. Such a situation certainly violates Lemma 1.1 as the point $(-p_1, H_L(-p_1))$ lies above the line connecting $(-1, H_L(-1))$ and $(p, H_L(p))$, $\forall\ p \in (-p_1, p_2]$.

Step 5: G_i must a) attribute some weight $\gamma \in (0, 1]$ b) over a single, closed interval $[t, 1]$ (party *R*) / $[-1, -t]$ (party *L*) where $t \geq 0$ c) in a uniform way:

a) Follows from Proposition 1.2[12]

b) By Steps *1-3*, $t < 1$. By *Step 4*, the lower bound of the interval (t) for *R* must be equal to $-t$ which is the upper bound of the interval for *L*. By a similar argument as applied in *Step 4*, the interval must also be closed.

c) Follows directly from Equation A.36: For Lemma 1.1 to hold for party *L*, G_R needs to be linear in p which is the case if, and only if, g_R follows a uniform distribution over the interval $[t, 1]$. A similar argument applies for party *L*.

Step 6: In equilibrium, $|E_i(p)| = \frac{1}{3}$.

Given *Step 5*, there exist two possible shapes of $G_i(p)$:

1. G_i places no atom on $p = 0$ such that $|sup(T_L)| = |inf(T_R)| = t \geq 0$
2. G_i places an atom of size α_i on $p = 0$

[12]Check the introductory example in Section 1.3 for some intuition.

In both scenarios, Lemma 1.4 needs to hold. Therefore, the best replying simple-cdf $G_i^{'}$ which places an atom of size α_i on $sup(T_L)(inf(T_R))$ and an atom of size $1-\alpha_i$ on $inf(T_L) = -1, sup(T_R) = 1$ must yield the same expected value as G_i, given the opponent's strategy. Suppose that for a given strategy G_R, $E_R(p) = \mu_R$.

In scenario 1) $sup(T_L) = -t$, $H_L(-1) = 0$, $H_L(-t) = 1$. By Corollary A.1.3:

$$\alpha_L^{opt.} = \frac{1+\mu_R}{2(1-x)} \tag{A.37}$$

Furthermore, it follows from Lemma 1.3 by a similar argument as outlined in Corollary 1.2 that, in equilibrium, $E_L(p) = \mu_L = -\mu_R$. Thus, for G_L to be a best reply *and* a potential equilibrium candidate it must hold that for $\alpha_L^{opt.}$ as outlined in (A.37), $E_L^{'}(p) = \mu_L = -\mu_R$:

$$\alpha_L^{opt.}(-t) + (1-\alpha_L^{opt.})(-1) = -\mu_R \rightarrow \mu_R = \frac{1}{3}$$

Thus, in equilibrium $|E_i(p)| = \frac{1}{3}\ \forall\ i \in \{L, R\}$.

In scenario 2) $sup(T_L) = 0$, $H_L(-1) = 0$, $H_L(0) = (1-\alpha_i) + \frac{1}{2}\alpha_i$. After applying identical steps as above-outlined for scenario 1) it follows, again, that $|E_i(p)| = \frac{1}{3}\ \forall\ i \in \{L, R\}$ in equilibrium.

Step 7: Considering the results from *Step 5* and Proposition 1.1, there exists no uniform distribution with $|inf(T_L)| = |sup(T_R)| = 1$ and $x \geq 0$ which satisfies $|E_i(p)| = \frac{1}{3}$, $\forall\ i \in \{L, R\}$. Therefore, scenario 1) outlined in the previous step cannot constitute an equilibrium.

The following two conditions ensure that Lemmas 1.1 & 1.4 are mutually satisfied in scenario 2) where G_i places an atom of size α_i on $p = 0$ and uniformly distributes the remaining weight over $[-1, t]$ (party L) and $[t, 1]$ (party R):

1. Lemma 1.1 is satisfied if and only if: $\frac{1-\alpha_R}{1-t} = H_L(0) = (1-\alpha_R) + \alpha_R\frac{1}{2}$
2. Lemma 1.4 is satisfied if and only if: $(1-\alpha_R) * \frac{1+t}{2} = \frac{1}{3}$

The first condition ensures that $H_L(0)$ lies on the same straight line connecting all $p \in T_L$.

The second condition follows from *Step 6* and ensures that the best-replying simple *cdf* also generates an average platform position equal to 1/3. Any pair of strategies which satisfies both conditions for both players constitutes an equilibrium by definition of a Nash Equilibrium.

After solving the second condition for t and then substituting the result into the first condition, the unique equilibrium is obtained:

$$\alpha_{L,R} = \frac{1}{2}, t = \frac{1}{3}$$

In equilibrium, party L plays the *cdf* G_L with support $T_L = \left[-1, -\frac{1}{3}\right] \cup \{0\}$ and party R plays the *cdf* G_R with support $T_R = \{0\} \cup \left[\frac{1}{3}, 1\right]$, where:

$$G_L(p) = \begin{cases} \frac{1}{2} * \frac{p+1}{\frac{2}{3}} & \text{if } p \leq -\frac{1}{3} \\ \frac{1}{2} & \text{if } -\frac{1}{3} < p < 0 \\ 1 & \text{if } 0 \leq p \end{cases} \qquad G_R(p) = \begin{cases} 0 & \text{if } p < 0 \\ \frac{1}{2} & \text{if } 0 \leq p < \frac{1}{3} \\ \frac{1}{2} + \frac{1}{2} * \frac{p-\frac{1}{3}}{\frac{2}{3}} & \text{if } \frac{1}{3} \leq p \leq 1 \end{cases}$$

Step 8: If $|X_i| < \frac{1}{3}$, the previously stated strategies cannot constitute an equilibrium as $|X_i| < |E_i(p)|$, which constitutes a violation of Lemma 1.2. However, *Steps 1-5* and the reasoning in *Step 7* that rules out scenario 1) from *Step 6* continue to apply. Thus, in any equilibrium party L places an atom on $sup(T_L) = 0$ and chooses $inf(T_L) = -1$. From Corollary A.1.4, which also applies here, it follows that any equilibrium strategy G_i must yield a 'corner solution' where $|E_i(p)| = |X_i|$ for Lemma 1.4 and 1.2 to mutually hold.

Step 9: I can again formulate two conditions that both need to be satisfied such that Lemmas 1.1 & 1.4 hold for $|X_i| < \frac{1}{3}$:

1. Lemma 1.1 is satisfied if, and only if: $\frac{1-\alpha_R}{1-t} = H_L(0) = (1 - \alpha_R) + \alpha_R \frac{1}{2}$

2. Lemma 1.4 is satisfied if, and only if: $(1 - \alpha_R) * \frac{1+t}{2} = |X_i|$

The first condition, again, ensures that $H_L(0)$ lies on the line connecting all $p \in T_L$. The second condition follows from *Step 8* and ensures that the best-replying simple *cdf* also generates an average platform position equal to $|X_i|$. After applying similar steps as in *Step 7*:

$$\alpha_{L,R} = \frac{1 - 2|X_i|}{1 - |X_i|}, t = 1 - 2|X_i|$$

In the limit, where $|X_i| = \frac{1}{3}$, both equilibria converge.

□

Appendix B

Appendix to Chapter 2

B.1 Proofs

We do not consider equilibria in weakly dominated pricing strategies. In addition, we apply the following tie-breaking rules: In case of indifference, consumers buy from R, M launches A and R and f are hosted. Overall welfare (W) is defined as $W = E(\pi_R) + E(\pi_M) + E(CS_1) + E(CS_{2A})$.

$inf(T_R), inf(T_M)$ describe the infimums of $R's$ and $M's$ supports T_R, T_M if playing a mixed strategy and $sup(T_R), sup(T_M)$ their supremums.

Let me start this section with establishing the following two Lemmas, where the reasoning was implicitly mentioned in the main body of the text.

Lemma B.1.1. *In any period, M never wants to host R and launch A on its own and never wants to not host R and not launch A on its own.*

Proof: M never wants to host R and launch A: If M hosts R and launches A and if equilibria in weakly dominated pricing strategies are ruled out, simultaneous Bertrand competition leads to $p^m_M = 0$ and $p^m_R = \Delta_R - \Delta_M$. Then, $M's$ profits are zero and M cannot recover the fixed costs F. Anticipating this, it is never optimal for M to launch A, if M wants to host R.

M never wants to not host R and not launch A: If M does not launch A, then M can at least generate transfer payments from hosting R.

□

Lemma B.1.2. *In $t = 2$, R always wants to be hosted.*

Proof: As the game ends after $t = 2$, R certainly generates additional profits from being hosted in $t = 2$ equal to $(1 - \phi)\xi(u_a + \Delta_R - \tau)$ by selling product A to *Type 2A* consumers without risking that the platform enters the market in a future period. Therefore, the specialist is strictly better off than if not being hosted.

□

Proof of Lemma 2.1

By Lemma B.1.1, the *Pooling Equ.* outcome is the unique equilibrium outcome in $t = 2$, if M does not launch A: If the parameter values are such that launching is not profitable for M, R has no incentive to decrease p_R^m and p_R^o because it already sells A to all potential consumers. R also has no incentive to increase p_R^m and p_R^o because demand would discretely jump to zero on- and off-platform. Thus, the *Pooling Equ.* is the unique equilibrium in such case.

We now continue with outlining the conditions where the *Pooling Equ.* is more profitable for M than the other two possible pricing equilibria.[1] By Lemma B.1.2, R wants to be hosted for sure in $t = 2$. Therefore, it is sufficient to check for which parameter values M finds launching A less profitable than hosting R. $M's$ profit from hosting R is $\pi_M = (1 - \phi)\xi\tau$. $M's$ profit from launching A is $\pi_M = (1 - \phi)\xi(u_A + \Delta_M) - F$. Thus, whenever:

$$(1 - \phi)\xi\tau > (1 - \phi)\xi(u_A + \Delta_M) - F$$

M does not want to launch A. Rearranging yields:

$$\xi < \frac{F}{(1 - \phi)(u_A + \Delta_M - \tau)}$$

[1] Their proofs follow in the subsequent lemmas.

If the *Pooling Equ.* is realized, the specialist is able to extract the entire consumer surplus from *Type 1* and *Type 2A* consumers. It immediately follows that $\pi_M = (1-\phi)\xi\tau$, $\pi_R = \phi(u_A+\Delta_R-s)+(1-\phi)\xi(u_A+\Delta_R-\tau)$, $W = (\phi+(1-\phi)\xi)(u_A+\Delta_R)-\phi s$ and $CS_1 = CS_{2A} = 0$.

□

Proof of Lemma 2.2

If M launches A, R is not hosted by Lemma B.1.1. In this case, the *Separating Equ.* outcome and the *Mixed Equ.* outcome are the only possible equilibrium outcomes in $t = 2$.

Furthermore, if the parameter values are such that any unilateral deviation from the *Separating Equ.* yields a lower profit for M or R than obeying to the strategies associated with in the *Separating Equ.*, it is the only possible equilibrium outcome by the following argument:

In the *Separating Equ.* R and M have no incentive to deviate from p_R^o and/or p_M^m, given the opponent's price. If they increase the prices, demand discontinuously jumps to zero. Decreasing p_R^o is also not reasonable since R already sells to all *Type 1* consumers. Since M already sells to all *Type 2A* consumers, it does not have an incentive to decrease p_M^m, conditional on the *Separating Equ.* being more profitable than additionally attracting *Type 1* consumers in the *Mixed Equ.* outcome. Thus, the *Separating Equ.* is the unique equilibrium in such case.

By reversing the reasoning provided in the proof of Lemma 2.1, the *Pooling Equ.* is less profitable for M than the *Separating Equ.* if:

$$\xi > \frac{F}{(1-\phi)(u_A+\Delta_M-\tau)}$$

We now continue with finding the parameter space where, conditional on launching A, the *Separating Equ.* is more profitable then the *Mixed Equ.*: Given that R sets $p_R^o = u_A+\Delta_R-s$ in the *Separating Equ.*, M can additionally attract *Type 1* consumers by setting $p_M^m =$

$u_A - s - \epsilon$ where ϵ is arbitrarily small. Such a deviation is not profitable for M if:

$$(1-\phi)\xi(u_A + \Delta_M) - F > (\phi + (1-\phi)\xi)(u_A + \Delta_M - s) - F$$

Rearranging yields:

$$\xi > \frac{\phi(u_A + \Delta_M - s)}{(1-\phi)s}$$

If the *Separating Equ.* is realized, the specialist is able to extract the entire consumer surplus from *Type 1* consumers and M is able to extract the entire consumer surplus from *Type 2A* consumers. It immediately follows that $\pi_M = (1-\phi)\xi(u_A + \Delta_M) - F$, $\pi_R = \phi(u_A + \Delta_R - s)$, $W = \phi(u_A + \Delta_R - s) + (1-\phi)\xi(u_A + \Delta_M) - F$ and $CS_1 = CS_{2A} = 0$.

□

Proof of Lemma 2.3

We subsequently demonstrate that the expected profit of $M's$ equilibrium strategy is equal to $(1-\phi)\xi(u_A + \Delta_M) - F$. Thus, by reversing the reasoning provided in the proof of Lemma 2.1, the *Pooling Equ.* is less profitable for M than the *Mixed Equ.* if:

$$\xi > \frac{F}{(1-\phi)(u_A + \Delta_M - \tau)}$$

By also reversing the reasoning provided in the proof of Lemma 2.2, the *Separating Equ.* cannot be an equilibrium, conditional on M launching A if:

$$\xi < \frac{\phi(u_A + \Delta_M - s)}{(1-\phi)s} \tag{B.1}$$

If this is the case, there exists no equilibrium in pure strategies as shown in the following steps:

1. Playing any $p_M^m > u_A + \Delta_M$ does not generate any sales for M. Playing $p_M^m \leq 0$ generates a (weakly) negative profit. With $p_M^m \in (u_A + \Delta_M - s, u_A + \Delta_M)$ M does not attract *Type*

1 consumers and sells to *Type 2A* consumers at a price below the monopoly price.

Hence, choosing $p_M^m = u_A + \Delta_M$, which allows M to extract monopoly rents from *Type 2A* consumers, yields a higher profit for M than all the pricing strategies listed above, independent of the pricing strategy of R. Let the above-listed strategies non-rationalizable as they are never played with positive probability as a best reply of M to any strategy of R.

2. Setting any $p_R^o < \Delta_R - \Delta_M$ or any $p_R^o > u_A + \Delta_R - s$ can never be optimal for R as setting $p_R^o = \Delta_R - \Delta_M$ yields in both cases a strictly higher profit, given the above non-rationalizable strategies of M and the consumers' tie-breaking rule.

3. If $p_M^m = u_A + \Delta_M$ is not optimal, M best replies to any pure strategy $p_R^o \in (\Delta_R - \Delta_M, u_A + \Delta_R - s]$ by setting $p_M^{m*}\left(p_R^o\right) = p_R^o - \Delta_R + \Delta_M - \epsilon$ where ϵ is infinitesimally small to slightly undercut the specialist.

4. $M's$ profit from playing $p_M^{m*}\left(p_R^o\right)$ is $(\phi + (1-\phi)\xi)p_M^{m*}\left(p_R^o\right) - F$. The profit from playing $p_M^m = u_A + \Delta_M$ is $(1-\phi)\xi(u_A + \Delta_M) - F$. Thus, if $\frac{\phi p_M^{m*}(p_R^o)}{(1-\phi)\left(u_A+\Delta_M - p_M^{m*}(p_R^o)\right)} > \xi$ for a given p_R^o, it is optimal for M to play $p_M^{m*}\left(p_R^o\right)$ instead of $p_M^m = u_A + \Delta_M$.

5. As playing $p_M^{m*}\left(p_R^o\right)$ yields a negative profit for the specialist if $p_R^o = \Delta_R - \Delta_M$ and as playing $p_M^{m*}\left(p_R^o\right)$ is certainly optimal in case $p_R^o = u_A + \Delta_R - s$ because of Condition B.1 and as the profit from playing $p_M^{m*}\left(p_R^o\right)$ is continuously increasing in p_R^o, there always exists a $\underline{p_R^o}$ where $\Delta_R - \Delta_M < \underline{p_R^o} < u_A + \Delta_R - s$ such that $\frac{\phi p_M^{m*}\left(\underline{p_R^o}\right)}{(1-\phi)\left(u_A+\Delta_M - p_M^{m*}\left(\underline{p_R^o}\right)\right)} = \xi$ by the *Intermediate Value Theorem*. For such a $\underline{p_R^o}$, M has no strict incentive to switch from playing $p_M^m = u_A + \Delta_M$ to playing $p_M^{m*}\left(\underline{p_R^o}\right)$.

6. For any $p_R^o > \underline{p_R^o}$, a strategy profile of the form $\left(p_R^o, p_M^{m*}\left(p_R^o\right)\right)$ cannot constitute an equilibrium as the specialist makes no sales at p_R^o. By lowering its price to $p_R^{o'} = p_R^o - \epsilon > 0$, where ϵ is arbitrarily small, R could attract all *Type 1* consumers, which strictly improves its payoff.

7. For any $p_R^o < \underline{p_R^o}$, a strategy profile of the form $\left(p_R^o, u_A + \Delta_M\right)$ cannot be an equilibrium as, given M plays $p_M^m = u_A + \Delta_M$ as prescribed by step (6), R would be better off by

playing $p_R^o = u_A + \Delta_R - s$, which still allows R to sell to all *Type 1* consumers.

8. For $p_R^o = \underline{p_R^o}$ and M best- replying with $p_M^m = u_A + \Delta_M$, a similar reasoning as in bullet-point (7) applies. Otherwise, if M best replies with $p_M^{m*}\left(\underline{p_R^o}\right)$ a similar argument as in bullet-point (6) applies. Therefore, we can conclude that there exists no stable equilibrium for a rationalizable, pure strategy profile (p_R^o, p_M^m).

We now continue with finding the mixed strategy equilibrium:

The *cdf* of R for p_R^o $(F_R(p_R))$ must be such that M is indifferent for all $p_M^m \in T_M$ and, given the specialist's strategy, all $p_M^m \notin T_M$ must yield a weakly lower (expected) profit than any $p_M^m \in T_M$. M can always guarantee itself a profit equal to $(1-\phi)\xi u_A + \Delta_M$ when charging $p_M^m = u_A + \Delta_M$ from *Type 2A* consumers, which defines its indifference condition:

$$p_M^m[(1-\phi)\xi + \phi(1-(F_R(p_M^m + \Delta_R - \Delta_M)))] - F = (1-\phi)\xi(u_A + \Delta_M) - F$$

Letting $p_M^m = p_R^o - \Delta_R + \Delta_M$ and rearranging:

$$F_R(p_R^o) = 1 - \frac{(1-\phi)\xi}{\phi}\left(\frac{u_A + \Delta_M}{p_R^o - \Delta_R + \Delta_M} - 1\right)$$

$F_R\left(p_R^o\right)$ is a *cdf* if, and only if:

1. $\lim_{p_R^o \to inf(T_R)} F_R\left(p_R^o\right) = 0$. This holds for $inf(T_R) = \Delta_R - \Delta_M + \frac{u_A+\Delta_M}{\frac{\phi}{(1-\phi)\xi}+1}$
2. $\lim_{p_R^o \to sup(T_R)} F_R\left(p_R^o\right) = 1$. However, $F_R\left(p_R^o\right) < 1,\ \forall\ p_R^o \in [inf(T_R), u_A + \Delta_R - s]$. Yet, any $p_R^o > u_A + \Delta_R - s$ cannot lie in the support of $F_R\left(p_R^o\right)$ because such prices are non-rationalizable as argued above and would contradict the specialist's indifference condition laid out in more detail below. Therefore, $\lim_{p_R^o \to sup(T_R)} F_R\left(p_R^o\right) = 1$ if, and only if, R places an atom of size $\frac{(1-\phi)\xi}{\phi}\left(\frac{u_A+\Delta_M}{u_A+\Delta_M-s} - 1\right)$ on $p_R^o = u_A + \Delta_R - s$.

The *cdf* of M for p_M^m $\left(F_M\left(p_M^m\right)\right)$ must be such that R is indifferent for all $p_R^o \in T_R$ and, given $M's$ strategy, all $p_R^o \notin T_R$ must yield a weakly lower (expected) profit than any $p_R^o \in T_R$. Notice that any $p_M^m < inf(T_R) - \Delta_R + \Delta_M$ certainly contradicts $M's$ indifference condition from above. If we assume for the moment that $F_M\left(p_M^m\right)$ does not place an atom on $p_M^m = inf(T_R) - \Delta_R + \Delta_M = \frac{u_A+\Delta_M}{\frac{\phi}{(1-\phi)\xi}+1}$, R is able to sell A to all *Type 1* consumers when charging $p_R^o = inf(T_R) = \Delta_R - \Delta_M + \frac{u_A+\Delta_M}{\frac{\phi}{(1-\phi)\xi}+1}$, which yields a profit equal to $\phi\left(\Delta_R - \Delta_M + \frac{u_A+\Delta_M}{\frac{\phi}{(1-\phi)\xi}+1}\right)$. Therefore, the specialist's indifference condition is given by:

$$\phi p_R^o\left(1 - F_M\left(p_R^o - \Delta_R + \Delta_M\right)\right) = \phi\left(\Delta_R - \Delta_M + \frac{u_A + \Delta_M}{\frac{\phi}{(1-\phi)\xi} + 1}\right)$$

Letting $p_R^o = p_M^m + \Delta_R - \Delta_M$ and rearranging:

$$F_M\left(p_M^m\right) = 1 - \frac{1}{p_M^m + \Delta_R - \Delta_M}\left(\Delta_R - \Delta_M + \frac{u_A + \Delta_M}{\frac{\phi}{(1-\phi)\xi} + 1}\right)$$

For $F_M\left(p_M^m\right)$ to be a *cdf*, we need the following:

1. $\lim_{p_M^m \to inf(T_M)} F_M\left(p_M^m\right) = 0$. This holds for $inf(T_M) = \frac{u_A+\Delta_M}{\frac{\phi}{(1-\phi)\xi}+1}$. Notice that $F_M\left(p_M^m\right)$ is atomless at $inf(T_M)$, which is what we assumed earlier.

2. $\lim_{p_M^m \to sup(T_M)} F_M\left(p_M^m\right) = 1$.

   However, $F_M\left(p_M^m\right) < 1,\ \forall\ p_M^m \in \{[inf(T_M), u_A + \Delta_M - s]\ \cup\ \{u_A + \Delta_M\}\}$. Yet, any $p_M^m > u_A + \Delta_M$ cannot lie in the support of $F_M\left(p_M^m\right)$ because such prices are non-rationalizable and would contradict $M's$ indifference condition laid out in detail in the previous step. Therefore, $\lim_{p_M^m \to sup(T_M)} F_M\left(p_M^m\right) = 1$ if, and only if, M places an atom of size $\frac{1}{u_a - s + \Delta_R}\left(\frac{u_A+\Delta_M}{\frac{\phi}{(1-\phi)\xi}+1} + \Delta_R - \Delta_M\right)$ on $p_M^m = u_A + \Delta_M$.

Next, we clarify whether $p_M^m = u_A + \Delta_M - s \in T_M$. This, in general, cannot be the case. Since R places an atom on $p_R^o = u_A + \Delta_R - s$, $M's$ indifference condition would be violated at $p_M^m = u_A + \Delta_M - s$.

Lastly, we need to show that, given the opponent's strategy, no price outside of the support of each player yields a strictly higher payoff than any price in the support of each player. We have already argued above that $p_M^m \leq 0$, $p_M^m \in [u_A + \Delta_M - s, u_A + \Delta_M)$ or $p_M^m > u_A + \Delta_M$ can never be optimal for M and that $p_R^o > u_A + \Delta_R - s$ or $p_R^o < 0$ can never be optimal for

R. As neither S nor M places a mass point on $inf(T_R), inf(T_M)$, $p_M^m < inf(T_M)$ or $p_R^o < inf(T_R)$ would violate their indifference conditions. By a similar argument, $inf(T_R) \in T_R$ and $inf(T_M) \in T_M$.

From our derivations above it becomes evident that the *Mixed Equ.* specified in Lemma 2.3 is unique if $\xi \in \left(C_{Launch}^{Monop.}, C_{Competition}^{Monop.}\right)$. Notice further that the transition from the *Mixed Equ.* to the *Separating Equ.* is smooth as whenever $\xi = C_{Competition}^{Monop.}$. In case $\xi = C_{Launch}^{Monop.}$, it directly follows from the rationale outlined in the proof of this Lemma and Lemma 2.1 that the *Mixed-* and the *Pooling Equilibrium* mutually co-exist. There might exist additional equilibria where the platform randomizes between launching and not launching product A. However, given our tie-breaking rule specified at the beginning of the appendix, we abstract from such equilibria.

The expected profit of the specialist and the platform directly follow from the indifference conditions of both players. Without further going into detail, it immediately follows from $F_M\left(p_M^m\right) / F_R\left(p_R^o\right)$ that $E\left(p_M^m\right) < u_A + \Delta_M \,/\, E\left(p_R^o\right) < u_A + \Delta_R - s$ such that $E(CS_1) > 0$ and $E(CS_{2A}) > 0$.[2].

□

Proof of Lemma 2.4

By assumption, M only launches A if R is hosted in $t = 1$. The proof of the pricing outcome follows from an identical argument as outlined in the proof of Lemma 2.1.

□

Proof of Proposition 2.1

If the specialist does not want to get hosted in $t = 1$, it serves only *Type 1* consumers at a price $p_R^o = u_A + \Delta_R - s$. Lowering the price is not profitable as R already serves all consumers it is able to. If R increases its price, demand discontinuously jumps to zero. As a consequence, it follows from Lemma 2.4 that the *Pooling Equ.* outcome is realized in

[2]This argument holds as long as $\xi \in \left(C_{Competition}^{Monop.}, \xi = C_{Launch}^{Monop.}\right)$ as already outlined.

$t = 2$. If R is not hosted in $t = 1$, the present value of its expected, discounted profits in both periods therefore is:

$$E(\pi_R|\text{no hosting in } t = 1) =$$
$$\phi\,(u_A + \Delta_R - s) + \delta\left[\phi\,(u_A + \Delta_R - s) + E(\xi)(1-\phi)(u_A + \Delta_R - \tau)\right]$$

If the specialist wants to be hosted in $t = 1$, the platform accepts such an inquiry by Lemma B.1.1, as the platform cannot launch A on its own in $t = 1$ by assumption. The pricing strategy known from the *Pooling Equ.* outcome is then realized in the first period by an identical reasoning as outlined in the proof of Lemma 2.1. With ξ being revealed in $t = 1$, the specialist forms the following expectations about the probabilities that the distinct pricing equilibria occur in $t = 2$:

1. $Pr(Pooling\ Equilibrium) = G\left(C^{Monop.}_{Launch}\right)$
2. $Pr(Separating\ Equilibrium) = \left(1 - G\left(C^{Monop.}_{Launch}\right)\right) \cdot \left(1 - G\left(C^{Monop.}_{Comp.}\right)\right)$
3. $Pr(Mixed\ Strategy\ Equilibrium) = \left(1 - G\left(C^{Monop.}_{Launch}\right)\right) \cdot G\left(C^{Monop.}_{Comp.}\right)$

If the specialist is hosted in $t = 1$, the present value of its expected, discounted profits in both periods is:

$$E(\pi_R|\text{hosting in } t = 0) = \phi\,(u_A + \Delta_R - s) + (1-\phi)E(\xi)\,(u_A + \Delta_R - \tau) + \delta\Bigg[$$
$$G\left(C^{Monop.}_{Launch}\right)\left(\phi\,(u_A + \Delta_R - s) + E(\xi|\text{Pooling Equ.})(1-\phi)(u_A + \Delta_R - \tau)\right)$$
$$\left(1 - G\left(C^{Monop.}_{Launch}\right)\right) \cdot \left(1 - G\left(C^{Monop.}_{Comp.}\right)\right)\phi\,(u_A + \Delta_R - s)$$
$$\left(1 - G\left(C^{Monop.}_{Launch}\right)\right) \cdot G\left(C^{Monop.}_{Comp.}\right)\phi\left(\Delta_R - \Delta_M + \frac{u_A + \Delta_M}{\frac{\phi}{(1-\phi)E(\xi|\text{Mixed Equ.})} + 1}\right)\Bigg]$$

The specialist wants to get hosted in $t = 1$ if its discounted gains from doing so outweigh potential losses in the second stage. This is the case if, and only if:

$$E(\pi_R|\text{hosting in } t = 1) > E(\pi_R|\text{no hosting in } t = 1)$$

Substituting in the expressions from above and rearranging yields:

$$\Delta_R > \frac{\delta G\left(C^{Monop.}_{Comp.}\right)\left(1 - G\left(C^{Monop.}_{Launch}\right)\right)\phi\left(\frac{\phi(u_A+\Delta_M)}{\phi+(1-\phi)E\left(\xi|\text{Mixed Equ.}\right)} - s\right)}{(1-\phi)\,E\left(\xi\right)\left(1 - \delta\left(1 - G\left(C^{Monop.}_{Launch}\right)\frac{E\left(\xi|\text{Pooling Equ.}\right)}{E(\xi)}\right)\right)} - u_A + \tau \qquad \text{(B.2)}$$

Thus, if Condition B.2 is satisfied, R is willing to get hosted in $t = 1$ and ξ is revealed such that one of the pricing equilibrium outcomes outlined in Lemmas 2.1 - 2.3 is realized in $t = 2$ (by a similar reasoning as also outlined in the respective Lemmas). However, if Condition B.2 is not satisfied, the specialist is not hosted in $t = 1$ and the *Pooling Equ.* outcome is realized in $t = 2$ by Lemma 2.4. If Condition B.2 holds with equality, both equilibria coexist. Furthermore, there might exist additional mixed strategy equilibria. However, given our tie-breaking rule specified at the beginning of the appendix, we abstract from such equilibria and assume that R is hosted in case of indifference

□

Proof of Lemma 2.5

With an additional fringe seller, the off-platform market becomes competititive such that any equilibrium outcome is associated with R charging $p^o_R \leq \Delta_R$ on the off-platform market and the fringe seller not being competitive. (Check the proof of the subsequent Lemma to understand this claim.)

If the specialist is not hosted in $t = 1$, it follows that it is a best reply for f to certainly get hosted as f is thereby able to sell A at monopoly prices on $M's$ platform. By Lemma B.1.1, the platform is going to accept the hosting inquiry of the fringe seller as it is not able to launch product A on its own in $t = 1$ by assumption.

Given this best reply of f, it follows that it cannot be optimal for R to not be hosted in $t = 1$ as R certainly faces the risk of M launching A in $t = 2$ with ξ being revealed, independent of its own hosting decision. If being hosted in $t = 1$, R generates additional profits by selling A to *Type 2A* consumers via $M's$ platform.

Given that the fringe seller is never able to serve *Type 1* consumers if the specialist charges

$p_R^o = \Delta_R$ on the off-platform market, the platform is indifferent between accepting just one or both hosting inquiries. Given our specified tie-breaking rule and given a similar reasoning as in the proof of Lemma B.1.1 and the above outlined reasoning, R and f are both willing to get hosted[3] and M accepts both hosting inquiries in $t = 1$.

□

Proof of Lemma 2.6

If M does not launch A, the unique market outcome is associated with the following equilibrium outcome. **Pooling Equ.(f)**: M does not launch A. R and f are hosted and charge $p_R^o = \Delta_R$ and $p_f^o = 0$ off-platform and $p_R^m = \tau + \Delta_R$ and $p_f^o = \tau$ on-platform. *Type 1* and *Type 2A* consumers both buy from R on- and off-platform. $\pi_R = (\phi + (1-\phi)\xi)\,\Delta_R$, $\pi_M = (1-\phi)\xi\tau$ and $\pi_f = 0$, $CS_1 = u_A - s$, $CS_{2A} = u_A - \tau$ and $W = (\phi + (1-\phi)\xi)\,(u_A + \Delta_R) - \phi s$. Notice that an equilibrium outcome cannot exist where f is competitive on the off- or the on-platform market. Let us start by proving this claim for the off-platform market. It is ruled out by assumption that R sets a price $p_f^{o'} < 0$ as such prices are weakly dominated pricing strategies. If we can further show that any (mixed) strategy with $sup(T_R) > \Delta_R$ cannot constitute an equilibrium, it immediately follows that f is not competitive on the off-platform market. The following argument applies independently of the pricing strategy that might be played by M:

1. Given that $p_f^o \geq 0$ and given the tie-breaking rule introduced in Section 2.3, R can always guarantee itself a profit equal to $\pi_R = \phi(\Delta_R)$ if setting $p_R^o = \Delta_R$. It directly follows that any $p_R^o > u_A + \Delta_R - s$ cannot be part of T_R, as such prices generate zero sales.

2. Suppose that $sup(T_R) \in (\Delta_R, u_A + \Delta_R - s]$ and the specialist's strategy places an atom on $sup(T_R)$. It follows that f never best-replies by playing any $p_f^o > sup(T_R) - \epsilon$, where ϵ is arbitrarily small, with positive probability as such prices generate zero sales and playing $p_f^o = sup(T_R) - \epsilon$ generates an (expected) profit strictly greater than zero.

[3] Check the proof of the subsequent Lemma 2.6 to understand this claim.

Given $f's$ best-reply, the expected profit from playing $sup(T_R)$ is equal to zero and therefore strictly smaller than $\pi_R = \phi(\Delta_R)$ such that there cannot exist an equilibrium.

3. The argument extends to situations where the specialist's strategy is continuous around $sup(T_R)$. By the *Intermediate Value Theorem*, there now exists a sufficiently small $\epsilon > 0$ such that f chooses $p^o_f = sup(T_R) - \epsilon$ as the upper bound of its pricing strategy.

Given the platform does not launch A, the fringe seller and the specialist are both hosted in $t = 2$, by a similar logic as outlined in Lemma 2.5. Furthermore, the just outlined reasoning extends to the on-platform market where R never sets a price $p^m_R > \tau + \Delta_R$ in equilibrium. Then, the pricing strategies outlined in the *Pooling Equ.(f)* indeed constitute an equilibrium as R already sells to all consumers and is, therefore, strictly worse off if reducing the off- or the on-platform price. It directly follows that the outcome that is associated with the *Pooling Equ.(f)* is the unique outcome of the game if the parameters are such that it can be realized. This is true, even if there exist additional equilibria where f plays some prices $p^o_f > 0$ with positive probability as the fringe seller cannot be competitive on the off- or on-platform market, given the above-outlined rationale.

Conditional on launching A, M again forecloses its platform to third-party sellers by a similar reasoning as in Lemma B.1.1. As M is, thereby, able to extract monopoly profits from *Type 2A* consumers after launching A and transfer payments are not impacted by the existence of a fringe seller, it follows from the identical argument as in the proof of Lemma 2.1 that it is not optimal for M to launch A in $t = 2$ if:

$$\xi < \frac{F}{(1-\phi)(u_A + \Delta_M - \tau)} = C^{Fringe}_{Launch} = C^{Monop.}_{Launch}$$

If this is the case, it follows from the above reasoning that the outcome that is associated with the *Pooling Equ.(f)* is certainly realized in $t = 2$.

□

Proof of Lemma 2.7

By the same reasoning as in the proof of Lemma B.1.1, M forecloses its platform to third-party sellers if launching A. After reversing the reasoning provided in the proof of Lemma 2.6, the *Pooling Equ.(f)* is less profitable for M than launching A if:

$$\xi > C^{Fringe}_{Launch}$$

If, furthermore, M has no incentive to additionally attract *Type 1* consumers for a given vector of parameters, any equilibrium outcome is associated with the following equilibrium:

Separating Equ.(f): M launches A. M charges $p^m_M = u_A + \Delta_M$ on-platform. R and f are not hosted and charge $p^o_R = \Delta_R$ and $p^o_f = 0$ off-platform. *Type 1* consumers purchase A from R directly. *Type 2A* consumers purchase A from M. $\pi_R = \phi\Delta_R$, $\pi_M = (1-\phi)\xi(u_A + \Delta_M) - F$ and $\pi_f = 0$. $CS_1 = u_A - s$ and $CS_{2A} = 0$ and $W = \phi(u_A + \Delta_R - s) + (1-\phi)\xi(u_A + \Delta_M) - F$

As the parameters are such that launching A is profitable for the platform, M launches A. $M's$ on platform pricing strategy follows from an identical argument as in the proof of Lemma 2.2. The off-platform prices follow from the same logic as for Lemma 2.6. Notice further that by a similar reasoning as outlined in the proof of Lemma 2.6, there cannot exist an equilibrium where R plays a price $p^o_R > \Delta_R$ with positive probability such that f is again not competitive on the off-platform market. Thus, any potential equilibrium in $t = 2$ generates an identical outcome as the *Separating Equ.(f)* if the parameters are such that setting $p^m_M = u_A + \Delta_M$ is the most profitable strategy for M

We now continue with finding the parameter space where, conditional on launching A, playing $p^m_M = u_A + \Delta_M$ is the most profitable strategy for M: Given that R sets $p^o_R = \Delta_R$ in the *Separating Equ.(f)*, the most profitable alternative for M is to set $p^m_M = \Delta_M - \epsilon$ where ϵ is negligibly small. Thereby, M additionally sells A to all *Type 1* consumers. M weakly prefers ending up in the *Separating Equ.(f)* outcome if:

$$(1-\phi)\xi(u_A+\Delta_M)-F \geq (\phi+(1-\phi)\xi)\Delta_M - F \tag{B.3}$$

Rearranging yields:

$$\xi > \frac{\Delta_M \phi}{u_A(1-\phi)} = C^{Fringe}_{Competition}$$

Where it follows from Lemma 2.2 that $C^{Fringe}_{Competition} < C^{Monop.}_{Competition}$.

Suppose Condition B.3 is not satisfied. In this case, there can again not exist an equilibrium in pure strategies by a similar argument as outlined in the proof of Lemma 2.3. The mixed strategy equilibrium **Mixed Equ.(f)** is also found in a similar way as in the proof of Lemma 2.3. The only substantial difference is that it follows from a similar argument as in the proof of Lemma 2.6 that R can never play any $p^o_R > \Delta_R$ with positive probability in equilibrium. As a consequence, it follows from a similar argument as applied in the proof of Lemma 2.3 that any $p^m_M \in [\Delta_M, u_A + \Delta_M)$ is a non-rationalizable strategy for M. After analogously exploiting the relevant indifference conditions that again follow from the assumption that M can foreclose its marketplace to charge monopoly prices from *Type 2A* consumers and R can sell to all *Type 1* consumers if at $p^o_R = inf(T_R)$, we find that R and M compete by applying the following mixed strategies:

p^m_M is distributed according to the $c.d.f.$ F^{Fringe}_M
with support $T_M = \left[\frac{u_A+\Delta_M}{\frac{\phi}{(1-\phi)\xi}+1}, \Delta_M\right) \cup \{u_A+\Delta_M\}$, where

$$F^{Fringe}_M(p^m_M) = \begin{cases} 1 - \frac{1}{p^m_M+\Delta_R-\Delta_M}\left(\frac{u_A+\Delta_M}{\frac{\phi}{(1-\phi)\xi}+1} + \Delta_R - \Delta_M\right) & \text{if} \quad \frac{u_A+\Delta_M}{\frac{\phi}{(1-\phi)\xi}+1} \leq p^m_M < \Delta_M \\ 1 - \frac{1}{\Delta_R}\left(\frac{u_A+\Delta_M}{\frac{\phi}{(1-\phi)\xi}+1} + \Delta_R - \Delta_M\right) & \text{if} \quad \Delta_M \leq p^m_M < u_A + \Delta_M \\ 1 & \text{if} \quad u_A + \Delta_M \leq p^m_M \end{cases}$$

p^o_R is distributed according to the $c.d.f.$ F^{Fringe}_R
with support $T_R = \left[\Delta_R - \Delta_M + \frac{u_A+\Delta_M}{\frac{\phi}{(1-\phi)\xi}+1}, \Delta_R\right]$, where

$$F_R^{Fringe}(p_R^o) = \begin{cases} 1 - \frac{(1-\phi)\xi}{\phi}\left(\frac{u_A+\Delta_M}{p_R^o-\Delta_R+\Delta_M} - 1\right) & \text{if} \quad \frac{u_A+\Delta_M}{\frac{\phi}{(1-\phi)\xi}+1} + \Delta_R - \Delta_M \leq p_R^o < \Delta_R \\ 1 & \text{if} \quad \Delta_R \leq p_R^o \end{cases}$$

By otherwise similar arguments as in the proof of Lemma 2.3, it directly follows from reversing the reasoning the initial reasoning that the outlined pricing strategies constitute a unique equilibrium outcome if $\xi \in \left(C_{Launch}^{Fringe}, C_{Comp.}^{Fringe}\right)$.[4]

The (expected) profits of R and M follow from their indifference conditions. $E(\pi_R) = \phi\left(\frac{u_A+\Delta_M}{\frac{\phi}{(1-\phi)\xi}+1} + \Delta_R - \Delta_M\right)$ and $E(\pi_M) = (1-\phi)\xi(u_A + \Delta_M) - F$.

Depending on the prices drawn from $F_M(p_M^m)$ and $F_R(p_R^o)$ *Type 1* consumers can purchase A from M or R. From comparing $F_R^{Fringe}(p_R^o)$ to $F_R(p_R^o)$ from Lemma 2.3 it follows that $inf(T_R)$ is identical. Furthermore, $F_R^{Fringe}(p_R^o) = F_R(p_R^o),\ \forall\ p_R^o \in \{[inf(T_R), \Delta_R)\ \cup\ \{u_A + \Delta_R - s\}\}$ and $F_R^{Fringe}(p_R^o) > F_R(p_R^o),\ \forall\ p_R^o \in [\Delta_R), u_A + \Delta_R - s)$. Thus, it immediately follows that $F_R(p_R^o)$ stochastically dominates $F_R^{Fringe}(p_R^o)$.

Furthermore, it directly follows from the above outlined strategies that *Type 1* consumers never buy product A from M if they sample a price $p_M^m \geq \Delta_M$. Also, notice that $F_M^{Fringe}(p_M^m) = F_M(p_M^m),\ \forall\ p_M^m < \Delta_M$. After combining these arguments, it immediately follows that (i) the expected price faced per unit of valuation for good A by *Type 1* consumers must be strictly lower than without a fringe seller and (ii) relatively more consumers must buy A from R since $M's$ (expected) profit remains unchanged compared to Lemma 2.3. Compared to a monopolistic market structure, it follows from (i) that $E(CS_1|\text{Mixed Equ.(f)}) > E(CS_1|\text{Mixed Equ.}) > 0$ and from (ii) that $E(W|\text{Mixed Equ.(f)}) > E(W|\text{Mixed Equ.})$ as R provides a higher quality product.

Extending the just outlined argument and also considering that $inf(T_M)$ remains unchanged, it follows from comparing the outlined pricing equilibria with and without a fringe seller that $F_M^{Fringe}(p_M^m) = F_M(p_M^m),\ \forall\ p_M^m \in \{[inf(T_M), \Delta_M)\ \cup\ \{u_A + \Delta_M\}\}$. Furthermore, $F_M^{Fringe}(p_M^m) < F_M(p_M^m),\ \forall\ p_M^m \in [\Delta_M, u_A + \Delta_M)$. Thus, it follows that the expected

[4] If $\xi = C_{Launch}^{Fringe}$, there exist equilibria that are associated with the outcome known from the *Pooling Equ.(f)* and the *Mixed Equ.(f)* might co-exist. If $\xi = C_{Competition}^{Fringe}$ a similar argument now applies as the convergence from the *Mixed Equ.(f)* to the *Separating Equ. (f)* is again smooth.

price faced by *Type 2A* consumers must be strictly higher than without a fringe seller. Therefore, $E(CS_{2A}|\text{Mixed Equ. (f)}) < E(CS_{2A}|\text{Mixed Equ.})$. Given the outlined pricing strategies, however, it still holds that $E(CS_{2A}|\text{Mixed Equ. (f)}) > 0$.

□

Proof of Proposition 2.2

The equilibrium follows directly from Lemmas 2.5, 2.6 and 2.7.

In $t = 1$, the *Pooling Equ.(f)* outcome occurs by a similar argument as outlined in Lemma 2.6 as M cannot launch A in $t = 1$ by assumption such that Lemma B.1.1 applies.

It follows from Lemmas 2.6 and 2.7 that, depending on a specific realization of ξ, one of the outcomes specified in these Lemmas is subsequently realized in $t = 2$.

□

Comment on Corollary 2.2

The subsequent reasoning takes scenarios (i)-(iv) as defined in Table 2.2 as given.

In scenario (i), the *Pooling Equ.* occurs with a fringe seller and the *Pooling Equ.(f)* without. The impact of the fringe seller directly follows from comparing the outcomes as defined in (the proofs of) Lemmas 2.1 and 2.6.

In scenario (ii), the *Mixed Equ.* outcome is realized without a fringe seller and the *Mixed Equ.(f)* outcome is realized with a fringe seller. The impact of the fringe seller directly follows from comparing all outcomes as defined in the proofs of Lemmas 2.3 and 2.7 and from the discussion at the end of Lemma 2.7.

In scenario (iii), the *Separating Equ.(f)* outcome is realized with a fringe seller and the *Mixed Equ.* without. The impact on $E(\pi_R)$ and $E(\pi_M)$ directly follows from comparing the payoffs as defined in Lemmas 2.3 and 2.7. Likewise, the impact on $E(CS_{2A})$ follows. The impact on $E(W)$ is also straightforward: Considering Lemmas 2.7 and 2.3, all *Type 1* consumers buy the product of the highest quality in the *Separating Equ.(f)*, whereas only a fraction of *Type 1* consumers strictly smaller than one buy the highest quality product in the *Mixed Equ.*. *Type 2A* consumers never buy the highest quality product in any of the two counterfactuals.

To understand the impact on $E(CS_1)$ consider the following argument: It follows from Lemma 2.3 that $inf(T_M)$ and $inf(T_R)$ are strictly increasing in ξ. Thus, they are both minimized at the lower bound of the interval that characterizes scenario (iii). In the limit, where $\xi = C^{Fringe}_{Competition}$, it follows from the pricing strategies outlined in Lemma 2.3 that $inf(T_M) = \Delta_M$ and $inf(T_R) = \Delta_R$ in the *Mixed Equ.*. Thus, it immediately follows from the outlined pricing strategies in Lemma 2.3 that the expected consumer surplus is strictly smaller than $u_A - s$. If there exists an additional fringe seller, *Type 1* consumers certainly pay $p^o_R = \Delta_R$ for a good manufactured by R and, therefore, experience a consumer surplus equal to $u_A - s$. Consequently, *Type 1* are better off if an additional fringe seller exists.

In scenario (iv), the *Separating Equ.(f)* outcome is realized with a fringe seller and the *Separating Equ.* outcome is realized without. The impact of the fringe seller directly follows from comparing all outcomes as defined in the proofs of Lemmas 2.2 and 2.7.

□

Proofs of section 2.6

To prove all subsequent Propositions, we outline in a first step the equilibria that prevail under the analyzed polices. In a second step, we prove the stated impact by comparing the market outcome conditional on a specific realization of ξ. As we only consider the impact on the expected outcome variables and as $G(\cdot)$ is continuous such that the ex-ante probability that ξ is equal to a specific value equals zero, we only consider scenarios where outlined inequalities are strict. For instance, if a statement is made conditional on $\xi > 0.5$, the term ‘otherwise’ implies $\xi < 0.5$.

Equilibrium outcome 'LOB' (without a fringe seller)

By definition, M cannot launch A. Thus, R does not risk to end up in the *Separating*- or the *Mixed Equ.* outcome in $t = 2$ if being hosted in $t = 1$. Hence, it follows from a similar reasoning as outlined in Lemma 2.1 and Proposition 2.1 that the *Pooling Equ.* outcome is the only possible outcome in $t = 1$ and $t = 2$.

□

Equilibrium outcome 'SEP' (without a fringe seller)

Remark: In $t = 1$ the policy is not implemented by assumption. Thus, there exists a *single* platform. This might also be the case in $t = 2$ if the policy is not binding. In both cases the letter 'M' still is used when referring to *the* platform. This changes if the policy is binding in $t = 2$. When speaking of the former retail arm of the platform, we use the letter 'M''. When speaking of the former platform arm, we use the letter 'M''' or the term 'platform'. When referring to both (combined profits), we continue to use the letter 'M'.

Step 1: It follows from Proposition 2.1 and Lemma 2.1 that, by definition, the policy can only be binding if the specialist is hosted in $t = 1$ and there occurs a realization of ξ such that:

$$\xi > C_{Launch}^{Monop.} \tag{B.4}$$

Otherwise, M' would never launch A in the first place. If this is the case, let me briefly summarize the timing of the game in $t = 2$:

1. M' decides whether to be hosted and M'' decides whether to accept an inquiry.
2. R decides whether to be hosted and M'' decides whether to accept an inquiry.
3. All prices are set simultaneously.
4. Consumers make their purchase decision.

As we are interested in the subgame perfect equilibrium (outcomes), we solve the game by backward induction.

Step 2: Let us first outline the pricing equilibrium outcome where M' and R are hosted. There exists the unique **Hosting Equ.(SEP)** outcome:

R and M are both hosted. R chooses $p^o_R = p^m_R = \Delta_R - \Delta_M + \tau$ and M chooses $p^m_{M'} = \tau$. Type 1 consumers purchase A from R directly. Type 2A consumers purchase A from R via the platform. As a consequence, $CS_1 = \phi(u_A + \Delta_M - \tau - s)$ and $CS_{2A} = (1-\phi)\xi(u_A + \Delta_M - \tau)$. $\pi_R = (\phi + (1-\phi)\xi)(\Delta_R - \Delta_M) + \phi\tau$, $\pi_{M'} = 0$, $\pi_M = \pi_{M''} = (1-\phi)\xi\tau - F$ and $W = (\phi + (1-\phi)\xi(u_A + \Delta_R) - F - \phi s$.

Proof: Consumers purchase the product that maximizes the value per money spent. Their purchase decision follows from the tie-breaking rules. Given that $\tau < u_A - s$, R has no incentive to increase p^o_R and/or p^m_R. A higher price would result in zero sales for R from the respective consumer type. Further reducing the prices is again not reasonable, as R already sells to all consumers. Prices below $p^m_{M'}$ yield a negative profit for M' and, therefore, constitute a weakly dominated strategy ruled out by assumption. It follows from a similar argument as outlined in the proof of Lemma 2.6 that there cannot exist an equilibrium where R chooses a price $p^o_R > \Delta_R - \Delta_M + \tau$ or $p^m_R > \Delta_R - \Delta_M + \tau$. Thus, conditional on R and M both being hosted, the *Hosting Equ.* outcome is unique even if there might exist additional equilibria.

Step 3: There cannot exist an equilibrium where neither M' nor R are hosted by a similar argument as outlined in Lemma 2.5 if an additional fringe seller exists.

It still needs to be clarified which equilibrium outcome occurs if only M' is hosted. By a similar reasoning as in the proof of Lemma 2.2, there exists a unique equilibrium outcome if the parameters are such that M' has no incentive to contest *Type 1* consumers such that markets are segmented. In the **Separating Equ.(SEP)** outcome:

M' charges $p^m_{M'} = u_A + \Delta_M$ on-platform. R is not hosted and charges $p^o_R = u_A + \Delta_R - s$ off-platform. Type 1 consumers purchase A from R directly. Type 2A consumers purchase A from M. $\pi_R = \phi(u_A + \Delta_R - s)$, $\pi_{M'} = (1-\phi)\xi(u_A + \Delta_M - \tau)$, $\pi_{M''} = (1-\phi)\xi\tau - F$,

$\pi_M = (1-\phi)\xi(u_A+\Delta_M)-F$, $CS_1 = CS_{2A} = 0$ *and* $W = \phi(u_A+\Delta_R-s)+(1-\phi)\xi(u_A+\Delta_M)-F$.

It follows from analogous arguments as outlined in the proof of Lemma 2.2 that the **Separating Equ.(SEP)** constitutes the unique equilibrium outcome if the parameters are such that R is not hosted after M' has launched A.

We continue with finding the parameter space where playing $p^m_{M'} = u_A + \Delta_M$ is the most profitable strategy for M'.

Given that R sets $p^o_R = u_A + \Delta_R - s$ in the *Separating Equ.(SEP)*, the most profitable alternative for M' is to set $p^m_{M'} = u_A + \Delta_M - s - \epsilon$ where ϵ is negligibly small. Thereby, M' additionally sells A to all *Type 1* consumers. M' prefers the *Separating Equ.(SEP)* if:

$$(1-\phi)\xi(u_A+\Delta_M-\tau) > (\phi+(1-\phi)\xi)(u_A+\Delta_M-s-\tau) \tag{B.5}$$

Rearranging yields:

$$\xi > \frac{\phi(u_A+\Delta_M-s-\tau)}{(1-\phi)s} = C^{SEP}_{Comp.} \quad \text{where} \quad C^{SEP}_{Comp.} < C^{Monop.}_{Comp.}$$

If Condition B.5 does not hold, a mixed strategy equilibrium occurs by a reasoning already familiar from Lemma 2.3. The **Mixed Equ.(SEP)** is found after analogously exploiting the relevant indifference condition of M':[5]

$$(p^m_{M'}-\tau)[(1-\phi)\xi+\phi(1-(F_R(p^m_{M'}+\Delta_R-\Delta_M)))] = (1-\phi)\xi(u_A+\Delta_M-\tau)$$

We find that R applies the following mixed strategy where p^o_R is distributed according to the *c.d.f.* F^{SEP}_R with support $T_R = \left[\Delta_R-\Delta_M+\frac{u_A+\Delta_M-\tau}{\frac{\phi}{(1-\phi)\xi}+1}, u_A+\Delta_R-s\right]$ where:

$$F^{SEP}_R(p^o_R) = \begin{cases} 1-\frac{(1-\phi)\xi}{\phi}\left(\frac{u_A+\Delta_M-\tau}{p^o_R-\Delta_R+\Delta_M-\tau}-1\right) & \text{if} \quad \dfrac{u_A+\Delta_M-\tau}{\frac{\phi}{(1-\phi)\xi}+1}+\Delta_R-\Delta_M+\tau \le p^m_{M'} \\ & \quad < u_A+\Delta_R-s \\ 1 & \text{if} \quad u_A+\Delta_R-s \le p^o_R \end{cases}$$

[5]Notice that now F are sunk.

It follows from a similar reasoning as outlined in the proof of Lemma 2.3 that any $p^m_{M'} < inf(T_R) - \Delta_R + \Delta_M$ is never played M'. Assuming again that the strategy of M' does not place an atom on $p^m_{M'} = inf(T_R) - \Delta_R + \Delta_M$, one finds the indifference condition of R:

$$p^o_R\phi(1 - (F_{M'}(p^o_R - \Delta_R + \Delta_M))) = \phi\left[\frac{u_A + \Delta_M - \tau}{\frac{\phi}{(1-\phi)\xi} + 1} + \Delta_R - \Delta_M + \tau\right]$$

After performing similar steps as in the proof of Lemma 2.3, one finds that $p^m_{M'}$ is distributed according to the $c.d.f.$ $F^{SEP}_{M'}$ with support $T_{M'} = \left[\frac{u_A+\Delta_M-\tau}{\frac{\phi}{(1-\phi)\xi}+1}, u_A + \Delta_M - s\right) \cup \{u_A+\Delta_M\}$, where

$$F^{SEP}_{M'}(p^m_{M'}) = \begin{cases} 1 - \frac{1}{p^m_{M'}+\Delta_R-\Delta_M}\left(\frac{u_A+\Delta_M-\tau}{\frac{\phi}{(1-\phi)\xi}+1} + \Delta_R - \Delta_M + \tau\right) & \text{if} \quad \frac{u_A + \Delta_M - \tau}{\frac{\phi}{(1-\phi)\xi} + 1} + \tau \leq \\ & \quad p^m_{M'} < u_A + \Delta_M - s \\ 1 - \frac{1}{u_A+\Delta_R-s}\left(\frac{u_A+\Delta_M-\tau}{\frac{\phi}{(1-\phi)\xi}+1} + \Delta_R - \Delta_M + \tau\right) & \text{if} \quad u_A + \Delta_M - s \leq p^m_{M'} \\ & \quad < u_A + \Delta_M \\ 1 & \text{if} \quad u_A + \Delta_M \leq p^m_{M'} \end{cases}$$

The (expected) profits of R and M' follow from their indifference conditions. $E(\pi_R) = \phi\left(\frac{u_A+\Delta_M-\tau}{\frac{\phi}{(1-\phi)\xi}+1} + \Delta_R - \Delta_M + \tau\right)$ and $E(\pi_{M'}) = (1 - \phi)\xi(u_A + \Delta_M - \tau)$.

Step 4: It directly follows from the reasoning in *Steps 5* and *6* that:

Conditional on $\xi > C^{SEP}_{Comp.}$, being hosted is more profitable for the specialist if:

$$\pi_R^{Separ.(SEP)} = \phi(u_A + \Delta_R - s) < (\phi + (1 - \phi)\xi)(\Delta_R - \Delta_M) + \phi\tau = \pi_R^{Host.}$$

If this is the case, the *Hosting Equ.* outcome is realized. Otherwise, the *Separating Equ.(SEP)* outcome is realized.

Conditional on $\xi < C^{SEP}_{Comp.}$, being hosted is more profitable for the specialist if:

$$\pi_R^{Mix.(SEP)} = \phi\left(\frac{u_A + \Delta_M - \tau}{\frac{\phi}{(1-\phi)\xi} + 1} + \Delta_R - \Delta_M + \tau\right) < (\phi + (1 - \phi)\xi)(\Delta_R - \Delta_M) + \phi\tau$$

If this is the case, the *Hosting Equ.(SEP)* outcome is realized. Otherwise, the *Mixed*

Equ.(SEP) outcome is realized.

Step 5: The open question is: Which hosting inquiries will the platform accept if both R and M' want to get hosted?

It follows from a straightforward argument that the platform is willing to accept the hosting inquiry of M' if R is not willing to get hosted subsequently since M'' can generate transfer payments by doing so.

If R is willing to get hosted subsequently, the incentive of M'' to accept a hosting inquiry depends on the number of customers served via the platform. In the *Hosting Equ.(SEP)* and the *Separating Equ.(SEP)* the profit of M'' is equal to $(1-\phi)\xi\tau$ which stems from *Type 2A* being served via the platform. Thus, whenever the *Separating Equ.(SEP)* is the relevant counterfactual, our tie-breaking rule ensures that M'' accepts the hosting inquiries of R and M.

If, however, the *Mixed Equ.(SEP)* is the relevant counterfactual, M'' rejects the hosting inquiry of R, given the following argument: As M' sells to all *Type 2A* consumers with certainty and also to *Type 1* consumers with a probability that is strictly greater than zero, it follows from a straightforward argument that the expected profit of the platform entity, $E(\pi_{M''})$, is bigger in the *Mixed Equ.(SEP)* than in the *Separating Equ. (SEP)*.

Step 6: To summarize the reasoning from all previous steps:

Conditional on $\xi > C^{SEP}_{Comp.}$, the *Hosting Equ.(SEP)* outcome is realized if $\phi(u_A + \Delta_R - s) < (\phi + (1-\phi)\xi)(\Delta_R - \Delta_M) + \phi\tau$. Otherwise, the *Separating Equ.(SEP)* outcome is realized.

Conditional on $\xi < C^{SEP}_{Comp.}$, the *Mixed Equ. (SEP)* outcome is realized with certainty.

□

Equilibrium outcome 'MAN' in $t = 2$ (without a fringe seller)

Step 1: In the subsequent reasoning we take the proofs and outcomes of the *Pooling Equ.* (see Lemma 2.1), the *Separating Equ.* (see Lemma 2.2) and the *Mixed Equ.* (see Lemma 2.3) as given. In the following *Steps 2-5* we implicitly assume that R is hosted in $t = 1$ such that ξ is revealed and M has the possibility to launch A in $t = 2$ since otherwise, the mandatory access regulation is not binding as M cannot launch A in $t = 2$ by assumption.

Step 2: Let us first outline the (unique) equilibrium outcome **Hosting Equ.(MAN)** that occurs if M launches A and R is additionally hosted in $t = 2$:

R chooses $p_R^o = p_R^m = \Delta_R - \Delta_M + \frac{(1-\phi)\xi\tau}{\phi+(1-\phi)\xi}$ and M chooses $p_M^m = \frac{(1-\phi)\xi\tau}{\phi+(1-\phi)\xi}$. Type 1 consumers purchase A from R directly. Type 2A consumers purchase A from R via the platform. As consequence, $CS_1 = u_A + \Delta_M - \frac{(1-\phi)\xi\tau}{\phi+(1-\phi)\xi} - s$ and $CS_{2A} = u_A + \Delta_M - \frac{(1-\phi)\xi\tau}{\phi+(1-\phi)\xi}$. $\pi_R = (\phi + (1-\phi)\xi)\left(\Delta_R - \Delta_M + \frac{(1-\phi)\xi\tau}{\phi+(1-\phi)\xi}\right) - (1-\phi)\xi\tau$ and $\pi_M = (1-\phi)\xi\tau - F$. $W = \phi(u_A + \Delta_R - s) + (1-\phi)\xi(u_A + \Delta_R) - F$.

The proof follows from a similar reasoning as for the *Hosting Equ.(SEP)* in *Step 5* of the Equilibrium 'SEP'. The only difference is that, without a structural separation, M does no longer have to make a transfer payment when selling via the platform such that no price $p_M^m \geq \frac{(1-\phi)\xi\tau}{\phi+(1-\phi)\xi}$ is a weakly dominated strategy for M, conditional on F being sunk. Only prices below this cutoff are ruled out by assumption.

Step 3: If M is not willing to launch A, it follows from Lemma B.1.1 that the *Pooling Equ.* outcome is realized with certainty in $t = 2$. Therefore, a straightforward comparison shows that M is unwilling to launch A if the *Hosting Equ.(MAN)* outcome is subsequently realized. $M's$ profit in the *Hosting Equ.(MAN)* outcome is strictly smaller than in the *Pooling Equ.* outcome as M does not incur fixed costs with the latter.

Step 4: It follows from a similar reasoning as outlined in the proof of Lemma 2.1 that M is not willing to launch A if $\xi < C_{Launch}^{Monop.}$. Otherwise, launching A might be profitable for M if R is not willing to get hosted subsequently.

The open question is: Which equilibrium outcome is realized if launching A is profitable for M, conditional on R not being willing to get hosted in $t = 2$?

- It follows from an identical reasoning as outlined in the proof of Lemma 2.2 that the *Separating Equ.* outcome is realized if $\xi > max\{C_{Launch}^{Monop.}, C_{Competition}^{Monop.}\}$.

- Otherwise, it follows from an identical reasoning as outlined in the proof of Lemma 2.3 that the *Mixed Equ.* outcome is realized if $\xi \in \left(C^{Monop.}_{Launch}, C^{Monop.}_{Competition}\right)$.

Step 5: After combining the above arguments, the *Mixed Equ.* outcome is realized in $t = 2$ if, and only if, a) $C^{Monop.}_{Launch} < \xi < C^{Monop.}_{Comp.}$ and b) $\pi^{Host.(MAN)}_{R} < \pi^{Mix.}_{R}$. If condition b) is not satisfied, the *Pooling Equ.* outcome occurs in $t = 2$ as M is not willing to launch A if subsequently ending up in the *Hosting Equ.(MAN)*.

The *Separating Equ.* outcome is realized in $t = 2$ if, and only if, c) $\xi > max(C^{Monop.}_{Launch}, C^{Monop.}_{Comp.})$ and d) $\pi^{Host.(MAN)}_{R} < \pi^{Separ.}_{R}$. If condition d) is not satisfied, the *Pooling Equ.* outcome occurs in $t = 2$ as, again, M is not willing to launch A if subsequently ending up in the *Hosting Equ.(MAN)*.

□

Proof of Proposition 2.3

Step 1: It follows from the proof of equilibrium outcome 'LOB' that the specialist is hosted with certainty in $t = 1$ with a line of business restriction.

Step 2: With a laissez-faire policy, it follows from the proof of Proposition 2.1 that the willingness to be hosted in $t = 1$ is determined by the probability that $\xi \in \left(C^{Monop.}_{Launch}, C^{Monop.}_{Comp.}\right)$ is realized after being hosted.

With a mandatory access regulation, it follows from *Step 5* of the proof of the *Equilibrium outcome B.1* that $R's$ concerns about expected future losses for a realization of $\xi \in \left(C^{Monop.}_{Launch}, C^{Monop.}_{Competition}\right)$ are equal to these in an unregulated market whenever condition b) is satisfied. However, if condition b) is not satisfied, these are strictly smaller. Furthermore, the profits that are attached to a realization of $\xi > max\{C^{Monop.}_{Launch}, C^{Monop.}_{Competition}\}$ are also strictly greater if condition d) is not satisfied and identical to an unregulated market if condition d) is satisfied.

Step 3: By definition, a structural separation is applied ex-post and does not impact $R's$ incentive to get hosted in $t = 1$.

It follows from *Step 2* and a similar reasoning as applied in the proof of Proposition 2.1 that the set of parameter vectors for which the specialist is willing to get hosted in $t = 1$ is weakly greater with a mandatory access regulation than with the laissez-faire policy.

As shown in *Step 1*, R is certainly hosted in $t = 1$ with a line of business restriction. Hence, the outlined ranking immediately follows.

□

Proof of Proposition 2.4

The proof is based on scenarios (ii)-(iv) as defined in Table 2.4 and Conditions (1)-(6) as defined in Table 2.5.

Step 1: Given scenario (ii) is the relevant counterfactual and Condition (1) applies, it follows from Lemma 2.3 that the *Mixed Equ.* outcome is realized with a laissez-faire policy. It follows from the proof of equilibrium outcome 'MAN' that the same applies with a mandatory access policy if Condition (1) applies. It follows from the proof of equilibrium outcome 'LOB' that the *Pooling Equ.* outcome is realized with a line of business restriction. Finally, it follows from the proof of equilibrium outcome 'SEP' that the *Mixed Equ.(SEP)* outcome is realized with a structural separation. The ranking then follows from a straightforward comparison of the outcomes that are associated with the respective equilibria. It follows from a non-trivial comparison that $E(W)$ is higher in the *Mixed Equ.(SEP)* than in the *Mixed Equ.* as $F^{SEP}_{M'}(p^m_{M'})$ stochastically dominates $F_M(p^m_M)$ and the former also places a bigger atom on $p^m_{M'} = u_A + \Delta_M$. Therefore, relatively fewer *Type 1* consumers buy product A of inferior quality from M' than in the *Mixed Equ.*.

Step 2: Given scenario (ii) is the relevant counterfactual and Condition (2) applies, it follows from Lemma 2.3 that the *Mixed Equ.* outcome is realized with a laissez-faire policy. It follows from the proof of equilibrium outcome 'LOB' that the *Pooling Equ.* outcome is realized with a line of business restriction. It follows from the proof of equilibrium outcome 'MAN' that the same is true with a mandatory access policy. Finally, it follows from the proof of equilibrium outcome 'SEP' that the *Mixed Equ.(SEP)* outcome is realized with a structural separation. The ranking then follows from a straightforward comparison of the outcomes that are associated with the respective equilibria. Notice that with the mandatory access regulation, no fixed cost from launching *A* accrue, which is why it outperforms the structural separation.

Step 3: Given scenario (iii) is the relevant counterfactual and Conditions (1) and (5) apply, it follows from Lemma 2.3 that the *Mixed Equ.* outcome is realized with a laissez-faire policy. It follows from the proof of equilibrium outcome 'MAN' that the same is true with a mandatory access policy. It follows from the proof of equilibrium outcome 'LOB' that the *Pooling Equ.* outcome is realized with a line of business restriction. Finally, it follows from the proof of equilibrium outcome 'SEP' that the *Separating Equ.(SEP)* outcome is realized with a structural separation. The ranking then follows from a straightforward comparison of the outcomes that are associated with the respective equilibria.

Step 4: Given scenario (iii) is the relevant counterfactual and Conditions (1) and (6) apply, the reasoning is vastly the same as in the previous step. There is one exception: It follows from the proof of equilibrium outcome 'SEP' that the *Hosting Equ.(SEP)* outcome is realized with a structural separation. It follows from the proof of equilibrium outcome 'SEP' that the *Hosting Equ.(SEP)* is only realized if it is more profitable for *R* than the *Separating Equ.(SEP)*. However, compared to the *Pooling Equ.*, the specialist is still worse off and fixed costs are incurred by the platform, which is why the ranking is the same as in the previous step. (This also directly follows from a straightforward comparison of the outcomes associated with the respective equilibria.)

Step 5: Given scenario (iii) is the relevant counterfactual and Conditions (2) and (5) apply, it follows from Lemma 2.3 that the *Mixed Equ.* outcome is realized with a laissez-faire policy. It follows from the proof of equilibrium outcome 'LOB' that the *Pooling Equ.* outcome is realized with a line of business restriction. It follows from the proof of equilibrium outcome 'MAN' that the same is true with a mandatory access policy. Finally, it follows from the proof of equilibrium outcome 'SEP' that the *Separating Equ.(SEP)* outcome is realized with a structural separation. The ranking then follows from a straightforward comparison of the outcomes that are associated with the respective equilibria.

Step 6: Given scenario (iii) is the relevant counterfactual and Conditions (2) and (6) apply, the only difference to the previous step is that the *Hosting Equ.(SEP)* outcome is realized with a structural separation, which follows from the proof of equilibrium outcome 'SEP'. It follows from a straightforward comparison that the ranking remains unchanged by a similar argument as outlined in *Step 4*.

Step 7: Given scenario (iv) is the relevant counterfactual and Conditions (3) and (5) apply, it follows from Lemma 2.3 that the *Separating Equ.* outcome is realized with a laissez-faire policy. It follows from the proof of equilibrium outcome 'MAN' that the same is true with a mandatory access policy. It follows from the proof of equilibrium outcome 'LOB' that the *Pooling Equ.* outcome is realized with a line of business restriction. Finally, it follows from the proof of equilibrium outcome 'SEP' that the *Separating Equ.(SEP)* outcome is realized with a structural separation. The ranking then follows from a straightforward comparison of the outcomes that are associated with the respective equilibria.

Step 8: Given scenario (iv) is the relevant counterfactual and Conditions (3) and (6) apply, the reasoning is vastly similar to the previous step, only that now the *Hosting Equ.(SEP)* outcome is realized with a structural separation. As this can only be the case if it yields a higher profit for the specialist than the *Separating Equ.(SEP)*. Notice also that compared to the *Separating Equ./(SEP)*, all consumers buy the higher quality product. The ranking, therefore, immediately follows.

Step 9: Given scenario (iv) is the relevant counterfactual and Condition (4) applies, it follows from the proof of equilibrium outcome 'SEP' that (4) also implies that the payoff in the *Hosting Equ.(SEP)* is certainly greater than the payoff in the *Separ. Equ.(SEP)* since the specialist's payoff in the *Separ. Equ.(SEP)* and the *Separ. Equ.* are identical and the payoff in the *Hosting Equ.(SEP)* is strictly greater than in the *Hosting Equ.(MAN)*, which additionally follows from a comparison to the proof of equilibrium outcome 'MAN'.

Thus, the *Hosting Equ.(SEP)* is certainly realized with a structural separation if (4) is satisfied. It also follows from Lemma 2.3 that the *Separating Equ.* outcome is certainly realized with a laissez-faire policy. It follows from the proof of equilibrium outcome 'LOB' that the *Pooling Equ.* outcome is realized with a line of business restriction. It follows from the proof of equilibrium outcome 'MAN' that the same is true with a mandatory access policy. The ranking then follows from a straightforward comparison of the outcomes that are associated with the respective equilibria.

□

Proof of Proposition 2.5

The proof is based on scenarios (ii)-(iv) as defined in Table 2.4 and Conditions (1)-(6) as defined in Table 2.6.

Step 1: Given scenario (ii) is the relevant counterfactual and Condition (1) applies, the realized outcomes are the same as defined in *Step 1* of the proof of Proposition 2.4. Since the distribution functions of R *and* M' in the *Mixed Equ.(SEP)* stochastically dominate these in the *Mixed Equ.*, both consumer types are in expectation worse off with a structural separation. However, it also directly follows from the strategies played in the *Mixed Equ.(SEP)* that the consumer surplus of both consumer types is, in expectation, still strictly greater than zero. The ranking then follows from a straightforward comparison of the distinct outcomes.

Step 2: Given scenario (ii) is the relevant counterfactual and Condition (2) applies, the realized outcomes are the same as defined in *Step 2* of the proof of Proposition 2.4. Furthermore, a similar reasoning as in the previous step applies. The ranking then follows from a straightforward comparison of the distinct outcomes.

Step 3: Given scenario (iii) is the relevant counterfactual and Conditions (1) and (5) apply, the realized outcomes are the same as defined in *Step 3* of the proof of Proposition 2.4. The ranking then follows from a straightforward comparison of the distinct outcomes.

Step 4: Given scenario (iii) is the relevant counterfactual and Conditions (1) and (6) apply, the realized outcomes are the same as defined in *Step 4* of the proof of Proposition 2.4. The ranking then follows from a straightforward comparison of the distinct outcomes.

Step 5: Given scenario (iii) is the relevant counterfactual and Conditions (2) and (5) apply, the realized outcomes are the same as defined in *Step 5* of the proof of Proposition 2.4. The ranking then follows from a straightforward comparison of the distinct outcomes.

Step 6: Given scenario (iii) is the relevant counterfactual and Conditions (2) and (6) apply, the realized outcomes are the same as defined in *Step 6* of the proof of Proposition 2.4. The ranking then follows from a straightforward comparison of the distinct outcomes.

Step 7: Given scenario (iv) is the relevant counterfactual and Conditions (3) and (5) apply, the realized outcomes are the same as defined in *Step 7* of the proof of Proposition 2.4. The ranking then follows from a straightforward comparison of the distinct outcomes.

Step 8: Given scenario (iv) is the relevant counterfactual and either Conditions (3) and (6) or Condition (4) applies, the realized outcomes are the same as defined in *Step 8* or *Step 9* of the proof of Proposition 2.4. The ranking then follows from a straightforward comparison of the distinct outcomes.

□

Proof of Corollary 2.4

Step 1: The reasoning behind Lemma 2.5 continues to hold such that, given our tie-breaking rule, the specialist is hosted with certainty in the first period, independent of which policy applies. Hence, it follows from a similar reasoning as outlined in Lemma 2.6 that the *Pooling Equ.(f)* outcome is realized in $t = 1$. As a consequence, all consumers experience a consumer surplus equal to $u_A - \tau$.

Step 2: With a line of business restriction, it follows from a similar reasoning as outlined in the proof of Lemma 2.6 that the *Pooling Equ.(f)* outcome is also realized with certainty if there exists an additional fringe seller. Again, all consumers experience a consumer surplus equal to $u_A - \tau$.

Step 3: With a mandatory access regulation, M never launches A if the specialist is subsequently hosted by a similar reasoning as outlined in proof of Equilibrium outcome 'MAN'. In this case, the *Pooling Equ.(f)* outcome is again realized by a similar reasoning as outlined in the proof of Lemma 2.6.

If it is more profitable for R to not be hosted after M launches A, it follows from a similar reasoning as in Lemma 2.5 and from our tie-breaking rule that at least the fringe seller is certainly hosted. Then, it follows from a similar reasoning as outlined in the proof of Lemma 2.6 that $p_M^m = \tau + \Delta_M$ constitutes an upper bound for any pricing strategy of M that is potentially played in equilibrium

In both scenarios *Type 2A* consumers experience a consumer surplus equal to $u_A - \tau$.

□

B.2 Further Discussion

B.2.1 Auxiliary Assumption on Fixed Costs

It can be made part of the players' equilibrium strategies that S and M can only learn about on-platform demand through hosting R, by assuming sufficiently high fixed costs that make launching A in expectation unprofitable. This is the case if:

$$F > (1+\delta)(1-\phi)E(\xi)(u_A + \Delta_M - \tau) + (1-\phi)E(\xi)(1 - Pr(\xi \textit{ is revealed in } t = 0))\tau$$

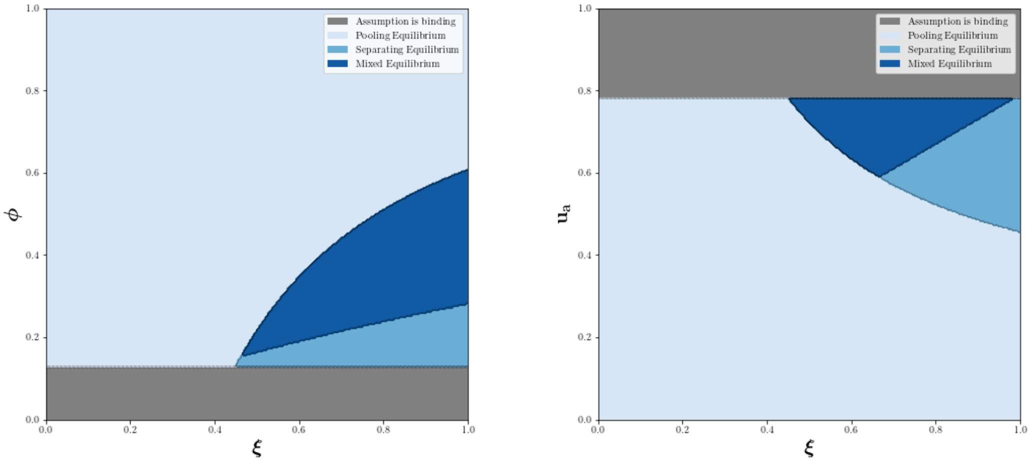
Where $Pr(\xi \textit{ is revealed in } t = 0)) \in \{0, 1\}$ and $Pr(\xi \textit{ is revealed in } t = 0)) = 1$ if Condition 2.4 from Proposition 2.1 is satisfied.

Figure B.1 shows that such an auxiliary assumption on F does not rule out any of the three mutually exclusive equilibria in $t = 2$ being realized. In both panels, we choose the same parameter values as in Figure 2.2.

We see from both panels that, ceteris paribus, the auxiliary assumption on F is binding for high levels of u_A or low levels of ϕ. Both parameters directly determine the revenue from *Type 2A* consumers.

However, we also observe that the auxiliary assumption on F is not binding for most parameter sets. Both panels show that for a given level of ϕ or u_A, where all pricing equilibria are possible, the *Pooling Equ.* outcome is realized for low levels of ξ, the *Separating Equ.*

Figure B.1: Numerical Simulation with Assumption on F



(a) Equilibrium outcome depending on ξ and ϕ

(b) Equilibrium outcome depending on ξ and u_A

outcome is realized for high levels of ξ and the *Mixed Equ.* outcome is realized for intermediate levels of ξ.

B.2.2 Comparative Statics: Second Stage

Lemmas 2.1-2.3 outline how the realization of second stage equilibrium outcomes depend on ξ. We now want to analyze the impact of all parameters on the realization of different euilibria in the second stage. In the following, we assume that Condition B.6 holds such that all pricing equilibria can be realized.

$$0 < C^{Monop.}_{Launch} < C^{Monop.}_{Battleground} < 1 \tag{B.6}$$

The impact of some parameters is straightforward to analyze. For instance, with increasing levels of the fixed costs F, the ex-ante likelihood that the *Pooling Equilibrium* outcome is subsequently realized increases. The same is true for higher levels of τ as profits from launching A must compensate the platform for higher foregone transfer payments from not hosting S.

From Lemmas 2.2 and 2.3 it follows that the shopping costs s impact the realization of the *Separating Equ.* and the *Mixed Equ.* conditional on M launching A. An increase in s reduces $M's$ incentive to contest *Type 1* consumers and, therefore, increases the set of parameter vectors for which the *Separating Equ.* outcome occurs.

The impact of ϕ, u_A and Δ_M on the realization of pricing equilibria is, however, equivocal as these parameters enter both cutoffs that determine $M's$ decision to launch A and to contest *Type 1* consumers.

$M's$ profit is inversely correlated with ϕ as the share of *Type 1* consumers directly impacts the guaranteed profits M can obtain when charging monopoly prices from *Type 2A* consumers. These determine the platform's (expected) profit under the *Separating Equ.* outcome and the *Mixed Equ.* outcome that is traded off against the fixed costs F from launching A. Conditional on launching A, the *Separating Equ.* outcome is realized for low levels of ϕ where additional demand from *Type 1* consumers is an insufficient incentive for M to lower its price below the willingness to pay of *Type 2A* consumers.

The impact of Δ_M is identical to the impact of u_A as both enter Lemmas 2.1-2.3 in a similar way. At low levels of u_A, the *Pooling Equ.* outcome occurs as the (expected) profit from launching product A is too small for M to recover F. Furthermore, u_A is scaling the (expected) profits from all sales. At high levels of u_A, M is willing to forego profits from *Type 2A* consumers by selling A at an expected price lower than the monopoly price to contest *Type 1* consumers such that the *Mixed Equ.* outcome occurs.

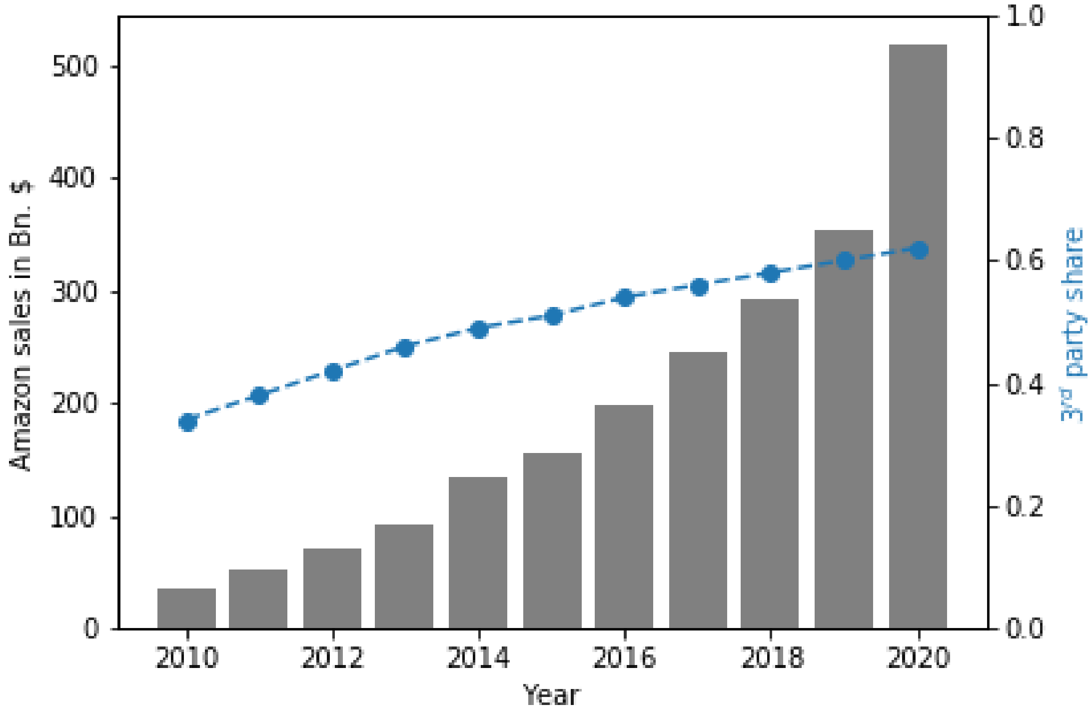
B.2.3 Comparative Statics: First Stage

Besides their indirect impact in the second period, some parameters also directly affect the specialist's incentives to get hosted in the first period.

With increasing levels of s, getting hosted in the first period becomes more attractive as the specialist's losses in the *Mixed Equ.* are relatively less harmful compared to a monopolistic off-platform market where the specialist needs to reimburse off-platform customers for their incurred shopping costs. The opposite is true for increasing levels of u_A and Δ_M. The parameter τ negatively impacts immediate profits and, therefore, the specialist's incentive to get hosted.

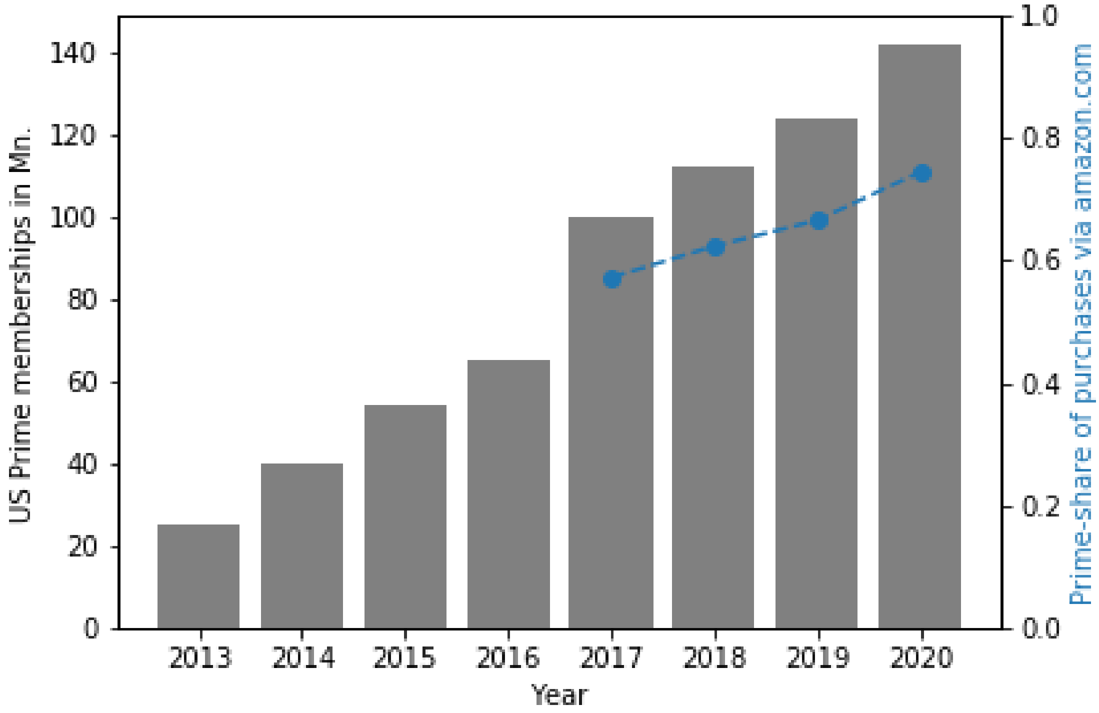
B.3 Graphs

Figure B.2: Amazon Third- and First-Party Share



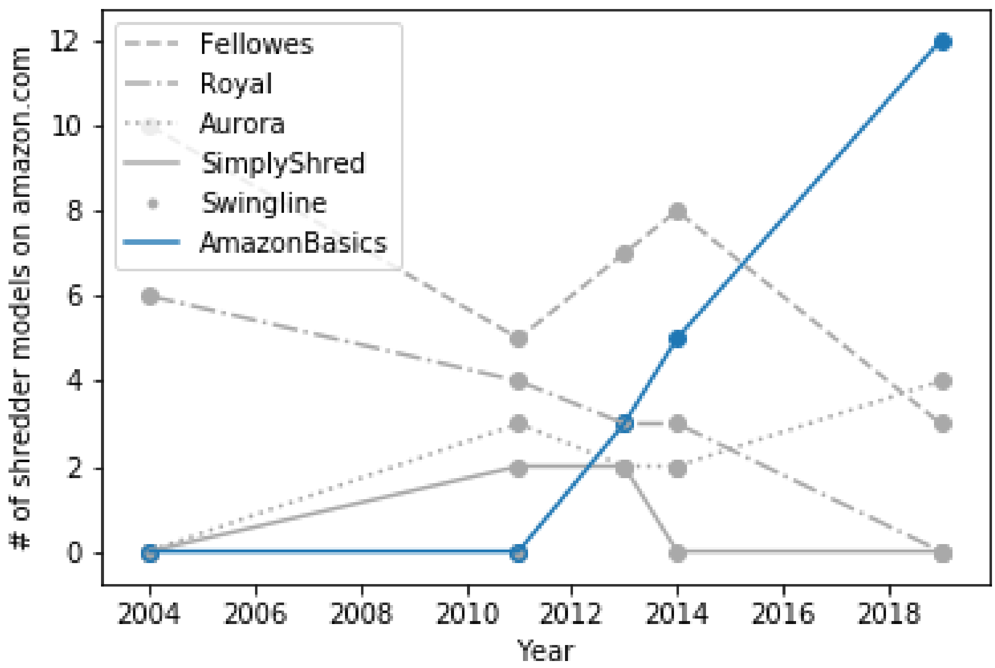
Data obtained from "Amazon annual report" (2018) and **marketplaces2020**.

Figure B.3: Amazon Prime Member Statistics



Data obtained from Lipsman (2020) and Kaziukenas (2020).

Figure B.4: Amazon Shredder Example



Data compiled by the author via web.archive.org.

Appendix C

Appendix to Chapter 3

C.1 Proofs

Throughout the appendix, $inf(T_R)$ denotes the infimum of the specialist's support T_R, and $sup(T_R)$ the supremum if the specialist applies a mixed pricing strategy. Likewise, $inf(T_M)$ and $sup(T_M)$ denote the infimum and the supremum of the platform's support T_M. Furthermore, (expected) overall welfare is defined as the sum of (expected) consumer surplus of both consumer types and the specialist's and the platform's expected profit: $E(W) = E(CS_1) + E(CS_{2A}) + E(\Pi_S) + E(\Pi_M)$.

Proof of Proposition 3.1

Step 1: Proof of pricing strategies in *Direct Competition Outcome*, given that no exclusive contract is entered, M launches A, fixed costs are sunk and the specialist is hosted.

Playing any strategy where some $p_R^{on} < \tau_H$ is chosen with a positive probability that induces some consumers to buy product A from R via the platform certainly yields a negative profit for R, which cannot be optimal. The specialist is indifferent between $p_R^{on} = \tau_H$ and playing any $p_R^{on} < \tau_H$ if nobody buys from R via the platform. Therefore, any $p_R^{on} < \tau_H$ is a weakly dominated strategy for the specialist, which is ruled out by assumption.

Furthermore, there cannot exist an equilibrium where some $p_M^{on} > \tau_H$ is played with positive probability. To show this, I demonstrate that there exists no equilibrium such that $sup(T_M) > \tau_H$ where $sup(T_M)$ characterizes the supremum of the support of $M's$ pricing strategy.

1. Given that any $p_R^{on} < \tau_H$ is weakly dominated and therefore ruled out by assumption, it follows from the tie-breaking rule introduced in Section 3.2 that M can always guarantee itself a profit equal to $\pi_M = (1-\phi)\xi\tau_H - F$ if setting $p_M^{on} = \tau_H$. As any $p_M^{on} > u_A$ does not generate any sales, it directly follows that playing such a price with positive probability is a non-rationalizable strategy for M, independent of the strategy of R.

2. Suppose that $sup(T_M) \in (\tau_H, u_A]$ and $M's$ pricing strategy places an atom on $sup(T_M)$. It follows that the specialist's expected profit from just *Type 2A* consumers if charging $p_R^{on} = sup(T_M) - \epsilon$, where ϵ is arbitrarily small, must be strictly greater than zero. Furthermore, playing any $p_R^{on} > sup(T_M) - \epsilon$ generates zero sales and can therefore never be optimal.[1] Given the specialist's best reply, the expected profit from playing $sup(T_M)$ is therefore equal to $-F$ such that there cannot exist an equilibrium.

3. The argument extends to situations where $M's$ strategy is continuous around $sup(T_M)$. By the intermediate value theorem, there must now exist a sufficiently small $\epsilon > 0$ such that the specialist chooses $p_R^{on} = sup(T_M) - \epsilon$ as the upper bound of its on-platform pricing strategy.

Given $sup(T_M) \leq \tau_H$, I continue by showing that there cannot exist an equilibrium in pure pricing strategies:

1. Given $b < \tau_H$, which follows from *A3*, any $p_M^{on} \leq b$ also is a non-rationalizable strategy for M as setting $p_M^{on} = \tau_H$ certainly yields a higher profit for M.

[1] Remark: If playing $p_R^{on} = sup(T_M) - \epsilon$ pushes some *Type 1* consumers that would otherwise buy from the specialist's off-platform distribution channel into buying on-platform in some conditions, the specialist's profit jumps discontinuously downwards. In this case, the specialist optimally reacts by deducting ϵ from all off-platform prices $p_R > 0$ that are played with positive probability. Since the (expected) off-platform profit is continuous in $E(p_R)$ and since ϵ is arbitrarily small, the change in the (expected) off-platform profit can be neglected. Notice that if no $p_R > 0$ is played with positive probability, inducing some additional *Type 1* consumers to buy on-platform when playing $p_R^{on} = sup(T_M) - \epsilon$ does not reduce the specialist's off-platform profit that is equal to zero in any case.

2. Given that, in equilibrium, $b < p_M^{on} \leq \tau_H$ and, again, given $b < \tau_H$, playing any $p_R > \tau_H - b$ or $p_R \leq 0$ can never be optimal for R as such prices either generate no sales or a profit (weakly) smaller than zero. Given the outlined tie-breaking rule, switching to $p_R = p_M^{on} - b$ generates a profit strictly greater than zero for R from *Type 1* consumers, independent of the pricing strategy of M.

3. If M has no incentive to compete for *Type 1*, an equilibrium can only exist if M plays $p_M^{on} = \tau_H$, given the initial reasoning and the specified tie-breaking rule.

4. If M has an incentive to contest *Type 1* consumers as playing $p_M^{on} = \tau_H$ is not optimal, M best replies to any pure strategy $p_R \in (0, \tau_H - b]$ by setting $p_M^{on*}\left(p_R\right) = p_R + b - \epsilon$ where ϵ is arbitrarily small.

5. $M's$ profit from playing $p_M^{on*}\left(p_R\right)$ is $(\phi + (1-\phi)\xi) p_M^{on*}\left(p_R\right) - F$. The profit from playing $p_M^{on} = \tau_H$ is $(1-\phi)\xi\tau_H - F$. Thus, if $\frac{\phi p_M^{on*}(p_R)}{(1-\phi)\left(\tau_H - p_M^{on*}(p_R)\right)} > \xi$ for a given p_R, it is optimal for M to play $p_M^{on*}\left(p_R\right)$ instead of $p_M^{on} = \tau_H$.

6. Since $M's$ profit is continuously increasing in $p_M^{on*}\left(p_R\right)$ and playing $p_M^{on*}\left(p_R\right)$ if $p_R = 0$ is certainly not optimal by the argument outlined in (1), there always exists a $\underline{p_R}$ where $0 < \underline{p_R} < \tau_H - b$ such that M rather charges $p_M^{on} = \tau_H$ instead of $p_M^{on*}\left(p_R\right)$ if $p_R < \underline{p_R}$ by the *Intermediate Value Theorem*.

7. For any $p_R > \underline{p_R}$ where M chooses to play $p_M^{on*}\left(p_R\right)$, a strategy profile of the form $\left(p_R, p_M^{on*}\left(p_R\right)\right)$ cannot be an equilibrium as the specialist makes no sales at p_R. By lowering its price to $p_R^{'} = p_R - \epsilon > 0$, R is able to attract all *Type 1* consumers and is therefore strictly better off.

8. For any $p_R < \underline{p_R}$, a strategy profile of the form $\left(p_R, \tau_H\right)$ cannot be an equilibrium as, given M plays $p_M^{on} = \tau_H$, R is better off by playing $p_R = \tau_H - b > \underline{p_R}$, where R is still able to sell to all *Type 1* consumers.

9. For any $p_R = \underline{p_R}$ and M best-replying with $p_M^{on} = \tau_H$, a similar reasoning as in the previous step applies. Otherwise, if M best-replies with $p_M^{on*}\left(p_R\right)$, a similar reasoning as in the second to last step applies.

I now continue with finding the mixed strategy equilibrium:

Given the above reasoning, only $p_M^{on} \leq \tau_H$ is played with positive probability in equilibrium. Hence, there cannot exist an equilibrium where some $p_R > \tau_H - b \in T_R$ since R would certainly generate no sales as already outlined above.

The *cdf* of R that describes the distribution of p_R $(F_{R,D}(p_S))$ must be such that M is indifferent for all $p_M^{on} \in T_M$ and, given the specialist's strategy, all $p_M^{on} \notin T_M$ must yield a weakly lower (expected) profit than any $p_M^{on} \in T_M$. M can always guarantee itself a profit equal to $(1-\phi)\xi\tau_H - F$ when charging $p_M^{on*} = \tau_H$ from *Type 2A* consumers. Therefore, $M's$ indifference condition is:

$$p_M^{on}[(1-\phi)\xi + \phi(1-(F_{R,D}(p_M^{on}-b)))] - F = (1-\phi)\xi\tau_H - F$$

Letting $p_M^{on} = p_R + b$ and rearranging yields:

$$F_{R,D}(p_R) = 1 - \frac{(1-\phi)\xi}{\phi}\left(\frac{\tau_H}{p_R + b} - 1\right)$$

$F_{R,D}\left(p_R\right)$ is a *cdf* if, and only if:

1. $\lim_{p_R \to inf(T_R)} F_{R,D}\left(p_R\right) = 0$. This holds for $inf(T_R) = \frac{\tau_H}{\frac{\phi}{(1-\phi)\xi}+1} - b$. Suppose for the moment that this is greater than zero. This assumption is verified ex-post in *Step 2* of the proof of Lemma 3.3, after outlining the realizations of ξ where M is willing to launch A.

2. $\lim_{p_R \to sup(T_R)} F_{R,D}\left(p_R\right) = 1$. This holds for $sup(T_R) = \tau_H - b$.

The cdf of M that describes the distribution of p_M^{on} $\left(F_{M,D}\left(p_M^{on}\right)\right)$ must be such that R is indifferent for all $p_R \in T_R$ and, given $M's$ strategy, all $p_R \notin T_R$ must yield a weakly lower (expected) profit than any $p_R \in T_R$. Notice that any $p_M^{on} < inf(T_R) + b$ would contradict $M's$ indifference condition outlined above. Let me assume for the moment that $F_{M,D}\left(p_M^{on}\right)$ does not place an atom on $p_M^{on} = inf(T_R) + b$ such that, given the specified tie-breaking rule, R is able to sell A to all *Type 1* consumers when charging $p_R = inf(T_R)$, which yields a profit of $\phi\ inf(T_R)$. Therefore, the specialist's indifference condition is equal to:

$$\phi p_R \left(1 - F_{M,D}\left(p_R + b\right)\right) = \phi \left(\frac{\tau_H}{\frac{\phi}{(1-\phi)\xi} + 1} - b \right)$$

Letting $p_R = p_M^{on} - b$ and rearranging yields:

$$F_{M,D}\left(p_M^{on}\right) = 1 - \frac{1}{p_M^{on} - b} \left(\frac{\tau_H}{\frac{\phi}{(1-\phi)\xi} + 1} - b \right)$$

For $F_{M,D}\left(p_M^{on}\right)$ to be a *cdf*, the following must apply:

1. $\lim_{p_M^{on} \to inf(T_M)} F_{M,D}M\left(p_M^{on}\right) = 0$. This holds for $inf(T_M) = \frac{\tau_H}{\frac{\phi}{(1-\phi)\xi}+1}$. Suppose that this is greater than b as required in equilibrium by the above arguments. This assumption is verified ex-post in Lemma 3.3, after outlining the realizations of ξ where M is willing to launch A. Notice that $F_{M,D}\left(p_M^{on}\right)$ is atomless at $inf(T_M)$, which is what I assumed earlier.

2. $\lim_{p_M^{on} \to sup(T_M)} F_{M,D}\left(p_M^{on}\right) = 1$. However, $F_{M,D}\left(p_M^{on}\right) < 1, \quad \forall \quad p_M^{on} \in [inf(T_M), \tau_H]$. Yet, any $p_M^{on} > \tau_H$ cannot be part of the support of $F_{M,D}\left(p_M^{on}\right)$ because such prices would contradict $M's$ indifference condition outlined in detail above. Therefore, M places an atom of size $\frac{1}{\tau_H - b}\left(\frac{\tau_H}{\frac{\phi}{(1-\phi)\xi}+1} - b\right)$ on $p_M^{on} = \tau_H$.

Next, I show that $inf(T_M) \in T_M$. Suppose that this is not the case. Then:

$$\frac{\partial F_M\left(p_R\right)}{\partial p_R}\Big|_{p_R = inf(T_R)} = 0$$

Thus, there exists a sufficiently small $\epsilon > 0$ where the difference $F_{M,D}\left(p_R + b + \epsilon\right) - F_{M,D}\left(p_R + b\right)$ is negligible. Consequently, the indifference condition of the specialist is violated, and the strategies outlined above do not constitute an equilibrium. By a similar argument, $inf(T_R) \in T_R$ and by an analogous argument $sup(T_R) \in T_R$. $sup(T_M) \in T_M$ by the definition of $F_{M,D}\left(p_M^{on}\right)$.

I directly follows from the above outlined strategies that any $p_R < inf(T_R)$ or $p_M^{on} <$

$inf(T_M)$ violate the outlined indifference conditions as both players' strategies are atomless at the infimum.

Given that in any equilibrium $sup(T_M) = \tau_H$ and given that any price $p_R^{on} < \tau_H$ is ruled out by assumption, the specialist is not competitive on the on-platform market if hosted. In this case, there certainly exists an equilibrium where R chooses the pure strategy $p_R^{on} = \tau_H$. Suppose there exist additional equilibria where a/some $p_R^{on} > \tau_H$ is/are played with positive probability by the specialist. In that case, all players' payoffs remain unchanged as $p_M^{on} \leq \tau_H$ in any alternative equilibria by the initially outlined argument. Thus, any potnetial equilibrium outcome is associated with the *Direct Competition Outcome*.

The specialist's and the platform's expected profit follow from the indifference conditions outlined above. It also directly follows that $E(C_1^D), E(C_{2A}^D) > 0$.

Step 2: Proof of pricing strategies in *Cross Competition Outcome*, given that no exclusive contract is entered, M launches R, fixed costs are sunk and the specialist is not hosted.

If the specialist is not hosted, M is able to charge monopoly prices from *Type 2A* consumers if setting $p_M^{on} = u_A$. By an analogous reasoning as in *Step 1*, there cannot exist an equilibrium in pure strategies.[2]

By exploiting that, in equilibrium, M needs to be indifferent between playing a mixed strategy and receiving a payoff equal to $(1 - \phi)\xi u_A - F$ when charging monopoly prices from just *Type 2A* consumers, $F_{R,C}$ follows from an analogous procedure as applied in the previous step. Furthermore, R can guarantee itself a payoff equal to $\phi \ inf(T_R)$ if playing $p_R = inf(T_R)$. By exploiting this indifference condition, $F_{M,C}$ follows from similar reasoning as in the previous step. Therefore, the proofs are omitted. The resulting mixed strategy equilibrium is unique as the specialist is not hosted. The equilibrium is associated with the following distribution functions:

$$F_{R,C}\left(p_R\right) = 1 - \frac{(1-\phi)\xi}{\phi}\left(\frac{u_A}{p_R + b} - 1\right)$$

[2] The only difference to the previous step is that M sets $p_M^{on} = u_A$ if not being incentivized to attract *Type 1* consumers and the specialist never chooses an off-platform price $p_R > u_A - b$. Otherwise, the analysis is similar.

Where $inf(T_R) = \frac{u_A}{\frac{\phi}{(1-\phi)\xi}+1} - b$. Let me suppose that this is greater than zero, which is again verified ex-post in Lemma 3.3, after outlining the realizations of ξ where M is willing to launch A. Furthermore, $sup(T_R) = u_A - b$.

$$F_{M,C}\left(p_M^{on}\right) = 1 - \frac{1}{p_M^{on} - b}\left(\frac{u_A}{\frac{\phi}{(1-\phi)\xi} + 1} - b\right)$$

Where $inf(T_M) = \frac{u_A}{\frac{\phi}{(1-\phi)\xi}+1}$. Suppose that this is greater than b as required in equilibrium. This assumption is also verified ex-post in *Step 2* of the proof of Lemma 3.3, after outlining the realizations of ξ where M is willing to launch A. Furthermore, M places an atom of size $\frac{1}{u_A-b}\left(\frac{u_A}{\frac{\phi}{(1-\phi)\xi}+1} - b\right)$ on $p_M^{on} = u_A$.

As the outlined strategies in the *Cross Competition Outcome* first-order stochastically dominate these in the *Direct Competition Outcome*, but still charge an expected price below the monopoly price, it directly follows that $E(C_1^D) > E(C_1^C) > 0$ and $E(C_{2A}^D) > E(C_{2A}^C) > 0$.

Step 3: With $u_A > \tau_H$, $E(\pi_R^C) = \phi\left(\frac{u_A}{\frac{\phi}{(1-\phi)\xi}+1} - b\right) > \phi\left(\frac{\tau_H}{\frac{\phi}{(1-\phi)\xi}+1} - b\right) = E(\pi_R^D)$ such that Proposition 3.1 immediately follows as the specialist is optimally not hosted after M launches A.

□

Proof of Lemma 3.1

Step 1: I start by showing that the pricing strategy in the *Hosting Outcome (H)* constitutes a unique equilibrium outcome in a setting where the platform does not launch A and the specialist is hosted in $t = 2$:

R has no incentive to decrease p_R^{on} and p_R because it already sells A to all potential consumers. R has no incentive to increase p_R^{on} because demand would discretely jump to zero. R also has no incentive to increase p_R as *Type 1* consumers would switch to buying product A via the platform, which cannot be profitable for R given $b < \tau_H$ by *A3*.

By the reversed reasoning the outcome constitutes a unique pricing equilibrium if $\xi > 0$. Given $\xi = 0$, however, there exist infinitely many pricing equilibria as R is indifferent between all possible on-platform prices. Nevertheless, all pricing equilibria generate an identical outcome as the outlined *Hosting Outcome (H)*.

Step 2: I verify ex-post that being hosted is more profitable for R than entering an exclusive contract if M does not launch A. Suppose it is the case for the moment and M is still not willing to launch A if an exclusive contract is rejected. Then, it is an optimal strategy of M to accept a hosting inquiry by a straightforward argument: Compared to not being hosted, M receives a profit of $(1-\phi)\xi\tau_H$, which is strictly greater than zero.

Step 3: As shown in Proposition 3.1, the *Cross Competition Outcome* is the unique equilibrium of the game if no exclusive contract is entered and M launches product A. Therefore, the platform is unwilling to launch product A and host R instead if the profit from hosting R is greater than the (expected) profit in the *Cross Competition Equilibrium*, which is the case if:

$$(1-\phi)\xi\tau_H > (1-\phi)\xi u_A - F \rightarrow \xi < \frac{F}{(1-\phi)(u_A-\tau_H)} = C_E \tag{C.1}$$

If Condition C.1 is satisfied, launching product A is an implausible threat of M such that the specialist infers that it is hosted after rejecting an exclusive contract.

Step 4: Notice that $C_E < 1$ if $F < (1-\phi)(u_A-\tau_H)$, which is satisfied by *A1*. Hence, given $\xi \in \{\xi_l, \xi_m, \xi_h\}$, Condition C.1 is satisfied if, and only if: $\xi = \xi_l$.[3]

For $\xi = \xi_l = 0$, being hosted is indeed more profitable than entering an exclusive contract if:

$$\phi(u_A-\tau_E) < \phi(u_A-b) \rightarrow b < \tau_E \tag{C.2}$$

It follows from combining *A2* and *A3* that...

$$b < \frac{\tau_H[F-(1-\phi)\alpha(u_A-\tau_H)]}{F+\phi(u_A-\tau_H)} < \frac{F\tau_H}{F+\phi(u_A-\tau_H)} < \tau_E$$

...such that Condition C.2 is satisfied certainly satisfied, by assumption. Therefore, the

[3] It follows from the subsequent proof of Lemma 3.2 that $C_E < \xi_m < \xi_h = 1$.

specialist rejects an exclusive contract if offered.

Given the tie-breaking rule specified in Section 3.2, the specialist is hosted in case of indifference between being hosted and not being hosted.

Step 5: It follows from the above-outlined arguments that any potential equilibrium is associated with the *Hosting Outcome (H)* outcome. The profit in the equilibrium outcome of R and M (π_R^H, π_M^H) directly follows after substituting $\xi = \xi_l$ and the above outlined pricing strategy into $\dot{\pi}_R^H$ and $\dot{\pi}_R^H$ outlined in Section 3.2. It also directly follows that $CS_1 = CS_{2A} = 0$. By the definition of welfare, $E(W) = \pi_R^H$ as $\pi_M^H = 0$ for $\xi_l = 0$.

□

Proof of Lemma 3.2

Step 1: Let me start by showing that the pricing strategy outlined in the *Exclusive Outcome* constitutes an equilibrium if an exclusive contract is entered in $t = 2$: By definition, the only parameter that can be chosen freely is the on-platform price p_R^{on}. R has no incentive to decrease $p_R^{on} = u_A$ since it already sells A to all potential customers. R also has no incentive to increase p_R^{on} because demand would discretely jump to zero.

Step 2: Up to the point where M decides whether to launch product A, the proof is identical to Lemma 3.1. For $\xi \geq C_E$, the platform launches product A instead of hosting the specialist if no exclusive contract is entered. Therefore, R certainly ends up in the *Cross Competition Equilibrium* if it rejects an exclusive contract (or if no such contract is offered) by Proposition 3.1. Thus, R optimally accepts an exclusive contract if:

$$(\phi + (1-\phi)\xi)(u_A - \tau_E) > \frac{\phi(1-\phi)\xi}{(1-\phi)\xi + \phi}u_A - \phi b$$

$$\rightarrow \tau_E < u_A\left(1 - \frac{\phi(1-\phi)\xi}{(\phi + (1-\phi)\xi)^2}\right) + \frac{\phi b}{(\phi + (1-\phi)\xi)^2} \qquad \text{(C.3)}$$

It follows from from $A2$ that $\tau_E < (1-\phi)u_A(-F)$. Thus, if Condition C.3 holds after substituting τ_E by $(1-\phi)u_A$, it directly follows that Condition C.3 holds for any τ_E that satisfies *A2*. Let me furthermore set $b = 0$ to minimize the right-hand-side of Inequality C.3 to show that:

$$(1-\phi)u_A < u_A\left(1 - \frac{\phi(1-\phi)\xi}{(\phi+(1-\phi)\xi)^2}\right)$$

$$\rightarrow 0 < [\phi + (1-\phi)\xi]^2 - (1-\phi)\xi \tag{C.4}$$

Straightforward calculus shows that the right-hand-side is minimized for $\xi^{min} = \frac{(1/2)-\phi}{1-\phi}$. Substituting ξ^{min} into Inequality C.4 yields:

$$\rightarrow 0 < \left(\frac{1}{2}\right)^2 - \frac{1}{2} + \phi \tag{C.5}$$

Which is satisfied by assumption as $\phi > \frac{1}{4}$. Hence, it follows that Inequality C.3 is satisfied for any parameter vector that potentially occurs, given *A2* and $\phi > \frac{1}{4}$. As a consequence, the specialist accepts an exclusive contract it the platform otherwise launches *A*.

Step 3: In turn, the platform offers an exclusive contract if ending up in the alternatively reached *Cross Competition Equilibrium* is less profitable. Thus, whenever:

$$(\phi + (1-\phi)\xi)(\tau_E) > (1-\phi)\xi u_A - F \rightarrow \xi < \frac{F + \phi\tau_E}{(1-\phi)(u_A - \tau_E)} = C_C \tag{C.6}$$

Notice that $C_C > C_E$ if $\tau_E > \frac{F\tau_H}{F+\phi(u_A-\tau_H)}$ and that $C_C < 1$ if $\tau_E < (1-\phi)u_A - F$, which is both ensured by *A2*.

Step 4: Given that $\xi_l < C_E < \xi_m < C_C < \xi_h$, there exists a unique equilibrium where the platform offers an exclusive contract and the specialist accepts such a contract and subsequently both players choose their strategies as outlined in the *Exclusive Outcome* if and only if $\xi = \xi_m$.

The profit of R and M (π_R^E, π_M^E) directly follows after substituting $\xi = \xi_m$ and $p_R^{on} = u_A$ into $\dot{\pi}_R^E$ and $\dot{\pi}_R^E$ outlined in section 3.2. It also directly follows that $CS_1 = CS_{2A} = 0$. Hence, $E(W) = \pi_R^H + \pi_M^H$ by the definition of welfare.

□

Proof of Lemma 3.3

Step 1: Up to the point where M decides whether to offer an exclusive contract, the proof is identical to Lemma 3.2. Since $\xi_h > C_C$, as already shown by *A2*, the platform is unwilling to offer an exclusive contract to the specialist. It follows from a similar logic as outlined in the proof of Lemma 3.2 that the *Cross Competition Outcome (C)* is the unique equilibrium outcome of the game.

Step 2: As just argued, M launches A if and only if $\xi = \xi_h = 1$. Let me now verify that for the pricing strategies in the *Direct-* and the *Cross Competition Equilibrium* that $inf(T_S) > 0$ as argued in the proof of Proposition 3.1. After substituting $\xi = 1$ into the relevant equation for the *Direct Competition Equilibrium* one finds $inf(T_S) = (1-\phi)\tau_H - b$. One finds from *A3* that b is at most $b^{max} = \frac{\tau_H[F-(1-\phi)\alpha(u_A-\tau_H)]}{F+\phi(u_A-\tau_H)}$. Therefore:

$$(1-\phi)\tau_H > \frac{\tau_H[F-(1-\phi)\alpha(u_A-\tau_H)]}{F+\phi(u_A-\tau_H)} = b^{max}$$

$$\rightarrow (1-\phi)\phi(u_A-\tau_H) > \phi F - (1-\phi)\alpha(u_A-\tau_H) \tag{C.7}$$

If Inequality C.7 holds in conditions where the right-hand side is maximized, it immediately follows that $inf(T_S) > 0$. After substituting in the maximum possible $F = (1-\phi)(u_A-\tau_H)$ from *A1*, it follows after rearranging that Inequality C.7 holds for all $\alpha \in (0,1]$ as $u_A > \tau_H$ and $\phi \in (1/4, 1)$ by assumption. It immediately follows that $inf(T_S) > 0$. Again, as $u_A > \tau_H$, the same is true in the *Cross Competition Equilibrium.* Likewise, it follows that $inf(T_M) > b$ in both outlined equilibria.

Step 3: It also follows from the mixed strategies outlined in Proposition 3.1 that $E(CS_1) > 0$ and $E(CS_{2A}) > 0$. The expected profit of R and M follows from the indifference conditions outlined in Proposition 3.1. Furthermore, $E(W^C_{t=2}) = E(CS_1) + E(CS_{2A}) + E(\pi^C_R) + E(\pi^C_M)$ by the definition of welfare.

□

Proof of Lemma 3.4

By similar reasoning as outlined initially in the proof of Lemma 3.1, the *Hosting Outcome* is the unique equilibrium outcome in $t = 2$ if the platform does not launch A and the specialist is hosted.

Step 1: Suppose the specialist is not hosted in $t = 1$ such that ξ is unknown when entering $t = 2$. In this case, the specialist is always rather hosted than not hosted: With $\alpha \in (0, 1)$, $E(\xi|\mu(\theta_s|\Psi)) > 0$, $\forall\ \mu(\theta_s|\Psi) \in [0, 1]$, M generates, in expectation, an additional on-platform profit if getting hosted without risking to create a future competitor as the game ends after $t = 2$ by assumption. M is going to accept such a hosting inquiry if not hosting R is the counterfactual by a similar argument.

Step 2: If M offers an exclusive contract, R rejects such a contract in favor of being hosted if:

$$(\phi + (1 - \phi)E(\xi|\mu(\theta_s|\Psi)))(u_A - \tau_E) < \phi(u_A - b) + (1 - \phi)E(\xi|\mu(\theta_s|\Psi))(u_A - \tau_H)$$

$$\rightarrow b < \tau_E + \frac{(1 - \phi)}{\phi}E(\xi|\mu(\theta_s|\Psi))(\tau_E - \tau_H) \tag{C.8}$$

It directly follows from comparing both expressions that the right-hand side is minimized for the minimum possible τ_E, which follows from *A2:* $\tau_E^{min} = \frac{F\tau_H}{F+\phi(u_A-\tau_H)}$. Given that $\tau_E^{min} < \tau_H$, the right-hand side is further minimized for the maximum possible $E(\xi|\mu(\theta_s|\Psi)) = \alpha$, which follows after setting $\mu(\theta_s|\Psi) = 1$.

Hence, if I can show that Condition C.8 holds, given τ_E^{min} and $E(\xi|\mu(\theta_s|\Psi) = 1) = \alpha$, accepting an exclusive contract is a strategy that is (in expectation) strictly dominated for R by getting hosted.

After substituting both expressions into Condition C.8, one finds that the inequality is satisfied as long as $b < \frac{\tau_H[F-(1-\phi)\alpha(u_A-\tau_H)]}{F+\phi(u_A-\tau_H)}$, which is the case by *A3*.

Step 3: It follows from the reasoning in *Steps 1 & 2* that R is hosted if M does not launch A. If launching A, the *Cross Competition Equilibrium* outcome is realized as argued in Proposition 3.1. Therefore, launching A is, in expectation, not profitable for M if:

$$(1-\phi)E(\xi|\theta)\tau_H > (1-\phi)E(\xi|\theta)u_A - F \rightarrow F > (1-\phi)E(\xi|\theta)(u_A - \tau_H) \tag{C.9}$$

Analogous to the logic applied in *Step 2*, launching product A is never optimal for M if Inequality C.9 is satisfied in a scenario where launching A is most profitable.

For a given vector of parameters, the right-hand side of the Inequality C.9 is maximized for $\theta = \theta_s$. After substituting $E(\xi|\theta_s) = \alpha$ into the inequality one finds that not launching A and subsequently hosting R is optimal for M if $F > (1-\phi)\alpha(u_A - \tau_H)$, which is the case by *A1*.

To summarize, given Assumptions *A1*, *A2* and *A3*, the *Hosting outcome* is certainly realized in $t = 2$ if R is not hosted in $t = 1$.

□

Proof of Lemma 3.5

In $t = 1$, M cannot launch product A by assumption. Conditional on R not being willing to get hosted, it follows from similar reasoning as outlined in the proof of Lemma 3.1 that the chosen pricing strategy constitutes a unique equilibrium that is associated with the *No Hosting Outcome* as outlined in Lemma 3.5. It follows from the outlined pricing strategy that $CS_1 = 0$ (and $CS_{2A} = 0$). Furthermore, the profit of R (and M) directly follows and $E(W) = \pi_R^N$ by the definition of welfare.

□

Proof of Lemma 3.6

Step 1: Given that R is not offered an exclusive contract and given that R holds an updated belief $\mu(\theta_s|\Psi) = q \in [0,1]$ that $\theta = \theta_s$, it follows from Corollary 3.1 that $R's$ expected profits from being hosted and from not being hosted are the following:

$$E(\Pi_R^H|q) = \phi(u_A - b) + (1-\phi)\alpha(q + (1-q)\xi_m)(u_A - \tau_H) + \delta\left[(1-\alpha)\phi(u_A - b)\right]$$
$$+ \delta\alpha\left[q(\phi(1-\phi)u_A - \phi b) + (1-q)(\phi + (1-\phi)\xi_m)(u_A - \tau_E)\right]\right]$$
$$E(\Pi_R^N|q) = \phi(u_A - b) + \delta\left[\phi(u_A - b) + \alpha(q + (1-q)\xi_m)(1-\phi)(u_A - \tau_H)\right]$$

Let me define $D \equiv E(\Pi_R^H|q) - E(\Pi_R^N|q)$ for the remainder of this proof. It follows that:

$$\frac{\partial D}{\partial q} \neq 0 \quad \text{and} \quad \frac{\partial^2 D}{\partial^2 q} = 0$$

Thus, D is either (a) strictly decreasing in q or (b) strictly increasing in q. Notice further that it follows from rearranging the above equations that $D\big|_{q=0} > 0$. Thus, the specialist certainly prefers being hosted if facing a weak-type platform. From these arguments, two statements can be derived that build on each other in the outlined order:

1. $D < 0$ can only occur if and only if scenario (a) applies.
2. If $D\big|_{q=1} > 0$, $D > 0$, $\forall\ q \in [0,1]$, independent of which of scenarios (a) or (b) applies.

The first statement follows from the definition of (a) and (b) and from $D\big|_{q=0} > 0$ as outlined earlier. The second statement follows from the reasoning in the first statement and the definition of (a) as outlined above.

The proof proceeds by outlining the conditions where $D\big|_{q=1} < 0$. Given $q = 1$ and the payoffs as defined above, it follows from rearranging D that $D\big|_{q=1} < 0$ if:

$$\delta > \frac{(1-\phi)(u_A - \tau_H)}{\phi^2 u_A + (1-\phi)(u_A - \tau_H)} \equiv \Delta_{no} \in (0,1) \tag{C.10}$$

If Condition C.10 is not satisfied, $D\big|_{q=1} > 0$ such that $D > 0,\ \forall\ q \in [0,1]$ under all circumstance as outlined in statement (b). Then, R is certainly hosted by the above reasoning.

Step 2: If $D\big|_{q=1} < 0, \exists\ \bar{\mu}_{no} \in (0,1)$ such that $D\big|_{q} < 0,\ \forall\ q > \bar{\mu}_{no}$:

If Condition C.10 from the previous step is satisfied, R is not hosted if facing a strong-type platform and scenarios (a) applies as argued in *Step 1*. Nevertheless, R wants to be hosted if facing a weak-type platform since $D\big|_{q=0} > 0$ as also shown in *Step 1*.

Thus, there must exist a cutoff value $\bar{\mu}_{no} \in (0,1)$ where the specialist wants to be hosted if $q < \bar{\mu}_{no}$ and does not want to be hosted if $q > \bar{\mu}_{no}$ as D is strictly decreasing in q, which was also shown in *Step 1*. The existence of such a threshold follows from the *Intermediate Value Theorem*, which applies here since D is continuous in q, which follows from the expected payoffs outlined in the previous step.

□

Proof of Lemma 3.7

Step 1: Given that the specialist is offered an exclusive contract and given that the specialist holds an updated belief $\mu(\theta_s|\Psi) = q \in [0,1]$ that $\theta = \theta_s$, it follows from Corollary 3.1 that, for a given vector of parameters Γ, $R's$ expected profit from accepting an exclusive contract is:

$$E(\Pi_R^E|q) = (1+\delta)(\phi + \alpha(1-\phi)(q + (1-q)\xi_m))(u_A - \tau_E)$$

$E(\Pi_R^H|q)$ is the same as outlined in *Step 1* of the proof of Lemma 3.6.

Let me define $D_1 \equiv E(\Pi_R^E|\mu(\theta|\Psi)) - E(\Pi_R^H|\mu(\theta|\Psi))$ for the remainder of this proof.

It follows from the outlined equations that $D_1\big|_{q=0} < 0$. From here onward, the proof follows the same logic as the proof of Lemma 3.6 when solving for Δ_o and $\bar{\mu}_o \in (0,1)$. It is therefore omitted. Δ_o follows from solving $D_1\big|_{q=1} > 0$ and the existence of $\bar{\mu}_o \in (0,1)$ if $\delta > \Delta_o$ again follows from the *Intermediate Value Theorem*.

Step 2: Suppose that $\delta < \Delta_{no}$ such that there exists $\bar{\mu}_{no} \in (0,1)$ by Lemma 3.6. I continue by showing that entering an exclusive contract is a better reply than not being hosted for all $\mu(\theta_s|\Psi) = q > \bar{\mu}_{no}$.

Given the relevant equations as outlined in *Step 1* and in the proof of Lemma 3.6, let me define $D_2 \equiv E(\Pi_R^E|q) - E(\Pi_R^N|q)$ for the remainder of this proof. It follows from Corollary 3.1 that:

$$\frac{\partial D_2}{\partial q} > 0, \ \forall \ q \in [0, 1]$$

Thus, if I can show that $D_2\big|_{q=\bar{\mu}_{no}} > 0$ it immediately follows that, compared to being hosted, accepting an exclusive contract is a better outside option than not being hosted for all $q > \bar{\mu}_{no}$. $D_2\big|_{q=\bar{\mu}_{no}} > 0$ if:

$$(1 + \delta) \left[(\phi + x(1 - \phi)\alpha)(u_A - \tau_E) - \phi(u_A - b)\right] - \delta x(1 - \phi)\alpha(u_A - \tau_H) > 0 \qquad \text{(C.11)}$$

where $x = (\bar{\mu}_{no} + (1 - \bar{\mu}_{no})\xi_m)$. After solving the inequality for τ_E, one finds:

$$\tau_E < \frac{\phi(1 + \delta)b + x(1 - \phi)\alpha[u_A + \delta\tau_H]}{(1 + \delta)[\phi + x(1 - \phi)\alpha)]}$$

This certainly is the case with *A2*.

Step 3: It follows from the previous step that entering an exclusive contract is a better outside option than not being hosted for $\mu(\theta_s|\Psi) = q > \bar{\mu}_{no}$: Thus, conditional on $\mu(\theta_s|\Psi) = q > \bar{\mu}_{no}$, $R's$ payoffs listed in Corollary 3.1 are ranked in the following way:

$$E(\Pi_R^E|q > \bar{\mu}_{no}) > E(\Pi_R^N|q > \bar{\mu}_{no}) > E(\Pi_R^H|q > \bar{\mu}_{no})$$

Suppose now that $\bar{\mu}_{no} < \bar{\mu}_o$. Then, it follows from the reasoning in Lemma 3.6 and the reasoning outlined in *Step 1* that for any $\mu(\theta_s|\Psi) \in (\bar{\mu}_{no}, \bar{\mu}_o)$, R prefers not being hosted over accepting an exclusive contract, which contradicts the above outlined preference ranking. It therefore immediately follows that $\bar{\mu}_{no} \geq \bar{\mu}_o$.

Consequentially, also $\Delta_{no} \geq \Delta_o$ since otherwise, it follows from *Step 1* that for $\delta \in (\Delta_o, \Delta_{no})$

$\bar{\mu}_o$ is not defined. However, it follows from Lemma 3.6 that $\bar{\mu}_{no}$ exists and $\bar{\mu}_{no} < 1$, which contradicts $\bar{\mu}_{no} \geq \bar{\mu}_o$.

□

Proof of Lemma 3.8

Given $\theta = \theta_w$, the platform prefers to enter an exclusive contract over hosting the specialist in $t = 2$ by the reasoning outlined in Lemma 3.2 if $\xi = \xi_m$ is realized. The same is certainly true for $\xi = \xi_l$ by a straightforward argument: With an exclusive contract, M can generate transfer payments from *Type 1* consumers, which is otherwise not the case if the specialist is hosted. Given $X_w = \{x_l, x_m\}$, the combination of both arguments implies that M is (in expectation) better off if M can enter an exclusive contract in $t = 1$ instead of hosting the specialist for any possible vector of parameters Γ. It directly follows from Corollary 3.1 that $E(\Pi_M^H|\theta_w) > E(\Pi_M^N|\theta_w)$, again, for any possible vector of parameters Γ. Hence, the following payoff ranking results for a weak-type platform:

$$E(\Pi_M^E|\theta_w) > E(\Pi_M^H|\theta_w) > E(\Pi_M^N|\theta_w)$$

$E(\Pi_M^H|\theta_w) > E(\Pi_M^N|\theta_w)$ follows from an analogous argument as outlined, e.g., in Lemma 3.4. Lemma 3.8 immediately follows.

□

Proof of Lemma 3.9

Given $\theta = \theta_s$, it depends on the specific vector of parameters whether $E(\Pi_M^E|\theta_s) \lesseqgtr E(\Pi_M^H|\theta_s)$. It follows from Corollary 3.1:

$$E(\Pi_M^E|\theta_s) = (1 + \delta)[\phi + (1 - \phi)\alpha]\tau_E$$
$$E(\Pi_M^H|\theta_s) = \alpha(1 - \phi)\tau_H + \delta\alpha\left[(1 - \phi)u_A - F\right]$$

Let me start by demonstrating that there exist scenarios where $E(\Pi_M^E|\theta_s) < E(\Pi_M^H|\theta_s)$ by assuming conditions where this most likely is the case. Therefore, I assume that F is minimized at $F = \alpha(1-\phi)(u_A - \tau_H)$ (from *A1*). In these conditions, $E(\Pi_M^E|\theta_s) < E(\Pi_M^H|\theta_s)$ if:

$$\alpha > \frac{\phi\tau_E}{(1-\phi)(\tau_H - \tau_E)}$$

This is only feasible if $\frac{\phi\tau_E}{(1-\phi)(\tau_H-\tau_E)} < 1$. This is the case if $\tau_E < (1-\phi)\tau_H$, which is possible with *A2*. Therefore, I can conclude that there exist parameter constellations where $E(\Pi_M^E|\theta_s) < E(\Pi_M^H|\theta_s)$.

After rearranging the payoffs as outlined above, one finds more generally that $E(\Pi_M^E|\theta_s) < E(\Pi_M^H|\theta_s)$ if:

$$\alpha > \frac{(1+\delta)\phi\tau_E}{\delta[(1-\phi)(u_A - \tau_E) - F] - (1-\phi)(\tau_E - \tau_H)} \equiv \underline{\alpha}$$

It follows from the reversed reasoning that $E(\Pi_M^E|\theta_s) > E(\Pi_M^H|\theta_s)$ if $\alpha < \underline{\alpha}$.

□

Proof of Proposition 3.2

Step 1: Let me start by defining $D_3 = E(\Pi_M^E|\theta_s) - E(\Pi_M^N|\theta_s)$. The relevant payoff functions are outlined in Corollary 3.1. I continue by showing that $D_3 > 0$ for any vector of parameters Γ.

Let me, therefore, proceed by demonstrating that $D_3 > 0$, even in the most adverse conditions. These are found by substituting the lower bound of *A2* (τ_E^{min}) for τ_E into D_3. One finds that $D\big|_{\tau_E=\tau_E^{min}} > 0$. I can therefore conclude that a strong-type platform certainly prefers entering an exclusive contract over not hosting R.

If additionally considering Lemma 3.8, it follows that:

$$E(\Pi_M^E|\theta) > E(\Pi_M^N|\theta),\ \forall\ \theta \in \Theta$$

.

Step 2: It follows from *Step 2* of Lemma 3.7 that for a given $\mu(\theta_s|\Psi) = q$:

$$E(\Pi_R^E|q) > E(\Pi_R^N|q), \ \forall \ q) \in [\bar{\mu}_{no}, 1]$$

Thus, if an exclusive contract is offered, not being hosted is a strictly dominated strategy for any possible belief where the specialist is not hosted if no exclusive contract is offered.

Step 3: It follows directly from combining the above-otlined arguments that a platform of any type can profitably deviate and offer an exclusive contract if the specialist is otherwise not hosted.

Consequentially, there cannot exist an equilibrium where the specialist does not offer product *A* via the platform in the first period as *R* is either hosted or an exclusive contract occurs.

□

Proof of Proposition 3.3

The proof relies on scenarios (i)-(v) outlined in Table 3.1.

Step 1: I start by showing that there cannot exist an equilibrium where an exclusive contract is entered in scenarios (i)-(ii) as outlined in Table 3.1.

An exclusive contract is entered if at least one platform type is willing to offer an exclusive contract and *R* is willing to enter such a contract, given its belief about the platform type. The following arguments demonstrate that both conditions never mutually hold in scenario (i):

1. Suppose that only the θ_w type offers an exclusive contract: *R* updates its belief according to $\mu(\theta_s|\omega_o) = 0$. By a similar reasoning as in the proof of Lemma 3.7, it is optimal for *R* to reject an exclusive contract.

2. Suppose that only the θ_s type offers an exclusive contract: *R* updates its belief according to $\mu(\theta_s|\omega_o) = 1$. If $1 \leq \Delta_o$, the specialist never accepts such a contract by Lemma 3.7.

Otherwise, the updated belief induces the specialist to accept an exclusive contract by Lemma 3.7. However, such a scenario cannot constitute an equilibrium since it follows from Lemma 3.8 that it cannot be an optimal signaling strategy for a weak-type platform not to offer an exclusive contract if it is subsequently accepted.

3. Assume now that both platform types offer an exclusive contract with positive probability that is also subsequently accepted by R with positive probability. Then, there cannot exist an equilibrium where the θ_w type platform does not always offer an exclusive contract by a similar argument as above. In this case, the specialist updates its belief to $\mu(\theta_s|\omega_o) \leq \mu_s$ such that it follows from Lemma 3.7 that R certainly rejects an exclusive contract and is subsequently hosted as $\mu_s < \bar{\mu}_o$ in scenario (i), which contradicts the initial assumption of R accepting an exclusive contract with positive probability.

Let me now continue with analyzing scenario (ii):

1. By a similar reasoning as in scenario (i), there cannot exist a separating equilibrium where *only* the θ_w or the θ_s platform type offers an exclusive contract and such a contract is subsequently accepted.

2. Assume that both platform types offer an exclusive contract with positive probability such that $\mu(\theta_s|\omega_o) > \bar{\mu}_o$ where it follows from Lemma 3.7 that R subsequently accepts such a contract with positive probability. Then, any signaling strategy that induces $\mu(\theta_s|\omega_o) > \bar{\mu}_o$ can never be optimal for a strong-type platform as $E(\Pi_M^E|\theta_s) < E(\Pi_M^H|\theta_s)$ by the following argument: Given $\delta < \Delta_{no}$, it follows from Lemma 3.6 that the specialist is hosted if not being offered an exclusive contract, independent of its belief about the platform type. Thus, a strong-type platform can always profitably deviate to not offering an exclusive contract.

Step 2: Given Proposition 3.2, there only exist an equilibrium where the specialist is hosted to sell product A via the platform in $t = 1$.

In scenario (i), exists a *Pooling equilibrium* where:

1. Both platform types offer an exclusive contract: $\Psi(\theta) = \omega_o, \ \forall \ \theta \in \Theta$ such that $\mu(\theta_s|\omega_o) = \mu_s$.

2. R rejects the offer and is subsequently hosted such that the *Hosting Equilibrium* occurs in $t = 1$ where ξ is revealed.

3. Given a specific realization of ξ, one of the outcomes from Lemmas 3.1 - 3.3 occurs.

The proof follows from backward induction: (3:) Directly follows from Lemmas 3.1 - 3.3. (2:) Directly follows from Lemma 3.7 since $\mu(\theta_s|\omega_o) = \mu_s < \bar{\mu}_o$. (1:) It follows from Corollary 3.1 that $E(\Pi_M^H|\theta) > E(\Pi_M^N|\theta), \ \forall \ \theta \in \Theta$. Thus, any platform type is weakly worse off if it does not offer an exclusive contract. Given that the specialist is hosted for the outlined strategies, they, therefore, constitute an equilibrium.

For scenarios (ii) (and (i) with $\delta < \Delta_o$ *and* $E(\Pi_M^E|\theta_s) < E(\Pi_M^H|\theta_s)$), exists a *Separating equilibrium* where:

1. The strong-type platform does not offer an exclusive contract and the weak-type platform type does offer an exclusive contract such that $\mu(\theta_s|\omega_{no}) = 1$.

2. R is hosted and the *Hosting Outcome* occurs in $t = 1$ where ξ is revealed.

3. Given a specific realization of ξ, one of the outcomes from Lemmas 3.1 - 3.3 occurs.

The proof follows from backward induction: (3:) Directly follows from Lemmas 3.1 - 3.3. (2:) With $\delta < \Delta_o$, (2) directly follows from Lemma 3.7. (1:) Given that $E(\Pi_M^E|\theta_s) < E(\Pi_M^H|\theta_s)$ by definition, a strong-type platform certainly has no incentive to deviate. Given the reasoning outlined in Lemma 3.8, a weak-type platform also has no (strict) incentive to deviate from its specified signaling strategy.

There might exist additional equilibria. If additionally considering the results from *Step 1*, however, the specialist is hosted with certainty in $t = 1$ in any potential equilibrium that exists in scenarios (i) and (ii) since otherwise either Proposition 3.2 or *Step 1* is contradicted. As consequence, ξ is revealed in $t = 1$ and Lemmas 3.1 - 3.3 specify the outcome of the game in $t = 2$ for any realization of $\xi \in \{\xi_l, \xi_m, \xi_h\}$. Thus, any alternatively reached equilibrium must yield an identical equilibrium outcome.

The proof of the first part of Proposition 3.3 is now complete.

Step 3: I continue with showing that there exists an equilibrium where an exclusive contract is entered in $t = 1$ for scenarios (iii) - (v) as outlined in Table 3.1.

For scenarios (iii) - (v), there exists a *Pooling equilibrium* where:

1. Both platform types offer an exclusive contract: $\Psi(\theta) = \omega_o, \ \forall \ \theta \in \Theta$ such that $\mu(\theta_s|\omega_o) = \mu_s$.
2. R accepts the offer such that the *Exclusive Outcome* occurs in $t = 1$ and $t = 2$.

The proof follows from backward induction.

(2:) If an exclusive contract is entered in $t = 1$, it continues to be valid in $t = 2$ by definition. Furthermore, R accepts an exclusive offer in $t = 1$ by Lemma 3.7 as $\mu(\theta_s|\omega_o) = \mu_s > \bar{\mu}_o$ by definition scenarios (iii)-(v).

(1:): By Lemma 3.8 and given $E(\Pi_M^E|\theta_s) > E(\Pi_M^H|\theta_s)$, the specified signal is optimal for both platform types in scenario (iii) as the first-best *Exclusive Outcome* is realized.

In scenario (iv) and (v), the same reasoning applies if $E(\Pi_M^E|\theta_s) > E(\Pi_M^H|\theta_s)$. Otherwise, it follows from Lemma 3.8 that only the strong-type platform has an incentive to deviate. Then, the divine criterion requires that R updates its off-equilibrium belief accordingly: $\sigma(\theta_s|\omega_{no}) = 1$. Since $\delta > \Delta_{no}$ in both scenarios (iv) and (v)[4], R is not willing to be hosted if offered no exclusive contract by Lemma 3.6. Therefore, it cannot be optimal not to offer an exclusive contract for a strong-type platform by a similar reasoning as outlined in *Step 1* of the proof of Proposition 3.2.

[4]It follows from Lemma 3.6 that this is implicitly given in scenario (v): $\mu_s > \bar{\mu}_{no}$ implies that $\bar{\mu}_{no}$ exists, which only can be the case if $\delta > \Delta_{no}$.

Step 4: In scenario (v), there cannot exist an equilibrium where only the θ_s type platform does not offer an exclusive contract with positive probability by a similar reasoning why the strong-type platform cannot profitably deviate from the pooling strategy in the previous step.

If only the θ_w type platform does not offer an exclusive contract with positive probability, R updates its belief to $\mu(\theta_s|\omega_o) > \mu_s > \bar{\mu}_o$ such that R accepts an exclusive contract if offered by Lemma 3.7. By a similar reasoning as outlined in *Step 1*, this cannot be optimal for a weak-type platform.

A scenario where both types do not offer an exclusive contract with positive probability can only constitute an equilibrium if the specialist is subsequently hosted by Proposition 3.2. For this to be the case, the θ_w type platform must be relatively more likely to not offer an exclusive contract since otherwise it follows from the definition of scenario (v) that $\mu(\theta_s|\omega_{no}) > \mu_s > \bar{\mu}_{no}$ where R is not hosted if not offered an exclusive contract by Lemma 3.6. If the θ_w type platform is relatively more likely to not offer an exclusive, $\mu(\theta_s|\omega_o) > \mu_s > \bar{\mu}_o$ as $\bar{\mu}_o < \bar{\mu}_{no}$ such that R accepts an exclusive contract if being offered as argued in Lemma 3.7. By an already familiar argument, this cannot constitute an equilibrium.

It follows from combining the above arguments that there can only exist an equilibrium in scenario (v) if both platform types certainly offer an exclusive contract. The uniqueness of the *Pooling Equilibrium* outlined in *Step 3* immediately follows.

Step 5: To prove the uniqueness of the *Pooling Equilibrium* outlined in *Step 3* for scenarios (iii) and (iv), I leverage the *CR* criterion outlined in Section 3.2: *The signaling function may only include signals for each platform type that are weakly dominant signals, independent of any possible (updated) belief the specialist could hold about the platform's type for a given vector of parameters* Γ*. The criterion only applies if such signals uniquely exist after removing strategies from the specialist's strategy set that are never-best responses.*

Given the reasoning in Corollary 3.2, it is a never-best response for R not to be hosted if offered an exclusive contract.[5] Therefore, it follows from Lemma 3.8 that offering an ex-

[5]By Lemma 3.6, this is not the case if not offered an exclusive contract.

clusive contract is a unique weakly dominant signal of a weak-type platform, independent of the specialist's belief about the platform type. The same is true for a high type platform as long as $E(\Pi_M^E|\theta_s) > E(\Pi_M^H|\theta_s)$. Thus, with the *CR* criterion, both platform types necessarily offer an exclusive contract in scenario (iii). Uniqueness immediately follows.

The same is true in scenario (iv) as long as $E(\Pi_M^E|\theta_s) > E(\Pi_M^H|\theta_s)$. Otherwise, R infers that only a strong-type platform does not offer an exclusive contract with positive probability and updates its belief accordingly. It follows from Lemma 3.6 that the specialist is subsequently not hosted as $\delta > \Delta_{no}$ in scenario (iv). By an already familiar argument, not offering an exclusive contract cannot be optimal for a strong-type platform. Again, uniqueness immediately follows.

Step 6: **Final remark:** Abstracting from the reasoning in the previous step and the 'CR' criterion, there exist additional equilibria where no exclusive contract is offered in the first stage and the specialist is subsequently hosted in scenarios (iii) and (iv).

If $E(\Pi_M^E|\theta_s) > E(\Pi_M^H|\theta_s)$ such equilibria rely on *both* platform types playing weakly dominated signaling strategies. In this case, the 'NITS' criterion proposed by Y. Chen, Kartik, and Sobel (2008) would also select the *Pooling equilibrium* proposed in *Step 3* as unique outcome in scenarios (iii) and (iv).

Otherwise, if $E(\Pi_M^E|\theta_s) < E(\Pi_M^H|\theta_s)$, which is only possible in scenario (iv) by definition, there does - to the best of my knowledge - not exist a satisfying equilibrium selection criterion that rules out an (implausible) equilibrium where the specialist is hosted in $t = 1$.

Formally outlining such an equilibrium selection rule is beyond the scope of this paper. However, it appears reasonable that the signaling function that is associated with an equilibrium should expose some notion of 'conditional rationalizability'(CR) for each platform type. This is captured by the 'CR-'selection rule outlined in Section 3.2. Assuming that scenario (iv) applies, the underlying idea is sketched in the following:

Let me abstract from the role nature has within Bayesian games. Imagine the two platform types being two distinct players that 'negotiate' over a signaling function Ψ. Suppose that a weak-type platform always offers an exclusive contract and that the specialist knows that. Then, it is a strict best reply for a strong-type platform to also offer an exclusive contract since otherwise the specialist would infer that the platform certainly is of the strong type if not being offered an exclusive contract and would subsequently not be hosted by the reasoning in Lemma 3.6 as $\delta > \Delta_{no}$ in scenario (iv). Then, it follows from an already familiar argument that it is indeed optimal for a strong-type platform to offer an exclusive contract as the specialist is otherwise not hosted.

Given the rationale outlined in *Step 4*, the specialist accepts an exclusive contract in $t = 1$ if both platform types offer an exclusive contract, which is the preferred outcome of a weak-type platform. Therefore, a weak-type platform has a strict incentive to restrict itself to always offer an exclusive contract and also make this public knowledge as it infers that the strong-type platform necessarily 'responds' by also offering an exclusive contract.

The reversed argument does not analogously hold if a strong-type platform restricts itself to always not offer an exclusive contract in $t = 1$. There exist beliefs of R in scenario (iv) where R is not hosted if not offered an exclusive contract such that a strong-type platform would have a strict incentive to deviate from not offering such a contract, given R updates its belief accordingly. This is, for instance, the case if a weak-type platform always offers an exclusive contract: As $\delta > \Delta_o$, R is not hosted if not offered an exclusive contract by Lemma 3.7. As opposed to before, it is indeed a credible 'best-reply' for a weak-type platform to offer an exclusive contract as this is a weakly dominant signal.

It therefore follows that an equilibrium selection criterion based on conditional rationalizability would only allow for equilibria where a weak-type platform certainly offers an exclusive contract and, therefore, uniquely selects the *Exclusive Equilibrium* in scenario (iv) (and (iii)), as shown in the previous step.

□

Proof of Proposition 3.4

Step 1: Given the implementation of the policy as outlined in section 3.6.1, the unique equilibrium outcome of the game is the following:

R is hosted in t = 1 *if:*

$$\mu_s < \bar{\mu}_{no}^{\mathbb{1}[\delta > \Delta_{no}]} \tag{C.12}$$

. *As consequence, the Hosting Outcome occurs in* $t = 1$. *Depending on the platform type and a specific realization of* ξ, *the Hosting Outcome occurs if* $\xi = \xi_l$ *and the Cross Competition Outcome occurs if* $\xi \in \{\xi_m, \xi_h\}$ *in* $t = 2$. *If Condition C.12 is not satisfied, the specialist is not hosted in* $t = 1$ *such that the No Hosting Outcome certainly occurs in* $t = 1$ *and the Hosting Outcome certainly occurs in* $t = 2$.

The proof follows from backward induction. Let me first assume that Condition C.12 holds and R is hosted such that ξ is revealed in $t = 1$. In $t = 2$, Lemma 3.1 still applies such that the *Hosting Outcome* certainly occurs if $\xi = \xi_l$. As Proposition 3.1 also still applies if M launches A, it follows from reversing the logic in the proof of Lemma 3.1 that the *Cross Competition Outcome* occurs if $\xi \in \{\xi_m, \xi_h\}$ is realized.

If the specialist is not hosted in $t = 1$, *Step 3* of the proof of Lemma 3.4 still applies such that the *Hosting Outcome* certainly occurs in $t = 2$.

If the specialist is hosted in $t = 1$, the *Hosting Outcome* is the unique outcome in $t = 1$ by Lemma 3.1. If the specialist is not hosted, the *No Hosting Outcome* occurs by Lemma 3.5 in $t = 1$.

Given all outcomes as just specified and that nature draws the θ_s platform type with probability $\mu_s \in (0, 1)$, the specialist is not willing to be hosted if $\mu_s > \bar{\mu}_{no}$ where $\bar{\mu}_{no}$ is only well defined if $\delta > \Delta_{no}$ by a similar arguments as outlined in (the proof of) Lemma 3.6. After reversing the logic, it directly follows that, the specialist is hosted if $\mu_s < \bar{\mu}_{no}$ where $\bar{\mu}_{no}$, again, is only well defined for $\delta > \Delta_{no}$. Otherwise, if $\delta < \Delta_{no}$, it follows from Lemma 3.6 that R is certainly hosted for all $\mu_s \in (0, 1)$.

Step 2: Given the counterfactual scenarios (i) and (ii) as defined in Table 3.1 where $\mu_s < \bar{\mu}_{no}^{\mathbb{1}[\delta > \Delta_{no}]}$, it follows from a comparison of the outcomes as specified in *Step 1* and Proposition 3.3 that:

The policy has no impact in $t = 1$ since R is hosted, independent of whether the policy applies or not.

It also follows from a straightforward comparison to Proposition 3.3 that the policy has no impact in $t = 2$ if $\xi \in \{\xi_l, \xi_h\}$ is realized.

It therefore immediately follows that the policy does not impact the outcome of the game if $\theta = \theta_s$ where the platform is of a strong-type.

For $\theta = \theta_w$, the policy has an impact in the second stage if $\xi = \xi_m$ is realized as an exclusive contract can no longer be entered and the *Cross Competition Outcome* is realized. It directly follows from the reasoning provided in Lemma 3.2 that a weak-type platform and R are (in expectation) worse off. Both consumer types, however, strictly profit as the (expected) consumer surplus is strictly greater than zero. (After substituting $\xi = \xi_m$ in the mixed strategies provided in Proposition 3.1, it even follows the pricing strategies applied in a setting with R and a weak-type platform launching A are first-order stochastically dominated by these applied in a setting with R and a strong-type platform launching A since $\xi_m < \xi_h$.)

For $\theta = \theta_w$, (expected) overall welfare decreases in $t = 2$: It follows from a comparison of the outcomes outlined in Lemmas 3.2 and 3.3 that the weak-type platform now incurs additional fixed costs from launching A and also more consumers buy (in expectation) via the off-platform market with positive probability where shopping costs equal to b accrue for $\xi = \xi_m$ in $t = 2$. Otherwise, it follows from a straightforward comparison that the policy has no impact on overall welfare as the policy has no impact for $\xi = \xi_l$.

In conclusion, for $\theta = \theta_w$, the policy has a negative impact on $E(W)$, $E(\Pi_R)$ and $E(\Pi_M|\theta_w)$ and a positive impact on $E(CS_1), E(CS_{2A})$.

Step 3: Given the counterfactual scenarios (iii) and (iv) as defined in Table 3.1 where $\mu_s < \bar{\mu}_{no}^{\mathbb{1}[\delta>\Delta_{no}]}$, it follows from a comparison of the outcomes as specified in *Step 1* and Proposition 3.3 that:

The policy is to the disadvantage of R by Lemma 3.7 and a weak-type platform by Lemma 3.8 as it is already binding in $t = 1$ since R is now hosted with the policy as an exclusive contract can no longer be entered. A strong-type platform is (in expectation) better off if $E(\Pi_M^E(\theta)) < E(\Pi_M^H(\theta))$, which is, by definition, only possible if scenario (iv) is the counterfactual. Otherwise, a strong-type platform is also worse off.

The (expected) impact of the policy of the players on the supply side, therefore, immediately follows.

From comparing the *Hosting*- and the *Exclusive Outcome*, it follows that, in $t = 1$, consumer surplus is not impacted and overall welfare decreases as the specialist is in both outcomes able to set monopoly prices and *Type 1* consumers buy via the off-platform market where shopping costs equal to b accrue.

In $t = 2$, it follows from a comparison to the *Exclusive Outcome* that both consumer types are (in expectation) strictly better off if the platform enters the market as a rivaling supplier of product A if $\xi \in \{\xi_m, \xi_h\}$ is realized where M launches A. Overall welfare, again, decreases as additional fixed costs accrue and relatively more consumers buy (in expectation) off-platform where shopping costs are incurred if the platform launches product A in $t = 2$ for $\xi \in \{\xi_m, \xi_h\}$. Otherwise, for $\xi = \xi_l$, consumer surplus and overall welfare are not impacted, which follows from a straightforward comparison of the *Exclusive*- and the *Hosting Outcome*.

In conclusion, the policy therefore has a negative impact on $E(W)$ and $E(CS_1), E(CS_{2A})$.

Step 4: Given the counterfactual scenario (v) as defined in Table 3.1 where $\mu_s > \bar{\mu}_{no}^{\mathbb{1}[\delta > \Delta_{no}]}$, it follows from a comparison of the outcomes as specified in *Step 1* and Proposition 3.3 that:

The *No Hosting Outcome* occurs in $t = 1$ and the *Hosting Outcome* in $t = 2$. It directly follows from Proposition 3.2 that the policy is to the disadvantage of R and M, independent of $M's$ type.

Furthermore, a straightforward comparison shows that (expected) overall welfare decreases as *Type 2A* consumers are not served in the first period and *Type 1* consumers buy off-platform in both periods where shopping costs are incurred.

Consumer surplus remains unchanged as R can set monopoly prices in $t = 1$ and $t = 2$ in the *Hosting*- and the *Exclusive Outcome*. Thus, independent of whether the policy applies or not.

The impact on all expected outcome variables immediately follows.

□

Proof of Proposition 3.5

Step 1: Given the regulation described in Section 3.6.2, the platform and the specialist have to reimburse consumers when buying via the platform. The maximum possible price they can charge if being the sole provider of product A that sells via the platform is $p_M^{on} = p_R^{on} = u_A - b$. From here onward, the equilibrium outcome is found from backward induction.

Step 2: The subsequent reasoning is based on the assumption that the specialist is hosted in $t = 1$.

By similar reasoning as in the proof of Proposition 3.1, the unique equilibrium in $t = 2$ if no exclusive contract is entered and M launches product A is the *Cross Competition Equilibrium' (C')* that is found in an analogous way as the *Cross Competition Equilibrium* outlined in Proposition 3.1. The formal proof is therefore omitted. By exploiting that M is indifferent between playing a mixed strategy and receiving a payoff equal to $(1 - \phi)\xi(u_A - b) - F$ when charging monopoly prices from just *Type 2A* consumers, $F_{R,C'}$ follows. Furthermore, R can guarantee itself a payoff equal to $\phi\, inf(T_R)$ if playing $p_R = inf(T_R)$. By exploiting this indifference condition, $F_{M,C'}$ follows.

The equilibrium is associated with the following distribution functions:

$$F_{R,C'}(p_R) = 1 - \frac{(1-\phi)\xi}{\phi}\left(\frac{u_A - b}{p_R} - 1\right)$$

Where $inf(T_R) = \frac{u_A - b}{\frac{\phi}{(1-\phi)\xi} + 1}$ and $sup(T_R) = u_A - b$.

$$F_{M,C'}(p_M^{on}) = 1 - \frac{1}{p_M^{on}}\left(\frac{u_A - b}{\frac{\phi}{(1-\phi)\xi} + 1}\right)$$

Where $inf(T_M) = \frac{u_A - b}{\frac{\phi}{(1-\phi)\xi}+1}$ and M places an atom of size $\frac{(1-\phi)\xi}{\phi+(1-\phi)\xi}$ on $p_M^{on} = u_A - b$.

The (expected) profits of R and M immediately follow from both players' indifference conditions:

$$E(\pi_R^{C'}) = \frac{\phi(1-\phi)\xi(u_A - b)}{\phi + (1-\phi)\xi} \quad // \quad E(\pi_M^{C'}) = (1-\phi)\xi(u_A - b) - F.$$

By a similar reasoning as in Lemmas 3.1 and 3.2, there exists a *Hosting' (H')* - and a *Exclusive Equilibrium Outcome' (E')* in $t = 2$ if M does not launch A and R is hosted or an exclusive contract is entered. The formal proofs are therefore omitted. Given *Step 1*, the corresponding profits of both players in these outcomes are the following:

$$\pi_R^{H'} = (\phi + (1-\phi)\xi)(u_A - b) - (1-\phi)\xi\tau_H \quad // \quad \pi_M^{H'} = (1-\phi)\xi\tau_H = \pi_M^H$$

$$\pi_R^{E'} = (\phi + (1-\phi)\xi)(u_A - b - \tau_E) \quad // \quad \pi_M^{E'} = (\phi + (1-\phi)\xi)\xi\tau_E = \pi_M^E$$

Step 3: If R is hosted in $t = 1$, there exist analogous cutoffs $C_E^{'}$ and $C_C^{'}$ that follow from a similar reasoning as in Lemmas 3.1 and 3.2 and from *A1'* and *A2'*.[6] The formal proofs are therefore omitted. These cutoffs determine which of the outcomes specified in *Step 2* is realized, given a specific realization of ξ:

$$C_E^{'} = \frac{F}{(1-\phi)(u_A - b - \tau_H)} > C_E \quad // \quad C_C^{'} = \frac{F + \phi\tau_E}{(1-\phi)(u_A - b - \tau_E)} > C_C$$

Given the definition of X_w and X_h in Section 3.2 and given *A1'* and *A2'*, it follows that:

$$\xi_l = 0 < \xi_m < C_E^{'} < C_C^{'} < 1 = \xi_h \qquad \text{(C.13)}$$

It therefore follows from an analogous reasoning as outlined in the proofs of Lemmas 3.1

[6]Notice that the specialist now certainly prefers to be hosted compared to entering an exclusive contract as consumers incur shopping costs on- and off-platform. An analogous assumption to *A3* is, therefore, *not* required.

that, if R is hosted in $t = 1$, in $t = 2$ the *Hosting' Outcome* is realized for $\xi \in \{\xi_l, \xi_m\}$. Likewise, it follows from a similar reasoning as in Lemma 3.3 that, if R is hosted in $t = 1$, the *Cross Competition Outcome'* is realized in $t = 2$ for $\xi = \xi_h$.

Hence, notice that with an ex-situ policy the *Exclusive Outcome'* is never realized in $t = 2$ if R is hosted in $t = 1$ as it is still not profitable for a strong-type platform to offer an exclusive contract for $\xi = \xi_h$ and the weak-type platform can no longer credibly threaten to launch A for $\xi = \xi_m$. Furthermore, it is an optimal strategy for R to get hosted if M does not launch A. This is ensured by the lower bound from *A2'* as $\tau_E > \frac{\tau_H(1-\phi)\xi_m}{\phi(1-\phi)\xi_m}$ and follows from a similar reasoning as outlined in the proof of Lemma 3.1.

Step 4: If R is not hosted in $t = 1$, it follows from an analogous reasoning as in Lemma 3.5 that identical unregulated market, the *No Hosting Outcome* occurs in $t = 1$. Furthermore, by a similar reasoning as in Lemma 3.4, the *Hosting Outcome'* outcome is realized $t = 2$ given *A1'* and the lower bound on τ_E from *A2'*.

Step 5: Given *Step 1*, it otherwise follows from analogous reasoning as in Corollary 3.1 that the ex-ante payoffs, if certainly facing a strong-type platform under an ex-situ policy, are the following:

$$
\begin{aligned}
E(\Pi_R^{H'}|\mu_s = 1) &= \phi(u_A - b) + (1-\phi)\alpha(u_A - \tau_H - b) + \\
&\qquad \delta\left[(1-\alpha)\phi(u_A - b) + \alpha(\phi(1-\phi)(u_A - b)\right] \\
E(\Pi_R^{E'}|\mu_s = 1) &= (1+\delta)(\phi + (1-\phi)\alpha)(u_A - \tau_E - b) \\
E(\Pi_R^{N'}|\mu_s = 1) &= \phi(u_A - b) + \delta\left[\phi(u_A - b) + \alpha(1-\phi)(u_A - \tau_H - b)\right]
\end{aligned}
$$

It follows from a comparison of the above-outlined payoffs to these outlined in Corollary 3.1 that accepting an exclusive contract becomes relatively less attractive compared to being hosted with an ex-situ policy. It follows from performing similar steps as in Lemma 3.7 that for $\mu_s = 1$, R is rather hosted if:

$$
\delta > \frac{\tau_E\left[(1-\phi) + \frac{\phi}{\alpha}\right] - (1-\phi)\tau_H}{\phi^2 u_A + (1-\phi)u_A - b[1-\phi(1-\phi)] - \tau_E\left[(1-\phi) + \frac{\phi}{\alpha}\right]} \equiv \Delta_o^{'} > \Delta_o \qquad \text{(C.14)}
$$

Given that accepting an exclusive contract is relatively less attractive with an ex-situ policy as $\Delta_o^{'} > \Delta_o$, it directly follows follows more generally from an identical reasoning as in the proof of Lemma 3.7 that $\bar{\mu}_o^{'} > \bar{\mu}_o$ where the specialist accepts an exclusive in $t = 1$ if $\mu_s > \bar{\mu}_o^{'}$.

When comparing $\Delta_{no}^{'}$ and Δ_{no} or $\bar{\mu}_{no}^{'}$ and $\bar{\mu}_{no}$, there can occur scenarios where $\Delta_{no}^{'} \geq \Delta_{no}$ // $\bar{\mu}_{no}^{'} \geq \bar{\mu}_{no}$ if hosting becomes relatively more attractive than accepting an exclusive contract or vice versa with an ex-situ policy.[7] Specifying the parameter spaces where the one or the other applies is beyond the scope of this paper.

However, *A2'* ensures, again, that not being hosted is still a dominated strategy for R if offered an exclusive contract by an analogous argument as outlined in *Step 2* of the proof of Lemma 3.7. Therefore, it follows from a similar argument as applied in *Step 3* of the proof of Lemma 3.7 that $\Delta_{no}^{'} > \Delta_o^{'}$ and $\bar{\mu}_{no}^{'} > \bar{\mu}_o^{'}$.

Furthermore, it follows from a similar reasoning as applied in the Proposition 3.2, that R is either hosted or an exclusive contract is entered in $t = 1$.

Step 6: Given that scenario (i) as outlined in Table 3.1 is the relevant counterfactual, it follows from *Step 5* and a similar reasoning as applied in the proof of Proposition 3.3 that R is again hosted with certainty in $t = 1$. Consequentially, it follows from *Step 3* that, depending on a specific realization of ξ either the *Hosting'*- or the *Cross Competition Equilibrium'* is realized in $t = 2$. Then:

- It follows from *Step 2* and Proposition 3.1 that $E(\pi_R^{C'}) > E(\pi_R^{C})$. In $t = 1$ and the *Hosting Outcome'* that is alternatively reached in $t = 2$, however, the specialist has to additionally reimburse consumers for b when selling product A via the platform. The impact on the specialist's (expected) profit is, therefore, ambiguous and depends on the specific vector of parameters.

- Both consumer types are (in expectation) worse off by the following argument: Given the reasoning in *Steps 1-5* and the equilibrium outcomes outlined in Lemmas 3.1 and 3.2,

[7]The underlying reason is that it follows from *Step 2* and Proposition 3.1 that *R's* expected profit in the *Cross Competition Outcome'* is higher than in the *Cross Competition Outcome*.

(expected) consumers surplus is only impacted if $\xi = \xi_h$ is realized in $t = 2$. Otherwise, R is the sole provider of product A, independent of the policy.

It follows from the outlined strategies in the proof of Proposition 3.1 and in *Step 2* that $F_{R,C'}$ first-order first-order stochastically dominates $F_{R,C}$.

Let now $x' = u_A - b - p_M^{on}$ describe the net utility that consumers experience when purchasing via the platform in the *Cross Competition Equilibrium'* and $x = u_A - p_M^{on}$ the net utility in the *Cross Competition Equilibrium*. Then, it follows from a straightforward change of variable that the distribution functions $F_{M,C'}(x')$ and $F_{M,C}(x)$ describe the probability that a consumer's net utility is at most x' (or x), conditional on buying from M. It follows from a straightforward comparison that $F_{M,C'}(x')$ first-order stochastically dominates $F_{M,C}(x)$ such that consumers are better off in the *Cross Competition Equilibrium*.

It directly follows from the last argument that *Type 2A* consumers are worse off with an ex-situ regulation if $\xi = \xi_h$ is realized. The same is true for *Type 1* consumers when combining both arguments.

- (Expected) Overall welfare decreases as consumers that buy via the platform now experience additional shopping costs equal to b.

- Conditional on $\theta = \theta_w$, it follows from reversing the reasoning applied in Lemma 3.2 that M can no longer enter an exclusive contract in $t = 2$ if R is hosted in $t = 1$ and $\xi = \xi_m$ occurs as $\xi_m < C_E^{'}$ as outlined in *Step 3*. As shown in *Step 2*, $M's$ payoffs if hosting R or if entering an exclusive contract does not depend on whether the policy applies. It follows that a weak-type platform is, in expectation, worse off.

- Conditional on $\theta = \theta_s$, a straightforward comparison shows that $E(\pi_M^{C'}) < E(\pi_M^{C})$. As shown in *Step 2*, the policy does not impact transfer payments. Hence, it follows that the policy negatively impacts a strong-type platform's (expected) profits.

Step 7: Given that scenario (ii) as outlined in Table 3.1 is the relevant counterfactual:

- It follows from a similar reasoning as outlined for the proof of Proposition 3.3 for scenario (v) that an exclusive contract is entered in $t = 1$ if $\mu_s > \bar{\mu}_{no}^{'}$, which is now also possible in scenario (ii) if $\bar{\mu}_{no}^{'} < \bar{\mu}_{no}$ as argued in *Step 5*.

- It follows from a similar reasoning as in Lemma 3.9 that it is optimal for the platform to offer an exclusive contract if $E(\Pi_M^{E'}|\mu_s) > E(\Pi_M^{H'}|\mu_s)$. With:

$$E(\Pi_M^{E'}|\mu_s) = (1+\delta)[\phi + (1-\phi)\alpha(\mu_s(1-\xi_m)+\xi_m)]\tau_E$$
$$E(\Pi_M^{H'}|\mu_s) = (1-\phi)\alpha(\mu_s(1-\xi_m)+\xi_m)\tau_H$$
$$+ \delta\alpha[\mu_s[(1-\phi)(u_A-b)-F] + (1-\mu_s)(\phi+(1-\phi)\xi_m)\tau_E]$$

It must therefore be that...

$$\rightarrow \alpha > \frac{(1+\delta)\phi\tau_E}{(\mu_s(1-\xi_m)+\xi_m)[\delta[(1-\phi)(u_A-b-\tau_E)-F]-(1-\phi)(\tau_E-\tau_H)]} \equiv \underline{\alpha}^{'}$$

...for M to not be willing to offer an exclusive contract. Given $(\mu_s(1-\xi_m)+\xi_m) < 1$, it follows from comparing $\underline{\alpha}^{'}$ and $\underline{\alpha}$ from Lemma 3.9 that $\underline{\alpha}^{'} > \underline{\alpha}$.

Thus, M is willing to offer an exclusive contract if $\alpha \in (\underline{\alpha}, \underline{\alpha}^{'})$. If additionally $\mu_s \in (\bar{\mu}_o^{'}, \bar{\mu}_{no})$, it follows from a similar reasoning as in the proof of Proposition 3.3 that R accepts the offer. Otherwise, R is again hosted with certainty in $t = 1$ by *Step 5* and a similar reasoning as applied in the proof of Proposition 3.3. Then, which outcome is realized depends on a specific realization of ξ.

If $\alpha > \underline{\alpha}^{'}$ and $\mu_s \in (\bar{\mu}_o^{'}, \bar{\mu}_{no})$ or $\mu_s < \bar{\mu}_o^{'}$, it follows from a similar reasoning as in the proof of Proposition 3.3 that R is hosted with certainty in $t = 1$. If R is hosted in $t = 1$, the policy's impact is the same as for scenario (i).

If $\alpha < \underline{\alpha}^{'}$ and $\mu_s \in (\bar{\mu}_o^{'}, \bar{\mu}_{no})$ or $\mu_s > \bar{\mu}_{no}^{'}$, it follows from a similar reasoning as in the proof of Proposition 3.3 that an exclusive contract is entered in $t = 1$. Then, a straightforward comparison to the equilibrium outcome in scenario (ii) as outlined in Proposition 3.3 shows that:

- The (expected) overall impact on welfare is ambiguous and depends on the specific vector of parameters. Consumers that buy via the platform now experience shopping costs equal to b, which negatively impacts welfare. However, M does not incur fixed costs from launching A.

- Both consumer types are (in expectation) strictly worse off as M no longer is able to launch A and the specialist can set monopoly prices with certainty.

- The impact on the specialist's profit is ambiguous and depends on the specific vector of parameters: As argued above, an exclusive contract is only offered if $\mu_s > \bar{\mu}_o^{'} > \bar{\mu}_o$ such that the specialist certainly prefers entering an exclusive contract compared to being hosted (in expectation) by a similar argument as in Lemma 3.7. However, the specialist has to additionally reimburse consumers for b when selling product A via the platform.

- A weak-type platform is (in expectation) better off if an exclusive contract is entered, which follows from the reasoning in Lemma 3.8.

- A strong-type platform is (in expectation) worse off with an exclusive contract as it follows from *Step 2* that in scenario (ii): $E(\Pi_M^{E'}|\theta_s) = E(\Pi_M^{E}|\theta_s) < E(\Pi_M^{H}|\theta_s)$.

Step 8: Given that scenario (iii) as outlined in Table 3.1 is the relevant counterfactual, it follows from the reversed reasoning of *Step 7* that $\underline{\alpha}^{'} > \underline{\alpha}$. Furthermore, $\underline{\alpha} > \alpha$ by definition of scenario (iii) such that the platform is certainly willing to offer an exclusive contract. R accepts the offer if additionally $\mu_s > \bar{\mu}_o^{'}$ as outlined in the previous step. Otherwise, R rejects the exclusive offer and is subsequently hosted by a similar reasoning as in Proposition 3.2.

If no exclusive contract is entered and R is hosted, a straightforward comparison to the equilibrium outcome in scenario (iii) as outlined in Proposition 3.3 shows that:

- Both consumer types are (in expectation) strictly better off as the *Cross Competition Equilibrium'* outcome potentially occurs in $t = 2$ where it directly follows from the applied pricing strategies outlined in *Step 2* that $E(CS_1) > 0$ and $E(CS_{2A}) > 0$. Otherwise, R is able to set monopoly prices such that consumer surplus is not impacted.

- A weak-type platform is (in expectation) worse off if no exclusive contract is entered. The same is true for a strong-type platform as $E(\Pi_M^{H'}|\theta_s) < E(\Pi_M^{E'}|\theta_s) = E(\Pi_M^{E}|\theta_s)$, which follows from *Step 2* and $\underline{\alpha}^{'} > \underline{\alpha}$ as shown in *Step 7*. Thus, if scenario (iii) is the relevant counterfactual, a strong-type platform is certainly worse off. This might, however, not be the case for scenarios (iv) and (v) as shown in the subsequent step.[8]

- The policy's overall impact on $R's$ (expected) profit is ambiguous and depends on the set of parameters: As already argued, $E(\pi_R^{C'}) > E(\pi_M^C)$. However, in all alternatively realized equilibrium outcomes, R is worse off since it also needs to reimburse consumers who buy via the platform.
- (Expected) overall welfare decreases as all consumers experience shopping costs equal to b when buying off-platform and fixed costs from launching A might occur.

If an exclusive contract is entered, the policy does not impact $M's$ profit as transfer payments are not affected by the policy as shown in *Step 2*. Consumer surplus remains unchanged as R can extract the entire consumer surplus, independent of whether an ex-situ policy applies. Since (expected) overall welfare decreases as all consumers experience shopping costs equal to b when purchasing A, the policy must, in expectation, be to the disadvantage of R, which directly follows from the definition of (expected) overall welfare.

Step 9: Given that scenario (iv) or (v) as outlined in Table 3.1 are the relevant counterfactuals:

- It follows from the reasoning outlined for scenarios (ii) and (iii) in *Steps 7 & 8* that the specialist is hosted if $\mu_s \in (\bar{\mu}_o, \bar{\mu}_o^{'})$ or if $\mu_s \in (\bar{\mu}_o^{'}, \bar{\mu}_{no}^{'})$ and $\alpha > \underline{\alpha}^{'}$.[9]

  Then, the policy has the same impact as shown in the previous step if R is hosted and scenario (iii) is the relevant counterfactual. There is one exception: The profits of a strong-type platform might increase with the policy if $\alpha > \underline{\alpha}^{'} > \underline{\alpha}$, which follows from a similar reasoning as outlined in Lemma 3.9.
- Otherwise, an exclusive contract is certainly entered by a similar reasoning as outlined for scenarios (ii) and (iii). Then, the policy has the same impact as shown in the previous step where scenario (iii) is the relevant counterfactual.

[8]The latter is the reason why Proposition 3.5 shows an ambiguous impact on the profit of a strong platform type for scenarios (iii)-(v).

[9]In scenario (v) $\mu_h < \bar{\mu}_{no}^{'}$ is only feasible if $\bar{\mu}_{no}^{'} > \bar{\mu}_{no}$, which cannot be ruled out by *Step 4*.

□

Proof of Proposition 3.6

Given the implementation of the policy as outlined in section 3.6.3, the equilibrium outcome for $\theta = \theta_w$ is the following:

- *If $\theta = \theta_w$, the Hosting Outcome occurs with certainty in $t = 1$. In $t = 2$, the Hosting Outcome occurs if $\xi = \xi_l$ and the Exclusive Outcome occurs if $\xi = \xi_m$.*

The proof follows from backward induction. The outcomes in $t = 2$ directly follow from Lemmas 3.1 and 3.2 as these continue to apply. Furthermore, given that $\theta = \theta_w$, it is an optimal strategy of R to reject an exclusive contract in $t = 1$ and to be hosted instead by the reasoning outlined in the proof of Lemma 3.7, which also implies that R is hosted if not offered an exclusive contract by Lemma 3.6. Thus, independent of whether M offers an exclusive contract or not, the *Hosting Outcome* occurs with certainty in $t = 1$.

Given the counterfactual scenarios as outlined in Table 3.1, the impact on (expected) overall welfare and consumer surplus is straightforward. Given that M is of the weak type, it would have never launched A in an unregulated market and also does not launch A with an ex-situ policy. Nevertheless, overall welfare reduces as now *Type 1* consumers are served via the less convenient off-platform distribution channel. However, R is still able to set monopoly prices to extract all surplus from consumers, independent of whether an in-situ policy applies or not.

The impact on $E(\Pi_S|\theta_w)$ and $E(\Pi_M|\theta_w)$ directly follows from comparing the equilibrium outcome as outlined above to the counterfactual outcomes in an unregulated market as defined in Proposition 3.3 for scenarios (i)-(v) from Table 3.1.

The policy has no impact if R is also hosted in $t = 1$ in an unregulated market, which is the case for scenarios (i) and (ii).

It follows from Corollary 3.1 that, conditional on $\theta = \theta_w$, R is strictly better off in scenarios (iii)-(v) where it would have entered an exclusive contract in $t = 1$ in an unregulated market. The opposite is true for a weak-type platform, which follows from Lemma 3.8.

□

Proof of Proposition 3.7

Given the implementation of the policy as outlined in section 3.6.3, the equilibrium outcome for $\theta = \theta_s$ of the game is the following:

- *If* $\theta = \theta_s$*, the Hosting Outcome occurs with certainty in* $t = 1$ *if either* $\delta < \Delta_o$ *or* $\{\delta \in (\Delta_o, \Delta_{no}) \ \wedge \ \alpha < \underline{\alpha}\}$.[10] *Otherwise, the Exclusive Outcome occurs with certainty in* $t = 1$*. If the Exclusive Outcome occurs in* $t = 1$*, it also is the unique outcome in* $t = 2$*. Otherwise, the Hosting Outcome occurs if* $\xi = \xi_l$ *and the Cross Competition Outcome occurs if* $\xi = \xi_h$ *in* $t = 2$*.*

The proof follows from backward induction. Let me first assume that R is hosted in $t = 1$. Then, the outcomes in $t = 2$ directly follow from Lemmas 3.1 and 3.3 that continue to apply. Otherwise, an exclusive contract is entered in $t = 1$ such that the outcome in $t = 2$ directly follows from the definition of an exclusive contract.

If $\delta < \Delta_o$, it directly follows from Lemma 3.7 that R rejects an exclusive offer in $t = 1$ and is subsequently hosted, even for $\theta = \theta_s$. Notice that this is only possible in scenario (i) by definition of scenarios (i)-(v) in Table 3.1 and by the reasoning outlined in *Step 1* of the proof of Lemma 3.7. It immediately follows that the policy has no impact in this case.

Furthermore, it directly follows from a similar reasoning as outlined in the proof of Lemma 3.3 for scenario (v) that M certainly offers an exclusive contract if $\delta > \Delta_{no}$ that is subsequently accepted by R. Notice that, with an in-situ policy, this is possible in scenario (i) and certainly is the case for scenarios (iv) and (v) by definition of scenarios (i)-(v) in Table 3.1. It immediately follows that the policy has no impact if scenario (iv) or (v) are the relevant counterfactual.

However, if scenario (i) is the relevant counterfactual, expected overall welfare increases

[10] Check Lemmas 3.6, 3.7 and Lemma 3.9 for the relevant cutoffs.

as all consumers buy via the platform where no shopping costs accrue and no fixed costs from launching A are incurred by M. Expected consumer surplus strictly decreases as the *Cross Competition Outcome* can no longer occur with an in-situ policy. It follows from Lemma 3.7 that $E(\Pi_M|\theta_s)$ increases if $\alpha < \underline{\alpha}$ and decreases otherwise such that the policy's impact depends on the specific vector of parameters. It follows from Lemma 3.7 that R is, in expectation, better off, given $\delta > \Delta_{no}$ and $\theta = \theta_s$.

Notice that $\delta \in (\Delta_o, \Delta_{no})$ is possible in scenario (i) and certainly is the case for scenarios (ii) and (iii) by definition of scenarios (i)-(v) in Table 3.1. If this is the case, it follows from Lemma 3.7 that R is going to accept an exclusive contract if offered and is hosted otherwise by the reversed reasoning as outlined in Lemma 3.6. Thus, if $\delta \in (\Delta_o, \Delta_{no})$, it depends on $M's$ incentive to offer an exclusive contract whether the *Exclusive Outcome* or the *Hosting Outcome* is realized. M does not offer an exclusive contract if the latter is more profitable. Thus, if $E(\Pi_M^E|\theta_s) < E(\Pi_M^H|\theta_s)$, which is true if $\alpha > \underline{\alpha}$ as argued in Lemma 3.9. This is only possible in scenarios (i) and (ii) such that the policy has no impact.

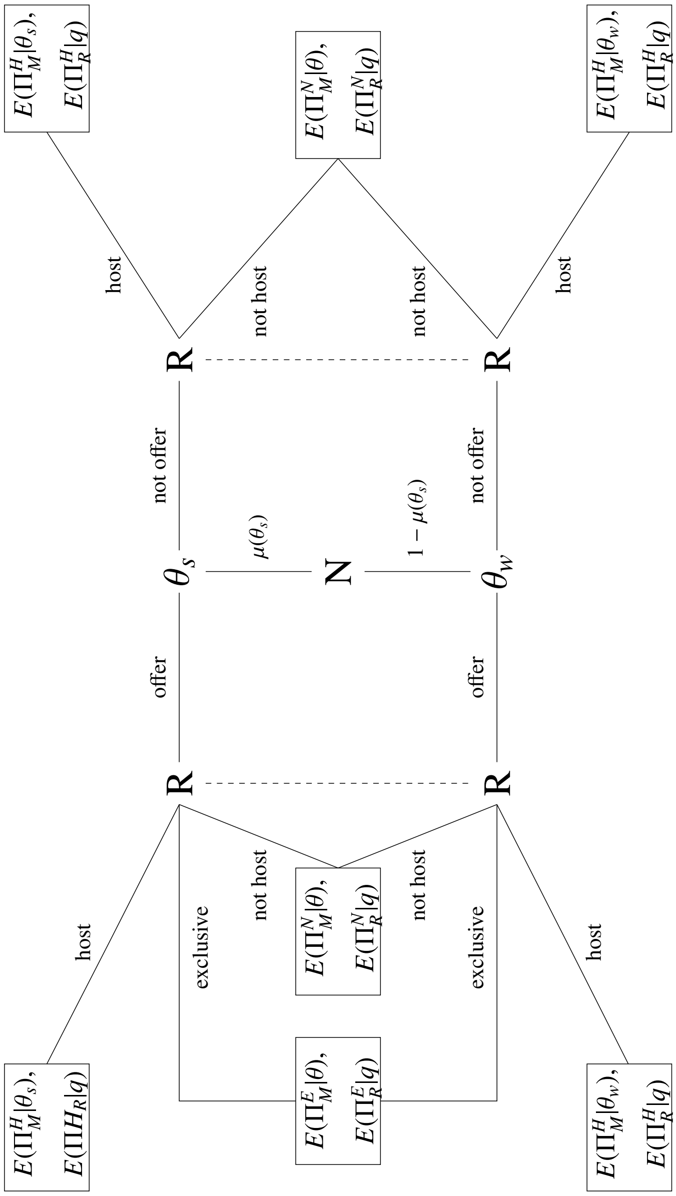
Otherwise, an exclusive contract is offered and subsequently accepted by the combination of the arguments outlined in Lemmas 3.6 and 3.9. Notice that this is only possible in scenarios (i) and (iii). In this case the policy has the same impact as outlined above for $\delta > \Delta_{no}$ where an exclusive contract is entered.

□

C.2 Game-Tree in First Period

Figure C.1 depicts the decision problem of both players in the first period. The expected payoffs that are associated with the distinct outcomes for a given platform type $\theta \in \Theta$ and a given (updated) belief $\mu(\theta|\Psi) = q \in [0, 1]$ are depicted in Corollary 3.1.

Figure C.1: Game-Tree of the Decision Problem in $t = 1$



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Ich versichere hiermit eidesstattlich, dass ich die vorliegende Arbeit selbständig und ohne fremde Hilfe verfasst habe. Die aus fremden Quellen direkt oder indirekt übernommenen Gedanken sowie mir gegebene Anregungen sind als solche kenntlich gemacht. Die Arbeit wurde bisher keiner anderen Prüfungsbehörde vorgelegt und auch noch nicht veröffentlicht. Sofern ein Teil der Arbeit aus bereits veröffentlichten Papers besteht, habe ich dies ausdrücklich angegeben.

München, den 14.09.2022

Maximilian Schader