Learning Processes in Political Economy and Financial Markets

Inaugural-Dissertation
zur Erlangung des Grades
Doctor oeconomiae publicae (Dr. oec. publ.)
an der Ludwig-Maximilians-Universität München

2004

vorgelegt von
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Promotionsabschlussberatung: 9. Februar 2005
Acknowledgements

I want to thank my supervisor Sven Rady for his support, helpful comments and discussions during my Ph.D. studies at the Seminar for Dynamic Modelling. Also I want to thank all colleagues and fellow students for interesting discussions during seminars, conferences and coffee-breaks. Special thanks go to my family, my girlfriend and to numerous friends and acquaintances in and outside Munich, who made this a really great time and gave me manifold moral support and opportunities to recover from hard research-days during travels, mountain-trips, parties or simply in the famous Grandma-Cafe next to the university. I owe all of you a beer or two!
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Preface

While the title of this dissertation is "Learning processes in political economy and financial markets" it could as well be "Learning in classical and behavioral economics" as the work presented not only covers different fields of application but also faces classical learning concepts in economics with an innovative and less conventional approach to this topic.

The importance of learning as a key characteristic of human behavior and the base for proper decision making is seldom denied among economists nowadays. However, the process of learning was a "black box" for a long time in the field of economic research. The focus was not on learning itself but on its outcome and its consequences, namely the perfect rational behavior that is usually assumed when describing decision processes of economic actors. Learning was treated as some sort of passive adjustment, i.e. it was assumed that actors adapt perfect and immediately to new circumstances without any frictions or delay.\(^1\)

In recent years more and more economists investigated the process of learning itself in economic environments.\(^2\) The main areas of research in economic learning theory are rational equilibrium models and game theory.\(^3\) A newer and less conventional

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\(^2\)Kirman and Salmon (1995) provide a general overview of research on learning in economic theory.

\(^3\)"Social learning" is another field that could be distinguished. It is based on both, game theory and rational expectations, and has developed new concepts like "herding" and "informational cascades" etc. See Vives (1996) for more details on social learning.
approach to learning in economic settings is done in the field of behavioral economics.

*Rational equilibrium* learning models work on the basis of statistical methods where agents update their beliefs about an uncertain environment based on observations they make and signals they receive. It is assumed that economic subjects have exact information about probability distributions of uncertain events, they interpret signals right and they have sufficient calculation capabilities to perform even complex updating tasks correctly. The common adjustment or updating mechanism is Bayesian learning. Learning in this sense is the addition of new information to a former set of knowledge and the adjustment of the agents best decision to this joint set of information using utility maximization concepts. The abilities to process the information and to calculate optimal results are assumed to be given from the beginning and are not changed over time. Thus the agents work in a mechanistic way and no deviations from optimal behavior are allowed.

*Game Theory* builds on rational learning but adds various additional mechanisms to develop more sophisticated equilibrium concepts and to cope with multiple equilibria, a common problem when dealing with rational equilibrium models. The refinement of equilibria allows for more precise results but requires additional assumptions on the behavior, the interactions and the potential errors agents make.\(^4\)

The strength of game theory and the rational equilibrium concept clearly lies in its relative homogeneity and perhaps more important in the precision and the determination of its results. More than that they have often been successfully applied to observed economic phenomena and were able to unify problems from very different areas within the roof of one theoretical framework. One example of applied game theory is political economy and we will present a classical learning model in this area in Chapter 1.

The other side of the medal is than economists internalized the rationality concept and its assumptions so deeply that some became unaware of phenomena where

\(^4\)Fudenberg and Tirole (1991) provide an overview of game theoretic concepts.
the concept of rational equilibria might not be the best solution or where behavior obviously is driven by other forces. Psychology, for instance, mentions many forces that influence human behavior besides rationality but most of them were persistently ignored, maybe because they are hard to grasp theoretically or because some of them are contradictory in their nature.

Anyhow there is an increasing number of facts from real life observations and experimental results that cannot be explained satisfactory by standard economic theory. Especially when looking for the micro foundation of large scale economic phenomena there seems to be no way around regarding actual individual behavior even if it differs from perfect rational behavior. This is where the growing discipline of behavioral economics steps in and tries to explain human decision making at the border line of rational economic choice and psychological evidence.\textsuperscript{5} Behavioral economics tries to set up experiments where the decision making of human actors is tracked in more or less realistic situations and identifies deviations from the predictions of classical theory. The theoretical line of behavioral economics then develops new methods or changes assumptions in order to describe observed phenomena more precisely.

In the case of learning models, behavioral economics attempts to view learning as a more complex process than classical concepts. Instead of focussing on the outcome of perfect rational behavior it allows for all kinds of individual misinterpretation of information, suboptimal decision making or inconsistencies in behavior as long as there is enough evidence for them from either psychological research or economic experiments and observations.\textsuperscript{6} Specifically, even heterogeneity and progress in information processing capabilities are allowed and we will draw on real life evidence of this phenomena when building up a behavioral learning model in Chapter 3.

While keeping a closer eye to the whole spectrum of human behavior and attempting to be more rigid with assumptions that are normally made without a thought, behavioral economics has its caveats too. The most important one is probably the

\textsuperscript{5}In Section 3.1, 3.1.1 and 3.1.2 we will describe the different approaches in the application case of financial markets more detailed.

\textsuperscript{6}See Slembeck (1997) for a review of learning in behavioral economics.
lack of a unifying framework which makes behavioral economics harder to grasp and less easy to set up rules to deal with a given problem. Some facets of human behavior - as behavioral economics sees it - seem to be arbitrary (or so complex that they look arbitrary to an observer) to a certain degree. This makes it hard to formulate precise mathematical models as economists are used to, and even harder to formulate them in such a way that they are compatible to standard economics.

Today, it seems to be still unclear whether the more reality-based behavioral approach or the theoretically sophisticated traditional concepts will prevail in economic research, or if it will be possible to combine both of them in a meaningful way.

In the following we will confront the different approaches to learning in economics. Therefore we present two applications of both of the mentioned learning concepts in two theoretical models in political economy and financial markets. Additionally we present an empirical study that was performed in order to confirm predictions made from the first of our models.

Thus, Chapter 1 sets up a game theoretic model in a political competition framework where agents learn by observing signals of economic outcome and politicians can hide corrupt activities behind exogenous shocks to growth. Chapter 2 provides an empirical cross-country study on the theoretical results of chapter 1. Chapter 3 follows the behavioral branch of learning theories. A model is set up where agents improve their quality of decision making and we show that asset pricing bubbles and other frequently observed financial markets phenomena can be explained within this setup. Chapter 4 extends the model of Chapter 3 to a dynamic setting and to more general price paths.
Chapter 1

Learning in Political Economy: A
Game Theoretical Model to Explain
Corruption in Unstable Economies

In this chapter we stick to a classical "learning"-model. Agents use Bayesian
updating in a political economy framework to gain information about politicians.
We describe how the level of corruption can be linked to the economic stability of a
country independent of the actual height of the country’s GDP.

1.1 Introduction

There is almost no country which has not been hit by some sort of corruption
at some time of its history. Even today, where corruption no longer appears to
play a major role in western economies, no one would claim that it has been totally
defeated. For developing and transition countries things are much worse. Corruption
is more pervasive there and in some countries it is blamed for having a severe negative
impact on economic activity. This impact of corruption on growth and stability is
widely studied in the literature but as far as we know there is little work yet in the
opposite direction, i.e. in examining the influence of the economic (in)stability of a
country on the level of corruption.
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In this chapter we present a model addressing this gap. Therefore we set up a two-period model of political competition with asymmetric information and two types of politicians, where incumbents have to decide about the level of corruption and economic output is affected by their decision. We will show that in a Bayesian equilibrium, the incumbent chooses a higher level of corruption if the variance of economic output is high and a lower level if it is low. Also we will show that high levels of corruption are more likely if regular legal remunerations for being in office are low. Thus we conclude that a high level of economic instability should foster corruption and we present an empirical test of our suggestions in Chapter 2.

This chapter is organized as follows: First we take a look at the related literature to define the notion of corruption and to track alternative explanations for the appearance of corruption. Then we present the model in Section 1.3 and work through the game and the propositions. A conclusion and an outlook are following paper and a variation of the model is discussed in Appendix A. In Chapter 2 the theoretical model is tested in a cross-country study.

1.2 The Notion of Corruption

Corruption is a more or less prevalent phenomenon in any society that runs a political apparatus to control the allocation and distribution of limited resources, rights and claims within its economic system. Whenever the control of public decision makers by the society either by direct observation or by moral norms is not absolute, there is room for the former to behave opportunistically and to take inefficient or unjust decisions in exchange for payments or other grants from the privileged parties. The sum of this socially undesirable behavior is what we call corruption in our paper.

To get a better understanding of the types of corruption and how they might lead to inefficient outcomes, we follow Rose-Ackerman (1999) and distinguish five genres of corruption:

1. Bribes to equate differences in supply and demand stemming from legal restric-
tions: In this case bribes raise the price of a good in excess demand and with fixed supply until demand is lowered to the available amount of supply. Therefore some rents are transferred and pocketed by the corrupt official. Inefficiency occurs if the considered party is different from the one with the highest valuation.

2. Bribes as incentive payments are payed when it depends on the goodwill of officials whether demanded work is done fast or slow ("speed money"). Alternatively bribes may be payed to slow down the work of officials concerning competitors. The second case is sometimes even more "effective" for the bribing party, as it is often easier for an official to slow down a process than to accelerate it. Inefficiency results from wrong incentives and the creation of additional "road blocks" by officers to increase their veto power.

3. Bribes to lower costs. This means that an official is payed to be indulgent when controlling for example safety standards. The inefficiency might result from ignoring external effects that have been internalized by laws.

4. Bribes to obtain limited concessions which otherwise would have been sold in an auction or a beauty contest. This leads to misallocation if not all parties have the same readiness to bribe. Otherwise the result would be the same as in an auction, but rents would go to the official and not to the state.

5. Bribes to buy political influence and votes. Here lobbies pay bribes to politicians so as to strengthen their position. In a broader way one could also put politicians' favors to special interest groups in exchange for votes or campaign contributions into this category.

Whereas probably hardly anybody doubted the negative effects of corruption in practice, from a theoretical point of view it was not clear for a long time whether corruption is really that distortionary. Some economists (see for example Leff (1964) or Huntington (1968) ) even defended the use of bribery as an efficient, welfare enhancing mechanism. The first argument was that bribes provide a motivation for officers to work harder in so far as they act at a piece rate. They also claimed that "speed money" avoids bureaucratic delays and bribes for concessions work as an auction-like
allocation device, where scarce resources are given to the parties with the highest valuation and thus the highest bribe offer. According to this school bribes should not have more negative effects on the economy than other transfers, for example taxes. Interesting as it is, this theory falls short of the fact that the access to such a bribe-driven market could very likely differ for various demanding parties either for differences in moral considerations or for different levels of trustworthiness, connections to officers and available information. It also ignores the negative incentives for officers induced by corruption, as they might try to strengthen their position by setting up additional hurdles. From a contract-theoretic perspective, bribe "contracts" have the disadvantage of not being enforceable by the trade partners.

Shleifer and Vishny argue that corruption is more costly than taxation because of the secrecy premise, i.e. the necessity to hide away corrupt activities from the public and the law. The secrecy premise thus allocates resources for setting up and covering secure information channels (see Shleifer and Vishny (1993)). Other empirical and theoretical studies as from the United Nations (1989) or from Klitgaard (1991) confirm the suspicion that corruption is a wasteful activity and should be banned wherever possible. Another implication of Leff and Huntington’s theory is that corruption should especially exert its pretended positive effect in inefficient bureaucratic environments. This argument can be cancelled out by a empirical study by Mauro (1995), who shows that the correlation between growth and corruption is far from being significantly different in countries with highly efficient and with less efficient bureaucracies, whereas according to Leff and Huntington growth should be closer correlated to corruption in less efficient bureaucracies, because they should work better with the "help" of corruption. In our work we stick to the view of corruption as a "bad" activity, as we refer to a negative effect of corruption on economic output in a common way.

In the terms of informational economics, corruption could be seen as a principal-agent-problem with the society as the principal and the politicians and officers as the agents. Under asymmetric information about the agents’ actions there is no way to
provide perfect incentives without handing over the entire surplus to them. Fixed remunerations and benefits given, only transparency and monitoring (reduction of asymmetry), means of punishment (deterrence), or some "moral codex" (altering of the agent’s utility function) can reduce the level of corruption.

All points play an important role in the determination of the levels of corruption observed in reality. For example the means of control should be higher in societies with a high level of democracy and free media than in autocratic countries with suppressed media. Also the society in autocratic systems has fewer possibilities to punish decision makers than in democracies. The model of Rasmussen and Ramseyer (1994) addresses this point and thus claims that corruption should be higher in autocratic systems.

But even in the most autocratic society the people has a possibility to discipline the government - for example by threatening a revolution (see Acemoglu and Robinson (2000)). Of course this threat is quite poor and probably less effective than the possible sanctions in a democratic country. Still it may play a role in the behavior of corrupt decision makers in autocratic countries and could well bound away the level of corruption from the maximum level. And as the level of corruption differs widely both among autocratic and among democratic countries, the degree of democratization cannot be the sole determinant of corruption.

The last point, the differences in moral norms, certainly also plays an important role in the explanation of corruption. Some even consider it to be the main reason for the degree of corruption and claim that the effects of different constitutional and economic circumstances are negligible compared to those of different social norms. Bardhan (1997) gives an example of the different views of corruption by Westerners and Asians. The former perceive the regular "baksheesh" payments in Asian countries as corrupt, whereas the latter find the high degree of monetarization even in personal transactions in western countries corrupt. The differences could be explained by the comparatively high level of individualization in western societies and by the long tradition of mutual gift exchange at all levels of society in many Asian countries.

Even if they sound somewhat tautological, it has to be admitted that explanations related to cultural differences are capable to explain differences in the level of corrup-
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tion even *within* one country or society, for example between northern and southern Italy. Other theories and examples of this type are provided by Putnam (1993) for Italy, by Yang (1989) for the Chinese society and by many others.

However, in our opinion cultural differences rather enforce existing tendencies of corruption stemming from the principal-agent-relationship, than being the sole source of it. An argument supporting this view is that there are many counter-examples where culturally related areas show huge differences in the level of corruption (e.g. Singapore compared to Malaysia or Indonesia, or many examples where corruption differs between urban and rural areas within one culturally homogeneous region).

Because corruption seems to be much more common in developing and poor countries, many studies concentrated on the connection of corruption to growth or to wealth. It is argued that economies with low output and slow growth are more susceptible to corruption because the controlling power of the executive is weak and people on all levels of societies are in great need for extra incomes.

Paldam (2002) ran a number of cross-country regressions and found empirical evidence for the influence of GDP per capita on corruption levels, though in the same paper he argues that the correlation is more a trend than of precise forecasting value.

Detailed ideas concerning the corruption-growth relationship are highlighted by Ehrlich and Lui (1999) or by DelMonte and Papagani (2001). Mauro (1995) looks for empirical evidence in this direction, and finds a negative association between perceived corruption and the investment rate in a cross-country study. The intuition is that it is expensive and risky to invest in highly corrupt countries, and therefore growth should be low, whereas the resulting poverty fosters corruption even further and so on.

But on the other hand, Bardhan (1997) points out that it cannot simply be inferred that low economic growth is the only source for corruption, as there are many cases where corruption is rising sharply although growth is relatively high and incomes are rising. Many of the eastern European transition economies as well as some south-east Asian states fall into this category.
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These examples also contradict the argument of many liberal economists that corruption is spawned by regulatory states because the level of corruption increased substantially after market reforms in recent years for example in post-communist Russia or China.

Other models stress the idea that corruption shows a sort of a reinforcing effect, i.e. that it pays more to be corrupt when everybody else is. Thus corruption is expected to spread fast once a certain critical level is reached and on the other hand it should be difficult to introduce corruption in an extremely "clean" country. For example Andvig and Moene (1990) come to this result in their model by pointing out that for a corrupt officer it is cheaper to get detected by a corrupt superior than by a clean one. Rasmusen and Ramseyer (1994) support this view of corruption as a collective action problem with the results of another model.

The intuition of this theory is quite appealing as it is easy to imagine that a newly installed and so far innocent officer in Bangladesh is more likely to accept a bribe than one in Finland. Nevertheless it fails in describing how the level of corruption was able to reach this critical point in some countries and not in others.

To summarize, there are some interesting and plausible approaches to the phenomenon of corruption, most of them addressing cultural, constitutional or economic differences between countries and some of them relating growth and wealth to the level of corruption. Yet none of the theories can fully explain corruption and, as far as we know, none examined the relationship between economic (in)stability and corruption. Our model wants to address this gap and show why it is plausible to think of economic instability as a possible additional source of corruption.

To investigate this issue we define stability as the amplitude of economic output growth around a long term drift rate. A country with a low variance of output growth (i.e. a relatively monotone output path) is called stable, one with a high variance (i.e. a relatively non-monotonic output path) is called unstable. We show that instability in economic growth can hamper the electorate in learning the politicians true type and
allow them to behave in a less social way. The stability of output enters exogenously in our model, and one can think of it as being an effect of the state of the world economy, exchange rates, foreign relationships, prices of raw materials, internal frictions from reorganization processes, natural disasters and the like. The idea of output stability being exogenous is supported by a paper of Easterly et al. (1993), where they find that most of the variation in growth rates is due to random shocks and not to some special policy.

1.3 The Model

We use a two period political competition model where an incumbent can decide on the level of corruption and faces some elections\(^1\) at the end of the first period.

1.3.1 Model Structure

There are two periods \(t = 1\) and \(t = 2\) and two possible levels of corruption in each period, \(l_t \in \{l, \bar{l}\}\). The normalization \(l_t = \bar{l} = 0\) means there will be "no corruption" in period \(t\) and \(l_t = \bar{l} > 0\) means that a high level of corruption is chosen in period \(t\).\(^2\)

\(^1\)Basically the instrument of election is only a democratic device to discipline politicians. If one thinks about the threat of revolution being a disciplining device in autocratic countries, as mentioned above, one can expand our model even to non-democratic countries. The "election" there would be the decision whether to revolt or not. Of course the disciplining effect of a threat of revolution should be small due to the high private costs of such a revolution, but in a different context Acemoglu and Robinson (2000) show how this threat can suffice to cause the ruling elite of an autocratic country to extend the franchise in order to calm the people. Similarly the ruling class might restrict corruption in our model to keep the people quiet. In Chapter 2 it is shown how the level of democracy influences the effects we show and it will come out that they are stronger in democratic countries but not negligible in non-democratic ones.

\(^2\)The level of corruption chosen can be thought of as the degree of the incumbent being corrupt himself or allowing his subordinate officers to be corrupt. If an incumbent decides to turn a blind eye on corruption in the governmental apparatus he will benefit from this by getting stronger support by his officers. In the other case, when he decides on low corruption he will not only lose the direct income of corruption, but his life will get harder because it is likely that support from his officers
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Definition of Players

There are two types of politicians, a "good" one ($\theta = g$) and a "bad" one ($\theta = b$). The overall utility function of politicians is

$$U_P = \tau_t (u_0 + a_\theta l_1) + \tau_2 \delta (u_0 + a_\theta l_2)$$  \hspace{1cm} (1.1)

$\tau_t$ indicates whether $P$ is in office in period $t$ ($\tau_t = 1$) or not ($\tau_t = 0$). $u_0$ is the "base" utility or ego-rent of being in office, i.e. the salary, social status and so on. This utility is fixed and cashed in for certain in every period the politician is in office. The "good" politician derives negative utility from being corrupt, as he might feel guilty or fear punishment, i.e. he will stick to his promise not to be corrupt. The bad politician does not care about external effects or morality considerations and gets direct positive utility from a high level of corruption. In the model this is expressed by $a_g < 0$ and $a_b > 0$. $a_g$ and $a_b$ denote the gains-factor from corruption for good respectively bad politicians.

All politicians are drawn from a large pool of politicians, where $\pi$ is the fraction of "good" politicians in this pool. Future utility is discounted by $\delta$.

The incumbent $I$ of period 1 has to decide on the level of corruption $l_1$ in period 1 and on the level of corruption $l_2$ in period 2 for the case that he gets reelected.

Citizens derive utility $U_V$ only from the performance of the economy, which is measured by economic output $e$. It is assumed that they all have strictly monotonic utility functions in output, so $U_V(e') > U_V(e) \iff e' > e$. Thus, citizens can be modeled by a representative voter $V$ with utility function $U_V$. The only action the representative citizen takes is to vote at the end of period 1. Then, $V$ can decide is lower when they experience stricter controls of their actions. Therefore in this model only the decision of the incumbent is considered and the officers are supposed to follow the decision of the government.
whether to confirm $I$ in office or to elect a challenger $C$.

$C$ is the challenger that is drawn out of the pool of politicians to face $I$ in the election. If he gets elected he will be in office in period 2, therefore his only action is to set a level of corruption in period 2 if he is elected.

**Economic Stability**

The economic output defines the voter’s utility and is crucial as a signal about $I$’s policy in period 1. The voter observes the change in economic output directly by comparing his utility at the beginning and at the end of period 1. However, the change in economic output is affected by three factors. First, there is a drift rate $d$ determining the long term growth rate of the economy. $d$ is of no importance for the theoretical model itself but nevertheless we keep it for the empirical part in Chapter 2. Second, there is an exogenous shock $s$ to the economy. $s$ brings in the variance to output. Third, we assume a negative impact of corruption on the performance of the economy. Therefore the change in economic output $e$ is modeled as follows:

$$\Delta e = d + s - l \quad (1.2)$$

$s$ has mean zero and is distributed with one of two possible density functions $h(\cdot)$ which have either high or low variance, where high and low variance of $s$ are equally likely. For the computation of the equilibrium we assume $h(a, b)$ to be a uniform distribution on the interval $[a; b]$ in the remainder of the model:\footnote{In appendix A, a version of the model with a more natural normally distributed shock to economic output is discussed. Unfortunately this leads to some computational problems due to the characteristics of the normal distribution function.}

$$s \sim h(-\bar{e}, +\bar{e}) \text{ with } \bar{e} \in \{e, \bar{e}\}, \quad p(\bar{e} = e) = p(\bar{e} = \bar{e}) = \frac{1}{2} \quad (1.3)$$

$\bar{e} = e$ stands for a relatively "stable" economy, i.e. shocks are comparatively small, whereas $\bar{e} = \bar{e}$ denotes an "unstable" economy which suffers large shocks to economic
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outcome. Think of a stable economy for example as a well diversified one, whereas an unstable economy could be a country highly dependent on one export good with large price fluctuations. We assume that \( \bar{I} < 2c \), i.e. the influence of the exogenous shock on the economy is not too small compared to the influence of corruption.

Information Structure

There are two types of uncertainty which the citizen is facing when making his electoral decision:

First, the level of \( l_1 \) chosen by \( I \) cannot be directly observed by the citizen. It is assumed that corruption mostly takes place between government and only a few of the citizens in special positions, e.g. firm managers, lobbyists etc. Therefore the great majority does not know whether much or little corruptive activity is executed by the government. However politicians and citizens observe \( \hat{e} \) at the beginning of period 1, as they know whether they live in a "stable" or "unstable" economy. Thus citizens know the maximum amplitude of the shock \( s \), but they do not know its actual size. Also citizens naturally can observe their own utility after \( e \) incarnates. Uncertain of the origin of an income shock, they only can calculate probabilities for the chosen size of \( l_1 \). Thus \( e \) acts as a noisy signal about the action of \( I \).

Second, the types of \( I \) and \( C \) are not known a priori, but both have an initial reputation of being of "good" type denoted by \( \alpha_I \) and \( \alpha_C \), i.e. \( \alpha_i = p(\theta(i) = g) \), \( i \in \{I, C\} \). \( \alpha_I \) and \( \alpha_C \) are independent of actual types and are drawn from the same cumulative distribution function \( F \). \( F \) is common knowledge.

The Game

The players of the game are the incumbent, the representative citizen and the challenger. The timing of the game is as follows:
First, nature chooses the type of the incumbent \( \theta(I) \), which is only observed by the incumbent \( I \). Then, nature chooses \( \hat{e} \), which is observed by all players. Now, \( I \) must
choose \( l_1 \), and nobody except herself knows her choice. Nature then draws \( s \) with 
\[ p(s = x) = h(x) \]  
and \( s - l_1 \) is computed as the net output, as we set \( d = 0 \) in the theoretical model. \( s - l_1 \) is observed by all players.

After the citizen and the incumbent receive their first-period utilities, an election is held, and the former has to choose between the incumbent \( I \) and the challenger \( C \) based on his beliefs about \( \theta(I) \) and on \( \alpha_C \). \( C \) is drawn by nature from the pool of politicians (and only himself knows his type) and as mentioned above his initial reputation is drawn from \( F \).

The winner of the election then chooses the second period \( l_2 \). Finally second-period payoffs are realized and the game ends. A timeline of the game can be found in Figure 1.1.

![Figure 1.1: Game Timing](image)

A strategy for the incumbent is the pair \( l := (l_1, l_2) \) of decision rules about the degree of corruption in the first and (if reelected) the second period. \( I \)'s choice is dependent on her type \( \theta(I) \) and the state of the world \( \tilde{e} \).

A strategy for the challenger is the decision rule \( l_2 \) if he is elected for the second period. His choice is dependent only on his type \( \theta(C) \).

A strategy for the citizen is the voting decision rule \( v \) that specifies whether he votes for \( I \) \( (v = I) \) or \( C \) \( (v = C) \). The choice is dependent on his beliefs \( \beta_I(e) \) about the incumbent’s type after observing her initial reputation \( \alpha_I \) and the signal \( e \) and on the
challenger’s initial reputation $\alpha_C$. $V$ compares $\alpha_C$ and $\beta_I$ and chooses the candidate with the higher value. Thus the probability for $I$ to get reelected is $F(\beta_I)$.

A perfect Bayesian equilibrium of the game is a set of optimal strategies for $I$, $C$ and $V$ and of consistent beliefs of the citizen about $I$’s type.

It must satisfy the following properties:

- $I$ chooses $l$ such that $l(\theta(I)) = \operatorname{argmax}_l EU_P$
- $C$ chooses $l_2$ such that $l_2(\theta(C)) = \operatorname{argmax}_{l_2}(u_0 + a_\theta l_2)$
- $V$ chooses a voting rule $v$ such that $v = \operatorname{argmax}_v EU_V$ where $V$’s second period utility is dependent on the type of the elected politician $v$
- $V$’s posterior belief about $I$’s type after observing the signal $e$, $\beta_I(e) = p(\theta(I) = g | e)$ is derived by updating using Bayes’ rule and is consistent

### 1.3.2 Equilibrium

As we face a two period game, we can solve it by backward induction.

**Proposition 1.1** In a perfect Bayesian equilibrium as defined above, a "good" incumbent chooses $l_2 = \underline{l}$, a "bad" one $l_2 = \overline{l}$.

**Proof**

In the second period the behavior of the politician in power is clear. She does not have to worry about being reelected and will simply maximize her second period utility $u_0 + a_\theta l_2$. This leads to $l_2 = \underline{l}$ for $a < 0$ ("good" politician in power) and to $l_2 = \overline{l}$ for $a > 0$ ("bad" politician in power).

\[\square\]

\footnote{Here we have an endgame effect, which allows us to solve the game by backward induction. However this assumption is not too unrealistic. First, many countries restrict the time for higher...}
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In the first period, a "good" incumbent maximizes her expected utility. Her behavior will obviously depend on citizens’ beliefs. In order to rule out equilibria based on unnatural out-of-equilibrium beliefs, such as "a low level of e indicates a low l_1" which could lead to self-fulfilling equilibria where even good politicians play l_1 = \bar{L}, we concentrate on equilibria with monotonic beliefs of citizens. This refinement was first used by Coate and Morris (1995) for situations where other refinements such as the equilibrium dominance argument by Cho and Kreps (1987) cannot be applied because of the noisy character of the signal. Basically it means that a higher economic output is believed to be produced more likely by a "good" politician and thus l_1 = 0 with a higher probability as if a low economic output is observed. Formally: e' > e \Rightarrow \beta(e') \geq \beta(e), other things held constant. This concept sounds rather plausible in our case, as citizens know about "bad" politician’s preferences and their negative impact on output. For further discussion see Coate and Morris (1995).

**Proposition 1.2** In a perfect Bayesian equilibrium with monotonic beliefs as defined above, a "good" incumbent chooses l_1 = \bar{L}.

**Proof**

Under the assumption of monotonic beliefs both the direct utility from engaging in corruption decreases because a_g < 0, and the chance of getting reelected is non-increasing, thus the "good" incumbent will choose l_1 = \bar{L}.

\[ \square \]

Foreshewing this, citizens prefer to have a good politician in office in the second period, which would result in a higher expected level of e. Therefore they will vote politicians to be in office, and second every politician faces a limited lifespan and an increasing probability of dying in each period, which should lead to the same results in a multi period game. Empirically it would be interesting to test whether corruption activities rise the closer incumbents get to their last possible period in office, in which - as in our 2nd period - they do not have to care about reelections anymore.
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for the candidate whom they consider to be good with a higher probability, thus $v = I \iff \beta_I(e) \geq \alpha_C$. Starting from their prior belief $\alpha_I$ they form $\beta_I(e)$ as follows:

$$\beta_I(e) = \frac{\alpha_I p(s - l = e \mid \theta(I) = g)}{\alpha_I p(s - l = e \mid \theta(I) = g) + (1 - \alpha_I) p(s - l = e \mid \theta(I) = b)}$$  \hspace{1cm} (1.4)

$$= \frac{\alpha_I h_I(e)}{\alpha_I h_I(e) + (1 - \alpha_I) h_I(e)}$$  \hspace{1cm} (1.5)

where $h_I(e)$ is the density function of $e$ if $l_1 = \underline{l}$ and $h_I(e)$ is the density for $e$ if $l_1 = \overline{l}$.

$h_I(e)$ and $h_I(e)$ are similar to the distribution function $h$ of $s$ but shifted by $l$ resp. $\overline{l}$ to the left. For the case of uniformly distributed $s$ this leads to

$$\beta_I(e) = \begin{cases} 0 & \text{if } e \in [-\bar{e} - \overline{l}; \bar{e}] , \\ \alpha_I & \text{if } e \in [-\bar{e}; \bar{e} - \overline{l}] , \\ 1 & \text{if } e \in [\bar{e} - \overline{l}; \bar{e}] . \end{cases}$$  \hspace{1cm} (1.6)

In this case there is either no learning or full revelation of $I$'s type.\(^5\) For $e < -\bar{e} - \overline{l}$ and $e > \bar{e}$ beliefs are not defined but these cases cannot occur in the model, so this beliefs can be set to any value.\(^6\)

The incumbent’s first period behavior is determined by the maximization of $I$'s overall expected utility which in turn depends both on $I$'s choice of $l_1$ and on the citizen’s belief. For a "good" incumbent it was easy to show that $l_1 = \underline{l}$ under monotonic beliefs, as both direct utility and the probability of getting reelected are decreasing in $l_1$. For the case of being "bad", $I$ has to make a tradeoff between increasing his income in the first period by choosing $l_1 = \overline{l}$ as $a_b > 0$ and maximizing the probability of being reelected to get in favor of second period incomes by choosing $l_1 = \underline{l}$. To find his actual behavior, we have to maximize overall expected utility as

\(^5\) Again, see the appendix for a case with imperfect revelation of types. Anyway, this is only a matter of "elegance", as uniformly distributed shocks perfectly suffice to describe the effect under consideration.

\(^6\) Note that the citizens can only punish the incumbent by voting her out of office and deprive her of second period benefits. However, if they could punish her arbitrarily hard, it would be possible to implement the first best, as there are cases where $V$ can be sure that he is facing a bad politician.
stated in equation 1.1, given that the citizen updates his beliefs according to equation 1.6.
If \( I \)'s decision is \( l_1 = \bar{l} \) for \( \theta(I) = b \), then we are in a pooling equilibrium, where behavior does not depend on \( I \)'s type, if \( I \)'s decision is \( l_1 = \bar{l} \) for \( \theta(I) = b \), we are in a separating equilibrium, where the good politician stays clean and the bad one fosters corruption.

### 1.3.3 Impact of Stability

While solving for the optimal first period behavior of "bad" incumbents, we come to the main topic of the Chapter, the question of whether the stability of an economy will have any impact on the degree of corruption within this country. To answer this question we set up a lemma and several additional propositions.

**Lemma 1.1** In a perfect Bayesian equilibrium with monotonic beliefs as defined above, an incumbent \( I \) with \( \theta(I) = b \) chooses \( l_1 = \bar{l} \) \( \iff \delta \left( \frac{u_0}{a_b} + \bar{I} \right) > \bar{e} \).

**Proof**

The incumbent maximizes her expected payoff as follows:

\[ l(\theta(I)) = \arg \max_{l_1} EU_{P}(\theta(I), l) \]

Thus, an incumbent with \( \theta(I) = g \) chooses \( l_1 = \bar{l} \). An incumbent with \( \theta(I) = b \) chooses \( l_1 \) such that

\[
\begin{align*}
    l_1 &= \arg \max_{l_1} E(u_0 + ab_l_1 + p(v = I)\delta(u_0 + ab\bar{l})) \\
    &= \arg \max_{l_1} E(u_0 + ab_l_1 + F(\beta(e))\delta(u_0 + ab\bar{l})) \\
    &= \arg \max_{l_1} E(ab_l_1 + \int_{-\bar{e}-\bar{l}}^{\bar{e}} \beta(e) \, de \, \delta(u_0 + ab\bar{l}))
\end{align*}
\]

(1.7)

Thus, an incumbent with \( \theta(I) = g \) chooses \( l_1 = \bar{l} \). An incumbent with \( \theta(I) = b \) chooses \( l_1 \) such that

\[
\begin{align*}
    l_1 &= \arg \max_{l_1} E(u_0 + ab_l_1 + p(v = I)\delta(u_0 + ab\bar{l})) \\
    &= \arg \max_{l_1} E(u_0 + ab_l_1 + F(\beta(e))\delta(u_0 + ab\bar{l})) \\
    &= \arg \max_{l_1} E(ab_l_1 + \int_{-\bar{e}-\bar{l}}^{\bar{e}} \beta(e) \, de \, \delta(u_0 + ab\bar{l}))
\end{align*}
\]

(1.8)

So, \( l_1 = \bar{l} \) if and only if

\[ EU_{P}(l_1 = \bar{l}) > EU_{P}(l_1 = \bar{l}) \iff \]

(1.9)
\[
\int_{-\tilde{e}}^{\tilde{e}} \left( p(e|l_1 = \bar{l}) \beta(e|l_1 = \bar{l}) - p(e|l_1 = \tilde{l}) \beta(e|l_1 = \tilde{l}) \right) \, de > \frac{a_b \bar{l}}{\delta(u_0 + a_b \bar{l})}
\]

Equation 1.9 states the general condition for a low corruption choice for an arbitrary density function \( h(\cdot) \) of shocks \( s \). For the case of uniformly distributed prior beliefs \( \alpha \) and uniformly distributed shocks \( s \) this yields:

\[
l_1 = \bar{l} \iff \frac{1}{2\tilde{e}} \left( \pi(2\tilde{e} - \bar{l}) + \bar{l} - \pi(2\tilde{e} - \bar{l}) \right) > \frac{a_b \bar{l}}{\delta(u_0 + a_b \bar{l})}
\]

\[
\iff \frac{1}{2\tilde{e}} > \frac{a_b}{\delta(u_0 + a_b \bar{l})}
\]

\[
\iff \frac{\delta}{2} \left( \frac{u_0}{a_b} + \bar{l} \right) > \tilde{e}
\]

(1.10)

Lemma 1.1 states the condition for a "bad" incumbent to behave properly in the first period. The probability to do so increases in \( \delta \) as this leads to a higher valuation of second period payoffs and thus gives an incentive to stay in office, and the probability to get reelected can be maximized by abstain from corruption. Clearly, increasing \( u_0 \) also has a positive impact on behaving properly in the first period, as it rises the payoff in the second period and thus the incentive to get reelected. Interestingly, rising the volume of corruption, namely \( \bar{l} \), has the same effect of lowering the incentive of first period corruption, though it affects both periods. Obviously, the disadvantage from the diminished reelection probability overweighs the advantage of higher first period gains. This issue will be discussed below Corollary 1.1. Increasing \( a_b \) rises the willingness to engage in first period corruption, as it increases immediate gains from corruption more than future gains, which has to be discounted by \( \delta \) and the probability of getting reelected. The probability of first period corruption is also increasing in \( \tilde{e} \), as it rises the right hand side of (1.10).

**Proposition 1.3** For fixed \( \bar{l}, a_b \) and \( \delta \), there exists a \( \bar{u}_0 \) s.t. \( u_0 > \bar{u}_0 \Rightarrow l_1 = 0 \) for all \( \theta \) in equilibrium, and \( \bar{u}_0 = a_b(\frac{\tilde{e}}{\delta} - \bar{l}) \).
Chapter 1

Proof

Rewriting (1.10) leads to

\[ u_0 > a_b \left( \frac{2\tilde{e}}{\delta} - \bar{l} \right) \]

thus

\[ \bar{u}_0 = a_b \left( \frac{2\tilde{e}}{\delta} - \bar{l} \right). \]

(1.11)

Therefore and because \( l_1 = 0 \) for \( \theta(I) = g \), \( l_1 = 0 \) for sure if and only if \( u_0 > \bar{u}_0 \), other things held constant.

\[ \square \]

Proposition 1.3 is a second result of the model and basically states that the amount of first period corruption is decreasing in the benefits or "wages" of being in office. This argument that officers are less in need for income from non-legal activities when they are sufficiently payed is often found in the officials and politicians salary discussion. \( \bar{u}_0 \) is the higher the more impatient politicians are, because then they increasingly prefer immediate gains from corruption to future gains from salary.

If we think of non-linear per period utility functions of politicians, the salary needs to be the lower, the higher the concavity of the utility function is, because politicians then might want to smoothen their income over periods and therefore more likely abstain from corruption in period 1. Note that this effect is affected by saving possibilities. If politicians are able to transfer wealth to the 2nd period, the critical salary rises because the dependency on constant income is falling. This plays an important role when thinking about non-democratic countries. The politician gets additional disciplined by the threat of revolution if this threat includes a risk of losing some saved money (because it is fixed in assets, land, and so on, which can be expropriated in case of a revolution). Hence the lower a politician's possibilities to save money in a secure way, the less he will risk to induce a rebellion by behaving too corrupt.

Corollary 1.1 For fixed \( a_b \) and \( \delta \), the critical "wage" \( \bar{u}_0 \) decreases in the size of \( \bar{l} \).
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Proof

trivial.

The statement of Corollary 1.1 appears rather counterintuitive at a first glance. Analogous to the phenomenon discussed in Lemma 1.1, it makes the assertion that, other things held constant, the critical salary to prevent the politician from getting into corruption decreases when the volume of potential corruptive activities increases. This is because an increase of volume of corruption leads both to a higher risk of getting identified as being the corrupt type and losing second period gains, and to a higher volume of second period benefits, making it more desirable to win elections in period 1. Therefore if the volume of corruption rises, the incumbent has increasing motivation to behave properly in period 1 to preserve his chances to get reelected and to extract the increasing second period benefits when corruption is riskless. However this only holds true if the politician has to decide between \( l \) and \( \tilde{l} \) in a discrete manner as in our model.

An even more interesting fact is that equation 1.11 contains the variance of economic output, which brings us to our main proposition, stating the negative correlation between output stability and the expected level of corruption within an economy. For this, we consider a world where the gains for "bad" politicians, \( a_b \), potential corruption volume \( \tilde{l} \) and base utility \( u_0 \) are different for different economies. More precisely we assume, that the \( u_0 \) for each country is drawn out of a distribution that assigns a positive probability mass to the range \( a_b(\frac{2e}{\sigma} - \tilde{l}); a_b(\frac{2e}{\sigma} - \tilde{l}) \).\(^7\) Now we can state

Proposition 1.4 Expected total corruption is monotonically decreasing in output stability.

\(^7\)This means there are cases where a given \( u_0 \) would suffice to deter politicians from corruption in a stable economy (\( \bar{e} = \underline{e} \)), but not in an unstable one (\( \bar{e} = \bar{e} \)).
Chapter 1

Proof

Because "good" politicians always choose \( l_1 = l_2 = \underline{l} \) and "bad" ones always \( l_2 = \bar{l} \), we can focus on the first period decisions of incumbents with \( \theta(I) = b \) and compare them for low and high variance of \( \epsilon \):

Clearly, \( \overline{u}_0(\epsilon) > \overline{u}_0(\bar{\epsilon}) \) for \( \epsilon > \bar{\epsilon} \), thus there exists a range \( [\overline{u}_0(\epsilon); \overline{u}_0(\bar{\epsilon})] \) with \( |[\overline{u}_0(\epsilon); \overline{u}_0(\bar{\epsilon})]| > 0 \), s.t. for \( u_0 \in [\overline{u}_0(\epsilon); \overline{u}_0(\bar{\epsilon})] \) the "bad" politician chooses \( l_1 = 0 \) if \( \bar{\epsilon} = \epsilon \) and \( l_1 = \bar{l} \) if \( \bar{\epsilon} = \bar{\epsilon} \).

As we assumed a positive probability for \( u_0 \in [\overline{u}_0(\epsilon); \overline{u}_0(\bar{\epsilon})] \), the probability that \( u_0 < \overline{u}_0 \) (i.e. high first-period corruption) is lower for the more stable economy. Second-period corruption stays unaffected, hence expected overall corruption decreases monotonically in output stability. 

Stated differently, with randomly chosen variables, equation 1.10 is more likely to be satisfied if \( \bar{\epsilon} = \epsilon \) than if \( \bar{\epsilon} = \bar{\epsilon} \).

\[\Box\]

Proposition 1.4 states our main thesis, i.e. high variance amplifies corruption and thus the channel between economic stability and corruption does not only work from corruption to stability but also from stability to corruption. The intuition for this is that it is easier for politicians to hide their dubious affairs away in the rather uncertain environment of an unstable economy than in the more deterministic case of a stable economy.

For a cross country comparison as presented in chapter 2, this means that the parameter space that leads to high corruption is larger for countries with low economic stability. Thus if one assumes that parameters are different between countries (e.g. \( u_0 \) or \( \bar{l} \) differs from country to country), one should expect a negative correlation between output stability and corruption when observing a larger number of countries.

\[\text{if we would allow for a choice of } l_1 \text{ out of a continuous set, a bad incumbent would always choose his optimal level of corruption and the proposition would strengthen to strict monotonicity.}\]
1.4 Conclusion

In our model we have shown that the possibility for extended corrupt activities of politicians should be higher in countries with an unstable economic output and that remuneration, risk-aversion and the level of control exercised by the voters play important roles in determining the incentives to engage in corruption. Therefore we should on average expect a higher level of corruption in countries with a high variance in economic output.

An interesting addition to the theoretical model would be the endogenization of economic growth by explicitly modeling the interaction between the level of corruption and the amount of investments in a economy as it is highlighted for example by Alesina and Perotti (1996). On the one hand this could provide insight in the triangle relationship between corruption, growth and stability and deliver additional arguments for the question of causality that we will face in Chapter 2. On the other hand this would require additional assumptions on the mechanics of the model economy and it is not clear if the clear cut results of our model would change for the better in the end.
Chapter 2


In this chapter we want to focus on a task many scientists working in economic theory put little weight on, namely the empirical checking of our model against reality.

This step is not as trivial and natural as most outsiders might expect. After all it includes the risk of reality disagreeing with our model predictions. This would somewhat deplete the persuasiveness of the model in the best case and make it absolutely untrustworthy in the worst case. Nevertheless we are convinced that - if possible - empirical verification of results from theoretical models is necessary before making a precise advice or stating well-defined declarations. Without a proper testing a theoretical model remains no more than a theory among a large set of possible theories.

Therefore we added this cross-country study to our model from Chapter 1, being aware that our abilities in empirical economics are limited compared to a scientist doing only empirical work. We will stick to basic tools from panel data analysis and of course we do not want to prove that variance of economic output is the only source of corruption. The main goal is to find empirical evidence confirming our view that it is plausible to think about instability as one reason for amplifying corruption.
2.1 Data

One of the problems when running a cross country regression concerning corruption is the measurability of corruption. Due to the partly subjective and secret character of corruption there is hardly any exact measure of it for any given country. Until the mid-1990s most empirical findings concerning corruption were of a mere anecdotal nature and cross-country comparisons were speculative and theoretical. Corruption was even cited as a classic example of a phenomenon that was observable but not quantifiable. Later on, the empirical research on corruption grew significantly because of increasing international public and private interest in determining and curbing it. Today, most major surveys use polls to obtain their data. This means that some personal perception is retained in the data. But for large numbers of observations the broad picture should at least give a somewhat realistic impression of the actual level of corruption in a country.

There is a number of different country risk surveys including ratings of corruption in there analysis. One is the Index of Business International (BI), a private firm now integrated in the Economist Intelligence Unit (EIU). It ranked countries on a range from 1 to 10 in the years 1980-1983 and is used for example by Mauro (1995). Another index using a notion of corruption is the "Civil Rights Index" of Freedom House. It uses a criterium called "Free of Corruption" and provides data for 192 countries, nevertheless it is problematic to isolate the actual influence of the corruption criterium on this index value.

Probably the most famous source which tries to rank countries according to their level of corruption is the Corruption Perception Index (CPI) of Transparency International, which is published yearly in a global corruption report (Transparency International 2003). Basically it is a survey that subsumes a larger number of cross country polls, most of which reflect the opinion of people working for multinational firms and institutions. The original polls are carried out by NGOs as well as by private institutions.\footnote{For a comprehensive list of the composition of the CPI see (Transparency International 2003).} Thus, they capture the degree of corruption from a mostly western
Chapter 2

(but also pretty homogeneous) point of view. The CPI assigns a score between 0 (severe corruption) and 10 (no corruption) to to each country. In our research we use the CPI of 2003 because it seems to provide the most independent and unbiased measure of corruption and is freely available to the public. Also it includes many other corruption indices in its composition and benefits from high correlation to those not included.\(^2\)

In order to check for the results of our model we ran some regression models to match the variance of a countries GDP per capita growth with its CPI value. The variance in GDP per capita growth is calculated from the cross-country GDP values from the Penn World Table (2001) for the time horizon of 20 years (1981-2000).\(^3\)

### 2.2 Regressions on Whole Data Set

To illustrate the correlation between GDP growth variance and the CPI without regard of the underlying causality, we first ran a simple OLS regression (model (1)) with the CPI as the dependent variable and only the standard deviation of growth on the explaining side:

\[
CPI = \alpha_1 + \alpha_2 \sigma
\]

where \(\sigma\) denotes the standard deviation of GDP per capita growth over the given time horizon. Graphically the correlation is depicted in Figure 2.1, with the bold line being the linear trend line of the correlation. Its slope is the highly significant coefficient from Table 2.2 and suggests a negative correlation between CPI and \(\sigma\) as a stylized fact.\(^4\)

\(^2\)A table of correlation coefficients between the CPI and other corruption indices is given in (Transparency International 2003).

\(^3\)Stability seems to be a somewhat persistent phenomenon as calculations with other time horizons (10, 30 years) led to similar results.

\(^4\)Remember that high CPI-values indicate low corruption.
Figure 2.1: Negative correlation between standard deviation of economic output and CPI in a simple regression
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To get a better understanding of the causality and the impact of $\sigma$ on CPI, we set up some more complex models. First we specify model (2), which includes some common explanatory variables on corruption additional to $\sigma$ (The papers using them are cited later on). Therefore we include the drift $d$ of the GDP growth rate, the democracy level of 1995 $dem95$, the investment level $inv$ as a percentage of GDP, the variable $school$ as the percentage of age 15+ population in secondary school and the variable $ethno$ that describes the ethnolinguistic fractionalization of a country. $d$ is the average growth rate in percent per year derived from the GDP data of 1981-2000, $dem95$ takes values from 0 to 1 (1 being very democratic) and is taken from (Barro 1999) to control for democratization of the countries. $inv$ is from the Penn World Table and $school$ is computed from the updated (Barro and Lee 2000) dataset on education, both averages from 1980-2000. $ethno$ is the ethnolinguistic fractionalization index calculated by Taylor and Hudson (1972). It measures the probability that two randomly chosen persons of a given country will not belong to the same ethnolinguistic group. Its value ranges from 100 (very high fractionalization) to 0 (total ethnolinguistic homogeneity).

Additionally we include the variable $protestant$ as the percentage of protestants in a society, the variable $import$ denoting the openness to trade (i.e. the goods and services imported as a percentage of GDP), the dummy variables $formerUK$ (former British colony or UK) and $federal$ (federal constitution). Finally we include the variable $absgovwage$ as the absolute wage of central government members in thousands of 1990 US-Dollars.\textsuperscript{5} $protestant$, $import$, $formerUK$ and $federal$ are found to explain a large amount of corruption in the models of Treisman (2000). $absgovwage$ is included because of its relation to our theory and is derived from Treisman’s data on relative income of central government members to GDP per capita which in turn is taken from (Schiavo-Campo, de Tommaso and Mukherjee 1997).

Another variable closely related to the level of corruption in a country is the level of GDP per capita. However, GDP per capita is very strongly correlated to the other explaining variables in the model (it can be explained with an $R^2 = 0.81$) and thus is

\textsuperscript{5} All data can be found in the appendix.
Chapter 2

not entered explicitly to avoid multicollinearity. It is an interesting question which way the causality between GDP per capita and the other explaining variables in model (2) works but here we decided to focus on the other variables. To be on the safe side we also did models that included GDP per capita explicitly using robust estimation techniques to correct for collinearity and σ still stays significant at the 5%-level. The same holds true for all following regression models.

The coefficients of model (2) can be found in Table 2.2. The CPI of 2003 is explained very well in the model, with an adjusted $R^2 = 0.8071$.\footnote{We are aware of the boundedness of our dependent variable, however, as the range is from 1-10, we still use an OLS model.} Note that we use percentage changes in GDP per capita to calculate the variance and therefore the standard deviation $σ = \sqrt{\text{var}(\Delta \text{GDP/capita})}$ is normalized. For low $σ$, the economy evolved in a relatively "stable" way over the last 20 years, whereas high values of $σ$ indicate an economy with large short term deviations from the long term drift in economic output growth, hence an "unstable" economy in our definition.

The correlation between CPI and $σ$ is negative as anticipated by Proposition 1.4 and the $t$-value indicates a result significant at the 5% level. In model (2) even under consideration of the many other explanatory variables, $σ$ still has a considerable impact on CPI. Take for example Cameroon, Uganda and Angola, which can be found on places 73, 72 and 67 out of 75 regarding the level of corruption and on places 72, 73 and 75 regarding stability. According to model (2) a reduction of their $σ$ to lets say the level of Thailand would increase their CPI level by 1.39, 1.73 and 1.96 respectively corresponding to positions 49, 37 and 40 among the countries under consideration.

Interesting is the fact that the drift $d$ in growth rates does not have any significant impact on corruption in model (2), whereas often it is suggested in the literature that high growth would decrease corruption. Also, including the absolute volume of GDP per capita does not display a significant coefficient thus the richness of a countries inhabitants does only play a minor role in explaining corruption in this model.

Also, absgovwage is significant at the 5%-level and displays the positive correla-
Table 2.1: Results of OLS regressions

dependent variable: CPI

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<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
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<td></td>
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<tr>
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<td>(0.0069)</td>
<td>(0.0078)</td>
<td>(0.0079)</td>
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<tr>
<td>formerUK</td>
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<td>(0.4230)</td>
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<tr>
<td>import</td>
<td>0.0223**</td>
<td>0.0342***</td>
<td>0.0290***</td>
<td>0.0280***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0091)</td>
<td>(0.0071)</td>
<td>(0.0076)</td>
<td>(0.0075)</td>
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</tr>
<tr>
<td>federal</td>
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<td>(0.4779)</td>
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<tr>
<td>absgovwage</td>
<td>0.0436**</td>
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<td>(0.0166)</td>
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<tr>
<td>constant</td>
<td>1.3895</td>
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<td>2.4281</td>
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</tr>
<tr>
<td></td>
<td>(0.9369)</td>
<td>(0.7143)</td>
<td>(0.7007)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.2784</td>
<td>0.8472</td>
<td>0.7878</td>
<td>0.8279</td>
<td>0.8418</td>
</tr>
<tr>
<td>adj. ( R^2 )</td>
<td>0.2685</td>
<td>0.8030</td>
<td>0.7688</td>
<td>0.8045</td>
<td>0.8192</td>
</tr>
</tbody>
</table>

Model (5) uses predictions \( \hat{\sigma} \) from the instrumental regression instead of actual \( \sigma \)-values.

*** denotes significance at the 1%-level, ** denotes significance at the 5%-level, * denotes significance at the 10%-level. Standard errors are given in parentheses.
tion to the CPI that we predicted in Proposition 1.3. High wages thus reduce the level of corruption in our sample of countries. The effect is strong if you note that the range of absgovwage is from 0 to 50, compared to 0-10 for the CPI. In fact the beta-value, i.e. the standardized regression coefficient, is -0.2367 for cpi03 and 0.2399 for absgovwage, thus both effects are roughly of the same magnitude.

Next we reduce model (2) to all significant variables to check whether the impact of σ stays unchanged. We set up two new models (3) and (4), both containing σ, dem95, school, protestant, and import as explaining variables and model (4) additionally containing absgovwage. School is included as it turns out to be the only variable which is highly correlated with absgovwage and which changes its t-value drastically if absgovwage is excluded from model (2).\textsuperscript{7}

The reduced models show that the coefficients of σ stay pretty much unchanged whereas its significance is even rising close to the 1%-level, indicating that the underlying relationship is not negligible. absgovwage became significant at the 1%-level in model (4). The impact of all predictors in model (4) has about the same magnitude, as can be seen in Table 2.2, where beta-coefficients of the regression are shown. Note that for model (4) it was even possible to increase the adjusted $R^2$ to 0.8045. Controlling models (2), (3) and (4) for heteroscedasticity by deriving robust standard errors using a White correction does not change the significance levels of any of the explaining variables.

This results are encouraging, still it is not easy to show the direction of the causality between σ and CPI. To test for that, one would need appropriate instruments for σ. We tried to explain σ by the investment level inv and dummies for intermediate and OECD countries, inter and OECD. These three can explain σ with an adjusted $R^2$ of 0.4153.\textsuperscript{8}

\textsuperscript{7}In fact, school gets significant at the 5%-level if absgovwage is excluded from model (2).

\textsuperscript{8}We also tried to use normalized terms of trade variance as taken from the World Development Indicators (2002) as an instrument for σ, but it turns out that - with a $\rho = 0.07$ - this variable is


### Table 2.2: Beta-coefficients of model (4)

<table>
<thead>
<tr>
<th>Variable</th>
<th>beta-coeff.</th>
<th>Variable</th>
<th>beta-coeff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>-0.1725</td>
<td>protestant</td>
<td>0.2967</td>
</tr>
<tr>
<td>dem95</td>
<td>0.2050</td>
<td>import</td>
<td>0.2536</td>
</tr>
<tr>
<td>school</td>
<td>0.2346</td>
<td>absgovwage</td>
<td>0.2653</td>
</tr>
</tbody>
</table>

Using predictions of $\hat{\sigma}$ in model (4) leads to model (5) where the coefficient of $\hat{\sigma}$ changes considerably compared to the use of non-instrumented values of $\sigma$ but at least it stays significant at the 1%-level. As in any empirical research on corruption with its many interacting factors it is still not easy to entirely reject the hypotheses that CPI affects $\sigma$ and not the other way round. Nevertheless we argue that a high level of corruption might well have a negative impact on the *volume* of growth in GDP but we do not see many reasons why it should increase the *variance* of growth in GDP, especially as it turns out that corruption levels do not change quickly over time for the most countries. Additional support for this point of view comes from Easterly, Kremer, Pritchett and Summers (1993). They show in their paper that much variation in growth rates is due to random shocks and not connected to country characteristics. Their empirical results are supported by the finding that country characteristics (and also corruption levels) are strongly autocorrelated and very persistent whereas growth rates are not. Therefore it seems much more plausible that $\sigma$ is the independent variable and not CPI and we stick to our theory that the causality of the significant relation between CPI and $\sigma$ works from $\sigma$ to CPI and not the other way round.

Another way to check for causality would be the analysis of global recessions or phases of high worldwide growth variance and their impact on corruption. Higher overall variance in growth should lead to higher mean levels of corruption according not sufficiently correlated to $\sigma$. 

34
to our theory. Unfortunately time series for corruption data are hardly available, partly because the quantification of corruption is a relatively new concept, and partly because the methods of compiling the indices are changing over time.9 Also, different measures of corruption can not be compared, because of the blurry nature of corruption definitions. Thus we had to abandon the idea of doing an additional time series analysis with respect to global recessions.

2.3 Regressions Differentiating by Level of Democratization

As said above, we expect our model to work best for democratic societies where the people has the best means to punish politicians for opportunistic behavior. Therefore we ran two additional regressions based on model (4). Model (6) is similar to model (4) but uses only "democratic" countries, model (7) uses only "non-democratic" countries. Again, it is not too easy to find a reliable variable for the level of democratization of a country, as many countries call themselves democratic or even run elections, that are definitely non-democratic from an objective point of view. For our study we follow Barro (1999) and refer to the indicator of political rights compiled by Gastil (1991) and followers. Originally, Gastil classified each country from 1 (highest level of political rights) to 7 (lowest level). We use the transformed data of Barro10, where 1 denotes the highest level of political rights, and 0 the lowest. Countries with a democracy index higher than 0.8 in 1995 (45 out of 75 observations) are classified as democratic, others as non-democratic. The results (along with the repeated results of model (4)) are shown in Table 2.3.11 The σ-coefficient is only significant for democratic countries and plays a minor role for non-democratic countries, confirming our theory. In contrast, school, import and absgovwage are only significant for non-

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9 Transparency International for example explicitly warns not to use its CPI reports as time series as the survey method and the sample changed many times.

10 Which we already included in model (2).

11 Robust errors are used to control for heteroscedasticity.
Figure 2.2: Negative correlation between standard deviation of economic output and CPI, differentiated for democratic (dcpi03) and non-democratic (ndcpi03) countries

democratic countries, indicating that these variables play a minor role for the accrualment of corruption in democratic countries. 12

To give a graphical representation, we ran two simple regressions as in model (1), for democratic and non-democratic countries respectively. Results (denoted by (8) for democratic and (9) for non-democratic) are shown in Table 2.3. In the simple regression, the σ-coefficient is, as expected, only significant for democratic countries. For them this simple model yields an $R^2$ of 0.4143. For non-democratic countries, the σ-coefficient is insignificant and the $R^2$ is pretty close to zero. In Figure 2.2, the CPI of both democratic and non-democratic countries is plotted against $σ$ additional

12We included $dem95$ in this regression to maintain comparability to model (4). It has a very high coefficient in model (6), but note that its range is only from 0.8-1 in the group of democratic countries. This relativizes its impact and makes it equal in magnitude to its impact in model (4).
## Table 2.3: Results of OLS regressions, differentiating between democratic and non-democratic countries

<table>
<thead>
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<th>Variable</th>
<th>(4)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
<th>(9)</th>
</tr>
</thead>
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<td>20</td>
<td>45</td>
<td>30</td>
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<td>$\sigma$</td>
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<td>-0.0311</td>
<td>-0.6672***</td>
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<td>(0.0805)</td>
<td>(0.0722)</td>
<td>(0.0491)</td>
<td>(0.1209)</td>
<td>(0.1494)</td>
</tr>
<tr>
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</tr>
<tr>
<td>dem95</td>
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<td>(0.7303)</td>
<td></td>
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<td>(0.0152)</td>
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<td>0.0270***</td>
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</tr>
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<tr>
<td>formerUK</td>
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</tr>
<tr>
<td>import</td>
<td>0.0290***</td>
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<td>(0.0080)</td>
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<tr>
<td>absgovwage</td>
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<td>(0.7007)</td>
<td>(4.6524)</td>
<td>(0.5847)</td>
<td>(0.5473)</td>
<td>(0.7814)</td>
</tr>
</tbody>
</table>

$R^2$ 0.8279 0.7656 0.8830 0.4143 0.0336

adj. $R^2$ 0.8045 0.4007 -0.0010

*** denotes significance at the 1%-level, ** denotes significance at the 5%-level, * denotes significance at the 10%-level. Standard errors are given in parentheses.
to the linear trend line of each group.

2.4 Conclusion

In this chapter we tested the results of the theoretical model of Chapter 1 in a cross-country study to confirm our predictions with the help of real world observations.

The data in the cross-country study confirms the main finding of Chapter 1 well, as standard deviation of per capita GDP growth displays a significant and non-negligible coefficient in all relevant regression models. Also, our second proposition, the negative correlation of politicians remunerations and the level of corruption, is confirmed by the data.

Summarizing, the fitting of the theoretical predictions in Chapter 1 and the empirical results in Chapter 2 seems to be good. Of course there could be other possible explanations for these results. An alternative explanation would be that in unstable economies politicians are more afraid of future stability and development because of a higher risk of institutional crisis in this case, therefore discounting the future with a lower \( \delta \), which in turn would lead to a more myopic behavior of "bad" politicians in the first period of being in office and hence to a higher degree of corruption. At least, this interpretation would be consistent with the theoretical model as can be seen from equation 1.9.

A further extension of the theoretical model as discussed in the Conclusion of Chapter 1 might help to rule out different possible explanations. On the other hand already the presented theoretical model and the additional empirical findings can hopefully give an interesting impulse for thinking not only about corruption affecting the path of economic output but also to take into account the characteristics of economic output paths as one additional source of corruption itself.
Chapter 3

Learning in Financial Markets: A Model of Asset Pricing with Heterogeneous Agents

In this chapter, we want to explore another notion of learning, which has hardly been explored in economic theory so far, namely learning in a sense of changing information processing strategies. We try to depart as little as possible from classical model building blocks but introduce heterogeneity in the quality of agents belief formation to explain frequently observed phenomena on financial markets. Besides providing alternative explanations for these phenomena, the model has a methodological motivation and wants to show that approaches from behavioral learning economics can well be used to deliver some microfoundation for classical economic problems within a highly formalized framework.

3.1 Introduction

Empirical research in recent years showed that asset prices often exhibit characteristic behavior that cannot satisfactory be explained by classical approaches like the capital asset pricing model (CAPM) by Sharp (1964) and Lintner (1965). Among the most frequent results obtained by econometric research are phenomena like short term
momentum in asset prices and long-run fundamental reversion. Also, excess volatility that persists after the arrival of shocks is frequently observed. Other findings are over- and underpricing when new information is arriving at the market.\footnote{See Jegadeesh and Titman (1993), Rouwenhorst (1998), Rouwenhorst (1999).}

One approach to cope with the observed puzzles is to introduce psychological evidence into economic modeling, e.g. by dropping the assumption of perfectly rational behavior. Psychological research supports this approach as it shows that human beings do not seem to be able to estimate and calculate all possible uncertainties at once and without time delay. Instead they tend to trade-off reasoning-time against optimality and often use heuristics to come to quicker and mostly sufficient decisions. Using heuristics means that a decision maker tries to roughly categorize a given problem, to split it in sub-problems and to reduce it to previously solved tasks. Thus humans often try to save on calculation effort by referring to their experience or to former solutions they figured out when they first encountered a somehow similar task.

When applying this theory on financial markets, it is obvious that gaining experience in trading should increase the quality of a subject’s decision as it can draw on an increasing base of reference cases and it should have adjusted his decision making process as a reaction to former feedbacks. Consequently the returns of a trader should partly depend on her experience and her life-cycle returns should exhibit certain patterns dependent on the combination of market condition and individual experience.

It is important to highlight the difference of learning in the sense described above to learning as it mostly is understood in classical economic modeling. There, learning mostly deals with the acquisition of new information and not with the improvement of one’s capability of correct information processing respectively of one’s progress of correct decision making. To make this point clear, let us state the issue in terms of a "reaction function" $y = f(x)$, where $x$ is the information available to the subject, $y$ is her response, and $f$ is the decision process. Classical theory assumes that $f$ is a "rational process", i.e. the $y$ is the best possible response given $x$. Learning would only take place via $x$, hence by acquisition of new information, whereas $f$ would not
change. In contrast, we argue that learning takes place both on $x$ and on $f$, such that the reaction $y$ improves both because the set of available information grows and the quality of the decision process $f$ increases over time.

In the model at hand we follow the goal to set up a unified behavioral model for financial markets that incorporates this idea of learning and that explains all the named phenomena in asset pricing. Thus, the motivation for this work is twofold:

First, we have an explanatory goal as to explain the phenomena of underpricing, overpricing, excess volatility, life cycle trading returns and the behavior of stock prices after shocks to fundamental values.

Second, we have a methodological intention as we want to introduce a new aspect to economic modeling by considering the improvement in personal decision strategies over time in an economic model. To the best of our knowledge this aspect has not been raised before and we think that exploring it could help to understand some puzzling patterns in observed human behavior. In particular we will try to shed light on how these heterogeneous and improving heuristics might influence the development of supply and demand and hence prices in an asset market.

More specifically, and in contrast to former models, our model features agents which are heterogeneous not with respect to the information they have available, but also to their belief formation heuristics. That means that young and inexperienced agents will process the available information in another (and less efficient) way, than experienced traders. Based on the same information set the former will come to an inferior decision that the latter.

This is modeled by equipping all agents with two kinds of information: First a noisy "fundamental" signal reflecting possible shocks to the fundamental value of an risky asset, and second an observation of past price moves or "momentum".² Depend-

²There is a lot of evidence that there is in fact momentum trading in asset markets. Kahneman, Slovic and Tversky (1982) widely discuss this issue, and Griffin and Tversky (1992) formed the notion of conservatism, e.g. the inertia of belief on the arrival of new information. De Bondt (1993) provides empirical research based on 38,000 forecasts of stock prices and exchange rates and comes
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ing on their age\(^3\), agents either put high weight on their momentum observation or on the signal observation when forming their final belief about the expected payoff of the asset. As agents get older their experience and thus their "degree of rationality" increases.

The interplay of the different belief formation strategies and the resulting differences in demand lead to interesting dynamics in the pricing of the risky asset and can explain the formation of pricing bubbles and the presence of excess volatility.

We show that instantly after the arrival of a (positive) shock to fundamentals there will be underpricing in our model as suggested by the empirical evidence. This is due to the fact that inexperienced market participants underestimate the relevance of the fundamental signal and do not adjust their demand sufficiently. In the next period, as additional to the new fundamental situation momentum is observed, there will be overpricing. In a positive feedback manner, overpricing will persist for some time until short selling by the experienced (rational) traders prevails and the bubble is bursting. Now, a downward cascade will take place as negative momentum is increasing. Eventually the mispricing cycle will fade out and the price will converge against the fair price.

During the mispricing period, inexperienced traders on average will lose money, as they misjudge the true value of the asset by incorporating their momentum observation in their decision. On the other hand, experienced traders are able to realize excess returns in expectations, as they can profit from the wrong positions of inexperienced traders. As every trader belongs to each group once in his lifetime, it is to the conclusion that "non-experts" expect the continuation of past "trends" in prices, whereas "experts" do not behave in such a way. Another strong indicator for real world momentum-like trade is the existence of the large industry of technical analysis one can observe at the financial markets and which tries to forecast future price paths by past observation. Clearly there must be some demand for this services to keep that industry alive. See also Frankel and Froot (1990) for empirical evidence on the existence of so-called "chartists".

\(^3\)Age is the time agents are active in the market and thus is set equivalent to experience in that market.
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not clear whether the losses in the first phase or the additional gains in later periods prevail. We show that the timing of market entrance is crucial for overall lifetime performance. When agents enter in times with high mispricing they are unlikely to recover from their first periods losses later in their life. In contrast, if they enter in calm times and are able to collect experience without losing to much, their later periods excess returns are likely to overcompensate for early losses.

The remainder of this chapter is organized as follows:
After some thoughts on the methodological classification of our model, a survey of related literature follows in this section. In Section 2 we introduce our theoretical model and state propositions for the basic setting. In Section 3 we present some results of numerical simulations, e.g. price paths and life-cycle performance of traders. In Chapter 4 we extend our model to the dynamic case. There we also present an outlook and an agenda for an possible empirical confirmation of our theoretical results.

3.1.1 Methodological Considerations

We think that our approach might be helpful to better understand the formation of trading strategies and its impact on asset market prices. Nevertheless we know that it is always risky to deviate from classical theory as one quickly enters a somewhat dusty area of scientific uncertainty. However, we are convinced that anyway it is necessary to explore that region in the hope of finding some new and possibly useful supplementary insights within. In any case, the departure from widely accepted assumptions should only be done with extreme care and only if no other way seems to be passable. Hong and Stein (Hong and Stein, 1999, p.2144) set up a list of "requirements" to justify theories departing from full rationality and unlimited computational capacity of investors:

"(1) [they have to...] rest on assumptions about investor behavior that are either a priori plausible or consistent with casual observation; (2) explain the existing evidence in a parsimonious and unified way; and (3) make a
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number of further predictions that can be subject to 'out-of sample' testing and that are ultimately validated."

We try to stick to this criteria and to classical modeling tools where ever possible. However there are of course different opinions about behavioral approaches. One somewhat harsh critique of behavioral methods in economic theory can by found in Rubinstein (2000). The cited paper surely hits some critical points. We nevertheless think that behavioral approaches will probably not be the magic bullet for all unsolved questions, but are worth to be as seriously considered as any classical attempt for the sake of more realistic economic models.

3.1.2 Learning and Decision Making

In this Chapter we focus on a maturing process of improving decision making over time, thus unexperienced traders use a suboptimal heuristic for their decisions and improve it over time until they finally reach perfectly optimal behavior. To represent suboptimal decision making, our model takes a behavioral approach, and we are convinced that this point of view will allow for explanations of puzzles that cannot be solved by pure standard theory in a satisfactory way.

In the behavioral literature there are a couple of approaches to explain decision making that seem to be suboptimal from a classical point of view. Psychological research and experiments support most of these approaches in some way and it is not entirely clear until now which approach suites best for which problem set or whether a unifying theory could be formulated to incorporate all directions of this research area.

One approach is to alter the shape of utility functions to incorporate psychological or behavioral findings in the process of decision making:

In 1979, Kahneman and Tversky (1979) introduced their "prospect theory", an alternative form of utility function including the psychological fact that people seem
to be "loss averse". The resulting function is not differentiable but has a kink at a certain reference point, making the subject excessively risk averse around this point. Benartzi and Thaler (1995) offered an related explanation of the equity premium puzzle first stated by Mehra and Prescott (1985). They showed that by frequent portfolio evaluations the effect of the kink accumulates and makes an equity premium necessary to compensate for psychological costs due to the high risk aversion around the reference point. Another approach is the hyperbolic discounting applied by Loewenstein and Prelec (1992), where preferences are inconsistent over time and actual decisions will be revised in the future. Work in that direction can also be found in Benabou and Tirole (2002) or in O’Donoghue and Rabin (1999). Fehr and Schmidt (1999) propose a utility function with an additional fairness term such that subjects bias their choice in a direction where payoff differences compared to a reference group are smaller. In Gebhardt (2001) a model is set up that allows diverging price paths in asset pricing by using Fehr-Schmidt preferences for investors.

Other approaches preserve the classical utility function but claim that systematic "errors" are made during the decision process. One approach of that kind is the mental accounting literature, e.g. Barberis and Huang (2001). Here it is argued that the same loss or gain can be valued differently depending on its framing. Another theory explains suboptimal behavior by investors overconfidence in private information. This idea was raised by Barber and Odean (2000) and then used for example in a model by Daniel, Hirshleifer and Subrahmanyam (1998) to explain over- and underpricing.

As a further example for systematic errors in a financial context, momentum trading is often cited. Momentum traders expect price changes to be positively auto-correlated and thus buy when prices are rising. Of course in the long run this does not make much sense but actually there are empirical hints that returns are indeed short-run positively auto-correlated (see Cutler, Poterba and Summers, 1990). Together with overconfidence (in identifying the right moment to leave the trend) it therefore seems imaginable that some traders might follow this strategy. A model
where momentum traders are used along with "news watcher" is set up by Hong and Stein (Hong and Stein, 1999). In their model, momentum traders are justified by the assumption that news spread slowly in the market and thus positive returns are possible by doing momentum trading. Hong, Lim and Stein (2000) confirm this assumption by carrying out an empirical test to show that gradual information flow in markets exist. Jegadeesh and Titman (1993) as well as Chan, Jegadeesh and Lakonishok (1996) propose some momentum strategies in their papers. Further discussion of this issue can also be found in Hong and Stein (1999).

In our model traders use a simple momentum strategy when first entering the market and then improve their strategy towards an "optimal" rational decision process over time. When classifying it, our model combines two features: Features of models with two types of traders as the Hong and Stein (1999) model mentioned above or the model by Grinblatt and Han (2002) where a group of "rational" traders and a group with "disposition affected" traders interact. And features of models where investors change their view of the world over time as there is for example Barberis, Shleifer and Vishny (1998), where representative investors consider the world switching between two "regimes", a mean reverting and a trend one following a Markov process. Also, Hong and Stein (2003) set up another model where investors switch between two forecasting models both of which use only a part of the available information. Investors change the model in use only if it did a particularly poor job in forecasting.

Note that none of the cited models exhibit monotonic increase of the quality of agents decision making processes. Rather agents either stay in their group and stick to a certain type of decision making strategy, or they can switch to and fro between the different modes arbitrarily often without continuity, as in Hong and Stein (2003). While this might well fit to map certain aspects of human decision making it is not what we would call leaning, as we think learning always leads from a suboptimal behavior to a more optimal one in a predominantly monotonic way.

This is the crucial new aspect in our model. There are heterogeneous groups of traders and they incrementally ameliorate their decision making strategies over time when getting experienced. A trader starts with a weak decision making strategy and
ends with a strong one, i.e. a perfectly rational one, such that each trader belongs to each group once in his lifetime.

The improvement of decision strategies is only dependent of time, i.e. it is not triggered by observations or outside processes but purely by collecting experience in signal processing and it is incremental in a sense that it improves from a pretty suboptimal momentum strategy to a fully rational fundamental signal observation strategy. Our motivation thereby is to fetch a gradual improvement of investors behavior with respect to optimality.

3.2 The Model

To capture the ideas developed above we set up an overlapping generations model with discrete time steps where agents improve the quality of their decisions over time. Each period traders get signals from the fundamental process and they can observe past price changes. When first entering the market they are not experienced and behave in a suboptimal way by forming their beliefs on the basis of a momentum strategy. In addition to possible rationalizations of momentum trading indicated in Section 3.1.2, the motivation behind this is that beginners are particularly susceptible to momentum behavior as they tend to follow the advise and the behavior of others since feeling unsure about their own judgement abilities.

When getting older and collecting experience, the traders learn to draw their own conclusions from the observed fundamental signal and older generations form beliefs more and more on the basis of signal observation. Thus belief formation is heterogeneous across generations.

The final belief $b_{it}$ of generation $i$ is therefore composed out of two "intermediate" beliefs $b_{it}^s$ (signal component) and $b_{it}^m$ (momentum component). The exact mechanism is described below in Section 3.2.5.
3.2.1 Assets

There is a risky asset $A$ which can be bought and sold by the agents once per period. More precisely $A$ represents a sequence of short time assets. It is liquidated at the end of each period and is issued again at the beginning of the next period. The price is formed by the market clearing condition and the asset is in zero net supply, thus each unit purchased by an agent has to be sold short by someone else. The asset has a liquidating dividend which can be $y_h > 0$ in the "good" case or $y_l = 0$ in the "bad" case. The probability of high payoff is denoted by $\pi_t$, the probability of low payoff $1 - \pi_t$. The fundamental variable $z_t$ is the log odds ratio of receiving the high payoff:

$$z_t \equiv \ln \frac{\pi_t}{1 - \pi_t}, \quad (3.1)$$

where $z_t > 0$ indicates $\pi_t > \frac{1}{2}$ and $z_t < 0$ indicates $\pi_t < \frac{1}{2}$.

The fundamental value $z$ can stay unchanged from $t - 1$ to $t$ with probability $\beta_0$ or it can increase or decrease by $+1$ or $-1$ with probabilities $\beta_+$ and $\beta_-$ respectively, with $\beta_- = \beta_+ = \frac{1 - \beta_0}{2}$. Whenever a change in $z$ occurs, all agents receive the same informative signal $s_t = \pm 1$ which is correct (i.e. $s_t = \Delta z_t$) with probability $\tau > \frac{1}{2}$ or incorrect ($s_t = -\Delta z_t$) with probability $1 - \tau$.

$\pi_t$ is calculated from the actual $z_t$ by the inverse of (3.1):

$$\pi_t = \frac{1}{e^{-z_t} + 1} \quad (3.2)$$

Besides investing in $A$, agents can invest in a riskfree asset $B$, yielding a payoff of $r$ for each unit invested, where we assume $y_h > 2r > 0$, such that the expected payoff is larger for the risky asset than for the riskfree asset at $\pi = \frac{1}{2}$.

With no discounting the fair price of $A$ for risk neutral investors would be

$$p_t^f = \pi_t y_h = \frac{1}{e^{-z_t} + 1} y_h. \quad (3.3)$$

---

4 Equations 3.1 and 3.2 are a way to transform a $z_t \in ]-\infty; +\infty[$ to the interval $[0; 1]$ of valid probabilities. It allows us to let $z_t$ follow a random walk process without generating unfeasible values of $\pi_t$. 

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As agents are risk averse in our model the fair paying willingness \( p^*_t \) will be lower than \( p^f \). It will be calculated below in Section 3.3.1.

### 3.2.2 Agents

The model is populated by \( n \) generations of agents. Each generation consists of a continuum of agents with mass 1 and should be thought of as a generation of traders in the market under consideration and not as a generation in a demographic sense. Thus, an "old" or "experienced" trader is one who entered the market long ago and had much time to collect experience in trading the related asset, whereas a "young" or "inexperienced" trader cannot look back on a trading history in that particular market. In total there is a mass of \( n \) possible traders in the world. Each time step one generation enters the world and one is leaving it, therefore each generation lives for \( n \) time periods. They start each period with the same endowment \( W \) which is supposed to be some regular income from other activities. They can now form a portfolio which is liquidated at the end of the period. The liquidation proceedings are not storable and are therefore consumed immediately.\(^5\) For the sake of simplicity, agents do not discount future payoffs.

All agents are risk-averse and have CARA utility functions of the style

\[
U(\chi_t) \equiv -e^{-\rho \chi_t}
\]  

(3.4)

with a constant coefficient of risk aversion \( \rho > 0 \) across generations and \( \chi_t = x_{At}p_{At} + rx_{Bt} \) being the cumulated returns from both the risky and the riskfree asset at the end of period \( t \) where \( x_{At} \), \( x_{Bt} \) and \( p_{At} \) are the amounts of assets \( A \) and \( B \) held and the price of \( A \) in period \( t \).

\(^5\)One advantage of this setting is that we avoid inconsistencies of consumption in the first period an agent enters the market.
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3.2.3 Non-Strategic Behavior

Until Chapter 4, where strategic trading is addressed, traders do not act strategically when making their decision, i.e. they do not actively exploit the potential weakness of others. We refer to this as the basic case. When examining strategic behavior in Chapter 4 we will find that there are excess returns to be realized by sophisticated traders if they manage to act strategically. Nevertheless the basic case is suitable for two situations: First, it describes the dynamics if information about belief formation is private, that is agents have no information about the belief formation strategies of other agents. In the model based on the basic case agents simply assume other agents use the same heuristics as themselves. Second, even when relaxing this assumption, the basic case holds if traders do not have any market power. Whenever they are acting as atomistic price-takers in a market, there is no room for strategically influencing the price as others would free ride on that occasion. I.e. sophisticated traders are caught in a public good situation where they wait for others to invest in order to move prices in the desired direction, and in equilibrium no one is willing to do so. In the expanded setting in Chapter 4 strategic trading takes place, but collusion or individual market power is necessary in that case.

Also, inter-period saving is not possible, that is the returns from investing are perishable and agents have to consume them at the end of each period. Thus they do not worry about balancing their lifetime utility paths. However, at the beginning of the period there is no credit constraint to the agents and they can borrow to finance their portfolios, where debts have to be repaid during the consumption phase at the end of the same period. Altogether as a result agents behave short-sighted in this Chapter and simply maximize the expected utility of their portfolio returns in a myopic way each period \( t \). In Chapter 4 the restrictions are relaxed and agents can set their demands strategically and in a dynamic way, considering future effects of their choices.
3.2.4 Timing

Each period \( t \) consists of three steps:

First, generations receive their endowment \( W \) and the fundamental signal \( s_t \) is revealed to all generations. The signal suggests whether the fundamental value \( z \) for high asset payoff has increased, decreased or stood unchanged. \( s_t = 0 \) indicates no change in the log odds ratio (this will be called "no signal" further on), \( s_t = +1 \) suggests an increase of \( z \) by one (if it wasn’t a wrong signal) and \( s_t = -1 \) a decrease of \( z \) by one ("positive" and "negative" signal further on). If a signal arrives, it is used to update \( b^s \) in a Bayesian way. A signal is only sent when a change occurs, thus no signal means there is no necessity to update \( b^s \). Also, traders observe the recent price change \( \Delta p_t \)\(^6 \) and interpret this as a signal of \( \pi \) to form their \( b^m \).

Second, generations form beliefs about the expected payoff \( Ey_t \) and maximize \( EU(x) \) accordingly, yielding a demand \( x_{it} \) of \( A \) for each period and each generation. The remainder \( W_{it} - x_{it} \) is invested in the riskfree asset \( B \). A market clearing price \( p_t \) is calculated.

Third, the actual \( z_t \) is revealed and a payoff \( y_t \) is drawn according to the probability \( \pi_t \) corresponding to \( z_t \). All agents receive their liquidation returns and consume them. At the end of the turn all generations are getting older, the oldest generation drops out and a new generation of traders enters the market with initial beliefs calculated from the observation of the recently revealed \( z_t \) and \( \Delta p_t \). Figure 3.1 graphically shows the timing of each period.

3.2.5 Belief Formation

To capture the idea that traders regard direct signals as well as market observation in a momentum manner when forming their beliefs about the fundamental value of a risky asset, we consider the belief \( b_{it} \) as the probability assigned to high payoff by generation \( i \) at time \( t \). It is composed out of two components \( b^s_{it} \) (signal component) and \( b^m_{it} \) (momentum component). The intermediate, non rational momentum compo-

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\(^6\) The exact definition of \( \Delta p_t \) is given in the following section.
ponent \( (b^m) \) is motivated by the consideration "If I look at the history, I would judge the probability for high payoff to be...". The intermediate signal component \( (b^s) \) belongs to the thought "If I interpret the signal I received, I would judge the probability of high payoff to be...". To compute the final belief \( b \), both intermediate beliefs are regarded with weights depending on the age of traders.\(^7\)

### 3.2.5.1 Signal Belief \( b^s \)

The direct signal observation belief \( b^s \) is deduced by updating the previous value \( \pi_{t-1} \) if a new signal arrives. \( \pi_{t-1} \) in turn can be exactly inferred by the observation of \( z_{t-1} \) at the end of period \( t - 1 \).

The corresponding estimated \( z_t \) is:

\[
E^s(z_t) = z_{t-1} + s_t(2\tau_t - 1)
\]  

We have,

\[
b_t^s = E^s(\pi_t) = \left( \frac{1}{e^{-(z_{t-1}+s_t)} + 1} + (1 - \tau) \frac{1}{e^{-(z_{t-1}-s_t)} + 1} \right) s_t^2 \\
\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad + \left( 1 - s_t^2 \right) \frac{1}{e^{-z_{t-1}} + 1}\]

Equation 3.6 guarantees that \( b^s \) stays unchanged if no signal arrives \( (s_t = 0) \), and is updated if \( s_t \neq 0 \).

\(^7\)Time indices are dropped if not necessary.
3.2.5.2  Momentum Belief $b^m$

The momentum belief $b^m$ is deducted from the observation of a possible recent price change, which is (incorrectly) interpreted as a signal on the log odds ratio of the asset. We define

$$\Delta p_t \equiv p_{t-1} - p_{t-2}. \quad (3.7)$$

Please note that $\Delta p_t$ describes the change in $p$ from the pre-preceding to the preceding period and not from the preceding to the actual period! This is for the sake of conformity in time indices in further equations.

In a typical momentum manner, estimates of fundamentals are linearly increasing in former price shifts. Therefore the point estimate for $z_t$, denoted by $\hat{z}_t$, is

$$\hat{z}_t = z_{t-1} + \lambda \Delta p_t. \quad (3.8)$$

Accordingly,

$$b^m_t = E^m_t(p_t) = \frac{1}{e^{-(z_{t-1} + \lambda \Delta p_t)} + 1}, \quad (3.9)$$

with the momentum intensity parameter $\lambda > 0$ describing the strength of the momentum effect. For simplicity, we will set $\lambda = 1$ for the following sections. All results will be independent of the precise value of $\lambda$, except the divergence/convergence behavior of cycles. In Chapter 4 we will study the impact of $\lambda$ on dynamic optimization of rational agents exploiting the less experienced traders.

3.2.5.3  Belief Composition

The final belief $b_i$ of generation $i$ is composed out of $b^s$ and $b^m$ in dependence on its age $i$. Younger generations are unexperienced in signal observation and rely to a higher degree on trend following strategies represented by the momentum belief $b^m$, with the youngest generation $i = 1$ being totally momentum driven and only regarding the momentum belief $b^m$. Older generations possess more "rational capabilities" and set their focus on $b^s$, with the oldest generation $i = n$ being perfectly rational and only regarding the signal belief $b^s$. 

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In general the final belief is formed as follows:

\[ b_t = \frac{1}{n-1} \left( (n-i)b_t^{n} + (i-1)b_t^* \right) \]  \hspace{1cm} (3.10)

Each period, each generation is advancing to a more mature state, such that generation \( i \) becomes generation \( i + 1 \). During this process, generations change their method of updating insofar as they now put a higher weight on signal observation and a lower weight on the observation of price changes. The oldest generation simply drops out and a new generation enters at position \( i = 1 \), whose initial beliefs are calculated by the observed \( z_{t-1} \) and \( \Delta p_t \).

Keep in mind that this is a behavioral model and thus an agent is making the same systematic error (namely including the momentum term in his belief formation) again and again. His internal model of the world is inflexible in this sense and any deviations of the price from his predicted values are assigned to some noise and not to the quality of the own heuristic. A justification could be that people tend to repress wrong predictions they made and that they seem to filter arriving information according to its fit to their initial beliefs. This effect is known as "anchoring" or "belief perseverance" and is well described in psychological and behavioral economics literature.\(^8\) Lord, Ross and Lepper (1979, p. 2099) describe some of the underlying cognitive mechanisms involved in such biases:

"...there is considerable evidence that people tend to interpret subsequent evidence so as to maintain their initial beliefs. The biased assimilation processes underlying this effect may include a propensity to remember the strengths of confirming evidence but the weaknesses of disconfirming evidence, to judge confirming evidence as relevant and reliable, but disconfirming evidence as irrelevant and unreliable, and to accept confirming evidence at face value while scrutinizing disconfirming evidence hypercrit-

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\(^8\) See for example Rabin (1996) for a detailed description of "anchoring" and generally an extensive compilation of psychological phenomena relevant for economics.
ically. With confirming evidence, we suspect that both lay and professional scientists rapidly reduce the complexity of the information and remember only a few well-chosen supportive impressions. With disconfirming evidence, they continue to reflect upon any information that suggests less damaging 'alternative interpretations'. Indeed, they may even come to regard the ambiguities and conceptual flaws in the data opposing their hypotheses as somehow suggestive of the fundamental correctness of those hypotheses. Thus, completely inconsistent or even random data - when 'processed' in a suitable biased fashion - can maintain or even reinforce one's preconceptions. 9

In the case of financial markets the trader might justify himself by seeing the asset as a long-term or growth investment, which - although it payed poor dividends today - will exhibit strongly increasing returns in future periods. 9 For a further discussion of this issue with regard to economic modeling see for example Barberis, Shleifer and Vishny (1998, p.319f).

However, the new feature of our approach is that agents’ heuristics do improve over time. In the model presented here, this happens in a exogenous way. But one could well think of models where the size of estimation errors determines the speed of the improvement of the belief formation process. Qualitatively the results should be the same in the latter case, thus we stick to the mechanism described here to keep the model trackable.

9This would match the observation that growth markets - where the mentioned psychological effect would work best - seem to be more susceptible for bubble behavior than well established markets with lower expected growth.
3.3 Equilibrium

3.3.1 Demand and Asset Price Formation

For the formation of asset prices we use a standard approach where agents can choose between a risky asset (the asset \( A \) under consideration) and a riskfree asset \( B \). Agents estimate the liquidation value of the assets, where the certain payoff of one unit of \( B \) is \( r \) and the payoff of one unit of \( A \) is believed to be \( y_h \) with probability \( b_i \) and \( y_i \) with probability \( 1 - b_i \).

Each generation \( i \) maximizes the expected utility of its portfolio with respect to \( x_{it} \), the amount of \( A \) bought at time \( t \):

\[
EU_{it}(x_{it}) = -b_i e^{-\rho(r(W-x_{it}p_t)+x_{it}y_h)} - (1-b_i) e^{-\rho(r(W-x_{it}p_t))}
\]

The FOC is:

\[
\frac{\partial}{\partial x_{it}} = -b_i \rho(r p_t - y_h) e^{-\rho(r(W-x_{it}p_t)+x_{it}y_h)} - (1-b_i) \rho(r p_t) e^{-\rho(r(W-x_{it}p_t))} = 0
\]

This yields

\[
x_{it} = \frac{1}{\rho y_h} \ln \frac{b_i (y_h - r p_t)}{(1-b_i) r p_t}.
\]

Aggregating demand and equalling to the supply of 0 results in

\[
p_t = \frac{K_t y_h}{r (1 + K_t)}
\]

with \( K_t \equiv \sqrt{\frac{\Pi_i b_{it}}{\Pi_i (1-b_{it})}} \).

Equation 3.14 can be seen as a general pricing function for two-asset models with CARA utility functions and heterogeneous beliefs about high payoff probabilities. The variable \( K \) can be interpreted as the market sentiment. It directly determines the final market price \( p \) in our model and will be very useful for the proofs in the following sections. A \( K > 1 \) means the overall belief of all market participants in high payoffs is positive (larger than \( \frac{1}{2} \)), thus \( p > \frac{y}{2} \); \( K < 1 \) indicates negative overall
belief implicating \( p < \frac{b}{2} \). Very high \( K \)'s represent strong overall belief in high future payoffs, leading to a price of \( y_h \) for \( K \to \infty \), very low \( K \)'s represent low aggregate beliefs in high future returns, thus yielding a price of 0 for \( A \) if \( K \to 0 \), as nobody wants to hold \( A \) if its expected value is 0.

Note also that even if \( K > 1 \), still a majority of traders can assign probabilities of \( b < \frac{1}{2} \) to a high payoff if this is counterbalanced by extremely high beliefs of the minority. For example a single private belief of \( 0 + \varepsilon \) or \( 1 - \varepsilon \) for \( \varepsilon \to 0 \) leads to a price of 0 resp. \( \frac{b}{r} \) because the agent under consideration then is willing to sell short or buy the whole market if not counterbalanced by an appropriately extreme opposing belief of another agent.

For \( n = 1 \) or with homogeneous beliefs and arbitrary \( n \) we get the pricing function for homogeneous traders as a special case, then it follows that \( K_i = \frac{b_n}{1-b_n} \) and equation 3.14 simplifies to \( p_t = \frac{b_t y_t}{r} \). Therefore, the fully rational price for correctly informed risk averse agents with a utility function described above would be

\[
p_t^* = \frac{\pi_t y_t}{r} = \frac{y_t}{r(e^{-z t} + 1)}. \tag{3.15}
\]

We define a sentiment function \( K(b_i) \) which describes the market sentiment dependent on \( b_i \) if \( b_{-i} \) (i.e. all \( b_j \) with \( j \neq i \)) are held fix:

\[
K(b_i|b_{-i}) \equiv \sqrt{\frac{b_i \prod_{j \neq i} b_j}{(1 - b_i) \prod_{j \neq i} (1 - b_j)}} \tag{3.16}
\]

Figure 3.2 depicts three examples of the sentiment function \( K(b_i|b_{-i}) \) for high, neutral and low \( b_{-i} \) (\( b_{-i} >, =, < \frac{1}{2} \)). Depending on other agents beliefs the belief of \( i \) has to be more or less extreme to shift overall sentiment in the other direction than suggested by the majority. The more extreme \( b_i \), the faster overall \( K \) is shifted away from the sentiment of the majority.

Thanks to the CARA utility functions, equations 3.13 and 3.14 are independent of \( W \), that means we face the same myopic decision problem each period even if we allow for accumulating wealth over periods.
Figure 3.2: Sentiment function for n=5 dependent on belief \( b_i \), with beliefs \( b_{-i} \) of all other agents \( j \neq i \) kept fix at 0.5 (bold line), 0.4 (dashed line) and 0.6 (dotted line). Depending on \( b_{-i} \), \( i \)'s belief has to be higher or lower in order to yield overall positive sentiment (\( K > 1 \)).

### 3.3.1.1 Properties of the Demand, Price and Sentiment Functions

We now want to state some properties of the demand function (equation 3.13), the price function (equation 3.14) and the sentiment function (equation 3.16). This properties help us to determine the eventual effect of a shock to fundamental value to the market price of asset \( A \). A shock is defined as a change in the fundamental variable \( z_t \) defined in Section 3.2.1 by +1 (positive shock) or -1 (negative shock). The causality works from \( \pi_t \) via the signal \( s_t \) and individual beliefs \( b_i \) to market sentiment \( K_t \) and finally to price \( p_t \) as depicted in Figure 3.3.

The price function (3.14) is downward sloping in \( r \) and upward sloping in \( K \) and \( y \), as

\[
\frac{\partial p}{\partial r} = -\frac{K y_h}{r^2(1 + K)} < 0,
\]

(3.17)
Figure 3.3: Causality from fundamental shock to market price.

\[
\frac{\partial p}{\partial K} = \frac{y_h}{r(1+K)^2} > 0 \forall r > 0; K, y_h > 0, \tag{3.18}
\]

\[
\frac{\partial^2 p}{\partial K^2} = -\frac{2y_h}{r(1+K)^3} < 0 \forall r > 0; K, y_h > 0, \tag{3.19}
\]

and

\[
\frac{\partial p}{\partial y_h} = \frac{K}{r(1+K)} > 0 \forall r > 0; K > 0. \tag{3.20}
\]

Individual demand \(x_i\) is downward sloping in \(p\) and upward sloping in \(b_i\) for prices smaller than \(\frac{y_h}{r}\):

\[
\frac{\partial x_i}{\partial p} = -\frac{1}{\rho p(y_h - rp)} < 0 \forall p < \frac{y_h}{r}, \tag{3.21}
\]

\[
\frac{\partial x_i}{\partial b_i} = \frac{1}{\rho y_h b_i(1 - b_i)} > 0 \forall 0 < b_i < 1; y_h, \rho > 0. \tag{3.22}
\]

Given the rational price of \(p = \frac{\pi y_h}{r}\), individual demand would be positive if

\[
\frac{b_i(y_h - rp)}{(1 - b_i)rp} > 1, \tag{3.23}
\]

\[
\Leftrightarrow b_i > \pi. \tag{3.24}
\]

The slope of market sentiment \(K\) with respect to \(b_i\) is determined by

\[
\frac{\partial K}{\partial b_i} = \frac{K}{nb_i(1 - b_i)}. \tag{3.25}
\]

This is positive for all \(b_i \in ]0; 1[\). Also:

\[
\frac{\partial^2 K}{\partial b_i^2} = \frac{K(1-n+2nb_i)}{n^2b_i^2(1-b_i)^2}, \tag{3.26}
\]

\[
\Rightarrow \frac{\partial^2 K}{\partial b_i^2} = 0 \Leftrightarrow b_i = \frac{n-1}{2n}. \tag{3.27}
\]
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For smaller \( b_i \), \( K \) is bent rightwards, for larger \( b_i \) leftwards in \( b_i \). In any case, for \( b_i > \frac{1}{2} \), \( K(b_i) \) is leftward bend as equation 3.27 is always smaller than \( \frac{1}{2} \).

The total differential of \( K \) with respect to \( b \) is thus

\[
dK = \sum_{i=1}^{n} \frac{\partial K}{\partial b_i} db_i. \tag{3.28}
\]

For the proofs of under- and overpricing following in the next section we state estimations for the change in \( b_i \)'s if a fundamental shock arrives. For that, suppose that the probability \( \pi_{t-1} \) for high payoff last period was \( \frac{1}{2} \), i.e. we faced an asset with equal high/low payoff-probability, and suppose further that the market was stable for at last two periods, i.e. no momentum is observed. Then, if a (correct/incorrect) fundamental signal arrives corresponding to a shift in \( \pi \) by \( \Delta \pi \), generation \( i \)'s belief is changing according to:

\[
\Delta b_{it} = \pm \frac{i - 1}{n - 1} \Delta \pi (2\tau - 1). \tag{3.29}
\]

For a proof see appendix C.

3.3.2 Existence of Bubbles

Using the results of the last section, we now analyze the impact of a positive fundamental shock on the price path in our model.\(^{10}\) We will show that a single positive shock to \( z \) (and thus to \( \pi \)) and an according positive signal will lead to underpricing immediately after the observation of the shock, to an overshooting price afterwards, to some periods of overpricing later on, and then to an endogenous breakdown of the asset pricing bubble and some more periods of underpricing. This cycle is repeated with diminishing amplitude until it converges to the new "fair" price defined by the altered \( \pi \).

During the cycle, young generations lose money by buying when prices are too high and old generations win money by selling short on high prices. The market will

\(^{10}\)Think of a shock to the profitability of the asset, e.g. the arrival of a new production technology that increases expected payoffs.
predominantly be held by young generations when prices fall and by old generations when prices rise. The overall lifetime performance of a trader depends mainly on the time he enters the market. If prices are low at that point in time, she is likely to obtain an overall gain from trading, if prices are high she will probably lose money.

Because the model displays complex dynamics as current beliefs are influenced by all former beliefs from past periods via the momentum term, it is not possible to provide proofs for price paths in any situation with arbitrary sequences of shocks. Nevertheless it is feasible to analytically characterize price paths under certain conditions. Then we can show exemplary how bubbles arise in an initially fair priced markets and how prices fade into the new stable state only after a number of mispricing-cycles after a shock.

To set up formal proofs, we restrict ourselves to a situation where a formerly fair priced market is hit by exactly one shock. The situation we have in mind is the arrival of a new technology, e.g. the information technologies in the late 90s, and its impact on the market. In Chapter 4 we will model more complex price paths with several shocks. This will be done with the help of numerical simulations and we will show that bubbles can overlap and intensify such that prices display strong excess volatility.

For the one-shock-case we assume a situation at time $t$ where $\pi_t = \frac{1}{2}$ and there is no momentum in the market, i.e. the price $p$ was constant for at least the last two periods. This implies that all traders beliefs about $\pi_t$ are correct in this initial state, i.e. $b_{it} = \pi_t = \frac{1}{2}$, $\forall i$. Thus $K_t = 1$ and price $p_t$ is equal to $p_t^*$. Moreover is follows that all generations hold zero units of the risky asset.

First we will give the intuitions and informal proofs for under- and overpricing in period $t$ and $t+1$ respectively, and we deduce limits for the change in $K$ when a shock hits the formerly stable market (Lemma 3.1). Later we will state formal proofs for under- and overpricing based on the estimations of $\Delta K$ and $\Delta p$ (Propositions 3.1 and 3.2).
3.3.2.1 Intuition for Underpricing in Period \( t \)

In the situation described above, the risky asset will be underpriced compared to the fully rational price in the first period following a positive shock to the fundamental variable. The intuition for this is as follows:

Consider a single shock \( \Delta z_t \) to the fundamental variable \( z \) and a corresponding change of \( \pi \) by \( \Delta \pi \) (see equation 3.2). Accordingly, traders receive a signal \( s_t \), which is assumed to be correct.\(^{11}\) As \( p \) was constant the last periods, \( b^m \) obviously does not change at time \( t \), but \( b^s \) increases and thus total \( b_i \) for generation \( i \) rises by \( \Delta b_i \) (as defined by equation 3.29) due to the signal \( s_t \).\(^{12}\) For young generations the increase in \( b \) is very small (or even zero for generation \( i = 1 \)) because they overweight \( b^m \) over \( b^s \), whereas it is high for old generations, with the oldest generation \( i = n \) incorporating the full effect of the increase in \( b^s \) in their \( b_n \). Clearly, the new price \( p_t \) will be above \( p_{t-1} \) (as almost all generations strictly increase their beliefs) and below \( p_t^* \), as only the oldest generation increases its belief to the fully rational level. In other words, there will be underpricing in period \( t \). Old generations will accumulate the risky asset as their demand will rise more strongly while young generations will sell it short, as they falsely assume it is too expensive.

3.3.2.2 Intuition for Price Overshooting in Period \( t + 1 \)

After \( p_t \) has formed in a way that clears the market, the true \( z_t \) is revealed to the traders. This has an effect on the signal beliefs \( b^s \), as the uncertainty about \( z_t \) is dissolved, thus \( b^s \) rises to \( \pi_t \). Also the basis for the estimation of \( b^m \) changes from \( z_{t-1} \) to \( z_t \). But \( b^m \) is even further increased by the observation of the positive price change in the previous period. As a consequence, overall \( b \) will at least be equal to \( \pi_t \) and is strictly higher than \( \pi_t \) for all generations \( i < n \) because they care for \( b^m \). Therefore, generational demand for the fair price of \( p_t^* \) would be zero for generation \( n \) and positive for all generations \( i < n \) (see equation 3.24). As net supply is zero, this is

\(^{11}\)If it was false, the first direction in the price path would be wrong. Nevertheless a bubble would arise later as the true value of \( z \) is revealed at the end of the period.

\(^{12}\)A proof can be found in Appendix C.
impossible and price \( p_t \) must rise above \( p^*_t \). In other words there will be overshooting of prices in period \( t + 1 \).

As the maximal slope \( \frac{1}{4} \) of \( \pi(z) \) is reached for \( z = 0 \), the increase in \( b_{it+1} \) is bounded by:\(^{13}\)

\[
\Delta b_{it+1} < \frac{n - i}{4(n - 1)}(\Delta z_t + \Delta p_{t+1}) + \frac{i - 1}{n - 1} \Delta b^*_t. \tag{3.30}
\]

It is particularly high for young generations, as they are heavily affected by the strongly positive momentum part. As their beliefs will now "overtake" the beliefs of old generations, they will accumulate the risky asset in this period whereas old generations will sell it at the new (and rationally too high) equilibrium price.

**Intuition for Further Periods**

In the subsequent period \( t + 2 \), \( b^* \) does not change anymore, as no new signal arrives and all fundamental information is already incorporated to all \( b^*_t \)’s. The sole changes in beliefs will now stem from changes in \( b^m \):

\[
\Delta b_{it+2} < \frac{n - i}{4(n - 1)} \Delta p_{t+2} - \frac{n - i}{4(n - 1)} \Delta p_{t+1} \tag{3.31}
\]

Note that, given \( b^* \) and \( z \) are constant now, an increase in \( p \) at time \( t + 2 \) only occurs if the change in prices was larger in the preceding period than in the pre-preceding period, i.e. if \( \Delta p_{t+2} > \Delta p_{t+1} \). Therefore, to show that \( p \) will again increase in period \( t + 2 \) it is sufficient to prove the increase in \( \Delta p \) in period \( t + 2 \) relative to that in period \( t + 1 \). We will do that below in Proposition 3.3.

**3.3.2.3 Formal Proofs for Under- and Overpricing**

We will now set up a lemma that defines the lower and upper bounds of changes in market sentiment \( K \) in the first period after the arrival of a shock. It will be used in the successive proofs.

\(^{13}\)Recall equations 3.2, 3.9 and 3.10.
Lemma 3.1  In a situation at time $t$ where all traders have neutral beliefs about $\pi_t$ and beliefs are correct, i.e. $b_i = \pi_t = \frac{1}{2} \equiv b \ \forall i$, and $p$ was constant for at least the last two periods, it holds that:

$$2\Delta\pi(2\tau - 1) < \Delta K_t < \frac{\Delta\pi(2\tau - 1)}{\frac{1}{2} - 2\Delta\pi^2(2\tau - 1)^2}. \quad (3.32)$$

Proof

From equations 3.25, 3.28 and 3.29 and respecting the fact that $b_i = \pi_t \ \forall i$ at the beginning of the period and that $K(b_i)$ is convex for $b_i > \frac{1}{2}$ by equation 3.26, it follows that $K_t$ will increase at least by

$$\Delta K_t > \sum_{i=1}^{n} \left( \left. \frac{\partial K}{\partial b_i} \right|_{b_i} \cdot i - \frac{1}{n} \Delta \pi (2\tau - 1) \right) \quad (3.33)$$

$$= \sum_{i=1}^{n} \frac{(i - 1)K_i \Delta \pi (2\tau - 1)}{n(n - 1)b_i(1 - b_i)} \quad (3.34)$$

$$= k_i \Delta \pi (2\tau - 1) \frac{2b(1 - b)}{2b(1 - b)}. \quad (3.35)$$

As $K_t = 1$ and $b = \frac{1}{2}$, it follows:

$$\Delta K_t > 2\Delta\pi(2\tau - 1). \quad (3.36)$$

Because $K(b)$ is convex, we also can deduct an upper bound for $\Delta K_t$ by assuming the highest possible slope in the sentiment function:

$$\Delta K_t < \sum_{i=1}^{n} \left( \left. \frac{\partial K}{\partial b_i} \right|_{b_i + \Delta b_i} \cdot i - \frac{1}{n} \Delta \pi (2\tau - 1) \right) \quad (3.37)$$

$$= \sum_{i=1}^{n} \frac{(i - 1)K_i \Delta \pi (2\tau - 1)}{n(n - 1)(b_i + \Delta b_i)(1 - b_i - \Delta b_i)}, \quad (3.38)$$

where $\Delta b_i$ denotes the increase in the beliefs of the particular generation.

Another (less strict) upper bound can be derived by replacing all changes of signal beliefs by their maximal possible increases $\Delta b_i$:

$$\Delta K_t < \sum_{i=1}^{n} \left( \left. \frac{\partial K}{\partial b_i} \right|_{b_i} \cdot i - \frac{1}{n} \Delta \pi (2\tau - 1) \right) \quad (3.39)$$

$$= \sum_{i=1}^{n} \frac{(i - 1)K_i \Delta \pi (2\tau - 1)}{n(n - 1)(b_i + \Delta b_i)(1 - b_i - \Delta b_i)}. \quad (3.40)$$
Although equation 3.40 is less strict than equation 3.38, it is more useful as we now can collapse the sum. With $K = 1$, $b = \frac{1}{2}$ and (3.29) it follows that

$$\Delta K_t < \frac{\Delta \pi (2\tau - 1)}{2(\frac{1}{2} + \Delta \pi (2\tau - 1))(\frac{1}{2} - \Delta \pi (2\tau - 1))}$$

(3.41)

$$= \frac{\Delta \pi (2\tau - 1)}{\frac{1}{2} - 2\Delta \pi^2 (2\tau - 1)^2}.$$  

(3.42)

\[\Box\]

The next lemma states the first aftershock-period price change triggered by the arrival of a positive shock. It will help to determine conditions for bubble formation later on.

**Lemma 3.2** Given the initial situation described in Lemma 3.1, $p$ rises by less than $\frac{y \Delta \pi (2\tau - 1)}{4r \frac{1}{2} - 2\Delta \pi^2 (2\tau - 1)^2}$ in period $t$.

**Proof**

As $p(K)$ is concave, we can define the upper bound $\Delta p_t$ in period $t$ as follows:

$$\Delta p_t < \left. \frac{\partial p}{\partial K} \right|_{K_t=1} \cdot \Delta K_t.$$  

(3.43)

From equations 3.18 and 3.42 it follows:

$$\Delta p_t < \frac{y}{4r} \frac{\Delta \pi (2\tau - 1)}{\frac{1}{2} - 2\Delta \pi^2 (2\tau - 1)^2}.$$  

(3.44)

\[\Box\]

Note also that the change of the rational price $p^*$ in period $t$ is

$$\Delta p^*_t = \frac{\Delta \pi y_h}{r}.$$  

(3.45)

We can now use the lemmas to set up propositions stating under- and overpricing following a positive fundamental shock.

We start by stating underpricing in the first period after the arrival of a positive shock.\(^{14}\)

\(^{14}\) All following statements have to be inverted for the case of a negative shock.
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**Proposition 3.1** Given the initial situation described in Lemma 3.1, there will be underpricing in the first period \( t \) after the arrival of a positive fundamental shock:

**Proof**

Underpricing occurs if the increase in \( p \) is smaller than in \( p^* \). Using equations 3.44 and 3.45, this is the case if

\[
\frac{y}{4r} \frac{\Delta \pi (2\tau - 1)}{0.5 - 2\Delta \pi^2 (2\tau - 1)^2} < \frac{\Delta \pi y_h}{r},
\]

which is equivalent to:

\[
|\Delta \pi| < \frac{\sqrt{6 - 4\tau}}{4(2\tau - 1)}.
\]

As \(|\Delta \pi| = 0.231\) given the initial situation described in Lemma 3.1,$^{15}$ and the maximum value for \( \frac{\sqrt{6 - 4\tau}}{4(2\tau - 1)} \) is \( \approx 0.35 \) for \( \tau = 1 \),\(^ {16} \) there will always be underpricing in period \( t \).

\[
\square
\]

Similar to Proposition 3.1 we derive a condition for \( \Delta p_t < \frac{1}{2} \Delta p^*_t \), which will later help to show overpricing in the third period after the arrival of a shock:

**Corollary 3.1** Given the initial situation described in Lemma 3.1, actual price \( p_t \) increases less than half as much as the rational price \( p^*_t \) in period \( t \), if \( \Delta \pi < \frac{\sqrt{2 - 2\tau}}{2(2\tau - 1)} \). Stated differently: \( \Delta p_{t+1} < \frac{1}{2}(p^*_t - p^*_t) \) if \( \Delta \pi < \frac{\sqrt{2 - 2\tau}}{2(2\tau - 1)} \).

**Proof**

Analogous to proof of Proposition 3.1 with an additional factor \( \frac{1}{2} \) on the right hand side.

\[
\square
\]

\(^{15}\) Note that by equation 3.2, the initial value \( \pi = \frac{1}{2} \) changes to 0.269 or 0.731 if \( z \) is shifted by \( \pm 1 \).

\(^{16}\) A smaller \( \tau \) leads to an even larger right hand side of equation 3.47, thus \( \tau = 1 \) is the most critical value. This is confirmed by intuition: A less reliable signal would lead to lower increases in \( b^* \) and thus to even lower prices.

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Corollary 3.1 provides only a sufficient and not a necessary condition for the statement that first period price increases will be less than half the increase in rational prices. For $\tau \leq \frac{9}{10}$ it is fulfilled for a standard unit shock to $z$, for $\tau \leq \frac{8}{10}$ it is already satisfied for the whole range of possible shock sizes.\(^{17}\)

Now we state overpricing in the second period after the arrival of the positive shock:

**Proposition 3.2** Given the initial situation described in Lemma 3.1, there will be overpricing of the risky asset in period $t + 1$.

**Proof**

To proof overpricing in the situation under consideration, note that

$$p_{t+1} > p_{t+1}^s$$

has to hold. This is equivalent to

$$K_{t+1} > \frac{\pi_{t+1}}{1 - \pi_{t+1}}.$$  \hspace{1cm} (3.49)

From the definition of $K$ on page 56 it follows that Proposition 3.2 holds if and only if $b_{it+1} \geq \pi_{t+1} \forall i$ and $\exists i$ s.t. $b_{it+1} > \pi_{t+1}$.\(^{18}\)

As $\forall i < n : b_{it+1}^s = \pi_t = \pi_{t+1}$ and $b_{it+1}^m > \pi_t$, and $\exists i : b_{it+1} \in [b_{t+1}^s; b_{t+1}^m]$, this holds true.

After stating that there is underpricing in the first period following a positive shock and price overshooting in the second, we now come to our main proposition, namely the description of the total price path following a shock to the fundamental variable.

\(^{17}\)Intuition and numerical simulations show that Corollary 3.1 in fact holds for the whole parameter range of $\tau$ and $\Delta z$.

\(^{18}\)This says that the odds ratio for all traders is at least as high as the rational odds ratio, with at least one trader having a strictly higher odds ratio than the rational one. Recall also equation 3.16.
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Proposition 3.3 Given the initial situation described in Lemma 3.1, at least for 
\[ \Delta \pi < \frac{\sqrt{2-2\pi}}{2(2\pi-1)} \] an asset pricing bubble will arise and deflate again after a single positive 
shock \( \Delta \pi \) and an according signal with precision \( \tau \) arrive at period \( t \).

Proof

To prove the emergence of a pricing bubble, we will describe the development of 
the asset price \( p \) after a positive observed shock at the beginning of period \( t \).

1st period after shock, time \( t \): Underpricing

As stated in Corollary 3.1, \( \Delta p_t \) will be smaller than \( \frac{1}{2} \Delta p^*_t \), and underpricing will 
occur in the first period after the observed positive shock.

2nd period after shock, time \( t + 1 \): Overpricing

As shown in Proposition 3.2, period \( t + 1 \) exhibits overpricing, thus \( p_{t+1} > p^*_{t+1} = p^*_t \) 
and, combining Corollary 3.1 with underpricing in period \( t \) and overpricing in period 
\( t + 1 \):

\[ \Delta p_{t+2} = p_{t+1} - p_t > \Delta p_{t+1}. \]  \hspace{1cm} (3.50)

In words, the price increases more from period \( t \) to period \( t + 1 \) than it increases from 
period \( t - 1 \) (the initial price) to period \( t \).

3rd period after shock, time \( t + 2 \): Overpricing continues

As the \( b^m \) stay constant from period \( t + 1 \) to \( t + 2 \) because all uncertainty about 
z was already resolved at the end of period \( t \) when \( z_t \) was revealed, and \( z \) did not 
change later, only \( b^m \) will matter for the overall belief formation of the generations.

\( b^m_{t+2} \) will increase compared to \( b^m_{t+1} \) if and only if \( \Delta p_{t+2} \) (which determines \( b^m_{t+2} \)) is 
larger than \( \Delta p_{t+1} \) (which determines \( b^m_{t+1} \)). Because this holds true by equation 3.50, 
b, increase for all generations except generation \( i = n \) (for which it stays unchanged).
thus $K$ increases and subsequently $p$ rises again and $\Delta p_{t+3}$ is positive.

$m^{th}$ period after shock, time $t + m$: Breakdown of Bubble

As the price function is asymptotically reaching $\frac{b}{t}$ for $K \to +\infty$ and $\Delta p_{t+1} > 0$, it is obvious that there exists a period $m$, such that: $\Delta p_m < \Delta p_{m-1}$. Thus in period $m$, $p$ will decrease if no other signal arrives. Another way to see this is to examine generational demand which increases less and less fast for high $p$ even if beliefs are rising further (see equation 3.22).

Further periods, time $t + m + 1, t + m + 2, ...$: Pricing cycle with diminishing amplitude

The decrease will be small in the first period $t + m$ and will trigger an accelerating downward cascade via the momentum term, as $\frac{dp}{dK}$ increases with falling $K$ and also the slope of $\pi(\dot{z})$ is higher for $\dot{z}$ close to $\frac{1}{2}$. By the same argument as before, the decline in $p$ will smoothen out when $p$ is approaching the minimal price of zero, and another upward cascade will start. Depending on the value of $\lambda$, the cycle flattens out faster or slower, and $p$ finally converges to the stable value $p^*$.

$\square$

3.4 Numerical Simulation of Price Paths

Although it was possible to prove that bubbles and a mispricing-cycle arise following a single exogenous shock, it is hardly possible to state a general closed form solution for exact values of an entire price path over more than a couple of periods. This is due to the price dependency on all former prices and beliefs. Nevertheless it is easily possible to calculate price paths for given parameter settings. To visualize the dynamics and get a better impression of the bubble behavior explained above, we

\footnote{Actually, simulations show that in the whole parameter range it holds that $m = t + 3$, that means already in $t + 4$ prices are reverting again.}
ran a couple of numerical simulations to calculate explicit price path developments after shocks. The simulations also allow us to track the portfolio dynamics of single traders in the market and to confirm related statements made above.

In Section 3.4.1 we describe simulations for the case of one exogenous shock arriving at the market. We will find the behavior described in Proposition 3.3 again, namely one period of underpricing, a phase of overpricing and flattening pricing cycles around the new fair price.

In Chapter 4 we also run simulations for the case of multiple shocks arriving at the market at several times. This helps us to illustrate long run pricing behavior and excess volatility displayed by our model.

3.4.1 The One Shock Case

As a first case we simulate a price path related to the situation described above in Lemma 3.1, i.e. a single positive fundamental shock hitting a formerly balanced market. The concrete parameter specification for the following figures is:\textsuperscript{20}

\begin{align*}
\text{number of generations } n &= 5, \\
\text{precision of signal } \tau &= \frac{7}{10}, \\
\text{payoff of risk free asset } r &= 1, \\
\text{high payoff of risky asset } y_h &= 3, \\
\text{coefficient of risk aversion } \rho &= \frac{1}{2}, \\
\text{momentum intensity parameter } \lambda &= 2.
\end{align*}

The other specifications are as stated in Lemma 3.1, that is agents’ beliefs before the arrival of the shock are correct and equal to $\frac{1}{2}$ and there is no momentum in the market at the time the shock arrives. Therefore, the price path described below for the given parameter values exactly corresponds to the path described by the theoretical analysis in the last section.

\textsuperscript{20}We tested the model behavior with several other specifications and the predicted patterns emerged in any case.
3.4.1.1 Price Path

The simulation ran for 20 periods with the shock and the signal arriving in period 4. The resulting price path is displayed in Figure 3.4. The rational price before the shock is $\frac{\mu_s}{\gamma} = 1.5$ and rises to 2.19 after the shock. The expected under- and overpricing as well as the cyclical pricing behavior afterwards can well be seen in the figure.

As stated in the intuition arguments in Section 3.3.2, we expect that young traders are losing money on average, as they buy the market in its downturns, whereas old generations profit by selling their shares when prices are high and by buying when prices are falling.

In our simulation we can track the amount of stock held by each generation at any time. The results for the specification given above is depicted in Figure 3.5, where the demand of generation 1 (youngest) and 5 (oldest) are shown. Next to that the divergence of $p_i^*$ from $p_i$ is drawn, positive values indicate underpricing, negative values

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21 The signal is correctly reflecting the change in fundamentals in the depicted simulation.
Figure 3.5: Generational demand and divergence of the rational price from the actual price. High positive divergence means the asset is underpriced and vice versa. Demand of the oldest generation ($i = 5$) is correlated with underpricing whereas the youngest generation ($i = 1$) buys when the asset is overpriced.
overpricing. As predicted by our considerations above, the old generation exhibits excess demand when prices are relatively low, whereas young generations demand the risky asset when its price is relatively high. To estimate the size of losses/gains made by young/old generations we tracked the per-period expected returns for each generation. From the numerical model we calculate the average gains/losses of each generation of traders and we find that indeed the young generations lose on average whereas the older win. However, the differences are especially dramatic when there is substantial mispricing, i.e. in the first periods after the shock. In Table 3.1, the end of period expected wealth and period utility is listed for all generations for the period where the bubble reaches its top (period 6 in Figure 3.4). Also, the relative expected wealth with respect to the highest reached expected wealth (that of generation 5) is denoted. Relative gains and losses are substantial in this one shock numerical example and with many shocks as in Section 4.1 they would even increase, as mispricing increases if bubbles overlap and add to each other.

<table>
<thead>
<tr>
<th>Generation #</th>
<th>end of period wealth</th>
<th>period utility</th>
<th>in % of top wealth</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.8657</td>
<td>-0.4207</td>
<td>78%</td>
</tr>
<tr>
<td>2</td>
<td>0.9488</td>
<td>-0.3872</td>
<td>85%</td>
</tr>
<tr>
<td>3</td>
<td>1.0119</td>
<td>-0.3635</td>
<td>91%</td>
</tr>
<tr>
<td>4</td>
<td>1.0640</td>
<td>-0.3450</td>
<td>95%</td>
</tr>
<tr>
<td>5</td>
<td>1.1095</td>
<td>-0.3297</td>
<td>100%</td>
</tr>
</tbody>
</table>

In our model momentum traders lose money, and experienced investors profit from their existence. Other models with heterogeneous investors often run into troubles justifying the survival of non-rational traders in the long run. One justification put forward by DeLong, Shleifer, Summers and Waldmann (1990) is the idea that irrational noise traders generate extra risk for which they are compensated, because
short sighted and risk averse arbitrageurs cannot bet aggressively against them due to the risk that the former might push prices even further away from fundamental values next period. In contrast, there is no need to justify survival of the "momentum traders" in our model, as they do not remain on the losing side for long, but gradually switch to the smart camp when getting older.

Thus a trader will have negative expected returns when he is inexperienced and positive expected returns when he is experienced. The overall actual life time performance mainly depends on the time an agent enters the market in our model. If he enters it in a phase of severe mispricing (i.e. directly after the arrival of a shock) he might suffer strong losses in his first periods of life which he will not be able to compensate later. On the other hand, if he enters in a calm phase and a shock arrives when he is already experienced, he will profit from the arising price divergence and his expected lifetime returns will be unusually high.

3.5 Conclusion

In classical asset pricing models, agents are either homogeneous or - if heterogeneous - static with respect to their belief formation heuristics. In contrast to that it seems sensible that complex decision making is a task where experience plays a critical role and one would expect that agents do better the more experienced they are. This model captures that idea and explains well known asset pricing puzzles, as over- and underpricing, bubble behavior and excess volatility in the framework of an overlapping generations model where young traders follow a simple non-rational momentum strategy and old traders behave as perfect rational decision makers. When getting older, young traders gain experience and gradually switch to the rational camp.

Moreover it is possible within the model to make predictions as to which traders hold the market in any particular phase, and in which time of their life-cycle traders will on average face positive or negative expected returns.

In the next chapter we will examine the case of multiple shocks and we present an extended version of the model that allows for strategic trading.
Chapter 3

Also in the next chapter, in Section 4.6, an outlook and suggestions for empirical testing are presented that refer to Chapter 3 as well as to Chapter 4.
Chapter 4

An Extended Model of Asset Pricing with Heterogeneous Agents

In Chapter 3 we restricted ourselves to the case of one fundamental shock only. This made it possible to treat the model analytically and clearly showed the basic mechanics of our model. The intuition for markets with multiple shocks is obvious: Each shock will generate its own mispricing cycle and these "waves" will interfere like waves in a physical context and will add to a much more complex price path with rough and calm periods. It is no longer possible to describe these general paths analytically, but in this chapter we will present some simulations to get at least some qualitative insights into a multi-shock environment.

In the previous chapter we also stuck to the assumption that agents are price takers in the market or that they have no knowledge about other agents decision making processes. Both assumptions are pretty plausible and should cover the vast majority of markets we observe. However, in this chapter we want to extend the basic model to the case of fully informed agents with pricing power. Clearly this changes the situation considerably as rational traders now will act strategically and will try to influence prices in their favor to profit from mispricing. We will find that bubble formation persists in principle, that incomes are even more unequally distributed and that excess volatility rises.
4.1 The Multi Shock Case

In real asset markets, there are numerous shocks to the fundamental value of an asset over time. In our model, each shock triggers a bubble and a pricing cycle, and different cycles will overlap if many shocks arrive. In Figure 4.1 a situation is simulated where shocks arrive with a probability of 10% each period. All other parameters are as in Section 3.4.1. It is clearly visible that actual prices in principle follow the rational price, but that they show price overshooting, bubble behavior and some excess volatility. Later, in Section 4.4.1, we take a closer look at the volume of excess volatility dependent on the intensity of momentum trading in the market.

The simulation shows that the "waves" of mispricing interfere and sometimes cancel out each other, whereas on other occasions they amplify each other and lead to strong moves in the market. Especially when strong down-(or up-)ward momentum is reinforced by another shock in the same direction, there can be severe crashes (or increases) in market price followed by periods of extensive mispricing.

Mispricing is most strong for $z_t$ around zero due to our process specification where the changes in $\pi_t$ are the higher the closer $z_t$ is to zero.\(^1\)

Also, the effect of market entrance timing can well be seen in Figure 4.1. Imagine a trader entering the market at a "calm" time, i.e. a point of time with little momentum, let’s say period 96. He then has a couple of periods with low volatility where he can gain experience without losing to much money due to mispricing. When the positive shock finally arrives in period 99, he well be acting rational enough to profit strongly from the subsequent periods of mispricing. He therefore has a good chance to make positive overall profits over his life-cycle.

\(^1\)One criticism of the price path in our simulation was that it follows the rational price path (and thus the fundamental signals) in time, and does not lead it. One probably would expect the latter, as financial markets are said to anticipate future developments. But note that we do not say anything about the nature of the fundamental shock. If you interpret it not as an direct shock but as a shock to expectations about the fundamental variables, the simulated price path would indeed anticipate fundamental changes in future periods.
Figure 4.1: Simulated price path for multiple shock. Price overshooting and excess volatility is clearly visible.

Contrary, look at a trader entering the market during volatile times, e.g. at period 87. This trader faces the opposite situation, namely high mispricing during her "learning phase" and lower mispricing during her "earning phase". Thus it is likely that she will on average lose money over her life-cycle.

For the multi shock case, we again tracked demand of the different generations. In analogy to the one shock case, the corresponding values for the oldest and the youngest generation are plotted for the first 40 periods in Figure 4.2. Note that demand again does not depend on the price level but on the divergence of the fair price from the actual price.

4.2 Strategic Trading

In the following we will extend our model such that experienced traders do have information about the behavior of other traders and have sufficient market power to
Figure 4.2: Generational demand and divergence of actual price from rational price for the multi-shock case.

engage in strategic trading. The dynamics in this case get even more interesting than in the non-strategic setting, but on the other hand they quickly lead to equations that are no longer solvable in closed form. It is possible to set up the maximization problem for the strategically acting rational traders and to formulate first order conditions. Also, propositions about pricing behavior are possible by solving the equations numerically for specified parameter settings.

4.2.1 Motivation

Let’s consider a market where the experienced, more "rational" traders have sufficient market power or can collude to actively exploit the systematic errors made by inexperienced traders. They are assumed to have full knowledge about the decision making process of the young traders.

Market power or collusion on the side of experienced traders is necessary to force the price away from $p_t$. While collusion seems to be unfeasible due to free-rider effects, there occur situations where some or one of the traders hold a large enough
share of the market. This might not be the case for an aggregated asset market but is well possible for single special assets. For example there are many companies whose shares are concentrated in the hand of one or a few large investors with only a smaller fraction floating freely. In many of these cases it seems plausible to think of the large investor as the experienced generation, trying to actively exploit weaknesses of inexperienced traders. Therefore we should expect the effects described here mainly in illiquid markets or in single assets, and less frequent in aggregate or very liquid markets.\(^2\)

One outcome of the extended model is that strategic trading even amplifies mis-pricing and excess volatility. This point is particulary interesting, as it is counter-intuitive to the idea that the introduction of additional "rationality" should lead to less mispricing in the market.

Again we will have to pick out a simple case that on the one side makes clear the intuition behind this extension, and on the other side does not lead into calculations too complex to be solved in a meaningful way. Our first restriction will be that we only allow for \(n = 2\) generations and two distinct groups of decision strategies. We therefore keep only pure momentum traders (denoted by \(\mathcal{M}\)) and pure rational traders (denoted by \(\mathcal{R}\)). We also want generations to stay more than 1 period in each group to allow for intertemporal optimization by rational agents. Therefore we let each generation stay for two periods in each state and thus set the lifespan of each trader to \(2n = 4\), i.e. each generation stays in the momentum group for 2 periods and then immediately switches to the rational group and stays there for another 2 periods.

### 4.2.2 Intuition

When optimizing in a dynamic way, rational agents will not only put total weight on signal beliefs \(b_{\mathcal{R}_t}\) but will also try to exploit the systematical judgement errors

\(^2\)For the case of no collusion and no concentration the market behavior will be as in the previous chapter, as price taking rational investors will not be able to coordinate on strategic prices in a competitive equilibrium.
made by inexperienced generations. The intuition for their reasoning is as follows:

Suppose we are in a balanced situation as defined in Lemma 3.1. Now the positive fundamental shock arrives at time $t$ and let's say the received signal $s_t$ is correct. Then prices will rise in period $t$ due to the increase of $b_{R_t}$. Now there are two effects to be considered by the rational group $R$ in the first period after the shock:

1) As $R$ knows that $M$ did not adjust their beliefs properly in response to the signal, they know that $M$ will underestimate the value of $A$. Therefore $R$ can strategically reduce its demand to lower $p_t$, as this decrease will not be fully absorbed by $M$, and for a marginal decrease in $x_{R_t}$ the gains from lower $p_t$ will be higher than the losses from a smaller $x_{R_t}$. This affects the immediate return at the end of period $t$. The optimal resulting price when $R$ considers only this effect is denoted by $\hat{p}_t$ and will be called "myopic exploiting optimum".3

2) As $R$ knows that $M$ will react to momentum in period $t+1$ by increasing its demand and $R$ anticipates that there will be overpricing in $t+1$, there is an incentive to create as much momentum as possible in period $t$ to make a maximum gain from selling the risky asset short next period. This effect is contrary to effect 1, therefore $R$ will increase its demand such that $p_t$ moves above the myopic exploiting optimum $\hat{p}_t$ to a point $\tilde{p}_t \geq \hat{p}_t$. It depends on the momentum intensity parameter $\lambda$ whether there is a positive shift and whether even $\tilde{p}_t > p_t$, with $p_t$ being the equilibrium price in the model without strategic trading. The price $\tilde{p}_t$ is called "dynamic exploiting optimum".4

In the following we will set up the optimization equations for a strategically acting generation $R$. We will first look at effect (1) and later at effect (2) described above.

---

3From now on we will take the standpoint of group $R$, hence "optimal" means optimal from $R$'s point of view.

4When considering strategic behavior, we treat $R$ as one actor in this chapter, therefore assuming that rational traders are able to cooperate perfectly. If they do not, it depends on the actual size and number of agents in this group whether the price impact is large or small.
4.3 Myopic Exploitation

As a benchmark case we first consider rational agents who do not care about future periods but optimize their utility myopically each period. As now only the first effect from above will work, we expect the price (compared to the non-exploiting case under the same conditions) to be lower than in the standard case in periods where underpricing exists and higher in periods where there is overpricing. To this benchmark we will later compare optimal prices when rational agents optimize in a dynamic way. As we are interested in the behavior of prices, and we will find that prices and \( R \)'s demand are directly linked (see equations 4.2 and 4.3 below), we directly state the maximization problem in terms of \( p \). This saves us the non-linear transformation from \( p \) to \( x_R \) afterwards.

The rational agent \( R \) has to solve

\[
\max_p E U_R(p) \\
\text{s.t.}
\]

\[
x_R = -x_M, \\
x_M = \frac{1}{\rho y_h} \ln \frac{b_M(y_h - rp)}{(1 - b_R)rp}.
\]

Equation 4.2 follows from net supply of zero in asset \( A \), equation 4.3 is the optimality condition stated in Section 3.3.1. The resulting maximization problem and the first order condition can be found in Appendix D.

As it is not possible to solve the FOC for \( p \) analytically, we have to fall back on numerical methods to maximize equation 4.1. We use an algorithm based on golden section search and parabolic interpolation as described in Forsythe, Malcolm and Moler (1976). This algorithm belongs to the group of constrained search algorithms and is well suitable because we know that prices outside the open interval \( ]0; \frac{\nu_0}{\nu'}[ \) cannot be enforced by \( R \).\(^5\) The algorithm converges fast and with any given precision to the inner solution of the maximization problem.

\(^5\) For \( p \to 0 \) the demand of \( M \) rises to infinity, for \( p \to \frac{\nu_0}{\nu'} \) to minus infinity.
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stage of the model. In Figure 4.3 (dashed line, left plot) the price is depicted for \( \tau = \frac{7}{10} \) and as expected, it is below the non-exploiting price \( p_t \).

In the second period the optimal myopic price will be higher than without exploiting rational agents because now the intuition from above will work the other way round.\(^6\) Depending on \( \lambda \) the optimal price will increase in \( \tau \). It also increases directly in \( \lambda \) as the latter determines the extra charge payed for momentum by \( \mathcal{M} \). A plot of second period myopic optimal pricing can also be found in Figure 4.3 (dashed line, right plot).

In Figure 4.4 the corresponding expected utilities are depicted for different \( \lambda \). Note that optimizing in a myopic way increases gains for \( \mathcal{R} \) in period 1, but also decreases gains in period 2, as the momentum effect is weakened when prices are pushed down in the first period. If we define \( \hat{\lambda} \) as the \( \lambda \) where myopic optimization yields the same total expected utility for \( \mathcal{R} \) than non-strategic optimization, then for \( \lambda > \hat{\lambda} \) total utility is actually higher with no optimization than with myopic optimization. Stated differently, if momentum has a very strong impact on the decisions of inexperienced traders, then experienced traders are worse off if they do myopic strategic optimization than when they do not act strategically at all.

In the right plot of Figure 4.4 the region where non-strategic trading would be superior to myopic optimization can clearly be seen. From the plot in the middle it can be seen, that this comes from the relative losses by neglecting the second period momentum effect, which for high \( \lambda \) eventually outweighs the constant additional gains from the first period.

4.4 Dynamic Optimization

If \( \mathcal{R} \) considers the consequences of her action in period 1 on prices in period 2 she additionally has to consider the second effect stated in Section 4.2.2. We then expect prices to be strictly above the myopic maximizing prices for \( \lambda > 0 \), and exactly matching the former for \( \lambda = 0 \).

\(^{6}\)Note that now we have overpricing, thus \( \mathcal{R} \) strategically reduces its short-selling.
Figure 4.3: Prices $p_1$ and $p_2$ for different $\lambda$ with no optimization, myopic and dynamic optimization by rational agents $\mathcal{R}$.

Figure 4.4: Period 1, period 2 and total expected utilities depending on $\lambda$ for no optimization, myopic and dynamic optimization by rational group $\mathcal{R}$.
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Therefore, in period 1 after a positive shock, a dynamically optimizing rational trader who has one more period to live faces the optimization task:

\[
\max_{p_t} \sum_{k=t}^{t+1} EU_R(x_{R_k}, p_k, p_{k-1}, p_{k-2})
\]  \hspace{1cm} (4.4)

s.t.

\[
x_{R_k} = -x_{M_k} = \frac{1}{py_h} \ln \frac{b_{M_k}(y_h - rp_k)}{(1 - b_{M_k})rp_k} \quad k = t, t + 1, \hspace{1cm} (4.5)
\]

\[
b_{M_{t+1}} = \frac{1}{e^{-z_t - \lambda(p_t - p_{t-1})} + 1}. \hspace{1cm} (4.6)
\]

Thus, given \(p_{t-1}\) the rational trader faces a two dimensional maximization problem where he has to choose an optimal triple \((p_t, p_{t+1}, p_{t+2})\). \(p_{t+1}\) and \(p_{t+2}\) are the optimal myopic exploiting prices in period \(t + 1\) for the case that at the end of \(t\) the signal \(s_t\) turns out to have been correct or incorrect.\(^7\) Equation 4.5 again represents the market clearing condition with zero net supply. Equation 4.6 states the anticipated belief of \(M\) in period \(t + 1\) and is important as it determines the demand of non-rational traders in the second period.

In a different notion, in period one (set \(t \equiv 1\)) \(R\) has to choose \(x_{R_1}\) such that her demand induces a price \(\tilde{p}_1\) that satisfies the equation:

\[
\frac{\partial EU_{R_1}(\tilde{p}_1)}{\partial p_1} + \frac{\partial EU_{R_2}(\tilde{p}_1)}{\partial p_1} = 0, \hspace{1cm} (4.7)
\]

The second period marginal ex ante expected utility for a change in \(p_1\) can be found in Appendix D. For the first period marginal expected utility with respect to \(p_1\), the derivative of equation 4.1, as stated for the myopic case in equation D.2 in the appendix, can be used as there is no momentum effect in the first period.

The resulting optimality condition is very complex and cannot be solved in closed form. But again it is feasible to calculate the marginal expected utilities for given parameters and then find the solution of equation 4.7 using numerical solvers. As it turns out (and is clear by intuition) that marginal expected utility of period 1 with

\^Note the rational exploiting agent optimizes differently in period \(t + 1\) dependent on the actual realization of \(z_t\) and that he chooses myopic optimization in \(t + 1\), as this is his last period to live. Also, \(p_2\) is determined by the choice of \(p_1\), thus there is no need to formulate an explicit choice of \(p_2\).
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respect to \( p_1 \) is downward sloping for prices \( p_1 > \hat{p}_1 \),\(^8\) coming from a value of zero for \( p = \hat{p}_1 \), and marginal expected utility of period 2 is also downward sloping for \( p_1 > \hat{p}_1 \) starting from a strictly positive value for \( p = \hat{p}_1 \) and \( \lambda > 0 \).\(^9\) the maximization problem has a unique solution \( \hat{p}_1 \) above the myopic optimum \( \hat{p}_1 \).

Thus, simple incremental increases in \( p_1 \) and a refining of iteration steps close to the solutions leads to a fast and accurate convergence to the unique solution \( \hat{p}_1 \). Optimized prices depending on \( \lambda \) can be found in Figure 4.3, corresponding expected utilities in Figure 4.4 (dotted lines). In the case of no momentum (\( \lambda = 0 \)), \( p_1 \) will be chosen equal to myopic optimization. It will be strictly higher for all positive \( \lambda \). But in contrast to \( \lambda \) vs. \( p_2 \), the relationship of \( \lambda \) and \( p_1 \) is not a monotonic one, as for very high \( \lambda \) additional momentum will yield only small additional returns because \( p_2 \) will then anyway be close to the maximum, such that costs in period 1 will outweigh gains in period 2. However, this can not be seen in Figures 4.3 and 4.4 as the scale of \( \lambda \) is too small there and this effect only takes place for very high \( \lambda \).

Expected utility goes down in period 1 for increasing \( \lambda \) as prices are chosen higher to take advantage of the stronger momentum effect. For the same reasons as above, this relationship is non-monotonic. In contrast, expected utility of period 2 rises monotonically in \( \lambda \) as well as total expected utility. Note that with respect to total expected utility, dynamic optimization is always superior to both myopic and no optimization, as it takes into account both the first period pricing effect and the second period momentum effect and it guaranties that first period relative gains outweigh second period relative losses in any case.

4.4.1 Excess Volatility

In numerical treatments one can relax the requisitions on the starting conditions and simulate optimal decisions for any situation. Thus, similar to Section 4.1, long-term price paths with multiple shocks can be calculated. The question we want to

---

\(^8\)Remember that \( \hat{p}_1 \) is the optimal price if ignoring period 2.

\(^9\)Inducing momentum by rising \( p_1 \) creates utility in period 2 if \( \lambda > 0 \). But the effect gets smaller for high \( p_1 \) as \( \mathcal{M} \) will have falling marginal demand in period 2 due to risk aversion.
state here is whether dynamic optimization will have an influence on the qualitative nature of the price path, especially on excess volatility. It is clear that there is excess volatility in both cases, dynamic and non-strategic trading, but we want to show that it is even increased by dynamic trading. Therefore we randomly chose 20 vectors of fundamental shocks with a length of 80 time periods each, and simulated price paths for both non-strategic and strategic trading for different $\lambda$'s between 0 and 3. This allowed us to compare price paths of both cases within the same external environment. We then checked the standard deviations of the divergence of actual from fundamental price for both cases as a measure of excess volatility $V$:

$$V_N \equiv \sigma(|p_t - p^*_t|) \quad (\text{excess volatility without strategic trading}),$$

$$V_D \equiv \sigma(|\tilde{p}_t - p^*_t|) \quad (\text{excess volatility with dynamic optimization}).$$

The mean excess volatility of the 20 runs is shown in Figure 4.5 for the dynamic and for the non-strategic case. Clearly, excess volatility is much higher for the dynamic optimization case regardless of the momentum intensity. Thus, the introduction of a higher grade of rationality for the sophisticated traders in the presence of other, boundedly rational, traders increases the divergence of prices from their fundamental value.

Obviously, more rationality in this case increases the mispricing effects. This is an interesting finding as it contradicts the first idea that, due to arbitrage, mispricing should be the lower the higher the degree of rationality is in the market. This does not seem to hold in cases where the rational agents even reinforce the non-optimal behavior of other, less rational, actors. The reason in our case is straightforward: As sophisticated traders benefit from higher mispricing, they have an incentive to amplify price deviations by engaging in strategic trading.

Thus, when dealing with heterogeneous agents, one cannot simply assume a "monotone relationship" between the "degree" of rationality and the proximity of the outcome to a rational benchmark.

For both cases, excess volatility is increasing in $\lambda$, suggesting more volatile markets.
if inexperienced traders put a high weight on momentum observations. In a more realistic setting, it should be discriminated between changes in individual demand of the members of a given generation of traders and changes in its population size, as both affect the total demand of that generation. In real markets this would mean that volatility should rise if more "inexperienced" traders are entering the market or if experienced traders start to exploit them more strongly. Volatility thus could reflect not only the frequency of new information but also the composition of the market participants. This could be an interesting point for further empirical research. In our opinion it also fits the anecdotal evidence that both stock markets grew more volatile, and - by easier access to financial markets - investing in stock became more popular among formerly inexperienced investors in the late 1990s.
4.5 Conclusion

In Chapter 4 the basic model of Chapter 3 was expanded to a multi-shock case and to strategic behavior of some market participants. We showed that a more realistic multi-shock environment preserves bubble formation and adds complexity to the price path as bubbles overlap and interfere. This makes the market entrance timing of a trader especially important for overall life-cycle performance.

Introducing strategic behavior of the rational traders led to an even more sophisticated formation of price paths. Although basic features like bubble-formation and over- and underpricing persist in principle, prices are heavily influenced by strategic decisions. Two contradicting effects play a role in this case, namely short and long term gains that can be made by rational and strategically acting agents. The actual price path depends on the power of rational agents to also behave dynamically, as well as on the importance non-rational traders ascribe to momentum observations.

With the help of simulations we showed that excess volatility is always increasing in markets where rational traders interact strategically with bounded rational agents, compared to markets where they have no capabilities or no market power to do so.

4.6 Outlook and Testable Implications

On the theoretical side there are many possible and interesting extensions to the models of Chapters 3 and 4.\textsuperscript{10} For example it would be desirable to endogenize the learning process. From psychological research it is known that positive feedback leads to the enforcement of certain actions. One could include this finding by increasing the impact of momentum observations on the final decision if they resulted in good predictions for several times, and vice versa. Another point would be to allow for entry and exit decisions in the market.

All these extensions would certainly be interesting, but on the other hand all of them bring the danger of intractability of the model with them.

\textsuperscript{10}This Outlook refers to Chapter 3 as well as to Chapter 4.
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Rather, as suggested by the Hong/Stein criteria in Section 3.1.1, one could take a look at testable implications that are made by our model and that go beyond the already observed phenomena. Especially suited for that would be the results of the portfolio dynamics. As there is little empirical research in this area so far, one could test whether the correlation between experience and asset ownership at market peaks and lows holds true. The most severe problem one is facing when addressing this question would be the notion of experience. From a practical point of view, there is hardly any variable that directly observes "experience". One proxy could be portfolio age or the "knowledge statements" customers have to make when opening portfolio accounts ("Depots") at German banks.

Another approach would simply assume newcomers to the market as "inexperienced". Therefore, one could observe the correlation of newly opened accounts and the proximity of prices to a local price peak. In this case, our model would suggest that a high number of account openings (as it is correlated with high demand of inexperienced traders) would indicate that prices will soon reach a peak and vice versa. A stylized hint in that direction is a report on equity ownership in Switzerland by the Swiss Banking Institute at the University of Zürich (Cocca and Volkart 2002). It states that the number of small shareholders drastically increased in Switzerland during the end of the 1990s (during rising prices and close to the price peak in 2000) and fell again during the period from 2000 to 2002 (during falling prices). In fact the number of newcomers developed proportionally to changes of the Swiss Market Index (SMI) in the observed period from 1995 to 2002.

However, a sound econometric check and closer investigation of the stated interrelations is still outstanding and could be the subject of a separate work. There, one could also test a filter strategy that buys assets if the number of newcomers is low and sells short if it is high, and check whether it would beat a simple buy and hold strategy in the long run.
Epilogue

In this dissertation we explored the notion of learning in economics from two different approaches.

The first part presented a model in the field of political economy that uses standard economic learning methodology in a game theoretic context. Learning takes place via information accumulation and all information is processed optimally in a perfect rational way. The findings are then checked in a cross-country empirical study.

The second part introduces a different type of "learning", namely learning as a process of improving decision making strategies. The suggested methodology is applied to a financial markets model using elements from the discipline of behavioral economics. An extension of the model introduces additional rationality to some of the traders in the model and explores more complicated price paths using numerical simulations.

These two approaches are quite different in nature, in particular with respect to the degree of rationality they impose on the interacting agents, and both clearly have their advantages and their downsides. In any case we think that there is still a lot of interesting work to be done in the field of learning and economics. Especially learning in behavioral-economic environments, and more precisely the mentioned notion of learning as a process of improving decision strategies still seems to be very unexplored. We hope that our work is able to add fruitfully to this exciting branch of study and we are curious which way learning theories will take in future economic research.
Appendix

A Technical Appendix of Chapter 1

In the technical appendix we want to vary the model in so far as we replace the uniform output distribution by a normal one, such that \( e \sim N(\mu, \sigma) \). In our opinion this has three advantages:

1. A normally distributed output seems to be more realistic than the quite unnatural edge knifed uniform distribution.

2. Voters’ beliefs can be derived for every outcome \( e \).

3. There are no regions where citizens can infer the type of the incumbent with certainty from observing \( e \), hence there is no perfect learning.

The stability of the economy is then expressed by the standard deviation \( \sigma \) and the drift by \( \mu \). Citizens form beliefs according to equation 1.6 but the probabilities now stem from the Gaussian density function for normally distributed random variables

\[
 f_{l,e}(e) = \frac{1}{\sqrt{2\pi} \sigma} e^{-\frac{(e-\mu)^2}{2\sigma^2}} \quad (A.1)
\]

where \( l \) can be \( \underline{l} \) or \( \bar{l} \) depending on the choice of the incumbent. So there are two possible distributions of economic outcome, depicted in Figure 4.6.

For the equilibrium choice of \( I \) this leads to the two conditions

\[
 \int_{-\infty}^{+\infty} \frac{f_{l,e}(e)(f_{l,e}(e) - f_{\bar{l},e}(e))}{f_{l,e}(e) + f_{\bar{l},e}(e)} \, de > \frac{a_bl}{\delta(u_0 + a_bl)} \quad (A.2)
\]
for $\tilde{e} = \mathcal{E}$ and $\tilde{e} = \mathcal{V}$, respectively, and $f(\cdot)$ being the distributions with the gaussian form of equation A.1.

To proof Proposition 1.4 with normal distributed density functions, one has to compare the left hand sides of equation A.2 for $\tilde{e} = \mathcal{E}$ and $\tilde{e} = \mathcal{V}$. If the left hand side of (A.2) is smaller for $\tilde{e} = \mathcal{V}$ than for $\tilde{e} = \mathcal{E}$, the proof would go through.

Unfortunately we are running into mathematical difficulties at this point, because there does not exist any closed form solution for the integral in equation A.2. Until now we did not even find any qualitative statement on comparison between two integrals of this type with different standard deviation values in the gaussian functions. One way to approach this problem is by numerical integration. At least there exists some quite efficient algorithms for that, and we did many sample calculations which confirmed our guess that Proposition 1.4 holds true even with norm distributed shocks.

But also a proof for the validity of Proposition 1.4 can be given when looking at the behavior of the components in equation A.2:
Appendix

The integral in equation A.2 consists of the citizens' belief $\beta_I(e)$ and the probability $p(x = e|l = l_1)$ that output $e$ occurs when $I$ chose $l_1$ and the standard deviation of shock $s$ is $\bar{c}$. Thus, the integrand in equation A.2 can also be written as

$$\frac{f_{l_1 \bar{c}}(e)}{f_{l_1 \bar{c}}(e) + f_{\bar{l}_1 \bar{c}}(e)} \cdot (f_{l_1 \bar{c}}(e) - f_{\bar{l}_1 \bar{c}}(e)).$$

(A.3)

The first factor of expression (A.3) is citizens' beliefs when observing $e$ and the second factor is the difference between the probabilities that this outcome occurs when the choice is $l_1 = l$ or $l_1 = \bar{l}$.

When looking at Figure 4.6, things get clear quickly:

The first factor is the dotted sigmoid-shaped line at the top, citizens' beliefs $\beta_I(e)$.

The second factor is the difference between the right Gaussian curve ($l_1 = 0$) and the left one ($l_1 = \bar{l}$). Thus expression A.3 gives us the marginal contribution to the integral for every $e$, depicted by the s-shaped light line around the abscissa.

Therefore the integral value we are searching is the integral over this s-shaped function. The areas between the Gaussian curves on the left and on the right of their intersection have the same absolute value, but their contribution to the searched integral is negative on the left side and positive on the right side.

Because the beliefs-curve is upward sloping and always has the value 0.5 for the $e^*$ where the gaussian curves intersect, the integral is positive in any case because positive contributions right of $e^*$ are weighted with an higher $\beta_I(e)$ than negative ones left of $e^*$.

Now comparing for two different $\sigma$ (a small $\sigma$ in the left picture of Figure 4.6 and a bigger one on the right side) yields a "flatter" curve $\beta_I(e)$, leading to a smaller difference between positive and negative contributions to the integral, and therefore to a lower positive integral value.\textsuperscript{11}

This shows us that the condition for the choice of high corruption changes with the

\textsuperscript{11}In the extreme case of $\sigma \to \infty$ the $\beta_I(e)$ curve gets horizontal and the integral value of equation A.2 converges to 0, meaning that there would be no chance for learning in this case. On the other hand, the simplified case with a discrete distribution of $e$ leads to a step function instead of the former sigmoid and opens the possibility for perfect learning.
Appendix

variance of output even in the normally distributed case and therefore a modified Proposition 1.4 holds true for this case as well.

\[ \square \]

B Data used in Chapter 2

Appendix B.1
Descriptive Statistics of Regression Variables

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\( d \) is the average growth rate in percent per year derived from the GDP data of 1981-2000. \( \sigma \) is the standard deviation of GDP per capita changes over the period 1981-2000. \textit{dem95} is the democracy level of Barro and ranges from 0 to 1. \textit{ethno} is the ethnolinguistic fractionalization index calculated by Taylor and Hudson (1972). \textit{gdp/cap} is in thousands of 1990 US$. \textit{gdpabs} is GDP in billions of US$. \textit{cpi03} is the corruption perception index 2003 of Transparency International. \textit{protestant} is the fraction of protestants in the population in percent. \textit{formerUK} and \textit{federal} are dummies for former colonies of the UK respectively federal constitution. \textit{import} is the percentage of GDP spent on the import of goods and services. \textit{absgovwage} is the wage of central government members in 1990 US$. \textit{inv} is the investment level as percentage of GDP and \textit{school} is the percentage of age 15+ population in secondary school from the Barro and Lee dataset on education.
Appendix

Appendix B.2
Data Set of Regression Variables

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### Appendix B.2

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Appendix

C  Appendix of Chapter 3

C.1 Proof of Equation 3.29 - $\Delta b_{it}$

Let $\pi^+$ and $\pi^-$ denote the new $\pi$-values for the case that the received signal $s_t$ was correct and $z$ changed from $z_{t-1} = 0$ to $z_t = s_t$. $\Delta \pi$ be the absolute change in $\pi$ which is equal for an increase and a decrease in $z$ if $z_{t-1}$ was zero. Note that $b_{it-1} = \pi_{t-1}$.

Generation $i$’s belief is then

$$b_{it} = \frac{i-1}{n-1}(\tau\pi^+ + (1-\tau)\pi^-) + \frac{n-i}{n-1}\pi_{t-1}$$  \hspace{1cm} (C.1)

$$= \pi_{t-1} + \frac{i-1}{n-1}\Delta\pi(2\tau - 1).$$  \hspace{1cm} (C.2)

Hence,

$$\Delta b_{it} = b_{it} - \pi_{t-1} = \frac{i-1}{n-1}\Delta\pi(2\tau - 1).$$  \hspace{1cm} (C.3)

□

D  Appendix of Chapter 4

D.1 Optimization problem of myopic exploiting rational agents

The optimization problem is thus\textsuperscript{12}

$$\max_p -b_R e^{-\rho(rW - \frac{1}{\rho_n} \ln(\frac{y_h}{y_p} - 1)(y_h - p))} - (1 - b_R)e^{-\rho(rW + \frac{1}{\rho_n} \ln(\frac{y_h}{y_p} - 1)p)}.$$  \hspace{1cm} (D.1)

The marginal expected utility in period one after stable prices for two periods

\textsuperscript{12}Note that we are in the first period after a shock, and thus $b_M = \frac{1}{2}$.\n
100
Appendix

\[ (p_0 = p_{-1}) \text{ is:} \]

\[
\frac{\partial EU_1(p_1)}{\partial p_1} = \]

\[
b_{R1} \left( \frac{y_h - p_1}{r p_1^2 (\frac{y_h}{r p_1} - 1)} + \frac{1}{y_h} \ln \left( \frac{y_h}{r p_1} - 1 \right) \right) e^{-\rho W} - \frac{1}{y_h} \ln \left( \frac{y_h}{r p_1} - 1 \right) (y_h - p_1)
\]

\[
+ (1 - b_{R1}) \left( - \frac{1}{r p_1 (\frac{y_h}{r p_1} - 1)} + \frac{1}{y_h} \ln \left( \frac{y_h}{r p_1} - 1 \right) \right) e^{-\rho W} + \frac{1}{y_h} \ln \left( \frac{y_h}{r p_1} - 1 \right) p_1, \quad (D.2)
\]

where

\[
b_{R1} = \tau \frac{1}{e^{-s_1} + 1} + (1 - \tau) \frac{1}{e^{s_1} + 1}.
\]

Setting equation D.2 to zero and solving for \( p_1 \) yields the optimal price and thus the optimal demand of the myopic exploiting rational traders.

D.2 Optimization problem of dynamic exploiting rational agents

From an ex ante perspective (uncertainty about first period \( z_1 \)), the second period expected utility dependent on \( p_1 \) is:

\[
EU_2(p_1) = \tau (\tilde{b}_{R2} U^*(s_1, y_h) + (1 - \tilde{b}_{R2}) U^*(s_1, 0))
\]

\[
+ (1 - \tau) (b_{R2} U^*(-s_1, y_h) + (1 - b_{R2}) U^*(-s_1, 0)), \quad (D.3)
\]

where \( U^*(c, d) \) is the myopic maximized utility when \( z_1 = c \) and \( y_2 = y_h \). For example \( U^*(-s_1, y_h) \) is the maximal possible utility of generation \( R \) if it exploits \( \mathcal{M} \), the signal was wrong \( (z_1 = -s_1) \) and the risky asset \( A \) yields high payoff at the end of period 2 \( (y_2 = y_h) \).

Therefore,
\[ \frac{\partial EU_2(p_1)}{\partial p_1} = \frac{r r p_2 (1 - b_{M2}(p_1))}{y h b_{M2}(p_1)(y h - r p_2)} \left( \frac{b_{M2}(p_1)(y h - r p_2)}{(1 - b_{M2}(p_1)) r p_2} + \frac{b_{M2}(p_1)(y h - r p_2) b'_{M2}(p_1)}{(1 - b_{M2}(p_1))^2 r p_2} \right) \]

\[ \cdot \left[ -b_{R2}(y h - p_2) e^{-\rho r W - \frac{1}{y h} \ln \frac{b_{M2}(p_1)(y h - r p_2)}{(1 - b_{M2}(p_1)) r p_2} (y h - p_2)} + (1 - b_{R2}) p_2 e^{-\rho r W + \frac{1}{y h} \ln \frac{b_{M2}(p_1)(y h - r p_2)}{(1 - b_{M2}(p_1)) r p_2} (y h - p_2)} \right] \]

\[ \cdot \left[ b_{R2}(y h - p_2) e^{-\rho r W - \frac{1}{y h} \ln \frac{b_{M2}(p_1)(y h - r p_2)}{(1 - b_{M2}(p_1)) r p_2} (y h - p_2)} + (1 - b_{R2}) p_2 e^{-\rho r W + \frac{1}{y h} \ln \frac{b_{M2}(p_1)(y h - r p_2)}{(1 - b_{M2}(p_1)) r p_2} (y h - p_2)} \right], \]

(D.4)

where

\[ b_{M2}(p_1) \equiv \frac{1}{e^{-s_1 - \lambda (p_1 - p_0)} + 1} \] (\( b_{M2} \) dependent on \( p_1 \) if \( s_1 \) was right),

\[ b_{M2}(p_1) \equiv \frac{1}{e^{+s_1 - \lambda (p_1 - p_0)} + 1} \] (\( b_{M2} \) dependent on \( p_1 \) if \( s_1 \) was wrong),

\[ b'_{M2}(p_1) = \frac{\lambda e^{-s_1 - \lambda (p_1 - p_0)}}{(e^{-s_1 - \lambda (p_1 - p_0)} + 1)^2}, \]

\[ b'_{M2}(p_1) = \frac{\lambda e^{+s_1 - \lambda (p_1 - p_0)}}{(e^{+s_1 - \lambda (p_1 - p_0)} + 1)^2}, \]

\[ \bar{p}_2 \equiv \text{argmax}_{p_2} EU_2(p_2|p_1, s_1 = +\Delta z_1) \] (optimal \( p_2 \) if \( s_1 \) was right),

\[ p_2 \equiv \text{argmax}_{p_2} EU_2(p_2|p_1, s_1 = -\Delta z_1) \] (optimal \( p_2 \) if \( s_1 \) was wrong),

\[ \bar{b}_{R2} \equiv \frac{1}{e^(-s_1) + 1} \] (\( b_{R2} \) if \( s_1 \) was right),

\[ b_{R2} \equiv \frac{1}{e^{(s_1)} + 1} \] (\( b_{R2} \) if \( s_1 \) was wrong).

In the square brackets of the main equation you can find the utilities of the four possible end states again.

Setting equation D.4 to zero and solving for \( p_1 \) yields the optimal price and thus the optimal demand of the dynamic exploiting rational traders.
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