Swampland Conjectures as Generic Predictions of Quantum Gravity

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Zusammenfassung

Im Rahmen des Swampland-Programms soll die Frage beantwortet werden, ob und wie effektive Quantenfeldtheorien UV-vervollständigt werden können, wenn sie an das Gravitationsfeld koppeln. Unser begrenztes Verständnis dieser Problematik spiegelt sich in einem stetig wachsenden Dschungel von Swampland-Vermutungen wider. Diese sollen notwendige Kriterien für solch eine UV-Vervollständigung darstellen und sind aus unserem Verständnis der Thermodynamik schwarzer Löcher, der Holographie und der Stringtheorie motiviert.

Falls der Beweis einer Version dieser Vermutungen in der Stringtheorie gelingt, stellen sie potenziell dramatische Auswirkungen für physikalische Modelle dar, die Phänomene in unserem Universum beschreiben sollen. Zum Beispiel könnten sie "large field"-Inflation, eine kosmologische Konstante, oder nichtverschwindende Massen des Photons sowie des Gravitons ausschließen. Zweck dieser Arbeit ist es, zu einem besseren Verständnis dieser Vermutungen beizutragen, indem wir sie in verschiedenen Bereichen der Stringtheorie testen. Weiterhin decken wir ein komplexes Netzwerk an Verbindungen zwischen den Swampland-Vermutungen auf. Dies deutet auf die Existenz einer tieferen zugrunde liegenden Struktur hin, welche es noch in vollem Umfang aufzudecken gilt.

Eine der vorgeschlagenen Swampland-Vermutungen ist die Distanzvermutung. Diese besagt, dass effektive Feldtheorien nur einen endlichen Gültigkeitsbereich im Skalarfeldraum besitzen, außerhalb dessen unendlich viele Zustände exponentiell leicht werden und die Beschreibung zusammenbrechen lassen. Wir quantifizieren diesen Gültigkeitsbereich im Kontext von Moduliräumen von Calabi-Yau-Kompaktifizierungen mit $\mathcal{N} = 2$ Supersymmetrie und identifizieren die zugehörigen Zustände. Wir behaupten, dass der gleiche Effekt eintritt, wenn wir versuchen die Masse eines Spin-2 Feldes gegen Null gehen zu lassen. Diese Erwartung konkretisieren wir in Form einer Spin-2-Swampland-Vermutung. Schließlich untersuchen wir die Fragestellung, ob die KKLT-Konstruktion von de Sitter-Vakua in der Stringtheorie konsistent ist. Auf diese Art und Weise testen wir eine kürzlich vorgeschlagene de Sitter-Swampland-Vermutung welche besagt, dass der de Sitter-Raum kein Vakuum der Stringtheorie sein kann.

Abstract

The swampland program aims at answering the question of whether and how effective quantum field theories coupled to gravity can be UV-completed. Our limited understanding of the issues that can arise in this process is reflected in a steadily growing zoo of swampland conjectures. These are supposed to be necessary criteria for such a UV-completion and are motivated from our understanding of black hole thermodynamics, holography and string theory.

The swampland conjectures can potentially have dramatic implications for physical models of real-world phenomena if they are proven in string theory. For example, they could rule out large field inflation, a cosmological constant, or non-vanishing masses of the standard model photon and graviton. The purpose of this work is to contribute to a better understanding of these conjectures by testing them in various corners of string theory. Furthermore, we reveal a complicated network of relations between the swampland conjectures. This hints at the existence of a deeper underlying structure, which is yet to be fully uncovered.

One of the suggested swampland conjectures is the distance conjecture. It states that effective field theories have a finite range of validity in scalar field space, after which they necessarily break down due to an infinite tower of states becoming light. We quantify this range and identify the tower of states in the context of moduli spaces of Calabi-Yau compactifications with $\mathcal{N} = 2$ supersymmetry. We claim that an analogous tower of states appears also in the limit where we send the mass of a spin-2 field to zero. We concretize this expectation in form of a spin-2 swampland conjecture. Finally, we investigate the question of whether the KKLT construction of de Sitter vacua in string theory is consistent. In this way, we challenge a recently proposed de Sitter swampland conjecture, which claims that de Sitter space cannot be a vacuum of string theory.

Acknowledgments

This work is inspired by the beauty and perfection of nature's being.

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Contents

Part I

Introduction

Towards a Unified Quantum Theory of Gravity and Particle Physics

String theory, which was originally developed as a theory of hadronic resonances in the strong interactions[1], can be viewed as a framework for achieving the perturbative UV completion of quantum field theories coupled to gravity. It is quite orthogonal to other approaches to quantum gravity that either postulate a non-trivial UV fixed point of the RG flow as in "asymptotic safety" or a fundamental discreteness of the spacetime as for example in "loop quantum gravity" or "causal dynamical triangulations".

The problem of unifying the renormalizable quantum field theory of the standard model with gravity, whose interactions are non-renormalizable and grow in the ultraviolet, is of course highly non-trivial. The ultraviolet divergences that arise in quantum field theory can be traced back to the point-like nature of the fundamental particle excitations – or equivalently to the *locality* of the interactions of quantum fields. It can thus be expected on general grounds that the fundamental description of our universe is to some degree *non-local* and smears out the interaction regions at the Planck scale.

If we want to refrain from including a breaking of the smooth Lorentz invariant structure of the spacetime itself as a fundamental ingredient of the theory, a rather natural way to cure the divergences of quantum field theory is to postulate that the particles we observe are not fundamentally point-like, but higher-dimensional extended objects, or membranes. The plenitude of possible vibrational modes of these branes would explain the different particle species that we observe in nature. The smearing out of the interactions of string-like objects is depicted in figure 0.1.

The idea that strings in particular, (1+1)-dimensional objects, should be the fundamental building blocks of a theory of quantum gravity is not an arbitrary one. Two-dimensional extended objects allow for a maximally ignorant approach to quantum gravity. First of all, in contrast to the case of higher-dimensional membranes, we do not need to worry about gravitational degrees of freedom induced on the world-volume of the string because two-dimensional

[1]The reader is advised to read the introduction of [1] for a review of the history of string theory.

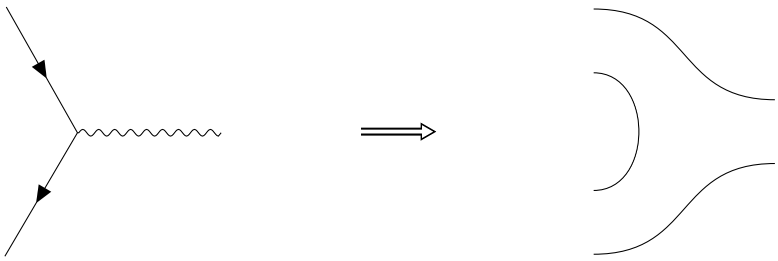

Figure 0.1: The extended nature of the fundamental string smears out the localized interactions of quantum fields.

quantum gravity is topological[2]. If the characteristic length scale ℓ_s of the fundamental string is by orders of magnitude larger than the Planck length ℓ_p, we also do not have to worry about the quantum structure of the spacetime on which the string propagates, as the string only experiences an averaged, smooth spacetime.

The reader might object at this point that we still need to specify the details of how the strings that describe the graviton interact, reintroducing the problem of non-renormalizability and non-predictivity due to the infinite number of Wilson coefficients in front of the higher-dimensional operators. It is a non-trivial miracle of string theory that the consistent interaction of strings is unique up to a single dimensionless parameter, the string coupling constant g_s. This is reflected in the simple pants decomposition of 2D surfaces compared to the infinite number of possible n-valent vertices in Feynman diagrams, see figure 0.2.

The apparent radical simplicity of string theory comes not without what would at first glance be considered a fatal flaw. The theory is found to be ghost-free only in the critical dimension D_{crit}, which is 10 for the superstring. It is in this dimension that the closed and open strings propagate a massless graviton and a gauge boson respectively. In order to make contact with phenomenology in the four-dimensional world that we experience, the extra dimensions have to be compactified on a small "internal" manifold, a generalization of the old idea of Kaluza [2] and Klein [3]. Whereas the fifth dimension of the classic Kaluza-Klein theory can only be a circle or line interval, the number of ways to curl up six extra dimensions is manifold. The physical outcome in four dimensions is encoded in the topology and geometry of the compactification. For example, the requirement of maximal SUSY at energy scales low enough to not probe the extra dimensions leads to the requirement that the compactification geometry should be a particular type of complex manifold, called a *Calabi-Yau*.

[2]This is at least true in the *critical dimension*, as it will be explained in section 1.1

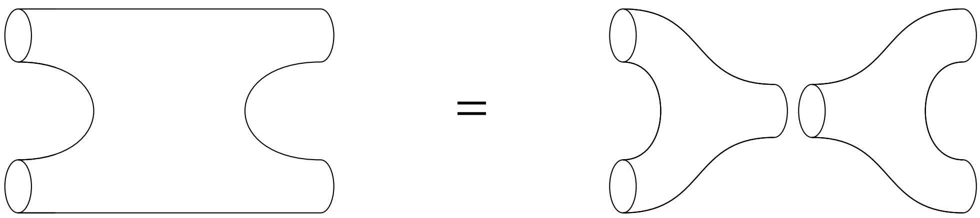

Figure 0.2: Decomposition of an apparent tetravalent closed string vertex into two trivalent ones.

Besides strings as the fundamental degrees of freedom at weak coupling, it was realized by Polchinski [4] that the consistency of the theory also mandates the inclusion of non-perturbative (p+1)-dimensional objects, which are called *Dp-branes*, on which open strings can end. This leads to the idea of brane-worlds [5] and the possibility that we might live on such a D-brane that is extended along the visible four dimensions. D-branes will in general extend, intersect or be localized in the internal dimensions leading to further arbitrariness in the resulting four-dimensional physics. Finally, on top of this, the fields of the ten-dimensional effective supergravity description of the string can have different vacuum expectation values, called *fluxes*, which can have a direct imprint on the energy scales of the resulting four-dimensional description.

This arbitrariness of the resulting four-dimensional effective field theory starting from the two fundamental parameters ℓ_s and g_s in ten dimensions can certainly be viewed as a disappointment if one had hoped for a single possible consistent UV completion of the standard model. A less defeatist perspective is that from the point of view of low energy effective field theory there is an uncountable infinity of possible theories that can accommodate the standard model, a fact that is reflected in the lively activity of the BSM-phenomenology community. String theory improves on this situation by promoting these effective theories to vacua of a single fundamental underlying description, thus making it in principle possible to talk in a well-defined way about the probability distribution of parameters in effective field theories.

The String Theory Landscape

The set of effective field theories produced by string theory has been termed the *string theory landscape*. In contrast to a landscape, we only see a single vacuum with definite values of the constants of nature around us. There are several ways to resolve this dichotomy [6]. One bold idea is that there could be some sort of vacuum selection mechanism that dynamically explains how our universe evolved from one of the simple string vacua in ten or eleven dimensions into a four-dimensional vacuum. Although notable attempts have been made to understand the emergence of four large spacetime dimensions [7], there has not been much progress to this end and it is not clear whether such a mechanism exists at all. Another set of ideas is relevant, especially in the context of the multiverse and eternal inflation [8], where our universe consists of locally expanding bubbles in which the physical laws can be different to the particular bubble we inhabit. On the one hand, as proposed by Steven Weinberg for the particular case of the cosmological constant problem [9], anthropic reasoning can be invoked in order to explain our position in the landscape of vacua. On the other hand, there are certainly many parameters that do not directly affect the existence of observers, such as the QCD theta angle or the tenth decimal place of the fine structure constant. In such cases anthropic reasoning cannot be invoked and we end up only being able to ask questions about the likelihood of our universe, given the distribution of effective field theory parameters in the string landscape [6].

In the last 50 years of string theory research, there have been several shifts of perspective. After realizing that the string naturally unifies gravity and Yang-Mills theories, there was a great hope for its uniqueness. This hope was soon shattered, when it was discovered that there are five different superstring theories even in ten dimensions. First insights into an emerging landscape of semi-realistic models of particle physics were obtained in the heterotic string theory [10] even though explicit realizations were found only more recently [11].

Although it was realized in the second superstring revolution of the 90s that the ten-dimensional string vacua are all connected by *dualities*, which allow one to interpolate from one theory to another by moving through a regime of strong coupling, the situation got worse in the early 2000s when very general classes of $\mathcal{N} = 1$ vacua of the type II strings were first constructed by Giddings, Kachru and Polchinski (GKP) [12]. This *flux landscape* is a class of four-dimensional AdS and Minkowski vacua with minimal $\mathcal{N} = 1$ supersymmetry that achieve a full stabilization of the dynamics of the extra dimensions. The flux landscape is parametrized by a choice of Calabi-Yau geometry and flux integers, which determine the values of the parameters in the low energy effective action. This picture of a flux landscape is illustrated in figure 0.3 for the vacuum expectation value of the string coupling. We currently still do not know much about the size of this landscape, and the possibility that it is countably infinite is not yet ruled out. This is mostly due to the large number of possible Calabi-Yau compactification geometries[3]. Even for a single Calabi-Yau four-fold, a huge number of

[3]One indication for the finiteness of their number is the fact that most known examples are

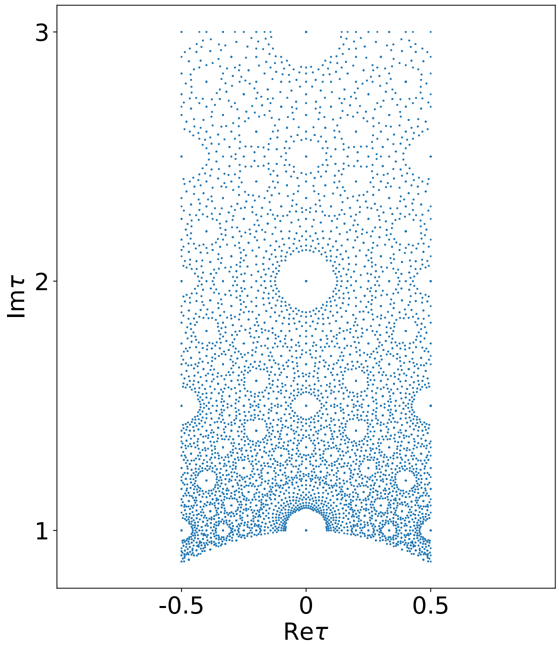

Figure 0.3: Distribution of the axio-dilaton τ (complexified string coupling) in the flux landscape of a rigid Calabi-Yau. Plot created using the methods of [17].

$10^{272,000}$ of so-called F-theory flux vacua were found in recent work [16].

One ingredient that was still missing in these early constructions was a cosmological constant. This was in part due to a widespread belief that the cosmological constant could be zero, a possibility that experiments slowly began to rule out [18, 19]. A model for the possibility of a landscape of vacua with different, closely spaced values of the cosmological constant was proposed by Bousso and Polchinski in [20], suggesting a potential realization of Weinberg's dream. A framework for the construction of such a de Sitter landscape, based on the work of GKP, was introduced by Kachru, Kallosh, Linde and Trivedi in their seminal paper [21]. At the time, this was a breakthrough in the field, leading to the widespread belief that indeed there may be a huge number of string vacua that are exponentially close to our universe. In retrospect, this attitude might have been too optimistic, in particular because there is to date not a single fully explicit realization of these ideas in terms of an actual compactification from 10D. Nonetheless, it seems hard to envision that at least the flux vacua leading to AdS and Minkowski space in four dimensions could be argued away.

elliptically fibered [13, 14]. It is known that there exists only a finite number of elliptically fibered Calabi-Yau manifolds [15]

The landscape idea is here to stay. But what are the implications of the string landscape? Can we construct every quantum field theory as a sufficiently complicated vacuum of string theory? Is the best we can hope for really to speculate about statistical predictions for the low energy parameters? We would like to answer these questions in the next section.

The Swampland – Predictions from String Theory?

In parallel to the development of the string landscape idea, it was emphasized by a group of researchers that the predictive power of string theory is infinitely stronger than that of effective quantum field theory and that string theory implies radically different predictions than the usual reasoning based on the renormalization group and technical naturalness. In a seminal paper of Cumrun Vafa, it was claimed that the set of string theory vacua is in fact only a measure zero subset of the set of all quantum field theories [22]. The complement of the string landscape in the set of quantum field theories was termed the *swampland*. This idea was initially overwhelmed by the landscape set of ideas, the work on which was fueled by the discovery of the flux vacua. As we can see in figure 0.4, it was only in 2014 that the swampland idea got some traction. The situation changed dramatically from that point and as of 2019 work on the swampland is completely dominating work on the landscape. The aim of this chapter is to explain why the swampland is an important concept for our understanding of quantum gravity and also for connecting string theory to observations.

In general, we should distinguish between the *string swampland*, as defined above, and a more general concept of a swampland of effective field theories that cannot arise from any non-perturbatively consistent, UV-complete quantum gravity theory. This general idea is depicted in figure 0.5. If string theory turns out to be the only such theory the two concepts coincide. If this is not true, it is important to keep the distinction and analyze the possible constraints on quantum field theories also from the perspective of what is known about the unification of quantum field theory and gravity on general grounds.

For the sake of exploring the possible consequences of the swampland, let us now assume that string theory is the only consistent quantum theory of gravity and particle physics. Under this premise, it is clear that our universe is part of the landscape rather than of the swampland. The use of the swampland is now that of constraining possible ideas for physics beyond the standard model. For example, we might be interested in a resolution of the hierarchy problem that involves large scalar field displacements [24]. If we determine that theories in which scalar fields traverse distances being larger than order one in Planck units are in the string swampland, we can rule this out and move on to something else.

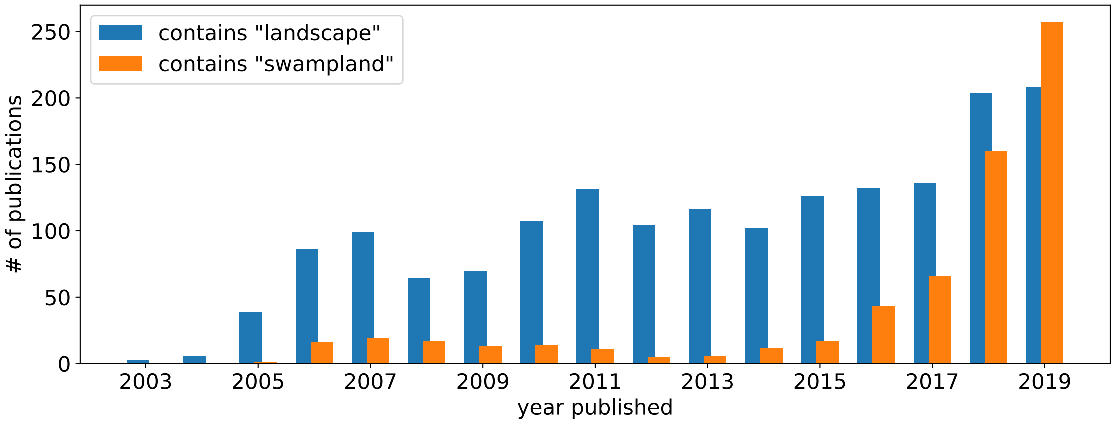

Figure 0.4: Bibliometric analysis of the high-energy physics literature, as indexed by INSPIRE [23], with respect to the occurrence of the words "landscape" and "swampland". The graphic shows the number of publications per year that contain the indicated keywords in the full text.

We can also assume that string theory is one of a few or even many consistent quantum gravity theories. Under this assumption, we can do something even more interesting. If we determine that a certain extension of the standard model is in the string swampland, but still observe it experimentally, we can rule out string theory! In other words, if we manage to understand the concept of the string swampland, we may be able to derive non-trivial predictions from string theory.

In order to understand why the swampland should exist at all, we find it useful to draw an analogy to geometry. It is a well-known general fact in mathematics that objects which live on compact spaces are very constrained. For example, any holomorphic function on a compact complex manifold (such as the two-sphere) is necessarily constant, so the vector space of such functions is $\mathbb{C}$. However, if we drop the compactness assumption, things become much more wild. The space of holomorphic functions on the complex plane is certainly not a finite-dimensional vector space. In the case of string theory compactifications, having non-trivial gravity in the lower-dimensional theory is intimately tied to the compactness of the internal space. From this it should be clear that coupling quantum field theories to gravity should give rise to powerful constraints.

The idea of a vast swampland, formalized in 2005, became very prominent after the year 2014 with the claimed discovery of tensor polarization in the CMB by the BICEP collaboration [25]. It was well-known at the time that such a discovery would have implied large field inflation in the early universe, in which

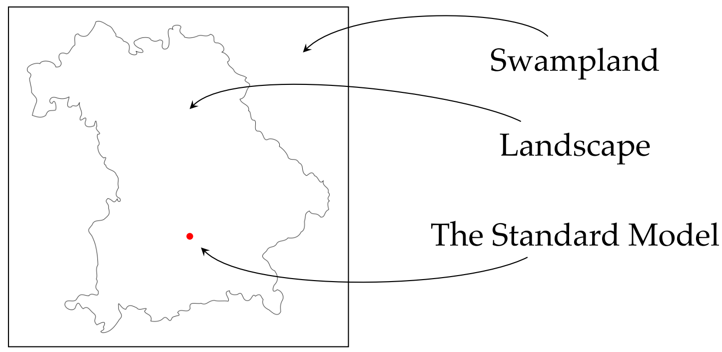

Figure 0.5: The set of all quantum field theories is divided into those which can be coupled to quantum gravity (the landscape) and those for which this is not possible (the swampland). We hope that the standard model of particle physics is part of the string landscape.

the inflaton traverses a super-Planckian distance in the field space [26]. In the end, it was found that the signal could be explained away by dust [27]. This is in a sense a victory for string theory as it was claimed much earlier that these large field displacements should be unnatural and may even be in the swampland [28–30]. Unfortunately, because there was such a widespread trust in the findings of the BICEP collaboration, what happened was that researchers tried to accommodate for large field inflation in string theory, ignoring the previous work that indicated its unnaturalness. After it became accepted that the BICEP results did not properly account for the background the pendulum swung back and arguments against large field inflation in string theory were taken much more seriously again. This sparked the still continuing exponential growth of interest in the swampland. In a process that could be called the *swampland revolution*, many new possible swampland constraints, dubbed *swampland conjectures*, were discovered. As of now, we are still in the process of establishing a more rigorous foundation for these conjectures and of understanding how they fit together in the big picture. Nevertheless, it also seems to be clear that, just as the landscape, the swampland is here to stay and might provide us with both deep insights into the structure of quantum gravity as well as with predictions from string theory.

Overview

In this work we aim to give both an introduction to the swampland in the context of string theory, as well as to summarize and extend the related work of several publications on which this thesis is based. The document is structured as follows. After this introduction, there are two main parts. Part II sets the stage by introducing the concepts that are used in part III, which contains the results of the aforementioned publications as well as some previously unpublished material. Parts II and III are divided into several chapters. After this, part IV summarizes the results obtained in this thesis and provides an outlook on future topics within the context of the swampland program.

Part II consists of several introductory chapters, which culminate in the review of some results of the swampland program in chapter 7. The reader familiar with the concepts introduced here may want to skip directly to part III. Chapter 1 will remind the reader of some basic facts about string theory in ten dimensions, with an emphasis on light cone quantization. In chapter 2, we review how to obtain lower-dimensional effective field theories coupled to gravity from the ten-dimensional string theories using the idea of compactification. In particular, we emphasize differences between compactifications of field theories and string theories. As simple enough compactifications of string theory often have non-trivial scalar moduli spaces, we introduce this concept in chapter 3. Moduli spaces play an important role in our results for the swampland distance conjecture. Stringy moduli spaces are constrained by dualities, which are introduced in chapter 4. Chapters 5 and 6 discuss results on Calabi-Yau compactifications of the type II strings. Chapter 5 explains the relation of these to $\mathcal{N} = 2$ supergravity in four dimensions and moves on to introduce mirror symmetry between the type IIA and type IIB strings. The introduction of orientifold and fluxes is explained in chapter 6, where we also introduce the KKLT scenario for constructing de Sitter vacua in string theory. Finally, we introduce several of the swampland conjectures that have been proposed in recent years in chapter 7.

Part III contains three chapters, each focusing on one of the publications [31–33] that are the basis of thesis. In chapter 8 we explain our work on testing the refined swampland distance conjecture in Calabi-Yau moduli spaces [31]. This chapter crucially relies on chapter 5 of the previous part II, where Calabi-Yau moduli spaces and mirror symmetry were introduced. In chapter 9 we review our work on swampland constraints on massive spin-2 particles [32]. Furthermore, we discuss several technical details that were omitted in this publication. The reader will find it useful to read chapter 2.2 of part II in conjunction with this, as it introduces the notation and conventions. The topic of chapter 10 is our work on the consistency of the KKLT scenario [33]. We focus in particular on the role that very light KK modes could play for the consistency of the effective field theory. We also clarify the consistency of the proposal of emergent kinetic terms under compactification.

This thesis is based on the following publications:

- **"The Refined Swampland Distance Conjecture in Calabi-Yau Moduli Spaces"**

  Ralph Blumenhagen, Daniel Kläwer, Lorenz Schlechter, Florian Wolf

  JHEP 1806 (2018) 052

- **"A Spin-2 Conjecture on the Swampland"**

  Daniel Kläwer, Dieter Lüst, Eran Palti

  Fortsch.Phys. 67 (2019) no.1-2, 1800102

- **"Swampland Variations on a Theme by KKLT"**

  Ralph Blumenhagen, Daniel Kläwer, Lorenz Schlechter

  JHEP 1905 (2019) 152

The following publication also grew out of research during my time at the Max-Planck-Institut für Physik in München:

- **"Machine Learning Line Bundle Cohomologies of Hypersurfaces in Toric Varieties"**

  Daniel Kläwer, Lorenz Schlechter

  Phys.Lett. B789 (2019) 438-443

Part II

Prerequisites

1 String Theory in Ten Dimensions

In this chapter we will introduce the basic concepts needed to understand superstring theory in its most simple background, flat ten-dimensional Minkowski space. We will see that while string theory is an incredibly constraining framework there are actually several different incarnations of the superstring, known as the type I, type IIA, type IIB and the two heterotic string theories. It turns out that all of them are related by so-called dualities, which are highly non-trivial equivalences of two seemingly different quantum theories. We will furthermore hint at the existence of M-theory, which is thought to be a strongly coupled quantum theory of gravity in eleven dimensions involving no strings as fundamental degrees of freedom but rather higher-dimensional extended objects called M-branes. The type IIA string can be thought of as arising from wrapping such an M2-brane on a circular eleventh dimension. A similar mechanism is at work for one of the heterotic string theories.

For understanding several of the swampland conjectures, to be introduced in chapter 7, it will be useful to have some basic knowledge of the zoology of different string theories. We will then focus mostly on the type II strings, as these will be mostly investigated in part III of this thesis. This chapter will lay the foundation for studying compactifications of the string to lower dimensions later on.

1.1 From Two to Ten Dimensions

This section reviews the basics of quantizing a relativistic string. For a more thorough introduction we refer the reader to one of the standard textbooks on the subject [1, 34–38].

In contrast to the second quantized quantum field theory, much progress can be made in string theory by first quantization of strings that are propagating in some ambient space.[1] The bosonic string propagating in D-dimensional Minkowski space is described by a map

$$X^M(\tau,\sigma): \quad \Sigma \to \mathbb{R}^{1,D-1}\ . \tag{1.1}$$

[1]String field theory – the second quantized version of string theory – is not discussed here.

Due to the extended nature of the string, the embedding coordinates $X^M(\tau, \sigma)$ themselves constitute a set of D quantum fields on the world-sheet Σ. Perturbative string theory is to a large extent the study of such two-dimensional world-sheet field theories. It is worth pointing out that with this shift of perspective from the embedding space to the world-sheet, spacetime symmetries become internal symmetries.

The classical dynamics of the string are determined by extremizing its volume. The corresponding action bears the names of Nambu and Goto [39, 40]

$$S_{\rm NG} = -T \int_\Sigma dV = -T \int_\Sigma d^2\sigma \left[-\det\left(\partial_\alpha X^M \partial_\beta X^N \eta_{MN} \right) \right]^{1/2} . \tag{1.2}$$

Here the *tension* of the string is the only dimensionful quantity that enters the theory. We also define several derived quantities – the *Regge slope* $\alpha' = 1/2\pi T$, the *string length* $\ell_s = 2\pi\sqrt{\alpha'}$ and the *string mass scale* $M_s = 1/\sqrt{\alpha'}$.

The action (1.2) is apparently non-polynomial in the fields X^M and hence difficult to quantize. It can be famously cast into the form of a free theory by introducing an auxiliary metric $h_{\alpha\beta}$ on the world-sheet.

$$S_{\rm P} = -\frac{T}{2} \int_\Sigma d^2\sigma \sqrt{-h}\, h^{\alpha\beta} \partial_\alpha X^M \partial_\beta X^N \eta_{MN} . \tag{1.3}$$

The resulting action conventionally bears the name of Polyakov, who used it as a basis for constructing a path integral formalism for the string [41, 42], although it was introduced earlier elsewhere [43–45]. Solving the equations of motion for $h_{\alpha\beta}$, it is identified with the pullback of the spacetime metric η_{MN} to the world-sheet, leading back to the Nambu-Goto action. The Polyakov action (1.3) is in principle easy to quantize although in practice a few subtleties arise from having to impose the constraint

$$\delta_{h_{\alpha\beta}} S_{\rm P} = T_{\alpha\beta} = 0 , \tag{1.4}$$

where $T_{\alpha\beta}$ is the world-sheet energy momentum tensor.

We observe that the action (1.3) has general coordinate invariance and Weyl symmetry as local symmetries

$$\delta_{\xi,\Lambda} X^M = -\mathcal{L}_\xi X^M , \qquad \delta_{\xi,\Lambda} h_{\alpha\beta} = -\mathcal{L}_\xi h_{\alpha\beta} + 2\Lambda h_{\alpha\beta} . \tag{1.5}$$

As a consequence of the Weyl symmetry the trace of the energy-momentum tensor vanishes

$$T_{\alpha\beta} h^{\alpha\beta}\big|_{\rm class.} = 0 , \tag{1.6}$$

a statement that holds for (1.3) off shell but will in general fail to hold at the quantum level.

The gauge invariance allows for a complete gauge fixing of the metric degrees of freedom, so that locally $h_{\alpha\beta} = \eta_{\alpha\beta}$. As the two-dimensional wave equation splits into left-moving and right-moving solutions, it is very convenient to introduce light-cone coordinates on the world-sheet according to $x^{\pm} = \tau \pm \sigma$. We will here only be concerned with closed strings, which satisfy the periodicity condition

$$X^M(\tau, \sigma) = X^M(\tau, \sigma + 2\pi) \ . \qquad (1.7)$$

As it is standard practice in quantum field theory, we expand the scalar fields X^M into modes α^M (right-moving) and $\overline{\alpha}^M$ (left-moving)

$$\partial_- X^M \simeq \sum_{n=-\infty}^{\infty} \alpha_n^M e^{-in\sigma_-} \qquad \partial_+ X^M \simeq \sum_{n=-\infty}^{\infty} \overline{\alpha}_n^M e^{-in\sigma_+} \ . \qquad (1.8)$$

When we canonically quantize the string, the α_n^M with $n < 0$ will act as raising operators on the Fock-vacuum, while the positive ones annihilate it[2]. The modes satisfy the usual harmonic oscillator algebra

$$[\alpha_m^M, \alpha_n^N] = m\delta_{m+n}\eta^{MN} \ . \qquad (1.9)$$

The operator α_0^M turns out to be proportional to the spacetime momentum p^M and commutes with the other α_n^M. Therefore, the Fock vacuum $|0, p\rangle$ carries quantum numbers of the Heisenberg algebra. Equivalent comments apply to the left-moving sector.

The energy momentum tensor (1.4) can also be expanded into modes as

$$L_n = -\frac{1}{2\pi}\int d\sigma e^{-in\sigma} T_{--} = \frac{1}{2}\sum_m \alpha_{n-m}\cdot\alpha_m \to \frac{1}{2}\sum_m :\alpha_{n-m}\cdot\alpha_m: \qquad (1.10)$$

and equally so for the left-movers $T_{++} \to \bar{L}_n$. The colons represent a normal ordering prescription, which has to be adopted in order to make sense out of the quadratic operators L_n when the theory is quantized. For L_0 there is an ambiguity in the normal ordering, which is taken into account by adding an a priori arbitrary *normal ordering constant* $L_0 \to L_0 + a$.

The L_n satisfy the *Virasoro algebra*

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}(m^3 - m)\delta_{m+n} \ . \qquad (1.11)$$

It is a central extension of the algebra of circle diffeomorphisms by the term proportional to c, the *central charge*. One can see that the central extension arises due to an anomaly of the Weyl symmetry at the quantum level, that is, the energy

[2] Due to the reality of the scalar fields X^M we have $\alpha_{-n}^M = (\alpha_n^M)^\dagger$.

momentum tensor (1.4) acquires a trace proportional to c. An explicit computation reveals that each boson X^M contributes with $c = 1$, so the total central charge is $c = D$. Free fermions on the world-sheet would contribute $c = \frac{1}{2}$.

In the so-called *covariant quantization* sketched above, the complicated constraint (1.4) has to be implemented in the form

$$L_n |\phi\rangle = (L_0 + a) |\phi\rangle = 0 , \qquad \forall\, |\phi\rangle \in \mathcal{H}_{\text{phys}} , \qquad \forall n > 0 . \tag{1.12}$$

We will in the following discuss only the conceptually easy route of *light-cone gauge quantization* as this will allow for an easy identification of physical states in section 9.4.2. A very extensive introduction to light-cone gauge quantization can be found in [37]. A more contemporary approach is based on the BRST quantization of the Polyakov path integral [35, 36, 41, 42].

Light-cone gauge is defined in terms of the target space light-cone coordinates $X^{\pm} = (X^0 \pm X^1)/\sqrt{2}$. The idea is to use the world-sheet reparameterization invariance in order to identify the world-sheet time with one of the light-cone coordinate fields X^+ and to solve the constraint (1.4) explicitly for X^- in terms of the transverse fields X^i, where $i = 2, \ldots, D-1$. This is achieved by the identification

$$X^+ = \alpha' p^+ \tau , \qquad \partial_\pm X^- = \frac{1}{\alpha' p^+} \left(\partial_\pm X^i\right)^2 . \tag{1.13}$$

As a result, there is no analogue of the condition (1.12) and all states obtained by applying transverse raising operators to the vacuum are physical

$$\alpha^i_{-n} |p, 0\rangle = \overline{\alpha}^i_{-n} |p, 0\rangle = 0 , \quad \forall n > 0 . \tag{1.14}$$

The masses of the resulting states turn out to be given by

$$m^2 = 4\alpha' (N - a) , \qquad N = \sum_{n=1}^{\infty} n\, \alpha_{-n} \cdot \alpha_n , \tag{1.15}$$

where N is the right-moving number operator. Note that due to the *level matching constraint* $N = \overline{N}$, resulting from invariance under spatial reparameterizations of the world-sheet equation (1.15) is in fact invariant under exchange of the left-moving and right-moving sector. By applying raising operators we obtain states of growing mass in increasingly complex tensor representations of the transverse rotation group $SO(D-2)$. If the mass is not equal to zero, these can be reassembled into representations of the massive little group $SO(D-1)$.

Because the light-cone formalism is not manifestly $SO(1, D-1)$ covariant, the existence of a set of operators satisfying the commutation relations of the Lorentz algebra has to be checked. The non-trivial calculation turns out to be the commutator of two generators in the $(i-)$ plane, which should vanish. It only does so if the spacetime is of the *critical dimension* and the normal ordering constant a has the correct value

$$\left[M^{i-}, M^{j-}\right] = 0 \qquad \Leftrightarrow \qquad D = 26 \quad \& \quad a = -1 \; . \tag{1.16}$$

The critical dimension is related to the central charge and depends on the field content of the world-sheet theory. For the superstring, which includes fermions on the world-sheet in a supersymmetric way, the critical dimension turns out to be $D = 10$.

The Fock vacuum of the world-sheet theory $|0, p\rangle$ is a scalar particle that has a negative mass-squared – a tachyon. While this is certainly unacceptable and expected to lead to a vacuum decay, it is absent in the superstring and we will not be concerned with this problem. The endpoint of the vacuum decay of the closed bosonic string is subject of ongoing research, see for example [46] and references therein.

At the first excited level, which is massless in the critical dimension according to equation (1.15), we find a traceless symmetric tensor G_{ij}, a two-form B_{ij} and a scalar field Φ

$$\begin{aligned} &G_{ij}\left(\alpha_{-1}^{(i}\bar{\alpha}_{-1}^{j)} - \frac{1}{D-2}\delta^{ij}\delta_{kl}\alpha_{-1}^{k}\bar{\alpha}_{-1}^{l}\right)|0,p\rangle \;, \\ &B_{ij}\alpha_{-1}^{[i}\bar{\alpha}_{-1}^{j]}\,|0,p\rangle \;, \qquad\qquad \Phi\,\delta_{ij}\alpha_{-1}^{i}\bar{\alpha}_{-1}^{j}\,|0,p\rangle \;. \end{aligned} \tag{1.17}$$

One of the great miracles of string theory is that it contains a perturbative massless graviton. In the open string sector there is similarly a tachyonic vacuum as well as a massless vector field at the first excited level. In this way string theory unifies gravity with the other fundamental forces.

As we did not second quantize the string, it is an important task to obtain the target space effective action for the massless degrees of freedom (1.17). The propagation of the string on a non-trivial background of the massless fields is described by a so-called *non-linear sigma model*

$$S_{\text{NLSM}} = -\frac{T}{2}\int_{\Sigma} G_{MN}(X)\, dX^M \wedge \star dX^N + B_{MN}(X)\, dX^M \wedge dX^N + \alpha'\Phi(X)\, R \star 1 \;, \tag{1.18}$$

where R is the Ricci scalar on the world-sheet. Thus, spacetime fields become coupling constants on the world-sheet. The free string theory described by the Polyakov action (1.3) can be viewed abstractly as a two-dimensional conformal field theory. Such a conformal field theory has vanishing beta functions, whereas for generic backgrounds G_{MN}, B_{MN}, Φ, the action (1.18) will not define a conformal field theory. The vanishing of the beta functions

$$\beta^{G}_{MN} = \beta^{B}_{MN} = \beta^{\Phi} \stackrel{!}{=} 0 \;, \tag{1.19}$$

is a non-trivial consistency requirement, known as the *string equations of motion* [38]. To leading order, these can be equivalently obtained from the action

$$S_{\rm eff} = \frac{1}{2\kappa_D^2}\int d^DX\sqrt{-G}e^{-2\Phi}\left[R - \frac{1}{12}H_{MNO}H^{MNO} + 4\partial_M\Phi\partial^M\Phi\right] + \mathcal{O}(\alpha')\ , \quad (1.20)$$

where $H = dB$. This concludes our journey from two to ten dimensions for the bosonic string.

We end with some remarks on the superstring. The superstring can be constructed by supersymmetrizing the Polyakov action (1.3). In the so-called superconformal gauge, the theory is described by the world-sheet action

$$S_{\rm RNS} = -\frac{T}{2}\int_\Sigma d^2\sigma\sqrt{-h}\,(\,\partial_\alpha X^M\partial^\alpha X^N + i\bar{\psi}^M\partial\!\!\!/\psi^N)\,\eta_{MN}\ . \quad (1.21)$$

The fields $\psi^M = (\psi_+^M, \psi_-^M)$ are Majorana fermions on the world-sheet but vector fields from the spacetime point of view. The world-sheet fermions can be consistently quantized in a periodic or anti-periodic way leading to the *Ramond* (R) and *Neveu-Schwarz* (NS) sectors respectively. In both sectors the fields ψ^M will contribute additional canonically anti-commuting modes b_r^M that will act as raising and lowering operators on the vacuum. Depending on the sector, different kinds of modes are present

$$\psi_-^M \simeq \sum_r b_r^M e^{-ir\sigma_-}\ , \qquad \begin{cases} r\in\mathbb{Z}+\frac{1}{2} & \text{NS sector} \\ r\in\mathbb{Z} & \text{R sector}\end{cases}\ . \quad (1.22)$$

An analogous expansion applies to the left-moving sector. The total closed string Hilbert space is then obtained by combining the different sectors for the left- and right-moving fermions

$$\mathcal{H} = \mathcal{H}_{NS/NS}\oplus\mathcal{H}_{NS/R}\oplus\mathcal{H}_{R/NS}\oplus\mathcal{H}_{R/R}\ . \quad (1.23)$$

One finds that the R sector ground state is a spacetime spinor because the zero mode oscillators b_0^M form a representation of the Clifford algebra Cliff$(1,9)$. At this point, the spacetime spectrum is not yet supersymmetric. As for the bosonic string, one can use the light-cone gauge quantization to find that the physical states in each sector are obtained from the vacuum by applying transverse raising operators α_{-n}^i and b_{-r}^i with n and r positive.

An important fact is that the NS-sector ground state is again a tachyon. One can show that consistency of the one-loop partition function of the superstring leads to the requirement to impose a so-called *GSO-projection*. This removes the tachyon from the spectrum as a side effect and leads to a target-space supersymmetric spectrum. The spectrum is projected according to the eigenvalues of the world-sheet fermion number operator

$$(-1)^F = (-1)^{\sum_{r\geq 0} b^i_{-r} b^i_r} . \tag{1.24}$$

While it is uniquely fixed to be $(-1)^F = 1$ in the Neveu-Schwarz sector[3], both projections are admissible in the Ramond sector and lead to different chiralities of the massless fermionic ground state. We obtain the two different ten-dimensional string theories with $\mathcal{N} = 2$ spacetime supersymmetry, called the type IIA (non-chiral) and the type IIB (chiral) string.

Let us briefly mention that there also exist string theories in ten dimensions with $\mathcal{N} = 1$ spacetime supersymmetry, namely the heterotic string with gauge groups $E_8 \times E_8$ or $SO(32)$, as well as the type I string with gauge group $SO(32)$.

The following two sections 1.2 and 1.3 will discuss the low-energy physics of the type II superstrings in ten dimensions. The material presented is standard and partially adapted from [47].

1.2 The Type IIA String

The ten-dimensional massless spectrum of the type IIA string is that of the 10D type IIA supergravity multiplet. The graviton G_{MN}, the Kalb-Ramond field B_{MN} and the dilaton Φ of the NS/NS sector are accompanied by two gravitini ψ^I_M and dilatini λ^I of opposite chiralities originating from the NS/R and R/NS sectors. The matching of the bosonic and fermionic degrees of freedom is provided by the inclusion of a one-form field C_1 and a three-form field C_3 from the R/R sector with the field-strengths $F_p = dC_{p-1}$.

The resulting action for the bosonic fields is given by [36]

$$S_{\text{IIA}} = S_{\text{NS/NS}} + S^{\text{IIA}}_{R/R} + S^{\text{IIA}}_{CS} . \tag{1.25}$$

Here the NS/NS sector action is the same as that for the bosonic string (1.20). The Ramond-Ramond and Chern-Simons terms are given by

$$\begin{aligned} S^{\text{IIA}}_{R} &= -\frac{1}{4\kappa^2_{10}} \int d^{10}X \sqrt{-G} \left(|F_2|^2 + |\tilde{F}_4|^2 \right) , \\ S^{\text{IIA}}_{CS} &= -\frac{1}{4\kappa^2_{10}} \int B \wedge F_4 \wedge F_4 , \end{aligned} \tag{1.26}$$

[3]The NS ground state is projected out.

form field	electric source	magnetic source
C_1	D0	D6
C_3	D2	D4
B	F1	NS5

Table 1.1: Brane spectrum of the type IIA string. F1 corresponds to the fundamental string.

where the "improved" four-form field strength is defined by $\tilde{F}_4 = F_4 + C_1 \wedge H_3$ and the 10D gravitational coupling constant is $\kappa_{10}^2 = \ell_s^8/4\pi$ [38].

The (p+1)-form fields couple to so-called Dp-branes, which are (p+1)-dimensional extended objects. The brane spectrum of the type IIA string is summarized in table 1.1. At weak coupling, the branes are heavy non-perturbative objects with masses proportional to $1/g_s$. They serve as Dirichlet boundary conditions for open strings, which have to end on them. Because the ends of open strings carry gauge degrees of freedom, branes are essential ingredients for building realistic string models, see for example [48–50]. The NS5-brane has a special status among the branes, because it is the magnetic dual of the fundamental string. Its mass scales as $1/g_s^2$ and therefore it is heavier than the D-branes in perturbation theory.

The low energy brane excitations of a single brane are described by the *Dirac-Born-Infeld (DBI) action* [36], which is a generalization of the Nambu-Goto action (1.2)

$$S_{\text{DBI}} = -T_p \int_{\Sigma_{p+1}} d^{p+1}\xi e^{-\Phi} \sqrt{-\det(i^*G)} + \mathcal{O}(\alpha') \,, \tag{1.27}$$

where i is the embedding of the brane into spacetime and the omitted terms are associated with the gauge field arising in the open string sector ending on the brane. The *brane tension* is given by $T_p = 2\pi/\ell_s^{p+1}$. The coupling to various p-forms is described by the brane Chern-Simons action [51]

$$S_{\text{CS}} = -T_p \int_{\Sigma_{p+1}} \text{ch}\,(2\pi\alpha'\mathcal{F}) \wedge \sqrt{\frac{\hat{A}(T\Sigma)}{\hat{A}(N\Sigma)}} \wedge \left(\sum_k C_k \right)\Bigg|_{p+1} \,. \tag{1.28}$$

Here the sum is understood to be only over all of the p-forms and their magnetic duals that appear in the type IIA theory. $\hat{A}$ denotes the A-roof genus of the tangent and normal bundles respectively and ch$(\dots)$ is the Chern character.

The D-branes are half-BPS states with respect to the 10D $\mathcal{N} = 2$ SUSY, which means that they preserve 1/2 of the SUSY generators. Furthermore, the $D0 - D4$ and $D2 - D6$ systems preserve the same supersymmetry charges and thus are half-BPS, too.

form field	electric source	magnetic source
C_0	D(-1)	D9
C_2	D1	D5
C_4	D3	D7
B	F1	NS5

Table 1.2: Brane spectrum of the type IIB string. F1 corresponds to the fundamental string. D(-1) is the so-called D-instanton.

1.3 The Type IIB String

Similar to the type IIA string, the ten-dimensional massless spectrum of the type IIB string is that of the 10D type IIB supergravity multiplet. Since the GSO projection acts the same way in the NS/NS sector, its spectrum is not altered. The first difference arises in the NS/R and R/NS sectors, where the gravitini and dilatini now have the same chirality. Because of this, other p-forms arise in the R/R sector. The type IIB supergravity contains a zero-form C_0, a two-form C_2 and a four-form C_4, with the associated field strengths F_{p+1}.

The bosonic part of the action is given by [36]

$$S_{\text{IIB}} = S_{\text{NS/NS}} + S^{\text{IIB}}_{\text{R/R}} + S^{\text{IIB}}_{\text{CS}} \,. \tag{1.29}$$

The NS/NS sector action is again the same as for the bosonic string (1.20), but the Ramond-Ramond and Chern-Simons terms differ and are given by

$$\begin{aligned} S^{\text{IIB}}_{R} &= -\frac{1}{4\kappa_{10}^2}\int d^{10}X\sqrt{-G}\left(|F_1|^2 + |\tilde{F}_3|^2 + \frac{1}{2}|\tilde{F}_5|^2\right) \,, \\ S^{\text{IIB}}_{CS} &= -\frac{1}{4\kappa_{10}^2}\int C_4 \wedge H_3 \wedge F_3 \,, \end{aligned} \tag{1.30}$$

with $\tilde{F}_3 = F_3 - C_0 \wedge H_3$, $\tilde{F}_5 = F_5 - \frac{1}{2}C_2 \wedge H_3 + \frac{1}{2}B \wedge F_3$ and the supplementary condition that $\star\tilde{F}_5 = \tilde{F}_5$. This self-duality condition for F_5 has to be imposed as a constraint together with the equations of motion resulting from (1.29).

The brane spectrum of the type IIB string is described in table 1.2. The brane effective actions are the same as for type IIA, the only difference being the particular p-forms appearing in the summation over k in (1.28). All of the type IIB branes are again half-BPS and so are the $D1 - D5 - D9$ and $D3 - D7$ systems.

2 Compactification

Establishing contact with our four-dimensional reality requires a mechanism that renders six of the ten dimensions in which the superstring propagates unobservable to our current experiments. A popular idea is to place the theory on a background that is a product manifold

$$\mathcal{M}_{1,3} \times \mathcal{M}_6 \,, \tag{2.1}$$

where $\mathcal{M}_6$ is a "small" compact six-dimensional manifold and $\mathcal{M}_{1,3}$ is a non-compact, maximally symmetric spacetime such as Minkowski or (A)dS space. The general idea of such a *compactification* goes back to the work of Kaluza [2] and Klein [3], who wanted to embed our four-dimensional universe into a five-dimensional one in order to unify electromagnetism and gravity. In string theory the situation is opposite in the sense that we already have a unified theory in ten dimensions. Here the compactification process is a mere necessity.

The compactness assumption is generically required for obtaining a finite value of the Planck mass in four dimensions, which we will see explicitly in section 2.1. Nevertheless, the study of string theory on non-compact manifolds, which can be considered a local approximation of a compact geometry, has been used intensively to construct lower-dimensional non gravitational field theories in a process known as geometric engineering [52]. A deeper understanding of gauge theories and the discovery of new dualities, which will be discussed in chapter 4, can often be achieved by taking their string theory origin seriously. In this thesis we will be primarily concerned with compact geometries because we care about constraints arising from the coupling to gravity. These are effects that can only be visible for a finite value of the Planck mass M_p.

In order to define a compactification, we have to specify the Riemannian structure of the manifold (2.1). The most general Ansatz that is compatible with all isometries of the four-dimensional theory is given by the *warped product*

$$G_{MN} dX^M dX^N = e^{2A(y)} g_{\mu\nu} dx^\mu dx^\nu + e^{-2A(y)} g_{mn} dy^m dy^n \,, \tag{2.2}$$

where the *warp factor* $e^{A(y)}$ only depends on the internal coordinates y^m.

In section 2.1 we will first review the standard Kaluza-Klein circle compactification of five-dimensional gravity. We will then explore some of the features of compactifications on higher-dimensional manifolds in section 2.2. Finally, we will discuss some qualitatively new features that arise when we compactify extended objects such as strings rather than point-like particles in section 2.3.

2.1 Kaluza-Klein Theory on the Circle and the Line Interval

The most simple compactification to four dimensions that includes gravity is that of the five-dimensional Einstein-Hilbert action

$$S = M_5^3 \int d^5X \sqrt{-G} R(G) \, , \tag{2.3}$$

on a one-dimensional manifold M_1. There are only two distinct topologies to choose from, namely the circle S^1 and the line interval $I = S^1/\mathbb{Z}_2$, where the circle is parameterized by $y \simeq y + 2\pi R$ and the orbifold $\mathbb{Z}_2$ acts by $y \to \pi R - y$.

For simplicity we will assume an unwarped background

$$G_{MN} dX^M dX^N = g_{\mu\nu} dx^\mu dx^\nu + dy^2 \tag{2.4}$$

and work on the circular topology. Here we have absorbed a possible non-trivial $g_{yy}(y)$ into the definition of the coordinate y. More general warped backgrounds of five-dimensional theories were famously investigated by Randall and Sundrum in [53, 54] but will not be discussed further here. In the unwarped case the five-dimensional Ricci scalar simply evaluates to the four-dimensional one, because the circle is flat. The resulting 4D action is

$$S = M_5^3 \int_{S^1} dy \int d^4x \sqrt{-g} R(g) = \underbrace{2\pi R M_5^3}_{M_4^2} \int d^4x \sqrt{-g} R(g) \, . \tag{2.5}$$

From this equation we see that finiteness of the four-dimensional Planck mass depends on the finiteness of the compactification volume

$$M_4^2 = M_5^3 \mathrm{Vol}(S^1) \, . \tag{2.6}$$

Consider now a 5D minimally coupled free real scalar field φ on the background (2.4) with Fourier expansion

$$\varphi(x, y) = \frac{1}{\sqrt{2\pi R}} \sum_n \varphi^{(n)}(x) e^{iny/R} \, . \tag{2.7}$$

Its five-dimensional action decomposes into

$$\begin{aligned} S_{\text{scalar}} &= -\frac{1}{2}\int d^5X\sqrt{-G}\left(\partial_M\varphi\partial^M\varphi + m^2\varphi^2\right) \\ &= -\frac{1}{2}\sum_n\int d^4x\sqrt{-g}\left(\partial_\mu\varphi^{(-n)}\partial^\mu\varphi^{(n)} + \left(\frac{n^2}{R^2}+m^2\right)\varphi^{(-n)}\varphi^{(n)}\right)\,, \end{aligned} \tag{2.8}$$

where $\varphi^{(-n)} = (\varphi^{(n)})^\dagger$. The spectrum of the 4D theory consists of a zero-mode, which can be massless if $m = 0$, as well as of an infinite tower of Kaluza-Klein excitations with mass gap

$$\Delta m_{\text{KK}} = \frac{1}{R} = \frac{M_4}{r^{3/2}}\,, \qquad r = RM_5 \equiv R/\ell_5\,, \tag{2.9}$$

where r is the circle radius in 5D Planck units.

A new phenomenon occurs for fields that have Lorentz indices in 5D. For example, in the case of a vector field V_M we get a scalar field in 4D if the index is aligned with the y-direction

$$V_M(x,y) = \begin{pmatrix} V_\mu(x,y) \\ V_y(x,y) \end{pmatrix} \equiv \begin{pmatrix} V_\mu(x,y) \\ v(x,y) \end{pmatrix}\,. \tag{2.10}$$

The lower-dimensional spectrum thus consists of a massless vector plus scalar, as well as an infinite tower of Kaluza-Klein excitations of them.

Let us also consider the decomposition of the 5D metric around this background. To this end we parameterize a general 5D metric as

$$G_{MN} = \phi^{-1/3}\begin{pmatrix} g_{\mu\nu}+\phi A_\mu A_\nu & \phi A_\mu \\ \phi A_\nu & \phi \end{pmatrix} \qquad G^{MN} = \phi^{1/3}\begin{pmatrix} g^{\mu\nu} & -A^\mu \\ -A^\nu & g_{\alpha\beta}A^\alpha A^\beta + \frac{1}{\phi} \end{pmatrix}\,. \tag{2.11}$$

Here $g^{\mu\nu}$ is the inverse of $g_{\mu\nu}$ and by definition we raise and lower 4D indices with g. One can easily check that G_{MN} and G^{MN} are in fact inverse to each other. The prefactor of $\phi^{-1/3}$ ensures that the resulting 4D action will be expressed in the so-called *Einstein frame*, with no ϕ-dependence in front of the 4D Einstein-Hilbert term. The metric determinant is

$$\sqrt{-G} = \phi^{-1/3}\sqrt{-g}\,. \tag{2.12}$$

The fields $A_\mu(x,y)$ and $\phi(x,y)$ in equation (2.11) are considered small fluctuations around the background (2.4). The Ansatz (2.11) can be Fourier expanded as

$$\begin{aligned} g_{\mu\nu}(x,y) &= \sum_n g_{\mu\nu}^{(n)}(x)e^{iny/R}\,, \qquad A_\mu(x,y) = \sum_n A_\mu^{(n)}(x)e^{iny/R}\,, \\ \phi(x,y) &= \sum_n \phi^{(n)}(x)e^{iny/R}\,. \end{aligned} \tag{2.13}$$

A classic and well-known result, see for example [55], is the action for the zero-modes $g^{(0)}$, $A^{(0)}$ and $\phi^{(0)}$

$$\begin{aligned} S &= M_4^2 \int d^4x \sqrt{-g^{(0)}} \left(R(g^{(0)}) - \frac{1}{6} \frac{\partial_\mu \phi^{(0)} \partial^\mu \phi^{(0)}}{(\phi^{(0)})^2} - \frac{1}{4} \phi_0 (F_{\mu\nu}^{(0)})^2 \right) \\ &= \int d^4x \sqrt{-g^{(0)}} \left(M_4^2 R(g^{(0)}) - \frac{1}{2} \partial_\mu \Theta \partial^\mu \Theta - \frac{1}{4 g_{\text{KK}}^2} (F_{\mu\nu}^{(0)})^2 \right) \end{aligned} , \tag{2.14}$$

where

$$\Theta = \frac{M_4}{\sqrt{3}} \log \left(\phi^{(0)} \right) \qquad g_{\text{KK}} = e^{-\frac{\sqrt{3}}{2M_4} \Theta} . \tag{2.15}$$

The field Θ has an interpretation as the proper distance on scalar field space, where the metric is determined by its kinetic term. We will discuss this further in the context of the swampland conjectures in section 7.

The Kaluza-Klein particles are charged under the Kaluza-Klein gauge field $F^{(0)} = dA^{(0)}$

$$\int d^5X \sqrt{-G} G^{MN} \partial_M \varphi^{(-n)} \partial_N \varphi^{(n)} \supset \int d^4x \sqrt{-g} \left(g^{\mu\nu} D_\mu \varphi^{(-n)} D_\nu \varphi^{(n)} \right) , \tag{2.16}$$

where the covariant derivative is defined as

$$D_\mu \varphi^{(n)} = \partial_\mu \varphi^{(n)} - i \frac{n}{R} A_\mu^{(0)} \varphi^{(n)} . \tag{2.17}$$

The corresponding gauge symmetry descends from the invariance under 5D diffeomorphisms

$$y \to y + \chi(x) \qquad A^{(0)} + dy \to A^{(0)} + d\chi + dy \quad \Rightarrow \quad \delta A^{(0)} = d\chi . \tag{2.18}$$

In section 9.4.1 it will become important to also keep track of the higher Kaluza-Klein modes of the metric in equation (2.13). Their interactions are constrained by the $5D$ diffeomorphism invariance, which descends into an infinite-dimensional Kac-Moody symmetry algebra in 4D [55].

Finally, let us mention that if we decide to compactify on the orbifold $S^1/\mathbb{Z}_2$, the spectrum is truncated to the states that are invariant under the $\mathbb{Z}_2$ identification. In particular for the vector components of the metric $A_\mu^{(n)}$ the even fields with $n \equiv 0 \bmod 2$ are projected out, while the odd ones survive.

2.2 Kaluza-Klein Theory on General Manifolds

We will now generalize the setup of section 2.1 to compactifications on higher-dimensional manifolds[1]. We gained first insights into what kind of general fea-

[1] A rather explicit review of simple Kaluza-Klein compactifications of the gravity sector can be found also in [56].

tures will emerge there. First of all, the higher-dimensional fields descend into a zero mode and an infinite tower of massive Kaluza-Klein replica in the lower dimensions. Second, fields that carry non-trivial Lorentz representations can produce fields in other representations by aligning some of their tensor indices with the compact directions.

Here we will mostly be concerned with the zero-mode sector because the massless fields determine the low energy effective field theory in 4D. Furthermore, we restrict the study to massless fields in $D > 4$ dimensions because we have seen that higher-dimensional mass terms carry through into the lower-dimensional theory. For now we restrict to trivial warping. Consider a massless p-form field C_p with field strength F_{p+1}. This will have an equation of motion of the form

$$\Delta_D C_p = (dd^\dagger + d^\dagger d)C_p = 0 \,. \tag{2.19}$$

The p-form Laplacian splits into a sum $\Delta_D = \Delta_4 + \Delta_{D-4}$ in a compactification of the form (2.2). We can then use a product Ansatz $C_p = \tilde{C}_q \wedge \omega_{p-q}$ and obtain

$$(\Delta_4 \tilde{C}_q) \wedge \omega_{p-q} + \tilde{C}_q \wedge (\Delta_{D-4}\omega_{p-q}) \,. \tag{2.20}$$

If furthermore we choose ω_{p-q} to be an Eigenfunction of Δ_{D-4} with eigenvalue λ, we find

$$(\Delta_4 + \lambda)\tilde{C}_q = 0 \,. \tag{2.21}$$

In order to be left with a massless field $\tilde{C}$ in 4D, λ has to vanish and hence we are led to the requirement that ω_{p-q} is *harmonic*.

Harmonic differential forms are in one-to-one correspondence with de Rham cohomology classes as a result of Hodge theory

$$\mathcal{H}^k_\Delta(\mathcal{M}_{D-4}) = H^k_{\mathrm{dR}}(\mathcal{M}_{D-4}) \,, \tag{2.22}$$

where $\mathcal{H}^k_\Delta$ is the space of Δ-harmonic k-forms. We come to the conclusion:

> Massless fields in a (string) compactification are in correspondence with generators of cohomology groups.

Massless scalar fields in particular, such as the radion ϕ_0 in equation (2.14) are known as moduli and will be discussed in chapter 3. The question of their existence is of great importance for the phenomenology of the lower-dimensional theory as they mediate long range forces.

2.3 Stringy Ingredients

One crucial difference that arises when we compactify string theory compared to an ordinary quantum field theory is the appearance of winding modes. Consider compactifying any of the 10D superstring theories from chapter 1 on a circle. Not only can the strings have Kaluza-Klein momentum along the compact circle, which we without loss of generality take to be the X^9 direction, but they can also wind

$$X^9(\tau, \sigma + 2\pi) = X^9(\tau, \sigma) + 2\pi R w \,, \tag{2.23}$$

where w is the integer *winding number*. The mass of a string that has KK momentum n and winding number w is [35]

$$\begin{aligned} m^2 &= \frac{n^2}{R^2} + \frac{w^2 R^2}{\alpha'^2} + \frac{2}{\alpha'}\left(N + \overline{N} - 2\right) \,, \\ 0 &= nw + N - \overline{N} \,. \end{aligned} \tag{2.24}$$

The first summand can be recognized as the ordinary Kaluza-Klein mass term, whereas the second term has an interpretation as the string tension times the radius. The last term is the usual oscillator mass term with an altered level matching constraint.

We have discussed in section 1.2 that string theory contains Dp-branes. In order not to break any 4D isometries, these branes have to extend along all of the four non-compact directions. In addition, depending on their dimensionality they will also extend along some of the compact directions. For stability reasons they will then wrap cycles in the geometry that are non-trivial in homology. For example in type IIA string theory, one typically considers D6-branes that wrap three-cycles in an internal geometry, see figure 2.1. Two such D6-branes generically intersect in a point of the compact space.

Another ingredient that is not stringy per se are fluxes. As in electrodynamics, magnetic fluxes of (p-1)-form gauge fields C_{p-1} can thread p-cycles Σ_p in the compact geometry. In string theory, all charges are quantized and so are the fluxes [57][2]

$$\frac{1}{(2\pi\alpha'^{1/2})^{p-1}} \int_{\Sigma_p} F_p \in \mathbb{Z} \,. \tag{2.25}$$

Due to the p-form kinetic terms, fluxes provide an important source of potential energy for other fields in the theory. They can lead to the stabilization of moduli fields, which will be discussed in sections 6.2, 6.3 and 6.4.

[2] In massive type IIA theory, there is a subtlety with equation (2.25), see [57].

	X^0	X^1	X^2	X^3	X^4	X^5	X^6	X^7	X^8	X^9
D6:	✗	✗	✗	✗				✗	✗	✗

Figure 2.1: A single D6-brane extended along the four macroscopic dimensions will wrap a three-cycle in the internal geometry.

Frame Conventions

The 10D SUGRA actions that describe the massless sector of the superstring arise naturally in the so-called *string frame*, where the Ricci scalar is multiplied by the dilaton

$$S \supset \frac{2\pi}{\ell_s^8} \int d^{10}X \sqrt{-G} e^{-2\Phi} R(G) \ . \tag{2.26}$$

We define the *Einstein frame* by diagonalizing the kinetic terms via the Weyl transformation $G = G_E \cdot \exp(\Phi/2)$

$$S \supset \frac{2\pi}{\ell_s^8} \int d^{10}X \sqrt{-G_E} R(G_E) + \dots \ . \tag{2.27}$$

It is also common practice to first extract the expectation value of the dilaton $\exp(\langle\Phi\rangle) = g_s$, then to absorb it into the definition of the 10D gravitational coupling, and finally to perform the Weyl transformation $G = G_{\tilde{E}} \exp(\Phi/2)$ such that

$$S \supset \frac{2\pi}{\ell_s^8 g_s^2} \int d^{10}X \sqrt{-G_{\tilde{E}}} R(G_{\tilde{E}}) + \dots \ . \tag{2.28}$$

This is also called the *modified Einstein frame*.

3 Moduli Spaces

Moduli such as the radion of Kaluza-Klein theory of section 2.1 arise in (string) compactifications as shape deformations of the compact geometry that leave the Einstein equations fulfilled. The term *modulus* refers to any direction in the scalar field space of a theory with a flat potential. In general, classical moduli of a theory will not be moduli of the quantum theory due to quantum corrections generating an effective potential. This can be avoided if there is a powerful symmetry such as a continuous shift symmetry or supersymmetry prohibiting a potential. Indeed, this is the case in string theory – string compactifications will often have exactly flat directions before supersymmetry breaking. The collection of all flat deformations of a given theory, called a *moduli space*, will often form a manifold and will be equipped with additional structure such as a metric and connection, which will then determine couplings in the action. We distinguish between the classical moduli space of a theory and its quantum moduli space.

Since massless scalar fields can mediate long-range interactions, so-called fifth forces, which could be observed as deviations from GR in experiments, there are stringent experimental bounds on the existence of such exact moduli[1]. The typically used compactification geometries are quite complicated and have many allowed massless deformations. The process of engineering them to be massive is called *moduli stabilization*. A phenomenologically interesting string compactification will often involve several different mechanisms in order to achieve the stabilization of all moduli.

In this chapter we will give a few simple toy examples of supersymmetric gauge theories with non-trivial quantum moduli spaces. We will then see how moduli spaces arise in string theory and its compactifications.

3.1 Moduli in Supersymmetric Field Theory

Moduli spaces are omnipresent in supersymmetric gauge theories because of SUSY non-renormalization theorems. Let us first have a look at the structure of the general theory of a chiral multiplet and vector multiplet in $\mathcal{N} = 1$ SUSY. A chiral multiplet contains a scalar Z, a chiral spinor χ and an auxiliary field F.

[1]See for example [58] and references therein.

The bosonic part of the action for N such fields Z^α with $\alpha = 1, \ldots, N$ is [59]

$$S = \frac{1}{2}\int d^4x \left(-g_{\alpha\bar{\beta}}\partial_\mu Z^\alpha \partial^\mu \bar{Z}^{\bar{\beta}} + g_{\alpha\bar{\beta}}F^\alpha \bar{F}^{\bar{\beta}} + \frac{\partial W}{\partial Z^\alpha}F^\alpha + \text{h.c.} \right) , \tag{3.1}$$

where

$$g_{\alpha\bar{\beta}} = \partial_\alpha \partial_{\bar{\beta}} K(Z, \bar{Z}) . \tag{3.2}$$

The Kähler potential $(Z, \bar{Z})$ determines the *Kähler metric* $g_{\alpha\bar{\beta}}$ and gives the scalar field space the structure of a complex Kähler manifold. The coupling to the fermions χ^α is determined by the geometry of this manifold. For example, there are four-fermion interactions proportional to the Riemann tensor associated to $g_{\alpha\bar{\beta}}$ [59].

Integrating out the auxiliary fields F^α, we obtain an *F-term* scalar potential for the Z^α

$$V_F = g_{\alpha\bar{\beta}}F^\alpha \bar{F}^{\bar{\beta}} = g^{\alpha\bar{\beta}} \frac{\partial W}{\partial Z^\alpha} \frac{\partial \overline{W}}{\partial \bar{Z}^{\bar{\beta}}} . \tag{3.3}$$

It is determined by both the Kähler potential and the *superpotential* $W(Z)$. An important property of the superpotential is its holomorphicity as a function of the scalar fields Z^α. Due to this fact it is not renormalized in perturbation theory and receives only non-perturbative corrections. This is not true for the scalar potential, as the Kähler potential will in general receive corrections at every order in perturbation theory.

The other fundamental multiplet of 4D $\mathcal{N} = 1$ SUSY is the gauge multiplet, which in Wess-Zumino gauge contains an auxiliary scalar field D, gaugino λ and gauge field $F_{\mu\nu}$. The bosonic action for M such abelian gauge multiplets with $A, B = 1, \ldots, M$ is [59]

$$\int d^4x \left(-\frac{1}{4}\text{Re}(f_{AB})F^A_{\mu\nu}F^{\mu\nu B} - \frac{1}{8}\text{Im}(f_{AB})F^A_{\mu\nu}\tilde{F}^{\mu\nu B} - \frac{1}{2}\text{Re}(f_{AB})D^A D^B \right) . \tag{3.4}$$

Here $f_{AB}(Z)$ encodes the gauge couplings and θ-angles of the gauge theory and can depend on the chiral multiplets in the theory. Just as the superpotential it is a holomorphic quantity. If the Kähler manifold spanned by the chiral multiplets in the theory has isometries, they can be gauged so that the chiral multiplets become charged under the gauge multiplets. All of this can be generalized to non-abelian gauge groups [59].

In $\mathcal{N} = 2$ supersymmetry, the vector multiplet consists of the $\mathcal{N} = 1$ chiral and vector multiplets such that there is an $SU(2)$ rotational invariance in the fermion sector. Because of this, the Kähler potential of the chiral multiplet is related to the gauge kinetic terms and hence both are determined in terms of a single holomorphic quantity, the *prepotential* $F(X)$. The bosonic part of the action of an $\mathcal{N} = 2$ gauge multiplet is given by [59]

$$S = i \int d^4x F_{IJ} \left(D_\mu X^I D^\mu \bar{X}^J + \frac{1}{4} F^{I-}_{\mu\nu} F^{\mu\nu J-} \right) + \text{h.c.} \,, \tag{3.5}$$

where $F_I, F_{IJ}, \ldots$ denote the derivatives of the prepotential and the $F^{I-}_{\mu\nu}$ are the anti self-dual parts of the field strengths[2]. The resulting geometry of the scalar field space is called *(rigid) special Kähler geometry* and has a Kähler potential

$$K = iX^I \bar{F}_I - i\bar{X}^I F_I \,. \tag{3.6}$$

$\mathcal{N} \geq 2$ SUSY does not allow for a superpotential and hence the only allowed source of a scalar potential are the D-terms which are further constrained by gauge invariance. For this reason, while moduli spaces can still be considered accidental in $\mathcal{N} = 1$ SUSY, they are ubiquitous in $\mathcal{N} \geq 2$. As a very simple example, let us consider the unique 4D $\mathcal{N} = 4$ supersymmetric gauge theory

$$S^{4D}_{\mathcal{N}=4} = \frac{1}{g^2} \text{tr} \int d^4x \left(-\frac{1}{4} F^2 + \frac{g^2 \theta}{8\pi^2} F\tilde{F} - \sum_i (D\phi^i)^2 + \sum_{i,j} \frac{1}{2} \left[\phi^i, \phi^j\right]^2 + \text{fermions} \right) . \tag{3.7}$$

Here the scalar fields ϕ^i with $i = 1, \ldots, 6$ are in the adjoint representation of the gauge group. The D-term potential

$$V = -\sum_{i,j} \frac{1}{2} \left[\phi^i, \phi^j\right]^2 \stackrel{!}{=} 0 \tag{3.8}$$

is fixed by gauge invariance and supersymmetry. It determines a classical moduli space of vacua, which is identical to the quantum moduli space because the potential is not renormalized. After taking into account gauge invariance by modding out adjoint orbits, the flat directions of (3.8) are given by six real parameters per Cartan generator [60]

$$\mathcal{M} = \mathbb{R}^{6r}/S_r \,, \tag{3.9}$$

where the discrete quotient arises because VEVs along the different Cartan generators are physically equivalent.

At a generic point of the moduli space, the gauge group is broken and the theory is in a Coulomb phase, whereas at the orbifold-singular point $\phi^i = 0$ of the moduli space the W-bosons become massless and the full non-abelian gauge symmetry is restored. This is in fact a general lesson:

[2] The two quantities should not be confused and we will distinguish them by including the spacetime indices of the field strengths when needed.

Singular loci in moduli space correspond to the appearance of new massless states in the theory.

The 4D $\mathcal{N} = 4$ theory is also an instructive example to see how moduli spaces arise in compactifications. The theory can be obtained as a toroidal T^6 compactification of the unique $\mathcal{N} = 1$ gauge theory in 10D

$$S^{10D}_{\mathcal{N}=1} = -\mathrm{tr} \int d^{10}X \left(\frac{1}{4g^2} F^2 + i\overline{\psi}\not{D}\psi \right) . \tag{3.10}$$

The scalar fields ϕ^i are simply the components of the 10D gauge field with index aligned along the toroidal directions $\phi^i \equiv A_i$.

Moduli spaces in theories with less supersymmetry, $\mathcal{N} = 2$ being the prime example, can have a much more interesting topological and differentiable structure than the $\mathcal{N} = 4$ example. In rigid supersymmetry, the study of moduli spaces of supersymmetric gauge theories has been an immensely active field of study since the publication of the seminal paper by Seiberg and Witten [61]. In chapter 5 we will see that moduli spaces in $\mathcal{N} = 2$ supergravity play a crucial role in understanding compactifications of the superstring.

3.2 Moduli in String Theory

Due to the maximal supersymmetry of the type IIA and type IIB theories in 10D many interesting $\mathcal{N} = 2$ and $\mathcal{N} = 1$ compactifications feature quite complicated classical moduli spaces. In the $\mathcal{N} = 1$ case the flat directions will often be lifted by non-perturbative effects. Let us first look at the moduli of the uncompactified theories. The moduli space of the type IIA theory is the real line $\mathbb{R}$ parameterized by the dilaton VEV Φ. As in the example of the $\mathcal{N} = 4$ gauge theory in 4D, one can give this moduli space a geometric interpretation. The type IIA supergravity can be thought of as arising from compactification of 11D supergravity on a circle. Conjecturally this extends to the whole string theory at the quantum level, the type IIA string theory being the compactification of a strongly coupled 11-dimensional quantum theory of gravity called *M-theory*. This will be discussed in more detail in chapter 4.

The type IIB string (1.29) has a moduli space parameterized by the *axio-dilaton* $S = C_0 + ie^{-\Phi}$. As we will discuss in chapter 4, the type IIB theory has a $\mathrm{SL}(2, \mathbb{Z})$ duality symmetry, which relates physically equivalent vacua of the theory. Under this duality, S transforms via the fractional linear transformation

$$S \to \frac{aS + b}{cS + d} \qquad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2, \mathbb{Z}) . \tag{3.11}$$

This can be made manifest by transforming the action (1.29) to *Einstein frame* [38]

$$S_{IIB} = \frac{1}{2\tilde{\kappa}_{10}^2} \int d^{10}X\sqrt{-G}\left(R - \frac{\partial S \partial \bar{S}}{2(\mathrm{Im}S)^2} - \frac{|G_3|^2}{2\mathrm{Im}S} - \frac{|F_5|^2}{4}\right) + \dots \,, \tag{3.12}$$

where $G_3 = F_3 - SH_3$. The manifestly SL$(2,\mathbb{Z})$-invariant form of the action reads [62, 63]

$$S_{IIB} = \frac{1}{2\tilde{\kappa}_{10}^2} \int d^{10}X\sqrt{-G}\left(R + 4\mathrm{tr}\partial_M \mathcal{M}\partial^M \mathcal{M}^{-1} - \frac{1}{12}\mathrm{tr}\mathcal{H}^T\mathcal{M}\mathcal{H}\right) + \dots \,. \tag{3.13}$$

Here $\mathcal{H}^T = (F_3, H_3)$ transforms as $\mathcal{H} \to \Lambda^{-1}\mathcal{H}$ under $\Lambda \in$ SL$(2,\mathbb{Z})$ and we define the $SL(2,\mathbb{R})$ matrix

$$\mathcal{M} = \frac{1}{S_2}\begin{pmatrix} 1 & S_1 \\ S_1 & |S|^2 \end{pmatrix} \qquad \mathcal{M} \longrightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix}^T \mathcal{M} \begin{pmatrix} a & b \\ c & d \end{pmatrix} \text{ under } \tau \to \frac{aS+b}{cS+d}\,. \tag{3.14}$$

As a result of this duality the quantum moduli space of type IIB string theory in 10D is the SL$(2,\mathbb{Z})$ fundamental domain depicted in figure 3.1. Again, one can give this moduli space a geometric interpretation by recognizing that it is precisely the space of *complex structures* of a two-torus T^2. The resulting theory, called *F-theory*, is quite powerful as it allows for a geometrical description of backgrounds with varying axio-dilaton, which occur in the presence of D7-branes [64].

Before discussing compactifications of the type IIA and type IIB string theories to 4D it is worthwhile to discuss at some length the moduli space of the torus. We can obtain the torus from the complex plane with the coordinate $z = x + iy$ by imposing the periodic identification

$$z \simeq z + 2\pi m + 2\pi n\tau \qquad (m,n) \in \mathbb{Z}^2\,, \tag{3.15}$$

where $\tau \in \mathbb{C}$ is the aforementioned complex structure parameter of the torus, see figure 3.2. This torus inherits the structure of a complex manifold from its ambient space and has a unique globally defined and non-vanishing holomorphic middle-dimensional form

$$\Omega = dz\,. \tag{3.16}$$

We now notice that SL$(2,\mathbb{Z})$ = Sp$(2,\mathbb{Z})$ and define a symplectic basis of (translation invariant) one-cycles A and B on the torus as in figure 3.2. These satisfy

$$\begin{pmatrix} A\cap A & A\cap B \\ B\cap A & B\cap B \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \tag{3.17}$$

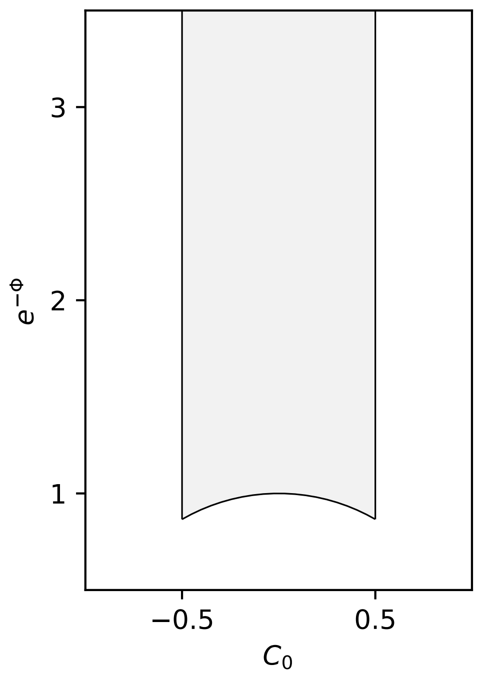

Figure 3.1: The SL$(2,\mathbb{Z})$ fundamental domain, parameterized by the axio-dilaton $S = C_0 + ie^{-\Phi}$, is the moduli space of type IIB string theory in ten dimensions. It is obtained from the upper half-plane by imposing the identifications $S \simeq -1/S$ and $S \simeq S+1$.

and transform as a symplectic vector. We can now give a definition of the complex structure modulus that has a manifest invariance under rescaling of the torus volume

$$\tau = \frac{\Pi_B}{\Pi_A} \qquad \Pi_\Sigma = \int_\Sigma \Omega \,, \tag{3.18}$$

where we have defined the *period* Π_Σ associated to a middle-dimensional cycle Σ. Given a basis of such cycles, we can define the associated period vector Π.

In the embedding space $\mathbb{C}$, the complex structure deformations of the torus are interpreted as deformations of the A- and B-cycles with fixed complex structure of the embedding space.

Forgetting about the embedding, from an intrinsic point of view we should define coordinates $(\tilde{x},\tilde{y})$ that are adapted to the periodicity of the torus. In terms of these

$$z(\tau) = \tilde{x} + \tau\,\tilde{y} \qquad (\tilde{x},\tilde{y}) \simeq (\tilde{x},\tilde{y}) + 2\pi(m,n)\,. \tag{3.19}$$

From this point of view, τ really parameterizes the transition from real to complex coordinates. It turns out that the complex structure moduli space is a Kähler

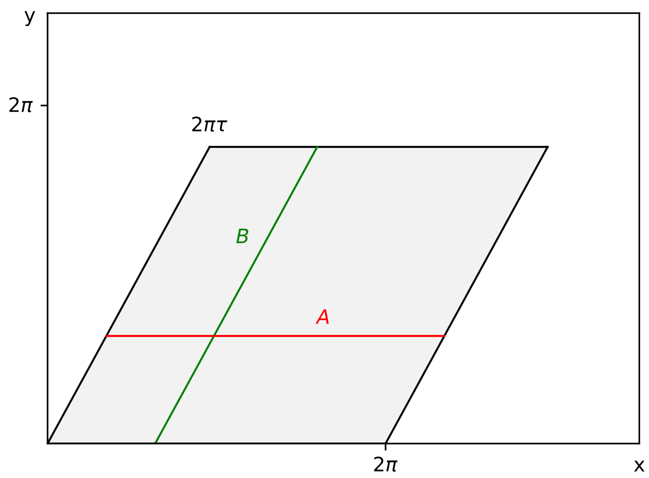

Figure 3.2: The flat torus with the complex structure parameter τ as a quotient of the complex plane. A choice of a symplectic basis of one-cycles is indicated.

manifold itself with natural Kähler potential and metric

$$\begin{aligned} K(\tau,\bar{\tau}) &= -\ln\left(i\int\Omega\wedge\bar{\Omega}\right) = -\ln\left(\tau-\bar{\tau}\right)+\text{const.} \\ g_{\tau\bar{\tau}} &= \frac{1}{4(\text{Im}(\tau))^2} \end{aligned} \quad . \tag{3.20}$$

Under the identification $\tau \leftrightarrow S$ the above coincides with the kinetic metric for the axio-dilaton in type IIB, see equation (3.12).

Besides the complex structure of the torus, we can also deform its volume or Kähler class by

$$ds^2 = dzd\bar{z} \to v\, dzd\bar{z} \tag{3.21}$$

without interfering with the complex structure. Due to the lack of a clear 12-dimensional origin of the type IIB theory, the volume of the torus v is unphysical in this case. We can also consider compactifying either one of the two type II theories on T^2. In this case, the moduli space is enhanced to $\text{SL}(2,\mathbb{R})/U(1) \times \text{SL}(2,\mathbb{R})/U(1)$. The metric and Kalb-Ramond field can be parameterized as[3]

$$\begin{aligned} G &= v\frac{1}{\tau_2}\begin{pmatrix} 1 & \tau_1 \\ \tau_1 & |\tau|^2 \end{pmatrix}, \qquad & B &= \begin{pmatrix} 0 & b \\ -b & 0 \end{pmatrix}, \\ ds^2 &= v dzd\bar{z}\,, & K &= iv dz\wedge d\bar{z}\,. \end{aligned} \tag{3.22}$$

[3]Note that the complex coordinates z here differ by a factor $1/\tau_2$ from those used above in order to keep the torus volume fixed under complex structure deformations.

The now physical Kähler modulus v pairs up with the Kalb-Ramond field b to form a second complex modulus $t = b + iv$. The Kähler modulus t is also subject to modular $\mathrm{SL}(2,\mathbb{Z})$ identifications because of T-duality $t \to -i/t$, which we will discuss in section 4.2 and shift symmetry of the Kalb-Ramond field $t \to t + 1$.

The point of dwelling on these basic points about the complex geometry of the torus is that the torus is a so-called Calabi-Yau (CY) manifold. The type II string on a complex three-dimensional Calabi-Yau manifold leads to a $\mathcal{N} = 2$ supergravity theory in four dimensions, which is the starting point for constructing phenomenologically interesting compactifications. As it turns out, the geometric moduli space of a Calabi-Yau splits into complex structure and Kähler deformations just as in the case of a torus. As we will note in chapter 5, the above discussion of the complex structure applies almost isomorphically to the type IIB theory with the replacement

$$\mathrm{SL}(2,\mathbb{Z}) \to \mathrm{Sp}(h^3(X_6),\mathbb{Z}) \;, \tag{3.23}$$

where h^3 is the third (middle-dimensional) Betti number.

4 Duality

An important and non-trivial property of the framework of string theory is that it incorporates highly non-trivial quantum mechanical equivalences, called *dualities*, between seemingly different consistent theories in ten dimensions. The basic idea goes back to the early days of quantum mechanics and the discovery of the wave/particle duality or duality between the position and momentum space representations of the Hilbert space of a free particle. A duality between two theories should comprise a precise map between the quantum observables in one theory and those in the other. Dualities appear not only in string theory but also in quantum field theories, although a deeper understanding of field theory dualities is often obtained when they are embedded into string theory. In the following we will recapitulate some basic facts about dualities in supersymmetric field theories and in string theory with a view towards applications in part III.

4.1 Duality in Supersymmetric Field Theory

In field theory it is important to distinguish exact dualities that are valid at all energy scales from infrared dualities that relate the infrared limits of two field theories [65]. The former ones usually require extended supersymmetry.

A very central example of a duality is electric-magnetic duality. It is based on the symmetry of the (source-free) Maxwell equations under exchange of the electric and magnetic fields $F \leftrightarrow \star F$

$$\begin{pmatrix} dF \\ d\star F \end{pmatrix} = 0 \,. \tag{4.1}$$

Plenty of research has been done to investigate whether this duality could also hold in interacting quantum field theories such as Yang-Mills theory coupled to matter. Such an equivalence has to be extremely non-trivial, as the duality operation can be seen to exchange weak and strong coupling $g \to 1/g$. Hence, the point-like perturbative electric sources have to be exchanged with magnetic monopoles, which are heavy and extended solitons at weak coupling.

It is conjectured – and generally believed due to many non-trivial consistency checks – that electric-magnetic duality is exact in $\mathcal{N} = 4$ super Yang-Mills theory

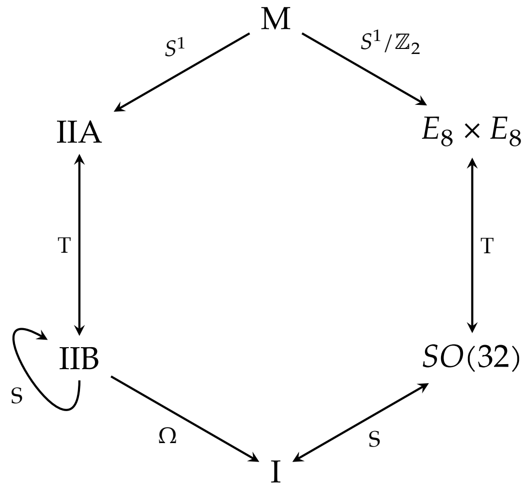

Figure 4.1: Dualities bind the different consistent superstring theories into a tight and constrained framework. In addition, the 11-dimensional M-theory leads to the type IIA and $E_8 \times E_8$ heterotic strings when compactified on a circle or line interval. Figure adapted from [47].

in four dimensions where it is the maximally supersymmetric field theory [66, 67]. The duality involves a precise matching of BPS operators in the theory. In the case of the $\mathcal{N} = 2$ supersymmetric gauge theory with gauge group $SU(2)$ there is a corresponding infrared duality, which is described by Seiberg-Witten theory [61, 68][1]. The duality has been extended to a duality of $\mathcal{N} = 1$ supersymmetric gauge theories in [65].

4.2 Duality in String Theory

Dualities are omnipresent in string theory. This is facilitated by the maximal supersymmetry of the type II strings but still very surprising considering that gravity is involved. The fact that all of the apparently different 10D string theories are connected by a *duality web*, see figure 4.1, was realized during the second superstring revolution [69]. The spectrum of possible dualities becomes richer under compactification.

We have already discussed the *S-duality* of the type IIB string to some extent in section 3.2. The type IIB theory contains not only the fundamental string $F1$, which is the electric charge of the Kalb-Ramond field B. It contains also the $D1$ brane of the same dimensionality, which is electrically charged under C_2. Their

[1] It is also conjectured to be exact in certain scale invariant $SU(2)$ theories with matter [68].

tensions scale inversely to each other with the string coupling

$$\frac{T_{F1}}{M_{10}^2} \sim \sqrt{g_s} \qquad \frac{T_{D1}}{M_{10}^2} \sim \frac{1}{\sqrt{g_s}} \,. \tag{4.2}$$

Furthermore, the spectrum of massless BPS states on the D1-brane reproduces the massless spectrum of the type IIB F1-string [36]. This strongly suggests a non-perturbative duality between the type IIB string theory at weak and strong coupling that exchanges the F-strings and the D-strings. There is a rather impressive body of evidence for this conjectured duality, see for example [36, 70, 71]. Finally, we note that the world volume gauge theory of a stack of D3-branes in type IIB is $\mathcal{N} = 4$ SYM and the S-duality reduces in this sector to the Montonen-Olive electromagnetic duality, which we discussed in section 4.1.

An example of a duality that relates two seemingly *different theories* is the *T-duality* between the type IIA and type IIB strings. We observe that if we compactify either one of these on a circle, the mass formula (2.24) is invariant under exchange of momentum and winding, provided we also invert the circle radius

$$(n, w) \leftrightarrow (w, n) \qquad R \leftrightarrow \frac{\alpha'}{R} \,. \tag{4.3}$$

The claim is that the type IIB theory on a small circle is equivalent to the type IIA theory on a large circle and vice versa. From the point of view of the free theory on the world-sheet, we see that this is in fact just electromagnetic duality of the world-sheet scalars X^M

$$dX^M \to \star dX^M \qquad \Leftrightarrow \qquad \partial_\pm X^M \to \pm \partial_\pm X^M \,. \tag{4.4}$$

One can then argue that the duality also has to involve a mapping of the D-branes in the theories. Because momentum and winding are exchanged, Dirichlet and Neumann boundary conditions are also exchanged along the circular direction. It is important for the reader to note that, while strong-weak coupling dualities are common in supersymmetric field theories, spacetime dualities such as T-duality are unique to string theory and seem to require the existence of extended objects.

The link to the heterotic string theories, which we have not discussed in detail, is established via the 11-dimensional *M-theory*. This is a quantum theory of gravity in 11 dimensions that has the unique 11D supergravity as its low energy limit. The 11D supergravity multiplet contains a three-form gauge field, which couples electrically to *M2-branes* and magnetically to *M5-branes*. These branes appear as soliton solutions of the supergravity and are important dynamical ingredients of the full M-theory. For example, by compactifying the 11D SUGRA on a circle we obtain the type IIA SUGRA in 10D. This is explained by the M2-brane giving rise to a string when wrapped on the circle that is precisely the type IIA string. The string coupling is determined by the radius of the circle such that the weakly coupled type IIA description is valid at small radius. Details can be found for example in [36, 38, 70].

As we have seen in section 2.1, the only other choice of compactification for M-theory to ten dimensions is a line interval, which breaks half of the supersymmetry. The resulting theory suffers an anomaly, which can only be cured by introducing 10D $\mathcal{N} = 1$ E_8 gauge sectors at both ends [72, 73]. At small radius this leads to the identification of the resulting theory with the weakly coupled *heterotic string* with gauge group $E_8 \times E_8$. Just as the type IIA string, the $E_8 \times E_8$ string does not have a D1-brane so there is no candidate for an S-dual. If we compactify on a circle, we can perform a T-duality and obtain the other known heterotic string theory with gauge group $SO(32)$. Under this duality, the D2-brane that descends from the M2-brane is mapped to a D1-brane and indeed there is an S-dual theory, the type I string with identical gauge group $SO(32)$.

Finally, the last arrow in the diagram 4.1 is explained the observation that the type I string can be obtained from the type IIB string by a so-called orientifold projection Ω. We will not discuss orientifold projections here in detail because they will reappear in the discussion of $\mathcal{N} = 1$ compactifications of the type II string in chapter 6. The fact that the relations between the theories form a closed circle means that one can perform many non-trivial consistency checks. This picture seems to be unexpectedly consistent and allows using the dualities to probe for example regions of strong coupling (S-duality) and non-geometry (T-duality) in moduli space by changing to a weakly coupled and geometric description.

> Dualities can be used as a tool for understanding regions of moduli space that are otherwise inaccessible using conventional field theory techniques.

We will use one of the many string dualities, *mirror symmetry*, in chapter 8 to test the swampland distance conjecture (sec. 7.2.1) in non-geometric regimes of string theory.

5 4D N=2 Vacua of the Superstring

The aim of this chapter is to introduce the reader to some aspects of Calabi-Yau compactifications of the type II superstring that lead to $\mathcal{N} = 2$ SUSY in four dimensions. We will review the basic structure of the bosonic sector of the corresponding 4D $\mathcal{N} = 2$ supergravity and show how string theory breathes life into this theory by filling the SUSY multiplets with massless fields that arise in the Kaluza-Klein compactification of the 10D effective theory (ch. 1). Assuming that the reader has some familiarity with basic concepts of complex Kähler geometry, we will introduce Calabi-Yau manifolds in section 5.2. We will then discuss the type IIA and type IIB string on such Calabi-Yau manifolds n sections 5.3.1 and 5.3.2. Finally, we introduce *mirror symmetry*. Like T-duality, it is a duality between type IIA and type IIB strings, in this case both compactified on a Calabi-Yau manifold.

5.1 4D N=2 Supergravity

The compactification of a type II string theory on a torus leads to maximal $\mathcal{N} = 8$ SUSY in 4D. Like the $\mathcal{N} = 4$ gauge theory, this theory is uniquely determined by its symmetries. More interesting compactifications arise when we break some of the supersymmetries. The moduli spaces of $\mathcal{N} > 2$ supergravities are always coset spaces [59]. Interesting moduli space geometries start to arise at $\mathcal{N} = 2$. It is also rather easy to find controlled backgrounds of the type II strings with $\mathcal{N} = 2$ by compactifying it on a Calabi-Yau. This construction will be the starting point for the construction of $\mathcal{N} < 2$ vacua in chapter 6. The discussion will mostly follow [59, 74, 75].

Although we have already briefly discussed the case of rigid $\mathcal{N} = 2$ SUSY in section 3.1, turning on gravity leads to several new features. Besides the supergravity multiplet, the basic field theory multiplets of $\mathcal{N} = 2$ are the vector, the hyper- and the tensor multiplet. Their field and helicity content is summarized in table 5.1. The hypermultiplet contains two complex scalars and two Weyl fermions, which one can think of as arising from the combination of an $\mathcal{N} = 1$ chiral and anti-chiral multiplet. The vector multiplet of $\mathcal{N} = 2$ arises from combining the $\mathcal{N} = 1$ vector and chiral multiplets in the same way. The tensor multiplet is a hypermultiplet in disguise where one exchanges one of the

Field	**d.o.f. by helicity content**								
	-2	-3/2	-1	-1/2	0	+1/2	+1	+3/2	+2
				SUGRA multiplet					
$g_{\mu\nu}$	1								1
ψ^i_μ		2						2	
G_μ			1				1		
				vector multiplet					
A_μ			1				1		
λ^i				2		2			
X					2				
				hypermultiplet					
$\zeta^1, \bar{\zeta}^2$				2		2			
$H^1, \bar{H}^2$					4				
				tensor multiplet					
$T_{\mu\nu}$					1				
χ^i				2		2			
S^{ij}					3				

Table 5.1: The massless multiplets with helicity $h \leq 2$ of $\mathcal{N} = 2$ SUSY. Latin and explicit numeric indices are for the $SU(2)$ R-symmetry.

scalar fields with its dual two-form, the main difference being the associated gauge symmetry [76].

The bosonic part of the action of $\mathcal{N} = 2$ SUGRA is given by [59, 74, 75]

$$\begin{aligned} S = \int \Bigg(& \frac{1}{2\kappa^2} R \star 1 - g_{i\bar{j}}\, dx^i \wedge \star d\bar{x}^{\bar{j}} - h_{uv}\, dq^u \wedge \star dq^v \Bigg) \\ & + \left(\frac{1}{2} \mathrm{Im}\left(\mathcal{N}_{IJ}\right) F^I \wedge \star F^J + \frac{1}{2} \mathrm{Re}\left(\mathcal{N}_{IJ}\right) F^I \wedge F^J \right) . \end{aligned} \tag{5.1}$$

Here $F^{I\pm}$ are the (anti) self dual parts of the field strengths, x^i are the vector multiplet scalars and q^u are the hypermultiplet scalars. The index ranges are $i = 1 \ldots n_V$, $I = 0, \ldots n_V$, $u = 1, \ldots, 4n_H$, where n_V and n_H are the numbers of vector and hypermultiplets respectively. The mismatch between the index ranges i and I arises because the graviphoton mixes with the vector multiplet gauge fields.

We see that on the one hand, the hypermultiplet scalars parameterize what is known as a quaternionic Kähler manifold. They will not play an important role in our discussion of $\mathcal{N} = 2$ compactifications and therefore be mostly disregarded. On the other hand, the vector multiplet scalars x^i live on a special Kähler manifold and will play an important role. The special Kähler manifolds that appear in SUGRA are different from the rigid special Kähler manifolds that we have encountered in section 3.1.

As it is reviewed for example in [59], electromagnetic duality transformations act in general by symplectic matrices $M \in \mathrm{Sp}(2n_V + 2)$. The field strengths and their duals transform as a vector under this group

$$\begin{pmatrix} F^{I+} \\ G_I^+ \end{pmatrix} \equiv \begin{pmatrix} F^{I+} \\ -i\mathcal{N}_{IJ} F^{J+} \end{pmatrix} \to M \begin{pmatrix} F^{I+} \\ G_I^+ \end{pmatrix} . \tag{5.2}$$

Accordingly, a special Kähler manifold is a Kähler-Hodge manifold equipped with a holomorphic flat vector bundle $\mathcal{V}$ with structure group $\mathrm{Sp}(2n_V + 2)$, of which the field strengths are sections [75]. The Kähler-Hodge property means that we also have a complex line bundle $\mathcal{L}$ with first Chern class equal to the Kähler class. A holomorphic section Π of the bundle $\mathcal{H} = \mathcal{V} \otimes \mathcal{L}$ can be decomposed as

$$\Pi = \begin{pmatrix} X^I \\ F_I \end{pmatrix} . \tag{5.3}$$

A (local) special Kähler manifold is then defined to be a manifold as above, such that the symplectic structure is compatible with the Kähler structure in the sense that

$$K = -\log\left(i \langle \Pi | \overline{\Pi} \rangle\right) \qquad \langle V|W \rangle \equiv V^T \Sigma W \qquad \Sigma = \begin{pmatrix} 0 & \mathbf{1} \\ -\mathbf{1} & 0 \end{pmatrix} . \tag{5.4}$$

Here Σ is the symplectic scalar product, which determines a hermitian scalar product of the sections of $\mathcal{H}$. Note that the logarithm arises here because the transition functions $\Pi \to e^{-f}\Pi$ of the bundle $\mathcal{L}$ are identified with Kähler transformations $K \to K + f + \bar{f}$.

The notation for the components of Π, which is reminiscent of the prepotential, is not accidental. One can often determine a prepotential F such that $F_I = \partial_{X^I} F$, although this is not the case in every electromagnetic duality frame. Presuming its existence, the prepotential is a homogeneous function of degree two of the X^I

$$F_I X^I = 2F . \tag{5.5}$$

As the Kähler transformations are acting on Π, the X^I only form projective coordinates on the vector multiplet moduli space. Provided that the Jacobian of the transformation does not vanish, we can introduce the *special coordinates* that appear in the action (5.1) by

$$x^I = \frac{X^I}{X^0} = \begin{pmatrix} 1 \\ x^i \end{pmatrix} . \tag{5.6}$$

In terms of these special coordinates the Kähler potential reads

$$K = -\log i \left(2(\bar{F} - F) - \left(\bar{x}^i - x^i\right)\left(\bar{F}_i + F_i\right)\right) . \tag{5.7}$$

The global SUSY limit of this can be taken by substituting $F \to F + M_p^2/4$ and then sending $M_p \to \infty$, see also [30]. In this way we get back to the rigid Kähler potential (3.6). Finally, the gauge kinetic terms can be obtained as [59]

$$\mathcal{N}_{IJ} = \bar{F}_{IJ} + 2i \frac{\mathrm{Im}(F_{IK}) X^K \mathrm{Im}(F_{JL}) X^L}{\mathrm{Im}(F_{KL}) X^K X^L} . \tag{5.8}$$

5.2 Calabi-Yau Geometry

Let us now see how to arrive at the concept of a Calabi-Yau manifold from the requirement that a compactification of the type II superstring should preserve $\mathcal{N} = 2$ supersymmetry in 4D. The 10D $\mathcal{N} = 2$ SUSY of the type II strings amounts to 32 real supercharges, which we have to compare with the 8 real supercharges of the 4D $\mathcal{N} = 2$ theory[1]. Thus, we want to retain only 1/4 of the supercharges.

In the following we are interested in backgrounds on which the fermion VEVs vanish, which is required if we want a maximally symmetric background in 4D, such as Minkowski or (A)dS. If our background should respect a certain combination $\bar{\epsilon}Q$ of the SUSY generators, we need to require

$$\langle \delta_\epsilon \mathcal{O} \rangle = \langle [\bar{\epsilon}Q, \mathcal{O}\} \rangle = 0 , \tag{5.9}$$

where $[\ ,\ \}$ is the graded commutator. As only fermions appear in the SUSY variation of a bosonic operator and their expectation value vanishes, the non-trivial condition here amounts to

$$\delta_{\mathrm{SUSY}}\,(\text{fermions}) \stackrel{!}{=} 0 . \tag{5.10}$$

[1]The Dirac representation of $\mathrm{Cliff}(1,9)$ has the complex dimension $2^5 = 32$. Hence, the fundamental Majorana-Weyl spinor representation in 10D has real dimension 16.

In the present context of the type II string, the relevant fermions are the gravitini and dilatini. The gravitino variation has the schematic form[2]

$$\delta_\epsilon \psi_M = \nabla_M \epsilon + f_M(H_3, F_i) + (\text{fermions})^2 \ , \tag{5.11}$$

where $f_M(H_3, F_i) \to 0$ if the fluxes vanish, see [57] for an explicit expression.

We will now specify the compactification geometry to be of the warped product form (2.2) and consider only solutions without fluxes, that is

$$\langle H_3 \rangle = \langle F_i \rangle = 0 \ . \tag{5.12}$$

On such backgrounds, equation (5.11) leads to

$$\langle \delta_\epsilon \psi_M \rangle = \langle \nabla_M \epsilon \rangle \stackrel{!}{=} 0 \ , \tag{5.13}$$

which implies that preserved supersymmetries correspond to covariantly constant spinors. A further examination of the commutator of two covariant derivatives on this covariantly constant spinor leads to an integrability condition, which requires [57]

> Any compactification of the type II superstring to a maximally symmetric space in 4D without fluxes that preserves at least $\mathcal{N} = 1$ supersymmetry is necessarily an unwarped compactification to Minkowski space.

We now split the two independent 10D SUSY parameters ϵ^A into four-dimensional and six-dimensional parts, while retaining the correct chiralities [57]

$$\begin{gathered} \epsilon^1_{\text{IIA}} = \xi^1_+ \otimes \eta_+ + \xi^1_- \otimes \eta_- \ , \qquad \epsilon^2_{\text{IIA}} = \xi^2_+ \otimes \eta_- + \xi^2_- \otimes \eta_+ \ , \\ \epsilon^A_{\text{IIB}} = \xi^A_+ \otimes \eta_+ + \xi^A_- \otimes \eta_- \ . \end{gathered} \tag{5.14}$$

Here the spinors ξ and η are of the Weyl type with chirality indicated by the $\pm$ indices. Because a covariantly constant spinor in Minkowski space is just a constant one, we get the non-trivial condition

$$\nabla_m \eta = 0 \ . \tag{5.15}$$

By parallel transporting the spinor η along the internal manifold this implies severe restrictions on the geometry. The transformation properties of sections of a vector bundle E on a Riemannian manifold $\mathcal{M}$ such as η under parallel transport are encoded in the so-called *holonomy group*

$$\text{Hol}_x(\mathcal{M}, \nabla) = \left\{ P_\gamma \in \text{GL}(E_x) \mid \gamma : S^1 \to \mathcal{M} \right\} \ , \tag{5.16}$$

[2]The dilatini variations do not give any further constraints.

where ∇ is a connection and P_γ is the result of parallel transport along the closed loop γ based at $x \in \mathcal{M}$. In the case of a connected manifold, we can drop the base point $x \in \mathcal{M}$. If ∇ is the Levi-Civita connection and $\mathcal{M}$ is orientable of dimension n, the holonomy groups are subgroups of $SO(n)$.

Spinors on a 6D manifold are sections of the spin bundle and hence have holonomies contained in $\text{Spin}(6) \simeq SU(4)$. Remembering that we want to break exactly one quarter of the supersymmetries, the holonomy group we are looking for should be a subgroup of $SU(4)$ under which

$$\mathbf{4} \to \mathbf{3} \oplus \mathbf{1} \,. \tag{5.17}$$

This determines the holonomy group to be $SU(3)$. We define:

> A compact Riemannian manifold $\mathcal{M}$ of dimension $d = 2n$ is a *Calabi-Yau* n-fold provided its Riemannian holonomy is $SU(n)$. It follows that it is a complex Kähler manifold with vanishing first Chern class $c_1(\mathcal{M}) = 0$.

Any such manifold admits a Ricci-flat Kähler metric by Yau's theorem.

In the following we will specialize to Calabi-Yau (CY) threefolds. Some general facts about the cohomology groups of a CY will turn out to be useful. The Hodge diamond of cohomology dimensions for a CY reads

$$\begin{matrix} & & & h^{3,3} & & & \\ & & h^{3,2} & & h^{2,3} & & \\ & h^{3,1} & & h^{2,2} & & h^{1,3} & \\ h^{3,0} & & h^{2,1} & & h^{1,2} & & h^{0,3} \\ & h^{2,0} & & h^{1,1} & & h^{0,2} & \\ & & h^{1,0} & & h^{0,1} & & \\ & & & h^{0,0} & & & \end{matrix} \quad = \quad \begin{matrix} & & & 1 & & & \\ & & 0 & & 0 & & \\ & 0 & & h^{1,1} & & 0 & \\ 1 & & h^{2,1} & & h^{2,1} & & 1 \\ & 0 & & h^{1,1} & & 0 & \\ & & 0 & & 0 & & \\ & & & 1 & & & \end{matrix} \,. \tag{5.18}$$

As we have seen in section 2.2, the fields in the four-dimensional $\mathcal{N} = 2$ SUGRA will be determined by the cohomology groups of the Calabi-Yau and hence their multiplicities will depend on the two numbers $h^{1,1}$ and $h^{2,1}$.

A Calabi-Yau is equipped with two special globally defined differential forms ω and Ω, which one can build out of the covariantly constant spinor. On the one hand, the (1,1)-form ω is known as the *Kähler form* and it is in principle determined by the (unknown) Kähler metric on the CY. It measures the volumes of holomorphic curves and surfaces via

$$\text{Vol}(C) = \int_C \omega \qquad \text{Vol}(S) = \int_S \omega \wedge \omega \,. \tag{5.19}$$

On the other hand, the holomorphic (3,0)-form determines the complex structure of the Calabi-Yau via the period integrals

$$\Pi_\Sigma = \int_\Sigma \Omega\,, \tag{5.20}$$

where Σ is a three-cycle in the CY. As we will see below, the similarity in notation to the discussion of the complex structure of the torus in section 3.2 is not a coincidence.

Independent of whether we consider type IIA or type IIB, there will be moduli in the 4D effective field theory that arise from deformations of the metric. These come in two types, for both of which one can again show that the massless moduli are in one-to-one correspondence with harmonic differential forms and thus cohomology groups. First, we have the *Kähler moduli*. These arise as deformations of the metric δg_{mn} that do not interfere with the complex structure and thus respect the split into holomorphic and anti-holomorphic indices. They can be parameterized by expanding the ω into basis elements ω_a of $H^{1,1}$

$$\omega = \sum_a v^a(x)\,\omega_a \qquad\qquad i = 1,\dots,h^{1,1}\,. \tag{5.21}$$

These are constrained to lead to positive volumes in equation (5.19). The v^a that satisfy this constraint are said to lie in the *Kähler cone*. Second, we have the *complex structure moduli*. These are such deformations of the metric that do mix holomorphic and anti-holomorphic indices and hence require a simultaneous redefinition of the complex structure to still satisfy the Kähler property. They can be mapped to elements of $H^{2,1}$ via [57]

$$\delta g_{ij} = i z^k(x) \left(\frac{(\bar{\chi}_k)_{i\bar{k}\bar{l}}\,\Omega^{\bar{k}\bar{l}}{}_j}{|\Omega|^2} \right) \qquad\qquad k = 1,\dots,h^{2,1}\,. \tag{5.22}$$

Here the χ_k are a basis of harmonic (1,2)-forms.

Examples of Calabi-Yau manifolds

The torus T^2 is a CY 1-fold and in fact the only one, as it is evident from the classification of Riemann surfaces. The only non-trivial CY 2-fold is a K3-surface. In three complex dimensions, which is the relevant case for string compactifications to 4D, there are in fact many examples. Some Calabi-Yau manifolds can be obtained as sub-varieties of complex projective spaces, such as the *quintic* hypersurface in $\mathbb{CP}^4$ given by the vanishing set $V(P_5)$ of a generic homogeneous polynomial P_5 of degree five. Such a hypersurface inherits a Kähler structure from its embedding space. We can furthermore calculate the total Chern class for such a degree d hypersurface in $\mathbb{CP}^n$ as

$$c\left(\mathrm{Proj}(\mathbb{C}[x_1,\dots,x_{n+1}]/(P_d))\right) = \frac{(1+H)^{n+1}}{(1+dH)} = 1+(n+1-d)H+\mathcal{O}(H^2)\ , \quad (5.23)$$

where H is the hyperplane class of $\mathbb{CP}^n$. For the quintic we see that the first Chern class vanishes and we can invoke Yau's theorem to argue that this space admits a Calabi-Yau metric.

We can also calculate the Hodge numbers $h^{1,1}$ and $h^{2,1}$ for the quintic. The projective space $\mathbb{CP}^4$ has a single Kähler modulus measuring its overall volume. This is inherited by the quintic, so we find $h^{1,1} = 1$[3]. The complex structure moduli are obtained as deformation parameters of the polynomial

$$P_5 = ax_1^5 + bx_2^5 + cx_3^5 + dx_4^5 + ex_5^5 + 5\psi x_1x_2x_3x_4x_5 + 120 \text{ other terms}\ . \quad (5.24)$$

Here used that the space of homogeneous polynomials of degree d in n variables has dimension $d+n-1$ choose d. The parameters $(a,b,c,d,e,\psi,\dots)$ are the complex structure moduli. Because we can in principle absorb some complex structure moduli into a redefinition of the coordinates, we have to subtract the dimension of $\mathrm{GL}(5,\mathbb{C})$ and find that the quintic has $h^{2,1} = 101$.

There are two well-known generalizations of this construction. First, we can consider so-called *complete intersections* or *CICYs* of several homogeneous polynomials in products of projective spaces [77]. Second, we can generalize the notion of a projective space by assigning different charges to the homogeneous coordinates, which leads to the notion of a *weighted* projective space [78]. The latter are a special case of *toric varieties*, which can be defined in a very similar manner to projective spaces. Hypersurfaces in toric varieties form the second large class of CY manifolds [79, 80].

Let us finally mention that, besides compact Calabi-Yau spaces, one is often also interested in non-compact or local Calabi-Yau geometries. For these one can often determine analytically a Calabi-Yau metric in contrast to the compact case. The non-compact case is interesting if we only want to construct 4D quantum field theories without coupling them to gravity. In two dimensions we can for example consider ALE spaces of the form $\mathbb{C}/G$, where $G \subset SU(2)$ is a finite subgroup. In three dimensions, we are interested in the (deformed) *conifold*. It can be constructed as a sub-variety of $\mathbb{C}^4$ as

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 = \epsilon^2\ , \quad (5.25)$$

[3] In more general cases hypersurfaces will be singular. Additional Kähler moduli may arise in the process of singularity resolution.

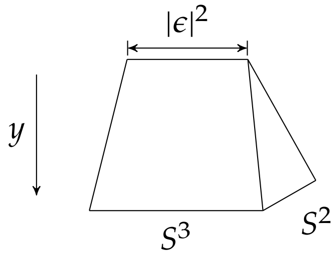

Figure 5.1: A sketch of the deformed conifold geometry.

where ϵ is the deformation parameter and $\epsilon = 0$ corresponds to the conifold. For finite ϵ, this space is diffeomorphic to the cotangent bundle of S^3, see for example [81]. Another way to look at it is that the resulting space for $\epsilon = 0$ is a cone over $S^2 \times S^3$. If we deform it by giving a finite value to ϵ, the S^3 at the tip of the cone is blown up to a finite volume determined by $|\epsilon|$. A sketch of the geometry is shown in figure 5.1. The cross-section $S^2 \times S^3$ can be given an Einstein metric [82]

$$ds^2_{T^{1,1}} = \frac{1}{9}(g^5)^2 + \frac{1}{6}\sum_{i=1}^{4}(g^i)^2 \,,. \tag{5.26}$$

It is then known as $T^{1,1}$. The basis of one-forms g^i is given by

$$\begin{gathered} e^1 = -\sin\theta_1 d\phi_1 \,, \qquad e^2 = d\theta_1 \,, \qquad e^3 = \cos\psi\sin\theta_2 d\phi_2 - \sin\psi d\theta_2 \,, \\ e^4 = \sin\psi\sin\theta_2 d\phi_2 + \cos\psi d\theta_2 \,, \qquad e^5 = d\psi + \cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2 \,, \\ g^1 = (e^1 - e^3)/\sqrt{2}\,, \qquad g^2 = (e^2 - e^4)/\sqrt{2}\,, \qquad g^3 = (e^1 + e^3)/\sqrt{2}\,, \\ g^4 = (e^2 + e^4)/\sqrt{2}\,, \qquad g^5 = e^5 \,, \end{gathered} \tag{5.27}$$

where ψ, θ_1, ϕ_1 are Euler angles on S^3 and θ_2, ϕ_2 parameterize S^2. This metric satisfies $R_{ij} = 8g_{ij}$. It is a simple fact that the metric cone

$$ds^2 = dy^2 + y^2 ds^2_{T^{1,1}} \,, \tag{5.28}$$

which is precisely the undeformed conifold, is a Calabi-Yau. This is because forming the metric cone over an n-dimensional manifold adds $2(1-n)g_{ij}$ to the Ricci curvature.

The conifold geometry naturally arises as a local geometry in compact Calabi-Yau manifolds. For example, the quintic (8.71) has a conifold point at $|\psi| = 1$ in its complex structure moduli space where

$$0 = \sum_{i=1}^{5} x_i^5 - 5\prod_{i=1}^{5} x_i \,. \tag{5.29}$$

We fix the projective invariance by $x_5 = 1$ and expand to quadratic order around the point $y_i = 1 - x_i = 0$ to find

$$0 = \sum_{i=1}^{4} y_i^2 + \frac{1}{2} \sum_{i<j} y_i y_j \; , \tag{5.30}$$

which is equivalent to (5.25) under a linear change of coordinates. Moving away from $|\psi| = 1$ locally realizes the deformed conifold geometry. In such an embedding, we naturally get a symplectic pair of three-cycles. The A-cycle is defined as being the three-sphere of the deformed conifold, whereas the B-cycle can be locally thought of as the cone over the S^2. After growing with y in the direction of the bulk of the CY, the S^2 will have to shrink again to a point somewhere, forming a three-cycle. This could be achieved for example by having two conifold regions in the CY [83].

5.3 Type II String Theory on Calabi-Yau Manifolds

We proceed now to match the fields that arise from a compactification of the type II strings on a Calabi-Yau to the general 4D $\mathcal{N} = 2$ SUGRA (5.1). There is a unique way to do so for the type IIA and the type IIB string. We follow [57]. The strategy is as follows. We choose a set of harmonic differential forms that generates the Dolbeault cohomology groups of the CY

$$\begin{aligned}
&\omega_a \in H^{1,1}(\mathcal{M}, \mathbb{C}) \; , && \tilde{\omega}^a \in H^{2,2}(\mathcal{M}, \mathbb{C}) \; , && a = 1, \dots, h^{1,1} \; , \\
&\chi_k \in H^{2,1}(\mathcal{M}, \mathbb{C}) \; , && && k = 1, \dots, h^{2,1} \; , \\
&(\alpha_K, \beta^K) \in H^3(\mathcal{M}, \mathbb{R}) \; , && && K = 1, \dots, h^3 = 2h^{2,1} + 2 \; , \\
&\Omega \in H^{3,0}(\mathcal{M}, \mathbb{C}) \; , && \overline{\Omega} \in H^{0,3}(\mathcal{M}, \mathbb{C}) \; . &&
\end{aligned} \tag{5.31}$$

The notation employed here is standard and adapted from [57]. The real generators α_K and β^K of H^3 are Poincaré dual to a basis of three-cycles B^K and A_K respectively, which have a symplectic intersection pairing

$$A_K \cap B^L = \delta_K^L \; , \tag{5.32}$$

just as it is the case for the two-torus (3.17).

Here we record the decomposition of the 10D NS-NS sector fields into massless four-dimensional ones

$$\begin{aligned}
G_{MN}(x, y) \to g_{\mu\nu} \, , v_a(x) \, , z^k(x) \qquad\qquad & B(x, y) = B(x) + b^a(x)\omega_a \\
\Phi(x, y) = \Phi(x) \; , &
\end{aligned} \tag{5.33}$$

Multiplet	#	Fields
SUGRA	1	$g_{\mu\nu}, A_1^0$
vector	$h^{1,1}$	A_1^a, v^a, b^a
hyper	$h^{2,1}$	$z^k, \xi^k, \tilde{\xi}_k$
tensor	1	$B, \Phi, \xi^0, \tilde{\xi}_0$

Table 5.2: Bosonic fields arising in a type IIA compactification on a Calabi-Yau manifold. Reproduced from [57].

where again v_a are the Kähler moduli and z^k are the complex structure moduli. In contrast to the 5D Kaluza-Klein reduction on the circle, no massless vector fields arise from the metric. The reason for this is that just like the orbifold $S^1/\mathbb{Z}_2$, a CY has no isometries that they could gauge. Because of the shift symmetry, B will give rise to a tensor multiplet in both type II theories. Outside of the NS-NS sector, we have to treat the type IIA and type IIB theories separately.

5.3.1 Type IIA

In the type IIA theory we get the following additional fields from the R-R sector

$$\begin{aligned} C_1(x,y) &= A_1^0(x) \ , \\ C_3(x,y) &= A_1^a(x)\omega_a + \xi^K(x)\alpha_K - \tilde{\xi}_K(x)\beta^K \ . \end{aligned} \tag{5.34}$$

Focusing on the vector multiplets, the $h^{1,1}$ vector fields are obtained from C_3. These need to be accompanied by $h^{1,1}$ complex scalars. The only available scalar fields are

$$t^a = b^a + iv^a \ . \tag{5.35}$$

The remaining vector field arising from C_1 has to correspond to the graviphoton. All other field assignments are displayed in table 5.2. We find

> Type IIA theory on a Calabi-Yau threefold gives rise to a 4D $\mathcal{N} = 2$ supergravity theory with $h^{1,1}$ vector multiplets and $h^{2,1}$ hypermultiplets.

The type IIA prepotential consists of a tree-level term, a one-loop contribution and an infinite number of instanton corrections

$$F^{1,1}(t) = \frac{1}{6}\kappa_{abc}\frac{t^a t^b t^c}{t^0} + \frac{1}{2}a_{ab}t^a t^b + b_a t^a t^0 + \frac{1}{2}c(t^0)^2 + F_{\text{inst}}(t) \ . \tag{5.36}$$

Multiplet	#	Fields
SUGRA	1	$g_{\mu\nu}, A_1^0$
vector	$h^{2,1}$	A_1^k, z^k
hyper	$h^{1,1}$	v^a, b^a, c^a, ρ_a
tensor	1	B, C_2, Φ, C_0

Table 5.3: Bosonic fields arising in a type IIB compactification on a Calabi-Yau manifold. Reproduced from [57].

Here κ_{abc} are the classical triple-intersection numbers of $\mathcal{M}$

$$\kappa_{abc} = \int \omega_a \wedge \omega_b \wedge \omega_c \; . \tag{5.37}$$

One can furthermore show that the constants a_{ab} and b_a do not enter the Kähler potential. The constant c can be determined in terms of the Euler characteristic of $\mathcal{M}$ as [84]

$$c = \frac{1}{(2\pi i)^3}\chi(M)\zeta(3) \; . \tag{5.38}$$

The resulting Kähler potential is, to leading order at large volume $v^a \to \infty$,

$$K = -\log\left(\frac{i}{6}\kappa_{abc}(t^a - \bar{t}^a)(t^b - \bar{t}^b)(t^c - \bar{t}^c)\right) \; . \tag{5.39}$$

5.3.2 Type IIB

In the case of type IIB, taking into account the self-duality of dC_4, the p-form fields give rise to

$$\begin{gathered} C_0(x,y) = C_0(x) \; , \qquad\qquad C_2(x,y) = C_2(x) + c^a(x)\omega_a \; , \\ C_4(x,y) = A_1^K(x)\alpha_K + \rho_a(x)\tilde{\omega}^a \; . \end{gathered} \tag{5.40}$$

We see that C_4 gives rise to $h^{2,1} + 1$ vector fields. Splitting off the graviphoton, there will be $h^{2,1}$ vector multiplets. The scalar components of these have to be the complex structure moduli. The remaining multiplet assignments are shown in table 5.3.

Type IIB theory on a Calabi-Yau threefold gives rise to a 4D $\mathcal{N} = 2$ supergravity theory with $h^{2,1}$ vector multiplets and $h^{1,1}$ hypermultiplets.

The prepotential of the type IIB theory can be obtained by identifying the holomorphic section (5) with the periods (5.20) of the Calabi-Yau and applying the relation (5.5). The resulting Kähler potential is given by (5.4). It is not corrected perturbatively or non-perturbatively.

5.4 Mirror Symmetry

We are now in the position to discuss mirror symmetry, which will be a central tool in chapter 8:

> Mirror symmetry is a duality between type IIA and type IIB on two different Calabi-Yau manifolds $\mathcal{M}$ and $\mathcal{W}$, which are said to be mirrors to each other. The pair of mirror manifolds is related by a flip of the Hodge diamond (5.18) along the diagonal
>
> $$h^{1,1}(\mathcal{M}) = h^{2,1}(\mathcal{W}) \,, \qquad\qquad h^{2,1}(\mathcal{M}) = h^{1,1}(\mathcal{W}) \,. \tag{5.41}$$

To see why such a duality is plausible, let us have a look at the rectangular torus $T^2 = S^1 \times S^1$ with radii R_1 and R_2. The Kähler and the complex structure moduli, which we discussed in section 3.2, are given in this case by

$$t = iR_1R_2 \qquad\qquad \tau = i\frac{R_1}{R_2} \,. \tag{5.42}$$

We pick out the second circle and perform a T-duality $R_2 \to 1/R_2$[4]. Under this, the Kähler and complex structure moduli interchange

$$t \leftrightarrow \tau \,, \tag{5.43}$$

giving a proof of mirror symmetry in the case of 1D CYs in terms of T-duality.

A similar interpretation of mirror symmetry for threefolds has been conjectured by Strominger, Yau and Zaslow [85]. They argued that a pair of mirror CYs should admit a description in terms of two three-torus fibrations over a special Lagrangian sub-manifold. As for the torus, which can be trivially presented as an S^1 fiber bundle over S^1, mirror symmetry would then be realized by T-duality on the fibers. The core of their argument is as follows. Any duality between two supersymmetric theories is expected to include in particular a one-to-one mapping of the BPS states in the theory as well as their moduli spaces[5]. Consider

[4] We measure the radii in units of ℓ_s.

[5] If we view the Calabi-Yau geometry itself as a BPS state of the 10D SUGRA, this matching of BPS moduli spaces is nothing else than equation (5.41)

type IIA compactified on a CY $\mathcal{M}$. D0-branes are point-like and can sit at any point in $\mathcal{M}$, leading to a BPS state in the 4D theory. The moduli space of such states is precisely $\mathcal{M}$. For the type IIB theory on the mirror $\mathcal{W}$, it turns out that the only BPS states one can build on top of the CY geometry are D3-branes that wrap special Lagrangian three-cycles. It follows that there must be such a special Lagrangian cycle in $\mathcal{W}$ that, when wrapped by a D3-brane, it leads to a BPS state with moduli space identical to the original CY $\mathcal{M}$. In the type IIB theory, the transverse deformations of the brane form a three real-dimensional moduli space B. In order to match the dimension of $\mathcal{M}$, for each point in B there have to be three more moduli. If the special Lagrangian cycle has $h^1 = 3$, these are provided by the moduli space of flat connections on it, which is a T^3. This leads to the conclusion that the manifold $\mathcal{M}$ is a torus fibration $T^3 \to \mathcal{M} \to B$. One can exchange type IIA with type IIB and see that also $T^3 \to \mathcal{W} \to \tilde{B}$. T-duality on the fibers turns the D3-brane into a D0-brane and hence mirror symmetry *is* T-duality.

The above is not a proof of mirror symmetry but merely an interpretation of it unless one can prove the existence of the torus fibration for a certain class of Calabi-Yau manifolds. This is hard in practice, in particular because of the special Lagrangian condition. Topological T^3 fibrations were obtained in [86]. Mirror symmetry can also be understood as a duality between world-sheet theories. In this context, one can also construct mirror theories for backgrounds that are not Calabi-Yau [87].

Mirror symmetry was first conjectured in the context of the 2D $N = 2$ world-sheet SCFTs that are associated to Calabi-Yau compactifications [88]. It was then later observed in [78] that the set of Calabi-Yau hypersurfaces in weighted four-dimensional projective spaces satisfies an approximate symmetry under flipping the Hodge numbers. This was unexpected because Kähler and complex structure moduli arise in a very asymmetric way in this setting. Nowadays, several classes of explicit mirror constructions are known and they can be used as a practical tool which we will see below. After the pioneering work of Greene and Plesser [89], a construction for mirror manifolds of Calabi-Yau manifolds that can be constructed as hypersurfaces in toric varieties was found in [79]. This covers all of the cases that we will encounter in this thesis.

On the one hand, because the dilaton is in the hypermultiplet moduli space, mirror symmetry acting on the vector moduli space should hold order by order in g_s. On the other hand, because mirror symmetry is related to T-duality we can expect it to mix different orders in α' perturbation theory and even mix effects that are perturbative in α' with non-perturbative ones. It happens to be the case that the vector multiplet moduli space of type IIB on a Calabi-Yau $\mathcal{M}$ receives no α' corrections, while the vector multiplet moduli space of type IIB on the mirror $\mathcal{W}$ receives both perturbative and non-perturbative ones. This is very powerful, because it allows us to relate a classical period computation on the type IIB side to an infinite number of world-sheet instanton corrections on the type IIA side [90].

The interested reader can find a good first introduction to the practical applications of mirror symmetry in [84]. For a more comprehensive exposition we refer to [91]. For our applications in chapter 8 it will be of great importance that we can relate the prepotential on the type IIB side directly to the prepotential on the type IIA side

$$F^{2,1}_{\mathcal{W}}(z) \equiv F^{1,1}_{\mathcal{M}}(t) \qquad\qquad t^a = t^a(z^k) \ . \tag{5.44}$$

The map between the complex structure moduli of type IIB and the Kähler moduli of type IIA is known as the *mirror map*. In order to compute the left-hand side we need to determine the holomorphic three-form Ω on the mirror manifold $\mathcal{W}$ and compute the periods for an integral symplectic basis of three-cycles. This can be done and is in principle a straightforward computation using tools from complex analysis. The right-hand side takes the complicated general form (5.36) with an infinite number of non-perturbative corrections. These are highly non-trivial to compute but needed to match the non-polynomial form of the type IIB prepotential on the left-hand side of equation (5.44). They have the interpretation as arising from world-sheet instantons wrapping holomorphic curves of various degrees in $\mathcal{M}$. Computing them directly in the type IIA picture constitutes a complicated counting problem. The comparative ease of classically computing $F(z)$ on the type IIB side and translating the result via the mirror map is what was initially very surprising to researchers in the field.

6 4D N≤ 1 Vacua of Type IIB

We now take first steps into the direction of phenomenologically interesting string vacua with $\mathcal{N} = 1$ SUSY in 4D. As we have seen in section 5.2, in absence of fluxes of the field strengths in the NS-NS and R-R sectors we arrive at $\mathcal{N} = 2$ compactifications to Minkowski space. Using fluxes and an orientifold projection, one can eliminate one half of the supersymmetries and end up in either Minkowski or AdS. In order to break SUSY completely and get de Sitter, it turns out that anti-branes are a useful ingredient. In the following sections we will restrict to the type IIB theory because its vacua form the basis of chapter 10. We will mostly follow the review [57], which also covers type IIA[1].

6.1 4D $\mathcal{N} = 1$ Supergravity

The 4D $\mathcal{N} = 1$ SUGRA has as its basic building blocks the gravity, the vector and the chiral multiplets. They consist of a graviton, a gauge field or a complex scalar accompanied by a single fermion and auxiliary fields. We have discussed the action for the chiral and vector multiplets in globally supersymmetric theories in section 3.1. The main novelty when coupling to gravity is the typically logarithmic structure of the Kähler potential for the chiral multiplet scalars and the addition of a negative contribution to the F-term potential. We will present here only the structure of the bosonic part of the action. More details can be found for example in [59]. The $\mathcal{N} = 1$ SUGRA action in 4D is (we follow [57])

$$\begin{aligned} S^{4D}_{\mathcal{N}=1} = \int \frac{1}{2} R \star 1 - K_{I\bar{J}} DM^I \wedge \star D\overline{M}^{\bar{J}} - V \star 1 \\ - \frac{1}{2}\mathrm{Re}(f_{\alpha\beta}) F^\alpha \wedge \star F^\beta - \frac{1}{2}\mathrm{Im}(f_{\alpha\beta}) F^\alpha \wedge F^\beta \ . \end{aligned} \tag{6.1}$$

In this equation M^I are the chiral multiplet scalars where $I = 1, \dots, n_C$ and F^α are the vector multiplet field strengths with $\alpha = 1, \dots, n_V$. The scalar potential

$$V = e^K \left(K^{I\bar{J}} D_I W D_{\bar{J}} W - 3|W|^2 \right) + \frac{1}{2} (\mathrm{Re} f)^{-1\,\alpha\beta} D_\alpha D_\beta \tag{6.2}$$

[1]The effective action of $\mathcal{N} = 1$ type IIA compactifications was first derived in [92].

is a sum of an F-term and D-term potential. Kähler covariant derivatives are defined by

$$D_I W = \partial_I W + (\partial_I K) W \tag{6.3}$$

and they are required because the superpotential is a section of a line bundle over the scalar field space with Kähler transformations as the transition functions [59]

$$W \sim e^{-f(M)} W \qquad\qquad K \sim K + f(M) + \bar{f}(\overline{M}) \ . \tag{6.4}$$

6.2 Orientifolds, Branes and Fluxes

Let us now see how type IIB compactifications on Calabi-Yau orientifolds fit into this picture. The orientifold projection is a combination of a geometric quotient and a world-sheet involution[2] [57]

$$(-)^{F_L} \Omega_p \sigma \ , \tag{6.5}$$

where Ω_p is the world-sheet parity, F_L is the left-moving fermion number and σ is a holomorphic isometric involution $\sigma^2 = 1$ of the Calabi-Yau $\mathcal{W}$. A consistent choice for the action of σ on the holomorphic three-form and Kähler form is

$$\sigma^* \Omega = -\Omega \qquad\qquad \sigma^* J = +J \ . \tag{6.6}$$

The cohomology groups of $\mathcal{W}$ split into even and odd sectors

$$H^{p,q} = H^{p,q}_+ \oplus H^{p,q}_- \ . \tag{6.7}$$

The spectrum is obtained as a truncation to the *orientifold even* states of the $\mathcal{N} = 2$ one from chapter 5. Whether a field is projected out is determined both by its world-sheet parity and its geometric parity if it arises from wrapping a cycle. The set of surviving fields under the above projection is listed in table 6.1.

The fixed point sets of the orientifold projection in $\mathcal{W}$ are called *orientifold planes*. These will be O3- and O7-planes in our setting. Orientifold planes enter the Einstein equations as localized objects with negative tension. This negative tension can now be balanced by positive tension from fluxes. Besides having to solve the Einstein equations, one has to satisfy a tadpole cancellation condition arising from the integrated F_5 Bianchi identity[3] [57]

$$N_{D3} - \frac{1}{4} N_{O3} + \frac{1}{(2\pi)^4 \alpha'^2} \int H_3 \wedge F_3 \ , \tag{6.8}$$

[2]This *choice* of projection corresponds to O3/O7-planes, as we discuss below.

[3]There is an analogous condition for D5-brane charge.

Multiplet	#	Fields
SUGRA	1	$g_{\mu\nu}$
vector	$h^{2,1}_{+}$	A^{α}_{1}
chiral	$h^{2,1}_{-}$	z^k
	$h^{1,1}_{+}$	$v^{\alpha}, \rho_{\alpha}$
	$h^{1,1}_{-}$	b^a, c^a
	1	Φ, C_0

Table 6.1: Bosonic fields in a Calabi-Yau orientifold of the type IIB string with O3/O7-planes. Adapted from [57].

so possible fluxes are constrained by the number of orientifold planes. The contribution from the orientifold planes is replaced by the Euler characteristic of the CY four-fold in the F-theory context. In applying the formula, one has to keep in mind that D7-branes wrapped on four-cycles induce a D3-charge.

The fluxes that can be turned on are those of the self-dual five-form field strength F_5 and the complexified three-form field strength G_3. In order to not interfere with 4D isometries, the three-form fluxes should be along elements of $H^3(\mathcal{W})$ of the compactification. In the case of F_5, there are no non-trivial five-cycles in a Calabi-Yau, so it will be of the general form

$$F_5 = (1 + \star) d\alpha(y) \wedge \mathrm{vol}_4 \ , \tag{6.9}$$

where vol_4 is the volume form of the non-compact space.

The branes will have to extend along the non-compact directions in order to preserve isometries. As shown in figure 6.1, D3-branes are point-like in $\mathcal{W}$, whereas D7-branes wrap elements of $H_4(\mathcal{W})$. We note here that a background with D7-branes leads to a running of the dilaton in the transverse directions in $\mathcal{W}$. The manifold $\mathcal{W}$ will acquire Ricci curvature and generically contain regions of strong coupling, which mandates an embedding into F-theory [64, 93]. By placing four D7-branes on top of an O7-plane, it is possible though to get a background with constant axio-dilaton. The so-called Sen limit or orientifold limit of F-theory [94] generalizes this situation. We will assume in the following that a type IIB orientifold description of our background is applicable.

Because the $\mathcal{N} = 2$ vector multiplet scalars already live on a (special) Kähler manifold, it turns out that the scalars surviving the projection inherit the Kähler potential from the $\mathcal{N} = 2$ theory.

We will specialize here to the simple case $h^{1,1}_{+} = 1$ and $h^{1,1}_{-} = 0$. In this case,

D3:	X^0	X^1	X^2	X^3	X^4	X^5	X^6	X^7	X^8	X^9
	✗	✗	✗	✗						

D7:	X^0	X^1	X^2	X^3	X^4	X^5	X^6	X^7	X^8	X^9
	✗	✗	✗	✗			✗	✗	✗	✗

Figure 6.1: D3-branes that extend along the four non-compact dimensions are point-like in the compactification geometry, whereas D7-branes wrap four-cycles.

the Kähler potential of $\mathcal{N} = 1$ reads [57]

$$K = -\ln\left(-i\int \Omega \wedge \bar{\Omega}\right) - \ln(-i(S-\bar{S})) - 3\ln\left(-i(T-\bar{T})\right) \ , \tag{6.10}$$

with $S = C_0 + ie^{-\Phi}$ the type IIB axio-dilaton and

$$T = \rho + i\tau \ , \qquad\qquad \tau = \kappa_{111} v^2 \sim \mathcal{V}^{2/3} \ . \tag{6.11}$$

The imaginary part of the single Kähler modulus measures the overall volume of the Calabi-Yau.

The other ingredient of the scalar part of the $\mathcal{N} = 1$ action is the Gukov-Vafa-Witten superpotential [95]

$$W = \int G_3 \wedge \Omega \ , \tag{6.12}$$

which depends only on the complex structure moduli and the axio-dilaton. Using this, we can compute the classical scalar F-term potential

$$V_F = e^K \sum_k |D_k W|^2 \ . \tag{6.13}$$

In this expression the Kähler moduli drop out due to the so-called no scale structure, except for the overall $\exp(K)$ factor, which cannot lead to a stabilization. This potential has Minkowski vacua where the F-term conditions

$$F_k = D_k W = 0 \tag{6.14}$$

are satisfied. Generically, because there is one equation for each complex structure modulus and the axio-dilaton, they are all stabilized. The F-term for the Kähler modulus determines whether the above vacuum is supersymmetric or not

$$F_T = D_T W = 0 \qquad \Leftrightarrow \qquad W = 0 \ . \tag{6.15}$$

In order to fix the Kähler modulus, one has to rely on non-perturbative corrections to the superpotential. These can be generated by brane instantons or gaugino condensation and are generally of the form [21]

$$\delta W = A(z^k)e^{iaT} \ . \tag{6.16}$$

We will see in section 6.4 how these can stabilize T.

6.3 Warping, GKP and Klebanov-Strassler

Once we include three-form fluxes, it was shown by Giddings, Kachru and Polchinski (GKP) [12] that, in the type IIB orientifold limit, the geometry of the CY reacts back mildly by acquiring a non-trivial warp factor as in equation (2.2). Hence, the field content of the SUGRA is still that of a Calabi-Yau compactification, but quantities such as the Kähler potential can receive corrections. One can often work in a limit where the fluxes are dilute and one can stay within the usual SUGRA framework of the preceding section.

GKP determined a solution to the 10D SUGRA equations of motion under the assumption that only certain localized sources like O3/D3, O7/D7 on four-cycles and D5-branes on shrinking two-cycles are included in the background[4]. They found that the 10D equations of motion require

$$\star_6 G_3 = iG_3 \qquad\qquad \alpha(y) = e^{4A(y)} \ , \tag{6.17}$$

where $\alpha(y)$ is the expectation value of the five-form field strength as in equation (6.9). This means that the three-form flux is imaginary self-dual (ISD) and the five-form sources determine the warp factor through the equation of motion for F_5. For closed $dH_3 = dF_3 = 0$, the three-form equations of motion are fulfilled because of the ISD condition. The remaining equations of motion are then the Einstein equations and axio-dilaton equation of motion.

At large radius of the Calabi-Yau one expects the fluxes to be sufficiently dilute so that one can trust the unwarped Kähler potential (6.10) and superpotential (6.12). Consistent with this, one finds from the 10D analysis that the overall scale of the Calabi-Yau is still a flat direction of the potential. However, there can still be corrections to the Kähler potential from warping that respect this no-scale structure, see [96, 97]. This will become important in chapter 10.

The global picture of a warped Calabi-Yau compactification is that there is a bulk geometry with warp factor of order one and which has *warped throat* regions where three-form flux generates strong warping. This is shown in figure 6.2.

[4]These all satisfy a certain BPS-like bound on their energy momentum tensor

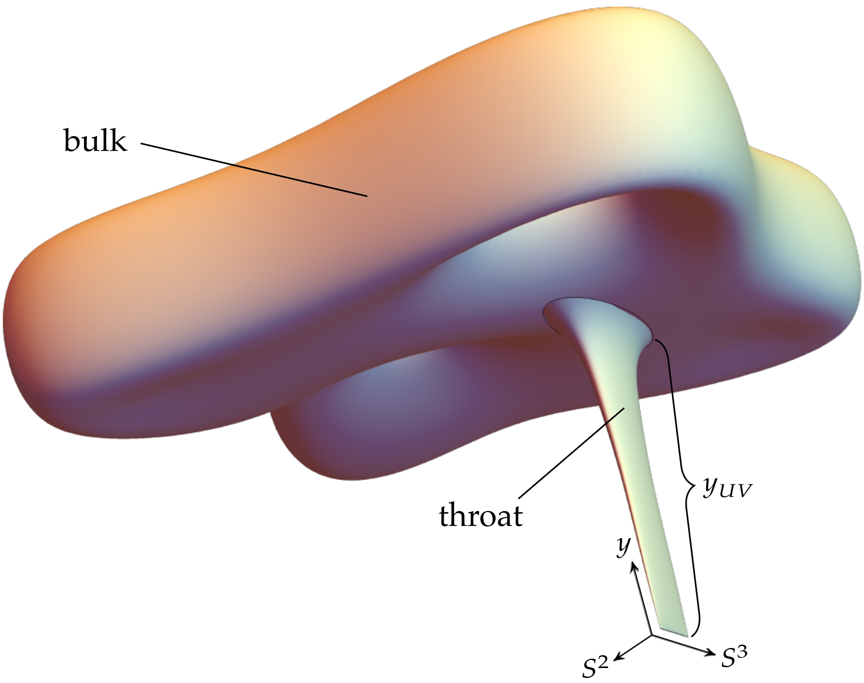

Figure 6.2: A warped throat region in a Calabi-Yau can be formed by adding three-form flux on the S^3 of a deformed conifold region. Figure reproduced from [33].

In order to understand the effects of warping on the EFT, one can use a local description of these warped throats. Such a throat can be constructed by placing N D3-branes and M fractional D3-branes[5] at the tip of the deformed conifold (sec. 5.2), as studied by [98]. The (fractional) D3-branes induce the following three-form flux on the conifold A-cycle and B-cycle

$$\frac{1}{(2\pi)^2\alpha'}\int_A F_3 = M \qquad \frac{1}{(2\pi)^2\alpha'}\int_B H_3 = N\,. \tag{6.18}$$

The 10-dimensional SUGRA solution for such a situation was found by Klebanov and Strassler [82] and is known as the KS throat. It is a warped solution of the form (2.2). From now on we conventionally denote the metric of the compactification geometry by g_{mn} if the warp factor is to be included and by $\tilde{g}_{mn}$ if we strip it off.

[5]These are D5-branes wrapped on the S^2 of the conifold and then moved to the tip where it shrinks to size zero.

The Einstein frame metric of the KS throat is

$$\widetilde{ds}^2 = \frac{1}{2}|\epsilon|^{\frac{4}{3}} K(y) \left[\frac{dy^2 + (g^5)^2}{3K^3(y)} + \cosh^2\left(\frac{y}{2}\right)\left((g^3)^2 + (g^4)^2\right) \right. \\ \left. + \sinh^2\left(\frac{y}{2}\right)\left((g^1)^2 + (g^2)^2\right) \right] . \tag{6.19}$$

The one-forms g^i can be found in equation (5.27) and $K(y)$ is the function

$$K(y) = \frac{(\sinh(2y) - 2y)^{1/3}}{\sqrt[3]{2}\sinh(y)} . \tag{6.20}$$

The warp factor of the Klebanov-Strassler solution is given by

$$e^{-4A(y)} = 2^{\frac{2}{3}} \frac{(\alpha' g_s M)^2}{|\epsilon|^{\frac{8}{3}}} \mathcal{I}(y) , \tag{6.21}$$

where

$$\mathcal{I}(y) = \int_y^\infty dx \frac{x \coth x - 1}{\sinh^2 x} (\sinh(2x) - 2x)^{\frac{1}{3}} . \tag{6.22}$$

6.4 The KKLT Scenario and de Sitter

We have now discussed all of the ingredients that are needed to explain the construction of de Sitter vacua with full moduli stabilization in the context of type IIB string theory on Calabi-Yau orientifolds, which was proposed by Kachru, Kallosh, Linde and Trivedi (KKLT) [21]. The KKLT scenario is a framework for how the typical ingredients of such compactifications can in principle be used rather than a top-down construction. It furthermore relies heavily on the use of four-dimensional effective supergravity, in contrast to for example the Klebanov-Strassler solution. Therefore, one is exposed to the danger of neglecting ten-dimensional effects such as a back-reaction of the 10D geometry. Nevertheless, because of the importance of constructing de Sitter solutions for cosmology, a lot of work has been dedicated to the task of putting KKLT on a firm foundation, elucidating also many possible 10D obstructions and their circumvention. As we will discuss in section 7.3.2, it was recently conjectured that de Sitter space cannot be a vacuum of string theory. Since then, even more effort has been put into the task of identifying a possible point of failure in KKLT. One such attempt will be presented in chapter 10.

The KKLT scenario can be thought of as a three-step procedure

1. Pick a Calabi-Yau orientifold and fluxes in order to *stabilize complex structure moduli* in a non-SUSY Minkowski minimum of the F-term potential.

2. Make sure that non-perturbative effects *stabilize the Kähler moduli*, leading to a shallow AdS minimum.

3. Add an exponentially small positive energy contribution to *uplift* this to a de Sitter minimum.

Each step should correspond to a different 4D effective supergravity description, at least in the favorable case where one can separate the different energy scales. We will now briefly discuss the three steps and some of the relevant literature.

In the first step, by estimating the size of the flux contributions to the kinetic terms in the 10D theory, KKLT find that the complex structure moduli and the dilaton are generically stabilized at a mass scale α'/R^3, which is polynomial in the Kähler modulus. For this estimate it is important that one can neglect the effects of warping, as we will see in chapter 10. In general SUSY will be broken and we end up with a non-zero value of the superpotential $W_0 \neq 0$.

In the second step, one considers the following effective theory for the Kähler modulus

$$K = -3\ln\left(-i(T-\bar{T})\right) \,, \qquad W = W_0 + A\,e^{iaT} \,, \tag{6.23}$$

where it is assumed that, due to the exponentially small T-dependent contribution, one can integrate out the complex structure moduli and dilaton such that W_0 and A are fixed constants. In order to find a minimum, one has to argue that the two terms in (6.23) are of comparable size, so one requires $|W_0| \ll 1$. Generically this is not the case and whether this can be achieved with integral fluxes is not clear. Under this assumption, with $\tau = \mathrm{Re}(T)$, one can confirm the existence of a SUSY AdS minimum of the potential with

$$\tau\,e^{-a\tau} \sim |W_0| \,, \qquad m_\tau^2 \sim -|W_0|^2/\ln|W_0| \,, \qquad V_{AdS} \sim -e^{-2a\tau}/\tau \,. \tag{6.24}$$

For exponentially small $|W_0|$, τ can be within its perturbative regime $\tau \gg 1$ while it has an exponentially small mass compared to the other moduli. In addition, the AdS minimum is exponentially shallow and very narrow.

In the third step, one would like to add an exponentially small amount of energy that can barely cancel the negative V_{AdS} and should lead to a de Sitter minimum. This step necessarily breaks SUSY. The authors of [21] proposed to use an anti-D3-brane. Because the positive energy contribution of such a brane is not small when placed at a generic point in the CY, one has to place it in a warped region such as the tip of the KS throat from section 6.3 where it is red-shifted. In doing so we get a contribution [21]

$$\delta V = 2\frac{e^{-4A_0}T_3}{g_s^4}\frac{1}{\rho^3} \,. \tag{6.25}$$

The constant $\exp(-A_0)$ represents the warp-factor at the tip of the KS throat. Under the assumption that we can freely tune the warp-factor without disturbing the previous moduli stabilization, this leads to a de Sitter minimum in 4D.

7 Swampland Conjectures

We would now like to introduce some of the swampland conjectures, which have been motivated in the introduction as potential means of distinguishing between effective field theories being in the landscape and those being in the swampland. Although the subject is still rapidly evolving, it has already been reviewed with an emphasis on the developments of the recent years in [99].

While there is no universally accepted definition of what constitutes a "good" swampland conjecture, there are several requirements that such a conjecture should fulfill. First, the statement of the conjecture should become trivial in the $M_p \to \infty$ limit where gravity decouples. Second, the conjecture should be universally valid in any effective field theory that can be consistently coupled to gravity. The second statement is controversial, as most of the evidence for this universality comes from string theory. Nevertheless, we will see that often we can also construct arguments that rely only on the consistency of quantum field theory with general assumptions about quantum gravity such as black hole thermodynamics or holography.

In the following we will roughly divide swampland conjectures into those conjectures that place constraints on the spectrum of a vacuum of quantum gravity, those that constrain the geometry of the moduli space of quantum gravity and those that constrain the admissible gravitational backgrounds. In section 7.5 we will then argue that there exists a non-trivial web of relations between these different types of conjectures, similar to the duality web of string theory in section 4.2. Finally, we will discuss how the idea of emergent fields in the context of quantum gravity could be used as an organizing principle in the cosmos of swampland conjectures.

The swampland conjectures have many possible far-reaching consequences for phenomenology. These have been explored to a great extent for the case of the weak gravity and distance conjectures applied to the idea of large field inflation or trans-Planckian field displacements [28, 29, 31, 100–141]. Even in this case, there is unfortunately no consensus on the stringency of the constraints because of certain order one fudge factors that appear in the conjectures. Furthermore, it is often unclear whether certain strong or weak forms of the conjectures are correct. It is thus of utmost importance that we can make these statements more precise by studying them rigorously within the realm of string theory. Examples of such precision science in the swampland can be found in the recent work on F-theory and the swampland [134, 142–146] but also in the context of holography [147].

7.1 Conjectures on the Spectrum

In this section we will introduce several conjectures that were put forward about the spectrum in vacua of quantum gravity. By this we mean the spectrum of different particle species, as well as possible symmetries of their interactions.

7.1.1 No Global Symmetries

An old "folk theorem" about quantum gravity states that it is incompatible with global symmetries (see for example [148] and references therein).

No Global Symmetries:
There are no global symmetries in a quantum theory of gravity.

This predates the notion of the swampland but we will still classify it as a swampland conjecture. It is also arguably the most rigorously understood and most universal swampland conjecture, because very general and powerful arguments for it exist also outside of the context of string theory.

A classic argument (see e.g. [29]) only relies on the existence of Hawking radiation, a notion of approximate locality and the validity of quantum mechanics. Consider for example a compact global $U(1)$ symmetry. If the symmetry acts locally on the field operators in a non-trivial way we can form a macroscopic black hole of mass M and some charge Q under this $U(1)$. Because the interaction Hamiltonian commutes with the symmetry charge, the evaporation will be a random walk in charge space. There are now two possibilities. In the first case, the Hawking evaporation continues until there is no black hole and we violate charge conservation. In the second case, the Hawking evaporation stops at the point where we cannot trust effective field theory anymore, so we get Planck-scale "remnants" of mass $m_{\text{remn.}} \simeq M_p$, the mass of which does not depend on Q due to the symmetry. By considering an ensemble of such evaporating black holes, we get an ensemble of remnants with arbitrary charge. The pathology arises in this case because we have to associate an infinite additional remnant entropy to the black hole

$$S = \frac{A}{4G} + S_{\text{remn.}} = \infty \; . \tag{7.1}$$

It was shown in [149] that among other issues this fact leads to a renormalization of the Newton constant to zero, hence gravity decouples[1]. This argument extends to any compact Lie group [147] but breaks down if the global symmetry is a discrete group, such as $\mathbb{Z}_2$. We will come back to this point later.

One can also argue within the context of the AdS/CFT correspondence that global symmetries in the bulk are inconsistent. The notion of a global symmetry was elaborated in [147], where it was also proven that such a symmetry in the AdS bulk leads to a contradiction. For the case of $U(1)$, the argument is again quite simple. A global $U(1)$ symmetry in the bulk implies the same global symmetry in the boundary CFT. The associated Noether current on the boundary is dual to a gauge boson in the bulk, so the global $U(1)$ can be at most the global part of a $U(1)$ gauge group [147, 152]. Again, this argument fails for discrete symmetries because there is no associated Noether current. It was shown in [148] that *any* global symmetry, whether discrete or continuous, is incompatible with the locality of bulk reconstruction in AdS/CFT.

Finally, we can examine the situation in string theory. Starting from the world-sheet point of view, it is similarly hard to envision how one would go about to construct a global symmetry in the target space. Assuming the existence of a continuous such symmetry acting on the target space fields, it would lead to a continuous global symmetry of the sigma model with associated world-sheet current j. One can then build a vertex operator from this current that constructs a massless spin-1 field that gauges the global symmetry. We refer the reader to [153] for a rigorous discussion of this point.

The case of discrete global symmetries is more subtle, but the AdS/CFT arguments of [147] imply their inconsistency. We would like to emphasize again that while the above discussion focused on the symmetry group $G = U(1)$, the arguments generalize also to non-abelian and p-form symmetries [147, 150].

7.1.2 Completeness and Compactness

In the following, we discuss two conjectures that were formulated for example in [150, 154]. The *completeness conjecture* is the statement that in a quantum theory of gravity with some gauge symmetry:

> **Completeness Conjecture**
> The lattice of allowed electric and magnetic charges is completely occupied by physical states.

To see why this could be true, observe that the (generalized) Reissner-Nordström solution exists for arbitrary charge independent of the existence of elementary

[1] See [150] for a contradiction with the covariant entropy bound [151].

particles of the same charge. It has been argued though that this argument is invalid because the Reissner-Nordström solution should be thought of as a wormhole connecting two opposite charges with a net charge equal to zero [147].

There is also a connection to the previously presented conjecture about global symmetries. Assume for example that a $U(1)$ gauge theory has no charged states, that is, we are dealing with the free Maxwell field

$$S = \int dA \wedge \star dA \, . \tag{7.2}$$

Independent of the gauge symmetry, this has an additional global one-form symmetry $A \to A + \omega$, where ω is a constant one-form. Demanding the absence of this global symmetry, we infer that there must be at least some charged state [147]. Now assume that in a $U(1)$ gauge theory only the sub-lattice $2\mathbb{Z} \subset \mathbb{Z}$ is occupied by physical states. Then analogously the theory will have a global $\mathbb{Z}_2$ one-form symmetry [155], which is forbidden.

The *compactness conjecture* states that

Compactness Conjecture
If G is an internal symmetry group, G must be compact.

In the example of $\mathfrak{g} = \mathfrak{u}(1)$, the global structure of the gauge group could also be the universal cover $\mathbb{R}$ of $U(1)$. In this case, we would have at least two mutually irrational charges, which we can use to generate all possible charges $q \in \mathbb{R}$. A theory with this property will again run into contradictions with entropy bounds [147, 150, 154].

Both the completeness conjecture and the compactness conjecture have been confirmed within the context of the AdS/CFT correspondence in [147].

7.1.3 Weak Gravity Conjecture(s)

The *weak gravity conjecture* (WGC) can be thought of as a strengthening of the statement that there should be no global symmetries. It states that in a consistent quantum theory of gravity, which contains a $U(1)$ gauge field with gauge coupling g, it should be impossible to simply make this gauge symmetry global by going to the limit $g \to 0$. More precisely, there are two statements [29]:

Electric WGC: There has to exist a particle with

$$m \lesssim gqM_p \, . \tag{7.3}$$

Furthermore, there is a magnetic verion:

Magnetic WGC: The cutoff of the EFT has to satisfy

$$\Lambda \lesssim g M_p \,. \tag{7.4}$$

Just as in the case of the global symmetry conjecture, one can approach this statement from several angles – black hole decay, AdS/CFT and string theory. In [29] evidence for the WGC was presented mainly in the form of a black hole decay argument, which was supplemented by string theory examples. In the first case, we observe that the $g \to 0$ limit corresponds to decoupling the gauge field from all charged states, which means that in this limit the gauge symmetry becomes global. Condition (7.4) prevents this limit as the EFT description necessarily breaks down.

Let us see how we arrive at equation (7.3) by considering evaporating charged black holes. Assuming that the lightest charged particle violates (7.3) by some factor $R > 1$

$$m = R \times g q M_p \,, \tag{7.5}$$

we find that the number of black holes of fixed mass M below the extremality bound $M \geq g Q M_p$ that cannot decay diverges with $1/g$ as

$$N_{\text{remn.}} \sim \left(1 - \frac{1}{R}\right) \frac{M}{g M_p} \,. \tag{7.6}$$

Thus, we get a mild version of the remnant problem. The statement (7.3) is equivalent to the statement that extremal black holes can decay and hence the complete absence of remnants in the theory. The magnetic version (7.4) can then be obtained by requiring also the decay of magnetically charged black holes. To this end, one estimates the monopole mass as

$$m_{\text{mag}} \sim p^2 \frac{\Lambda}{g^2} \,. \tag{7.7}$$

Note that in the above, we have not been careful about the (theory-dependent) order one factors in the extremality bound, which are also reflected in the precise form of the WGC, see for example [156]. One should adjust the inequality (7.3) such as to require the existence of a super-extremal particle state. In connection with this, one has to be careful when extending the WGC to theories with multiple $U(1)$-factors. In this case, the statement that is equivalent to the decay of extremal black holes is that the convex hull of all charge-to-mass ratios in the theory should contain the region corresponding to sub-extremal black holes [157].

There are several interesting generalizations of the WGC that have been proposed in the literature. In the original paper [29], it was already stated that one can naturally generalize the WGC to p-form gauge fields. The WGC will then require the existence of super-extremal branes and an associated cutoff. The case $p = 0$ is subtle and corresponds to the so-called *axion WGC*. Here the interpretation is that the zero-dimensional objects to which the zero-form field "couples" is an instanton. The WGC then requires that in a theory with an axion decay constant f (f^2 being the coefficient of the kinetic term), there exists an instanton with action S such that

$$fS \lesssim M_p \,. \tag{7.8}$$

Because instanton effects contribute to the effective potential with terms of order $\exp(-nS)$, where n is the instanton number, a controlled instanton expansion requires $S > 1$, and hence

$$f \lesssim M_p \,. \tag{7.9}$$

This statement was in fact conjectured to be true in string theory independently of the WGC [28]. It has an important implication for moduli spaces in string theory. Because its continuous shift-symmetry has to be broken to a discrete gauge symmetry by the arguments of section 7.1.1, an axion φ with decay constant f should be thought of as living on a field space with periodicity $\varphi \simeq \varphi + 2\pi f$. This means that

Axion WGC Periodic directions in moduli space should be sub-Planckian when measured with the moduli space metric.

In section 7.2.1 we will discuss how this idea generalizes to non-periodic directions.

A further important generalization is related to the completeness conjecture of section 7.1.2. In [156], the authors studied whether the WGC is consistent under dimensional reduction. It was shown that it is in fact inconsistent and could be repaired by requiring a state satisfying (7.3) at every point in the charge lattice. It was then later argued in [158] that this requirement could be weakened to a sub-lattice. These statements were backed up by examples from string theory. Nevertheless, in [144], concrete examples in string theory were found where there is a tower of super-extremal states that does not form a sub-lattice. We adopt here the point of view advocated in [159] that in general the weakest requirement for consistency is a *tower WGC*:

Tower WGC The electric WGC is satisfied by an infinite tower of states with growing mass and charge.

A simple example realizing this is given by Kaluza-Klein theory. The Kaluza-Klein states are extremal with respect to the Kaluza-Klein $U(1)$

$$m_n = \frac{n}{R}\,, \qquad g_{\text{KK}} = \frac{1}{RM_p}\,, \tag{7.10}$$

so they correspond to an infinite tower of states that marginally satisfy the corresponding WGC.

In the context of string theory, it is important to realize that the WGC is only satisfied in fully consistent backgrounds and that seemingly small deformations can lead to its apparent violation. Consider a single D3-brane in type IIB on a 10D flat space background. The world-volume theory does not have any charged particles because all of the states transform in the (trivial) adjoint representation. This is not in contradiction with the WGC only because the four-dimensional theory on the brane world-volume is non-gravitational due to the non-compact transverse space. We can make it gravitational by compactifying the transverse space. The apparent violation of the WGC in this case is resolved by noticing that the D3-brane now creates a tadpole that has to be canceled by introducing for example an anti-brane. This produces stretched strings that, from the point of view of the brane, are precisely the WGC states satisfying (7.3). One finds

$$\frac{m_{\text{open}}}{g_{\text{open}}M_p} \sim \frac{M_s \Delta x}{M_s \mathcal{V}^{1/2}/\sqrt{g_s}} \sim \frac{\sqrt{g_s}}{\mathcal{V}^{1/3}} \ll 1\,, \tag{7.11}$$

where the brane separation Δx and the volume $\mathcal{V}$ are measured in string units, see also [29]. We have assumed for simplicity that $\Delta x \sim \mathcal{V}^{1/6}$ scales homogeneously with the volume and that we work in the perturbative regime where $g_s \ll 1$ and $\mathcal{V} \gg 1$.

Finally, we note that there are proposed generalizations of the WGC to include scalar fields [127, 142, 160]. It seems plausible that some sort of generalization has to exist because properties of black holes like the extremality bound can also depend on forces mediated by scalar fields.

7.2 Conjectures on Moduli Space Geometry

The set of quantum field theories that can consistently be UV-completed into a theory that contains gravity is called the moduli space of quantum gravity. Swampland conjectures that constrain the topology and geometry of this moduli space were pioneered by Ooguri and Vafa in [30]. While it seems hard to make this idea concrete outside of string theory, one can give general arguments for this type of conjecture by relating them to one of the "spectrum conjectures" of section 7.1. The axion WGC of section 7.1.3 was only a first glimpse of this idea, which we will further develop in section 7.5.

A foundational hypothesis for the discussion of the following section is that the moduli space of quantum gravity is (locally) described by the expectation values of scalar fields, which is in fact one of the conjectures proposed by [30]. In the following we will focus on the so-called distance conjecture.

7.2.1 Distance Conjecture

The swampland *distance conjecture* (SDC) [30] is the statement that the moduli space of quantum gravity has points at infinite distance, at which an infinite tower of states necessarily becomes massless. More precisely, comparing the effective field theory at some points A and B:

Swampland Distance Conjecture:
As the distance $\Theta = d(A, B)$ between two points in the moduli space diverges, there must be an infinite tower of states becoming exponentially light

$$m_n|_B \sim m_n|_A e^{-\alpha\Theta} , \tag{7.12}$$

where α is of order one. Because an effective field theory can only contain a finite number of light particles[a], any effective description must break down for $\Theta \to \infty$.

[a] We will make this precise in section 7.4.

Here the distance on moduli space is naturally given by the distance function induced by the metric determined from the kinetic terms of the moduli

$$S \supset -\int G_{ij} d\phi^i \wedge \star d\phi^j . \tag{7.13}$$

To be precise

$$d(A, B) = \inf\left\{ \int_\gamma \sqrt{\gamma^*(G)} \;\middle|\; \gamma : [0, 1] \to \mathcal{M} , \gamma(0) = A , \gamma(1) = B \right\} , \tag{7.14}$$

where

$$\gamma^*(G) = G_{ij} \frac{d\phi^i(\gamma(\tau))}{d\tau} \frac{d\phi^j(\gamma(\tau))}{d\tau} d\tau^2 \tag{7.15}$$

is the pull-back of the moduli space metric onto a curve $\gamma(\tau)$ connecting the points A and B.

For the moduli space of a circle compactification of string theory we see that the SDC is indeed fulfilled due to the Kaluza-Klein states becoming exponentially light for $R \to \infty$, as well as the winding modes, which become light for $R \to 0$[2], as can be seen in equation (2.15). In fact, in this case a much stronger statement holds – the exponential behavior is not only valid at infinite distance loci but also for infinitesimal displacements. This is an artifact of the maximal supersymmetry-preserving nature of toroidal compactifications and will not hold in general as we will see in chapter 8. The departure from the asymptotic behavior is quantified by the *refined SDC* (RSDC) [119, 122]:

Refined Swampland Distance Conjecture:
The exponential decay of the mass scale of an infinite tower of states predicted by the swampland distance conjecture is a good approximation to $m_n|_B/m_n|_A$ for any two points that are separated by a critical distance Θ_c that is of order one in Planck units.

The swampland distance conjecture is not a wild speculation. First of all, it is in line with the general expectation from $\mathcal{N} = 2$ gauge theories that singularities of the moduli space correspond to massless states. Furthermore, it seems to hold in all controlled top-down string constructions and can even be partially proven if one has more than eight supercharges [161]. We will find evidence for the refined version in chapter 8.

As with the WGC it is important to realize that an apparent violation of the SDC is often an illusion. Consider for example the $\mathcal{N} = 4$ SYM theory described in section 3.1. From the 4D point of view, this is a perfectly consistent quantum field theory. We can take the moduli to infinity without any states becoming light. In a string theory realization though, for finite M_p, the fields ϕ^i are always periodic and the moduli space becomes compact. For example, if we compactify the type I string to four dimensions on a torus, the ϕ^i are secretly components of a 10D gauge field and hence subject to large gauge transformations. In this way the violation of the SDC is censored.

7.3 Conjectures on Admissible Vacua

Other swampland conjectures aim to rule out certain types of vacua of the effective potential. We will concentrate here on two conjectures that discriminate among vacua by the value of the cosmological constant.

[2]In principle Kaluza-Klein monopoles also become light in the limit, so the consistency of this particular case with the SDC does not seem to necessarily depend on a string theory embedding.

7.3.1 Anti-de Sitter Conjecture

Anti-de Sitter space forms the basis of one of our best approaches to quantum gravity, namely holography [162]. The arguably best understood examples of the AdS/CFT correspondence arise in string theory and are of the simple form $AdS_d \times S^{D-d}$, where D is ten or eleven. For instance, the canonical example of $AdS_5 \times S^5$ is formed by placing a large number of D3-branes on top of each other and considering the near horizon geometry. Because the D3 couples to F_5, there is a flux along the AdS_5.

These stringy examples of the AdS/CFT correspondence are supersymmetric. It has been conjectured that [163, 164]:

No non-SUSY AdS:
Non-supersymmetric AdS holography is not realizable in a consistent quantum theory with low energy description in terms of the Einstein gravity coupled to a finite number of matter fields.

We may also call this the *AdS swampland conjecture*.

This was based mainly on two observations. The first one applies to AdS spaces that are supported by flux, like the $AdS_5 \times S^5$ example. If one interprets the electric WGC (7.3) in a strong way, such that equality can only hold in the supersymmetric BPS case, then the WGC predicts a decay channel for the AdS space where spherical brane instantons nucleate in the AdS bulk, reducing the flux as they expand towards the AdS boundary [165].

If this is not the case, there are often generalized "bubble of nothing" instabilities in non-SUSY AdS examples [163, 164] where the AdS arises from compactification of some higher-dimensional theory. This can happen for example if the transverse space to the AdS is not simply connected where the analysis is similar to the original bubble of nothing of Witten [166]. There it was shown that there exists a five-dimensional instanton solution to Kaluza-Klein theory that under Wick rotation maps to a non-singular geometry appearing like a bubble expanding at the speed of light to the 4D observer. The space terminates at the surface of the bubble where the Kaluza-Klein circle shrinks to size zero to form a smooth cigar-shaped geometry.

The idea of [163] is that these instabilities can be detected instantaneously on the AdS boundary because the volume of AdS diverges there. Hence, either there can be no AdS/CFT in the presence of such an instability, or more conservatively there can be no smooth AdS spacetime near the boundary.

7.3.2 De Sitter Conjecture

It has also been speculated very recently that there can be no de Sitter vacua in string theory [167]. The *de Sitter conjecture* differs from the AdS conjecture of the last section in that there is no proposed decay mechanism but rather a bound on the allowed effective potential that would completely forbid these vacua. We quote here the refined version of the conjecture [168]:

de Sitter Conjecture:
The effective potential for the scalar fields should satisfy either

$$|\nabla V| \geq \frac{c}{M_p} \cdot V \, , \tag{7.16}$$

or

$$\min\left(\nabla_i \nabla_j V\right) \leq -\frac{c'}{M_p^2} \cdot V \, . \tag{7.17}$$

Here the absolute value in (7.16) is computed with the moduli space metric and the left-hand side of (7.17) is the smallest eigenvalue of $\nabla_i \nabla_j V$ in an orthonormal frame. The constants c and c' are of order one.

The left-hand side of equation (7.17) computes the smallest eigenvalue of the mass-squared matrix. Thus, if it is obeyed there is a tachyon in the spectrum that has a sub-horizon wavelength if the spacetime is approximately de Sitter. Hence it is expected to lead to an instability. If the second inequality is violated, the conjecture postulates that (7.16) is fulfilled, which forbids a minimum of the potential at positive vacuum energy. Therefore, a meta-stable de Sitter vacuum is forbidden if the conjecture were true.

We would like to mention that related results, which are independent of a string theory embedding, were obtained in [169, 170]. There it was argued that if we think of de Sitter space as a condensate of gravitons, the semi-classical description necessarily has to break down after the *quantum break time*. The precise relation of this idea to the de Sitter conjecture was described in [171, 172].

7.4 Emergence of Kinetic Terms

It has been argued that in many instances the kinetic terms of fields in quantum gravity can be thought of as arising from one-loop corrections that occur in the presence of a large number of particle species coupling to them [130, 131, 173, 174]. In this context, the tower of states predicted by the distance conjecture is dual to the geometry of the moduli space in the sense that one can obtain the latter by integrating out the former. To get an understanding of this, we first have to understand the impact of an infinite tower of particles on a low-energy effective field theory containing gravity.

While the whole tower should be important for the UV-completion of the theory, only a finite number N_{sp} of the particles in the tower will be visible in an effective field theory with a cutoff Λ. At one loop, the graviton propagator is renormalized by the N_{sp} light particle species as

$$M_p^2|_{\text{tree}} + c\, N_{\text{sp}}\, \Lambda^2\,, \tag{7.18}$$

where c is an order one constant. We can at most trust our perturbative calculations up to the point where both terms are of comparable magnitude. Thus, it is expected that the scale at which quantum gravity effects become important is lowered to the *species scale* [175]:

Species Scale:
In a theory with N_{sp} particle species, the quantum gravity scale is lowered relative to the classical coefficient M_p^2 of the Einstein-Hilbert term to

$$\Lambda_{\text{sp}}^2 = \frac{M_p^2}{N_{\text{sp}}}\,. \tag{7.19}$$

Given the mass spectrum of the tower $m(q_n)$ one can then solve equation (7.19) together with the requirement $m(q_{N_{\text{sp}}}) \leq \Lambda_{\text{sp}}$ to get a concrete formula for Λ_{sp}.

If we have a tower of particles with equal spacing Δm, it follows that $N_{\text{sp}} = \Lambda_{\text{sp}}/\Delta m$ leading to

$$\Lambda_{\text{sp}}^3 = \Delta m M_p^2\,, \qquad\qquad N_{\text{sp}} = \left(\frac{M_p}{\Delta m}\right)^{2/3}\,. \tag{7.20}$$

For example, in Kaluza-Klein theory we have $\Delta m = 1/R$ and we recover the 5D Planck scale

$$\Lambda_{\text{sp}} = \sqrt[3]{\frac{1}{R}M_4^2} = M_5\,. \tag{7.21}$$

The counting of excited string states is tricky because at finite g_s they are in general not stable. It has been argued that the string oscillators contribute a factor of $1/g_s^2$ to the effective number of species [176, 177]. Using this, one indeed recovers the string scale as the species scale

$$N_{\text{sp}} \sim \frac{1}{g_s^2} \times \underbrace{\left(\Lambda_{\text{sp}}^6 \mathcal{V}_6\right)}_{\text{KK species}} \qquad \Rightarrow \qquad \Lambda_{\text{sp}}^8 = \frac{g_s^2}{\mathcal{V}_6} M_4^2 \simeq M_s^8\,. \tag{7.22}$$

The infinite tower of states of the swampland distance conjecture gives rise to an infinite distance singularity in moduli space as follows. Suppose we have either a scalar field ψ or a fermion λ whose mass $m(\phi)$ depends on a scalar field ϕ. Expanding the mass term around the expectation value of ϕ produces Yukawa couplings

$$S_{\text{Bose}} \supset -\int d^4x\, m\left(\partial_\phi m\right)\, \phi\, \psi^2\,, \qquad S_{\text{Fermi}} \supset -\int d^4x \left(\partial_\phi m\right)\, \phi\, \bar{\lambda}\lambda\,. \quad (7.23)$$

When we integrate out ψ or λ, we get a one-loop contribution to the kinetic term of ϕ [130]

$$\delta g_{\phi\phi}\Big|^{\text{Bose}}_{\text{1-loop}} = \frac{|\partial_\phi m|^2}{8\pi^2}\left(\frac{2\pi}{3\sqrt{3}} - 1\right)\,, \qquad \delta g_{\phi\phi}\Big|^{\text{Fermi}}_{\text{1-loop}} = \frac{|\partial_\phi m|^2}{4\pi^2}\log\left(\frac{\Lambda_{\text{UV}}}{m}\right)\,. \quad (7.24)$$

Suppose now that we have not only a single field, but rather a whole infinite tower of scalar fields ψ_n or fermions λ_n with mass $m_n(\phi) \simeq nm(\phi)$, where ϕ is a canonically normalized scalar field. In this case, the contribution of the tower to the kinetic term of ϕ can become large. In the scalar case

$$\delta g_{\phi\phi}\Big|_{\text{1-loop}} \simeq \sum_{n=0}^{N_{\text{sp}}} n^2 \left(\partial_\phi \Delta m\right)^2 \simeq N_{\text{sp}}^3 \left(\partial_\phi \Delta m\right)^2 = \left(\partial_\phi \log \Delta m\right)^2 M_p^2\,. \quad (7.25)$$

For the fermion case, the sum over the logarithm in (7.24) gives the same result up to an overall factor and sub-leading corrections that vanish in the limit $N_{\text{sp}} \to \infty$.

Assume now that $\Delta m(\phi) \simeq \phi^\alpha$ vanishes to some polynomial order α at $\phi = 0$, so that an infinite tower of particles becomes massless at this point. Naively, measured with the classical kinetic term, this point is at finite distance in the moduli space parameterized by ϕ. However, the one-loop correction (7.25) diverges logarithmically at this point and it follows that the point is at infinite distance in the quantum moduli space. Because of the logarithmic structure of the kinetic term, it follows that the particles become massless exponentially fast in the proper distance.

The same argument can be again applied to gauge fields if we assume an infinite tower of *charged* states with $m(q) \simeq q\Delta m$. In this case, the one-loop correction to the kinetic term is

$$\delta\left(\frac{1}{g^2}\right)\Big|_{\text{1-loop}} \simeq \sum_{q=0}^{Q_{\text{max}}} q^2 \simeq Q_{\text{max}}^3 = N_{\text{sp}}^3 = \left(\frac{M_p}{\Delta m}\right)^2 \sim e^{2\alpha\Theta}\,, \quad (7.26)$$

where in the last step we have assumed again that Δm is a function of some canonically normalized modulus. The dependence of Δm on this modulus must be exponential according to the preceding discussion. The one-loop contribution becomes dominant for $\Theta \to \infty$. The interpretation of this is that vanishing gauge couplings are at infinite distance in the moduli space.

In string theory, as we have seen, there are usually also points at infinite distance in the classical moduli space metric. In this light the above discussion allows for two different interpretations, see also [139]:

1. **One-loop consistency:** The functional form of the one-loop correction to the propagator induced by the tower is the same as the tree-level result.

2. **Emergence:** The propagator is zero in the UV and the field becomes dynamical by integrating out the tower of states.

That the second, more radical interpretation is not totally absurd can be seen by studying toy models, such as the $\mathbb{CP}^n$ model [173].

Finally, we would like to point out that similar ideas about emergent dynamics were put forward in the context of Sakharov's induced gravity [178] (see [179] for a review). Sakharov originally advocated the idea that a quantum field theory on a non-dynamical manifold "induces" gravitational dynamics at one-loop. This idea is close in spirit to option two above and was termed "one-loop dominance" in [179].

7.5 Disentangling a Web of Conjectures

The conjectures we have introduced are not at all independent. Rather they form a web, similar to the duality web connecting the different string theories that we have discussed in section 4.2. Some of the connections between the conjectures that we have introduced are summarized in figure 7.1.

We have already discussed in section 7.1.3 how the weak gravity conjecture can be thought of as a deformation of the conjectured absence of global symmetries. In fact, the same is true for the swampland distance conjecture. At the infinite distance locus in field space $\Theta = \infty$, a continuous global shift symmetry $\Theta \to \Theta + \delta\Theta$ emerges. The analogy between the two situations is not an accident. It was shown in [122] that the (lattice or tower) weak gravity conjecture can be used to argue for the distance conjecture if the gauge coupling in equations (7.3), (7.4) is determined by the expectation value of a modulus. In such a setting the WGC has to hold locally on the moduli space. By considering black hole backgrounds with spatial gradients of the gauge coupling, one can then argue that for super-Planckian displacements of the modulus the gauge coupling has to vary exponentially and the tower of WGC states becomes exponentially light. This is precisely the statement of the refined SDC.

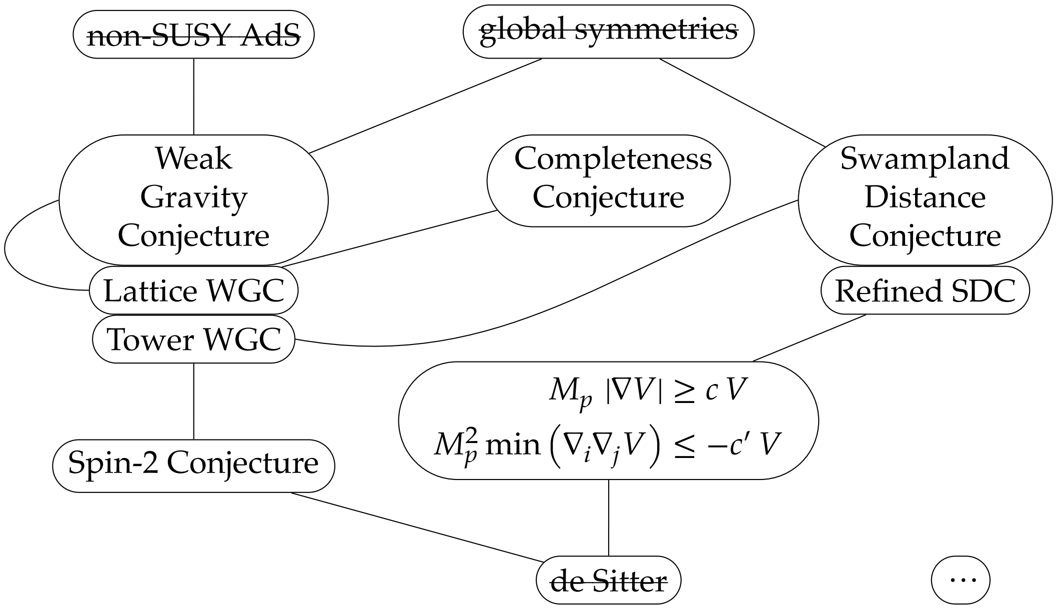

Figure 7.1: Swampland Conjectures and their relations.

While the lattice WGC trivially implies the completeness conjecture, it was shown to be false in general [144, 158]. Nevertheless, there seems to be an interesting connection between the tower WGC and the completeness conjecture. As discussed in [150], while it is possible to remove charges from the spectrum in QFT by sending the mass of the corresponding field to infinity, this procedure fails in a gravitational theory because the corresponding particle would transition into a black hole. The tower WGC states that something stronger is true – under any deformations of the theory, an infinite subset of the charge lattice must be populated with super-extremal particle-like states.

The WGC furthermore leads directly to a spin-2 conjecture, as we will discuss in chapter 9, which in turn has a connection with the de Sitter conjecture [32]. In [168] it was shown that also from the point of view of the SDC one can argue for the refined de Sitter conjecture (7.16), (7.17). Because this connection will become relevant in chapter 10, we briefly describe the argument here. The authors argued that the entropy $S \lesssim R^2 = 1/V^{1/2}$ within an expanding universe is bounded by the area of its apparent horizon. Assuming that the tower of SDC states dominates the Hilbert space in the infinite distance limit, so that total entropy grows as $S_{\text{tower}} \sim N^\gamma R^\delta$ and saturates the bound, one obtains

$$V(\phi) \sim N^{-\frac{2\gamma}{2-\delta}} \sim e^{-c\phi}\,, \tag{7.27}$$

which implies (7.16). Hence, at infinite distance points in field space, the SDC seems to lead to evidence for the dS conjecture.

The idea of emergent kinetic terms from section 7.4 can be viewed as the fundamental principle behind the seemingly different swampland conjectures. There we have presented mainly the argument for the SDC. We have also seen that, from this perspective, the exponential scaling of gauge couplings at infinite distance points of the moduli space, which connects the SDC and WGC, is quite natural. It was argued in [131, 173, 174] that emergent gauge kinetic terms can imply the WGC in certain cases. In chapter 9 we will furthermore show that also the spin-2 conjecture has connections to this idea. Even though these are hints that the idea of emergence could be a deep principle underlying the swampland, much remains to be understood.

Part III

Results

8 The Swampland Distance Conjecture in Calabi-Yau Moduli Spaces

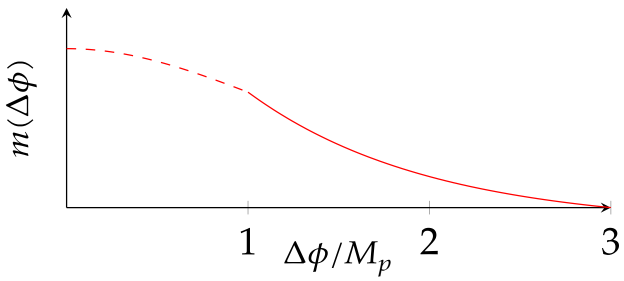

Figure 8.1: Exponential decrease of masses for $\Delta\phi > M_p$.

The Refined Swampland Distance Conjecture (RSDC) supplements the Swampland Distance Conjecture (SDC) by the statement that the predicted behavior can be avoided locally in moduli space for distances smaller than $\mathcal{O}(1)$ in Planck units (see also sec. 7.2.1). This is illustrated in figure 8.1.

Historically, such a behavior was first observed in the context of $\mathcal{N} < 2$ flux compactifications where the moduli fields obtain a potential and hence cannot be considered as moduli in the strict sense, see chapter 3. This naturally leads to the question whether the observed evasion of the behavior predicted by the SDC can be meaningfully interpreted as a violation of the conjecture as it was originally meant to apply only to theories with more than one supercharge[1].

Another obstruction to a clear interpretation of the results in this setting is the fact that in a theory with a scalar potential, the physical trajectories followed by fields (for example in inflation) are largely determined by the potential itself and not by the kinetic terms, see figure 8.2. The trajectories are in general completely unrelated to the geodesics that enter the definition of distance

[1]It is only in this case that one expects to have a true moduli space parameterized by massless scalar fields as quantum corrections generically induce mass terms for scalar fields in theories with less supersymmetry.

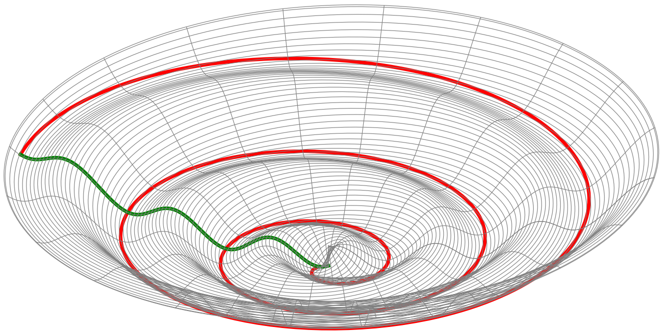

Figure 8.2: In higher-dimensional field spaces, a potential can lead to physical trajectories (red) that are parametrically longer than the distance (green) between the start point and end point. The particular potential plotted can be found in [180].

in the (R)SDC. This begs the question whether the RSDC, if true for displacements along geodesics, has any non-trivial implications for phenomenology at all. Adding further confusion to this puzzle, in [119, 126] evidence for the RSDC was gathered along trajectories in field space that started in local minima and continued along a trough of the potential. Nevertheless, other early arguments for a refined version of the SDC were also obtained in absence of an explicit scalar potential [122] and did not seem to rely directly on the amount of supersymmetry.

The objective of [31], which we discuss in this chapter, was to take a step back into the world of eight supercharges ($\mathcal{N} = 2$ SUSY in 4D) and provide evidence for the RSDC in this more simple setting. $\mathcal{N} = 2$ supergravity theories can be obtained from string theory by compactifying any one of the two type II superstring theories on a Calabi-Yau threefold.

Our discussion will focus here mostly on one-dimensional Calabi-Yau moduli spaces. Details on two-dimensional moduli spaces are covered in the doctoral thesis of Florian Wolf [181]. Parts of the following sections of this chapter have been quoted, up to minor changes, in verbatim from [31].

8.1 Calabi-Yau Manifolds as a Testing Ground

Let us now see how a possible testing ground for the RSDC could arise within the context of type II string theory on a Calabi-Yau threefold. Early works on the SDC focused on displacements around the large volume (LV) or large complex structure (LCS) points in the Kähler and complex structure moduli spaces respectively. In the case of large volume, the Kähler potential for the Kähler moduli in type IIA string theory can be cast into the form (5.36). This leads to a logarithmic structure of the kinetic term of $r = \mathrm{Im}(t)$

$$S \supset -\int d^4x\sqrt{-g}\left(\frac{\partial r}{r}\right)^2 . \tag{8.1}$$

Hence, one concludes that Kaluza-Klein modes, whose masses scale with inverse powers of r as

$$M_{\text{KK}} \sim \frac{M_s}{\sqrt{r}} \sim \frac{M_p}{r^2} \sim M_{\text{KK},0}\, \exp(-2\lambda\Theta) , \tag{8.2}$$

fall off exponentially in the distance Θ along the trajectory $r \to \infty$ and one is unlikely to encounter a violation of the SDC here. A similar story can be told about the complex structure moduli space and its LCS point on the type IIB side as it is related to the large volume point by mirror symmetry.

The conclusion is that we should look for violations of the SDC in regions of the moduli space that are far away from the LV or LCS points and where corrections to the asymptotic form of the Kähler potential (5.39) become important. A few words of caution are in order here. Based on what we have seen in section 7.2.1 for the example of a toroidal compactification it is not immediately clear that we will be able to obtain an interesting result even for $t \ll 1$. In this case we saw that the SDC is exactly fulfilled for every point in the moduli space due to T-duality.

Fortunately, we will see that mirror symmetry for Calabi-Yau manifolds is more complicated than T-duality on the torus and there are indeed regions in the Kähler moduli space of a Calabi-Yau manifold that look very much unlike large volume. The resulting picture will heuristically be the following: As we move away from an infinite distance point in the moduli space there are two options. On the one hand, we can be moving towards another infinite distance point, which could be dual to the first one, with an associated second tower of light states (this is the case for the torus). On the other hand it can also be the case that the moduli space terminates in a Planck-size bulk, where the SDC is possibly violated. The two different situations are depicted in figure 8.3.

To see how this works in a concrete example, let us discuss the Kähler moduli space of the most canonical Calabi-Yau threefold – the quintic hypersurface in $\mathbb{CP}^4$, which was discussed in section 5.2. As the quintic has $h^{1,1} = 1$, the moduli space is complex one-dimensional and parameterized by the overall volume modulus. It is depicted in figure 8.4 and features two different regions labeled as large volume (LV) and Landau-Ginzburg (LG)[2].

[2]This is the terminology appropriate for the type IIA description.

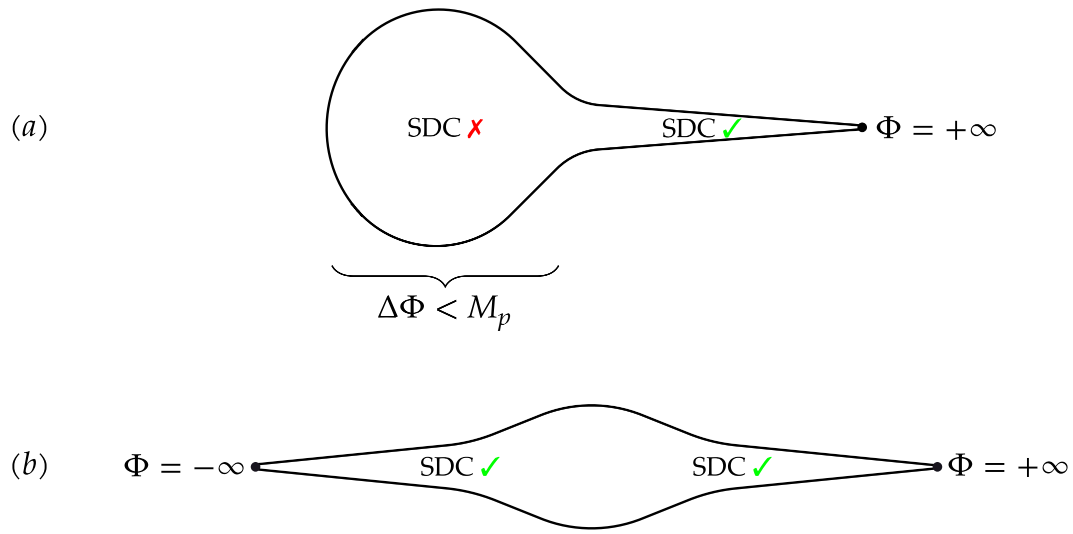

Figure 8.3: Two possible structures of the moduli space according to the RSDC. In (a) the moduli space is cut off by a finite size bulk region. In (b) the moduli space has two "opposite" infinite distance points with two towers of states whose masses scale inversely to each other with respect to the distance.

In the large volume phase the compactification is appropriately described by a 10D supergravity approximation with the additional inclusion of world-sheet instanton corrections. In the one-dimensional case, the general form of the type IIA prepotential (5.36) specializes to [90]

$$F = -\frac{5}{6}t^3 + at^2 + bt + i\underbrace{\frac{\zeta(3)}{2(2\pi)^3}\chi(\mathcal{M})}_{\equiv -c} + \mathcal{O}\left(e^{2\pi i t}\right). \tag{8.3}$$

The terms proportional to t, t^2 do not enter the moduli space metric and we will neglect them in the following discussion. The prepotential converges only in the LV region and the instanton expansion breaks down below the dashed line in figure 8.4. The region below is accessible by analytic continuation and can be interpreted as a different phase of the same system where the geometric description as a Calabi-Yau compactification breaks down, but a CFT description is still available [182]. Another special point in this moduli space is the conifold one, which under mirror symmetry maps to the point $\psi = 1$ in the complex structure moduli space of the mirror quintic, at which it develops a conifold singularity (see the discussion around (5.30)).

While we have seen that the SDC holds in the large volume phase, we can now ask the question whether it also holds in the small volume or Landau-Ginzburg

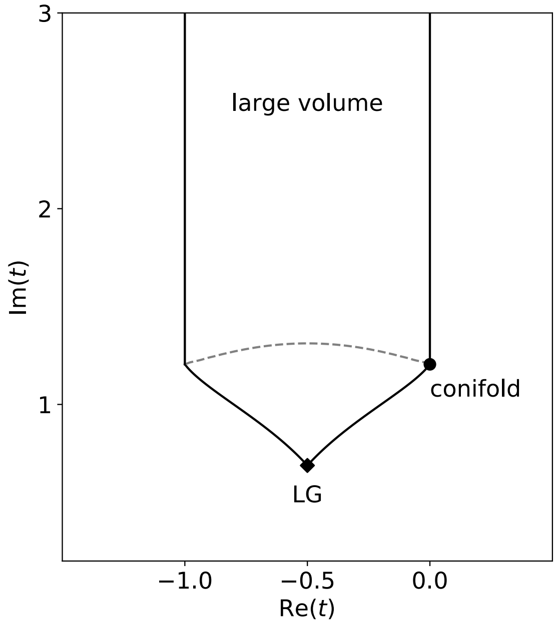

Figure 8.4: Sketch of the Kähler moduli space of the quintic. The dashed line represents the boundary between the geometric and non-geometric phases.

phase. We do not expect any discontinuity in the mass spectrum while following a geodesic that passes through the phase boundary. Hence the transition between the behavior in the LG phase and the asymptotic exponential decrease in the LV phase predicted by the SDC is necessarily somewhat fuzzy, see figure 8.5. While the (non-refined) SDC holds strictly only at the large volume *point*, as the instanton corrections are never strictly zero elsewhere, we want to have an operationally meaningful definition of when the SDC is approximately fulfilled and when not.

In the following analysis it will turn out to be useful to identify the distance between the LG point and the phase boundary as the critical distance Θ_0 that enters the RSDC. This quantity is closely correlated to the maximal distance of two points within the LG phase. Let us check that this quantity is indeed a good proxy for when the exponential behavior becomes relevant. From equation (8.3) we find that the asymptotic form of the proper distance is

$$\Delta\Theta = \frac{\sqrt{3}}{2}\log(t) + \frac{\sqrt{3}c}{5}\frac{1}{t^3} + \mathcal{O}\left(\frac{1}{t^6}\right) . \tag{8.4}$$

The point where the perturbative $1/t^3$ contribution is of the same order as the

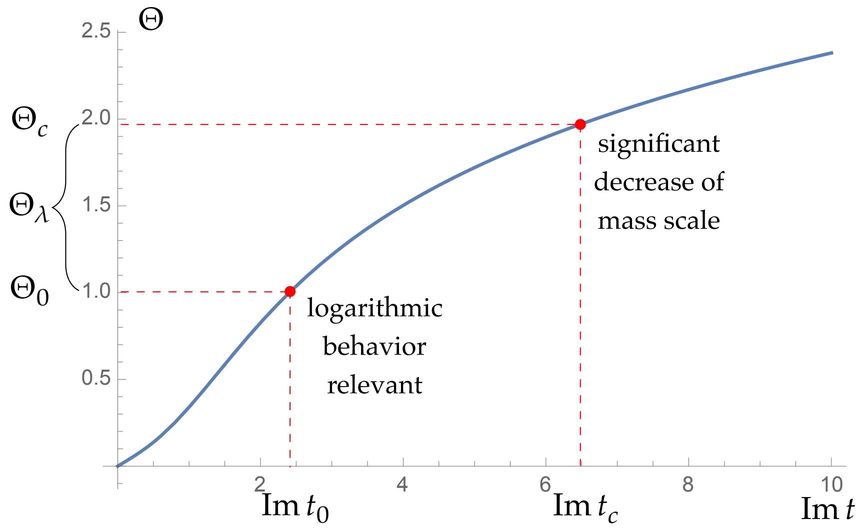

Figure 8.5: Expected relation between proper field distance Θ and $\mathrm{Im}\, t$.

asymptotic logarithm is found to be at

$$t_{\mathrm{eq}} \simeq e^{\frac{1}{3}W(2c/5)} \,, \tag{8.5}$$

where $W(x)$ is the Lambert W-function. For the quintic with Euler characteristic $\chi = -200$, this is approximately at $t_{\mathrm{eq}} \simeq 1.06$, a value well below the minimal value of t in the large radius phase $t_{\mathrm{min}} \simeq 1.21$. Indeed, at the phase boundary, the logarithm is approximately twice the size of the $1/t^3$ term, which can be considered as being a small correction for all t larger than this. This is the case for all models that we analyzed and of course consistent with the fact that the $1/t$ expansion is a perturbative expansion that breaks down in the non-geometric regions of the moduli space.

As a result, the logarithm is a good approximation for the behavior of the proper distance over the whole large volume phase and only breaks down at the boundary to non-geometric regions. In other words, the only relevant contribution to Θ_0 comes from inside the non-geometric phases and the behavior predicted by the SDC sets in immediately after crossing the phase boundary.

In the following chapters we will introduce the relevant mathematics needed to compute the moduli space metrics and distances on the moduli space, as well as discuss mostly the one-parameter models $\mathbb{P}^4_{11111}[5]$, $\mathbb{P}^4_{11112}[6]$, $\mathbb{P}^4_{11114}[8]$ and $\mathbb{P}^4_{11125}[10]$[3] that form the basis of the analysis in [31]. We will briefly summarize

[3]These are certain Calabi-Yau manifolds with only a single volume modulus. The particular subset discussed are the only such ones that can be obtained as hypersurfaces in weighted projective space.

also the results for two-dimensional moduli spaces and one high-dimensional example. Finally, we will discuss the relation of our work [31] to other tests of the SDC in Calabi-Yau moduli spaces.

8.2 Computation of the Kähler Potential and the Mirror Map

In order to proceed with testing the RSDC in the setting of type IIA Calabi-Yau moduli spaces, we first need to compute the metric on the whole Kähler moduli space. This can be done directly on the Kähler side using the GLSM techniques of [182–186] or by using mirror symmetry to map the calculation to the complex structure moduli space of type IIB compactified on the mirror Calabi-Yau manifold. In the second case we also need the explicit form of the mirror map. Both methods were used in [31] to provide a crosscheck of our results in the case of the quintic. For other Calabi-Yau spaces with $h^{1,1} = 1$ the GLSM method was used. For the examples with $h^{1,1} = 2$ it turned out to be easier to compute the metric from the periods. Here we will describe only the computation of the metric and mirror map using mirror symmetry.

In order to compute an independent set of periods over the whole complex structure moduli space there are two basic methods. One approach is based on the fact that the periods satisfy the so-called Picard-Fuchs equations, see for example [187]. One can solve this system of differential equations numerically in the vicinity of singular points in the moduli space. In this way one can patch together the periods and metric from their local series expansions. Even if we manage to find a symplectic basis of periods locally, the process is complicated by the fact that the patching can involve coordinate transformations as well as symplectic transformations. As it turns out, there exists a second approach that avoids this. The procedure is as follows

- Find an analytic expression for a distinguished *fundamental period* at the large complex structure point.
- Analytically continue it to a point where it undergoes a monodromy. Moving around this point generates additional periods, which are in general not linearly independent.
- From this set, pick a basis of periods and perform the analytic continuation of this set over the whole moduli space.
- Turn it into a symplectic basis by requiring that all monodromies act as symplectic matrices.

In practice, the special point in the second step will be the Landau-Ginzburg point.

If the Calabi-Yau threefold $\mathcal{M}$ is defined by a degree $d = k_1 + \cdots + k_5$ vanishing polynomial P in a weighted projective space $\mathbb{P}^4_{k_1 \ldots k_5}$, then the mirror $\mathcal{W}$ can be constructed as a quotient $\mathcal{M}/G$, where G is a product of $\mathbb{Z}_n$ symmetries [89]. This is the so-called Greene-Plesser construction. The polynomial P is split into a defining (Fermat type) polynomial P_0, a fundamental deformation $\Phi_0 \cdot \prod_{i=1}^5 x_i = \Phi_0 \, e_0$ and all other possible deformations $\Phi_\alpha e_\alpha$. The Φ_i can be considered as variables on the complex structure moduli space of $\mathcal{W}$.

The holomorphic three-form is given by the residue

$$\Omega(\Phi_\alpha) = \mathrm{Res}_{\mathcal{W}} \left[\frac{\prod_{i=1}^5 dx_i}{P(x_i, \Phi_\alpha)} \right] , \tag{8.6}$$

and the fundamental period is defined as

$$\omega_0(\Phi_\alpha) = -\Phi_0 \oint_{B_0} \Omega(\Phi_\alpha) = -\Phi_0 \frac{C}{(2\pi i)^5} \int_\Gamma \frac{\prod_{i=1}^5 dx_i}{P(x_i, \Phi_\alpha)} , \tag{8.7}$$

where C is an arbitrary constant and the sign as well as factors of $2\pi i$ can be reabsorbed into it. Moreover, B_0 is the *fundamental cycle*, which is a T^3 in the limit $\Phi_0 \to \infty$, while Γ is an auxiliary contour in $\mathbb{C}^5$ that allows for a rewriting as a residue integral. In [188], in the large complex structure limit $\Phi_0 \to \infty$, the residue integral (8.7) has been carried out perturbatively in $1/\Phi_0$ to all orders.

Given the fundamental period in the large Φ_0 region of the moduli space, one can analytically continue the fundamental period to small Φ_0. In this region, one can obtain a complete set of periods by

$$\omega_j(\Phi) = \omega_0 \left(A^j \Phi \right) , \tag{8.8}$$

where A is an element of the symmetry group of the Fermat type polynomial P_0. These periods can be analytically continued back to the large Φ_0 region.

8.2.1 One-Parameter Models

The most simple CY threefolds that we will analyze are given by non-singular hypersurfaces in weighted projective spaces $\mathbb{P}^4_{k_1 \ldots k_5}$ and have $h^{1,1} = 1$. Their mirror duals are given by a discrete quotient of the vanishing set of

$$P = \sum_{i=1}^5 x_i^{d/k_i} + \Phi_0 \prod_{i=1}^5 x_i . \tag{8.9}$$

Periods for one-parameter models

In this case (8.7) evaluates to

$$\omega_0(\Phi_0) = \sum_{r=0}^{\infty} \frac{\Gamma(d\,r+1)}{\prod_{j=1}^{5}\Gamma(k_j\,r+1)\,\Phi_0^{d\,r}} \;. \tag{8.10}$$

A complete basis of the periods for the one-parameter CY hypersurfaces can be found in [189, 190].

Mirror map for one-parameter models

To obtain the mirror map, we take the known form of the fundamental period around the large complex structure point from the literature [187, 188]

$$\omega_0(z) = \sum_{n=0}^{\infty} c_n z^n \;. \tag{8.11}$$

The coordinate z is related to the deformation parameter of the fundamental deformation via

$$z = \frac{\prod_{j=1}^{5} k_j^{k_j}}{d^d}\psi^{-d} \;, \qquad \Phi_0 = d\psi \;. \tag{8.12}$$

For complete intersections, d is replaced by the sum of hypersurface degrees. The Picard-Fuchs equation admits exactly one solution linear in logarithms

$$\tilde{\omega}(z) = \frac{1}{2\pi i}\omega_0 \log(z) + \sum_{n=0}^{\infty} \tilde{c}_n z^n \;. \tag{8.13}$$

The series coefficients $\tilde{c}_n$ can be determined algorithmically from ω_0 as [187]

$$\tilde{c}_n = \frac{1}{2\pi i}\frac{\partial}{\partial\rho}c_{n+\rho}\bigg|_{\rho\to 0} \;, \tag{8.14}$$

where implicitly the coefficients c_n have to be analytically continued in n. The mirror map in the large complex structure/large radius regime is determined by its monodromy properties as

$$t(z) = \frac{\tilde{\omega}(z)}{\omega_0(z)} \;. \tag{8.15}$$

The continuation to small ψ is achieved by finding a Mellin-Barnes integral representation for the power series over c_n and $\tilde{c}_n$.

For degree d hypersurfaces in $\mathbb{P}^4_{k_1\dots k_5}$, we have

$$\begin{aligned} c_n &= \frac{\Gamma(dn+1)}{\prod_{j=1}^5 \Gamma(k_j n+1)} \,, \\ \tilde{c}_n &= \frac{c_n}{2\pi i}\left(\Psi(dn+1) - \sum_{j=1}^5 \Psi(k_j n+1)\right) . \end{aligned} \tag{8.16}$$

Here Ψ denotes the polygamma function. From the coefficients one can see that the series converge in the region $|z| < \prod_j k_j^{k_j}/d^d$. The corresponding Mellin-Barnes integrals are

$$\begin{aligned} \omega_0(z) &= \int_\gamma \frac{d\nu}{2i\sin(\pi\nu)} c_\nu(-z)^\nu \,, \\ \sum_{n=0}^\infty \tilde{c}_n z^n &= \int_\gamma \frac{d\nu}{2i\sin(\pi\nu)} \tilde{c}_\nu(-z)^\nu \,. \end{aligned} \tag{8.17}$$

For small z, corresponding to large ψ, we pick up the residues of the sine at $\nu \in \mathbb{N}_0$. The residue integral for $\omega_0(z)$ only gets contributions from the simple poles of the gamma function at $\nu = -n/d,\ n \in \mathbb{N}$. The second integrand also has poles at the same values of ν but now up to second order from the combination of the gamma and polygamma functions. For the analytic continuation of $\omega_0(z)$ into the Landau-Ginzburg regime we find

$$\omega_0(z) = -\frac{\pi}{d}\sum_{n=1}^\infty \frac{(-1)^n}{\sin(\pi n/d)} \frac{(-z)^{-n/d}}{\Gamma(n)\prod_{j=1}^5 \Gamma(1-nk_j/d)} \,, \qquad |z| > \frac{\prod_j k_j^{k_j}}{d^d} \,. \tag{8.18}$$

Furthermore, for the period containing the logarithm in the LCS phase we obtain

$$\tilde{\omega}(z) = \frac{\pi}{2id}\sum_{n=1}^\infty \frac{(-1)^n}{\sin(\pi n/d)} \frac{(-z)^{-n/d}}{\Gamma(n)\prod_{j=1}^5 \Gamma(1-nk_j/d)} \Big(\cot(\pi n/d) + i\Big) \,. \tag{8.19}$$

From this information one can now determine the mirror map via (8.15).

An example: The Quintic

For the quintic, that is, $\mathbb{P}^4_{11111}[5]$ with $\Phi_0 = 5\psi$, the fundamental period in the large complex structure regime $|\psi| > 1$ reads

$$\omega_0(\psi) = \sum_{r=0}^\infty \frac{\Gamma(5r+1)}{\Gamma^5(r+1)(5\psi)^{5r}} = \sum_{n=0}^\infty \frac{(5n)!}{(n!)^5(5\psi)^{5n}} \,. \tag{8.20}$$

Starting from this, one can analytically continue it to the Landau-Ginzburg point in order to find a basis of periods [90]. On the other hand, analytic expressions for a (different) basis of periods were found in [90] by solving the Picard-Fuchs equation around the LG point

$$\omega_k(\psi) = (5\psi)^k \sum_{n=0}^{\infty} \left(\frac{\Gamma(k/5+n)}{\Gamma(k/5)}\right)^5 \frac{\Gamma(k)}{\Gamma(k+5n)} (5\psi)^{5n} \,, \qquad k = 1,\dots,4 \,. \tag{8.21}$$

The analytic continuation of these to the LCS region $|\psi| > 1$ is straightforward as they can be represented as hypergeometric functions. The transformation to a symplectic basis $\Pi = \mathrm{m}\,\omega$ can be found in [191]. Using this, one finds

$$\begin{aligned} X^0 &= (2.021600 - 1.468779i)\,\psi - (0.696854 - 2.144697i)\,\psi^2\,, \\ &\quad - (0.237613 + 0.731300i)\,\psi^3 + O(\psi^4)\,, \\ X^1 &= 2.125637i\,\psi - 1.185559i\,\psi^2 + 0.404253i\,\psi^3 + O(\psi^4)\,, \\ F_0 &= 2.937558i\,\psi - 4.289394i\,\psi^2 + 1.462601i\,\psi^3 + O(\psi^4)\,, \\ F_1 &= (7.314220 - 11.691003i)\,\psi - (0.963029 - 6.520577i)\,\psi^2\,, \\ &\quad - (0.328374 + 2.223391i)\,\psi^3 + O(\psi^4)\,, \end{aligned} \tag{8.22}$$

from which one can compute the Kähler potential around the LG point

$$\begin{aligned} K &= -\log\left(-i(X^I\bar{F}_I - \bar{X}^I F_I)\right) \\ &= -\log\left(19.217617\,|\psi|^2 - 3.694710\,|\psi|^4 + 0.429576\,|\psi|^6\right. \\ &\qquad \left. + 0.320294\,|\psi|^7\cos(5\theta) + O(|\psi|^8)\right)\,. \end{aligned} \tag{8.23}$$

Here we have decomposed $\psi = |\psi|\exp(i\theta)$. This leads to a metric that is constant to leading order $g_{\psi\bar{\psi}} \sim 0.44$.

We can also compute the mirror map

$$t(\psi) = \int B + i\int J = \frac{X^1(\psi)}{X^0(\psi)}\,, \tag{8.24}$$

where we have used the fact that X^0 and X^1 in equation (8.22) correspond to the fundamental period and the period with a simple logarithm at the LCS point. We compute

$$t_{\text{LG}}(\psi) = -\frac{1}{2} + 0.688\,i + (0.279 + 0.384\,i)\;\psi + \dots \qquad \text{for} \qquad |\psi| < 1\,. \tag{8.25}$$

Figure 8.6 shows t_{LG} for different $\text{Arg}(\psi)$, which agrees with the more qualitative

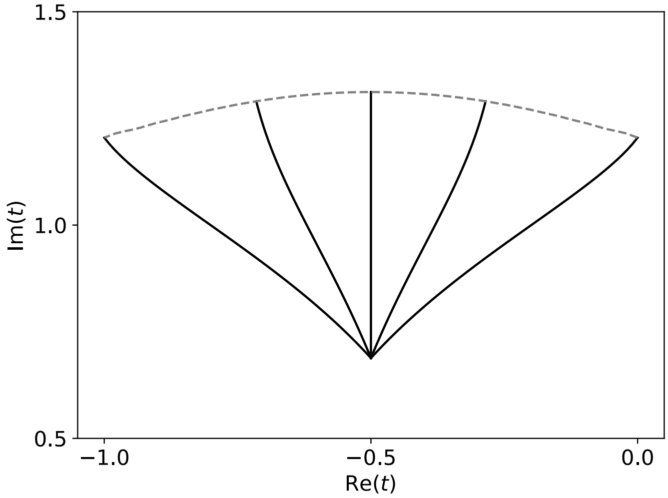

Figure 8.6: The fundamental domain of the Landau-Ginzburg region in the coordinate $t_{\text{LG}} = X_1/X_0$ for various $\text{Arg}(\psi)$.

plot in figure 8.4.

Using the results from section 8.2.1, we can also compute the mirror map around the LCS point

$$t_{\text{M}}(\psi) = \frac{5}{2\pi}\log(5) + \frac{5}{2\pi}i\log\psi + \ldots \qquad \text{for} \qquad |\psi| > 1\,. \tag{8.26}$$

Here we observe that the phase symmetry $\psi \to \alpha\,\psi$ maps to gauge transformations of the Kalb-Ramond field on the type IIA side, which is a general fact for such hypersurfaces.

8.2.2 Two-Parameter Models

The second class of CYs is a set of five two-parameter ($h^{1,1} = 2$) Fermat hypersurfaces in $W\mathbb{CP}^4$ for which the full set of periods in the Landau-Ginzburg phase was calculated in [188]. These are the manifolds

$$\begin{gathered}\mathbb{P}^4_{11222}[8]^{86,2}_{-168}\,,\quad \mathbb{P}^4_{11226}[12]^{128,2}_{-252}\,,\quad \mathbb{P}^4_{11169}[18]^{272,2}_{-540}\,,\\ \mathbb{P}^4_{14223}[12]^{74,2}_{-144}\,,\quad \mathbb{P}^4_{17222}[14]^{122,2}_{-240}\,.\end{gathered} \tag{8.27}$$

Their mirrors are given by the vanishing set of the polynomial

$$P = \sum_{j=1}^{5} x_j^{d/k_j} - \psi\, x_1x_2x_3x_4x_5 - \frac{d}{q_1}\phi\, x_1^{q_1}x_2^{q_2}x_3^{q_3}x_4^{q_4}x_5^{q_5}\,, \tag{8.28}$$

after modding out an appropriate discrete symmetry group. Here d denotes the degree of the polynomial, the k_i are the projective weights, $D = d/q_1$ is always an integer and the q_i ($i \neq 1$) can be computed from the projective weights and D through

$$\frac{q_i k_i}{q_1} = \begin{cases} 0\,, & i \geq D \\ 1\,, & i < D \end{cases} \quad . \tag{8.29}$$

Periods for two-parameter models

For the computation of the periods it suffices to know that $D = 2$ for all of the above models, except for the case of $\mathbb{P}^4_{11169}$, where $D = 3$. Let us first focus on the case where $D = 2$. The fundamental period ω_0 in the large complex structure/large volume regime has been computed in [188] to be given by

$$\omega_0(\psi,\phi) = \sum_{l=0}^{\infty} \frac{(q_1 l)!(d\psi)^{-q_1 l}(-1)^l}{l!\prod_{i=2}^{5}\left(\frac{k_i}{d}(q_1-q_i)l\right)!} U_l(\phi)\,, \tag{8.30}$$

where the function $U_\nu(\phi)$ can be written in terms of hypergeometric functions as

$$U_\nu(\phi) = \frac{e^{\frac{i\pi\nu}{2}}\Gamma\left(1+\frac{\nu}{2}(k_2-1)\right)}{2\Gamma(-\nu)} \left[2i\phi \frac{\Gamma(1-\nu/2)}{\Gamma\left(\frac{1+\nu k_2}{2}\right)}\ {}_2F_1\left(\frac{1-\nu}{2}, \frac{1-k_2\nu}{2}; \frac{3}{2}; \phi^2\right) + \right.$$
$$\left. + \frac{\Gamma(-\frac{\nu}{2})}{\Gamma\left(\frac{2+\nu k_2}{2}\right)}\ {}_2F_1\left(-\frac{\nu}{2}, -\frac{k_2\nu}{2}; \frac{1}{2}; \phi^2\right)\right]\,. \tag{8.31}$$

For fixed values of ϕ the series converges for sufficiently large ψ. The actual convergence criterion is model-dependent.

In order to obtain a full set of periods, this expression has to be analytically continued to small ψ. The result is [188]

$$\omega_0(\psi,\phi) = -\frac{2}{d}\sum_{n=1}^{\infty} \frac{\Gamma(\frac{2n}{d})\,(-d\psi)^n\, U_{-\frac{2n}{d}}(\phi)}{\Gamma(n)\,\Gamma(1-\frac{n}{d}(k_2-1))\,\prod_{i=3}^{5}\Gamma(1-\frac{k_i n}{d})}\,, \tag{8.32}$$

which converges for sufficiently small ψ. By acting with the phase symmetry of the polynomial one derives the remaining periods

$$\omega_j(\psi,\phi) = \omega_0(\alpha^j\psi, \alpha^{jq_1}\phi)\,, \tag{8.33}$$

where α is a d-th root of unity.

As this set of periods is over-complete, we have to choose a linearly independent subset[4]. This form of the periods is useful for performing the analytic continuation to large ϕ, since it can be done by standard techniques for the hypergeometric function. In order to continue the periods back to the region where ψ is large, we will find it useful to use an alternative form, where the principal summation runs over powers of ϕ times a certain generalized hypergeometric function in ψ. The result of a rather lengthy computation is that $\omega_j(\psi, \phi)$ can be decomposed into eigenfunctions $\eta_{j,r}(\psi, \phi)$ of the phase symmetry $(\psi, \phi, j) \to (\alpha\psi, -\phi, j+1)$ as

$$\omega_j(\psi, \phi) = -\frac{2}{d} \sum_{r=1}^{d} (-1)^r \, e^{2\pi i j r/d} \, \eta_{j,r}(\psi, \phi) \tag{8.34}$$

with

$$\eta_{j,r}(\psi, \phi) = \frac{1}{2} \sum_{n=0}^{\infty} e^{i\pi n(j+1/2)} \, \frac{(2\phi)^n}{n!} \, V_{n,r}(\psi) \, . \tag{8.35}$$

Now, the full ψ-dependence is contained in the functions $V_{n,r}(\psi)$

$$V_{n,r}(\psi) = N_{n,r} \, (d\,\psi)^r \, H_{n,r}(\psi) \, , \tag{8.36}$$

which are analogues of the function $U_\nu(\phi)$ in equation (8.31). They consist of a generalized hypergeometric function $H_{n,r}(\psi)$ and a numerical prefactor $N_{n,r}$. The first is explicitly given by

$$H_{n,r}(\psi) = {}_{(d+1)}F_d\Big(1, \frac{n}{2}+\frac{r}{d}, \underbrace{1+\frac{r}{d} - \frac{l_2+1-\frac{n}{2}}{k_2}, 1+\frac{r}{d} - \frac{l_i+1}{k_i}}_{i=3,\dots,5 \quad l_i=0,\dots,k_i-1}; \underbrace{\frac{r+l}{d}}_{l=0,\dots,d-1}; \prod_{j=1}^{5} k_j^{k_j} \, \psi^d\Big) \tag{8.37}$$

where the under-brackets indicate that for each value in the allowed index range for i, l_i, l we have to insert the corresponding parameter in the hypergeometric function. For the relevant models we have $k_1 = 1$, so we indeed obtain a hypergeometric function with $(p, q) = (2 + k_2 + \cdots + k_5, d) = (d+1, d)$.

The numerical prefactor can explicitly be expressed as

$$\begin{aligned} N_{n,r} = \pi^{d-3} \, d^{\frac{1}{2}-r} &\left(\prod_{j=2}^{5} k_j^{-\frac{1}{2}+\frac{k_j r}{d}} \right) k_2^{\frac{n}{2}} \\ \times &\frac{\Gamma\left(\frac{n}{2}+\frac{r}{d}\right)}{\prod_{l=0}^{d-1} \Gamma\left(\frac{l+r}{d}\right) \prod_{l_2=0}^{k_2-1} \Gamma\left(\frac{l_2+1-n/2-k_2 r/d}{k_2}\right) \prod_{i=3}^{5} \prod_{l_i=0}^{k_i-1} \Gamma\left(\frac{l+1-k_i r/d}{k_i}\right)} \, . \end{aligned} \tag{8.38}$$

[4]In the cases of interest to us, one can take the first six periods.

This expression is valid for $\phi < 1$ and arbitrary ψ with implicit analytic continuation of the hypergeometric function understood. In practice the degree of the generalized hypergeometric function ${}_pF_q$ is reduced in all these examples to at most $(p,q) = (13,12)$.

The periods for the mirror of $\mathbb{P}^4_{11169}[18]$ have a different form. Its periods in the Landau-Ginzburg phase are given by

$$\begin{aligned} \omega_j(\psi,\phi) &= -\frac{1}{6}\sum_{n=1}^{\infty} \frac{\Gamma(n/6)(-2\cdot 3^{11/6}\psi)^n e^{2\pi i n j/18}}{\Gamma(n)\Gamma(1-n/3)\Gamma(1-n/2)} U_{-\frac{n}{6}}\left(e^{2\pi i j/3}\phi\right) \ , \\ U_\nu(\phi) &= 3^{-\frac{3}{2}-\nu}\frac{2\pi e^{2\pi i\nu/6}}{\Gamma(-\nu)} \sum_{m=0}^{\infty} \frac{\Gamma\left(\frac{m-\nu}{3}\right)\left(e^{2\pi i/3}\phi\right)^m}{\Gamma^2\left(1-\frac{m-\nu}{3}\right)\prod_{i=1}^{3}\Gamma\left(\frac{i+m}{3}\right)} \end{aligned} \tag{8.39}$$

which converge for $|\phi| < 1$ and $|2^2 3^8 \psi^6| < |\phi - \alpha^{-6\tau}|$, where α is an 18th root of unity and $\tau = 0,\ldots,2$. The U-function can be rewritten in terms of generalized hypergeometric functions as follows

$$\begin{aligned} U_\nu(\phi) = \frac{3^{-1-\nu}}{2}\frac{e^{2\pi i\nu/6}}{\Gamma(-\nu)} \Bigg(& 2\frac{\Gamma\left(\frac{-\nu}{3}\right)}{\Gamma^2\left(\frac{3+\nu}{3}\right)} {}_3F_2\left(\frac{-\nu}{3},\frac{-\nu}{3},\frac{-\nu}{3};\frac{1}{3},\frac{2}{3};\phi^3\right) \\ & + 9e^{4\pi i/3}\phi^2 \frac{\Gamma\left(\frac{2-\nu}{3}\right)}{\Gamma^2\left(\frac{1+\nu}{3}\right)} {}_3F_2\left(\frac{2-\nu}{3},\frac{2-\nu}{3},\frac{2-\nu}{3};\frac{4}{3},\frac{5}{3};\phi^3\right) \\ & + 6e^{2\pi i/3}\phi \frac{\Gamma\left(\frac{1-\nu}{3}\right)}{\Gamma^2\left(\frac{2+\nu}{3}\right)} {}_3F_2\left(\frac{1-\nu}{3},\frac{1-\nu}{3},\frac{1-\nu}{3};\frac{2}{3},\frac{4}{3};\phi^3\right)\Bigg) \end{aligned} \tag{8.40}$$

which allows for analytic continuation to $|\phi| > 1$. Alternatively, if we first sum over powers of ψ we get the representation

$$\begin{aligned} \omega_j(\psi,\phi) &= \frac{-\pi}{3^{5/2}} \sum_{r=1}^{18} e^{2\pi i j r/18} e^{-i\pi r/18} \eta_{j,r}(\psi,\phi) \ , \\ \eta_{j,r}(\psi,\phi) &= \sum_{m=0}^{\infty} \frac{e^{2\pi i m(j+1)/3}\phi^m}{\prod_{l=1}^{3}\Gamma\left(\frac{l+m}{3}\right)} V_{m,r}(\psi) \ , \\ V_{m,r}(\psi) &= (-18\psi)^r \frac{\Gamma\left(\frac{m}{3}+\frac{r}{18}\right) {}_7F_6\left(a;b;6^6 9^9\psi^{18}\right)}{\Gamma^2\left(1-\frac{m}{3}-\frac{r}{18}\right)\Gamma(r)\Gamma(1-\frac{r}{2})\Gamma(1-\frac{r}{3})} \ , \\ a &= \left(1,\frac{r+6}{18},\frac{r+12}{18},\frac{r+6m}{18},\frac{r+6m}{18},\frac{r+6m}{18},\frac{r}{18}\right) \ , \\ b &= \left(\frac{r+1}{18},\frac{r+5}{18},\frac{r+7}{18},\frac{r+11}{18},\frac{r+13}{18},\frac{r+17}{18}\right) \end{aligned} \tag{8.41}$$

which again converges for $|\phi| < 1$ and can be analytically continued to the region $|2^2 3^8 \psi^6| > |\phi - \alpha^{-6\tau}|$.

To compute the metric on moduli space, we first have to transform a linearly independent set of periods into a symplectic basis $\Pi = (X^I, F_I)$ in order to apply equation (5.4). The basis transformation can be found case by case through a monodromy calculation [192, 193] or the algorithmic procedure of [194]. In our analysis of the RSDC for two parameter models, we will focus on the three manifolds $\mathbb{P}^4_{11222}[8]$, $\mathbb{P}^4_{11169}[18]$ and $\mathbb{P}^4_{11226}[12]$. For the first two, the mirror map and change to symplectic basis can be found in [192, 193]. For $\mathbb{P}^4_{11226}[12]$, we will compute the mirror map in the following.

An Example: $\mathbb{P}^4_{11226}[12]$

To illustrate the calculation of the periods, the mirror map and the metric on the whole moduli space, we consider the CY that is defined by the mirror of the hypersurface $\mathbb{P}^4_{11226}[12]$. A cousin of this model, $\mathbb{P}^4_{11222}[8]$, has first been analyzed in great detail in [192], although the emphasis has been mostly on the LCS region. The defining polynomial is

$$P(x) = x_1^{12} + x_2^{12} + x_3^6 + x_4^6 + x_5^2 - 12\psi\, x_1 x_2 x_3 x_4 x_5 - 2\phi\, x_1^6 x_2^6 \,, \tag{8.42}$$

where we have to mod out a $H = \mathbb{Z}_6^2 \times \mathbb{Z}_2$ phase symmetry [192]. The transformation $(\phi, \psi) \to (-\phi, \alpha\psi)$ can be absorbed into a coordinate redefinition of the ambient space that leaves the hypersurface constraint $P(x)$ invariant, so the actual (uncompactified) moduli space becomes the corresponding $\mathbb{Z}_{12}$ quotient of $\mathbb{C}^2$. The manifold develops a conifold singularity at

$$864\psi^6 + \phi = \pm 1 \,. \tag{8.43}$$

Before analytic continuation, a detailed analysis of the asymptotic behavior of $U_\nu(\phi)$ and application of the Cauchy root test shows that the periods (8.32) converge in the region $|\phi| < 1$ and $|864\psi^6| < |\phi \pm 1|$, where the "$\pm$" indicates the minimum of the two values. Upon analytical continuation one finds four distinct regions of the moduli space as summarized in table 8.1.

The LG and LCS regions are familiar from the quintic. In addition we get two hybrid regions that share properties of the LG and LCS regions. The hybrid region where $\phi \to \infty$ and ψ stays small will be called the $\mathbb{P}^1$(-fibration)-phase, whereas the region with $\psi \to \infty$ and ϕ small will be referred to as the orbifold(-hybrid)-phase, for reasons that are explained in detail in [31].

The reduced set of periods $(\omega_0, \ldots, \omega_5)$ form a basis. We can calculate these in the LG phase by expanding the hypergeometric function in equation (8.32) around $\phi = 0$. The result is polynomial in both ϕ and ψ. In the $\mathbb{P}^1$ fibration

Region	Convergence Criterion
Landau Ginzburg	$\lvert\phi\rvert < 1$ and $\lvert 864\psi^6\rvert < \lvert\phi \pm 1\rvert$
Hybrid: $\mathbb{P}^1$ fibration	$\lvert\phi\rvert > 1$ and $\lvert 864\psi^6\rvert < \lvert\phi \pm 1\rvert$
Hybrid: orbifold	$\lvert\phi\rvert < 1$ and $\lvert 864\psi^6\rvert > \lvert\phi \pm 1\rvert$
LCS	$\lvert\phi\rvert > 1$ and $\lvert 864\psi^6\rvert > \lvert\phi \pm 1\rvert$

Table 8.1: Different physical regions in the complex structure moduli space of the mirror of $\mathbb{P}^4_{11226}$. The $\pm$ is to be interpreted as a logical "and".

region we expand the hypergeometric function around $i\infty$. We find that the even periods ω_{2j} now contain simple logarithms in ϕ

$$\frac{\vec{\omega}_{\mathbb{P}^1}(\psi,\phi)}{\psi} = \begin{pmatrix} (4.39+0.00i) \\ (5.61+0.70i)+(1.21+0.70i)\log\phi \\ (2.20+3.80i) \\ (2.20+5.21i)+(0.00+1.40i)\log\phi \\ (-2.20+3.80i) \\ (-3.41+4.51i)-(1.21-0.70i)\log\phi \end{pmatrix} \phi^{-1/6} + \mathcal{O}\big(\phi^{-5/6}\big)\ . \quad (8.44)$$

In a similar fashion, in order to obtain an expression for the periods in the orbifold hybrid phase, we expand the generalized hypergeometric function in equation (8.34) around $i\infty$, upon which all of the periods except ω_0 acquire logarithmic terms up to third order $\log(\psi)^3$.

For the LCS region, standard tools are available to compute the metric and periods such as INSTANTON [187]. For this reason we will not further pursue the analytic continuation of the periods into the LCS region.

Mirror Map and Intersection Matrix for Two-Parameter Models

We now explain the computation of the mirror map and intersection matrix. These can both be determined by calculating the monodromy matrices of the periods around certain boundary divisors of the compactified moduli space. For the manifolds $\mathbb{P}^4_{11222}[8]$ and $\mathbb{P}^4_{11169}[18]$ this has been done in [192] and [193], respectively. Next, we analyze the case $\mathbb{P}^4_{11226}[12]$ for which the procedure is analogous to $\mathbb{P}^4_{11222}[8]$ (see [192]).

Using the just determined explicit form of the periods, it is straightforward to calculate the monodromy transformations. For the monodromy obtained by moving around $\phi = 1$, we find $\vec{\omega} \to \mathrm{B}\,\vec{\omega}$ with

$$\mathrm{B} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 & 1 \\ -1 & 1 & 1 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & 2 & -2 \\ 0 & 0 & -1 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} . \tag{8.45}$$

The monodromy matrix T about the conifold is determined to be

$$\mathrm{T} = \begin{pmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 1 & 0 & 0 & 0 \\ -2 & 2 & 0 & 1 & 0 & 0 \\ 2 & -2 & 0 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 0 & 1 \end{pmatrix} . \tag{8.46}$$

Finally, we calculate the monodromy matrix A corresponding to the monodromy around $\psi = 0$

$$\mathrm{A} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} . \tag{8.47}$$

Following [192] we define the matrix $\mathrm{T}_\infty = (\mathrm{AT})^{-1}$. The monodromies around the boundary divisors whose intersection is the large complex structure point are then $\mathrm{S}_1 = \mathrm{T}_\infty^2$ and $\mathrm{S}_2 = \mathrm{B}^{-1}\mathrm{T}_\infty$. We also define $\mathrm{R}_i = \mathrm{S}_i - 1$. We check that the triple products between the R_i reproduce the triple intersection numbers of $\mathbb{P}^4_{11226}[12]$. Using the Ansatz

$$t^i = \frac{\vec{A}^i \cdot \vec{\omega}}{\omega_0} , \tag{8.48}$$

where $\vec{A}^i$, $i = 1, 2$ are row vectors and demanding that monodromies around the LCS boundary divisors correspond to shifts of the B-field, hence gauge transformations

$$\vec{A}^i \cdot \mathrm{R}_j = \delta^i_j \,(1, 0, 0, 0, 0, 0) , \tag{8.49}$$

we can solve for the mirror map up to a constant shift. The result for the A-vectors is

$$\vec{A}^1 = \left(c_1, 0, +\frac{1}{2}, 0, +\frac{1}{2}, 0\right) , \qquad \vec{A}^2 = \left(c_2, +\frac{1}{2}, -\frac{1}{2}, +\frac{1}{2}, -\frac{1}{2}, +\frac{1}{2}\right) , \tag{8.50}$$

where we fix the constants to be $c_1 = -\frac{1}{2}$ and $c_2 = \frac{1}{2}$. By requiring that the monodromy in the symplectic basis of periods are in fact integral and symplectic, we determine the basis transformation to be

$$\Pi = \begin{pmatrix} -1 & 1 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{3}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{3}{2} \\ 2 & 0 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{2} & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{pmatrix} \omega \, . \tag{8.51}$$

This can then be used to compute the Kähler potential (5.4).

8.3 Geodesic Distance in 1D Moduli Spaces

In this section we investigate the manifestation of the RSDC for regions of one-dimensional Calabi-Yau Kähler moduli spaces beyond the large volume phase. As our prototype example, we will discuss the quintic in very much detail. Besides the large volume point, there also exist the conifold and Landau-Ginzburg orbifold points. Recall that in proper distance, the large volume point was infinitely far away from any other point in moduli space, but that for field distances larger than $\Theta_\lambda = \sqrt{3/4}M_p < M_p$ a logarithmic scaling sets in, which renders infinitely many states exponentially light. Thus, at distances larger than M_p the effective field theory could not be trusted anymore. The question is whether proper distances Θ_0 accumulated before by traversing non-geometric phases are also smaller than the Planck-scale. Besides the quintic we also check the RSDC for the other three one-parameter Calabi-Yau manifolds given as smooth hypersurfaces in $\mathbb{P}^4_{k_1 \ldots k_5}$.

8.3.1 An Illustrative Example: The Quintic

We proceed now with testing the RSDC for the quintic $\mathbb{P}^4_{11111}[5]$. After first investigating local properties of the moduli space, we proceed to compute distance between points in different coordinate patches. Due to the complicated form of the metric, described in terms of hypergeometric functions, we solve the geodesic equation numerically.

Local Analysis

Let us now see whether there is any potential trouble with the RSDC that we can already detect by studying the metric locally around the LG and conifold points

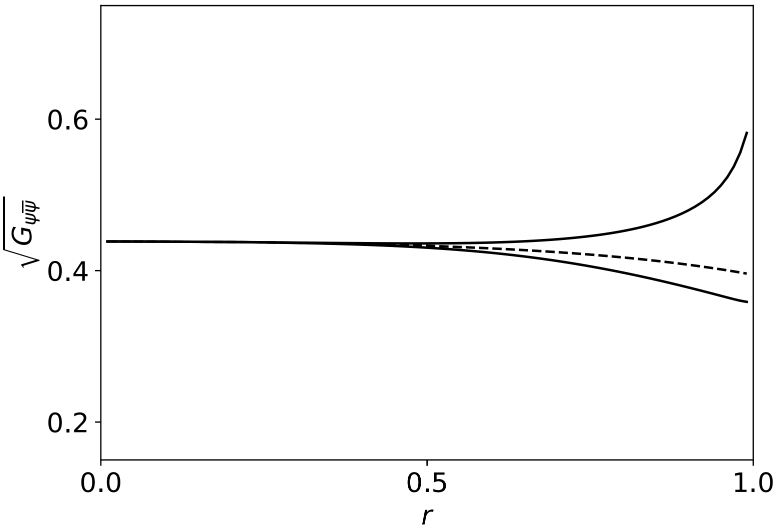

Figure 8.7: Plot of the square root of the metric in the Landau-Ginzburg phase. Dashed line: up to order r^6. Solid upper line: up to order r^{100} for $\theta = 0$. Solid lower line: up to order r^{100} for $\theta = \frac{2\pi}{10}$.

in figure 8.4. The first question that we can ask is about the distance between the LG point $\psi = 0$ and an arbitrary point $\psi = re^{i\theta}$ that is still in the LG phase $r < 1$. As we can see in equation 8.23, the three leading terms in the Kähler potential are symmetric under shifts of θ, so geodesics starting at the origin will be approximately lines of constant θ. This approximation is good for $r < 0.5$. Approximating the metric by its leading constant behavior, we can approximate

$$\Delta\Theta = \int_0^1 dr\,\sqrt{G_{\psi\bar{\psi}}(r)} \sim 0.44\,. \tag{8.52}$$

Evaluating the periods up to higher orders, we find that the integrand in (8.52) behaves as in figure 8.7. Even though the metric diverges at the conifold point $r = 1$, $\theta = 0$, the divergence is mild enough so that the distance stays finite

$$\Delta\Theta(\theta = 0) = 0.45\,, \qquad \Delta\Theta(\theta = \tfrac{2\pi}{10}) = 0.42\,. \tag{8.53}$$

Because all of the distances that we find are smaller than one in Planck units, the RSDC cannot possibly be violated in the convergence region of the LG point of the quintic.

The conifold point is not as interesting as the Landau-Ginzburg point with regards to violating the RSDC because it sits right at the boundary between the two phases. It is at maximum distance to the Landau-Ginzburg point, but this distance is still finite and below the Planck scale. The distance to the large volume

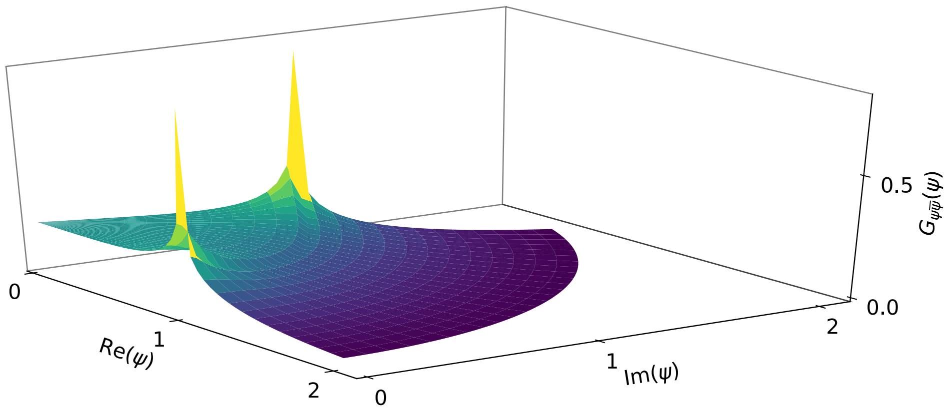

Figure 8.8: The metric on the Kähler moduli space of the quintic.

phase is infinitesimal. This means that independent of the direction in which we displace, there will be no tension with the RSDC. In the following discussion we will therefore focus on geodesics that start at the point deepest inside the non-geometric phase, that is, the Landau-Ginzburg point, and continue into the large volume phase.

Trajectories Traversing Multiple Patches

After locally checking some necessary conditions for the Refined Swampland Distance Conjecture to hold, we will now consider geodesics that traverse multiple coordinate patches. For this purpose, we use the following general procedure:

- Determine the metric on the moduli space and compute the corresponding geodesics $x^\mu(\Theta)$ parameterized by proper distance Θ.
- Identify a tower of states whose mass should decrease along the geodesics.
- Find the mass $M_{\text{KK}}(\psi)$ of this tower as a function of the position in moduli space.
- Express the mass $M_{\text{KK}}(\Theta)$ in terms of the distance along the geodesics.

The moduli space metric of the quintic is obtained patch-wise from the periods in the Landau-Ginzburg and large volume regions respectively up to order ψ^{50}, or alternatively by the GLSM construction [31]. The resulting metric is illustrated in figure 8.8.

The geodesics can be obtained numerically by solving the geodesic equation

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} = 0 \,, \tag{8.54}$$

where $\tau = a\Theta + b$ is an affine parameter and in our case $x = (r, \theta)$. We set $b = 0$ without loss of generality. The parameter a can then be determined from the initial conditions by computing the square root of the pullback of the metric onto the geodesic

$$a = \frac{d\Theta}{d\tau} = \sqrt{G_{\alpha\beta}(x(\tau))\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau}}\Bigg|_{\tau=0} \,. \tag{8.55}$$

We first want to investigate the fate of geodesics going radially outward from the Landau-Ginzburg point $\psi = 0$. While the coordinate $\psi = r\exp(i\theta)$ is periodic, $\theta \equiv \theta + 2\pi/5$, the metric additionally enjoys an enhanced $\mathbb{Z}_2$ reflection symmetry along the ray $\theta = \pi/5$. This allows us to restrict to geodesics in the angular region $\theta \in (0, \pi/5)$. We can also directly infer that the rays $\theta = 0, \pi/5$ are exact geodesics.

The behavior of the radially outgoing geodesics can be qualitatively deduced as follows. The geodesic equation implies

$$\ddot{\theta} = -\Gamma^\theta_{rr}\,\dot{r}^2 - 2\Gamma^\theta_{r\theta}\,\dot{r}\,\dot{\theta} = \frac{1}{2}G^{\theta\theta}G_{rr,\theta}\,\dot{r}^2 - G^{\theta\theta}G_{\theta\theta,r}\,\dot{r}\,\dot{\theta} \,. \tag{8.56}$$

For small r the metric is approximately constant so that, with the initial condition $\dot{\theta}(0) = 0$, θ stays approximately constant while r increases. When $r \simeq 1$, the θ gradient of the metric becomes important and equation (8.56) implies that the initially straight line is attracted towards the region of increasing G_{rr}, i.e. the conifold point. Once the geodesic passes into the region $r \gg 1$, the metric becomes approximately flat in the θ direction, but the r gradient becomes important. Since now $\dot{\theta} < 0, \dot{r} > 0, G_{\theta\theta,r} < 0$, the minus sign in the second term of equation (8.56) implies that the geodesic continues towards decreasing θ until it hits the $\mathrm{Re}(\psi) = 0$ axis. It then re-enters the moduli space from the $\theta = 2\pi/5$ ray.

Since we are interested in the geodesic distance and hence the shortest geodesics between points, we can stop integrating the geodesic when it hits the axis. This is because, due to the symmetry properties of the metric, we will always find a shorter geodesic connecting the relevant points in the "upper" half-cone of the moduli space. Figure 8.9 shows a few representative geodesics.

We can see directly from the discussion of the radius of the Landau-Ginzburg region that geodesics passing too close to the conifold point will not be interesting, since they hit the $\mathrm{Re}(\psi) = 0$ axis shortly after crossing the phase boundary.

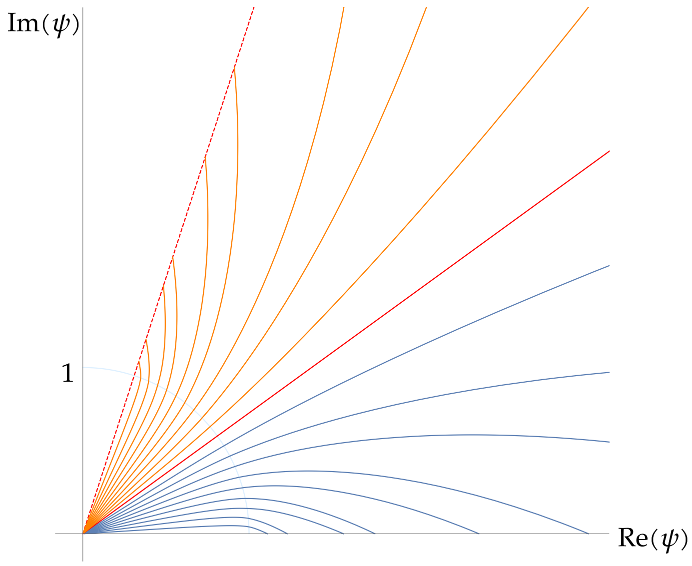

Figure 8.9: Geodesics for the initial data $(r, \dot{r}, \theta, \dot{\theta}) = (0, 1, i \cdot \pi/50, 0)$, for $i = 1, \dots, 10$. The orange geodesics are the $\mathbb{Z}_2$ images.

This means that they will not have a total length much larger than the distance between the Landau-Ginzburg and the conifold point $\Delta\Theta \simeq 0.45$. In fact, for the geodesic with $\theta = \pi/50$ we find the numerical result $\Delta\Theta \approx 0.5$.

In order to test the RSDC in this moduli space we will consider another set of geodesics with a slightly finer scanning of the angle, $\theta = \pi/5 - i\pi/60$, for $i = 0, \dots, 11$. Disregarding the geodesic $\theta = \pi/5$, which continues straight to the large volume point, the longest geodesic in this family has $\theta = \frac{11}{60}\pi$, which hits the axis at $\text{Re}(\psi) \approx 110$, after traveling for a total proper distance of $\Delta\Theta \approx 1.53$.

By performing an asymptotic expansion of the metric in the large volume phase, we realize that it has the asymptotic form

$$g_{rr}(r) \approx \frac{3}{4\,(r \log r)^2} \tag{8.57}$$

with $\lambda = 2/\sqrt{3}$. Using this, one can see that the geodesic distance from the Landau-Ginzburg point asymptotically grows as the double logarithm

$$\Theta(r) \simeq \frac{1}{\lambda} \log\left(\log(r)\right) . \tag{8.58}$$

After identifying a family of relevant geodesics, the next step is to identify a tower of states whose mass we expect to display the exponential behavior predicted by the Refined Swampland Distance Conjecture. Through mirror symmetry, the complex structure moduli space of the mirror quintic is mapped to the Kähler moduli space of the quintic. The single complex structure modulus ψ is mapped to the overall volume modulus $t = \int B + i\int J$ of the quintic. Working in the Kähler moduli space of the quintic, we have as a natural candidate the Kaluza-Klein tower associated to the overall volume. As we have computed in section 8.1, the associated mass scale is then

$$M_{\rm KK}(t) \sim \frac{1}{(\mathrm{Im}(t))^2} \,. \tag{8.59}$$

In order to express this in terms of the proper field distance, one needs the mirror map $t = t(\psi)$ of section 8.2.

Using our results, we can immediately verify the exponential relation between $M_{\rm KK}$ and Θ. Combining the doubly logarithmic behavior of $\Theta(r)$ (8.58) with the logarithmic one of $t_M(\psi)$ (8.26), we get[5]

$$\Theta \simeq \frac{1}{\lambda}\log\left(\mathrm{Im}(t_M)\right) \qquad \Rightarrow \qquad M_{\rm KK} \simeq \frac{1}{(\mathrm{Im}(t_M))^2} \simeq e^{-2\lambda\,\Theta} \,. \tag{8.60}$$

This is precisely the behavior predicted by the RSDC.

Since we know both the proper distance Θ and the value of the complexified Kähler modulus $t = \int B + iJ$ along the geodesics, we can plot the logarithm of $\mathrm{Im}(t)$ against Θ.

The Refined Swampland Distance Conjecture predicts a linear behavior after some critical distance $\Theta_0 \lesssim 1$. Figure 8.10 shows that this is precisely the case. The expected linear behavior is reached for $\Theta = \Theta_0 \lesssim \mathcal{O}(1)$. The depicted red graph corresponds to the central geodesic with initial angle $\theta = \pi/5$. We find that this is the geodesic for which Θ_0 takes the largest value. The dotted blue line shows the fit to the asymptotic linear behavior, while the dashed gray fit also captures corrections to $\Theta \simeq \frac{1}{\lambda}\log(\mathrm{Im}(t))$ up to order $1/\mathrm{Im}(t)^3$.

The parameters λ and Θ_0 are determined as follows. The value of Θ_0 is defined to be the value of the proper distance along the geodesics from the Landau-Ginzburg point to the phase boundary at $|\psi| = 1$. We also define t_0 to be the value of the Kähler modulus at the phase boundary, $t_0(\theta) \equiv t(r = 1, \theta)$. To determine λ, we perform a fit of the asymptotic behavior of the proper distance as a function of the Kähler modulus according to the leading order terms

$$\Theta(t) \simeq \frac{1}{\lambda}\log(t) + \alpha_0 + \frac{\alpha_1}{t^3} \,. \tag{8.61}$$

[5] The factor of two in the exponential should not be taken too seriously, as we just gave a rough estimate for the KK mass scale, $M_{\rm KK} \sim M_s/\mathrm{Vol}^{1/6}$. We note that in [130] another proposal for the infinite tower of exponentially light states has been given. There, these were BPS wrapped D-branes, i.e. non-perturbative states.

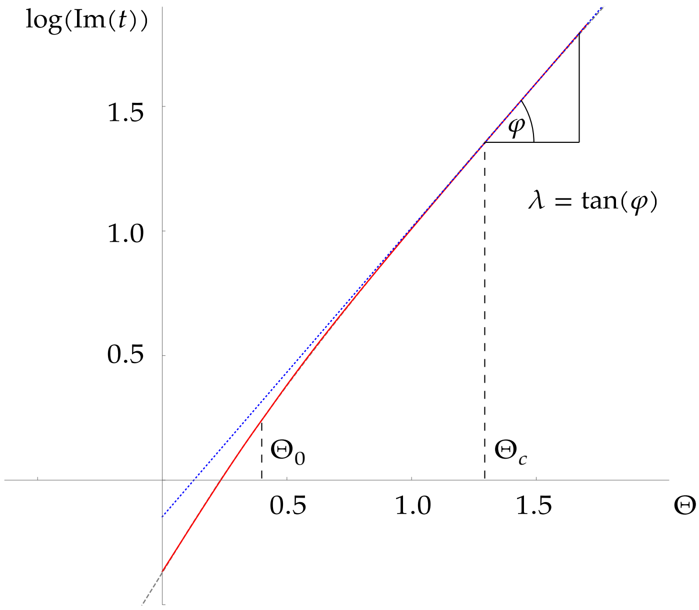

Figure 8.10: The logarithm of $\mathrm{Im}(t)$ against Θ.

The angular distributions of the fit-parameters as well as Θ_λ, Θ_0 and Θ_c are shown in table 8.2.

We have excluded the geodesics with $i = 10, 11$ from the analysis because the fact that they hit the $\mathrm{Re}(\psi)$ axis almost immediately after the phase transition did not allow for a good fit. As a result we find that the critical distance is always of order one and satisfies the bound

$$\Theta_c \leq 1.413 \,. \tag{8.62}$$

The amount of variation of Θ_0 is minimal, while λ deviates noticeably between the different geodesics.

As a crosscheck of our method, for the central geodesic with $\theta = \pi/5$ we can compare the result for λ and α_1 with the analytic result (8.4). The value for $\Theta_\lambda \simeq 0.866$ agrees perfectly with the expected value of $\Theta_\lambda = \sqrt{3/4}$. For α_1, we insert the Euler characteristic of the quintic ($\chi = -200$) into the formula (8.4) and obtain $\alpha_1 \simeq 0.168$, which deviates from the fit value only by one percent. This can be explained by the fact that we neglected the higher order $1/t$ corrections in our fit.

We observe that those geodesics passing closer to the conifold and thus deviating the most from being straight lines in the ψ-plane have the largest Θ_λ. This is because of the fact that while both the real and the imaginary part of the complexified Kähler modulus contribute to the proper distance, only the imaginary

$\theta_{\text{init}} \cdot 60/\pi$	α_0	α_1	λ^{-1}	Θ_0	Θ_c
3	0.1315	0.2043	0.9605	0.4262	1.3866
4	0.1127	0.2099	0.9865	0.4261	1.4125
5	0.0998	0.2213	0.9780	0.4260	1.4040
6	0.0955	0.2294	0.9567	0.4259	1.3827
7	0.0818	0.2475	0.9611	0.4259	1.3869
8	0.0877	0.2592	0.9275	0.4258	1.3533
9	0.0808	0.2825	0.9253	0.4257	1.3510
10	0.0929	0.3093	0.8969	0.4257	1.3226
11	0.0998	0.3497	0.8845	0.4257	1.3102
12	0.1234	0.1662	0.8657	0.4256	1.2914
					

Table 8.2: Values of the fit-parameters $\alpha_0, \alpha_1, \lambda^{-1}$, critical distance Θ_0 and combined critical distance Θ_c for the family of geodesics with initial angles $\theta_{\text{init}} = \pi/5 - i\pi/60$, for $i = 2, \dots, 11$. We see that Θ_0 is approximately constant for the quintic. The total critical distance varies mostly because of the angular dependence of λ.

part controls a mass scale. The imaginary part of the Kähler modulus is (asymptotically) mapped to the absolute value $|\psi|$ through the mirror map, while the real part is mapped to $\text{Arg}(\psi)$. Curving into the "axionic" direction in moduli space thus decreases the rate of the exponential mass fall-off. The fact that we still find $\Theta_\lambda < M_p$ for all geodesics is a non-trivial test of the RSDC. It seems to be not unrelated to the statement that periodic directions of the moduli space should have a sub-Planckian periodicity.

For the average over all sampled geodesics of t_0, the value of the Kähler modulus at the phase transition, we obtain the value $\text{Im}(t_0) \simeq 1.31$. The average values of the characteristic proper distances of the geodesics turn out to be

$$\Theta_0 \simeq 0.4259\,, \qquad \Theta_\lambda \simeq 0.9343\,, \qquad \text{and} \qquad \Theta_c \simeq 1.3601\,, \tag{8.63}$$

in perfect agreement with the RSDC.

8.3.2 Other Smooth Hypersurfaces

As pointed out in section 8.1 there are only four Calabi-Yau manifolds with $h^{1,1} = 1$ defined by a single polynomial constraint in a weighted projective space, namely $\mathbb{P}^4_{11111}[5]$, $\mathbb{P}^4_{11112}[6]$, $\mathbb{P}^4_{11114}[8]$ and $\mathbb{P}^4_{11125}[10]$[6]. After having discussed the quintic rather detailed in the last section, we are now turning towards the other three manifolds. These are per construction very similar to the quintic so that the results we are going to compute will be qualitatively equivalent and only differ by slightly altered numerical values.

In particular, the structure of the moduli space agrees exactly with the one of the quintic. That is, there exists a conifold singularity at $\psi = 1$ with ψ being the coordinate of the moduli space. The regime $|\psi| > 1$ is covered by the large volume chart, whereas the region $|\psi| < 1$ corresponds to the (orbifolded) Landau-Ginzburg phase. There is again a residual $\mathbb{Z}_d$ symmetry depending on the degree d of the analyzed projective space.

The behavior of the metric for $\mathbb{P}^4_{11112}[6]$ is qualitatively the same as for the quintic. For these two CY threefolds the metric is approximately flat around the origin $G_{\psi\overline{\psi}} \simeq \text{const}$, whereas the asymptotic behavior of the metric is like $G_{\psi\overline{\psi}} \simeq 1/(|\psi|^2 \log|\psi|^2)$. For the other two threefolds $\mathbb{P}^4_{11114}[8]$ and $\mathbb{P}^4_{11125}[10]$ the metrics differ slightly around the origin, in that they have $G_{\psi\overline{\psi}} \simeq \text{const} \cdot |\psi|^2$ but show the same asymptotic behavior.

In the following we shall determine average values for λ, Θ_0 and Θ_c based on characteristic geodesic trajectories for each of the three moduli spaces. Analogously to figure 8.9 we will investigate geodesics starting close to $\psi = r\exp(i\theta) = 0$ and moving outwards in radial direction. All of them will transit from the Landau-Ginzburg phase into the large volume regime. More precisely, we analyze 12 geodesics $\gamma_j, j = 0, \dots, 11$ with start points $(r_i, \theta_i) = \left(r_i, \frac{\pi}{d}\left(1 - \frac{j}{12}\right)\right)$ and choose an initial velocity $(r'_i, \theta'_i) = (1, 0)$. Note that r_i has to be adjusted model by model since numerical fluctuations disturb the metric near the origin. Apparently γ_0 corresponds to the angle bisector and γ_{11} comes close to the conifold singularity.

Before presenting the results for each model, let us point out that we determined the Kähler metric from the partition function of the corresponding GLSM, as described in [31]. A formula to calculate the mirror map of one-parameter models was given in section 8.2.1. The evaluation of the geodesics follows the procedure described for the quintic. Hence, in order to determine the slope parameter λ, we will again fit the Ansatz

$$\Theta(t) \simeq \frac{1}{\lambda}\log(t) + \alpha_0 + \frac{\alpha_1}{t^3}\,. \tag{8.64}$$

Now, let us present our results case by case.

[6]We have also analyzed complete intersections in a single projective space with $h^{1,1}$ [77] in early stages of the project. These turned out to be not too interesting because they have no degeneration points that are at finite distance. They were subsequently also studied in [140].

$\theta_{\text{init}} \cdot 72/\pi$	α_0	α_1	λ^{-1}	Θ_0	Θ_c
1	-0.081	0.414	0.957	0.405	1.362
2	-0.058	0.347	0.934	0.404	1.338
3	-0.057	0.329	0.929	0.402	1.331
4	-0.052	0.319	0.911	0.400	1.311
5	-0.056	0.327	0.906	0.399	1.305
6	-0.067	0.347	0.914	0.398	1.312
7	-0.074	0.368	0.914	0.397	1.311
8	-0.068	0.389	0.896	0.396	1.292
9	-0.060	0.423	0.882	0.396	1.278
10	-0.060	0.459	0.881	0.395	1.276
11	-0.060	0.502	0.879	0.395	1.274
12	-0.042	0.283	0.866	0.395	1.260
					

Table 8.3: Fitting the Ansatz (8.64) to a plot of the proper length of geodesics γ_j depending on the mirror map coordinate t. The table lists all fitting parameters including the critical distance $\Theta_c = \Theta_0 + \lambda^{-1}$ for the model $\mathbb{P}^4_{11112}[6]$.

$\mathbb{P}^4_{11112}[6]$: We take the initial value $r_i = 0.01$ and add to every geodesic an initial length of

$$\Theta_i = \int_0^{r_i} dr \sqrt{G_{\psi\bar{\psi}}} \simeq 0.0039 \,. \tag{8.65}$$

By computing the proper distance for each geodesic and fitting the Ansatz (8.64), we obtain the values listed in table 8.3. Notice that our fits led to an average threshold $\operatorname{Im} t_0 \simeq 1.49$ calculated as the transition point to the large volume phase.

The average proper distance collected in the non-geometric phase is $\Theta_0 \simeq 0.40$. For the distance between the Landau-Ginzburg point and the conifold point we find the value $\Delta\Theta \simeq 0.41$. These results are not in contradiction with the RSDC as all values are $O(1)$

$$\Theta_0 \simeq 0.3984 \,, \qquad \Theta_\lambda \simeq 0.9056 \,, \qquad \text{and} \qquad \Theta_c \simeq 1.3041 \,. \tag{8.66}$$

$\mathbb{P}^4_{11114}[8]$: For this moduli space we have computed the geodesics starting from $r_i = 0.1$ and as a consequence had to add $\Theta_i \simeq 0.0023$ to the proper lengths of

$\theta_{\text{init}} \cdot 96/\pi$	α_0	α_1	λ^{-1}	Θ_0	Θ_c
1	-0.426	0.747	0.933	0.225	1.158
2	-0.409	0.673	0.920	0.224	1.144
3	-0.399	0.629	0.908	0.223	1.130
4	-0.393	0.619	0.893	0.221	1.115
5	-0.398	0.640	0.892	0.221	1.113
6	-0.409	0.668	0.900	0.220	1.120
7	-0.417	0.700	0.904	0.219	1.123
8	-0.414	0.737	0.893	0.218	1.112
9	-0.409	0.782	0.883	0.218	1.101
10	-0.410	0.827	0.882	0.218	1.100
11	-0.408	0.885	0.878	0.218	1.096
12	-0.388	0.613	0.865	0.217	1.082

Table 8.4: Fitting the Ansatz (8.64) to a plot of the proper length of geodesics γ_j depending on the mirror map coordinate t. The table lists all fitting parameters including the critical distance $\Theta_c = \Theta_0 + \lambda^{-1}$ for the model $\mathbb{P}^4_{11114}[8]$.

the geodesics. The critical value of the Kähler modulus at the phase transition is on average $\operatorname{Im} t_0 \simeq 1.77$.

The distance between the Landau-Ginzburg point and the conifold point is $\Delta\Theta \simeq 0.23$. All results can be found in table 8.4.

The RSDC is thus in agreement with the analyzed geodesics in the moduli space of $\mathbb{P}^4_{11114}[8]$. On average we end up with the following values:

$$\Theta_0 \simeq 0.2201\,, \qquad \Theta_\lambda \simeq 0.8961\,, \qquad \text{and} \qquad \Theta_c \simeq 1.1162\,. \tag{8.67}$$

$\mathbb{P}^4_{11125}$[10]: Here we have assumed $r_i = 0.14$, leading to $\Theta_i \simeq 0.0040$ that needs to be added to the proper lengths. For this model the proper distance between the Landau-Ginzburg point and the conifold point has the smallest value, $\Delta\Theta \simeq 0.21$. Fitting the Ansatz (8.64) gives the values summarized in table 8.5. Note that the critical Kähler modulus is $\operatorname{Im} t_0 \simeq 2.17$. Also in this model the average values agree with the RSDC

$$\Theta_0 \simeq 0.2086\,, \qquad \Theta_\lambda \simeq 0.8911\,, \qquad \text{and} \qquad \Theta_c \simeq 1.0997\,. \tag{8.68}$$

$\theta_{\text{init}} \cdot 120/\pi$	α_0	α_1	λ^{-1}	Θ_0	Θ_c
1	-0.655	1.482	0.949	0.213	1.162
2	-0.616	1.289	0.919	0.212	1.131
3	-0.593	1.179	0.899	0.210	1.109
4	-0.583	1.151	0.885	0.210	1.094
5	-0.587	1.182	0.885	0.209	1.094
6	-0.597	1.219	0.891	0.208	1.099
7	-0.602	1.263	0.892	0.208	1.100
8	-0.598	1.322	0.884	0.207	1.091
9	-0.593	1.392	0.876	0.207	1.083
10	-0.594	1.449	0.875	0.207	1.082
11	-0.592	1.522	0.873	0.207	1.080
12	-0.578	1.195	0.865	0.206	1.071
					

Table 8.5: Fitting the Ansatz (8.64) to a plot of the proper length of geodesics γ_j depending on the mirror map coordinate t. The table lists all fitting parameters including the critical distance $\Theta_c = \Theta_0 + \lambda^{-1}$ for the model $\mathbb{P}^4_{11125}[10]$.

8.4 Summary of Results for 2D Moduli Spaces

The two-parameter case is more difficult for several reasons. First, the analytic continuation becomes more subtle as the periods are now expressed as double sums, so that the convergence condition for a given variable will in general be a complicated function of all of the other variables. The more fundamental difference is the fact that now there will be "hybrid" regions in the moduli space in which one of the Kähler moduli is in the perturbative limit but another one is not. It also becomes much more complicated to find the shortest geodesic between two given points in the moduli space, so that we will not be able to precisely determine the distance entering the SDC.

The last problem can be partially avoided by considering not true geodesics but only certain trajectories between two points. The length of these trajectories will give us upper bounds on the actual distance of the points because geodesics are locally distance minimizing. We can thus verify the RSDC by checking that an upper bound on the diameter of a non-geometric phase, determined in this

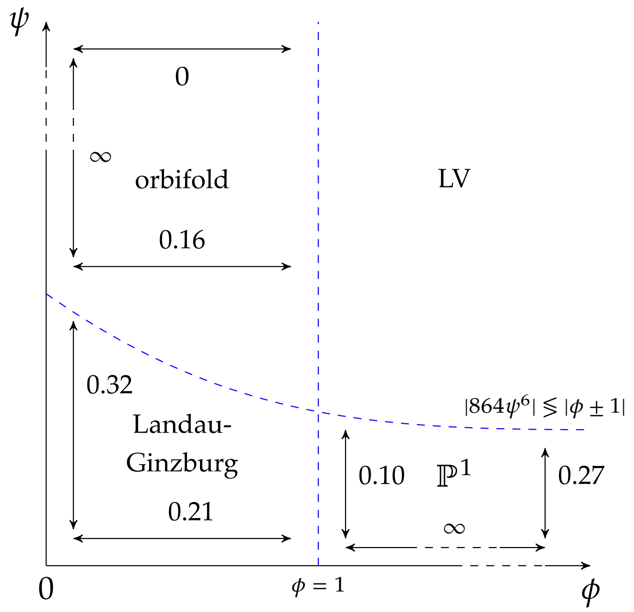

Figure 8.11: Schematic plot of the moduli space of the mirror $\mathbb{P}^4_{11226}[12]$. Characteristic diameters of the non-geometric and hybrid phases are indicated.

way, is sub-Planckian. On the contrary, if we find that the upper bound on the diameter exceeds a value of order one in Planck units we can not immediately declare a violation of the RSDC, because we can blame the excessive length on the non-geodesic nature of the trajectory. Hence, such an analysis can at most serve as a useful indicator to know where we have to look more carefully.

Let us now illustrate the new features that appear in higher-dimensional moduli spaces based on the example of the degree 12 hypersurface in the weighted projective space $\mathbb{P}_{11226}$. The mirror of this is a discrete quotient of the hypersurface defined by the following polynomial with two complex parameters ψ, ϕ

$$P = x_1^{12} + x_2^{12} + x_3^6 + x_4^6 + x_5^6 - 12\psi\, x_1 x_1 x_3 x_4 x_5 - 2\phi\, x_1^6 x_2^6\ . \tag{8.69}$$

One can see that the actual moduli space of this family of manifolds is given by the space of all possible ψ, ϕ subject to the $\mathbb{Z}_{12}$ identification

$$(\psi, \phi) \sim (\alpha\psi, -\phi)\ , \qquad \alpha^{12} = 1\ . \tag{8.70}$$

As before we will work in the covering space. The resulting moduli space is depicted in figure 8.11 which also summarizes the most important results of our analysis. Just as in the one-parameter case, the moduli space has a large volume and a Landau-Ginzburg region. In addition, there are two other hybrid regions, which we denote the "orbifold" and $\mathbb{P}^1$ regions[7]. The boundary between these regions is again determined by convergence criteria for the periods as in table 8.1. Note that the convergence criteria are again related (but not identical) to the conifold condition (8.43).

The proper distances that we found inside the LG phase are indicated in figure 8.11 and significantly smaller than one in Planck units. They are indeed smaller than the distances in the one-dimensional cases, which seems to be a general pattern [31]. The hybrid regions show an interesting hybrid behavior with respect to the RSDC. They feature an infinite direction, along which only one of the Kähler moduli becomes large and leads to an infinite tower of KK modes becoming light. The orthogonal direction is finite with a characteristic length scale smaller than $0.27M_p$ before one reaches the large volume region, where both Kähler moduli have large expectation values. In this way, we expect that the behavior predicted by the SDC will set in for any two points separated by a distance larger than one also in this complex two-dimensional moduli space although we are short of a proof of this claim.

Other two-dimensional moduli spaces discussed in [31] were the weighted projective hypersurfaces $\mathbb{P}_{11222}[8]$ and $\mathbb{P}_{11169}[18]$. The results are qualitatively similar as for the degree 12 hypersurface discussed here. The RSDC passed all tests in these examples. Further details on these two moduli spaces can also be found in the doctoral thesis of Florian Wolf [181].

8.5 High-dimensional Example

It is worth pointing out that one can also probe the diameter of the Landau-Ginzburg phase for very high-dimensional moduli spaces, although the global picture of the moduli space is elusive. A natural example to study is the complex structure moduli space of the quintic, which is complex 101-dimensional [31]. The periods can in this case be obtained using the methods developed in [195, 196]. The defining polynomial of the quintic (8.71) can be parameterized as

$$P = x_1^5 + x_2^5 + x_3^5 + x_4^5 + x_5^5 + \sum_{s=0}^{100} \Phi_s e^s \tag{8.71}$$

such that Φ_s is the complex structure modulus associated to the deformation $x_1^{s_1} \cdots x_5^{s_5}$. It is useful to group deformations into orbits of the permutation symmetry of the homogeneous coordinates x_i. There are five different orbits generated by the s-vectors $(1,1,1,1,1)$, $(2,1,1,1,0)$, $(2,2,1,0,0)$, $(3,1,1,0,0)$ and

[7] See [31] for an explanation of these names.

direction	$\Delta\Theta$
Φ_0	0.4656
Φ_1	0.0082
Φ_2	0.0670
Φ_3	0.0585
Φ_4	0.0089

Table 8.6: Proper distances between the conifold and the Landau-Ginzburg point in the complex structure moduli space of the quintic.

$(3, 2, 0, 0, 0)$. These orbits have the size $(1, 20, 30, 30, 20)$ respectively. By an abuse of notation, we will denote the moduli associated to coherent deformations along these orbits by $\Phi_0, \dots, \Phi_4$[8].

Starting at $\Phi_i = 0$ and displacing along any one of the Φ_i we will at some point $\Phi_{i,\text{crit.}}$ violate transversality of the polynomial (8.71). We expect that, as in the lower-dimensional examples, at this point the expansion of the periods in Φ_i breaks down and we are entering a different phase of the moduli space. By calculating the distance

$$\Delta\Theta_i = \int_0^{\Phi_{i,\text{crit.}}} \sqrt{\sum_s g_{\Phi_s \Phi_t} \frac{\partial \Phi_s}{\partial \Phi_i} \frac{\partial \Phi_t}{\partial \Phi_i}} d\Phi_i \tag{8.72}$$

we can thus probe the characteristic length scales of the Landau-Ginzburg phase. The results are shown in table 8.6. We see that they are all much smaller than order one and in particular that displacing along a naively long diagonal direction in field space does not lead to larger but rather to a shorter distance. This is because displacing many moduli means that transversality of the polynomial is violated faster and this effect dominates. It was speculated in [31] that this effective decrease in size of the non-geometric phases with the dimension of the moduli space could save the RSDC from being violated in such a naively very large moduli space with many different hybrid phases.

8.6 Relation to Other Work

In [130, 135] it was argued that, in the type IIB picture, the light states at infinite distance points predicted by the SDC should be BPS states, which can be thought

[8] By this we mean that for example each deformation parameter Φ_s in the orbit of $x_1^2 x_2 x_3 x_4$ will be set equal to Φ_1.

of as D3-branes wrapping special Lagrangian three-cycle in the compactification. For BPS states, showing that the mass decreases exponentially fast is equivalent to demonstrating that the central charge of the state does so

$$Z = e^{K/2} Q \cdot \Pi = \frac{Q \cdot \Pi}{|\Pi|} \,, \tag{8.73}$$

where the scalar products and norms are the usual symplectic ones.

We can easily see that because at the Landau-Ginzburg points the periods have a polynomial expansion without any logarithms, this will be a rational function of the complex structure moduli and hence never exponential in the proper distance. The importance of this point is that the Landau-Ginzburg region also violates the SDC for the BPS states as tentative tower states and hence there is a genuine need for the Refined Swampland Distance Conjecture, based on solid evidence from $\mathcal{N} = 2$ string compactifications.

In [197] it was recently shown that along certain trajectories in the *hypermultiplet* moduli space an infinite number of unsuppressed instanton corrections to the field space metric can render a classical infinite distance finite. This is in some sense opposite to the behavior that we see at large volume / large complex structure, where the infinite tower of states seems to generate a contribution to the kinetic terms that reproduces the same infinite distance as it is present in the tree-level metric.

9 The Spin-2 Swampland Conjecture

Interactions of fields of spin larger or equal than two are known to be very constrained in four dimensions. For example, the couplings of a massless spin-2 particle in flat space are known to be required to satisfy the equivalence principle [198]. For interacting massless particles of spin $s > 2$ there are no-go theorems [198–205]. The only known way to construct a non-trivial interacting theory of massless spin $s > 2$ fields must include an infinite number of such fields, for example $s = 0, 1, 2, 3, 4, \dots$, an infinite number of higher derivative interactions and a non-zero cosmological constant [206–210], see [211] for a review.

The gauge invariance is not needed in the massive case, but perturbative unitarity of scattering amplitudes usually breaks down at some scale Λ, the strongest constraints typically arising from the scattering of lower helicity degrees of freedom. Above this scale the EFT description has to be modified. In the case of massive spin-1 fields, this is precisely what happens in the Higgs mechanism of the standard model. Unfortunately, a Higgs mechanism is not known for spins $s > 1$ [212]. This is related to the fact that fields transforming in non-trivial representations of the Lorentz group cannot obtain a non-trivial VEV.

From the discussion of massless particles it is clear that the limit $m \to 0$ should be censored and thus the maximum cutoff of such a theory has to scale in proportion with some power of the mass of the particle. For massive $s > 1$ fields, bounds on the EFT cutoff have been determined in the case in which they are charged under a gauge force [213] and when they interact with gravity [214], see also [215]. It was recently argued that scattering of any finite number of massive $s > 2$ fields with gravitons violates causality above the mass scale of the lightest higher spin fields and thus the cutoff of the theory cannot be parametrically separated from it [216]. Any such theory that is causal must necessarily contain an infinite tower of higher spin resonances.

The case of massive spin-2 is particularly interesting because the arguments of [216] do *not* necessitate the inclusion of an infinite tower of higher spin resonances. Nevertheless, as we will explain in the next section, such a singular massive spin-2 particle is not known within the framework of string theory and these are always accompanied by an infinite tower. In section 9.2, based on the original work [32], we will explain that the weak gravity conjecture applied to the helicity-1 mode of the massive spin-2 particle can be used to argue for the existence of a cutoff and tower of resonances.

While interactions with gravity provide constraints on the cutoff of theories with massive spin-2 fields, it has also been speculated that the fundamental graviton itself has a non-vanishing mass. The theoretical and phenomenological consistency of such "massive gravity" theories has been reviewed in [217]. While this is a very interesting idea, it is not completely clear how such a setup can be realized within string theory, as all known perturbative string vacua in 10D possess the massless spin-2 excitation of the closed string. It has been recently argued that the massless sector of the heterotic or the type II superstring can be consistently projected out by an S-fold construction, leading to a non-Lagrangian strongly coupled theory [218]. The resulting theories contain several mass-degenerate spin-2 fields but no massless ones. Their mass scale is invariably tied to the string scale. In section 9.5 we will argue that a natural strong form of our conjecture implies this mass/cutoff relation in general and point out an apparent clash with observational constraints.

Massive spin-2 fields are in particular very interesting for phenomenology if they have gravity-like interactions. For example, the scalar mode of a massive graviton provides an infrared modification of the gravitational potential, while at short distances it becomes strongly coupled and the so-called Vainshtein mechanism [219] can make the theory compatible with local tests of GR. This means that the massive graviton could conceivably give rise to a possible explanation of dark matter and dark energy on large scales. In contrast, bimetric theories [220–222] contain both a massive and a massless graviton degree of freedom, where the interaction basis is not equal to the mass basis. While the gravitational interaction is in principle mediated by both of them, a large mass of the massive spin-2 can be used to Yukawa-suppress any long-range interaction of it [223].

For these reasons it is important to investigate how massive spin-2 particles can be embedded in string theory and whether there are any swampland constraints on effective field theories containing them.

The material presented in the following sections builds largely on the publication [32], but significantly extends it by adding detailed calculations that were left out therein. We will not cover the relation to the de Sitter conjecture (sec. 7.3.2) described in [32].

9.1 Massive Fields with Spin, Stückelberg and the WGC

The aim of our work was to use the WGC, which a priori only applies to p-form fields (which have a spin of at most one), in order to constrain theories of massive spin $s > 1$ fields. The key step in doing this involves realizing that while massless spin s representations of the Poincaré group contain only the highest helicity component $h = \pm s$, the corresponding massive representation contains also lower helicity degrees of freedom. This can be made explicit by introducing so-called Stückelberg fields [224]. In the following we will discuss the case of spin $s = 1$ and spin $s = 2$.

Spin-1

The most simple example of this is the case of a massive spin-1 field, governed by the Proca action

$$S = -\int \frac{1}{g^2} F \wedge \star F + f^2 A \wedge \star A + A \wedge \star j \,, \tag{9.1}$$

where we have introduced a coupling to a source j. The A^2 term gives a mass $m = gf$ to the vector field and breaks the would-be gauge invariance $A \to A + d\omega$. We can reinstate it by performing the gauge transformation at the level of the action and by reinterpreting the ω as dynamical "Stückelberg" fields $\phi = f\,\omega$. The action

$$S = -\int \frac{1}{g^2} F \wedge \star F + f^2 A \wedge \star A + A \wedge \star j + d\phi \wedge \star d\phi + 2f\, d\phi \wedge \star A - \frac{1}{f}\phi\, d \star j \tag{9.2}$$

is then invariant under the gauge transformation $(A, \phi) \to (A + d\omega, \phi - f\omega)$. The Stückelberg field should be thought of as the longitudinal helicity-0 degree of freedom needed to complete the massless Poincaré multiplet into a massive one.

Already here we can see a constraint on the spin-1 mass arising from the WGC applied to ϕ [225]. Since ϕ is a shift symmetric scalar with "decay constant" f one could speculate for example that the axionic WGC (7.8) should apply to it. Imposing that the decay constant should be sub-Planckian, we obtain

$$m = gf \lesssim gM_p \,. \tag{9.3}$$

In order to argue for a cutoff of the effective field theory we should employ the magnetic side of the weak gravity conjecture. In this case this means that we need to study string charged under the two-form field dual to ϕ. The WGC implies that their tension obeys $T \lesssim fM_p$. We thus expect that the cutoff of a local QFT describing the "Stückelberg"-massive spin-1 field should be subject to the bound [29, 225]

$$\Lambda \lesssim \sqrt{fM_p} = \sqrt{\frac{mM_p}{g}} \,. \tag{9.4}$$

Based on this and the species scale from the WGC applied directly to the vector field it was then argued in [225] that the standard model photon mass should be exactly zero.

Before we move on to the case of spin-2, some words of caution are in order. The cutoff (9.4) can be understood as censoring the limit $m \to 0$. If the vector field obtains a mass through the Higgs mechanism, there is nothing wrong with sending the mass to zero by adjusting the Higgs VEV to zero and the bound should not apply. As explained in [225] the defining difference is that for the Stückelberg case the mass is non-vanishing over the entire moduli space, except for singular points at infinite distance. From the formula $m = gf$ it is clear that besides the global symmetry limit $g \to 0$, which is censored by the WGC, the only way to send m to zero is by $f \to 0$. In a SUSY setting, axion decay constants are usually given by the VEV of a scalar field s from the same multiplet in such a way that $f \to 0$ when $s \to \infty$. This implies a breakdown of the EFT because of the distance conjecture.

The subtle distinction is not relevant for the spin-2 case as there is no known Higgs mechanism [217] for spin-2 and we expect all such points where a spin-2 mass vanishes to be at infinite distance in the string moduli space, see also [226]. If such a Higgs mechanism and corresponding finite distance points do exist after all, the following work should only be thought of as applying in the infinite distance case.

Spin-2

At the linearized level, massive spin-2 fields propagating in flat space are described by the Fierz-Pauli action [228]

$$S_{\mathrm{FP}} = -\int d^4x \left(\frac{1}{4} w^{\mu\nu} L_{\mu\nu}{}^{\rho\sigma} w_{\rho\sigma} + \frac{1}{8} m^2 \left(w_{\mu\nu} w^{\mu\nu} - w^2 \right) \right) , \tag{9.5}$$

where $w = w_{\mu\nu}\eta^{\mu\nu}$ and the (flat space) Lichnerowicz kinetic operator $L_{\mu\nu}{}^{\rho\sigma}$ is given by

$$\begin{aligned} L_{\mu\nu}{}^{\rho\sigma} w_{\rho\sigma} \;=\; & -\frac{1}{2} \Big[\Box\, w_{\mu\nu} - 2\partial_{(\mu}\partial_\alpha w^\alpha_{\nu)} + \partial_\mu\partial_\nu w \\ & -\eta_{\mu\nu} \left(\Box\, w - \partial_\alpha\partial_\beta w^{\alpha\beta} \right) \Big] \,. \end{aligned} \tag{9.6}$$

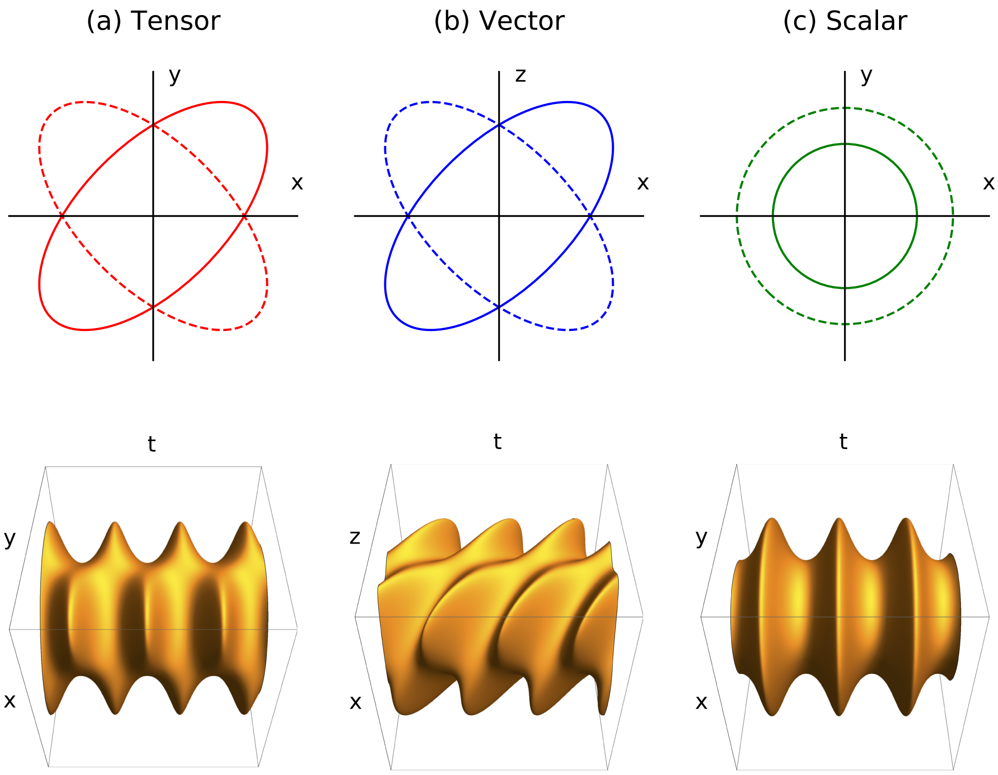

Figure 9.1: Top: Different physical polarizations of a massive spin-2 field propagating along the z-axis (adapted from [227]). The tensor '+' and vector 'yz' polarizations are suppressed. Bottom: Distortion of a circle in the 'xy'- or 'xz'-plane by a gravitational wave with the above polarization.

The free massive spin-2 field described by (9.5) has five physical degrees of freedom. As discussed before, these can be divided into two tensor, two vector and one scalar polarizations. If the massive spin-2 field is the graviton we can visualize the different polarizations by the distortion of a circle in Euclidean three-space upon the impact of a polarized gravitational wave. This is illustrated in figure 9.1. The polarizations can be captured in a three-tensor as [227]

$$\begin{pmatrix} S + T_+ & T_\times & V_{xz} \\ T_\times & S - T_+ & V_{yz} \\ V_{xz} & V_{yz} & L \end{pmatrix} , \tag{9.7}$$

where $T_{+/\times}, V_{xz/yz}$ and S are the physical tensor, vector and scalar polarizations. The longitudinal polarization L is a possible polarization for massive spin-2 fields, but it is a classical ghost with the wrong sign in front of its kinetic term.

It is absent in the linear Fierz-Pauli theory (9.5) due to a tuning of the precise coefficient between the two $\mathcal{O}(w^2)$ terms. Interactions generically reintroduce this ghost degree of freedom [217, 228, 229].

Just as in the spin-1 case, it is possible to make the different helicity components more explicit by introducing Stückelberg fields. To this end we perform a fake linearized diffeomorphism $w_{\mu\nu} \to w_{\mu\nu} + 2\partial_{(\mu}\xi_{\nu)}$. The two fields w, ξ are subject to the Stückelberg gauge transformations

$$\begin{pmatrix} w_{\mu\nu} \\ \xi_\mu \end{pmatrix} \to \begin{pmatrix} w_{\mu\nu} + 2\partial_{(\mu}\epsilon_{\nu)} \\ \xi_\mu - \epsilon_\mu \end{pmatrix} , \tag{9.8}$$

where $\epsilon_\mu(x)$ is the gauge parameter.

The difference to the spin-1 case is that w and ξ_μ still contain a helicity-1 admixture. We can fully disentangle the helicity components by replacing

$$w_{\mu\nu} \to h_{\mu\nu} + 2\partial_{(\mu}A_{\nu)} + \left(4\partial_\mu\partial_\nu - m^2\eta_{\mu\nu}\right)\pi \, . \tag{9.9}$$

The resulting action can be cast into the form [217, 230]

$$\begin{aligned} S = -\int d^4x \Bigg(& \frac{1}{4}h^{\mu\nu}L_{\mu\nu}{}^{\rho\sigma}h_{\rho\sigma} + \frac{3m^2}{4}(\partial\pi)^2 - \frac{m^2}{8}F_{\mu\nu}^2 \\ & + \frac{1}{8}m^2\left(h_{\mu\nu}^2 - h^2\right) - \frac{3}{2}m^4\pi^2 - \frac{3}{2}m^3\pi h \\ & + \frac{1}{2}m^2\left(h^{\mu\nu} - h\eta^{\mu\nu}\right)\partial_{(\mu}A_{\nu)} - 3m^3\pi\partial_\alpha A^\alpha \Bigg) \, . \end{aligned} \tag{9.10}$$

It is invariant under the two-parameter (ξ_μ, Λ) gauge transformations

$$\begin{pmatrix} h_{\mu\nu} \\ A_\mu \\ \pi \end{pmatrix} \to \begin{pmatrix} h_{\mu\nu} + 2\partial_{(\mu}\epsilon_{\nu)} + m^2\Lambda\eta_{\mu\nu} \\ A_\mu - \epsilon_\mu + 2\partial_\mu\Lambda \\ \pi - \Lambda \end{pmatrix} . \tag{9.11}$$

Note that the fields A_μ, π are not canonically normalized at this point.

So far we have neglected interactions with sources. At the linearized level, $w_{\mu\nu}$ will couple to some symmetric tensor source $T_w^{\mu\nu}$ as

$$S_{\text{source}} \sim \int d^4x \, w_{\mu\nu} T_w^{\mu\nu} \, . \tag{9.12}$$

After integrating by parts, the coupling to the helicity components is

$$S_{\text{source}} \sim \int d^4x \left(h_{\mu\nu}T_w^{\mu\nu} - 2A_\mu \underbrace{\partial_\nu T_w^{\nu\mu}}_{\sim J_w^\mu} + 4(\partial_\mu\partial_\nu\pi)T_w^{\mu\nu} - m^2\pi T^\mu_{w,\mu} \right) . \tag{9.13}$$

Note that while the helicity-0 mode π necessarily couples to the source itself, the helicity-1 mode A_μ couples only to the non-conservation $J_w^\mu \sim \partial_\nu T_w^{\nu\mu}$ of it. We will see in the next section that the precise relation between the tensor source $T_w^{\mu\nu}$ and the vector source J_w^μ will involve an additional mass scale. Let us emphasize that while many of the known bounds on the cutoff of massive spin-2 theories arise from studying the interactions of π we will focus on the coupling $A_\mu J_w^\mu$ in the following. We are now in the position to formulate the conjectured cutoff in analogy to what we described for the case of massive spin-1.

9.2 The Spin-2 Conjecture

As discussed in the last section, the helicity $h = \pm 1$ component of a massive spin-2 field can be described by a Stückelberg vector field A_μ coupling to the non-conservation of a symmetric tensor source. In the case of a massive graviton, this would be the matter energy-momentum tensor. We will now couple the effective field theory of the massive spin-2 field (9.5) to Einstein gravity

$$\begin{aligned} S_G = \int d^4x\, \sqrt{-G}\Big[& M_p^2 R\,(G) - \frac{1}{4} w^{\mu\nu} L_{\mu\nu}{}^{\rho\sigma} w_{\rho\sigma} \\ & -\frac{1}{8} m^2 \left(w_{\mu\nu} w^{\mu\nu} - w^2 \right) + ... \Big] \,. \end{aligned} \tag{9.14}$$

In order to apply the WGC, we need to identify a gauge coupling for A_μ. In order to do so we need to be careful about parameterizing its coupling to matter, since the notion of a gauge coupling is arbitrary without matter couplings. Since the kinetic term for A_μ is proportional to m^2, it is a field of mass dimension zero. This means that, if it couples to a quantized current of mass dimension three, there should be an additional dimensionful coupling constant M

$$S \supset \int d^4x \sqrt{-G} \left(-\frac{m^2}{8} F_{\mu\nu}^2 - M A_\mu J_w^\mu \right) \,. \tag{9.15}$$

Because the coupling is associated to the non-conservation of the tensor source $T^{\mu\nu}$, it has to vanish in the limit $m^2 \to 0$ by gauge invariance, so we parameterize $M = m^2/M_w$, where M_w is an interaction scale, which is roughly analogous to the Planck scale for massless spin-2 fields

$$S \supset \int d^4x \sqrt{-G} \left(-\frac{m^2}{8} F_{\mu\nu}^2 - \frac{m^2}{M_w} A_\mu J_w^\mu \right) \,. \tag{9.16}$$

We read off the coupling constant of the canonically normalized gauge field as

$$g_m = \frac{m}{\sqrt{2} M_w} \, . \tag{9.17}$$

Since our theory contains an Einstein-Maxwell sector coupled to gravity, we can now invoke the (magnetic) weak gravity conjecture to arrive at [32]

Spin-2 Conjecture: An effective theory with a self-interacting massless spin-2 field (Einstein gravity) that additionally contains a field of spin 2, mass m, and associated interaction mass scale M_w has a universal cutoff Λ_m with

$$\Lambda_m \sim \frac{m \, M_p}{M_w} \, . \tag{9.18}$$

This cutoff is associated with the mass scale of an infinite tower of states.

We immediately verify that, inherited from the WGC, this fulfills one of the basic requirements of a swampland conjecture – it is only non-trivial in the case of a finite Planck mass and trivializes for $M_p \to \infty$. Before gathering evidence for this statement and exploring the consequences of it, let us first address some possible objections.

It is clear that the application of the WGC to a Stückelberg field rather than a fundamental gauge field is somewhat speculative. One can hope to clarify the plausibility of this in several ways. First, one can look for string theory examples in order to check whether such a statement could be true. Because higher spin interactions are quite constrained, we will see in the next sections that this is a non-trivial task away from the most simple examples. An easier way would be to check the analogous situation of massive spin-1 to see whether the WGC can be meaningfully applied there. It would also be important to investigate whether the usual black hole decay arguments for the weak gravity conjecture can be applied to Stückelberg fields.

One objection, raised in [231], is that the assumed scaling proportional to m^2 of the interaction (9.16) in the limit of vanishing mass is not necessary and that a linear proportionality to m might suffice. Here we simply want to mention that (9.16) serves as a *definition* of M_w and that in principle M_w and m might not be independent quantities[1]. Nevertheless, the proportionality to m^2 holds at least in KK theory, as we will see in the following.

Relatedly, it was claimed in [231] that the identification of the gauge coupling in (9.17) is largely meaningless because it does not capture the true m-scaling of

[1]Further comments on this point can be found in [99].

the interactions, as generically interactions of a massive spin-2 field blow up in the limit $m \to 0$. This is true in particular for the helicity-0 mode. The blowing up of other interactions is certainly an issue that deserves attention and is extensively discussed in the existing literature. It is precisely the point here that limits of vanishing coupling constants are very constrained in quantum gravity. We would like to add that the precise behavior of the interactions is theory-dependent. This theory-dependence is captured by a possible scaling of M_w with powers of m.

A further possible problem with the spin-2 conjecture raised in [231] is the purported absence of physical states charged under the "fake" Stückelberg $U(1)$ gauge symmetry. This is in general not true. At least in the case of massive spin-2 arising as KK excitations of the massless graviton, the Stückelberg fields can be interpreted as physical $U(1)$ gauge fields that arise from the higher-dimensional polarizations of the metric tensor. In section 9.4 we will explicitly write down the coupling of these to matter with momentum along the KK circle. In the context of quantum gravity, a gauge field without a matter coupling is certainly in conflict with the completeness conjecture, so one could very well expect the behavior showcased in the KK case to be a template for all possible appearances of massive spin-2 fields in string theory.

While our understanding of the relevance of the WGC for Stückelberg fields is certainly incomplete, we think that the results and the evidence presented in the following sections are sufficiently intriguing to entertain the possibility that a statement such as the spin-2 conjecture could be true.

Let us end this section by briefly discussing a possible application of the spin-2 conjecture to massive gravity, that is, a theory where the graviton itself is massive. The action of (ghost-free) massive gravity is given by

$$\int d^4x \sqrt{-G} \left[M_p^2 R(G) + \mathcal{I}(m, G) + \ldots \right] , \tag{9.19}$$

where $\mathcal{I}(m, G)$ is the mass term for the graviton about a flat space background, see [217] for an explicit expression.

It was argued in [32] that in this case it is natural to identify the interaction scale M_w with the Planck scale M_p. This results in the following statement

Strong Spin-2 Conjecture: An effective theory that contains a massive graviton with mass m has a universal cutoff Λ_m with

$$\Lambda_m \sim m \, . \tag{9.20}$$

This cutoff is associated with the mass scale of an infinite tower of states.

While the general structure WGC reasoning still applies, it is not totally clear whether a form of the WGC should hold for any consistent UV-complete quantum theory of massive gravity because black hole solutions are modified by the graviton mass [217]. One can in fact see that the argument for the WGC based on the emergence of gauge symmetries [130, 173, 174] will still go through.

From the point of view of string theory it is very natural to expect such a strong spin-2 conjecture. Indeed, the unique mass parameter of the string world-sheet is α' and hence, if we want to make the leading spin-2 excitation massive, the only possible mass seems to be of order the string scale, which also serves as the maximal cutoff of any derived EFT. Evidence for this picture is provided by the S-fold W-superstring constructions of [218].

9.3 Relation to Higher Derivative Terms

Another possible approach to the spin-2 conjecture is to consider the addition of higher derivative terms to the Einstein-Hilbert action. One such particular term is given by the square of the Weyl tensor [232]

$$S_G = \int d^4x \sqrt{-G} \left[M_p^2 R + \frac{1}{2g_W^2} W_{\mu\nu\rho\sigma} W^{\mu\nu\rho\sigma} + \ldots \right] , \tag{9.21}$$

with

$$W_{\mu\nu\rho\sigma} = R_{\mu\nu\rho\sigma} + G_{\mu[\sigma} R_{\rho]\nu} + G_{\nu[\rho} R_{\sigma]\mu} + \frac{1}{3} R G_{\mu[\rho} G_{\sigma]\nu} \, .$$

The addition of this term leads to the propagation of an additional massive spin-2 mode. This can be seen for example from the propagator which has two poles

$$\Delta_{\mu\nu\rho\sigma}(k) = \frac{1}{k^2 \left(\frac{k^2}{2g_W^2} - M_p^2 \right)} P_{\mu\nu\rho\sigma} \, , \tag{9.22}$$

where

$$P_{\mu\nu\rho\sigma} = \frac{1}{2} \left(\theta_{\mu\rho}\theta_{\nu\sigma} + \theta_{\mu\sigma}\theta_{\nu\rho} \right) - \frac{1}{3} \theta_{\mu\nu}\theta_{\rho\sigma} \, , \tag{9.23}$$

with

$$\theta_{\mu\nu} = \eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \, . \tag{9.24}$$

The massive pole is located at

$$m = \sqrt{2} g_W M \, . \tag{9.25}$$

The Weyl-squared term by itself is conformally invariant and as such is quite analogous to the F^2 term of Yang-Mills theory. One can thus speculate whether a form of weak gravity conjecture could apply to the coupling constant g_W in front of W^2, that is, to postulate the existence of a cutoff of order

$$\Lambda \lesssim g_W M_p \sim m \ . \tag{9.26}$$

We see that this would be equivalent to the *strong* spin-2 conjecture.

The above reasoning is of course very naive because closer inspection of (9.21) reveals that the residue of the massive pole has the wrong sign and thus it represents a ghost. It was argued in [233] that this is an artifact of truncating a theory that should really be thought of as containing an infinite series of higher derivative terms at a finite order. The authors considered a bimetric theory

$$\int d^4x \left[M_p^2 \sqrt{-G} R\,(G) + M_w^2 \sqrt{-W} R\,(W) + V(G,W) \right] \ . \tag{9.27}$$

The potential term has a rather constrained structure[2] and gives a particular linear combination of the (linearized) fields G and W a non-zero mass while leaving the orthogonal combination massless. When the background for G and W is chosen to be proportional, one can try to integrate out the massive linear combination from the theory. This has been done in the regime of parameter space where G and W approximately coincide with the mass eigenstates, so one can approximately integrate out W in a perturbative expansion in M_w/M_p. The result is to leading order the action (9.21), which is supplemented by infinitely many terms higher in the derivative expansion that render the ghostly pole physical.

In the approximation where the massive mode is roughly aligned with W, M_w in (9.27) is the mass scale associated to its self-interactions. This justifies the identification with the interaction energy scale in the spin-2 conjecture. From the general form of the spin-2 conjecture 9.18 we see that the interpretation as the WGC applied to the Weyl-squared term in the action only makes sense when $M_w = M_p$. We will see later that this is in fact how the Weyl-squared action is realized in string theory.

9.4 Massive Spin-2 in String Theory

Let us now see in detail how massive spin-2 excitations may arise in string theory and check whether there is indeed evidence that the conjecture presented in the last section could hold.

[2]See for example [220–222].

Massive spin-2 particles are usually not the focus of phenomenological studies within the context of string theory. This is grounded in the fact that in order to make contact with experimental particle physics it is of primary importance to first understand how particles of spin 0, $\frac{1}{2}$, 1 can arise – these are the ingredients for chiral gauge theories and their Higgsings in four dimensions. Furthermore, problems like moduli stabilization, the cosmological constant or inflation are at first glance spin-0 problems as they require an understanding of the scalar potential landscape in string theory.

The two primary examples that we will investigate are given by Kaluza-Klein gravitons which arise naturally when we compactify string theory from ten to lower dimensions, as well as more intrinsically string spin-2 states in the string oscillator tower. It is worth pointing out that massive spin-2 particles also arise as composite objects in confining gauge theories. We will leave studying this within the context of string theory for future work.

9.4.1 Kaluza-Klein Gravitons

The primary example of massive spin-2 fields, which can arise in any gravitational theory in $d > 4$ dimensions, are the Kaluza-Klein excitations of the graviton. They are in a sense the harmonic oscillator of massive spin-2 theories[3] as they already provide us with a great number of consistent examples parameterized by the different possible compactification geometries. In the following we will focus on the KK reduction from five to four dimensions, which we have introduced in chapter 2.1.

In the Kaluza-Klein reduction of five-dimensional gravity on a circle, the massless spectrum is well-known to consist of a massless scalar, a vector and a traceless symmetric tensor arising from the $5 \to 4$ decomposition of the metric tensor. This corresponds to the decomposition of $SO(1,4)$ representations

$$\mathbf{15} \to \mathbf{10} + \mathbf{4} + \mathbf{1} \ . \tag{9.28}$$

It is important though to count only the physical degrees of freedom. For a massless particle in D dimensions these are given by the dimensions of irreducible representations of the little group $SO(D-2)$. We conclude that the metric fluctuation, being a traceless symmetric tensor of this $SO(3)$, propagates five degrees of freedom. This matches the number of degrees of freedom of the massless spectrum described, since $5 = 2 + 2 + 1$. For the excited KK modes the counting is modified. Massive particles in four dimensions have the same little group $SO(3)$ as massless particles in one dimension higher. For this reason

[3]This term was coined by Eran Palti.

the excited KK modes of the five-dimensional graviton assemble into a single massive spin-2 multiplet in the four-dimensional theory and there are no vector or scalar excitations at the massive levels.

Let us now see how the Stückelberg mechanism works in the case of Kaluza-Klein theory and determine the coupling constant (9.17) associated to the non-conservation of the four-dimensional energy-momentum tensor. We adopt the conventions of section 2.2. In the following we will occasionally neglect numerical factors like 2π, since we care about the parametric scaling with the dimensionful coupling constants which we will carefully track. There are now two equivalent ways to derive the Stückelberg gauge coupling (9.17). The first approach is to directly interpret the Fourier modes of the off-diagonal components of the 5D metric A_μ as the helicity-1 modes of the massive KK gravitons. The second approach consists of first going to a gauge where $\partial_y A_\mu = 0$, then deriving the lower-dimensional Fierz-Pauli Lagrangian and finally reinstating the vector fields by the Stückelberg trick. We will see that both methods lead to the same result.

Method 1: Direct KK reduction

We decompose the five-dimensional metric as in (2.11). From now on we will set all the excited modes of g and ϕ to zero retaining only their zero modes $g^{(0)}_{\mu\nu}$ and $\phi^{(0)}$. In the following we omit the KK indices on $g_{\mu\nu}$ and ϕ. The proper length of the extra dimension in 5D Planck units is then given by

$$l_{S^1} = \text{Length}(S^1)/\ell_5 = \int_0^{2\pi R} \sqrt{-G_{55}} = 2\pi r \left(\phi^{(0)}\right)^{1/3} , \qquad r = R/\ell_5 \equiv R M_5 \tag{9.29}$$

which is the physical quantity that will replace the radius R if we properly express all four-dimensional quantities in 4D Planck units. Now consider a 5D scalar field φ with Fourier modes

$$\varphi = \frac{1}{\sqrt{2\pi R}} \sum_n \varphi^{(n)} e^{iny/R} . \tag{9.30}$$

Note that the normalization factor accounts for the fact that the mass dimension of a canonically normalized scalar in 5D is 3/2, while it has to be one in 4D. The compactified kinetic term will again be canonically normalized. We use the metric Ansatz (2.11):

$$\begin{aligned} S_\varphi &= \frac{1}{2} \int d^5X \sqrt{-G} G^{MN} \partial_M \varphi \partial_N \varphi \\ &= \frac{1}{2} \int d^5X \sqrt{-g} \left[g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - 2A^\mu \partial_y \varphi \partial_\mu \varphi + \left(A^2 + \frac{1}{\phi} \right) (\partial_y \varphi)^2 \right] . \end{aligned} \tag{9.31}$$

The current interaction has the form

$$\begin{aligned} S_{\rm int} &= \int d^5X\sqrt{-g}A^\mu\partial_y\varphi\partial_\mu\varphi \\ &= \frac{1}{2\pi R}\int d^4x dy\sqrt{-g}\sum_{m,l,n}A_\mu^{(m)}\left(\frac{in}{R}\right)\varphi^{(n)}\partial^\mu\varphi^{(l)}e^{iy(m+n+l)/R} \\ &= \sum_{m,n}\left(\frac{in}{R}\right)\int d^4x\sqrt{-g}A_\mu^{(m)}\left(\varphi^{(n)}\partial^\mu\varphi^{(-n-m)}\right) \\ &\equiv \frac{1}{R}\sum_{m,n}\int d^4x\sqrt{-g}A_\mu^{(-m-n)}j^{(m,n)\mu} \end{aligned}, \qquad (9.32)$$

where the current $j_\mu^{(m,n)}$ is implicitly defined through the last equality.

If we want to reinstate integral charges, we thus have to define a renormalized gauge field $\tilde{A} = A/R$ such that

$$\mathcal{L}_{\rm int} = \sum_{m,n}\int d^4x\sqrt{-g}\tilde{A}_\mu^{(-m-n)}j^{(m,n)\mu} . \qquad (9.33)$$

Now we work out the masses and kinetic terms. The mass term for KK modes of φ is given by (9.31)

$$\frac{1}{2}\int d^5X\sqrt{-g}\left[\frac{1}{\phi}(\partial_y\varphi)^2\right] = \frac{1}{2}\sum_n\int d^4x\sqrt{-g}\left(\frac{n^2}{\phi R^2}\right)\varphi^{(n)}\varphi^{(-n)} , \qquad (9.34)$$

so we read off

$$m_{\rm KK}^2 = \frac{1}{R^2\phi} . \qquad (9.35)$$

To obtain the kinetic term for $A^{(n)}$ we observe that it is a component of G, which is dimensionless, so $A^{(n)}$ itself is dimensionless. The kinetic term of G is reduced as

$$\frac{M_5^3}{2}\int d^5X\sqrt{-G}R(G) = \frac{M_5^3R}{2}\int d^4x\sqrt{-g}\left[R(g) + \sum_n\phi F^{(n)\mu\nu}F_{\mu\nu}^{(-n)} + \ldots\right] . \qquad (9.36)$$

From this, if $\tilde{F}^{(n)} = d\tilde{A}^{(n)}$, we see that

$$\begin{aligned} S_{{\rm kin},\tilde{A}} &= -\frac{M_4^2}{2}\sum_n\int d^4x\sqrt{-g}\left(\phi R^2\right)\tilde{F}^{(n)\mu\nu}\tilde{F}_{\mu\nu}^{(-n)} \\ &= -\frac{1}{2}\sum_n\int d^4x\sqrt{-g}\left(\frac{M_4^2}{m_{\rm KK}^2}\right)\tilde{F}^{(n)\mu\nu}\tilde{F}_{\mu\nu}^{(-n)} . \end{aligned} \qquad (9.37)$$

So finally we can identify the Stückelberg gauge coupling (9.17) as

$$g = \frac{m_{\rm KK}}{M_4} = \frac{1}{r^{3/2}\phi^{1/2}} = \frac{1}{l_{S^1}^{3/2}} . \qquad (9.38)$$

Method 2: Stückelberg mechanism and current non-conservation

We will now confirm this analysis by directly applying the Stückelberg trick as in section 9.1. In order to simplify the discussion, we set the full $A^\mu = 0$ and keep only the zero mode of $\phi = \phi^{(0)}$. In doing so, we assume that the remaining $g^{(n)}_{\mu\nu}$ have eaten up the $A^{(m)}, \phi^{(m)}$ by the Stückelberg mechanism[4]. We will then obtain the 4D action for the metric perturbation $h^{(n)}_{\mu\nu}$ and as in section 9.1 reinstate the Stückelberg vectors by the transformation

$$h^{(n)}_{\mu\nu} \mapsto h^{(n)}_{\mu\nu} + \partial_{(\mu}\chi^{(n)}_{\nu)} \ . \tag{9.39}$$

The 5D metric specializes to

$$G_{MN} = \phi^{-1/3}\begin{pmatrix} g_{\mu\nu} & 0 \\ 0 & \phi \end{pmatrix} \qquad G^{MN} = \phi^{1/3}\begin{pmatrix} g^{\mu\nu} & 0 \\ 0 & \frac{1}{\phi} \end{pmatrix} \ . \tag{9.40}$$

We split this into a background metric plus perturbation

$$G_{MN} = \langle G_{MN}\rangle + \frac{2}{M_5^{3/2}} H_{MN} \ , \tag{9.41}$$

$$\langle G_{MN}\rangle = \phi^{-1/3}\begin{pmatrix} \langle g_{\mu\nu}\rangle & 0 \\ 0 & \phi \end{pmatrix} \ , \qquad H_{MN} = \phi^{-1/3}\begin{pmatrix} \frac{1}{R^{1/2}} h_{\mu\nu} & 0 \\ 0 & 0 \end{pmatrix} \ . \tag{9.42}$$

For simplicity of presentation, we only pick out particular index contractions from the full action in the following. All other terms can be worked out in the same fashion. From [56] we obtain the kinetic term for the perturbation H_{MN}:

$$\begin{aligned} S &= -\frac{1}{2}\int d^5X\sqrt{-\langle G\rangle}\langle G^{OP}\rangle\nabla_O H_{MN}\nabla_P H^{MN} \supset -\frac{1}{2R}\int d^5X\sqrt{-\langle g\rangle}\langle g^{\gamma\delta}\rangle\nabla_\gamma h_{\mu\nu}\nabla_\delta h^{\mu\nu} \\ &= -\frac{1}{2}\sum_n\int d^4x\sqrt{-\langle g\rangle}\langle g^{\gamma\delta}\rangle\nabla_\gamma h^{(n)}_{\mu\nu}\nabla_\delta h^{(-n)\mu\nu} \ . \end{aligned} \tag{9.43}$$

From this, we see that $h_{\mu\nu}$ is indeed the proper mass dimension one Fierz-Pauli field. Furthermore, we also get the mass term from the analogous term with y derivatives

$$\begin{aligned} S &= -\frac{1}{2}\int d^5X\sqrt{-\langle G\rangle}\langle G^{OP}\rangle\nabla_O H_{MN}\nabla_P H^{MN} \supset -\frac{1}{2R}\int d^5X\sqrt{-\langle g\rangle}\frac{1}{\phi}\partial_y h_{\mu\nu}\partial_y h^{\mu\nu} \\ &= -\frac{1}{2}\sum_n\int d^4x\sqrt{-\langle g\rangle}\left(\frac{n^2}{R^2\phi}\right)h^{(n)}_{\mu\nu}h^{(-n)\mu\nu} \ . \end{aligned} \tag{9.44}$$

[4]In fact, this is not only a gauge choice but also a choice of vacuum for the zero mode.

We see that the Fierz-Pauli mass of $h_{\mu\nu}^{(n)}$ is given by

$$\left(m_{FP}^{(n)}\right)^2 = \frac{n^2}{R^2\phi} \ . \tag{9.45}$$

Consequently, $\chi_\mu^{(n)}$ will have a kinetic term

$$S = -\frac{1}{2}\sum_n \int d^4x\sqrt{-\langle g\rangle} m_{FP}^2 \left(F_{\chi^{(n)}}^{\mu\nu}\right)^2 \ . \tag{9.46}$$

Because h has mass dimension one and $h \sim d\chi$, the field χ_μ will have mass dimension zero.

Now we want to investigate the coupling of χ_μ to the energy momentum tensor of 5D matter. For concreteness, let us take for example a scalar field φ with mode expansion as in the previous subsection. The coupling can be worked out as follows[5]

$$\begin{aligned} S &= -\int d^5X\sqrt{-G}G^{MN}\partial_M\varphi\partial_N\varphi \supset -\int d^5X\sqrt{-g}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi \\ &= -\int d^5X\sqrt{-\langle g\rangle}\delta g_{\mu\nu}\left(\partial^\mu\varphi\partial^\nu\varphi - \frac{1}{2}\langle g^{\mu\nu}\rangle\mathcal{L}_\varphi\right) = -\int d^5X\sqrt{-\langle g\rangle}\frac{h_{\mu\nu}}{M_5^{3/2}R^{1/2}}T_\varphi^{\mu\nu} \\ &= -\int d^5X\sqrt{-\langle g\rangle}\frac{h_{\mu\nu}}{M_4}T^{\varphi,\mu\nu} \supset -\int d^5X\sqrt{-\langle g\rangle}\frac{\partial_{(\mu}\chi_{\nu)}}{M_4}T^{\varphi,\mu\nu} \\ &= -\int d^5X\sqrt{-\langle g\rangle}\frac{\chi_\mu}{M_4}\partial_\nu T^{\varphi,\nu\mu} \ . \end{aligned} \tag{9.47}$$

The last line is completely general and does not only hold for a 5D scalar field, so let us proceed in full generality. We use the fact that the 5D energy momentum tensor is conserved:

$$\partial_\mu T^{\mu\nu} = -\partial_y T^{y\nu} \ . \tag{9.48}$$

It follows that

$$\begin{aligned} &\int d^5X\sqrt{-\langle g\rangle}\frac{\chi_\mu}{M_4}\partial_\nu T^{\nu\mu} \\ &= -\int d^5X\sqrt{-\langle g\rangle}\frac{\chi_\mu}{M_4}\partial_y T^{y\mu} \\ &= -\sum_m \int d^4x\sqrt{-\langle g\rangle}\frac{m}{M_4 R}\chi_\mu^{(m)}T^{(-m)y\mu} \\ &= -\sum_m \int d^4x\sqrt{-\langle g\rangle}\frac{m}{M_4 R\phi}\chi^{(m)\mu}T_{y\mu}^{(-m)} \ . \end{aligned} \tag{9.49}$$

[5]We focus on the kinetic coupling and neglect the coupling to potential energy, as it is analogous.

Here T^m are the modes of the energy momentum tensor

$$T^{\mu\nu} = \frac{1}{2\pi R} \sum_m T^{(m)\mu\nu} e^{imy/R} \, . \tag{9.50}$$

At this point it is convenient to rescale the fields $\chi^{(m)} \mapsto \chi^{(m)}/m$, so we obtain the following kinetic term and interaction

$$\begin{aligned} S_{\text{kin}} &= -\frac{1}{2} \sum_m \int d^4x \sqrt{-\langle g \rangle} \frac{1}{R^2 \phi} \left(F^{\mu\nu}_{\chi^{(m)}} \right)^2 \\ S_{\text{int}} &= -\sum_m \int d^4x \sqrt{-\langle g \rangle} \frac{1}{M_4 R \phi} \chi^{(m)\mu} T^{(-m)}_{y\mu} \end{aligned} \, . \tag{9.51}$$

Here the factor of m in the Fierz-Pauli mass of (9.46) has been canceled together with the factor m in the interaction by the field redefinition. The $(y\mu)$-component of the energy momentum tensor for a scalar is

$$T^{\varphi}_{y\mu} = \partial_y \varphi \partial_\mu \varphi - \frac{1}{2} \langle g_{y\mu} \rangle \mathcal{L}_\varphi = \partial_y \varphi \partial_\mu \varphi \, , \tag{9.52}$$

because the metric does not have the off-diagonal term. This universal term $\sim \langle g_{y\mu} \rangle$ arises from the variation of $\sqrt{-\langle g \rangle}$ in the action and vanishes for every field, not only scalars.

We want to evaluate the modes of the energy momentum tensor for the scalar

$$\begin{aligned} T_{y\mu} = \partial_y \varphi \partial_\mu \varphi = \frac{1}{2\pi R} \sum_{l,n} \frac{l}{R} \varphi^{(l)} \partial_\mu \varphi^{(n)} e^{i(l+n)y/R} &= \frac{1}{2\pi R} \sum_{l,m} \frac{l}{R} \varphi^{(l)} \partial_\mu \varphi^{(m-l)} e^{imy/R} \, , \\ T^m_{y\mu} = \sum_l \frac{l}{R} \varphi^{(l)} \partial_\mu \varphi^{(m-l)} \, . \end{aligned} \tag{9.53}$$

Putting everything together, the interaction term is

$$S_{\text{int}} = \sum_{m,l} \int d^4x \sqrt{-\langle g \rangle} \frac{m^2_{\text{KK}}}{M_4} \chi^{(m)\mu} \left(l \varphi^{(l)} \partial_\mu \varphi^{(-m-l)} \right) \, . \tag{9.54}$$

This is precisely of the same form as (9.32) being linear in the integral charges l.

In order to determine the coupling constant to integrally quantized charges, we again need to rescale the gauge field. For the gauge coupling we obtain

$$\frac{1}{g^2} = m^2_{\text{KK}} \left(\frac{M_4}{m^2_{\text{KK}}} \right)^2 = \frac{M_4^2}{m^2_{\text{KK}}} \, . \tag{9.55}$$

This coincides with the previously obtained result (9.38).

Relation to WGC Applied to the KK Gauge Field

It was suggested in [231] that the WGC applied to the "unphysical" KK Stückelberg gauge fields only gives a physical tower of states because it coincides with the WGC applied to the physical zero mode gauge field $A_\mu^{(0)}$. That this is not the case can be seen by considering the theory not on a circle S^1 but on the orbifold $S^1/\mathbb{Z}_2$. As we have discussed briefly in section 2.1, in this case the zero mode is projected out. Nevertheless, our conjecture (9.18) is still satisfied in this case.

9.4.2 Massive Spin-2 from Excited Strings

A second way to obtain massive spin-2 fields from string theory is directly as excitations of the fundamental string. Let us first look at the open bosonic string in light-cone quantization. Recall that the vacuum state $|0,k\rangle$ of the open bosonic string is a tachyon. The first excited state $\alpha^i_{-1}|0,k\rangle$ is a massless vector field, consistent with its transformation as a vector under the little group $SO(24)$. At the first massive level we have the states

$$\underbrace{\alpha^i_{-1}\alpha^j_{-1}|0,k\rangle}_{(D-2)(D-1)/2} \qquad \underbrace{\alpha^i_{-2}|0,k\rangle}_{D-2}, \qquad M^2 = \frac{2}{\alpha'}, \tag{9.56}$$

where $D = 26$ for the bosonic string.

These add up precisely to the traceless symmetric tensor representation of the little group $SO(25)$ for massive particles

$$(D-2)(D-1)/2 + (D-2) = \binom{(D-1)+1}{2} - 1\,. \tag{9.57}$$

The Nth excited level will contain, among other representations, the rank N symmetric traceless tensor which in four dimensions would correspond to a spin N particle.

This result generalizes rather straightforwardly to the superstring. Since we are interested only in bosons, we can restrict to the NS sector. Here the first states above the massless vector that are not killed by the GSO projection are

$$\underbrace{b^i_{-\frac{1}{2}}b^j_{-\frac{1}{2}}b^k_{-\frac{1}{2}}|0,k\rangle}_{(D-2)(D-3)(D-4)/6}, \qquad \underbrace{\alpha^i_{-1}b^j_{-\frac{1}{2}}|0,k\rangle}_{(D-2)^2}, \qquad \underbrace{b^i_{-\frac{3}{2}}|0,k\rangle}_{D-2}, \qquad M^2 = \frac{2}{\alpha'}\,. \tag{9.58}$$

The decomposition of these states under the massive little group $SO(9)$ is

$$\mathbf{R} = \mathbf{84} + \mathbf{44}\,, \tag{9.59}$$

where **44** is the rank two symmetric traceless tensor representation. It is spanned by the subset of states given by

$$|p,k\rangle = \left(p_{ij}\, b^{(i}_{-\frac{1}{2}} \alpha^{j)}_{-1} + p_i\, b^{i}_{-\frac{3}{2}} \right) |0,k\rangle \ , \tag{9.60}$$

with p_{ij} and p_i independent tensors.

In general, the maximally spinning bosonic state on the leading Regge trajectory can be constructed as

$$|p,k\rangle = \left(p_{i_1 \dots i_N}\, b^{(i_1}_{-\frac{1}{2}} \alpha^{i_2}_{-1} \dots \alpha^{i_N)}_{-1} + p_{i_1 \dots i_{N-1}}\, b^{(i_1}_{-\frac{3}{2}} \alpha^{i_2}_{-1} \dots \alpha^{i_{N-1})}_{-1} + \dots + p_i\, b^{i}_{-\frac{N}{2}} \right) |0,k\rangle \ . \tag{9.61}$$

These states assemble into a rank N symmetric traceless tensor of $SO(9)$. The counting of dimensions works because of the identity

$$\binom{(D-1)+N-1}{N} - 1 = \sum_{n=1}^{N} \binom{(D-2)+n-1}{n} \ . \tag{9.62}$$

From this leading Regge trajectory one can infer that in the closed string spectrum every massive level contains a rank two symmetric traceless tensor representation of $SO(9)$, because the tensor product of representations necessarily produces this irreducible summand[6]

$$\underbrace{\square\square\cdots\square}_{N} \otimes \underbrace{\square\square\cdots\square}_{N} = \square\square \oplus \dots \ . \tag{9.63}$$

We thus find that the whole tower of massive closed string states is populated with rank two symmetric traceless tensors at each level. Upon compactification this leads to an infinite tower of massive spin-2 fields which are not of the KK type and provide our second example of massive spin-2 in string theory. We mention in passing that the Stückelberg gauge symmetry of the open string massive spin-2 state is closely related to the Virasoro symmetry of the string [234].

For all of these states the spin-2 conjecture (9.18) is fulfilled in the sense that the mass of even the lowest lying massive string states is precisely the string scale, at which local quantum field theory breaks down and we need to include the whole infinite tower of excited string states with masses

$$m_n \sim \sqrt{n} M_s \ \ \forall\, n \in \mathbb{N} \tag{9.64}$$

[6]We can explicitly construct it as

$$T^{A_1 \dots A_{N-1}(A_N} \tilde{T}_{A_1 \dots A_{N-1}}{}^{B_N)} - \frac{1}{9} \delta^{A_N B_N}\, T \cdot \tilde{T} \ ,$$

where T and $\tilde{T}$ are rank N symmetric traceless $SO(9)$ tensors and the dot is contraction.

in order to get a consistent theory. Thus, it seems natural to set $\Lambda_w \sim M_p$ for these states such that the fundamental cutoff predicted by the conjecture is identical to the string scale. We leave an explicit verification of this for future work.

Let us briefly sketch how one would go about obtaining a four-dimensional effective description of these modes along the lines of [235]. We will assume a $\mathbb{R}^{1,3} \times T^6$ compactification of the type II superstring with a stack of N D3-branes extended in the non-compact directions. This will lead to an uncanceled tadpole which we will ignore in the following[7]. In a more detailed analysis this tadpole could be cured by the inclusion of O3-planes.

The authors of [235] argued that the four-dimensional description on the D3-brane of the open string massive spin-2 state is given by a supersymmetric extension of the Weyl-squared action (9.21) constructed in [218]. We can thus map this setup directly to the discussion of section 9.3. Because the natural scale of self-interactions of the spin-2 state in string theory is given by $M_w = M_p$, we expect $g_m = g_W$. The coupling in front of the Weyl-squared term is explicitly given by [235]

$$g_W = \frac{g_{\mathrm{D3}}^2}{\sqrt{\mathcal{V}}}\,, \tag{9.65}$$

where $\mathcal{V}$ is the volume of the torus. In the perturbative limit $\mathcal{V}, g_{\mathrm{D3}}^{-1} \gg 1$ we see that $g_w \ll g_{\mathrm{D3}}$ and hence the spin-2 conjecture provides a stronger constraint on the EFT than just the WGC applied to the D3-brane gauge theory. The interpretation of the difference is that the coupling g_W is an open-closed coupling rather than an open-open coupling like g_{D3} [32].

In order to put the spin-2 conjecture on a firm footing it will be crucial to study more extensively the possible embeddings of massive spin-2 fields in string theory. For example, [236, 237] studied holographic constructions of massive spin-2 in AdS backgrounds to which tentative stringy embeddings have also been proposed [238, 239]. Understanding massive spin-2 fields in the context of the AdS/CFT correspondence might lead to a better understanding or even a proof of the spin-2 conjecture, as it is expected for the WGC itself [240].

9.5 Possible Implications for Massive Gravity

We will now briefly discuss possible observable implications of what has been discussed in the previous sections. In particular the strong spin-2 conjecture 9.20 has striking phenomenological consequences [32]. This is because the mass of the graviton itself is tightly constrained by experiments. The recent detection of

[7] As we have discussed in section 7.1.3, tadpole cancellation can be important for verifying swampland conditions although here this would only be relevant for D3-brane gauge field.

gravitational waves emitted by two in-spiraling black holes by the LIGO collaboration provides the upper bound on the graviton mass [241]

$$m < 10^{-22}\,\mathrm{eV}\,. \tag{9.66}$$

A review of bounds on the graviton mass can be found in [242]. Clearly we have so far not observed an infinite tower of states at this mass scale. As a result the assumption of a non-vanishing graviton mass, together with the strong spin-2 conjecture, is in sharp contradiction with experimental evidence. Either the graviton is exactly massless or the spin-2 conjecture is wrong.

Similar to the connection of the claimed detection of large tensor modes in the CMB [25, 27] and large field inflation [26], this can lead in principle to a prediction from string theory that is experimentally falsifiable at low energies.

10 Consistency of the KKLT Scenario with Swampland Conjectures

The swampland conjecture having attracted the most attention in the last year is certainly the de Sitter conjecture [167] which we have introduced in section 7.3.2. The flood of papers following up on this publication is not only explained by the fundamental theoretical importance of the cosmological constant problem and its resolution in string theory but also by the provocative nature of the claim that there could be no de Sitter vacua in string theory.

In fact, many models of de Sitter space of varying rigor and explicitness have been constructed by practitioners of string phenomenology. These models certainly deserve the label "string inspired" [99], because they use string theory as a well-defined framework to go beyond effective quantum field theory. However, they often combine ingredients that are predicted by string theory in intricate ways so that the consistency is not always immediately obvious. Most known constructions of de Sitter space crucially rely on the inclusion of quantum effects together with SUSY breaking leading to significant challenges. Au contraire, AdS backgrounds can be supersymmetric and easily generated by simple classical ingredients, as for example in the $AdS_4 \times S^7$ compactification of M-theory. In the context of classical flux compactifications of M-theory on smooth manifolds, there is a celebrated no-go theorem against obtaining de Sitter [243]. Similar results have been obtained for type IIA orientifolds [244][1].

To date one of the most popular construction of a potentially large family of de Sitter vacua in string theory is given by the KKLT scenario [21], which we reviewed in section 6.4. Simply put, de Sitter is achieved through weakly breaking the SUSY of a very shallow AdS vacuum by "uplifting" it with a tunable small source of positive vacuum energy. The detailed description within the context of type IIB superstring theory involves a delicate balance between classical and non-perturbative contributions to the potential energy, the tuning of flux quantum numbers, as well as the back-reaction of SUSY-breaking ingredients on the

[1] The validity of the type IIA no-go result seems to be tied to the cancellation of K-theory charge [245].

background, which has to be carefully taken into account. The claim to fame of this model is of course that all of this can be achieved at least in principle with a rather explicit description of most of the steps in a set of effective field theories, while maintaining a stable hierarchy of energy scales and stabilizing all of the moduli fields.

Ideally, the hierarchy that one would like to achieve is

$$M_{\text{Kähler}} < M_{\text{cs}} < M_{\text{KK}} < M_s < M_p \, . \tag{10.1}$$

While the last two inequalities are the usual conditions allowing for a consistent geometric description of the Calabi-Yau background in the context of perturbative string theory, the first three of them correspond to the different steps constituting the KKLT construction. The requirement that the (smallest) KK mass scale is heavier than the moduli masses means that their stabilization can be understood completely within a truly four-dimensional effective description. The hierarchy between the mass scale of the complex structure moduli and the Kähler moduli arises because the former are stabilized by classical effects (fluxes), while the latter only acquire an exponentially small non-perturbative mass at the quantum level.

It has been under debate whether this uplift mechanism in the last step is controlled (see [246] for a recent review). In particular, it was recently attempted to construct a Maldacena-Nuñez type no-go theorem [247] which was subsequently strongly debated (see also [248] for another criticism). Arguments for a flattening of the uplift were given from a 10D perspective as well as a possible four-dimensional interpretation in terms of the nilpotent superfield description of the uplift. An inconsistency in the four-dimensional picture was pointed out in [249], to which the authors of [247] responded both by stressing the importance of their 10D analysis and by suggesting a modified 4D description [250]. This was again refuted in [251] and later in [252] where it was shown that the proposed flattening effects do not exclude the existence of de Sitter minima.

Another, more direct criticism attacked the 10D arguments against the uplift. At the core of the analysis of [247] was a computation of the on shell 4D potential by deriving it from the (trace of the) 10D Einstein equations. To do so requires implementing the effects of gaugino condensation on a D7-brane stack in the ten-dimensional framework. While [253] agreed with the analysis of [247], it was later argued that one has to take into account certain four fermion interactions that lead to a perfect square form of the action [254, 255]. The conclusion of [256, 257] is that the argument against the de Sitter uplift presented in [247] breaks down, whereas the authors of [258] seem to reconfirm the no-go. Here we do not want to enter this discussion.

An entirely different possible point of failure for the uplift was pointed out in [259]. In order to control the uplift by an anti D3-brane, its energy density has to be exponentially suppressed to be comparable to the exponentially small mass scale of Kähler moduli stabilization. This is usually achieved through warping. A warped compactification can be constructed in the vicinity of a conifold singularity of a generic Calabi-Yau by turning on three-form fluxes [12]. While the warping is a relatively mild deformation of the geometry, one has to use a modified effective field theory that accounts for the effects of warping in the Kähler potential. This was studied by [96, 97, 260–262] already right after the seminal paper [12] by Giddings, Kachru, Polchinski (GKP).

The main result of [259] is that the uplift can significantly disturb the stabilization of the conifold modulus Z. Avoiding a destabilization requires turning on a certain minimal amount of three-form flux $M > M_{\text{min}}$. Whether this is feasible depends on the tadpole conditions of the specific compactification under consideration. The fact that the conifold modulus Z has an exponentially suppressed mass due to warping, a priori comparable in magnitude to the Kähler modulus T[2], motivated us to study more closely the consistency of the (warped) effective field theory including both light moduli prior to the uplift.

Even before the uplift, the KKLT construction features a very desirable property that is notoriously difficult to achieve in tree-level supergravity compactifications. This is the scale separation of the AdS length scale and the moduli masses [263]. Thus, it seems to violate an *AdS scale separation swampland conjecture* saying that AdS minima of string theory satisfy

$$m^2 L_{\text{AdS}}^2 \geq c'' \,, \tag{10.2}$$

where c'' is an order one coefficient and m is the lightest non-vanishing (moduli) mass [33, 263].

In the KKLT scenario we expect

$$m_\tau L_{\text{AdS}} \sim a\tau \sim -\log W_0 \,. \tag{10.3}$$

Here W_0 is the value of the superpotential after complex structure moduli stabilization. The right-hand side of (10.3) can become large for exponentially small values of W_0. These are in fact also needed in order to stabilize the Kähler modulus in the perturbative regime. Whether such small values can be achieved at all in a concrete Calabi-Yau compactification is not at all clear.

In our work [33] we modified the KKLT procedure as follows. Similar to [259] we assume that the bulk complex structure moduli are fixed at some high energy scale E_{bulk}, with the exception of a single modulus describing the degeneration

[2]Consistency of the uplift requires the mass of the Kähler modulus T to be suppressed by two additional powers of the warp factor compared to Z leading to an a posteriori justification of stabilizing them in two separate steps. Nevertheless, the results of [259] seem to indicate that this indirect reasoning might be dangerous.

to a conifold locus. The resulting warped effective action is studied under the assumption that the bulk stabilization leads to an arbitrary value of $W_{0,\text{bulk}} \equiv W_{\text{cs}}$. For the special value $W_{\text{cs}} = 0$ we find that the stabilization of the conifold modulus Z is able to dynamically generate an exponentially small effective W_0 for the overall Kähler modulus T due to the warping.

The procedure just outlined generates a KKLT-like scale separated AdS minimum without the assumption of fine-tuning $0 < |W_0| \ll 1$ violating (10.2). Provided that there is nothing wrong with the anti-brane uplift we also reproduce the KKLT de Sitter minimum, violating the de Sitter conjecture (7.16). To see what might still go wrong besides the assumption of a tuned $|W_0|$ we set out to calculate the masses of warped KK modes, which can possibly violate the hierarchy of energy scales (10.1). We find that a finite number of these can indeed be lighter than the mass scale of the conifold modulus.

Strictly speaking, this means that the four-dimensional effective field theory is not under control and we should analyze the setup in 10D[3]. Even though these modes are not taken into account in the effective field theory, we find that integrating them out has a strikingly mild effect. Their one-loop contribution to the moduli kinetic terms is proportional to the tree-level functional form which is a behavior that the reader will recognize from our discussion of emergent kinetic terms in section 7.4. This could lead to the interpretation that the setup is self-consistent and not completely out of control. We also find that the recent derivation of the de Sitter conjecture [168] from the distance conjecture, discussed in section 7.5, fails here.

In section 10.1, we will first briefly review the warped effective field theory employed by [33]. The following sections 10.2 and 10.3 will then discuss the calculation of the warped Kaluza-Klein masses and the emergence of the classical kinetic terms from integrating out a tower of light modes. Section 10.4 will investigate in some detail the relevance of KK modes in such a scenario where kinetic terms can be considered to be emergent.

With the exception of section 10.4, the material presented in the rest of this chapter is reproduced from [33].

10.1 Modified KKLT Effective Field Theory

We consider a type IIB CY orientifold compactification with three-form fluxes that stabilize the complex structure moduli (see sec. 6.2). We furthermore assume that the complex structure moduli adjust to the fluxes such that the CY

[3] In fact, a 5D description that integrates in the radial direction of the warped throat might suffice.

is close to a conifold singularity. Locally, we can describe the geometry by the deformed conifold from section 5.2. In such a setup, we have two distinguished moduli – the *Kähler modulus T* that describes the overall volume of the compactification and the *conifold modulus Z* that parameterizes the degeneration to the conifold singularity. One can show that locally the period vector is of the form

$$\Pi = X^0 \begin{pmatrix} 1 \\ Z \\ -\frac{1}{2\pi i} Z \log Z + C + DZ + O(Z^2) \\ \vdots \end{pmatrix} , \tag{10.4}$$

where $\Pi^T = (X^0, X^1, F_1, \ldots)$, $A > 0$ is a real constant and $X^1 = \int_A \Omega$, $F_1 = \int_B \Omega$ are determined as integrals over the conifold A-cycle and B-cycle. The other periods are analytic functions of Z.

We will now discuss the effective field theory in the cases where the flux-induced warping can and cannot be neglected.

Unwarped Case

The periods lead to a universal form of the Kähler potential in the conifold limit

$$\begin{aligned} K_{\text{cs}} &= -3 \log\left(-i(T - \bar{T})\right) - \log\left(-i(S - \bar{S})\right) - \log\left(-i \overline{\Pi} \Sigma \Pi\right) \\ &= -3\left(-i(T - \bar{T})\right) - \log\left(-i(S - \bar{S})\right) - \log\left(\frac{1}{2\pi} |Z|^2 \log(|Z|^2) + A + O(|Z|^2)\right) , \end{aligned} \tag{10.5}$$

where $T = \rho + i\tau$ and $S = C_0 + i \exp(-\Phi)$ are the Kähler modulus and axio-dilaton. In addition, we parameterize the conifold modulus as $Z = \zeta \cdot \exp(i\sigma)$. The moduli space metric is diagonal and in the Z-plane is given by

$$G_{Z\bar{Z}} \sim -\log(|Z|^2)/A . \tag{10.6}$$

The Gukov-Vafa-Witten superpotential (6.12) evaluates in this case to

$$W = -\frac{M}{2\pi i} Z(\log Z - 1) + KSZ + \ldots , \tag{10.7}$$

where the A-cycle and B-cycle are threaded by M units of F_3 flux and K units of H_3 flux respectively.

$$\frac{1}{(2\pi)^2 \alpha'} \int_A F_3 = M \qquad \frac{1}{(2\pi)^2 \alpha'} \int_B H_3 = -K \tag{10.8}$$

One derives the following potential for Z using the standard supergravity formula (6.2)

$$V \approx e^K G^{Z\bar{Z}} D_Z W D_{\bar{Z}} \overline{W} \approx M_p^4 \frac{g_s}{\tau^3} \left(-\log(|Z|^2)\right)^{-1} \left|\frac{M}{2\pi} \log Z + KS\right|^2 , \qquad (10.9)$$

where we assumed that the axio-dilaton is fixed at some VEV $S = g_s^{-1}$. This has a minimum at which

$$Z = e^{-\frac{2\pi K}{g_s M}} , \qquad m_Z^2 \sim \frac{M_p^2}{\mathcal{V}^2 |Z|^2} \sim \frac{M_s^2}{\mathcal{V}|Z|^2} . \qquad (10.10)$$

The hierarchy of scales (10.1) can only be achieved if the volume of the A-cycle in string units is much larger than one

$$\text{Vol}(A) = \mathcal{V}^{\frac{1}{2}} \left|\int_A \Omega\right| = (\mathcal{V}|Z|^2)^{\frac{1}{2}} \gg 1 . \qquad (10.11)$$

This is the so-called dilute flux limit, where the 4D effective SUGRA can be effectively trusted and the backreaction through the warp factor neglected. We want to study the opposite regime where the warping is strong, so the effective description needs to be modified. In the limit that we consider here, the mass scale of the complex structure moduli and the axio-dilaton scales as

$$m_{\text{cs}}^2 \sim \frac{M_p^2}{\mathcal{V}^2} \sim \frac{M_s^2}{\mathcal{V}} . \qquad (10.12)$$

Warped Case

In the case of strong warping, we proceed along the lines of [97]. If the fluxes do not stabilize the Kähler modulus T, rescaling the internal metric $\tilde{g} \to \lambda^2 \tilde{g}$ should be an unconstrained deformation. Using this, the authors of [97] showed that the warp factor at the tip of the KS throat

$$e^{-4A} = 1 + \frac{e^{-4A_{\text{con}}}}{\lambda^4} \sim 1 + \frac{c}{(\mathcal{V}_w |Z|^2)^{\frac{2}{3}}} + \ldots \qquad (10.13)$$

has to scale with the warped volume $\lambda \sim \mathcal{V}_w^{1/6}$ which is given by

$$\mathcal{V}_w = \frac{1}{g_s^{3/2} (\alpha')^3} \int d^6 y \, e^{-4A} \sqrt{\tilde{g}} \sim \tau^{\frac{3}{2}} . \qquad (10.14)$$

Using this scaling, we match the conifold modulus Z to the parameter ϵ of the KS solution

$$\epsilon^2 \to (\alpha')^{3/2}\sqrt{g_s^{3/2}\,\mathcal{V}_w}Z\ . \tag{10.15}$$

This finally allows us to express the warp factor in the vicinity of the conifold locus as

$$e^{-4A(y)} \approx 2^{\frac{2}{3}}\frac{g_s M^2}{(\mathcal{V}_w|Z|^2)^{\frac{2}{3}}}\mathcal{I}(y)\ . \tag{10.16}$$

We note that there is a smooth limit to the dilute flux regime, where one keeps Z small and blows up the volume $\mathcal{V}_w$. In this limit, $\exp(-2A) \to 1$ and $\mathcal{V}_w \to \mathcal{V}$. Here we want to work in the regime of strong warping

$$\mathcal{V}_w|Z|^2 \ll 1\ . \tag{10.17}$$

There are several bounds on the effective field theory parameters that one should consider and which are obtained by requiring that certain length scales of the solution are large compared to the string length. Some relevant length scales are the size of the A-cycle at the tip of the KS throat

$$R^2_{S^3} \sim e^{-2A(0)}|\epsilon|^{\frac{4}{3}} \sim \alpha' g_s|M|\ , \tag{10.18}$$

as well as the length of the KS throat

$$L_{\text{throat}} = \int_0^{y_{\text{UV}}} dy\sqrt{G_{yy}} \sim (\alpha' g_s|M|)^{\frac{1}{2}}\int_0^{y_{\text{UV}}} dy\,\frac{\mathcal{I}^{\frac{1}{4}}(y)}{K(y)} \sim (\alpha' g_s|M|)^{\frac{1}{2}}\ y_{\text{UV}}\ . \tag{10.19}$$

Here y_{UV} is value of the coordinate y at which the throat is glued onto the bulk Calabi-Yau, see figure 6.2. To summarize, we should require that the parameters g_s, M, y_{UV} satisfy

$$g_s|M| \gg 1 \qquad\qquad g_s|M|y^2_{\text{UV}} \gg 1\ . \tag{10.20}$$

Let us now see how the warping affects moduli stabilization. While the superpotential is unmodified, the Kähler potential in the regime of strong warping is given by [97, 259]

$$K = -3\log\left(-i(T-\bar{T})\right) + i\frac{c'\xi|Z|^{\frac{2}{3}}}{(T-\bar{T})} + O(\xi^2)\ . \tag{10.21}$$

The expansion is in $\xi = g_s M^2$ and we take the constant $c' \approx 1.18$ from [259]. Despite the mixing between T and Z, to leading order in ξ, the potential will still be of the no-scale type because

$$G^{I\bar{J}}K_I K_{\bar{J}} = 3 + \mathcal{O}(\xi^2)\ , \qquad\qquad G^{Z\bar{J}}\partial_{\bar{J}}K = \mathcal{O}(\xi)\ . \tag{10.22}$$

We are now ready to consider the stabilization of the complex structure moduli in this setting. Because the conifold modulus is supported on the A-cycle of the KS throat, we expect it to be significantly lighter than the rest of the complex structure moduli and the axio-dilaton due to warping. Hence, we can imagine to first stabilize the heavy moduli at a high scale and consider an effective field theory for T and Z. Their stabilization will in general introduce some value $W_{\rm cs}$ in the effective superpotential

$$W_{\rm eff} = W_{\rm cs} - \frac{M}{2\pi i} Z(\log Z - 1) + KSZ\,, \tag{10.23}$$

where we have assumed that Z gets stabilized at a value close to the conifold locus in the end. The scalar potential is

$$\begin{aligned} V &= e^K G^{Z\bar{Z}} \partial_Z W \partial_{\bar{Z}} \overline{W} = -\frac{18 g_s}{c'\xi} \frac{|Z|^{\frac{4}{3}}}{(T-\bar{T})^2} \left| \frac{M}{2\pi} \log Z + KS \right|^2 \\ &= \frac{9}{2c' M^2} \frac{\zeta^{\frac{4}{3}}}{\tau^2} \left[\left(\frac{M}{2\pi} \log \zeta + \frac{K}{g_s} \right)^2 + \left(\frac{M}{\pi} \right)^2 \sigma^2 \right]. \end{aligned} \tag{10.24}$$

As in the unwarped case the potential has a Minkowski minimum, where

$$Z_0 = e^{-\frac{2\pi K}{g_s M}}\,, \qquad W_0 = W_{\rm cs} - \frac{M}{2\pi i} Z_0 = W_{\rm cs} - \frac{M}{2\pi i} e^{-\frac{2\pi K}{g_s M}}\,. \tag{10.25}$$

Importantly, we see that if $W_{\rm cs} = 0$ in the first step, the stabilization of the conifold modulus naturally generates an effective exponentially small W_0.

Because of the warping, the mass of the conifold modulus scales differently than in the unwarped case

$$m_Z^2 \simeq \frac{\left(\mathcal{V}_w |Z|^2\right)^{\frac{1}{3}}}{g_s M^2} \frac{M_p^2}{\mathcal{V}_w} \simeq \frac{\left(\mathcal{V}_w |Z|^2\right)^{\frac{1}{3}}}{g_s^{3/2} M^2} M_s^2\,. \tag{10.26}$$

In the limit of exponentially strong warping this expression is smaller than the string scale and also exponentially lighter than the mass scale of the remaining complex structure moduli and the axio-dilaton (10.12). This is the a posteriori justification for integrating them out before deriving the effective potential.

Finally, we can try to stabilize the Kähler modulus using non-perturbative effects just as in the KKLT scenario. One has to be careful because, due to its exponential lightness, it is a priori not guaranteed that the conifold modulus can be integrated out. This motivates us to study the combined effective field theory defined by

$$K = -3\log\left(-i(T-\bar{T})\right) + i\frac{c'\xi|Z|^{\frac{2}{3}}}{(T-\bar{T})}\,,$$
$$W = W_{\rm cs} + W^{(Z)} + W^{(T)} = W_{\rm cs} - \frac{M}{2\pi i}Z(\log Z - 1) + i\frac{K}{g_s}Z + Ae^{iaT}\,. \tag{10.27}$$

In contrast to KKLT, the idea is now to try to achieve $W_{\rm cs} = 0$ with the stabilization of the complex structure moduli. Without having to implicitly tune the effective superpotential exponentially close to zero, we would then hope for a combined SUSY AdS minimum for T and Z.

We can minimize the potential in a two-step procedure, by assuming that the complex structure modulus is still stabilized at Z_0 from equation (10.25) and then performing the KKLT stabilization of T using the effective superpotential generated by Z, or directly look for minima numerically. While the first option offers more analytical control, we should then crosscheck the results numerically. The consistency of this procedure will only be guaranteed if we find in the end that the mass scale of T is significantly lower than that of Z. Using the two-step procedure, we find a KKLT-like SUSY AdS minimum

$$\begin{array}{lll} \text{no-scale minimum:} & \zeta = e^{-\frac{2\pi}{g_s}\frac{K}{M}}\,, & \sigma = 0 \\ \text{KKLT minimum:} & A(2a\tau+3) - 3|W_0|e^{a\tau} = 0\,, & \rho = -\pi/2 \end{array} \tag{10.28}$$

with gravitino mass and value of the potential

$$m_{3/2} = e^{K/2}|W| \sim \frac{g_s^{1/2}M|Z_0|}{(4\pi)\tau_0^{3/2}}M_p\,, \qquad V_0 \sim -m_{3/2}^2 M_p^2 \sim -\frac{g_s M^2|Z_0|^2}{16\pi^2\tau_0^3}M_p^4\,. \tag{10.29}$$

The KKLT result for the mass of the Kähler modulus applied to our case is

$$m_\tau^2 \sim \frac{a^2|W_0|^2}{\tau_0}M_p^2 \sim \frac{a^2M^2\,|Z_0|^2}{\tau_0}M_p^2\,. \tag{10.30}$$

We can thus confirm that the Kähler modulus is exponentially lighter than the conifold modulus

$$\frac{m_\tau^2}{m_Z^2} \sim \left(M^3\,|Z_0|\right)^{\frac{4}{3}} \ll 1\,. \tag{10.31}$$

The numerical analysis confirms this minimum [33].

Uplift

As we have discussed in section 6.4, the KKLT AdS minimum can be uplifted to de Sitter space by placing an anti-D3-brane at the tip of the KS throat. In doing so, the energy of the anti-D3-brane has to be of the same order of magnitude as the AdS depth. The contribution to the scalar potential is given by [21, 264]

$$S_{\overline{\rm D3}} \sim 2\,\frac{M_s^4}{g_s}\int d^4x\sqrt{-g}\,e^{4A(y)} \sim \int d^4x\sqrt{-g}\;\frac{2\,M_p^4}{\tau^3}e^{4A(y)} = \frac{9c''}{2g_sM^2}\frac{\zeta^{\frac{4}{3}}}{\tau^2}\,. \tag{10.32}$$

In the last step, we have evaluated the potential at the point $y = 0$, which is the energetically most favorable configuration of the anti-D3-brane. The constant $c'' = 2^{1/3}/\mathcal{I}(0) \approx 1.75$ was taken from [259]. The warping changes the polynomial behavior with respect to τ from τ^{-3} to τ^{-2}.

This gives rise to a further constraint

$$\frac{|Z|^{\frac{2}{3}}}{\mathcal{V}^{\frac{4}{3}}} \sim \frac{1}{(g_sM^2)^2} \tag{10.33}$$

which requires large values of the flux M. Let us also note that the contribution of the anti-D3-brane (10.32) has the same parametric scaling with $\zeta^{4/3}/\tau^2$ as the flux-induced potential (10.24). As observed in [259], this means that in general we cannot neglect the uplift when dealing with the stabilization of ζ. Adding the anti-brane after integrating out the complex structure moduli, as it is done in the original KKLT analysis [21], is thus not a valid approach. The result of [259] is that a de Sitter minimum can only be achieved for a sufficiently large value of the Ramond-Ramond flux

$$g_sM^2 > 12\,. \tag{10.34}$$

Our findings indicate so far that if the fluxes can be chosen sufficiently large within the limits of the tadpole cancellation condition (6.8), there seems to be no obstruction for both scale-separated AdS-minima, as well as dS minima of the potential. Since this would violate the scale separation (10.2) and de Sitter (7.16) conjectures, it motivates us to look for other possible points of failure of the construction.

10.2 Light Modes in Warped Throats

A natural possibility for invalidating the hierarchy (10.1) needed for the consistency of the sequence of Wilsonian EFTs that are employed in the KKLT construction, are KK modes becoming light due to the warping. These can arise for example from the graviton whose internal wave-function can localize in the region of the KS throat, because this minimizes the energy. Such warped KK modes have been investigated for example in [260, 262, 265, 266]. In [33] we have analyzed numerically and analytically the mass spectrum of warped KK modes localized in the KS throat that arise from a 10D scalar field with action

$$S \sim \int d^{10}X\sqrt{-G}\left(G^{MN}\partial_M\Phi\partial_N\Phi + m^2\Phi^2\right) \, . \tag{10.35}$$

The KK modes that we found have masses which scale as

$$m^2_{\text{KK},n} = \frac{2\cdot 3^{1/3}}{\kappa}\frac{f_n^2\,(\mathcal{V}_w|Z|^2)^{\frac{1}{3}}}{g_s^{3/2}(M\,y_{\text{UV}})^2}M_s^2 \, , \tag{10.36}$$

where $\kappa \approx 0.72$ is a numerical constant from the KS solution and $f_n \approx (2n+1)\pi/2$. We want to compare this mass-scale to the mass m_Z of the conifold modulus

$$\frac{m^2_{\text{KK}}}{m^2_Z} = \mathcal{O}(1)\cdot\frac{f_n^2}{y^2_{\text{UV}}} \, . \tag{10.37}$$

For $y_{\text{UV}} \gtrsim 1$, the KK modes can become dangerously light and there will be a finite number of KK modes that we would in principle have to include in the EFT for Z. While this parameter regime is not strictly required by the constraint (10.20), because y_{UV} is dynamically generated in a global analysis of the Calabi-Yau, it motivates us to further study this region of parameter space.

10.3 Emergence of Kinetic Terms in the KS Throat

Let us first take a step back and consider the unwarped conifold geometry (5.26). It is a classic result of Strominger [267] that the singularity of the moduli space metric for the conifold modulus as $Z \to 0$ in equation (10.6) can be interpreted as arising from having integrated out a state that becomes massless. To be precise, the relevant state is the 4D particle that arises if we wrap a D3-brane on the A-cycle of the conifold. Without the orientifold projection, this will be a state in a 4D $\mathcal{N} = 2$ SUGRA, and it can be seen that it corresponds to a hypermultiplet. The mass of this multiplet is given by integrating the DBI-action over the A-cycle[4]

$$S_{\text{D3}} = -\frac{1}{g_s}M_s^4\int_{\mathbb{R}\times S^3}\sqrt{-g} = -\underbrace{g_s^{-1/4}M_s(\mathcal{V}|Z|^2)^{\frac{1}{2}}}_{m_{\text{D3}}}\int_{\mathbb{R}} d\tau \, . \tag{10.38}$$

[4] Note that in our conventions the volume $\mathcal{V}$ also contains a factor $g_s^{-3/2}$.

Using the formula (7.24), we find that the effect of integrating out this hypermultiplet on the kinetic term of Z is

$$\delta g_{Z\bar{Z}} \sim |\partial_Z m_{\text{D3}}|^2 \left[1 + \alpha \log\left(\frac{\Lambda_{\text{UV}}^2}{m_{\text{D3}}^2}\right)\right] , \tag{10.39}$$

where α is an order one constant. Using the Planck scale as the UV-cutoff $\Lambda_{\text{UV}} = M_p = M_s \mathcal{V}^{1/2}/g_s^{1/4}$ we find

$$\delta g_{Z\bar{Z}} \sim -\log(|Z|^2) \tag{10.40}$$

which is precisely the parametric behavior of the tree-level metric (10.6). The interpretation is that in writing down the effective field theory for Z, we have neglected non-perturbative states, such as the wrapped D3-brane. This state cannot be neglected in the limit $Z \to 0$ leading to the singularity in $g_{Z\bar{Z}}$. This singularity would in principle not be expected in the full theory where we also include the non-perturbative states [267].

The question is now whether we have a similar interpretation in the warped case, where the metric for the conifold modulus can be derived from the Kähler potential (10.21)

$$g_{Z\bar{Z}} \sim \frac{g_s M^2}{(\mathcal{V}_w |Z|^2)^{2/3}} . \tag{10.41}$$

Is the singularity in the strongly warped regime $\mathcal{V}_w |Z|^2 \ll 1$ connected to the fact that we have neglected the modes which become massless in this limit? The warped KK modes provide an example of such states and we will now analyze the corrections for the kinetic terms that are induced by integrating them out. We will restrict our attention to scalar KK modes, as we have seen in section 7.4 that a KK tower of fermions gives parametrically the same contribution.

Applying formula (7.24) to the case at hand, we find

$$\begin{aligned} g_{Z\bar{Z}}^{\text{1-loop}} &\sim M_p^{-2} \sum_{n=1}^{N_{\text{sp}}} \left(\partial_Z m_n(Z)\right)^2 \sim \sum_{n=1}^{N_{\text{sp}}} n^2 \left(\frac{1}{\sqrt{g_s M^2 y_{\text{UV}}}} \frac{1}{(\mathcal{V}_w |Z|)^{1/3}}\right)^2 \\ &\sim N_{\text{sp}}^3 \frac{1}{g_s M^2 y_{\text{UV}}^2} \frac{1}{(\mathcal{V}_w |Z|)^{2/3}} . \end{aligned} \tag{10.42}$$

If we indeed require that the kinetic term (10.41) arises purely from integrating out these states, we need to have the parametric relationship

$$N_{\text{sp}} \sim \left(g_s M^2 y_{\text{UV}}\right)^{2/3} \gtrsim M^{2/3} , \tag{10.43}$$

where in the last step we used the bound (10.20). In addition one can check that the scaling (10.43) leads to the fact that also the kinetic term $g^{\text{1-loop}}_{T\bar{T}}$ for T and the mixing term $g^{\text{1-loop}}_{T\bar{T}}$ correctly reproduce the tree-level result obtained from (10.21). As a result, a consistent effective description of the warped throat should accommodate at most N_{sp} light KK modes and thus should have a cutoff of at most

$$\tilde{\Lambda}_{\text{sp}} \sim N_{\text{sp}} \Delta m \sim \left(\frac{g_s M^2}{y_{\text{UV}}^2}\right)^{\frac{1}{6}} \left(\frac{|Z|}{\mathcal{V}_w}\right)^{\frac{1}{3}} M_p \,. \tag{10.44}$$

In analogy to the "gravitational" species scale $\Lambda_{\text{sp}} = M_p/\sqrt{N_{\text{sp}}}$ we can interpret this scale as a generalized species scale[5]

$$\tilde{\Lambda}_{\text{sp}} = \frac{\Lambda}{\sqrt{N_{\text{sp}}}} \tag{10.45}$$

for an effective gravity theory with a cutoff

$$\Lambda \sim \sqrt{g_s M^2} \left(\frac{|Z|}{\mathcal{V}_w}\right)^{\frac{1}{3}} M_p \,. \tag{10.46}$$

In contrast to the emergence of the SDC at large volume, here the ultimate cutoff Λ is also field-dependent in Planck units. This implies a finite distance of the conifold point in the complex structure moduli space

$$\Phi = d(0, |Z_0|) \sim \int_0^{|Z_0|} \sqrt{g_{Z\bar{Z}}} \sim \sqrt{g_s M^2} \left(\frac{|Z_0|}{\mathcal{V}_w}\right)^{\frac{1}{3}} \sim \frac{\Lambda}{M_p} \,, \tag{10.47}$$

where $\Phi < 1$ is the canonically normalized field corresponding to Z. In terms of Φ the relevant quantities become

$$\Lambda \sim \Phi M_p \,, \qquad \Delta m \sim \frac{\Phi}{g_s M^2 y_{\text{UV}}} M_p \,, \qquad \tilde{\Lambda} \sim \frac{\Phi}{(g_s M^2 y_{\text{UV}})^{\frac{1}{3}}} M_p \tag{10.48}$$

with still $N_{\text{sp}} \sim (g_s M^2 y_{\text{UV}})^{2/3}$. The mass of the conifold modulus Z scales as $m_Z \sim \Phi/(g_s M^2)$ and the coefficient in the three-point vertex $\gamma \phi h_n^2$ reads

$$\gamma \sim m(\Phi) \partial_\Phi m(\Phi) \sim \frac{\Phi}{(g_s M^2 y_{\text{UV}})^2} \ll 1 \,, \tag{10.49}$$

[5]At this scale the one-loop correction to the Planck-scale $M_p^2(\mu) = M_p^2(0) - \frac{\mu^2}{12\pi} N_{\text{sp}}$ becomes of the order of the cutoff scale Λ.

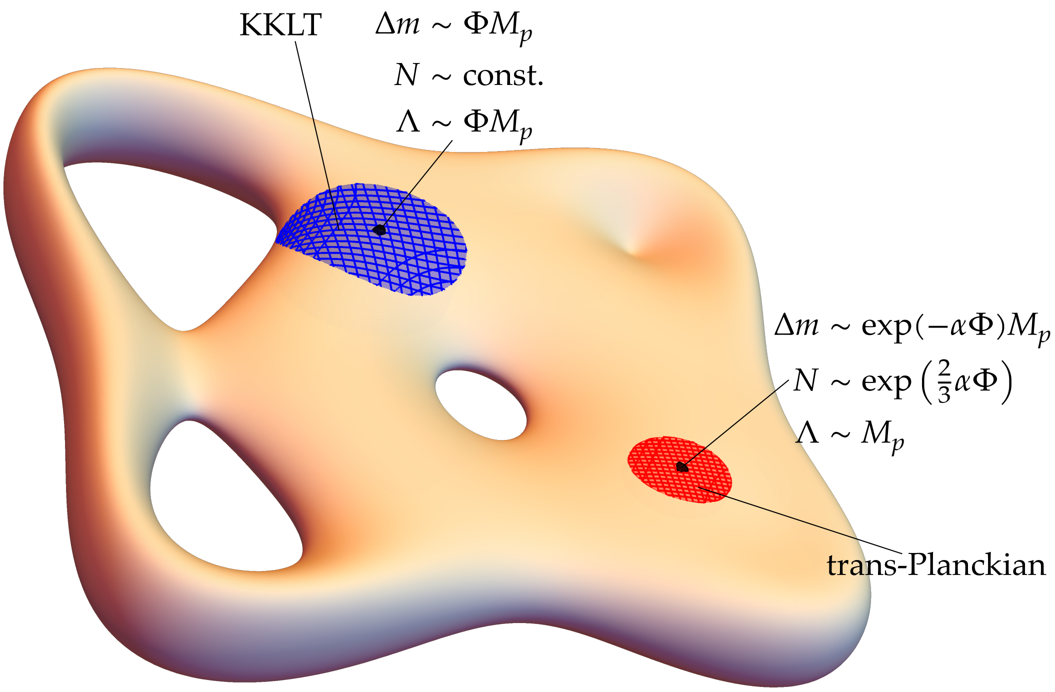

Figure 10.1: A sketch of the complex structure "moduli space" of a warped Calabi-Yau. The red/blue regions are a neighborhood of the large complex structure and conifold points where towers of modes become lighter than the cutoff scale.

so that perturbation theory makes sense. We notice that, in contrast to the SDC for infinite field distances, at the conifold point Δm does not scale exponentially with the proper field distance but only linearly. In addition, the number of light species does not increase exponentially but stays constant. The differences between the two cases are summarized in Figure 10.1. As indicated, the warped KKLT scenario lies in the blue region where KK modes are lighter than the cutoff scale. In this respect, KKLT is analogous to for example large field inflationary models, which require trans-Planckian field distances.

Meaning of the Cutoff

Because we have derived the cutoff scale Λ in equation (10.46) rather indirectly by requiring proportionality of the one-loop 1PI effective kinetic term with the tree-level one, the interpretation of this scale is not clear at this point. We will now proceed to show that it can be thought of as the mass scale of a non-perturbative D3-brane state that is wrapping the A-cycle of the conifold.

The Cutoff and the D3-Brane

Depending on the orientifold projection, the wrapped D3-brane state, which formed a hypermultiplet in the unwarped case (10.38), descends to a chiral multiplet in the $\mathcal{N} = 1$ theory. Before the orientifold projection, the conifold modulus Z is in a vector multiplet with the gauge field that arises from F_5 reduced on the A-cycle. Because the D3 is charged under F_5, the hypermultiplet will be charged under the vector multiplet in the 4D theory. We now want to consider an orientifold projection that leaves both the chiral multiplet as well as the vector multiplet intact. Specifically, we consider a holomorphic involution σ that interchanges the A- and A'-cycles of two local conifold regions in the geometry[6]. In this case one finds that the orientifold odd cycle $A - A'$ supports a conifold modulus Z, whereas the orientifold even cycle supports the RR gauge field. We can estimate the mass of the D3-brane state also in the warped case

$$S_{\text{D3}} \sim \frac{M_s^4}{g_s} \int dt \int_{S^3} d^3y \sqrt{-G} \sim \frac{M_s^4}{g_s} \int dt \int_{S^3} d^3y \, e^{-2A} \sqrt{\tilde{g}_{\text{CY}}} \,, \tag{10.50}$$

from which we extract using the KS-metric (6.19) and warp factor (10.16)

$$m_{\text{D3}}^2 \sim g_s^{\frac{1}{2}} M^2 (\mathcal{V}_w |Z|^2)^{\frac{1}{3}} M_s^2 \sim g_s M^2 \left(\frac{|Z|}{\mathcal{V}_w} \right)^{\frac{2}{3}} M_p^2 \,. \tag{10.51}$$

Note that this features exactly the same parametric scaling as the cutoff (10.46). We observe that while it scales in the same manner as the mass of the conifold modulus and the warped KK modes with respect to $\mathcal{V}_w$ and Z, the scalings with respect to g_s, M and y_{UV} differ. In particular, in the regime (10.34) where we can have a consistent uplift to de Sitter it is significantly lighter than the KK mass-scale.

The Cutoff and the Length of the Throat

There is also a neat interpretation of the cutoff (10.46) in terms of the length of the KS throat. Consistency requires that the total warped volume $\mathcal{V}_w$ is larger than the volume of the throat alone. Let us see what the constraints are that we

[6]The appearance of several conifold regions in a Calabi-Yau is the generic case, as it was pointed out for example in [136].

can derive from this. The warped volume contribution from the throat is

$$
\begin{aligned}
\mathcal{V}_w^{\text{throat}} &= \frac{1}{(\alpha')^3 g_s^{\frac{3}{2}}} \int d^6y \sqrt{\tilde{g}_{\text{CY}}}\, e^{-4A} \\
&\sim \frac{1}{(\alpha')^3 g_s^{\frac{3}{2}}} \left((\alpha')^3 g_s^{\frac{3}{2}} \mathcal{V}_w |Z|^2 \right) \left(\frac{g_s M^2}{(\mathcal{V}_w |Z|^2)^{\frac{2}{3}}} \right) \int_0^{y_{\text{UV}}} dy \, \sinh^2(y) I(y) \\
&\sim \mathcal{V}_w \left(g_s M^2 \left(\frac{|Z|}{\mathcal{V}_w} \right)^{\frac{2}{3}} \right) \int_0^{y_{\text{UV}}} dy \, \sinh^2(y) I(y) \\
&= \mathcal{V}_w \left(\frac{\Lambda}{M_p} \right)^2 \int_0^{y_{\text{UV}}} dy \, \sinh^2(y) I(y) \overset{!}{>} \mathcal{V}_w \, .
\end{aligned}
\tag{10.52}
$$

By approximating the integral for large values of y_{UV}, we find the bound

$$
y_{\text{UV}} \lesssim 3 \log\left(\frac{M_p}{\Lambda} \right) \qquad \Rightarrow \qquad \Lambda \lesssim M_p e^{-\frac{1}{3} y_{\text{UV}}} \, . \tag{10.53}
$$

We see that making the throat region longer lowers the cutoff scale of the effective field theory exponentially fast.

The D3-Brane and Emergence?

We now seek to clarify the different role of the wrapped D3-brane in the unwarped and warped case. While the wrapped D3-brane was responsible for the kinetic term of the conifold modulus Z in the unwarped case, it seems to play quite a different role in the warped case, where it simply sets the cutoff of the effective field theory. As we have seen, the kinetic term of Z can then be thought of as arising from integrating out the warped KK modes within the EFT defined below $\Lambda = m_{\text{D3}}$. To test whether this is the correct interpretation, let us consider also the option that the wrapped D3 could be directly responsible for generating the kinetic term of Z as in the unwarped setting. Its contribution is

$$
\begin{aligned}
g_{Z\bar{Z}}^{\text{1-loop, D3}} &\sim \left(\partial_{|Z|} m_{\text{D3}} \right)^2 \left(1 + \alpha \log\left(\frac{\Lambda^2}{m_{\text{D3}}^2} \right) \right) \\
&\sim \frac{g_s M^2}{(\mathcal{V}_w |Z|^2)^{\frac{2}{3}}} \left(1 + \alpha \log\left(\frac{\Lambda^2}{m_{\text{D3}}^2} \right) \right) .
\end{aligned}
\tag{10.54}
$$

If we assume, as in the unwarped calculation, that the cutoff of the theory is the Planck scale $\Lambda = M_p$, this does not reproduce the kinetic term of Z (10.41). A

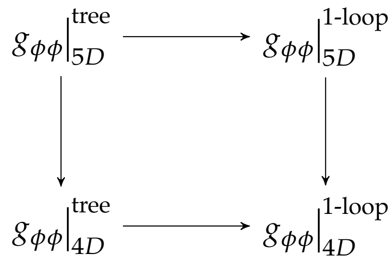

Figure 10.2: In compactifying from 5D to 4D, we can either first integrate out the 5D tower and then compactify or reverse the order. The Kaluza-Klein modes of the 5D tower make this diagram commute.

possible interpretation of this is that the kinetic term is indeed not generated by integrating out the D3-brane but rather from the warped KK modes. Because the warped KK modes can be much lighter than the D3-brane, they are the first states that become massless in the limit of infinite warping. In this sense they are the analogue of the D3-brane in the unwarped case.

10.4 The Role of KK Modes for Emergence

Clarifying the role of KK modes for the emergence of kinetic terms in string theory is of general interest. Here we will give a different perspective that is oriented more towards a higher-dimensional interpretation. The material presented is based on unpublished work together with Eran Palti [268][7]. The basic setup will be a 5D theory

$$S_{5D} = \int \sqrt{-G} d^5 X \left(M_5^3 R(G) - \frac{1}{2}(\partial\phi)^2 - \sum_n \overline{\psi}_n \left(i\partial\!\!\!/ + m_n(\phi) \right) \psi_n \right) , \quad (10.55)$$

where for simplicity $m_n(\phi) = n\Delta m(\phi)$. In the KK decomposition of the 4D metric (2.11) we will take the metric to be 4D Minkowski with trivial radion and KK gauge field VEVs. The interesting feature upon compactification will be the KK modes of the fermions ψ_n – we will ignore the KK modes of ϕ. The action after compactification reads

$$S_{4D} = \int \sqrt{-g} d^4 x \left(M_4^2 R(g) - \frac{1}{2} R(\partial\phi)^2 - \sum_{a,n} \overline{\psi}_n^{(a)} \left(i\partial\!\!\!/ + m_n(\phi) + \gamma^* \frac{a}{R} \right) \psi_n^{(a)} \right) , \quad (10.56)$$

[7] Our results are also summarized in the review [99].

where we defined $M_4^2 = M_5^3 R$ and performed a Kaluza-Klein reduction of the fermions

$$\psi_n(x,y) = \frac{1}{\sqrt{R}} \sum_a \psi_n^{(a)}(x)\, e^{iay/R} \, . \tag{10.57}$$

A chiral rotation reveals that the mass of the KK modes is given by

$$\left(m_n^{(a)}\right)^2 = m_n^2(\phi) + \left(\frac{a}{R}\right)^2 \, . \tag{10.58}$$

Crucially, the Yukawa-coupling $n(\partial_\phi \Delta m(\phi))\phi\overline{\psi}_n\psi_n$ is the *same* for all KK species of a given fermion ψ_n, despite their different masses.

We now compute corrections to the kinetic term of ϕ from the point of view of the five-dimensional theory. The five-dimensional species scale due to the tower of fermions is [176]

$$\Lambda_5 = \frac{M_5}{\sqrt[3]{N_5}} = \sqrt[4]{\Delta m M_5^3} \qquad\qquad N_5 = \frac{\Lambda_5}{\Delta m} \, . \tag{10.59}$$

The one-loop diagram correcting the kinetic term of ϕ has a linear divergence in 5D

$$\begin{aligned} \delta g_{\phi\phi}|_{5D} &\sim \Lambda_5 \sum_n^{m_n<\Lambda_5} \left(\frac{\partial m_n}{\partial \phi}\right)^2 = \Lambda_5 (\partial_\phi \Delta m)^2 \sum_n^{m_n<\Lambda_5} n^2 \\ &\simeq \Lambda_5 (\partial_\phi \Delta m)^2\, N_5^3 = \Lambda_5 (\partial_\phi \Delta m)^2\, \frac{\Lambda_5^3}{m^3} \\ &= M_5^3 \left(\partial_\phi \log \Delta m\right)^2 \, . \end{aligned} \tag{10.60}$$

Compactifying to four dimensions, we find that the kinetic term is of the form

$$\delta g_{\phi\phi} \sim M_5^3 R \left(\partial_\phi \log \Delta m\right)^2 = M_4^2 \left(\partial_\phi \log \Delta m\right)^2 \, . \tag{10.61}$$

We want to reverse the order of these steps and first compactify, then include the loop effects of the fermions and their KK states. The results should agree. A first puzzle is that the species scale should not be affected by compactification. For example, in string theory it should be always fundamentally related to the string scale. However, the number of particles running in the loop increases because we now have the additional KK states with the same Yukawa-couplings. The resolution is simply that the running of the kinetic term with the loop momentum k is slower in lower dimensions and both effects cancel precisely. The KK particles running in the loop are dual to the additional running induced by adding an extra dimension.

Let us check this at the level of equations. First we need the number of species in the lower-dimensional theory. We are interested in the number of states of mass $m_{n,a}$ below some scale Λ_4. This is equivalent to the counting of lattice points (n, a) in the interior of an ellipsoid and the result is (approximately for large number of species) given by

$$N_4 \sim \frac{\Lambda_4}{\Delta m} (\Lambda_4 R) \ . \tag{10.62}$$

We can solve the relation $\Lambda_4 = M_4/\sqrt{N_4}$ using this and obtain the 4D species scale

$$\Lambda_4 = \left(M_4^2 \frac{\Delta m}{R} \right)^{1/4} \equiv \Lambda_5 \tag{10.63}$$

which is consistently the same as the 5D species scale. Now we take the KK species into account in the 4D computation of $\delta g_{\phi\phi}$

$$\begin{aligned} \delta g_{\phi\phi}|_{4D} &= \sum_{n,a}^{m_{n,a}<\Lambda_4} \left(\frac{\partial m_n^{(a)}}{\partial \phi} \right)^2 = (\partial_\phi \Delta m)^2 \sum_{n,a}^{m_{n,a}<\Lambda_4} n^2 \\ &\simeq (\partial_\phi \Delta m)^2 \ (\Lambda_4 R) \left(\frac{\Lambda_4}{\Delta m} \right)^3 = (\partial_\phi \log \Delta m)^2 \, \frac{\Lambda_4^4 R}{\Delta m} \\ &= M_4^2 (\partial_\phi \log \Delta m)^2 \ . \end{aligned} \tag{10.64}$$

This result is in perfect agreement with the previous calculation.

It is interesting to see whether we can make one tower of states dominant over the other in the lower-dimensional description. We can never completely remove any of the towers. Let us assume we have

$$\Delta m > \Lambda_{4,KK} \equiv \sqrt[3]{m_{\rm KK} M_4^2} \qquad \Leftrightarrow \qquad \Delta m > M_5 \ . \tag{10.65}$$

On the other hand we have that

$$m_{\rm KK} > \Lambda_{4,\psi} \equiv \sqrt[3]{\Delta m M_4^2} \qquad \Leftrightarrow \qquad m_{\rm KK} > \Lambda_5 \ . \tag{10.66}$$

Both situations are inconsistent. So the ellipse in the (n, a)-plane should be approximately a circle and cannot be parametrically compressed along any of the two axes. As a result we expect

$$\frac{1}{R} \sim \Delta m \ . \tag{10.67}$$

We can thus say that the KK tower becoming light at large distance in moduli space is a consequence of the fermion tower consistently becoming light in this limit!

Connection to KKLT

The difference of the above to the KKLT setting is that there the modulus Z is not already present in the 10D theory, explaining the different role that the KK modes play. In order to make a direct connection, one should first compactify the 10D SUGRA background of KKLT to five dimensions. The fifth dimension would be the radial direction of the KS throat. In such a 5D description, the conifold modulus is already present, whereas the warped KK modes arise only upon further compactification to 4D, where $y_{\rm UV}$ becomes finite. A 5D description of the throat has been analyzed for example in [269]. Further investigating these issues was beyond the scope of this thesis.

10.5 The Fate of KKLT, de Sitter and AdS Scale Separation

We will now discuss the implications of the observations made in section 10.3.

Swampland Distance Conjecture & Emergence

The main question that one has to address is whether the towers of light states that become light at certain points in the moduli space are harmful or not. Let us formalize our observations. We set out to construct an effective field theory for some light scalar fields with an action S and a tree-level metric $g^{(0)}_{\phi\phi}$. We then observe the following:

> There exist points in moduli space with singular tree-level metric $g^{(0)}_{\phi\phi}$ at which towers of modes become lighter than the species scale $\tilde{\Lambda}_{\rm sp} = \Lambda/\sqrt{N}$ and whose mass approaches zero. Adding these states to the action, they induce a one-loop correction $\hat{g}^{(1)}_{\phi\phi}$ to the field space metric whose functional form is always proportional to the former tree-level metric $g^{(0)}_{\phi\phi}$.

When we extend our effective field theory to an action $\hat{S}$ that includes the tower of light states, this will be associated with a different tree-level metric $\hat{g}^{(0)}_{\phi\phi}$. As we have discussed in section 7.4, there are two different possibilities for the relation between $g^{(0)}_{\phi\phi}$ and $\hat{g}^{(0)}_{\phi\phi}$:

Emergence: The singularity in the tree-level metric $g^{(0)}_{\phi\phi}$ is entirely emerging from the one-loop correction $\hat{g}^{(1)}_{\phi\phi}$ in the extended effective theory $\hat{S}$, so $\hat{g}^{(0)}_{\phi\phi} = 0$.

One-loop consistency: In $\hat{S}$ there also exists a non-vanishing tree-level metric $\hat{g}^{(0)}_{\phi\phi} \simeq g^{(0)}_{\phi\phi}$ and it is a peculiar property of effective actions of quantum gravity that the one-loop correction is also proportional to $g^{(0)}_{\phi\phi}$.

Both phenomena seem to occur in compactifications of string theory. On the one hand, in the case of the unwarped conifold, which we discussed in section 10.1, the kinetic term is supposed to be truly emergent. On the other hand, for Kaluza-Klein states we have seen that their induced one-loop corrections are the same as the non-vanishing tree-level value. One difference between the two cases is the nature of the states that are being integrated out. In the first case we are dealing with a non-perturbative state, whereas it is an infinite tower of perturbative states in the second case.

In the present case of the warped throat we can draw two very different conclusions. First, we could dismiss the effective field theory on the grounds that we did not include the light KK modes. Second, due to the "one-loop consistency" property, we could argue that on the contrary the effective field theory will only be mildly corrected through some irrelevant numerical factors in the Kähler potential. In order to settle which of these interpretations is correct, a more detailed analysis is needed. Let us discuss the implications of both possibilities.

The interpretation that the light tower of states destroys the effective field theory was followed in recent applications of the distance conjecture to large field inflation and the relaxion mechanism (see chapter 7). The difference between the two cases is that here we are dealing with a static situation, so the number of light states is finite and does not increase. If we follow this interpretation, we would conclude that both the AdS and the dS minima of the warped KKLT construction are not under control.

The point of view that the effective field theory is under control might be justified as well. This is because the superpotential in the SUSY AdS minimum is not corrected and should only receive small numerical corrections due to the correction to the Kähler potential. This would be a striking result because it implies that we can trust the classical calculation even close to the conifold singularity. We would conclude that the AdS/dS minima could possibly survive in the action $\hat{S}$.

Settling these questions conclusively is of great importance. If it turns out that the effect of the KK modes is not deadly for the 4D EFT, the KKLT AdS and dS minima demonstrate that the scale separation conjecture (10.2) and de Sitter conjecture (7.16) do not hold everywhere in the moduli space of string theory. We have not explicitly analyzed the stabilization of the heavy complex structure moduli. The possibility remains that it is either impossible to generate a small

W_0 in the original KKLT scenario, or a vanishing W_0 after stabilizing all of the complex structure moduli except for the conifold modulus in our scenario. In this case, both conjectures could be saved.

De Sitter Conjecture from the Distance Conjecture

Because we have "experimentally" found a de Sitter minimum, we should check whether the general argument for the de Sitter conjecture using the distance conjecture from [168] fails in our case. We have discussed a sketch of the argument in section 7.5. The argument relied on the fact that one is in a perturbative regime close to an infinite distance point in the moduli space, where one can then apply the distance conjecture. In our case we are neither at the large complex structure point, nor at a very large volume. Furthermore, the number of light states in the tower is always finite in the case at hand, so these states are unlikely to dominate the Hilbert space. To summarize, the argument of [168] seems not applicable.

Part IV

Summary and Outlook

Summary

In this work we have discussed three main topics:

1. A test of the refined swampland distance conjecture in the vector multiplet moduli space of the type II string theories compactified on a CY threefold
2. The spin-2 swampland conjecture, which states that the massless limit of a massive spin-2 field is plagued by the appearance of an infinite tower of states with an associated cut-off scale
3. An analysis of the consistency of the warped effective field theory underlying the KKLT scenario with an emphasis on the effects of light KK modes

We review our results in the following.

The RSDC in Calabi-Yau Moduli Spaces

In chapter 8, we have restricted our attention to Calabi-Yau compactifications of the type II string theories. Because these have eight real supercharges, the classically massless moduli remain massless at the quantum level. This setup allowed us to address finite distance aspects of the original swampland distance conjecture, which was formulated for true moduli without a potential [30].

The swampland distance conjecture predicts a universal asymptotic behavior at infinite distance points of the vector multiplet moduli space. The mass splitting of an infinite tower of states should get compressed exponentially fast in the distance. In the type IIA theory, where the vector multiplet moduli are the complexified Kähler moduli, we have identified the Kaluza-Klein states as a tentative tower. At large volume, these are indeed expected to have an exponentially vanishing mass scale. Our aim was to examine whether this asymptotic behavior holds also for small volumes of the compactification. Due to mirror symmetry, we have control over the vector multiplet moduli space even in the case where the size of the Calabi-Yau is of the same order as the string scale. The result was negative. The asymptotic behavior was violated in particular near the so-called Landau-Ginzburg point of the moduli space. By means of examples we have examined how far the asymptotic behavior could be delayed.

Our results were consistent with the refined version of the distance conjecture in all examples that were studied.

There were two main classes of examples. First, we discussed mirror pairs of manifolds with a one-dimensional vector multiplet moduli space. In this case, we were able to globally construct the metric and mirror map. This allowed us to solve the geodesic equation and to compute globally shortest distances in the moduli space. Equipped with this method of computing distances we found that the swampland distance conjecture is violated only for distances smaller than one in Planck units. Second, we analyzed a class of toric hypersurface Calabi-Yau manifolds with a two-dimensional moduli space. In this case we, were not able to globally solve for the shortest path between two arbitrary points. Nevertheless, we showed that certain characteristic diameters of the non-geometric phases, where we expected a violation of the SDC, are sub-Planckian in diameter. We also analyzed certain hybrid phases where the asymptotic statement of the SDC was found to be violated for a finite distance in one direction but satisfied even for infinitesimal distances in the other direction. In the case of a 101-dimensional example we were able to confirm that the characteristic diameters of the Landau-Ginzburg phase of the moduli space were smaller than order one.

The Spin-2 Swampland Conjecture

The main result of chapter 9 was the introduction of the spin-2 conjecture (9.18) as well as its strong version (9.20). In order to argue for these, we pointed out the fact that a massive spin-2 field in Minkowski space propagates a helicity one mode, which can be described explicitly by introducing a Stückelberg vector boson. In the case that the massive spin-2 field couples to gravity, we proceeded by identifying a gauge coupling for this vector field and applied the magnetic weak gravity conjecture to this. The resulting cut-off scale was proportional to the mass of the spin-2 field divided by a characteristic interaction scale. If a tower version of the weak gravity conjecture is true, we expect an infinite tower of states to appear at this scale, with typical mass splitting of order of the cut-off. As a result, the massless limit of a massive spin-2 field that couples to gravity should be accompanied by an infinite tower of states becoming massless as well.

We have checked this statement in two examples that are known to have a UV-completion into a consistent theory of quantum gravity. The first example, which we discussed in great detail, was that of Kaluza-Klein excitations of a four-dimensional graviton. In this case, we were able to identify the interaction scale explicitly. As a result, the predicted cut-off turned out to be identical with the Kaluza-Klein scale. This is of course associated with an infinite tower of states, as suggested by our conjecture. The second example were massive spin-2 fields that appear as excitations of the string. Also in this case we are clearly dealing with an infinite tower and not a massive spin-2 field in isolation.

Finally, if the fundamental graviton itself had a non-vanishing mass, experimental constraints on this mass together with our strong spin-2 conjecture lead to significant trouble for massive gravity model builders, as they now have to deal with an infinite tower of states of exponentially small mass.

Consistency of the KKLT Scenario

The objective of chapter 10 was to look for possible points of failure in the KKLT construction of de Sitter vacua. This was important because the KKLT vacua feature scale separation as well as a positive cosmological constant, both of which are features that have been conjectured to be impossible to be realized in string theory.

To this end, we carefully took into account the effects of warping on the 4D effective field theory. Our analysis focused on the effective field theory for the conifold modulus and the overall volume modulus, both of which are expected to be exponentially light. First results were encouraging for KKLT. We found a simultaneous SUSY AdS minimum for the conifold and the volume modulus. This could be consistently uplifted to de Sitter space with the inclusion of an anti-brane term in the effective potential. The main new result was the fact that we found an explicit realization of an exponentially small value of the superpotential in the AdS vacuum, which was one ingredient of the KKLT scenario that received criticism in the literature. The smallness in our case could be viewed as a result of the warping.

We proceeded by calculating the mass of Kaluza-Klein modes becoming dangerously light due to the warping. This was done by locally solving the Laplace equation in a warped throat region. As a result we found that a finite number of KK modes becomes lighter than the conifold modulus, independent on the warp factor. Integrating out these Kaluza-Klein modes leads to a one-loop correction to the moduli kinetic terms that is proportional to their tree-level value. It was pointed out that this might have a rather mild impact on the effective field theory. We did not identify a definite point of failure in the KKLT construction.

Outlook

Because our knowledge of the swampland mostly boils down to conjectures, there are many open problems. In the end we would like to promote these conjectures, or suitably modified versions of them, to swampland theorems. There is hope for doing this both within the context of holography and string theory. The hardness of this task is expected to increase as we decrease the number of preserved supercharges. For example, a lattice version of the weak gravity conjecture was proven for maximally SUSY-preserving compactifications of the heterotic string theory in [156]. The author believes that much progress can be made in terms of rigorously establishing some of the swampland conjectures for string vacua with extended supersymmetry. One reason for this optimism is the fact that extended supersymmetry connects fields of different spin and thus naturally also different swampland conjectures. Once we map a swampland conjecture into a statement about the moduli space geometry, we can use the fact that possible scalar target spaces are scarce in extended SUSY. Similarly interesting is the possibility that one could be able to prove the weak gravity conjecture via the AdS/CFT correspondence (see [240] for a recent approach). Another important future direction would be to understand the significance of the emergence of kinetic terms from integrating out infinite towers of states. While we are starting to understand this phenomenon for compactifications of the ten-dimensional string theories, we should start asking also fundamental questions about the situation in eleven dimensions. Can the 11D graviton and the three-form be thought of as emergent gauge fields?

Swampland Distance Conjecture

Most of the phenomenological interest in the SDC refers to scalars with a potential. While this instance is admittedly not very well understood, we think that recent progress on the swampland distance conjecture in compactifications with eight supercharges [31, 130, 134, 135, 139, 142] should motivate us to take a step back and try analyzing systematically the highly supersymmetric case. For theories with more than eight supercharges, a sketch of a proof of the SDC is contained in [161]. There it was shown that the SDC is equivalent to the non-compactness, the completeness and the finite volume of the moduli space. Assuming this, it was then shown that the central charge of the theory decreases exponentially in the distance at infinite distance loci. This is possible because for such theories the moduli space is constrained to be a double coset space $\Gamma\backslash G/H$ by supersymmetry – we have explicit control over the moduli space geometry. These results seem encouraging and one could hope being able to prove the asymptotic statement of the SDC for all vacua of string theory with eight or more supercharges. We finally comment that the SDC should also have a holographic interpretation which is yet to be explored.

Spin-2 Conjecture

The spin-2 conjecture was formulated for a massive spin-2 field coupling to Einstein gravity in four-dimensional flat space. It is natural to weaken these assumptions by either allowing for other maximally symmetric spacetimes or by changing the spacetime dimensionality. Our argument is expected to break down in lower dimensions where the massive spin-2 field does not propagate a vector field. It would also be interesting to check the conjecture explicitly in more examples where the massive spin-2 interaction scale is not identical to the Planck scale.

De Sitter Space and String Theory

It is the author's opinion that settling the question of the existence of de Sitter space in string theory should be of an extremely high priority within the field. The KKLT scenario has been lacking a concrete realization in terms of an explicit compactification from ten dimensions for a long time. While the different ingredients needed for the construction, such as anti-branes in warped throats, or brane stacks leading to Kähler moduli stabilization, seem to exist individually, it is crucial to ensure also their mutual consistency and most importantly their coexistence in the landscape. This has not been demonstrated so far. If an explicit realization of de Sitter space is not possible, we should look for a convincing physical principle behind this impossibility.

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