# ON DOMAIN UNIFYING CATEGORICAL KINDS

## THE FORMATION OF A PSYCHOPHYSICAL ALGEBRA

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Not without Rini

# Contents

# Abstract

*DUCKs – Domain Unifying Categorical Kinds* – provides an (onto)logical framework, within which the mental is conditionally captured as an emergent object via the inductive limes in the logical language of categories, known as category theory. DUCKs is a proposal for unification on three levels: (i) epistemic unification of scientific domains, i.e. physics, biology, psychology, as well as mathematics and logic; (ii) ontological unification of the existent, i.e. of matter and the mental; (iii) meta-unification of the epistemological and the ontological themselves.

*Category theory* is examined as a meta-mathematical structuralist language for its suitability to formalize dynamic hierarchies of natural systems.

The *Hyperstructure* approach of Baas from the field of theoretical biology determines which structures and new „properties“ can be viewed as existent when dependent on a classifier. The role of the observer is examined, which is itself constituted as a higher-order structure.

Using the *Memory Evolutive Neural Systems* construction developed by Ehresmann and Vanbremeersch, we examine how the colimit can have a real correspondence in nature, first by using neural networks. On this basis, we also clarify the individual aspects of mental „properties“, such as multiple realization, reductionism, causality, and necessity trying a category theoretical reconstruction particularly of the central concepts of systems, emergence, and functions.

As a deterministic interpretation of quantum mechanics, *Bohmian Mechanics* is introduced as an established experimental metaphysics with the aim of giving an ontological foundation to the mathematical determiners by their spatially integration into DUCKs. We clarify the need for the local observer (the mental), as an inductive limes via the *non-locality* of his quantum mechanical substrate, to be *locally* emergent, which determines the aimed at psychophysical algebraic structure.

With the rational and ontological reconstruction of mental (bi-directional determination, DUCKs is developed as the framework. Given that, it becomes clear why the reduction of the mental to the physical must fail. Instead of reduction, the necessary and sufficient conditions of existence and the changes in these conditions must be identified. The result is a materialization of the conditioning and the proposed unification.

The problem of the mental is not solved by DUCKs, rather it is reformulated and made more precise. The mental as an *emergent object* seems in principle to be outside the limits of human understanding. It is treated as a physical object, i.e. as an object in space and time. And as such, the reformulation requires a new physics that must include the mental if it purports to be complete, since at the least, the latter is determined by the physical.

# 1 Introduction

*Domain Unifying Categorical Kinds*, in short *DUCKs*, is a reductionist proposal to sort our intuitions on the existent and its change. In the case that reductionism fails, we are emergentists. The crucial point in the attempt to unify the domains, i.e. all the sciences, in particular those concerned with natural phenomena, the so-called *mind-body problem* shows to be persistently recalcitrant. The mental seems to be impossible to integrate into a physicalist worldview, i.e. to identify with, or to reduce to, the physical. And thus, we are forced to search for a new framework by which this problematic case can at least be reformulated such that the inconsistencies become clearer.

Therefore, the actual endeavor is to deliver both perceptual and conceptual beliefs to Ockham, and he shall tell us where the axe should fall between the ontological and epistemological domain. It may be that Ockham proves to be a psychologist giving us fair warning not to fall victim to our early childhood imprinting. What is good for the routine of worming our way through the world's obstacles might manifest an obstacle in understanding the world. But is it really about understanding? We don't plan on explaining the phenomena, but rather to coherently describe them. Our objective is to find the right objects to yield the right phenomena *and* to find the right phenomena to yield the right objects.

Setting out to investigate *existence*, one is challenged with defining these objects that are considered to be candidates of what as a result is then called the *existent*. The objective is to catch sight of the epistemological more than the ontological status of those objects in keeping with the constraint that every existent entity must be conceptionalized, i.e. put into a conceptional framework, which lets one go „absent without leave“ to some extent, accepting that nothing can be done about it. But there is no need to desert yet. What remains to be done is to find a kind of framework that is sufficiently *minimalist* to get closest to reality, which immediately prompts the question, what are the criteria for considering a theory minimalist enough to be closest to reality? Imagining the world that naturally comes to our minds, it *consists* of objects, and it's the objects we have to construct a minimalist theory to or of. That seems to be a reasonable endeavor, and we continue asking what kind of thing these objects are and how they *interrelate*.

Thus, we have objects and interrelations, and the most obvious thing to seek is a theory that is concerned with objects and their interrelations. Adding associative composition of the interrelations and the neutral element, which is the identity, we have swiftly defined a *category*. And with these, we now have available a rigorous relational language, which is up to the challenge of clarifying the epistemological and ontological status of the concepts and phenomena to be analyzed. We seek a theory that is capable of dealing with structures and structures of structures, and it is a certain theory of categories that will be proposed as the requested minimalist account for initiating an *aufbau* of the world. And *category theory* will be this fundamental language.

But before we define the categories we will work with, we should reflect further on their morphisms. Amenable to the slogan ‘It’s the arrows that really matter!’, we investigate to what extent *time-delayed* and *non-time-delayed determinations* can be said to be key for the evolution of the world, or the universe, not only the objects themselves.

The right ontology, i.e. one that is vague enough not to be possibly convicted of being none but significant enough not to be trivial, is the basis for the definition of the categories that will be provided by the *domain unifying categorical kinds* evolving in time. These kinds will be the objects and morphisms in the category of ontology, and category theory will give the algebraic structure to them for the *formation of a psychophysical algebra*.

The language of category theory, which is a language rather than a theory, would then be „spoken“ and with the programmatic speaking hopefully automatically revealing new, i.e. yet unknown, insights. Thus, which category to use is not arbitrary but of arbitrative importance to get the right „feeling“ of the possible, that is the necessarily occurring *NEXUS* in (chap 4), which is the fundamental process of *changing existents*.

With the adopted ontology, we have the ingredients to define the category of existence which is owed to our intuition of that being existent that is *now*, endorsing *presentism*. The existents, the category theoretical objects, are *quality particulars* and their *necessary conditions for change* at the same temporal and spacial location.Those categorical kinds are real, but what about their interrelations? They are not real but *true*. Inter-time determination should not be seen as some dubious force or even causal power, determination is not a pressing into existence or change, it is rather a true relation about states of affairs, holding scepticism at a minimum due to serious metaphysical investigation. The objects *behave* in a deterministic manner: they are necessitated to be there, and the necessitation resides in their existence. Thus, the morphisms only state the objects’ existence (ontological intra-time determination) or their change in existence (nomological inter-time determination). There are no additional natural laws needed.

Thus, DUCKs intends to provide an (onto)logical grounding of existents and change of existents and its necessity. And in order to give it a serious basis, it has to show its consistency with a well-established, fundamental physical theory. Since the ontology that has been developed by then is about existents and their change in position, it stands to reason to look for the physical theory that best allows for one-to-one correspondences, and of course, we will have to ask whether this ‘best’ is good enough. This procedure in (chap 4) could be termed an *experimental metaphysics* approach, with the emphasis on ‘metaphysics’ to be experimentally confirmed. The theory is called *Bohmian mechanics*.

This provides us with two ingredients for our minimalist theory: primitive objects and their primitive interrelations. But can that be all? Maybe considerable benefits accrue, if we go for *gathering* and introduce families of primitive objects and their primitive interrelations representing the building blocks necessary to construct higher orders or levels of objects and interrelations by some complexification process. Thus, one could allow for composing the objects to some object with composite interrelations yielding new, i.e. *emergent objects*. And that is the idea: the *mental*, formerly known as ‘mental phenomenon’, doesn’t show as ‘properties’, ‘states’, or ‘events’ but as some existing object with existing necessary conditions for change. In (chap 4), the consequences of experiencing and thinking the *existing* mental will be discussed.

But before this is done, in (chap 2) DUCKs will be established by building on the elaborations on the construction of hierarchies of higher orders that are already formulated in the language and logics of category theory. The starting point is the *hyperstructure* construction set in which the mathematician Nils Baas explicitly introduces the *observer* as both a result of and a condition for the emergence of new properties showing in new interrelations, the so-called bonds. The difficulties of the conditioned result will be pointed out and made explicit in a computer simulation performed a by him and others of the evolution of proto-cells that are on the way to becoming living matter.

In (chap 3) the mathematician Andrée Ehresmann and the physician Jean-Paul Vanbremeersch try to overcome those issues of coming into existence by introducing the mental object as some *representation* of lower structures resulting from the colimit construction process. But it turns out that the complexification process in their *Memory Evolutive Neural Systems (MENS)* is also based on troublesome building blocks, which renders it to be more an epistemic model than an ontology. This forces us to go further since we are interested in preparing for unifying the mental and the physical by referring to experimental constraints provided by the—at least for today—most fundamental theory of existence, which is the theory of guided matter points introduced in (chap 4).

# 2 Hyperstructures

## 2.1 The Emergence of Structures

Since the early 90s, Nils Baas has been developing a theory of hierarchical or higher order structures. The aim in this branch of his explorations is to have on hand a framework suitable to allow for, register, or set up new entities and their properties. Baas' theory of hyperstructures is about the *observed emergence* of new structures with the final objective of providing the transition from mere self-assembled matter to living systems[1].

An important aspect of his construction set is the *observer* begging the question of the ontology of the evolution of *novelty*. Without an observer, i.e. without some kind of perceiving and conceiving classifier, would there exist something like higher-order structures? The observer is considered an "internal or external mechanism"[2] that declares some aggregate as new iff some property is detected conditioned by a certain *composition* of the primitives. And Baas distinguishes between self-assembly and biological evolution. But what are the criteria for that kind of discrimination, and are they admissible? For isn't evolution nothing but self-replicating self-assembly under natural selection in the sense that selecting for *existence* is implicitly given in the self-assembly process—no external, „downward causal" intruding needed?

These questions will be investigated but not without first having delineated his grounding construction set while already modifying the theory where necessary for our purpose of sufficiently integrating the observer to see how real higher-order structures could be registered.

Baas' idea of something new coming into existence is that its emergence from the old is or is also an appearance, i.e. it has to be *observed* to be an emergent structure with emergent properties. There is a number of approaches to the (de)complexification process[3] that are necessarily in a way akin—they all assert a kind of hierarchy of interrelated structures of subsequent order.[4] The unifying concept here is the construction of so-called *hyperstructures* as the most fundamental hierarchy, which is the result of an iteration of structures beginning at a defined order 1:[5]

**Definition 1** (First-order structure)**.** *First-order structures* $S^1_{i_1}$ *are primitives.* They constitute a family of structures

$$(S^1_{i_1})_{i_1 \in I_1} \tag{2.1.1}$$

with $I_1$ denoting some finite or infinite index set. ▲

[1] Significantly, his grounding article has been published in an anthology on artificial life.
[2] Cf. [60, 516].
[3] Cf. e.g. [50], [1], and especially [3] which we refer to intensively in (chap 3).
[4] Cf. [60] and [63].
[5] Cf. [60, 517].

The properties of the "observational mechanism" or "mechanism of registration" are necessary for these first-order structures to be obtained. It has to be discussed to what extent they are part of or woven into the properties and interrelations of the structures themselves. In particular, it is questionable what is being observed when one—an observer—states the existence of those primitives as such and as constituents of higher-order structures:

> *[T]o register that something new has come into existence, we need mechanisms to observe the entities.*[6]

That could mean that it is a fact that the novel phenomenon emerged, and it simply has to be recognized. But it is also said:

> *[W]e apply our observational mechanisms—Obs—to obtain properties of the structures $S_i$, $Obs(S_i)$. Next we subject the $S_i$'s to a family of interactions—Int—using the properties registered under the observation [...].*[7]

Hence, the next higher order, the second-order structure, would be *generated* by the primitives in interaction with the observer, and the primitives would be both the observed primitive first-order objects $(S^1_{i_1})$ *and* their observed primitive first-order interactions $(Int^1_{i_1})$. The "role" of the observer to register or to generate remains unclear.

We first give the definition of Baas' "construction process" of second-order structures and then proceed to his theory in favour of our motivation of stating the nature's entities as ontological:[8]

**Definition 2** (Second-Order Structure)**.** A *second-order structure* is a relation

$$S^2 = R((S^1_{i_1}), (Int^1_{i_1}), Obs^1) \tag{2.1.2}$$

with

(i) $R$: result of the construction process,

(ii) $(S^1_{i_1})$: first-order structures,

(iii) $(Int^1_{i_1})$: first-order interrelations,

(iv) $Obs^1$: observer of $(S^1_{i_1})$ and $(Int^1_{i_1})$.

▲

The crucial point of introducing emergent structures and properties is the process of *observing* properties of both the first-order structures and the interrelations between them given the constraints being imposed from the outside.[9] The observer itself being included in these constraints from the outside *interacts* with the first-order structures. The result $R$ of the process, which is the second-order structure $S^2$, depends on the primitives

[6] Cf. [60, 516].
[7] Cf. [60, 517].
[8] Cf. [63, 554].
[9] These constrains will be one of the key issues to be reflected on in (chap 4.2) in the ontological grounding referring to the experimental metaphysics of the fundamental physical theory of *existence* which is chosen to be Bohmian mechanics.

$(S^1_{i_1})$, their primitive interrelations $(Int^1_{i_1})$, and the observer $Obs^1$ observing the $(S^1_{i_1})$ interacting with $Obs^1$, which then are the objects' properties $Obs^1(S^1_{i_1})$ and the objects' interactions $Obs^1(Int^1_{i_1})$. However, Baas points out that "[i]n some cases the structures and the interactions may not be separable; Obs will then act upon them jointly."[10] This is of course a major problem and brings to mind dispositionalism—the structures showing up, more precisely their (observed) properties, just by interaction.[11] And we have to ask: interaction of what? Obviously, it is the observer that is acted on by „watching“ the first-order interacting first-order structures, in a way captured by the ominous $R$, such that the result is the observer's stating the second-order structure via having observed the alleged first-order properties $Obs^1(S^1_{i_1})$ and interactions $Obs^1(Int^1_{i_1})$. What are the dependencies then for the process $R$ such that the outcome $S^2$ is either an ontological or an epistemic entity? Given the primitives $(S^1_{i_1})$ and $(Int^1_{i_1})$[12] and the observer $Obs^1$ in (2.1.2) are real, the question for ontology then would be down to $R$. It would have to describe how first-order structures interact with a non first-order structure—or an aggregate of first-order structures—$Obs^1$ which renders two uncertainties given that the primitives are seen to be unproblematic: $Obs^1$ and $R$, i.e. the existence of an observer and his real interactions such that a construction process $R$ brings about a new structure $S^2$.

A first idea to avoid the difficulties is to drop the observer or to have it included in the families of the primitives:

$$S^2 = R((S^1_{i_1}), (Int^1_{i_1})) \tag{2.1.3}$$

That would give us an important advantage over (2.1.2). The higher-order structures were observer-independent unless the process of construction, or its result, $R$ embraced the non-primitive observer.[13]

Another attempt to face the problem is to substitute the ontological observer in (2.1.2) by an epistemic observation of the first-order structures and their first-order interactions, which are then families of observed properties:

$$S^2 = R((S^1_{i_1}), (Int^1_{i_1}), Obs^1(S^1_{i_1}), Obs^1(Int^1_{i_1})) \tag{2.1.4}$$

The task of the process $R$ then is to describe the emergence of the real stated second-order structure $S^2$ from the primitives and their observational properties. But without the observer $Obs^1$ explicitly taken into consideration, the observed structures $Obs^1(S^1_{i_1})$ and observed interactions $Obs^1(Int^1_{i_1})$ would be „living on their own“, meaning the second-order structure would be the result of the construction process on the primitives and observed primitives only. But why then wouldn't we just leave the entities alone and pack the observing into $R$ again?

Ontologically speaking, for the second-order structure to evolve or emerge or just be, why should we dwell on the observation of the primitives and not just make do with

[10] Cf. [60, 517].

[11] Cf. [51] for dispositionalism in the context of quantum properties necessitating change in the domain unifying process in DUCKs (chap 4).

[12] We take the first-order interrelations also to be primitives in anticipation of the condition for the change underlying "interaction".

[13] This is what Ehresmann and Vanbremeersch favor in their colimit construction. Cf. [60, 532], [63], and [4].

looking at $S^2$:

$$Obs(S^2) = Obs(R((S^1_{i_1}), (Int^1_{i_1}))) \tag{2.1.5}$$

Firstly, it is because we again wouldn't know about the ontological status of the observer and secondly we would just see $Obs(S^2)$, even without knowing that the observation is due to a second-order structure $S^2$. We would see a structure $S$ having properties $Obs(S)$. But we observe first-order structures *and* second-order structures, and we want to relate the observed second-order structure to the observed first-order primitives:

$$Obs(S^2) = R(Obs^1(S^1_{i_1}), Obs^1(Int^1_{i_1})) \tag{2.1.6}$$

having neglected the primitives for the observations while still expecting some objectivity. But that wouldn't work either unless the relation $R$ again included the observer in addition to everything necessary for the observer to observe, i.e. further boundary conditions.

Focussing on $Obs^1$, if we wish to stay as vague as possible and admit the observer as one determinant for the result of the construction process $R$, we have to explicitly add the observer to the observed in equation (2.1.4), yielding the observed second-order structure:

$$Obs(S^2) = R((S^1_{i_1}), (Int^1_{i_1}), Obs^1(S^1_{i_1}), Obs^1(Int^1_{i_1}), Obs^1) \tag{2.1.7}$$

with the observed second-order structure being *identified* with its observed properties ($S^2 := Obs(S^2)$). In words, this puts emphasis on the observer: the observer is necessary for the observed primitives to obtain their observation that are proceeded to result in an observed second-order structure which *is* its observed properties.

Focussing on $R$, with (2.1.7), the construction process is quite reduced then, but we still have to ask what is left for it if it isn't the primitives, the observed *properties* of the primitives, the observer, *and* their interrelating? Obviously, it is the remainder, the boundary conditions, e.g. including an experimental apparatus Baas was referring to. And one must still find a *criterion* as to where to draw the line between the process, the observer, the observed, and their complement. If one wished to emphasize the observer to be the process of construction itself but without neglecting the necessary remainder, call it $\bar{R}$, the „equation" (2.1.7) would turn into

$$Obs(S^2) = Obs^1((S^1_{i_1}), (Int^1_{i_1}), Obs^1(S^1_{i_1}), Obs^1(Int^1_{i_1}), \bar{R}) \tag{2.1.8}$$

Despite significant effort, we sense that these considerations do not really achieve success, and that is just the beginning. We have an issue already with separating the process from the observer, let alone the observer from the observed. It indeed appears to be unpromising to further attempt to create new entities with such vague tools at hand, and it is not necessarily the complexity of the subject but, as is often the case, our intuitions as human beings that lead us astray. But because of the highly intuitive elaboration[14] and exemplification[15] of Baas' concept of hyperstructures and the clarity of how they fail, we will follow his procedure of complexification. We want to show the redundancy of giving the process $R$ and the observer of whatever order *Obs* disjunctive treatments. And we will uncover which of the difficulties are homemade and which are genuine. Thus, we follow

[14] Cf. (chap 2.1.2).
[15] Cf. (chap 2.2).

Baas in his definition of $R$ covering (the result of) the process and the boundary conditions in the observation of higher-order structures in the confidence that the segregation of $R$ and $Obs$ appears obsolete, and it will be interesting why.

For now, $R$ is the result of some kind of process not further understood ontologically which has to be introduced in order to follow the intuition of higher-order structures to exist.

Before we further elaborate our main interest in the emergence of novelty, there is still a remark to be made on the order of the observer in (2.1.7). If $Obs^1$ itself is not meant to be a structure of order 1, it must be seen to be an observer of some higher order „looking at" the primitives, i.e. operating or interacting at order 1. Here, as said, we do not want to inquire after the origin of the observer, but we state that an observer at order 1 cannot possibly detect some higher order structure, e.g. $S^2$. Thus, whether the observed properties of the primitives can be still seen to be of order 1 is also doubtful—they could be externally imposed:

> *The interactions may be caused by the structures themselves or imposed by external factors.*[16]

And with the interactions may come the assigned properties.[17]

We do not want to distance ourselves too far from Baas' process of „producing" higher-order properties and again change the dependencies in (2.1.8) the "internal or external observational mechanism" being one factor in the construction process. Thus, the relation we work with instead of Baas' (2.1.2) is

$$Obs(S^2) = R((S^1_{i_1}), (Int^1_{i_1}), Obs(S^1_{i_1}), Obs(Int^1_{i_1}), Obs) \qquad (2.1.9)$$

Again, what is the difference? Although Baas addressed the dependence of the properties on the observer, he is not very strict in his notation—the structures and their interactions, even those of first order, are not indicated as being observed. Of course, one could state the aggregation $(S^1_{i_1})$ with the interactions $(Int^1_{i_1})$ was what is by the observer being acted on, and all observed higher-order properties did intrinsically relate to these primitives, and thus it wasn't necessary to refer to the observation of the primitives $Obs(S^1_{i_1})$ and $Obs(Int^1_{i_1})$. $R$ then wouldn't proceed on the observations but on the observer only. But as long as $R$ is not specified, one can just admit higher-order structures are constructively observed or *assumed*. Without an observer higher-order structures wouldn't exist, and with an observer those structures are epistemic. Therefore, Baas' elaboration is on *epistemic emergence*, and he makes the uncovering easy for us since he explicitly introduces the observer as a necessary ingredient in the procedure of construction or registration such that the properties—including the interactions—can be stated as mere observations. That is a merit which can't be valuated highly enough since in the endeavor of searching for an appropriate framework for construction procedures the observer, or the observer's ontology, doesn't explicitly attract attention, for the most part.

[16] Cf. [60, 517].

[17] Cf. (chap 2.1.2) in detail in the category theoretical framework.

### 2.1.1 Emergent Properties

In order to build a hierarchy on subsequent structures of increasing complexity, new structures have to be identified, and that is done by $R$ and $Obs$ together—the process of observing their emergent properties. Thus, Baas' definition of emergence is:[18]

**Definition 3** (First-order emergence)**.** $P$ is an *emergent property* of $S^2$ iff

$$\forall i_1 \in I_1 : \; P \in Obs(S^2), \text{ and } P \notin Obs(S^1_{i_1}) \tag{2.1.10}$$

$S^2$ is called an *emergent structure*, and $P$ an emergent property of *level* 2.[19] ▲

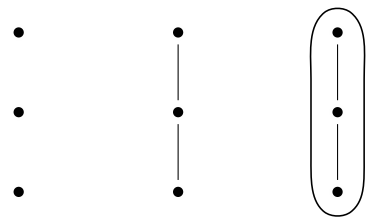


Figure 2.1: Left: The figure shows a given family of first-order structures $(S^1_{i_1})_{i_1 \in I_1}$. Nothing is being observed yet. Middle: In addition, their first-order interactions $(Int^1_{i_1})$ are shown. It is still a mere aggregation without an observed property $P \in Obs(S^1_{i_1})$. Right: The edging signifies the second-order structure $S^2$ has been observed: the observer $Obs$ has detected a new property $P$ with $P \in Obs(S^2)$, and $P \notin Obs(S^1_{i_1})$ $\forall i_1 \in I_1$.

This kind of observational complexification process defines structures of consecutive order with the family $(S^1_{i_1})_{i_1 \in I_1}$ being the primitive structures and $(S^2_{i_2})_{i_2 \in I_2}$ the first complex structures emerging.[20] The procedure can be continued by an observer which can be considered a *selection mechanism* somehow producing, according to Baas, new higher-order structures possibly corresponding to new levels.[21] "The adaption of higher-order observers" then is said to be regarded as an or the evolutive process itself. The result of the adaption is a hierarchy of structures, the so-called *hyperstructure*:[22]

**Definition 4** (Hyperstructure)**.** A *hyperstructure of order N* is a relation

$$S^N = R((S^{N-1}_{i_{N-1}}), (Int^{N-1}_{i_{N-1}}), Obs^{N-1}(S^{N-1}_{i_{N-1}}), Obs^{N-1}(Int^{N-1}_{i_{N-1}}), Obs^{N-1}, ...) \tag{2.1.11}$$

with $i_{N-1} \in I_{N-1}, ...$ ▲

And the observer or the process of observation is still the weak point. In the ontological view, a first-order token observation is not a first-order observable, i.e. an instantiation of

[18] Cf. [63, 554].
[19] With Baas, $Obs$ would be $Obs^2$ but, as said, the order of the observed primitive assigns the order of the observations.
[20] Cf. (fig 2.1).
[21] Differentiation order–level see below.
[22] Cf. [60, 524].

a variable but the very observed property itself. The observer $Obs^1$ observing the primitive $S^1_{i_1}$ resulting in the property $Obs^1(S^1_{i_1})$ would result in the same property observed by a higher-order observer, e.g. $Obs^{N-1}$: $Obs^1(S^1_{i_1}) = Obs^{N-1}(S^1_{i_1})$.[23] But what is $Obs^1$ in terms of the primitives? Is it a primitive itself or some higher-order structure just observing a primitive? It is not some higher-order structure, since they are explicitly noted, namely $Obs^{N-1}$ for instance. Hence, it is a first-order structure observing a first-order structure, and the observation could just be a first-order interrelation. Thus there is no need to mention all the observers explicitly in (2.1.11) as just being $i_j$-order structures. But on the other hand, those structures give the structures their observed properties, and we shouldn't omit them all. Given that $Obs^1(S^1_{i_1}) = Obs^{N-1}(S^1_{i_1})$ for all index sets, it may suffice to admit the highest-order or *the* observer equation (2.1.11) resulting in:[24]

$$Obs(S^N) = R((S^1_{i_1}), (Int^1_{i_1}), Obs(S^{N-1}_{i_{N-1}}), Obs(Int^{N-1}_{i_{N-1}}), ..., Obs) \tag{2.1.12}$$

The hyperstructure $S^N$ is merged of lower-order structures not forming some kind of

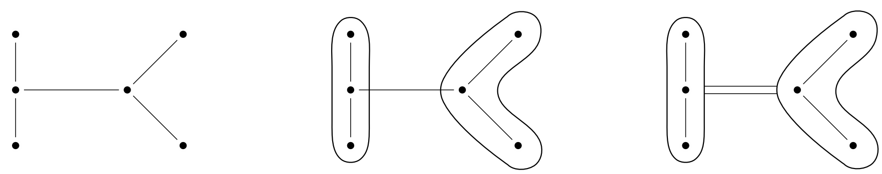


Figure 2.2: The evolution of a structure of order 3. Left: Six first-order structures $S^1_{i_1}$ and their first-order interrelations $Int^1_{i_1}$ are identified. Middle: Two second-order structures $S^2_{i_2}$ are detected but no additional interrelation. Right: Those two second-order structures interrelate through $Int^2_{i_2}$ (double line), which is observed by $Obs^2$. The result is structure $S^3$.

new substance but remaining the old substance and is only detectable by showing a new property or interrelation due to the property and vice versa. Then according to definition (def 3), $Obs(S^N)$ is a set of *$N$th order emergent properties* exclusively shown by hyperstructures of order $N$, but the family $(S^N_{i_N})$ can also admit preceding lower order properties.

Before we show the difference between (2.1.11) and (2.2) in a biochemical example in order to make clear the necessity of sorting the epistemic from the ontological part, we first proceed with Baas' more refined framework on *abstract matter* as a generalisation of the hyperstructure concept in which the connection between properties and interactions are especially elaborated—properties to be observed due to the *bonding* of the observer with the observed.

## 2.1.2 Emergent Interactions

As described, at the time that Baas was forming a hyperstructural framework for hierarchies, higher categories—so-called $n$-categories—came in vogue again. The result was

[23]It is not included in equation (2.1.11) since the properties are already detected at lower orders.
[24]In (chap 4) the structures will be called systems and the observer identified with the mental.

that he re-formulated his conceptual findings in the language of higher categories, which emphasize the interrelations of entities of levels or layers rather than the objects themselves in the sense of the category theory's motto: 'It's the bonds that really matter!' Again, we want to call to mind that we are urged to leave out mathematics, because the objective is still to stay as ontological as possible. And we are encouraged to look closely at whether Baas' formulation suits our needs. The emphasis on relations—and relations of relations as it will turn out—rather than the related real or observed objects could lead us astray as elegant as this construction set is. But this begs the question: Is focussing on arrows—the "bonds"—more suitable in the endeavor to get us as close as possible to reality? And we will see that it might not be, although it is a tool we human beings cannot go without:

> *The Hyperstructure Principle: In order to study or use a collection of objects it is useful to put on a suitable Hyperstructure to reach a goal. Hyperstructures are tools for thought and tools for organizing complex information.*[25]

The suspicion that hyperstructures is all about *cognition* and that the relations refined in this chapter do not exist outside the mind shouldn't keep us from trying to integrate the *imagined* in proper diagrams—and see how we demonstrate this.

In his newly couched framework, Baas calls the unfolding of higher structures a construction, but we call it what it is—a constructive definition. The primitives here are of order 0:[26]

**Definition 5** (Objects of order 0)**.** *Objects of order 0* are basic units, and $X_0$ is their set. ▲

**Definition 6** (Properties of order 0)**.** To each subset $S_0 \subset X_0$ it is assigned a set of *properties or states of order 0*:

$$\begin{aligned} \Omega_0 : \mathcal{P}(X_0) &\to \text{Sets} \\ S_0 &\mapsto \Omega_0(S_0) \end{aligned} \tag{2.1.13}$$

with $\mathcal{P}(X) = \{A \mid A \subset X\}$ is the power set and Sets a suitable set of sets.[27] ▲

Interestingly, Baas doesn't distinguish between properties and states that the primitives—allocated to a subset of $X_0$—taken together or individually have or are in. And it is true. Is it possible for a primitive or a set of primitives to change the state without differing in at least one property? Being in a state means having a certain property.

The interrelations will now be introduced very differently. They depend on the observed first-order properties of the subsets of the first-order structures, called bonds here:

**Definition 7** (Bonds of order 0)**.** Given a set of basic units $X_0$, to each pair of subsets of basic units and properties $(S_0, \omega_0) \in \Gamma_0 = \{(S_0, \omega_0) \mid S_0 \in \mathcal{P}(X_0), \omega_0 \in \Omega_0(S_0)\}$, it is assigned a set of *bonds* $B_0(S_0, \omega_0)$ *of order 0*:

$$\begin{aligned} B_0 : \Gamma_0 &\to \text{Sets} \\ (S_0, \omega_0) &\mapsto B_0(S_0, \omega_0) \end{aligned} \tag{2.1.14}$$

[25] Cf. [62, 21].
[26] Cf. [61, 158].
[27] In the language of category theory, $\mathcal{P}(X_0)$ would refer to the category of subsets and Sets to the category of sets.

▲

Comparing these definitions of the set/category theoretical framework with those of the former construction set already shows what Baas is aiming for. He wants to distract our attention away from the materially existent towards their interactions:

(i) $X_0$: set of first-order structures $(S_i^1)$,
(ii) $S_0$: family of first-order structures $(S_{i_1}^1)$,
(iii) $\Omega_0(S_0)$: set of all first-order observed properties $Obs^1((S_{i_1}^1))$ of the family of first-order structures $(S_{i_1}^1)$,
(iv) $\omega_0$: first-order observed property of $(S_{i_1}^1)$,
(v) $B_0(S_0, \omega_0)$: set of all interrelations $Obs^1((Int_{i_1}^1))$ of $(S_{i_1}^1)$ given an observed first-order property of $(S_{i_1}^1)$,
(vi) $b_0(S_0, \omega_0) \in B_0(S_0, \omega_0)$: a specific choice of above interrelations being the resultant $R$ of an observational process.

The properties become apparent from the interactions—an intuition we will return to repeatedly.

In the following complexification process, the higher order objects are bonds:[28]

**Definition 8** (Objects of order 1)**.** Given a subset $S_0 \subset X_0$ of objects of order 0, their observed properties or states $\Omega_0(S_0)$, and their bonds $B_0$, then $X_1 = \{b_0 \mid b_0 \in B_0(S_0, \omega_0), S_0 \in \mathcal{P}(X_0), \omega_0 \in \Omega_0(S_0)\}$ is the set of objects of order 1 with

$$\begin{aligned} \Omega_1 : \mathcal{P}(X_1) &\to \text{Sets} \\ S_1 &\mapsto \Omega_1(S_1) \end{aligned} \tag{2.1.15}$$

and the projection map $\pi_0$

$$\begin{tikzcd} X_1 \arrow[d, "\pi_0"] \\ \mathcal{P}(X_0) \end{tikzcd} \tag{2.1.16}$$

with $\pi_0(b_0) = S_0$: the objects of order 1 are observed bonds of families of objects of order 0. ▲

With the bonds $X_1$ of families of objects of order 0, their source and target is implicitly given so that they need not be carried along for further discussions. The bonds define the bonded, or to put it another way, without a new bond emerging, no new structure can be observed.

**Definition 9** (Bonds of order 1)**.** Given a set of units $X_1$, each pair of subsets of units and properties $(S_1, \omega_1) \in \Gamma_1 = \{(S_1, \omega_1) \mid S_1 \in \mathcal{P}(X_1), \omega_1 \in \Omega_1(S_1)\}$ is assigned a set of *bonds* $B_1(S_1, \omega_1)$ *of order 1*:

$$\begin{aligned} B_1 : \Gamma_1 &\to \text{Sets} \\ (S_1, \omega_1) &\mapsto B_1(S_1, \omega_1) \end{aligned} \tag{2.1.17}$$

▲

[28] Cf. [61, 158,159].

And with it, just as important for our considerations, a vertical unfolding of projection maps again suggests a hierarchical organization:

**Definition 10** (Objects of order 2)**.** Given a subset $S_1 \subset X_1$ of objects of order 1, their observed properties or states $\Omega_1(S_1)$, and their bonds $B_1$. Then $X_2 = \{b_1 \mid b_1 \in B_1(S_1, \omega_1), S_1 \in \mathcal{P}(X_1), \omega_1 \in \Omega_1(S_1)\}$ is the set of objects of order 2 with

$$\begin{aligned} \Omega_2 : \mathcal{P}(X_2) &\to \text{Sets} \\ S_2 &\mapsto \Omega_2(S_2) \end{aligned} \tag{2.1.18}$$

and the projection map $\pi_1$

$$\begin{array}{c} X_2 \\ \downarrow{\scriptstyle \pi_1} \\ \mathcal{P}(X_1) \end{array} \tag{2.1.19}$$

with $\pi_1(b_1) = S_1$. ▲

Further observations of new properties set up new observed objects, i.e. bonds, with the iteration process resulting in level $N$:

(i) Observations: $\Omega_{N-1} : \mathcal{P}(X_{N-1}) \to \text{Sets}$,

(ii) Bonds: $B_{N-1} : \Gamma_{N-1} \to \text{Sets}$,

(iii) Objects: $X_N = \{b_{N-1} \mid b_{N-1} \in B_{N-1}(S_{N-1}, \omega_{N-1}), S_{N-1} \in \mathcal{P}(X_{N-1}), \omega_{N-1} \in \Omega_{N-1}(S_{N-1})\}$.

Compared to (def 4), with this procedure of construction we can now define hyperstructures in a somewhat more integrative way:

**Definition 11** (Hyperstructure of order $N$)**.** The system $\mathcal{H} = (\mathcal{X}, \Omega, \mathcal{B}, \pi)$ is a *hyperstructure of order* $N$ with

$$\begin{aligned} \mathcal{X} &= \{X_0, ..., X_N\} \\ \Omega &= \{\Omega_0, ..., \Omega_{N-1}\} \\ \mathcal{B} &= \{B_0, ..., B_{N-1}\} \\ \pi &= \{\pi_0, ..., \pi_{N-1}\} \end{aligned} \tag{2.1.20}$$

▲

Thus, a hyperstructure of order $N$ is all there is up to a level $N$ *including* the observers: “An Observer mechanism is implicit in the $\Omega_i$’s.”[29] And they also “represent the emergent properties”.[30] In this framework then, one should not compare observed properties of a subset of primitive existents $\Omega_0(S_0)$ with the observed properties of a subset of bonds $\Omega_1(S_1)$. Rather, due to an observed new bond, some criterion for a subset has been found.

[29] Cf. [61, 160].
[30] Cf. [61, 159].

In a further step of the construction process, Baas introduces a second criterion for selection, besides the observation. Only those sets of units are assembled for which certain bonds are observed—"bond-type structures":[31]

$$\mathcal{P}(X_i) \to \mathrm{Coll}(X_i) \tag{2.1.21}$$

meaning that only some sets of units are allowed, resulting in a collection of assemblies, "and then define $\Omega_i$ and $B_i$ on these." This step however appears to be a simple act of bookkeeping after having already assigned properties to those that are of special interest. But "[t]his is just a useful refinement to be aware of", as Baas states himself.

Still, the assemblies in $\mathrm{Coll}(X_i)$ are said to have no "internal structure". But this could be remedied by allowing a sequence of families of structure types $\mathcal{S} : \mathcal{S}_0, \mathcal{S}_1, ..., \mathcal{S}_N$. The types in $\mathcal{S}_i$ at every level could then be seen as a classification of the objects of the set $X_i$ due to some criterion, and the property and state assignment $\Omega_i$ would be classifiable in properties and states of structures of some certain type. And the bond assignment $B_i(S_i, \omega_i)$ on structures in types $\mathcal{S}_i$ „creates“ the higher level $X_{i+1}$, resulting again in an iterating process. Baas includes these types in the hyperstructures:[32]

**Definition 12** (Hyperstructure of order $N$ and structure type $\mathcal{S}$)**.** The system $\mathcal{H} = (\mathcal{X}, \Omega, \mathcal{B}, \pi, \mathcal{S})$ is a *hyperstructure of order* $N$ with *structure type* $\mathcal{S} = \{\mathcal{S}_0, ..., \mathcal{S}_N\}$. ▲

Summing up, the two refinements of hyperstructures are due to a separation into bond types and in types of internal structure. An example for hyperstructures with structure types would be "multilevel systems like biological structures bound together level by level, for example, molecules, cells, tissues, organs, organisms."[33] Generalizing physical entities Baas shows how to build "life-like matter" starting with objects in a category theoretical sense—so-called categorical or *abstract* matter. Hyperstructures then are constructed to be higher order diagrams.[34]

Finally, Baas tries to define the stable result of the observational process by families of *rules*. The key concept of the construction procedure is the emergence of new bonds $b_N \in B_N(S_N, \omega_N)$[35] but which means that they evolve in time. Baas doesn't explicitly take the sets of structures $S_N$ to be time-variant. Rather, he concentrates on the states/properties $\omega_N(t)$ and bonds $b_N(t)$ and gives a definition for a discrete time evolution of hyperstructures:[36]

**Definition 13** (Hyperstructural Dynamics)**.** The dynamics of a hyperstructure of order $N$ is given by families of rules $\mathcal{R} = \{R_i\}$ at each level $i$

$$\begin{aligned} R_i : \omega_i(t) &\mapsto \omega_i(t+1) \\ b_i(t) &\mapsto b_i(t+1) \end{aligned} \tag{2.1.22}$$

such that the change is compatible with the states/properties $\Omega = \{\Omega_0, ..., \Omega_i, ..., \Omega_{N-1}\}$ and the bonds $\mathcal{B} = \{B_0, ..., B_i, ..., B_{N-1}\}$ in $\mathcal{H} = (\mathcal{X}, \Omega, \mathcal{B}, \pi)$. ▲

[31] Cf. [61, 160].
[32] Cf. [61, 161].
[33] Cf. [61, 161]. The ingredients for artificial life will be extensively discussed in the next section.
[34] Cf. [61, 168].
[35] and not of sets of basic units due to the emergence of those bonds
[36] Cf. [61, 172].

In this dynamical hyperstructural construction process, the change of the observed states/properties and the observed bonds, i.e. the observables, is ruled by the basic units $X_0$ *and* the observer, and naturally the question arises: isn't the observer itself some existent? And the rules $\mathcal{R} = \{R_i(t)\}$ themselves are not time-invariant since they are not separable from the *behavior* of the existent and the observational mechanism. On this, Baas says, "[T]he rules could be" and even "[t]he whole dynamics may be organized as a hyperstructure."[37] And again, the question arises: what is change and what is changing, anyhow? In the hyperstructure construction set, the line between epistemics and ontology is not clearly drawn. Especially, intermediate levels appear to be mere conceptual, i.e. epistemological. In the transition to the ontological view in (chap 4), there will be objects only—and their necessary conditions for change. And the task will be to investigate whether some objects are „special", of course still employing fundamental epistemics which will be the so-called *colimes* construction.

But first we want to give a clarifying, concrete example of dynamical hyperstructures resulting in dynamical hierarchies of "abstract matter represented by stable equilibria or fixed points." Now we are on the way to "living matter".

## 2.2 The Observational Mechanism

In a physicochemical example, the physicist Rasmussen and Baas elaborate the conception of the spontaneous formation of structures in a dynamical hierarchy showing novel properties. They develop a computer simulation[38] based on the framework introduced of interrelating *levels* in addition to orders showing so-called observed *multiple functionalities*. As will be seen, the collective dynamics of the structures result in higher-level behavior that is actually *defined* by the observation of their higher-order properties.

Therefore, they present a relatively easy-to-handle physicochemical paradigm of a dynamical hierarchy that is comparable with experimental self-assembly systems: micelles[39] as third-order structures together with water molecules as first-order structures showing *functional* higher-level properties of the *aggregation*.

However, in a reply to the process of micellation, two other fellow soldiers of the artificial life community argue for micelles to be of only order 2. And it will be analyzed whether they are right and whether their arguments can be used to up the ante and reduce the order down to 1—the micelles not showing new properties at all, with all the implications.

### 2.2.1 A Computer Simulation

The reason why one would simulate some dynamics in a computer is either to generate unpredicted or unpredictable phenomena grounded on certain input data or to uncover the process resulting in well-known ones. In this case, both of these apply. The phenomenon is supposed to be micellation and all that which is observed in the biochemistry being assumed an exemplar of hyperstructural emergents. And the goal of this simulating

[37] Cf. [61, 172].
[38] Cf. [76] and [75].
[39] Cf. [29, 146], [46], and [54, 45].

epistemics is to confirm the rules for the structures occurring and see how they fit the advocated ontology.[40]

Thus, there is ontology, epistemology, and simulation, and the reason for counting the latter in addition to the epistemic domain is that the epistemics has to be modified to become computable.[41] And this computer-adequate adaption could prove to be inadequate to represent proper knowledge, possibly laying a false trail in getting the ontology right. We will see that, not just here, it might well be so.

Rasmussen et al. exercise a concrete *descriptive*[42] physicochemical example for spontaneously formed structures at three levels, i.e. length scales:

(i) Level 1: water and monomers,

(ii) Level 2: water and polymers (linear monomer aggregates),

(iii) Level 3: water and micelles (polymer aggregates).

Building up the simulation, we need a set of axiomatically given basic objects at level 1 and a set of dynamical rules[43] which determine that these objects behave in certain ways. The objects might have an intrinsic structure according to which their properties and higher-level functionalities emerge. It is acknowledged

> *that the specification of observable properties is somewhat arbitrary. This is quite analogous to the arbitrariness involved in the specification of the primitive objects and their interactions.*[44]

But for the simulation and the final conjecture—the *ansatz*[45]—the possibility of *zooming in* on the structures and revealing new structures doesn't pose a problem.

The primitive objects—the *monomers* and *water molecules*—are equipped with these intermolecular interactions: attraction and repulsion.[46] More precisely, there are hydrophilic monomers (hydrophilics) which tend to attract water molecules and hydrophobic monomers (hydrophobics), which tend to repulse them or attract them more weakly.[47] Both monomers are mutually attractive but in a weaker way than they bind water molecules if they bind. And the strongest attraction is between water molecules.

In a simplified version for the sake of clarity in the 2-dimensional model, water is just implicitly involved by interpreting vacant lattice sites as a mixture of vacuum and water resulting in mean field interactions reducing the effects of the many bodies to a single averaged effect providing for heat bath „kicks“. In the more sophisticated 3-dimensional model, water is simulated explicitly with all the additional individual interrelations, which allows for more realistic behaviors to evolve while the principle of emerging properties of

[40] Cf. (chap 4).
[41] Cf. [58], [9].
[42] To what extent they want to build an ontology is not clear at this stage.
[43] Cf. (def 13).
[44] Cf. [76, 334].
[45] Cf. (con 1).
[46] Monomer-monomer bonds are also listed and actually used in the simulation. And we already see that they might be just an add-on due to observation and simplification. Cf. [76, 333].
[47] Attraction and repulsion are here considered to be a symmetric relations: the objects attract or repulse each other equally.

higher-order structures stays the same.

What is not the same is the notation of the hyperstructural complexification process, and we will see whether the change is a shift again.[48] In the example, the objects, the monomers and water molecules constitute our first-order family of $n$ primitives:[49]

$$S_r^1 = S_r^1(s_r, f_{rs}, \tau_r), \quad r, s = 1, ..., n \tag{2.2.1}$$

with

(i) $s_r$: the object's state (e.g. type, position, orientation of the molecule),

(ii) $f_{rs}$: the object's interactions,

(iii) $\tau_r$: the object's local time.

The novel element in the theory $s_r$ defines the *state* of the first-order object $S_r^1$ comprising e.g. the "aspect" of being in the state of being a certain molecule or being in the state of having or being at a certain position. So far, in the hyperstructure context, states did not arise explicitly[50], rather they were included in the observation function $\Omega_0$, or $Obs^1$ respectively, of the structures yielding the observables, i.e. properties:

> *For example, being hydrophobic or hydrophilic are observable properties of monomers that can be observed using an observation function $O^1$.*[51]

But then one wonders whether being a hydrophobic or hydrophilic monomer is considered being a type, i.e. being in a state, or a property. This is a difficulty that stems from the vagueness of the definition of the structure $S_r^1$ itself. Is it a kind of a carrier or a substance? We will return to that soon. Here we will just point out: states are *not* properties.

Another novelty is that the interactions $f_{rs}$ directly determine the structure and are not explicitly listed[52] or, in a subsequent step, allocated by certain observed properties[53]. This is especially a problem where the property of being e.g. hydrophobic, if it is a property, is the result of an observed interaction[54] with other first-order structures, the water molecules. The reason that the interactions are introduced as part of the primitive will be also addressed soon.

Probably the most important innovation is to provide the structures with a local time $\tau_r$, which is indeed necessary to make the change computable: the structures now come with a *spacio-temporal location.*[55]

To build a dynamics, one has to choose the constituents, i.e. what is going to change, and what are the conditions for change? The "updating" is executed on the above object's states and interrelations between them that are itself considered objects: in the mathematical framework, called *molecular dynamics lattice gases*[56], matter and force fields

[48] Cf. (def 1).
[49] Cf. [76, 333].
[50] Cf. (def 1).
[51] Cf. [76, 333].
[52] Cf. (def 2).
[53] Cf. (def 9).
[54] Cf. (def 7).
[55] Cf. (chap 13).
[56] Cf. [55].

propagate locally as force-communicating *information particles* between neighboring lattice sites indexed by integers: $(i, j),\ i, j \in N$. An object $r$ then corresponds to a location in the lattice $\mathcal{L}$, i.e. given the object $r$ is at $(i, j)$, it transfers its values of the state variables to the lattice:[57]

$$s_r = (i, j, x_1, ..., x_q) \in \mathcal{S} \tag{2.2.2}$$

with

(i) $\mathcal{S}$: state space of the $n$ objects,

(ii) $i, j$: position of the internal states of $r$ in the triangular lattice representing physical space,

(iii) $x_l \in X_l,\ l = 1, ..., q$: $q$-many aspects of $r$ at lattice site $(i, j)$.

Considering the object-view, i.e. spot-lightening the position of the $n$ objects, the total state of the „system“ then results in

$$z^{obj} \in \mathcal{Z}^{obj} = \prod_{obj} \mathcal{S} \tag{2.2.3}$$

with the dynamics being proceeded by the update operator $U^{obj} : \mathcal{Z}^{obj} \to \mathcal{Z}^{obj}$ sweeping over the objects:

$$z^{obj}(t + \Delta t) = U^{obj}(z^{obj}(t)) \tag{2.2.4}$$

These are the preparations made for the following computer simulation, and it is said that they are still in correspondence with equation (2.2.1):

> *So, the reidentification and movement of each primitive object is represented implicitly by the local propagation of state information about each site, just as a glider's reidentification and movement is represented in a cellular automata like the Conway's game of life.*[58]

Thus, the claim they make is that there has been no loss of realism on the way from the epistemology $S_r^1 = S_r^1(s_r, f_{rs}, \tau_r)$ to „its“ simulation $s_r = (i, j, x_1, ..., x_q)$, resulting in the $q$-many aspects' re-formulation of the objects' being and behavior, but that has to be discussed. Firstly, we inspect the transition from the structure $S_r^1$ to its state $s_r$, on which seemingly all attention is being concentrated—letting go of $S_r^1$ and also the interrelations $f_{rs}$. And secondly, we examine why the point of view will be changed from the moving objects to fixed spacial locations.

### Unifying States and Bonds

Referring again to equation (2.2.1), the primitive $S_r^1$ is only being specified by further information, i.e. it is in *dependence on* and not *identified with* its determinants: the object is in the state of being a monomer, it is not the monomer itself. But what is left for $S_r^1$ then? Why not commit to $S_r^1 := (s_r, f_{rs}, \tau_r)$ leaving behind the „carrier“ as just being a superfluous specifier or designator? In their own notation, the fact of being in the state of being a monomer is treated equally with other states, like being in a certain position.

[57] Cf. [76, 335].
[58] Cf. [76, 335].

And furthermore, is it appropriate to add the interrelations to the object in the sense of belonging to it—and not to the state? They are related to force particles, as indicated by the authors, and should be treated as such, especially where the interrelations are covered separately in the original framework.[59] Of course, they somehow determine the object, in the manner of what it is, a *sink* and a *source*[60] for changing the object's state—the monomer having a new position. And we will see in the simulation, the force particles are indeed not separated from the object's aspects. If we drop the self-evident time of existence to be a specification on top, the object would result in the state itself:[61]

$$S_r^1 = s_r^t \tag{2.2.5}$$

That is perfectly fine and almost ontological.[62] In our example[63], the specific object or structure $r$ (of order 1) is a monomer at time $t$ at a certain position, for example, $(i, j)$ having properties $Obs(S_r^1)$ to be observed, e.g. a monomer being hydrophobic.

Thus, we can indeed endorse the equation (2.2.2)

$$S_r^1 = s_r^t = (i, j, x_1, ..., x_q, t) \tag{2.2.6}$$

with time-dependent "aspects" $x_l$ of $r$ at time-dependent position $(i, j)$. Now certainly, it is key to specify the aspects properly. In the first place, they were said to be a state or a bunch of states, e.g. the type of object and its position. However, if they are not states but aspects, what are the aspects of a monomer, and those for a second-order structure, a polymer? It dawns on us that the state-changing force particles are woven in, the aspects just being some explication tool for the dynamics. We will discuss that decisive point in detail.

But first we concentrate on position to prepare for the simulation. On the left side in (2.2.6), the integer $r$ is an indexical and refers to the state or family of states, and on the right, there are given integers $(i, j)$ assigning some position, but the position of what? There are two readings of the "transfer of the variables of the values". Firstly, the position follows the states of object $r$, i.e. the spotlight is thrown on a particular object's trajectory: $(i, j)(t) = (i, j)(r(t))$. Then the equation (2.2.6) turns into

$$s_{i,j}^t = ({}_1x_{i,j}^t, ..., {}_qx_{i,j}^t) \tag{2.2.7}$$

Thus, the raison d'être of the conceived „carrier“ $S_r^1$ in (2.2.1)—it never makes an appearance—is just to mark particular states to be dropped again with the consequence that clustered states inherit a certain position—$r$-bounded tokens. And that is what is lost in the second, the lattice data structure view, but which enables the glider's game interpretation. It is not $(i, j)(t) = (i, j)(r(t))$ any longer, but the integers are fixed, constituting a lattice site $\mathcal{D}_{i,j}$, and the tokens simply become variables gliding through it, allowing for coordinates pointing at nothing:

> *The internal states have now been modified to enable representation of vacant lattice sites.*[64]

[59] Cf. (def 2).
[60] Cf. (chap 4).
[61] The order 1 is implicitly given by the designator $r$, and we give the objects a common time.
[62] We refer to the ontology put forward in (chap 3).
[63] We take the easy case: 2-dimensional lattice and no orientation in space.
[64] Cf. [76, 335].

This is indeed a shift in the focus on existence: if one doesn't stick to the objects but to adjacent sites of certain length scales, there is now room for spotting higher-order structures.

Looking at the whole volume available, the total system is

$$z^{lat} \in \mathcal{Z}^{lat} = \prod_{lat} \mathcal{D} \tag{2.2.8}$$

where $\mathcal{D}_{i,j} \in \mathcal{D} = X_1 \times ... \times X_q$ represents the data structure at $(i, j)$ which is the $r$'s cluster state $s^t_{i,j}$ if it just happens to be there at $t = \tau_r$. And the corresponding update operator $U^{lat} : \mathcal{Z}^{lat} \to \mathcal{Z}^{lat}$—now sweeping over the lattice sites—is

$$z^{lat}(t + \Delta t) = U^{lat}(z^{lat}(t)) \tag{2.2.9}$$

Those two state-space formulations are said to be "equivalent"[65], and indeed, why should the traditional, more intuitive molecular dynamics framework be more suitable just because the focus is on some particular entity drawing a *trajectory*? That brings us back to the question as to whether there is a loss of realism in choosing the lattice gas view. The objective is clear, the *bias* is on (slowly) moving gatherings to be observed. While looking at a grid, we define new length scales—*levels*—at which higher-order structures are ready for observation.

The update operators then represent the *internal* dynamics, and one could state there is no other. And indeed in a strict sense, there is no dynamics but that which is brought along with the $s_r$'s interactions and certain properties:

> *A family $S^1_r$ of objects with specified interactions and properties represents a process, and the stable outcome or result of this process—in some situations the attractor for the system—will be represented by $R(S^1_r)$.*[66]

And, as we stated before, "by properties we mean the resulting values of the observables applied to the system"[67], here obtained by the observation function $\Omega_0$, or $Obs^1$ respectively yielding the second-order structure[68]

$$S^2_v = R(S^1_r), \quad r = 1, ..., n, \quad v = 1, ..., m \quad (n > m) \tag{2.2.10}$$

which again is being observed by the observation function $Obs^2$ yielding properties $P$ that are emergent iff

$$P \in Obs^2(S^2_v) \;\text{ and }\; P \notin Obs^2(S^1_r) \tag{2.2.11}$$

Those properties $P$ will be now discussed, and in the simulation, we will see that in order to gain additional structure some extra sort of dynamics has to be included—a *higher-level* observer mechanism, which is nothing but a pattern classifier on the basis of distinct dwell times of the particulars in *neighboring* voxels which define a new length scale by the asymptotic behavior of the objects' gluing together. Therefore, we need a *new* higher-order higher-level data structure.

[65] Cf. [76, 335].
[66] Cf. [76, 335].
[67] Cf. [76, 335].
[68] Cf. [76, 335].

### Higher-Level Data Structure

We have to distinguish between the data structures $\mathcal{D}_{i,j}$, i.e. all "information" at position $(i, j)$ including the update operator $U^{lat}$—and that is all there is—and structures that, at a higher scale, allow for identifications of higher-order structures at higher levels. Those higher-order structures are not observable at the lattice sites, since at $(i, j)$, there is only the information gathered at the position itself and the "information-particles" arriving from its very next neighbors. However, Rasmussen et al. state that "data structures [are] acting as internal observers [as opposed to external observers] generating second-order interactions."[69] But setting up a second-order bond is a property of a second-order structure and, according to (def 2.2.11), an exemplar of first-order emergence, which means that there must be at least a second-order observer, which is an *external* observer at a higher level, i.e. higher scale. And we can take them at their word saying: "Second-order interactions are, for instance, the interactions *we find* between polymers."[70] It is the human being that literally steps out of the picture, i.e. data structure $\mathcal{D}_{i,j}$, and identifies the polymer due to some criterion at a higher length scale, probably its constituents, the monomers, staying together for a significant amount of time, assigning it a new bond between structures of all levels capturing the structures of order $N$, which in fact are between first-order structures just extending the neighborhood. In other words, in order to detect higher $N$-order structures, we need additional data structures $\mathcal{D}^N_{i+\Delta i,j+\Delta j}$ with new matter and force fields as "information particles"—the higher-order structures and their higher-order interrelations. And of course, these data structures are at higher levels.

Rasmussen et al. seem to be reasonably aware of their transition from the "first-order epistomological" $\mathcal{D}^1_{i,j}$ to the "higher-order epistomological" $\mathcal{D}^N_{i+\Delta i,j+\Delta j}$ saying: "The second-order structures are therefore only *implicitly* given."[71] And with that confession the description language changes completely:

> *The local use—or* interpretation*—of the different kinds of communicated information defines the* operational semantics *of the information. [...] The interpretation of the information depends on which hyper-structural level the communicating objects belong to.*[72]

However, those higher levels, the level of the polymers and the level of the aggregates of the polymers do not appear in the data structure closer to ontology, and indeed all new hyper-structures and hyper-structural levels are a matter of *semantics*.

This shows even more plainly when we return to the topic of emergent properties. For instance, the polymers are said to exhibit the property of being *elastic*, which is due to "a unique combination of the transmitted information", but who is the sender and who is the receiver? The answer is that the information comes from the monomers (plus the solvent water) and arrives at the solute *and* the external observer, the „investigator", whatever he constitutes. For example, elasticity is defined as the tendency to fold again when the chain of the monomers, which is the polymer, is stretched out. This is due to the fact that the polymer is in a heat bath—the water molecules giving random kicks—, and it is more likely that the chain takes an unfolded geometrical configuration. A measure for elasticity

[69] Cf. [76, 343].
[70] Cf. [76, 342], inverse italics added.
[71] Cf. [76, 343].
[72] Cf. [76, 343].

is said to be the average end-to-end length and the radius of gyration as a function of the polymer length.[73] And that "emergent observable"—the measure for elasticity—is outside the data structure $\mathcal{D}^1$. Indeed, Rasmussen et al. state that elasticity is not a property of the monomers or monomer-monomer bonds, but it is also not a property of the polymer floating in the heat bath. 'Elasticity' is the property of the observer *together* with all that is necessary to identify certain behaviors of the first-order structures, which is at least in bonds with them and the first-order structures constituting the experiment apparatus that is in bonds with them in turn, such that the result of the process is having observed a token suitable to the *concept* of 'elasticity'.

The same applies to aggregates of polymers and water molecules—micellar third-order structures. They are assigned emergent properties or "functionalities" like bringing about the dichotomy of inside/outside, permeability, i.e. the possibility that, for example, some hydrophobic monomer inside the micelle is able to leave the single-layer "membrane", or even self-reproduction by adding a surface-induced reaction as long as „fuel" molecules are available.

As is seen, the transition from data structure to semantic talk is somewhat abrupt. We will arrive at that at the end of the chapter to see how the transition can be justified and motivate the category theoretical framework in the following chapter. But first, we will focus on the „experimental" data structure, the observer receiving a closer treatment, and end with a modification of the authors' *ansatz*.

The data structures $\mathcal{D}^1_{i,j}$ are seen to act as internal observers, explicitly opposed to external observers, and that is a view one could indeed hold, since the first-order object is a kind of interpreter of what happened to him, including an automatic updating, which simply is the execution of conditionalized change. And so far, there is no need to change to information and its semantics. But in the undertaking of installing a hyperstructural hierarchy the crucial point would be to find a criterion for the next higher order or meaningful length scale, i.e. level. It must be found intrinsically by the next level observer, but how could that be accomplished? There are first-order objects showing first-order interactions with variations in *strength*, and the imperative must be to observe a second-order interaction due to the *selection* of first-order interactions. Rasmussen et al. content themselves with:

> *These interactions are generated from a* composition *of first-order interactions, since each pairwise interaction always occurs between the first-order objects.*[74]

Higher-order interactions were *generated by composition* of lower-order interactions. In the case of polymer-polymer second-order bonds[75], the composition would be directed towards the monomer-monomer interactions as is delineated in (fig 2.3), for instance.

As is said, the data structures $\mathcal{D}^1_{i,j}$ are acting as internal observers—the primitives just receiving information-particles constituting the first-order interactions. But given that there were data structures covering an appropriately bigger length scale, what would be

[73] Cf. [76, 338].
[74] Cf. [76, 342].
[75] Interactions are not restricted to bonds in the sense of spacial attachment in general, but in the simulation, it's about aggregation of higher levels, and that is staying together due to bonding.

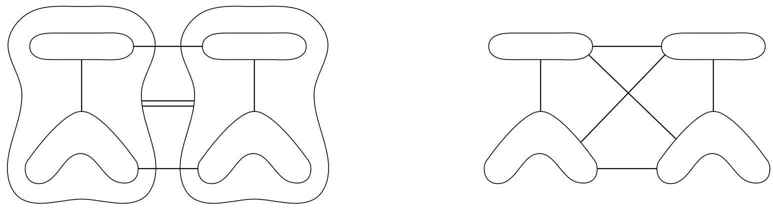


Figure 2.3: Left: polymer-polymer second-order interaction (double line), the crossing interactions being neglected. Right: first-order interactions. Cf. (fig 2.1) for zooming in to the atomic level.

the information-particles of the polymer-polymer interactions being „generated“ by the composition of the primitives' „information-particles“? Is it just the *superposition* of the first-order interactions, and if so, of all of them or just a selection? Rasmussen et al. simply state:

> *The only way a unique, concurrent means of communication can be obtained is by a unique combination of the transmitted information. Each inter-level communication needs to have at least a partially independent information channel.*[76]

Now they make a clean breast of it—at least only semantic terms are being used, and obviously, they talk about themselves, the observers. But we are not too quick on the trigger, we still want to stay close to the physical and ask: is there a unique combination of first-order interactions or interchanging „force particles“ conceivably resulting in a second-order interaction? The situation is delineated in (fig 2.4).

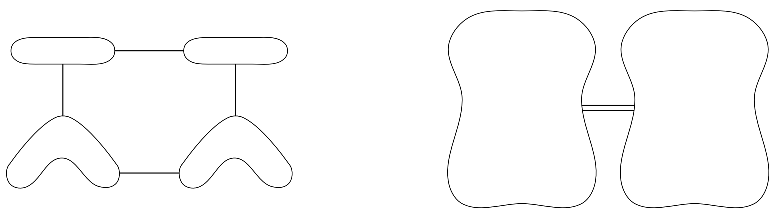


Figure 2.4: Left: four monomers interrelating (the crossing interactions being neglected again). Right: two polymers interacting.

The square on the left is to be uniquely combined or composed to the single interrelation on the right, and it is more than just some ordinary composition since it is an emergent[77].

[76] Cf. [76, 343].
[77] Cf. (def 3).

Formally, the figure reduces to the diagrams below

$$\begin{tikzcd} s_1^1 \arrow[r, dash, "Int_4^1"] \arrow[d, dash, "Int_1^1"'] & s_4^1 \arrow[d, dash, "Int_3^1"] & & \\ s_2^1 \arrow[r, dash, "Int_2^1"'] & s_3^1 & \end{tikzcd} \xrightarrow[\text{for}]{\text{necessary}} \quad s_1^2 \xrightarrow{Int_1^2} s_2^2 \tag{2.2.12}$$

neglecting the dynamics and the spacial aspects. This basic example already raises fundamental questions which will be extensively discussed in (chap 3), especially analyzing the consequences of focusing on time and space separately and finally making a *process* of (2.2.12). *Composition* turns out to become the key concept worthy enough to seek for a new framework in which it takes the center stage.

And of course, another grounding assessment for the framework is how it can handle *necessity* and *sufficiency*. The four first-order interactions in (2.2.12) are necessary for the second-order interaction to emerge. More precisely, $Int_1^1$ and $Int_3^1$ are necessary for the polymers $S_1^2$ and $S_2^2$ to emerge and in the next time step together with $Int_2^1$ and $Int_4^1$ for the higher aggregate with the polymer-polymer interaction to evolve in accordance with the hyperstructural dynamics (def 13). But the result of the process further depends on the external, which is an observer or observers and the remainder, called the boundary conditions. They are also necessary to render the conditions for existence complete, and we have to discuss whether or not they are sufficient.

What is the overall goal in the end? To make the objects move, at least in the simulation on display. That is what the lattice gas framework is good for. But given that there were no observers watching the grid, a cellular automaton[78], the first-order structures would behave the same, because by definition, no higher-order structures and bonds would have emerged. By the same token, if the second-order structures, the polymers, brought about new „communication channels“ within the lattice sending and receiving not only new information but force particles, we would have a serious case of *overdetermination* just by watching.

And indeed, Rasmussen et al. recognize some necessitation in the constitution of the higher-order structures by lower-order ones: “[A] two-way causation exists in these dynamical hierarchies.”[79] In addition to the upward causal composing of the whole, they spot *downward causal* constraints on the constituents:[80]

> *The dynamics of the monomers are more restricted once they form a polymer and polymers are more restricted once they form an aggregate. Thus, there is a clear downward causation in such systems as well.*[81]

It is not at all clear how the parts cause the whole and how those restrictions of the parts by the whole are being executed, and how the asymmetry in the direction of inter-order action or sending and receiving of information is to be conceived. Looking at (fig

[78] Cf. [76, 335], [55].
[79] Cf. [76, 345].
[80] Cf. figure (fig 2.5).
[81] Cf. [76, 345].

2.5), one would naively ask where, for example, the points of origin of the information-particles or force-particles are.[82] And how many objects are on the right, anyway: two or three?

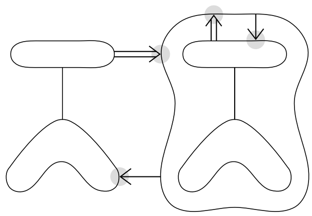


Figure 2.5: Asymmetric upward and downward causation: the first-order to second-order causation is of second order whereas the second-order to first-order causation is of first order.

The real, i.e. spacial, situation is depicted in (fig 2.6), where the edging is being dropped and the ordered direction doesn't make sense any more. Left on the plate, it is shown a monomer binding with two monomers, and an additional unbound monomer. Now the question is, how does the free monomer „effect" $Int_4^1$ while interacting with the monomer $S_1^1$[83] resulting in the aggregation right on the plate? It might be, which is an empirical question, that the "first-order"-interactions $Int_4^1$ in the left and in the right figure are not identical, meaning that there is a *restriction* by $Int_1^1$, as is said in the above quotation. That is what is meant by 'composition'—the two monomers $S_1^1$ and $S_2^1$ binding[84] together result in a different interaction $Int_4^1$ as it were the case without having bound. In this reading of 'restriction', one could except downward causation, e.g. the monomers $S_2^1$, $S_3^1$, and $S_4^1$ imposing a necessary condition for change or boundary condition on $S_1^1$, or one would just state that the composition is not a superposition.

Figure 2.6: The physical view: the primitives occupying spacial positions. Left: four monomers, $S_2^1$ hypothetically free. Right: $S_2^1$ being in bond with $S_1^1$ effecting $Int_1^1$. Cf. diagram (2.2.12).

Naturally, one would ask why the aggregation in the figure on the right is not four monomers but two polymers? It is because one *observes* that $Int_4^1$ is weaker than $Int_1^1$

[82] The points of origin are assigned by grey circles.
[83] Cf. (2.2.12).
[84] or, more neutrally, interacting

and $Int_3^1$, i.e. given slightly different boundary conditions, the geometrical distance between the monomers $S_2^1$ and $S_4^1$ might become greater, where the ones between $S_1^1$ and $S_2^1$, $S_4^1$ and $S_3^1$ respectively stay pretty much the same, the criterion for selection being just metrical.[85]

Another criterion for the observer to classify structures is *tracking back* the objects. It is more likely that it is not the situation depicted on the left side in (fig 2.6) that is registered, but rather the process in (fig 2.7)—two „polymers"[86] coming close and docking.

Figure 2.7: Classifying structures by back tracking of the constituents: the result on the right side appears to be the same (cf. (fig 2.6)) but the starting structures being different.

## 2.2.2 The Ansatz

Rasmussen et al. define: "An ansatz is a hypothesis taken to be true but acknowledged to be unproven that is used to reach further conclusions."[87] This German loanword is normally taken to refer to heuristically guessed solutions of differential equations.[88] But here perhaps it is meant to be more than an "educated guess"—it is rather a conjecture. The question is: is a certain degree of complexity of the primitives necessary or even sufficient to gain any order of structure in the hyperstructural hierarchy? And in which way does the chosen framework of description—including simulation—determine the emerging „functional" properties of the higher-order structures?

Coming back to all that specifies the object $S_r^1 = S_r^1(s_r, f_{rs}, \tau_r)$, which is the states or aspects $s_r$ and the interactions $f_{rs}$ together with the update operator $U$ that determine the primitive object $S_r^1$ at some local time $\tau_r$. Sorting the specifications we find that the aspects embrace the interactions, more precisely the corresponding "informative particles" that have the first-order structure as target or receiver.[89] In the 2-dimensional lattice gas

[85] The same holds for „functionalities" just being certain observed sequences of spacial behavior.

[86] We see that it is quite convenient to refer to „new" structures observed just for the sake of convenience, which is the reason why these concepts or theoretical terms needn't be eliminated along with the theories .

[87] Cf. [76, 347].

[88] "An important technique for solving differential equations is to guess the functional form of a solution (called an ansatz, or trial answer), substitute it in, and then see if the free parameters can be adjusted to make the solution work." Cf. [64, 10].

[89] That will match the duality proposed in (chap 4) defining two kinds of objects: those that send and receive and those that are sent and received.

simulation at position $(i, j)$, there are seven aspects in $s^t_{i,j} = (i, j, x_1, ..., x_7)$:[90]

$x_1$: scheduling color

$x_2$: type of matter (if any)

$x_3$: incoming excluded-volume particles (“repellons”)

$x_4$: incoming force particles (“forceons”)

$x_5$: velocity

$x_6$: bond directions

$x_7$: incoming binding-force particles (“bondons”)

and six update steps

$U_1$: propagate information particles (repellons, forceons, bondons)

$U_2$: create new bond (if any)

$U_3$: compute move direction (if any)

$U_4$: move monomer

$U_5$: clear lattice

$U_6$: repeat steps 1 to 5 for the other scheduling color

Those aspects and update cycles are sufficient, for example, for a monomer at an instance of time $t$ to exist at position $(i, j)$ and move with a certain velocity. That would be the framework also sufficient for some observer to identify higher-order structures, including their functional properties, and of course, “[i]f a different framework were chosen, these details would also be different[...]”[91]—Rasmussen et al. referring to the above seven aspects being enabled or disabled, resulting in the observed or not observed properties, e.g. a monomer being hydrophilic.[92] There might be, or indeed are, frameworks with less complex primitives resulting in equivalent behavior and properties of e.g. the first-, second-, and third-order structures in the given simulation, but Rasmussen et al. quite rightly ask: “[W]hat would that give us?”[93] As we see, the above aspects already strike us as odd, strange particles like “repellons”, “forceons”, and “bondons” being computed, diminishing the *explanatory power*. The notion of ‘force particles’ is weird enough, but inventing aspects to make them do what the observer commands conforms to Columbus banging his egg on the table.[94]

But that is not the crucial point—let the simulation follow the epistemics and make the particles move. It will just stay moving particles, all the other being „within“ the observer. Thus, there are *two* object complexities to be taken in consideration, but first we quote their *ansatz*:

**Conjecture 1** (Ansatz)**.** Given an appropriate simulation framework, an appropriate increase of the object complexity of the primitives is necessary and sufficient for generation of successively higher-order emergent properties through aggregation.[95] ♦

[90] Cf. [76, 338-340] for details.
[91] Cf. [76, 346].
[92] Cf. [76, 345]: Figure 5.
[93] Cf. [76, 346].
[94] “When the egg came round to the hands of Columbus, by beating it down on the table he fixed it.” Cf. [31, 17].
[95] Cf. [76, 347].

The simulation framework is set to do only the epistemic-like simulation as far as the moving is concerned, and the suitability simply reflects the object complexity bringing about the observed aggregations. Thus, there is no need to separately mention the framework but the complexity of the observer:

**Conjecture 2** (Ansatz*)**.** An appropriate increase of the object complexity of the primitives (ontology$^1$) and an appropriate increase of the object complexity of the observer (ontology$^2$) is necessary and sufficient for the generation of observations of successively higher-order emergent properties through aggregation. ♦

And this is almost trivially true. No matter whether the higher-order emergent properties are epistemic or ontological, if there is no external law that comes on top given a certain order of structure then it is sufficient for the generation of the higher-order observations. A major issue of course is the remainder, if there is any. As was said, it is not clear what the result of the process $R$ is meant to include. Obviously, the observation of the aggregation needs some stage to perform on, and again we have to ask: what is the observed, what is the observer, and what is the stage?

The more or less unwelcome dichotomy of ontology$^1$ and ontology$^2$ is what DUCKs will be about: what objects will be necessary and sufficient to be followed by the very observation *itself*—letting go of the common association of an observer.

### Composition

As we have seen, Baas assigns a subset of structures showing new properties to the higher-order interactions or bonds that are *meaningful* for the observer.[96] These emergent structures and properties could be now called on to build a hierarchical system of sets of structures having properties at a certain *level*, which is in accordance with the general view what cognition is to carry out:

> *Intuitively we would like to see higher levels and their properties emerge from the lower levels.*[97]

In the hyperstructure context, the level structure would then constitute of sets of units showing properties at the subsequent levels[98]

$$X_1, X_2, ..., X_i, ..., X_n \tag{2.2.13}$$

with an ordering of „production“

$$X_1 < X_2 < ... < X_i < ... < X_n \tag{2.2.14}$$

where $X_n$ denotes the highest level.

The corresponding properties and functionalities then define a system of “semantic representations” $S_i$ with a *compositional* ordering

$$S_1 \leftarrow S_2 \leftarrow ... \leftarrow S_i \leftarrow ... \leftarrow S_n \tag{2.2.15}$$

[96] Cf. (def 7).
[97] Cf. [60, 530].
[98] Cf. [60, 530].

such that the allocation

$$X_i \mapsto S_i \tag{2.2.16}$$

is by construction intrinsically given.

The key aspect of course is the kind of compositionality principle, which here is the composition of mappings of the sets of properties and functionalities, which "corresponds to Frege-type compositionality of meaning as used in linguistics"[99], showing a syntactic/structural part (2.2.14) and a semantic/functional part (2.2.15) with an evolution (2.2.16) corresponding to the hyperstructural dynamics (def 13).

In the next chapter, *memory evolutive systems*, a certain powerful compositionality principle will be employed to introduce new objects that are the result of gluing objects together in the manner that this new object is *identified* with the "observed"—the gluing—such that it does exactly the same as the compound system does. Thus, the difference to hyperstructures is that there are no emergent properties but emergent objects with emergent bonds that are just as good as the corresponding compound bonds of the composite.

Rasmussen et al. regard the observer as being organized as a dynamic hyperstructure, but as we concluded, there is no emergent structure without emergent properties and emergent interactions. Thus the observer as a higher-order structure observing lower-order structures cannot possibly show emergent properties and interactions, since a still higher-order structure is necessary.[100] The observer has to be external—or there is no observer.

With the memory evolutive systems, the situation is different, since in this framework the observers have no bonds in their own right. They are just a certain *representation* of already existing ones. But that means we will encounter unsolvable difficulties again, motivating us to go on, arriving at DUCKs, and one already senses that these will make trouble as well, but not due to the observation or cognition as much as the *experiencing* of the cognition if it *exists*.

[99] Cf. [60, 529].
[100] Cf. (def 3).

# 3 Memory Evolutive Neural Systems

*Memory Evolutive Neural Systems (MENS)*[1] has been developed as an application of the more general model *Memory Evolutive Systems (MES)*.[2] It is deemed appropriate to describe the emergence of animal behavior-like pattern recognition and classification of objects into invariance classes up to and including human capacities, like the evolution of semantics, the development of language, and even the phenomenon of consciousness. Since all behavior is assumed to emerge from pattern formation of a neural network, i.e. the brain, in MENS the philosophical position is suggested to be an "emergentist monism"[3] or "emergentist reductionism"[4] claiming ontological emergentism. Again as with the hyperstructures, the formation of a *hierarchy* of increasingly complex objects renders the construction process on the neuronal constituents to be "at the root of the emergence of mental objects"[5], therefore allowing for the physical, biological, psychological, and even sociological domain to be unified in an overarching theory—MENS. This approach of an integrative theory of human behavior seems promising, even though it rests on a concept called *multiple realization (MR)*.[6] But it turns out to be dubious whether MR has any physical reality and that general memory evolutive systems depend on it. This and further discussions of the assumptions in MENS will again lead to a reconsideration of the hierarchy of levels and finally result in the realization that something else has to be called for, i.e. a more fundamental, or ontological, theory of *Domain Unifying Categorical Kinds (DUCKs)*.

## 3.1 The Category Theoretical Framework

MENS is introduced to be a *category theoretical* modeling of brain[7] and mental states, thus the components are *objects* and *morphisms* with *composition*. Another theoretical term will be the definition of a *colimit*, which is actually the key concept of MENS along with *multiplicity*, i.e. more than one pattern admitting the same colimit. Those category theoretical notions have sharp definitions.[8] If the rigor proves to be too rigid to be clarifying (at first sight), the definition is given in a more descriptive manner. But one has to be on guard, since the definitions are only valid within categories, which will be further explored in (chap 4).

[1] Cf. [4].
[2] Cf. [3].
[3] Cf. [4, 154].
[4] Cf. [4, 172].
[5] Cf. [4, 130].
[6] In fact, MR is argued to be the only reason that reductionism fails.
[7] brain meaning all physical
[8] Cf. [70] and [39].

MENS, just like Baas' hyperstructures, argues for a hierarchy of levels:

> *The main idea is that these higher levels emerge from the basis through iterative binding processes, so that a* mental object *appears as a family of synchronous assemblies of neurons, then of assemblies of assemblies of neurons, and so on.*[9]

It might be true that assemblies of assemblies of neurons, i.e. neurons, are necessary for a certain mental object to emerge, but it also has to be clarified sufficiency for the mental "to appear as" is not identical to the real reductionist account of 'to be identically equal to' neurons. This is after all the *mind-body problem*, and some philosophers think there is a whole science behind that appearance—and make a living off it.

In the following, the picture of MENS will be redrawn insofar as the concepts are essential to motivate DUCKs. Therefore, we mainly concentrate on the existence and interrelation of its category theoretical objects in order to obtain a minimalist ontology that could help to reconstruct what this *psychophysical algebra* is thought to refer to.

### The Category of Neurons

Biological neurons are at the basis of MENS. All constructions in the complexification process are grounded on the category $\mathcal{N}^t$, the objects being neurons living in a time interval $[t, t + \Delta t]$ and the morphisms being their activating relation:

> *The operations are not instantaneous but require some period of time; thus what is particularly interesting is the category of neurons and their links existing during such a period, and we generally operate in this category.*[10]

Thus, a neural system or *diagram* consists of neurons $N_i(t)$ at an instant of time $t$, with their internal state of being active or not as a function of time. Or alternatively, they can be thought of as *states* of neurons[11] being enumerated with an ordering (time) index $t$: $N_i^t$. These states can be, for example, a conjunction of firing rates, thresholds, depolarisations, etc., all variables that classify the entities as active or non-active neurons:

> *[A]n item (external object or neuron)* activates $N_i^t$ *[notion modified] at t if it causes an increase in the activity of* $N_i^t$ *at this date; and we think of the resulting activation as a kind of information transmitted by the item to* $N_i^t$*.*[12]

In MENS, the neurons and their *synapses*[13], also called *links* $s_{i,j}^t$, are modelled in a graph and "with the composition defined by concatenation, this graph becomes the *category* $\mathcal{N}^t$ of *neurons* at $t$":[14]

[9] Cf. [4, 129], inverse italics added.
[10] Cf. [4, 133].
[11] rather than the neurons themselves
[12] Cf. [4, 131].
[13] 'Synapse' here is to be considered an activating connection rather than the single physiological synaptic bouton. For the sake of clarity, all the boutons shall be w.l.o.g. embraced by a unique directed connection between two neurons.
[14] Cf. [4, 132].

**Conjecture 3.** Neurons and their activation links build a category $\mathcal{N}^t$ with

(i) The *objects* $\mathrm{ob}(\mathcal{N}^t)$ are neurons $N_i^t$.

(ii) For every pair $N_i^t, N_j^t \in \mathrm{ob}(\mathcal{N}^t)$, the set of *morphisms* $\hom(N_i^t, N_j^t)$ is the synapse from $N_i^t$ to $N_j^t$.[15]

(iii) For every three objects $N_i^t, N_j^t, N_k^t \in \mathrm{ob}(\mathcal{N}^t)$, the *composition* is a concatenation of synapses, called a synaptic path.

(iv) For every $N_i^t \in \mathrm{ob}(\mathcal{N}^t)$, there exists the *identity* on $N_i^t$.

♦

Our guideline is to stay ontological as long as possible, since the objective is to approach the existence of "mental objects" as close as possible. Thus we need to examine whether the category $\mathcal{N}^t$ is indeed built on existent entities, because biological neurons are the basis of MENS.

### (i) The Objects

Are the neurons $N_i^t$ objects in a category theoretical sense? They are introduced to be "determined by [their] activity around $t$"[16] and therefore constitute a *set* of neurons $(N_i^{t'})_{t' \in [t, t+\Delta t]}$, which on that basis alone renders the objects not physical, but *conceptional.* In the first place, the objects being sets should do no harm to a category theoretical modeling[17]. But with the sets, we become detached from ontology without good reason. It's obvious that the sets don't exist in spacetime.

Thus those neurons $N_i^t$ do not exist, and the reason for their circuity is to introduce the idea of *action* and *activation.* An active neuron contributing to the activity of another neuron is popular parlance in the neurosciences, and there is a good reason for that, since it proves to be very convenient not to talk of several neurons being necessary for several neurons. It's *one* neuron activating another *one.* And that talk of single neurons doesn't cause confusion, because what really matters in the hard science is doing time series analysis—recording or modeling change following change.

But that is a problem that can be easily resolved, albeit awkwardly, meaning in a rigorous ontological process view, the separation of time and space of the „neurons" enables us to refrain from counting on sets of sufficiently similar neurons to be *classified* as one but to unfold them in a time series which admittedly appears to be somewhat clumsy. The transition between the categories $\mathcal{N}^t$ would then only be refined. This process of unpleasant unraveling is necessary to avoid inconsistencies that inevitably follow from the construction process of higher levels in MENS, as we will see. In DUCKs, however, this difficulty is evaded, because neurons aren't an issue any longer.

### (ii) The Morphisms

The synapses, which physically connect the neurons, are characterized by their strength and propagation delay and activate the subsequent neuron due to the transmission of

[15] More than one synapse between every pair of neurons won't be taken into account in MENS.

[16] Cf. [4, 131].

[17] Instead, the question could be, is $(N_i^{t'})_{t' \in [t, t+\Delta t]}$ a family of one neuron $N_i^t$ changing in time or several neurons taking over in time: $(N_i^{t_i})_{i \in T}$, with $T$ being an ordered time-index set. That would hold for the morphisms, too. This kind of distinction will be explored extensively in (chap 4).

so-called action potentials. And again, the activating link $(s_{i,j}^{t'})_{t'\in[t,t+\Delta t]}$ is itself actually a *family* of synapses—or states of the synapse varying in time. Thus, there are both active neurons and active synapses at a time around $t$, but only the „material“ neurons are objects, while the synapses, which are also „material“, are considered to be morphisms. But firstly, there is no need to separate the axon, including its presynaptic endings, from the neuron, since the whole device—constituting the object then—is necessary for „activating“ or „inhibiting“ the subsequent neuron. And secondly, a more serious objection, these synaptic connections are indeed seen both as material and *conceptual* activating links[18], which leads to difficulties concerning the composition of synapses as follows.

### (iii) The Composition

The composition of synapses has to satisfy

$$\mathcal{N}^t((N_j^t),(N_k^t)) \times \mathcal{N}^t((N_i^t),(N_j^t)) \to \mathcal{N}^t((N_i^t),(N_k^t)) \tag{3.1.1}$$
$$((s_{j,k}^t),(s_{i,j}^t)) \mapsto (s_{i,j}^t)(s_{j,k}^t) \tag{3.1.2}$$

In a diagram the neural network would be

$$\begin{tikzcd} (N_i^t) \arrow[r, "(s_{i,j}^t)"] \arrow[rr, bend right, "(s_{i,j}^t)(s_{j,k}^t)"'] & (N_j^t) \arrow[r, "(s_{j,k}^t)"] & (N_k^t) \end{tikzcd} \tag{3.1.3}$$

according to the definition of a synaptic path:

> *A synaptic path from $(N_i^t)$ to $(N_k^t)$ [notion modified][19] is* activated *at $t$ if $(N_i^t)$ activates $(N_k^t)$ at $t$ along it.*[20]

The activation process being executed via the sequence $\langle (s_{i,j}^t),(s_{j,k}^t)\rangle$ would result in exactly the same (activated) neuron $(N_k^t)$ as the composition $(s_{i,j}^t)(s_{j,k}^t)$ would have resulted in. The set $\hom((N_i^t),(N_k^t))$ of all possible paths between the neurons $(N_i^t)$ and $(N_k^t)$ would include a path that is equal to the composition in respect to strength and propagation delay: $(s_{i,k}^t) = (s_{i,j}^t)(s_{j,k}^t)$. But of course, $(s_{i,j}^t)(s_{j,k}^t)$ does not exist in general as a *real* path, it is only conceptual satisfying (3.1.1) for the neural system qualifying as a category. Thus, the threefold interpretation of $(s_{i,k}^t)$ as activation, synaptic path, and synapse doesn't allow for the physical neural network to live within the category $\mathcal{N}^t$. Furthermore, what could indeed happen to exist is $(s_{i,k}^t)$ being a material connection between $(N_i^t)$ and $(N_k^t)$, creating the graph

$$\begin{tikzcd} (N_i^t) \arrow[r, "(s_{i,j}^t)"'] \arrow[rr, bend left, "(s_{i,k}^t)"] & (N_j^t) \arrow[r, "(s_{j,k}^t)"'] & (N_k^t) \end{tikzcd} \tag{3.1.4}$$

[18] and not as objects in addition to neurons or as components of neurons
[19] Notions will be modified from now on if necessary.
[20] Cf. [4, 132].

Then the situation would be

$$\begin{tikzcd}
(N_i^t) \arrow[rr, bend left, "(s_{i,k}^t)"] \arrow[rr, bend right, "(s_{i,j}^t)(s_{j,k}^t)"'] & & (N_k^t)
\end{tikzcd} \tag{3.1.5}$$

both activating links being morphisms without distinction from reality. As said, a recommendation to modify the approach could be to refrain from thinking of synapses as some material sort in addition to neurons, but belonging to them and keeping up interpreting the activating link conceptually as a *condition* for the following neuron to be active or to *change* to activity or non-activity.[21]

### (iv) The Identity

The identity on $(N_i^t)$ is

$$(N_i^t) \xrightarrow{id_{(N_i^t)}} (N_i^t) \tag{3.1.6}$$

The interpretation of this diagram remains completely unclear, and no assistance is provided in the treatise of Memory Evolutive Systems.[22] Every neuron showing self-activation—synaptic or just conceptual—isn't what we find.

As an intermediate result, $\mathcal{N}^t$ being a category is a misleading concept and does not reflect physical entities without inconsistency, especially concerning the interrelations of the counterparts in nature. Modeling the material world would instead remain a graph, because no composition and no identity can be reasonably motivated. In DUCKs, a suggestion will be given for another interpretation of the concepts 'neuron' and 'activation' that is consistent with the concept of natural laws *and* the category theoretical framework.

However, by reason of important results of these explorations, in the next sections $\mathcal{N}^t$ will be rehabilitated and the category will be expanded by objects being colimits with different interpretations: physical neurons, mental objects, and (mental) concepts. We will see whether new inconsistencies arise or whether they accrue from the old. The finely, but maybe not appropriately, wrought framework will be probed to make out what can remain for and in the formation of an alternative Memory Evolutive (Neural) System without neurons or sets of neurons.

The key idea so far is local change necessitating local change, i.e. an active neuron at a certain position being necessary for a certain activity of a certain neuron at another position, which is actually activity moving in space, and which all happens within a time period $t + \Delta t$. And the category theoretical framework is considered to be tailor-made for that kind of motion:

[21] The claim here is that the objects that matter are active neurons and not, for example, active synapses. It could be discussed whether the synapses aren't the critical piece, the neurons just being their „provider".

[22] Cf. [3].

> *[A] diagrammatic representation [...] allows one to imagine motion along it. For instance, paths in a graph are a way of jumping from one object to another.*[23]

On the other hand, in $\mathcal{N}^t$, *synchronously* activated assemblies of neurons are said to exist during the time slice $t + \Delta t$. That raises the question how long $\Delta t$ can be at most not to risk being interrupted by a transition into the next category $\mathcal{N}^{t'}$. Otherwise put, what is the criterion for an assembly of active neurons to live within the same category? In MENS, it is more the *representation* of being, i.e. the active or activated assembly of neurons is represented in the simple case by a physical neuron and in a more complex by a so-called *cat-neuron* ($\widehat{N}^t$).[24] The focus here is clearly not placed on the *process* of activation.

## 3.2 The Binding Problem

We now want to go a little deeper into the concept "of an object, which is itself a system of systems" relating "the properties of the overall system to those of its component systems"[25] to be brought to a categorical setting in order to evaluate the achievements in trying to solve the *hierarchy problem* and the *emergence problem*, and we begin with the core problem, which is the *binding problem.*

### 3.2.1 Patterns and Collective Links

We don't spoil the ending when we say that for the rest of the inquiry, it will be all about representation, because firstly, it is already clear with (chap 2), and secondly, it is already clear that there *is* no such a thing. But before we substantiate this allegation we first define what is to be represented, the so-called *patterns*:[26]

**Definition 14 (Pattern).** A *pattern* $P^t$ is a diagram in $\mathcal{N}^t$ consisting of

(i) a family $(N_i^t)_{i \in I}$ of objects $N_i^t \in \mathrm{ob}(\mathcal{N}^t)$ with $I$ being a finite index set. The objects $N_i^t$ are the *components* of the pattern.

(ii) a family $(d_{i,j}^t)_{i,j \in I}$ of arrows $d_{i,j}^t \in \mathrm{hom}(N_i^t, N_j^t)$ called the *distinguished links* of $P^t$.

▲

Now to begin the next step, all information concerning a *representing object* is given by the morphisms, and the question is raised:

> *How can we use the links to recognize that a given object is complex, in the sense of having an internal organization that allows its components to operate synergistically?*[27]

[23] Cf. [3, 30].
[24] Cf. [4, 134].
[25] Cf. [3, 49].
[26] Cf. [3, 52]. In the following, for the sake of clarity, we again refrain from the set notation of the neurons and synapses and keep in mind that activity and activation takes time.
[27] Cf. [3, 50].

In order to understand what is meant by saying that a complex category-theoretical object has "components", we need to define the "information giving links" that are directed towards that object. It is the so-called *collective link* that allows the components to synergistically operate on the object, which then could possibly function as the pattern's representation. But first, we look at an arbitrary object that is not the representative:[28]

**Definition 15 (Collective Link).** Let $P^t$ be a pattern in the category $\mathcal{N}^t$. A *collective link* from $P^t$ towards an object $N^t \in \mathrm{ob}(\mathcal{N}^t)$ is defined as a family $(s_i^t)_{i\in I}$ of individual links $s_i^t \in \hom(\mathcal{N}^t)$, such that:

(i) every component $N_i^t$ of the pattern $P^t$ has a synapse $s_i^t$ to $N^t$,

(ii) for every distinguished link $d_{i,j}^t$ from $N_i^t$ to $N_j^t$, the correlating equation $s_i^t = d_{i,j}^t s_j^t$ is satisfied.

▲

On the basis of a simple pattern, we want to discuss the implications of the collective link:[29]

$$
\begin{tikzcd}
 & N^t & \\
N_i^t \arrow[ur, bend left, "s_i^t"] \arrow[rr, "d_{i,j}^t"'] & & N_j^t \arrow[ul, bend right, "s_j^t"']
\end{tikzcd}
\tag{3.2.1}
$$

In the diagram, the following boundary conditions are to be covered:

> *[T]he behaviour of each component must be coherent with that of the components to which it is connected in the pattern, so that the constraints imposed by the distinguished links are respected.*[30]

The neuron $N_j^t$ in the period of time $t + \Delta t$ activates the neuron $N^t$ via the synapse $s_j^t$, while it is subject to the constraints imposed by the neuron $N_i^t$ given by the distinguished link $d_{i,j}^t$. In short, $N_j^t$ activates $N^t$ while being activated (restricted) by $N_i^t$.[31] With the additional activation of $N^t$ by $N_i^t$, it is that the neurons $N_i^t$ and $N_j^t$ together activate the neuron $N^t$, while the neuron $N_j^t$ is being influenced by $N_i^t$. But that's not all. There is the correlation equation:

> *[...] $N_i^t$ interacts with $N_j^t$ along $d_{i,j}^t$, and so $N_i^t$ must coordinate its action $s_i^t$ on $N^t$ with the action $s_j^t$ of $N_j^t$ on $N^t$, the coordination being done along $d_{i,j}^t$, and thus $s_i^t$ must be the composite $d_{i,j}^t s_j^t$ (in conformity with the correlation equations). If there is also a distinguished link from $N_j^t$ to $N_i^t$, both components must reach an accord.*[32]

[28] Cf. [3, 53,54].
[29] Cf. [3, 54].
[30] Cf. [3, 53].
[31] Cf. (fig 2.6) as an example for the correlating equation to be satisfied in the hyperstructure context.
[32] Cf. [3, 54], notation changed.

It should be mentioned that the pattern consists of $N_i^t \xrightarrow{d_{i,j}^t} N_j^t$ and is not to be identified with the whole diagram (3.2.1), meaning that the neural net is restricted to the pattern with a special focus on the object $N^t$. These are the preparations for the concept of representation, which follows below.[33]

### 3.2.2 Cat-Neurons

To construct an expanded category of neurons $\mathcal{K}^t$, the category of real neurons $\mathcal{N}^t$ are to be coherently enlarged by so-called *cat-neurons* and their activating links. In particular, the interpretation of these *new* objects and morphisms must be examined carefully to see whether *composition* can possibly uncover certain features of nature. And we again don't give away the ending when we say that the whole investigation of (epiphenomenal) representation will be about exactly that: composition.

The following exploration is concerned with the question of whether cat-neurons could be physical neurons, or at least a physical object, and thus an element of $\mathrm{ob}(\mathcal{N}^t)$, if we allowed all physical to be included, or just a means for an intuitive, more or less metaphoric, modeling of cognition. This will be the groundwork for considering the ontological status of mental objects, e.g. the experiencing of cognition or the "mental image" of physical objects as Ehresmann and Vanbremeersch (EV) call it in their *Memory Evolutive Neural Systems*.

A cat-neuron is semantically defined to be a representation[34] of a (neural) pattern and formally a colimit in the category $\mathcal{K}^t$. "[H]owever, in a category, the objects themselves have no distinguishing features, and the only information, we have about them, comes from their links."[35] Thus, it's more the binding collective links from the objects constituting the pattern to the colimit, the cat-neuron, that give the significance. The cat-neuron is also said to have an "internal organization"[36] but, as cited, "no distinguishing features". If the binding collective link is added to the colimit object, one can accept both taken together exhibiting some kind of structure. But looking at the neural net to be represented, there is neither such a collective link nor an object to be called a cat-neuron. There are only neurons, i.e. an „active" assembly of neurons to be followed by another „active" assembly of neurons. That is what we want to analyse now.

In MENS, the basis for all representations is an active assembly or *pattern* $P^t$ of neurons, say $N_i^t$ and $N_j^t$ with an activation link $d_{i,j}^t$, as depicted in the diagram:

[33] We will return to the troublesome correlating equation $s_i^t = d_{i,j}^t s_j^t$. Cf. (3.1.4).
[34] Cf. (def 16).
[35] Cf. [3, 50].
[36] Cf. [3, 50].

$$
\begin{tikzcd}
& N^t & \\
& C^t \arrow[u, dashed, "s^t"'] & \\
N^t_i \arrow[ruu, bend left, "s^t_i"] \arrow[ru, "c^t_i"'] \arrow[rr, "d^t_{i,j}"'] & & N^t_j \arrow[luu, bend right, "s^t_j"'] \arrow[lu, "c^t_j"]
\end{tikzcd}
\tag{3.2.2}
$$

But there is something else, too. It is said that if the pattern $P^t$ could collectively activate a cat-neuron $C^t$ and a (cat-)neuron $N^t$, and the cat-neuron $C^t$ and only $C^t$ could activate $N^t$ in the same way as $P^t$ could, then $C^t$ is called the *binding* of $P^t$. In short, "the *collective links* $(s^t) = (s^t_i, s^t_j)$ from $P^t$ to any (cat-)neuron $N^t$ are in 1-1 correspondence with the links $s^t$ from $C^t$ to $N^t$."[37] The binding $C^t$ can be seen as the *unique* representative of $P^t$. It is said that "$P^t$ as a whole and its binding $C^t$ have the same functional role."[38]

Thus, the objective is to find some kind of justification for an object to be the representation of an active assembly of neurons. Here are the conditions for the object $C^t$ in (3.2.2) to represent the inner structure of the pattern $P^t$:[39]

**Definition 16** (**Representation**)**.** The necessary and sufficient conditions for $C^t$ being *the representation* for a pattern $P^t$ are

(i) the *collective link* $(c^t) := (c^t_i, c^t_j)$ to the binding $C^t$ coherently takes into account the *distinguished links*[40] $(d^t) = d^t_{i,j}$ between the components of the pattern $P^t$,

(ii) the binding $C^t$ via $s^t$ and the pattern $P^t$ via $(s^t) = (s^t_i, s^t_j)$ perform an equivalent function with respect to $N^t$.

▲

The representation for a pattern $P^t$ is defined to be *unique* meaning that there is no other cat-neuron having a function equivalent to $P^t$. On the contrary, the binding $C^t$ could represent more than one pattern having the same functional role which will be the basis for the *irreducibility* of higher-order cat-neurons in MENS.

Now this definition of a representation is quite similar to a well-known construction in category theory.[41] In MENS, it is claimed that the representation $C^t$ together with the collective link $(c^t)$ is indeed a *colimit*, which is actually the motivation for applying the algebraic framework.

[37] Cf. [4, 135], notations modified.
[38] Cf. [4, 134].
[39] Cf. [3, 57].
[40] which impose restrictions or boundary conditions on the objects in $P^t$
[41] Cf. [3, 57].

**Definition 17 (Colimit).** In a category[42] $\mathcal{C}$, the colimit of a morphism $v : A \to B$ is a triple $(Q, q_1, q_2)$, with for any $(Z, z_1, z_2)$, there is a unique morphism $u : Q \to Z$, such that the following diagram commutes:

$$\begin{tikzcd}
& Z & \\
& Q \arrow[u, dashed, "u"'] & \\
A \arrow[ur, "q_1"'] \arrow[uur, bend left, "z_1"] \arrow[rr, "v"'] & & B \arrow[ul, "q_2"] \arrow[uul, bend right, "z_2"']
\end{tikzcd} \tag{3.2.3}$$

▲

To identify whether the diagrams (3.2.2) and (3.2.3) are equivalent, the commutativity of the triangles and the uniqueness in (3.2.2) has to be proven. In MENS, *cat-neurons* being a colimit are said to satisfy the conditions for being a representation of a pattern $P^t$:[43]

**Conjecture 4 (Cat-neuron).** If a pattern $P^t$ admits a representation $C^t$, it is its colimit and is called the *cat-neuron* of $P^t$. ♦

With the new objects and their activation links, the category of physical neurons $\mathcal{N}^t$ can be extended:[44]

**Conjecture 5 (category $\mathcal{K}^t$).** Cat-neurons and their activation links build a category $\mathcal{K}^t$, with $\mathcal{N}^t$ being a subcategory. ♦

And also the patterns are no longer restricted to the physical basis:[45]

**Definition 18 (Pattern).** A pattern $P^t$ is a finite diagram in $\mathcal{K}^t$. ▲

These are the necessary concepts to introduce higher-level cat-neurons that bind so-called homologous patterns, and now we have to investigate whether the higher-level categories actually do have ontological counterparts.

### 3.2.3 Representism, or something near enough

We come back to the kind of representation one might have in mind and which puts us up to perceive a colimit construction here: why are we not content with diagram (3.2.1) and still seeking (3.2.2)? There can be manifold reasons for setting up something to represent and something representing. In MENS, it is intended to capture

(i) the same internal organization or structure of the pattern $P^t$,

(ii) the same functional role of the pattern $P^t$,

[42] Cf. [71, 108].
[43] Cf. [3].
[44] Cf. [3].
[45] Cf. [3].

(iii) the same internal organization or structure of patterns $P^t$ and $Q^t$,

(iv) the same functional role of patterns $P^t$ and $Q^t$.

Thus, the old distinction is made between form and function but without consequences in this case. If an object shows the same structure as the corresponding pattern, they admit the same „functional role". We recognize and remember that one can go without functions but still describe and predict the „same" behavior of the „constituents", and also that the entire structure of a real pattern can't be represented by an object. There is always some remainder neglected assumed to be not necessary for the same to follow. And that share is sufficient that results in a similar enough follow up, just as definition (def 16) says, if one substitutes 'a diagram and an object to *have* the same function for' by 'a diagram and an object to *be* a similar sufficient condition for'. That transition is key to showing that the hierarchy problem and the emergence problem aren't problems any more.

But first, we want to explicitly define the *binding problem*, which is said to be solved by the colimit construction (3.2.2):[46]

**Definition 19** (**Binding Problem**)**.** The problem of yielding an object which binds the pattern $P^t$ into a single object *having* the same coherent functional role is called the *binding problem.* ▲

Replacing 'functional role' while keeping the colimit's 'sufficient condition' for „near enough sameness" in the transition from epistemology towards ontology gives us:

**Definition 20** (**Binding Problem***)**.** The problem of yielding an object which binds the pattern $P^t$ into a single object *being* the same coherent sufficient condition is called the *binding problem.* ▲

We now want to examine whether the binding problem in the ontological reading (def 20) can be „realized", in the best sense of the word. To accomplish the task, we first have to translate the problem into the physical realm, which means we have to give the building blocks both a spacial and a temporal location.

### The Physical View

For a pattern to be a condition for a (cat)neuron to be activated takes some time, hence the *configuration category at time t* $\mathcal{K}^t$ is meant to include all transitions necessary to give the involved objects the status of being activated. However, for EV "it is a structural or relational concept, not a spatio-temporal one (nearer to Leibniz than to Newton)."[47] But that confession becomes troublesome in two respects: firstly, as already indicated, the „intended" boundary between epistemics and ontology, or modeling and reality, is not so clearly drawn. In MENS, in reference to Kim[48], it is assumed that the systems "make mental causation possible while preserving the physical closure of the world."[49] After all in the framework, it often seems that the objects and arrows were located in space and time, saying, for example:

[46] Cf. [3, 59].
[47] Cf. [3, 151].
[48] Cf. [44]: multiple realization.
[49] Cf. [4, 172].

> *A mental object modeled by such a cat-neuron "supervenes" on physical brain processes via the stepwise construction of a ramification from the neuron level up; later it will cause physical brain states through the unfolding of this ramification down to the neuron level, leading to a synchronous hyper-assembly of neurons.*[50]

Secondly, on the other hand, it might be the case that the concept of an evolutive system based on configurations of objects and their links "in the neighbourhood of $t$" is not appropriate to capture the behavior of natural systems, where the spatio-temporal location of the objects is not seen to be the delicate part:

> *The problem is more difficult for the links which must represent the interactions between these components (e.g. attachment of a protein to a receptor). Since these interactions are not instantaneous; what we should represent are the interactions around $t$. If the time scale $T$ is continuous, the links represent not just events occurring at $t$, but, in the terminology of Whitehead (1925), "germs" of interactions in small intervals of time around $t$.*[51]

In the following, we make the process explicit and execute an unfolding of the „realizing“ patterns and first concentrate on the change in perspective of the binding problem from the epistemic 'having a function' to the more ontological 'being a sufficient condition'. Still supporting the *coherence condition* (commuting triangles), the diagram in (def 20) then unfolds in time given the interval $\Delta t = t_4 - t_1 =: t_{1,4}$:

$$\begin{tikzcd}
 & & C^{t_3} \arrow[ddr, dashed, "s^{t_{3,4}}"] & \\
N_i^{t_1} \arrow[urr, "c_i^{t_{1,3}}"] \arrow[dr, "d_{i,j}^{t_{1,2}}"'] \arrow[drrr, bend right, "s_i^{t_{1,4}}"] & & & \\
 & N_j^{t_2} \arrow[uur, "c_j^{t_{2,3}}"'] \arrow[rr, "s_j^{t_{2,4}}"'] & & N^{t_4}
\end{tikzcd} \tag{3.2.4}$$

A condition for the above diagram to still obey the colimit-construction is that the arrows follow the time arrow. So imagining a coordinate system, the y-axis in the diagram could be regarded as referring to the spacial position of the objects, whereas the x-axis refers to the arrow of time. The pattern $P^{t_{1,2}} := (N_i^{t_1}, N_j^{t_2}, d_{i,j}^{t_{1,2}})$ and the object $C^{t_3}$ having the same functional role, i.e. now being the same sufficient condition for the object in the spacio-temporal coordinate system $N^{t_4}$ to be „activated“, yields a decomposed physical system now following *spatio-temporal relations.*

[50] Cf. [4, 172].
[51] Cf. [3, 151].

Since the neuronal diagram in the category of neurons $\mathcal{N}^{t_1+\Delta t}$

$$\begin{tikzcd}
N_i^{t_1} \arrow[dr, "d_{i,j}^{t_{1,2}}"'] \arrow[ddrr, bend left, "s_i^{t_{1,4}}"] & & \\
& N_j^{t_2} \arrow[dr, "s_j^{t_{2,4}}"'] & \\
& & N^{t_4}
\end{tikzcd} \tag{3.2.5}$$

is supposed to refer to physical[52] entities that can activate a mental object which “later will cause physical brain states”, we have to ask:

(i) How can the mental object, say $C^{t_3}$ along with the „activations“ $c_i^{t_{1,3}}$, $c_j^{t_{2,3}}$, and $s^{t_{3,4}}$,

$$\begin{tikzcd}
& & C^{t_3} \arrow[ddr, dashed, "s^{t_{3,4}}"] & \\
{} \arrow[urr, "c_i^{t_{1,3}}"] & & & \\
& {} \arrow[uur, "c_j^{t_{2,3}}"'] & & {}
\end{tikzcd} \tag{3.2.6}$$

be located in space and time?

(ii) How can the sufficient[53] conditions $P^{t_{1,2}}$ and $C^{t_3}$ along with the activation links for the neuron $N^{t_4}$ to be in a certain way active be identical?

Given the object $N^{t_4}$ in (3.2.5) is activated by the pattern $P^{t_{1,2}}$ via the synapses $s_i^{t_{1,4}}$ and $s_j^{t_{2,4}}$. $C^{t_3}$ being activated via the collective link $(c_i^{t_{1,3}}, c_j^{t_{2,3}})$ is then said also to be a sufficient condition for synchronously activating $N^{t_4}$ in the same way. This is provided by the commuting triangles $s_i^{t_{1,4}} = c_i^{t_{1,3}} s^{t_{3,4}}$ and $s_j^{t_{2,4}} = c_j^{t_{2,3}} s^{t_{3,4}}$. In order for $N^{t_4}$ not to be doubly activated in the same way by $N_j^{t_2}$, for example (activation meaning $N_j^{t_2}$ being a sufficient condition for $N^{t_4}$ to exist in the way it does for which $N_j^{t_2}$ is a sufficient condition), the sufficient condition coming from $N_j^{t_2}$ via $C^{t_3}$ $(c_j^{t_{2,3}} s^{t_{3,4}})$ is not the same as directly from $N_j^{t_2}$ $(s_j^{t_{2,4}})$ by computation or logical relation $(s_j^{t_{2,4}} = c_j^{t_{2,3}} s^{t_{3,4}})$ but by physical *existence*. The *necessitating* conditions have to be identical in space and time at least when they execute the activation of $N^{t_4}$. Thus, both ways of activating $N^t$ cannot

[52] Cf. (3.2.1).

[53] Cf. sufficient but not necessary conditions in [47, 62]: “[I]t is an insufficient but non-redundant part of an unnecessary but sufficient condition: it will be convenient to call this (using the first letters of the italicized words) an inus condition.” Also compare (def 28).

be executed synchronously, but rather in sequence, as shown in diagram (3.2.7):

$$
\begin{tikzcd}
 & & C^{t_3} & & & & & C^{t_7} & \\
N_i^{t_1} \arrow[dr, "d_{i,j}^{t_1,2}"'] \arrow[ddrrr, bend left, "s_i^{t_1,4}"] & & & & & N_i^{t_5} \arrow[urr, "c_i^{t_5,7}"] \arrow[dr, "d_{i,j}^{t_5,6}"'] & & & \\
 & N_j^{t_2} \arrow[drr, "s_j^{t_2,4}"'] & & & & & N_j^{t_6} \arrow[uur, "c_j^{t_6,7}"'] & & \\
 & & & N^{t_4} & & & & & N^{t_8}
\arrow[from=1-8, to=4-9, "s^{t_7,8}"]
\end{tikzcd}
\tag{3.2.7}
$$

Either the pattern actives $N^{t_4}$ or its representation, which renders the colimit construction inappropriate. The representation doesn't exist in the same way as the physical diagram (3.2.5) does; it has no place in space and time. What then is the colimit, the cat-neuron, a representation for? Does it have to be brought into existence in the same way the neurons obtain their presence? These questions shall prepare for the investigation on the existence of mental objects in DUCKs, where *the mental* is to be treated as a more or less physical object in space and time, and we will see how far we can go.

What have we gained so far? Firstly, a cat-neuron to fulfill the same functional role as a pattern seems highly intuitive, at first glance. But as we have seen, the cat-neuron is only capable of doing the same that the neural network does anyway. And we intended to clarify this misunderstanding of functions being not just conceptual by setting the focus on their sufficiency in conditionalizing activity, which is done by localizing the conditions in space and time.[54] Secondly, we wanted to introduce the framework of category theory as a powerful tool to analyze what is possible and what is not, or at least to see what strikes us as odd. One example will be the problem of *overdetermination*, which has been formulated and resolved in terms of category theoretical composition. Commuting diagrams show how sufficient conditions are to be split, and we have to deal with the question of whether there is a counterpart for composition in nature.

The above considerations on representation apply to any objects and morphisms in a category theoretical framework using the colimit construction and are of course not restricted to neural networks. Furthermore, the framework appears to be an appropriate means for defining representation, and hence in the following, we stay with the construction set and make the claim:

**Conjecture 6 (Representation).** *Representations* do not exist in space and time. ♦

This already gives us an indication of how the mental will be treated, but before we turn our attention to this part of the psychophysical, we still want to labor on the concept of the 'coherent sameness' in (def 20). As we have seen, particular conditions (and they are always particular) are unique in the sense that any two conditions cannot be conditions

[54] Later on we will call those conditions 'necessary conditions for change' and see that giving them a location in space and time can be helpful in defining the conditions for the objects' changing. There is still no distinction being made here between 'being a condition' and 'having something as condition': cf. (con 11).

for the same, but for a similar. This is a motivation to modify the definition of the binding problem again:

**Definition 21** (**Binding Problem****)**.** The problem of yielding an object which binds the pattern $P^t$ into a single object being a sufficient condition for a *similar* successor is called the *binding problem.* ▲

This binding object then, say $C^{t_3}$ in diagram (3.2.4), and „its" pattern $P^{t_{1,2}}$[55] are able to activate the successor $N^{t_4}$. However, there are three sufficient conditions for $N^{t_4}$ to be activated:

(i) the pattern $P^{t_{1,2}}$,

(ii) the object $C^{t_3}$,

(iii) both simultaneously.

Now in all three cases, the activation is just *similar*, and the coherence condition, i.e. the commutativity of the triangles, does not apply. If the object $C^{t_3}$ was to bind the pattern $P^{t_{1,2}}$, then in the sense that $P^{t_{1,2}}$ and $C^{t_3}$ activate $N^{t_4}$ such that if one of the both conditions was omitted the activation would be similar enough such that it would be a similar sufficient condition again for some successor, possibly resulting in a cascade of similar sufficient conditions. But one has to be cautious, even though the problem of overdetermination does not appear any longer, the object $C^{t_3}$ has to be correlated to the pattern $P^{t_{1,2}}$ in the above manner having *similarity* as a criterion, otherwise it wouldn't count as a similar enough representation of the pattern. Of course one could argue: what's the matter with being a representation? The object $C^{t_3}$ is only to be activated such that it happens that (and this is not functionalism) $C^{t_3}$ could similarly activate $N^{t_4}$ while the pattern being omitted wouldn't make a big enough difference for the future.

And one could play around with the diagram to see what else could be achieved. For example, the object $C^{t_3}$ might be activated by the pattern in such a way that $C^{t_3}$ activating $N^{t_4}$ is not at all similar to $P^{t_{1,2}}$ activating $N^{t_4}$ but nevertheless results in something else such that... Or the synapse $s^{t_{3,4}}$ could be so weak that it doesn't exist, rendering $C^{t_3}$ an epiphenomenon as far as this pattern is concerned. One could also set $C^t$ at the beginning of the concatenation of activations, $C^{t_1}$, and turn the collective link around, rendering it no longer a collective link.

Before we come back to the similarity condition, we should replace the 'binding into' in (def 21) less metaphorically by 'being a sufficient condition for' yielding the final definition that could have a counterpart in nature:

**Definition 22** (**Binding Problem*****)**.** The problem of yielding an object which is a sufficient condition for the pattern $P^t$ to activate being a sufficient condition for a *similar* successor is called the *binding problem.* ▲

[55] If the links are apparent from the context, they won't be mentioned in the following.

The ‘being a sufficient condition for some similar’ can be proceeded in different ways, for example in diagram (3.2.7), $N^{t_4}$ and $N^{t_8}$ are the similar successors, or:

$$\begin{tikzcd}
Q^{t_1} & & & & Q^{t_5} \arrow[drr, "(q^{t_{5,7}})"] & & & \\
& & C^{t_3} \arrow[dddr, "s^{t_{3,4}}"] & & & & C^{t_7} \arrow[dddr, "s^{t_{7,8}}"] & \\
N_i^{t_1} \arrow[urr, "c_i^{t_{1,3}}"] \arrow[dr, "d_{i,j}^{t_{1,2}}"'] & & & & N_i^{t_5} & & & \\
& N_j^{t_2} \arrow[uur, "c_j^{t_{2,3}}"'] & & & & N_j^{t_6} & & \\
& & & N^{t_4} & & & & N^{t_8}
\end{tikzcd} \tag{3.2.8}$$

with $Q^{t_5}$ being another pattern and $(q^{t_{5,7}})$ its collective link and $N^{t_4}$ and $N^{t_8}$ being similar again.

Before we define the critical *similarity-condition* in (chap 3.3), there is still something to say about the colimit construction in the binding problem, which is crucial for ongoing considerations. It is important to see what can be saved for the alternative approach to patterns capturing, to some extent, the identity now being replaced by some kind of measure of similarity. We ask what is so special about colimits that they are recognized in this context? It is that the pattern and the alleged colimit often happen to activate similarly and, at least at the beginning, close to simultaneously.[56] And that representation need not be a single object but can also stand for another pattern being a sufficient condition for similar activation.

It was said that “the colimit is a solution of the binding problem”, and “the binding link $c_i^t$ [...] in some way identifies the component $N_i^t$ to a „piece“ of $C^t$.”[57] It is given an example: “A molecule integrates the pattern formed by its spatial configuration, with its atoms and the chemical bonds between them.”[58] And that is exactly what we considered in the response to Baas’ hyperstructures. The “pieces” of the molecule are the atoms themselves, there being no extra higher-order object which “involves both local and global properties”[59] where the former was due to “structure” and the latter due to “function”.

We now want to reflect upon the properties, that they are “locally on the structure” and “globally on the function”. Therefore, we again consider the colimit construction, more precisely the reason it appears so attractive.

[56] Cf. [58] which presents ‘similarity’ as a powerful means to capture model-world relations.
[57] Cf. [3, 59].
[58] Cf. [3, 59].
[59] Cf. [3, 60].

### Local and global properties of a colimit

“[T]he organization of the pattern is made more robust and efficient [...].”[60] And the reason is that it admits a colimit. But what is the „mechanism“ or behavior behind that effectiveness? First, one has to ask what it is more effective for? The only thing the pattern does is activate an object $N^t$, and $C^t$, but the latter only if it admits a colimit, which renders the pattern, i.e. the components and their distinguished links, most efficient. Now effectiveness is a matter of natural selection, but what has the evolutionary process been selecting for while both the pattern and its colimit simply do the same, namely activating $N^t$ identically. There must be some mechanism to identify identical synchronous activation, whatever the colimit constitutes. And after all, the colimit can’t be selected for, since it doesn’t exist before. So, what is it that happens to be most effective? It is said to be

(i) the functional role of the constituents of the pattern, e.g. the neuron $N_i^t$, that contribute to $C^t$,

(ii) the local property of the colimit of being integrative of e.g. the activating neuron $N_i^t$, and

(iii) the global, “universal” property “that the colimit is the object which best implements the operative functions of the pattern.”[61]

We already mentioned that ‘having the function to’ is to be substituted by ‘being sufficient to’. And with it, we reformulate the natural efficiency problem, which doesn’t only reside in the capability of the pattern and its colimit to activate an object $N^t$ but also in the object $N^t$ itself to have it be „best“ activated. The efficiency lies in the most effortless sufficiency of the activation of the overall diagram (3.2.4):

**Conjecture 7 (Efficiency).** In natural selection, the efficiency of a diagram is driven by its minimal sufficiency over time to activate similarly. ♦

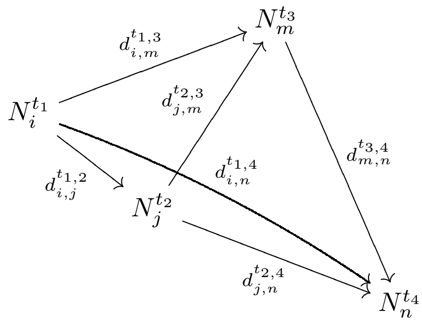
And efficiency in function, i.e. “best” or “robust”, is no longer a property of something, but rather is the fact of minimal sufficiency of a diagram over a relevant time span in activating similarly.

### Hypothesis on the genesis of an efficient diagram

As shown, one reason that colimits, i.e. objects that behave like a colimit, don’t exist in nature is that they can’t be selected for. But in terms of efficiency and similarity, it could happen that different conditions for being activated, or rather for being what it is, can be driven. Finally, we want to delineate a possible evolutionary process and start with a

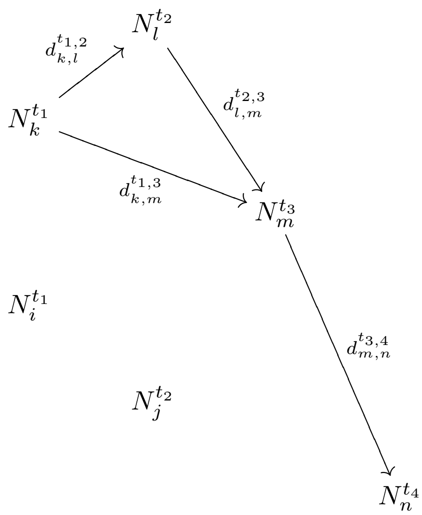
[60] Cf. [3, 60].
[61] Cf. [3, 60].

diagram similar to (3.2.4):




(3.2.9)

No neuron is standing out; each one only activating or being activated via the distinguished links. But that is not the whole story. They are in temporal order ($N_i^{t_1}, N_j^{t_2}, N_m^{t_3}, N_n^{t_4}$), which is a necessary condition to separate the neurons due to perceived functions. Take the last activated neuron $N_n^{t_4}$. If it was the object, the successors would depend on, in terms of efficiency, the sub-pattern ($N_i^{t_1}, N_j^{t_2}, N_m^{t_3}$), including their distinguished link, the synapses, would evolve with the pattern's sufficient condition be minimal in the end. And that could happen in the following way. The distinguished link $d_{m,n}^{t_{3,4}}$ and the *anticipated* collective link ($d_{i,n}^{t_{1,4}}, d_{j,n}^{t_{2,4}}$) both activate neuron $N_n^{t_4}$ such that each of them is sufficient to reach the neuron's threshold for transmitting nerve impulses. If that happened to be the case, it would no longer be necessary for the pattern ($N_i^{t_1}, N_j^{t_2}$) to be active in order for $N_n^{t_4}$ to be activated. Neuron $N_m^{t_3}$, being activated second to last, could activate $N_n^{t_4}$ on its own. This then is the first step for $N_m^{t_3}$ to be regarded a representation of the pattern ($N_i^{t_1}, N_j^{t_2}$), or something near enough. But of course that would only make sense if there were more than one pattern being involved, e.g. ($N_i^{t_1}, N_j^{t_2}$) and ($N_k^{t_1}, N_l^{t_2}$) both being able to activate $N_n^{t_4}$ in a similar enough way:[62]




(3.2.10)

[62]In these diagrams, the links depict active synaptic paths, and thus, they have to be made invisible to show when they are inactive.

It could happen due to some mechanism that even some single "grandmother" neuron $N_m^{t_3}$ could activate some successor, here $N_n^{t_4}$, in a similar way as the whole pattern $(N_i^{t_1}, N_j^{t_2})$ would do. This kind of „backup" neuron $N_m^{t_3}$ could of course also be a more complicated pattern.

In MES, it is said that both patterns $(N_i^{t_1}, N_j^{t_2})$ and $(N_k^{t_1}, N_l^{t_2})$ are "homologous", meaning that they are represented by the same colimit. But as we have seen, $N_m^{t_3}$ isn't such an object, i.e. both patterns are not homologous but similar. This crucial concept of multiplicity will be discussed in the next chapter, in order to substantiate that there are no levels one has to fall back on.

## 3.3 The Hierarchy Problem

Before we investigate the hierarchy problem, we will have a look at the category of neurons $\mathcal{N}^t$ and the category of colimits $\bar{\mathcal{K}}^t$ again, which together make up the category $\mathcal{K}^t$. It is said:

> *The global property shows that the existence of a colimit imposes constraints on all the objects of the category, not only on the components of the pattern. It explains that the existence of a colimit depends in an essential way on the given category $\mathcal{K}^t$ in which the pattern is considered.*[63]

That is the universal property of colimits, i.e. each link from the constituents of a colimit basis, a pattern, to any object of the category $\mathcal{K}^t$ binds to one and only one link from the colimit to the object, yielding a commutative triangle.[64] As shown, however, that condition doesn't satisfy the real circumstances of general neural networks, the synapses being highly restricted. It is right that the category essentially determines the possibility of the existence of colimits, and here, in the naturalistic view, they are excluded, as we have seen.

However, the idea of colimits having multifold patterns subverting *reductionism* is interesting and deserves a continuation of the inspection of the modeling of mental representation, which MES finally targets.

In the following, in MES, the notion of 'evolving systems' is used more loosely than in dynamical systems theory which is characterized by treating all constituents as if they were on equal footing. Here, systems are constituted of lower levels that are subordinate to higher levels, and we are now to define the hierarchy of levels of increasing complexity by linking objects that are "complex". The failure of reductionism, from the point of view of MES, needs some preparation, which will at first relate to diagrams like (3.2.2), which are conceptual without taking space and time into account.

We start with a diagram showing a pattern and its binding colimit and first add some non-complex object $A^t$. That object can be seen as anything that is able to activate or change with no further specification.[65] For that situation, MES gives a definition and calls $g_i^t$ a $P^t$-factor of $g^t$:[66]

[63] Cf. [3, 60].
[64] Cf. [3, 57].
[65] Later on we will refrain from any concept of activity and only refer to existence and change.
[66] Cf. [3, 74,75].

**Definition 23** (**$P^t$-Factor**)**.** A link $g^t$ from $A^t$ to the colimit $\widehat{P}^t$ of the pattern $P^t$ is said to be mediated by $P^t$ if it is of the form $g_i^t c_i^t$ for at least one component $N_i^t$ of $P^t$ and one link $g_i^t$ from $A^t$ to $N_i^t$, where $c_i^t$ is the binding link to the colimit associated to the index $i$. In this case, $g_i^t$ is called a $P^t$-factor of $g^t$:

$$\begin{tikzcd}
A^t \arrow[rr, "g^t"] \arrow[dr, "g_i^t"'] \arrow[drrr, "g_j^t"] & & \widehat{P}^t & \\
& N_i^t \arrow[ur, "c_i^t"] \arrow[rr, "d_{i,j}^t"'] & & N_j^t \arrow[ul, "c_j^t"']
\end{tikzcd} \tag{3.3.1}$$

▲

Thus, an arbitrary object $A^t$ activates the colimit $\widehat{P}^t$ with restrictions given by the above commutativity. As we said, identically "mediated activation" isn't possible, but that shouldn't prevent us from acknowledging the intuition's merits for the domain unifying constructions in (chap 4). We fall into line with the composition conception but not into the trap of spotting something ontological in it. When a more exact consideration is needed, it will be provided, even though the diagrams could appear a little convoluted. And that is probably one reason why time and space isn't separated here. Another is of course the idea of functionality, which again rests on the first, the great complexity of the processes.

The reason for defining a $P^t$-factor will become obvious soon. We want to gain interacting patterns in order to define interacting colimits, and $A^t$ will then be part of such a pattern. Fleshing out the composition conception, if $A^t$ via $g^t$ admits a $P^t$-factor $g_i^t$, and the object $N_i^t$ is connected with $N_j^t$ by a distinguished link $d_{i,j}^t$, $g^t$ yields another $P^t$-factor with[67]

$$(g_i^t d_{i,j}^t) c_j^t = g_i^t (d_{i,j}^t c_j^t) = g_i^t c_i^t = g^t \tag{3.3.2}$$

which says the information transmitted by $g^t$ is also transmitted by $g_i^t c_i^t$ and transitively transmitted by $g_i^t d_{i,j}^t c_j^t$. The commutativity of the diagrams yields the following proposition:[68]

**Proposition 1.** If $g^t$ admits a $P^t$-factor $g_i^t$, it also admits as a $P^t$-factor any link correlated with $g_i^t$ by a zigzag of distinguished links of $P^t$. ◊

## 3.3.1 Interacting Cat-neurons

In order to define the interrelation between „higher-order“ objects that are colimits, we first have to approach their patterns' exchange. Diagram (3.3.1) already illustrated two interrelating patterns: $(N_i^t, N_j^t)$ and $(A^t)$. In the following, we want to consider the more

[67] Cf. [3, 75].
[68] Cf. [3, 76].

general case with two patterns $P_1^t = (N_i^t, N_j^t)$ and $P_2^t = (N_m^t, N_n^t)$:

$$
\begin{tikzcd}
N_m^t \arrow[rr, "d_{m,n}^t"] \arrow[dr, "d_{m,i}^t"'] \arrow[drrr, "d_{m,j}^t"] & & N_n^t \arrow[dl, "d_{n,i}^t"'] \arrow[dr, "d_{n,j}^t"] & \\
& N_i^t \arrow[rr, "d_{i,j}^t"'] & & N_j^t
\end{tikzcd}
\tag{3.3.3}
$$

In the first step of introducing interacting cat-neurons, these two patterns are connected by a so-called cluster:[69]

**Definition 24 (Cluster).** Given two patterns $P_1^t$ and $P_2^t$ in a category, a *cluster* from $P_2^t$ to $P_1^t$ is a maximal set $G_{2,1}^t$ of links between components of these patterns satisfying the following conditions [cf. (3.3.3)]:

(i) For each index $k$ of $P_2^t$, the component $N_k^t$ of $P_2^t$ has at least one link to a component of $P_1^t$; and if there are several such links, they are correlated by a zigzag of distinguished links of $P_1^t$.

(ii) The composite of a link of the cluster with a distinguished link of $P_1^t$, or of a distinguished link of $P_2^t$ with a link of the cluster, also belongs to the cluster.

▲

Thus, a cluster can be thought of as a maximal set of restrictions on the constituents of the pattern $P_1$ by the pattern $P_2$. Combining the interacting patterns with their representational colimits yields the following diagram:

$$
\begin{tikzcd}
& \widehat{P}_2^t \arrow[rrrr, "g_{2,1}^t"] & & & & \widehat{P}_1^t & \\
N_m^t \arrow[ur, "c_{m,2}^t"] \arrow[urrrrr, "g_{m,1}^t"] \arrow[rr, "d_{m,n}^t"] \arrow[rrrr, bend right, "g_{m,i}^t"'] \arrow[rrrrrr, bend right=40, "g_{m,j}^t"'] & & N_n^t \arrow[ul, "c_{n,2}^t"'] \arrow[urrr, "g_{n,1}^t"] \arrow[rr, bend left, "g_{n,i}^t"] \arrow[rrrr, bend left, "g_{n,j}^t"] & & N_i^t \arrow[ur, "c_{i,1}^t"] \arrow[rr, "d_{i,j}^t"] & & N_j^t \arrow[ul, "c_{j,1}^t"']
\end{tikzcd}
\tag{3.3.4}
$$

with $\widehat{P}_2^t$ denoting the colimit of the pattern $P_2^t$, and $\widehat{P}_1^t$ of $P_1^t$, respectively. The purpose seems to be clear. We wish to connect the interrelations between the patterns with the interrelation between their colimits, fusing the *levels*, which consist of the physical

[69] Cf. [3, 81].

neural network and its representation. This way of merging the colimit constructions is achieved by a "simple" link: "A simple link from $\widehat{P}_2^t$ to $\widehat{P}_1^t$ is a link which is mediated by decompositions $P_2^t$ of $\widehat{P}_2^t$ and $P_1^t$ of $\widehat{P}_1^t$."[70] We call the link $g_{2,1}^t$ the *binding of the cluster* $G_{2,1}^t$, and the links constituting the cluster the *factors* from $P_2^t$ to $P_1^t$. A more compact definition of simple links between colimits is:[71]

**Definition 25** (**Simple Link**)**.** If $\widehat{P}_2^t$ is the colimit of $P_2^t$ and $\widehat{P}_1^t$ the colimit of $P_1^t$, a link from $P_2^t$ to $P_1^t$ is called a $(P_2^t, P_1^t)$-*simple link* if it binds a cluster from $P_2^t$ to $P_1^t$. Otherwise, it is said to be $(P_2^t, P_1^t)$-*complex*:

$$\begin{tikzcd} \widehat{P}_2^t \arrow[r, "g_{2,1}^t", "(P_2^t,P_1^t)-\text{simple}"'] & \widehat{P}_1^t \\ P_2^t \arrow[u, "C_2^t"] \arrow[r, "G_{2,1}^t"'] & P_1^t \arrow[u, "C_1^t"'] \end{tikzcd} \tag{3.3.5}$$

with $C_1^t$ and $C_2^t$ being the collective links[72]. ▲

It can be shown that the simple link $g_{2,1}^t$ is unique:[73]

**Proposition 2.** Let $G_{2,1}^t$ be a cluster from a pattern $P_2^t$ to $P_1^t$. If $P_2^t$ has a colimit $\widehat{P}_2^t$ in the category $\mathcal{K}^t$ and $P_1^t$ a colimit $\widehat{P}_1^t$, there exists a unique link $g_{2,1}^t$ from $\widehat{P}_2^t$ to $\widehat{P}_1^t$ binding the cluster, in the sense that the links of the cluster are $P_1^t$-factors of the composites of a binding link to $\widehat{P}_2^t$ with $g_{2,1}^t$ [cf. (3.3.4)]. ◊

The commuting square (3.3.5) is a shortcut of diagram (3.3.4), in which all pathways are said to commute. But as we have seen, these kinds of diagrams are confusing. Firstly, time and space have to be taken into account, and secondly, there is no material basis for composition. The entities in the diagrams ought to be physical, or something close to that, e.g. neurons, but not conceptual. In short, still ignoring the spatial relation, when we tilt the whole diagram to the right such that all arrows point conformal to the flow of time, the slimmed down diagram might look like

$$\begin{tikzcd} & & \widehat{P}_2^{t_3} \arrow[rrr, "g_{2,1}^{t_3,6}"] & & & \widehat{P}_1^{t_6} \\ N_m^{t_1} \arrow[r, "d_{m,n}^{t_1,2}"] & N_n^{t_2} \arrow[ur, "c_{n,2}^{t_2,3}"] \arrow[rr, "g_{n,i}^{t_2,4}"] & & N_i^{t_4} \arrow[r, "d_{i,j}^{t_4,5}"] & N_j^{t_5} \arrow[ur, "c_{j,1}^{t_5,6}"] \end{tikzcd} \tag{3.3.6}$$

[70] Cf. [3, 83].
[71] Cf. [3, 84].
[72] Cf. (def 16).
[73] Cf. [3, 84].

both pathways transporting the same „information“ from $N_m^{t_1}$ to $\widehat{P}_1^{t_6}$.[74]

Concerning the second objection, the two paths obviously do not result in the same „difference making“, regardless which material object is referred to, as has been pointed out.

We now approach the second core concept of Memory Evolutive Systems: multiplicity. Along with the colimit construction set, the means is then provided to discuss *emergence* in the complexification process promoted in the categorical framework.

### 3.3.2 Multiplicity

According to MES, “the notion of a simple link per se has no meaning; it is necessary to indicate with respect to which decompositions it is considered.”[75] Thus, the binding $g^t_{2,1}$ of the cluster in diagram (3.3.5) is not isomorphic to $G^t_{2,1}$, and that is a consideration that directly leads us to the conception of *multiple realizability (MR)* or *multiplicity*.

But before we can discuss the principle of MR, there is another decisive step to take, i.e. the composition of clusters resulting in the composition of simple links. It can be proved that two adjacent clusters can be composed such that (def 24) is satisfied:[76]

$$
\begin{tikzcd}
\widehat{P}_4^t \arrow[r, "g^t_{4,2}", "(P_4^t{,}P_2^t)-\text{simple}"'] \arrow[rr, bend left, "g^t_{4,1}", "(P_4^t{,}P_1^t)-\text{simple}"'] & \widehat{P}_2^t \arrow[r, "g^t_{2,1}", "(P_2^t{,}P_1^t)-\text{simple}"'] & \widehat{P}_1^t \\
P_4^t \arrow[u, "C_4^t"] \arrow[r, "G^t_{4,2}"'] \arrow[rr, bend right, "G^t_{4,1}"'] & P_2^t \arrow[u, "C_2^t"'] \arrow[r, "G^t_{2,1}"'] & P_1^t \arrow[u, "C_1^t"']
\end{tikzcd}
\tag{3.3.7}
$$

But patterns are in general non-connected and hence don’t compose. We now examine the case where they are non-connected but admit the same cat-neuron. The concept of multiple patterns adopting “the same functional role” is the second reason why colimits appear so attractive. They render composed bindings of clusters *complex.* We first recapitulate the findings and then discuss them.

The multiplicity principle is often mentioned in the same breath with *robustness* and *adaptivity.*[77] This seems to be a core concept of organisms: If something happens not to be perfectly adequate for a system to maintain its structure in the long run, it doesn’t

[74] That is due to the definition of a colimit (def 17) and the proposition (prop 2).
[75] Cf. [3, 85].
[76] Cf. [3, 82], [3, 85].
[77] Cf. [59], [45], [36].

necessarily imply its exitus. There is some kind of *redundancy*[78] that protects life from catastrophic consequences. Moreover, some capacities, such as the consolidation of memory, are highly dependent on „sufficient vagueness". Indeed, the behavior of a system to allow small variances near attractors, like stabile fixed points, to converge to similar outcomes is adapted within the natural selection process. Similar enough input is then *not* followed by an identical output, i.e. colimit. There is only the imperative for similar enough output, and that is similar enough that it doesn't imply the organism's death—or the fall of its sexual performance. Before we proceed with a similarity measure, we still stay with identity, i.e. different decompositions admitting the same colimit, and refer to the following definition:[79]

**Definition 26** (**Complex Switch**)**.** Two decompositions $P_2^t$ and $P_3^t$ of an object[80] $\widehat{P}_{2;3}^t$ are said to be *connected* if there is a cluster from $P_2^t$ to $P_3^t$, or a cluster from $P_3^t$ to $P_2^t$, which binds into an isomorphism. If $\widehat{P}_{2;3}^t$ admits at least two decompositions $P_2^t$ and $P_3^t$, which are not connected, we say that $\widehat{P}_{2;3}^t$ is a *multifold object*, and the passage between $P_2^t$ and $P_3^t$ is called a *complex switch*:

$$\begin{tikzcd}
 & \widehat{P}_{2;3}^t & \\
P_3^t \arrow[ur, "C_3^t"] \arrow[rr, Leftrightarrow, "\text{complex switch}"'] & & P_2^t \arrow[ul, "C_2^t"']
\end{tikzcd} \tag{3.3.8}$$

▲

Those decompositions are particularly emphasized:

**Definition 27** (**Homologous**)**.** Two patterns $P_2^t$ and $P_3^t$ admitting colimits $\widehat{P}_2^t$ and $\widehat{P}_3^t$ are said to be *homologous* iff $\widehat{P}_2^t = \widehat{P}_{2;3}^t = \widehat{P}_3^t$. ▲

The interchangeability of the two patterns $P_2^t$ and $P_3^t$ delineates a global property, since the complex switch cannot be observed at the level of the homologous patterns $P_2^t$ and $P_3^t$ in the category of neural systems $\mathcal{N}^t$ but only at the level of the binding. The colimit $\widehat{P}_{2;3}^t$ is said to be a *multifold component* of MENS and its binding homologous patterns an *emergent* property of $\widehat{P}_{2;3}^t$.[81] The redundancy then follows from the random fluctuation at the micro-level:

> *Switching between such non-connected decompositions can be seen as a random fluctuation in the internal organization of $\widehat{P}_{2;3}^t$ which does not modify its functionality on a higher level, where the fluctuation is not observable: different micro-states lead to the same macro-equilibrium.*[82]

It is even "a 'global' property of the *category* and not a 'local' property of the patterns":[83]

[78] Cf. [33], [49].
[79] Cf. [3, 90].
[80] In the notation, $\widehat{P}_{2;3}^t$ says that the colimit posses at least the two decompositions $P_2^t$ and $P_3^t$.
[81] Cf. [4, 136].
[82] Cf. [3, 90], notation adjusted.
[83] Cf. [3, 90], italics included.

**Definition 28** (**Multiplicity**)**.** The category $\mathcal{K}^t$ satisfies the *multiplicity principle* if it admits at least two patterns which are homologous but not connected. ▲

These are the prerequisites for the emergence conjecture that has already been touched upon. Looking at the level of bindings, one couldn't determine which decomposition has realized the colimit, and therefore, reduction has to fail.

Referring to the complex switch, one has to note that figure (3.3.8) is strictly speaking not one diagram but two showing the equivalence of the alternatives in activation. This ambiguity can be considered one of the major merits of MENS, and since it is indeed crucial for further constructions, it will be subject to more in-depth examination.

In MENS, the next step towards the formation of a *psychophysical algebra* is to combine the foregoing definitions, that is to compose complex switches with simple links yielding *complex links*:

**Definition 29** (**Complex Link**)**.** A $(P_4^t, P_1^t)$-complex link is defined as the composite of a path of simple links binding clusters between non-connected patterns, the intermediate objects being multifold, so that the link is not $(P_4^t, P_1^t)$-simple.

$$\begin{tikzcd}
\widehat{P}_4^t \arrow[rr, "g^t_{4,2;3}", "(P_4^t, P_{2;3}^t)-\text{simple}"'] \arrow[rrrr, bend left, "g^t_{4,2;3}g^t_{2;3,1}", "(P_4^t, P_1^t)-\text{complex}"'] & & \widehat{P}_{2;3}^t \arrow[rr, "g^t_{2;3,1}", "(P_{2;3}^t, P_1^t)-\text{simple}"'] & & \widehat{P}_1^t \\
P_4^t \arrow[u, "C_4^t"] \arrow[r, "G^t_{4,3}"'] & P_3^t \arrow[ur, "C_3^t"] \arrow[rr, Leftrightarrow, "\text{complex switch}"'] & & P_2^t \arrow[ul, "C_2^t"'] \arrow[r, "G^t_{2,1}"'] & P_1^t \arrow[u, "C_1^t"']
\end{tikzcd} \tag{3.3.9}$$

▲

It follows that the composite of complex links can only be simple or complex.

Since the simple links are in 1-1 correspondence to their clusters, all *properties* at level $\mathrm{L}_1$, i.e. the category $\mathcal{K}_1^t$, could be reduced to the lower level $\mathrm{L}_0$, i.e. the category $\mathcal{K}_0^t = \mathcal{N}^t$, if no homologous pattern was involved. The complex link would just turn into a simple link being the binding of the compositions of the corresponding clusters, that is diagram (3.3.7) for $P_2^t$. But with the complex switch, the complex link "conveys more 'global' information":

> *[I]t 'emerges' at the level of cat-neurons, but it does not appear 'ex machina', it just actualizes at the higher level a* global *property of the lower level.*[84]

Summing up, the global property of level $\mathrm{L}_1$ consists of the cat-neurons binding homologous patterns of level $\mathrm{L}_0$, which is the principle of multiplicity. This construction set can be applied to any higher level, resulting in a *hierarchy* of higher level cat-neurons.

[84] Cf. [4, 139].

### 3.3.3 The Hierarchy of Categories

So far it's all about colimits admitting patterns that are homologous or not. On this ground, we now want to build a categorical hierarchy, or hierarchical category:[85]

**Definition 30** (**Hierarchical Category**)**.** A *hierarchical category* is a category $\mathcal{K}^t$, the objects of which are partitioned into a finite sequence of levels $0, 1, ..., N$, so that any object $A^t$ of the level $n+1$ is the colimit in $\mathcal{K}^t$ of at least one pattern $P^t$ included in the levels $< n+1$ (*i.e.* each component $N_i^t$ of $P^t$ is of a level lower or the same as $n$). ▲

For the complexification process, the basic objects and morphisms have to be chosen somehow, without loss of generality. Here we start with the level $\mathrm{L}_0$ of neurons and activation links constituting patterns and clusters: $({}^0P_i^t)$. The objects of the neuron level bind into cat-neurons $({}^1\widehat{P}_j^t)$ with their simple and complex links yielding level $\mathrm{L}_1$. In an iterating construction process, they again raise patterns binding, for example, into a cat-neuron ${}^2\widehat{P}^t$ at level $\mathrm{L}_2$:

$$\begin{tikzcd} {}^2\widehat{P}^t \\ ({}^1\widehat{P}_j^t) \arrow[u, "{}^1C^t"'] \\ ({}^0P_i^t) \arrow[u, "({}^0C_i^t)"'] \end{tikzcd} \tag{3.3.10}$$

Since (3.3.10) includes the multiplicity principle, it is not, strictly speaking, a diagram in the category theoretical sense but a class of diagrams. They are called *ramifications*, and the various realizations *micro-components*, i.e. objects and links of lower levels building the highest-level colimits:[86]

**Definition 31** (**Ramification**)**.** We define a *ramification of length $k$* of an object $\widehat{P}^t$ of a category by recurrence as follows:

(i) A ramification of length 1 is a pattern $P^t$ admitting $\widehat{P}^t$ for a colimit (thus, it reduces to a decomposition of $\widehat{P}^t$).

(ii) A ramification of length $k$ of $\widehat{P}^t$ consists of a pattern $P^t$ having $\widehat{P}^t$ for colimit and, for each one of the components $P_i^t$ of $P^t$, of a ramification $R_i^t$ of length $k-1$ of $P_i^t$. In this case, we also say that $\widehat{P}^t$ is a *$k$-iterated colimit* of the ramification $(P^t, (R_i^t))$.

▲

Thus, a ramification $(P^t, (R_i^t))$ of a colimit $\widehat{P}^t$ is a set of families of decompositions sufficient to bind into $\widehat{P}^t$ admitting as many ramifications as multifold realizations can be specified. In (3.3.9) for instance, the possible two ramifications of length 2 for a further

[85] Cf. [3, 95].
[86] Cf. [3, 100].

cat-neuron ${}^2\widehat{P}^t$, being the binding of the cat-neurons at level $\mathrm{L}_1$, would be

$$
\begin{tikzcd}
 & {}^2\widehat{P}^t & \\
{}^1\widehat{P}^t_4 \arrow[ur, "{}^1c^t_4"] \arrow[r, "g_{4,2}", "(P^t_4{,}P^t_2)-\text{simple}"'] & {}^1\widehat{P}^t_2 \arrow[u, "{}^1c^t_2"'] \arrow[r, "g_{2,1}", "(P^t_2{,}P^t_1)-\text{simple}"'] & {}^1\widehat{P}^t_1 \arrow[ul, "{}^1c^t_1"'] \\
{}^0P^t_4 \arrow[u, "{}^0C^t_4"] \arrow[r, "G^t_{4,2}"'] & {}^0P^t_2 \arrow[u, "{}^0C^t_2"'] \arrow[r, "G^t_{2,1}"'] & {}^0P^t_1 \arrow[u, "{}^0C^t_1"']
\end{tikzcd}
\qquad
\begin{tikzcd}
 & {}^2\widehat{P}^t & \\
{}^1\widehat{P}^t_4 \arrow[ur, "{}^1c^t_4"] \arrow[r, "g_{4,3}", "(P^t_4{,}P^t_3)-\text{simple}"'] & {}^1\widehat{P}^t_3 \arrow[u, "{}^1c^t_3"'] \arrow[r, "g_{3,1}", "(P^t_3{,}P^t_1)-\text{simple}"'] & {}^1\widehat{P}^t_1 \arrow[ul, "{}^1c^t_1"'] \\
{}^0P^t_4 \arrow[u, "{}^0C^t_4"] \arrow[r, "G^t_{4,3}"'] & {}^0P^t_3 \arrow[u, "{}^0C^t_3"'] \arrow[r, "G^t_{3,1}"'] & {}^0P^t_1 \arrow[u, "{}^0C^t_1"']
\end{tikzcd}
\tag{3.3.11}
$$

These elaborated concepts yield the toolbox for further considerations concerning the complexification process running into the paradigm of *reductionism*. The question is what are the conditions that levels can be jumped over in a certain ramification? The distinguished links in the decomposition of the colimit have to be simple links binding the clusters between the objects of lowest level, the atoms:

> *If some of the distinguished links of $P^t$ are complex, the reduction fails and $\widehat{P}^t$ really has emergent properties.*[87]

### 3.3.4 Reductionism

It is said that Descartes' conducting from simple to complex "is the stance modern science has adopted through the paradigm of reductionism, which tries to reduce the study of an object to that of its lower level elementary components."[88] In the hierarchical hyperstructure context (chap 2), those elements are subject to an observer *Obs*, or an observer function $\Omega$, that assigns to an aggregation of lower-order structures a *new* structure by interaction due to *new* bonds[89]. We have discussed an artificial life paradigm to see how emergence could show up in the natural evolution of higher-order structures, the micellars for example. The new bonds and properties are to be *registered.*[90] Now, Ehresmann and Vanbremeersch also ask whether organisms are nothing but the constituents: "For example, molecular biology would like to reduce the study of a living organism, or at least of a cell, to the level of its molecular organization."[91]

However, in the category theoretical framework, the observer doesn't explicitly appear, and we will see how the emergent properties nevertheless make an appearance. As regards the bonds, they are already introduced as emergent in the beginning, since they are new, along with the colimit $C^t$, say a cat-neuron in MENS, which represents a pattern $P^t$ in the way that each collective link from $P^t$ to an arbitrary neuron $N^t$ binds into that bond

[87] Cf. [3, 103].
[88] Cf. [3, 102].
[89] Cf. (def 8) in the hyperstructure context.
[90] Cf. (1.2.11).
[91] Cf. [3, 102].

$s^t$ from $C_i^t$ to $N^t$.[92]

A number of obstacles to reductionism can be seen stemming from the representative construction based on colimits, e.g. cat-neurons, that span the hierarchical category of order $N$: ${}^N\mathcal{K}^t$. In MES, the doubts don't arise so much from the question of whether those complex objects do physically *exist*, but how they and their "global" properties can *emerge due to composition*, how the complex objects can be built *bottom up* from level $L_0$, which is the neural network in MENS:

> *Reductionism Problem: can a complex object $\widehat{P}^t$ be reconstructed from atomic components up in only one step, as the colimit of a (perhaps large) pattern included in level $L_0$?*[93]

EV mainly enter two reasons for the possible failure of reductionism.[94] Firstly, a complex object cannot be reduced to lower level objects alone, say the ultimate (atomic) neurons at level $L_0$, for their constraints, the distinguished links, must also be taken into account for their "synergistic action". This situation has been already tackled in the hyperstructure context (chap 2), where Rasmussen et al. in (fig 1.6) deny mere superposition of the first-order interactions. But only being reducible to diagrams rather than solely to the objects, the neurons, shouldn't be an extra barrier. Of course *in praxi*, it is hard to know which links are necessary to have all constraints covered, but the same holds for the objects being linked, i.e. the entire pattern. Thus, the additional synapses are not the problem; rather, the colimit construction of the cat-neuron itself is, and that, not to forget, includes the binding links $c_i^t$ and $c_j^t$ as well, and also the binding link $s^t$ to the neuron $N^t$. Thus in (3.2.2), one had to reduce the diagram of the colimit construction $(C^t, c_i^t, c_j^t, s^t)$ to the diagram of the physical neural network $(N_i^t, N_j^t, N^t, d_{i,j}^t, s_i^t, s_j^t)$. And that is not reducible, or trivially by definition reducible, since the new bonds and the new objects at the higher level $L_1$ and the bonds between the two levels $L_0$ and $L_1$ are merely *postulated* in such a way that the diagram commutes, e.g. $s_j^t = c_j^t s^t$.[95]
Secondly, the alleged existence of homologous patterns disables reductionism at least to the atomic level, and this again is, as we will see, a homemade obstacle involving the mistaken intuition of multiple realization in terms of "knowledge":

> *The knowledge of one of its decompositions $P$ does not determine its other decompositions, and this will have important consequences [...].*[96]

Thus, there is knowledge that cannot be deduced or predicted by watching an actual ramification, and the severe consequences are the failure of reductionism.

However, the above reductionism problem is very well posed if the focus is on epistemic representation. Of course, if one wants to reconstruct a colimit from the atoms at level $L_0$[97], the existence of colimits has to be presupposed, and also the basic patterns at that level being sufficiently interconnected to yield the highest-level colimit. The reduction

[92] Cf. (3.2.2).
[93] Cf. [3, 103].
[94] Cf. [3, 102,103].
[95] The real problems start to surface when the colimit object is not seen to be a mere model for cognition but to exist, for example, as a mental object, say the *experiencing* of some cognition.
[96] Cf. [3, 103].
[97] which in MENS are the biological neurons

theorem then states which inter-level conditions have to be met such that reduction succeeds:[98]

**Theorem 1** (**Reduction Theorem**). *Let ${}^2\widehat{P}^t$ be the 2-iterated colimit of a ramification $({}^1P^t, ({}^0P^t_i))$. If each distinguished link $d^t$ from ${}^1\widehat{P}^t_i$ to ${}^1\widehat{P}^t_j$ in ${}^1P^t$ is $({}^1\widehat{P}^t_i, {}^1\widehat{P}^t_j)$-simple, then ${}^2\widehat{P}^t$ is also the colimit of the following pattern ${}^0R^t$: its components are those of the various ${}^0P^t_i$ and its distinguished links are those of the patterns ${}^0P^t_i$, as well as the links of the clusters $G^t_d$ that the distinguished links $d^t$ of ${}^1P^t$ bind. Moreover, ${}^0R^t$ and ${}^1P^t$ are connected by the cluster generated by the binding links of the various ${}^0_kP^t_i$ to ${}^1\widehat{P}^t_i$. On the other hand, if certain distinguished links of ${}^1P^t$ are complex links, ${}^2\widehat{P}^t$ may not have any such decomposition.*

In the following example, the $L_1$ diagram can be jumped over, meaning that the $L_2$ colimit reduces to the micro-components and their distinguished links at $L_0$ (right diagram):

$$
\begin{tikzcd}
 & & & {}^2\widehat{P}^t & & & \\
 & {}^1\widehat{P}^t_i \arrow[urr, "{}^1c^t_i"] \arrow[rrrr, "{}^1d^t_{i,j}"] & & & & {}^1\widehat{P}^t_j \arrow[ull, "{}^1c^t_j"'] & \\
{}^0_kP^t_i \arrow[ur, "{}^0c^t_{k,i}"] \arrow[rr, "{}^0d^t_{k,l}"] \arrow[rrrr, bend right, "{}^0g^t_{k,m}"] \arrow[rrrrrr, bend right=40, "{}^0g^t_{k,n}"'] & & {}^0_lP^t_i \arrow[ul, "{}^0c^t_{l,i}"'] \arrow[rr, bend left, "{}^0g^t_{l,m}"] \arrow[rrrr, bend left=40, "{}^0g^t_{l,n}"] & & {}^0_mP^t_j \arrow[ur, "{}^0c^t_{m,j}"] \arrow[rr, "{}^0d^t_{m,n}"] & & {}^0_nP^t_j \arrow[ul, "{}^0c^t_{n,j}"']
\end{tikzcd}
\qquad
\begin{tikzcd}
 & & & {}^2\widehat{P}^t & & & \\
{}^0_kP^t_i \arrow[urrr, "{}^0c^t_k"] \arrow[rr, "{}^0d^t_{k,l}"] \arrow[rrrr, bend right, "{}^0g^t_{k,m}"] \arrow[rrrrrr, bend right=40, "{}^0g^t_{k,n}"'] & & {}^0_lP^t_i \arrow[ur, "{}^0c^t_l"] \arrow[rr, bend left, "{}^0g^t_{l,m}"] \arrow[rrrr, bend left=40, "{}^0g^t_{l,n}"] & & {}^0_mP^t_j \arrow[ul, "{}^0c^t_m"'] \arrow[rr, "{}^0d^t_{m,n}"] & & {}^0_nP^t_j \arrow[ulll, "{}^0c^t_n"']
\end{tikzcd}
\tag{3.3.12}
$$

With the reduction theorem, an intuitively and rigorously formulated criterion of an object's complexity is now available. If it is not the case that all levels of the hierarchy have to be taken into account to yield the same colimit, the complexity of a higher level object ${}^{n+1}\widehat{P}$ isn't really represented by $n$ but can be reduced in the sense above. To approach this issue adequately, some measure for *complexity* has to be defined:[99]

**Definition 32** (**Complexity order**). The *complexity order* of an object ${}^{n+1}\widehat{P}^t$ of level $n+1$ is defined as the smallest $m$ such that there exists a pattern ${}^mP^t$ whose colimit is ${}^{n+1}\widehat{P}^t$ and which is included in the levels $\leq m$. And ${}^{n+1}\widehat{P}^t$ is said to be $q$-reducible for any $q$ equal to or higher than its order.[100] ▲

[98] Cf. [3, 104].
[99] Cf. [3, 105].
[100] Cf. Baas: higher-order structure with the complexity order of a colimit.

This suggests a definitional follow-up:[101]

**Definition 33 (*m*-simple Link).** A link is *m-simple* iff it binds a cluster between patterns included in the levels $\leq m$. ▲

With the links between the objects of the levels $\leq m$ being $m$-simple, the following theorem follows:[102]

**Theorem 2 (Reducibility).** *An object* ${}^{n+1}\widehat{P}^t$ *of the level* $n+1$ *can be* $(n-1)$*-reducible if it admits a decomposition* ${}^{n}P^t$ *of which all the distinguished links are* $(n-1)$*-simple. If not,* ${}^{n+1}\widehat{P}^t$ *is generally not* $(n-1)$*-reducible. This result extends to lower levels, allowing a reduction to these lower levels.*

This program for checking for reducibility focuses on simple links, i.e. links that bind clusters between patterns. If distinguished links between their bindings, i.e. the colimits, do not bind clusters, they are not simple but complex. Simple links cannot bind clusters if they don't exist, which is the case if the patterns are non-connected. According to theorem (2), if a distinguished link between colimits is not simple, it cannot be reduced. Thus, reductionism or reducibility is a matter of homologous patterns existing. If the patterns are not disjunctive, i.e. at least one object along with others admits more than one colimit, again by definition no cluster is possible. These two cases are taken into account in the following definition:[103]

**Definition 34 (*n*-Multifold).** We say that a hierarchical category *satisfies the multiplicity principle* if, for each $n$:

(i) there are objects of level $n+1$ which are $n$*-multifold* in the sense that they are the colimit of at least two non-connected patterns included in levels less than or equal to $n$.

(ii) an object of level $n$ can belong to several patterns having different colimits at the level $n+1$.

▲

## 3.4 The Emergence Problem

Thus, the „mechanism" of emergentism is the complexification process of binding objects, and for reducibility to hold, the complexity has to be of order 0.[104] And that means all links are 0-simple. Then with multiple patterns being involved, which is said to be ubiquitous at least for living matter, the evolutionary emergence process is defined:[105]

**Theorem 3 (Emergence Process).** *The root of emergence is the existence of multifold objects (modeled by the multiplicity principle). If it is satisfied, the emerging objects and links are explicitly constructed through sequences of complexification processes.*

[101] Cf. [3, 106].
[102] Cf. [3, 106].
[103] Cf. [3, 108].
[104] Cf. (def 32).
[105] Cf. [3, 139].

Summing up and setting our stance for emergentism, our claim is that this theorem, which is more of a definition, does not hold for three reasons: (i) the category is not valid, (ii) cat-neurons as colimits do not physically exist, and (iii) homologous patterns do not physically exist. So far, there is nothing to be ontologically reduced in nature—it is all about *epistemic* emergence and *functional roles* providing a highly sophisticated and elaborated model for complexification and cognition.

The view that a cat-neuron could admit multiple decompositions, i.e. homologous patterns of (cat-)neurons, will be called homo-emergence, or *homergence* for short:

**Definition 35** (**Homergence**). *Homergence* is the assumption that a colimit ${}^{q}\widehat{N}$ at level $\mathrm{L}_q$ is the binding of *homologous* objects or patterns at level $\mathrm{L}_{q-1}$. ▲

In contrast to homergence, we will opt for *co-emergence*, or *comergence*, as the interesting part:

**Definition 36** (**Comergence**). *Comergence* is the assumption that a colimit exists. ▲

Since homergence is set as the key precondition for Memory Evolutive Systems to evolve, in chapter (chap 4) the following have to be inspected carefully: first, category theoretical consistency and, second, its plausibility concerning its correspondence to existents and phenomena in nature. Thus, it will be all about the question: Is it *thinkable* to find some kind of physical composition that is in correspondence with the category theoretical counterpart.

### Degeneracy and Redundancy

The complexification processes, in biological systems for example, satisfying the multiplicity principle is seen as a problem "of an epistemological nature".[106] We now have to discuss whether or not this is true, i.e. whether there are not additional ontological claims being made that are not warrantable. And we have to be cautious not to be inappropriately strict in rendering multifold patterns problematic. We talk about equivalent but non-identical structures in biology as a paradigm for functional roles systems to be built on, with 'equivalent' meaning only that, i.e. performing the same function.

Multiplicity, in biology better known as *degeneracy*, is said to be ubiquitous in natural selection. "[D]egeneracy is not a property simply selected by evolution, but rather is a prerequisite for and an inescapable product of the process of natural selection itself."[107] Degeneracy is not to be equated with functional redundancy, since two types can be differentiated, which perform a function being specified by *identity* and *similarity*:

> *Degeneracy is the ability of elements that are structurally different to perform the same function or yield the same output. Unlike redundancy, which occurs when the same function is performed by* identical *elements, degeneracy, which involves* structurally different *elements, may yield the same or different functions depending on the context in which it is expressed.*[108]

[106] Cf. [3, 109].
[107] Cf. [49, 13763].
[108] Cf. [49, 13763].

The criterion is always behavior, the output of some behavior, the conceived function of some behavior to be fulfilled finally resulting in motion. It is crucial to notice that those "identical elements" are of course not identical in existence but fairly identical in structure, e.g. two molecules being indistinguishable (from a distance) while the "structurally different elements" also appear not alike from afar.

This same is said to hold for the function of neural networks: "[...] many different patterns of neural architecture are functionally equivalent, i.e., functional neuroanatomy is highly degenerate", and "[t]he pattern of connectivity [...] depends on the presence of a large number of different, alternative reentrant circuits that dynamically yield a *similar* output, i.e., such circuits are degenerate."[109] Thus, it's not as much about function as behavior, e.g. motor neuron activation resulting in muscular contractions sufficiently similar to be effective for what the process is or will be selected.

### Mental Object

To sum up what has been achieved so far in MENS, the iterating construction process of hierarchical categories, or their objects, the cat-neurons, is considered to "generate an expanding algebra of mental objects."[110] It is built on the ($q$-)reducibility of representations of perceptual or cognitive processes. And the fact that a mental object is not generally 0-reducible, meaning reducible down to the neuron level, is adopted due only to the assumption that the multiplicity principle holds, which again is considered the "source of emergence".[111]

The essence of the analysis so far is that no clear line between the epistemological and the ontological has been drawn. Thus, before one can venture on to the mental object, one should first be clear about the basic building blocks and their interrelations. In MENS, the mental objects, that are complex cat-neurons, are seen both to comprise "formal units"[112] as some sort of "representation" and to be physically activatable "through the unfolding of one of their hyper-assemblies down to the neuronal level".[113]

In the next chapter, the notion of 'mental objects' will be maintained, which describes it fairly precisely as what it then is treated as: point-like. The non-local "holistic new property"[114], that emerges at levels defined by emerging cat-neurons, will turn, in the ontological view, into a local property itself, or *quality particular* as we call it, and as a result, we refrain from thinking in properties at all. Therefore, of course, we have to build a new ontology that will be grounded on, of course, cognition, since that is what we do: pointer-wise cognizing which is pointing at the existent. This „content-driven“ stating the existent thus becomes, as a proposition, the existent, i.e. that which is all that is necessary to propose. With the transition from MENS to DUCKs, we will have finally unified the epistemological and the ontological leaving "emergentist reductionism"[115], respectively "emergentist monism"[116] behind, only stating the dependencies.

[109] Cf. [49, 13765,13766], italics added.
[110] Cf. [3, 145].
[111] Cf. [3, 145].
[112] Cf. [3, 287].
[113] Cf. [3, 349].
[114] Cf. [3, 138].
[115] Cf. [3, 138].
[116] Cf. [3, 302].

# 4 Domain Unifying Categorical Kinds

What are the categorical kinds that in some way unify which domains? This is a question that should be answered by doing, meaning by unifying the domains by using categorical kinds—*in principle*. The objective here is not to detect the mechanisms of or behind certain behaviors but to give them an ontological grounding.

MES is concerned with those mechanisms, and it is useful in some way but inconsistent. The question is, could it be made consistent without loosing its value? And again, what is it good for if it is inconsistent? There will be concepts that won't be so easily made accessible with DUCKs. And that is one reason why the following work will focus on a more fundamental basis, which is more minimally ontological, i.e. consistent and coherent with *fundamental* theories of nature.

In this chapter the objective is to elaborate a minimalist ontology—DUCKs—which can be compared and contrasted with a physical theory on primitives that can be considered sufficiently serious *and* fundamental. This theory will be shown to be Bohmian mechanics, and we will see whether something is missing that would prevent encompassing all *phenomena*.

## 4.1 The Minimalist Ontology

What is a minimalist ontology? An ontology should be *the* ontology, since that which is existent shouldn't depend on any ontology other than itself. At the very least, the ontologies shouldn't interfere with each other as long as they are about the existent. And that is the salient point: is it possible to *think* the existent? And with that, the case is brought to its point: thinking the albeit minimalist ontology is *epistemic* and thus obviously depends on the cognizer stating *his* ontology.

The question then would be, are there at least invariances in cognizing the existent, and if so, do they correspond to invariances in nature, i.e. are there objective correlates? But this seems to be an inadmissible question, since cognizing is natural, i.e. within nature. Thus, there must be some natural cognizing the natural, and hence, it is to *define* and *describe* the process of (natural) cognizing.

An often profitable procedure for tracking down the unknown is calling for *necessary* and *sufficient* conditions. A necessary condition for all observations or cognitions is, as shown in (chap 2), the process of *composing* interrelating parts into emergent wholes due to some *criterion*. Thus, pattern formation, e.g. the evolution of hyperstructures, suggests that there are indeed by definition parts that form observed patterns. It seems to be true that there isn't just one „thing“ but several, and that they are localized in space and time. Whatever those constituents constitute, they are at least distinguishables, in the minimalist view non-universal, non-instantiable „properties“, or rather *quality particulars*, since they are not seen to be properties of something but properties in their own right. We refer to Lewis' view on Humean supervenience but without endorsing Humean supervenience, just challenging his word: "[T]he world is a vast mosaic of local matters of particular fact [...] And at those points we have local qualities [...]"[1]

In metaphysics, the combination of 'qualities' and 'particulars' is not well received. The former count as abstract individuals, i.e. qualities to be instantiated, while the latter is a concrete individual being spatially and temporally located. But a quality particular is meant to be exactly that: (i) some individual quality having, i.e. being at, a spacetime position, and (ii) something that is not just distinguishable by its location. In short, universals are not seen to require discussion here, although they are intrinsically given due to the alleged quality particulars' interdependency regarding change.

In the following, these entities, which will compose and be decomposed to, are more neutrally called *objects*, still playing a hunch for a looming depuration of some embosomed heirlooms of traditional intuitional thinking.

## The Objects

**Definition 37** (Object)**.** An *object* ${}_jO^t_{x(j,t)}$ at an instance of time $t$ and a spatial location $x(j,t)$ is a fundamental quality particular.[2] ▲

So far, there is no need to introduce more classes to sort the objects into, although, as it will turn out, we will have to admit at least a dualism of quality particulars.[3] Furthermore, in the ontological minimalist view, the observer should not mistakenly bind these real objects and arrive at the result of having them „composed“ to real, so-called *ordinary objects*[4]. These compounds or hyperstructures or colimits, that are actually concepts, appear only because of attributed properties by the linguistic usage of 'having them'. But as we will see soon, the retention of the thinking of ordinary objects possessing properties is a major fallacy in the transition from the epistemic to the ontological—*to be* and not *to have* makes the turn towards the realm of the existent.

'Having' signifies that the ordinary object $\widetilde{O}$ serves as some kind of *carrier* for properties: $\widetilde{O}^t_j = \widetilde{O}^t_j({}_{j_1}o^t, ..., {}_{j_n}o^t, x^t_j)$ with the properties ${}_{j_i}o^t$ having no location in space but the object $\widetilde{O}^t_j$, namely $x^t_j$. Instead, in the case of 'being', the ordinary object would be identified with a set: $\bar{O}^t_j = \{{}_{j_1}O^t_{x(j_1,t)}, ..., {}_{j_n}O^t_{x(j_n,t)}\}$ with the quality particulars ${}_{j_i}O^t_{x(j_i,t)}$ being located in space and time. But we have to be cautious for these second case ordinary objects $\bar{O}^t_j$ are also not real, but the ensembles are just necessary conditions for the observer to classify. Defining $\bar{O}^t_j$ is nonetheless meant to be a supporting intermediate step on the way to the diagrammatic formulation of the observational process we are striving for. Besides the objects, it is also crucial to take their interrelations into account, moving away from *elements* towards *diagrams*.

**Definition 38** (Ordinary Object)**.** An *ordinary object* $\bar{O}^t_j$ at an instance of time $t$ is an assemblage $\bar{O}^t_j = \{{}_{j_1}O^t_{x(j_1,t)}, ..., {}_{j_i}O^t_{x(j_i,t)}, ..., {}_{j_n}O^t_{x(j_n,t)}\}$ of objects ${}_{j_i}O^t_{x(j_i,t)}$ being compiled by some observer to be classified as a whole due to some criterion. ▲

[1] Cf. [25, x] and (chap 4.1.2).

[2] For the sake of simplicity in the notation, the spacial location of the object could be replaced by its indexical signature $j$: $O^t_j := {}_jO^t_{x(j,t)}$.

[3] Cf. (con 8).

[4] Cf. [48] and [2].

This preliminary definition of ordinary objects is of course too vague to be operational, because the vital components 'observer', 'whole', and 'criterion' lack clarification. Actually, in the following, one of the objectives is to answer the question of how to treat these definitional determinants. At this stage, we still rely on so-called folk intuitions, and certainly, we will do some work to find more rigorous definitions, or to resist them.

The idea of detecting properties and considering them as belonging together *without* taking an additional carrier into account is not so strange; rather, it is another powerful intuition. For example, the positron, the antiparticle of the electron that was first detected in a cloud chamber[5] by Carl Anderson in 1933, is, in the traditional view, a point-like particle *having* properties that are located at the very same spatio-temporal position. In the experiment, the "positron's mass and charge" are deduced from the curvature of the trajectory in the chamber filled with supersaturated vapor. However, with definition (def 38), the positron, an ordinary object, would e.g. *be* the mass and the charge, $\bar{O}_j^t = [\beta^+]_j^t = \{_{j_1}O^t_{x(j_1,t)}, {}_{j_2}O^t_{x(j_2,t)}\} = \{_j m^t_{x(j,t)}, {}_j q^t_{x(j,t)}\}$ with $[\beta^+]_j^t$ just being the designator for the particular set.

A water molecule $[H_2O]_j^t$ could serve as an example for a *non point-like* ordinary object being a spatially distributed assemblage of objects.[6] If one was to fall just short of arguing about identity with the positron, one now probably would abstain from denying that such a molecule is not identical with the objects: for example, in addition to the constituents, it is a polar molecule with an electrical dipole moment qualifying for intermolecular hydrogen bonds. But that is a misconception—dipoles and hydrogen bonds are just theoretical terms of a certain domain of description following the observer's criterion for classifying near-enough invariances of the dynamics, i.e. time transition rules for predicting the object's future behavior.

Obviously, it could turn out in an experiment that an object $_{j_i}O^t_{x(j_i,t)}$ shows itself not to be fundamental but rather an assemblage of objects. But that is unproblematic, since in the conception of ordinary objects, zooming in is not only explicitly allowed but called for or just necessary to go further if the scope of application of a theory is left, especially with regard to the objects' bonds, i.e. the underlying criterion for their gathering to ordinary objects. Losing their bonds and changing over to other ordinary objects will be key in the description of the dynamics of objects, and the way they lose their bonds will give some indication of the ordinary objects' importance for the minimalist ontology.

And of course we go on to ask: could there still be more than the objects themselves, some kind of *natural law* that „externally“ interrelates the objects? The interrelations could also be „intrinsically“ given by the objects, and the whole at some instance of time $t$ would then solely consist of the objects at $t$ with their spacial arrangement. Trying to define the whole then, it could be the objects, the quality particulars, that are the building blocks of the universe, and it will be interesting to see whether and, if so, to what extent the observer jibs at his integration into the ordinary. To venture a guess, it won't be the observing but the *experiencing* of the observation or registration that will be troublesome. But we will see.[7]

[5] Cf. [13].

[6] Cf. the computer simulation in (chap 2.2.1).

[7] Later on, we will take the experience attending the observation—or the cognition, as the observational process will be also called—for existent, more precisely also for some kind of quality particular,

In the following, we will be concerned with the existents, which are for now the quality particulars at an instance of time $t$, and define:

**Definition 39** (Universe-instance)**.** A *universe-instance* at an instance of time $t$ is the set of all objects $\mathbb{U}^t = \{{}_jO^t_{x(j,t)} \mid j \in \mathbb{I}\}$, $\mathbb{I} \subset \mathbb{N}$ being an index set. ▲

And assuming there is change, the whole evolving in time, i.e. there is a subsequent whole that replaces the former, we consolidate the instances:

**Definition 40** (Universe)**.** The *universe* is the set of all universe-instances $\mathbb{U} = \{\mathbb{U}^t \mid t \in \mathbb{T}\}$, $\mathbb{T} \subset \mathbb{N}$ being an index set, with a strict total order $<^t$ defining the direction of time $t$: $(\mathbb{U}, <^t)$. ▲

So far, these are just definitions, and there is no claim being made that our universe is so. However, in (chap 4), an experimental metaphysics will be presented to evaluate whether there is some connection to reality. Moreover, the definition (def 39) simply says that there are objects ${}_jO^t_{x(j,t)}$ around at time $t$, and with (def 40) at $t'$ with $t < t'$ there again are objects ${}_jO^{t'}_{x(j,t')}$ around, i.e. a *temporal concatenation of conjunctions*.

We are now interested in how one would introduce regularity, law-like behavior, to the quality particulars.

### The Interrelations

Having declared the objects, which altogether constitute the universe, the remainder—if there is any—hinges on the question regarding the *conditions* for their *existence* and *change*. In the following, we are not interested in „interactions“, which were substantial in the context of hyperstructures[8] and Memory Evolutive Systems[9], but rather *conditionalized interrelations*. What are the conditions for an object ${}_jO^t_{x(j,t)}$ to exist, and if it changes, what are the conditions for the quality particular to change? The first that can be thought of is that those conditions be *necessary* or *not necessary*. The existence of ${}_jO^t_{x(j,t)}$ to be necessary, trivially means that it is not possible for ${}_jO^t_{x(j,t)}$ at the instance of time $t$ not to exist. And what should prevent ${}_jO^t_{x(j,t)}$ from existing? Obviously, that it was necessitated, i.e. it is necessary, not to exist.

In this *deterministic* setting, all objects had been put in place at some instance of time $t^*$, that is at $t^*$ every object is a necessary condition for the universe-instance to be $\mathbb{U}^{t^*}$. Thus, for instance, for the object $O_j^{t^*}$ to exist, it is necessary for the object $O_i^{t^*}$ to exist:

$$\begin{tikzcd} O_i^{t^*} \arrow[d, "\Delta_{i,j}^{t^*}"] \\ O_j^{t^*} \end{tikzcd} \tag{4.1.1}$$

and see if we can get anywhere with it.

[8]Cf. (chap 2).

[9]Cf. (chap 3).

with $\Delta_{i,j}^{t^*}$ denoting 'the existence of the object $O_i^{t^*}$ is necessary for the existence of the object $O_j^{t^*}$'. This is trivially true, since every object existing at $t^*$ is necessary for every object to exist at $t^*$. And whatever happened before $\mathbb{U}^{t^*}$ cannot be worrisome, especially because, for our purposes, it is dispensable to hypothesize about a very first beginning or whether or not some different universe-instance, say $\tilde{\mathbb{U}}^{t^*} \in \tilde{\mathbb{U}}$, could have been „instantiated" instead. Given $\mathbb{U}^{t^*}$ with an arbitrary $t^*$ as initiation, it follows that the existence of $O_i^{t^*}$ and $O_j^{t^*}$ are mutually ontologically necessary for their existence, with $\Delta_{i,j}^{t^*}$ being a symmetric relation, and that is true for all $i, j \in \mathbb{I}$.[10]

And that should be true for any instance of time $t$ with $t^* < t$:

$$\begin{tikzcd} O_i^t \arrow[d, "\Delta_{i,j}^t"] \\ O_j^t \end{tikzcd} \tag{4.1.2}$$

Thus, *given* $\mathbb{U}^t$, all objects are again mutually ontologically necessary for their existence.

One can gather a first good impression of what lies ahead. Does the above interrelation $\Delta_{i,j}^{t^*}$ come within the limits of *ontology* or *epistemology*? Given $\mathbb{U}^{t^*}$, is the fact '$O_i^{t^*}$ to exist is necessary for $O_j^{t^*}$ to exist' meant to be ontologically or epistemically true? One should reformulate to 'knowing $O_i^{t^*}$ to exist is sufficient for knowing $O_j^{t^*}$ to exist' and ask again: what could ontologically be sufficient for the mere existence of the objects without any observer stating the sufficiency?[11]

And the same holds for '$O_i^{t^*}$ to exist is necessary for $O_j^{t^*}$ to exist'. It is again obvious that, given $\mathbb{U}^{t^*}$, if $O_i^{t^*}$ didn't exist, the object $O_j^{t^*}$ also wouldn't exist. It is just that, at an instance of time $t^*$, all existing quality particulars exist, and that should be true without an observer paying attention.

This effort is made, because later on, the existence of an object will be necessary for another object to exist in a certain unique way, and these are the preparations for a follow-up definition of *necessarily executed conditionalizing*.

We now give the necessitation a definition and shift a tighter discussion to (chap 4), where we hope to gain some insight into $\mathbb{U}^{t^*}$, still without knowing the objects:

**Definition 41** (Intradet)**.** The *intra-time determination (intradet)*

$$\Delta_{i,j}^t : {}_iO_{x(i,t)}^t \to {}_jO_{x(j,t)}^t \tag{4.1.3}$$

[10] For an observer, the object $O_i^{t^*}$ would also be epistemically *sufficient* for $O_j^{t^*}$ to exist.

[11] In (chap 4.2), we will see that there is good reason to state that there is no sufficiency for objects at all but the whole universe. If we know $O_i^{t^*}$ we certainly know $O_j^{t^*}$, and this seems to be possible with only both objects existing at $t^*$. But of course in reality, there is no way to know $O_i^{t^*}$, thus to know $O_i^{t^*}$ *would* be sufficient to know $O_j^{t^*}$; $\mathbb{U}^{t^*}$ is nevertheless, without further specification, ontologically true.

states the existence of the object $O_i^t$ with position $x(i,t)$ being necessary for the existence of the object $O_j^t$ with position $x(j,t)$.[12] ▲

Now an actual and undeniable existence of $\mathbb{U}^t$ is admittedly of poor potency concerning considerations of relations within the universe-instance. They are logical rather than metaphysical, and not at all nomological. Finding a more sophisticated *structure* in a universe-instance will be approached next. And a very crucial question will be, can there be structure just taking one instance of time into account? The answer will be given by the definition of structure itself.

A first step to looking at sequences of universe-instances is to determine the conditions for these to follow from prior ones. The *transition* between universe-instances would then state the existence of an object $O_i^t \in \mathbb{U}^t$ being necessary for the existence of an object $O_j^{t'} \in \mathbb{U}^{t'}$ with $t < t'$:

**Definition 42** (Interdet)**.** The *inter-time determination (interdet)*

$$\Delta_{i,j}^{t,t'} : {}_iO_{x(i,t)}^t \to {}_jO_{x(j,t')}^{t'} \tag{4.1.4}$$

states the existence of the object $O_i^t$ with position $x(i,t)$ being necessary for the existence of the object $O_j^{t'}$ with position $x(j,t')$. ▲

And again, it holds for any two objects $O_i^t \in \mathbb{U}^t$ and $O_j^{t'} \in \mathbb{U}^{t'}$ that they are mutually necessary to exist. In the case of the objects living in the same (actual) universe-instance where there is no change, mutual necessity is unproblematic, even somehow trivial at first sight, as mentioned. Here, we have to be cautious stating conditions of existence, since in (def 42) at most one universe-instance can be real. Be $\mathbb{U}^{t_0}$ the actual universe-instance at $t = t_0$ with objects that are being existing just *now*. Then $O_i^{t_0}$ with $t_0 < t'$ cannot be ontologically necessary for $O_j^{t'}$ to exist, for the latter is a future object and therefore doesn't exist yet. 'The object $O_i^{t_0}$ exists' is an *ontological* statement, and 'the object $O_i^{t_0}$ is necessary for some later instance of time $t'$ for ...' is just an *epistemic*, though true, in our reconstruction. Thus, if one only drew the existent, the picture would be

$$O_i^{t_0} \tag{4.1.5}$$

and if one presumed it an actual precursor for say $O_j^{t'}$

$$\begin{tikzcd} O_i^{t_0} \arrow[d, "\Delta_{i,j}^{t_0}"] \\ O_j^{t_0} \end{tikzcd} \tag{4.1.6}$$

[12] It's not that the emphasis is put on the object *at* the position but on the object together *with* the position, which is the object *and* the position.

which is quite similar to (4.1.2). And mounting $O_j^{t'}$ again would result in the *diagram*

$$\begin{tikzcd}
O_i^{t_0} \arrow[d, "\Delta^{t_0}_{i,j}"'] \arrow[dr, dotted, "\Delta^{t_0,t'}_{i,j}"] & \\
O_j^{t_0} \arrow[r, dotted, "\Delta^{t_0,t'}_{j,j}"'] & O_j^{t'}
\end{tikzcd} \tag{4.1.7}$$

where the dashed arrows signify the non-existent part of the triangle, including $O_j^{t'}$.

Now what is gained by integrating a precursor, or a successor, taking only all at $t_0$ to exist? The picture is reminiscent of a *commutative* diagram, and the transition $\Delta^{t_0,t'}_{i,j}$ can be seen as being *factorized* by the „existent“[13] part $\Delta^{t_0}_{i,j}$ and the future resultant $\Delta^{t_0,t'}_{j,j}$:

$$\Delta^{t_0,t'}_{i,j} = \Delta^{t_0}_{i,j}\Delta^{t_0,t'}_{j,j} \tag{4.1.8}$$

That kind of *composing* arrow will be key for further considerations, and also one reason for changing from the set-theoretic to the *category-theoretic* description language. But does it make sense here just having those intradet and interdet on hand? The truth value of 'the existence of $O_i^{t_0}$ at $t_0$ is necessary for $O_j^{t'}$ at $t'$ to exist' is the same as 'the existence of $O_i^{t_0}$ at $t_0$ is necessary for $O_j^{t_0}$ at $t_0$ to exist' „followed by“ 'the existence of $O_j^{t_0}$ is at $t_0$ necessary for $O_j^{t'}$ at $t'$ to exist'. But what could that ominous 'followed by' stand for other than mere conjunction with temporal order, there being a logical prior, which is in accordance with the arrows' directions? With the direction of time, we would gain temporal logics:

$$\Delta^{t_0,t'}_{i,j} = \Delta^{t_0}_{i,j} \wedge^{<} \Delta^{t_0,t'}_{j,j} \tag{4.1.9}$$

with '$\wedge^{<}$' meaning simply 'and then' ($t_0 < t'$).[14] In (chap 4), we will discuss whether there can be more logics than just conjunctive concatenation in nature using category theoretical „concepts“, which is actually what this language has been chosen for.

So far, there is no *law* involved besides basic *determination due to necessity by existence*, and we have to ask whether we can think of some refinement of the arrows that describes necessity conditions, including some kind of connectivity and continuity between certain objects resulting in change? We can achieve that by introducing a second sort of object travelling through space and time that is, alongside the introduced existents, a further *condition for change.*

[13] It will become explicit; those arrows don't exist in the same manner as one would regard the corresponding objects to exist.

[14] There could be two sorts of conditionalizing existence at a new position here: time-delayed and non time-delayed.

### 4.1.1 Condition for Change

Let us first consider an easy change, some lonely object at time $t$ about to move in space:

$$ {}_jO^t_{x(j,t)} \xrightarrow{\Delta^{t,t'}_{j,j}} {}_jO^{t'}_{x(j,t')} \tag{4.1.10} $$

i.e. the object ${}_jO^t_{x(j,t)}$ at temporal location $t$ and spacial location $x(j,t)$ existing being necessary for the object ${}_jO^{t'}_{x(j,t')}$ at temporal location $t'$ and spacial location $x(j,t')$ to exist. So far it's mere *existence* that is the necessary condition for a later object to exist. But there is also *change*, and what has been changed, or rather what has changed, is at least the spatio-temporal coordinates of the object ${}_jO^t$: $x(j,t) \neq x(j,t')$. And we must ask for the condition that must be met for the spacial position to change. There must be some *condition for change* at time $t$ that is *in addition* to the existence of the quality particular necessary for the object's new position $x(j,t')$, a condition that is to be postulated for the apparent change. And that condition for change seems to be tightly connected to the quality particular ${}_jO^t_{x(j,t)}$, saying it has the same position $x(j,t)$. Thus, there must be some spatially located condition for change that happens to travel along with the object or just to be at its position at a minimum of one instance of time.

And obviously, there must be a condition for the condition for change; in particular, there are conditions for change that eventually change. But since they can be integrated into the existences and positions of the conditions for change and objects, there is no need for higher resolution here. Actually, it could be that the quality just consists of providing for the conditions for change to move.

Thus, the ongoing considerations will be about objects of course, but especially about necessary conditions for existence and (their) change, and the objective is now to find a criterion to differentiate between *ontological* necessary conditions (for existence) and *nomological* necessary conditions (for change).

#### The Minimal Epistemic

Conditions here are not put to use as a handy academic tool in the struggle to define notions in as precise a way as possible. Rather, they are at the core of metaphysical considerations concerning necessity of existence and change. Little can be said about necessary conditions. Those are, or more precisely, this is a fundamental concept, or rather a fundamental that is hard to couch in other terms. If one wishes to eliminate the notions 'necessary' and 'condition' in 'necessary condition', one seems to be forced to fall back on counterfactual thinking, which is very inconvenient, since in the deterministic setting, everything is factual, and a non-factual to be followed by another non-factual is intuitionally not more enlightening than stating the state of affairs directly. But in order to get 'necessary conditions' on the right path, we permit possible-worlds thinking as auxiliary, not without reserving that they not be possible.

First, since all that can be done is making statements, we consider the relata to be *statements* $A_i$ with value to be true and then to be statements about *existents* with value to be truly existent. The result is that these meditations may appear a little convoluted. One might think why not start straightaway with existent objects, but one also might wish to get the transition between the epistemic and ontological, assuming there is any here, to refer only to necessary conditions that will in addition necessarily be met, i.e. *necessarily necessary conditions.*

**Definition 43** (Necessarily Necessary Condition)**.** $A_i$ being true is a *necessarily necessary condition (NNC)* for $A_j$ to be true iff $A_j$ being true is necessarily a sufficient condition for $A_i$ to be true, *and* $A_j$ is necessarily true:

$$A_i \xrightarrow{\text{NNC}} A_j \;:\Leftrightarrow\; \Box(A_j \Rightarrow A_i) \wedge \Box A_j \tag{4.1.11}$$

▲

Thus, here we restrict ourselves to propositions that are necessarily true, i.e. it is not possible that they are not true. This restriction is due to considerations about experimental metaphysics we contemplate in (chap 4). For the sake of convenience, we immediately skip the modal operators again but bear in mind that we have an attitude towards contingency, defining:

**Definition 44** (Necessary Condition)**.** $A_i$ being *true* is a *necessary condition (NC)* for $A_j$ to be true iff $A_j$ being true is a sufficient condition for $A_i$ to be true, *and* $A_j$ is true:

$$A_i \xrightarrow{\text{NC}} A_j \;:\Leftrightarrow\; (A_j \Rightarrow A_i) \wedge A_j \tag{4.1.12}$$

▲

There are still three remarks to be made: firstly, a conditioned proposition being true, here $A_j$, is certainly sufficient for the conditioning, here $A_i$, to be true and thus, strictly speaking, there is no need to apply the material conditional. One could just state instead: '$A_i$ is a necessary condition for $A_j$, and $A_j$ is true', or just '$A_i$ necessitates $A_j$, and $A_i$ is true' following our default that all propositions are made to be true. With that, we can still shorten the definition to '$A_i$ necessitates $A_j$' and just make a notation of it. Secondly, one might be misled to think that the arrow is the necessary condition in (4.1.12), in the sense of being necessary to *make* the following be true, but 'to be necessary for ... to be true' doesn't contain any activity. It is just about statements, and it is a statement of $A_i$ being true *and* $A_i$ being true to be a NC, or more explicitly, a NC for $A_j$ to be true.[15] And thirdly, one could ask again why not start straightaway with the antecedent $A_j$ in (4.1.12)? The reason is that, in the end, we are aiming for a transition of determination as transportation of necessary conditions or, more precisely, necessary conditions for change referring to the sender-receiver principle, which is close to the concept of causation.[16]

Now we want to go for the transition from general true statements to certain true statements, namely true statements on existence, or existents. Therefore, we build the following attributive succession:[17]

[15] We will later see that the arrows are mere epistemic, and hence, although true, there is no counterpart in the realm of spatio-temporal existence.

[16] Cf. Aronson's, resp. Fair's transference theory and Dowe's quantity conservation theory: [41], [20], [66], and [65].

[17] The index is shifted to the left just to clear space for further specifications.

${}_iA$ : 'this exists' (pointing at something),

${}_iA$ : '${}_iA$ exists' (${}_iA$ being the pointed at),

${}_iA^t_{x(i,t)}$ : '${}_iA$ exists at time $t$ and position $x(i,t)$' (${}_iA^t_{x(i,t)}$ being the pointed at).

Thus, we identify the epistemically true statement on existence with the ontologically adopted fact of existents ${}_iA^t_{x(i,t)}$. Then, if ${}_iA^t_{x(i,t)}$ and ${}_jA^t_{x(j,t)}$ are true statements on existents located in space and time, i.e. they are ontologically true, the NC will turn into an ontological necessary condition.[18]

**Definition 45** (Ontological Necessary Condition)**.** ${}_iA^t_{x(i,t)}$ being *existent* at time $t$ and position $x(i,t)$ is an *ontological necessary condition (oNC)* for ${}_jA^{t'}_{x(j,t')}$ to be existent at time $t'$ and position $x(j,t')$ iff ${}_jA^{t'}_{x(j,t')}$ being existent at time $t'$ and position $x(j,t')$ is sufficient for ${}_iA^t_{x(i,t)}$ to be existent at time $t$ and position $x(i,t)$, *and* ${}_jA^{t'}_{x(j,t')}$ is existent:

$$ {}_iA^t_{x(i,t)} \xrightarrow{\text{oNC}} {}_jA^{t'}_{x(j,t')} \;:\Leftrightarrow\; \left({}_jA^{t'}_{x(j,t')} \Rightarrow {}_iA^t_{x(i,t)}\right) \wedge {}_jA^{t'}_{x(j,t')} \tag{4.1.13} $$

▲

Here, it must be mentioned that existence is claimed to be restricted to presence, i.e. there is an instance of time $t_0$ which shall signify the existent, and obviously, just that can ontologically be true, which carries this signifier. Thus, e.g. for $t = t_0 < t'$ (4.1.13) should be read as follows: ${}_iA^{t_0}_{x(i,t_0)}$ is (now) an oNC for the future existent ${}_jA^{t'}_{x(j,t')}$. Or for $t < t' = t_0$, one would say: ${}_iA^t_{x(i,t)}$ has been an oNC for the (now) existing ${}_jA^{t_0}_{x(j,t_0)}$. And then one could ask, what is so ontological about the necessary condition where future or past (non)-existences are being involved? Isn't it that just $t = t' = t_0$ could qualify to be oNCs? It is inevitable that we encounter epistemic necessary conditions again, which we wanted to avoid. But it shows to be safe, even if $t > t'$, since every ontological fact is an ontological necessary condition for every ontological fact.

Now here is the question we already announced: in addition to the defined necessity of existence, is there the need to capture so-called lawful behavior, e.g. physical determination? There is indeed the wish to capture *change*, or more precisely, the necessity of change, of becoming. Mere ontological necessary conditions (oNCs) are too meager to fruitfully interrelate true statements of existence, i.e. existents.

Instead of ruminating on the debate on (natural) laws[19], we immediately introduce the idea of nomological in contrast to ontological necessity:

**Definition 46** (Nomological Necessary Condition)**.** ${}_iA^t_{x(i,t)}$ being existent at time $t$ and position $x(i,t)$ is a *nomological necessary condition (nNC)* for ${}_jA^{t'}_{x(j,t')}$ to be existent at time $t'$ and position $x(j,t')$:

$$ {}_iA^t_{x(i,t)} \xrightarrow{\text{nNC}} {}_jA^{t'}_{x(j,t')} \tag{4.1.14} $$

iff

[18]Strictly speaking, it is a necessarily ontological necessary condition (NoNC).
[19]We will actually do it while seeking for physical permission in (chap 4).

(i) $t < t'$ (*directedness* of time),

(ii) $\exists k$ such that $x(k,t) = x(i,t)$ and $x(k,t') = x(j,t')$:

$$_kA^t_{x(i,t)} \xrightarrow{\text{oNC}} {}_kA^{t'}_{x(j,t')} \tag{4.1.15}$$

(iii) $_iA^t_{x(i,t)}$ is the *source* of (for) $_kA^t_{x(k,t)}$ and $_jA^{t'}_{x(j,t')}$ the *sink* of (for) $_kA^{t'}_{x(k,t')}$.

▲

Remarks again: (i) For a necessary condition to not just be ontological but nomological, a directed ordering is included, which is in accord with the *experience* of the directedness of flow of time. (ii) There is some existent that departs from the position of the existent that is said to be a nomological necessary condition and arrives at the position of a subsequent existent. With that one might think, all said, it is needed. But needed for what? For moving through space and time? That is not satisfying since it is not clear why one should tag those existents that just show a passage through the other two existents. Or to refer to (4.1.14), what could be the reason to distinguish those existents that are passed through or reached by the very same existent? If one wishes to classify the existents, it is to find some special behavior of the existents, giving rise to at least stating an elementary *law*. The first that could come to mind would just be to specifically *relate* those existents that pass through the same location in space and time: $x(i,t') = x(j,t')$. It is, if not again an experience, at least a matter of *intuition* due to experience that existents that are at distinct position at the same instance of time could also be nomologically necessary, in addition to ontologically, for *change* of position, for example. Thus, it's now about change and its conditions, and it's our task to find some criterion, which should be satisfied for a condition to count as nomological rather than just ontological, and here, it is existents changing their position from one existent to the other. But as we notice, in (def 46 (ii)) there is no talk of change, only of positions being *doubly* occupied. One now could claim that some existent finding itself at some spacial location already taken, or „critically near enough", could be sufficient, besides the directedness of time, to fulfill nomological necessity. But one could imagine an existent passing through or sufficiently by another existent without a change due to, i.e. in a certain way followed by, the passage. Thus, (i) and (ii) couldn't be sufficient, and we need at least a third. In (iii) then, one could concentrate on either change or passage: the condition for existents passing by sufficiently resulting in certain change or change at all. In the above definition, we shift our attention to the prior to change claiming the *nomological invariance* of there being no change without passing, such that the existent $_kA^t_{x(k,t)}$ stems from a location where the existent $_iA^t_{x(i,t)}$ happens to be, which is called its *source*, and goes in or into the existent $_jA^{t'}_{x(j,t')}$, called its *sink*. Of course, now the obligation to provide is to that duality, in particular whether being the sink for $_kA^{t'}_{x(j,t')}$ is a necessary condition for $_jA^{t'}_{x(j,t')}$ to change in a certain way. Here are two possible objections. Firstly, it could be argued that there was still only conditionalized existence at certain spacial and temporal locations, i.e. moving followed by moving. It is correct that the definition only includes existent ontological necessary conditions[20] with time directedness, but the invariance doesn't just consist of the persistence of the existent, instead in addition, there is the claim of departure and

[20]Cf. (4.1.15).

arrival to be met while watching the happening, or at least we set it as a definition. And that is obviously a more sophisticated law.[21]

And secondly, one could remark that there is still counterfactual thinking involved while stating conditionalized existence: if the necessary condition wasn't met, the future existence would be different. Why not be content with the following: it is that, and then it is that, and then it is that, ..., i.e. the (temporal) concatenation of conjunctions already mentioned? But that is exactly how the ontological necessary conditions are defined. They always hold.[22] The point is somewhat different. If there is no counterfactual possible, how can one epistemically separate the change? Given oNC and nNC are true, one will not be able to state more than 'the existent ${}_iA^t_{x(i,t)}$ is an nNC for the existent ${}_jA^{t'}_{x(j,t')}$ with change in position, which *in a certain way* ${}_iA^t_{x(i,t)}$ has been necessary for'. In short, one might think this is a case for Hume, except that he is against necessity: the actual necessary change is a change for which there has been a fulfilled condition, that's all.

This is another crucial point: why was the definition (def 46) of nomological necessary conditions restricted to $t < t'$? The answer is that it seems to be imperative to avoid *action at a distance*. This is a valid point, and it will be one of the most striking issues here. We prepare for that by giving a kind of nomological necessary conditioning for $t = t'$, which will be taken up again:

**Definition 47** (Non-time delayed nomological necessary condition)**.** ${}_iA^t_{x(i,t)}$ being existent at time $t$ and position $x(i,t)$ is a *non-time delayed* or *instantaneous nomological necessary condition (inNC)* for ${}_jA^t_{x(j,t)}$ to be existent at time $t$ and position $x(j,t)$:

$$ {}_iA^t_{x(i,t)} \xrightarrow{\text{inNC}} {}_jA^t_{x(j,t)} \tag{4.1.16} $$

iff $\exists t^\dagger < t$ such that

$$ {}_iA^{t^\dagger}_{x(i,t^\dagger)} \xrightarrow{\text{nNC}} {}_jA^t_{x(j,t)} \tag{4.1.17} $$

▲

The situation described is better conceived if one focusses on the present: $t = t_0$. Then the definition just says that the present existent ${}_iA^{t_0}_{x(i,t_0)}$ had been an existent before, ${}_iA^{t^\dagger}_{x(i,t^\dagger)}$, as a source for the present sink ${}_jA^{t_0}_{x(j,t_0)}$. This simple conception will be of major interest in (chap 4.2), where we want to follow the imperative of proper metaphysics.

We are getting closer to our ontology. Mere existence didn't satisfy, and the crucial idea was at least to couple existents. Some existents belong together due to some criterion, and we had to find the criterion. And we found that in general there are at least two classes of conditionalized existents. This is the conjecture:

**Conjecture 8** (Duality of Existence)**.** There are at least two kinds of existents:

(i) There are existents that *are* sources and sinks.

[21] That is all a law can contribute, i.e. classifying sequences of invariant happenings. And the invariance with the oNCs was just persistence in existence, which couldn't be sparser.

[22] Cf. (def 45).

(ii) There are existents that *have* existents as sources and sinks.

♦

Now, it is to find invariances that are consistent within the theory and also match nature or experimental metaphysics. And the conception of sources and sinks is ubiquitous in the description of flow out of or into a given point. Powerful tools from vector analysis, such as divergence, curl, and gradient of a vector field, can now be introduced, and it will actually be done intrinsically in (chap 4.2), investigating a grounding physical theory called Bohmian mechanics, but the geometrical implications are not the point here. Instead, the conditionalizing has to be assigned a location in space along with the direction of time, and we define a source to be a *place*, where something becomes visible by means of some criterion or observational skill, and a sink to be another place where it vanishes again but without great emphasis on the disappearance but rather on that something's arrival being followed by the sink's certain change:

**Conjecture 9** (Change of Sinks)**.** Certain existents of sort (ii) in (con 8) arriving at sinks are followed by changes of the sinks, which the existents are uniquely necessary for. ♦

We further prepare for our minimal ontology combining the written-out diagram (4.1.14)

$$\begin{tikzcd}
{}_iA^t_{x(i,t)} \arrow[r, "\text{oNC}"] \arrow[d, "\text{oNC}"'] \arrow[dr, "\text{nNC}"] & {}_iA^{t'}_{x(i,t')} \arrow[d, "\text{oNC}"] \\
{}_jA^t_{x(j,t)} \arrow[r, "\text{oNC}"'] & {}_jA^{t'}_{x(j,t')}
\end{tikzcd} \tag{4.1.18}$$

with (4.1.15), yielding

$$\begin{tikzcd}
{}_iA^t_{x(i,t)} : {}_kA^t_{x(i,t)} \arrow[r, "\text{oNC}"] \arrow[d, "\text{oNC}"'] \arrow[dr, "\text{nNC}"] & {}_iA^{t'}_{x(i,t')} \arrow[d, "\text{oNC}"] \\
{}_jA^t_{x(j,t)} \arrow[r, "\text{oNC}"'] & {}_jA^{t'}_{x(j,t')} : {}_kA^{t'}_{x(j,t')}
\end{tikzcd} \tag{4.1.19}$$

or, in short, again

$$ {}_iA^t_{x(i,t)} : {}_kA^t_{x(i,t)} \xrightarrow{\text{nNC}} {}_jA^{t'}_{x(j,t')} : {}_kA^{t'}_{x(j,t')} \tag{4.1.20}$$

saying 'the existent ${}_iA^t_{x(i,t)}$ (via) being a source for the existent ${}_kA^t_{x(i,t)}$ is a nomological necessary condition for the existent ${}_jA^{t'}_{x(j,t')}$ (via) being a sink for the existent ${}_kA^{t'}_{x(j,t')}$' resulting in the sink's change. Two remarks need to be noted: firstly, the first colon signifies 'being a source of', whereas the second signifies 'being a sink of', and in both, the parenthetical 'via' simply suggests there is some additional lawlike behavior only in the manner of how the source/sink consideration has been introduced, and we continue to refrain from keeping any activity or even causation in mind. Adding some „causal power“ here would indeed not be enlightening. In the following, the inconvenient usage of sources and sinks will be abbreviated by ‘having’ with (4.1.20), then saying: ‘the existent ${}_iA^t_{x(i,t)}$ having ${}_kA^t_{x(i,t)}$ is a nomological necessary condition for the existent ${}_jA^{t'}_{x(j,t')}$ having ${}_kA^{t'}_{x(j,t')}$’. This connects to the second remark. The above ‘resulting in the sink’s change’ referring to (4.1.20) seems to be somewhat superfluous, since ‘being a sink’ is only introduced to „capture“ the change which ${}_kA^{t'}_{x(j,t')}$ is necessary for. Both perspectives, the arrival of the existent and the resultant change, are equivalent in respect to nomological necessary conditioning. But since we first want to take account of the natural intuition to focus on transmission and change, and not so much on transmission and sources and sinks, we concentrate on the perspective of an existent ${}_iA^t_{x(i,t)}$ being a nomological necessary condition for change of (the sink) ${}_jA^{t'}_{x(j,t')}$. In the following, it is more about change and not so much about sinks.

Nonetheless, there is the misleading intuition of something *having* or possessing something, and in order to direct the convenient, but false, friend onto the right path, we make it explicit and define:

**Definition 48** (Having (:))**.** *Having (:)* is an abbreviation for ‘being a source for’ and ‘being a sink for’. ▲

After these preparations, we now conjecture that the existents behave in the following way:

**Conjecture 10** (Continuity)**.** All nomological necessary conditions are instantaneous nomological necessary conditions:

$$\begin{tikzcd}
{}_iA^t_{x(i,t)} \arrow[r, "\text{oNC}"] \arrow[d, "\text{inNC}"'] \arrow[dr, "\text{nNC}"] & {}_iA^{t'}_{x(i,t')} \arrow[d, "\text{inNC}"] \\
{}_jA^t_{x(j,t)} \arrow[r, "\text{oNC}"'] & {}_jA^{t'}_{x(j,t')}
\end{tikzcd} \tag{4.1.21}$$

♦

It says that, given some existent is a nNC, it is also a inNC, albeit not for the same universe-instance. In the example above, ${}_iA^t_{x(i,t)}$ is an nNC for ${}_jA^{t'}_{x(j,t')}$ and an inNC for its precursor ${}_jA^t_{x(j,t)}$.

The diagram in (con 10) can be expanded to

$$\begin{tikzcd}
{} \arrow[r, dotted, no head] & {}_iA^t_{x(i,t)} \arrow[r, "\mathrm{oNC}"] \arrow[d, "\mathrm{inNC}"'] \arrow[dr, "\mathrm{nNC}"] & {}_iA^{t'}_{x(i,t')} \arrow[r, dotted, no head] \arrow[d, "\mathrm{inNC}"] & {} \\
{} \arrow[r, dotted, no head] & {}_jA^t_{x(j,t)} \arrow[r, "\mathrm{oNC}"'] & {}_jA^{t'}_{x(j,t')} \arrow[r, dotted, no head] & {} \\
{} \arrow[r, dotted, no head] & {}_lA^t_{x(l,t)} \arrow[r, "\mathrm{oNC}"'] \arrow[u, "\mathrm{inNC}"] \arrow[ur, "\mathrm{nNC}"'] & {}_lA^{t'}_{x(l,t')} \arrow[r, dotted, no head] \arrow[u, "\mathrm{inNC}"'] & {}
\end{tikzcd} \tag{4.1.22}$$

where ${}_iA^t_{x(i,t)}$ and ${}_lA^t_{x(l,t)}$ are nNCs for the same existent ${}_jA^{t'}_{x(j,t')}$.

Maybe the ground is now prepared to ask whether the *conceptual* framework of true statements that we introduced on spatio-temporal existents behaving in the defined and conjectured way can be applied to the existents themselves, i.e. our quality particulars that are sources and sinks, or have sources and sinks. In a second step, we present a physical theory that explicitly relies on a "primitive ontology", face to face with the objects that will be called *domain unifying categorical kinds*.

### The Minimal Ontological

Referring to the final remarks of (chap 4.1.1), these considerations of *general* necessity of existents suggest there are at least two *kinds* of existents, which are our quality particulars and describe conditionalized change in space and time:

**Conjecture 11** (Ontological Duality)**.** There are at least two kinds of quality particulars:

(i) There are quality particulars that *are* sources and sinks. They are still called *objects* ${}_iO^t_{x(i,t)}$.

(ii) There are quality particulars that *have* the objects as sources and sinks. They are called *necessary conditions for change (NCCs)*: ${}_k\Box^t_{x(i,t)}$.

♦

The reason the quality particulars in (ii) are called 'necessary conditions for change' is obvious: the name reflects the aim. The NNCs are certain quality particulars that move in space and time, and what makes them special is that they are necessary conditions for change. Now, where we have identified the source and sink, we go for the interrelations that are the determinations defined in (def 41) and (def 42). We assume that the natural deterministic relations between the quality particulars behave in the same way as in diagram (4.1.22):

**Conjecture 12** (Dets are NCs)**.** Intra-time and inter-time determinants are necessary conditions, i.e. the *ontological* diagram on the left is isomorphic to the *epistemic* diagram on the right:

$$
\begin{tikzcd}
{}_iO^t_{x(i,t)} \arrow[r, "\Delta^{t,t'}_{i,i}"] \arrow[d, "\Delta^t_{i,j}"'] \arrow[dr, squiggly, "\Delta^{t,t'}_{i,j}"] & {}_iO^{t'}_{x(i,t')} \arrow[d, "\Delta^{t'}_{i,j}"] \\
{}_jO^t_{x(j,t)} \arrow[r, "\Delta^{t,t'}_{j,j}"'] & {}_jO^{t'}_{x(j,t')} \\
{}_lO^t_{x(l,t)} \arrow[r, "\Delta^{t,t'}_{l,l}"'] \arrow[u, "\Delta^t_{l,j}"] \arrow[ur, squiggly, "\Delta^{t,t'}_{l,j}"'] & {}_lO^{t'}_{x(l,t')} \arrow[u, "\Delta^{t'}_{l,j}"']
\end{tikzcd}
\cong
\begin{tikzcd}
{}_iA^t_{x(i,t)} \arrow[r, "\text{oNC}"] \arrow[d, "\text{inNC}"'] \arrow[dr, "\text{nNC}"] & {}_iA^{t'}_{x(i,t')} \arrow[d, "\text{inNC}"] \\
{}_jA^t_{x(j,t)} \arrow[r, "\text{oNC}"'] & {}_jA^{t'}_{x(j,t')} \\
{}_lA^t_{x(l,t)} \arrow[r, "\text{oNC}"'] \arrow[u, "\text{inNC}"] \arrow[ur, "\text{nNC}"'] & {}_lA^{t'}_{x(l,t')} \arrow[u, "\text{inNC}"']
\end{tikzcd}
\tag{4.1.23}
$$

♦

Here, we must direct our attention to the fact that there is a tension in the emphasis on the elements of the two diagrams in (4.1.23). On the right, the focus is on the object being a necessary condition, where on the left, the arrows are more decorated: the whole process, which is the determination, is represented by the mapping itself, e.g. $\Delta^{t,t'}_{i,i}$:

$$ {}_iO^t_{x(i,t)} \xrightarrow{\Delta^{t,t'}_{i,i}} {}_iO^{t'}_{x(i,t')} \tag{4.1.24} $$

And there is indeed a reason why we don't just note

$$ {}_iO^t_{x(i,t)} \xrightarrow{\Delta} {}_iO^{t'}_{x(i,t')} \tag{4.1.25} $$

in correspondence to

$$ {}_iA^t_{x(i,t)} \xrightarrow{\text{oNC}} {}_iA^{t'}_{x(i,t')} \tag{4.1.26} $$

Firstly, we want to refer to processes that are mapped to processes, and secondly, a crucial new notion will be introduced: the *composition* of processes. Thus, later on we want to deal with the relations in order to apply category theory as a not yet very well established language to refer to a more sophisticated ontology. Whether or not it works out will be evaluated by looking for something that comes in addition to the existent, e.g. natural laws. With that, not only objects and NCCs would be necessary but also some „entity“ that „operates“ on these.[23]

At least, the indices of the determinations can be simplified by reducing the redundancy that doesn't help to keep track of the existent.

[23] Cf. the universal wave function: (chap 4.2.1).

We now combine the *principle of having* (def 48) with conjecture (con 12) and yield the continuous diagram

$$
\begin{tikzcd}[column sep=huge, row sep=huge]
{}_iO^t_{x(i,t)} : {}_k\square^t_{x(i,t)} \arrow[r, "\Delta_i^{t,t'}"] \arrow[d, "\Delta^t_{i,j}"'] \arrow[dr, squiggly, "\Delta^{t,t'}_{i,j}"] & {}_iO^{t'}_{x(i,t')} \arrow[d, "\Delta^{t'}_{i,j}"] \\
{}_jO^t_{x(j,t)} \arrow[r, "\Delta_j^{t,t'}"'] & {}_jO^{t'}_{x(j,t')} : {}_k\square^{t'}_{x(j,t')} \wedge {}_r\square^{t'}_{x(j,t')} \\
{}_lO^t_{x(l,t)} : {}_r\square^t_{x(l,t)} \arrow[u, "\Delta^t_{l,j}"] \arrow[r, "\Delta_l^{t,t'}"'] \arrow[ur, squiggly, "\Delta^{t,t'}_{l,j}"'] & {}_lO^{t'}_{x(l,t')} \arrow[u, "\Delta^{t'}_{l,j}"']
\end{tikzcd}
\tag{4.1.27}
$$

and emphasize for a corresponding physical theory the two interesting parts:

1. The object ${}_iO^t_{x(i,t)}$ having the necessary condition for change ${}_k\square^t_{x(i,t)}$ and the object ${}_lO^t_{x(l,t)}$ having the necessary condition for change ${}_r\square^t_{x(l,t)}$ time-delayed determine the object ${}_jO^{t'}_{x(j,t')}$ having the necessary conditions for change ${}_k\square^{t'}_{x(j,t')}$ and ${}_r\square^{t'}_{x(j,t')}$.
2. The object ${}_iO^t_{x(i,t)}$ having the necessary condition for change ${}_k\square^t_{x(i,t)}$ and the object ${}_lO^t_{x(l,t)}$ having the necessary condition for change ${}_r\square^t_{x(l,t)}$ non-time-delayed determine the object ${}_jO^t_{x(j,t)}$.

With the principally denotable and traceable existents of the objects, i.e. the quality particulars, the ground is now prepared to apply an experimental metaphysics that intrinsically includes „the pointing at“, i.e. the observation.[24]

### 4.1.2 Guided Matter Point Theory

DUCKs intends to provide an (onto)logical grounding of existents and change of existents and its necessity. In order to give it a serious basis, it has to show its consistency with a well-established, fundamental physical theory. Since our ontology developed so far is about existents and their change in position, it stands to reason to look for the physical theory that best allows for one-to-one correspondences, and of course, we will have to ask whether this „best“ is good enough. This procedure could be termed an *experimental metaphysics* approach, with the emphasis on ‘metaphysics’ to be confirmed.

Since the task is to describe the behavior of the existents, one has to deal with natural laws. In the nomological reading, these are said not to move the entities, but what could do so, anyway? ‘Moving’ here is meant to be causal, transitive, i.e. having an

[24] Neither the experiencing of the pointer or of some pointing is included and is not necessary to register experimental outcomes: cf. [68] for the metaphysical foundations of physics. One goal on the way for a unifying theory to embrace more than these objects is to uncover the necessary conditions for change.

object to be moved. But actually there is no need to assume causality to state a law. Rather, it is necessity to be referred to. And of course, there are advocates who even deny this evaporated connectivity, one of whom is David Hume, who is fond of *bookkeeping*.[25] According to him, laws are merely descriptive, and what then can be more rigorous than just stating what has happened? Despite the *problem of induction*, one wants to look for laws that also state future happenings, and that can be done just by assuming necessity.

Regarding arrangements of particles or quality particulars without necessity, there is another proponent that is often started with on the perception of existents and change of existents and all other depending on it:

> *Humean supervenience is named in honor of the greater denier of necessary connections. It is the doctrine that all there is to the world is a vast mosaic of local matters of particular fact, just one little thing and then another. (But it is no part of the thesis that these local matters are mental.) We have geometry: a system of external relations of spatio-temporal distance between points. Maybe points of spacetime itself, maybe point-sized bits of matter or aether or fields, maybe both. And at those points we have local qualities: perfectly natural intrinsic properties which need nothing bigger than a point at which to be instantiated. For short: we have an arrangement of qualities. And that is all. There is no difference without difference in the arrangement of qualities. All else supervenes on that.*[26]

Two contentious points are addressed, one of which is having such a bad reputation in the matter of matter points that, in the citation, it is often not even mentioned in its parenthesis: the mental supervening on all there is—the arrangement of qualities. The second one is Humeanism, and we begin with that since the other will take a bit more work.

One could be satisfied with the end of all days' bookkeeping, i.e. contenting oneself with the registration of the happenings. That alone is what the Humeans admit to be objectively true—and deny any causal force and also non-causal necessity or determination. But what about similar repetitions, called 'structures'? If there were no conditionalized arising, i.e. natural laws, what then placed the particles in those structures? Humeanism appears to be all too sceptical to be satisfying. It is not that we crave for laws but for structure, if not ontological structure then still structure to be perceived or experienced by some observer or cognizer. Now if we seek a physical theory that is against subsystems—and they are necessary to approach structures—but endorses necessity in terms of determination, one could think that a guided matter point theory isn't the right one to spend time on. Actually, far from it, as we will see—it shows features we endorse, and that it is not structural is not a flaw but an imperative in matching our considerations. We make no bones about our desire to rediscover the elements of DUCKs in a sophisticated enough scientific theory. And of course, we ask whether Bohmian mechanics, a theory of guided matter points, became such a sufficiently recognized theory in the scientific community by then because it offered attractive promises—determinism, localizability, and even nonlocality. And that matches quite well with what we find attractive, too: (i) nomological inter-time determination as necessity, (ii) change only in the arrangement of the existents.

[25] Cf. [24, VII] and [34]: Hume on Causation, and [52] for circularity.
[26] Cf. [25, x].

Humeanism states the contingency of the arrangements: it is metaphysically possible to hold some entities fixed and change the others. If this is a claim, Humeanism is false, or it is just a definition, for it cannot be assumed to be *true*, first lacking evidence and second itself imposing denied necessity—the necessity of no necessity is not contingency. Moreover, there is the imagination that "[i]t is a contingent matter of fact that the distribution of the fundamental physical properties throughout space-time in the actual universe manifests certain regularities."[27] Those regularities are meant to be only conceived as Hume states that the "repetitions" (due to regularities)

> *has an influence only on the mind, by that customary transition it produces: that this customary transition is, therefore, the same with the power and necessity; which are consequently qualities of perceptions, not of objects, and are internally felt by the soul, and not perceived externally in bodies[.]*[28]

That means that there is no physical regularity inducing regularity in mind—the mental *not* supervening on the physical. Before we proceed further in that direction, we first, as mentioned above, try a metaphysical grounding on Bohmian mechanics (BM). Therefore, below is a brief introduction to non-relativistic BM without spin to ascertain first whether Humeanism can be satisfactory, second whether dispositionalism can do better, and third whether DUCKs can assist in this battle.

## 4.2 Bohmian Mechanics

For some, there is the ultimate wish not to let one's sight or even mind be clouded but to have good visibility, i.e. to have a theory that provides an „explanation“ of vision of the existent. Heisenberg's sentiment on those reactionary:

> *They would prefer to come back to the idea of an objective real world whose smallest parts exist objectively in the same sense as stones or trees exist, independently of whether or not we observe them. This, however, is impossible or at least not entirely possible[.]*[29]

Yes, not entirely possible—as we will see, there is indeed good reason that stones and trees do not exist, but their subjectivity is not the reason why one might be better off not believing in the objective world of at least the constituents, the smallest parts.

There are mayor inconsistencies that have to be overcome in handling the small. Probably the most prominent and far-reaching is the *measurement problem* in orthodox quantum mechanics, which is expressed in the following trilemma concerning the completeness of the wave function:

> *The following three claims are mutually inconsistent.*
>
> 1. *The wave-function of a system is complete, i.e. the wave-function specifies (directly or indirectly) all of the physical properties of a system.*

[27] Cf. [51, 781].
[28] Cf. [23, 166].
[29] Cf. [83, 129].

2. *The wave-function always evolves in accord with a linear dynamical equation (e.g. the Schrödinger's equation).*
3. *Measurements [...] have determinate outcomes [.]*[30]

There are several quantum theories focussing on one of the claims to be suspended to resolve the measurement problem. The orthodox Copenhagen QT denies the second while voting for an observer bringing about the knowledge about the actual outcome of a measurement which is done by „epistemically“ *reducing the state*, i.e. *collapsing the wave function*, whereby the sole reign of mathematics comes to an abrupt end.[31] Another way to abrogate the unrestricted unitary operator necessary for a continuous time evolution of the wave function is to add a non-linear term in the Schrödinger's equation to provide an intrinsic „mechanism“ for the collapse of the wave function making an „active“ observer unnecessary and the outcome ontological, i.e. the GRW equation, now allowing objective discontinuous and indeterministic quantum behavior due to spontaneous wave function collapse.[32]

The third claim has been questioned by bringing forward the concept of the *relativity of states*: "Deductions are drawn about the state of the observer relative to the state of the object system"[33], saying the observer and the measuring apparatus and the measured are constituent subsystems composing a whole, i.e. a system. This means that *all* of the subsystems are not in a single well-defined state, and the observer is not independent of but correlated to the remainder:

> *[A]ll measurements and observation processes are to be regarded simply as interactions between the physical systems involved—interactions which produce strong correlations.*[34]

Thus the observer does not have determinate knowledge of the outcomes. This theory has been further developed into *The Many-Worlds Interpretation of Quantum Mechanics*[35], which states again a realistic view—at a measurement outcome the world splitting into as many worlds as superpositioned states existing, including states of the observer.[36] This approach is called a 'no-collapse' interpretation, because the state reduction of the whole system is not determinate in a *single* universe—the world splits or the observer is indecisive. But this is not the only possible denial of collapse, there is a second one, and that doubts the first claim—the wave function being complete.

A lot has been written on that issue. Actually, from the very beginning, *completeness* of the future *orthodox* quantum mechanics appeared suspicious for some, and among those were, after all, most of the founders.[37] We follow Bohm's proposal from *A Suggested Interpretation of the Quantum Theory in Terms of 'Hidden' Variables*[38] admitting

[30] Cf. [78, 7].
[31] Here, we already sense a looming conflict between the epistemic and the ontological realm. Does the observer play an active role, and what is 'activity'? Cf. (chap 9.1) in [19] regarding pointer positions and their detection.
[32] Cf. [10] and [84] for an overview.
[33] Cf. [35, 455].
[34] Cf. [35, 455].
[35] Cf. [74].
[36] Cf. [53] for the *many minds* interpretation.
[37] Cf. [5].
[38] Cf. [15] and [16].

quantum particles and their having positions at any time—the *hidden parameters*[39] that come in addition to the wave function. The idea of there being point-like particles being guided by a wave, the so-called *pilot wave*, which is nothing but the solution of the Schrödinger's equation itself, i.e. the wave function, was already introduced at the 5th Solvay conference in Brussels in 1927 by de Broglie[40] and then fell off the radar and had to be rediscovered—or better, discovered once again.[41]

After having introduced the basic specifications of BM, we will go on to one of the probably most puzzling questions of all the quantum theories—the „role" of the wave function, which is indeed key for our ontology and the real reason why we attend to this theory seeking an empirical grounding of DUCKs. In BM, the situation is clear and for our purposes comes in quite handy: *the wave function makes the particles move*, and the attracting feature here is that the moving of one particle—or its changing as we want it—is „mediated", i.e. has been made necessary, as we take it, by all the other particles. What this mediation looks like will first be outlined in the following by introducing three more or less convincing „physico-mathematical" intuitions and then an intuition that keeps complexity low by trying to excerpt the underlying ontology by taking the mathematics serious by leaving it behind.

This is then non-relativistic BM for $N$ particles. The physical reality is seen to be describable by the pair $(\psi, Q(t))$ of the wave function *and* the configuration of the actual positions of all particles in the universe at an instance of time $t$: $Q(t) = (Q_1(t), ..., Q_N(t))$ with

$$\begin{aligned} Q_i : \mathbb{R} &\to \mathbb{R}^3 \\ t &\mapsto Q_i(t) \end{aligned} \tag{4.2.1}$$

being the *trajectories* of the particles in three-dimensional space.[42]

The „law of motion", i.e. the necessitating of change of the particles' positions, is mathematically represented by the wave function $\psi$ with

$$\begin{aligned} \psi : \mathbb{R}^{3N} \times \mathbb{R} &\to \mathbb{C}^k \\ (q,t) &\mapsto \psi(q,t) \end{aligned} \tag{4.2.2}$$

which is a mapping of the generic configuration space variable $q = (q_1, ..., q_N)$ onto a complex number having $k$ complex components. $\psi$ is said to "guide" the particles in physical space, which is nothing but determining the particles' velocities $v_1^\psi, ..., v_N^\psi$ in $\mathbb{R}^3$.[43] But how is it done? The wave function defines a velocity vector field $v^\psi \equiv v^\psi(q)$ on the configuration space $\mathbb{R}^{3N}$ with $v_i^\psi \equiv v_i^\psi(q_1, ..., q_N) \in \mathbb{R}^3$ being the velocity of the $i$-th particle in physical space:[44]

$$\frac{\mathrm{d}Q_i}{\mathrm{d}t} = v_i^\psi(Q_1, ..., Q_N) \tag{4.2.3}$$

[39] Another deterministic view on hidden parameters obeying Bell's theorem is proposed by G. t'Hooft: a cellular automaton interpretation of moving in space following moving in Conway's game of life. Cf. [81].
[40] Cf. [26].
[41] Cf. [30] for an anthology of the very beginning of pilot wave mechanics and also of its disregard.
[42] Cf. [18, 29].
[43] Cf. [19, 144]. Is velocity a second categorical property in addition to position? We will come to that later. So far, velocity is just a mathematical way of describing change of position.
[44] Cf. [15] and [16].

with $Q_1, ..., Q_N$ being the particles' actual positions.

In addition to the wave function $\psi$ specifying the vector field $v_i^\psi$ it solves, as in orthodox quantum mechanics, the Schrödinger's equation (SE)[45]

$$i\hbar \frac{\partial \psi}{\partial t}(q,t) = -\sum_{i=1}^{N} \frac{\hbar^2}{2m_i} \Delta_i \psi(q,t) + V(q)\psi(q,t) = H\psi(q,t) \tag{4.2.4}$$

with the Laplace operator $\Delta_i = \partial^2/\partial x_i^2 + \partial^2/\partial y_i^2 + \partial^2/\partial z_i^2$ and the so-called potential $V$[46] both providing further specifications of the universe that are necessary to determine $\psi$.[47] The form of the SE is, as with (4.2.3), a first-order differential equation that determines the time evolution of $Q_i(t)$ given the wave function $\psi$:

$$\frac{\mathrm{d}Q_i}{\mathrm{d}t} = \frac{\hbar}{m_i} \operatorname{Im}\left(\frac{\psi^* \partial_i \psi}{\psi^* \psi}\right)(Q_1, ..., Q_N), \qquad i = 1, ..., N \tag{4.2.6}$$

with $\partial_i = (\partial/\partial x_i, \partial/\partial y_i, \partial/\partial z_i)$ and $m_i$ being the so-called mass of the $i$-th particle.

That is the physical theory. Now one could think that the considerations here are all about physics, but strictly speaking, it is just logics on moving entities, or more specifically, on changing parameters. That $m_1, ..., m_N$ are called 'masses' is

> *[...] because in certain physical situations the particles will move along Newtonian trajectories and then these masses are Newtonian masses, and there is no point in inventing new names here.*[48]

Now we want to sketch the three ways that yield the *pilot equation* (4.2.6) for the particles' trajectories.

### Hamilton-Jacobi Continuation

Louis de Broglie was the first to suggest a deterministic quantum theory, the first who was fond of the idea of piloting the particles on a trajectory, consequently calling his account *Pilot Wave Theory.*[49] But unfortunately, he and his theory weren't well received in those days in Brussels, and so the deterministic theory of quantum objects got a second try in 1952 by David Bohm.[50] Unaffected by de Broglie's forgotten studies, he was looking for some kind of continuation—at least, it came to that—of the Hamilton-Jacobi equation of

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[45] Cf. [19, 146].

[46] For example, a static electromagnetic interaction with the particles $Q_i$ „having" a charge $e_i$ the Coulomb potential

$$V(q_1, ..., q_N) = \sum_{i<k} \frac{e_i e_k}{\|q_i - q_k\|} \tag{4.2.5}$$

has to be filled in the Schrödinger's equation.

[47] The Laplace operator includes a metrics, i.e. specifications on the spacial geometry.

[48] Cf. [19, 144]. Of course one could also doubt 'mass' in Newtonian mechanics to be more than sources and targets for the conditions for the particles moving.

[49] Cf. [26].

[50] Cf. [30] for a further analysis of the perspectives on the early quantum physics and the discord that has been set out of the gate.

classical mechanics to be a limes contained in the new theory.

In short, given the Schrödinger's equation for one particle[51]

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V(x)\psi \tag{4.2.7}$$

with the solution

$$\psi(x,t) = Re^{iS/\hbar} \tag{4.2.8}$$

where $R(x,t)$ and $S(x,t)$ are real-valued numbers, the SE splits into

$$\frac{\partial R}{\partial t} = -\frac{1}{2m}[R\nabla^2 S + 2\nabla R\cdot\nabla S] \tag{4.2.9}$$

$$\frac{\partial S}{\partial t} = -\left[\frac{(\nabla S)^2}{2m} + V(\mathbf{x}) - \frac{\hbar^2}{2m}\frac{\nabla^2 R}{R}\right] \tag{4.2.10}$$

With these equations, two substantially different approximations can be made that combine the classical world with the quantum world:[52]

(i) $\hbar\to 0$:
This is the so-called classical limit. Substituting $\rho(x,t) = R^2(x,t)$ yields

$$\frac{\partial\rho}{\partial t} + \nabla\left[\rho\frac{\nabla S}{m}\right] = 0 \tag{4.2.11}$$

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V(x) = 0 \tag{4.2.12}$$

where the first equation is gained using the derivative $\frac{\partial\rho}{\partial t} = 2R\frac{\partial R}{\partial t}$. It is the sort of a *continuity equation* with the interpretation of $\frac{\nabla S(x)}{m}$ being the velocity vector and $\rho\frac{\nabla S}{m}$ the mean current of particles if an ensemble of particles is being considered. $\rho(x,t)$ then is the *probability density* for the particles, and (4.2.11) describes the conservation of probability.

The second equation—having executed the limes—is just the Hamilton-Jacobi equation for the momentum $p = \nabla S$. But that isn't really the interesting part here for this approach is also excepted by the orthodox interpretation of quantum theory restricting itself to probability distributions.

(ii) Not $\hbar\to 0$:
In the second approximation, in which $\hbar$ does not converge to zero, the difference between the classical and the „non-classical" Hamilton-Jacobian resides in the extra „potential"

$$U(x) = -\frac{\hbar^2}{2m}\frac{\nabla^2 R}{R} \tag{4.2.13}$$

[51] Cf. [15, 169].
[52] Cf. [15, 170].

> This term will again effectively vanish, and it does so if it is negligible in comparison with $\frac{(\nabla S)^2}{2m}$, which happens if the change in the amplitude $R$ is much smaller than in the phase $S$. If it can not be ignored, we "[...] assume that each particle is acted on, not only by a 'classical' potential, $V(x)$ but also by a 'quantum-mechanical' potential[.]"[53]

The very next step then to yield a deterministic interpretation of quantum mechanics would be to conserve the definition of the Hamilton-Jacobian velocity of the particle:

> *Then Eq. [(4.2.10) with $\rho(x,t) = R^2(x,t)$] can still be regarded as the Hamilton-Jacobi equation for our ensemble of particles, $\nabla S(x)/m$ can still be regarded as the particle velocity, and Eq. [(4.2.11)] can still be regarded as describing conservation of probability in our ensemble. Thus, it would seem that we have here the nucleus of an alternative interpretation for Schroedinger's equation.*[54]

Still advancing the classical analogue, *Newton's law of motion* is now applied:

$$m\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\nabla\left[V(x) - \frac{\hbar^2}{2m}\frac{\nabla^2 R}{R}\right] \tag{4.2.14}$$

But this very last step has been subject to yet unsolved controversy from the beginning, for there seems to be no need to introduce a second derivative, i.e. an acceleration due to a force. The first-order derivative

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = \frac{\nabla S(x,t)}{m}|_{x=Q} \tag{4.2.15}$$

suffices since $S$ is already determined by the SE and the initial conditions for the particle positions.

The discussion of the merit of a "quantum force" evolved into group formation with protagonists differering on believing in motion being *causally* brought about by a force or *nomologically* by only the wave function itself. This will be debated in detail after having shortly shown a modification of the idea of conserving probability from quantum hydrodynamics again yielding the *Bohmian law of motion* (4.2.15) but now without referring to the Hamilton-Jacobi equation and the controversial "quantum force"(4.2.13).

#### Conserved Quantity

This approach exploits an analogy between Schrödinger's quantum theory and hydrodynamics by using the intuition that probability of position could, like mass, be considered a conserved „quantity" while moving in space. The procedure is to compare the hydrodynamical continuity equation

$$\frac{\partial \rho_m}{\partial t} + \nabla j_m = 0 \tag{4.2.16}$$

that equates the time derivative of the mass density $\rho_m$ with the negative of the divergence of the mass current $j_m$ with the continuity equation which can be derived from the

[53] Cf. [15, 170].
[54] Cf. [15, 170].

Schrödinger's equation relating the probability density $\rho = |\psi|^2$ to the probability current $j$.[55] Then thinking of the probability density as referring to real positions of the particles, consequently applying fluid mechanics again ($j = \rho v = \rho \mathrm{d}Q/\mathrm{d}t$), and using polar coordinates in $\psi$ immediately yields the law of motion for the particle positions (4.2.15), which is also known as the *guidance equation* of Bohmian mechanics.[56]

#### Minimality, Simplicity, and Symmetry

The last approach to the trajectory theory of quantum mechanics relies on "first principles".[57] Dürr and Teufel also have a high opinion of perceptions—stating in their textbook *Bohmian mechanics*:

> *Moving particles agree with our experience of the microscopic world. In a double slit experiment, a particle is sent onto a double slit and later caught on a screen. We see the spot on the screen (where the particle hits the screen), and since the particle came from somewhere else it had to move.*[58]

And the most natural way to theoretically catch the particles' moving is by referring the change of the actual configuration space vector $Q(t)$ to a vector field on $\mathbb{R}^{3N}$ determined by the wave function $\psi$:

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = v^{\psi}(Q,t) \tag{4.2.17}$$

Now having found the equation of motion for the particle, the system's specification $(\psi, Q(t))$ would be complete.

In short, the minimal simplicity of $v^{\psi}$ demands:[59]

(1) Physical equivalence of $\psi$ and $c\psi$.

Since a nonzero constant multiple of the WF $\psi$ solves the same SE, the velocity vector field $v^{\psi}$ should be a homogeneous function of degree 0:

$$v^{c\psi} = v^{\psi} \tag{4.2.18}$$

(2) Galilean invariance.

Galilean invariance (esp. invariance under rotation) holds for the gradient of a scalar function:

$$v^{\psi} \propto \nabla\psi \tag{4.2.19}$$

with (4.2.18) yielding

$$v^{\psi} = \alpha \frac{\nabla\psi}{\psi} \tag{4.2.20}$$

with $\alpha$ being a constant scalar in the simplest case.

[55] Cf. [27, 323].
[56] Cf. [28, 40 ff.].
[57] Cf. [19, 147].
[58] Cf. [19, 147].
[59] Cf. [19, 147,148] and [18, 29 ff.].

(3) Time-reversal invariance.

Setting for the complex conjugate of the wave function $\psi(q,t) \mapsto \psi^*(q,-t)$ for $t \mapsto -t$ demands for the equation of motion (4.2.17):

$$v^{\psi^*} = -v^{\psi} \tag{4.2.21}$$

yielding for the imaginary part in (4.2.20)

$$v^{\psi} = \alpha \ \mathrm{Im}\frac{\nabla\psi}{\psi} \tag{4.2.22}$$

(4) Galilean invariance under boosts.

Searching for $v^{\psi}$ under the transformation $v^{\psi} \mapsto v^{\psi} + u$ then determines the constant $\alpha$:

$$v^{\psi} = \frac{\hbar}{m} \ \mathrm{Im}\frac{\nabla\psi}{\psi} \tag{4.2.23}$$

Finally, these considerations on simplicity and symmetry result in the guiding equation (4.2.6) for a system of $N$ particles.

Before we turn to our original question on primitive ontology, we should summarize the results. This is Bohmian mechanics, i.e. the physical theory we refer to:

A system is described by

$$(\psi, Q) \tag{4.2.24}$$

with the wave function $\psi$ being a solution of

$$i\hbar\frac{\partial\psi}{\partial t} = -\sum_{i=1}^{N}\frac{\hbar^2}{2m_i}\Delta_i\psi + V\psi \tag{4.2.25}$$

and the positions $Q_i$ of the primitives obeying

$$\frac{\mathrm{d}Q_i}{\mathrm{d}t} = \frac{\hbar}{m_i} \ \mathrm{Im}\left(\frac{\nabla_i\psi}{\psi}\right)(Q_1, ..., Q_N) \tag{4.2.26}$$

So far, the obvious existent is that which moves with $Q_i(t)$, the matter points as *point particles*. And we wonder whether, at those positions, there might be more than just these particles, perhaps there „is“ the wave function. What that could mean has been a significant point of contention in the debate about *wave function ontology*.

Closely connected with the interpretation of $\psi$ is the discussion on quantum behavior being mechanical or non-mechanical. Bohm emphasizes the indivisibility of the universe, which was simply the opposite of the classical view:

> *The entire universe must, on a very accurate level, be regarded as a single indivisible unit in which separate parts appear as idealizations permissible only on a classical level of accuracy of description. This means that the view of the world as being analogous to a huge machine, the predominant view from the sixteenth to nineteenth centuries, is now shown to be only approximately correct. The underlying structure of matter, however, is not mechanical.*[60]

[60] Cf. [14, 167].

And he does not feel at all flattered by Dürr et al. assigning his name to their theory of guided matter points:

> *It should be noted that the views expressed in our book differ very substantially from those of Dürr et al. (1992) who have developed an alternative theory. It was very unfortunately that they chose the term 'Bohmian mechanics' to describe their work. When Bohm first saw the term he remarked, "Why do they call it 'Bohmian mechanics'? Have they not understood a thing that I have written?" He was referring not only to our article Bohm and Hiley (1987), but also to a footnote in his book Quantum Theory in which he writes, "This means that the term 'quantum mechanics' is a misnomer. It should, perhaps, be called 'quantum nonmechanics'" (Bohm 1951). It would have been far better if Dürr et al. (1992) had chosen the term 'Bell mechanics'. That would have reflected the actual situation far more accurately.*[61]

But this appears to be possibly a misunderstanding. Both „schools" refer to the same thing—the trajectory view of moving particles, and they mainly differ in the way the change in position is brought about. Bohm advocates a causal view by referring to a quantum potential yielding a quantum force while Dürr holds that referring to the classical view of the Hamilton-Jacobi description is not necessary and nomological considerations on the underlying logics of spatio-temporal postulates can do as well. And one could ask, what is more mechanical: referring to a quantum force or calling the theory a mechanics? The crucial point here seems to be that there are localized particles, and they behave deterministically, and that is which both are interested in. Thus, being causal but non-mechanical versus being non-causal but mechanical is not the point. There are particles and something drives them, and the driving being *causal* or *nomological*. That is the actual point of discord, and we will see how far it goes.

### 4.2.1 Wave Function Ontology

Wave function ontology is of major interest to us, since we assume an at least dyadic ontology. We have ascertained two kinds of entities: the objects being the sources and targets and the NCCs having them as sources and targets. And to say it straight away—we have a suspect for the role of the wave function, and with that comes its ontological status. And naturally, it comes with Bohm, but in the interpretation of the „nomological school".

In order to understand the wave function, we investigate what is embraced or contained by it. In particular, we have to look at the different kinds of wave functions referring to the different quantum theories.

#### The Universal Wave Function

There are three possible attitudes towards the wave function. The wave function is[62]

(1) *everything*, as with orthodox quantum mechanics with the observer being included,[63]

[61] Cf. [21, 117].
[62] Cf. [18, 264].
[63] Cf. e.g. [35, 457]: The observers "are considered as purely physical systems and are treated within the theory."

(2) *something*, as with Bohmian mechanics, with the observer being also included by assuming subsystems as such,

(3) *nothing*.[64]

The game becomes evident: play both ends against the middle. It is indeed an epistemic endeavor with Bohm leaving a good impression: the road is seeking *the* wave function, which is intrinsically Bohmian.

In orthodox quantum mechanics, there are wave functions of systems, and they are considered epistemic by the proponents. That is the traditional view of quantum mechanics—it's all about measurement outcomes. But if one was all too meticulous, one would have opposed that saying epistemic is not ontological. That means there must be something behind the wave function of a system, call it $X$. Thus, saying the wave function is only epistemic admits something hidden, some hidden variable, rendering orthodox quantum mechanics no longer merely epistemic.[65]

In BM, however, there is only one wave function and only one system—the universe. All other kinds of wave functions must be stated in terms of this *universal wave function* $\Psi$—and the "hidden" $Q$, the *actual universal configuration*, as will be shown. These so-called *conditional wave functions* are approximations and rule epistemically suitably decoupled systems, which then are called *subsystems*. But regarding part-whole relations, in BM objectively without there being an additional „collapsing" observer at place, there is just one thing that rules, and that is the total system $(\Psi, Q)$:

> *The behavior of the parts of a big system are already determined by the behavior of the whole. And what you have for the whole is the wave function $\Psi$ of the universe, together with its configuration $Q$.*[66]

Thus, the question is raised how concrete subsystems can at least be defined. We take a subsystem to be identically equal to the conditional wave function $\psi(x)$ and the configuration $X$: $(\psi(x), X)$.

How to make BM work, i.e. how to decouple the subsystem suitably from its environment such that BM could have been found anyway, is shown elsewhere in detail[67], and also how the typical collapse behavior of these subsystems in orthodox quantum mechanics can be reproduced without collapse.[68] Here, it is only relevant that there is no *external* observer needed for state reduction and trajectile behavior of one world, i.e. the deterministic observer is itself decoupled along with the measured and the measuring apparatus, if all together suitably, a subsystem of the universe, and there is no measurement problem any more.

For this strong claim, we need to show how the splitting of the universe into subsystems is done in principle.[69] The configuration of the universe is to be divided into the above

[64] Even if there was nothing, one would take the moderate line that it is at least epistemic, because there should be something that makes us believe.

[65] Cf. [18, 263].

[66] Cf. [18, 264].

[67] Cf. e.g. [18, 37 ff.] and [19, 216 ff.].

[68] Cf. e.g. (chap 2) in [18] and (chap 11) in [19].

[69] Cf. [18, 264,265].

$x$-subsystem's actual configuration $X$ and the environment $Y$:

$$Q = (Q_{sys}, Q_{env}) = (X, Y) \tag{4.2.27}$$

For the conditional wave function of the subsystem, the universal wave function $\Psi(q) = \Psi(x, y)$ must be freed from the complement's generic variable since $\psi(x)$ is just a function on its configuration space. But it would be false to neglect its environment's presence $Y$ so that:

$$\psi(x) = \Psi(x, Y) \tag{4.2.28}$$

Now we have to be cautious, since there are two equations involved in the Bohmian subsystem's time evolution. Its pilot equation appears to be quite evident:[70]

$$\frac{dQ}{dt} = v^{\Psi}(X, Y) \implies \frac{dX}{dt} = v^{\psi}(X) \tag{4.2.29}$$

for $\psi(x) = \Psi(x, Y)$.

But for the conditional wave function, it is not so clear at first sight that it is Bohmian, since it must obey the Schrödinger's equation, which says that $\Psi$ in (4.2.28) has to be time dependent. And also the universe's particles do not stand still in general, so we gain the following time dependence:

$$\psi_t(x) = \Psi_t(x, Y_t) \tag{4.2.30}$$

with $Q_t = (X_t, Y_t)$.

That the conditional wave function obeys the Schrödinger's equation such that all standard quantum measurement outcomes are reproduced can be found elsewhere[71]—and is not our issue. It is just that the subsystem's, or system's wave function in the orthodox reading, can be asymptotically derived from the universal wave function. And we then ask: what does the *universal* wave function stand for?

In BM, the necessity of subsystems to emerge from the universe should give us a hint as to what to think of the universal wave function. According to Bohm, who puts forward a *causal* interpretation of quantum mechanics[72], it is „mediating“ a quantum force[73] *actively* guiding the particles in three-dimensional space. According to the second, anticausal school, "it's really more in the nature of a law than a concrete physical reality."[74] This "nature of a law" is not meant to be a natural law but a means of description via mathematical, logical formalism. And we endorse, as we will see, their *nomological* and *passive* stance, i.e. the function of the wave function being a function.

What could the arguments be for the view that the WF is only nomological? It is said that a lack of "back action" of the particles' position on the WF could violate causality. Following the guidance equation (4.2.17), it is $Q = Q(\psi)$ but not $\psi = \psi(Q)$, the WF

[70] Cf. [18, 265].
[71] Cf. e.g. [18, 23 ff].
[72] Cf. [15, 175]: "Our interpretation of the quantum theory describes all processes as basically causal and continuous."
[73] Cf. (4.2.14).
[74] Cf. [18, 266].

simply obeying the Schrödinger's equation. This is a major point, since as we will soon see, it does not—in the ontological reading, the universal WF being time independent. But nevertheless, it still holds that there is no "back action", and one wonders why. It is that $\psi = \psi(q)$, i.e. a function of the generic configuration space variable, which means that the universe has to be tautological over time, saying at any instance of time $t$ the actual configuration $Q(t)$ must be irrelevant for the WF, as long as the initial setup had determined $\Psi$ such that $Q(t)$ couldn't have been otherwise. And this is a matter of *big bang* and flooding the becoming universe with the necessary conditions for the particles' change, which we called 'continuation'.

Another line of argumentation is finding the WF „strange", meaning it is a field on configurations space $q(t) = (q_1(t), ..., q_N(t))$ guiding the particles in physical space. It makes it especially strange where, as shown above, the nonzero multiple of a solution of SE yields the same velocity vector field in $\mathbb{R}^3$. What kind of thing other than a law could it be if a system's behavior can just be described by an equivalence class of fields?[75]

But there are also reasons not to believe in the WF's nomology, and their power depends on whether the WF is thought to *be* a natural law or just to *show* the nature or characteristics of a law. In the epistemic view, one can "prepare" a system by postulating a certain Hamiltonian and solving the corresponding SE. The resulting WF then is man-made and in this sense artificial and as noted does not exist, neither as some physical reality nor as a natural law. It is an approximation and as such changes in time, which shouldn't be the case if it was a real natural law. But in fact, this isn't a proper objection since ontologically the conditional wave functions do not interest us. All that counts is $\Psi$, the universal wave function, and that should be time independent, and it is. The subsystems are defined in terms of the one and only „existent" WF, which should not bother us further.

One could take the view that now entering into the business of canonical quantum cosmology by applying the Wheeler-De Witt equation[76]

$$\mathscr{H}\Psi = 0 \tag{4.2.31}$$

is overdoing things. It is not. Rather, this is key for our investigation.

The point is that the generalized Hamiltonian does not involve a time evolution, and with the time derivative being substituted by 0, it follows that the solutions, the „laws", are time-independent. The next thing then would be to make the law unique which could be indeed done.[77] This is the *problem of time* in quantum cosmology, but not a problem in Bohmian mechanics. It is

> *[...] what the doctor ordered because laws are not supposed to change with time, so we don't want the fundamental wave function to change with time. It's good that it doesn't change with time.*[78]

Nevertheless, two questions remain:

[75] Cf. [18, 266].
[76] Cf. [73].
[77] Cf. [7].
[78] Cf. [18, 269].

1. How can a non-changing $\Psi$ determine the velocity vector field on configuration space such that the configuration $Q(t)$ changes?

2. How can the conditional wave function of a subsystem be defined in terms of the universal wave function?

The answer to the first question is: it cannot, at least alone. There must be change elsewhere, and that is the configuration $Q(t)$ itself. And that also answers the question at the beginning: $\Psi$ is not everything but something.

The answer to the second question is: it cannot; it is just a limit, i.e. it describes an asymptotic behavior. The wave function $\psi_t(x)$ enlarged towards the totality is stationary and does not solve the Schrödinger's equation. Thus the SE is just *phenomenal*, and so is $\psi_t(x)$.

So far, we have learned our law, the universal wave function, firstly does not change with time, secondly is unique, and thirdly is something. Given our domain unifying goal, we are interested in having the observer as a subsystem in the sense of $(\psi(x_{obs}), X_{obs})$ included in the universe without remainder, which would render $(\Psi, Q)$ *everything*. The whole would be the parts and nothing but the parts, ruling out emergentism and reductionism. There would be pure *identity* if there wasn't some experiencing cognizer—not experiencing wouldn't be enough. But before we leave the realm of Bohmian mechanics, we should first continue with BM—which is honest matter—to ground our logically abduced ontological theory DUCKs.

### 4.2.2 The Transition

"You start with just $\psi$, you end with just $Q$."[79] That is the result of „eliminating" the WF by making it universal, i.e. time-independent and unique—a law. Thus, the question of ontology not only refers to introducing particles, that which is to be moved, but also to exercising a transition from the prepared approximate to the observer-absent exactly. And the question now is what is the ontological difference between $\psi$ and $\Psi$ other than the latter being complete, i.e. everything having been taken into account that determines the universal WF, and that is the *whole* configuration $Q$?

Goldstein and Zanghì in *Quantum Physics Without Quantum Philosophy* ask

> *Why should the law of motion governing the behavior of the constituents of the universe be of such a form that there is a wave function in terms of which the motion can be compactly expressed?*[80]

and answer

> *We should read off from the state of the primitive ontology, whatever it may be, what the relevant wave function is. There should be some algorithm connecting the state of the primitive ontology, for Bohmian mechanics the relevant configuration $Q(t)$ over, say, some suitable time interval, with the relevant wave function.*[81]

[79] Cf. [18, 271].
[80] Cf. [18, 271].
[81] Cf. [18, 271].

An algorithm connecting $Q(t)$ with $\Psi$ over some suitable time interval—that is doubted to exist since one "wouldn't expect there to be an algorithm for theory formation" for "specifying the wave function amounts to specifying the theory."[82] That there is a theory, call it $\Psi$, including a theory for building, i.e. epistemically preparing a subsystem, call it $(\psi_{obs}, X_{obs})$ or $(\psi_{alg}, X_{alg})$[83], such that $\Psi$ is being built is precisely the problem we are concerned with in DUCKs, wishing to integrate the mental into a unifying theory, although it not necessarily being nomologically necessary for theory formation. We keep these „mind games" for more detailed elaboration later.

Here, we ask what if one took $\Psi$ to be only *quasi-nomological* or *quasi-material*, there being again some hidden variable now reflecting the necessary conditions for change the particles mutually impose? And what if those conditions had a position? With that one would have "exclusively local beables"—the particles being the source and target for their necessarily conditioned change by some additional kind of "primitive ontology". And those primitives again move.

The goal then is to find an interpretation of $\Psi$, the universal wave function, to impose its condition for change on the particles in $\mathbb{R}^3$, which is first and foremost the attempt to ontologically connect the configuration space to physical space and, best, to eliminate configuration space as a necessary ingredient of quantum mechanics. Actually, this has been the desire of wave function theory from the outset. Those endorsing particles being guided by waves—satisfying particle/wave dualism—thought or at least hoped that one could have all of the physical entities living in 3- or 4-space.[84] The guiding wave function being defined on configuration space seemed to be inherited only by Schrödinger's time evolution of $\psi$. There ought to be $N$ (local) particles being guided by their allocated local wave functions, which makes $N$ pairs of corresponding local entities.[85]

But it turned out that those pairs failed to describe the intrinsic quantum mechanical phenomenon of entanglement. The "non-local beable"[86] $\psi$ in $\mathbb{R}^{3N}$ could not be transformed into $N$ local beables in $\mathbb{R}^3$ without loosing the nature's feature of non-locality. How it could be achieved to at least leave configuration space behind is shown by Norsen in a self-declared "un-serious toy model" of two spinless, non-relativistic particles moving in one spatial dimension—*The Theory of (Exclusively) Local Beables (TELB)*[87]:

> *it* contains *N particles and N pilot-wave fields on physical space, but includes also a number of* additional *fields (also on physical space) which capture what is described in ordinary QM as 'entanglement' between particles.*[88]

More precisely, the number of additional fields shows to be infinite to exactly reproduce Schrödinger's wave function, and Norsen asks what the more fundamental description is—

[82] Cf. [18, 272].

[83] 'alg' for 'algorithm'

[84] Cf. [22, 83]: Einstein for instance thought that "Schrödinger's works are wonderful—but even so one nevertheless hardly comes closer to a real understanding. The field in a many-dimensional coordinate space does not smell like something real."

[85] Cf. [30, 62]: "[D]e Broglie asserts that configuration space is 'purely abstract', and that a wave propagating in this space cannot be a physical wave: instead, the physical picture of the system must involve $N$ waves propagating in 3-space."

[86] Cf. [72].

[87] Cf. [79, 1879,1880]: Adding the two missing dimensions is just a computational extra effort, spin and relativistic generalizations to be more serious endeavors as with „ordinary" BM.

[88] Cf. [79, 1878].

maybe the ordinary wave function appears to be just an approximation of a TELB. And this question precisely matches our discussion of how to valuate the (background) ontology of scientific theories, here quantum mechanics. He goes on:

> *Unfortunately, [...] the most dominant approach to trying to uncover the reality behind quantum mechanics, is to let the theory itself tell us—as people sometimes say, to simply 'read the correct ontology off' from the equations of the theory.*[89]

With that, he alludes to the 'many world' or *'Relative State' Formulation of Quantum Mechanics*[90], which indeed seems a little obsessed over completeness of wave function ontology. But is it so unfortunate to look at the equations and look for possible correspondences in reality if they are available? Of course, Norsen appreciated the founders' having directed their attention to sense data of, for example, the "smallness of the scintillation on the screen" of a double slit experiment proposing the existence of waves and particles[91], calling the view of the latter being "some kind of mere epiphenomenon [...] a bizarre suggestion"[92]. And then he tries this transformation of the resulting non-local wave function being defined on configuration space to a number of local wave functions on physical space. Thus, what he does is, first, *retroduction*:

> *[O]ne tries to work out the correct ontology, the correct slate of beables, first—by making abductive inferences from the qualitative behaviors observed in certain key experiments—and then one worries about how to formulate/infer the mathematical laws governing the behavior of those beables.*[93]

Given that abductive reasoning worked in that way, i.e. starting from a set of observations yielding at "the correct slate of beables" and then giving their time evolution a mathematical law, Norsen adopts orthodox Bohmian mechanics. And then second, he accomplishes his *reconstruction* of a system, here consisting of $N = 2$ particles in 1-dimensional physical space with their generic configuration $(x_1, x_2)$, the potential energy $V(x_1, x_2, t)$ at time $t$ due to "some kind of contact interaction"[94], the system's wave function $\Psi(x_1, x_2, t)$, and—with BM—the actual configuration $X = (X_1(t), X_2(t))$ yielding the conditional wave functions for the two particles[95]

$$\psi_1(x,t) = \Psi(x, x_2, t)|_{x_2 = X_2(t)} \tag{4.2.32}$$

and

$$\psi_2(x,t) = \Psi(x, x_1, t)|_{x_1 = X_1(t)} \tag{4.2.33}$$

The non-local system $(\Psi, X)$ then would be transformed into the local beables $(\psi_1, X_1)$ and $(\psi_2, X_2)$, i.e. the particle $i$ being guided by the local wave function $\psi_i(x,t)$ with the position's time evolution

$$\frac{\mathrm{d}X_i(t)}{\mathrm{d}t} = \frac{\hbar}{m_i} \,\mathrm{Im} \left( \frac{\partial \psi_i(x,t)/\partial x}{\psi_i(x,t)} \right) \Bigg|_{x = X_i(t)} \qquad , \; i = 1, 2 \tag{4.2.34}$$

[89] Cf. [79, 1881].
[90] Cf. [35].
[91] Cf. [72].
[92] Cf. [79, 1879].
[93] Cf. [79, 1882].
[94] Cf. [79, 1867].
[95] Cf. [79, 1866].

But that is shown not to work. The time evolution of the conditional wave function for particle 1, for example, is *not* of the form

$$i\hbar\frac{\partial\psi_1(x,t)}{\partial t} = -\frac{\hbar^2}{2m_1}\frac{\partial^2\psi_1(x,t)}{\partial x^2} + V(x, X_2(t), t)\psi_1(x,t) \tag{4.2.35}$$

but there are additional $x_2$-derivatives of $\Psi(x, x_2, t)|_{x_2=X_2(t)}$ that appear and have to appear to obtain *determinate behavior* for the initial particle positions and conditional wave functions.[96] The point is, the particle's locality is exclusively given by its local position in physical space and its local conditional wave function, but the dynamics is manifestly non-local:

> *In the end, we have a (countable) infinity of fields associated with each particle: a conditional wave function which (directly) determines the particle's velocity at each instant, and then a hierarchy of additional fields which influence the conditional wave function (and one another)[.]*[97]

This kind of new local beable or field hierarchically conditioning fields Norsen calls "non-local causation", non-locality now being defined by an infinity of *local* fields. One could reply that this then is not a TELB, since, although all beables are now local, the dynamics, i.e. the conditional wave functions' time evolution, is not. Nevertheless, the goal is achieved, and one could gain new insights if one was to investigate non-locality. Before we discuss the possible connection to the DUCKs' *continuity*, we now quickly go into a bottom-up approach, leaving ordinary pilot wave theory behind and following Norsen starting from scratch.

His ingredients are again two moving points assigned by something existing—particles—in 1-dimensional space without spin and relativistic complications and their dynamics, which is given by the two associated pilot wave fields and again an infinite set of what he calls "entanglement fields", all local and no configuration space from the beginning.[98] Of course, one is tempted to ask why use such strange things like pilot wave and entanglement fields and not ordinary classical fields, especially where "pilot wave and entanglement fields are *invisible*". Certainly, classical fields are invisible, too; they just manifest with test particles. Nevertheless, there is the abduction again, the observations forced to introduce an appropriate construct, and entanglement is an obvious observation, and thus the opposite approach of forgetting about ordinary wave functions but still adopting the equations of motion, i.e. the beables' time evolution, probably could yield new insights.

The three equations for particle 1 (same as for particle 2), now without conditional wave functions, are „again“[99]

$$\frac{\mathrm{d}X_1(t)}{\mathrm{d}t} = \frac{\hbar}{m_1}\,\mathrm{Im}\left(\frac{\partial\psi_1(x,t)/\partial x}{\psi_1(x,t)}\right)\Bigg|_{x=X_1(t)} \tag{4.2.36}$$

[96] Cf. [79, 1867,1868]: The initial particle positions and conditional wave functions can be identical still the dynamics of the particles differing. This is when the „system's“ wave function does not factorize, meaning the two particles are entangled.
[97] Cf. [79, 1871].
[98] Cf. [79, 1872].
[99] Cf. [79, 1872].

$$
\begin{aligned}
i\hbar\frac{\partial\psi_1(x,t)}{\partial t} =& -\frac{\hbar^2}{2m_1}\frac{\partial^2\psi_1(x,t)}{\partial x^2} + V(x, X_2(t), t)\psi_1(x,t) \\
& - \frac{\hbar^2}{2m_2}\psi_1''(x,t) + i\hbar\frac{\mathrm{d}X_2(t)}{\mathrm{d}t}\psi_1'(x,t)
\end{aligned}
\tag{4.2.37}
$$

$$
\begin{aligned}
i\hbar\frac{\partial\psi_1^{(n)}(x,t)}{\partial t} =& -\frac{\hbar^2}{2m_1}\frac{\partial^2\psi_1^{(n)}(x,t)}{\partial x^2} - \frac{\hbar^2}{2m_1}\psi_1^{(n+2)}(x,t) \\
& + i\hbar\frac{\mathrm{d}X_2(t)}{\mathrm{d}t}\psi_1(x,t)^{(n+1)} + P_n
\end{aligned}
\tag{4.2.38}
$$

with the potential being involved in

$$
P_n \equiv \sum_{i=0}^{n}\binom{n}{i}\frac{\partial^i V}{\partial x_2^i}(x, X_2(t), t)\psi_1^{(n-i)}(x,t)
\tag{4.2.39}
$$

The pilot wave field $\psi_1(x,t)$, entangle fields $\psi_1^{(n)}(x,t)$, and potential $V(x, X_2(t), t)$ properly chosen determine the future particle's position $X_1(t)$ effectively the way BM does.

With that, we have local beables only, which now allows us to return to the key problem of *back action*, or the *action-reaction principle*, in more detail:

> *In the "dualist" version of the Bohm theory, reality is attributed both to the particle trajectories and to the wave funtion, the latter being treated as a real "$\psi$-field" which causally acts on the particle, "informing" it as to where to go. But the particle does not react back on the wave function. So, they do not satisfy the AR principle between them.*[100]

### Back Action

As said, it is often interposed that there is action or at least influence of the wave function on the particles resulting in change but not the opposite direction, and sometimes this fact is put forward to identify particles as non-existent. But maybe the wave *function* is the mere epiphenomenal part. On the way to finding the real things behind the wave function, with TELB one might have moved one step further. Now every particle $X_i(t)$ is directed by its associated pilot wave field $\psi_i(x,t)$. But this field's evolution is not just determined by the usual Schrödinger's equation but also by the „entanglement fields" $\psi_i'(x,t)$ and $\psi_i''(x,t)$[101], which in turn obey equation (4.2.38), including the other particle's instantaneous position and motion and the potential field, where one particle moves as a function of the actual position of the second particle.[102]

Thus, TELB being equivalent to BM indeed suggests that there is back action, i.e. full interaction between particles:

[100] Cf. [40, 358].
[101] Cf. (4.2.37).
[102] Cf. (4.2.39).

(1) The pilot wave fields depend on the other particles' actual positions.[103]

(2) The particles' actual positions are influenced by the other particles' actual positions.[104]

Now of course with that, we don't know what "action" is conceived of. So far, it is just a logico-mathematical description of beables necessitating beables, and since the necessitation is being executed, the theory has to be furnished with a time direction or flow.[105]

But that is exactly what we were aiming for. The non-locality of BM crumbling into infinite locality of TELB admit, though both being mathematically equivalent theories, completely different interpretations as regards ontology and causation. Norsen's lesson from that is

> *[I]t seems quite naive to think that one is going to learn anything useful about external reality by staring at some one particular mathematical formulation of a theory[.]*[106]

Here, staring at the two formulations could strengthen the view that the wave function $\psi$ should be treated more as nomological rather than ontological. Nonetheless, pessimism is not indicated: if one was to seek ontology, one way to find it rich would be to make use of the combination of time-honored concepts, here the mathematically precise non-locality of the conditional wave function and the intuition of localized particles, and give the necessary conditions for the particles' change a position, too. And that is what the conditional, and finally the universal, wave function had hoped to capture: *local conditioning.*

In the case of TELB, at the spatio-temporal position $X_1(t)$, there are two pieces of information about the second particle:

(1) The spatio-temporal position of the second particle $X_2(t)$.

(2) The way the second particle's spatio-temporal position $X_2(t)$ determines the first particle's change in spatio-temporal position.

The question is how can this kind of non-local determination be possible, or now practically, how can it be achieved—by local beables? Therefore, we field our continuity NEXUS (4.1.21) from (con 10) with the propositions of actual position already inserted:

$$\begin{tikzcd}
X_2(t^*) \arrow[r, "\text{oNC}"] \arrow[d, "\text{inNC}"'] \arrow[dr, "\text{nNC}"] & X_2(t) \arrow[d, "\text{inNC}"] \\
X_1(t^*) \arrow[r, "\text{oNC}"'] & X_1(t)
\end{tikzcd} \tag{4.2.40}$$

Of course, this is not exactly what we introduced as a proposition of existence, a spacial location to exist in space, but in BM, the coordinates are seen to refer to primitives, the particles. Nevertheless, it is an instructive inception to integrate the BM's

[103] Cf. (4.2.37).
[104] Cf. (4.2.36).
[105] Actually, we will see that time flow is inherited by the fact of non-locality.
[106] Cf. [79, 1881].

or—equivalently—TELB's ingredients, and we come back to that issue soon.

Then also propositionalizing the mathematics, the second particle's actual position $X_2(t)$ is an instantaneous necessary condition for particle 1 at time $t$ to exist at spacial location $X_1(t)$ and to change (location). But this is not the only necessary condition. In our NEXUS, there are two more. $X_1(t)$ is then necessitated due to

(1)

$$X_2(t) \xrightarrow{\text{inNC}} X_1(t) \tag{4.2.41}$$

(2)

$$X_1(t^*) \xrightarrow{\text{oNC}} X_1(t) \tag{4.2.42}$$

(3)

$$X_2(t^*) \xrightarrow{\text{nNC}} X_1(t) \tag{4.2.43}$$

And the question is how do these determinations correlate?

TELB (and also BM) refers to (1): $X_2(t)$ is an instantaneous necessary condition for $X_1(t)$. And it is so by „instantaneous influence“ of the second particle's position on the structure of the associated wave function $\psi_1(x,t)$ given by its „equation of motion“ (4.2.37). In $\psi_1(x,t)$, as in the time evolution (4.2.38) of the entanglement fields $\psi_1^{(n)}(x,t)$, there is the actual position $X_2(t)$ involved in the potential $V(x, X_2(t), t)$ and the velocity $\mathrm{d}X_2(t)/\mathrm{d}t$ of the second particle at the same time $t$. With (4.2.36), it holds that (4.2.41).

The necessary conditions at time $t^*$ in (2) and (3) are not explicitly involved in $X_1(t)$. But in the derivative (4.2.36), there is a prior position necessary, say w.l.o.g. at $t^*$, if only in its limit $\lim_{t^*\to t} X_1(t^*) = X_1(t)$, where for every real $\epsilon > 0$, there exists a real $\sigma > 0$ such that for all real $t^*$, $0 < |t^* - t| < \sigma$ implies $|X_1(t^*) - X_1(t)| < \epsilon$ which mathematically states the continuity conjecture. With that, (2) and (3), the NEXUS (4.2.40) is thus implicitly given in (4.2.36). But still, how can it be executed—the instantaneous "incredible fine-tuning of initial conditions"[107] at an instance of time $t$?

The answer is (3): the proposition '$X_2(t^*)$ is a nomological necessary condition for $X_1(t)$' is equivalent to 'the successor of $X_2(t^*)$ at time $t$ will be an instantaneous necessary condition for $X_1(t)$'.

The underlying ontology of the logico-mathematical model NEXUS (4.2.40) is then

[107] Cf. [79, 1873].

$$\begin{tikzcd}[column sep=huge, row sep=huge]
{}_2m^{t^*}_{x(2,t^*)} \arrow[r, "\Delta_2^{t^*,t}"] \arrow[d, "\Delta_{2,1}^{t^*}"'] \arrow[dr, squiggly, "\Delta_{2,1}^{t^*,t}"] & {}_2m^{t}_{x(2,t)} \arrow[d, "\Delta_{2,1}^{t}"] \\
{}_1m^{t^*}_{x(1,t^*)} \arrow[r, "\Delta_1^{t^*,t}"'] & {}_1m^{t}_{x(1,t)}
\end{tikzcd} \tag{4.2.44}$$

saying the successor of the quality particular ${}_1m^{t^*}_{x(1,t^*)}$ at location $x(1,t^*) \equiv X_1(t^*)$ will find at location $x(1,t)$ the $k$-th (necessarily) necessary condition for change (NCC) ${}^{t^*}_{k_2}\square^t_{x(1,t)}$ having ${}_2m^{t^*}_{x(2,t^*)}$ as source. Our convention of naming by indexing the quality particulars obtains[108]

$$\begin{tikzcd}[column sep=huge, row sep=huge]
{}_2O^{t^*}_{x(2,t^*)} : {}_{k_2}\square^{t^*}_{x(2,t^*)} \arrow[r, "\Delta_2^{t^*,t}"] \arrow[d, "\Delta_{2,1}^{t^*}"'] \arrow[dr, squiggly, "\Delta_{2,1}^{t^*,t}"] & {}_2O^{t}_{x(2,t)} \arrow[d, "\Delta_{2,1}^{t}"] \\
{}_1O^{t^*}_{x(1,t^*)} \arrow[r, "\Delta_1^{t^*,t}"'] & {}_1O^{t}_{x(1,t)} : {}^{t^*}_{k_2}\square^t_{x(1,t)}
\end{tikzcd} \tag{4.2.45}$$

Now for a „system“ consisting of two quality particulars only—and the NCCs being enroute—the situation seems to be almost easy. Still, where is the ontology? All the arrows are just epistemic. Without them and with only the NCC that comes on top at time $t$, the NEXUS appears quite „liberal“:

$$\begin{tikzcd}[column sep=huge, row sep=huge]
{}_2O^{t^*}_{x(2,t^*)} : {}_{k_2}\square^{t^*}_{x(2,t^*)} & {}_2O^{t}_{x(2,t)} \\
{}_1O^{t^*}_{x(1,t^*)} & {}_1O^{t}_{x(1,t)} : {}^{t^*}_{k_2}\square^t_{x(1,t)}
\end{tikzcd} \tag{4.2.46}$$

Or, even worse, when presentism is taken seriously, i.e. no $t^*$ being involved

$$\begin{tikzcd}[row sep=huge]
{}_2O^{t}_{x(2,t)} \\
{}_1O^{t}_{x(1,t)} : {}^{t^*}_{k_2}\square^t_{x(1,t)}
\end{tikzcd} \tag{4.2.47}$$

[108] Assigning the particulars a ‘mass’ isn’t relevant here.

And again, this is an easy toy system of two quality particulars mutually intradetting, meaning there is just one source to be mutually „taken into account“[109]. The two-directional situation then is this

$$ {}_2O^t_{x(2,t)} : {}^{t^*}_{l_1}\Box^t_{x(2,t)} \tag{4.2.48}$$

$$ {}_1O^t_{x(1,t)} : {}^{t^*}_{k_2}\Box^t_{x(1,t)} $$

And that is the situation the observer has to capture by calculating

$$\begin{tikzcd} X_2(t) \arrow[d, shift left, "\text{inNC}"] \\ X_1(t) \arrow[u, shift left, "\text{inNC}"] \end{tikzcd} \tag{4.2.49}$$

with the mathematics given in (4.2.36) and (4.2.37). The task would be to *predict* the behavior of $X_1(t)$ and $X_2(t)$ without knowing about the NCCs—especially, where and when they were „emitted“. And that is the „functional role“ of the associated wave functions $\psi_1$ and $\psi_2$. They provide the two particles with the „information“[110] of the other's spacial position, and the „effect“ they have is given by the two equations, the Schrödinger's equation and the guiding equation.

The question now is can the NCCs capture all the conditions for the particles' change that are present at the objects' positions and are implicated in the local associated and entanglement fields? The particles are guided by their pilot wave fields, and those include the information of the second particle's instantaneous position, the "'conditional potential', i.e., the potential field in which, for example, particle 1 moves *given the actual location of particle 2*"[111] being integrative. Thus, the particles' positions depend on the particles' positions, and the local associated and entanglement fields' *structure* of how to place the necessary conditions for change. This structure emerges due to spatio-temporal considerations, i.e. derivatives. What we have in the end are positions *sufficiently* determining positions in geometrical physical spacetime, and that is what we hoped to capture with the second sort of beables, the NCCs.

Coming back again to the role of the universal wave function in determining the particles' trajectories leads to the reasonable presumption: generalizing TELB to BM yields DUCKs as the overall theory of conditionalizing[112].

[109] Certainly, the toy model has to be constructed by some observer. But we already neglected the relations, and referring to Norsen's two interacting particulars is seen to be just an intermediate state on the way to the universe.

[110] Cf. [67]: active information.

[111] Cf. [79, 1879].

[112] The focus is traditionally on the particles and their positions with the necessary conditions for change

## 4.3 DUCKs and Bohmian Mechanics

In the guidance equation (4.2.3), $\mathrm{d}Q_i(t)/\mathrm{d}t$ is strictly speaking the time derivative of a coordinate that is simply traditionally assigned to a quality that is called the mass $m_i$. It is a spacial location that is being tagged by a certain quality, thus it is a quality particular—the position $Q_i(t) \in \mathbb{R}^3$ to be assumed to be „filled“ by a quality connected to $m_i$, and which quality it realy refers to does not matter. But that could correspond to our object $O_i^t := {}_iO^t_{x(i,t)}$ with $\mathrm{d}x(i,t)/\mathrm{d}t$ if we allowed time derivatives, but of course we don't, because we only need change over time. Furthermore, the position $Q_i(t)$ is determinately guided by the wave function $\psi(q_1, ..., q_N, t)$, and it is a solution of the Schrödinger's equation (4.2.4). As a result, even the „free“ particle $Q_i(t)$, meaning that there is no additional potential ($V(q_1, ..., q_N) = 0$), depends on all the spatially distributed qualities $m_j$ of the universe. And in a way, that makes all ontological intradets between objects nomological in the pilot equation (4.2.3), which is key for the correspondence between BM and DUCKs[113].

And here then is the ontology of the wave function. Where $Q(t)$ describes the actual configuration of all particles in the universe at time $t$, the wave function $\Psi_t$ assigns to the generic configuration of all NCCs in the universe at time $t$. Mathematically speaking, it integrates all information that can be found at any location in the configuration space $\mathbb{R}^{3N}$, and if a particle resides at a certain place in $\mathbb{R}^3$, all the information that is needed to *calculate* the next particle's position is given. The universal wave function is then *holistic* in the sense that it corresponds to the *totality* of all necessary conditions for change that at time $t$ happen to be located at all positions in physical space. The correlation between BM and DUCKs is then

**Conjecture 13** (DUCKs' BM)**.** DUCKs correlates to BM in

(1) Quality particular:
$$O_i^t \mathrel{\widehat{=}} m_i(t) \tag{4.3.1}$$

(2) Spatio-temporal position:
$$x(i,t) \mathrel{\widehat{=}} Q_i(t) \tag{4.3.2}$$

(3) Exogenous NCC:
$${}_i\check{\boxplus}^t_{x(i,t)} \mathrel{\widehat{=}} \psi(x_i, t) \tag{4.3.3}$$

with $\psi(x_i, t) = \Psi(x_i, Y_t, t)$ being the conditional wave function.[114] ♦

Some remarks on the correlations:
(1) Quality particular:
Concerning the Schrödinger's and pilot equations, it seems that there is no particle or primitive or beable without there being a correlated „mass“ $m_i$. If there was, for example, an extra determiner added in the form of a so-called charge, this would be integrated into the potential $V$, providing the circumstances the particles find themselves in. Those

departure and arrival. It will have to stand up to further scrutiny whether it would be revealing to invert the perspective—watching the NCCs collide with the objects (cf. (4.3.7)).

[113] Cf. (4.2.44).

[114] Cf. [18, 42, 265] and (chap 4.2.1).

charges then would again be primitives that happen to be at the same position some primitive is fixed to correlating to some „mass“. As said, an electron $e_i$ wouldn't be a particle having mass $m_i$ and charge $q_i$, but at position $x(i,t) = x(j,t)$, there was the quality particular $O_i^t$ and the quality particular $O_j^t$. Thus, the potential would be given quality particulars.[115]

(2) Spatio-temporal position:
The matter point $Q_i(t)$ is seen to be located at what is called the mass $m_i(t)$ by reason of its sole appearance in the guiding equation (4.2.3) as some kind of particular quality. Thus, there is an intuitive connection to 'matter points', but we will return to these relations.

(3) Total NCC:

**Definition 49** (Total NCC)**.** The *total NCC* in ${}_iO^t_{x(i,t)} : {}_i\boxplus^t_{x(i,t)}$ is the conjunction of all *endogenous* and *exogenous* necessary conditions for change:

$$ {}_i\boxplus^t_{x(i,t)} := {}_i\hat{\boxplus}^t_{x(i,t)} \wedge {}_i\check{\boxplus}^t_{x(i,t)} \tag{4.3.4}$$

▲

**Definition 50** (Total Endogenous NCC)**.** The *endogenous* conditions (*endocons*) are necessary conditions for objects to change, which have the quality particular ${}_iO^t_{x(i,t)}$ as source:

$$ {}_i\hat{\boxplus}^t_{x(i,t)} := \bigwedge_{\hat{k}_i=1}^{\hat{I}} {}_{\hat{k}_i}\hat{\Box}^t_{x(i,t)} \tag{4.3.5}$$

▲

**Definition 51** (Total Exogenous NCC)**.** The *exogenous* conditions (*exocons*) are necessary conditions of objects to change, which have the quality particular ${}_iO^t_{x(i,t)}$ as sink:

$$ {}_i\check{\boxplus}^t_{x(i,t)} := \bigwedge_{\check{k}_i=1}^{\check{I}} {}_{\check{k}_i}\check{\Box}^t_{x(i,t)} \tag{4.3.6}$$

▲

Summing up, in Bohmian mechanics, there are (mass) particles (4.3.1), positions (4.3.2) seen to be marked by the particles, and conditional wave functions (4.3.3) that impose conditions on how to change. Thus, BM is about marked positions moving in space, and no activity is involved in the manner of some causal guidance:

> *The relationship between the wave function and the motion of the particles is more appropriately conceived as a nomic one, rather than as a causal one in which one physical entity acts on the other.*[116]

[115] Cf. (4.2.5).
[116] Cf. [51, 779].

And indeed, there is no „acting on“ in the equations (4.2.3) and (4.2.4), both just saying what is going to happen. And there is no understanding besides stating the state of affairs that correspond to the modelling. And what about the law? The nomological intra-time determination is just another way of describing the happening but with less provision as far as geometric circumstances are concerned. There is no need for the wave function to belong to the ontology, but there is one for the conditions for change of the objects, and those are *mathematically* described by $\psi$.

In our experimentally informed intuition, we refer to localizable beables that change and are the source and targets of the conditions of their change. In this picture, a non-local beable would be preferably desirable not to come up, and if it came up, one would like to have it transformed into a local one, hopefully living in physical space as we do. Now it is a fact that Schrödinger came up with his non-local wave function in configuration space and that it has been adopted in BM. Maybe it was a historical necessitation, but it also could have been a psychological one, which is of course not a contradiction upon closer inspection. Norsen's TELB gave us a good example of how things could become complicated if one wished to refrain from a second, configurational realm, and he is probably right in saying:

> *In fact, we think in fact that if someone gave us a Bohmian kind of theory, involving a complicated collection of exclusively local beables, and then someone else pointed out to us that the complicated local beables can be repackaged into a simple mathematical object of a nonlocal character—like a wave function on configuration space—our reaction would likely be that we would prefer to regard the wave function in that simpler though more unfamiliar way, just because of its mathematical simplicity.*[117]

The wave function does not refer to the positions of the necessitators but of their „launching pads“. The reason why this indirect determination nonetheless works is that, given all information about how the other objects change at the changing object ${}_iO^t_{x(i,t)}$, the "repacking" can be reversed. Due to the initial becoming of the universe from a singularity—the big bang—the exocons ${}_{\check{k}_i}\check{\square}^t_{x(i,t)}$ are sufficient to give indication of all particulars' actual position and quality at any time $t$. But it might turn out that the description in configuration space is indeed the most elegant, i.e. the easiest one and maybe just one that is convenient for most practical purposes.[118]

**Conjecture 14** (Ockham's DUCK)**.** The universal wave function is a mathematical representation of all NCCs. ♦

Having discussed the „role“ of the wave function, one started with the orthodox quantum theory (OQT), which says that $\psi$ is complete in describing a prepared experiment. As an extension, the orthodox Bohmian mechanics (OBM) then keeps the wave function as a means to guide added point-like entities, the so-called *primitive ontology*. In a second transition, in universal Bohmian mechanics, the system's wave function may be abandoned for its being defined in terms of the universal wave function only having nomological and no ontological, physical status. In a final step of rehabilitation, the physical reality of the universal wave function $\Psi$ remains with conjecture (con 14)—the necessary

[117] Cf. [18, 275].
[118] Cf. [79, 1879,1880].

conditions for change also being assigned local beable status. Thus, we started with non-local wave functions and end with extended local beables in a universal Bohmian theory (UBT*), which corresponds to DUCKs:[119]

$$\begin{array}{ccccccc} \text{OQT} & & \text{OBM} & & \text{UBT} & & \text{UBT}^{*} \\ \psi & \longrightarrow & (\psi, Q) & \longrightarrow & Q & \longrightarrow & (Q^{\boxplus}, Q) \end{array}$$

And with it, the focus changes. Quantum theory is no longer about the behavior of wave functions but about the entities underlying them, and these are all necessary conditions for change sufficient to describe the *perceived* quantum phenomena. And if one was to refer only to the main phenomenon of the invention of Schrödinger's (epistemic) wave function, the overall transition would finally be

$$\begin{array}{ccc} \text{OQT} & & \text{UBT}^{**} \\ \psi & \longrightarrow & Q^{\boxplus} \end{array} \tag{4.3.7}$$

Now, all that effort is made to make further considerations tempting for considering more than there is according to a physical theory with some sophisticated experimental metaphysics, which is Bohmian mechanics. There are no subsystems, end of story. But what if there are objects that are emergent by transformation processes of the objects and NCCs if it holds that nothing evolves out of nothing? Then there has to be some criterion for the existent to be transformed into some emergent, and that is something which could be rated a subsystem.

That kind of transformation process is not captured by the Bohmian mechanics. The physically necessitated spatio-temporal *mental object* just doesn't appear in the theory, and it should if the mental exists. In the following chapter, we will therefore consider the relations between emergent and non-emergent objects, and that in the language of category theory.

[119] Cf. [18, 271].

## 4.4 Aggregations, Systems, and Substrates

An aggregation is commonly seen to be some collection of a number of objects with or without interrelations, and it is a matter of *organization* whether aggregations turn into systems with or without extra-systemic interrelations. If those objects cluster due to some organization, can they then still be considered aggregations? From the viewpoint of an agglomerating object, i.e. an object that attaches more or less tightly to some other object, is it really a matter of organization or even the mode[120] of organization that it is now more than it was before, the parts being more than the sum?

*Aggregativity* is not definable without taking into account its opposite, i.e. non-aggregativity, which could be viewed as more or less than aggregativity. More would be some kind of at least new properties coming into existence, suggesting an emergentist approach: "Emergence can be analyzed as a failure of aggregativity"[121], saying that objects just aggregating are in their natural state in some way. But what about less than aggregating? That would be single objects of whatever kind not even aggregating, i.e. there is no criterion for them to gather. That means that stating some aggregation of objects without having some criterion of *selection* in *mind* is nothing but the single objects themselves. In short, the ability of some gathering being perceived as an aggregation and also the principle of aggregation itself—aggregativity—is not possible without some property that restricts the gathering, and that property is obviously not in the aggregates themselves but necessary for the restricting criterion, and being necessary for a criterion is in a way *emergent*. To the point, the aggregation along with the, let's call it, non-aggregation$^{\text{plus}}$, which is also necessary for the criterion, necessitate (together) an emergent something. And that is what needs to be specified.

Now, as indicated the considerations of existence will take place within the category $\mathcal{U}^t$ of the universe-instance at time $t$ with the objects being the quality particulars and the morphisms the intra-time determinations.

### Aggregations

In the following, the objects stay the objects, and whatever mode may have arrived, it is still said to be an aggregation but now with certain *restrictions* on them, and obviously it's these restrictions that separate them from other objects. Without these still nebulous constraints, there is just one aggregation at an instance of time $t$, which is the universe-instance $\mathcal{U}^t$ itself. But before these restrictions are explored, a rigorous definition of an aggregation has to be given in the category theoretical setting.

**Definition 52** (Aggregation)**.** An *aggregation* $A^t$ in $\mathcal{U}^t$ with *restriction* $\mathcal{R}^t$ at an instance of time $t$ is a functor

$$A^t : \mathcal{R}^t \to \mathcal{U}^t \tag{4.4.1}$$

▲

Thus, an aggregation is a diagram in $\mathcal{U}^t$:

$$A^t(i, j : i \to j) = A^t(i, j) : A^t(i) \to A^t(j) = \Delta^t_{i,j} : O^t_i \to O^t_j \tag{4.4.2}$$

[120] Cf. [12] for modes of organization of systems and their decompositions.
[121] Cf. [11, 372].

and the restriction on the diagram in $\mathcal{U}^t$ is determined by the category $\mathcal{R}^t$. For $\mathcal{R}^t$ not to be arbitrary, there has to be a *criterion* that allows for the subdiagram $A^t$ to separate from the total diagram $\mathcal{U}^t$, and then to be called a *system*.

### 1st-Order Systems

People say that systems are aggregates with some duty. And if not, they think of systems as aggregates that perform a function, or, which is pretty much the same, that accomplish a task. Though opaque, they have got a point here: systems are aggregates that show a more or less predictive behavior with some *perceived* property all constituents are necessary for. The criterion to define, or better which defines, a system is that it is necessary and sufficient for some new *quality particular* to emerge, which naturally leads to the definition of *1st-order emergence*:

**Definition 53** (1st-Order Emergence)**.** A *1st-order emergent object* is the colimit of an aggregation ${}^1S_k^t : {}^1\mathcal{S}_k^t \to \mathcal{U}^t$, i.e. an object ${}^1\widehat{S}_k^t \in ob(\mathcal{U}^t)$ and a natural isomorphism

$$\mathcal{U}^t({}^1\widehat{S}_k^t, O_l^t) \cong [{}^1\mathcal{S}_k^t, \mathcal{U}^t]({}^1S_k^t, C_{O_l^t}) \tag{4.4.3}$$

▲

And with it comes the definition of *1st-order systems*:

**Definition 54** (1st-Order System)**.** A *1st-order system* ${}^1S_k^t : {}^1\mathcal{S}_k^t \to \mathcal{U}^t$ is an aggregation with restriction ${}^1\mathcal{S}_k^t$ at an instance of time $t$ for which the colimit ${}^1\widehat{S}_k^t \in ob(\mathcal{U}^t)$ exists. ▲

The definition of 1st-order emergence is an application of the general concept of colimits, including the naturality condition of commutative squares of natural transformations.[122] The right part of the isomorphic relation in (4.4.3) refers to the totality of cocones to be taken under an aggregation in the category $\mathcal{U}^t$. A cocone here is some vertex $O_l^t$ together with a set of injection maps that are the intra-time determinations with the source being the objects of the 1st-order system ${}^1S_i^t$. In other words, the totality of the cocones is the set of natural transformations between the functors from the restriction ${}^1\mathcal{S}_k^t$ to the universe $\mathcal{U}^t$ that start at the 1st-order system ${}^1S_k^t$ and end at an object $O_l^t \in ob(\mathcal{U}^t)$ with $C_{O_l^t}^t$ being the constant functor that maps the restriction to $O_l^t$. The left part of the natural isomorphism is the set of morphisms in $\mathcal{U}^t$ that start at the object ${}^1\widehat{S}_k^t$ and end at some object $O_l^t$.

Now, the isomorphic equation (4.4.3) says that, for every cocone of the system ${}^1S_k^t$, there is a *unique* intradet, the so-called factor, from the 1st-order emergent object ${}^1\widehat{S}_k^t$ to the cocone's vertex, the object $O_l^t$. That means, all intradets from the components of the system to $O_l^t$ factorize, and the corresponding triangles with the intradets of the *universal* cocone commute.

In (fig 4.1), a simple situation is depicted with four objects that have to be specified. In order to make out a *meaning* of them, morphisms have to be spread. A suggestion is given in figure (fig 4.2). The aggregation consists of the objects $O_i^t$ and $O_j^t$, and in combination with the intradet $\Delta_{i,j}^t$, they yield the 1st-order system ${}^1S_k^t$. And according

[122] Cf. [39, 267].


Figure 4.1: Four objects in $\mathcal{U}^t$.

to definition (def 54), there is no system without an emergent object that is necessitated by the aggregation. And finally for the factor to exist, an additional object with its cocone is necessary. Thus, a universal cocone to the 1st-order emergent object $O_k^t = {}^1\widehat{S}_k^t$ and an non-universal one to $O_l^t$ are entered.

### 1st-Order Structures

In general, if one aims to define or detect some kind of *structure*, objects have to be looked at, and looking at objects is done by looking at their morphisms. It has to be examined how the intradets interrelate, i.e. how they commute at an instance of time $t$. Hence, the recipe for spotting a structure is looking for *commutativity*, and an suggestion for a definition would be:

**Definition 55** (1st-Order Structure)**.** A *1st-order structure* in the category $\mathcal{U}^t$ is a triple

$$ {}^1\Sigma_{k,l}^t = \langle {}^1S_k^t, {}^1\widehat{S}_k^t, O_l^t \rangle \tag{4.4.4}$$

The object $O_l^t$ is called the *determinate* of the structure ${}^1\Sigma_{k,l}^t$. ▲

In this definition, structure should be ontic, and therefore, it is said to be of 1st-order. Although there is still no cognizer involved, it is required a little more than moving objects. What is required is moving objects plus something new having come into existence. The whole plays at an instance of time $t$, and therefore, so far there is no dynamics being considered. Structures are not patterns which are identified by having been assigned to a

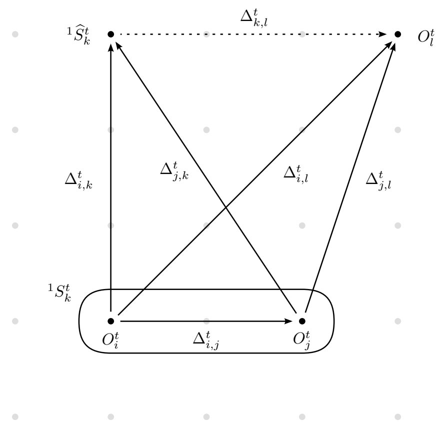


Figure 4.2: The four objects determining a structure with the system $^1S^t_k$, its emergent object $^1\widehat{S}^t_k$, and an arbitrary object $O^t_l \in ob(\mathcal{U}^t)$.

class or type due to perceived similarity or invariance. 1st-order structures are typically tokens and therefore ontic, although they almost don't exist, as far as we know.

But before we finally arrive at that ‘almost’, another major issue needs to be tackled here, that of *multiple realization (MR)*. In the definition of 1st-order emergence (def 53), nothing is explicitly said about multiple systems $^1S^t_{k_r}$ having the same colimit $^1\widehat{S}^t_k$.[123] Obviously, with the intra-time determinations in mind, that is impossible. Either it is not the case that both are necessary, or one is not sufficient. If objects are not necessary, according to the definition of the intradets, they are not part of the system, and if they are not sufficient, they don't shape the system. Thus, trivially there can be only one system necessitating an emergent object at an instance of time $t$:

**Proposition 3** (Unique Realization)**.** In the category $\mathcal{U}^t$, it holds that

$$(^1\widehat{S}^t_k) \cong [^1\mathcal{S}^t_k, \mathcal{U}^t](^1S^t_k, C^t_{O^t_l}) \cong (^1S^t_k) \tag{4.4.5}$$

◊

The parlance of ‘unique realization’ is a little unfortunate, since ‘realization’ purports some kind of activity, but the emergent object just appears to be—under certain conditions.[124] The notion is enforced by the entrenched usage of the technical term ‘multiple realization’. So far, at the ontic 1st-order stage, MR is not possible by definition of emergent objects.

[123] The situation is depicted in figure (fig 4.3): two homologous systems $^1S^t_{k_1}$ and $^1S^t_{k_2}$ having the same colimit $^1\widehat{S}^t_k$, meaning, for both it holds that they are necessary and sufficient for the emergent object to exist. Cf. (def 27).

[124] There is even nothing anticipatory about it, in contrast to what is claimed in [69] and [38], where life, when it turns to it, is built on anticipation.

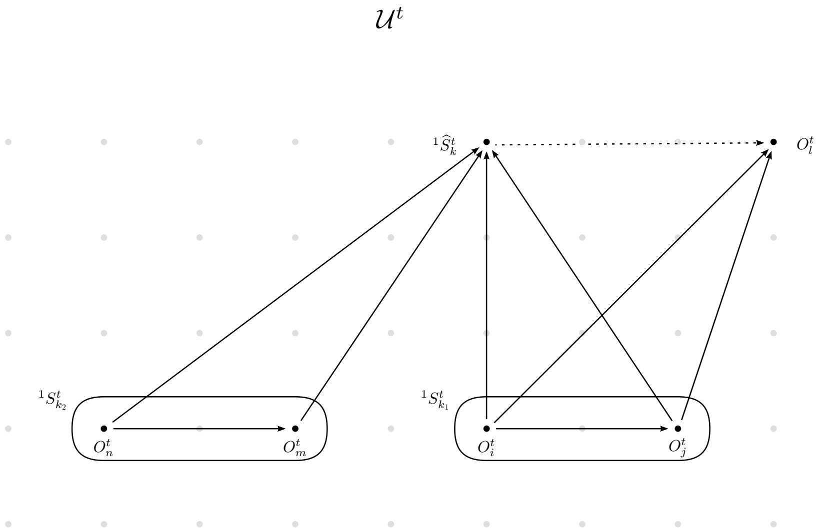


Figure 4.3: Two structures $^1\Sigma^t_{k_1,l} = \langle {}^1S^t_{k_1}, {}^1\widehat{S}^t_k, O^t_l \rangle$ and $^1\Sigma^t_{k_2,j} = \langle {}^1S^t_{k_2}, {}^1\widehat{S}^t_k, O^t_l \rangle$ with two homologous 1$^{\text{st}}$-order systems $^1S^t_{k_1}$ and $^1S^t_{k_2}$ ($\Delta^t_{n,l}$ and $\Delta^t_{m,l}$ not drawn).

### 4.4.1 Emergence

'Emergence' is also a theoretical term, and therefore, what is to be understood by it is a matter of definition. There are several features that can be woven in or rejected, with the result that more than one definition of emergence is circulating.

A feature that qualifies to be necessary for emergence is said to be

(i) new, i.e. it didn't exist before its coming into existence,
(ii) novel, i.e. it is of a new fundamental type,
(iii) unpredictable,
(iv) without structure, i.e it is not composed of parts,
(v) irreducible,
(vi) accompanied by new, novel, unpredictable, and irreducible causal powers or laws.

The first distinction that is being made concerns the degree of compatibility with physicalism meaning whether the emergent property of the system can be reduced to the system's parts. If the reduction can be accomplished in principle, people speak of *weak* emergence, and it becomes *strong* if the systemic property is irreducible. By introducing a time evolution and with it the possibility of *novelty* for a property, a further distinction is opened: *synchronic* versus *diachronic* emergence. Within a time-slice $t$, synchronic considerations don't allow for novelty. Trivially, a time transition is necessary for a property to be novel. Thus, the case that is most interesting is the systemic property that is novel and irreducible—strong diachronic emergence. But obviously, when the novel property has emerged at some instance of time, the situation is synchronic again, and the question is raised as to whether this new property is novel, meaning of a fundamental new type, *and* irreducible. And the question arises: What is a novel systemic property that is not irreducible? Novelty and irreducibility are, by definition of systems, features of systemic properties. Better still, every novel property is irreducible and vice versa. Thus, what remains to be presented is a or the rigorous description of how systemic properties could come about.

First, it is conventional that the emergence of some irreducible property of a system seems to be a matter of being composed of properties that don't show that systemic property in isolation or other compositions:

> *Put in abstract terms the emergent theory asserts that there are certain wholes, composed (say) of constituents A, B, and C in a relation R to each other; that all wholes composed of constituents of the same kind as A, B, and C in relations of the same kind as R have certain characteristic properties; that A, B, and C are capable of occurring in other kinds of complex where the relation is not the same kind as R; and that the characteristic properties of the whole R(A,B,C) cannot, even in theory, be deduced from the most complete knowledge of the properties of A, B, and C in isolation or in other wholes which are not of the form R(A,B,C).*[125]

In a more contemporary paper, the following definition for an emergent property P of a whole w is given, using mereological supervenience:

> *If P is a property of w, then P is emergent if and only if*
>
> (i) *P supervenes with nomological necessity, but not with logical necessity, on properties the parts of w have taken separately or in other combinations; and*
>
> (ii) *some of the supervenience principles linking properties of the parts of w with w's having P are fundamental laws.*[126]

On the way to a revision of the above definition of an emergent property that makes for the colimit's interpretation of emergent objects, the somewhat pedestrian restriction to parts that have their properties "taken separately or in other combinations"[127] can be dropped, since the „properties" in $\mathcal{U}^t$ can't be taken separately or in other combinations, and again, there is no having but being, and not of properties but particular qualities. A whole or system having a property P is just epistemic talk and could and should in the first place be replaced by a whole coming up is accompanied by some property necessitated by the whole—and the worry "for any property P of any whole w, there

[125] Cf. [17, 61].
[126] Cf. [8, 16].
[127] Cf. [42, 223].

will always be properties of the parts from which P may be deduced"[128] is unfounded.[129] Thus, the distinction between "taken separately" and "in other combinations" can be dropped, because, in order to change a combination, the constituents have to change and if the constituents change, their combination change, too.

A more substantial concern is the question whether determination is a logical or nomological necessitation? Is the intra-time determination a law or just a logical relation? The intradet $\Delta^t_{i,j} : O^t_i \to O^t_j$, meaning 'the existence of $O^t_i$ is a necessary condition for $O^t_j$ to exist', results in the existence of $O^{t'}_j$ at an instance of time $t < t'$ and states at $t$ the *tendency for change* $\Delta^{t,t'}_{j,j}|_t : O^t_j \to O^{t'}_j$. The necessity condition defined in (chap 4.1) is not at all lawlike. It just says that it is going to happen without any need for further assumptions in addition to what is being included in the objects as sources and sinks, there being no extra existence of something. This is an ontological statement and true by premise, as discussed in (chap 4). Thus, there are ontological necessities only, and it is just the question what exactly is necessary for what.

Then as the first step, the above definition adjusted and revised in a clearly and solely ontological reading would be:

If ${}^1\widehat{S}^t_k$ is a property of ${}^1S^t_k$, then ${}^1\widehat{S}^t_k$ is emergent if and only if

(i) ${}^1\widehat{S}^t_k$ supervenes with ontological necessity on the properties in ${}^1S^t_k$; and

(ii) some of the supervenience principles linking the properties in ${}^1S^t_k$ with ${}^1\widehat{S}^t_k$ are necessary for partial tendencies for change.

The content stays the same; only the implicated contrafactuals are eliminated—almost, since supervenience is intrinsically contrafactual: no difference in ${}^1\widehat{S}^t_k$ without a difference in ${}^1S^t_k$. It is more straightforward to say: the system ${}^1S^t_k$ is necessary and sufficient for the single tendencies for change of the emergent object ${}^1\widehat{S}^t_k$, and with that, the superfluous notion of supervenience can be dropped as well:

If ${}^1\widehat{S}^t_k$ is an object necessitated by ${}^1S^t_k$, then ${}^1\widehat{S}^t_k$ is emergent if and only if

(i) ${}^1\widehat{S}^t_k$ is determined with ontological necessity by the objects in ${}^1S^t_k$; and

(ii) the determinations linking the objects in ${}^1S^t_k$ with ${}^1\widehat{S}^t_k$ are necessitations for partial tendencies for change.

All three formulations state the same if ontological determination is taken seriously, and truth be told, the outcome is fairly meager. The comparison of the latter definition with (def 53) shows the overlay. What is missing here is the relevance of emergent objects for the „outer world“. A description is needed for how the emergent object can be related to other objects. The most interesting considerations are $1^{\text{st}}$-order structures, which now will be discussed: is the above definition of emergence appropriate anyway? Does it capture the circulating ideas of systems and systems' properties?

[128] Cf. [42, 223].

[129] For reasons of convenience, we are sticking with 'properties' but already thinking 'quality particulars', until we have replaced all the troublesome notions.

### 1st-Order Emergence

In the colimit definition of emergence, the necessity conditions have to be made up for and of emergent objects, and then what the intradets perform has to be tested, or to couch it in correct terms, what they necessitate, i.e. existence only or existence *and* tendency for change. To explore the nature of emergent objects, the basic structure ${}^1\Sigma^t_{k,l} = \langle {}^1S^t_k, {}^1\widehat{S}^t_k, O^t_l \rangle$ depicted in figure (fig 4.2) will be inspected: the system ${}^1S^t_k$, i.e. its components[130] $O^t_i$ and $O^t_j$, being necessary for $O^t_l$ to exist and to tend to change.

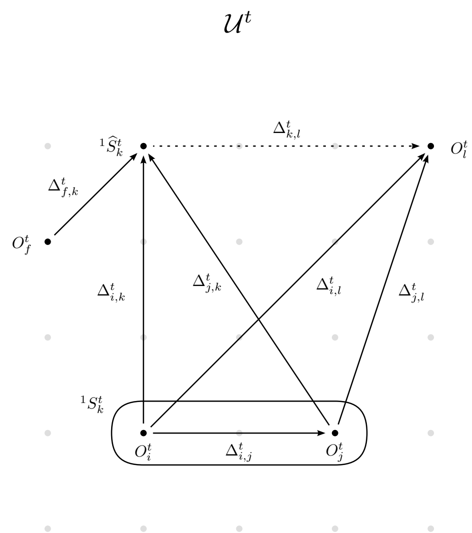


Figure 4.4: A structure ${}^1\Sigma^t_{k,l} = \langle {}^1S^t_k, {}^1\widehat{S}^t_k, O^t_l \rangle$: is ${}^1\widehat{S}^t_k$ necessary for the existence of the object $O^t_l$ only or also for the ${}^1\widehat{S}^t_k$-related tendency for partial change $\Delta^{t,t'}_{k,l}|_t$?

In the first instance, five major cases can be identified concerning the kind of existence of an emergent object:

(i) It does't exist.

(ii) It exists and imposes no condition for tendency for partial change $\Delta^{t,t'}_{i,j}|_t$, just for existence at time $t$.

(iii) It exists and imposes the same condition for tendency for partial change as an object in the system does.

(iv) It exists and imposes the same condition for tendency for partial change as all objects in the system in conjunction do.

(v) It exists and imposes a condition for tendency for partial change in addition to the conditions for tendency for partial change the objects in the system impose.

To use category theory then in the discussion of emergent objects is not meant to be metaphorical. Conjectures that are at odds with this formal framework should be put

[130] The intradets come with the objects.

to the test as to whether they are to be discarded. Thus, CT is considered more than a language, it is introduced to correspond to natural, i.e. existent, structures.

With that doing the inspection:

(i) In this case, the state of affairs is simple. In the small section of the universe $\mathcal{U}^t$ in figure (fig 4.4), ${}^1\widehat{S}_k^t$ doesn't exist. The objects remaining will just sort out the conditions between them. The definition of $1^{\text{st}}$-order emergence (def 53) is perfectly consistent with the non-existence of emergent objects. There is no structure.

(ii) The emergent object ${}^1\widehat{S}_k^t$ exists, but the intradet heading towards $O_l^t$ would just be the *logical* necessity condition for existence: $\Delta_{k,l}^t$. This would be quite unspectacular, since every object in $\mathcal{U}^t$ is necessary for $O_l^t$ to exist. Moreover, obviously with logical necessity, ${}^1\widehat{S}_k^t$ is also, in conjunction with existence, necessary for the object's $O_l^t$ total tendency for change $\Delta_l^{t,t'}|_t$:

$$ {}^1\widehat{S}_k^t \xrightarrow{\Delta_{k,l}^t} O_l^t : \Delta_l^{t,t'}|_t \tag{4.4.6}$$

Since tendency implies existence, the object can be dropped and the emergent object is necessary for the total tendency for change of $O_l^t$:

$$ {}^1\widehat{S}_k^t \xrightarrow{\Delta_{k,l}^t} \Delta_l^{t,t'}|_t \tag{4.4.7}$$

Equally, for ${}^1\widehat{S}_k^t$ to be sufficiently necessitated by the system ${}^1S_k^t$, intradets for existence and hence for tendency for total change are not enough. Necessity conditions for tendency for partial change are necessary. The systemic object ${}^1\widehat{S}_k^t$ doesn't exist without a tendency for total change $\Delta_k^{t,t'}|_t$ with the components being sufficient for the *superposed* partial tendencies for change $\Delta_{i,k}^{t,t'}|_t$ and $\Delta_{j,k}^{t,t'}|_t$. Any other object in $\mathcal{U}^t$ is necessary for existence and tendency for total change and any tendency for partial change of ${}^1\widehat{S}_k^t$ but not for its own induced tendency for partial change, since it doesn't exist. For example, in figure (fig 4.4) the intradet

$$ O_f^t \xrightarrow{\Delta_{f,k}^t} {}^1\widehat{S}_k^t \qquad \cong \qquad O_f^t \xrightarrow{\Delta_{f,k}^t} \Delta_{f,k}^{t,t'}|_t \tag{4.4.8}$$

isn't assigned. That excludes this object from being a part of the system ${}^1S_k^t$, although for instance, it certainly holds that

$$ O_f^t \xrightarrow{\Delta_{f,k}^t} {}^1\widehat{S}_k^t : \Delta_{i,k}^{t,t'}|_t \tag{4.4.9}$$

the object $O_f^t$ being necessary for the object $O_i^t$ being necessary for the tendency for partial change $\Delta_{i,k}^{t,t'}|_t$ of the emergent object ${}^1\widehat{S}_k^t$.

Coming back to the colimit discussion, the emergent property would come into existence necessitated by some aggregation, but without announcing itself to the object $O_l^t$. That kind of appearance, an allocation which it hardly deserves, is typically called *epiphenomenal*: ${}^1\widehat{S}_k^t$ doesn't contribute to the tendency for total change $\Delta_l^{t,t'}|_t$ of the object $O_l^t$. Now the question arises, whether it is possible in the colimit definition of emergent objects that ${}^1\widehat{S}_k^t$[131] isn't necessary for $O_l^t$ to partially change with $\Delta_{k,l}^{t,t'}|_t$, while the system's objects induce a tendency for partial change? In short, is it possible that ${}^1\widehat{S}_k^t$ is an epiphenomenon for $O_l^t$, but $O_i^t$ and $O_j^t$ are not?

Looking at the commutative diagram $\Delta_{i,l}^t = \Delta_{i,k}^t\Delta_{k,l}^t$, ${}^1\widehat{S}_k^t$ not imposing any necessity condition on $O_l^t$ to change, or better yet, in order not to go into rapture of thinking of any kind of activity, not being necessary for a tendency of partial change $\Delta_{k,l}^{t,t'}|_t$, entails that $O_i^t$ is, concerning $O_l^t$, also an epiphenomenon. That means, in the category theoretical definition of 1st-order emergence (def 53), it is not possible for the emergent object ${}^1\widehat{S}_k^t$ not to contribute a condition of necessity to the object's $O_l^t$ tendency for change with the system ${}^1S_k^t$ consisting of non-epiphenomenal objects. In short, if ${}^1\widehat{S}_k^t$ is an epiphenomenon for $O_l^t$, so is ${}^1S_k^t$. For the constituents of the system in figure (fig 4.4), it holds that

$$\Delta_{i,l}^t = \Delta_{i,k}^t\Delta_{k,l}^t \quad\text{and}\quad \Delta_{j,l}^t = \Delta_{j,k}^t\Delta_{k,l}^t \tag{4.4.10}$$

By definition of systems, $\Delta_{i,k}^t$ and $\Delta_{j,k}^t$ have to determine partial change, but with the factor $\Delta_{k,l}^t$ the non-time-delayed determinations $\Delta_{i,l}^t$ and $\Delta_{j,l}^t$ cannot.

(iii) In this case, ${}^1\widehat{S}_k^t$ is no longer an epiphenomenon. Without loss of generality, if ${}^1\widehat{S}_k^t$ is necessary for the same tendency for partial change as the object $O_i^t$ in figure (fig 4.4)

$$\begin{aligned}\Delta_{i,l}^t &= \Delta_{i,k}^t\Delta_{k,l}^t &&(4.4.11)\\ &= \Delta_{k,l}^t &&(4.4.12)\end{aligned}$$

which is equivalent to

$$\Delta_{k,l}^{t,t'}|_t = \Delta_{i,l}^{t,t'}|_t \tag{4.4.13}$$

That means, $\Delta_{i,k}^t$ is the identity on $O_i^t$: $\Delta_{i,k}^t = id_{O_i^t}$. And from that follows that ${}^1\widehat{S}_k^t$ is not the colimit of ${}^1S_k^t$ and therefore not an emergent object.

(iv) Here, the necessity condition for change of the emergent object ${}^1\widehat{S}_k^t$ for $O_l^t$ is the totality of all necessity conditions for change of the objects constituting the system ${}^1S_k^t$:

$$\Delta_{k,l}^t = \bigwedge_{\nu\in|ob({}^1\mathcal{S}_k^t)|} \Delta_{\nu,l}^t \tag{4.4.14}$$

That seems to be a classical case of a blatant category mistake, since in mathematics the symbol '=' means quantities having the same value, or expressions representing the same mathematical object. Since quantities have so far not been defined and the

[131] the total tendency for change $\Delta_i^{t,t'}|_t$ included

objects are not mathematical, the identity here should be read as 'is necessary for the same as', namely is necessary for the same as the conjunction of all necessities of the system regarding the determinant in the structure. And the result is the same tendency for total change as the emergent object is necessary for:

$$\Delta_{k,l}^{t,t'}|_t = \bigcup_{\nu \in |ob(^1\mathcal{S}_k^t)|} \Delta_{\nu,l}^{t,t'}|_t \tag{4.4.15}$$

Then ${}^1\widehat{S}_k^t$ can be seen as an epiphenomenon again, but this time with necessity of tendency for partial change. Besides overdeterminating the determinant $O_l^t$, ${}^1\widehat{S}_k^t$ is, as in (ii), perfectly consistent with the colimit definition of emergent objects.

(v) This is key, since it is the only case with the emergent object not being an epiphenomenon. The system ${}^1S_k^t$, i.e. its components $O_i^t$ and $O_j^t$, and the systemic object ${}^1\widehat{S}_k^t$ are necessary for tendency for partial change of the object $O_l^t$, but they are not fully congruent, because ${}^1\widehat{S}_k^t$ imposes a condition for tendency for partial change of $O_l^t$ in addition to the system ${}^1S_k^t$.

Further explorations could be performed, for example, on whether there are additional „structures“ or something or some thing that can be *thought of*[132] to exist in the universe $\mathcal{U}^t$.[133] So, the idea or notion of 'downward' could be put in perspective.

### 4.4.2 The Mental

In order to finally arrive at the mental object, in short the mental, it is not necessary to know exactly what it refers to, as long as it is an object in $\mathcal{U}^t$. Stating the mental to be existent, before we turn to its determination, i.e. *mental determination*, it is necessary to reflect on the possible determinants of the mental.

We ask: what is the merit of an emergent object if it isn't good for downward determination? Which conditions must be met for a relation to be justified as downward directed? A prerequisite for top-down determination is that there is indeed a top and a down, that is, some kind of *hierarchy of levels* must be assumed. The intradet between two objects with neither being emergent could hardly serve as a candidate. We consider the two possibilities with the source being emergent and also the determinant (fig 4.5), or the latter being a constituent of a system (fig 4.6).

We recognize that the circulating notion of 'downward determination' doesn't yield much in the framework of DUCKs. It is only that the determinant is either emergent or part of a system, unless it is neither. The special case shown in (fig 4.7) will become clear having defined $2^{\text{nd}}$-order systems.[134]

#### Mental Objects

Before we finally reach the crucial definition of $2^{\text{nd}}$-order systems, we want to be clear in our minds that emergence seems to be more special than is commonly thought. Mental objects appear to take a prominent position:

[132] It is still thinking, i.e. $2^{\text{nd}}$-order dynamics albeit about the existence.
[133] We keep in mind that there is still no dynamics.
[134] Cf. (chap 4.4.2).

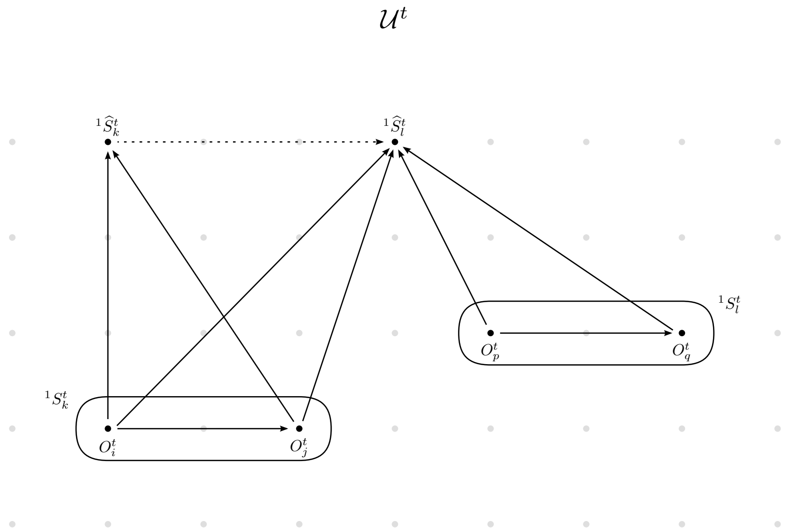


Figure 4.5: The determinant being an emergent object: $^1\widehat{S}_l^t$.

**Conjecture 15** (Mental Object)**.** The universe $\mathcal{U}^t$ is complete with respect to existence.

(i) If a mental object exists, it is an object $M_l^t \in ob(\mathcal{U}^t)$.

(ii) If an object $O_l^t \in ob(\mathcal{U}^t)$ is a mental object, it is a 1$^{\text{st}}$-order emergent object:

$$O_l^t = M_l^t \implies O_l^t = {}^1\widehat{S}_l^t. \tag{4.4.16}$$

Its 1$^{\text{st}}$-order system $^1S_l^t = {}^1M_l^t$ is called the *substrate*, and its colimit $colim(^1S_l^t) = colim(^1M_l^t) = {}^1\widehat{M}_l^t$ its mental object, in short, its *mental*.

♦

No statement will be made about the existence of the mental. The claim is that, if it exists, it is an emergent object. And there seem to be no other emergent objects besides mental objects, but that question has to remain unanswered. It wouldn't even occur to us that there are emergent objects at all if first-person perspectives were not experienced. In the known world, all phenomena can be reduced to the constituents—and their time evolution—of mere aggregations, except the mental.

Consequently, there are only structures involving mental objects:

**Conjecture 16** (1$^{\text{st}}$-Order Structure)**.** A *1$^{st}$-order structure* in the category $\mathcal{U}^t$ is a triple $^1\Sigma_{k,l}^t = \langle {}^1M_k^t, {}^1\widehat{M}_k^t, O_l^t \rangle$. ♦

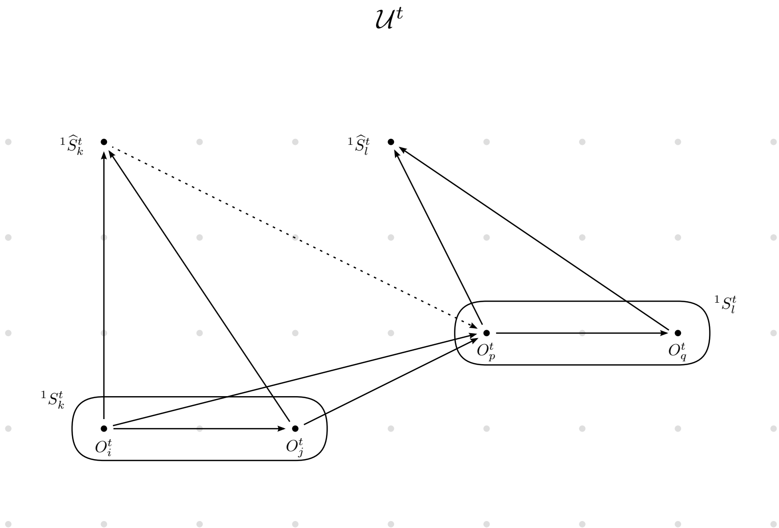


Figure 4.6: The determinant being a constituent of a system: $O^t_p$.

The mental is said to be a process.[135] That is not quite correct, since processes do not exist, only their interim stages, which are the (emergent) objects. But, obviously what is meant is that the mental follows some time evolution, for to come into existence precursors are necessary. And that is what is done now: the execution of the time separation of the substrate and its mental.

#### 2nd-Order Systems

What is the connection between what the mind is and what the mind is about? When a human substrate—or any other 1st-order system—claims some „property“ of a system to be emergent, it just states its own emergent object of its system. And this is what is called *epistemic*, i.e. there is no such system or emergent object, which would be the criterion to define that system. Is there an aggregation at some instance of time $t$ that is necessary and sufficient for some 1st-order system at $t' > t$ being accompanied by its 1st-order emergent object, which then is an aggregation that could be called the ‘material object’ that is perceived with the perception being the „representation“ of the „thing“ or the „content“ of the mental? Again, there must be some sufficiency condition for the aggregation in $\mathcal{U}^t$ to result in the mental object at the instance of time $t'$. And similar to the definition of 1st-order systems, this is already the needed criterion in a kind of back-tracing manner, which means that inter-time determination comes into play: an aggregation in $\mathcal{U}^t$ interdets a 1st-order system in $\mathcal{U}^{t'}$ to have a 1st-order emergent object.

[135] Cf. [29, 252].

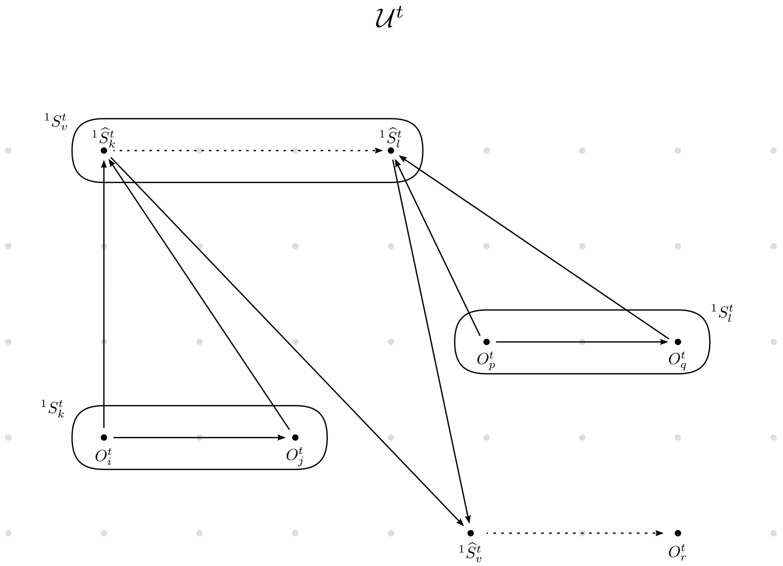


Figure 4.7: Downward determination: three structures ${}^1\Sigma^t_{k,l} = \langle {}^1S^t_k, {}^1\widehat{S}^t_l, {}^1\widehat{S}^t_l \rangle$, ${}^1\Sigma^t_{l,k} = \langle {}^1S^t_l, {}^1\widehat{S}^t_l, {}^1\widehat{S}^t_k \rangle$, and ${}^1\Sigma^t_{v,r} = \langle {}^1S^t_v, {}^1\widehat{S}^t_v, O^t_r \rangle$ with the factor $\Delta^t_{v,r}$ being considered downward directed.

Thus, due to the fact that there is a criterion for a subdiagram in $\mathcal{U}^t$, although it doesn't have an emergent object, it contributes to an emergent object of a 1st-order system in $\mathcal{U}^{t'}$. Therefore, in the definition of 2nd-order systems, emergent objects are also involved, even though they occur time delayed, and that serves as the justification to call them a system, too. Answering the question, the 2nd-order system is not a „thing“ that exists at $t$ and results in its perception at $t'$, but it is all that is necessary and sufficient to bring that perception about. This situation is depicted in figure (fig 4.8).

**Definition 56** (2nd-Order System)**.** A *2nd-order system* ${}^2S^t_k : {}^2\mathcal{S}^t_k \to \mathcal{U}^t$ is an aggregation with restriction ${}^2\mathcal{S}^t_k$ at an instance of time $t$ being nomologically necessary and sufficient for a 1st-order system ${}^1M^{t'}_k : {}^1\mathcal{M}^{t'}_k \to \mathcal{U}^{t'}$ at $t'$ with $\mathrm{colim}({}^1M^{t'}_k) = {}^1\widehat{M}^{t'}_k$. ▲

The 1st-order system ${}^1M^{t'}_k$ in (fig 4.8) is the substrate for the mental object ${}^1\widehat{M}^{t'}_k$, being determined with some time delay by the 2nd-order system ${}^2S^t_k$ which is called a *selection.* More generally, selections are subdiagrams that are not accompanied by a mental object at a particular instance of time, but they have been or will be.

Now, it is often claimed that a mental „entity“, consciousness as a special case, has an internal structure in some way. For example, in [56][2] five “central axioms, which are taken to be immediately evident” are listed, and the second says, “consciousness is compositional (structured): each experience consists of multiple aspects in various

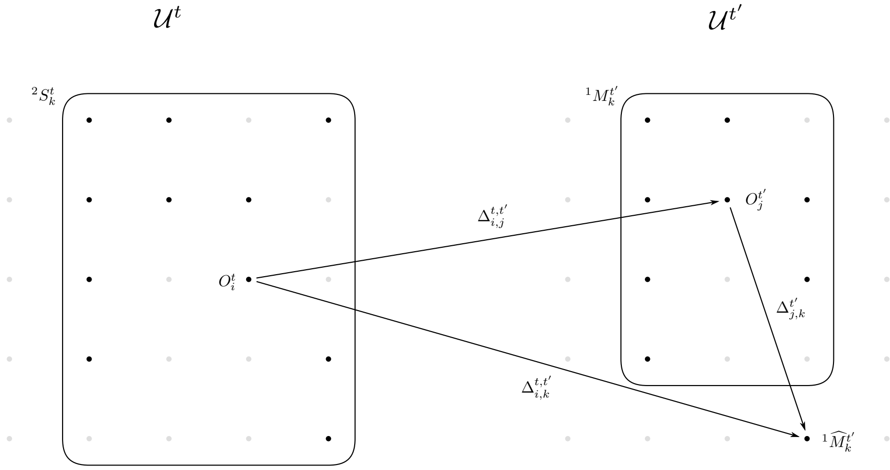


Figure 4.8: A 2nd-order system ${}^2S_k^t$ in $\mathcal{U}^t$ which interdets a 1st-order system ${}^1M_k^{t'}$ in $\mathcal{U}^{t'}$ with $\mathcal{U}^t$ signifying all universe-instances prior to $\mathcal{U}^{t'}$ and the NCCs simultaneously arriving at ${}^1\widehat{M}_k^{t'}$.

combinations. Within the same experience, one can see, for example, left and right, red and blue, a triangle and a square, a red triangle on the left, a blue square on the right, and so on."

It is indeed an intriguing question how this kind of compositionality could be brought into accordance with the category theoretical formalization of the domains unifying the logical relations of their entities. What kind of structure could be hiding within the substrate such that ${}^1\widehat{M}_k^{t'}$ could be subdivided into „parts" still being the emergent object, i.e. the colimit, of a system? One natural answer would be that there is no hiding structure but that there is more than one system and thus more than one colimit coming into existence at the same time $t'$ and the same place $x_k$.[136] Thus, if there is more than one mental object, they coexist and somehow overlay the category theoretical object as introduced in definition (def 38): ${}^1\widehat{M}_k^{t'} = \langle {}^1\widehat{M}_{k,1}^{t'}, {}^1\widehat{M}_{k,2}^{t'}, ..., {}^1\widehat{M}_{k,n}^{t'} \rangle$.

For further explorations, two definitions for the totality of diagrams are given:

**Definition 57** (Total Substrate)**.** The *total substrate* ${}^1M_k^{t'}$ for a mental object ${}^1\widehat{M}_k^{t'} = \langle {}^1\widehat{M}_{k,1}^{t'}, {}^1\widehat{M}_{k,2}^{t'}, ..., {}^1\widehat{M}_{k,n}^{t'} \rangle$ is the aggregation

$$ {}^1M_k^{t'} = \bigcup_{j=1}^{n} {}^1M_{k,j}^{t'} \tag{4.4.17}$$

with ${}^1M_{k,j}^{t'} : {}^1\mathcal{M}_{k,j}^{t'} \to \mathcal{U}^{t'}$ and $\mathrm{colim}({}^1M_{k,j}^{t'}) = {}^1\widehat{M}_{k,j}^{t'}$. ▲

[136] The mental seen to be describable by a function of spatio-temporal coordinates with some spacetime metrics.

**Definition 58** (Total Selection)**.** The *total selection* ${}^2S^t_k$ for a mental object ${}^1\widehat{M}^{t'}_k = \langle {}^1\widehat{M}^{t'}_{k,1}, {}^1\widehat{M}^{t'}_{k,2}, ..., {}^1\widehat{M}^{t'}_{k,n} \rangle$ is the aggregation

$$ {}^2S^t_k = \bigcup_{j=1}^{n} {}^2S^t_{k,j} \tag{4.4.18}$$

with ${}^2S^t_{k,j} : {}^2\mathcal{S}^t_{k,j} \to \mathcal{U}^t$ and $\operatorname{colim}(\Delta^{t,t'}({}^2S^t_{k,j})) = {}^1\widehat{M}^{t'}_{k,j}$. ▲

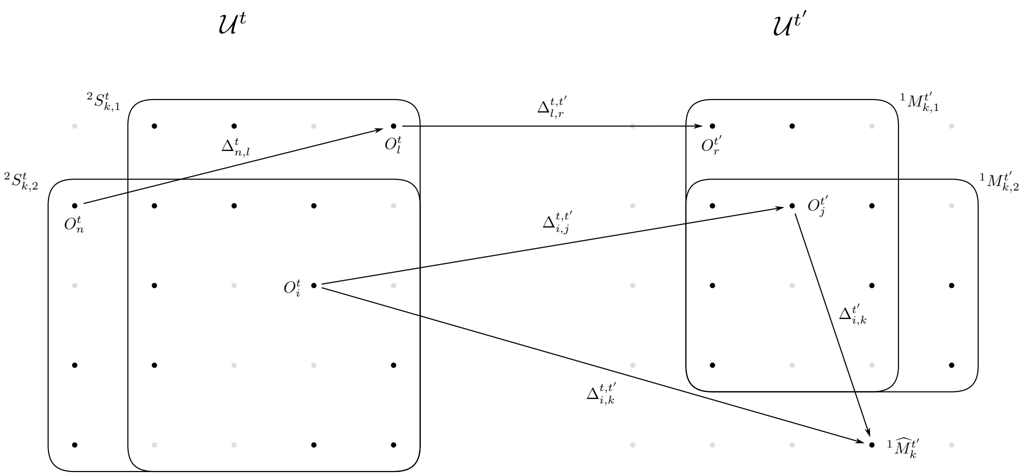


Figure 4.9: Two 2nd-order systems ${}^2S^t_{k,1}$ and ${}^2S^t_{k,2}$ in $\mathcal{U}^t$ which interdet two 1st-order systems ${}^1M^{t'}_{k,1}$ and ${}^1M^{t'}_{k,2}$ in $\mathcal{U}^{t'}$.

The question of whether mental objects „compose“, or rather their substrates that interdepend, can be now formulated more rigorously. In figure (fig 4.9), two sample selections and substrates are shown, and we have to clarify their common necessity conditions.

There are two possible readings:

(1) Both selections ${}^2S^t_{k,1}$ and ${}^2S^t_{k,2}$ interdet the substrates ${}^1M^{t'}_{k,1}$ and ${}^1M^{t'}_{k,2}$, which in turn intradet their corresponding colimits ${}^1\widehat{M}^{t'}_{k,1}$ and ${}^1\widehat{M}^{t'}_{k,2}$. Both 1st-order systems might have some objects in common (e.g. $O^{t'}_k$), meaning they are nomologically necessary for both emergent objects to come about. Thus, given that those two substrates are not disjoint, their colimits are not independent of one another. In this sense, they are composed, as suggested in [56]: <redness> and <triangle> yields in a <red triangle>. And e.g. the mental object <triangle> could itself be composed by colimits in that way.

(2) The selection ${}^2S^t_k$ interdets the substrate ${}^1M^{t'}_k$, which necessitates in turn the mental object ${}^1\widehat{M}^{t'}_k$, but which is also separable, meaning that the diagram ${}^2S^t_k$ is a total selection consisting of subdiagrams ${}^2S^t_{k,1}$ and ${}^2S^t_{k,2}$, interdetting the substrates ${}^1M^{t'}_{k,1}$ and ${}^1M^{t'}_{k,2}$ to have colimits ${}^1\widehat{M}^{t'}_{k,1}$ and ${}^1\widehat{M}^{t'}_{k,2}$. Hence, it comes out to three colimits in total versus two in (1), and this kind of „composition“ would amount to <redness> and <triangle>, and <red triangle>.

Coming back to the assumed structure including the mental, e.g. a certain consciousness, case (1) could hardly be considered to be or to have any such. The total substrate would just be an aggregation, the overlay of the single substrates and colimits, although they are not nomologically independent from one another. The red triangle might no longer be a triangle without the substrate that results in the mental object <redness>. Case (2) could more easily serve as a structure, since here the substrate is effectively decomposable.

Now, the question arises whether it is possible in principle to identify the one conception that is at odds with the real world. To ascertain which one is the right one, one could examine the assumption that nature behaves in accordance with category theory as its overall grounding and that mental objects are emergent objects obeying the „behavior" of colimits, or look for some experimental evidence that excludes an alternative.[137]

[137] The so-called binding problem is at its core not understood. Different brain areas contributing to an "integrated" sensation could appear to be the key for experiments to investigate spatio-temporal dependencies of the mental. Cf. e.g. [77], [82], and [57].

# 5 Conclusion

The current study deals with the existent and its dependencies, and promotes a thinking or underlying logic of thinking that is introduced as the best to cope with phenomena that persistently resist its inclusion into the existent. The most prominent, or the only one we so far surely know of, is the mental. It *obviously* depends on the known physical, and it obviously is not the known physical itself, rather it is some emergent.

Building on first principles then, which is the logics of necessary and sufficient conditions for existence and change of existence *and* the fact of experiencing itself, this is the attempt to ascertain the conditions for that which there is more than the fundamental objects which were said to be quality particulars and their materialized necessary conditions for change.

But before we came to the minimal ontology, we first looked at the emergence of proto-micelles in the artificial life debate, in order to see how emergent properties are introduced in the sciences. They are the result of some observation—and the whole survey then is on him, the observer, that in a way „produces“ the emergent higher-order structures like the proto-micelles and their emergent properties—manifesting itself as a so-called hyperstructure necessitated by objects of different orders and their relations.

The next step on the way to a minimal ontology then was the reformulation of the findings in the language of category theory, which led to the model of Memory Evolutive Systems with its application on neural networks where the observer becomes a mental object with the properties of a category theoretical construct which is called the colimit. As the ontological status of the mental remained unclear, we were seeking a new, experimental metaphysics that rests on findings from the foundational sciences—or is at least compatible with them: Bohmian mechanics.

The objective then was to integrate the mental into the so-called Bohmian NEXUS built on the introduced duality of localized primitives, i.e. quality particulars and NCCs, with the condition that it remains an object that obeys the „behavior“ of colimits. And it of course proved to be difficult, since emergent objects do not appear in Bohmian mechanics. Bohmian Mechanics is superpositional where category theory is compositional.

Is it possible that composition has a counterpart in nature and is not just a cognitive, conceptual tool as in the model of advanced hyperstructures and Memory Evolutive Systems in the manner of combining rigid arrows saying some arrow *following* some arrow? In physics, it is all about being ‘followed by’: if some conditions for some state of affair are met, a new state of affair is followed. Even transformation processes like annihilation only describe the constituents under certain conditions. What is happening or about to happen is never captured. Is it the „electrons“ that in the „brain“ does the job, or rather the „photons“? It is not that the quality particulars ontologically compose—they keep a safe standoff. It is the NCCs that overcome the distances.

The most interesting part of the mental is that it exists and it is localized given our knowledge of the natural world and action-at-a-distance not to be reanimated. The mental being at least physically necessitated gave us the obvious next idea to treat it as some entity in spacetime—a quality particular. It is locally existent, and in order to concentrate on locality we even made some assiduous efforts to motivate a new interpretation of a theoretical term, that is traditionally seen to reflect non-locality, as local as well—the wave-function. It mathematically describes the usual suspects that are responsible for time evolution, which are the necessary conditions for change. And as we have seen they stem from somewhere and go to somewhere. And here, they go to the mental:

$$O_i^t \xrightarrow{\Delta_{i,k}^{t,t_0}} \widehat{M}_k^{t_0} \xleftarrow{\Delta_{j,k}^{t,t_0}} O_j^t \tag{5.0.1}$$

the constraints in the substrate being neglected.

Again, who is in charge of executing the tendency for change? The objects $O_i^t$ and $O_j^t$ are only the sources for their conditions for change and existence of the mental $\widehat{M}_k^{t_0}$. The situation of the ontological composition is

$$\Box_i^{t_0} \xrightarrow{\Diamond_{i,k}^{t_0}} \widehat{\Box}_k^{t_0} \xleftarrow{\Diamond_{j,k}^{t_0}} \Box_j^{t_0} \tag{5.0.2}$$

the diamonds stating the NCCs $\Box_i^{t_0}$ and $\Box_j^{t_0}$ at the position $x(k, t_0)$ being necessary for $\widehat{\Box}_k^{t_0}$ if the mental was to be the source for an NCC heading for some other object.

This is the kind of thinking to cope with conditions for existence and change of existence—and not only with respect to the mental but to every kind of conditioning. And things become even more inconvenient when the whole diagram is taken into account: the mental being identically necessary for the same object the substrate is necessary for.

The question becomes evident: Which language is appropriate to analyze and describe ontology? If category theory is just the way all thinking is intrinsically happening, saying the „brain“ and the mental just works like that, in objects and morphisms, how would one overcome the limitations with respect to emergentism? Maybe the only way to understand emergentism is its or the experience.

Finally, cognizing is not to be confused with mentalizing. It is not necessary that the human abilities to refer, to count, or to learn always have to be accompanied by what we call the mental—unless it is a fact that certain cognitions don't come without experience. Cognition itself could be seen as nothing but moving particles in certain ways. Thus the *execution* of categorical theorizing at an instance of time $t$, which is obviously some concrete cognition, is identical to trajectile behavior that is alike. But that being alike need not necessarily be subject to scrutiny if one considers certain ways of moving particles to count as thinking in categories. It is what it is: they are objects, morphisms, domains of morphisms, codomains of morphisms, identities, which are morphisms, and compositions of morphisms. One might think it's all about morphisms, but it is not. There are still objects that are, after all, the sources and targets of the arrows. And as we put it, the arrows ontologically only reflect the materialized conditions for change, saying 'It's the objects that really matter!'

Following the attempt to leave the natural, i.e. objects and (materialized) morphisms in $\mathcal{U}^t$, behind and discover or develop anything over and above the natural, *concepts* seem to be most suitable to show that the endeavor is doomed to failure at the outset. Certainly one could reply, it is the *first-person experience* that seems to be the hard problem, but what are concepts without qualia? Cognition. And cognition is, as seen, not at all striking, meaning it hasn't got any touch of supernatural at all. Thus, what has to be done is risk an attempt to probe a natural, i.e. an ontological, approach to 'cognition with an experience', or *experienced cognition*: pattern classification resulting in or being accompanied by an experience.

The starting point for defining concepts is to be clear about *representations*, and then again, once we have a working substitution. And the ingredients ought to be in DUCKs, since it was the goal to unify the domains of the mental and the physical, for example, using minimalist, but well defined notions. Naturally, sometimes one has to fall back on established concepts, which hopefully can be overcome over the course.

A representation can be seen as a binary relation $R = (G_R, A, B)$ over sets $A$ and $B$ with the directed graph $G_R = Graph(R) \subseteq A \times B = \{(a,b)|a \in A, b \in B\}$ and $aRb := (a,b)$ being the infix notation of '$a$ is a representation of $b$'. Now, the intuition for the representation $aRb$ is that $a$ and $b$ have some structure in common, and $R$ has to allocate the appropriate attributes they share.

In the compilation of a proper notion of what is meant by 'is a representation of', one can refer to [43, 102]. Three candidates for relations are debated for specifying *structural commonality*:

(i) Similarity: $A$ represents $B$ iff $A$ is similar to $B$.

(ii) Isomorphism: $A$ represents $B$ iff there is an isomorphism between $A$ and $B$, that is to say $A$ and $B$ are two isomorphic structures.

(iii) Homomorphism: $A$ represents $B$ iff there is a homomorphism between $A$ and $B$.

For further considerations, it has to be clarified what exactly $A$ and $B$ relate to? Here, we compared objects and diagrams in DUCKs, and the assignment was: $A$ relates to the 1st-order system ${}^1M_k^{t'}$ in $\mathcal{U}^{t'}$ being accompanied by its 1st-order emergent object ${}^1\widehat{M_k^{t'}}$, and $B$ to the 2nd-order system ${}^2S_k^t$ in $\mathcal{U}^t$. And as seen, there is no structure other than the 1st-order structure ${}^1\Sigma_{k,l}^t$ that can be ontologically referred to.

Finally, the process of *categorization* is tightly connected to the perception of *similarity*. And it is an intriguing question to ask what it is that is being perceived anyway. Is it structure that is immanent in the universe and then being reflected in the mental or is it structure that is genuinely produced in or by the mental without having an objective precursor in the universe. In [37, 1], it is stated

> *Whether category structure determines how similarity is perceived or whether perceived similarity dictates the existence of categories is more a question of metaphysics than of empirical investigation.*

But what is this structure the author talks about? In our view, we assume that all there metaphysically is, is objects moving in space: $\Delta_{i,j}^{t,t'} : O_i^t \longrightarrow O_j^{t'}$, which is pure geometry enriched with some necessity conditions determining the future geometry. And sometimes the eternal objects necessitate some emerging mental object, and then there is the mental, too. The consequence is the emergence of objective $1^{\text{st}}$-order structures. However, those structures are not meant in the above citation.

In general, structures are supposed to be certain *organizations* of entities which are then called the components which is actually more than entities since it is said that they make a whole being "instances of concepts and therefore share a structure specified by the concept".[1] Thus, concepts come first, and no structure without concepts for them. And the natural next question would then be: what is the criterion for a structure being an instance of a concept? First of all it has to be a perceived structure, whether or not it is one. And the claim in DUCKs is that the perception itself is the criterion—in particular perceived similarity. That eventually opens the key question whether concepts are nothing but just that: *perceived similarities*. The goal then would be to find a kind of *similarity measure* that in turn results in some conceptual distance measure that obviously has to be connected—again by some 'nothing but'-relation—to geometrical distances of the existents, in the way: that is a concept due to its exemplars are similar due to a certain measure, and that is due to some spacial geometry of parts. But that had to be discussed after having had recapitulated the traditional view of concepts, but that already in a category theoretical formulation.

In order for a structure to be approachable for scientific study it has to be determined by structural properties, so-called features, but these features are perceived structures themselves. Thus in order to detect some similarity for structures, first a similarity measure for their features has to be at hand. It is obvious, the process lacks a starting point. And anyhow, if one wishes to build a model for concepts, axioms of a model theory have to be satisfied by an instance of the concept. Well then, axioms are themselves concepts, and something is lacking again.

If one wishes to state an ontology for concepts, the point of departure has to be located in the existent, i.e. in the above defined objects. In short, we are not allowed to recruit any kind of symbolic treatment unless the logics refers to an inherent behavior of the universe. And here is the starting point: the ontic universe is consistent with the category $\mathcal{U}^t$ which allows us to use the symbolic apparatus of category theory with the defined objects and morphisms, and the challenge will be to find some similarity measure within $\mathcal{U}^t$ that at the core will be some equivalence class that has to be sufficiently vague in order for exemplars not to be classified similar. That seeming insuperability of ontic *subjectivity* of the mental could give a first hint that concepts, or better the instances, are indeed just certain processes that finally get lost in deep space.

Thus, searching for an *objective* measure for similarity of concepts is a taxing endeavor since similarity itself is a concept and therefore not existent. If one tries to reduce similarity to the existent, one must finally refer to the existent, and those are the mental objects and their underlying substrates. Aiming then, as an outlook, at a criterion for a measure for similarity, we find that there are two kinds of comparison: intra-personal and inter-personal comparison of mental objects. With that, a quite important new notion cames into play—the intuitive concept of *person*[2], and one could use the above measure

[1] Cf. [37, 1].
[2] Cf. [80].

for similarity of mental objects to mark out persons. And yet we are debating their persistence and identity.[3] In the following then it would be to provide additional tools for identifying persons and give them some near enough identity.[4] A first step then would be to pass on the criterion of similarity: Two mental objects ${}^{1}\widehat{M}_i^{t'}$ and ${}^{1}\widehat{M}_k^{t'}$ are similar, and thus their substrates, iff their 2nd-order systems ${}^{2}S_i^{t}$ and ${}^{2}S_k^{t}$ are. And that would be some objective anthropic and anthropocentric means of dividing up the universe—into persons.

[3] Cf. [6].
[4] Cf. [32].

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