How Gravity Shapes the Low-Energy Frontier of Particle Physics

Neutrino Masses and the Domestic Axion

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Zusammenfassung

Das Standardmodell der Teilchenphysik und seine kosmologischen Implikationen lassen einige fundamentale Fragen unbeantwortet, insbesondere die Abwesenheit von CP-Verletzung in der starken Wechselwirkung sowie die Ursprünge von Neutrinomassen, Dunkler Materie und Dunkler Energie. Innerhalb der Modellentwicklung jenseits des Standardmodells konzentrieren sich die populärsten Forschungsrichtungen üblicherweise auf neue Strukturen bei hohen Energien bzw. kleinen Abständen. Als eine alternative Richtung präsentieren wir in dieser Dissertation eine neue Klasse von niederenergetischen Lösungen der Neutrinomassen- und starken CP-Probleme. Diese Klasse manifestiert sich auf einer neuen infraroten Gravitationsskala, welche numerisch übereinstimmt mit der Skala der Dunklen Energie. Wir zeigen, wie sich ein Neutrinokondensat, kleine Neutrinomassen und ein Axion aus einer topologischen Formulierung der chiralen Gravitationsanomalie ergeben können. Zuerst rekapitulieren wir, wie ein gravitativer θ-Term zur Entstehung eines neuen gebundenen Neutrinozustands η_ν führt, analog zum η'-Meson in der QCD. Auf dieser Basis leiten wir her, dass sich ein niederenergetisches Neutrino-Vakuumskondensat bildet, welches kleine Neutrinomassen generiert. Im Rahmen eines darauf aufbauenden Modells, in welchem auch die Masse des Up-Quarks durch das Neutrinokondensat erzeugt wird, identifizieren wir ein Axion, welches ausschließlich aus Fermionen des Standardmodells besteht: dem η'-Meson plus einer winzigen Beimischung des η_ν-Bosons bestehend aus Neutrinos. Diese neue niederenergetische Modellklasse hat einige außergewöhnliche Konsequenzen für Kosmologie, Astrophysik, Gravitation, und Teilchenphänomenologie. Zum Beispiel zeigen wir, dass aufgrund eines späten kosmischen Phasenübergangs im Neutrinosektor die kosmologischen Grenzen für die Neutrinomassen verschwinden. Darüber hinaus untersuchen wir die Auswirkungen der vorhergesagten topologischen Defekte und der verstärkten kosmischen Neutrino-Selbstwechselwirkungen auf Dunkle Materie und Dunkle Strahlung im späten Universum. Im astrophysikalischen Bereich ist die wichtigste Modellvorhersage die Verstärkung von Neutrinozerfällen, welche in extraterrestrischen Neutrinoflüssen beobachtbar sind. In Bezug auf Gravitationsmessungen implizieren unsere Modelle verschiedene Polarisationsintensitäten von Gravitationswellen sowie eine neue kurzreichweitige Kraft zwischen Nukleonen, konkurrierend mit der Gravitationskraft. Im Hinblick auf Teilchenphänomenlogie erläutern wir mögliche Signaturen von flavor-verletzenden Prozessen, Licht-durch-die-Wand-Signalen, und etwaigen sterilen Neutrinos in Short-Baseline-Experimenten. Wir kommentieren, wie diese Modellvorhersagen mit laufenden und zukünftigen Experimenten getestet werden können, insbesondere mit Euclid, IceCube, KATRIN und PTOLEMY.

Abstract

The Standard Model of particle physics and its implications for cosmology leave several fundamental questions unanswered, including the absence of CP violation in strong interactions and the origins of neutrino masses, dark matter, and dark energy. The most popular directions of model building beyond the Standard Model usually focus on new physics at short distances corresponding to high-energy scales. As an alternative direction, we present a novel class of low-energy solutions to the neutrino mass and strong CP problems at a new infrared gravitational scale, which is numerically coincident with the scale of dark energy. We demonstrate how a neutrino condensate, small neutrino masses, and an axion can emerge from a topological formulation of the chiral gravitational anomaly. First, we recapitulate how a gravitational θ-term leads to the emergence of a new bound neutrino state η_ν analogous to the η' meson of QCD. On this basis, we show that a low-energy neutrino vacuum condensate forms and generates small neutrino masses. In the context of a follow-up model in which also the up-quark mass is generated by the neutrino condensate, we identify an axion that is composed entirely out of Standard Model fermion species: the η' meson plus a minuscule admixture of the neutrino-composite η_ν boson. This new low-energy class of models has several unusual consequences for cosmology, astrophysics, gravity, and particle phenomenology. For example, we show that the cosmological neutrino mass bound vanishes due to a late cosmic phase transition in the neutrino sector. Moreover, we investigate the impact of the predicted topological defects and enhanced relic neutrino self-interactions on the dark matter and dark radiation content of the late Universe. On the astrophysics side, the key model prediction is the enhancement of neutrino decays observable in extraterrestrial neutrino fluxes. Concerning gravitational measurements, our models imply different polarization intensities of gravitational waves and a new attractive short-distance force among nucleons with a strength comparable to gravity. With regard to particle phenomenology, we explain potential signatures of flavor-violating processes, shining-light-through-walls signals, and possible sterile neutrinos in short-baseline experiments. We comment on how these model predictions can be tested with current and future experiments, in particular Euclid, IceCube, KATRIN, and PTOLEMY.

Projects and Publications

This thesis is based on completed and ongoing projects to which I contributed during my research conducted at the Ludwig–Maximilians–Universität München and the Max Planck Institute for Physics from October 2015 to May 2018.

A substantial part of this thesis was originally published in two papers [1, 2] written in collaboration with Prof. Dr. Georgi Dvali. The authors of these publications share the principal authorship and are listed alphabetically by convention in particle physics. The presentation in this thesis, including figures and tables, closely follows these papers:

1. G. Dvali and L. Funcke, "Small neutrino masses from gravitational θ-term," *Phys. Rev. D* **93** (2016) 113002, `arXiv:1602.03191[hep-ph]`.

2. G. Dvali and L. Funcke, "Domestic Axion," `arXiv:1608.08969[hep-ph]`.

Further parts of this thesis are based on ongoing projects [3–7], which are carried out in different collaborations and address the following topics:

3. Soft topological defects from a late cosmic phase transition in the relic neutrino sector. In collaboration with A. Vilenkin.

4. Resolving tensions between high- and low-redshift data with time-varying relic neutrino masses. In collaboration with C. S. Lorenz, E. Calabrese, and S. Hannestad.

5. Relic neutrino overdensity on Earth with strong neutrino self-interactions. In collaboration with L. Mirzagholi and E. Vitagliano.

6. Impact of late neutrino mass generation and neutrino self-interactions on cosmic structure formation. In collaboration with J. Stadler, C. Bœhm, and S. Pascoli.

7. Extending the low-energy frontier to nonperturbative BSM models beyond gravity. In collaboration with M. Shifman and A. Vainshtein.

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Contents

List of Abbreviations

ABJ	Adler-Bell-Jackiw
ALP	Axion-like particle
BBN	Big Bang nucleosynthesis
$B-L$	Baryon number minus lepton number
BSM	Beyond the Standard Model
CL	Confidence level
CMB	Cosmic microwave background
CP	Charge conjugation and parity transformation
DA	Domestic Axion
DE	Dark energy
DFSZ	Dine-Fisher-Srednicki-Zhitnisky
DM	Dark matter
EW	Electroweak
GR	General Relativity
GUT	Grand Unified Theory
GW	Gravitational wave
IR	Infrared ($\widehat{=}$ low-energy)
KiDS	Kilo Degree Survey
KSVZ	Kim-Shifman-Vainshtein-Zakharov
LH	Left-handed
LHC	Large Hadron Collider
PQ	Peccei-Quinn
PQWW	Peccei-Quinn-Weinberg-Wilczek
QCD	Quantum Chromodynamics
RH	Right-handed
SBL	Short-baseline
SM	Standard Model
SN	Supernova
$SO(n)$	Special orthogonal group of order n
$SU(n)$	Special unitary group of order n
SUSY	Supersymmetry
SZ	Sunyaev-Zeldovich
$U(n)$	Unitary group of order n
UV	Ultraviolet ($\widehat{=}$ high-energy)
VEV	Vacuum expectation value
Z_n	Cyclic group of order n
$0\nu\beta\beta$	Neutrinoless double beta

Chapter 1

Introduction

The Standard Model (SM) of particle physics provides an exceptionally successful description of the most fundamental laws of Nature [8–12]. From the first stages of development in the 1960s [13–15] until the discovery of the last SM particle (the Higgs boson) at the Large Hadron Collider (LHC) in 2012 [16, 17], its continuous success has stemmed from three complementary factors:

- *Content:* The SM classifies all known elementary particles, such as electrons, quarks, and neutrinos. It fully describes three of the four known fundamental forces (electromagnetic, weak, and strong interactions), and can be consistently coupled to low-energy quantum gravity [18].

- *Range:* The SM plus General Relativity (GR) [19, 20] describe physical phenomena over a large distance range of 44 orders of magnitude (from 10^{-18} to 10^{26} m, see Fig. 1.1) without any contradiction to experiment.

- *Precision:* The predictions of the SM have been experimentally confirmed up to 11 digits of precision [21]. It might therefore be considered the most precisely tested theory in the history of science.

Despite its remarkable achievements, the SM leaves open several highly fundamental questions. These include: What is the theoretical mechanism that generates the observed small neutrino masses [22, 23]? Why do strong interactions as described by Quantum Chromodynamics (QCD) not violate charge conjugation parity (CP) symmetry [24,25]? What are the origins of dark energy (DE) [26, 27], dark matter (DM) [28, 29], and cosmic inflation [30, 31]? How can we incorporate high-energy quantum gravity into the SM [18,32]? Why do we observe significantly more matter than antimatter in our Universe [33,34]? These puzzles indicate that our understanding of the Universe is incomplete, which causes vast interest in the exploration of physics beyond the SM (BSM).

The research presented in this thesis focuses on two of these fundamental puzzles: the neutrino mass and strong CP problems, which will be introduced in sections 2.1.1 and 2.2.1, respectively. Before going into the technical details

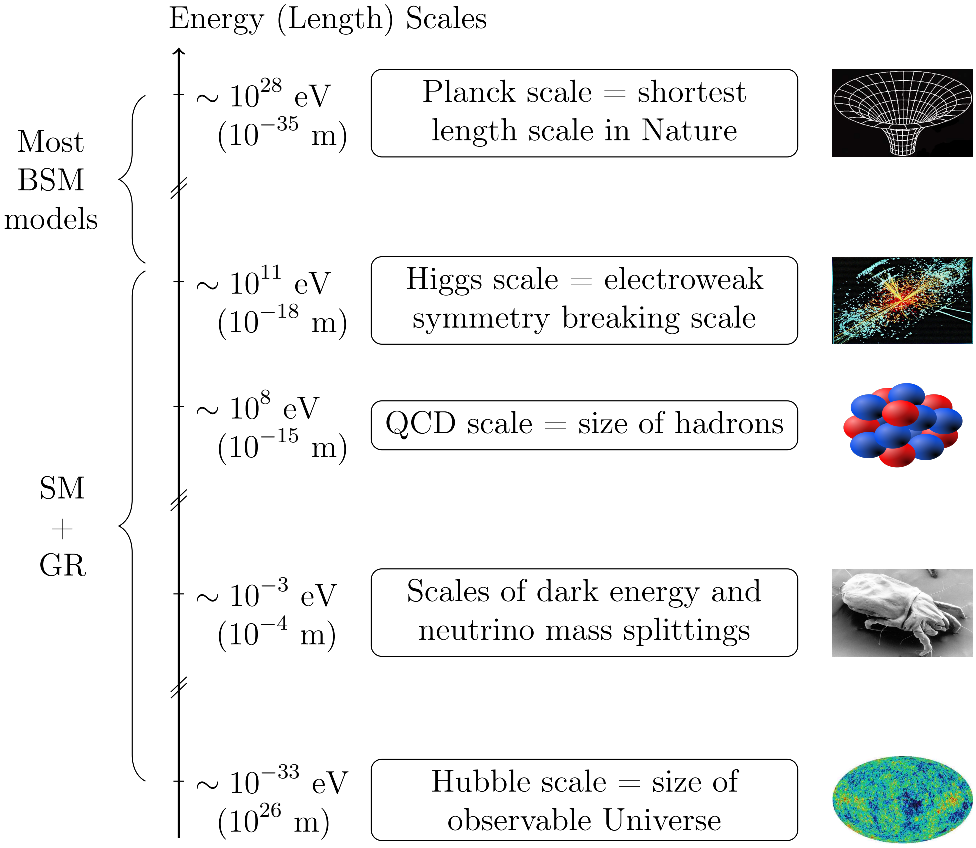

Figure 1.1: Important energy (length) scales in Nature, visualized by objects of the respective sizes (figures taken from [35–39]). Except for the open questions mentioned in the text, the entire regime from 10^{-18} to 10^{26} m is described by the SM plus GR. Most BSM models focus on new physics at shorter distances.

of these two puzzles, let us ask a more general question: Why do particle physicists devote such intense theoretical and experimental research activities to these issues? One fundamental motivation is that the neutrino mass and strong CP problems hint at the existence of a *new fundamental energy scale* at which novel physical phenomena might become important. While the two fundamental energy scales of the SM are the electroweak (EW) symmetry breaking scale $\Lambda_{\rm EW} \sim 100$ GeV and the QCD scale $\Lambda_{\rm QCD} \sim 100$ MeV of chiral symmetry breaking, these open SM issues hint at possible new physics at drastically different energy scales (cf. Fig. 1.1).

The most popular directions of BSM model building, such as supersymmetry (SUSY) [40–42] or Grand Unified Theories (GUTs) [43–45], usually focus on potential new physics at *high-energy scales* $\Lambda_{\rm BSM} \gtrsim$ TeV. Typically, the terms "particle physics" and "high-energy physics" are even used interchangeably in the research community. However, despite intensive experimental effort during the past decades, none of the high-energy model predictions, e.g., new heavy particles or proton decay, have been confirmed so far.

Moreover, most high-energy BSM models fail to account for the origin of DE, which was hypothesized to explain the observation that our Universe is expanding at an accelerated rate [26, 27]. While one theoretically expected contribution to DE is the zero-point energy of quantum fields, the corresponding energy density of $\rho_{\mathrm{DE}} \sim 10^{76}$ GeV^4 is more than 120 orders of magnitude larger than the observed value of $\rho_{\mathrm{DE}} \sim (\mathrm{meV})^4$, which is considered "the worst theoretical prediction in the history of physics" [46]. Perhaps even more surprisingly, the observed DE scale is numerically close to the neutrino mass scale, $\Lambda_{\mathrm{DE}} \sim \mathrm{meV} \sim m_\nu$. However, low-energy scales in high-energy BSM models, such as the neutrino and axion mass scales, are usually not fundamental new-physics scales but emerge from an interplay between SM and higher-energy scales (see sections 2.1.2 and 2.2.3). Therefore, these models yield no fundamental low-energy scales of new physics that could explain why the meV-scale of DE is so small compared to either the Planck scale or the SM scales.

As an alternative direction, this thesis presents experimentally viable BSM solutions to the neutrino mass and strong CP problems at a new fundamental *low-energy scale* inherent to gravity, $\Lambda_G \sim$ meV–eV. The neutrino and axion mass scales in these BSM models are intrinsically set by the gravitational new-physics scale, $\Lambda_G \sim m_\nu \sim m_a$, which coincides numerically with the DE scale. Thus, our models suggest a common gravitational low-energy origin of neutrino masses, the axion, and potentially DE.

Our novel class of gravitational low-energy models is based on a mathematical formalism called topological three-form language [47–49]. Differential forms, such as the mentioned three-form, provide a coordinate-independent approach to multivariable calculus, which is a useful tool in geometry, topology and several areas of physics (see, e.g., [50–52] for reviews). To give one example, the electromagnetic four-potential can be written as a differential one-form, the field strength tensor as a two-form, and the current density as a three-form. For gravity and QCD, the topological three-form formalism [47–49] reveals possible similarities between the topological sectors of these two theories. These similarities appear in the presence of physical θ-terms in gravity and QCD, which can be written as total derivatives of topological gauge fields called Chern-Simons three-forms [53]. In the following three paragraphs, we will briefly introduce the essence of this three-form formalism, whose technical details will be treated in the description of our neutrino mass model (see section 3.1).

The essence of the topological three-form formalism can be best explained by means of the following example: the generation of the η'-meson mass in QCD, which will be reviewed in section 2.2.1. In QCD, the topologically nontrivial vacuum spontaneously breaks the approximate chiral symmetry of the three lightest quark flavors, which gives rise to eight pseudo-Goldstone bosons, the light mesons [54, 55]. Due to the Adler-Bell-Jackiw (ABJ) anomaly [56, 57] of the isospin singlet axial current, the ninth meson η' gets a large mass through topologically nontrivial field configurations [58–60].

The effect that serves as an important analogy for our gravitational models takes place in case when at least one of the quarks has vanishing bare mass. In this case, the vacuum θ-angle of QCD becomes unphysical, as it can be rotated away by an anomalous chiral rotation of the massless quark field [61–64]. This case is important for us because of its alternative description in a topological language. As pointed out in [48] using the power of gauge redundancy, the elimination of the vacuum θ-angle can be understood in terms of a "Higgs effect" of a topological three-form: the η' meson is eaten up by the topological Chern-Simons three-form of QCD, and the two combine into a single massive pseudoscalar particle. Thus, the initially massless three-form field, which propagates no degrees of freedom, acquires one degree of freedom and becomes massive by eating up η'. This phenomenon can be formulated in a model-independent way [47], entirely in terms of topology and anomaly, without needing to know the underlying microscopic structure of the theory. Whenever a theory contains a vacuum θ-angle that can be eliminated by chiral transformation, a mass gap is necessarily generated. Consequently, there exists a pseudo-Goldstone boson, which is eaten up by a corresponding Chern-Simons three-form. The generality of this phenomenon makes it applicable to other systems, such as gravity coupled to neutrinos.

Indeed, in case of the existence of physical vacuum θ-angles, gravity and QCD have a similar topological structure [48]: the gravitational Chern-Simons three-form [65] enters the Higgs phase provided the theory contains a fermion with zero bare mass, such as the neutrino. As shown in [49], an important consequence of this three-form Higgs effect follows for the neutrinos: they are the analogon to the light quarks in QCD, and consequently the neutrino sector delivers a pseudo-Goldstone boson of broken axial neutrino symmetry, called the η_ν boson. This pseudo-Goldstone boson becomes a longitudinal component of the Chern-Simons gauge three-form and generates a mass gap in the theory. The η_ν boson represents a bound neutrino state triggered by the chiral gravitational anomaly [65–68], analogous to the η' meson triggered by the ABJ anomaly of QCD.

These previous insights about the η_ν-boson generation through the chiral gravitational anomaly pave the way for our gravitational neutrino mass and axion models. As presented in this thesis, an order parameter, which is most obviously a neutrino vacuum condensate triggered by nonperturbative gravitational effects, is required in order to deliver the new degree of freedom η_ν. The same neutrino vacuum condensate that delivers this degree of freedom can also generate small effective neutrino masses. This low-energy neutrino mass mechanism can yield both Dirac and/or Majorana masses and allows for the experimentally observed neutrino mass hierarchy.

Going one step further, we can consider the possibility that the neutrino condensate also generates the mass of the up quark spontaneously. In such a scenario, a Peccei-Quinn (PQ) symmetry [69] arises as a combination of axial symmetries acting on the up quark and on the neutrinos. This PQ symmetry is free of the chiral gravitational anomaly, it is anomalous only with respect to QCD, and it gets spontaneously broken by both the QCD up-quark condensate and the gravitational neutrino condensate. The corresponding "domestic axion" predominantly consists of the QCD η' meson with a small admixture of η_ν, while the orthogonal combination is a gravitational axion analogue called "graviaxion", which consists mostly of η_ν with a small admixture of η'.

Even though our two gravitational low-energy models are *a priori* separate scenarios, their combination is especially economical: the solution of the strong CP problem can be connected to the origin of the neutrino masses, without the need for any new species and with a built-in protection mechanism of the axion solution against potential gravitational threats.

Outline

Chapter 2 reviews the most puzzling aspects of neutrino and axion physics. First, we will present the neutrino mass problem and give an overview of the most common high-energy BSM neutrino mass models. Second, we will treat the origin and current status of axion physics by explaining the strong CP problem, discussing its potential resolution within the SM, and elucidating how high-energy BSM axion models attempt to solve this problem.

Chapter 3 contains the theoretical concepts behind our proposed gravitational neutrino mass mechanism. First, we will review the emergence of bound states from anomalies in QCD and in gravity. Then, we will demonstrate how these bound states are related to neutrino condensation and the emergence of small effective neutrino masses. In this context, we will briefly comment on the matters of Dirac versus Majorana masses, the neutrino mass hierarchy, and the numerical coincidence of the neutrino mass and DE scales.

Chapter 4 is dedicated to the theoretical foundations of our Domestic Axion model. In order to grasp our model's new aspects, we will first compare it with the original axion scenario. Then, we will illustrate how the chiral gravitational anomaly can jeopardize common high-energy axion solutions and how the neutrino can eliminate this gravitational threat. Afterwards, we will gradually build up our model by elucidating its anomalous $U(1)$ symmetries, its up-quark mass generation, and its axion and graviaxion content. Finally, we will comment on the consistency of the model with chiral perturbation theory.

Chapter 5 treats the experimental consequences of our gravitational neutrino mass and axion models. After examining the observational bounds on the new low-energy gravitational scale Λ_G, we will present our models' implications for cosmology, astrophysics, gravity, and particle phenomenology. With regard to cosmology, we will examine the predicted late cosmic phase transition in the neutrino sector, which has a severe impact on the relic neutrino background and can give rise to topological defects, dark radiation, and DM. On the astrophysical side, we will treat enhanced neutrino decays and light particle emission in stellar neutrino processes. The section on gravity is devoted to the modified propagation of gravitational waves and a new gravity-competing short-distance force. Concerning particle phenomenology, we will cover a variety of different experimental fields, which includes photon conversion, neutrinoless double beta decay, possible sterile neutrinos in short-baseline experiments, and flavor-violating processes.

Chapter 6 provides overall conclusions, which complement the separate summaries and discussions provided by each of the chapters 3–5. Furthermore, this final chapter gives an outlook to future theoretical studies, which build upon the two low-energy models presented in this thesis.

Chapter 2

Review of Neutrino and Axion Physics

This chapter provides a review of the neutrino mass and strong CP problems, as well as the most popular approaches to solve these puzzles. Within section 2.1 on neutrino physics, we will first present the neutrino mass problem and then give an overview of common high-energy neutrino mass models. In the subsequent section 2.2 on axion physics, we will explain the essence of the strong CP problem, discuss its potential resolution within the SM, and finally point out how high-energy invisible axion models attempt to resolve this problem.

2.1 Neutrino Physics

2.1.1 The Neutrino Mass Problem

Even though neutrinos are the second-most abundant particles in our Universe after photons, most of their characteristics are still experimentally undetermined. The experimental discovery of neutrino flavor oscillations [22, 23] was one of the major breakthroughs for particle physics in the last two decades. Consequently, the Nobel Prize in Physics 2015 was awarded to Takaaki Kajita from the Super-Kamiokande (SK) Collaboration and Arthur B. McDonald from the Sudbury Neutrino Observatory (SNO) Collaboration "for the discovery of neutrino oscillations, which shows that neutrinos have mass" [70].

To be more precise, SK [22] and SNO [23] detected flavor conversion in two different channels, which however could have been caused by phenomena other than neutrino masses (see, e.g., [71] for more details). While SK observed flavor oscillations of atmospheric neutrinos, SNO discovered the almost nonoscillatory effect of adiabatic neutrino flavor conversion in the density-varying matter of the Sun [72, 73]. Since the original SNO data was energy independent within the measurement uncertainties, it could have also been explained by, e.g., resonant spin-flavor precession, violation of the equivalence principle, or nonstandard neutrino interactions [74]. Also the original SK data excluded alternative explanations, such as neutrino decay or neutrino decoherence, only at 3.4σ [75]. However, various subsequent data sets from solar, atmospheric, and reactor neutrino experiments [76–94] confirmed the characteristic sinusoidal flavor transition probability induced by nonzero neutrino mass differences and mixing angles. Thus, we can definitely conclude "that neutrinos have mass".

Can the SM account for nonzero neutrino masses? The answer is no, because the SM particle content does not allow for any renormalizable neutrino mass terms [12]. On the one hand, nonzero Dirac neutrino masses cannot be accommodated in the SM due to the absence of right-handed (RH) neutrino states. On the other hand, nonzero Majorana masses for the left-handed (LH) neutrinos are not allowed as the SM Higgs sector only contains an $SU(2)_L$ doublet and no triplets. Thus, it is widely believed that renormalizable neutrino mass terms in the SM require the postulation of new elementary particles.

Consequently, the discovery of neutrino oscillations hints at fundamentally new physics beyond the SM, which has triggered intense research activities on both the theoretical and the experimental side. In the following section, we will provide a brief summary of the currently most popular theoretical approaches to resolve the neutrino mass problem. For a detailed overview of the current state of the art of neutrino physics, we refer the reader to the numerous extensive review articles available on the market, e.g., [95–99].

2.1.2 High-Energy Neutrino Mass Models

The SM is a renormalizable theory with at most dimension-four operators. This forbids higher-dimensional operators, such as the Weinberg operator of dimension five [100],

$$\mathcal{L}_{\text{Weinberg}} = \frac{c_{\alpha\beta}}{\Lambda_{\text{UV}}} \left(\overline{(L_{\alpha L})^c}\tilde{H}^*\right) \left(\tilde{H}^\dagger L_{\beta L}\right) + \text{h.c.} \tag{2.1.1}$$

Here, $c_{\alpha\beta}$ is a model-dependent coefficient, $L_{\alpha L} = (\nu_{\alpha L}, e_{\alpha L})^T$ are the LH lepton doublets of the SM with flavor indices $\alpha, \beta = e, \mu, \tau$, and $H = (H^+, H^0)^T$ is the SM Higgs $SU(2)_L$-doublet, where $\tilde{H} = i\sigma_2 H^*$.

As illustrated in the Introduction, the standard path of BSM model building is to propose new physics at some high-energy scale Λ_{UV}, where the terms "high-energy" and "ultraviolet" (UV) can be used interchangeably. Thus, it is generally expected that the SM is only an effective "low-energy" or "infrared" (IR) theory of some yet unknown "UV-completed" full theory. For neutrino physics, this standard assumption implies that some high-energy new-physics effects suppress the neutrino masses with respect to the Higgs scale. Indeed, after integrating out the model-dependent new physics, all high-energy Majorana mass models reduce to the five-dimensional Weinberg operator (2.1.1) or some higher-dimensional equivalent [101].

In the following, we will summarize the basic ideas behind the most popular high-energy neutrino mass models, while we refer the reader to [102] for a more extensive review. In short, the most common paths to naturally generate small neutrino masses are the *seesaw mechanisms* and *radiative models*, which are based on the concepts of scale suppression and loop suppression, respectively.

In the case of scale suppression, the Weinberg operator (2.1.1) can already be generated at tree level with either an $SU(2)_L$-singlet fermion, a triplet scalar, or a triplet fermion as the mediator. These famous three possibilities are called type-I [103–106], type-II [107–113], and type-III [114] seesaw mechanisms (see [115] for a review). As one can infer from (2.1.1), the neutrino mass scale $m_\nu \sim \langle H^0 \rangle^2 / \Lambda_{\mathrm{UV}}$ is suppressed by the high-energy scale Λ_{UV} with respect to the vacuum expectation value (VEV) of the Higgs doublet, $\langle H^0 \rangle$. The most appealing aspect of the seesaw mechanisms is their possible embedding in GUTs [43–45], where Λ_{UV} corresponds to the GUT scale, typically ranging between 10^{14} and 10^{16} GeV. The eventual unification of the fundamental SM forces in a GUT is one of the most dominant BSM concepts, and the quantitative success of gauge coupling unification in SUSY GUTs [116, 117] has added much support to this idea. The minimal GUT scheme in which the seesaw mechanism can be realized is $SO(10)$ with both 10 and 126-dimensional Higgs representations (see [118] for a review).

In the case of loop suppression, neutrino masses are forbidden at tree level and thus only arise at loop level, e.g., at one-loop level [119], two-loop level [120–122], or three-loop level [123]. The neutrino masses generated at n-loop level can be estimated as

$$m_\nu \propto \frac{\langle H^0 \rangle^2}{\Lambda_{\mathrm{UV}}} \times \left(\frac{1}{16\pi^2} \right)^n . \tag{2.1.2}$$

Consequently, the UV scales of new physics in radiative models are usually lower than in standard seesaw mechanisms. In combination with other possible suppression mechanisms, such as small lepton-number-violating couplings [124], the new-physics scale Λ_{UV} can even be as low as the TeV scale, which makes radiative models testable at the LHC (see, e.g., [125–131]).

Another type of high-energy neutrino mass models testable at particle colliders are scenarios that do not extend the SM by RH neutrinos but, for example, postulate a scalar $SU(2)_L$ Higgs triplet whose neutral component yields LH Majorana masses at tree level [132]. However, such models require either a very small Yukawa coupling of the lepton doublets to the Higgs triplet and/or a tiny VEV of the neutral Higgs component, which need to be motivated. Notice here that our gravitational *low-energy* neutrino mass model could give rise to LH Majorana neutrino masses without any fine-tuning, which corresponds to the minimal case of our model presented in chapter 3.

The mentioned fine-tuning of neutrino Yukawa couplings is also the reason why the following minimal Dirac solution of the neutrino mass problem is commonly considered less elegant than the concepts of scale and/or loop suppression. As a minimal extension of the SM, one could simply add RH neutrinos and generate Dirac neutrino masses similar to the charged-fermion masses, i.e., via Yukawa couplings to the SM Higgs doublet. However, there is a huge gap between the small neutrino masses and the masses of the charged fermions, which requires tiny neutrino Yukawa couplings of $Y_\nu \lesssim 10^{-11}$ [97]. Even though small Yukawa couplings are technically natural as their absence increases the symmetry of the Lagrangian [133], the question remains why the neutrino Yukawa couplings should be so much smaller than the charged lepton couplings ranging between 10^{-6} and 1. Moreover, the observed neutrino mixing pattern differs drastically from the quark mixing pattern [97]. In addition to these phenomenological issues, the RH gauge-singlet neutrinos could *a priori* have large Majorana masses, even though the absence of such lepton-number-breaking masses can also be considered technically natural [133] as lepton number is only nonperturbatively broken in the SM. These facts do not exclude but render unlikely the possibility that neutrinos have the same mass origin as the charged fermions. Note that in our gravitational neutrino mass model (see sections 3.3 and 5.5.2), small Dirac neutrino masses could be generated without fine-tuned couplings through a neutrino-composite "second Higgs" field with a small VEV, which would correspond to the minimal Dirac case of our model.

In summary, there exist numerous high-energy neutrino mass mechanisms that are based on the concepts of scale suppression, loop suppression, and/or various types of fine-tuning. Let us emphasize that there are many more examples than those we mentioned, such as models based on R-parity violating SUSY [134–137], string theory [138], or large extra dimensions [139–141].

In addition to the common high-energy BSM scenarios, there also exist a few *low-energy* neutrino mass proposals. These are mostly motivated by experimental anomalies hinting towards the existence of light ($\sim$ eV) sterile neutrino states (see [142] for a recent review). In case of growing experimental evidence for light sterile neutrinos, high-scale neutrino mass mechanisms would get into difficulties, because they usually predict sterile neutrinos of much larger masses. On the other hand, scenarios that include light sterile neutrinos, such as the low-scale seesaw proposal with six light Majorana neutrinos [143], commonly face severe conflicts with cosmology (see section 5.5.3). For resolving these conflicts, such low-energy scenarios have to be further complicated to suppress the light sterile neutrino abundance in the early Universe. Another problematic aspect of the mentioned low-scale seesaw proposal is that there is no theoretically compelling reason to assume a small Majorana mass scale, tiny Yukawa couplings for generating a small Dirac mass scale, and the coincidence of having those two scales numerically so close [143]. Notice that the low-scale seesaw proposal could be "IR-completed" by the mixed Dirac-Majorana case of our gravitational neutrino mass model, which could explain the coincidence of the Dirac and Majorana mass scales and render the light sterile neutrinos completely consistent with cosmology (see section 5.5.3).

While none of the previously proposed neutrino mass models have been experimentally substantiated so far, their predictions get continuously tested in various particle and nuclear physics experiments, such as collider [144] and neutrinoless double beta decay experiments [145]. In addition, there are two research areas beyond particle and nuclear physics that allow for a complementary test of BSM neutrino scenarios: cosmology and astrophysics [146]. First, cosmic neutrinos play an important role for the evolution of the Universe, in particular for cosmic structure formation [147]. Second, neutrinos are emitted in great abundance by many astrophysical sources [148]. Therefore, BSM neutrino physics can drastically modify cosmological observables as well as astrophysical processes, which in turn allows to test BSM neutrino scenarios with cosmological and astrophysical data. This provides a prime example of the close connection between the research areas of nuclear physics, particle physics, astrophysics, and cosmology.

For our gravitational low-energy neutrino mass model (see chapter 3), this interdisciplinarity will become particularly clear when examining the numerous model predictions that arise for cosmology (see section 5.2), astrophysics (see section 5.3), gravity (see section 5.4), as well as nuclear and particle physics (see section 5.5). Our follow-up gravitational axion model (see chapter 4) will also yield several nontrivial connections between "the very small and the very large", i.e., fundamental particle physics phenomena and the properties of our entire Universe. In order to motivate this second gravitational low-energy model, we will briefly review the current status of axion physics in the following section: first, we will explain the essence of the strong CP problem, then we will discuss a potential solution of this problem within the SM, and finally we will show how high-energy BSM axion models attempt to solve this problem.

2.2 Axion Physics

2.2.1 The Strong CP Problem

In the early development of the SM, it was expected to be a fundamental symmetry of Nature that one can simultaneously exchange particles with antiparticles (abbreviated "C" for charge conjugation) and spatially reflect the coordinate system (abbreviated "P" for parity) without altering the physical equations. One of the most important breakthroughs for particle physics of the last century was the surprising experimental discovery that EW interactions violate this CP symmetry and therefore distinguish matter from antimatter [149]. For this discovery, the Nobel Prize in Physics 1980 was awarded jointly to the experimental physicists James W. Cronin and Val L. Fitch [150]. Later, it was established that also the theory of strong interactions contains a term for CP violation, but strong CP violation is experimentally excluded with high accuracy [24, 25]. Consequently, the question why strong interactions do not violate CP and thus do not distinguish matter from antimatter has been one of the greatest, long-standing open puzzles of the SM.

In more technical terms, the CP-violating term in QCD,

$$\mathcal{L}_{\text{QCD}} \supset -\theta G\tilde{G}, \tag{2.2.1}$$

originates from the super-selection sectors of the topologically nontrivial QCD vacuum, which are labeled by an angular parameter θ [58,151]. Here, the gluon field strength G and its Hodge dual $\tilde{G}$ are defined as

$$G^a \equiv dA^a + f^{abc}A^b A^c, \tag{2.2.2}$$
$$\tilde{G}^{a\,\alpha\beta} \equiv \epsilon^{\alpha\beta\mu\nu}G^a_{\mu\nu}, \tag{2.2.3}$$

where $G = G^a T^a$, T^a are the generators and f^{abc} are the structure constants of the appropriate Lie algebra, the superscript a denotes the gauge group index, d is the exterior derivative, A is the gluon field matrix, and we omitted several numerical factors as throughout the entire thesis.

Since the Levi-Civita pseudo-tensor $\epsilon^{\alpha\beta\mu\nu}$ in (2.2.3) changes sign under parity transformations, the so-called "θ-term" (2.2.1) violates the combined CP symmetry. Note that the θ-term also violates the time reversal symmetry T due to the CPT invariance of QCD [152–156].

The topologically nontrivial QCD vacuum does not only give rise to the CP-violating θ-term, it also spontaneously breaks the approximate chiral symmetry of the three light (u, d, s) quarks,

$$U(3)_L \times U(3)_R \to U(3)_V = SU(3)_V \times U(1)_V, \tag{2.2.4}$$

which in principle should give rise to $8+1$ light pseudo-Goldstone bosons [54,55]. However, in Nature we only observe eight light mesons, while the ninth singlet pseudoscalar meson η' is much heavier than its companions and cannot be regarded as a pseudo-Goldstone boson. This is due to the fact that the isospin-singlet axial $U(1)$ current

$$j^{(q)}_\mu = \bar{q}\gamma_\mu\gamma_5 q \tag{2.2.5}$$

corresponding to the axial quark symmetry

$$q \to e^{i\gamma_5\chi}q \tag{2.2.6}$$

has an anomalous divergence,

$$\partial^\mu j^{(q)}_\mu = G\tilde{G} + m_q\bar{q}\gamma_5 q, \tag{2.2.7}$$

which was found by Adler, Bell, and Jackiw (ABJ) [56, 57]. Note that the second term in (2.2.7) originates from explicit chiral symmetry breaking by the nonzero quark masses m_q. For simplicity, we only consider a single quark q.

Because the term $G\tilde{G}$ is a total derivative (see section 3.1.1 for more details), one may naively expect that the ABJ anomaly (2.2.7) is only a mathematical artifact without any physical meaning. Correspondingly, the θ-term (2.2.1) would disappear after the integration over the Lagrangian and there would be no observable CP violation in QCD.

However, 't Hooft showed that certain topologically nontrivial field configurations can yield nonzero contributions to the integral $\int d^4x\, G\tilde{G} = N$ [58].

Here, N is the integer-valued topological charge [157] of these finite-action field configurations, which are called "instantons" because they describe instantaneous tunneling between the different QCD vacua. Later, Witten argued that 't Hooft's semi-classical instanton result is incomplete because quantum corrections yield the dominant nonperturbative contribution to the integral over $G\tilde{G}$ [158]. This can be seen in the large-N_c limit of an $SU(N_c)$ gauge theory with N_c colors [159], which represents a remarkably good field-theoretic description of QCD even though its number of colors $N_c = 3$ might not seem particularly large. In this limit, Witten showed that instantons contribute only at order e^{-N_c}, while the θ-dependence of the QCD vacuum is present at leading order of the $1/N_c$ expansion [158]. Consequently, the mass square of the η' meson is proportional to $1/N_c$, as shown by Witten [59] and Veneziano [60],

$$m_{\eta'}^2 = \frac{\langle G\tilde{G}, G\tilde{G}\rangle_{q\to 0}}{f_{\eta'}^2} \propto \frac{1}{N_c}, \tag{2.2.8}$$

where $f_{\eta'} \propto N_c^{-1/2}$ is the η' decay constant and $\langle G\tilde{G}, G\tilde{G}\rangle_{q\to 0} \propto N_c^{-2}$ is the topological vacuum susceptibility of QCD as defined in (3.1.8).

To summarize, nonperturbative QCD effects spoil the $U(1)_A$ symmetry via the ABJ anomaly and give a large mass to the η' meson. Since the same effects also render the θ-term physical, we expect to observe strong CP violation.

However, there are severe experimental constraints on CP-violating effects in QCD, which stem from the electric dipole moment (EDM) of the neutron n,

$$\mathcal{L}_{\text{EDM}} = -d_n \bar{n} i\gamma^5 \sigma^{\mu\nu} n F_{\mu\nu}, \tag{2.2.9}$$

where $F_{\mu\nu}$ is the photon field strength. The neutron EDM is experimentally excluded down to $|d_n| < 3.0 \times 10^{-13}\,\text{e fm}$ [24,25], which translates into a strong upper bound on the angle $|\theta| < 1.3 \times 10^{-10}$ as $d_n \propto m_q \theta$ [160]. To be more precise, the physical (i.e., measurable) quantity is not θ alone, but a particular combination of two different angles,

$$\bar{\theta} = \theta - \arg\det M_q, \tag{2.2.10}$$

where $\det M_q$ is the determinant of the complex quark mass matrix M_q. This shift of the θ-angle to the physical quantity $\bar{\theta}$ stems from the chiral quark transformation (2.2.6) on the path integral (see [161] and section 2.2.2 for more details). Because $\bar{\theta}$ could *a priori* take a wide range of values, $-\pi \leq \bar{\theta} \leq \pi$, the natural theoretical expectation without any fine-tuning would be a substantial violation of CP in QCD, $\bar{\theta} \sim 1$. The drastic deviation of this expectation from the experimental constraints on $\bar{\theta}$ is the essence of the strong CP problem.

2.2.2 Massless Up-Quark Solution with η'-Axion

One solution to the infamous strong CP problem is the celebrated Peccei-Quinn (PQ) mechanism [69], which relies on the existence of a spontaneously broken chiral $U(1)_{\mathrm{PQ}}$ symmetry that is anomalous under the QCD gauge group. In such a case, the QCD θ-term can be absorbed by rephasing the pseudo-Goldstone boson a_{PQ} of the $U(1)_{\mathrm{PQ}}$ symmetry,

$$\frac{a_{\mathrm{PQ}}}{f_a} \to \frac{a_{\mathrm{PQ}}}{f_a} - \theta, \tag{2.2.11}$$

which was found by Weinberg and Wilczek [162, 163]. Here, f_a is the decay constant of the pseudo-Goldstone boson a_{PQ}, which Wilczek named "axion" after a detergent brand, because it "clean[s] up a problem with an axial current" [164]. The absorption (2.2.11) can happen because the $U(1)_{\mathrm{PQ}}$ symmetry, which acts on the axion as a shift symmetry,

$$a_{\mathrm{PQ}} \to a_{\mathrm{PQ}} + \text{const.}\,, \tag{2.2.12}$$

is explicitly broken by the ABJ anomaly of QCD (2.2.6). In colloquial terms, the constant vacuum angle θ has been "promoted to a dynamical field", which interacts with gluons via the term

$$\mathcal{L}_{\text{axion}} \supset \frac{a_{\mathrm{PQ}}}{f_a} G\tilde{G}, \tag{2.2.13}$$

where we again omitted numerical factors. This interaction conserves CP as the pseudoscalar boson a_{PQ} changes sign under P transformations. According to a theorem by Vafa and Witten [165], also the potential for the shifted field (2.2.11) is minimized at the CP-conserving value. Therefore, the PQ mechanism solves the strong CP problem.

Within the SM, the simplest realization of an anomalous chiral PQ symmetry $U(1)_{\mathrm{PQ}}$ could have been achieved if one of the quark flavors, say the up quark, had no Yukawa coupling to the Higgs doublet. One may argue that setting this coupling to zero creates another naturalness problem. However, this is a spurious argument, as setting a number protected by a symmetry to zero is not more unnatural than choosing it to be $\sim 10^{-5}$, putting aside that the gain of solving the strong CP problem is enormous. Note that the $U(1)$ symmetry in question is perturbatively safe and only nonperturbatively broken, i.e., it is a true symmetry with regard to 't Hooft's technical naturalness argument [133].

In case of a vanishing up-quark Yukawa coupling to the Higgs doublet, the anomalous chiral PQ symmetry would have been an axial $U(1)_{Au}$ symmetry acting on the up quark,

$$u \to e^{i\alpha\gamma_5} u\,, \tag{2.2.14}$$

where we combined the LH (u_L) and the RH (u_R) components of the up quark into a single Dirac fermion u. The corresponding current

$$j_\mu^{(u)} \;=\; \bar{u}\gamma_\mu\gamma_5 u \tag{2.2.15}$$

exhibits the anomalous ABJ divergence (2.2.7) with respect to QCD [56, 57],

$$\partial^\mu j_\mu^{(u)} \;=\; G\tilde{G}\,. \tag{2.2.16}$$

Consequently, the vacuum θ-angle can be removed by performing the chiral transformation (2.2.14) and becomes unphysical.

Although sometimes this scenario is presented as being different from the PQ case, in reality it represents a particular version of the PQ solution [48]: the chiral symmetry is spontaneously broken by the QCD up-quark condensate and the role of the axion is played by the η' meson. This degree of freedom describes the fluctuations of the phase of the quark condensate $\langle \bar{u}_L u_R \rangle \equiv V^3 e^{i\eta'/V}$, where V^3 is the VEV of its absolute value, which is set by the QCD scale, $|\langle \bar{u}_L u_R \rangle| = V^3 \sim \Lambda^3_{\mathrm{QCD}}$. In the absence of the up-quark Yukawa coupling constant, and when ignoring all other quark flavors, the η' meson is getting its mass solely from the QCD anomaly [58–60].

Needless to say, such a solution to the strong CP problem would be highly economical. However, chiral perturbation theory indicates the need for a nonzero up-quark mass that breaks the chiral symmetry (2.2.14). It was proposed that nonperturbative QCD effects might generate this nonzero up-quark mass [61–64], but so far there is no evidence that the QCD contribution is large enough to make this proposal phenomenologically viable (see [166] for a detailed treatment of this issue). Therefore, it is usually assumed that within the SM, the only possible source for a large enough up-quark mass is the Yukawa coupling to the Higgs doublet, which of course is incompatible with the solution to the strong CP problem as it breaks the chiral PQ symmetry (2.2.14) explicitly. Consequently, the axion needs to be implemented in form of a degree of freedom from beyond the SM.

2.2.3 High-Energy Invisible Axion Models

By taking the standard path of a nonzero up-quark Yukawa coupling, one needs to hypothesize the axion as a new degree of freedom. In the first BSM axion model proposed by Peccei, Quinn, Weinberg, and Wilczek (PQWW) [69, 162, 163], the SM Higgs sector gets replaced by two Higgs doublets H and H'. The VEVs of these doublets provide masses to the SM fermions, while their neutral phases mix to yield an axion and a Higgs boson. We will further discuss this PQWW model in section 4.1. At this point, it suffices to mention that this simple and elegant theoretical idea was soon ruled out by experiments. The proposed new physics emerged at energy scales close to the EW symmetry breaking scale, and the resulting significant interactions of the axion with SM particles were not observed experimentally [167, 168]. Therefore, the quest arose for "invisible" axion models whose new-physics effects are hard to catch in experiments. Similar to the conventional paths of neutrino mass model building (cf. section 2.1.2), the idea was to hide the proposed BSM axion physics at very high energy scales, so that the new-physics effects efficiently decouple from SM processes.

The two most popular high-energy axion scenarios are the Kim-Shifman-Vainshtein-Zakharov (KSVZ) [169, 170] and Dine-Fisher-Srednicki-Zhitnisky

(DFSZ) [171, 172] models. The DFSZ model [171, 172] extends the PQWW proposal by including a new complex SM-singlet scalar S with a coupling $\propto H_d^\dagger H_u^* S$. This term violates the original PQ symmetry of the PQWW model, but respects a new PQ symmetry, whose pseudo-Goldstone is called the DFSZ axion. This DFSZ axion becomes "invisible" if the VEV of the scalar S is much larger than the SM Higgs VEV, $f_a \gg \Lambda_{\rm EW}$.

In the KSVZ model [169, 170], the SM is also enlarged by a new SM-singlet complex scalar S, but an extra heavy SM-singlet quark doublet Q is added instead of a second Higgs doublet. The PQ symmetry is spontaneously broken by the VEV of the scalar S, which is again much larger than the EW scale. Since the VEV of S gives a large mass $m_Q \sim f_a \gg \Lambda_{\rm EW}$ to the extra quark doublet, this BSM particle species can be integrated out.

In some variants of the KSVZ model, the new scalar S can be used to generate large RH neutrino masses, which allows one to unify the axion and seesaw proposals. In these models, the axion can be identified with the majoron [173–175], which is a pseudo-Goldstone boson related to the spontanous breaking of baryon minus lepton number symmetry $(B - L)$ [176]. Such an "axi-majoron" can also emerge when generating the small neutrino masses radiatively [177]. Moreover, invisible axions can be embedded into flavor symmetry models [178], SUSY frameworks [179], and GUTs [180] (see [181] for a review).

To summarize, the most popular attempts of solving the strong CP problem require the introduction of at least one scalar field with a very large VEV plus either a hypothetical heavy quark doublet or an additional Higgs doublet. Thus, several newly postulated particle species are needed in order to nullify one single parameter of the SM. In this light, one of the original KSVZ papers from 1979 stated that "the model discussed here is in a sense extreme and does not pretend to be a true one" [170]. In contrast, present reviews agree that the KSVZ model is "the simplest case" [182] with "the virtue of simplicity " [161]. This demonstrates how the notion of extremes and simplicity has changed over several decades of high-energy axion model building, and how complicated it actually is to identify experimentally viable origins of strong CP conversion.

Common invisible axion models do not only suffer from this new-species problem, but also face a naturalness issue: the VEV of the hypothesized scalar field S is constrained by astrophysical arguments to be much larger than the SM Higgs VEV, $f_a \gtrsim 10^9$ GeV $\gg \Lambda_{\rm EW}$ (see [183] for a review). Therefore, the mass square term of the SM Higgs boson has to be tuned relative to the PQ scale by at least a factor of 10^{-14}, which is smaller than the $\bar{\theta}$-parameter itself (see section 4.3.2 for more details). One positive consequence of this unexplained large hierarchy of scales is that a large axion decay constant f_a implies that the axion is a stable particle, which makes it a possible cold DM candidate [184–186]. The small axion mass m_a originates from the axion mixing with the η' meson through the axion-gluon coupling (2.2.13) and reads

$$m_a = \frac{\left(\langle G\tilde{G}, G\tilde{G}\rangle_{q\to 0}\right)^{1/2}}{f_a} \sim 5.70 \ \mu\text{eV} \left(\frac{10^{12} \text{ GeV}}{f_a}\right), \tag{2.2.17}$$

analogous to the Witten-Veneziano formula (2.2.8) for the η' mass. Here, the topological vacuum susceptibility $\langle G\tilde{G}, G\tilde{G}\rangle_{q\to 0}$ was computed to be $(75.5\,\mathrm{MeV})^4$ with lattice QCD methods [187]. Notice that the same axion-gluon coupling (2.2.13) that yields the small axion mass (2.2.17) also induces an axion coupling to protons, neutrons, and two photons, which opens up the window for direct axion detection.

A third potential problem that standard invisible axion models face is a gravitational threat: following the folk theorem that gravity violates global symmetries, nonperturbative gravitational effects were claimed to spoil the global PQ symmetry [188]. Many models have been developed to solve this axion quality problem, which are based on additional discrete gauge symmetries [189–192], SUSY [193–195], axion compositeness [196–198], or new continuous gauge symmetries [199, 200]. However, it was shown in [48] that the only gravitational operator that can actually invalidate the axion solution is the gravitational Chern-Simons term (3.1.18). All other nonperturbative gravitational effects that explicitly break the PQ symmetry can only yield axion mass contributions that are CP conserving [48]. Therefore, the actual gravitational danger to the axion solution stems from the chiral gravitational anomaly, as we will explain in detail in sections 3.1.2 and 4.2.

In the light of these three common problems of invisible axion models, one may ask whether there is any BSM scenario that solves the strong CP problem without any new species, without a new high-energy scale, and with a built-in protection mechanism against gravity? As we will explain in detail in chapter 4, the very gravitational anomaly that spoils the standard invisible axion solutions can yield an alternative low-energy solution to the strong CP problem that is free of all the mentioned problems.

Chapter 3

Gravitational Neutrino Mass Model

This chapter is devoted to the theoretical concepts behind our proposed gravitational neutrino mass mechanism. First, we will review the emergence of bound states from chiral anomalies in section 3.1, where sections 3.1.1 and 3.1.2 will treat chiral anomalies in QCD and in gravity, respectively. The subsequent section 3.2 will illustrate how these bound states are related to neutrino condensation triggered by nonperturbative gravity. In section 3.3, we will discuss how the neutrino condensate yields small effective neutrino masses. In this context, we will briefly comment on the matters of Dirac versus Majorana masses, the neutrino mass hierarchy, and the numerical coincidence of the neutrino mass and DE scales. We will summarize and discuss our results in section 3.4.

3.1 Review of Bound States from Anomalies

In [47] it was shown that in the presence of a chiral anomaly, the topological formulation of any model immediately generates a mass gap in the theory, rendering the corresponding vacuum θ-angle unphysical. In particular, this generic concept illuminates the origin of the massive η' degree of freedom in QCD in model-independent terms. In the formulation of [48], the power of gauge redundancy allows one to understand this phenomenon as a "Higgs effect" of the corresponding Chern-Simons three-form of QCD.

Furthermore, it was pointed out in [48] that the similar effect of a mass gap generation must be exhibited by gravity in the presence of neutrinos with zero bare mass. Based on this insight, the authors of [49] showed that in the presence of a chiral anomaly, gravity gives rise to a new degree of freedom in the neutrino sector: a pseudo-Goldstone boson of spontaneously broken axial neutrino symmetry. This η_ν particle is analogous to the η' meson of QCD.

In the following two subsections, we will recapitulate the theoretical foundations of this mass gap generation from chiral anomalies in the cases of QCD and gravity, respectively.

3.1.1 Topological Mass Gap Generation in QCD

As pointed out in section 2.2.1, the topologically nontrivial vacuum in QCD spontaneously breaks the approximate chiral symmetry of the three light (u, d, s) quarks,

$$U(3)_L \times U(3)_R \to U(3)_V = SU(3)_V \times U(1)_V, \tag{3.1.1}$$

which in principle should give rise to $8 + 1$ pseudo-Goldstone bosons [54, 55]. However, in Nature we only observe eight light mesons, and the ninth singlet pseudoscalar meson η' is much heavier than its companions. This is due to the fact that the corresponding isospin singlet axial $U(1)$ current

$$j_\mu^{(q)} = \bar{q}\gamma_\mu\gamma_5 q \tag{3.1.2}$$

has an anomalous ABJ divergence [56, 57],

$$\partial^\mu j_\mu^{(q)} = G\tilde{G} + m_q \bar{q}\gamma_5 q. \tag{3.1.3}$$

For the sake of simplicity, we here only considered a single quark flavor q and omitted several numerical factors as throughout the entire thesis. The gluon field strength G and its Hodge dual $\tilde{G}$ are defined as

$$G^a \equiv dA^a + f^{abc}A^bA^c, \tag{3.1.4}$$

$$\tilde{G}^{a\;\alpha\beta} \equiv \epsilon^{\alpha\beta\mu\nu}G^a_{\mu\nu}. \tag{3.1.5}$$

where $G = G^aT^a$, T^a are the generators and f^{abc} are the structure constants of the appropriate Lie algebra, the superscript a denotes the gauge group index, d is the exterior derivative, and A is the gluon field matrix.

It is well known (see, e.g., [47, 48, 201–203]) that one can equivalently and more elegantly formulate QCD in terms of topological entities: the Chern-Simons three-form C and the Chern-Pontryagin density E,

$$C \equiv AdA - \frac{3}{2}AAA, \tag{3.1.6}$$

$$E \equiv G\tilde{G} = \mathrm{d}\tilde{C}, \tag{3.1.7}$$

where $\tilde{C}^\mu \equiv \varepsilon^{\mu\alpha\beta\gamma}C_{\alpha\beta\gamma}$. The Chern-Simons three-form C (3.1.6) obtains the meaning of a field in QCD and plays a decisive role in the infamous strong CP problem. QCD is θ-dependent only if its topological vacuum susceptibility $\langle G\tilde{G}, G\tilde{G}\rangle$ does not vanish in the limit of zero momentum,

$$\langle G\tilde{G}, G\tilde{G}\rangle_{q\to 0} \equiv \lim_{q\to 0}\int d^4x\, e^{iqx}\langle T[G\tilde{G}(x)G\tilde{G}(0)]\rangle \tag{3.1.8}$$

$$= \text{const} \neq 0. \tag{3.1.9}$$

Expressed in terms of the field C, the nonvanishing correlator (3.1.9) implies that C has a massless pole for vanishing momentum,

$$\langle C, C\rangle_{q\to 0} = \frac{1}{q^2}, \tag{3.1.10}$$

since $G\tilde{G} = \mathrm{d}\tilde{C} \sim q\tilde{C}$. This means that C is a massless gauge field, which does not propagate any physical degree of freedom but mediates a constant long-range electric field E in the vacuum. Notice that we use the term "electric field" by analogy with electrodynamics, with C being analogous to the electromagnetic vector potential.

The fact that C is a massless field, combined with the gauge symmetry of C, leads to the following effective Lagrangian describing the topological structure of the QCD vacuum, which is simply a gauge theory of the three-form C [48]:

$$\mathcal{L}_{3-\mathrm{form}} = \frac{1}{2\Lambda_{\mathrm{QCD}}^4} E^2 + \theta E + \text{higher order terms}, \tag{3.1.11}$$

where the QCD scale Λ_{QCD} takes care of the dimensionality. Notice that in this formulation, the famous θ-term (2.2.1)

$$\mathcal{L}_{\mathrm{QCD}} \supset \theta G\tilde{G} \tag{3.1.12}$$

in the vacuum is nothing but the VEV of the zero-form electric field strength E, i.e., $\theta\langle G\tilde{G}\rangle = \theta\langle E\rangle = \theta\Lambda_{\mathrm{QCD}}^4$.

Inserting (3.1.7) into (3.1.12) shows that the θ-term in the QCD Lagrangian is a total derivative. As 't Hooft, Witten, and Veneziano pointed out [58–60], only nonperturbative effects give this term a physical significance (see section 2.2.1 or the review [204] for more details).

If we now introduce massless quarks or axions [162, 163] into the theory in order to make QCD independent of θ (see sections 2.2.2 and 2.2.3), the topological susceptibility of the QCD vacuum (3.1.8) vanishes and the electric field E (3.1.7) gets screened. Consequently, the massless pole of the low-energy correlator $\langle C, C\rangle_{q\to 0}$ (3.1.10) has to be eliminated, which is only possible through introducing a mass gap into the theory [48],

$$\langle C, C\rangle_{q\to 0} = \frac{1}{q^2 + m^2}, \tag{3.1.13}$$

as $\langle G\tilde{G}, G\tilde{G}\rangle_{q\to 0} = \langle d\tilde{C}, d\tilde{C}\rangle_{q\to 0} \sim q^2/(q^2+m^2)|_{q\to 0} = 0$. By gauge symmetry, the only way to account for this phenomenon in the language of the effective Lagrangian (3.1.11) is by introducing a massive pseudoscalar degree of freedom, which generates the mass gap. If the chiral symmetry was due to massless quarks, the corresponding massive pseudo-Goldstone boson would be η'. The lowest-order terms are then uniquely fixed to be

$$\mathcal{L}_{3-\mathrm{form}} = \frac{1}{2\Lambda_{\mathrm{QCD}}^4} E^2 - \frac{1}{f_{\eta'}}\eta' E + \frac{1}{2}\partial_\mu\eta'\partial^\mu\eta', \tag{3.1.14}$$

where the decay constant $f_{\eta'}$ of the η' meson is given by the QCD scale.

Solving the equation of motion for C,

$$\mathrm{d}\left(E - \frac{\Lambda_{\mathrm{QCD}}^4}{f_{\eta'}}\eta'\right) = 0, \tag{3.1.15}$$

we obtain for the electric field

$$E = \Lambda_{\mathrm{QCD}}^4 \left(\frac{\eta'}{f_{\eta'}} - \theta \right), \tag{3.1.16}$$

where θ is an integration constant.

Inserting this in the equation of motion for η', we get

$$\Box \eta' + \frac{\Lambda_{\mathrm{QCD}}^4}{f_{\eta'}} \left(\frac{\eta'}{f_{\eta'}} - \theta \right) = 0. \tag{3.1.17}$$

Here we immediately observe two important points:

(1) The fields η' and C combine and form a single propagating massive bosonic field of mass $m_{\eta'} = \Lambda_{\mathrm{QCD}}^2 / f_{\eta'}$.

(2) The VEV of this field is exactly where the electric field E vanishes.

It is clearly seen that this low-energy perspective of screening the electric field E, rendering the θ-term unphysical, and solving the strong CP problem matches the high-energy perspective: θ becomes an unobservable quantity, since a change in the effective value of θ can always be induced by making a chiral rotation of the massless fermion fields.

Let us emphasize this point again: expressed in the topological three-form language, the solution of the strong CP problem equals the generation of a mass gap for C [48]. This is exactly what happens when introducing massless quarks or an additional PQ symmetry [69] into the theory. The emerging massive pseudoscalar degree of freedom, which is either the η' in the massless quark case ($m = m_{\eta'}$) or the axion in the PQ solution ($m = m_a$), is eaten up by C. This means that the pseudoscalar provides a mass for the three-form and is the origin of the massive pole of the correlator $\langle C, C \rangle_{q \to 0}$ (3.1.13).

3.1.2 Topological Mass Gap Generation in Gravity

It has been a long-open question whether not only QCD but also quantum gravity effects could induce CP violation (see also sections 2.2.3 and 4.2). While perturbative gravity is known to conserve CP, arguments based on gravitational instanton and wormhole solutions (see, e.g., [188, 205–210] and references therein) suggest that nonperturbative gravity might violate CP and destroy the massless up-quark or axion solutions of the strong CP problem. This widespread belief is supported by the folk theorem that gravity breaks global symmetries because black holes or wormhole solutions can swallow global charges [211]. However, so far there is no completely consistent description of the (non-)violation of global symmetries by quantum gravity. In contrast, local symmetries are generally considered to be conserved by gravity, since local charges cannot be absorbed by black holes or wormholes due to Gauss's law.

This different behavior of global and local symmetries becomes especially important when noticing that spontaneously broken Abelian *global* symmetries can be dualized to *local* shift symmetries that are gauged by three-form fields [48]. For example, the anomalous $U(1)$ symmetries in the massless up-quark or PQ cases can be transformed into local symmetries gauged by the QCD Chern-Simons three-form (3.1.6). For gravity, this general statement implies that any possible CP-violating quantum correction must enter the dualized theory in a gauge-invariant way, which can only happen if gravity provides an additional massless three-form field [48]. The most obvious candidate for this three-form is the unique gravitational Chern-Simons term [65]

$$\mathcal{L}_G \supset \theta_G R\tilde{R}, \tag{3.1.18}$$

where R is the Riemann tensor and $\tilde{R}$ is its Hodge dual. If we extend Einstein gravity by this gravitational analog of the QCD θ-term, the general considerations of the previous subsection can be directly applied to gravity [48, 49]. Analogous to QCD, we can formulate the nonperturbative sector of gravity in terms of topological quantities: a gravitational Chern-Simons three-form C_G and a gravitational Chern-Pontryagin density E_G [47, 212, 213],

$$C_G \equiv \Gamma \mathrm{d}\Gamma - \frac{3}{2}\Gamma\Gamma\Gamma, \tag{3.1.19}$$

$$E_G \equiv R\tilde{R} = \mathrm{d}\tilde{C}_G, \tag{3.1.20}$$

where $\tilde{C}^{\mu}_{G} \equiv \varepsilon^{\mu\alpha\beta\gamma} C_{G\,\alpha\beta\gamma}$ is the dual of C_G and Γ is the Christoffel connection.

Since the term $R\tilde{R}$ is a total derivative just as in QCD, it might only be a mathematical artifact without any physical significance. However, gravity offers many nonperturbative phenomena, which are not totally understood so far. Therefore, in this thesis we shall not be concerned with the possible microscopic origin of this term, which can easily be intrinsically quantum gravitational rather than semi-classical. Instead, we will take its physicality as a starting assumption and examine the ensuing implications for the SM.

If the gravitational vacuum angle θ_G is physical in the absence of massless fermions, for example in a theory with pure gravity, the gravitational topological vacuum susceptibility would not vanish

$$\langle R\tilde{R}, R\tilde{R}\rangle_{q\to 0} = \text{const} \neq 0, \tag{3.1.21}$$

similar to the QCD case explained above. The strength of the vacuum correlator $\langle R\tilde{R}, R\tilde{R}\rangle_{q\to 0}$ is given by a scale Λ_G, which will appear as an effective cutoff scale in a low-energy theory of the gravitational three-form C_G. The scale Λ_G is unknown and will be treated as a parameter, solely fixed from phenomenological requirements (see section 5.1). One thing that we can expect about this scale is that it must be strongly suppressed with respect to the Planck scale, which is normal for the IR scales generated by nonperturbative effects.

In the following, we will consider gravity coupled to the lightest known fermions, the neutrinos, in analogy to considering QCD with light quarks. Just as in the case of quarks, there exists a neutrino chiral symmetry, which is anomalous with respect to gravity. If no RH neutrinos are introduced, i.e., if neutrinos are purely LH Majorana particles, this anomalous symmetry coincides with the neutrino lepton number. In case when neutrinos have RH partners, they carry the opposite charges under the anomalous symmetry, which is different from the standard lepton number charge assignment. Hence, to stress this difference we will call this symmetry an *axial* neutrino lepton number.

In what follows, we will outline how the anomalous axial neutrino symmetry leads to a massive new pseudoscalar degree of freedom. For simplicity, we will first consider a single massless neutrino species and afterwards add a small neutrino mass as a perturbation.

As already mentioned, the axial $U(1)$ neutrino current

$$j_\mu^{(\nu)} = \bar{\nu}\gamma_\mu\gamma_5\nu \tag{3.1.22}$$

corresponding to the axial neutrino symmetry

$$\nu \to e^{i\gamma_5\chi}\nu \tag{3.1.23}$$

has an anomalous divergence [65–68]

$$\partial^\mu j_\mu^{(\nu)} = R\tilde{R} = E_G. \tag{3.1.24}$$

Due to this anomaly, the effective Lagrangian of the gravitational three-form field C_G is given by

$$\mathcal{L}_{3-\mathrm{form}} = \frac{1}{2\Lambda_G^4}E_G^2 + \frac{1}{f_\nu^2}E_G\frac{\partial^\mu}{\Box}j_\mu^{(\nu)}. \tag{3.1.25}$$

The first term accounts for treating the field C_G in an effective low-energy theory, where terms of higher order in E_G and its derivatives can be neglected. The unique contact interaction between $j^{(\nu)}$ and E_G with strength $f_\nu \sim \Lambda_G$ is generated by the triangle diagram of the gravitational ABJ anomaly [48].

The equation of motion for C_G,

$$\left(\Box + \Lambda_G^2\right)E_G = 0, \tag{3.1.26}$$

shows that there are no massless modes in E_G. This means that massless neutrinos can screen the gravitational electric field E_G and hence make the gravitational θ-term vanish, completely analogous to the massless quark case in QCD. The corresponding generation of the mass gap for C_G, analogous to Eq. (3.1.13), automatically implies that the current (3.1.22) must be identified with a pseudoscalar degree of freedom. We shall call it η_ν in analogy to the η' meson of QCD.

It is important to notice that the existence of η_ν is required for generating the mass gap in the presence of the chiral gravitational anomaly [47–49]. The correlator (3.1.21) has to be screened, which can only be accomplished if C_G eats up a Goldstone-like degree of freedom and gets massive, i.e., the Stückelberg field η_ν *has* to arise in order to preserve gauge symmetry. As already mentioned in the previous subsection, this "three-form Higgs effect" is the low-energy perspective of rendering the gravitational θ-term unphysical. From the high-energy point of view, θ_G is made unobservable because it can be arbitrarily shifted by a chiral rotation of the massless neutrino field.

By analogy with η', the degree of freedom η_ν can be regarded as a pseudo-Goldstone boson arising because nonperturbative gravity breaks axial neutrino number symmetry (3.1.23). It can be written as the effective low-energy limit of the neutrino bilinear operator

$$\eta_\nu \to \frac{1}{\Lambda_G^2}\bar{\nu}\gamma_5\nu \tag{3.1.27}$$

with the corresponding axial singlet current (3.1.22)

$$j_\mu^{(\nu)} \to \Lambda_G \partial_\mu \eta_\nu . \tag{3.1.28}$$

The effect of the mass gap generation is readily explained by the effective Lagrangian for η_ν, which is obtained by inserting (3.1.28) into (3.1.25),

$$\mathcal{L}_{3-\text{form}} = \frac{1}{2\Lambda_G^4}E_G^2 - \frac{1}{\Lambda_G}\eta_\nu E_G + \frac{1}{2}\partial_\mu\eta_\nu\partial^\mu\eta_\nu , \tag{3.1.29}$$

analogous to the QCD Lagrangian (3.1.14).

Since the decay constant of the η' meson is given by the QCD scale, we here analogously identify the decay constant f_ν of η_ν with Λ_G [49].

The equation of motion for η_ν,

$$\Box\eta_\nu + \frac{1}{\Lambda_G}E_G = 0, \tag{3.1.30}$$

immediately implies that $E_G = R\tilde{R}$ must vanish in any state in which η_ν is constant and, in particular, in the vacuum.

After integrating the equation of motion for E_G,

$$\mathrm{d}\left(E_G - \Lambda_G^3\eta_\nu\right) = 0, \tag{3.1.31}$$

we obtain for the electric field

$$E_G = \Lambda_G^3(\eta_\nu - \theta_G\Lambda_G), \tag{3.1.32}$$

where θ_G is an integration constant.

Inserting this expression for E_G into (3.1.30), we get

$$\Box\eta_\nu + \Lambda_G^2(\eta_\nu - \theta_G\Lambda_G) = 0. \tag{3.1.33}$$

We see that η_ν is a massive field with the VEV exactly at the point $\eta_\nu = \theta_G \Lambda_G$, which makes E_G zero in the vacuum.

From the effective Lagrangian (3.1.29) it is clear that an axial rotation of the neutrinos results in the shift of η_ν by a constant. This confirms the self-consistency of the statement that η_ν is a pseudo-Goldstone boson of the spontaneously broken axial neutrino symmetry. It is most obvious to identify the order parameter of this symmetry breaking with a neutrino condensate, similar to the quark condensate in QCD. The validity of this identification will be further discussed in the next section, but at this point is only a secondary consideration. What is important here is that there exists an order parameter, which breaks axial neutrino symmetry, and that the corresponding pseudo-Goldstone boson η_ν makes the Chern-Pontryagin density E_G zero.

The similarity of the gravity and QCD stories continues also in the presence of a small bare neutrino mass, which breaks axial neutrino symmetry explicitly. In order to see this, we now introduce a bare neutrino mass m_ν into the picture, which provides a small explicit mass for η_ν in the effective low-energy Lagrangian. Analogous to the small explicit η' mass given by the u, d, and s quark masses, the small explicit η_ν mass is not related to the anomaly and vanishes in the chiral limit.

In the presence of a bare neutrino mass, the axial neutrino number is explicitly broken, and the divergence of the singlet axial current (3.1.22) obtains a second contribution, just as in the QCD case (2.2.7),

$$\partial^\mu j_\mu^{(\nu)} = R\tilde{R} + m_\nu \bar{\nu}\gamma_5 \nu. \tag{3.1.34}$$

Again replacing the current (3.1.34) in the initial Lagrangian (3.1.25) with the pseudo-Goldstone boson (3.1.27), the effective Lagrangian (3.1.29) acquires an additional explicit mass term for η_ν,

$$\mathcal{L}_{3-\text{form}} = \frac{1}{2\Lambda_G^4} E_G^2 - \frac{1}{\Lambda_G} \eta_\nu E_G + \frac{1}{2} \partial_\mu \eta_\nu \partial^\mu \eta_\nu - \frac{1}{2} m_\nu \Lambda_G \eta_\nu^2. \tag{3.1.35}$$

The equation of motion and the corresponding solution for E_G are again given by (3.1.31) and (3.1.32), but the equation for η_ν changes to

$$\Box \eta_\nu + \Lambda_G^2 (\eta_\nu - \theta_G \Lambda_G) + m_\nu \Lambda_G \eta_\nu = 0, \tag{3.1.36}$$

so that the VEV of η_ν is shifted to

$$\eta_\nu = \frac{\theta_G \Lambda_G}{1 + \frac{m_\nu}{\Lambda_G}}. \tag{3.1.37}$$

Inserting this into (3.1.32), we get in the leading order in m_ν / Λ_G expansion

$$\langle R\tilde{R} \rangle_{q \to 0} = \langle E_G \rangle_{q \to 0} \simeq -\theta_G m_\nu \Lambda_G^3. \tag{3.1.38}$$

3.2 Low-Energy Neutrino Condensation

In the previous section, we mentioned that the order parameter of the spontaneous axial neutrino symmetry breaking is most obviously identified with a neutrino condensate, similar to the quark condensate in QCD. Notice that many possible condensation channels have been discussed both for QCD [214–217] and for fermions coupled to gravity (see appendix A and [218]). By analogy with QCD, where the nonzero bilinear quark condensate $\langle \bar{q}q \rangle \neq 0$ is known to trigger chiral symmetry breaking, we will in the following express the order parameter of spontaneous axial neutrino symmetry breaking with a bilinear neutrino condensate, $\Lambda_G^3 = \langle \bar{\nu}\nu \rangle$. Then, we can write the VEV (3.1.38) of the Chern-Pontryagin density in the form

$$\langle R\tilde{R} \rangle_{q\to 0} = -\theta_G m_\nu \langle \bar{\nu}\nu \rangle. \tag{3.2.1}$$

Let us compare this result with the topological vacuum susceptibility in QCD, which was computed in [170] to linear order in the u and d quark masses,

$$\langle G\tilde{G} \rangle_{q\to 0} = \theta \langle G\tilde{G}, G\tilde{G} \rangle_{q\to 0} = -\theta \frac{m_u m_d}{(m_u + m_d)^2} \langle m_u \bar{u}u + m_d \bar{d}d \rangle. \tag{3.2.2}$$

In the limit of a single fermion flavor, transferring the expression (3.2.2) to the gravitional neutrino sector yields a topological vacuum susceptibility that coincides with our derivation (3.2.1). As we will discuss below, incorporating more neutrino flavors is straightforward and extends our result (3.2.1).

Thus, with a single basic assumption that in the absence of massless chiral fermions the gravitational θ-term Chern-Pontryagin density would be physical, we uniquely arrive at a story very similar to QCD. Namely, once massless neutrinos couple to gravity, they must condense and deliver a pseudo-Goldstone boson η_ν, which generates a mass gap and screens the Chern-Pontryagin density, rendering the gravitational θ-term unphysical.

The absence of the condensate in a theory with massless fermions would cause an obvious contradiction. On the one hand, as the θ-term can be rotated away by an axial transformation, the Chern-Pontryagin density must vanish. On the other hand, this requires the generation of a mass gap for the Chern-Simons three-form field, for which the existence of a to-be-eaten-up pseudo-Goldstone boson η_ν is necessary. But in the absence of a condensate, which breaks axial symmetry spontaneously, the origin of such a Goldstone boson is impossible to explain. Notice that this phenomenon of neutrino condensation also persists in the potential presence of other massless fermions, such as a massless up quark (see chapter 4) or a massless BSM fermion.

At this point it is important to notice that the neutrino bound state η_ν differs from bound states arising due to universal confining dynamics. The η_ν is forced upon us by the Goldstone theorem and the three-form Higgs effect, which is due to the same nonperturbative gravitational dynamics that are responsible for the $\langle R\tilde{R}, R\tilde{R} \rangle$ correlator in the massive-fermion theory and for its screening in the massless one. These dynamics are not obliged to produce universal gravitational confinement among other particles.

Naively, one might expect that the neutrino condensate requires neutrinos to be bound below the symmetry breaking scale Λ_G, due to 't Hooft's anomaly matching condition [133, 219]. If this were the case, one could distinguish two options. The first one is the binding of only low-energy neutrinos below Λ_G energies. In the next section, we will identify Λ_G with the neutrino mass scale; thus, it would be consistent with current observations that no free neutrinos exist below this low-energy scale. The second option is to consider a classic picture in which neutrinos are connected by nonperturbative flux tubes of tension of order $(\text{meV})^2$. Such tubes would be hard to rule out based on existing observations; for example, neutrinos of MeV energy can stretch the flux tube to a huge macroscopic length $L \sim 10^7$ cm. In a recent paper [218], it was finally shown that gravitational anomaly matching does not force neutrinos to be bound. This leaves open the possibility that low-energy neutrinos are strongly coupled but do not form bound states.

So far, we have only considered one single neutrino species. Incorporating three neutrino flavors is straightforward and extends the result for the gravitational Chern-Pontryagin density (3.2.1). In this case, we obtain additional pseudo-Goldstone bosons ϕ_k if we assume the neutrinos to have hard masses smaller than the symmetry breaking scale Λ_G. If neutrinos have no hard but only effective masses, i.e., if the only source of all the observed neutrino masses is the spontaneous breaking of neutrino flavor symmetry by the neutrino condensate, as proposed in the next section, then the ϕ_k's become massless Goldstones. Even though it might seem counterintuitive to write a massless Goldstone particle as a bilinear of effectively massive constituents, one should be reminded that Goldstone bosons cannot simply be regarded as bound states of elementary particles. In case of neutrinos with no Yukawa couplings, the chiral symmetry is not explicitly broken. We consequently have to obtain several massless and one massive new degree of freedom (see [220] for analogous considerations). In this discussion, we ignore small corrections to some of the Goldstone masses generated by weak effects (see section 5.2.3).

The condensate of Dirac neutrino flavors transforms as bifundamental under the $U(3)_L \times U(3)_R$ symmetry. In order to generate hierarchical neutrino masses, the condensate needs to break this neutrino flavor symmetry down to a diagonal $U(1)_1 \times U(1)_2 \times U(1)_3$ subgroup of individual neutrino number symmetries for each flavor, which is further broken explicitly by weak effects. The spontaneous symmetry breaking results in 14 massless Goldstones ϕ_k and one massive pseudo-Goldstone η_ν. The off-diagonal Goldstones can induce neutrino-flavor-changing transitions. In the following, we will denote the massless Goldstone bosons together with the massive η_ν boson as $\phi \equiv \{\phi_k, \eta_\nu\}$.

Here at the end of the section, we want to face the question why the analogy between QCD and gravity works so well (see Table 3.1 for a concise summary), even though these two theories seem to be completely different at first sight.

Quantity	QCD	Gravity
An. $U(1)_A$ symmetry	$q \to \exp(i\gamma_5\chi)q$	$\nu \to \exp(i\gamma_5\chi)\nu$
An. $U(1)_A$ current	$j_\mu^{(q)} = \bar{q}\gamma_\mu\gamma_5 q$	$j_\mu^{(\nu)} = \bar{\nu}\gamma_\mu\gamma_5\nu$
Anomalous divergence	$\partial^\mu j_\mu^{(q)} = G\tilde{G} + m_q\bar{q}\gamma_5 q$	$\partial^\mu j_\mu^{(\nu)} = R\tilde{R} + m_\nu\bar{\nu}\gamma_5\nu$
Pseudoscalar boson	$\eta' \to \bar{q}\gamma_5 q/\Lambda^2_{\text{QCD}}$	$\eta_\nu \to \bar{\nu}\gamma_5\nu/\Lambda_G^2$
Chern-Simons three-form	$C \equiv AdA - \frac{3}{2}AAA,$	$C_G \equiv \Gamma\mathrm{d}\Gamma - \frac{3}{2}\Gamma\Gamma\Gamma$
Chern-Pontryagin density	$E \equiv G\tilde{G} = \mathrm{d}\tilde{C}$	$E_G \equiv R\tilde{R} = \mathrm{d}\tilde{C}_G$
Vacuum correlator	$\langle G\tilde{G}\rangle_{q\to 0} = -\theta m_q\langle\bar{q}q\rangle$	$\langle R\tilde{R}\rangle_{q\to 0} = -\theta_G m_\nu\langle\bar{\nu}\nu\rangle$

Table 3.1: Overview of the analogy between QCD and gravity. For simplicity, only a single fermion flavor is considered.

How can we at all perform computations in quantum gravity and consider the analogy to QCD, even though quantum gravity still requires a consistent UV completion? The answer is simple: anomalies are only sensitive to the massless sector of a theory and hence are insensitive to its energy scale [68, 221]. Therefore, we do not need to understand the UV regime of quantum gravity and can safely work in the well-understood effective low-energy regime.

3.3 Effective Neutrino Mass Generation

With the previous discussions of a bilinear neutrino vacuum condensate $\langle\nu\bar{\nu}\rangle \equiv v$ and the fluctuations around it, the pseudoscalar degrees of freedom ϕ, we can write our neutrino-composite field as

$$\nu\bar{\nu} = \langle\nu\bar{\nu}\rangle e^{i\phi} = ve^{i\phi}. \tag{3.3.1}$$

We obtain an interaction between the $\phi = \{\phi_k, \eta_\nu\}$ bosons and the neutrinos,

$$\mathcal{L}_{\text{int}} \supset g_\phi \sum_k (\partial_\mu\phi_k\bar{\nu}\gamma^\mu\gamma_5\nu) + g_{\eta_\nu}\eta_\nu\bar{\nu}\gamma_5\nu, \tag{3.3.2}$$

and an effective neutrino mass term

$$\mathcal{L}_{\text{mass}} \supset g_v v\bar{\nu}\nu. \tag{3.3.3}$$

There are four important points to notice:

(1) The mass term is not forbidden by any symmetries because chiral symmetry is broken by the chiral neutrino condensate formed through non-perturbative gravitational effects, as shown in Fig. 3.1.

(2) It was demonstrated in [218] that gravitational anomaly matching requires all fermions in the low-energy spectrum to be massive. Therefore, our neutrino mass mechanism predicts a nonzero mass of the lightest neutrino.

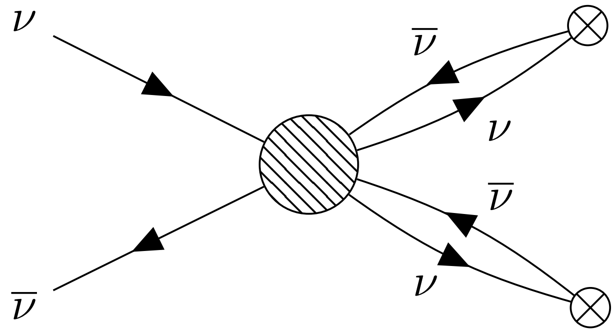

Figure 3.1: Neutrino mass generation through the condensate (crossed circles) via nonperturbative interaction (striped circle).

(3) We are not making any extra assumption when pointing out that the neutrino condensate provides effective neutrino masses. This anomalous neutrino mass generation is equivalent to the quark mass generation from the QCD condensate: the 't Hooft determinant $e^{i\theta}\det(\bar{q}_L q_R) + c.c.$ [222] provides an effective anomalous coupling due to integrated instantons, which can be written as an effective four-fermion interaction.

(4) By now we have treated the neutrinos as Dirac particles. If no RH neutrinos are introduced in the SM, the whole analysis remains similar, except the neutrino condensate is formed in the Majorana channel, $\langle \nu_L C \nu_L \rangle$, which breaks lepton number spontaneously. Correspondingly, the resulting neutrino masses are of Majorana type. If neutrinos are identical to their antiparticles and the SM is extended by RH neutrinos, both Dirac and Majorana masses are generated (see section 5.5.3 for more details).

Since the gravitational topological vacuum susceptibility (3.2.1) only depends on the hard neutrino masses and not on our effectively generated ones, the correlator gets screened by $m_\nu = 0$. Consequently, our model predicts that we will not observe any *CP* violation in the gravitational vacuum. By analogy, it has been considered that the strong *CP* problem could be resolved by assuming that the up quark has a vanishing bare mass and only obtains an effective mass through the 't Hooft vertex (see section 2.2.2 and [61–64]). Notice here that in QCD, we can treat the quark masses coming from the Higgs VEV as hard masses because the EW symmetry breaking occurs on a higher scale than the chiral symmetry breaking of QCD. This is not the case in our modified neutrino sector, where the scale Λ_G of chiral symmetry breaking coincides with the scale of the vacuum condensate generating the masses.

In the following, we will denote m_ν as an *effective* neutrino mass rather than a hard one. In order to naturally provide an effective neutrino mass at the observed low-energy scale [22, 23], the neutrino condensate needs to have a VEV v of the order of the neutrino masses. As we will show in section 5.1, this requirement is in accordance with several model-independent constraints, which yield $v \sim$ meV–eV. The symmetry breaking scale Λ_G coincides with v as in the QCD analog, and hence also the η_ν mass has to be of the same order,

$$\Lambda_G \sim v \sim m_\nu \sim m_{\eta_\nu}, \tag{3.3.4}$$

where the largest neutrino mass can only be larger than the neutrino condensate by a maximal factor of $g_v = 4\pi$.

It is interesting to notice that our neutrino vacuum condensate provides a vacuum energy at the scale of DE. The numerical coincidence of the neutrino mass scale and the DE scale $\Lambda_{\mathrm{DE}} \sim$ meV has been pointed out before (see, e.g., [223, 224]). Certainly, there is no obvious reason why the neutrino condensate should be more physical than other SM vacuum energy contributions, such as the Higgs condensate. However, our neutrino condensate is the only one triggered by gravitational effects, and it is inherently connected to the new IR gravitational scale Λ_G (see section 5.6 for further discussion).

If we finally include all three neutrino flavors, as already broached in the previous section, one immediate issue that we face in trying to generate all neutrino masses effectively is how to generate the hierarchy of masses. In analogy with QCD, it may be expected that in the limit of zero bare neutrino masses, the condensate should be universal in all the flavors and break $U(3)_L \times U(3)_R$ chiral symmetry to a diagonal $U(3)_V$ subgroup. That means the condensate would be a unit matrix in flavor space, $\langle \bar{\nu}_L \nu_R \rangle = \Lambda_G^3 \,\mathrm{diag}(1,1,1)$.

We would like to stress that this is a detailed dynamical question, and a different pattern is equally possible. We can parameterize the patterns of symmetry breaking in very general terms by denoting the neutrino condensate order parameter as a matrix in flavor space, $\langle \bar{\nu}_{\alpha_L} \nu_{\alpha_R} \rangle \equiv \hat{X}^{\alpha_R}_{\alpha_L}$, where $\alpha_L = 1, 2, 3$ and $\alpha_R = 1, 2, 3$ stand for the LH and RH flavor index, respectively. The effective potential for $\hat{X}$ is then some generic function of all possible invariants, for example,

$$V(\hat{X}) = \sum_n \frac{1}{n} c_{2n} \,\mathrm{Tr}[(\hat{X}^+ \hat{X})^n], \tag{3.3.5}$$

where c_{2n} are some coefficients. For simplicity we have excluded other invariants, which are treated in appendix A. The matrix $\hat{X}$ can always be brought to a diagonal form by a $U(3)_L \times U(3)_R$ rotation, $\hat{X} = \mathrm{diag}(x_1, x_2, x_3)$. The extremum values are then determined by the following set of equations:

$$\frac{\partial V}{\partial x_j} = x_j^* \left(\sum_n c_{2n} |x|_j^{2(n-1)} \right) = 0. \tag{3.3.6}$$

It is clear that the VEVs are determined as the roots of the polynomial in brackets and can be different and even hierarchical depending on the parameters c_{2n}. The symmetry breaking $U(3)_L \times U(3)_R \to U(3)_V$ corresponds to a particular choice of a single root, $x_1 = x_2 = x_3$. While this choice is conventionally assumed to be realized in QCD, there is no *a priori* reason to expect the same in other cases. Consequently, in our case of generating neutrino masses through the low-energy condensate, hierarchical neutrino masses can emerge. Notice that these masses do not imply flavor-dependent gravitational couplings but are determined by the effective potential. Moreover, notice that the different mass origins of neutrinos and charged leptons can account for the observed large neutrino mixing angles [97] due to the mismatch of the lepton mass bases.

3.4 Summary and Discussion

In this chapter, we presented how neutrino condensation and small neutrino masses directly emerge from a topological formulation of the chiral gravitational anomaly. In order to clarify our argument, we first outlined the analogy to well-known QCD effects. Based on [47–49], we recapitulated that gravity and QCD have a very similar topological and anomaly structure. This similarity relies on only one assumption: that the gravitational θ-term is physical in the absence of massless fermions. In the presence of chiral fermions, the elimination of the vacuum θ-angle through the chiral gravitational anomaly then amounts to the generation of a mass gap. Consequently, there exists a bound neutrino state η_ν triggered by the chiral gravitational anomaly, analogous to the η' triggered by the ABJ anomaly of QCD.

As we showed, this predicted new bound neutrino state implies important consequences for the neutrino sector: a neutrino vacuum condensate has to emerge for consistency reasons. While numerous theoretical motivations to consider neutrino condensation have been previously discussed in the literature, these were mainly based on cosmological and astrophysical backgrounds. Already in 1967 [225], the possibility of a neutrino superfluid in the Universe was investigated, but the particle physics origin of the required neutrino-neutrino interaction was admitted to be unknown. Other works [226, 227] proposed that a strongly coupled RH neutrino condensate may be the scalar inflaton field, which drives cosmic inflation and gives a large Majorana mass to the RH neutrino. Moreover, a possible link between a neutrino condensate and DE has been suggested (see, e.g., [223, 224]). While most of these previous models assumed BSM neutrino physics in order to solve open cosmological problems, our proposed neutrino condensation intrinsically emerges from a topological formulation of the chiral gravitational anomaly when coupling the SM to gravity.

Without making any additional assumptions, we pointed out that the neutrino vacuum condensate can generate effective small neutrino masses, through interactions mediated by the same nonperturbative gravitational effects that are responsible for the chiral gravitational anomaly. We emphasized that our mass generation mechanism allows for the observed neutrino mass hierarchy and is independent of the Majorana or Dirac nature of the neutrinos. Furthermore, we explained that in the case where all neutrino masses are exclusively generated by our effective mechanism, not only does one new degree of freedom η_ν emerge, but 14 massless Goldstone bosons ϕ_k emerge as well, analogous to the pseudoscalar mesons in QCD.

We reminded the reader that the connection between the topological vacuum susceptibility and the quark condensate in QCD was proven to be unrelated to confinement. Furthermore, we found that the QCD computations of the topological vacuum susceptibility are in full accordance with our derivation in the gravity sector. We also commented on the results of [218] that the gravitational anomaly matching condition does not require low-energy neutrinos to form bound states, but requires all neutrinos to have nonzero masses.

Finally, let us discuss the new scale Λ_G, which we introduced into gravitational physics through the vacuum correlator $\langle R\tilde{R}\rangle$ (3.2.1). Concerning this scale, we observe a crucial difference of the gravitational scenario to the QCD analog: the gravitational scale M_P where gravitational interactions become strong cannot be equal to the scale of symmetry breaking Λ_G, which we identify with the neutrino mass scale (see section 5.1). This seems to be different in QCD, where the symmetry-breaking scale is not far from the scale where perturbative gluon-gluon interactions get strong. In sharp contrast, the coupling of gravitons of wavelength Λ_G^{-1}, given by $\alpha_G \sim \Lambda_G^2/M_P^2$, is minuscule. This naive difference however should not confuse the reader. First, our analogy was based not on perturbative analogies, but rather on striking similarities between the topological and anomaly structures of the two theories. Second, the scale Λ_G has to be understood not as a scale of perturbative strong coupling, but as the scale where collective nonperturbative phenomena become important.

Chapter 4

Gravitational Domestic Axion Model

This chapter is dedicated to the second cornerstone of our novel class of low-energy models: the Domestic Axion (DA) model. In order to grasp our model's new aspects, we will first compare it in section 4.1 with the original PQWW axion scenario. Then we will explain in section 4.2 how the chiral gravitational anomaly can jeopardize common high-energy axion solutions and how the neutrino can eliminate this gravitational threat. Afterwards, we will gradually build up our model in section 4.3 by elucidating its anomalous $U(1)$ symmetries in section 4.3.1, its up-quark mass generation in section 4.3.2, and finally its axion and graviaxion content in section 4.3.3. The subsequent section 4.4 is devoted to checking the consistency of our model with chiral perturbation theory. We will summarize and discuss our findings in section 4.5.

4.1 Comparison with PQWW Axion Model

As reviewed in section 2.2.3, the common way of solving the strong CP problem is to assume a nonzero Yukawa coupling of the up quark to the Higgs doublet and to add the axion as a hypothetical degree of freedom. We shall not take this standard path, but instead consider whether the nonzero up-quark mass indicated by lattice QCD can be provided by a low-energy neutrino condensate in such a way that it could break the chiral PQ symmetry (2.2.14) *spontaneously*. As explained in the previous chapter, the existence of such a neutrino condensate follows from a very general assumption about the topological structure of the vacuum due to the chiral gravitational anomaly.

In order to better capture its novel aspects, it is useful to confront the present scenario with the original PQWW axion case [69, 162, 163], which we introduced in section 2.2.3. For the existence of chiral PQ symmetry, it is a necessary condition that different quarks get their masses from different Higgs doublets. In the original axion scenario, this is accomplished by coupling the up and down quarks to two distinct Higgs doublets H and H',

$$\mathcal{L}_{\text{PQ}} = H'\bar{Q}_L u_R + H\bar{Q}_L d_R + \dots , \qquad (4.1.1)$$

where $Q_L \equiv (u_L, d_L)$ is the doublet of LH quarks. This decoupling of some quarks from a particular Higgs doublet is justified by the chiral PQ symmetry, $H \to e^{i\alpha}H,\ H' \to e^{i\alpha}H',\ (\bar{Q}_L u_R) \to e^{-i\alpha}(\bar{Q}_L u_R),\ (\bar{Q}_L d_R) \to e^{-i\alpha}(\bar{Q}_L d_R)$. In this scenario, the axion comes predominantly from the phase of the neutral Higgs with a smaller VEV, but as the VEV is around the weak scale, such an axion is ruled out experimentally [167, 168].

In our model, it remains true that the different quarks get masses from different Higgs doublets, but the additional doublet is provided by the SM itself: it is a neutrino condensate. The SM fermion composition of the emerging PQWW axion is the reason why we will call this axion a "domestic axion".

The simplest prototype effective Lagrangian describing the DA idea is

$$\mathcal{L}_{\rm DA} = \frac{f}{\Lambda_G^2}(\bar{\nu}_R L)\bar{Q}_L u_R + H\bar{Q}_L d_R + \dots , \tag{4.1.2}$$

where $L \equiv (\nu_L, e_R)$ is the lepton doublet. The IR scale Λ_G and the invariant function f are provided by gravity and will be discussed below.

Thus, the additional doublet H' of the original PQWW model is replaced by an effective doublet composed of the lepton doublet and the RH neutrino, $H' \to (\bar{\nu}_R L)$. In this minimal realization, the PQ symmetry is the chiral symmetry acting both on quarks as well as on neutrinos, $(\bar{\nu}_R L) \to e^{i\alpha}(\bar{\nu}_R L)$, and is spontaneously broken by both condensates. A similar scenario with a purely RH neutrino doublet was investigated in [228, 229], but without specifying the origin of the RH neutrino condensate and with assuming a symmetry breaking scale just as high as in conventional invisible axion models.

The crucial ingredient in our model is the low-energy condensate (3.2.1) of the composite doublet $\langle \bar{L}\nu_R\rangle = \langle \bar{\nu}_L \nu_R\rangle \neq 0$, imposed by the gravitational chiral anomaly. The role of this condensate is to spontaneously generate the mass of the up quark, but the contribution from its phase, i.e., the η_ν boson (3.1.27), to the axion is negligible. Instead, the axion comes almost entirely from the η' meson of QCD, because the breaking of chiral PQ symmetry is predominantly accomplished by the QCD condensate of quarks, which is much larger than the gravitationally induced neutrino condensate.

The roles of the pseudo-Goldstone bosons are split in the following way: the η' meson gets its mass from the QCD anomaly and becomes an axion, whereas the η_ν boson gets its mass from the chiral gravitational anomaly and "sacrifices" itself to protect the shift symmetry of the η' meson against the chiral gravitational anomaly via the mechanism of [48, 49]. The crucial point that makes our neutrino-composite doublet compatible with experimental bounds is that it is very "fat": its extremely low compositeness scale makes it contribute only to very soft processes and decouple efficiently from hard high-energy processes.

Such a DA scenario has the following obvious advantages:

(1) It provides the axion without any need of postulating the existence of hypothetical particle species.

(2) The axion is automatically immune to the chiral gravitational anomaly and its shift symmetry is broken exclusively by QCD effects [48, 49].

(3) The neutrino condensate that breaks PQ symmetry is also the source of neutrino masses, via the mechanism presented in section 3.3.

Thus, the present scenario connects the solution of the strong CP problem to the origin of neutrino masses, without the need for new species, and simultaneously protects the axion solution against gravity. Note here that the gravitational protection mechanism in the presence of a massless fermion is applicable to any axion model [49], even beyond our proposed DA scenario.

Before presenting our complete model in section 4.3, in the following section we will briefly elaborate on each of the above three topics and review the previous results that we shall use. The main focus will be on the second topic of the axion protection against the chiral gravitational anomaly.

4.2 Elimination of Gravitational Threat

Let us first review how the chiral gravitational anomaly can threat standard invisible axion models and thereafter explain how the axion can be protected by the neutrino. For a more detailed treatment of some of the following aspects, we refer the reader to sections 2.2 and 3.1.2, as well as to [48, 49].

In order for the axion to relax the θ-term to zero and solve the strong CP problem, the axion shift symmetry (2.2.12) must be explicitly broken exclusively by QCD effects via the ABJ anomaly. However, there is an old belief that quantum gravity effects can generate an additional breaking of the axion shift symmetry and therefore ruin the axion solution of the strong CP problem (see, e.g., [188, 210]). The necessary and sufficient conditions for the possibility of such an explicit breaking were identified in [48], where – by reformulating the axion solution in the language of a three-form Higgs effect – the breaking of the axion shift symmetry by gravity was linked to the chiral gravitational anomaly and to the gravitational topological susceptibility of the vacuum. Namely, the condition is that the gravitational topological vacuum susceptibility is nonzero in the absence of massless fermions or axions, e.g., in a theory with pure gravity,

$$\langle R\tilde{R}, R\tilde{R}\rangle_{q\to 0} \equiv \lim_{q\to 0} \int d^4x\, e^{iqx} \langle T[R\tilde{R}(x) R\tilde{R}(0)]\rangle = \text{const} \neq 0\,, \tag{4.2.1}$$

where R is the Riemann tensor and $\tilde{R}$ is its dual. Note that this condition is equivalent to the statement that the gravitational analog of the θ-term [65],

$$\mathcal{L}_G \supset \theta_G R\tilde{R}\,, \tag{4.2.2}$$

is physical. As shown in [47] and explained in section 3.1, the direct connection between the topological susceptibility and the generation of the mass gap in the anomalous current is a very general phenomenon and goes well beyond gravity.

The existence of a nonvanishing topological vacuum susceptibility in pure gravity is currently an open question. If it is zero, then the chiral gravitational anomaly poses no danger to the axionic shift symmetry [48]. However, if it is nonzero, one has to face the consequences. What we want to show is that in such a case the gravitational danger comes with a built-in protection mechanism, which does not only eliminate itself, but as a bonus identifies the viable axion candidate within the SM in form of the η' meson.

Thus, we shall assume that the above condition, i.e., gravity gives rise to (4.2.1) in the absence of an anomalous current, is fulfilled and consequently the threat to the axion solution of the strong CP problem from gravity is real. As pointed out in section 3.1.2, this introduces a new gravitational scale in the problem, Λ_G, which sets the scale of the correlator (4.2.1). At the level of our discussion, Λ_G is a free parameter, solely constrained by phenomenological requirements (see section 5.1). One thing that we can expect about this scale is that it must be strongly suppressed with respect to the Planck scale. This is normal for the IR scales generated by nonperturbative effects, such as instantons or virtual black holes. However, in this thesis we shall not commit to any particular microscopic origin of the correlator (4.2.1), which can easily be *intrinsically quantum gravitational* rather than semi-classical. Notice that the effective low-energy interactions generated by this IR physics must be assumed to become irrelevant in short-distance processes at energies $E \gg \Lambda_G$, i.e., their contribution must sharply diminish for $\Lambda_G/E \ll 1$. Later, for the phenomenological estimates (see chapter 5) we shall parameterize the high-energy behavior of the effective interactions generated by nonperturbative gravitational physics by a power-law dependence on Λ_G/E.

In case of a physical gravitational θ-term in the absence of axions or massless fermions, there exist two physically observable theta-parameters: one from QCD (2.2.1) and one from gravity (4.2.2) [48]. Consequently, after the axion is introduced, it can only cancel a single combination of the two θ-terms, whereas the other combination remains physically observable. Hence, the strong CP problem is not solved. In this situation, as a possible protection mechanism it was suggested in [48] to take into account some fermions, e.g., neutrinos, with zero bare mass. In such a case, there always exists a chiral symmetry that is anomalous with respect to gravity. For example, for a single Dirac neutrino flavor, we have an axial $U(1)_{A\nu}$ symmetry

$$\nu \to e^{i\alpha\gamma_5}\nu \tag{4.2.3}$$

with the corresponding axial current

$$j_\mu^{(\nu)} = \bar{\nu}\gamma_\mu\gamma_5\nu\,. \tag{4.2.4}$$

Due to the chiral gravitational anomaly [65–68], the current has an anomalous divergence,

$$\partial^\mu j_\mu^{(\nu)} \;=\; R\tilde{R}, \tag{4.2.5}$$

and – just like in QCD with a massless quark – the gravitational θ-term (4.2.2) can be eliminated by an axial transformation of the neutrino (4.2.3). As a result, the gravitational topological susceptibility (4.2.1) vanishes and gravity generates a mass gap in the neutrino sector, so that the axion potential is not affected. This mechanism was implemented in detail as the axion protection mechanism against gravity in [49]. One of the predictions of this scenario is the existence of a pseudo-Goldstone boson, η_ν, which corresponds to the neutrino axial current (4.2.4). The η_ν boson represents a collective excitation of the neutrino condensate phase and plays a role closely analogous to the η' meson of QCD, which gets its mass from the QCD anomaly (2.2.16).

The next step was undertaken in our gravitational neutrino mass model presented in chapter 3, where we suggested to identify the neutrino condensate triggered by the chiral gravitational anomaly as the unique source of all the neutrino masses. This model, as well as several model-independent constraints (see section 5.1), fixes the scale of the neutrino condensate in the ~ 0.1 eV range.

In all the above studies, it was assumed that the axion that solves the strong CP problem comes from some unspecified BSM sector. With the DA model presented in this chapter, we would like to suggest a much more economical possibility: we would like to propose a scenario in which the neutrino condensate also generates the mass of the up quark spontaneously.

The attractive feature of such a scenario is that the role of the PQ symmetry is played by a combination of the axial symmetries (2.2.14) and (4.2.3) acting on the up quark and on the neutrinos, respectively. This symmetry is free of the chiral gravitational anomaly and is anomalous solely with respect to QCD. It is spontaneously broken by the QCD up-quark condensate as well as by the neutrino condensate. Since the quark condensate dominates, the corresponding axion mostly consists of the QCD η' meson with a small admixture of η_ν. The orthogonal combination, which consist mostly of η_ν with a small admixture of η', gets its mass from the chiral gravitational anomaly.

4.3 Composition of Domestic Axion Model

4.3.1 Anomalous $U(1)_G$ and $U(1)_{\text{PQ}}$ Symmetries

Let us now describe our model in more detail. The key postulate is that the masses of some quarks are generated by their couplings to the neutrino condensate as opposed to the Yukawa couplings to the SM Higgs. The neutrino condensate acts as an additional composite Higgs doublet, and this allows the Lagrangian to be invariant under a chiral PQ symmetry that is anomalous with respect to QCD. For solving the strong CP problem in this way, it is unimportant which quarks get their masses from the neutrino condensate, but it would be natural to employ the light quarks.

We shall start with a minimal scheme in which only the up quark and a single neutrino flavor are involved. We thus set to zero the Yukawa coupling constants of the Higgs doublet to the up quark and to one of the three neutrino flavors. We shall assume that the masses of all the other fermions are generated in a standard way through their Yukawa couplings to the Higgs VEV and we shall exclude them from our considerations. These additional fermions can be easily integrated back in without affecting the essence of the DA scenario, and we will discuss this possibility later. Note that in case the down-quark Yukawa coupling is set to zero as well, the resulting flavor symmetry would be exact on both the perturbative and the nonperturbative level. Thus, such a scenario would evade the naturalness debate mentioned in section 2.2.2.

Let us now discuss the new global symmetries in the minimal scenario of decoupling only the up quark and neutrino from the Higgs doublet. At the perturbative level, gravity treats all fermion species democratically; thus, it effectively sees the three colors of the LH and RH up-quark pairs and one LH and RH neutrino pair, forming a representation of the $U(4)_L \times U(4)_R$ flavor symmetry group. Since all the fermions can be written in the LH basis with RH fermions ψ_R replaced by LH anti-fermions ψ_{cL}, they can be viewed as a fundamental representation of the $U(8)$ flavor group. However, note that the Lorentz and gauge invariant bilinear order parameters of the type $\bar{\psi}_L \psi_R$ form the bifundamental representations of the $U(4)_L \times U(4)_R$ group.

Before taking into account the quantum anomalies, the QCD and electromagnetic gauge interactions break this symmetry explicitly down to the following subgroup:

$$\mathcal{G} \equiv \; SU(3)_{\text{color}} \times U(1)_{\text{EM}} \times U(1)_{Vu} \times U(1)_{V\nu} \times U(1)_{Au} \times U(1)_{A\nu} \,, \tag{4.3.1}$$

where $SU(3)_{\text{color}}$ is a color group and $U(1)_{Vu}$ and $U(1)_{V\nu}$ are the vector-like quark (baryon) and neutrino (lepton) number symmetries, respectively. Since we have ignored other fermion species, the electromagnetic symmetry $U(1)_{\text{EM}}$ acts essentially as the gauged version of the up-quark number symmetry $U(1)_{Vu}$.

The asymmetry between the LH and RH fermion species in the SM is only created by the weak gauge interaction. Since we are interested in low-energy phenomena, we shall ignore the effects that break the left-right symmetry.

Finally, $U(1)_{Au}$ and $U(1)_{A\nu}$ are the quark and neutrino axial symmetries given by (2.2.14) and (4.2.3), respectively. The following combination of these symmetries,

$$u \to e^{i\alpha\gamma_5} u \,, \quad \nu \to e^{i\alpha\gamma_5} \nu \,, \tag{4.3.2}$$

is anomalous with respect to gravity, and we shall denote it by $U(1)_G$. The corresponding current

$$j_\mu^{(G)} \; = \; \sum_a \bar{u}^a \gamma_\mu \gamma_5 u_a \, + \, \bar{\nu} \gamma_\mu \gamma_5 \nu \tag{4.3.3}$$

exhibits the anomalous divergence (4.2.5)

$$\partial^\mu j_\mu^{(G)} \; = \; R\tilde{R} \,. \tag{4.3.4}$$

Note that the anomalous $U(1)_G$ symmetry contains an anomaly-free Z_8 subgroup corresponding to the discrete values of the phase parameter $\alpha = \frac{\pi}{4}n$ with n being an arbitrary integer.

Another important symmetry is the orthogonal combination of $U(1)_{Au}$ and $U(1)_{A\nu}$,

$$u_a \to e^{i\alpha\gamma_5} u_a\,, \quad \nu \to e^{-i3\alpha\gamma_5}\nu\,, \tag{4.3.5}$$

which we shall denote by $U(1)_{\mathrm{PQ}}$. This symmetry is free of the chiral gravitational anomaly, but it is anomalous with respect to QCD, i.e., the corresponding current

$$j_\mu^{(\mathrm{PQ})} \;=\; \sum_a \bar{u}^a\gamma_\mu\gamma_5 u_a \;-\; 3\,\bar{\nu}\gamma_\mu\gamma_5\nu \tag{4.3.6}$$

exhibits the anomalous divergence (2.2.16)

$$\partial^\mu j_\mu^{(\mathrm{PQ})} \;=\; G\tilde{G}\,. \tag{4.3.7}$$

Thus, this symmetry is the right candidate for the PQ symmetry. Notice that, although both symmetries (4.3.2) and (4.3.5) include a $U(1)_{Au}$ component and therefore are anomalous with respect to QCD, we identify (4.3.5) as the PQ symmetry because it is the one that is anomaly-free with respect to gravity.

4.3.2 Generation of Up-Quark Mass

Let us now discuss the effective interaction that is induced by the chiral gravitational anomaly and is responsible for generating the mass gaps for the Goldstone bosons as well as for the fermions. Since we use the anomaly-free symmetries as a guideline, we shall consider interactions that are invariant with respect to (4.3.1).

The pattern of chiral symmetry breaking is determined by minimization of an effective potential for the quark and the neutrino order parameters, $X_u \equiv (\bar{u}_L u_R)$ and $X_\nu \equiv (\bar{\nu_L}\nu_R)$. This effective potential consists of the ordinary QCD part and the part generated by gravity. The QCD part consists of the effective potential that induces the quark condensate and breaks the axial $U(1)_{Au}$ symmetry (2.2.14) spontaneously, as well as the 't Hooft-type interactions that break this symmetry explicitly and contribute to the mass of the η' meson. Likewise, the effective potential induced by gravity can be split into the part that breaks $U(1)_G$ symmetry spontaneously and the one that breaks it explicitly.

The parts that are responsible for spontaneous breaking are given by some unknown polynomial consisting of a generally infinite series of phase-independent invariants, such as $X_u^+ X_u$ and $X_\nu^+ X_\nu$. Its explicit form is unimportant for our purposes. It suffices to know that the minimum of this effective potential is achieved for a nonvanishing VEV of the neutrino condensate, $\langle X_\nu\rangle = v^3 e^{i\langle\eta_\nu\rangle/v}$, where v is the characteristic scale of the condensate and $\langle\eta_\nu\rangle$ is the VEV of its phase. The scale v is *a priori* unknown, and we must treat it as a free parameter. We do not expect it to be very far from the scale Λ_G that sets the scale of the correlator (4.2.1), although it can be parametrically different. Thus, we assume $v \sim \Lambda_G$. In addition, gravity triggers a condensate for X_u of similar order of magnitude, but this is just a tiny correction to the condensate of X_u triggered by QCD, $\langle X_u \rangle = V^3 e^{i\langle \eta' \rangle / V}$, where V is of the order the QCD scale.

If the considered effective potential consisted solely of the spontaneous-breaking part, the phase degrees of freedom η_ν and η' would be *exactly massless* Goldstone bosons. However, from the chiral anomaly and topology we know that mass gaps in both of these Goldstones must be generated. In particular, QCD generates a mass gap in η'. At the level of the effective potential, this can be modeled by a 't Hooft-type vertex, which for a single quark case is just a linear term in X_u multiplied by an arbitrary function of the phase-independent invariant $X_u^+ X_u$. In this context, we have to emphasize that we do not commit to the assumption that the main source of the η' mass in QCD are instantons. As it is well known (see section 2.2.1), the Witten-Veneziano mechanism [59,60] is expected to give the dominant contribution for a large number of colors. For us, the 't Hooft like structure – regardless of its underlying origin – is a useful parameterization of the symmetry properties of the effective vertex that explicitly breaks the anomalous $U(1)_{Au}$ chiral symmetry to an anomaly-free discrete subgroup and generates the pseudo-Goldstone mass. In case of a single quark flavor, the anomaly-free symmetry is Z_2, which uniquely fixes the structure of the vertex in form of a linear term in X_u times an arbitrary function of phase-independent invariants.

Similar to the QCD case, gravity generates a mass gap for a particular superposition of Goldstones corresponding to the $U(1)_G$ symmetry. Correspondingly, the effective Lagrangian generated by gravity on top of the standard QCD effects must contain additional interaction terms among X_u and X_ν, which break the anomalous $U(1)_G$ symmetry explicitly and generate the pseudo-Goldstone masses. The same interaction contributes into the spontaneous generation of the masses of the up quark and the neutrino. Let us show how this happens.

We can model the gravity-induced interaction by the following vertex:

$$\mathcal{L}_G \,=\, \frac{1}{\Lambda_G^2}(X_u X_\nu)\, f(X_u^+ X_u, X_\nu^+ X_\nu, ...) \,+\, \text{h.c.}, \tag{4.3.8}$$

where Λ_G is the IR scale of gravity. For example, this interaction can be thought of as being generated from a $SU(8)$-invariant 't Hooft-type gravitational vertex,

$$\frac{1}{\Lambda_G^8}(\bar{u}_L^1 u_{1R})(\bar{u}_L^2 u_{2R})(\bar{u}_L^3 u_{3R})(\bar{\nu}_L \nu_R) \,+\, \text{h.c.}, \tag{4.3.9}$$

which explicitly breaks (4.3.2) but respects (4.3.5) after dressing it by QCD effects, which alone would break $U(1)_{Au}$ but not $U(1)_{A\nu}$. The combined effects of the generated terms do not leave any unbroken continuous chiral symmetry. Here, we need to stress again that we invoke the analogy with the 't Hooft vertex exclusively because of the Z_8-symmetry structure of the vertex. We do not assume that the gravitational vertex is necessarily generated from semi-classical physics, such as gravitational instantons, but rather can be of deeply quantum gravitational origin.

The function $f(X_u^+ X_u, X_\nu^+ X_\nu, ...)$ in (4.3.8) is some unknown dimensionless function of the phase-independent quark and neutrino invariants, $X_u^+ X_u$ and $X_\nu^+ X_\nu$, scaled by the parameter Λ_G (see appendix A for more details about f). For the phenomenological consistency of our scenario, we need to impose the following constraint on this function:

$$\langle X_u \rangle \left\langle \frac{\partial^2 \mathcal{L}_G}{\partial \bar{u}_L \partial u_R} \right\rangle = \xi \, \langle \mathcal{L}_G \rangle \sim \xi \langle X_\nu \rangle \left\langle \frac{\partial^2 \mathcal{L}_G}{\partial \bar{\nu}_L \partial \nu_R} \right\rangle , \tag{4.3.10}$$

with $\xi \sim 10^7$. This means that the VEV of the derivatives of the function f with respect to X_u must be much larger than the other expectation values, i.e., the function f must be steep in the X_u direction. As we shall see in a moment, this condition replaces the fine tuning of the up-quark Yukawa coupling constant in the standard scenario. In the standard case, the tuning of the Yukawa coupling constant sets the hierarchy between the up-quark mass and the Higgs VEV, whereas in our case, ξ sets the hierarchy between the up-quark and the neutrino masses. As it will become clear later, the same condition also guarantees that the Goldstone bosons do not enter the strong coupling regime.

Since the neutrino condensate contributes into the spontaneous breaking of the $U(1)_{\mathrm{PQ}}$ symmetry, it provides an additional non-QCD contribution to the up-quark mass through the vertex (4.3.8). An effective up-quark mass term is obtained by replacing all fermion bilinears by their VEVs while keeping the two quark legs free. Consequently, we get

$$\left\langle \frac{\partial^2 \mathcal{L}_G}{\partial \bar{u}_L \partial u_R} \right\rangle \bar{u}_L u_R \simeq \xi \, \frac{v^3}{\Lambda_G^2} \, \langle f \rangle \, \bar{u}_L u_R + \, ... \, , \tag{4.3.11}$$

where we took into account the condition (4.3.10). The resulting up-quark mass thus reads

$$m_u \simeq \xi \, v \left(\frac{v}{\Lambda_G} \right)^2 \langle f \rangle. \tag{4.3.12}$$

By the same estimate, the contribution to the neutrino masses from the above vertex is

$$m_\nu \simeq v \left(\frac{v}{\Lambda_G} \right)^2 \langle f \rangle. \tag{4.3.13}$$

Notice that, even though the QCD-induced up-quark condensate is large, it must be effectively cut-off around the scale v when inserted into the gravitational vertex. This is because v is the softness scale of the effective vertex (4.3.8), implying that we have to effectively represent all the condensates by the scale $v \sim \Lambda_G$. Then, the hierarchy between the neutrino and the up-quark masses is controlled by the parameter ξ. For a phenomenologically acceptable value of the up-quark mass, we need to choose $\xi \sim 10^7$ for an up-quark mass of $m_u \sim$ MeV and neutrino masses of, e.g., $m_\nu \sim 0.1$ eV.

Although the above choice of the parameter ξ may seem somewhat unnatural, it is much milder than the fine tuning of parameters required for achieving a more modest goal in the original KSVZ and DFSZ axion models (see section 2.2.3). In these models, first, one needs to fine-tune the Yukawa coupling constant of the up quark to the value $\sim 10^{-5}$. Second, given the phenomenological lower bound on the PQ scale $\sim 10^9$ GeV [183], one has to fine-tune the mass-square term of the Higgs boson relative to the PQ scale by a factor of $\sim 10^{-14}$. Apart from this, the minimal KSVZ and DFSZ scenarios neither address the protection of the axion solution against gravity nor the origin of the neutrino masses. In this light and given the goals we aim to achieve, the choice of a relatively large ξ may not be such a big price to pay after all. Also when comparing to the fine tuning of the up-quark Yukawa constant in common invisible axion scenarios, it is important to stress that ξ is a coefficient of a very high dimensional operator and its tuning amounts to much milder tuning when translated in terms of mass scales, because of high-power sensitivity.

4.3.3 Emergence of Domestic Axion and Graviaxion

The vertex (4.3.8) in combination with the standard QCD contribution explicitly breaks all continuous chiral symmetries. Correspondingly, both would-be Goldstone bosons η' and η_ν become massive pseudo-Goldstones. In order to evaluate their masses, we shall replace the absolute values of the fermion bilinears by their VEVs and express their phases though the corresponding pseudo-Goldstone modes. We thus write $\langle \bar{u}_L u_R \rangle = V^3 e^{i\eta'/V}$ and $\langle \bar{\nu}_L \nu_R \rangle = v^3 e^{i\eta_\nu/v}$, where V and v are the scales of the two condensates introduced above.

As already mentioned, the neutrino condensate forms due to nonperturbative gravitational effects, whereas the up-quark condensate is dominantly triggered by QCD effects with a negligible gravitational contribution. Thus, the scale V is given by the QCD scale, $V = \Lambda_{\text{QCD}}$. Nevertheless, when inserted into the vertex (4.3.8), we have to set the VEV of the absolute value also for X_u to be of the order of v^3. The reason is the same UV softness of the vertex (4.3.8) as explained earlier. When we terminate the external legs of the vertex into the VEVs, we should keep in mind that the contribution freezes out above a certain critical value of the VEV. This value corresponds to the momentum above which the vertex (4.3.8) is resolved and "melts". We assumed this scale to be given by $v \sim \Lambda_G$. Hence, when we plug the quark condensate into the vertex, we must effectively replace it with $\langle \bar{u}_L u_R \rangle = v^3 e^{i\eta'/V}$. Notice that the decay constant of the η' is still given by V, because this is just an information about the canonical normalization of the pseudo-Goldstone mode.

Inserting the above expressions for the fermion bilinears into (4.3.8) yields

$$\mathcal{L}_G = \frac{v^6}{\Lambda_G^2}\langle f\rangle \cos\left(\frac{\eta'}{V} + \frac{\eta_\nu}{v}\right). \tag{4.3.14}$$

When expanding the cosine, we obtain an effective mass term

$$\mathcal{L}_{\text{mass}} = -\frac{1}{2} m_G^2\, a_G^2 \tag{4.3.15}$$

for one combination of the Goldstone modes,

$$a_G \equiv \frac{\eta_\nu + \eta'\epsilon}{\sqrt{1+\epsilon^2}}\,, \tag{4.3.16}$$

with the mass

$$m_G^2 = \frac{v^4}{\Lambda_G^2}\,(1+\varepsilon^2)\langle f\rangle \simeq v^2 \left(\frac{v}{\Lambda_G}\right)^2 \langle f\rangle\,. \tag{4.3.17}$$

Here, we have taken into account the smallness of the parameter $\varepsilon \equiv v/V$. Since the function f only depends on real invariants, it does not break any of the $U(1)$ symmetries and contributes to the Goldstone potential only in form of an overall factor.

From (4.3.15) it is clear that the mode a_G is the pseudo-Goldstone boson that gets its mass from the chiral gravitational anomaly and screens the gravitational θ-term. It consists mostly of the neutrino-composite pseudoscalar η_ν with a small ($\sim \epsilon$) admixture from the η' meson of QCD. In the absence of the QCD anomaly, the above mode would be a true mass eigenstate, while the orthogonal combination, $(\eta' - \eta_\nu\epsilon)/\sqrt{1+\epsilon^2}$, which is a Goldstone boson of the spontaneously broken $U(1)_{\text{PQ}}$ symmetry, would remain exactly massless.

However, this is not the case, since the $U(1)_{\text{PQ}}$ symmetry is anomalous with respect to QCD. Thus, the Goldstone bosons also get a mass from this chiral anomaly. Note that it is only the η' component that couples to QCD and contributes into the QCD anomaly. This component gets a mass through the QCD mechanism and solves the strong CP problem. Since a_{PQ} is mostly η', its mass generation can be described in terms of the Witten-Veneziano mechanism [59, 60] (see section 2.2.1). The only difference is that a_{PQ} screens the θ-term entirely and solves the strong CP problem, since in our case the up-quark mass is generated spontaneously.

As a result, the mass matrix takes the form

$$\mathcal{L}_{\text{mass}} = -\frac{1}{2} m_{\eta'}^2 \eta'^2 - \frac{1}{2} m_G^2 (\eta_\nu + \epsilon\eta')^2\,, \tag{4.3.18}$$

which can also be derived using the three-form formalism (see appendix B). As we can see, the mixing angle between η' and η_ν is minuscule ($\sim \epsilon\, m_G^2/m_{\eta'}^2 \sim \epsilon^3$).

Correspondingly, up to the mixing of order ϵ^3, they are the true mass eigenstates,

$$a_{\mathrm{PQ}} = \eta' + \mathcal{O}(\epsilon^3)\eta_\nu\,, \quad a_G = \eta_\nu + \mathcal{O}(\epsilon^3)\eta'\,, \tag{4.3.19}$$

with the masses equal to $m_{\eta'}$ and m_G, respectively.

The boson a_{PQ} represents a perfect domestic PQ axion, but with one advantage: unlike ordinary PQ symmetries, the $U(1)_{\mathrm{PQ}}$ symmetry shifting the domestic axion (2.2.12) is free of the chiral gravitational anomaly and hence is protected against gravitational destabilization [48,49]. This is also demonstrated in appendix B in the language of a Chern-Simons gauge three-form [48].

The a_G boson, which is mostly composed of η_ν and gets its mass from the chiral gravitational anomaly, will be referred to hereafter as a *graviaxion*. For $\langle f\rangle \sim 1$, the mass (4.3.17) of the graviaxion is of the same order as the neutrino mass and is given by the scale Λ_G. Since the function f is independent of phases, it only contributes to the Goldstone masses as an overall factor. Therefore, in order to create a hierarchy between the neutrino and the up-quark masses without simultaneously pushing m_G above the scale v, we need to take a large ξ while keeping the VEV of f to be of order one.

4.4 Consistency with Chiral Perturbation Theory

In this section, we will examine whether our model is consistent with chiral perturbation theory (see [166] for a review). As we will explain in the following, low-scale elementary axion models are ruled out by chiral perturbation theory, because they predict too small a pion mass contribution coming from the up-quark mass. However, our *composite* axion model promises to cure this problem due to the off-shellness of the experimentally observed pions.

Concerning the pions' off-shellenss, notice that the most precise tests of the up-quark contribution to the neutral pion mass are S-wave $\pi\pi$ scattering lengths, where both the experimental and the theoretical accuracy are currently at the few percent level [166]. The predictions by chiral perturbation theory [230] have been confirmed by a series of low-energy precision experiments, e.g., of the decays $K^\pm \to \pi^0\pi^0 e^\pm\nu$ [231], $K^\pm \to \pi^\pm\pi^0\pi^0$ [232], and pionic atoms [233]. All neutral pions involved in these processes are off-shell by at least eight orders of magnitude more than the scale v of our neutrino condensate.

Therefore, the up-quark mass contribution to the pion mass in our DA model can be phenomenologically viable, as we will argue in the following. First, we will remind the reader of the problem that arises in the case of a low-scale *elementary* axion. Afterwards, we will promote this low-scale axion into a composite object to show that, in this case, the highly off-shell pions must be assumed to obtain the correct mass contribution.

Let us consider a toy model of a low-scale elementary axion a. To illustrate the different pion mass contributions, we now take into account the down quark with a hard mass m_d. The up-quark mass $m_u = gf_a$ is assumed to be generated by a second elementary Higgs doublet with a very small VEV f_a and a Yukawa coupling g. The toy model Lagrangian reads

$$\mathcal{L}_{\text{toy}} = \left(g f_a \, e^{ia/f_a}\right) \bar{u}u + m_d \, \bar{d}d. \tag{4.4.1}$$

The spontaneous breaking of the $U(2)_L \times U(2)_R$ flavor symmetry of the up and down quarks gives rise to an η' meson plus three pions, π^0, π^+, and π^-. The neutral pion mixes with both the η' and the axion a, which we explicitly show by writing the neutral arguments of the two quark condensates as

$$\arg(\langle \bar{u}u \rangle) = \frac{\eta'}{f_{\eta'}} + \frac{\pi^0}{f_{\pi^0}} \tag{4.4.2}$$

and

$$\arg(\langle \bar{d}d \rangle) = \frac{\eta'}{f_{\eta'}} - \frac{\pi^0}{f_{\pi^0}}, \tag{4.4.3}$$

where $f_{\eta'}$ and f_{π^0} are the decay constants of η' and π^0, respectively. Then the mass terms for the three neutral pseudo-Goldstone bosons read

$$\mathcal{L}_{\text{toy}} = m_u V^3 \left(\frac{a}{f_a} + \frac{\eta'}{f_{\eta'}} + \frac{\pi^0}{f_{\pi^0}}\right)^2 + m_d V^3 \left(\frac{\eta'}{f_{\eta'}} - \frac{\pi^0}{f_{\pi^0}}\right)^2 + V^4 \left(\frac{\eta'}{f_{\eta'}}\right)^2, \tag{4.4.4}$$

where V is the scale of the quark condensates, $|\langle \bar{u}u \rangle| = |\langle \bar{d}d \rangle| = V^3$. The first two terms are the up- and down-quark mass contributions coming from (4.4.1). The last term is generated from the QCD 't Hooft determinant, which we put approximately equal to V^4.

Let us now specify the elementary axion scale f_a and consider the limit of $f_a \to 0$ and $g \to 0$, while keeping g/f_a fixed. In this limit of a very low axion decay constant, we see from (4.4.4) that the up-quark mass only contributes into the axion mass,

$$m_a^2 = \frac{m_u V^3}{f_a^2} = \frac{g V^3}{f_a}, \tag{4.4.5}$$

but not into the η' and π^0 masses. Thus, the neutral pion gets its only mass contribution from the down-quark mass, which stands in conflict with the Gell-Mann-Oakes-Renner relation [234],

$$m_{\pi^0}^2 = \frac{(m_d + m_u) V^3}{f_{\pi^0}^2}. \tag{4.4.6}$$

One may argue against our DA scenario that this conclusion could also hold true if the axion is elementary and not composite, which is currently under discussion [235]. However, the low-scale composite "axion" η_ν in our model must be assumed to dissolve for off-shell energies larger than v (see section 5.5.2 for related discussions). Consequently, a pion that is off-shell by more than the scale v can be expected to receive the full mass contribution from both the up- and the down-quark masses, in accordance with (4.4.6). This argument renders the resulting value of the neutral pion mass in our DA model most likely to be phenomenologically viable.

4.5 Summary and Discussion

In the Introduction, we already emphasized the standard lore that new physical effects can hide and decouple if the energy scale of their origin is very high. In particular, the usual invisible axion is decoupled because of an extremely high scale of PQ symmetry breaking. Apart from the new naturalness problem in form of the hierarchy between the PQ and EW scales, this leaves us with the questions why a whole new high-energy sector should be designed in order to nullify one particular parameter of the SM?

In this chapter, we have proposed an alternative hiding place for axion physics within the SM in form of a deep-infrared scale, without the need of postulating any new particle species. This IR scale is related to the neutrino masses. Our axion consists of the η' meson with a minuscule admixture of the neutrino composite η_ν. The latter is a pseudo-Goldstone of the neutrino condensate triggered by nonperturbative gravity and gets its mass from the chiral gravitational anomaly.

The neutrino condensate does several jobs. On the one hand, it generates the mass for the up quark *spontaneously*. This is the key that in our scenario allows the η' meson to act as an axion and cancel the θ-term. On the other hand, the Goldstone boson η_ν originating from the neutrino condensate protects the shift symmetry of the η'-axion from being broken by the chiral gravitational anomaly [49]. At the same time, the neutrino condensate is a natural source for generating the neutrino masses via the scenario proposed in chapter 3.

However, the latter possibility is not tied to the DA model presented here, for which it is enough that only a single neutrino flavor gets its mass from the chiral gravitational anomaly, whereas the other flavor masses can be generated in conventional ways. In such a case, the field content of the model is reduced to the scenario investigated in [49], in which the bare mass of a single neutrino is set to zero. In this situation, the DA model would be fully realized, but the possibility of explaining the masses of all the neutrinos from the gravitational mechanism would not be used.

Conversely, the neutrino mass model presented in chapter 3 can be used without the need of fine-tuning in the neutrino sector, even if one is not willing to address the strong CP problem. The introduction of the parameter $\xi \sim 10^7$ is only needed if we want to spontaneously generate the up-quark mass by the neutrino condensate and thus solve the strong CP problem by the DA scenario described in the present chapter. As explained in the text, this does not increase the number of required tunings: we trade the tuning of the Yukawa coupling constant for the tuning of ξ, but with the big bonus of solving the strong CP

problem. In this light, it is natural as well as beneficiary to unify the two scenarios that nicely complement each other and to connect the solution of the strong CP problem and the origin of neutrino masses to a single gravitational source.

From a broader perspective, what we have observed is that very low-scale compositeness can mask new physical effects not less and in some cases even more efficiently than the phenomenon of high-energy decoupling. The low-scale compositeness of our neutrino-composite doublet makes it contribute only into very soft processes and decouple efficiently from hard high-energy processes. This is the crucial point that makes both our gravitational low-energy models consistent with experimental bounds (see chapter 5 for more details). Moreover, the low compositeness scale most probably makes our DA scenario compatible with chiral perturbation theory. Even beyond our two specific gravitational models, the possibility of an alternative low-energy hiding place for BSM physics is a very general message that we believe should be payed more attention to when looking for new physical effects.

Chapter 5

Phenomenological Constraints and Detection Opportunities

This chapter treats the phenomenological consequences of our gravitational neutrino mass and Domestic Axion (DA) models. First, we will show that the new IR gravitational scale Λ_G is constrained to lie within a narrow low-energy regime (section 5.1). This has wide-ranging implications for cosmology (section 5.2), astrophysics (section 5.3), gravity (section 5.4), as well as particle and nuclear physics (section 5.5). The most important aspect of the cosmological model predictions is a phase transition in the very late Universe, which substantially modifies the relic neutrino background and gives rise to dark radiation, dark matter (DM), as well as soft topological defects. On the astrophysical side, the key model predictions are enhanced neutrino decays and the strongly suppressed emission of light particles in stellar neutrino processes. Concerning gravity measurements, we will comment on the modified propagation of gravitational waves and a new gravity-competing short-distance force. The particle and nuclear physics section will cover a variety of different fields, including photon conversion, neutrinoless double-beta decay, possible light sterile neutrinos, and flavor-violating processes.

Note that this chapter will not cover the experimental implications of the axion from our DA model. Since this axion is predominantly the η' meson of QCD with a tiny admixture of the η_ν boson, its phenomenological implications are similar to the well-known ones of the SM η' particle (see, e.g., [97, 236, 237] for reviews). This identification of the QCD axion with the η' meson implies that the pseudo-Goldstone boson associated with the PQ symmetry was already discovered in 1964 [238, 239]. Thus, all experiments designated for the discovery of invisible axions [240] can only discover *axion-like* particles (ALPs). In our DA case, such an ALP is the graviaxion, which predominantly consists of the η_ν boson and gets its mass from the chiral gravitational anomaly.

Before moving on to the phenomenological implications of our neutrino mass and DA models, let us emphasize again that these two gravitational low-energy models are *a priori* separate scenarios, but it is both natural and beneficiary to unify them (see section 4.5 for further discussions). While the minimal version of the DA model requires only the up quark and a single neutrino flavor, the minimal version of the neutrino mass model includes exclusively neutrino species. In the combined scenario, which contains all neutrino flavors and the up quark, most of the cosmological, astrophysical, and phenomenological consequences carry over from the minimal neutrino mass model. Therefore, in the following sections we will mainly focus on the predictions of our neutrino mass model and will only comment on our DA model in case its predictions substantially differ from the pure neutrino mass case.

5.1 Bounds on Symmetry Breaking Scale

Various BSM interactions and (pseudo)scalar degrees of freedom in the neutrino sector have been investigated to date. There are several constraints on the consequences of such models, which have been frequently updated in the past and will be further enhanced by future experiments. To present one example, the SM predicts an effective number of neutrino species in the early Universe of $N_{\text{eff}} = 3.046$ [241]. While earlier observational values (e.g., $N_{\text{eff}} = 3.14^{+0.70}_{-0.65}$ [242] or $N_{\text{eff}} = 4.34^{+0.86}_{-0.88}$ [243]) still left open the window for many BSM predictions for N_{eff}, recent Planck data ($N_{\text{eff}} = 3.15 \pm 0.23$ [244]) narrowed the range around the SM value and excluded many of the investigated scenarios.

As discussed in the following, our symmetry breaking scale Λ_G is constrained by the EW Higgs effect, cosmological data, and short-distance gravity measurements. For the sake of simplicity, we will assume that Λ_G is equal to the scale v of the condensate and to the temperature T_{Λ_G} of the cosmic neutrino phase transition. However, just as in the QCD case, we should keep in mind that these scales can be parametrically different (see, e.g., section 5.2.1).

A universal upper bound on the symmetry breaking scale Λ_G comes from the fact that the gravitationally triggered chiral fermion condensate contributes to the SM Higgs condensate. Such a chiral fermion condensate must also involve other fermion flavors besides the light neutrinos, including quarks and charged leptons. This is because the leading-order gravity effects should distinguish the different fermions of the SM only by their masses, implying that all fermion flavors condense. If we assume that the heavy flavors f of mass $m_f \gg \Lambda_G$ decouple in the same way as in QCD, then their condensates scale as $\langle \bar{f}_L f_R \rangle \sim \Lambda_G^4/m_f$, whereas the condensates of the light flavors with $m_f \ll \Lambda_G$ must be of order Λ_G. The immediate bound on Λ_G comes from the fact that such a condensate, similarly to the quark condensate in QCD, contributes to the Higgs effect of the W and Z bosons, requiring that Λ_G must be below the QCD scale. For $\Lambda_G \sim m_\nu$, the contribution to the SM Higgs effect is negligible and the small contamination of the Higgs condensate by the condensate of the quarks and leptons is in accordance with EW constraints.

Further bounds on the symmetry breaking scale and on the BSM modifications of the neutrino sector come from cosmology. The most important cosmological restriction is the neutrino free-streaming constraint at the photon decoupling epoch, where the temperature of the Universe was $T \sim 256$ meV [245]. The authors of [246] found that neutrino free-streaming is favored by the data over a relativistic perfect neutrino fluid with high significance. Also secret Yukawa couplings between neutrinos and light majoron-like (pseudo)scalars, such as our ϕ bosons, are strongly restricted in the early Universe [247–249]. For example, if the neutrinos already coupled to the ϕ particles before $T \sim 256$ meV, their diagonal interactions would have to be smaller than 1.2×10^{-7}, and their off-diagonal couplings would be even more strongly constrained [249]. Note that these constraints should still apply if neutrinos get small hard masses $m_\nu \lesssim \Lambda_G$ through other mechanisms, because also in this case the relic neutrinos condense in the presence of the chiral gravitational anomaly and thus experience strong self-interactions.

In view of these cosmological constraints, we assume Λ_G and thus the temperature of the phase transition to be below 256 meV. *A priori*, Λ_G could also be larger than 256 meV. However, even if we consider neutrino masses of up to 2.2 eV [250], which would be allowed in our neutrino mass model due to the inapplicability of cosmological mass limits (see section 5.2.1), generating such masses through couplings to a condensate of scale $\Lambda_G \lesssim 256$ meV would be perfectly consistent. This is because the diagonal couplings of the neutrinos to the condensate are observationally unconstrained and only the off-diagonal couplings are experimentally restricted (see section 5.3.1), leaving open all possible diagonal couplings $g_v \leq 4\pi$. Therefore, we assume a phase transition in the very late Universe after photon decoupling, $\Lambda_G \lesssim 256$ meV, which makes our modifications of the relic neutrino sector phenomenologically viable.

The smallest possible mass of the heaviest neutrino, $m_{\nu_{\text{heavy}}} \sim 50$ meV [97], and the maximal coupling of the condensate to this neutrino, $g_v = 4\pi$, requires at least a scale of $\Lambda_G \sim 4$ meV. Moreover, a neutrino-independent lower bound on the scale Λ_G arises from the possibility that the nonperturbative gravitational effects might induce corrections to Newtonian gravity, which are experimentally excluded down to distances corresponding to inverse meV energies [251] (see section 5.4.2 for more details).

All in all, we observe that several model-independent arguments constrain the IR gravitational scale Λ_G to lie in the range of meV to eV. This fact is intriguing due to the numerical proximity to both the neutrino mass splittings and the DE density $\rho_{\text{DE}} \sim (3 \text{ meV})^4$. As we will further discuss in section 5.6, our models therefore open up the possibility for a common gravitational low-energy origin of neutrino masses, new light ALPs, and potentially DE.

5.2 Implications for Cosmological Models

After having fixed the scale Λ_G with several phenomenological constraints, let us now turn to the possible implications of the predicted phase transition for cosmological models. In the following subsections, we will investigate the evolution of the relic neutrino background as well as the emerging dark radiation, DM, and topological defects after the late transition. Notice that the precise impact of this transition on cosmological observables is still under investigation by means of numerical simulations and data analyses [4, 6].

5.2.1 Neutrino Masses from Cosmology

First, let us briefly examine how much energy density is available in the relic neutrino sector for generating the late neutrino masses and the new dark degrees of freedom. For the sake of simplicity, we assumed in our original paper [1] that the phase transition takes place instantaneously, i.e., at a temperature $T_{\Lambda_G} \sim \Lambda_G \sim v \sim m_\nu$. However, in general, the phase transition can also be delayed and thus become *apparent* only at lower temperatures. Such a supercooling mechanism is well known from inflationary and other cosmological scenarios (e.g. [30, 252, 253]) and can drastically increase the energy density in an expanding Universe. In our case, it could give rise to relatively large neutrino masses even at a low apparent transition temperature, $T_{\Lambda_G} \lesssim \Lambda_G \sim v \sim m_\nu$. Moreover, it allows for substantial energy densities of the pseudoscalar bosons and the topological defects after the transition (see sections 5.2.3 and 5.2.4).

The relevant factor characterizing the possible delay of the phase transition is the self-coupling of the neutrino-bilinear field Φ, which is a free parameter of the theory. For a small self-coupling, e.g., $\lambda_\Phi \sim 0.01$, the temperature-dependent part of the potential stabilizes the wrong metastable vacuum at $\langle\Phi\rangle = 0$ (see, e.g., Fig. 4.10 in [254]). Therefore, the neutrino sector supercools in the wrong symmetric state until tunneling becomes significant at lower temperatures, which enables the vacuum decay to the true minimum at $\langle\Phi\rangle \neq 0$. Since the energy density in the relic neutrino sector is frozen in the false vacuum during the supercooling phase, it continuously increases compared to the other diluting energy densities in the Universe, e.g., of the photons. Consequently, our gravitational neutrino mass model implies that the energy density in today's neutrino sector can be significantly larger than expected by standard cosmology.

This increase of the energy density in the late Universe might substantially alter cosmological observables and yield a potential resolution of recently observed cosmological tensions, as we are currently testing within the framework of different numerical projects [4, 6]. There are several discrepancies between cosmological parameters inferred from early and late Universe data, e.g., (i) between the parameter σ_8 inferred from Planck CMB data [244] and from weak lensing data by the Kilo Degree Survey (KiDS) [255], (ii) between the value of the Hubble parameter H_0 inferred from Planck [244] and local observations of Cepheid stars [256], and (iii) between parameters inferred from Planck [257] and Sunyaev-Zeldovich (SZ) cluster counts. While these tensions could be due to systematical errors, they might also hint at new physics (see, e.g., [258–262]). For example, it was found that time-varying DE might reduce the discrepanices between KiDS and Planck [262] and that nonzero neutrino masses might resolve the tensions between CMB, lensing, and SZ cluster counts [263]. Intriguingly, the authors of [263] found that cosmic neutrino mass bounds shift to higher values when CMB data is combined with low-redshift data such as from weak lensing, which potentially hints towards an increase of relic neutrino masses with time. These observations might be explained by our neutrino mass model, as will become clear in the following (see also sections 5.2.2 and 5.6).

In standard cosmology, large-scale structure provides a strong upper bound on the sum of the neutrino masses, which is currently stated to lie around $\sum_i m_i \lesssim (0.1 - 0.3)$ eV [264–266]. Note that authors of the analysis discussed above [263] even claimed to have cosmologically detected nonzero neutrino masses, $\sum_\nu m_\nu = (0.320 \pm 0.081)$ eV, for the degenerate mass scenario with 4σ.

The recent idea of a neutrinoless Universe [267] sparked a lot of excitement among cosmologists because it eludes these cosmological neutrino mass bounds. However, the considered model was finally ruled out by neutrino free-streaming in the early Universe [268] and the aforementioned precision measurements of the effective number of neutrino species in the early Universe [244].

In our gravitational neutrino mass model, the cosmological mass bounds are also invalid, independently of any other neutrino property, such as the Dirac or Majorana nature or possible BSM sterile neutrino states (see section 5.5.3). The reasons for this invalidity are threefold. First, the relic neutrinos are predicted to be massless until the cosmic phase transition in the very late Universe after photon decoupling (see section 5.1). Note here that generating large neutrino masses at relatively low temperatures requires a substantial amount of false vacuum energy from the supercooled phase transition, which can in turn be constrained by cosmological observables. Second, the relic neutrinos rapidly decay into the lightest neutrino mass eigenstate after the transition (see section 5.3.1). Therefore, any possible cosmological mass bound is only applicable to the lightest neutrino. Third, the relic neutrinos become strongly coupled after the phase transition and substantially annihilate into massless or very light (pseudo)Goldstone bosons. This almost complete annihilation could only be evaded in the presence of large neutrino asymmetries (see section 5.2.2). Notice here that the authors of [269] recently analyzed cosmological constraints on late neutrino masses and self-interactions as predicted by our model, but their analysis neither included the annihilation nor the false vacuum energy [270].

In the late Universe, the neutrinos are not in thermal equilibrium anymore, and thus one might expect that the phase transition and the mass generation may not happen as in the thermal case. However, the neutrino energy density should affect the order parameter in a way similar to the temperature. As long as the energy density scales as T^4, the neutrinos will obtain their masses at the scale Λ_G or at lower scales in case of a delayed transition.

Consequently, our neutrino mass model gives rise to the same phenomenological consequence as the original neutrinoless Universe scenario, i.e., the inapplicability of the usual cosmological mass limits. Depending on the existence of neutrino asymmetries, either no information on the neutrino masses can at all be inferred from cosmology or the cosmological mass bounds only apply to the lightest neutrino species, which can in principle be undetectably light. Here, note that our model predicts all neutrinos to be massive due to gravitational anomaly-matching conditions [218], while experiments still allow for $m_{\text{lightest}} = 0$. We can conclude that in the absence of neutrino asymmetries in the Universe, our model could be falsified by a cosmological neutrino mass detection, e.g., by the upcoming DESI or Euclid surveys [271].

According to our model, only beta-decay experiments are currently suitable to determine the absolute neutrino mass scale. The endpoint of the kinetic electron energy spectrum measured in beta-decay experiments like KATRIN [272] depends on the effective electron neutrino mass m_β, parametrized in terms of the phase-space factor $[(E_0 - E)^2 - m_\beta^2]^{1/2}$. Here, E is the variable kinetic energy of the electron and E_0 is the maximal electron energy, i.e., the endpoint of the spectrum if we had no neutrino mass. Now the crucial question to ask is whether this phase space factor could be altered by our modification of the low-energy neutrino sector [273], because neutrinos with energies below the symmetry breaking scale $\Lambda_G \sim$ meV–eV might need to form bound states (see sections 3.2 and 5.2.2 for discussions). In this regard, we notice that assuming neutrinos to be in bound states below Λ_G would not constrain processes with single low-energy neutrino emission, since the singly emitted neutrinos would directly "mesonize" in form of massless ϕ_k-Goldstones by picking up partners from the neutrino sea. This may happen with the emitted antineutrinos at the endpoint of the beta-decay spectrum at KATRIN, where the neutrinos have energies below Λ_G. We expect that possible modifications of the electron energy spectrum due to such a bound state formation could not be experimentally resolved by KATRIN, since the beta-decay process happens on much shorter timescales. Therefore, the emitted low-energy neutrinos can be treated as free particles, irrespective of whether or not they need to be bound below Λ_G.

The latest beta-decay experiment still allows for an effective electron neutrino mass of up to $m_\beta \sim 2$ eV [250]. Due to the invalidity of the cosmological bounds in our model, the KATRIN experiment probing m_β down to 0.2 eV at 90% CL [272] has the potential to discover a relatively large absolute neutrino mass scale soon. To our knowledge, our gravitational mechanism is the only one that intrinsically evades *all* cosmological constraints on neutrino masses. Therefore, under the assumption that the standard cosmological ΛCDM model is valid, the detection of an unexpectedly large absolute neutrino mass scale in beta-decay experiments would provide a strong hint towards our model.

5.2.2 Fate of Relic Neutrino Background

As discussed in section 3.2, one might naively expect that the neutrino vacuum condensate requires neutrinos to form bound states below the symmetry breaking scale Λ_G, due to 't Hooft's anomaly matching condition [133, 219]. However, in a recent paper [218] it was shown that gravitational anomaly matching does not force low-energy neutrinos to be bound, which leaves open the possibility that the relic neutrinos are strongly coupled but do not form bound states. In this case, depending on potential neutral lepton asymmetries in our Universe, we can distinguish two possible fates of the cosmological neutrino background: after the relic neutrinos become strongly coupled after the phase transition, they either completely or only partially annihilate into massless Goldstone bosons.

Complete Relic Neutrino Annihilation

According to our gravitational neutrino mass model, relic neutrinos and antineutrinos can annihilate into massless Goldstone bosons through the process $\nu + \bar{\nu} \to \phi_k + \phi_k$, with an annihilation rate in the nonrelativistic limit of [267]

$$\Gamma_\nu(T) = \langle \sigma_\nu v_\nu \rangle n_{\mathrm{eq},\nu} = \frac{g^4 T}{64\pi m_\nu^3} \left(\frac{m_\nu T}{2\pi} \right)^{3/2} e^{-m_\nu/T}, \tag{5.2.1}$$

where σ_ν is the annihilation cross section, v_ν is the neutrino velocity, $n_{\mathrm{eq},\nu}$ is the neutrino equilibrium density, T is the neutrino temperature, and g is the diagonal or off-diagonal neutrino-Goldstone coupling. Note that in the minimal one-neutrino scheme of our DA model, only a single neutrino flavor could annihilate into massive η_ν pseudo-Goldstone bosons. The corresponding annihilation rate is expected to be strongly suppressed for $m_{\eta_\nu} > m_\nu$.

In the multi-flavor scheme of our neutrino mass model, the enhanced neutrino decays (see section 5.3.1) imply that all the relic neutrinos decay immediately after the phase transition into the lightest mass eigenstate ν_1, where we suppose normal mass hierarchy for simplicity. In the following, we will present two scenarios that make clear how crucially the freeze-out temperature of the cosmic neutrino background depends on the explicit scales of m_1 and Λ_G.

Let us in the first scenario assume a quite heavy lowest-mass eigenstate, $m_1 \sim 50$ meV, and a phase transition at $T_{\Lambda_G} = \Lambda_G \gtrsim 50$ meV. As argued in [267], our enhanced interactions after T_{Λ_G} keep the neutrinos in equilibrium until $T_\nu \lesssim m_1$. Afterwards, the neutrino abundance will undergo exponential suppression until the annihilation rate $\Gamma(T)$ (5.2.1) becomes equal to the Hubble expansion rate $H(T)$, i.e., the neutrinos freeze out. If the freeze-out temperature is $T_f < \mathcal{O}(m_1/7)$, the neutrino abundance becomes negligible due to exponential suppression by a factor $\gtrsim 100$. Solving $\Gamma(T_f) = H(T_f)$ on condition that $T_f < m_1/7$ hence provides a constraint on the minimal coupling g that is necessary for obtaining a "neutrinoless Universe". We find that a coupling of $g \gtrsim 3 \times 10^{-6}$ is required in order to annihilate a significant amount of neutrinos into Goldstone bosons by T_f, leaving only a negligible fraction of relic neutrinos behind. If we consider a second scenario in which the mass of the lightest neutrino is negligibly small, $m_1 \lesssim 1$ meV, we find that for a coupling of $g \gtrsim 2 \times 10^{-6}$, the neutrino annihilation rate would still be higher than the expansion rate today, $\Gamma(T_0) > H(T_0)$. In this case, the relic neutrinos of the late Universe have not frozen out until today.

From (5.2.1) it becomes evident that the relic neutrino density always becomes strongly suppressed after the phase transition, independently of whether or not the neutrinos have frozen out until today. Consequently, our neutrino mass model predicts an (almost) neutrinoless present-day Universe and thus evades all cosmological neutrino mass bounds, as we already pointed out in section 5.2.1.

Are there any possibilities to evade the substantial relic neutrino annihilation in our neutrino mass model or is this prediction inevitable? One naive approach might be to consider a strongly delayed phase transition (see section 5.2.1), by analogy with thermal inflation scenarios [274,275]. In such scenarios, superheavy DM is generated in a supercooled phase transition such that it immediately freezes out after its production, without substantial annihilation. However, these models crucially depend on very large DM masses and a small reheating temperature after supercooling. In our case, an analogous scenario would require neutrino masses larger than ~ 100 keV for an unsuppressed neutrino number density of $n_\nu \sim T_0^3 \sim (10^{-4}\ \text{eV})^3$, as can be deduced from (5.2.1) with $\Gamma(T_0) < H_0 \sim 10^{-33}$ eV and $\langle\sigma_\nu v_\nu\rangle \sim T_0/m^3 \lesssim 10^{-21}/\text{eV}^2$. Thus, this first suggested option to evade a neutrinoless Universe disagrees with experimental upper bounds on the absolute neutrino mass scale.

A second possibility is to consider subtantial neutral lepton asymmetries in the early Universe. In the following, we shall not commit to the unknown origin of such asymmetries but assume their existence and investigate their impact on the relic neutrino background in our neutrino mass model.

Relic Neutrino Survival in Presence of Asymmetries

The main constraints on large neutrino asymmetries stem from Big Bang nucleosynthesis (BBN), especially from the helium abundance [276]. In particular, BBN puts the strong bounds $-4.5 \lesssim 10^3 L_{\nu_e} \lesssim 2.0$ [277] on the electron-neutrino asymmetry $L_{\nu_e} \equiv (n_{\nu_e} - n_{\bar{\nu}_e})/n_\gamma$, where n_{ν_e}, $n_{\bar{\nu}_e}$, and n_γ are the number densities of ν_e, $\bar{\nu}_e$, and photons, respectively. Other bounds on neutrino asymmetries come from ΔN_{eff}, which is constrained by both BBN and CMB observations. Recent Planck CMB data yields relatively weak constraints on the muon- and tau-neutrino asymmetries, $|L_{\mu,\tau}| \lesssim 0.24$ [244]. According to standard neutrino cosmology, neutrino oscillations in the early universe mix the neutrino flavors such that any asymmetry $L_{\mu,\tau}$ that is present well before BBN is converted subtantially to L_e [278–281]. However, the resulting strong constraints on $L_{\mu,\tau}$ can be evaded in our gravitational neutrino mass model: the relic neutrinos are massless in the early Universe, and all flavor-violating couplings only turn on abruptly when approaching the late-time phase transition (analogous to, e.g., axion couplings [282]). Therefore, the relic neutrinos are unmixed at BBN, which implies that the total neutrino asymmetry can still be quite large in our model, even though the ν_e asymmetry must be small enough to be in accordance with the BBN bound.

The above-mentioned weak bounds on the $\nu_{\mu,\tau}$ asymmetries can be derived from constraints on ΔN_{eff}, which currently read $\Delta N_{\text{eff}} < 0.38$ at 68% CL (Planck TT+lowP+BAO) [244] and yield the bounds

$$\left| \frac{n_{\nu_\alpha} - n_{\bar{\nu}_\alpha}}{n_{\nu_\alpha}} \right| \lesssim 0.17 \times \frac{11}{3} \sim 0.62, \tag{5.2.2}$$

where $\alpha = \mu, \tau$. This implies that up to $\sim 62\%$ of the ν_μ and ν_τ flavors could have survived the annihilation after the late phase transition, which corresponds to $62\% \times 2/3 \sim 42\%$ of the relic neutrinos.

To sum up, a cosmological detection of nonzero neutrino masses would imply the following consequences according to our neutrino mass model:

(1) There exists a substantial neutrino asymmetry in our Universe.

(2) Neutrinos are Dirac particles, since an asymmetry of nonrelativistic neutrinos would not survive in the Majorana case [273, 283].

(3) The current relic neutrino number density is at most $\sim 42\%$ of the value expected by standard cosmology.

(4) The lightest neutrino has a nonzero mass, since the relic neutrino background has completely decayed to the lightest mass eigenstate.

Concerning point (4), note that the relic neutrino background in the late Universe consists only of the lightest mass eigenstate, even though the assumed neutrino asymmetry is predominantly in the early-Universe ν_μ and ν_τ sectors. Thus, in case of a normal mass hierarchy and a dominance of neutrino matter over antimatter, today's relic neutrinos could sufficiently consist of ν_e states to be detectable by beta-decay experiments, as discussed below. Notice here that these strongly coupled low-energy neutrinos behave as a superfluid, implying that the phase transition in the late Universe is a transition of massless neutrino radiation to neutrino cold DM. Additional DM is required in our Universe to explain, *inter alia*, earlier structure formation and galaxy rotation curves [28,29].

Relic Neutrino Detection

The unusual cosmological implications of our neutrino mass model affect experiments that aim at detecting the relic neutrino background via induced beta decay. In such experiments, the abundant relic neutrinos may be detectable through neutrino capture on the radioactive tritium nuclei, so that a disctinct monoenergetic electron energy peak is measured above the initial endpoint of the electron energy spectrum, $E = E_0 + m_{\nu_e}$. Here, E is the variable kinetic energy of the electron and E_0 is the maximal electron energy, i.e., the endpoint of the spectrum if we had no neutrino mass. This idea was first elaborated in [284] and further evaluated in [285] and [286], among others.

The distinct electron energy peak at $E = E_0 + m_{\nu_e}$ from relic neutrino capture should in principle be detectable and separable from the end of the continuous decay spectrum. However, according to standard neutrino cosmology, the sensitivity of the KATRIN experiment to measure this deviation is not high enough for the current low density of relic neutrinos in the Universe [285]. Also the gravitational clustering of the neutrinos in our Galaxy cannot enhance the local neutrino density enough for a detection in the near future [287], since KATRIN would require an overdensity $n/\langle n\rangle$ as large as 2×10^9 [285]. In our neutrino mass model, the predicted relic neutrino self-interactions could further enhance the local neutrino density, as we are currently investigating by means of numerical simulations [5]. In our original paper [1], we argued that these neutrino self-interactions might result in $n/\langle n\rangle$ being high enough for a detection at KATRIN. However, our estimates were based on the claim in [288] that gravitational clustering yields $n/\langle n\rangle \lesssim 10^6$, which was shown by more recent work [287] to be too optimistic.

The recently proposed PTOLEMY experiment [289] aims to achieve the sensitivity required to detect relic neutrinos. However, such a detection would only be feasible in case of degenerate or quasi-degenerate neutrino masses due to the proposed energy resolution of ~ 0.15 eV [290]. While such large masses are ruled out by the conventional cosmological neutrino mass bounds, they are still allowed in our gravitational neutrino mass model (see section 5.2.1). Let us remember once again that in the absence of substantial neutrino asymmetries, our model predicts either a massless bound-state formation or a complete annihilation of the self-interacting relic neutrino background. Therefore, a detection at PTOLEMY would only be possible in the presence of an unbound and strongly asymmetric neutrino background, as discussed above. Notice that the resulting relic neutrino detection rate would be enhanced (suppressed) in case of a normal (inverted) neutrino mass hierarchy, because the relic neutrinos have all decayed into the lightest mass eigenstate (see section 5.3.1), which contains a large (small) fraction of the electron neutrino flavor eigenstate.

5.2.3 Late Dark Radiation or Dark Matter

We now leave the relic neutrino sector and move on to the massive and massless (pseudo)Goldstone excitations ϕ of the neutrino-composite field

$$\Phi = \bar{\nu}\nu = \langle\bar{\nu}\nu\rangle e^{i\phi} \tag{5.2.3}$$

after the phase transition. Each of the dark boson species $\phi \equiv \{\phi_k, \eta_\nu\}$ is created with an approximate energy density of $\sim \Lambda_G^4$ in the transition.

As explained in section 3.1.2, the η_ν pseudoscalar acquires its mass through the chiral gravitational anomaly, $m_{\eta_\nu} \sim \Lambda_G$. Naively, one might expect that the other ϕ_k pseudoscalars are massless, since the neutrinos have no hard masses in our model but only effective ones. However, the original $U(3)_L$ neutrino flavor symmetry is not an exact symmetry but only approximate due to small weak corrections: box diagrams involving W-boson and charged lepton exchange give rise to tiny mass contributions for some of the ϕ_k pseudoscalars,

$$m_{\phi_k} \sim \frac{g_W^4}{m_W^4} m_l^2 \Lambda_G^3. \tag{5.2.4}$$

When incorporating all relevant numerical factors and inserting $m_l \sim m_e$ and $\Lambda_G \sim 0.1$ eV, we observe that the mass contributions are of the order of the Hubble constant, $m_{\phi_k} \sim 10^{-33}$ eV $\sim H_0$. This estimate is highly sensitive to the unknown exact value of Λ_G.

Now let us consider the evolution of the dynamical (pseudo)Goldstone excitations after the late cosmic phase transition. When denoting the different masses of the ϕ pseudoscalars as m_ϕ, we see from

$$\ddot{\phi} + 3H\dot{\phi} + m_\phi^2 \phi = 0 \tag{5.2.5}$$

that the (pseudo)Goldstone excitations are frozen in the vacuum if their masses are much smaller than the Hubble parameter, $m_\phi \ll H$. Since our mass estimates above show that the nonzero ϕ masses are of the order of the Hubble expansion rate at the time of the phase transition ($H_0 \sim 10^{-33}$ eV $\lesssim H_{\Lambda_G} \lesssim H_{\text{CMB}} \sim 10^{-29}$ eV [244]), the (pseudo)Goldstone modes can be frozen in the vacuum for a substantial amount of time before starting to oscillate.

Since the massless and extremely light ϕ_k pseudoscalars scale as radiation, $\rho_{\phi_k} \propto T^4$, they redhift away quickly after starting to oscillate. Thus, they are expected to give only a significant contribution to today's total energy density if the phase transition happened very recently. In contrast, the energy density of the massive η_ν excitations scales as matter, $\rho_{\eta_\nu} \propto T^3$. In the one-neutrino scenario of our minimal DA model, η_ν is the only pseudo-Goldstone emerging in the relic neutrino sector, so that for $m_{\eta_\nu} < m_\nu$ the η_ν particles would be stable and thus contribute to today's DM density. However, in the three-flavor neutrino mass model, the η_ν bosons quickly decay into the lighter ϕ_k. Because the very light ϕ_k decay further into the massless ϕ_k, our multi-flavor neutrino mass model predicts that only massless radiation is left over in today's neutrino-composite pseudoscalar sector.

5.2.4 Soft Topological Defects

In addition to the dark radiation and DM discussed in the previous subsection, the cosmic neutrino phase transition is expected to yield soft topological defects. Topological defects, such as domain walls, are ubiquitous in condensed matter physics but only hypothetical in cosmology. These massive objects are assumed to emerge in various cosmic phase transitions due to the Kibble mechanism [291] and could have a severe impact on the evolution of the Universe [292]. In our neutrino mass and DA models, we expect the topological defects to be very light and thus difficult to detect directly. However, they could have important implications for late cosmology due to their huge abundance. Therefore, we are currently examining their formation and evolution [3] and will present some of our preliminary results in the current subsection.

In the following, we will first investigate the different symmetry breaking steps in the neutrino phase transition. Then, we will treat the resulting formation and evolution of various defects, such as textures, domain walls, and strings. Most importantly, we will observe that the defects rapidly annihilate into dark radiation and DM in our neutrino mass and minimal DA scenarios, respectively (cf. section 5.2.3). In case of a delayed transition (see section 5.2.1), this can result in substantial energy densities in the respective late dark sectors. The resulting impact on cosmological observables will be examined in future studies (see chapter 6), building on our current numerical simulations of cosmological models with time-varying neutrino masses [4, 6].

Neutrino flavor symmetry breaking pattern

In the following discussion of the neutrino flavor symmetry breaking, we will for simplicity only treat the three-flavor Dirac neutrino case of our neutrino mass model. In this case, the initial flavor symmetry of the neutrinos reads

$$U(3)_L \times U(3)_R = SU(3)_V \times SU(3)_A \times U(1)_V \times U(1)_A. \tag{5.2.6}$$

The vacuum of our theory has a N-fold degeneracy, where $N = N_F = 3$. Therefore, the $U(1)_A$ part of the initial flavor symmetry (5.2.6) spontaneously breaks down in two steps via the discrete Z_N subgroup of $U(1)_A$,

$$U(1)_A \to Z_N \to 1. \tag{5.2.7}$$

Due to the neutrino mass hierarchy (see section 3.3 for more details), the remaining parts of the initial flavor symmetry (5.2.6) break in several steps. The generation of hierarchical nonzero Dirac neutrino masses induces the breaking

$$\begin{aligned} SU(3)_V \times SU(3)_A \times U(1)_V &\xrightarrow{m_3 \neq 0} SU(2)_V \times U(1)_V \times SU(2)_A \times U(1)_V \\ &\xrightarrow{m_2 \neq 0} U(1)_V \times U(1)_V \times U(1)_A \times U(1)_V, \\ &\xrightarrow{m_1 \neq 0} U(1)_V \times U(1)_V \times U(1)_V, \end{aligned} \tag{5.2.8}$$

for $m_1 < m_2 < m_3$. The $U(1)_V$ symmetries after the different breaking steps are mixtures of the initial $U(1)_V$ symmetries, some of which are the $U(1)_V$ subgroups of $SU(3)_V$ and $SU(2)_V$, respectively. For example, the initial $U(1)_V$ symmetry in (5.2.8) corresponds to lepton number conservation, while the residual $U(1)_V \times U(1)_V \times U(1)_V$ symmetry corresponds to the separate conservation of the three neutrino flavors. In other words, the latter is a diagonal group of individual neutrino number symmetries for each flavor,

$$U(1)_1 \times U(1)_2 \times U(1)_3. \tag{5.2.9}$$

There are two important points to notice:

(1) In case of nearly degenerate neutrino masses, the $SU(3)_V \times SU(3)_A \times U(1)_V$ symmetry (5.2.8) breaks within timescales much smaller than the Hubble scale, so that topological defects from the intermediate breaking steps only exist for a short time. However, in case of a nondelayed generation of strongly hierarchical neutrino masses, e.g., $m_{(1,2,3)} \sim (1, 10, 50)$ meV, the three subsequent breaking steps can take place at very different redshifts, e.g., $z_{(1,2,3)} \sim (4, 54, 268)$.

(2) It is well known that the residual symmetry (5.2.9) cannot be further broken by neutrino masses alone, since the neutrino mass matrix has only three different nonzero elements in the diagonal basis, $m_{ij} = \mathrm{diag}(m_1, m_2, m_3)$ (see section 3.3). The observed neutrino mixing only appears due to the mismatch of the mass bases of the neutrinos and the charged leptons. Thus, the individual neutrino flavor symmetries (5.2.9) are further broken explicitly by weak effects.

In the following, we will discuss the different kinds of topological defects, which arise in our neutrino mass and DA models. First, we will demonstrate that monopole formation is not supported. Then, we will show that the breaking of the $SU(3)_V \times SU(3)_A \times U(1)_V$ symmetry (5.2.8) in our neutrino mass model gives rise to neutrino skyrmions. Finally, we will explain how the breaking of the $U(1)_A$ symmetry (5.2.7) in both our neutrino mass and DA scenarios yields hybrid topological defects of cosmic strings and domain walls.

Non-Existence of Global Monopoles

Let us explain why the spontaneous breaking of $SU(3)_V \times SU(3)_A \times U(1)_V$ flavor symmetry (5.2.8) in our neutrino mass model does not support global monopole formation. Monopoles form if the second homotopy group $\pi_2(\mathcal{M})$ is nontrivial, where $\mathcal{M}$ is the vacuum manifold [291]. While $\pi_2(G)$ is trivial for any compact connected Lie group G, monopoles can arise if the group G is broken down to a subgroup $H = H' \times H_0$, where H' is the simply connected component of H and H_0 is the component of H connected to the identity [292]. The extent to which a manifold $\mathcal{M}$ fails to be simply connected can be measured by the first homomotopy group $\pi_1(\mathcal{M})$, which is trivial if $\mathcal{M}$ is simply connected. Intuitively, $\pi_1(\mathcal{M})$ detects holes in a manifold, e.g., a sphere is simply connected but a torus is not. Most importantly, $SU(N)$ groups are simply connected, while $U(N)$ groups are not.

If G is a simply connected group, the second fundamental theorem implies

$$\pi_2(G/H) = \pi_1(H) = \pi_1(H' \times H_0) = \pi_1(H_0) \neq 1, \tag{5.2.10}$$

which indicates the formation of monopoles [292].

In our neutrino mass model, one may naively think that the cosmological generation of the largest neutrino mass m_3 induces a breaking of the type $G_1 \to H_1' \times U(1)$, where $G_1 \equiv SU(3) \times SU(3)$ and $H_1' \equiv SU(2) \times SU(2)$ are simply connected groups. Therefore, one may expect that global monopoles form in the first stage of flavor symmetry breaking,

$$\pi_2(SU(3) \times SU(3)/SU(2) \times SU(2) \times U(1)) = \pi_1(U(1)) = Z. \qquad (5.2.11)$$

However, we already noticed above that there are two different $U(1)_V$ symmetries after the first breaking step in (5.2.8), which are mixtures of the initial $U(1)_V$ symmetry and the $U(1)_V$ subgroup of $SU(3)_V$. Therefore, we have to consider the second homotopy group of the full flavor symmetry under consideration, $\pi_2(SU(3) \times SU(3) \times U(1)/SU(2) \times SU(2) \times U(1) \times U(1))$. Since this homotopy group is trivial, no global monopoles form in the breaking sequence (5.2.8). This situation is similar to the case of EW symmetry breaking, $SU(2)_W \times U(1)_Y \to U(1)_{\mathrm{EM}}$, where magnetic monopoles are absent because the $U(1)_{\mathrm{EM}}$ symmetry is a mixture of the initial $U(1)_Y$ group and the $U(1)$ subgroup of $SU(2)_W$ [292].

Dark Radiation from Skyrmions

In our neutrino mass model, one type of topological defect that definitely forms in the cosmic phase transition are textures, which are classified by the third homotopy group $\pi_3(\mathcal{M})$ [291]. When spontaneously breaking large symmetry groups, such as our $SU(3)$ flavor groups, textures arise due to

$$\pi_3(SU(3)) = Z. \qquad (5.2.12)$$

These defects do not arise in our minimal DA scenario, which only involves global $U(1)$ symmetries. In the three-flavor neutrino mass scenario, the third homotopy group of the full coset, $\pi_3(SU(3) \times SU(3) \times U(1)/SU(2) \times U(1) \times SU(2) \times U(1))$, can be computed from the exact sequence of homotopy groups,

$$\pi_n(H) \to \pi_n(G) \to \pi_n(G/H) \to \pi_{n-1}(H) \to ..., \qquad (5.2.13)$$

where $H \subset G$ is a closed subspace and $\pi_n(G/H)$ is called the nth relative homotopy group of the pair (G/H). Equation (5.2.13) implies that $\pi_n(G/H) = \pi_n(G)$ for $\pi_{n-1}(H) = 1$, which yields

$$\begin{aligned}&\pi_3(SU(3) \times SU(3) \times U(1)/SU(2) \times U(1) \times SU(2) \times U(1))\\ =&\,\pi_3(SU(3) \times SU(3) \times U(1)) = Z \times Z,\end{aligned} \qquad (5.2.14)$$

where we used $\pi_2(SU(2) \times SU(2) \times U(1) \times U(1)) = 1$, $\pi_n(\mathcal{M}_1 \times \mathcal{M}_2) = \pi_n(\mathcal{M}_1) \times \pi_n(\mathcal{M}_2)$, and $\pi_3(U(1)) = 1$.

Unless the textures are stabilized by the Skyrme term [293], they are unstable, i.e. the topological texture knots quickly unwind with a rate $dn/dt \sim \Lambda_G^4$ and dissipate into Goldstone bosons [294]. In our model, the textures are stabilized skyrmions, i.e. spin-1/2 bound states of $N = N_F = 3$ neutrinos.

Analogous to treating the pions as the fundamental fields in the Skyrme model [293], in our effective low-energy neutrino theory we can consider the

ϕ pseudoscalars as the fundamental fields. Similar to the Eightfold Way of QCD [54, 55], whose baryons can be modeled as skyrmions in the large-N_c limit of QCD [295–297], we can infer the number of different neutrino (anti)skyrmions from symmetry considerations. The antisymmetric total wave function of the fermionic neutrino skyrmions requires an antisymmetric spin-flavor part, since the spatial part of the wave function is symmetric for zero orbital angular momentum. This is different from QCD, where the wave function from spin and isospin states has to be symmetric due to the existence of the antisymmetric color wave function. Totally symmetric three-neutrino flavor states are not allowed to form for zero orbital angular momentum because no totally antisymmetric spin wave function exists for three neutrinos. The entire spin-(3/2) spin-flavor symmetric "baryon decouplet" does not exist in the neutrino case, only the spin-(1/2) mixed symmetry "baryon octet" and the totally antisymmetric "baryon singlet". In total, we obtain nine different spin-(1/2) three-neutrino bound states and thus 18 different types of neutrino (anti)skyrmions.

The initial energy density ρ_T of these 18 stabilized textures is proportional to the self-coupling λ_Φ of the neutrino-bilinear Φ field (see [292] and section 5.2.1). For $\lambda_\Phi \sim 1$, each of the (anti)skyrmions is produced with an initial energy density of $\rho_T(t = t_{\Lambda_G}) \sim \xi^{-4}$ [291] in the phase transition, where the correlation length $\xi \sim \Lambda_G^{-1}$ corresponds to almost micrometer distances. Afterwards, the skyrmions and antiskyrmions annihilate with a geometrical cross section [298],

$$\sigma_T \sim \frac{\pi}{\Lambda_G^2}, \tag{5.2.15}$$

where we assumed (anti)skyrmion masses of the order of the chiral symmetry breaking scale, $m_T \sim \Lambda_G$. This annihilation into massless (or very light) ϕ_k bosons is much less efficient than, e.g., for global monopoles, due to the negligible long-range force between skyrmions and antiskyrmions [299]. Therefore, one might naively expect that not all skyrmions find a partner for annihilation. However, similar to the annihilation of relic neutrinos (see section 5.2.2), the skyrmion annihilation rate in the nonrelativistic limit is nevertheless large [267],

$$\Gamma_T(T) = \langle \sigma_T v_T \rangle \, n_{\mathrm{eq,T}} \sim \frac{\pi}{\Lambda_G^2} \left(\frac{3T}{\Lambda_G} \right)^{1/2} \left(\frac{\Lambda_G T}{2\pi} \right)^{3/2} e^{-m_T/T}. \tag{5.2.16}$$

Here, v_T is the thermal texture velocity and $n_{\mathrm{eq,T}}$ is the texture equilibrium density. Just as in the relic neutrino case without asymmetries, we deduce from (5.2.16) that the skyrmions rapidly annihilate into the massless Goldstone bosons. For the exemplary values of $\Lambda_G \sim T \sim m_T \sim 0.1$ eV, this happens within timescales of $\Gamma_T^{-1} \sim 10^{-15}$ s. Thus, the immediate annihilation of these massive topological defects increases the dark radiation energy density in the late Universe by $\rho_r \sim \rho_T(t = t_{\Lambda_G})$ after the neutrino phase transition.

Dark Radiation or Dark Matter from String-Wall Network

It is well known that a phase transition with PQ symmetry breaking can form axionic cosmic strings. These strings later become boundaries of domain walls [300] and decay producing axions. In our DA scenario, everything happens at the same scale: the generation of the quark condensate and the generation of the η' mass take place in the early Universe around QCD temperatures. Thus, the axionic cosmic strings are produced in form of small loops spanned by membranes, i.e., domain walls, and they decay very quickly.

In the late Universe, the second phase transition at temperatures around the neutrino mass scale – which is predicted by both our DA and neutrino mass models – also gives rise to cosmic strings bounded by walls. In the following, we will demonstrate how this string-wall network forms and annihilates into DM or dark radiation in our DA or neutrino mass scenarios, respectively.

Cosmic strings form if the vacuum manifold $\mathcal{M}$ is not simply connected, i.e., it contains incontractible loops, implying $\pi_1(\mathcal{M}) \neq 1$ [291]. This is given in both our neutrino mass and DA models when spontaneously breaking the $U(1)_A$ neutrino axial symmetry (5.2.7) down to the discrete subgroup Z_N, where $N = 3$ due to the three-fold degeneracy of the vacuum [301],

$$\pi_1(U(1)_A/Z_3) = Z_3. \tag{5.2.17}$$

Notice that this relation follows from (5.2.18) via the first fundamental theorem, $\pi_1(G/H) = \pi_0(H)$, which is given because $G \equiv U(1)_A$ is a simply-connected covering group of $H \equiv Z_N$ [292]. The formation of more exotic cosmic strings, such as Alice strings [302], is not supported by the symmetry breaking patterns in both our neutrino mass and DA scenarios.

Domain walls are produced if the manifold $\mathcal{M}$ of degenerate vacua after symmetry breaking consists of two or more disconnected components, making it impossible to pass continuously between the vacuum states in different regions of coordinate space. Thus, since the Z_3 symmetry gets further broken, domain walls characterized by the zeroth homotopy group $\pi_0(\mathcal{M}) \neq 1$ arise [291],

$$\pi_0(Z_3) = Z_3. \tag{5.2.18}$$

Since $N = 3$ of these domain walls get attached to each string (see Fig. 5.1), a hybrid string-wall network forms in the neutrino phase transition [300, 303].

Which are the most interesting cosmological features of such a network? First, the string tension is scale dependent, $\mu_S \sim \eta_S^2 \ln(d_S/\delta_S)$, where $\delta_S \sim \Lambda_G^{-1}$ is the thickness of the string core and d_S is the distance to the nearest string in the network [304]. Second, the domain walls have a thickness of $\delta_W \sim \Lambda_G^{-1}$ and a mass per unit area of $\sigma_W \sim \Lambda_G^3$, more precisely of $\sigma_W \sim \Lambda_G^3/N$ for a Z_N symmetry [300]. Third, the relic neutrinos get reflected at the domain walls, because their wavelengths $\lambda \sim T^{-1}$ are larger than the thickness of the walls, $\Lambda_G^{-1} \sim T_{\Lambda_G}^{-1}$ [305]. However, this reflection is not expected to yield any cosmological consequences apart from friction slowing the domain walls down.

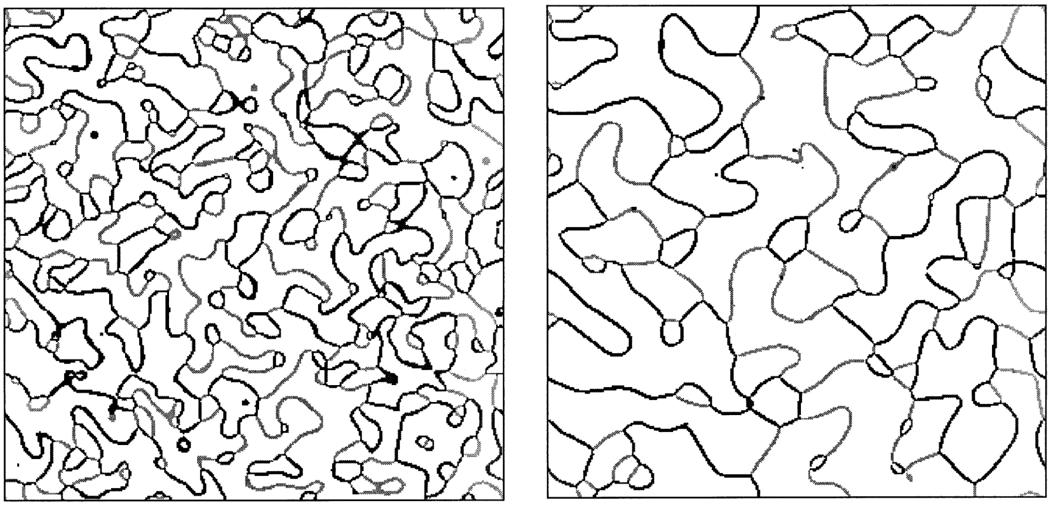

Figure 5.1: Simulated evolution of the string-wall network for $N = 3$ at early (left) and late (right) times. Figure taken from [306].

Most importantly, global strings and domain walls have repulsive instead of attractive gravitational fields. While the equation of state parameter of global strings in the nonrelativistic limit reads $\omega_S = -1/3$ [307], the domain walls with $\omega_W = -2/3$ [308] give rise to an accelerated expansion of our Universe. In case that the string-wall energy density does not dissipate too efficiently, the network could therefore mimic a dynamical equation of state parameter of DE. Initially, the string-wall energy density could be larger than the cosmological constant and thus we could have $-1/3 \lesssim \omega_{\mathrm{DE}} \lesssim -2/3$ immediately after the phase transition. Eventually, the cosmological constant with $\omega_{\mathrm{DE}} = \omega_\Lambda = -1$ could take over. In order to determine the cosmological significance of such a possible effect, in the following we must examine how the energy density of the string-wall network evolves after the phase transition.

At the time of formation, both the domain wall and string energy densities are comparable. Since their tension straightens out the defects very efficiently, the string energy density soon becomes negligible compared to the wall energy density. Therefore, the domain walls determine the dynamics of the string-wall network [300]. If we omit all numerical factors and assume the scale Λ_G to be the only scale in the scenario, the initial energy density of these topological defects reads $\rho_{\mathrm{SW}}(t = t_{\Lambda_G}) \sim \Lambda_G^4$. Early numerical simulations (see Fig. 5.1 and [306]) show that for a Z_N symmetry with $N = 3$, the string-wall energy density decreases proportional to $\eta^{-0.89}$, where η is the conformal time. This implies a deviation from scaling, which can approximately be written as

$$\rho_{\mathrm{SW}}(\eta) \propto \left[a + b \ln\left(\frac{\eta}{\delta_S}\right)\right] \eta^{-1}, \tag{5.2.19}$$

as discussed in [309]. However, this deviation only yields an increase of ρ_{SW} by a factor of ~ 20 compared to the scaling solution, if we insert the values

$a \sim 1.87$ and $b \sim 0.30$ inferred from the numerical simulations for the case $N = 3$ [306]. Furthermore, more recent numerical investigations [310] suggest that the deviations from scaling appeared due to the limited range of the early simulations. When neglecting the small deviation from scaling, the wall-dominated energy density of the hybrid network evolves as

$$\rho_{\rm SW}(t) \sim \sigma_W t^{-1} \sim \Lambda_G^3 t^{-1}. \tag{5.2.20}$$

If we compare this energy density to the matter density ρ_M in the Universe, we observe that the string-wall network contribution is negligible today,

$$\frac{\rho_{\rm SW}}{\rho_M}(t = t_0) \sim 30\, t_0 \frac{\Lambda_G^3}{M_P^2} \sim 10^{-25}, \tag{5.2.21}$$

where $t_0 \sim 4\times10^{17}$ s is the cosmic time today and we again assumed $\Lambda_G \sim 0.1$ eV. Our estimates imply that the efficient string-wall annihilation gives rise to an additional η_ν energy density of $\rho_{\eta_\nu} \sim \rho_{\rm SW}(t = t_{\Lambda_G})$. Since η_ν can in principle be substantially lighter than the VEV of the neutrino condensate, the string-wall network can be parametrically longer-lived before they decay into η_ν bosons and neutrinos. Just like in the case of standard axionic strings, this could be a way of populating the Universe by an even larger number of η_ν particles.

In our neutrino mass model, the abundant η_ν bosons further decay into ϕ_k and thus contribute to dark radiation in the late Universe. In our minimal DA scenario, the η_ν bosons are stable. In this case, the precise η_ν density and thus its contribution to the DM abundance in the late Universe strongly depends on the free parameters of the model, such as the exact value of the scale Λ_G.

5.3 Implications for Neutrino Astrophysics

5.3.1 Enhanced Neutrino Decays

After having discussed various cosmological model predictions in the previous section, we now turn to the astrophysical implications of our low-energy BSM scenarios. In this first subsection, we will discuss the enhanced neutrino decays predicted by our multi-flavor neutrino mass model, which are absent in the minimal one-neutrino scheme of our DA model.

Conventional SM neutrino interactions imply decays of heavier neutrinos into lighter ones, which are suppressed by the W and Z-boson masses. This strong suppression leads to neutrino lifetimes that exceed the lifetime of the Universe. In our gravitational neutrino mass model, the massive η_ν and the massless Goldstone particles ϕ_k open up new decay channels for the neutrinos,

$$\mathcal{L}_{\rm int} \supset \sum_k \partial_\mu \phi_k \sum_{ij} g_{\phi,ij} \bar{\nu}_i \gamma^\mu \gamma_5 \nu_j + \eta_\nu \sum_{ij} g_{\eta_\nu,ij} \bar{\nu}_i \gamma_5 \nu_j, \tag{5.3.1}$$

as we already pointed out in section 3.3. Here, it is important to notice that the Dirac and Majorana neutrino cases yield different decay channels [273].

Therefore, the composition of the final neutrino states after the decay crucially depends on the type of masses generated through our gravitational mechanism, as we will further investigate in future phenomenological studies.

For the neutrino decays $\nu_i \to \nu_j + \phi$ and $\nu_i \to \bar{\nu}_j + \phi$ (where $m_i > m_j$), the pseudoscalar and derivative couplings in (5.3.1) are equivalent [247]. Therefore, we will for simplicity assume only pseudoscalar couplings and will denote all couplings between the ϕ bosons and the neutrinos as g_{ij} in the following.

Our enhanced neutrino decays can happen via intermediate ϕ states, i.e., via box diagrams or Fermi-like interactions. However, the resulting decay widths are suppressed by four powers of the off-diagonal couplings, g_{ij}^4.

The process in which a physical ϕ particle is emitted, is only suppressed by g_{ij}^2. The decay rate Γ_i of the sum of the two processes $\nu_i \to \nu_j + \phi$ and $\nu_i \to \bar{\nu}_j + \phi$ in the rest frame of ν_i reads [247, 311]

$$\Gamma_i = \frac{g_{ij}^2}{16\pi} m_i \tag{5.3.2}$$

if we neglect the masses of the final states. In the medium frame, the rate is reduced by a Lorentz factor of m_i/E. For the lowest possible normal-ordered masses of $m_1 = 0$ meV, $m_2 = 9$ meV, and $m_3 = 50$ meV [97], the rate (5.3.2) transfers into the neutrino rest-frame lifetimes $\tau_i = 1/\Gamma_i$ of

$$\frac{\tau_3}{m_3} \simeq \frac{1 \times 10^{-11}}{g_{3j}^2} \frac{\mathrm{s}}{\mathrm{eV}}, \tag{5.3.3}$$

$$\frac{\tau_2}{m_2} \simeq \frac{4 \times 10^{-10}}{g_{21}^2} \frac{\mathrm{s}}{\mathrm{eV}}, \tag{5.3.4}$$

As already mentioned, for $T_{\Lambda_G} > 256$ meV, such a modification by secret majoron-type interactions would be highly constrained by CMB data [247, 249], but for our considered symmetry breaking after photon decoupling at temperatures $T_{\Lambda_G} < T_{\mathrm{CMB}}$, these cosmological constraints do not hold true. Our ν-ϕ couplings are also not constrained by leptonic decays of mesons [312] due to the high off-shellness of the virtual neutrino states (see sections 4.4 and 5.5.2 for discussions of off-shellness). However, several constraints from accelerator, atmospheric, and solar neutrino experiments play an important role, since our enhanced decays take place on all energy scales.

The current noncosmological experimental constraints on the neutrino mass eigenstate lifetimes for a normal nondegenerate mass hierarchy are [313, 314]

$$\frac{\tau_3}{m_3} \geq 9.3 \times 10^{-11} \frac{\mathrm{s}}{\mathrm{eV}}, \tag{5.3.5}$$

$$\frac{\tau_2}{m_2} \geq 7.2 \times 10^{-4} \frac{\mathrm{s}}{\mathrm{eV}}, \tag{5.3.6}$$

at 99% CL. Here, the first bound stems from atmospheric and long-baseline neutrinos, while the second one comes from solar neutrinos.

It is important to notice that these bounds only apply to invisible neutrino decays, i.e., the decay products are assumed not to cause significant signals in the detectors under consideration [313, 314]. Especially the strong solar neutrino limit (5.3.6) is based on the assumption of sterile daughter neutrinos or active daughter neutrinos of substantially lower mass than the parent neutrinos (see, e.g., [311, 315] for discussions of this assumption). Under this condition of invisible decays, the constraints (5.3.5) and (5.3.6) enforce our off-diagonal couplings to be

$$g_{3j} \lesssim 4 \times 10^{-1} \quad \text{and} \quad g_{21} \lesssim 8 \times 10^{-4}, \tag{5.3.7}$$

which translates into bounds on Λ_G via $g_{ij} = m_j/\Lambda_G$ [316]. However, we must stress that these constraints do not necessarily apply to our neutrino mass model. This is because the predicted invalidity of the cosmological neutrino mass bounds (see section 5.2.1) still leaves open the window for degenerate neutrino masses. In this case, an active daughter neutrino would carry the full energy of the parent neutrino and thus could obscure the typical signatures of decay [273]. This would apply in particular to the solar neutrino decay of ν_2 into ν_1, since both these mass eigenstates have large ν_e proportions [311].

Our enhanced decays of the heavier into the lightest neutrino state would lead to the dominant presence of a distinct flavor composition in long-traveling extraterrestrial neutrino fluxes. As observed in [247], with an assumed neutrino flux of energy $E = 10$ TeV coming from a source at distance $D = 100$ Mpc, a strong decay effect would be visible if $\Gamma_i(m_i/E) \gtrsim D^{-1}$. Taking into account (5.3.2), this means that couplings of

$$g_{ij} \gtrsim 1 \times 10^{-7} \left(\frac{50 \text{ meV}}{m_i}\right) \left(\frac{E}{10 \text{ TeV}}\right)^{1/2} \left(\frac{100 \text{ Mpc}}{D}\right)^{1/2} \tag{5.3.8}$$

would already lead to observable effects. Our constraints on the couplings (5.3.7) therefore imply that the expected deviation from an equal neutrino flavor ratio, $(\nu_e : \nu_\mu : \nu_\tau) = (1 : 1 : 1)$, could be measured in extraterrestrial neutrino fluxes detected, for example, with the IceCube experiment.

With the three-year data of the IceCube experiment, an equal flavor composition is excluded at 92% CL by one analysis [317], and the best fit is obtained for a ratio $(\nu_e : \nu_\mu : \nu_\tau)$ of $(1 : 0 : 0)$. Another analysis gives the best-fit ratio of $(0 : 0.2 : 0.8)$ but also an equal flavor ratio or a ratio of $(1 : 0 : 0)$ are not significantly excluded [318]. A dominance of ν_μ and ν_τ over ν_e would match our decay picture in case of an inverted mass hierarchy, while a normal mass hierarchy would imply a dominance of ν_e [319]. In order to obtain significant results for a deviation from an equal flavor ratio, more data is needed. If our model is true, equal flavor ratios are ruled out because the state ν_2 with nearly equal flavor content cannot be the lightest mass state [320]. Therefore, our predicted enhanced neutrino decays can probably be verified in the near future.

If enhanced neutrino decays will be observed, this modification of neutrino physics will also play an important role in modeling supernova (SN) events.

Most crucially, the enhanced decays would imply that the neutrinos from SN 1987A [321, 322] have decayed into the lightest mass eigenstate on their way to Earth. Since SN 1987A was about $D = 50$ kpc away from Earth [323] and the neutrino flux energy was in the range of $E = 10$ MeV [324], decay effects would have already occurred for off-diagonal couplings of (5.3.8)

$$g_{3j} \gtrsim 4 \times 10^{-9} \quad \text{and} \quad g_{21} \gtrsim 2 \times 10^{-8}, \tag{5.3.9}$$

where we again assume the lowest possible normal-ordered neutrino mass scheme. If our proposed neutrino decays are mediated by off-diagonal couplings in the range given by (5.3.7) and (5.3.9), the analyses of the original neutrino spectra of SN 1987A and specifically the constraints on the flavor composition of the observed neutrinos [325] have to be substantially modified [326]. This decay scenario is not excluded so far because the SN 1987A data restricts only the lowest mass eigenstate to be stable, $\tau_1/m_1 > 10^5$ s/eV [327], and the simulations of SN explosions still exhibit many uncertainties [328–332].

In this context, we note that neutrino decay can in principle be probed through the future detection of the SN relic neutrino flux, i.e., the redshifted neutrino background from all past supernovae. In [332] it was argued that our model's complete decay scenario can potentially enhance the SN relic neutrino background density up to the current experimental detection bound, so that its measurement might be feasible with future experiments.

5.3.2 Particle Emission in Stellar Neutrino Processes

As we pointed out in sections 3.3 and 5.2.3, the masses of the ϕ bosons in our neutrino mass and DA models are either determined by the low-energy scale $\Lambda_G \sim$ meV–eV for η_ν or are negligibly small or zero for ϕ_k. Thus, these new particles could in principle be created in every high-energy neutrino process, i.e., at energies much larger than the scale Λ_G. Analogous to pion bremsstrahlung [333], the ϕ radiation spectrum would be continuous and would have a peak at small energies above m_ϕ. Furthermore, by analogy with the disoriented chiral condensate in QCD [334], one might also have such condensates in high-energy neutrino processes. These condensates could finally decay into the real vacuum by emission of coherent low-energy ϕ bosons.

One might naively expect that the production of ϕ particles could have a strong impact on high-energy stellar neutrino processes. For example, the diagonal coupling of conventional majoron-like BSM particles to neutrinos is severely restricted by SN energy loss [335–338]. However, introducing the ϕ bosons does not open up a new emission channel. In high-energy processes, such as the ϕ production in stars, the ν-ϕ coupling is expected to be strongly suppressed because of the very low compositeness scale of the ϕ bosons and the high-energy softening of the gravitational vertex. Therefore, none of the current astrophysical constraints hold true for our couplings of the ϕ bosons to neutrinos. Notice that similar arguments also evade astrophysical bounds on the two-photon couplings of the ϕ particles, as we will further discuss in section 5.5.1. In both cases, the suppression is due to the large momentum transfer in the processes under consideration. Thus, these arguments do not apply to the neutrino decays considered in the previous subsection.

In the minimal one-neutrino scheme of our DA model, the η_ν boson would immediately decay and thus be generically unobservable in astrophysical processes if this boson is heavier than the neutrinos. The hidden numerical parameter of the chiral gravitational anomaly in (4.3.4) and the exact scale Λ_G are unknown, implying that the absolute mass of η_ν cannot be predicted. If the mass is lower than the lightest neutrino mass, this new degree of freedom could potentially be detected in future experiments. In our multi-flavor neutrino mass model, the heavy η_ν boson will always decay within timescales of (sub)picoseconds into the lighter ϕ_k bosons because $\tau \sim \Lambda_G^{-1} \sim 10^{-12}$ s for $\Lambda_G \sim$ meV, which makes the massive η_ν boson undetectable in astrophysical processes.

5.4 Implications for Gravity Measurements

5.4.1 Chern-Simons Modified Gravitational Waves

The neutrino-composite η_ν boson is not only relevant for neutrino astrophysics as discussed in the previous section, but it also affects gravitational physics. In the current subsection, we will discuss the implications of the η_ν particle for CP violation in gravity, which are given in both our neutrino mass and DA models. As we extensively discussed in chapters 3 and 4, the pseudoscalar η_ν plays the role of the axion for the gravitational analog of the QCD θ-term [49]. The gravitational chiral anomaly generates the coupling $\eta_\nu R\tilde{R}$ in the Lagrangian, which relaxes the gravitational θ-term to zero, preventing the manifestation of CP violation by the gravitational vacuum. This effect is fully analogous to the QCD axion scenario, in which the pseudoscalar axion suppresses strong CP violation by the vacuum θ-angle.

The promotion of the gravitational θ-angle to a dynamical field η_ν provides us with CP-violating effects in out-of-vacuum processes, such as gravitational waves (GW). For example, backgrounds with time-dependent η_ν can modify GW propagation as suggested in [339] in the context of Chern-Simons modified GR. More precisely, the vacuum is promoted to a “birefringent” medium that induces different polarization indensities of GWs (see [340] for a review).

So far, the GW events observed by the LIGO and Virgo detectors have yielded no constraints on either the dynamical or the nondynamical Chern-Simons modifications of GR [340–343]. This is because there is a lack of related theoretical predictions for black-hole merger and neutron star signals, making such studies not yet feasible. However, first analyses of simplified Chern-Simons modified black-hole systems imply that our predicted deviations from GR are beyond current experimental sensitivity on astrophysical scales [344] (see [345] for a review). Nevertheless, it is interesting to notice that the gravitational out-of-vacuum effects predicted by our models are in principle testable by GW detectors.

5.4.2 Gravity-Competing Short-Distance Forces

In addition to testing gravity on astrophysical scales, potential deviations from GR are also under examination at short-distance scales, i.e., in the laboratory. The connection between the scale of the neutrino Compton wavelength and the experimental short-distance frontier of gravitational force measurements was already established in the past [141]. In the previously considered scenario, the physics that set the neutrino mass simultaneously modified Newton's law due to large extra dimensions [346]. Our gravitational low-energy models, which generate neutrino masses of the order of the scale of nonperturbative gravity Λ_G, offer another way of realizing a connection between the neutrino mass and the current experimental frontier of short-distance tests of gravity [347]. Now it is natural to ask whether we can predict any observable corrections to Newtonian gravity at distances shorter than Λ_G^{-1}.

In general, it would be hard to make a concrete prediction due to the lack of knowledge of a direct relation between the gravitational topological vacuum susceptibility (4.2.1) and the modification of the graviton propagator. Therefore, our minimal neutrino mass model can only yield gravity-competing forces in the hypothetical case of weakly gauged axial lepton number, e.g., in form of $B-L$ local symmetry. In this case, the neutrino condensate would trigger a mass for the $B-L$ gauge boson. This could result in some interesting experimental prospects by looking for signatures of the $B-L$ force in short-distance measurements, such as the ones presented in [347]. The existence of a gravity-competing force in form of a gauged $B-L$ symmetry was originally suggested in the context of large extra dimensions [348]. In the present context, however, the allowed parameter range is different.

In our DA scenario, one concrete prediction emerges that is directly tied to the generation of the up-quark mass by the neutrino condensate. Indeed, we predict a new force mediated by the Higgs-like excitation(s) of the neutrino condensate, which describe(s) small fluctuations of its absolute value. For illustrating this point, it is enough to consider one such mode, which we shall denote by σ_ν. Then, the expansion of the neutrino condensate around its VEV can be written as

$$\bar{\nu}_L \nu_R = (v + \sigma_\nu) e^{i\eta_\nu / v}. \tag{5.4.1}$$

Due to the UV-softening of the gravitational vertex (4.3.8), the coupling of σ_ν to a constituent up quark inside the proton is suppressed by powers of the ratio (v/m_p), where m_p is the proton mass. However, at the same time it is enhanced by the same parameter ξ that is responsible for the generation of the relatively large up-quark mass. Hence, the effective coupling to a proton, up to an unknown coefficient of order one, is expected to be given by $\xi(v/m_p)^\beta$, where β is a positive number that parameterizes the softening of the gravitational vertex in high-energy processes. Notice that for processes at energies $E \ll m_p$ such that $\xi(v/E)^\beta \gtrsim 1$, this large effective coupling would render impossible any precise computation beyond tree level. Therefore, such a computation could not be trusted from an effective field theory point of view.

At distances r around or larger than the Compton wavelength of the σ_ν boson, its exchange will result in a gravity-like potential between two protons of the order $V(r)_\nu \sim \xi^2(v/m_p)^{2\beta}(e^{-rm_{\sigma_\nu}}/r)$, where m_{σ_ν} is the mass of the σ_ν boson. Since $\xi\, v \sim m_u$ and $v \sim m_\nu$, we can rewrite the new force in terms of the quark and neutrino masses as

$$V(r)_\nu \sim \left(\frac{m_u}{m_\nu}\right)^2 \left(\frac{m_\nu}{m_p}\right)^{2\beta} \frac{e^{-rm_{\sigma_\nu}}}{r} \,. \tag{5.4.2}$$

Putting this into the conventional expression for gravity-competing forces,

$$V(r) = -G_N \frac{m_1 m_2}{r}\left(1 + \alpha e^{-r/\lambda}\right), \tag{5.4.3}$$

we obtain for two protons, $m_1 = m_2 = m_p$, the parameters $\alpha \sim 10^{128-58\beta}$ and $\lambda = m_{\sigma_\nu}^{-1}$. For $m_{\sigma_\nu} \sim v \sim 0.1$ eV corresponding to $v^{-1} \sim \mu$m, the existing experimental measurements [349] put the bound $\beta \gtrsim 2.1$. The case $\beta = 2$, which corresponds to a simplest minimal suppression that one can obtain based on very general scaling arguments, is compatible with the current bounds and can lead to observable effects for a slightly higher mass of σ_ν.

The interesting message we would like to take from here is that one place to look for the effects of the σ_ν boson is in searches for a new force at micron and sub-micron distances, which can exceed the strength of Newtonian gravity by many orders of magnitude. The force is highly sensitive to the parameter β, which we cannot predict, but for values of $\beta \simeq 2$ such a force can be just within the reach of the planned improved measurements [350, 351]. The modification of Newton's law must appear as a threshold effect, which should diminish both above and below the scale $r \sim m_{\sigma_\nu}^{-1}$. As it is clear from (5.4.2), for $r \gg m_{\sigma_\nu}^{-1}$ the force diminishes exponentially. Instead, for distances $r \ll m_{\sigma_\nu}^{-1}$ it is expected to diminish as a power-law, e.g., for $\beta = 2$,

$$V(r)_\nu \sim \left(\frac{m_u m_\nu}{m_p^2}\right)^2 v^4 r^3 \,, \tag{5.4.4}$$

due to the decoupling of IR physics from short-distance effects.

Notice that, since the σ_ν boson has a dominant coupling to the up quark, the resulting force depends on the number of up quarks in the source and thus is predicted to be *isotope-dependent.* Namely, the coupling to a proton is by a factor of two larger than the coupling to a neutron. The force continues to be isotope-dependent even in a nonminimal scenario in which also the down-quark mass is generated from the neutrino condensate, since the relative strength of the coupling to up and down quarks is set by the ratio of the quark masses. Thus, in this case the coupling to the down quark is larger and correspondingly the coupling to a neutron is stronger than to a proton.

5.5 Implications for Particle Physics Experiments

5.5.1 Shining Light Through Walls

After having examined our various model predictions for cosmology, astrophysics, and gravity in the previous sections, we now come to the last phenomenological section devoted to particle physics experiments. Both our neutrino mass and DA models allow for several new signals in different terrestrial experiments. One of these predicted low-energy processes is the conversion of photons into ϕ bosons in the background of a magnetic field. This process is analogous to the SM process of two-photon conversion into neutrino-antineutrino pairs via a virtual electron and a Z boson, when replacing the neutrino-antineutrino pair by a ϕ boson and the weak interaction by the soft gravitational vertex.

In our minimal DA scenario, the only ϕ boson is the graviaxion consisting predominantly of η_ν, wherefore we here call this process "graviaxion-to-photon" conversion. The direct contribution from η' into the graviaxion is very strongly suppressed in the DA model. Therefore, as mentioned above, the dominant communication of the graviaxion to the photon is through virtual charged particles, i.e., quarks and charged leptons, to which the graviaxion couples through the soft gravitational vertex. If we assume a maximally generic form of such a vertex, the least suppression factor we get can be estimated to be $(v/m_e)^3$, where m_e is the electron mass. For $v \sim 0.1$ eV, this imitates the two-photon coupling strength of a standard invisible axion with a decay constant of order 10^{10} GeV.

This ϕ-photon conversion predicted by both our neutrino mass and DA models is interesting for future experimental searches with Shining Light Through Walls type experiments (see [352] for a review). These experiments are searching for ALPs with a current sensitivity of $\sim 10^7$ GeV for ALP masses of $\lesssim 10^{-3}$ eV [353]. The next generation of experiments will reach a sensitivity of $\sim 10^{11}$ GeV for very light ALPs with masses of $\lesssim 10^{-4}$ eV [354, 355]. Consequently, the heavy graviaxion in our minimal DA model is out of experimental reach, but the light and massless ϕ particles in our multi-flavor neutrino mass model can be probed by these future ALP searches.

Notice that the above estimate for the strength of the effective coupling is only valid for very low energy ϕ-photon processes. In high-energy processes, for example, in the ϕ production in stars, the coupling is expected to be much stronger suppressed because the low compositeness scale of the ϕ bosons and the high-energy softening of the gravitational vertex (see section 5.3.2). Therefore, we should not expect the standard axion-type correlation between the predictions for Shining Light Through Walls and solar axion experiments (see, e.g., [356]).

5.5.2 Majorana Versus Dirac Neutrino Nature

Up to now, our analysis has focused on Dirac neutrinos for the sake of simplicity. In this section, we want to approach the question whether our models predict either the Dirac or the Majorana nature of neutrinos. Unfortunately, we cannot make a definite prediction because both our neutrino mass and DA models can work for Dirac and Majorana neutrinos. Indeed, even if only one active LH neutrino ν_L is introduced in the DA model, there still exists a chiral symmetry anomalous under gravity, which acts on ν_L and thus leads to neutrino condensation. Since ν_L is a part of a lepton doublet, $L \equiv (\nu_L, e_L)$, the neutrino condensate $\langle \nu_L C \nu_L \rangle$ transforms as a triplet under the weak $SU(2)$ symmetry. Nevertheless, an effective doublet can be composed by convoluting it with a doublet quark condensate, and the up-quark mass can still be generated through the following operator:

$$(\bar{Q}_L^j u_R)(L_j C L_m)(\bar{Q}_L^m u_R)\,, \tag{5.5.1}$$

where $j, m = 1, 2$ are the indexes of the weak $SU(2)$ gauge symmetry and $Q \equiv (u_L, d_L)$ is the quark doublet.

Also our neutrino mass model works for both the Dirac and the Majorana cases, as we pointed out in section 3.3 and will further discuss in section 5.5.3. Therefore, the crucial question is how we can experimentally distinguish between these two possibilities. As mentioned in section 5.3.1, the Dirac and Majorana neutrino cases yield different neutrino decay channels in our model, whose phenomenological implications we will further investigate. Independently of that, neutrinoless double-beta ($0\nu\beta\beta$) decay is commonly expected to provide the most promising opportunity to test the neutrino nature. In the following, we will examine whether our neutrino mass model might modify this decay.

The $0\nu\beta\beta$ transition is the key prediction of the most popular Majorana neutrino mass models based on high-energy seesaw mechanisms, radiative corrections, or large extra dimensions (see section 2.1.2). The nonobservation of $0\nu\beta\beta$ decay has put so far an upper bound on the effective Majorana mass of $\langle m_{\beta\beta} \rangle < (61 - 165)$ meV at 90% CL [357]. Under the assumption that the decay is predominantly mediated by a pure-Majorana SM neutrino, this result corresponds to a 90% CL upper limit on the lightest neutrino mass of $m_{\text{lightest}} < (180 - 480)$ meV [357]. Several future experiments aim to completely explore the allowed Majorana neutrino mass region in case of an inverted mass hierarchy [145]. In total, more than 20 different experiments have been completed, are currently running, or will prospectively search for this decay. The nondetection of $0\nu\beta\beta$ decay would rule out the most popular Majorana neutrino mass models in case $m_{\text{lightest}} \gtrsim 10$ meV [358] and thus provide an indication for either Dirac masses or nonstandard neutrino mass origins.

Notice that not only Majorana neutrinos can mediate $0\nu\beta\beta$ decay, but also other BSM effects, such as higher-dimensional operators or RH currents [145]. These effects are expected to induce minuscule [359] Majorana neutrino masses due to the Schechter-Valle (black box) theorem [360]. However, for simplicity,

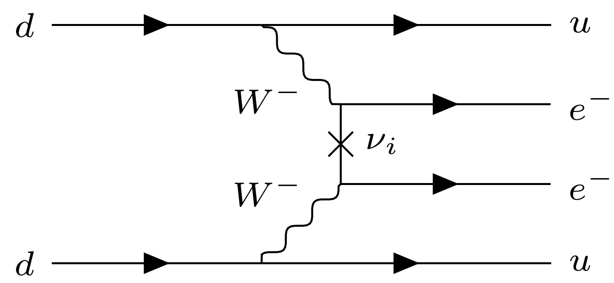

Figure 5.2: Neutrinoless double beta decay, mediated by a light neutrino with a hard Majorana mass (indicated by the cross). This transition was hypothesized to become impossible in our gravitational neutrino model, where the Majorana mass is effective. Below, we will present an argument against this hypothesis.

we will focus in the following on the common expectation that the leading mechanism behind $0\nu\beta\beta$ decay is the exchange of light Majorana neutrinos.

In the context of our gravitational neutrino mass model, it was hypothesized by Arkani-Hamed [361] that neutrino-mediated $0\nu\beta\beta$ decay might become unobservable, because the effective gravitational mass vertex potentially dissolves at the intersection of the off-shell neutrinos in the decay (see Fig. 5.2). In order to check the validity of this hypothesis, let us take a closer look at the theoretical requirements for the neutrino-mediated $0\nu\beta\beta$ transition. This process is allowed if the two off-shell neutrinos can merge in a lepton number violating Majorana mass vertex. The Majorana neutrino propagator $\Delta(x_i)$, sandwiched between two chiral projectors P_L, reads [145]

$$P_L\,\Delta(x_i)P_L = P_L \int \frac{d^4q}{(2\pi)^4}\,\frac{i\,m_i}{q^2 - m_i^2 + i\varepsilon}\,C\,e^{-iqx_i}, \tag{5.5.2}$$

where C is the charge conjugation matrix and q and m_i denote the virtual neutrino momentum and mass. For light neutrinos, m_i^2 can be neglected in the denominator because $m_i^2 \ll q^2 \sim \langle q^2\rangle \sim 1/R^2 \sim (100\text{ MeV})^2$, where R is the nuclear radius. Due to the final state electrons (see Fig. 5.2), the propagator mass m_i is connected to the effective Majorana neutrino mass $\langle m_{\beta\beta}\rangle$ by

$$\langle m_{\beta\beta}\rangle \equiv \left|\sum_{i=1,2,3} U_{ei}^2\,m_i\right|, \tag{5.5.3}$$

where U_{ei} are the usual mixing matrix elements relating the flavor eigenstate ν_e to the mass eigenstates ν_i.

In standard high-energy Majorana neutrino models, m_i is momentum independent and can be pulled out of the integral's numerator (5.5.2). This results in the $0\nu\beta\beta$ decay rate [145]

$$(T_{1/2}^{0\nu})^{-1} = G^{0\nu}|M^{0\nu}|^2\left(\frac{\langle m_{\beta\beta}\rangle}{m_e}\right)^2, \tag{5.5.4}$$

where $G^{0\nu} \propto 1/R^2$ is the phase space factor, $M^{0\nu} \propto R$ the nuclear matrix element, and m_e the electron mass.

According to the mentioned hypothesis by Arkani-Hamed, the effective neutrino masses of our gravitational mechanism should melt for off-shell momenta much larger than the scale Λ_G. In other words, the off-shell Majorana neutrino mass vertex (see Fig. 5.2) would dissolve for high-momentum transfer, which can be modeled by

$$m_i(q^2) = m_i(0) \begin{cases} 1 & \text{for } q^2 < \Lambda_G^2 \\ (\Lambda_G^2/q^2)^\beta & \text{for } q^2 \geq \Lambda_G^2, \end{cases} \tag{5.5.5}$$

or by the approximate form factor

$$m_i(q^2) = m_i(0) \left(\frac{\Lambda_G^2}{q^2 + \Lambda_G^2} \right)^\beta , \tag{5.5.6}$$

where β parametrizes the neutrino mass softening and $\beta = 0.5$ corresponds to the minimal suppression case.

Let us now check how this mass dissolution hypothesis would change the standard $0\nu\beta\beta$ decay rate (5.5.4). Replacing the neutrino mass m_i in (5.5.2) by the form factor (5.5.5) would suppress the neutrino potential in the nuclear matrix element $M^{0\nu}$ [362] by a factor of $\sim \Lambda_G^2 R^2$, while all other parts of the decay rate (5.5.4) would remain unchanged. Thus, the decay rate (5.5.4) would diminish by approximately $\Lambda_G^4 R^4 \lesssim 10^{-32}$ for $\Lambda_G \lesssim$ eV and $\beta = 1$. In other words, this would imply that the off-shell neutrinos in $0\nu\beta\beta$ decay would effectively behave as massless particles, since $m_i(\langle q^2 \rangle \sim 1/R^2) \lesssim 10^{-16}\, m_i(0)$ (5.5.5). Or to put it differently, the $0\nu\beta\beta$ transition would be phase-space suppressed because the phase space is dominated by momenta close to $\langle q^2 \rangle \sim 1/R^2$ but the transition would solely be allowed if q^2 is not much larger than Λ_G^2. Thus, only a negligibly small phase space region would be available for the process to happen, and $0\nu\beta\beta$ decay would be far beyond experimental reach. The importance of such an effect would increase with the off-shellness of the process under consideration. Therefore, the on-shell neutrinos in high-energy neutrino oscillations would be unaffected by such a hypothetical neutrino mass vertex softening and such would be the observed neutrino mass splitting.

In the following, we will present an argument against the mass dissolution hypothesis, which is based on the insight [363] that form factors like (5.5.6) violate unitarity (see, e.g., [364] for similar considerations). In order to understand the argument, notice that the hypothesized neutrino mass modeling (5.5.6) indicates that the neutrino propagator has a physical pole at some momentum $q^2 = m_i^2$ and at the same time behaves as a massless propagator for $q^2 \gg m_i^2$. As follows from a spectral representation argument, this seems to be impossible unless (i) there are additional poles and (ii) at least some of them are ghost-like, i.e., they give a negative contribution to the spectral function. This can be seen from the spectral representation of the propagator $\Delta(q^2)$,

$$\Delta(q^2) = \int_0^\infty ds\, \frac{\rho(s)}{q^2 + s}, \tag{5.5.7}$$

where $\rho(s)$ is a bounded spectral function, which has to be semi-positive definite in the absence of negative-norm states. When equating (5.5.7) with (5.5.2) in position space and inserting the form factor (5.5.6), we see that the nonnegativity of $\rho(s)$ implies that β cannot be positive for $m_i(q^2) \propto q^{-\beta}$. For example, in case of $\beta = 1$, the modified neutrino propagator would split into two parts with opposite sign,

$$\Delta(q^2) \propto \frac{1}{q^2} \frac{\Lambda_G^2}{q^2 + \Lambda_G^2} \longrightarrow \frac{1}{q^2} - \frac{1}{q^2 + \Lambda_G^2}, \tag{5.5.8}$$

which implies that a theory with such a high-energy neutrino mass softening (5.5.6) would propagate additional ghost-like degrees of freedom that cancel the neutrino exchange for values of q^2 relevant for the $0\nu\beta\beta$ transition. In other words, neutrino mass dissolution at high energies appears to be impossible in a ghost-free theory, and unitarity enforces all (Majorana or Dirac) neutrino masses in our gravitational model to be present at every energy scale.

Here, there are two crucial points to notice. First, the validity of the mass dissolution hypothesis or the counterargument above is still under discussion [235] (see section 4.4 for related arguments). Second, while unitarity seems to forbid the mass dissolution of *elementary* neutrinos at high energies, the interaction vertexes of *composite* ϕ bosons should definitely be suppressed for large momentum transfer. As we discussed in sections 5.3.2 and 5.5.1, the couplings of ϕ bosons to neutrinos, photons, and other particles in high-energy processes are expected to be strongly suppressed because of the very low compositeness scale of the bosons and the high-energy softening of the interaction vertex. Therefore, the majoron-like ϕ particles in our models are not constrained by $0\nu\beta\beta$ decay [365, 366] and leptonic decays of mesons [312], which usually put strong bounds on the emission of majoron-like particles by the virtual intermediate neutrino states.

To conclude, the arguments above imply that our neutrino mass model supports the standard approach to test the Majorana or Dirac neutrino nature with $0\nu\beta\beta$-decay experiments. The current bound on the lightest Majorana neutrino mass, $m_{\text{lightest}} < (180 - 480)$ meV [357], becomes especially important in our model due to the absence of cosmological neutrino mass limits (see section 5.2.2). While the KATRIN experiment has the potential to soon detect an effective electron neutrino mass scale of $m_\beta \gtrsim 0.5$ eV without any cosmological conflicts, such a large-mass detection would rule out the minimal Majorana scenario of our gravitational neutrino mass model. However, as we will show in the following section, $0\nu\beta\beta$ decay could vanish in the presence of three light sterile neutrinos and therefore in a nonminimal Majorana case of our model.

5.5.3 Light Sterile Neutrinos

Based on the above discussion on the Majorana or Dirac neutrino nature, we now want to move on to different possible mass generation schemes in our gravitational neutrino mass model. Depending on the number of LH and RH neutrino states, we can in principle distinguish three different possible scenarios:

(1) *The pure Dirac case:* If neutrinos are distinct from their antiparticles, Majorana mass terms are not allowed. In the minimal scenario of three neutrino flavors with zero Yukawa couplings to the SM Higgs doublet, the gravitationally induced Dirac masses break the $SU(6)$ symmetry of the six Weyl fermions, which pair up to three massive Dirac fermions.

(2) *The pure LH Majorana case:* If neutrinos are identical to their antiparticles, the most natural option would be to consider the SM without RH neutrinos. In this case, neutrinos are massless in the SM due to the impossibility of Dirac or LH Majorana masses provided by the SM. Our gravitational mechanism generates LH Majorana mass terms, which violate isospin by one unit but are allowed after EW symmetry breaking.

(3) *The mixed case:* If neutrinos are identical to their antiparticles and we extend the SM by RH neutrinos, our gravitational mechanism is in general expected to give rise to Dirac masses m_D as well as LH and RH Majorana masses, m_L and m_R. Since the chiral gravitational anomaly triggers all the different neutrino condensates close to the scale Λ_G, the resulting active and sterile neutrino flavors naturally have masses of order $m_D \sim m_L \sim m_R \sim \Lambda_G$ and thus are substantially mixed.

While in the previous sections we focused on case (1) for simplicity, let us in the following consider the phenomenological consequences of case (3). In other words, let us go beyond the three-flavor scenario by assuming the possible existence of additional sterile neutrino states. Postulating sterile neutrino oscillations has the potential to resolve several different anomalies observed in short-baseline (SBL) neutrino experiments (see [142, 367] for recent reviews). While simple attempts to resolve these anomalies with only one light sterile neutrino are not sufficient to simultaneously explain all different experimental data sets, a $(3 + 3)$ scenario with three light sterile neutrinos yields a high compatibility among all data sets and thus is favored by experiment [368].

However, light sterile neutrinos usually cause severe cosmological discrepancies, which get increasingly problematic the more states are added: in case of significant mixing with the active neutrinos, the sterile states would be copiously produced in the early Universe, resulting in a conflict with cosmological bounds on the primordial radiation density and neutrino masses [369]. For the sterile neutrino solution of the SBL anomalies to survive, usually additional mechanisms are required to suppress the sterile neutrino abundance in the early Universe, such as, large primordial neutrino asymmetries or secret interactions among sterile neutrinos (see, e.g., [370–373]). While these modifications drastically complicate the respective models, all cosmological conflicts vanish if the neutrinos gain their masses through our gravitational mechanism.

As explained above, the case (3) of our neutrino mass model implies naturally light and substantially mixed active and sterile neutrinos in the late Universe, as required for resolving the SBL anomalies. In the early Universe, the relic (active) neutrinos are massless and thus have vanishing couplings to their sterile partners (see section 5.2.1). These couplings only turn on abruptly when approaching the late-time phase transition, analogous to, e.g., axion couplings [282]. Therefore, the sterile neutrinos are not produced in the early Universe and do not stand in conflict to the observed number of effective relativistic degrees of freedom, which usually rules out already one fully thermalized sterile neutrino at more than 99% CL [374]. After the cosmic neutrino phase transition in the late Universe, the LH and RH neutrinos mix due to the effective Dirac mass term. On top of that, effective late-time LH and RH Majorana masses arise, so that immediately after the transition, the relic neutrino background contains both active and sterile states with parametrically different masses. Within timescales of orders of (sub)picoseconds, the relic neutrinos then rapidly decay into the lightest neutrino mass eigenstate and hence evade the standard cosmological bounds on the sum of the neutrino masses (see section 5.2.1 for more details).

This elimination of light sterile neutrino problems with cosmology continues to hold true in case the sterile states are not the RH (Majorana) partners of the active neutrinos, but additional sterile (Dirac) neutrino flavors. However, one might argue that a hint towards the Majorana nature of neutrinos exists in case sterile neutrinos will be discovered, since the most plausible sterile neutrino candidates are the RH partners of the active LH neutrinos. If both RH and LH neutrinos exist and neutrinos are Majorana particles, our gravitational mechanism would generate both active and sterile neutrino masses in the range of meV to eV. Therefore, this case (3) of our model would naturally provide sterile-active mass differences in the range required for resolving the SBL anomalies. Note that astrophysical bounds, e.g., from SN 1987A neutrinos, restrict the lightest neutrino to be active in our model (see section 5.3.1).

Future experiments will further probe the hypothesized sterile neutrino solution to the SBL anomalies (see, e.g., [375] and references therein). As mentioned in section 2.1.2, in case of growing evidence for light sterile neutrinos, high-scale seesaw mechanisms would get into difficulties because they usually predict sterile neutrinos of much larger masses. Also scenarios allowing for light sterile neutrinos, such as, the low-scale seesaw proposal [143], could not provide a satisfactory explanation of such an observation due to severe conflicts with cosmology. Interestingly, if we interpret our model's case (3) as an "IR-completion" of this experimentally motivated low-scale seesaw proposal, all possible cosmological conflicts are immediately evaded. Moreover, our gravitational mechanism can rectify the proposal's lack of a theoretically compelling reason to assume a small Majorana mass scale, tiny Yukawa couplings for generating a small Dirac mass scale, and the coincidence of having those two scales numerically so close [143]. In our model, this coincidence is explained by the common gravitational origin of all neutrino masses, which are naturally close to the infrared scale Λ_G. Thus, a detection of light sterile neutrinos might provide a hint towards our gravitational neutrino mass mechanism, since it ensures perfect compatibility with cosmology and can naturally account for small active-sterile neutrino mass splittings.

At this point, it is important to notice that light sterile neutrinos would strongly distort the parameter space for $0\nu\beta\beta$ decay [376]. In the presence of three light sterile neutrinos, the contribution to the $0\nu\beta\beta$ transition from the heavy, mostly sterile and the lighter, mostly active SBL neutrino states would completely cancel if $m_L = 0$ and $m_R \ll 1$ MeV [143]. Notice that this prediction only holds true in our model's case (3) with zero or very small LH Majorana masses. In this case, neutrinos could still be Majorana particles with an effective electron neutrino mass scale of $m_\beta \gtrsim 0.5$ eV (see section 5.5.2).

Before finishing this section, let us comment on the pros and cons of different possible mass scales of the hypothetical sterile neutrino species. First, high-scale seesaw mechanisms can explain the observed matter-antimatter asymmetry in our Universe: they can account for leptogenesis by spontaneously generating a lepton asymmetry that converts into a baryon asymmetry via SM sphaleron processes [377]. However, it has been argued that the absence of new particles between the EW and Planck scales, such as heavy RH neutrinos, might be useful for ensuring the Higgs mass stability against radiative corrections [378–380]. Second, intermediate-scale seesaw mechanisms can provide a DM candidate in the early Universe: a keV-scale sterile neutrino [369]. However, similar to low-scale seesaw mechanisms, the sterile neutrino mass scale is hard to motivate theoretically. Third, one attractive feature of eV-scale sterile neutrino models, in addition to the phenomenological aspects mentioned above, is that SN nucleosynthesis can be explained by resonant oscillation of active into light sterile neutrinos [381]. For a concise overview of the advantages and disadvantages of different sterile neutrino mass scales, see, e.g., Fig. 3 in [382].

5.5.4 Flavor-Violating Processes

So far, our analysis has mainly focused on the neutrinos and the up quark. However, other charged fermions can be easily incorporated by adding additional fermion legs to the effective gravitational vertex (4.3.8), as we briefly mentioned in sections 4.3 and 5.1. Such a vertex will generate an additional contribution to the masses of all the fermions once the gravity-induced fermion condensates are taken into account. This raises the question whether we can generate the masses of other light charged fermions entirely via the mechanism considered in this thesis, as an alternative of generating their masses from the coupling to the Higgs doublet. However, since all the effective masses generated through the neutrino condensate are not present in the early Universe before CMB formation, our effective mass generation mechanism can only account for the entire masses of neutrinos, up and down quarks, while all other fermions need to have additional nongravitational mass sources.

Without taking extra care, the gravitational corrections to the masses will naturally be of the order of the neutrino masses. A phenomenologically interesting possibility from the point of view of flavor physics opens up in case when these mass contributions are not diagonal in the eigenbasis of the SM Higgs Yukawa couplings. In such a case, new flavor-changing processes emerge from the IR neutrino sector. The current subsection is devoted to examining the appearance of such processes in both our neutrino mass and DA models.

As we will see in the following, flavor-violating processes are absent in the minimal DA scenario but present in the multi-flavor scheme of our neutrino mass model. Remarkably, even if the IR flavor violation at the scale of the neutrino masses is of order one, such a possibility can nevertheless be fully viable phenomenologically and potentially testable. This may come as a surprise, since generating masses from sources other than a single Higgs condensate is normally associated with severe problems, such as flavor-changing neutral currents. In our case, the role of the second Higgs doublet with a tiny VEV is played by the neutrino condensate. The reason why this condensate *a priori* is not causing the usual problems, such as flavor-changing neutral currents mediated by the exchange of a σ_ν boson (see section 5.4.2), is because its compositeness scale is extremely low. Even if the σ_ν boson has order-one flavor-nondiagonal couplings, it decouples very efficiently from the high-energy processes. Correspondingly, the contribution of the neutrino composites into the high-energy flavor-changing processes, such as e.g., $K^0 - \bar{K}^0$ transitions or $\mu \to e + \gamma$ decays, is small, but can be potentially interesting for future measurements.

Let us now estimate the strength of these flavor-violating neutral currents. From the point of view of such flavor violation, the story effectively reduces to the introduction of additional Higgs doublets, which are the composites of the LH lepton doublets $L^\alpha \equiv (\nu_L^\alpha, e_L^\alpha)$ and the RH neutrinos ν_R^α, where $\alpha, \beta = 1, 2, 3$ are generation (family) indexes. In order to understand the essence of flavor violation, it is enough to consider only one such doublet, $\equiv (\bar{L}^\alpha \nu_R^\beta)$. Notice that the effective doublet is not necessarily diagonal in family space.

The neutral component of this composite doublet is the neutrino bilinear, which develops a VEV. Expanding around its VEV, we can write $\bar{\nu}_L \nu_R = (v + \sigma_\nu) e^{i\eta_\nu / v}$, where σ_ν describes excitations of the absolute value and plays the role analogous to the neutral Higgs particle, h_0. If the couplings of h_0 and σ_ν to quarks of the same charge are not diagonal in the mass-eigenstate basis, there will be flavor-changing neutral currents mediated due to their exchange.

Absence of Flavor Violation in Minimal DA Model

In order to trace the origin of flavor violation more explicitly, let us first consider our minimal DA model and see that it is not leading to flavor violation. In this minimal scheme, it is enough to consider the case in which only one neutrino transforms under the $U(1)_{\mathrm{PQ}}$ symmetry, for instance, the ν_τ neutrino.

Consider a gravity-generated coupling,

$$\frac{1}{\Lambda_G^2}(\bar{u}_R Q_L)(\bar{L}^3 \nu_{\tau R}) = \frac{1}{\Lambda_G^2}(\bar{u}_R u_L)(\bar{\nu}_{\tau L}\nu_{\tau R}) + \frac{1}{\Lambda_G^2}(\bar{u}_R d_L)(\bar{\tau}_L \nu_{\tau R}), \qquad (5.5.9)$$

where $Q \equiv (u_L, d_L)$ is the first-generation LH quark doublet. For the purpose of the discussion of flavor conservation, the function f of the invariants introduced in (4.3.8) is not important and we drop it for simplicity. The anomalous PQ symmetry in this case can be identified as the chiral symmetry acting on u_R and ν_R species only,

$$u_R \to e^{i\alpha} u_R, \quad \nu_R \to e^{i\alpha} \nu_R. \qquad (5.5.10)$$

This symmetry is incompatible with the Yukawa couplings of u_R and ν_R to the Higgs doublet H. Correspondingly, unlike the rest of the fermions, the up quark and the ν_τ neutrino are not getting any mass from the VEV of the Higgs.

In such a case, the couplings of both the neutral Higgs h_0 as well as of the σ_ν are diagonal in the mass-eigenstate basis and no flavor-violating neutral currents appear. Notice that the last term in (5.5.9) can contribute to the decay of the τ lepton into a pion and a neutrino, but since the vertex is strongly suppressed at high energies, the rate is expected to be negligible. For instance, already for a suppression by a factor of v^2/m_τ^2, the rate is way beyond current experimental sensitivity. This suppression of an effective vertex in high-energy processes is the main reason for making this new IR physics compatible with present experimental bounds.

Presence of Flavor Violation in Neutrino Mass Model

Let us now turn to the generic nonminimal case of our neutrino mass model, in which all three generations are involved in the effective gravitational vertex. Before illustrating in details, let us summarize the story. As mentioned above, the flavor-violating neutral currents will appear if the Yukawa coupling matrixes of the σ_ν bosons and the h_0 are not diagonal in the fermion mass eigenbasis. In such a case, it is useful to split the potential flavor-violating contributions into the ones mediated by the SM neutral Higgs h_0 and the ones mediated by the σ_ν bosons. Note that we will neglect additional contributions from some of the ϕ_k pseudoscalars, since they are essentially similar to the ones of σ_ν.

Both contributions from h_0 and σ_ν are suppressed, but because of different reasons: The Higgs-mediated flavor violation is typically suppressed by a factor of $\delta m_{\alpha\beta}^2/|m_\alpha - m_\beta|^2$, where m_α are the fermion mass eigenvalues coming from the Higgs Yukawa couplings and $\delta m_{\alpha\beta}$ is the off-diagonal mass generated by the neutrino condensate. The flavor violation mediated by σ_ν is universally suppressed due to the suppression of the effective gravitational vertex in high-energy processes. We assume here that this suppression goes as powers of v^2/E^2, although it could in principle be stronger.

All the above can only take place if gravity violates flavor, that is, if gravity generates off-diagonal effective couplings for the σ_ν bosons in the basis in which

Higgs Yukawa couplings are diagonal. Even though there is no way of predicting this *a priori*, we can perform a useful parameterization of this breaking.

The generation of off-diagonal couplings by gravity can be explicit or spontaneous. Since the fermion flavor group is not anomalous with respect to gravity, the explicit breaking must come from other quantum gravity effects, which we can only parameterize. Spontaneous breaking is simpler to visualize. For spontaneous generation, it is necessary that the condensates of charged leptons and quarks are off-diagonal in the basis in which the Higgs Yukawa couplings are diagonal. This depends on the minimization of the effective potential for these order parameters, and it is easy to come up with prototype potentials that would result in disoriented condensates in flavor space.

Quark-Flavor Violation

In the following, we will estimate the flavor violation in an example of the down-quark sector. The effective Yukawa coupling matrixes are

$$(V_h + h_0) g_{\alpha\beta} \bar{d}_L^\alpha d_R^\beta + (v + \sigma_\nu) g^\sigma_{\alpha\beta} \bar{d}_L^\alpha d_R^\beta \,, \tag{5.5.11}$$

where $\alpha, \beta = 1, 2, 3$ are flavor indexes and $V_h \sim 100$ GeV is the Higgs VEV. Let us work in the basis in which the SM Higgs Yukawa coupling matrix $g_{\alpha\beta}$ is diagonal. Then, if the down-quark condensate can have off-diagonal values in this basis, $g^\sigma_{\alpha\beta}$ will develop off-diagonal elements. The standard QCD condensate of quarks is diagonal in the mass-eigenstate basis, so the off-diagonal contribution must come from gravity. We do not know how strong such a contribution is, so we can parameterize it as unknown.

Let us consider the $1-2$ transition via the condensate $\langle \bar{d}_L^1 d_R^2 \rangle \equiv \langle \bar{d}_L s_R \rangle$. We shall assume that the condensate as well as the Yukawa matrixes are left-right symmetric. The resulting off-diagonal Yukawa coupling of σ_ν is $g^\sigma_{12} \sim v^{-3} \langle \bar{d}_L s_R \rangle$ and this induces a shift in the off-diagonal mass, $\delta m_{12} \sim v^{-2} \langle \bar{d}_L s_R \rangle$. This generates the flavor-changing neutral currents via exchanges of h_0 and σ_ν.

The currents mediated by h_0 are controlled by the effective off-diagonal coupling to the quarks that h_0 acquires after we re-diagonalize the small off-diagonal mass term, δm_{12}, induced by the neutrino condensate. The new mixing angle is suppressed by the ratio of the this off-diagonal mass to the diagonal mass difference, $\delta m_{12}/(m_s - m_d) \simeq \delta m_{12}/m_s$, so that we obtain $g_{sd} \sim (m_s/V_h)(\langle \bar{d}_L s_R \rangle/(m_s v^2))$. Thus, the $(\bar{s}d)^2$-operator induced by the Higgs exchange has the form

$$(\bar{s}d)^2 \frac{1}{m_h^2} \left(\frac{\langle \bar{d}_L s_R \rangle}{V_h v^2} \right)^2 , \tag{5.5.12}$$

where m_h is the Higgs mass. Even if we assume that the off-diagonal condensate is of the same order as the diagonal one, the condensate must be suppressed by the masses of the quarks relative to the scale v. For example, for

$$\langle \bar{d}_L s_R \rangle \sim v^3 \frac{v}{\sqrt{m_s m_d}} \,, \tag{5.5.13}$$

the operator (5.5.12) is hugely suppressed.

The similar operator generated by the exchange of a σ_ν boson has the form

$$(\bar{s}d)^2 \frac{1}{m_\sigma^2} \left(\frac{\langle \bar{d}_L s_R \rangle}{v^3} \right)^2 \sim (\bar{s}d)^2 \frac{1}{m_\sigma^2} \frac{v^2}{m_s m_d} \,. \tag{5.5.14}$$

Since $m_\sigma \sim v$, this operator looks very strong, but we have to remember that this is an effective interaction valid only at energies below the neutrino mass scale v. Thus, the contribution into high-energy processes, such as $K^0 - \bar{K}^0$ transitions, is additionally suppressed by the ratio of the scales v^2/m_K^2, which gives another factor of order 10^{-20}. Overall, we are down to an effective scale of $(\bar{s}d)^2/(10^{16}\ \mathrm{GeV}^2)$, which although suppressed is stronger than the previous one and lies at the current edge of experimental sensitivity [383].

Lepton-Flavor Violation

Analogously, we can estimate the processes with lepton-flavor violation. Consider a leptonic fragment of the gravitational vertex that involves charged leptons of the first two generations and neutrinos of the third generation, with all other fermion pairs being replaced by their masses and VEVs,

$$\frac{1}{\Lambda_G^5} (\bar{e}_L \mu_R)(\bar{\mu}_L e_R)(\bar{\nu}_{\tau L} \nu_{\tau R}) \,. \tag{5.5.15}$$

We assume that this interaction is written in the basis in which the Higgs Yukawa couplings to the charged leptons are diagonal. If in this basis the condensate $\langle \bar{\mu}_L e_R \rangle$ is nonzero, this results in the following strength of the off-diagonal couplings of the Higgs boson and σ_ν with charged leptons:

$$h_0 \bar{\mu}_L e_R \left(\frac{\langle \bar{\mu}_L e_R \rangle}{V_h v^2} \right) + \sigma_\nu \bar{\mu}_L e_R \left(\frac{\langle \bar{\mu}_L e_R \rangle}{v^3} \right) . \tag{5.5.16}$$

The first coupling at one loop can result in $\mu \to e + \gamma$ decay, whereas the second one into a direct decay of a muon into an electron plus a σ_ν or a neutrino-antineutrino pair. Again, we have to take into account the additional suppression by a factor of $v^2/m_\mu^2 \sim 10^{-18}$, due to the decoupling of IR physics in high-energy processes. This decoupling is the key of putting even a maximal IR flavor violation into a potentially phenomenologically interesting domain.

5.6 Summary and Discussion

In the current chapter, we investigated the numerous predictions of our neutrino mass and DA models for cosmology, astrophysics, gravity, as well as particle and nuclear phenomenology. Concerning the cosmological model predictions, the most interesting aspects only apply to our neutrino mass model with multiple neutrino flavors and thus not to our minimal one-flavor DA model. Only the point (v) of the following list implies intriguing cosmological consequences already in the minimal DA case. For more detailed information on the differences between our minimal model schemes and the combined neutrino mass and DA cases, we refer the reader to the respective sections above.

(i) The symmetry breaking scale Λ_G is fixed by several model-independent, mainly phenomenological requirements to lie within the low-energy range of $\Lambda_G \sim$ meV–eV. Thus, this fundamental new IR gravitational scale is numerically coincident with the DE and neutrino mass scales.

(ii) The phase transition in the cosmic neutrino sector happens after photon decoupling, either instantaneously at temperatures $T_{\Lambda_G} \sim \Lambda_G \sim v \sim m_\nu$ or at lower temperatures $T_{\Lambda_G} \ll \Lambda_G \sim v \sim m_\nu$ in case of a supercooled transition. The latter case could substantially increase the energy density in the cosmic neutrino sector and thus could yield large neutrino masses.

(iii) Cosmological neutrino mass bounds vanish, since the neutrinos (a) are massless until the late phase transition, (b) decay into the lightest eigenstate after the transition, and (c) completely or partially annihilate into massless bosons. Thus, neutrino masses of up to 2.2 eV are still cosmologically allowed and could be detected by KATRIN in the near future.

(iv) In the absence of neutrino asymmetries, the relic neutrino background annihilates, resulting in a neutrinoless Universe. In case of substantial neutrino asymmetries, which are still allowed by our model, up to $\sim 42\%$ of the neutrinos could survive. Due to their large possible mass, these strongly self-interacting neutrinos might be detected by PTOLEMY.

(v) The late phase transition in the neutrino sector can give rise to various topological defects, such as skyrmions, domain walls, and cosmic strings. The predicted new (pseudo)Goldstone bosons $\phi = \{\phi_k, \eta_\nu\}$ as well as the annihilating topological defects yield an additional late dark radiation (dark matter) contribution in our neutrino mass (minimal DA) model.

With regard to astrophysics, both our neutrino mass and DA models yield unusual implications. Note that point (i) in the following list only applies to our neutrino mass model, while point (ii) holds true for both scenarios:

(i) While enhanced neutrino decays in extraterrestrial neutrino fluxes could be observed in future IceCube data, a nonobservation has the potential to rule out our mass model. The predicted enhanced decays also necessitate modified analyses of the original neutrino spectra of SN 1987A.

(ii) The production of ϕ bosons in high-energy neutrino processes, such as star cooling, is expected to be strongly suppressed because of the very low ϕ-compositeness scale and the high-energy softening of the gravitational vertex. Thus, there are no ϕ-ν coupling constraints from, e.g., SN cooling.

Concerning gravity measurements, both our low-energy gravitational models predict deviations from GR, which appear on large as well as small scales:

(i) The promotion of the gravitational θ-angle to the dynamical η_ν-field modifies out-of-vacuum processes in gravity, such as gravitational waves. The coupling $\eta_\nu R\tilde{R}$ predicted by both of our models represents a special instance of dynamical Chern-Simons modified GR in the late Universe.

(ii) Our DA model predicts a new gravity-competing isotope-dependent force among nucleons at (sub)micron distances. If axial lepton number is $B-L$ gauged, the neutrino condensate would provide a mass for the $B-L$ gauge boson, yielding yet another gravity-competing short-distance force.

For particle and nuclear phenomenology, our neutrino mass and DA models yield rather different predictions, as summarized in the following:

(i) The axion-like ϕ bosons couple to photons through the gravitational vertex and virtual charged leptons, with a decay constant of $\sim 10^{10}$ GeV. The lightest ones of these ALPs, which are absent in the minimal DA model, can be detected in future Shining Light Through Walls experiments.

(ii) Both our neutrino mass and DA models are independent of the Dirac or Majorana nature of the neutrinos. For highly off-shell processes, the gravitational vertex becomes strongly suppressed at large momentum transfer while the neutrino masses remain unsuppressed at all energy scales. Therefore, our ϕ-ν couplings are unconstrained by leptonic meson decays, while neutrinoless double beta decay stays unaltered in our models.

(iii) When hypothesizing additional sterile (Majorana or Dirac) neutrino states to resolve experimental short-baseline anomalies, our neutrino mass model naturally allows for small active-sterile mass splittings. Moreover, it resolves all the usual conflicts of light sterile neutrinos with cosmology.

(iv) Our neutrino mass model yields the first low-energy origin of flavor-violating processes beyond the SM, while these are absent in the minimal DA scenario. Generic estimates put the strength of these processes close to the current edge of experimental sensitivity, e.g., of the LHC.

In the remaining part of this section, we want to discuss the role of vacuum energy in our neutrino mass model and emphasize again how this model can be experimentally distinguished from other neutrino mass mechanisms.

As we pointed out in section 5.2.1, the late phase transition in the cosmic neutrino sector could be a strongly delayed second-order transition. In this case, the neutrino sector supercools in the wrong symmetric vacuum state until decaying to the true minimum by releasing a substantial amount of false vacuum energy. The resulting increase of the energy density in the late Universe might yield a possible resolution of the recently reported Hubble-parameter discrepancies. Local measurements of the Hubble parameter H_0 [256] deviate with more than 3σ from the value inferred from CMB observations [244], which can be resolved by an increase of $H(t)$ at a redshift of $z \sim 0.8$ [384].

Our neutrino mass model potentially yields this late enhancement of $H(t)$ if the predicted neutrino phase transition takes place at $z \sim 0.8$. However, in case the neutrino vacuum condensate might be the origin of DE, the phase transition has to happen at larger redshifts $z \gtrsim 5$ in order to be in accordance with observations of baryonic acoustic oscillations. As we mentioned in section 3.3, the reason to possibly consider the vacuum condensate as DE is the surprising numerical coincidence of the scales of DE and the observed neutrino mass splitting, $\Lambda_{\mathrm{DE}} \sim \Delta m_\nu$. Since the neutrino condensate is generated at the energy scale $v \sim m_\nu$, it thus might account for DE without substantial tuning. Even though there is no obvious reason why the neutrino condensate should be more physical than other SM vacuum energy contributions, such as the Higgs condensate, our neutrino condensate is the only one triggered by gravitational effects and is inherently connected to the new IR gravitational scale Λ_G.

Let us now turn to the question how our gravitational neutrino mass model can be experimentally distinguished from other neutrino mass mechanisms. To our knowledge, our mechanism is the only one that intrinsically evades *all* cosmological constraints on neutrino masses. Therefore, under the assumption that the standard cosmological ΛCDM model is valid, the detection of an unexpectedly large absolute neutrino mass scale in upcoming beta-decay experiments would provide a strong hint towards our model. This statement holds true in the absence of substantial neutrino asymmetries, in which case our model yields a late neutrinoless Universe. Notice that this scenario could be falsified by a cosmological neutrino mass detection in the next decade, e.g., by the upcoming DESI or Euclid surveys [271]. In the presence of large asymmetries, a substantial fraction of relic neutrinos might still exist today, but any cosmological mass bound can only apply to the lightest neutrino mass eigenstate.

Another possible experimental hint towards our neutrino mass model could be provided by the detection of light sterile neutrinos in short-baseline experiments. In this case, our model would be the only neutrino mass mechanism that intrisically evades all cosmological conflicts of these sterile neutrinos and could naturally account for the small active-sterile neutrino mass splittings. However, notice that sterile neutrinos are not a prediction of our model, since our gravitational mass mechanism is independent of the number of neutrino flavors. The minimal scenario of our model would actually be a purely LH Majorana case, in which gravity naturally generates small LH Majorana neutrino masses $m_L \sim$ meV–eV, without the need of any new species added to the SM.

To summarize this phenomenological section from a broader perspective, we observe that a very low-scale compositeness can mask new physical effects not less and in some cases even more efficiently than the standard phenomenon of high-energy decoupling. This is a very general message that we believe should be payed more attention to when looking for new physical effects.

Chapter 6

Conclusions and Outlook

In this thesis, we presented a novel class of models beyond the SM. While the most popular BSM models usually focus on high-energy scales, we developed alternative *low-energy* solutions to the neutrino mass and strong CP problems at a new IR gravitational scale, $\Lambda_G \sim$ meV–eV. Thus, we demonstrated that new-physics effects may not only be hidden at very high energy scales, but also in the widely unexplored low-energy sector of particle physics. Our models furthermore differ from the most popular BSM scenarios, since they do not require any hypothetical particle species, high-energy scales, grand unified symmetry groups, or extra dimensions. Instead, after minimally coupling the SM to gravity, we identified a possible origin of neutrino mass generation and strong CP conservation in the form of nonperturbative gravitational effects.

To set up the first cornerstone of this low-energy frontier of model building, we discussed how a neutrino condensate and small neutrino masses emerge from a topological formulation of the chiral gravitational anomaly. We started by recapitulating how a physical gravitational θ-term, analogous to the famous θ-term of strong interactions, can lead to the emergence of a new bound neutrino state η_ν similar to the η' meson of QCD. On this basis, we showed that a neutrino vacuum condensate forms, which can generate small effective neutrino masses. This neutrino mass model can yield both Dirac and/or Majorana masses and allows for the experimentally observed neutrino mass hierarchy.

As the second cornerstone of our novel class of BSM models, we proposed a phenomenologically viable solution of the strong CP problem in which the axion is composed entirely of SM fermions. This Domestic Axion (DA) model is based on the assumption that not only the neutrino masses but also the up-quark mass is spontaneously generated by the neutrino condensate. We demonstrated that the resulting PQ symmetry is an axial symmetry, which acts on the up quark as well as the neutrinos and is spontaneously broken by both the quark and neutrino condensates. Consequently, the axion consists predominantly of the η' meson with a tiny admixture of the η_ν boson. Finally, we presented a strong argument why this low-energy composite axion solution is consistent with chiral perturbation theory.

Even though our two gravitational low-energy models are *a priori* separate scenarios, their combination is especially economical: the solution of the strong CP problem can be connected to the origin of the neutrino masses, without the need for any new species and with a built-in protection mechanism of the axion solution against potential gravitational threats. While the minimal version of the DA scenario requires only a single neutrino flavor that gets its mass from the chiral gravitational anomaly, the minimal version of the neutrino mass model only yields neutrino masses and thus does not address the strong CP problem. The natural as well as beneficiary inclusion of all neutrino flavors and the up quark unifies the two scenarios, which nicely complement each other.

Let us highlight the most important predictions of the combined neutrino mass and axion scenario for cosmology, astrophysics, gravity, and particle phenomenology. First, late cosmology becomes considerably modified due to a post-recombination phase transition in the neutrino sector. Most crucially, neutrino masses can be much larger than permitted by standard cosmology and could be detected in the upcoming KATRIN beta-decay experiment. In addition, the relic neutrino background as well as the predicted topological defects almost completely annihilate into massless bosons. Hereby, a substantial fraction of relic neutrinos can only survive in the hypothetical case of large neutral lepton asymmetries, which are allowed in the framework of our models. On the astrophysics side, the key model prediction is the enhancement of neutrino decays, which is weakly constrained by solar neutrinos, strongly affects supernova neutrino modeling, and is potentially measurable at the IceCube neutrino observatory in the future. Concerning gravity measurements, our models predict different polarization intensities of gravitational waves and a new short-distance force among nucleons with a strength comparable to gravity. With regard to particle phenomenology, we demonstrated that our models provide the first possible low-energy origin of flavor-violating processes beyond the SM. Furthermore, our axion-like neutrino-composite pseudoscalars yield shining-light-through-wall signals, while their production in stellar processes is strongly suppressed because of their low compositeness scale and the high-energy softening of the gravitational vertex. Finally, our low-energy models can yield a natural resolution of the observed short-baseline anomalies without causing the standard conflicts of light sterile neutrinos with cosmology.

Our models demonstrate that gravity can substantially shape the low-energy frontier of particle physics. What began as a basic hypothesis that I proposed at the beginning of my PhD, i.e., generating neutrino masses through a mechanism similar to effective quark mass generation in QCD, has resulted in potential wide-ranging implications for phenomenological aspects and energy scales of BSM physics. During the subsequent years of my PhD, an increasing number of researchers, especially cosmologists, have started to further explore our model predictions and studying possible extensions of our proposed scenarios. This has resulted in several completed, ongoing, and planned projects, which will be briefly sketched in the following three paragraphs.

Motivated by our neutrino mass model, the authors of [269] analyzed some cosmological constraints on late neutrino mass generation and self-interactions. Based on their analysis, we are currently investigating [4] whether the late phase transition predicted by our model might explain several recently observed cosmological discrepancies (see section 5.2.1). These discrepancies between cosmological parameters inferred from early and late Universe data were shown to be partially reduced by time-varying DE and late neutrino masses. In this light, we developed the hypothesis that our model might explain some of these tensions, since it gives rise to (i) late neutrino masses and self-interactions (see section 5.2.1), (ii) dark radiation from the partial annihilation of relic neutrinos and topological defects (see sections 5.2.2–5.2.4), and (iii) a potential late DE emergence or contribution from the neutrino vacuum condensate (see section 5.6). In order to test this hypothesis, we are currently examining whether the tensions between the Planck and KiDS data sets could be explained by late neutrino mass generation, assuming only modest relic neutrino annihilation due to neutral lepton asymmetries. In future studies, we plan to incorporate further aspects of our neutrino mass and DA models, in particular the cosmological impact of the predicted topological defects, whose formation and evolution we examined in the framework of a separate project [3].

In two further collaborations with theoretical cosmologists, we recently started to investigate further cosmological predictions of our neutrino mass model in the presence of neutrino asymmetries. While the first project [6] is devoted to simulating the impact of late neutrino masses and self-interactions on cosmic structure formation, the second project [5] focuses on the predictions of our neutrino mass model for relic neutrino detection with beta-decay experiments. Concerning the latter, let us emphasize that standard neutrino cosmology predicts too low a relic neutrino density on Earth for a near-future detection with KATRIN [288], which motivates the proposal of new experiments, such as PTOLEMY [289] (see section 5.2.2). In the presence of large neutral lepton asymmetries, our nonstandard neutrino cosmology can imply a substantial enhancement of the local relic neutrino density due to the clustering of the strongly self-interacting relic neutrino fluid in the Earth's galactic region.

While the projects above focus on testing our gravitational low-energy models in the "cosmological laboratory", a fourth ongoing project [7] is devoted to exploring possible nonperturbative neutrino mass and axion origins beyond gravity. Since EW instanton contributions to Majorana neutrino masses appear be too small to be experimentally relevant (see appendix C), we recently began to examine nonperturbative mass contributions in gauged flavor theories. If our results turn out to be phenomenologically viable, further projects may involve the investigation of a related flavor solution to the strong CP problem.

To summarize, both previous and ongoing studies demonstrate that our gravitational low-energy models have many unusual implications for various fields of physics, such as cosmological evolution, neutrino decay, gravitational waves, and flavor-violating processes. These can be tested at several currently running and near-future experiments, such as the KATRIN beta-decay experiment, the IceCube neutrino observatory, and the high-energy proton collider LHC. Most importantly, our neutrino mass model opens up the possibility of a near-future detection at KATRIN, which would not be expected based on standard high-energy models. From a wider perspective, our gravitational models open up a new hiding place for BSM phenomena at the low-energy frontier of fundamental physics. As is well known, the common high-energy BSM approach increasingly suffers from a lack of experimental confirmation, despite intensive testing effort during the past decades. If this lack continues, BSM model builders might need to rethink the possible regimes of fundamental new-physics scales. Our low-energy models demonstrate that viable solutions of SM puzzles can also emerge at sub-eV scales and yield a common origin of neutrino masses, the axion, and possibly DE. Therefore, this novel direction of model building can potentially lay the foundation for a new BSM research area.

Appendix **A**

Generic Structure of Gravitational Fermion Condensates

This appendix is devoted to explaining the structure of gravitationally induced fermion condensates, which applies to both our neutrino mass and DA models. To start with, let us first ignore all the SM gauge and Higgs interactions and consider gravity coupled to a certain number N_F of fermion flavors, ψ_i and $\psi_{c\bar{i}}$ with $i, \bar{i} = 1, 2, ..., N_F$, where we wrote all the fermions in the LH basis and the subscript c stands for anti-fermion. For example, in the massless limit of the SM with three RH neutrinos included, we have $N_F = 24$.

From the anomaly and topological arguments [47, 49] discussed in the text, we know that the fermions must condense and spontaneously break the anomalous chiral symmetry

$$\psi_i \to e^{i\alpha}\psi_i\,, \quad \psi_{c\bar{i}} \to e^{i\alpha}\psi_{c\bar{i}}\,. \tag{A.0.1}$$

However, we do not have any definite information about the flavor structure of the condensate. This structure must be determined dynamically by minimization of the effective potential for the following order parameters:

$$\hat{X}_{i\bar{j}} \equiv \psi_i\psi_{c\bar{j}}\,,\ X_{ij} \equiv \psi_i C\psi_j\,,\ \bar{X}_{\bar{i}\bar{j}} \equiv \psi_{c\bar{i}}C\psi_{c\bar{j}}\,, \tag{A.0.2}$$

where C is the matrix of charge conjugation. Notice, although the fermions ψ_i, ψ_{ci} can be embedded as a fundamental representation of the $U(2N_F)$ group, the Lorentz-invariant bilinear order parameters form the representations of the $U(N_F)_L \times U(N_F)_R$ group acting on indexes i and $\bar{i}$, respectively: $\hat{X}_{i\bar{j}}$ is bifundamental, whereas X_{ij} and $\bar{X}_{\bar{i}\bar{j}}$ transform as symmetric tensors under $U(N_F)_L$ and $U(N_F)_R$, respectively.

We can classify various terms in the effective potential according to their transformation properties with respect to the $U(N_F)_L \times U(N_F)_R$ flavor group. Namely, we split all possible terms in two categories: the terms that are flavor invariants and the terms that break one part or an entire flavor group explicitly.

There is a finite number of independent invariants, which have the form of various traces, such as, $\mathrm{Tr}(\hat{X}^+\hat{X})$, $\mathrm{Tr}(\hat{X}^+\hat{X}\hat{X}^+\hat{X})$, ..., $\mathrm{Tr}(X^+X)$, $\mathrm{Tr}(\bar{X}^+\bar{X})$,

$\mathrm{Tr}(\hat{X}X^{+}\bar{X}\hat{X})$, ... The effective potential can in general represent an infinite polynomial of such invariants scaled by powers of Λ_G.

In order to characterize the terms that break the flavor group explicitly, we need some guideline. As such, we are going to use the anomaly. It is reasonable to expect that pure gravitational effects only explicitly break the anomalous chiral symmetry (A.0.1), and leave invariant the anomaly-free subgroup Z_{2N_F} as well as the discrete symmetry under the exchange of fermions and anti-fermions. An operator with such transformation properties is

$$\epsilon^{i_1 \cdots i_{N_F}} \epsilon^{\bar{j}_1 \cdots \bar{j}_{N_F}} \hat{X}_{i_1 \bar{j}_1} \dots \hat{X}_{i_{N_F} \bar{j}_{N_F}} \,, \tag{A.0.3}$$

which is analogous to the 't Hooft vertex in QCD. This operator will in general be multiplied by an arbitrary function f of the phase-independent invariants.

Here, we must stress that we are *not* making any assumption about the possible origin of the above vertex from gravitational instantons. The analogy with the instanton-induced 't Hooft vertex in QCD is purely from the point of view of its symmetry properties: if the gravitational anomaly generates a mass gap for the η_ν pseudo-Goldstone, the effective potential of the order parameters must contain terms that break the $U(1)_G$ symmetry explicitly down to Z_{2N_F}. This uniquely fixes the structure of the minimal vertex (A.0.3), irrespective of its underlying origin, which can be fully quantum rather than semi-classical.

After including all possible terms, we get an effective potential invariant under $SU(N_F)\times SU(N_F)\times U(1)_V\times Z_{2N_F}$ symmetry. The form of the condensate that spontaneously breaks this symmetry group is determined by minimization of the potential. It is well accepted that the analogous potential in case of QCD breaks the flavor group down to a diagonal subgroup $U(N_F)_V$. However, *a priori* there is no reason that gravity should follow the same pattern of symmetry breaking. In fact, as also discussed in section 3.3, it is easy to see that already an effective potential that includes up to quartic order invariants in the order parameters $\hat{X}$, X, and $\bar{X}$ allows for a rich variety of patterns of flavor symmetry breaking. The possibility of spontaneous breaking of the flavor group is important due to resulting new flavor-violating phenomena in our neutrino mass and nonminimal DA models (see section 5.5.4).

If we switch on the SM gauge and Higgs interactions, these break the flavor group explicitly down to a much smaller subgroup. In particular, after "dressing" the effective gravitational vertex (A.0.3) by effects of QCD and EW interactions, we can integrate out all the heavy species of masses $\gg \Lambda_G$ and obtain an effective vertex for the species that are getting masses from the gravitational effects. In our minimal DA model, the resulting effective vertex has the form (4.3.8) and is enough for reducing this solution to the strong CP problem to its bare essentials. However, for precision phenomenology, taking into account other species is important, as discussed in section 5.5.4.

Appendix **B**

(Gravi-)Axion Mass Matrices from Three-Form Formalism

In this appendix, we will explicitly show how the η' meson and the η_ν boson cancel both the QCD and the gravitational θ-terms in our DA model, and we will diagonalize their mass matrix. We will achieve this by using the three-form formalism [48]. A detailed discussion of the diagonalization of the mass matrix in case of mixing the η_ν meson with a conventional axion is given in [49]. The only difference in our case is that the standard axion is replaced by η'.

The three-form formalism uses the fact that a nonzero topological vacuum susceptibility both in gravity and in gauge theory implies that the topological density can be interpreted as the gauge-invariant field strength of a massless three-form field: $R\tilde{R} \equiv d\tilde{C}_G \equiv E_G$ and $G\tilde{G} \equiv d\tilde{C} \equiv E$, where $\tilde{C}$ and $\tilde{C}_G$ are the Hodge duals of the QCD and gravitational Chern-Simons three-form fields $C \equiv AdA - \frac{2}{3}AAA$ and $C_G \equiv \Gamma d\Gamma - \frac{2}{3}\Gamma\Gamma\Gamma$, respectively (see section 3.1).

The low-energy effective theory that fully captures the details of the mass-gap generation is a gauge invariant theory of these three-forms coupled to pseudo-Goldstone bosons of anomalous currents. The gauge invariance and anomaly fully determines the form of this effective Lagrangian.

In the present case of our DA model, the Lagrangian reads

$$\begin{aligned} \mathcal{L}_{3-\text{form}} &= \frac{1}{2V^4}E^2 + \frac{1}{2v^4}E_G^2 - \frac{\eta'}{V}E - \left(\frac{\eta'}{V} + \frac{\eta_\nu}{v}\right)E_G \\ &\quad + \frac{1}{2}\,\partial_\mu\eta'\partial^\mu\eta' + \frac{1}{2}\,\partial_\mu\eta_\nu\partial^\mu\eta_\nu\,, \end{aligned} \tag{B.0.1}$$

analogous to the Lagrangian (3.1.29) in our neutrino mass model.

As shown in [48], the higher-order polynomial terms in E and E_G can easily be taken into account and they only affect the form of the resulting pseudo-Goldstone potentials for large field values, but cannot affect the mass gap. The terms with higher derivatives are irrelevant, since they vanish for constant field values, i.e., in the zero momentum limit, and thus cannot affect the form of the scalar potentials (see [48] for details). We do not explicitly display the numerical coefficients that absorb irrelevant combinatoric factors, especially in the light of the strong hierarchy between the scales, $V \gg v$.

The equations of motion for the fields C and C_G are

$$\begin{aligned} \mathrm{d}\left(E - V^3\eta'\right) &= 0\,, \\ \mathrm{d}\left(E_G - v^4\left(\frac{\eta'}{V} + \frac{\eta_\nu}{v}\right)\right) &= 0\,, \end{aligned} \tag{B.0.2}$$

and the ones for η' and η_ν read

$$\begin{aligned} \Box\eta' + \frac{1}{V}(E_G + E) &= 0\,, \\ \Box\eta_\nu + \frac{1}{v}E_G &= 0\,. \end{aligned} \tag{B.0.3}$$

Integrating (B.0.2) we get

$$\begin{aligned} E &= V^4\left(\frac{\eta'}{V} + \theta\right), \\ E_G &= v^4\left(\frac{\eta'}{V} + \frac{\eta_\nu}{v} + \theta_G\right), \end{aligned} \tag{B.0.4}$$

where θ and θ_G appear as two arbitrary integration constants. Notice, in the absence of the η_ν boson, there would be no way to compensate both θ-terms by a shift of η' alone. This is a simple manifestation of how gravity ruins the solution to the strong CP problem by "destructing" the axion – in the present version η' – from its job of compensating the θ-angle of QCD.

However, as we can easily see, the problem is solved by η_ν. Namely, both integration constants θ and θ_G can be eliminated by the appropriate shifts of η' and η_ν, i.e., $\eta' \to \eta' - V\theta$ and $\eta_\nu \to \eta_\nu - v(\theta - \theta_G)$. Moreover, the vacuum of the theory is at

$$\eta' = -V\theta\,, \quad \eta_\nu = v(\theta - \theta_G)\,, \tag{B.0.5}$$

where $E = E_G = 0$ and both topological susceptibilities vanish. The physical meaning of this is that both three-forms C and C_G become massive by eating up the corresponding pseudo-Goldstone bosons, η' and η_ν (see section 3.1).

After eliminating the two integration constants, we can plug the expressions (B.0.4) for E and E_G into (B.0.3) and get the following effective mass terms:

$$\begin{aligned} \Box\eta' + V^2(1+\epsilon^4)\eta' + \epsilon v^2\eta_\nu &= 0\,, \\ \Box\eta_\nu + \epsilon v^2\eta' + v^2\eta_\nu &= 0\,. \end{aligned} \tag{B.0.6}$$

Ignoring terms of order ϵ^4, the corresponding mass terms in the Lagrangian are

$$\mathcal{L}_{\text{mass}} = -\frac{1}{2}V^2\eta'^2 - \epsilon v^2\eta'\eta_\nu - \frac{1}{2}v^2\eta_\nu^2\,. \tag{B.0.7}$$

As one can deduce from the mass matrix

$$M^2 = V^2 \begin{pmatrix} 1 & \epsilon^3 \\ \epsilon^3 & \epsilon^2 \end{pmatrix}, \tag{B.0.8}$$

the mixing between the two states is absolutely minuscule ($\sim \epsilon^3$). Therefore, the eigenvalues of the mass matrix are approximately given by

$$m_{1,2}^2 \simeq \frac{1}{2}(V^2 + v^2) \pm \frac{1}{2}(V^2 - v^2), \tag{B.0.9}$$

and the corresponding eigenstates up to a mixing of $\sim \epsilon^3$ are η' and η_ν,

$$a_{\rm PQ} \, = \, \eta' + \mathcal{O}(\epsilon^3)\eta_\nu \, , \quad a_G \, = \, \eta_\nu \, + \, \mathcal{O}(\epsilon^3)\eta' \, , \tag{B.0.10}$$

with masses $m_{\eta'}^2 = V^2$ and $m_{\eta_\nu}^2 = v^2$, respectively.

Appendix C

Neutrino Mass Contribution from Electroweak Instantons

This appendix treats a possible contribution of EW instantons to LH Majorana neutrino masses [7]. As we will demonstrate, this contribution can only be present in GUTs and is too small to be phenomenologically relevant.

As we briefly reviewed in section 2.1.2, most GUTs include a mechanism to account for the observed small neutrino masses, except for the minimal SUSY and non-SUSY $SU(5)$ frameworks. The standard seesaw mechanism cannot work in these minimal schemes, because it needs both the existence of RH neutrinos as well as the Higgs multiplet that generates a large Majorana mass for them (see [118] for a review). Thus, one motivation for investigating LH Majorana neutrino mass contributions from EW instantons is their possible existence already in minimal $SU(5)$, without involving RH neutrino states.

Before going into the details of this mass contribution, let us briefly comment on the experimental constraints on $SU(5)$ GUTs due to nucleon decay predictions. The measured bounds on the proton lifetime, such as $\tau_p/\mathrm{Br}(p \to e^+\pi^0) > 1.67 \times 10^{34}$ years at 90% CL [385], rule out the minimal non-SUSY $SU(5)$ scenario. Even though the minimal SUSY $SU(5)$ case is also widely considered to be falsified [386], there are several possibilities to circumvent or weaken the proton decay constraints [387–392].

Now, let us consider the basic EW $SU(2)_L$ instanton, which generates an effective vertex containing nine quarks and three leptons, i.e., three baryons and one lepton per generation [393]. In order to generate a neutrino mass operator from such an instanton, one has to leave two neutrino legs open and annihilate the remaining nine quark and one lepton legs in B and L-violating vertexes: one four-fermion and one six-fermion vertex. Since there are no such vertexes in the SM, they can only be generated in SM extensions such as GUTs [43–45]. Notice, the considered four-fermion vertex is not necessarily accounting for proton decay, since the quarks can come from all three generations.

The EW instanton generates several diagrams each time with two neutrino legs open from two generations. Even though the instanton-mediated interactions are the same for all neutrino flavors having the same weak hypercharge, the entries in the emerging Majorana mass matrix differ from each other, since the four-fermion vertex produces different numerical parameters for different flavors. Therefore, the traceless, off-diagonal 3×3 matrix M_{EW} of the generated hard Majorana masses has different eigenvalues and thus can yield hierarchical neutrino masses. The resulting Majorana neutrino mass term reads

$$\mathcal{L}_{\mathrm{EW}} = -M_{\mathrm{EW}} \nu_i^T C \nu_j, \tag{C.0.1}$$

where C is the charge conjugation matrix and ν denotes the LH neutrino. Note that the Majorana mass term is no longer forbidden by any symmetry, since $SU(2)$ is now explicitly broken.

In order to estimate the energy scale of the neutrino mass operator M_{EW}, we have to evaluate the instanton integral [393]

$$M_{\mathrm{EW}} \propto \int \frac{d\rho}{\rho^5} \frac{1}{M_{4f}^2 M_{6f}^5 \rho^4} \, e^{-2\pi/\alpha(\rho)} \tag{C.0.2}$$

at the instanton scale M_I, where ρ is the instanton size, $\alpha(\rho)$ is the EW coupling, and M_{4f} and M_{6f} are the scales of the four- and six-fermion vertexes, respectively. After taking into account the $\alpha(\rho)$-dependent zero-modes factor, we obtain

$$M_{\mathrm{EW}} \sim \left(\frac{2\pi}{\alpha(M_I)} \right)^4 \frac{M_I^8}{M_{4f}^2 M_{6f}^5} \, e^{-2\pi/\alpha(M_I)} \tag{C.0.3}$$

in a non-SUSY scenario, while the zero-modes factor in a SUSY scenario contributes not to the fourth but to the tenth power.

In non-SUSY $SU(5)$ theories, the gauge couplings do not unify and the weak coupling $\alpha_W(M_G) \simeq 1/42$ [394] would obviously yield a too strong suppression of the resulting neutrino mass scale. Therefore, let us consider SUSY $SU(5)$ with a larger grand unified coupling of $\alpha_G(M_G) \simeq 1/24.3$ [97]. In the minimal SUSY $SU(5)$ scheme, the four- and six-fermion vertexes are generated by the exchange of the scalar color-triplet partner of the Higgs doublet, which has a mass of $M_{H_c} \leq 3.6 \times 10^{15}$ GeV [386]. Thus, the four- and six-fermion vertexes can be evaluated at the scale $M_{4f} = M_{6f} = M_{H_c} \simeq 3.6 \times 10^{15}$ GeV, while the EW instanton is generated at the Planck scale, $M_I = M_P$.

At first sight, the EW instanton seems to yield a phenomenologically viable neutrino mass scale of $M_{\mathrm{EW}} \sim$ meV (C.0.3), while choosing slightly different values for M_{4f}, M_{6f}, M_I, and α can change the result by several orders of magnitude. However, for the considered case of $M_I \gg M_H$, the factor $1/(M_{4f}^2 M_{6f}^5)$ actually reduces to ρ^7, implying that the EW instanton only generates a factor of M_I. Consequently, the induced LH Majorana neutrino mass scale reads $M_{\mathrm{EW}} \lesssim 10^{-16}$ eV. Thus, we can conclude that the EW instanton contribution to LH Majorana neutrino masses in GUTs is many orders of magnitude too small to be of experimental relevance.

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