The calibration of economic growth: An application to carbon emission scenarios and to the DICE model

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I am grateful to my husband, my little daughter,
my parents and my sister.
I love you.

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Preface

In the centuries to come, climate change will be one of the biggest, if not the biggest challenge for mankind. Already in 1992, when the 'United Nations Framework Convention on Climate Change' (UNFCCC) was adopted, the international community of the member states agreed that in the interest of human safety the threat of global warming has to be counteracted. At the time, this pledge was made in the face of far greater scientific uncertainty than we know today. At present, the majority of scientists agrees that global warming is caused and accelerated by anthropogenic emissions of greenhouse gases and we begin to observe that climate change is impacting the livelihood of millions of people worldwide[1].

Anthropogenic greenhouse gas emissions are primarily caused by the use of fossil energy sources. Although many countries strive to substitute fossil energies by renewables, especially in the developing world economic growth is still reliant on the use of fossil energy sources. While many progressive countries continue to reduce their emissions, in the medium run some developing countries are expected to cause even higher carbon emissions in their struggle to reduce poverty. In the Paris Agreement, which was adopted at the 21st COP (conference of the parties - the principal decision making body of the UNFCCC) in 2015 in Paris, these differences are recognized. The aim of this agreement is to keep the global atmospheric temperature rise in this century below $2°C$. One major advancement compared to the Kyoto Protocol from 1997 is that it is based on voluntary action plans or so-called 'nationally determined contributions' (NDCs), which are determined by a bottom-up process. In this way more countries have voluntarily committed to pursue climate mitigation measures than under the Kyoto Protocol. For instance, the member states of the European Union have pledged to reduce their carbon emissions by 2030 by 40 % compared to 1990 levels. On the other hand, numerous countries in Africa and south

[1]See for instance the Fifth Assessment Report by the Intergovernmental Panel on Climate Change (IPCC) in Field et al. (2014).

Asia have committed to reduce their emissions relative to the business as usual, such that in fact their emissions are expected to rise while their economies grow.
The negotiations which have lead to the Paris Agreement have shown that in the medium run it will not be possible to separate world economic growth from the use of fossil energy. On the contrary, economic growth will be an important determinant of future carbon emissions and its magnitude will have a decisive impact on the climate.
In this thesis, I systematically study the relationship between economic growth and climate change and I put some emphasis on the inter-dependencies between the two. In addition, I investigate the degree of uncertainty that is associated with future economic growth and how it translates into uncertainty over future climate damages. As instruments I use carbon emission scenarios as well as integrated models of the economy and the climate - so called Integrated Assessment Models (IAMs)[2]. Both help to anticipate potential temperature increases and climate damages. The main contributions of this thesis to the literature are threefold. First, it carefully examines better representations of economic growth dynamics for the use in carbon emission scenarios and IAMs. Second, it evaluates the uncertainty of future economic growth and relates it to the expected future atmospheric temperature increase. Third, it studies the effects of climate change on endogenous economic growth through changing investment incentives that often go overlooked in the literature.

This thesis is organized in four chapters which all look at economic growth and climate change from a different angle and with a different emphasis on theoretical and methodological approaches. At this point, I want to give a summary of the most important results in this thesis. A short summary of each chapter on its own can be found at the end of this preface.
To find a better representation of economic growth, I calibrate a model of endogenous growth to data in Europe that span one and a half centuries, and simulate growth trajectories into the future (chapter 1). The central question in this exercise is by how much the energy and emissions intensities have to be reduced in Europe in order to offset additional carbon emissions from future economic growth. The key quantitative finding is that because Western Europe is expected to grow at a higher rate, it has to reduce its energy and carbon emissions intensities by about twice as much as Eastern Europe.
Because of the very complex nature of climate change, a very high degree of uncertainty is

[2]Integrated Assessment models are combined models of the economy and the climate, where both, economic growth and climate change, are inter-dependent and endogenous to the model.

tied to all modeling approaches. Point forecasts have only a limited informative value and it is just as important to quantify expected variations from the mean. I therefore estimate confidence intervals of future economic growth and I assess how the uncertainty that is tied to future economic growth translates into uncertainty regarding carbon emission pathways (chapter 1). In addition, I dedicate a whole chapter to the development of a new Bayesian approach, which can be used for the calibration of deterministic growth models of the long run (chapter 2). The aim of this chapter is to develop a standardized approach towards the calibration of deterministic growth models, which can be used for the construction of carbon emission scenarios or IAMs. The appeal of this new Bayesian approach is that it transforms the stochastic residual between the simulated and the observed data, which comprises technological and other shocks, and turns it into confidence intervals of future economic growth.

The relation between economic growth and climate change runs both ways. Not only carbon emissions drive global warming, but climate change also affects our future income. In recent years, there has been a rising debate on how strongly climate damages will affect GDP and whether they will have negative level or lasting growth effects. In the Integrated Assessment literature, this question has predominantly been studied based on the notion that climate damages might directly hit productivity growth or the accumulation of productive assets. In this thesis, I identify and discuss a channel that is often overlooked in the literature, through which global warming affects GDP (chapter 3). More specifically, I estimate the adverse effect of climate change on the incentive to invest in research and development, which ultimately leads to negative growth rather than level effects. In the process, I replace the growth component of a workhorse IAM (the DICE model by Nordhaus (2008)) by an endogenous growth model and re-calibrate growth towards its original growth trajectory. Colloquially speaking, in this model, climate damages incentivize forward looking households to invest less into economic growth. In an empirical exercise I find that in its 'Optimal Scenario' the original DICE model with exogenous growth overestimates gross income in 2100 by 2.3 %. The difference gets larger the further time progresses.

In 2000, the IPCC Special Report on Emission Scenarios clearly showed that one major source for uncertainty regarding the scale of global warming is future economic growth. To quantify its scale, I estimate the uncertainty that is tied to economic growth using the Bayesian approach developed in this thesis and I implement it in the DICE model (chapter 4). In this way, I can show how the uncertainty regarding future economic growth translates into carbon emissions within the framework of an IAM. An interesting result from this exercise is that, in the 'Optimal Scenario', even though the estimated confidence interval on gross income in 2100 is rather big, with an expected variation from the mean of roughly ± 36 % within the 90 % confidence interval, the expected variation of the temperature increase by 2100 is much smaller, with an expected variation from the mean of roughly ± 4 % within the 90 % confidence interval. This is first, because the effects of carbon emissions on the atmospheric temperature fully unfold with a delay and, second, because in the 'Optimal Scenario' households mitigate nearly 100 % of their carbon emissions by 2100.

Altogether, this thesis underlines the inter-dependency between economic growth and climate change. It gives a new perspective on the calibration of growth models and how this changes our expectations of future carbon emission and climate damages. Central to the analysis are not only point estimates, but also measures regarding the uncertainty of future growth. In addition, this thesis investigates a rarely discussed channel through which climate change might have a lasting and negative impact on growth.

Chapter 1 In this chapter, I construct carbon dioxide emission scenarios for Europe until 2100. The three most important ways in which this chapter contributes to the literature are, first, that economic growth is driven by endogenous investments into research and development. Second, the model is formally calibrated, using data that span a period (1850-2008) longer than the projection period (2008-2100). Third, this work provides statistically valid confidence intervals of economic growth, which translate into a measure of uncertainty regarding future carbon emissions. Forecasts are made on the regional level for Europe as a whole, Western Europe, Eastern Europe and the former USSR. The calibration of economic growth yields higher future annual growth rates in Western Europe than in Eastern Europe and the former USSR. In order to offset this higher economic growth (or equivalently in order to keep future carbon emissions until 2100 at the same level as today), Western Europe would have to reduce its energy and emission intensities by roughly 1 % annually, while in Eastern Europe and former USSR countries approximately 0.5 % of annual reductions would be sufficient. Because of past economic turmoil the estimated uncertainty that is tied to future economic growth is larger in Eastern Europe and former USSR countries than in Western Europe.

An older version of this chapter has been published together with Prof. Dr. Richard S.J.

Tol as CESifo Working Paper No. 4971 (see Ciesielski and Tol (2014)).

Chapter 2 In the Integrated Assessment literature there has been a growing need for sound calibration techniques of growth models which go beyond the medium run and aid as a forecasting device of macroeconomic trends. To model the future costs of climate change, we need to know more about carbon emissions in the long run, and they will crucially depend on future economic growth and technological advancements. Because the majority of Integrated Assessment Models is deterministic, in this chapter I develop a robust calibration technique for deterministic models of long run growth. The aim is to present a standardized approach towards calibration, which is straight forward and easy to adapt. I suggest a Bayesian inversion technique to elicit the distributions of all parameters of calibration and to project the confidence intervals of future income and consumption shares. Since in this chapter I propose a technique to calibrate deterministic models of economic growth in the long run, I set my self apart from the Bayesian calibration literature of stochastic macro-economic models (see for instance Fernández-Villaverde and Rubio-Ramírez (2007) and Fernández-Villaverde (2010)). To integrate over Bayes' law I use a Markov chain Monte Carlo (MCMC) algorithm. The likelihood function is derived from a stochastic process, which describes the residuals between the observed and the simulated data. Since the majority of Integrated Assessment models is based on a Ramsey type growth model, I demonstrate this procedure by calibrating a standard Ramsey model of exogenous growth as well as an endogenous growth model by Aghion and Howitt (1999). The resulting growth trajectories until 2050 from both models are similar with an average growth rate of the median projection of 2.3 % in the Ramsey model and 2.2 % in the Aghion & Howitt model. All parameters of calibration are well identified and have clear-cut distributions. Therefore, I conclude that this approach is highly flexible for the calibration of the growth component in Integrated Assessment Models and that it can be adopted for a wide range of different growth models.
This chapter is the result of joint work with Dr. David Anthoff.

Chapter 3 This chapter analyzes the negative impact of climate change on economic growth caused by a reduction of the return on general R&D and consequently of investments into the same. The framework is based on an Integrated Assessment Model, the DICE model by Nordhaus (2008). The DICE model builds on Ramsey type growth where environmental damages cause a negative level effect on GDP. In a version of this model, I substitute the growth component by endogenous Schumpeterian type growth and calibrate it to the original. In the socially 'Optimal Scenario', the social planner is able to mitigate climate change in two ways. First, he can invest in the reduction of carbon emissions and, second, he can shift his spending away from the carbon-emitting capital stock. In addition, in the endogenous Schumpeterian growth setting, the return on investment in R&D declines due to environmental damages and thus investments into the general R&D sector are reduced. Since endogenous investments into R&D are what drives economic growth in a Schumpeterian model, global warming has a lasting and negative impact on GDP growth through this channel. It can be understood as an additional effect which adds to the channel of directed technical change as described in Acemoglu et al. (2012). In both model versions, the reallocation of resources reduces future total output. However, the negative effect in the endogenous growth setting is stronger, since investments into R&D are allowed to go down. Comparing the Ramsey and the Schumpeterian version of the DICE model, this long-lasting, negative growth effect is even stronger in a 'Constrained Optimum Scenario', where households cannot actively mitigate to reduce climate damages. On the contrary, in a 'Business as Usual Scenario', where the climate externality is not internalized and the private return on investment is not affected by climate change, there are no negative growth effects due to the reallocation of resources. In this scenario, however, higher growth rates cause more climate damages, which eventually overcompensate an initially higher rate of economic growth.

Chapter 4 In the Integrated Assessment literature, economic growth is a major determinant of projected carbon emissions and climate damages. Nevertheless, its importance is often overlooked. While most macroeconomic models of growth are run for a couple of decades at best, in an environmental context these same growth models are often solved for centuries. This increases the dependency of all growth projections on their underlying model assumptions. In this chapter, I carefully recalibrate the growth component of the DICE-2016R model as described in chapter 3 using the Bayesian calibration approach developed in chapter 2. One major advantage of this Bayesian calibration technique is that it quantifies the uncertainty that is tied to economic growth, given the model assumptions and the observed hostorical data. In the DICE model, the link between economic growth

and carbon emissions is particularly intense, since GDP translates directly into carbon emissions at an exogenously given proportion, which shrinks over time as fossil energy is gradually substituted by clean energies.
The expected mean temperature increase compared to pre-industrial levels, in the 'Optimal Scenario' in the re-calibrated version of the DICE model, amounts to 3.8°C. Interestingly, the results show that even though the expected variation of future gross income is very high, the expected variation from the mean temperature increase by 2100 is relatively low, with only 4 % within the 90 % confidence interval. This is because in the 'Optimal Scenario' the mitigation of carbon emissions amounts to almost 100 % in 2100. In the 'Constrained Optimum Scenario', where households have no instrument of direct mitigation, the mean temperature increase of the atmosphere compared to pre-industrial levels amounts to 4.7°C, with an expected variation of 11 % in the 90 % confidence interval in 2100. Thus, the implementation of effective climate change policies aimed at reducing carbon emissions does not only lower the level of the future temperature increase significantly, but also the uncertainty over the magnitude of future climate damages.

Chapter 1

Carbon emissions scenarios in Europe based on an endogenous growth model

1.1 Introduction

Carbon emission scenarios help to anticipate potential temperature increases and consequential damages to the climate and the environment. The IPCC Special Report on Emission Scenarios by Nakicenovic et al. (2000) comprises a wide range of plausible projections from near zero emissions worldwide in 2100 to an over tenfold increase compared to 1990. In these scenarios, income per capita growth is identified as a major determinant of future emissions, and a great source of uncertainty. Nevertheless, in the scenario literature economic growth is not systematically studied, and the underlying growth models do not reflect the rapid development of economic theory.

The aim of this study is to construct carbon dioxide emission scenarios for Europe until 2100. Our main contributions to the literature are, first, that economic growth is modeled using a model of endogenous growth by C. I. Jones (1995a), rather than a model of exogenous growth. In the Jones model, economic growth is based on an increasing product variety as first suggested by P. M. Romer (1990). Second, the model is formally calibrated using data that span a period (1850-2008) longer than the projection period (2008-2100). Third, this work provides statistically valid confidence intervals of economic growth, which translate into a measure of uncertainty regarding future carbon emissions. Forecasts are made on the regional level for Europe as a whole, for Western Europe, Eastern Europe and the Former USSR.

We use the Kaya identity to decompose carbon emissions[1]. Carbon emissions, X_t, are subdivided into four components: population size, P_t, income per capita, Y_t/P_t, the energy intensity, E_t/Y_t, and the emissions intensity, X_t/E_t:

$$X_t = P_t * \frac{Y_t}{P_t} * \frac{E_t}{Y_t} * \frac{X_t}{E_t} \tag{1.1}$$

Our focus is on calibrating income per capita using a model of endogenous economic growth. In addition, we quantify the uncertainty that is tied to economic growth, which translates into an uncertainty regarding future carbon emissions. Future population sizes are taken from UN forecasts and energy and emission intensities are assumed to decrease at constant rates, which will be part of the analysis in section 1.5. We discuss the implications of future population growth as well as changes in the energy and emission intensities, however, they are exogenous to our growth model.

The United Nations Framework Convention on Climate Change (UNFCCC) channels global political efforts by 197 member states towards the common goal of keeping the atmospheric carbon concentration below a level that could endanger the human livelihood. In the face of scientific uncertainty this goal has been identified with a temperature increase of below 2°C compared to pre-industrial times. One of the UNFCCC's principles is to burden developed countries with the larger part of emission reductions, since these countries are also responsible for the majority of current and past carbon emissions. In the Kyoto Protocol the member states of the European Union have committed to reduce their domestic carbon emissions between 2008 and 2012 by 8 % below 1990 levels. The Kyoto Protocol was adopted in 1997 and it entered into force in 2005. It included greenhouse gas emission reduction plans by 37 industrialized countries and the European Union. In 2012, this was followed by the Doha Amendment, where the member states of the European Union committed to reduce carbon emissions until 2020 by 20 % below 1990 levels. A subsequent agreement was reached in Paris in 2015. In this agreement, the member states of the European Union have forwarded a 'nationally determined contributions' plan (NDC), where they commit to reduce domestic carbon emissions by 2030 by 40 % below 1990 levels.

[1] As has for instance been done in Hoffert et al. (1998) and Tol, Pacala, and Socolow (2009).

In this work, we investigate the impact of expected future economic growth on carbon emissions in Europe. We focus on the question by how much the energy and emissions intensities would have to be reduced in order to achieve an absolute decoupling of carbon emissions from economic growth. Put differently, we derive necessary reductions in both intensities such that carbon emissions stay constant despite expected future growth. In fact, both would have to be lowered much further, to reach the emission reductions as targeted for by the NDC of the member states of the European Union. In addition, we show at the regional level by how much more regions which grow at a higher rate have to engage in the reduction of their energy and emissions intensities. However, both intensities are exogenous to our growth model. Therefore, we are not able to depict the impact of these efforts on future growth.
Our calibrations yield higher annual growth rates of income per capita in Western Europe than in Eastern Europe and former USSR countries. Because past income in Western Europe has grown at a higher rate than in Eastern Europe and the Former USSR, the Jones model picks up on these growth trends and projects them into the future. Consequently, we find that in order to keep future carbon emissions until 2100 constant, Western Europe would have to reduce its' energy and emission intensities by roughly 1 % annually, while in Eastern Europe and former USSR countries 0.5 % of annual reductions would be sufficient. According to the Jones model, these reductions will be necessary to offset future economic growth.
Our confidence intervals on future economic growth turn out to be significantly large. Because of past economic turmoil, the confidence intervals of future income per capita and, thus, carbon emissions are larger in Eastern Europe and former USSR countries than in Western Europe.

The calibration of endogenous growth is potentially interesting for implementation in Integrated Assessment Models (IAMs). These are combined models of the economy and the climate. For instance, there have been proposed endogenized growth versions of the DICE model by Nordhaus (2008). Because of its tractability, the DICE model is a workhorse model in the environmental economics literature. However, while Moyer et al. (2014) and Dietz and Stern (2015) endogenize economic growth in the DICE model, they recalibrate growth to the original model and not to observed time series on gross income. In this work, we calibrate an endogenous growth model towards data that span one and a half centuries and we show how projected growth and its confidence intervals translate into future carbon emissions.

In the following section we present and discuss our choice of data sets employed in the calibration. Section 1.3 proceeds with a description of the Jones model and section 1.4 summarizes our technical approach. All results will be discussed in section 1.5. This entails carbon emission projections and their corresponding confidence intervals on the regional level. In section 1.6 we compare our income pathways to the SSP scenario framework which was developed for the Fifth Asessment Report of the Intergovernmental Panel on Climate Change. In section 1.7 we conduct a sensitivity analysis regarding the discount rate and the capital share. Finally, section 1.8 concludes.

1.2 The data

The data set comprises four regions in Europe. These are Europe as a whole, Western Europe, Eastern Europe and the former USSR. In addition, we collect data for 22 countries in Europe. The data set includes annual observations from 1850 to 2008. Thus, it covers the turmoil of two world wars. To pick up historical time trends, the time frame for calibration starts well before the rise of the 19th century. Several boundary changes in Europe's past turn the collection of data into a cumbersome business. Because we are interested in time series which are not influenced by the changing geographical size of a country, the data was compiled as if today's borders had been in place since 1850. Except for former Czechoslovakia, the former USSR and former Yugoslavia, which were each aggregated into one region. The collection of regional data is not affected by these complications.
In the remainder of this section, we will describe our data on population, income, energy use and carbon emissions and our assumptions regarding the output elasticity.

Population Historical data on population by country is available in Angus Maddison's historical statistics (see Maddison (2010)). Coverage is complete by region (respectively country) and year starting in 1920. For six Eastern European countries and the former USSR there are some missing values before 1920. However, data are complete in 1850. This allows for cubic interpolation to fill in gaps[2]. Annual population forecasts by country from 2011 to 2100 are provided by the United Nations (see United Nations (2011)). Figure 1.1 illustrates past and future population growth of the whole sample in Europe and Russia. A population maximum is expected to be reached shortly after 2000.

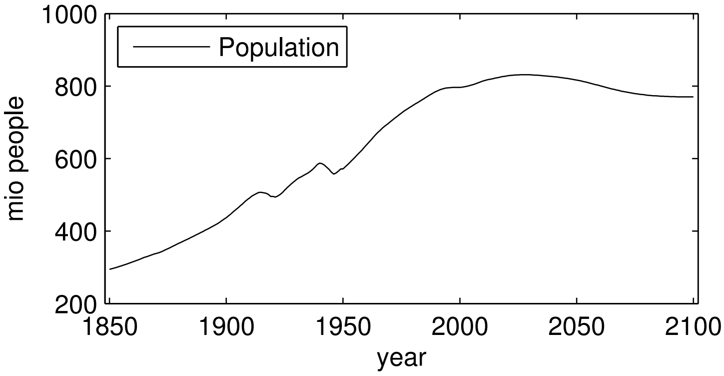


Figure 1.1: Sample population in total Europe and Russia

Income Income between 1850 and 2008 was taken from Angus Maddison's historical statistics as well. The data are complete from 1950 onwards. As before, lacking observations for 12 countries and all four regions were filled in by cubic interpolation. For seven countries, including the former USSR, there are no observations available before 1870. In those cases missing observations were filled in by linear data extrapolation[3]. For the other three regions in Europe data in 1850 is available.
Since 1850 the sample income in Europe and Russia has increased by a factor of 34 (see figure 1.3). Growth has been relatively constant except for some kinks during the First and Second World War and the collapse of the communist system in 1990.

The output elasticity In a Cobb-Douglas production setting, the output elasticity with respect to the productive assets, capital and labor, determines their marginal productivity. A higher output elasticity with respect to capital, which is concurrent with a lower

[2]In those countries where boundary changes led to a sudden change in the population size, we construct a replacement which reflects the current geographical size of the country.

[3]Again, in those cases where the boundaries of a country have changed, historical GDP was adjusted to the current geographical size of the country.

output elasticity with respect to labor, increases the marginal productivity of capital and, therefore, raises the capital share in the economy. Historical capital shares in Europe are controversially discussed. The Kaldor facts (Kaldor (1961)) state that long run capital shares have been relatively constant over time. For a discussion see for instance Kongsamut, S. Rebelo, and Xie (2001), Foellmi and Zweimüller (2008) and Acemoglu (2009). Thus, the majority of carbon emission scenarios and also Integrated Assessment Models assumes the output elasticity to be constant. However, recent empirical findings suggest that in a number of developed countries capital shares have been strongly increasing for some decades. In Blanchard, Nordhaus, and Phelps (1997) the estimated capital share in some European countries increased from 0.32 in 1980 to roughly 0.4 in 1995. C. I. Jones (2003) provides empirical data on capital shares in OECD countries since 1960. The data shows that capital shares have been non-constant over time and in a number of countries in Europe they have been increasing significantly. Since, in this study, the time frame for calibration alone covers 150 years, we give credit to these findings and assume that the output elasticity with respect to capital has been growing[4]. To be more precise, we assume that in Europe and before 1850 the output elasticity with respect to capital was constant at 25 %. After 2100 we assume the output elasticity to stagnate at 45 %. In between we assume that the elasticity has been increasing. To keep our results comparable between countries, we assume the same evolution of output elasticities, $(1-\sigma)$, in all countries as is given in equation (1.2) (see also figure 1.2).

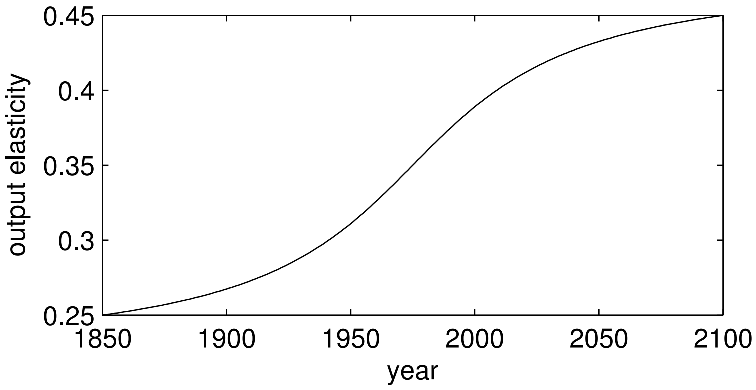


Figure 1.2: Assumed output elasticity with respect to capital

$$1-\sigma = \arctan((year-1975)*0.02)*0.084+0.35 \tag{1.2}$$

[4]Vice versa we assume that the output elasticity with respect to labor has been decreasing.

In a sensitivity analysis in section 1.7 we present a continuum of assumptions regarding the output elasticity and find that our results are relatively sensitive towards these assumptions. This is an interesting result and could be a fruitful topic for future research, given that output elasticities in the scenario literature are often assumed to be constant even over long periods of time.

Energy use and carbon emissions Annual data on energy use by country since 1960 is available at the World Bank (2014) online data base. Data on annual carbon emissions from fossil fuel by country since 1751 is available in a very comprehensive collection by Boden, Marland, and Andres (2017).

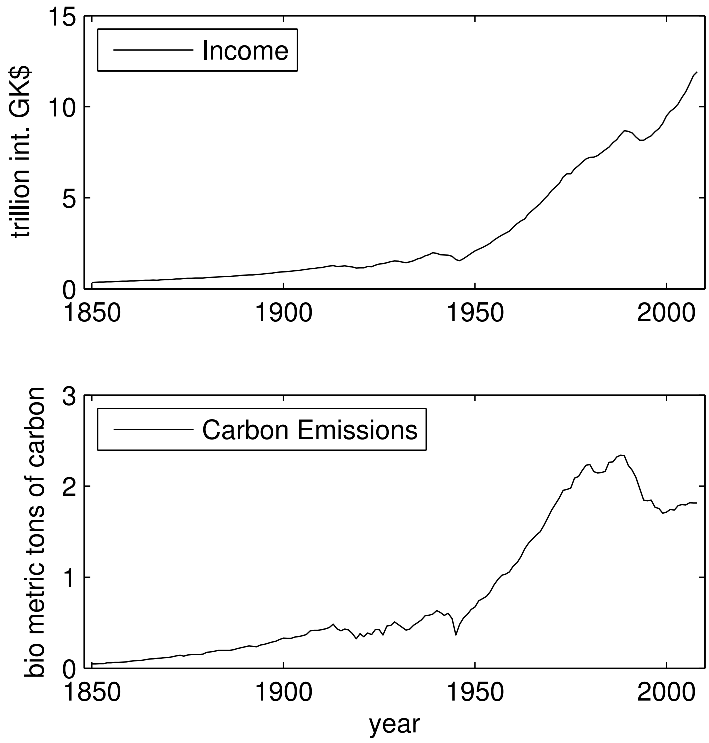


Figure 1.3: Sample income and carbon emissions in total Europe and Russia

Figure 1.3 illustrates how past carbon emissions in Europe and Russia have been increasing until 1990. Thereafter, we observe their sudden decline, while income in figure 1.3 continues to grow. For this reason, we assume that future energy and emission intensities until 2100 will continue to decrease. When presenting our results in section 1.5, we will vary the rate at which both are assumed to decrease. In this way we are able to differentiate the impact of future economic growth on carbon emissions depending on potential future energy and emission intensities.

1.3 The Jones model

We calibrate the C. I. Jones (1995a) model of endogenous growth. This model is particularly suited for calibration, because it does not entail economies of scale. Economies of scale imply that GDP growth depends on the population size of an economy. As C. I. Jones (1995b) shows, empirically, economies of scale cannot be maintained. In the Jones model, economic growth depends on the growth rate of the labor force, rather than on its level as in the P. M. Romer (1990) model. Although the Romer model is a seminal model in the theoretical literature on new growth theory and it is often cited as an example for horizontal innovations driving economic growth, because it entails economies of scale, it is less suited for calibration.

We do not make any changes to the Jones model. Economic growth in this model is innovation-based and it is driven endogenously by profit maximizing agents who invest in the creation of new technologies and thereby increase overall productivity. Technological change is characterized as an increasing variety in intermediate products.

The economy consists of three sectors, a final goods sector, a sector producing intermediate product varieties and a research sector. The latter develops blueprints, needed for the production of additional variants of intermediate goods, which are employed in the production of final output. Labor is divided between the final goods sector and the research sector. Economic growth is endogenous in the sense that growth derives from the invention and pursuit of new technologies by profit maximizing agents.

The final goods sector Final output, Y, is produced under perfect competition. It is derived from labor and a variety of intermediate inputs[5]:

$$Y = (\phi L)^{\sigma} \int_0^A {x_i}^{1-\sigma} \, di \tag{1.3}$$

$0 < \phi < 1$ represents the share of labor, L, which is allocated to the production of final output. The number of blueprints that are available for the production of intermediate

[5] For convenience we drop the time index. Otherwise every variable would have an additional index such as Y_t.

products is denoted by A. It is equivalent to the knowledge stock and it represents technological progress in the economy. x_i denotes the amount of variant i that is employed in the production of final output and σ is the production elasticity.
The productive assets, labor and intermediate goods, earn their marginal product. w_Y stands for wages in the final goods sector and p_i for the price of an intermediate good:

$$w_Y = \sigma \frac{Y}{\phi L} \tag{1.4}$$

$$p_i = \frac{\partial Y}{\partial x_i} = (1-\sigma)(\phi L)^{\sigma} x_i^{-\sigma} \tag{1.5}$$

The intermediate sector Each firm in the intermediate sector acts as a profit maximizing monopolist for the production of its own variant. Thus, intermediate firms are able to choose their profit maximizing price. Blueprints for the production of intermediate goods are purchased from the research sector. Production costs in the intermediate sector derive from the cost of capital. One unit of an intermediate good requires one unit of capital, for which intermediate firms have to pay interest, r. In addition, capital is depreciated at rate δ.

$$\max_{x_i} p_i(x_i)x_i - rx_i - \delta x_i \tag{1.6}$$

Using equation (1.5), we derive the optimal quantity in which each intermediate product is produced. Because all firms in the intermediate sector are symmetric, all variants are produced in the same quantity.

$$\bar{x} = x_i = \left(\frac{(1-\sigma)^2(\phi L)^{\sigma}}{r+\delta} \right)^{\frac{1}{\sigma}} \tag{1.7}$$

Now equation (1.5) can be simplified to:

$$\bar{p} = p_i = \frac{r+\delta}{1-\sigma} \tag{1.8}$$

Because intermediate product variants are produced in equal amounts, the capital stock accumulates to: $K = Ax$ and the production function in equation 1.3 can be rewritten as follows:

$$Y = (A\phi L)^{\sigma} K^{1-\sigma} \tag{1.9}$$

In addition, all intermediate firms yield the same profits. Using equations (1.6), (1.7) and (1.8), we can rewrite these profits:

$$\pi = \sigma(1-\sigma)\frac{Y}{A} \tag{1.10}$$

and the interest rate on capital:

$$r = (1-\sigma)^2\frac{Y}{K} - \delta \tag{1.11}$$

To maximize their profits, intermediate firms produce less goods and sell them at a higher price than what would be socially optimal. When equation (1.11) is re-arranged into the sum of the interest and depreciation rate, they exceed the marginal product of capital by a factor $(1-\sigma)$. Consequently, there is underinvestment into physical capital.

The research sector The research sector operates under perfect competition. The creation of new blueprints depends on the size of the stock of intangible knowledge and on the labor share $(1-\phi)L$, which was allocated to the research sector:

$$\frac{dA}{dt} = \dot{A} = \alpha_J A^{\eta_A}[(1-\phi)L]^{\eta_L} \tag{1.12}$$

η_A and η_L denote the production elasticities and α_J is an exogenous technological parameter.
The patent price P_A of a blueprint is bid up among firms in the intermediate goods sector until it equals the present value of all profits that a monopolist is able to extract from the production of its intermediate good:

$$P_A(t) = \int_t^\infty e^{-\int_t^\tau r(s)\,ds}\pi(\tau)\,d\tau \tag{1.13}$$

All earnings, which are derived from the sale of new blueprints to the intermediate goods sector, are evenly allocated among all researchers in the form of wages, w_L:

$$w_L = P_A\frac{\dot{A}}{(1-\phi)L} \tag{1.14}$$

Hence, there are no profits in the research sector. In a labor market equilibrium, wages in the final output sector and in the research sector are equal.

Households Households draw their instantaneous utility, u, from consumption. Per capita consumption, c, is inter-temporally additive and is discounted at rate ρ. The discounted lifetime utility is denoted as U.

$$U_t = \int_0^\infty u\left(c(t)\right) e^{\rho t} dt, \qquad \rho > 0 \tag{1.15}$$

Households have constant elasticity preferences:

$$u(c) = \frac{c^{1-\theta} - 1}{1-\theta}, \qquad \theta > 0 \tag{1.16}$$

θ is the elasticity of marginal utility.
They maximize their lifetime utility given their budget constraint:

$$\max \int_0^\infty u\left(c(t)\right) e^{-\rho t} dt \tag{1.17}$$

$$\text{subject to:} \qquad \dot{b} = rb + w - \frac{P_A \dot{A}}{L} + \frac{A\pi}{L} - c - nb \tag{1.18}$$

$$\text{with:} \qquad b(0) = b_0$$

where w denotes the equilibrium wage in the research and the final goods sector.
We can thus set up the Present-Value-Hamiltonian:

$$H = \left(\frac{c^{1-\theta} - 1}{1-\theta}\right) e^{-\rho t} + \lambda \left(rk + w - \frac{P_A \dot{A}}{L} + \frac{A\pi}{L} - c - nk\right) \tag{1.19}$$

and the first order conditions:

$$H_c = 0 \qquad \Leftrightarrow \qquad c^{-\theta} e^{-\rho t} = \lambda \tag{1.20}$$

$$H_b = -\dot{\lambda} \qquad \Leftrightarrow \qquad (r-n)\lambda = -\dot{\lambda} \tag{1.21}$$

This leaves us with the Keynes-Ramsey rule:

$$\frac{\dot{c}}{c} = \frac{1}{\theta}(r - n - \rho) \tag{1.22}$$

Steady State Growth On the balanced growth path, which is also called the steady state, the growth rates of all variables in the model are constant. For instance, the rate of total factor productivity growth is constant:

$$\frac{\frac{\partial\left(\frac{\dot{A}}{A}\right)}{\partial t}}{\frac{\dot{A}}{A}} = 0 \tag{1.23}$$

and, thus:

$$\frac{\dot{A}}{A} = g_A = \frac{\eta_L}{1-\eta_A}n \tag{1.24}$$

In the steady state, the rate of total factor productivity growth in equation (1.24) equals the growth rate of income per capita, y, of the capital stock per capita, k, and of consumption per capita: $g_A = g_y = g_k = g_c$ (see C. I. Jones (1995a)). The growth rate of the economy does not depend on its size as in P. M. Romer (1990), but on population growth[6].

The system of four differential equations and one static constraint To sum up, the model is fully determined by a system of four differential equations, which describe the evolution of the stock of intangible knowledge, A, the physical capital stock, K, household consumption, C, and the patent price for new blueprints, P_A. All these equations hold at every point in time. This includes time periods outside the steady state.
The evolution of the intangible knowledge stock is given in equation (1.12). The evolution of the physical capital stock is determined by household saving. Households spend their income either on consumption or invest into the physical capital stock. Gross investments are reduced by the depreciation of existing capital.

$$\dot{K} = Y - C - \delta K \tag{1.25}$$

The evolution of total consumption can be derived from the Keynes-Ramsey rule in equation (1.22).

$$\dot{C} = \frac{C}{\theta}(r - \rho - n) + nC \tag{1.26}$$

[6]See Eicher and Turnovsky (1999) for a growth model based on which the authors derive general conditions for non-scale balanced growth.

The transition of the patent price can be derived by differentiation of equation (1.13) with respect to time[7].

$$\dot{P}_A = rP_A - \pi \tag{1.27}$$

Using equations (1.4) and (1.14), a static equation guarantees wage equality:

$$\sigma \frac{Y}{\phi L} = P_A \alpha_J A^{\eta_A} [(1-\phi)L]^{(\eta_L - 1)} \tag{1.28}$$

In order to solve this system of four simultaneous differential equations and one static constraint, A, K, C and P_A have to be transformed into stationary variables whose growth rates on the balanced growth path are zero (see appendix A for the transformed system of differential equations and static constraints).

1.4 The technical approach

We calibrate the Jones model such that the sum of the squared residuals between the observed and the simulated income is minimized. Numerically, we use the relaxation algorithm by Trimborn, Koch, and Steger (2008) to solve equations (1.12) and (1.25) through (1.28) simultaneously. Using this algorithm, the growth model is solved in continuous time from minus infinity to infinity. Before our time frame of calibration sets in, the model is assumed to be in a steady state. In 1850 transitional dynamics set in due to a changing population growth rate and a changing output elasticity. After 2100 these parameter values stay constant again, such that the model converges into a new steady state. A big advantage of this algorithm is that we do not need to impose initial and end-point conditions on the model. Yearly changes in the population growth rate and the production elasticity are like a series of small, periodical and exogenous shocks that we impose on the model.

All parameter values from the Jones model are listed in table (1.1). We choose our parameters of calibration to be η_A and η_L, which determine the growth rate of the economy (see equation (1.24)). Both are constrained by the common non-negativity and non-increasing-returns-to-scale assumptions. In addition, the choice of the growth model itself poses some assumptions on future projected growth. For instance, on the balanced growth

[7] The derivation of equation (1.27) involves the application of the product rule and the fundamental theorem of calculus.

path the Jones model predicts constant rates of growth, while in the scenario literature growth rates especially in developed countries are often assumed to decline.

Table 1.1: Parameters of the model

Symbol	Description	Value	Source
ρ	discount factor	0.015	DICE 2013
θ	elasticity of marginal utility	1.45	DICE model 2013
(1-σ)	capital share in final output production	non-const.	see section (1.2)
δ	capital depreciation rate	0.1	DICE model 2013
n	population growth rate	non-const.	own calculations
α_J	technological shift factor	1	assumption
$0 \leq \eta_A \leq 1$	productive elasticity of technology	-	parm. of calibration
$0 \leq \eta_L \leq 1$	productive elasticity of labor	-	parm. of calibration

For the construction of confidence intervals of future economic growth trajectories, we need to derive the joint distribution of η_A and η_L. We assume that both follow a multivariate normal distribution. From this distribution we draw a representative sample of random values for both parameters. We solve the Jones model for every drawn combination of η_A and η_L and derive the corresponding income per capita projection. The 95 % confidence interval of future income per capita represents all forecasts except for the 2.5 % highest and lowest.

The variance-covariance matrix of η_A and η_L is derived from the Hessian of the log-likelihood function, as in Amemiya (1985). We assume that the observed value of past income is a composite of a long-run time trend, which is described by the Jones model, and an identical, independent and normally distributed error term, ϵ_i. Consequently, income is also normally distributed. It is rather bold to assume that the error term is white noise, given that observed income is a time series.
The density function of income, y_i, with an expected value of μ_i and variance σ^2, is thus[8]:

$$y_i \sim N(\mu_i, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{\epsilon_i^2}{2\sigma^2}} \tag{1.29}$$

The likelihood of the whole sample is:

[8]The residual in equation (1.29) is: $\epsilon_i = y_i - \mu_i$. It is the difference between observed and simulated income. The variance, σ^2, is also the variance of ϵ_i.

$$\mathcal{L} = \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{\epsilon_i^2}{2\sigma^2}} \tag{1.30}$$

with N representing the number of forecasts. We derive the corresponding log-likelihood function:

$$ln\mathcal{L} = \frac{-N}{2} ln(2\pi) - \frac{N}{2} ln(\sigma^2) - \frac{1}{2\sigma^2}(\epsilon'\epsilon) \tag{1.31}$$

and the hessian, H, of the log-likelihood function. The Hessian measures the curvature of the log-likelihood function. It is a matrix of second derivatives with regard to our parameters of calibration, η_A and η_L, and to the variance of the error term. We summarize these values in κ and refer to η_A and η_L as η:

$$\kappa = \begin{bmatrix} \eta \\ \sigma^2 \end{bmatrix} \tag{1.32}$$

$$\begin{aligned} H = \frac{\partial^2 ln\mathcal{L}}{\partial\kappa\partial\kappa'} &= \begin{bmatrix} \frac{\partial^2 ln\mathcal{L}}{\partial\eta\partial\eta'} & \frac{\partial^2 ln\mathcal{L}}{\partial\eta\partial\sigma^2} \\ \frac{\partial^2 ln\mathcal{L}}{\partial\sigma^2\partial\eta'} & \frac{\partial^2 ln\mathcal{L}}{\partial\sigma^2\partial\sigma^2} \end{bmatrix} \\ &= \begin{bmatrix} -\frac{1}{2\sigma^2}\frac{\partial^2(\epsilon'\epsilon)}{\partial\eta\partial\eta'} & \frac{1}{2\sigma^4}\frac{\partial(\epsilon'\epsilon)}{\partial\eta} \\ \frac{1}{2\sigma^4}\frac{\partial(\epsilon'\epsilon)}{\partial\eta'} & \frac{N}{2\sigma^4} - \frac{(\epsilon'\epsilon)}{\sigma^6} \end{bmatrix} \end{aligned} \tag{1.33}$$

The covariance matrix of κ is the inverse of the negative expected value of the Hessian:

$$Cov = \left[-E[H]\right]^{-1} \tag{1.34}$$

Since the expected value of ϵ_i is zero, the expected value of the first derivative of the sum of the squared residuals with respect to η is zero, too. Thus, η is not correlated with the variance of the residual, σ^2. In order to calculate the Hessian and the Covariance matrix, we determine all gradients numerically.
Once we have estimated η_A and η_L, by minimizing the residual between the observed and the simulated data, and their variance-covariance matrix as described in this section, we draw a large sample of 150 random parameter values for η_A and η_L and derive their corresponding income per capita projection as well as their confidence interval until 2100[9].

[9] η_A and η_L are truncated. However, this is not problematic, since our estimates for η_A and η_L, including their expected variation, are not close to their boundary values.

1.5 Results

We calibrate the Jones model of economic growth by minimizing the squared residual between observed and simulated income from 1850 to 2008. Our parameters of calibration are the production elasticities η_A and η_L in the research sector. Table 1.2 shows all point estimates for η_A and η_L in each region and their corresponding variances and covariances. We achieve a good fit for all four regions: Europe as a whole, Eastern Europe, Western Europe and the Former USSR. This means that the average values of the residuals between observed and simulated income are close to zero. The variances of our parameter estimates give us a crude idea of the uncertainty that is tied to future income per capita growth. Interestingly, the variances of η_A and η_L in Europe are smaller than when Western and Eastern Europe are calibrated on their own. This might indicate that the sources of variation in both regions offset rather than amplify each other.

Moreover, we have undertaken an attempt to calibrate income on the country level. However, in 10 out of 22 countries we achieve a dis-satisfactory fit (see table A.2 in appendix A for our calibration results). In those countries calibrated income remains below observed income between 1850 and 2008 and η_A and η_L are underestimated. This is the case, because for these countries our model does not converge for higher values of η_A and η_L. Given the observed and expected paths of population growth and the assumed path of capital shares between 1850 and 2100, there is no optimal path of consumption that provides a better fit to past income. For these countries, a different set of exogenously given parameter values would allow for a better fit between simulated and observed income. Clearly, capital shares and depreciation rates are not the same in all countries all over Europe. However, to maintain comparability between countries, we do not adjust parameter values for particular countries. This aspect of country-specific calibrations could be a potential target for further research.

Table 1.2: Coefficients, variances and covariances

Country	η_A	$Var(\eta_A)$	η_L	$Var(\eta_A)$	Cov
Western Europe	0.71	3.919e-04	0.73	5.561e-04	-3.759e-04
Eastern Europe	0.52	0.0021	0.80	0.0013	-7.21e-04
Europe	0.67	2.738e-04	0.75	3.141e-04	-1.513e-04
Former USSR	0.37	0.0025	0.78	0.0027	1e-05>Cov>-1e-05

In the Fromer USSR, the Covariance between eta_A and eta_L is not distinguishable from numerical imprecision.

In addition, we construct future carbon emission scenarios according to the Kaya identity in equation (1.1). These are based on UN population forecasts, our own income per capita forecasts, derived from the calibration of the Jones model, and energy as well as emission intensity projections. The resulting future emissions and their components are summarized for Western Europe, Eastern Europe, Europe as a whole and the Former USSR in figures 1.4 to 1.7. Future economic growth substantially differs between these regions. Table 1.3 summarizes the mean, upper and lower bound of annual growth rates (averaged over the 21st century). Projected growth is clearly strongest in Western Europe, followed by Eastern Europe and the Former USSR. Since 1850 Western European economies have, on average, been growing at higher rates than Eastern European economies and the Jones model projects similar growth patterns into the future. In addition, according to UN forecasts of population growth, the Eastern European population will be strongly reduced after 2000 (see figure 1.5) while the Western European population size will stay relatively constant (see figure 1.4). In the Jones model this has a negative impact on Eastern Europe's growth rate compared to Western Europe's growth rate (see equation (1.24)).
The confidence intervals on income are not symmetric. In Western Europe, the distribution of income displays a strong positive skewness[10], while in Eastern Europe it has a smaller but negative skewness. In addition, the confidence intervals do not have the same width. The variation around the mean in Eastern Europe is much larger than in Western Europe, reflecting greater economic (and other) turmoil in the past.

Table 1.3: Forecasted annual average growth rates in %

Region	*Mean*	*Upper bound*	*Lower bound*
Western Europe	1.81	1.83	1.58
Eastern Europe	1.06	1.49	0.85
Europe	1.61	1.83	1.39
Former USSR	1.05	1.31	0.71

Since energy and emission intensities are outside of our model, we choose three constant rates of their annual reduction for which we calculate future paths of carbon emissions. These rates are 0.5 %, 0.75 % and 1 %.
In Eastern Europe (figure 1.5) and the former USSR (figure 1.7), we observe a decline in the

[10]The very strong and positive skewness of the distribution of income in Western Europe might be partially driven by the circumstance that for some variations of the free parameter values there is no solution to the Jones model. However, since the Jones model does not have a solution for these parameter values, it is not possible to quantify this effect. The asymmetry of the other confidence intervals is data driven.

past energy use and also in carbon emissions from around 1989 to 2000, which is probably linked to the collapse of the communist system. However, to sustain comparability between regions, we use the same rates of reduction of future energy and emission intensities for all regions and assume that the collapse of the communist system was only a temporary shock.

Overall, we find that, compared to current levels and given our choices regarding the energy and emission intensity, carbon emissions in Western Europe and Europe as a whole are projected to increase throughout the next century, while in Eastern Europe and the Former USSR their path declines. Thus, we conclude that, according to the Jones model, Western Europe would have to decrease its energy and emission intensities by approximately 1 % per year, while in Eastern Europe and the Former USSR an annual reduction of 0.5 % would be sufficient, in order to keep future carbon emissions roughly constant and to offset the effects of future economic growth. Since Western Europe is predicted to grow faster, it has to engage in heavier emission reduction schemes than Eastern Europe, in order to keep its future carbon emissions constant. Notably, Western Europe's energy and emission intensities are below the ones in Eastern Europe in 2008, however, its carbon emissions per capita in 2008 are approximately 21 % higher. In addition, emission forecasts in Eastern Europe and Former USSR countries reveal larger confidence intervals due to higher uncertainty in income per capita growth.

In those countries where we are able to achieve a good fit of simulated to observed income, the average confidence interval of income per capita is smaller than those of the regional forecasts. Consequently, the derived variation in future carbon emissions on the country level is also smaller than on the regional level. In these cases, the usage of country level data could add information to the process of calibration and thus lead to more precise forecasts.

We do not take trade and carbon leakage into account. Peters and Hertwich (2008), for instance, find that in 2001 in a number of European countries emissions which were embodied in imports were significantly higher than in exports. Thus, we expect that our point estimates underestimate the true value of future carbon emissions caused by consumption.

1.6 A comparison with the SSP framework

Together with the Fifth Assessment Report of the Intergovernmental Panel on Climate Change (IPCC AR5) a new framework of so-called Shared Socioeconomic Pathways (SSPs)

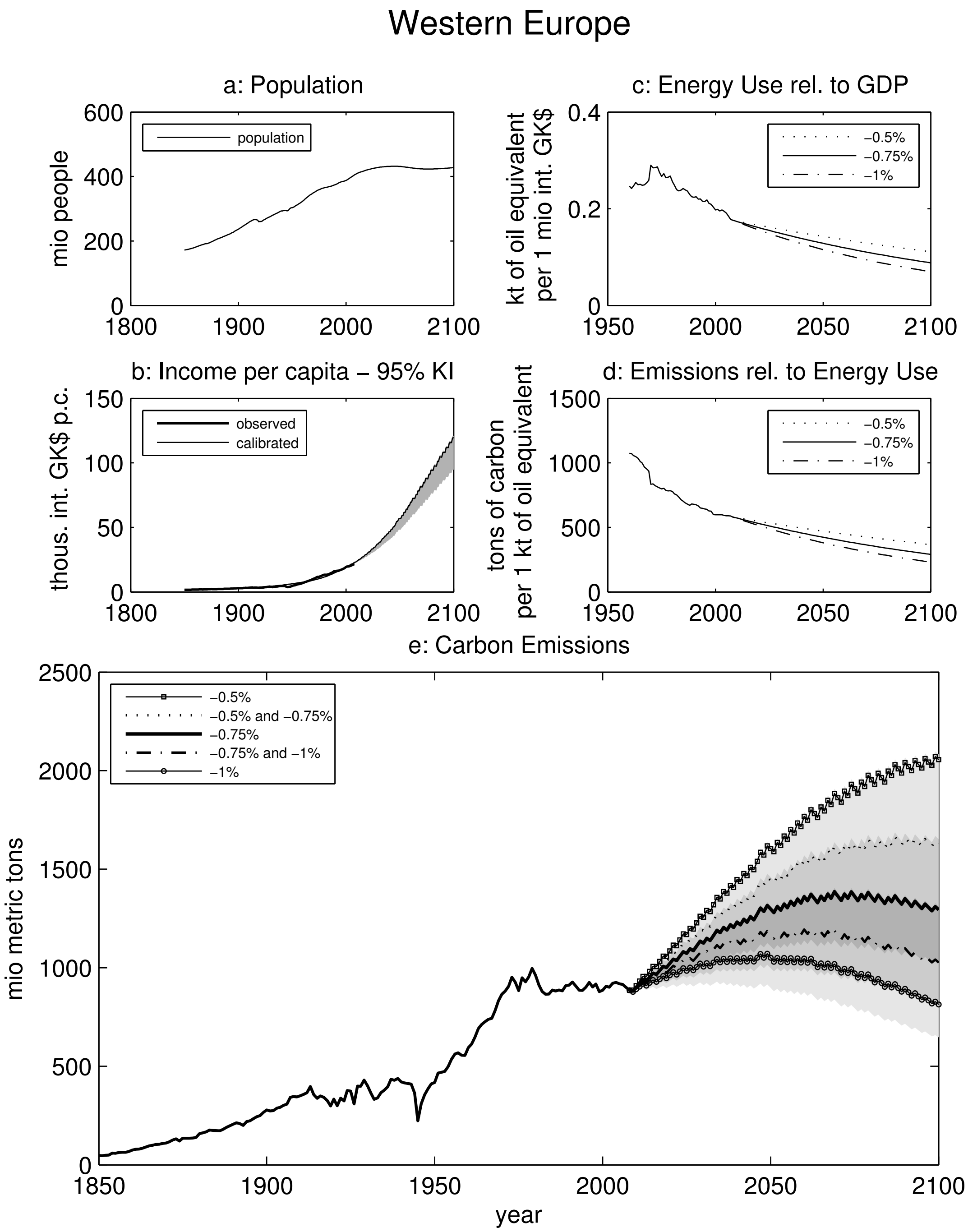


Figure 1.4: Kaya decomposition of carbon emissions in Western Europe
b: Income per capita after 2008 is surrounded by a shaded area representing the 95 % confidence interval. c, d: Energy use rel. to GDP and emissions rel. to energy use after 2008 were extrapolated assuming annual reductions of 0.5 %, 0.75 % (solid line) and 1 %. e: Carbon emissions after 2008 were calculated for the following annual energy and emission intensity reductions: 0.5 %, one 0.5 % and the other 0.75 %, 0.75 % (solid line), one 0.75 % and the other 1 % and finally 1 % (top to bottom). The shaded areas represent the 95 % confidence intervals of the respective annual energy and emission intensity reduction with regard to the uncertainty in income per capita. Thus, the darkest area represents the 95 % confidence interval for an annual reduction of the emissions and energy intensity by 0.75 %.

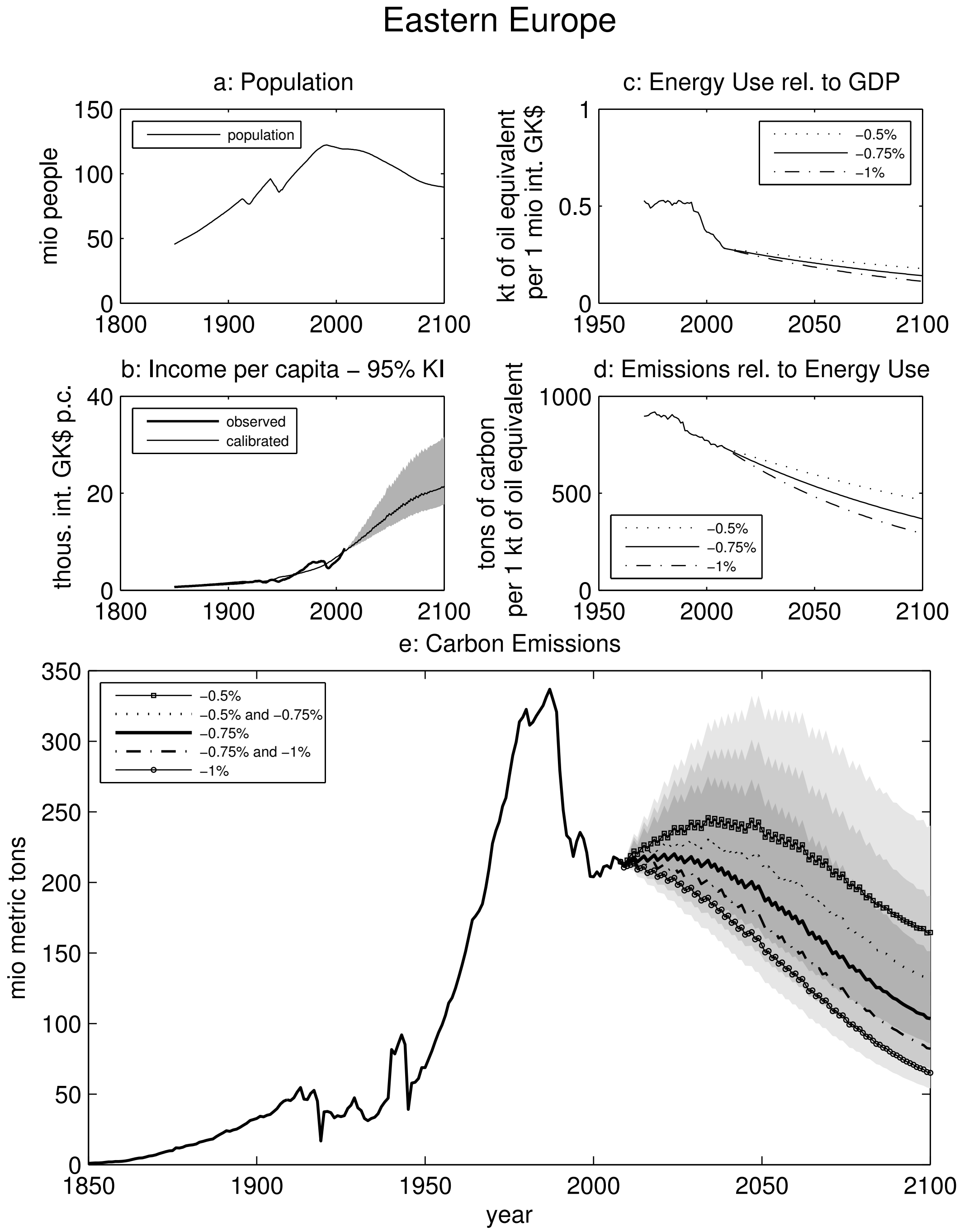


Figure 1.5: Kaya decomposition of carbon emissions in Eastern Europe
See figure 1.4 for explanations.

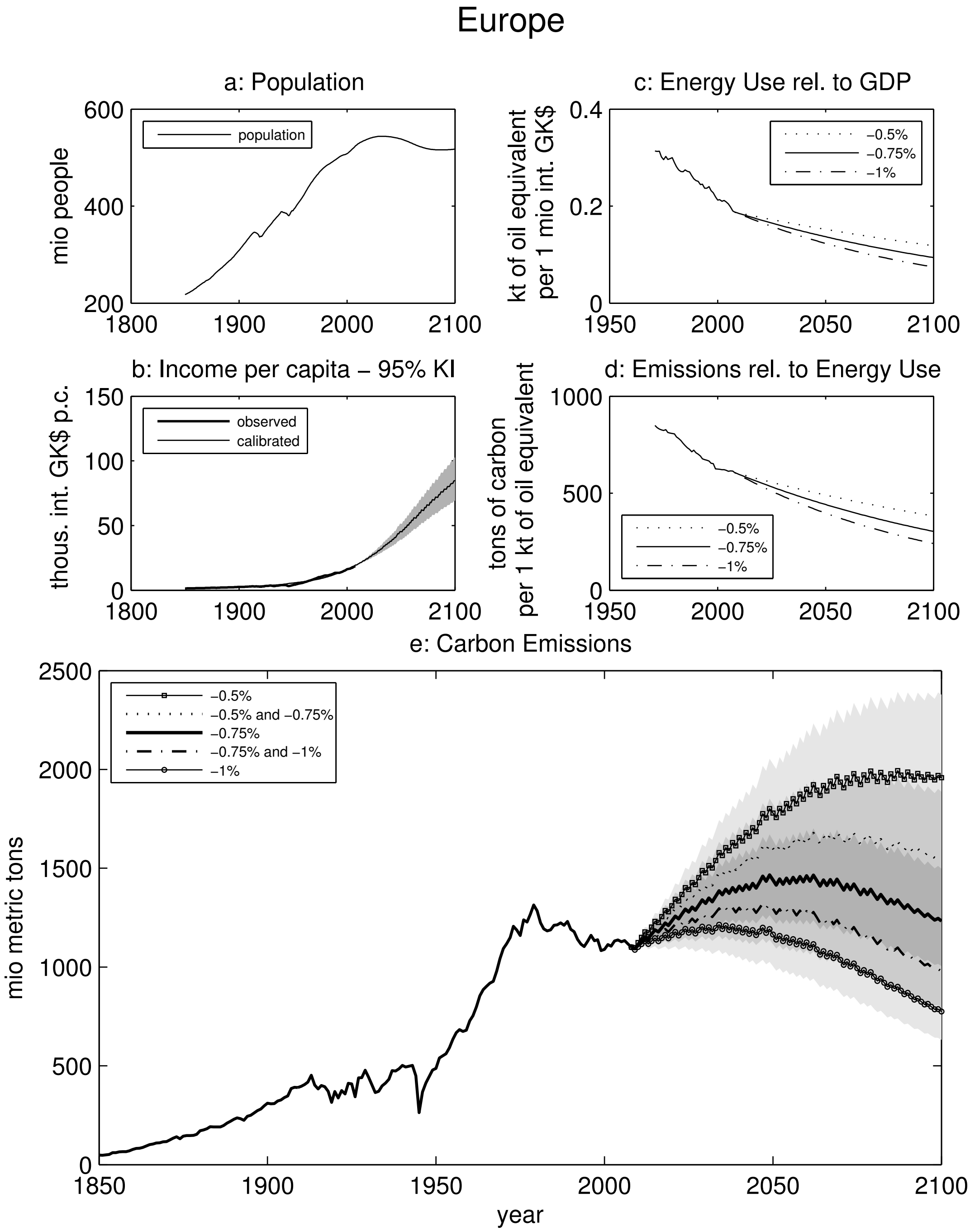


Figure 1.6: Kaya decomposition of carbon emissions in Europe
See figure 1.4 for explanations.

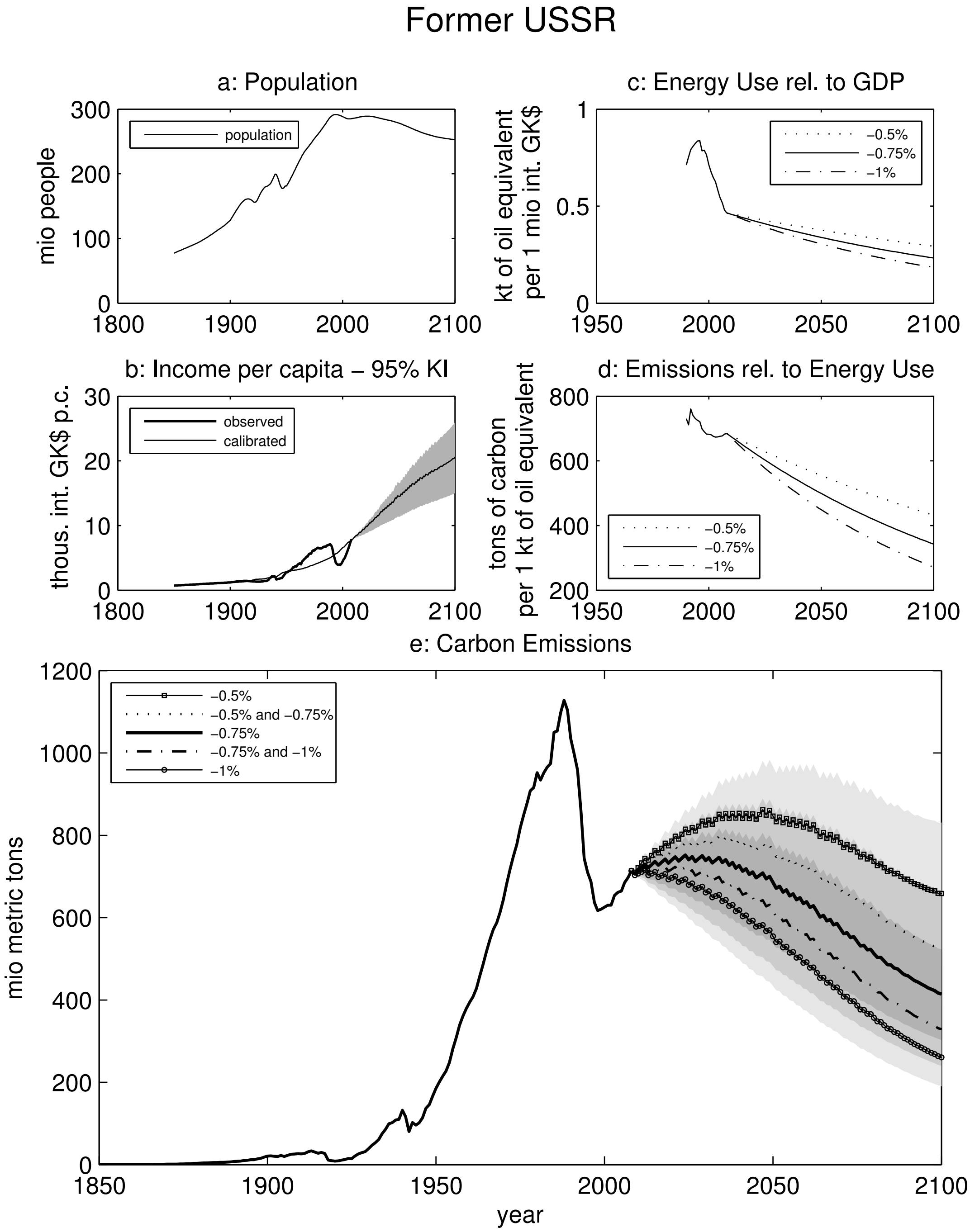


Figure 1.7: Kaya decomposition of carbon emissions in the former USSR
See figure 1.4 for explanations.

and Representative Concentration Pathways (RCPs) was adopted (see Field et al. (2014) and SSP Public Database (2014)). In this section we compare our own income projections with the SSP pathways.
The SSPs comprise of worldwide GDP, population and urbanization data until 2100 grouped into 32 regions. All three time series are available for 5 different scenarios. SSP2 is the 'Middle of the Road' scenario, with which we compare our results. Three modeling teams, one each at IIASA, OECD and PIK, have made projections for GDP. Each team used models with different growth dynamics, although they are comparable in the sense that in all three models growth is driven by an increase in primary inputs, labor-augmenting efficiency improvements and total factor productivity improvements (O'Neill et al. (2017)). However, the degree to which these forces influence economic growth differs between models. The IIASA model places more emphasis on economic growth induced by human capital growth, while the PIK model places more weight on long run total factor productivity growth. In the OECD model, after 2016, growth is modeled in a similar way as in the PIK model with total factor productivity being the dominant source of future economic growth. Before this date, economic growth is calibrated towards projections made by the OECD, the IMF and the World Bank. In the SSP2 scenario, all three groups implemented medium total factor productivity growth and a medium speed of income convergence between countries. In figures 1.8 through 1.10 we compare the three GDP pathways with our own income projections for Western Europe, Eastern Europe and the Former USSR[11].

For Western Europe, the income projection of this paper lies above all three GDP pathways projected by IIASA, OECD and PIK. In Eastern Europe and the Former USSR, on the contrary, our own projection lies below the other three pathways. Since we calibrate the growth dynamics of the Jones model to data since 1850 and project these into the future, we pick up stronger growth rates in Western Europe than in the other two regions.

Altogether, while the SSP2 pathways are constructed such that income across countries and regions converges, the growth dynamics of the Jones model lead do diverging incomes.

[11]The SSP data on GDP is corrected for purchasing power parities and given in 2005 US \$. Since our own projections are made in 1995 GK \$, we normalize our own time series such that income in 2010 is the same as in the SSP data. Our own region specifications are very similar to those which were used for the SSP pathways. See figure A.2 in appendix A for a comparison.

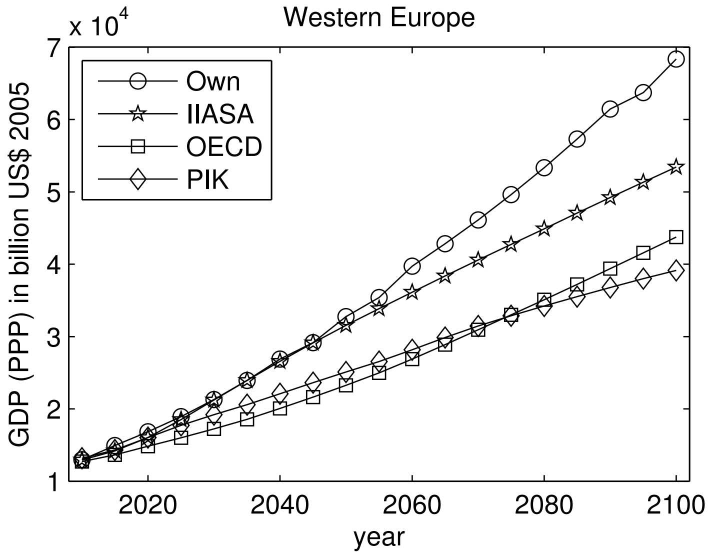


Figure 1.8: A comparison of the SSP2 pathways ('middle of the road scenario') in Western Europe by IIASA, OECD and PIK with our own results. (Source: SSP Public Database (2014) and own calculations)

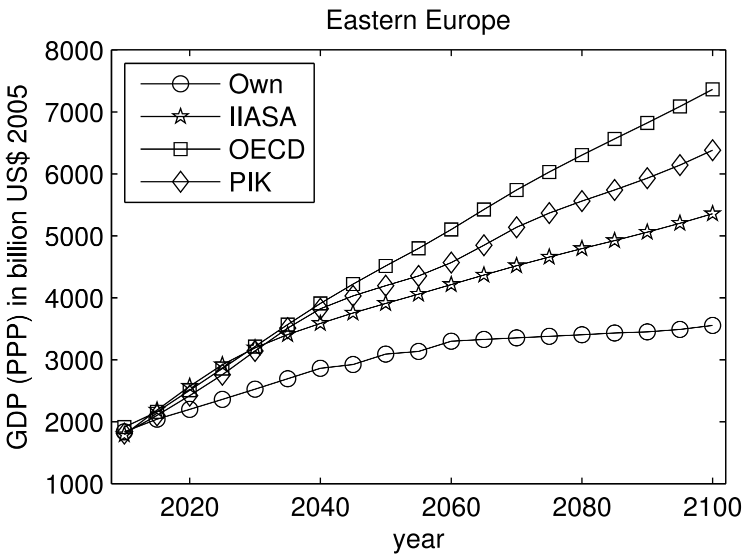


Figure 1.9: A comparison of the SSP2 pathways ('middle of the road scenario') in Eastern Europe by IIASA, OECD and PIK with our own results. (Source: SSP Public Database (2014) and own calculations)

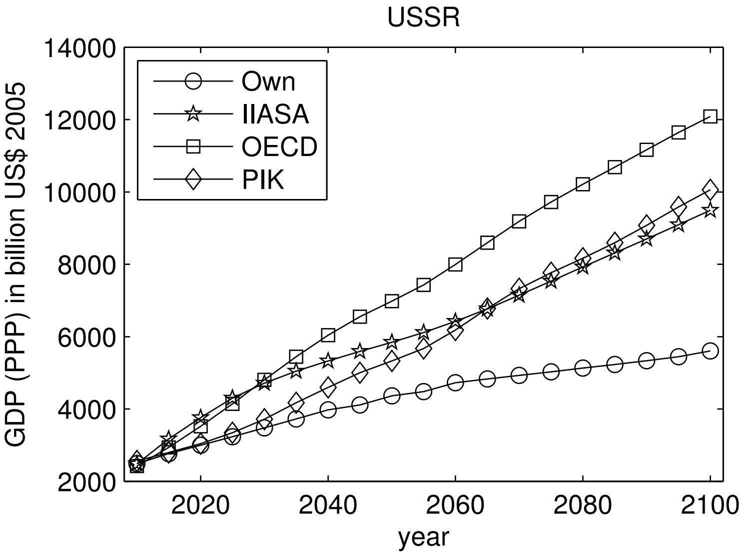


Figure 1.10: A comparison of the SSP2 pathways ('middle of the road scenario') in the Former USSR by IIASA, OECD and PIK with our own results. (Source: SSP Public Database (2014) and own calculations)

1.7 Sensitivity analysis

So far, we have discussed carbon emission projections in Europe and former USSR countries depending on a certain set of parameter choices. In this section, we will evaluate the sensitivity of our results towards the discount rate, ρ, and the output elasticity with respect to capital, $(1 - \sigma)$.

In the past, there has been an intense debate about different parameterizations of discount rates (see for instance Weitzman (1994)) and their moral dimension. In this study, we refrain from a discussion of this sort, but we do want to provide insights into the sensitivity of our results towards the discount rate. For high discount rates, households are less inclined to invest into the capital stock in order to raise their future income. A high discount rate, thus, leads to more consumption and lower savings today and to a lower level of future income per capita. Correspondingly, in figure 1.11, we observe that a higher discount rate leads to lower income per capita in 2100. This will also imply lower emissions in the future. Overall, the sensitivity towards the discount rate is rather low. For a variation of discount rates between 1 % to 3 %, η_A and η_L in figure A.1 (in appendix A) stay entirely constant, income per capita in figure 1.11 decreases by 4.7 % and the fit of our model, measured by the root mean squared error (RMSE) in figure A.2 (in appendix A), deteriorates slightly. The RMSE decreases by 1.8 %.

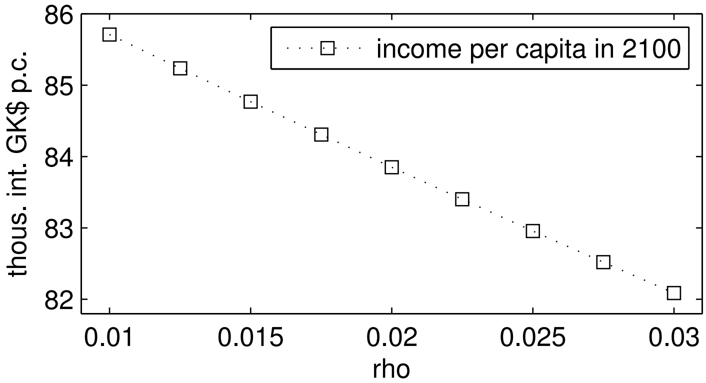


Figure 1.11: Sensitivity analysis towards the discount rate - Europe

In section 1.2, we have discussed some evidence which can be found in the literature for the assumption that, in the past, capital shares in developed countries have been increasing. The choice for the exact path of the output elasticity with respect to capital in this work, however, was rather ad-hoc, even if within a range of reasonable trajectories. We assume that the output elasticity with respect to capital had a size of 25 % in 1850 and that it increases to 45 % in 2100. In the literature, sensitivity analyses towards the output elasticity with respect to capital are rare. In this section, we increase the output elasticity with respect to capital from 25 % in 1850 to 30 % up to 50 % in 2100. We find that this choice has strong impacts on income per capita growth in figure 1.12. Income per capita in 2100 increases by roughly 60 % if the capital share amounts to 50 % in 2100 compared to 30 %. Taking into account that population growth rates in Europe will continue to decline, it is straightforward that increasing capital shares have a strong and positive impact on growth. Furthermore, the size of the output elasticity with respect to capital also has a strong impact on the optimal size of our free parameters η_A and η_L in figure A.3 and on the fit of our model to the data in figure A.4 (in appendix A). The fit of our model improves with a lower output elasticity with respect to capital in 2100. Output elasticities of more than 0.45 in 2100 yield an unproportionally large error.

1.8 Conclusions

In this chapter, we endogenize economic growth in the construction of carbon dioxide emission scenarios. Our calibration period is longer than our projection period and we derive statistically valid confidence intervals of growth. We show that the calibration of

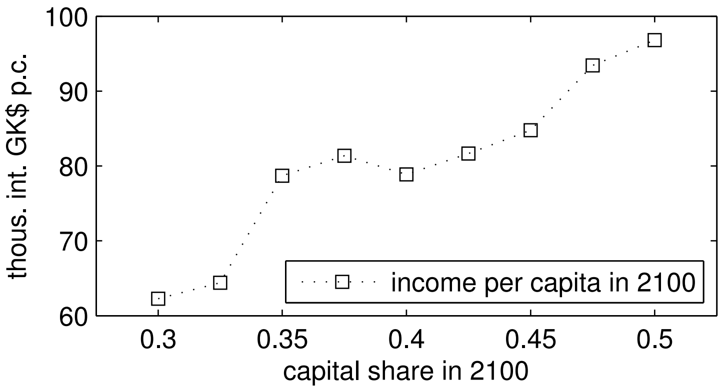


Figure 1.12: Sensitivity analysis towards the capital share - Europe

the Jones model leads to stronger annual average growth rates in countries with a higher current income per capita. In these countries the model picks up higher growth rates from the past and projects them into the future. In addition, in the Jones model steady state growth is exponential. This is in contrast to the SSP framework in the Fifth Assessment Report, where incomes are assumed to converge. As a consequence, we find that countries with a higher income have to lower their annual energy and emission intensities by more in order to offset the effects of their future economic growth and to keep their total carbon emissions constant. While Western Europe would have to decrease its energy and emission intensities by approximately 1 % per year, in Eastern Europe and the former USSR 0.5 % per year seem to suffice. If we regard both regions independently from each other, the same goal of keeping carbon emissions constant would require more radical policy measures to be taken in Western Europe than in Eastern Europe. Taking into account past reductions in the energy intensity in Western Europe (see figure 1.4), a continued yearly reduction by 1 % seems achievable. A yearly reduction of the emissions intensity by 1 % however, entails a reduction of the same by more than 50 % by 2100. In other words, in order to keep future carbon emissions in Western Europe relatively constant, it would have to substitute 50 % of its fossil energy supplies by non-fossil sources. This will only be feasible with severe political intervention. However, the NDC which was forwarded by the member states of the European Union in response to the Paris Agreement goes even further and voluntarily commits to reduce carbon emissions by 40 % below 1990 levels by 2030. This certainly shows that there exist very ambitious joint political efforts in Europe.
Further, we find weak evidence that conducting forecasts on the country level might add to the overall precision of our projections. When income per capita is calibrated on the

regional level, there is more uncertainty tied to the corresponding projections of future carbon dioxide emissions than to the sum of country-wise forecasts. However, since we made the same parameter choices for each country, our calibration of the Jones model to country-wise data is only plausible for a small number of countries. The sensitivity analysis reveals that our income and carbon emission projections are rather robust to the discount rate, while they are strongly dependent on the output elasticity with respect to capital. Although there has been an intense debate about the former, in the scenario literature, capital shares are typically assumed to stay at a constant level. This aspect could be worth considering in future research.

Chapter 2

A Bayesian approach towards the calibration of deterministic models of economic growth

2.1 Introduction

Models of long-run growth have initially been developed to explore the origins of growth and to explain cross-country differences in growth trajectories. The literature has suggested numerous potential sources of growth such as the availability of human capital and natural resources or a country's political and institutional stability. Originally, macroeconomic models of long-run growth were not so much developed for the prediction of growth paths into the distant future. On the contrary, Millner and McDermott (2016) have recently cast serious doubt on the credibility of the predictive power of long-run growth models in the context of environmental economics using the example of the Ramsey model. Nevertheless, in the environmental economics literature there has been a growing need for sound calibration techniques of growth models which go beyond the medium run and act as a forecasting device of macroeconomic trends. To model the future costs of climate change, we need to know more about carbon emissions in the long run and they will crucially depend on future economic growth and technological advancements. Growth models can serve as a stepping stone for more complex environmental models such as Integrated Assessment Models (IAMs). These models estimate the cost of future carbon emissions and as such integrate a climate model with a socio-economic component. Hence, it is important to develop a robust calibration technique for models of long run economic growth.

Since the majority of IAMs is deterministic, we develop a calibration technique for deterministic growth models. The intuition is that deterministic models of growth describe a long-run trend. Stochastic shocks to the economy, which may or may not be correlated, are absorbed by the residual between the observed and the simulated data. Therefore, we assume that the residuals follow a stochastic process. We suggest a Bayesian inversion technique to elicit the distributions of our parameters of calibration and to project the confidence intervals of future income and consumption shares. To integrate over Bayes' law we use a Markov chain Monte Carlo (MCMC) algorithm. Our aim is to present a standardized approach towards the calibration of deterministic models of long-run growth, which is tractable and easy to adapt. Our contribution is therefore methodological. Bayesian approaches towards model calibration have already been described in the Real Business Cycle (RBC) and the Dynamic Stochastic General Equilibrium (DSGE) literature (see for instance Fernández-Villaverde (2010)). The focus of this literature, however, lies on the calibration of stochastic macroeconomic models, which are mostly used to assess stochastic moments of time series and to project economic activity into the near future.

The origins of economic growth are complex and they are studied in a long tradition of growth models. The first model of economic growth, which had a neo-classical production function at its heart was the Solow (1956) and Swan (1956) model. In this model capital and labor are used as inputs to produce a uniform good for consumption and reinvestment. This approach has sparked a whole new thread of research, which seeks to explore how growth is fostered and maintained. These are, for instance, the Ramsey (1928), Cass (1965) and Koopmans (1965) type of neoclassical growth models, which emphasize the optimization of household utility and endogenize the savings rate. Models of endogenous technological change followed later. A very prominent example is by P. M. Romer (1990), who endogenizes investments into R&D, which expand the horizontal variety of inputs and thereby raise productivity. An other class of thoroughly investigated endogenous growth models are Schumpeterian type models of creative destruction such as in Aghion and Howitt (1992). In these models vertical innovations replace incumbent products and thereby increase productivity.

We show how our procedure is applied to a standard Ramsey model of exogenous growth as well as the endogenous growth model by Aghion & Howitt. Because in the past there have been attempts to endogenize the growth component in IAMs (see for instance the

ENTICE model by Popp (2004) or an extension of the DICE model by Dietz and Stern (2015)), we also show as an example how a model of endogenous growth can be calibrated. As a side-effect we can demonstrate that our Bayesian approach can be applied to a variety of deterministic growth models without modifying its essence. Since the majority of IAMs is deterministic, we believe that this is a useful exercise. The resulting growth trajectories until 2050 from both models are similar with an average growth rate of the median projection of 2.3 % in the Ramsey model and 2.2 % in the Aghion & Howitt model. Our parameters of calibration are well identified and have clear-cut distributions. Hence, we conclude that our approach is highly flexible and can be adopted for a wide range of different growth models.

2.2 Calibration versus estimation

Throughout the past approximately three decades model estimation has constantly gained in importance, while calibration procedures remain vague and although they are applied very frequently, the precise calibration technique and its accompanying assumptions are fully and explicitly specified in a few cases only. Therefore, in this article we propose a standardized Bayesian approach towards calibration, which is applicable to a big variety of deterministic models of economic growth. Due to its schematic structure and its very high degree of adaptability even to complex models of economic growth, we believe that this approach will help to render future research in macroeconomic modeling and in particular in the Integrated Assessment Modeling literature more transparent.

The gist of model calibration as well as model estimation is to make a parameter choice such that a selection criterion is optimized. The selection criterion is based on the goodness of fit of the model solution to the observed data. However, both differ in their choice of selection criteria and in their methods. Estimation relies on statistical inference to determine how strongly we can believe in the ability of a particular econometric model to resemble the real world given a set of observed data. In contrast, what is commonly understood when talking about calibration, is to strive for consistency of a theoretical model with observed data. It involves setting parameter values such that a benchmark data set can be reproduced as model solution, or such that particular moments in the model solution are close to analogous moments in the benchmark data. To reduce the degrees of freedom it is common practice to take some parameter values from unrelated studies. However, in economics there are no collections of universally true parameter values available. Therefore, any calibration exercise with exogenous parameter values is conditional on these same parameters.

Dawkins, Srinivasan, and Whalley (2001) observe that the economic model that is underlying an estimation model is often very limited in its structure, while the estimation model's statistical extension can be very sophisticated. This allows econometricians to perform numerous statistical tests to assess a model's performance. Calibrators, on the other hand, strive to maintain the richer model structure of economic models which are well established in the theoretical literature. These models may be highly complex or non-linear. In a way, the growing popularity of calibration results stems from an increasing need by policy makers in the application of more complex and non-linear models which prohibit estimation or testing. Dawkins, Srinivasan, and Whalley (2001) claim that, in the past, the economics underlying econometrics and pure theory have drifted apart and that calibration techniques may be a way to build a bridge between both.
Hoover (1995) describes this dichotomy as follows: Estimators rely on statistical inference and pursue horse races between competing models and theories. If a model is rejected, this casts doubt on the validity of its underlying theory. On the contrary, calibrators derive parsimonious, stylized models from theory. They are interested in extracting information from their model as best they can. If simulated model outcomes don't match actual data to a desired degree, this wont lead to a rejection of the model but rather to more refinements until the match is satisfying. Since calibrated models are not subject to formal testing, they hold little if any information regarding their validity. As Canova (2007) puts it, when calibrating a theoretical model, we do not believe, that the model reflects the data-generating process of the observed data. On the contrary, we acknowledge that a theoretical model is highly stylized and does not capture all relevant properties of the observed data and their stochastic components. Consequently, model structure can have an unquantifiable impact on the inferred implications and policy recommendations. Hence, the choice of model is critical in calibration. However, as long as it is common practice to base model selection on the theoretical literature, standardized criteria for model elicitation will be unattainable.

In the macroeconomic literature, the frequent use of calibration procedures came up together with real business cycle (RBC) models. These models constitute a flexible approach towards the analysis of fluctuations and business cycles in the short and medium run. The essence of the traditional calibratory approach of RBC models is outlined in a seminal article by Kydland and Prescott (1982) - although at the time they did not call it calibration. In their pioneering work the authors suggest to use a Neoclassical growth model and combine it with stochastic technological shocks to study business cycles. Their ideas have sparked a whole new field of research, the essence of which is described in Kydland and Prescott (1991) and Kydland and Prescott (1996). More recently, practitioners have turned to Bayesian estimation and inference, which are better suited for a larger number of endogenous parameter values and which give rise to new perspectives on the assessment of a models empirical validity. Karagedikli et al. (2010) give a coherent historical account on the evolution of estimation techniques of RBC and DSGE models and how they were followed by calibratory approaches, especially in the Bayesian strand of the literature.

Taking the calibration of macroeconomic models from the medium to the long run, the literature is less abundant. In the long run we are foremost interested in persistent growth effects. While in the RBC and DSGE literature these growth trends are filtered from the data (see King and S. T. Rebelo (1993) for a discussion), they are exactly the object of interest in the long run. Stokey and S. Rebelo (1995) is an insightful meta-analysis of growth effects of flat-rate taxes. In the process the authors compare the calibratory approaches towards endogenous growth models in Lucas (1990), L. E. Jones, Manuelli, and Rossi (1993) and King and S. Rebelo (1990). However, there is no unified calibration technique apparent from these works.

2.2.1 A Bayesian approach

In every estimation or calibration exercise there are three sources of uncertainty: the model specification itself, its parameterization and the observed data (see DeJong, Ingram, and Whiteman (2000)). In frequentist statistics the model parameterization is perceived as fixed, while the observed data is treated as random. The aim of conventional statistics is to undertake inference about model parameters, which leads to their acceptance or rejection at a specifically expressed degree of uncertainty. In Bayesian statistics, on the other hand, the model parameters are random variables and the aim is to infer their likeliest distribution, given the data and some prior information about the parameterization of the model. Thus, the Bayesian approach turns away from being a selection device to find the "true" model or parameter value. It rather determines an optimal parameter space given a certain model. This characteristic improves our ability to apply a particular model for policy analysis.

Among the first authors who emphasize the uncertainty which is tied to the parameterization of a theoretical model and therefore turn to a Bayesian approach for the calibration of an RBC model, outlined by King, Plosser, and S. T. Rebelo (1988), are DeJong, Ingram, and Whiteman (1996). They explicitly draw a line to a common practice in calibration, which applies point-mass priors to parameters of the theoretical model and therefore implicitly assumes that the only source of uncertainty lies in the sampling error. On the contrary they suggest to express a researcher's uncertainty over his prior beliefs about model parameters of the theoretical model by stipulating a prior distribution. In addition they stipulate priors for a statistical model which is assumed to reflect the data generating process of the observed time series. Thereupon, they assess model fit by assessing the proximity and degree of overlap of the implied posterior distributions of those parameter values describing the real data and those describing the simulated data from the model.

The fact that the formulation and calibration of RBC and DSGE models has become a systematic discipline was strongly supported by the fast development of Bayesian techniques and more importantly their computability. Fernández-Villaverde (2010) and An and Schorfheide (2007) give a stringent overview of their evolution and how economists have solved theoretical and empirical challenges along the way. There are two distinct problems, which arise specifically in a Bayesian approach: first that the likelihood function of a fully specified macroeconomic model is very likely to be unknown and second that the posterior distribution of the model parameters cannot be expressed analytically.

In their work DeJong, Ingram, and Whiteman (2000) show the essence of how these problems are overcome in the literature. They calibrate a neoclassical RBC model with a stochastic component and approximate the likelihood function of the non-stochastic steady state of their model numerically. Using a Markov chain Monte Carlo algorithm the authors can compute a large random sample, which is representative for their posterior. They compare the ability of their model to forecast macroeconomic time series with that of a Bayesian VAR and conclude that the predictive qualities of both are remarkably similar.

In comparison to these previous approaches, in this chapter we calibrate deterministic models of long run growth. For this reason, the likelihood function is not derived from the economic model itself, but from its in-sample forecasting error. Since deterministic growth models neglect the stochastic nature of their growth engine, this stochastic process is reflected in the gap between the observed data and the data generated from the model. As in DeJong, Ingram, and Whiteman (2000) we integrate over the resulting likelihood function using a Markov chain Monte Carlo algorithm.

2.3 The growth models

We choose to demonstrate our calibration technique based on the Ramsey (1928), Cass (1965) and Koopmans (1965) model. This is a well established workhorse model of exogenous growth with amenable characteristics for calibration. As D. Romer (2012) points out, the Ramsey-Cass-Koppmans model serves as a natural benchmark model with no market imperfections, homogeneous households and no links among generations. In addition, we show the flexibility of our approach by calibrating the Aghion and Howitt (1999) model of endogenous growth. However, there are potential pitfalls tied to the choice of an endogenous growth model.

Models of long-run growth have been developed to explore the origins of growth and to explain cross-country differences of growth trajectories. Their predominant purpose has not so much been for calibration and forecasting. This explains why a large number of growth models, especially in the recent literature on endogenous growth, are, due to their inherent characteristics, unsuitable for calibration. While many models lead to fruitful insights and policy advice in the theoretical literature, some of them are contradicted by time series evidence and are empirically problematic.

C. I. Jones (1999) divides the endogenous growth literature into three groups. In the first group technology is a nonrival good, because the number of people using a technology does not influence the cost of inventing it. Hence, holding the share of household spending into R&D fixed, a larger population size raises the number of researchers which bring about more technological change and thus economic growth. This group of model entails economies of scale, which means that the rate of income per capita growth accelerates as the population size and, consequently, the labor force grows or, equivalently, that larger countries have a higher GDP per capita growth. Well-known representatives of this type of model are P. M. Romer (1990), Grossman and Helpman (2001) and Aghion and Howitt (1992). In his earlier work C. I. Jones (1995b) had already argued that empirically economies of scale are undesirable. In various western countries since 1960 we have observed that even a strong increase in population size did not have a persistant impact on economic growth. In his subsequent work C. I. Jones (1995a) proposes a second class of model where the income per capita growth rate is proportional to the rate of population growth. Thus, this model entails a level effect of population size on income per capita, rather than a growth effect. As a consequence economic growth cannot be sustained without population growth and even turns negative as population sizes go down. Again, in the face of a diminishing population growth in western countries, this is not concurrent with the observed evidence. However, this issue is overturned by a third class of model where an increase in scale raises the number of product lines, but the economies growth depends on the amount of R&D invested into each product line. Thus, growth can be sustained even as the population shrinks. Prominent representatives are Aghion and Howitt (1999), Young (1998) and Dinopoulos and Thompson (1998). For this reason this last class of endogenous growth models is very suitable for calibration. The Aghion and Howitt (1999) model is a representative of this last type of model and since it is very accessible, we choose to base the demonstration of our calibration technique on this model.

Both models are set in continuous time, t. Capital letters denote flow or stock variables. For instant K denotes the capital stock. Lower case letters represent per capita units, such as k denotes the capital stock per capita. Cases with a hat represent efficiency units. $\hat{k}$ denotes the capital stock per effective unit of labor. The number of effective units per worker increases over time as a result of technological progress. Dots represents the derivative with respect to time.

2.3.1 The Ramsey-Cass-Koopmans model

There are two agents, households and firms, in the Ramsey-Cass-Koopmans (RCK) model. Homogenous households consume final output Y and invest in firms in order to maximize their current and future stream of discounted utilities. Final output is produced by a large number of identical, profit maximizing firms which sell their products in a competitive market. All firms are owned by households and thus their profits (if there were any) fully accrue to households. They all have access to the same Cobb-Douglas production function, $Y = F(K, AL)$, with constant returns to scale:

$$Y_t = (A_t L_t)^{1-\alpha} K_t^{\alpha} \tag{2.1}$$

Firms hire labor (L) and rent capital (K) from households in competitive factor markets. Labor enters multiplicatively with labor-augmenting A, which is commonly interpreted as the effectiveness of labor, a state of knowledge or as the overall technological progress taking place in an economy. In this chapter it will be referred to as the latter. The initial values of these three variables are strictly positive. L_t grows exogenously at rate n and A_t at rate g_A. Firms take both as given. The output elasticity of capital, α, determines the elasticity of substitution between one unit of technology augmented labor and one unit of capital. Output can be converted into units of effective labor:

$$\hat{y}_t = \hat{k}_t^{\alpha} \tag{2.2}$$

Households Households divide their income at every point in time between consumption, C, and savings, S. Savings are fully re-invested into the capital stock of firms. Utility is drawn from consumption and is intertemporally additive. Households maximize their discounted lifetime utility, denoted as U_t at discount rate ρ[1]:

$$U_t = \int_0^{\infty} u\left(c(t)\right) e^{(n-\rho)t} dt, \qquad n \geq 0, \qquad \rho > 0 \tag{2.3}$$

Households derive their per-period utility at time t from per capita consumption, $c_t = C_t/L_t$. The utility function carries the form of *constant-relative-risk-aversion* (CRRA). ϵ denotes the elasticity of marginal utility of consumption and, thus, $1/\epsilon$ is the elasticity of substitution between consumption at different points in time. The higher ϵ is, the lower is the marginal utility of consumption as consumption rises and the stronger is the incentive for households to exercise consumption smoothing over time.

$$u\left(c(t)\right) = \frac{c_t^{1-\epsilon}}{1-\epsilon}, \qquad \epsilon > 0 \tag{2.4}$$

Households maximize the sum of their future discounted period utilities in equation (2.5) subject to a budget constraint in equation (2.6), which accounts for household spending and income. Per capita wealth, b, rises with income from assets and labor and is reduced by expenditures for consumption and the reallocation of assets to new members of the overall population. All assets are invested into firms in the form of capital and earn

[1] In equation (2.3), we consider population growth, n. This means that households maximize their lifetime utility which they draw from per capita consumption. This equation is the same in the Ramsey model and in the Aghion & Howitt model, which is described in the following section 2.3.2. In the equivalent equation (1.15) for the Jones model in chapter 1, n is not considered, because this is how the original model by C. I. Jones (1995a) was set up. This means that households maximize their lifetime utility which is drawn from per capita consumption. However, in the utility function, per capita consumption is not weighed by population size.

an interest r. Wages are denoted with w. Both equations are in per capita terms, such that each household has only one person supplying assets and labor and consuming final output.

$$\max \int_0^\infty u\left(c(t)\right) e^{(n-\rho)t} dt \tag{2.5}$$

$$\text{subject to:} \quad \dot{b_t} = r_t b_t + w_t - c_t - n b_t \tag{2.6}$$

$$\text{with:} \quad b(0) = b_0$$

We can thus derive the Present-Value-Hamiltonian:

$$H \equiv \frac{c_t^{(1-\epsilon)}}{1-\epsilon} e^{(n-\rho)t} + \chi_t \left(r_t b_t + w_t - c_t - n b_t\right) \tag{2.7}$$

And the first order conditions:

$$H_c = 0 \quad \Leftrightarrow \quad c_t^{-\epsilon} e^{(n-\rho)t} = \chi_t \tag{2.8}$$

$$H_b = -\dot{\chi}_t \quad \Leftrightarrow \quad (r_t - n)\chi_t = -\dot{\chi}_t \tag{2.9}$$

This leaves us with the Keynes-Ramsey-Rule, which describes the growth trajectory of consumption in efficiency units:

$$g_{\hat{c}} = \frac{\dot{\hat{c}}}{\hat{c}} = \frac{r_t - \rho}{\epsilon} - g_A \tag{2.10}$$

Firms behavior Firms all have access to the same production function in equation (2.1) and sell their final output to households. Because there is perfect competition among all firms and because the production function exhibits constant returns to scale, they earn zero profits. Since factor markets are competitive, labor and capital are paid respectively earn their marginal product. The net rate of return on capital, r, thus equals the marginal product of capital reduced by the depreciation rate.

In the remaining part of this chapter the sum of all firms in the economy is represented by one big firm. This firm maximizes its profits:

$$\max_{K,L} \pi_t = K_t^{\alpha}(A_t L_t)^{(1-\alpha)} - r_t K_t - w_t L_t - \delta K_t \tag{2.11}$$

The first order conditions follow:

$$\pi_K = 0 \quad \Leftrightarrow \quad \alpha k_t^{(\alpha-1)} A^{(1-\alpha)} = r_t + \delta \tag{2.12}$$

$$\pi_L = 0 \quad \Leftrightarrow \quad (1-\alpha) k_t^{\alpha} A_t^{(1-\alpha)} = w_t \tag{2.13}$$

Next we insert equation (2.12) into the Keynes-Ramsey-Rule in equation (2.10):

$$\dot{\hat{c}}_t = \left(\frac{\alpha \hat{k}_t^{(\alpha-1)} - \delta - \rho}{\epsilon} - g_A \right) \hat{c}_t \tag{2.14}$$

The financial assets of all households in the economy in equation (2.6) have to add up to the amount of physical capital that is available[2]. The transition of the capital stock is determined by household savings and the depreciation rate:

$$\dot{\hat{k}}_t = \hat{y}_t - \hat{c}_t - (\delta + n + g_A)\, \hat{k}_t \tag{2.15}$$

As was shown in D. Romer (2012), equations (2.14) and (2.15) constitute a system of differential equations, which together with the initial values c_0 and k_0 describes a unique path for the capital stock and consumption along which the economy evolves. Eventually the economy converges towards a balanced growth path, where output, consumption and the capital stock, all in per capita terms, grow at the rate of technological progress g_A.

2.3.2 The Aghion-Howitt model

The endogenous growth model by Aghion and Howitt (1999) is an augmented Schumpeterian type model with no scale effects. Thus, a larger population size does not accelerate economic growth. As in Young (1998), this is achieved by the assumption that, as the population grows, research has to be spread over more variants of intermediate products.

[2] This is, because the economy is a closed economy with no governmental or inter-generational lending and borrowing.

Final output can be used interchangeably for consumption, as an investment into the capital stock or as an input to research. It is produced employing labor, L_t, and a continuum of intermediate products x_{it}:

$$Y_t = Q_t^{(\alpha-1)} \left(\int_0^{Q_t} A_{it} x_{it}^{\alpha} di \right) L_t^{(1-\alpha)} \tag{2.16}$$

Each variant is produced with its own specific productivity A_{it}. New (horizontal) product variants originate from pure imitation of existing product variants. They are produced with a random productivity level which is within the range of existing productivity levels at time t. We assume that the number of intermediate products Q_t at every point in time is proportional to population size. Thus as the population grows, each household will give rise to a new firm which produces its own new product variant. By making this assumption we depart from the original model by Aghion and Howitt (1999) where population size is constant. For this special case the authors assume that every household has the same constant propensity to imitate existing variants and thus the number of products per worker asymptotically converges toward a constant, supposedly before the analysis sets in (for more details see Aghion and Howitt (1999)). This setting is certainly easier to justify, because every household has the same small propensity to innovate, rather than not being horizontally innovative at all except for new households, who have a propensity to innovate of 100 %. However, when calibrating a growth model, population growth naturally is not constant. For this reason, we make these alterations to the original model. Nevertheless, for simplicity we assume that the number of product variants per worker is constant and $Q_t = L_t$. As product variety increases, expenditures towards research have to be spread over more variants and thus any productivity gain from an expanding product variety is entirely offset by this increasing cost. This model is thus at the opposite extreme of those which stipulate horizontal innovation to be the main source of growth, such as in P. M. Romer (1990). Aghion and Howitt (1999) argue that, while we may observe both in reality, productivity gains from horizontal innovation are less obvious than from vertical innovation which constitutes real quality improvements. While a larger variety of intermediate products enhances the scope of specialization, it also renders production more complicated and susceptible to errors and increases thin-market transaction costs. In this model growth is solely fostered by vertical innovation, which improves the quality of goods.

Intermediate firms Product variants are produced by monopolistic firms, who each produce output x_{it} of their own variant i. The number of monopolistic firms is proportional to the population. Vertical product innovations are targeted at specific variants. Their propensity of occurrence depends positively on the amount of resources allocated towards research. Vertical innovations replace the earlier vintage of the product and with it the incumbent monopolist. Intermediate products are produced using capital only. The higher the quality of a firm's product variant, the more capital intense is its production process:

$$x_{it} = \frac{K_{it}}{A_{it}} \tag{2.17}$$

Firms aim to maximize their profits given the cost of one unit of capital ζ_t:

$$\zeta_t = r_t + \delta \tag{2.18}$$

The profit maximizing price p_{it} for product variant i equals its marginal product in the final goods sector.

$$p_{it} = \alpha A_{it} x_{it}^{(\alpha-1)}$$

Firms thus solve the following optimization problem:

$$\max_{x_{it}} \pi_{it} = p_{it}x_{it} - \zeta_t A_{it} x_{it} \tag{2.19}$$

From the first order condition we derive the optimal quantity of each product variant:

$$\pi_{x_{it}} = 0 \qquad \Leftrightarrow \qquad x_t = \left(\frac{\zeta_t}{\alpha^2}\right)^{\frac{1}{\alpha-1}} \tag{2.20}$$

$$\Leftrightarrow \qquad p_{it} = \frac{A_{it}\zeta_t}{\alpha} \tag{2.21}$$

Note that the optimal quantity in which product variants are produced in equation (2.20) is independent of variant i, because the cost for one unit of capital is the same for all firms. Defining an average productivity parameter $A_t = \frac{\int_0^{Q_t} A_{it} di}{L_t}$ across all intermediate products, we can rewrite the capital stock,

$$K_t = \int_0^{Q_t} K_{it} di \tag{2.22}$$
$$= x_t \int_0^{L_t} A_{it} di$$
$$= x_t L_t A_t \tag{2.23}$$

and thus we can also reformulate the production function as in the RKC model: $\hat{y}_t = \hat{k}_t^\alpha$. Assuming that labor earns its marginal product, it is paid the same wages as in the Ramsey-Cass-Koopmans model in equation (2.13).

Using equations (2.18), (2.20) and (2.23), we can derive the interest rate:

$$r_t = \alpha^2 k_t^{(\alpha-1)} A_t^{(1-\alpha)} - \delta \tag{2.24}$$
$$= \alpha f'(\hat{y}_t) - \delta$$

Note that the gross interest rate equals the marginal product of capital times α. Monopolistic firms can increase their profits by producing less intermediate goods, than what would be socially optimal, in order to raise the price of their product variant. Hence, there is less demand for capital which induces households to underinvest. Using equations (2.20) and (2.21) we can derive firms profits:

$$\pi_{it} = \alpha(1-\alpha) A_{it} x_t^\alpha \tag{2.25}$$

Using equation (2.23) we can rewrite a firms profit into a productivity adjusted value:

$$\pi_{it} = A_{it} \pi_t \tag{2.26}$$
$$\text{with:} \qquad \pi_t = \alpha(1-\alpha)\hat{k}_t^\alpha \tag{2.27}$$

The Research sector Innovations are firm-specific. Successful innovators each introduce a new generation of a particular product variant to the market and produce with the leading productivity parameter A_t^{max}, which represents the cutting edge technology. Although innovations are firm specific, they exert knowledge spillovers and add to the

publicly available knowledge stock. Thus, current innovators have a positive externality on future research and development. On the other hand, they also exert a negative externality, since innovators render the previous version of their product variant obsolete and replace the incumbent producer of this product variant.
Households respectively firms give up a certain share of final output for investment into research to enhance technological progress and to promote new innovations. These individual research expenditures are divided between all product variants. The average Poisson arrival rate ϕ_t of an innovation in each firm, into which the amount R_t of final output has been invested into, is specified as follows:

$$\phi(h_t) = \lambda h_t^{\gamma} \tag{2.28}$$

$$\text{with:} \qquad h_t \equiv \frac{R_t}{A_t^{max}}, \quad 0 < \gamma < 1, \quad 0 < \lambda \tag{2.29}$$

Thus, the sum of all research expenditures amounts to $L_t R_t$. The exponent γ in equation (2.28) causes decreasing returns to research with respect to the innovation propensity. This assumption represents research congestion within a firm. In addition, in equation (2.29) research expenditures are normalized by the leading edge parameter A_t^{max}. This manifests what Aghion and Howitt (1999) call the curse of complexity. The further technology advances, the higher is the increase in resource costs for new innovations.
The amount of resources allocated to research is determined by the arbitrage condition that the marginal cost of research equal its marginal expected benefit (see equation (2.31)). The marginal benefit from research on the sectoral level equals the marginal effect of research on the propensity to innovate times the value of an innovation, V_t. Research firms face an arrival rate of innovations which is proportional to the amount of research undertaken in their own firm as compared to all other firms: $\phi(h_t)h_{it}/h_t$. The marginal propensity to innovate is thus the derivative of the individual propensity to innovate with regard to individual research expenditures: $\phi(h_t)/(h_t A_t^{max})$. Expressed in units of final output the arbitrage equation thus reads as follows:

$$1 = \frac{\phi(h_t)}{h_t A_t^{max}} V_t \tag{2.30}$$

$$\text{equivalent to:} \qquad h_t = (\lambda v_t)^{\frac{1}{1-\gamma}} \tag{2.31}$$

$$\text{with:} \qquad v_t \equiv \frac{V_t}{A_t^{max}}$$

To describe how research expenditures evolve over time, we determine the first derivative of equation (2.31) with respect to t:

$$\dot{h}_t = \frac{1}{1-\gamma} (\lambda v_t)^{\frac{\gamma}{1-\gamma}} \lambda \dot{v}_t \tag{2.32}$$

where the productivity-adjusted value of an innovation equals the discounted sum of all productivity adjusted future profits accrued by the respective innovation (see equation (2.33)). The instantaneous discount rate in equation (2.33) equals the sum of the interest rate, which reflects the opportunity cost of an investment, and the propensity to innovate, which reflects the risk of a product variant to be replaced by a new innovation:

$$v_t = \int_t^\infty e^{-\int_t^\tau (r(s)+\phi(s))ds} \pi(\tau) d\tau \tag{2.33}$$

Taking the first derivative with respect to time and using the product rule as well as the fundamental theorem of calculus we obtain the following asset equation:

$$\dot{v}_t = (r_t + \phi_t) v_t - \pi_t \tag{2.34}$$

Using equations (2.24), (2.26), (2.28) and (2.31) we can rewrite equation (2.32):

$$\dot{h}_t = \frac{\lambda}{1-\gamma} \left[\left(\alpha^2 \hat{k}_t^{(\alpha-1)} - \delta + \lambda h_t^\gamma \right) \frac{h_t}{\lambda} - \alpha(1-\alpha) \hat{k}_t^\alpha h_t^\gamma \right] \tag{2.35}$$

In the Aghion and Howitt (1999) model the growth rate of the leading technology parameter is driven by an endogenous decision by households to allocate resources to research. The more research firms undertake, the higher is their propensity to innovate and a higher aggregate propensity to innovate, leads to more knowledge spillovers produced by innovations which add to a publicly available stock of knowledge, A_t^{max}. Therefore, the growth rate of the leading technology parameter is proportional to the aggregate propensity to innovate $Q_t\phi(h_t)$ by a factor of σ. Because innovations have a smaller impact on the aggregate economy, if there is a larger number of product varieties in place, the growth rate of the leading-edge-technology-parameter is divided by Q_t and, thus, reads as follows:

$$g_A = \frac{\dot{A}_t^{max}}{A_t^{max}} = \sigma\phi(h_t) \tag{2.36}$$

Further, Aghion and Howitt show that the ratio between the leading-edge parameter and the average productivity parameter converges asymptotically:

$$A_t^{max} = A_t(1+\sigma) \tag{2.37}$$

Households optimization problem Households face the same CRRA utility function as in the Ramsey-Cass-Koopmans model in equation (2.4). The budget constraint, however, compared to the one above is altered in such a way that households not only earn wages and interest rates on the capital they lent to firms, but also from running monopolistic firms. On the other hand, their income now is divided between consumption, savings and investments into research and development. Their optimization program is as follows:

$$\begin{aligned} & \max \int_0^\infty u\left(c(t)\right) e^{(n_t-\rho)t} dt && (2.38)\\ \text{subject to:} \quad & \dot{b}_t = r_t b_t + w_t + \pi_t A_t - c_t - R_t - n_t b_t && (2.39)\\ \text{with:} \quad & b(0) = b_0 \end{aligned}$$

Deriving the fist order conditions and solving for $\hat{g}_c$ leaves us with the Keynes-Ramsey-Rule as given in equation (2.10). Inserting equation (2.24) results in the simultaneous equation of households and firms:

$$\dot{\hat{c}} = \left(\frac{\alpha^2\hat{k}_t^{(\alpha-1)} - \delta - \rho}{\epsilon} - g_A\right)\hat{c} \tag{2.40}$$

The change of the capital stock per efficiency unit from one point in time to an other is equivalent to the gross capital accumulation throughout the same period of time net of the depreciation of the existing capital stock and adjusted for growth in efficiency and population size:

$$\dot{\hat{k}}_t = \hat{y}_t - \hat{c}_t - h_t(1+\sigma) - (\delta + n_t + g_A)\hat{k}_t \tag{2.41}$$

While in the Ramsey-Cass-Koopmans model gross capital formation is the share of GDP, which is not spent for consumption purposes, in the Aghion and Howitt model GDP can also be allocated to the research sector.
Equations (2.35), (2.40) and (2.41) are the equations of motion which describe the transition of the capital stock, consumption and research activity over time. In the steady state balanced growth prevails in the usual sense (see Aghion and Howitt (1999)). The constant steady state rate of growth equals the growth rate of technological progress, which is denoted by g_A. The system is stable in the sense that for any initial amount of physical capital in efficiency units there exists a unique path of consumption and research which is a stable trajectory converging to the steady state.

2.4 The calibration procedure

2.4.1 The data

The aim of calibration is to optimize the fit of the simulated data resulting from a theoretical model to a set of observed time series. This is done by choosing optimal values for a predetermined set of free parameters or as we call them parameters of calibration. We calibrate two deterministic models of growth: The Ramsey model and the Aghion & Howitt model. Our parameters of calibration have a direct impact on these models' rate of economic growth. In the Ramsey model this is the rate of technological advancements g_A and in the Aghion & Howitt model this is γ which is a scaling parameter to the propensity to innovate and thus has a strong impact on the rate of technological advancements (see equations (2.28) and (2.36)). In addition, we calibrate the initial values of the stock variables (in $t = 0$). These are in both models the initial capital stock, K_0, and the initial set of technological skills, A_0. In the Aghion & Howitt model there are two extra parameter values, connected to the rate of technological growth, σ and λ, which, in order to achieve a good identification of the free parameters, have to remain at a predetermined value. All other parameters are the same in both models and we keep the parameterization of both models, as summarized in table 2.1, identical.

Table 2.1: Model parameterization

Parameter		Value	Source
Both models:			
ρ	discount rate	0.015	Nordhaus and Sztorc (2013)
α	production elasticity	0.3	Nordhaus and Sztorc (2013)
ϵ	intertemporal elasticity of substitution	1.45	Nordhaus and Sztorc (2013)
δ	capital depreciation rate	0.039	Penn World Table (Feenstra, Inklaar, and Timmer (2015a))
n	population growth rate	non-constant	Maddison (2010) and Nordhaus and Sztorc (2013)
Aghion & Howitt model:			
σ	variation in technologies	0.5	this article
λ	rescaling factor	0.2	this article

For ρ, α and ϵ we lean on the DICE model, because it is a widely known and very transparent Integrated Assessment Model, and we expect that the calibration of long-run growth models will be especially interesting to environmental economists and Integrated Assessment modelers. The depreciation rate we calculate from the Penn World Tables, since this is also the source for our time series on consumption shares. Because there is comprehensive data on past population growth rates available, we prefer those over a constant population growth rate. In the Aghion and Howitt model σ and λ are scaling parameters to the propensity to innovate and thus their optimal values are strongly correlated with each other and to the free parameter γ. For this reason they have to remain exogenous and are set to values for which, depending on the other exogenous parameter values, there is a large solution set.

We calibrate both models towards world GDP and the consumption share, which reflects on the overall household decision to split income into consumption and investment. In addition to past time series which we feed into the model for calibration, we also use population forecasts in the calibrated model to predict future values of world GDP and the consumption share.

In the remainder of this section we provide a detailed description of all three time series and their sources.

Population There are a number of historical data sets as well as forecasts of varying quality and time stretches available. Naturally, we are interested in those which go as far back and forward in time as possible. Since our Bayesian calibration technique is based on auto-regressive time series statistics, we cannot use time series with missing or interpolated data.

With regard to past observations of population size, we use the Maddison (2010) data set. Maddison constructs historical population estimates from year 1 to 2009, where possible on the country level. Starting in 1950 his world aggregate is available for every year. Some exceptions aside, starting in 1950, Maddison derived his estimates from the U.S. Census Bureau (2016). The U.S. Census Bureau (USBC) in turn provides an estimate of midyear population size worldwide and where available on the country level from 1950 to 2050.

The United Nations (2015) offer an equally comprehensive data set on population sizes on the country level starting in 1950 with forecasts until 2100 for different scenarios. Figure 2.1 shows the medium range projection. In addition, the United Nations (1999) provide a dozen data points of world population before 1950 starting in year 0. According to Maddison (2010), the differences between both, the USBC and the UN data sets, are very limited. While the UN resort to interpolations for those years which lie in between censuses, the USBC accounts for drastic events such as warfare and natural disasters even in those years. Hence, we refrain from using the UN data set and prefer the Maddison data, which also provides comprehensive data on world GDP. In figure 2.1 it is apparent that the Maddison and UN time series are more or less the same until 2010. What is striking, however, is the very strong exponential population growth starting in the fifties in contrast to a rather low population growth in earlier years. We will pick up this growth spurt in our growth model as well. But, as our growth models do not exhibit economies of scale, this will not cause any problems.

In order to use our calibrated models to make forecasts, we are also dependent on population forecasts. The United Nations (2004) go so far as to construct five different scenarios until 2300. At the same time they emphasize that: *"Given the enormous uncertainties of the character of demographic trends over such an extended period, the information content of these projections is somewhat elusive."* In their zero growth scenario population growth will level out shortly after 2100 at 8.3 bio people.

Since we anticipate that our model calibrations will primarily be used by climate economists, we use the same population projection as William Nordhaus did in his DICE model in

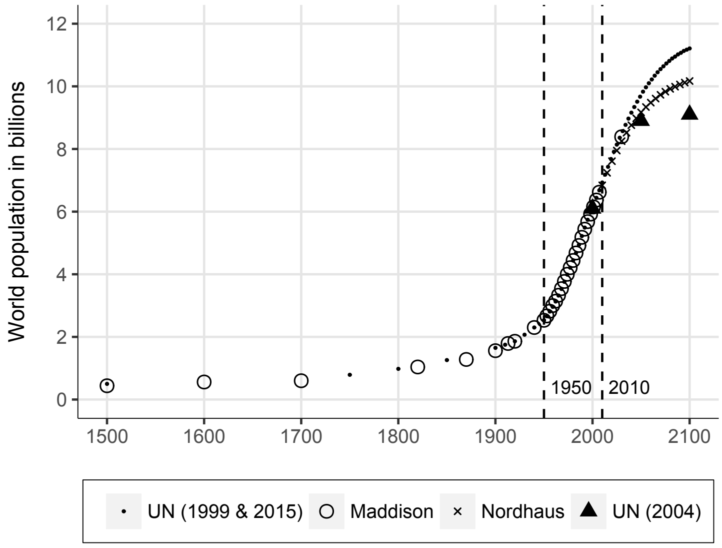


Figure 2.1: World population estimates/forecasts

Note: Population estimates/forecasts by the United Nations (1999) from year 1500 to 1949 and the United Nations (2015) from year 1950 to 2100, by Maddison (2010) from year 1500 to 2030, by Nordhaus and Sztorc (2013) from year 2010 to 2100 and by the United Nations (2004) from 2000 to 2100.

Nordhaus and Sztorc (2013). He assumes that population growth will go down to zero in the farther future and that the population size will stagnate at 10.5 bio people. His data points follow a continuous, logistic type function which was calibrated such that population size in 2050 approximates the medium range estimate of the 2010 revision in the United Nations (2011). In the United Nations (2015) revision numbers have been raised upward by roughly 10 %. Hence the gap between the UN and the Nordhaus forecast of population size in figure 2.1. But even the United Nations (2015) revision shows decreasing rates of growth the closer the data gets to 2100. For this work it does not matter too much, whether the population size will level off at 8.3, 10.5 or even 11 bio people. However, what does matter is the assumption that its growth rate will go down to zero. There are models of economic growth, which stipulate that zero population growth is accompanied by zero GDP per capita growth. Other models cannot comply with zero population growth in their steady state at all. Hence, there is a number of models of economic growth which should not be calibrated for their lacking ability to handle the prevailing forecasts of population growth. Together with the data set at hand the choice of model has strong implications for the outcome of any calibration exercise.

GDP We continue to use the Maddison (2010) data for historical GDP estimates. Again, Maddison constructs his data on the country level from year 1 to 2008. Starting in 1950 he provides a world aggregate on a yearly basis. To measure the level of economic performance rather than exchange rates GDP is adjusted by Purchasing Power Parity converts (PPP)[3]. He selects 1990 as the benchmark year and thus arrives at 1990 international dollars. Starting in the fifties, the rate at which GDP per capita is growing is astonishing (see figure 2.2), even more so if one takes into account that historical time series of GDP growth are suspected to underestimate the true growth of economic performance. Nordhaus (1997) argues that standard price indices, including the Geary/Khamis-indice employed by Maddison, have difficulties to capture the improving quality and increasing range of goods and services over time. Thus, he concludes that due to this mismeasurement of prices, economic growth has been significantly underestimated since the Industrial Revolution.
There are other comprehensive sources for GDP growth, although their yearly estimates do not reach as far back as in the Maddison data. For instance the World Bank (2016) provides yearly observations, where available on the country level, starting in 1960. The Penn World Table by Feenstra, Inklaar, and Timmer (2015a) provide yearly estimates on the country level, starting in 1950 where available. Yet they do not include a world aggregate. Palgrave Macmillan Ltd (2013) contains an extensive collection of historical data including GDP by sector, where available starting in 1750, however not on a yearly basis. Hence, for the purpose of this work and to maintain coherency with the population estimates, we use the Maddison data.
Looking at the whole data starting in 1500 it may be that we start calibrating our growth models in the midst of a kink. After 1950 GDP growth is suddenly very steep. If we were able to start calibrating our growth models at an earlier point in time, our growth projections would be flatter. Depending on whether one beliefs that this sudden growth spurt will not be continued, we may overestimate future growth considerably.

Consumption share At a very high level of abstraction macroeconomists divide household spending into consumption and savings. In the growth literature, households can invest their savings into a physical capital stock or, depending on the economies' growth engine, into some form of knowledge, technological progress or innovations. There are manifold terms in the growth literature all leading to an increasing productivity of labor

[3]To derive PPP's Maddison chooses the Geary-Khamis method developed in Geary (1958) and Khamis (1972).

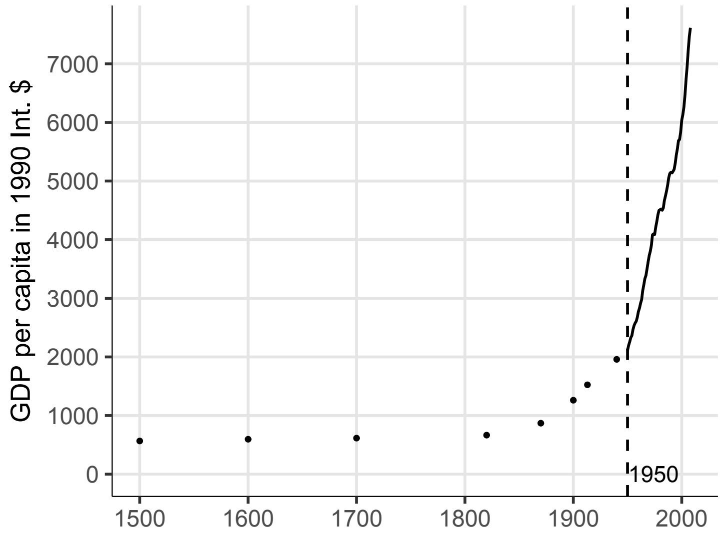


Figure 2.2: World GDP per capita

Note: World GDP per capita estimates by Maddison (2010) from year 1500 to 2008.

and/or capital depending on the productivity of the research and development sector itself. In this work we calibrate models of economic growth not only to household income, but also to the share of income that is allocated to consumption versus savings.
We use the Penn World Table by Feenstra, Inklaar, and Timmer (2015a), documented in Feenstra, Inklaar, and Timmer (2015b), to derive the consumption share from separate income and consumption time series on the country level. The tables cover 167 countries. Where available the data starts in 1950. The tables do not include world aggregates. We therefore constructed aggregates which miss 97 countries in 1950, but which are complete by 2010. The majority of the in 1950 missing countries are low income countries, which in 2011 accounted for 13 % of world GDP. Since, on average, low income countries have a lower savings rate, this might bias our time series of consumption shares in the first half of the time series and amplify its negative trend. Figure 2.3 shows the resulting world consumption share from 1950 to 2011. To our knowledge there is no other data available which could overcome this problem. However, this potential bias is very limited, since missing observations comprise of one eighth of total GDP in 2011 only.

To summarize, since we need all three time series on an annual basis and aggregated to the world level, this leaves us at the smallest common denominator with GDP and the consumption share both from 1950 to 2008 and the population size from 1950 until 2100 and beyond.

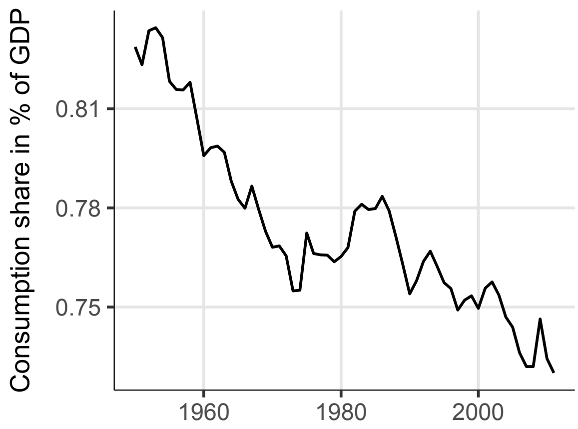


Figure 2.3: World consumption share

Note: World consumption share in % of GDP derived from Feenstra, Inklaar, and Timmer (2015a) from 1950 to 2011.

2.4.2 The methodology

The aim is to find a set of parameter values which optimizes the fit of the simulated data, derived from the economic models, to the observed time series. Both are deterministic models of growth, which we solve for 401 time steps from 1950 until 2350. Population growth is non-constant. However, after year 2100 it goes rapidly down until it is roughly zero in year 2200 and remains zero thereafter. Consequently, both growth models cannot remain in a steady state throughout their whole evaluation period, but we can determine the economy's optimal transition path into its steady state to avoid the end point problem. Thus, we condition the final value of all flow variables to match their steady state values and solve both growth models for enough time periods to ensure that the end point does not interrupt the economy's transition path.

The parameter values which are to be identified are denoted as parameters of calibration or free parameters. They directly influence the rate of technological progress and the initial conditions on the stock variables. The observed time series are GDP and the consumption share, which reflects on a households decision to split its income into consumption and investment.

The calibration procedure is based on the Bayesian evaluation of the linear regression model with auto-correlated errors (for details see Zellner and Tiao (1964)), only that the linear regression model is replaced with non-linear models of macroeconomic growth. This approach stems from the geophysical literature and is outlined in Urban and Keller (2010), Urban, Holden, et al. (2014) and Ruckert et al. (2017) based on the calibration of climate models.

The aim of calibration is to center the observational time series on the model output. Real world time series are subject to recurrent shocks and can thus best be described as a stochastic process fluctuating around its trend. In contrast, the simulated data, which is the solution to a deterministic model, is comparable to a smooth and non-constant time trend. In order to calibrate our growth model we aim to produce those simulated time series which best describe the trending behavior of our observed data. The residual between these two time series follows a stochastic process with no time trend. Hence, observations for GDP and the consumption share are interpreted as the sum of model output and an unknown stochastic process, which we subsequently also call residual:

$$\underset{\text{observations}}{q_t} = \underset{\text{model}}{f(\Theta, t)} + \underset{\text{residual}}{Z_t} \tag{2.42}$$

Here Θ contains the uncertain model parameters. The residuals are assumed to follow an auto-regressive AR(1) statistical process with no drift and white-noise error terms.

$$Z_t = \rho Z_t + U_t, \tag{2.43}$$

with $U_t \sim i.i.d.N(0, \sigma_u^2)$. We assume that the residuals are wide-sense stationary with $\rho < 1$, $E[Z_t] = 0$ and $Var[Z_t] = \sigma_u^2/(1-\rho^2)$. Hence, their expected value is zero, which reflects our aim to calibrate the growth models such that the expected residual between the simulated and the observed data is zero. The residual AR(1) process represents a combination of two error terms, first, an observation error which is sometimes also referred to as measurement error and, second, an error of model misfit and other structural errors. We do not attempt to disentangle these two sources of variability, but rather assume that as one they can be modeled as an auto-regressive process. In this way we account for the auto-correlation in the model-data residuals. While the observation errors are likely to be uncorrelated, this is not true for potential model misfit, which is likely strongly correlated over time. Hence, we believe that the unknown stochastic process of our residuals is best described by an auto-regressive process of order one.

The notion to describe the discrepancy between simulated and observed data as a statistical process in the context of calibration was developed by Watson (1993). According to him, a deterministic growth model is a highly stylized approximation of the stochastic

data generating process generating the real data and the residual "... represents the degree of abstraction of the model from the data."
The AR(1) is a parsimonious stochastic process, however it neglects possible correlations between the estimated residuals of GDP and the consumption share. Regarding developed countries it is a stylized fact that, as income grows, the consumption share persists at a relatively constant level and the correlation between both is very small. In developing countries on the other hand, there is a strong tendency that growing incomes lead to smaller consumption shares and more savings. However, given the relatively small share in world GDP that developing countries have, this is a weak argument for implementing a vector-auto-regressive-process (VAR) compared to its cost, as it would double the number of unknown parameter values and thus heavily deteriorate identification. Calibration always is conditional on the choice of model for both, the theoretical and the statistical model, as well as on the observed data. Thus, to draw conclusions on the true stochastic processes resulting from the residuals between the simulated and the observed data would be circular reasoning.

A strong advantage of Bayesian calibration techniques is that it enables the modeler to elicit the likeliest distribution of the parameters of calibration rather than point estimates in frequentist statistics. Parameters of calibration stem from the theoretical model as well as the statistical model which describes the residual. We summarize both in κ and aim to elicit their joint distribution. A crude outline of our approach is the following: We estimate the joint distribution of the parameters of calibration using Bayesian inference. Since there is no analytical solution for the so-called posterior distribution of the free parameters, we resort to Monte Carlo integration using Markov chains (MCMC). The Monte Carlo algorithm draws representative samples from the posterior distribution by running Markov chains. In the remainder of this section we will lay out Bayes law and the Monte Carlo algorithm in more detail.

Bayes law The parameters of calibration stem from the theoretical model of economic growth as well as from the statistical model, formalizing the evolution of the residuals between the simulated and the observed data. In this section both are represented by vector κ. When calibrating an economic model we strive to determine the likeliest joint distribution of κ given the observed data. In order to derive this entity, we first note that the joint distribution of the observed data (D) and the parameters of calibration (κ) can be computed by combining the likelihood of the observed data given κ, $p(D|\kappa)$, with its prior distribution $p(\kappa)$ and vice versa:

$$p(D, \kappa) = p(D|\kappa)p(\kappa) = p(\kappa|D)p(D) \tag{2.44}$$

This relation can be reformulated into Bayes' rule, which gives us what we are mainly interested in: the probability of the parameter values, given the data. This probability, $p(\kappa|D)$, is called the posterior distribution of κ.

$$p(\kappa|D) = \frac{p(D|\kappa)p(\kappa)}{p(D)} \tag{2.45}$$

The denominator, $p(D)$, is also called the normalization constant. In Bayesian inference it is a common problem that, because the normalization constant is an integral over the marginal probabilities of the data, there is no analytical solution for the posterior in equation (2.45). Hence we resort to the following proportionality:

$$Posterior \propto Likelihood \times Prior \tag{2.46}$$

$p(\kappa)$ represents a researcher's prior beliefs about the distribution of the unknown parameter values. This so-called prior opens the door to a subjective influence on the estimation procedure. Bayesian inference is about letting the data tell us how to transform these prior beliefs into a posterior distribution of the unknown parameter values given the observed data. Suitable priors and their degree of neutrality are extensively discussed in the literature. In this work we prefer to be as neutral as possible and therefore we use uniform prior distributions.

The MCMC algorithm Since there is no analytical solution for the joint posterior distribution of κ, we use the Monte Carlo algorithm to generate a representative sample $\{K_t, t = 1, ..., n\}$ of the joint posterior distribution of the elements of κ conditional on the observed data. From this sample we derive the marginal kernel densities of each parameter of calibration. The sample is drawn as a sequence, a so-called Markov chain, where the distribution of the next realization K_{t+1} is sampled from a distribution $p(K_{t+1}|K_t)$, which depends on the latest realization in the chain. The distribution $p(.|.)$ is called the transition kernel of the sequence. There are numerous kernels discussed in the literature, however all of them are special cases of the general framework by Metropolis et al. (1953) and Hastings (1970). For efficiency reasons we use the Robust Adaptive Metropolis (RAM) sampler developed by Vihola (2012). The subsequent enumeration gives a rough intuition on how the RAM sampler determines K_{t+1}, the next segment in the chain[4]. For the computational realization of the sampler we use the Klara package usable within the programming environment of Julia.

- Make a guess for the initial chain value K_1, which is the same as an initial guess about the free parameters of calibration in κ.
- Sample a candidate point $\Pi_t := K_t + S_{t+1}$, where $S_{t+1} \sim q$ constitutes the proposal distribution. It is distributed with q, a spherically symmetric probability density.
- Accept the candidate point Π_t with probability $\alpha(K_t, \Pi_t)$:

$$\alpha(K_t, \Pi_t) = min\left(1, \frac{\pi(\Pi_t)}{\pi(K_t)}\right)$$

  $\pi(.)$ represents the density of K_t and Π_t. It is the posterior probability of K_t and Π_t given the data which is determined using to Bayes' law. If the proposal is accepted, $K_{t+1} = \Pi_t$. Otherwise $K_{t+1} = K_t$. In words, a candidate point is always accepted if its density is higher than that of the previous chain element. Otherwise a candidate point is accepted with a probability that equals the density of the candidate point divided by the density of the previous chain element.
- Update the proposal distribution S_{t+1} such that if the acceptance probability α_t is smaller (higher) than the pre-determined target rate of acceptance α^*, the proposal distribution is shrunk (enlarged).

To ensure convergence of the RAM algorithm, the proposal distribution should be either a Gaussian or a Student distribution. The first realization of the chain, K_1, has to be supported by the target distribution $\pi(K_1) > 0$. With a growing chain size n, the influence of the first segment on K_n will diminish. With a reasonably sized burn-in, this influence can be eradicated from the chain.

[4]See Vihola (2012) for an in depth description of the sampler.

The likelihood function According to Bayes' law the posterior distribution, which is the target distribution of the RAM sampler, is the product of two components: the likelihood of the data given the parameters of calibration (κ) and the prior distributions of all parameters of calibration. Since we assume all priors to be uniform distributions, the joint posterior distribution equals the likelihood rescaled to be a density.

When sampling from the target distribution, for every realization of the Markov chain, we have to determine the likelihood of the data, given the proposal value Π. Hence we solve our growth models for every proposal value. The residuals between the simulated and the observed data are assumed to follow an AR(1) with no drift and white-noise error terms.

Given these assumptions, we can determine the likelihood of the AR(1)'s given the observed data and Π, where Π also contains the parameter values of the auto-regressive processes, ρ and σ_{u^2}. This is done for every segment of the Markov chain. Since the disturbance term, U_t, is Gaussian, so is Z_1: $Z_1 \sim N(0, \sigma_u^2/(1-\rho^2))$. Thus, the density of the first observation takes the form:

$$f_{Z_1}(z_1; \rho, \sigma_u^2) = \frac{1}{\sqrt{2\pi}\sqrt{\sigma_u^2/(1-\rho^2)}} exp\left[-\frac{z_1^2}{2\sigma_u^2/(1-\rho^2)}\right] \tag{2.47}$$

Conditioning on $Z_{t-1} = z_{t-1}$, for any consecutive realization of Z_t holds: $E[Z_t] = \rho z_{t-1}$ and $Var[Z_t] = \sigma_u^2$. Hence the density of observation t conditional on observation $t-1$ is given by:

$$f_{Z_t|Z_{t-1}}(z_1|z_{t-1}; \rho, \sigma_u^2) = \frac{1}{\sqrt{2\pi\sigma_u^2}} exp\left[-\frac{u_t^2}{2\sigma_u^2}\right] \tag{2.48}$$

$$= f_{U_t}(u_t; \sigma_u^2) \tag{2.49}$$

Hence, the joint density of $Z_1, Z_2, ...Z_T$ amounts to:

$$f_{Z_T, Z_{T-1},...Z_1}(z_T, z_{T-1}, ...z_1; \rho, \sigma_u^2) = f_{Z_1}(z_1; \rho, \sigma_u^2) \prod_{t=2}^{T} f_{U_t}(u_t; \sigma_u^2) \tag{2.50}$$

If the sum of the log-likelihoods of GDP and the consumption share for parameter values Π is greater than for parameter values K_n from the previous segment in the Markov chain, then Π is accepted. Otherwise it is only accepted with probability $\alpha(K_n, \Pi)$.

To summarize the MCMC procedure

1. Stipulate an arbitrary starting value for the Markov chain which is within the support of the posterior distribution (e.g. for which there is a solution to the model of economic growth).
2. Solve the growth model using this starting value and simulate data for GDP and the consumption share.
3. Determine the likelihood of the resulting residuals between the simulated and the observed data, assuming that they follow an AR(1) and given the parameter values of the AR(1) by the starting value of the Markov chain.
4. Let the RAM sampler determine the next random segment of the chain and repeat steps 2 and 3 for this realization of parameter values.
5. Let the RAM sampler decide whether this next chain value is accepted.
6. Continue with step 4 and 5 until the desired length of the Markov chain is reached.

A major advantage of this procedure is that it is independent of the economic growth model which is to be calibrated. The theoretical model has no impact on the algebraic form of the likelihood function and the MCMC algorithm is the same for any number of free parameters. Even if we wanted to calibrate a growth model towards more than two observed time series, this could be done without changing the essence of this procedure. However, one would have to calibrate over a higher number of free parameters, which might raise identification issues. In this article, for instance, it might seem expedient to calibrate both models towards the real interest rate, too. However, our prediction is that this would reduce the goodness of fit immensely and pose problems on chain convergence[5].

[5]For instance, the Aghion and Howitt model strongly underestimates the real interest rate while at the same time it overestimates the households return on investment in order to create a sufficiently strong incentive to invest in R&D. This is necessary to foster those strong rates of economic growth within the model, which we have observed throughout the past, and still meet the observed consumption share. Because of its inherent structure the model is incapable of adapting to all three time series at the same time

The Markov chains are representative, if they explore the full range of the target distribution and do so in the right proportions. They should not get stuck or contain sudden spikes. In theory, if a chain meet these criteria, we say it has converged. Unfortunately, in practice there are no objective criteria for convergence of a realized chain. Suppose for instance a chain has several isolated modes and it was ended before it was able to explore more than one mode. This chain would seem to have converged, although it has not. Strictly speaking, if we wanted to be absolutely sure about the convergence of a chain, it would have to be infinitely long. The optimal length of a chain is discussed controversially in the literature. Gelman and Rubin (1992a) and Gelman and Rubin (1992b) propose to compute several long chains with different starting values, which, if they have converged, should have the same sampling distributions at the end. Geyer (1992) on the other hand argues that with one very long chain one has the best chance to discover if this chain slides into a new mode. We resort to the former approach and compute several long chains with very different starting values. For both models we sample 100.000 parameter vectors and discard the first half as burn-in with a thinning of 100, such that we compute six chains of 500 segments each.

As a check for convergence we compute auto-correlation functions and the Gelman-Rubin statistic as modified by Brooks and Gelman (1998). The intuition behind the Gelman-Rubin statistic is to run several chains and to compare their between and within variation. If a chains starting point is over-dispersed relative to the target distribution, the between variation initially overstates the chains variance and vice versa, while the within variation understates it, because early draws will not yet have fully explored the state space of the target distribution. With an increasing number of realizations the influence of the starting value diminishes, such that if a chain converges the Gelman-Rubin statistic approaches unity.

A final note To conclude this section we will discuss how this calibration technique relates to the RBC literature. The RBC literature aims to assess the nature of business cycles. As such it models fluctuations of macroeconomic time series in the medium run. For this reason the observed time series are detrended. In the past, stochastic models with random shocks have been in the center of attention. These models are generally complex and non-linear, such that they are linearized and in some cases a filter is applied in order to derive a likelihood function directly from the model. Model calibration can be based

on either frequentist or bayesian statistics. Model fit is assessed by comparing a choice of statistical moments of the simulated data with the ones of the observed time series.
In this paper on the contrary, we calibrate models of long-run growth and thus we are interested in replicating the time trend of the observed data. The models we calibrate are deterministic and they are solved without prior linearization. The simulated data is thus smooth while the observed time series follow a statistical process. Using the simulated data we aim to replicate the time trend of the observed data as best we can, assuming that the residual between the observed and simulated data follows a statistical process. Consequently, we can derive the likelihood function of these residuals, given the theoretical growth model, the statistical model and the observed data. Our framework is Bayesian. Since there is no analytical solution to the posterior distribution of our parameters of calibration we integrate over this distribution using an MCMC algorithm. This procedure has numerous advantages. It allows for a profound statistical framework for calibration, even for models without any stochastic components. We can abstain from model linearization and filtering to derive the likelihood function, which potentially improves identification. In addition, our method is very accessible and can be used for the calibration of a broad variety of models without any alterations to the essence of this approach.
Since our goals are entirely different from those in the RBC literature, we do not see this article as an addition to this literature.

2.5 Results

This chapter proposes a Bayesian approach towards calibration and demonstrates its application to a Ramsey type model of exogenous growth and to the Aghion and Howitt model of endogenous growth. In this way we are able to show the broad applicability of this calibration technique. In this article the choice of theoretical models was narrowed down by the prerequisite that the growth model's inherent rate of economic growth should be independent of population growth. Even though world population growth is expected to cease after the 21 century, we do not expect that this will be a cause for economic stagnation.
Our results show that all parameters of calibration are well identified, although their priors were assumed to be uninformative, except for some truncations which were implied by the economic models and the stochastic processes. In table 2.2 we summarize all priors.

Stipulating uninformative priors yields a convenient way for assessing the sensitivity of the simulated data towards the parameters of calibration.

Parameter		Prior
Both models:		
K_0	initial capital stock	uniform, $K_0 > 0$
A_0	initial stock of skills	uniform, $A_0 > 0$
ρ_Y	parameter in AR(1) regarding GDP	uniform, $0 \leq \rho_Y < 1$
ω_Y	variance of disturbances in AR(1) regarding GDP	uniform, $\omega_Y > 0$
ρ_C	parameter in AR(1) regarding consumption share	uniform, $0 \leq \rho_C < 1$
ω_C	variance of disturbances in AR(1) regarding consumption share	uniform, $\omega_C > 0$
Ramsey model:		
g_A	growth rate of technological skills	uniform, $g_A > 0$
Aghion & Howitt model:		
γ	scale parameter on propensity to innovate	uniform, $0 < \gamma \leq 1$

Table 2.2: Assumed priors

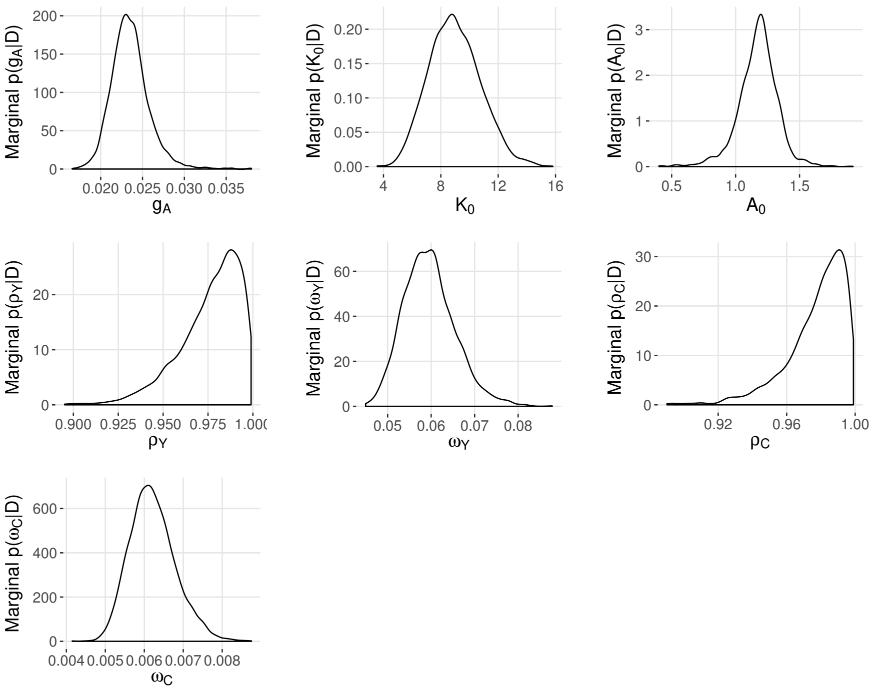
The marginal posterior distributions of all parameters are presented in figure 2.4. The only binding prior truncations are on ρ_y and ρ_c, which are forced below one to rule out unit root shocks to the residuals between the simulated and the observed data on GDP and the income share. Our prior assumption is that the expected value of these residuals is zero. This prior assumption is a precondition for this calibration procedure. Otherwise the simulated and the observed data could drift apart without bounds. In Bayesian statistics the treatment of non-stationary time series does not differ from the treatment of stationary time series. If one beliefs in the stationarity of a time series, then one may want to reflect these beliefs with appropriate priors. Contrary to frequentistic statistics, which require specific methods to tackle non-stationarity, the Bayesian theorem applies to both cases alike and likelihood functions stay the same with and without a unit root (see Sims and Uhlig (1991)).

Furthermore, the parameter values ρ_y and ρ_c measure the correlation between different realizations of the stochastic processes over time and as such the persistence of disturbances to the residuals between the simulated and the observed data. For both growth models the distributions of ρ_y and ρ_c are skewed to the left and they bend towards their maximum value. The likelihood functions have reshaped our priors and the data tells us that there is a strong correlation between the elements of the stochastic processes. Interpreting ρ_y and ρ_c as a measure for auto-correlation, they can be used to assess our growth models ability to reproduce the observed time series. Lower values would correspond to a lower auto-correlation between the AR(1) realizations and thus with more intersections of the simulated and the observed data. Although, ρ_y and ρ_c do not constitute an absolute measure of fit, an extremely narrow distribution with a median just below one points to a bad model fit. This is not the case for the models calibrated in this article. The model fit is depicted in figure 2.5 where the observed data on GDP and the cosumption share between 1950 and 2008 are plotted against their respective median in the simulated data. The confidence intervals are derived by choosing 1000 random vectors of parameter values from the Markov chain, which captures the joint distribution of all parameters of calibration. Then both economic models are solved repeatedly for the whole sample and to their resulting simulated data we add the stochastic realization of an AR(1), which is parametrized using the respective parameter values from the Markov chain. GDP is well captured by the 90 % confidence intervals. Regarding the consumption share, some observed values from before 1970 lie outside the confidence interval. We simulate both time series until 2050.

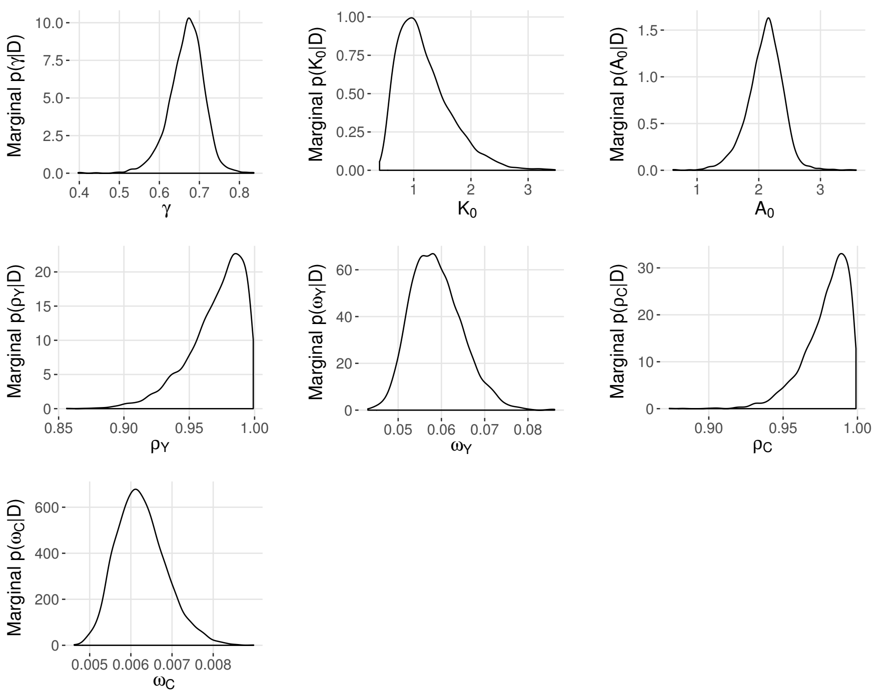
The marginal posterior distributions of the AR(1) parameter values in the Ramsey model are almost identical to the ones in the Aghion and Howitt model in figure 2.4. This comes as no surprise. In both models we have the same number of free parameters and the initial conditions on the stocks lead to the same initial income in both models. This also contributes to the similarity of both models' confidence intervals of the calibrated time series and their projections in figure 2.5. In the Ramsey model the annual average growth rate of the median trajectory of GDP is 2.3 % and in the Aghion and Howitt model the annual growth rate amounts to 2.2 %.

Figure 2.6 depicts the two-dimensional marginal probabilities of those free parameters which stem from the growth models. In both models the initial stock of technological skills, A_0, is strongly correlated with the respective parameter value driving technological change. In the Ramsey model this is g_A and it is negatively correlated with A_0. To fit the observed data on GDP, an initially high stock of technological skills compensates for a subsequently lower rate of technological change. In the Aghion and Howitt, model an initially high stock of technological skills has a lasting and negative impact on the returns to future research investments. The authors call this the curse of complexity. The further technological progress advances, the higher are the resource cost of new innovations. Thus, to hit the average observed rate of GDP growth, the scale parameter γ, which has a positive influence on the propensity to innovate, has to rise, too. As a consequence, in the Aghion and Howitt model γ and A_0 are positively correlated. Figure 2.6 shows that all parameter values which stem from the theoretical models are well identified.

In the remainder of this section we assess the convergence of our Markov chains versus their computational cost. We say that a Markov chain has converged, if it consists of a representative sample for its target distribution. However, because the target distribution is unknown, in practice we resort to a number of formal and semi-formal criteria, which have to be met in order for us to believe that a chain has converged. Roughly speaking, these criteria involve that a chain should be stable and not show any big lumps or spikes. This is the case for all our chains and we support this result with the aid of the auto-correlation functions and the Gelman-Rubin statistics below.
However, although longer Markov chains are a better representation of their target distribution, longer chains also induce higher computational costs. Thus, in practice one needs to determine the optimal length of a chain in order to make sure that it, on the one hand, has converged towards its target distribution and is independent of its starting value and, on the other hand, is as short as possible to save computational time. We compute six chains for both growth models, which each contain 100.000 draws. We reduce the lumpiness of our chains by a thinning of 100, such that the auto-correlation functions of all free parameters remains roughly below 5 % (see figure (B.1) in appendix B). Further, we divide all chains into two and compute the Gelman and Rubin statistic for all parameters of calibration (see figure (B.2) in appendix B). This statistic compares the chains within variation with their variation between chains and converges towards unity when chain convergence is achieved. In both models this is the case after approximately 150 out of 500 observations. To be on the save side, we disregard the first half of each chain as burn-in. Thus we are left with 500 draws in six converged chains, amounting to 3000 draws in toal, which we can use to compute the kernel densities of our parameters of calibration.

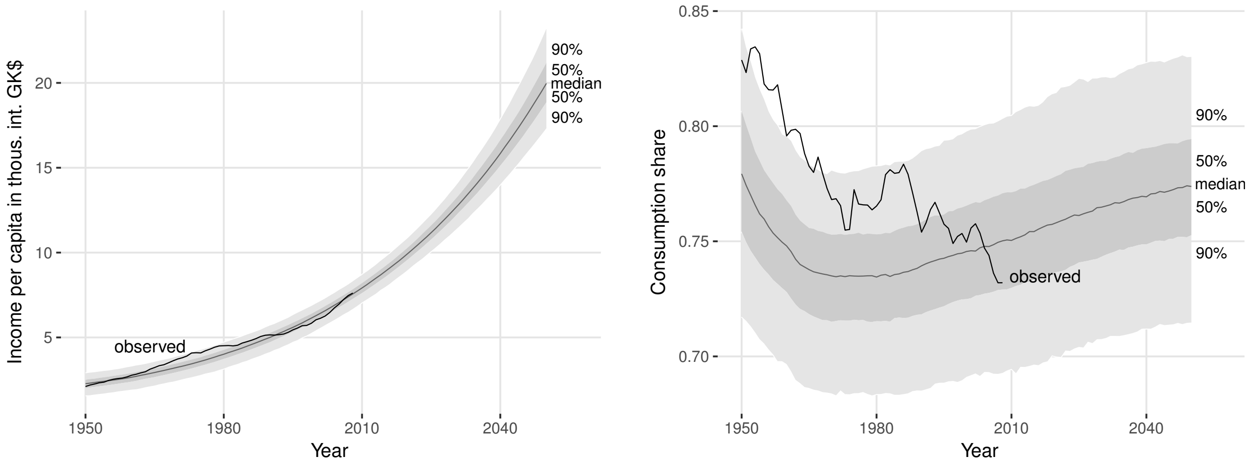


(a) Ramsey-Cass-Koopmans model

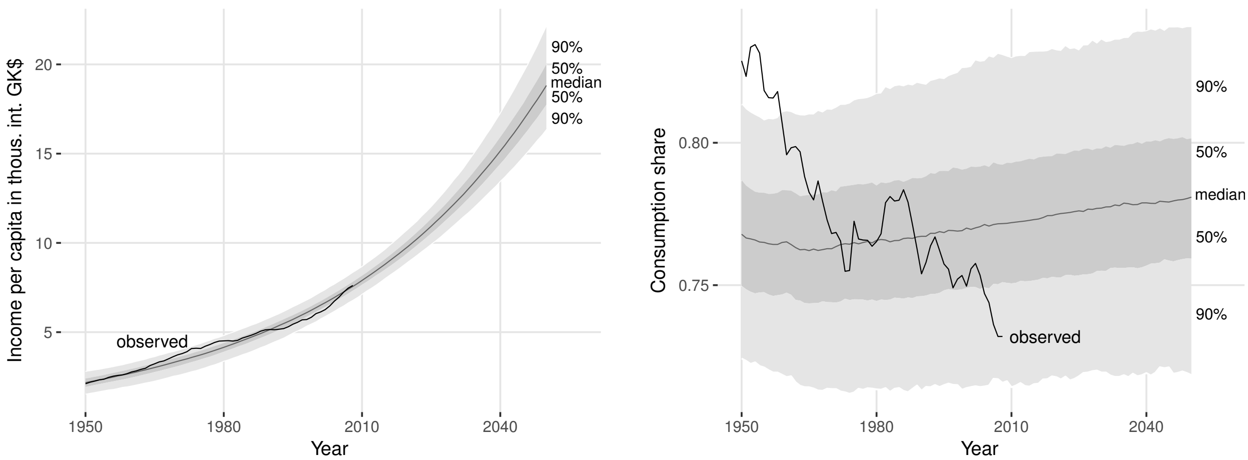


(b) Aghion & Howitt model

Figure 2.4: Marginal density functions of the estimated parameters

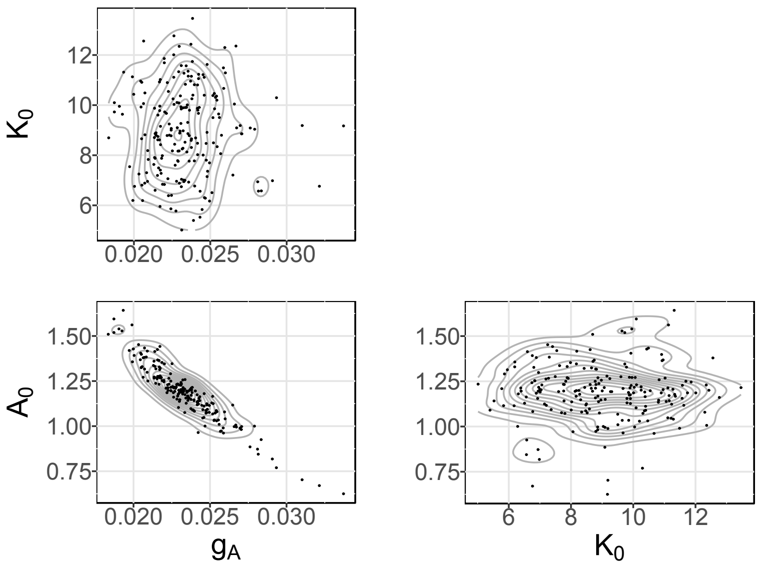


(a) Ramsey-Cass-Koopmans model

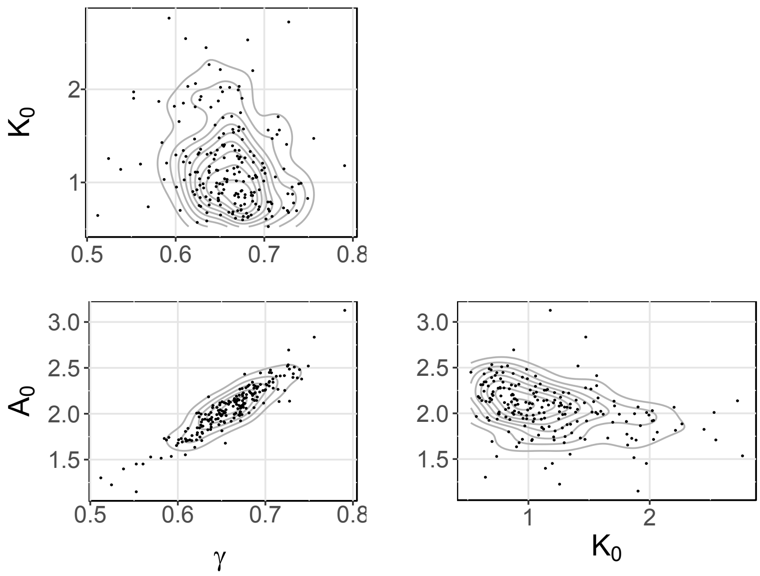


(b) Aghion & Howitt model

Figure 2.5: Income per capita and consumption share projection



(a) Ramsey-Cass-Koopmans model



(b) Aghion & Howitt model

Figure 2.6: Two-dimensional marginal probabilities

2.6 Conclusions

The aim of this article is to introduce a transparent approach towards the calibration of deterministic models of long-run growth, which allows for a statistically sound derivation of the calibration parameters' joint distribution and of the confidence intervals for the simulated data. The approach is a Bayesian inversion technique and to integrate over Bayes' law we apply a Markov chain Monte Carlo algorithm. To show that this approach is applicable to a wide range of growth models without changing the essence of the proposed technique, we exemplarily calibrate a Ramsey type model of exogenous growth and the Aghion and Howitt model of endogenous growth. We find that all parameter values of calibration are well identified. We are able to replicate the time series trend of GDP per capita very well. The consumption share, however, has a tendency to be underestimated by both growth models at least in the early years of the calibration exercise. Numerous Integrated Assessment Models, such as for instance the DICE model by William Nordhaus, are based on a Ramsey model. However, since there is a growing interest to endogenize growth in the Integrated Assessment literature, we also demonstrate the calibration of an endogenous growth model. A logical continuation of this article would be to replace the growth component of a suitable Integrated Assessment model by our calibrated version of the Aghion and Howitt model of endogenous growth and to test how the predictions of this IAM change.

Two quantitative results from our calibration approach are the confidence intervals of the simulated data and the distributions of our free parameters. Both depend on our choice regarding the economic as well as the statistical model and on the parameterization of the fixed parameters. A full sensitivity analysis would require to vary over these three impact factors. In this article we show how the choice of growth model changes our quantitative results. While the simulated confidence intervals of GDP are very similar between the Ramsey and the Aghion and Howitt model, the confidence intervals of the consumption share differ in their course and width between both models. These differences are not tremendous, but it appears that the endogenous growth model by Aghion and Howitt is able to achieve a better fit towards the consumption share. To expand the sensitivity analysis a logical step would be to vary the statistical model and, for instance, to assume that the residuals between the simulated and the observed data between different time series are correlated and follow a vector-auto-regressive process. However, even for a vector-auto-regressive model of order one, this would raise the number of free parameters by four and deteriorate the identification of the individual stochastic processes. This could be a connecting point for further research.

We have to acknowledge that we do not know what the true values of our fixed parameters are. Our prediction is that, if the fixed parameters are varied within a reasonable range, most of this variation will be picked up by the free parameters and change their distribution accordingly. The simulated data will vary only marginally. Since the aim of this paper is foremost to propose a new methodology of calibration and not to derive the most credible parameter values for our parameters of calibration, we leave this exercise to future research.

An advantage of our Bayesian approach towards calibration is that as a side effect it reveals the sensitivity of the simulated data towards the free parameters. This is reflected in the simulated data's confidence intervals, which are derived from the joint distribution of the free parameters.

Chapter 3

The DICE model with endogenous technological change driving economic growth

3.1 Introduction

Climate change is widely understood as a dynamic problem. It entails various direct as well as indirect effects on economic welfare. An atmospheric temperature increase affects, for instance, agricultural productivity through droughts and water shortages instantaneously. But it also has indirect effects on capital accumulation and savings which accrue over time. In the literature, there is a rising awareness for potentially lasting and path-dependent growth effects of climate change. Fankhauser and Tol (2005) describe two main dynamic effects that connect climate change to economic growth. First, a capital accumulation effect: when income is reduced, there are fewer resources available for investments into the capital stock. Second, a savings effect: in a world with perfect foresight, it is likely that households adjust their inter-temporal savings plan to the occurrence of damages caused by climate change. A priori it is not clear whether the savings effect is a positive or a negative effect. On the one hand, households may save more to make up for future expected losses. On the other hand, the return on capital investments decreases, such that the incentive to invest is lowered. In a reduced model of climate change and Ramsey type growth, Frankauser and Tol show analytically that under a relatively broad set of assumptions both, the capital accumulation effect and the savings effect, are unambiguously negative. In addition, the authors investigate the magnitude of these effects in a numerical analysis

based on four variations of the DICE model by Nordhaus (2008)[1]. The authors find that the capital accumulation effect by far dominates the savings effect.
In the original version of the DICE model the capital accumulation effect as well as the savings effect have relatively small impacts on the social cost of carbon and on economic growth even under extreme climatic conditions. However, this is not in accordance to the expectations raised by climate experts and has been criticized on numerous occasions. There is a growing number of studies which suggest additional channels of climatic impacts on the economy. This study and a handful of other studies argue that global warming also affects investments into R&D, the formation of knowledge stocks and, thus, technological progress in a very broad sense. In this way, climate change would have a negative impact on the economy's productivity. If this is the case, then there is an additional accumulation and savings effect which influences the size of the knowledge stock, and climate change would have a stronger and more lasting negative impact on economic growth. Gross income would be even more path dependent.
There are different ways to investigate the effects of global warming on the formation of knowledge stocks and economic growth. Dietz and Stern (2015), Moyer et al. (2014) and Moore and Diaz (2015) construct different versions of the original DICE model, where climate damages have a direct and negative impact on the level of knowledge stocks. Unsurprisingly, they find a much higher social cost of carbon and a stronger negative impact on economic growth than in the original DICE model. The magnitude of these results is strongly dependent on model assumptions. In contrast, Fankhauser and Tol (2005) and Dietz and Stern (2015) suggest introducing endogenous growth to the DICE model. In these model versions either overall investments are equally divided between physical and human capital or the size of the knowledge stock is tied to the size of the capital stock, such that investments into the latter also increase the size of the former. Again, these model versions yield higher negative impacts of climate change on growth, while their quantitative size is strongly dependent on model assumptions. In these model versions the savings effect and the accumulation effect regarding physical and human capital are inseparably connected to each other.
This paper goes beyond Fankhauser and Tol (2005) and Dietz and Stern (2015) by introducing exclusive investments into the knowledge stock which drives economic growth. Households can decide endogenously how much they want to invest into human capital independently from physical capital. In this way, climate change is modeled to have sepa-

[1] In the literature, the DICE model is an integrated workhorse model of the climate and the economy.

rable effects on both stocks. However, since the knowledge stock of the economy is directly linked to the economies productivity and to its rate of growth, in a world with perfect foresight, households implicitly decide on their optimal pathway of economic growth. In this way, I am able to abstain from making an arbitrary assumption regarding the share of damages which might directly or indirectly affect the size of the knowledge stock.
The framework is based on the DICE model by Nordhaus (2008). The model will be described in detail in section 3.3. In a version of this model the growth component is substituted by endogenous Schumpeterian type growth and calibrated towards the original. I solve three scenarios as have previously been described by Rezai (2011), each with an exogenous as well as an endogenous growth component. The introduction of an endogenous growth component is new to the literature. Rezai's social optimum scenario is identical to the original by William Nordhaus. In this scenario the social planner is able to mitigate climate change in two ways. First, he can invest in the reduction of carbon emissions and, second, he can shift his spending away from the carbon-emitting capital stock. In addition, in the endogenous growth setting, this reallocation of resources is accompanied by a reduction of investments into the R&D sector. In both growth model versions this reallocation reduces future total output and thus carbon emissions. However, in the endogenous growth setting there are two additional negative growth effects. First, the reduction of income leads to a negative accumulation effect regarding the knowledge stock and, second, as the return on investments in R&D decreases, the allocation of resources towards R&D declines. Hence, there is also a negative savings effect. The accumulation effect is even stronger in Rezai's constrained optimum scenario, where households cannot actively mitigate to reduce climate damages. This scenario is equivalent to the "business as usual scenario" by William Nordhaus. Yet, as in this scenario the climate externality is still fully internalized, Rezai (2011) argues that it does not correspond to a business as usual scenario. On the contrary, in the business as usual scenario in the spirit of Rezai (2011), where the climate externality is not internalized, and the private return on investment is not affected by climate change, there are no negative growth effects due to the reallocation of resources.

3.2 Literature review

The potential channels through which climate change might affect economic growth as well as the magnitude of their effects have been discussed controversially in the past. Dietz and Stern (2015) criticize that under standard assumptions current Integrated Assessment Models (IAMs) do not support strong emission controls. The authors question this fact and, therefore, introduce the endogeneity of economic growth, convexity of damages and an increased climate risk to the DICE model. In particular, Dietz and Stern emphasize that in the DICE model the damage multiplier on the level of gross output is the only mechanism through which climate change has an effect on growth. This effect operates indirectly through a reduction of investments into the capital stock, which is a rather narrow story on how climate change affects economic growth. As mentioned in the previous section, Dietz and Stern suggest two variations of the growth component in the DICE model which entail endogenous growth. In both models the damage multiplier is partitioned between the level of gross output and either the capital stock or the total factor productivity term. In the model version where damages additionally hit the capital stock, the capital stock is assumed to be proportional to an economy-wide productivity term, which, apart from necessary investments into the capital stock, is costless. Investments into this productivity term are consequently endogenous. In the model version where damages affect the level of total factor productivity, Dietz and Stern assume a positive knowledge externality in the form of knowledge spillovers from capital investments towards total factor productivity. In both models, climate damages are interpreted as having a negative effect on the economy's productivity, which has a direct and negative effect on economic growth. The authors find that these growth assumptions lead to an increase in the optimal emission control rate, even if the effects on their own are not very strong - especially before 2100. All results strongly depend on the fraction of damages which affects the capital stock or total factor productivity. In both models, the social planner's choice to invest into the knowledge stock is tied to capital investments. There is no standalone and endogenous decision to invest into R&D, as I will suggest in this chapter. A very interesting finding from the overall exercise with convex damages and an increased climate risk is that the importance of growth assumptions diminishes when climate damages are more severe.

In the same context, Moyer et al. (2014) note that the three IAMs which are used for the cost-benefit-analysis of carbon emissions by the US Interagency Working Group on the social cost of carbon (SCC) produce very narrow SCC pathways and thus do not reflect the true scale of uncertainty that they entail. Therefore, the authors build a model variant of the DICE model where the damages caused by global warming affect the economy's level of productivity or even its growth rate. As expected, both approaches lead to an increase in damages by many orders of magnitude. Naturally, when productivity growth is affected, the effects are stronger as compared to negative level effects. Moyer et al. (2014) thus conclude that the resulting SCC pathways are in general very sensitive to model assumptions. Moore and Diaz (2015) calibrate a two-regional version of the DICE model, where one region is poor and the other is rich. In one version of this two-regional DICE model global warming is modeled to have negative effects on total factor productivity growth, and in a second version it is modeled to accelerate capital depreciation. Both models lead to modest negative growth effects in rich countries and to strong effects in poor countries. This result could be caused by non-linear effects of climate damages on growth as suggested in Burke, Hsiang, and Miguel (2015). This would mean that even if the economies of rich countries in temperate climatic zones are less affected by global warming today, they would be increasingly affected if the atmospheric temperature continues to increase. In addition, it could be that poor countries have a higher share of climate sensitive production. Consequently, as poor countries grow richer, they might become less affected by global warming. Moore and Diaz (2015) underline that both possibilities have implications on future SSC pathways and are therefore highly policy relevant.

To my best knowledge, potential negative growth effects of climate change in combination with endogenous technological change driving economic growth, have not been addressed so far in the context of other IAMs. There are at least three IAMs which entail endogenous technological change, while economic growth is still driven by an exogenously given time series of total factor productivity. Their economies include two or more competing energy technologies. In the spirit of directed technical change, investments into the efficiency of these energy technologies is endogenous. These models are the ENTICE model by Popp (2004), which is a modified version of the DICE model, the WITCH model by Bosetti, Massetti, and Tavoni (2007) and the DEMETER model by Gerlagh and van der Zwaan (2003) and Gerlagh, van der Zwaan, et al. (2004).

In a second strand of the literature, the interdependency of economic growth and climate change is analyzed econometrically. Dell, B. F. Jones, and Olken (2012) assess historical panel data on within-country temperature fluctuations and find that temperature shocks overall have a lasting effect on income per capita and economic growth, but especially so in developing countries. In a meta analysis Dell, B. F. Jones, and Olken (2014) substantiate this result by analyzing potential drivers of this negative growth effect, such as agricultural output, labor productivity and mortality rates. In a similar context, Hsiang and Jina (2014) find empirically valid evidence that windstorms exert negative growth rather than level effects on income.

As mentioned above, Burke, Hsiang, and Miguel (2015) find a global non-linear effect of temperature on productivity levels. Their findings suggest that country-level production is concave in temperature and that it peaks at an average annual temperature of 13C°. Therefore, while cold countries are expected to be impacted by climate change only modestly, and in very cold countries this impact may even be positive, hot countries are expected to suffer more damages from climate change. As a result, the income-distribution between countries world wide may become more unequal. Further, the authors estimate a distributed lag model and find that only temperatures at the hot end of the temperature distribution suggest negative growth effects. Looking at the whole temperature distribution, this study does reject neither level nor growth effects of temperature on GDP.

3.3 The DICE model

The DICE model is a widely used IAM. It is very accessible and tractable. It is based on a Ramsey type economy, whose output causes carbon emissions, which in turn, through climate forcing, increase the global surface temperature as well as the temperature of the water systems. Damages due to global warming are modeled as a fraction of economic output which is lost to household income. In all scenarios the agents have perfect foresight. In the *social optimum scenario* the social planner is able to mitigate climate change in two ways. First, he can invest in the reduction of carbon emissions and second, he can shift his spending away from the carbon-emitting capital stock.
Following Rezai (2011), I rename the original base scenario by Nordhaus and call it the *constrained optimum scenario*. This scenario solves for the social optimum, where direct investments into the mitigation of carbon emissions are not possible, but otherwise the externality is fully internalized. The only way for the social planner to mitigate climate change in this scenario, is to reallocate resources away from growth enhancing capital stocks. The third scenario is the *business as usual scenario*, where the climate externality is not internalized. Consequently, there is no reallocation of resources and the private return on investment exceeds the social return on investment. When introducing endogenous growth to the DICE model, growth is no longer driven by an exogenously given parameter of technological advancements, but it is driven by the endogenous decision of the economy's agents to invest in their own productivity. Hence, agents divide their income between consumption and savings, which are sub-divided between investment into the physical capital

stock and investments into research and development. Future output is increased by higher investments in general, be it into the physical capital stock or into the knowledge stock. As a consequence, in the optimal and in the constrained optimal scenario, the savings and the capital accumulation effects are even stronger in the endogenous growth setting and, thus, negative growth effects caused by climate change become larger. In the business as usual scenario with endogenous growth, there is only a negative savings effect. In this scenario, households over-invest into capital and, therefore, have to endure more damages from climate change. Eventually, the damages become so big that they overcompensate any positive effects on income.

3.3.1 The original DICE model

Social planner problem Since the DICE model has been described in the literature numerous times, this section will provide a short overview of the model. For this study I use the latest version of DICE, which is DICE2016R. There is not yet a manual to this model available, but its code can be downloaded from the web page by Nordhaus (2017). The model is solved in discrete time and for sixty time intervals of five years. The first time period starts in 2015. The social planner problem is a fully centralized problem where the climate externality is fully internalized. At the heart of the model is neo-classical growth. The social planner maximizes the discounted sum of utilities from consumption per capita, c_t:

$$max\sum_{t=0}^{T}\left\{\frac{1}{(1+\rho)^{T_{\Delta}t}}T_{\Delta}L_tU\left(c_t\right)\right\} \tag{3.1}$$

Time intervals are denoted by t. T_{Δ} represents the length and T the absolute number of time intervals. The social discount rate ρ equals 1.5 %. Population size, L, grows over time at an exogenously given rate (see the appendix for more detail). Consumption, c_t, is expressed in per capita terms.

Per-period utility, U, carries the form of constant relative risk aversion (CRRA). The inter-temporal elasticity of substitution, ϵ, is 1.45.

$$U\left(c_t\right)=\frac{c_t^{1-\epsilon}-1}{1-\epsilon}-1 \tag{3.2}$$

The production function is Cobb-Douglas with labour-augmenting productivity, A. The production value, Y, equals gross income. The output elasticity of capital, K, is denoted by α and equals 0.3.

$$Y_t = (A_t L_t)^{1-\alpha} K_t^{\alpha} \tag{3.3}$$

The capital stock is raised by investments and is lowered by depreciation at a rate, δ, of 10 %.

$$K_{t+1} = I_t T_\Delta + (1-\delta)^{T_\Delta} K_t \tag{3.4}$$

A certain fraction of gross income is lost to damages, Ω, which are convex in the atmospheric temperature, $TATM$. A further fraction is lost to the cost of abatement, Λ, which increases exponentially in the emission control rate, MIU. Investments, I, thus, equal the difference between net income and consumption.

$$I_t = Y_t \left(1 - \Omega(TATM_t) - \Lambda(MIU_t)\right) - C_t \tag{3.5}$$

$$\Lambda_t = \varphi_t MIU_t^{2.6} \tag{3.6}$$

$$\Omega_t = 0.00236\, TATM_t^2 \tag{3.7}$$

Nordhaus assumes that due to exogenous and costless technological advancements, the efficiency of abatement, φ_t, in figure 3.6[2], increases over time until eventually the cost of abatement reaches zero.

Carbon emissions, E, are the sum of industrial emissions, E_{Ind}, and exogenous emissions from deforestation, E_{Land}[3]. Industrial emissions are caused in the process of final output production and they can be lowered in an endogenous effort by investing into the control rate MIU.

$$E_{Ind\,t} = sigma_t Y_t \left(1 - MIU_t\right) \tag{3.8}$$

[2]See appendix C for an exact representation of the efficiency of abatement term.
[3]See appendix C.

$sigma_t$ denotes the emissions output ratio[4].

$$E_t = E_{Ind\,t} + E_{Land\,t} \tag{3.9}$$

The carbon cycle consists of three reservoirs, whose transition is interdependent. Carbon emissions first flow into the lower atmosphere and from there they are passed on to the lower and deeper oceans. In equation (3.10) MAT stands for the carbon concentration in the lower atmosphere, MU represents the concentration in the upper and ML in the lower oceans.

$$\begin{pmatrix} MAT_{t+1} \\ MU_{t+1} \\ ML_{t+1} \end{pmatrix} = \begin{pmatrix} 0.2728 \\ 0 \\ 0 \end{pmatrix} E_t T_\Delta + \begin{pmatrix} 0.88 & 0.196 & 0 \\ 0.12 & 0.797 & 0.0015 \\ 0 & 0.007 & 0.9985 \end{pmatrix} \begin{pmatrix} MAT_t \\ MU_t \\ ML_t \end{pmatrix} \tag{3.10}$$

There is no limit to the accumulation of carbon in either of the reservoirs and there is no decay rate. Once carbon is added to the carbon cycle, it remains there.
Atmospheric carbon which is above the equilibrium concentration of $MAT_{EQ} = 588GtC$, drives the temperature of the atmosphere through radiative forcing, $FORC$. Forcing due to other greenhouse gases, $FORC_{EXt}$, evolves exogenously[5].

$$FORC_t = 3.6813 \frac{log\left(\frac{MAT_t}{MAT_{EQ}}\right)}{log(2)} + FORC_{EXt} \tag{3.11}$$

The climate system is modeled as a cycle as well, where energy is passed between the atmosphere, $TATM$, to the oceans, $TOCEAN$.

$$\begin{pmatrix} TATM_{t+1} \\ TOCEAN_{t+1} \end{pmatrix} = \begin{pmatrix} 0.1005 \\ 0 \end{pmatrix} FORC_t + \begin{pmatrix} 0.8718 & 0.0088 \\ 0.025 & 0.975 \end{pmatrix} \begin{pmatrix} TATM_t \\ TOCEAN_t \end{pmatrix} \tag{3.12}$$

Constrained optimal problem In the base scenario by Nordhaus (2008), the social planner has perfect foresight and fully internalizes the climate externality. However, he has no instrument for direct climate change mitigation at hand. Thus, Nordhaus sets $MIU_t = 0$. Consequently, in this scenario the economy arrives at the social optimum under the

[4]See appendix C.
[5]See appendix C.

constraint that direct investments into mitigation are not possible. The problem remains the same as the social planners problem with the additional constraint that $MIU_t = 0$. As mentioned above, I follow the terminology of Rezai (2011) and name this scenario the constrained optimal problem.

Business as usual problem In this scenario, the agent perceives carbon emissions, caused by economic activity, as exogenous. Consequently, although industrial emissions are still caused in the process of production, they are substituted by an exogenous emissions term as in equation (3.13). Therefore, the agent has no incentive to mitigate, neither through direct investments into the reduction of carbon emissions, nor by avoiding the use of carbon-emitting capital, and, in the endogenous model version by avoiding investments into the knowledge stock. The private return on capital and R&D investments exceeds the social return and, as a consequence, the agent over-invests.

$$E_t = E_{Land\,t} + E_{Exg\,t} \tag{3.13}$$

The computation of this scenario is carried out iteratively. The vector of initial carbon emissions over time is set to an arbitrary, but plausible, value. Using these values, climate damages are calculated accordingly and the growth model is solved. In the following round, carbon emissions are adjusted to those values, which households would have emitted given their gross output from the first round. The model is solved sequentially and in rounds until the vector of exogenous carbon emissions has converged to the industrial emissions.

The economic intuition behind the business as usual scenario and its computational solution are a simplified version of the business as usual scenario suggested in Rezai (2011) and Shiell and Lyssenko (2008). These authors suggest to divide the economy into N dynasties, who are each endowed with equal capital stocks and labor. All dynasties are aware of the externality caused by their own emissions and, therefore, mitigate. However, the larger the number of dynasties is, and since all foreign emissions are taken to be exogenous, the smaller is the amount of the externality which is internalized. From the perspective of each dynasty, as $N \to \infty$, the social marginal cost of carbon emissions within the dynasty goes down to zero and, thus, households effectively do not mitigate.
Although the economic intuition behind the business as usual scenario by Rezai (2011) is very plausible, for simplicity, I choose to solve the scenario as described above, where the economy has one representative agent who takes carbon emissions as fully exogenous.

Both economies evolve along the same path. Yet, the business as usual scenario by Rezai is computationally not as stable, as the one stated above. When dynasties become very small in order for mitigation to converge to zero, their gross income and consumption become very small, too. Combined with the discounting of future values, at some point the savings rate becomes arbitrary.

3.3.2 Endogenous growth and the DICE model

In the endogenous growth scenario, labor augmenting productivity, A_t, is explained within the model. The social planner has the additional option to invest into R&D, which fosters the development of new skills and technologies. To introduce endogenous growth to the DICE model, two alterations are necessary. First, it has to be clarified how the cost of investments into R&D is deducted from the budget and, second, one needs to define a functional form of how R&D investments translate into growth. I base those modifications on the Schumpeterian growth model described in chapter 2. In this model economic growth is driven by vertical innovations targeted at intermediate product variants. The likelihood of an innovation increases in R&D investments. However, while in the original DICE model innovations are costless, in the Schumpeterian growth model, R&D investments are costly and reduce the social planners budget for investments into the capital stock and for consumption. The explicit trade-off between the marginal cost and the marginal benefit of R&D investments in this study is new to the literature and it opens up an additional channel through which climate change affects economic growth. First, climate damages reduce the budget, which is available for re-investment versus consumption, and, second, they reduce the return on investments.

Social planner problem In this scenario the social planner internalizes the entire climate externality. He has two instruments at hand. He can mitigate climate change through direct emission control or by shifting resources away from carbon-emitting capital and knowledge stocks. The transition equation of the capital stock is identical to the original DICE model in equation (3.4). However, overall investments are now reduced by the social planner's spending on R&D, which is denoted by R.

$$I_t = Y_t \left(1 - \Omega(TATM_t) - \Lambda(MIU_t)\right) - C_t - L_t\, R_t \tag{3.14}$$

Since R&D investments are targeted at product variants, with L_t representing the number of product variants available at time t, total investments into research equal $L_t R_t$. Equation (3.15) is the discretized version of the transition of labor augmenting productivity in equation (2.36) in chapter 2.

$$A_{t+1} = \left(1 + \sigma\lambda \left(\frac{R_t}{(1+\sigma)A_t}\right)^{\gamma}\right)^{T_\Delta} A_t, \quad 0 < \gamma < 1, \quad 0 < \lambda, \sigma \tag{3.15}$$

In the Schumpeterian growth model σ describes the bandwidth of productivity levels which are attached to the production of intermediate products. The highest productivity level equals: $A_t^{max} = (1+\sigma)A_t$. Research expenditures in equation (3.15) are normalized by the highest available productivity level, because resource costs for new innovations increase the further technology advances. Aghion and Howitt (1999) call this the curse of complexity. The exponent γ causes decreasing returns to research with respect to R&D investments. λ is a rescaling factor to the propensity to innovate and σ is a factor of proportionality between the propensity to innovate and productivity growth. In equation (3.15), the term in the outer bracket, minus one, equals the arrival rate of an innovation in chapter 2 [6]. Torn from the original model described in chapter 2, both constants loose some of their meaning. From a technical point of view, together all three parameter values (λ, σ and γ) determine how R&D translates into productivity growth. Therefore, in this chapter, they are the free parameter values used to calibrate the endogenous growth component to the original growth component of the DICE model. What is important in equation (3.15) are the curse of complexity and the decreasing returns to scale in R&D. Other than that, the exact formulation of equation (3.15) is not essential to this study.

Constrained optimal problem This scenario is equivalent to the constrained optimal scenario with exogenous growth. Again, the social planner is capable of fully internalizing the externality. However, he is constrained and unable to mitigate, such that $MIU_t = 0$.

Business as usual problem This scenario is equivalent to the business as usual scenario with exogenous growth, where the agent anticipates future climate damages, but does not acknowledge that these are caused by his own carbon emissions. Therefore, the agent takes climate change as a given obstacle and not as an externality of production, which may be internalized.

[6] In chapter 2, this was denoted as ϕ_t, the average Poisson arrival rate of an innovation.

3.4 Calibration

The original version of the DICE model has a Ramsey-type economy at its core and economic growth is driven by the growth rate of total factor productivity. In DICE, this productivity term follows an exogenously given path and its growth rate declines over time. For this reason, the growth rate of GDP goes down as well. After some years, gross income follows an approximately linearly ascending path, which is visualized in figure 3.1 (a). This is in contrast to the endogenous growth component suggested in this paper, which follows an exponential growth path with a constant rate of growth once the steady state is reached. For this reason, it is not possible to achieve a perfect fit between the endogenous growth component and the original growth component from the DICE model. This is why it is necessary to construct not only a new version of DICE with an endogenous growth component, but also one with a Ramsey model, which essentially is identical to the original version except for its path of total factor productivity growth. Total factor productivity growth in the new Ramsey model is calibrated such that it produces the same path of output as the endogenous growth model.

For calibration, I divide the original DICE model as well as the new exogenous and endogenous model versions into their growth and climate components. Ignoring the climate component for now, I first calibrate the endogenous growth component to the original growth component by minimizing the sum of squared errors between both models' gross output. In this calibration the free parameters are those which determine how investments into R&D translate into the propensity to innovate. In equation (3.15) these are σ, γ and λ. The resulting growth trajectory is plotted in figure 3.1 (a). In a second step, I calibrate the new and exogenous growth component with exponential growth towards the endogenous growth component. Again, I do this by minimizing the sum of the squared residuals between gross income in both growth models. This time, however, I use the level of total factor productivity in each time period as the parameters of calibration, such that I am able to generate a near-perfect match between gross income in both models (apart from a numerical error). In this way, gross output of the recalibrated growth model versions is in the ballpark of the original growth model in DICE. Since gross output of the recalibrated model versions is almost identical, I will be able to compare the climate impacts of the endogenous growth setting to the exogenous counterpart in DICE. The climate component of the DICE model remains unchanged.

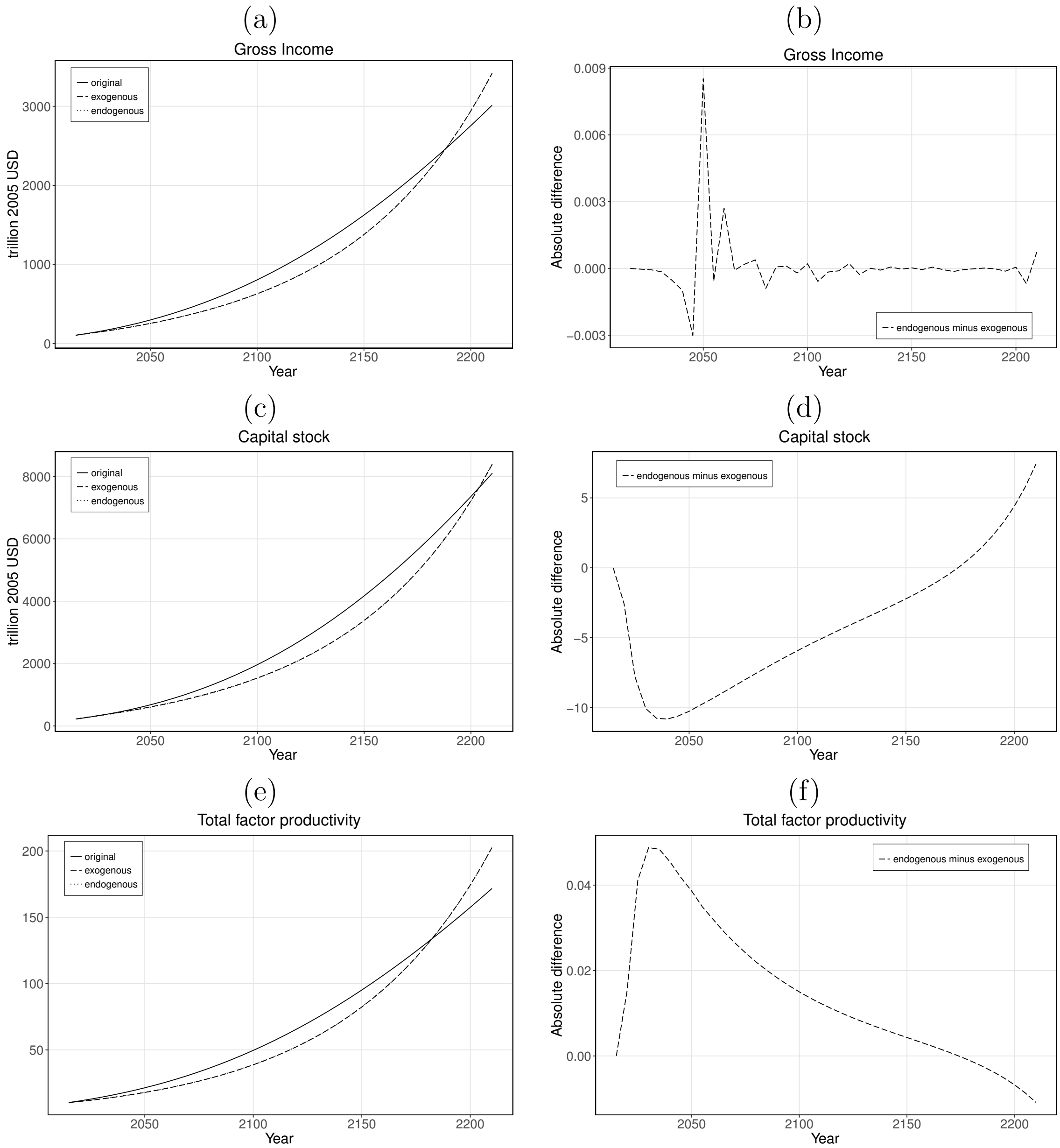


Figure 3.1: Left panels: Comparison of the original Ramsey model with an endogenous growth model and its new exogenous counterpart. The lines for the endogenous and the new exogenous growth model are on top of each other, since they are calibrated to one another. Right panels: Absolute differences between the endogenous growth model and its exogenous counterpart.

The transition between growth models As described above, I calibrate the exogenous growth model with exponential growth towards the endongenous growth model, such that gross income in both models is identical. Population sizes remain unchanged. I use

the path of total factor productivity, A_t, as free parameter values. Looking at the production function in equation (3.3), it is clear that for every path of A_t there exists only one path for the capital stock, K_t, which leads to a match of gross income, Y_t, in both models. Since the transition of K_t is uniquely determined by the social planner's optimization of social welfare over the savings rate, the adjustment of the exogenous to the endogenous growth model is a convex problem and there is only one combination of A_t and K_t which minimizes the difference in gross income between both models.
To better understand the transition from the endogenous to the exogenous growth model, think of the following thought experiment. What happens if I insert the same path of A_t from the endogenous growth model into the exogenous growth model? From the production function in equation (3.3) follows that in this case K_t would also have to be the same in both models in order to match gross income. But it wont be, because in this case all expenditures on R&D in the endogenous growth model would have to be added to consumption in the exogenous growth model. This, however, changes the utility in consumption over time and, thus, the same path of K_t from the endogenous growth model is not necessarily welfare optimal in the exogenous growth model.

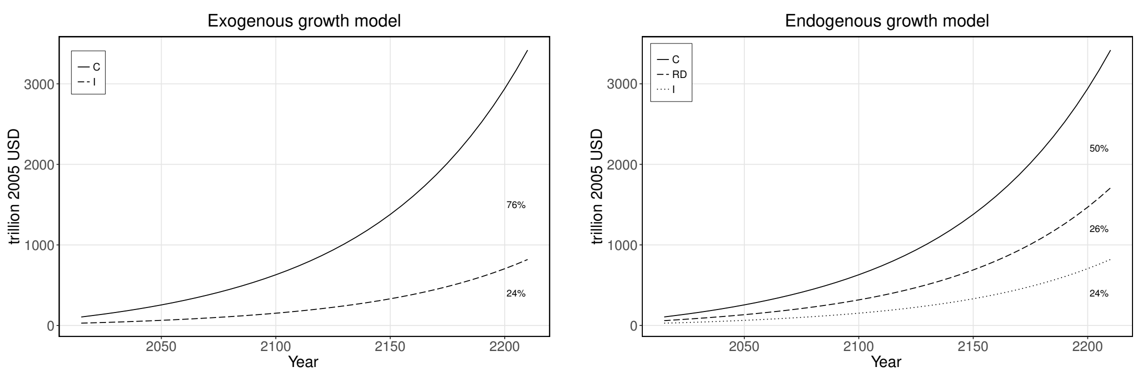


Figure 3.2: Stacked distribution of income

Ex-ante, it is not possible to say what the welfare optimal path for K_t in the exogenous model would be. Since households have an incentive to smooth their consumption over time, this will depend on the relative allocation of the additional units of consumption, which were freed from R&D investments. Therefore, I solve the exogenous growth model with the same path of A_t from the endogenous growth model. I find that this leads to a higher welfare optimizing capital stock in early time periods than what would be necessary to match gross income. Apparently, households postpone some of their consumption into the future. However, since the resources for R&D investments in the endogenous growth model are very evenly distributed over time (see figure 3.2), this effect is rather small. The capital stock in the exogenous model differs by less than 3 % from the capital stock in the endogenous growth model.

The result of this thought experiment is also reflected in the calibration of the exogenous growth model to the endogenous growth model. In figure 3.1 (d), the welfare optimizing capital stock is initially slightly higher in the exogenous model and it is offset by an initially smaller total factor productivity which is depicted in figure 3.1 (f). As a result, gross-income in both models in figure 3.1(b) is a near-perfect match. Both differ only by a numerical error [7].

To conclude, since the paths for K_t and A_t in both growth models are very close, nearly all resources, which are freed from investments in the endogenous growth model, are used for consumption in the exogenous growth model as shown in figure 3.2.

3.5 Results

After having calibrated the exogenous and the endogenous growth models with no climate externality towards each other and towards the original growth component of the DICE model, they are now put back together with the climate cycle from the original DICE model. Each version of the DICE model, the exogenous and the endogenous growth model version, is solved in three different scenarios, the *social planner optimum* (OPT), the *constrained optimum* (COPT) and the *business as usual* (BAU) problem, as described in section 3.3.

This section will focus on the impact of climate change on economic growth in an exogenously versus an endogenoulsy growing economy. This impact crucially depend on two economic mechanisms. First, it depends on how economic agents redistribute their savings between investments into physical capital versus knowledge stocks and, second, on how they reallocate their savings and consumption over time. In addition, in the OPT scenario this impact depends on the re-allocation of direct mitigation efforts.

To disentangle all growth effects which occur due to the climate externality, the endoge-

[7] To see whether the small differences in K_t and A_t in figures 3.1 (d) and (f) are driven by the end-point conditions of both models, I have also run the calibration of both growth models with 90 instead of 60 time steps. However, the curvature of both graphs for K_t and A_t remain. The results start to differ slightly after 35 time periods. Since I evaluate only the first 20 time periods of the DICE model, which correspond to 100 years, the end-point conditions do not drive my the results.

nization of the growth component and the three different scenarios, the following discussion is subdivided into paragraphs each targeted at a distinct model comparison.

Comparison of the exogenous OPT, COPT and BAU scenarios In this paragraph, I focus on the differences between the OPT, COPT and BAU scenarios in the exogenous growth setting. These are best understood when looking at the return on capital. Figure 3.3 depicts the return on capital in all six versions of the DICE model in different metrics. The absolute return on capital in the OPT scenario based on exogenous growth is depicted in sub-plot (a). This plot serves as a reference point to the other sub-plots. Subplot (b) depicts the returns on capital in the exogenous COPT and BAU scenarios plotted against the OPT scenario. As Rezai (2011) argues, the absence of a mitigation instrument in the COPT scenario, increases carbon emissions and climate damages relative to each unit of capital investments compared to the OPT scenario and, therefore, lowers the private return on capital. For this reason, in figure 3.3 (b) in the first time period the return on capital sets out at a lower level in the COPT compared to the OPT scenario. Because the only way to internalize the climate externality in the COPT scenario, is to avoid investments into carbon-emitting physical capital, the capital stock in figure 3.4 (b) remains at a lower level. In a world with decreasing returns to capital this increases the marginal return to capital relative to the OPT scenario. Therefore, after only a few years, the return on capital in the COPT scenario is slightly above the one in the OPT scenario (figure 3.3 (b)). In the long run, as climate damages kick in, the return on capital goes down way below the optimal scenario. In the BAU scenario, the private return on capital does not reflect the social cost of the productive assets. It neglects the negative climate externality and, therefore, in early time periods, it exceeds by far the socially optimal return in the OPT scenario. Hence, economic agents over-invest. Eventually, climate damages dominate all other effects and the return on investment under BAU adjusts to the return under COPT. This is also reflected in the behavior of the capital stock in figure 3.4 (b). While the capital stock under BAU is above the capital stock in the OPT scenario for a century, eventually, climate damages become so big, that it falls below the OPT benchmark. Figure C.2 (b) in appendix C shows that gross income follows a similar behavior as the capital stock and figure 3.6 (b) shows that consumption per capita goes down whenever investments go up and vice versa. In the exogenous growth scenarios, total factor productivity is exogenously given. Thus, their pathways are identical as shown in figure 3.5 (b).

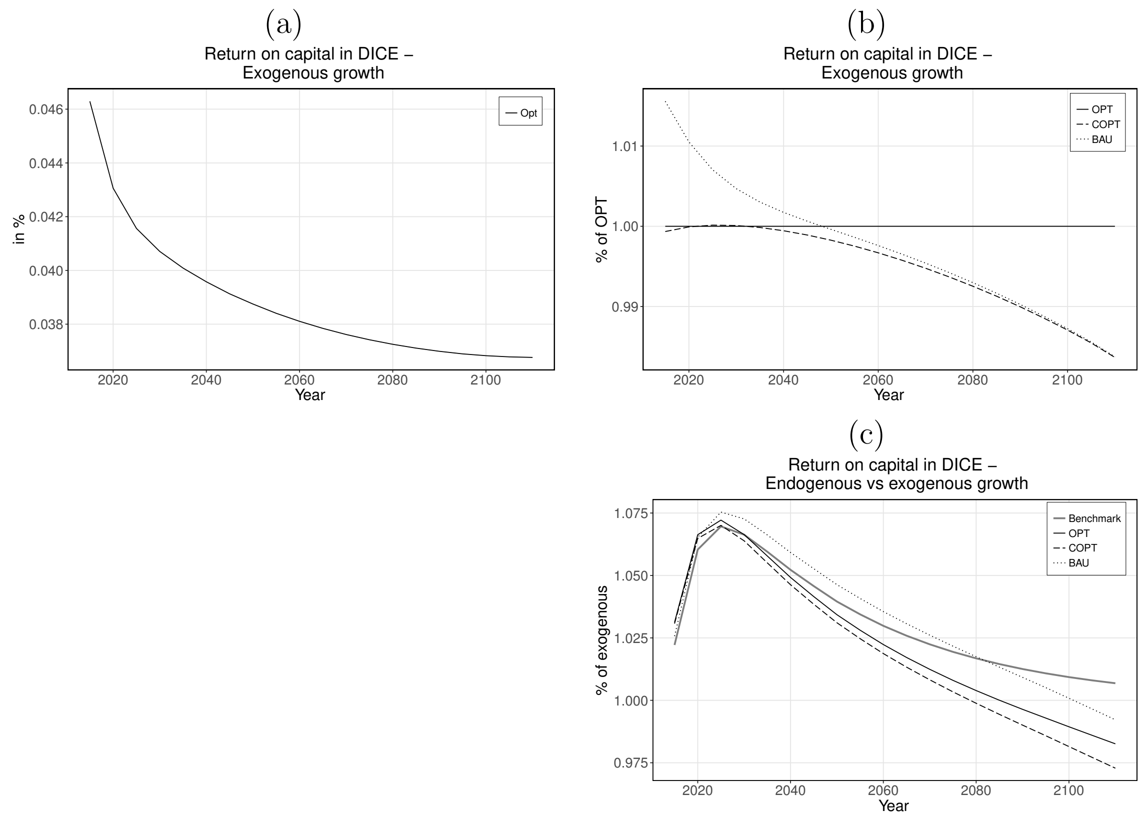


Figure 3.3: Comparison of DICE model versions: Return on capital

The endogenization of growth The question remains how these results change when growth in DICE is endogenized. To answer this question, I will first revisit the endogenization of growth in general as discussed in the previous section 3.4. In the literature, it is often overlooked that, the effects regarding the transition from exogenous to endogenous growth without the climate externality remain, when the climate externality is added. However, now they are accompanied by effects which are specifically brought about by climate damages. Figures 3.3 (c) to 3.7 (c) depict the relative differences between all three scenarios, when moving from the exogenous to the endogenous model version. The benchmark line illustrates the relative difference between the exogenous and the endogenous growth model component without a climate externality. Hence, it depicts those changes which are purely due to the transition between growth models. As was argued in section 3.4, since gross output is calibrated to be the same in both model versions (see the strictly horizontal benchmark line in figure C.2 (c)), additional investments into the knowledge stock in early time periods are, to a very small degree, compensated for by a reduction of investments into physical capital. Thus, in figure 3.4 (c) the benchmark line depicts a reduction in the capital stock in the first decade, which does not fully recover within the upcoming century. As a result the marginal benefit of capital investments increases and, thus, the benchmark for the return on capital in figure 3.3 (c) goes up. The labor force in both model versions is the same. Therefore, as the physical capital stock goes down, it is offset by a slightly higher knowledge stock, respectively total factor productivity, as depicted in figure 3.5 (c). Since the additional expenditures on R&D are paid for by a reduction in consumption, the benchmark for consumption per capita in figure 3.6 (c) is significantly lower than one. Overall, the endogenization of growth in the DICE model has similar effects on the stocks, on consumption and on the return on capital, as has the endogenization of the pure growth component.

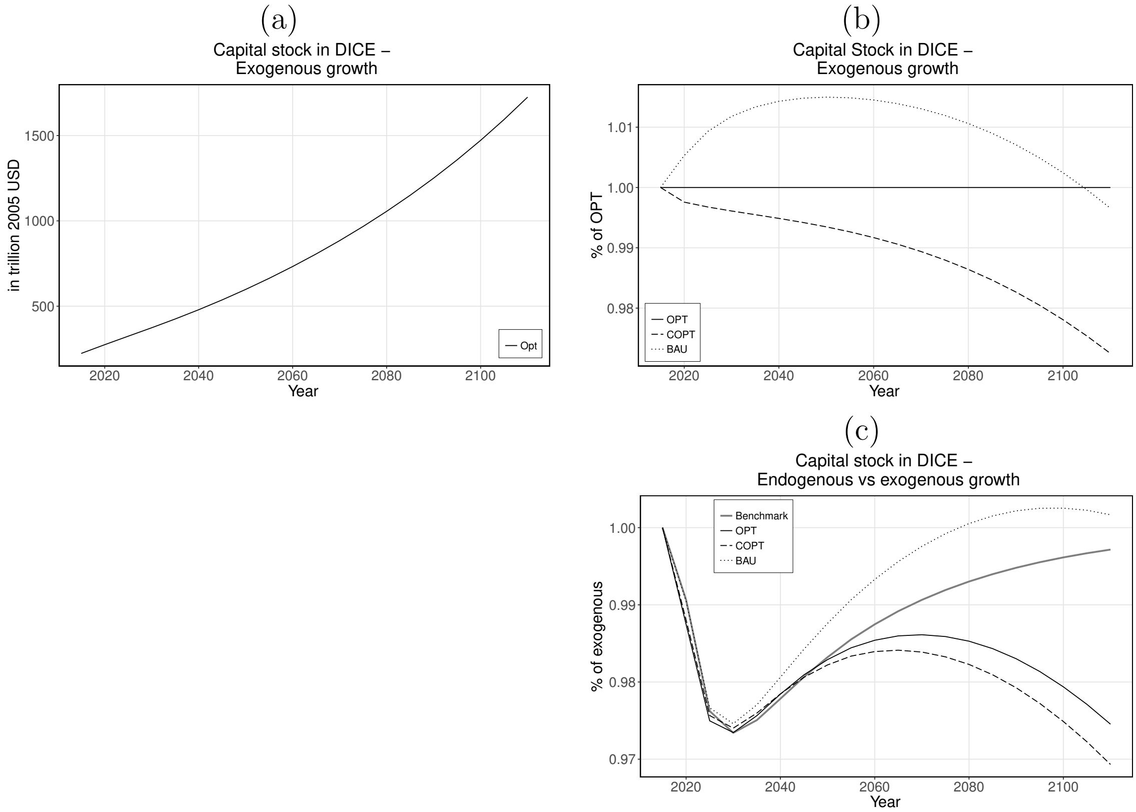


Figure 3.4: Comparison of DICE model versions: Capital stock

Transitioning from a pure growth model to DICE Transitioning from a growth model without a climate externality to a growth model which entails a climate externality, due to climate damages, the savings effect and the capital accumulation effect kick in. In section 3.1 I argued that both effects are negative. First, the capital accumulation effect is negative, because climate damages lower the budget that is available for investments. Second, the savings effect is negative, because, due to climate damages, the return on investment decreases and, therefore, the economic agent has an incentive to shift some of his consumption towards earlier time periods. Using the DICE model, Fankhauser and Tol (2005) show in a numerical example that this incentive is stronger than the one of consumption smoothing, where households have an incentive to save more in earlier time periods in order to make up for future losses. In the endogenous growth model version of DICE both effects are even stronger than under exogenous growth, because here climate damages not only reduce the return on investment into physical capital, but also into the knowledge stock. This leads to a negative savings effect on the knowledge stock and, therefore, the relative knowledge stock in figure 3.5 (c) in all three scenarios of the DICE model eventually falls below the benchmark. At this point, the negative savings effect on the capital stock is not relevant, because it occurs in the exogenous growth model version of DICE, too. A lower level of total factor productivity causes a lower growth rate of gross income, which also falls below its benchmark in all three scenarios in figure C.2 (c). This, in turn, leads to a negative capital accumulation effect. If gross income is smaller, the budget which is available for investment is also smaller. Consequently, in the long run, the relative capital stocks in all three scenarios are eventually below the benchmark line in figure 3.4 (c) [8]. In accordance to this, after some decades, in the endogenous growth model version relative consumption in figure 3.6 (c) is lower than the benchmark [9].

Comparing the endogenization of OPT, COPT and BAU When comparing the endogenization of the OPT, COPT and BAU scenarios with each other, it is striking that the same effects that we observed for the exogenous growth model scenarios are now magnified when growth is endogenized. In the COPT scenario with endogenous growth, when there is no instrument of direct mitigation available, the return on investment for both stocks falls below the return on investment in the OPT scenario with endogenous growth. This means that the knowledge stock also grows at a slower rate than in the OPT scenario with endogenous growth and, thus, gross income growth and the accumulation of physical capital are slower. Consequently, when the COPT scenario is endogenized, gross income and the capital stock in relative terms in figures C.2 (c) (in appendix C) and 3.4 (c) are lower compared to when the OPT scenario is endogenized. Households shift

[8] Even the relative capital stock in the BAU scenario falls below the benchmark shortly after year 2100.

[9] The relative return on capital in all three scenarios in figure 3.3 (c) falls below its benchmark, because its marginal product depends on the size of the knowledge stock.

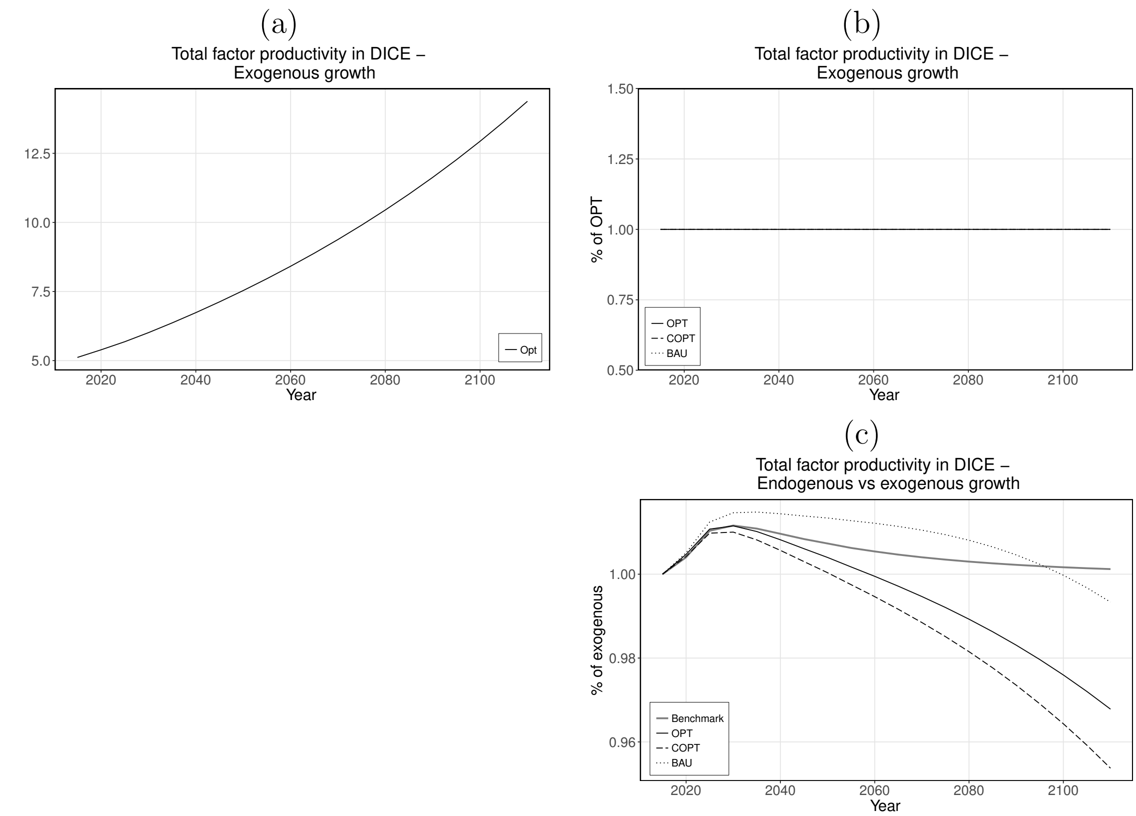


Figure 3.5: Comparison of DICE model versions: Total factor productivity

their spending accordingly towards consumption (see figure 3.6 (c)). To sum up, when growth is endogenized the climate externality has a stronger, negative impact on economic growth in the COPT scenario than in the OPT scenario in relative terms[10].

The same line of argument with opposite signs holds for the BAU scenario. In this scenario, the climate externality is perceived as exogenous and, therefore, the private return on investment exceeds the social return. This increases the rate at which the knowledge stock grows in the BAU scenario with endogenous growth compared to the OPT scenario with endogenous growth. Initially, this raises the pace at which gross income grows. As a result, the physical capital stock also grows at a higher pace, due to the accumulation effect. Therefore, when the BAU scenario is endogenized, gross income and the capital stock in relative terms are initially higher compared to when the OPT scenario is endogenized (see figure C.2 (c) in appendix C and figure 3.4 (c)). To finance the over-investment into capital, consumption in figure 3.6 (c) is reduced. However, since a higher gross output

[10]Whether the absolute reduction in gross income in the COPT scenario is bigger than in the OPT scenario due to the endogenization of growth, is not deducible from figure C.2 (b) and (c).

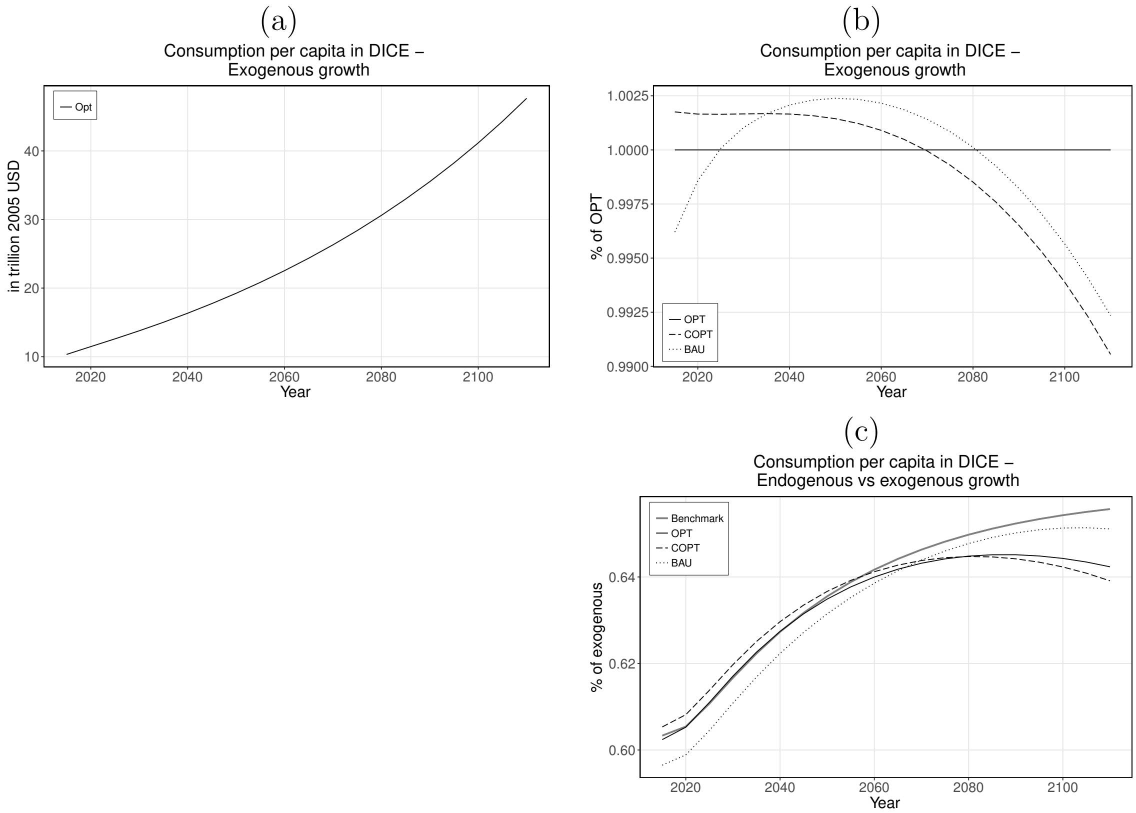


Figure 3.6: Comparison of DICE model versions: Consumption per capita

causes stronger climate damages, this initially positive growth effect compared to the OPT scenario is eventually reversed around year 2200.

The temperature increase of the atmosphere As shown in figure 3.7 (a), in the OPT scenario with exogenous growth the temperature increase in 2100 relative to pre-industrial times amounts to roughly 3.5°C. In figure (b), the temperature increase in the COPT and BAU scenarios with exogenous growth, compared to the OPT scenario with exogenous growth is even higher, because in these scenarios the agents do not have an instrument of direct mitigation at hand. The temperature increase in the BAU scenario is slightly higher than in the COPT scenario, as gross-income under BAU is higher than under COPT and consequently the economy emits more carbon into the atmosphere. When growth is endogenized in sub-figure (c), temperatures increase even further in the BAU scenario, because in this scenario gross income is higher than in the BAU scenario with exogenous growth. Vice versa, in the COPT scenario with endogenous growth the temperature increase is smaller, because gross income in this scenario is smaller than in the

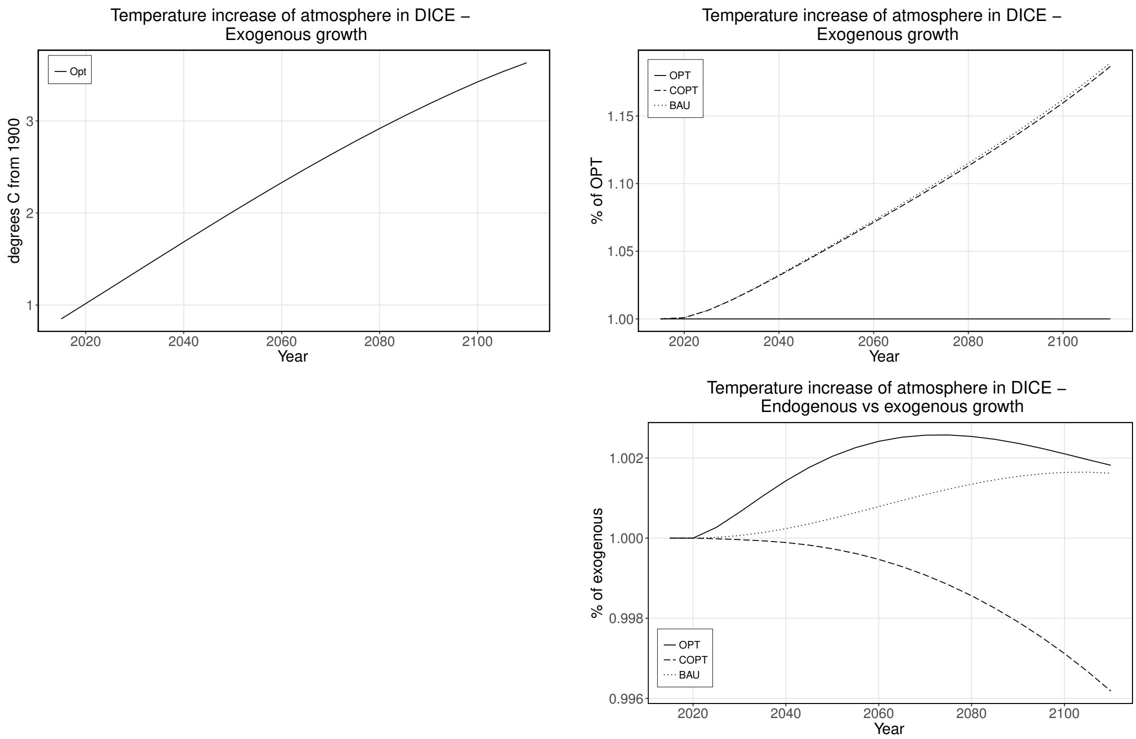


Figure 3.7: Comparison of DICE model versions: Temperature increase of atmosphere

COPT scenario with exogenous growth. In the OPT scenario, the endogenization leads to a smaller gross income compared to the exogenous growth version of this scenario. Therefore, the social planner's resources for climate mitigation shrink and thus the social planner reduces its mitigation efforts. This effect even over-compensates the fact that a lower gross income also leads to less carbon emissions. Therefore, the temperature increase in the OPT scenario with endogenous growth is higher than in the OPT scenario with exogenous growth. Yet, figure 3.7 (c) shows that these effects are rather small. The differences in the atmospheric temperature increase between the scenarios are brought about by similar pathways of carbon emissions (see figure C.4 in appendix C) and they lead to equivalent damages as a fraction of gross income (see figure C.5 in appendix C).

3.6 Conclusions

Climate change impacts economic growth through various channels. In the literature, this has recently lead to multiple suggestions on how this inter-dependent relation could be modeled. Prominent ideas are to have climate damages not only reduce gross income levels, but also physical capital or even knowledge stocks. These models predict by far more severe reductions of economic growth and a higher social cost of carbon. However, these results are strongly driven by their underlying assumptions. To date there is only punctual data available on how climate change affects gross income growth, capital stocks and total factor productivity. In addition, empirically, it is not clear that climate damages have a persistent effect on future outcomes.

At this point this chapter can contribute to the literature. The modeling approach in this chapter does not rely on an arbitrary assumption on how climate damages might affect productivity directly. It rather introduces an endogenous growth model to open new channels for indirect dynamics, affecting investments not only into a physical capital stock, but also into the knowledge stock. These dynamics turn out to have a substantial negative and, in particular, lasting impact on the accumulation of capital stocks and thereby on economic growth.

Using a recalibrated version of the DICE model with endogenous growth, I find that in the *Optimal Scenario*, where the climate externality is fully internalized, the exogenous growth model version of DICE over-estimates gross income by 2.3 % in 2100 and by 6.8 % in 2150. In the very long run, this gap gets even larger. This result, however, is driven by the choices of the social planner. The social planner chooses to invest less into capital and labor-augmenting productivity as a measure to reduce carbon emissions. Put differently, welfare-optimizing growth might be lower than what has previously been found. As long as our societies have not yet achieved the transition towards a carbon-free economy, it might be optimal to reduce economic growth intentionally by more than has been found so far, until new and clean technologies allow for higher rates of economic growth again.

Chapter 4

A re-calibration and Monte-Carlo analysis of different growth trajectories in the DICE model

4.1 Introduction

While most macroeconomic growth models are run for a couple of decades at best, in an environmental context, these same growth models are often solved for centuries. The resulting projections are, consequently, very sensitive to the model assumptions. In this chapter, I will show what historical data on GDP and consumption shares tell us about the uncertainty that is tied to future growth trajectories and how these add to the uncertainty regarding future climate change. To give an example, I recalibrate the growth component of the DICE-2016R model as described in chapter 3 using the Bayesian calibration approach developed in chapter 2.

While economic growth is a major determinant of projected carbon emissions and climate damages, its importance is often overlooked in the Integrated Assessment literature. In the DICE model, this link is especially intense, since GDP translates directly into carbon emissions at an exogenously given proportion, which shrinks over time as fossil energy is gradually substituted by clean energies.

One major advantage of this Bayesian calibration technique in chapter 2 is that it quantifies the uncertainty associated with plausible parameterization of economic growth, given a particular growth model and the historical data. The procedure involves sampling Markov chains of all parameters of calibration. These are then used to run a Monte-Carlo analysis

on the DICE model. The resulting confidence intervals will show how different growth trajectories within the DICE model translate into confidence intervals of atmospheric temperature increases and climate damages.
The FUND model is a prominent IAM on which Monte-Carlo analyses have been run before. These analyses are run with respect to a variety of parameter values which stem from both the growth as well as the climate component (see for instance Anthoff, Tol, and Yohe (2009a), Anthoff, Tol, and Yohe (2009b) and Anthoff and Tol (2009))[1]. The density functions of these parameter values are predominantly based on expert guesses. In contrast to this approach, in this chapter, I specifically focus on the uncertainty of future economic growth and how it translates into the uncertainty over future climate outcomes. The novelty in this chapter is that a Bayesian approach allows for a statistically sound estimation of the probability density functions of those parameters which determine economic growth in DICE. However, the recalibrated version of the DICE model in this chapter is still a deterministic model. The uncertainty over parameter values is explored in a Monte-Carlo analysis. Thus, we repeat the same optimization problem with different, but fixed parameter values. An other conceptual approach towards uncertainty in IAMs would be to include the uncertainty over certain parameter values directly in the optimization problem. Implementations of a corresponding recursive dynamic programming approach in DICE can be found in Crost and Traeger (2013), Cai, Judd, and Lontzek (2013) and Traeger (2014).

4.2 Model calibration and data

It is not fully clear how the DICE-2016R model and its growth component were calibrated. The calibration of previous model versions has been addressed fragmentarily in numerous publications as for instance in Nordhaus (2008) and Nordhaus and Sztorc (2013). According to these items, economic growth is calibrated towards historical data from the International Financial Statistics (IFS) of the IMF (see Nordhaus (2007)) starting in 1960. Total factor productivity is chosen such that the model can reproduce observed data on world GDP and capital stocks. Projections of future economic growth starting in 2015 are made assuming that total factor productivity declines exogenously following a logistic equation. Labor supply is exogenous and follows a logistic-type equation as well, which is fitted towards the United Nations (2015) projections. World population reaches a limit of

[1]The documentation of the FUND model and further information on Monte-Carlo runs can be found on the web page: http:www.fund-model.org

11.5 billion in 2300.

In this chapter, I recalibrate the growth component of the original DICE model using the same Bayesian approach which I have described in chapter 2. In a nutshell, this approach assumes that macroeconomic models of long-run growth describe a smooth trend of gross income growth, while they neglect business cycles and seasonal fluctuations in the medium and short run. Thus, the difference between the observed and simulated data picks up all stochastic shocks which occur in the economy. Therefore, it is assumed to follow a stochastic process, an auto-regressive process of order one. The aim is to maximize the likelihood of the simulated data to describe the true growth trend of the economy, by maximizing the likelihood of the residuals between the observed and simulated data to follow an AR(1) process. The maximization takes place over a pre-determined range of free parameter values. Along the way, I draw Markov chains, which constitute a representative sample of the kernel-densities of the free parameter values. These can then be used to run a Monte-Carlo analysis on the DICE model.

At the core of the original DICE model lies a Ramsey-type economy. Since the original DICE model is solved in discrete time with time intervals of five years, I cannot use the Ramsey model in continuous time like I have already calibrated in chapter 2. Therefore, for the purpose of this chapter, I recalibrate the Ramsey model in discrete time and specifically with time intervals of five years. Parameters of calibration are those parameters, which have a direct impact on the level and slope of gross income. These are total factor productivity growth and the initial stocks of capital and total factor productivity. I draw one Markov chain of 100k realizations for each parameter of calibration. All chains converge within the first 25k realizations. To be on the safe side, I disregard the first half of each chain. Further, I apply a thinning of 50. This pushes the auto-correlation of the chain elements in each chain down to below 5% after a few realizations each. The final chain length is thus 1k. Consequently, this is also the number of Monte-Carlo runs on the DICE model.

The DICE-2016R model starts in 2015 and it is solved for sixty time periods. It consequently runs until year 2310, which is long enough for reasonable end-point conditions to not affect the model's outcome within the first century at a discount rate of 1.5%. I calibrate the growth component of the model towards GDP and the consumption share from 1950 to 2010. This means that the model is calibrated towards 13 observations of

GDP and consumption shares each. Consumption shares from 1950 to 2010 are calculated using the Penn World Tables, described in Feenstra, Inklaar, and Timmer (2015a) and GDP was taken from the Maddison data (see Maddison (2010)) and rescaled to meet the unit requirement of the original DICE-2016R model. For population sizes from 1950 to 2010 I use the Maddison data set as well. The newly calibrated Ramsey model is able to reproduce GDP per capita very well. The observed data lies within the 50% confidence interval. The consumption share is less accurately reproduced and lies within the 90% confidence interval.

4.3 Results

After having recalibrated the growth component of the DICE model, I compare its model output to the original DICE model. In addition, I evaluate the uncertainty over future economic growth and how it translates into confidence intervals of an atmospheric temperature increase in the recalibrated model version.

The Social Optimum Scenario Since total factor productivity growth is a parameter of calibration, I implicitly assume it to be a constant. The annual growth rate of total factor productivity which best fits the data is 8.5%. In the original DICE model, this growth rate is initially 8.2% and it is assumed to gradually decrease thereafter. The initial values for the capital and knowledge stocks are very similar to those in the original DICE model (see figures 4.1 and D.2 in appendix D). Since total factor productivity in the recalibrated version of the DICE model is strictly above the one in the original model, while the labor force in both models is identical and the initial capital stocks are very close, the median projection of gross output in the recalibrated model version is also higher than in the original model in figure D.1 in appendix D. The same holds for the path of future capital stocks and consumption in figures D.2 and D.3 [2].

The confidence interval of total factor productivity in figure 4.1 is rather large. The expected variation from the mean in 2100 within the 90 % confidence interval amounts to 26 %[3]. This has a strong effect on the uncertainty of future economic growth in figure D.1

[2] The return on capital in figure D.5 is higher than in the original version of the DICE model, because a higher total factor productivity increases the marginal productivity of the capital stock.

[3] In 2100, the median projection of total factor productivity amounts to 19.4 units and the 90 % confidence interval is 10.3 units wide.

in appendix D. The expected variation from the mean within the 90 % confidence in 2100 interval amounts to 36 %[4].

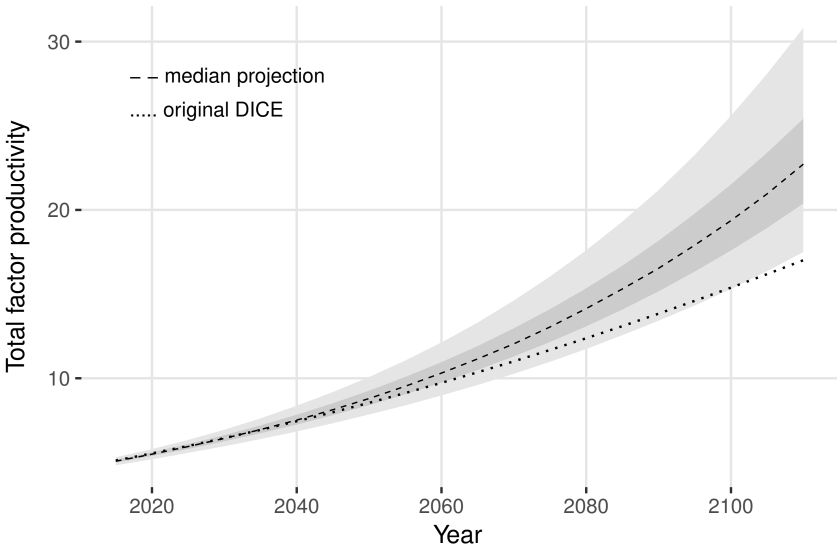


Figure 4.1: Total factor productivity

Note: The light gray shaded area corresponds to the 90% confidence interval. The dark gray shaded area corresponds to the 50% confidence interval of all projections.

Compared to the original DICE model, the persistently higher gross income in the recalibrated DICE model leads to more carbon emissions and climate damages in figures D.7 and D.8 (in appendix D) and to a stronger temperature increase of the atmosphere in figure 4.2. In 2100, the estimated mean atmospheric temperature increase amounts to $3.6°C$. Interestingly, while the confidence interval on total factor productivity is rather wide, the expected variation from the mean temperature within the 90% confidence interval in 2100 reaches only 4%. This is for two reasons. First, carbon emissions have a delayed impact on atmospheric temperatures and, second, households are able to mitigate. Overall, since mitigation efforts reach 100% shortly after 2100, the relatively high uncertainty in gross income has only a small impact on the uncertainty regarding the temperature increase.

In addition, the emission control rate and the savings rate in figures 4.3 and D.4 in appendix D are lower than in the original DICE model. This is because in the recalibrated version of DICE income per capita grows faster. Thus, households are able to consume

[4] The median projection of gross income in 2100 is 1057 units and the width of its 90 % confidence interval is 760 units.

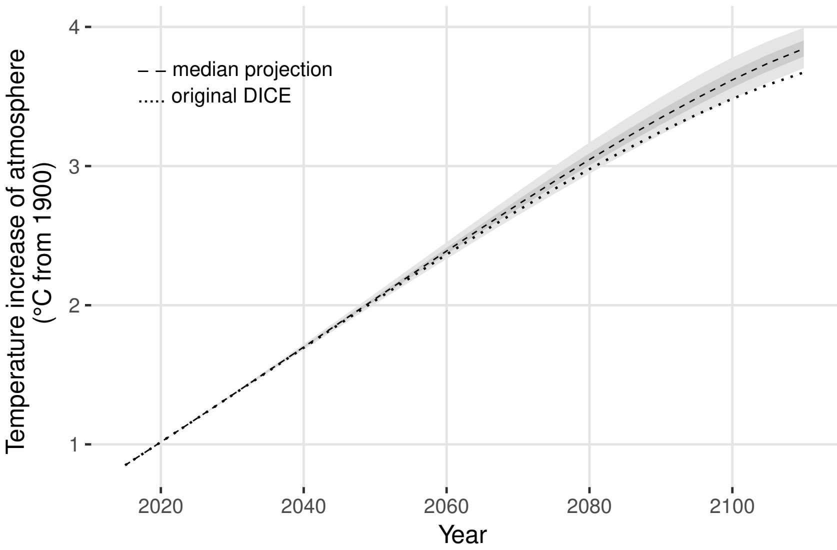


Figure 4.2: Temperature increase of atmosphere

Note: The light gray shaded area corresponds to the 90% confidence interval. The dark gray shaded area corresponds to the 50% confidence interval of all projections.

more, which leads to a steeper reduction of the marginal benefit of consumption over time. Therefore, households are less inclined to give up consumption in early time periods in order to increase future income growth.

The social cost of carbon (SCC) is a widely used concept for understanding the cost of global warming and for implementing climate change policies. It is the social cost of an additional ton of carbon dioxide emissions into the atmosphere. In the *Optimal Scenario* this is equivalent to the marginal cost of reducing carbon dioxide emissions by one ton:

$$SCC = -\frac{d\Lambda_t\left(MIU_t\right)Y_t}{dE_{Ind\,t}} = -\frac{\frac{\partial \Lambda_t(MIU_t)Y_t}{\partial MIU_t}}{\frac{\partial E_{Ind\,t}}{\partial MIU_t}} \tag{4.1}$$

The variation in the SCC is purely driven by the probability distribution of total factor productivity growth. In 2020 in figure 4.4 (a) this variation is rather small, but it almost doubles by 2050 in figure 4.4 (b).

The Constrained Optimum Scenario In this scenario households have no instrument of direct mitigation at hand. The only way to reduce carbon emissions is to emit less in

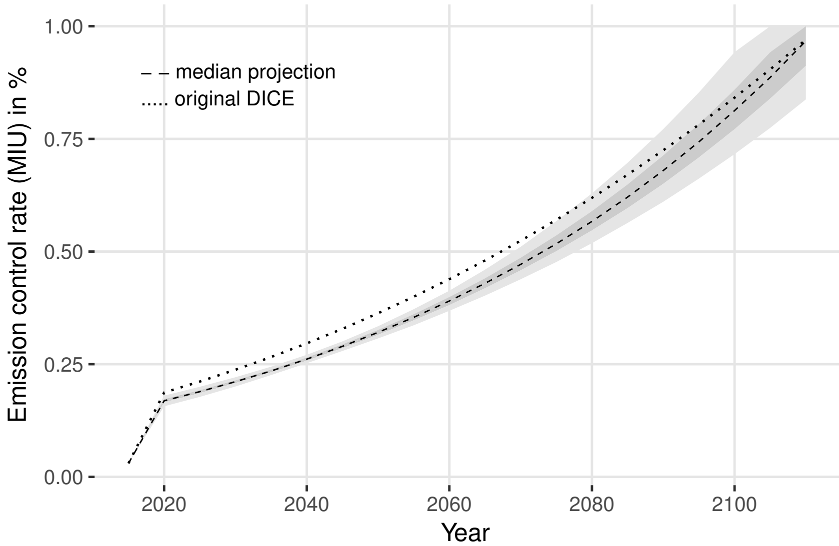


Figure 4.3: Emission control rate

Note: The light gray shaded area corresponds to the 90% confidence interval. The dark gray shaded area corresponds to the 50% confidence interval of all projections.

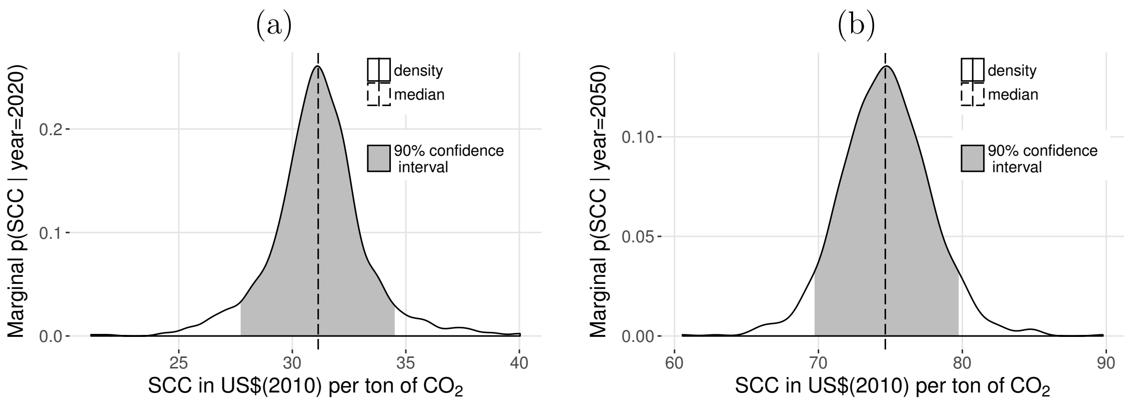


Figure 4.4: Marginal probability density functions of the social cost of carbon in 2020 (a) and 2050 (b) in 2100.

the first place by reducing gross production. As discussed in chapter 3, in this scenario gross income and the capital stock are lower compared to the *Optimal Scenario*, while carbon emissions and climate damages are higher. This scenario is identical to the original *Business as usual Scenario* by William Nordhaus.

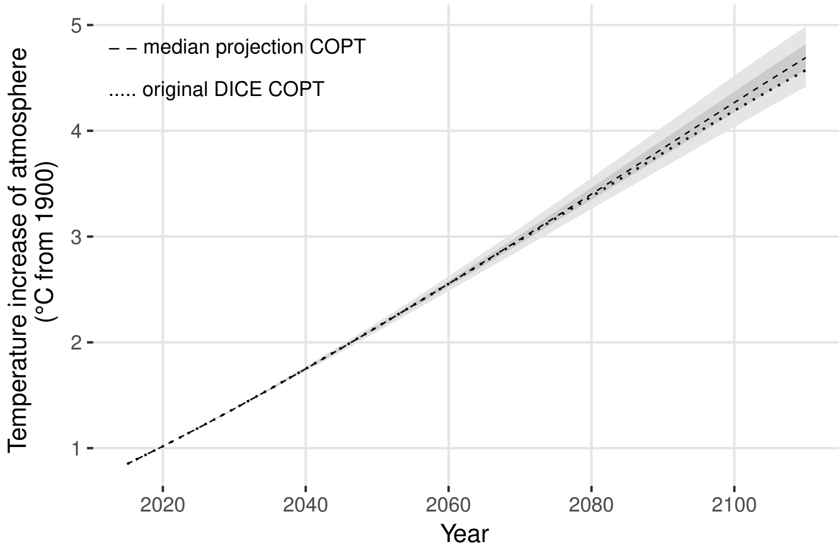


Figure 4.5: Temperature increase of atmosphere in the *Constrained Optimal Scenario*

Note: The light gray shaded area corresponds to the 90% confidence interval. The dark gray shaded area corresponds to the 50% confidence interval of all projections.

The atmospheric temperature increase in this scenario compared to pre-industrial times amounts to 4.7C°. In the *Optimal Scenario*, the temperature increase is lower with 3.8C°. For the *Optimal Scenario* I have argued that the confidence interval around the atmospheric temperature increase, even in 2100, is rather narrow, although the uncertainty associated with the parameterization of economic growth is relatively high. This is the case for two reasons. First, carbon emissions have a delayed impact on atmospheric temperatures and, second, households are able to mitigate. After one century, mitigation efforts in the *Optimal Scenario* reach almost 100%.
In the *Constrained Optimal Scenario*, households cannot directly mitigate carbon emissions. While gross income in this scenario is smaller, its variation translates unmitigated into the temperature increase of the atmosphere. Consequently, in this scenario, the atmospheric temperature increase is expected to vary by 11% from the mean within the 90% confidence interval in 2100, compared to only 4% of variation in the *Optimal Scenario*.

Thus, if climate change is unmitigated in the DICE model, the uncertainty concerning future economic growth renders future climate outcomes almost three times as uncertain compared to the social optimum with direct mitigation.

4.4 Conclusions

In this chapter I demonstrate how a Bayesian approach towards the calibration of growth models, as described in chapter 2, can be used to recalibrate the growth component of an Integrated Assessment model. Since the approach is very flexible, it can be used to recalibrate any Integrated Assessment model which has a model of economic growth at its core. In this chapter, I have recalibrated the growth component of the DICE-2016R model. The growth component was calibrated towards observed data on GDP and consumption shares since 1950. The resulting parameters of calibration were used to project economic growth, together with the original climate component of the DICE model, into the future. The Bayesian approach allows for a statistically sound derivation of confidence intervals of future economic growth. By running a Monte-Carlo analysis on the DICE model, I am able to assess how these translate into an expected variation of the climate variables. A considerable complication of this approach is that it requires observed time series on those variables towards which the model is calibrated. Because of the statistical nature of this approach, these time series have to be without gaps. This may be problematic for unobservable variables or for multi-regional models where the gathering of regional data can be cumbersome or even impossible.

A major contribution of this chapter is to derive the uncertainty over future GDP growth from a stochastic model and observed data and to show how it translates into uncertainty over future climate damages and the social cost of carbon. Since Nordhaus assumes in his *Optimal Scenario* relatively low costs of mitigation, which decrease sharply over time, mitigation efforts reach 100% shortly after 2100. Therefore, the large uncertainty over future income which I find in this chapter has only a small impact on the uncertainty over the temperature increase and climate damages. In the *Constrained Optimum Scenario*, where carbon emissions remain unmitigated, this impact is almost three times as large.

The median projection of gross income in the recalibrated version of DICE is above the one in the original model. This is because I project past observed growth into the future using a constant rate of growth of total factor productivity, while Nordhaus assumes a gradual decay of total factor productivity growth. Nevertheless, from a modeling point of view it would be possible to transfer Nordhaus' growth decay to the recalibrated version of the DICE model.

Appendix A

Appendix to chapter 1

A.1 Transformation of the Jones model into stationary variables

All state and control variables are transformed into stationary values using:
$\hat{A} = \frac{A}{L^{\beta_A}}$, $\hat{K} = \frac{K}{L^{\beta_K}}$, $\hat{C} = \frac{C}{L^{\beta_K}}$ and $\hat{P}_A = \frac{P_A}{L}$ with $\beta_A = \frac{\eta_L}{1-\eta_A}$ and $\beta_K = \frac{1-\eta_A+\eta_L}{1-\eta_A}$.

such that in the steady state $\dot{\hat{A}} = \dot{\hat{K}} = \dot{\hat{C}} = \dot{\hat{P}}_A = 0$.
To derive β_A:

$$\dot{A} = \frac{\eta_L}{1-\eta_A} nA \tag{A.1}$$

$$\begin{aligned}\left(\frac{\dot{A}}{L^{\beta_A}}\right) &= \frac{\dot{A}L^{\beta_A} - A\beta_A L^{\beta_A - 1}\dot{L}}{L^{2\beta_A}} \\ &= \frac{\hat{A}\eta_L n}{1-\eta_A} - \hat{A}\beta_A n = 0\end{aligned} \tag{A.2}$$

if

$$\beta_A = \frac{\eta_L}{1-\eta_A} \tag{A.3}$$

To derive β_K:

$$\dot{K} = \frac{\eta_L}{1-\eta_A} nK + nK \tag{A.4}$$

$$\begin{aligned} \left(\frac{\dot{K}}{L^{\beta_K}}\right) &= \frac{\dot{K}L^{\beta_K} - K\beta_K L^{\beta_K-1}\dot{L}}{L^{2\beta_K}} \\ &= \frac{\hat{K}\eta_L n}{1-\eta_A} + n\hat{K} - \hat{K}\beta_K n = 0 \end{aligned} \tag{A.5}$$

if

$$\beta_K = \frac{\eta_L}{1-\eta_A} + 1 \tag{A.6}$$

Thus, for the equation of motion for $\dot{\hat{A}}$ follows:

$$\dot{\hat{A}} = \frac{\dot{A}}{L^{\beta_A}} - \frac{A}{L^{\beta_A}}\beta_A \frac{\dot{L}}{L} \tag{A.7}$$

$$\dot{\hat{A}} = \alpha_J \hat{A}^{\eta_A}(1-\phi)^{\eta_L} - \hat{A}\beta_A n \tag{A.8}$$

For $\dot{\hat{K}}$:

$$\dot{\hat{K}} = \frac{\dot{K}}{L^{\beta_K}} - \frac{K}{L^{\beta_K}}\beta_K \frac{\dot{L}}{L} \tag{A.9}$$

$$\dot{\hat{K}} = \hat{Y} - \hat{C} - (\delta + n\beta_K)\hat{K} \tag{A.10}$$

For $\dot{\hat{C}}$:

$$\dot{\hat{C}} = \frac{\dot{C}}{L^{\beta_K}} - \frac{C}{L^{\beta_K}}\beta_K \frac{\dot{L}}{L} \tag{A.11}$$

$$\dot{\hat{C}} = \frac{\hat{C}}{\theta}(r - \rho - n) + n\hat{C} - \hat{C}\beta_K n \tag{A.12}$$

For $\dot{\hat{P}}_A$:

$$\dot{\hat{P}}_A = \frac{\dot{P}_A}{L} - \hat{P}_A \frac{\dot{L}}{L} \tag{A.13}$$

$$\dot{\hat{P}}_A = \hat{r}\hat{P}_A - \hat{\pi} - \hat{P}_A n \tag{A.14}$$

with

$$\hat{r} = (1-\sigma)^2 \frac{\hat{Y}}{\hat{K}} - \delta = r \tag{A.15}$$

and

$$\hat{\pi} = \sigma(1-\sigma)\frac{\hat{Y}}{\hat{A}} = \frac{\pi}{L} \tag{A.16}$$

In the steady state r is constant and π grows at rate n. Consequently P_A does also grow at rate n.

The wage equality is given by:

$$\sigma\frac{\hat{Y}}{\phi} = \hat{P}_A \alpha_J \hat{A}^{\eta_A}(1-\phi)^{(\eta_L - 1)} \tag{A.17}$$

and $\hat{Y}$ by:

$$\hat{Y} = \frac{Y}{L^{\beta_K}} = (\hat{A}\phi)^\sigma \hat{K}^{1-\sigma} \tag{A.18}$$

The system of differential equations transformed into stationary variables is, thus, given by:

$$\dot{\hat{A}} = \alpha_J \hat{A}^{\eta_A}(1-\phi)^{\eta_L} - \hat{A}\beta_A n \tag{A.19}$$

$$\dot{\hat{K}} = (\hat{A}\phi)^\sigma \hat{K}^{1-\sigma} - \hat{C} - (\delta + \beta_K n)\hat{K} \tag{A.20}$$

$$\dot{\hat{C}} = \frac{\hat{C}}{\theta}\left((1-\sigma)^2(\hat{A}\phi)^\sigma \hat{K}^{-\sigma} - \delta - \rho - n + \theta n\right) - n\beta_K \hat{C} \tag{A.21}$$

$$\dot{\hat{P}}_A = \Big((1-\sigma)^2(\hat{A}\phi)^\sigma \hat{K}^{-\sigma} - \delta - n\Big)\hat{P}_A - \sigma(1-\sigma)\hat{A}^{\sigma-1}\phi^\sigma \hat{K}^{1-\sigma} \tag{A.22}$$

and the static constraint is given by the equality of wages:

$$\sigma\hat{A}^\sigma\phi^{\sigma-1}\hat{K}^{1-\sigma} = \hat{P}_A\alpha_J\hat{A}^{\eta_A}(1-\phi)^{\eta_L-1} \tag{A.23}$$

A.2 Region definitions

Table A.1: Region definitions in this paper and in the SSP database

Region	This paper	SSP database
Western Europe	Austria, Belgium, Denmark, Finland, France, Germany, Italy, Netherlands, Norway, Sweden, Switzerland, UK, Ireland, Greece, Portugal, Spain	Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal, Spain, Sweden, UK, Iceland, Norway, Switzerland
Eastern Europe	Bulgaria, Hungary, Poland, Romania, Former Czechoslovakia (Czech-Republik, Slovakia), Former Yugoslavia (Yugoslavia, Bosnia, Croatia, Macedonia, Slovenia, Serbia/ Montenegro/ Kosovo)	Albania, Bosnia and Herzegovina, Croatia, Montenegro, Serbia, The former Yugoslav Republic of Macedonia, Cyprus, Czech Republik, Estonia, Hungary, Malta, Poland, Slovakia, Slovenia, Bulgaria, Latvia, Lithuania, Romania
Former USSR	Former USSR (Armenia, Azerbaijan, Belarus, Estonia, Georgia, Kazakhstan, Kyrgyzstan, Latvia, Lithuania, Moldova, Russia, Tajikistan, Turkmenistan, Ukraine, Uzbekistan)	Russian Federation, Belarus, Republic of Moldova, Ukraine

A.3 Parameter estimates from a model calibration on the country level

Table A.2: Coefficients, variances and covariances

Country	η_A	$Var(\eta_A)$	η_L	$Var(\eta_A)$	Cov
Western Europe:					
Austria *	0.55	0.0096	0.55	0.0038	1e-05>Cov>-1e-05
Belgium	0.66	0.012	0.74	0.0041	-0.0069
Denmark	0.66	7.607e-05	0.75	1.044e-04	1e-05>Cov>-1e-05
Finland	0.62	1.77e-04	0.77	2.058e-04	1e-05>Cov>-1e-05
France *	0.58	0.0027	0.81	0.0037	1e-05>Cov>-1e-05
Germany	0.63	5.149e-04	0.78	3.56e-04	-2.367e-04
Italy	0.66	1.634e-04	0.82	2.419e-04	1e-05>Cov>-1e-05
Netherlands	0.37	1.373e-04	0.78	154e-04	1e-05>Cov>-1e-05
Norway *	0.60	9.997e-04	0.74	0.0012	1e-05>Cov>-1e-05
Sweden *	0.65	5.575e-04	0.73	7.649e-04	-3.323e-04
Switzerland	0.64	0.001	0.76	0.001	1e-05>Cov>-1e-05
UK	0.59	2.202e-04	0.64	4.025e-04	1e-05>Cov>-1e-05
Ireland *	0.68	0.0039	0.68	0.0301	1e-05>Cov>-1e-05
Greece	0.58	0.0023	0.75	0.0017	-0.0018
Portugal *	0.62	0.0014	0.64	6.985e-04	1e-05>Cov>-1e-05
Spain *	0.55	0.0122	0.55	0.0049	9.919e-04
Eastern Europe:					
Former Czechoslovakia *	0.60	0.0018	0.63	0.0018	1e-05>Cov>-1e-05
Former Yugoslavia *	0.37	0.0017	0.78	0.0017	1e-05>Cov>-1e-05
Bulgaria *	0.60	0.0011	0.74	8.944e-04	1e-05>Cov>-1e-05
Hungary	0.71	8.533e-04	0.74	0.002	-7.795e-04
Poland	0.43	9.436e-04	0.77	0.0012	-6.786e-04
Romania	0.34	0.0011	0.82	0.0011	1e-05>Cov>-1e-05

In countries marked with an asterisk calibrated income is more than 5% lower than observed income in 2008.

A.4 Results from the sensitivity analysis

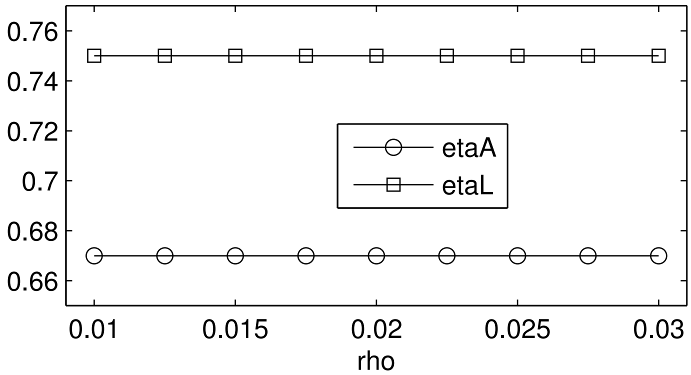


Figure A.1: Sensitivity of η_A and η_L towards the discount rate - Europe

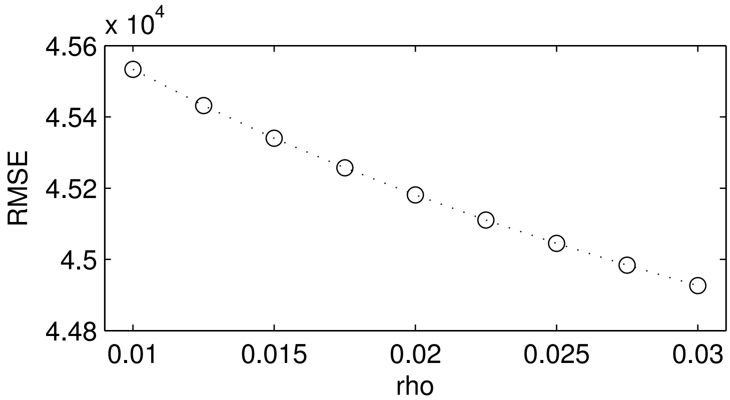


Figure A.2: Sensitivity of the RMSE towards the discount rate - Europe

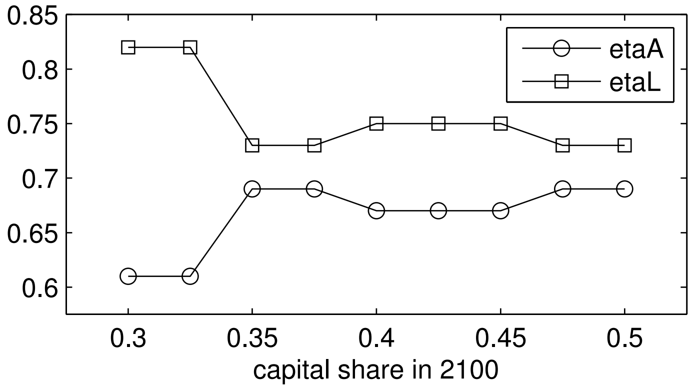


Figure A.3: Sensitivity of η_A and η_L towards the capital share - Europe

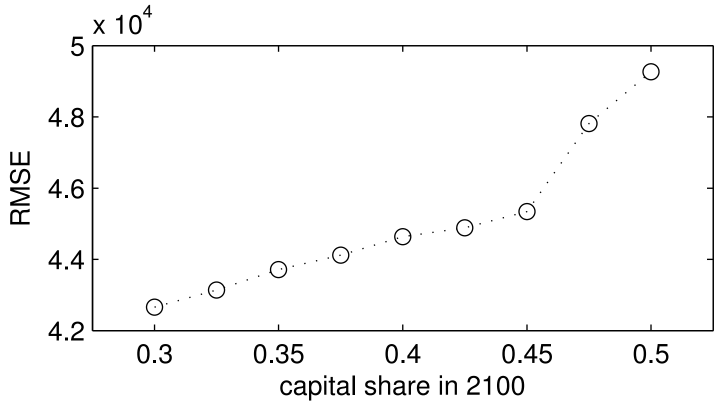
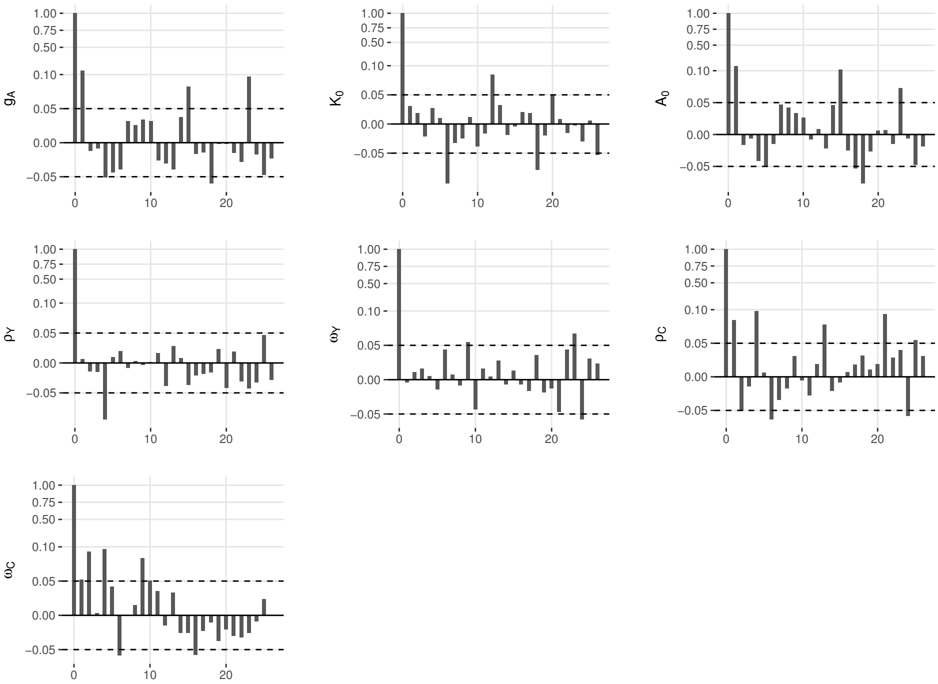


Figure A.4: Sensitivity of the RMSE towards the capital share - Europe

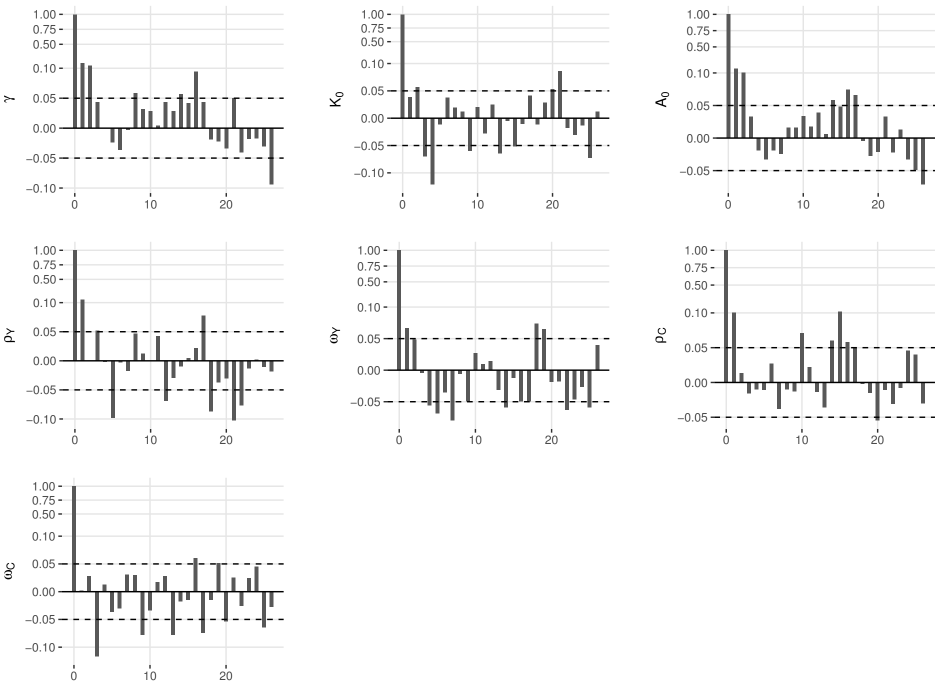
Appendix B

Appendix to chapter 2

B.1 Figures related to the convergence of the Markov chains

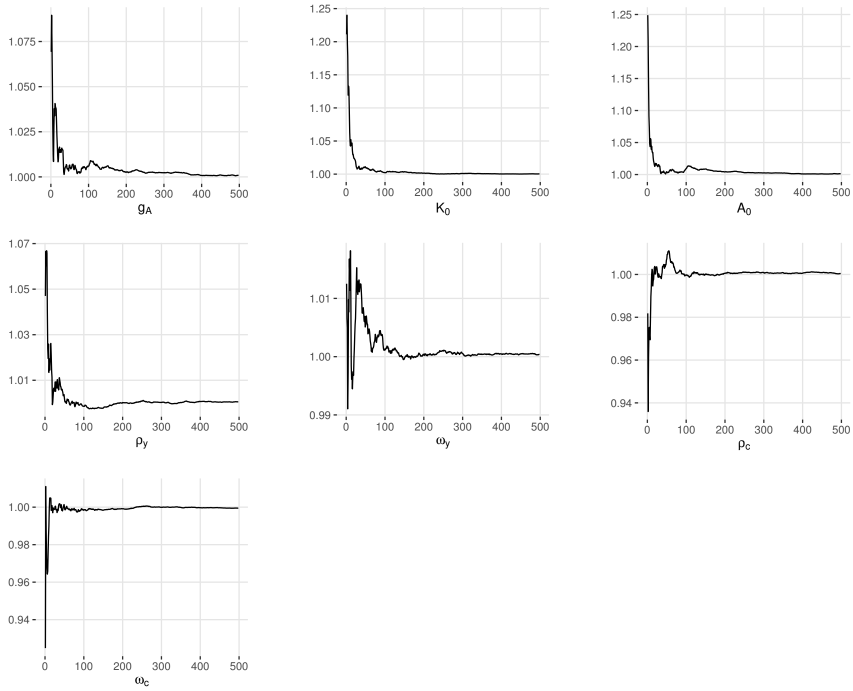


(a) Ramsey-Cass-Koopmans model

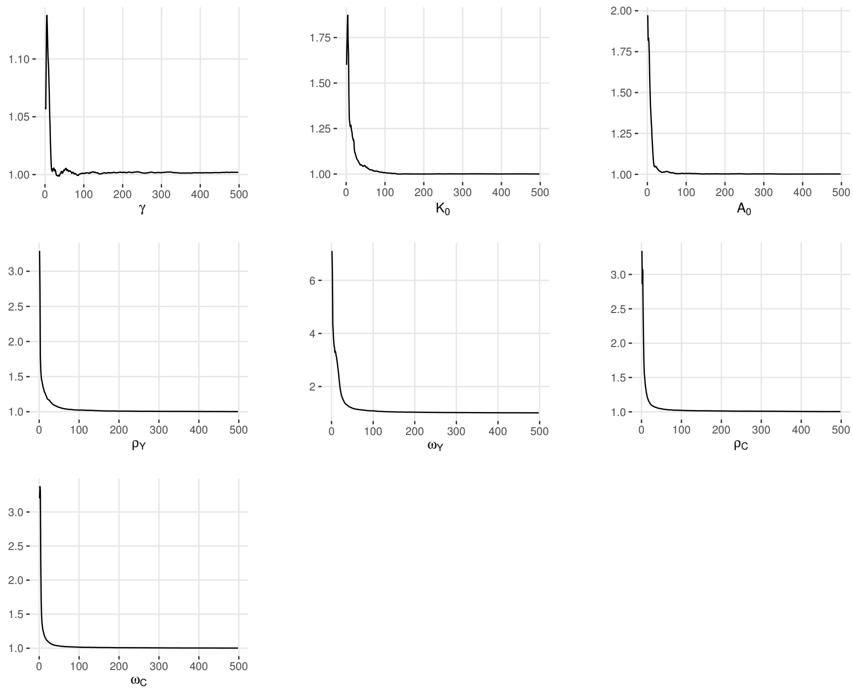


(b) Aghion & Howitt model

Figure B.1: Autocorrelation functions (lags 0 to 26)



(a) Ramsey-Cass-Koopmans model



(b) Aghion & Howitt model

Figure B.2: Gelman and Rubin statistics

Appendix C

Appendix to chapter 3

C.1 The DICE model in detail

The growth model The population size, L_t, evolves exogenously according to:

$$L_1 = 7403 \qquad \text{(million people)} \tag{C.1}$$

$$L_{t+1} = \frac{L_t \left(\frac{11500}{L_t}\right)^{0.134}}{1000} \tag{C.2}$$

The initial value of the capital stock, K_1, is:

$$K_1 = 223 \qquad \text{(in trillion 2010 USD)} \tag{C.3}$$

Total factor productivity, al_t, evolves exogenously according to:

$$al_1 = 5.115 \tag{C.4}$$

$$al_{t+1} = \frac{al_t}{1 - ga_t} \tag{C.5}$$

with ga_t representing productivity growth:

$$ga_t = 0.076e^{-0.005\Delta T(t-1)} \tag{C.6}$$

The return on investment, r_t, is derived from the Keynes-Ramsey rule. ρ denotes the rate of time preference, ϵ the elasticity of marginal utility from consumption and C_t denotes consumption in absolute terms:

$$r_t = (1+\rho)\left(\frac{C_{t+1} * 1000}{L_{t+1}} \frac{L_t}{C_t * 1000}\right)^{\frac{\epsilon}{T_\Delta}} - 1 \tag{C.7}$$

The climate model The adjusted backstop price φ_t is:

$$\varphi_t = 0.212 * 0.975^{(t-1)} * \, sigma_t \tag{C.8}$$

with an emissions output ratio $sigma_t$ of:

$$\begin{aligned} sigma_1 &= 0.35 \\ sigma_{t+1} &= sigma_t \, e^{gsig_t * T_\Delta} \end{aligned} \tag{C.9}$$

The change in $sigma_t$, named $gsig_t$, equals the cumulative improvement of the energy efficiency:

$$\begin{aligned} gsig_1 &= -0.0152 \\ gsig_{t+1} &= gsig_t * 0.999^{T_\Delta} \end{aligned} \tag{C.10}$$

Emissions from deforestation, $E_{Land\,t}$, amount to:

$$E_{Land\,t} = 2.6 * 0.885^{(t-1)} \tag{C.11}$$

Exogenous forcing of other greenhouse gases is denoted by $FORC_{EX\,t}$:

$$FORC_{EX\,t} = 0.5 + \frac{0.5}{17} * (t-1) \qquad \text{for} \qquad t \leq 17 \tag{C.12}$$

$$FORC_{EX\,t} = 1 \qquad \text{for} \qquad t \geq 18 \tag{C.13}$$

C.2 Graphs

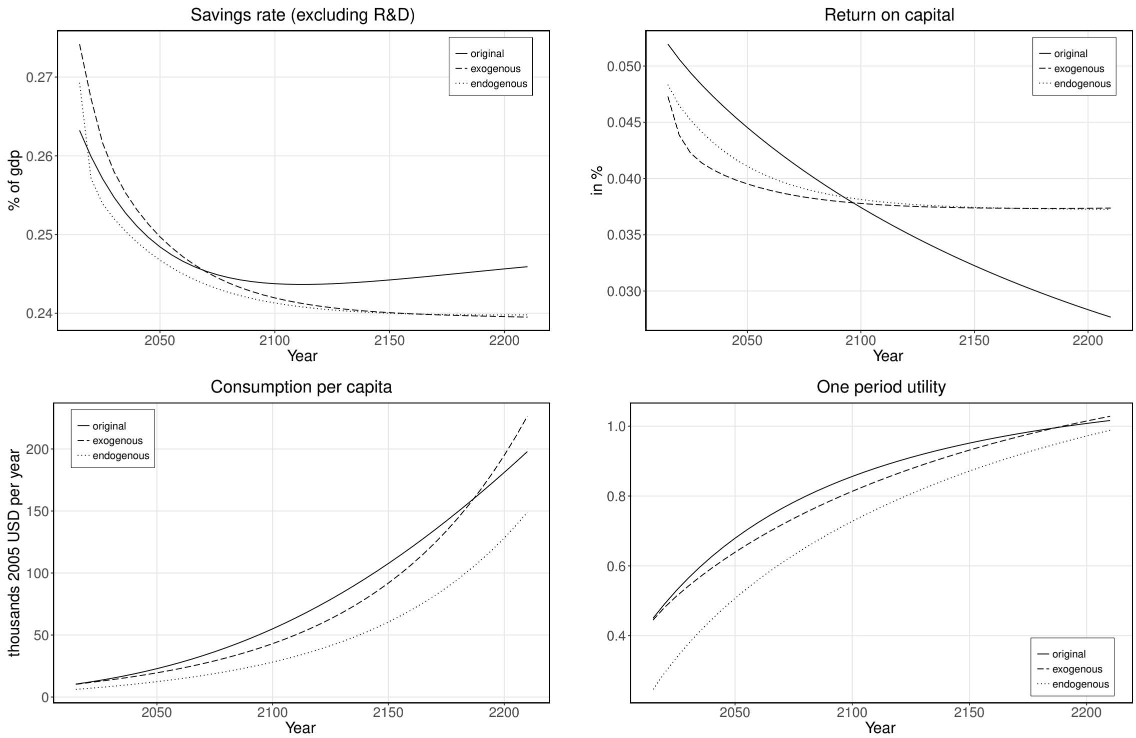


Figure C.1: Comparison of the original Ramsey model in DICE with an endogenous growth model and its new exogenous counterpart.

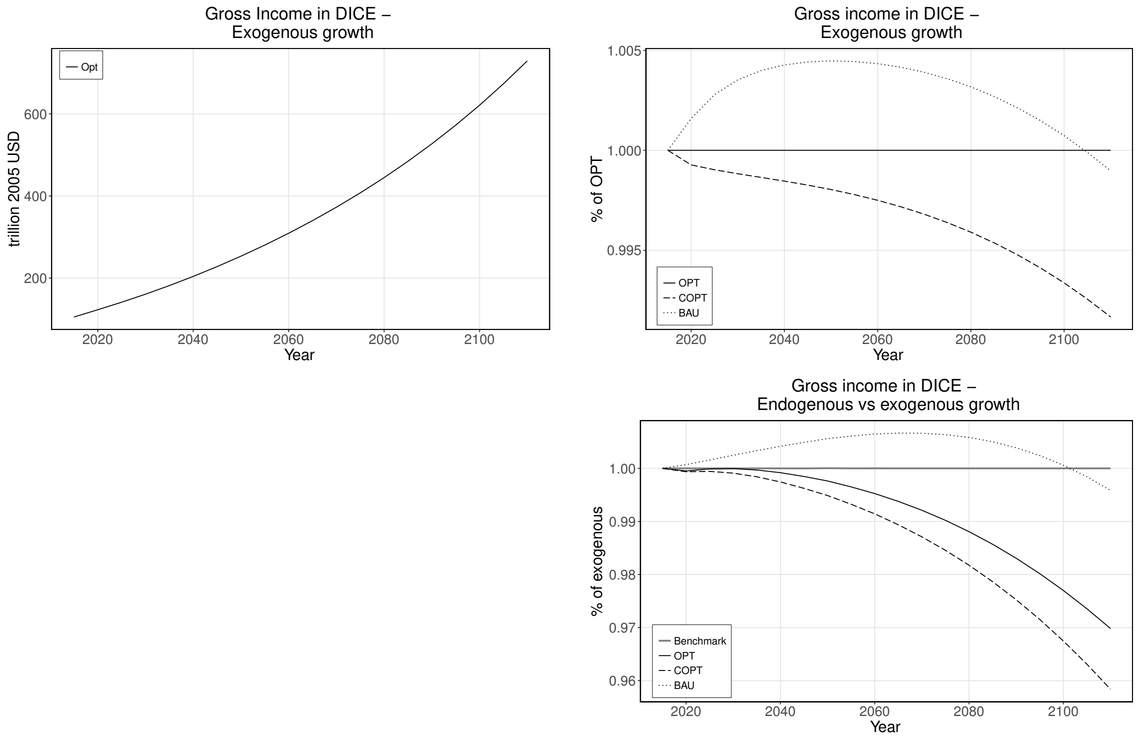


Figure C.2: Comparison of DICE model versions: Gross income

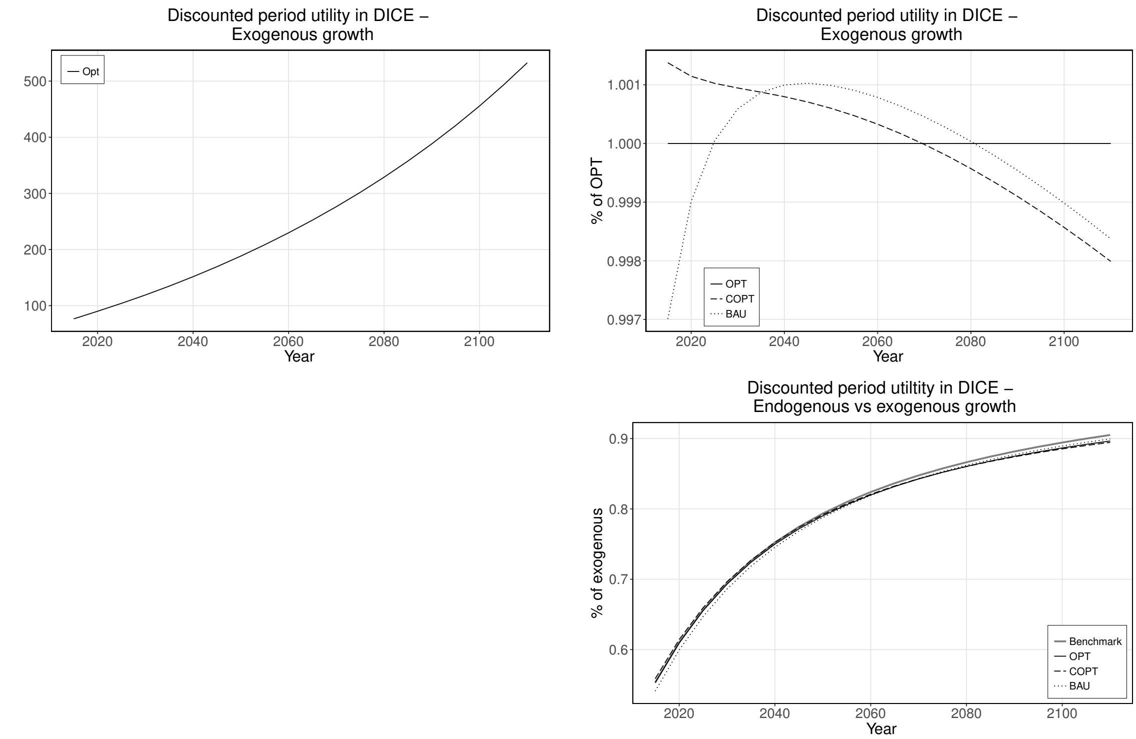


Figure C.3: Comparison of DICE model versions: Discounted period utility

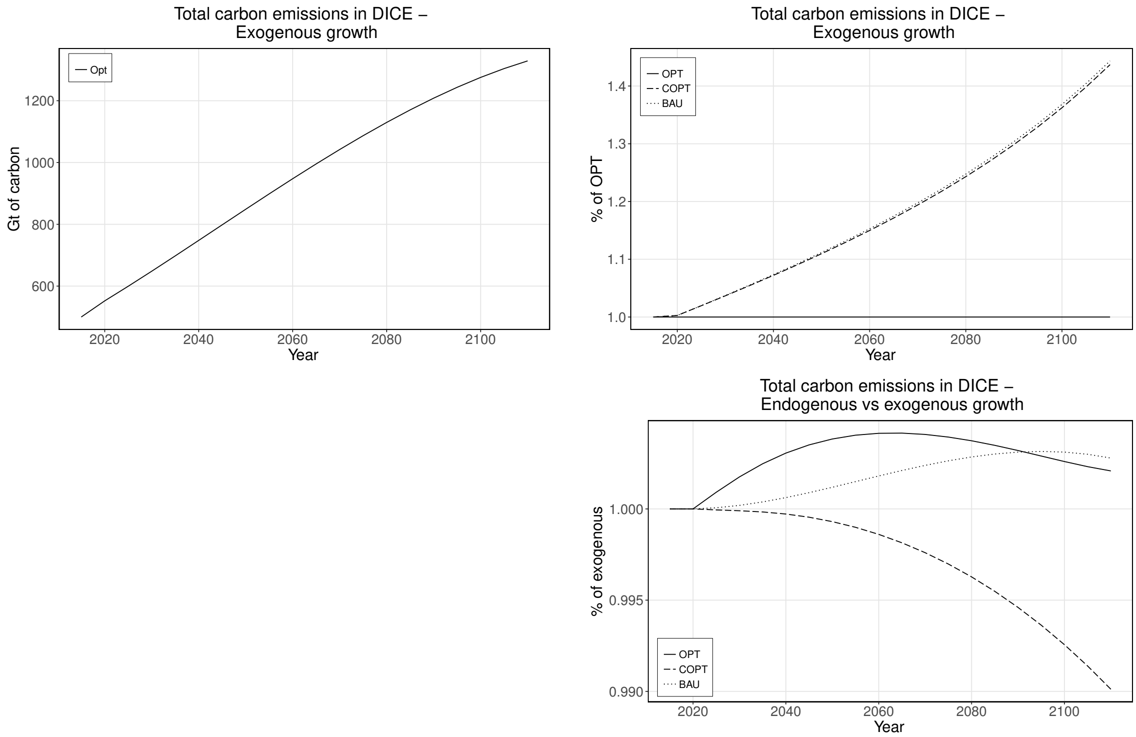


Figure C.4: Comparison of DICE model versions: Total carbon emissions

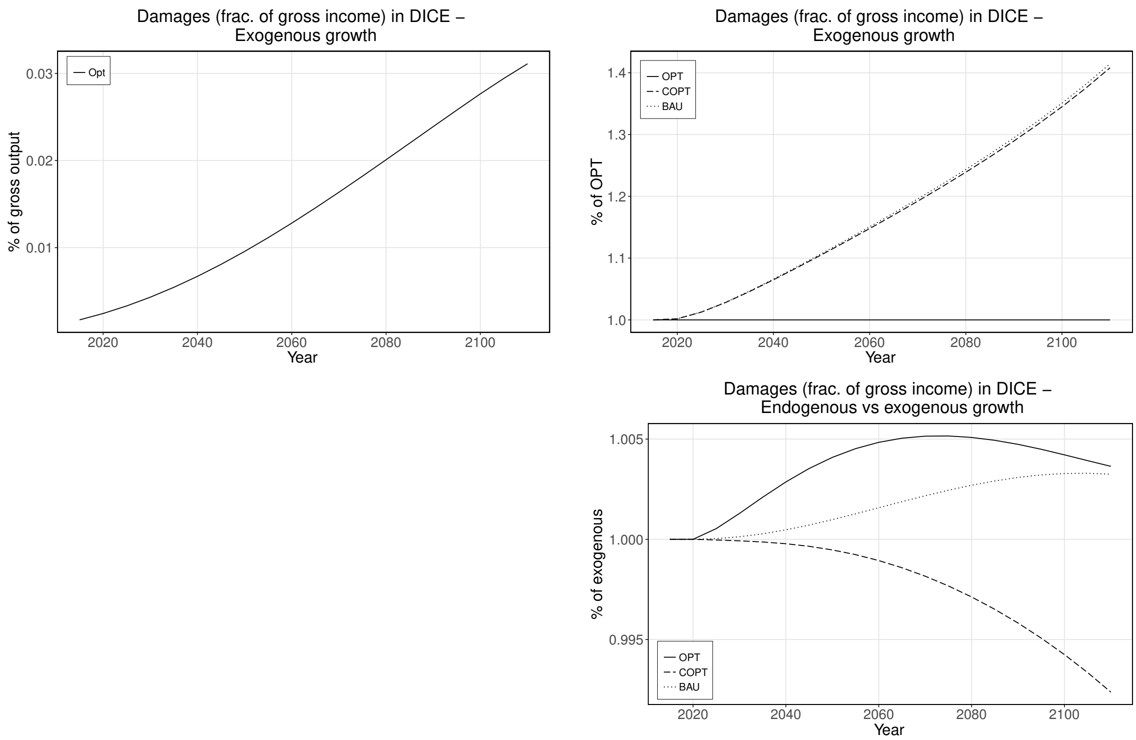


Figure C.5: Comparison of DICE model versions: Damages as fraction of income

Appendix D

Appendix to chapter 4

D.1 Model output of the re-calibrated DICE-2016R model

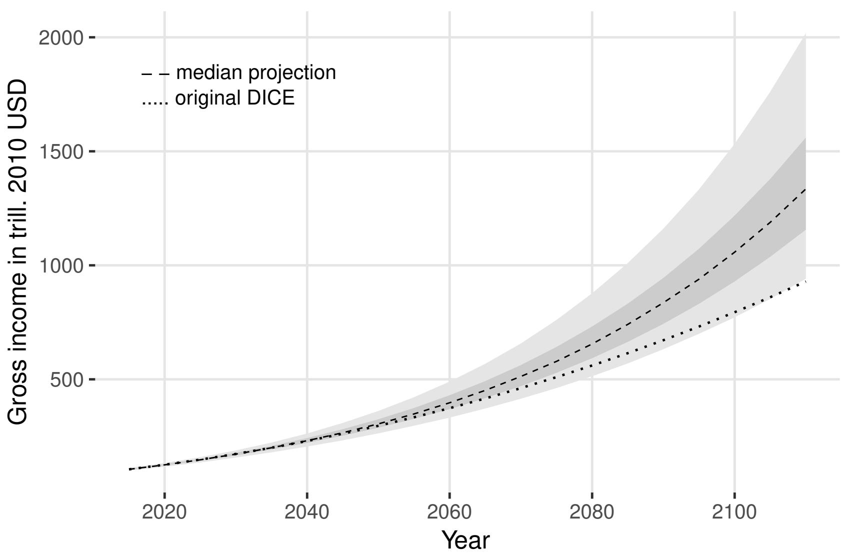


Figure D.1: Gross income

Note: The light gray shaded area corresponds to the 90% confidence interval. The dark gray shaded area corresponds to the 50% confidence interval of all projections.

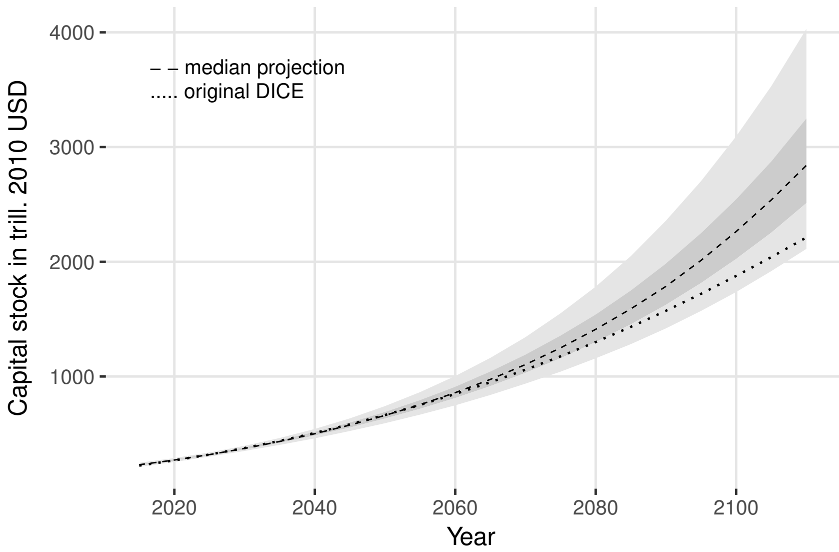


Figure D.2: Capital stock

Note: The light gray shaded area corresponds to the 90% confidence interval. The dark gray shaded area corresponds to the 50% confidence interval of all projections.

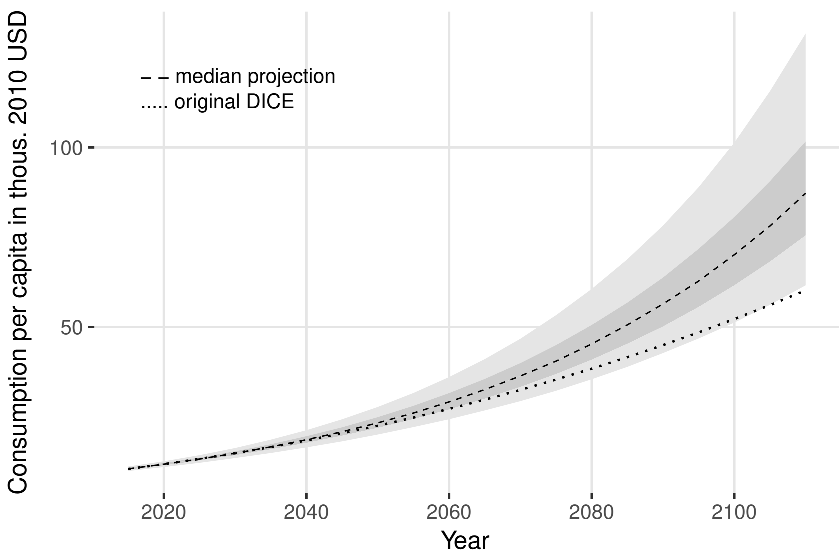


Figure D.3: Consumption per capita

Note: The light gray shaded area corresponds to the 90% confidence interval. The dark gray shaded area corresponds to the 50% confidence interval of all projections.

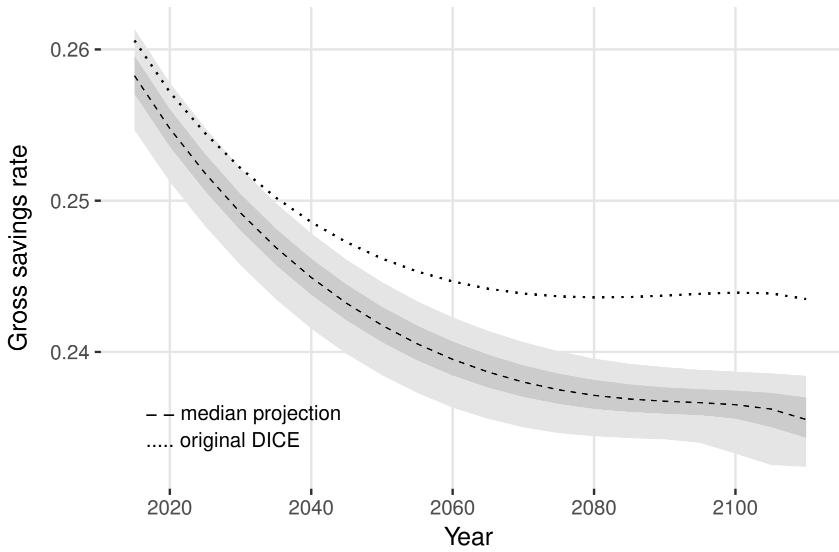


Figure D.4: Gross savings rate

Note: The light gray shaded area corresponds to the 90% confidence interval. The dark gray shaded area corresponds to the 50% confidence interval of all projections.

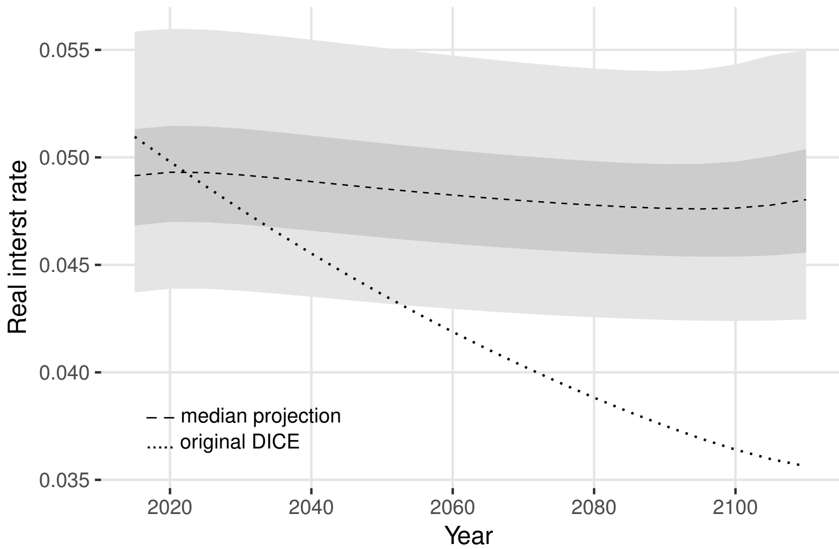


Figure D.5: Real interest rate

Note: The light gray shaded area corresponds to the 90% confidence interval. The dark gray shaded area corresponds to the 50% confidence interval of all projections.

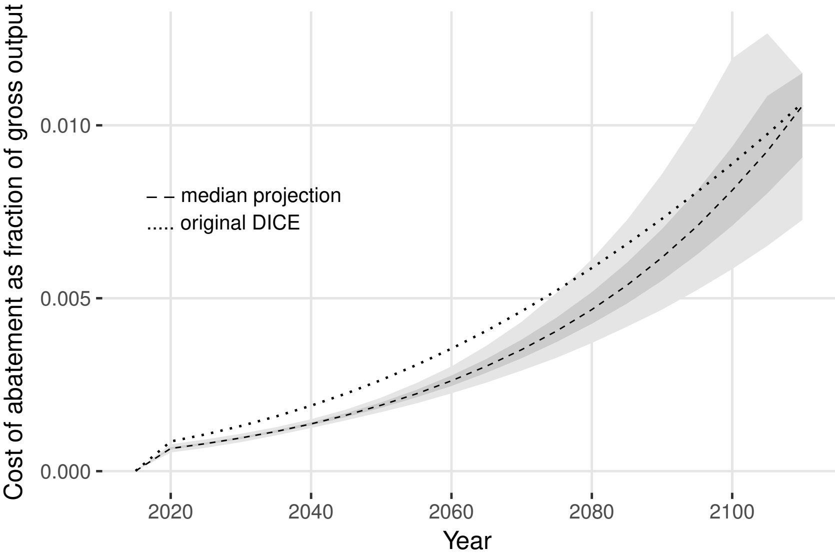


Figure D.6: Cost of emission reductions

Note: The light gray shaded area corresponds to the 90% confidence interval. The dark gray shaded area corresponds to the 50% confidence interval of all projections.

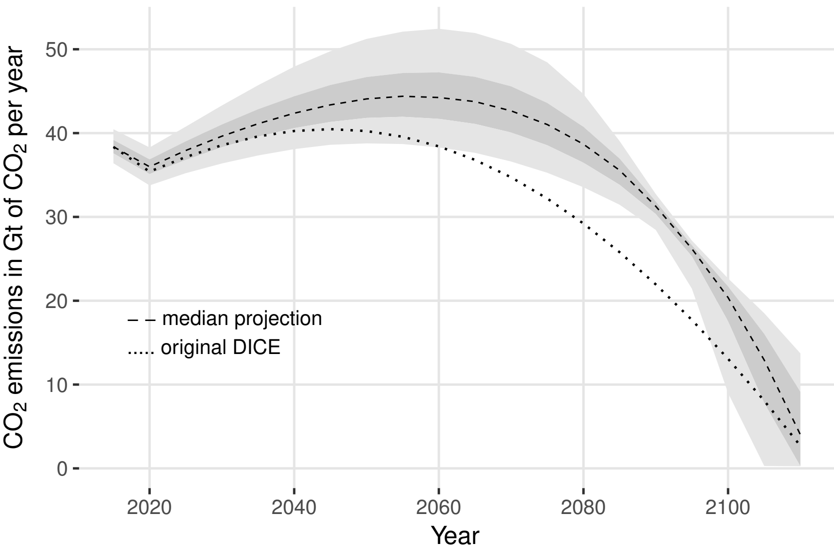


Figure D.7: Total CO_2 emissions

Note: The light gray shaded area corresponds to the 90% confidence interval. The dark gray shaded area corresponds to the 50% confidence interval of all projections.

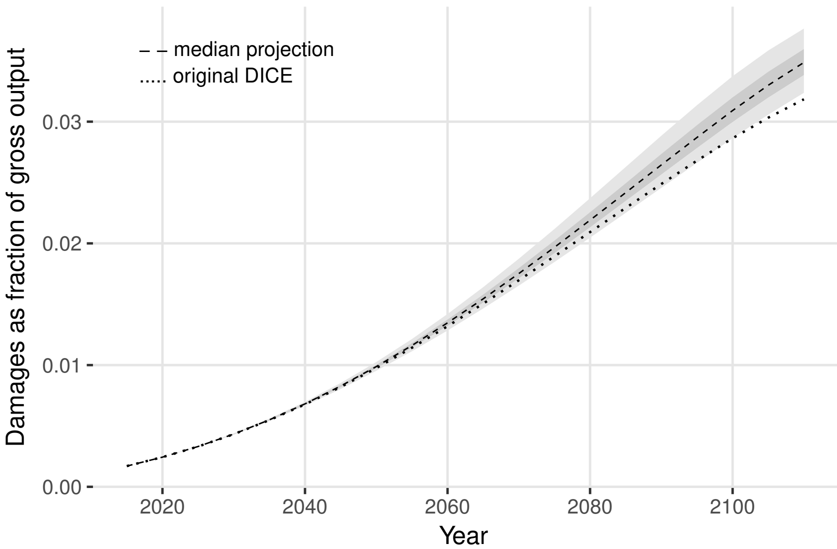


Figure D.8: Damages as a fraction of gross output

Note: The light gray shaded area corresponds to the 90% confidence interval. The dark gray shaded area corresponds to the 50% confidence interval of all projections.

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Eidesstattliche Versicherung

Ich versichere hiermit eidesstattlich, dass ich die vorliegende Arbeit selbständig und ohne fremde Hilfe verfasst habe. Die aus fremden Quellen direkt oder indirekt übernommenen Gedanken sowie mir gegebene Anregungen sind als solche kenntlich gemacht.
Die Arbeit wurde bisher keiner anderen Prüfungsbehörde vorgelegt und auch noch nicht veröffentlicht. Sofern ein Teil der Arbeit aus bereits veröffentlichten Papers besteht, habe ich dies ausdrücklich angegeben.

Datum: 20.09.2017

Unterschrift: