Multivariate GARCH and Dynamic Copula Models for Financial Time Series

With an Application to Emerging Markets

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Zusammenfassung

Diese Arbeit stellt verschiedene nicht-parametrische und parametrische Modelle zur Schätzung von dynamischen Abhängigkeiten zwischen Finanzzeitreihen vor und überprüft die Qualität der verschiedenen Modelle mehrere Risikomaße präzise zu bestimmen. Des Weiteren werden die unterschiedlichen Abhängigkeitsstrukturmodelle benutzt, um die Integration von Schwellenländern und entwickelten Märkten zu analysieren. Um eine Vielzahl dynamischer Abhängigkeitsstrukturen und insbesondere mögliche Asymmetrien darstellen zu können, werden zwei in ihren Grundcharakteristika verschiedene parametrische Modellklassen untersucht: die multivariaten GARCH und Copula-Modelle. Für die Archimedischen Copulas wird eine neue dynamische Abhängigkeitsstruktur eingeführt, welche die bisherige Beschränkung der dynamischen Archimedischen Copulas auf zwei Dimensionen aufhebt und auf den mehrdimensionalen Fall erweitert. Darauf aufbauend wird eine Mixture-Copula vorgestellt, welche die neue multivariate Abhängigkeitsstruktur der Archimedischen Copulas nutzt, diese mit multivariaten elliptischen Copulas mischt und gleichzeitig für die Modellierung der zeitabhängigen Gewichte in der Mixture-Copula einen neuen Prozess vorschlägt: somit kann innerhalb eines Modells eine breites Spektrum möglicher Abhängigkeitsstrukturen untersucht werden.
Die Analyse verschiedener Portfolios zeigt zum einen, dass alle Aktienportfolios sowie die Rentenportfolios der Schwellenländer negative Asymmetrien aufweisen, d.h. steigende Abhängigkeiten bei sinkenden Kursen. Das Rentenportfolio welches die entwickelten Länder repräsentiert zeigt hingegen keine negativen Asymmetrien. Grundsätzlich zeigen die Analysen der verschiedenen Risikomaße, dass die parametrischen Modelle die Portfoliorisiken besser darstellen als die nicht-parametrischen Modelle. Jedoch gibt es nicht ein einziges Modell welches allen anderen über die verschiedenen Portfolios und Risikomaße hinweg überlegen ist.
Die Untersuchung der Abhängigkeiten zwischen Aktien- und Rentenportfolios entwickelter Länder, proprietärer und sekundärer Schwellenländer zeigt, dass sekundäre Schwellenländer in die Weltmärkte weniger integriert sind als proprietäre und sich damit zur Portfoliodiversifikation eines Portfolios bestehend aus Aktien- oder Rentenindizes entwickelter Länder besser eignen.

Abstract

This thesis presents several non-parametric and parametric models for estimating dynamic dependence between financial time series and evaluates their ability to precisely estimate risk measures. Furthermore, the different dependence models are used to analyze the integration of emerging markets into the world economy. In order to analyze numerous dependence structures and to discover possible asymmetries, two distinct model classes are investigated: the multivariate GARCH and Copula models. On the theoretical side a new dynamic dependence structure for multivariate Archimedean Copulas is introduced which lifts the prevailing restriction to two dimensions and extends the multivariate dynamic Archimedean Copulas to more than two dimensions. On this basis a new mixture copula is presented using the newly invented multivariate dynamic dependence structure for the Archimedean Copulas and mixing it with multivariate elliptical copulas. Simultaneously a new process for modeling the time-varying weights of the mixture copula is introduced: this specification makes it possible to estimate various dependence structures within a single model.

The empirical analysis of different portfolios shows that all equity portfolios and the bond portfolios of the emerging markets exhibit negative asymmetries, i.e. increasing dependence during market downturns. However, the portfolio consisting of the developed market bonds does not show any negative asymmetries. Overall, the analysis of the risk measures reveals that parametric models display portfolio risk more precisely than non-parametric models. However, no single parametric model dominates all other models for all portfolios and risk measures. The investigation of dependence between equity and bond portfolios of developed countries, proprietary, and secondary emerging markets reveals that secondary emerging markets are less integrated into the world economy than proprietary. Thus, secondary emerging markets are more suitable to diversify a portfolio consisting of developed equity or bond indices than proprietary.

Vorwort

Zum Gelingen dieser Arbeit haben viele Leute in unterschiedlichster Weise beitragen. Ihnen Allen bin ich zum Dank verpflichtet. Besonders danken möchte ich meinem Doktorvater Herrn Prof. Stefan Mittnik, PhD: er hat mir die Erstellung dieser Arbeit überhaupt erst ermöglicht. Seine Unterstützung und wertvollen Ratschläge waren für das Gelingen dieser Arbeit unerlässlich. Mein Dank gilt außerdem Herrn Prof. Dr. Marc Paolella der sich freundlicherweise bereit erklärte das Zweitgutachten für diese Arbeit zu übernehmen, sowie Herrn Prof. Dr. Göran Kauermann für die Übernahme des Vorsitzes der Promotionskommission. Ebenso möchte ich mich bei allen Arbeitskollegen für die fachlichen und insbesondere praxisnahen Diskussionen bedanken. Besonders hervorzuheben sind die IPC-Kollegen von der Deka Investment GmbH, die mich in die Welt der Finanzökonometrie eingeführt haben und so letztendlich den Anstoß für diese Arbeit gaben. Entscheidend zum Erfolg dieser Arbeit beigetragen hat Dr. Valentin Braun von der Goethe-Universität Frankfurt. Unsere Diskussionsrunden haben der Arbeit einige entscheidende Impulse gegeben. All meinen Freunden und meiner Familie gilt der Dank für den Rückhalt, die Geduld und das Verständnis während der Promotionszeit. Besonderer Dank gebührt meinen Eltern, die mir in allen Lebenslagen die größtmögliche Unterstützung gewährt haben und da wo es nötig ist immer noch gewähren.

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Chapter 1

Introduction

One of the pillars of modern finance theory - as founded by Markowitz (1952) - is that low or negative dependence between assets can lead to a superior risk-return relationship. Therefore, the method of estimating dependence between several variables is a critical issue for portfolio allocation and risk management. For a number of years the standard method of estimating dependence has been Pearson's correlation coefficient, which is based on the multivariate Gaussian distribution. However, as Mandelbrot (1963) and Fama (1965) noted, financial time series do not fulfill the assumption of normality. It then took another thirty years until the seminal paper of Embrechts, McNeil, and Straumann (1999) proved that Pearson's correlation coefficient is not sufficient to depict the dependence between variables not belonging to the family of elliptical distributions. In addition, the dependence between financial time series varies through time- as has been pointed out by Hamao, Masulis, and Ng (1990). Furthermore, the work of Erb, Harvey, and Viskanta (1994) and Longin and Solnik (2001), among others, showed that dependence tends to increase during bear markets.

Taking these so called 'stylized facts' into account it is clear that there was a need for the establishment of new methods to overcome the drawbacks of Person's correlation coefficient. Multivariate GARCH models constituted such an attempt. The main task of this model class is the investigation of co-movements between several assets via the modelling of the conditional covariance or conditional correlation matrix. A popular multivariate GARCH model is the Dynamic Conditional Correlation model of Engle (2002), who decomposes the conditional covariance matrix into conditional standard deviations and conditional correlations. Since the original DCC-model disregards asymmetric correlations Cappiello, Engle, and Sheppard (2006) enhanced Engle's model in this direction. Today, a commonly used second alternative can be found in the so called copulas introduced by Sklar (1959). As will be explained shortly, a multivariate distribution can be decomposed into its marginal distributions and the dependence structure, at which the copula represents the dependent part. One important feature of copulas is the possibility of modelling the occurrence of joint positive and- in particular- joint negative events in the tails of distributions. Joint negative events tie in with the 'stylized fact' of asymmetric dependence during bear markets. Until Patton (2006) copulas were useful in modelling asymmetries but neglected the dynamic behavior of dependence. Patton resolved this by developing time-varying copulas.

Naturally, this research impacts on the work of practitioners. In the asset management industry multivariate dependence plays a crucial role for both portfolio and the risk managers. Of particular importance to portfolio mangers is the detection of diversification benefits and therefore an investigation of the dependence structure between several variables. Often proposed for the diversification of a developed market portfolio are the emerging markets (see Errunza (1977) for example). The risk manager meanwhile is interested in the best possible display of portfolio risk.

In short, this analysis tries to satisfy the needs of both portfolio and risk managers. The portfolio manager's needs are accommodated by an analysis of the time-varying multivariate dependence structure between the emerging and developed equity and bond markets. The needs of the risk manager are met by investigating the ability of the different models to estimate the Value-at-Risk, a popular risk measure.

On the theoretical side this analysis contributes to the existing literature in the following ways. In cooperation with Valentin Braun new time-varying copulas are proposed. Until now time-varying Archimedean copulas have been restricted to bivariate cases. The dynamic structure proposed here makes it possible to examine the time-varying tail dependence with more than two dimensions. The second innovation extends the class of mixture copulas and has two elements. Firstly it builds on the time-varying multivariate Archimedean copulas to provide the possibility of estimating a multivariate dynamic mixture copula consisting of an elliptical and an Archimedean copula with more than two dimensions. Secondly, a new modelling scheme for time-varying weights of the mixture copulas is proposed. Furthermore, the elliptical copulas are estimated with a diagonal matrix structure of Cappiello, Engle, and Sheppard (2006) to incorporate possible asymmetric dependence. The scalar version of this structure is applied to D-vine copulas which also gives them a time-varying structure with possible asymmetries.

On the empirical side a multivariate GARCH and several time-varying copula models are compared for their ability to estimate the Value-at-Risk of developed and emerging market equity and bond indices. A second empirical task is an in-depth analysis of the dependence between emerging and developed markets. In this area a great deal of research is devoted to the analysis of equity indices. We extend this line of analysis to bond indices. Furthermore, the newly developed multivariate copula models allow an appraisal of time-varying tail dependence: this is also novel.

The analysis is structured as followed.

Chapter 2 presents the model of the financial returns that is used throughout the analysis. A brief overview of autoregressive models for the conditional mean and univariate GARCH models for the conditional variance are given. Autoregressive Moving Average theory is also presented since it is used to specify the time-varying behaviour of some copulas. Chapter 3 introduces multivariate GARCH models. As noted above these are alternative for estimating time-varying correlations with asymmetric dependence. Chapter 4 is devoted to copulas. Firstly, the multivariate elliptical copulas are explained, followed by Archimedean copulas. The last copula class introduced is the vine copulas. Thereafter, the theoretical concepts of Kendall's tau and tail dependence are explained. Kendall's tau plays a crucial role for the time-varying vine copulas whilst tail dependence is one of the main features of copulas. It is thus important to present them in more detail. Following this, copula theory is enhanced to include time-varying cases. In the final step of the theoretical part a copula goodness-of-fit test is illustrated.

Chapter 5 is devoted to the empirical analysis. The first part engages with the ability of the miscellaneous models to estimate risk. Definitions of some risk measures are given and the different risk estimation results are compared via various tests. This is accompanied by a description of the different model risk-characteristics and a conclusion concerning which model displays the risk of the respective portfolio best.

We focus on two popular asset classes: equity and bond indices. In order to cope with a broad range of risk and return characteristics both asset classes are sub-divided into developed and emerging markets. It is often said that emerging market and developed market returns behave quite differently. For example Harvey (1995) argues that emerging market returns are more driven by local information. Furthermore Bekaert and Harvey (1997) points out that emerging market returns are more predictable and exhibit higher asymmetric volatility than developed markets. These different characteristics of emerging and developed markets make them useful in highlighting the different risk estimation abilities. Special attention is given to risk-model behaviour through the recent financial crisis. The second empirical section is an in-depth analysis of the dependence structure between emerging and developed markets. Again several portfolios are built. Where the risk was previously estimated only for advanced emerging markets now secondary emerging markets are added to the analysis. Advanced and secondary emerging markets differentiate in their economic power and may also distinguish in their level of financial integration with the developed markets. The more the emerging markets are integrated the less diversification benefits they offer. To test this hypothesis, multivariate GARCH and different copulas are estimated and the goodness-of-fit test is applied to each estimated model. Comparing the results of the goodness-of-fit tests while considering the properties of the different dependence models provide the possibility to deduce the dependence structure. Attention is given to shifts in dependence over time, especially during the recent financial crisis.

Chapter 2

Marginal Models

2.1 Preliminaries

At this point it is necessary to introduce some basic definitions used in the following chapters.

This thesis is primary concerned with financial time series variables which belong in general to the group of economic variables. We will assume that the variables are described by a stochastic process. Therefore, it is useful to define a probability space $(\Omega, \mathscr{F}_t, Pr)$ where Ω represents the set of all elementary events, $\mathscr{F}_t$ is a sigma-algebra containing all information up to time t and Pr is a probability measure conditional on the information set $\mathscr{F}_t$. Then a random variable x is defined as a real valued function on Ω.

A $d \times 1$ vector will be denoted $\mathbf{x} = (x_1, \ldots, x_d)'$, x_t denotes a process observed in sequence over time $t = 1, \ldots, T$, and $\mathbf{x}_t$ a d-dimensional multivariate time series process $\mathbf{x}_t = (x_{1,t}, \ldots, x_{d,t})'$.

The majority of the time we will follow the usual notation in econometrics, e.g. random and real valued numbers will both be denoted by lower-case letter. Variations from this notation will be explicitly mentioned.

2.2 Model Structure

In this section we will present in detail the models used to specify the marginal distributions of the multivariate models.

Consider y as a real valued variable then in the context of this analysis y_t is a financial return at time t and is calculated as $y_t = \ln\left(\frac{p_t}{p_{t-1}}\right)$, where p_t is the price of the financial time series. The variable y_t will then be modeled as

$$\begin{aligned} y_t &= \mu_t + \varepsilon_t & (2.1) \\ \varepsilon_t &= h_t^{1/2} z_t, & (2.2) \end{aligned}$$

where μ_t describes the conditional mean $(E\{y_t|\mathscr{F}_{t-1}\} = \mu_t)$, h_t the conditional variance $\left(E\{y_t^2|\mathscr{F}_{t-1}\} = h_t\right)$, and z_t is an *i.i.d.* process with zero mean and unit variance. The conditional mean will be specified through an Autoregressive (hereafter, AR) model and the conditional variance through an Generalized Conditional Heteroscedasticity (hereafter, GARCH) model. Both are explained in the following section.

2.3 Autoregressive Moving Average Processes

The following chapter is based on the results of Hamilton (1994), Enders (1995), and Brockwell and Davis (2002).

The family of Autoregressive Moving Average (hereafter, ARMA) models proved useful in capturing the first-moment dynamics of an univariate financial time series. In the following we will introduce the basic features of the AR, ARMA, and GARCH processes. Although we do not use the ARMA model in the marginal specification several of its properties will be used in the copula section. Since the simple ARMA model belongs to the class of univariate models we will present the necessary theory in this chapter.

Crucial for MA and ARMA models is the white noise process and so a definition needs to be given. In this analysis a process v is called white noise if it satisfies

$$\begin{aligned} E\{v\} &= 0 \\ E\{v^2\} &= h, \qquad h < \infty \\ E\{v_t v_\tau\} &= 0, \qquad \text{for} \quad t \neq \tau, \end{aligned}$$

where $E\{\}$ is the usual expectations operator. If v_t and v_τ are also independent for $t \neq \tau$ the process is called strict white noise. The MA model needs only a brief introduction as it appears in this analysis only in the context of the ARMA model and is not treated directly. Consider a real valued vector time series $\mathbf{x}$, then an MA process of order q is described by

$$x_t = v_t - \varphi_1 v_{t-1} - \ldots - \varphi_q v_{t-q}, \tag{2.3}$$

where v_t is white noise. This process satisfies the stationarity condition if

$$\sum_{i=0}^{\infty} |\varphi_i| < 0.$$

The Autoregressive model will be considered in a little more detail because the conditional mean of the marginal model is estimated as an AR(p_1) process. A first order autoregressive process is described by

$$x_t = \phi_1 x_{t-1} + v_t. \tag{2.4}$$

This model may deduced by backward substitution

$$\begin{aligned} x_t &= v_t + \phi_1 v_{t-1} + \phi_1^2 v_{t-2} + \ldots \\ &= \sum_{i=0}^{\infty} \phi_1^i v_{t-i}. \end{aligned}$$

Only if $|\phi_1| < 1$, x_t is said to be stationary and ergodic. The moments of x_t are computed using the infinite sum

$$\begin{aligned} E\{x_t\} &= \sum_{i=0}^{\infty} \phi_1^i E\{v_{t-i}\} = 0 \\ var(x_t) &= \sum_{i=0}^{\infty} \phi_1^{2i} var(v_{t-i}) = \frac{h}{1-\phi_1^2}. \end{aligned}$$

If (2.4) is estimated with a constant

$$x_t = \phi_0 + \phi_1 x_{t-1} + v_t$$

the expected value of the mean changes to

$$E\{x_t\} = \frac{\phi_0}{1-\phi_1}.$$

Sometimes it might be necessary to estimate a model with a lag length greater than one. The stationary conditions for an AR(p_1) model with $p_1 > 1$

$$x_t = \phi_1 x_{t-1} + \ldots + \phi_{p_1} x_{t-p_1} + v_t \tag{2.5}$$

are derived via the complex roots of the process. Therefore, it is useful to define the lag operator L

$$\begin{aligned} L(x_t) &= x_{t-1} \\ L^2 &= LL \\ L^2(x_t) &= x_{t-2} \\ \vdots &= \vdots \end{aligned}$$

Thus, the lag operator L shifts the time index t one period backward. With these definitions an AR(p_1) assumes the form

$$x_t - \phi_1 L x_t - \ldots - \phi_{p_1} L^{p_1} x_t = v_t$$

or

$$\phi(L) x_t = v_t,$$

where

$$\phi(L) = 1 - \phi_1 L - \ldots - \phi_{p_1} L^{p_1}.$$

Now it is possible to derive the complex roots via the fundamental theorem of algebra which states that any polynomial can be factored into

$$\phi(z) = (1 - \lambda_1^{-1} z) \cdots (1 - \lambda_{p_1}^{-1} z),$$

where $\lambda_1, \ldots, \lambda_z$ are the complex roots of z. An AR(p_1) model is stationary and ergodic if all the complex roots lie outside the unit circle, i.e. $|\lambda_i| > 1$, for all i, where $|\lambda|$ denotes the modulus of the complex number λ. The moment conditions for an AR(p_1) process are

$$E\{x_t\} = 0$$
$$var\{x_t\} = h \sum_{i=1}^{p_1} \phi_i.$$

When estimating (2.3) with a constant the expected value of the mean changes to

$$E\{x_t\} = \frac{\phi_0}{1 - \sum_{i=1}^{p_1} \phi_i}.$$

Next, we will present the theoretical results for the ARMA process. In general ARMA processes exhibit the same stationary conditions as AR(p_1)-processes. In this analysis we will use ARMA process to model the time-varying behavior of copulas which will be introduced later. Since the time-varying copulas will always be modelled through an ARMA(1,1) process we present only the results for this kind of process. An ARMA(1,1) model might be written

$$x_t = \phi_1 x_{t-1} + \varphi_1 v_{t-1} + v_t \tag{2.6}$$

and in lag-operator notation as

$$x_t = (\phi(L))^{-1} \varphi(L) v_t$$
$$v_t = (\varphi(L))^{-1} \phi(L) x_t,$$

if $\varphi(L)$ or $\phi(L)$ are invertible. In the case of an ARMA(1,1) process invertibility and stationarity are guaranteed if

$$|\varphi| < 1 \tag{2.7}$$

and

$$|\phi| < 1. \tag{2.8}$$

The first two moment conditions of an ARMA(1,1) are

$$E\{x_t\} = 0$$
$$var\{x_t\} = \frac{1 + \varphi_1^2 - 2\phi_1 \varphi_1}{1 - \phi_1^2} h,$$

where again in case of a constant the expected mean changes to

$$E\{x_t\} = \frac{\phi_0}{1 - \phi_1}.$$

Another important aspect of this analysis is the calculation of the Value-at-Risk (hereafter, VaR). Since the VaR is always predicted forecast properties of the different models need to be deduced. The forecast is conditional on all the information available up to time t. All variables treated in this thesis will only be conditioned on their own past values

$$\mathscr{F}_t = \sigma\{x_t, \ldots, x_{t-T+1}\} \quad \text{for} \quad t = 1, \ldots, T.$$

The general forecast function of an j-step ahead forecast for an AR(1) process with a constant takes the form

$$E\{x_{t+j}|\mathscr{F}_t\} = \phi_0(1 + \phi_1 + \phi_1^2 + \ldots + \phi_1^{j-1} + \phi_1^j x_t). \tag{2.9}$$

The second important model is the ARMA$(1,1)$ model with j-step ahead forecast of

$$E\{x_{t+j}|\mathscr{F}_t\} = \phi_0 + \phi_1 E\{x_{t+j-1}|\mathscr{F}_t\}, \quad \text{for } j \geq 2.$$

As mentioned above we will only make one period ahead forecasts and in the ARMA case this takes the simple form

$$E\{x_{t+1}|\mathscr{F}_t\} = \phi_0 + \phi_1 x_t + \varphi_1 v_t.$$

2.4 Univariate ARCH Processes

The following chapter is based on the results of Hamilton (1994), Tsay (2005), and Greene (2008).

While the above introduced family of AR- and ARMA models successfully describes the conditional mean they assume a time invariant variance. This assumption has been challenged empirically by the observation that volatility varies through time. High and low volatility seems to arise in clusters. This empirical observation leads to a class of models able to gather the time-varying behavior of the variance - the so called Autoregressive Conditional Heteroscedasticity (hereafter, ARCH) model.

Recall the general model structure of the financial return $y_t = \mu_t + \varepsilon_t$, where in this analysis $\mu_t = \phi_0 + \sum_{i=1}^{p_1} \phi_i y_{t-i}$. Then, the ARCH$(p_2)$ invented by Engle (1982) may be written

$$\begin{aligned} h_t &= \alpha_0 + \sum_{j=1}^{p_2} \alpha_j \varepsilon_{t-j}^2, \qquad (2.10)\\ \varepsilon_t &= h_t^{1/2} z_t, \end{aligned}$$

where z_t is a strict white noise process (with zero mean and unit variance) and h_t the conditional variance. If z_t is assumed to be strict white noise then the ARCH model is referred to strong ARCH. Thus, the time varying variance modeled by an ARCH process is described by a linear function of the past squared residuals. The conditional moments of an ARCH(1) model are defined as

$$\begin{aligned} E\{\varepsilon_t|\mathscr{F}_{t-1}\} &= 0\\ var\{\varepsilon_t^2|\mathscr{F}_{t-1}\} &= h_t. \end{aligned}$$

Another important condition in ARCH models is the finiteness of the variance, i.e. $h < \infty$. Due to the autoregressive structure of ARCH models, a large $|\varepsilon|$ in one period leads to large $|\varepsilon|$ in the following periods. This captures the time-varying behavior of volatility. In principle the white noise variable z_t might be distributed with any zero mean and unit variance distribution. We will make use of this when specifying the different GARCH models. Bollerslev (1986) and Taylor (1986) generalized Engle's ARCH model to the famous GARCH model. The ARCH model in (2.10) captures the heteroscedasticity adequately, but only with a very long lag order q. The GARCH model is a more parsimonious version of the ARCH model. Many studies have shown that low order GARCH models capture the empirical properties of the volatility of asset returns relatively well, see e.g. Engle, Hong, and Kane (1990), Day and Lewis (1990), Lamoureux and Lastrapes (1990), and Anderson and Bollerslev (1998). In particular, the simple GARCH(1,1) model captures the heteroscedasticity well (Bollerslev, Engle, and Nelson (1994) and Hansen and Lunde (2001)). Bollerslev (1986) introduced the GARCH(p_2, q_2) model

$$h_t = \alpha_0 + \sum_{i=1}^{p_2} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q_2} \beta_j h_{t-j}. \tag{2.11}$$

The GARCH process is weakly stationary if- and only if- $\sum_{i=1}^{p_2} \alpha_i + \sum_{j=1}^{q_2} \beta_j < 1$. To be identified, at least one α_i has to be greater 0. One feature of the GARCH model is the persistence of high volatility periods because either a large $|\varepsilon_t|$ or a large h_{t-1} can lead to a large h_t. Another interesting feature of a GARCH(1,1) process is the kurtosis

$$Kurt\{\varepsilon_t\} = 3 + \frac{6\alpha_1^2}{1 - \beta_1^2 - 2\alpha_1\beta_1 - 3\alpha_1^2}. \tag{2.12}$$

For general conditions of higher moments of GARCH processes see He and Teräsvirta (1999). Even with Gaussian innovations the kurtosis will always be greater than three if $\alpha_1 > 0$. This implies that it is possible to capture some fat-tailed behavior of a time series, even with Gaussian innovations. This should be kept in mind since it will play a role when comparing the different multivariate models in later chapters.

GARCH models are broadly divided into two classes: the symmetric and the asymmetric GARCH models. To handle as many features of a financial time series as possible we make use of both classes. The symmetric GARCH models we use for specifying the variance are

- Symmetric GARCH models:
  - GARCH - Bollerslev (1986)
    $h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}$
  - AVGARCH Absolute Value - Taylor (1986)
    $h_t^{1/2} = \alpha_0 + \alpha_1 |\varepsilon_{t-1}| + \beta_1 h_{t-1}^{1/2}$.

Symmetric GARCH models can be critiqued because they treat negative and positive shocks in the same way. Black (1976) was one of the first to recognize that after a negative shock the volatility seemed to increase more than after a positive shock of the same magnitude. Black (1976) and Christie (1982) give an explanation for this phenomena, noting that a decline in a firm's stock price would raise the debt to equity ratio of the firm and that the larger the debt to equity ratio the larger the risk of a default of the firm would be. This leads to an increase of volatility of the stock's return. This is today known as the 'leverage effect'. Another explanation is given by Campbell and Hentschel (1992) and Wu (2001), the so called 'volatility feedback effect'. When the volatility of the market is expected to increase an investor requires the return of the stock to increase and thus lowers stock prices. Studies which reviewed time varying risk premiums where given by Pindyck (1984) and French, Schwert, and Stambaugh (1987). As a result a whole new class of GARCH models evolved trying to take the leverage or volatility feedback effect into account: the asymmetric GARCH-models.

- Asymmetric GARCH models:
  - EGARCH Exponential - Nelson (1991)
    $\log(h_t) = \alpha_0 + \alpha_1 \frac{|\varepsilon_{t-1}|}{\sqrt{h_{t-1}}} + \gamma \frac{\varepsilon_{t-1}}{\sqrt{h_{t-1}}} + \beta_1 \log(h_{t-1})$
  - ZARCH Threshold - Zakoian (1994)
    $h_t^{1/2} = \alpha_0 + \alpha_1 |\varepsilon_{t-1}| + \gamma I\{\varepsilon_{t-1} < 0\} |\varepsilon_{t-1}| + \beta_1 h_{t-1}^{1/2}$
  - GJRGARCH Glosten, Jagannathan, and Runkle (1993)
    $h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \gamma I\{\varepsilon_{t-1} < 0\} \varepsilon^2 + \beta_1 h_{t-1}$

The miscellaneous reactions of the different models to shocks are best described by the news impact curve, originally developed by Pagan and Schwert (1990). For a detailed survey of the news impact curve from most of the models described above see Hentschel (1995). For example the standard GARCH model, e.g. is characterized by a symmetric news impact curve, the EGARCH by a rotated news impact curve, and the AVGARCH model by a re-centered news impact curve.

We will now describe describe the main features of the miscellaneous GARCH models briefly. This is important as in the empirical section of this thesis we estimate volatility of the different equity and bond indices by GARCH models. An understanding of the main features of each model might lead to an better understanding of the properties of the respective time series.

The AVGARCH dampens the effect of the residuals because it models the absolute value of the residual instead of the squared value. Recent research suggests that absolute returns show more autocorrelation than squared returns (Granger, Spear, and Ding (2000)). This would make a parameterization as in the AVGARCH more useful as the fundamental logic that builds the whole ARCH class is an autoregressive structure of the data. The most celebrated model which incorporates asymmetric effects is the EGARCH model of Nelson (1991). A unique feature of the EGARCH model is that it models the log-variance instead of the variance. This ensures the positivity of the variance and thus no restrictions on the parameters are needed. The ZARCH and GJRGARCH belong to the class of threshold models. In threshold models only innovations below or above a certain threshold will be modelled separately. The threshold in the ZARCH and GJRGARCH is assumed to be zero. The ZARCH model adopts the idea of the AVGARCH and models the absolute innovations. Additionally it models the square root of the variance instead of the variance. The GJRGARCH combines the threshold idea with the squared innovations of the Bollerslev-GARCH. Thus, the simplest specification of the financial return y_t would be an AR(1)-GARCH(1,1) model

$$y_t = \phi_0 + \phi_1 y_{t-1} + \sqrt{\alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}} z_t.$$

Multiperiod forecasting for GARCH models sometimes require fairly complicated formulas. As Zivot (2009) offers a derivation of multiperiod forecasting functions for several GARCH models these formulas are omitted. The for this analysis relevant one-period ahead forecasts are as in the AR and ARMA case relatively simple, requiring only a shift in the time index one-period ahead.

Surveys of the mathematical and statistical properties of the different models are provided by Bera and Higgins (1995), Diebold and Lopez (1996), Pagan (1996), and Palm (1996). For an introductory and concise survey of univariate GARCH models see Teräsvirta (2009).

2.4.1 Estimation

The class of ARCH models can be estimated by the Maximum Likelihood Estimation (hereafter, MLE) method.

When explaining the MLE method in the context of GARCH models it is necessary to introduce the concept of conditional likelihood. As a result of the recursive nature of GARCH models a starting value y_0 at $T+1$ must be added. Following McNeil, Frey, and Embrechts (2005) the density of a random vector $\mathbf{y}_t$ can then be written

$$f(y_0, \ldots, y_T) = f(y_0) \prod_{t=1}^{T} f(y_t | y_{t-1}, \ldots, y_0). \tag{2.13}$$

Since the marginal density in (2.13) is not known for ARCH and GARCH models the following conditional likelihood is used

$$\begin{aligned} f(y_1, \ldots, y_T | y_0, h_0) &= \prod_{t=1}^{T} f(y_t | y_{t-1}, \ldots, y_0, h_0) \\ L(y_1, \ldots, y_T; \alpha_0, \alpha_1, \beta_1) &= \prod_{t=1}^{T} \frac{1}{h_t} g\left(\frac{y_t}{h_t}\right), \end{aligned}$$

where $h = h_0$, i.e. the unconditional variance is used as the starting value. The variable g is the so called density generator. We estimate the conditional variance and conditional mean in one step and therefore the conditional likelihood has to be modified to

$$L(y_1, \ldots, y_T; \alpha_0, \alpha_1, \beta_1, \boldsymbol{\gamma}) = \prod_{t=1}^{T} \frac{1}{h_t} g\left(\frac{y_t - \mu_t}{h_t}\right),$$

where $\boldsymbol{\gamma}$ is the parameter vector of the AR(p_1) conditional mean specification. We use four different distributions for the density generator g:

- Gaussian

$$g(y_1,\ldots,y_T;y_t|\mu_t,h_t) = \frac{1}{\sqrt{2\pi h_t}} \exp\left(\frac{-(y_t-\mu_t)^2}{2h_t}\right)$$

- Student-t

$$\begin{aligned} g(y_1,\ldots,y_T;y_t|\mu_t,h_t,\nu) =& \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\pi^{1/2}\Gamma\left(\frac{\nu}{2}\right)}(\nu-2)^{-1/2}h_t^{-1/2} \\ &\cdot\left(1+\frac{(y_t-\mu_t)^2}{h_t(\nu-2)}\right)^{-\frac{\nu(\nu+1)}{2}} \end{aligned}$$

- GED

$$\begin{aligned} g(y_1,\ldots,y_T;y_t|\mu_t,h_t,\nu) &= \frac{\nu}{\lambda 2^{\left(\frac{\nu+1}{\nu}\right)}\Gamma\left(\frac{1}{\nu}\right)} \exp\left(-\frac{1}{2}\left|\frac{y_t-\mu_t}{h_t^{1/2}}\lambda^{-1}\right|^{\nu}\right), \\ \lambda &= \left(\frac{2^{(-2/\nu)}\Gamma\left(\frac{1}{\nu}\right)}{\Gamma\left(\frac{3}{\nu}\right)}\right)^{1/2} \end{aligned}$$

- *skew* $-$ *t*

$$g(y_1,\ldots,y_T;z_t|\nu,\lambda) = \begin{cases} bc\left(1+\frac{1}{\nu-2}\left(\frac{bz_t+a}{1-\lambda}\right)^2\right)^{-\frac{\nu+1}{2}}, & z<-\frac{a}{b}, \\ bc\left(1+\frac{1}{\nu-2}\left(\frac{bz_t+a}{1+\lambda}\right)^2\right)^{-\frac{\nu+1}{2}}, & z\geq-\frac{a}{b}, \end{cases}$$

where $2<\nu<\infty$, $-1<\lambda<1$, and $z_t=\frac{y_t-\mu_t}{h_t^{1/2}}$. The constants are given by

$$\begin{aligned} a &= 4\lambda c\left(\frac{\nu-2}{\nu-1}\right), \\ b^2 &= 1+3\lambda^2-a^2, \\ c &= \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\pi(\nu-2)}\Gamma\left(\frac{\nu}{2}\right)}. \end{aligned}$$

Engle (1982) and Bollerslev (1986) both used the Gaussian distribution to estimate the ARCH and GARCH models. Bollerslev, Engle, and Wooldrige (1988) were the first to use the t-distribution in estimating a GARCH model whilst Nelson (1991) proposed the GED to estimate his EGARCH model. The *skew* $-$ *t* distribution was first used by Hansen (1994) in the context of modeling a financial time series. Since estimating with products is frequently a complicated task it is more convenient to use the log-likelihood

$$\ln L(y_1,\ldots,y_T;\boldsymbol{\theta}) = \sum_{t=1}^{T} l_t(\boldsymbol{\theta}), \tag{2.14}$$

where l_t describes the log-likelihood of the tth observation and $\boldsymbol{\theta} = (\alpha_0, \alpha_1, \beta_1, \mu)'$ is a vector of parameters. As the name suggests the likelihood will be maximized

$$\frac{\partial}{\partial \boldsymbol{\theta}} \ln L(y_1, \ldots, y_T; \boldsymbol{\theta}) = \sum_{t=1}^{T} \frac{\partial l_t(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} = 0. \tag{2.15}$$

Even if the true data process of the innovations is not described by the distributions above it is still possible to obtain consistent estimates of the parameters (White (1982), Weiss (1984), Weiss (1986), or Bollerslev and Wooldridge (1992)). This is known as quasi maximum likelihood procedure and is essentially the same as maximum likelihood with the exception of standard errors which have to be estimated in a different way. To account for these QML-properties We use the robust standard errors of White (1980) in case of estimating univariate GARCH models. The robust covariance matrix $\boldsymbol{\Omega}$ is estimated as

$$\boldsymbol{\Omega} = \mathbf{A}^{-1}\mathbf{B}\mathbf{A}^{-1}, \tag{2.16}$$

where

$$\begin{aligned} \mathbf{A} &= -E\left\{\frac{\partial^2 \ln L}{\partial \boldsymbol{\theta} \partial \boldsymbol{\theta}'}\right\} \\ \hat{\mathbf{A}} &= \frac{1}{n}\sum_{t=1}^{n}\left(\frac{\partial l_t(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\frac{\partial l_t(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}'}\right). \end{aligned}$$

Here $\hat{\mathbf{A}}$ denotes the sample estimate of $\mathbf{A}$ and $\frac{\partial \mathbf{l}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}$ the score vector. $\mathbf{B}$ and the associated sample estimate $\hat{\mathbf{B}}$ looks like

$$\begin{aligned} \mathbf{B} &= E\left\{\frac{\partial \ln L}{\partial \boldsymbol{\theta}}\frac{\partial \ln L}{\partial \boldsymbol{\theta}'}\right\} \\ \hat{\mathbf{B}} &= -\frac{1}{n}\sum_{t=1}^{n}\frac{\partial^2 l_t(\boldsymbol{\theta})}{\partial \boldsymbol{\theta} \partial \boldsymbol{\theta}'}. \end{aligned}$$

For an introduction of ML estimation and heteroscedasticity-consistent standard errors, see Greene (2008). More technical details are found in Straumann (2005).

Chapter 3

Multivariate GARCH Models

3.1 Preliminaries

Having explained the importance of taking the time varying behavior into account when modelling the (daily) returns of a financial time series, we will now focus on the comovements between several variables. The discovery of a time varying variance for a single financial time series led to the assumption that the comovements between two or more variables may also be time varying. So it was natural to enhance the univariate ARCH models to multivariate ones.

The time-varying behavior of comovements between several variables has attracted a lot of attention ins science. Among the first to investigate the time-varying dependence of stock markets were Hamao, Masulis, and Ng (1990), Susmel and Engle (1994), and Bekaert and Harvey (1995). Longin and Solnik (1995) was among the first to recognize that correlation increases through periods of high volatility. Also of interest for portfolio allocation purposes- and in the line of Longin and Solnik (1995)- are the empirical findings that correlation seems to increase during bear markets (Erb, Harvey, and Viskanta (1994), De Santis and Gerard (1997), Das and Uppal (2001), Longin and Solnik (2001), and Ang and Bekaert(2002a)). Tse (2000) invented a test for time-varying correlations: he tested three asian stock markets and found time-varying correlations for all of them.

3.2 Multivariate ARCH and GARCH Processes

The first to propose the class of multivariate ARCH (hereafter MVARCH) models were Kraft and Engle (1982) and Engle, Granger, and Kraft (1984). The multivariate models took the same route as the univariate models and were refined to multivariate GARCH (hereafter MVGARCH) models by Bollerslev, Engle, and Wooldrige (1988). The modelling of the conditional covariance matrix has attracted some attention and several different specifications have evolved. A comprehensive survey of MVGARCH models is delivered by Bauwens, Laurent, and Rombouts (2006) whilst Gouriéroux (1997) and Lütkepohl (2005) treat a number of theoretical aspects. A concise introductory survey is given by Silvennionnen and Teräsvirta (2009). To give a better understanding of the models, we first present briefly the original MVARCH and MVGARCH. Following this, the enhanced models of Engle (2002) and Cappiello, Engle, and Sheppard (2006) will be introduced. These are the time-varying models we use in the empirical section.

In spirit of the univariate specification in (2.2) we will assume that the ith element of a d-dimensional vector of zero mean residuals $\boldsymbol{\varepsilon}_t = (\varepsilon_{1,t}, \ldots, \varepsilon_{d,t})'$ is described by

$$\varepsilon_{i,t} = h_{i,t}^{1/2} \eta_{i,t}, \qquad \text{for } i = 1, \ldots, d, \tag{3.1}$$

where $h_{i,t}$ is the ith element on the main diagonal of the conditional covariance matrix $\mathbf{H}_t$ and $\eta_{i,t}$ is a strict white noise process. The covariance matrix is again estimated conditional on its own past values summarized in the information set $\mathscr{F}_{t-1} = (x_{1,t-1}, x_{2,t-1}, \ldots, x_{d,t-1}, x_{1,t-2}, \ldots, x_{d,t-2}, \ldots, x_{d,T-t+1})$. The MVARCH($P$) of Kraft and Engle (1982) and Engle, Granger, and Kraft (1984) model may then be written as

$$\text{vech}(\mathbf{H}_t) = \mathbf{A}_0 + \sum_{i=1}^{P} \mathbf{A}_i \text{vech}(\boldsymbol{\varepsilon}_{t-i}\boldsymbol{\varepsilon}_{t-i}'), \tag{3.2}$$

where *vech* denotes the half-vectorization operator stacking only the different elements of a square matrix in a $\frac{1}{2}d(d+1)$ dimensional vector. $\mathbf{A}_i$ is the coefficient matrix containing $\left(\frac{d(d+1)}{2}\right)^2$ elements and $\mathbf{A}_0$ is a $\frac{1}{2}d(d+1)$ vector of constants.

With this notation the MVGARCH(P, Q) of Bollerslev, Engle, and Wooldrige (1988) model is characterized by

$$\text{vech}(\mathbf{H}_t) = \mathbf{A}_0 + \sum_{i=1}^{P} \mathbf{A}_i \text{vech}(\boldsymbol{\varepsilon}_{t-i}\boldsymbol{\varepsilon}_{t-i}') + \sum_{j=1}^{Q} \mathbf{B}_j \text{vech}(\mathbf{H}_{t-j}), \tag{3.3}$$

where $\mathbf{B}$ is a symmetric coefficient matrix containing $\left(\frac{d(d+1)}{2}\right)^2$ elements. As mentioned before $\mathbf{H}_t$ has to be a positive definite matrix. Engle and Kroner (1995) showed that $\mathbf{H}_t$ is positive definite for the MVGARCH model in (3.3) if all eigenvalues of

$$\sum_{i=1}^{P} \mathbf{A}_i + \sum_{j=1}^{Q} \mathbf{B}_j$$

are smaller than one in modulus.

3.3 Estimation

The estimation method of MVARCH is exact the same as in the univariate case: the MLE method. Since in this analysis only MVGARCH models are considered only the theory relevant for these models will be presented.

Again, following McNeil, Frey, and Embrechts (2005) consider a first order model ($P = 1$ and $Q = 1$) in (3.3) with conditional joint density

$$f(\boldsymbol{\varepsilon}_1, \ldots, \boldsymbol{\varepsilon}_T | \boldsymbol{\varepsilon}_0, \mathbf{H}_0) = \prod_{t=1}^{T} f_t(\boldsymbol{\varepsilon}_t | \mathscr{F}_{t-1}, \boldsymbol{\varepsilon}_0, \mathbf{H}_0), \tag{3.4}$$

where $\boldsymbol{\varepsilon}_1 = (\varepsilon_{1,1}, \ldots, \varepsilon_{d,1})'$, f_t is the density of the vector $\boldsymbol{\varepsilon}_t$ conditioned on the sigma algebra $\mathscr{F}_{t-1}$, the vector of starting values $\boldsymbol{\varepsilon}_0$, and the staring value for the conditional covariance matrix $\mathbf{H}_0$. As can be seen by (3.4) the multivariate ARCH representation is easily derived from the multivariate GARCH results. The conditioning must be done only on $\boldsymbol{\varepsilon}_0$ instead of $\boldsymbol{\varepsilon}_0$ and $\mathbf{H}_0$ since in (M)ARCH models only lagged squared residuals are used as explanatory variables. Defining the multivariate error density of $\boldsymbol{\varepsilon}_t$ by $d(\boldsymbol{\varepsilon}_t)$, then the conditional joint density may be written

$$f(\boldsymbol{\varepsilon}_1, \ldots, \boldsymbol{\varepsilon}_T | \boldsymbol{\varepsilon}_0, \mathbf{H}_0) = |\mathbf{H}_t|^{-\frac{1}{2}} d(\mathbf{H}_t^{-\frac{1}{2}} \boldsymbol{\varepsilon}_t),$$

where $\mathbf{H_t}$ is described by (3.3). Assuming that $d()$ is generated by a Gaussian distribution then the conditional likelihood is given by

$$L(\boldsymbol{\varepsilon}_1, \ldots, \boldsymbol{\varepsilon}_T; \boldsymbol{\theta}) = \prod_{t=1}^{T} |\mathbf{H}_t|^{-\frac{1}{2}} d(\boldsymbol{\varepsilon}_t' \mathbf{H}_t^{-1} \boldsymbol{\varepsilon}_t), \tag{3.5}$$

where $\boldsymbol{\theta}$ contains all the estimated parameters. As in the univariate case it is often easier to use the log-likelihood instead of the likelihood function. The conditional log-likelihood of (3.5) turns out to be

$$\begin{aligned} \ln L(\boldsymbol{\varepsilon}_1, \ldots, \boldsymbol{\varepsilon}_T; \boldsymbol{\theta}) &= \sum_{t=1}^{T} \left(-\frac{1}{2} \ln |\mathbf{H}_t| + \ln d(\boldsymbol{\varepsilon}_t' \mathbf{H}_t^{-1} \boldsymbol{\varepsilon}_t) \right) \qquad (3.6) \\ &= \sum_{t=1}^{T} l_t(\boldsymbol{\theta}). \end{aligned}$$

For Maximum Likelihood estimation it is necessary to specify multivariate probability distribution functions. As the multivariate normal distribution belongs to the family of spherical distributions (3.5) can be used with the multivariate normal plugged in for the density generator $d()$. The multivariate GARCH(1,1) likelihood equation based on a multivariate normal then takes the form

$$L(\boldsymbol{\varepsilon}_1, \ldots, \boldsymbol{\varepsilon}_T; \mathbf{A}_0, \mathbf{A}, \mathbf{B}) = \prod_{t=1}^{T} \frac{1}{2\pi^{\frac{d}{2}} |\mathbf{H}_t|^{\frac{1}{2}}} \exp\left\{ -\frac{1}{2} (\boldsymbol{\varepsilon}_t' \mathbf{H}_t^{-1} \boldsymbol{\varepsilon}_t) \right\},$$

where $\mathbf{H}_t$ follows an process described by (3.3). The Quasi Maximum Likelihood Procedure might also be applied to the multivariate normal MLE and again a robust covariance estimation method has to be implemented. The robust covariance method we use was especially developed for the MVGARCH model and it is this we will now explain presenting first the MVGARCH model and thereafter then the standard errors that belong to it.

3.4 Dynamic Conditional Correlation

The fundamental logic behind conditional correlation models is that the positive-definite conditional covariance matrix $\mathbf{H}_t$ can be decomposed into the conditional standard deviations and conditional correlation matrix. In the following the Dynamic Conditional Correlation (hereafter DCC) developed by Engle (2002) will be introduced making use of this decomposition. The DCC model tries to shape possible time-varying behavior of the correlation between time series by a GARCH-based procedure. From a practical point of view an advantage of the DCC model is the two-step procedure. In the first step univariate GARCH models are estimated for each time series and in the second step the covariance matrix with standardized residuals from the first step. Any univariate GARCH model which assumes Gaussian innovations can be used in the first stage. This allows great flexibility and makes it possible to allow for example asymmetric GARCH models. On the other side, even GARCH processes with Gaussian innovations depict a kurtosis greater than 3 and so the fat-tailed behavior of financial time series returns can be embraced in part. As has been shown by numerous studies this might still prove insufficient (see Bollerslev (1987) for example), but using GARCH models with Gaussian innovations assumption is the only possibility using the two-step procedure. As always, multi-step estimation procedures comes with a loss of efficiency but the advantages- especially in higher dimensions- seems to outweigh this. One of the first to use the decomposition of the correlation matrix was Bollerslev (1990) who invented the Constant Conditional Correlation (hereafter CCC) model. The specification of the CCC model may be written

$$
\begin{aligned}
\mathbf{H}_t &= \mathbf{D}_t \mathbf{R} \mathbf{D}_t \qquad (3.7) \\
\mathbf{D}_t &= diag\left(\sqrt{h_{i,t}}\right) \quad \text{for } i = 1, \ldots, d \\
\boldsymbol{\eta}_t &= D_t^{-1} \boldsymbol{\varepsilon}_t,
\end{aligned}
$$

where *diag* is an operator which projects a vector onto the main diagonal of a square matrix and **R** is a positive definite correlation matrix. The CCC incorporates the time-varying behavior of (univariate) volatility but neglects the possibility of a time varying behavior of the correlation. Bera and Kim (2002) and Tse and Tsui (2002) for example both reject the hypothesis of constant correlation in international equity markets. These findings raised the need to develop a model which incorporates time-varying correlations. Engle (2002) developed such a model: the DCC model. This uses the same decomposition of the covariance matrix as the CCC model in (3.7) but models the correlation matrix as time varying

$$
\mathbf{H}_t = \mathbf{D}_t \mathbf{R}_t \mathbf{D}_t. \qquad (3.8)
$$

Furthermore, it assumes conditional normality for the zero mean residuals

$$
\boldsymbol{\varepsilon}_t | \mathscr{F}_{t-1} \sim \mathscr{N}(0, \mathbf{D}_t \mathbf{R}_t \mathbf{D}_t).
$$

Engle (2002) also noted that the conditional correlation matrix equals the conditional covariance matrix of the standardized returns

$$
\mathbf{R}_t = E\{\boldsymbol{\eta}_t \boldsymbol{\eta}_t' | \mathscr{F}_{t-1}\}.
$$

To give a better understanding of the DCC-model the derivation of the DCC structure following Engle (2002) will be briefly explained. The dynamic structure of the correlation matrix can be derived via the exponential smoother used by RiskMetrics

$$\rho_{ij,t} = \frac{\sum_{s=1}^{t-1} \lambda^s \eta_{i,t-s} \eta_{j,t-s}}{\sqrt{\left(\sum_{s=1}^{t-1} \lambda^s \eta_{j,t-s}^2\right)\left(\sum_{s=1}^{t-1} \lambda^s \eta_{j,t-s}^2\right)}},$$

where λ describes the smoothing parameter. For daily returns RiskMetrics set $\lambda = 0.94$ based on their empirical evidence. This might be an aspect to criticize since λ is kept constant no matter what kind of sample is used in the analysis although stock and bond indices might need quite different values. Exponential smoothing leads to

$$\begin{aligned} q_{ij,t} &= (1-\lambda)(\eta_{i,t-1}\eta_{j,t-1}) + \lambda(q_{ij,t-1}) \\ \rho_{ij,t} &= \frac{q_{ij,t}}{\sqrt{q_{ii,t}q_{jj,t}}}. \end{aligned} \tag{3.9}$$

Equation (3.9) can be written as a GARCH(1,1)

$$q_{ij,t} = \overline{\rho}_{i,j} + \alpha\left(\eta_{i,t-1}\eta_{j,t-1} - \overline{\rho}_{i,j}\right) + \beta\left(q_{i,j,t-1} - \overline{\rho}_{i,j}\right), \tag{3.10}$$

where $\overline{\rho}$ denotes the unconditional correlation. In matrix form (3.10) may be written as

$$\mathbf{Q}_t = \mathbf{S}(1-a-b) + a(\boldsymbol{\eta}_{t-1}\boldsymbol{\eta}'_{t-1}) + b\mathbf{Q}_{t-1},$$

where $\mathbf{S}$ represents the unconditional sample correlation of $\boldsymbol{\eta}$. With the definition of $\mathbf{Q}_t$ the conditional correlation matrix is given by

$$\mathbf{R}_t = diag(\mathbf{Q}_t^*)^{-1}\ \mathbf{Q}_t^*\ diag(\mathbf{Q}_t^*)^{-1},$$

where $\mathbf{Q}_t^*$ is a diagonal matrix with the square root of the ith diagonal element of Q_t on its ith diagonal position

$$\mathbf{Q}_t^* = \begin{bmatrix} \sqrt{q_{11,t}} & & \\ & \ddots & \\ & & \sqrt{q_{ii,t}} \end{bmatrix}.$$

Note that different lag length of the standardized residuals and Q_ts are also possible

$$\mathbf{Q}_t = \mathbf{S}\left(1 - \sum_{i=1}^{P_2} a_i - \sum_{j=1}^{Q_2} b_j\right) + \sum_{i=1}^{P_2} a_i\left(\boldsymbol{\eta}_{t-i}\boldsymbol{\eta}'_{t-i}\right) + \sum_{j=1}^{Q_2} b_j\mathbf{Q}_{t-j}. \tag{3.11}$$

This is the original DCC(P_2, Q_2) model as it has been developed by Engle (2002). Both univariate and multivariate GARCH give geometric declining weights to information contained in the sample, i.e. the most recent data point achieves the highest weight and the most distant the least weight. This a huge difference to Pearson's correlation coefficient which applies the same weight to every data point.

As explained above the correlation matrix needs to be positive-definite and this implies a positive-definite covariance matrix. Ding and Engle (2001) worked out some sufficient conditions for $\mathbf{Q}_t$ to be positive definite. Firstly, the unconditional covariance matrix $\mathbf{Q}_0$ has to be positive definite. Further the intercept $\mathbf{S}(1-\sum_{i=1}^{P_2} a_i - \sum_{j=1}^{Q_2} b_j)$ has to be positive definite, too. As a result of the two step procedure the sufficient conditions for $\mathbf{H}_t$ in (3.8) to be positive definite are the same as the conditions for GARCH models, i.e. at least one a_j has to be greater 0 and $1-\sum_{i=1}^{p_2} a_i - \sum_{j=1}^{q_2} b_j > 0$. If this criteria is not fulfilled the unconditional covariance matrix of $\boldsymbol{\eta}_t$ would not exist. Additional $\sum_{i=1}^{P_2} a_i$ and $\sum_{j=1}^{Q_2} b_j$ in (3.11) have to be greater than zero and $\sum_{i=1}^{P_2} a_i + \sum_{j=1}^{Q_2} b_j < 1$. For more details see Engle and Sheppard (2001).

3.4.1 Asymmetric Dynamic Conditional Correlation

The DCC model met with the same criticism as the original univariate GARCH model developed by Bollerslev - the symmetrical response to positive and negative shocks. One explanation for the asymmetric volatility is the volatility feedback effect- the increase in expected returns from investors leads to lower stock prices. As Kroner and Ng (1998) notes this could lead to a change in the correlation between assets if the volatility feedback back applies to one asset but not to the others. Cappiello, Engle, and Sheppard (2006) developed in the spirit of the univariate GARCH specification of Glosten, Jagannathan, and Runkle (1993) an asymmetric version of the DCC model - the Asymmetric Generalized Dynamic Conditional Covariance (AG-DCC(P_2,Q_2)) model here presented with $P_2 = 1$ and $Q_2 = 1$

$$\mathbf{Q}_t = \left(\overline{\mathbf{Q}} - \mathbf{A}'\overline{\mathbf{Q}}\mathbf{A} - \mathbf{B}'\overline{\mathbf{Q}}\mathbf{B} - \mathbf{G}'\overline{\mathbf{N}}\mathbf{G}\right) + \mathbf{A}'\boldsymbol{\eta}_{t-1}\boldsymbol{\eta}_{t-1}'\mathbf{A} + \mathbf{B}'\mathbf{Q}_{t-1}\mathbf{B} + \mathbf{G}'\mathbf{n}_{t-1}\mathbf{n}_{t-1}'\mathbf{G}, \quad (3.12)$$

where $\overline{Q}$ is the unconditional covariance matrix, $\mathbf{A},\mathbf{B}$, and $\mathbf{G}$ are parameter matrices, and $\mathbf{n}_t = R[\boldsymbol{\eta_t} < 0] \circ \boldsymbol{\eta}_t$. R describes the $d \times 1$ indicator function and $\circ$ the Hadamard product. The indicator function takes the value 1 if the condition in the brackets is fulfilled, 0 otherwise. In this analysis we restrict $\mathbf{A},\mathbf{B}$, and $\mathbf{G}$ to diagonal matrices as otherwise there could be to many parameters to estimate. The AG-DCC model in (3.12) clearly nests the simple DCC-model if $\mathbf{G}$ is replaced by the zero matrix $\mathbf{0}_{dd}$ and $\mathbf{A} = I_d\sqrt{a}$ and $\mathbf{B} = I_d\sqrt{b}$, where I_d is the d-dimensional identity matrix. The A-DCC model is obtained by keeping the DCC specification and furthermore by setting $\mathbf{G} = I_d\sqrt{g}$. The so-called G-DCC model is obtained by setting $\mathbf{G} = \mathbf{0}_{dd}$ in (3.12). As with the univariate GARCH models there might be some considerable advantages in specifying an asymmetric term especially if the phenomena of increasing correlation during bear markets exists in the sample. In the DCC model of Engle all variables react in the same way to the arrival of new information. When using matrices instead of scalars every time series can respond to their own specific news. This might be advantageous especially when estimating portfolios with low dimension and different dynamic structures.

Again the one-period ahead forecast of the covariance and therefore also the forecast of the correlation matrix are fairly simple. The multiperiod forecasts for DCC models are explained in Engle and Sheppard (2001).

The time-varying behavior modelling through MVGARCH models has attracted some interest. For example Tse and Tsui (2002), Billio and Caporin (2006), Silvennionnen and

Teräsvirta (2005), and Silvennionnen and Teräsvirta (2005) model either directly $\mathbf{R}_t$ or $\mathbf{Q}_t$ through different specifications.

3.4.2 Estimation

To complete the theoretical section of the MVGARCH models we must explain the estimation procedure of the DCC models following Engle (2002). The DCC model is estimated via the MLE method just as for other multivariate GARCH models. A unique feature of the DCC model is that it can be estimated in two steps. Therefore the (log-)likelihood in equation (3.6) has to be modified. Although efficiency is lost with this estimation method especially when estimating high dimensional portfolios the computational advantage cannot be neglected.

The first step consists of estimating univariate GARCH models for the variance of each asset. The second step estimates the covariance matrix using residuals standardized by the standard deviation of the variance estimated in the first step. The complete likelihood may then be written

$$L(\boldsymbol{\theta}^{DCC}) = L_V(\boldsymbol{\kappa}) + L_C(\boldsymbol{\upsilon}, \boldsymbol{\kappa}),$$

where $\boldsymbol{\kappa}$ represents the parameter vector of the univariate GARCH models, $\boldsymbol{\upsilon}$ the parameter vector of the correlation part, and $\boldsymbol{\theta}^{DCC} = (\boldsymbol{\kappa}, \boldsymbol{\upsilon})'$ is the parameter vector of all estimated parameters. It can be clearly seen that the correlation part is conditioned on the volatility part. Under the normality assumption the log-likelihood of the volatility term

$$\ln L_V(\boldsymbol{\varepsilon}_1, \ldots, \boldsymbol{\varepsilon}_T; \boldsymbol{\kappa}) = -\frac{1}{2} \sum_{t=1}^{T} \left(d \ln(2\pi) + \ln |\mathbf{D}_t|^2 + \boldsymbol{\varepsilon}_t' \mathbf{D}_t^{-2} \boldsymbol{\varepsilon}_t \right),$$

is just the sum of the univariate GARCH estimates as can be seen by

$$\ln L_V(\boldsymbol{\varepsilon}_1, \ldots, \boldsymbol{\varepsilon}_T; \boldsymbol{\kappa}) = -\frac{1}{2} \sum_{t=1}^{T} \left(\ln(2\pi) + \sum_{i=1}^{d} \ln h_{i,t} + \sum_{j=1}^{d} \frac{\varepsilon_{j,t}^2}{h_{j,t}} \right),$$

where $|\mathbf{D}_t|$ depicts the determinant of $\mathbf{D}_t$. The log-likelihood of the correlation term is given by

$$\ln L_C(\boldsymbol{\eta}_1, \ldots, \boldsymbol{\eta}_T; \boldsymbol{\upsilon} | \hat{\boldsymbol{\kappa}}) = -\frac{1}{2} \sum_{t}^{T} \left(\ln |\mathbf{R}_t| + \boldsymbol{\eta}_t' \mathbf{R}_t^{-1} \boldsymbol{\eta}_t - \boldsymbol{\eta}_t' \boldsymbol{\eta}_t \right),$$

where $\hat{\boldsymbol{\kappa}}$ is the estimated parameter vector of the univariate GARCH models. Under standard regularity conditions consistency and asymptotic normality will hold for the parameters of the two-step procedure, see Newey and McFadden (1994) and Engle and Sheppard (2001). Since the full likelihood combines the likelihood of the univariate GARCH models and the likelihood of the correlation part the same distribution has to be assumed for both estimation stages. Thus, when the above method is used the DCC model is limited to the Gaussian distribution in the MLE. Pesaran and Pesaran (2007) points out that estimating the DCC model for example with a multivariate t-distribution implies that the univariate GARCH models have to be estimated with an univariate t-distribution, too. This would not be a problem were it not for the fact that the degree-of-freedom parameters for each univariate GARCH and the multivariate GARCH must be the same, something that is extremely unlikely in practice.

If the standardized residuals are not Gaussian distributed then the DCC estimator preserves QML properties. Similarly to univariate GARCH models this implies that the DCC GARCH process is able to model some of the fat-tailed behavior often found in daily returns of financial time series which might still be enclosed in the standardized residuals. Again, as in the univariate case, this requires modifications of the standard errors. The formula for the DCC standard errors is fairly complicated because they depend on the standard errors of the first stage univariate GARCH standard errors. The standard errors for both stages must be the Bollerslev and Wooldridge (1992) robust standard errors. To guarantee asymptotic normality

$$\sqrt{T}\left(\hat{\boldsymbol{\theta}}^{DCC} - \hat{\boldsymbol{\theta}}_0\right) \overset{A}{\sim} N(0, \boldsymbol{\Omega}_*),$$

the Bollerslev-Wooldridge standard errors need to be modified. The asymptotic robust Bollerslev-Wooldridge covariance matrix $\boldsymbol{\Omega}_*$ involves all covariance terms, i.e. covariance estimates from the first and the second stage

$$\boldsymbol{\Omega}_* = \mathbf{A}_*^{-1}\mathbf{B}_*\mathbf{A}_*^{-1},$$

where

$$\mathbf{A}_* = \begin{bmatrix} E\left\{\dfrac{\partial^2 \ln L_V(\boldsymbol{\kappa})}{\partial \boldsymbol{\kappa}' \partial \boldsymbol{\kappa}}\right\} & 0 \\ E\left\{\dfrac{\partial^2 \ln L_C(\boldsymbol{\upsilon}, \boldsymbol{\kappa})}{\partial \boldsymbol{\upsilon}' \partial \boldsymbol{\kappa}}\right\} & E\left\{\dfrac{\partial^2 \ln L_C(\boldsymbol{\upsilon}, \boldsymbol{\kappa})}{\partial \boldsymbol{\upsilon}' \partial \boldsymbol{\upsilon}}\right\} \end{bmatrix}$$

and

$$\mathbf{B}_* = var\left[E\left\{\frac{\partial L_V(\boldsymbol{\kappa})}{\partial \boldsymbol{\kappa}}\right\}, E\left\{\frac{\partial \ln L_C(\boldsymbol{\upsilon}, \boldsymbol{\kappa})}{\partial \boldsymbol{\upsilon}}\right\}\right].$$

The derivation of the standard errors and all proofs can be found in Bollerslev and Wooldridge (1992) and Engle and Sheppard (2001).

The class of multivariate GARCH models (and especially the DCC model of Engle) offers a relatively simple way to estimate time-varying correlations. A major advantage of the DCC models compared to the rolling window linear correlation coefficient is that the weighting of the different point in time is not arbitrary. A drawback of Engle's approach is the necessity of the multivariate Gaussian distribution in the MLE procedure. Although due to the QML properties of the DCC estimator and the implied modelling of a kurtosis greater than the multivariate Gaussian distribution this might not be enough when modelling daily returns of financial time series. Another disadvantage might be the symmetry of the multivariate Gaussian distribution. To overcome these drawbacks a new class able to model dependence in more flexible ways has to be introduced: the copula.

Chapter 4

Copulas

4.1 Preliminaries

As this chapter deals mostly with (pure) mathematics the notation departs from the style used in standard econometrics. In accordance with proper mathematical practice random variables are labelled with upper-case letters. Real valued numbers continue to be denoted by lower-case letters whilst real valued vectors and matrices are denoted by bold lower-case letters.

Generally speaking, copulas are mathematical objects able to describe the dependence structure between random variables. Since their development in the 1950s they have gained a great deal of attention particularly in the field of finance. Here we introduce some basic copula theory before discussing the static version of copulas and the dynamic copula- enhanced for time-varying purposes. Finally some useful dependence measures related to copulas will be explained.

The use of copulas in financial applications became well known due to Li (2000) who used the Gaussian copula in CDO pricing. The work of Embrechts, McNeil, and Straumann (1999,2002) is highly relevant for the use of copulas in finance whilst Mikosch (2006) provides a nice discussion about the advantages and disadvantages using copulas to estimate dependence. For a concise survey of copula models and their use in financial time series context see Patton (2009).

Sklar (1959) was the first to model dependence through a copula. The basic idea that underlies every copula is that it is possible to decompose a joint distribution function for a random vector into its marginal functions and their corresponding dependence structure. A copula describes this dependence structure and belongs to the class of joint cumulative distribution functions (c.d.f., hereafter).

Consider some random numbers $X_1,\dots X_d$ with marginal distributions $F_1(x_1) = Pr[X_1 < x_1]$ and $F_d(x_d) = Pr[X_d < x_d]$. Their joint distribution function F may be written as

$$F(x_1,\dots,x_d) = Pr[X_1 \le x_1,\dots,X_d \le x_d] \tag{4.1}$$

$$F(x_1,\dots,x_d) = Pr[F_1(X_1) \le F_1(x_1),\dots,F_d(X_d) \le F_d(x_d)]. \tag{4.2}$$

Sklar (1959) shows in his theorem that the copula can be depicted as

$$F(x_1,\dots,x_d) = C(F_1(x_1),\dots,F_d(x_d)), \qquad x_1,\dots,x_d \in \mathbb{R}^d. \tag{4.3}$$

If the marginal distributions $F_1,\ldots,F_d$ are continuous than C is unique. In contrast, F is a multivariate distribution function with univariate distributions $F_1,\ldots,F_d$ if C is a copula.

A multivariate distribution function consists of marginal distributions and a dependence structure. As shown by (4.3) the copula describes the dependence structure and binds the univariate marginal distributions together to a multivariate distribution function. The copula itself can be deduced from (4.3) directly via

$$C(u_1,\ldots,u_d) = F\left(F_1^{-1}(u_1),\ldots,F_d^{-1}(u_d)\right). \tag{4.4}$$

In the spirit of equation (4.1) it is possible to show that the copula is the distribution function of the (continuous) marginal distributions

$$C(u_1,\ldots,u_d) = Pr\left[F_1(X_1) \le u_1,\ldots,F_d(X_d) \le u_d\right].$$

Fisher (1932) and Rosenblatt (1952) introduced the concept of probability integral transform. A random variable X_1 with a continuous distribution function F_1 may transformed into an uniform distributed random variable by applying the distribution function to the variable

$$U_1 = F_1(X_1) \sim Uniform(0,1), \tag{4.5}$$

where $Uniform(0,1)$ denotes the uniform distribution on the interval $[0,1]$. By using the quantile function F_1^{-1}, X_1 is re-extracted by

$$X_1 = F_1^{-1}(U_1) \Rightarrow X_1 \sim F_1.$$

If all marginal distributions are assumed continuous, then the copula C is unique and represents a mapping for the d-dimensional unit hypercube into the unit interval

$$C : [0,1]^d \to [0,1].$$

If the following properties hold for a distribution function it is a copula

1. $C(u_1,\ldots,u_d)$ is increasing in every component u_i
2. $C(1,\ldots,1,u_i,1,\ldots,1) = u_i, \quad \forall i \in \{1,\ldots,d\}, u_i \in [0,1]$
3. $\forall (a_1,\ldots,a_d),(b_1,\ldots,b_d) \in [0,1]^d, \qquad a_i \le b_i :$
   $\sum_{i_1=1}^{2} \cdots \sum_{i_d=1}^{2} (-1)^{i_1+\cdots+i_d} C(u_{1,i},\ldots,u_{d,i}) \le 0,$

where $u_{j,1} = a_j$, $u_{j,2} = b_j$ for all $j = 1,\ldots,d$, see McNeil, Frey, and Embrechts (2005, p.185). The first property is required by every multivariate density function. A function with this property is called grounded. The second property is a prerequisite of every uniform marginal density. The third property is the rectangle inequality and a function with this property is 2-increasing. Since the density of a multivariate distribution is given by

$$f(x_1,\ldots,x_d) = \frac{\partial^d F(x_1,\ldots,x_d)}{\partial x_1 \cdots x_d}. \tag{4.6}$$

the density of a copula may be written as

$$\begin{aligned} f(x_1,\ldots,x_d) &= \frac{\partial^d C(F_1(x_1),\ldots,F_d(x_d))}{\partial x_1 \cdots \partial x_d} \\ &= \frac{\partial^d C(F_1(x_1),\ldots,F_d(x_d))}{\partial F_1(x_1)\cdots \partial F_d(x_d)} \cdot \prod_{i=1}^{d} \frac{\partial F_i(x_i)}{\partial x_i} \qquad (4.7) \\ &= \frac{\partial^d C(u_1,\ldots,u_d)}{\partial u_1 \cdots \partial u_d} \cdot \prod_{i=1}^{d} \frac{\partial F_i(x_i)}{\partial x_i} \\ &= c(u_1,\ldots,u_d) \cdot \prod_{i=1}^{d} f_i(x_i), \qquad (4.8) \end{aligned}$$

where the last term in (4.8) represents the density of the marginals. Equation (4.7) is also called the canonical representation of a multivariate density, (Cherubini, Luciano, and Vecchiato (2004, p.145)). With elementary algebra the copula density derived from (4.8) is given by

$$c(u_1,\ldots,u_d) = \frac{f(x_1,\ldots,x_d)}{\prod_{i=1}^{d} f_i(x_i)} \tag{4.9}$$

and in quantile transformation form as

$$c(u_1,\ldots,u_d) = \frac{f\left(F_1^{-1}(u_1),\ldots,F_d^{-1}(u_d)\right)}{f_1\left(F_1^{-1}(u_1)\right)\cdots f_d\left(F_d^{-1}(u_d)\right)}.$$

Among the copula's many attractive properties is the fact that it is invariant under strictly increasing transformations of the margins (for more details on copula properties see Embrechts, Lindskog, and McNeil (2003)). This is of great use in finance in finance applications because often logarithmic returns are frequently used instead of arithmetical returns. A number of different copulas have now been developed and useful theoretical summaries of many of these are given in Nelsen (2006) and Joe (1997) whilst Cherubini, Luciano, and Vecchiato (2004) treats the empirical side. In the following we will introduce two special copulas which are not used directly in this analysis but are nontheless relevant as they own some properties which most copulas share.
Hoeffding (1940) and Fréchet (1951) showed that all multivariate distribution functions are bounded by their marginal distributions. The Fréchet-Hoeffding lower bound is also known as the countermonotonic copula and is defined by

$$W^d(u) = \max\left\{\sum_{i=1}^{d} u_i + 1 - d, 0\right\}. \tag{4.10}$$

It represents perfect negative dependence between random variables but it is not a copula for $d \geq 3$. The Fréchet-Hoeffding upper bound, also known as the comonotonic copula, may be stated as

$$M^d(u) = \min(u_1,\ldots,u_d) \tag{4.11}$$

and describes perfect positive dependence between random variables. The third copula is the independence copula. As the name suggests it covers the case of independence between random variables

$$\Pi^d(u) = \prod_{i=1}^{d} u_i. \tag{4.12}$$

Finally, (4.10) and (4.11) imply that for every d-dimensional copula

$$W^d(u) \leq C(u) \leq M^d(u) \tag{4.13}$$

must hold.

4.2 Copula Classes

4.2.1 Elliptical Copulas

A copula that belongs to the elliptical family is the Gaussian copula

$$C^{GA}(u_1, u_2, \ldots, u_d; \mathbf{R}) = \Phi_R\left(\phi^{-1}(u_1), \ldots, \phi^{-1}(u_d)\right), \tag{4.14}$$

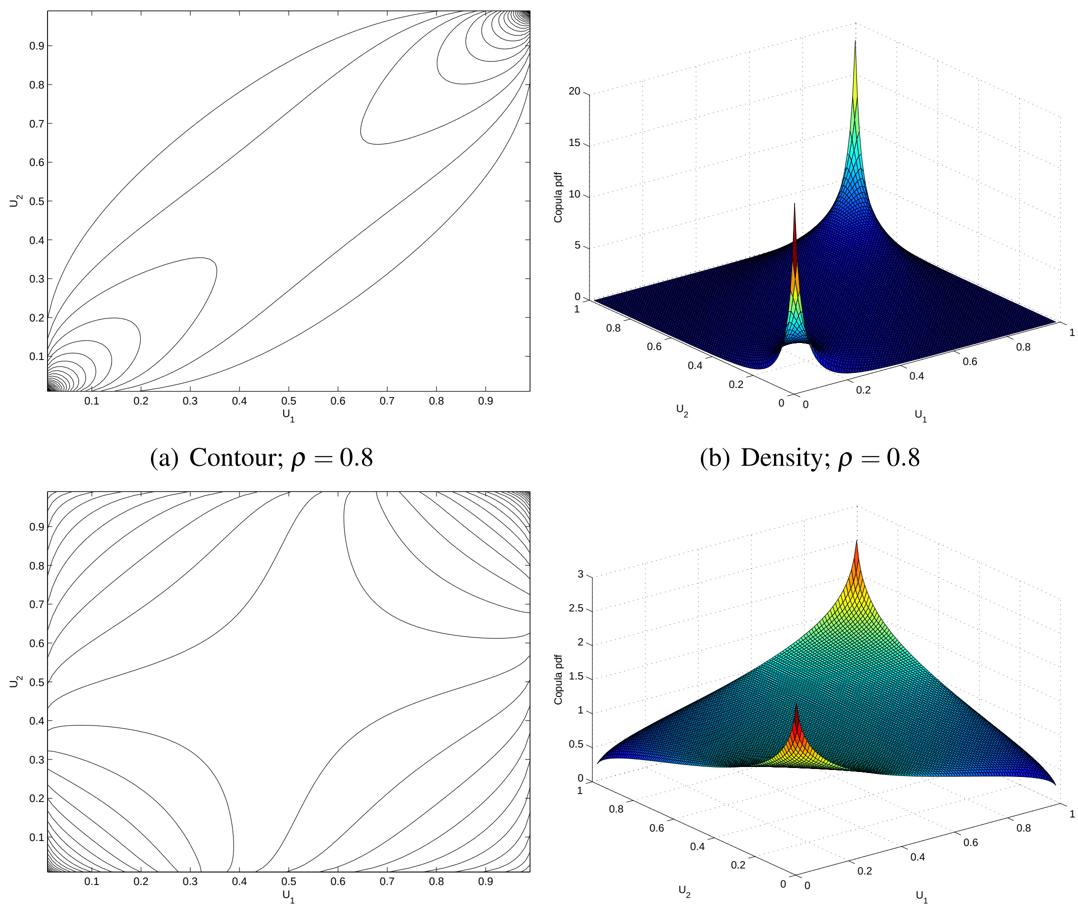
where Φ_R denotes the joint distribution of the multivariate standard normal distribution with the usual positive definite linear correlation matrix $\mathbf{R}$. The inverse of the standard univariate Gaussian distributions is denoted by ϕ^{-1}. The complete multivariate Gaussian copula may be written as

$$\begin{aligned} C^{GA}(u_1, \ldots, u_d; \mathbf{R}) = \int_{-\infty}^{\Phi^{-1}(u_1)} \cdots \int_{-\infty}^{\Phi^{-1}(u_d)} & \frac{1}{2\pi^{d/2}|\mathbf{R}|^{1/2}} \\ & \cdot \exp\left\{-\frac{1}{2}\mathbf{x}'\mathbf{R}^{-1}\mathbf{x}\right\} \mathrm{d}x_1, \ldots \mathrm{d}x_d. \end{aligned} \tag{4.15}$$

According to equation (4.9) the Gaussian copula density is derived by[1]

$$\begin{aligned} c^{GA}(u_1, \ldots, u_d) &= \frac{f^{GA}\left(\phi_1^{-1}(u_1), \ldots, \phi_d^{-1}(u_d)\right)}{\prod_{i=1}^{d} f_i\left(\phi_i^{-1}(u_i)\right)} \\ &= \frac{\frac{1}{(2\pi)^{d/2}|\mathbf{R}|^{1/2}} \exp\left\{-\frac{1}{2}\boldsymbol{\zeta}'\mathbf{R}^{-1}\boldsymbol{\zeta}\right\}}{\prod_{i=1}^{d} \frac{1}{(2\pi)^{1/2}} \exp\left\{-\frac{1}{2}\phi_i^{-1}(u_i)^2\right\}} \\ &= \frac{\frac{1}{(2\pi)^{d/2}|\mathbf{R}|^{1/2}} \exp\left\{-\frac{1}{2}\boldsymbol{\zeta}'\mathbf{R}^{-1}\boldsymbol{\zeta}\right\}}{\frac{1}{(2\pi)^{d/2} \exp\left\{-\frac{1}{2}\zeta\zeta'\right\}}} \\ &= \frac{1}{|\mathbf{R}|^{1/2}} \exp\left\{-\frac{1}{2}\boldsymbol{\zeta}'(\mathbf{R}^{-1} - I)\boldsymbol{\zeta}\right\}, \end{aligned}$$

[1]The analytical expressions for the elliptical copulas are based on Bouyé et al (2000).



(a) Contour; $\rho = 0.8$ (b) Density; $\rho = 0.8$

(c) Contour; $\rho = 0.2$ (d) Density; $\rho = 0.2$

Figure 4.1: Contour and Density Gaussian Copula. This figure shows contour and density plots of the Gaussian copula.

where $\boldsymbol{\zeta} = \left(\phi^{-1}(u_1), \ldots, \phi^{-1}(u_d)\right)'$, f^{GA} represents the multivariate density of the normal distribution, and f_i the density of the margin. Figure 4.1 shows the contour and density plots of a Gaussian copula with a bivariate linear correlation coefficient of $\rho = 0.8$ and $\rho = 0.2$. For the copula density and contour plots all margins are constructed in the following way. We simulate 100 values from a $Uniform(0,1)$ distribution with equal distance between each point. The data points are then sorted in ascending order and plugged into the copula. The Gaussian copula is comprehensive because its lower bound is given by $\lim_{R \to -1} C^{GA} = W$, its upper bound by $\lim_{R \to 1} C^{GA} = M$ and the independence case is depicted by $\lim_{R \to 0} C^{GA} = \Pi$.

Another copula that belongs to the class of elliptical copulas is the t-copula

$$C^T\left(u_1, \ldots, u_d; \mathbf{R}, \nu\right) = T_{\mathbf{R},\nu}(t_\nu^{-1}(u_1), \ldots, t_\nu^{-1}(u_d)),$$

where $T_{\mathbf{R},\nu}$ denotes the joint distribution of the multivariate t-distribution with the correlation matrix $\mathbf{R}$ and the degree-of-freedom parameter ν. The univariate inverse c.d.f. of the

t-distribution is denoted by t_ν^{-1}. The t-copula in the multivariate case may be written then as

$$C^T(u_1,\ldots,u_d;\mathbf{R},\nu) = \int_{-\infty}^{t_\nu^{-1}(u_1)} \cdots \int_{-\infty}^{t_\nu^{-1}(u_d)} \frac{\Gamma\left(\frac{\nu+d}{2}\right)|\mathbf{R}|^{-1/2}}{\Gamma\left(\frac{\nu}{2}\right)(\nu\pi)^{d/2}}$$
$$\cdot\left(1+\frac{1}{\nu}\mathbf{x}'\mathbf{R}^{-1}\mathbf{x}\right)^{-\frac{\nu+d}{2}} \mathrm{d}x_1 \ldots \mathrm{d}x_u$$

and the associated density as

$$c^T(u_1,\ldots,u_d) = |\mathbf{R}|^{-1/2}\frac{\Gamma\left(\frac{\nu}{2}\right)}{\Gamma\left(\frac{\nu+1}{2}\right)}\left(\frac{\Gamma\left(\frac{\nu+d}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)}\right)^d \frac{\left(1+\frac{1}{\nu}\boldsymbol{\zeta}'\mathbf{R}^{-1}\boldsymbol{\zeta}\right)^{-\frac{\nu+d}{2}}}{\Pi_{j=1}^d\left(1+\frac{\zeta_j^2}{\nu}\right)^{-\frac{\nu+d}{2}}},$$

where $\boldsymbol{\zeta} = (t_\nu^{-1}(u_1),\ldots,t_\nu^{-1}(u_d))'$. Figure 4.2 shows the t-copula contour and density plots with two different bivariate correlations coefficients. Compared to the Gaussian copula both figure highlight that the t-copula has more mass in the tails for the respective correlation coefficient. A key drawback of elliptical copulas when they are used for financial time series is their inability to model asymmetric behavior. As noted above it seems to be a so-called stylized fact that dependence increases with large losses more than with large wins. This should resemble in asymmetric tail dependence.

4.2.2 Archimedean Copulas

The inability of elliptical copulas to model asymmetric dependence in the tails can be overcome by Archimedean copulas. These are able to model positive dependence in either the left or the right tail. Unlike Elliptical copulas the Archimedean copulas have closed-form expressions which makes them in most cases simpler to handle.

Fundamental to the construction of Archimedean copulas is the generator function

$$\varphi : [0,1] \to [0,\infty]$$

and the pseudo-inverse of the generator function defined by

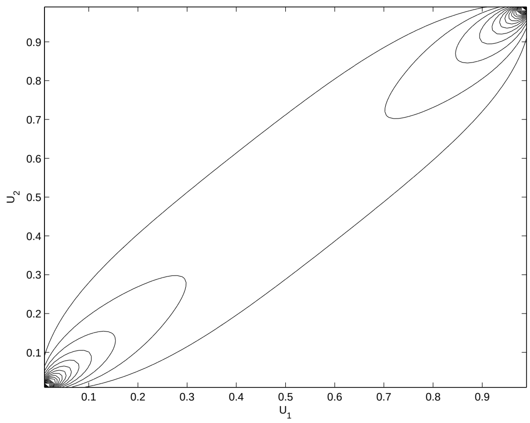
$$\varphi^{-1} = \begin{cases} \varphi^{-1}, & 0 \le t \le \varphi(0), \\ 0, & \varphi(0) < t \le \infty. \end{cases}$$

Nelsen (2006, p. 152) derived the conditions for the generator to guarantee that C in (4.17) is d-dimensional copula. The generator φ must be a continuous, strictly decreasing function, i.e. $\varphi(1) = 0$ and $\varphi(0) = \infty$. The inverse generator φ^{-1} must be completely monotonic on $[0,\infty)$. Genest and MacKay(1986a, 1986b) define a bivariate Archimedean copula as

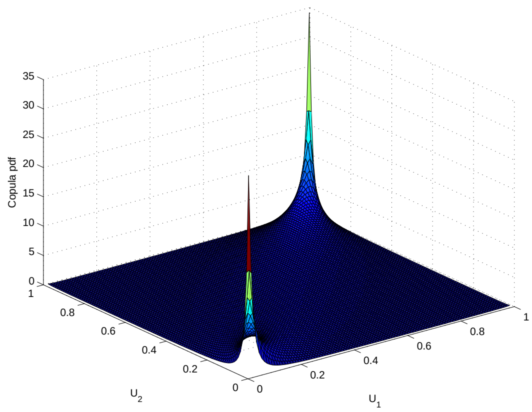
$$C(u_1,u_2) = \varphi^{-1}\left(\varphi(u_1)+\varphi(u_2)\right). \tag{4.16}$$

Bivariate Archimedean copulas are easily extended to the multivariate case

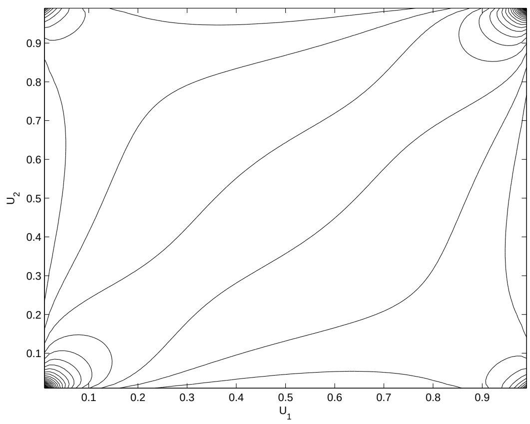
$$C(u_1,\ldots,u_d) = \varphi^{-1}\left(\varphi(u_1)+\varphi(u_2)+\cdots\varphi(u_d)\right). \tag{4.17}$$



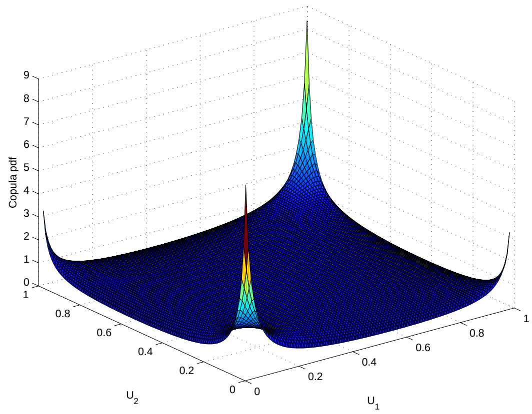
(a) Contour; $\rho = 0.8$



(b) Density; $\rho = 0.8$



(c) Contour; $\rho = 0.2$



(d) Density; $\rho = 0.2$

Figure 4.2: Contour and Density Student-*t* Copula. This figure shows contour and density plots of the *t*-copula.

The density of an Archimedean copula may be deduced by means of the generator function

$$c(u_1,\ldots,u_2) = \varphi^{-1[d]}\left(\varphi(u_1)+\ldots+\varphi(u_d)\right)\cdot\prod_{i=1}^{d}\varphi'(u_i), \tag{4.18}$$

where $\varphi^{-1[d]}$ depicts the d-derivative of the inverse generator function.

The first Archimedean copula to be introduced is the Gumbel copula (Gumbel (1958)). The d-dimensional Gumbel copula has generator and pseudo-inverse

$$\begin{aligned}\varphi(t) &= (-\ln t)^{\theta}, \qquad \theta \geq 1\\ \varphi^{-1}(t) &= \exp\left(-t^{1/\theta}\right).\end{aligned}$$

In the multivariate case it is described by

$$C^{GU}(u_1,\ldots,u_d) = \exp\left\{-\left[(-\ln u_1)^{\theta}+\cdots+(-\ln u_d)^{\theta}\right]^{\frac{1}{\theta}}\right\}, \qquad \theta \geq 1. \tag{4.19}$$

The Gumbel copula contains the Frèchet upper bound $\lim_{\theta\to\infty} C^{GU}(\theta) = M$ as defined in (4.11) and the independence copula $\lim_{\theta\to 1} C^{GU}(\theta) = \Pi$ described in (4.12) as limiting cases. It effectively models joint positive events as demonstrated by Figure 4.3. The dependence between the elliptical and the Archimedean copulas is not directly comparable. Kendall's τ is a measure that is able to map the elliptical dependence into Archimedean dependence. An Gaussian dependence of $\rho = 0.8$ implies $\theta^{GU} = 2.4410$ and $\rho = 0.2$, $\theta^{GU} = 1.4470$. Since the analytical expressions of Archimedean densities in the multivariate case can be rather complicated we present the densities only for the bivariate case. Venter (2001) derived the density of the bivariate Gumbel copula

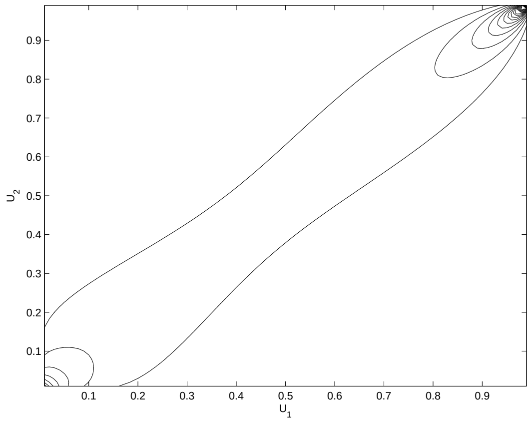
$$\begin{aligned}c(u_1,u_2) \;=\; &\exp\left[-\left((-\ln u_1)^{\theta}+(-\ln u_2)^{\theta}\right)^{1/\theta}\right]\cdot(u_1\cdot u_2)^{-1}\\ &\cdot\left((-\ln u_1)^{\theta}+(-\ln u_2)^{\theta}\right)^{-2+2/\theta}\cdot(\ln u_1\cdot\ln u_2)^{\theta-1}\\ &\cdot\left((1+\theta-1)\left((-\ln u_1)^{\theta}+(-\ln u_2)^{\theta}\right)^{-1/\theta}\right).\end{aligned}$$

The second Archimedean copula presented is the Clayton copula (Clayton (1978)). It has the generator and the pseudo-inverse

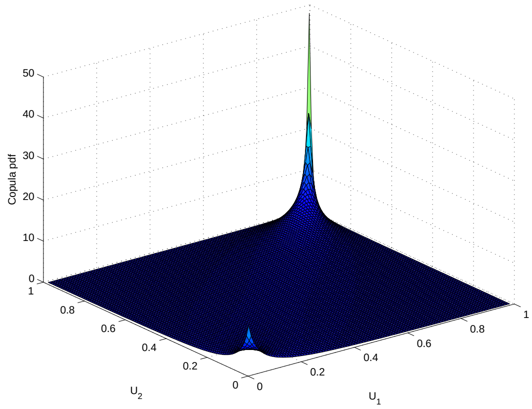
$$\begin{aligned}\varphi(t) &= \frac{1}{\theta}(t^{-\theta}-1), \qquad \theta \geq 0\\ \varphi^{-1}(t) &= (\theta t+1)^{-1/\theta}.\end{aligned}$$

The Clayton copula in the multivariate case takes the form

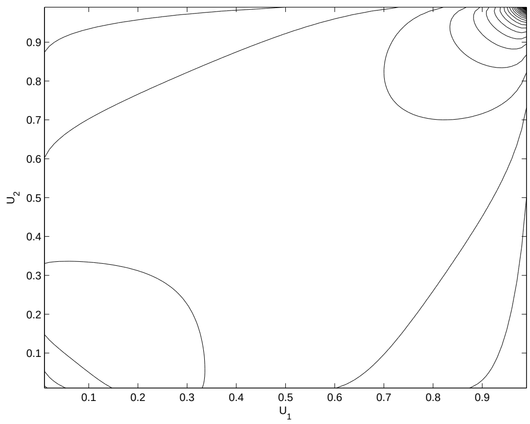
$$C^{CL}(u_1,\ldots,u_d) = \left(u_1^{-\theta}+\cdots+u_d^{-\theta}+1\right)^{-\frac{1}{\theta}}. \tag{4.20}$$



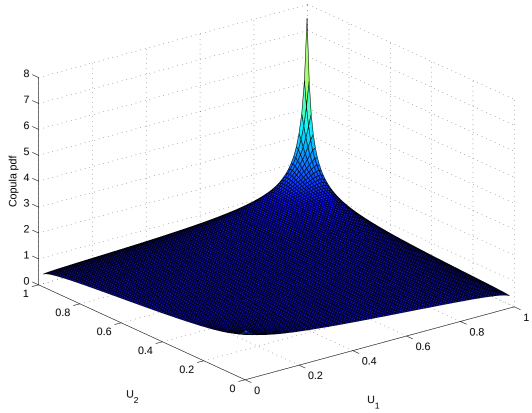
(a) Contour $\theta^{GU} = 2.4410$



(b) Density $\theta^{GU} = 2.4410$

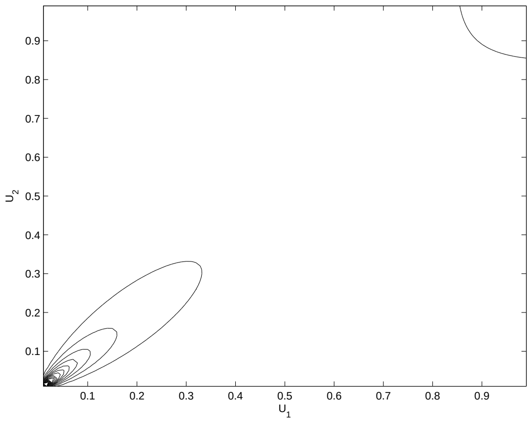


(c) Contour $\theta^{GU} = 1.1470$

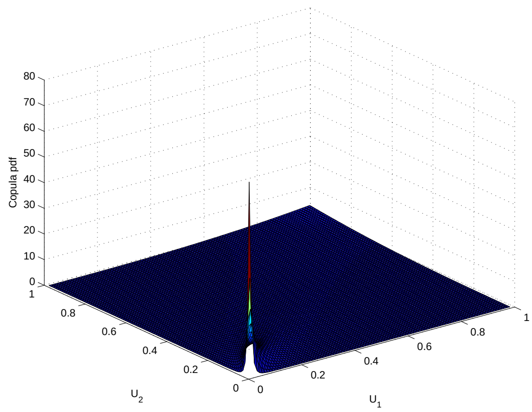


(d) Density $\theta^{GU} = 1.1470$

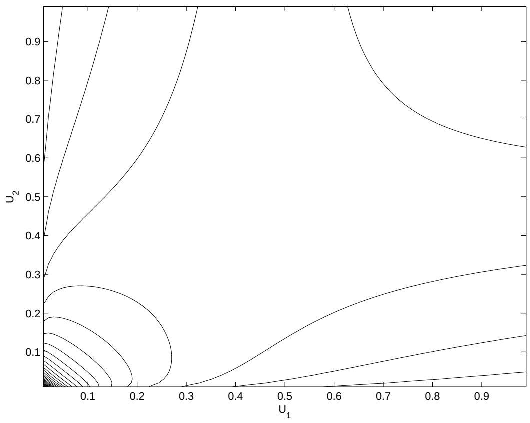
Figure 4.3: Contour and Density Gumbel Copula. This figure shows contour and density plots of the gumbel copula.



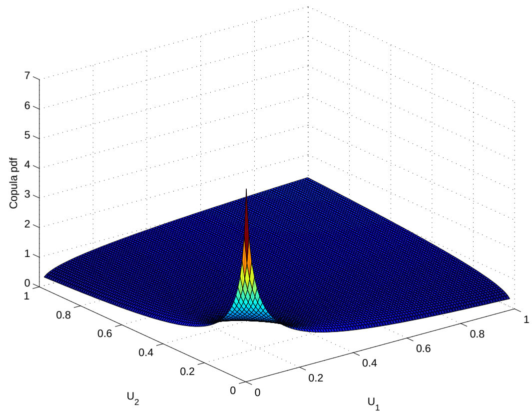
(a) Contour $\theta^{CL} = 2.8820$



(b) Density $\theta^{CL} = 2.8820$



(c) Contour $\theta^{CL} = 0.2941$



(d) Density $\theta^{CL} = 0.2941$

Figure 4.4: Contour and Density Clayton Copula. This figure shows contour and density plots of the clayton copula.

Its characteristic feature is the modelling of joint negative events, see Figure 4.4. The Clayton copula contains as borderline cases the independence copula $\lim_{\theta \to 0} C^{CL}(\theta) = \Pi$ and the comonotonic copula $\lim_{\theta \to \infty} C^{CL}(\theta) = M$. The density of a bivariate Clayton copula is

$$c(u_1, u_2) = (1+\theta) \cdot (u_1 \cdot u_2)^{-1-\theta} \cdot \left(u_1^{-\theta} + u_2^{-\theta} - 1 \right)^{-1/\theta - 2},$$

see Venter (2001).

Closely related to the Clayton copula is the rotated Clayton copula, also called survival Clayton copula. Nelsen (2006, p.32) defines a survival function $\overline{F}$ of a random variable X by

$$\overline{F}(x) = Pr[X > x] = 1 - F(x).$$

The name 'survival function' owes to the time an individual survives beyond time t. Based on the survival function a d-dimensional rotated Copula is given by

$$\overline{C} = u_1 + \dots + u_d - d + C(u_1, \dots, u_d). \tag{4.21}$$

With the definition of (4.21) the multivariate survival (or rotated) Clayton copula can be depicted by

$$C^{RCL}(u_1,\dots,u_d) = u_1 + \cdots + u_d - d + 1 + C^{CL}(1-u_1,\dots,1-u_d) \tag{4.22}$$

with associated density

$$c^{RCL}(u_1,\dots,u_d) = c^{CL}(1-u_1,\dots,1-u_d), \tag{4.23}$$

see Kaishev and Dimitrova (2006) for example. Thus, the rotated Clayton copulas is a Clayton copula rotated by 180° and therefore models events in the right tail which is similar to the Gumbel copula. The main advantage of the rotated Clayton copula compared to the Gumbel copula is its less complicated density which makes MLE much easier.

For many purposes (Monte-Carlo applications for example) it is necessary to simulate random numbers. One of the most famous Monte-Carlo applications in risk management is the estimation of the Value-at-Risk: this will be presented in detail later. In the following section we describe some simulation algorithms for the copulas introduced so far. For more details on the simulation algorithm for elliptical copulas, see McNeil, Frey, Embrechts (2005, p.193). The simulation algorithm for the Gaussian copula builds on the positive definiteness of the correlation matrix. The general rule is that if a symmetric matrix is positive-definite the Cholesky decomposition can be applied

$$\mathbf{R} = \mathbf{J}\mathbf{J}',$$

where $\mathbf{J}$ is a lower triangular matrix with positive diagonal elements. If a vector $\mathbf{W} = (W_1,\dots,W_d)'$ exists, where $W_i \overset{\text{i.i.d.}}{\sim} N(0,1)$ for $i = 1,\dots,d$ then $\mu + \mathbf{JW} \sim N_d(\mu,\mathbf{R})$. After introducing these building blocks the instruction to simulate some random number from a Gaussian copula looks like this: first determine the Cholesky decomposition of the correlation matrix $\mathbf{R}$, then generate $\mathbf{z} = (z_1,\dots,z_d)'$ where every z_i, for $i = 1,\dots,d$ is from a $N(0,1)$ distribution. Thereafter, create a new vector $\mathbf{y} = \mathbf{Jz}$. Finally apply the standard normal c.d.f. ϕ to every y_i. Then $(u_1,\dots,u_d)$ are simulated from the Gaussian-copula with correlation matrix $\mathbf{R}$.

The simulation algorithm for the t-copula follows basically the same idea. The core idea is to generate a vector $\mathbf{W} \sim T_{\mathbf{R},\nu}(\boldsymbol{\mu}, \frac{\nu}{\nu-2}\mathbf{R}, \nu)$. $\mathbf{W}$ may be obtained by

$$\mathbf{W} = \mu + \frac{\sqrt{\nu}}{\sqrt{V}}\mathbf{Z},$$

where $V \sim \chi^2_\nu$, $\mathbf{Z} \sim N_d(0,\mathbf{R})$, and $\frac{\nu}{\nu-2}\mathbf{R}$ describes the scale matrix. The sequence of steps to simulate random numbers from the t-copula equal the steps of the Gaussian copula number generation process. First, apply the Cholesky decomposition to the correlation matrix $\mathbf{R}$ then generate $\mathbf{z}$ from a standard normal. Thereafter, simulate a random number V from a χ^2_ν and set $\mathbf{y} = \mathbf{Jz}$. Finally create a new vector $\mathbf{x} = \frac{\sqrt{\nu}}{\sqrt{V}}\mathbf{y}$ and apply to every variable x_i of this new vector the c.d.f. of a Student's t. Then $(u_1,\dots,u_d)$ are simulated from a t-copula with correlation matrix $\mathbf{R}$ and d.o.f. parameter ν.

McNeil, Frey, Embrechts (2005, p.224) developed simulation algorithms for the Gumbel and Clayton copula based on the discovery of Joe (1997, chapter 4.2) that it is possible to construct Archimedean copulas using the Laplace-Stiltjes transform of distribution functions. In the following the simulation algorithm for both copulas and the necessary distribution functions will be introduced. Define a function L with $L(0) = 0$ and Laplace-Stiltjes transformation

$$\hat{L}(w) = \int_0^\infty e^{-wx} \, \mathrm{d}G(x), \qquad t \leq 0.$$

Assume a random variable V with distribution function L and a sequence of random variables $U_1, \dots, U_d$ with conditional distribution function $F_{U_i|V}(u|v) = \exp\left\{-v\hat{L}^{-1}(u)\right\}$. Then an Archimedean copula with generator $\varphi = \hat{L}^{-1}$ has the form

$$Pr[U_1 \geq u_1, \dots, U_d \geq u_d] = \hat{L}\left(\hat{L}^{-1}(u_1) + \cdots + \hat{L}^{-1}(u_d)\right).$$

To simulate random numbers from an Archimedean copula first it is necessary to simulate some i.i.d. $Uniform(0,1)$ variables $X_1, \dots, X_d$. Following this only one more step is needed. One just has to calculate

$$\mathbf{U} = \left(\hat{L}(-\ln(X_1)/V, \dots, \hat{L}(-\ln(X_d)/V\right)',$$

where V depends on the copula to be simulated. For the Gumbel copula

$$V \sim St\left(\frac{1}{\theta}, 1, \gamma, 0\right), \qquad \text{where } \gamma = \left(\cos\left(\frac{\pi}{2\theta}\right)\right)^\theta,$$

St describes the α-stable distribution with characteristic function

$$\phi(t) = E\left[\exp(itX)\right] = \begin{cases} \exp\left(-\gamma^\alpha |t|^\alpha \left(1 - i\beta\,\mathrm{sign}(t)\tan\left(\frac{\pi}{2}\right)\alpha\right) + i\delta t\right), & \alpha \neq 1, \\ \exp\left(-\gamma|t|\left(1 - i\beta\,\mathrm{sign}(t)\left(\frac{2}{\pi}\right)\ln|t|\right) + i\delta t\right), & \alpha = 1 \end{cases}$$

where $\alpha \in (0,2]$, $\beta \in [-1,1]$, $\gamma > 0$, and $\delta \in \mathbb{R}$. A simulation algorithm for a variable from an α-stable distribution is provided by Nolan (2010). For the Clayton copula V is simulated from the gamma distribution

$$V \sim \Gamma\left(\frac{1}{\theta}, 1\right), \quad \theta > 0.$$

A random variable Y has a gamma distribution, written $Y \sim \Gamma(\alpha, \beta)$ if its density is described by

$$f(y) = \frac{\beta^\alpha}{\Gamma(\alpha)} y^{\alpha-1} \exp(-\beta y), \quad y > 0, \; \alpha > 0, \; \beta > 0,$$

where Γ represents the usual gamma function

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} \, \mathrm{d}x, \quad \alpha > 0.$$

A drawback of the t-copula and the Archimedean copulas is their modelling of tail-dependence through a single parameter. The t-copula estimates a whole correlation matrix but tail dependence is related to the degrees of freedom (d.o.f.) and the multivariate t-copula shares just one d.o.f. parameter. Although both the Gumbel and the Clayton copula models have opposite dependence they just have one dependence parameter. The higher the dimension of the portfolio the lower the likelihood that all assets share the same degree of dependence. Now the question arises as to whether it is possible to keep the characteristics of the respective copula whilst making them a better fit to portfolios with more than two dimensions.

4.2.3 Vine Copulas

Bedford and Cooke (2001,2002) developed the vine copula by building on the findings of Joe (1996) regarding conditional functions. Aas et al (2009) introduced the concept of vine copula to finance. The fundamental idea of a vine copula is to decompose a multivariate density function and thus a multivariate copula density into univariate margins and several bivariate copulas. These bivariate copulas are often called pair-copulas. With the concept of vine copulas it is now possible to build a multivariate copula model based on the well-known characteristics of bivariate copulas. Furthermore, as the name suggests, a pair copula only has to estimate the dependence between two variables: this is beneficial for a model with just one dependence parameter. To fully understand the concept of vine-copulas it is necessary to introduce the pair-copula decomposition of a multivariate density. Following Aas et al (2009) we will explain the concept of vine copulas in detail. Consider some random numbers $X_1, \dots, X_d$ with density function $f(x_1, \dots, x_d)$. This density function may factorized into a series of univariate (conditional) densities by

$$f(x_1, \dots, x_d) = f(x_d) \cdot f(x_{d-1}|x_d) \cdot f(x_{d-2}|x_{d-1}, x_d) \cdots f(x_1|x_2, \dots, x_d). \tag{4.24}$$

According to (4.9) a multivariate joint density is constructed using a multivariate copula density via

$$f(x_1, \dots, x_d) = c_{12\cdots d}\left(F_1(x_1), \dots, F_d(x_d)\right) \cdot f_1(x_1) \cdots f_d(x_d), \tag{4.25}$$

where $c_{12\cdots d}$ depicts a d-dimensional copula density. A pair copula density then may be written

$$f(x_1, x_2) = c_{12}(F_1(x_1), F_2(x_2)) \cdot f_1(x_1) \cdot f_2(x_2).$$

This result is fairly intuitive since- by definition- the copula is the 'extracted' dependence structure of a multivariate distribution. The conditional function is elementary for vine copulas. If the variable x_1 is conditioned on the variable x_2 the conditional density function in the view of pair copulas takes the form

$$f(x_1|x_2) = c_{12}(F_1(x_1), F_2(x_2)) \cdot f_1(x_1). \tag{4.26}$$

Since we will deal with more than two dimensions, we will describe one more vine copula in detail to clarify the concept of the conditional density. Consider a conditional density where

x_1 is conditioned on x_2 and x_3

$$\begin{aligned} f(x_1|x_2,x_3) &= \frac{f(x_1,x_2|x_3)}{f(x_2|x_3)} \\ &= \frac{c_{12|3}\left(F_{1|3}(x_1|x_3),F_{2|3}(x_2|x_3)\right)\cdot f(x_1|x_3)\cdot f(x_2|x_3)}{f(x_2|x_3)} \\ &= c_{12|3}\left(F_{1|3}(x_1|x_3),F_{2|3}(x_2|x_3)\right)\cdot f(x_1|x_3). \end{aligned} \tag{4.27}$$

With these examples it can be shown that a d-dimensional conditional density $f(x_{d-1}|x_d)$ may be decomposed into a pair copulas of the form $c_{(d-1)d}(F_{d-1}(x_{d-1}),F_d(x_d))$ and the marginal densities $f_{d-1}(x_{d-1})$. Now it becomes clear that the last term in (4.27) can be further factorized into $f(x_1|x_3) = c_{13}(F_1(x_1),F_3(x_3))\cdot f_1(x_1)$. With these prerequisites the three dimensional vine copula of (4.27) takes the form

$$f(x_1|x_2,x_3) = c_{12|3}(F_{1|3}(x_1,x_3),F_{2|3}(x_2|x_3))\cdot c_{13}(F_1(x_1),F_3(x_3))\cdot f_1(x_1).$$

This decomposition is not the only possible, however. Another factorization is

$$\begin{aligned} f(x_1|x_2,x_3) &= \frac{f(x_1,x_3|x_2)}{f(x_3|x_2)} \\ &= \frac{c_{13|2}\left(F_{1|2}(x_1|x_2),F_{3|2}(x_3|x_2)\right)\cdot f(x_1|x_2)\cdot f(x_3|x_2)}{f(x_3|x_2)} \\ &= c_{13|2}\left(F_{1|2}(x_1|x_2),F_{3|2}(x_3|x_2)\right)\cdot f(x_1|x_2). \end{aligned}$$

Taking into account (4.24) and(4.9) a threedimensional multivariate joint density can be first decomposed into

$$\begin{aligned} f(x_1,x_2,x_3) &= f_3(x_3)\cdot f_{2|3}(x_2|x_3)\cdot f_{1|23}(x_1|x_2,x_3) \\ &= f_3(x_3)\cdot c_{23}(F_2(x_2),F_3(x_3))\cdot f_2(x_2)\cdot c_{13|2}(F_{1|2}(x_1|x_2),F_{3|2}(x_3|x_2)) \\ &\quad \cdot f_{1|2}(x_1|x_2), \end{aligned}$$

and finally into:

$$\begin{aligned} f(x_1,x_2,x_3) = \; & f_1(x_1)\cdot f_2(x_2)\cdot f_3(x_3)\cdot c_{12}(F_1(x_1),F_1(x_2))\cdot c_{23}(F_2(x_2),F_3(x_3)) \\ & \cdot c_{13|2}(F_{1|2}(x_1|x_2),F_{3|2}(x_3|x_2)). \end{aligned} \tag{4.28}$$

These examples show that each component of equation (4.28) may be factorized into a pair-copula and an appropriate conditional margin via

$$f(s|\mathbf{v}) = c_{s\mathbf{v}_{-j}}(F(s|\mathbf{v}_{-j}),F(\mathbf{v}_j|\mathbf{v}_{-j}))\cdot f(s|\mathbf{v}_{-j}),$$

where $\mathbf{v}$ is a d-dimensional vector with $j = 1,\ldots,d$ elements. The notation $\mathbf{v}_{-j}$ denotes the vector $\mathbf{v}$ without element j. With the theory presented above it is possible to factorize a d-dimensional multivariate density into a sequence of pair copulas. To complete the theory of vine copulas a thorough definition of conditional margins must be given. The conditional margins $F(s|\mathbf{v})$ developed by Joe (1996) are determined via

$$F(s|\mathbf{v}) = \frac{\partial C_{sv_j|\mathbf{v}_{-j}}\left(F(s|\mathbf{v}_{-j}), F(v_j|\mathbf{v}_{-j})\right)}{\partial F(v_j|v_{-j})}, \tag{4.29}$$

where $C_{ij|k}$ denotes a bivariate conditional copula. In the univariate case the vector $\mathbf{v}$ reduces to the variable v and (4.29) may be re-written as

$$F(s|v) = \frac{\partial C_{s|v}\left(F(s), F(v)\right)}{\partial F(v)}.$$

The following notation was introduced to simplify the delineation of a conditional distribution given s and v are uniform distributed, i.e. $f(s) = f(v) = 1$, $F(s) = s$, and $F(v) = v$. Aas et al (2009) defined the so called h-function as

$$h(s, v; \theta) = F(s|v) = \frac{\partial C_{s,v}(s, v; \theta)}{\partial v}, \tag{4.30}$$

where θ denotes the parameter vector of the copula. The function $h^{-1}(u, v; \theta)$ is the inverse function of h with respect to the variable u. The example of the 3-dimensional pair-copula density may now be continued, taking the definition of the h-function into account. It can be clearly seen that $F(x_1|x_2) = h(x_1, x_2; \theta_{12})$ where θ_{12} describes the parameter of the first pair-copula. Therefore, $F(x_1|x_2, x_3)$ can be reduced to

$$\begin{aligned}\frac{\partial C_{13|2}(F(x_1|x_2), F(x_3|x_2))}{\partial F(x_3|x_2)} &= h(F(x_1|x_2), F(x_3|x_2); \theta_{13|2}) \\ &= h(h(x_1, x_2; \theta_{12}), h(x_3, x_2; \theta_{32}); \theta_{13|2}).\end{aligned}$$

Based on the results of the factorization of a 3-dimensional density a 3-dimensional copula density now may be decomposed into

$$c(u_1, u_2, u_3) = c_{12}(u_1, u_2) \cdot c_{23}(u_2, u_3) \cdot (c_{13|2}(h(u_1, u_2; \theta_{12}), h(u_3, u_2; \theta_{23})).$$

In the following we will introduce the derivation of the Clayton h-function as an example. The Clayton copula is defined in (4.20) and with partial differentiation the $h-$ function is given by

$$h(u_1, u_2; \theta_{12}) = \frac{\partial}{\partial u_2} C^{CL}(u_1, u_2; \theta_{12}) = u_2^{-\theta_{12}-1}\left(u_1^{-\theta_{12}} + u_2^{-\theta_{12}} - 1\right)^{-1-1/\theta_{12}},$$

where θ_{12} in this case depicts the dependence parameter of the bivariate Clayton copula. The inverse function h^{-1} of the Clayton copula takes the form

$$h^{-1}(u_1, u_2; \theta_{12}) = \left[\left(u_1 \cdot u_2^{\theta_{12}+1}\right)^{-\frac{\theta_{12}}{\theta_{12}+1}} + 1 - u_2^{-\theta_{12}}\right]^{-1/\theta_{12}}.$$

Derivation of h-functions and the associated inverse h-functions for the Gaussian, Student-t and Gumbel copula are found in Aas et al (2009). As noted above the decomposition of a multivariate density function is not univocal. Aas et al (2009) showed that it is possible to construct 240 different pair-copula decompositions for a 5-dimensional distribution function. Therefore, they focus on two special cases, namely the canonical vine and the D-vine. In the following analysis we concentrate on the D-vine copula as the canonical vine requires a lead variable, i.e. a variable that captures some common behavior of all variables. The D-vine copula was originally introduced by Kurowicka and Cooke (2004) and consists of several levels. In the first level bivariate copulas are applied to the original data. Thereafter, the data is generated by the conditional distribution functions introduced in (4.29). In Figure 4.5, the

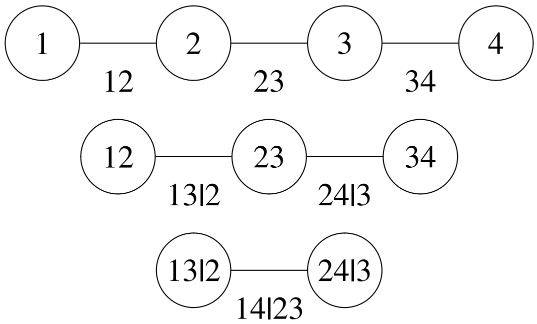


Figure 4.5: D-Vine Copula. This figure shows a 4-dimensional D-vine copula. The variables are denoted by circles and the connection between the circles represent the pair-copulas.

sequence of a 4-dimensional D-vine copula is shown. The circles denote the data, e.g. 1 describes the first vector of a financial time series. The numbers under the cross-connections between the circles connote the respective pair-copula. The data in the first level is ordered by degree of dependence. Therefore, bivariate copulas are estimated for all possible bivariate compositions. The pair with the highest dependence will be in first place. Now the rank order of the first two variables has to be determined: this will depend on the dependence of the first two variables with all other variables. Again the data might be re-ordered according to the highest dependence as in Figure 4.5 where the highest dependence was found between the variables 1 and 2 and the second highest dependence between the variables 2 and 3 determining the position of variable 4. In the second level the first conditional copulas ($C_{13|2}$ and $C_{24|3}$) are estimated with the conditional marginals ($F_{1|2}$, $F_{2|3}$, and $F_{3|4}$). Based on the second level conditional margins, $F_{13|2}$ and $F_{24|3}$ are created and estimated by the conditional copula $C_{14|23}$. An algorithm to simulate (conditional) random numbers from a D-vine copula was introduced by Aas et al (2009). First, independent uniform random numbers are generated

$w_i \stackrel{\text{i.i.d.}}{\sim} Uniform(0,1)$, for $i = 1\ldots,d$. Thereafter,

$$x_1 = w_1 \tag{4.31}$$
$$x_2 = F_{2|1}^{-1} = (w_2|x_1) \tag{4.32}$$
$$\vdots = \vdots \tag{4.33}$$
$$x_d = F_{d|1,2,\ldots,d-1}^{-1}(w_d|x_1,\ldots,x_{d-1}), \tag{4.34}$$

where x are the random numbers simulated from a D-vine and

$$F(x_j|x_1,\ldots,x_{d-1}) = \frac{\partial C_{j,1|2\ldots,j-1}\left(F(x_j|x_2,\ldots,x_{j-1}),F(x_1|x_2,\ldots,x_{j-1})\right)}{\partial F(x_1|x_1,\ldots,x_{j-1})}. \tag{4.35}$$

4.2.4 Mixture Copulas

Each of the copulas introduced so far has both advantages and disadvantages. The Archimedean copulas are restricted to show either dependence of negative or positive joint events. It may be that the more dimensions the data set has, the more unlikely it is that all variables share the same dependence: a prerequisite for a good fit of copulas with just one dependence parameter. The elliptical copula model meanwhile models dependence through a greater number of parameters, but is limited to symmetric dependence. To combine the advantageous features of both a new class of copulas has been introduced. These are known as 'mixture copulas'. The mixture allows the use of two or more copulas to describe the dependence structure of a data set. Clearly, the mixing of different copulas can generate a wide range of dependence structures. In this analysis we restrict ourself to bivariate mixture copulas. A bivariate mixture copula may be written

$$\begin{aligned} C^{MIX}(u_1,\ldots,u_d;\theta_1,\theta_2,w_1,w_2) &= w_1 \cdot C^{MIX_1}(u_1,\ldots,u_d;\theta_1) \\ &\quad + w_2 \cdot C^{MIX_2}(u_1,\ldots,u_d;\theta_2), \end{aligned} \tag{4.36}$$

where $w_2 = 1 - w_1$, C^{MIX} represents the mixture copula, C^{MIX_1} the first copula in the mixture copula and C^{MIX_2} the second. The mixture is estimated in this analysis the same way as the other copulas - by the two-step MLE. Following Hu (2006) we always mix one elliptical copula with one of the Archimedean copulas. This has several advantages. As noted above, the elliptical copulas model dependence on a bivariate basis while even multivariate Archimedean copulas describe dependence only through one parameter. Thus, mixing an elliptical with an Archimedean copula preserves the bivariate dependence analysis of elliptical copulas but overcomes the drawback of symmetric dependence. Furthermore, the elliptical copulas (particularly the Gaussian) remain in frequent use in finance. Thus, it might be interesting to investigate the behavior of the weight of these copulas when they are mixed with the Archimedean copulas. If the elliptical copulas are a bad choice for the data sets used here they should have only little weight in the mixing copula. The upper part of Figure 4.6 shows scatter-plots of a t-copula with $\rho = 0.8$, a clayton copula with dependence parameter $\theta^{CL} = 2.881$, a mixture copula where the t-copula has a weight of $w_1 = 0.7$, and the clayton

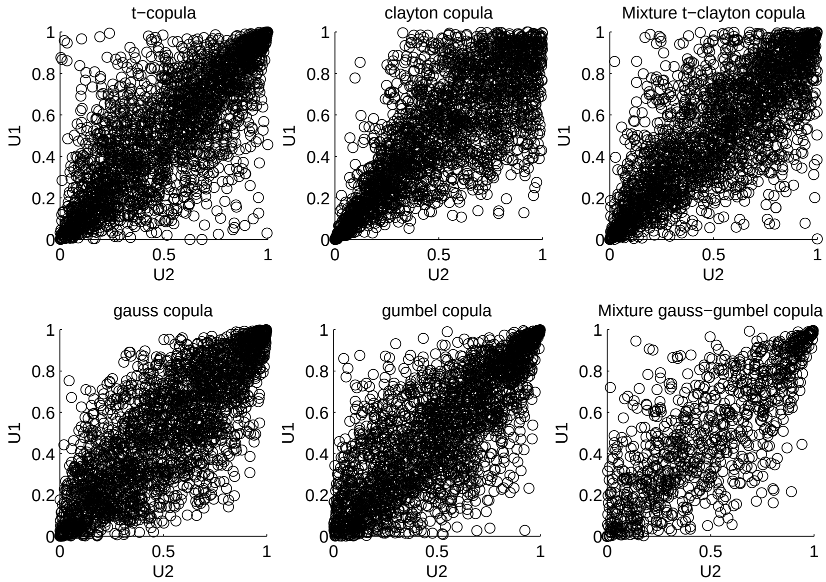


Figure 4.6: Scatter Plots Mixture Copulas. This figure shows scatter plots of the t, Clayton, t-Clayton mixture, Gaussian, Gumbel, and Gaussian-Gumbel Mixture Copula

copula of $w_2 = 1 - w_1 = 0.3$. The mixture copula has more mass in the left tail as the t-copula and more mass in the right tail as the Clayton copula. In the lower part of the figure a Gaussian copula is mixed with a Gumbel copula. Again, the linear correlation coefficient is $\rho = 0.8$. The dependence parameter of the Gumbel copula θ^{GU} equals 2.441. To make a comparison possible the weight of the Gaussian copula equals 0.7 and the weight of the Gumbel copula 0.3. It can clearly be seen that the Gaussian-Gumbel mixture has more mass in the right tail than the mixture copula and more mass in the left tail than the gumbel copula. To summarize, the mixture approach is able to generate a wide range of dependence structures. the greater the number of dimensions the lower the likelihood that the dependence structure can be described by a single parameter.

4.3 Copula-related Dependence Measures

4.3.1 Kendall's tau

Embrechts, McNeil, and Straumann (1999) discuss several pitfalls using Pearson's correlation coefficient. If the distribution of the variables does not belong to the elliptical family it is no longer true that the marginal distributions and their pairwise correlation determine their joint distribution. Furthermore it is not anymore always possible to construct a joint distribution F with given marginal distributions F_1 and F_2 and correlation coefficient ρ because in the elliptical case correlation always depends on the marginal distributions.

Kendall's tau is a dependence measure that does not depend on the marginal distribution. It is based on the rank of the variable and might be interpreted as a concordance measure for dependence vectors. We will only present the results for the bivariate case because we use Kendall's tau only for the time-varying D-vine copulas which are build on bivariate pair-copulas. Nelsen (2006, p.158) defines Kendall's tau as

$$\tau = Pr[(X_1 - X_2) - (Y_1 - Y_2)] > 0] - Pr[(X_1 - X_2)(Y_1 - Y_2) < 0],$$

where $(X_1, Y_1)'$ and $(X_2, Y_2)'$ are two vectors of random variables. Thus, Kendall's tau resembles the difference of the probability of concordant and discordant pairs. A pair of random variables (X_1, Y_1) and (X_2, Y_2) is said to be concordant if $(X_1 - X_2)(Y_1 - Y_2) > 0$. In contrast, a pair of random variables (X_1, Y_1) is said to be discordant if $(X_1 - X_2)(Y_1 - Y_2) < 0$. If all pairs of variables are concordant (discordant) (X_1, Y_1) and (X_2, Y_2) are identified as comonotone (anti-comonotone). Kendall's tau lies always in the interval $[-1, 1]$and takes the value 0 if the variables are independent. But as Embrechts, McNeil, and Straumann (1999) remarks a dependence parameter of 0 does not necessarily mean independence. Schweizer and Wolff (1981) showed that Kendall's tau is related to every copula via

$$\begin{aligned} \tau(X_1, X_2) &= 4E[C(F_1(X_1), F_2(X_2))] - 1 \\ &= 4\int_0^1 \int_0^1 C(u_1, u_2)\, \mathrm{d}C(u1, u2) - 1 \\ &= 1 - 4\int_0^1 \int_0^1 \frac{\partial C(u_1, u_2)}{\partial u_1} \frac{\partial C(u_1, u_2)}{\partial u_2}\, \mathrm{d}u_1\, \mathrm{d}u_2. \end{aligned}$$

Like copulas Kendall's tau keeps its properties under monotone increasing transformations of the random variables. Lindskog, McNeil, and Schmock (2003) showed that Kendall's tau for elliptical copulas is given by

$$\tau^{GA} = \tau^{T} = \frac{2}{\pi} \arcsin \rho, \tag{4.37}$$

where ρ denotes the bivariate correlation coefficient. Genest and MacKay (1986a) and Genest and Rivest (1986) derived for the Archimedean case Kendall's tau via the generator function $\varphi(w)$

$$\tau = 1 + 4\int_0^1 \frac{\varphi(w)}{\varphi'(w)}\, \mathrm{d}w.$$

If one takes into account that $\frac{\varphi(w)}{\varphi'(w)} = \frac{1}{\theta} w \ln w$ then for the Gumbel copula Kendall's tau is calculated as

$$\begin{aligned} \tau^{GU} &= 1 + 4\int_0^1 \frac{1}{\theta} w \ln w \, \mathrm{d}w \\ &= 1 + f\frac{1}{\theta}\left(\left[\frac{1}{2}w^2 \ln w\right]_0^1 - \int_0^1 \frac{1}{2} w \, \mathrm{d}w\right) \\ &= 1 + 4\frac{1}{\theta}\left(0 - \frac{1}{4}\right) \\ &= 1 - \frac{1}{\theta}. \end{aligned} \tag{4.38}$$

Kendall's tau for the Clayton copula is derived by

$$\begin{aligned}
\tau^{CL} &= 1+4\int_0^1 \frac{1}{\theta}\left(w^{\theta+1}-w\right)\mathrm{d}w \\
&= 1+4\frac{1}{\theta}\left[\frac{1}{\theta+2}w^{\theta+2}-\frac{1}{2}w^2\right]_0^1 \\
&= 1+4\frac{1}{\theta}\left(\frac{1}{\theta+2}-\frac{1}{2}\right). \\
&= \frac{\theta}{\theta+2}. \qquad (4.39)
\end{aligned}$$

Nelsen (2006) delivers a thorough introduction and many proofs for dependence measures related to copulas.

4.3.2 Tail Dependence

A measure of the dependence of extreme values is the tail dependence. Here some general results concerning tail dependence are presented. Almost all theoretical derivations of tail dependence are on a bivariate level. Therefore, we will present only some general results and the tail dependence for the Gaussian and t-copula in detail. First of all, it should be noted that there exist two different measures of tail dependence. Joe (1997) defined strong tail dependence which explained below. Coles et al (1999) defined a measure that is often called tail dependence, too. Related to the strong tail dependence of Joe it is sometimes known as 'weak tail dependence' and describes the speed the strong tail dependence converges to zero.

The upper asymptotic tail dependence coefficient is defined by Joe (1997, p.33) as

$$\lambda_U = \lim_{u\to 1} Pr\left[X_2 > F_2^{-1}(u)|X_1 > F_1^{-1}(u)\right]$$

assuming $\lambda_U \in [0,1]$ exists. The lower asymptotic tail dependence coefficient may be written as

$$\lambda_L = \lim_{u\to 0} Pr\left[X_2 < F_2^{-1}(u)|X_1 \leq F_1^{-1}(u)\right]$$

assuming $\lambda_L \in [0,1]$ exists. Thus, the tail dependence shows how probable it is that an extreme event of one variable occurs conditional on an extreme event of another variable. Nelsen (2006, pp.214, 215) proved that if the margins of the random variable are continuous, the upper tail dependence is connected with the copula via

$$\begin{aligned}
\lambda_U &= \lim_{u\to 1} Pr\left[X_1 > F_1^{-1}(u), X_2 > F_2^{-1}(u)\right] \\
&= \lim_{u\to 1} \frac{Pr\left[X_1 > F_1^{-1}(u), X_2 > F_2^{-1}(u)\right]}{Pr\left[X_2 > F_2^{-1}(u)\right]} \\
&= \lim_{u\to 1} \frac{1-2u+C(u,u)}{1-u} \qquad (4.40)
\end{aligned}$$

and the lower tail dependence by

$$\begin{aligned}
\lambda_L &= \lim_{u \to 0} Pr\left[X_1 \leq F_1^{-1}(u), X_2 \leq F_2^{-1}(u)\right] \\
&= \lim_{u \to 0} \frac{Pr\left[X_1 \leq F_1^{-1}(u), X_2 \leq F_2^{-1}(u)\right]}{Pr\left[X_2 \leq F_2^{-1}(u)\right]} \\
&= \lim_{u \to 0} \frac{C(u,u)}{u}. \qquad (4.41)
\end{aligned}$$

The tail dependence parameters for the Gaussian copula have been derived by Embrechts, Lindskog, and McNeil (2003). Following equation (4.40), the upper tail dependence parameter of the Gaussian copula is given by

$$\begin{aligned}
\lambda_U^{GA} &= \lim_{u_1 \to 1} \frac{1 - 2u_1 + C^{GA}(u_1,u_1)}{1-u_1} = \lim_{u_1 \to 1} \frac{\overline{C}^{GA}(u_1,u_1)}{1-u_1} = \lim_{u_1 \to 1} \frac{d\overline{C}^{GA}(u_1,u_1)}{du_1} \\
&= -\lim_{u_1 \to 1} \left(-2 + \frac{\delta}{\delta s} C^{GA}(s,t)|_{s=t=u_1} + \frac{\delta}{\delta t} C^{GA}(s,t)|_{s=t=u_1}\right) \\
&= \lim_{u_1 \to 1} \left(Pr[U_2 > u_1 | U_1 = u_1] + Pr[U_1 > u_1 | U_2 = u_1]\right),
\end{aligned}$$

where $\overline{C}^{GA}$ denotes the Gaussian survival copula. The Gaussian copula belongs to the class of exchangeable copulas , e.g. $C(u_1,u_2) = C(u_2,u_1)$. This simplifies the derivation of the upper tail dependence parameter to

$$\lambda_U^{GA} = 2 \lim_{u_1 \to} Pr[U_2 > u_1 | U_1 = u_1].$$

The lower tail dependence parameter may be calculated the same way

$$\begin{aligned}
\lambda_L^{GA} &= \lim_{u_1 \to 0} \frac{C(u_1,u_1)}{u_1} = \lim_{u_1 \to 0} \frac{d\overline{C}(u_1,u_1)}{du_1} \\
&= 2 \lim_{u_1 \to 0} Pr[U_2 < u_1 | U_1 = u_1].
\end{aligned}$$

Finally, the upper tail dependence of the Gaussian copula is given by

$$\begin{aligned}
\lambda_U^{GA} &= 2 \lim_{x_1 \to \infty} Pr\left[\phi_2^{-1}(U_2) > u_1 | \phi_1^{-1}(U_1) = u_1\right] = 2 \lim_{x_1 \to \infty} Pr\left[X_2 > x_1 | X_1 = x_1\right] \\
&= 2 \lim_{x_1 \to \infty} \overline{\phi}\left(\frac{x_1 - \rho x_1}{\sqrt{1-\rho^2}}\right) \\
&= 2 \lim_{x_1 \to \infty} \overline{\phi}\left(\frac{x_1\sqrt{1-\rho}}{\sqrt{1+\rho}}\right),
\end{aligned}$$

where $\overline{\phi}(\cdot) = 1 - \phi(\cdot)$ denotes the survival function of the standard normal distribution. As a result of the symmetric properties: $\lambda_U^{GA} = \lambda_L^{GA}$. Furthermore, as long as $\rho < 1$ upper and lower tail dependence equal zero ($\lambda_U^{GA} = \lambda_L^{GA} = 0$). Thus, events in the tails of the Gaussian copula appear independent of each other and are not related to the correlation coefficient. In contrast to the Gaussian copula, the t-copula shows some tail dependence. Due to Demarta and McNeil (2005), the upper tail dependence of the t-copula is given by

$$\begin{aligned}\lambda_U^T &= 2\lim_{x_1\to\infty} Pr\left[t_\nu^{-1}(U_2) > u_1 | t_\nu^{-1}(U_1) = u_1\right] = 2\lim_{x_1\to\infty} Pr\left[X_2 > x_1 | X_1 = x_1\right] \\ &= 2\lim_{x_1\to\infty} t_{\nu+1}\left(-\left(\frac{\nu+1}{\nu+x_1^2}\right)^{1/2}\cdot\frac{x_1-\rho x_1}{\sqrt{1-\rho^2}}\right) \\ &= 2\, t_{\nu+1}\left(-\frac{\sqrt{1-\rho}\sqrt{\nu+1}}{\sqrt{1+\rho}}\right),\end{aligned}$$

where $X_2|X_1 \sim t_{\nu+1}\left(\rho x_1, \left(\frac{\nu+x_1^1}{\nu+1}\right)\left(1-\rho^2\right)\right)$. Thus, the tail dependence parameter of the t-copula depends not only on the correlation coefficient but also on the degree of freedom parameter ν. The tail dependence decreases as the degree-of-freedom parameter increases and increases as the correlation parameter increases. Like the Gaussian copula the t-copula is also symmetric, i.e. $\lambda_U^T = \lambda_L^T$.

For bivariate tail dependence of Archimedean copulas see e.g. Savu and Trede (2004) and Nelsen (2006).

4.4 Dynamic Copulas

4.4.1 Theory

The copulas introduced above do not capture any comovements over time between several variables. This means that every point in time is given the same weight when estimating the dependence parameter. Extensive research in the multivariate GARCH area (see chapter 3) has shown the time-varying dependence behavior of multivariate financial time series. The multivariate GARCH models discussed above are able to model time-varying correlations but are rooted in the Gaussian world for the marginals and the dependence structure. The advantage of the copula approach is the separate modelling of the marginal distributions and the multivariate dependence with neither limited to the Gaussian world anymore. Patton (2006) provided the necessary theory to take the static copula to a time-varying one. To give some theoretical background we will begin in spirit of Patton (2006) by developing the general theory of a multivariate distribution function conditioned on a variable.

Assume a conditioning variable W which is of dimension one. Then the conditional bivariate distribution may be written

$$F(x_1,x_2|w) = f(w)^{-1}\cdot\frac{\partial F(x_1,x_2,w)}{\partial w}, \quad w\in\mathscr{W},$$

where $f(w)$ denotes the unconditional density of W, $\mathscr{W}$ is the support of W, and $F(x_1,x_2)$ the bivariate distribution function of x_1 and x_2. As the copula is a distribution function it is possible to apply the theory developed above to it

$$F(x_1,x_2|w) = C(F_1(x_1|w),F_2(x_2|w)|w), \quad \forall(x_1,x_2)\in\mathbb{R}^2 \tag{4.42}$$

If the conditioning variable is assumed to be a sigma algebra $\mathscr{F}_{t-1}$

$$\mathscr{F}_{t-1} = \sigma\{x_{1,t-1},\ldots,x_{d,t-1},\ldots,x_{1,t-T+1},\ldots,x_{d,t-T+1}\},$$

Sklar's theorem may be extended to

$$F_t\left(x_{1,t},\ldots,x_{d,t}|\mathscr{F}_{t-1}\right) = C_t\left(F_{1,t}(x_{1,t}|\mathscr{F}_{t-1}),\ldots,F_{d,t}(x_{d,t}|\mathscr{F}_{t-1})|\mathscr{F}_{t-1}\right),$$

where C_t denotes the conditional Copula of $(X_1,\ldots,X_d)$ conditional on the information set $\mathscr{F}_{t-1}$ and the conditional distribution function F_t. The random variable X (conditional on the information set $\mathscr{F}_{t-1}$) is distributed as $X_i|\mathscr{F}_{t-1} \sim F_{i,t}$. It is important to note that the conditioning variable has to be the same for the marginal models and the copula, see Fermanian (2005) and Fermanian and Wegkamp (2012). As described above the marginal distributions are estimated as AR-GARCH models. This implies that each random variable is conditioned only on its own past values and not on the past values of other variables. One can still estimate the conditional copula, but subsets of the sigma algebra have to be created. Thus, $\mathscr{F}_{i,t-1}$ must be defined as the smallest subset of $\mathscr{F}_{t-1}$ so that $X_{i,t}|\mathscr{F}_{i,t-1} \overset{\mathrm{D}}{=} X_{i,t}|\mathscr{F}_{t-1}$. Now each marginal distribution is conditioned on their own past values $\mathscr{F}_{i,t-1}$ but the copula is conditioned on the past values of all variables $\mathscr{F}_{t-1}$. The density of the conditional copula is derived in the same manner as in (4.6) and (4.7)

$$\begin{aligned} f(x_1,\ldots,x_d|w) &= \frac{\partial^d F(x_1,\ldots,x_d|w)}{\partial x_1\cdots\partial x_d} \\ &= \frac{\partial^d C(F_1(x_1|w_1),\ldots,F_d(x_d|w_d)|w)}{\partial x_1\cdots\partial x_d} \\ &= c(u_1,\ldots,u_d|w)\cdot\prod_i^d f_i(x_i|w_i), \end{aligned} \tag{4.43}$$

where W_i is a subset of W.

Having engaged with the necessary theory we will now introduce a range of dynamic copulas.

4.4.2 Multivariate Dynamic Elliptical Copulas

As the primary innovator of the dynamic copula, the dependence structure of Patton (2006) will be considered first. For the bivariate Gaussian copula his dynamic structure takes the form

$$\rho_t = \Lambda\left(\omega+\beta\cdot\rho_{t-1}+\alpha\cdot\frac{1}{10}\sum_{j=1}^{10}\Phi^{-1}(u_{1,t-j})\cdot\Phi^{-1}(u_{2,t-j})\right), \tag{4.44}$$

where $\Lambda(x)$ is the modified logistic transformation $\Lambda(x) = \frac{(1-e^{-x})}{(1+e^{-x})}$. The transformation is necessary to ensure that $-1 \leq \rho_t \leq 1$ which are the bounds of the Gaussian dependence parameter.

It can clearly be seen that the dynamic structure is similar to an ARMA(1,10) process[2]. The lagged dependence parameter is imposed to capture any persistence in the dependence. The final term depicts the forcing variable- an average over the last ten observations to incorporate any short-term variation in dependence.

With the dependence structure of the MVGARCH models multivariate elliptical copulas are easily enhanced to multivariate dynamic elliptical copulas. Jin (2009) was among the first to apply Engle's DCC-approach to the Gaussian and the t-copula: a fairly simple extension of this is to apply the (diagonal) parameter matrix approach of Cappiello, Engle, and Sheppard (2006) to both elliptical copulas. For this extension, all that has to be done is the re-transformation of the the variables $\mathbf{u}_t = u_{1,t}, \ldots, u_{d,t}$ into standardized residuals $\boldsymbol{\eta_t}$ and plugging them into (3.12). The transformation of the $Uniform(0,1)$ variables is achieved by $\boldsymbol{\eta}_t = \Phi^{-1}(\mathbf{u}_t)$ in the Gaussian case, where Φ^{-1} is the inverse c.d.f. of the multivariate standard normal Gaussian distribution. When estimating the t-copula the transformation is done via $\boldsymbol{\eta}_t = T^{-1}_{\mathbf{R},\nu}(\mathbf{u}_t)$, where $T^{-1}_{\mathbf{R},\nu}$ is the inverse c.d.f. of the multivariate t-distribution with correlation matrix $\mathbf{R}$ and degree of freedom parameter ν[3].

4.4.3 Multivariate Archimedean Copulas

Archimedean copulas are an useful tool for estimating asymmetric dependence. Time-varying Archimedean copulas are until now limited to the bivariate analysis. The following makes an attempt to enhance this line of research to the multivariate case. For the multivariate Archimedean copulas [4] we propose a structure similar to Patton's original specification of the bivariate symmetrized Joe-Clayton copula. The original dynamic structure of Patton (2006) for the Archimedean copulas is given by

$$\theta_t^{PA} = \Lambda\left(\omega^{PA} + \beta^{PA}\theta_{t-1}^{PA} + \alpha^{PA}\frac{1}{10}\sum_{j=1}^{10}|u_{1,t-j} - u_{2,t-j}|\right), \tag{4.45}$$

where $\Lambda(x) = (1+e^{-x})^{-1}$ to keep θ_t^{PA} in $[0,1]$. Intuitively explained: when the variables are comonotone the distance between them is zero whilst with countermonotone variables the distance between them is maximized and thus equals one. This implies that the forcing variable lies always in $[0,1]$. The approach of Patton is limited to bivariate dependence this makes him useful with regards to multivariate estimation only through the use of D-vine copulas. To use multivariate Archimedean copulas 'directly' equation (4.45) has to be expanded to the multivariate case. Instead of using the absolute distance between variables we propose the use of the so called $K-means$ algorithm which belongs to the class of multivariate distance measures. As a distance measure we choose the 'cityblock' measure which calculates the absolute distance from every point to the median within the cluster. Consider a random vector in time $\mathbf{X}_t = (X_{1,t}, \ldots, X_{d,t})'$ then the random variables $X_{1,t}, \ldots, X_{d,t}$ are defined as a cluster at time t,

[2] The process has been described by Patton as an ARMA(1,10) process and we will stick to this notation although this might be irritating since there is only one parameter to estimate the effect of the forcing variable.

[3] As a starting value for ν in the estimation procedure the unconditional d.o.f. parameter of the t-copula is used.

[4] The multivariate extension of the Archimedean copulas is developed in cooperation with Valentin Braun.

there exists one cluster at each point in time. The $K-means$ algorithm with cityblock distance measure now calculates the median of this cluster and measures the absolute distance of every point in this cluster to the median

$$\theta_t^{MA} = \omega^{MA} + \beta^{MA} \cdot \theta_{t-1}^{MA} + \alpha^{MA} \frac{1}{10} \sum_{j=1}^{10} \sum_{i=1}^{d} |u_{i,t-j} - m_{t-j}|, \tag{4.46}$$

where m_t is the median at time t. Clearly, this models the time-varying behavior akin to an ARMA(1,10) process and thus resembles the idea of Patton. The forcing variable still takes the value of zero if the variables are comonotone and one if they are countermonotone. However, although Patton's transformation keeps the copula dependence parameters in their desired bounds, it does not make use of the forecasting properties of an ARMA process. To be able to do so the restrictions of ARMA-process needs to be applied to (4.46). Although the dynamic dependence structure are referred to as an ARMA(1,10), the restrictions are the same as for an ARMA(1,1) process. This leads to the following restrictions $|\beta^{MA}| < 1$ and $|\alpha^{MA}| < 1$ (see (2.7) and (2.8) in section 2.3). The constant ω^{MA} remains unrestricted. The restrictions introduced so far ensure that (4.46), has the properties of an ARMA process but this does not guarantee that the dependence parameter of the respective copulas stay in their defined bounds. The dependence parameter of the Clayton and rotated Clayton copula is always in the interval $[0,\infty]$ and the dependence parameter of the Gumbel copula in $[1,\infty]$. Therefore, θ_t^{MA} in the (rotated) Clayton case is transformed by $e^{\theta_t^{MA}}$ and for the Gumbel copula by $1+e^{\theta_t^{MA}}$ (as in Hafner and Manner (2012)). Thus, for example the time-varying dependence parameter of the Clayton copula may be written as

$$\exp\left(\theta_t^{MA}\right) = \exp\left(\omega^{MA} + \beta^{MA} \cdot \theta_{t-1}^{MA} + \alpha^{MA} \frac{1}{10} \sum_{j=1}^{10} \sum_{i=1}^{d} |u_{i,t-j} - m_{t-j}|\right).$$

4.4.4 Dynamic Vine Copulas

The only chance to use Patton's original time-varying dependence structure for copulas in the multivariate case is through the use of D-vine copulas, since vine copulas cascades a multivariate distribution function into several pair-copulas. But instead of using Patton's dynamic structure we follow the approach of Heinen and Valdesogo (2009) who criticize Patton's approach because ρ_t is a nonlinear function of lagged values of conditional dependence and the dynamics are not easily comparable across different copulas. Instead they propose modelling the time-varying behavior through Engle's DCC-approach (see section 3.4). Since Engle's DCC-model is embedded in the Gaussian world again some transformations need to be made. First, the uniform variables u_1 and u_2 are transformed to standardized residuals $\boldsymbol{\eta_t}$ via the inverse Gaussian c.d.f. Thereafter, these standardized residuals are plugged into the DCC-structure and finally a 2×2 positive definite correlation matrix $\mathbf{R}_t$ is achieved. But since the Gumbel and Clayton copulas do not belong to the elliptical copula family, some mapping of the bivariate Gaussian correlation parameter is necessary. Since vine copulas estimate at every stage, $\mathbf{R}_t$ contains only one secondary diagonal with one element- the bivariate Gaussian correlation coefficient ρ_t. This correlation coefficient is mapped into the dependence parameter off the different copulas via the respective Kendall's tau. The Gaussian and Student-t dependence parameter are mapped into Kendall's tau via equation (4.37)

$$\tau_t = \frac{2}{\pi} \arcsin \rho_t,$$

where arcsin denotes the inverse of the sinus function. Thereafter, Kendall's tau is mapped into the copula specific dependence parameter. In case of the Gaussian and the t-copula this is done through

$$\theta_t^{GA} = \sin(\frac{\tau_t \pi}{2}) = \theta_t^T$$

where sin depicts the sinus function. Formulas for Kendall's tau for the different copulas were derived in section 4.3.1. Now Kendall's tau is given and the respective copula parameter needs to be inferred. For the Clayton copula Kendall's tau were defined in (4.39). The dependence parameter of the Clayton copula related to Kendall's tau then is calculated as

$$\theta_t^{CL} = \frac{2\tau_t}{1-\tau_t}.$$

Kendall's tau of the Gumbel copula is found in equation (4.38) and the dependence parameter is obtained by

$$\theta_t^{GU} = \frac{1}{1-\tau_t}.$$

The advantage of this approach is the comparability of the different dependence parameters since every dependence parameter is estimated with a Gaussian time-varying structure. A drawback of this approach is that for the Clayton and Gumbel copula ρ_t must be restricted. Both copulas just model positive dependence, the Clayton copula positive dependence in the left tail and the Gumbel copula positive dependence in the right tail. That is for both copulas ρ_t needs to be redefined to $\rho_t = max(0, \rho_t)$. Heinen and Valdesogo (2009) argues that if the best model is chosen by the MLE method or a related criterion as the AIC or BIC criterion the restriction does not matter since copulas which allow for negative dependence will still be chosen if the data set contains periods with negative dependence of the data, i.e. the copulas which fit's the data best should be those one's with the highest MLE values.

4.4.5 Dynamic Mixture Copula

We explained the benefits of mixture copulas in section 4.2.4, above. In this section we will introduce dynamic mixture copulas. These incorporate time-varying copulas and time-varying weights. Time-varying weights seem to be a useful enhancement for mixture copulas. Consider, for example, a data sample that at times inheres dependence in the left tail and fit a mixture copula consisting of an elliptical and the Clayton copula to this sample. Then the dependence parameter of the Clayton copula should increase during times with left tail dependence. It might be the case that during times of low left tail dependence the weight of the

Clayton copula is fairly low and then when left tail dependence increases dramatically the low weight might not reflect the asymmetric dependence enough despite an increase in the dependence parameter of the Clayton copula. Figure 4.7 shows a t-Clayton mixture copula with different dependence parameters and weights for the Clayton copula. In sub-image 4.7(a), the weight of the t-copula is $w_T = 0.8$ and the correlation parameter $\rho = 0.8$. The weight of the Clayton copula in the mixture copula is $w_{Cl} = 0.1$ and $\theta^{CL} = 2$. Sub-image 4.7(b) shows a t-Clayton mixture copula where $\theta^{CL} = 10$. The weight and the correlation parameter of the t-copula stay the same. In subimage 4.7(c) the weight of the t-Copula changes to $w_T = 0.6$ and the weight of the Clayton copula to $w_{CL} = 0.4$. Finally in subimage 4.7(d) $w_T = 0.6$, $w_{Cl} = 0.4$, $\rho = 0.8$ and $\theta^{CL} = 10$. It can be clearly seen that the weight has a great impact to the dependence structure. Ng (2008) proposed as one of the first dynamic weights for a

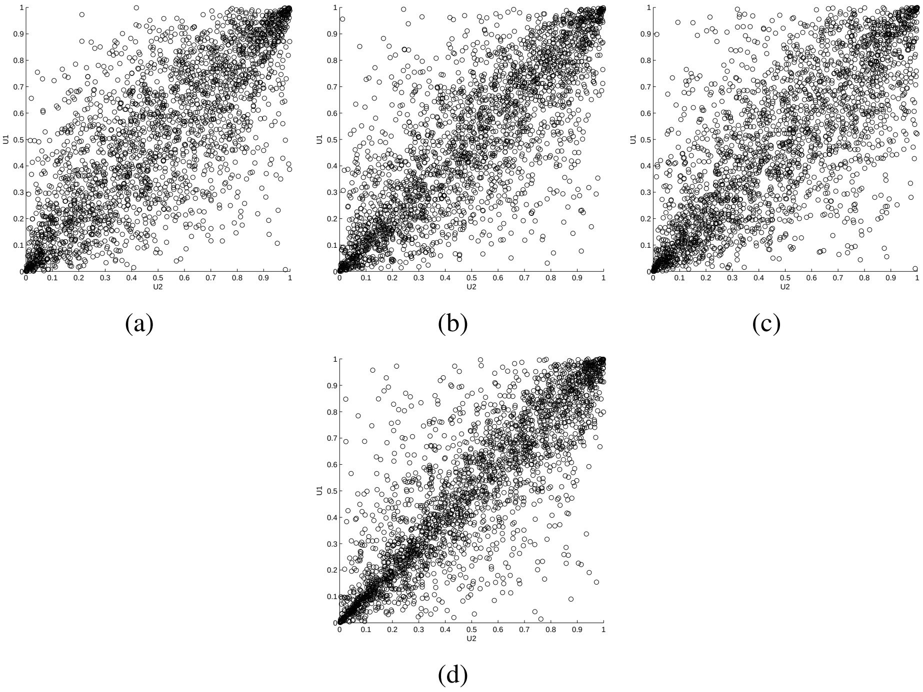


Figure 4.7: Scatter Plots t-Clayton Mixture Copula. This figure shows the scatter plots of a t-Clayton mixture copula.

mixture copula

$$w_{i,t}^{NG} = \left[1 + \exp\left(-\left(\omega_i^{NG} + \alpha_i^{NG} w_{i,t-1}^{NG} + \beta_i^{NG} \frac{1}{L} \sum_{j=1}^{L} |u_{1,t-j} - u_{2,t-j}|\right)\right)\right]^{-1}, \tag{4.47}$$

for $i = 1, \ldots, n$ where n represents the numbers of copulas in the mixing structure. The weight of the copula i at time t is represented by $w_{i,t}^{NG}$. A drawback of Ng's approach is that only the weights are modelled as time-varying. The dependence parameter of the copulas are estimated only once, i.e. they are kept constant all the time. This clearly restricts the dependence structure. To overcome this we[5] introduce a procedure where the copula parameters are also

[5]The dynamic mixture copulas are developed in cooperation with Valentin Braun.

time-varying. The dynamic structure of the Archimedean copulas follow (4.46) whereas for the elliptical copulas the dynamic structure of equation (3.12) developed by Cappiello, Engle, and Sheppard (2006) is chosen. The weights are modelled again by an ARMA(1,10) structure

$$w_{i,t}^{DW} = \omega_i^{DW} + \alpha_i^{DW} w_{i,t-1}^{DW} + \beta_i^{DW} \frac{1}{10} \sum_{j=1}^{10} \left(\frac{c_{i,t-j}\left(u_1, \ldots, u_d, \theta_{i,t-1}\right)}{\sum\limits_{k=1}^{n} c_{k,t-j}\left(u_1, \ldots, u_d, \theta_{k,t-j}\right)} \right), \tag{4.48}$$

for $i = 1, \ldots, n-1$. The forcing variable is defined as an average of the respective copula density at time t relative to sum of all copula densities at time t. The MLE procedure implies that the higher the density of the copula the better the fit to the data sample. For the dynamic structure in (4.48) this implies that the higher the density of the respective copula the higher the weight in the mixing structure should be. This is assured by the use of the relative density. To ensure that (4.48) exhibits the properties of an ARMA process several parameter constraints have to be implemented. Again we can make full use of the results developed in section 2.3. To guarantee stationarity and invertibility of the dynamic process some parameters have to be restricted: $|\alpha_i^{DW}| < 1$ and $|\beta_i^{DW}| < 1$. The weights have to sum up to one, i.e. $\sum_{i=1}^{n} w_{i,t}^{DW} = 1$ and need to be in the interval $[0,1]$. Negative weights are ruled out since they would not make any sense here. To ensure $0 \leq w_{i,t} \leq 1$ the following restrictions need to be imposed:

$$|\alpha_i^{DW}| + |\beta_i^{DW}| \leq \omega_i^{DW}$$

and

$$\omega_i^{DW} + |\alpha_i^{DW}| + |\beta_i^{DW}| \leq 1.$$

Since the weight is modelled through an ARMA-like equation the expected value appears to be

$$E\{w_{i,t}\} = \frac{\omega_i^{DW}}{1 - \alpha_i^{DW}}.$$

To guarantee the existence of the expected value and to keep the weight in $[0,1]$ two more conditions are needed, namely $0 \leq \omega_i^{DW} \leq 1$ and $|\alpha_i^{DW}| < 1$.

A major advantage of this approach is the possibility of multiperiod forecasts as (4.48) gets along without any exponential transformations. Furthermore, the restrictions to bivariate copulas and constant copula parameters (as in Ng (2008)) are abolished. Modelling the forcing variable as relative densities takes advantage of the MLE feature such that the better the fit of the respective copula, the better the likelihood value. Due to the use of time-varying copulas it is possible to give the copula with the best fit at time t the most weight at that point in time. The benefit in contrast to Patton (2006) is clearly in the mixing composition giving far more flexibility to model the dependence structure and again the enhancement from the bivariate to the multivariate case.

According to the static mixture in (4.36) the representation of the dynamic mixture is given by

$$C_t^{DW}(u_1, \ldots, u_d; \boldsymbol{\theta}_t) = \sum_{i=1}^{n} w_{i,t}^{DW} C_{i,t}(u_1, \ldots, u_d; \theta_{i,t}). \tag{4.49}$$

Several other authors have also investigated dynamic copulas. Jondeau and Rockinger (2006) employed a similar strategy as Patton whilst Chollete (2005), Garcia and Tsafack (201), Rodriguez (2007), Okimoto (2008), Chollete, Heinen, and Valdesogo (2009), and Kenourgios, Samitas, and Paltalidis (2011) use regime-switching copulas to specify the time-varying behavior. Panchenko (2005) used a semi-parametric copula and Lee and Long (2009) consider a combination of MVGARCH and copulas. Bodnar and Hautsch (2012) uses a multiplicative error model for modelling the conditional mean of high frequency data in combination with a DCC-GARCH Gaussian Copula. and DCC-GARCH Cherubini et al (2012) gives a thorough introduction to the so called convolution-based copulas. A survey of bivariate time-varying copulas is provided by Manner and Reznikova (2012). A survey reviewing copula especially for economic time series is given by Patton (2012).

4.5 Estimation

The MLE method is a useful tool in estimating parametric copulas. Since the copula is a distribution function itself one needs only to specify the corresponding density and estimation can be carried out. Let $\mathbf{y}_t = (y_{1,t}, \ldots, y_{d,t})'$ be a vector of real valued variables with marginal densities $f_1, \ldots, f_d$ and joint density f.

The first method introduced is the 'Exact Maximum Likelihood' (hereafter EML) and involves estimating the univariate margins and the copula parameter together in one step. Therefore, the parametric univariate margins must be known. The log-likelihood for the EML may be written as

$$\ln L(\mathbf{u}_1, \ldots, \mathbf{u}_T; \boldsymbol{\theta}^{EML}) = \sum_{t=1}^{T} \ln c\left(F_1(y_{1,t}), \ldots, F_d(y_{d,t})\right) + \sum_{t=1}^{T} \sum_{j=1}^{d} \ln f_j(y_{j,t}),$$

where $\boldsymbol{\theta}^{EML}$ denotes the complete set of estimated parameters. Since the estimation of the copula with ELM is computationally very extensive the estimation in this analysis is done as a two-step procedure. This two step procedure is sometimes referred to as the 'Inference Functions for Margins' (hereafter IFM) method and was developed by Joe and Xu (1996). Similarities to process of estimation in Engle's DCC model should be noted. First, the parametric univariate margins are estimated and then the copula is conditioned on the parameters of the univariate margins. The canonical density form of a copula in (4.7) clarifies the possibility of a two-step estimating procedure. This special embodiment of a copula density demonstrates the possibility of the disjunction between the copula and the marginal densities. The completed likelihood for the IFM method is given by

$$L^{IFM}(\boldsymbol{\theta}^M, \boldsymbol{\theta}^C) = L_M^{IFM}(\boldsymbol{\theta}^M) + L_C^{IFM}(\boldsymbol{\theta}^C, \boldsymbol{\theta}^M), \tag{4.50}$$

where $L_M^{IFM}(\boldsymbol{\theta}^M)$ denotes the likelihood of the margins and $L_C^{IFM}(\boldsymbol{\theta}^C, \boldsymbol{\theta}^M)$ the likelihood of the copula[6]. First, every univariate margin is estimated separately

$$\ln L_M^{IFM}(\mathbf{y}_1,\ldots,\mathbf{y}_T;\boldsymbol{\theta}^M) = \sum_{t=1}^{T}\sum_{j=1}^{d} \ln f_j(y_{jt}). \tag{4.51}$$

The second step incorporates the estimation of the copula, conditioned on the parameter of the univariate margins $\boldsymbol{\theta}^M$.

$$\ln L_C^{IFM}(\mathbf{u}_1,\ldots,\mathbf{u}_T;\boldsymbol{\theta}^C,\hat{\boldsymbol{\theta}}^M) = \sum_{t=1}^{T} \ln c(F_1(y_{1,t}),\ldots,F_d(y_{d,t}),\hat{\boldsymbol{\theta}}^M). \tag{4.52}$$

Finally, the IFM estimator takes the form

$$\hat{\boldsymbol{\theta}}^{IFM} = \left(\hat{\boldsymbol{\theta}}^M,\hat{\boldsymbol{\theta}}^C\right)',$$

where $\hat{\boldsymbol{\theta}}^M = \arg\max L_M^{IFM}(\boldsymbol{\theta}^M)$ and $\hat{\boldsymbol{\theta}}^C = \arg\max L_C^{IFM}(\boldsymbol{\theta}^C,\hat{\boldsymbol{\theta}}^M)$ are the estimated parameter vectors of the univariate margins and the copula. This procedure leads to inefficient results compared to a method were all parameters are estimated in one step, see Joe (2005). Regardless of the inefficiency it is used here since it significantly reduces the number of parameters to be estimated at each step.

Joe (1997) discusses several issues of the IFM estimation procedure- asymptotic covariance estimation among them. Joe and Xu (1996) and Joe (2005) proved the asymptotical normality of the two-step procedure

$$\sqrt{T}\left(\hat{\boldsymbol{\theta}}^{IFM} - \boldsymbol{\theta}_0\right) \to N\left(0,V^{-1}(\boldsymbol{\theta}_0)\right),$$

where V is the Godambe Information Matrix which is given by

$$V(\boldsymbol{\theta}_0) = \mathbf{D}^{-1}\mathbf{M}\,\mathbf{D}^{-1}.$$

According to Durrlemann, Nikeghbali, and Roncalli (2000) when estimating the complete score vector $\mathbf{D}$ it is useful to first define a score vector $\mathbf{g}$: $\mathbf{g} = (\mathbf{g}_1,\ldots,\mathbf{g}_d,\mathbf{g}_m)'$, where $m = d+1$ and $\mathbf{g}_j(\boldsymbol{\theta}^{IFM}) = \left(\frac{\partial L_{M,j}^{IFM}}{\partial \theta_j^M},\frac{\partial L_C^{IFM}}{\partial \theta^C}\right)'$. Then $\mathbf{D}$ is the score vector $E\{\partial \mathbf{g}(\boldsymbol{\theta}^{IFM})/\partial(\boldsymbol{\theta}^{IFM})'\}$ and $\mathbf{M} = E\{\mathbf{g}\mathbf{g}'\}$. In a final step we will explain the construction of the mixture copula: this is the weighted sum of the single copula likelihoods

$$L^{MIX} = w_1\cdot L_1 + (1-w_1)\cdot L_2. \tag{4.53}$$

A third method to estimate copulas is the Canonical Maximum Likelihood (hereafter CML) method. Fermanian and Scaillet (2005) showed that for misspecified marginal distributions, the estimate of the copula dependence parameter might be biased. The CML method tries

[6]Note that in this analysis the standardized residuals $\boldsymbol{\eta}_t$ are always used when fitting copulas with the IFM method.

to overcome this by replacing the parametric margins. The first alternative to the parametric margins is the empirical c.d.f.. This usually takes the form

$$\hat{F}_d(y_i) = \frac{1}{d+1} \sum_{i=1}^{d} \mathbf{1}(Y_i \leq y), \tag{4.54}$$

where $\mathbf{1}(A)$ is the indicator function that takes on value 1 if event A happened and zero if event A did not happen. The second alternative is the semiparametric estimation method. McNeil and Frey (2000) argued that the empirical c.d.f. is a poor estimator for the marginals. In their approach, univariate GARCH models are fitted to financial time series and standardized returns are constructed. Following this, two thresholds are defined- one for the left tail and one for the right tail. Values above (right tail) and below (left tail) the threshold are then modelled according to the Generalized Pareto Distribution. Values between the two thresholds are modelled either through the empirical c.d.f. or a kernel. Thereafter, the copula is estimated based on these semiparametric margins.

According to McNeil and Frey (2000) the generalized pareto density function may be written

$$G(t) = \begin{cases} 1-(1+\xi t/\beta)^{-1/\xi}, & \text{if } \xi \neq 0 \\ 1-\exp\{-t/\beta\}, & \text{if } \xi = 0, \end{cases} \tag{4.55}$$

where $\beta > 0$, $t \geq 0$ when $\xi \geq 0$, and $0 \leq t \leq -\beta/\xi$ when $\xi \leq 0$. The estimate of the upper tail (y>s) is given by

$$\hat{F}_u(y) = 1 - \frac{n}{T}\left(1+\hat{\xi}_n \frac{y-s}{\beta_n}\right)^{-1/\xi_n},$$

where n/T represents the proportion of data in the tail. The lower tail estimator is defined in a similar manner. We model the middle part through a Gaussian kernel

$$\hat{F}_m(y) = \frac{1}{\sqrt{2\pi}} \exp\left(\frac{y^2}{2}\right).$$

The log-likelihood function then is given by

$$\ln L_C^{CML}(\hat{\mathbf{u}}_t;\boldsymbol{\theta}^C) = \sum_{t=1}^{T} \ln c(\hat{F}_1(y_{1,t}),\ldots,\hat{F}_d(y_{d,t})),$$

where this time $\hat{F}$ is composed of three parts - the two tail estimators and the Gaussian kernel. Again the estimator will be maximized through $\hat{\boldsymbol{\theta}}^C = \arg\max L_C^{CML}(\boldsymbol{\theta}^C)$. For properties of the CML estimator see Genest, Ghoudi, and Rivest (1995), Chen and Fan (2006), and Chen, Fan, and Tsyrennikov (2006). Embrechts, Klüppelberg, and Mikosch (1997) provides a reference for extreme value theory. Several more kernel functions are explained in Franke, Härdle, and Hafner (2008). The D-Vine copula is also estimated by the Maximum Likelihood Estimation method. Therefore, a likelihood function has to be derived. For a d-dimensional D-vine copula the density is given by

$$f(y_1,\dots,x_d) = \prod_{k=1}^{d} f(y_k) \prod_{j=1}^{d-1}\prod_{i=1}^{d-j}$$
$$c_{i,i+j|i+1,\dots,i+j-1}(F(y_i|y_{i+1},\dots,y_{i+j-1}),F(y_{i+j}|y_{i+1},\dots,y_{i+j-1})), \quad (4.56)$$

see Aas et al (2009). Acar, Genest, and Něslehová (2012) and Stoeber, Joe, and Czado (2013) criticize the "simplified" estimation method of (4.56) since all pair copulas do not depend on the variables they are conditioned on. This makes the estimation of the copulas more tractable but has the drawback of - under certain conditions - misleading inference about dependence.

4.6 Goodness-of-fit Test

After estimating several different parametric copulas, the question of which one is the best arises. Miscellaneous methods to determine this have emerged in the literature. First of all there are tests built solely for testing the goodness-of-fit of single copulas. For example Malevergne and Sornette (2003) tests the null hypothesis of a Gaussian copula and Cui and Sun (2004) the null of a Clayton copula. Other tests are based on kernels, weight functions and other smoothing parameters, as in Berg and Bakken (2005) or Fermanian (2005). Some belong to the class of so called ‘blanket tests’ developed by Genest and Rémillard (2008) and Genest, Rémillard, and Beaudoin (2009). Especially concerned with time-varying copulas is Busetti and Harvey (2011), who test the null that the copula is changing over time - based on indicator variables. The goodness-of-fit test we use has been developed by Breymann, Dias and Embrechts (2003).

In general, the null hypothesis of a copula goodness-of-fit test may be stated as

$$H_0: C_i(U_{1,t},\dots,U_{d,t};\hat{\theta}_t) = C_0(U_{1,t},\dots,U_{d,t};\theta_t^0),$$

where C_0 denotes the true copula and C_i the copula the null hypothesis is tested against. The test of Breymann, Dias and Embrechts (2003) is based on the probability integral transform (hereafter PIT) introduced in section 4.1. Since copulas are multivariate functions, the PIT theory must be taken to the multivariate case. Fundamental is the construction of so called pseudo-variables Z_t. The multivariate conditional PIT is given by

$$\begin{aligned}
Z_{1,t} &= F_1(X_{1,t}) \\
Z_{2,t} &= F_{2|1}(X_{2,t}|X_{1,t}) \\
\vdots &= \vdots \\
Z_{d,t} &= F_{d|1,\dots,d-1}(X_{d,t}|X_{1,\dots,d-1,t}),
\end{aligned}$$

where $F_{d|1,\dots,d-1}$ denotes the conditional distribution function. For (closed form) copula models these conditional distributions are derived via

$$C(u_{k,t}|u_{1,t},\dots,u_{k-1,t}) = \frac{\partial^{k-1}C(u_{1,t},\dots,u_{k,t})}{\partial u_{1,t}\cdots\partial u_{k-1,t}} \cdot \left(\frac{\partial^{k-1}C(u_{1,t},\dots,u_{k-1,t})}{\partial u_{1,t}\cdots\partial u_{k-1,t}}\right)^{-1},$$

for $k = 2, \ldots, d$, see Cherubini, Luciano, and Vecchatio (2004, p.182). Since the Gaussian and t-copula have no closed form it is not possible to derive the conditional distribution of by dividing partial derivatives (as it is for Archimedean copulas). For the Gaussian copula we fall back on the results of Chen, Fan, and Patton (2004) and for the t-copula on the findings of Box and Jenkins (1976, pp.262-264). According to Chen, Fan, and Patton (2004) the conditional distribution of the ith element of the Gaussian distribution may be depicted as

$$Z_{i,t} = \Phi\left(\frac{X_{i,t} - \mathbf{X}_{[1:i-1],t} \cdot \mathbf{R}^{-1}_{[1:i-1,1:i-1],t} \cdot \mathbf{R}_{[1:i-1,i],t}}{\sqrt{1 - \mathbf{R}_{[i,1:i-1],t} \cdot \mathbf{R}^{-1}_{[1:i-1,1:i-1],t} \cdot \mathbf{R}_{[1:i-1,i],t}}}\right),$$

for $i = 2, \ldots, d$, where $\mathbf{R}_t$ is the correlation matrix at time t and $\mathbf{R}_{[1:i,1:i],t}$ is a submatrix of $\mathbf{R}_t$ with dimension $i \times i$. The formula for the multivariate t-copula is a little bit more complicated. First, the partitioning of a matrix has to be introduced: let $\mathbf{X}$ be a vector of dimension k then $\mathbf{X} = (\mathbf{X}_1, \mathbf{X}_2)'$ where $\mathbf{X}_1 = \mathbf{X}_{[1:k_1]}$ is of dimension k_1 and $\mathbf{X}_2 = \mathbf{X}_{[k_1+1:k]}$ of dimension $k_2 = k - k_1$. Then a symmetric matrix $\mathbf{S}$ is in a similar manner partitioned into

$$\mathbf{S} = \begin{bmatrix} \mathbf{S}_{11} & \mathbf{S}_{12} \\ \mathbf{S}_{21} & \mathbf{S}_{22} \end{bmatrix},$$

where $\mathbf{S}_{11}$ is of dimension $k_1 \times k_1$. Subsequently the ith pseudo-variable is given by

$$Z_{i,t} = T_{\mathbf{R},\nu}\left(\mathbf{S}_{21,t}\mathbf{S}^{-1}_{11,t}\mathbf{X}_{2,t}, \frac{\nu + \mathbf{X}'_{2,t}\mathbf{S}^{-1}_{11,t}\mathbf{X}_{2,t}}{\nu + k_1} \cdot \mathbf{S}_{22,t} - \mathbf{S}_{21,t}\mathbf{S}^{-1}_{11,t}\mathbf{S}_{12,t}, \nu + i - 1\right),$$

for $i = 2, \ldots, d$. The vector $\mathbf{X}_{2,t} = \mathbf{X}_{[1:i-1],t}$ is the vector composed of the conditioning variables, $T_{R,\nu}$ denotes the c.d.f. of the non-standardized Student-t distribution, and $\mathbf{S}_t = \mathbf{R}_t \cdot \frac{\nu-2}{\nu}$ is the so called scale-matrix. For the mixture copulas we generate the pseudo variables by the weighted sum of the respective copula in the mixture

$$\begin{aligned} C^{MIX}(u_{k,t}|u_{1,t}, \ldots, u_{k-1,t}) = &w_1 \cdot C^{MIX_1}(u_{k,t}|u_{1,t}, \ldots, u_{k-1,t}) \\ &+ w_2 \cdot C^{MIX_2}(u_{k,t}|u_{1,t}, \ldots, u_{k-1,t}), \qquad \text{for } k = 1, \ldots, d. \end{aligned}$$

Liu (2006) generated conditional variables of a mixture copula consisting of the Gumbel and rotated Gumbel copula in this way. Conditional variables from a D-vine are generated as in (4.31) and (4.35) with $w_d = F_d(X_d)$.

The definition of the probability integral transforms the variables $Z_{1,t}, \ldots, Z_{d,t}$ should be distributed as $Uniform(0,1)$ and Breymann, Dias and Embrechts (2003) proposes the transformation $G_t = \sum_{i=1}^{d} \left(\Phi^{-1}(Z_{i,t})\right)^2$, where Φ is the standard normal distribution and $G_t \overset{\text{i.i.d.}}{\sim} \chi^2_d$. This procedure makes it possible to apply several statistical distance measures between G_t and the χ^2_d distribution. we make use of the following distance measures:

$$\begin{aligned}
KS &= \sqrt{T}\max_{t=1,\ldots,T}\left(\left|F_{\chi^2_d}(G_t)-\hat{F}(G_t)\right|\right), &(4.57)\\
AKS &= \frac{1}{\sqrt{T}}\sum_{t=1}^{T}\left|F_{\chi^2_d}(G_t)-\hat{F}(G_t)\right|, &(4.58)\\
AD &= \sqrt{T}\max_{t=1,\ldots,T}\left(\frac{\left|F_{\chi^2_d}(G_t)-\hat{F}(G_t)\right|}{\sqrt{\hat{F}(G_t)\left(1-\hat{F}(G_t)\right)}}\right), &(4.59)\\
AAD &= \frac{1}{\sqrt{T}}\sum_{t=1}^{T}\frac{\left|F_{\chi^2_d}(G_t)-\hat{F}(G_t)\right|}{\sqrt{\hat{F}(G_t)(1-\hat{F}(G_t))}}, &(4.60)
\end{aligned}$$

where $F_{\chi^2_d}$ is the chi-square distribution function with d d.o.f., $\hat{F}$ the empirical distribution function, AKS the average Kolmogorov-Smirnov distance, AD the Anderson-Darling distance (Anderson and Darling (1952)), and AKS the average Anderson-Darling distance. The main difference between the AD and KS tests is that the AD test gives more weights to the tails of the distribution. We use these distance measures just as another quality measure in the manner of the AIC and BIC criteria. Therefore, we do not assume any null hypotheses and do not report any critical values. Instead, we compare the different statistics: the smaller the reported statistic of the respective copula is, the better the copula fits the data. This procedure is recommended by Berg (2007) and avoids the critique of Dobric and Schmid (2007) who argued that the test performs poorly in rejecting a false null hypothesis. Another advantage is that the fit of the different copulas are directly comparable amongst one another.

Chapter 5

Empirical Results

5.1 Emerging Markets

The World Bank classification of developing countries lead to the term ‘emerging markets’ in the 1980s. According to the World Bank definition all countries with a Gross National Income (GNI) of US$ 975 per capita or less (in 2008) belong to the group of developing countries. The group of developing countries is then divided into several subgroups. There are some developing countries (e.g. Ethiopia, Cambodia, Uganda, etc.) whose financial markets barely exist. By World Bank standards such countries do not belong to the emerging markets. The International Finance Corporation (IFC) tried to create a definition that separates between emerging and developing countries. By their definition a country is classified as emerging if it belongs to the group of developing countries; shows high potential for economic growth; has macroeconomic and political stability; and shows signs of financial and economic reforms. Its financial markets must also be relatively liquid and accessible to foreign investors and they must show significant changes in terms of its relative size compared to Gross Domestic Product (GDP).

FTSE, meanwhile, defines twenty-four countries as emerging markets and these are divided into two categories: advanced and secondary emerging markets. In the group of ‘advanced emerging markets’ are Brazil, Hungary, Mexico, Poland, and South Africa. The group of ‘secondary emerging markets’ is much broader and consists of Argentina, Chile, China, Colombia, Czech Republic, Egypt, India, Indonesia, Malaysia, Morocco, Pakistan, Peru, Philippines, Russia, Thailand, and Turkey. This breakdown makes sense since there are large disparities in the level of economic and financial development. For example, Brazil is an emerging market in the ‘maturity phase’ which means it is relatively close to the developed countries whereas countries such as Colombia and Indonesia still have a long way to go [1]. In the context of this analysis, it seems to make sense to follow the FTSE definition of advanced and secondary emerging markets and to investigate the financial integration of each group with some developed markets [2].

[1] For an introduction to the stock markets and related data of emerging markets see Arouri, Jawadi, and Nguyen (2010).

[2] For a discussion of the economic differences between several emerging markets among each other and

5.2 Data Description

As defined by FTSE the category of advanced emerging markets that we use in this analysis consists of: Brazil (BRA), Hungary (HUN), Mexico (MEX), Poland (POL), and South Africa (RSA)[3]. The secondary emerging markets defined by FTSE and used in this analysis are: Chile (CHI), Czech Republic (CZE), Indonesia (IND), India (INA), and Thailand (THA). FTSE defines several more countries as a secondary emerging market. We have chosen these seven because they provide enough data to conduct a realistic VaR backtest. The countries representing the developed markets are: Australia (AUS), Denmark (DEN), Sweden (SWE), Italy (ITA), Japan (JPN), Great Britain (GBR), and the United States of America (USA). All stock indices are provided by MSCI and all bond indices by J.P. Morgan. All emerging market bond indices bar India's belong to the subdivision J.P. Morgan Government Bond Emerging Markets-Index Global (GBI-EM) which consists of investable indices. India belongs to the group of GBI-EM Broad and so might not be accessible to all investors. All developed market indices belong to the Government Bond Index Global (GBI Global) series and are also investable. Furthermore, all indices are total return indices and denominated in US$. The full sample runs from 05/03/2003 to 08/10/2010 and contains 1826 daily returns.

5.2.1 Risk, Return and Correlation Characteristics

The returns of emerging and developed market indices show different characteristics. For example Harvey (1995) argues that emerging market returns are more driven by local information. Furthermore, Bekaert and Harvey (1997) point out that emerging market returns are more predictable and exhibit higher asymmetric volatility. Claessens, Dasgupta, and Glen (1995) and Bekaert et al (1998) highlight the non-normality and the time-varying skewness and kurtosis of emerging market returns. The above mentioned volatility of these markets is characterized by several shifts which are linked to country specific social-economic events, as has been pointed out by Aggarwal, Inclan, and Leal (1999). Thus it might be the case that empirical results for the emerging and market indices differ. Furthermore, the more the investigated indices differ the more empirical properties of the different risk models can be highlighted.

Figures A.1, A.2, A.3, and A.4 show the price and return series of all indices. By simple graphical inspection, heteroscedastic behaviour of the return series can be detected- this is important for the determination of the marginals via the AR-GARCH models. It is also a first indication that the return series departs from the normality assumption. Furthermore, it is often said that emerging markets offer higher expected returns but also higher volatility in contrast to developed markets. To rectify this we make use of some statistical measures. Nevertheless, these measures are useful in highlighting some important properties of the respective time series. Table B.1 shows these summary statistics for the bond indices used in this analysis. All series show departures from normality which can be seen by their respective kurtosis and skewness. The normal distribution has a kurtosis of 3 and a skewness of 0. All indices exhibit a

between developed markets see e.g. Bekaert and Harvey (2003).

[3] Taiwan does belong to this group too, but J.P. Morgan does not provide any bond prices so it is here omitted.

kurtosis greater than three indicating a higher probability of large negative and positive returns than would be expected under normality. Interestingly, all emerging market bond series have negative skewness which indicates more frequent large negative returns than large positive returns. Another interesting feature is the wide spread between the individual skewness of the particular series. A simple statistical test to confirm the departure from normality is the Jarque-Bera(JB) test invented by Jarque and Bera (1980)

$$JB = \frac{T}{6}\left(SK^2 + \frac{(KU-3)^2}{4}\right),$$

where SK denotes the sample skewness, KU the sample kurtosis, and T the sample size. The null hypothesis states that the sample is drawn from a normal distribution. The appropriate test statistic is calculated as $JB \sim \chi^2_2$. All series clearly reject the assumption of normality.

The summary statistics for all stock indices are found in table B.2. Again all indices show a kurtosis greater than three and a skewness other than zero. The Jarque-Bera test rejects the assumption of normality for all stock indices.

Often the unconditional correlation (also called mean correlation) is still used in finance. Therefore, we report the unconditional correlations for the full sample. The lower the correlation the lower the integration between these two markets. Interestingly, all stock markets show positive dependence on each other, the lowest between the United States and Japan (0.167) and the highest between Denmark and Sweden (0.794) as can be seen by table C.1. When comparing bond markets unconditional correlations in table C.2 it stands out that the bond markets of Japan and the United States show negative correlation to several other markets. Japan and Mexico show the highest negative correlation (-0.231) whereas Sweden and Italy show the highest positive (0.854).

Since one part of this analysis is the detection of asymmetries in correlation it makes sense to introduce a test capable of discovering these asymmetric correlations. This is particularly the case for vine copulas which are based on bivariate pair copulas. The tests might give a hint if an asymmetric dependence structure is given. Ang and Chen (2002b) and Hong, Tu, and Zhou (2007) both provide two simple tests of asymmetric correlations. We use the test of Hong, Tu, and Zhou (2007) since this does not presume a statistical model for the data. Ang and Chen (2002b) tests the null hypothesis to determine whether the quantile dependence of the data sample resembles the quantile dependence of the Gaussian distribution. Since there are more symmetric distributions than the Gaussian, the test does not rule out symmetric dependence by rejection of the null hypothesis. Hong, Tu, and Zhou (2007) overcome this drawback by introducing a distribution free test. Their test is based on bivariate data samples. First, they define an exceedance level c as standard deviations away from the mean. The correlation at this exceedance level is the correlation between two real valued variables (e.g. x and y) when both variables exceed c

$$\rho^+(c) = \mathrm{corr}(x_{1,t}, y_{1,t} | x_{1,t} > c, y_{1,t} > c), \tag{5.1}$$

$$\rho^-(c) = \mathrm{corr}(x_{1,t}, y_{1,t} | x_{1,t} < -c, y_{1,t} < -c). \tag{5.2}$$

Therefore, the null hypothesis can be stated as

$$H_0: \ \rho^+(c) = \rho^-(c), \quad \forall c > 0$$

and the alternative hypothesis as

$$H_1: \rho^+(c) \neq \rho^-(c), \quad \text{for some } c > 0.$$

The test is based on the intuitive idea that the difference of the $m \times 1$ difference vector $\boldsymbol{\rho}^+ - \boldsymbol{\rho}^- = (\boldsymbol{\rho}^+(c_1) - \boldsymbol{\rho}^-(c_1), \ldots, \boldsymbol{\rho}^+(c_m) - \boldsymbol{\rho}^-(c_m))$ must be close to zero. The covariance needed for the test statistic is estimated by the Newey and West (1994) covariance estimator $\hat{\boldsymbol{\Sigma}}$. With these definitions the test statistic for the null of symmetric dependence is given by

$$J_\rho = T \cdot \left(\hat{\boldsymbol{\rho}}^+ - \hat{\boldsymbol{\rho}}^-\right) \hat{\boldsymbol{\Sigma}} \left(\hat{\boldsymbol{\rho}}^+ - \hat{\boldsymbol{\rho}}^-\right),$$

and $J_\rho \overset{A}{\to} \chi_m$ as $T \to \infty$. We follow Ang and Chen (2002b) and Hong, Tu, and Zhou (2007) in choosing four different values for c: $(c_1 = 0,\ c_2 = 0.5,\ c_3 = 1,\ c_4 = 1.5)$. At 10% confidence level eighteen significant asymmetric correlations are found when evaluating all possible pairwise bond returns. Only three are when comparing all possible stock returns (see Tables C.3 and C.4).

5.3 Marginal Models

Fantazzini (2009) studied the effect of misspecified marginal models on the estimation of VaR using copulas modelling the dependence structure. As in this analysis he uses AR-GARCH models and concludes that misspecified marginals do have an effect. This effect diminishes as the sample size increases in relation to the effect a misspecified copula has on the VaR. In order to keep the effect of the possibly misspecified marginals as low as possible we put much emphasis on finding the correct marginal model.

As explained above, for every time series AR(p_1)-GARCH(1,1) with four different distribution-assumptions for the innovations (Gauss, t, GED, and $skew-t$) and $p_1 = 1, \ldots, 5$ will be estimated. Since we use 5 different GARCH models (AVGARCH, GARCH, EGARCH, GJRGARCH, ZARCH) this leaves one hundred possible specifications for every marginal distribution. To give a first hint as to which model might be the best we use the Schwarz (1978) criteria, also known as Bayesian Information Criteria (hereafter BIC)

$$BIC = -2 \cdot \ln L + \log(T) \cdot k,$$

where k denotes the numbers of parameters estimated and T the sample size. The GARCH model which minimizes the BIC criterion is put to further investigations.

To begin, we check the residuals of the AR-part for (G)ARCH effects. Only when the residuals show heteroscedastic behavior does the use of GARCH models makes sense. Engle (1982) constructed a simple Lagrange multiplier test to test for heteroscedasticity. The squared residuals of (2.1) are regressed on p own lags

$$\varepsilon_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \cdots + \alpha_p \varepsilon_{t-p}^2 + e_t.$$

Then,

$$T \cdot R^2 \overset{A}{\sim} \chi_p^2$$

under $H_0 : \varepsilon_t \overset{i.i.d.}{\sim} N(0,h)$, where R^2 is the usual sample correlation coefficient and T denotes the sample size. Since the above described test is related to ARCH models and only ARCH models of a high lag order are able to capture heteroscedastic features of a time series we use $p = 10$ and a confidence level of $\alpha = 0.05$. For all time series the null hypothesis of no ARCH effects is rejected. Furthermore, the properties of the standardized residuals should show the behavior of a white-noise process, i.e. they must not show any autocorrelation. This can be checked by the so called Portmanteau test. We use the test of Ljung and Box (1978) (hereafter LB) which may be stated as

$$Q = n(n+2)\sum_{j=1}^{l}\frac{\hat{\rho}(j)^2}{n-j},$$

where $\hat{\rho}$ denotes the sample autocorrelation function and l the degrees of freedom. The statistic Q then has an asymptotic χ_l^2 distribution under the null hypothesis of no autocorrelation. Thereafter a goodness-of-fit test is performed to check if the right distribution for the MLE method was chosen. The goodness-of-fit test used here is the Kolmogorov-Smirnov (hereafter KS) test which makes use of the Rosenblatt (1952) transformation which is discussed extensively in the copula chapter, see (4.5). The null hypothesis thus can be stated as

$$H_0 : F_S(x) = F_0(x), \quad \forall x \in R$$

where $F_S(x)$ is the sample c.d.f. and $F_0(x)$ is the hypothesized c.d.f. with respect to which $F_S(x)$ is being evaluated for. This test is of importance for the VaR-Analysis since in that section we use the IFM approach to estimate the copulas. The IFM approach uses exactly the transformation of equation (4.5). Any misspecification of the $Uniform(0,1)$ variables could lead to series biases in the copula estimation. If the model with the minimum BIC does not fail either the KS test or the LB test no further tests are applied. If one of the two tests is failed we do a graphical inspections as in the work of Diebold, Gunther, and Tay (1998). They define a variable z as $z_1 = F_1(x_1), \quad z \overset{i.i.d.}{\sim} U(0,1)$ and $\bar{z} = \sum_{i=1}^{d} z_i$. The graphical test is based on the correlograms of $(z-\bar{z}), (z-\bar{z})^2, (z-\bar{z})^3$, and $(z-\bar{z})^4$. These tests are useful to reveal dependence through the conditional mean, conditional variance, conditional skewness, and conditional kurtosis. Under the assumption of a correctly specified model the autocorrelation of these measure should be zero. Any significant autocorrelation implies a misspecified model.

Table D.1 shows the GARCH models for the emerging market stocks used to estimate copulas. For the copula evaluation all four distributions are possible specifications for the innovations. First of all, the BIC-criterion selected $p_1 = 1$ as the appropriate lag length for all models implying that the return series does not show any significant autocorrelation. For the ten emerging market stock indices values four times the GED and six times the $skew-t$ are chosen. All GED distributions are heavier than the normal, since ν is always smaller than two. Brazil shows the highest ν (10.1868) for indices estimated with the $skew-t$. Thus, it can be concluded that all indices show heavier tails than could be expected under Gaussian assumption. For all indices the null hypothesis of the KS test is not rejected which implies that the transformed standardized residuals are $Uniform(0,1)$ distributed. The marginal distributions for use with the MVGARCH models has to be estimated with Gaussian innovation assumption. Table D.2 reports the chosen GARCH models. The KS-test rejects the assumption of

$Uniform(0,1)$ variables at a 5% confidence level on six out of ten occasions, indicating that the Gaussian innovation assumption is not suitable for the data sample. Figure D.3 reports the graphical tests for the different indices which failed the KS-test. Since no autocorrelation is detected we again stay with the selected models.

For the emerging stock market indices either the GJRGARCH or the ZARCH are chosen, implying that all indices exhibit asymmetric volatility. The results of the emerging market bonds marginal models for use with copulas estimation are reported in table D.3. Interestingly, the chosen distribution is always the *skew*−t for the advanced emerging market indices, whereas for the secondary emerging markets either the GED or the Student-*t* are selected. Furthermore, for the advanced emerging bond indices always an asymmetric GARCH model is selected whereas for the secondary emerging markets this is only once the case. Either the KS-test or the LB-test fails for India, Indonesia, and Thailand. Therefore, we check the correlograms of the z-variables in Figure D.4. Again no serious autocorrelation is detected.

For the emerging market bonds estimated with Gaussian distribution assumption either the KS- or the LB-test failed on ten out of twelve occasions (see table D.4). Figure D.4 shows the z-plots for all indices which failed either one of the two tests. Again, no autocorrelation can be recognized and so the chosen models are used for the estimation with the multivariate GARCH models. Interestingly, now the symmetric GARCH model is selected only for the Czech Republic.

Table D.5 shows the marginal models for the developed bond indices. The upper part of the table describes the models with Gaussian assumption used for MVGARCH estimation and the lower part the GARCH models with all possible innovation assumptions. The simple GARCH model clearly dominates all other models. It is chosen eleven out of fourteen times emphasizing the symmetric volatility structure of daily bond returns for the developed markets. The KS-test fails only in the cases of Italy and Japan but as Figure D.1 reveals the z-plots do not show any autocorrelation. The marginal models for the developed stocks are reported in table D.6. Only asymmetric GARCH models are chosen as the appropriate volatility model. In contrast to the bond market returns this clearly indicates a higher volatility after negative shocks than after positive shocks of the same magnitude. Only Italy and Great Britain with Gaussian distribution assumption for the innovations fail the KS-test. Again the results of the z-plots in Figure D.2 do not provide an indication of any serious autocorrelation.

To sum up: whenever it is possible to estimate the GARCH models with any other innovation assumption than the Gaussian the non-Gaussian distribution is selected as the appropriate one. This seems to confirm the stylized fact that financial time series returns do not follow a Gaussian distribution (although the GARCH models with Gaussian assumption does account for some kurtosis, see equation (2.12)). Furthermore, the conditional mean of all models is estimated as an AR(1) process which confirms again a stylized fact, namely that (daily) financial time series returns do not reveal any significant autocorrelation in their first moment. Almost all stock indices seem to show some asymmetry in the variance, whereas for the bond indices the simple GARCH model is chosen more often than the asymmetric versions.

After introducing the marginal model we will now turn to the main part of this analysis: the different methods to estimate dependence between financial time series.

5.4 Multivariate Models

One aspect of this analysis is to find the best risk model for different portfolios. For this purpose, we consider four different equally weighted portfolios which combine different features. Table B.3 reports some summary statistics for the portfolios considered in detail. We introduce another model choosing criteria based on the log-likelihood, the so called Akaike (1974) (hereafter AIC) criteria

$$AIC = -2(\ln L) + 2 \cdot k.$$

The difference between both likelihood-criteria is that the BIC criterion penalizes models with more estimation parameters heavier than the AIC criterion. First, we will present the different statistical features of the miscellaneous portfolios. This makes it possible to infer later on which multivariate model characteristic best suits the different portfolio characteristics. The first portfolio considered consists of developed market stocks (AUS, DEN, SWE, ITA, JAP, GBR, USA). This portfolio shows the highest kurtosis (16.5211) of all portfolios, i.e. the fattest tails. In line with this finding is that it has the largest Jarque-Bera statistic (13925.1). The developed bonds portfolio shows quite different characteristics and is investigated at second place. It shows the lowest positive daily return (0.0461), the smallest negative daily return (-0.0296), the lowest annualized volatility (0.0880), and the lowest kurtosis (8.0296). As the only portfolio considered in the VaR analysis it exhibits positive skewness (0.3221). The third portfolio considered consists of advanced emerging market stocks (BRA, HUN, MEX, POL, RSA). This portfolio shows the highest (0.1565) and the smallest daily returns (-0.1311) of all portfolios and also the highest annualized volatility (0.2922) but its kurtosis (12.8311) is below that of the developed market stocks. The final portfolio investigated in detail is the emerging market bond portfolio. This portfolio exhibits the highest negative skewness (-0.4202) but the rest of its statistics are in the 'middle' range of all portfolios.

It can be seen, then that the four different portfolios exhibit quite different features. This makes them interesting for comparing the ability of the different time-varying models to estimate their dependence structures and their respective Value-at-Risk. Several working hypotheses should be considered. First of all, it might be interesting to analyze how well the MVGARCH models capture the fat-tailed behavior of the stock markets. Within MVGARCH models the AR-GARCH margins are estimated with Gaussian innovation assumption. The MVGARCH itself is also estimated with multivariate Gaussian distribution assumption. Due to their QML properties both models are able to capture some fat-tailed behavior and the question arises if this is enough for the portfolio consisting of developed stock indices since this shows the highest kurtosis? Furthermore, a comparison between the MVGARCH and the Gaussian copula is of particular interest because the only difference between them is their marginal model. Since the copula models depict a wide range of different dependence structures it is only natural to ask if the dependence structure makes a difference in VaR estimation.

Models of time-varying dependence only make sense if the investigated data sample shows changing correlations. Therefore, we implement a simple test of time-varying correlation developed by Engle and Sheppard (2001). The null hypothesis of this test can be stated as

$$\mathbf{H}_0 : \mathbf{R}_t = \overline{\mathbf{R}} \quad \forall t \in T$$

and the alternative hypothesis as

$$\mathbf{H}_1 : vech^u(\mathbf{R}_t) = vech^u(\overline{\mathbf{R}}) + \beta_1 vech^u(\mathbf{R}_{t-1}) + \beta_2 vech^u(\mathbf{R}_{t-1}) + \dots$$
$$+ \beta_p vech^u(\mathbf{R}_{t-1}),$$

where $vech^u$ is an operator that selects only elements above the diagonal and projects them onto a vector. In short the procedure of the test is described like this: First estimate the univariate GARCH models and standardize the residuals. Thereafter, the correlation matrix of the standardized residuals is estimated by the MVGARCH model and the vector of univariate standardized residuals is jointly standardized by the square root decomposition of $\overline{\mathbf{R}}$. As Engle and Sheppard (2001) points out the use of the correlation matrix in finite samples weakens further the power of the test. Using the correlation matrix, the test is also sensitive to the standardized variance of the univariate GARCH processes not being unity. Under the null of constant correlation now the residuals should be *i.i.d.* with covariance matrix I_d. We use the covariance matrix instead of the correlation matrix and so the necessary vector autoregression is done with the (lagged) outer products of the standardized residuals

$$r_t = vech^u \left[\left(\overline{\mathbf{R}}^{-1/2} \mathbf{D}_t^{-1} \boldsymbol{\varepsilon}_t \right) \left(\overline{\mathbf{R}}^{-1/2} \mathbf{D}_t^{-1} \boldsymbol{\varepsilon}_t \right)' - I_d \right].$$

The test statistic is estimated as $\frac{\hat{\boldsymbol{\Psi}}' \mathbf{X}\mathbf{X}' \hat{\boldsymbol{\Psi}}}{\hat{h}}$ which is asymptotically χ^2_{s+1}, where $\mathbf{X}$ depicts the regressor matrix and $\hat{\boldsymbol{\Psi}}$ is a $d \times 1$ vector of the estimated parameters from the vector autoregression. We assume $s = 10$ although different lag length are possible. For all portfolios the assumption of constant correlation is rejected as can be seen by table 5.1 and thus the analysis of time-varying correlations make sense.

Table 5.1: Engle and Sheppard (2001) test of time-varying correlation.

Portfolio	Developed Bonds	Developed Stocks	EM advanced Bonds	EM advanced Stocks
Stat	8217.0	8954.4	3667.1	4008.2

19.6751 is the respective critical value from a χ^2_{s+1} distribution, where $s = 10$.

We estimate the dynamic structure of the MVGARCH and the elliptical copula models according to equation (3.12) with restrictions of the respective model. In all tables concerning the DCC models we report the squared values as Cappiello, Engle, and Sheppard (2006) because this makes it easier to compare results with other studies. To avoid any confusion we will not report standard errors as they are estimated with the original coefficients which are not squared. Significant values are marked with an asterisk.

5.5 Value-at-Risk Analysis

5.5.1 Value-at-Risk Theory

The Value-at-Risk is a popular measure of risk for portfolios and was used first for financial applications by J.P. Morgan in 1994. The VaR at confidence Level α is defined as

$$VaR^{\alpha}(L) = -\inf\{l \in \mathbb{R} : Pr(L > l) \leq 1-\alpha\}, \quad \alpha \in (0,1) \tag{5.3}$$

where inf denotes the infimum, L the loss[4], see e.g. Artzner et al (1999).

Literally, the VaR is the number that will not be exceeded by a loss with a probability of $1-\alpha$. We report the VaR as a positive number since this is the usual convention, see e.g Christoffersen and Pelletier (2004) . The 1% 1 day ahead VaR ($\alpha = 0.01$) is often written VaR(1/1) and we follow this notation. However, VaR has significant drawbacks and we will discuss these below. A first drawback is that the VaR measure has nothing to say about the severity of the loss that occurs with probability $\leq \alpha$. Another related critique is the non-subadditivity of VaR, i.e. the VaR of a portfolio can be greater than the sum of the single VaRs, see Artzner et al (1997) and Artzner et al (1999). This contradicts a main proposition of modern finance theory, namely that there should be diversification benefits when combining several assets if they are not perfectly dependent. A risk measure that is sub-additive is the Expected Shortfall (hereafter ES)

$$ES^{\alpha}(L) = -E(L|L < -VaR^{\alpha}) \tag{5.4}$$

The ES can be interpreted as the expected value of the loss beyond the VaR and is also reported as a positive number. Common in practice are the VaR(1/1), the VaR(5/1), and finally the VaR(10/1). We use these three different levels since they highlight the ability of the model to estimate different risk levels. The VaR(1/1) can be interpreted as the capability of the model to capture 'extreme' behavior, the VaR(5/1) displays more the 'intermediate' range and the VaR(10/1) the 'lower' one. We estimate the VaR via the Monte Carlo method, which implies full repricing of the portfolio at every point in time. To keep a balance between the computational costs and the accuracy of the results we simulate five thousand portfolio values for each point in time. As the benchmark model we choose the Historical Simulation (hereafter HS) method. In this approach the loss function L is the empirical distribution of the equal weights portfolio returns $y_{t-n+1,1}, \ldots, y_t$, where we use $n = 100$.

The first parametric model we use is the so called Delta-Normal (hereafter DN) method. The VaR estimated by the DN method is described by

$$VaR^{\alpha}_{t+1} = -\left((w_{1,t}, \ldots, w_{d,t}) \begin{bmatrix} h_{11,t} & \cdots & h_{1d,t} \\ \vdots & \cdots & \vdots \\ h_{d1,t} & \cdots & h_{dd,t} \end{bmatrix} (w_{1,t}, \ldots, w_{d,t})' \right)^{1/2} \cdot s^{\alpha},$$

[4] The Loss L is calculated as $L = V_{t+1} - V_t$ where V_t denotes the portfolio value at time t.

where s^{α} is the quantile at confidence level α from a standard normal distribution. The variance and the covariance this time are the simple sample variance and covariance estimates. To make a comparison between the HS and DN method possible we estimate both on the past 100 returns. Clearly the Delta-Normal method is based on the multivariate Gaussian distribution. Since this does not own any QML properties it is interesting to see how it compares to the DCC model.

Backtesting the VaR is useful to detect any misspecification of the risk model. Below we will introduce some common backtesting techniques, following Christoffersen (2003), Christoffersen and Pelletier (2004), and Christoffersen (2008).

First of all a visual inspection of the VaR violations is done. The 1% confidence level one-day ahead VaR predicts one VaR violation every 100 days. A VaR violation occurs if the realized loss is greater than the predicted VaR. Thus, a first check might be to see if the number of occurred VaR violations equals the number of expected violations. Another common check when backtesting VaR's is defining a so called 'hit-sequence'

$$Hit_{t+1} = \begin{cases} 1, & \text{if } L_{t+1} < -VaR_{t+1}^{\alpha} \\ 0, & \text{if } L_{t+1} > -VaR_{t+1}^{\alpha}. \end{cases}$$

The 'hit-sequence' is kind of a indicator function that returns a 1 if the loss exceeds the predicted VaR and 0 if not for each point in time. Following Christoffersen, we use the notation $t+1$ to make clear that the VaR is predicted from period t to period $t+1$. It has been mentioned above that- in a correctly specified risk model- the VaR should be violated with probability α. It could be the case that the number of violations indicates a perfect risk model: 75 violations for a VaR(10/1) model for this analysis. However, this assumption would be misleading since the violations needs to be independently distributed. This intuitively makes sense since 75 violations in a time period of 75 days clearly leads to the rejection of the risk model although in a sample of 750 observations the number of violations is as expected for the VaR(10/1). The information that the violations of a perfect risk model must be independent can be used to construct a formal test. Completely unpredictable variables (that can only take two values) are modelled with a Bernoulli distribution and thus the null hypothesis of the test can be stated as

$$H_0 : Hit_{t+1} \sim Bernoulli(p),$$

where the Bernoulli distribution function may be written

$$f(Hit_{t+1}) = (1-p)^{1-Hit_{t+1}} p^{Hit_{t+1}}.$$

The variable p describes the probability of an event to occur. In the case of VaR backtesting p takes the value of α, i.e. 0.01, 0.05, or 0.10.

Based on this theory Christoffersen develops several VaR-tests. The first is the 'unconditional coverage' test which verifies if the numbers of violations is significantly different from p. That is to say, it checks if the VaR model delivers violations across the sample with probability p. Therefore the likelihood of an *i.i.d.* Bernoulli hit sequence needs to be defined as

$$L(\pi) = \prod_{t=1}^{T} (1-\pi)^{Hit_{t+1}} \pi^{Hit_{t+1}} = (1-\pi)^{T_0} \pi^{T_1},$$

where T_1 represents the number of violations and T_0 the number of non-violations in the back-testing sample. The variable $\hat{\pi}$ is estimated by T_1/T, i.e. it is just the unconditional probability of a violations. This leads to a sample likelihood of

$$L(\hat{\pi}) = \left(1 - \frac{T_1}{T}\right)^{T_0} \cdot \left(\frac{T_1}{T}\right)^{T_1}$$

and with the previously stated null hypothesis, i.e. $\pi = p$

$$L(p) = \prod_{t=1}^{T} (1-p)^{1-Hit_{t+1}} p^{Hit_{t+1}} = (1-p)^{T_0} p^{T_1}.$$

The test statistic is calculated by an likelihood ratio test

$$LR_{uc} = -2\ln\left[\frac{L(p)}{L(\hat{\pi})}\right] \overset{A}{\sim} \chi_1^2.$$

A second test is concerned with the independence of the violations. To take a look only at the sheer number of observation could be very misleading in determining the perfect risk model since the violations should not be predictable. This prerequisite makes it necessary to construct a test that is able to give some information about the independence of the violations. Fortunately this task can be easily performed by an Ljung-Box test

$$LB(k) = T(T+2)\sum_{k=1}^{m} \frac{\gamma^2}{T-k} \sim \chi_m,$$

where m denotes the d.o.f. and the γ the autocorrelation at lag length k for the hit sequence Hit_{t+1}. The null hypothesis states that the first m autocorrelations are zero. Berkowitz, Christoffersen, and Pelletier (2009) finds that $m = 5$ delivers good testing power for daily data and so we use this lag-length.

Another simple test is the means test

$$MT = \sqrt{T}\frac{\hat{\pi} - p}{\sqrt{var(Hit_{t+1})}} \sim N(0,1).$$

The intuition behind this test is that the standardized unconditional violation probability behaves like a standard normal.

Copulas and VaR analysis have also been considered by Fantazzini (2008), Giacomini and Härdle (2005), Huang et al (2009), and Ozun and Cifter (2007). A D-vine copula for VaR estimation in a Bayesian setting has been considered by Hofmann and Czado (2011). Especially with emerging markets occupied is e.g. Bao, Lee, and Saltoğlu (2006) who investigate the VaR behavior of different models through the Asian crisis, thus giving insight into emerging markets whilst Dimitrakopoulos, Kavussanos, and Spyrou (2010) compares various VaR approaches for 16 different emerging markets and finds no outperformance for a specific model. Gençay and Selçuk (2004) uses extreme value theory models to estimate the VaR and finds that the generalized pareto distribution fits the data well.

5.5.2 Multivariate Model Determination

In this section we examine the ability of the miscellaneous models to estimate the VaR of the different portfolios.

The model of Cappiello, Engle, and Sheppard (2006) incorporates symmetric (DCC, G-DCC) and asymmetric (A-DCC, AG-DCC) specifications for the time-varying $\mathbf{Q}_t$ matrix. These different dynamic structures are applied to the MVGARCH and several copula models. Since estimating the VaR for all different specifications would be an elaborate task we explain in the following the procedure we have chosen to determine the best dynamic structure within a given model.

First of all, we estimate all models with different dynamic structures for the full sample. Based on these results and the AIC and BIC criterion we choose the best fitting dynamic structure for the VaR estimation. Another criterium is the amount of significant parameter. When- for whatever reason- the AIC and BIC criteria choose an asymmetric model where most of the asymmetry parameters are insignificant we investigate the corresponding symmetric model. Thereafter, we decide which model fits better. For the D-vine copulas we restrict ourselves to the scalar dynamics DCC and A-DCC. The vine copulas consist of several pair-copulas which in turn are based on the data sample and ordered according to their (unconditional) dependence. As a result of this procedure it can be concluded that when the data is ordered in a bivariate manner the respective data pair displays a similar dependence structure and so reacts to news in the same way. The multivariate dynamic mixture copula models consists of an elliptical copula and a multivariate archimedean one. For the elliptical copula in the mixture we use the optimal dynamic structure of the elliptical copula estimated alone. All copula models are estimated by the IFM procedure explained in section 4.5. The next chapter covers a complete dependence structure analysis so we will keep the analysis of this process here as short as possible.

The first portfolios we consider are the single developed market stock and bond indices. Table E.1 shows the results of the AG-DCC MVGARCH estimation for the developed market stocks for the full sample. The BIC criterion favors the A-DCC model and the AIC criterion the AG-DCC model. Since only three out of five asymmetric parameters in the AG-DCC model are significant we choose the more parsimonious A-DCC model as the best fitting one. Table E.2 shows the parameter estimates for the developed market bonds estimated with the MVGARCH models. Here the BIC criterion favors the DCC model while the AIC criterion prefers the AG-DCC model. Since no asymmetric parameter of the AG-DCC model is significant we choose the DCC model as the better fit. Table E.5 reports the results of the Gaussian copula for the developed markets stocks. Again, the BIC criterion favors the A-DCC model while AIC favors the AG-DCC one. Based on the results above (again only three out of five asymmetric parameters are significant) we choose the A-DCC dynamic structure as the best fit.

Table E.6 shows the results for the developed market bonds estimated with a Gaussian copula. Here it can be seen that the BIC criterion prefers the DCC model and the AIC criterion the G-DCC model. Since all G-DCC parameters are significant we conclude the G-DCC structure is the best fit. After comparing the Gaussian dependence structures we consider the results of the t-copula. As above we first investigate the developed market stocks (see Table

E.9) for which both criteria favor the A-DCC model. To complete the comparisons between the developed markets stock and bond indices and the Gaussian and t-copula the parameter estimates for the developed market bonds estimated with a t-copula are missing. This time the BIC criterion favors the DCC while the AIC favors the G-DCC. Since again all parameters for the G-DCC structure are significant we select G-DCC as our favorite dynamic structure for the developed bond indices. Table E.9 reports the parameter estimation results.

The final copula class that exhibits different dynamic dependence structures is the vine copula class introduced in subsection 4.2.3. Table E.13 shows the bivariate dependence parameters of the developed stock indices for the Gaussian, t-, Clayton, and rotated Clayton D-vine copulas. We follow Aas et al (2009) and order the data according to their bivariate unconditional dependence. The pair with the highest dependence then ascertains the first two variables in Figure 4.5. The order of the first variables follows from the dependence of each of these variables with all other variables. For example for the t-copula the highest dependence (measured by the lowest d.o.f.) is between Sweden and Italy ($\nu = 6.7416$). The highest dependence between one of these two countries and all other countries is between Sweden and Denmark ($\nu = 10.2089$). Therefore, the first variable is Denmark, the second Sweden, and the third Italy. This routine is repeated until the whole sample is ordered. The same procedure is applied to the developed bond indices. Table E.14 shows the results.

Table E.15 shows the log-likelihood, AIC, and BIC values of the vine estimation for the developed market stock and bonds. The developed stocks prefer the A-DCC model while for the developed bond the DCC model is selected for all vine copulas. The first conclusion is that the asymmetric correlation test does not give any hints about the dependence structure. The test of Hong, Tu, and Zhou (2007) indicates a more asymmetric correlation in the bonds than the stock indices case. The vine model selection criteria reveal a different picture. They choose an asymmetric model only in the case of stock indices. After explaining the different models of the developed stock and bond indices we turn now to the emerging market stock and bond indices. The procedure to determine the best model is the same as with the developed markets.

For the emerging market stock indices estimated with the MVGARCH model the AIC and BIC criteria both favor the A-DCC dynamic structure- as for the emerging bond indices (see tables E.3 and E.4). Table E.7 shows the results of the Gaussian copula estimation for the emerging market stocks. Again, the A-DCC model structure is chosen by both criteria as for the emerging market bonds (see Table E.8). Table E.11 reports the estimation results for the t-copula emerging market stocks. In contrast to the d.o.f. parameter of the developed market stocks in Table E.9 the d.o.f. parameter across the different dynamic structures is pretty close to each other. A significant difference is only detected between the symmetric and asymmetric models. Nevertheless, the A-DCC model is the best fitting one. Table E.12 reports the results of the emerging market bond t-copula estimation. Again, the A-DCC model is chosen as the best fit. Table E.18 reports the log-likelihood values and the AIC and BIC criteria for the advanced emerging stock and bond indices estimated with the D-vine copula. For the advanced emerging markets stocks we choose the DCC model as the best fitting one for all vine copulas. The Gaussian vine performs better with the asymmetric A-DCC structure when estimating the advanced emerging market bonds. Tables E.16 and E.17 show the unconditional pair dependence and the data-ordering for the pair-copulas.

Having explained the full sample properties of the different portfolios and the selection of the best model we will now to examine the Value-at-Risk estimation abilities of the miscellaneous models.

5.5.3 VaR Empiricism

In order to find out if the fit of a risk model is adequate a backtest is needed. The VaR-backtesting procedure can be explained as follows: for all parametric models, i.e. for all models except the HS, we estimate the coefficients of the dependence models with the data sample running from May 02, 2003 to Aug 08, 2007. The backtesting routine then runs from August 09, 2007 to Aug 09, 2010. This gives 1075 data points for the sample that is used to estimate the multivariate model coefficients and 750 data points for the VaR backtest sample. Within this backtest sample we keep the coefficients of the multivariate time-varying dependence models constant. Through the backtesting period we update the AR-GARCH models five times, i.e. every 150 days. In total, this procedure gives 750 VaR predictions for each model. To these predictions we apply the set of tests described above with a view to finding the best risk model. Within this backtest sample the optimal risk model should show 7.5 VaR violations for the 1% confidence level VaR, 37.5 violations for the 5%, and 75 violations for the 10% level.

We also report other measures ancillary to the tests and VaR violations. One of them is the maximum VaR exceedance, which describes the distance between $|VaR_{t+1}|$ and $|L_{t+1}|$ if a VaR violation occurred[5]. If different risk models have the same number of VaR violations and perform identically on the VaR tests a conservative risk manager might choose the risk model with the smallest VaR exceedance. If an unpredicted loss appears, the difference between the loss and the VaR should be as small as possible. We report only the maximum VaR exceedance for the 1% VaR since this is the most common VaR used in practice. We also test for the Expected Shortfall defined in (5.4) and follow McNeil and Frey (2000) who defines empirical residuals $er_{t+1} = L_{t+1} - (-ES^{\alpha}_{t+1}(L_{t+1} < -VaR^{\alpha}_{t+1}))$, where L_{t+1} is the realized portfolio return on day $t+1$ and $ES^{\alpha}_{t+1}(L_{t+1} < -VaR^{\alpha}_{t+1})$ the expected shortfall at time $t+1$ predicted at time t when the realized loss is greater than the VaR at the respective confidence level [6]. We report the minimum exceedance residual ($\min er_{t+1}$) for the 1% confidence level VaR. A negative number indicates that the expected shortfall has been violated, i.e. the loss is greater than the expected shortfall. On the contrary, whenever $\min(er_{t+1})$ is positive the expected shortfall is not violated. In both cases a risk manager might prefer a model where the estimated ES is as close as possible to the realized loss. Banks for example have to deposit capital for risky positions. When a risk model overestimates the true risks the bank has to build to much accruals, decreasing their profits. When the risk model underestimates the risk to much capital is allocated to risky positions and that might hurt the profits, too. Thus in both cases a conservative risk manager might prefer a risk model where $|\min(er_{t+1})|$ is smaller compared to other risk models.

The final number we consider is a rather simple one and shows the absolute difference

[5] We report the distance since in every model tested at least one VaR violation occured.

[6] Note that the expected shortfall is reported as a positive number, see equation (5.4).

between expected VaR violations and VaR violations which occur when comparing the one-day-ahead VaR with the realized loss. The lower this number, the better the model fits (when looking only at the number of violations). We do this for each VaR confidence level under the heading 'error'. To compare the different models over all confidence levels we sum these errors under the heading 'error sum'. Both numbers are found in the Table where the backtest sample is partitioned into three periods. We calculate the absolute difference for every period. The column 'error' shows the arithmetic mean of the errors calculated for each period.

To recall, there are four different classes of risk models we test: First, a naive risk model is applied to the data. This is the HS method and is a pretty simple model which acts as the benchmark. The second class is the Delta-Normal (DN) method which is a simple parametric method. The other two classes of dependence models need to beat the naive and the DN VaR forecast otherwise it would not make any sense to invest the time (and probably money) building more sophisticated models. The third model class consists of the MVGARCH model. It is more sophisticated than HS but is limited to symmetric dependence and Gaussian marginals. The final class contains the copulas who are able to capture a wide range of dependence structures and marginal models. It is important to mention that the marginal models of the different copulas for the respective portfolio are always the same but the marginals between the MV-GARCH and copula models differ. Therefore, the ability of the copula models to predict the VaR is solely dependent on the dependence structure of the respective copula. In general it should be noted that the tests presented below does not give any direct information about the dependence structure of the data. It is the backtest which considers the complete risk model, encompassing both the marginal and dependence models. Section 5.6 below contains an analysis of determining the (true) dependence structure.

Table 5.2 gives an overview of the different VaR backtest results for the developed stock and bond indices. According to Christoffersen a 10% confidence level is appropriate for the VaR-tests and so all p-values in the table are calculated at this level. The first portfolio we investigate in detail is the developed market stocks. This is the portfolio with the highest kurtosis of any we investigate. For the VaR(1/1) the MVGARCH model delivers 7 violations which is pretty close to the 7.5 expected but the Ljung-Box (LB) test rejects the null hypothesis that the violations are independently distributed. All copula models deliver three violations when 7.5 are expected for the 1% confidence level VaR. Hence in contrast to the MVGARCH model the copulas overestimates the risk. Both, the unconditional coverage (LR_{uc}) and the LB test are rejected for all copula models, too. The HS method has fourteen violations and the DN 26 for the VaR(1/1) which is far above the 7.5 expected. Figure 5.1 shows the VaR(1/1) estimations of the developed stocks portfolio for different models. This pictures depicts the differences between the copula estimation, the MVGARCH, the DN, and the HS method. The different copula models does not show any significant differences in their VaR estimates, so we only show the newly developed t-Clayton mixture Copula. A significant difference can be detected between the copula and MVGARCH model. Most of the time the MVGARCH VaR estimate is below the estimate derived from the copula models although the MVGARCH model shows the highest peak. As might be expected the VaR increases dramatically after the default of the Lehman Brothers bank on Sep 15, 2008- indicated by the vertical line. Since all of the marginal models for the copula are estimated with either a $skew-t$ or Student-t innovation assumption it is no surprise that most of the time the VaR derived from the Gaussian

Table 5.2: VaR Backtest Developed Markets.

	Model	VaR Violations			VaR Tests LR_{uc} p-value			LB p-value			MT p-value			$\min er_{t+1}$	max VaR exceed
VaR Confidence Level		1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	1%
Expected no. of Viol		7.5	37.5	75											
Developed Stocks															
Historical Sim		14	48	86	0.0000	0.0000	0.0000	0.0332	0.0911	0.1897	0.0797	0.1175	0.2077	—-	0.0420
Delta-Normal		26	55	84	0.0000	0.0143	0.1405	0.0000	0.0060	0.1225	0.0000	0.0000	0.0000	—-	0.0521
MVGARCH	A-DCC	7	32	64	0.8528	0.3450	0.1707	0.0155	0.2713	0.0507	0.8495	0.3207	0.1508	-0.0100	0.0158
MVCopula (GA)	A-DCC	3	28	69	0.0603	0.0963	0.4517	0.0000	0.0106	0.0253	0.0093	0.1077	0.4487	0.0048	0.0108
MVCopula (T)	A-DCC	3	28	68	0.0603	0.0963	0.3874	0.0000	0.0106	0.0142	0.0093	0.0675	0.3737	0.0036	0.0100
Vine (GA)	A-DCC	3	28	68	0.0603	0.0963	0.3874	0.0000	0.0106	0.0550	0.0093	0.0675	0.3737	0.0028	0.0111
Vine (T)	A-DCC	3	28	68	0.0603	0.0963	0.3874	0.0000	0.0106	0.1402	0.0093	0.0675	0.3737	0.0036	0.0106
Vine (CL)	A-DCC	3	28	62	0.0603	0.0963	0.1039	0.0000	0.0106	0.0336	0.0093	0.0675	0.0850	0.0032	0.0134
Vine (RCL)	DCC	3	27	66	0.0603	0.0646	0.2644	0.0000	0.0273	0.0215	0.0093	0.0397	0.2463	0.0043	0.0116
Mix Copula (GA-CL)	A-DCC	3	28	67	0.0603	0.0963	0.3222	0.0000	0.0106	0.0330	0.0093	0.0675	0.3061	0.0038	0.0116
Mix Copula (T-CL)	A-DCC	3	28	68	0.0603	0.0963	0.3874	0.0000	0.0106	0.0697	0.0093	0.0675	0.3737	0.0053	0.0110
Mix Copula (GA-GU)	A-DCC	3	27	66	0.0603	0.0946	0.2644	0.0000	0.0042	0.0212	0.0093	0.0675	0.5305	0.0032	0.0134
Mix Copula (T-GU)	A-DCC	3	29	68	0.0603	0.1387	0.3874	0.0000	0.0234	0.0550	0.0093	0.1077	0.3737	0.0048	0.0129
MV Archimed Copula (GU)		3	29	68	0.0603	0.1387	0.3874	0.0000	0.0234	0.0142	0.0093	0.1077	0.3737	0.0036	0.0138
MV Archimed Copula (CL)		3	28	68	0.0603	0.0963	0.3847	0.0000	0.0106	0.0439	0.0093	0.0675	0.3737	0.0062	0.0124
Developed Bonds															
Historical Sim		12	49	86	0.0000	0.0000	0.0000	0.1288	0.0625	0.1897	0.1906	0.0895	0.2077	—-	0.0083
Delta-Normal		13	37	70	0.1241	0.9329	0.5305	0.0675	0.9331	0.5387	0.9464	0.0517	0.2251	—-	0.00909
MVGARCH	DCC	2	11	30	0.0000	0.0000	0.0000	0.0165	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0034
MVCopula (GA)	G-DCC	1	12	32	0.0027	0.0000	0.0000	1.0000	0.9621	0.8442	0.0000	0.0000	0.0000	0.0001	0.0040
MVCopula (T)	G-DCC	1	12	33	0.0027	0.0000	0.0000	1.0000	0.9621	0.5588	0.0000	0.0000	0.0000	-0.0001	0.0037
Vine (GA)	DCC	1	12	33	0.0027	0.0000	0.0000	1.0000	0.9621	0.5873	0.0000	0.0000	0.0000	0.0001	0.0042
Vine (T)	DCC	1	12	32	0.0027	0.0000	0.0000	1.0000	0.9621	0.8442	0.0000	0.0000	0.0000	-0.0001	0.0036
Vine (CL)	DCC	2	12	27	0.0165	0.0000	0.0000	1.0000	0.9621	0.6663	0.0000	0.0000	0.0000	-0.0027	0.0015
Vine (RCL)	DCC	2	12	28	0.0165	0.0000	0.0000	1.0000	0.9621	0.6701	0.0000	0.0000	0.0000	-0.0026	0.0015
Mix Copula (GA-CL)	G-DCC	1	12	32	0.0027	0.0000	0.0000	1.0000	0.9621	0.8442	0.0000	0.0000	0.0000	-0.0001	0.0037
Mix Copula (T-CL)	G-DCC	1	12	34	0.0027	0.0000	0.0000	1.0000	0.9621	0.5873	0.0000	0.0000	0.0000	-0.0026	0.0042
Mix Copula (GA-GU)	G-DCC	1	12	33	0.0027	0.0000	0.0000	1.0000	0.9621	0.5588	0.0000	0.0000	0.0000	0.0001	0.0035
Mix Copula (T-GU)	G-DCC	1	12	32	0.0027	0.0000	0.0000	1.0000	0.9621	0.5617	0.0000	0.0000	0.0000	0.0001	0.0040
MV Archimed Copula (GU)		1	12	33	0.0027	0.0000	0.0000	1.0000	0.9621	0.5588	0.0000	0.0000	0.0000	-0.0001	0.0037
MV Archimed Copula (CL)		1	12	32	0.0027	0.0000	0.0000	1.0000	0.9621	0.8442	0.0000	0.0000	0.0000	-0.0001	0.0033

Notes to table: This table reports the VaR violations and p-values for the different test based on the violations for the developed stock and bond portfolios. All p-values are calculated at 10% confidence level.

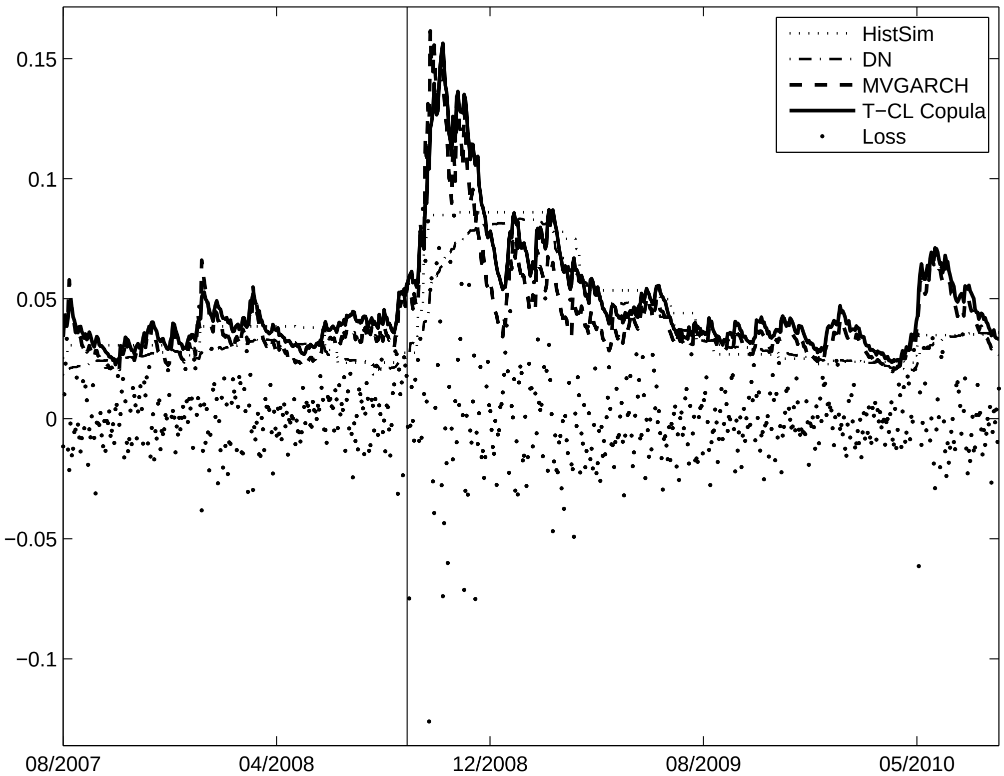


Figure 5.1: VaR Developed Stocks. This figure shows the 1% confidence level VaR estimates for the developed stock indices portfolio. The vertical line shows the default of the Lehman Brothers bank.

copula is higher than the VaR derived from the MVGARCH model. But for the 1% confidence level VaR the fat-tailed innovation assumption does not improve the VaR estimation. The fat tails generated through the QML properties of the Gaussian univariate DCC-GARCH seems to be enough for the developed stocks portfolio. For the VaR(5/1) the MVGARCH model beats all other models with 34 VaR violations (against an expectation of 37.5) This time all null hypotheses are accepted. The violations of the copula models are between 27 and 29. Only the t-Gumbel mixture and the multivariate Gumbel copula do not reject the LR_{uc} test. The LB independence test is rejected by all copula models and the same goes for the means test (MT) with the exception of the t-Gumbel and Gumbel copulas. The HS delivers 48 violations and only the null hypothesis of the MT test is accepted. The same applies to the DN with 55 violations. The final VaR we consider is the VaR(10/1) with 75 expected violations. This time the MVGARCH falls short of some copula models with only 64 violations where the copula models range from 62 (Clayton vine) to 69 (Gaussian copula). This time all copula models pass the LR_{uc} test, reject the independence test and pass the MT test. The HS has 86 violations, passing the LB and MT test whereas the DN method shows 84 violations but passes none of the tests.

Another interesting set of results derives from the maximum VaR exceedance at 1% confidence level. Here the DN delivers the highest number with 0.00521. The MVGARCH displays the third highest number with 0.00158. The t-vine copula denotes the lowest VaR exceedance value with 0.00106. Of note is the minimum exceedance residual ($\min er_{t+1}$) for which only the MVGARCH shows a positive number. This implies that the expected shortfall is at least once not violated. This is in start contrast to the copula models where the expected shortfall is violated whenever the VaR is violated.

So far it can be concluded that the MVGARCH delivers the best results for the developed stocks portfolio with a 1% confidence level VaR. In Table 5.3 we break down the backtesting sample into three periods each with 250 trading days. The backtesting period covers a relatively quiet period for the first 250 trading days from Aug 09, 2007 to Aug 11, 2008 a turbulent second period from Aug 12, 2008 until Aug 11, 2009- with the breakdown of Lehman Brothers on the Sep 15, 2008 -and another tranquil period from Aug 12, 2009 to Aug 09, 2010. we have chosen this kind of break down because it divides the sample in a relatively quiet period (the first one), a turbulent period (the second period with the default of the Lehman Brothers bank) and a third 'intermediate' section. It might be interesting to see if the VaR violations are distributed equally across these three periods or if e.g. in the turbulent period more violations occur. The MVGARCH model performed best for the VaR(1/1) with 7 violations. Of these 7 violations 2 occurred in the first backtesting period, 5 in the second and 0 in the third. Since every period has 250 trading days 2.5 VaR violations would be expected within each of the three backtesting periods.

By analyzing the results it can be seen that for the VaR(1/1) the MVGARCH model performs well in the first period underestimates the risk in the turbulent second period and overestimates the risk in the third period. For the 5% confidence level VaR the MVGARCH performs well in the crisis period with 13 violations (where 12.5 would be expected). In the final period it overestimates the risk showing 9 violations when 12.5 would be expected. For the VaR(1/1) all copula models show 1 violation in the first, 2 violations in the second and no violations in the third period. For the 5% VaR most copula models perform relatively well in the first and second periods but fail in the third period. For the 10% VaR the picture is reversed: here the copula models show good performance in the first and second periods but perform poorly in the third period.

Considering the sum of the violations of the different VaR levels the Gaussian copula performs best for the developed stocks portfolio with an error sum of 7. The DN method shows the worst performance with an error sum of 16.33.

The next portfolio we investigate is the developed market bonds. This is the portfolio with the lowest annualized volatility and kurtosis, and the only one with positive skewness. All parametric models perform poorly for all confidence levels because they overestimate the risks dramatically (see Table) 5.2. Only the HS and DN methods shows different results. The HS underestimates the risk for all confidence levels whereas the DN does this only for the 1% confidence level. All violation numbers are far away from their expected values. Figure 5.2 shows the 1% confidence level VaR estimates for the developed bonds. The first thing to note is that it seems curious that the VaR for the MVGARCH model peaks at a higher level than the VaR of the copula models, but this might be due to the different selection of GARCH models for the marginal distributions. Again, it can be noted that the VaR(1/1) of the different copula models is pretty close together and a difference is only detected between the DN, MVGARCH, the copula models, and the nonparametric HS method. We do not consider any further investigations of the developed bond portfolio since all models underestimate the true risk by a considerable amount. One might conclude that this is because of the specification of the margins by AR-GARCH models. Considering the error sum the DN method performs best with 9.00: all parametric models have an error sum higher than 20.

The next portfolio we investigate is the emerging markets stocks (see table 5.4). The main characteristic of this portfolio is that it has the highest annualized volatility and the smallest and highest daily returns. This time the MVGARCH seems to underestimate the VaR with ten violations at the 1% confidence level whereas most copula models are around seven. The HS method shows nine violations, and the DN method overestimates enormously, showing twenty-one violations. For the 5% VaR the MVGARCH has thirty-six VaR violations just as most copula models only the Clayton-vine copula falls short with just thirty-one violations. Again, the HS method underestimates the VaR with forty-eight violations, far more than the thirty-seven and a half expected. The same is true for the DN method with fifty-three violations. For the 10% VaR the picture differs a little. Now the MVGARCH has seventy-three violations while the copula models vary between sixty-five (Clayton vine) and eighty-one (t-copula). The HS method meanwhile reports eighty-three and the DN eighty-five. All of the models accept the null hypothesis of the unconditional coverage test bar the HS (for all confidence levels) and DN (with 1% and 5% confidence levels). The independence test for the 1% and 5% confidence level are passed by all models whereas for the 10% confidence level only the multivariate t-copula and Gumbel copula pass the test. The MT test is successfully completed by all models for all confidence levels except for the HS VaR(10/1) and the DN method, which fails to pass at any confidence level. This time $\min(er_{t+1})$ is positive for all models implying that whenever the 1% confidence level VaR is violated the ES is also violated. The MVGARCH shows the greatest maximum exceedance residual and the greatest max VaR exceedance.

It is also necessary to analyze how the VaR violations partition into the three periods we defined above by looking at Table 5.5. For the 1% confidence level VaR all copula models

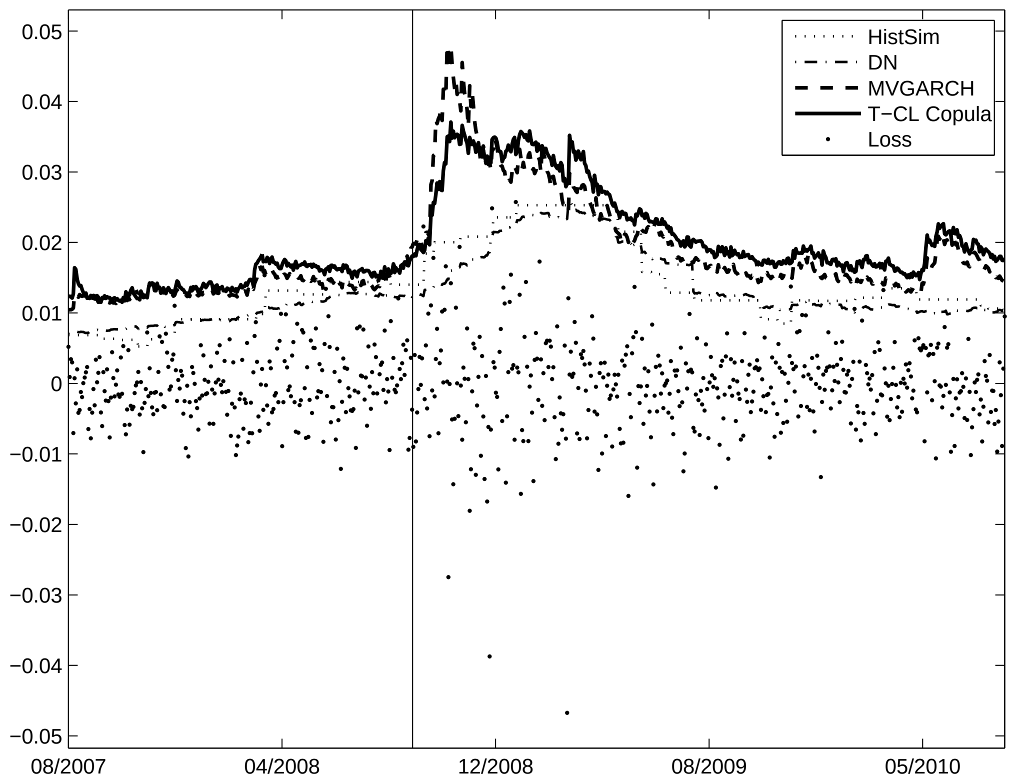


Figure 5.2: VaR Developed Bonds. This figure shows the 1% confidence level VaR estimates for the developed bonds portfolio. The vertical line shows the default of the Lehman Brothers bank.

Table 5.3: VaR Backtest II Developed Markets.

	Model	1% VaR Period 1	1% VaR Period 2	1% VaR Period 3	error	5% VaR Period 1	5% VaR Period 2	5% VaR Period 3	error	10% VaR Period 1	10% VaR Period 2	10% VaR Period 3	error	sum error
Expected no. of Viol		2.5	2.5	2.5		12.5	12.5	12.5	12.5	25	25	25		
							Developed Stocks							
Historical Sim		2	7	5	2.50	12	19	7	3.83	26	31	29	3.76	10.00
Delta-Normal		5	13	8	6.17	14	22	18	5.83	28	32	28	4.33	16.33
MVGARCH	A-DCC	2	5	0	1.83	10	13	9	2.17	20	22	22	3.67	7.67
MVCopula (GA)	A-DCC	1	2	0	1.50	10	12	7	2.83	18	25	26	2.67	7.00
MVCopula (T)	A-DCC	1	2	0	1.50	10	12	6	3.17	17	24	27	3.67	8.33
Vine (GA)	A-DCC	1	2	0	1.50	10	12	6	3.17	18	25	26	2.67	7.33
Vine (T)	A-DCC	1	2	0	1.50	10	12	6	3.17	18	25	25	2.33	7.00
Vine (CL)	A-DCC	1	2	0	1.50	10	12	6	3.17	16	25	21	4.33	9.00
Vine (RCL)	DCC	1	2	0	1.50	9	12	6	3.50	17	24	25	3.00	8.00
Mix Copula (GA-CL)	A-DCC	1	2	0	1.50	10	12	6	3.17	16	25	21	4.33	9.00
Mix Copula (T-CL)	A-DCC	1	2	0	1.50	10	12	6	3.17	18	24	25	2.67	7.33
Mix Copula (GA-GU)	A-DCC	1	2	0	1.50	10	11	6	3.5	18	22	26	3.67	8.67
Mix Copula (T-GU)	A-DCC	1	2	0	1.50	10	13	6	3.17	17	25	26	3.00	7.67
MV Archimed Copula (GU)		1	2	0	1.50	10	12	7	2.83	18	24	26	3.00	7.33
MV Archimed Copula (CL)		1	2	0	1.50	10	12	6	3.17	19	24	25	2.33	7.00
							Developed Bonds							
Historical Sim		7	3	2	1.83	20	14	15	3.83	36	25	25	3.67	9.33
Delta-Normal		4	6	3	1.83	15	14	8	2.83	29	22	19	4.33	9.00
MVGARCH	DCC	1	1	0	1.83	4	4	3	8.83	15	8	7	15.00	25.67
MVCopula (GA)	G-DCC	0	1	0	2.17	4	5	3	8.50	14	11	7	14.33	25.00
MVCopula (T)	G-DCC	0	1	0	2.17	4	5	3	8.50	14	12	7	15.00	24.67
Vine (GA)	DCC	0	1	0	2.17	4	5	3	8.50	14	12	7	14.00	24.67
Vine (T)	DCC	0	1	0	2.17	4	5	3	8.50	14	11	7	14.33	25.00
Vine (CL)	DCC	1	1	0	1.83	4	6	2	8.50	10	11	6	16.00	26.33
Vine (RCL)	DCC	1	1	0	1.83	4	6	2	8.50	11	10	7	15.67	26.00
Mix Copula (GA-CL)	G-DCC	0	1	0	2.17	4	5	3	8.50	14	11	7	14.33	25.00
Mix Copula (T-CL)	G-DCC	0	1	0	2.17	4	5	3	8.50	15	12	7	13.67	24.33
Mix Copula (GA-GU)	G-DCC	0	1	0	2.17	4	5	3	8.50	14	12	7	14.00	24.67
Mix Copula (T-GU)	G-DCC	0	1	0	2.17	4	5	3	8.50	13	12	7	14.33	25.00
MV Archimed Copula (GU)		0	1	0	2.17	4	5	3	8.50	14	12	7	14.00	24.67
MV Archimed Copula (CL)		0	1	0	2.17	4	5	3	8.50	14	11	7	14.33	25.00

This table reports the expected number of violations for the 750 trading days lasting backtest sample. Period 1 runs from 08/13/2007 to 08/11/2008, Period 2 from 08/12/2008 to 11/08/2009 and Period 3 from 08/12/2009 to 08/10/2020 so that every period covers 250 trading days. The column ‘error’ reports the absolute deviation of the VaR violations that occurred in the respective period and the expected number of VaR violations. The column ‘error sum’ reports the sum of the errors for every period. model.

Table 5.4: VaR Backtest Emerging Markets.

	Model	VaR Violations			VaR Tests LR_{uc} P-value			LB P-value			MT P-value			$\min er_{t+1}$	max VaR exceed
VaR Confidence Level		1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	1%
Expected no. of Viol		7.5	37.5	75											
Advanced Emerging Market Stocks															
Historical Sim		9	48	83	0.0000	0.0000	0.0000	0.5936	0.0911	0.3376	0.8590	0.3207	0.0616	—-	0.0448
Delta-Normal		21	53	85	0.0046	0.0273	0.2497	0.0000	0.0142	0.2323	0.0000	0.0000	0.0000	—-	0.0667
MVGARCH	A-DCC	10	36	73	0.3827	0.8003	0.8069	0.9832	0.3692	0.0887	0.4264	0.7979	0.8055	-0.0205	0.0300
MVCopula (GA)	A-DCC	7	36	78	0.8528	0.8003	0.7166	0.0155	0.5844	0.0450	0.8495	0.7979	0.7199	-0.0045	0.0143
MVCopula (T)	A-DCC	7	36	81	0.8528	0.8003	0.4703	0.0155	0.5844	0.1321	0.8495	0.7979	0.4806	-0.0061	0.0148
Vine (GA)	DCC	7	36	80	0.8528	0.8003	0.5467	0.0155	0.5844	0.0746	0.8495	0.6654	0.4130	-0.0035	0.0188
Vine (T)	DCC	7	37	79	0.8528	0.9331	0.6290	0.0155	0.6521	0.0622	0.8495	0.9337	0.5545	-0.0046	0.0168
Vine (CL)	DCC	5	31	77	0.3288	0.2622	0.2140	0.9996	0.1827	0.0692	0.8495	0.6654	0.7199	-0.0027	0.0163
Vine (RCL)	DCC	7	34	77	0.8528	0.5516	0.8084	0.0155	0.2525	0.1452	0.8495	0.5393	0.8100	-0.0034	0.0141
Mix Copula (GA-CL)	A-DCC	7	37	78	0.8528	0.9331	0.7166	0.0155	0.6521	0.0683	0.8495	0.9329	0.7199	-0.0053	0.0169
Mix Copula (T-CL)	A-DCC	7	36	79	0.8528	0.8003	0.6390	0.0155	0.5409	0.0542	0.8495	0.7979	0.6344	-0.0047	0.0151
Mix Copula (GA-GU)	A-DCC	7	36	79	0.8528	0.8003	0.6290	0.0155	0.5844	0.0622	0.8495	0.7979	0.6344	-0.0041	0.0159
Mix Copula (T-GU)	A-DCC	7	35	76	0.8528	0.6720	0.6290	0.0155	0.5058	0.0548	0.8495	0.6654	0.6344	-0.0027	0.0156
MV Archimed Copula (GU)		7	37	77	0.8528	0.9331	0.8084	0.0155	0.6521	0.1425	0.8495	0.9329	0.5545	-0.0103	0.0183
MV Archimed Copula (CL)		7	37	80	0.8528	0.9331	0.5467	0.0155	0.6521	0.0975	0.8495	0.9329	0.8100	-0.0054	0.0175
Advanced Emerging Market Bonds															
Historical Sim		18	42	80	0.0000	0.0000	0.0000	0.0011	0.4591	0.5467	0.0123	0.4751	0.5545	—-	0.0340
Delta-Normal		25	41	65	0.0000	0.5742	0.1946	0.0000	0.5632	0.2140	0.0000	0.0000	0.0000	—-	0.0395
MVGARCH	A-DCC	7	35	62	0.8528	0.6720	0.1039	0.0159	0.7712	0.3677	0.5378	0.6654	0.0616	-0.0066	0.0121
MVCopula (GA)	A-DCC	6	31	68	0.5684	0.2622	0.3874	0.0016	0.1827	0.0467	0.5389	0.2334	0.3737	-0.0066	0.0160
MVCopula (T)	A-DCC	6	32	68	0.5684	0.3450	0.3874	0.0016	0.2483	0.1988	0.5389	0.3207	0.3737	-0.0098	0.0186
Vine (GA)	A-DCC	6	31	69	0.5684	0.2622	0.4597	0.0016	0.1827	0.1169	0.5389	0.2334	0.4487	-0.0084	0.0179
Vine (T)	DCC	6	31	70	0.5684	0.2622	0.5387	0.0016	0.1827	0.1026	0.5389	0.2334	0.5305	-0.0105	0.0191
Vine (CL)	DCC	5	31	65	0.3288	0.2622	0.2410	0.9996	0.1827	0.0692	0.2623	0.2334	0.1946	-0.0052	0.0172
Vine (RCL)	DCC	5	31	69	0.3288	0.2622	0.4597	0.9996	0.1827	0.0873	0.2623	0.2334	0.4597	-0.0087	0.0183
Mix Copula (GA-CL)	A-DCC	6	32	70	0.5684	0.3450	0.5387	0.0016	0.2483	0.0331	0.5389	0.3207	0.5305	-0.0105	0.0196
Mix Copula (T-CL)	A-DCC	5	32	70	0.3288	0.3450	0.5387	0.9996	0.2483	0.1368	0.2623	0.3207	0.5305	-0.0084	0.0181
Mix Copula (GA-GU)	A-DCC	5	31	67	0.3288	0.2622	0.3222	0.9996	0.1827	0.1330	0.2623	0.2334	0.3061	-0.0089	0.0184
Mix Copula (T-GU)	A-DCC	6	31	69	0.5684	0.2622	0.4597	0.0016	0.1827	0.1160	0.2623	0.2334	0.3061	-0.0110	0.0196
MV Archimed Copula (GU)		5	31	67	0.3288	0.2622	0.3222	0.0000	0.1827	0.1330	0.2623	0.2334	0.3061	-0.0103	0.0198
MV Archimed Copula (CL)		6	34	70	0.5684	0.5516	0.5387	0.0016	0.0307	0.0673	0.5389	0.5393	0.5305	-0.0095	0.0194

Notes to table: This table reports the VaR violations and p-values for the different test based on the violations for the advanced emerging stock and bond portfolios. All p-values are calculated at 10% confidence level.

deliver one violation in the first period. For the second period all deliver four violations with the exception of the multivariate Gumbel which delivers two violations. In the third period all copula models deliver two violations. The MVGARCH delivers three violations in the first period, four in the second and three in the third. The HS method shows two violations in the first period, five in the second and two in the third. The DN method shows four in the first period, twelve in the second and again four in the third. For the 5% confidence level VaR all parametric models are pretty close together in all three periods. They show eight or nine in the first, between eighteen and twenty in the second and eight in the third period. Only the HS and DN method differ- the former showing fourteen violations in the first period, twenty in the second and fourteen in the third and the latter showing fifteen, twenty-one, and seventeen in the respective periods. At the 10% confidence level the parametric models differ a little bit more. In the first period the copula models report between nineteen and twenty-two violations whereas the MVGARCH has only sixteen, the HS twenty-five, and the DN twenty-three. In the second (turbulent) period all parametric models are between thirty-one and thirty-three violations whilst the HS produces thirty-four violations and the DN thrity-seven. In the final period the parametric models show twenty-six or twenty-seven violations, the HS method twenty-four violations and the DN twenty-five.

Figure 5.3 shows the 1% confidence level VaR for the different models. From this it can be seen that all models seem to underestimate the risk during the turbulent second period for all confidence levels. The MVGARCH VaR estimate is slightly below the copula models which are again fairly close together. After the default of the Lehman Brothers Bank the VaR increases for all models but for the HS method it stays on high level for a much longer period. No model passes all VaR tests: most pass either seven or eight out of nine tests. The $\min er_{t+1}$ is positive for all models, whereas the Clayton vine and t-Gumbel mixture show the smallest difference.

The last portfolio considered in detail is the emerging market bonds. First of all it should be noted that all models perform far better than for the developed bond indices portfolio. For the 1% VaR the HS method has eighteen violations, the DN twenty-five, the MVGARCH seven, and the copula models either five or six violations. For the 5% VaR the MVGARCH has thirty-five violations, the HS method fourty-two, and the DN forty-one. All copula models stay within twenty-nine (rotated Clayton vine) or thirty-four (multivariate Clayton). For the 10% VaR the MVGARCH shows only sixty-two violations when seventy-five are expected. Three copulas (Gaussian-Clayton, t-Clayton, and Clayton) show seventy violations and are closest to the number of expected violations along with the HS method with eighty violations. The DN method reports only sixty-five violations. Only four copulas do not pass all tests. The MVGARCH rejects only the LB independence test for the 1% confidence level VaR whilst the HS method passes only the 5% and 10% confidence level LB independence tests.

The null hypothesis of the MT means test is accepted for all parametric models at all confidence levels. All copula models underestimate the risk in the second period for the 5% and 10% confidence levels (see table 5.5). The MVGARCH model delivers here better results. Taking again a look at the sum of all errors the MVGARCH model performs best with 7.33: it also shows the lowest max exceedance VaR. For all parametric models the maximum exceedance residual is positive and again the MVGARCH shows the smallest number, implying the smallest difference between the realized loss and the violated expected shortfall. Figure

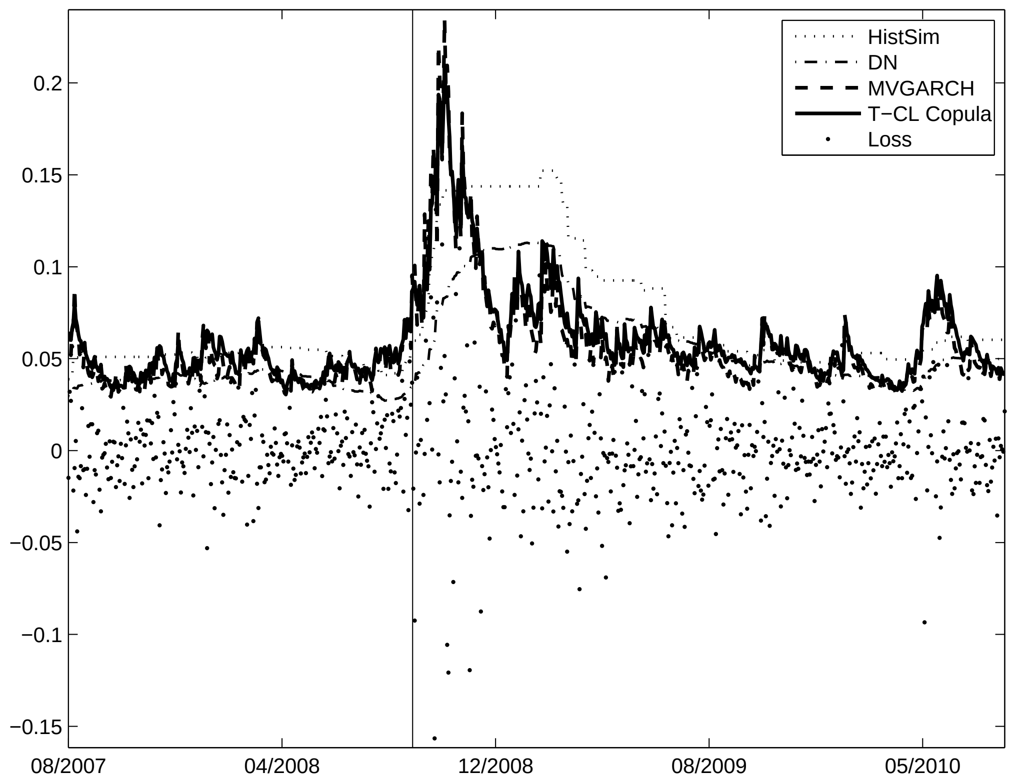


Figure 5.3: VaR Emerging Market Stocks. This figure shows the 1% confidence level VaR estimates for the advanced emerging market stock indices portfolio. The vertical line shows the default of the Lehman Brothers bank.

5.4 shows the 1% confidence level VaR estimations for the advanced emerging market bonds. The MVGARCH model show the highest peak which might be again through the different determination of the marginal models.

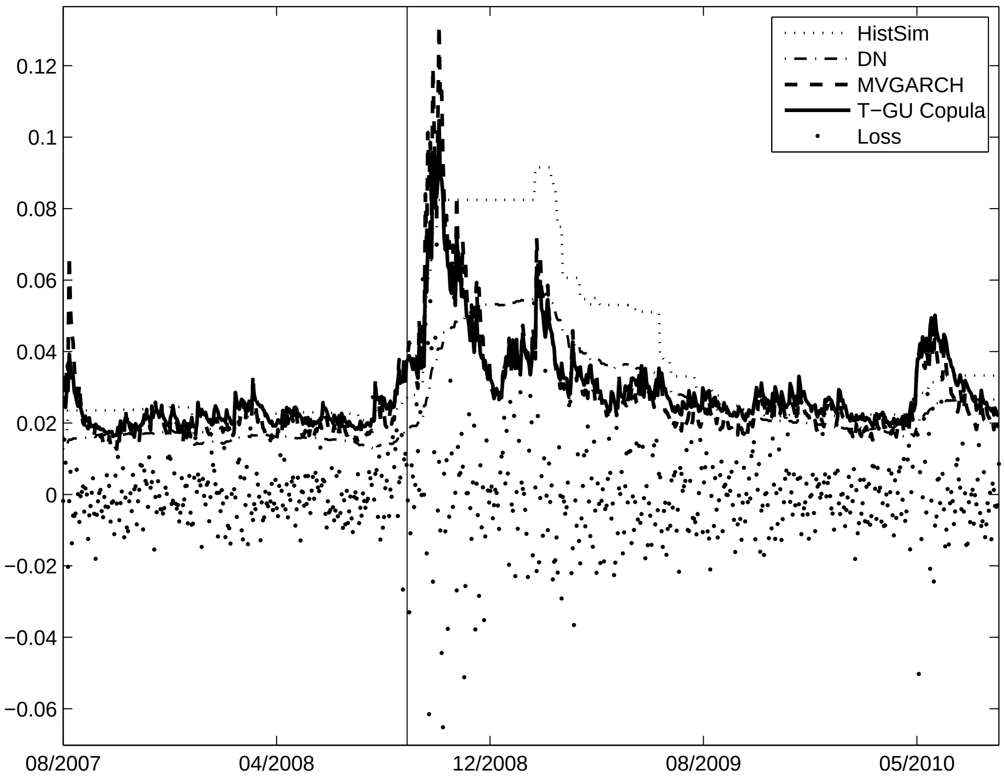


Figure 5.4: VaR Emerging Market Bonds. This figure shows the 1% confidence level VaR estimates for the advanced emerging market bond indices portfolio. The vertical line shows the default of the Lehman Brothers bank.

We will now sum up the results discussed above and find the best risk model for each portfolio.

For the developed stocks portfolio the MVGARCH seems to perform best for the 1% and 5% confidence levels. The t-vine copula is the only model that does not fail a test for the 10% confidence level VaR and thus seems to be the best model for this confidence level. It should be noted that the HS and DN methods overestimate the risk for all confidence levels. Especially for the 1% confidence level the DN method gives a particularly bad performance. For the developed bond indices all parametric models except for the DN method overestimate the risk and deliver far fewer VaR violations than expected. For these indices the DN method shows a relatively good performance for the 5% and 10% confidence levels. Overall the DN method performs better than the more sophisticated parametric models. For the advanced emerging market stocks several copulas show a good overall performance accepting eight out of nine null hypotheses that the VaR risk model is correctly specified. The only model that accepts all null hypotheses is the Clayton D-vine copula. However, it falls a little bit short when

Table 5.5: VaR Backtest II Emerging Markets.

	Model	1% VaR Period 1	1% VaR Period 2	1% VaR Period 3	error	5% VaR Period 1	5% VaR Period 2	5% VaR Period 3	error	10% VaR Period 1	10% VaR Period 2	10% VaR Period 3	error	sum error
Expected no. of Viol		2.5	2.5	2.5		12.5	12.5	12.5	12.5	25	25	25		
		Advanced Emerging Market Stocks												
Historical Sim		2	5	2	1.67	14	20	14	3.50	25	34	24	3.33	8.00
Delta-Normal		4	12	4	4.17	15	21	17	5.17	23	37	25	4.67	14.00
MVGARCH	A-DCC	3	4	3	0.83	8	20	8	5.50	16	31	26	5.33	11.67
MVCopula (GA)	A-DCC	1	4	2	1.17	9	19	8	4.83	21	31	26	4.83	10.83
MVCopula (T)	A-DCC	1	4	2	1.67	9	19	8	4.83	21	33	27	4.67	10.67
Vine (GA)	DCC	1	4	2	1.67	9	19	8	4.83	21	33	26	4.33	10.33
Vine (T)	DCC	1	4	2	1.17	9	20	8	5.17	21	32	26	4.00	13.33
Vine (CL)	DCC	1	4	2	1.67	9	18	8	4.50	20	31	26	4.00	10.33
Vine (RCL)	DCC	1	4	2	1.17	8	18	8	4.83	19	31	27	4.67	10.67
Mix Copula (GA-CL)	A-DCC	1	4	2	1.17	9	20	8	5.17	20	31	27	4.33	10.67
Mix Copula (T-CL)	A-DCC	1	4	2	1.17	8	20	8	5.50	22	31	26	3.33	10.00
Mix Copula (GA-GU)	A-DCC	1	4	2	1.17	9	19	8	4.83	20	33	26	4.67	10.67
Mix Copula (T-GU)	A-DCC	1	2	2	1.17	9	18	8	4.50	21	32	26	4.00	9.67
MV Archimed Copula (GU)		1	4	2	1.17	9	20	8	5.17	21	32	27	4.33	10.67
MV Archimed Copula (CL)		1	4	2	1.17	9	20	8	5.17	19	32	26	4.67	11.00
		Advanced Emerging Market Bonds												
Historical Sim		6	5	7	3.50	9	17	16	3.83	22	31	27	3.67	11.00
Delta-Normal		6	13	6	5.83	9	17	15	3.50	20	24	21	3.33	12.67
MVGARCH	A-DCC	2	2	3	0.50	9	14	12	1.83	12	26	24	5.00	7.33
MVCopula (GA)	A-DCC	2	2	2	0.50	5	17	9	5.17	14	33	21	7.67	13.33
MVCopula (T)	A-DCC	2	2	2	0.50	5	17	10	4.83	14	32	22	7.00	12.33
Vine (GA)	A-DCC	2	2	2	0.50	5	17	10	4.83	14	32	22	7.00	12.33
Vine (T)	DCC	2	2	2	0.50	5	17	9	5.17	14	34	22	7.67	13.33
Vine (CL)	DCC	1	2	2	0.83	5	16	8	5.17	14	31	20	7.33	13.33
Vine (RCL)	DCC	2	2	1	0.83	5	17	9	5.17	14	33	22	7337	13.33
Mix Copula (GA-CL)	A-DCC	2	2	2	0.50	6	17	9	4.83	15	34	21	7.67	13
Mix Copula (T-CL)	A-DCC	2	2	1	0.83	5	17	10	4.83	14	34	22	7.67	13.33
Mix Copula (GA-GU)	A-DCC	2	2	1	0.83	5	17	9	5.17	14	32	21	7.33	13.33
Mix Copula (T-GU)	A-DCC	2	2	2	0.50	5	17	9	5.17	14	33	22	7.33	13
MV Archimed Copula (GU)		1	2	2	0.83	5	17	9	5.17	14	32	21	7.33	13.33
MV Archimed Copula (CL)		2	2	2	0.50	5	19	10	5.50	14	34	22	7.67	13.67

This table reports the expected number of violations for the 750 trading days lasting backtest sample. Period 1 runs from 08/13/2007 to 08/11/2008, Period 2 from 08/12/2008 to 11/08/2009 and Period 3 from 08/12/2009 to 08/10/2020 so that every period covers 250 trading days. The column "error" reports the absolute deviation of the VaR violations that occurred in the respective period and the expected number of VaR violations. The column ‘error sum’ reports the sum of the errors for every period. model.

the number of violations are considered, showing only five for the VaR(1/1). This time most copulas perform better than the MVGARCH even for the 1% confidence level VaR. The HS does well for the 10% confidence level but overall rejects five out of nine null hypotheses. This however, is better than the DN, which rejects seven. For the advanced emerging market bond indices the MVGARCH results are good for the 1% and 5% confience level VaR. In general it accepts seven out of nine null hypothesis and shows the lowest maximum exceedance VaR. Some copula models pass all tests but have either a higher max exceedance VaR or more violations than expected which results in a higher error sum.

The task of finding the best risk model is a difficult one. The four different models classes display different characteristics for different portfolios and VaR confidence levels. The miscellaneous copula models deliver more or less the same VaR estimates across the different portfolios and VaR confidence levels. In contrast, the MVGARCH produces a diverging performance the majority of the time. Of further interest is that the MVGARCH model for the 10% confidence level VaR has fewer violations than the copula models for all portfolios. For the 5% confidence level the evidence is mixed, however, with the MVGARCH model and copula models alternating with regards to which produces more VaR violations. For the VaR(1/1) the MVGARCH shows at least as many VaR violations as the copula models and sometimes produces more. In the case of the developed stock portfolio this is beneficial because it is closer to the expected number of violations, but this is not the case with regards to the emerging market stocks.

Even looking at the table where the entire backtesting period is broken down into three sub-periods it is not possible to detect constant behavior in the respective models, meaning it would be a challenging task for a risk manager to determine favorite. The only clear result is that the HS and DN methods perform poorly compared to the sophisticated parametric models for three out of four portfolios: only for the developed bond portfolio do they give a better performance. The Delta-Normal method performs particularly poorly dramatically underestimating the 1% confidence level VaR. Any risk manager using this method should pay great attention to this. It can be concluded that it would be worth investing in the development of a sophisticated parametric model. Until now we have compared the different risk models. It might also be worth comparing the emerging and developed market VaRs within each asset class. Figure 5.5 compares the 1% confidence level VaR estimated with a Gaussian copula for the developed and emerging markets. It can be seen that, in general, the VaR increased after the Lehman Brothers default for all portfolios. Of particular interest is the fact that the VaR of the emerging stock markets increased more than that of the developed markets, despite the default occurring in the US- one of the developed markets. A major component of the VaR in the introduced framework is the volatility, which reacts to positive and negative shocks. As can be seen in the figure the emerging markets show much more larger positive and negative returns than the developed markets around the default of the Lehman bank. Clearly this leads to a higher volatility and explains the higher VaR. In principle the emerging stock markets' VaR seems to be above the VaR of the developed markets most of the time. This is particularly the case for the bond indices, as can be seen in Figure 5.6.

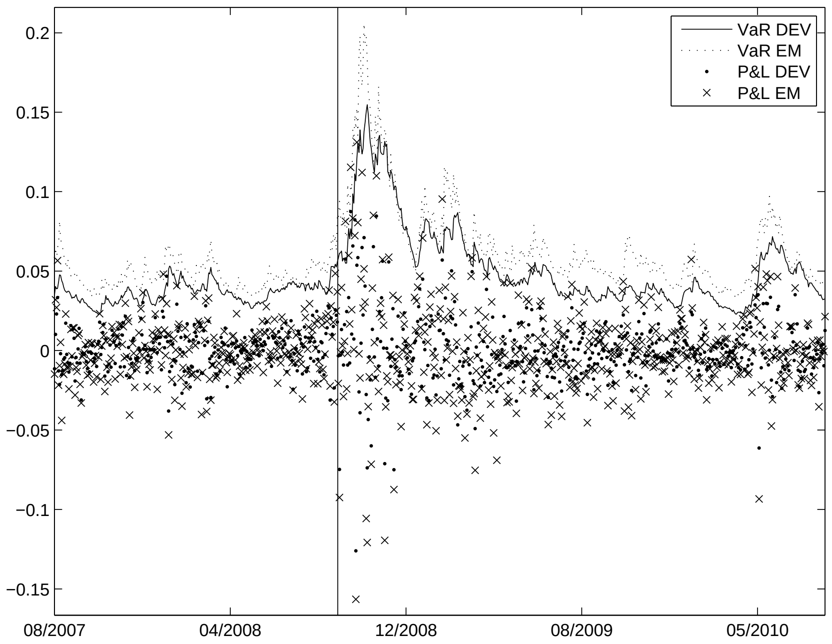


Figure 5.5: VaR Emerging and Developed Stocks. This figure compares the 1% confidence level VaR for the emerging and developed stock indices. The vertical line shows the default of the Lehman Brothers bank.

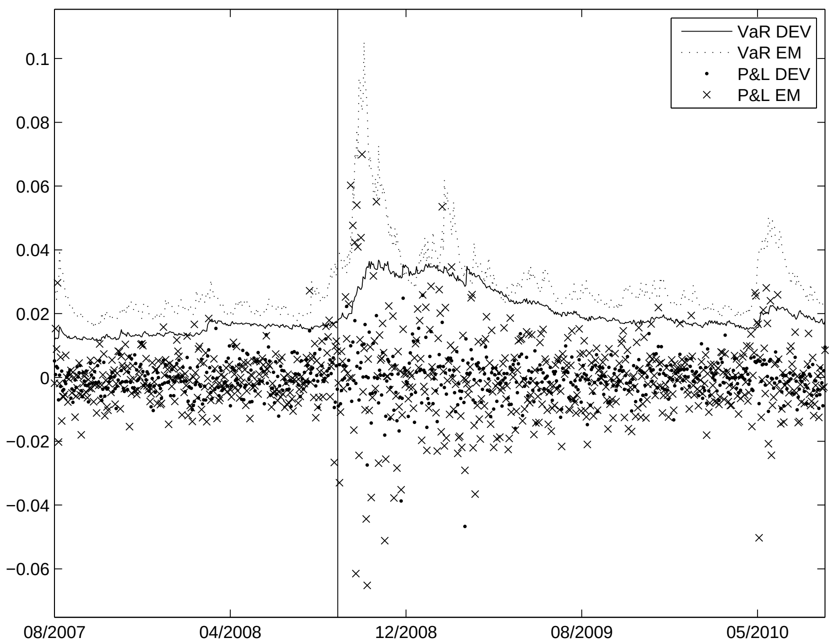


Figure 5.6: VaR Emerging and Developed Bonds. This figure compares the 1% confidence level VaR for the emerging and developed bond indices. The vertical line shows the default of the Lehman Brothers bank.

5.6 Dependence Structure Analysis

The final part of this analysis is devoted to the analysis of the dependence structure between emerging and developed markets. The characteristic of the dependence provides an indication as to how integrated the financial markets are and might offer further clues as to possible diversification benefits.

First of all, the term ‘financial integration’ should be clarified. Kindleberger (1988, p.75) offers a broad definition, defining (financial) markets as integrated if they share the same price for the same asset. In this analysis assets are either equity or bond indices. Thus the ‘same price’ in Kindleberger’s definition should be redefined here to ‘level’ or- in a time-varying context- to ‘movement’. Clearly the different indices will not share *exactly* the same price but the respective levels should move together if they are integrated. To build on Kindleberger’s definition of integration it might be said, that two financial markets are integrated if financial assets with the same characteristics return the same performance (Arouri, Jawadi, and Nguyen (2010, p.150)), or in the words of Bekaert and Harvey (2003, p.4): "In finance, markets are considered integrated when assets of identical risk command the same expected return irrespective of their domicile."

According to Markowitz (1952) a diversified portfolio yields the best risk-return relationship. The lower the dependence between the financial assets the better the possibilities of building a portfolio with superior risk-return relationship. For modern portfolio theory, then the identification of levels of financial market integration is always connected to possible diversification benefits.

As a result there has been a great deal of research on the international diversification benefits of equities. Among the first to implement this kind of analysis based on dependence patterns were Grubel (1968), Levy and Sarnat (1970), and Solnik (1974).

In general recent research can be subdivided into two categories: To the first category belongs research finding diminishing international diversification benefits and increasing correlations. Of these Errunza, Hogan, and Hung (1999) examine return correlations and volatility spanning: they conclude that international diversification benefits diminish. Forbes and Rigobon (2002) uses heteroscedasticity biases tests to find comovements for market crash periods. Goetzmann, Li, and Rouwenhorst (2005) investigates correlations between the world equity markets and documents changes through the whole period; and Carrieri, Errunza, and Sarkissian (2007) examines industrial linkages and equity correlations between the US and 16 OECD countries, finding increasing correlations from the 1990s on. Lewis (2007) considers a U.S. investor’s point-of-view and explores the correlation and volatility to foreign equity markets, finding increasing correlation but decreasing volatilities. Baele and Inghelbrecht (2009) estimates a multifactor model for twenty-one developed markets during the period from 1973-2007, and reports increasing correlations for this time period. This line of research suggests that for a developed market equity investor investing in other (foreign) developed equity markets does not make much sense since the miscellaneous developed markets are to dependent among themselves. Then it makes sense to look for diversification opportunities outside the developed markets.

The second category is comprised of researchers that have not found increasing correlations or diminishing diversification benefits between developed markets. Among the first in this category are King, Sentana, and Wadhwani (1994) also estimating a multifactor model. Karolyi and Stulz (1996) explores the dependence between the U.S. and Japanese equities and finding no evidence for the hypothesis that U.S. macroeconomic announcement have an effect on the U.S. and Japanese equity return correlations; and Brooks and Del Negro (2004) investigates the international diversification benefits of industrial sectors as against the diversification benefits of countries, finding that industrial sectors are more integrated than countries.

There is much less research to be found regarding bond indices. For example Levy and Lerman (1988) studied the benefits of international diversification for an US bond holder. Cappiello, Engle, and Sheppard (2006) finds asymmetric correlations for equity and bond indices concluding that diversification effects diminish within both groups for the world markets.

Of particular relevance for this analysis are the integration and diversification benefits between emerging and developed markets. Several studies have already investigated this: For example Errunza (1977) considers the general possibilities of emerging markets for international diversification benefits as one of the first. Among those who focus on the correlation are e.g. Bekaert and Harvey (2000) who tests for shifts in correlation, finding that the correlation between emerging markets after the liberalization period are significantly higher. This is an important finding since increasing correlations between the emerging markets themselves reduces also the probability of diversification benefits of a portfolio comprised of emerging and developed markets. Fujii (2005) meanwhile, studies the integration of Latin American equities with those of the rest of the world using residual cross-correlation function tests finding significant linkages. Carrieri, Errunza, and Hogan (2007) investigates integration between eight emerging markets for the time period 1977-2000 and detects differences of the degree of integration. Clacher et al (2006) analyzes the diversification benefits of an US investor diversifying into twelve emerging markets and finds an improvement of the risk-return relationship. Kuecuek (2009) finds that emerging market debt denominated in local currency adds significant alpha to a bond portfolio consisting of developed countries. Brunda, Hamann, Lall (2010) investigates the co-movement in emerging market bond returns and the influence of external and domestic factors, finding that following the collapse of the Lehman Brothers the low correlation period between the emerging markets ended. Turgutlu and Ucer (2010) use a static mixed copula approach to analyze the dependence between emerging and developed equity indices. Eiling and Gerard (2011) discover highly significant positive time trends in cross-country correlations between twenty-four developed and thirty-two emerging equity markets for the period from 1973 to 2009. Chollete, de la Peña, and Lu (2011) documents asymmetric dependence for Latin American countries and less downside risk for the G5 and East Asia using several static copulas. Christoffersen et al (2012) uses MVGARCH and multivariate copulas to find an uptrending correlation between emerging and developed equity markets. Kenourgios, Samitas, and Paltalidis (2011) estimates dependence between between the BRIC markets and the developed markets of the UK and the USA for the period 1995-2006 using a Gaussian regime-switching copula and the AG-DCC model of Cappiello, Engle, and Sheppard (2006). They find that contagion effects spread from the country where a crisis originates to all other countries implying significant linkages between all countries. Dimitriou, Kenourgios, and Simos (2013) investigates contagion effects between the US and BRIC equity markets using a FIAPARCH-DCC GARCH framework. They find no signs of contagion in the early stage of the crisis but significant linkages after the collapse of the Lehman Brothers bank. Chen, Firth, and Rui (2002) does not directly investigate the dependence but studies the integration via an error-correction vector autoregressive model finding a cointegration vector for equity prices. Cointegration, vector error correction analysis, and the AG-DCC model has been used by Kenourgios and Padhi (2012).They focus on the behaviour of bond and equity indices between three emerging markets and the US and two global indices and find that equity indices seem to haver stronger links than bond indices.

Briefly, the already existing literature have not led to an unanimous view concerning diversification benefits of emerging markets. Some researcher found significant linkages between emerging and developed markets implying only minor diversification benefits. Others showed the existence of diversification benefits.

This analysis tries to enhance the already existing literature in the following ways: First of all, the newly developed multivariate Archimedean and dynamic mixture copulas present a novel view on (tail) dependence in general and hence also on emerging and developed market dependence. Furthermore, the dependence between emerging and developed markets through the recent financial crisis is investigated. Since the roots of the crisis are found in the developed markets it might be interesting to see how the emerging markets react to this situation. Especially for bond indices only few research in this direction can be found. In addition the emerging markets are split into advanced and secondary emerging markets allowing to investigate if economic power and dependence are linked.

Our first working hypothesis is that the advanced emerging markets are fairly well integrated into the (financial) world economy while the secondary emerging markets are still decoupled. If this is the case there should be a low dependence between the secondary emerging and developed markets and a higher dependence between the advanced emerging and developed markets.

In the following section we concentrate on the analysis of the dependence between the emerging and developed equity and bond markets. We build six different portfolios wherein all indices are equal weighted. As in the VaR analysis chapter, all indices are total return indices and denominated in US$. The first portfolio consists of the developed markets stock indices. This is the benchmark portfolio for the equity portfolios. The second portfolio is comprised of the developed markets and the advanced emerging markets stocks. The last stock portfolio contains the developed market and the secondary emerging market stocks. After the investigation of the stock indices we explore the behavior of the bond indices. The bond portfolios are built in the same way as the stock portfolios. This procedure allows me to analyze the effect the advanced and secondary emerging market indices have on the portfolio consisting of the developed markets.

The copula estimation method used in the last section was the IFM method. In this section we try to determine the dependence structure via a goodness-of-fit test. Since the IFM method has been criticized as unsuitable for this purpose we use the CML method instead. Again, we estimate AR(p_1)-GARCH(1,1) models for all indices and construct standardized residuals based on the estimated variances. Due to their theoretical construction the models based on the DCC dynamic dependence need standardized residuals as input. Therefore, we estimate all models with standardized residuals. To these standardized residuals we apply the generalized pareto distribution defined in (4.55) with a Gaussian kernel. We appoint 10% of the data to each tail. The estimation method of the MVGARCH models stays the same as in the previous sections. We use Breymann, Dias and Embrechts's (2003) goodness-of-fit test. We apply the different statistical measures defined in (4.57) to the generated pseudo-variables. For some models it is necessary to determine the optimal dynamic structure (DCC, A-DCC, G-DCC, or AG-DCC). Hence, first we estimate the possible dynamic structures for each model and apply the goodness-of-fit test to each of these. If the various statistical distance measures prefer different structures we use the AIC and BIC criterion to determine the best one. Thereafter, we compare the different MVGARCH and copula models to determine the best fitting one, again via the goodness-of-fit test.

5.6.1 Stock Portfolios Analysis

First, we will analyze the equity indices. It seems to makes sense to examine whether the new portfolios exhibit time-varying dependence via the test of Engle and Sheppard (2001). Table 5.6 shows the results. The null hypothesis of no dynamic correlation is rejected in all cases. The first portfolio investigated thoroughly is the one containing the developed stock indices.

Table 5.6: Engle and Sheppard (2001) Test of Time-Varying Correlation for the Composed Portfolios.

Portfolio	Stat
Developed Stocks + Advanced Emerging Market Stocks	21532.3
Developed Stocks + Secondary Emerging Market Stocks	15056.6
Developed Bonds + Advanced Emerging Market Bonds	17877.1
Developed Bonds + Secondary Emerging Market Bonds	11565.0

Notes to Table: 19.6751 is the respective critical value from a χ^2_{s+1} distribution, where $s = 10$.

For the MVGARCH the AG-DCC dynamic structure fits best as can be seen in Table F.1. Three out of the four statistical distance measures and the AIC criterium favor this dynamic dependence structure. The A-DCC structure seems to be the best fit for the Gaussian copula. Only the AD measure suggests the AG-DCC structure as preferable (see table F.7). This is similar to the results of the AIC and BIC criterion which also preferred the AG-DCC and A-DCC dynamic structure. The estimation results for the t-copula are found in Table F.18. Again, the AIC criterion favors the AG-DCC structure while the BIC favors the A-DCC one. This time however, the statistical measures have a clear favorite, namely the G-DCC dynamic structure.

Of particular interest is the comparison between the Gaussian and t-copula. Both belong to the class of elliptical copulas and the (linear) correlation matrix resembles the dependence of each of the copulas. The t-copula seems to deliver the better fit since all statistical measures are smaller than in the Gaussian case (see Table 5.7). When taking tail dependence into account the preferred dynamic structure changes from an asymmetric (A-DCC for the Gaussian copula) to a symmetric one (G-DCC for the t-copula). This seems quite surprising since both copulas are symmetric and one might think that they share the same dynamic structure. A first conclusion is that the best fitting structure of Cappiello, Engle, and Sheppard (2006) is

Table 5.7: Goodness-of-fit test for the Stock Portfolios.

	Dynamic Structure	Log-l	AIC	BIC	AD	AAD	KS	AKS
				Developed Stocks				
MVGARCH	AG-DCC	41026.0	-81972.0	-81961.0	7.0189	0.1039	3.2835	0.0407
GA	A-DCC	41177.2	-82348.4	**-82331.9**	4.3694	0.0606	1.8360	0.0234
T	G-DCC	41216.2	-82402.5	-82319.9	3.0337	0.0423	1.4800	0.0175
CL		39749.3	-79492.6	-79476.1	9.8589	0.1466	4.6259	0.0619
RCL		39541.8	-79077.6	-79061.0	10.9092	0.1611	5.1449	0.0674
T-CL	G-DCC	41221.9	**-82397.9**	-82271.1	3.0945	0.0421	1.5211	0.0175
Vine GA	A-DCC	41273.0	-82333.5	-81986.4	4.5685	0.0323	1.0962	0.0110
Vine T	A-DCC	41280.6	-82393.2	-81930.5	**2.3425**	**0.0141**	**0.5647**	**0.0051**
				Advanced Emerging Markets Stocks & Developed Stocks				
MVGARCH	A-DCC	67910.9	-135815.8	-135799.2	9.6646	0.1422	4.1796	0.0549
GA	A-DCC	68186.1	-136366.2	-136349.7	6.6969	0.0952	2.6991	0.0359
T	G-DCC	68276.3	-136502.6	**-136364.9**	3.8536	0.0616	1.9037	0.0262
CL		65431.7	-130931.6	-130915.1	13.7441	0.2141	6.6425	0.0908
RCL		65046.8	-130246.8	-130230.3	15.1044	0.2263	7.4122	0.0957
T-CL	G-DCC	68289.1	**-136512.2**	-136330.4	2.8986	0.0436	1.4254	0.0188
Vine GA	A-DCC	68312.7	-136229.5	-135138.6	5.8802	0.0749	1.9198	0.0271
Vine T	DCC	68435.4	-136474.8	-135384.0	**2.4251**	**0.0271**	**1.0785**	**0.0103**
				Secondary Emerging Markets Stocks & Developed Stocks				
MVGARCH	A-DCC	67914.4	-135806.2	-135822.8	10.7141	0.1491	5.4844	0.0643
GA	G-DCC	68325.6	-136603.3	-136471.1	5.9954	0.0730	2.0633	0.0272
T	G-DCC	68409.7	-136769.4	**-136631.6**	3.4980	0.0466	1.6932	0.0194
CL		66200.5	-132447.9	-132431.4	9.8500	0.1369	4.6508	0.0563
RCL		65821.6	-131735.2	-131718.6	13.1320	0.1949	6.2211	0.0822
T-CL	G-DCC	68423.0	**-136780.0**	-136598.2	2.5751	0.0248	1.2569	0.0105
Vine GA	DCC	68390.7	-136385.5	-135294.6	5.8747	0.0648	1.7393	0.0238
Vine T	DCC	68482.0	-136568.1	-135477.3	**2.1343**	**0.0194**	**0.8809**	**0.0073**

Notes to Table: This table reports the log-likelihood values, log-likelihood based criteria, and four statistical tests for the investigated stock portfolios. The statistical tests are: Anderson-Darling (AD), Average Anderson-Darling (AAD), Kolmogorov-Smirnov (KS), and Average Kolmogorov-Smirnov (AKS). All tests are defined in (4.57). In bold letters denoted is the smallest value of the different test for each model since this suggests the best fit of the respective model.

related to tail dependence. Figure 5.7 shows the differences in the average correlation of the Gaussian and the t-copula. The Gaussian copula is estimated with the A-DCC structure and the t-copula with the G-DCC one according to the best fit. Thus, the average correlation of the Gaussian copula seems to be lower than the average correlation of the t-copula for the majority of the time. As both are estimated with different dynamic structures we plot the Gaussian copula with the two different dynamic structure (A-DCC and G-DCC, see Figure5.8). This

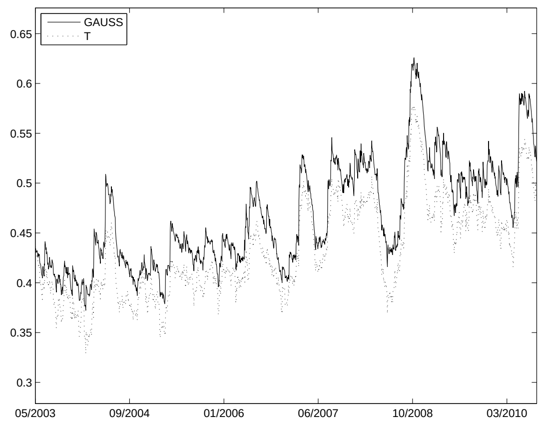


Figure 5.7: Gaussian and t-copula Correlation Developed Stocks. This figure compares the Gaussian (A-DCC) and t-copula (G-DCC).

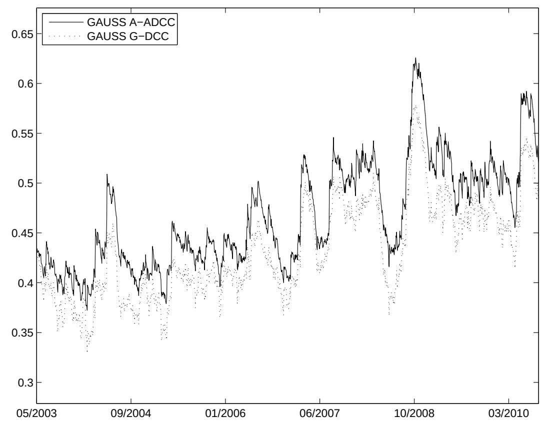


Figure 5.8: Gaussian Copula A-DCC and G-DCC Dependence Structure. This figure compares the Gaussian copula with A-DCC and G-DCC structure.

plot makes it clear that the major difference comes from the chosen dynamic structure and not from the copula since the time-varying dependence of both copulas with the G-DCC structure is almost the same.

A further comparison of interest is between the MVGARCH and the Gaussian copula. The dependence structure of both is Gaussian and the only difference lies in their marginal distributions. When comparing the different statistical measures it is apparent that the Gaussian copula seems to fit better than the MVGARCH: the log-likelihood value is higher and all distance measures are smaller for the Gaussian copula. The graphical inspection of Figure 5.9 shows that correlation estimated with the MVGARCH is lower than the one of the Gaussian copula for the majority of the time. Both are estimated with the A-DCC structure.

Now, we investigate the two multivariate Archimedean copulas, the Clayton and rotated Clayton copula, whose parameter estimation results can be found in Table F.28. Compared with other copulas used on the developed stock indices these two do not perform very well and the multivariate rotated Clayton copula clearly shows the worst results, when looking at the different goodness-of-fit measures. The first conclusion to be drawn is that the developed equity indices do not share common features which the time-varying multivariate Archimedean copulas can capture. However, the discovery that the Clayton copula performs better than the rotated Clayton copula indicates that the developed stock indices share more dependence in the left tail than in the right one. Much better results than the multivariate Archimedean copulas-regarding the goodness-of-fit- show the dynamic mixture copula consisting of a t-copula and

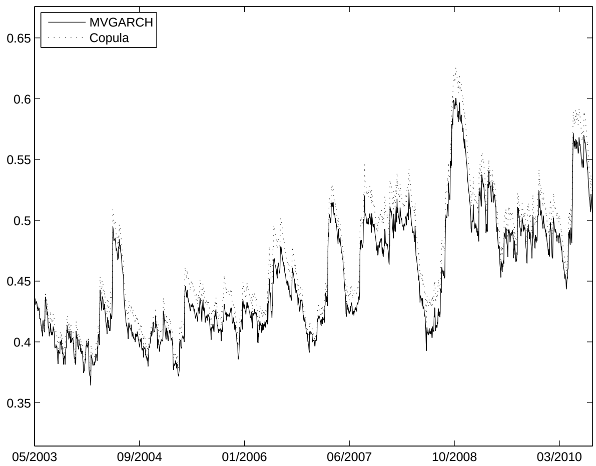


Figure 5.9: Average Correlation MVGARCH vs. Gaussian Copula Developed Stocks. This figure shows the average correlation of the MVGARCH and Gaussian copula for the developed stocks portfolio estimated with the A-DCC structure.

a Clayton copula (see Table F.30 for the parameter estimation results). Since the Clayton copula performs better than the rotated Clayton copula and the t-copula better than the Gaussian copula, this is the only mixture copula we estimate for the developed stock indices.

The last models we estimate for each portfolio are the Gaussian- and t-vine copulas. The results of the goodness-of-fit statistics for the different dynamic structures of the vine copulas are found in table F.24. In general the A-DCC structure is preferred for both copulas. The t-vine with the A-DCC structure seems to perform better than the Gaussian vine and every other model for the developed stock indices. The data-ordering for the bivariate pairs of the vine copula are based on the unconditional dependence parameters (see Table F.26). Investigating the different portfolios it can be concluded that the developed stock indices share asymmetric dependence: the correlation between them increases after negative shocks more than after positive shocks of the same magnitude.

Now we turn to the composed portfolios. In this part we will only analyze the fit of the different models. Later on we will try to give the findings an economic interpretation. First we investigate the portfolio consisting of the advanced emerging[7]and developed market stocks[8]. Again, the models of the Gaussian world, the MVGARCH and the Gaussian copula, all prefer an asymmetric dependence structure, namely the A-DCC one (see Tables F.2 and F.10). However the picture is complicated by the fact that both of the measures based on averages (AAD and AKS) favor the AG-DCC structure. Since only one asymmetric parameter is significant and the AIC and BIC both prefer A-DCC we choose this as the best fitting one. The t-copula again yields the best results concerning the different distance measures when estimated with the symmetric G-DCC structure (see Table F.19). Furthermore, the Clayton copula seems to

[7]To recall: The advanced emerging markets consists of Brazil (BRA), Hungary (HUN), Mexico (MEX), Poland (POL), and the Republic of South Africa (RSA)

[8]To recall: The developed markets consists of Australia (AUS), Denmark (DEN), Sweden (SWE), Italy (ITA), Japan (JAP), Great Britain (GBR), and the Unites States of America (USA).

perform better than the rotated Clayton copula and so the data seems to share more dependence in the left tail than in the right tail: see Table F.28 for the parameter estimation results. Again, the t-Clayton mixture copulas perform better than either the t-copula or the Clayton copula estimated alone. Parameter estimation results for the mixture copula are found in Table F.30. When comparing the different vine copulas for the portfolio in Table F.24 it can be seen that the t-vine with a symmetric dependence structure fits best. This is also the best model comparing all of them for the portfolio, see again Table 5.7.

The second composed portfolio contains the secondary emerging markets[9]. This time all copulas give the best results with a symmetric structure. The elliptical copulas perform best when estimated with the G-DCC structure related to the different distance measures- this is shown in Tables F.11 and F.20. To find the best dynamic structure for the MVGARCH model is again a complicated task: the statistical distance measures choose the AG-DCC MVGARCH model as the best fitting one. Since only two asymmetric parameter within this model are significant we choose again the A-DCC structure as the better fit (see Table F.3). The rotated Clayton copula again performs worse than the Clayton copula and has the worst overall performance. The t-vine copula seems to fit best overall when accounting for the different statistical distance measures in Table 5.7. The second best performance comes from the t-Clayton (T-CL) mixture copula: estimation results are found in Table F.31.

At this point we want to take a closer look at the Dynamic Mixture model and display some of its properties. Figure 5.10 shows the time-varying weights and the Clayton copula in the dynamic mixture t-Clayton copula. Taking a look at the figure it can be seen that the weight of the respective copula remains relatively constant through time and the t-copula clearly dominates with a weight > 90% all of the time. The lower part of the figure shows the dynamic multivariate Clayton copula in the mixture structure: after the default of Lehman Brothers the dependence increases dramatically. Thus, although the weight stays the same the increased dependence of the Clayton copula parameter leads to a different dependence structure at that point in time. The analysis of tail dependence above showed that when tail dependence is taken into account the preferred dynamic structure is a symmetric one. The dynamic mixture shows that although the symmetric G-DCC t-copula fits better than either an A-DCC or AG-DCC t-copula there still might be some asymmetries in the data. Within the mixture structure these are ‘carried’ by the Clayton copula.

A further interesting comparison is between the multivariate Clayton copula and the Clayton copula in the mixing structure. Figure 5.11 shows the multivariate dynamic Clayton copula. Compared to the Clayton copula in the mixing structure the time-varying dependence behavior of the Clayton copula overall seems to be similar overall but the dependence level is much lower. This can be explained by the fact that within the mixing structure the t-copula has much more weight than the Clayton copula and so the Clayton copula in the mixing structure can concentrate on the events with dependence in the left tail. In contrast the multivariate Clayton copula (estimated without a mixing structure) has to capture all events. Thus a mixing structure seems preferable especially to an Archimedean copula for multivariate data sets. Only within the mixing structure the multivariate Archimedean copula can play out its strength.

[9] To recall: The secondary emerging markets consists of Chile (CHI), Czech Republic (CZE), India (IND), Indonesia (INA), Thailand (THA).

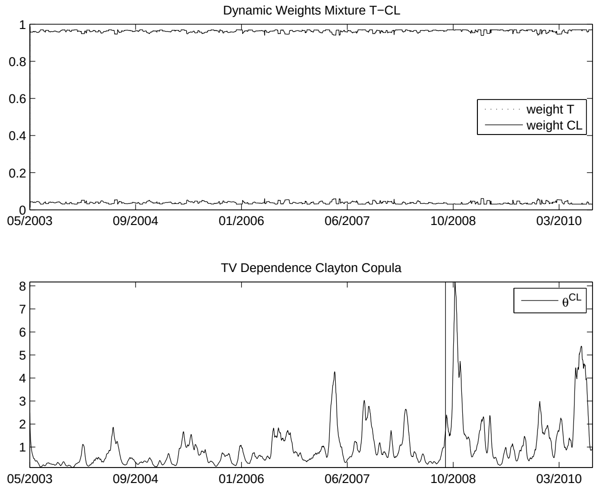


Figure 5.10: t-Clayton Mixture Copula Secondary and Developed Market Stocks. This figure shows the dynamic t-Clayton mixture copula for the portfolio consisting of the Secondary Emerging Market and Developed Market Stocks. The upper panel shows the weights of the respective copula in the mixing structure. The lower panel shows the dependence parameter of the Clayton copula in the mixing structure. The vertical line shows the default of the Lehman Brothers bank.

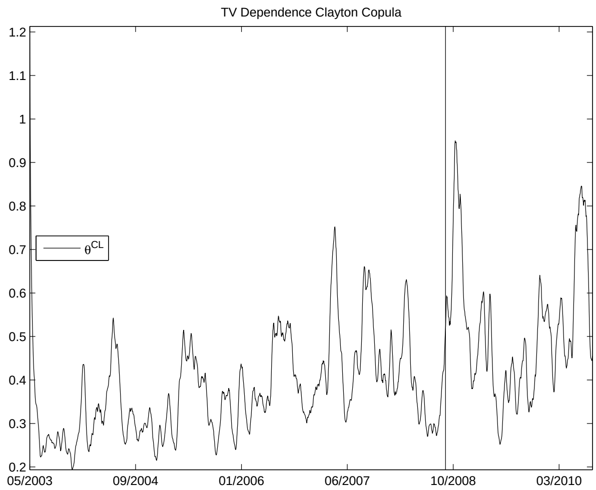


Figure 5.11: Dynamic Clayton Copula Secondary and Developed Market Stocks. This figure shows the multivariate dynamic Clayton copula for the portfolio consisting of the Secondary Emerging Market and Developed Market Stocks. The vertical line shows the default of the Lehman Brothers bank.

5.6.2 Bond Portfolios Analysis

In the next section we do the same analysis as in the previous section for portfolios composed of bond indices. Initially, we investigate the portfolio consisting of developed market bonds. Table 5.8 summarizes the results of the goodness-of-fit tests. For the developed bond indices, the t-vine copula with a symmetric dynamic dependence structure is clearly the best fitting one: Table F.25 shows the estimation results. All statistical measures favor this copula over all other copulas. This time only the MVGARCH model prefers an asymmetric structure while all other models favor a symmetric one. Parameter estimation results for the Gaussian and t-copula can be found in Tables F.12 and F.21. Interestingly, this time the rotated Clayton copula performs better than the Clayton copula (see Table F.29). This implies that the developed bond indices share more joint negative than positive events. Until now the AG-DCC structure defined in (3.12) has incorporated only asymmetric negative dependence. The finding that the rotated Clayton copula fits better than the Clayton copula implies possible positive asymmetric dependence. In this case $\mathbf{n}_t = N[\boldsymbol{\eta_t} < 0] \circ \boldsymbol{\eta}_t$ should be redefined to $\mathbf{n}_t = N[\boldsymbol{\eta_t} > 0] \circ \boldsymbol{\eta}_t$. Table F.13 shows the parameter estimation results. Interestingly, for the A-DCC structure the asymmetric parameter is now significant and the different statistical distance measures indicate a better fit of the positive asymmetries.

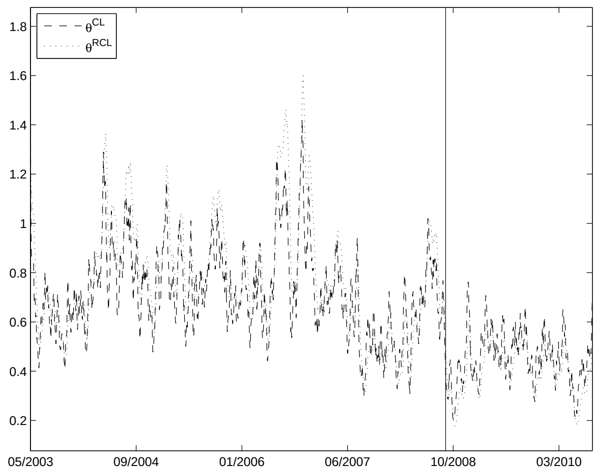


Figure 5.12: Dynamic Clayton and Rotated Clayton Copula Developed Bonds. This figure shows the dependence parameter of the dynamic Clayton (θ^{CL}) and Rotated Clayton copula (θ^{RCL}) for the Developed Bond Indices. The vertical line shows the default of the Lehman Brothers bank.

Figure 5.12 compares the Clayton and rotated Clayton copula for the developed bond indices. Note that the dependence parameter does not have to be mapped into Kendall's tau because both copulas share the same tau. Most models point in the direction of symmetric returns and therefore it is not a surprise that both seem to follow almost similar patterns. This can also be seen by the different statistical measures since they are pretty close together for both copulas. Again, the t-copula with a symmetric dynamic structure (G-DCC) performs much better than the two elliptical models from the Gaussian world. The data sample esti-

Table 5.8: Goodness-of-fit test for the Bond Portfolios.

	Dynamic Structure	Log-l	AIC	BIC	AD	AAD	KS	AKS
				Developed Bonds				
MVGARCH	A-DCC	53149.3	-106292.6	-106276.1	9.8848	0.1515	4.7230	0.0614
GA	DCC	53380.4	-106756.9	-106745.9	7.1453	0.1033	3.3736	0.0417
T	G-DCC	53546.9	-107063.9	-106981.2	6.1951	0.1099	3.0923	0.0454
CL		48921.2	-97836.4	-97819.9	15.2962	0.2292	7.4330	0.0961
RCL		49020.7	-98035.5	-98019.0	14.9765	0.2295	7.1511	0.0961
T-RCL	G-DCC	53560.4	-107074.9	-106948.2	4.2896	0.0460	1.2926	0.0171
Vine GA	A-DCC	53461.4	-106796.9	-106449.8	6.8211	0.1034	3.2475	0.0420
Vine T	DCC	53591.1	**-107056.3**	**-106709.2**	**4.1549**	**0.0292**	**0.9808**	**0.0102**
				Advanced Emerging Bonds & Developed Bonds				
MVGARCH	A-DCC	85565.0	-171124.0	-171107.5	11.7444	0.1758	5.4639	0.0704
GA	A-DCC	86195.5	-172385.0	-172368.5	8.3731	0.1190	3.4922	0.0464
T	G-DCC	86431.4	**-172812.9**	-172675.1	5.3101	0.0903	2.5568	0.0373
CL		79417.9	-158829.8	-158813.2	17.7846	0.2663	8.7157	0.1124
RCL		79363.6	-158721.3	-158704.8	18.8260	0.2870	9.3003	0.1217
T-CL	G-DCC	86442.8	-172757.9	**-172735.8**	**4.6052**	0.0717	2.1584	0.0298
Vine GA	A-DCC	86360.7	-172325.4	-171234.6	8.8116	0.1198	3.9088	0.0473
Vine T	DCC	86547.6	-172699.3	-171608.4	5.2452	**0.0638**	**1.8403**	**0.0254**
				Secondary Emerging Bonds & Developed Bonds				
MVGARCH	G-DCC	88661.0	-177315.9	-177304.9	13.0645	0.1965	6.2846	0.0789
GA	A-DCC	89437.7	-178869.4	-178852.9	7.7395	0.1120	3.5206	0.0439
T	G-DCC	89735.3	-179420.7	-179282.9	4.9053	0.0849	2.4123	0.0348
CL		83314.4	-166606.3	-166622.8	15.0180	0.2243	7.2336	0.0944
RCL		83340.1	-166674.3	-166657.8	15.1875	0.2351	7.4150	0.0988
T-CL	G-DCC	89751.5	**-179437.0**	**-179255.2**	**4.0192**	0.0635	**1.9830**	0.0259
Vine GA	A-DCC	89719.6	-177952.4	-179043.2	8.2059	0.1151	3.8458	0.0463
Vine T	DCC	89911.7	-179427.5	-1.783367	4.6422	**0.0593**	2.1419	**0.0242**

Notes to Table: This table reports the log-likelihood values and four statistical tests for three different bond portfolios. The statistical tests are: Anderson-Darling (AD), Average Anderson-Darling (AAD), Kolmogorov-Smirnov (KS), and Average Kolmogorov-Smirnov (AKS). All tests are defined in (4.57). In bold letters denoted is the smallest value of the different test for each model since this suggests the best fit of the respective copula.

mated with the Gaussian vine copula favors the A-DCC structure and performs better than the multivariate Gaussian copula.

After the investigation of the developed bond indices we turn now to the analysis of the composed portfolios. The first composed portfolio consists of the advanced emerging market and developed market bond indices.

This time the two models from the Gaussian world favor different dynamic structures. While the MVGARCH model prefers a symmetric one (G-DCC) the Gaussian copula yields the best fit when estimated with the A-DCC structure. The t-copula favors the G-DCC structure (see Tables F.16 and F.22 for the parameter estimation results). This time the Clayton copula displays a better fit than the rotated Clayton copula. Again the t-vine copula with a symmetric structure performs best overall when looking at the different statistical measures. Three out of four measures choose the t-vine over all other models. Only the AD measure favors the t-Clayton mixture copula. The AIC criterion prefers the t-copula while the BIC criterion prefers the t-Clayton mixture copula. This implies that the data sample shows more negative joint dependence than positive joint dependence.

The last portfolio consists of the secondary emerging markets and the developed bond indices. The MVGARCH model favors the G-DCC dynamic structure while the Gaussian copula favors the asymmetric scalar one (A-DCC). The t-copula with the G-DCC structure again fits better than both models from the Gaussian world. The Clayton copula performs better than the rotated Clayton and the t-vine copula better than the Gaussian vine copula. Two out of four statistical distance measures choose the t-vine copula as the best fitting one. The other two distance measures favor the t-Clayton dynamic mixture copula. Since the AIC criterion also favors this copula we conclude that the t-Clayton dynamic mixture copula is the best fitting one.

5.6.3 Economic Analysis

Having determined the most adequate model via the goodness-of-fit test for all portfolios we will now try to give the results some economic interpretation. The data sample ranges from 2003 to 2010 and so the recent financial crisis is incorporated. It might be especially interesting to see what kind of dependence the emerging markets and the developed markets show in this period.

In the following we will analyze the different portfolios and their dependence pattern estimated by the miscellaneous models. Based on these results we will try to determine the degree of financial integration, which also implies possible diversification benefits. Since the financial integration is measured against a portfolio composed of developed assets we will start with a short analysis of the developed markets; specifically with the equity indices. According to the proposed goodness-of-fit test the best fitting copula is the t-vine copula with an asymmetric dynamic structure. By the construction of the asymmetric dependence structure this implies that the developed stock indices tend to fall together in bear markets. Since especially in downward markets diversification should take effect this kind of behavior is not what a portfolio manager is looking for. The same conclusion can be draw from a comparison of the multivariate Archimedean copulas. Although the overall fit is not very good, the multivariate Clayton copula performs better than the multivariate rotated Clayton copula. This gives further indication of greater dependence in the left tail than in the right : that is to say (extreme) joint negative events tend to occur more often than (extreme) joint positive events.

Another number that characterizes the risk of the portfolio is the d.o.f. of the multivariate t-copula. The lower the d.o.f. parameter, the more fat-tailed the data sample is resulting in a greater extremity of events shown. A portfolio manager (and in particular a risk manager) might prefer a portfolio where the d.o.f. parameter is high relative to another portfolio with the same risk-return relationship. All descriptive statistics for the different equally weighted portfolios can be found in Table B.3. The d.o.f. parameter of the t-copula for the benchmark portfolio composed of developed stocks is $\nu = 20.7376$. Briefly: if the best fitting copula for the developed stock indices is a t-vine with an asymmetric dynamic dependence structure this indicates fairly strong asymmetries.

We will now investigate the portfolio composed of the developed and advanced emerging market indices in order to attempt to identify what kind of benefits the addition of the advanced emerging market stocks- if any- generates. Firstly, we will take a look at the advanced emerging market stock indices estimated with a Gaussian copula. This is the only copula we estimate for this portfolio because the only purpose of this analysis is a short characterization of the dependence behavior of the advanced emerging stock indices. Different statistical distance measures prefer either the A-DCC of the AG-DCC structure (see Table F.8) with both structures pointing to the direction that the correlation between the advanced emerging markets increases during bear markets.

Now we come back to the main analysis, the investigation of the composed portfolio. The first composed portfolio consists of the advanced emerging and developed market stock indices. The best fitting copula for this portfolio is the t-vine copula but this time with a symmetric dynamic dependence structure. In contrast to the portfolio composed of the developed stocks, this indicates a symmetric reaction to the arrival of good and bad news. This, in turn, means that the markets move upwards and downwards together to the same extent. In contrast, the developed stock indices seem to move together more during downward moves than during upward ones. Thus, this model points in the direction of diversification during bear markets when adding advanced emerging markets to a portfolio of developed markets. When looking at the tail dependence the picture is unclear: In comparing the Clayton and rotated Clayton copula it can be seen that the Clayton copula again performs better than the rotated Clayton copula (see table 5.7). Figure 5.13 compares the time-varying Clayton dependence parameters and shows that the variation of the dependence parameters for both portfolios is almost the same. Sometimes the composed portfolio seems to share more left tail dependence than the portfolio consisting only of the developed markets. Thus, from a (left) tail dependence view, the advanced emerging and developed stock indices seem to be integrated, attenuating diversification benefits.

Since the best fitting copula- by the different statistical distance measures- is a vine copula and these kind of copulas are a multivariate distribution cascaded into several pair copulas it makes sense to take a look at the results of the multivariate t-copula in Table F.18 for the developed stocks and Table F.19 for the portfolio consisting of the advanced emerging market and developed stocks. The asymmetric parameter of the A-DCC dynamic structure for the developed stock indices portfolio (0.0127) is much higher than for the advanced emerging and developed market indices portfolio (0.0089). This might explain why- in the case of the

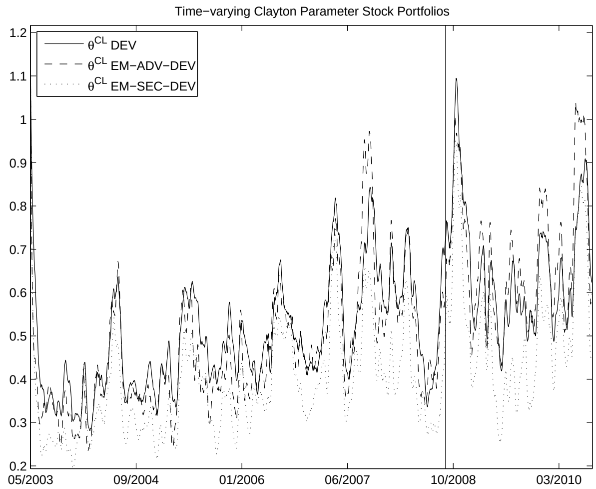
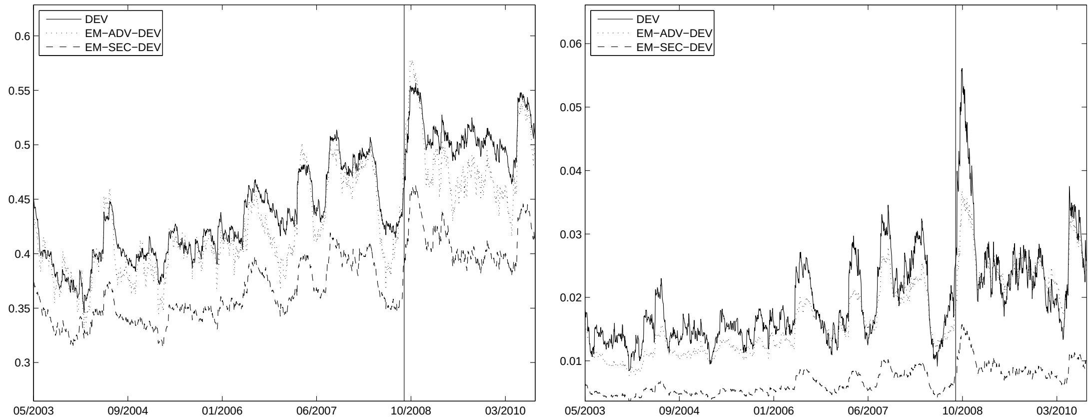


Figure 5.13: Dynamic Clayton Copula Stocks. This figure shows the dependence parameter of the Clayton copula for the stock indices portfolios. The vertical line shows the default of the Lehman Brothers bank.

developed stocks portfolio- the t-vine copula performs better with an A-DCC structure and for the composed portfolio with the DCC structure. The d.o.f. parameter of the t-copula is $\nu = 20.4885$ almost the same as for the developed stock market indices portfolio.

The last stock portfolio is composed of the secondary emerging market and developed market stock indices. Firstly, we will take a look at the estimation results of the Gaussian copula for the secondary emerging markets indices. The different statistical measures prefer either the A-DCC or the AG-DCC structure, see Table F.9. For the portfolio consisting of the secondary emerging and developed stock indices again the t-vine copula with the DCC structure again proves to be the best fitting dependence model. Only the MVGARCH model prefers an asymmetric structure (AG-DCC) clearly indicating that the secondary emerging markets display a different behavior to the advanced emerging markets when added to a portfolio of developed stock indices. Figure 5.13 seems to prove this for tail dependence: a graphical inspection reveals that the time-varying dependence parameter of the Clayton copula for the portfolio of the secondary emerging markets is below the other two for the majority of the time. This implies that the negative returns of the secondary emerging market and developed market stock indices do not coincide as frequently as those of the developed indices alone, nor the portfolio composed of the advanced emerging market and developed indices.

Figure 5.14(a) shows the average correlation for the stock portfolios estimated with the t-copula the best fitting of the elliptical copulas. It reaffirms our findings thus far with the dependence level of the portfolio containing the secondary emerging markets stocks at a much lower level than the other two. However, it has to be mentioned that shortly after the Lehman Brothers crash the dependence of all portfolios increased. In addition, there seems to be an upward trend for all portfolios until the Lehman default. Returning to the secondary and developed stock indices, the d.o.f. parameter of the t-copula for the last stock portfolio is $\nu = 23.3114$: slightly higher than for the other two portfolios. Thus, another feature of the



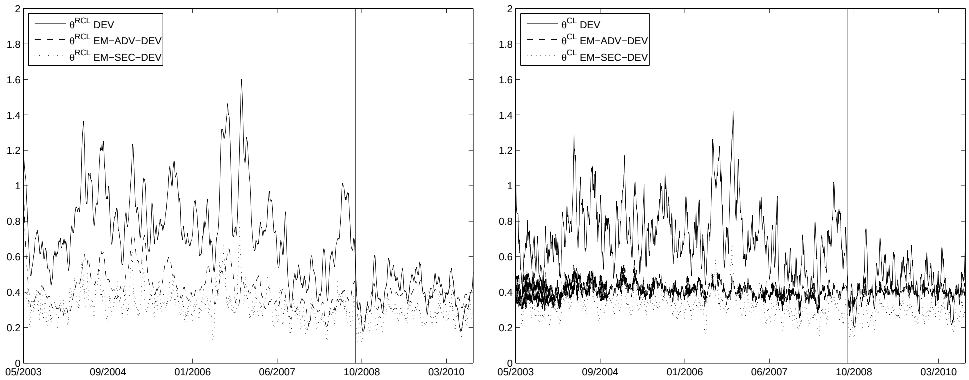
(a) Average Correlation (b) Average Tail Dependence

Figure 5.14: Average Correlation and Tail Dependence Stocks. This figure shows the average bivariate correlation and the average bivariate tail coefficient for the stock portfolios estimated with the t-copula. The vertical line shows the default of the Lehman Brothers bank.

portfolio is that both positive and negative returns are smaller.

Figure 5.14(b) shows the bivariate average tail dependence of the t-copula for the stock indices. After the default of the Lehman Brothers bank, the tail dependence of the developed markets and the portfolio consisting of the advanced emerging and developed markets increases more than the average tail dependence of the portfolio composed of the secondary emerging and developed market stocks. After the default the tail dependence between the advanced emerging and developed stock indices converges to almost the same level. Thus, to return to the issue of financial integration of stock markets, it seems that the secondary emerging markets are less integrated into the developed markets than the advanced emerging markets. The advanced emerging stock indices display very similar behavior to the developed stock indices. In general, there seems to be an upward trend in the dependence for all portfolios. This kind of behavior applies to left tail dependence, the average correlation and the average t-copula tail dependence coefficient.

The next task is to determine the financial integration and diversification benefits of the different bond indices. Again, we first analyze the benchmark portfolio- this time illustrated by the developed bond indices. The best fitting copula is the t-vine with a symmetric dynamic structure (DCC, see Table 5.8). Only the MVGARCH and the Gaussian-vine copula favor an asymmetric dependence structure. The A-DCC dependence parameter for the MVGARCH is very low and not significant (see Table F.4). Thus, although all statistical distance measures favor the A-DCC structure, in this case it might prove sensible to choose the more parsimonious DCC models as the best fitting one for the MVGARCH. Figure 5.15(a) shows the time-varying dependence parameter of the rotated Clayton copula. It can be seen that the developed bond indices share a considerably larger tail dependence than the other two portfolios. Interestingly, there seems to be a break after the default of the Lehman Brothers bank, with dependence continuing on a much lower and smoother level. The dependence parameter of the Clayton



(a) Rotated Clayton (b) Clayton

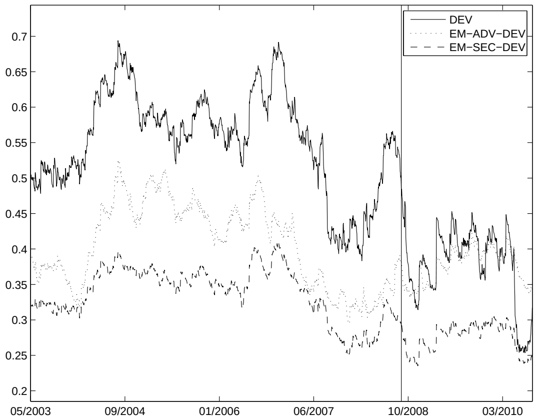
Figure 5.15: Dynamic Clayton and Rotated Clayton Copula Bonds. This figure shows the rotated Clayton and Clayton time-varying dependence parameter for the bond portfolios. The vertical line shows the default of the Lehman Brothers bank.

has a maximum value of $max(\theta^{CL}) = 1.4235$ and the maximum value of the rotated Clayton parameter is $max(\theta^{RCL}) = 1.5996$, indicating stronger right tail than left tail dependence. A graphical inspection of Figure 5.15 seems to confirm that the peaks of the rotated Clayton copula seems to be marginally higher than the one for the Clayton copula. After investigating the positive joint tail dependence it is only natural to compare the joint negative dependence of the portfolios in Figure 5.15(b). Since the statistical measures indicated that the rotated and Clayton copula exhibit almost the same fit it is no surprise that the dependence parameter for the Clayton copula and the developed bond portfolio is also higher than for the other two portfolios. The figure also reveals that the dependence parameter of the Clayton copula is more volatile than that of the rotated Clayton copula. However, there seems to be a break after the Lehman Brothers crash for the Clayton copula, too, although this is not as clear as in the case of the rotated Clayton copula. In summary, the comparison of the Clayton and rotated Clayton- and the Gaussian copula with negative and positive asymmetries- both point to the conclusion that the developed bond indices seem to share more joint positive than joint negative dependence.

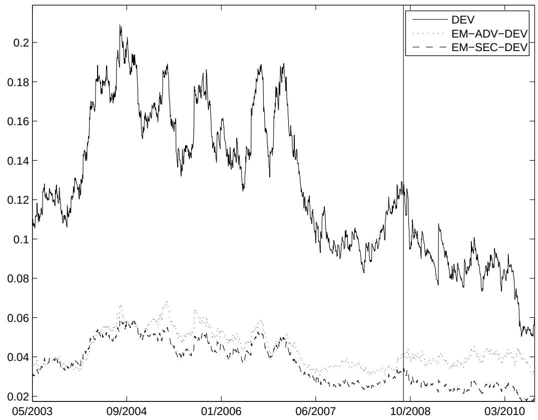
Now, we turn our focus to the first combined bond portfolio, composed of the advanced emerging market and developed market bonds. The best fitting model is again the t-vine copula with the DCC structure, with three out of the four statistical measures choosing this model. In second place is the t-Clayton dynamic mixture copula, chosen by the BIC criterion and the Anderson-Darling (AD) statistic. This time the MVGARCH, Gaussian copula and the Gaussian vine copula favor an asymmetric dependence structure, whereas for the developed bond portfolio only the MVGARCH and Gaussian vine do so. This suggests that the advanced emerging market bond indices tend to fall together with the developed market bonds. This is confirmed by the fact that the Clayton copula fits the sample better than the rotated Clayton copula.

When comparing the estimation results of the A-DCC Gaussian copula it is striking that the asymmetric parameter for the advanced emerging market bond portfolio is significant: it is significantly higher than for the developed bond portfolio (parameter estimation results can be found in table F.12 and F.14). Since the A-DCC parameter of the portfolio consisting of advanced emerging and developed bond indices is also significant it can be concluded that both seem to move together in bear markets.

Figure 5.16(a) shows the average correlation estimated with the t-copula for the bond index portfolios. Again, we choose this copula because it yields the best fit of the elliptical



(a) Average Correlation



(b) Average Tail Dependence

Figure 5.16: Average Correlation and Tail Dependence Bonds. This figure shows the average bivariate correlation and the average bivariate tail coefficient for the bond portfolios estimated with the t-copula. The vertical line shows the default of the Lehman Brothers bank.

copulas. Before the Lehman crash the average correlation of the developed bond indices is permanently higher than for the other two portfolios. Shortly after the default however, there is a drop and the average correlation continues on a lower level. The tail dependence of the (rotated) Clayton copula displays the same pattern. Following the crash the portfolio of the developed and advanced emerging bond indices converges to the average correlation level of the developed bond indices. This implies that the bond markets of the advanced emerging economies became more integrated into the world economy after the Lehman default. Figure 5.16(b) shows the average bivariate tail dependence coefficient of the bond portfolios. First of all, the figure confirms the decreasing dependence of the developed bond indices for the tail dependence, though there is no sharp drop after the Lehman default. It is also apparent that the average tail dependence of the portfolio composed of the advanced emerging and developed markets is constantly below the one of the developed markets- there is no upward trend even after the Lehman default.

The last portfolio we investigate consists of the secondary emerging market and developed market bonds. For this portfolio the t-Clayton mixture copula seems to fit the data sample best. The two log-likelihood based criteria (AIC and BIC), the AD, and the KS distance measures all favor this copula whilst the AAD and AKS criteria prefer the t-vine copula with a symmetric dynamic dependence structure (DCC). Again, the Clayton copula performs slightly better than the rotated Clayton copula for three out of the four statistical distance measures.

Since all statistical distance measures between these two copulas are very close together, it might be concluded that the joint positive and joint negative events occur almost to the same degree. Of the copulas with possible different dependence structures only the Gaussian and Gaussian-vine copula favor an asymmetric dependence structure.

Table F.15 shows the parameter estimation results for secondary emerging market bonds estimated with a Gaussian copula. Here the asymmetric parameter of the A-DCC structure is not significant, it's low level indicating that the secondary emerging market bonds do not move together during bear markets. Figure 5.16(a) confirms that the average correlation of the secondary emerging bond markets with the developed markets is much lower than that of the advanced emerging and the developed markets. A look at the bivariate average tail dependence coefficient of the t-copula in figure 5.16(b) supports this from a tail dependence view, although the difference is relatively small. A graphical inspection of figures 5.15(b) and 5.15(a) meanwhile reveals that the secondary emerging market bonds share fewer joint positive and less joint negative events with the developed bond indices than the advanced emerging markets.

Taking all information accumulated so far into account, it can be concluded that the secondary emerging market bonds are less integrated than the advanced emerging market bonds with all models suggesting this is the case. In contrast to the stock portfolios, the average correlation of the bond portfolios does not seem to increase during the time period. Furthermore, for the majority of the time, the three portfolios display different kinds of behavior, whereas the stock portfolios share the same behavior for the majority of the time- only on a different level. In fact, the average correlation and average tail dependence for the bond portfolios seems to decrease up until the default of the Lehman Brothers bank. Thereafter, the average correlation between the advanced emerging and the developed bond indices increases until they are on the same level. The bivariate tail dependence between these two increases too, although not as considerably as the average correlation.

It is also important to consider possible diversification benefits from the level of financial integration. The comparison of the multivariate Archimedean copulas shows a better fit of the rotated Clayton copula with regards to developed bond indices portfolio. The Gaussian copula with positive asymmetries supports these results. When mixed with emerging market bonds the Clayton copula is more appropriate, implying that the portfolios share more joint negative than positive events. Since the dependence has been altered through the addition of the emerging markets it can be concluded that they are not integrated into the developed financial markets. However it is important to note that from the persepctive of a portfolio manager the increased occurrence of joint negative events might have had a negative impact on the portfolio's characteristics.

Chapter 6

Conclusions

Coping with multivariate dependence is a major issue in portfolio and risk management. For a long time Pearson's correlation coefficient dominated the field. Recent research showed that (i) Pearson's correlation coefficient is not suitable for financial time series as a result of its statistical properties, (ii) dependence varies over time and (iii) dependence of financial time series often shows asymmetries, i.e. the dependence increases during bear markets. The main purpose of this analysis is the development and comparison of different multivariate time-varying dependence models that take these features into account. The multivariate dependence concepts introduced can be sub-classified into multivariate GARCH and copulas, with the copulas then grouped into elliptical, Archimedean and D-vine copulas. The risk management requirements are taken care of by a comparison of Value-at-Risk estimates and the portfolio management requirements with an in-depth dependence structure analysis of different portfolios.

Copulas used for estimating time-varying dependence of more than two dimensions are dominated by the elliptical copulas which means analysts much accept their known limitations to symmetrical dependence. Time-varying Archimedean copulas are able to capture asymmetric dependence but research in this area is dominated by bivariate analysis. Of most practical relevance for portfolio and risk managers are the cases concerned with more than two dimensions. To allow for time-varying dependence analysis with multivariate Archimedean copulas I have proposed in co-operation with Valentin Braun a new dependence structure. Building on the multivariate time-varying Archimedean copulas, time-varying dynamic mixture copulas are introduced: these combine the features of Elliptical and Archimedean copulas. Additionally, a new modelling scheme for the time-varying weights of the copulas in the mixture is proposed. To take the elliptical copulas to time-varying asymmetric dependence, a model structure originally proposed for multivariate GARCH model is applied to them. A scalar version of this dependence structure is applied to D-vine copulas.

After these theoretical considerations the focus is turned to empirical applications of the multivariate models introduced. First of all, the ability to estimate the Value-at-Risk is tested. Four different but equally weighted portfolios of different asset classes and countries are created. The countries are sub-divided into developed and emerging countries and the asset classes into equity and bond indices. This gives a broad range of different portfolio characteristics and raises the question as to whether there is single, superior risk model for all portfolios or if the different portfolio characteristics require different risk models. The VaR-backtest sample includes the recent financial crisis- with the default of the Lehman Brothers bank- and thus it is possible to compare the risk behavior of emerging and developed markets during the crisis. To justify the use of parametric models the non-parametric historical simulation method acts as a benchmark model. The comparison of different VaR tests shows that the two parametric model classes differ in their results.

In general, the different copula models showed only minor differences although they depict quite different dependence structures. For the developed stocks portfolio the Gaussian copula seems to have the best overall performance, although the multivariate GARCH model gives a superior result for the 99% confidence level VaR. All parametric models but the Delta-Normal method overestimate the VaR of the developed bond indices portfolio for all confidence levels by a considerable amount: this might be through misspecification of the marginals. Several copulas show similar performance for the emerging market stocks portfolio, although the results of the rotated Clayton D-vine copula suggest that it outperforms the other models slightly. The MVGARCH displays a good VaR estimation overall performance for the emerging market bond portfolio. For all portfolios bar the developed bond indices, the parametric risk models outperform the historical simulation method: devoting time and money to the development of parametric models would therefore seem to make sense. The main result is that no model displays a superior performance for all portfolios and VaR confidence levels. In fact, the risk manager should clarify which level of risk is most important to him and thereafter decide which model suits his risk needs and the portfolio characteristic best.

The second empirical section of this analysis is devoted to an in-depth analysis of the dependence structure between emerging and developed markets. This is guided by the consideration that emerging markets are not fully integrated into the developed financial markets and therefore turn out to be useful diversifying a portfolio of developed countries. A further point to note is that there are differences in the economic development of emerging markets: some already exhibit the growth levels of the developed countries whilst others remain such levels of growth. Therefore, emerging equity and bond markets are subdivided into advanced and secondary emerging markets. The latter resemble countries that are one level below the economic development of the advanced emerging markets. The dependence structure analysis also includes the findings of the best-fitting model for each portfolio via a goodness-of-fit test.

The analysis of all possible dependence models results in the finding that the secondary emerging markets are less integrated into the developed markets than the advanced emerging markets. This applies to the equity as well as bond indices. However, the equity and bond portfolios display quite different behaviors through time: the average correlation and left tail dependence seems to increase for all stock portfolios over time. Furthermore, the behavior of the two mixed portfolios frequently resembles the behavior of the developed stocks portfolio, only on different dependence levels. In contrast, left and right tail dependence for the developed bond indices decreases until the default of the Lehman Brothers bank. The Lehman crash seems to mark a structural break. After the default, left and right tail dependence of the developed bond indices remains continuous on a lower level. Both mixed portfolios permanently display a much lower tail dependence at almost the same level. After the default of the Lehman Brothers bank, the average correlation between the advanced emerging and the developed bond indices converge to the same level, while the average correlation between the secondary emerging and developed indices continuous on a much lower level. The goodness-of-fit test reveals that the time-varying D-vine t-copula exhibits the best fit for most portfolios. For one portfolio the newly developed t-Clayton dynamic mixture copula fits best. The worst fit across all portfolios comes from the multivariate time-varying Archimedean copulas. These copulas describe the dependence structure only through one parameter and it is likely that financial time series do not show enough common features to be described through a single dependence parameter. Nevertheless, they are useful in highlighting the time-varying tail dependence. For example the comparison of the Clayton and rotated Clayton copula for the developed bond shows that the developed bond indices exhibit more dependence in the right than in the left tail. This is an important finding for the use of diversification benefits since in the case of bond indices the addition of the emerging market indices changes the predominant right tail dependence to a left tail one.

Future research following this line might apply the newly developed time-varying dependence structure to copulas other than Archimedean copulas, potentially circumventing the disadvantage of describing the whole dependence through a single parameter. Furthermore, it would seem that further research on multivariate tail dependence would be fruitful. The goodness-of-fit section shows that the dynamic mixture copulas seem to be a promising approach displaying dependence. Since several more multivariate copulas other than those introduced in this research exist it might be useful to mix other Archimedean copulas or more than two copulas at a time. The analysis also shows that time-varying D-vine copulas are a useful tool to analyze financial time series: it might also be promising to mix two or more of these.

Appendix A

Price and Return Series

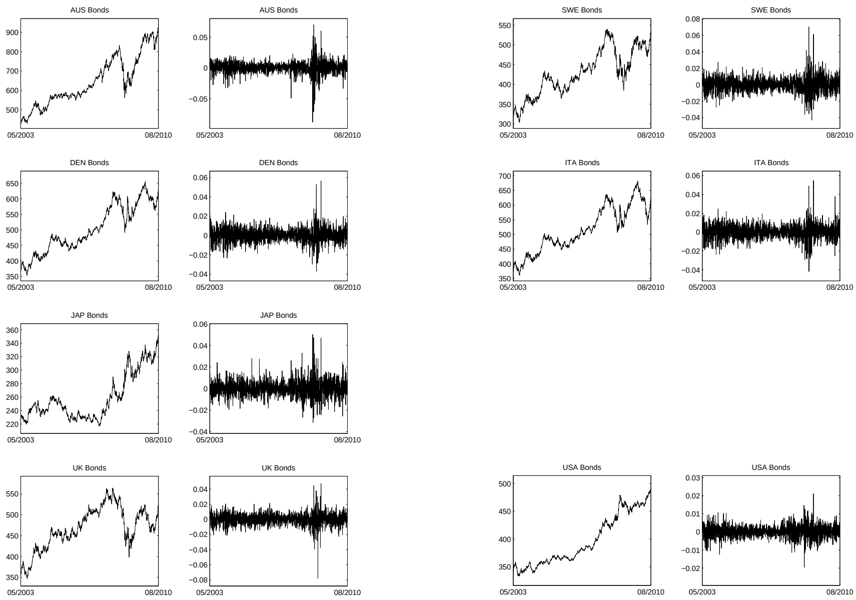


Figure A.1: Developed Bond Indices Characteristics. This figure shows AUS, DEN, SWE, ITA, JAP, UK, and USA Bond Characteristics.

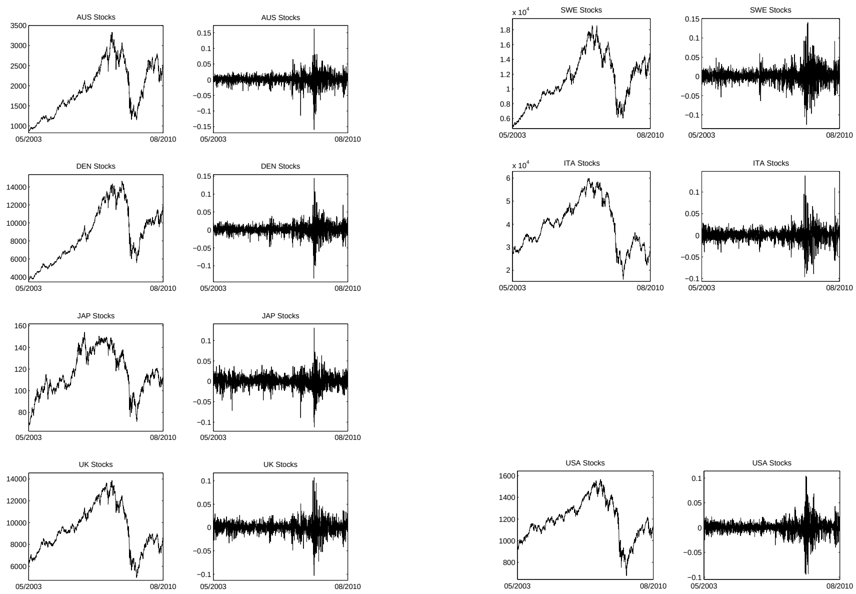


Figure A.2: Developed Stock Indices Characteristics. This figure shows AUS, DEN, SWE, ITA, JAP, UK, and USA Stock Characteristics.

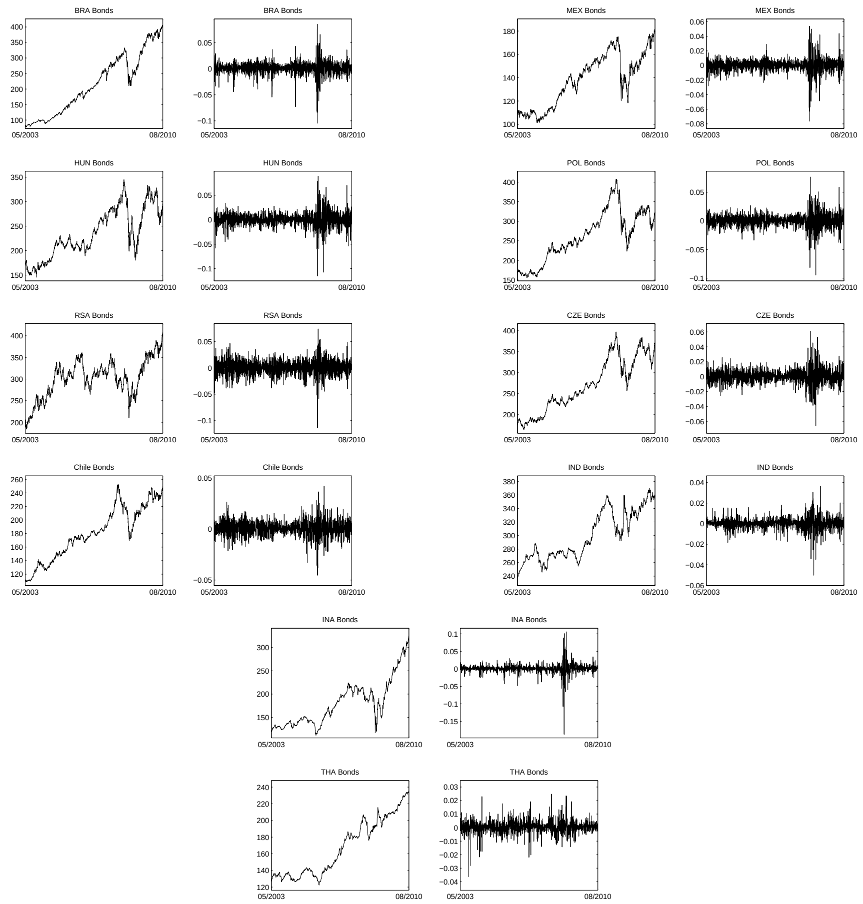


Figure A.3: Emerging Bond Indices Characteristics. This figure shows BRA, HUN, MEX, POL, RSA, CHI, CZE, IND, INA, and THA Bond Characteristics.

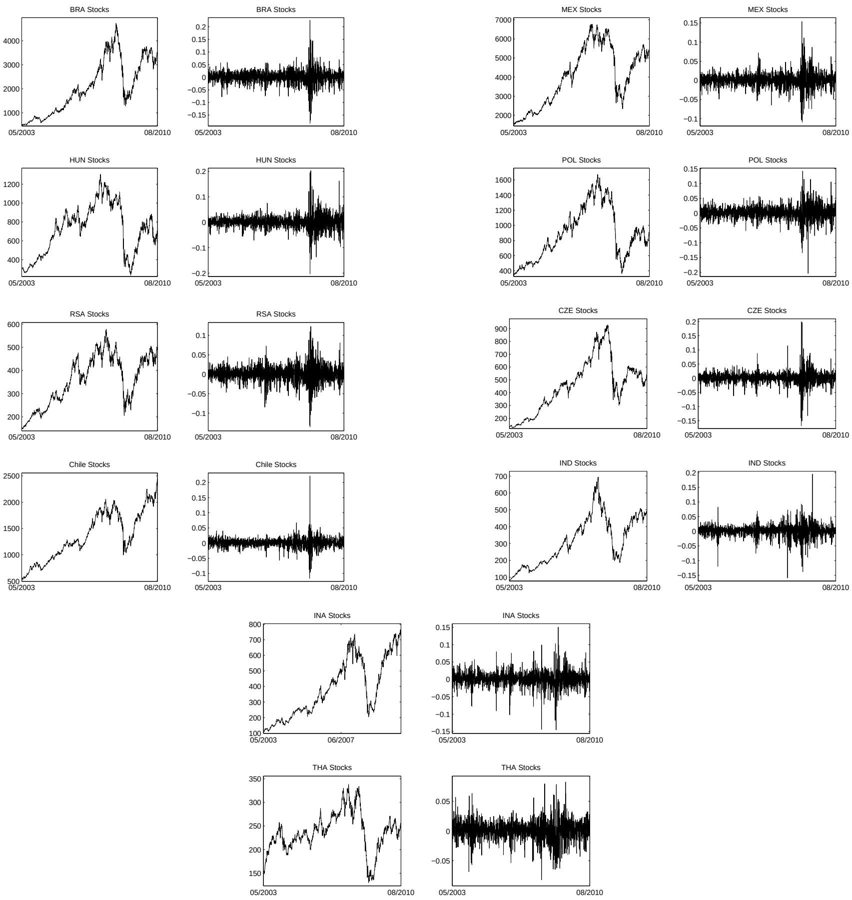


Figure A.4: Emerging Stock Indices Characteristics. This figure shows BRA, HUN, MEX, POL, RSA, CHI, CZE, IND, INA, and THA Stock Characteristics.

Appendix B

Descriptive Statistics

Table B.1: Descriptive Statistics Bonds.

Asset	Min Return	Max Return	Annualized Mean Return	Annualized Volatility	Kurtosis	Skewness	JB
			Advanced Emerging Markets				
BRA	-0.1048	0.0857	0.1414	0.1818	15.6840	-0.8587	12465
HUN	-0.1152	0.0894	0.0614	0.2135	12.2443	-0.5743	6602
MEX	-0.0764	0.0538	0.0570	0.1376	14.9295	-0.7565	11002
POL	-0.0947	0.0767	0.0689	0.1739	11.5303	-0.4910	5610
RSA	-0.1138	0.0741	0.0739	0.2166	7.1795	-0.4355	1387
			Secondary Emerging Markets				
CHI	-0.0454	0.0421	0.0854	0.1138	6.8715	-0.3010	1167
CZE	-0.0660	0.0613	0.0782	0.1407	8.6259	-0.0073	2408
IND	-0.0502	0.0365	0.0475	0.0838	12.7805	-0.0534	7364
INA	-0.1874	0.1060	0.0999	0.1834	58.4229	-2.1669	23513
THA	-0.0363	0.0248	0.0686	0.0664	11.5614	-0.3604	5616
			Developed Markets				
AUS	-0.0883	0.0701	0.0812	0.1525	16.6034	-0.9106	14332
DEN	-0.0371	0.0567	0.0590	0.1108	8.1187	0.2886	2019
SWE	-0.0431	0.0705	0.0581	0.1363	8.0959	0.4353	2033
ITA	-0.0418	0.0548	0.0588	0.1170	7.0360	0.1421	1245
JAP	-0.0318	0.0502	0.0495	0.1152	7.2314	0.5713	1461
GBR	-0.0781	0.0471	0.0425	0.1202	11.8293	-0.5655	6028
USA	-0.0196	0.0212	0.0427	0.0522	5.5064	0.0007	477

Notes to Table: This table shows summary statistics for daily bond returns. JB denotes the test statistic of the Jarque-Bera test (critical value=5.9606). The annualized volatility is based on the sample variance.

Table B.2: Descriptive Statistics Stocks.

Asset	Min Return	Max Return	Annualized Mean Return	Annualized Volatility	Kurt	Skew	JB
			Advanced Emerging Markets				
BRA	-0.1832	0.2263	0.1596	0.4000	12.8131	-0.2051	7339
HUN	-0.2035	0.2031	0.0845	0.4127	13.1874	0.0271	7896
MEX	-0.1090	0.1539	0.1162	0.2925	10.4417	-0.0679	4214
POL	-0.2040	0.1423	0.0952	0.3605	10.1189	-0.4584	3919
RSA	-0.1357	0.1235	0.1136	0.3273	8.0642	-0.3263	1983
			Secondary Emerging Markets				
CHI	-0.1162	0.2215	0.1342	0.2435	32.1645	0.6672	64849
CZE	-0.1675	0.1998	0.1299	0.3305	19.7189	-0.0281	21267
INA	-0.1595	0.1949	0.1503	0.3241	13.2630	-0.3613	8054
IND	-0.1458	0.1504	0.1600	0.3368	9.8333	-0.3904	3599
THA	-0.0821	0.0823	0.0615	0.2561	6.0098	-0.2325	705
			Developed Markets				
AUS	-0.1597	0.1630	0.1019	0.2952	15.0768	-0.7800	11282
DEN	-0.1351	0.1446	0.1110	0.2574	14.1861	-0.1971	9532
SWE	-0.1256	0.1405	0.1034	0.3198	9.4659	0.0604	3182
ITA	-0.0968	0.1380	0.0064	0.2546	12.5227	-0.0152	6899
JAP	-0.1122	0.1311	0.0593	0.2547	10.0835	-0.2982	3844.6
GBR	-0.1031	0.1071	0.0391	0.2245	11.5291	-0.0444	5535
USA	-0.0947	0.1042	0.0255	0.2150	14.1638	-0.3890	9524

Notes to Table: This table shows summary statistics for daily stock returns. JB denotes the test statistic of the Jarque-Bera test (critical value=5.9606). The annualized volatility is based on the sample variance.

Table B.3: Descriptive Statistics Portfolios.

Asset	Min Return	Max Return	Annualized Mean Return	Annualized Volatility	Kurtosis	Skewness	JB	
Dev Stocks	-0.0874	0.12603	0.0771	0.2029	16.5211	-0.2262	13925.1	
EM Adv Stocks	-0.1311	0.1565	0.1257	0.2997	12.8311	-0.1769	7363.0	
EM Sec Stocks	-0.1007	0.1331	0.1396	0.2180	14.8435	-0.6583	10798.1	
EM Adv	Dev Stocks	-0.0990	0.1389	0.1008	0.2344	15.3167	-0.1739	11551.2
EM Sec	Dev Stocks	-0.0930	0.1290	0.1073	0.1980	17.5171	-0.4867	16106.4
Dev Bonds	-0.0257	0.0467	0.0583	0.0865	8.1573	0.3388	2058.6	
EM Adv Bonds	-0.0699	0.0652	0.0874	0.1478	12.0769	-0.4202	6322.2	
EM Sec Bonds	-0.0284	0.0273	0.0849	0.0656	10.1983	-0.3604	3981.8	
EM Adv	Dev Bonds	-0.0347	0.0434	0.0722	0.0999	8.4309	-0.0052	2244.0
EM Sec	Dev Bonds	-0.0266	0.0373	0.0681	0.0717	9.4235	0.2513	3158.5

Notes to Table: This table shows summary statistics for equal weighted portfolios. JB denotes the test statistic of the Jarque-Bera test (critical value=5.9606). The annualized volatility is based on the sample variance.

Appendix C

Unconditional and Asymmetric Correlation Analysis

Table C.1: Unconditional Correlation Stocks.

	BRA	HUN	MEX	POL	RSA	CHI	COL	CZE	IND	INA	MAS	THA	AUS	DEN	SWE	ITA	JPN	GBR	USA
BRA	1																		
HUN	0.547	1																	
MEX	0.772	0.544	1																
POL	0.548	0.729	0.557	1															
RSA	0.615	0.652	0.587	0.667	1														
CHI	0.666	0.489	0.644	0.490	0.534	1													
COL	0.496	0.433	0.489	0.445	0.479	0.470	1												
CZE	0.540	0.679	0.514	0.703	0.637	0.510	0.479	1											
IND	0.387	0.410	0.373	0.450	0.461	0.357	0.364	0.436	1										
INA	0.325	0.349	0.312	0.374	0.431	0.308	0.323	0.411	0.489	1									
MAS	0.310	0.358	0.296	0.361	0.434	0.312	0.286	0.377	0.446	0.533	1								
THA	0.309	0.331	0.280	0.348	0.414	0.265	0.266	0.365	0.425	0.506	0.506	1							
AUS	0.477	0.547	0.444	0.548	0.649	0.483	0.408	0.609	0.505	0.521	0.560	0.547	1						
DEN	0.626	0.657	0.602	0.657	0.698	0.579	0.489	0.682	0.447	0.439	0.420	0.389	0.661	1					
SWE	0.633	0.664	0.631	0.678	0.688	0.555	0.496	0.639	0.422	0.334	0.378	0.350	0.572	0.794	1				
ITA	0.602	0.579	0.627	0.585	0.581	0.549	0.452	0.561	0.427	0.365	0.366	0.325	0.506	0.656	0.711	1			
JPN	0.320	0.365	0.300	0.346	0.446	0.317	0.296	0.418	0.392	0.446	0.448	0.541	0.636	0.413	0.345	0.396	1		
GBR	0.619	0.569	0.629	0.575	0.604	0.560	0.482	0.569	0.425	0.357	0.370	0.348	0.524	0.677	0.711	0.842	0.405	1	
USA	0.651	0.404	0.703	0.379	0.358	0.525	0.320	0.328	0.284	0.146	0.134	0.149	0.230	0.422	0.501	0.527	0.167	0.516	1

Notes to Table: This table shows the unconditional correlation for daily stock returns.

Table C.2: Unconditional Correlation Bonds.

	BRA	HUN	MEX	POL	RSA	CHI	CZE	IND	INA	THA	AUS	DEN	SWE	ITA	JPN	GBR	USA
BRA	1																
HUN	0.456	1															
MEX	0.608	0.457	1														
POL	0.469	0.800	0.494	1													
RSA	0.489	0.589	0.472	0.602	1												
CHI	0.087	0.064	0.111	0.092	0.104	1											
CZE	0.350	0.732	0.345	0.793	0.526	0.072	1										
IND	0.211	0.276	0.193	0.271	0.242	0.009	0.215	1									
INA	0.266	0.303	0.287	0.323	0.267	0.079	0.238	0.212	1								
THA	0.160	0.218	0.171	0.234	0.213	0.082	0.265	0.203	0.245	1							
AUS	0.527	0.593	0.498	0.634	0.565	0.083	0.618	0.280	0.398	0.282	1						
DEN	0.230	0.579	0.201	0.641	0.428	0.122	0.778	0.204	0.212	0.289	0.586	1					
SWE	0.317	0.645	0.280	0.702	0.506	0.109	0.784	0.254	0.219	0.258	0.619	0.845	1				
ITA	0.256	0.618	0.231	0.682	0.452	0.127	0.802	0.229	0.223	0.280	0.593	0.965	0.854	1			
JPN	-0.22	-0.00	-0.23	0.017	-0.06	0.073	0.179	-0.016	-0.143	0.260	-0.040	0.370	0.248	0.323	1		
GBR	0.198	0.447	0.212	0.529	0.367	0.109	0.582	0.165	0.192	0.206	0.520	0.725	0.624	0.722	0.248	1	
USA	-0.17	-0.08	-0.11	-0.06	-0.10	0.054	0.004	-0.063	-0.063	0.069	-0.074	0.252	0.089	0.222	0.331	0.233	1

Notes to Table: This table shows the unconditional correlation for daily bond returns.

Table C.3: Asymmetric Correlation Stocks.

	HUN	MEX	POL	RSA	CHI	CZE	IND	INA	THA	AUS	DEN	SWE	ITA	JPN	GBR	USA
BRA	0.1485	0.8502	0.6840	0.7899	0.6565	0.2302	0.4215	0.5904	0.0880	0.5805	0.8908	0.8615	0.4098	0.7533	0.5744	0.9155
HUN		0.6863	0.7853	0.5684	0.7587	0.2816	0.1838	0.5129	0.7560	0.6199	0.6767	0.9681	0.7454	0.5893	0.5623	0.3640
MEX			0.7514	0.7798	0.8938	0.1491	0.9749	0.9513	0.5509	0.4226	0.8177	0.9546	0.8819	0.6508	0.8284	0.8366
POL				0.3427	0.4018	0.6270	0.4951	0.4408	0.4353	0.5185	0.8868	0.7477	0.5035	0.1814	0.3266	0.8370
RSA					0.2007	0.5381	0.3003	0.4202	0.8206	0.2420	0.8167	0.7496	0.6572	0.3657	0.2657	0.2811
CHI						0.8815	0.8349	0.7385	0.4733	0.9381	0.7878	0.3136	0.2919	0.5197	0.7276	0.8766
CZE							0.2139	0.2928	0.1293	0.8117	0.9412	0.2723	0.8275	0.6637	0.8364	0.6126
IND								0.4079	0.3271	0.6267	0.2831	0.3423	0.2745	0.1355	0.1153	0.6180
INA									0.2894	0.2475	0.7365	0.6798	0.0681	0.1134	0.4133	0.3109
THA										0.0889	0.8177	0.7596	0.7375	0.4000	0.8651	0.4397
AUS											0.8240	0.9410	0.2670	0.8252	0.4887	0.3370
DEN												0.9826	0.3803	0.6301	0.7239	0.5461
SWE													0.8932	0.4154	0.7461	0.5549
ITA														0.2147	0.8247	0.8215
JPN															0.1965	0.6585
GBR																0.4140

Notes to Table: This table reports p-values for the test of Hong, Tu, and Zhou (2007) for asymmetric correlation.

Table C.4: Asymmetric Correlation Bonds.

	HUN	MEX	POL	RSA	CHI	CZE	IND	INA	THA	AUS	DEN	SWE	ITA	JPN	GBR	USA
BRA	0.3966	0.4747	0.3908	0.4165	0.2300	0.0912	0.0547	0.9253	0.2149	0.7797	0.7407	0.8887	0.6490	0.4667	0.2111	0.0402
HUN		0.0052	0.8950	0.5685	0.2357	0.9453	0.8430	0.5563	0.0012	0.3049	0.2401	0.8851	0.2803	0.1446	0.5642	0.7023
MEX			0.5559	0.8961	0.4001	0.1836	0.4565	0.9927	0.8408	0.9896	0.0218	0.6922	0.1275	0.3452	0.0661	0.0001
POL				0.9143	0.2399	0.9747	0.4142	0.0923	0.3057	0.7917	0.5962	0.9981	0.8459	0.1238	0.7947	0.0138
RSA					0.6094	0.3661	0.6876	0.7901	0.1490	0.4981	0.5380	0.3882	0.7721	0.3665	0.4429	0.5375
CHI						0.2058	0.8115	0.0468	0.2622	0.4193	0.4933	0.2925	0.8849	0.2567	0.5701	0.3846
CZE							0.8064	0.1277	0.3450	0.2112	0.8234	0.8644	0.9898	0.6327	0.7795	0.6288
IND								0.5150	0.4643	0.3250	0.0266	0.3066	0.2509	0.5123	0.2254	0.0171
INA									0.0072	0.5973	0.0600	0.7911	0.4020	0.0006	0.5563	0.0452
THA										0.3397	0.0981	0.2696	0.3531	0.2340	0.2337	0.1982
AUS											0.1232	0.5377	0.1603	0.6212	0.3803	0.3686
DEN												0.6201	0.7862	0.4965	0.7512	0.6920
SWE													0.8826	0.5236	0.3802	0.1933
ITA														0.4180	0.7076	0.1290
JPN															0.3522	0.7027
GBR																0.1291

Notes to Table: This table reports p-values for the test of Hong, Tu, and Zhou (2007) for asymmetric correlation.

Appendix D

Estimation Results Univariate AR-GARCH Models

Table D.1: Marginal Models Emerging Market Stocks (Copula).

	Model selected	Distribution	α_0	α_1	γ	β_1	ϕ_0	ϕ_1	ν	λ	KS p-value	LB p-value	LM p-value
BRA	GJRGARCH	skew-t	0.0000	0.0065	0.1432	0.8831	0.0011	0.0731	10.1868	-0.1430	0.6028	0.6371	0
			(0.0000)	(0.0258)	(0.0657)	(0.0436)	(0.0005)	(0.0279)	(2.2273)	(0.0336)			
HUN	ZARCH	GED	0.0004	0.0610	0.0701	0.9039	0.0014	0.0462	1.5215		0.3769	0.5478	0
			(0.0002)	(0.0148)	(0.0200)	(0.0165)	(0.0004)	(0.0331)	(0.0706)				
MEX	ZARCH	skew-t	0.0004	0.0081	0.1351	0.9151	0.0006	0.0844	8.2150	-0.1447	0.5751	0.6796	0
			(0.0002)	(0.0400)	(0.0471)	(0.0307)	(0.0003)	(0.0257)	(1.4753)	(0.0326)			
POL	GJRGARCH	GED	0.0000	0.0160	0.0857	0.9214	0.0009	0.0271	14719		0.7419	0.7909	0
			(0.0000)	(0.0099)	(0.0265)	(0.0175)	(0.0004)	(0.0210)	(0.0746)				
RSA	ZARCH	skew-t	0.0005	0.0229	0.1108	0.9138	0.0008	0.0348	12.0492	-0.1113	0.7688	0.4770	0
			(0.0002)	(0.0224)	(0.0383)	(0.0260)	(0.0004)	(0.0387)	(3.0721)	(0.0316)			
CHI	GJRGARCH	skew-t	0.0000	0.0358	0.1296	0.8574	0.0009	0.1085	9.4775	-0.0747	0.7145	0.2737	0
			(0.0000)	(0.0306)	(0.0619)	(0.0491)	(0.0003)	(0.0276)	(1.9827)	(0.0334)			
CZE	GJRGARCH	skew-t	0.0000	0.0470	0.1281	0.8409	0.0012	0.0347	7.4159	-0.0769	0.4690	0.1945	0
			(0.0000)	(0.0324)	(0.0752)	(0.0462)	(0.0004)	(0.0276)	(1.0923)	(0.0330)			
IND	ZARCH	skew-t	0.0007	0.0488	0.1529	0.8640	0.0016	0.0616	5.9679	-0.1173	0.5209	0.1248	0
			(0.0003)	(0.0291)	(0.0639)	(0.0378)	(0.0004)	(0.0303)	(0.8586)	(0.0382)			
INA	GJRGARCH	GED	0.0000	0.0465	0.1357	0.8416	0.0012	0.0701	1.1465		0.2555	0.1247	0
			(0.0000)	(0.0169)	(0.0563)	(0.0536)	(0.0005)	(0.0329)	(0.0637)				
THA	ZARCH	GED	0.0002	0.0404	0.0446	0.9372	0.0006	0.0118	1.2369		0.4197	0.1018	0
			(0.0001)	(0.0125)	(0.0175)	(0.0156)	(0.0004)	(0.0024)	(0.0624)				

Notes to Table: Univariate GARCH parameter estimates of the emerging market stocks for use with multivariate copula models. Standard errors are in parenthesis.

Table D.2: Marginal Models Emerging Markets Stocks (MVGARCH).

	Model selected	Distribution	α_0	α_1	γ	β_1	ϕ_0	ϕ_1	KS p-value	LB p-value	LM p-value
BRA	GJRGARCH	Gaussian	0.0000	0.0113	0.1406	0.8754	0.0010	0.0943	0.0186	0.6924	0
			(0.0000)	(1.8518)	(1.3328)	(1.1342)	(0.0081)	(0.6903)			
HUN	ZARCH	Gaussian	0.0005	0.0598	0.0743	0.9030	0.0012	0.0550	0.1441	0.3940	0
			(0.0001)	(0.0141)	(0.0193)	(0.0162)	(0.0000)	(0.0001)			
MEX	ZARCH	Gaussian	0.0004	0.0147	0.1133	0.9192	0.0005	0.0893	0.0406	0.7342	0
			(0.0001)	(0.0192)	(0.0250)	(0.0151)	(0.0003)	(0.0237)			
POL	GJRGARCH	Gaussian	0.0000	0.0110	0.0968	0.9195	0.0008	0.0623	0.0045	0.8561	0
			(0.0000)	(0.0214)	(0.0278)	(0.0266)	(0.0004)	(0.0236)			
RSA	GJRGARCH	Gaussian	0.0000	0.0272	0.1179	0.8882	0.0009	0.0403	0.0756	0.4450	0
			(0.0000)	(0.0155)	(0.0299)	(0.0199)	(0.0004)	(0.0283)			
CHI	GJRGARCH	Gaussian	0.0000	0.0463	0.1102	0.8684	0.0009	0.1158	0.3107	0.2728	0
			(0.0000)	(0.0173)	(0.0306)	(0.0193)	(0.0003)	(0.0254)			
CZE	GJRGARCH	Gaussian	0.0000	0.0502	0.1343	0.8397	0.0011	0.0595	0.1234	0.2888	0
			(0.0000)	(0.0151)	(0.0384)	(0.0229)	(0.0003)	(0.0280)			
IND	ZARCH	Gaussian	0.0007	0.0768	0.1408	0.8511	0.0016	0.0820	0.0252	0.1851	0
			(0.0001)	(0.0215)	(0.0174)	(0.0288)	(0.0001)	(0.0276)			
INA	GJRGARCH	Gaussian	0.0000	0.0327	0.1268	0.8707	0.0010	0.1224	0.0064	0.6170	0
			(0.0000)	(0.0156)	(0.0466)	(0.0427)	(0.0004)	(0.0280)			
THA	ZARCH	Gaussian	0.0003	0.0452	0.0451	0.9323	0.0005	0.0357	0.0555	0.1688	0
			(0.0001)	(0.0126)	(0.0201)	(0,0158)	(0.0003)	(0.0224)			

Notes to Table: Univariate GARCH parameter estimates of the emerging market stocks for use with multivariate GARCH models. Standard errors are in parenthesis.

Table D.3: Marginal Models Emerging Markets Bonds (Copula).

	Model selected	Distribution	α_0	α_1	γ	β_1	ϕ_0	ϕ_1	ν	λ	KS p-value	LB p-value	LM p-value
BRA	ZARCH	skew-t	0.0004	0.0522	0.1411	0.8613	0.0009	0.0501	9.0707	-0.1500	0.9429	0.7127	0
			(0.0003)	(0.0305)	(0.0625)	(0.0528)	(0.0002)	(0.0321)	(1.7492)	(0.0361)			
HUN	GJRGARCH	skew-t	0.0000	0.0240	0.0759	0.9223	0.0001	0.0735	9.0209	-0.0860	0.9922	0.8075	0
			(0.0000)	(0.0229)	(0.0479)	(0.0333)	(0.0002)	(0.0245)	(1.7652)				
MEX	ZARCH	skew-t	0.0002	0.0386	0.1184	0.8991	0.0002	0.1221	9.2745	-0.1862	0.9119	0.8530	0
			(0.0001)	(0.0304)	(0.0444)	(0.0413)	(0.0001)	(0.0272)	(1.8635)	(0.0327)			
POL	ZARCH	skew-t	0.0001	0.0399	0.0531	0.9336	0.0006	0.0728	10.9640	-0.0871	0.9547	0.1143	0
			(0.0001)	(0.0212)	(0.0362)	(0.0264)	(0.0002)	(0.0272)	(2.3592)	(0.0361)			
RSA	GJRGARCH	skew-t	0.0000	0.0232	0.0557	0.9275	0.0004	0.0318	15.5608	-0.0686	0.9970	0.4657	0
			(0.0000)	(0.0204)	(0.0410)	(0.0299)	(0.0003)	(0.0235)	(4.5730)	(0.0312)			
CHI	GARCH	Student-t	0.0000	0.0756		0.9205	0.0005	0.1020	10.1319		0.1053	0.0226	0
			(0.0000)	(0.0161)		(0.0163)	(0.0001)	(0.0230)	(2.2578)				
CZE	GARCH	Student-t	0.0000	0.0395		0.9583	0.0005	0.0207	9.9768		0.0894	0.0920	0
			(0.0000)	(0.0068)			(0.0063)	(0.0002)	(2.0775)				
IND	GARCH	GED	0.0000	0.0679		0.9261	0.0003	-0.0080	1.0500		0.6587	0	0
			(0.0000)	(0.0149)		(0.0140)	(0.0003)	(0.0735)	(0.0545)				
INA	GARCH	Student-t	0.0000	0.2574		0.7426	0.0008	0.1054	3.2126		0	0	0
			(0.0000)	(0.0448)		(0.0439)	(0.0001)	(0.0245)	(0.2101)				
THA	ZARCH	GED	0.0001	0.0936	0.0001	0.9063	0.0003	0.0576	1.0100		0.9648	0	0
			(0.0001)	(0.0747)	(0.1251)	(0.0542)	(0..0002)	(0.0019)	(0.0544)				

Notes to Table: Univariate GARCH parameter estimates of the emerging market bonds for use with multivariate copula models. Standard errors are in parenthesis.

Table D.4: Marginal Models Emerging Market Bonds (MVGARCH).

	Model selected	Distribution	α_0	α_1	γ	β_1	ϕ_0	ϕ_1	KS p-value	LB p-value	LM p-value
BRA	ZARCH	Gaussian	0.0004	0.0703	0.1413	0.8450	0.0009	0.08754	0.0266	0.6775	0
			(0.0001)	(0.0180)	(0.0229)	(0.0182)	(0.0002)	(0.0248)			
HUN	GJRGARCH	Gaussian	0.0000	0.02324	0.0911	0.9157	0.0004	0.0801	0.0388	0.8070	0
			(0.0000)	(0.0134)	(0.0199)	(0.0123)	(0.0002)	(0.0247)			
MEX	GJRGARCH	Gaussian	0.0000	0.04660	0.14364	0.8430	0.0002	0.1473	0.0055	0.8336	0
			(0.0000)	(0.00173)	(0.0331)	(0.0299)	(0.0001)	(0.0261)			
POL	ZARCH	Gaussian	0.0002	0.0374	0.0684	0.9272	0,0006	0.0847	0.4631	0.1294	0
			(0.0000)	(0.0111)	(0.0152)	(0.0112)	(0.0001)	(0.0251)			
RSA	GJRGARCH	Gaussian	0.0000	0.0273	0.0602	0.9182	0.0005	0.0405	0.2899	0.4998	0
			(0.0000)	(0.0126)	(0.0193)	(0.0145)	(0.0003)	(0.0244)			
CHI	ZARCH	Gaussian	0.0001	0.0869	0.0008	0.9126	0.0006	0.1177	0.3308	0.0298	0
			(0.0001)	(0.0169)	(0.0184)	(0.0266)	(0.000)	(0.0014)			
CZE	GARCH	Gaussian	0.0000	0.0445		0.9525	0.0006	0.0362	0.0089	0.1352	0
			(0.0000)	(0.0082)		(0.0076)	(0.0002)	(0.0236)			
IND	ZARCH	Gaussian	0.0001	0.0673	0.0079	0.9288	0.0003	0.0131	0	0	0
			(0.0001)	(0.0215)	(0.0174)	(0.0288)	(0.0001)	(0.0276)			
INA	GJRGARCH	Gaussian	0.0000	0.0715	0.0857	0.8687	0.0005	0.1664	0	0.0034	0
			(0.0000)	(0.0286)	(0.0324)	(0.0399)	(0.0002)	(0.0294)			
THA	ZARCH	Gaussian	0.0001	0.0937	0.0013	0.9056	0.0004	0.1331	0	0	0
			(0.0006)	(0.01677)	(0.0209)	(0.2842)	(0.0004)	(0.0094)			

Notes to Table: Univariate GARCH parameter estimates of the emerging market stocks for use with multivariate GARCH models. Standard errors are in parenthesis.

Table D.5: Marginal Models Developed Market Bonds.

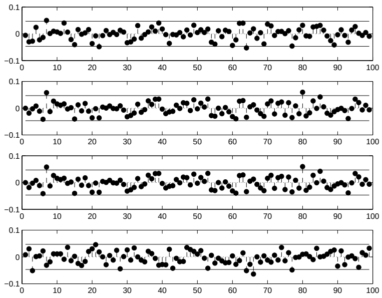
	Model selected	Distribution	α_0	α_1	γ	β_1	ϕ_0	ϕ_1	ν	λ	KS p-value	LB p-value	LM p-value
AUS	ZARCH	Gaussian	0.0001	0.0298	0.0651	0.9377	0.0005	-0.0122			0.2079	0.3213	0
			(0.0001)	(0.0103)	(0.0240)	(0.0172)	(0.0002)	(0.0297)					
DEN	GARCH	Gaussian	0.0000	0.0301		0.9684	0.0004	-0.0134			0.2100	0.3345	0
			(0.0000)	(0.0059)		(0.0054)	(0.0001)	(0.0075)					
SWE	GARCH	Gaussian	0.0000	0.0335		0.9633	0.0004	-0.0116			0.2388	0.3157	0
			(0.0000)	(0.0151)		(0.0132)	(0.0002)	(0.3012)					
ITA	GARCH	Gaussian	0.0000	0.0362		0.9617	0.0004	-0.0096			0.6587	0.2327	0
			(0.0000)	(0.0068)		(0.0070)	(0.0002)	(0.0276)					
JPN	GARCH	Gaussian	0.0000	0.0438		0.9469	0.0001	0.0194			0.0366	0.4548	0
			(0.0000)	(0.0142)		(0.0170)	(0.0002)	(0.0149)					
GBR	GARCH	Gaussian	0.0000	0.0385		0.9562	0.0003	0.0167			0.3107	0.6609	0
			(0.0000)	(0.0079)		(0.0068)	(0.0002)	(0.0158)					
USA	GARCH	Gaussian	0.0000	0.0263		0.9737	0.0002	-0.0127			0.6307	0.1220	0
			(0.0000)	(0.0035)		(0.0043)	(0.0001)	(0.0063)					
AUS	ZARCH	skew-t	0.0001	0.0263	0.0480	0.9497	0.0005	-0.0304	7.2800*	-0.1317	0.5478	0.2416	0
			(0.0001)	(0.0168)	(0.0270)	(0.0200)	(0.0002)	(0.0292)	(1.2127)	(0.0324)			
DEN	EGARCH	skew-t	-0.0772	-0.0022	0.0729	0.9979	0.0003	-0.0226	9.6022	0.0103			0
			(0.0392)	(0.0094)	(0.0259)	(0.0039)	(0.0002)	(0.0216)	(2.2249)	(0.0634)			
SWE	GARCH	Student-t	0.0000	0.0303		0.9657	0.0003	-0.0173	12.2908		0.2388	0.2774	0
			(0.0000)	(0.0062)		(0.0060)	(0.0002)	(0.0121)	(3.1041)				
ITA	GARCH	Student-t	0.0000	0.0362		0.9589	0.0004	-0.0195	10.8319		0.0336	0.2101	0
			(0.0000)	(0.0069)		(0.0069)	(0.0001)	(0.0193)	(2.5799)				
JPN	GARCH	Student-t	0.0000	0.0449		0.9430	0.0000	-0.0052	6.6034		0.0012	0.4216	0
			(0.0000)	(0.0200)		(0.0221)	(0.0001)	(0.2364)	(1.3922)				
GBR	GARCH	GED	0.0000	0.0370		0.9548	0.0003	0.0168	1.5311		0.4946	0.6572	0
			(0.0000)	(0.0081)		(0.0079)	(0.0001)	(0.0707)	(0.0837)				
USA	EGARCH	skew-t	-0.0684	0.0013	0.0684	0.9987	0.0002	-0.0188	10.2036	0.0020	0.4440	0.1008	0
			(0.0388)	(0.0173)	(0.0309)	(0.0032)	(0.0001)	(0.0164)	(2.2474)	(0.0140)			

Notes to Table: Univariate GARCH parameter estimates of the developed market bonds. The upper part of the table shows the models for use with multivariate GARCH, the lower part the models for use with copulas.

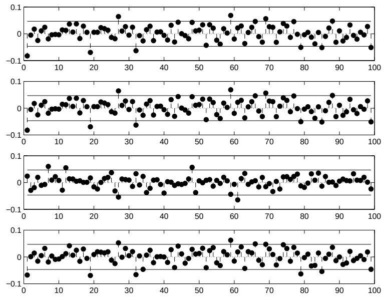
Table D.6: Marginal Models Developed Market Stocks.

	Model selected	Distribution	α_0	α_1	γ	β_1	ϕ_0	ϕ_1	ν	λ	KS p-value	LB p-value	LM p-value
AUS	GJRGARCH	Gaussian	0.0000*	0.0286*	0.1432*	0.8774*	0.0008*	0.0408			0.0971	0.9820	0
			(0.0000)	(0.0121)	(0.0494)	(0.0270)	(0.0003)	(0.0330)					
DEN	GJRGARCH	Gaussian	0.0000*	0.0228	0.0972*	0.9095*	0.0009*	-0.0021			0.5290	0.8405	0
			(0.0000)	(0.0229)	(0.0299)	(0.0195)	(0.0005)	(0.2563)					
SWE	GJRGARCH	Gaussian	0.0000*	0.0038	0.1060	0.9323*	0.0006	-0.0005			0.8665	0.1982	0
			(0.0000)	(0.3515)	(0.2519)	(0.2114)	(0.0005)	(1.4346)					
ITA	ZARCH	Gaussian	0.0002*	0.0261*	0.0927*	0.9275*	0.0003	-0.0188			0.0111	0.8503	0
			(0.0001)	(0.0113)	(0.0187)	(0.0128)	(0.0003)	(0.0634)					
JPN	GJRGARCH	Gaussian	0.0000*	0.0359*	0.0951*	0.8969*	0.0003	-0.0936*			0.3517	0.5753	0
			(0.0000)	(0.0116)	(0.0319)	(0.0153)	(0.0003)	(0.0240)					
GBR	ZARCH	Gaussian	0.0002*	0.0200*	0.0894*	0.9341*	0.0004*	-0.0471*			0	0.4397	0
			(0.0001)	(0.0108)	(0.0167)	(0.0129)	(0.0002)	(0.0239)					
USA	GJRGACH	Gaussian	0.0000*	0.0000	0.1389*	0.9085*	0.0001	-0.0855*			0.4440	0.8863	0
			(0.0000)	(0.0243)	(0.0313)	(0.0259)	(0.0002)	(0.0240)					
AUS	ZARCH	skew-t	0.0003	0.0341	0.1046	0.9134	0.0008	0.0176	9.6447	-0.1681	0.2388	0.9642	0
			(0.0003)	(0.0604)	(0.0739)	(0.0868)	(0.0005)	(0.3108)	(4.2653)	(0.0573)			
DEN	ZARCH	skew-t	0.0002	0.0341	0.0854	0.9232	0.0009	-0.0068	10.8948	-0.0811	0.3517	0.7499	0
			(0.0001)	(0.0247)	(0.0355)	(0.0302)	(0.0000)	(0.0030)	(2.7093)	(0.0331)			
SWE	GJRGARCH	Student-t	0.0000	0.0000	0.1056	0.9383	0.0008	-0.0050	10.3733		0.0288	0.1633	0
			(0.0000)	(0.0115)	(0.0236)	(0.0134)	(0.0003)	(0.0405)	(2.1492)				
ITA	ZARCH	skew-t	0.0002	0.0253	0.0845	0.9318	0.0003	-0.0121	10.2766	-0.1481	0.6587	0.8848	0
			(0.0001)	(0.0189)	(0.0297)	(0.0230)	(0.0003)	(0.0123)	(2.2715)	(0.0309)			
JPN	GJRGARCH	skew-t	0.0000	0.0341	0.0772	0.9115	0.0003	-0.1085	9.7553	-0.0469	0.8439	0.5077	0
			(0.0000)	(0.0200)	(0.0466)	(0.0274)	(0.0003)	(0.0245)	(2.1463)	(0.0318)			
GBR	ZARCH	skew-t	0.0002	0.0179	0.0925	0.9342	0.0004	-0.0467	10.3411	-0.1158	0.9399	0.8876	0
			(0.0001)	(0.0161)	(0.0328)	(0.0223)	(0.0002)	(0.0276)	(2.2403)	(0.0306)			
USA	ZARCH	skew-t	0.0002	0.0000	0.1238	0.9381	0.0001	-0.0967	9.6382	-0.1735	0.3308	0.4067	0
			(0.0001)	(0.0625)	(0.0507)	(0.0406)	(0.0002)	(0.0260)	(2.5373)	(0.0270)			

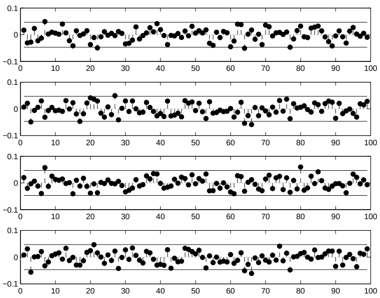
Notes to Table: Univariate GARCH parameter estimates of the developed market stocks. The upper part of the table shows the models for use with multivariate GARCH, the lower part the models for use with copulas.



(a) JAP (GAUSS)

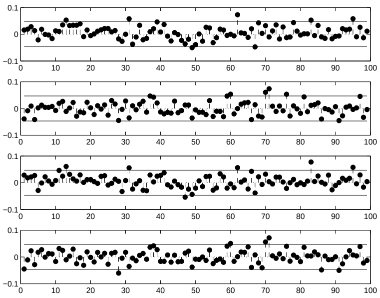


(b) ITA (STUDENT T)

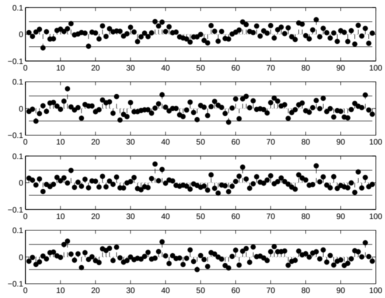


(c) JAP (STUDENT T)

Figure D.1: Z-plots Developed Market Bonds. This figure shwos Diebold, Gunther, and Tay (1998) z-plots developed market bonds.

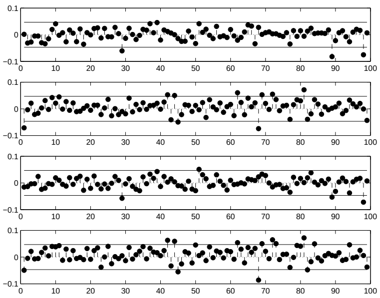


(a) ITA (GAUSS)

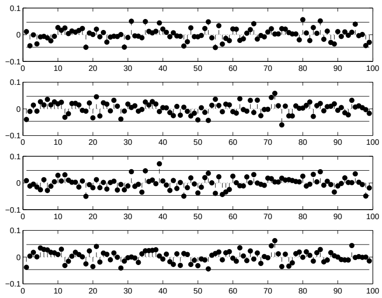


(b) GBR (GAUSS)

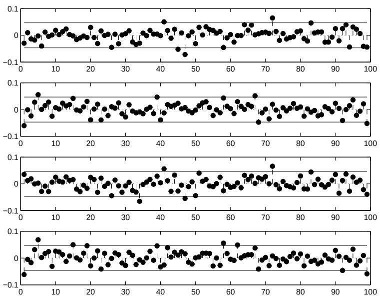
Figure D.2: Z-plots Developed Market Stocks. This figure shows Diebold, Gunther, and Tay (1998) z-plots developed market stocks.



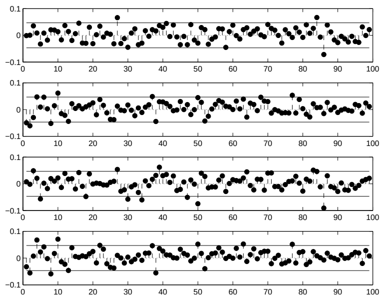
(a) BRA (GAUSS)



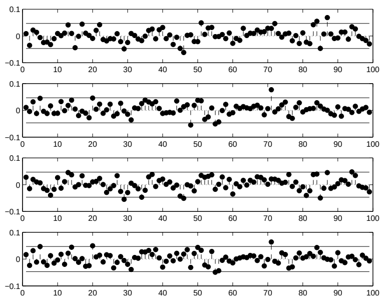
(b) MEX (GAUSS)



(c) POL (GAUSS)

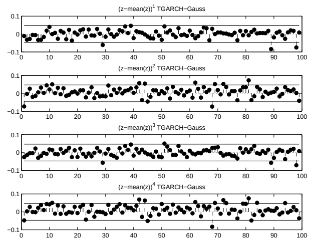


(d) IND (GAUSS)

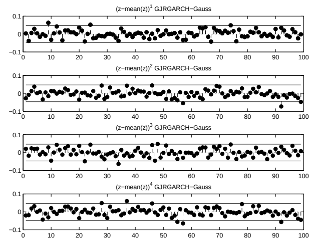


(e) INA (GAUSS)

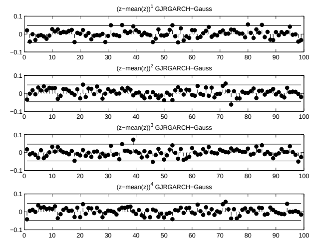
Figure D.3: Z-plots Emerging Market Stocks. This figure shows Diebold, Gunther, and Tay (1998) z-plots emerging market stocks.



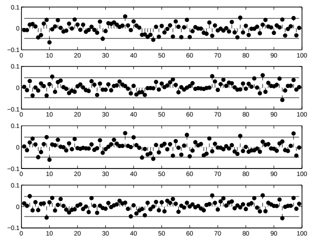
(a) BRA (GAUSS)



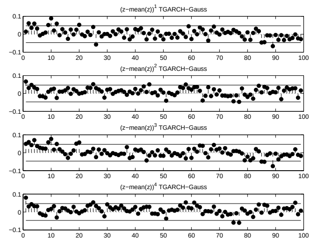
(b) HUN (GAUSS)



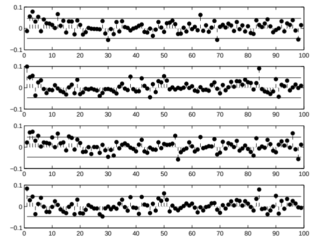
(c) MEX (GAUSS)



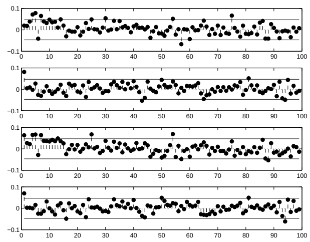
(d) CHI (GAUSS)



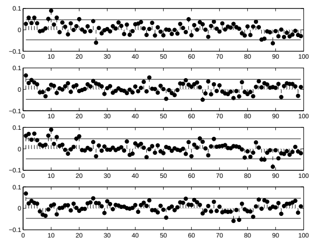
(e) IND (GAUSS)



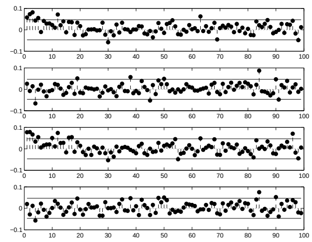
(f) INA (GAUSS)



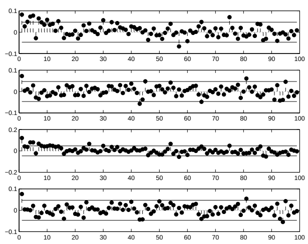
(g) THA (GAUSS)



(h) IND (GED)



(i) INA (STUDENT T)



(j) THA (GED)

Figure D.4: Z-plots Emerging Market Bonds. This figure shows Diebold, Gunther, and Tay (1998) z-plots emerging market bonds.

Appendix E

Estimation Results Multivariate Models VaR Analysis

Table E.1: DCC MVGARCH Developed Market Stocks (VaR Analysis).

	a_i^2	b_i^2	a_i^2	g_i^2	b_i^2
AUS	0.0099*	0.9813*	0.0068	0.0070*	0.9810*
DEN	0.0238	0.9400*	0.0167	0.0254*	0.9318*
SWE	0.0249	0.9391*	0.0135	0.0271	0.9371*
ITA	0.0241*	0.9478*	0.0179*	0.0218	0.9382*
JPN	0.0068	0.9898*	0.0078	0.0018	0.9879*
GBR	0.0298*	0.9358*	0.0156*	0.0304*	0.9376*
USA	0.0046*	0.9834*	0.0037	0.0014	0.9819*
Scalar Model	0.0133*	0.9695*	0.0104*	0.0086*	0.9661*
	Log-l	BIC	AIC		
DCC	40988.0	-81961.0	-81972.0		
A-DCC	40995.2	**-81967.9**	-81984.5		
G-DCC	41011.6	-81918.1	-81995.2		
AG-DCC	41026.0	-81894.3	**-82010.0**		

Notes to Table: The upper part of this table reports the parameter estimates for the symmetric and asymmetric DCC MVGARCH models. Parameter estimates significantly different from zero at 5% confidence Level are marked with an asterisk. The lower part reports the different goodness-of-fit criteria.

Table E.2: DCC MVGARCH Developed Market Bonds (VaR Analysis).

	a_i^2	b_i^2	a_i^2	g_i^2	b_i^2
AUS	0.0235*	0.9718*	0.0246*	0.0000	0.9704*
DEN	0.0200*	0.9783*	0.0214*	0.0002	0.9763*
SWE	0.0208*	0.9762*	0.0216*	0.0028	0.9747*
ITA	0.0213*	0.9770*	0.0226*	0.0005	0.9752*
JPN	0.0258*	0.9653*	0.0253*	0.0011	0.9656*
GBR	0.0171*	0.9791*	0.0178*	0.0000	0.9782*
USA	0.0093*	0.9896*	0.0097*	0.0013	0.9881*
Scalar Model	0.0183*	0.9797*	0.0183*	0.0001	0.9798*
	Log-l	BIC	AIC		
DCC	53149.3	**-106283.6**	-106294.6		
A-DCC	53149.3	-106276.1	-106292.6		
G-DCC	53176.8	-106248.6	-106325.7		
AG-DCC	53187.9	-106218.1	**-106333.8**		

Notes to Table: The upper part of this table reports the parameter estimates for the symmetric and asymmetric DCC MVGARCH models. Parameter estimates significantly different from zero at 5% confidence Level are marked with an asterisk. The lower part reports the different goodness-of-fit criteria.

Table E.3: DCC MVGARCH Emerging Market Stocks.

	a_i^2	b_i^2	a_i^2	g_i^2	b_i^2
BRA	0.0200	0.9659*	0.0200	0.0063	0.9604*
HUN	0.0207	0.9692*	0.0170	0.0133*	0.9635*
MEX	0.0142	0.9757*	0.0127	0.0025	0.9785*
POL	0.0121	0.9803*	0.0085	0.0185	0.9784*
RSA	0.0092*	0.9859*	0.0079	0.0109	0.9772*
Scalar Model	0.0144*	0.9759*	0.0115*	0.0077	0.9747*
	Log-l	BIC	AIC		
DCC	25535.5	-51055.9	-51067.0		
A-DCC	25414.5	**-51058.4**	**-51075.0**		
G-DCC	25539.3	-51003.5	-51058.6		
AG-DCC	25546.7	-50980.8	-51063.4		

Notes to Table: The upper part of this table reports the parameter estimates for the symmetric and asymmetric DCC MVGARCH models. Parameter estimates significantly different from zero at 5% confidence Level are marked with an asterisk. The lower part reports the different goodness-of-fit criteria.

Table E.4: DCC MVGARCH Advanced Emerging Market Bonds (VaR Analysis).

	a_i^2	b_i^2	a_i^2	g_i^2	b_i^2
BRA	0.0238	0.9550*	0.0195*	0.0301*	0.9295*
HUN	0.0265	0.9611*	0.0272*	0.0061	0.9610*
MEX	0.0166*	0.9637*	0.0061*	0.0260*	0.9786*
POL	0.0146	0.9690*	0.0108*	0.0084*	0.9701*
RSA	0.0153	0.9822*	0.0185*	0.0105	0.9613*
Scalar Model	0.0188*	0.9686*	0.0151*	0.0103*	0.9659*
	Log-l	BIC	AIC		
DCC	31647.8	-63280.7	-63291.7		
A-DCC	31654.1	**-63285.8**	**-63302.3**		
G-DCC	31654.3	-63233.6	-63288.7		
AG-DCC	31665.4	-63218.1	-63299.8		

Notes to Table: The upper part of this table reports the parameter estimates for the symmetric and asymmetric DCC MVGARCH models. Parameter estimates significantly different from zero at 5% confidence Level are marked with an asterisk. The lower part reports the different goodness-of-fit criteria.

Table E.5: DCC Gaussian Copula Developed Market Stocks.

	a_i^2	b_i^2	a_i^2	g_i^2	b_i^2
AUS	0.0093*	0.9838*	0.0072	0.0112	0.9774*
DEN	0.0227*	0.9513*	0.0140*	0.0270*	0.9464*
SWE	0.0258*	0.9428*	0.0104	0.0287*	0.9474*
ITA	0.0242*	0.9498*	0.0171*	0.0243	0.9396*
JPN	0.0059*	0.9909*	0.0083	0.0027	0.9838*
GBR	0.0287*	0.9398*	0.0128*	0.0293*	0.9449*
USA	0.0043*	0.9838*	0.0032	0.0035	0.9760*
Scalar Model	0.0123*	0.9734*	0.0089*	0.0127*	0.9681*
	Log-l	BIC	AIC		
DCC	41146.0	-82277.0	-82288.0		
A-DCC	41163.3	**-82304.1**	-82320.6		
G-DCC	41167.8	-82230.4	-82307.6		
AG-DCC	41191.8	-82226.0	**-82341.7**		

Notes to Table: The upper part of this table reports the parameter estimates for the symmetric and asymmetric DCC Gaussian copula models. Parameter estimates significantly different from zero at 5% confidence Level are marked with an asterisk. The lower part reports the different goodness-of-fit criteria.

Table E.6: DCC Gaussian Copula Developed Market Bonds.

	a_i^2	b_i^2	a_i^2	g_i^2	b_i^2
AUS	0.0240*	0.9714*	0.0248*	0.0001	0.9708*
DEN	0.0181*	0.9786*	0.0194*	0.0002	0.9768*
SWE	0.0200*	0.9755*	0.0200*	0.0011	0.9750*
ITA	0.0189*	0.9771*	0.0196*	0.0007	0.9759*
JPN	0.0281*	0.9696*	0.0260*	0.0011	0.9708*
GBR	0.0153*	0.9790*	0.0158*	0.0002	0.9781*
USA	0.0102*	0.9889*	0.0103*	0.0010	0.9880*
Scalar Model	0.0181*	0.9784*	0.0180*	0.0001	0.9785*
	Log-l	BIC	AIC		
DCC	53107.1	**-106210.2**	-106210.2		
A-DCC	53107.1	-106191.7	-106208.2		
G-DCC	53122.4	-106139.7	**-106216.8**		
AG-DCC	53128.7	-106099.7	-106215.4		

Notes to Table: The upper part of this table reports the parameter estimates for the symmetric and asymmetric DCC Gaussian copula models. Parameter estimates significantly different from zero at 5% confidence Level are marked with an asterisk. The lower part reports the different goodness-of-fit criteria.

Table E.7: DCC Gaussian Copula Advanced Emerging Market Stocks.

	a_i^2	b_i^2	a_i^2	g_i^2	b_i^2
BRA	0.0235*	0.9620*	0.0212*	0.0088*	0.9588*
HUN	0.0184*	0.9732*	0.0149*	0.0122*	0.9694*
MEX	0.0137*	0.9785*	0.0130*	0.0039*	0.9777*
POL	0.0108*	0.9838*	0.0087*	0.0103	0.9791*
RSA	0.0098*	0.9854*	0.0084*	0.0095	0.9794*
Scalar Model	0.0137*	0.9785*	0.0114*	0.0081*	0.9762*
	Log-l	BIC	AIC		
DCC	25630.0	-51245.0	-51256.0		
A-DCC	25636.8	**-51251.2**	**-51267.7**		
G-DCC	25634.3	-51193.5	-51248.6		
AG-DCC	25642.2	-51171.8	-51254.5		

Notes to Table: The upper part of this table reports the parameter estimates for the symmetric and asymmetric DCC Gaussian copula models. Parameter estimates significantly different from zero at 5% confidence Level are marked with an asterisk. The lower part reports the different goodness-of-fit criteria.

Table E.8: DCC Gaussian Copula Advanced Emerging Market Bonds.

	a_i^2	b_i^2	a_i^2	g_i^2	b_i^2
BRA	0.0297*	0.9454*	0.0237	0.0264	0.9342*
HUN	0.0185*	0.9784*	0.0194	0.0081	0.9702*
MEX	0.0221*	0.9533*	0.0117	0.0244	0.9616*
POL	0.0127*	0.9802*	0.0106	0.0085	0.9772*
RSA	0.0269*	0.9569*	0.0210	0.0149	0.9567*
Scalar Model	0.0197*	0.9682*	0.0158*	0.0127*	0.9652*
	Log-l	BIC	AIC		
DCC	31823.6	-63632.3	-63643.3		
A-DCC	31832.6	**-63642.7**	**-63659.2**		
G-DCC	31829.7	-63584.3	-63639.4		
AG-DCC	31840.0	-63567.3	-63650.0		

Notes to Table: The upper part of this table reports the parameter estimates for the symmetric and asymmetric DCC Gaussian copula models. Parameter estimates significantly different from zero at 5% confidence Level are marked with an asterisk. The lower part reports the different goodness-of-fit criteria.

Table E.9: DCC t-Copula Developed Market Stocks.

	a_i^2	b_i^2	a_i^2	g_i^2	b_i^2
AUS	0.0094*	0.9837*	0.0070	0.0116	0.9771*
DEN	0.0230*	0.9477*	0.0136	0.0286*	0.9434*
SWE	0.0248*	0.9441*	0.0092	0.0302	0.9482*
ITA	0.0236*	0.9506*	0.0171	0.0238	0.9400*
JPN	0.0056*	0.9912*	0.0087	0.0023	0.9833
GBR	0.0286*	0.9400*	0.0135	0.0295	0.9425*
USA	0.0043*	0.9841*	0.0035	0.0030	0.9746*
Scalar Model	0.0123*	0.9734*	0.0089*	0.0127*	0.9681*
	v	Log-l	BIC	AIC	
DCC	17.5553*	41185.8	-82349.1	-82365.6	
A-DCC	17.5553*	41197.4	**-82364.8**	**-82386.8**	
G-DCC	20.8899*	41205.5	-82298.4	-82381.1	
AG-DCC	22.9212*	41227.6	-82290.1	-82411.3	

Notes to Table: The upper part of this table reports the parameter estimates for the symmetric and asymmetric DCC t-copula models. Parameter estimates significantly different from zero at 5% confidence Level are marked with an asterisk. The lower part reports the different goodness-of-fit criteria.

Table E.10: DCC t-Copula Developed Market Bonds.

	a_i^2	b_i^2	a_i^2	g_i^2	b_i^2
AUS	0.0227*	0.9733*	0.0231*	0.0005	0.9734*
DEN	0.0180*	0.9788*	0.0189*	0.0003	0.9777*
SWE	0.0209*	0.9747*	0.0208*	0.0013	0.9742*
ITA	0.0182*	0.9780*	0.0188*	0.0008	0.9769*
JPN	0.0228*	0.9763*	0.0226*	0.0016	0.9748*
GBR	0.0144*	0.9797*	0.0147*	0.0004	0.9793*
USA	0.0101*	0.9893*	0.0106*	0.0013	0.9876*
Scalar Model	0.0175*	0.9793*	0.0174*	0.0003	0.9791*
	v	Log-l	BIC	AIC	
DCC	15.7129*	53212.0	**-106401.5**	-106418.0	
A-DCC	15.7933*	53212.1	-106394.1	-106416.2	
G-DCC	15.7841*	53225.0	-106337.5	**-106420.1**	
AG-DCC	15.8121*	53228.7	-106292.2	-106413.4	

Notes to Table: The upper part of this table reports the parameter estimates for the symmetric and asymmetric DCC t-copula models. Parameter estimates significantly different from zero at 5% confidence Level are marked with an asterisk. The lower part reports the different goodness-of-fit criteria.

Table E.11: DCC t-Copula Emerging Market Stocks.

	a_i^2	b_i^2	a_i^2	g_i^2	b_i^2
BRA	0.0219*	0.9638*	0.0190	0.0091	0.9625
HUN	0.0190*	0.9716*	0.0149	0.0113	0.9699
MEX	0.0152*	0.9767*	0.0135	0.0049	0.9764
POL	0.0105*	0.9850*	0.0086	0.0087	0.9807
RSA	0.0092*	0.9880*	0.0082	0.0076	0.9820
Scalar Model	0.0136*	0.9790*	0.0111*	0.0082*	0.9771*
	v	Log-l	BIC	AIC	
DCC	14.9582*	25669.7	-51316.9	-51333.4	
A-DCC	16.3916*	25675.3	**-51320.7**	**-51342.7**	
G-DCC	14.9948*	25673.5	-51264.5	-51325.1	
AG-DCC	16.3658*	25679.4	-51238.8	-51326.9	

Notes to Table: The upper part of this table reports the parameter estimates for the symmetric and asymmetric DCC t-copula models. Parameter estimates significantly different from zero at 5% confidence Level are marked with an asterisk. The lower part reports the different goodness-of-fit criteria.

Table E.12: DCC t-Copula Emerging Market Bonds.

	a_i^2	b_i^2	a_i^2	g_i^2	b_i^2
BRA	0.0248	0.9535	0.0189	0.0300*	0.9426*
HUN	0.0262	0.9646	0.0172	0.0146	0.9663*
MEX	0.0184	0.9633	0.0135	0.0216	0.9569*
POL	0.0156	0.9746	0.0114	0.0097	0.9747*
RSA	0.0215	0.9722	0.0232	0.0138	0.9582*
Scalar Model	0.0208*	0.9669*	0.0155*	0.0151*	0.9650*
	v	Log-l	BIC	AIC	
DCC	14.4355*	31875.1	-63727.8	-63744.3	
A-DCC	15.2105*	31885.3	**-63740.6**	**-63762.6**	
G-DCC	14.3769*	31879.9	-63677.3	-63737.9	
AG-DCC	15.1651*	31891.1	-63662.2	-63750.3	

Notes to Table: The upper part of this table reports the parameter estimates for the symmetric and asymmetric DCC t-copula models. Parameter estimates significantly different from zero at 5% confidence Level are marked with an asterisk. The lower part reports the different goodness-of-fit criteria.

Table E.13: Vine Copula Unconditional Dependence Parameter Estimates Developed Stocks.

	DEN	SWE	ITA	JAP	GBR	USA	
	Developed Stocks Gaussian Copula (ρ)						
AUS	0.6612	0.5722	0.5066	0.6364	0.5247	0.2305	
DEN		0.7948	0.6570	0.4136	0.6771	0.4226	
SWE			0.7111	0.3456	0.7110	0.5013	
ITA				0.3962	0.8420	0.5278	
JAP					0.4059	0.1675	
GBR						0.5166	
	Developed Stocks Student-t Copula (ν)						
AUS	26.0020	21.6683	18.8055	7.0431	37.3075	265.7061	
DEN		6.7416	10.2089	300.0000	11.3455	22.7083	
SWE			12.6179	182.5938	18.1244	15.9005	
ITA				64.2327	8.3917	8.1042	
JAP					300.0000	300.0000	
GBR						21.1044	
	Developed Stocks Clayton Copula (θ^{CL})						
AUS	0.8312	0.6929	0.4167	0.7709	0.4412	0.2005	
DEN		1.4425	0.6597	0.3277	0.7070	0.4029	
SWE			1.0089	0.3477	1.0379	0.6109	
ITA				0.3778	1.9355	0.7000	
JAP					0.4178	0.1395	
GBR						0.6288	
	Developed Stocks Rotated Clayton Copula (θ^{RCL})						
AUS	0.7067	0.7362	0.3155	0.6325	0.3591	0.2009	
DEN		1.3794	0.5666	0.2338	0.6348	0.3920	
SWE			0.9429	0.3085	0.9283	0.6549	
ITA				0.3227	1.8138	0.6988	
JAP					0.3588	0.1434	
GBR						0.6407	
	Data ordering						
GA	GBR	ITA	SWE	DEN	AUS	JAP	USA
T	SWE	DEN	ITA	GBR	USA	AUS	JAP
CL	ITA	GBR	SWE	DEN	AUS	JAP	USA
RCL	GBR	ITA	SWE	DEN	AUS	JAP	USA

Notes to Table: The upper part of this table reports the unconditional bivariate dependence parameter estimates for the Gaussian, t-, Clayton and rotated Clayton copula. The lower part describes the data ordering according to the dependence of the bivariate variables.

Table E.14: Vine Copula Unconditional Dependence Parameter Estimates Developed Bonds.

	DEN	SWE	ITA	JAP	GBR	USA	
			Developed Bonds Gaussian Copula (ρ)				
AUS	0.5869	0.6198	0.5933	-0.0407	0.5203	-0.0741	
DEN		0.8458	0.9652	0.3709	0.7254	0.2523	
SWE			0.8541	0.2483	0.6243	0.0898	
ITA				0.3233	0.7229	0.2224	
JAP					0.2488	0.3316	
GBR						0.2336	
			Developed Bonds Student-t Copula (ν)				
AUS	8.4491	13.4541	10.4817	2.5141	12.2714	10.6884	
DEN		6.1282	4.0758	5.3779	7,.8689	11.7071	
SWE			8.0779	3.8437	8.0588	7.6554	
ITA				4.0704	6.9955	7.1840	
JAP					5.1374	9.3053	
GBR						11.0590	
			Developed Bonds Clayton Copula (θ^{CL})				
AUS	0.9859	1.1402	1.0751	0.3605	0.7743	0.0221	
DEN		2.7980	7.4404	0.7607	1.4158	0.3196	
SWE			3.1253	0.6378	1.1404	0.1870	
ITA				0,8036	1,5386	0.3412	
JAP					0.5440	0.4782	
GBR						0.3238	
			Developed Bonds Rotated Clayton Copula (θ^{RCL})				
AUS	0.9737	1.1344	1.0767	0.3269	0.7980	1.4508	
DEN		2.7817	7.4319	0.6621	1.6270	0.3461	
SWE			3.0720	0.5319	1.3115	0.1868	
ITA				0.7141	1.7957	0.3876	
JAP					0.5404	0.4377	
GBR						0.4102	
			Data ordering				
GA	DEN	ITA	SWE	GBR	AUS	JAP	USA
T	AUS	JAP	SWE	DEN	ITA	GBR	USA
CL	DEN	ITA	SWE	GBR	AUS	JAP	USA
RCL	DEN	ITA	SWE	GBR	AUS	JAP	USA

Notes to Table: The upper part of this table reports the unconditional bivariate dependence parameter estimates for the Gaussian, t-, Clayton and rotated Clayton copula. The lower part describes the data ordering according to the dependence of the bivariate variables.

Table E.15: DCC Vine Copulas Log-likelihood Developed Markets.

	Dynamic structure	Log-l	AIC	BIC
	Developed Stocks			
GA	DCC	41194.0	-82304.0	-82072.6
GA	A-DCC	41221.6	-82317.2	-81970.1
T	DCC	41238.6	-82351.3	-82004.2
T	A-DCC	41268.8	-82369.7	-81906.9
CL	DCC	39901.5	-79719.1	-79487.8
CL	A-DCC	39928.1	-79730.2	-79383.1
RCL	DCC	40321.6	-80559.3	-80327.9
RCL	A-DCC	40323.4	-80520.8	-80173.7
	Developed Bonds			
GA	DCC	53171.2	-106258.5	-106027.1
GA	A-DCC	53160.4	-106194.9	-105847.8
T	DCC	53311.1	-106496.2	-106149.1
T	A-DCC	53316.0	-106464.0	-106001.2
CL	DCC	50601.6	-101119.3	-100887.9
CL	A-DCC	50634.4	-101142.9	-100795.8
RCL	DCC	51840.2	-103596.5	-103365.1
RCL	A-DCC	51918.9	-103711.9	-103364.8

Notes to Table: This table reports the log-likelihood values estimated with four different vine copulas: Gaussian (GA), t- (T), Clayton (CL), and Rotated Clayton (RCL).

Table E.16: Vine Copula Unconditional Dependence Parameter Estimates Emerging Market Stocks.

	HUN	MEX	POL	RSA	
	Emerging Markets Stocks Gaussian Copula (ρ)				
BRA	0.5475	0.7725	0.5480	0.6159	
HUN		0.5448	0.7293	0.6525	
MEX			0.5570	0.5874	
POL				0.6672	
	Emerging Markets Stocks Student-t Copula (ν)				
BRA	9.6861	5.7650	11.4720	17.2026	
HUN		14.3216	6.6208	16.7160	
MEX			11.2632	13.1293	
POL				8.2697	
	Emerging Markets Stocks Clayton Copula (θ^{CL})				
BRA	0.5592	1.2227	0.6462	0.7239	
HUN		0.4828	1.1606	0.8507	
MEX			0.6088	0.6571	
POL				0.8979	
	Emerging Markets Stocks Rotated Clayton Copula (θ^{RCL})				
BRA	0.5139	1.1160	0.6297	0.6453	
HUN		0.4534	1.1384	0.7589	
MEX			0.5772	0.5891	
POL				0.8322	
	Data ordering				
GA	MEX	BRA	USA	POL	HUN
T	MEX	BRA	HUN	POL	RSA
CL	MEX	BRA	HUN	POL	RSA
RCL	HUN	POL	RSA	BRA	MEX

Notes to Table: The upper part of this table reports the unconditional bivariate dependence parameter estimates for the Gaussian, t-, Clayton and rotated Clayton copula. The lower part describes the data ordering according to the dependence of the bivariate variables.

Table E.17: Vine Copula Unconditional Dependence Parameter Estimates Emerging Market Bonds.

	HUN	MEX	POL	RSA	
	Emerging Markets Bonds Gaussian Copula (ρ)				
BRA	0.4562	0.6084	0.4694	0.4892	
HUN		0.4571	0.8008	0.5892	
MEX			0.4943	0.4725	
POL				0.6026	
	Emerging Markets Bonds Student-t Copula (ν)				
BRA	6.9739	9.6858	11.2837	10.4950	
HUN		10.8005	5.2684	14.0518	
MEX			11.9087	9.0906	
POL				10.9558	
	Emerging Markets Bonds Clayton Copula (θ^{CL})				
BRA	0.5492	0.7535	0.5659	0.6148	
HUN		0.4551	1.7567	0.8531	
MEX			0.4982	0.5548	
POL				0.8290	
	Emerging Markets Bonds Rotated Clayton Copula (θ^{RCL})				
BRA	0.5538	0.6358	0.5122	0.5432	
HUN		0.4730	1.8065	0.8930	
MEX			0.4709	0.5133	
POL				0.8092	
	Data ordering				
GA	POL	HUN	RSA	BRA	MEX
T	POL	HUN	BRA	MEX	RSA
CL	POL	HUN	RSA	BRA	MEX
RCL	POL	HUN	RSA	BRA	MEX

Notes to Table: The upper part of this table reports the unconditional bivariate dependence parameter estimates for the Gaussian, *t*-, Clayton and rotated Clayton copula. The lower part describes the data ordering according to the dependence of the bivariate variables.

Table E.18: DCC Vine Copulas Log-likelihood Emerging Markets.

	Dynamic structure	Log-l	AIC	BIC
	Advanced Emerging Market Stocks			
GA	DCC	25655.6	-51271.2	-51161.0
GA	A-DCC	25662.1	-51264.2	-51098.9
T	DCC	25686.3	-51312.6	-51147.4
T	A-DCC	25697.6	-51315.3	-51094.9
CL	DCC	24906.5	-49773.0	-49662.9
CL	A-DCC	24908.9	-49757.8	-49592.5
RCL	DCC	25043.6	-50047.2	-49937.0
RCL	A-DCC	25047.3	-50034.7	-49869.4
	Advanced Emerging Market Bonds			
GA	DCC	31846.4	-63652.8	-63542.6
GA	A-DCC	31860.4	-63660.8	-63495.5
T	DCC	31896.7	-63733.4	-63568.1
T	A-DCC	31906.0	-63732.0	-63511.6
CL	DCC	30960.9	-61881.9	-61771.7
CL	A-DCC	30960.1	-61860.2	-61694.9
RCL	DCC	31420.0	-62800.1	-62689.9
RCL	A-DCC	31427.0	-62794.1	-62628.8

Notes to Table: This table reports the log-likelihood values estimated with different four vine copulas: Gaussian (GA), *t*- (T), Clayton (CL), and Rotated Clayton (RCL).

Appendix F

Estimation Results Multivariate Models Dependence Analysis

Table F.1: DCC MVGARCH Models Developed Market Stocks.

	a_i^2	b_i^2	a_i^2	g_i^2	b_i^2
AUS	0.0099*	0.9813*	0.0068	0.0070*	0.9810*
DEN	0.0238	0.9400*	0.0167	0.0254*	0.9318*
SWE	0.0249	0.9391*	0.0135	0.0271	0.9371*
ITA	0.0241*	0.9478*	0.0179*	0.0218	0.9382*
JPN	0.0068	0.9898*	0.0078	0.0018	0.9879*
GBR	0.0298*	0.9358*	0.0156*	0.0304*	0.9376*
USA	0.0046*	0.9834*	0.0037	0.0014	0.9819*
Scalar Model	0.0133*	0.9695*	0.0104*	0.0086*	0.9661*
	Log-l	BIC	AIC		
DCC	40988.0	-81961.0	-81972.0		
A-DCC	40995.2	**-81967.9**	-81984.5		
G-DCC	41011.6	-81918.1	-81995.2		
AG-DCC	41026.0	-81894.3	**-82010.0**	-	
	AD	AAD	KS	AKS	
DCC	8.1708	0.1231	3.7200	0.0498	
A-DCC	7.2613	0.1068	**3.2480**	0.0419	
G-DCC	8.1731	0.1200	3.7543	0.0485	
AG-DCC	**7.0189**	**0.1039**	3.2835	**0.0407**	

Notes to Table: The upper part of this table reports the parameter estimates for the symmetric and asymmetric DCC MVGARCH models. Parameter estimates significantly different from zero at 5% confidence Level are marked with an asterisk. The lower part reports the different goodness-of-fit criteria.

Table F.2: DCC MVGARCH Advanced Emerging Market & Developed Market Stocks.

	a_i^2	b_i^2	a_i^2	g_i^2	b_i^2
BRA	0.0107	0.9761*	0.0120	0.0022	0.9722*
HUN	0.0108*	0.9786*	0.0107*	0.0069	0.9703*
MEX	0.0082*	0.9809*	0.0089*	0.0012	0.9784*
POL	0.0133*	0.9698*	0.0113	0.0111	0.9605*
RSA	0.0112*	0.9714*	0.0087*	0.0126	0.9608*
AUS	0.0042*	0.9880*	0.0015	0.0114	0.9812*
DEN	0.0156	0.9602*	0.0121	0.0134*	0.9520*
SWE	0.0200	0.9521*	0.0164	0.0121	0.9458*
ITA	0.0147*	0.9646*	0.0114	0.0102	0.9602*
JPN	0.0033	0.9935*	0.0006	0.0095	0.9923*
GBR	0.0173*	0.9518*	0.0113*	0.0152	0.9500*
USA	0.0055	0.9869*	0.0073	0.0001	0.9857*
Scalar Model	0.0087*	0.9796*	0.0074*	0.0055*	0.9759*
	Log-l	BIC	AIC		
DCC	67898.8	-135782.6	-135793.7		
A-DCC	67910.9	**-135799.2**	-135815.8		
G-DCC	67923.1	-135666.1	-135798.3		
AG-DCC	67944.2	-135618.1	**-135816.4**		
	AD	AAD	KS	AKS	
DCC	10.9352	0.1569	5.0052	0.0624	
A-DCC	9.6646	0.1422	4.1796	0.0549	
G-DCC	10.6943	0.1535	4.8189	0.0609	
AG-DCC	**9.5956**	**0.1394**	**4.0926**	**0.0537**	

Notes to Table: The upper part of this table reports the parameter estimates for the symmetric and asymmetric DCC MVGARCH models. Parameter estimates significantly different from zero at 5% confidence Level are marked with an asterisk. The lower part reports the different goodness-of-fit criteria.

Table F.3: DCC MVGARCH Secondary Emerging Market & Developed Market Stocks.

	a_i^2	b_i^2	a_i^2	g_i^2	b_i^2
CHI	0.0073	0.9795*	0.0086	0.0014	0.9828*
CZE	0.0127*	0.9628*	0.0094	0.0089*	0.9581*
IND	0.0041	0.9761*	0.0009	0.0543*	0.8800*
INA	0.0031	0.9781*	0.0000	0.0484	0.9019*
THA	0.0035	0.9916*	0.0000	0.0150	0.9732*
AUS	0.0070	0.9870*	0.0013	0.0186	0.9756*
DEN	0.0172	0.9480*	0.0146	0.0135	0.9373*
SWE	0.0215	0.9472*	0.0190	0.0067	0.9496*
ITA	0.0163	0.9536*	0.0187	0.0098	0.9411*
JPN	0.0030	0.9925*	0.0000	0.0107	0.9783*
GBR	0.0173	0.9496*	0.0139*	0.0090	0.9534*
USA	0.0059	0.9835*	0.0059*	0.0011	0.9854*
Scalar Model	0.0079*	0.9751*	0.0063*	0.0073*	0.9674*
	Log-l	BIC	AIC		
DCC	67901.9	-135788.9	-135799.9		
A-DCC	67914.4	**-135806.2**	-135822.8		
G-DCC	67929.1	-135678.1	-135810.3		
AG-DCC	67956.4	-135642.6	**-135840.9**		
	AD	AAD	KS	AKS	
DCC	11.8398	0.1612	5.4844	0.0643	
A-DCC	10.7141	0.1494	4.9002	0.0583	
G-DCC	11.6885	0.1593	5.4492	0.0635	
AG-DCC	**10.0172**	**0.1461**	**4.6100**	**0.0568**	

Notes to Table: The upper part of this table reports the parameter estimates for the symmetric and asymmetric DCC MVGARCH models. Parameter estimates significantly different from zero at 5% confidence Level are marked with an asterisk. The lower part reports the different goodness-of-fit criteria.

Table F.4: DCC MVGARCH models Developed Market Bonds.

	a_i^2	b_i^2	a_i^2	g_i^2	b_i^2
AUS	0.0235*	0.9718*	0.0246*	0.0000	0.9704*
DEN	0.0200*	0.9783*	0.0214*	0.0002	0.9763*
SWE	0.0208*	0.9762*	0.0216*	0.0028	0.9747*
ITA	0.0213*	0.9770*	0.0226*	0.0005	0.9752*
JPN	0.0258*	0.9653*	0.0253*	0.0011	0.9656*
GBR	0.0171*	0.9791*	0.0178*	0.0000	0.9782*
USA	0.0093*	0.9896*	0.0097*	0.0013	0.9881*
Scalar Model	0.0183*	0.9797*	0.0183*	0.0001	0.9798*
	Log-l	BIC	AIC		
DCC	53149.3	**-106283.6**	-106294.6		
A-DCC	53149.3	-106276.1	-106292.6		
G-DCC	53176.8	-106248.6	-106325.7		
AG-DCC	53187.9	-106218.1	**-106333.8**		
	AD	AAD	KS	AKS	
DCC	9.9268	0.1518	4.7447	0.0615	
A-DCC	9.8848	**0.1515**	**4.7230**	**0.0614**	
G-DCC	**9.8363**	0.1552	4.7524	0.0633	
AG-DCC	10.2421	0.1595	4.8755	0.0654	

Notes to Table: The upper part of this table reports the parameter estimates for the symmetric and asymmetric DCC MVGARCH models. Parameter estimates significantly different from zero at 5% confidence Level are marked with an asterisk. The lower part reports the different goodness-of-fit criteria.

Table F.5: DCC MVGARCH Advanced Emerging Market & Developed Market Bonds.

	a_i^2	b_i^2	a_i^2	g_i^2	b_i^2
BRA	0.0144*	0.9710*	0.0154*	0.0006	0.9680*
HUN	0.0122*	0.9804*	0.0123*	0.0112	0.9777*
MEX	0.0106*	0.9795*	0.0106*	0.0007	0.9788*
POL	0.0106*	0.9836*	0.0103*	0.0020	0.9824*
RSA	0.0116*	0.9810*	0.0115*	0.0022	0.9800*
AUS	0.0176*	0.9755*	0.0185*	0.0000	0.9740*
DEN	0.0126	0.9839*	0.0125	0.0014	0.9829*
SWE	0.0157*	0.9823*	0.0160*	0.0012	0.9810*
ITA	0.0160	0.9819*	0.0163	0.0016	0.9806*
JPN	0.0193*	0.9740*	0.0202*	0.0002	0.9731*
GBR	0.0118*	0.9858*	0.0119*	0.0003	0.9854*
USA	0.0084*	0.9890*	0.0085*	0.0017	0.9881*
Scalar Model	0.0122	0.9849*	0.0118	0.0011	0.9846*
	Log-l	BIC	AIC		
DCC	85562.8	**-171110.7**	-171121.7		
A-DCC	85565.0	-171107.5	-171124.0		
G-DCC	85626.5	-171072.9	-171205.1		
AG-DCC	85643.6	-171016.9	**-171215.2**		
	AD	AAD	KS	AKS	
DCC	11.9663	0.1800	5.6853	0.0725	
A-DCC	11.7444	**0.1758**	**5.4639**	**0.0704**	
G-DCC	11.8929	0.1791	5.6167	0.0722	
AG-DCC	**11.6641**	0.1774	5.5348	0.0714	

Notes to Table: The upper part of this table reports the parameter estimates for the symmetric and asymmetric DCC MVGARCH models. Parameter estimates significantly different from zero at 5% confidence Level are marked with an asterisk. The lower part reports the different goodness-of-fit criteria.

Table F.6: DCC MVGARCH Secondary Emerging Market & Developed Market Bonds.

	a_i^2	b_i^2	a_i^2	g_i^2	b_i^2
CHI	0.0026*	0.9736*	0.0025*	0.0000	0.9757*
CZE	0.0128*	0.9844*	0.0137*	0.0000	0.9827*
IND	0.0052*	0.9835*	0.0051*	0.0002	0.9843*
INA	0.0047*	0.9816*	0.0046*	0.0000	0.9826*
THA	0.0033*	0.9867*	0.0031*	0.0002	0.9878*
AUS	0.0178*	0.9751*	0.0182*	0.0000	0.9747*
DEN	0.0169*	0.9810*	0.0184*	0.0001	0.9790*
SWE	0.0165*	0.9804*	0.0177*	0.0022	0.9786*
ITA	0.0173*	0.9806*	0.0186*	0.0001	0.9789*
JPN	0.0229*	0.9683*	0.0220*	0.0017	0.9690*
GBR	0.0140*	0.9832*	0.0147*	0.0000	0.9821*
USA	0.0086*	0.9889*	0.0089*	0.0003	0.9884*
Scalar Model	0.0101*	0.9879*	0.0101*	0.0007	0.9876*
	Log-l	BIC	AIC		
DCC	88659.9	-177304.9	-177315.9		
A-DCC	88661.0	-177299.6	-177316.1		
G-DCC	88791.2	**-177402.2**	-177534.4		
AG-DCC	88803.9	-177337.4	**-177535.8**		
	AD	AAD	KS	AKS	
DCC	13.3417	0.2003	6.3698	0.0808	
A-DCC	**13.0645**	**0.1965**	**6.2846**	**0.0789**	
G-DCC	13.6027	0.2025	6.5761	0.0822	
AG-DCC	13.7924	0.2059	6.7041	0.0839	

Notes to Table: The upper part of this table reports the parameter estimates for the symmetric and asymmetric DCC MVGARCH models. Parameter estimates significantly different from zero at 5% confidence Level are marked with an asterisk. The lower part reports the different goodness-of-fit criteria.

Table F.7: DCC Gaussian Copula Developed Market Stocks.

	a_i^2	b_i^2	a_i^2	g_i^2	b_i^2
AUS	0.0097*	0.9831*	0.0066	0.0142	0.9756*
DEN	0.0234*	0.9500*	0.0145*	0.0288*	0.9450*
SWE	0.0243*	0.9463*	0.0119	0.0268*	0.9472*
ITA	0.0250*	0.9489*	0.0172	0.0264*	0.9394*
JPN	0.0065*	0.9904*	0.0070	0.0055	0.9832*
GBR	0.0295*	0.9378*	0.0129	0.0315*	0.9433*
USA	0.0043*	0.9841*	0.0036	0.0032	0.9749*
Scalar Model	0.0128*	0.9730*	0.0088*	0.0141*	0.9684*
	Log-l	BIC	AIC		
DCC	41156.2	-82297.5	-82308.5		
A-DCC	41177.2	**-82331.9**	-82348.4		
G-DCC	41177.3	-82249.5	-82326.6		
AG-DCC	41204.0	-82250.3	**-82366.0**		
	AD	AAD	KS	AKS	
DCC	6.1306	0.0973	2.8488	0.0408	
A-DCC	4.3694	**0.0606**	**1.8360**	**0.0234**	
G-DCC	6.0999	0.0946	2.7655	0.0396	
AG-DCC	**4.2127**	0.0624	1.8856	0.0246	

Notes to Table: The upper part of this table reports the parameter estimates for the symmetric and asymmetric DCC Gaussian copula models. Parameter estimates significantly different from zero at 5% confidence Level are marked with an asterisk. The lower part reports the different goodness-of-fit criteria.

Table F.8: DCC Gaussian Copula Advanced Emerging Markets Stocks.

	a_i^2	b_i^2	a_i^2	g_i^2	b_i^2
BRA	0.0254*	0.9598*	0.0200*	0.0150*	0.9566*
HUN	0.0188*	0.9738*	0.0138*	0.0169*	0.9683*
MEX	0.0148*	0.9774*	0.0127*	0.0069*	0.9755*
POL	0.0111*	0.9837*	0.0085*	0.0145	0.9757*
RSA	0.0099*	0.9859*	0.0085*	0.0152	0.9744*
Scalar Model	0.0145*	0.9775*	0.0105*	0.0128*	0.9746*
	Log-l	BIC	AIC		
DCC	25626.8	-51238.6	-51249.6		
A-DCC	25640..5	**-51258.5**	**-51275.1**		
G-DCC	25631.2	-51187.4	-51242.5		
AG-DCC	25645.5	-51178.4	-51261.0		
	AD	AAD	KS	AKS	
DCC	5.7738	0.0874	2.6966	0.0367	
A-DCC	3.7528	**0.0536**	**1.7372**	**0.0210**	
G-DCC	5.6331	0.0868	2.6437	0.0365	
AG-DCC	**4.1503**	0.0567	1.9070	0.0225	

Notes to Table: The upper part of this table reports the parameter estimates for the symmetric and asymmetric DCC Gaussian copula models. Parameter estimates significantly different from zero at 5% confidence Level are marked with an asterisk. The lower part reports the different goodness-of-fit criteria.

Table F.9: DCC Gaussian Copula Secondary Emerging Markets Stocks.

	a_i^2	b_i^2	a_i^2	g_i^2	b_i^2
CHI	0.0290*	0.9415*	0.0196*	0.0044*	0.9704*
CZE	0.0142*	0.9836*	0.0157*	0.0142*	0.9540*
IND	0.0085*	0.9815*	0.0135*	0.0510*	0.8945*
INA	0.0031	0.9903*	0.0001	0.0860*	0.8909*
THA	0.0026	0.9870*	0.0000	0.0248*	0.9422*
Scalar Model	0.0106*	0.9737*	0.0086*	0.0107*	0.9641*
	Log-l	BIC	AIC		
DCC	26116.6	-52218.2	-52229.2		
A-DCC	26122.6	**-52222.7**	-52239.2		
G-DCC	26121.4	-52167.8	-52222.9		
AG-DCC	26134.9	-52157.1	**-52239.8**		
	AD	AAD	KS	AKS	
DCC	4.7851	0.0676	2.1753	0.0281	
A-DCC	**3.8597**	0.0470	**1.7633**	0.0190	
G-DCC	4.9667	0.0684	2.2703	0.0286	
AG-DCC	4.0668	**0.0423**	1.7655	**0.0168**	

Notes to Table: The upper part of this table reports the parameter estimates for the symmetric and asymmetric DCC Gaussian copula models. Parameter estimates significantly different from zero at 5% confidence Level are marked with an asterisk. The lower part reports the different goodness-of-fit criteria.

Table F.10: DCC Gaussian Copula Advanced Emerging Market Stocks and Developed Market Stocks.

	a_i^2	b_i^2	a_i^2	g_i^2	b_i^2
BRA	0.0112*	0.9762*	0.0075	0.0092	0.9783*
HUN	0.0108*	0.9810*	0.0083	0.0106	0.9776*
MEX	0.0078*	0.9826*	0.0051	0.0063	0.9830*
POL	0.0127*	0.9743*	0.0095	0.0110	0.9708*
RSA	0.0123*	0.9732*	0.0082	0.0166*	0.9649*
AUS	0.0041*	0.9892*	0.0006	0.0266	0.9679*
DEN	0.0159*	0.9653*	0.0102	0.0194	0.9600*
SWE	0.0190*	0.9588*	0.0124	0.0144	0.9606*
ITA	0.0143*	0.9672*	0.0111	0.0180	0.9575*
JPN	0.0030*	0.9939*	0.0000	0.0258	0.9604*
GBR	0.0166*	0.9543*	0.0092	0.0224	0.9535*
USA	0.0053*	0.9876*	0.0044	0.0027	0.9880*
Scalar Model	0.0085*	0.9820*	0.0062*	0.0102*	0.9770*
	Log-l	BIC	AIC		
DCC	68148.3	-136281.6	-136292.6		
A-DCC	68186.1	**-136349.7**	**-136366.2**		
G-DCC	68169.7	-136159.2	-136291.4		
AG-DCC	68215.3	-136160.3	-136358.6		
	AD	AAD	KS	AKS	
DCC	9.2068	0.1324	4.2003	0.0545	
A-DCC	**6.6969**	0.0952	**2.6991**	0.0359	
G-DCC	9.2381	0.1293	4.2012	0.0531	
AG-DCC	6.7408	**0.0928**	2.8091	**0.0349**	

Notes to Table: The upper part of this table reports the parameter estimates for the symmetric and asymmetric DCC Gaussian copula models. Parameter estimates significantly different from zero at 5% confidence Level are marked with an asterisk. The lower part reports the different goodness-of-fit criteria.

Table F.11: DCC Gaussian Copula Secondary Emerging Market Stocks and Developed Market Stocks.

	a_i^2	b_i^2	a_i^2	g_i^2	b_i^2
CHI	0.0070*	0.9806*	0.0080	0.0042	0.9823*
CZE	0.0113*	0.9666*	0.0067	0.0133*	0.9626*
IND	0.0039*	0.9822*	0.0003	0.0521	0.8917*
INA	0.0034	0.9823*	0.0000	0.0569*	0.9075*
THA	0.0032	0.9925*	0.0000	0.0180*	0.9686*
AUS	0.0067*	0.9883*	0.0006*	0.0263*	0.9670*
DEN	0.0169*	0.9555*	0.0095*	0.0253*	0.9504*
SWE	0.0214*	0.9516*	0.0133*	0.0147*	0.9594*
ITA	0.0164*	0.9561*	0.0153*	0.0168*	0.9483*
JPN	0.0026*	0.9931*	0.0000	0.0141*	0.9682*
GBR	0.0168*	0.9526*	0.0096	0.0154*	0.9626*
USA	0.0050*	0.9845*	0.0048*	0.0028	0.9850*
Scalar Model	0.0071*	0.9802*	0.0048*	0.0106*	0.9728*
	Log-l	BIC	AIC		
DCC	68303.0	-136590.9	-136602.0		
A-DCC	68335.3	**-136648.2**	-136664.7		
G-DCC	68325.6	-136471.1	-136603.3		
AG-DCC	68373.3	-136476.2	**-136674.6**		
	AD	AAD	KS	AKS	
DCC	8.9280	0.1231	4.2127	0.0510	
A-DCC	6.5168	0.0890	2.6813	0.0341	
G-DCC	**5.9954**	**0.0730**	**2.0633**	**0.0272**	
AG-DCC	6.3432	0.0862	2.5902	0.0330	

Notes to Table: The upper part of this table reports the parameter estimates for the symmetric and asymmetric DCC Gaussian copula models. Parameter estimates significantly different from zero at 5% confidence Level are marked with an asterisk. The lower part reports the different goodness-of-fit criteria.

Table F.12: DCC Gaussian Copula Developed Market Bonds.

	a_i^2	b_i^2	a_i^2	g_i^2	b_i^2
AUS	0.0238*	0.9727*	0.0245*	0.0000	0.9715*
DEN	0.0194*	0.9792*	0.0197*	0.0002	0.9787*
SWE	0.0194*	0.9782*	0.0197*	0.0009*	0.9777*
ITA	0.0200*	0.9784*	0.0203*	0.0003	0.9779*
JPN	0.0231*	0.9726*	0.0232*	0.0008	0.9724*
GBR	0.0159*	0.9801*	0.0162*	0.0000	0.9796*
USA	0.0101*	0.9893*	0.0103*	0.0002	0.9889*
Scalar Model	0.0180*	0.9805*	0.0180*	0.0008	0.9801*
	Log-l	BIC	AIC		
DCC	53380.4	**-106745.9**	**-106756.9**		
A-DCC	53380.6	-106738.8	-106755.3		
G-DCC	53398.6	-106692.2	-106769.3		
AG-DCC	53401.5	-106645.4	-106761.1		
	AD	AAD	KS	AKS	
DCC	**7.145**3	0.1033	**3.3736**	0.0417	
A-DCC	7.2173	**0.1012**	3.3753	**0.0407**	
G-DCC	7.2803	0.1087	3.4199	0.0443	
AG-DCC	7.3693	0.1102	3.4600	0.0450	

Notes to Table: The upper part of this table reports the parameter estimates for the symmetric and asymmetric DCC Gaussian copula models. Parameter estimates significantly different from zero at 5% confidence Level are marked with an asterisk. The lower part reports the different goodness-of-fit criteria.

Table F.13: DCC Gaussian Copula Developed Bonds Positive Asymmetries.

	a_i^2	b_i^2	a_i^2	g_i^2	b_i^2
AUS	0.0238*	0.9727*	0.0222*	0.0026	0.9731*
DEN	0.0194*	0.9792*	0.0195*	0.0006	0.9786*
SWE	0.0194*	0.9782*	0.0190*	0.0013	0.9779*
ITA	0.0200*	0.9784*	0.0202*	0.0004	0.9777*
JPN	0.0231*	0.9726*	0.0214*	0.0036	0.9732*
GBR	0.0159*	0.9801*	0.0155*	0.0006	0.9797*
USA	0.0101*	0.9893*	0.0107*	0.0000	0.9882*
Scalar Model	0.0180*	0.9805*	0.0169*	0.0027*	0.9801*
	Log-l	BIC	AIC		
DCC	53380.4	-106745.9	-106756.9		
ADCC+	53371.8	-106721.2	-106737.7		
GDCC	53398.6	-106692.2	-106769.3		
AGDCC+	53391.5	-106625.3	-106741.0		
	AD	AAD	KS	AKS	
DCC	7.1453	0.1033	3.3736	0.0417	
ADCC+	6.6176	**0.0680**	**2.0795**	**0.0253**	
GDCC	7.2803	0.1087	3.4199	0.0443	
AGDCC+	**6.3886**	0.0765	2.3940	0.0296	

Notes to Table: The upper part of this table reports the parameter estimates for the symmetric and asymmetric DCC Gaussian copula models. Parameter estimates significantly different from zero at 5% confidence Level are marked with an asterisk. The lower part reports the different goodness-of-fit criteria.

Table F.14: DCC Gaussian Copula Advanced Emerging Market Bonds.

	a_i^2	b_i^2	a_i^2	g_i^2	b_i^2
BRA	0.0320*	0.9425*	0.0227	0.0323	0.9342*
HUN	0.0206*	0.9755*	0.0190	0.0124	0.9665*
MEX	0.0219*	0.9558*	0.0116	0.0263	0.9604*
POL	0.0131*	0.9798*	0.0106	0.0119	0.9743*
RSA	0.0252*	0.9619*	0.0181	0.0179	0.9614*
Scalar Model	0.0204*	0.9679*	0.0146*	0.0173*	0.9654*
	Log-l	BIC	AIC		
DCC	31822.8	-63630.6	-63641.6		
A-DCC	31838.6	**-63654.7**	**-63671.2**		
G-DCC	31829.1	-63583.2	-63638.3		
AG-DCC	31845.6	-63578.5	-63661.2		
	AD	AAD	KS	AKS	
DCC	7.0699	0.0987	3.0099	0.0403	
A-DCC	**6.0131**	**0.0653**	**2.3790**	**0.0247**	
G-DCC	7.0801	0.0965	2.9959	0.0394	
AG-DCC	6.0764	0.0678	2.5458	0.0260	

Notes to Table: The upper part of this table reports the parameter estimates for the symmetric and asymmetric DCC Gaussian copula models. Parameter estimates significantly different from zero at 5% confidence Level are marked with an asterisk. The lower part reports the different goodness-of-fit criteria.

Table F.15: DCC Gaussian Copula Secondary Emerging Market Bonds.

	a_i^2	b_i^2	a_i^2	g_i^2	b_i^2
CHI	0.0253*	0.9157*	0.0045*	0.0019	0.9821*
CZE	0.0101*	0.9866*	0.0067*	0.0015	0.9594*
IND	0.0090*	0.9810*	0.1234*	0.0026	0.8386*
INA	0.0061*	0.9884*	0.0123*	0.0242*	0.9400*
THA	0.0024*	0.9969*	0.0044	0.1611*	0.5422*
Scalar Model	0.0075*	0.9834*	0.0081*	0.0008	0.9801*
	Log-l	BIC	AIC		
DCC	34902.9	**-69790.8**	**-69801.8**		
A-DCC	34902.8	-69783.1	-69799.7		
G-DCC	34907.3	-69739.5	-69794.6		
AG-DCC	34908.0	-69703.5	-69786.1		
	AD	AAD	KS	AKS	
DCC	4.7392	0.0738	2.1991	0.0311	
A-DCC	4.6355	0.0719	2.1521	0.0302	
G-DCC	4.8284	0.0728	2.2530	0.0307	
AG-DCC	**4.5970**	**0.0681**	**2.1322**	**0.0286**	

Notes to Table: The upper part of this table reports the parameter estimates for the symmetric and asymmetric DCC Gaussian copula models. Parameter estimates significantly different from zero at 5% confidence Level are marked with an asterisk. The lower part reports the different goodness-of-fit criteria.

Table F.16: DCC Gaussian Copula Advanced Emerging Market and Developed Market Bonds.

	a_i^2	b_i^2	a_i^2	g_i^2	b_i^2
BRA	0.0130*	0.9768*	0.0134*	0.0020	0.9746*
HUN	0.0120*	0.9822*	0.0118*	0.0043	0.9808*
MEX	0.0102*	0.9812*	0.0097*	0.0036	0.9806*
POL	0.0094*	0.9868*	0.0090*	0.0022	0.9860*
RSA	0.0120*	0.9810*	0.0112*	0.0039	0.9806*
AUS	0.0173*	0.9774*	0.0178*	0.0003	0.9760*
DEN	0.0142*	0.9842*	0.0139*	0.0010	0.9838*
SWE	0.0140*	0.9835*	0.0137*	0.0020	0.9829*
ITA	0.0146*	0.9837*	0.0142*	0.0013	0.9831*
JPN	0.0170*	0.9802*	0.0172*	0.0009	0.9798*
GBR	0.0103*	0.9874*	0.0102*	0.0004	0.9873*
USA	0.0084*	0.9905*	0.0088*	0.0011	0.9891*
Scalar Model	0.0118*	0.9860*	0.0138*	0.0042*	0.9801*
	Log-l	BIC	AIC		
DCC	86196.9	**-172378.9**	**-172389.9**		
A-DCC	86195.5	-172368.5	-172385.0		
G-DCC	86250.2	-172320.2	-172452.4		
AG-DCC	86260.2	-172250.1	-172448.4		
	AD	AAD	KS	AKS	
DCC	9.2094	0.1351	3.9819	0.0542	
A-DCC	**8.3731**	**0.1190**	**3.4922**	**0.0464**	
G-DCC	9.4650	0.1361	4.0557	0.0549	
AG-DCC	9.1469	0.1313	4.0432	0.0525	

Notes to Table: The upper part of this table reports the parameter estimates for the symmetric and asymmetric DCC Gaussian copula models. Parameter estimates significantly different from zero at 5% confidence Level are marked with an asterisk. The lower part reports the different goodness-of-fit criteria.

Table F.17: DCC Gaussian Copula Secondary Emerging Market Bonds and Developed Market Bonds.

	a_i^2	b_i^2	a_i^2	g_i^2	b_i^2
CHI	0.0031*	0.9717*	0.0034*	0.0000	0.9674*
CZE	0.0122*	0.9856*	0.0126*	0.0000	0.9847*
IND	0.0052*	0.9850*	0.0050*	0.0007	0.9856*
INA	0.0050	0.9836*	0.0050*	0.0000	0.9839*
THA	0.0029	0.9921*	0.0029*	0.0002	0.9919*
AUS	0.0178*	0.9758*	0.0182*	0.0000	0.9753*
DEN	0.0159	0.9824*	0.0167*	0.0000	0.9815*
SWE	0.0154*	0.9820*	0.0158*	0.0006	0.9814*
ITA	0.0162*	0.9819*	0.0168*	0.0000	0.9811*
JPN	0.0201	0.9758*	0.0194*	0.0009*	0.9763*
GBR	0.0131*	0.9841*	0.0134*	0.0000	0.9836*
USA	0.0095*	0.9890*	0.0096*	0.0001	0.9887*
Scalar Model	0.0098*	0.9886*	0.0138*	0.0026*	0.9801*
	Log-l	BIC	AIC		
DCC	89466.5	-178918.0	-178929.1		
A-DCC	89437.7	-178852.9	-178869.4		
G-DCC	89579.8	**-178979.4**	**-179111.7**		
AG-DCC	89584.8	-178899.3	-179097.6		
	AD	AAD	KS	AKS	
DCC	8.0854	0.1175	3.7337	0.0466	
A-DCC	**7.7395**	**0.1120**	**3.5206**	**0.0439**	
G-DCC	8.2339	0.1242	3.9243	0.0502	
AG-DCC	8.2720	0.1259	3.9536	0.0510	

Notes to Table: The upper part of this table reports the parameter estimates for the symmetric and asymmetric DCC Gaussian copula models. Parameter estimates significantly different from zero at 5% confidence Level are marked with an asterisk. The lower part reports the different goodness-of-fit criteria.

Table F.18: DCC t-Copula Developed Markets Stocks.

	a_i^2	b_i^2	a_i^2	g_i^2	b_i^2
AUS	0.0097*	0.9832*	0.0063	0.0142	0.9757*
DEN	0.0232*	0.9470*	0.0137*	0.0304*	0.9426*
SWE	0.0230*	0.9478*	0.0102	0.0290*	0.9480*
ITA	0.0244*	0.9495*	0.0178	0.0255*	0.9385*
JPN	0.0062*	0.9907*	0.0077	0.0046	0.9831*
GBR	0.0291*	0.9384*	0.0134	0.0310*	0.9415*
USA	0.0042*	0.9846*	0.0039	0.0026	0.9735*
Scalar Model	0.0127	0.9732*	0.0085*	0.0127*	0.9705*
	ν	Log-l	BIC	AIC	
DCC	17.5171*	41197.4	-82372.4	-82388.9	
A-DCC	23.0426*	41214.3	**-82398.6**	-82420.6	
G-DCC	20.7376*	41216.2	-82319.9	-82402.5	
AG-DCC	23.1134*	41239.9	-82314.6	**-82435.8**	
	AD	AAD	KS	AKS	
DCC	4.2276	0.0624	2.0923	0.0261	
A-DCC	4.7122	0.0727	2.3475	0.0302	
G-DCC	**3.0337**	**0.0423**	**1.4800**	**0.0175**	
AG-DCC	4.4361	0.0669	2.2019	0.0277	

Notes to Table: The upper part of this table reports the parameter estimates for the symmetric and asymmetric DCC t-copula models. Parameter estimates significantly different from zero at 5% confidence Level are marked with an asterisk. The lower part reports the different goodness-of-fit criteria.

Table F.19: DCC t-Copula Advanced Emerging Market Stocks and Developed Market Stocks.

	a_i^2	b_i^2	a_i^2	g_i^2	b_i^2
BRA	0.0109*	0.9751*	0.0063	0.0097*	0.9797*
HUN	0.0102*	0.9797*	0.0082*	0.0101*	0.9771*
MEX	0.0084*	0.9819*	0.0051	0.0069*	0.9827*
POL	0.0129*	0.9722*	0.0105*	0.0105*	0.9678*
RSA	0.0120*	0.9722*	0.0081*	0.0168*	0.9637*
AUS	0.0037*	0.9892*	0.0005	0.0249	0.9689*
DEN	0.0151*	0.9637*	0.0098*	0.0191*	0.9581*
SWE	0.0177*	0.9595*	0.0112*	0.0151*	0.9617*
ITA	0.0129*	0.9701*	0.0102*	0.0164*	0.9605*
JPN	0.0026	0.9942*	0.0001	0.0194	0.9630*
GBR	0.0152*	0.9545*	0.0086*	0.0211*	0.9534*
USA	0.0049*	0.9885*	0.0038	0.0027	0.9890*
Scalar Model	0.0087*	0.9801*	0.0057*	0.0089*	0.9790*
	v	Log-l	BIC	AIC	
DCC	20.3006*	68257.9	-136493.3	-136509.8	
A-DCC	23.1984*	68285.3	**-136540.6**	**-136562.6**	
G-DCC	20.4885*	68276.3	-136364.9	-136502.6	
AG-DCC	23.5057*	68310.8	-136343.7	-136547.6	
	AD	AAD	KS	AKS	
DCC	3.9416	**0.0613**	1.9200	**0.0260**	
A-DCC	5.8357	0.1028	2.8749	0.0436	
G-DCC	**3.8536**	0.0616	**1.9037**	0.0262	
AG-DCC	5.8805	0.1008	2.9200	0.0427	

Notes to Table: The upper part of this table reports the parameter estimates for the symmetric and asymmetric DCC t-copula models. Parameter estimates significantly different from zero at 5% confidence Level are marked with an asterisk. The lower part reports the different goodness-of-fit criteria.

Table F.20: DCC t-Copula Secondary Emerging Market Stocks and Developed Market Stocks Portfolio.

	a_i^2	b_i^2	a_i^2	g_i^2	b_i^2
CHI	0.0062*	0.9809*	0.0078	0.0041	0.9820*
CZE	0.0106*	0.9664*	0.0063*	0.0127*	0.9619*
IND	0.0045*	0.9800*	0.0012	0.0505*	0.8871*
INA	0.0041	0.9758*	0.0000	0.0548*	0.9058*
THA	0.0034*	0.9926*	0.0000	0.0172	0.9697*
AUS	0.0063*	0.9889*	0.0005*	0.0246	0.9684*
DEN	0.0149*	0.9574*	0.0080*	0.0253*	0.9518*
SWE	0.0185*	0.9565*	0.0115*	0.0157*	0.9611*
ITA	0.0145*	0.9605*	0.0139*	0.0165*	0.9499*
JPN	0.0026*	0.9931*	0.0000	0.0128	0.9707*
GBR	0.0151*	0.9568*	0.0087*	0.0153*	0.9634*
USA	0.0045*	0.9863*	0.0045*	0.0025	0.9855*
Scalar Model	0.0069*	0.9801*	0.0045*	0.0094*	0.9753*
	v	Log-l	BIC	AIC	
DCC	22.9195*	68390.9	-136759.3	-136775.8	
A-DCC	25.8858*	68414.6	**-136799.1**	**-136821.2**	
G-DCC	23.3114*	68409.7	-136631.6	-136769.4	
AG-DCC	26.7674*	68447.4	-136616.9	-136820.8	
	AD	AAD	KS	AKS	
DCC	3.7129	0.0478	1.7333	0.0200	
A-DCC	5.5892	0.0883	2.6531	0.0369	
G-DCC	**3.4980**	**0.0466**	**1.6932**	**0.0194**	
AG-DCC	5.5768	0.0869	2.7337	0.0362	

Notes to Table: The upper part of this table reports the parameter estimates for the symmetric and asymmetric DCC t-copula models. Parameter estimates significantly different from zero at 5% confidence Level are marked with an asterisk. The lower part reports the different goodness-of-fit criteria.

Table F.21: DCC t-Copula Developed Bonds.

	a_i^2	b_i^2	a_i^2	g_i^2	b_i^2
AUS	0.0231*	0.9747*	0.0232*	0.0004	0.9745*
DEN	0.0195*	0.9797*	0.0199*	0.0001	0.9791*
SWE	0.0210*	0.9777*	0.0209*	0.0005	0.9777*
ITA	0.0201*	0.9789*	0.0205*	0.0001	0.9783*
JPN	0.0185*	0.9783*	0.0187*	0.0000	0.9781*
GBR	0.0155*	0.9818*	0.0158*	0.0000	0.9811*
USA	0.0107*	0.9886*	0.0110*	0.0003	0.9878*
Scalar Model	0.0189*	0.9801*	0.0177*	0.0028*	0.9801*
	ν	Log-l	BIC	AIC	
DCC	13.5750*	53529.5	**-107036.5**	-107053.0	
A-DCC	13.4748*	53532.3	-107034.6	-107056.7	
G-DCC	13.4326*	53546.9	-106981.2	**-107063.9**	
AG-DCC	13.4161*	53547.9	-106930.7	-107051.9	
	AD	AAD	KS	AKS	
DCC	6.8617	0.1206	3.4186	0.0499	
A-DCC	7.3395	0.1299	3.6610	0.0538	
G-DCC	**6.1951**	0.1099	**3.0923**	0.0454	
AG-DCC	6.2134	**0.1083**	3.1009	**0.0447**	

Notes to Table: The upper part of this table reports the parameter estimates for the symmetric and asymmetric DCC t-copula models. Parameter estimates significantly different from zero at 5% confidence Level are marked with an asterisk. The lower part reports the different goodness-of-fit criteria.

Table F.22: DCC t-Copula Advanced Emerging Market Bonds and Developed Market Bonds.

	a_i^2	b_i^2	a_i^2	g_i^2	b_i^2
BRA	0.0125*	0.9781*	0.0126*	0.0018	0.9767*
HUN	0.0136*	0.9808*	0.0131	0.0043	0.9798*
MEX	0.0102*	0.9820*	0.0094*	0.0041	0.9821*
POL	0.0097*	0.9866*	0.0093	0.0023	0.9861*
RSA	0.0117*	0.9826*	0.0106*	0.0036	0.9829*
AUS	0.0165*	0.9791*	0.0170*	0.0005	0.9778*
DEN	0.0144*	0.9843*	0.0141	0.0011	0.9836*
SWE	0.0145*	0.9832*	0.0140	0.0021	0.9826*
ITA	0.0145*	0.9839*	0.0141	0.0013*	0.9833*
JPN	0.0155*	0.9821*	0.0156*	0.0010	0.9818*
GBR	0.0103*	0.9877*	0.0105	0.0005	0.9873*
USA	0.0090*	0.9898*	0.0095	0.0015	0.9881*
Scalar Model	0.0159*	0.9801*	0.0137*	0.0054*	0.9801*
	ν	Log-l	BIC	AIC	
DCC	18.8403*	86364.7	-172706.8	-172723.4	
A-DCC	19.2566*	86382.9	**-172735.8**	-172757.9	
G-DCC	18.7610*	86431.4	-172675.1	**-172812.9**	
AG-DCC	19.0032*	86440.2	-172602.5	-172806.4	
	AD	AAD	KS	AKS	
DCC	5.5230	0.0922	2.6346	0.0382	
A-DCC	6.9818	0.1188	3.3076	0.0494	
G-DCC	**5.3101**	**0.0903**	**2.5568**	**0.0373**	
AG-DCC	5.4338	0.0938	2.6651	0.0388	

Notes to Table: The upper part of this table reports the parameter estimates for the symmetric and asymmetric DCC t-copula models. Parameter estimates significantly different from zero at 5% confidence Level are marked with an asterisk. The lower part reports the different goodness-of-fit criteria.

Table F.23: DCC t-Copula Secondary Emerging Market Bonds and Developed Market Bonds.

	a_i^2	b_i^2	a_i^2	g_i^2	b_i^2
CHI	0.0043*	0.9654*	0.0044*	0.0010	0.9637*
CZE	0.0123*	0.9861*	0.0124*	0.0000	0.9859*
IND	0.0044*	0.9881*	0.0043*	0.0002	0.9881*
INA	0.0043*	0.9893*	0.0044	0.0064	0.9866*
THA	0.0028	0.9924*	0.0027	0.0002	0.9922*
AUS	0.0172*	0.9774*	0.0174*	0.0001	0.9772*
DEN	0.0157*	0.9830*	0.0158*	0.0000	0.9829*
SWE	0.0160*	0.9817*	0.0161*	0.0000	0.9816*
ITA	0.0162*	0.9822*	0.0162*	0.0000	0.9821*
JPN	0.0168*	0.9802*	0.0167*	0.0014	0.9803*
GBR	0.0126*	0.9855*	0.0127*	0.0001	0.9853*
USA	0.0096*	0.9888*	0.0098*	0.0004	0.9883*
Scalar Model	0.0151*	0.9801*	0.0137*	0.0039*	0.9801*
	ν	Log-l	BIC	AIC	
DCC	20.0962*	89588.8	-179155.1	-179171.6	
A-DCC	20.3452*	89599.1	-179168.1	-179190.2	
G-DCC	20.1995*	89735.3	**-179282.9**	**-179420.7**	
AG-DCC	20.1705*	89739.6	-179201.4	-179405.2	
	AD	AAD	KS	AKS	
DCC	5.1454	0.0884	2.4637	0.0365	
A-DCC	6.4296	0.1085	3.0644	0.0450	
G-DCC	**4.9053**	0.0849	**2.4123**	0.0348	
AG-DCC	5.0191	**0.0836**	2.4799	**0.0343**	

Notes to Table: The upper part of this table reports the parameter estimates for the symmetric and asymmetric DCC t-copula models. Parameter estimates significantly different from zero at 5% confidence Level are marked with an asterisk. The lower part reports the different goodness-of-fit criteria.

Table F.24: DCC Vine Stock Portfolios Dependence Analysis.

	Model	Log-l	AIC	BIC	AD	AAD	KS	AKS
				Developed Stocks				
Vine GA	DCC	41202.9	-82321.8	**-82090.4**	6.0503	0.0938	2.8762	0.0394
Vine GA	A-DCC	41273.0	-82333.5	-81986.4	**4.5685**	**0.0323**	**1.0962**	**0.0110**
Vine T	DCC	41255.3	-82384.7	**-82037.6**	2.5232	0.0315	1.1229	0.0129
Vine T	A-DCC	41280.6	**-82393.2**	-81930.5	**2.3425**	**0.0141**	**0.5647**	**0.0051**
				Advanced Emerging Markets & Developed Stocks				
Vine GA	DCC	68274.7	**-136285.5**	**-135558.3**	6.9350	0.1000	3.1467	0.0399
Vine GA	A-DCC	68312.7	-136229.5	-135138.6	**5.8802**	**0.0749**	**1.9198**	**0.0271**
Vine T	DCC	68435.4	**-136474.8**	-135384.0	**2.4251**	**0.0271**	**1.0785**	**0.0103**
Vine T	A-DCC	68482.1	-136436.2	-134981.7	5.0319	0.0753	2.3437	0.0317
				Secondary Emerging Markets & Developed Stocks				
Vine GA	DCC	68347.3	-136430.7	**-135703.5**	**5.7015**	0.0805	2.6568	0.0319
Vine GA	A-DCC	68390.7	-136385.5	-135294.6	5.8747	**0.0648**	2.6568	**0.0238**
Vine T	DCC	68482.0	**136568.1**	**-135477.3**	**2.1343**	**0.0194**	**0.8809**	**0.0073**
Vine T	A-DCC	68525.9	-136523.9	-135069.5	5.3970	0.0820	2.4419	0.0343

Notes to Table: This table reports the different goodness-of-fit criteria.

Table F.25: DCC Vine Bond Portfolios Dependence Analysis.

	Model	Log-l	AIC	BIC	AD	AAD	KS	AKS
				Developed Bonds				
Vine GA	DCC	53463.1	**-106842.2**	**-106610.8**	7.3213	0.1104	3.5264	0.0453
Vine GA	A-DCC	53461.4	-106796.9	-106449.8	**6.8211**	**0.1034**	**3.2475**	**0.0420**
Vine T	DCC	53591.1	**-107056.3**	**-106709.2**	**4.1549**	**0.0292**	**0.9808**	**0.0102**
Vine T	A-DCC	53565.4	-106962.8	-106500.0	4.4671	0.0357	1.1665	0.0130
				Advanced Emerging Markets & Developed Bonds				
Vine GA	DCC	86349.7	**-172435.4**	**-171708.2**	9.5905	0.1402	4.2927	0.0570
Vine GA	A-DCC	86360.7	-172325.4	-171234.6	**8.8116**	**0.1198**	**3.9088**	**0.0473**
Vine T	DCC	86547.6	**-172699.3**	**-171608.4**	**5.2452**	**0.0638**	**1.8403**	**0.0254**
Vine T	A-DCC	86564.8	-172601.6	-171147.1	6.2374	0.1029	3.0049	0.0428
				Secondary Emerging Markets & Developed Bonds				
Vine GA	DCC	89714.0	**-179164.1**	**-178436.8**	8.5232	0.1252	3.8934	0.0510
Vine GA	A-DCC	89719.6	-179043.2	-177952.4	**8.2059**	**0.1151**	**3.8458**	**0.0463**
Vine T	DCC	89911.7	**-179427.5**	-178336.7	**4.6422**	**0.0593**	**2.1419**	**0.0242**
Vine T	A-DCC	89897.1	-179266.2	-177811.7	5.3282	0.0780	2.5918	0.0322

Notes to Table: This table reports the different goodness-of-fit criteria.

Table F.26: DCC Gaussian and t-Copula Vine Stock Portfolios Unconditional Correlation and Data Ordering.

	HUN	MEX	POL	RSA	AUS	DEN	SWE	ITA	JAP	GBR	USA	
						EM adv Stocks & Dev Stocks (ρ))						
BRA	0.5475	0.7725	0.5480	0.6159	0.4771	0.6269	0.6330	0.6027	0.3207	0.6191	0.6517	
HUN		0.5448	0.7293	0.6525	0.5477	0.6570	0.6643	0.5791	0.3650	0.5690	0.4045	
MEX			0.5570	0.5874	0.4449	0.6027	0.6312	0.6270	0.3003	0.6297	0.7036	
POL				0.6672	0.5482	0.6577	0.6783	0.5853	0.3461	0.5759	0.3797	
RSA					0.6498	0.6988	0.6882	0.5813	0.4465	0.6047	0.3581	
AUS						0.6611	0.5722	0.5065	0.6364	0.5246	0.2305	
DEN							0.7947	0.6569	0.4136	0.6770	0.4226	
SWE								0.7111	0.3455	0.7110	0.5013	
ITA									0.3961	0.8420	0.5278	
JAP										0.4059	0.1674	
GBR											0.5165	
						EM adv Stocks & Dev Stocks (ν)						
BRA	9.1845	5.6599	10.4690	16.7095	19.6308	20.2577	11.7406	8.9391	300.00	15.4086	6.6709	
HUN		13.3549	6.3959	16.2283	18.1036	8.9127	8.3694	10.4481	122.3940	15.7041	13.2964	
MEX			10.3150	12.7886	20.2686	14.5815	10.3543	9.1640	300.00	9,7089	8,0937	
POL				7.6632	12,6276	5,7773	5,8513	12,1547	152,247	9.7820	19.7694	
RSA					19.0006	8.9234	17.5751	17.3673	300.00	31.1335	88.2094	
AUS						21.7067	18.7580	17.4982	6.8057	31,3412	86,8539	
DEN							6.8608	9.7885	300.00	11,1715	25,3829	
SWE								11.6420	141.6391	15.5566	12,5509	
ITA									71.9571	8.1171	7.6044	
JAP										279.4552	300.00	
GBR											23.4678	
						EM sec Stocks & Dev Stocks (ρ)						
BRA	0.5101	0.3570	0.3083	0.2654	0.4835	0.5799	0.5556	0.5494	0.3176	0.5606	0.5250	
HUN		0.4368	0.4115	0.3655	0.6093	0.6820	0.6398	0.5613	0.4180	0.5694	0.3280	
MEX			0.4897	0.4257	0.5056	0.4478	0.4222	0.4278	0.3920	0.4259	0.2845	
POL				0.5069	0.5215	0.4390	0.3341	0.3658	0.4467	0.3577	0.1464	
RSA					0.5470	0.3892	0.3506	0.3255	0.5412	0.3486	0.1496	
AUS						0.6611	0.5722	0.5065	0.6364	0.5246	0.2305	
DEN							0.7947	0.6569	0.4136	0.6770	0.4226	
SWE								0.7111	0.3455	0.7110	0.5013	
ITA									0.3961	0.8420	0.5278	
JAP										0.4059	0.1674	
GBR											0.5165	
						EM sec Stocks & Dev Stocks (ν)						
BRA	12.7830	18.7878	14.8056	138.2488	28.8751	12.3511	17.3920	14.7873	56.7096	15.7213	8.38825	
HUN		18.4411	21.3287	80.9601	14.3557	10.1653	10.5385	9.0643	19.5071	16.8414	12.0406	
MEX			12.6719	15.7341	16.6992	13.3094	23.0041	20.6857	44.4444	14.3771	300.00	
POL				8.2532	11.0709	16.7659	11.0867	25.8274	23.1177	300.00	29.5523	
RSA					10.1244	131.7387	30.1386	15.9287	18.3288	36.3969	300.00	
AUS						26.0630	19.8787	18.5874	6.9168	40.5057	125.3212	
DEN							7.2165	10.0804	300.00	11.3060	23.7024	
SWE								11.8309	111.0146	15.6574	14.0761	
ITA									60.6625	8.9026	8.5809	
JAP										396.2766	300.00	
GBR											27.3584	
						Data ordering						
						EM adv Stocks & Dev Stocks						
GA	GBR	ITA	SWE	DEN	RSA	POL	HUN	AUS	JPN	BRA	MEX	USA
T	MEX	BRA	USA	ITA	GBR	POL	DEN	SWE	HUN	RSA	AUS	JPA
						EM sec Stocks & Dev Stocks						
GA	GBR	SWE	DEN	AUS	JPN	THA	INA	IND	CZE	USA	CHI	
T	MEX	BRA	USA	ITA	GBR	POL	DEN	SWE	HUN	RSA	AUS	JPA

Notes to Table: The upper part reports the unconditional dependence parameters for the bivariate stock pairs. The lower part reports the ordering of the indices according to their bivariate unconditional dependence. In parenthesis are the bivariate dependence parameter of either the Gaussian (ρ) or t-Copula Vine (ν)

Table F.27: DCC Gaussian and t-Copula Vine Bond Portfolios Unconditional Correlation and Data Ordering.

	CHI	CZE	IND	INA	AUS	DEN	SWE	ITA	JAP	GBR	USA
						EM adv Bonds & Dev Bonds (ρ)					
CHI	0.4562	0.6084	0.4694	0.4892	0.5271	0.2305	0.3176	0.2560	-0.2288	0.1981	-0.1766
CZE		0.4571	0.8008	0.5892	0.5935	0.5790	0.6453	0.6188	-0.0089	0.4476	-0.0871
IND			0.4943	0.4725	0.4984	0.2019	0.2803	0.2318	-0.2312	0.2120	-0.1108
INA				0.6026	0.6349	0.6415	0.7026	0.6823	0.0178	0.5297	-0.0628
THA					0.5659	0.4281	0.5063	0.4528	-0.0645	0.3675	-0.1020
AUS						0.5869	0.6197	0.5932	-0.0407	0.5203	-0.0740
DEN							0.8457	0.9651	0.3708	0.7254	0.2523
SWE								0.8540	0.2483	0.6242	0.0897
ITA									0.3233	0.7228	0.2223
JAP										0.2488	0.3315
GBR											0.2335
						EM adv Bonds & Dev Bonds (v)					
CHI	6.9853	9.6433	11.1423	10.2770	7.3951	9.8746	9.3510	9.6339	6.9720	29.0170	14.5151
CZE		11.0167	5.4222	14.0358	10.0994	7.8331	9.1208	6.8904	6.1290	14.5260	7.6601
IND			11.8844	8.9035	9.0636	13.0106	12.4250	9.4055	8.8874	22.5339	9.6754
INA				11.0657	24.6067	16.2621	18.4199	12.9801	6.1631	22.9926	10.1289
THA					12.8015	11.7184	12.7418	11.1010	6.7117	16.4393	13.9066
AUS						8.4754	14.4616	10.4539	2.8544	12.2450	10.3099
DEN							5.8328	2.0112	5.3433	8.5144	11.6072
SWE								8.4707	5.3184	8.2407	7.9915
ITA									5.4690	7.4028	7.5563
JAP										5.7240	9.5693
GBR											10.4833
						EM sec Bonds & Dev Bonds (ρ)					
CHI	0.0723	0.0090	0.0790	0.0824	0.0830	0.1222	0.1090	0.1271	0.0735	0.1099	0.0540
CZE		0.2155	0.2384	0.2652	0.6183	0.7784	0.7846	0.8027	0.1797	0.5821	0.0047
IND			0.2123	0.2034	0.2804	0.2042	0.2541	0.2298	-0.016	0.1650	-0.063
INA				0.2459	0.3983	0.2120	0.2195	0.2235	-0.143	0.1920	-0.063
THA					0.2822	0.2890	0.2584	0.2800	0.2601	0.2061	0.0693
AUS						0.5869	0.6197	0.5932	-0.040	0.5203	-0.074
DEN							0.8457	0.9651	0.3708	0.7254	0.2523
SWE								0.8540	0.2483	0.6242	0.0897
ITA									0.3233	0.7228	0.2223
JAP										0.2488	0.3315
GBR											0.2335
						EM sec Stocks & Dev Stocks (v)					
CHI	18.7132	22.9493	135.3992	12.1660	19.4628	34.6312	58.3860	43.4683	16.9226	24.5056	95.4968
CZE		22,5389	12.2371	20.5493	9.5307	19.1696	11.1683	14.9231	6.6700	13.3119	7.4889
IND			18.8735	29.5825	26.7493	87.3365	28.0198	35.2982	10.8658	73.2140	300.00
INA				11.5983	11.5763	24.1590	16.6753	16.8758	6.8541	39.0748	300.00
THA					18.3110	22.4569	15.0592	24.3669	8.1015	13.7496	26.5087
AUS						8.4754	14.4616	10.4539	2.8544	12.2450	10.3099
DEN							5.8328	2.0112	5.3433	8.5144	11.6072
SWE								8.4707	5.3184	8.2407	7.9915
ITA									5.4690	7.4028	7.5563
JAP										5.7240	9.5693
GBR											10.4833

						Data ordering						
						EM adv Bonds & Dev Bonds						
GA	DEN	ITA	SWE	POL	HUN	AUS	RSA	BRA	MEX	JAP	GBR	USA
T	ITA	DEN	JAP	AUS	BRA	HUN	POL	USA	SWE	GBR	RSA	MEX
						EM sec Bonds & Dev Bonds						
GA	DEN	ITA	SWE	CZE	AUS	GBR	JAP	USA	THA	INA	IND	CHI
T	DEN	THA	SWE	JAP	AUS	CZE	USA	GBR	THA INA	IND	CHI	

Notes to Table: The upper part reports the unconditional dependence parameters for the bivariate bond pairs. The lower part reports the ordering of the indices according to their bivariate unconditional dependence. In parenthesis are the bivariate dependence parameter of either the Gaussian (ρ) or t-Copula Vine (v).

Table F.28: Archimedean Copulas Stock Portfolios.

	ω^{MA}	α^{MA}	β^{MA}	Log-l	AIC	BIC	AD	AAD	KS	AKS
					Developed Stocks					
CL	0.1107*	0.9334*	-0.1465*	39749.3	**-79492.6**	**-79476.1**	**9.8589**	**0.1466**	**4.6259**	**0.0619**
	(0.0398)	(0.0307)	(0.0552)							
RCL	0.3108	-0.0253	-0.9999	39541.8	-79077.6	-79061.0	10.9092	0.1611	5.1449	0.0647
	(0.2585)	(0.8855)	(0.7580)							
					Advanced Emerging Markets & Developed Stocks					
CL	0.1963*	0.8801*	-0.1478*	65431.7	**-130931.6**	**-130915.1**	**13.7441**	**0.2141**	**5.6425**	**0.0908**
	(0.0520)	(0.0358)	(0.0396)							
RCL	0.1792	-0.9730*	-0.8894*	65046.8	-130246.8	-130230.3	15.1044	0.2263	7.4122	0.0957
	(0.2416)	(0.0124)	(0.1207)							
					Secondary Emerging Markets & Developed Stocks					
CL	0.2165*	0.8884*	-0.1554*	66200.5	**-132447.9**	**-132431.4**	**9.8500**	**0.1369**	**4.6508**	**0.0563**
	(0.0553)	(0.0301)	(0.0392)							
RCL	0.1636	0.7734*	-0.1918	65821.6	-131735.2	-131718.6	13.1320	0.1949	6.2211	0.0822
	(0.1032)	(0.1931)	(0.1398)							

Notes to Table: The left part of the table reports the parameter estimates, the right part the log-likelihood values and goodness-of-fit tests.

Table F.29: Archimedean Copulas Bond Portfolios.

	ω^{MA}	α^{MA}	β^{MA}	Log-l	AIC	BIC	AD	AAD	KS	AKS
					Developed Bonds					
CL	0.6296	0.4092	-0.9999	48921.2	-97836.4	-97819.9	15.2962	**0.2292**	7.4330	**0.0961**
	(0.6105)	(0.5789)	(0.9665)							
RCL	0.1992*	0.8828*	-0.2779*	49020.7	**-98035.5**	**-98019.0**	**14.9765**	0.2295	**7.1511**	**0.0961**
	(0.0511)	(0.0311)	(0.0713)							
					Advanced Emerging Markets & Developed Bonds					
CL	-0.7105*	-0.9972*	-0.5718*	79417.9	**-158829.8**	**-158813.2**	**17.7846**	**0.2663**	**8.7157**	**0.1124**
	(0.1141)	(0.0025)	(0.0619)							
RCL	0.0939*	0.9271*	-0.0845	79363.6	-158721.3	-158704.8	18.8260	0.2870	9.3003	0.1217
	(0.0542)	(0.0671)	(0.0589)							
					Secondary Emerging Markets & Developed Bonds					
CL	0.5194	0.0142	-0.8145*	83314.4	-166606.3	-166622.8	**15.0180**	**0.2243**	**7.2336**	**0.0944**
	(0.3181)	(0.2919)	(0.2623)							
RCL	0.8852*	0.0033	-0.9999*	83340.1	**-166674.3**	**-166657.8**	15.1875	0.2351	7.4150	0.0988
	(0.2070)	(0.0048)	(0.1034)							

Notes to Table: The left part of the table reports the parameter estimates, the right part the log-likelihood values and goodness-of-fit tests.

Table F.30: Dynamic Mixture Copulas Stock Portfolios.

	Developed Stocks		***Adv EM & Dev Stocks***			***Sec EM & DevStocks***		
	a^2	b^2		a^2	b^2		a^2	b^2
AUS	0.0096*	0.9834*	BRA	0.0117*	0.9737*	CHI	0.0060*	0.9823*
DEN	0.0225*	0.9483*	HUN	0.0106*	0.9793*	CZE	0.0105*	0.9682*
SWE	0.0226*	0.9484*	MEX	0.0092*	0.9814*	IND	0.0050*	0.9791*
ITA	0.0255*	0.9478*	POL	0.0132*	0.9715*	INA	0.0045	0.9747*
JPN	0.0062*	0.9906*	RSA	0.0118*	0.9725*	THA	0.0038*	0.9923*
GBR	0.0302	0.9381*	AUS	0.0038*	0.9894*	AUS	0.0068*	0.9888*
USA	0.0042*	0.9844*	DEN	0.0155*	0.9625*	DEN	0.0133*	0.9598*
			SWE	0.0175*	0.9595*	SWE	0.0169*	0.9601*
			ITA	0.0123*	0.9716*	ITA	0.0122	0.9667*
			JPN	0.0026	0.9943*	JPN	0.0028*	0.9930*
			GBR	0.0145	0.9566*	GBR	0.0146*	0.9597*
			USA	0.0052	0.9883*	USA	0.0045*	0.9878*

	ν	ω^{MA}	α^{MA}	β^{MA}	ω^{DW}	α^{DW}	β^{DW}
			Developed Stocks				
T-CL	20.7408*	0.5318*	0.9328	-0.4426	0.9425	0.0189	0.0381
	(5.9099)	(0.0573)	(0.4824)	(-0.0610)	(1.2230)	(0.0273)	(0.1593)
			Advanced Emerging Market & Developed Stocks				
T-CL	20.4379*	1.4651*	0.7418*	-0.7798*	0.5455*	0.4545	-0.0177
	(1.8415)	(0.3079)	(0.0745)	(0.1816)	0.5679	0.5467	0.0566
			Secondary Emerging Market & Developed Stocks				
T-CL	23.5284*	0.9122*	0.8684*	-0.4627*	0.9688*	-0.0309	0.0312
	(0.0024)	(0.2646)	(0.0324)	(0.1317)	(0.1830)	0.1792	0.0586

	Log-l	BIC	AIC	AD	AAD	KS	AKS
			Developed Stocks				
T-CL	41221.9	-82397.9	-82271.1	2.8866	0.0390	1.4212	0.0162
			Advanced Emerging Market & Developed Stocks				
T-CL	68289.1	-136512.2	-136330.4	2.8805	0.0424	1.3581	0.0183
			Secondary Emerging Market & Developed Stocks				
T-CL	68423.0	-136780.0	-136598.2	2.5871	0.0238	1.2628	0.0101

Notes to Table: The upper part of the table shows the parameter estimates of the elliptical copula in the mixture structure. The middle part reports (i) the parameters estimates of the archimedean copula in the mixture and (ii) the time-varying weights. Parameter estimates significant at 5% confidence level are marked with an asterisk. Standard errors are in parenthesis. The lower part of the table reports the goodness-of-fit tests.

Table F.31: Dynamic Mixture Copulas Bond Portfolios.

	Developed Bonds		***Adv EM & Dev Bonds***			***Sec EM & Dev Bonds***		
	a^2	b^2		a^2	b^2		a^2	b^2
AUS	0.0239*	0.9750*	BRA	0.0120*	0.9793*	CHI	0.0047	0.9644*
DEN	0.0203	0.9791*	HUN	0.0136*	0.9813*	CZE	0.0131	0.9856*
SWE	0.0221	0.9772*	MEX	0.0098*	0.9828*	IND	0.0044	0.9883*
ITA	0.0212	0.9781*	POL	0.0098*	0.9871*	INA	0.0042	0.9893*
JPN	0.0187*	0.9776*	RSA	0.0112*	0.9838*	THA	0.0029	0.9921*
GBR	0.0167	0.9816*	AUS	0.0167*	0.9793*	AUS	0.0175*	0.9776*
USA	0.0111*	0.9880*	DEN	0.0145*	0.9844*	DEN	0.0162	0.9827*
			SWE	0.0148*	0.9832*	SWE	0.0164	0.9818*
			ITA	0.0148*	0.9839*	ITA	0.0169	0.9817*
			JPN	0.0154*	0.9821*	JPN	0.0166	0.9802*
			GBR	0.0107*	0.9878*	GBR	0.0132	0.9852*
			USA	0.0094*	0.9893*	USA	0.0101	0.9882*

Model	ν	ω^{MA}	α^{MA}	β^{MA}	ω^{DW}	α^{DW}	β^{DW}
				Developed Bonds			
T-CL	15.6410*	0.8571	0.6571*	-0.9999	0.9800*	0.0091	0.0019
	(0.0030)	(0.5981)	(0.1614)	(0.6829)	(0.0593)	(0.0729)	(0.0185)
				Advanced Emerging Market & Developed Bonds			
T-CL	19.9121*	0.1116	-0.9878*	-0.3825	0.8584	0.1416	-0.0052
	(1.6593)	(5.0700)	(0.5406)	(3.6079)	(7.7271)	(7.4851)	(0.2935)
				Advanced Emerging Market & Developed Bonds			
T-CL	22.0674	1.8850	0.7168*	-0.9999	0.9800	0.0200	-0.0069
	(32.2821)	(1.8930)	(0.3864)	(0.8495)	(0.0186)*	(0.2463)	(0.2323)

	Log-l	BIC	AIC	AD	AAD	KS	AKS
				Developed Bonds			
T-CL	53562.7	-106952.7	-107079.4	4.6924	0.0781	2.3448	0.0322
				Advanced Emerging Market & Developed Bonds			
T-CL	86442.8	-172757.9	-172735.8	4.6052	0.0717	2.1584	0.0298
				Advanced Emerging Market & Developed Bonds			
T-CL	89751.5	-179437.0	-179255.2	4.0192	0.0635	1.9830	0.0259

Notes to Table: The upper part of the table shows the parameter estimates of the elliptical copula in the mixture structure. The middle part reports (i) the parameters estimates of the archimedean copula in the mixture and (ii) the time-varying weights. Parameter estimates significant at 5% confidence level are marked with an asterisk. Standard errors are in parenthesis. The lower part of the table reports the goodness-of-fit tests.

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Eidesstattliche Versicherung

(Siehe Promotionsordnung vom 12.07.11, § 8, Abs. 2 Pkt. .5.)

Hiermit erkläre ich an Eidesstatt, dass die Dissertation von mir selbstständig, ohne unerlaubte Beihilfe angefertigt ist.

Grziska, Martin

Name, Vorname

Frankfurt, 20.10.2013

Ort, Datum

Unterschrift Doktorand/in

Formular 3.2