Inflation from Field Theory and String Theory Perspectives
– Matter Inflation and Slow-Walking Inflation

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Zusammenfassung

Diese Dissertation befasst sich mit Aspekten von Inflation sowohl aus Sicht von Feldtheorie als auch von Stringtheorie. Sie hat zum Ziel neue Ansätze zu untersuchen um das Problem der Moduli-Destabilisierung und das η-Problem anzugehen und um Inflation im Materiesektor zu realisieren.

Der erste Teil widmet sich der Untersuchung von Inflationsmodellen im Rahmen von vierdimensionaler $\mathcal{N} = 1$ Supergravitation. Wir beginnen damit einen neuen Vorschlag zur Lösung des Problems der Moduli-Destabilisierung zu untersuchen, dass uns dazu zu zwingen scheint zwischen Niederenergie-Supersymmetrie und einer hohen Inflationsskala zu wählen. Dieser neue Ansatz basiert auf einer bestimmten Art den Modulus an den F-Term zu koppeln der Inflation antreibt. Wir benutzen chaotische Inflation mit einer Shiftsymmetrie als Beispiel und zeigen, dass wir erfolgreich Niederenergie-Supersymmetrie mit einer hohen Inflationsskala verbinden können.

Danach konstruieren wir eine Klasse von Inflationsmodellen in $\mathcal{N} = 1$ Supergravitation bei denen Inflation in nicht-trivialen Darstellungen der Eichgruppe realisiert wird. Dabei handelt es sich um Erweiterungen einer speziellen Klasse von Hybridinflationsmodellen, sogenannte Tribridinflation, bei denen das η-problem durch eine Heisenbergsymmetrie gelöst werden kann. Verglichen mit bisher untersuchten Modellen haben wir unsere Modelle mit etwas Inspiration aus der Stringtheorie verallgemeinert. Wir untersuchen die Stabilisierung der Moduli während Inflation und identifizieren Situationen in denen die Steigung des Inflatonpotentials durch Strahlungskorrekturen dominiert wird. Wir skizzieren unter welchen Bedingungen man diese Klasse von Materieinflationsmodellen in heterotische Orbifoldkompaktifizierungen einbetten kann. Dabei schlagen wir einen neuen Mechanismus vor mit dem man einige Kählermoduli durch die F-Terme von Materiefeldern stabilisieren kann.

Im zweiten Teil betrachten wir Modelle von gewarpter D-Branen-Inflation in einer Familie von zehndimensionalen Supergravitationshintergründen. Wir betrachten Inflation entlang der radialen Richtung in der Nähe des Endes des gewarpten Throats und zeigen, dass das Inflationspotential generisch einen Wendepunkt enthält, der mit einem Wendepunkt im Dilatonprofil zusammenhängt. Wir nutzen ein universelles Skalierungsverhalten mit den Modellparametern aus um Vorhersagen für Inflationsobservablen zu erhalten.

Abstract

This thesis is concerned with aspects of inflation both from a field theory and a string theory perspective. It aims at exploring new approaches to address the problem of moduli destabilization and the η-problem and to realize inflation in the matter sector.

The first part is devoted to studying models of inflation in the framework of four-dimensional $\mathcal{N} = 1$ supergravity. We begin with investigating a new proposal to solve the problem of moduli destabilization, which seems to force us to choose between low-energy supersymmetry and high-scale inflation. This new approach is based on a particular way to couple the modulus to the F-term driving inflation. Using chaotic inflation with a shift symmetry as an example, we show that we can successfully combine low-energy supersymmetry and high-scale inflation.

We construct a class of inflation models in $\mathcal{N} = 1$ supergravity where the inflaton resides in gauge non-singlet matter fields. These are extensions of a special class of hybrid inflation models, so-called tribrid inflation, where the η-problem can be solved by a Heisenberg symmetry. Compared to previously studied models, we have generalized our models with some inspiration from string theory. We investigate moduli stabilization during inflation and identify situations in which the inflaton slope is dominated by radiative corrections. We outline under which conditions this class of matter inflation models could be embedded into heterotic orbifold compactifications. In doing so, we suggest a new mechanism to stabilize some Kähler moduli by F-terms for matter fields.

In the second part, we consider models of warped D-brane inflation on a family of ten-dimensional supergravity backgrounds. We consider inflation along the radial direction near the tip of the warped throat and show that generically an inflection point arises for the inflaton potential, which is related to an inflection point of the dilaton profile. A universal scaling behaviour with the parameters of the model is exploited to extract the predictions for inflationary observables.

Publications

This thesis is based on work done in collaboration with Johanna Erdmenger, Stefan Antusch, Koushik Dutta, Carlos Nuñez and Gianmassimo Tasinato. Part of the covered material has already been published in [1,2], while the remaining part was in its final stages and by now has been published in [3].

[1] S. Antusch, K. Dutta, J. Erdmenger and S. Halter, "Towards Matter Inflation in Heterotic String Theory", JHEP **1104** (2011) 065 [arXiv:1102.0093 [hep-th]].

[2] S. Antusch, K. Dutta and S. Halter, "Combining High-scale Inflation with Low-energy SUSY", JHEP **1203** (2012) 105 [arXiv:1112.4488 [hep-th]].

[3] J. Erdmenger, S. Halter, C. Nuñez, G. Tasinato and , "Slow-walking inflation," JCAP **1301** (2013) 006 [arXiv:1210.4179 [hep-th]].

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Part I

Introduction

Chapter 1

Motivation

From the ongoing quest for a deeper fundamental understanding of our universe two standard pictures have emerged for the laws of physics at small and large length scales. The physics at very small subatomic scales is the realm of *particle physics*, which describes our world in terms of elementary particles and their (non-gravitational) interactions. Zooming out to very large length scales above the size of galaxies, we enter into the realm of *cosmology*, which is based on the theory of general relativity to understand the evolution of our universe from the geometry of space-time. These two seemingly unrelated pictures are actually deeply connected. Going back in time, the energy density of the universe increases more and more and thus the physics at smaller and smaller length scales is probed. Therefore, to understand the early universe it is inevitable to consider particle physics and cosmology in a common framework.

The *Standard Model* (SM) of particle physics encompasses all known elementary particles and their *strong* and *electroweak* (EW) interactions. It has passed numerous high-precision tests up to the EW scale of $\mathcal{O}(100\,\mathrm{GeV})$ and is currently tested to even higher energies with the ongoing experiments at the Large Hadron Collider (LHC). Sooner or later the LHC will unveil the mechanism underlying ElectroWeak Symmetry Breaking (EWSB).

The *concordance model* (or ΛCDM model) is the standard model of cosmology. With only a few parameters it describes the history of our universe from the formation of light elements during Big Bang Nucleosynthesis (BBN) until the present day phase of accelerated expansion – including the decoupling of the Cosmic Microwave Background (CMB) radiation during recombination and the formation of structure (stars, galaxies etc.) via gravitational collapse.

Notwithstanding their remarkable agreement with experiments, both of these standard models suffer from deficiencies which demand for new physics. For instance, we know from observations that the universe is homogeneous, isotropic and spatially flat on very large scales. Using the time evolution of the universe in the hot big bang scenario, both the homogeneity and the spatial flatness would require a high degree of fine-tuning of the initial conditions.

These two issues are referred to as the *horizon problem* and the *flatness problem.* A way to *dynamically* resolve these fine-tuning problems is to introduce a phase of very rapid accelerated expansion called *inflation* in the very early universe [4–7]. The universe is then driven towards a homogeneous, isotropic and spatially flat geometry. Furthermore, inflation naturally provides an *initial seed* for structure formation [8–11] since due to quantum fluctuations inflation ends at different places in the universe at different times.

Similarly, in the SM, if EWSB is described by the Higgs mechanism, perturbative unitarity of the scattering of the longitudinal modes of the W bosons implies an upper bound on the Higgs mass of roughly $m_h \lesssim \mathcal{O}(1\,\mathrm{TeV})$. However, the mass of an elementary scalar field is not protected from receiving large quantum corrections of the order of the cutoff scale of the effective field theory. Assuming the SM to be valid up to the Planck-scale, one would need an incredible cancellation between a *classical* versus a *quantum* contribution to a very high precision – this is often called the *hierarchy problem.*

Solving the hierarchy problem is (one of) the most important motivation(s) for introducing new physics beyond the standard model. The most popular solution is low-energy *supersymmetry* (SUSY). It protects the Higgs mass from receiving large quantum corrections by a cancellation between loop contributions from bosons and fermions above a scale $M_{\mathrm{SUSY}} \sim \mathcal{O}(\mathrm{TeV})$. Alternatively, one can lower the cutoff of the effective field theory. This can be achieved, for instance, by lowering the fundamental scale of gravity to $\mathcal{O}(\mathrm{TeV})$ assuming *extradimensions* which are either *large* [12–14] or *warped* [15].[1] Newton's constant G_N which we measure in four dimensions is then only an *effective* parameter and is weakened either due to a "dilution" by the large volume of the extradimensions or due to a "redshift" in the warped space-time.

In the light of these considerations, it is important to incorporate both, a way to realize inflation and a solution to the hierarchy problem into a common framework. A suitable framework is *supergravity* (SUGRA) which is obtained by promoting supersymmetry to a *local* symmetry. Ultimately, these ideas should be embedded into a more fundamental theory unifying gravity and quantum field theory – a theory of everything. At present, *string theory* is arguably the best candidate for such a theory. Moreover, at low energies where the finite size of the string cannot be resolved, string theory can be described by an effective supergravity theory. Thus, supergravity is a good interface between effective field theory and string theory approaches.

The aim of this thesis is to consider inflation from both a field theory and a string theory perspective, *i. e.* from a bottom-up and a top-down perspective. We now motivate the ideas underlying the effective field theory models which we will construct and why it is necessary to embed inflation into string theory.

[1] We do not consider alternative solutions such as walking [16–18] or extended technicolor [19, 20]. Via gauge/gravity duality, they may admit an interpretation in terms of warped extradimensions, see *e. g.* [21, 22].

Why Inflation in the Matter Sector?

Even though the paradigm of cosmic inflation fits very nicely with the observed CMB power spectrum [23], we have so far no idea about the "nature" of the mechanism behind inflation. In the standard *slow-roll inflation* approach, inflation is driven by a scalar condensate with *negative* pressure [24–27]. This is realized by a scalar field whose potential energy dominates over its kinetic energy. This scalar field, the *inflaton*, effectively acts as a "clock" telling us when inflation ends. Its quantum fluctuations are stretched out to macroscopic scales and can directly be related to the temperature fluctuations of the CMB.

But which particle is the inflaton, *i. e.* what are its quantum numbers and interactions? To get a handle on this problem, it is inevitable to consider inflation in a particle physics framework. It is particularly appealing to embed the inflaton into the matter sector. Then its interactions are not only constrained by cosmology but also by particle physics and astroparticle physics.

For instance, it is tempting to identify the inflaton with the Higgs scalar responsible for electroweak symmetry breaking. To match the observed amplitude of the CMB fluctuations we would need a quartic Higgs coupling $\lambda \sim 10^{-13}$ [28], which implies $m_h \sim 10^{-4}\,\mathrm{GeV}$. However, we have a lower bound from the Higgs searches at LEP: $m_h > 114.4\,\mathrm{GeV}$ (95% C.L.) [29].[2] Obviously, the SM Higgs boson is ruled out as the inflaton candidate and we need some new physics to account for inflation.[3]

An interesting class of inflation models are models of *hybrid inflation* [34], where inflation ends via a phase transition when the so-called *waterfall* fields acquire expectation values. Typically, matching the observed CMB fluctuations requires these expectation values to be rather close to the scale where the SM gauge couplings seem to (almost) unify. Thus, it is tantalizing to relate the phase transition at the end of hybrid inflation with the breaking of the gauge group of a *Grand Unified Theory* (GUT) down to the SM gauge group. In a similar spirit, one can relate the phase transition to the generation of masses for right-handed neutrino masses, which explain the small masses of the observed left-handed neutrinos in a seesaw scenario. This opens up the intriguing possibility to relate hybrid inflation with leptogenesis to generate the observed baryon asymmetry, which then puts constraints on the parameters in the game, see *e. g.* [35, 36].

With this motivation in mind, we will construct models of inflation where the inflaton resides in a *gauge non-singlet matter* field and we refer to this class of models as *matter inflation*.

[2] The LHC has already increased this lower bound somewhat (and found a Higgs-like boson with $m \sim 125\,\mathrm{GeV}$), but it is the order of magnitude which matters.

[3] It has been proposed recently that by assuming a non-minimal coupling to gravity the Higgs boson can act as the inflaton [30]. However, some criticism about the naturalness of this class of models has been raised in [31–33].

Why Inflation in String Theory?

To sustain prolonged slow-roll inflation, we must have a "sufficiently *flat*" potential, which is measured by the *smallness* of the *slow-roll parameters*. One of them, the *η-parameter*, can be viewed as the "mass" of the inflaton m_ϕ measured in units of the *expansion rate* $\mathcal{H}$ of the universe. Slow-roll inflation requires $\eta \sim m_\phi^2/\mathcal{H}^2 \ll 1$. However, in the absence of any symmetry protecting it, m_ϕ is driven towards the cut-off of the Effective Field Theory (EFT). That is, a small inflaton mass $m_\phi \ll \mathcal{H}$ is *radiatively unstable* and receives large quantum corrections – this is often called the *η-problem*.

Here, the situation is very similar to the hierarchy problem for the Higgs – both problems occur since the mass of an elementary scalar is sensitive to the UV completion of the EFT – and one might hope that we can also solve it by using supersymmetry. However, during inflation supersymmetry must be broken. Generically, this induces masses for all the scalars of the order $m \sim \mathcal{H}$, in particular, also for the inflaton, thereby posing a serious threat for inflation commonly referred to as the *supergravity η-problem* [37–39]. Solutions within effective field theory either require some tuning among various (model-dependent) contributions or imposing a global symmetry.

In technical terms, the crucial point is that through the η-problem inflation is sensitive to *Planck-suppressed* dimension-6 operators. This is remarkable since such operators are typically irrelevant for most particle physics considerations (one exception being gravity-mediated supersymmetry breaking). In other words, inflation allows us to probe aspects of the *UV completion* which unifies quantum field theory and gravity. String theory, being a candidate for such a UV completion, is therefore a natural playground for building models of inflation. In particular, one would like to check whether field theory solutions for the η-problem can be realized or not.

In many attempts to realize inflation in string theory, one encounters a version of the η-problem which is intrinsically related to the issue of *moduli stabilization*. Moduli are light scalar fields controlling the expansion parameters of the four-dimensional effective field theory. The physics responsible for generating a potential for the moduli then induces potentially large corrections to η (see for instance [40–42]). Moreover, the very presence of an inflationary sector may even *destabilize* the moduli, *i. e.* destroy the minimum of the effective moduli potential [43–45].

Despite (or because of) the above problems, inflation offers a rather unique window to gain insights into string theory or more generally quantum gravity. As advocated earlier, in this dissertation we will employ both bottom-up and top-down approaches to address these issues. That is, we consider both string-inspired effective field theory models and models obtained from genuine string theory compactifications.

Chapter 2

Introduction

Keeping in mind the motivation discussed above, we now move on to explain some of the underlying ideas and the broader context of the work presented in this dissertation. In particular, we will try to put more meaning into some of the key words which popped up in the previous chapter. In doing so, we intend to be as less technical as possible.

We begin with a short introduction into the aspects of string theory relevant for this thesis in Sec. 2.1. Namely, that the low-energy limit of string theory is described by a *supergravity* theory in *ten dimensions*, the presence of *branes* and *fluxes* and the *AdS/CFT-correspondence*. Moreover, the need for compactification to four dimensions has profound implications for the four-dimensional effective action, in particular, the presence of *moduli* which have to be stabilized.

Next, we give try to motivate why a phase of inflation should be introduced at all in Sec. 2.2. That is, we state what the horizon and flatness problems are and explain how inflation solves them and how it provides a seed for the formation of structure.

In Sec. 2.3, we begin by revisiting the η-problem in a bit more detail and discuss its possible solutions in effective (supergravity) theories. The idea of this approach is to view four-dimensional effective supergravity theories as an interface to a more fundamental theory of quantum gravity which allows us to parametrize the effects of Planck-scale physics. Then one searches for viable and phenomenologically interesting models of inflation in supergravity setups whose structure is inspired, for instance, by string theory.

Afterwards, we explain the motivations for considering inflation in string theory in more detail and outline the major problems one faces in Sec. 2.4. We will be particularly interested in models of *warped brane inflation* which are among the best-understood examples of inflation in string theory.

Finally, in Sec. 2.5, we give an outline of the structure of this thesis.

2.1 A Short Introduction to String Theory

String theory aims at being a fundamental theory of nature capable of unifying quantum field theory and gravity into a single framework. We have argued above why inflation probes aspects of the UV completion of the effective field theory by a theory of quantum gravity. As a candidate for such a theory, we choose string theory. Given the limited amount of space, we try to be as brief as possible and refer to the given references for more detailed explanations and, in particular, to the textbooks [46–52].[1] For example, we will not discuss the quantization of the string at all since we are interested in the effective supergravity description.

2.1.1 Supergravity as the Low-Energy Limit of String Theory

In field theory, the fundamental excitations are those of point-like objects (aka particles) whose action is governed by the length of their world-lines. In string theory, however, the fundamental object is a one-dimensional object – a *string* – whose action is then governed by the volume of its *world-sheet*, *i. e.* by the surface in space-time swept out by the moving string. The theory on the world-sheet is a *two-dimensional Conformal Field Theory* (CFT) (cf. *e. g.* [62–65]) and the field content includes, in particular, the fields describing the embedding of the string world-sheet into space-time.

Consistency of the theory, namely avoiding *anomalies*[2] of the world-sheet theory and the absence of *tachyons* in the spectrum, forces us to consider supersymmetric strings – *superstrings* – living in *ten space-time dimensions* (see *e. g.* [46–52]). As it turns out, there are five distinct ways to formulate superstring theory in ten dimensions involving *closed* and sometimes *open* strings. The *type II* string theories involve closed strings and can have either a chiral (*type IIB*) or non-chiral (*type IIA*) spectrum [67, 68]. The *type I* string theory is a theory of closed and open strings since their world-sheets are required to be unoriented [69–71]. Finally, there are the two *heterotic* string theories. These are theories of closed strings which can be *charged* under either an $SO(32)$ or $E_8 \times E_8$ gauge group [72–74].

The only dimensionful parameter in string theory is the *length* of the strings ℓ_s which, in particular, sets the scale for the massive string excitations that have masses at least of the order of ℓ_s^{-1}. Experimentally, we have a lower bound on

[1] Unfortunately, we have to leave out many interesting research areas in string theory to which we could not do justice here. This includes topics such as M-theory [53–56], F-theory [57], understanding black hole entropy [58], the string landscape [59] or cosmic strings [60,61].

[2] The word "anomaly" refers to the fact that a symmetry of the classical theory need not be a symmetry of the full quantum theory. If this happens to a local symmetry, the theory is *inconsistent* (cf. *e. g.* [66]).

the string scale $\ell_s^{-1} \gtrsim 4.00\,\text{TeV}$ (95% C.L.) from searches at the LHC [75]. But in principle, the string scale ℓ_s^{-1} can be anywhere from this bound up to the Planck-scale $\sim 10^{19}\,\text{GeV}$.

If we are looking for a low-energy description valid for energies $E \ll \ell_s^{-1}$, *i. e.* at energies where we cannot resolve the finite size of the string anymore, we have to keep only the *massless* modes. The standard logic of effective field theory dictates that we should integrate out all fields which are heavier than a specified mass scale (see *e. g.* [76–79]). The effective theory is then obtained as a series in $E\ell_s \ll 1$. But to get the *lowest* order terms in the effective field theory for $E \ll \ell_s^{-1}$, the following argument provides a shortcut. The low-energy description of the five string theories must preserve the two conditions required for consistency – ten space-time dimensions and supersymmetry – and include the graviton in its spectrum. Thus, the low-energy limit of string theory is ten-dimensional *Supergravity* (SUGRA) and there are exactly five such supergravity theories, one for each formulation of string theory [80–86]. To get the higher order corrections one has to work much harder. For instance, one strategy to obtain the low-energy effective action would be to reconstruct it from the S-matrix of string scattering amplitudes expanded in $E\ell_s \ll 1$ (see *e. g.* [87, 88] for the heterotic case[3]).

Finally, two conceptually important comments are in order. First, since all of the five string theories contain a closed string sector they also contain a (massless) spin-2 state which acts as the graviton. This is why string theory is considered as a candidate theory of quantum gravity [73, 74, 80, 97, 98].

Second, string theory is intrinsically defined as a *perturbative* expansion in terms of the *string coupling* g_s, which counts the number of "handles" of the world-sheet (*i. e.* it is related to the *topology* of the world-sheet). The validity of such an expansion requires g_s to be small, but this is a non-trivial constraint since the value of g_s is dynamically determined by the theory (cf. *e. g.* [46–52]). The low-energy supergravity limit is then actually a *double* perturbative expansion in small g_s and ℓ_s. The string length ℓ_s controls the loop expansion of the fields living on the string world-sheets whose different topologies count the powers of g_s.

2.1.2 Branes, Fluxes and the AdS/CFT-Correspondence

Branes

In addition to the one-dimensional strings, string theory contains also higher-dimensional objects, so-called *Dp-branes*, on which *open strings* end [99–101] (see also [102–104]). The name stems from the fact that the strings ending on such a surface obey Dirichlet boundary conditions, *i. e.* they are stuck to

[3]The same strategy can also be applied to four-dimensional string compactifications, see for instance [89–96] for both, heterotic and type II string theory.

this surface and may only move along it but not away from it. The number p counts the number of *spatial* dimensions of these objects, *i. e.* a Dp-brane is a $(p+1)$-dimensional object. These objects are present for both type I and type II theories. There are also similar objects, so-called NS5-branes, which are present in all types of string theory and related to closed strings [105].

D-branes have become a basic ingredient of modern string theory compactifications, in particular, because they introduce *non-Abelian* gauge symmetries into the type II theories. That is, a D-brane has a non-Abelian gauge theory living on its world-volume originating from the quantization of the open strings which end on it. Typically, one obtains a $U(N)$ gauge group for a stack of N D-branes all on top of each other, see *e. g.* [102–104].

Fluxes

In addition to the graviton and the dilaton, the bosonic field content of all of the ten-dimensional supergravity theories contains *p-form field strengths* with $(p-1)$-form potentials. All supergravity theories contain the *Neveu-Schwarz* (NS) 2-form field B_2 with field strength H_3, but type IIA and IIB supergravity contain also the *Ramond-Ramond* (RR) forms C_{p-1} with field strengths F_p. The type IIA theory contains all the even p's while the type IIB theory contains all the odd p's. These p-form field strengths are generalizations of the electromagnetic field strength F_2 with a gauge field potential A_1, just with more indices to antisymmetrize.

We can switch on non-trivial background values for these field strengths, so-called *fluxes*. Branes act as *sources* for fluxes, but fluxes can exist also in the absence of any sources. Moreover, the fluxes turn out to be quantized [106–108], *i. e.* the integral of F_p over a p-dimensional compact surface is quantized. Just like D-branes, fluxes have become a standard ingredient of modern string theory and we will explain soon why fluxes are particularly important for the issue of moduli stabilization.

The AdS/CFT-Correspondence

D-branes are also the cornerstone of a recent important development in string theory, the *AdS/CFT-correspondence* [109–112]. By looking at two different ways to describe a stack of N_c D3-branes in ten-dimensional flat space one arrives at a surprising statement. Namely that two seemingly unrelated theories, $\mathcal{N}=4$ $SU(N_c)$ Super-Yang-Mills (SYM) theory in four dimensions and type IIB string theory on $\mathrm{AdS}_5 \times S^5$, are *dual* to each other, *i. e.* that they describe exactly the *same* physics. One can make this equivalence precise in the sense that there exists a dictionary (a one-to-one map) between the correlation functions computed in one theory and those computed in the other.

The idea of the AdS/CFT-correspondence is believed to be valid in more general examples. This goes under the name of *gauge/gravity duality*, cf. *e. g.* [113–115] and D-branes are useful to construct explicit examples.

The AdS/CFT-correspondence has triggered an enormous amount of work using it as a tool to understand gauge theory dynamics at *strong coupling*. By "strong coupling" we actually mean the limit of large *'t Hooft coupling* $\lambda \equiv g^2 N_c$ and large number of colors N_c [116]. On the gauge theory side, this corresponds to a limit where only *planar* diagrams contribute, while in the dual gravity theory this corresponds to a limit where only *weakly-coupled classical* (super-)gravity is taken into account. The converse is also true – when the gauge theory is weakly-coupled, the dual gravity theory is strongly-coupled. This very remarkable fact lies at the heart of the success of the AdS/CFT-correspondence.

Let us very briefly mention a few important results and applications and refer to the given literature for details and a more extensive list of references. Perhaps the most striking result obtained from gauge/gravity duality is that the ratio of the shear viscosity to the entropy density of a fluid takes a *universal* value $1/(4\pi)$ in the strong coupling limit [117–122]. One of the most important applications of the AdS/CFT-correspondence is to QCD and the Quark Gluon Plasma (see *e. g.* [123–126]). It has also helped to gain insight into scattering amplitudes in $\mathcal{N} = 4$ SYM (see *e. g.* [127–129]) and models of strongly-coupled electroweak symmetry breaking (see *e. g.* [21, 22]). Very recently gauge/gravity duality was also applied to condensed matter systems [130–134].

In this thesis, we will make use of the AdS/CFT-correspondence in the context of inflation. We consider inflation in scenarios which are a generalization of the original AdS/CFT-correspondence. It involves backgrounds which are *confining* in the IR, the Klebanov-Strassler solution [135]. This is a supergravity solution dual to an $\mathcal{N} = 1$ $SU(N+M) \times SU(N)$ gauge theory in four dimensions. We will be concerned with the supergravity solutions [136] dual to a state where the gauge theory is on the *baryonic branch*, *i. e.* to a state where baryonic operators acquire non-zero expectation values. More precisely, we will consider deformations of these supergravity solutions which are dual to a one parameter family of deformations of the baryonic branch [137]. We add a probe D3-brane to these backgrounds and study the induced potential.

2.1.3 Compactification and Moduli Stabilization

String theory "likes" to live in ten space-time dimensions which in particular implies that gravity should live in ten-dimensions. But experiments measuring deviations of the gravitational force from Newton's law tell us that we live in *four* space-time dimensions and put an upper bound on the size of any other spatial dimensions of the order $\lesssim \mathcal{O}(40\,\mu m)$ [138, 139].[4] In other words, we need to find an explanation why gravity "looks" four-dimensional even though it fundamentally is not. To achieve this, six out of the nine spatial dimensions of string theory have to be coiled up into something small – this process is called *compactification*.[5]

Let us denote the "typical" length scale of the compact extradimensions by L. We can obtain an *effective* theory in four dimensions which is valid for energies $E \ll L^{-1}$ by performing a "dimensional reduction" (or Kaluza-Klein (KK) reduction [142, 143]) from ten down to four dimensions. Dimensional reduction is essentially a sort of a generalized Fourier expansion. Since the extradimensions are compact, the momenta of particles along them are quantized. From the four-dimensional point of view, this corresponds to an extra contribution to the mass of the particle (as measured in four dimensions). For each field one obtains a set of so-called *KK modes* which can be arranged into "towers" with increasing masses. The mass of the lightest KK mode is of the order of L^{-1}. This step is crucial to make contact with our four-dimensional world and to understand the low-energy consequences of string theory. Note that the resulting effective field theory in four dimensions is valid only for energies $E \ll L^{-1} \ll \ell_s^{-1}$. Nonetheless, the compactification is much easier described in the effective ten-dimensional supergravity theory than in the world-sheet theory.

There are many ways to compactify six dimensions and thus this is not an unambiguous process. That is, the low-energy limit of string theory is *not* a unique four-dimensional effective field theory. More importantly, the *size* and *shape* of the compact space are a priori undetermined – they are controlled by the expectation values of four-dimensional scalar fields, so-called *moduli*. The overall volume of the compact space is always a modulus. For instance, in the type IIB theory, the four-dimensional Planck-mass M_P is determined by the volume $\mathcal{V}_6$ of the compact space as[6]

$$M_P^2 \sim \frac{\mathcal{V}_6}{g_s^2\,\ell_s^8}\,. \tag{2.1}$$

The Planck-mass M_P is fixed to be $\approx 2 \times 10^{18}$ GeV since we have measured the strength of four-dimensional gravity, but the quantities on the right-hand side are practically unconstrained.

Similarly, the moduli control also the size of other couplings in the low-energy effective action such as gauge or Yukawa couplings. Hence, it is of utmost

[4]There are also bounds on extradimensions from *e. g.* collider physics [139]. However, only gravity *must necessarily* propagate in all space-time dimensions – particles may live on lower-dimensional *branes*. This is precisely how the hierarchy problem is solved in [12–14] with large extradimensions.

[5]For an alternative suggestion using strongly-warped extradimensions see [140], which can be realized in string theory along the lines of [141].

[6]The (reduced) Planck-mass M_P is related to the Newton constant as $M_P^2 = (8\pi G_N)^{-1}$.

importance to understand the potential for the moduli and, in particular, find minima within the domain of validity of the effective theory. That is, we have to understand the *"stabilization" of moduli*.

Unfortunately, the potential for the moduli obtained from just invoking compactification is typically *flat*, *i. e.* the expectation values of the moduli are completely undetermined since it does not cost any energy to change their field values. That is, the moduli correspond to *massless* scalar fields in four dimensions. However, massless scalar fields would mediate so-called *fifth forces* which are excluded by experiments [144].

On top of these problems, there is also the *cosmological moduli problem* [145–151]. Light moduli can cause a couple of severe cosmological problems. For instance, late decays of moduli can spoil the successful predictions of BBN for the production of light elements or lead to an overproduction of gravitinos (the superpartner of the graviton). In practice, this means that they typically have to be heavier than about $\mathcal{O}(30\,\mathrm{TeV})$. Thus, the cosmological moduli problem puts constraints on the minima of the moduli potential, but of course we need to find some minima in the first place.

There has been tremendous progress to identify mechanisms for moduli stabilization, for instance the presence of non-trivial *fluxes* [152–155], *perturbative corrections* [156,157] and *non-perturbative effects* [158]. The two currently best-understood scenarios are the *"KKLT scenario"* [40] and the *"LARGE Volume Scenario"* (LVS) [159] in the context of "type IIB orientifold compactifications with fluxes".[7]

To put some more meaning into these catchphrases, let us try to understand qualitatively what happens. What we need to do to stabilize the moduli is "associate an energy cost to changing their field value". In the case of fluxes (of which one should think as "magnetic fields" along the internal (compact) directions), the energy density stored in the flux depends on the values of the moduli and thus deforming the compact space costs energy.[8] Stabilization using perturbative effects makes use of loop corrections involving massive particles whose masses depend on the values of the moduli (*e. g.* KK modes with masses depending on the geometry of the extradimensions), while non-perturbative (instanton) corrections $\propto e^{-\#/g^2}$ use the dependence of the gauge couplings g on the moduli. In all three cases one generates a non-trivial dependence of the effective potential on the moduli.

Generically, the presence of fluxes strongly *warps* the compactification manifold, which introduces a dependence of the four-dimensional metric on the

[7]For reviews and many more references see [96, 160–163] (see also Chap. 6) and for a systematic study of examples cf. [164,165]. For recent attempts in heterotic compactifications (without and with fluxes) see *e. g.* [166–170].

[8]One may think of a sphere whose radius R we interpret as a modulus and put a magnetic field on the surface of the sphere. The energy density stored in the magnetic field depends on R.

position in the extradimensions characterized by a *warp factor*, see *e. g.* [96, 160–163]. In type IIB, there are certain situations in which this *backreaction* is rather "mild" in the sense that the metric of the compact directions changes also only by introducing an overall warp factor.

To conclude this short introduction to string compactifications, we would like to very briefly comment on the status of two important aspects of string phenomenology which are also connected to the issue of moduli stabilization. Namely, the construction of potentially realistic low-energy field theories and the string theory version of solutions to the hierarchy problem. Both of which are important to make contact with low-energy particle physics. The ambition of string theory is to unify the concepts of gravity (in the form of general relativity) with quantum field theory (in the form of the SM), *i. e.* to provide a *UV completion* of the SM.

Constructing MSSM-like Models One of the major goals of string phenomenology is to find compactifications whose low-energy spectrum is as close as possible to the spectrum of the Minimally Supersymmetric Standard Model (MSSM), see *e. g.* [171]. For instance, we would like to find the low-energy gauge group of the SM, $SU(3)_C \times SU(2)_L \times U(1)_Y$, or a gauge group such as $SU(5)$ or $SO(10)$ which contains the SM gauge group if one pursues the idea of grand unification. In addition, there should be matter fields in the appropriate representations to accommodate quarks and leptons as well as the Higgs.

Even though one starts with supergravity in ten dimensions, requiring that one preserves some amount of supersymmetry in four dimensions in the process of compactifications places some constraints on the structure of the internal space, see *e. g.* [98, 172–174]. A class of six-dimensional spaces which fulfills these constraints are so-called *Calabi-Yau* (CY) 3-folds (see *e. g.* [175, 176]) and for this reason they have become a standard ingredient in string compactifications.

There has been a great amount of progress in finding potentially realistic models, especially in the last decade. For our purposes, it is important that it is quite plausible that something similar to the MSSM can arise from string theory (recall that one of our motivations is to realize inflation in the matter sector). For some examples in the heterotic case (with only the MSSM spectrum) see [177–187] and for the case of type II theories cf. the reviews [95, 96, 188–190] and the references therein.[9]

Solutions to the Hierarchy Problem Assume we find some realization of the SM in string theory. Since string theory aims at being a theory of quantum gravity, it should have some answer to the hierarchy problem.

[9] For an estimate of the probability to find MSSM-like models with intersecting D-branes in toroidal orientifolds see [191] and for MSSM-like GUT models in the context of F-theory see [190, 192, 193].

As it turns out, the subject of moduli stabilization is intimately connected to a string theory solution of the hierarchy problem. We can in principle find string theory versions of all the aforementioned solutions.

- **Low-Energy Supersymmetry**: Supersymmetry solves the hierarchy problem by cancellations between the loops involving fermions and bosons *if* it is broken at low enough energies $\sim \mathcal{O}(\text{TeV})$. Moduli stabilization generically requires spontaneous supersymmetry breaking and thus the moduli comprise a hidden sector. How this breakdown of supersymmetry is mediated to the visible sector depends on many details, but gravity mediation often contributes significantly. For studies in the KKLT scenario see *e. g.* [194–199] and for recent studies in the context of heterotic orbifolds see *e. g.* [166, 200].
- **Large Extradimensions**: In the ADD scenario of large extradimensions [12–14] four-dimensional gravity appears weak compared to the other forces but only because it is "diluted" in the large volume of the extradimensions. A very large volume of the extradimensions is what gives the LARGE Volume Scenario [159] its name and by exploiting Eq. (2.1) we can lower the string scale to $\ell_s^{-1} \sim \mathcal{O}(\text{TeV})$. This would have fascinating implications such as the possibility to discover strings at the LHC [201–208].
- **Warped Extradimensions**: In the RS scenario [15], one solves the hierarchy problem via "gravitational redshift" using a strongly warped extradimension. A string theory version of this scenario has emerged from compactifications with fluxes [155] involving a *warped throat* [135]. Sufficiently strong warping seems possible on statistical grounds [209].

Thus, string theory does in principle contain the ingredients necessary to find the SM and to solve the hierarchy problem. Of course putting everything together, *i. e.* combining a realization of the MSSM and a solution to the hierarchy problem within a fully stabilized compactification is very challenging.[10] So far, no fully realistic, explicit and compelling model with a realistic low-energy spectrum, a viable phenomenology and cosmological evolution and so on has been constructed. But one has found a variety of promising mechanisms by which moduli can be stabilized and ways to realize the MSSM spectrum. Hence, there is hope to find realistic compactifications.

2.2 Motivation for Inflation

Cosmic inflation [4–7] is a *paradigm* solving many problems related to the standard *hot big bang cosmology* by assuming a phase of rapid accelerated ex-

[10] Also sometimes some tension may arise, for instance, between moduli stabilization and chirality in the type IIB context [210] (but see [211]).

pansion in the very early universe. Specifically, it addresses the *horizon problem* and the *flatness problem*. In addition, it provides a *seed for structure formation* via quantum fluctuations of the so-called inflaton field. Let us now review these problems and their solution by inflation one by one.[11]

The Horizon Problem

On very large scales, the universe is well-approximated by a *homogeneous* and *isotropic* space. Therefore, the space-time is characterized by a *scale factor* $a(t)$ which encodes the relative size of space-like slices at fixed times and thus encodes the *expansion* of the universe. The characteristic scale of these space-times is set by the *expansion rate* (or *Hubble scale*)

$$\mathcal{H} \equiv \frac{\dot{a}}{a} \,. \tag{2.2}$$

The time-evolution of the expanding space-time is determined by the Einstein equations from the *energy density* $\rho(t)$ stored in all forms matter. In the hot big bang scenario, at early times the universe is dominated by *radiation* (*i. e. relativistic* particles) and then undergoes a transition to a universe dominated by *non-relativistic matter*.[12] Without going into the technical details, this implies a time-dependence $a(t) \sim t^{\alpha}$ for some $\alpha < 1$ and therefore $\mathcal{H}(t) \sim t^{-1}$ always *decreases* with time.

In other words, for radiation and non-relativistic matter, gravity always *decelerates* the expansion, $\ddot{a} < 0$. Consequently, the size of the *particle horizon* is determined by $d(t) \sim \mathcal{H}^{-1} \sim t$ which *increases* with time.

Today, at t_0, a patch of the size $d(t_0) \sim t_0$ must be homogeneous and isotropic. Now going back in time we encounter a serious problem. Namely, our entire observable universe today seems to originate from many causally disconnected regions, the number of which keeps increasing the further we go back in time. This is because in a decelerating universe the patch of size $d(t_0)$ is *scaled down* while the maximal size of a causal region is at most $\ell_c \sim t$ at any given time since this is the maximal distance light can travel. Therefore, the observed homogeneity and isotropy of our universe would require a huge amount of fine-tuning of the initial conditions – this is the *horizon problem.*

Inflation being a phase of *accelerated* expansion, $\ddot{a} > 0$, solves this problem since the particle horizon is forced to *shrink* during this phase. As a consequence, the universe could have been created out of a single causal patch if a sufficiently long phase of inflation took place in the very early universe.

[11] For technical details and further explanations see Chap. 3 and [24–27].

[12] We have evidence [23, 212] that there has been yet another transition to a universe dominated by "dark energy" or a "cosmological constant". However, this phase is irrelevant for the sake of the arguments in this section.

Now what does "inflation lasted sufficiently long" mean? Quantitatively, one expresses this in the *number of e-folds* N_e which is defined by

$$N_e(t) \equiv \ln \frac{a_{\text{end}}}{a(t)} = \int_t^{t_{\text{end}}} \mathcal{H}(t)\, dt\,. \tag{2.3}$$

"Sufficiently long inflation" typically means that inflation should have lasted long enough to produce roughly $N_e \gtrsim 60$. Hence, this corresponds to a relative growth of the universe by at least a factor of $e^{60} \sim 10^{26}$!

The Flatness Problem

The second problem of the hot big bang cosmology is that the universe appears to be *spatially flat* on large scales. This again translates into a large fine-tuning of initial conditions. Let us try to understand why.

Defining a *"critical" energy density*[13] $\rho_{\text{crit}} \equiv 3\mathcal{H}^2 M_P^2$ and the ratio of the total energy density $\rho(t)$ to the critical energy density ρ_{crit}, $\Omega \equiv \rho/\rho_{\text{crit}}$, one can show that the following equation must hold:

$$\Omega(t) - 1 = \frac{k}{(a\mathcal{H})^2}\,. \tag{2.4}$$

Here, k is a measure of the spatial curvature of space-like slices.

In the hot big bang evolution one has $(a\mathcal{H})^{-1}$ *increasing* with time. That is, the quantity $|\Omega - 1|$ must *diverge* with time and thus $\Omega = 1$ corresponds to an *unstable* fixed point. That is, even if we start very close to $\Omega = 1$ at early times, the time evolution drives us away from this value at later times. To have a nearly flat universe such as the one we observe today, *i. e.* a universe which has $\Omega(t_0) \sim 1$, would therefore require a huge amount of fine-tuning of the initial condition for Ω at early times. This issue is commonly referred to as the *flatness problem.*

Inflation solves this problem *automatically* since for an accelerated expansion the quantity $(a\mathcal{H})^{-1}$ *shrinks* with time. Hence, the system is *dynamically driven towards* a spatially flat universe with $\Omega \approx 1$ at late times. This can be understood also by considering the length scale at which spatial curvature becomes important, the *physical* curvature scale $R_{\text{phys}}(t) = a(t)|k|^{-1/2}$. Increasing the scale factor $a(t)$ by a large amount during inflation also increases R_{phys} by a large amount. Hence, the universe can become practically flat today.

[13] From Eq. (2.4) one can see that the critical energy density essentially measures the energy density which corresponds to a spatially flat universe ($k = 0$) at a given time t.

A Seed for Structure Formation

Gravitational collapse of small initial inhomogeneities is a nice explanation how all the structure we observe in the universe, *i. e.* all the stars, galaxies and galaxy clusters, could have formed. However, this explanation requires an initial *seed* for structure formation. To account for the structure we observe today, the initial inhomogeneities $\delta\rho$ of the energy density on galactic scales must have had a *density contrast* $\delta\rho/\rho \sim 10^{-5}$.

The horizon and flatness problems are solved by inflation automatically by the accelerated expansion. Does inflation provide also an initial seed for structure formation? As it turns out it does: inflation has a particularly appealing mechanism to address this issue by *quantum fluctuations* [8–11]. The accelerated expansion then stretches these small-scale quantum fluctuations to macroscopic scales. This picture lies at the heart of the success of cosmic inflation. One can perform a detailed calculation and *predict* the statistical properties of the initial perturbations which are in very nice agreement with the data from observations of the CMB and of the large-scale structure of our universe [23].

An interesting signature of inflation are *gravitational waves*. Similar to the spin-0 inflaton fluctuations which translate into density perturbations, one can produce spin-2 fluctuations which correspond to gravitational waves. The PLANCK satellite is an experiment searching for a signature of gravitational waves in the CMB.

2.3 Inflation in Supergravity

Inflation is a *paradigm*, not a concrete theory, and thus there are many different models in which a phase of accelerated expansion can arise. The standard approach to building a model of inflation is *slow-roll inflation*, where a scalar field ϕ slowly rolls down its potential, *i. e.* the potential energy dominates over the kinetic energy, $V(\phi) \gg \dot{\phi}^2$. Virtually *any* potential which fulfills the conditions given below in Eq. (2.5) will do the job.

Roughly, we can classify slow-roll inflation models into *small-field* and *large-field* models according to whether during inflation ϕ travels over a distance $\Delta\phi \ll M_P$ or $\Delta\phi \gg M_P$ in field space, respectively. This essentially is a distinction by whether the model will produce observable gravitational waves or not [213]. The prototypical examples for models of inflation are (large-field) *chaotic inflation* [214] and (small-field) *hybrid inflation* [34].

2.3.1 The Eta-Problem

Frustratingly, inflation is very effective in washing out signatures of high-energy physics (*e. g.* relics from a GUT-scale phase transition). But the η-problem does provide us with some (indirect) information about physics at very high energies. Therefore, let us now discuss the η-problem in a bit more detail from an effective field theory point of view.

To sustain inflation over a "sufficiently long" period of time, *i. e.* sufficient to solve the horizon and flatness problems, the potential must be very flat. The flatness is measured in terms of the slow-roll parameter ϵ and η, $\epsilon, |\eta| \ll 1$, which are obtained from the derivatives of V as

$$\epsilon \equiv \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2 , \quad \eta \equiv M_P^2 \frac{V''}{V} \approx \frac{m_\phi^2}{3\mathcal{H}^2} , \tag{2.5}$$

where $\mathcal{H}^2 \approx V/(3M_P^2)$ is the expansion rate of the universe.

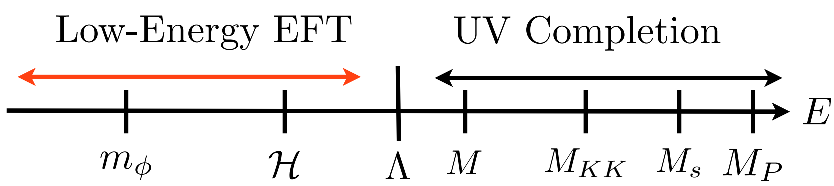

Figure 2.1: Hierarchy of energy scales in the effective field theory of inflation.

Now suppose we have an effective field theory (EFT) for inflation with a cut-off scale Λ determined by the mass of the lightest field which is not part of the low-energy spectrum and has been integrated out (see Fig. 2.1). Such an approach requires a hierarchy of energy scales $m_\phi \ll \mathcal{H} \lesssim \Lambda$. However, in the absence of any symmetry protecting it, the mass of any elementary scalar is driven towards the cut-off. That is, a small inflaton mass $m_\phi \ll \mathcal{H}$ is *radiatively unstable* and quantum corrections generically yield a large correction. Thus,

$$\Delta\eta = \frac{m_\phi^2}{3\,\mathcal{H}^2} \gtrsim 1 , \tag{2.6}$$

because we need the cut-off to be at least $\Lambda \gtrsim \mathcal{H}$.

One example for degrees of freedom we have integrated out are those which are relevant to render graviton-graviton scattering meaningful at very high energies. From the point of view of the low-energy effective field theory, we can parametrize their impact by adding higher-dimensional operators which are suppressed by the Planck-scale M_P. Now the flatness of the potential is particularly sensitive to *Planck-suppressed dimension-6* operators of the form[14]

$$\frac{\mathcal{O}_6}{M_P^2} = \frac{\mathcal{O}_4}{M_P^2}\phi^2 \,. \tag{2.7}$$

If $\langle \mathcal{O}_4 \rangle \sim V$, such terms yield dangerous contributions $\Delta\eta \sim 1$.

For the other slow-roll parameter, ϵ, the situation is less problematic. The largest contributions would arise from dimension-5 operators of the form

$$\frac{\mathcal{O}_5}{M_P} = \frac{\mathcal{O}_4}{M_P}\phi \,, \tag{2.8}$$

but such operators can be forbidden by a discrete $\mathbb{Z}_2$ symmetry which renders it somewhat less problematic. This is *not* possible for the contributions of the form in Eq. (2.7).

The phrasing of the η-problem with Planck-suppressed dimension-6 operators is *precisely* what happens in the context of supergravity models of inflation. During inflation supersymmetry must be broken, the amount of which is determined by the vacuum energy driving inflation, $\langle F_X^2 \rangle = 3M_P^2\mathcal{H}^2 = V$. Generically (aka using gravity mediation), this induces masses for the scalars of the order

$$m \sim \frac{\langle F_X \rangle}{M_P} \sim \mathcal{H} \,. \tag{2.9}$$

This is the essence of the *supergravity η-problem* [37–39] where a contribution of this form induces also an inflaton mass $m_\phi \sim \mathcal{H}$, thereby posing a serious threat to inflation.

2.3.2 Solutions to the Eta-Problem in Supergravity

Because of the η-problem described above, inflation in supergravity requires either some tuning of the parameters, an accidental cancellation or an additional (approximate) global symmetry protecting the inflaton. Actually, this is true for all models of inflation since the above reasoning applies to any model of slow-roll inflation. But supersymmetric models of inflation do have some advantages. For example, the required tuning tends to be somewhat less severe than in a generic effective field theory since supersymmetry still takes care of some part of the dangerous quantum corrections. Moreover, supersymmetry often helps to ensure a "technically natural" [215] effective action for the inflaton (see *e. g.* [216, 217]). The discussion below is presented with supergravity models in mind, but actually it essentially carries over straightforwardly to non-supersymmetric models of inflation.

[14] This is in full analogy to the relation between Fermi's theory of β-decay an the electroweak theory with W-bosons. In the limit $E \ll M_W$, we can integrate out the W-bosons and obtain an effective theory with four fermion operators of the form $\bar{\psi}\psi\bar{\psi}\psi/M_W^2$. To render this theory meaningful at energies $E \sim M_W$ we have to introduce the W-bosons.

Accidental Cancellations and Tuning of Parameters

In some circumstances, there can be accidental cancellations between model-dependent terms in the supergravity potential. For instance, this is what happens in the standard realization of F-term hybrid inflation in supergravity [218–220]. However, this cancellation only works if some assumptions about higher order corrections of precisely the form in Eq. (2.7) are made. For models of inflation where the field values are always below M_P, which is the case in hybrid inflation, one can perform an expansion in ϕ_i/M_P. In this way, one can quantify the required tuning of the parameters, see for instance [221–224].

Approximate Global Symmetries

By far the most popular way to get rid of the η-problem in field theory is to assume the existence of a global symmetry which protects the inflaton direction. The most prominent example is a continuous global *shift symmetry* under which the inflaton ϕ transforms as $\phi \to \phi + \alpha$ and which is only weakly broken by the potential $V(\phi)$. This type of symmetry is powerful enough to forbid dangerous terms of the form Eq. (2.7). Note that for models of chaotic inflation where $\Delta\phi \gg M_P$ a symmetry is the only option to render the inflaton potential "technically natural" since a tuning of coefficients in an expansion is impossible. There are many models of supergravity inflation with a shift symmetry on the market belonging either to the chaotic or the hybrid case, for some examples see *e. g.* [35, 216, 225–232].

2.4 Inflation in String Theory

Unfortunately, the above solutions to the η-problem are not the end of the story. The reason is that essentially both ways "solve" the problem somewhat by *assumption*. In the case of the fine-tuning of parameters it is by no means clear that the favorable choices of parameters are consistent with a UV completion of the theory by quantum gravity.

In the case of symmetry solutions to the η-problem, the situation is a bit more subtle. Proposing a shift symmetry for the inflaton is basically equivalent to identifying the inflaton with the *Pseudo-Nambu-Goldstone-Boson* (PNGB) of a spontaneously broken global $U(1)$ symmetry. The conceptual problem with this approach is that on fairly general grounds one can argue that a "generic" theory of quantum gravity does *not* respect *any* continuous global symmetries [233–238]. Thus, using a symmetry to forbid operators of the form in Eq. (2.7) places strong constraints on a UV completion of such a model. For instance, models with a controlled shift symmetry even for large field values have been found only recently in string theory [239–241].[15]

Since string theory is currently the best-developed candidate for a theory of quantum gravity, it is of course interesting to look for models of inflation in string theory. It is in principle possible to compute corrections to the inflaton potential and hence one may test whether the assumptions made on the UV completion in an effective field theory model can be fulfilled in string theory.

Conversely, inflation provides us with a window to probe some aspects of string theory. For example, one may hope to be able to constrain the compactification. Together with other constraints from *e. g.* finding a viable low-energy phenomenology this could help to reduce the vast number of possible compactifications. Thus, it is worthwhile to study inflation (and also more general ideas in cosmology) from a string theory point of view.

Inflation is notoriously difficult to achieve in string theory mostly due to the issue of moduli stabilization. Even though one often finds nice structures for some fields which seem to solve the problem, as soon as moduli stabilization is included one usually finds corrections which ruin slow-roll inflation. Nonetheless, there are sometimes favorable structures present in the beginning and one can try to expand around these.

From the point of view of bottom-up model building in four-dimensional supergravity, there are not many constraints on the form of the action. But the structure of four-dimensional supergravity theories obtained from string theory is much more restrictive. Therefore, we can use string theory as an inspiration (or motivation) for deciding which terms to write down. In this way, we obtain a string-inspired (or string-motivated) effective supergravity theory. This is the spirit of the approach we pursue in Part III of this thesis.

2.4.1 Interplay between Inflation and Moduli Stabilization

As we mentioned in our motivation, there is an important *interplay* between inflation and moduli stabilization. We will now explain this interplay and its implications in a bit more detail.

Moduli Stabilization and the Eta-Problem

For slow-roll inflation to occur, there should be in particular no runaway direction for any of the fields and thus the moduli must be fixed in some minimum. Moreover, we will throughout this work require that all fields except the inflaton acquire masses of at least $m \gtrsim \mathcal{H}$. That is, we assume only a single field ϕ is dynamically relevant during inflation, *i. e.* has a mass $m_\phi \ll \mathcal{H}$. Note

[15] For interesting recent attempts of "low-energy solutions" of the η-problem see [242, 243].

that this is *not* required by any physical argument, but just serves to simplify the effective field theory.[16] Therefore, we have to adjust the parameters in the effective potential for the moduli fields accordingly.[17]

Upon integrating out all the fields with masses $m \gtrsim \mathcal{H}$, in particular all the moduli, we obtain the effective potential for our inflaton candidate ϕ. By the same logic we used to arrive at Eq. (2.7), integrating out the moduli generically induces corrections to the inflaton potential. These corrections are particularly dangerous for moduli which are stabilized non-perturbatively via the superpotential. Thus, a solution to the η-problem is always intimately connected to the physics of moduli stabilization.

Moduli Destabilization by Inflation

When the issue of moduli stabilization is discussed in conjunction with inflation, however, another severe problem emerges. Namely, the very presence of an inflationary sector may *destabilize* the moduli [43–45], *i. e.* destroy their minimum.[18] To avoid this destabilization, one often finds an *upper bound* on the Hubble scale during inflation in terms of the present-day gravitino mass,

$$\mathcal{H}_{\rm inf} \leq m_{3/2}^{\rm today} \, . \tag{2.10}$$

But in order to solve the hierarchy problem with supersymmetry one typically needs $m_{3/2} \sim \mathcal{O}(\text{TeV})$ and thus the scale of inflation is bound to be very small, much below the scale required for many model building approaches (and also very much below observational sensitivities for gravitational waves).

Basically, the above tension arises since there is effectively only one scale in the problem, which sets both the gravitino mass today and the height of the barrier towards decompactification [40, 45]. Typically, inflation in such a setup induces a runaway-type of potential, in particular, for the modulus controlling the overall volume. Thus, if the contribution from inflation becomes too large, the barrier and hence the minimum disappear. We will explain this in more detail in Sec. 10.1 in a concrete setting.

2.4.2 A Brief Overview of Some Possibilities

Scalar fields are abundant in string theory compactifications. We have already introduced the notion of *geometric moduli* controlling the size and shape of

[16] For observable effects of additional fields with $m \sim \mathcal{H}$ (which is generic in supergravity) see *e. g.* [244, 245] and for an introduction into multi-field inflation see *e. g.* [246].

[17] Strictly speaking, all of these parameters should be computed from an explicit underlying string theory compactification. However, in practice, one typically treats them as free parameters and varies them within a reasonable range. In this way, one places constraints on possible compactifications.

[18] For earlier work discussing problems of moduli related to inflation, cf. *e. g.* [247–249].

the compact space. In addition to these, there are also *axions* – pseudo-scalar fields which receive a potential only via non-perturbative corrections. And as if these were not enough there are also the (relative) *positions* of D-branes inside the compactification which show up as scalar fields.

In the following, we give an incomplete and very brief overview of some string theory approaches to inflation. For recent reviews on string theory models of inflation cf. *e. g.* [250–252] and the textbook [253].

The existing proposals can be classified according to whether the inflaton resides in the closed string or in the open string sector of the theory. The discussion below is focused on type IIB models of inflation since in this situation the problem of moduli stabilization is arguably best-understood at present.

With respect to solving the η-problem, the idea in the models below is to either find an underlying mechanism to protect the inflaton from dangerous corrections (which happens rarely) or to find a way to systematically write down all the possible corrections, albeit with in general unknown coefficients.

Closed String Inflation

Inflation in the closed string sector means to either identify the inflaton with a modulus or an axion. The models in this class are typically stringy versions of the chaotic inflation scenario.

Kähler Moduli Inflation After invoking fluxes to stabilize many geometric moduli and the dilaton with large masses, only a certain class of moduli, so-called *Kähler moduli* survive as light fields. There is always at least one of them which controls the overall volume $\mathcal{V}_6$.

The success of these models in solving the η-problem relies crucially on the structure of the manifolds which allow for the LVS scheme of moduli stabilization [159]. The large volume $\mathcal{V}_6$ of the compact space helps to suppress potentially dangerous corrections, together with a cancellation. In this setup, the inflaton potential can be systematically expanded in powers of $1/\mathcal{V}_6$. The precise structure of the terms often requires an educated guess [157, 254, 255] and the coefficients of the terms are in general unknown. For examples of this class see *e. g.* [256–259].

Axion Inflation Axions are a priori nice candidates to realize inflation since at the perturbative level they enjoy a *continuous* shift symmetry $\phi \to \phi + \alpha$ which is broken to a *discrete* subgroup by non-perturbative effects. This is the idea of *natural inflation* [260]. The flatness of the potential is then controlled by the axion decay constant f. What one would like to have is $f > M_P$, but this does not seem to be possible for axions arising in string theory which have $f \ll M_P$ [261, 262]. Several solutions have been proposed to overcome this problem and still solve the η-problem. For instance, to invoke a second non-perturbative correction and fine-tune the coefficients [263, 264], to consider *N-flation* which uses a large number of axions and considers the collective excitation of all the axions as the inflaton [265–267] or to use a *monodromy*[19] which enhances the effective range of field values protected by the shift symmetry in a subtle way [239–241].

Open String Inflation

Inflation in the open string sector amounts to identifying the inflaton with the position of a D-brane in the compact internal space which corresponds either to the distance between two D-branes or the distance between a D-brane and an anti-D-brane.[20] The models of this class are typically a stringy version of hybrid inflation.

The models we briefly mention here use D3 and D7-branes both of which always span all four external (non-compact) directions and the D7-branes are in addition extended along four other internal directions in the compact space.

D3-D7 Inflation As the name suggests, this model identifies the distance between a D3 and a D7-brane with the inflaton and the D3-brane is attracted towards the D7-brane. These models feature a *geometrically* realized shift symmetry [268], *i. e.* its existence is tied to a special form of the compact space. However, this shift symmetry is broken upon including string-loop corrections and non-perturbative moduli stabilization and the model requires some fine-tuning to work [42].[21]

Fluxbrane Inflation (or D7-D7 Inflation) This model is conceptually similar to the D3-D7 setup, but uses two D7-branes which attract each other in the transversal directions due to the presence of world-volume gauge flux (*i. e.* "magnetic fields" along the internal directions of the D7-brane) [270]. An advantage of this class of models compared to D3-D7 inflation is that it yields a larger field range for the inflaton and that constraints from the formation of *cosmic strings*[22] seem easier to fulfill. Some tuning is required to make the model work (related to a large volume) and the corrections from moduli stabilization have not been systematically explored yet.

[19] The basic idea of a monodromy is that by going around in a loop in configuration space we do not end up at the point we started. A useful simple example of this effect is a spiral staircase on which we move around the center by 360° along the angular direction.

[20] Recall from Sec. 2.1.2 that D-branes source fluxes. Similar to the electromagnetic case, there are "positively" and "negatively" charged objects, D-branes and anti-D-branes.

[21] Within a similar setup, it was proposed in [269] to identify the inflaton with a combination of an axion and the brane position.

[22] The formation of cosmic strings at the end of inflation is a generic problem in many models of hybrid inflation [60, 61] (see [271] for a recent review).

Warped D-brane Inflation The original proposal for warped D-brane inflation [272], or more precisely "D3-anti-D3-brane inflation[23] in a warped throat", attempted to achieve a sufficiently flat potential by making use of a strongly warped geometry. Unfortunately, as it turned out, this is not the leading contribution to the inflaton potential [272] precisely because of moduli stabilization. However, a lot of information is available in some cases which has triggered a systematic study of various corrections in this class of models, see *e. g.* [41, 279–296]. As a result, the structure of the corrections to the inflaton potential and their impact on the η-parameter has been determined, albeit with a priori unknown coefficients. Using this knowledge, one can study the system quantitatively in a statistical approach [295, 296]. The generic picture of a working inflationary model in this class is that there is an *inflection point*, *i. e.* a point where V'' changes sign and V' is also small. This inflection point arises by chance because of cancellations between various terms.

In this thesis, we will focus on models of inflation in the open string sector and more specifically on the warped D-brane inflation scenario. The motivation for this is twofold. First, by having inflation in the open string sector, one may expect more flexibility in the sense that the tunings required for stabilizing the geometric moduli and those required to yield succesful inflation are separated. Second, we choose the warped D-brane inflation scenario since it has been more widely studied and seems to be applicable in more general situations. In Part IV of this thesis, we will consider a version of warped D-brane inflation in a particular class of ten-dimensional warped backgrounds.

2.5 Outline

In this thesis, we explore new approaches to the problem of moduli destabilization and the η-problem, and to realize inflation in the matter sector. The remaining parts of this thesis are organized as follows.

In Part II, we review the basics of some assorted topics at a technical level and collect some facts and results which might be useful for understanding the main part of this thesis. We begin with slow-roll inflation (Chap. 3), writing down 4d supergravity actions (Chap. 4) and their application to some simple models of inflation in supergravity (Chap. 5). Then we introduce the basics of moduli stabilization (Chap. 6) and of warped geometries (Chap. 7) as well as their application to warped D-brane inflation (Chap. 8). Finally, we turn to the 4d effective supergravity action of heterotic orbifolds (Chap. 9).

The main body of this thesis is split into two parts. In the first part, Part. III, we pursue a bottom-up approach and consider some interesting supergravity models whose structure is motivated/inspired by string theory.

[23] For earlier work on brane anti-brane inflation see [273–278].

In Chap. 10, we discuss a new solution to evade the aforementioned bound on the Hubble scale during inflation, Eq. (2.10), which allows us to successfully combine high-scale inflation and low-energy supersymmetry breaking. It is based on using two different mechanism for moduli stabilization during and after inflation. We demonstrate this new solution first in an explicit example using a shift symmetric model of chaotic inflation and a KKLT mechanism for moduli stabilization and then in a less explicit example where we replace the KKLT setup by a more general one.

In Chap. 11, we perform a generalization of the inflation models of [297], which use gauge non-singlet matter fields, to a much broader class of genuine models of matter inflation in supergravity. These models belong to a particular class of hybrid inflation models dubbed “tribrid inflation”. This class of models is well-suited for solving the η-problem by some symmetry property [38]. We discuss moduli stabilization and the possible sources of corrections to the inflaton potential from an effective supergravity point of view.

We outline how to embed this class of matter inflation models into the framework of heterotic orbifold compactifications in 12. This particular class of compactifications is chosen because of a certain structure (a “Heisenberg symmetry”) which helps to solve the η-problem and is present in the tree-level Kähler potential. We propose a new way to stabilize some of the moduli during inflation using supersymmetry breaking contributions to the inflaton potential driven by matter fields. We again discuss moduli stabilization and some sources for an inflaton slope in this setup.

In the second part, Part IV, we make use of some very recently obtained warped throat geometries. We first review these backgrounds in Chap. 13, in particular, how they are obtained using a solution generating technique and a master equation. We then discuss their application in the context of warped D-brane inflation in Chap. 14. This is a top-down approach to inflation and, as we will explain, an *inflection point* arises very generically in these models. Using some approximate scaling behaviour, we show that realistic values for the inflationary observables can be obtained.

Finally, we give our conclusions in Part V and present an outlook of further directions of research. The appendix, Part VI, contains some information about notations and conventions used in this thesis and some useful formulas.

Part II

Theoretical Basics

CHAPTER 3

Slow-Roll Inflation

The first concept we need to introduce is *slow-roll inflation*. This term refers to a phase of accelerated expansion which is controlled by a scalar field slowly rolling down its potential, the “inflaton”. During the accelerated expansion, the energy density of the universe is then dominated by the potential energy of the scalar field(s).

This phase ends when the inflaton picks up speed and subsequently starts to oscillate around a potential minimum. Subsequently, the inflaton decays and there is a transition to a radiation dominated universe which is commonly called reheating.

We begin by deriving the Friedmann equations which describe the evolution of a homogeneous and isotropic universe in Sec. 3.1. In Sec. 3.2, we summarize some observational evidence in favor of the concordance or ΛCDM model (with w_Λ fixed from the data). We review slow-roll inflation in Sec. 3.3. In particular, we define the slow-roll parameters ϵ, η in terms of the slope and the curvature of the potential. Next, in Sec. 3.4, we review a few results on the perturbations generated by inflation, which leads to predictions for the observed CMB spectrum. Finally, we illustrate the setup in a simple but very explicit example of chaotic inflation in Sec. 3.5.

The discussion presented in this chapter follows [27]. For textbook treatments of inflation see for example [24–26].

3.1 Friedmann-Robertson-Walker Universe

We are interested in the cosmological evolution of the universe on very large scales, where we can assume *homogeneity* and *isotropy*. A space is *homogeneous* if it is translationally invariant, *i. e.* the same at every point, while a space is *isotropic* if it is rotationally invariant, *i. e.* the same in every direction. Any space which is everywhere isotropic is necessarily also homogeneous. However, the converse is not true. Consider *e. g.* a space filled with a uniform electric field which is translationally invariant but breaks invariance under rotations. It can be shown that such spaces are described by the *Friedmann-Robertson-Walker* (FRW) metric

$$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1 - k\, r^2} + r^2 \left(d\theta^2 + sin^2\theta \, d\phi^2 \right) \right) . \tag{3.1}$$

Here, the *scale factor* $a(t)$ encodes the relative size of *spacelike* hypersurfaces at different points in time. The parameter k is related to the (sign of the) *spatial curvature* of these hypersurfaces. Expansion of the universe corresponds to a growth of the scale factor $a(t)$.

An important quantity characterizing the FRW spacetimes is the *expansion rate* or *Hubble paramater*,

$$\mathcal{H} \equiv \frac{\dot{a}}{a} . \tag{3.2}$$

$\mathcal{H}$ sets the fundamental scale of the FRW universe. For example, the characteristic time and length-scales are given by $t \sim \mathcal{H}^{-1}$ and $d \sim \mathcal{H}^{-1}$ (in units where $c = 1$).

The dynamics of the FRW universe is governed by the Einstein equations,

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = M_P^2 \, T_{\mu\nu} , \tag{3.3}$$

where $g_{\mu\nu}$ denotes the spacetime metric while $R_{\mu\nu}$ and R denote the Ricci tensor and Ricci scalar, respectively, which describe the curvature of space time, and $M_P^2 \equiv 8\pi G_N$ is the Planck mass. The right hand side is the energy-momentum tensor associated with all forms of "matter" in the universe. Here, we assume $T_{\mu\nu}$ to be a homogeneous and isotropic fluid and thus in a frame comoving with the fluid we may choose $u^\mu = (1, 0, 0, 0)$ such that

$$T^\mu{}_\nu = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & -p \end{pmatrix} , \tag{3.4}$$

where ρ and p denote the energy density and the pressure in the rest frame of the fluid, respectively. The Einstein equations then take the form of two coupled, non-linear but ordinary differential equations often referred to as the *Friedmann Equations*:

$$\mathcal{H}^2 = \left(\frac{\dot{a}}{a} \right)^2 = \frac{\rho}{3} - \frac{k}{a^2} , \tag{3.5}$$

and

$$\dot{\mathcal{H}} + \mathcal{H}^2 = \frac{\ddot{a}}{a} = -\frac{1}{6} \left(\rho + 3p \right) , \tag{3.6}$$

with dots denoting derivatives with respect to t. Note that these Eqs. already tell us something very interesting: In an expanding universe ($\dot{a} > 0$) filled with ordinary matter – satisfying the strong energy condition $\rho + 3p \geq 0$ – Eq. (3.6) implies that $\ddot{a} < 0$, i.e. the expansion decelerates. Most importantly, this indicates the existence of a singularity in the finite past, *i. e.* $a(t \equiv 0) = 0$. However, such a conclusion requires General Relativity and the Friedmann Equations to be valid all the way down to arbitrarily small length scales (or correspondingly arbitrarily high energies). Thus, it is more likely that the singularity signals the break down of General Relativity and the necessity to develop a quantum theory of gravity.

By assuming an equation of state of the form

$$p \equiv w\,\rho\,, \tag{3.7}$$

and assuming a flat universe ($k = 0$), we can solve the equations for the time evolution of the scale factor:

$$a(t) \propto \begin{cases} t^{2/3(1+w)} & \text{for } w \neq -1\,, \\ e^{\mathcal{H}t} & \text{for } w = -1\,. \end{cases} \tag{3.8}$$

	w	$\rho(a)$	$a(t)$
MD	0	a^{-3}	$t^{2/3}$
RD	$\frac{1}{3}$	a^{-4}	$t^{1/2}$
Λ	-1	a^0	$e^{\mathcal{H}t}$

Table 3.1: FRW solutions for a spatially flat universe ($k = 0$) and dominated either by non-relativistic matter (MD), radiation (RD) or a cosmological constant (Λ).

To describe more realistic systems, such as our current universe, it is necessary to allow more than one species of matter to contribute significantly to the energy density and pressure. In this case, ρ and p refer to the sum of all components, *i. e.*

$$\rho = \sum_i \rho_i\,, \quad p = \sum_i p_i\,. \tag{3.9}$$

It is convenient to define the quantities Ω_i at the *present* time t_0 as the ratio of the current energy density $\rho_i(t_0)$ to the *critical energy density* $\rho_{\rm crit} \equiv 3\mathcal{H}_0^2 M_P^2$, *i. e.*

$$\Omega_i \equiv \frac{\rho_i(t_0)}{\rho_{crit}}\,, \tag{3.10}$$

with the scale factor normalized such that $a_0 \equiv a(t_0) \equiv 1$. The critical energy density is the energy density required to have a spatially flat universe. Each of the species has an equation of state of the form

$$p_i = w_i\,\rho_i\,, \tag{3.11}$$

and thus we can write the Friedmann Equation, Eq. (3.5), as

$$\left(\frac{\mathcal{H}}{\mathcal{H}_0}\right)^2 = \sum_i \Omega_i a^{-3(1+w_i)} + \Omega_k a^{-2}\,, \tag{3.12}$$

where $\Omega_k \equiv -k/a_0^2\mathcal{H}_0^2$. Evaluating this equation at t_0 implies the consistency relation

$$\sum_i \Omega_i + \Omega_k = 1\,, \tag{3.13}$$

while evaluating Eq. (3.6) yields

$$\frac{1}{a_0^2\mathcal{H}_0^2}\frac{d^2a_0}{dt^2} = -\frac{1}{2}\sum_i \Omega_i(1+3w_i)\,. \tag{3.14}$$

Before we discuss how to realize a phase of inflation dynamically in Sec. 3.3, we briefly review a few observational results on the parameters Ω_i and Ω_k for a particular cosmological model in the next section.

3.2 The Concordance Model

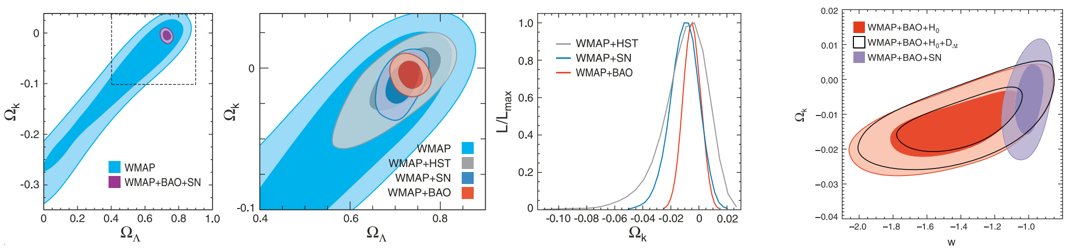

Figure 3.1: Constraints on the vacuum energy density Ω_Λ and spatial curvature parameter Ω_k from a combination of CMB and LSS observations (*left three figures taken from [298]*) . This provides evidence for a spatially flat geometry of our universe. However, note that the constraint depends on assumptions about the dark energy equation of state parameter w_Λ (*rightmost figure taken from [23]*).

Observations of the Cosmic Microwave Background (CMB) are well-described by the so-called "concordance model" (or ΛCDM model). It contains four components of "matter": radiation (rad), atoms (or "baryons", b), (cold) dark matter (dm) and vacuum energy (or dark energy, Λ) with $w_\Lambda \approx 1$. Combining probes of the CMB such as the WMAP experiment with Large-Scale Structure (LSS) observations, one can fit the parameters of the model to the data, which yields the following values for the Ω_i [23]:

$$\Omega_{\rm rad} \approx 3.4\times10^{-5}\,,\quad \Omega_{\rm b} \approx 0.04\,,\quad \Omega_{\rm dm} \approx 0.23\,,\quad \Omega_\Lambda \approx 0.73\,, \tag{3.15}$$

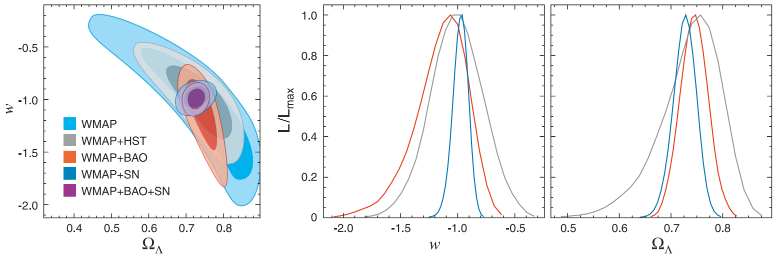

Figure 3.2: Constraints on the dark energy ratio Ω_Λ and equation of state parameter w_Λ from combined observations of the CMB and LSS. Figures taken from [298].

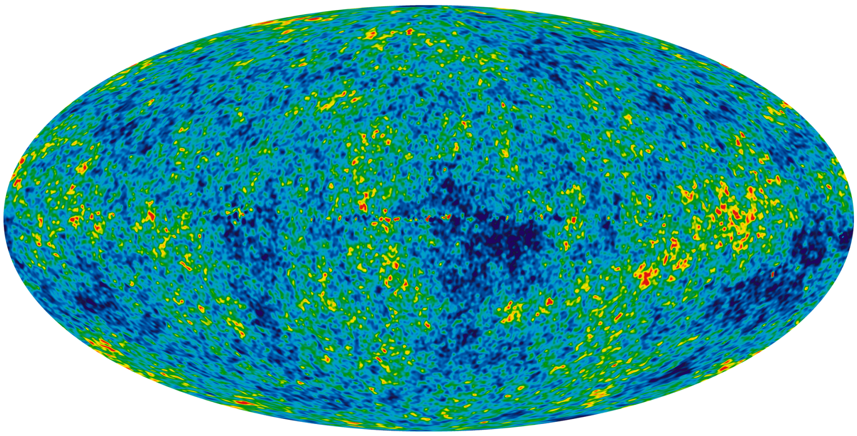

Figure 3.3: WMAP 7yr foreground-reduced Full Sky Map. Figure taken from [299].

with $w_\Lambda \approx -1$. The spatial curvature is constraint to be $\Omega_k \approx 0.0023^{+0.0054}_{-0.0056}$ [23] for $w_\Lambda = -1$. However, the constraint depends on the particular dark energy model considered (cf. Fig. 3.2).

Moreover, one observes tiny ripples in the temperature fluctuations at the level of $\delta T/T \sim 10^{-5}$ around the average CMB temperature of $T \approx 2.7\,K$ (cf. Fig. 3.3). These fluctuations are adiabatic, (nearly) scale-invariant and Gaussian. It is these observed, tiny structures which can be interpreted as the as quantum fluctuations of the inflaton field.

3.3 Slow-Roll Inflation

Inflation corresponds to an accelerated expansion, $\ddot{a} > 0$. From Eq. (3.6), we see that this requires $p < -3\rho$. We will be moreover interested in *slow-roll inflation* where the space-time is *approximately* de Sitter. That is, we are interested in solutions of the form $a \sim e^{\mathcal{H}t}$ with $\mathcal{H} \approx \text{const}$. As we can see from Tab. 3.1, to realize an exponential expansion of the universe, we must have an equation of state parameter $w \approx -1$. This can be achieved by considering a scalar field ϕ

which is slowly rolling down its potential, *i. e.* when the potential energy, $V(\phi)$, dominates over the kinetic energy, $\frac{1}{2}\dot{\phi}^2$.

Consider a scalar field ϕ minimally coupled to gravity. The dynamics are described by the sum of the Einstein-Hilbert action $\mathcal{S}_{\rm EH}$ and the action $\mathcal{S}_\phi$ with the canonical kinetic term and the potential $V(\phi)$,

$$\mathcal{S} = \mathcal{S}_{\rm EH} + \mathcal{S}_\phi = \int d^4x\sqrt{-g}\left(\frac{M_P^2}{2}R + \frac{1}{2}g^{\mu\nu}\nabla\mu\phi\nabla_\nu\phi - V(\phi)\right) . \tag{3.16}$$

The self-interactions of ϕ are encoded in $V(\phi)$. The energy-momentum tensor associated with this scalar is

$$T_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}}\frac{\delta\mathcal{S}_\phi}{\delta g^{\mu\nu}} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left(\frac{1}{2}\partial^\sigma\phi\partial_\sigma\phi + V(\phi)\right) . \tag{3.17}$$

Restricting to the case of a homogeneous scalar field $\phi(\vec{x},t) \equiv \phi(t)$ and to the FRW metric for $g_{\mu\nu}$, the energy-momentum tensor takes the form of a perfect fluid, Eq. (3.4), with

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) , \tag{3.18}$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) . \tag{3.19}$$

The equation of state parameter w_ϕ becomes

$$w_\phi \equiv \frac{p_\phi}{\rho_\phi} = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} . \tag{3.20}$$

Thus, if the potential energy $V(\phi)$ dominates over the kinetic energy $\frac{1}{2}\dot{\phi}^2$, a scalar field can cause accelerated expansion of the universe. The system consisting of the homogeneous scalar field and the FRW spacetime is described by

$$\ddot{\phi} + 3\mathcal{H}\dot{\phi} + \frac{\partial V}{\partial\phi} = 0 , \tag{3.21}$$

and

$$\mathcal{H}^2 = \frac{1}{3M_P^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right) . \tag{3.22}$$

Note that the second term in Eq. (3.21) causes significant *Hubble friction* due to the expanding universe if the value of the $\mathcal{H}$ is large enough.

3.3.1 Slow-Roll Parameters

The equation for the acceleration can be written as

$$\frac{\ddot{a}}{a} = -\frac{1}{6}\left(\rho_\phi + 3p_\phi\right) \equiv \mathcal{H}^2(1 - \epsilon_{\mathcal{H}}) , \tag{3.23}$$

with

$$\epsilon_{\mathcal{H}} \equiv \frac{3}{2}\left(w_\phi + 1\right) = \frac{1}{2}\frac{\dot{\phi}^2}{\mathcal{H}^2}\,. \tag{3.24}$$

This *slow-roll parameter* $\epsilon_{\mathcal{H}}$ is related to the evolution of the Hubble parameter $\mathcal{H}$ as

$$\epsilon_{\mathcal{H}} = -\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}\,. \tag{3.25}$$

Accelerated expansions occurs as long as $\epsilon_{\mathcal{H}} < 1$. This phase of accelerated expansion will last long if

$$|\ddot{\phi}| \ll |3\mathcal{H}\dot{\phi}|\,,\ \left|\frac{\partial V}{\partial \phi}\right|\,, \tag{3.26}$$

which is ensured if also the second slow-roll parameter $\eta_{\mathcal{H}}$ is sufficiently small,

$$\eta_{\mathcal{H}} \equiv -\frac{\ddot{\phi}}{\mathcal{H}\dot{\phi}} < 1\,. \tag{3.27}$$

The slow-roll conditions $\epsilon_{\mathcal{H}}$, $|\eta_{\mathcal{H}}| < 1$ can also be expressed in terms of the potential:

$$\epsilon(\phi) \equiv \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2\,, \tag{3.28}$$

and

$$\eta(\phi) \equiv M_P^2\frac{V''}{V}\,, \tag{3.29}$$

where the primes denote derivatives of V with respect to ϕ. Note that ϵ characterizes the *slope* of the potential while η characterizes the *curvature* of the potential. Slow-roll inflation requires both ϵ and $|\eta|$ to be $\ll 1$.

The equations of motion in the slow-roll regime, $\epsilon, |\eta| \ll 1$, are given by

$$\mathcal{H}^2 \approx \frac{V(\phi)}{3M_P^2} \approx \text{const.}\,, \tag{3.30}$$

and

$$\dot{\phi} \approx -\frac{V'}{3\mathcal{H}}\,, \tag{3.31}$$

such that the scale factor is

$$a(t) \sim e^{\mathcal{H}t}\,. \tag{3.32}$$

Note that using the slow-roll equation of motion, $\mathcal{H}^2 \sim V/M_P^2$ and the definition of the mass of a scalar as $m_\phi^2 = V''$, we can see that the slow-roll parameter η is essentially the ratio of the inflaton mass to the Hubble scale,

$$\eta \sim \frac{m_\phi^2}{\mathcal{H}^2}\,. \tag{3.33}$$

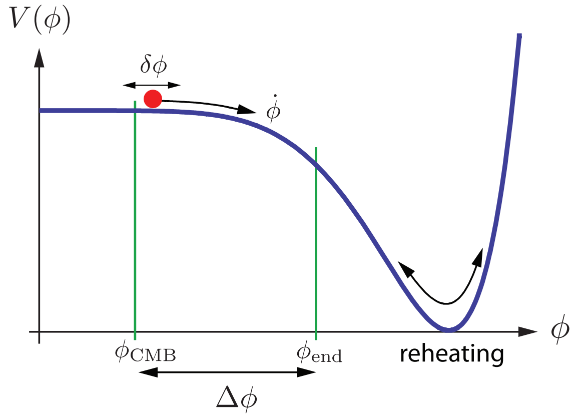

Figure 3.4: Prototype potential for canonical, single-field, slow-roll inflation. Figure taken from [27].

In the slow-roll regime, the two sets of slow-roll parameters $\epsilon_{\mathcal{H}}, \eta_{\mathcal{H}}$ and ϵ, η are related by

$$\epsilon_{\mathcal{H}} \approx \epsilon \,, \quad \eta_{\mathcal{H}} \approx \eta - \epsilon \,; \tag{3.34}$$

A prototypical example for a potential suitable for single-field inflation is shown in Fig. 3.4.

Inflation ends at a value $\phi_{\rm end}$ satisfying

$$\epsilon_{\mathcal{H}}(\phi_{\rm end}) \equiv 1 \,, \quad \epsilon(\phi_{\rm end}) \approx 1 \,. \tag{3.35}$$

Another important quantity which we have to define is the number of e-folds before the end of inflation N_e,

$$N_e(\phi) \equiv \ln \frac{a_{\rm end}}{a} = \int\limits_t^{t_{\rm end}} \mathcal{H} \, dt \,. \tag{3.36}$$

N_e measures by how much the scale factor increases in powers of e, *e. g.* $N_e = 60$ means that $a(t)$ grows by a factor of $e^{60} \sim 10^{26}$.

Using the slow-roll equations Eqs. (3.30) and (3.31), this can be written as an integral over $d\phi$,

$$N_e(\phi) = \int\limits_{\phi}^{\phi_{\rm end}} \frac{\mathcal{H}}{\dot{\phi}} \, d\phi \approx \int\limits_{\phi_{\rm end}}^{\phi} \frac{V}{V'} \, d\phi \,, \tag{3.37}$$

or in terms of the slow-roll parameters

$$N_e(\phi) = \int\limits_{\phi_{\rm end}}^{\phi} \frac{d\phi}{\sqrt{2\epsilon_{\mathcal{H}}}} \approx \int\limits_{\phi_{\rm end}}^{\phi} \frac{d\phi}{\sqrt{2\epsilon}} \,. \tag{3.38}$$

To solve the horizon and flatness problems puts a lower bound on the total number of e-folds during inflation,

$$N_{\rm tot} \equiv \ln \frac{a_{\rm end}}{a_{\rm start}} \gtrsim 60\,. \tag{3.39}$$

The observed CMB fluctuations are created at roughly $N_{\rm CMB} \approx 40-60$ e-folds before the end of inflation. The precise values for $N_{\rm tot}$ and $N_{\rm CMB}$ depend on both the energy scale of inflation and the details of reheating.

3.3.2 Classes of Inflationary Models

Inflationary models can be classified according to the number of active fields involved and the distance of field space traversed during inflation. The former classifies the models into *(effectively) single-field* and *multi-field* models, while the latter can be classified as *small-field* or *large-field* models.

Single-Field Models of Inflation

Any inflationary model is specified by the inflaton kinetic term and potential and by its coupling to gravity. Up to now, we have restricted ourselves to single-field slow-roll inflation models with actions of the form Eq. (3.16). There is a useful criterion to classify models of this type by checking whether the potential $V(\phi)$ allows the inflaton ϕ to travel a large or small distance $\Delta\phi$ (in Planck units) between the creation of CMB fluctuations and the end of inflation.

Small-Field Inflation All small-field models of inflation have $\Delta\phi < M_P$ and therefore predict a very small amplitude for the production of gravitational waves, as we will see in Sec. 3.4.4. This class of models is typically obtained from the spontaneous breaking of some symmetry and inflation takes place in an unstable vacuum displaced from a stable minimum of the potential. A simple example is a Higgs-type potential

$$V(\phi) = V_0 \left[1 - \left(\frac{\phi}{\mu}\right)^2\right]^2 . \tag{3.40}$$

Phenomenological models are often based on the *Coleman-Weinberg potential* of the form

$$V(\phi) = V_0 \left[\left(\frac{\phi}{\mu}\right)^4 \left(\ln\left(\frac{\phi}{\mu}\right) - \frac{1}{4}\right) + \frac{1}{4}\right] , \tag{3.41}$$

which arises for the radiative breaking of a symmetry, *e. g.* in electroweak or grand unified theories and remains popular for inflationary model-building (cf. *e. g.* [300]). We will see a supersymmetric version of this potential later on, when we discuss hybrid models of inflation in supergravity in Secs. 5.2.1 and 5.2.2.

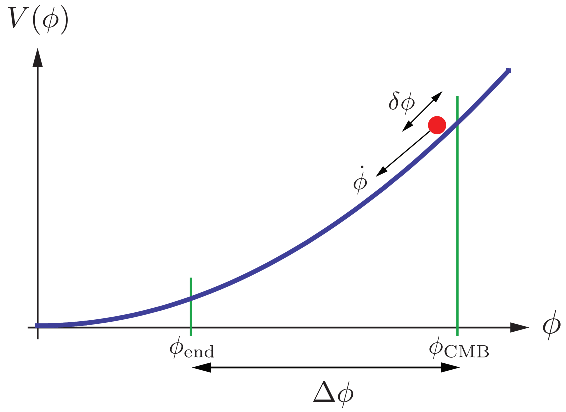

Figure 3.5: Typical potential for large-field chaotic inflation. Figure taken from [27].

Large-Field Inflation Large-field models are interesting since $\Delta\phi > M_P$ can produce amounts of gravitational waves observable in the near future. The prototype examples are *chaotic inflation* models based on a potential dominated by a single monomial

$$V(\phi) = \lambda\,\phi^p\,. \tag{3.42}$$

A typical potential of this type is shown in Fig. 3.5.

An important feature of chaotic inflation is that the slow-roll conditions are independent of the coupling constant λ and are only satisfied for $\phi \gg M_P$. However, to produce the observed amount of density fluctuations requires $\lambda \ll 1$, thereby guaranteeing $V \ll M_P^4$ such that quantum gravity corrections are not necessarily important. We will discuss the simplest of these models, $V(\phi) \sim m^2\phi^2$, in Sec. 3.5.

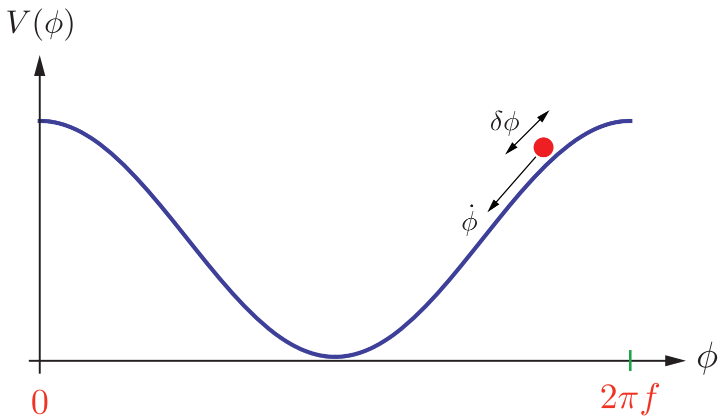

Figure 3.6: Typical potential for natural inflation. Figure taken from [27].

Natural Inflation A very elegant inflationary model, *natural inflation* [260], relies on potentials of the form

$$V(\phi) = V_0 \left[\cos\left(\frac{\phi}{f}\right) + 1 \right] , \tag{3.43}$$

which arise if the inflaton is identified with an axion. Whether this leads to small-field or large-field inflation depends of course on the value of f. Even though it is perhaps more appealing to have natural inflation with $2\pi f > M_P$, in particular, because for axions a shift symmetry can protect the above form of the potential over a large range of field values.

Beyond Single-Field Models

All of the model types presented so far are based on a few common assumptions.

1. Only a single field ϕ has important dynamics during inflation,
2. the kinetic terms for ϕ are canonical,
3. ϕ is minimally-coupled to gravity,
4. and gravity is described by the Einstein-Hilbert action.

In this thesis, we will focus on models which satisfy these assumptions at least approximately. Nevertheless, we now briefly comment on some ways to go beyond those assumptions to some extent. For a thorough discussion of inflationary model building cf. *e. g.* [217].

Multi-Field Models of Inflation First, we could consider models where more than one field contributes significantly to the dynamics during inflation. However, this leads to many possible inflationary trajectories through field space and different ways to produce fluctuations. Thereby, the theory loses a lot of predictive power. For some examples see *e. g.* [246].

From a string theory perspective, models with multiple dynamical fields are not uncommon. For example, in the closed string sector one often has multiple axions or Kähler moduli which may contribute or in the open string sector the positions of moving D-branes may also be relevant (*e. g.* a D3-brane moving in a six-dimensional compact space gives rise to six scalar fields parametrizing its position inside the compact space). However, depending on the details of (bulk) moduli stabilization one may assume that all but one of the many scalar fields are effectively fixed to some value during inflation. This is the viewpoint we will take throughout this thesis: Even though there are multiple scalar fields around inflation proceeds along a single direction in field space.

Non-Canonical Kinetic Terms One can generalize the kinetic term of the scalar field ϕ such that the Lagrangian takes the form

$$\mathcal{L}_\phi = F(\phi, X) - V(\phi)\,, \tag{3.44}$$

for some function $F(\phi, X)$ with

$$X \equiv \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi\,. \tag{3.45}$$

This can lead to the very interesting possibility that inflation is actually driven by the kinetic term and occurs even for potentials which are too steep for slow-roll inflation. A particular example coming from string theory is so-called *DBI inflation* [301,302], where the form of the function $F(\phi, X)$ is determined from the DBI action of a D3-brane moving in a warped throat background. It has a very distinct phenomenology since it leads to rather large non-gaussianities (compared to single-field slow-roll inflation).

Non-Minimal Couplings to Gravity or Modified Gravity Other options to modify the models include to either introduce non-minimal couplings of ϕ to gravity or modify the gravity action itself. The first option, introducing non-minimal couplings to gravity, is in principle a possibility, but in practice one can employ a field redefinition to get back a minimally-coupled scalar. The same is true for the simplest example of a UV modification of gravity, the so-called $f(R)$ theories: They can also be transformed into a theory with a minimally-coupled scalar ϕ and some potential $V(\phi)$. Thus, these two possibilities are essentially useful to obtain a potential suitable for slow-roll inflation after a field redefinition. This is the central idea behind the recently proposed new models of Higgs inflation, cf. *e. g.* [30, 303–310].[1]

3.4 Inflationary Perturbations

One part of the big success of the inflationary paradigm hinges crucially on the prediction of the primordial spectrum of density fluctuations [8–11], which can be compared with the temperature fluctuations we observe in the CMB today. The details of this important calculation are reviewed *e. g.* in [24–27]. Here, we only outline the basic steps to setup the notation. What we will be interested in are the predictions for models of slow-roll inflation (cf. Sec. 3.4.2). Moreover, we discuss the so-called *Lyth bound* and its implications in Sec. 3.4.4.

The basic idea behind cosmological perturbation theory is to split all fields $X(t, \vec{x})$ into a homogeneous background part $\bar{X}(t)$ depending only on cosmic time and a perturbation $\delta X(t, \vec{x}) \equiv X(t, \vec{x}) - \bar{X}(t)$ depending also on $\vec{x}$. Here

[1] However, see [31–33] for some critical remarks on the naturalness of these models of Higgs inflation and the validity of the effective field theory approach.

$X(t, \vec{x})$ represents any of the fields, *i. e.* the metric $g_{\mu\nu}$ or matter fields ($T_{\mu\nu}$ can be written in terms of ϕ, ρ, p etc.). Such a perturbative treatment is justified since at the time of decoupling the universe was very homogeneous with inhomogeneities at the order of $\delta\rho/\rho \sim 10^{-5}$. For small perturbations, one can use *linear* perturbation theory by expanding the Einstein equations to linear order in the perturbations δX,

$$\delta G_{\mu\nu} = M_P^2 \delta T_{\mu\nu} \,. \tag{3.46}$$

However, this procedure suffers from an important complication since the split into background value and perturbation depends on the particular choice of coordinates, *i. e.* on a *choice of gauge*. One can trade metric fluctuations for density fluctuations and vice versa via gauge transformations. Thus, one has to be very careful in order to obtain meaningful, physical results. In particular, it is useful to consider *gauge-invariant* combinations of perturbations.

3.4.1 Scalar-Vector-Tensor Decomposition

A universe which is spatially flat and in addition homogeneous and isotropic has a lot of symmetries. As a consequence, we can decompose the perturbations of the metric $g_{\mu\nu}$ and stress-energy tensor $T_{\mu\nu}$ into representations of these symmetries. More precisely, we can decompose them into scalar (S), vector (V) and tensor (T) representations under rotations. Additionally, it is convenient to go into Fourier space, *i. e.*

$$\delta X_{\vec{k}} = \int d^3\vec{x}\, X(t, \vec{x})\, e^{i\vec{k}\cdot\vec{x}} \,, \tag{3.47}$$

for all perturbations, because the translation invariance of the equations of motion implies that the different Fourier modes decouple. Thus, we can treat each Fourier mode independently and, in particular, consider rotations around a single wavevector $\vec{k}$. For modes with well-defined helicity m, rotating around $\vec{k}$ by an angle ψ leads to a multiplication by a factor of $e^{im\psi}$, *i. e.*

$$\delta X_{\vec{k}} \to e^{im\psi}\, \delta X_{\vec{k}} \,. \tag{3.48}$$

The helicities for scalars, vectors and tensors are 0, ± 1 and ± 2, respectively.

The upshot of the SVT decomposition of the Fourier modes is that each set of perturbations evolves independently at the linear level. This is an important simplification since we can treat them separately. We will now introduce the notations for the decompositions of metric and density perturbations.

Metric Perturbations

During inflation we have perturbations around the homogeneous background values of the inflaton $\bar{\phi}(t)$ and metric $\bar{g}_{\mu\nu}(t)$,

$$\phi(t, \vec{x}) = \bar{\phi}(t) + \delta\phi(t, \vec{x}) \,, \quad g_{\mu\nu}(t, \vec{x}) = \bar{g}_{\mu\nu}(t) + \delta g_{\mu\nu}(t, \vec{x}) \,. \tag{3.49}$$

We parametrize the metric as follows

$$\begin{aligned} ds^2 &= g_{\mu\nu} dx^\mu dx^\nu \\ &= -(1+2\Phi)dt^2 + 2aB_i dx^i dt + a^2 \left[(1-2\Psi)\delta_{ij} + E_{ij}\right] dx^i dx^j \,. \end{aligned} \tag{3.50}$$

In position space, the SVT decomposition is given by

$$B_i \equiv \partial_i B - S_i \,, \tag{3.51}$$

where $\partial^i S_i = 0$ (*i. e.* transversal vector), and by

$$E_{ij} \equiv 2\partial_i \partial_j E + 2\left(\partial_i F_j + \partial_j F_i\right) + h_{ij} \,, \tag{3.52}$$

with $\partial^i F_i$ and $h^i_i = \partial^i h_{ij} = 0$ (*i. e.* traceless, transversal symmetric tensor).

Inflation does not produce vector perturbations S_i, F_i and they both decay with the expansion of the universe. Thus, in the following, we concentrate on scalar and tensor perturbations, which lead to observable density fluctuations and gravitational waves. The scalar fluctuations are *not* gauge invariant (as opposed to the tensor perturbations). In a similar fashion, we can obtain perturbations of the stress-energy tensor in such a decomposition.

Gauge-Invariant Combinations

To avoid problems with gauge-dependencies, it is very useful to introduce gauge-invariant combinations formed out of the metric and matter perturbations [311]. There are two such combinations which are commonly used. The first one is [312]

$$-\zeta \equiv \Psi + \frac{\mathcal{H}}{\dot{\bar{\rho}}} \delta\rho \,, \tag{3.53}$$

which can be interpreted geometrically as measuring the spatial curvature of hypersurfaces with constant density ρ. ζ has the nice property that it remains constant outside the horizon for *adiabatic matter perturbations*, *i. e.* perturbations satisfying

$$\delta p_{\rm en} \equiv \delta p - \frac{\dot{\bar{p}}}{\dot{\bar{\rho}}} \delta\rho = 0 \,. \tag{3.54}$$

This condition is always satisfied for single-field models of slow-roll inflation and thus the modes $\zeta_{\vec{k}}$ do not evolve outside the horizon, *i. e.* for $k \ll a\mathcal{H}$. Note that during slow-roll inflation one has

$$-\zeta \approx \Psi + \frac{\mathcal{H}}{\dot{\bar{\phi}}} \delta\phi \,. \tag{3.55}$$

By choosing a gauge with spatially flat hypersurfaces, the perturbation ζ is the dimensionless density perturbation $\sim \delta\rho/(\bar{\rho}+\bar{p})$. Thus, ζ is the fluctuation one can relate to observations of the CMB and LSS (which is however quite a non-trivial task and beyond the scope of the discussion presented here).

The second gauge-invariant combination often used is

$$\mathcal{R} \equiv \Psi - \frac{\mathcal{H}}{\bar{\rho} + \bar{p}} \delta q \,, \tag{3.56}$$

where δq is the scalar part of T^0_i, *i. e.* $T^0_i = \partial_i \delta q$. During inflation, one has $T^0_i = -\dot{\bar{\phi}} \partial_i \delta\phi$ such that

$$\mathcal{R} = \Psi + \frac{\mathcal{H}}{\dot{\bar{\phi}}} \delta\phi \,. \tag{3.57}$$

$\mathcal{R}$ can also be interpreted geometrically as measuring the spatial curvature of comoving hypersurfaces, *i. e.* hypersurfaces with constant ϕ.

One can show via the linearized Einstein equations that (cf. *e. g.* Appendix A of [27])

$$-\zeta = \mathcal{R} + \frac{k^2}{(a\mathcal{H})^2} \frac{2\bar{\rho}}{3(\bar{\rho} + \bar{p})} \Psi_B \,, \tag{3.58}$$

where Ψ_B is one of the *Bardeen potentials* [311]. Now ζ and $\mathcal{R}$ are equal both on superhorizon scales $k \ll a\mathcal{H}$ and during slow-roll inflation (cf. Eqs. (3.55) and (3.57)). Hence, the correlation functions of ζ and $\mathcal{R}$ are equal at horizon crossing and, more importantly, they do not evolve on superhorizon scales. This latter point is important since it allows one to make predictions without having to go into the details of the reheating phase.

3.4.2 Predictions from Slow-Roll Inflation

Definitions: Scalar and Tensor Power Spectrum

We are interested in the *statistical properties* of the fluctuations of $\mathcal{R}$ or ζ. The (scalar) *power spectrum* is defined as

$$\left\langle \mathcal{R}_{\vec{k}} \mathcal{R}_{\vec{k}'} \right\rangle \equiv (2\pi)^3 \delta(\vec{k} - \vec{k}') P_{\mathcal{R}}(k) \,, \tag{3.59}$$

and

$$\Delta_s^2 \equiv \Delta_{\mathcal{R}}^2 = \frac{k^3}{2\pi^2} P_{\mathcal{R}}(k) \,, \tag{3.60}$$

where $\langle \ldots \rangle$ is the ensemble average of the fluctuations. The *scale-dependence* of the power spectrum is expressed in terms of the *scalar spectral index*

$$n_s - 1 \equiv \frac{d \ln \Delta_s^2}{d \ln k} \,. \tag{3.61}$$

Scale invariance corresponds to $n_s = 1$. Similarly, one can define the running of the spectral index

$$\alpha_s = \frac{d n_s}{d \ln k} \,. \tag{3.62}$$

Typically, a power law form of the primordial power spectrum is assumed, *i. e.*

$$\Delta_s^2(k) = A_s(k_\star)\left(\frac{k}{k_\star}\right)^{n_s(k_\star)-1+\frac{1}{2}\alpha_s(k_\star)\ln(k/k_\star)}\,, \tag{3.63}$$

with $k_\star$ denoting some arbitrary reference scale.

For Gaussian fluctuations, the power spectrum encodes the entire information. Non-Gaussianity shows up only in higher-order correlation functions and no large non-Gaussianties are expected for single-field models of slow-roll inflation [313, 314]. Therefore, we do not consider non-Gaussianities in this thesis.

The power spectrum of the tensor fluctuations is defined in a similar way, but with taking into account the two independent polarizations of h_{ij}. Thus, for $h \equiv h^+, h^\times$, we define their power spectrum as

$$\left\langle h_{\vec{k}} h_{\vec{k}'}\right\rangle = (2\pi)^3\delta(\vec{k}+\vec{k}')P_h(k)\,, \quad \text{and}\Delta_h^2 = \frac{k^3}{2\pi^2}P_h(k)\,. \tag{3.64}$$

The *tensor power spectrum* is now obtained as

$$\Delta_t^2 \equiv 2\Delta_h^2\,, \tag{3.65}$$

and the *tensor spectral index* is defined analogously to the scalar spectral index by

$$n_t \equiv \frac{d\ln\Delta_t^2}{d\ln k}\,. \tag{3.66}$$

Slow-Roll Predictions

After a very lengthy calculation one arrives at the following expressions for the scalar and tensor power spectra in single-field models of slow-roll inflation in terms of $V(\phi)$ and the two slow-roll parameters ϵ and η

The scalar and tensor power spectrum are fully specified in terms of $V(\phi)$ and ϵ as

$$\Delta_s^2(k) \approx \frac{1}{24\pi^2}\frac{V}{M_P^4}\frac{1}{\epsilon}\bigg|_{k=a\mathcal{H}}\,, \tag{3.67}$$

and

$$\Delta_t^2(k) \approx \frac{2}{3\pi^2}\frac{V}{M_P^4}\bigg|_{k=a\mathcal{H}}\,. \tag{3.68}$$

The spectral indices n_s and n_t at leading order in the slow-roll parameters are given by

$$n_s - 1 \approx 2\,\eta_\star - 6\,\epsilon_\star\,, \tag{3.69}$$

and

$$n_t \approx -2\,\epsilon_\star\,, \tag{3.70}$$

respectively. The running α_s of the scalar spectral index n_s starts at second order in the slow-roll parameters,

$$\alpha_s = -16\,\epsilon_\star\eta_\star + 24\,\epsilon_\star^2 + 2\,\xi_\star\,, \tag{3.71}$$

where ξ is another slow-roll parameter defined as

$$\xi \equiv M_P^4 \frac{V'V'''}{V^2}\,, \tag{3.72}$$

with the primes denoting again derivatives of V with respect to ϕ.

The *tensor-to-scalar ratio* is given by

$$r \equiv \frac{\Delta_t^2}{\Delta_s^2} = 16\,\epsilon_\star\,. \tag{3.73}$$

The $(\ldots)_\star$ indicates that the quantity has to be evaluated at horizon crossing, *i. e.* when $k = (a\mathcal{H})_\star$. Notice that Eqs. (3.70) and (3.73) imply a *consistency relation*,

$$r = -8\,n_t\,, \tag{3.74}$$

for all models of slow-roll inflation with a single active field.

To summarize the above, single-field slow-roll inflation with $\epsilon, |\eta| \ll 1$ *generically* predicts

- a nearly scale-invariant spectral index $n_s \approx 1 + \mathcal{O}(\epsilon, \eta)$,
- a small tensor-to-scalar ratio $r \approx \mathcal{O}(10\,\epsilon) \ll 1$,
- a very small running $\alpha_s \approx \mathcal{O}(10\,\epsilon\,\eta, 10\,\epsilon^2, \xi^2) \ll 1$.

3.4.3 Current Observational Evidence for Inflation

We summarize the current constraints on inflationary models from a combination of CMB and LSS data [23] in Tab. 3.2.

A very important lesson from the observational data is that the CMB fluctuations are *scale-invariant*, *Gaussian* and *adiabatic.*

Scale-Invariance As we can see from the current constraints of n_s obtained from the CMB and LSS data, cf. Tab. 3.2, the spectrum is nearly *scale-invariant* $n_s \approx 1$, consistent with the expectation from slow-roll inflation. Moreover, the data now excludes a perfectly scale-invariant spectrum by more than 2σ and seems to prefer a spectral index $n_s < 1$ (*i. e.* slightly tilted red). Actually, if inflation ends at some point, $\mathcal{H}$ is *time-dependent* which affects the time at which each Fourier mode exits the horizon. Therefore, a phase of inflation in the early universe *predicts* a deviation from perfect scale-invariance.

Parameter	WMAP 7-year Mean	WMAP+BAO+H$_0$ Mean
amplitude of scalar power spectrum		
$\Delta^2_{\mathcal{R}}(k_0)$	$(2.43 \pm 0.11) \times 10^{-9}$	$(2.430 \pm 0.091) \times 10^{-9}$
no running, $\alpha_s = 0$		
n_s	0.967 ± 0.014	0.968 ± 0.012
r	< 0.36 (95%CL)	< 0.24 (95%CL)
tensors & running, $r \neq 0$ & $\alpha_s \neq 0$		
n_s	1.076 ± 0.065	1.070 ± 0.060
r	< 0.49 (95%CL)	< 0.49 (95%CL)
α_s	-0.048 ± 0.029	-0.042 ± 0.024
non-Gaussianities f_{NL}		
Local	$-10 < f_{NL}^{\rm local} < 74$ (95%CL)	–
Equilateral	$-214 < f_{NL}^{\rm equil} < 266$ (95%CL)	–
Orthogonal	$-410 < f_{NL}^{\rm orth} < 6$ (95%CL)	–

Table 3.2: Constraints on inflationary models from a combination of CMB and LSS data [23]. The reference scale is $k_0 = 0.002$ Mpc^{-1} and the errors indicate the 68% confidence levels (CL).

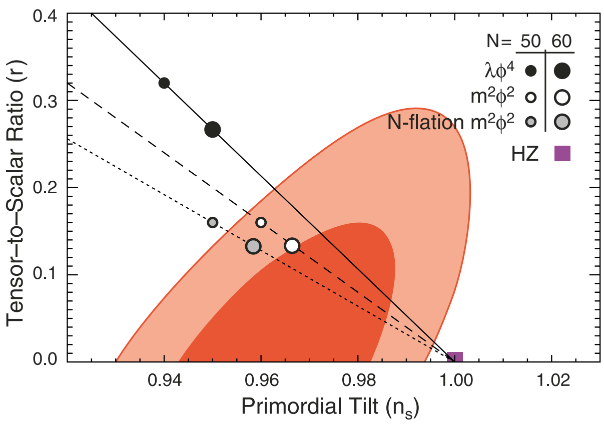

Figure 3.7: Contour plot with 68% and 95% confidence level constraints in the n_s-r-plane. The symbols and lines indicate the predictions from two models of single-field chaotic inflation (solid and dashed lines) and a model of multi-axion inflation (dotted line). Figure taken from [23].

Gaussianity To get a feeling for the constraints on possible non-Gaussianities, let us consider the following parametrization which is *local* in position space [315]

$$\mathcal{R}(\vec{x}) = \mathcal{R}_g(\vec{x}) + \frac{3}{5} f_{NL}^{\rm local}\, \mathcal{R}_g(\vec{x})^2\,, \tag{3.75}$$

where $\mathcal{R}_g$ is a Gaussian fluctuation. Now for $\mathcal{R}_g \sim 10^{-5}$ even a value as large as $f_{NL}^{\rm local} \sim 100$ would only lead to a deviation from a Gaussian spectrum of

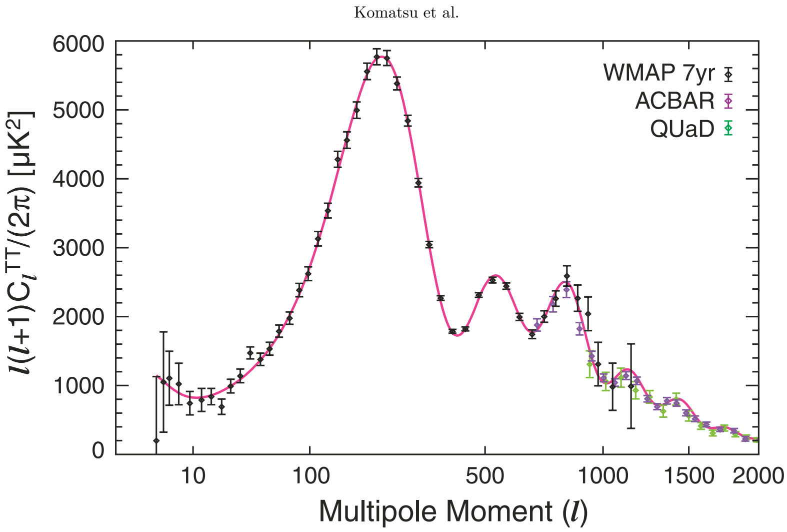

Figure 3.8: Temperature power spectrum expanded in multipole moments. Figure taken from [23].

$\sim 0.1\%$. We quoted the current constraint on f_{NL}^{local} (and on two other possible forms of non-Gaussianities) in Tab. 3.2, which show that the CMB spectrum is indeed *Gaussian* to a very high degree. This fits nicely with the way inflation produces density fluctuations via quantum fluctuations of the inflaton.[2] Moreover, non-Gaussianities essentially measure how strongly the inflaton field interacts. Slow-roll inflation, however, requires the self-interactions of the inflaton (encoded in its potential) to be very small and thus predicts only a very small non-Gaussianity $f_{NL} \sim \mathcal{O}(\epsilon, \eta) \ll 1$ [314].

Adiabaticity Single-field inflation generically leads to primordial density perturbations which are *adiabatic*: All density perturbations of "matter" (including photons, neutrinos, baryons and dark matter) originated from the *same* curvature perturbation $\mathcal{R}$ and thus there is *no relative* difference in the density for different "matter" components, only a variation in the total density. Adiabaticity is sometimes expressed as $\delta(n_{\text{non-rel}}/n_{\text{rel}}) = 0$ or

$$\frac{\delta\rho_{\text{non-rel}}}{\rho_{\text{non-rel}}} = \frac{3}{4}\frac{\delta\rho_{\text{rel}}}{\rho_{\text{rel}}}\,, \tag{3.76}$$

with the indices "non-rel" and "rel" collectively denoting all non-relativistic (*e. g.* cold dark matter) and relativistic species (*e. g.* photons), respectively. The current CMB data shows no violation of this condition [23], but the search for a violation is interesting since it would signal multiple fields were dynamically relevant during inflation [316].

[2]This is true in a specific gauge, the *spatially flat gauge*, where $\Psi = 0$ and $\mathcal{R}$ is entirely determined by the fluctuations $\delta\phi$.

3.4.4 Lyth Bound and Energy Scale of Inflation

There is a huge number of inflationary models consistent with current data on the CMB fluctuations. An important quantity for discriminating between different models is the tensor-to-scalar ratio r, because the *Lyth bound* [213] implies that r is related to the distance $\Delta\phi$ traversed in field space during inflation as

$$\frac{\Delta\phi}{M_P} = \mathcal{O}(1) \times \left(\frac{r}{0.01}\right)^{1/2} , \tag{3.77}$$

Thus, large values of $r > 0.01$ at the sensitivity of current CMB experiments such as Planck require $\Delta\phi > M_P$, *i. e.* are only possible for *large-field* models of slow-roll inflation.

In addition, $\Delta_s^2 \sim 10^{-9}$ is fixed by observation and $\Delta_t^2 \propto \mathcal{H}^2 \sim V$. Thus, an observation of gravitational waves would allow us to constrain the energy scale of inflation:

$$V^{1/4} \sim \left(\frac{r}{0.01}\right)^{1/2} 10^{16}\ \text{GeV} . \tag{3.78}$$

Hence, if we would observe gravitational waves in the near future (*e. g.* in the Planck experiment), inflation would have occurred at energy scales around the GUT scale.

If only a single field has relevant dynamics during inflation, the scalar spectral index and the tensor-to-scalar ratio provide direct information about the inflaton potential $V(\phi)$ driving inflation. One may try to use CMB data to determine the coefficients of a series expansion of $V(\phi)$ around $\phi \approx \phi_{\text{CMB}}$, which denotes the value of ϕ when the fluctuations in the CMB became superhorizon.[3]

3.5 Simple Example: $m^2\,\phi^2$ Chaotic Inflation

To conclude this chapter, we compute the predictions for the simplest model of single-field slow-roll inflation: Chaotic inflation with potential [214]

$$V(\phi) = \frac{m^2}{2}\phi^2 . \tag{3.79}$$

We can now straightforwardly apply the formulas reviewed in Sec. 3.3 and 3.4.

The slow-roll parameters then become

$$\epsilon(\phi) = \eta(\phi) = 2\left(\frac{M_P}{\phi}\right)^2 , \tag{3.80}$$

which both are small only for values of ϕ well above the Planck scale,

$$\phi > \sqrt{2}M_P \equiv \phi_{\text{end}} . \tag{3.81}$$

[3]Classically, the inflaton monotonically evolves in time and thus one may use ϕ and cosmic time t interchangeably.

The number of e-folds as a function of ϕ turns out to be given by

$$N_e(\phi) = \frac{\phi^2}{4M_P^2} - \frac{1}{2}\,. \tag{3.82}$$

Thus, since we must have around ~ 60 e-folds of inflation after the CMB fluctuations were created, inflation must have occurred at field values $\gg M_P$,

$$\phi_{\rm CMB} = 2\sqrt{N_{e,\rm CMB}}M_P \sim 15 M_P \gg M_P\,. \tag{3.83}$$

It is at this value that the slow-roll parameters ϵ and η need, *i. e.*

$$\epsilon_\star = \eta_\star = 2\left(\frac{M_P}{\phi_{\rm CMB}}\right)^2 = \frac{1}{2N_{e,\rm CMB}}\,. \tag{3.84}$$

Note that both are *independent* of the mass parameter m. Actually, this is true for all of the simplest models of chaotic inflation based on monomial potentials $V = \lambda_p\,\phi^p$ have this property. The value of m (or λ_p for the other models) is fixed by matching the prediction for the amplitude of the scalar power spectrum,

$$\Delta_s^2 = \frac{m^2}{M_P^2}\frac{N_{e,\rm CMB}^2}{3}\,. \tag{3.85}$$

To get $\Delta_s^2 \sim 10^{-9}$ requires $m \sim 10^{-6} M_P$ for $N_{e,\rm CMB} \sim 60$. After m is fixed, the model predicts

$$n_s = 1 + 2\,\eta_\star - 6\,\epsilon_\star = 1 - \frac{2}{N_{e,\rm CMB}} \approx 0.96\,, \tag{3.86}$$

and

$$r = 16\,\epsilon_\star = \frac{8}{N_{e,\rm CMB}} \approx 0.1\,. \tag{3.87}$$

Comparing these predictions to observations (cf. Tab. 3.2 and Fig. 3.7), we see that the scalar spectral index n_s fits quite well with observations and the tensor-to-scalar ratio $r \sim 0.1$ is in a range in which it could be excluded by the Planck satellite.

Chapter 4

Basics of 4d $\mathcal{N} = 1$ Supergravity

The next concept we introduce is *supersymmetry*, an extension of the Poincaré symmetry group of spacetime relating bosonic and fermionic degrees of freedom. In particular, if supersymmetry would be unbroken, we would have to observe for example a scalar version of the electron, the *selectron*, with exactly the same quantum numbers except for the spin, *i. e.* the same mass and the same electric charge. This clearly contradicts what we observe. Therefore, if supersymmetry is realized in nature, it must be broken. As we will see later on, we also need to break supersymmetry to realize a phase of inflation driven by a positive vacuum energy in the very early universe. Moreover, recall from Sec. 2.3 that we will focus on supersymmetric models of inflation, because they offer a better control over quantum corrections to the scalar potential. The aim of this chapter is to briefly introduce the ideas and notions we need to describe a huge class of supersymmetric models of inflation.

First, we introduce the algebra of global supersymmetry in Sec. 4.1.1. Next, in Sec. 4.1.2, we introduce first the concept of superspace and afterwards discuss the chiral superfield and real superfield representations of the supersymmetry algebra. The focus will be on writing down supersymmetric actions. Afterwards, we briefly outline the changes required to include gravity by moving on to locally supersymmetric theories in Sec. 4.2. Finally, in Sec. 4.3, we show how spontaneous supersymmetry breaking is encoded and give a few simple examples.

The discussion presented in this chapter mostly follows [317]. We do not intend to give a complete and comprehensive review of supersymmetric theories and for more details we refer to the excellent textbooks, lecture notes and review articles available, for example cf. [171, 317–320]. For instance, we do not discuss how to embed the standard model into a supersymmetric theory (a nice review of the minimally supersymmetric standard model can be found in [171]).

4.1 Basics of 4d $\mathcal{N} = 1$ Global Supersymmetry

4.1.1 Supersymmetry Algebra

Supersymmetry is an extension of the Poincaré algebra of space-time transformations. Due to the Coleman-Mandula theorem [321], such an extension of the space-time symmetry group is only possible via fermionic generators.

The combined algebra of Poincaré and supersymmetry transformations is

$$\begin{aligned} \{Q^A_\alpha, \bar{Q}_{\dot{\beta},B}\} &= 2\, (\sigma^m)_{\alpha\dot{\beta}}\, P_m\, \delta^A_B\,, \\ \{Q^A_\alpha, Q^B_\beta\} &= \epsilon_{\alpha\beta} Z^{AB}\,, \\ \left[P_m, Q^A_\alpha\right] &= 0\,, \\ [P_m, P_n] &= 0\,, \end{aligned} \tag{4.1}$$

and similarly for the ones with $\bar{Q}$ not written down explicitly. Here, P_m denotes the generators of space-time translations, while Q^A_α and $\bar{Q}_{\dot{\beta},B}$ denote the extra fermionic generators. As usual, $[a, b]$ and $\{a, b\}$ denote commutator and anti-commutators, respectively. The Greek indices $(\alpha, \beta, \ldots, \dot{\alpha}, \dot{\beta}, \ldots)$ run from one to two and transform as two-component Weyl spinors under the Poincaré algebra. The Latin indices $(m, n, \ldots)$ running from one to four transform as Lorentz four-vectors. The capital indices $(A, B, \ldots)$ label possible sets of $Q, \bar{Q}$'s and take values form 1 to some number $\mathcal{N} \geq 1$. Algebras with $\mathcal{N} > 1$ are called extended supersymmetry algebras (and for these the Z^{AB} play a role). In this thesis, however, we will be mostly concerned with $\mathcal{N} = 1$ supersymmetry since only $\mathcal{N} = 1$ supersymmetry has chiral representations, which are required to describe *e. g.* electroweak interactions.

4.1.2 Superspace & Superfields

The smallest irreducible representations of the $\mathcal{N} = 1$ supersymmetry algebra Eq. (4.1) are the following multiplets.

- **Chiral Supermultiplet** (ϕ, ψ_α): it consists of a complex scalar ϕ and a two-component Weyl spinor ψ_α.
- **Real Supermultiplet** (λ_α, A_m): it consists of a gauge field A_m and a spin-1/2 spinor λ_α; in general both fields transform in the adjoint representation of the gauge group.
- **Gravity Supermultiplet** $(\psi_{m,\alpha}, g_{mn})$: it consists of a spin-2 symmetric tensor g_{mn}, the graviton, and a spin-3/2 spinor $\psi_{m,\alpha}$, the gravitino.

For us, the chiral multiplets will be particularly important since they contain scalar fields, *i. e.* candidates for the inflaton field. The other multiplets are used to supersymmetrize gauge interactions and gravity. We now briefly review how to write down supersymmetric actions for chiral superfields in global supersymmetry and supergravity.

Superspace

A very elegant concept is the *superspace* (see *e. g.* [318]) which greatly simplifies writing down supersymmetric actions. It allows us to organize the multiplets of supersymmetry into so-called *superfields*.

The starting point is to add to the coordinates x^μ, $\mu = 0, \dots 3$, a set of "fermionic coordinates" θ^α, $\alpha = 1, 2$, transforming as Weyl spinors as well as a set $\bar{\theta}^{\dot{\alpha}} = (\theta^\alpha)^*$. These coordinates are *anti-commuting Grassmann numbers*, *i. e.* they satisfy

$$\left\{\theta^\alpha, \theta^\beta\right\} = \left\{\theta^\alpha, \bar{\theta}^{\dot{\alpha}}\right\} = \left\{\bar{\theta}^{\dot{\alpha}}, \bar{\theta}^{\dot{\beta}}\right\} = 0\,. \tag{4.2}$$

Note that this implies, in particular, $(\theta^1)^2 = (\theta^2)^2 = 0$, and analogously for $\bar{\theta}^{\dot{\alpha}}$.

The basic idea is to realize the supersymmetry algebra as differential operators on superspace, completely analogous to what is done for the Poincaré Algebra, *e. g.* $P_m = -i\partial_m$. One can now define $Q, \bar{Q}$ as

$$Q_\alpha = \partial_\alpha - i\left(\sigma^m\right)_{\alpha\dot{\alpha}} \bar{\theta}^{\dot{\alpha}} \partial_m\,, \tag{4.3}$$

$$\bar{Q}_{\dot{\alpha}} = -\bar{\partial}_{\dot{\alpha}} + i\theta^\alpha \left(\sigma^m\right)_{\alpha\dot{\alpha}} \partial_m\,. \tag{4.4}$$

With this definition the anti-commutation relations of $Q, \bar{Q}$ are given by

$$\left\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\right\} = 2i\left(\sigma^m\right)_{\alpha\dot{\alpha}} \partial_m\,, \tag{4.5}$$

$$\left\{Q_\alpha, Q_\beta\right\} = \left\{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\right\} = 0\,. \tag{4.6}$$

The most general representation one may consider is a generic field in superspace, $A = A(x, \theta, \bar{\theta})$, which can be Taylor-expanded in the θ's since the expansion ends because of anticommuation:

$$\begin{aligned} A(x,\theta,\bar{\theta}) = {} & f(x) + \theta\phi(x) + \bar{\theta}\bar{\chi}(x) + \theta^2 m(x) + \bar{\theta}^2 n(x) \\ & + \theta\sigma^m\bar{\theta} v_m(x) + \theta^2\bar{\theta}\bar{\lambda}(x) + \bar{\theta}^2\theta\psi(x) + \theta^2\bar{\theta}^2 d(x)\,. \end{aligned} \tag{4.7}$$

The supersymmetry transformations of the components $f(x), \phi(x), \dots$ are computed by applying $\delta_\epsilon \equiv (\epsilon Q + \bar{\epsilon}\bar{Q})$ to each term in A and then organizing the result into an expansion in $\theta, \bar{\theta}$, which is then interpreted as

$$\delta_\epsilon A = (\delta_\epsilon f)(x) + \theta(\delta_\epsilon \phi)(x) + \dots\,. \tag{4.8}$$

However, this representation is pretty complicated and, moreover, reducible. To reduce the general superfield above, we introduce another set of differential operators $D_\alpha, \bar{D}_{\dot{\alpha}}$ defined as

$$D_\alpha = \partial_\alpha + i\left(\sigma^m\right)_{\alpha\dot{\alpha}} \bar{\theta}^{\dot{\alpha}} \partial_m\,, \tag{4.9}$$

$$\bar{D}_{\dot{\alpha}} = -\bar{\partial}_{\dot{\alpha}} - i\theta^\alpha \left(\sigma^m\right)_{\alpha\dot{\alpha}} \partial_m\,. \tag{4.10}$$

They satisfy the anti-commutation relations

$$\left\{D_\alpha, \bar{D}_{\dot\alpha}\right\} = -2i\left(\sigma^m\right)_{\alpha\dot\alpha}\partial_m\,, \tag{4.11}$$

$$\left\{D_\alpha, D_\beta\right\} = \left\{\bar{D}_{\dot\alpha}, \bar{D}_{\dot\beta}\right\} = 0\,. \tag{4.12}$$

In addition, all D's anti-commute with all Q's, *i. e.*

$$\left\{D_\alpha, \bar{Q}_{\dot\alpha}\right\} = 0\,. \tag{4.13}$$

These additional operators are useful because if $A(x,\theta,\bar\theta)$ obeys $\bar{D}A = 0$ so does $\delta_\epsilon A$. Thus, the differential operators $D, \bar{D}$ provide a way to project out or relate components of the general superfield.

Chiral Superfields

We now first consider *chiral superfields* (or *scalar superfields*), which are general superfields Φ satisfying $\bar{D}\Phi = 0$. One may check that this implies that $\bar\Phi$ is an anti-chiral superfield satisfying $D\bar\Phi = 0$.

Let us define the quantity

$$y^m = x^m + i\theta\sigma^m\bar\theta\,, \tag{4.14}$$

which fulfills $\bar{D}y = 0$. Then any function $\Phi = \Phi(y,\theta)$ automatically satisfies $\bar{D}\Phi = 0$.[1] We can expand Φ into components which yields

$$\Phi = \phi(y) + \sqrt{2}\theta\psi(y) + \theta^2 F(y)\,. \tag{4.15}$$

The component fields have the following supersymmetry transformations:

$$\delta_\epsilon\phi = \sqrt{2}\epsilon\psi\,, \tag{4.16}$$

$$\delta_\epsilon\psi = i\sqrt{2}\sigma^m\bar\epsilon\partial_m\phi + \sqrt{2}\epsilon F\,, \tag{4.17}$$

$$\delta_\epsilon F = i\sqrt{2}\bar\epsilon\bar\sigma^m(\partial_m\psi)\,. \tag{4.18}$$

To write down an action for chiral superfields, first note that if Φ is a chiral superfield, any analytic function $W(\Phi)$ is also a chiral superfield. From the supersymmetry transformations of chiral superfields, one can check that $\delta_\epsilon F$ is a total derivative ($\bar\epsilon$ and σ are constant). Hence, defining the projection onto the θ^2 component of $W(\Phi)$ as $W(\Phi)|_{\theta^2}$, any term of the form

$$\int d^4x\, W(\Phi)\Big|_{\theta^2}\,, \tag{4.19}$$

is invariant under supersymmetry transformations.[2] If we define the integration over a Grassmann number θ as

$$\int d\theta\, 1 = 0\,, \qquad \int d\theta\,\theta = 1\,, \tag{4.20}$$

[1] By abuse of notation, the same symbol is used for both $\Phi(x,\theta,\bar\theta)$ and $\Phi(y,\theta)$.

[2] Assuming that space-time has no boundaries.

the projection on the θ^2 component can actually be written as

$$W(\Phi)\Big|_{\theta^2} = \int d\theta^2\, W(\Phi)\,. \tag{4.21}$$

In addition to the analytic function $W(\Phi)$, we can also consider a function $K(\Phi,\bar{\Phi})$ which, however, is *not* chiral. But one can show that the highest component $\theta^2\bar{\theta}^2 d(x)$ of such a general superfield transforms into a total derivative.[3] Therefore, also any term of the form

$$\int d^4x\, K(\Phi,\bar{\Phi})\Big|_{\theta^2\bar{\theta}^2} = \int d^4x d^2\theta d^2\bar{\theta}\, K(\Phi,\bar{\Phi})\,, \tag{4.22}$$

is invariant under the supersymmetry transformations. The full general Lagrangian for a single chiral superfield Φ is then given by

$$\mathcal{L} = K(\Phi,\bar{\Phi})\Big|_{\theta^2\bar{\theta}^2} + \left(W(\Phi)\Big|_{\theta^2} + h.c.\right)\,, \tag{4.23}$$

with $K(\Phi,\bar{\Phi})$ some *real* function of Φ and $\bar{\Phi}$, the so-called *Kähler potential* and $W(\Phi)$ some *holomorphic* function of Φ, the so-called *superpotential.*

Let us consider the simplest example, the *Wess-Zumino* model [322]

$$K = \Phi\bar{\Phi}\,, \quad W = \frac{m}{2}\Phi^2 + \frac{\lambda}{3}\Phi^3\,. \tag{4.24}$$

The Lagrangian in component form becomes[4]

$$\begin{aligned}\mathcal{L} = &-\partial_m\phi\partial^m\bar{\phi} - i\bar{\psi}\bar{\sigma}^m\partial_m\psi + |F|^2\\ &+ m(\phi F - \frac{1}{2}\psi^2) + h.c.\\ &+ \lambda(\phi^2 F + \psi^2\phi) + h.c.\,.\end{aligned} \tag{4.25}$$

The crucial point to notice now is that no derivatives of F are involved, *i. e.* it is *not* propagating and just an *auxiliary field*, which we can integrate out by solving its equations of motion. This yields

$$\mathcal{L} = -|\partial\phi|^2 - i\bar{\psi}\bar{\sigma}^m\partial_m\psi - \frac{m}{2}\psi^2 + \lambda\psi^2\phi - V(\phi,\bar{\phi})\,, \tag{4.26}$$

where

$$V(\phi,\bar{\phi}) = |F|^2\,, \quad \text{and} \quad F = -m\bar{\phi} - \lambda\bar{\phi}^2\,. \tag{4.27}$$

$V(\phi,\bar{\phi})$ is the scalar potential and it is determined by the F-component of the chiral superfield Φ.

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[3] This can be understood, for instance, by counting the mass dimensions. In units where $\hbar = c = 1$ we have $[\partial_m] = 1$. Now $Q,\bar{Q} \sim \partial_\alpha, \bar{\partial}_{\dot{\alpha}} + \ldots$ satisfy $\{Q,\bar{Q}\} \sim \partial_m$ and hence $[\partial_\alpha] = \frac{1}{2}$ such that $[\theta^\alpha] = -\frac{1}{2}$. A canonical scalar field has mass dimension $[\phi] = 1$, and thus for a chiral superfield $\Phi \sim \phi + \theta\psi + \theta^2 F$ we must have $[\psi] = \frac{3}{2}$ and $[F] = 2$. Now $QF \sim \frac{\partial}{\partial\theta}F$ has mass dimension $\frac{5}{2}$ and therefore must involve a derivative ∂_m. The same line of arguments now also applies to the highest component $\theta^2\bar{\theta}^2 d(x)$ of a general superfield.

[4] The kinetic term for ϕ comes from $K = \Phi\bar{\Phi}$. To understand this, recall that $\Phi = \Phi(y,\theta)$ with $y^m = x^m + i\theta\sigma^m\bar{\theta}$. Taylor expanding the lowest component $\phi(y)$ then yields the kinetic term.

Non-Renormalization Theorem

The superpotential W enjoys a *non-renormalization theorem*, *i. e.* it does *not* receive perturbative loop corrections [323] (see also [324] for a simple argument). Though, in general, there can be non-perturbative corrections to the superpotential and those will become important later on for the subject of *moduli stabilization* in Sec. 6.2. The Kähler potential K, however, does *not* enjoy such a non-renormalization theorem and this for example has consequences for inflationary model building in supegravity, cf. Sec. 5.3.

Real Superfields

The second type of superfields we consider here are *real superfields* (or *vector superfields*), which are general superfields $V(x,\theta,\bar\theta)$ satisfying the constraint $V=\bar V$.[5] This is again compatible with the supersymmetry transformations. Expanding V yields

$$\begin{aligned} V(x,\theta,\bar\theta) &= c(x)+\theta\chi(x)+\bar\theta\bar\chi(x)+\theta^2M(x)+\theta^2\bar M(x) && (4.28)\\ &\quad -\theta\sigma^m\bar\theta A_m(x)+i\theta^2\bar\theta\bar\lambda(x)-i\bar\theta^2\theta\lambda(x)+\frac{1}{2}\theta^2\bar\theta^2D(x)\,. && (4.29)\end{aligned}$$

where c, D, and A_m are real. To build consistent models involving vector field A_m it is usually necessary to require an action invariant under a gauge transformation of the form

$$A_m \to A_m+\partial_m\lambda\,. \tag{4.30}$$

To find the supersymmetric version of this, we first replace λ by a chiral superfield Λ with components

$$\Lambda(x,\theta,\bar\theta)=\omega(y)+\sqrt{2}\theta\psi(y)+\theta^2F(y)\,, \tag{4.31}$$

which at the component level reproduces Eq. (4.30). However, $V+\Lambda$ is *not* real. Therefore, the gauge transformation of a real superfield V is instead defined as

$$2V\to 2V+\Lambda+\bar\Lambda\,, \tag{4.32}$$

with Λ a chiral superfield. It is convenient to partially fix the gauge such that the three lowest components of $2V+\Lambda+\bar\Lambda$ vanish. This is the Wess-Zumino gauge which breaks supersymmetry, but leaves us with the gauge transformation in Eq. (4.30) instead of the full one in Eq. (4.32). In this gauge, the real superfield is determined by the component fields A_m, λ and D as

$$V=-\theta\sigma^m\bar\theta A_m(x)+i\theta^2\bar\theta\bar\lambda(x)-i\bar\theta^2\theta\lambda(x)+\frac{1}{2}\theta^2\bar\theta^2D(x)\,. \tag{4.33}$$

[5] Note that often a † is used to indicate complex conjugation, *i. e.* the real superfield is then defined by $V=V^\dagger$.

To build an action for a gauge field, one usually constructs the field strength F_{mn} out of derivatives of A_m. The supersymmetric analogue of F_{mn} is obtained using $D, \bar{D}$ to define the supersymmetric field strength as

$$\mathcal{W}_\alpha = -\frac{1}{4}\bar{D}^2 D_\alpha V \,. \tag{4.34}$$

One can show that $\mathcal{W}_\alpha$ is chiral and gauge-invariant. In components, $\mathcal{W}$ is given by

$$\mathcal{W} = -i\lambda(y) + (D(y) - i\sigma^{mn}F_{mn}(y)) \cdot \theta + \theta^2\sigma^m\partial_m\bar{\lambda}(y) \,, \tag{4.35}$$

where as usual $F_{mn} = \partial_m A_n - \partial_n A_m$ and $\sigma^{mn} = \frac{1}{4}(\sigma^m\bar{\sigma}^n - \sigma^n\bar{\sigma}^m)$. The lowest-order gauge-invariant and Lorentz-invariant Lagrangian built out of $\mathcal{W}_\alpha$ is given by

$$\mathcal{L} = \frac{1}{2g^2}\left(\mathcal{W}^\alpha\mathcal{W}_\alpha\Big|_{\theta^2} + h.c.\right) \,, \tag{4.36}$$

which in terms of component fields becomes[6]

$$\mathcal{L} = -\frac{1}{4g^2}F_{mn}F^{mn} - i\bar{\lambda}\bar{\sigma}^m\partial_m\lambda + \frac{1}{2g^2}|D|^2 \,. \tag{4.37}$$

D is again an auxiliary field like the F for the chiral superfield, A_m describes a massless gauge field and λ is a Weyl spinor, the *gaugino*.

Coupling to Matter Matter is usually described by chiral superfields. The gauge transformation for a complex scalar $\varphi(x)$ under a $U(1)$ is

$$\varphi(x) \to e^{-iq\lambda(x)}\varphi(x) \,, \tag{4.38}$$

together with $A_m(x) \to A_m(x) + \partial_m\lambda(x)$. Then the "minmally coupled" Lagrangian involves $(D_m\varphi)^*(D^m\varphi)$ with $D_m = \partial_m + iA_m$. The supersymmetric analogue for a chiral superfield Φ is

$$\Phi \to e^{-\Lambda}\Phi \,. \tag{4.39}$$

However, now the kinetic term $\Phi\bar{\Phi}|_{\theta^2\bar{\theta}^2}$ is no longer gauge-invariant. To fix this, it is replaced by

$$\bar{\Phi}e^{2V}\Phi\Big|_{\theta^2\bar{\theta}^2} \,. \tag{4.40}$$

The simplest Lagrangian then consists of

$$\mathcal{L} = \bar{\Phi}e^{2V}\Phi\Big|_{\theta^2\bar{\theta}^2} + \frac{1}{2g^2}\left(\mathcal{W}^\alpha\mathcal{W}_\alpha\Big|_{\theta^2} + h.c.\right) \,. \tag{4.41}$$

[6]In general, there is also a term $F_{mn}\tilde{F}^{mn} \equiv \epsilon^{mnpq}F_{mn}F_{pq}$. However, we ignore this possibility for now and give the more general expression later in Sec 4.2.

In components, this Lagrangian reads

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4g^2}F_{mn}F^{mn} - i\bar{\lambda}\bar{\sigma}^m D_m\lambda + \frac{1}{2g^2}|D|^2 \\ &- |D_m\phi|^2 - i\bar{\psi}\bar{\sigma}^m D_m\psi + |F|^2 \\ &+ i\sqrt{2}\left(\phi^\dagger\lambda\psi - \psi^\dagger\bar{\lambda}\phi\right) + \phi^\dagger D\phi\,.\end{aligned} \tag{4.42}$$

After integrating out the auxiliary fields F and D, there are contributions to the scalar potential for ϕ from both the F- and the D-component, $V = V_F + V_D$. V_F and V_D are called the *F-term* and *D-term potential*, respectively. They are given by

$$V_F = |F|^2\,, \quad \text{and} \quad V_D = \frac{1}{2g^2}|D|^2\,, \tag{4.43}$$

with

$$F = \frac{\partial W}{\partial\bar{\Phi}}\,, \quad \text{and} \quad D = g^2\phi^\dagger\phi\,. \tag{4.44}$$

This concludes our overview of $\mathcal{N} = 1$ global supersymmetry. We will give the general expressions involving multiple chiral superfields Φ_i and for non-Abelian gauge symmetries below only for the case of local supersymmetry.

4.2 Basics of 4d $\mathcal{N} = 1$ Supergravity

Our aim is to embed the concept of inflation into a supersymmetric theory. Therefore, we need to couple a supersymmetric theory to gravity. This amounts, in particular, to promoting supersymmetry to a *local* symmetry and the resulting theories are so-called *supergravity* theories. We do not go into the details of how this procedure is done exactly, but merely quote the results which are important for this thesis.

Namely, similar to globally supersymmetric theories, the terms in the effective action obtained from a set of chiral superfields Φ_i, which include up to two derivatives or four fermions, are fully determined by the following functions of the chiral superfields.

- The *Kähler potential*, a *real* function $K(\Phi, \bar{\Phi})$.
- The *superpotential*, a *holomorphic* function $W(\Phi_i)$.
- The *gauge-kinetic functions*, a set of *holomorphic* functions $f_{ab}(\Phi_i)$, one for each gauge group factor $\mathcal{G}_a$.

What we are interested in is mainly the form of the Lagrangian for the scalar components of the chiral superfields in terms of these functions.

In a frame in which the Einstein-Hilbert term takes the canonical form, *i. e.*

$$\mathcal{L}_{\text{grav}} = \frac{M_P^2}{2}\sqrt{-g}\,R\,, \tag{4.45}$$

the Lagrangian for the scalar fields is given by

$$\frac{1}{\sqrt{-g}}\mathcal{L}_{\text{scalar}} = -K_{i\bar{\jmath}}\,\partial_m\phi^i\partial^m\bar{\phi}^{\bar{\jmath}} - V_F - V_D + \text{terms with fermions}\,, \tag{4.46}$$

where V_F and V_D are the supergravity *F-term* and *D-term potentials*, respectively. Here, the index i denotes a derivative with respect to ϕ_i, while the index $\bar{\jmath}$ denotes a derivative with respect to $\bar{\phi}_{\bar{\jmath}}$.

The F-term scalar potential for a set of chiral superfields $\{\Phi_i\}$ is given by the following expression:

$$V_F = e^{K/M_P^2}\left((K_{i\bar{\jmath}})^{-1}\,D_iW D_{\bar{\jmath}}\overline{W} - \frac{3}{M_P^2}|W|^2\right)\,, \tag{4.47}$$

where

$$D_iW \equiv W_i + \frac{1}{M_P^2}\,K_i\,W\,. \tag{4.48}$$

The kinetic terms for the scalar components ϕ_i of the Φ_i are determined by $K_{i\bar{\jmath}}$. This quantity is the so-called *Kähler metric.* It is the metric of the space in which the ϕ_i take their values. For $\mathcal{N}=1$ supergravity theories, this space must be a Kähler manifold [325, 326], for which the metric is obtained from a real function K as

$$g_{i\bar{\jmath}} = \partial_i\bar{\partial}_{\bar{\jmath}}K\,. \tag{4.49}$$

This is precisely why $K_{i\bar{\jmath}}$ and K are called the Kähler metric and the Kähler potential, respectively.

The simplest choice for the Kähler potential is the one for which the kinetic terms are canonical, *i. e.*

$$K = \sum_i |\Phi_i|^2\,. \tag{4.50}$$

But in principle any real function is allowed.

An important consequence of the Kähler geometry is an invariance of the action under *Kähler transformations*, that is transformations of the form

$$\begin{aligned} K(\Phi,\bar{\Phi}) &\to K(\Phi,\bar{\Phi}) + f(\Phi) + \bar{f}(\bar{\Phi})\,,\\ W(\Phi) &\to e^{-f(\Phi)}\,W(\Phi)\,. \end{aligned} \tag{4.51}$$

Defining the quantity G, which is invariant under Kähler transformations, as (setting $M_P \equiv 1$)

$$G \equiv K + \ln W + \ln\overline{W}\,, \tag{4.52}$$

the F-term potential can be written as

$$V_F = e^G\left(G^{i\bar{\jmath}}G_iG_{\bar{\jmath}} - 3\right)\,. \tag{4.53}$$

Note that since W is independent of $\bar{\Phi}$ we have $G^{i\bar{\jmath}} = K^{i\bar{\jmath}} \equiv (K_{i\bar{\jmath}})^{-1}$.

If some Φ_i are charged under some gauge group $\mathcal{G}_a$, the partial derivatives ∂_m in the kinetic term in Eq. (4.46) have to be replaced by covariant derivatives D_m. The kinetic terms for the gauge fields are given by

$$\frac{1}{\sqrt{-g}}\mathcal{L}_{\text{gauge}} = -\frac{1}{4}\mathrm{Re}\,(f_{ab})\,F_{mn}^{(a)}F^{mn(b)} - \frac{1}{8}\mathrm{Im}\,(f_{ab})\,\epsilon^{mnpq}F_{mn}^{(a)}F_{pq}^{(b)}\,. \tag{4.54}$$

Thus, the gauge-kinetic functions f_a encode the gauge couplings g_a as $\mathrm{Re}\,f_a \sim g_a^{-2}$ and $\mathrm{Im}\,f_a \sim \theta_a$ encodes the possible θ-terms or axions. To get the standard kinetic terms for the gauge fields, one may choose $f_{ab} = g_a^{-2}\delta_{ab}$.

In addition to the F-term contribution, there is now also a contribution to the scalar potential from the D-terms which is given by

$$V_D = \sum_a \frac{1}{2}\,(\mathrm{Re}\,f_{ab})^{-1}\,D_a D_b\,, \tag{4.55}$$

with

$$D_a = K_i\,(T_a)^{ij}\,\Phi_j\,, \tag{4.56}$$

where T_a is a generator in the appropriate representation.

The above is all we need to know for the purposes of this thesis. From the point of view of supergravity, *any* choice of functions G, which is typically specified in terms of K and W, and f_{ab} is allowed. However, as we will see later on, string theory imposes some restrictions on the form and functional dependence of these functions.

4.3 Spontaneous Breaking of Supersymmetry

If supersymmetry is unbroken, the vacuum energy in a globally (locally) supersymmetric theory is vanishing (negative) and thus one obtains a Minkowski (Anti-de-Sitter) spacetime.[7] Inflation requires a large positive vacuum energy and thus we have to break supersymmetry. However, we do not like to break it explicitly but *spontaneously*, *i. e.* we like to have a supersymmetric theory with non-supersymmetric ground state(s).[8]

But let us first understand the statements about the supersymmetric ground states of supersymmetric theories. We begin by noting that in a globally supersymmetric theory the Hamiltonian $H = P_0$ can be written as

$$H = \frac{1}{4}\left(\bar{Q}_1 Q_1 + Q_1\bar{Q}_1 + \bar{Q}_2 Q_2 + Q_2\bar{Q}_2\right)\,, \tag{4.57}$$

[7]The value of the scalar potential at the minimum acts like a cosmological constant and spaces with negative cosmological constant are Anti-de-Sitter.

[8]In general, there can be multiple ground states which are either supersymmetric or not.

as a result of the supersymmetry algebra. Unbroken supersymmetry corresponds to $Q|0\rangle = 0$, $\bar{Q}|0\rangle = 0$ which is the case if and only if $\langle 0|H|0\rangle = 0$. Now recall that the scalar potential is given by

$$V \equiv V_F + V_D = \sum_i |F_i|^2 + \sum_a \frac{1}{2g^2}|D_a|^2 \,, \tag{4.58}$$

where i runs over the set of chiral superfields Φ_i and a runs over the different gauge groups $\mathcal{G}_a$. This is a positive definite quantity and thus a vanishing ground state energy corresponds to vanishing expectation values for all F_i and D_a. In other words, $\langle F_i\rangle$ and $\langle D_a\rangle$ are the order parameters of spontaneous supersymmetry breaking. This statement remains true also in supergravity, but with a few important changes: The supergravity generalization of the F_i's are the D_iW's and more importantly V_F contains a *negative* term $\sim -|W|^2$ (cf. Eq. (4.47)). This is important since it allows us to fine-tune the cosmological constant Λ to a small value. Constructing a model of spontaneous supersymmetry breaking therefore amounts to finding a model with a minimum for the scalars ϕ_i such that not all F_i and D_a vanish simultaneously.

The spontaneous breaking of a continuous global symmetry gives rise to a Goldstone boson, a massless boson. Similarly, spontaneous breaking of global supersymmetry gives rise to the *goldstino*, a massless fermion, since the supersymmetry variations δ_ϵ are fermionic. In locally supersymmetric theories, *i. e.* supergravities, the goldstino is eaten by the gravitino to provide the degrees of freedom required for a massive spin-3/2 particle. This is in full analogy to the Higgs mechanism for (electroweak) symmetry breaking, where the would-be Goldstone modes become the longitudinal degrees of freedom of the $W^\pm$ and Z spin-1 gauge fields, and hence it is often called the super-Higgs mechanism. Specifically, for the case of F-term supersymmetry breaking, we have

$$\delta_\epsilon\psi \sim F \neq 0\,. \tag{4.59}$$

Note that we do not want do break supersymmetry by VEVs for other fields, *e. g.* for ψ, since this would break Lorentz invariance.

A particular consequence of supersymmetry breaking are mass splittings among the different components of a supermultiplet.

We conclude this section by briefly introducing a few simple models of F-term and D-term supersymmetry breaking.

4.3.1 F-term Supersymmetry Breaking

In the following we briefly discuss three simple models for F-term supersymmetry breaking. The first two are models of the spontaneous breaking of global supersymmetry, while the third one is a model of spontaneous supersymmetry breakdown in supergravity.

O'Raifeartaigh Model The O'Raifeartaigh model [327] has three chiral superfields Φ_i with the following choice of K and W:

$$K = \sum_{i=1}^{3} \Phi_i \, \bar{\Phi}_i \,, \tag{4.60}$$

$$W = \Phi_1 \left(m^2 + \frac{\lambda}{2} \Phi_2^2 \right) + \mu \Phi_2 \Phi_3 \,. \tag{4.61}$$

The scalar potential is then given by

$$V = V_F = |m^2 + \frac{\lambda}{2} \phi_2^2|^2 + |\mu \phi_3 + \lambda \phi_1 \phi_2|^2 + |\mu \phi_2|^2 \,, \tag{4.62}$$

which is extremized by $\phi_2 = \phi_3 = 0$ and any value of ϕ_1. This is a minimum if μ is sufficiently large, as can easily be seen by expanding the scalar potential to quadratic order around the extremum. At the minimum, supersymmetry is broken since

$$F_1 = \frac{\partial W}{\partial \Phi_1} = m^2 = \sqrt{V_{\min}} \neq 0 \,. \tag{4.63}$$

A Non-Renormalizable Model If we would like to have a model with only one chiral superfield, the simplest globally supersymmetric model is probably obtained from

$$K = \Phi \bar{\Phi} - \frac{\left(\Phi \bar{\Phi} \right)^2}{\Lambda^2} \,, \quad W = m^2 \Phi \,, \tag{4.64}$$

where Λ is some cutoff scale. Assuming m to be real, this yields

$$F = - \frac{m^2}{1 - 4 \frac{|\phi|^2}{\Lambda^2}} \quad \Rightarrow \quad V_F = \frac{m^4}{\left(1 - 4 \frac{|\phi|^2}{\Lambda^2} \right)^2} \,. \tag{4.65}$$

The minimum is at $\phi = 0$ and breaks supersymmetry. Note that this model can be viewed as an effective description of the O'Raifeartaigh model after integrating out $\Phi_{2,3}$ assuming they are heavy enough (*i. e.* $\mu^2 \gg \lambda m^2$) [328]

Polonyi Model The simplest model for supersymmetry breaking in supegravity is the Polonyi model [329]. It assumes the following Kähler and superpotential:

$$K = |Z|^2 \,, \tag{4.66}$$

$$W = m^2 (Z + \beta) \,. \tag{4.67}$$

Due to the non-renormalizable gravitational couplings, the resulting potential has a minimum at

$$Z = \left(\sqrt{3} - 1 \right) M_P \,. \tag{4.68}$$

If we now fine-tune the coefficient

$$\beta = \left(2 + \sqrt{3}\right) M_P \,, \tag{4.69}$$

this minimum has a *vanishing* cosmological constant, $\langle V \rangle = 0$, due to the $-|W|^2$ piece in the supergravity F-term potential, cf. Eq. (4.47).

The idea of *gravity mediated supersymmetry breaking* is to basically exploit the fact that all other fields present will couple to the F-term of Z via the e^K prefactor. Thus, by expanding this prefactor one finds terms which are of the form

$$\int d^4\theta \, \frac{\bar{Z} Z \, \bar{Q} Q}{M_P^2} \,, \tag{4.70}$$

where Q denotes for instance some quark superfields. Such a term induces in particular a mass for the squarks of the form

$$m_{\tilde{q}} \sim \frac{F_Z}{M_P} \,. \tag{4.71}$$

For details about supersymmetry breaking mediation see *e. g.* the lecture notes [320].

4.3.2 D-term Supersymmetry Breaking

We may also break supersymmetry spontaneously via non-vanishing D-terms. The following model works both in globally and locally supersymmetric models.

Fayet-Iliopoulos Model If we have a $U(1)$ supersymmetric gauge theory with the real superfield denoted by V, we usually have $D = 0$ in the vacuum. However, we may add a *Fayet-Iliopoulos* (FI) term [330],

$$\mathcal{L}_{\mathrm{FI}} = \int d^2\theta d^2\bar{\theta}^2 \, \xi \, V = \xi \, D \,. \tag{4.72}$$

This *only* works for *Abelian* gauge symmetries since the gauge-invariant generalization to the non-Abelian case vanishes due to the tracelessness of the non-Abelian generators. If we add the piece $\mathcal{L}_{\mathrm{FI}}$ to the standard Lagrangian obtained from

$$\mathcal{L} = \int d^2\theta \frac{1}{2g^2} \mathcal{W}^2 + h.c. \,, \tag{4.73}$$

one can see that the system is extremized for $D = -g^2\xi \neq 0$ and thus supersymmetry is broken. D-term supersymmetry breaking directly affects the chiral superfields charged under the $U(1)$. Adding two chiral superfields $\Phi_\pm$ with charges ± 1, the fermion masses are unchanged and the charged scalars acquire a mass splitting,

$$m_\pm^2 = m^2 \pm g^2 \xi \,, \tag{4.74}$$

where m is a supersymmetric mass from a superpotential term $W = m\Phi_+\Phi_-$ to stabilize the chiral fields at $\Phi_\pm = 0$.

Chapter 5

Inflation in 4d $\mathcal{N} = 1$ Supergravity

After having reviewed both slow-roll inflation and supergravity, it is time to bring these two concepts together. As we outlined in Sec. 4.2, specifying a supergravity model with chiral superfields amounts to specifying the Kähler potential $K(\Phi, \bar{\Phi})$ and the superpotential $W(\Phi)$ (and also the gauge kinetic function $f_{ab}(\Phi)$ if gauge interactions are important).

As we have seen in Sec. 4.3, spontaneous breaking of supersymmetry is either due to a non-zero F-term or a non-vanishing D-term (or both). From the point of view of inflation model building, this corresponds to two different ways of generating the vacuum energy driving inflation.

In this chapter, we first discuss a few simple but explicit models. In Sec. 5.1, we outline how to construct a supergravity version of the $m^2\phi^2$ chaotic inflation model introduced in Sec. 3.5. Next, we introduce two ways of getting another interesting class of inflationary models, so-called *hybrid inflation* models, from D-term and F-term breaking of supersymmetry models in Secs. 5.2.1 and 5.2.2, respectively. All these three models merely serve as toy models or illustrative examples of inflationary model building in supergravity. We conclude this chapter by a discussion of the supergravity η-problem, how it is evaded in the simple toy models of Secs. 5.1 to 5.2.2 and some features of more general models of F-term inflation related to solutions of the η-problem.

The aim of this chapter is only to provide an illustration of how to build models of inflation in supergravity and the problems one faces while doing so. For a recent review and many details on supergravity models of inflation and their predictions, cf. [232] and references therein (see also [217]). Note that we will work in units where $M_P \equiv 1$ unless stated otherwise.

5.1 F-Term Chaotic Inflation

The first model we would like to consider is the supergravity version of chaotic inflation with $V(\phi) = m^2\phi^2$ (cf. Sec. 3.5). The inflaton ϕ is considered to be the lowest component of a chiral superfield Φ, which is also assumed to be a singlet under gauge interactions. Thus, ϕ receives its entire potential via F-terms.

Recall that the main feature of these chaotic inflation models is that inflation occurs for field values $\phi \gg M_P$. However, the F-term potential in supergravity, cf. Eq. (4.47), has an overall prefactor e^{K/M_P^2}. For a canonical choice of the Kähler potential, $K = \bar{\Phi}\Phi$, this prefactor tends to prevent fields from having field values $\gg M_P$. In [225], Kawasaki, Yamaguchi and Yanagida introduced a continuous global shift symmetry for the Kähler potential K to resolve this issue.[1] Under this symmetry, the chiral superfield Φ transforms as

$$\Phi \to \Phi + i\alpha\,. \tag{5.1}$$

This symmetry restricts K to be a function of $\Phi + \bar{\Phi}$ only and the imaginary part of Φ can take values $\gg M_P$ and thus act as the inflaton. In other words, the inflaton is like a (pseudo) Nambu-Goldstone boson from the spontaneous breaking of a continuous global symmetry. However, if the symmetry remains intact, the potential for the inflaton would be exactly flat. Therefore, we introduce a small breaking of the shift symmetry. The simplest option is to introduce a small mass term in the superpotential,

$$W = m\Phi^2\,. \tag{5.2}$$

Unfortunately, this leads to a potential which is unbounded from below as $\phi \to \infty$ [225].[2] This can be overcome if a second field X is involved [225]:[3]

$$W = mX\Phi\,. \tag{5.3}$$

This model is "natural" in the sense of 't Hooft since the shift symmetry is restored in the limit $m \to 0$. The Kähler potential is assumed to be of the form

$$K(\Phi, \bar{\Phi}, X, \bar{X}) = K(\Phi + \bar{\Phi}, X\bar{X})\,, \tag{5.4}$$

which is invariant under the shift symmetry as well as under a $U(1)_R \times \mathbb{Z}_2$ symmetry.[4] Actually, one usually considers a more specific choice of Kähler potential, namely [225]

$$K = \frac{1}{2}\left(\Phi + \bar{\Phi}\right)^2 + X\bar{X} - \gamma\left(X\bar{X}\right)^2 + \dots\,, \tag{5.5}$$

[1] As we will explain in Sec. 5.3, this shift symmetry also protects the model from the supergravity η-problem.

[2] It is easy to see this problem if one considers the simplest possibilty, $K = \frac{1}{2}\left(\Phi + \bar{\Phi}\right)^2$, because then $V \propto -\phi^4 \to -\infty$ as $\phi \equiv \operatorname{Im}\Phi \to \infty$ with $\operatorname{Re}\Phi = 0$.

[3] Note that this superpotential has a $U(1)_R$ symmetry under which X is charged and a $\mathbb{Z}_2$ symmetry under which X and Φ simultaneously change signs.

[4] This is valid as long as the corrections induced by the small breaking term Eq. (5.3) are negligible, cf. footnote 1 in [225].

where the dots represent possible higher order terms. As we will see soon, the third term is required to keep X fixed at zero during inflation.

While the D-term potential is assumed to vanish, the F-term potential is determined from inserting the expressions Eqs. (5.3) and (5.5) into Eq. (4.47), which yields

$$\begin{aligned} V_F &= e^K m^2 \left[|X \left(1 + (\Phi + \bar{\Phi})\Phi\right)|^2 + \frac{\Phi\,(1 + |X|^2 - 2\gamma|X|^4)}{1 - 4\gamma|X|^2} - 3|X\Phi|^2 \right] \\ &\simeq m^2 e^{|X|^2 - \gamma|X|^4 + \frac{1}{2}\left(\Phi+\bar{\Phi}\right)^2} \Big[|\Phi^2| \left(1 + (1 + 4\gamma + 16\gamma^2)|X|^4\right) \\ &+ |X|^2 \Big(1 + (1+4\gamma)|\Phi|^2 + \left(\Phi^2 + \bar{\Phi}^2\right) + \left(\Phi + \bar{\Phi}\right)^2 |\Phi|^2\Big) \Big] , \end{aligned} \tag{5.6}$$

where in the second line we expanded the F-term potential for $|X| \ll 1$.

We parametrize the real and imaginary parts of the scalar component of Φ as

$$\Phi \equiv \frac{\phi_R + i\,\phi_I}{\sqrt{2}} , \tag{5.7}$$

where ϕ_I acts as the inflaton. During inflation, we will have $\phi_I \gg 1$ while $\phi_R, |X| \ll 1$ and thus we can expand the potential, Eq. (5.6), which yields

$$V(\phi_R, \phi_I, X) \simeq \frac{1}{2} m^2 \phi_I^2 + \frac{1}{2} m^2 \phi_R^2 \left(1 + \phi_I^2\right) + m^2 |X|^2 \left(1 + 2\gamma\phi_I^2\right) . \tag{5.8}$$

The first term is precisely the potential for the chaotic $m^2\phi^2$ inflation model we were looking for. From the second term we see that during inflation ϕ_R receives a large mass of the order of the Hubble scale $\mathcal{H} \sim m^2\phi_I^2$ and thus it quickly settles to its minimum at $\phi_R = 0$ and stays fixed during inflation. Considering the last term, we see that if $\gamma \gtrsim \mathcal{O}(1)$ the same is true for X.[5] Hence, even though we started out with four real scalar fields (recall that a chiral superfield has a *complex* scalar as its lowest component), in the end we have a model which *effectively* describes single-field chaotic inflation. Similarly, the models we consider in Sec. 5.2 (and actually all models considered in this thesis) effectively reduce to a single-field model of inflation, even though they are formulated using multiple fields.

5.2 D-Term and F-Term Hybrid Inflation

The next models we want to consider are models of *hybrid inflation* [34] in supergravity, where inflation ends via a phase transition: Once the inflaton reaches a certain critical value, some direction in field space (the *waterfall* field) becomes tachyonic, thereby ending inflation. Such models are popular

[5] This requirement can be understood nicely from a geometric point of view in terms of the sectional curvature along the Goldstino direction, cf. [331–336].

since they allow inflation to occur while all field values are $\ll M_P$ and since their potentials are nicely related to the ones used for the spontaneous breaking of symmetries.

Basic Idea of Hybrid Inflation

The way a typical model of hybrid inflation [34,337] works is that a second field, say h, has a mass which depends on the inflaton ϕ in such a way that $m_h^2(\phi)$ changes sign at some value $\phi \equiv \phi_{\rm crit}$. This turns a formerly stable minimum with $h = 0$ into an unstable one and the inflationary vacuum energy vanishes since h develops an expectation value.

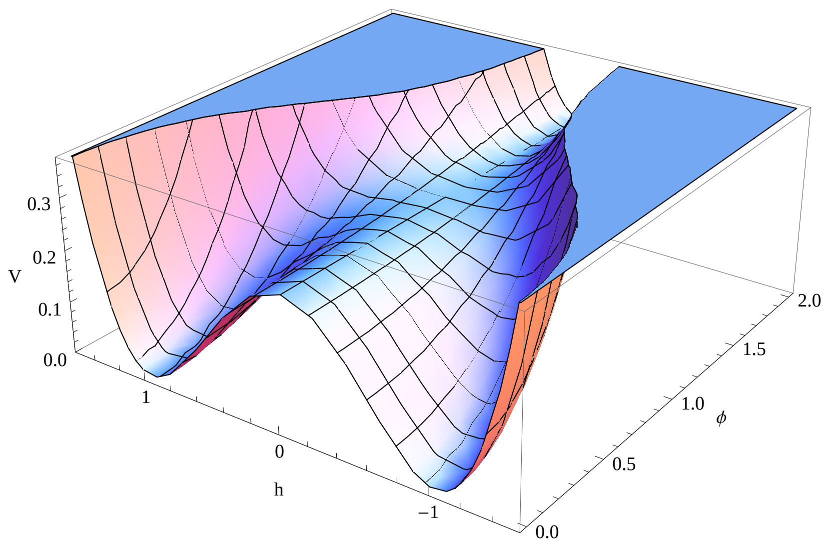

Figure 5.1: Schematic plot of the potential in Eq. (5.9) with $m = 0$ and all other parameters set equal to 1.

The prototype model of hybrid inflation is based on the potential [34]

$$V(\phi, h) = \frac{m^2}{2}\phi^2 + \frac{\kappa^2}{4}\left(h^2 - M^2\right)^2 + \frac{\lambda^2}{2}\phi^2 h^2 \,. \tag{5.9}$$

The mass of h depends on ϕ, namely $m_h^2(\phi) = -\kappa^2 M^2 + \lambda^2\phi^2$. Thus, for $\phi > \phi_{\rm crit} \equiv \kappa M/\lambda$, the waterfall field is fixed at $h = 0$ and inflation can proceed along the ϕ direction. During inflation, the vacuum energy is $V_0 \approx \kappa^2 M^4$ if $m^2\phi^2 \ll \kappa M^4$. Once ϕ reaches $\phi_{\rm crit}$, h becomes tachyonic and then the system rolls quickly to its true minimum at $h = M, \phi = 0$, thereby ending inflation. A schematic plot of the potential in Eq. (5.9) can be found in Fig. 5.1.

The $m^2\phi^2$ term in Eq. (5.9) induces a tree-level slope for the inflaton. However, hybrid inflation models are often designed to have ϕ as a classically (almost) flat direction, *i. e.* terms such as $m^2\phi^2$ are negligible. Then the dominant contribution to the slope V' arises from quantum corrections which makes it naturally small. This also explains why many models of this class predict a spectral index $n_s \approx 1$ (typical values are $n_s \approx 0.98$) and a very small tensor-to-scalar ratio $r \ll 0.1$ (since the field variation during inflation is small).

5.2.1 D-Term Hybrid Inflation

We now review a supergravity realization of hybrid inflation using D-term supersymmetry breaking [338, 339]. The model contains three chiral superfields Φ, H_+, H_- which are charged under a $U(1)$ gauge symmetry with charges $0, +1, -1$, respectively. The superpotential is assumed to be

$$W = \kappa \, \Phi H_+ H_- \,, \tag{5.10}$$

while the Kähler potential is taken to be canonical, *i. e.*

$$K = |\Phi|^2 + |H_+|^2 + |H_-|^2 \,. \tag{5.11}$$

The $U(1)$ symmetry is assumed to have a constant FI-term $\xi > 0$ [6] and a gauge coupling g.

Unlike in the F-term chaotic inflation model discussed previously in Sec. 5.1, we now have a contribution to the scalar potential from the D-term,

$$V_D = \frac{g^2}{2} \left(|H_+|^2 - |H_-|^2 + \xi\right)^2 \,. \tag{5.12}$$

The F-term contribution V_F is as usual determined by inserting the expressions for K and W in Eqs. (5.10) and (5.11) into Eq. (4.47), which yields

$$\begin{aligned} V_F = \kappa^2 e^{|\Phi|^2+|H_+|^2+|H_-|^2} \Big(&|H_+H_-|^2 + |H_+\Phi|^2 + |H_-\Phi|^2 \\ &+ \left(3 + |\Phi|^2 + |H_+|^2 + |H_-|^2\right) |\Phi H_+ H_-|^2 \Big) \,. \end{aligned} \tag{5.13}$$

The combined scalar potential $V = V_F + V_D$ has a unique global minimum at $V = 0$ for

$$\Phi = H_+ = 0 \,, \quad H_- = \sqrt{\xi} \,. \tag{5.14}$$

More interestingly for us, if Φ is large, we can have a local minimum with $V > 0$ and

$$H_+ = H_- = 0 \,. \tag{5.15}$$

In this case, $V_F = 0$ and $V = V_D \sim g^2\xi^2 > 0$. Above a certain critical value Φ_{crit} the two fields $H_\pm$ are fixed at zero and the potential $V > 0$ drives the inflationary expansion. Once $\Phi = \Phi_{\text{crit}}$, H_- develops a tachyonic mass, *i. e.* $H_\pm = 0$

[6] As was shown in [340, 341], a constant FI-term is actually *inconsistent* with a coupling to supergravity. However, this is not the case if ξ is actually field-dependent. This is precisely the case for the FI-terms that arise in string theory, which depend on the moduli. Thus, one should think of the D-term inflation model presented here only as an effective description after fixing the moduli.

is no longer a minimum and H_- starts to develop an expectation value while moving to the true minimum at $\sqrt{\xi}$. To determine the critical value Φ_{crit}, we have to look at the mass matrix of $H_\pm$. It turns out that they have masses given by

$$m^2_{H_\pm} = \kappa^2|\Phi|^2 e^{|\Phi|^2} \pm g^2\xi^2\,. \tag{5.16}$$

The first term is a supersymmetric mass term coming from the superpotential, Eq. (5.10), and the second term is the mass splitting induced by the supersymmetry breaking D-term contribution $\pm g^2\xi^2$. Since inflation occurs only as long as $\Phi > \Phi_{\text{crit}}$ and we would like to consider inflation with all field values $\Phi \lesssim 1$, we may approximate $\Phi_{\text{crit}} \approx g\sqrt{\xi}/\kappa$. The potential along Φ turns out to be classically flat for $H_\pm = 0$. However, the mass splitting for $H_\pm$ induced by the D-term generates a Coleman-Weinberg-type potential [342] via quantum corrections at the 1-loop level. For $\Phi_{\text{crit}} \ll \Phi \lesssim 1$, the effective potential during inflation is approximately given by

$$V_{\text{eff}}(\Phi) = V_0 + V_{\text{1-loop}} \simeq \frac{g^2\xi^2}{2} + \frac{g^4\xi^2}{16\pi^2}\ln\left(\frac{\kappa^2|\Phi|^2}{Q^2}\right)\,, \tag{5.17}$$

where Q is some renormalization scale. This is independent of the phase of Φ and therefore we may identify the real part of Φ as the inflaton.

The predictions for the CMB observables, however, are much more complicated in this model since the breaking of the $U(1)$ gauge symmetry at the end of inflation leads to the formation of cosmic strings. These also contribute to the CMB fluctuations, but their contribution is severely constrained. The energy scale for ξ is set by matching to the observed amplitude of scalar fluctuations, $\Delta_s^2 \sim 2 \times 10^{-9}$, and the required number of e-folds, $N_e \sim 60$, which typically leads to $\sqrt{\xi}$ a bit lower than the GUT scale. However, this depends also on g and κ which cannot be too large, *e. g.* in [343, 344] the following bounds on the parameters ξ, g and κ are computed[7]

$$\sqrt{\xi} \lesssim 2 \times 10^{15}\ \text{GeV}\,, \quad g \lesssim 2 \times 10^{-2}\,, \quad \kappa \lesssim 3 \times 10^{-5}\,. \tag{5.18}$$

Basically, the constraints come from reducing the tension of the cosmic strings to levels consistent with observations.

5.2.2 F-Term Hybrid Inflation

Hybrid inflation can also be obtained when supersymmetry is broken by an F-term during inflation. The simplest model of this sort is very similar to the D-term model just discussed. However, the D-term now vanishes and the vacuum energy is due a non-zero F-term.

The model has the same field content and Kähler potential as before in Sec. 5.2.1, *i. e.* the chiral superfields Φ, H_+, H_- which are charged under a $U(1)$

[7]For ways to relax these constraints somewhat see *e. g.* [345–347].

gauge symmetry with charges $0, +1, -1$, respectively, and a canonical Kähler potential,

$$K = |\Phi|^2 + |H_+|^2 + |H_-|^2 \,. \tag{5.19}$$

But now the superpotential is given by [218–220]

$$W = \kappa\,\Phi\left(H_+H_- - M^2\right)\,, \tag{5.20}$$

and there is no FI-term for the $U(1)$, $\xi = 0$.

Using again the standard formula, Eq. (4.47), to compute the F-term potential, we find

$$\begin{aligned} V_F = \kappa^2 e^{|\Phi|^2+|H_+|^2+|H_-|^2}\Big[&\left(1-|\Phi|^2+|\Phi|^4\right)|H_+H_- - M^2|^2 \\ &+|\Phi|^2\left(|(1+|H_+|^2)H_- - M^2\bar{H}_+|^2 + |(1+|H_-|^2)H_+ - M^2\bar{H}_-|^2\right)\Big]\,. \end{aligned} \tag{5.21}$$

The D-term contribution V_D vanishes along the *D-flat* direction $H_+ = \bar{H}_-$ (to which the system is actually forced by the D-term potential). Neglecting M_P-suppressed corrections and expanding for $\Phi \ll 1$, the two mass eigenvalues are

$$M_{1,2}^2 \simeq \kappa^2|\Phi|^2 \pm \kappa^2 M^2\,. \tag{5.22}$$

Thus, now $\Phi_{\rm crit} \approx M$ and we again have a mass splitting due to supersymmetry breaking, which will again induce a 1-loop correction of the Coleman-Weinberg-type. Unlike in the D-term hybrid inflation model discussed, though, there is now also a classical contribution. During inflation, when $H_\pm = 0$, we can expand the F-term potential for $\Phi \ll 1$ and find

$$V_F \approx \kappa^2 M^4 + \frac{\kappa^2 M^4 |\Phi|^4}{2M_P^4}\,, \tag{5.23}$$

where we have reinserted the powers of M_P. However, this contribution could be subleading with respect to the Coleman-Weinberg (CW) potential induced via quantum corrections since $\Phi \ll 1$. For $\Phi \gg \Phi_{\rm crit}$, we can approximate the CW potential as

$$V_{\text{1-loop}} \simeq \frac{\kappa^4 M^4}{8\pi^2}\ln\left(\frac{\phi}{\phi_{\rm crit}}\right)\,, \tag{5.24}$$

where we have switched to $\Phi = \frac{\phi}{\sqrt{2}}$ without loss of generality since the phase of Φ does not enter into the potential. Now the effective scalar potential during inflation is approximately given by

$$V_{\rm eff} \simeq \kappa^2 M^4\left(1 + \frac{\kappa^2}{8\pi^2}\ln\left(\frac{\phi}{\phi_{\rm crit}}\right) + \frac{\phi^4}{8M_P^4}\right)\,. \tag{5.25}$$

Which of the two terms dominates the slope of the potential depends on the value of κ.

An advantage of F-term hybrid inflation over D-term hybrid inflation is that one has more possibilities for the two fields $H_{\pm}$. Here, they are charged under a $U(1)$ symmetry, but in general they just have to be in conjugate representations of some gauge group $\mathcal{G}$, *e. g.* fundamental and anti-fundamental representations. Thus, from a phenomenological point of view, this combines nicely with the breaking of *e. g.* a GUT gauge group in the early universe. However, one again has constraints from formation of topological defects and/or cosmic strings. Values up to $\kappa \lesssim 10^{-2}$ seem to be consistent according to [348–350].

5.3 Supergravity η-problem and General F-Term Inflation Models

Now that we have presented three rather simple supergravity models of inflation, it is time to understand in more detail why these models actually work and what the problems are one faces when going beyond these examples.

The Supergravity η-problem

The main obstacle for building a working model of slow-roll inflation is the so-called *supergravity η-problem* [37–39]. It is a fundamental obstacle in the sense that it is not about agreeing with observations, but really about slow-roll inflation occurring *at all*.

To understand the problem and its origin, let us recall the general structure of the supergravity F-term potential, Eqs. (4.47) and (4.48), which is

$$V_F = e^{K/M_P^2}\left(\left(K_{i\bar{j}}\right)^{-1} D_iW D_{\bar{j}}\bar{W} - \frac{3}{M_P^2}|W|^2\right) , \tag{5.26}$$

where the supergravity F-terms D_iW are given by

$$D_iW = W_i + \frac{1}{M_P^2}K_iW , \tag{5.27}$$

Now let us assume that the vacuum energy during inflation is provided by the F-term of a chiral superfield, say X, which may or may not be the one which contains the inflaton. To keep things general, the superfield containing the inflaton is denoted by Φ. Moreover, we assume the Kähler potential for Φ and X starts with the canonical terms, *i. e.*

$$K = |\Phi|^2 + |X|^2 + \dots . \tag{5.28}$$

Then, since during inflation only $D_XW \neq 0$, we can expand the F-term potential as

$$\begin{aligned} V_F &= e^{K/M_P^2}\left((K_{X\bar{X}})^{-1}\,|D_X W|^2 - \frac{3}{M_P^2}|W|^2\right) \\ &\approx e^{K/M_P^2} V_0 + \dots\,, \\ &= \left(1 + \frac{|\Phi|^2}{M_P^2} + \dots\right) V_0 + \dots\,, \end{aligned} \tag{5.29}$$

with $V_0 \equiv |D_X W|^2$. Now we immediately face a problem: The slow-roll parameter η (cf. Eq. (3.29)) obtained from this potential is given by

$$\eta \equiv M_P^2 \frac{V''}{V} \approx 1 + \dots\,, \tag{5.30}$$

where the prime denotes again a derivative with respect to the inflaton contained in Φ and the 1 comes precisely from the $|\Phi|^2 V_0/M_P^2$ term in the last line of Eq. (5.29). Thus, the e^K prefactor *generically* gives rise to masses for the scalar fields which are $\sim \mathcal{H}^2$. To prevent these generic terms from spoiling slow-roll inflation, there are basically two options at the level of effective field theories:

1. Fine-tune the additional corrections denoted by the ellipsis in the last line of Eq. (5.29) to (almost) cancel the 1.

2. Forbid the mass term arising from the e^K prefactor, *e. g.* by imposing an (approximate) symmetry.

We have already seen three simple models for slow-roll inflation in supergravity, so let us now examine why they actually worked in the first place.

F-Term Chaotic Inflation This is an example of the second way of solving the η-problem, namely by imposing a continuous global shift symmetry to make e^K actually *independent* of the inflaton. If we would have added a correction term to K of the form $|\Phi|^4/M_P^2$ to Eq. (5.5) with an $\mathcal{O}(1)$ coefficient, we would have spoiled slow-roll inflation even though this corresponds to adding a M_P-suppressed dimension-six operator to the Lagrangian!

D-Term Hybrid Inflation Here, the vacuum energy was due to the D-term, which has *no* e^K prefactor and therefore does not suffer from the above version of the η-problem. However, these models inevitably lead to the formation of cosmic strings whose contribution to the CMB fluctuations is strongly constrained by observations. Moreover, the η-problem may reappear via threshold corrections of the gauge-kinetic function, see *e. g.* the nice explanation in [41] in the context of string theory models of inflation.

F-Term Hybrid Inflation This is an example of the first way of solving the η-problem. Since the superpotential, Eq. (5.20), is linear in Φ and the Kähler potential, Eq. (5.19), is assumed to be canonical, the potentially dangerous term discussed above cancels miraculously against the contributions coming from the dots, leaving only a term $V_F \supset V_0|\Phi|^4/M_P^4$, which is much less dangerous for $\Phi \ll M_P$. However, if we would have added a term $|\Phi|^4/M_P^2$ with $\mathcal{O}(1)$ coefficient to the Kähler potential in Eq. (5.19), we would again have spoiled this cancellation and reintroduced the η-problem.

The upshot of this section is that the supergravity η-problem is deeply related to corrections to the Kähler potential. Unlike the superpotential, K is *not* protected by a non-renormalization theorem and therefore it can receive all kinds of possible corrections. To have slow-roll inflation occur at all, one must assume either some special structure of the terms in K or fine-tune the coefficients. For small-field models of inflation, the most relevant corrections to K are the dimension-six operators induced by terms of the form

$$\int d^4\theta \, \frac{c}{M_P^2} \, \bar{X} X \bar{\Phi} \Phi \,, \tag{5.31}$$

where $d^4\theta \equiv d^2\theta d^2\bar{\theta}$ and X and Φ are some chiral superfields.

A Guideline for F-Term Model Building

For more general models, if we intend to solve the η-problem via some special structures (aka symmetries) in the Kähler potential, it has been shown that the following conditions are preferable [38, 225, 227–230, 351, 352]

$$D_X W \neq 0 \,, \quad D_\Phi W \approx 0 \,, \quad W \approx 0 \,, \tag{5.32}$$

where $X \neq \Phi$ does *not* contain the inflaton field ϕ. That is, we should *not* break supersymmetry along the inflaton direction. This is precisely what has been used in the F-Term Chaotic Inflation model of [225] presented above, which introduced the extra field X to break supersymmetry along another direction. This idea has been used recently in [229, 230] to construct more general models of F-Term chaotic inflation based on superpotentials of the form

$$W = X f(\Phi) \,, \tag{5.33}$$

with $f(\Phi)$ some general function. This can be supplemented, for example, by the Kähler potential in Eq. (5.5), or by a more general choice for the Kähler potential of Φ. The important constraint on the Kähler potential for these models is to ensure $X \approx 0$ during inflation such that $W \approx D_\Phi W \approx 0$.

We will discuss hybrid inflation models in supergravity based on these guidelines later on in this thesis, cf. Chap. 11.

Chapter 6

Basics of Moduli Stabilization

In this chapter, we briefly introduce a few basic aspects of moduli stabilization in string theory. For definiteness, we focus on flux compactifications of type IIB string theory from ten to four dimensions. But the schemes for moduli stabilization in type IIA or heterotic string theory are based on the same basic ingredients, namely *fluxes* and *non-perturbative effects*.

Fluxes have become a basic ingredient of modern string compactifications since they are important for *moduli stabilization* and can lead to *warped geometries*.

The term "moduli" originally referred to strictly massless scalar fields which correspond to a motion in the vacuum manifold of the theory, *i. e.* by changing the expectation value of a modulus field we can move "for free" to a new ground state of the theory. However, we have not observed any massless scalar fields in nature and there are strong constraints on light and weakly interacting scalar fields. In this sense, "moduli stabilization" corresponds to finding a mechanism (*i. e.* a non-trivial potential) which freezes the modulus such that we can describe the low-energy theory in a fixed vacuum.

In string theory compactifications, there is always the dilaton which sets the string coupling g_s and in addition there are geometric moduli which parametrize the "size and shape" of the compactification manifold. For Calabi-Yau 3-folds, the geometric moduli can be divided into two classes, *complex structure* and *Kähler* moduli. The complex structure moduli control the sizes of 3-cycles while the Kähler moduli control the sizes of 2- and 4-cycles (due to Poincaré duality).[1] For a Calabi-Yau 3-fold $\mathcal{M}$, the number of inequivalent 3-cycles (*i. e.* complex structure moduli) and 2-cycles (*i. e.* Kähler moduli) is encoded in two numbers, the two dimensions of the (co-)homology groups denoted $h_{(2,1)}(\mathcal{M})$ and $h_{(1,1)}(\mathcal{M})$, respectively, which are determined by the *topology* of $\mathcal{M}$. For a CY 3-fold these are the only two numbers which can vary.

[1] Roughly speaking, a p-cycle of a compactification manifold $\mathcal{M}$ is a (sub-)manifold $\Sigma_p \subseteq \mathcal{M}$ with no boundary and which is *not* the boundary of some $(p+1)$-dimensional manifold.

By "warped geometries" we mean compactifications to four dimensions for which the metric in the four external directions depends quite strongly on the internal directions, which are interesting for various reasons. For example, warped compactifications provide an alternative solution to the hierarchy problem [15]. Moreover, they typically also play an important role in gauge-gravity dualities (see Sec. 7).

For textbook treatments of string theory see *e. g.* [46–52]. Here, we will not provide an introduction into string theory and only work in the low-energy supergravity limit.

For comprehensive reviews on flux compactifications and their application to moduli stabilization (especially in type II supergravity) cf. *e. g.* [96,160–163]. Here, we will mostly follow the original papers, but see the reviews for many details and a more extensive list of references. The following sections are mainly to setup the notation and state the important results.

6.1 Moduli Stabilization via Fluxes

The ten-dimensional supergravity descriptions of both type II and heterotic string theory contain the 2-form[2] B_2. Its *field strength* is denoted by H_3, *i. e.* $H_3 \equiv dB_2$. In addition to B_2, the type II supergravity theories also contain other p-forms C_p with field strengths $F_{p+1} \equiv dC_p$. Which values of p are allowed depends on whether type IIA or type IIB is considered. The term "fluxes" refers to non-trivial background configurations of these (anti-symmetric) tensor field strengths. Of course, when compactifying to four dimensions, we would like to preserve 4d Poincaré invariance[3] and thus these field strengths must either be only along the internal directions or along all four external directions.

The field strengths F_{p+1} fulfill the corresponding Bianchi identity, $dF_{p+1} = 0$. One can show that similar to Dirac's famous charge quantization [353] also the fluxes are quantized [106–108]. Namely, if we integrate F_{p+1} over a $(p+1)$-dimensional manifold Σ_{p+1} without a boundary, the "charge" must be an integer, *i. e.*

$$\frac{1}{\ell_s^p} \int\limits_{\Sigma_{p+1}} F_{p+1} \in \mathbb{Z} \,. \tag{6.1}$$

Since Σ_{p+1} has no boundary and due to the Bianchi identity, only the *cohomology class* of F_{p+1} is relevant.

[2]The appearance of a 2-form B_2 is actually very generic since a string world-sheet may always source a two-form field. This is in complete analogy to the worldline of a charged point particle sourcing a 1-form (or gauge field) $A_1 \equiv A_\mu dx^\mu$.

[3]More generally, we are interested in compactifications to 4d maximally symmetric spaces, *i. e.* to Minkowski, Anti-de-Sitter or de-Sitter spacetimes, and demand that all of their respective symmetries are preserved.

A Quick Note on (Co-)Homology We do not discuss the interesting beautiful mathematical theory behind (co-)homology classes of manifolds here, but merely state the results important for us. In the language of differential forms, the Bianchy identity $dF_{p+1} = 0$ means that F_{p+1} is a *closed* form. If F_{p+1} is an *exact* form, *i. e.* is obtained as $F_{p+1} = dC_p$ *globally*, the Bianchi identity is trivially fulfilled. However, the more interesting possibilities are the non-trivial ones which depend on the topology of the manifold $\mathcal{M}$. These non-trivial possibilities are counted by the (dimension of) the cohomology groups of $\mathcal{M}$. Important for our purposes is that one can specify a basis for the p-forms on p-cycles in terms of harmonic p-forms ω_p. The ω_p also specify a basis of the p-cycles. It should be noted that (co-)homology defines *equivalence classes* and we are actually talking about *representatives* of these equivalence classes.

As a simple example, consider the two-dimensional torus T^2. It has a 0-cycle (the equivalence class of all point p in T^2), a 2-cycle (T^2 itself) and two 1-cycles a (red) and b (blue) corresponding losely speaking to the two inequivalent ways of drawing a circle on the torus, cf. Fig. 6.1.

For the CY 3-fold, there are two types of geometric moduli [354]. *Kähler moduli* which control the sizes of 2-cycles and *complex structure moduli* which control the sizes of 3-cycles. They are usually defined as parametrizations of the Kähler $(1,1)$-form $\mathcal{J}$ and the holomorphic $(3,0)$-form Ω, respectively. A CY 3-fold is a six-dimensional manifold but it is also a manifold with *three complex dimensions*. That is, out of the six coordinate we can form 3 complex coordinates and 3 complex conjugate coordinates. This "splitting" is encoded in Ω. The quantities of the form (p,q) refer to forms which are comprised out of p holomorphic and q anti-holomorphic coordinates.

For an introduction into (co-)homology and complex geometry for physicists, see *e. g.* [175, 176, 355].

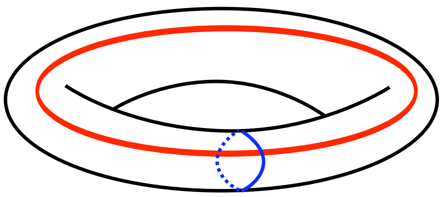

Figure 6.1: Simplified illustration of the concept of "cycles" using a torus.

Basic Idea of Moduli Stabilization via Fluxes Roughly speaking, the basic idea underlying moduli stabilization by fluxes is the following. Without fluxes, the moduli corresponding to the volumes of some cycles have either a runaway potential (either to 0 or ∞) due to a curvature contribution which is minimized when the volume of the cycle shrinks to zero or blows up (depending on the sign of the curvature contribution). When fluxes are switched on, the energy density stored in the fluxes depends on the volumes of the cycles. Thus, by adding appropriate contributions from fluxes, we may fix the volumes of the cycles supporting the fluxes at some finite values. This fixing of the cycle volumes is the *stabilization of geometric moduli.*

Flux Compactifications of Type IIB Supergravity

From now on, we assume the six-dimensional compactification manifold $\mathcal{M}_6$ to be a (conformal) Calabi-Yau manifold. Moreover, we consider only flux compactifications of type IIB supergravity with background 3-form flux since this is the setup which is reasonably well-understood and under control. In particular, because under certain assumptions the backreaction of the fluxes on the internal space only leads to a non-trivial warp factor and the internal space is a *conformally-Calabi-Yau* manifold. That is, it is still a CY up to a common overall warp factor.

In this subsection, we closely follow [155] (for earlier work on moduli stabilization via fluxes see [153, 154]).

The starting point is the low-energy limit of type IIB string theory which is type IIB supergravity with the ten-dimensional action (in Einstein frame)

$$\begin{aligned} \mathcal{S} = \frac{M_{10}^8}{2} \int d^{10}x\sqrt{-g} \left(\mathcal{R}_{10} - \frac{|\partial\tau|^2}{2\,(\mathrm{Im}\,\tau)^2} - \frac{|G_3|^2}{12\mathrm{Im}\,\tau} - \frac{\tilde{F}_5^2}{4\cdot 5!} \right) \\ + \frac{M_{10}^8}{8i} \int \frac{C_4 \wedge G_3 \wedge \bar{G}_3}{\mathrm{Im}\,\tau} + \mathcal{S}_{\mathrm{local}} + \text{terms involving fermions}\,, \end{aligned} \tag{6.2}$$

where M_{10} is the (reduced) Planck mass in 10d and g is the 10d Einstein frame metric and $\mathcal{R}_{10}$ is the 10d Ricci scalar. The *axio-dilaton* S is a particular combination of the dilaton ϕ and the Ramond-Ramond (RR) axion C_0,

$$S \equiv e^{-\phi} + iC_0\,. \tag{6.3}$$

C_4 denotes the RR 4-form potential which gives rise to a field strength F_5. The field strengths G_3 and $\tilde{F}_5$ appearing in the action are constructed out of F_5 as well as the RR 2-form C_2 and the Neveu-Schwarz (NS) 2-form B_2 and their respective field strengths F_3 and H_3. They are defined as follows:

$$\tilde{F}_5 \equiv F_5 - \frac{1}{2}C_2 \wedge H_3 + \frac{1}{2}B_2 \wedge F_3\,, \tag{6.4}$$

$$G_3 \equiv F_3 - iSH_3\,. \tag{6.5}$$

Moreover, $\tilde{F}_5$ is required to be self-dual and this has to be imposed as an additional constraint at the level of the equations of motion. Finally, $\mathcal{S}_{\mathrm{local}}$ contains contributions from various types of localized sources such as D-brane and orientifold planes. In particular, these localized contributions may source the metric and p-form fields.

Let us consider ten-dimensional spacetimes which are a warped product of a maximally symmetric four-dimensional spacetime $\mathcal{M}_4$ (for definiteness we take 4d Minkowski space with metric $\eta_{\mu\nu}$) and a six-dimensional manifold $\mathcal{M}_6$, *i. e.* a metric of the form

$$ds^2 = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{-2A(y)}\tilde{g}_{mn}dy^m dy^n \,. \tag{6.6}$$

Here, $A(y)$ is the *warp factor* and $\tilde{g}_{mn}$ is the metric on the internal space with coordinates y^m. To preserve 4d Poincaré invariance, the background values of the fields must fulfill certain conditions. As mentioned already above, the fluxes must either extend only along the six internal directions or span all four external dimensions. For the concrete case of type IIB supergravity this means that G_3 is only extended in the internal directions and the self-dual $\tilde{F}_5$ must be of the form

$$\tilde{F}_5 = (1 + \star_{10})\, d\alpha(y) \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \,, \tag{6.7}$$

where $\star_{10}$ denotes the 10d Hodge star operator. Moreover, the axio-dilaton can only be a function of the internal directions, *i. e.* $S = S(y)$. By taking the trace of the Einstein equations, one can show that in the absence of localized sources the fluxes must vanish and the warp factor is constant [356, 357]. However, this no-go theorem can be evaded if sources with *negative* tension are included [155]. These objects are indeed present in string theory, for example orientifold planes or anti-branes, but they are subject to certain consistency conditions, in particular the tadpoles of the D-brane charges must vanish. Also note that both anti-branes and orientifold planes break supersymmetry. Type IIB on a Calabi-Yau manifold preserves $\mathcal{N} = 2$ supersymmetry in 4d, but orientifold branes break it down to $\mathcal{N} = 1$ while anti-branes completely break supersymmetry.

We may also add D3-branes, which either fill all four external dimension or wrap a 4-cycle (so-called Euclidean D3-branes), or D7-branes filling all four external dimensions. In [155], it was shown that any type of localized sources satisfying a certain BPS-like condition on their energy-momentum tensor (which all sources mentioned so far do satisfy) lead to solutions with

$$\star_6 G_3 = iG_3 \,, \tag{6.8}$$

$$e^{4A(y)} = \alpha(y) \,, \tag{6.9}$$

with $\star_6$ denoting the Hodge star operator in the internal dimensions. The first line states that the flux G_3 is *imaginary self-dual* (ISD), while the second line relates the 5-form flux and the warp factor. In addition, F_3 and H_3 must satisfy their Bianchi identities

$$dF_3 = 0 \,, \quad dH_3 = 0 \,. \tag{6.10}$$

Moreover, if we would like to preserve 4d $\mathcal{N} = 1$ supersymmetry, the flux G_3 should be a $(2, 1)$-form [358, 359].

From the point of view of the 4d effective $\mathcal{N}=1$ supergravity theory, the flux G_3 gives rise to a superpotential of the Gukov-Vafa-Witten form [152],

$$W_{\text{flux}} = \int\limits_{\mathcal{M}_6} \Omega \wedge G_3 \,, \tag{6.11}$$

where Ω is the holomorphic $(3,0)$-form. This superpotential depends on the complex structure moduli U_α via Ω and on the dilaton S via G_3. In the 4d effective supergravity theory, the axio-dilaton S is the lowest component of a chiral multiplet S (by the "standard" abuse of notation we denote the superfield and its lowest component with the same symbol).

The tree-level Kähler potentials for the complex structure moduli and the dilaton are given by [354]

$$K_{\text{cs}} = -\ln\left(-i\int\limits_{\mathcal{M}} \Omega \wedge \bar{\Omega}\right) \,, \tag{6.12}$$

and

$$K_{\text{S}} = -\ln(S+\bar{S}) \,, \tag{6.13}$$

respectively. For simplicity, we will focus on the case of a single Kähler modulus here, *i. e.* $h_{(1,1)}=1$. Its tree-level Kähler potential is given by

$$K = -3\ln(T+\bar{T}) \,, \tag{6.14}$$

where $\mathrm{Re}\, T \propto \mathcal{V}^{2/3}$. Note that this Kähler potential for the T_i satisfies the *no-scale* property

$$K^{i\bar{j}} K_i K_{\bar{j}} = 3 \,, \tag{6.15}$$

which is trivially fulfilled for a single T, but its holds generically at tree-level also for $h_{(1,1)}>1$. Due to this no-scale property, the F-term potential simplifies significantly and becomes a sum over contributions from only the F-terms of the dilaton S and the complex structure moduli U_α. Through fluxes, we may then stabilize both U_α and S in a supersymmetric way, *i. e.* such that

$$D_{U_\alpha} W = D_S W = 0 \,. \tag{6.16}$$

For an explicit example, namely the complex structure modulus of a conifold, see Sec. 7.3.

Henceforth, we assume throughout this work that the dilaton and the complex structure moduli are stabilized with sufficiently large masses by an appropriate choice of fluxes. However, note that the above superpotential does *not* depend on the Kähler moduli T_i. Their stabilization requires additional ingredients described in the next section.

6.2 Moduli Stabilization via Non-Perturbative Effects

The scenario proposed by Kachru, Kallosh, Linde and Trivedi (KKLT) in [40] uses additional non-perturbative corrections to the superpotential to stabilize the Kähler moduli in a type IIB flux compactification. This leads to a 4d supersymmetric AdS minimum and they proposed to slightly overcompensate the negative cosmological constant of the AdS vacuum by adding anti-D3-branes. In the following, we briefly review the KKLT setup for the case of a single Kähler modulus T (essentially following [40]).

The starting point is a flux compactification where all complex structure moduli U_α and the dilaton S have acquired masses around the string scale[4] $M_{\rm st}$. After integrating them out, the flux-induced superpotential $W_{\rm flux}$ (cf. Eq. (6.11)) reduces to a constant contribution

$$W_0 \equiv \langle W_{\rm flux}(S, U_\alpha) \rangle \,. \tag{6.17}$$

Assuming a single Kähler modulus T, it has a no-scale Kähler potential (at leading order in the α'-expansion),

$$K = -3 \ln(T + \overline{T}) \,. \tag{6.18}$$

We may now add a stack of $N_{\rm D7}$ D7-branes which extend along the four external directions and wrap the 4-cycle whose size is controlled by T. On the world-volume of this stack of D7-branes one has a $SU(N_{\rm D7})$ Super-Yang-Mills (SYM) gauge theory. The 4d gauge coupling of the wrapped branes is set by T as

$$\frac{8\pi^2}{g_{\rm YM}^2} = 2\pi \,{\rm Re}\, T \,. \tag{6.19}$$

In the absence of light charged matter, this SYM theory undergoes *gaugino condensation*, *i. e.* it develops an expectation value of the form

$$\langle \lambda\lambda \rangle \sim \Lambda^3 \,, \tag{6.20}$$

which gives rise to a *non-perturbative* contribution to the superpotential [158, 360–362]

$$W_{\rm gc} = \Lambda^3_{N_{\rm D7}} = A e^{-\frac{2\pi}{b} T} \,, \tag{6.21}$$

where $b \equiv b_0/3$ and $b_0 = 3N_{\rm D7}$ is the coefficient of the 1-loop β-function. A similar contribution to the superpotential can also arise from Euclidean D3-brane instantons which then has $b = 1$.

[4]If the overall volume of the compactification manifold is $\mathcal{V} \sim R^6$, the complex structure moduli and the dilaton acquire masses of the order $m \sim \alpha'/R^3$ [40], where $\alpha' = \ell_s^2$.

In summary, due to the non-perturbative correction from either a gaugino condensation on a stack of D7-branes or from Euclidean D3-brane instantons, we end up with a superpotential of the form

$$W_{\mathrm{KKLT}} = W_0 + Ae^{-aT}\,, \tag{6.22}$$

where $a = \frac{2\pi}{b}$. The F-term scalar potential following from the above K and W reads

$$V_F = \frac{a^2A^2e^{-a\left(T+\overline{T}\right)}}{3\left(T+\overline{T}\right)} + \frac{aAW_0e^{-a\overline{T}} + h.c.}{\left(T+\overline{T}\right)^2} + \frac{2aA^2e^{-a\left(T+\overline{T}\right)}}{\left(T+\overline{T}\right)^2}\,. \tag{6.23}$$

Without loss of generality, we now take W_0, A and a to be real, and $A > 0, W_0 < 0$. Then there exists a *supersymmetric* minimum determined by

$$D_TW = 0 \quad \Rightarrow \quad W_0 = -Ae^{-aT}\left(1 + \frac{a}{3}\left(T+\overline{T}\right)\right)\,. \tag{6.24}$$

Denoting the real and imaginary parts of T as $T \equiv \sigma + i\alpha$, one can see that for our choice of phases we have $\alpha = 0$ and σ_0 determined by solving

$$W_0 = -Ae^{-a\sigma_0}\left(1 + \frac{2a\sigma_0}{3}\right)\,. \tag{6.25}$$

At the minimum, the potential energy is *negative*,

$$V_{\mathrm{AdS}} = -3e^K|W|^2\Big|_{\sigma=\sigma_0} = -\frac{a^2A^2e^{-2\,a\,\sigma_0}}{6\sigma_0}\,, \tag{6.26}$$

i. e. we have a supersymmetric AdS-minimum. To have a controlled supergravity approximation and to justify keeping only the single instanton contribution in the superpotential, we require $\sigma \gg 1$ and $a\sigma > 1$, respectively. In terms of the parameters of W_{KKLT}, this means $|W_0| \ll 1$ and $a < 1$. Note that the assumption of $W_0 \ll 1$ can be relaxed in the LARGE Volume Scenario [159], which adds the leading order α'-correction to the potential [156] and involves at least two Kähler moduli and a particular structure in the Kähler potential. But here we confine ourselves to the case of a single Kähler modulus for simplicity.

Including the contributions from fluxes and non-perturbative effects, we end up with an AdS-minimum which needs to be *uplifted* to a Minkowski or dS-minimum. In the KKLT construction this uplifting is achieved by a stack of anti-D3-branes at the tip of a warped throat[5] such as the Klebanov-Strassler solution (cf. Sec. 7.2). The supergravity solution corresponding to anti-D3-branes at the tip of Klebanov-Strassler is a metastable state which breaks supersymmetry [363] and the backreacted solution has been found only recently [364–368] (for earlier work see [369–373]). In the literature, also other possibilities for uplifting exits, see *e. g.* [199, 328, 374–378]. For instance, [374] makes use of D-terms induced by world-volume flux on D7-branes.

[5]The anti-D3-brane in a warped throat of a flux compactification with only ISD flux experiences a force which drives it to the tip.

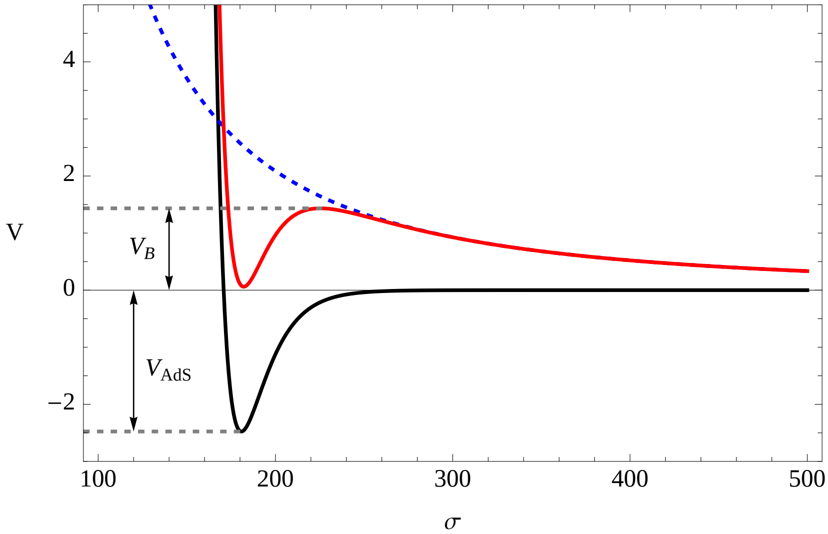

Figure 6.2: Plot of the KKLT modulus potential as a function of $\sigma = \operatorname{Re} T$. The solid black line is the F-term contribution V_F, the dotted blue line is the uplifting contribution V_{up} and the solid red line is the sum of the two contributions. The parameters haven been chosen as follows: $W_0 = -10^{-4}$, $A = 1$, $a = \frac{2\pi}{100}$ and $C_{\text{up}} = 6.68 \times 10^{-11}$. All three contributions have been rescaled by an overall factor 5×10^{15}. The positions of the minima with and without uplifting are $\sigma_{\text{up}} \approx 182.1$ and $\sigma_0 \approx 180.8$, respectively. We have also indicated the depth V_{AdS} of the potential prior to uplifting and the barrier V_B towards decompactification.

It gives rise to an extra contribution to the scalar potential of the form [272]

$$V_{\text{up}} = \frac{C_{\text{up}}}{\left(T + \overline{T}\right)^2}\,, \tag{6.27}$$

where C_{up} is a constant which depends both on the number of anti-D3-branes and on the warp factor at the tip of the throat. By adjusting these two parameters, one can (almost) cancel the negative contribution V_{AdS} to obtain a vacuum with (almost) vanishing vacuum energy, *i. e.* obtain a Minkowski vacuum or a dS-vacuum with a small cosmological constant. Including the uplifting term, the full scalar potential is

$$V \equiv V_F + V_{\text{up}}\,. \tag{6.28}$$

The position of the minimum receives only a small correction due to the uplifting term, *i. e.*

$$\sigma_{\text{up}} \approx \sigma_0\,. \tag{6.29}$$

See Fig. 6.2 for a plot of the resulting potential V and its individual contributions V_F and V_{up}. The resulting vacuum is only metastable since the vacuum energy vanishes completely as $\sigma \to \infty$ and as a consequence the uplifted minimum is not a global minimum. Thus, the vacuum can decay quantum mechanically via tunneling. To estimate the decay rate (or lifetime) of a metastable vacuum, one usually considers the Coleman-de Luccia instanton [379]. In [40], the lifetime of the uplifted vacuum was estimated in this way and it exceeds the age of our universe by many orders of magnitude for realistic parameter choices.

The above construction can be applied also to the case with more than one Kähler modulus by adding appropriate terms to the superpotential. However, note that the presence of such non-perturbative effects is related to certain properties of the 4-cycles associated to the corresponding (combination of) Kähler moduli, see *e. g.* [158]. In particular, it was shown in [210] that there is generically some tension between having non-perturbative corrections and having a chiral MSSM-like sector.[6] We will revisit the KKLT scenario when combined with inflation later on in Chap. 10.

[6] In [211], a possible way out was suggested which uses instantons carrying flux and 2-forms with negative parity under the orientifold involution.

Chapter 7

Basics of Warped Conifold Throats

An important consequence of flux compactifications are *warped throats*, *i. e.* regions of the compact manifold which are strongly warped. Such strongly warped regions have been argued to be a generic feature of type IIB flux compactifications [209, 380–384]. This chapter is devoted to reviewing some basic facts about a special class of warped throats based on *conifold singularities* and their resolutions.

We begin by briefly recalling some basic facts about the AdS/CFT- or gauge/gravity correspondence [109–112] in Sec. 7.1. Next, we review the Klebanov-Strassler (KS) solution [135] in Sec. 7.2. Finally, in Sec. 7.3, we review how the KS throat is embedded into type IIB flux compactifications [155].

This chapter is essentially following the lecture notes [113] as well as the review [385] and we refer to these for many details and an extensive list of references. In the following, we will introduce the very basics of the AdS/CFT-correspondence albeit in a brief way. For a more thorough introduction see for instance [114, 115, 386, 387].

7.1 Basics of the AdS/CFT-Correspondence

Putting a stack of N D3-branes on top of each other in flat space, one obtains an $\mathcal{N} = 4$ Super-Yang-Mills (SYM) theory with $U(N)$ gauge group on the world-volume of the D3-branes. This theory describes a *conformal* field theory in four-dimensions.

As an alternative point of view, we can "integrate out" the D3-branes by computing their backreaction on the space-time geometry. The result is a strong deformation of space-time which in the "near-horizon limit" described below is $\mathrm{AdS}_5 \times S^5$ with type IIB closed superstrings propagating in it. AdS is the abbreviation for *Anti-de-Sitter space* and refers to a space of constant *negative* curvature. AdS-spaces are highly symmetric spaces. For instance, the isometries of AdS_5 are equivalent to the superconformal group in four dimensions. This fact is one of the important tests of the AdS-CFT correspondence.

Putting the N D3-branes into 10d flat space, their backreaction induces a metric of the form

$$ds^2 = h(r)^{-1/2}\eta_{\mu\nu}dx^\mu dx^\nu + h(r)^{1/2}ds_6^2\,, \tag{7.1}$$

$$h(r) = 1 + \frac{4\pi g_s N\alpha'^2}{r^4}\,. \tag{7.2}$$

The *decoupling limit* or near-horizon limit corresponds to $\alpha' \to 0$ while N, g_s and r/α' are kept fixed. This limit essentially amounts to dropping the 1 in the expression in Eq. (7.2), *i. e.* approximating h as

$$h(r) = 1 + \frac{4\pi g_s N\alpha'^2}{r^4} \approx \frac{4\pi g_s N\alpha'^2}{r^4}\,. \tag{7.3}$$

The resulting geometry of this space is $\mathrm{AdS}_5 \times S^5$. Defining $u \equiv r/\alpha'$, the metric becomes

$$ds^2 = R^2\frac{du^2}{u^2} + \frac{\alpha'^2 u^2}{R^2}\eta_{\mu\nu}dx^\mu dx^\nu + R^2 d\Omega_5^2\,, \tag{7.4}$$

with $R^4 \equiv 4\pi g_s N\alpha'^2$ setting both the curvature scale of the AdS factor and the radius of the S^5. The trace left of the N D3-branes are N units of F_5 flux on the S^5 which simultaneously determines the scale of the curvature of the AdS_5 and the radius of the S^5.

The matching between the parameters on the gravity (or string theory) side and in the dual gauge theory is as follows

$$\begin{aligned} 4\pi g_s &= g_{\mathrm{YM}}^2 = \frac{\lambda}{N}\,, \\ \frac{R^4}{\alpha'^2} &= g_{\mathrm{YM}}^2 N = \lambda\,, \end{aligned} \tag{7.5}$$

where the *'t Hooft coupling* [116] is defined as $\lambda \equiv g_{\mathrm{YM}}^2 N$. The perturbative string theory expansion in powers of g_s (*i. e.* string loops) and α' (*i. e.* higher derivative corrections) is related to the $1/N$ and $1/\lambda$ expansions for fixed N of the gauge theory, respectively. For large-N and $\lambda \gg 1$, the string theory side is weakly coupled while the gauge theory side is strongly coupled. This is why the AdS/CFT-correspondence has become useful for studying aspects of the dynamics of strongly coupled gauge theories. There is a (one-to-one) mapping between *bulk fields in AdS* and *operators in the dual CFT*, albeit it is in general not known explicitly. That is, unless one constructs the backgrounds explicitly, *e. g.* using D-branes.

Let us write the metric of AdS_5 in conformally flat coordinates

$$ds^2 = \frac{R^2}{z^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2\right)\,, \tag{7.6}$$

and perform a KK reduction on the S^5. This yields, in particular, a bunch of scalars ϕ with 5d masses m. The solutions to the equations of motion for $z \to 0$ take the form

$$\phi(z) \sim z^{4-\Delta}\phi_0 + z^{\Delta}\langle\mathcal{O}\rangle\,, \tag{7.7}$$

where $m^2 = \Delta(\Delta - 4)$. The coefficients ϕ_0 and $\langle\mathcal{O}\rangle$ correspond to the *source* and the *expectation value* of the CFT operator $\mathcal{O}$ which is dual to the bulk field ϕ.

One can then state the AdS/CFT-correspondence in the following way:

$$\left\langle e^{\int d^4x\,\phi_0(x)\mathcal{O}(x)}\right\rangle_{\mathrm{CFT}} = Z_{\mathrm{SUGRA}}\Big|_{\phi(0,x)=\phi_0(x)}\,, \tag{7.8}$$

which is valid in the limit of large-N and large-λ.

An important point to notice is that the radial direction of AdS_5 can be related to the renormalization scale or energy scale in the dual gauge theory. Thus, one can interpret the equations for the radial dependence of the bulk fields as the renormalization group equations in the dual field theory.

To reduce the amount of supersymmetry, one way is to place the D3-branes at conical singularities, *i. e.* spaces C_6 with a metric of the form

$$ds_6^2 = dr^2 + r^2 ds_{X_5}^2\,. \tag{7.9}$$

If the space X_5 is a five-dimensional Sasaki-Einstein manifold[1], the space C_6 is a Ricci-flat Calabi-Yau manifold [394–398]. The AdS/CFT correspondence then uses $\mathrm{AdS}_5 \times X_5$ instead of $\mathrm{AdS}_5 \times S^5$. The resulting theory is less supersymmetric than the S^5 case and the amount of supersymmetry preserved depends on X_5. However, there is no general way known so far to derive the world volume theory (*i. e.* the dual gauge theory) from the geometry of X_5. The cases where the world volume theory is known explicitly are constructed by placing stacks of D-branes at the singularities and considering the decoupling (or near horizon) limit. In general, the (effective) number of D-branes shows up as fluxes threading the various cycles of X_5 and it is roughly related to the rank of the gauge groups involved.[2]

The two systems introduced so far have an AdS-factor which implies that both dual gauge theories are *conformal*. This can be understood by noting that the isometries of AdS_5 coincide with the superconformal symmetry group in

[1]The simplest examples known are S^5 and $T^{1,1} = SU(2)\times SU(2)/U(1)$, but see *e. g.* [388–393] for more general examples.

[2]For example, in the Klebanov-Witten solution [394] which has $X_5 = T^{1,1}$ and N D3-branes at the conifold singularity, the dual gauge theory is an $SU(N)\times SU(N)$ gauge theory. In the Klebanov-Tseytlin solution [399] there are in addition M D5-branes which act as fractional D3-branes such that the dual gauge theory is $SU(N+M)\times SU(N)$. The Klebanov-Strassler solution (cf. Sec. 7.2) is essentially a deformation of the Klebanov-Tseytlin solution which is regular everywhere.

4d. In the next section, we discuss an example of a *non-conformal* gravity background. The breaking of conformal invariance manifests itself as a deviation from AdS_5 in "the IR" (*i. e.* "small" r) while in "the UV" (*i. e.* "large" r) the space is approximately AdS_5.

7.2 Klebanov-Strassler Solution

The Klebanov-Strassler (KS) solution [135] is based on the *warped deformed conifold*. The conifold (see *e. g.* [400]) is a cone over the manifold $T^{1,1} = (SU(2)\times SU(2))/U(1)$, which we can write as the set of points $w \in \mathbb{C}^4$ satisfying

$$\sum_{a=1}^{4} w_a^2 = 0\,. \tag{7.10}$$

$T^{1,1}$ has a $SO(4)\times U(1)_R \simeq SU(2)\times SU(2)\times U(1)_R$ isometry and an Einstein metric on it is given by

$$ds^2_{T^{1,1}} = \frac{1}{9}\left(d\psi + \sum_{i=1}^{2}\cos\theta_i\, d\phi_i\right)^2 + \frac{1}{6}\sum_{i=1}^{2}\left(d\theta_i^2 + \sin^2\theta_i\, d\phi_i^2\right)\,, \tag{7.11}$$

with $\psi \in [0, 4\pi)$ and (θ_i, ϕ_i) each parametrizing an S^2. Concerning the isometries, each $SU(2)$ acts on one of the S^2's while the $U(1)_R$ corresponds to shifts in ψ. From the metric Eq. (7.11), one can see that $T^{1,1}$ is an S^1 bundle over $S^2\times S^2$. Moreover, it can also be written as an S^3 bundle over S^2 which is topologically trivial (see *e. g.* [394]) such that $T^{1,1}$ is essentially $S^3\times S^2$. That is, $T^{1,1}$ has a non-trivial 2-cycle and a non-trivial 3-cycle on which we can wrap branes. At the tip of the cone, both the S^3 and the S^2 shrink to zero size.

Placing N D3-branes in the conifold yields the Klebanov-Witten solution [394], an $SU(N)\times SU(N)$ gauge theory[3] with two sets of chiral superfields A_i and B_j which transform in the $(N,\bar{N})$ and $(\bar{N},N)$ representations, respectively and a superpotential

$$W = h\,\epsilon_{ij}\epsilon_{pq}\,\mathrm{Tr}\left(A_iB_pA_jB_q\right)\,. \tag{7.12}$$

The dual gravity theory is type IIB supergravity on the background $\mathrm{AdS}_5\times T^{1,1}$. The complete KK spectrum of type IIB supergravity compactified on $T^{1,1}$ is known [401, 402] and it agrees with the spectrum of operators in the CFT. To turn this into a non-conformal background, one way is to add *fractional* D3-branes which are D5-branes wrapped around the vanishing S^2 (*i. e.* the 2-cycle of $T^{1,1}$). These fractional branes are stuck at the singularity and make

[3] Actually, one find a $U(N)\times U(N)$ gauge theory, but in general any $U(1)$ factors are not captured by the dual gravity solution in the AdS/CFT-correspondence [112] and hence we ignore them here.

the theory non-conformal. The resulting gauge theory for N D3-branes and M fractional branes at the conifold singularity is $SU(N+M)\times SU(N)$ with again two sets of chiral superfields A_i and B_j transforming in the $(N+M,\overline{N})$ and $(\overline{N+M},N)$ representations, respectively, and with i,j denoting $SU(2)\times SU(2)$ indices.

The gauge couplings of the two factors of the gauge group are determined by the dilaton ϕ and the NS 2-form B_2 as[4]

$$\frac{1}{g_1^2}+\frac{1}{g_2^2}\sim e^{-\phi}\,, \tag{7.13}$$

$$\frac{1}{g_1^2}-\frac{1}{g_2^2}\sim e^{-\phi}\left[\left(\int_{S^2} B_2\right)-\frac{1}{2}\right]. \tag{7.14}$$

In the corresponding supegravity solution, also known as the Klebanov-Tseytlin (KT) solution [399], the fractional branes source M units of F_3 flux through the S^3 (*i. e.* the 3-cycle of $T^{1,1}$) and via the equations of motions then B_2 becomes a non-trivial function of the radial direction. The latter can be understood as a running of the two gauge couplings in the dual field theory. As a consequence of the 3-form flux, the warp factor has an additional logarithmic factor such that the metric becomes [399]

$$\begin{aligned} ds^2 &= h(r)^{-1/2}dx_4^2+h(r)^{1/2}\left(dr^2+r^2ds^2_{T^{1,1}}\right)\,,\\ h(r) &= \frac{81}{8}\left(g_s\alpha' M\right)^2\frac{\ln(r/r_s)}{r^4}\,, \end{aligned} \tag{7.15}$$

which is valid for the region above the tip, *i. e.* for $r\gtrsim r_s$. Due to the extra logarithmic factor, the metric in the UV is only approximately AdS$_5$ which signals the breaking of conformal invariance.

The KT solution, however, has a naked singularity at the tip of the cone. The Klebanov-Strassler (KS) solution [135] resolves this singularity by moving to the *deformed conifold* which is obtained from

$$\sum_{a=1}^{4} w_a^2=\varepsilon^2\,. \tag{7.16}$$

This way of resolving the singularity is called the *deformation* and blows up the S^3 to a finite size at the tip [400]. The alternative way of resolving the singularity by making the size of the S^2 finite is called the *(small) resolution*. However, since we have 3-form flux on the S^3, we opt for the deformation.

The metric of the deformed conifold can be written as

$$ds_6^2=\frac{\varepsilon^{4/3}}{2}K(\tau)\left(\frac{d\tau^2+(\tilde{\varepsilon}_3)}{3K(\tau)^3}+\frac{\cosh\tau}{2}\left(e_1^2+e_2^2+\varepsilon_1^2+\varepsilon_2^2\right)+\frac{1}{2}\left(e_1\varepsilon_1+e_2\varepsilon_2\right)\right)\,, \tag{7.17}$$

[4]These relations hold with and without fractional D-branes.

where

$$K(\tau) = \left(2^{1/3}\sinh\tau\right)^{-1}(\sinh(2\tau)-2\tau)^{1/3} \tag{7.18}$$

and the e_i, ε_i are defined as follows: $e_{1,2}$ are given by

$$e_1 = d\theta_1\,, \quad e_2 = -\sin\theta_1\, d\phi_1\,, \tag{7.19}$$

while $\varepsilon_i, \tilde{\varepsilon}_3$ are given by

$$\varepsilon_1 = \sin\psi\sin\theta_2\, d\phi_2 + \cos\psi\, d\theta_2\,, \tag{7.20}$$
$$\varepsilon_2 = \cos\psi\sin\theta_2\, d\phi_2 - \sin\psi\, d\theta_2\,, \tag{7.21}$$
$$\varepsilon_3 = -d\psi + \cos\theta_2\, d\phi_2\,, \tag{7.22}$$
$$\tilde{\varepsilon}_3 = \varepsilon_3 + \cos\theta_1\, d\phi_1\,, \tag{7.23}$$

as well as $d\varepsilon_i = -\frac{1}{2}\varepsilon_{ijk}\varepsilon_j\wedge\varepsilon_k$.

The *regular* supergravity solution obtained from the deformed conifold is the KS solution [135], which has a metric of the form

$$ds_{10}^2 = h(\tau)^{-1/2}dx_4^2 + h(\tau)^{1/2}ds_6^2\,, \tag{7.24}$$

where ds_6^2 is given by Eq. (7.17) and

$$h(\tau) = (g_s M\alpha')^2\, 2^{2/3}\varepsilon^{-8/3}\int_\tau^\infty dx\frac{x\coth x - 1}{\sinh^2 x}(\sinh(2x)-2x)^{1/3}\,. \tag{7.25}$$

The axio-dilaton $S \equiv$ const and the antisymmetric p-form background fields are given by

$$\begin{aligned}
B_2 &= \frac{g_s M\alpha'}{4}\Big[(f+k)\,(e_2\wedge e_1 + \varepsilon_2\wedge\varepsilon_1) + (k-f)\,(e_2\wedge\varepsilon_1 - e_1\wedge\varepsilon_2)\Big]\,,\\
F_3 &= \frac{M\alpha'}{4}\Big[\tilde{\varepsilon}_3\wedge(e_2\wedge e_1+\varepsilon_2\wedge\varepsilon_1) + (1-2F)\,\tilde{\varepsilon}_3\wedge(e_2\wedge\varepsilon_1 - e_1\wedge\varepsilon_2)\\
&\quad + F'd\tau\wedge\left(e_1^2+e_2^2+\varepsilon_1^2+\varepsilon_2^2\right)\Big]\,,\\
F_5 &= (1+\star)\,\mathcal{F}_5\,,\\
\mathcal{F}_5 &= B_2\wedge F_3 = \frac{g_s M^2\alpha'^2}{16}\Big(f\,(1-F)+kF\Big)e_1\wedge e_2\wedge\varepsilon_1\wedge\varepsilon_2\wedge\varepsilon_3\,.
\end{aligned} \tag{7.26}$$

The functions $f(\tau), k(\tau)$ and $F(\tau)$ are given by

$$\begin{aligned}
F(\tau) &= \frac{\sinh\tau - \tau}{2\sinh\tau}\,,\\
f(\tau) &= \frac{\tau\coth\tau - 1}{2\sinh\tau}(\cosh\tau - 1)\,,\\
k(\tau) &= \frac{\tau\coth\tau - 1}{2\sinh\tau}(\cosh\tau + 1)\,.
\end{aligned} \tag{7.27}$$

The radial coordinate r used previously is related to the radial coordinate τ used here. For large τ, one has $r \sim e^{\tau/3}$.

The KS solution is dual to an $SU(N+M) \times SU(N)$ gauge theory and it exhibits a *cascade of Seiberg dualities* [403] (see also [404] and references therein) and in the deep IR one eventually ends up with a pure $SU(M)$ gauge theory. The reduction of the effective number of D3-branes can be seen from the RR 5-form field strength which in the UV limit is given by

$$\mathcal{F}_5 \sim N_{\text{eff}}(r)\, vol\left(T^{1,1}\right)\,, \quad N_{\text{eff}}(r) = N + \frac{3g_sM^2}{2\pi}\ln\left(r/r_0\right)\,, \tag{7.28}$$

with the reference scale r_0 chosen such that $N_{\text{eff}}(r_0) = N$. N_{eff} decreases while moving towards the IR and this is interpreted as a lowering of the rank of the dual gauge group via a cascade of Seiberg dualities [399].[5]

An important property of the KS solution is that it is the supergravity dual of a *confining* gauge theory, which is signaled by the warp factor approaching a finite, non-zero value at the tip ($\tau = 0$).[6] One can then show that the deformation parameter ε sets the (IR) scale of the confining gauge theory. In addition to confinement, the KS solution also shows chiral symmetry breaking. Namely, the $U(1)_R$ symmetry is first broken to $\mathbb{Z}_{2M}$ by non-perturbative effects and this $\mathbb{Z}_{2M}$ symmetry is then broken spontaneously to $\mathbb{Z}_2$.

7.3 Klebanov-Strassler in Type IIB Flux Compactifications

The KS solution belongs to the class of type IIB flux compactifications described in Sec. 6.1 since the dilaton S is constant and one may show that the combination $G_3 = F_3 - iSH_3$ satisfies the ISD condition

$$\star_6\, G_3 = iG_3\,. \tag{7.29}$$

We now review how to embed the non-compact KS solution into a type IIB flux compactification as shown in [155].

A Calabi-Yau manifold may develop singularities at special points in its moduli space. The most common type of singularity is the conifold [400]. Let us denote the S^3-cycle of the conifold cycle A and its Poincaré-dual cycle B. In the conifold, the cycle B is non-compact (it is essentially the S^2 times the radial

[5] Actually, the last step in the cascade is presumably on the so-called *baryonic branch* [135, 405]. We will give more details on the baryonic branch of KS later on in Chap. 13.

[6] Confinement is usually investigated by considering Wilson loops (see *e. g.* [112]) and if the warp factor $h^{-1/2}$ of the 4d part of the 10d metric approaches a finite, non-zero value one has fundamental strings with a finite tension. These strings will be localized in the region where the warp factor has its minimum, *i. e.* at the tip.

direction), but it becomes a compact cycle in the full flux compactification. We then place F_3 and H_3 integer fluxes on the cycles A and B, respectively, *i. e.*

$$\frac{1}{(2\pi)^2\alpha'}\int\limits_A F_3 = M\,, \quad \frac{1}{(2\pi)^2\alpha'}\int\limits_B H_3 = -K\,. \tag{7.30}$$

As in [155], we describe the deformed conifold by the equation

$$\sum_{a=1}^{4} w_a^2 = z\,. \tag{7.31}$$

Note that z is related to the parameter ε introduced earlier by $z = \varepsilon^2$.

The flux-induced superpotential Eq. (6.11) is given by

$$W_{\text{flux}} = (2\pi)^2\alpha' \left(M\int\limits_B \Omega - KiS\int\limits_A \Omega \right)\,. \tag{7.32}$$

The two integrals here are called *periods* and define the complex structure of the conifold. For the collapsing cycle A one defines the coordinate z as

$$z = \int\limits_A \Omega\,. \tag{7.33}$$

Then one can show that the integral over the Poincaré-dual cycle B yields

$$\int\limits_B \Omega \equiv \mathcal{G}(z) = \frac{z}{2\pi i}\ln z + \text{holomorphic}\,. \tag{7.34}$$

With these two results, the superpotential Eq. (7.32) becomes

$$W_{\text{flux}} = (2\pi)^2\alpha' \left(M\mathcal{G}(z) - KiSz \right)\,. \tag{7.35}$$

Assuming $K/Mg_s \gg 1$, the minimization condition $D_z W = 0$ is essentially given by

$$0 \approx \frac{M}{2\pi i}\ln z - i\frac{K}{g_s}\,, \tag{7.36}$$

which then stabilizes z at an exponentially small value

$$z \sim e^{-2\pi K/Mg_s}\,. \tag{7.37}$$

This exponentially small value implies a large hierarchy between "the UV", where the KS throat is glued into the compact manifold, and "the IR" described by the tip region of the KS solution. The KS solution is actually the prime example of a warped throat geometry in type IIB flux compactifications.

We will revisit the KS solution (or more specifically its baryonic branch) later on in Chap. 13.

Chapter 8

Basics of Warped Brane Inflation

Warped throat geometries such as the Klebanov-Strassler solution discussed in the previous chapter have many applications in particle physics and cosmology. In this chapter, we briefly introduce the application of warped throat geometries to realize a particular string theory scenario for inflation often referred to as *warped brane inflation* [272] (for further developments see *e. g.* [41, 279–296] or the reviews [27, 250, 251] as well as references therein).[1] We have illustrated the basic idea in Fig. 8.1.

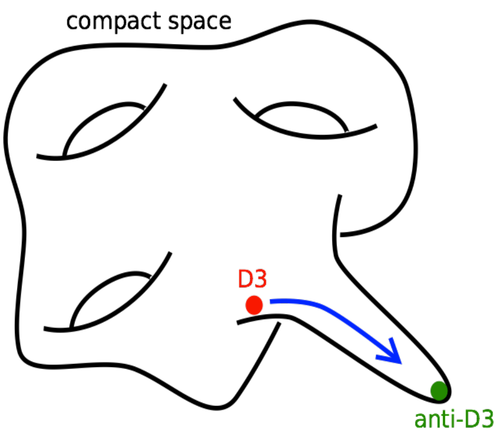

Figure 8.1: Illustration of warped D-brane inflation. There is an anti-D3-brane sitting at the tip of a warped throat region inside of a compact space and the D3-brane falls towards the anti-D3-brane.

The discussion presented here essentially follows [27, 251] and is mainly meant to introduce the concept of identifying the inflaton field with the position of a Dp-brane in a warped throat geometry.

[1]For earlier work on brane anti-brane inflation see [273–278].

8.1 Basic Idea of Warped Brane Inflation

We will be concerned with probe D3-branes which fill out all four external space-time dimensions and move along the radial direction r of a warped throat background in a type IIB flux compactification. These backgrounds have a metric of the form

$$ds^2_{\rm st} = e^{2A(y)} g_{\mu\nu} dx^\mu dx^\nu + e^{-2A(y)} \tilde{g}_{mn} dy^m dy^n \,, \tag{8.1}$$

where $\tilde{g}_{mn}$ is a Calabi-Yau metric. We assume that in some region $\tilde{g}_{mn}$ can be approximated as a cone over some Einstein manifold X_5, *i. e.*

$$\tilde{g}_{mn} dy^m dy^n \approx dr^2 + r^2 ds^2_{X_5} \,. \tag{8.2}$$

The prime example for such a warped throat geometry is the KS solution with $X_5 = T^{1,1}$ (cf. Sec. 7.2) since such throats are indeed present and actually quite common in type IIB flux compactifications (cf. Sec. 7.3). The radial coordinate r ranges from the tip at $r_{\rm IR}$ to some value $r_{\rm UV}$ where the throat is glued into a compact space. In the regime $r_{\rm IR} \ll r < r_{\rm UV}$, the warp factor for a KS warped throat is approximately given by [399]

$$e^{-4A(r)} \approx \frac{R^4}{r^4} \ln\left(\frac{r}{r_{\rm IR}}\right) \,, \quad R^4 \equiv \frac{81}{8} \left(g_s M \alpha'\right)^2 \,, \tag{8.3}$$

and $r_{\rm UV}$ and $r_{\rm IR}$ are related by

$$\ln\left(\frac{r_{\rm UV}}{r_{\rm IR}}\right) \approx \frac{2\pi K}{3 g_s M} \,. \tag{8.4}$$

There is also a 5-form background, *i. e.* a non-trivial C_4 background, to which the probe D3-brane couples. The self-dual 5-form background is assumed to be of the form

$$\tilde{F}_5 = (1 + \star_{10})\, d\alpha(y) \wedge \sqrt{-g_4}\, dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \,. \tag{8.5}$$

The Einstein frame action for a probe D3-brane in such backgrounds is then given by[2]

$$S_{\rm D3} = -T_3 \int d^4x \sqrt{-g_4}\, e^{4A} \sqrt{1 - e^{-4A} \tilde{g}_{mn} \dot{y}^m \dot{y}^n} + T_3 \int C_4 \,, \tag{8.6}$$

where the dots represent derivatives with respect to time t. Expanding this action to quadratic order in the time-derivatives, one can see that the potential for the D3 is determined by

$$V_{\rm D3} = T_3 \left(e^{4A} - \alpha\right) \equiv T_3 \Phi_- \,. \tag{8.7}$$

[2]Here we assume a "static gauge" for the worldvolume coordinates ξ^a.

Note that in a background with only ISD fluxes (cf. Sec. 6.1) the quantity Φ_- *vanishes* such that a probe D3-brane in those backgrounds experiences *no* force (at leading order in g_s and α').

In the warped brane inflation scenario of [272], there is an anti-D3-brane at the tip of a KS throat which leads to a Coulomb-type contribution to the potential for the D3-brane. The leading terms in a multipole expansion are given by [272]

$$V_c = D_0 \left(1 - \frac{27}{32\pi^2 T_3} \frac{h_0^{-1}}{r^4} \right) , \tag{8.8}$$

where $D_0 \equiv 2T_3 h_0^{-1}$ with h_0^{-1} denoting the warp factor at the tip (notice that $h(y)^{-1} \equiv e^{4A(y)}$). Due to the suppression by the warp factor this potential is sufficiently flat. Inflation ends once the distance between the D3-brane and the anti-D3-brane drops below a critical value a tachyonic mode develops and the brane and anti-brane annihilate each other and subsequently the universe is reheated [406–412].

Unfortunately, the Coulomb potential is not the full story and inflation is actually generically spoiled by an important contribution coming from stabilizing the overall volume as in the KKLT scenario [272]. In the $\mathcal{N} = 1$ supergravity language, this can be understood as follows. The overall volume $\mathcal{V}$ is no longer given by the real part of the Kähler modulus T, but instead depends also on the position y of the D3-brane such that the Kähler potential is of the form [413]

$$K = -3 \ln \left(T + \overline{T} - k(y, \bar{y}) \right) . \tag{8.9}$$

Stabilizing T via contributions to the superpotential $W(T) \sim e^{-aT}$, gives rise to a mass term for the inflaton $\mathcal{H}^2\varphi^2$ (φ is the canonically normalized inflaton). Alternatively, this contribution can be understood as arising from the coupling $\frac{1}{12}\varphi^2\mathcal{R}_4$ — φ is a conformally coupled scalar. During inflation $\mathcal{R}_4 \approx 12\mathcal{H}^2$ and thus this coupling precisely leads to a term $\mathcal{H}^2\varphi^2$ in the scalar potential [272]. However, there are also additional contributions, *e. g.* from various other kinds of bulk effects. Thus, the full potential and the corresponding contributions to the slow-roll parameter η may be written as

$$\begin{aligned} V(\varphi) &= V_c(\varphi) + \mathcal{H}^2\varphi^2 + \Delta V(\varphi) && (8.10)\\ \eta &= \eta_c + \frac{2}{3} + \Delta\eta \,. && (8.11) \end{aligned}$$

The contribution η_c from the Coulomb potential V_c is typically very small, $|\eta_c| \ll 1$. Therefore, $\eta \sim \mathcal{O}(1)$ unless the extra corrections $\Delta\eta$ from *e. g.* bulk effects conspire to cancel the generic $\frac{2}{3}$ contribution. This is essentially a string theory version of the supergravity η-problem introduced in Sec. 5.3.

8.2 Capturing Effects from Bulk Physics

In the this section, we briefly outline how to capture the corrections to the D3-brane potential from bulk effects along the lines of [282–284] (see also [279–281, 287, 288, 291, 294–296]).

The supergravity equation for determining Φ_- is of the form

$$\nabla^2 \Phi_- \sim |\mathcal{G}_-|^2 + \mathcal{R}_4 \,, \tag{8.12}$$

where $\mathcal{G}_- \equiv (\star_6 - i)\, G_3$ is the *imaginary anti-self-dual* (IASD) component of the 3-form flux G_3 and $\mathcal{R}_4$ is the 4d curvature. This equation is then solved as a perturbative expansion around the KS solution[3] by expanding all fields into the harmonic functions on the base $T^{1,1}$ of the conifold. For example,

$$\Phi_-(\varphi, \Psi) = \sum_{L,M} \Phi_{LM} \left(\frac{\varphi}{\varphi_{\rm UV}} \right)^{\Delta(L)} Y_{LM}(\Psi) + c.c. \,. \tag{8.13}$$

Here, Ψ collectively denotes the angular coordinates of $T^{1,1}$ and most importantly the φ-dependence by the eigenvalues $\Delta(L)$ of the Laplace operator in the angular directions. The possible values of $\Delta(L)$ are known from the KK-reduction of type IIB supergravity on $T^{1,1}$ [401, 402]. Thus, the structure of the D3-brane potential are of the form

$$V_{\rm D3} = T_3 \Phi_- = \sum_{\Delta} \varphi^{\Delta} f_{\Delta}(\Psi) \,. \tag{8.14}$$

The perturbations of Φ_- are either *normalizable* or *non-normalizable*[4] modes. It is also interesting to note that the corrections to the inflaton potential can also be computed on the gauge theory side via the AdS/CFT-correspondence [282–284]. The normalizable modes are deformations of the *state* of the gauge theory while the non-normalizable modes correspond to deformations of the gauge theory Lagrangian by adding *sources* for operators. The eigenvalues Δ are then related to the dimensions of the operators in the dual gauge theory.

Considering only the *non-normalizable modes* (which are sourced by gluing the throat into a compact space), the possible values for Δ are [282–284]

$$\Delta = 1, \frac{3}{2}, 2, \ldots \,. \tag{8.15}$$

The $\Delta = 1$ term does not contribute to η. Situations where either the $\Delta = \frac{3}{2}$ or the $\Delta = 2$ case dominate have been investigated in the literature. If the

[3] In practice, one expands around $\mathrm{AdS}_5 \times T^{1,1}$ which is approximately the same as the UV of KS up to a logarithmic correction.

[4] Actually, these modes are also normalizable but only once the throat is glued into a compact space. The difference between normalizable and non-normalizable modes is essentially the difference between IR and UV localized modes.

first contribution with $\Delta = \frac{3}{2}$ dominate, inflation occurs around an inflection point [280–282, 291, 294]. Or if the $\Delta = \frac{3}{2}$ mode can be projected out (see [282]) and the dominant contribution is from $\Delta = 2$. This essentially leads to a scenario with a tuneable mass term [272, 287, 288].

Recently, in [295,296] a more sophisticated approach was employed. Namely, by not taking limits where only a few terms dominate the potential, but actually using all terms up to a certain order and drawing their coefficients from a random distribution. Then if inflation works it is typically an inflection point scenario where the inflection point arises since many terms in the potential conspire to yield a flat enough potential in a certain region.

We will consider a version of warped brane inflation later in this thesis in Chap. 14, which uses the baryonic branch of the Klebanov-Strassler solution.

Chapter 9

4d Effective Supergravity for Heterotic Orbifold Compactifications

Both the type II A/B string theory and the heterotic string theory are theories of *closed strings*. The excitations of the string can be decomposed into *left-moving* and *right-moving* excitations. In the type II case, both the left-moving and the right-moving sector are supersymmetric, while in the heterotic case only one (say the left-moving sector) is supersymmetric (see *e. g.* [46–52]). As a consequence, the heterotic string contains non-Abelian gauge fields in 10d. That is, unlike for type II string theory, there is no need to introduce open strings (*i. e.* stacks of D-branes) to introduce a non-Abelian gauge group.

The low-energy limit of heterotic string theory is heterotic supergravity. The bosonic field content is given by the 10d metric G_{MN}, the NS 2-form B_{MN}, the dilaton ϕ and a gauge field A_M with either $E_8 \times E_8$ or $SO(32)$ gauge group. At leading order in α', the effective action in ten dimensions is the sum of a $\mathcal{N} = 1$ supergravity action and a $\mathcal{N} = 1$ Super-Yang-Mills action (see *e. g.* [414] or the textbooks [46–52]).

Orbifold compactifications of heterotic string theory are among the simplest possible compactifications, but they already lead to models with MSSM-like spectra, *e. g.* the "heterotic mini-landscape" models [177, 178, 415–417]. Also these models already contain a surprisingly rich phenomenology, see *e. g.* [182, 200, 418–421]. In some sense, they can be viewed as "toy models" for more complicated Calabi-Yau compactifications via *resolution* of the orbifold singularities, cf. *e. g.* [180, 422–429].[1]

The aim of this chapter is to review some facts about the 4d $\mathcal{N} = 1$ effective supergravity description of such models which we will need later on in Chap. 12. For an excellent and comprehensive review of orbifold compactifications see [432].

[1]For some MSSM-like models obtained from Calabi-Yau compactifications see *e. g.* [183–187, 430, 431].

The structure of this chapter is as follows. In Sec. 9.1, we give a brief introduction into heterotic orbifold compactifications. Next, in Sec. 9.2, we review the $\mathcal{N} = 1$ supergravity action, in particular, the constraints on the low-energy effective action. Afterwards, we briefly comment on non-perturbative corrections and on how to generate expectation values in Sec. 9.3. Finally, Sec. 9.4 contains a qualitative discussion on how moduli stabilization is achieved in heterotic orbifold compactifications.

9.1 Heterotic Orbifold Compactifications

In orbifold compactifications of the heterotic string, the six internal directions are compactified on a torus T^6 modulo a discrete symmetry group, *e. g.* a $\mathbb{Z}_N$ group. The compact dimensions can be organized into three complex coordinates:

$$Z_1 \equiv X_4 + iX_5\,,\ Z_2 \equiv X_6 + iX_7\,,\ Z_3 \equiv X_8 + iX_9\,. \tag{9.1}$$

Each can be viewed as parametrizing a torus T^2, which is obtained from the complex plane parametrized by Z_i upon imposing the identifications $Z_i \sim Z_i+1$ and $Z_i \sim Z_i + \tau$ for some $\tau \in \mathbb{C}$.

The orbifold is characterized by a three dimensional 'twist' vector v, which encodes the twist acting on the coordinates Z_i as $Z_i \to e^{2\pi i v_i} Z_i$ for $i = 1, 2, 3$. For example, the heterotic 'mini-landscape' models [177,178,415–417] based on $\mathbb{Z}_{6-II}$ have the twist vector $v = \frac{1}{6}(1, 2, -3)$, *i. e.* a rotation by $(60°, 120°, 180°)$ of the first, second and third torus, respectively. The vector v defines the first twisted sector of the theory, and the k-th twisted sector is defined by the twist vector

$$\eta_i(k) \equiv k\, v_i \mod 1\,, \tag{9.2}$$

where $0 \leq \eta_i(k) < 1$ and $k = 1, \ldots, N-1$ for $\mathbb{Z}_N$ orbifolds and one requires in addition

$$\sum_i \eta_i(k) \equiv 1\,. \tag{9.3}$$

Field Content

The field content of heterotic orbifolds is therefore devided roughly into two classes: *untwisted* and *twisted* sector fields. Roughly speaking, untwisted states come from strings which are both closed on the orbifold and on the original toroidal compactifications while twisted states come from strings which are closed only on the orbifold, *i. e.* which are closed up to a twist in the torus.

Geometrically, this classification distinguishes fields propagating in all 10 dimensions (untwisted) from those propagating only in 6 or 4 dimensions (twisted). The latter are the two types of twisted fields which can arise: they can be either confined to a fixed plane[2] or to a fixed point. This depends on the particular twisted sector, *i. e.* on whether the twist leaves one torus unrotated or rotates all three of them. Note that these sectors also have a different amount of supersymmetry: the untwisted sector has $\mathcal{N} = 4$ supersymmetry, while the two types of twisted sectors have $\mathcal{N} = 2$ and $\mathcal{N} = 1$ supersymmetry, respectively.

Dilaton and Geometric Moduli

Orbifold models have various moduli, in particular, there are always the dilaton S, which controls the strength of the string coupling g_s, and three untwisted Kähler moduli T_i associated to the volumes of the three orbifold planes. In principle, there can be also complex structure moduli U_α for the three planes, if they are not fixed by the orbifold projection. For example, the "mini-landscape" models have three Kähler moduli, *i. e.* $h_{(1,1)} = 3$, and one complex structure modulus U_3, for the third complex plane, *i. e.* $h_{(2,1)} = 1$.

Denoting the metric on a T^2 by G_{ij}, the geometric moduli T and U associated to this torus are given by (see *e. g.* [433])

$$T = \frac{1}{2}\left(\sqrt{G} + iB_{12}\right) \ , \ U = \frac{1}{G_{11}}\left(\sqrt{G} + iG_{12}\right) . \tag{9.4}$$

B_{ij} denotes the components of the 2-form inside the torus. If one introduces an explicit parametrization of the metric G_{ij} on this torus as follows

$$G_{ij} = \begin{pmatrix} R_1^2 & R_1 R_2 \cos\theta_{12} \\ R_1 R_2 \cos\theta_{12} & R_2^2 \end{pmatrix} , \tag{9.5}$$

with R_i measured in string units, then the moduli T and U are given by

$$T = \frac{1}{2}\left(R_1 R_2 \sin\theta_{12} + iB_{12}\right) \ , \ U = \frac{R_2}{R_1}\sin\theta_{12} + i\frac{R_2}{R_1}\cos\theta_{12} . \tag{9.6}$$

Note that U depends only on the *ratio* of the radii and thus determines the *shape* of the torus, while T determines the overall *size* of the torus.

In general, there are also *twisted moduli* corresponding *e. g.* to blow-up modes of the orbifold singularities (cf. *e. g.* [180, 422–429]), but we neglect them here for simplicity.

9.2 Effective 4d $\mathcal{N} = 1$ Supergravity

In this section, we review some facts about the "generic" structure of the 4d $\mathcal{N} = 1$ supergravity action for heterotic orbifold models. We begin with the

[2] This fixed plane can be either a torus or an orbifold.

structure of the (tree-level) Kähler potential K in Sec. 9.2.1. Afterwards, we introduce target space modular invariance and see how it constraints the moduli dependence of terms in the superpotential W in Sec. 9.2.2. Then in Sec. 9.2.3 the gauge kinetic function f_a and the Green-Schwarz counterterm are introduced. For this section, we are following the review of the general structure of heterotic orbifolds presented in Sec. 2 of [166] to a large extent (see also [432]).

9.2.1 Tree-level Kähler Potential and Heisenberg Symmetry

The matter fields in 4d arise from components of the gauge fields A_m in the compact internal dimensions while the moduli fields arise from the internal components of the metric G_{mn} and 2-form B_{mn} as well as from the dilaton S. Both sets of fields can be described by chiral superfields.

If we ignore the matter fields, the Kähler potential for the geometric moduli is given by [434–437]

$$K = -\sum_{i=1}^{h_{(1,1)}} \log(T_i + \overline{T}_i) - \sum_{\alpha=1}^{h_{(2,1)}} \log(U_\alpha + \overline{U}_\alpha)\,, \tag{9.7}$$

where $h_{(1,1)}$ and $h_{(2,1)}$ count the number of untwisted Kähler moduli T_i and complex structure moduli U_α, respectively. There are at least three Kähler moduli, *i. e.* $h_{(1,1)} \geq 3$ corresponding to the volumes of the three orbifold planes. The number of untwisted complex structure moduli, $h_{(2,1)}$, is model-dependent since some or all of them might already be fixed by the orbifold projection. Note that we do not consider any twisted moduli and focus on the three Kähler moduli, which parametrize the volumes of the three orbifold planes.

An important property of the tree-level Kähler potential for the untwisted matter fields $\Phi_{a,i}$ is that it enjoys a so-called "Heisenberg symmetry" [438] once the complex structure of the i-th torus is fixed such that the tree-level Kähler potential depends only on the combination [434–437]

$$\rho_i = T_i + \overline{T}_i - \sum_a |\Phi_{a,i}|^2\,. \tag{9.8}$$

Note that this is only a symmetry of the Kähler potential and is *not* preserved by the superpotential. The ρ_i are related to the radii R_i of the i-th torus in the presence of a non-trivial background for the $\Phi_{a,i}$, *i. e.* $\rho_i \sim R_i^2$, because the moduli T_i have to be redefined (see *e. g.* [433]). The Heisenberg symmetry acts on T_i and the $\Phi_{a,i}$ simultaneously as [438]:

$$T \to T + i\alpha \quad , \quad \alpha \in \mathbb{R}\,, \tag{9.9}$$

and

$$\begin{aligned} T &\to T + \bar{\beta}\Phi + \frac{1}{2}\bar{\beta}\beta\,, \\ \Phi &\to \Phi + \beta \quad , \quad \beta \in \mathbb{C}\,. \end{aligned} \tag{9.10}$$

In [439], the 10d origin of the Heisenberg symmetry was discussed. The symmetry appears in the limit of vanishing superpotential and gauge coupling and can be traced back to a shift of the 10d gauge fields A^α_M by a harmonic form λ^α_M,

$$A^\alpha_M \to A^\alpha_M + \lambda^\alpha_M \,, \tag{9.11}$$

and a corresponding shift of the 2-form B_{MN} by

$$B_{MN} \to B_{MN} - \sqrt{\frac{1}{2}}\, A^\alpha_{[M}\, \lambda^\alpha_{N]} \,. \tag{9.12}$$

Upon compactification on an orbifold these transformations induce the Heisenberg symmetry transformations on the fields T_i and $\Phi_{a,i}$.

For fixed complex structure moduli U_α, the tree-level Kähler potential including the Kähler moduli T_i and both untwisted matter fields $\Phi_{a,i}$ and twisted matter fields Ψ_a is of the following form

$$K_0 = -\sum_i \log \rho_i + \sum_a \left(\prod_i \rho_i^{-q_i^a} \right) |\Psi_a|^2 \,, \tag{9.13}$$

where the exponents $q_{a,i}$ are determined by the corresponding twist vector (cf. Eq. (9.22)). In general, these are rational numbers and there can be two cases: either all three q_i's are non-zero or exactly one of them vanishes [91]. If one expands the Kähler potential for the untwisted matter fields $\Phi_{a,i}$ to quadratic order, the tree-level contribution has the same form as for the twisted matter fields with $q_{a,j} = \delta^i_j$. Note that the Kähler potential in Eq. (9.13) is valid in the limit where $\mathrm{Re}\, T_i$ is much larger than the matter fields $\Phi_{a,i}$ and Ψ_a, *i. e.* $\langle \mathrm{Re}\, T_i \rangle \gg \langle |\Phi_{a,i}| \rangle, \langle |\Psi_a| \rangle$.

There are two different formalisms to describe the dilaton which are closely related: the string spectrum contains the antisymmetric tensor field $B_{\mu\nu}$. We can combine this field with the dilaton ϕ in a linear multiplet L. Alternatively, we can perform a duality transformation to implement this tensor field as an axion and describe it together with the dilaton as a chiral multiplet S. Both formalisms are believed to be equivalent even at the non-perturbative level [440]. In the chiral multiplet formalism, the tree-level Kähler potential for the dilaton is given by

$$K_{\mathrm{ch}} = -\log(S + \overline{S}) \,, \tag{9.14}$$

while in the linear multiplet formalism it is instead given by

$$K_{\mathrm{lin}} = \log L \,. \tag{9.15}$$

At tree level, the two formalisms are related by

$$\ell = \frac{1}{s + \bar{s}} \,, \tag{9.16}$$

where ℓ and s denote the lowest components of the linear multiplet and chiral multiplet, respectively. Thus, the weak-coupling limit, $g_s \to 0$, corresponds to $\ell \to 0$ or $s \to \infty$. However, Eq. (9.16) is subject to both perturbative and non-perturbative corrections and we will see the required modifications of Eq. (9.16) in Sec. 9.3.1.

9.2.2 Target Space Modular Invariance

The low-energy effective supergravity action for heterotic orbifolds is subject to strong constraints from *target space modular invariance*, which is preserved to all orders in perturbation theory. We will now discuss these transformations and the restrictions they impose, in particular, on the superpotential.

The *modular transformations* of the Kähler moduli T_i and complex structure moduli U_α are often elements of $SL(2,\mathbb{Z})$ [434–437,441,442]. There is one such $SL(2,\mathbb{Z})$ group for each modulus $M \in \{T_i, U_\alpha\}$ which acts on M as

$$M \to \frac{aM - ib}{icM + d}\,, \quad ad - bc = 1\,, \quad a,b,c,d \in \mathbb{Z}\,. \tag{9.17}$$

Hence, the $\log(M + \overline{M})$ terms appearing in the Kähler potential are not invariant but instead transform as

$$\log(M+\overline{M}) \to \log\left(\frac{M+\overline{M}}{(icM+d)(-ic\overline{M}+d)}\right)\,. \tag{9.18}$$

Therefore, the modular group induces a transformation of the Kähler potential:

$$K \to K + \sum_{i=1}^{h_{(1,1)}} \log|ic_i T_i + d_i|^2 + \sum_{\alpha=1}^{h_{(2,1)}} \log|ic_\alpha U_\alpha + d_\alpha|^2\,. \tag{9.19}$$

Since the F-term scalar potential is necessarily invariant and depends only on the combination $G = K + \log|W|^2$ (cf. Sec. 4.2), the superpotential must also transform under modular transformations. The transformaton Eq. (9.19) has the form of a Kähler transformation and thus W transforms as

$$W \to \prod_{i=1}^{h_{(1,1)}} \prod_{\alpha=1}^{h_{(2,1)}} (ic_i T_i + d_i)^{-1} (ic_\alpha U_\alpha + d_\alpha)^{-1}\, W\,. \tag{9.20}$$

Moreover, in addition to the moduli, also the matter fields $\Phi_a \equiv \{\Phi_{a,i}, \Psi_a\}$ transform under the modular group as

$$\Phi_a \to \prod_{i=1}^{h_{(1,1)}} \prod_{\alpha=1}^{h_{(2,1)}} (ic_i T_i + d_i)^{-q_{a,i}} (ic_\alpha U_\alpha + d_\alpha)^{-p_{a,\alpha}}\, \Phi_a\,. \tag{9.21}$$

The exponents $q_{a,i}, p_{a,\alpha}$ in Eq. (9.13) and (9.21) are called *modular weights* [91, 92]. They are determined by the orbifold twist vector of the given sector $\eta_i(k)$, cf. Eq. (9.2), as follows

$$q_{a,i} \equiv (1 - \eta_i(k)) + N_i - \bar{N}_i \qquad \text{for } \eta_i(k) \neq 0\,, \tag{9.22a}$$
$$q_{a,i} \equiv \qquad N_i - \bar{N}_i \qquad \text{for } \eta_i(k) = 0\,, \tag{9.22b}$$

where the N_i and $\bar{N}_i$ are integer oscillator numbers of left-moving oscillators $\widetilde{\alpha}_i$ and $\bar{\widetilde{\alpha}}_i$, respectively. Similarly, the $p_{a,j}$ are given by

$$p_{a,\alpha} \equiv (1 - \eta_\alpha(k)) - N_\alpha + \bar{N}_\alpha \qquad \text{for } \eta_\alpha(k) \neq 0\,, \tag{9.23a}$$
$$p_{a,\alpha} \equiv \qquad -N_\alpha + \bar{N}_\alpha \qquad \text{for } \eta_\alpha(k) = 0\,. \tag{9.23b}$$

For a given polynomial in the matter fields to be used in the superpotential, the correct transformation of W can be ensured by appropriate powers of the Dedekind η-function multiplying this polynomial. Under modular transformations, the η-function transforms (up to a phase) as

$$\eta(M) \to (icM + d)^{1/2}\, \eta(M)\,, \tag{9.24}$$

with $\eta(M)$ defined as

$$\eta(M) = e^{-\pi M/12} \prod_{n=1}^{\infty} \left(1 - e^{-2\pi n M}\right)\,. \tag{9.25}$$

Thus, a generic term in the superpotential has the following structure

$$W \supset \prod_{i=1}^{h_{(1,1)}} \eta(T_i)^{2\sigma_i} \prod_{\alpha=1}^{h_{(2,1)}} \eta(U_\alpha)^{2\widetilde{\sigma}_\alpha} \prod_a \Phi_a^{n_a}\,, \tag{9.26}$$

where Φ_a denotes both untwisted and twisted matter fields and the exponents of the η-functions are given by $\sigma_i = -1 + \sum_a n_a q_{a,i}$ and $\widetilde{\sigma}_\alpha = -1 + \sum_a n_a p_{a,\alpha}$.

For $\mathrm{Re}\, T_i \gtrsim 1$, we can approximate $\eta(T_i)$ by $\exp(-\frac{\pi T_i}{12})$ and hence if a term in the superpotential depends on T_i it is generically of the form $\sim e^{-cT_i}$ at large radius (*i. e.* for large $\mathrm{Re}\, T_i$) with some constant c. The interpretation of this e^{-T} dependence is that the corresponding interaction in the superpotential is generated by a non-perturbative effect: The strings have to stretch over the i-th torus to reach each other which then leads to a suppression of this interaction by the volume of the torus. In general, there are also term in W which do not depend on the moduli[3]. For instance, the coupling of three untwisted fields associated to three different planes or of three twisted fields living at the same fixed point will not have any moduli dependence.

The superpotential for the matter fields starts at cubic order in the fields (see *e. g.* [434]). Thus, for instance any mass term has to be generated by some other fields acquiring non-zero expectation values.

[3]Up to modular invariant functions which we do not consider here.

9.2.3 Gauge Kinetic Function and Green-Schwarz Counterterm

So far, we have reviewed the structure of the Kähler potential K and the superpotential W. The last function required to specify the action of the chiral superfields is the gauge kinetic function f_a. At string one-loop level, the modular transformations are anomalous and this anomaly is cancelled by the Green-Schwarz mechanism and threshold corrections from massive string modes, which in turn modifies the effective action. In particular, in the chiral multiplet formalism the dilaton S will generically mix with the Kähler moduli T_i and the complex structure moduli U_α.

The gauge couplings are determined by the gauge kinetic function f_a as $g_a^{-2} = \mathrm{Re}\, f_a$ with a labeling the different gauge groups $\mathcal{G}_a$. At tre-level, one has universally $f_a = k_a S$, but at string one-loop level f_a is given by [443–447]

$$f_a = k_a S + \sum_{i=1}^{h_{(1,1)}} \left(\alpha_a^i - k_a \delta_{GS}^i\right) \log(\eta(T_i))^2 + \sum_{\alpha=1}^{h_{(2,1)}} \left(\alpha_a^\alpha - k_a \delta_{GS}^\alpha\right) \log(\eta(U_\alpha))^2 \,, \tag{9.27}$$

where k_a is the Kac-Moody level of the group (typically $k_a = 1$), and the model-dependent constants α_a^i are defined as

$$\alpha_a^i \equiv \ell(\mathrm{adj}) - \sum_{\mathrm{rep}_A} \ell_a(\mathrm{rep}_A)(1 + 2q_{A,i}) \,. \tag{9.28}$$

Here, $\ell(\mathrm{adj})$ and $\ell_a(\mathrm{rep}_A)$ are the Dynkin indices of the adjoint and matter field representations of the corresponding gauge group factor $\mathcal{G}_a$, respectively.[4] The Green-Schwarz coefficients δ_{GS}^i are given by [446]

$$\alpha_a^i - k_a \delta_{GS}^i = \frac{b_{a,i}^{\mathcal{N}=2}}{|D|/|D_i|} \,, \tag{9.29}$$

where $b_{a,i}^{\mathcal{N}=2}$ is a beta function coefficient of the gauge group $\mathcal{G}_a$ for the i-th torus. These coefficients are non-zero only if there is some twisted sector with $\mathcal{N} = 2$ supersymmetry and if this twisted sector does not rotate the i-th torus. The factors $|D|$ and $|D_i|$ are the degree of the twist group D and the little group D_i, which leaves the i-th unrotated, respectively. For example, the mini-landscape models have $D = \mathbb{Z}_{6-II}$ and then $|D| = 6$, $|D_2| = 2$ and $|D_3| = 3$ since the little groups under which the second and third torus are fixed are $\mathbb{Z}_2$ and $\mathbb{Z}_3$, respectively. The first torus is rotated in all twisted sectors.

The δ_{GS}^i terms are introduced to cancel a sigma-model and Kähler anomaly of the modular group. This anomaly induces a non-trivial modular transfor-

[4]The Dynkin indices are determined from the normalization condition $Tr(\mathcal{T}_i\mathcal{T}_j) = \ell_a(\mathrm{rep})\delta_{ij}$ of the generators $\mathcal{T}_i$ in the given representation.

mation of the dilaton in the chiral formalism:

$$S \to S + \sum_{i=1}^{h_{(1,1)}} \delta_{GS}^{i} \log(ic_i T_i + d_i) + \sum_{\alpha=1}^{h_{(2,1)}} \delta_{GS}^{\alpha} \log(ic_\alpha U_\alpha + d_\alpha)\,. \tag{9.30}$$

The anomaly is cancelled (partially) by the so-called Green-Schwarz counterterm, which modifies the Kähler potential at string one-loop level. Neglecting the matter fields, the modified Kähler potential is given by

$$K = -\log Y - \sum_{i=1}^{h_{(1,1)}} \log(T_i + \overline{T}_i) - \sum_{j=1}^{h_{(2,1)}} \log(U_\alpha + \overline{U}_\alpha)\,, \tag{9.31}$$

where

$$Y = S + \overline{S} - \sum_{i=1}^{h_{(1,1)}} \delta_{GS}^{i} \log(T_i + \overline{T}_i) - \sum_{\alpha=1}^{h_{(2,1)}} \delta_{GS}^{\alpha} \log(U_\alpha + \overline{U}_\alpha)\,. \tag{9.32}$$

In general, the Green-Schwarz mechanism will not cancel the complete modular anomaly completely. The remaining part of the anomaly is cancelled by threshold corrections from massive string modes [446]. These threshold corrections are moduli-dependent since the masses of *e. g.* the Kaluza-Klein and winding states depend on the radii.

9.3 Non-Perturbative Corrections and Expectation Values

9.3.1 Non-Perturbative Corrections

So far, we have described the structure of the effective supergravity theory at tree-level and introduced perturbative corrections. Now we will introduce also non-perturbative corrections. These are an important ingredient for successful moduli stabilization in string theory and in heterotic models they are in particular crucial for stabilizing the dilaton S (cf. Sec. 9.4).

Non-perturbative corrections can be either of a field-theoretic [448] or a stringy origin [449] (see also [450–452]). Field-theory instantons (such as gaugino condensates) scale like e^{-1/g^2}, while string theory instanton effects scale like $e^{-1/g}$, where g is the coupling constant. Corrections to the Kähler potential are much harder to compute than corrections to the superpotential. But we may parametrize the non-perturbative corrections and treat the coefficients as essentially free parameters.

In the following, we first briefly comment on non-perturbative effects in the chiral multiplet formalism and then we turn to their description in terms of the linear multiplet. The discussion here follows [453–456].

The Chiral Multiplet Formalism

In the chiral multiplet formalism, one has non-perturbative corrections to both the Kähler and the superpotential,

$$\begin{aligned} K &= K_{\text{tree}} + K_{\text{pert}} + K_{\text{np}}\,, \\ W &= W_{\text{tree}} + W_{\text{np}}\,. \end{aligned} \tag{9.33}$$

The non-perturbative superpotential is due to the presence of a gaugino condensate and thus one has [447, 457–459]

$$W_{\text{np}} = A e^{-bS} \prod_{i=1}^{3} \eta(T_i)^{-2}\,, \tag{9.34}$$

where b is related to the beta-function coefficient of the condensing gauge group and the η-functions are introduced to ensure covariance of the superpotential under modular transformations. The non-perturbative corrections to the Kähler potential are typically parametrized in terms of $\operatorname{Re} S$, cf. *e. g.* [460, 461] for some examples.

The kinetic mixing between the dilaton and the Kähler moduli, cf. Eq. (9.31), however makes finding flat directions more complicated in the chiral multiplet formalism. Thus, we will consider the linear multiplet formalism and since the two formalisms are believed to be equivalent there should be no physical difference.

The Linear Multiplet Formalism

In the linear multiplet formalism, the dilaton is invariant under modular transformations.[5] The Green-Schwarz counterterm is then implemented somewhat differently [443, 462, 463]. Neglecting the complex structure moduli, it is given by

$$V_{GS} = -\sum_i \delta^i_{GS} \log(T_i + \overline{T}_i)\,. \tag{9.35}$$

In this thesis, we will assume that V_{GS} preserves the Heisenberg symmetry, *i. e.* that it is actually given by the tree-level Kähler potential [464]:

$$V_{GS} = -\sum_i \delta^i_{GS} \log \rho_i + \sum_a p_a |\Psi_a|^2 \left(\prod_i \rho_i^{-q_i^a} \right)\,, \tag{9.36}$$

with the unknown contribution of the twisted matter fields Ψ_a to the Green-Schwarz term parametrized by the coefficients p_a. Upon including this term, the effective Kähler metric for the fields is modified to

$$K^{\text{eff}}_{a\bar{b}} = K_{a\bar{b}} + \ell V^{GS}_{a\bar{b}}\,. \tag{9.37}$$

[5] One can make a field redefinition of the dilaton in the chiral formalism in order to make it invariant under modular transformations, cf. *e. g.* [443].

In the linear multiplet formalism, the superpotential is independent of the dilaton since it is not a chiral superfield. The non-perturbative corrections to the Kähler potential may be parametrized by a function $g(L)$ as

$$K = \log L + g(L) + \dots\,, \tag{9.38}$$

where the dots denote the terms involving the other moduli and matter fields. The gauge coupling constant (at the string scale) also receives non-perturbative corrections which we parametrize by another function $f(\ell)$:

$$g^2 = \frac{2\ell}{1+f(\ell)}\,. \tag{9.39}$$

The relation between the linear and the chiral multiplet formalism gets modified by both perturbative and non-perturbative effects:

$$\frac{\ell}{1+f(\ell)} = \frac{1}{s+\bar{s}+V_{GS}}\,, \tag{9.40}$$

where V_{GS} is given by Eq. (9.36).

The two functions $g(\ell)$ and $f(\ell)$ in Eqs. (9.38) and (9.39) are related by

$$\ell\frac{dg}{d\ell} = f - \ell\frac{df}{d\ell}\,, \quad f(\ell=0) = g(\ell=0) = 0\,. \tag{9.41}$$

The differential equation and the boundary conditions ensure a canonical normalization of the Einstein-Hilbert term and the correct behaviour in the weak-coupling limit $\ell \to 0$, respectively.

Following [453–456], we parametrize $f(\ell)$ as

$$f(\ell) = B\left(1 + A\frac{1}{\sqrt{a\ell}}\right)e^{-1/\sqrt{a\ell}}\,. \tag{9.42}$$

The function $g(\ell)$ is then determined by solving Eq. (9.41).

During inflation, the gaugino condensate is expected to be negligible and hence the effective scalar potential for vanishing D-terms is given by [464]

$$V = e^K\left(\left(\ell\frac{dg}{d\ell}+1\right)|W|^2 - 3|W|^2 + \sum_{a\bar{b}}\left(K^{\text{eff}}_{a\bar{b}}\right)^{-1}F_a\bar{F}_{\bar{b}}\right)\,, \tag{9.43}$$

where the indices a, b run over the scalar components of the Kähler moduli T_i, the untwisted matter fields $\Phi_{a,i}$ and the twisted matter fields Ψ_a. The supergravity F-terms F_m are given by

$$F_a = W_a + K_a W\,. \tag{9.44}$$

The effective Kähler metric in the last term of Eq. (9.43) is given by

$$K^{\text{eff}}_{a\bar{b}} = K^{\text{tree}}_{a\bar{b}} + \ell K^{\text{1-loop}}_{a\bar{b}}\,, \tag{9.45}$$

while the K to be used in Eqs. (9.43) and (9.44) is given by

$$K = \log(\ell) + g(\ell) - \sum_i \log \rho_i + \sum_a \left(\prod_i \rho_i^{-q_i^a} \right) |\psi_a|^2 \,, \tag{9.46}$$

which is obtained by replacing all superfields with their scalar components (denoted by lower case letters) and dropping the perturbative corrections. Note also that in Eq. (9.46) we use $\rho_i = t_i + \bar{t}_i - \sum_a |\phi_a^i|^2$.

9.3.2 Anomalous $U(1)_A$ and Generating Expectation Values

In this section, we review how to generate expectation values for matter fields via D-terms of an anomalous $U(1)_A$ and subsequently through F-terms. For more details and examples in the present context of inflationary model building, see *e. g.* [464, 465], which we will closely follow here.

D-term Expectation Values

In many orbifold models there exists an anomalous $U(1)_A$. The anomaly is cancelled via a Green-Schwarz counterterm, which gives rise to a Fayet-Iliopoulos contribution to the D-term D_A. Thus, we have a contribution to the scalar potential from the D-term,

$$V_D = \frac{g^2}{2} \left(\sum_a q_{A,a} K_a \phi_a + \xi_A \right)^2 \,, \tag{9.47}$$

where the index a runs over both twisted and untwisted matter fields, $q_{A,a}$ denotes the charge under the anomalous $U(1)_A$ (not to be confused with the modular weights $q_{a,i}$), the gauge coupling g^2 is given in Eq. (9.39), ϕ_a denotes the scalar component of the $\Phi_a = \{\Phi_{a,i}, \Psi_a\}$ and the Fayet-Iliopoulos D-term ξ_A (in the linear multiplet formalism) is given by

$$\xi_A = \frac{2\ell \,\mathrm{Tr}\, Q_A}{192\pi^2} \,, \tag{9.48}$$

with Q_A the generator of the anomalous $U(1)_A$. Using the Kähler potential of Eq. (9.13), the D-term potential in Eq. (9.47) becomes

$$V_D = \frac{1}{2} g^2 \left[\sum_a \left(\prod_i \rho_i^{-q_{a,i}} \right) q_{A,a} |\phi_a|^2 + \xi_A \right]^2 \,. \tag{9.49}$$

Cancellation of the D-term requires some matter fields to pick up non-zero expectation values of the form

$$\frac{|\langle \phi_a \rangle|^2}{q_{A,a}} = const \cdot \ell \cdot \left(\prod_i \rho_i^{q_{a,i}} \right) \,. \tag{9.50}$$

These expectation values driven by the cancellation of the D-term of an anomalous $U(1)_A$ are important also for the phenomenology of heterotic orbifold models. For example, they are required to get rid of extra exotic states in the mini-landscape models [177, 178, 415–417].

F-term Expectation Values

Via the superpotential, D-term expectation values can induce other non-zero expectation values. To illustrate this, let us review the example of [464, 465]. Consider the following modular-invariant expression of the three fields χ, ϕ, ϕ':

$$\Gamma = \chi\phi\phi' \prod_i \eta(T_i)^{2\sigma_i}\,, \tag{9.51}$$

where $\sigma_i = \sum_a q_{a,i}$, $a = \chi, \phi, \phi'$. Assume also that ϕ and ϕ' acquire non-zero expectation values, *e. g.* through the cancellation of a D-term as described above. Using this expression, we may build a possible superpotential contribution of the form

$$W(\Gamma) = \left(\psi\phi\phi' \prod_i \eta(T_i)^{2\sigma'_i}\right)(c_0 + c_1\Gamma)\,, \tag{9.52}$$

with some constants $c_{0,1} \neq 0$ and $\sigma'_i = -1 + \sum_b q_{b,i}$, $b = \psi, \phi, \phi'$. Such a term is allowed if the products $\psi\phi\phi'$ and $\chi\phi\phi'$ are gauge invariant. We can find a critical point[6] with a non-zero F-term for ψ if $\langle\psi\rangle = 0$ and $\Gamma = -c_1/c_0 = const$. Hence, the non-zero D-term expectation values for ϕ, ϕ' induce a non-zero expectation value for χ

$$|\langle\chi\rangle|^2 = const \cdot \left|\left\langle \phi\phi' \prod_i \eta(T_i)^{2\sum_a q_{a,i}} \right\rangle\right|^{-2}\,, \tag{9.53}$$

with $a = \chi, \phi, \phi'$. Note that if $\langle\phi\rangle$ and $\langle\phi'\rangle$ are induced by the D-term cancellation as above, we see from Eqs. (9.53) and (9.50) that $|\langle\chi\rangle|^2 \propto \ell^{-2}$. Also note that in principle $\langle\chi\rangle$ can involve $\eta(T_i)$ to some power.

The important lesson from the above considerations is that in general the VEVs induced by either D-terms or F-terms carry some dependence on the geometric moduli T_i, U_α and the dilaton ℓ.

9.4 Moduli Stabilization in Heterotic Orbifolds

Moduli stabilization in heterotic orbifold compactifications works somewhat different than in the type IIB picture presented in Chap. 6. This is due to the absence of fluxes which in the heterotic case would strongly deform the

[6]Whether this critical point is actually a minimum depends also on the other matter fields present [465].

compact manifold and are therefore not switched on in this class of models. Hence, for heterotic orbifold models, the moduli have to be stabilized entirely by invoking non-perturbative effects. The discussion here will use the chiral multiplet formalism for the dilaton to make the similarities to the type IIB case clearer.

As reviewed in Sec. 9.1, the moduli of a heterotic orbifold compactification are the dilaton S as well as $h_{(1,1)}$ Kähler moduli T_i and $h_{(2,1)}$ complex structure moduli U_α. In the simplest orbifold models, their tree-level Kähler potential is

$$K = -\ln\left(S + \overline{S}\right) - \sum_{i=1}^{h_{(1,1)}} \ln\left(T_i + \overline{T}_i\right) - \sum_{\alpha=1}^{h_{(2,1)}} \ln\left(U_\alpha + \overline{U}_\alpha\right) . \tag{9.54}$$

In general, there are corrections to both the superpotential and the Kähler potential. While the former receives only non-perturbative corrections, the latter receives both perturbative and non-perturbative corrections. In the phenomenologically interesting regime, the dilaton is most likely stabilized in a regime where both kinds of corrections are important [448]. Indeed, stabilizing the dilaton crucially relies upon non-perturbative corrections to either W or K or both. For two recent approaches on moduli stabilization in heterotic orbifold models see [166, 167]. Here, we only want to briefly outline the qualitative picture.

Non-Perturbative Superpotential Terms The geometric moduli T_i, U_α enter the superpotential via worldsheet instantons which induce certain couplings between matter fields. The dependence of the superpotential terms on these geometric moduli is constrained by target space modular invariance, cf. Sec. 9.2.2.

The dilaton does not enter the superpotential at any order in perturbation theory, but may enter at the non-perturbative level through *gaugino condensates*. The gaugino condensate depends on the dilaton S since the dilaton sets all tree-level gauge couplings [360, 361, 466, 467] and it in general also carries a dependence on the geometric moduli T_i, U_α due to threshold corrections [91, 445, 446]. Thus, a typical contribution from a gaugino condensate is of the form

$$W_{\text{gc}} = A e^{-bS - a_i T_i} , \tag{9.55}$$

where b is determined by the β-function coefficient of the condensing gauge group and a_i, c_j are determined by modular invariance.[7]

In [419], it was noted that the superpotentials of the heterotic "mini-landscape" models [177, 178, 415–417] have an approximate R-symmetry which is only broken at high orders in the matter fields. Based on this observation,

[7]There is a sublety here which has to do with *redefinitions* of the chiral superfield S containing the dilaton beyond tree-level. If we define a *modular-invariant* dilaton S', all the $a_i = -2$ [447].

one may construct KKLT-like setups for moduli stabilization for the dilaton of the form [166, 419]

$$W \supset w_0 e^{-aT} + e^{-bS-cT} \,. \tag{9.56}$$

But note that the stabilization of T is of a racetrack form. The e^{-T}-dependence is required again by modular invariance and $w_0 = w_0\left(\langle\Phi_a\rangle\right)$ arises due to a set of matter fields acquiring VEVs.

Non-Perturbative Kähler Potential Terms In addition to the inevitable loop corrections, the Kähler potential may also receive non-perturbative corrections. These can be of a similar for as the gaugino condensates, *i. e.* depend on the dilaton as $\sim e^{-bS}$, but there can be also stringy non-perturbative effects of the form $\sim e^{-b\sqrt{S+\overline{S}}}$ [43, 448, 449, 453–456, 460, 461]. And this latter type of correction is typically the more important one of the two.

Qualitative Picture

Depending on which terms are considered, we may roughly distinguish two classes of models: Racetrack-like and KKLT-like models or "Kähler Stabilization" models. In both classes, the superpotential W must depend on the moduli and then the parameters controlling the moduli dependence of W and/or K are adjusted such that a stable minimum is obtained.

Racetrack-like and KKLT-like models Neglecting the non-perturbative corrections to the Kähler potential, there are two options for stabilizing the moduli. The resulting potential for the dilaton can be of a racetrack form, where two gaugino condensates contribute [458, 459, 468, 469],

$$W \sim Ae^{-b_1 S} + Be^{-b_2 S} \,, \tag{9.57}$$

or of a KKLT-like form with only a single condensate [166, 419],

$$W \sim w_0 + Ae^{-bS} \,. \tag{9.58}$$

In both cases, the dilaton can be stabilized for suitable parameter choices. As noted above, the gaugino condensates carry also some dependence on (some of) the geometric moduli T_i, U_α. This dependence may already be sufficient to stabilize them [457, 459, 469–471] and the remaining ones may then be fixed by worldsheet instantons involved in twisted field Yukawa couplings [447]. Alternatively, the T_i, U_α are also fixed by a racetrack-like structure, see *e. g.* [166].

Kähler Stabilization Including the non-perturbative corrections $K_{\rm np}$ leads to the scheme of *Kähler stabilization* [43, 448, 453–456, 460, 461], which requires the presence of only a single gaugino condensate. Without $K_{\rm np}$, a single gaugino condensate is not sufficient to stabilize the dilaton, but we may include non-perturbative corrections to K parametrized for example as

$$e^K = e^{K_0} + e^{K_{\rm np}}\,, \tag{9.59}$$

$$e^{K_{\rm np}} = c\left(S+\overline{S}\right)^{p/2} e^{-q\sqrt{S+\overline{S}}}\,. \tag{9.60}$$

The parameters in $K_{\rm np}$ are chosen such that $p, q > 0$ and $K'' > 0$ (the prime denotes a derivative with respect to $\mathrm{Re}\, S$). Suitable parameter choices allow for stabilization of the dilaton [453–455, 460, 461]. Stabilization of the Kähler moduli T_i and complex structure moduli U_α works as in the models with only non-perturbative contributions to the superpotential. We will employ a version of this stabilization scheme later on in this thesis in Chap. 12.

The authors of [166] considered a model where the set of bulk moduli S, T were stabilized with non-zero F-terms and in addition there is an uplifting contribution from a non-zero F-term for a matter field driven by the cancellation of the D-term of an anomalous $U(1)_A$. They then argued that within such a framework generically all flat directions of heterotic orbifold models, including those corresponding to twisted (matter) fields, should be stabilized.

In [167], the authors tried to take into account all bulk moduli and the dilaton for a few examples from the heterotic mini-landscape models [177, 178, 415–417] and searched for stable de Sitter vacua. So far, they did not succeed in finding any explicit examples of a stable dS vacuum but only unstable ones. However, their analysis did not include non-perturbative corrections to K and some other kinds of moduli-dependent corrections. Thus, at present, the possible existence of metastable dS vacua for the mini-landscape models remains an open question.

Part III

Inflation in 4d Effective Supergravity Theories

Chapter 10

Combining Low-Energy Supersymmetry and High-Scale Inflation

After having reviewed the required basic material, we now move on to the actual work done in the course of this thesis. First, we discuss results for models of inflation in effective supergravity theories in this part. Later on, in Part IV, we discuss a new model of D-brane inflation.

This chapter is based on Ref. [2], where my collaborators and I proposed a new solution to simultaneously combining low-energy supersymmetry and high-scale inflation. These two seemingly uncorrelated subjects actually turn out to be deeply connected via *moduli stabilization*, especially in the context of string theory compactifications.[1]

Apart from other implications of moduli stabilization for supersymmetry breaking and single-field slow-roll inflation, there is a very profound one during inflation. Namely, the very presence of an inflationary sector may *destabilize* the moduli, as was pointed out by Buchmüller, Hamaguchi, Lebedev and Ratz and by Kallosh and Linde [43–45].[2] In this chapter, we are concerned with a particular version of this problem sometimes called the Kallosh-Linde (KL) problem [45]. That is, to avoid destabilization of the volume modulus during inflation one often has an *upper bound* on the Hubble scale during inflation determined by todays gravitino mass,

$$\mathcal{H}_{\text{inf}} \leq m_{3/2}^{\text{today}} \,. \tag{10.1}$$

Thus, a gravitino mass $m_{3/2} \sim \mathcal{O}(\text{TeV})$ would allow only for a very small Hubble scale during inflation, which is much below the values needed for many inflationary models and in addition also much below the sensitivity of experiments searching for primordial gravitational waves.

Basically, the problem appears since there is effectively only one scale involved, which sets both the gravitino mass today and the height of the bar-

[1] See Chap. 6 and references therein for a review on moduli stabilization.

[2] For earlier work discussing problems of moduli related to inflation cf. *e. g.* [247–249].

rier towards decompactification. Invoking only fluxes and non-perturbative effects, one ends up with an AdS-minimum which is then turned into a (nearly) Minkowski minimum via a supersymmetry breaking uplift. Thereby, a barrier towards decompactification is created and its height is related to todays gravitino mass. Inflation in such a setup can often be viewed as an additional uplifting which induces a runaway potential for the modulus. If the contribution of the inflationary sector becomes too large, the barrier and hence the minimum disappear. We will review the problem in more detail in Sec. 10.1.

In Ref. [2], my collaborators and I proposed a new resolution of the problem which uses two *different* mechanisms to stabilize the moduli during and after inflation. During inflation, the modulus receives a large mass proportional to the inflationary vacuum energy. This is achieved, for example, by a suitable moduli-dependence of the Kähler metric of the field driving supersymmetry breaking during inflation.[3] At the end of inflation, the vacuum energy goes to zero and we invoke a KKLT-like stabilization mechanism relying on non-perturbative terms in the superpotential. The general idea is illustrated in a simple example: chaotic inflation protected by a shift symmetry combined with a KKLT-type superpotential. However, let me stress that the framework can also be applied to more general setups.

This chapter is organized as follows. We review the KL problem in more detail in Sec. 10.1. Afterwards, we outline our general framework for solving the KL problem in Sec. 10.2. This general framework is then illustrated in an explicit example in Sec. 10.3 and in a less explicit example in Sec. 10.4. Finally, we summarize the results and discuss the relation to other works in Sec. 10.5.

10.1 Review of the Kallosh-Linde Problem

The Kallosh-Linde (KL) problem was discussed in [45] in the context of moduli stabilization in type IIB string theory within the KKLT scenario [40]. Below the scale where the complex structure moduli and the dilaton are stabilized via fluxes as in [153–155], the Kähler potential K and the superpotential W for the volume modulus $T \equiv \sigma + i\,\alpha$, which controls the overall size of the compact space, are given by [40]

$$K = -3\ln(T + \overline{T}) \quad \text{and} \quad W = w_0 + Ae^{-a\,T}\,, \tag{10.2}$$

respectively. Note that throughout this part of the thesis we work in units where $M_P \equiv 1$. In the following, we consistently set the imaginary part $\alpha = 0$, which corresponds to a particular choice of phases for w_0 and A, and consider only the potential for the real part σ. The resulting F-term potential has a *supersymmetric* Anti de Sitter (AdS) minimum and consequently the depth of

[3] Such couplings have been first used to stabilize moduli during inflation in [351].

this minimum is given by

$$V_{\rm AdS} = -3\, e^{\langle K\rangle_{\sigma_{\rm AdS}}}\, |\langle W\rangle_{\sigma_{\rm AdS}}|^2\,, \tag{10.3}$$

where $\sigma_{\rm AdS}$ is the position of the AdS minimum. To turn this into a Minkowski or de Sitter (dS) minimum, one has to add an uplifting term to the potential, which is typically of the form $\Delta V \sim \frac{C_{\rm up}}{\sigma^2}$.[4] The value of $C_{\rm up}$ is fine-tuned such that the value of the potential at the new minimum is equal to the present value of the cosmological constant. The uplifting usually induces only a small shift in the position of the minimum which is negligible, *i. e.* one has $\sigma_0 \approx \sigma_{\rm AdS}$. The uplifting procedure also creates a barrier that prevents the field from running away to the Minkowski vacuum at $\sigma \to \infty$, *i. e.* towards decompactification. The height V_B of this barrier turns out to be $V_B \simeq \mathcal{O}(1)|V_{\rm AdS}|$.

The gravitino mass in the uplifted minimum is given by

$$m_{3/2}^2(\sigma_0) = e^{\langle K\rangle_{\sigma_0}}|\langle W\rangle_{\sigma_0}| \approx e^{\langle K\rangle_{\sigma_{\rm AdS}}}|\langle W\rangle_{\sigma_{\rm AdS}}| = \frac{1}{3}|V_{\rm AdS}|\,. \tag{10.4}$$

Thus, the height of the barrier is related to the gravitino mass in the present vacuum,

$$V_B \sim |V_{\rm AdS}| \sim m_{3/2}^2\,. \tag{10.5}$$

The gravitino mass $m_{3/2}$ is directly related to the scale of supersymmetry breaking since the almost vanishing of the cosmological constant,

$$V_{\rm mod} = V_F + V_D = |F|^2 - 3m_{3/2}^2 + \frac{1}{2}D^2 \approx 0\,, \tag{10.6}$$

automatically implies

$$3m_{3/2}^2 \approx |F|^2 + \frac{1}{2}D^2\,. \tag{10.7}$$

When an inflationary sector is added to the moduli stabilizing sector, the generic form of the potential becomes[5]

$$V_{\rm tot} = V_{\rm mod}(\sigma) + \frac{V_{\rm inf}(\phi)}{\sigma^3}\,. \tag{10.8}$$

Even for a perfectly suitable inflationary potential $V_{\rm inf}(\phi)$, once the value of $V_{\rm inf}(\phi)$ becomes large enough, the second term in Eq. (10.8) dominates and σ

[4]This choice for $V_{\rm up}$ is motivated by introducing $\overline{D3}$-branes at the tip of a warped throat. The constant $C_{\rm up}$ is tuneable by adjusting the strength of the warping at the tip.

[5]If we add an F-term driving inflation, the scalar potential during inflation is

$$V = e^K\,\left(K^{X\bar{X}}\,|D_X W|^2 + K^{T\overline{T}}\,|D_T W|^2 - 3|W|^2\right) + \frac{C_{\rm up}}{\sigma^2}\,.$$

For all known candidate inflation models, the inflationary potential $V_{\rm inf} \sim e^K\, K^{X\bar{X}}\,|D_X W|^2$ vanishes as some inverse power of σ for $\sigma \to \infty$. Typically, this power is σ^{-3} from the e^K prefactor. Thus, adding inflation to $V_{\rm mod}$ can be viewed as an additional uplifting.

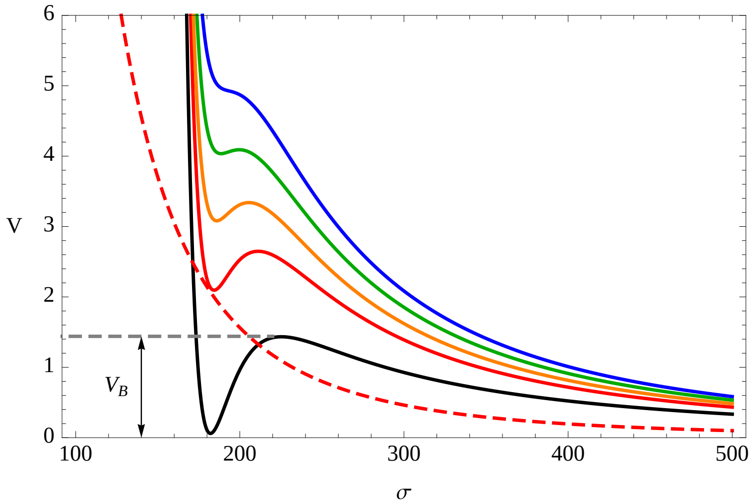

Figure 10.1: Schematic picture of the destabilization of the uplifted minimum due to adding an inflationary sector. The moduli potential after inflation is given by the solid black line. The dashed red line corresponds to the contribution from only $V_{\rm inf}(\phi)/\sigma^3$, while the solid red line corresponds to $V_{\rm mod}(\sigma) + \sigma^{-3}\, V_{\rm inf}(\phi)$. The other solid colored lines correspond to different increasing values of $V_{\rm inf}(\phi)$. The dashed grey line indicates the height of the barrier V_B. The scale on the vertical axis is arbitrary.

becomes a run-away direction. This is actually independent of the particular form of the moduli stabilizing sector (for now, it is KKLT), *i. e.* there is always an upper bound for the inflationary energy scale. Empirically, it has been argued that to avoid decompactification for the KKLT moduli stabilization scheme, we have to require [45]

$$V_{\rm tot} \lesssim \mathcal{O}(1) V_B\,. \tag{10.9}$$

We illustrate the destabilization of the uplifted minimum for the modulus due to inflation in Fig. 10.1. Obviously, by increasing $V_{\rm inf}(\phi)$, at a certain critical value of $V_{\rm inf}(\phi)$ the minimum disappears and the modulus runs away to infinity.

For the KKLT scenario, however, the height of the barrier is related to the gravitino mass in the present vacuum, cf. Eq. (10.5), and thus with $\mathcal{H}_{\rm inf}^2 \sim V_{\rm tot}$ the upper bound becomes

$$\mathcal{H}_{\rm inf} \lesssim m_{3/2}^{\rm today}\,. \tag{10.10}$$

This is at odds with having a high scale of inflation and low-energy supersymmetry, which requires $\mathcal{H}_{\rm inf} \gg m_{3/2}^{\rm today}$.[6]

The solution suggested by KL [45] is to add a second non-perturbative contribution to the superpotential. Then the value of the gravitino mass in the

[6] In the context of the LARGE Volume Scenario [159], the upper bound becomes even more severe, namely $\mathcal{H}_{\rm inf} \lesssim \left(m_{3/2}^{\rm today}\right)^{3/2}$ in Planck units [259].

present vacuum can be disentangled from the height of the barrier. To achieve this decoupling, the terms in the superpotential must be fine-tuned.

The crucial issue of the KL problem arises from the notion of having a single, common scale to the moduli stabilization mechanism *during* and *after* inflation. In this thesis, we propose a scenario where two *different* mechanisms stabilize the modulus during and after inflation. In particular, we consider models where a certain term in the Kähler potential is responsible for stabilizing the volume modulus during inflation, whereas a standard KKLT-type superpotential stabilizes the modulus after inflation, and thereby sets the gravitino mass in the present vacuum. In the next section, we will outline the general setup of our scenario, followed by an explicit example.

10.2 Resolution: A General Framework

We consider generic superpotentials of the following form

$$W = W_{\rm inf}(X, \Phi, \dots) + W_{\rm mod}(T)\,, \tag{10.11}$$

with X denoting the field whose F-term drives the inflationary vacuum energy and Φ containing the inflaton. The dots in the argument of $W_{\rm inf}$ represent possible other fields, *e. g.* some waterfall fields to realize hybrid inflation. The F-term potential during inflation takes the form

$$V_F = e^K\, K^{X\bar{X}} |D_X W_{inf}|^2 + V_{\rm mod}(\sigma) + V_{\rm mix}\,, \tag{10.12}$$

where $V_{\rm mod}(\sigma)$ originates solely from the modulus sector $W_{\rm mod}(T)$ and is responsible for moduli stabilization only *after inflation* when the F-term of X (the vacuum energy) - the first term in Eq. (10.12) - has vanished. Since the Hubble scale at the end of inflation is much smaller than the Hubble scale during inflation, $W_{\rm mod}(T)$ need not be necessarily large in contrast to the usual setup where $W_{\rm mod}(T)$ is responsible for moduli stabilization both during and after inflation. $V_{\rm mix}$ denotes possible additional mixing terms, in particular, the contributions due to $K_{T\bar{X}} \neq 0$. In principle, there can be also other mixing terms, *e. g.* due to $K_{\Phi\overline{T}} \neq 0$, but we assume those to be negligible in the following.

During inflation, moduli stabilization is achieved by a suitable moduli-dependence of the first term. Moreover, since we focus on setups where $W_{\rm inf}$ satisfies [38, 225, 227–230, 351, 352] (see Sec. 5.3)

$$W_{\rm inf} \approx 0 \quad , \quad W_{{\rm inf},X} \neq 0 \ \text{ and } \ W_{{\rm inf},\, i\neq X} \approx 0\,, \tag{10.13}$$

the minimum for T during inflation has to be generated by the prefactor of the first term, $e^K\, K^{X\bar{X}}$. Below we give a simple example (on which we will be more explicit later in section 10.3), where we keep a no-scale Kähler potential for T and modify the Kähler metric for X in an appropriate way to stabilize the modulus during inflation. But keep in mind that our framework allows for more general possibilities, *e. g.* if one breaks the no-scale structure in the Kähler potential by including α'-corrections [156].

$W_{\text{mod}}(T)$ is responsible for stabilizing the modulus at the end of inflation. Upon including the necessary uplifting term, SUSY is broken in the present vacuum. As long as the Hubble scale after inflation is smaller than the TeV-scale, the modulus will always remain stable since TeV-scale SUSY breaking generically induces moduli masses which are of the same order or heavier. Therefore, we can decouple low-energy SUSY breaking from the inflationary scale, thereby evading the KL problem, that is we can generically assume $|W_{\text{mod}}|, |D_T W_{\text{mod}}| \ll |W_{\text{inf},X}|$.

From now on, we restrict ourselves to a particular form of the Kähler potential, which contains the aforementioned coupling between T and X, namely

$$K = -3\ln(T+\overline{T}) + |X|^2\left(1-\beta(T+\overline{T})-\gamma|X|^2\right)+\dots\,. \tag{10.14}$$

It has been argued in [1] that couplings (qualitatively) similar to the second term in the brackets in Eq. (10.14) can arise as moduli-dependent string-loop corrections in heterotic orbifold compactifications (see also Sec. 12.2). For simplicity, we have dropped a possible overall factor of $(T + \overline{T})^{-p}$ (with p some rational number) for the terms in the brackets in Eq. (10.14), which does not change the results qualitatively. A discussion of the origin of such terms in type IIB string theory is beyond the scope of the present work and we defer it to the future.

Due to the $-\gamma|X|^4$ term, X generically acquires a large mass during inflation [225] for $\gamma \gtrsim \mathcal{O}(1)$[7] and remains near zero.[8] Schematically, for $|X| \ll 1$, V_F is given by

$$V_F \sim \frac{|W_X|^2}{\sigma^3\,(1-2\,\beta\,\sigma)} + V_{\text{mod}}(\sigma) + \mathcal{O}(X)\,. \tag{10.15}$$

In the limit $W_{\text{mod}} \to 0$, we have $X = 0$ and the potential during inflation is entirely given by the first term. Thus, let us concentrate on the first term for a moment: If $\beta > 0$, this term stabilizes σ at $\sigma_{\text{inf}} \sim \beta^{-1}$ with a large mass proportional to $\mathcal{H}^2_{\text{inf}} \sim |W_X|^2/\sigma^3_{\text{inf}}$. More precisely, taking into account the non-canonical kinetic term for T, we find $m_T^2 \simeq \mathcal{O}(10)\,\mathcal{H}^2_{\text{inf}}$. At the end of inflation, the vaccum energy goes away and the modulus would become unstable. However, in this phase, $V_{\text{mod}}(\sigma)$ begins to dominate and stabilizes the modulus. The novel feature of this setup is that the mechanisms for moduli stabilization during and after inflation are a priori unrelated. Therefore, the

[7]There exists a nice geometric interpretation for this requirement in terms of the sectional curvature along the Goldstino direction, cf. [331–336].

[8]Note that X is not stabilized exactly at zero since the non-vanishing gravitino mass $m^2_{3/2} \propto |W_{\text{mod}}|^2$ induces a shift away from zero. However, this shift is parametrically small for small $|W_{\text{mod}}|$, cf. Eq. (10.34).

gravitino mass in the present vacuum is independent of the inflationary scale, which evades the KL problem.

The presence of the moduli sector induces potentially dangerous corrections to the inflationary trajectory. However, for low-energy SUSY breaking (in particular for TeV-scale SUSY breaking) and high-scale inflation, these corrections are parametrically small, as we show in section 10.3.3 for a KKLT-type superpotential and in appendix 10.4.2 for a generic choice of $W_{\mathrm{mod}}(T)$.

The price we have to pay is that the minima during and after inflation do not have to be the same. Since there seems to be no dynamical mechanism involved, we choose them to (almost) coincide by tuning the parameters appropriately. This imposes a relation between the parameter β and the parameters controlling W_{mod}, but does not affect the possiblity of having low-energy SUSY breaking and high-scale inflation at the same time.

10.3 Resolution: An Explicit Example

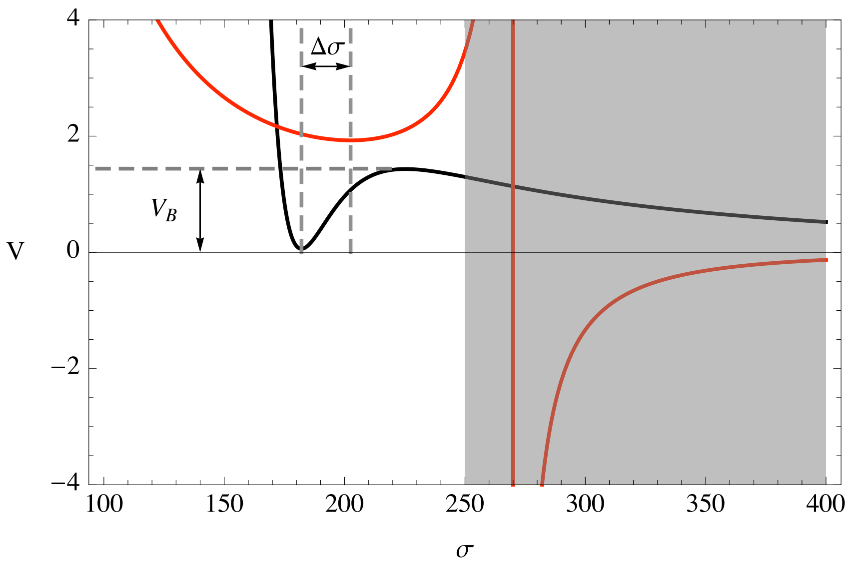

Figure 10.2: Schematic plot of the two moduli stabilizing potentials during inflation (red) and after inflation (black). The grey dashed lines indicate the displacement $\Delta\sigma$ of the two minima and the height of the barrier of the minimum after inflation. The grey region in the right part marks the regime where our effective field theory at second order in the derivatives ceases to be valid. The scale on the vertical axis is arbitrary.

In this section, we illustrate our general idea in a simple toy model: shift symmetric chaotic inflation combined with a KKLT-type superpotential. Results for hybrid (or tribrid) models and inflationary scenarios based on a Heisenberg symmetry will appear elsewhere [472].

For simplicity, we consider a chaotic inflation model [225] based on the superpotential

$$W_{\rm inf}(\Phi, X) = m\, X\, \Phi\,, \tag{10.16}$$

where Φ contains the inflaton and X is the field whose F-term drives the inflationary vacuum energy. For the modulus sector, we consider a KKLT-type superpotential [40],

$$W_{\rm mod}(T) = w_0 + A e^{-a\,T}\,. \tag{10.17}$$

We consider a no-scale Kähler potential for T and the coupling between T and X introduced in Eq. (10.14). To solve the supergravity η-problem, we assume a shift symmetry for the inflaton direction.[9] That is, we consider a Kähler potential

$$K = -3\ln\left(T+\overline{T}\right) + \frac{1}{2}\left(\Phi+\bar{\Phi}\right)^2 + |X|^2 - \beta\left(T+\overline{T}\right)|X|^2 - \gamma|X|^4\,. \tag{10.18}$$

The first term is invariant under a shift symmetry for the imaginary part of Φ, which protects this direction from the supergravity η-problem. Recall that the last term ensures that X is stabilized near $X=0$ with $m_X \gtrsim \mathcal{H}_{\rm inf}$ during inflation if $\gamma \gtrsim \mathcal{O}(1)$. The coupling between T and X in Eq. (10.18) stabilizes T during inflation, while the superpotential $W_{\rm mod}$ fixes T after inflation. As noted in section 10.1, since the minimum generated by $W_{\rm mod}$ is a supersymmetric AdS minimum, we need to uplift it to a dS minimum with a tiny cosmological constant. To achieve this, an uplifting contribution is added to the potential,

$$V_{\rm up} = \frac{C_{\rm up}}{\left(T+\overline{T}\right)^2}\,, \tag{10.19}$$

which is motivated by introducing anti-D3-branes at the tip of a warped throat in a string theory realization of such a setup. Here, however, we do not refer to a particular string theory embedding of our scenario and simply view the above setup as an effective parametrization for the potential of the modulus T and the inflationary sector with X and Φ.

Note that we assume the absence of any mixing between T and Φ and only a mixing between T and X given by the second term in the Kähler potential in Eq. (10.18). We comment on possible effects of such a mixing later on.

We denote the real and imaginary parts of the scalar components of the chiral superfields as follows

$$T \equiv \sigma + i\,\alpha\,,\; \Phi \equiv \phi_R + i\,\phi_I\,,\; X \equiv x_R + i\,x_I\,. \tag{10.20}$$

The vacuum expectation values of σ during and after inflation are denoted by $\sigma_{\rm inf}$ and σ_0, respectively. In addition, we choose the phases of the parameters in

[9] We do not discuss the naturalness of the shift symmetry with respect to quantum (gravity) corrections here. For our purposes, we assume a solution to the η-problem and effectively parametrize the resulting inflaton potential by $W_{\rm inf}$ in Eq. (10.16) and a negligible breaking of the shift symmetry in the Kähler potential Eq. (10.18).

$W_{\rm mod}$ such that the minimum is at $\alpha = 0$. More precisely, for the superpotential in Eq. (10.17), we choose w_0 to be real and negative and A to be real and positive.

A schematic plot of the two moduli stabilizing potentials generated by the F-term of X (cf. the first term in Eq. (10.15)) and the one induced by the KKLT-type superpotential and the uplifting term (cf. Eqs. (10.17) and (10.19)) can be found in Fig. 10.2.

To analyze the effects of the presence of a general $W_{\rm mod}$ during inflation, the basic idea is to perform a perturbative expanson in $W_{\rm mod}$, $D_T W_{\rm mod}$, $W''_{\rm mod}$ etc., with dimensionless expansion parameters given by $W_{\rm mod}/W_X$ etc. (up to appropriate powers of M_P). They parametrize the impact of the modulus sector on the inflationary trajectory and for low-energy SUSY breaking and high-scale inflation these expansion parameters are parametrically small, thereby making our treatment self-consistent. For the KKLT-type superpotential in Eq. (10.17), this procedure is essentially equivalent to an expansion in small $|w_0| \ll 1$ and large $a\,\sigma \gg 1$. Results for a general $W_{\rm mod}(T)$ are presented in Sec. 10.4.

10.3.1 Stability of the Vacuum after Inflation

We now discuss two possible problems which lead to constraints on the parameter space. First, since the two minima during and after inflation generically do not coincide, we may not end up in the true minimum. In particular, we may overshoot the minimum and roll away to infinity. Second, without $W_{\rm mod}$ and $V_{\rm up}$, the vacuum after inflation is $\Phi = X = 0$ and there are two flat directions corresponding to the real and imaginary parts of T. If we add $V_{\rm mod}$, these two flat directions are stabilized. However, we may induce some instabilities for Φ and X. The implications of these two issues for the model parameters are discussed below.

Overshoot Problem

To avoid overshooting of the modulus after inflation (as long as there is no dynamical mechanism ensuring a smooth transition between the two minima), we will require that the positions of the minima are sufficiently close to each other, *i. e.* $\sigma_0 \approx \sigma_{\rm inf}$ (c.f. Fig. 10.2). This relates the parameters controlling $W_{\rm mod}$ to the parameter β in the Kähler potential. For the specific case of the KKLT superpotential, Eq. (10.17), if we ignore the shift due to the uplifting potential Eq. (10.19), we can estimate σ_0 as[10]

$$\sigma_0 \simeq -\mathcal{O}(1)\frac{1}{a}\ln\left|\frac{w_0}{A}\right| . \tag{10.21}$$

During inflation, V_{mod} induces only a tiny shift which we can neglect and the position of the minimum is approximately given by

$$\sigma \simeq \sigma_{\text{inf}} \equiv \frac{3}{8\beta} . \tag{10.22}$$

For the KKLT superpotential, one typically uses $A \simeq 1$ and $a \simeq \frac{2\pi}{N}$ for some integer[11] N such that effectively w_0 determines the value of σ_0 and thus also the size of the gravitino mass after inflation.

For the two minima to be roughly at the same position, $\sigma_0 \approx \sigma_{\text{inf}}$, we have to tune the parameters such that

$$\frac{1}{\beta} \simeq -\mathcal{O}(1)\frac{1}{a}\ln\left|\frac{w_0}{A}\right| . \tag{10.23}$$

We will assume this condition to be satisfied to a good approximation.

Stability Bound on w_0

There is yet another condition leading to a bound on w_0, as we now discuss. Let us start by noting that, due to the absence of any mixings, the masses for ϕ_R and ϕ_I are simply given by

$$\frac{\partial^2 V}{\partial \phi_R^2} = \frac{m^2}{8\,\sigma_0^3\,(1-2\,\beta\,\sigma_0)} - \frac{|W_{\text{mod}}(\sigma_0)|^2}{4\,\sigma_0^3} , \tag{10.24}$$

and

$$\frac{\partial^2 V}{\partial \phi_I^2} = \frac{m^2}{8\,\sigma_0^3\,(1-2\,\beta\,\sigma_0)} , \tag{10.25}$$

respectively. To avoid a tachyonic mass for ϕ_I, we have to require that σ_0 satisfies

$$\sigma_0 < \frac{1}{2\beta} . \tag{10.26}$$

[10]Actually, there is a slighlty better approximation for the position of the AdS-minimum, namely

$$\sigma_0 \simeq -\frac{1}{a}\ln\left|\frac{w_0}{A}\frac{1}{1-\frac{2}{3}\ln|\frac{w_0}{A}|}\right| .$$

However, for our purposes here, the approximation in Eq. (10.21) is sufficient since we neglect the uplifting and only need a rough estimate for σ_0.

[11]For the effective field theory to be valid, we must have $N \gg 1$ such that $\sigma \gg 1$.

Note that this condition is also necessary to avoid a wrong sign for the kinetic term of X,[12] independently of the stability condition for ϕ_I. From also avoiding an instability for ϕ_R, we get an upper bound on the size of $W_{\rm mod}$ at the minimum in terms of m:

$$|W_{\rm mod}(\sigma_0)|^2 \lesssim \frac{m^2}{2\,(1-2\,\beta\,\sigma_0)} \approx 2\,m^2\,, \tag{10.27}$$

where the last step assumes $\sigma_0 \approx \sigma_{\rm inf} \equiv \frac{3}{8\beta}$. There is no further constraint from the stability of $x_{R,I}$. Using the superpotential in Eq. (10.17), the bound in Eq. (10.27) becomes a bound on w_0 in terms of m:

$$w_0^2 \lesssim \frac{(3+2\,a\,\sigma_0)^2}{8\,a^2\,\sigma_0^2\,(1-2\,\beta\,\sigma_0)}\,m^2 \approx\, 2\,m^2 + \mathcal{O}\left(\frac{m^2}{a\,\sigma_0}\right)\,, \tag{10.28}$$

where in the last step we again assumed $\sigma_0 \approx \sigma_{\rm inf}$ and expanded for $a\,\sigma_0 \gg 1$. Note that this bound will not be affected by adding the uplifting sector as long as the uplifting term $V_{\rm up}$, Eq. (10.19), does not depend on Φ. Moreover, if any mixing between T and Φ would be present, the bound is not directly on $W_{\rm mod}$, but on a particular combination of $W_{\rm mod}$ and $D_T W_{\rm mod}$ depending on the mixing terms. For the KKLT superpotential and the no-scale Kähler potential, one typically has $|D_T W_{\rm mod}(\sigma_0)| \sim |W_{\rm mod}(\sigma_0)|$ at the uplifted minimum and thus the bound would be essentially again on the size of $|W_{\rm mod}(\sigma_0)|$.

The important lesson here is that vacuum stability puts some constraint on the size of supersymmetry breaking after inflation, but for high-scale inflation it still allows for a large range of possible values of w_0, in particular those leading to low-energy SUSY breaking.

10.3.2 Comment on the Cosmological Moduli Problem

Since the moduli stabilization mechanisms during and after inflation are of completely different origin, the minima for the modulus field during and after inflation generically do not exactly coincide. This means the modulus field will oscillate and eventually dominate the universe. If it decays too late in the history of the universe, in particular if it decays after Big Bang Nucleosynthesis (BBN), it causes the well-known cosmological moduli problem [145–151]. However, since for the KKLT case there is a little hierarchy of scales [194–199],

$$m_T \sim 16\pi^2\,m_{3/2} \sim (16\pi^2)^2\,m_{soft}\,, \tag{10.29}$$

we can assume that the modulus (and also the gravitino) is heavier than about 30 TeV such that it will decay before BBN and thus the cosmological moduli problem is avoided. We leave a more detailed investigation of both the non-thermal history and the issue of relaxing the tuning $\sigma_0 \approx \sigma_{\rm inf}$, *e. g.* by implementing a dynamical mechanism, for the future.

[12] For values too close to the upper bound, *i. e.* $\sigma_0 \approx \frac{1}{2\beta}$, it is no longer justified to work only at second order in the derivatives and our effective field theory ceases to be valid.

10.3.3 Corrections to the Inflationary Trajectory

The inflationary trajectory is shifted due to the presence of the modulus sector. If we neglect W_{mod} and V_{up}, the trajectory is given by $\sigma = \sigma_{\text{inf}} \equiv \frac{3}{8\beta}$, $\alpha = 0$, [13] $X = 0$, $\phi_R = 0$ and $\phi_I \neq 0$. Except for the derivative with respect to ϕ_I, the only other non-vanishing derivatives along this trajectory are

$$\frac{\partial V}{\partial \sigma} = -\frac{A^2\, a\, e^{-2\,a\,\sigma}\,(6 + 7\,a\,\sigma + 2\,a^2\,\sigma^2)}{6\,\sigma^3} - \frac{w_0\, A\, c\, e^{-a\,\sigma}\,(2 + a\,\sigma)}{2\,\sigma^3} - \frac{3\,a^2\,\sigma_0\,w_0^2}{\sigma^3\,(3 + 2\,a\,\sigma_0)^2}\,, \tag{10.30}$$

and

$$\frac{\partial V}{\partial x_I} = \frac{m\,\phi_I}{4\,\sigma^3}\left(w_0 + Ae^{-a\,\sigma}\right)\,. \tag{10.31}$$

To (slighlty over-) compensate the negative cosmological constant in the would-be AdS-vacuum, we tune the uplifting potential V_{up} by choosing (recall that $\sigma_0 \approx \sigma_{\text{AdS}}$)

$$C_{\text{up}} \simeq \frac{3|W_{\text{mod}}(\sigma_0)|^2}{2\,\sigma_0} = \frac{6\,a^2\,\sigma_0\,w_0^2}{(3 + 2\,a\,\sigma_0)^2}\,. \tag{10.32}$$

Hence, we see that the last term in Eq. (10.30) is precisely the contribution from V_{up}.

As mentioned above, we compute the shifts in a perturbative expansion for $|w_0| \ll 1$ and $a\,\sigma \gg 1$. At leading order, the shifts in σ and x_I during inflation are given by

$$\begin{aligned}\delta\sigma \simeq\; & \frac{\sigma_{\text{inf}}\,w_0^2}{2\,m^2\,(1 + 32\,\gamma\,\phi_I^2)} \\ & + \frac{A^2\,e^{-2\,a\,\sigma_{\text{inf}}}(3\phi_I^2 + a\,\sigma_{\text{inf}}\,(2 + (3 + 64\gamma)\phi_I^2))}{6\,m^2\,\phi_I^2\,(1 + 32\,\gamma\,\phi_I^2)} \\ & + \frac{A\,e^{-a\,\sigma_{\text{inf}}}\,\sigma_{\text{inf}}\,w_0\,(6\phi_I^2 + a\,\sigma_{\text{inf}}\,(2 + (3 + 64\gamma)\phi_I^2))}{6\,m^2\,\phi_I^2\,(1 + 32\,\gamma\,\phi_I^2)}\,,\end{aligned} \tag{10.33}$$

and

$$\delta x_I \simeq -\frac{\phi_I\left(W_{\text{mod}} + \overline{W}_{\text{mod}}\right)}{m\,(1 + 32\,\gamma\,\phi_I^2)} \simeq -\frac{2\,\phi_I\,(w_0 + Ae^{-a\,\sigma_{\text{inf}}})}{m\,(1 + 32\,\gamma\,\phi_I^2)}\,. \tag{10.34}$$

Note that the shift δx_I vanishes as $\phi_I \to 0$ such that $\Phi = X = 0$ at the end of inflation. More interestingly, this shift is entirely controlled by the value of the gravitino mass during inflation — for $\phi_I \gg 1$:

$$\delta x_I \simeq -\frac{\phi_I\,(W_{\text{mod}} + \overline{W}_{\text{mod}})}{m\,(1 + 32\,\gamma\,\phi_I^2)} \sim -\frac{m_{3/2}\,\sigma_{\text{inf}}^{3/2}}{m\,\phi_I}\,, \tag{10.35}$$

[13] Without W_{mod}, α is not fixed at zero, but effectively frozen since it becomes massless in the limit $W_{\text{mod}} \to 0$. As noted above, we choose the phases in W_{mod} such that it has a minimum at $\alpha = 0$. Thus, once we include W_{mod}, the minimum both during and after inflation is at $\alpha = 0$.

with $m_{3/2}$ given by the usual expression

$$m_{3/2}^2 = e^{\langle K\rangle} \, |\langle W\rangle|^2 \, . \tag{10.36}$$

Due to the suppression by the large inflationary F-term $W_X \simeq m\phi_I$, δx_I is parametrically small for small values of the gravitino mass.

Including these shifts, the potential along the inflationary trajectory is given by

$$V_{\rm inf} \simeq \frac{m^2\,\phi_I^2}{4\sigma_{\rm inf}^3} + V_{\rm mod}(\sigma_{\rm inf}) + \frac{\phi_I^2\,(w_0 + Ae^{-a\,\sigma_{\rm inf}})^2}{4\,\sigma_{\rm inf}^3\,(1+32\,\gamma\,\phi_I^2))}\,, \tag{10.37}$$

where the first term is the standard contribution (up to the factor of $\sigma_{\rm inf}^3$) and the last term corresponds to the contribution from $\sim m^2\,x_I^2$ since now $x_I \neq 0$. The other terms combine to the moduli potential $V_{\rm mod}$, which is induced by $W_{\rm mod}$ and $V_{\rm up}$, evaluated at $\sigma_{\rm inf}$. Recall that to ensure $m_X \gtrsim \mathcal{H}$ during inflation we require $\gamma \gtrsim \mathcal{O}(1)$. Thus, the inflaton-dependence of the potential at large values of ϕ_I is not significantly affected by the addition of the last term in Eq. (10.37). Together with the bound from Eq. (10.28), $w_0^2 \lesssim 2\,m^2$, and the assumption $\sigma_0 \approx \sigma_{\rm inf}$, we can already anticipate at this stage that no large corrections are expected.

The next step is to compute the corrections to the inflationary observables. Note that the parameter m has to be redefined, both due to the factor of $\sigma_{\rm inf}^3$ in the first term of Eq. (10.37) and due to the other additional terms. As usual, it is fixed by matching the amplitude of the scalar perturbations to the observed value.

Assuming inflation ends at $\phi_I \approx 0$, we can compute the number of e-folds N_e in terms of ϕ_I. The two slow-roll parameters which determine the inflationary predictions are

$$\epsilon = \frac{1}{2}M_P^2\left(\frac{V'}{V}\right)^2 \quad \text{and} \quad \eta = M_P^2\frac{V''}{V}\,. \tag{10.38}$$

The amplitude of the scalar power spectrum is given by

$$\mathcal{P}_{\mathcal{R}}^{1/2} = \frac{1}{2\sqrt{3}\pi}\frac{V^{3/2}}{|V'|} \tag{10.39}$$

As usual, the parameter m is fixed by matching the observed value for the amplitude of the scalar power spectrum to the predicted one. To leading order, the amplitude of the scalar power spectrum $\mathcal{P}_\mathcal{R}^{1/2}$ is given by

$$\begin{aligned}\mathcal{P}_\mathcal{R}^{1/2} \simeq \frac{m\,N_e}{2\sqrt{3}\,\pi\,\sigma_{\rm inf}^{3/2}}\Bigg(1 &+ \frac{w_0^2}{m^2\,N_e}\left(\frac{\sigma_{\rm inf}}{\sigma_0}\left(\frac{9}{16}-\frac{3}{8}\ln(4N_e)\right)+\frac{2\ln(128\,\gamma\,N_e)-3}{256\,\gamma}\right)\\ &+ \frac{w_0Ae^{-a\,\sigma_{\rm inf}}}{m^2N_e}\left(-\frac{a\,\sigma_{\rm inf}}{4}\left(2\ln(4N_e)-3\right)+\frac{2\ln(128\,\gamma\,N_e)-3}{128\,\gamma}\right)\\ &+ \frac{A^2e^{-2\,a\,\sigma_{\rm inf}}}{m^2\,N_e}\Bigg(-\frac{a^2\,\sigma_{\rm inf}^2}{12}\left(2\ln(4N_e)-3\right)-\frac{a\,\sigma_{\rm inf}}{4}\left(2\ln(4N_e)-3\right)\\ &+ \frac{2\ln(128\,\gamma\,N_e)-3}{256\,\gamma}\Bigg)\Bigg)\,.\end{aligned} \tag{10.40}$$

Obviously, compared to the inflationary scenario without the modulus sector, we only need to redefine m to account for the prefactor $\sigma_{\rm inf}^{3/2}$. The extra terms are all negligible and only would give rise to higher order corrections in the expressions for ϵ and η so we can ignore those.

Having fixed the parameter m, we can calculate the predictions for the observables. For example, the scalar spectral index $n_s = 1 - 6\,\epsilon + 2\,\eta$ is given by

$$\begin{aligned}n_s - 1 \simeq &-\frac{2}{N_e}+\frac{w_0}{m^2\,N_e^2}\left(-\frac{3\,\sigma_{\rm inf}\,\left(2\ln(4N_e)-5\right)}{8\,\sigma_0}+\frac{2\ln(128\,\gamma\,N_e)}{128\,\gamma}\right)\\ &+\frac{w_0Ae^{-a\,\sigma_{inf}}}{m^2\,N_e^2}\left(-\frac{a\,\sigma_{\rm inf}}{2}(2\ln(4N_e)-5)+\frac{2\ln(128\,\gamma\,N_e)}{64\,\gamma}\right)\\ &\frac{A^2e^{-2\,a\,\sigma_{\rm inf}}}{m^2\,N_e^2}\Bigg(\frac{a^2\,\sigma_{\rm inf}^2}{6}\left(2\ln(4N_e)-5\right)-\frac{a\,\sigma_{\rm inf}}{2}\left(2\ln(4N_e)-5\right)\\ &+\frac{2\ln(128\,\gamma\,N_e)}{128\,\gamma}\Bigg)\,,\end{aligned} \tag{10.41}$$

and the tensor-to-scalar ratio $r = 16\,\epsilon$ is given by

$$\begin{aligned}r \simeq &\frac{8}{N_e}+\frac{w_0^2}{m^2\,N_e^2}\left(\frac{3\,\sigma_{\rm inf}(\ln(4N_e)-2)}{\sigma_0}-\frac{\ln(128\,\gamma\,N_e)-2}{16\,\gamma}\right)\\ &+\frac{w_0Ae^{-a\,\sigma_{\rm inf}}}{m^2\,N_e^2}\left(4\,a\,\sigma_{\rm inf}(\ln(4N_e)-2)-\frac{\ln(128\,\gamma\,N_e)-2}{8\,\gamma}\right)\\ &+\frac{A^2e^{-2a\,\sigma_{\rm inf}}}{m^2\,N_e^2}\Bigg(\frac{4\,a^2\,\sigma_{\rm inf}^2}{3}(\ln(4N_e)-2)+4\,a\,\sigma_{\rm inf}(\ln(4N_e)-2)\\ &-\frac{\ln(128\,\gamma\,N_e)-2}{16\,\gamma}\Bigg)\,.\end{aligned} \tag{10.42}$$

Note that all the correction terms start at order N_e^{-2} (up to some logarithms). Thus, they are suppressed with respect to the leading contribution $\sim N_e^{-1}$, as expected since we perform a perturbative expansion in $W_{\rm mod}/W_X$ etc. and

$W_X^2 = m^2\,\phi_I^2 \sim m^2\,N_e$. This suppression is sufficient to keep the corrections induced by the modulus sector small even if we saturate the bound in Eq. (10.28). Most importantly, for high-scale inflation and low-energy supersymmetry, the inflationary predictions are not significantly affected: the corrections are suppressed by $m_{3/2}^2/F_X^2$ and F_T^2/F_X^2.

10.4 Resolution: A Less Explicit Example

In this section, we present some results for a generic choice of $W_{\text{mod}}(T)$. The moduli superpotentials we have in mind are of the form

$$W_{\text{mod}}(T) = w_0 + \sum_n A_n\, e^{-a_n T}\,, \tag{10.43}$$

but their precise form is irrelevant for our discussion. We still restrict ourselves to the chaotic inflation model from section 10.3, *i. e.* we consider

$$W = m\,X\,\Phi + W_{\text{mod}}(T)\,, \tag{10.44}$$

and

$$K = -3\ln\left(T+\overline{T}\right) + \frac{1}{2}\left(\Phi+\bar{\Phi}\right)^2 + |X|^2 - \beta\left(T+\overline{T}\right)|X|^2 - \gamma|X|^4\,. \tag{10.45}$$

If necessary, we allow for the possibility of adding an uplifting term of the form

$$V_{\text{up}} = \frac{C_{\text{up}}}{\left(T+\overline{T}\right)^2}\,, \tag{10.46}$$

where the constant C_{up} is tuned to have an (almost) vanishing cosmological constant. Here, we do not consider other possibilites for uplifting such as those discussed *e. g.* in [199, 328, 374–378]. As before, we choose the phases of the parameters in W_{mod} such that the minimum for the imaginary part of T is at $\alpha = 0$.

The general strategy is to perform a perturbative expansion in W_{mod}, $D_T W_{\text{mod}}$, W''_{mod} and higher derivatives to determine the effect of the modulus sector during inflation. Recall that this is an expansion with dimensionless expansion parameters given by W_{mod}/W_X etc. (up to appropriate powers of M_P), which parametrize the impact on the inflationary trajectory by adding the modulus sector. Note that this expansion breaks down towards the end of inflation since W_X vanishes as $\Phi \to 0$. For simplicity, we assume the absence of any mixing between T and Φ and only a mixing betweeen T and X given by the second term in Eq. (10.45).

In many schemes for moduli stabilization, at the minimum σ_0 one has an upper bound $|D_T W_{\text{mod}}(\sigma_0)| \lesssim |W_{\text{mod}}(\sigma_0)|$, *e. g.* for the KKLT mechanism one has $|D_T W_{\text{mod}}(\sigma_0)| \sim |W_{\text{mod}}(\sigma_0)|$, while for the KL scenario one has $|D_T W_{\text{mod}}(\sigma_0)| \ll |W_{\text{mod}}(\sigma_0)|$. We restrict our discussion to moduli stabilization scenarios which obey such an upper bound.

10.4.1 Stability of the Vacuum after Inflation

If $W_{\rm mod}$ and $V_{\rm up}$ are not present, the vacuum after inflation is given by $\Phi = X = 0$ and both the real and the imaginary part of T remain as flat directions. These two flat directions are stabilized upon adding $W_{\rm mod}$. However, we may induce some instabilities for Φ and X.

Since there are no mixings, the masses for ϕ_R and ϕ_I are simply given by

$$\frac{\partial^2 V}{\partial \phi_R^2} \simeq \frac{m^2}{8\,\sigma_0^3\,(1-2\,\beta\,\sigma_0)} - \frac{|W_{\rm mod}(\sigma_0)|^2}{4\sigma_0^3}\,, \tag{10.47}$$

and

$$\frac{\partial^2 V}{\partial \phi_I^2} \simeq \frac{m^2}{8\,\sigma_0^3\,(1-2\,\beta\,\sigma_0)}\,, \tag{10.48}$$

respectively. As in the explicit example we discussed in section 10.3.1, σ_0 has to satisfy the upper bound

$$\sigma_0 < \frac{1}{2\beta}\,, \tag{10.49}$$

which is necessary to avoid both a tachyonic mass for ϕ_I and a wrong sign for the kinetic term of X. From the stability condition for ϕ_R, we get an upper bound on the size of $W_{\rm mod}$ at the minimum in terms of m:

$$|W_{\rm mod}(\sigma_0)|^2 \lesssim \frac{m^2}{2\,(1-2\,\beta\,\sigma_0)} \approx 2m^2\,, \tag{10.50}$$

where the last step assumes $\sigma_0 \approx \sigma_{\rm inf} \equiv \frac{3}{8\beta}$. There is again no further constraint from the stability of $x_{R,I}$. Note that this constraint will not be affected by adding the uplifting sector as long as the uplifting term $V_{\rm up}$, Eq. (10.46), does not depend on Φ. Furthermore, if any mixings between T and Φ would be present, the bound is not directly on $W_{\rm mod}$, but on a particular combination of $W_{\rm mod}$ and $D_T W_{\rm mod}$ depending on the mixing terms. However, since we consider only setups where $|D_T W_{\rm mod}(\sigma_0)| \lesssim |W_{\rm mod}(\sigma_0)|$, the upper bound does not change qualitatively.

10.4.2 Corrections to the Inflationary Trajectory

The presence of the moduli stabilizing sector shifts the inflationary trajectory. Without $V_{\rm mod}$, it is given by $\sigma = \sigma_{\rm inf} \equiv \frac{3}{8\beta}$, $\alpha = 0$,[14] $X = 0$, $\phi_R = 0$ and $\phi_I \neq 0$. Upon adding $V_{\rm mod}$, only σ and x_I receive shifts since except for the derivative with respect to ϕ_I the only non-vanishing first derivatives along the would-be inflationary trajectory are

$$\frac{\partial V}{\partial \sigma} = \left(\frac{D_T W_{\rm mod}\,\overline{W}_{\rm mod}}{2\,\sigma^3} + \frac{D_T W_{\rm mod}\,\overline{W}''_{\rm mod}}{6\,\sigma} + \text{c.c.}\right) - \frac{2|D_T W_{\rm mod}|^2}{3\,\sigma^2} - \frac{3|W_{\rm mod}(\sigma_0)|^2}{4\,\sigma^3\,\sigma_0}\,, \tag{10.51}$$

[14] See foonote 13.

and

$$\frac{\partial V}{\partial x_I} = \frac{m\,\phi_I}{8\,\sigma^3}\left(W_{\rm mod} + \overline{W}_{\rm mod}\right) \,. \tag{10.52}$$

Note that we have tuned the constant $C_{\rm up}$ in $V_{\rm up}$ such that it (slighlty over-) compensates the negative cosmological constant assuming one obtains a supersymmetric AdS vacuum without $V_{\rm up}$, which yields

$$C_{\rm up} \simeq \frac{3|W_{\rm mod}(\sigma_0)|^2}{2\sigma_0} \,. \tag{10.53}$$

Thus, the last term in Eq. (10.51) is precisely the contribution from $V_{\rm up}$. If no uplifting is necessary, the corresponding terms in Eq. (10.51) and in all of the following equations simply have to be dropped. If the minimum is a non-supersymmetric AdS or Minkowski minimum, $C_{\rm up}$ is fixed in terms of $W_{\rm mod}(\sigma_0)$ and $D_T W_{\rm mod}(\sigma_0)$ instead of just $W_{\rm mod}(\sigma_0)$ and it is straightforward to change to the correct approximate expression for $C_{\rm up}$ in all the equations.

In the following, we present the results of a perturbative expansion in $W_{\rm mod}$, $D_T W_{\rm mod}$ and $W''_{\rm mod}$. Note that unless stated otherwise it is implicit that these functions are evaluated at $\sigma = \sigma_{\rm inf} \equiv \frac{3}{8\beta}$. To leading order in the expansion, the shifts δx_I and $\delta\sigma$ are given by

$$\delta x_I \simeq -\frac{\phi_I\left(W_{\rm mod} + \overline{W}_{\rm mod}\right)}{m(1+32\gamma\phi_I^2)} \sim \frac{\sigma_{\rm inf}^{3/2}\, m_{3/2}}{m\phi_I} \,, \tag{10.54}$$

and

$$\begin{aligned}\delta\sigma \simeq\ & \frac{\sigma_{\rm inf}^2 |W_{\rm mod}(\sigma_0)|^2}{4m^2\sigma_0\phi_I^2} - \frac{\sigma_{\rm inf}\left(W_{\rm mod} + \overline{W}_{\rm mod}\right)^2}{16m^2\,(1+32\gamma\phi_I^2)^2} + \frac{2\,\sigma_{\rm inf}^3\,|D_T W_{\rm mod}|^2}{9m^2\phi_I^2} \\ & - \left(\frac{\sigma_{\rm inf}^4\, D_T W_{\rm mod}\,\overline{W}''_{\rm mod}}{18m^2\phi_I^2} + \text{c.c.}\right) + \left(-\frac{\sigma_{\rm inf}^2\, D_{\overline{T}}\overline{W}_{\rm mod}}{8m^2(1+32\gamma\phi_I^2)} \overline{W}_{\rm mod} + \text{c.c.}\right) \\ & + \left(\left(\frac{4}{\phi_I^2} - \frac{3}{1+32\gamma\phi_I^2}\right)\frac{\sigma_{\rm inf}^2\, D_T W_{\rm mod}}{24m^2}\overline{W}_{\rm mod} + \text{c.c.}\right) \,.\end{aligned} \tag{10.55}$$

The first term in $\delta\sigma$ is induced by the uplifting potential $V_{\rm up}$. Note that the shift δx_I is entirely controlled by $W_{\rm mod}$, *i. e.* by the gravitino mass during inflation. These shifts induce some changes to the potential along the ϕ_I-direction, in particular, the shift of x_I away from zero is potentially dangerous. To leading order, the inflaton potential including the shifts is now given by

$$V_{\rm inf} \simeq \frac{m^2\phi_I^2}{4\,\sigma_{\rm inf}^3} + V_{\rm mod}(\sigma_{\rm inf}) - \frac{\phi_I^2\left(W_{\rm mod} + \overline{W}_{\rm mod}\right)^2}{16\,\sigma_{\rm inf}^3\,(1+32\,\gamma\,\phi_I^2)} \,, \tag{10.56}$$

where the second term is the pure moduli potential evaluated at $\sigma = \sigma_{\rm inf}$, *i. e.*

$$V_{\rm mod}(\sigma_{\rm inf}) = \frac{|D_T W_{\rm mod}|^2}{6\,\sigma_{\rm inf}} - \frac{3|W_{\rm mod}|^2}{8\,\sigma_{\rm inf}^3} + \frac{3|W_{\rm mod}(\sigma_0)|^2}{8\,\sigma_{\rm inf}^2\sigma_0} \,, \tag{10.57}$$

with the last term in Eq. (10.57) coming from the uplifting potential $V_{\rm up}$, Eq. (10.46), with $C_{\rm up}$ tuned as in Eq. (10.53). The first term in Eq. (10.56) is the standard chaotic inflation potential, which is simply rescaled by a factor $\sigma_{\rm inf}^{-3}$ from the e^K prefactor in V_F and the last term is simply $\sim m^2\,\delta x_I^2$.

Since we must have $\gamma \gtrsim \mathcal{O}(1)$ to ensure that X has a mass $m_X \gtrsim \mathcal{H}$ during inflation, the ϕ_I-dependence of the potential at large values of ϕ_I is not significantly affected. Moreover, recall that from the stability of the vacuum after inflation and with $\sigma_{\rm inf} \approx \sigma_0$, we must obey the upper bound $|W_{\rm mod}| \lesssim \sqrt{2}m$, cf. Eq. (10.50). Thus, already at this stage, we see that no large corrections should be expected in the regime where our treatment is valid.

In the next step, we have to compute the impact of the extra terms on the inflationary predictions. Since we perform a perturbative expansion, we do not expect to get very large effects, but the bound on $W_{\rm mod}$ might change. Note that the parameter m has to be redefined from its "standard" value, both due to the $\sigma_{\rm inf}^{-3}$ factor in the first term of Eq. (10.56) and due to the extra terms. However, as we will see below, the latter turns out to be irrelevant.

Assuming inflation ends at $\phi_I \approx 0$, the number of e-folds N_e in terms of the initial value of ϕ_I at leading order in our perturbative expansion is given by

$$\begin{aligned} N_e \simeq\ & \frac{\phi_I^2}{4} + \frac{\sigma_{\rm inf}^2 |D_T W_{\rm mod}|^2 \ln\phi_I}{3m^2} + \frac{3\,\sigma_{\rm inf}|W_{\rm mod}(\sigma_0)|^2 \ln\phi_I}{4m^2\sigma_0} \\ & - \left(\frac{1+(1+32\,\gamma\,\phi_I^2)\ln(1+32\,\gamma\,\phi_I^2)}{512\,\gamma(1+32\,\gamma\,\phi_I^2)}\right)\frac{W_{\rm mod}^2+\overline{W}_{\rm mod}^2}{m^2} \\ & + \left(-\frac{1}{\gamma(1+32\,\gamma\,\phi_I^2)} - 192\ln(\phi_I) - \frac{\ln(1+32\,\gamma\,\phi_I^2)}{\gamma}\right)\frac{|W_{\rm mod}|^2}{256\,m^2}\,. \end{aligned} \tag{10.58}$$

There is some additional ϕ_I-dependence, but it is rather weak at large values of ϕ_I. Thus, we again perform a perturbative analysis to determine the required initial value ϕ_I as a function of N_e, which yields for $N_e \gg 1$

$$\begin{aligned} \phi_I(N_e) \simeq\ & 2\sqrt{N_e} - \frac{3\sigma_{\rm inf}\,|W_{\rm mod}(\sigma_0)|^2\ln(4N_e)}{8m^2\sqrt{N_e}\sigma_0} - \frac{\sigma_{\rm inf}^2|D_T W_{\rm mod}|^2\ln(4N_e)}{6m^2\sqrt{N_e}} \\ & + \frac{\left(W_{\rm mod}^2+\overline{W}_{\rm mod}^2\right)\ln(128\,\gamma\,N_e)}{512\,m^2\,\sqrt{N_e}} + \frac{(96\,\gamma\,\ln(4N_e)+\ln(128\,\gamma\,N_e))\,|W_{\rm mod}|^2}{256\,\gamma\,m^2\,\sqrt{N_e}}\,. \end{aligned} \tag{10.59}$$

Plugging this into the definitions for ϵ and η yields at leading order

$$\begin{aligned}\epsilon \simeq\ & \frac{1}{2N_e} + \frac{\sigma_{\rm inf}^2\,|D_T W_{\rm mod}|^2(\ln(4N_e)-2)}{12\,m^2\,N_e^2} + \frac{3\,\sigma_{\rm inf}|W_{\rm mod}(\sigma_0)|^2(\ln(4N_e)-2)}{16\,\sigma_0\,m^2\,N_e^2}\\ & - \frac{-2+96\,\gamma(\ln(4N_e)-2)+\ln(128\,\gamma\,N_e)|W_{\rm mod}|^2}{512\,\gamma\,m^2\,N_e^2}\\ & - \frac{(\ln(128\,\gamma\,N_e)-2)\left(W_{\rm mod}^2+\overline{W}_{\rm mod}^2\right)}{1024\,\gamma\,m^2\,N_e^2}\,,\end{aligned} \tag{10.60}$$

and

$$\begin{aligned}\eta \simeq\ & \frac{1}{2N_e} + \frac{\sigma_{\rm inf}^2\,|D_T W_{\rm mod}|^2(\ln(4N_e)-1)}{12\,m^2\,N_e^2} + \frac{\sigma_{\rm inf}|W_{\rm mod}(\sigma_0)|^2(\ln(4N_e)-1)}{16\,\sigma_0\,m^2\,N_e^2}\\ & - \frac{-1+96\,\gamma(\ln(4N_e)-2)+\ln(128\,\gamma\,N_e)|W_{\rm mod}|^2}{512\,\gamma\,m^2\,N_e^2}\\ & - \frac{(\ln(128\,\gamma\,N_e)-1)\left(W_{\rm mod}^2+\overline{W}_{\rm mod}^2\right)}{1024\,\gamma\,m^2\,N_e^2}\,,\end{aligned} \tag{10.61}$$

respectively. Note that all the corrections start at order N_e^{-2} (up to some logarithms): They are suppressed with respect to the leading contribution by $W_{\rm mod}^2/W_X^2$ etc. since $m^2 N_e \sim m^2\phi_I^2 = W_X^2$.

Before we continue, we have to fix the parameter m by matching the prediction to the observed amplitude of the scalar power spectrum $\mathcal{P}_{\mathcal{R}}^{1/2}$. We find

$$\begin{aligned}\mathcal{P}_{\mathcal{R}}^{1/2} \simeq\ & \frac{mN_e}{2\sqrt{3}\pi\sigma_{\rm inf}^{3/2}}\left(1 - \frac{\sigma_{\rm inf}^2|D_T W_{\rm mod}|^2(2\ln(4N_e)-3)}{12\,m^2\,N_e} - \frac{3\,\sigma_{\rm inf}|W_{\rm mod}(\sigma_0)|^2(2\ln(4N_e)-3)}{16 m^2 N_e \sigma_0}\right.\\ & + \frac{(2\ln(128\,\gamma\,N_e)-3)\left(W_{\rm mod}^2+\overline{W}_{\rm mod}^2\right)}{1024\,\gamma\,m^2\,N_e}\\ & \left. + \frac{(-3+96\,\gamma\,(2\ln(4N_e)-3)+2\ln(128\,\gamma\,N_e))\,|W_{\rm mod}|^2}{512\,\gamma\,m^2\,N_e}\right).\end{aligned} \tag{10.62}$$

Thus, except for the factor $\sigma_{\rm inf}^{3/2}$, the mass parameter m needs to be redefined only at second order in $W_{\rm mod}$ and $D_T W_{\rm mod}$. Consequently, this affects the above expressions for ϵ and η only at higher orders and we drop these corrections in the following. What matters, however, is the rescaling of the mass parameter m by $\sigma_{\rm inf}^{3/2}$ compared to the standard chaotic inflation scenario.

Now we can calculate the scalar spectral index $n_s = 1 - 6\,\epsilon + 2\,\eta$ and the tensor-to-scalar ratio $r = 16\,\epsilon$, for which we find

$$\begin{aligned} n_s - 1 \simeq &- \frac{2}{N_e} - \frac{\sigma_{\rm inf}^2 |D_T W_{\rm mod}|^2 (2\ln(4N_e) - 5)}{6m^2 N_e^2} - \frac{3\,\sigma_{\rm inf} |W_{\rm mod}(\sigma_0)|^2 (2\ln(4N_e) - 5)}{8m^2 N_e^2 \sigma_0} \\ &+ \frac{(-5 + 96\,\gamma(2\ln(4N_e) - 5) + 2\ln(128\,\gamma\,N_e))\,|W_{\rm mod}|^2}{256\,\gamma\,m^2\,N_e^2} \\ &+ \frac{(2\ln(128\,\gamma\,N_e) - 5)\left(W_{\rm mod}^2 + \overline{W}_{\rm mod}^2\right)}{512\,\gamma\,m^2\,N_e^2}\,, \end{aligned} \tag{10.63}$$

and

$$\begin{aligned} r \simeq &\frac{8}{N_e} + \frac{4\,\sigma_{\rm inf}^2\,|D_T W_{\rm mod}|^2 (\ln(4N_e) - 2)}{3\,m^2\,N_e^2} + \frac{3\,\sigma_{\rm inf} |W_{\rm mod}(\sigma_0)|^2 (\ln(4N_e) - 2)}{\sigma_0\,m^2\,N_e^2} \\ &- \frac{-2 + 96\,\gamma(\ln(4N_e) - 2) + \ln(128\,\gamma\,N_e)|W_{\rm mod}|^2}{32\,\gamma\,m^2\,N_e^2} \\ &- \frac{(\ln(128\,\gamma\,N_e) - 2)\left(W_{\rm mod}^2 + \overline{W}_{\rm mod}^2\right)}{64\,\gamma\,m^2\,N_e^2}\,, \end{aligned} \tag{10.64}$$

respectively. Obviously, the corrections with respect to the leading term are small. They also appear at a higher order in the large N_e expansion since the corrections are suppressed by $W_X^2 = m^2\Phi^2 \sim m^2 N_e$ with respect to the leading contribution. Low-energy supersymmetry and high-scale inflation correspond to $D_T W_{\rm mod}$ and $W_{\rm mod}$ being parametrically small compared to the F-term $D_X W$. Consequently, all the corrections are parametrically small as well since we have to require $\sigma_0 \approx \sigma_{\rm inf}$ to avoid the cosmological moduli problem and we assume that $D_T W_{\rm mod}$ and $W_{\rm mod}$ do not vary too strongly between σ_0 and $\sigma_{\rm inf}$.

In summary, the upshot of the somewhat lengthy calculation above is that the predictions for $n_s - 1$ and r are essentially unaffected from what one would obtain from standard chaotic inflation. In short, the general analysis above tells us that the corrections due to the presence of a sector responsible for moduli stabilization and low-energy supersymmetry breaking (encoded in $W_{\rm mod}(T)$) are completely negligible. The reason is that the scalar spectral index n_s and the tensor-to-scalar ratio r receive only corrections of the form

$$n_s - 1 = -\frac{2}{N_e}\left(1 + \mathcal{O}\left(\left(\frac{F_T}{F_X}\right)^2, \left(\frac{m_{3/2}}{F_X}\right)^2\right)\right)\,, \tag{10.65}$$

$$r = \frac{8}{N_e}\left(1 + \mathcal{O}\left(\left(\frac{F_T}{F_X}\right)^2, \left(\frac{m_{3/2}}{F_X}\right)^2\right)\right)\,, \tag{10.66}$$

with the terms in brackets being parametrically smaller than 1 precisely when we have high-scale inflation and low-energy supersymmetry breaking.

10.5 Summary and Discussion

We have proposed a general scenario for moduli stabilization where low-energy supersymmetry breaking can be accommodated together with a high scale of inflation. In our proposal, the KL problem is resolved because the stabilization of the modulus field *during* and *after* inflation is not associated with a single, common scale, but instead relies on two *different* mechanism to stabilize the modulus during and after inflation.

More explicitly (c.f. Sec. 10.2), we suggest to consider a Kähler potential which features a coupling between the modulus field and the field whose F-term drives inflation in such a way that the term $V_{\rm inf} \sim e^K\, K^{X\bar{X}} |D_X W|^2$ creates a minimum for the modulus which stablizes it with a large mass during inflation. After inflation, when $D_X W$ vanishes, a "standard" mechanism involving non-perturbative terms in the superpotential can take over to stabilize the modulus as usual. The way we avoid the KL problem in this setup works essentially as follows: The gravitino mass $m_{3/2}^{\rm today}$ now only sets the scale for moduli stabilization *after* inflation and therefore remains small all the time. *During* inflation, the scale for moduli stabilization is set by the inflationary energy scale $\mathcal{H}_{\rm inf}$ itself and no longer also by $m_{3/2}^{\rm today}$. This allows us to consistently combine low scale SUSY and high scale inflation.

There is of course a price to pay: Since the two minima for the modulus during and after inflation generically do not coincide, we have to make sure that there is no overshoot problem after inflation. This requires, for instance, that the two minima for the modulus are not too far apart (c.f. Sec. 10.3.1). Without a dynamical mechanism to guarantee a smooth transition between the two minima, achieving $\sigma_0 \approx \sigma_{\rm inf}$ may require some amount of tuning of the model parameters. However, notice that also the KL solution requires some tuning to disentangle the height of the barrier from the gravitino mass today. Also note that, for a KKLT-type stabilization mechanism after inflation, the mass of the modulus is $m_T \sim 16\pi^2\, m_{3/2}$, which means it can be heavy enough to decay before BBN, avoiding the standard cosmological moduli problem.

We have illustrated our general strategy in a simple model of chaotic inflation with a shift symmetry supplemented by a KKLT-type superpotential and an uplifting term (c.f. Sec. 10.3). Moduli stabilization during inflation is achieved considering a certain type of couplings in the Kähler potential between the field X which drives the inflationary vacuum energy by its F-term and the modulus T. We also showed that in the limit of high-scale inflation and low-energy supersymmetry breaking, *i. e.* for $W, D_T W \ll D_X W$, the corrections to the inflationary observables from the modulus sector become negligible. We also presented the results for a general moduli stabilizing superpotential $W_{\rm mod}(T)$ combined with the simple chaotic inflation model (cf. Sec. 10.4) which also lead to the same conclusion that one can successfully combine high-scale inflation and low-energy supersymmetry breaking.

Even though many ingredients of our scenario are motivated from a string theory perspective, we did not consider a particular embedding in a string theory compactification here and defer this discussion to the future. It is plausible that something at least qualitatively similar to our proposal could be realized.

We emphasize that our general strategy may work for more general scenarios. Results for hybrid (or tribrid) inflation models and models based on a Heisenberg symmetry instead of a shift symmetry will appear elsewhere [472].

To conclude this summary, we comment on the relation of our proposal to previous works on solutions to the KL problem.

As a solution to the problem, KL [45] suggested to add a second non-perturbative contribution to the potential. This allows one to disentangle the gravitino mass in the present vacuum from the height of the barrier. That is, choosing the gravitino mass at the TeV-scale one can always independently increase the height of the barrier such that high-scale inflation models do not destabilize the modulus. This is done by a fine-tuning of the terms in the superpotential.

Following the approach of KL, in the context of volume modulus inflation in a racetrack setup, the problem has been addressed when one of the exponents of the non-perturbative terms is positive and/or introducing extra terms in W and/or K [198, 473–476]. However, this can be done only at the expense of introducing more parameters in the theory.

In the large volume scenario, attempts have been made to accommodate a small gravitino mass with a high scale of inflation, when inflation happens exponentially far away from the present Minkowski vacua. Other than some inevitable fine-tuning, the working models have several phenomenological difficulties [259]. Dynamical avenues for the case of chaotic inflation [477] and hybrid inflation [478] have also been explored, where the gravitino mass becomes inflaton-dependent in a suitable way.

Difficulties related to the realizations of high-scale inflation together with low-energy SUSY breaking have been discussed in [479, 480] and some resolutions have been proposed in [479, 481]. However, this has been successfully achieved only for the superpotential of [45]. Combining chaotic inflation and supersymmetry breaking within the KL scheme has recently been discussed in [482] for the general chaotic inflation models of [229–231]. For other approaches to the KL problem see *e. g.* [483].

In summary, all solutions known so far require some amount of fine-tuning to solve the problem or introduce extra terms beyond the minimal KKLT setup. The same is true for our solution, but one may imagine to find a dynamical mechanism to smoothly transfer the modulus from the minimum during inflation to the post-inflationary minimum, thereby relaxing the required fine-tuning for our model.

Chapter 11

Matter Inflation in Supergravity

We now come to the results from Ref. [1], in which my collaborators and I studied models of *matter inflation*. First, we consider matter inflation from an effective supergravity point of view in this chapter. The subject of Chap. 12 is to discuss the conditions under which such models can be embedded into heterotic orbifold compactifications.

This phenomenologically motivated idea for inflationary model building has been applied also to other situations. For instance, the particular models studied in [297] used a superpotential that is well-suited for sneutrino hybrid inflation [221]. Their model has two phenomenologically interesting properties associated to the waterfall phase transition at the end of inflation. Namely, a dynamical breaking of some larger gauge group $\mathcal{G}_{\text{GUT}}$ to $\mathcal{G}_{\text{SM}}$ and a dynamical generation of the masses for the right-handed neutrinos.[1] Phenomenologically it is particularly appealing to identify the phase transition at the end of hybrid inflation with the breaking of some GUT group or flavor group, see *e. g.* [217, 218, 232].

Here, we consider a generalization of the models of [297] in various aspects. First, the form of the superpotential couplings is extended to include a more general tribrid structure. Second, the constant couplings are replaced by moduli-dependent functions. Finally, the energy scale for inflation is assumed to be set dynamically, *e. g.* driven by the cancellation of the D-term of an anomalous $U(1)_A$ gauge symmetry.

The outline of this chapter is as follows. We begin by explaining/reviewing the basic ideas underlying our supergravity models in Sec. 11.1, in particular, the ingredients required to realize inflation in the matter sector. Afterwards, in Sec. 11.2, we present our generalization of the models of Ref. [297]. Moduli stabilization during inflation for our setup is discussed in Sec. 11.3 and we discuss sources for a slope of the inflaton in Sec. 11.4. Finally, we summarize the findings of this chapter in Sec. 11.5.

[1]For other models of sneutrino inflation see *e. g.* [484, 485].

11.1 Basic Ideas for Matter Inflation

11.1.1 Tribrid Inflation

In Sec. 5.3, we have given an important guideline for F-term inflation model building in supergravity. Namely, that the following conditions should hold during inflation:

$$D_X W \neq 0\,, \quad D_\Phi W \approx 0\,, \quad W \approx 0\,, \tag{11.1}$$

where $X \neq \Phi$ is some direction other than the inflaton. Then the supergravity η-problem can be solved by assuming some form of the Kähler potential [38].

An interesting class of supergravity hybrid inflation models[2], dubbed "tribrid" inflation, is based on superpotentials of the following form [228, 351][3]

$$W = \kappa X(H^2 - M^2) + g(\Phi)H^2\,, \tag{11.2}$$

where the three fields X , H and Φ play different roles:

- The *driving field* X provides the large vacuum energy driving inflation.
- The *waterfall field* H allows inflation to end via a phase transition.
- The *inflaton* Φ provides the "clock" determining when inflation ends.

This class of models fulfills the conditions in Eq. (11.1) since inflation takes place while $X = H = 0$ and Φ is above its critical value. The scale M sets both the expectation value of H at the end of inflation and the size of the F-term of X during inflation (*i. e.* the vacuum energy). The $g(\Phi)H^2$ term in the superpotential provides a positive Φ-dependent mass squared for the waterfall field H. Possible choices for $g(\Phi)$ include, for example, $g(\Phi) = \lambda\Phi^n/\Lambda^{n-1}$ for any $n \geq 1$ and some cutoff scale Λ. The two most common choices in the literature use either the renormalizable choice $g(\Phi) = \lambda\Phi$ (see *e. g.* [216]) or the choice $g(\Phi) = \lambda\Phi^2/\Lambda$ (see *e. g.* [35, 221, 227, 228, 351]). Note that both of these choices provide a mass term for Φ *after* inflation.

11.1.2 Heisenberg Symmetry

The tribrid superpotential can be supplemented by a Kähler potential with a shift symmetry for Φ [35, 216, 227, 228] or a "Heisenberg symmetry" [351]. Here,

[2] For a review of hybrid inflation in supergravity see Sec. 5.2 and references therein.

[3] Note that in "conventional" hybrid inflation models H^2 is often replaced by H_+H_- such that a $U(1)$ symmetry is spontaneously broken at the end of inflation when one of the fields acquires an expectation value. More generally, instead of H_+H_- one may use any pair of fields HH^c in conjugate representations of some gauge group $\mathcal{G}$. But to understand the basic physical picture, it is best to set this slight complication aside for a moment

we opt for the Heisenberg symmetry for reasons which will become clear soon. That is, we consider Kähler potentials of the form

$$K = k(\rho) + |X|^2 + |H|^2 + d(\rho)|X|^2 + \dots\,, \tag{11.3}$$

where the dots represent all kinds of higher order terms such as $|X|^4$ or $|X|^2|H|^2$ and ρ contains the inflaton Φ and a modulus T in a combination invariant under the Heisenberg symmetry:

$$\rho \equiv T + \overline{T} - |\Phi|^2\,. \tag{11.4}$$

Some working examples for the functions $d(\rho)$ and $k(\rho)$ can be found below in Sec. 11.3 and in [351]. The Heisenberg symmetry acting on T and Φ consists of the following two transformations [438]:

$$T \to T + i\alpha \quad , \quad \alpha \in \mathbb{R}\,, \tag{11.5}$$

and

$$\begin{aligned} T &\to T + \bar{\beta}\Phi + \frac{1}{2}\bar{\beta}\beta\,, \\ \Phi &\to \Phi + \beta \quad , \quad \beta \in \mathbb{C}\,. \end{aligned} \tag{11.6}$$

The Kähler potential in Eq. (11.3) is a general expansion in H and X and is assumed to preserve the Heisenberg symmetry. The field X provides the vacuum energy by its F-term and the function $k(\rho)$, in combination with a suitable choice of $d(\rho)$, can stabilize ρ during inflation (see Sec. 11.3).

Since these models fulfill $W = W_\Phi = W_H = 0$, $W_X \neq 0$ during inflation when $X = H = 0$ and since K preserves the Heisenberg symmetry, the inflaton direction is protected from the supergravity η-problem [38, 351]. A slope is induced via quantum corrections to the tree-level flat potential due to the symmetry breaking term $g(\Phi)H^2$ in the superpotential [351].

11.1.3 Inflation from the Matter Sector

An interesting aspect of the Heisenberg symmetry is that unlike a shift symmetry it allows the inflaton Φ to be a *gauge non-singlet matter* field. We allow for this possibility following [297], where an explicit example of matter inflation in the context of supersymmetric Grand Unified Theories was constructed. This requires a modification of Eqs. (11.2) and (11.3). In particular, one has to introduce multiple matter fields Φ_a in order to satisfy the constraints required to impose *D-flatness* (*i. e.* vanishing D-terms). Note also that now the parameter β in the symmetry transformation in Eq. (11.6) has to be replaced by a set of parameters β_a.

We replace the field H by by two matter fields H, H^c in conjugate representations of some gauge group $\mathcal{G}$. The Φ_a are also matter fields charged under some gauge group $\mathcal{G}$.[4] Then the term $g(\Phi_a)HH^c$ in the superpotential Eq. (11.2) must be *gauge-invariant* and this restricts the possible forms of $g(\Phi_a)$. For instance, the choice made in [297] is $g(\Phi_a) = \lambda\Phi\Phi^c/\Lambda$ with Φ, Φ^c in conjugate representations of the gauge group $\mathcal{G}$. The simplest possibility would be $\Phi_\pm$ and $H_\pm$ for a $U(1)$ gauge group. Note that since the Φ_a are gauge non-singlets, $g(\Phi_a)$ is a least quadratic in the Φ_a.

Note that in general there can be more possibilities for gauge-invariant terms which lead to inflaton-dependent mass terms for H, H^c. To illustrate this and the constraints imposed by D-flatness, let us consider the superpotential used in [297] to embed inflation into a SUSY model with a Pati-Salam (PS) or an $SO(10)$ GUT gauge group:

$$W = \kappa X \left(HH^c - M^2\right) + \frac{\lambda_{ij}}{\Lambda} F_i F_j H^c H^c + \frac{\zeta_i}{\Lambda} F_i F^c H H^c + \frac{\gamma}{\Lambda} F^c F^c H H + \dots \,, \tag{11.7}$$

where F_i, H and F^c, H^c are in conjugate representations of the same gauge group $\mathcal{G}$. For instance, in the $\mathbf{16}$ and $\overline{\mathbf{16}}$ of $\mathcal{G}_{\mathrm{GUT}} = SO(10)$ or in the $(\overline{\mathbf{4}}, \mathbf{1}, \overline{\mathbf{2}})$ and $(\mathbf{4}, \mathbf{1}, \mathbf{2})$ (or vice versa) of $\mathcal{G}_{\mathrm{PS}} = SU(4)_C \times SU(2)_L \times SU(2)_R$, respectively.

The Kähler potential is assumed to be of the form

$$K = k(\rho) + \left(1 + d(\rho) - \kappa_X |X|^2\right) |X|^2 + |H|^2 + |H^c|^2 + \dots \,, \tag{11.8}$$

with $\rho \equiv T + \overline{T} - \sum_i |F_i|^2 - |F^c|^2$ and $k(\rho), d(\rho)$ some functions of ρ only, which are suitable for stabilizing ρ.

Assuming a constant diagonal gauge kinetic function $f_{ab} = g_a^{-2}\delta_{ab}$, the D-term potential during inflation when $H = H^c = 0$ is of the form

$$V_D = \frac{g^2}{2} k'(\rho) \sum_a \left(\bar{F}_i \mathcal{T}^a F_i - \bar{F}^c \mathcal{T}^{a*} F^c\right) \,, \tag{11.9}$$

where $\mathcal{T}_a$ denotes the generators of the gauge group $\mathcal{G}$ in the appropriate representation. Inflation is supposed to proceed along a D-flat direction, *i. e.* along a direction where $V_D = 0$.

The explicit examples considered in [297] have only $F \equiv F_1, F^c \neq 0$ and $F_{i\neq 1} = 0$. In addition, the authors consider inflation along the (right-handed) sneutrino direction, *i. e.* $F, F^c = 0$ except along ν_F, ν_{F^c} and then D-flatness imposes a relation between ν_F and ν_{F^c}. Finding the unstable direction of the waterfall field in such models is more involved than in the simplest hybrid inflation models due to the extra terms in Eq. (11.7), but it is straightforward. For generic choices of the parameters, the unstable direction is (a combination of) the sneutrino directions of H, H^c [297].

[4] A priori, H, H^c and Φ_a could be charged only under orthogonal subgroups of the full gauge symmetry. However, this might lead to problems with the generation of topological defects.

The Heisenberg symmetry is an approximate symmetry in the limit of vanishing superpotential and gauge couplings. We will discuss corrections to the inflaton potential later in Sec. 11.4. The predictions in [297] were obtained assuming the slope of the inflaton potential to be dominated by the Coleman-Weinberg 1-loop corrections.

Summary: Basic Requirements for Matter Inflation

To summarize this section, the important points that lead to a working model are the following requirements.

- A D-flat and F-flat direction of matter fields acting as the inflaton.
- An approximate symmetry protecting the inflaton from the η-problem.
- During inflation, $D_\Phi W \approx 0$ and $W \approx 0$.

The aim of the next section is to generalize the above ideas in various respects.

11.2 A More General Class of Models

We now turn to generalizing the class of matter inflation models described above. The generalization is done keeping in mind what might be viable in heterotic orbifold compactifications, but the models described below are themselves genuine supergravity models of inflation.

The superpotentials and Kähler potentials we consider from now on are a further generalization of the "tribrid" structure introduced in the last section:

$$\begin{aligned} W &= a(T_i)\, X\, \big(b(T_i)\, HH^c - \langle\Sigma\rangle^2\big) + c(T_i)\, g(\Phi_{a,3})\, HH^c \\ &\quad + W_{\text{mod}}(T_i) + \dots\,, \end{aligned} \tag{11.10}$$

$$\begin{aligned} K &= -\sum_{i=1}^{3} \log\rho_i + \left(\prod_{i=1}^{2} \rho_i^{-q_{X,i}}\right) |X|^2 \big(1 + d(\rho_3) - \kappa_X |X|^2\big) \\ &\quad + \left(\prod_{i=1}^{3} \rho_i^{-q_{H,i}}\right) |H|^2 + \left(\prod_{i=1}^{3} \rho_i^{-q_{H^c,i}}\right) |H^c|^2 + \dots\,. \end{aligned} \tag{11.11}$$

where the ρ_i are given by

$$\rho_i \equiv T_i + \overline{T}_i - \sum_a |\Phi_{a,i}|^2\,. \tag{11.12}$$

Structure of the Kähler Potential

The first term in the Kähler potential Eq. (11.11) is the analog of $k(\rho)$ used in the previous section. Notice that we have introduced three combinations ρ_i instead of the single one used previously. The T_i, $i = 1, 2, 3$ are moduli fields and their Kähler potential satisfies the no-scale property, *i. e.*

$$K^{i\bar{\jmath}}\, K_i\, K_{\bar{\jmath}} = 3\,, \quad i, j = T_1, T_2, T_3\,, \tag{11.13}$$

and in principle each of them comes with a set of associated matter fields $\Phi_{a,i}$. With out loss of generality we associate the inflaton with ρ_3, *i. e.* the inflaton is a D-flat combination of the $\Phi_{a,3}$, and from now on we set $\Phi_{a,i} = 0$ for $i = 1, 2$ and $\Phi_a \equiv \Phi_{a,3}$.

The terms in the second line of Eq. (11.11) are non-canonical terms for the waterfall fields H, H^c with $q_{H,i}, q_{H^c,i}$ some rational numbers — typically positive and $\mathcal{O}(1)$. The remaining part of the first line is the Kähler potential for the driving field X which is of a somewhat different non-canonical form and, in particular, the couplings $\sim d(\rho_3)|X|^2$ and $\sim -\kappa_X|X|^4$ will be important for stabilizing both ρ_3 and X during inflation (see Sec. 11.3). The dots denote further terms denote possible higher order terms and other fields which can be present (but not relevant for the inflationary dynamics).

Structure of the Superpotential

Concerning the superpotential, Eq. (11.10), first notice that we have replaced all constant couplings in the superpotential by functions depending on the T_i (compare to Eq. (11.2) or (11.7)). Additionally, the scale M has been replaced by the expectation value of a (collection of) field(s) $\langle\Sigma\rangle$. This expectation value is assumed to be driven for example by the cancellation of the D-term of an anomalous $U(1)_A$ or induced through other terms in the superpotential.

The function $a(T_i)$ must depend only on $T_{1,2}$ in order not to spoil the flatness of the potential at tree-level. But the functions $b(T_i)$ and $c(T_i)$ may depend on all three moduli since $H, H^c = 0$ during inflation. We have added also a piece $W_{\text{mod}}(T_i)$ to the superpotential which is responsible for moduli stabilization after inflation (and may lead to low-energy supersymmetry breaking).

The waterfall fields H, H^c are matter fields in conjugate representations of some gauge group $\mathcal{G}$. The Φ_a are gauge non-singlet matter fields and $g(\Phi_a)$ is supposed to be a gauge-invariant product of the Φ_a such that a D-flat combination of these fields can act as the inflaton. That is, the inflaton is a certain combination of the Φ_a specified by the requirement of vanishing D-terms. As noted above, the simplest choice would be $g(\Phi_a) = \lambda\Phi_+\Phi_-/\Lambda$, but we allow for more general possibilities.[5] The cutoff scale Λ required in $g(\Phi_a)$ is a priori

[5] For example, the product of $\mathbf{N}$ and $\overline{\mathbf{N}}$ of $SU(N)$ as well as the product of three $\mathbf{27}$'s of E_6 form a gauge-invariant combination.

undetermined, but in a heterotic orbifold model it would be given by the string scale, *i. e.* $\Lambda \sim M_s$. Note that F-flatness is ensured by construction since for sufficiently large values of the Φ_a we have $H = H^c = 0$ during inflation.

11.2.1 Effective F-Term Potential During Inflation

The field X receives a large F-term during inflation when $H = H^c = 0$, thereby driving the inflationary vacuum energy, and it is fixed at $X = 0$. This is enforced by the term $\kappa_X |X|^4$ with $\kappa_X \gtrsim \mathcal{O}(1)$ [225] (see also Secs. 5.1 and 10.2 as well as references therein). Inflation ends as usual in hybrid inflation once a combination of H, H^c becomes tachyonic and develops an expectation value. The first term in the superpotential Eq. (11.10) then provides a mass for X such that it is at $X = 0$ also after inflation.[6]

Throughout the remainder of this chapter, we assume that the parameters in the superpotential are adjusted such that during inflation

$$D_X W \neq 0\,, \quad D_\alpha W \approx 0\,, \quad W \approx 0\,, \tag{11.14}$$

where the index α runs over all fields other than X. In particular, there should be no large F-terms for Φ_a and T_3. Therefore, in this limit, the F-term potential during inflation is given by

$$\begin{aligned} V_F &\approx e^K \left(K^{-1}_{X\bar{X}} |W_X|^2 \right) \\ &= \left(\prod_{i=1}^{2} \left(T_i + \overline{T}_i\right)^{p_i} \right) \frac{|a(T_1,T_2)|^2}{1+d(\rho_3)} \frac{|\langle \Sigma \rangle|^4}{\rho_3}\,, \end{aligned} \tag{11.15}$$

where $p_i \equiv -1 + q_{X,i}$ and the overall scale of the potential is set by $\langle \Sigma \rangle$.

11.3 Moduli Stabilization during Inflation

The moduli fields T_i have to be stabilized during inflation. Since none of them is the inflaton, we would like to stabilize them with a high mass at least of the order of the Hubble scale $\mathcal{H}_{\text{inf}}$. For T_1 and T_2, stabilization is achieved by a suitable form of the function $a(T_1, T_2)$ in Eq. (11.10), which enters the F-term of X. Moreover, the modulus T_3, or rather the combination ρ_3 in Eq. (11.12), is fixed by an appropriate moduli dependence of the Kähler metric of X, *i. e.* by an appropriate choice of $d(\rho_3)$. We now discuss how both stabilization mechanisms work in a simple toy model. Note that due to the product structure of the moduli dependence in Eq. (11.15) we can discuss the stabilization of T_1, T_2 and ρ_3 separately.

[6] Note that $X = 0$ actually means $X \approx 0$ since the sector responsible for moduli stabilization after inflation generically induces a small shift of X away from zero, cf. *e. g.* Sec. 10.34.

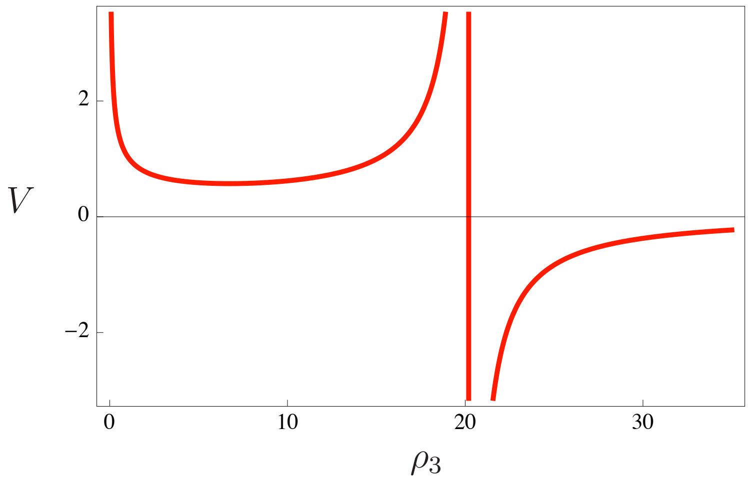

Figure 11.1: Form of the potential following from Eq. (11.16) for the example values $p = -\frac{1}{2}$, $\gamma = 0.01$ and $\beta = -0.05$, which has a minimum at $\langle\rho_3\rangle \approx 6.73$. The pole is at $\rho_3 \approx 20.2$. The overall scale of the potential has to be set by $\langle\Sigma\rangle$.

11.3.1 Stabilization of ρ_3

We stabilize ρ_3 during inflation with the F-term of X combined with a suitable kinetic term for X in order to give it a mass $m \gtrsim \mathcal{H}_{\text{inf}}$. The moduli dependence of the F-term potential Eq. (11.15) not only depends on $K_{X\bar{X}}$ and $a(T_i)$ but also on $\langle\Sigma\rangle$. Due to the moduli-dependent non-canonical Kähler potentials (and superpotential) terms, the expectation value $\langle\Sigma\rangle$ typically inherits some moduli dependence (cf. Sec. 9.3.2). For example, assuming that the ρ_3 dependence is inherited from the Kähler potential, we have $\langle\Sigma\rangle \propto \rho_3^q$ for some rational number $q \geq 0$.

The function $d(\rho_3)$ is assumed to preserve the Heisenberg symmetry to a sufficient amount and must be of a suitable form to stabilize ρ_3. That is, we assume that $d(\rho_3)$ is such that we can stabilize ρ_3 without inducing a large mass for the inflaton.[7] To illustrate what "suitable form" means, consider the ρ_3-dependence of the F-term potential, which is of the form[8]

$$V \propto \frac{\rho_3^p}{1 + d(\rho_3)}\,, \tag{11.16}$$

with some rational number $p \geq -1$. In order to get a minimum suitable for inflation, let us make the simple ansatz $d(\rho_3) = \gamma + \beta\rho_3$. If $p < 0$, $\beta < 0$ and $\gamma > -1$, this yields a minimum at

$$\langle\rho_3\rangle = -\frac{p\,(1+\gamma)}{(p-1)\,\beta} > 0\,. \tag{11.17}$$

[7]We will comment on relaxing this assumption somewhat later on in Sec. 11.4.
[8]If $\langle\Sigma\rangle$ is independent of ρ_3, we have $p = -1$. Otherwise it is some rational number ≥ -1.

For this choice of $d(\rho_3)$, the potential has a pole at $\rho_3 = -(1+\gamma)/\beta$, which provides the barrier towards $\rho_3 \to \infty$. For $p < 0$, the ρ_3^p factor prevents the field from rolling to $\rho_3 \to 0$. Therefore, the parameter p is constrained to be $-1 \leq p < 0$. Fig. 11.1 shows a schematic plot of the ρ_3-dependence of the potential (in arbitrary units) for an illustrative choice of parameters.

We expect the pole in the potential to be an artefact of our approximation: we work at second order in the derivatives and the pole appears when $K_{X\bar{X}} \to 0$ (recall that $V_F \propto (K_{X\bar{X}})^{-1}$). Therefore, this approximation breaks down close to the pole and higher derivative corrections become important. In particular, if one would like to address issues such as stability of the minimum with respect to tunneling, higher derivative terms have to be included. Note that in the region to the right of the pole, $K_{X\bar{X}} < 0$ and thus X has a kinetic term with the wrong sign. For our present purpose we only need that the potential given by Eq. (11.16) is a good approximation if we are not too close to the pole and we will assume that we are confined within this region.

To determine the physical mass of ρ_3 around its minimum in units of the Hubble scale $\mathcal{H}_{\text{inf}}$ during inflation, we have to take into account that the Kähler potential Eq. (11.11) leads to a non-canonical kinetic term for ρ_3, namely $\rho_3^{-2}(\partial_\mu \rho_3)^2$. Hence, the physical mass is given by

$$m_{\rho_3}^2 = 2p(p-1)V_0\,, \tag{11.18}$$

where V_0 denotes the value of the potential at the minimum — recall that we have set $M_P = 1$ and thus $\mathcal{H}_{\text{inf}}^2 \sim V_0$. For $p \lesssim -0.15$, we have $m_{\rho_3} \gtrsim \mathcal{H}_{\text{inf}}$, which should be heavy enough such that ρ_3 settles to its minimum sufficiently fast.

11.3.2 Stabilization of $T_{1,2}$

As noted above, stabilization of T_1 and T_2 requires a "suitably chosen" function $a(T_1, T_2)$. Assuming again that a possible moduli-dependence of $\langle\Sigma\rangle$ is only due to the non-canonical Kähler metric, the dependence of the scalar potential on T_1 and T_2 takes the form[9]

$$V \propto \left(T_1 + \overline{T}_1\right)^{p_1} \left(T_2 + \overline{T}_2\right)^{p_2} |a(T_1, T_2)|^2\,, \tag{11.19}$$

with p_1 and p_2 rational numbers ≥ -1. A simple choice for $a(T_1, T_2)$ which does the job is $a(T_1, T_2) = e^{a_1 T_1 + a_2 T_2}$. If $a_i > 0$ and $p_i < 0$, this will yield a minimum for $\operatorname{Re} T_1$ and $\operatorname{Re} T_2$: The exponentials diverge as $\operatorname{Re} T_i \to \infty$ and similarly the power law factors diverge as $\operatorname{Re} T_i \to 0$. The minima are at

$$\langle \operatorname{Re} T_i \rangle = -\frac{p_i}{\sqrt{2}a_i} > 0\,, \tag{11.20}$$

[9]If $\langle\Sigma\rangle$ is independent of T_i, we have $p_i = -1 + q_{X,i}$ and otherwise it is some other rational number.

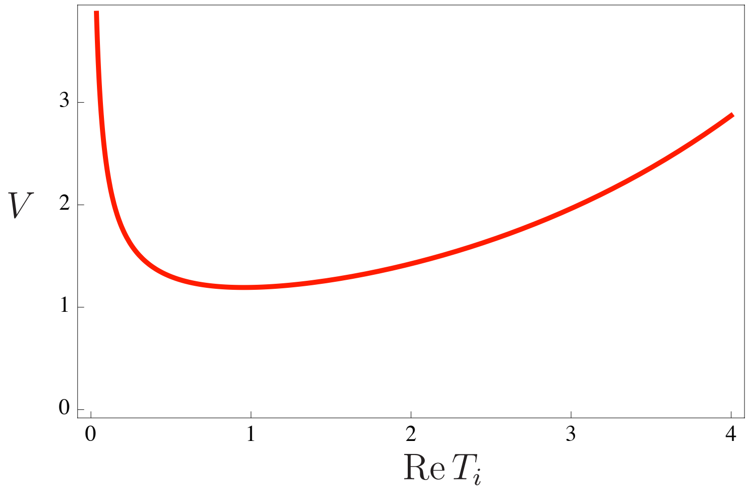

Figure 11.2: Form of the potential following from Eq. (11.19) with respect to T_i for the example values $p_i = -\frac{1}{2}$ and $a_i = \frac{\pi}{12} \approx 0.26$, which has a minimum at $\langle \mathrm{Re}\, T_i \rangle \approx 0.96$. The overall scale of the potential has to be set by $\langle \Sigma \rangle$.

which is typically $\mathcal{O}(1)$. Again, taking into account canonical normalization, also T_1 and T_2 are stabilized with masses $m \sim \mathcal{H}_{\mathrm{inf}}$. A plot of the T_i-dependence of the potential (in arbitrary units) for a sample choice of parameters is shown in Fig. 11.2.

Some Remarks

Note that in both situations, Eq. (11.16) and Eq. (11.19), for our choices of $d(\rho_3)$ and $a(T_1, T_2)$ the axions associated to $\mathrm{Im}\, T_i$ only receive a potential from the terms contained in $W_{\mathrm{mod}}(T_i)$. However, this is not a problem for the inflationary scenario discussed here: The axions are effectively frozen during inflation due to the strong Hubble damping [351].

Once the moduli have settled to their minima, the functions $a(T_i), b(T_i)$ and $c(T_i)$ can be effectively treated as constants. Similarly, the non-canonical kinetic terms only amount to a rescaling of the fields by a constant.

We will later interpret $\mathrm{Re}\, T_{1,2}$ and ρ_3 to parametrize the volume of some compact extradimensions, see Eq. (9.6), as follows (in units of the fundamental scale)

$$\mathcal{V} = (T_1 + \overline{T}_1)(T_2 + \overline{T}_2)\, \rho_3\,. \tag{11.21}$$

The illustrative choice of parameters used for Figs. 11.1 and 11.2 yields $\langle \mathrm{Re}\, T_{1,2} \rangle \approx 0.96$ and $\langle \rho_3 \rangle \approx 6.73$ and thus $\langle \mathcal{V} \rangle \approx 24.81$.

Actually, the choices for $a(T_1, T_2)$ and $d(\rho_3)$ in this section are not made purely for illustrative purposes, but are motivated from what we expect to find in heterotic orbifold compactifications, cf. Chap. 12.

Note that after inflation a different mechanism for moduli stabilization is required which gives rise to $W_{\text{mod}}(T_i)$. We have seen in Chap. 10 how this can be achieved for a model of chaotic inflation using essentially the same mechanism to stabilize the modulus ρ_3 during inflation. But we do not discuss this here in detail and leave this issue for future work along the lines of Chap. 10.

11.4 Hybrid Mechanism and a Slope for the Inflaton

So far, we have generated a large vacuum energy and stabilized all the moduli during inflation. But if the Heisenberg symmetry was exactly preserved, there would be no slope for the inflaton and hence no way to end inflation. In this section, we briefly discuss how we expect inflation to end via the hybrid mechanism and what sources could generate a slope within our setup.

11.4.1 Hybrid Mechanism

Once the inflaton ϕ reaches a critical value ϕ_{crit}, one of the waterfall fields H, H^c becomes tachyonic and triggers the waterfall phase transition, thereby ending inflation. This assumes that the inflaton slope drives ϕ towards ϕ_{crit}.[10]

During the phase transition, topological defects such as cosmic strings could be formed if the waterfall fields are charged under a gauge symmetry *e. g.* a $U(1)$ (if they are charged under a larger gauge group also other kinds of topological defects could form). Without a specific model it is difficult to decide whether these are problematic or not. First, if the symmetry is broken also during inflation, *i. e.* if the inflaton is also charged under this gauge symmetry, we expect that as in [297] topological defects could be avoided due to corrections to the inflaton potential, which lift the degeneracy. Second, the analysis of [348–350], who considered the somewhat similar scenario of standard F-term hybrid inflation, finds that the consistency of cosmic strings with WMAP data depends not only on the value of the gauge coupling but also of a parameter κ. This parameter is related to the inflationary superpotential used in standard F-term hybrid inflation $W = \kappa\,\Phi\,(HH^c - M^2)$, where Φ is the gauge singlet inflaton field. Values up to $\kappa \lesssim 10^{-2}$ seem to be consistent with the bounds. In any case, the issue of topological defects should only be addressed in a more specific model and therefore is beyond our present scope.

[10] If this is not the case, we will at some point leave the regime of validity of our effective field theory when ρ_3 becomes too small.

11.4.2 A Slope for the Inflaton

A slope for ϕ can be induced by various types of sources, for example

- by 1-loop CW corrections induced by $g(\Phi_a)HH^c$,
- through loops involving gauge fields,
- through a violation of $W \approx 0$, $W_\alpha \approx 0$ $(\forall \alpha \neq X)$ *e. g.* induced by W_{mod},
- or via Heisenberg symmetry breaking terms in K.

For the latter two cases, achieving a sufficiently small slope and, in particular, a sufficiently small $|\eta| \ll 1$ can arise essentially in two ways. First, the corrections induced by these terms are parametrically small compared to the contribution controlled by the F-term of X (see Sec. 10.4), which generically requires some tuning of parameters. Second, there can be accidental cancellations with many terms conspiring to give a small value $|\eta| \ll 1$. If this is the dominant source for the slope, one would expect a scenario somewhat similar to inflection point inflation. A detailed investigation of this scenario is left for the future.

If instead the Coleman-Weinberg corrections dominate, we expect similar inflationary dynamics as those discussed in [297] (with parameters appropriately rescaled). This is probably also the most predictive scenario since it involves the fewest parameters.

Finally, we now comment on the corrections to the inflaton potential from taking into account loops involving gauge bosons and gauginos following the discussion [297], where the one-loop and two-loop corrections to the inflaton mass have been computed in a specific model.

The one-loop Coleman-Weinberg potential is given by the expression

$$V_{\text{1-loop}} = \frac{1}{64\pi^2}\,\mathrm{STr}\left[\mathcal{M}^4(\Phi_a)\left(\log\left(\frac{\mathcal{M}^2(\Phi_a)}{Q^2}\right) - \frac{3}{2}\right)\right], \tag{11.22}$$

where Q is a renormalization scale and STr denotes the supertrace, which is taken over all bosonic and fermionic degrees of freedom with alternating signs. We are interested in the Φ_a dependence of all the masses since this dependence can induce a slope for the inflaton. However, only sectors with a mass splitting between the bosonic and fermionic degrees of freedom contribute to the supertrace in Eq. (11.22). The waterfall sector has such a mass splitting induced by the non-zero F-term for X (the $g(\Phi_a)HH^c$ term is effectively a supersymmetric mass term for the waterfall fields).

In addition to the waterfall sector, the gauge sector may contribute to the one-loop effective potential since the inflaton is a associated with a gauge non-singlet matter field. Its expectation value induces masses for some of the gauge fields. However, unlike the waterfall sector, the gauge sector has no mass splittings unless direct supergravity gaugino masses are present. The supergravity gaugino masses are given by

$$\mathcal{L}_{\text{gaugino}} \sim \left\langle e^{G/2}\, G_i \left(G^{-1}\right)^{i\bar{\jmath}} \frac{\partial \bar{f}_{ab}}{\partial \bar{\phi}^{\bar{\jmath}}} \right\rangle + h.c.\,, \tag{11.23}$$

where $G = K + \log|W|^2$, f_{ab} denotes the gauge kinetic function and a, b label different gauge groups while i, j label different scalar fields. In our case, during inflation we have $X \approx 0$, $W \approx 0$ and only $W_X \neq 0$. Thus, the gravitino mass $m_{3/2} \sim e^{\langle G \rangle/2} \sim e^{\langle K \rangle/2}\langle |W| \rangle \approx 0$, which already suppresses most contributions to the gaugino masses in Eq. (11.23). Namely, since $X \approx 0$ and only $W_X \neq 0$, the only contribution which survives in the limit $W \to 0$ (*i. e.* which is not suppressed due to the small gravitino mass) vanishes if we assume

$$\left\langle \frac{\partial \bar{f}_{ab}}{\partial \bar{X}} \right\rangle = 0\,. \tag{11.24}$$

More precisely, we actually only have to require that this expectation value does not depend on the inflaton, *i. e.* there should be no terms such as $X\tilde{g}(\Phi_a)$ contained in f_{ab}. Note that we also have to forbid this kind of terms in the superpotential. This can be achieved for example by discrete symmetries which either forbid $X\tilde{g}(\Phi_a)$ at all or force it to appear only together with some additional field(s) whose expectation value(s) vanish. Thus, the corrections from the gauge sector at one-loop are expected to be under control (they are essentially controlled by the small value of the gravitino mass $m_{3/2} \propto |\langle W \rangle|$).

There are also potentially dangerous corrections from the gauge sector at the two-loop level [486]. In [297], it was shown that in the large gauge boson mass limit the various two-loop corrections to the inflaton mass are suppressed by a universal factor $\delta m^2/\mathcal{H}^2 \propto \kappa^2/(4\pi)^4$, where in our case $\kappa \equiv a(\langle T_i \rangle)\, b(\langle T_i \rangle)$. For $\kappa \ll 1$ we the two-loop contributions are negligible. This suppression is also related to the fact that the non-zero inflaton VEV $\langle \phi \rangle$ already breaks the gauge symmetry during inflation to some subgroup $\mathcal{G}' \subset \mathcal{G}$ (*i. e.* some of the gauge bosons acquire masses $M_A \sim g\langle \phi \rangle$). But the inflaton is a *gauge singlet* under the unbroken gauge symmetry $\mathcal{G}'$.

11.5 Summary and Discussion

Let us summarize what we have obtained so far. We have explained the basic ideas for constructing models of matter inflation in Sec. 11.1. The important ingredients are a D-flat and F-flat direction of matter fields Φ_a which is protected from the η-problem by some approximate symmetry and during inflation $D_{\Phi_a} W \approx 0$, $W \approx 0$.

Based on these ingredients, we have constructed a class of supergravity models of matter inflation, *i. e.* models of inflation where the inflaton resides in the matter sector of the theory, which are a generalization of the models presented in [297] in various points (cf. Sec. 11.1):

1. We allow for a more general form of the tribrid structure in the superpotential, *i. e.* a more general choice of $g(\Phi_a)$ than $\Phi\Phi^c$.

2. The couplings in the superpotential are determined by the expectation values of a set of moduli fields T_i.

3. The scale of the F-term driving the inflationary vacuum energy is generated dynamically.

We have discussed in detail under which conditions the moduli T_i are successfully stabilized during inflation in Sec. 11.3. To stabilize the two moduli $T_{1,2}$ not associated with the inflaton, we introduced a suitable dependence of the F-term of X on them, namely $W_X \propto e^{a_1 T_1 + a_2 T_2}$ (cf. Sec. 11.3.2). The modulus ρ_3 associated to the inflaton is stabilized differently via a suitable moduli dependence of the Kähler metric $K_{X\bar{X}}$ (cf. Sec. 11.3.1).[11] Therefore, $T_{1,2}$ and ρ_3 (as well as X) are stabilized with masses $m \gtrsim \mathcal{H}_{\text{inf}}$ during inflation.

We briefly commented on the hybrid mechanism and the potentially dangerous production of topological defects in the phase transition (cf. Sec. 11.4.1). Whether these are problematic or not requires a more specific model and is beyond our present scope.

The models presented here are (by assumption) essentially flat at tree-level. We discussed sources for generating a slope at tree-level and loop-level in Sec. 11.4.2. In corners of the parameter space where the slope of the inflaton is dominated by the Coleman-Weinberg one-loop potential, the inflatonary phenomenology is expected to be similar to [297]. The corrections induced form the gauge sector are shown to be generically small if a certain condition is fulfilled (cf. Eq. 11.24 and the discussion below). The slope could also be dominated by a breaking of the Heisenberg symmetry in both K and W. We leave a detailed investigation of this corner of the parameter space for the future.

In the next section, we discuss under which circumstances matter inflation can be successfully embedded into heterotic orbifold compactifications. This was in fact the main motivation for the work published in Ref. [1].

[11] This is a version of the mechanism we proposed in Chap. 10 to solve the KL problem.

Chapter 12

Towards Matter Inflation in Heterotic Orbifolds

The aim of this chapter is to determine the conditions which allow the matter inflation models of the previous chapter to be realized in orbifold compactifications of the heterotic string. For example, since these models feature a Heisenberg symmetry protecting (to some extent) the inflaton direction, it is important to check whether such symmetries indeed arise in a string theory compactification. We have reviewed the structure of the effective supergravity theory of heterotic orbifolds in Chap. 9 and indeed a Heisenberg symmetry arises at tree-level for the untwisted matter fields [438, 439].

We will identify the moduli T_i with Kähler moduli of the orbifold compactification. The form of the superpotential in heterotic orbifolds is constrained by target space modular invariance, in particular, its dependence on the moduli T_i (cf. Sec. 9.2.2). However, in addition to the T_i, there is at least one extra modulus, the dilaton which determines the size of the string coupling and therefore also of the 4d gauge couplings. Its stabilization is quite difficult and here we employ a moduli stabilization scheme called Kähler stabilization [43, 448, 453–456, 460, 461] which allows to stabilize the dilaton during inflation [464, 487]. What is different in our models is the stabilization of the modulus T_3 (or ρ_3) associated to the inflaton. We argue that moduli-dependent loop corrections to the Kähler metric of the driving field X may lead to a mechanism for moduli stabilization similar to the one presented in Sec. 11.3.1.

This chapter is organized as follows. We begin by describing how to embed the field content and the constraints on the superpotential in Sec. 12.1. Next, we suggest a form of the loop corrections to the Kähler metric of the driving field in Sec. 12.2 and in Sec. 12.3 we will use these loop corrections to stabilize ρ_3 together with the dilaton, which is stabilized by non-perturbative corrections to the Kähler potential. After discussing some sources for the inflaton slope in Sec. 12.4, we summarize the results of this chapter in Sec. 12.5.

12.1 Field Content and Superpotential

We now begin to identify the field content of our general class of models of matter inflation within heterotic orbifold compactifications. We also discuss the constraints imposed on the superpotential by target space modular invariance.

Field Content

The identification of the field content of Sec. 11.2 is straightforward: The moduli T_i are identified with the three untwisted Kähler moduli present in any heterotic orbifold, which determine the radii of the three tori. $g(\Phi_a)$ is a product of untwisted matter fields Φ_a, associated without loss of generality with the third torus with modulus T_3. We neglect the complex structure moduli U_j here, assuming that they are either fixed by the orbifold projection or in a similar way to the T_i. We also neglect any twisted moduli or additional "off-diagonal" Kähler moduli.

We take X to be a twisted matter field in a twisted sector with $\mathcal{N} = 2$ supersymmetry. This is necessary to have a non-trivial moduli dependence in its Kähler metric, which is parametrized by $d(\rho_3)$ in Eq. (11.11) (cf. Sec. 12.2). We expect this moduli dependence to arise from string threshold corrections, cf. Sec. 12.2, and we assume here that it preserves the Heisenberg symmetry up to terms which are exponentially small in the large radius limit. In order to receive moduli-dependent corrections to its Kähler metric, X must be charged under (part of) the gauge group of the $\mathcal{N} = 2$ subsector. We will for simplicity assume that the inflaton (and the waterfall) fields are neutral with respect to this gauge group factor. Recall also that the inflationary setup of Sec. 11.2 requires a negative quartic $|X|^4$ term in the Kähler potential and we assume that this term exists. However, the Kähler potential of twisted matter fields is only known to quadratic order so far and our two assumptions on the Kähler potential terms involving X need to be checked in the future.

Constraints on the Superpotential

The superpotential starts at cubic order in the matter fields and thus the F-term of X has to arise from non-vanishing expectation values for some other fields, which were collectively denoted by Σ in Eq. (11.10). As reviewed in Sec. 9.3.2, such expectation values are generically moduli-dependent and therefore modify the moduli dependence of the effective scalar potential. In order to generate expectation values, such as $\langle\Sigma\rangle$, we require the presence of an anomalous $U(1)_A$. Moreover, we assume that $g(\Phi_a)$ carries a zero net charge under $U(1)_A$. Note that such a $U(1)_A$ is essential to the phenomenological success of the mini-landscape models [177, 178, 415–417] since it allows to give masses to unwanted exotic states via expectation values for SM singlet fields.

An important property of the setup presented in Sec. 11.2 is that during inflation only $W_X \neq 0$ while in particular $W_\Phi \approx 0$, $W \approx 0$. This property is well-suited for realizations of inflation in heterotic orbifolds [38, 464]. We now require that a superpotential of the form in Eq. (11.10) is present. It remains a task for the future to find explicit compactifications where this structure is realized and in particular whether this is possible in phenomenologically interesting setups such as the mini-landscape models [177, 178, 415–417]. However, this seems plausible since we can *e. g.* allow for terms of the form $\Phi_a \Psi \Psi'$ in W if $\Psi, \Psi' \approx 0$ during inflation.

The functions $a(T_i), b(T_i)$ and $c(T_i)$ in the superpotential, Eq. (11.10), are constrained by modular invariance: They are given by appropriate powers of the η-function $\eta(T_i)$, cf. Eq. (9.26). For example, we must have (for $\mathrm{Re}\, T_3 \gtrsim 1$)

$$c(T_i) \propto \eta(T_3)^{c_3} \sim e^{-\frac{\pi}{12} c_3 T_3}\,, \tag{12.1}$$

since $g(\Phi_a)$ involves at least two untwisted fields from the same sector. Note that in general it will also depend on T_1 and T_2 in a similar way. During inflation, the waterfall fields vanish, $H, H^c = 0$, and therefore the moduli dependence of $b(T_i)$ and $c(T_i)$ enters the effective scalar potential only through loops involving the waterfall fields.

There is a complication compared to the scenario of Sec. 11.2, namely that the functions $a(T_i), b(T_i)$ and $c(T_i)$ in principle may involve the expectation values $\langle \Psi \rangle$ of some matter fields, which can alter their moduli dependence. In particular, $\langle \Sigma \rangle$ directly affects the moduli dependence of the scalar potential and thus the stabilization of the moduli because it can depend on the moduli, as explained in Sec. 9.3.2. Therefore, achieving successful moduli stabilization imposes some constraints on the functional form of $|a(T_i)|^2 |\langle \Sigma \rangle|^4$ (which encodes the moduli dependence of the F-term of X). As we will see below in Secs. 12.3 and 12.4, we require it to dependend on T_3 only via ρ_3, *i. e.* to be independent of $\eta(T_3)$, and to depend only on inverse powers of $\eta(T_{1,2})$.

12.2 Loop Corrections to the Kähler Potential

We have seen in Sec. 11.3 that ρ_3 is stabilized during inflation for a suitable form of the function $d(\rho_3)$. In this section, we propose a functional form of $d(\rho_3)$ which we expect to arise in heterotic orbifolds. First, we consider known results for the string one-loop corrections to the Kähler metric of untwisted matter fields. Based on these results, we suggest a generalization in the presence of background values for matter fields. It will be of a form similar to the simple example discussed in Sec. 11.3, but in addition to a part linear in ρ_3 there is also a logarithmic contribution. Moreover, the corrections are proportional to the dilaton ℓ which controls the string-loop expansion. We also argue why the Heisenberg symmetry might be broken only by exponentially suppressed terms in the large radius limit.

The result of [488] for the string one-loop corrections to the Kähler metric of untwisted matter fields in $\mathcal{N}=2$ orbifolds has the following form:

$$K^{\text{eff}}_{a\bar{a}} = K^{\text{tree}}_{a\bar{a}} + \ell K^{\text{1-loop}}_{a\bar{a}}\,, \tag{12.2}$$

with

$$K^{\text{1-loop}}_{a\bar{a}} = K^{\text{tree}}_{a\bar{a}}\left(\gamma + \beta\, Y(T,\overline{T})\right)\,, \tag{12.3}$$

where

$$Y(T,\overline{T}) = \log\left[|\eta(T)|^4(T+\overline{T})\right]\,, \tag{12.4}$$

and ℓ is the loop counting parameter, the lowest component of $L \sim (S+\overline{S})^{-1}$. β is related to the $\mathcal{N}=2$ beta function coefficient of the torus associated with T and the gauge group which acts non-trivially on the matter field under consideration. The moduli-independent constant γ is the effect of the $\mathcal{N}=1$ subsectors. The moduli-dependence of $Y(T,\overline{T})$ in Eq. (12.3) originates from loops involving massive string states, namely Kaluza-Klein and winding modes, whose masses depend on the moduli, especially on the radius. Note that the moduli-dependent correction $Y(T,\overline{T})$ arises only from $\mathcal{N}=2$ sectors, which leave the plane associated to the matter field unrotated and therefore only depends on the moduli of that plane.

For $\operatorname{Re} T \gtrsim 1$, we can approximate $Y(T,\overline{T})$, Eq. (12.4), by

$$Y(T,\overline{T}) \approx \log(T+\overline{T}) - \frac{\pi}{6}(T+\overline{T}) + \mathcal{O}(e^{-2\pi T}) + c.c.\,, \tag{12.5}$$

as can be easily seen from Eq. (9.25). The dependence on $\operatorname{Im} T$ is only through the additional terms $\sim e^{-2\pi T}$, which are exponentially suppressed for large $\operatorname{Re} T$, *i. e.* for a large compactification radius. In other words, the continuous shift symmetry $T \to T + i\alpha$ (which is broken to a discrete one by worldsheet instantons [489, 490]) survives as an approximate symmetry in the large radius limit $\operatorname{Re} T \to \infty$.

Based on these results, we propose a generalization involving background values for untwisted matter fields as described below. It remains a task for the future to check our proposal by calculating the relevant string amplitudes.

As a working hypothesis, we consider the case where we simply replace $T+\overline{T}$ in the large radius limit of $Y(T,\overline{T})$, Eq. (12.5), by $\rho \equiv T+\overline{T}-\sum_a |\Phi_a|^2$, *i. e.* we assume

$$Y(T,\overline{T},\Phi_a,\bar{\Phi}_a) \approx \log\rho - \frac{\pi}{6}\rho + \lambda\sum_a |\Phi_a|^2 + \mathcal{O}(e^{-\pi\rho})\,, \tag{12.6}$$

where we have parameterized a breaking of the Heisenberg symmetry by the term $\lambda\sum_a |\Phi_a|^2$. The coefficient λ would have to be computed directly from string amplitudes and in general may depend on T.

If the Heisenberg symmetry is badly broken by such corrections, one would expect $\lambda \sim \mathcal{O}(1)$. But in the large radius limit the continuous shift symmetry

$T \to T + i\alpha$, which is part of the Heisenberg symmetry group, cf. Eqs. (11.5) and (11.6), is broken only by exponentially small terms. Thus, we might assume that the same happens for the Heisenberg symmetry, *i. e.* $\lambda \sim e^{-\pi\rho}$ is exponentially small for large radius (*i. e.* large ρ). We will discuss the implications of these two different possibilities for the size of λ in more detail later on in Sec. 12.4.

In the following, we apply this working hypothesis to the setup considered in Sec. 11.2, *i. e.* we consider

$$K_{X\bar{X}} = \left(\prod_{j=1}^{2}(T_j + \overline{T}_j)^{-q_j}\right)\left[1 + \ell\gamma + \ell\beta_3\left(\log\rho_3 - \frac{\pi}{6}\rho_3 + \lambda\sum_a|\Phi_a|^2\right)\right], \tag{12.7}$$

which has the same form as the Kähler metric for X considered previously, cf. Eq. (11.11), with $d(\rho_3) = Y(T_3, \overline{T}_3, \Phi_a, \bar{\Phi}_a)$ given by Eq. (12.6).

12.3 Stabilization of the Dilaton and the Kähler Moduli

We again have to stabilize all the moduli during inflation, preferably with masses $m \gtrsim \mathcal{H}_{\mathrm{inf}}$, since none of them is considered as the inflaton. The stabilization of $T_{1,2}$ and ρ_3 works essentially the same way as in the more phenomenological approach in Sec. 11.3, even though the dilaton complicates the situation significantly. We will now discuss the moduli dependence of the scalar potential and argue that all moduli can be stabilized. In particular, we show that one can stabilize ρ_3, as defined in Eq. (11.12), at a rather large value. Note that the moduli dependence we expect for the potential during inflation is again of a seperable form such that we can discuss the stabilization of ρ_3 and ℓ and the stabilization of $T_{1,2}$ independent of each other in Secs. 12.3.1 and 12.3.2, respectively.

12.3.1 Stabilization of the Dilaton and ρ_3

The typical scale of the gaugino condensate is around $\sim 10^{11}$ GeV and thus it is negligible during inflation [464] (it is however crucial for stabilizing the dilaton and the pattern of supersymmetry breaking after inflation). Since our setup has the properties $W \approx 0$ and $W_\alpha \approx 0$ for all fields Φ_α except X, we need to stabilize the dilaton solely by the contribution from the F-term of X. This is done by invoking also non-perturbative corrections to the Kähler potential in a moduli stabilization scheme called Kähler stabilization similar to what is done in [464, 487], see also cf. Sec. 9.4. Therefore, we expect that the dependence of the scalar potential on the dilaton ℓ and ρ_3 (as defined in Eq. (11.12)) can be

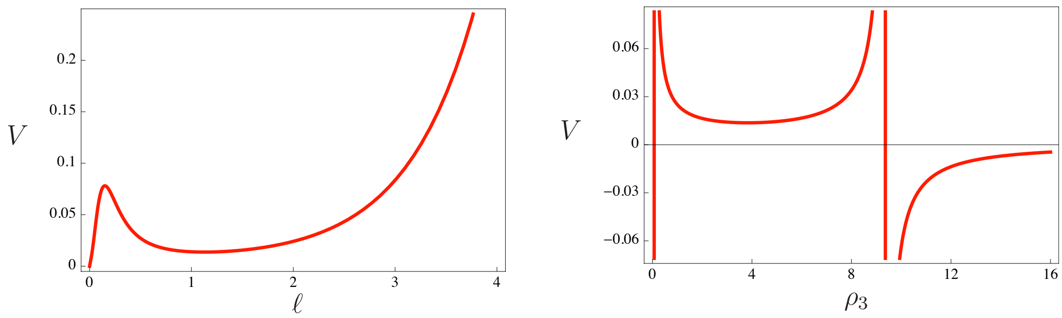

Figure 12.1: Dependence of the potential Eq. (12.8) on ℓ with ρ_3 at its minimum, and vice versa. For the example values $n = 1$, $q = -\frac{1}{2}$, $\gamma = \frac{10}{8\pi^2} \approx 0.13$, $\beta_3 = \frac{30}{8\pi^2} \approx 0.38$, $A = -0.7$, $B = 20$ and $a = 1$, there is a minimum at $\langle\ell\rangle \approx 1.13$ with $g^2 \approx 0.62$ and $\langle\rho_3\rangle \approx 3.83$. There is a pole at $\ell \approx 8.03$ for $\langle\rho_3\rangle \approx 3.83$, outside of the region shown in the figure left figure, and at $\rho_3 \approx 9.37$ for $\langle\ell\rangle \approx 1.13$ in the right figure. The overall scale of the potential has to be set by $\langle\Sigma\rangle$.

parametrized as follows

$$V \propto \frac{\rho_3^q \, \ell^n \, e^{g(\ell)}}{1 + \ell\gamma + \ell\beta_3(\log\rho_3 - \frac{\pi}{6}\rho_3 + \lambda\sum_a|\Phi_a|^2)} \,. \tag{12.8}$$

Recall that the F-term of X has a moduli dependence encoded in $|a(T_i)|^2|\langle\Sigma\rangle|^4$ and also $K_{X\bar{X}}$ depends on the moduli, cf. Eq. (11.15). If for example $\langle\Sigma\rangle$ is independent of ℓ, we have $n = 1$, while otherwise n can be either enhanced or reduced, but it is always an integer number (see also Sec. 9.3.2). Similarly, q is some model-dependent rational number typically ≥ -1. Whether ℓ and ρ_3 are stabilized or not depends on the interplay of various parameters. We have depicted the potential (in arbitrary units) as a function of ℓ and ρ_3 in Figs. 12.1 and 12.2 for an illustrative choice of parameters, which demonstrates that one can indeed stabilize ℓ and ρ_3.

Similar to [464, 487], if $n \leq 1$ the dilaton can be stabilized during inflation at $\langle\ell\rangle \sim \mathcal{O}(1)$ and with reasonable values for the gauge coupling g.[1] By analogy to the discussion in Sec. 11.3, we expect a minimum for ρ_3 if $q < 0$ and $\beta_3 > 0$, which indeed occurs. Interestingly, assuming γ to be negligible, the values of ℓ and ρ_3 at their minima appear to be parametrically related by $\langle\rho_3\rangle \sim (\beta_3\langle\ell\rangle)^{-1}$, up to a numerical factor which is roughly $\mathcal{O}(1)$. Hence, a minimum at rather large values of ρ_3 requires $\beta_3\langle\ell\rangle < 1$ and since β_3 is related to the beta function coefficient of an $\mathcal{N} = 2$ theory by $\beta_3 = b^{\mathcal{N}=2}/8\pi^2$, this can indeed be fulfilled if $\langle\ell\rangle \sim \mathcal{O}(1)$. This requirement is important, because we expect the Heisenberg symmetry to be preserved only in the large radius limit. Both ℓ and ρ_3 can be

[1]Note that as in [464, 487] at least one field contained in Σ, which collectively denotes a product of fields, has to receive an expectation value through an F-term such that the net dilaton dependence in the scalar potential satisfies $n \leq 1$.

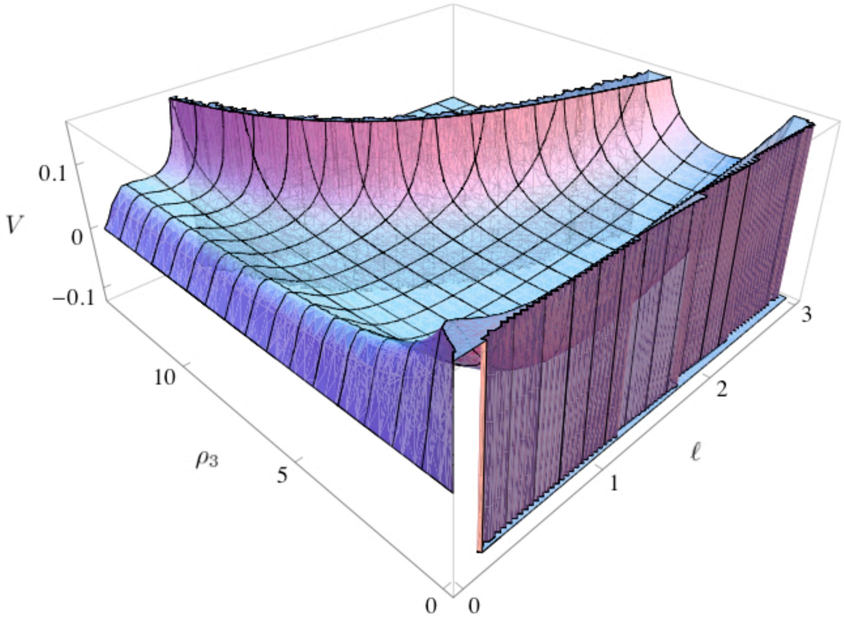

Figure 12.2: Dependence of the potential Eq. (12.8) on ℓ and ρ_3. For the example values $n = 1$, $q = -\frac{1}{2}$, $\gamma = \frac{10}{8\pi^2} \approx 0.13$, $\beta_3 = \frac{30}{8\pi^2} \approx 0.38$, $A = -0.7$, $B = 20$ and $a = 1$, there is a minimum at $\langle\ell\rangle \approx 1.13$ with $g^2 \approx 0.62$ and $\langle\rho_3\rangle \approx 3.83$. The overall scale of the potential has to be set by $\langle\Sigma\rangle$.

stabilized at masses $\sim \mathcal{H}_{\text{inf}}$. Note that analogous to the situation in Sec. 11.3, the potential has poles along a line in the ℓ and ρ_3 plane.

Concerning the pole in Fig. 12.1, the same discussion as under Fig. 11.1 in Sec. 11.3 applies.

12.3.2 Stabilization of $T_{1,2}$

So far, the dilaton ℓ and ρ_3 can be stabilized during inflation, but we still need to show that the remaining Kähler moduli T_1 and T_2 can also be fixed during inflation. They are defined as in Eq. (9.4) since we assume $\Phi_{a,1} = \Phi_{a,2} = 0$. The dependence of the scalar potential on T_1 and T_2 is then typically of the form [37]

$$V \propto \left[(T_i + \overline{T}_i)\,|\eta(T_i)|^4\right]^{-p_i} , \tag{12.9}$$

for some (in general rational) model-dependent numbers p_i. For this form of the potential, if $p_i > 0$, the T_i get stabilized at the self-dual value $T_i = e^{i\pi/6}$ with masses $\sim \mathcal{H}_{\text{inf}}$. Note that the η-function also provides a potential for $\operatorname{Im} T_i$ (via the terms $\sim e^{-2\pi T}$).

The stabilization of $\operatorname{Re} T_i$ can be also understood from the simple example of Sec. 11.3: for $\operatorname{Re} T_i \gtrsim 1$, we can approximate $\eta(T_i) \sim \exp(-\frac{\pi}{12}T_i)$. Thus, we expect that even if $T_i + \overline{T}_i$ and $|\eta(T_i)|^4$ do not enter with the same power

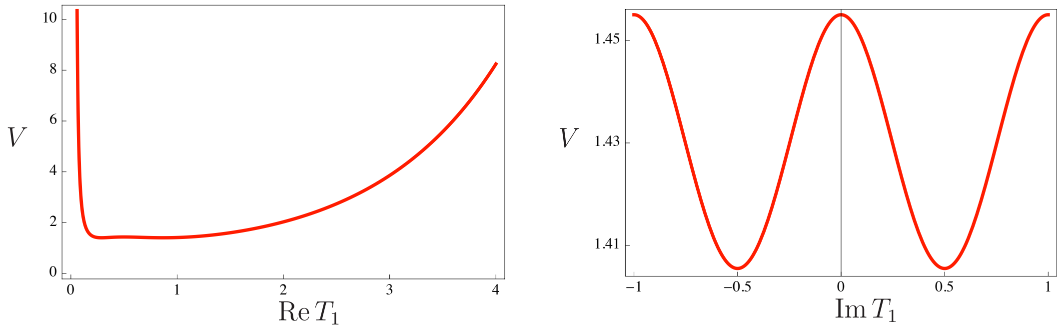

Figure 12.3: Form of the potential Eq. (12.9) for the example values T_i for $p_i = -1$, which yields a minimum at $T_i = e^{i\pi/6}$, *i. e.* $\operatorname{Re} T_i \approx 0.87$ and $\operatorname{Im} T_i = 0.5$. The overall scale of the potential has to be set by $\langle\Sigma\rangle$.

into the scalar potential, we get a minimum at $\operatorname{Re} T_i \sim \mathcal{O}(1)$ as long as both powers are negative. Note that if the two factors do not appear with the same power, T_i does not get stabilized exactly at the self-dual point. Fig. 12.3 shows a plot of the T_i dependence of the potential Eq. (12.9) for a sample choice of parameters (in arbitrary units).

In the light of the considerations above, we may now assume that all moduli are stabilized with masses $\sim \mathcal{H}_{\text{inf}}$ and regard them as effectively constant in the following discussion of the inflaton slope.

A Remark on α'-corrections

Due to the $\mathcal{O}(1)$ values for the Kähler moduli, one might worry about higher string-loop and α'-corrections which are controlled by them. The issue of α'-corrections is a difficult question in the context of orbifolds. They may or may not respect the Heisenberg symmetry to a sufficient amount (similar to the string-loop corrections to $K_{X\bar{X}}$ in Sec. 12.2), but in any case their precise form may affect the stabilization of the moduli.

At the orbifold point, *i. e.* at a point where the expectation values of all matter fields (in particular those of the twisted fields) vanish, one has a description in terms of an exact CFT. In the presence of an anomalous $U(1)_A$, however, some fields must acquire expectation values to cancel the FI-term, as is the case in all phenomenologically interesting orbifold models found so far. If twisted fields acquire expectation values, some (or all) of the orbifold singularities get resolved or "blown-up". Further investigations of this issue are required, *e. g.* along the lines of [428, 429, 491] by using gauged linear sigma models, but are beyond our present scope.

12.4 Contributions to the Inflaton Slope

So far, we have explained how to stabilize the moduli $T_{1,2}, \rho_3$ and ℓ. Now we discuss a few sources for a slope for the inflaton direction, which according to the classification given in Sec. 11.4.2 are either violations of the Heisenberg symmetry in the Kähler potential or the conditions $W \approx 0$, $W_\alpha \approx 0$ for all $\Phi_\alpha \neq X$.

We have already introduced a possible violation of the Heisenberg symmetry in the potential Eq. (12.8), namely the $\lambda|\Phi_a|^2$ piece, which we use to parametrize the amount of breaking of the Heisenberg symmetry by string-loop corrections. With the moduli at their minimum (and before considering further corrections), the inflaton potential has the form

$$V \simeq \frac{V_0}{1 + \langle\ell\rangle\gamma + \langle\ell\rangle\beta_3\left(\log\langle\rho_3\rangle - \frac{\pi}{6}\langle\rho_3\rangle + \lambda\sum_a|\phi_a|^2\right)}\,, \tag{12.10}$$

where V_0 depends on the expectation values of $\langle T_{1,2}\rangle$, $\langle\rho_3\rangle$ and $\langle\ell\rangle$ as well as $\langle\Sigma\rangle$ and we have replaced each Φ_a with its lowest component ϕ_a.

The kinetic terms of the ϕ_a are $\sim \rho_3^{-1}|\partial_m\phi_a|^2$. Due to the constraints from D-flatness, the inflaton ϕ is a certain combination of the ϕ_a and to trigger the waterfall phase transition, the inflaton has to roll towards $\phi = 0$. If the λ-term dominates the slope, this requires $\lambda < 0$ since $\beta_3 > 0$ is necessary to ensure stabilization of ρ_3 (cf. Sec. 12.3.1). Expanding around $\phi_a \approx 0$ and canonically normalizing, we can estimate the corresponding contribution to the slow-roll parameter[2] η as

$$|\eta| \sim |\lambda|\,, \tag{12.11}$$

where we used $\langle\rho_3\rangle \sim (\beta_3\langle\ell\rangle)^{-1}$ and $\beta_3\langle\ell\rangle \lesssim 1$. Since slow roll inflation occurs only if $|\eta| \ll 1$, we would then have to require that $|\lambda| \ll 1$. If the Heisenberg symmetry is broken only by non-perturbative effects such that $\lambda \sim e^{-\pi\rho_3}$ (see Sec. 12.2), this condition can be fulfilled with $\langle\rho_3\rangle$ somewhat larger than 1. The discussion of moduli stabilization in Sec. 12.3.1 implies that this indeed could be achieved. This would then result in a weak violation of the Heisenberg symmetry in the Kähler potential according to our classification in Sec. 11.4.2.

If λ is indeed exponentially small, however, the slope might actually turn out to be dominated by other sources. One example would be the slope induced by the one-loop corrections involving the waterfall fields. Another important source is related to the necessity to stabilize the moduli also after inflation. That is, there are additional terms present in the superpotential, denoted by W_{mod} in Eq. (11.10). Recently, a moduli stabilization scheme for heterotic orbifolds was proposed in [166], which includes a gaugino condensate and a superpotential term of the form $w_0 e^{-aT}$ for some constant w_0. The latter term is

[2]The slow-roll parameter η should not be confused with the Dedekind η-function $\eta(T)$ introduced above.

motivated from the breaking of an approxiamte R-symmetry at a high order in the superpotential [419], which leads to $\langle W \rangle \neq 0$ but parametrically small. Such terms would induce a small slope for the inflaton due to a parametrically small violation of $W \approx 0$ (the gaugino condensate may also induce non-zero F-terms for W_S, W_{T_i}).

Note that α'-corrections might give rise to a violation of the Heisenberg symmetry in the Kähler potential. Note also that if $a(T_i)$ (or more precisely $|a(T_i)|^2 |\langle \Sigma \rangle|^4$) would depend on T_3 not only through the combination ρ_3, *e. g.* if a term $a(T_i) \propto e^{-aT_3}$, this would generically lead to a large mass $\sim \mathcal{H}_{\text{inf}}$ of the inflaton which would spoil slow-roll inflation unless it is cancelled by some other contribution. We therefore assumed the absence of such a term here since studying potential cancellations between various terms in detail is beyond our present scope and left for the future. Other possible sources for a slope could be violations of $D_a = 0$ or $X \approx 0$ and the stabilization of the complex structure moduli.

To summarize, it appears in principle possible to find situations with the slope dominated by a single source if the Heisenberg symmetry is not too strongly broken and the conditions $W \approx 0$, $W_\alpha \approx 0$ $(\Phi_\alpha \neq X)$ are not too strongly violated. In general, one may also imagine situations where various sources contribute significantly to the slope for the inflaton and slow-roll inflation arises due to (accidental) cancellations between them. A detailed investigation of the phenomenologically interesting parameter space region of the models presented here is left for the future.

12.5 Summary and Discussion

In this chapter, we have presented the requirements for embedding the matter inflation models of Sec. 11.3 into heterotic orbifold compactifications. The untwisted matter fields in heterotic orbifolds enjoy a Heisenberg symmetry and thus they are a good starting point for a string theory embedding. Also there has been a lot of progress constructing MSSM-like models from heterotic orbifolds, see *e. g.* [177, 178, 181, 415–417]. Additionally, the condition that during inflation $W \approx 0$, $W_\alpha \approx 0$ for all $\Phi_\alpha \neq X$ is well-suited for heterotic orbifold models since the superpotential starts at cubic order in the matter fields.

Let us summarize the requirements necessary to successfully embed our model into a heterotic orbifold.

We require that a tribrid-like structure in the superpotential of the matter fields emerges which may involve other fields taking expectation values (cf. Sec 12.1). Also the existence of a D-flat and F-flat direction of untwisted matter fields in a torus with fixed complex structure modulus has to be verified. However, it seems plausible that these conditions can be fulfilled. It is then also particularly interesting if there is an overlap with models where spectra close to the MSSM can be achieved.

Constraints from target space modular invariance determine the T_i-dependence of the terms in the superpotential, which turns out to be of the form $\sim e^{\pm a_i T_i}$ for $\mathrm{Re}\, T_i \gtrsim 1$. The coefficients a_i are further constraint by allowing for successful moduli stabilization and avoiding large contributions to the inflaton slope. Actually, these constrains are most important for the function $a(T_i)$ in Eq. (11.10) (cf. Secs. 12.3.2 and 12.4).

Compared to previous studies of embedding inflation into heterotic orbifolds (see *e. g.* [37, 38, 464, 487, 492]), we proposed a new way to stabilize the Kähler modulus associated to the inflaton. This way is based on an ansatz for the string-loop corrections to the Kähler metric of the field X, which we assumed to live in a twisted sector, more precisely an $\mathcal{N} = 2$ twisted sector. In the presence of sectors with $\mathcal{N} = 2$ supersymmetry, there are known moduli-dependent threshold corrections to the matter Kähler metrics of untwisted matter fields [488]. As a working hypothesis, we assumed that these corrections have the same form for the twisted matter field Kähler metrics and parametrized a possible breaking of the Heisenberg symmetry (cf. Secs. 12.2 and 12.3.1). It allows us to stabilize the radius of the corresponding torus at rather large values and if the Heisenberg symmetry is only broken by non-perturbative effects $\sim e^{-a\rho_3}$ the induced correction to the inflaton slope is small. Moreover, in our scenario we provide an explicit way to end inflation via a phase transition.

We have arrived at the following conditions necessary for realizing matter inflation in heterotic orbifolds.

- There exists a (tree-level) D-flat and F-flat direction of untwisted matter fields in a torus with fixed complex structure modulus.
- The relevant part of the superpotential has a tribrid-like structure as defined in Eq. (11.10).
- There are suitable expectation values for the fields collectively denoted by $\langle\Sigma\rangle$, cf. Eq. (11.15).
- The Kähler potential of X has a moduli-dependence which leads to the stabilization of ρ_3 and $X \approx 0$.
- The dilaton ℓ can be stabilized *e. g.* by non-perturbative corrections to the Kähler potential.

It seems plausible that the above scenario can occur in heterotic orbifold compactifications, even though it remains to be checked whether our requirements and assumptions are fulfilled in an explicit and phenomenologically interesting model.

We discussed sources for an inflaton slope in Sec. 12.4. In particular, we argued that some of these sources can in principle lead to a parametrically small inflaton slope as is required for slow-roll inflation. For instance, if the string-loop corrections to $K_{X\bar{X}}$ preserve the Heisenberg symmetry up to terms which are exponentially small in the large radius limit. Unfortunately, the Kähler potential for twisted matter fields is typically only known at tree-level and to quadratic order in the fields. We have proposed a form which is interesting from the point of view of inflation and seems reasonable. However, it remains to be seen whether our hypothesis can be fulfilled in an explicit and phenomenologically interesting model.

Note that even if various contributions give large corrections, a sufficiently small inflaton slope may still arise due to (accidental) cancellations. We leave a detailed investigation of all kinds of corrections to the inflaton slope for the future since such questions should be addressed in a more concrete model.

In summary, we have proposed a framework for realising inflation in the matter sector of heterotic orbifold compactifications. Scenarios with a matter field as the inflaton are phenomenologically attractive since they relate models of inflation and particle physics. Our present work should be viewed as a first step towards this goal and we have discussed the conditions which have to be fulfilled in an explicit heterotic orbifold model.

Part IV

Brane Inflation on the Baryonic Branch

Chapter 13

The Baryonic Branch, a Master Equation and a Rotation

In the preceding part, we have considered inflation in effective 4d $\mathcal{N} = 1$ supergravity theories from a mostly bottom-up perspective. In this part, we turn things around and consider a top-down approach. That is, we consider inflationary potentials which are obtained from probe D3-branes moving in 10d supergravity background solutions. More precisely, we will consider models of inflation where the inflaton is associated with the (radial) position of a probe D3-brane in a warped throat geometry. We have provided a brief introduction into this idea in Chap. 8 and we have reviewed warped throat geometries, in particular, the Klebanov-Strassler (KS) solution in Chap. 7.

The work presented here is based on [3], where my collaborators and I consider a certain class of supergravity backgrounds which are deformations of the KS throat. In this chapter, we provide some general remarks about these supergravity backgrounds and describe how to systematically construct those we are interested in. The inflationary phenomenology is discussed in Chap. 14.

Before we discuss the brane inflation models obtained from certain deformations of the KS solution in the next chapter, we first review the material required to construct the supergravity backgrounds later on. All the backgrounds we consider there belong to the *baryonic branch* of Klebanov-Strassler. That is, the dual gauge theory is in a state where non-zero VEVs for baryonic operators are switched on. We review some facts about this branch of the moduli space of the dual gauge theory in Sec. 13.1.

There is a fairly general procedure to obtain solutions to the type IIB equations of motion, including those belonging to the baryonic branch of KS. The starting point is a *master equation* derived from a system of wrapped D5-branes (cf. Sec. 13.2). That is, solving the equations of motion for this system boils down to solving a single second order equation for one function. Afterwards, a solution generating technique referred to as *rotation* is employed to generate a whole family of supergravity solutions starting with a given solution to the master equation (cf. Sec. 13.3). Finally, we briefly introduce the the type of deformation of the baryonic branch of KS we will be interested in later on (cf. Sec. 13.4). Namely, we modify the IR part of the geometry completely.

13.1 The Baryonic Branch of Klebanov-Strassler

The Klebanov-Strassler solution, which we reviewed in Chap. 7, is dual to an $SU(N+M) \times SU(N)$ gauge theory where $N = MK$ with M and K specifying the F_3 and H_3 fluxes threading the pair of Poincaré-dual cycles A and B, respectively. This theory has a global $SU(2) \times SU(2) \times U(1)_R$ symmetry inherited from the isometries of $T^{1,1}$. In addition, there is also a $U(1)_B$ symmetry obtained from the RR 4-form C_4 by dimensional reduction using the harmonic form ω_3 associated to the S^3 in $T^{1,1}$ as $C_4 \sim A_1 \wedge \omega_3$ [401, 402]. This $U(1)_B$ is interpreted as baryon number symmetry. In the non-compact infinite throat limit the $U(1)_B$ is a global symmetry, but it is expected to become gauged upon gluing the throat into a compact space [493].

Flowing from the UV to the IR, this theory exhibits a cascade of Seiberg dualities reducing the rank of the gauge groups by M units in each step. The last step in the duality cascade, when the gauge group is $SU(2M) \times SU(M)$, is believed to be on the so-called *baryonic branch* [135, 405] with the $SU(2M)$ factor becoming strongly coupled. On this branch of the moduli space, the $U(1)_B$ global symmetry, which in the dual field theory acts on the superfields A_i, B_j as $A_i \to e^{i\alpha} A_i$, $B_j \to e^{-i\alpha} B_j$, is broken by VEVs for *baryonic operators* $\mathcal{B}, \bar{\mathcal{B}}$. We will now review some facts about the moduli space of the $SU(2M) \times SU(M)$ moduli space, in particular, about its baryonic branch, following the discussion in section 4.5 of [113].

As mentioned in Sec. 7.2, the dual gauge theory has a set of chiral matter fields, the $SU(2) \times SU(2)$ doublets A_i and B_j (each $SU(2)$ acts only on one of them) transforming in the $(\mathbf{2M}, \overline{\mathbf{M}})$ and $(\overline{\mathbf{2M}}, \mathbf{M})$ representations, respectively. They interact via a superpotential of the form (cf. Eq. (7.12))

$$W = h\, \epsilon_{ij} \epsilon_{pq} \operatorname{Tr}\left(A_i B_p A_j B_q\right) . \tag{13.1}$$

The baryonic operators $\mathcal{B}, \bar{\mathcal{B}}$ are formed out of the A_i, B_j as

$$\mathcal{B} = \epsilon_{\alpha_1 \dots \alpha_{2M}} \left(A_1\right)_1^{\alpha_1} \left(A_1\right)_2^{\alpha_2} \dots \left(A_1\right)_M^{\alpha_M} \left(A_2\right)_1^{\alpha_{M+1}} \left(A_2\right)_2^{\alpha_{M+2}} \dots \left(A_2\right)_M^{\alpha_{2M}} , \tag{13.2}$$

$$\bar{\mathcal{B}} = \epsilon^{\alpha_1 \dots \alpha_{2M}} \left(B_1\right)_{\alpha_1}^1 \left(B_1\right)_{\alpha_2}^2 \dots \left(B_1\right)_{\alpha_M}^M \left(B_2\right)_{\alpha_{M+1}}^1 \left(B_2\right)_{\alpha_2}^{M+2} \dots \left(B_2\right)_{\alpha_{2M}}^M . \tag{13.3}$$

These are invariant under the $SU(2) \times SU(2)$ global symmetry and under the $SU(M)$ flavor. Analogously, there are also *mesonic operators* $\mathcal{M}$ which we can form out of the A_i, B_j as

$$\left(\mathcal{M}_{ij}\right)_\alpha^\beta = A_{i\alpha} B_j^\beta , \tag{13.4}$$

which transform under $SU(2) \times SU(2)$ and $SU(M)$. The expectation values of $\mathcal{M}, \mathcal{B}, \bar{\mathcal{B}}$ parametrize the moduli space of the theory. At the classical level, they are subject to the constraint $\det \mathcal{M} - \mathcal{B}\bar{\mathcal{B}} = 0$, but at the quantum level this constraint receives (non-perturbative) corrections and reads [404, 494] (see also [495]).

$$\det \mathcal{M} - \mathcal{B}\bar{\mathcal{B}} - \Lambda^{4M} = 0\,, \tag{13.5}$$

with Λ denoting the UV scale of the $SU(2M)$ gauge group factor. Expressing the superpotential, Eq. (7.12), in terms of $\mathcal{M}, \mathcal{B}, \bar{\mathcal{B}}$ leads to

$$h\, \epsilon_{ip}\epsilon_{jq} \mathrm{Tr}\left(\mathcal{M}_{ij}\mathcal{M}_{pq}\right) + \lambda \left(\det \mathcal{M} - \mathcal{B}\bar{\mathcal{B}} - \Lambda^{4M}\right)\,, \tag{13.6}$$

with λ a Lagrange multiplier enforcing the constraint. From this superpotential one sees that the moduli space has two distinct branches.

- A *mesonic branch* characterized by $\mathcal{B} = \bar{\mathcal{B}} = 0$ and $\det \mathcal{M} = \Lambda^{4M}$ with complex dimension M.

- A *baryonic branch* characterized by $\mathcal{M} = \lambda = 0$ and $\mathcal{B}\bar{\mathcal{B}} = -\Lambda^{4M}$ with complex dimension one.

On the mesonic branch, the $SU(2) \times SU(2)$ symmetry is generically broken which implies that the Klebanov-Strassler solution corresponds to a point on the baryonic branch. A parametrization of the baryonic branch is given by

$$\mathcal{B} = i\zeta\Lambda^{2M}\,, \quad \bar{\mathcal{B}} = \frac{i}{\zeta}\Lambda^{2M}\,, \tag{13.7}$$

and the $U(1)_B$ corresponds to changing ζ by a phase. Along the baryonic branch, the $U(1)_B$ is spontaneously broken and the associated Goldstone boson has been identified in the dual supergravity solution as a massless pseudo-scalar state [493,496]. Due to $\mathcal{N} = 1$ supersymmetry, the Goldstone mode is in a chiral multiplet with a massless scalar which corresponds to changing the magnitude of ζ. This scalar is a modulus of the theory and corresponds to moving along the baryonic branch. Consequently, there should exist a one-parameter family of dual supergravity solutions and indeed a linearized deformation around KS was found in [493, 496] and the full family in [136]. It interpolates between the Klebanov-Strassler [135] and Maldacena-Nuñez solutions [497]. In the field theory, the dual operator is (see *e. g.* [495])

$$\mathcal{U} = \mathrm{Tr}\left(\sum_i A_i A_i^\dagger - \sum_j B_j^\dagger B_j\right)\,. \tag{13.8}$$

In the dual supergravity backgrounds, it controls the *resolution* of the conifold singularity [498], *i. e.* the size of the S^2 at the tip, and hence, these supergravity backgrounds have been termed *warped resolved deformed conifolds* in [493,496].

The KS solution corresponds to a special point on the baryonic branch where $\mathcal{U} = 0$ or $\mathcal{B} = \bar{\mathcal{B}} = i\Lambda^{2M}$.

In the next two sections, we review a technique which is useful to obtain (among many other solutions) the dual supergravity solution for the baryonic branch of Klebanov-Strassler.

13.2 The Master Equation

To describe solutions on the baryonic branch of Klebanov-Strassler and some further deformations (as well as a much broader class of solutions), it has proven to be useful to introduce a formalism based on a *master equation* combined with a solution generating technique. We will now review this procedure following [137] (cf. section 2).

13.2.1 Wrapped D5-branes and Master Equation

The idea is to start with a stack of N_c D5-branes and use those to construct a background dual to an $\mathcal{N} = 1$ SYM theory. The stack of D5-branes is wrapped on a 2-cycle in a very specific way such that precisely four supercharges are preserved. Details on this interesting procedure can be found *e. g.* in [385, 499–503], but for our purposes it is sufficient to know the form of the backgrounds arising from such wrapped D5-brane stacks for CY-cones with topology $\mathbb{R} \times S^2 \times S^3$. These solutions include a *warped* metric, RR 3-form flux F_3 and a *non-constant* dilaton Φ and can be parametrized by the following ansatz, where all functions are assumed to depend only on the radial coordinate ρ and the ranges of the angular coordinates are $0 \leq \theta, \tilde{\theta} < \pi$, $0 \leq \varphi, \tilde{\varphi} < 2\pi$ and $0 \leq \psi < 4\pi$.

$$
\begin{aligned}
ds_E^2 &= (\alpha' g_s N_c)\, e^{\Phi/2} \left((\alpha' g_s N_c)^{-1}\, ds_{1,3}^2 + ds_6^2 \right) , && (13.9)\\
ds_6^2 &= e^{2k} d\rho^2 + e^{2q} \left(d\theta^2 + \sin^2\theta\, d\varphi^2 \right) && (13.10)\\
&\quad + \frac{e^{2g}}{4} \left((\tilde{\omega}_1 + a\, d\theta)^2 + (\tilde{\omega}_2 - a\, \sin\theta\, d\varphi)^2 \right) + \frac{e^{2k}}{4} \left(\tilde{\omega}_3 + \cos\theta\, d\varphi \right)^2 , \\
F_3 &= \frac{\alpha' g_s N_c}{4} \Bigg[- (\tilde{\omega}_1 + b\, d\theta) \wedge (\tilde{\omega}_2 - b\, \sin\theta d\varphi) \wedge (\tilde{\omega}_3 + \cos\theta\, d\varphi) && (13.11)\\
&\quad + \partial_\rho b\, d\rho \wedge (-d\theta \wedge \tilde{\omega}_1 + \sin\theta\, d\varphi \wedge \tilde{\omega}_2) + \left(1 - b^2\right) \sin\theta\, d\theta \wedge d\varphi \wedge \tilde{\omega}_3 \Bigg] ,
\end{aligned}
$$

the $\tilde{\omega}_i$'s are defined as

$$
\begin{aligned}
\tilde{\omega}_1 &= \cos\psi\, d\tilde{\theta} + \sin\psi\, \sin\tilde{\theta}\, d\tilde{\varphi} , \\
\tilde{\omega}_2 &= - \sin\psi\, d\tilde{\theta} + \cos\psi\, \sin\tilde{\theta}\, d\tilde{\varphi} , \qquad (13.12)\\
\tilde{\omega}_3 &= d\psi + \cos\tilde{\theta}\, d\tilde{\varphi} .
\end{aligned}
$$

The set of functions $\{a, b, \Phi, k, g, q\}$ is then determined by solving the equations of motion.

The F_3 flux is quantized and satisfies

$$\frac{1}{2\kappa_{10}^2} \int\limits_{S^3} F_3 = N_c T_5 \,, \tag{13.13}$$

with the S^3 parametrized by $\tilde{\theta}, \tilde{\varphi}, \psi$.

Note that henceforth we will work in units where $\alpha' g_s = 1$ unless stated explicitly otherwise.

Requiring that the above ansatz preserves four supercharges leads to a set of BPS equations, which are a set of coupled non-linear first order equations for the functions $\{a, b, \Phi, k, g, q\}$ (see *e. g.* appendix B of [504]). Now one can introduce another set of functions $\{P, Q, \tau, \Phi, Y, \sigma\}$ to rewrite the BPS equations into a set of (partially) *decoupled* non-linear first order equations [505]. The two sets of functions are related as follows:

$$\begin{aligned}
4e^{2q} &= \frac{P^2 - Q^2}{P\cosh\tau - Q} \,, \\
e^{2g} &= P\cosh\tau - Q \,, \\
e^{2k} &= 4Y \,, \\
a &= \frac{P\sinh\tau}{P\cosh\tau - Q} \,, \\
b &= \frac{\sigma}{N_c} \,.
\end{aligned} \tag{13.14}$$

The BPS equations can now be rearranged in such a way that only a second order equation for P needs to be solved, while all other functions are determined in terms of P:

$$\begin{aligned}
Q &= (Q_0 + N_c)\cosh\tau + N_c\,(2\rho\cosh\tau - 1) \,, \\
\sinh\tau &= \frac{1}{\sinh(2\rho - 2\rho_0)} \,, \\
\cosh\tau &= \coth(2\rho - 2\rho_0) \,, \\
Y &= \frac{P'}{8} \,, \\
\sigma &= \tanh\tau(Q + N_c) = \frac{(2N_c\rho + Q_0 + N_c)}{\sinh(2\rho - 2\rho_0)} \,, \\
e^{4\Phi} &= e^{4\Phi_0}\frac{\cosh^2(2\rho_0)}{(P^2 - Q^2)Y\sinh^2\tau} \,,
\end{aligned} \tag{13.15}$$

where Q_0, ρ_0, Φ_0 are integration constants and the primes denote derivatives with respect to the radial coordinate ρ. The function P satisfies a second order non-linear differential equation,

$$P'' + P'\left(\frac{P'+Q'}{P-Q} + \frac{P'-Q'}{P+Q} - 4\coth(2\rho - 2\rho_0)\right) = 0\,, \tag{13.16}$$

which acts as a *master equation*. That is, by solving this equation we can generate a broad class of solutions belonging to the more general Papadopoulos-Tseytlin ansatz [506] (see also [507]). The entire remaining background functions are then obtained *algebraically* from Eqs. (13.14) and (13.15).

To avoid a bad singularity[1] in the IR, we will always set $Q_0 = -N_c$. Moreover, we will also set $\rho_0 = 0$ such that the dynamical scale is equal to 1.

13.2.2 Some Solutions of the Master Equation

There are many solutions to the master equation, Eq. (13.16), in particular, there are solutions which can be related to the baryonic branch of the Klebanov-Strassler solution (and deformations of it). In the next section, we introduce a solution generating technique which allows us to make this relation precise. But before let us briefly mention some solutions of the master equation (cf. the given references given for more details).

An Exact Solution A simple and *exact* solution to the master equation is

$$P(\rho) = 2N_c\rho\,, \tag{13.17}$$

which after inserting into Eqs. (13.14) and (13.15) yields the background already found in [497, 508].

IR and UV Expansions One can consider functions with an IR expansion ($\rho \to 0$) of the form [504]

$$\begin{aligned} P(\rho) = {} & h_1\rho + \frac{4h_1}{15}\left(1 - \frac{4N_c^2}{h_1^2}\right)\rho^3 \\ & + \frac{16h_1}{525}\left(1 - \frac{4N_c^2}{3h_1^2} - \frac{32N_c^4}{3h_1^4}\right)\rho^5 + \mathcal{O}(\rho^7)\,, \end{aligned} \tag{13.18}$$

where $h_1 > 2N_c$ (choosing $h_1 = 2N_c$ we obtain Eq. (13.17)). By numerically solving the master equation, one can show that such solutions are *smoothly* connected to solutions which have a totally different behaviour in the far UV ($\rho \to \infty$), namely

$$\begin{aligned} P(\rho) \sim {} & c\, e^{4\rho/3} + \frac{e^{-4\rho/3}}{64c}\left(256\rho^2 + 256Q_0\rho + 144N_c^2 + 64Q_0^2\right) \\ & - \frac{8}{3}c\, e^{-8\rho/3} + \mathcal{O}\left(e^{-8\rho/3}\right)\,. \end{aligned} \tag{13.19}$$

[1] This singularity is "bad" in terms of the criteria given in [357].

Recursive Solutions Instead of an IR or UV expansion of P, one can *recursively* construct solutions to the master equation as described in [509]. This relies on first rewriting the master equation, Eq. (13.16), as

$$\partial_\rho \left(s(P^2 - Q^2)P' \right) + 4sP'QQ' = 0\,, \quad s(\rho) = \sinh^2 \tau = \frac{1}{\sinh^2(2\rho)}\,, \tag{13.20}$$

and then integrating it two times to obtain

$$P^3 - 3Q^2 P + 6 \int_{\rho_2}^{\rho} d\rho' QQ'P + 12 \int_{\rho_2}^{\rho} d\rho' s^{-1} \int_{\rho_1}^{\rho} d\rho'' sP'QQ' = c^3 R(\rho)^3\,, \tag{13.21}$$

with

$$R(\rho) = \left(\cos^3 \alpha + \sin^3 \alpha \left(\sinh(4\rho) - 4\rho \right) \right)^{1/3}\,, \tag{13.22}$$

and c, α denoting the two integration constants of the master equation. The function P is then expressed as a formal expansion in $1/c$ [505],

$$P(\rho) = \sum_{n=0}^{\infty} c^{1-n} P_{1-n}\,, \tag{13.23}$$

and the P_i are then determined by inserting this expansion into Eq. (13.21), from which one finds *e. g.* $P_1 = R$ and $P_0 = 0$.

Backgrounds of the above form have interesting applications. For instance, solutions of the above form can describe the gravity duals of *"walking"* gauge theories [137, 509–512] or the cascades of quivers on the baryonic branch [513, 514]. Later, in Chap. 14, we will discuss applications of such backgrounds in the context of warped brane inflation. But first let us finish the discussion of how we actually generate warped throat backgrounds such as those dual to the baryonic branch of KS.

13.3 The Rotation - Generating Solutions

where In [513], a solution generating technique was proposed which is based on using a U-duality to map a background of the form in Eqs. (13.9) to (13.11) (which as we have seen previously is equivalent to a solution of the master equation Eq. (13.16)) into a new background where additional fluxes are present (namely F_5 and H_3 flux). The proposed U-duality can be regarded as a particular rescaling of the Kähler form $\mathcal{J}$ and complex structure form Ω describing the background geometry (see [514–517] for details). This technique has been termed *rotation* in [137] and we will follow the notation used in [137, 514].

Let us define a basis of 1-forms as

$$
\begin{aligned}
e^{x_m} &= h^{-\frac{1}{4}} e^{\frac{\Phi}{4}} dx_m \,, \quad e^{\rho} = h^{\frac{1}{4}} e^{\frac{\Phi}{4}+k} d\rho \,, \\
e^{\theta} &= h^{\frac{1}{4}} e^{\frac{\Phi}{4}+q} d\theta \,, \quad e^{\varphi} = h^{\frac{1}{4}} e^{\frac{\Phi}{4}+q} \sin\theta \, d\varphi \,, \\
e^{1} &= h^{\frac{1}{4}} \frac{e^{\frac{\Phi}{4}+g}}{2} (\tilde{\omega}_1 + a d\theta) \,, \quad e^{2} = h^{\frac{1}{4}} \frac{e^{\frac{\Phi}{4}+g}}{2} (\tilde{\omega}_2 - a \sin\theta \, d\varphi) \,, \\
e^{3} &= h^{\frac{1}{4}} \frac{e^{\frac{\Phi}{4}+k}}{2} (\tilde{\omega}_3 + \cos\theta d\varphi) \,,
\end{aligned}
\tag{13.24}
$$

where the $\tilde{\omega}_i$ are given by Eq. (13.12) and the function h is defined as

$$
h \equiv \kappa_1^{-2} \hat{h} \,, \quad \hat{h} \equiv 1 - \kappa_2^2 \, e^{2\Phi} \,. \tag{13.25}
$$

The rotated solution has a metric (in Einstein frame) and flux background which in the above basis is parametrized as follows.

$$
\begin{aligned}
ds_E^2 &= ds_4^2 + \kappa_1 ds_6^2 \,, \quad ds_4^2 = \sum_{m=0}^{3} (e^{x_m})^2 \,, \quad ds_6^2 = \sum_{i=\rho,\Psi} \left(e^{i}\right)^2 \,, \\
F_3 &= \frac{e^{-\frac{3\Phi}{4}}}{h^{\frac{3}{4}}} \Big[f_1 e^{123} + f_2 e^{\theta\varphi 3} - f_3 \left(e^{\varphi 13} + e^{\theta 23}\right) + f_4 \left(e^{\rho 1\theta} + e^{\rho\varphi 2}\right) \Big] \,, \\
B_2 &= \kappa_2 \frac{e^{\frac{3\Phi}{2}}}{h^{\frac{1}{2}}} \Big[e^{\rho 3} + \cos\mu \left(e^{\theta\varphi} + e^{12}\right) + \sin\mu \left(e^{\varphi 1} + e^{\theta 2}\right) \Big] \,, \\
H_3 &= -\kappa_2 \frac{e^{\frac{5\Phi}{4}}}{h^{\frac{3}{4}}} \Big[- f_1 e^{\theta\varphi\rho} - f_2 e^{12\rho} + f_3 \left(e^{\theta 2\rho} + e^{\varphi 1\rho}\right) - f_4 \left(e^{\theta 13} - e^{\varphi 23}\right) \Big] \,, \\
F_5 &= \kappa_2 \frac{d}{d\rho} \left(\frac{e^{2\Phi}}{h} \right) h^{\frac{3}{4}} e^{-k-\frac{5\Phi}{4}} \Big[- e^{x_0 x_1 x_2 x_3 \rho} + e^{\theta\varphi 123} \Big] \,,
\end{aligned}
\tag{13.26}
$$

where

$$
\cos\mu = -\frac{P - Q\coth(2\rho)}{P\coth(2\rho) - Q} \,, \tag{13.27}
$$

and the functions f_i specifying the fluxes are given by

$$
\begin{aligned}
f_1 &= -2N_c e^{-k-2g} \,, \quad f_2 = \frac{N_c}{2} \left(a^2 - 2ab + 1\right) e^{-k-2q} \,, \\
f_3 &= N_c (b-a) e^{-k-q-g} \,, \quad f_4 = \frac{N_c}{2} b' e^{-k-q-g} \,.
\end{aligned}
\tag{13.28}
$$

We have introduced $\Psi \equiv \{\psi, \theta, \varphi, \tilde{\theta}, \tilde{\varphi}\}$ to collectively denote all angular directions and the shorthand notation

$$
e^{ij\ldots l} \equiv e^{i} \wedge e^{j} \wedge \cdots \wedge e^{l} \,. \tag{13.29}
$$

The parameter κ_1 is an integration constant which is essentially the deformation parameter ϵ of the conifold we used to describe the KS solution in Chap. 7.

The parameter κ_2 characterizes the effect of the rotation and the backgrounds obtained from the wrapped D5-brane system correspond to $\kappa_2 = 0$. The rotation procedure requires that the dilaton e^{Φ} is bounded from above and monotonically increasing, *i. e.* the dilaton profile e^{Φ} grows towards $\rho \to \infty$ and its maximum value is $e^{\Phi(\infty)}$. Note that the dilaton Φ and the F_3 flux are *not* affected by the rotation, but we generate extra warping in the metric as well as switching on additional H_3 and F_5 flux (see *e. g.* appendix A of [137]).

Finally, a conceptually very important comment is in order. Namely that in particular the dilaton Φ and the function $P(\rho)$ are *not* affected by the rotation at all.

In principle, any value of κ_2 in the range $0 \leq \kappa_2 \leq e^{-\Phi(\infty)}$ corresponds to a valid background solution.[2] In this thesis, however, we will focus on backgrounds with $\kappa_2 \equiv e^{-\Phi(\infty)}$ since these are the ones which for $\rho \to \infty$ asymptote (at least at leading order) to the Klebanov-Strassler solution, or more precisely its baryonic branch. For reasons explained in [137], the tuning $\kappa_2 \equiv e^{-\Phi(\infty)}$ is equivalent to adiabatically switching off a dimension-8 operator. Effectively, this amounts to keeping only the subleading term in the UV expansion of h and dropping a possible constant part. Note that this is very similar to the near-horizon limit of the D3-branes considered in the AdS/CFT-correspondence (cf. Sec. 7.1), where one has a warp factor of the form

$$h(r) = 1 + \frac{R^4}{r^4}\,, \tag{13.30}$$

and drops the 1 in the near-horizon limit.

13.4 Baryonic Branch and Deformations

All the explicit case studies of solutions to the master equation we consider in the next chapter have a common leading behaviour in the UV ($\rho \to \infty$), namely

$$P \sim c_+ e^{4\rho/3} + \mathcal{O}(e^{-4\rho/3})\,. \tag{13.31}$$

This choice of UV behaviour together with the tuning of $\kappa_2 = e^{-\Phi(\infty)}$ yields supergravity backgrounds which are on the baryonic branch of KS. The expectation value of the dimension-2 operator $\mathcal{U}$ is controlled by the parameter c_+. More precisely, the expectation value is $\langle \mathcal{U} \rangle \sim N_c/c_+$.

We will not consider the pure baryonic branch solution but focus on an interesting type of deformation.

[2]The upper bound arises because the warp factor $\hat{h}$ should be positive. Note also that the bound is actually on the magnitude of κ_2, not its sign.

Modifying the IR Geometry

We strongly modify the IR geometry while keeping the UV geometry of KS to leading order, *i. e.* we consider a solution $P(\rho)$ which is essentially given by Eq. (13.31) in the UV but in the IR deviates from the one leading to the baryonic branch of KS. These backgrounds have been constructed recently in [137] and the modification can be understood as switching on an expectation value for a dimension-6 operator, in addition to the expectation value for $\mathcal{U}$.

The gauge theories dual to these supergravity backgrounds display a walking behaviour, *i. e.* there exists a region of energy scales over which the gauge coupling almost does not run. The function P characterizing these backgrounds has an expansion in the IR ($\rho \to 0$) of the form

$$P \sim c_0 + c_0 k_3 \rho^3 + \mathcal{O}(\rho^5)\,. \tag{13.32}$$

The original baryonic branch of the Klebanov-Strassler solution would instead be obtained from a solution which in the IR behaves as

$$P \sim h_1 \rho + \frac{4h_1}{15}\left(1 - \frac{4N_c^2}{h_1^2}\right)\rho^3 + \dots\,. \tag{13.33}$$

Chapter 14

Slow-Walking Inflation

The idea of this chapter is to realize inflation in string theory in terms of the (radial) motion of a probe D3-brane moving in the class of supergravity backgrounds we introduced previously, which are deformations of the baryonic branch of the Klebanov-Strassler throat.

Brane inflation on the baryonic branch of KS [136] was first discussed in [495], but the authors focused on the UV region. Here instead, we intend to focus on the IR regime where, as we will see, an inflection point arises quite generically due to the non-trivial dilaton profile along the radial direction.

We first derive the expressions for the potential along the radial direction and the relation between the radial coordinate ρ we use to describe the backgrounds and the canonically normalized field ϕ in Sec. 14.1.

To construct the supergravity backgrounds dual to the baryonic branch [136] and (IR) deformations of it, we use the method with the master equation and the rotation described in the previous chapter. We consider as an explicit case study in Sec. 14.2 the background obtained recently in [137].

In Sec. 14.2, we first use three different analytic approximations which are valid in the far UV (Sec. 14.2.2), the deep IR (Sec. 14.2.3) and for large values of c_+ (Sec. 14.2.4), respectively. We use these to extract a *universal scaling behaviour* in the limit of large c_+ for all the quantities of interest for inflation. Afterwards, we solve the master equation numerically in Sec. 14.2.5. In particular, we will show the existence of an inflection point at a geometrically defined location.

In Sec. 14.3, we discuss the qualitative picture which emerges from the combined analytical and numerical results. We also show that one can find potentially realistic values for the inflationary observables. Some comments about the dual field theory interpretation of the backgrounds can be found in Sec. 14.4. Finally, we summarize our findings in Sec. 14.5.

14.1 Probe D3-branes & Inflaton Potential

We are interested in the dynamics of a probe D3-brane which extends along all four non-compact directions and is moving along the radial direction in a warped throat geometry.

Suppose that we start with a metric which in string frame reads

$$ds^2_{\rm st} = H_1(\rho)dx^2_{1,3} + H_2(\rho)d\rho^2 + \dots\,, \tag{14.1}$$

where ρ is the radial coordinate introduced in the previous chapter and the dots indicate the angular directions which we will ignore throughout this chapter. The metric in Einstein frame is then given by a rescaling by $e^{-\Phi/2}$ with the dilaton Φ, *i. e.*

$$ds^2_E = H_1 e^{-\Phi/2}dx^2_{1,3} + H_2 e^{-\Phi/2}d\rho^2 + \dots\,. \tag{14.2}$$

Suppose also that there is a RR 5-form flux whose 4-form potential is

$$C_4 = \mathcal{C}(\rho)dt\wedge dx_1\wedge dx_2\wedge dx_3\,. \tag{14.3}$$

Now consider a probe D3-brane emedding of the form

$$\Sigma_4 = (t, x_1, x_2, x_3)\,,\quad \rho=\rho(t)\,. \tag{14.4}$$

Then the induced metric on such a brane reads

$$ds^2_{\rm ind} = e^{-\Phi/2}H_1\left(dx_1^2+dx_2^2+dx_3^2\right) - e^{-\Phi/2}H_1\left(1-\frac{H_2}{H_1}\dot\rho^2\right)e^{-\Phi/2}dt^2\,. \tag{14.5}$$

The action for such probe branes is given by

$$\begin{aligned}\mathcal{S}_{\rm D3} &= -T_3\int d^4x\left(\sqrt{-g_{\rm ind}}-\mathcal{C}\right)\\ &= -T_3\int d^4x\left(e^{-\Phi}H_1^2\sqrt{1-\frac{H_2}{H_1}\dot\rho^2}-\mathcal{C}\right)\,,\end{aligned} \tag{14.6}$$

with $T_3\equiv(2\pi)^{-3}(\alpha')^{-2}$. Expanding this to second order in $\dot\rho^2$, we find

$$\mathcal{S}_{\rm D3} = T_3\int d^4x\left(\frac{H_1H_2e^{-\Phi}}{2}\dot\rho^2-\left(1+\frac{e^{\Phi}\mathcal{C}}{H_1^2}\right)e^{-\Phi}H_1^2\right)\,, \tag{14.7}$$

and thus the potential for the probe D3-brane is given by

$$V_{\rm D3}\equiv T_3\,e^{-\Phi}H_1^2\left(1+\frac{e^{\Phi}\mathcal{C}}{H_1^2}\right)\,. \tag{14.8}$$

Introducing a "canonical" radial variable r in which the metric takes the form

$$ds^2 = f(r)^{-1/2}dx^2_{1,3} + f(r)^{1/2}dr^2 + \dots\,, \tag{14.9}$$

the relation between r and ρ turns out to be

$$dr = e^{-\Phi/2}\sqrt{H_1 H_2}\, d\rho\,, \tag{14.10}$$

and the canonically normalized inflaton is given by

$$\phi = \sqrt{T_3}\, r\,. \tag{14.11}$$

Note that we set$\alpha' g_s = 1$ unless stated explicitly otherwise.

For the backgrounds of the previous chapter, we have

$$H_1 = h^{-1/2} e^{\Phi}\,, \quad H_2 = \kappa_1 h^{1/2} e^{2k+\Phi}\,, \quad \mathcal{C} = -\kappa_2 \frac{e^{2\Phi}}{h}\,, \tag{14.12}$$

where $h = \kappa_1^{-2}(1 - \kappa_2^2 e^{2\Phi})$. Inserting these expressions into Eqs. (14.8) and (14.10) yields

$$dr = \sqrt{\kappa_1} e^{k+\Phi/2} d\rho\,, \tag{14.13}$$

and

$$V_{\text{D3}} = \frac{T_3\, \kappa_1^2}{e^{-\Phi} + \kappa_2}\,, \tag{14.14}$$

respectively. The slow-roll parameters are defined in terms of ϕ but we construct our backgroudns in terms of ρ. Using Eqs. (14.13) and (14.14) as well as (14.11), we can express the slow-roll parameters ϵ, η, cf. Eqs. (3.28) and (3.29),

$$\epsilon \equiv M_P^2 \frac{1}{2\, V^2} \left(\frac{dV}{d\phi}\right)^2\,, \quad \eta \equiv M_P^2 \frac{1}{V} \frac{d^2 V}{d\phi^2}\,, \tag{14.15}$$

in terms of derivatives with respect to ρ as follows

$$\epsilon = M_P^2 \left(T_3 \kappa_1 e^{2k+\Phi}\right)^{-1} \frac{1}{2V^2} \left(\frac{dV}{d\rho}\right)^2\,, \tag{14.16}$$

$$\eta = M_P^2 \left(T_3 \kappa_1 e^{2k+\Phi}\right)^{-1} \frac{1}{V} \left(\frac{d^2 V}{d\rho^2} - \left(\frac{dk}{d\rho} + \frac{1}{2}\frac{d\Phi}{d\rho}\right)\frac{dV}{d\rho}\right)\,. \tag{14.17}$$

The number of e-folds N_e and the amplitude of the curvature perturbations P_ζ are obtained as (cf. Eq. (3.38))

$$N_e \simeq -\frac{1}{M_P} \int_{\phi_i}^{\phi_e} \frac{d\phi}{\sqrt{2\epsilon}} = -\frac{1}{\sqrt{2} M_P} \int_{\rho_i}^{\rho_e} d\rho \frac{\sqrt{\kappa_1 T_3} e^{k+\Phi/2}}{\epsilon(\rho)}\,, \tag{14.18}$$

and

$$P_\zeta = \frac{V}{24\pi^2\, M_P^4\, \epsilon}\,, \tag{14.19}$$

respectively

The integration constant κ_1 is fixed below by demanding that the warp factor $f(r(\rho)) = 1$ at some arbitrarily chosen UV cutoff $\rho_{\text{UV}} \gg 1$ where we assume our warped throat to be glued into a compact space.[1]

[1]The constant κ_1 is essentially related to the deformation parameter ε of the conifold introduced in Sec. 7.2.

Why choosing $\kappa_2 = e^{-\Phi(\infty)}$? At this point, a technical comment is in order. Namely, why did we choose $\kappa_2 = e^{-\Phi(\infty)}$ if we intend to re-introduce gravity via an explicit UV cutoff? The reasoning behind this choice is as follows. If we make this choice for κ_2, the throat is strongly warped in its UV. Upon introducing a finite UV cutoff and gluing the throat into a compact space, the graviton zero mode will have most of its support in the essentially unwarped compact space since it is exponentially suppressed in strongly warped regions (see *e. g.* the discussion around Eq. (C.10) of [272]). That is, we can approximate the (four-dimensional Planck-mass) M_P as

$$M_P^2 \approx \frac{2\,\mathcal{V}_6}{(2\pi)^7\;\alpha'^4\,g_s^2}\,, \tag{14.20}$$

where $\mathcal{V}_6$ is the volume of the compact space. Using this formula, we can express the brane tension T_3 as[2]

$$\frac{T_3}{M_P^4} \approx \frac{(2\pi)^{11}\,g_s^4\alpha'^6}{4\mathcal{V}_6^2}\,. \tag{14.21}$$

The numbers on the right-hand side depend somewhat on the details of moduli stabilization, but reasonable values are for example $g_s \sim 0.1$ and $\mathcal{V}_6^{1/6} \sim 5\sqrt{\alpha'}$ which yield $T_3/M_P^4 \sim 10^{-4}$ [272].

The important point to takeaway from this section is that there is a non-zero potential for the radial position of a probe D3-brane since the dilaton $\Phi(\rho)$ has a non-trivial profile along the radial direction. We now move on to discuss the potentials obtained for some explicit background solutions of the form discussed in Chap. 13.

14.2 Case Study – Multi-Scale Solutions

As a case study, we consider the supergravity background obtained in [137]. In these backgrounds, the function $Q(\rho)$ is given by

$$Q(\rho) = N_c(2\rho\coth(2\rho) - 1)\,. \tag{14.22}$$

The class of backgrounds we consider now is obtained from a "seed solution". Namely, by observing that if $P \gg Q$, the master equation is approximately solved by

$$P_0(\rho) = c\left(\cos^3\alpha + \sin^3\alpha\,(\sinh(4\rho) - 4\rho)\right)^{1/3}\,, \tag{14.23}$$

with two integration constansts (c, α). The full solution for P is then constructed by expanding in powers of N_c/c as

$$P(\rho) = \sum_{n=0}^{\infty}\left(\frac{N_c}{c}\right)^{2n} P_n(\rho)\,, \tag{14.24}$$

[2]Note that unlike [272] we do not include a factor of g_s^{-1} in the brane tension.

and then iteratively solving for each P_n as a function of (c, α). This procedure yields a smooth solution for P provided that $P > Q$ for all $\rho > 0$. This yields a constraint

$$\cot\alpha \sim \exp\left(\frac{2^{4/3}\, c}{3N_c}\right) . \tag{14.25}$$

Now if α is small, the solution for P is approximately constant for $\rho < \rho_*$ while for $\rho > \rho_*$ one has $P \propto e^{4\rho/3}$. Approximately, one finds that α and ρ_* are related by

$$4\rho_* \approx \log(2\cot^3\alpha) . \tag{14.26}$$

We can derive analytic expressions in the far UV and deep IR regions as asymptotic solutions to the master equation, Eq. (13.16). We find that the asymptotic expansions for large and small ρ are given by

$$\begin{aligned} P = {} & 3\, c_+\, e^{4\rho/3} + \frac{4N_c^2}{3c_+}\left(\rho^2 - \rho + \frac{13}{16}\right) e^{-4\rho/3} \\ & + \left(-8\, c_+\rho - \frac{c_-}{192\, c_+^2}\right) e^{-8\rho/3} + \mathcal{O}(e^{-4\rho}) , \end{aligned} \tag{14.27}$$

and

$$\begin{aligned} P = {} & c_0 + c_0\, k_3\, \rho^3 + \frac{4}{5} c_0\, k_3\, \rho^5 - c_0\, k_3^2\, \rho^6 \\ & + \left(\frac{32\, c_0\, k_3}{105} - \frac{16\, k_3\, N_c^2}{21\, c_0}\right) \rho^7 - \frac{8}{5}\, c_0\, k_3^2\, \rho^8 + \mathcal{O}(\rho^9) , \end{aligned} \tag{14.28}$$

respectively. They are characterized by the two sets of integration constants (c_+, c_-) and (c_0, k_3), which are of course related to the integration constants (c, α) of the seed solution. The relations between these pairs of integration constants is not known in analytical form, but it can be determined numerically. However, in [137] some approximate relations among them have been determined by looking at the IR and UV expansions of the seed solution, Eq. (14.23), which yields

$$\begin{aligned} & c_0 \simeq c\cos\alpha , \qquad k_3 \simeq \frac{2^6}{3^2}\frac{1}{(2\cot^3\alpha)}\left(1 + \frac{N_c^2 \log^2(2\cot^3\alpha)}{2c_0^2}\right) , \\ & c_+ \simeq \frac{c\sin\alpha}{2^{1/3}\, 3} , \qquad c_- \simeq -192 c_+^3 2\cot^3\alpha . \end{aligned} \tag{14.29}$$

Moreover, for $\rho_* \gg 1$, one has $\log(2\cot^3\alpha) \sim 4\rho_*$ and we can express c_0, k_3 and c_- in terms of c_+ and ρ_* as follows

$$\begin{aligned} & c_0 \sim 3\, c_+\, e^{4\rho_*/3} , \qquad k_3 \sim \frac{64}{9} e^{-4\rho_*} + \frac{512\, N_c^2\, e^{-20\rho_*/3} \rho_*^2}{81\, c_+^2} , \\ & c_- \sim -192\, c_+^3\, e^{4\rho_*} . \end{aligned} \tag{14.30}$$

These relations should be a good approximation if

$$\frac{N_c}{c_+} < \frac{3\, e^{4\rho_*/3}}{2^{2/3}\, \rho_*} , \tag{14.31}$$

and we will use them frequently below to derive some analytic results.

Recall from Chap. 13 that we only need to obtain the function $P(\rho)$ for given $Q(\rho)$ (in the absence of flavor branes). The rest is just a straightforward application of the formalism.

The information on the geometry relevant for the probe D3-brane moving along the radial direction ρ is encoded in the two functions $e^{4\Phi}$ and e^{2k}. They are determined by the functions $P(\rho)$ and $Q(\rho)$ as[3]

$$e^{4\Phi-4\Phi_0} = \frac{2\sinh^2(2\rho)}{(P^2-Q^2)P'} \quad \text{and} \quad e^{2k} = \frac{P'}{2}\,, \tag{14.32}$$

respectively.

14.2.1 Fixing Integration Constants

Before we can proceed, we need to fix a couple of integration constants related to the dilaton and the warp factor, namely Φ_0 and $\kappa_{1,2}$ (recall that we have already fixed $Q_0 \equiv -N_c$ and $\rho_0 \equiv 0$).

Fixing Φ_0 and κ_2

As mentioned already in Chap. 13, we will always demand $\kappa_2 \equiv e^{-\Phi(\infty)}$ and we fix Φ_0 by demanding that $\Phi(\infty) \equiv 0$. Thus, we have $\kappa_2 \equiv 1$ and Φ_0 is determined in terms of c_+ as

$$\Phi_0 = \frac{1}{4}\ln(72\,c_+^3) \;\Leftrightarrow\; e^{4\Phi_0} = 72\,c_+^3\,. \tag{14.33}$$

This can be seen by expanding Φ for $\rho \to \infty$ which yields

$$\Phi(\rho) \simeq 4\Phi_0 - \ln\left(72\,c_+^3\right) - \frac{N_c^2}{12\,c_+^2}(8\rho-1)\,e^{-8\rho/3} + \dots\,. \tag{14.34}$$

These two choices, $\kappa_2 = 1$ and $\Phi(\infty) = 0$, were also made in [495] and yield a solution which in the UV asymptotes to the KS solution.

Fixing κ_1

As anticipated above, the integration constant κ_1 is fixed by demanding that at some arbitrary UV cutoff $\rho_{\rm UV} \gg 1$ the warp factor of the 4d part is ~ 1.

[3]Note that there is a factor of 4 difference to the expression for $e^{4\Phi}$ provided previously, which just amounts to a redefinition of Φ_0.

Using the UV asymptotic expansion, we can determine κ_1 in terms of c_+ and $\rho_{\rm UV}$ as

$$\kappa_1 \simeq \frac{N_c\, e^{-4\rho_{\rm UV}/3}\,\sqrt{8\rho_{\rm UV}-1}}{2\sqrt{6}c_+} \sim \frac{N_c\, e^{-4\rho_{\rm UV}/3}}{c_+}\,, \tag{14.35}$$

where in the last step we have dropped the $\sqrt{\rho_{\rm UV}}$ piece and kept only the most important scaling with N_c/c_+ and $e^{-4\rho_{\rm UV}/3}$. Note that this normalization κ_1 of the warp factor enters into the potential and the relation between ρ and the canonical field only as an overall factor, but does not affect the functional form.

14.2.2 Far UV Asymptotics

As a first cross check, we consider the far UV asymptotics. In this limit, since our background approaches the baryonic branch of Klebanov-Strassler (the effect of the dimension-6 VEV parametrized by c_- enters only in the subleading terms, cf. Eq. (14.27)), we should reproduce the results of [495].

In the far UV, *i.e.* for $\rho \to \infty$, the functions $P(\rho)$ and $Q(\rho)$ have the asymptotic expansions (cf. Eqs. (14.27) and (14.22))

$$\begin{aligned} P &\simeq 3\,c_+\,e^{4\rho/3} + \frac{4N_c^2}{3c_+}\left(\rho^2-\rho+\frac{13}{16}\right)e^{-4\rho/3} \\ &\quad + \left(-8\,c_+\rho - \frac{c_-}{192\,c_+^2}\right)e^{-8\rho/3} + \dots\,, \\ Q(\rho) &\simeq N_c(2\rho-1) + 4\,N_c\,e^{-4\rho} + \dots\,. \end{aligned} \tag{14.36}$$

Using these expressions, one can find the UV expansions of the relevant functions are given by (recall that $h=\kappa_1^{-2}\hat{h}$)

$$e^{4\Phi} \simeq 1 - \frac{N_c^2\,(8\rho-1)}{12\,c_+^2}e^{-8\rho/3} + \dots\,, \tag{14.37}$$

$$e^{2k} \simeq 2\,c_+ e^{4\rho/3} - \frac{N_c^2\,(4\rho-5)^2}{18\,c_+}e^{-4\rho/3} + \dots\,, \tag{14.38}$$

$$\hat{h} \simeq \frac{N_c^2\,(8\rho-1)}{24\,c_+^2}e^{-8\rho/3} + \dots\,. \tag{14.39}$$

Canonical Radial Coordinate and Warp Factor

The relation between the coordinate ρ and the canonical radial coordinate r is given by

$$r \sim \frac{3\sqrt{c_+}}{\sqrt{2}}\,e^{2\rho/3} \quad \text{and} \quad \rho \sim \frac{3}{2}\ln\left(\frac{\sqrt{2}}{3\sqrt{c_+}}\,r\right)\,, \tag{14.40}$$

respectively. Including the fixed normalization, the warp factor is obtained to leading order as

$$h(\rho) \simeq \frac{8\rho - 1}{8\rho_{\rm UV} - 1} e^{-8(\rho - \rho_{\rm UV})/3}\,, \tag{14.41}$$

or schematically in terms of the radial variable r

$$h(r) \simeq \frac{\alpha + \beta \ln(r)}{r^4}\,. \tag{14.42}$$

Induced Potential & Slow-Roll Parameters

Putting everything together, we arrive at a potential for the D3-brane in the UV which is of the form

$$V_{\rm UV}(\rho) \sim \frac{T_3 N_c^2\, e^{-8\rho_{\rm UV}/3}}{c_+^2} - \frac{T_3 N_c^4\, e^{-8\rho_{\rm UV}/3}}{c_+^4} (8\rho - 1) e^{-8\rho}\,. \tag{14.43}$$

Schematically, the slow-roll parameters are of the form

$$\epsilon_{\rm UV} \sim \frac{N_c^3\, e^{4\rho_{\rm UV}/3}}{c_+^4} e^{-20\rho/3} (2\rho - 1)^2\,, \tag{14.44}$$

and

$$\eta_{\rm UV} \sim -\frac{N_c\, e^{4\rho_{\rm UV}/3}}{c_+^2} e^{-4\rho} (10\rho - 8)\,. \tag{14.45}$$

Now let us compare this to what has been found in [495]. The authors find that the potential experienced by a probe D3-brane due to the non-trivial dilaton profile in the UV is given by (cf. their Eq. (15.9)):

$$V_{\rm UV}(t) \simeq \frac{T_3 U^2}{2\gamma} - \frac{3 T_3 U^4}{256\gamma} (4t - 1) e^{-4t/3}\,, \tag{14.46}$$

where $\gamma^{-1} \sim e^{-4t_{UV}/3}$. Using that the canonically normalized radial coordinate in their notation is $r \sim \varepsilon^{2/3} e^{t/3}$ (see Eq. (15.6)), they calculate η as (cf. Eqs. (15.10) and (15.12)):

$$\eta_{\rm UV}(t) \sim -U^2 \varepsilon^{-4/3} (5t - 8) e^{-2t} \sim U^2 e^{2t_{UV}/3} e^{-2t}\,, \tag{14.47}$$

and note that $\epsilon \propto U^4$ but do not give an explicit expression. To translate this into our notation, we make use of the following dictionary between the radial coordinates t and ρ and the parameters U and N_c/c_+ (see footnote 7 of [514]).[4]

[4] In [514], the notation is slightly different from ours. To convert it to our notation we have to make the replacements $c \to 3c_+$ and $\widetilde{N}_c \to N_c$, as can be seen from comparing the UV expansion of $P(\rho)$ in Eq. (3.17) of [514] with our UV expansion in Eq. (14.27).

We have the relations[5]

$$t = 2\rho\,, \quad U = \frac{2N_c}{c_+}\,, \tag{14.48}$$

and thus comparing their results, Eqs. (14.46) and (14.47), to our results in Eqs. (14.43) and (14.45), we see that we find the same parametric and functional dependence.

Including numerical factors, our expression for $V_{\rm UV}$ at leading order is

$$V_{\rm UV} \simeq \kappa_1^2 T_3 \left(\frac{1}{2} - \frac{N_c^2}{192\, c_+^2} e^{-8\rho/3}(8\rho - 1) + \dots \right)\,, \tag{14.49}$$

and the slow-roll parameters $\eta_{\rm UV}$ and $\epsilon_{\rm UV}$ are given by

$$\eta_{\rm UV} \simeq -\frac{M_P^2}{\kappa_1 T_3 \alpha' g_s} \frac{N_c^2}{27\, c_+^3}\, e^{-4\rho}(10\rho - 8)\,, \tag{14.50}$$

and

$$\epsilon_{\rm UV} \simeq \frac{M_P^2}{\kappa_1 T_3 \alpha' g_s} \frac{N_c^4}{324\, c_+^5}\, e^{-20\rho/3}(2\rho - 1)^2\,, \tag{14.51}$$

respectively.

14.2.3 Deep IR Asymptotics

Now that we passed this cross-check, we can look at the deep IR region, which is quite different from that of the backgrounds considered in [495].

In the deep IR, *i. e.* for $\rho \to 0$, the asymptotic expansions of $P(\rho)$ and $Q(\rho)$ are given by (cf. Eqs. (14.28) and (14.22))

$$\begin{aligned} P &\simeq c_0 + c_0\, k_3\, \rho^3 + \frac{4}{5} c_0\, k_3\, \rho^5 - c_0\, k_3^2\, \rho^6 \\ &\quad + \left(\frac{32\, c_0\, k_3}{105} - \frac{16\, k_3\, N_c^2}{21\, c_0} \right) \rho^7 - \frac{8}{5}\, c_0\, k_3^2\, \rho^8 + \dots\,, \\ Q &\simeq \frac{4N_c}{3}\rho^2 - \frac{16N_c}{45}\rho^4 + \frac{128N_c}{945}\rho^6 - \frac{256N_c}{4725}\rho^8 + \dots\,, \end{aligned} \tag{14.52}$$

respectively. Therefore, the IR asymptotic expansions of the relevant functions

[5]The full mapping between the functions $H, x, v, \tilde{g}$ used in [495] and ours is the following:

$$H = he^{-2\Phi}\,, \qquad 3\frac{e^x}{v} = he^{2k+\Phi}\,,$$
$$e^{2x} = he^{2\Phi+2q+2g}\,, \qquad e^{2\tilde{g}} = 4e^{2q-2g}\,.$$

can be determined as (recall again that $h = \kappa_1^{-2}\hat{h}$)

$$e^{4\Phi-4\Phi_0} \simeq \frac{8}{3c_0^3 k_3} + \frac{256N_c^2}{27c_0^5 k_3}\rho^4 - \frac{2048N_c^2}{405c_0^5 k_3}\rho^6 - \frac{2048N_c^2}{189c_0^5}\rho^7 + \dots\,, \tag{14.53}$$

$$e^{2k} \simeq \frac{3}{2}c_0 k_3 \rho^2 + 2c_0 k_3 \rho^4 - 3c_0 k_3^2 \rho^5 + \left(\frac{16c_0 k_3}{15} - \frac{8k_3 N_c^2}{3c_0}\right)\rho^6 + \dots\,, \tag{14.54}$$

$$\hat{h} \simeq 1 - \frac{8\sqrt{3}\,c_+^{3/2}}{c_0^{3/2}k_3} - \frac{128c_+^{3/2}N_c^2}{3\sqrt{3}\,c_0^{7/2}\sqrt{k_3}}\rho^4 + \frac{1024c_+^{3/2}N_c^2}{45\sqrt{3}\,c_0^{7/2}\sqrt{k_3}}\rho^6 + \dots\,. \tag{14.55}$$

Canonical Radial Variable and Warp Factor

The relation between the canonical radial variable r and the coordinate ρ is for small r given by

$$r \simeq \frac{3^{5/8}\,c_0^{1/8}(c_+k_3)^{3/8}}{2^{3/4}}\rho^2 + \dots \quad \text{and} \quad \rho \simeq \frac{2^{3/8}}{3^{5/16}c_0^{1/16}(c_+k_3)^{3/16}}\sqrt{r} + \dots\,. \tag{14.56}$$

The warp factor in the deep IR is schematically of the form

$$h(\rho) \simeq h_0 + h_4\rho^4 + \dots\,, \tag{14.57}$$

or equivalently

$$h(r) \simeq \alpha + \beta r^2 + \dots\,. \tag{14.58}$$

Induced Potential & Slow-Roll Parameters

The potential in the deep IR region is then given by

$$\begin{aligned} V_{\mathrm{IR}} \simeq \kappa_1^2 T_3 \Bigg(& \frac{12c_+^{3/4}}{\left(12c_+^{3/4} + \sqrt{2}\,3^{3/4}\,(c_0^3 k_3)^{1/4}\right)} \\ & + \frac{64\sqrt{3}\,c_+^{3/4}\,\left(c_0^{3/4}\sqrt{k_3} + 2\sqrt{2}3^{1/4}\,\left(c_+^3 k_3\right)^{1/4}\right)}{c_0^{5/4}\,\left(12c_+^{3/4} + \sqrt{2}\,3^{3/4}\,(c_0^3 k_3)^{1/4}\right)^3}\left(\rho^4 - \frac{8}{15}\rho^6 + \dots\right)\Bigg)\,. \end{aligned} \tag{14.59}$$

and the slow-roll parameters ϵ_{IR} and η_{IR} turn out to be

$$\epsilon_{\mathrm{IR}} \simeq \frac{M_P^2}{\kappa_1 T_3 \alpha' g_s}\,\beta(c_+, c_0, k_3)\,\left(\rho^4 - \frac{44}{15}\rho^6\right)\,, \tag{14.60}$$

and

$$\eta_{\mathrm{IR}} \simeq \frac{M_P^2}{\kappa_1 T_3 \alpha' g_s}\,\gamma(c_+, c_0, k_3)\,\left(1 - \frac{18}{5}\rho^2 - \frac{3k_3}{2}\rho^3 + \dots\right)\,, \tag{14.61}$$

respectively. Here, β and γ are some functions of the parameters c_+, c_0, k_3 which can be explicitly computed from the above but their precise form is not too enlightening. Using the relations between the various expansions coefficients in Eq. (14.30), we can perform the expansion in powers of $1/c_+$ to find

$$V_{\text{IR}} \simeq \kappa_1^2 T_3 \Bigg(\left(\frac{1}{2} - \frac{e^{-8\rho_*/3} N_c^2 \rho_*^2}{18\, c_+^2} + \dots \right) \\ + \left(\frac{2e^{-8\rho_*/3} N_c^2}{81\, c_+^2} + \dots \right) \left(\rho^4 - \frac{8}{15}\rho^6 + \dots \right) \Bigg) . \tag{14.62}$$

as well as

$$\epsilon_{\text{IR}} \simeq \frac{M_P^2}{\kappa_1 T_3 \alpha' g_s} \frac{4e^{-8\rho_*/3} N_c^4}{6561\, c_+^5} \left(\rho^4 - \frac{44}{15}\rho^6 + \dots \right) , \tag{14.63}$$

and

$$\eta_{\text{IR}} \simeq \frac{M_P^2}{\kappa_1 T_3 \alpha' g_s} \frac{N_c^2}{81\, c_+^3} \left(1 - \frac{18}{5}\rho^2 - \frac{32e^{-4\rho_*}}{3}\rho^3 + \frac{1128}{175}\rho^4 + \dots \right) . \tag{14.64}$$

The important observation is now to notice that for $\rho \to \infty$ we have $\eta_{\text{UV}} < 0$ (cf. Eq. (14.50)) while for $\rho \to 0$ we have $\eta_{\text{IR}} > 0$ (cf. Eq. (14.64)). That is, η changes sign and thus there is at least one zero somewhere in between. since $e^{-\Phi}$ is a monotonic function of ρ there should be exactly one zero. Hence, this setup automatically provides us with an *inflection point* (and as we will see later at this point ϵ has its maximum value).

Another important thing to notice which we will heavily exploit later on is that there is an *approximate scaling behaviour* in the limit of large c_+. For instance, comparing the expressions for η in the UV and IR, cf. Eqs. (14.50) and (14.64), one see that for large values of c_+

$$\eta(\rho) \simeq \frac{M_P^2\, N_c^2}{\kappa_1\, T_3\, c_+^3} \tilde{\eta}(\rho) + \dots \sim \frac{N_c\, e^{4\rho_{\text{UV}}/3}}{c_+^2} \tilde{\eta}(\rho) \, , \tag{14.65}$$

where the dots denote higher order terms and $\tilde{\eta}(\rho)$ is a function which does *not* depend on c_+ and in the last step we have used the expression for κ_1 in terms of ρ_{UV}, Eq. (14.35) and ignored the prefactor M_P^2/T_3. The relation for ϵ is of a similar form,

$$\epsilon(\rho) \simeq \frac{M_P^2\, N_c^4}{\kappa_1\, T_3\, c_+^5} \tilde{\epsilon}(\rho, \rho_*) + \dots \sim \frac{N_c^3\, e^{-4\rho_{\text{UV}}/3}}{c_+^4} \tilde{\epsilon}(\rho, \rho_*) \, . \tag{14.66}$$

Similar scaling relations will hold for all the other quantities of interest for inflation, in particular, for the amplitude of scalar perturbations P_ζ and for the number of e-folds N_e. We will summarize them later in Sec. 14.3. But first let us try to understand the origin of the scaling with c_+ analytically.

14.2.4 Analytic Derivation of c_+ Scaling Behaviour

It will prove useful to use the recursive technique of finding analytic solutions to the master equation in terms of an infinite series of non-explicit integrals. To this extent, we rewrite the master equation as

$$\partial_r \left(\frac{(P^2 - Q^2)}{\sinh^2(2\rho)} P' \right) + \frac{4}{\sinh^2(2\rho)} P' Q Q' = 0 \,. \tag{14.67}$$

Integrating this expression twice and taking into account the asymptotics described above, we find

$$\begin{aligned} P^3 - 3PQ^2 + 6 \int_0^\rho d\tilde{\rho}\, PQQ' - 12 \int_0^\rho d\tilde{\rho}\, \sinh^2(2\tilde{\rho}) \int_{\tilde{\rho}}^\infty d\hat{\rho}\, \frac{P'QQ'}{\sinh^2(2\hat{\rho})} \\ = 16\, c_+^3 R(\rho) \left(\int_0^\rho d\tilde{\rho} \sinh^2(2\tilde{\rho}) \right) \,. \end{aligned} \tag{14.68}$$

Now make an ansatz for $P(\rho)$ as an inverse series expansion in c_+,

$$P = c_+ P_1 + P_0 + \frac{P_{-1}}{c_+} + \frac{P_{-2}}{c_+^2} + \dots \,, \tag{14.69}$$

where all the P_n are independent of c_+. Inserting this ansatz into the integrated equation and matching powers of c_+ yields

$$\begin{aligned} P_1 &= \left(16 \int_0^\rho d\tilde{\rho} \sinh^2(2\tilde{\rho}) \right)^{1/3} \,, \\ P_0 &= P_{-2} = \cdots = P_{-2k} = 0 \,, \\ P_{-1} &= -\frac{1}{P_1^2} \Bigg(- P_1 Q^2 + 2 \int_0^\rho d\tilde{\rho}\, P_1 Q Q' \\ &\qquad - 4 \int_0^\rho d\tilde{\rho}\, \sinh^2(2\tilde{\rho}) \int_{\tilde{\rho}}^\infty d\hat{\rho}\, \frac{P_1' Q Q'}{\sinh^2(2\hat{\rho})} \Bigg) \,. \end{aligned} \tag{14.70}$$

A recurrence relation for $P_{-(2k+1)}$ can be found in Eq. (B.7) of [505]. This series converges quite rapidly to the numerical solution and is a very good approximation especially for large values of the radial coordinate.

We are interested in the expressions for the inflationary potential and the slow-roll parameters at leading order in an expansion in (inverse) powers of c_+. Using the above series expansion, we can find an expression for the dilaton Φ at leading order as

$$e^{4\Phi} = \frac{144 \sinh^2(2\rho)}{P_1' P_1^2} \left[1 - \frac{1}{c_+^2} \left(\frac{P_{-1}}{P_1} - \frac{Q^2}{P_1^2} + \frac{P_{-1}'}{P_1'} \right) + \dots \right] \,, \tag{14.71}$$

where the dots indicate the higher order corrections. Now let us define

$$N_c^2 \mathcal{M}(\rho) = \frac{P_{-1}}{P_1} - \frac{Q^2}{P_1^2} + \frac{P'_{-1}}{P'_1}\,, \tag{14.72}$$

and introduce the equality

$$\left(\frac{144\,\sinh^2(2\rho)}{P'_1 P_1^2}\right)^{-1/4} = 3^{-3/4}\,. \tag{14.73}$$

Using these two expressions, we can find the D3-brane potential at leading orders to be

$$V = \frac{N_c^2\, T_3\, e^{-8\rho_{\rm UV}/3}}{c_+^2\,(1+3^{-3/4})}\left(1 + \frac{N_c^2 3^{-3/4}}{4c_+^2}\,\frac{\mathcal{M}(\rho)}{1+3^{-3/4}}\right)\,. \tag{14.74}$$

Similarly, we obtain the relation between the canonical radial coordinate r and the coordinate ρ which does not depend on c_+ and reads

$$dr = 3^{3/8}\,\sqrt{T_3\,N_c\,P'_1}\,e^{-2\rho_{\rm UV}/3}\,d\rho\,. \tag{14.75}$$

Using these two expressions, it is straightforward to verify the above scaling behaviour (keeping in mind that κ_1 is given by Eq. (14.35)). For instance, we can easily determine ϵ at leading order in $1/c_+$ to be

$$\epsilon \simeq \frac{3^{-3/4}\,N_c^3\,e^{4\rho_{\rm UV}/3}}{16(1+3^{-3/4})\,T_3\,c_+^4}\,\frac{\mathcal{M}'^2}{P'_1}\,. \tag{14.76}$$

Similarly, we can obtain the scaling behaviour for all quantities of interest. Therefore, the origin of the scaling behaviour is that for large c_+ the solution $P(\rho)$ is determined essentially by $P_{\pm 1}$ only.

14.2.5 Numerical Solutions

To make the picture more precise we need to interpolate between the UV and IR expansions by solving the master equation numerically. But before we present the results, we briefly comment on the numerical method used to obtain them, because obtaining accurate results turned out to be surprisingly difficult.

Comment on the Numerical Method

The first step is of course to solve the master equation numerically using the built-in `NDSolve` routine of `Mathematica` to construct a numerical solution for $P(\rho)$ over a finite range of ρ values from say 10^{-3} to 15. This was particularly challenging since in order to achieve decent results for ϵ and η one needs to determine P'' and P''' very precisely. To achieve this goal, we used the `NDSolve`-Method `Extrapolation` for high-precision numerics. Choosing the following options for `NDSolve` produced sufficiently good results: `AccuracyGoal` $= 40$, `WorkingPrecision` $= 80$ and `MaxStepSize` $= 10^{-5}$. Afterwards, a second numerical interpolation step is performed to reduce the memory consumption by getting rid of unnecessary points. The outcome of this numerical method is a set of interpolating functions for P and its first three derivatives. Accurate input values at $\rho = 10^{-3}$ are achieved by constructing the IR asymptotic expansion to a sufficiently high order in ρ.

Using the leading term in the UV asymptotic expansion for $P(\rho)$, we determine the expansion coefficient c_+ from the numerical value of P at some point where the behaviour is very close to $P \sim c_+ e^{4\rho/3}$. We fix $\kappa_2 = 1$ by hand and the remaining integration constants Φ_0 and κ_1 are then fixed using their analytic expressions in terms of c_+ and $\rho_{\rm UV}$, cf. Eqs. (14.33) and (14.35). We checked that this yields accurate results by comparing it to fixing these integration constants directly from the numerical solution.

The rest is to simply use the explicit analytic expressions to determine all the other functions and repeat the procedure for varying input parameters.

Background Solution

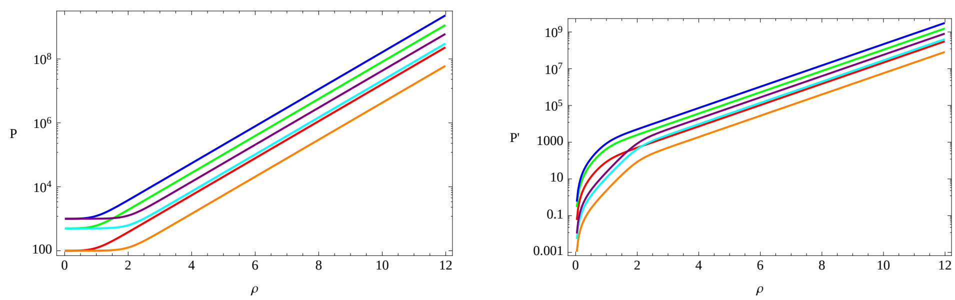

Figure 14.1: Plots of numerical solutions P to the master equation and its first derivative P' for different values of c_0 and k_3. The color coding is such that the red and orange curves, the light blue and green curves and the dark blue and purple curves have the same value of c_0, respectively, but differ by their value of ρ_*.

We show the results for a set of numerical solutions $P(\rho)$ obtained by the procedure described above in Fig. 14.1 for different values of c_+ and ρ_*. As can be seen from these plots, the value of ρ_* characterizes the point where the solution changes from $P \sim$ const to $P \sim e^{4\rho/3}$.

In Fig. 14.2, we show plots of the relevant background functions, Φ and k as well as their derivatives. We also show the warp factor $f(\rho)$. The important points to notice here are that k' does not depend on c_+ but only on ρ_* and that the value of the warp factor at the tip is determined entirely by $\rho_{\rm UV}$ and ρ_*, where the later marks the point where the warp factor starts to become

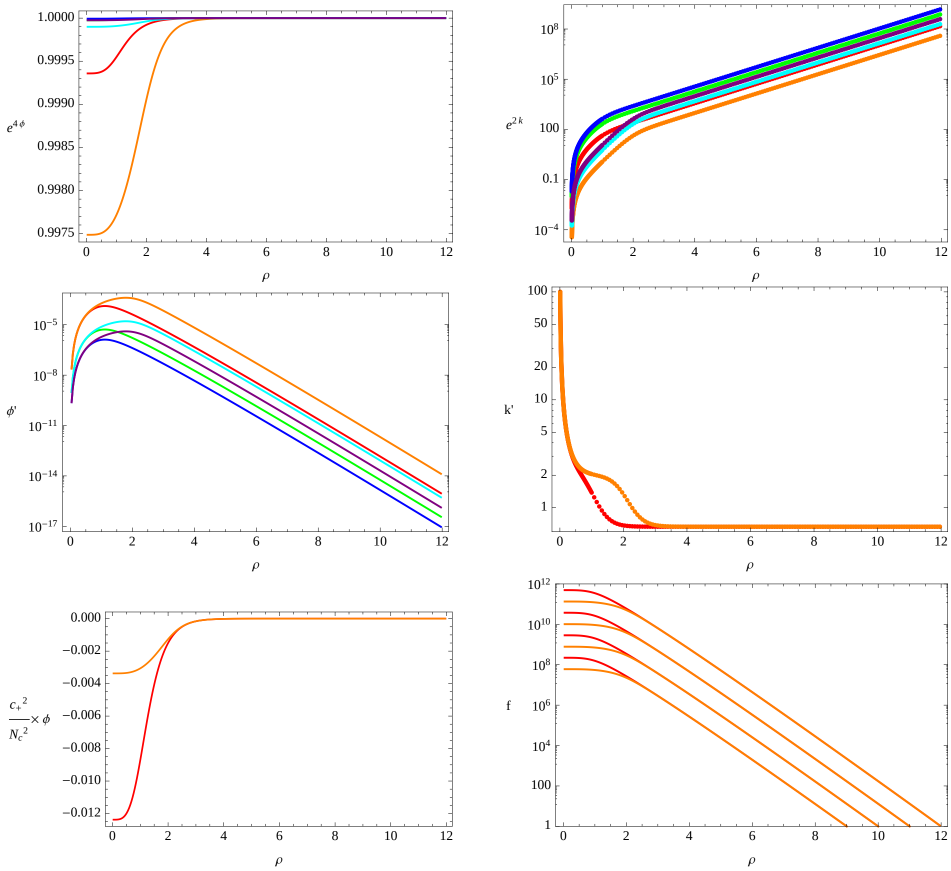

Figure 14.2: Plots of the relevant background functions obtained from numerical solutions to the master equation. From upper left to lower right we plot $e^{4\Phi}$, e^{2k}, Φ', k', Φ (with an overall factor of N_c^2/c_+^2 factored out) and the warp factor f for three different values of c_0 and two different values of k_3. The color coding is such that the red and orange curves, the light blue and green curves and the dark blue and purple curves have the same value of c_0, respectively, but differ by their value of ρ_*. The warp factor f is shown for four different values of the UV cutoff $\rho_{\rm UV}$ at which $f(\rho_{\rm UV}) = 1$.

practically constant. Note also that once we factor out an overall prefactor of N_c^2/c_+^2, Φ depends only on ρ_* (lower left plot in Fig. 14.2).

The divergence of k' as $\rho \to 0$ is related to a curvature singularity for $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ (with all other curvature invariants finite) in the deep IR [137]. The appearance of the singularity is connected to the fact that $P \sim$ const for $\rho \to 0$. However, this singularity does not show up in any of the quantities we compute (or those computed in [137]) and is rather "mild", which is why it was suggested in [137] that this singularity might be resolvable.[6] For this work,

[6]Moreover, in [518], we will consider backgrounds where we deform the baryonic branch by adding flavor D5-branes, but keep the $P \sim h_1\rho$ behaviour in the IR. With some well-

we do not worry about this issue too much and assume that the singularity is resolvable and that the resolution does not strongly affect the quantities we are interested in.

Dilaton Induced D3-Potential

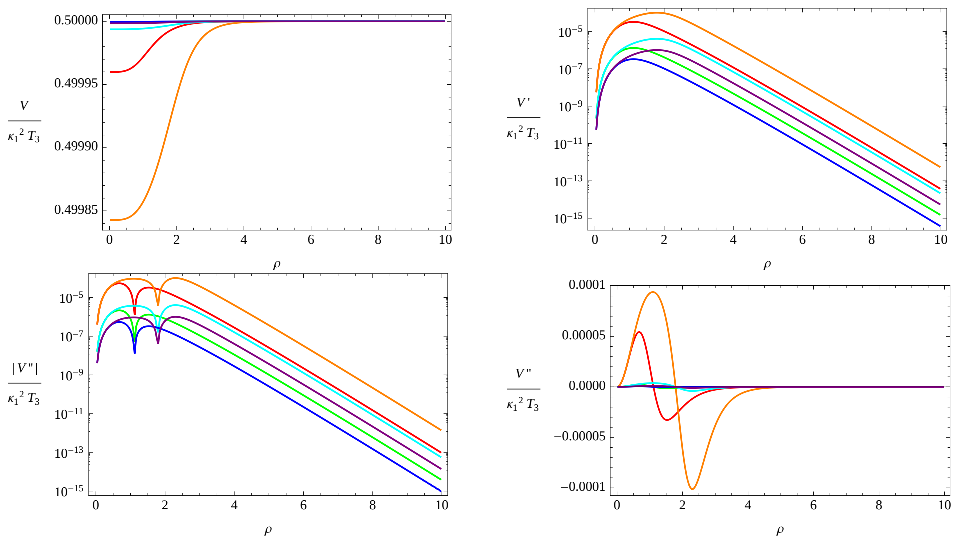

Figure 14.3: Plots of the D3-brane potential induced by the dilaton profiles obtained from the numerical solutions. Note that we have factored out a universal prefactor $\kappa_1^2 T_3$. From upper left to lower right we plot V, V', $|V''|$ and V for three different values of c_0 and two different values of k_3. The color coding is such that the red and orange curves, the light blue and green curves and the dark blue and purple curves have the same value of c_0, respectively, but differ by their value of ρ_*.

The resulting D3-brane potential induced by the dilaton is shown in Fig. 14.3. We have factored out the overall prefactor $\kappa_1^2 T_3$. What is important to notice from these plots are the following things. First, the potential is monotonically increasing towards the UV and thus the force V' is always positive and becomes maximal at $\rho \approx \rho_*$. Second, at the point where the force is maximal of course V'' changes sign – it is positive in the IR and negative in the UV. This is important since it will translate into η changing sign, albeit at a somewhat shifted position because of the required canonical normalization.

motivated assumptions about the flavor profiles (see *e.g.* section 2 of [519]), this yields perfectly regular backgrounds. We do not include the flavor branes here, but note that we recover the baryonic branch for $N_f = 0$. From the point of view of the inflationary phenomenology the pure baryonic branch behaves qualitatively similar to the case study we present here, except that we have to drop the terms involving ρ_* and change the actual numbers somewhat.

Slow-Roll Parameters

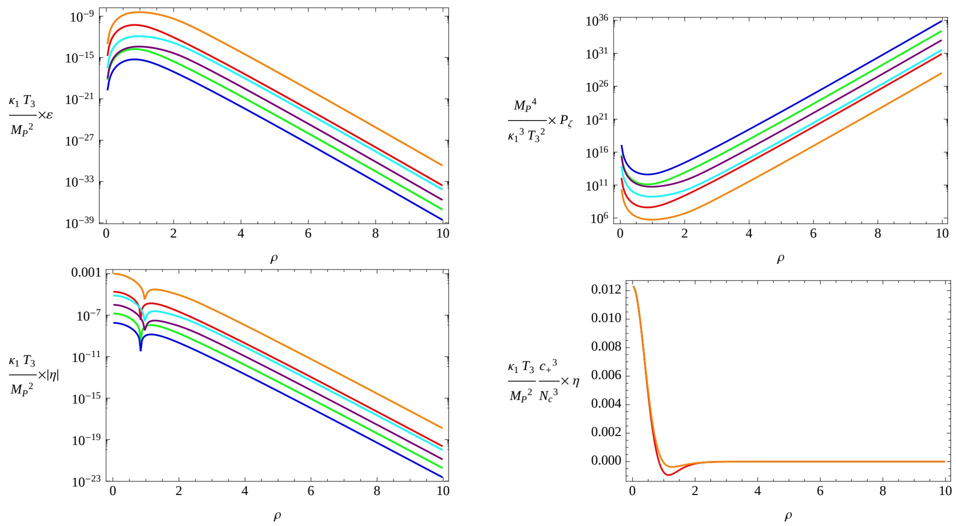

Figure 14.4: Plots of the slow-roll parameters ϵ and η as well as the amplitude of scalar perturbations P_ζ obtained from the numerical solutions. Note that we have factored out some universal prefactors of either $M_P^2/(\kappa_1 T_3)$ for ϵ and η or $M_P^4/(\kappa_1^3 T_3)$ for P_ζ. From upper left to lower right we plot ϵ, P_ζ, $|\eta|$ and η with an additional N_c^3/c_+^3 factored out for three different values of c_0 and two different values of k_3. The color coding is such that the red and orange curves, the light blue and green curves and the dark blue and purple curves have the same value of c_0, respectively, but differ by their value of ρ_*.

The resulting the slow-roll parameters ϵ and η as well as the amplitude of scalar perturbations P_ζ are shown in Fig. 14.4. We have factored out some universal prefactors of either $M_P^2/(\kappa_1 T_3)$ for ϵ and η or $M_P^4/(\kappa_1^3 T_3)$ for P_ζ. Perhaps the most important of these plots is that of $\eta(\rho)$ in the lower right. It shows that η starts out negative in the UV and then becomes positive at some value which depends on ρ_*. As anticipated, η does change sign when going from the UV to the IR.

The position of the inflection point depends on ρ_*, but *not* on c_+. The role of c_+ is to control the strength of the force. For the shown solutions, the inflection points are located at $\rho \approx 0.844$ and $\rho \approx 0.973$ for $\rho_* \approx 1$ and $\rho_* \approx 2$, respectively. Note also that the change of sign takes place in a rather small region of ρ values – η is changing sign rather abruptly.

Correlation between maxima of Φ and V

We have found something very interesting. Namely, we have an *inflection point* where η does change sign at a *geometrically* determined point. Let us now make more precise what we mean by "geometrically determined".

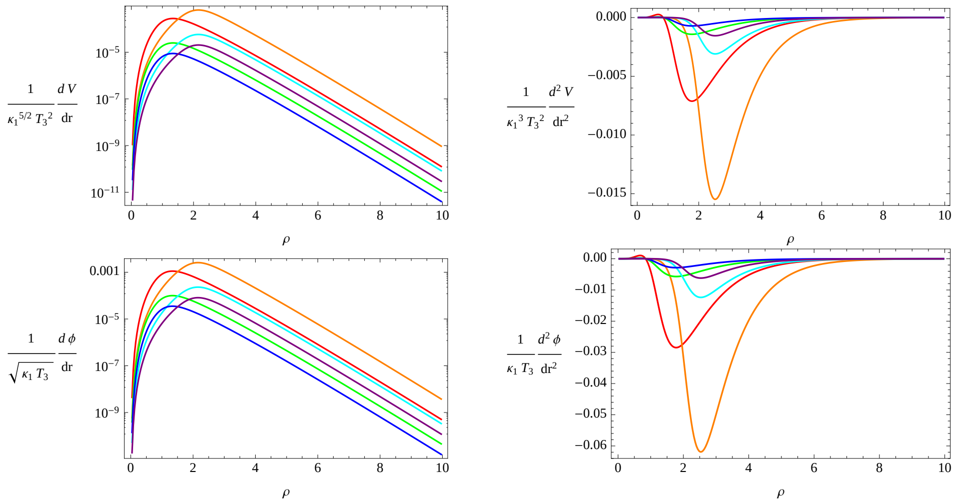

Figure 14.5: Plots of the derivatives of V and Φ with respect to the canonically normalized field $\phi = \sqrt{T_3}r$. We have again factored out some universal overall prefactors depending on κ_1 and T_3 as indicated. From upper left to lower right we plot $\frac{dV}{dr}$, $\frac{d^2V}{dr^2}$, $\frac{d\Phi}{dr}$ and $\frac{d^2\Phi}{dr^2}$ for three different values of c_0 and two different values of k_3. The color coding is such that the red and orange curves, the light blue and green curves and the dark blue and purple curves have (roughly) the same value of c_+, respectively, but differ by the value of ρ_*.

We have visualized in Fig. 14.5, the correlation of the maximum of $\frac{dV}{dr}$ with the maximum of $\frac{d\Phi}{dr}$. As we will explain now, this is a consequence of the large c_+ limit. Note that we are taking derivatives with respect to r, *not* with respect to ρ.

The D3-brane potential is proportional to

$$V \propto \frac{1}{e^{-\Phi} + \kappa_2} \,. \tag{14.77}$$

If we are looking for an inflection point, one can show that the following equation must hold.

$$\frac{d^2V}{dr^2} = 0 \quad \Rightarrow \quad \frac{d^2\Phi}{dr^2} = \left(\frac{d\Phi}{dr^2}\right)^2 \frac{\kappa_2 e^{\Phi} - 1}{\kappa_2 e^{\Phi} + 1} \,. \tag{14.78}$$

Now, we know that for large-c_+ we can expand Φ as

$$\Phi = \Phi_0 + \frac{1}{c_+^2}\tilde{\Phi}(\rho) + \dots \,. \tag{14.79}$$

Then one can check that the correlation is indeed enforced by making c_+ large. This is because for large-c_+ the right-hand side is much stronger suppressed by c_+ than the left-hand side (recall that we have set $\kappa_2 = e^{-\Phi(\infty)}$).

Finally, we can use the numerical solutions to confirm the approximate scaling behaviour of all the quantities (see Sec. 14.3). We have verified the scalings obtained from the asymptotic expansions and they provide a very good approximation.

14.3 Qualitative Picture and Phenomenology

Let us briefly summarize the most important points we have learned in the previous sections. First, we have used the IR and UV expansions of η and found that it changes sign between the IR and the UV. This implies the existence of an inflection point. Using numerical solutions we have confirmed this picture and located the inflection point.

Second, from the IR and UV asymptotic expansions, we also managed to extract some universal scaling behaviour of all the quantities of interest. We have confirmed this scaling behaviour both by an analytical argument and by comparing to the numerical solutions. We will make use of these scalings in a moment.

Most importantly, we have shown both numerically and by an analytic argument that the origin inflection point is directly related to the fact that the profile of Φ has an inflection point, cf. the lower left plot in Fig. 14.2.

In the following, we will heavily exploit the scaling behaviour in a semi-analytic way to obtain predictions.

Approximate Scaling Behaviour

We now summarize the scaling behaviour of the quantities relevant for the inflation phenomenology:[7]

$$\epsilon \simeq \frac{M_P^2}{T_3}\, e^{4\rho_{\rm UV}/3}\, \frac{N_c^3}{c_+^4} \times \begin{cases} \widetilde{\epsilon}_{\rm UV}(\rho)\,, & \rho \gtrsim \rho_* \\ e^{-8\rho_*/3}\,\widetilde{\epsilon}_{\rm IR}(\rho)\,, & \rho \lesssim 1 \end{cases} \tag{14.80}$$

$$\eta \simeq \frac{M_P^2}{T_3}\, e^{4\rho_{\rm UV}/3}\,, \frac{N_c}{c_+^2}\,\widetilde{\eta}(\rho,\rho_*)\,, \tag{14.81}$$

$$P_\zeta \simeq \frac{T_3^2}{M_P^6} e^{-4\rho_{\rm UV}}\, \frac{c_+^2}{N_c} \times \begin{cases} \widetilde{P}_{\zeta,{\rm UV}}(\rho)\,, & \rho \gtrsim \rho_* \\ e^{+8\rho_*/3}\,\widetilde{P}_{\zeta,{\rm IR}}(\rho)\,, & \rho \lesssim 1 \end{cases}\,, \tag{14.82}$$

$$N_e \simeq \frac{T_3}{M_P^2} e^{-4\rho_{\rm UV}/3}\, \frac{c_+^2}{N_c}\, \widetilde{N}_e\left(\rho_{\rm start},\rho_{\rm end}\right)\,. \tag{14.83}$$

Note that ϵ and thus also P_ζ have a scaling which depends on ρ_* in the deep IR but not in the UV. For $\widetilde{\eta}(\rho,\rho_*)$, we see from Fig. 14.4 (lower two figures)

[7] For the pure baryonic branch the scalings would look similar – one just has to drop the ρ_*-dependence.

that the main effect of ρ_* is to shift around the precise location of the inflection point a bit. The profiles for $\widetilde{\eta}$ etc. are determined numerically (but see also the analytic derivation of the c_+ scaling in Sec. 14.2.4).

The number of e-folds N_e is given by the overall factor and a numerical factor which depends on where inflation starts and ends, $\rho_{\rm start}$ and $\rho_{\rm end}$, but the dependence is rather weak. We have checked numerically that if we fix $\rho_{\rm start} = 1$ but vary $\rho_{\rm end}$ between 0.5 and 0.01, $\widetilde{N}_e$ changes by only about a factor of 4. Hence, where inflation ends will affect the actual number in the end, but not strongly. Moreover, note that we will want to start somewhat above the inflection point since close to the inflection point the spectral index $n_s - 1 \approx 0$ and it is negative slightly above the inflection point. We also do not want to start too far away from the inflection point since we do not expect our potential to be the correct description in the UV due to corrections from gluing the throat into a compact space.

With the above potential, inflation could proceed until $\rho = 0$. However, there are two reasons why we expect inflation to end earlier. First, we have a singularity at $\rho = 0$ and one may expect curvature corrections to the probe brane action will end inflation. Second, in the deep IR also the backreaction of the probe D3-brane becomes important which may also end inflation earlier. In any case, because of the weak dependence of the number of e-folds on the value of $\rho_{\rm end}$ this will affect the numerical values slightly, but not significantly affect our conclusions.

One final comment: Since ϵ is suppressed by an additional factor of $\epsilon^{-8\rho_*/3}$ one might expect that N_e is enhanced by a factor of $e^{4\rho_*/3}$. This is, unfortunately, not the case since the dependence of $1/\sqrt{\epsilon}$ on $e^{4\rho_*/3}$ cancels against a contribution from the canonical normalization involving $e^{k+\Phi/2}$.

Semi-Analytic Predictions

We will now exploit the above scalings to make some semi-analytic predictions. By "semi-analytic" we mean that the scalings are known analytically, but we have to extract the numbers from the universal part from a numerical solution.

It is useful to rewrite the above expressions in terms of the number of e-folds N_e. This yields

$$\epsilon \simeq \frac{1}{N_e} \frac{N_c^2}{c_+^2} \, e^{-8\rho_*/3} \, \widetilde{N}_e \, \widetilde{\epsilon}_{\rm IR} \tag{14.84}$$

$$\eta \simeq \frac{1}{N_e} \, \widetilde{N}_e \, \widetilde{\eta}(\rho_*) \tag{14.85}$$

$$P_\zeta \simeq \frac{T_3}{M_P^4} \, N_e \, e^{-8(\rho_{\rm UV}-\rho_*)/3} \, \frac{\widetilde{P}_{\zeta,{\rm IR}}}{\widetilde{N}_e} \, , \tag{14.86}$$

where N_e as above is given by

$$N_e \simeq \frac{T_3}{M_P^2}\alpha' g_s e^{-4\rho_{\rm UV}/3}\frac{c_+^2}{N_c}\,\widetilde{N}_e\,, \tag{14.87}$$

but we have reinstated a factor of $\alpha' g_s$ to get the units correct (recall that we have set $\alpha' g_s = 1$).

To illustrate the procedure, assume that we start at $\rho_{\rm start} = 1$ and inflation ends at $\rho_{\rm end} = 0.1$. Using the numerical solutions and the (IR) scaling behaviour of ϵ, N_e etc. we find the following universal values at $\rho = 1$.

$$\begin{aligned}
\widetilde{\epsilon}(1) &\sim 2\times 10^{-5}\,,\\
\widetilde{\eta}(1) &\sim -10^{-3}\,,\\
\widetilde{P}_{\zeta,\rm IR}(1) &\sim 3\times 10^{2}\,,\\
\widetilde{N}_e(1,0.1) &\sim 10^{3}\,.
\end{aligned} \tag{14.88}$$

For the following numerical estimates we will use Eqs. (14.20) and (14.21) with $g_s \sim 0.1$ and $\mathcal{V}_6 \sim \left(5\sqrt{\alpha'}\right)^{1/6}$ such that $T_3/M_P^4 \sim 6\times 10^{-5}$.

First, we need to match the observed amplitude of the scalar perturbations, $P_\zeta \sim 2\times 10^{-9}$ at $N_e \approx 60$ e-folds before the end of inflation. Note that from Eq. (14.86) we see that P_ζ depends on c_+ only via N_e and since we fixed T_3/M_P^4 by hand, P_ζ only depends on $\Delta\rho \equiv \rho_{\rm UV} - \rho_*$. Let us estimate which value we would need to match observations. With $T_3/M_P^4 \sim 6\times 10^{-5}$ and $N_e = 60$ and the values in Eq. (14.88), requiring $P_\zeta \sim 2\times 10^{-9}$ at $\rho = 1$ yields

$$\Delta\rho \equiv \rho_{\rm UV} - \rho_* \approx 4.96\,. \tag{14.89}$$

Note that $\Delta\rho$ is the distance over which the warp factor changes significantly, cf. Fig. 14.2 which has $\rho_* \approx 1$ and $\rho_* \approx 2$ and four different choices of $\rho_{\rm UV}$. This is also what is done in appendix C of [272] to match the observed amount of density perturbations.

Let us now try to estimate which values for c_+ and $\rho_{\rm UV}$ we need to choose to get $N_e \approx 60$. Using Eq. (14.87), for a given $\rho_{\rm UV}$ we can always adjust c_+ to get $N_e \sim 60$ by making c_+ large enough. For instance, using $\Delta\rho \approx 4.96$ and $\rho_* \approx 2$ as in some of the numerical examples we constructed, we should have $\rho_{\rm UV} \approx 6.96$.[8] Putting everything together with Eq. (14.87), we find that $N_e \sim 60$ is achieved for $c_+ \sim 3\times 10^3$.

This choice of parameters would imply that ϵ at $\rho = 1$ is roughly[9]

$$\epsilon \sim 10^{-13}\,, \tag{14.90}$$

[8] Note that this is sufficient to have $e^{-8\rho_{\rm UV}/3} \ll 1$ such that it is justified to estimate κ_1 as in Eq. (14.35).

[9] Note that the maximum value of ϵ is reached precisely at the inflection point.

and thus the tensor-to-scalar ratio $r = 16\,\epsilon$ is

$$r \sim 2 \times 10^{-12}\,. \tag{14.91}$$

Hence, the production of gravitational waves is completely negligible. Note that this is a generic feature of warped D-brane inflation models (see [295, 296] for a recent systematic analysis). As an estimate for the energy scale of inflation we find from the above values

$$V_{\rm inf}^{1/4} \sim \left(\frac{1}{2}\,\frac{T_3}{M_P^4}\,\frac{N_c^2}{c_+^2}\,e^{-8\rho_{\rm UV}/3}\right)^{1/4} M_P \sim 10^{-5}\,M_P \sim 2\times 10^{-13}\,{\rm GeV}\,. \tag{14.92}$$

Which is considerably lower than the GUT-scale and again a quite generic feature of warped D-brane inflation models

For η at $\rho = 1$ we find

$$\eta \sim -0.02\,, \tag{14.93}$$

such that the scalar spectral index $n_s = 1 + 2\eta - 6\epsilon$ is roughly

$$n_s \sim 1 - 3\times 10^{-2} \sim 0.97\,. \tag{14.94}$$

The tensor-to-scalar ratio r is consistent with the non-observation of gravitational waves in the CMB data by WMAP (cf. Tab. 3.2), but the model would be ruled out if the ongoing PLANCK satellite detect gravitational waves. The value of the spectral index n_s we obtained is consistent with the WMAP data (cf. again Tab. 3.2).

Note, however, that the above picture is too simplistic. We did not take into account corrections to the potential sourced in the UV from gluing the throat into a compact space. In particular, one expects a correction sourced by the non-zero curvature in 4d dimensions which gives rise to a contribution $\Delta\eta = \frac{2}{3}$ [272]. Hence, to make reliable predictions a systematic study of various corrections is necessary. This requires to extend the analysis performed for the regime $r_{\rm IR} \ll r < \rho_{\rm UV}$ in [295] (see also [296] for a study of multi-field effects) over the entire throat. And, in particular, include also the evolution of the angular modes and the perturbations sourced by them. Nonetheless, the inflection point appears so generically in the IR due to the dilaton profile and η changes quite drastically around it. Hence, there is hope that the inflection point may not be completely destroyed by corrections sourced in the UV. Note that in the language of [284] (on which the construction of the potentials used in [295, 296] rests) the UV corrections correspond to non-normalizable modes, while our potential is sourced by a normalizable mode which is similar to the anti-D3-brane at the tip in [272]. This is why [495] suggested to use the baryonic branch instead of an anti-D3-brane at the tip. A detailed investigation of this issue is beyond our present scope and we leave it for the future. Let us now comment about the dual field theory interpretation of the backgrounds we considered here.

14.4 Dual Field Theory Interpretation

The dual field theory analysis for all the backgrounds is usually done using the Klebanov-Witten solution [394], whose UV is $\mathrm{AdS}_5 \times T^{1,1}$, as an approximation since the Klebanov-Strassler solutions all have a UV which is $\mathrm{AdS}_5 \times T^{1,1}$ up to a logarithmic factor. The KK-reduction of type IIB on $\mathrm{AdS}_5 \times T^{1,1}$ is known explicitly [401, 402] and thus one knows the dimensions of the dual operators (see also [385] for a summary).

A probe D3-brane moving in a warped throat background dual to the baryonic branch of KS corresponds to a *meson* in the dual field theory [495]. In a full-fledged flux compactification, which contains the baryonic branch of KS as an approximate description of some region, the $U(1)_B$ symmetry is expected to become *gauged* [493] and may acquire a non-zero Fayet-Iliopoulos (FI) term[10] $\xi_b \neq 0$. This FI-term would then force the VEV of the dimension-2 operator $\langle \mathcal{U} \rangle \sim 1/c_+$ to a non-zero value and thus also c_+ as in [495].

The backgrounds of [137] which we studied above have in addition to the dimension-2 VEV also a VEV for a dimension-6 operator. For a thorough discussion of the dual field theory interpretation we refer to [137], in particular section 3 (see also [495, 513]). Here we will be rather brief and first start to collect a few important points.

The 'rotation' with fine-tuned $\kappa_2 = e^{-\Phi(\infty)}$ corresponds to switching off a dimension-8 source (in a somewhat subtle way) and thereby yields a healthy UV completion of the wrapped D5-brane system by turning it into a quiver gauge theory as in KS with the correct field content to cancel the dimension-8 operator.

The dimension-6 VEV affects the IR but leaves the UV unaffected. From the viewpoint of the geometry, it introduces a second scale ρ_* – the dimension-6 VEV is controlled by c_-/c_+^3 [137], which is $\propto e^{4\rho_*}$, cf. Eq. (14.30). Since the backgrounds are asymptotically KS, the dual gauge theory is a quiver theory which undergoes a cascade of Seiberg dualities. However, the cascade does not last until the end. At a scale determined by ρ_*, the cascade of Seiberg dualities stops and the gauge group gets *higgsed* by a non-trivial VEV, $SU(N_c+M) \times SU(N_c) \to SU(N_c)$ with $M = qN_c$. Below this scale, the effective field theory with gauge group $SU(N_c)$ is dual to the wrapped D5-brane system. The higgsing replaces the last steps of the cascade, which is also indicated by the fact that B_2, H_3 and F_5 are strongly suppressed relative to F_3 below ρ_* in the ten-dimensional background.[11] Consequently, using the standard logic of the

[10]Recently, it has been shown that a *constant* FI cannot be consistently coupled to supergravity. However, Fayet-Iliopoulos terms arising in string theory are typically moduli-dependent, see for instance [210, 520–523] for some discussions of D-terms from string theory and their consequences *e. g.* for moduli stabilization.

[11]In this sense, these supergravity backgrounds may be interpreted as interpolating between the KS and the wrapped D5-brane system.

gauge/gravity correspondence, the dual gauge theory is in a *Higgsed phase* and the Higgs VEV is controlled by the dimension-6 operator.

Due to the dimension-6 operator, the dual gauge theory shows a *'walking' behaviour* below the scale corresponding to ρ_*, *i. e.* there is a range of scales where it almost does not run. One phenomenological application of these backgrounds (besides that to inflation which we discuss here) would therefore be to describe strongly-coupled versions of electroweak symmetry breaking such as walking technicolor [16–18] or extended technicolor [19, 20] (see *e. g.* [21, 22]).

Let us now try to see if we can understand/interpret the existence of the inflection point from a field theory point of view. In the dual supergravity background, it was related to the particular shape of the dilaton profile. The dilaton Φ starts out constant in the IR, then grows but stabilizes again at a scale ρ_* and becomes practically constant in the UV.

Geometrically, this is related to the following. Let us consider the situation before the rotation. The position of the inflection point is related to ρ_*. It signals the region in which throat starts to unwarp and the space becomes flat space. From the IR until ρ_*, the force on a moving D3 keeps increasing since the cumulative effects of the F_3 fluxes continue to warp the geometry (recall that $V_{\mathrm{D3}} \propto h^{-1} - C_4$). After ρ_*, however, the fluxes become too weak to warp the geometry significantly and the magnitude of the force on the D3 starts decreasing. But note that the force is *always* positive, only its magnitude starts to decrease. The inflection point around which we are doing inflation is at ρ_*. In the dual field theory, this corresponds to the presence of the irrelevant dimension-8 operator which starts to control the dynamics towards the UV.

The dimension-8 operator is the reason why the dilaton stabilizes in the UV. Its effect is to "recouple" gravity to the system. Consider the warp factor sourced by a stack of D3-brane,

$$h = 1 + \frac{R^4}{r^4} \,. \tag{14.95}$$

For a solution which in the deep IR this solution is $\mathrm{AdS}_5 \times S^5$ and in the far UV it is flat space, the "1" represents a dimension-8 operator in the Lagrangian – namely F_{ab}^4. In the case of the D5's it is also a dimension-8 operator which brings the geometry back to flat space in the UV.

After the rotation, the interpretation remains more or less similar. What is changing is that now the effect of the new F_5 fluxes prevent the geometry from unwarping completely (*i. e.* to open up into flat space in the UV). The geometry starts to open up around ρ_* where both the dilaton and the warp factor (after the rotation with $\kappa_2 = e^{-\Phi(\infty)}$) change their profile. But at this point the D3 Maxwell charge turning on rather abruptly at ρ_* acts such that it decouples gravity in the UV and warps back the throat. Nevertheless, the inflection point for the dilaton remains and we can use it do to inflation.

Note that the origin of the inflection point is not related to the presence of the VEV for the dimension-6 operator and thus will be present in all backgrounds belonging to the baryonic branch.

In the dual field theory, what happens is that the dimension-8 operator which is starting to take over at the energy scale corresponding to ρ_* is tamed by the VEV of the dimension-2 operator. That is, its coupling is controlled by the VEV of the dimension-2 operator. A very nice analogy of this effect to the UV completion of the Fermi theory (or more precisely its generalization including $SU(3) \times U(1)$ interactions), which has higher-order operators suppressed by M_W from integrating out the W-bosons, was given in [137]. In this situation, the couplings of these higher-order operators, *i. e.* M_W, are controlled by the VEV of the Higgs.

In other words, to switch of the dimension-8 operator in the UV, we introduce a large number of D3-branes. This can be seen from the Maxwell charge growing like $Q_{\mathrm{D3}} \sim \log \rho$ in the UV while $Q_{\mathrm{D5}} = N_c$. Hence, the system becomes dominated by D3-branes in the UV which implies a constant dilaton and a space which is almost asymptotically AdS_5.

14.5 Summary and Discussion

Let us summarize what we have found in this part. We have considered warped D3-brane inflation in a class of recently constructed supergravity backgrounds which are dual to a *walking* gauge theory [137]. These backgrounds are a modification of the baryonic branch whose application to inflation was first studied in [495], but focused on the UV region. Here, we considered the IR regime. The difference between the original Klebanov-Strassler solution [135] and its baryonic branch [136] is encoded in the VEV of a dimension-2 operator $\mathcal{U}$ which controls the resolution of the conifold singularity. In our notation, the VEV is controlled by a parameter c_+ related to $\mathcal{U}$ as $\langle \mathcal{U} \rangle \sim 1/c_+$. Hence, for $c_+ \to \infty$ one recovers the KS solution. However, for any finite value of c_+, a D3-brane in this background breaks supersymmetry (see Eq. (15.3) of [495] for a nice check of this by computing the Killing spinors in 10d). A finite value for c_+ is assumed to be due to the presence of a Fayet-Iliopoulos term for the $U(1)_B$ symmetry, which is expected to become gauged once the throat is glued into a compact space [493]. Our backgrounds are a deformation of both KS and its baryonic branch in the IR due to the presence of the VEV of a dimension-6 operator. This operator generates us a new scale ρ_*.

To construct the supergravity backgrounds, we used a formalism using a *master equation* and a solution generating technique called *'rotation'*, which we reviewed in Secs. 13.2 and 13.3, respectively. This formalism describes a much broader class of solutions which includes the one we discussed and the baryonic branch of KS. We have picked the solution of [137] as a case study to investigate inflation in the IR region in Sec. 14.2. Using a combination of analytic expansions and numerical solutions, we arrived at the following qualitative picture (see Sec. 14.3).

The potential generically has an inflection point which is related to a particular feature of the dilaton potential. Namely, that it starts out constant, then starts to grow until some value of the radial coordinate $\rho \sim \rho_*$, where it stabilizes again to a constant. We provided both numerical evidence and an analytic argument why this statement is correct. This is interesting since this inflection point does not arise by chance but is instead at a *geometrically* defined position, cf. Sec. 14.2.5.

Interestingly, the quantities of interest for inflation obey a universal scaling behaviour in the limit of large-c_+, which we derived analytically in three different limits in Secs. 14.2.2, 14.2.3 and 14.2.4 and verified numerically to be valid on the entire throat (cf. Sec. 14.2.5). Geometrically, the limit of large-c_+ corresponds to pushing the background close to KS. We have exploited this scaling behaviour in Sec. 14.3 to study the inflationary phenomenology implied by the presence of the contribution to the potential we have studied. This serves as an illustration how one can derive predictions for inflationary observables and we found reasonable values.

In Sec. 14.4, we commented on the field theory interpretation of the qualitative picture. The dilaton profile relevant for the inflection point is before the rotation related to the presence of a dimension-8 operator. After the rotation, where the dimension-8 operator is adiabatically switched off, the picture nevertheless stays the same since in this process we induce a large number of D3 charge which dominates the UV and ensures a healthy UV completion [137].

Note that despite the rather nice numbers we presented as an illustrative example, the above should be viewed as a first step. To derive reliable predictions, especially for η, a more sophisticated analysis is necessary. This presumably requires to extend the analysis of the structure of the potential [284] and its phenomenological consequences [295, 296], which made use of the limit $\rho_{\mathrm{IR}} \ll r < \rho_{\mathrm{UV}}$ to describe inflation in this "near UV" regime and included the evolution of the angular modes. In contrast, we are interested in inflation in the IR of the baryonic branch (for previous work on brane inflation at the tip of the KS solution see *e. g.* [292]). However, since η changes quite drastically around the inflection point, we have hope that the inflection point will not be completely destroyed by adding UV corrections. Thus, it can serve as a starting point for trying to find viable models in an expansion around this inflection point and we do not have to rely on an inflection point arising by chance.

Part V

Conclusions and Outlook

Chapter 15

Conclusions and Outlook

We have now reached the end of this thesis and it is time to wrap up. In the following, we will briefly recall the important results obtained in Parts III and IV and provide an outlook on further directions of research.

The aims of this dissertation were to explore new approaches to address the problems discussed in Chap. 2 and to realize inflation in the matter sector. To gain new insights, we employed both a bottom-up and a top-down approach. Let us now go through the results of our different approaches one by one. For more complete summaries and detailed discussions confer the individual summary sections at the end of the appropriate chapters – for Part III see Secs. 10.5, 11.5 and 12.5 and for Part IV see Sec. 14.5.

Combining Low-Energy SUSY and High-Scale Inflation

The first problem we have addressed was related to moduli destabilization in a KKLT-type framework by the presence of an inflationary sector. Avoiding moduli destabilization then puts an upper bound on the Hubble scale during inflation in terms of todays gravitino mass, $\mathcal{H}_{\text{inf}} \lesssim m_{3/2}^{\text{today}}$. We studied a 4d effective supergravity model to cope with this problem in a novel way.

We have proposed a general scenario where low-energy supersymmetry and high-scale inflation can be accommodated simultaneously in Chap. 10. The idea is to stabilize the modulus *during* and *after* inflation by two *different* mechanisms. In this way, we evade the upper bound on $\mathcal{H}_{\text{inf}}$ since the scale for moduli stabilization is now set by $\mathcal{H}_{\text{inf}}$ itself and no longer by $m_{3/2}^{\text{today}}$. To avoid an overshooting problem at the end of inflation, we have to tune some coefficients to ensure that the minima during and after inflation are not too far apart.

We have illustrated that our general idea works in two explicit examples. In both cases, we modelled the inflationary sector by a simple toy model, shift symmetric F-term chaotic inflation. After inflation, the modulus is stabilized by a superpotential contribution $W_{\mathrm{mod}}(T)$, *e. g.* of the KKLT-type. But we have shown that even for general $W_{\mathrm{mod}}(T)$ the corrections to the inflationary observables are small. Both the spectral index n_s and the tensor-to-scalar ratio r receive only corrections which are schematically of the form

$$\begin{aligned} n_s - 1 &= -\frac{2}{N_e}\left(1 + \mathcal{O}\left(\left(\frac{F_T}{F_X}\right)^2, \left(\frac{m_{3/2}}{F_X}\right)^2\right)\right), \\ r &= \frac{8}{N_e}\left(1 + \mathcal{O}\left(\left(\frac{F_T}{F_X}\right)^2, \left(\frac{m_{3/2}}{F_X}\right)^2\right)\right). \end{aligned} \tag{15.1}$$

These corrections are negligible when we have high-scale inflation and low-energy supersymmetry breaking, *i. e.* when $F_T, m_{3/2} \ll F_X$. Thus, we recover the standard predictions of chaotic inflation: for $N_e \approx 60$ one finds $n_s \approx -0.97$ and $r \approx 0.13$, which are consistent with the constraint from the WMAP-7yr data, cf. Tab. 3.2.

Outlook

Despite the nice phenomenological picture for inflation we have obtained, there are some open questions left. Let us point out some interesting directions for future research.

An interesting aspect which deserves further investigations is to quantify the tuning required to avoid an overshooting of the modulus after inflation. One may also imagine to modify our setup somewhat to embed a dynamical mechanism which smoothly transfers the modulus from its inflationary minimum to its post-inflationary minimum.

It is also worthwhile to study the non-thermal history which emerges from our scenario. Typically, the modulus T is the lightest field and oscillates the longest. Thus, the reheating of the visible sector and the subsequent cosmological history are typically determined by the decays of the modulus and not by decays of the inflaton. This should allow us to constrain some parameters of the model *e. g.* by demanding successful BBN or generating enough baryon asymmetry.

Since our model is strictly speaking string-inspired but not string-derived, it is certainly interesting to find a string theory realization of our scenario. This is particularly interesting for models of large-field inflation due to their enhanced sensitivity to the physics of moduli stabilization.

It would also be nice to demonstrate explicitly that our approach works for hybrid models of inflation as well. There are good indications that this is possible. That is, the suppression of corrections to the inflationary observables seems to rely only on the hierarchy $F_T, m_{3/2} \ll F_X$ and not on the fact that we used a large-field chaotic inflation model.

Matter Inflation in Supergravity

Let us now move on to one of the guiding themes of this thesis – the phenomenologically motivated framework for supergravity models of inflation which we have termed *matter inflation*.

We have constructed a fairly general class of supergravity models which describe matter inflation in Chap. 11. Our starting point were the models discussed in [297], which feature a *tribrid* structure in the superpotential [227, 228, 351]. Unlike the standard F-term hybrid inflation model [218–220], they allow a solution to the η-problem with a specific form of the Kähler potential enforced by a symmetry [38].

In matter inflation models, where the inflaton resides in *gauge non-singlet matter* fields, one generates the inflaton-dependent mass term for the water fall fields by non-renormalizable terms, as *e. g.* in sneutrino hybrid inflation [221]. A Heisenberg symmetry protects the inflaton since a shift symmetry cannot be used for gauge non-singlets. We have generalized the models of [297] with some inspiration from string theory. In particular, we replaced all constant couplings in the superpotential by functions of a set of moduli T_i and generate the scale of the F-term driving inflation dynamically. All fields except the inflaton can be stabilized with masses of at least $m \gtrsim \mathcal{H}_{\text{inf}}$ and one of the T_i is stabilized by essentially the same mechanism we used in Chap. 10 to combine low-energy SUSY and high-scale inflation.

We discussed the sources for generating a small slope both at tree-level and loop-level. Assuming possible tree-level corrections to be parametrically small, the loop corrections dominate the inflationary predictions. This is the case if the sector responsible for moduli stabilization during inflation does not break the Heisenberg symmetry strongly. Then generically the loop corrections from the gauge fields are negligible and the loop corrections from the waterfall fields dominate such that the inflationary phenomenology is expected to be similar to [297]. If, however, there are large tree-level corrections, *e. g.* corrections induced by moduli stabilization, one may still find viable inflationary models due to cancellations between various terms. The phenomenology is then presumably similar to inflection point inflation.

Towards Matter Inflation in Heterotic Orbifolds

One of the main motivations for the generalization of the models of [297] considered in Chap. 11 was to study the conditions under which models of matter inflation can be embedded into heterotic orbifolds in Chap. 12. As we explained in Sec. 2.4, this is important since quantum gravity corrections may spoil an effective field theory solution to the η-problem.

Heterotic orbifolds constitute an interesting playground to search for matter inflation models. First, they feature a Heisenberg symmetry for a certain class of matter fields. Second, the structure of their matter superpotential makes it plausible that a tribrid structure can arise, *e. g.* after other fields acquire suitable VEVs. Third, MSSM-like models have been found [177–182, 415–417, 427, 524]. Moreover, the moduli potentials in heterotic orbifolds are often too steep to allow for slow-roll inflation and thus looking for inflation in the matter sector seems attractive.

We proposed a new way to stabilize the modulus associated to the inflaton, which is a string-inspired version of the mechanism used in Chap. 10 (albeit in a somewhat different context). We suggest an ansatz for the form of the string-loop corrections to the Kähler metric of the driving field X living in an $\mathcal{N} = 2$ twisted sector in the background of non-trivial untwisted matter fields. This ansatz allows us to parametrize a possible breaking of the Heisenberg symmetry due to moduli stabilization. The dilaton is stabilized entirely by non-perturbative corrections to the Kähler potential as in [464] since the gaugino condensates are typically negligible during inflation.

We used our ansatz for the parametrization of the Heisenberg symmetry breaking effects by moduli stabilization via the string-loop corrections. The inflationary phenomenology depends on whether or not these loop corrections respect the Heisenberg to a sufficient amount. In any case, our parametrization of the breaking effects allows for a systematic investigation of the phenomenology in 4d effective supergravity.

Outlook

Our present work is a first step towards realizing inflation in the matter sector of heterotic orbifold compactifications. Hence, it is of course very interesting to find explicit realization of the our scenario. Moreover, one can then check if there is an overlap with models which yield a MSSM-like spectrum.

The new way to stabilize one of the Kähler moduli by the F-term of a matter field and its Kähler metric might be relevant also for moduli stabilization in heterotic orbifolds, independently of the issue of inflation. For instance, in [166] a non-zero matter F-term provides an uplifting contribution.

From the phenomenological point of view, it is interesting to explore the parameter space of our matter inflation models in more detail. Especially, by systematically searching for regions where cancellations between various terms yield a small inflaton slope. Moreover, we typically have a couple of fields with masses around $\mathcal{H}$ so it would be nice to explore their phenomenological implications along the lines of [245].

Slow-Walking Inflation

In the second part of this thesis, we considered a genuine top-down approach and considered warped D3-brane inflation in a class of supergravity backgrounds dual to a walking gauge theory [137]. These backgrounds are interesting for applications of gauge/gravity duality to beyond the standard model physics (see *e. g.* [21, 22]).

The backgrounds we considered are deformations of the baryonic branch [136] of the Klebanov-Strassler throat [135]. They feature a non-trivial dilaton profile which induces a potential for a probe D3-brane. We studied the implications of this potential for brane inflation in the IR region, where the walking backgrounds deviate from both KS and its baryonic branch.

We found that there is generically an inflection point whose position is determined geometrically in terms of the dilaton profile. Around this inflection point, η changes sign and its magnitude varies quite strongly. Exploiting a universal scaling behaviour of the quantities of interest, we explained a way to make semi-analytical predictions. That is, we use the analytically derived scaling behaviour and a numerical solution to extract the required information. We demonstrated this procedure for an illustrative example for which we found reasonable values for n_s and r. The particular feature of the dilaton profile, which causes the inflection point to appear so generically, can be understood also from the point of view of the dual field theory using the AdS/CFT-correspondence.

A nice but crude physics analogy Finally, let us mention a physics analogy for the D3-brane force. Moving from the IR to the UV, it grows, reaches a maximum and then decreases. This behaviour is qualitatively similar to the force exerted on a probe charge by a uniformly charged sphere. There the force grows as r inside the charged sphere, but outside it drops as r^{-2}. The inflection point is then at the surface of the sphere where the solutions inside and outside have to be matched. A smooth transition as in the probe D3 case would correspond to a “fuzzy” spherical charge distribution.

Outlook

Even though we find that the potential induced by the non-trivial dilaton profile yields a nice phenomenology, the study presented above is a first step. Namely, one expects important corrections from bulk moduli stabilization which are particularly important for the value of η and thus the spectral index n_s. Hence, it would be interesting to extend the analysis of the structure of the potential for the regime $r_{\rm IR} \ll r < \rho_{\rm UV}$ which was completed in [284] to our case where inflation takes place in the IR. The phenomenological consequences then should be explored as in [295, 296] by sampling over the a priori unknown coefficients of the perturbations sourced in the UV. Since η changes quite strongly around the inflection point, we may expect that it will not be completely destroyed by adding corrections in the UV. Thus, our geometrically defined inflection point can serve as a starting point for a search for viable models in a systematic expansion around our setup.

D-brane inflation may also take place at very high velocities for the inflation due to the so-called DBI effect [301, 302], which is related to the particular form of the kinetic term for the D-branes. It would certainly be interesting to explore the possibilities for DBI inflation in our scenario. Moreover, including the angular modes may also lead to interesting effects.

Another interesting research direction would be to explore reheating after inflation in such a setup.

To conclude, we have explored some novel ideas to address inflation and its related problems both from the point of view of effective supergravity theories and from a string theory perspective. They are a step in the right direction and deserve further investigations.

Part VI

Appendix

Chapter A

Notations & Conventions

Metric Conventions

Throughout this work, we will use the "mostly minus" convention for the metric, *i. e.* the flat 4d Minkowski space metric is given by

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} , \tag{A.1}$$

and all other metrics $g_{\mu\nu}$ considered have the same signature.

Spinor Conventions

The matrices σ_m are defined as

$$\sigma_m = (\mathbb{1}, \vec{\sigma}) , \tag{A.2}$$

with indices $(\sigma_m)_{\alpha\dot{\alpha}}$. The convention for the Pauli matrices are

$$\begin{aligned} \sigma^0 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} , \quad \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} , \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} . \end{aligned} \tag{A.3}$$

The matrices $\bar{\sigma}_m$ are related to the σ_m as

$$(\bar{\sigma}_m)_{\dot{\alpha}\alpha} = \left(\sigma^T_m\right)_{\dot{\alpha}\alpha} = (\sigma_m)_{\alpha\dot{\alpha}} , \tag{A.4}$$

such that

$$(\bar{\sigma}_m)_{\dot{\alpha}\alpha} = (\mathbb{1}, -\vec{\sigma})_{\dot{\alpha}\alpha} . \tag{A.5}$$

The matrices $\sigma^m, \bar{\sigma}^n$ satisfy

$$\mathrm{Tr}\,(\sigma^m \bar{\sigma}^n) = -2\eta^{mn}\,, \tag{A.6}$$

and

$$\sigma^m_{\alpha\dot{\alpha}} \bar{\sigma}^{\dot{\beta}\beta}_m = -2\delta^\beta_\alpha \delta^{\dot{\beta}}_{\dot{\alpha}}\,. \tag{A.7}$$

Unit Conventions

We work in natural units where

$$\hbar = 1\,, \quad \text{and} \quad c = 1\,. \tag{A.8}$$

We use the *reduced* Planck-mass (in 4d)

$$M_P^2 = \frac{1}{8\pi G_N}\,, \tag{A.9}$$

where G_N is the Newton constant and we conveniently set it equal to 1 unless stated explicitly otherwise.

Chapter B

Actions for Dp-branes

The action of a probe Dp-brane is given by the sum of the *Dirac-Born-Infeld* (DBI) and *Wess-Zumino* (WZ) (or *Chern-Simons* (CS)) actions. $\mathcal{S}_{\text{DBI}}$ encodes the couplings to the metric, the dilaton and the NS 2-form, while $\mathcal{S}_{\text{WZ}}$ encodes the couplings to the RR p-forms under which the branes are charged.

In string frame, the DBI and WZ actions of a Dp-brane are given by

$$\mathcal{S}_{\text{DBI}} = -T_p \int\limits_{\mathcal{W}_{p+1}} d^{p+1}\xi \, e^{-\phi} \sqrt{-\det\left(G_{ab} + B_{ab} + 2\pi\alpha' \mathcal{F}_{ab}\right)}\,, \tag{B.1}$$

and[1]

$$\mathcal{S}_{\text{WZ}} = \mu_p \sum_q \int\limits_{\mathcal{W}_{p+1}} C_q \wedge e^{2\pi\alpha'\mathcal{F}+B} \tag{B.2}$$

respectively. Here, G_{ab} and B_{ab} denote the metric and 2-form induced on the brane, respectively. ϕ is the dilaton and $\mathcal{F}$ is the field strength of the worldvolume gauge theory. C_q are the RR q-forms, while T_p and μ_p are the tension and charge of the Dp-brane, respectively. They are given by

$$T_p \equiv |\mu_p| \equiv (2\pi)^{-p}\,\alpha'^{-\frac{p+1}{2}}\,. \tag{B.3}$$

The sign of μ_p depends on whether we are considering a Dp- or an anti-Dp-brane. In our conventions, the sign is positive for a Dp-brane.

Throughout this thesis, we will consider only vanishing background values for the worldvolume field strength $\mathcal{F}$. Then the action of a Dp-brane simplifies somewhat to

$$\mathcal{S}_{\text{D}p} = -T_p \int\limits_{\mathcal{W}_{p+1}} d^{p+1}\xi e^{-\phi} \sqrt{-\det\left(G_{ab} + B_{ab}\right)} + \mu_p \int\limits_{\mathcal{W}_{p+1}} C_{p+1}\,. \tag{B.4}$$

[1] In general, there are also couplings to curvature, see *e. g.* [49,50,103,104], but we neglect those here since they arise at higher orders in the α'-expansion.

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