Firm Heterogeneity, Financial Development, Foreign Direct Investment, and Monetary Policy

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To My Parents
Contents

Acknowledgments

Acronyms

Chapter 1  Introduction  1
   1.1 The Issue  2
   1.2 Brief Survey of Literature  6
   1.3 Main Contribution  7

Chapter 2  Multiple Sources of Finance, Margins of FDI, and Aggregate Industry Productivity  9
   2.1 Introduction  10
   2.2 The Model  14
      2.2.1 Closed Economy  15
      2.2.2 Open Economy  18
3.5 Conclusion

Appendix

A.3.1 Proofs
A.3.2 Calculation of Financial Structure of FDI
A.3.3 Calculation of Aggregate Risk of FDI

Chapter 4 Firm Heterogeneity, Endogenous Entry and Exit, and Monetary Policy

4.1 Introduction
4.2 The Model
  4.2.1 Producers
  4.2.2 Banking Sector
  4.2.3 Households
  4.2.4 Aggregation
  4.2.5 Shocks and Policy
4.3 Analyzing the Model
  4.3.1 New Keynesian Phillips Curve and a new Tradeoff
  4.3.2 Calibration and Impulse Responses
  4.3.3 Second Moment
4.4 Conclusion

Appendix
A.4.1 Aggregation 111
A.4.2 Steady State Equations 114
A.4.3 Log-Linearized System 116

References 117
List of Figures

2.1 Complementary Effect and Substitution Effect 27
A.2.1.1-A.2.1.4 Simulation of Firms’ Reserve for FDI 33
A.2.2.1-A.2.2.4 Simulation of Intensive Margin of FDI 35
A.2.3.1-A.2.3.4 Simulation of Extensive Margin of FDI 37
A.2.3.5 Simulation of Profit of FDI 38
A.2.4.1-A.2.4.3 Simulation of Cutoff Productivity Gap 38
3.1 Financial Structure and Volatility of Outward FDI 44
3.2 Production and Financing Choice 49
3.3 Financing Cost and Firm’s Productivity 60
3.4 Comparison of Expected Profit under Different Finance 61
3.5 Segmentation of Firms in Production and Financing 64
3.6 The Effects of an Increase in FDI Risk 65
3.7 Productivity Distribution and Financial Structure 66
3.8 The Evolution of Number of FDI Destinations and Productivity 68
3.9 The Rising Average Risk per Destination of FDI

3.10 Financial Structure and Aggregate Risk of FDI Portfolio

3.11 Financial Structure and Productivity

4.1 Cyclical Behavior of Entry and Exit

4.2 Correlation between Entry(t+k) and GDP(t); Exit(t+k) and GDP(t)

4.3 Impulse Responses to a One Percent Positive Technology Shock

4.4 Impulse Responses to a One Percent Contractionary Money Supply Shock

4.5 Simulated Entry and Exit (HP-Filtered Log-Deviation From Steady State)
List of Tables

3.1 Summery Statistics of Our Data 67

3.2 Empirical Estimation of Volatility of FDI (Financial Structure Plays a Significant Role) 72

A.3.1 Denotations of Variables in Empirical Analysis 76

4.1 Estimation for Money Supply Shocks 107

4.2 Moments for Data, Benchmark RBC model (King and Rebelo, 1999), Bilbiie, Ghironi and Melitz (2007a)’s model with Endogenous Entry, and Our Model with Both Endogenous Entry and Exit 108
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Acronyms

CES: Constant Elasticity of Substitution
CPI: Consumer Price Index
DSGE: Dynamic Stochastic General Equilibrium
FDI: Foreign Direct Investment
GDP: Gross Domestic Product
NPV: Net Present Value
PPI: Producer Price Index
RBC: Real Business Cycles
TFP: Total Factor Productivity
ZCP: Zero Cutoff Profit
Chapter 1

Introduction
1.1 THE ISSUE

The impact of financial development (financial constraint) on firms’ internationalization has receiving growing attentions among economists and policy makers. As widely believed, better access to external finance facilitates global activities. In the recent financial crisis, when credit suddenly dried out, we did observe sharp decline of global foreign direct investment flow as well as trade. For instance, as World Investment Report 2009 tells, global FDI inflow fell 14% in 2008, amount to 1.697 billion dollar. This triggered out the emergence of a huge body of literature which uses various data sets to re-emphasize the importance of external finance’s availability to multinational firms.

Nevertheless, most of them focus on the size effect of financial availability while neglect the structure effect of financial development. One obvious fact is that firms are heterogeneous, and they react to shocks and policies differently. It is important to notice that also during the current financial crisis, a significant fraction of firms reallocate capital structure and their sales remain unchanged or even expanded (reported by World Bank Financial Crisis Survey, 2010). Therefore, this thesis addresses the question that how heterogeneous firms behave differently in terms of making investment decisions and choosing types of external finance. Moreover, I extend the heterogeneous firms set-up into a dynamic stochastic general equilibrium model, embedded with New Keynesian model features, to analyze the transmission of shocks and make implications for policies.

Particularly, in chapter 2, I study the impact of financial development on foreign direct investment with multiple sources of external finance. It is motivated directly by the fact that facing crunch of bank credit, not all the firms are left helpless. Some less productive firms do suffer from less availability of credit, yet a bunch of
productive firms resort to alternative finance, e.g., bond market, to restore their investment. As former chairman of Federal Reserve Mr. Greenspan argues, the development of alternative financing channels helped to fill the funding gap and stabilize business financing, although people with disagreement point out that the shortage of liquidity in one financial market dries out other market. The chapter 2 contributes to the discussion and investigates firms’ choices among internal fund, bank credit and bond market credit in a very simple framework. Firms are heterogeneous in productivity, hence the ability of generating profit from FDI. We find that with a cut of bank credit, productive firms switch to bond finance to stabilize the investment. We call this result substitution effect between bank finance and bond finance, which is emphasized by Mr. Greenspan. However, the increased demand for bond finance of these productive firms bids up the bond rate, making it more expensive for others. As a consequence, less productive firms are forced to exit FDI market. This is called complementary effect between bank and bond finances in the sense that a cut in bank credit is associated with a higher cost of bond credit. The rising bond rate induces the reallocation of financial resources from less efficient firms towards more efficient ones and thus increases the aggregate industry productivity of the producing firms through a Meltiz-type selection effect.

Continue with this work, I further discuss firms’ choices of different sources of external finance and the impact of financial structure on the performance of FDI in chapter 3. This research is motivated by two observations: first, countries are different in financial systems. For example, as discussed by Fiore and Uhlig (2005), Germany (or Japan) has a bank based financial system while U.S (or U.K) has a market-based system; and second, FDI flows from countries with market-based financial system are more volatile relative to that from countries with bank-based system. As risk is a main driving force for volatility of investment and a key determinant for choosing capital structure, I therefore explicitly investigate the
relationship between financial structure and the risk of FDI.

Precisely, in chapter 3, I model firms’ choices of lenders when they engage in FDI. They can choose indirect finance as borrowing from banks or direct finance as issuing bond to bondholders. There are many differences between direct finance and indirect finance, as emphasized by different economists. For example, Russ and Valderrama (2009) argue that the fixed cost of underwriting bond finance is higher than bank finance while the marginal cost is lower. Most of others, agree on the characteristics of banks as costly middleman or delegated monitor compared to direct finance (Holstrom and Tirole, 1997, Fiore and Uhlig, 2005, etc.). I take Holstrom and Tirole (1997)’s specification in the model. Particularly, as FDI is risky, firms with lowest productivities will be unable to do FDI, and those with intermediate productivities choose bank finance while those most productive firms use direct finance. The partition of firms results from banks’ role as monitor: on one hand, it reduces the risk (and the moral hazard problem) of FDI; on the other hand, monitor is costly for firms. For less productive firms, finance through banks is better because they are more fragile to risk. However, those most productive firms will find it not attractive to hire an intermediary when financing the investment.

Based on firms’ choice of their lenders, the financial structure of the economy is therefore calculated as the ratio of aggregated bond finance over aggregate bank finance. We discuss the relationships between the financial structure and risk of FDI. Our model predicts that, ceteris paribus, if the risk of destination country is higher, more firms use bank finance relative to bond finance. Moreover, in case of productivity growth, other things equal, more firms will use bond finance and they will invest in riskier countries. The first prediction exams the relationship between financial structure and expected risk of FDI; while the second prediction exams the effect of productivity growth on both financial structure and risk-taking of FDI.
Both predictions are supported by our empirical analysis. In particular, we find that higher ratio of bond finance relative to bank finance is associated with higher risk of FDI per destination country, which is consistent with Germany and U.S example.

Finally, I embed firm heterogeneity and therefore their endogenous entry and exit into a DSGE framework to analyze the transmission of shocks and discuss monetary policy in chapter 4. This work is strongly motivated by the facts that entry is pro-cyclical while exit is counter-cyclical, and they are more volatile than output (see figure 4.1 and 4.2). Moreover, the entry and exit account significant share of output volatility, as suggested by Broda and Weinstein (2010) that in each unit increase in output, 35% of which comes from introduction of new products. Bernard, Redding and Schott (2010) also report that the value of newly introduced products accounts for 33.6% of total output while the value of destructed accounts for 30.4%.

However, most of the traditional DSGE models assume constant number of producers, and the fluctuations of the economy in these models simply reflect the reactions of producers’ intensive margin to shock, i.e., producers react by cutting or increasing sales. These models do not capture the cyclical behavior of entry and exit, and they face a lot of well know challenges in predicting the impulse responses of variables compared to what data suggests. For example, the counter-cyclical behavior of markups with pro-cyclical behavior of profit that are observed by data can not be generated by tradition RBC models or New Keynesian models. Moreover, traditional RBC models depend heavily on the persistence of shocks to explain the observed persistence of total factor productivity (TFP). In addition, in the second moment evaluation, these models generate too smooth consumption and labor and too pro-cyclicality of all the variables (too high correlation between variables and output).
By introducing endogenous entry and exit in a New Keynesian framework, we are able to make substantial improvements on the performance of the model in many aspects. The aggregate output depends on number of producers, and we find a new mechanism of the transmission of shocks: through the dynamics of firms. Moreover, in our model, the New Keynesian Phillips curve has additional tradeoff for policy makers such that the number of producers has impact on inflation. This opens the door for optimal policy analysis. We explicitly discuss the implications of our model in chapter 4.

1.2 Brief Survey of Literature

Since Melitz (2003) and Helpman et al. (2004), it is widely believed and empirically supported (somehow) that firm’s productivity is a key determinant for its internationalization. Particularly, Melitz (2003) is the workhorse for analyzing international trade with heterogeneous firms. Manova (2007) introduces export-oriented bank credit and takes credit constraint as another important determinant for firms’ export. Her research is followed by a growing empirical analysis, such as Muuls (2008), Berman and Hericourt (2008), etc. Buch et al. (2009) focus on the impact of financial constraint on FDI with German firm level data.

Regarding the financial structure, we focus on the structure of private finance and public finance, i.e., the choices of lenders, although there is large body of literature on firms’ choice between equity and debt. Holstrom and Tirole (1997) model a moral hazard problem with firms heterogeneous in initial wealth. They find a pecking order of the external finance that firms with largest initial wealth can borrow from market finance while those with intermediate initial wealth borrow from banks who monitor the firms to reduce the moral hazard problem. Fiore and Uhlig (2005) discuss the differences of financial system between Europe and U.S,
using a model with continuous level of shocks. Antràs, et al. (2009) analyze the impact of imperfect capital market on FDI flows which predicts that the cost of financial contracting and weak investor protection increases the reliance on FDI flows.

Finally, regarding the DSGE model with endogenous entry and exit, Bilbiie, Ghironi and Melitz (2007a, 2007b) introduced the endogenous entry but assume a constant exit rate of firms. Their models make some progress in bringing the dynamics of firms into real business cycle analysis and monetary policy analysis. However, their models generate some counter-intuitive impulse responses, e.g., inflation reacts positively to an expansionary productivity shock. And their models do not perform better than traditional RBC models in terms of second moment. Nevertheless, their models are important for understanding the effect of endogenous entry, as a lot of authors are emphasizing the importance of firms dynamics: Campbell (1998), Jaimovich and Floetotto (2008) etc.

1.3 MAIN CONTRIBUTION

This thesis contributes to the growing literature on financial development and firms internationalization in respect to that we are the first to address the impact of multiple sources of external finance on firms FDI. We discuss the selection effect through financial market such that besides the positive impact of technology spill over of multinationals on host countries, FDI can bring productivity gains in sourcing country through competition in financial market.

Moreover, the thesis is the earliest research that has close look at the risk of FDI and links it with the financial structure of the sourcing countries. We emphasize the impact of the type other than availability of external finance on performance of FDI.
Such an examination is important when we make policy implications on the development of certain type of financial system to facilitate FDI. The strategy of modeling firms heterogeneity in continuous manner also brings benefit for addressing related questions.

Last but not least, this thesis proposes a DSGE model with endogenous entry and exit of firms. It is more close to the reality that entry and exit exhibit cyclical behaviors. Moreover, we substantially improve the performance of the model in terms of impulse responses and second moment compared to traditional New Keynesian model as well as models with only endogenous entry (Bilbiie, Ghironi and Melitz (2007a, 2007b)). Finally, the model with endogenous entry and exit finds a new mechanism of transmission of shocks, which opens the door for further optimal policy studies.
Chapter 2

Multiple Sources of Finance, Margins of FDI, and Aggregate Industry Productivity
2.1 INTRODUCTION

An emerging body of literature documents the impact of financial development on facilitating firm internationalization. While its function through providing a larger scale of external finance and relaxing firms’ financial constraints is widely accepted, it is not clear whether the diversification of financial channels and access to alternative finance accompanied by financial development play a role. Attention was drawn to the significance of multiple sources of financing by Chairman Alan Greenspan after the 1997–98 Asian financial crisis (Greenspan, 2000). He argued that the development of alternative financing channels helped to fill the funding gap and stabilize business financing, which are especially important when either banks or capital markets freeze up in a crisis. Following this argument and motivated by the observations of credit crunch and simultaneous drawdown in foreign direct investment (henceforth FDI) in the recent financial crisis, we address the question of whether the availability of alternative financing sources could help reduce the size of the collapse and influence welfare.

Multinational firms have better access to multiple sources of finance than their domestically oriented peers. Firstly, multinational firms are usually large and productive ones (Helpman et al., 2004; Mayer and Ottaviano, 2007). Thus, they have a better chance of accessing market finance other than bank borrowing (Cantillo and Wright, 2000). Moreover, some firms can gain additional financial support from business partners or from the government in the form of trade credit or special policy loans. Secondly, multinational firms have access to finance from different locations. They can obtain finance from their parent country, raise funds from their host country locally or in some cases explore lower-cost finance on a worldwide basis (Antras et al., 2009; Marin and Schnitzer, 2006). Meanwhile, the internal capital market among the parent company and its foreign affiliates plays an important role
for multinational firms. The allocation of funds through the internal capital market extensively substitutes for external financing when the latter is costly (Desai et al., 2004). Finally, firms tend to keep a precautionary fund reserve to adapt to potential risks and uncertainty (Bates et al., 2009; Riddick and Whited, 2009), which is particularly the case for multinational firms considering the extra cost and higher risk in foreign operations.

Basing on a heterogeneous firm set-up, we model firms’ access to the internal capital market, bank finance as well as bond finance and investigate how firms’ adjustment among multiple sources of finance affects their performance in foreign direct investment and the aggregate industry productivity. We find that given exogenous contraction in the supply of bank finance, firms with different productivities react differently. Some less productive firms exit from the foreign market due to less access to bank finance and the unaffordable high cost of bond finance as a result of tougher competition in the bond market. In comparison, some relatively more productive firms can resort to bond finance as compensation for decreased bank finance to sustain their multinational status. The increased demand for bond finance as a substitute for bank finance by the surviving multinationals exacerbates the competition in the bond market and bids up the bond return rate, which triggers a Melitz-type selection effect through the bond market and brings aggregate industry productivity gains. However, the divestment of those failing FDI firms and thus their reduced bond financing demand mitigate this effect.

The contribution of this chapter is threefold. Firstly, it complements the quickly growing literature on credit constraint and firm internationalization by firstly proposing the impact of alternative financing and differentiating firm responses to the worsening financial condition. Manova (2007) introduces credit constraint into Melitz’s (2003) research and argues that credit constraint restricts firms’
participation and performance in cross-border activity. Arndt et al. (2009), Berman and Hericourt (2008), Buch et al. (2009), Li and Yu (2009) and Muuls (2008) provide supportive evidence for this argument using firm-level data from different countries. We reproduce this result that bad credit conditions impede firms from engaging in FDI. Furthermore, we show that this effect could be mitigated with the existence of alternative financing and could vary across firms with different productivities. Compensation from bond finance and the reallocation of the available funds stabilize firm financing and facilitate FDI. However, only the most productive firms are able to take advantage of multiple sources of finance in smoothing foreign investment.

Secondly, this chapter contributes to the work on financial systems by analyzing the complementary and substitution effects of bank finance and bond finance. Precisely, we find that more productive firms use more alternative finance as substitution to reduce the risk of credit shortage and risk of investment; hence the failure rate of firms’ FDI is endogenized in our model. The less productive firms, on the contrary, being unable to afford more expensive alternative finance, will choose to exit FDI market facing credit crunch; hence we also observe complementary effects. In existing literature, Datta et al. (1999) and Diamond (1991) document the complement of bank finance to bond finance by monitoring. Davis and Mayer (1991) show that the bank and bond markets can be alternatives to each other but they are not perfect substitutes. Saidenberg and Strahan (1999) focus on the role of bank finance in providing a back-up source and liquidity insurance for bond finance against market shocks. The complementary and substitution effects coexist in our model, which vary across firms. Although the substitution of multiple sources of finance could reduce the sensitivity of FDI to adverse shocks, only a fraction of more productive firms benefit from it. The complementary effect of bond finance on bank finance for those less productive firms implies that bond finance cannot fully
substitute for bank finance when the banking sector faces a crisis. In our model, it is the higher cost of bond finance over bank finance that hinders less productive firms from employing alternative financing, thus leading to the limited substitutability between the two sources. Our result suggests the importance of reducing the cost of bond finance and developing multi-layers of the financial system to satisfy the financing demand of various firms, especially those lower-quality firms.

Thirdly, we propose FDI-induced aggregate productivity gains for the parent country through the selection effect in the capital market. Although the question of whether FDI benefits its host country in productivity through technology spillover to local firms is widely discussed (Aitken and Harrison, 1999; Bitzer and Görg, 2005; Haskel et al., 2002; Javorcik, 2004; Keller and Yeaple, 2003), the impact of FDI on the parent country is rarely considered. Compared with Pottelsberghel and Lichtenberg (2001), who present evidence that a country gains from outward FDI through technology sourcing, we show that FDI could bring aggregate productivity gains for the parent country through the reallocation of financial resources towards more productive firms. The tougher competition in the bond market induced by the large FDI financing demand selects the least productive firms out of production and enhances the aggregate productivity. However, this effect is dampened due to firms' adjustment among multiple sources of finance.

This chapter is organized as follows: section 2.2 starts with the model in a closed economy as a benchmark case. After that, we introduce multiple sources of finance in an open economy setting, allowing firms to go abroad where the interaction of bank finance and bond finance and its impact on the margins of FDI are investigated. Section 2.3 characterizes the general equilibrium and discusses the aggregate outcome on industry productivity. Section 2.4 concludes.
2.2 THE MODEL

Consider a world with two countries. We call one country the home (domestic) country and the other the host (foreign) country for FDI. There is a continuum of firms, indexed by $i$, producing differentiated varieties in each country.

Firm $i$ is born with initial internal fund $N_i$, which is a random number from a common distribution $\Gamma(N_i)$. After paying an entry cost of $f_i(f_i < N_i)$, the firm draws productivity $\phi_i$ from a common distribution $g(\phi)$ (Melitz, 2003). With the knowledge of its own productivity, the firm makes the investing decision among three potential options: (1) purchasing corporate bonds $B_i$; (2) investing in domestic production, i.e. producing and selling a distinct product $\omega$ in the home country, the output being denoted by $q_{iD}$; (3) engaging in FDI, i.e. producing and selling $\omega$ in the host country, the output being denoted by $q_{iF}$. Note that the subscript $D$ denotes variables for domestic production whereas $F$ denotes those for foreign production; these apply to the whole chapter.

There is a perfect bond market in the economy in which firms can either buy or issue bonds, $B$, being positive or negative accordingly. Upon a draw of very low productivity, producing is not as profitable as buying bonds. The firm therefore invests all its internal funds in bond holdings to achieve a safe return. Upon a draw of high productivity, on the contrary, the firm will produce. If its internal fund is not enough to pay the production cost, the firm will raise the working capital by issuing corporate bonds through bond markets.

There is no fixed cost for the firm to invest in the bond market. In contrast, if the firm engages in production, regardless of whether it is domestic production or FDI, it must pay a fixed overhead cost $f$ to set up the factory. In addition, there is an extra
fixed cost $C_F$ for FDI. $f$ and $C_F$ are measured in labor units.

2.2.1 CLOSED ECONOMY

This subsection provides the closed economy case as a benchmark in which firms only serve the domestic market and obtain external finance merely by issuing corporate bonds.

2.2.1.1 Demand

The utility function of a representative consumer is

$$U = \left[ \int_{\omega \in \Omega} q(\omega)^{\varepsilon-1} \, d\omega \right]^{\varepsilon-1}$$

where the set $\Omega$ represents the mass of available varieties and $\varepsilon$ denotes the elasticity of substitution between any two varieties. Defining the aggregate good $Q=U$ with the aggregate price

$$P = \left[ \int_{\omega \in \Omega} p(\omega)^{1-\varepsilon} \, d\omega \right]^{\frac{1}{1-\varepsilon}}$$

and solving the expenditure minimization problem of the consumer, we have the demand function for every variety $\omega$.

$$q(\omega) = \left( \frac{P}{p(\omega)} \right)^{\varepsilon} Q$$  \hspace{1cm} (2.1)

2.2.1.2 Production

Each firm $i$ produces a distinct variety $\omega$ and its output for the domestic market is denoted as $q_{iD}$. Labor is the only input. Define the cost function for producing $q_{iD}$ as:

$$l_{iD} = \frac{q_{iD}}{\varphi_i} + f$$  \hspace{1cm} (2.2)

where $f>0$ is the fixed cost for production, which is the same for any single firm. $\varphi_i$ is the firm-specific productivity. The domestic nominal wage is denoted as $w_D$. Assume that labor must be prepaid.
2.2.1.3 Bond Market

Assume that the bond market is perfect in the sense that it is competitive and there is no information asymmetry, and the equilibrium bond rate is \( r \). Firms can invest their internal funds in buying a bond and achieve a return rate of \( i + r \). In comparison, firms for which the domestic production is confined by limited internal funds can also issue bonds at the rate of \( i + r \). In the general equilibrium setting, the bond return rate \( r \) is determined by the condition that there is no aggregate net demand for bonds. For a single firm, however, \( r \) is given.

2.2.1.4 Firms' Optimal Decision

In a closed economy, firm \( i \) allocates its own disposable internal fund after entry cost is paid between bond holding \( B_i \) and domestic production \( q_{iD} \) (if it produces) and maximizes the total profit from the investment portfolio. Firm \( i \) solves

\[
\max_{p_{iD}, B_i} \pi_{iD} = p_{iD}q_{iD} - w_{iD}l_{iD} + rB_i
\]

s.t. \( w_{iD}l_{iD} + B_i \leq N_i - f_e \); (2.1); (2.2)

where \( p_{iD} \) is the product price in the home country. We have:

\[
p_{iD} = \frac{\varepsilon}{\varepsilon-1} \frac{w_{iD}}{\varphi_i} (1 + r)
\]

(2.3)

\[
q_{iD} = \left[ \frac{(\varepsilon-1)\varphi_i P}{\varepsilon w_{iD} (1 + r)} \right] Q
\]

(2.4)

\[
l_{iD} = \varphi_i^{\varepsilon-1} \left[ \frac{(\varepsilon-1)P}{\varepsilon w_{iD} (1 + r)} \right] Q + f
\]

(2.5)

Bond holdings \( B_i \) can be calculated from the budget constraint.

\[
B_i = N_i - f_e - w_{iD} \left( \varphi_i^{\varepsilon-1} \left[ \frac{(\varepsilon-1)P}{\varepsilon w_{iD} (1 + r)} \right] Q + f \right)
\]

(2.6)
Proposition 2.1 (composition of pricing under limited internal funds): Both the financing cost (bond rate $r$) and the labor cost (wage rate $w_D$ over firm-specific productivity $\varphi_i$) compose the product price. Other things being equal, the higher $r$, higher $w_D$ or lower $\varphi_i$, the higher the product price and the lower the output.

In our setting, the derived price $p_{iD}$ consists of three parts: labor cost $w_D/\varphi_i$, markup $c/(c-1)$ and an additional part $1+r$, where $1+r$ reflects the extra external financing cost. If a firm does not have sufficient internal funds for production, it issues a bond with a cost of $1+r$ to raise working capital. Therefore, the limited internal fund set-up results in a higher price and lower output compared to traditional set-up (e.g., Melitz 2003). To focus on the discussion on productivity in this chapter, we do not model firm heterogeneity in terms of internal fund $N$, though the effect of $N$ on firm financing and production works through aggregation. If all the firms have more internal funds ($N$ increases), they will issue fewer (or hold more) bonds, hence the bond demand increases relative to the supply and the bond return rate $r$ declines. Other things being equal, the decreased financing cost results in a lower price and the supply of each variety will increase.

2.2.1.5 Cutoff Productivity for Domestic Production

As in Melitz (2003), a firm’s profit from domestic production depends on its productivity. The less productive the firm is, the less profit it earns from production. Therefore, only those firms with productivities above a certain threshold will produce because of the existence of outside option. In our model, safe return rate from bond market is the outside option, and firms compare the profits from production and those from investing all their internal funds in purchasing bonds and choose to produce if and only if the former is greater than the latter; therefore, the cutoff productivity for domestic production $\varphi^*_{iD}$ is determined by equation (2.7) below:
\[
p_{iD}q_{iD} - w_{iD}l_{iD} + rB_i = r(N_i - f_i) \quad (2.7)
\]

Using (2.3), (2.4), (2.5) and the binding budget constraint, we have

\[
\varphi^*_{iD} = \left\{ \frac{f(\varepsilon - 1)}{Q} \left[ \frac{w_{iD}(1 + r)}{(\varepsilon - 1)p} \right]^\varepsilon \right\}^{\frac{1}{\varepsilon - 1}} \quad (2.8)
\]

**Proposition 2.2 (cutoff productivity for domestic production):** The cutoff productivity for domestic production \( \varphi^*_{iD} \) is higher with a higher fixed production cost \( f \), higher labor wage \( w_{iD} \) or higher financing cost \( r \).

\( f \) and \( w_{iD} \) measure the real cost while \( r \) measures the financial cost of production. Intuitively, proposition 2.2 says that higher cost requires higher productivity for firms to be able to produce. The shapes of the increasing relationships depend on elasticity of substitution \( \varepsilon \). For example, when \( \varepsilon \) is less than 2, the cutoff productivity is convex in \( f \), while when \( \varepsilon \) is larger than 2, it is concave in \( f \). As for the impact of the firm’s internal fund, it only works through the bond market in aggregation. As we discussed in proposition 2.1, firms’ bond holding increases with their internal funds. More aggregate internal funds could pull down the bond rate and result in a lower cutoff productivity. However, in partial equilibrium, the bond rate is exogenous for a single firm. Therefore, the internal fund is not directly related to the firm-level cutoff productivity.

**2.2.2 OPEN ECONOMY**

In this subsection, we consider the case of an open economy in the sense that firms are interested in producing domestically as well as expanding production to a foreign country by means of FDI. Meanwhile, we introduce going-abroad-oriented bank credit as alternative financing and reconsider the above firm’s investment portfolio decision. The cutoff productivity for a firm to become a multinational is
also derived. Moreover, the interaction of borrowing from a bank and issuing corporate bonds and the overall effect of multiple sources of finance are discussed.

2.2.2.1 Demand

For simplicity and without loss of generality, we assume the aggregate price index and aggregate goods index in the host country are the same as those in the home country, and are denoted again as $P$ and $Q$, respectively. We impose further the assumption that when the economy shifts from autarky to openness, $P$ and $Q$ will not change. In other words, the new varieties coming in as the result of openness will not affect the aggregate indices. The demand function for each variety in the host country is given by:

$$ q_{if} = \left( \frac{P}{P_{if}} \right)^\varepsilon Q $$

(2.9)

2.2.2.2 Production

Assume firm $i$'s productivity spills over to its foreign affiliate and it produces in the foreign country with the same productivity as in the home country but it has to shoulder an extra fixed cost $C_F$ to carry out FDI. This foreign expansion-induced fixed cost includes the expenses for building up foreign affiliates and distribution channels, collecting information about the foreign market and foreign regulations, etc. Regardless of the form of such a cost, it is independent of the firm's output and must be paid before the firm's revenue in the foreign market is generated. This cost $C_F$ is assumed to be uncertain for the firm at the moment when a firm arranges its investment portfolio. The distribution of $C_F$ is common knowledge and the FDI decision is made based on firm's expectation for $C_F$. $C_F$ is revealed when the firm sets foot on the foreign land. FDI is successful (hence FDI profit is received) only if $C_F$ is fully covered.

In an open economy, the domestic production function is the same as equation (2.2),
whereas the production function for FDI is given as:

$$ l_{iF} = \frac{q_{iF}}{\varphi_i} + f + C_F $$

where $q_{iF}$ and $l_{iF}$ are respectively output and labor input in the foreign country. Here assume that the extra fixed cost $C_F$ follows a concave distribution $f(C_F)$ with support $[0, \infty]$. The $f(C_F)$ has the cumulative distribution $F(C_F)$.

### 2.2.2.3 Going-Abroad-Oriented Loans and Probability of FDI Success

To cover $C_F$, the firm can obtain finance from banks. Assume that a going-abroad-oriented bank loan is available for all FDI firms. Such loans aim to release firms’ financial constraints due to the substantial upfront costs of FDI and are therefore assumed to be used only to shoulder $C_F$.\(^1\) Collateral is required by banks. Firm $i$ pledges a fraction $\tau, \tau \in (0,1]$, of the overhead fixed cost $f$ as collateral to obtain a bank loan of the amount of $\mu \tau f$, where $\mu$ is the multiplier over the collateral. Here we use $\mu$ to measure the availability of external bank credit, which is an indicator of country-specific financial development. The higher $\mu$ implies better access to bank credit and better financial development of a country. For simplicity, we further assume that borrowing from banks is costless as bankers are competitive and have no access to the bond market.

Moreover, to guarantee the sufficiency of funds to cover $C_F$ and thus the success of FDI, firms may keep some reserve funds $A$ besides the bank borrowing $\mu \tau f$ to pay the extra fixed cost. $A$ could be a fraction of the internal fund or financed from the bond market. Therefore, before $C_F$ is revealed, the firm has $A+\mu \tau f$ prepared. Hence, the probability of the FDI’s success is $\text{Prob}(C_F \leq A+\mu \tau f) = F(A+\mu \tau f)$, which is the endogenous decision of firms. As we shall see, for FDI firms, the more productive

\(^1\) By this assumption, we rule out the case that firms use this loan to pay for domestic production so that we can obtain results in an open economy that are comparable to those in a closed economy and focus on the effect of the bank loan on firms’ financing strategy and FDI decisions.
the firm is, the larger \( A \) is kept and the more likely that the FDI will be successful. Our model thus is related to the observation that productive multinational firms issue corporate bonds to raise capital for FDI since the profits from FDI are sufficiently large and they have higher incentive to guarantee the success.

### 2.2.2.4 Firms’ Optimal Decision

Firm \( i \) maximizes the expected total profit from bond holding, domestic production and FDI.

\[
\max_{p_{D}, p_{F}, A, \mu \varphi} E[\pi_i] = p_{iD}q_{iD} - w_{iD} l_{iD} + (p_{iF} q_{iF} - w_{iF} l_{iF}) F(A_{i} + \mu \varphi) + rB_{i}
\]

s.t. \( w_{iD} l_{iD} + w_{iF} (l_{iF} - C_{F}) + A_{i} + B_{i} \leq N_{i} - f_{i}; \) (2.1); (2.2); (2.9); (2.10);

Note that the profit from FDI is multiplied by the probability of its success. Also note that in the budget constraint, \( C_{F} \) is covered by \( A \) and \( \mu \varphi \). Denoting the expected value of \( C_{F} \) as \( C \), we have:

\[
p_{iD} = \frac{\varepsilon}{\varepsilon - 1} \frac{w_{iD}}{\varphi_{i}} (1 + r)
\]

(2.11)

\[
p_{iF} = \frac{\varepsilon}{\varepsilon - 1} \frac{w_{iF}}{\varphi_{i}} \left(1 + \frac{r}{F(A_{i} + \mu \varphi)}\right)
\]

(2.12)

\[
B_{i} = N_{i} - f_{i} - w_{iD} l_{iD} - w_{iF} (l_{iF} - C) - A_{i}
\]

(2.13)

and \( A_{i} \) is determined by:

\[
(p_{iF} q_{iF} - w_{iF} l_{iF}) F(A_{i} + \mu \varphi) = r
\]

(2.14)

Equations (2.11)–(2.14) characterize the optimal choices of an FDI firm. We can compare the prices in the home country and the host country by comparing (2.11) and (2.12), noticing that \( F(A + \mu \varphi) \leq 1 \).

The price for the domestic market has the same expression as that in the closed economy benchmark (equation (2.3) in section 2.2.1.4), which means that firms do not change their pricing strategy for the home market when they start foreign
business. Nevertheless, the actual nominal value of the domestic price may be different. When the economy shifts from autarky to openness, firms of high productivity adjust their investment portfolios: purchase fewer bonds (or issue more bonds) and allocate funds to FDI. The adjustment, as will be discussed in aggregation in section 2.3, induces a tougher competition in the bond market and drives the bond return rate up. Hence, the actual price in the home market under an open economy setting will be higher than in a closed economy, although they share the same mathematical expression.

As the reserve fund $A$ is endogenously determined by firms, the probability of successful FDI is also endogenized. Hence we have a look at what affect the choice of the reserve fund. An implicit solution of $A$ is given by equation (2.14). The simulation results are provided in Appendix 2.1 (where propositions 2.3 to 2.6 are also simulated). We have the following proposition:

**Proposition 2.3 (reserve fund for FDI):** Given that a firm maintains FDI, its reserve fund for FDI $A_i$ is higher with higher productivity $\phi$, lower credit access $\mu$, lower production fixed cost $f$ or lower bond financing cost $r$.

The relationship between $A$ and $\mu$ suggests a firm’s substitution in multiple sources of finance. When bank credit is tighter, a firm increases $A_i$ as the alternative source to cover $C_F$, so that it can maintain FDI. This finding supplements the existing literature in which firms are left helpless but exit production when bank credit is tight (Buch et al., 2009; Manova, 2007). In our model, however, firms can resort to alternative finance and keep production unaffected.

Note that borrowing from a bank has no cost but $A_i$ has a cost of $(1+r)$, because $A_i$ is raised either from internal funds or from the bond market. If the bond return rate is
higher, it is more attractive to buy bonds rather than producing, hence the firm will cut \( A_i \).

As for the negative relationship between fixed cost \( f \) and \( A \), it works in two ways. On one hand, \( f \) is a real cost of FDI. The higher the cost is, the less incentive there is for firms to undertake FDI, and hence the smaller the reserve fund firms keep for FDI projects. On the other hand, \( f \) could be used as collateral: firms can obtain greater bank loans against a larger \( f \), so they could reduce the amount of the reserve fund.

An important finding is that more productive firms keep more reserve funds and thus have a higher probability of success in producing abroad. As FDI is more profitable with higher productivity, those firms have incentives to guarantee the FDI’s success. This result differs from the previous literature, in which the probability of success or the probability of firms’ default is assumed to be exogenous and independent of firm productivity (e.g. Buch et al., 2009; Manova, 2007). In Li and Yu (2009), more productive firms have a higher probability of success but such a relationship is ex ante given without a micro foundation. In our model, however, the probability is firm-specific and firms themselves choose how much to “invest” to increase the probability of success.

**Proposition 2.4 (intensive margin of FDI):** The more productive a firm is (higher \( \varphi \)), the larger is its affiliate sale. The sale is also larger if the wage cost \( w_F \) is lower or the bond financing cost \( r \) is lower. If a firm can maintain FDI after a credit crunch\(^2\) (decrease in credit multiplier \( \mu \)), it raises working capital from issuing bonds and keeps its affiliate sale unaffected.

The first three arguments on \( \varphi_i, w_F \) and \( r \) are intuitive and easily verified through

\(^2\) We will discuss the condition for firms to maintain FDI in section 2.2.2.5.
equation (2.12). Higher productivity or a lower cost, either the wage cost or the financial cost, results in more output and sales. However, the change in bank credit availability $\mu$ triggers firms’ adjustment to their financing strategy and affects affiliate sales indirectly. In partial equilibrium, when bank credit suddenly becomes tight, firms raise more funds from the bond market to substitute for bank credit in order to keep their working capital. In our model, when $\mu$ decreases such that borrowings from banks are less, and if a firm can maintain FDI, it will increase $A$ (proposition 2.3) to keep the probability of the FDI’s success. Therefore, according to equation (2.12), as long as the bond return rate does not change in partial equilibrium, the affiliate sale $q_{iF}$ will not be affected. This result is consistent with the evidence that during the recent financial crisis, a non-negligible fraction of firms reallocate more funds to finance working capital and their sales remain unchanged or even expand, especially in domestic-oriented or non-tradable sectors (World Bank Financial Crisis Survey 2010; 2010 Survey on Current Conditions and Intention of Outbound Investment by Chinese Enterprises).

### 2.2.2.5 Cutoff Productivity for FDI

To see how productive should a firm be to be profitable to do FDI, we calculate the cutoff productivity for FDI by equation (2.15), the LHS of which is the profit when the firm engages in domestic production as well as FDI while the RHS is the profit when the firm merely serves the domestic market. The firm will expand production to the foreign country if and only if its total profit is higher than that from only serving the domestic market.

$$ p_{iD} q_{iD} - w_{pD} l_{iD} + (p_{iF} q_{iF} - w_{pF} l_{iF}) F(A + \mu \tau) + r B_{iF} = p_{iD} q_{iD} - w_{pD} l_{iD} + r B_{iD} $$

(2.15)

$B_{iF}$ comes from (2.13) and $B_{iD}$ comes from (2.6). Then we derive the expression of cutoff productivity for FDI:
where $F$ denotes $F(A+\mu \tau)$.

**Proposition 2.5 (extensive margin of FDI):** The cutoff productivity for FDI $\varphi_{iF}^*$ is lower when firms face better access to credit (higher credit multiplier $\mu$), lower bond financing cost $r$, lower production fixed cost $f$ or $C$, and lower labor wage $w_F$. The expected profit of undertaking FDI is larger with a higher $\mu$.

With the support of better availability of bank credit, more firms are able to go abroad. Meanwhile, the induced higher expected profits make FDI more attractive to firms. This result implies that better credit conditions as a result of the financial development in a country play a positive role in facilitating firm internationalization. On the contrary, various costs, such as the labor wage, overhead cost and financial cost, impede firms from going abroad.

Moreover, we have a look at the difference between the cutoff productivity for FDI and that for domestic production in order to investigate the question whether FDI firms are necessarily more productive than domestic firms. We have the following proposition:

**Proposition 2.6 (cutoff gap):** The gap between the cutoff productivity for FDI and the cutoff for domestic production ($\varphi_{iF}^* - \varphi_{iD}^*$) is lower facing lower bond rate $r$, larger credit multiplier $\mu$ and lower expected fixed cost $C$.

Comparing equation (2.16) with equation (2.8), and knowing that $F(A+\mu \tau)\leq 1$, we immediately conclude that $\varphi_{iF}^* > \varphi_{iD}^*$. Due to the existence of extra fixed costs, firms require higher productivity to attain positive profits from FDI. The two cutoffs are
equal if and only if $C = 0$. In this case, the probability of a successful FDI is 1 and firms will not keep any $A$ as it is not necessary and $A$ is costly. Proposition 2.6 states that a better credit condition (higher $\mu$) or lower bond financing cost (lower $r$) can reduce the productivity requirement for FDI and promote domestic firms’ growth into multinationals. Note that when facing a lower bond return rate $r$, both cutoffs decrease, while that for FDI declines faster, indicating the higher sensitivity of FDI to financing conditions compared with domestic production.

2.2.3 Complementarity and Substitution of Multiple Sources of Finance

FDI firms have access to two external sources of finance, i.e. borrowing from banks and issuing corporate bonds. When facing a bank credit shock, firms adjust their financing strategy and fund allocation among investment projects but firms with different productivities react differently. Take a bad credit shock as an example.

When bank credit suddenly becomes tight, i.e. $\mu$ suddenly decreases, then $\varphi_{iF}^*$ increases (proposition 2.5) and hence some relatively less productive FDI firms are forced to exit. As a result of withdrawing capital from FDI, these firms issue fewer bonds. In this case, deteriorative bank credit results in shrinking bond issuance, which we call the **complementary effect** of bond issuing and bank borrowing.

In contrast, however, those firms that are productive enough to maintain FDI under a worse credit condition issue more bonds as a substitution for reduced bank credit and keep the working capital for foreign production unchanged (proposition 2.3 and 2.4), which we call the **substitution effect** of bond issuing and bank borrowing. For the existence of the possibility to issue bonds as an alternative form of finance, firms do not necessarily experience production contraction when facing credit tightness, which implies the significance of multiple sources of finance in smoothing investment.
Figure 2.1 depicts intuitively the change in $A$ with decreased $\mu$ (from $\mu$ to $\mu'$) and the differentiation of firms in financing. As we mentioned above, only those firms with productivities that are higher than the cutoff productivity for FDI keep reserve fund $A$. The more productive the firm is, the more $A$ it raises (proposition 2.3). Therefore, $A$ is 0 for the firms with productivities lower than $\varphi'_{IF}(\mu)$, and $A$ jumps to positive at the cutoff value $\varphi'_{IF}(\mu)$ and keeps increasing with $\varphi$ after that.

![Complementary and Substitution Effect](image)

**Figure 2.1: Complementary effect and substitution effect**

Facing a bank credit shock ($\mu$ decreases to $\mu'$), the cutoff of carrying out FDI increases from $\varphi'_{IF}(\mu)$ to $\varphi'_{IF}(\mu')$. The firms with productivities in between exit from FDI and hence do not reserve $A$ anymore, while those firms with productivity higher than $\varphi'_{IF}(\mu')$ maintain FDI and raise more $A$ from issuing bonds. As the adjustment of $A$ responding to the alteration of the bank credit condition is through bond finance, Figure 2.1 shows the complementary and substitution effect of bond finance and bank finance.
2.3 AGGREGATION

2.3.1 CHARACTERIZATION OF EQUILIBRIUM IN AN OPEN ECONOMY

In an open economy, stationary general equilibrium is characterized as follows: (1) there is an aggregate cutoff productivity for domestic production $\varphi_D^*$, which is determined by equalizing the profit from purely holding bonds and that from producing domestically; (2) there is an aggregate cutoff productivity for FDI $\varphi_F^*$, which is determined by equalizing the total profit from engaging in domestic production as well as FDI with that from merely domestic production; (3) a mass $M$ of incumbent firms is partitioned into three groups in terms of productivity. Firms with productivity higher than $\varphi_F^*$ produce domestically as well as abroad. Firms with productivity lower than $\varphi_D^*$ do not produce but invest in purchasing bonds. Firms with productivity in between produce and serve the domestic market; (4) a firm's entry decision is made by equalizing the present value of the expected average profit flows $\bar{\pi}$ of all types of firms and the sunk cost for entry $f_e$; (5) in each period, a mass $M_e$ of new entrants replaces the mass of $\delta M$ of incumbent firms that exit, where $\delta$ is the probability of being hit by the “forced-exit” shock; (6) product markets clear such that the consumers’ demand is met by the firms’ supply; (7) the labor market clears to determine the wage $w$ (we assume the inelastic supply of labor $L$); (8) the bond market clears in a sense that there is no aggregate net demand for bonds, where the bond rate $r$ is determined; (9) the resource constraint is satisfied such that the total income equals the total expenditure. The derivation of the general equilibrium is given in Appendix 2.2.

2.3.2 THE COMPLEMENTARY AND SUBSTITUTION EFFECTS REVISITED
As we discussed above, when an adverse shock on bank credit occurs, the complementary effect implies that firms divest from FDI and purchase more bonds, whereas the substitution effect means that firms issue more bonds to finance FDI. In general equilibrium, the complementary effect and substitution effect influence the equilibrium in the bond market and thus the bond rate oppositely. The overall outcome is a result of the relative scale of the two effects, which further relies on the distribution of firm productivity and the severity of shocks on bank credit.

In a country where the firm distribution skews towards high productivity, facing the same contractionary bank credit shock, more firms will sustain FDI and the substitution effect will be dominant. As a result, the bond rate will increase, and vice versa.

Moreover, when facing a more severe adverse shock, more firms exit from FDI and transfer internal fund to purchase bonds. On the other hand, the survivors in FDI will issue more bonds to compensate for the reduced bank finance. Consequently, both the complementary effect and the substitution effect are stronger and the overall effect is ambiguous.

2.3.3 SELECTION EFFECT IN THE BOND MARKET AND AGGREGATE INDUSTRY PRODUCTIVITY

When an economy opens, those productive firms that go abroad will issue more bonds from the parent country to finance foreign production. The increased demand in the bond market will bid up the bond return rate and thus increase the financing cost for all the producing firms, either FDI or non-FDI firms. Facing a higher financing cost, the least productive producing firms are forced to exit from production and become bond holders. Thus, the aggregate productivity of producing firms increases. Therefore, outward FDI triggers the selection effect
through the bond market and brings aggregate industry productivity gains for the parent country.

As was previously discussed in Section 2.3.2, a shock to the bank credit supply can also influence the bond return rate and hence further the aggregate productivity gains. However, whether the change in bank credit conditions will intensify or weaken such gains relies on the relative importance of the above complementary effect and substitution effect. As a response to an adverse shock to bank finance, the rising bond rate as a result of the substitution effect will shuffle the deck and wash out less productive firms. However, the existence of the complementary effect pulls down the bond rate and mitigates this selection.

2.4 **CONCLUSION**

This chapter introduces the internal fund, bank finance and bond finance into a heterogeneous firm set-up and analyzes firms’ adjustment among multiple sources of finance and its impact on the performance of FDI and the aggregate industry productivity. We show that with access to the bond market as an alternative source of financing, firms suffering from bank lending tightness could stabilize their financing and maintain FDI. However, only the more productive firms benefit from the substitution of bond finance for bank finance. In comparison, the less efficient firms could not afford the higher cost of bond finance due to the increased competition in the bond market when economy opens, and thus exit from production. Therefore, the rising bond rate induces the reallocation of financial resources from less efficient firms towards more efficient ones and thus increases the aggregate industry productivity of the producing firms. Nevertheless, the
decreased financing demand of divesting firms helps to pull down the bond rate and thus weakens the above effect.

Our results suggest the importance of the diversification of financial channels and significance of the availability of alternative financing in smoothing foreign direct investment, which is particularly important for low-quality firms. Moreover, the selection through the bond market implies the role of the capital market in reshuffling firms, which also proposes a mechanism of FDI-induced welfare change for parent countries.

To focus on the role of alternative financing in stabilizing investment, we did not discuss the difference between bank finance and bond finance in this chapter. However, modeling their differences in restructuring, monitoring and screening will help us to understand better the limited substitutability of the two sources of finance and might generate more fruitful results. Moreover, modeling the financing sources of bank sectors and investigating the co-movement of the bank sector and the bond market constitute another direction for future research. In addition, relaxing the perfect competition assumption for the bond market and introducing a firm-specific bond rate are also interesting extensions. This is what I do in the next chapter of the thesis.
**APPENDIX 2.1: NUMERICAL SIMULATIONS OF PROPOSITIONS**

Propositions 2.1 and 2.2 are straightforward, so here we only provide the simulation results for propositions 2.3, 2.4, 2.5 and 2.6.

**Distribution of the fixed cost for FDI:** Assume $C_F$ follows Pareto distribution

$$F(x) = \Pr(C_F \leq x) = 1 - \left( \frac{b}{x} \right)^k$$

(A.2.1)

with the support of $[b, \infty]$, where $b$ and $k$ are parameters of the distribution. The probability density function of $C_F$ is therefore given by

$$f(x) = \frac{kb^k}{x^{k+1}} = \frac{k}{x} \left( \frac{b}{x} \right)^k$$

(A.2.2)

Denote the mean of $C_F$ as $c$, then $c = E(C_F) = \frac{kb}{k-1}$.

**A.2.1.1 Simulation of Proposition 2.3**

The optimal reserve fund $A$ for an FDI firm is given by equation (2.14):

$$(p_F q_F - w_F l_{IF}) f(A_i + \mu \varphi) = r$$

By inserting equations (2.9), (2.10), (2.12), (A.2.1) and (A.2.2) into equation (2.14) we obtain equation A.2.3 for the simulation.

$$\left[ \frac{\varepsilon q_F}{\varepsilon - 1} \left( 1 + \frac{r}{1 - (b(A + \mu \varphi))^f} \right) \right]^{1-d} \cdot P^Q \left[ 1 - \frac{\varepsilon - 1}{\varepsilon} \frac{1}{1 + \frac{r}{1 - (b(A + \mu \varphi))^f}} \right] - w_F (f + c) \frac{k}{A_i + \mu \varphi} \left( \frac{b}{A_i + \mu \varphi} \right)^k = r$$

(A.2.3)

Figures A.2.1.1–A.2.1.4 depict the change in $A$ with $\mu$, $r$, $f$ and $\varphi$, respectively.

**Parameter values**
Figure A.2.1.1: \( r = 0.05, \ \varphi_i = 0.5, \ f = 10, \ \varepsilon = 2, \ w_F = 1, \ P = 10, \ Q = 10, \)
\( \tau = 0.5, \ b = k = 3, \ c = 4.5. \)

Figure A.2.1.2: \( \mu = 1.4, \ \varphi_i = 0.5, \ f = 10, \ \varepsilon = 2, \ w_F = 1, \ P = 10, \ Q = 10, \)
\( \tau = 0.5, \ b = k = 3, \ c = 4.5. \)

Figure A.2.1.3: \( r = 0.05, \ \mu = 1.4, \ \varphi_i = 0.5, \ \varepsilon = 2, \ w_F = 1, \ P = 10, \ Q = 10, \)
\( \tau = 0.5, \ b = k = 3, \ c = 4.5. \)

Figure A.2.1.4: \( r = 0.05, \ \mu = 1.4, \ f = 10, \ \varepsilon = 2, \ w_F = 1, \ P = 10, \ Q = 10, \)
\( \tau = 0.5, \ b = k = 3, \ c = 4.5. \)

A.2.1.2 Simulation of Proposition 2.4
We derive the solution for $q_{IF}$ (A.2.4) by inserting equation (2.12) and the distribution of $C_F$ into equation (2.9), where variable $A$ is determined by equation (A.2.3).

$$q_{IF} = P^c Q^c \left\{ \frac{\varepsilon}{\varepsilon - 1} \frac{w_F}{\varphi_i} \left[ 1 + \frac{r}{1 - \left( \frac{b}{A + \mu \tau} \right)} \right] \right\}^{-\varepsilon}$$  \hspace{1cm} (A.2.4)

Figures A.2.2.1–A.2.2.4 show the change in $q_{IF}$ with $r$, $w_F$, $\varphi$ and $\mu$, respectively.

Parameter values

Figure A.2.2.1: $\mu = 1.4$, $\varphi_i = 0.5$, $w_F = 1$, $\varepsilon = 2$, $f = 10$, $P = 10$, $Q = 10$, $\tau = 0.5$, $b = k = 3$, $c = 4.5$.

Figure A.2.2.2: $r = 0.05$, $\mu = 1.4$, $\varphi_i = 0.5$, $w_F = 1$, $\varepsilon = 2$, $f = 10$, $P = 10$, $Q = 10$, $\tau = 0.5$, $b = k = 3$, $c = 4.5$.

Figure A.2.2.3: $r = 0.05$, $\mu = 1.4$, $w_F = 1$, $\varepsilon = 2$, $f = 10$, $P = 10$, $Q = 10$, $\tau = 0.5$, $b = k = 3$, $c = 4.5$.

Figure A.2.2.4: $r = 0.05$, $\varphi_i = 0.5$, $w_F = 1$, $\varepsilon = 2$, $f = 10$, $P = 10$, $Q = 10$, $\tau = 0.5$, $b = k = 3$, $c = 4.5$. 
A.2.1.3 Simulation of Proposition 2.5

Inserting \( F = 1 - \left( \frac{b}{A + \mu f} \right)^k \) into equation (2.16), we obtain equation (A.2.5) for simulation relating to \( \varphi_{iF}^* \). Variable \( A \) in equation (A.2.5) is determined by equation (A.2.3) in which \( \varphi_i \) takes the value of \( \varphi_{iF}^* \). Hence the result is the solution to the simultaneous equations (A.2.3) and (A.2.5).

\[
\varphi_{iF}^* = \left\{ \frac{f(x-1)}{Q} \left[ \frac{\omega_{iF}}{1 + \left( \frac{b}{A + \mu f} \right)^x} \right]^{r-1} \left[ 1 - \left( \frac{b}{A + \mu f} \right)^x + \frac{rA}{\omega_{iF}} \right] \right\}^{\frac{1}{r-1}}
\]  
\( \text{(A.2.5)} \)
The total profit of FDI firms is
\[
\pi_i = p_{d,i}q_{i,d} - w_{d,i}l_{i,d} + (p_{f,i}q_{i,f} - w_{f,i}l_{i,f})F(A_i + \mu \varphi) + rB_i \quad (A.2.6).
\]
Inserting the optimal solutions of firms' profit maximization problem given by equations (2.1), (2.2), (2.9), (2.10), (2.11), (2.12) and (2.13) into (A.2.6) and rearranging, we obtain the final simulation equation for \( \pi \).

\[
\pi_i = \left[ \frac{\varepsilon}{\varepsilon - 1} w_{d,i} (1 + r) \right]^{1-\varepsilon} P^c Q - (1 + r) w_{d,i} \left[ \frac{\varepsilon}{\varepsilon - 1} w_{d,i} (1 + r) \right]^{1-\varepsilon} P^c Q \left[ 1 - \left( \frac{b}{A + \mu \varphi} \right)^k \right] + r(N - f_i + w_i c - A) + \left[ \frac{\varepsilon}{\varepsilon - 1} w_{f,i} \left( 1 + \frac{r}{1 - \left( \frac{b}{A + \mu \varphi} \right)^k} \right) \right]^{1-\varepsilon} \left[ \frac{\varepsilon}{\varepsilon - 1} w_{f,i} \left( 1 + \frac{r}{1 - \left( \frac{b}{A + \mu \varphi} \right)^k} \right) \right]^{1-\varepsilon} P^c Q \left[ 1 - \left( \frac{b}{A + \mu \varphi} \right)^k \right] + f + c \quad (A.2.7)
\]

Variable \( A \) in equation (A.2.7) is determined by equation (A.2.3).

Figures A.2.3.1–A.2.3.4 show the change of \( \varphi_i^* \) with \( \mu, r, f \) and \( w_{f,i} \), respectively. Figure A.2.3.5 depicts the increasing relationship of \( \pi \) with \( \mu \).

**Parameter values**

Figure A.3.1: \( r = 0.05, \ f = 10, \ w_{f,i} = 1, \ \varepsilon = 2, \ \varphi_i = 0.5, \ P = 10, \ Q = 10, \)
\( \tau = 0.5, \ b = k = 3, \ c = 4.5. \)
Figure A.3.2: $\mu = 1.4, \ f = 10, \ w_F = 1, \ \varepsilon = 2, \ \varphi_i = 0.5, \ P = 10, \ Q = 10, \ \tau = 0.5, \ b = k = 3, \ c = 4.5$.

Figure A.3.3: $r = 0.05, \ \mu = 1.4, \ w_F = 1, \ \varepsilon = 2, \ \varphi_i = 0.5, \ P = 10, \ Q = 10, \ \tau = 0.5, \ b = k = 3, \ c = 4.5$.

Figure A.3.4: $r = 0.05, \ f = 10, \ \mu = 1.4, \ \varepsilon = 2, \ \varphi_i = 0.5, \ P = 10, \ Q = 10, \ \tau = 0.5, \ b = k = 3, \ c = 4.5$.

Figure A.3.5: $r = 0.05, \ f = 10, \ \varepsilon = 2, \ \varphi_i = 0.5, \ P = 10, \ Q = 10, \ \tau = 0.5, \ b = k = 3, \ c = 4.5, \ w_D = w_F = 1, \ f_e = 10, \ N = 500$. 

![Figures](image-url)
The simulation equation for $\phi_{sD}^* - \phi_{iD}^*$ is derived by equation (A.2.5) minus equation (A.2.8). Figures A.2.4.1–A.2.4.3 describe the change in the cutoff gap $\phi_{sD}^* - \phi_{iD}^*$ with $r$, $\mu$, and $c$, respectively. Note that given $k$, the relationship of $\phi_{sD}^* - \phi_{iD}^*$ and $c$ is indirectly represented by the change in $\phi_{sD}^* - \phi_{iD}^*$ with $b$.

**Parameter values**

Figure A.4.1: $\mu = 1.4$, $f = 10$, $\epsilon = 2$, $\phi_i = 0.5$, $P = 10$, $Q = 10$, $\tau = 0.5$, $b = k = 3$, $c = 4.5$, $w_D = w_F = 1$. 
Figure A.4.2: $r = 0.05$, $f = 10$, $\varepsilon = 2$, $\varphi_i = 0.5$, $P = 10$, $Q = 10$, $\tau = 0.5$,

$b = k = 3$, $c = 4.5$, $w_D = w_F = 1$.

Figure A.4.3: $r = 0.05$, $\mu = 1.4$, $\varepsilon = 2$, $\varphi_i = 0.5$, $P = 10$, $Q = 10$, $\tau = 0.5$,

$k = 3$, $w_D = w_F = 1$. 
APPENDIX 2.2: SKETCH OF THE GENERAL EQUILIBRIUM IN AN OPEN ECONOMY

Following Melitz (2003), we assume that there is an unlimited number of prospective firms waiting to enter our model. Each firm was born with an initial fund $N$. To enter, they first have to pay entry cost $f_e$ with their initial fund to draw their own productivities from a common distribution $g(\varphi)$. $g(\varphi)$ is Pareto distribution with cumulative density function $G(\varphi)$ and the support of $[b, \infty]$ (Helpman et al., 2004). Firms with high productivity produce, among which the higher ones also engage in FDI, while those with low productivity hold bonds only. All the firms face a constant probability $\delta$ of forced exit in each period. The forced exit firms can pay $f_e$ to draw new productivity again.

Denotations of endogenous variables: $M$ number of incumbent firms; $M_e$ number of new entrants in each period; $\pi$ average profit across all types of firms; $\varphi^*_D$ cutoff productivity for domestic production; $\varphi^*_F$ cutoff productivity for FDI; $P$ price index; $Q$ aggregate goods; $w$ wage; and $r$ bond rate.

The steady-state equilibrium is characterized by the following equations.

Zero cutoff profit for domestic production:

$$\varphi^*_D = \left\{ \frac{f (\varepsilon - 1)}{Q} \left[ \frac{\delta w (1 + r)}{(\varepsilon - 1)P} \right]^\frac{1}{\varepsilon - 1} \right\}^{1/\varepsilon - 1}$$

(A.2.8)

Zero cutoff profit for FDI:
\[
\varphi^* = \frac{f(\varepsilon - 1)}{Q} \left[ \frac{d\nu}{\nu} \left( 1 + r/lF \right) \right]^{\frac{1}{\varepsilon - 1}} \left( 1 + \frac{FC + rA/w}{(F + r)f} \right)\]  

(A.2.9)

Expected average profit:

\[
\bar{\pi} = G(\varphi^*_D)\nu(N - f_e) + \left[ G(\varphi^*_D) - G(\varphi^*_D) \right]\nu \pi_D u_D(\varphi)\lambda_\varphi + \left[ 1 - G(\varphi^*_F) \right]\nu \pi_F u_F(\varphi)\lambda_\varphi
\]

where \( \pi_D = p_D q_D - w l_D + r B_D \),

\[
\pi_F = p_F q_D - w l_D + (p_F q_F - w l_F)F(A + \mu \varphi') + r B_F
\]

Free entry condition:

\[
\frac{\bar{\pi}}{\delta} = f_e
\]

(A.2.11)

Firm entry equals firm exit:

\[
M_e = \delta M
\]

(A.2.12)

Labor market clearing condition:

\[
L = M \left[ G(\varphi^*_D) - G(\varphi^*_D) \right]\nu \lambda_D u_D(\varphi)\lambda_\varphi + M \left[ 1 - G(\varphi^*_F) \right]\nu \lambda_F u_F(\varphi)\lambda_\varphi + M \left[ 1 - G(\varphi^*_F) \right]\nu \lambda_F u_F(\varphi)\lambda_\varphi
\]

where \( L \) is the exogenous total supply of the economy, and labor demands for domestic production and FDI are given by:

\[
\lambda_D = \varphi^{\varepsilon - 1} \left[ \left( \varepsilon - 1 \right) P \right]^{\frac{1}{\varepsilon - 1}} Q + f \quad \text{and} \quad \lambda_F = \varphi^{\varepsilon - 1} \left[ \frac{\varepsilon - 1}{\varepsilon - 1} P \right]^{\frac{1}{\varepsilon - 1}} Q + f + c
\]

Bond market clearing condition:

\[
MG(\varphi^*_D)\nu(N - f_e) +
\]

\[
M \left[ G(\varphi^*_F) - G(\varphi^*_D) \right]\nu B_D u_D(\varphi)\lambda_\varphi + M \left[ 1 - G(\varphi^*_F) \right]\nu B_F u_F(\varphi)\lambda_\varphi = 0
\]

(A.2.14)
where $B_D = N - f_e - wD$ and $B_F = N - f_e - wD - w(l_F - c) - A$

Price index:

$$
P = \left[ M \left[ G(\varphi^*_r) - G(\varphi^*_o) \right] p_D^{-1} u_D(\varphi) u_F + M \left[ 1 - G(\varphi^*_r) \right] p_D^{-1} u_F(\varphi) u_F + M \left[ 1 - G(\varphi^*_r) \right] p_D^{-1} u_F(\varphi) u_F \right]^{1/3}
$$

(A.2.15)

Resource constraint:

$$(N - f_e)M_e + PQ = wL \quad \text{(A.2.16)}$$

We thus have 9 equilibrium conditions as well as 9 unknowns.

In equation (A.2.8)–(A.2.16),

$$u_D(\varphi) = \frac{g(\varphi)}{G(\varphi^*_r) - G(\varphi^*_D)} \quad \text{if} \quad \varphi \geq \varphi^*_D \quad \text{and} \quad u(\varphi) = 0 \quad \text{if} \quad \varphi < \varphi^*_D.$$  

$$u_F(\varphi) = \frac{g(\varphi)}{1 - G(\varphi^*_F)} \quad \text{if} \quad \varphi \geq \varphi^*_F \quad \text{and} \quad u_F(\varphi) = 0 \quad \text{if} \quad \varphi < \varphi^*_F.$$  

$$G(\varphi) = 1 - \left( \frac{b}{\varphi} \right)^k$$  

$$g(\varphi) = \frac{k}{\varphi} \left( \frac{b}{\varphi} \right)^k$$

and $A$ is an implicit function of $\varphi$, which is determined by equation (2.14):

$$(p_{iF} q_{iF} - w_F l_{iF}) f(A_i + \mu \varphi) = r.$$
Chapter 3

Financial Structure,
Productivity, and
Risk of Foreign Direct Investment
3.1 INTRODUCTION

Risk is an important element in the theory of capital structure. Firms have incentives to reduce the costs associated with various risks by adjusting their capital structure (Desai et al. 2008). Meanwhile, risk is a key driving force for the volatility of investments and returns, which is particularly the case for FDI comparing to domestic investment. When comparing the FDI performances in countries with different financial systems, we find that outward FDI flows from countries with the market-based financial system like U.S. and U.K. are more volatile than those from countries with bank-based financial system like Germany and Japan (see Figure 3.1). Hence in this chapter, we investigate the question that facing business risks in foreign direct investment, how multinational firms choose their sources of financing and whether financial structure influences the volatility of foreign direct investment. Answering this question will illuminate the potential link of financial system and volatility of FDI, and further provide policy implications about how to structure the financial system to stabilize FDI and assist firms’ internationalization.

![Figure 3.1 Financial Structure and Volatility of Outward FDI](image)

**Note:** This graph shows the annual outward FDI flow (deviations from trend) of Japan, United Kingdom, United States and Germany over 1990-2009. Standard deviation: Japan 18.7; United Kingdom 72.1; United States 68.6; and Germany 35.8. The data is in billions of US dollars at current prices and current exchange rates. Data source: UNCTAD.
In this chapter, we develop a partial equilibrium model based on information asymmetry. The hidden information is the productivity shock, which happens when firms engage in FDI. A firm enters the model with a given amount of initial wealth as internal fund and draws its productivity. After knowing its own productivity, the firm makes two decisions, one is on whether investing abroad or not and the other is on the mean of financing if it does invest. There are two types of external finance: borrowing from bank or issuing corporate bonds from a group of bondholders.

The productivity shock of FDI is ex ante unknown to all the parties (either banks, bondholders or firms), and it is only freely observable by the firm ex post. However, banks are willing to spend some resources to monitor the risk and convey the information to the borrowing firms after they pay an information acquisition fee (Fiore and Uhlig 2005). The role of the banks as delegated monitors is also assumed by Diamond (1984), Holström and Tirole (1997), etc. The underlying motivation for banks to actively participate in monitoring investment is their private relationship with the lenders. Therefore, banking finance is usually the priori choice for less productive firms or firms with high agency cost. The bondholders, in contrast, have no incentive to do so since the risk is shared by each individual holder. Therefore, the cost of financing with bond is lower due to the monitor cost associated with banks. However, bond financing faces additional risk in the sense that under financial distress, firms are completely liquidized and left with nothing.

Particularly, if a firm borrows from a bank, it can be told about the information of potential risk before making production decision. If the bank tells that a good shock will happen, the firm will engage in FDI and get positive profit; while if a bad shock is coming such that FDI is not profitable, the firm has the option to abstain from FDI trial. Thus, when firms choose bank financing, they pay an extra fee to protect
themselves from the risk of productivity shock. In contrast, if the firm uses bond financing, it saves the information acquisition fee but expose itself to the risk. When facing a good shock, the firm gets positive net profit from FDI abstracting a fixed repayment to bondholders. However, it could happen that the firm is not able to repay the bondholders when suffering from a bad shock. In this case, the firm defaults and gets nothing whereas the bondholders completely seize all the generated revenues of the firm.

The first result that our model delivers is firms’ partition in financing in terms of productivity. Those firms trying to do FDI but with relatively low productivities use bank finance to reveal the information on productivity shock ex ante to reduce the potential risks, and this is similar to purchasing insurance. In comparison, those firms with high productivities and thus able to counterweigh bad productivity shocks prefer to skip the costly middleman and issue bond directly.

Secondly, the variance of productivity shocks (the indicator of risks) also impacts firms’ financing choices. Firms investing in low-risk host countries prefer bond finance since in this case the insurance from banks is not worthwhile. By contrast, firms who engage in FDI in more risky locations are more likely to use bank finance. This result links the financial structure of FDI sourcing country with the characteristics of its host countries as well as the volatility of its FDI flows. Higher ratio of bond finance relative to bank finance is associated with safer and less volatile foreign investment.

Thirdly, the relative cost of bank finance and bond finance matters for firms’ financing decision. Intuitively, firms are inclined to use relatively cheaper finance.

This chapter contributes to the rare research on the impact of financial development
on FDI. What distinguishes us is the investigation on the structure effect of financial development. Besides reproducing the results that reduction of financing cost facilitates FDI as discussed in existing literatures, we set up a link between financial structure and FDI locations as well as volatilities based on the fact that foreign investment faces significant risks and firms have incentive to reduce such risks by choosing different financing instruments. By doing so, we suggest a new direction of policy on reforming the structure of financial systems to promote firms’ internationalization.

It also contributes to a huge body of capital structure literature in the following two aspects: first, we use productivity as a reference to segment firms in the choice of financing. We argue that productivity, besides leverage, size or cash flow focused in previous literatures, could be a key indicator for firm’s profitability and default probability, and affect its financing choices. Second, we incorporate product market into a financial structure model. Instead of calculating return of investment as in prior studies, we derive firms’ pricing and the revenues generated in product market such that the impact of financing on the intensive margin of FDI is discussed. In addition, we introduce the continuous stochastic states to calculate the cutoff productivities and derive the aggregated results for the whole economy.

The remainder of this chapter is organized as follows. Section 3.2 derives the model and firm-level predictions. Section 3.3 derives the aggregation results and discusses the relationships between financial structure, productivity and FDI risks. Section 3.4 provides some facts and evidences. Section 3.5 concludes.

### 3.2 THE MODEL

Consider a world with two countries, one home country and one potential host
country for FDI.\(^1\) We focus on the behavior of firms from home country.

A continuum of firms is born with internal fund \(n\), and they are heterogeneous in terms of productivity \(\varphi_i\). Following Melitz (2003), we assume that firm \(i\) draws its idiosyncratic productivity \(\varphi_{ii}\) from a common distribution \(G(\varphi_i)\). After the productivity is revealed, the firm decides whether to engage in FDI or not. If it does not invest in foreign country, the firm can invest all its funds in a safe asset to get gross return of \(Rn\) where \(R\) is the exogenous safe return rate in the economy. Instead, if the firm decides to carry out FDI, it faces a productivity shock \(\varphi_2\), which brings uncertainty for the FDI revenue. The property of the shock will be specified in details when we introduce production.

Assume that labor is the only input in production, which must be prepaid. Also assume that firms’ internal funds are not enough to fully finance the production, hence they need to borrow. There are two types of external creditors: one is banks and the other is a group of bondholders. Both of them have access to the safe return \(R\), but they differ in the following aspects:

As the delegated monitor of investors (Diamond, 1984), banks are willing to collect information on investment projects of their borrowers. In our model, we assume that banks spend resources to acquire information about the productivity shock \(\varphi_2\). Then conditional on the information obtained, banks offer firms the option to get loans and do FDI or abstain from FDI and keep their initial wealth. However, the reduced uncertainty comes at a cost, namely that an information acquisition fee is paid by firms to banks, which is assumed to be a share of the internal fund: \(m\).

\(^1\) When we look at the data later in section 3.4, particularly when we examine the relationship between financial structure and average risk of FDI per destination country, we extend the model to multiple host countries setting.
In comparison, the bondholders also offer the firms options to obtain funds, but there is no ex ante information acquisition about the risk of FDI. This assumption can be justified by the idea that there might be free-riding problems among bondholders since the risk is shared. As a consequence, bond finance saves the intermediary cost but it is a more risky choice because in a situation of financial distress (a very low $\varphi_2$ is realized), firms will be fully liquidized by bondholders and completely lose their initial wealth.

Intuitively, firms that have bad draws of initial productivities earn no more profit in FDI with either type of external finance than the safe return, and these firms immediately choose no FDI option. Firms with intermediate productivities go to banks and spend some initial wealth to “buy security” as they are more likely to suffer from financial distress even under the same risk. Those most productive firms would rather skip the costly middleman and issue finance from bondholders directly. The structure is summarized in Figure 3.2:

![Diagram](image-url)
3.2.1 Demand

The utility function of a representative household in the host country is:

\[ U = \left( \int_{i \in \Omega} q_{iF}^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{1}{\varepsilon-1}} \]

where the set \( \Omega \) denotes the mass of available varieties and \( \varepsilon \) denotes the elasticity of substitution between any two varieties. Defining the aggregate good \( Q = U \) with aggregate price

\[ P = \left( \int_{i \in \Omega} p_{iF}^{1-\varepsilon} di \right)^{1-\varepsilon} \]

and solving the expenditure minimization problem of the consumer, we have the demand function for every variety \( i \):

\[ q_{iF} = \left( \frac{P}{p_{iF}} \right)^{\varepsilon} Q \]

(3.1)

3.2.2 Production

Each firm \( i \) produces a distinct variety in FDI, and labor is the only input. The cost function is given as:

\[ l_{iF} = \frac{q_{iF}}{\phi_0 \phi_2} + f \]

(3.2)

where \( l_{iF} \) is the labor input, \( q_{iF} \) output and \( f \) the fixed cost for production (measured in units of labor). The labor wage is normalized to 1.

\( \phi_2 \) is the productivity shock coming from a distribution \( F(\phi_2) \). \( F \) has a non-negative support and without loss of generality, we assume \( E[\phi_2]=1 \). Following Bernanke, Gertler and Gilchrist (1999)’s proof for the interior solution, we also require that the hazard rate of \( F(\phi_2) \) is non-decreasing in \( \phi_2 \):
\[ \frac{\partial h(\varphi_2)}{\partial \varphi_2} \geq 0 \]

where \[ h(\varphi_2) = \frac{dF(\varphi_2)}{1 - F(\varphi_2)} . \] We take uniform distribution as an example.

\[ F(\varphi_2) = \frac{\varphi_2 - (1 - C)}{2C}, \quad \varphi_2 \in [1 - C, 1 + C] \]

The mean of \( F(\varphi_2) \) is 1 and the variance is \((1/3)C^2\). This variance, indicated by parameter \( C \), is the measure of the potential risk of FDI.

### 3.2.3 No FDI

The firm is unlucky to draw a very low productivity such that FDI is not profitable. In this case, the firm chooses route ① and deposits all its internal fund to get a safe return \( Rn \). The profit of this route is \( \pi_3 = (R - 1)n \).

### 3.2.4 FDI with Bank Finance

The firm has an intermediate productivity such that it could make more profit in FDI than that from route ①. FDI has an additional risk \( \varphi_2 \) due to, for example, unanticipated institution or policy change or systematic risk in foreign economy. As we mentioned above, when a firm goes to a bank, the bank is willing to spend resources on monitoring the productivity shocks. For simplicity, we assume that bank monitoring is so efficient such that the uncertainty in FDI could be completely eliminated\(^2\). The bank then conveys the information about \( \varphi_2 \) to the borrower, allowing the firm to decide whether to continue with FDI or abstain from it. However, the firm has to pay a fee for the monitoring. Here we assume the information acquisition fee is a fixed share of its internal fund. Denote the share for

---

\(^2\) Relaxing such assumption does not change our results. What we are emphasizing is the role of banks’ monitor compared to bond finance.
the fee as $r$ so that after the payment, the firm has disposable fund $(1-r)n$ left. Banks have access to the safe return rate $R$ and they are perfectly competitive.

3.2.4.1 Abstain from FDI

If it is told that a bad shock will happen, i.e., $\varphi_2$ is below some threshold value, the firm will abstain from FDI and invest its remaining internal funds to get the safe return. In this case, the firm ends up with the profit in route $\circ2$-1:

$$\pi_{A1} = (R - 1)(1 - r)n$$  \hspace{1cm} (3.3)

3.2.4.2 Engage in FDI

If it is told that a good shock will happen, i.e., $\varphi_2$ is above a certain threshold value and FDI is profitable, the firm will engage in FDI and end up in route $\circ2$-2. After paying the information acquisition fee, the firm needs to borrow:

$$X_A = l_f - (1 - r)n$$  \hspace{1cm} (3.4)

As there is no uncertainty for bank finance anymore, the participation constraint of banks is given by:

$$M_A = RX_A$$  \hspace{1cm} (3.5)

where $M_A$ denotes the amount of repayment. The profit of the firm in route $\circ2$-2 is:

$$\pi_{A2} = p_{IF}q_{IF} - l_f + X_A - M_A$$

The firm maximizes $\pi_{A2}$ subjected to the demand (3.1), technology (3.2), borrowing (3.4) and repayment (3.5), which gives the optimal price:

$$p_{IF} = \frac{\varepsilon R}{\varepsilon - 1 \varphi_1 \varphi_2}$$  \hspace{1cm} (3.6)

The price is composed with markup and marginal cost where $R$ is financing cost and labor wage is normalized to 1. The optimal output, labor input, borrowing and
repayment can be calculated with this price.

3.2.4.3 Expected Profit of Route ②

The expected profit of route ② depends on both payoffs in sub-route ②-1 and ②-2 and the corresponding probability of ending up with each route. Firms with different initial productivities \( \phi_{i} \) have the different corresponding probabilities. Precisely, firm \( \phi_{i} \) choose sub-route ②-2 instead of ②-1 if and only if:

\[
\pi_{A2} \geq \pi_{A1}
\]

which gives a threshold value of the productivity shock \( \phi_{2} \):

\[
\phi_{2}^* = \left( \frac{(\varepsilon - 1)f \left( \frac{R \varepsilon}{(\varepsilon - 1)P} \right)^{\varepsilon}}{Q} \right)^{\frac{1}{\varepsilon - 1}} \phi_{1}^{-1}
\]

(3.7)

The firm will actually do FDI only if the realization of \( \phi_{2} \), told by the bank, is greater than \( \phi_{2}^* \). Note that \( \phi_{2}^* \) is inverse in \( \phi_{1} \), which implies that more productive firms are able to bear worse shocks and thus more likely to engage in FDI.

In our example distribution of \( \phi_{2} \), we require that \( \phi_{2}^* \in \left[ 1-C, 1+C \right] \). Accordingly, we derive the range of \( \phi_{1} \) from (3.7) and define the lower and upper bound of \( \phi_{1} \) as \( A_L \) and \( A_H \) respectively.

\[
A_L \equiv \frac{1}{1+C} \left\{ \left( \frac{(\varepsilon - 1)f \left( \frac{R \varepsilon}{(\varepsilon - 1)P} \right)^{\varepsilon}}{Q} \right)^{\frac{1}{\varepsilon - 1}} \phi_{1}^{-1} \right\} \leq \phi_{1} \leq \frac{1}{1-C} \left\{ \left( \frac{(\varepsilon - 1)f \left( \frac{R \varepsilon}{(\varepsilon - 1)P} \right)^{\varepsilon}}{Q} \right)^{\frac{1}{\varepsilon - 1}} \phi_{1}^{-1} \right\} \equiv A_H
\]

For a firm with initial productivity \( \phi_{i} < A_L \), even the best shock \( \phi_{2}=1+C \) can not bring it profit in FDI, hence the firm will definitely end up with route ②-1. On the contrary, if its initial productivity \( \phi_{i} > A_H \), then even the worst shock \( \phi_{2}=1-C \) can not stop the firm from doing FDI (end up with route ②-2). Only those firms whose productivities are between \( A_L \) and \( A_H \) might end up with either route ②-1 or ②-2.
Therefore the ex ante expected profit of route \( \circ \) is derived in the following three cases:

\[
E[\pi_A] = \begin{cases} 
\pi_{A1} = \int_{\phi_1}^{\phi_2} \pi_{A1}dF(\phi_2) 
+ \int_{\phi_2}^{C} \pi_{A2}dF(\phi_2) & \text{if } \phi_i < A_L \\
\pi_{A2} = \int_{\phi_2}^{C} \pi_{A2}dF(\phi_2) & \text{if } A_L \leq \phi_i \leq A_H \\
\pi_{A1} = \int_{\phi_1}^{C} \pi_{A1}dF(\phi_2) & \text{if } \phi_i > A_H
\end{cases}
\]

Substituting \( \pi_{A1} \) and \( \pi_{A2} \) by previous results, and using the uniform distribution of \( F \), we have:

\[
E[\pi_A] = \begin{cases} 
(R-1)(1-\tau)n & \text{if } \phi_i < A_L \\
(R-1)(1-\tau)n - \frac{1+\varepsilon}{2C} Rf + A_1 \phi_i^{\varepsilon-1} + A_2 \phi_i^{-1} & \text{if } A_L \leq \phi_i \leq A_H \\
(R-1)(1-\tau)n - Rf + A_3 \phi_i^{\varepsilon-1} & \text{if } \phi_i > A_H
\end{cases}
\]

(3.8)

where

\[
A_1 = \frac{(1+C)^\varepsilon Q R (\varepsilon-1)^P}{2C \varepsilon (\varepsilon-1)} \left( \frac{\varepsilon-1}{\varepsilon R} \right)^\varepsilon,
\]

\[
A_2 = \frac{(\varepsilon-1) Rf}{2C \varepsilon} \left[ (\varepsilon-1) f \left( \frac{R \varepsilon}{Q} \right)^\varepsilon \left( \frac{\varepsilon-1}{\varepsilon R} \right) \right]^{\frac{1}{\varepsilon-1}},
\]

\[
A_3 = \frac{[(1+C)^\varepsilon - (1-C)^\varepsilon] Q R (\varepsilon-1)^P}{2C \varepsilon (\varepsilon-1)} \left( \frac{\varepsilon-1}{\varepsilon R} \right)^\varepsilon
\]

are positive constants that determine the “slope” of the expected profit as a function of initial productivity \( \phi_i \).

### 3.2.4.4 Cutoff Productivity of Route \( \circ \)

The firm will choose route \( \circ \) rather than route \( \bullet \) if and only if the expected profit of route \( \circ \) is larger than that of route \( \bullet \):

\[
E[\pi_A] \geq \pi_S
\]

(3.9)
When condition (3.9) is binding, a unique cutoff productivity $\phi_{1A}^*$ is determined. Firms with initial productivity $\phi_i < \phi_{1A}^*$ will not do FDI with bank finance considering a high probability of failing besides the information acquisition fee charged by banks. Therefore, we have the following proposition:

**Proposition 3.1**: the cutoff productivity for firms to do FDI with bank finance $\phi_{1A}^*$ is increasing with the bank cost $\tau$ and firm size $n$.

Proof: see Appendix A.3.1

$\tau$ is the share of firm’s initial wealth that is paid for monitoring. A higher $\tau$ means a higher cost for bank finance, which leads to a higher threshold for firms to access bank loans. Moreover, the cutoff $\phi_{1A}^*$ is increasing with firm size $n$ since larger firms pay higher information fee $\tau n$ with a given $\tau$. Therefore, bank finance is less attractive for larger firms. This result is consistent with the one delivered by capital structure literature (Cantillo & Wright, 2000).

**Proposition 3.2**: the cutoff productivity for firms to do FDI with bank finance $\phi_{1A}^*$ is decreasing with the uncertainty in foreign investment $C$.

Proof: see Appendix A.3.1.

When a firm goes to a bank, it pays a fee to eliminate the uncertainty in future investment, which is similar to purchasing insurance with a fixed payment. If the investment is not risky (lower $C$ and lower variance of $\phi_2$), it is not worth for the fixed fees. Hence going to banks is a less attractive choice; on the other hand, if the investment is risky, (higher $C$ and higher variance of $\phi_2$), it is more worthwhile to pay a fixed fee to reduce the risk in foreign production.
3.2.5 FDI with Bond Finance

A large number of bondholders provide direct finance for firms. They have no incentive to monitor the risk of FDI as a result of free riding problem. Therefore, if firms borrow from bondholders, they save the intermediation cost but keep unknown ex ante about the potential shocks. When a firm draws a high productivity \( \varphi_i \) such that it feels “confident” to overcome possible bad shocks, it would rather borrow from bondholders directly.

Firms are assumed to be risk-neutral and the optimal lending contract is similar to debt contract where firms take all the risk. The firm and bondholders negotiate the amount of lending and corresponding repayment. The productivity shock is realized after conducting production and it is only observable by the firm. If the profit after repayment to bondholders is non-negative, the firm repays the borrowing and collects the remained profit. Otherwise, the firm defaults and its revenue from FDI is completely liquidized and taken by bondholders. Similar to banks, bondholders also have access to the safe return rate \( R \) and they are perfectly competitive.

3.2.5.1 Optimal Contract with Bond Finance

As labor must be prepaid, the firm with \( \varphi_i \) decides how much labor \( l_{iF} \) to hire for FDI ex ante. According to the cost function (3.2), with input \( l_{iF} \), the actual output of FDI will be:

\[
\tilde{q}_{iF} = \varphi_i \varphi_2 (l_{iF} - f) \tag{3.10}
\]

And the firm’s expectation of output (ex ante target output) is:

\[
q_{iF} = E[\tilde{q}_{iF}] = \varphi_i (l_{iF} - f) \tag{3.11}
\]

Thus, we have
\[ \tilde{q}_{IF} = \varphi_2 q_{IF} \] (3.12)

and the actual price (ex post realized price) is given by the inverse demand function (3.1):

\[ \tilde{p}_{IF} = \left( \frac{Q}{q_{IF}} \right)^{\frac{1}{\varepsilon}} P = p_{IF} \varphi_2^{\varepsilon} \] (3.13)

where \( p_{IF} \) denotes the ex ante target price.

To finance FDI, the firm needs to borrow

\[ X_B = l_{IF} - n \] (3.14)

Denote the repayment as \( M_B \), which is negotiated by the firm and bondholders. Then the actual profit of the firm after repayment to bondholders is given by:

\[ \pi_B = \tilde{p}_{IF} \tilde{q}_{IF} - l_{IF} + X_B - M_B \] (3.15)

The firm will repay \( M_B \) if and only if \( \pi_B \geq 0 \).

The optimal lending contract specifies borrowing \( X_B \) and repayment \( M_B \) and the payoffs are distributed according to the following plan:

- If \( \pi_B \geq 0 \), the firm gets \( \pi_B \) and bondholders get \( M_B \).

- If \( \pi_B < 0 \), the firm defaults and get 0 while bondholders get liquidized value of FDI total revenue.

Note that \( \pi_B = 0 \) determines a threshold level of shock \( \varphi_2^{B*} \) shown in expression (3.16) such that if the firm encounters a shock \( \varphi_2 < \varphi_2^{B*} \), it will default.

\[ \varphi_2^{B*} = \left( \frac{M_B + n}{p_{IF} q_{IF}} \right)^{\frac{1}{\varepsilon - 1}} \] (3.16)
The threshold level of shock $\phi_2^{B^*}$ depends on the repayment, firm initial wealth as well as target price and output, which are further positively determined by the firm’s initial productivity $\phi_1$. Hence $\phi_2^{B^*}$ is decreasing with $\phi_1$, implying that the more productive firms are less likely to default.

Similarly, we require $\phi_2^{B^*}\in[1-C, 1+C]$, which gives some partitions on $\phi_1$. Analogous to the case in bank finance, if $\phi_2^{B^*}>1+C$, i.e., $\phi_1$ is below some certain level $B_L$, this firm will default even if it has the best productivity shock when the firm borrows from bondholders; on the other hand, if $\phi_2^{B^*}<1-C$, i.e., $\phi_1$ is above some level $B_H$, this firm will never default even if it encounters the worst shock. Only those firms whose productivities are between $B_L$ and $B_H$ have both possibilities.

If $\phi_2^{B^*}\in[1-C, 1+C]$, the expected profit of FDI with bond finance is:

$$E[\pi_B] = \int_{\phi_2^{B^*}}^{1+C} \tilde{p}_{\phi_1} \tilde{q}_{\phi_1} - n - M_\phi dF(\phi_2)$$

(3.17)

The participation constraint of bondholders is given by:

$$\int_{\phi_2^{B^*}}^{1+C} M_\phi dF(\phi_2) + \int_{1-C}^{\phi_2^{B^*}} \tilde{p}_{\phi_1} \tilde{q}_{\phi_1} - n dF(\phi_1) = RX_B$$

(3.18)

Maximizing (3.17) subject to (3.18) gives the ex ante target price of FDI

$$P_{\phi_1} = \frac{\varepsilon}{\varepsilon-1} \frac{R}{\phi_1}$$

(3.19)

and the optimal amount of lending:

$$X_B = \left( \frac{\kappa(\varepsilon-1)P}{\varepsilon R} \right)^{\varepsilon} Q_{\phi_1^{\varepsilon-1}} + f - n$$

(3.20)

where

$$\kappa = \frac{\varepsilon \left( (1+C)^{\frac{\varepsilon-1}{\varepsilon}} - (1-C)^{\frac{\varepsilon-1}{\varepsilon}} \right)}{2C(2\varepsilon-1)}$$
Compared to (3.6), the optimal price includes an extra cost $1/\kappa$ induced by potential risk. Note that $\kappa$ is decreasing in $C$, meaning that a higher potential risk results in a higher price.

Meanwhile, the optimal repayment is given by:

$$M_B = \left\{ \frac{\varphi_{f,1} \varphi_{f} (1 - C)^{\varphi_{f}} + 2C(2\varphi_{f} - 1)(Rl_{f} - (R - 1)n)}{(3\varphi_{f} - 1)(p_{f} q_{f})^{\varphi_{f}}} \right\}^{\frac{1}{\varphi_{f}}} - n \quad (3.21)$$

Hence the repayment rate on bond finance is:

$$R_B = M_B / X_B \quad (3.22)$$

**Proposition 3.3:** the repayment rate on bond finance $R_B$ is decreasing in productivity $\varphi_{f,1}$ and increasing in FDI risk $C$.

Proof: see Appendix A.3.1.

Comparing to the constant cost of bank finance (fixed monitor cost as well as fixed marginal cost), the cost of bond finance is firm-specific, which is increasing with firm’s default probability and thus decreasing with firms’ own productivity (See Figure 3.3). When FDI is more risky (higher $C$), the firm has a higher default probability, therefore bondholders charge a higher bond rate. On the other hand, if the firm has a higher productivity, it is less likely to default facing the same risk, thus its repayment rate is lower.
3.2.5.2 Expected Profit of Bond Finance

The expected profit in route ③ is given by equation (3.17) if $\phi_{1i} \in [B_L, B_H]$. Moreover, if $\phi_{1i} < B_L$, regardless of how high $\phi_2$ is, the firm will default. If $\phi_{1i} > B_H$, the firm will never default and bondholders charge the repayment rate $R_B = R$. The result is summarized by equation (3.23):

\[
E[\pi_B] = \begin{cases} 
-n & \text{if } \phi_{1i} < B_L \\
(R-1)n - Rf + B_i \phi_{1i}^{\epsilon-1} & \text{if } B_L \leq \phi_{1i} \leq B_H \\
(R-1)n - Rf + B_2 \phi_{1i}^{\epsilon-1} & \text{if } \phi_{1i} > B_H
\end{cases}
\]

(3.23)

where

\[
B_i = B_2 = \frac{QR}{\epsilon - 1} \left( \frac{(\epsilon-1)kP}{\epsilon R} \right)^\epsilon
\]

are positive constants that determine the slope of the expected profit of bond finance.

3.2.6 The Choice between Bank Finance and Bond Finance

Based on the results derived above, we summarize the relationships between
expected profits and firm’s initial productivity $\varphi_i$ in route ① (green dashed line), ② (red curve) and ③ (black curve) in Figure 3.4.

Three things are worth mentioning in this figure. First, if productivity $\varphi_i$ is sufficiently low (lower than $\varphi_{A^*}$), the profit of FDI trial with either bank finance or bond finance is lower than safe return $\pi_S$ due to the monitor cost and existence of risk respectively. When $\varphi_i < B_L$, firms using bond finance will lose all their initial wealth $n$ because of the liquidation under default. These can be seen from the equations (3.8) and (3.23).

Second, when productivity is above certain value ($A_L$ and $B_L$ for bank finance and bond finance respectively), the expected profits with both bank finance and bond finance are increasing with productivity and that with bond finance $E[\pi_B]$ increases faster. This is because the cost of bank finance is constant while the cost of bond finance is decreasing with productivity, as figure 3.3 shows. Note that $B_L$ needs not to be higher than $A_L$. For example, $B_L < A_L$ when $C=0$, i.e., there is no risk associated with FDI. In this case, no firm uses bank finance. We rule out this uninteresting case by assuming a certain level of risk.
Third, firms make their decisions on production and financing choice by comparing the expected profits of each route. Finally, firms are segmented into three types by the two cutoff productivities \( \phi_{1A}^* \) and \( \phi_{1B}^* \). Those firms whose initial productivities are below \( \phi_{1A}^* \) will not engage in FDI but get safe return as in route ①. Those whose productivities are between \( \phi_{1A}^* \) and \( \phi_{1B}^* \) borrow from banks and do FDI trials since the expected profit is higher than safe return (red curve is above the green dashed line). And those whose productivities are higher than \( \phi_{1B}^* \) borrow from bondholders and engage in FDI as now the black curve is above the red curve.

3.3 AGGREGATION

After specifying firm-level decisions, we now aggregate over individual firms to form country-wide predictions and take them to the data in next section.

3.3.1 Financial Structure of FDI Sourcing Country

In the economy, a continuum of firms (the total number is normalized to 1) draws productivity \( \phi_i \) from a common distribution \( G(\phi_i) \). Denote the number of firms who do not engage in FDI, borrow from banks and borrow from bondholders on the aggregate level as \( N^S \), \( N^A \) and \( N^B \) respectively. Then we have

\[
N^S + N^A + N^B = 1, \quad N^S = G(\phi_{1A}^*),
\]

\[
N^A = G(\phi_{1B}^*) - G(\phi_{1A}^*), \quad N^B = 1 - G(\phi_{1B}^*)
\]

(3.24)

We define the financial structure of the economy as the ratio of total bond finance over total bank finance:

\[
FinStr = \frac{BOND_T}{BANK_T}
\]

(3.25)
To calculate the financial structure, we integrate firms’ borrowings from banks and from bondholders respectively based on their productivities. First we derive the total amount of bank finance. Note that not all the firms whose productivities are between $\phi_{iA}^*$ and $\phi_{iB}^*$ borrow from banks. Some of them, upon with a bad luck of productivity shock, abstain from FDI and do not borrow (route ②-1). Only those firms with productivity shock $\phi_i > \phi_i^*$ will borrow $X_A$. As $X_A$ is given by (3.4), which further depends on the realization of productivity shock $\phi_i$, we have the ex post amount of borrowing (substituting the labor demand by the optimal price (3.6) and the corresponding demand (3.1)):

$$X_A = \left( \frac{P(e-1)}{eR} \right) \phi_{iA}^c \phi_i^c - 1 + f - (1 - \tau) n$$

Hence the ex ante conditional expected amount of borrowing from banks by firm with productivity $\phi_i$ is given by:

$$E[X_A] = \frac{1}{1 - F(\phi_i^*)} \left( \frac{P(e-1)}{eR} \right) \phi_{iA}^c \phi_i^c - 1 \int_{\phi_i^c}^{\phi_i} \phi_i^c - 1 dF(\phi_i) + \left[ f - (1 - \tau) n \right]$$

By integration on $\phi_i$, the total amount borrowed from banks (expected value) is given by:

$$BANK_T = \int_{\phi_{iA}}^{\phi_{iB}} N^A E[X_A] [1 - F(\phi_i^*)] dG(\phi_i) \quad (3.26)$$

where $\phi_i^*$ is given by (3.7), and $1 - F(\phi_i^*)$ is the probability of borrowing.

Similarly, the total amount borrowed from bondholders is

$$BOND_T = \int_{\phi_{iA}}^{\phi_{iB}} N^B X_B dG(\phi_i) \quad (3.27)$$

where $X_B$ is given by (3.20).

As we can see from (3.26) and (3.27), the aggregate financial structure depends on the two cutoff productivities $\phi_{iA}^*$ and $\phi_{iB}^*$ as well as the distribution of $G(\phi_i)$, which is intuitively depicted in Figure 3.5. Given distribution of initial productivity $G(\phi_i)$, the aggregate financial structure is determined by the relative position of the two
cutoffs, since the integration is simply the area between the distribution and the horizontal axis.

![Productivity Distribution $G(\phi_i)$](image)

**Figure 3.5: Segmentation of Firms in Production and Financing**

### 3.3.2 Financial Structure and Risk of FDI

According to the above argument, the aggregate financial structure depends on the cutoff productivities for bank finance and bond finance. Therefore, we derive the relationship between FDI risk and financial structure by examining the impact of risks on two cutoff productivities. Note that $\phi_{1A}^*$ is calculated by equalizing $E[\pi_A]$ and $\pi_2$ while $\phi_{1B}^*$ is derived by equalizing $E[\pi_A]$ and $E[\pi_B]$ (see Figure 3.4). We have the following lemma:

**Lemma 3.1:** if $C > 1/(\varepsilon - 1)$, in the expression for the expected profit from bank finance (3.8), $A_1$ is increasing in $C$, $A_2$ is decreasing in $C$ and $A_3$ is increasing in $C$. In the expression for the expected profit from bond finance (3.23), $B_1$ and $B_2$ are decreasing in $C$.

Proof: see Appendix A.3.1.

Lemma 1 says that with a higher risk of FDI (higher $C$), the slope of $E[\pi_A]$ as a function of initial productivity is steeper while the slope of $E[\pi_B]$ is flatter (see respectively the expressions (3.8) and (3.23)). Therefore with a higher risk, Figure 3.4
changes to Figure 3.6.

![Figure 3.6 The Effects of an Increase in FDI Risk](image)

When the risk is higher, bank finance becomes more attractive since bank monitoring largely reduces the uncertainty and the expected profit of bank finance is therefore higher, resulting in a lower cutoff $\varphi_{A*}$. On the contrary, bond finance is more expensive as bondholders charge higher risk premiums. In comparison to bank finance, an increase in risk results in a much higher cutoff $\varphi_{B*}$. This result is driven by the slopes change of expected profits of both types of financing and it is independent of the initial positions of the two curves. We therefore make the following proposition:

**Proposition 3.4:** other things equal, the higher risk of FDI, the lower financial structure of the economy.

Proof: by Figure 3.6 and Lemma 1.
3.3.3 Financial Structure and Productivity

As Helpman et al. (2004), we assume firms’ productivities in an economy follow Pareto distribution. Comparing the distribution of $G(\phi_1)$ and $G(\phi_1)'$ in Figure 3.7, we see that the average productivity of $G(\phi_1)'$ is higher than $G(\phi_1)$. Meanwhile, fixing the two cutoffs $\phi_{A^*}$ and $\phi_{B^*}$, more firms use bond finance in the economy with $G(\phi_1)'$. Hence we expect a higher financial structure under $G(\phi_1)'$ than $G(\phi_1)$.

![Figure 3.7 Productivity Distribution and Financial Structure](image)

**Proposition 3.5:** other things equal, the higher productivity of the home economy, the more bond finance relative to bank finance is used.

With proposition 3.5, we extend our discussion to multiple destination countries case. Assuming country-specific risk $C$, it is easily to conclude a pecking order of FDI destinations, that is, firms start FDI in countries with lower risks and then go further to countries with higher risks. The more productive the firm is, the more destinations it can invest and hence the average risk per destination is increasing. On aggregate level, with the increase of productivity, a country invests in more destinations, which bring higher risks. We thus observe a positive relationship between the financial structure (bond finance over bank finance) and the risk of FDI. Importantly, the risk of FDI in current discussion is the average risk per destination rather than the risk of one particular destination. Hence this result does not
contradict proposition 3.4. Interestingly, both proposition 3.4 and 3.5 are supported by our empirical analysis, with risk measured respectively by “per-portfolio” and “per-destination country”.

3.4 FACTS AND EVIDENCE

3.4.1 Data

In the section, we examine the relationship of financial structure with productivity, outward FDI performance at aggregate country level using the panel data including 24 countries (Australia, Austria, Belgium, Canada, Czech Republic, Denmark, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Japan, Korea Republic, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, United States) over 1990-2009 period. The share of outward FDI flow of the 24 countries in the total world amounts to 80% in 2006. All the relevant variables are summarized in Table 3.1.

<table>
<thead>
<tr>
<th>Table 3.1 Summary Statistics</th>
<th></th>
<th></th>
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<tr>
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<td>Std. Dev.</td>
<td>Min</td>
</tr>
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<td>.5307063</td>
<td>.0034382</td>
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<td>Prod</td>
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<td>5.585721</td>
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<td>Agg.Risk</td>
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<td>.0159558</td>
<td>.0061285</td>
<td>.0053529</td>
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<td>FDI Vol.</td>
<td>FDI Volatility</td>
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<td>23432.59</td>
<td>17.01059</td>
</tr>
<tr>
<td>Nr.of Dest.</td>
<td>Number of Total Destinations</td>
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<td>44.65885</td>
<td>29.54044</td>
<td>2</td>
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<tr>
<td>Ave_Risk_per_Dest.</td>
<td>Average Risk per Destination</td>
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<td>.0159541</td>
<td>.0023777</td>
<td>.0117178</td>
</tr>
</tbody>
</table>

Note: Financial structure is measured as the ratio of bond finance over bank finance.
Productivity is measured by GDP per hour. Aggregate risk is the grade for destination country risk weighted by its share in a sourcing country’s total outward FDI flow. FDI volatility is the absolute value of deviation from trend (HP-Filtered). Number of destinations is counted by authors. Average risk per destination is the sum-up risk of all destination countries divided by the number of destinations. Risk data is from Euromoney Country Risk Dataset. We take the reverse of the original data, therefore, in this chapter, higher value indicates higher risk. Original data for calculating financial structure is from Beck (2010). The data of FDI flows is from UNCTAD dataset. All other data are from OECD Dataset. For the calculation of financial structure and aggregate risk see the appendix.

3.4.2 Productivity and Location Pecking Order of FDI

Evidence 1: Countries tend to invest in more destinations over time and the average risk per destination of outward FDI increases.³

![Figure 3.8 The Evolution of Number of Destinations and Productivity. Data source: OECD.](image)

³ Alternatively, we take the distance between FDI sourcing country and its destination country into account and calculate the average risk per distance, and we find similar pattern, namely, with the increase of the total distance of all destinations, the average risk per distance increase as well.
With the productivity growing over time, countries invest in more foreign destinations. As depicted in Figure 3.8, productivity and number of FDI destinations of a country are increasing simultaneously. They are significantly positively correlated except for Austria, Canada, Czech Republic, Norway, and Spain. The correlation coefficient is higher than 0.9 for Belgium, Finland, France, Greece, Ireland, Italy, Korea, and Sweden. The average correlation of the 24 countries is 0.61.

With investing in more destination countries, the average risk per destination is increasing (see Figure 3.9), which implies a pecking order of countries in choosing FDI destinations from low risk countries to high risk countries.

Figure 3.9 The Rising Average Risk per Destination of FDI
3.4.3 Aggregate Risk and Financial Structure of FDI

From the original country-pair FDI data, we find that the amount of investment varies across destinations that have different level of risks. FDI sourcing country adjusts its investment in each destination to reduce the aggregate risk of the portfolio. We therefore define the aggregate risk of FDI as the weighted risk of all the destinations by the share of outward FDI flow to each destination in the total amount outward FDI flow. When linking it to the financial structure of the sourcing country, we have the following observation:

**Evidence 2: The higher FDI aggregate risk, the less bond finance relative to bank finance is exploited.**

As our model predicts, facing higher risk in foreign investment, bank finance is more preferred. In reality, the sourcing country divests from more risky country and invests more in safer locations. In aggregation across all the destinations, the negative relationship between aggregate risk and financial structure ratio holds (see figure 3.10).

![Figure 3.10 Financial Structure and Aggregate Risk of FDI Portfolio](image_url)

**Figure 3.10 Financial Structure and Aggregate Risk of FDI Portfolio**

**Note:** This graph shows the relationship between a country’s financial structure for FDI and
its aggregate risk of FDI location portfolio. The aggregate risk is the grade for destination country risk weighted by its share in a sourcing country’s total outward FDI flow. It is the pooled data for 24 FDI sourcing countries over 1990-2009. Number of observation = 377. corr. = -0.29, coeff. = -24.99***

3.4.4 Financial Structure and Productivity

The impact of productivity on financial structure works in two ways. When the productivity distribution skews towards higher productivity, more firms will use bond finance, leading to a higher ratio of bond finance over bank finance. Nevertheless, as evidence 1 shows, more firms will tap more risky countries and in that case bank finance is more preferred by some firms to reduce uncertainty. The data shows a positive relation between productivity and financial structure of FDI, meaning the first effect dominates the second one.

Note: This graph shows the relationship between a country’s financial structure for FDI and its productivity. Financial structure is measured as the ratio of bond finance over bank finance. The x-axis gives the GDP per hour as a country-level measurement of productivity. It is the pooled data for 24 countries over 1990-2009. Number of observation = 388, coeff. = .0032409*.
3.4.5 Financial Structure and Volatility of FDI

Table 3.2 Financial Structure and Volatility of FDI

<table>
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<td>6791.108*</td>
<td>5237.271*</td>
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<td>(2242.533)</td>
<td>(3113.537)</td>
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<tr>
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<td>219.2699**</td>
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</table>

Note: FDI volatility is the absolute value of deviation from trend. Financial structure is the ratio of bond finance over bank finance.*p<0.1, ** p<0.05, *** p<0.01. Standard errors are in parentheses. Year dummies and country dummies are not reported.

We implement simple regressions of FDI volatility on financial structure. The coefficient of financial structure is positive and significant before and after controlling for productivity and average risk of FDI, which implies the advantage of bank-based financial system in reducing FDI volatility and is consistent with the pattern showed in figure 3.1.

3.5 CONCLUSION

Countries with different financial structures vary in the performance of FDI, especially in volatility and locations. We develop a theory on how heterogeneous firms choose financing instrument between borrowing bank loans and issuing
corporate bonds to finance FDI, and investigate the link of financial structure and country-level FDI performance. We establish an asymmetric information model where the hidden information is the productivity shock that happens when the firms engage in FDI. As the delegated monitors, banks are willing to spend resources to acquire information about the coming shocks while bondholders are not motivated to do so as a result of free riding problem. Our model predicts that firms with higher productivity, hence with more resistance to bad shocks, are more likely to use corporate bonds whereas firms with lower productivities resort to bank finance since banks help reduce the uncertainty ex ante. On the other hand, the risk expectation in potential FDI host countries is a key determinant on firm’s financing choice. Firms investing in more risky countries prefer bank finance to bond finance.

We test the theory with the panel data including 24 large FDI sourcing countries over 1990-2009. We find that countries with higher aggregate productivity, less risky investment portfolio of locations have higher ratio of bond finance over bank finance, which are consistent with the model’s predictions. Meanwhile, after controlling for productivity and risk, more employment of bond finance relative to bank finance leads to higher volatility of FDI.

This chapter contributes to the emerging literature on financial development and firms’ internationalization with emphasis on the impact of the type other than the availability of external financial resources on FDI. It also differs from the existing capital structure literatures by proposing productivity as a determinant of financing choices.
APPENDIX 3.1: PROOF OF PROPOSITIONS 3.1, 3.2, 3.3 AND LEMMA 3.1

We begin with proof of lemma 3.1. Note that

\[
A_1 = \frac{(1 + C)^\varepsilon QR}{2C \varepsilon (\varepsilon - 1)} \left( \frac{(\varepsilon - 1)P}{\varepsilon R} \right)^\varepsilon, \quad A_2 = \frac{(\varepsilon - 1)Rf}{2C \varepsilon} \left( \frac{R \varepsilon}{(\varepsilon - 1)P} \right)^\varepsilon \]

\[
A_3 = \frac{[1 + C]^\varepsilon - (1 - C)^\varepsilon}{2C \varepsilon (\varepsilon - 1)} QR \left( \frac{(\varepsilon - 1)P}{\varepsilon R} \right)^\varepsilon
\]

Denote \( \Delta = \frac{QR}{2\varepsilon (\varepsilon - 1)} \left( \frac{(\varepsilon - 1)P}{\varepsilon R} \right)^\varepsilon \), then \( A_1 = \frac{(1 + C)^\varepsilon}{C} \Delta \). Hence:

\[
\frac{\partial A_i}{\partial C} = \Delta \left( \frac{(\varepsilon - 1)(1 + C)^\varepsilon - \varepsilon(1 + C)^{\varepsilon - 1}}{C^2} \right)
\]

\[
(\varepsilon - 1)(1 + C)^\varepsilon - \varepsilon(1 + C)^{\varepsilon - 1} > 0 \iff C > \frac{1}{\varepsilon - 1}
\]

Therefore, when \( C > \frac{1}{\varepsilon - 1} \), \( A_1 \) is increasing in \( C \).

Secondly, it is obvious that \( A_2 \) is decreasing in \( C \).

Thirdly, \( A_3 = \Delta \frac{(1 + C)^\varepsilon - (1 - C)^\varepsilon}{C} \), and

\[
\frac{\partial A_3}{\partial C} = \Delta \left[ \frac{(\varepsilon - 1)(1 + C)^\varepsilon - \varepsilon(1 + C)^{\varepsilon - 1}}{C^2} \right] + \frac{\varepsilon(1 - C)^{\varepsilon - 1} C + (1 - C)^\varepsilon}{C^2}
\]

As \( \frac{\varepsilon(1 - C)^{\varepsilon - 1} C + (1 - C)^\varepsilon}{C^2} > 0 \) because \( C \) is between 0 and 1, then \( C > \frac{1}{\varepsilon - 1} \) is also sufficient to guarantee that \( \frac{\partial A_3}{\partial C} > 0 \), using the result from \( \frac{\partial A_1}{\partial C} \). Hence we complete the proof that \( A_1 \) is increasing in \( C \), \( A_2 \) is decreasing in \( C \) and \( A_3 \) is increasing in \( C \).

Meanwhile, note that
\[ B_1 = B_2 = \frac{QR}{\varepsilon - 1} \left( \frac{(\varepsilon - 1)kP}{\varepsilon R} \right)^\varepsilon, \quad \text{where} \quad \kappa = \frac{\varepsilon \left( (1 + C)^{\frac{\varepsilon - 1}{\varepsilon}} - (1 - C)^{\frac{\varepsilon - 1}{\varepsilon}} \right)}{2C(2\varepsilon - 1)} \]

Since

\[ \frac{\partial \kappa}{\partial C} = \left[ \frac{\varepsilon - 1}{\varepsilon} (1 + C)^{\frac{\varepsilon - 1}{\varepsilon}} - \left( 1 + \frac{\varepsilon - 1}{\varepsilon} \right) \left( 1 + C \right)^{\frac{\varepsilon - 1}{\varepsilon}} \right] \left( 1 - \frac{\varepsilon - 1}{\varepsilon} \right) \left( 1 - C \right)^{\frac{\varepsilon - 1}{\varepsilon}} - \frac{\varepsilon - 1}{\varepsilon} \left( 1 - C \right)^{\frac{\varepsilon - 1}{\varepsilon}} \right] \frac{\varepsilon}{2(2\varepsilon - 1)} \]

then \( \frac{\partial \kappa}{\partial C} < 0 \) if and only if \( -\frac{\varepsilon}{\varepsilon - 1} < C < \frac{\varepsilon}{\varepsilon - 1} \). As \( \varepsilon \) is the elasticity of substitutions which is greater than 2, (and the empirical analysis shows \( \varepsilon = 3.8 \), Bernard, Eaton, Jensen and Kortum, 2003), and \( C \) is between 0 and 1, condition \( -\frac{\varepsilon}{\varepsilon - 1} < C < \frac{\varepsilon}{\varepsilon - 1} \) is satisfied. Therefore \( B_1 \) and \( B_2 \) are decreasing in \( C \).

With the result of Lemma 3.1, proposition 3.2 that \( \Phi_{A*} \) is decreasing in \( C \) can be shown by Figure 3.6.

Proposition 3.1 can be seen from the expression of the profit (3.8). When \( \tau \) is higher, ceteris paribus, the expected profit curve of bank finance in figure 3.4 (the red curve) is moved down while the expected profit of no FDI (the green line) is intact. Hence the cutoff productivity for bank finance is higher. When the initial wealth \( n \) increases, both the green line and the red curve move up but the green line moves more due to the fact that \( m \) is paid as monitoring cost. Hence the cutoff productivity \( \Phi_{A*} \) is also higher.

Proposition 3.3 discusses the cost of bond financing. Substitute \( M_B \) and \( X_B \) by the results from optimal contract and optimal target price and labor demand, and take the partial derivatives with respect to \( \Phi_i \) or \( C \) to complete the proof.
APPENDIX 3.2: CALCULATION OF FINANCIAL STRUCTURE OF FDI

We have financial structure data for the whole economy of each FDI sourcing country which includes the finance for FDI as well as for domestic investment. Remember that we are trying to build a relationship between the financial structure and FDI risk where the financial structure is the one for FDI only. Therefore, we need to isolate the finance for FDI from that for domestic investment and figure out the financial structure of only FDI firms (aggregate FDI firms’ financial structure). Our data remains at macro level.

Table A.3.1 Denotations of Variables for Constructions

<table>
<thead>
<tr>
<th>Variable</th>
<th>Label</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>national total investment</td>
<td>Gross Capital Formation as proxy, data available directly</td>
</tr>
<tr>
<td>$F$</td>
<td>total outward FDI flow</td>
<td>data available directly</td>
</tr>
<tr>
<td>$I$</td>
<td>total inward FDI flow</td>
<td>data available directly</td>
</tr>
<tr>
<td>$D$</td>
<td>domestic firms’ total investment</td>
<td>$D = T - I + F$, investment of domestic firms, both FDI firms and non FDI firms, in both home country and foreign country, data available by calculation</td>
</tr>
<tr>
<td>$R_F$</td>
<td>risk of OFDI</td>
<td>the inverse of the above index of Agg.Risk, data available by calculation</td>
</tr>
<tr>
<td>$R_D$</td>
<td>risk of domestic production</td>
<td>the inverse of sourcing country risk, data available directly</td>
</tr>
<tr>
<td>$S_F$</td>
<td>financial structure for FDI</td>
<td>variable of interest</td>
</tr>
<tr>
<td>$S_D$</td>
<td>financial structure for domestic production</td>
<td>intermediate variable</td>
</tr>
<tr>
<td>$S$</td>
<td>financial structure of the whole economy, including the finance for both domestic production and FDI.</td>
<td>data available directly</td>
</tr>
</tbody>
</table>

Since we have assumed that all firms raise their finance at their home countries, the investment that has impact on $S$ of home country is just $D$ while Inward FDI $I$ is financed from foreign country. Remember $D$ includes investment in home country
as well as in foreign country. Hence, the financial structure $S$ is the overall outcome of $S_D$ and $S_F$ where the weight on $S_D$ is $(T-I)/D$ and the weight on $S_F$ is $F/D$. We then have:

$$\frac{T-I}{D}S_D + \frac{F}{D}S_F = S$$

(A.3.1)

The relationship between $S_D$ and $S_F$ is tricky. According to the model and theories on financial structure, the higher the investment risk is, the more bank finance will be used compared to bond finance, which suggests an inverse relationship between risk and financial structure where financial structure means the ratio of bond finance over bank finance. For simplicity, we assume the relationship follows equation (A.3.2):

$$S_D R_D = S_F R_F$$

(A.3.2)

Inserting it into equation (A.3.1) we have the financial structure for FDI

$$S_F = \frac{S}{\frac{T-I}{D}R_F + \frac{F}{D}}$$
APPENDIX 3.3: CALCULATION OF AGGREGATE RISK OF FDI

We get the country-specific risk rating data in grade ranging from 0 to 100, which takes four categories of risk: economic, political, structural and credit access risk into account. Higher grade implies lower risk.

Consider a country \( i \) investing in \( N \) foreign countries. Its risk in FDI is the aggregate risk of location portfolio. To assess the aggregate risk, we construct an index for FDI sourcing country which is the weighted average risk of its host countries, the weight being the share of outward FDI flow of each host country in the total outward FDI flow of the sourcing country.

For example, consider country \( i \) as an FDI sourcing country which invests in \( N \) foreign countries. Denote the outward FDI flow to each foreign country as \( F_1, F_2, \ldots, F_N \) and the risk grade of each corresponding destination as \( R_1, R_2, \ldots, R_N \). Then the aggregate FDI risk for country \( i \) is

\[
AggRisk_i = \sum_{j=1}^{N} R_j \frac{F_j}{\sum_{j=1}^{N} F_j}
\]

Assume country risk grade \( R_j \) is constant over time during the period we examine. Because of the change of the share \( \frac{F_j}{\sum_{j=1}^{N} F_j} \), the weighted average risk is time variant. Also note that although \( R_j \in [0,100] \), it is not necessary that \( AggRisk_i \in [0,100] \) because FDI flow can be negative.
Chapter 4

Firm Heterogeneity,
Endogenous Entry and Exit,
and Monetary Policy
4.1 INTRODUCTION

As Woodford (2003) writes in his book, “the development of such a theory is an urgent task, for rule-based monetary policy ... is possible only in the case that the central banks can develop a conscious and articulate account of what they are doing”, many central banks now employ micro founded, dynamic, stochastic, general equilibrium (DSGE) models as the “theory” for their policy decisions. Most of these existing models have similar features that firms are monopolistic competitive, hence they have pricing power and they generate positive profit. Monetary policy plays a role given that there are rigidities in the economy, and now we have fruitful development of the theories, each of which incorporates different rigidities respectively (for example, the real rigidities are introduced by Akerlof and Yellen (1985) for real cost of price adjustment (also Rotemberg 1982); Mankiw and Reis (2002) for cost of acquiring information (sticky information); Woodford (2003) for capital adjustment cost; Ravn, Schmitt-Grohé and Uribe (2006) for Habit formation; the nominal rigidities are analyzed by Taylor (1979, 1980) for wage rigidities; Calvo (1983) for sticky price).

Most of these models, however, assume a constant number of producers, and the fluctuations of the economy in these models simply reflect the reactions of producers’ intensive margin to shocks, i.e., producers react by increasing or cutting output. The extensive margin, that firms’ decision of entry or exiting the market, on the other hand, is neglected. However, empirical studies have found strong evidence of pro-cyclical behavior of firms’ entry and counter-cyclical behavior of exit. For example, Campbell (1998) shows that the entry of either new firms or new establishments is significantly positively correlated with GDP. Meanwhile, the correlation between exit and business cycle is even larger (negative correlation). To justify whether such empirical observations are driven by merely a few large

1 Here it refers to theoretical foundation for a rule-based approach to monetary policy
industries or not, Jaimovich and Floetotto (2008) assemble a new data set at industry level and confirm that each industry has such significant observations. Using data from Bureau of Economic Analysis and Business Dynamics Statistics, I draw two figures to illustrate the behaviors of entry and exit along with GDP. Figure 4.1 presents the real GDP of U.S together with its entry (measured by new business establishment) and its exit (Annual Data from 1977 to 2009). We observe the positive (negative) co-movements of entry (exit) with real GDP and that the volatility of entry and exit are larger than that of GDP. To show the pro-cyclical and counter-cyclical properties, Figure 4.2 depicts the corresponding cross-correlations between GDP and entry or exit for different leads and lags.

![Figure 4.1: Cyclical behavior of entry and exit. All series are HP-filtered log deviation from the trend.](image1)

![Figure 4.2: Correlation between Entry(t+k) and GDP(t), Exit(t+k) and GDP(t)](image2)

The entry and exit have cyclical behaviors, which certainly attract our attentions. But to explain the importance of embedding these features into theoretical models for monetary policy analysis, it is better to see how large the entry and exit account for volatility of GDP. Broda and Weinstein (2010) provide evidence that for a given
amount of increase in aggregate sales, 35% of such increase is associated with newly introduced products. Meanwhile, as Bilbiie, Ghironi and Melitz (2007b) document, “the contribution of new products (including those produced at existing firms) is substantially important enough to be a major source of aggregate output fluctuations”. They also find support from Bernard, Redding and Schott (2010) who show that 68% of firms change their product mix within each 5-year. Of these firms, 66% introduce new products as well as destruct old ones. The value of newly introduced products accounts for 33.6% of total output while the lost value of destructed products accounts for 30.4%.

Moreover, analyzing endogenous entry and exit has theoretical advantages. Besides the reports above, we also observe counter-cyclical behavior of markup. Within the framework of fixed number of producers, Rotemberg and Woodford (1992, 1995, 1999) explain such phenomenon with implicit collusion among oligopolistic behaved firms. Gali (1995) assumes that firms face demands from two sources and variation in composition of aggregate demand leads to variation of markup. The newly developed idea stems from introduction of endogenous entry, see Bilbiie et al (2007a), that pro-cyclical behavior of entry increases the competition of firms, which in turn generates counter-cyclical property of markup.

The models that incorporate endogenous entry were initialed by Bilbiie, Ghironi and Melitz (henceforth BGM, 2007a, 2007b). They first study the business cycles with endogenous entry and then add price adjustment cost to study monetary policy. However, one assumption of their (and some related literature, e.g., Bergin and Corsetti, 2008; Lewis, 2009b) models is that the exit rate of firms is constant. This assumption contradicts the observations mentioned above that the exit of firms exhibits an even more significant negative co-movement with business cycles. Moreover, their models do not perform better than traditional Real Business Cycle

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2 In Bergin and Corsetti (2008), firms are assumed to depreciate 100% each period, i.e., the value of the firm is the discounted profit of next period (no further profits).
models in respect to second moment, and some of their impulse responses to technology shock is not consistent with empirical evidences (see detail in section 4.3). Adding the feature of endogenous exit not only enables us to generate more plausible impulse responses of variables, but also substantially improves the performance of the model.

This chapter of the thesis therefore develops a fully micro-founded DSGE model with endogenous entry and exit of firms. To enable monetary policy a role, we add nominal price rigidities a-la-Calvo (1983). The challenge is that since exit is endogenous decision of firms, we cannot maintain homogenous-firms’ setup, otherwise whenever bad shocks happen to reach some “threshold” that one firm wants to exit the market, all firms quit at the same time. We therefore assume three types of firms: intermediary goods producers, wholesale firms and retailers. Intermediary goods producers are heterogeneous in productivities and they face entry and exit decisions. To make our model tractable in aggregation, price rigidities are associated with wholesale firms whose inputs are intermediary goods and outputs are sold to retailers under monopolistic competition. Retailers are perfect competitive and sell final goods to households. The numbers of wholesale firms and retailers are fixed and normalized to 1 (continuum of firms with measure 1).

Intermediary goods producers are financed by households subject to a fixed entry cost. After entry, they have to pay a fixed producing cost each period thereafter to be able to produce in the next period until they exit. We thus have a time-to-built lag of firms in our model which is in line with the observation by Devereux, Head and Lapham (1996a) that entries take place slightly prior to an increase in GDP while exit takes place contemporaneously. The fixed cost of production is financed by borrowing from banks who are perfect competitive. Banks attract deposit from households and issue loans to intermediary goods producers. Idiosyncratic productivity implies that some firms will generate negative profit after repayment
of the loans; hence it is optimal for banks to bankrupt these firms due to moral hazard problems. We therefore also incorporate the financial accelerator effects (Bernanke et al, 1999) in our model in a sense that during the economic downturn, for example, as the default expectation is higher, the banking sector will ask a higher interest rate which intensifies the severity of the downturn.

This chapter has the following main findings. First, we show that an expansionary technology shock causes number of firms to increase and inflation to decrease. The former result is due to an increase in number of entry and a decrease in exit. The latter result stems from our new version of New Keynesian Phillips curve: inflation is determined by marginal cost, expectation of next period’s inflation and also number of firms. Precisely, expansionary technology shock benefits the incumbent firms because they are able to generate higher profit. This leads to a lower cutoff productivity below which firms are bankrupted by banks, hence exit is reduced. Meanwhile, as the prospect of the economy is better, more firms enter the market. The reactions of entry and exit causes the number of producers to increase, which brings higher competition and lower market share of each firm. We thus have the observation of counter-cyclical behavior of markups without losing the pro-cyclical behavior of profit. Nevertheless, more firms in the market make the production more efficient. Therefore when number of firms increases, there are two oppositional effects that affect inflation. The first is cost effect that higher competition drives up marginal cost which has positive impact on inflation. The second is efficiency effect that has negative impact on inflation. When exit is constant, as in BGM (2007b)’s model, the cost effect dominates the efficiency effect, and inflation reacts positively to expansionary technology shocks. This is actually inconsistent with the empirical findings reported by Dedola and Neri (2006) and Smets and Wouters (2007). When exit is endogenized, as in our model, the efficiency effect dominates the cost effect because exit decision is made along with the cost effect; hence inflation reacts negatively to the shock.
Second, our model predicts that following an increase in aggregate productivity, hours worked is lower in the beginning (compared to steady state level). It then rebounds and after about 2 years surpasses the steady state level. Thus we have a negative correlation between productivity and hours worked. This prediction is supported by many empirical evidences such as Galí (1999), Galí and Rabanal (2004) and Francis and Ramey (2004). Traditional RBC models fail to provide such prediction due to their mechanism that technology shocks shift the labor demand while labor supply is not affected, and therefore hours always move in the same direction as productivity. To be in line with the data, some augmented RBC models try to incorporate other driving forces to be able to shift labor supply under the circumstances. For example, Christiano and Eichenbaum (1992) propose a model with government purchases and Bencivenga (1992) analyzes households’ preference shocks. Our model proposes a new mechanism that households react to a positive technology shock by investing in new firms and reducing labor supply to incumbent firms. The labor supply is gradually restored because investing in new firms is less attractive when number of firms increases.

Third, we find that both technology shocks and monetary policy shocks (money supply shocks) have persistent effect on total factor productivity (TFP). The direct mechanism in our model is that total output is affected by number of producers, and so is TFP. When there is a transitory technology shock, TFP responses positively and converges slower than the shock, hence the shock’s effect is amplified. Similarly, money supply shock also has real effect on TFP through its impact on firms’ dynamics. This result has important implications on empirical estimation: endogenous dynamics of firms explain part of TFP measured changes, and ignoring firms’ entry and exit may results in overestimate of exogenous shocks.

In the fourth place, we observe that the impulse responses of output and labor to a
contractionary monetary policy shock are negative but they reach the bottoms some quarters after the shock and then start to rebound. Under our calibration, the model shows that the bottoms are reached two quarters after the shock (for both output and hours), exactly the same as what observed by Christiano, Eichenbaum and Evans (1994), although Christina Romer and David Romer (2004) report that the bottom of output is reached after 1 year and a half under a new measurement of the monetary policy shock. The reason behind the sluggish reaction and lagged bottom of output in our model is that in the short run, contractionary money supply “cleans” the market such that the cutoff productivity of bankruptcy is higher (followed immediately by more exit) and the expected profit of survivals is higher (the survivals are more productive firms). This generates incentives for households to temporarily increase the investment on new firms, which further results in sluggish reaction of output. The incentives fade away and totally disappear when the policy shock gets momentum after some time, and that is when the output reaches the bottom.

Regarding the second moment, in the fifth place, our model performs much better than benchmark RBC models (e.g., King and Rebelo, 1999) and BGM (2007a, 2007b) in respect to the absolute as well as relative (to output) volatility of consumption. As a well known problem for benchmark RBC models, consumption and hours are too smooth relative to output. Our model makes substantial improvement in performance of consumption and minor contribution to hours. The additional volatility of consumption comes from households’ choice of establishing new firms, which should have been essentially similar to the choice of investment. Yet in our model, the “depreciation rate” of the investment is endogenized as we have endogenous exit of firms. This provides households better motivation and incentives to do investment, leading to a higher tolerance of consumption volatility. Moreover, concerning the correlation between variables and output, our model does not generate “too pro-cyclical” results which are also well-known problems for
standard DSGE models. Precisely, for example, both benchmark RBC model and BGM (2007b)’s model report the correlation between investment and output is 0.99 while the data is 0.80 (King and Rebelo, 1999). In our model with endogenous exit, the correlation is 0.85, which is unambiguously more close to the data.

The rest of this chapter is organized as follows. Section 4.2 presents our model. We talk about three types of producers in subsection 4.2.1, introduce banking sector and nominal contract in subsection 4.2.2, describe households’ behavior in subsection 4.2.3, and aggregate the economy in subsection 4.2.4. Section 4.3 analyzes the model where in subsection 4.3.1 we log-linearize the model to discuss the New Keynesian Phillips curve, in subsection 4.3.2 we calibrate the model and do impulse responses to technology shocks and money supply shocks, and in subsection 4.3.3 we test our model in terms of second moment. Section 4.4 concludes.

4.2 THE MODEL

4.2.1 Producers

We assume there are three types of producers in the market: the retailers, the wholesale goods producers and the intermediary goods producers. The retailers are perfect competitive who compose wholesale goods via a Constant Elasticity of Substitution (CES) technology and sell the final goods to households. There are no rigidities associated with retailers. The wholesale goods producers are identical and they input intermediary goods and produce also through a CES technology. They are monopolistic competitive and sell wholesale goods to retailers. In addition, we assume price adjustment rigidities a-la Calvo (1983) that in each period, wholesale goods producers have probability $1-\theta$ of changing the price. Finally, the intermediary goods producers are heterogeneous in productivity. They hire labor from households and produce through a linear technology (no physical capital is
assumed). There are potentially continuums of prospective intermediary goods producers who want to enter the market if the present value of entry is no less than the fixed entry cost. The incumbents, on the other hand, exit the market if their profit is non-positive (given that banks bankrupt them in case of default). The intermediary goods producers are monopolistic competitive and they sell to wholesale goods producers.

4.2.1.1 Retailers

Retailers are final goods producers. They differentiate the wholesale goods through a CES technology (Dixit and Stiglitz, 1977):

\[
Y_t = \left[ \int Y_t(z)^{\frac{\varepsilon-1}{\varepsilon}} dz \right]^{\frac{\varepsilon}{\varepsilon-1}}
\]  

(4.1)

where \( Y_t(z) \) denotes the input (demand) of wholesale goods \( z \), and \( \varepsilon \) is the elasticity of substitution. By solving expenditure minimization problem, we have the demand for wholesale goods:

\[
Y_t(z) = \left( \frac{P_t(z)}{P_t} \right)^{-\varepsilon} Y_t
\]  

(4.2)

\( P_t \) is the Consumer Price Index (CPI) defined as:

\[
P_t = \left[ \int P_t(z)^{1-\varepsilon} dz \right]^{\frac{1}{1-\varepsilon}}
\]  

(4.3)

4.2.1.2 Wholesale Goods Producers

Identical wholesale goods producers are under monopolistic competition, facing the demand by (4.2). They demand available intermediary goods as input, and the technology for production is also assumed as CES function. For simplicity, we assume the elasticity of substitution is the same as \( \varepsilon \):

\[
Y_t(z) = \left[ \int Y_t(\omega)^{\frac{\varepsilon-1}{\varepsilon}} d\omega \right]^{\frac{\varepsilon}{\varepsilon-1}}
\]  

(4.4)
\( \omega \) denotes the variety of intermediary goods while \( \Omega_t \) denotes the set of available ones at \( t \). As production function of type (4.4) is also widely used in international trade literatures because it exhibits the “love of variety” (Melitz, 2003; Melitz and Ottaviano, 2008), we use it here to borrow its property that the more varieties of intermediary goods as input, the more efficient of the production.

By expenditure minimizing, the real marginal cost of production is given by (4.5), which is exactly the Producer Price Index (PPI):

\[
m_{c_t}(z) = \left[ \int_{\Omega_t} P_t(\omega)^{-z} d\omega \right]^{\frac{1}{1-z}} = P_{t,z}
\]  

(4.5)

It is worthwhile noticing that the PPI is not necessarily the same as CPI in our framework, and they are different because of the rigidities in price setting of wholesale firms.

The demand for intermediary goods is given by:

\[
Y_t(\omega) = \left( \frac{P_t(\omega)}{P_{t,z}} \right)^{-z} Y_{t,z}
\]  

(4.6)

Furthermore, we assume that at each period the wholesale firms have probability of 1-\( \theta \) to be able to adjust the price, i.e., with probability \( \theta \), they maintain the price from previous period. Therefore, the expected net present value (NPV) with price \( P^*_t(z) \) is:

\[
V_t(z) = \sum_{i=0}^{\infty} \left[ \frac{\theta^i}{R_{t+i}} P^*_t(z) - P_{t+i}mc_{z+i}(z)Y_{t+i}(z) \right]
\]  

(4.7)

where \( P^*_t(z) \) is the optimal price set at period \( t \) and it is expected to be intact at time \( t+i \) with probability \( \theta^i \). The real interest rate \( R_{t+i} \) is the discount factor applied by households who own the wholesale firms and collect their profit. The firms maximize (4.7) subject to the demand (4.2), where the demand at time \( t+i \) is:
This gives us the optimal price of \( z \) as markup times the weighted future marginal costs:

\[
P_i^*(z) = \frac{\varepsilon}{\varepsilon - 1} \sum_{i=0}^{\infty} \psi_{t+i} P_{t+i} mc_{t+i}(z)
\]

\[
\psi_{t+i} = \frac{E_t \left[ \frac{\theta^i}{R_{t+i}} \left( \frac{1}{P_{t+i}} \right)^{1-\varepsilon} \right]}{E_t \sum_{i=0}^{\infty} \frac{\theta^i}{R_{t+i}} \left( \frac{1}{P_{t+i}} \right)^{1-\varepsilon} Y_{t+i}}
\]

If there are no price rigidities, namely, if \( \theta = 0 \), then (4.8) reduces to:

\[
P_i^*(z) = \frac{\varepsilon}{\varepsilon - 1} P mc_i(z)
\]

Because we have assumed the CES production technology of retailers, the markup of wholesale firms is constant if there are no rigidities as in (4.8'). However, with rigidities, as equation (4.8) indicates, the price takes future marginal costs into account, meaning the markup over the current period marginal cost can be written as:

\[
\rho_i = \frac{\varepsilon}{\varepsilon - 1} \sum_{i=0}^{\infty} \psi_{t+i} P_{t+i} mc_{t+i}(z)
\]

\[
\rho_i = \frac{\varepsilon}{\varepsilon - 1} \frac{\sum_{i=0}^{\infty} \psi_{t+i} P_{t+i} mc_{t+i}(z)}{P_i mc_i(z)}
\]

We will see that it is this markup that is counter-cyclical without losing the fact that profit is pro-cyclical.

### 4.2.1.3 Intermediary Goods Producers

There are continuum of intermediary goods producers, each of which has a single production line and produces a differentiated goods \( Y(\omega) \). To enter the market, it
must pay a fixed cost $f^F$ (measured in consumption, and financed by households) and then it draws its idiosyncratic productivity $\varphi$ from a common distribution $G(\varphi)$. Following the estimation by Helpman et al (2004), $G(\varphi)$ is assume to be Pareto distribution with the form:

$$G(\varphi) = 1 - \left( \frac{b}{\varphi} \right)^k$$  \hspace{1cm} (4.10)

where $b$ is the lower bound of the distribution and $k$ measures its shape. Since each firm produces differentiated goods, we can denote the goods by productivity, i.e., $Y_t(\varphi) = Y_t(\varphi)$.

Firms cannot produce immediately after the entry; instead, they must pay a fixed cost of production $f^F$ (also measured in consumption) each period thereafter until exit happens. Production takes place one period after fixed cost is paid. We thus model a time-to-built lag for production to capture the observation by Devereux, Head and Lapham (1996) as mentioned in introduction. The fixed cost of production is paid by borrowing from banks, who are perfect competitive and ask an interest rate $R_t^m$ based on zero-profit participation constraint (Detail will be discussed in Banking Sector subsection of the chapter). Firms then hire labor from households and produce through a linear technology:

$$Y_t(\varphi) = A_t \varphi L_t(\varphi)$$  \hspace{1cm} (4.11)

$A_t$ denotes the aggregate technology level, and $L_t(\varphi)$ is the labor demand by firm $\varphi$. Denote $W_t$ as the nominal wage, the real marginal cost of production is given by:

$$mc_t(\varphi) = \frac{1}{A_t \varphi} \frac{W_t}{P_t}$$  \hspace{1cm} (4.12)

As borrowing takes place one period before production, the real profit of firm $\varphi$ at period $t$ is therefore given by:

$$D_t(\varphi) = \frac{P_t(\varphi) - P_t mc_t(\varphi)}{P_t} Y_t(\varphi) - \frac{R^m_{t-1} f^F}{P_t}$$  \hspace{1cm} (4.13)
Since there are no rigidities for the intermediary goods producers, they can
costlessly adjust their prices each period. Therefore, firm $\phi$ maximizes (4.13)
subjected to the demand (4.6), which gives the optimal price (standard as markup
times nominal marginal cost):

$$P_t(\phi) = \frac{\varepsilon}{\varepsilon - 1} A_t \phi$$

(4.14)

4.2.1.4 Entry, Exit and Number of Firms

Equation (4.14) indicates that the optimal price depends on idiosyncratic
productivity $\phi$. Those less productive firms might result in non-positive profit and
they fail to repay the loans. These firms were bankrupted by banks and exit the
market. Therefore, there exist a cutoff productivity $\phi_t^*$ below which firms exit. $\phi_t^*$ is
determined by Zero Cutoff Profit (ZCP) condition, namely:

$$D_t(\phi_t^*) = 0$$

(4.15)

In our model, the cutoff $\phi_t^*$ is endogenous variable that determines the number of
exiting firms each period.

Entry decision is made under Free Entry condition, i.e., the real value of the firm ($V_t$)
after entry is no less than the entry cost $f^E$:

$$V_t = f^E$$

(4.16)

As entry is financed by households, the real value of a firm is therefore discounted
sum of expected future real profit given that the firm survives. The expression is
provided in Households subsection of the chapter (see equation 4.25).

Regarding the evolution of the number of firms, assume there are $N_{t-1}$ firms that
borrow from banks at $t-1$ and produce at $t$. As the cutoff productivity of exit is given
by (4.15) and the productivity distribution is given by (4.10), the number of
survivors at $t$ is therefore $(b/\phi_t^*)^k N_{t-1}$. Together with the new entry $N_t^E$, the number of
firms that borrow at $t$ for production in period $t+1$ is therefore given by:

$$N_t = \left( \frac{b}{\varphi_t} \right)^k N_{t-1} + N_t^E \quad (4.17)$$

### 4.2.2 Banking Sector

The banking sector is assumed to be perfectly competitive. Banks issue liabilities from households with nominal interest rate $R_t^n$, and extend loans to intermediary goods producers with rate $R_t^m$. Banks are not allowed to participate in other financial activities.

The inefficiency exists because of the moral hazard of firms. Explicitly, we assume banks have no idea of the idiosyncratic productivities of the borrowing firms. The lending takes place under banks’ expectation of firms’ probability of solvency next period. Lending contract is one-period contract and the only signal that banks observe is whether the firms repay the loan or not in the next period. Therefore, as there are $N_t$ firms borrowing for period $t+1$’s production, only $(b/\varphi_{t+1})^k N_t$ of them will be able to repay the loan with $R_t^m$. The rest default and banks collect their liquidized value. Hence the banks participation constraint is given by:

$$\left( \frac{b}{\varphi_{t+1}} \right)^k R_t^m B_t + \left( 1 - \left( \frac{b}{\varphi_{t+1}} \right)^k \right) N_t D_{t+1}^F P_{t+1} = R_t^n B_t \quad (4.18)$$

The first term in the left hand side of (4.18) represents the repayment from survivals, where $B_t$ is the total amount of credit. The second term in the left hand side of (4.18) represents the expected amount that can be recovered from default firms, where $D_{t+1}^F$ denotes the average liquidity of the default. The right hand side of (4.18) is the repayment to households. In addition, the financial market must clear such that the total credit equals to the total demand (fixed cost of production) of all $N_t$ firms:

$$B_t = P_t N_t f^F \quad (4.19)$$
The financial contract is written in nominal terms such that when there are policy shocks, the real value of the contract will be affected, which generates an even more persistent effect of the policy through financial market. Moreover, the banking sector provides a financial accelerator mechanism in a way that when the economy is in the downturn where the expected likelihood of default is larger, banks ask higher $R_t^m$ that intensify the recession, and vice versa.

The contract is essentially similar to the Costly State Verification (CSV) debt contract (Townsend, 1979; Gale and Hellwig, 1985), where we have the modification that the amount of lending is assumed to be the amount of deposit (see also Fiore and Uhlig, 2005 for a similar modification). Such modification simplifies the calculation, as banks in our model only decide the rate $R_t^m$ instead of the package $(R_t^m, B_t)$. Given up such assumption (see Bernanke et al., BGG, 1999, Carlstrom and Fuerst, 1997, for examples) adds computational burden yet the effects of the financial sector is enhanced rather than dampened because the financial accelerator not only works through price of the loan but also the quantity of loan.

### 4.2.3 Households

Households hold two types of assets: saving to the banking sector and shares in a mutual fund of intermediary goods producers. The mutual fund pays a total profit in each period that is equal to the aggregate profit of surviving firms. Denote $D_t$ as the average profit of surviving firms, then the mutual fund pays $D_t \left( b / \varphi_t \right)^k N_{t-1}$ to households at $t$ (where $\left( b / \varphi_t \right)^k N_{t-1}$ is the number of survivals). The households can also sell the shares with the price that is equal to the value of the firm ($V_t$) on a stock market. They also collect the saving in banks ($B_{t-1}$) and money holding ($M_{t-1}$) in previous period, together with their earned wage from labor supply to intermediary goods producers and profit from wholesale firms ($\Pi_{c,t}$), and decide how much to consume, how much money to be kept in the pocket, how
much savings in banks and how many shares of the mutual fund to purchase. Since the households do not know which firms will exit the market next period, they continue to finance all the surviving firms as well as new entries. The period budget constraint is given by:

\[
B_{t-1} + \frac{M_{t-1}}{P_t} + \frac{W_t}{P_t} L_t + \Pi_{t,z} + (D_t + V_t) \left( \frac{b}{\phi_t^z} \right)^k N_{t-1} = C_t + \frac{M_t}{P_t} + \frac{B_t}{R^n_t P_t} + V_t N_t
\]

(4.20)

It is worth noticing that by introducing mutual fund, the households are not facing heterogeneous firms. The mutual fund aggregates the heterogeneity and simplifies households’ budget constraint. Writing the problem in terms of share holding of individual firms will result in the same equilibrium (e.g., Ghironi and Melitz, 2005). Moreover, on the right hand side of (4.20), \( R^n_t \) is the nominal interest rate paid by banks. And according to (4.17), \( N_t \) is equal to surviving firms plus the new entry, meaning households finance all the entry.

Assume that households gain utility from consumption and real money holding but suffer from labor supply. Then they maximize expected life time utility function (4.21) subject to (4.20):

\[
E_t \left[ \sum_{t=0}^{\infty} \beta^t \left[ \ln C_t + \frac{1}{1-\sigma} \left( \frac{M_t}{P_t} \right)^{1-\sigma} - \chi \frac{L_t^{1+1/\phi}}{1+1/\phi} \right] \right]
\]

(4.21)

Here \( E_t \) denotes the expectation operator; \( \beta \) is the subject discount factor; \( \sigma \) measures the inverse of the consumption elasticity of real money demand; \( \phi \) is the Frisch elasticity of labor supply to wage and \( \chi \) is chosen to normalize the steady state labor supply. The first order conditions are given as:

\[
\chi L_t^{1/\phi} = \frac{W_t}{P_t} \frac{1}{C_t}
\]

(4.22)

\[
\left( \frac{M_t}{P_t} \right)^{-\sigma} = \frac{1}{C_t} \frac{R^n_t - 1}{R^n_t}
\]

(4.23)
\[ R_i^n = E_i \left[ \frac{C_{t+1}}{\beta C_t} \left( 1 + \pi_{t+1} \right) \right] \quad (4.24) \]

\[ V_i = E_i \left[ \beta \frac{C_t}{C_{t+1}} \left( \frac{b_{t+1}}{\varphi_{t+1}} \right)^k \left( D_{t+1} + V_{t+1} \right) \right] \quad (4.25) \]

(4.22) is the labor supply condition; (4.23) is the optimal money demand condition; (4.23) is the Euler Equation for saving, where \( \pi_{t+1} \) is the inflation rate defined as \( \pi_{t+1} = \frac{P_{t+1} - P_t}{P_t} \); (4.25) is the optimal condition for mutual fund purchase which gives us the expression of the value of the firm. With the free entry condition (4.16), the number of entry that households are willing to finance is determined. Although we don’t have physical capital in our model, the number of firms plays the role as physical capital. Households’ purchasing of new entry acts like the investment while the dynamics of the number of firms is essentially the same as the dynamics of the physical capital.

4.2.4 Aggregation

The economy resource constraint is given by (4.26), where the total final output is used for consumption, covering fixed cost of production and fixed cost of entry.

\[ Y_t = C_t + N_t f^E + N_t^E f^E \quad (4.26) \]

As mentioned earlier that number of firms acts like physical capital, \( f^E \) therefore represents “capital depreciation” while \( f^E \) represents investment.

In our model, the intermediary goods producers exit the market (if they default) after their production is complete. In other words, all the borrowing firms produce. Then the aggregate goods for wholesale firms is given by (transformation of (4.4) where the set of the available goods are substituted by the possible productivity distribution interval):

\[ Y_{t, z} = \int_{b}^{\infty} N_{t-1} Y_t(\phi) \zeta^{-1} \frac{dG(\phi)}{\phi} \bigg[ \frac{\zeta}{\zeta - 1} \bigg]^{\zeta - 1} = N_{t-1}^z Y_t(\phi^z) \quad (4.27) \]
\( \varphi^A \) is defined as:

\[
\varphi^A = \left[ \int_{b}^{\varphi^A} \varphi^{c-1} dG(\varphi) \right]^{\frac{1}{c-1}} = \left[ \frac{k}{k+1-\varepsilon} \right]^{\frac{1}{c-1}} b
\]  

(4.28)

Equation (4.27) says that the aggregate goods for wholesale firm can be represented by the average output times the number of producers (powered by \( \varepsilon/(\varepsilon-1) \)). The average output is the output of the intermediary goods producer whose productivity is \( \varphi^A \). Similarly, we can define the (weighted) average productivity of surviving firms and default firms respectively by (4.29) and (4.30):

\[
\varphi^s_t = \left[ \int_{\phi^s_t}^{\infty} \varphi^{c-1} u^s(\varphi) d\varphi \right]^{\frac{1}{c-1}} = \left[ \frac{k}{k+1-\varepsilon} \right]^{\frac{1}{c-1}} \varphi^*_t
\]  

(4.29)

\[
\varphi^d_t = \left[ \int_{0}^{\varphi^d_t} \varphi^{c-1} u^d(\varphi) d\varphi \right]^{\frac{1}{c-1}} = \left[ \frac{k}{k+1-\varepsilon} \right]^{\frac{1}{c-1}} \frac{b^k}{G(\varphi^*_t)} \left( b^{c-1} - \varphi^*_t \varphi^{c-1} \right)^{\frac{1}{c-1}}
\]  

(4.30)

where \( u^s(\varphi) \) is the productivity distribution of the surviving firms while \( u^d(\varphi) \) is that of the defaulting firms:

\[
u^s(\varphi) = \begin{cases} 
  \frac{g(\varphi)}{1-G(\varphi^*_t)} & \text{if } \varphi \geq \varphi^*_t \\
  0 & \text{if } b \leq \varphi < \varphi^*_t 
\end{cases} 
\]

\[
u^d(\varphi) = \begin{cases} 
  0 & \text{if } \varphi \geq \varphi^*_t \\
  \frac{g(\varphi)}{G(\varphi^*_t)} & \text{if } b \leq \varphi < \varphi^*_t 
\end{cases} 
\]

Similar to the transformation in (4.27) and with the definitions of (4.28), (4.29) and (4.30), we can write the PPI as:

\[
P_{t,z} = N_{t-1} \frac{1}{\varphi^A} P_r(\varphi^A)
\]  

(4.31)

The total labor demand is given by:

\[
L_t = N_{t-1} L_r(\varphi^A)
\]  

(4.32)

The average profit of surviving firms is the profit of the firm with productivity \( \varphi^s_t \):
$D_t = D_t(\varphi^0)$ and the expected liquidity of defaulting firms (collected by banks) is the liquidity of the firm with productivity $\varphi^0$: $D_t^E = D_t(\varphi^0)$.

Regarding the CPI, we note that each period there is only $1-\theta$ share of wholesale firms that adjust the price to the new optimal level while $\theta$ share of them keep the old price. Hence CPI is given by (transformation of (4.3)):

$$P_t = \left[\theta P_{t-1}^{1-\varepsilon} + (1-\theta)P_t^*(z)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$ (4.33)

The aggregate final output $Y_t$ is given by:

$$Y_t = \frac{1}{s_t} Y_{t,z} = \frac{1}{s_t} N_{t-1}^{\frac{1}{1-\varepsilon}} A_{t} \varphi^A L_t$$ (4.34)

$$s_t = (1-\theta) \left(\frac{P^*_t(z)}{P_t}\right) + \theta (1+\pi_t)^s s_{t-1}$$ (4.35)

Equation (4.34) says that a rising number of producers causes aggregate final output to increase. This effect stems from the CES production technology of wholesale firms that more varieties as input, more efficient the production will be.

The total factor productivity (TFP) is defined as:

$$TFP_t = \frac{Y_t}{L_t} = N_{t-1}^{\frac{1}{1-\varepsilon}} A_{t} \varphi^A / s_t$$ (4.36)

### 4.2.5 Shocks and Policy

Following standard real business cycle models, we assume the natural log level of aggregate technology is an AR(1) process:

$$\ln A_t = \left(1-\Psi^A\right) \ln A + \Psi^A \ln A_{t-1} + \zeta_t^A$$ (4.37)

where $\Psi^A$ is the autocorrelation coefficient, $\ln A$ is the steady state level of technology and $\zeta_t^A$ is i.i.d shocks.

Money supply, as the policy instrument in our model, is assumed as an AR(1)

---

3 See Appendix for proof
process too. Instead of arguing for the optimal monetary policy so far, we take such process as given and analyze the effect of the money supply shock. This means we have no feedbacks from the economy to policies.

\[
\ln M_t = \left(1 - \Psi^M\right) \ln M + \Psi^M \ln M_{t-1} + \zeta^M_t
\]

(4.38)

The shock \(\zeta^M_t\) is assumed to be i.i.d and independent of \(\zeta^A_t\).

4.3 ANALYZING THE MODEL

4.3.1 New Keynesian Phillips Curve and Additional Trade-off

To study the propagation of shocks and understand the mechanism of monetary policy, we log-linearize the model around the efficient steady state\(^4\). It is worthwhile to mention that by log-linearizing equation (4.33) and the optimal price (4.8), the new Keynesian Phillips curve with endogenous entry and exit it given by\(^5\):

\[
\pi_t = \kappa \left[ \hat{W}_t - \hat{A}_t \right] - \frac{\kappa}{\varepsilon - 1} \hat{N}_{t-1} + \beta E_t \left[ \pi_{t+1} \right]
\]

(4.39)

where parameter \(\kappa = (1 - \theta)(1 - \theta)/(\theta)\).

Compared to the benchmark New Keynesian model, equation (4.39) also relates the inflation dynamics with marginal cost of production. The difference is that with endogenous number of producers, we have an additional persistence in inflation dynamics. Such difference is important when we talk about the policy implications because disregarding the endogenous number of firms leads to an “endogeneity bias” when estimating the New Keynesian Phillips curve (a similar argument can be found by BGM, 2007b).

More precisely, the number of firms, as well as marginal cost, is affected by the cutoff productivity \(\varphi_0^*\) determined by Zero Cutoff Profit condition (4.15). An

\(^4\) See Appendix for the complete log-linearized model

\(^5\) Variable \(\xi\) denotes the log-deviation of \(x\) from its steady state.
increase in $\phi^\ast$ plays a role as cost-push shock which on one hand increases the marginal cost of production, and on the other hand decreases the number of producers. The two effects have impact on the inflation dynamics in the same direction. Therefore, policy makers face an additional trade-off in our framework.

4.3.2 Calibration and Impulse Response

In order to have a look at the impulse responses of endogenous variables to aggregate productivity shock and money supply shock, we calibrate our model and solve it by the method of undetermined coefficients. The period in our model is a quarter.

In the utility function of households, discount factor $\beta$ is set to 0.99 which is standard in Real Business Cycle models and implies an annual interest rate of 4% in steady state. Following Mankiw and Summers (1986) that the consumption elasticity of real money demand is 1, we set $\sigma = 1$. The Frisch elasticity of labor supply $\phi$ is set to 2 as widely applied by literatures; and $\chi$ is set to 3 to match the steady state level of labor supply to 0.36 (where total labor is normalized to 1).

The elasticity of substitution is set to $\varepsilon = 3.8$, following Bernard, Eaton, Jensen and Kortum (2003) and BGM (2007b) to fit U.S plant and macro trade data. The price rigidities is set to $\theta = 0.75$, following the estimation by Angelloni et al (2006). Regarding the productivity distribution of firms, we adopt Helpman, Melitz and Yeaple (2004)'s Pareto distribution setting; and following Ghironi and Melitz (2005)'s calculation based on the standard deviation of sales, we set the shape parameter $k = 3.4$ and the lower bound $b = 1$. The entry cost $f^E$ is normalized to 1 while the fixed cost of production is set to $f^F = 0.015$ in order to capture the average job destruction (death) rate of 5.6%.

\footnote{See data from “Longitudinal Business Database” 1977–2009. Total job destruction rate is around 15% per year, but that is induced by firms exit as well as contraction. The job destruction (death) rate measures the exit behavior.}
The parameters of shock process is given by $\Psi^A = 0.875$ and $\Psi^M = 0.85$, following Prescott (1986) and Schmitt-Grohe and Uribe (2005), where the standard deviations of $\zeta_t^A$ and $\zeta_t^M$ are all set to 0.01.

4.3.2.1 Impulse Responses to Aggregate Technology Shock

Figure 4.3 shows the impulse responses to a one percent positive aggregate technology shock. The vertical axes measure the percentage deviation of arguments from their respective steady states while the horizontal axis is the year after the shock (the period in our model is still a quarter).

Figure 4.3: Impulse Responses to a one percent positive technology shock
As expected, the positive aggregate technology shock increases output (4.3-4) and consumption (4.3-5). The households smooth the consumption by increasing investment in new firms (4.3-2). The incumbent firms benefit the positive technology shock at the beginning because they are having higher profit (4.3-10). This results in a lower cutoff productivity above which firms are able to survive (4.3-11) and the exit of firms is lower. However, as there are more and more entries, the real wage is pushed up (4.3-12), the market share of each firm is lower, and borrowing interest rate asked by banking sector begins to increase. The consequence of all these changes is that the exit of the firms starts to increase and surpasses its steady state level after about 3 quarters (4.3-3). These results are consistent with empirical findings of pro-cyclical behavior of entry and counter-cyclical behavior of exit.

In addition, as TFP depends positively on number of firms (see equation 4.36), its impulse response is hence positive and converges slower than the transitory technology shock (4.3-9), meaning the impact of the shock is amplified (even if the shock is non-persistent, firm dynamics can still generate persistent responses of labor productivity, see Vilmi (2011) for a discussion).

The inflation begins with negative reaction as the production is more efficient and number of firms is higher. It then rises and becomes positive after about one year when real wage is at peak and number of firms starts to fall (4.3-6). Our result is supported by the empirical analysis of Dedola and Neri (2006) and Smets and Wouters (2007). It is noteworthy that in BGM (2007b)'s work where only firms’ entry is endogenized while exit is assumed as constant, the inflation reacts positively in the beginning (see their figure 1). We argue that by endogenizing firms’ exit, we can correct such counter-intuitive behavior of inflation. The underlying mechanism is that with endogenous exit, the effect of endogenous entry is much stronger. Precisely, if exit rate is assumed as constant, then facing an expansionary technology
shock (even if with 0.979 persistency), the number of firms increases sluggishly from 0 and the prospect of entry is deteriorated very fast. On the contrary, with endogenous exit, the expansionary technology shock immediately decreases the number of exit, meaning the number of firms reacts stronger from a positive number. Meanwhile, the prospect of entry does not fade away that soon because higher competition is compensated by endogenous exit of firms, which generates more persistence in number of firms. Hence in our Phillips curve (equation 4.39), the effect of number of firms dominates the effect of marginal cost and inflation reacts negatively to expansionary technology shock.

Regarding the well known challenge for benchmark RBC models as well as New Keynesian models, namely modeling the counter-cyclical behavior of markups with pro-cyclicality of profit, our model works fine, as Figure 4.3 (4.3-7 and 4.3-10) shows. The result is in line with empirical findings of Rotemberg and Woodford (1999) and Gali, Gertler and López-Salido (2007). The mechanism works through the number of firms, although the stories can be different: Jaimovich and Floetotto (2008)'s story is based on supply side such that more producing firms generate higher competitions and lower markups while BGM (2007a)'s story is based on demand side in a way that counter-cyclical markups is based on preferences of households where more available varieties induce pricing complementarities.

Last but not least, our model predicts that the hours worked is negatively correlated with aggregate productivity, as (4.3-8) shows. Admittedly, there are still debates about relationship between productivity shocks and total hours worked, and our result contributes to the debate by providing a new mechanism that relates the aggregate productivity and total hours worked. Especially, as Gali (1999) points out, traditional RBC models predict a high positive correlation between hours worked and aggregate productivity while empirical data points out a negative correlation. The failure of the traditional RBC models regarding this particular prediction is due
to their mechanism: technology shocks shift the labor demand while labor supply is not affected. To match the data, RBC economists resort to other driving forces to be able to shift the labor supply, such as government purchases (Christiano and Eichenbaum, 1992) or preference shocks of households (Bencivenga, 1992). Our model, instead, finds the negative correlation of productivity and hours worked without relying on other shocks. The underline new mechanism is that households react to a positive technology shock by establishing new firms and reducing labor supply to incumbent firms. The output increases due to higher productivity and more producers in the market, rather than increasing labor input. Interestingly, the total hours worked rebound and surpass the steady state level after about 2 years in our simulation because labor demand is increasing and the labor supply is gradually restored. The dynamics of labor supply and labor demand generate the performance of real wage as in (4.3-12): it reaches the peak 1 year later. Our result is supported by empirical observations of Gali (1999), Gali and Rabanal (2004) and Francis and Ramey (2004).

4.3.2.2 Impulse Responses to Money Supply Shock

Figure 4.4 shows the impulse responses to a one percent contractionary money supply shock. The vertical axes measure the percentage deviation of arguments from their respective steady states while the horizontal axis is the year after the shock.

7 Initially, labor supply is decreased. Labor demand increases gradually because there are more and more producers. The real wage is increasing in the beginning until supply is gradually restored.
8 Bilbiie, Ghironi and Melitz (2007a) also generate a negative correlation between labor in production and aggregate productivity. Their explanation is similar to ours in a sense that households increase investment in new firms and decrease labor supply for producing sector. However, our model differs from theirs in a way that they assume that labor is used in either production or setting up new firms, hence reduced labor supply to production is overestimated by their assumption. Our model, on the contrary, by endogenizing exit, allows households a better environment to build up new firms. And we don’t assume the labor is divided in either production or building up new firms.
Our model predicts that a contractionary money supply shock generates deflation (4.4-6), although there are still debates about the impact of contractionary monetary policy on price and inflation. Some VAR analysis report a rising GDP deflator over two year after the contractionary shock and they call this observation “Price Puzzle”, e.g., as mentioned by Eichenbaum (1992) and Sims (1992). However, including the “commodity prices” in the VAR model, as emphasized by Christiano, Eichenbaum and Evans (1994), the shock leads to sharp and persistent declines in price level, which supports our prediction here. Because the borrowing contract between firms and banking sector is written in nominal terms, the real value of the contract increases and more firms are unable to repay the loan, resulting in more exit (4.4-3).
and a higher cutoff productivity of survival (4.4-7).

Meanwhile, output and labor decline but they reach the bottom about two quarters after the shock (4.4-4 and 4.4-10). This prediction is consistent with Christiano, Eichenbaum and Evans (1994), although C. Romer and D. Romer (2004) report that the output reaches the bottom almost 1 year and a half after a contractionary monetary shock by using a new measurement of the shock. The reason behind this “sluggish” reaction of output and labor in our model is as follows: on one hand, with contractionary shock, the economy reacts immediately with higher number of exit. This leads to less producing firms in the market. One important impact of less producing firms is that the labor productivity is lower, which in turn reduces output and consumption. The monetary policy shocks therefore have real effect on the labor productivity through the channel of dynamics of firms. On the other hand, nevertheless, in the short run, the expected profit of surviving firms is higher (because the less profitable firms choose to exit the market). This generates incentives for households to invest in new firms, i.e., the entry is temporarily higher (4.4-2). Thus the behavior of households slows down the decrease of output. The second effect is fading out and totally disappears after 2 quarters when the recession induced by contractionary policy shock gains momentum.

4.3.3 Second Moment

To further evaluate the necessity of embedding firms’ endogenous exit in the model, we compute the implied unconditional second moments of our model for some key macroeconomic variables and compare them with stylized facts (data reported by King and Rebelo, 1999), baseline RBC model without entry and exit (King and Rebelo, 1999), and RBC model with endogenous entry but exogenous exit (BGM 2007a).
In order to be comparable, we use the same aggregate productivity process as King and Rebelo (1999) where $\Phi^A = 0.979$ and $\zeta_t^A = 0.0072^9$. These two parameters measure the stochastic property of technology: it is substantial serial correlated. Although our model assumes no physical capital, the investment in new firms plays the substitutable role as investment in physical capital. Hence, similar to BGM (2007b), we use the real value of total entry to represent the investment each period. Moreover, as money supply in our model follows a random walk process, we maintain this assumption and estimate the serial correlation of money supply and the variance of the shocks. Table 4.1 reports the estimated autocorrelation coefficient and the variance of the residual of money supply M1. The original data comes from OECD data sets and we transform it to the log-deviation from the trend by HP-Filter to estimate equation (4.38).

**Table 4.1: Estimation for money supply (Quarterly Data, HP-Filtered Log-Deviation from Trend)**

<table>
<thead>
<tr>
<th>Country</th>
<th>U.S</th>
<th>Euro Zone</th>
<th>Japan</th>
<th>Switzerland</th>
<th>China</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Autocorrelation</strong></td>
<td>0.932</td>
<td>0.836</td>
<td>0.844</td>
<td>0.869</td>
<td>0.646</td>
</tr>
<tr>
<td><strong>Variance</strong></td>
<td>0.0084</td>
<td>0.0076</td>
<td>0.015</td>
<td>0.027</td>
<td>0.012</td>
</tr>
</tbody>
</table>

As we are comparing our model with U.S data, we also use U.S’ money supply process, i.e., $\Phi^M = 0.932$ and $\zeta_t^M = 0.0084$. Table 4.2 presents the results. For each moment, the first number (bold fonts) is the stylized facts implied by U.S data reported by King and Rebelo (1999), the second number is the moment generated by King and Rebelo (1999)’s benchmark RBC model, the third number is the moment generated by BGM (2007b)’s model with endogenous entry, and the last number (in bracket) is the moment generated by our model with endogenous entry and exit. The moment is calculated by HP-filtered series as standard practice.

---

9 In King and Rebelo’s benchmark RBC model, technology grows at positive rate ($\gamma = 1.004$), while in our current model we don’t have such feature.
Table 4.2: Moments for: Data, Benchmark RBC, BGM (2007b) and (Our Model)

<table>
<thead>
<tr>
<th>Variable $X_t$</th>
<th>$\sigma_{X_t}$</th>
<th>$\frac{\sigma_{X_t}}{\sigma_{Y_t}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_t$</td>
<td>1.81 1.39 1.36 (1.43)</td>
<td>1.00</td>
</tr>
<tr>
<td>$C_t$</td>
<td>1.35 0.61 0.66 (1.24)</td>
<td>0.75 0.44 0.48 (0.86)</td>
</tr>
<tr>
<td>Investment $V_tN_t^E$</td>
<td>5.30 4.09 5.20 (4.75)</td>
<td>2.93 2.94 3.82 (3.32)</td>
</tr>
<tr>
<td>$L_t$</td>
<td>1.79 0.67 0.63 (0.72)</td>
<td>0.99 0.48 0.46 (0.50)</td>
</tr>
</tbody>
</table>

Concerning the absolute as well as relative standard deviation, one of the most obvious improvements in our model is that consumption has higher volatility, which is more close to the data. Benchmark RBC models as well as BGM (2007b)'s endogenous entry model face the same well-known difficulties that consumption and hours are too smooth relative to output. Our model makes substantial improvement in consumption and minor contribution to hours. The additional volatility of consumption comes from households’ choice of setting up new firms. This should have been essentially similar to the tradeoff between consumption and investment as benchmark models, yet in our model, the “depreciation rate” of investment (capital) is endogenized as firms’ exit is endogenous. Therefore, households are better motivated and have higher incentives to do investment, resulting in a higher tolerance of consumption volatility.

Concerning the persistence of each variable, our model is able to generate higher persistence of output and consumption, although, admittedly, we do not perform better regarding to investment and hours, which is a general problem of New Keynesian and RBC models.

Our model also has a substantial improvement concerning the correlation between variables and output. The well-known problem for standard DSGE models is that
all real variables are too pro-cyclical relative to the data. For instance, as table 4.2 shows, both benchmark RBC model and BGM (2007b)'s model report that the correlation between each variable and output is almost 1. By embedding endogenous exit of firms, we are able to relieve such “too pro-cyclical” problem. Figure 4.5 shows the simulated entry and exit (HP-filtered log-deviation from steady state). Compared to the data (figure 4.1), our model is successful to capture the behavior of entry and exit.

![Figure 4.5: simulated entry and exit (HP-filtered log-deviation from steady state)](image)

### 4.4 CONCLUSION

In this chapter of the thesis, I extend the framework of heterogeneous firms to a dynamic stochastic general equilibrium setting and analyze the effects of technology shocks and monetary policy shocks. As entry and exit of firms have strong pro-cyclical and counter-cyclical behavior, this chapter shows that embedding the endogenous entry and exit is not only more close to the reality but also provides a new mechanism to explain many empirical findings that are challenges for traditional RBC models and New Keynesian models.

Particularly, compared to traditional RBC models and BGM (2007a)' model, our model predicts that facing an expansionary technology shock, the inflation and hours worked react negatively. Meanwhile, we find the counter-cyclical markups...
and pro-cyclical profit. And we find that non-persistent shocks have persistent impact on labor productivity. In term of money supply shock, our model predicts a “sluggish” reaction of output. Moreover, our model performs better in the second moment regarding either absolute or relative variances as well as autocorrelations of variables. As we have shown, all these predictions and improved performance benefit from the underline mechanism of both endogenous entry and exit of the firm.

Moreover, our model generates a new trade-off for monetary policy. As the number of producers affects inflation dynamics, labor productivity and aggregate output, we have a new policy transmission mechanism. Precisely, as there are nominal rigidities in lending contract and price setting, the fact of endogenous exit of firms implies a role for policy to stabilize the economy by stabilizing number of producers. This opens the door for future research on optimal monetary policy as well as possible fiscal policy.
APPENDIX 4.1: AGGREGATION

Producer Price Index $P_{t,z}$: according to the definition (4.5),

$$P_{t,z} = \left[\int_{\Omega} P_t(\omega)^{1-\varepsilon} d\omega\right]^{\frac{1}{1-\varepsilon}}$$

If we denote the productivity distribution of producing firms as $u^A(\varphi)$, then $P_{t,z}$ can be transformed to the aggregated price of all firms of different productivity ($N_{t-1}$ number of producers):

$$P_{t,z} = \left[\int_b^\infty N_{t-1} P_t(\varphi)^{-\varepsilon} u^A(\varphi) d\varphi\right]^{\frac{1}{1-\varepsilon}}$$

$$u^A(\varphi) = g(\varphi)$$

Use the optimal pricing strategy (4.14):

$$P_{t,z} = \left[\int_b^\infty N_{t-1} \left(\frac{\varepsilon W_t}{\varepsilon - 1 A_t}\right)^{-\varepsilon} \varphi^\varepsilon u^A(\varphi) d\varphi\right]^{\frac{1}{1-\varepsilon}}$$

Define the “representative” firm whose productivity is given by:

$$\varphi^A = \left[\int_b^\infty \varphi^{-\varepsilon+1} u^A(\varphi) d\varphi\right]^{\frac{1}{\varepsilon-1}}$$

Then

$$P_{t,z} = \left[N_{t-1} \left(\frac{\varepsilon W_t}{\varepsilon - 1 A_t}\right)^{-\varepsilon} \varphi^A\right]^{\frac{1}{1-\varepsilon}}$$

$$P_{t,z} = N_{t-1}^{\frac{1}{1-\varepsilon}} P_t(\varphi^A)$$

This gives the equation (4.31).

By the similar methodology, we transform the aggregate intermediary goods defined by equation (4.4), using the optimal pricing strategy (4.14) and the corresponding demand (4.6): (same definition of $\varphi^A$):

$$Y_{t,z} = N_{t-1}^{\frac{\varepsilon}{1-\varepsilon}} Y_t(\varphi^A)$$

This is exactly the same as equation (4.27).

---

Note that all entering firms produce (including those who might default in the next period), so the productivity distribution of producing firm is the same as the ex ante distribution $g(\varphi)$. 

Aggregate Labor Demand is defined as the total labor input for producing intermediary goods, which is given by:

$$L_t = \int_0^\infty N_t^{-1}L_t(\phi)\mu^A(\phi)d\phi$$

$$L_t = N_t^{-1}L_t(\phi^A)$$

This is the same as (4.32).

Then according to the production technology (4.11),

$$Y_{t,z} = N_t^{\frac{\epsilon}{\epsilon-1}}A_t\phi^A L_t(\phi^A)$$

Use the result of aggregate demand to get:

$$Y_{t,z} = \frac{1}{N_t^{\frac{1}{\epsilon-1}}}A_t\phi^A L_t$$

The aggregate final output is defined by (4.1):

$$Y_t = \left[ \int_0^1 Y_t(z)^{-\epsilon}dz \right]^{\frac{1}{\epsilon-1}}$$

Note that because of the price rigidities, the aggregation is difficult if we have “weight” on different firms. Therefore we start from a “no weight” definition and try to find the relationship between this and the true final output $Y_t$. Particularly, define:

$$Y_{t,s} = \int_0^1 Y_t(z)dz$$

$$Y_{t,s} = Y_{t,z}$$

Using the demand function (4.2):

$$Y_{t,z} = \int_0^1 \left( \frac{P_t(z)}{P_t} \right)^{-\epsilon}dz$$

Define $s_t$ as:

$$s_t = \int_0^1 \left( \frac{P_t(z)}{P_t} \right)^{-\epsilon}dz$$

$$Y_{t,z} = s_t Y_t$$

Some manipulation of $s_t$ gives (knowing that 1-$\theta$ of wholesale firms are able to adjust the price at $t$ to the new optimal level given by equation (4.8)):

$$s_t = \int_0^{1-\theta} \left( \frac{P_t(z)}{P_t} \right)^{-\epsilon}dz + \int_{1-\theta}^1 \left( \frac{P_t(z)}{P_t} \right)^{-\epsilon}dz$$
\[
= (1 - \theta) \left( \frac{P_t^*(z)}{P_t} \right)^{-\varepsilon} + \left( \frac{P_{t-1}}{P_t} \right)^{-\varepsilon} \int_{1-\theta}^{1} \left( \frac{P_{t-1}(z)}{P_{t-1}} \right)^{-\varepsilon} \, dz
\]

\[
= (1 - \theta) \left( \frac{P_t^*(z)}{P_t} \right)^{-\varepsilon} + \theta(1 + \pi_t) s_{t-1}
\]

This is equation (4.35).

Finally, the “representative” surviving firm \( \phi_t^S \) and “representative” defaulting firm \( \phi_t^D \) are given by (4.29) and (4.30) such that the average profit of surviving firms is given by (note that there is no number of survival multiplied because we are calculating the average profit instead of aggregate profit):

\[
D_s = \int_{\phi_t^S}^{\infty} D_s(\varphi) u^s(\varphi) \, d\varphi
\]

Then use the expression of profit (4.13) and definition (4.29), we can get

\[
D_s = D_s(\phi_t^S)
\]

Similarly, the average liquidity of default firms is given by:

\[
D_t^F = D_t(\phi_t^D)
\]
APPENDIX 4.2: STEADY STATE EQUATIONS

IS Block:

\[ Y = C + N^E f^E + Nj^F \]  
\[ \pi = 0 \]  
\[ \left( \frac{M}{P} \right)^{-\sigma} = \frac{1}{C} \left( \frac{R^n - 1}{R^n} \right) \]  
\[ R^n = \frac{1}{\beta} \]  
\[ V = \frac{\beta(b / \phi^* \bar{A})}{1 - \beta(b / \phi^* \bar{A})} D \]  
\[ B = PNj^F \]  
\[ R^n = \frac{R^n}{(b / \phi^* \bar{A})} - \frac{1}{(b / \phi^* \bar{A})} D^F \]

AS Block:

\[ Y = N^{\frac{1}{\tau - 1}} A\phi^A L \]  
\[ \chi L^{\frac{1}{\phi}} = \frac{W}{CP} \]  
\[ D = \left( \frac{\phi^A}{\phi^S} \right)^{1-\varepsilon} \frac{WL}{(\varepsilon-1)NP} - R^n f^E \]  
\[ D^F = \left( \frac{\phi^A}{\phi^F} \right)^{1-\varepsilon} \frac{WL}{(\varepsilon-1)NP} \]  
\[ \left( \frac{\varepsilon}{\varepsilon-1} \right)^2 WN^{\frac{1}{\tau-1}} = A\phi^A \]  
\[ V = f^E \]  
\[ s = 1 \]  

Number of Firms:

\[ N = \frac{N^E}{1 - (b / \phi^*)^k} \]
\[ \varphi^* = \varphi^A \left[ \frac{(\varepsilon - 1)R^m NPf^F}{WL} \right]^{\frac{1}{\varepsilon - 1}} \]  
(SS-16)

\[ \varphi^S = \left[ \frac{k}{k + 1 - \varepsilon} \right]^{\frac{1}{\varepsilon - 1}} \varphi^* \]  
(SS-17)

\[ \varphi^D = \left[ \frac{k}{k + 1 - \varepsilon} \frac{b^{\varepsilon}}{1 - (b / \varphi^*)^{\varepsilon}} \left( b^{\varepsilon - k - 1} - \varphi^{* - k - 1} \right) \right]^{\frac{1}{\varepsilon - 1}} \]  
(SS-18)

**Exogenous Variables Block:**

\[ A = 1 \]  
(SS-19)

\[ M = \overline{M} \]  
(SS-20)

**Note:**

In the above steady state system, we eliminate the wholesale sector’s equations by substitutions. For example, equation (SS-12) comes from the following steps:

\[ P = \left[ \theta P^{1 + \varepsilon} + (1 - \theta) P^* (z)^{-\varepsilon} \right]^{\frac{1}{\varepsilon - 1}} \]

\[ \Rightarrow P = P^* (z) \]

The optimal price of wholesale firm is given by (4.8), and in steady state it becomes:

\[ P^* (z) = \frac{\varepsilon}{\varepsilon - 1} Pmc(z) \]

Meanwhile, the marginal cost \( mc(z) \) equals to the producer price index:

\[ mc(z) = P^*_z = \frac{1}{N^{1 - \varepsilon}} P(\varphi^A) \]

And the optimal price of the firm with productivity \( \varphi^A \) is:

\[ P(\varphi^A) = \frac{\varepsilon}{\varepsilon - 1} \frac{W}{A \varphi^A} \]

Combine the above equations and eliminate \( P(\varphi^A), mc(z), P^* (z) \) and \( P \), we therefore have

\[ \left( \frac{\varepsilon}{\varepsilon - 1} \right)^2 WN^{1 - \varepsilon} = A \varphi^A \]  
(SS-12)

Similar substitutions and eliminations of wholesale sector’s variables are also applied to get equation (SS-10) and (SS-11).
APPENDIX 4.3: LOG-LINEARIZED SYSTEM

Variable $x$ denotes the steady state value calculated by Appendix 4.2 and $\hat{x}_t$ denotes its log-deviation from $x$. Note that $\hat{N}_t^x$ denotes the deviation of number of exit at period $t$. Technology and money supply follow (4.37) and (4.38) respectively.

$$
\sigma \dot{M}_t - \sigma \dot{P}_t - \dot{C}_t + \frac{1}{R^n - 1} \hat{R}_t^n = 0
$$

$$
\dot{R}_t - \dot{C}_{t+1} + \dot{C}_t - \pi_{t+1} = 0
$$

$$
\frac{1}{\phi} \dot{L}_t - \dot{W}_t + \dot{C}_t + \dot{P}_t = 0
$$

$$
- \dot{V}_t + \dot{C}_t - \dot{C}_{t+1} - k \hat{\phi}_t^* + [1 - \beta(b / \varphi^*)^k] \dot{D}_{t+1} + \beta(b / \varphi^*)^k \hat{V}_t = 0
$$

$$
Y \dot{Y}_t - C \dot{C}_t - Nf^F \hat{N}_t - N^F f^E \hat{N}^E_t = 0
$$

$$
- \left( \frac{b}{\varphi} \right)^k R^n \hat{R}_t^n + R^n \hat{R}_t^n + k \left( \frac{b}{\varphi} \right)^k \left( R^n \frac{D^n}{f^n} \right) \hat{\phi}_t^* - \frac{D^n}{f^n} \left( 1 - \left( \frac{b}{\varphi} \right)^k \right) \left( \dot{D}_t^F + \pi_t \right) = 0
$$

$$
- \pi_t + \frac{(1 - \theta) \theta}{\theta} \left[ \dot{W}_t - \dot{A}_t \right] - \frac{(1 - \theta)(1 - \theta \beta)}{\theta} \frac{1}{\varepsilon - 1} \hat{N}_t - \beta \varepsilon_{t+1} + \beta \varepsilon_{t+1} = 0
$$

$$
\dot{Y}_t - \frac{1}{\varepsilon - 1} \hat{N}_t - \dot{A}_t - \dot{L}_t = 0
$$

$$
- D \dot{D}_t + (D + R^n f^F) \left[ \dot{W}_t + \dot{L}_t + (\varepsilon - 1) \hat{\phi}_t^* - \hat{N}_t - \dot{P}_t \right] - R^n f^F \left[ \hat{R}_t^n - \pi_t \right] = 0
$$

$$
- \dot{D}_t^F + \dot{W}_t + \dot{L}_t + (\varepsilon - 1) \hat{\phi}_t^F - \hat{N}_t - \dot{P}_t = 0
$$

$$
(1 - \varepsilon) \hat{\phi}_t^F + \hat{R}_t^n + \hat{N}_t - \dot{P}_t - \hat{W}_t - \dot{L}_t - \pi_t = 0
$$

$$
\hat{\phi}_t^F - k \left( \frac{b / \varphi^*}{\varphi^*} \right)^k \left[ (\varphi^* / \varphi^D)^{\varepsilon - 1} - 1 \right] \hat{\phi}_t^* = 0
$$

$$
\hat{V}_t = 0
$$

$$
\hat{N}_t - \left( \frac{b}{\varphi} \right)^k \hat{N}_t + k \left( \frac{b}{\varphi} \right)^k \hat{\phi}_t^* - \left( 1 - \left( \frac{b}{\varphi} \right)^k \right) \hat{N}_t^E = 0
$$

$$
\hat{N}_t^x - \hat{N}_t - k \left( \frac{b / \varphi^*}{\varphi^*} \right)^k \hat{\phi}_t^* = 0
$$

$$
T \hat{F}_t - \frac{1}{\varepsilon - 1} \hat{N}_t - \dot{A}_t = 0
$$


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