Fermion masses and Higgs Physics in Grand Unified Theories

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Munich 2010

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Munich
February 5, 2010

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Date of Examination: March 12, 2010

This thesis
is dedicated to my wonderful father
Mian Chiragh-ud-Din *who is in the heaven*
and
to my loving mother
Sughra Chiragh *who is back in Pakistan*
praying for me and whom I miss alot here in Germany

Abstract

The Standard model of particle physics is a very successful theory of strong weak and electromagnetic interactions. This theory is perturbative at sufficiently high energies and renormalizable thus it describes these interactions at quantum level. However it has a number of limitations, one being the fact that it has 28 free parameters assuming massive neutrinos. Within the Standard model these parameters can not be explained, however they can be accommodated in the standard theory. Particularly the masses of the fermions are not predicted by the theory. The existence of the neutrino masses can be regarded as the first glimpse of the physics beyond the standard model.

In this thesis we have described the quark and lepton masses and mixings in context of non-SUSY SO(10) and four zero texture (FZT). In the four zero texture case the fermion masses and mixing can be related. We have made some predictions using tribimaximal mixing, the near tribimaximal (TBM) mixing and the triminimal parameterization. Our results show that under the TBM the neutrinos have normal, but weak hierarchy. Under near tri-bimaximal mixing and the triminimal parameterization we find that the neutrino masses in general increase, if the value of solar angle increases from its TBM value and vice versa.

It appears that the neutrinos become more and more degenerate for solar angle values higher than TBM value and hierarchical for lower values of solar angle. We also briefly discuss neutrino parameters in the SUSY SO(10) theories. An overview of SUSY SO(10) theories and proton decay is also presented.

Zusammenfassung

Das Standardmodell der Teilchenphysik ist eine sehr erfolgreiche Theorie der starken, schwachen und elektromagnetischen Wechselwirkungen. Bei hohen Energien kann diese Theorie mit perturbativen Methoden beschrieben werden. Sie ist renormierbar und beschreibt die Wechselwirkungen im Rahmen der Quantentheorie. Jedoch sind die Grenzen der Theorie durch die Tatsache gegeben, dass 28 freie Parameter eingeführt werden müssen, darunter 8 Parameter für die massiven Neutrinos. Diese Parameter können im Standardmodell nicht berechnet werden. Die Neutrinomassen müssen als erste Hinweise auf die Physik jenseits des Standardmodells interpretiert werden.

In dieser Arbeit haben wir die Massen der Quarks und Leptonen und ihre Mischungen in der SO(10) – Theorie beschrieben. In diesem Modell gibt es Relationen zwischen den Fermionenmassen und den Mischungswinkel auf der Grundlage einer durch Symmetrien festgelegten Textur der Massenmatrizen. Speziell konnten wir diese Relationen im Fall der „tribimaximal“ und der „tribiminimal“ Mischungen angeben. Unsere Resultate ergeben, dass die Neutrinos eine normale, allerdings nicht sehr ausgeprägte Massenhierarchie besitzen. Wir finden, dass die Neutrinomassen größer werden, wenn der solare Mischungswinkel anwächst. In diesem Fall werden die Neutrinomassen fast gleich. Falls der solare Mischungswinkel kleiner ist, wird die Massenhierarchie ausgeprägter.

Wir diskutieren auch Details der supersymmetrischen SO(10) – Theorie, insbesondere die Neutrinoparameter und den Protonzerfall.

Acknowledgement

I first of all pay may humble thanks to ALLAH the almighty who among other things, bestowed me with good health to complete this work. I pay my special regards to the prophet Muhammad whose life and deeds are constant source of inspiration for me.

I pay my deepest gratitude to may supervisor Professor Dr. Harald Fritzsch for providing me a chance to work with him and for his help and support in writing this thesis. This work would have been impossible without his guidance. I am also very much grateful to Prof. Dr. Gerhard Buchalla for invaluable discussions and suggestions.

I am especially grateful to my mother Sughra Chiragh, my brothers and sisters M. Aslam, M. Akram, G. Mustafa, G. Sarwar, A. Qayyum, A. Majeed, A. Rasheed, Shehzad, Maryam, Ayesha , Khadeja. I can not forget their continuous support and prayers for me throughout my education career.

In particular I pay my homage to my friends Kanwal, Faree, Farhan, Afzaal, Arfat, Sajjad, Ismatullah and Atif for encouragement and for the wonderful time we spent together.

Many thanks to may friends and colleagues Gamal Eldahoumi, Volker Pilipp, Matthäus Bartsch, Guido Bell, Dao-Neng Gao, Valentina Aquila, Michaela Albrecht, Marc Lietzow and Alexander Kubelsky for useful discussions during weekly journal club.

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Introduction

The Standard Model (SM) of electroweak and strong interactions is extremely successful theory of elementary particles [1],[2]. The electroweak theory put forward by Glashow, Salam and Weinberg [1], describing electromagnetic [3] and weak [4] interactions betweens the quarks and leptons, is based on an $SU(2)_L \times U(1)_Y$ gauge symmetry group of weak isospin and hypercharge. Combined with the $SU(3)_C$ gauge symmetry group for Quantum Chromodynamics (QCD)[2], which is the theory for the strong interactions, the SM describes the three forces of nature. This theory is perturbative at sufficiently high energies and renormalizable [5], and thus describes these interactions at quantum level [6]. One of the key ingredients of the SM is the phenomenon of the spontaneous electroweak symmetry breaking (EWSB)[7],[6], called the Higgs Mechanism, which respects renormalizability [5] and unitarity of the theory [8]. In this mechanism the neutral component of an $SU(2)$ doublet of complex scalar field assumes a non-zero expectation value. Hence the electroweak symmetry $SU(2)_L \times U(1)_Y$ is spontaneously broken to the $U(1)_Q$ symmetry. The $W^{\pm}$ and Z bosons absorb three of the four degrees of freedom of doublet scalar field to form their longitudinal polarizations and acquire masses. The fermions masses are generated through the Yukawa interaction with the same scalar field and its conjugate field. The remaining degree of freedom corresponds to a scalar particle called Higgs Boson, which is yet to be discovered.

Excluding neutrinos there are 19 free parameters in the standard model [9]. Out of these 19, 3 gauge couplings, the Higgs quartic coupling λ and Higgs mass squared μ^2 are flavor universal. The others are flavor parameters. They include 6 quark masses, 3 charged lepton masses, 4 quark mixing parameters and a strong CP phase. The existence of the neutrino mass can be regarded as the first glimpse of the physics beyond the standard model. Including small neutrino masses and mixings, 9 more parameters, 3 neutrino masses, 3 mixing angles and 3 phases have to be introduced. One may ask, why are there so many free parameters? Why do they show hierarchical structure spanning six order of magnitude? Are the mixing parameters and mass ratios related to each other? What is origin of CP violation? Answers to these questions necessarily leads one to the physics beyond the Standard model as the origin of these parameters is still unknown. Within the Standard model these parameters cannot be explained however they can be accommodated in the standard theory.

The grand unified theories (GUTs) can help to reduce these parameters. Some of the beyond standard model scenarios can be confirmed or refuted by the forthcoming experiments, especially at LHC. If the new physics exists near the TeV scale, it would be accessible at LHC, but if it exists at a much higher scale, it will not be accessible.

This thesis is organized as follows: in chapter 1 we review the generation of fermions masses through the Higgs mechanism. The mixing and mass matrices in the quak and lepton sectors are discussed. In the second chapter using zero texture, we discuss the possibility of relating the mixing angles and mass ratios of fermions. Some predictions are also made about neutrino masses. In chapter 3 we discuss the fermion mass and mixing problem in non- SUSY $SO(10)$ grand unified theory. The symmetry breaking and the Yukawa sector of the $SO(10)$ theory is described. In chapter 4 we describe the two explicit models of fermions masses in non-SUSY $SO(10)$ theory. Some predictions about neutrino mass are made. Chapter 5 is dedicated to the same problem in SUSY $SO(10)$ theories and beyond.

Chapter 1

Fermion masses in the Standard Model

The high precision measurement tests [10],[11] performed at LEP, SLC, Tevatron etc. has clearly established that the standard model of particle physics is the correct effective theory of the strong and electroweak interactions at the present energies.The tests have probed the quantum corrections and the structure of the $SU(3)_c \times SU(2)_L \times U(1)_Y$ local gauge symmetry. The theory has precisely predicted the measured values of the couplings of the electroweak gauge bosons with the leptons and quarks. The $SU(3)_c$ gauge symmetric sector of the theory has also been tested at LEP, however the scalar sector of the Standard Model(SM) has yet to be tested satisfactorily. The missing and the most important ingredient of the model has not been observed yet [11],[12]. Only indirect constraints from high precision data have been obtained [6],[10]. The three generations of weak isospin doublets are given by

$$L'_{eL} = \begin{pmatrix} \nu'_{eL} \\ e'_L \end{pmatrix}, \qquad L'_{\mu L} = \begin{pmatrix} \nu'_{\mu L} \\ \mu'_L \end{pmatrix}, \qquad L'_{\tau L} = \begin{pmatrix} \nu'_{\tau L} \\ \tau'_L \end{pmatrix} \tag{1.1}$$

$$Q'_{1L} = \begin{pmatrix} u'_L \\ d'_L \end{pmatrix}, \qquad Q'_{2L} = \begin{pmatrix} c'_L \\ s'_L \end{pmatrix}, \qquad Q'_{3L} = \begin{pmatrix} t'_L \\ b'_L \end{pmatrix} \tag{1.2}$$

and the singlets are

$$\ell'_{eR} = e'_R, \qquad \ell'_{\mu R} = \mu'_R, \qquad \ell'_{\tau R} = \tau'_R \tag{1.3}$$

$$q'^u_{uR} = u'_R, \qquad q'^u_{cR} = c'_R, \qquad q'^u_{tR} = t'_R \tag{1.4}$$

$$q'^D_{dR} = d'_R, \qquad q'^D_{sR} = s'_R, \qquad q'^D_{bR} = b'_R \tag{1.5}$$

The primes in (1.1) — (1.5) represent the fermion fields discussed above. They do not have a definite mass, but they are linear combinations of the fermions having definite mass. The three generation electroweak SM lagrangian is [13]

$$
\begin{aligned}
\mathcal{L} = {} & i \sum_{\alpha=e,\mu,\tau} \overline{L}'_{\alpha L}\gamma^{\mu} D_{\mu} L'_{\alpha L} + i \sum_{\alpha=1,2,3} \overline{Q}'_{\alpha L}\gamma^{\mu} D_{\mu} Q'_{\alpha L} + i \sum_{\alpha=e,\mu,\tau} \overline{\ell}'_{\alpha R}\gamma^{\mu} D_{\mu} \ell'_{\alpha R} \\
& + i \sum_{\alpha=d,s,b} \overline{q}'^{D}_{\alpha R}\gamma^{\mu} D_{\mu} q'^{D}_{\alpha R} + i \sum_{\alpha=u,c,t} \overline{q}'^{U}_{\alpha R}\gamma^{\mu} D_{\mu} q'^{U}_{\alpha R} - \frac{1}{4}\underline{A}_{\mu\nu}\underline{A}^{\mu\nu} - \frac{1}{4}\underline{B}_{\mu\nu}\underline{B}^{\mu\nu} \\
& + \left(D_{\rho}\Phi\right)^{\dagger}\left(D^{\rho}\Phi\right) - \mu^{2}\Phi^{\dagger}\Phi - \lambda\left(\Phi^{\dagger}\Phi\right)^{2} - \sum_{\alpha,\beta=e,\mu,\tau}\left(Y'^{\ell}_{\alpha\beta}\overline{L}'_{\alpha L}\Phi\ell'_{\beta R} + Y'^{\ell *}_{\alpha\beta}\overline{\ell}'_{\beta R}\Phi^{\dagger}L'_{\alpha L}\right) \\
& - \sum_{\alpha=1,2,3}\sum_{\alpha,\beta=d,s,b}\left(Y'^{D}_{\alpha\beta}\overline{Q}'_{\alpha L}\Phi q'^{D}_{\beta R} + Y'^{D *}_{\alpha\beta}\overline{q}'^{D}_{\beta R}\Phi^{\dagger}Q'_{\alpha L}\right) \\
& - \sum_{\alpha=1,2,3}\sum_{\alpha,\beta=d,s,b}\left(Y'^{U}_{\alpha\beta}\overline{Q}'_{\alpha L}\widetilde{\Phi} q'^{U}_{\beta R} + Y'^{U *}_{\alpha\beta}\overline{q}'^{U}_{\beta R}\widetilde{\Phi}^{\dagger}Q'_{\alpha L}\right)
\end{aligned}
\tag{1.6}
$$

The Higgs-Yukawa fermions couplings responsible for the generation of fermions masses, are given by last three entries of above lagrangian. The charged-current weak interaction lagrangian can be obtained from the first line of the equation above and is given by

$$
\mathcal{L}_{I}^{(CC)} = -\frac{g}{2\sqrt{2}} j^{\rho}_{W} W_{\rho} + h.c. \tag{1.7}
$$

The fermion charged current j^{ρ}_{W} represents the sum of the leptonic and quark charged weak currents

$$
j^{\rho}_{W} = j^{\rho}_{W,L} + j^{\rho}_{W,Q} \tag{1.8}
$$

They are given as

$$
j^{\rho}_{W,L} = 2\left(\overline{\nu}'_{eL}\gamma^{\rho}e'_{L} + \overline{\nu}'_{\mu L}\gamma^{\rho}\mu'_{L} + \overline{\nu}'_{\tau L}\gamma^{\rho}\tau'_{L}\right) \tag{1.9}
$$

$$
j^{\rho}_{W,Q} = 2\left(\overline{u}'_{L}\gamma^{\rho}d'_{L} + \overline{c}'_{L}\gamma^{\rho}s'_{L} + \overline{t}'_{L}\gamma^{\rho}b'_{L}\right) \tag{1.10}
$$

We can write the fermionic charged current in a compact form

$$
j^{\rho}_{W,L} = 2\sum_{\alpha=e,\mu,\tau}\overline{\nu}'_{\alpha L}\gamma^{\rho}\ell'_{\alpha L} \tag{1.11}
$$

with $\ell'_{eL} = e'_L$, $\ell'_{\mu L} = \mu'_L$, $\ell'_{\tau L} = \tau'_L$ and using raising and lowering operators

$$j^{\rho}_{W,L} = 2 \sum_{\alpha=e,\mu,\tau} \overline{L}'_{\alpha L}\gamma^{\rho} I_{+} L'_{\alpha L} \tag{1.12}$$

$$j^{\rho\dagger}_{W,L} = 2 \sum_{\alpha=e,\mu,\tau} \overline{L}'_{\alpha L}\gamma^{\rho} I_{-} L'_{\alpha L} \tag{1.13}$$

$$j^{\rho}_{W,Q} = 2 \sum_{\alpha=1,2,3} \overline{Q}'_{\alpha L}\gamma^{\rho} I_{+} Q'_{\alpha L} \tag{1.14}$$

$$j^{\rho\dagger}_{W,Q} = 2 \sum_{\alpha=1,2,3} \overline{Q}'_{\alpha L}\gamma^{\rho} I_{-} Q'_{\alpha L} \tag{1.15}$$

Raising and lowering operators are

$$I_{+} \rightarrow \frac{\tau_{+}}{2} = \frac{\tau_1 + \iota\tau_2}{2} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$I_{-} \rightarrow \frac{\tau_{-}}{2} = \frac{\tau_1 - \iota\tau_2}{2} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

where τ_1,τ_1 are the Pauli matrices. The equations containing raising operators represent the corresponding currents, and equations containing lowering operators represent corresponding complex conjugate currents. The neutral current weak interaction lagrangian obtained from (1.6) is

$$\mathcal{L}^{(Z)} = -\frac{g}{2\cos\theta_W} j^{\rho}_{Z} Z_{\rho} \tag{1.16}$$

The neutral current is

$$j^{\rho}_{Z} = j^{\rho}_{Z,L} + j^{\rho}_{Z,Q} \tag{1.17}$$

with the leptonic neutral current

$$j^{\rho}_{Z,L} = 2g^{\nu}_{L} \sum_{\alpha=e,\mu,\tau} \overline{\nu}'_{\alpha L}\gamma^{\rho}_{L}\nu'_{\alpha L} + 2 \sum_{\alpha=e,\mu,\tau} \left(g^{l}_{L}\overline{\ell}'_{\alpha L}\gamma^{\rho}\ell'_{\alpha L} + g^{l}_{R}\overline{\ell}'_{\alpha R}\gamma^{\rho}\ell'_{\alpha R} \right) \tag{1.18}$$

and the quark neutral current

$$\begin{aligned} j^{\rho}_{Z,Q} = {} & 2 \sum_{\alpha=u,c,t} \left(g^{U}_{L}\overline{q}'^{U}_{\alpha L}\gamma^{\rho} q'^{U}_{\alpha L} + g^{U}_{R}\overline{q}'^{U}_{\alpha R}\gamma^{\rho} q'^{U}_{\alpha R} \right) \\ & + 2 \sum_{\alpha=d,s,b} \left(g^{D}_{L}\overline{q}'^{D}_{\alpha L}\gamma^{\rho} q'^{D}_{\alpha L} + g^{U}_{R}\overline{q}'^{D}_{\alpha R}\gamma^{\rho} q'^{D}_{\alpha R} \right) \end{aligned} \tag{1.19}$$

coefficients $g_L^{\nu,l,U,D}$, $g_R^{\nu,l,U,D}$

Fermions	g_L	g_R	g_V	g_A
ν_e,ν_μ,ν_τ	$g_L^\nu = \frac{1}{2}$	$g_R^\nu = 0$	$g_V^\nu = \frac{1}{2}$	$g_A^\nu = \frac{1}{2}$
e,μ,τ	$g_L^\ell = -\frac{1}{2} + \sin^2\theta_W$	$g_R^\ell = \sin^2\theta_W$	$g_V^\ell = -\frac{1}{2} + 2\sin^2\theta_W$	$g_A^\ell = -\frac{1}{2}$
u,c,t	$g_L^U = \frac{1}{2} - \frac{2}{3}\sin^2\theta_W$	$g_R^U = -\frac{2}{3}\sin^2\theta_W$	$g_V^U = \frac{1}{2} - \frac{4}{3}\sin^2\theta_W$	$g_A^U = \frac{1}{2}$
d,s,b	$g_L^D = -\frac{1}{2} + \frac{1}{3}\sin^2\theta_W$	$g_R^D = \frac{1}{3}\sin^2\theta_W$	$g_V^D = -\frac{1}{2} + \frac{2}{3}\sin^2\theta_W$	$g_A^D = -\frac{1}{2}$

Table 1.1: Values for coefficients of the quark fields

with $q'^U_{uL} = u'_L$, $q'^U_{cL} = c'_L$, $q'^U_{tL} = t'_L$, $q'^D_{dL} = d'_L$, $q'^D_{sL} = s'_L$, $q'^D_{bL} = b'_L$ values of coefficients $g_L^{\nu,l,U,D}$ and $g_R^{\nu,l,U,D}$ are given in table 1.1.

1.1 The Higgs Mechanism

In the Standard Model the masses of the fermions and gauge bosons are generated through the Higgs mechanism. We consider the Higgs doublet

$$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \phi^0(x) \end{pmatrix}$$

Here $\phi^+(x)$, and $\phi^0(x)$ represent the charged and neutral complex fields, and their gauge quantum numbers are given by the table 1.2.

The Higgs part of the SM Lagrangian which is invariant under the $SU(2)_L$ $\times U(1)_Y$ gauge symmetry is

$$\mathcal{L}_{Higgs} = (D_\mu\Phi)^\dagger (D^\mu\Phi) - \mu^2\Phi^\dagger\Phi - \lambda\left(\Phi^\dagger\Phi\right)^2 \tag{1.20}$$

The corresponding potential is

$$V(\Phi) = \mu^2\Phi^\dagger\Phi - \lambda\left(\Phi^\dagger\Phi\right)^2 \tag{1.21}$$

The coefficient λ of quartic self- coupling of the Higgs fields must be positive ($\lambda > 0$) in order to have the Higgs potential which is bounded from below. We assume the mass squared parameter $\mu^2 < 0$ for spontaneous breakdown of the gauge symmetry

$$SU(2)_L \times U(1)_Y \to U(1)_Q \tag{1.22}$$

$U(1)_Q$ is the symmetry group for electromagnetic interaction. Defining

$$\upsilon = \sqrt{-\frac{\mu^2}{\lambda}}$$

Higgs Doublet				
	I	I_3	Y	Q
$\phi^+(x)$	$\frac{1}{2}$	$\frac{1}{2}$	$+1$	1
$\phi^0(x)$	$\frac{1}{2}$	$-\frac{1}{2}$	$+1$	0

Table 1.2: Quantum numbers of the Higgs Doublet

and neglecting the term $\frac{\upsilon^4}{4}$, the Higgs potential can be written as

$$V(\Phi) = \lambda\left(\Phi^\dagger\Phi - \frac{\upsilon^2}{2}\right)^2$$

the minimum of the potential is at

$$\Phi^\dagger\Phi = \frac{\upsilon^2}{2}$$

Unlike the fermions fields and charged scalar fields, the neutral field $\phi^0(x)$ can have non zero vacuum expectation value.

$$\langle\Phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ \upsilon\end{pmatrix}$$

So in unitary gauge the Higgs doublet can be written as

$$\langle\Phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ \upsilon + H(x)\end{pmatrix} \tag{1.23}$$

where $H(x)$ is a field which is obtained by the excitations of the neutral Higgs field above the vacuum. It represents the physical Higgs boson. Now we can write the Higgs lagrangian in the unitary gauge

$$\begin{aligned}\mathcal{L}_{Higgs} = &\frac{1}{2}(\partial H)^2 - \lambda\upsilon^2H^2 - \lambda\upsilon H^3 - \frac{\lambda}{4}H^4 + \frac{g^2\upsilon^2}{4}W_\mu^\dagger W^\mu \\ &+ \frac{g^2\upsilon^2}{8\cos^2\theta_W}Z_\mu Z^\mu + \frac{g^2\upsilon}{2}W_\mu^\dagger W^\mu H + \frac{g^2\upsilon}{4\cos^2\theta_W}Z_\mu Z^\mu H \\ &+ \frac{g^2}{4}W_\mu^\dagger W^\mu H^2 + \frac{g^2}{8\cos^2\theta_W}Z_\mu Z^\mu H^2\end{aligned} \tag{1.24}$$

From the second term we get the mass for the Higgs boson.

$$m_H = \sqrt{2\lambda\upsilon^2} = \sqrt{-2\mu^2} \tag{1.25}$$

$\triangle\chi^2 = \chi^2 - \chi^2_{\text{min}}$ versus m_H, from global fit to electroweak data

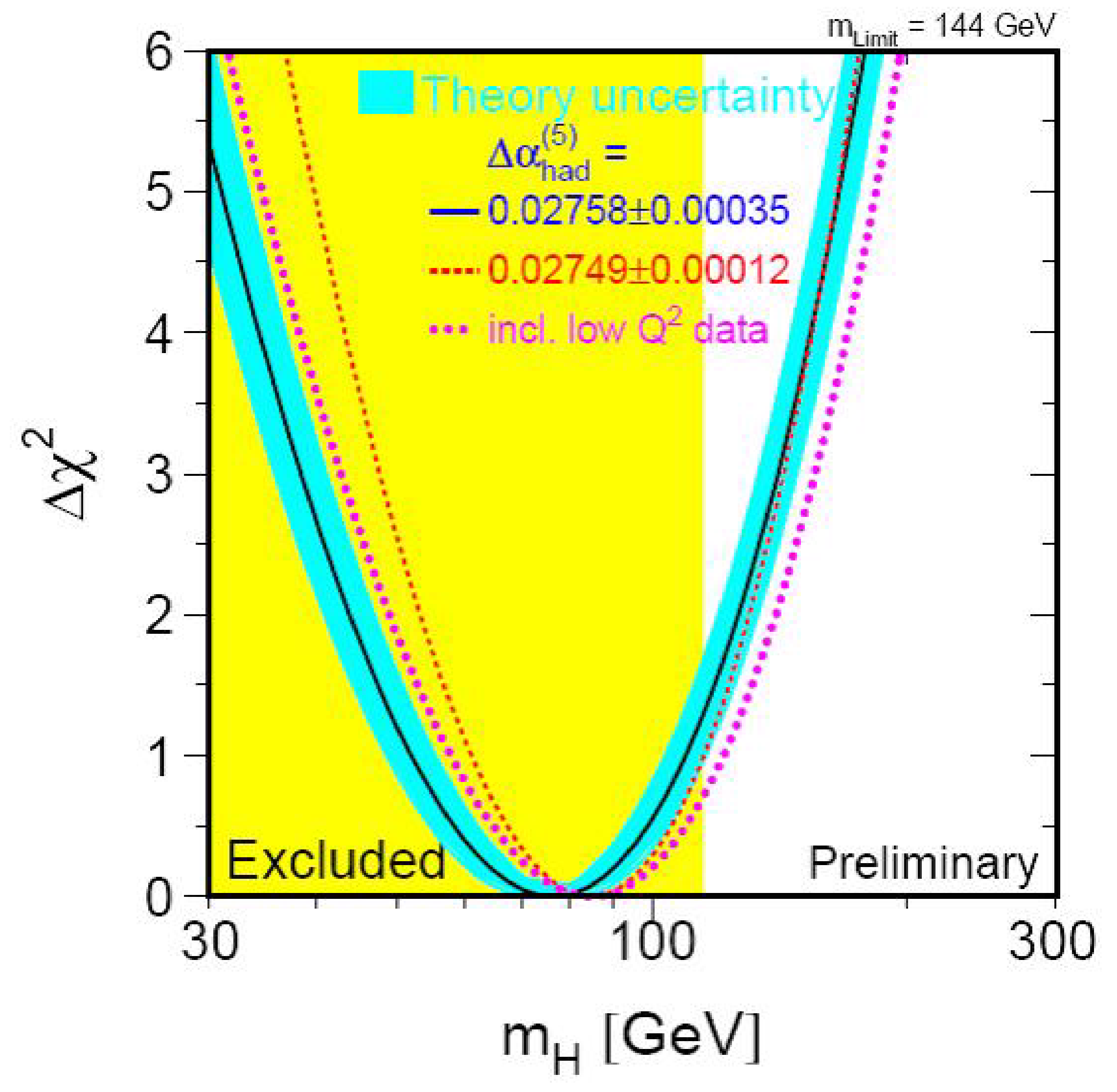

Figure 1.1: Bounds on the Higgs mass from electroweak data

Since μ^2 is not connected to the other quantities in the Standard Model, the value of the Higgs boson is not predicted in the Standard Model. It must be determined from experiments. One of the aims of LEP was to find the Higgs boson . Although LEP played a key role in establishing SM as the effective theory of the weak and electromagnetic interactions, it failed to detect any particle like the Higgs Boson. However these results put a very strong limit on SM-like Higgs boson, $m_H \gtrsim 114.4$ GeV[12]. The global electroweak fit results in $\triangle\chi^2 = \chi^2 - \chi^2_{\text{min}}$ curve is shown in the figure 1.1. At 95% C.L. one gets [14],[15] .

$$114.4 \;\; \text{GeV} < m_H < 144 \;\; \text{GeV}$$

The fifth term in the above lagrangian gives the mass of the W boson. The sixth term gives the Z boson mass:

$$m_W = \frac{gv}{2} \tag{1.26}$$

$$m_Z = \frac{gv}{2\cos\theta_W} \tag{1.27}$$

The experiments give $m_W = 80.398 \pm 0.025$ GeV and $m_Z = 91.1875 \pm 0.0021$ GeV. These masses agrees with the theoretical prediction.[14],[15]

1.2 Fermion Masses and Mixings

In the Standard Model the masses of the fermions arise through Yukawa couplings of the fermions fields with the Higgs doublet. As a fermion mass term must involve a left handed and a right handed field, the neutrinos are massless in the Standard Model, since they do not have right handed components.

1.2.1 Lepton Masses

The Yukawa lagrangian is given by the ninth term in (1.6)

$$\mathcal{L}_{H,L} = -\sum_{\alpha,\beta=e,\mu,\tau} Y'^{\ell}_{\alpha\beta}\overline{L}'_{\alpha L}\Phi\ell'_{\beta R} + h.c \tag{1.28}$$

The products $\overline{L}'_{\alpha L}\ell'_{\beta R}$ are isospin doublets and they have hypercharge $Y = -1$. The Higgs doublet has hypercharge $Y = +1$. Therefore the above mentioned lagrangian is invariant under the $SU(2)_L \times U(1)_Y$ gauge symmetry. After symmetry breaking the Higgs doublet is given by (1.23), and the Higgs-Lepton Yukawa lagrangian is

$$\mathcal{L}_{H,L} = -\left(\frac{\upsilon + H}{\sqrt{2}}\right) \sum_{\alpha,\beta=e,\mu,\tau} Y'^{\ell}_{\alpha\beta} \overline{\ell}'_{\alpha L} \ell'_{\beta R} + h.c \qquad (1.29)$$

In the above expression the term proportional to υ (VEV) is the mass term for the charged fermion. Because of the fact that Y'^{ℓ} is usually a non diagonal complex 3×3 matrix. e', μ', τ'do not have definite mass. Charged lepton fields can be obtained, if we diagonalize the Yukawa matrix Y'^{ℓ}.In order to do this lets define an array of charged lepton fields

$$\ell'_L = \begin{pmatrix} e'_L \\ \mu'_L \\ \tau'_L \end{pmatrix} \quad , \qquad \ell'_R = \begin{pmatrix} e'_R \\ \mu'_R \\ \tau'_R \end{pmatrix}$$

The Yukawa lagrangian is

$$\mathcal{L}_{H,L} = -\left(\frac{\upsilon + H}{\sqrt{2}}\right) \overline{\ell}'_L Y'^{\ell} \ell'_R + h.c$$

We can diagonalize the Y'^{ℓ} through a biunitary transformation

$$V_L^{\ell\dagger} Y'^{\ell} V_R^{\ell} = Y^{\ell}, \quad \text{with} \quad Y^{\ell}_{\alpha\beta} = y^{\ell}_{\alpha} \delta_{\alpha\beta} \qquad (\alpha, \beta = e, \mu, \tau)$$

(V_L^{ℓ} are V_R^{ℓ} are appropriate 3×3 matrices). Redefining the charged lepton fields

$$\ell_L = V_L^{\ell\dagger} \ell'_L = \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix} \quad , \qquad \ell_R = V_R^{\ell\dagger} \ell'_R = \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}$$

where ℓ_L,ℓ_R are charged lepton fields with definite mass, the Yukawa lagrangian is given by

$$\mathcal{L}_{H,L} = -\left(\frac{\upsilon + H}{\sqrt{2}}\right) \overline{\ell}_L Y^{\ell} \ell_R + h.c$$

$$\mathcal{L}_{H,L} = -\sum_{\alpha=e,\mu,\tau} \frac{y^{\ell}_{\alpha} \upsilon}{\sqrt{2}} \overline{\ell}_{\alpha} \ell_{\alpha} - \sum_{\alpha=e,\mu,\tau} \frac{y^{\ell}_{\alpha}}{\sqrt{2}} \overline{\ell}_{\alpha} \ell_{\alpha} H$$

($\ell_{\alpha} = \ell_{\alpha L} + \ell_{\alpha R}$ are charged lepton fields with definite mass.)

$$\ell_e = e, \qquad \ell_{\mu} = \mu \ , \qquad \ell_{\tau} = \tau$$

The mass term for the charged leptons is given by the first term of the lagrangian

$$m_\alpha = \frac{y_\alpha^\ell \upsilon}{\sqrt{2}} \qquad \text{with} \qquad (\alpha = e, \mu, \tau) \tag{1.30}$$

Since y_e^ℓ,y_μ^ℓ,y_τ^ℓ are free parameters in the standard model, the charged lepton masses are not predicted in the SM. If we define

$$\nu_L = V_L^{\ell\dagger}\nu_L' = V_L^{\ell\dagger}\begin{pmatrix}\nu_{eL}'\\ \nu_{\mu L}'\\ \nu_{\tau L}'\end{pmatrix} = \begin{pmatrix}\nu_{eL}\\ \nu_{\mu L}\\ \nu_{\tau L}\end{pmatrix}$$

Then the leptonic charged current can be written as

$$j_{W,L}^\rho = 2\overline{\nu}_L\gamma^\rho\ell_L = 2\sum_{\alpha,\beta=e,\mu,\tau}\overline{\nu}_{\alpha L}\gamma^\rho\ell_{\alpha L}$$

The neutrino fields ν_{eL}, $\nu_{\mu L}$, $\nu_{\tau L}$ still stay massless as they are linear combinations of the massless (primed) fields. They are called flavor neutrino fields as they only couple to the corresponding charged lepton fields. In the standard model the neutrino flavor eigenstates are also the neutrino mass eigenstates, however in theories beyond the standard model such as $SO(10)$, right handed neutrino field also exist, and neutrinos are massive particles. Therefore in these theories neutrino flavor eigenstates are not in general mass eigenstates. The leptonic neutral current is given by:

$$j_{Z,L}^\rho = 2g_L^\nu\overline{\nu}_L\gamma^\rho\nu_L + 2g_L^l\overline{\ell}_L\gamma^\rho\ell_L + 2g_R^l\overline{\ell}_R\gamma^\rho\ell_R$$

So the leptonic neutral current remains invariant. This phenomenon is called the GIM mechanism.

1.2.2 Quark Masses

As the mass term for the fermions possesses both the left handed and the right handed fields of the corresponding fermion, we can have two types of such products

$$\overline{Q}'_{\alpha L}q'^{D}_{\beta R}\,, \qquad \text{where} \quad \alpha = 1,2,3 \quad \text{and} \quad \beta = d,s,b$$
$$\overline{Q}'_{\alpha L}q'^{U}_{\beta R}\,, \qquad \text{where} \quad \alpha = 1,2,3 \quad \text{and} \quad \beta = u,c,t$$

The first term $\overline{Q}'_{\alpha L} q'^{D}_{\beta R}$ has hypercharge -1 and can be combined to the Higgs doublet with hypercharge $+1$ to form an $SU(2)_L \times U(1)_Y$ gauge invariant term

$$-\sum_{\alpha=1,2,3}\sum_{\alpha,\beta=d,s,b} Y'^{D}_{\alpha\beta}\overline{Q}'_{\alpha L}\Phi q'^{D}_{\beta R}$$

where Y'^{D} is a 3×3 complex Yukawa couplings matrix, given by the fourth line of (1.6) (mass term for the down type of quarks). In the unitary gauge we can write the mass term as

$$-\left(\frac{v+H}{\sqrt{2}}\right)\sum_{\alpha,\beta=d,s,b} Y'^{D}_{\alpha\beta}\overline{q}'^{D}_{\alpha L} q'^{D}_{\beta R}$$

where $Y'^{D}_{d,s,b} = Y'^{D}_{1,2,3}$. The term proportional to v belongs to the mass term for the down type quarks. For up type quarks, as $\overline{Q}'_{\alpha L} q'^{U}_{\beta R}$ has hypercharge $+1$, a Higgs doublet with hypercharge -1 is needed in order to have a $SU(2)_L \times U(1)_Y$ gauge invariant term. Therefore we define

$$\widetilde{\Phi} = \iota\tau_2\Phi^*$$

and write the gauge invariant term as

$$-\sum_{\alpha=1,2,3}\sum_{\alpha,\beta=u,c,t} Y'^{U}_{\alpha\beta}\overline{Q}'_{\alpha L}\widetilde{\Phi} q'^{U}_{\beta R}$$

In the unitary gauge we have

$$\widetilde{\Phi} = \frac{1}{\sqrt{2}}\begin{pmatrix} v+H(x) \\ 0 \end{pmatrix}$$

The mass term for the up type of quarks can be written as

$$-\left(\frac{v+H}{\sqrt{2}}\right)\sum_{\alpha,\beta=u,c,t} Y'^{U}_{\alpha\beta}\overline{q}'^{U}_{\alpha L} q'^{U}_{\beta R}$$

Here $Y'^{U}_{u,c,t} = Y'^{U}_{1,2,3}$. The term proportional to v belongs to the masses of the up type of quarks. In the unitary gauge the quark Yukawa lagrangian can be written as

$$\mathcal{L}_{H,Q} = -\left(\frac{v+H}{\sqrt{2}}\right)\left[\sum_{\alpha,\beta=d,s,b} Y'^{D}_{\alpha\beta}\overline{q}'^{D}_{\alpha L} q'^{D}_{\beta R} + \sum_{\alpha,\beta=u,c,t} Y'^{U}_{\alpha\beta}\overline{q}'^{U}_{\alpha L} q'^{U}_{\beta R}\right] \tag{1.31}$$

The complex Yukawa coupling matrices $Y'^{D}_{\alpha\beta}$, $Y'^{D}_{\alpha\beta}$ are non diagonal in general. In order to get mass terms for the quarks, we must diagonalize the Yukawa matrices. We define

$$q'^{U}_{L} = \begin{pmatrix} u'_L \\ c'_L \\ t'_L \end{pmatrix}, \qquad q'^{U}_{R} = \begin{pmatrix} u'_R \\ c'_R \\ t'_R \end{pmatrix}, \qquad q'^{D}_{L} = \begin{pmatrix} d'_L \\ s'_L \\ b'_L \end{pmatrix}, \qquad q'^{D}_{R} = \begin{pmatrix} d'_R \\ s'_R \\ b'_R \end{pmatrix}$$

This allows us to write the quark Yukawa lagrangian in matrix form as

$$\mathcal{L}_{H,Q} = -\left(\frac{\upsilon + H}{\sqrt{2}}\right)\left[\overline{q}'^{D}_{L} Y'^{D} q'^{D}_{R} + \overline{q}'^{U}_{L} Y'^{U} q'^{U}_{R}\right] + h.c \tag{1.32}$$

The Yukawa matrices can be diagonalized by a biunitary transformation:

$$\begin{aligned} V^{D\dagger}_{L} Y'^{D} V^{D}_{R} &= Y^{D}, \quad \text{with} \quad Y^{D}_{\alpha\beta} = y^{D}_{\alpha}\delta_{\alpha\beta} \;\; (\alpha,\beta = d,s,b) \\ V^{U\dagger}_{L} Y'^{U} V^{U}_{R} &= Y^{U}, \quad \text{with} \quad Y^{U}_{\alpha\beta} = y^{U}_{\alpha}\delta_{\alpha\beta} \;\; (\alpha,\beta = d,s,b) \end{aligned}$$

(V^{D}_{L}, V^{D}_{R}, V^{U}_{L}, V^{U}_{R} are four suitable 3×3 matrices). Now we define unprimed fields in terms of primed fields:

$$\begin{aligned} q^{U}_{L} &= V^{U\dagger}_{L} q'^{U}_{L} = \begin{pmatrix} u_L \\ c_L \\ t_L \end{pmatrix}, \qquad & q^{U}_{R} &= V^{U\dagger}_{R} q'^{U}_{R} = \begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix} \\ q^{D}_{L} &= V^{D\dagger}_{L} q'^{D}_{L} = \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}, \qquad & q^{D}_{R} &= V^{D\dagger}_{R} q'^{D}_{R} = \begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix} \end{aligned} \tag{1.33}$$

The Yukawa lagrangian becomes:

$$\begin{aligned} \mathcal{L}_{H,Q} &= -\left(\frac{\upsilon + H}{\sqrt{2}}\right)\left[\overline{q}^{D}_{L} Y^{D} q^{D}_{R} + \overline{q}^{U}_{L} Y^{U} q^{U}_{R}\right] + h.c \\ &= -\sum_{\alpha=d,s,b} \frac{y^{D}_{\alpha}\upsilon}{\sqrt{2}} \overline{q}^{D}_{\alpha} q^{D}_{\alpha} - \sum_{\alpha=u,c,t} \frac{y^{U}_{\alpha}\upsilon}{\sqrt{2}} \overline{q}^{U}_{\alpha} q^{U}_{\alpha} \\ &\quad - \sum_{\alpha=d,s,b} \frac{y^{D}_{\alpha}}{\sqrt{2}} \overline{q}^{D}_{\alpha} q^{D}_{\alpha} H - \sum_{\alpha=u,c,t} \frac{y^{U}_{\alpha}}{\sqrt{2}} \overline{q}^{U}_{\alpha} q^{U}_{\alpha} H \end{aligned}$$

where the unprimed quarks fields

$$q^{D}_{\alpha} = q^{D}_{\alpha L} + q^{D}_{\alpha R}\,, \qquad q^{U}_{\alpha} = q^{U}_{\alpha L} + q^{U}_{\alpha R}$$

have a definite mass. Their masses are given by:

$$m_\alpha = \frac{y_\alpha^D \upsilon}{\sqrt{2}} \qquad \text{with} \qquad (\alpha = d, s, b) \tag{1.34}$$
$$m_\alpha = \frac{y_\alpha^U \upsilon}{\sqrt{2}} \qquad \text{with} \qquad (\alpha = u, c, t)$$

Similar to the leptons case the values of y_d^D,y_s^D,y_b^D,y_u^U,y_c^U,y_t^U are unknown parameters of the SM. Therefore the quark masses cannot be calculated. Due to mismatch between the (massive) unprimed quark fields and primed quark fields, we are led to a particular phenomenon called quark mixing. The quark weak current (1.10) can be written in matrix form

$$j^\rho_{W,Q} = 2\overline{q}'^U_L \gamma^\rho q'^D_L$$

Now expressing unprimed quark fields in terms of primed quark fields (1.33), we can write the quark weak current as

$$j^\rho_{W,Q} = 2\overline{q}^U_L V_L^{U\dagger} \gamma^\rho V_L^D q_L^D = 2\overline{q}^U_L \gamma^\rho V_L^{U\dagger} V_L^D q_L^D$$

The charged current depends on the combination of $V_L^{U\dagger}$,V_L^D called quark mixing matrix or Cabibbo-Kobayashi-Maskawa (CKM) matrix, carrying the physical effects of quark mixing.

$$V = V_L^{U\dagger} V_L^D \tag{1.35}$$

Neutral currents in unprimed quark fields are given by

$$j^\rho_{Z,Q} = 2\left(g_L^U \overline{q}^U_L \gamma^\rho q^U_L + g_R^U \overline{q}^U_R \gamma^\rho q^U_R\right) + 2\left(g_L^D \overline{q}^D_L \gamma^\rho q^D_L + g_R^U \overline{q}^D_R \gamma^\rho q^D_R\right)$$

This shows that the neutral current in terms of unprimed quark fields is identical to the neutral current, written in terms of the primed quark fields (GIM mechanism). Thus in the Standard Model there are no flavor changing neutral currents.

1.2.3 Quark Mixing

The Cabibbo-Kobayashi-Maskawa (CKM) matrix is a 3×3 unitary matrix given by

$$V_{CKM} = V_L^{U\dagger} V_L^D = \begin{pmatrix} V_{11} & V_{12} & V_{13} \\ V_{21} & V_{22} & V_{23} \\ V_{31} & V_{32} & V_{33} \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \tag{1.36}$$

Here V_L^U and V_L^D unitary matrices which are used to diagonalize the up type and the down type of quark mass matrices respectively. An arbitrary unitary $N \times N$ has N^2 real parameters, out of which

$$\frac{N(N-1)}{2} \quad \text{are mixing angles and} \quad \frac{N(N+1)}{2} \quad \text{are phases}$$

where N represents the number of generations of quarks. If we take $N = 3$, then we will have

$$\frac{N(N-1)}{2} = 3 \quad \text{mixing angles,} \quad \frac{N(N+1)}{2} = 1 \quad \text{phase}$$

For $N = 2$ we have:

$$\frac{N(N-1)}{2} = 1 \quad \text{mixing angles,} \quad \frac{N(N+1)}{2} = 0 \quad \text{phase}$$

So in case of 3 generations we have 1 phase (δ_{13}) and 3 mixing angles ($\theta_{12}, \theta_{23}, \theta_{13}$) where as in case of 2 generations, we do not have any phase however we have one mixing angle called the Cabibo angle θ_W[18]. Parameterization of a 3×3 mixing matrix is given in [16],[17]

$$V_{CKM} = \begin{pmatrix} c_{12}c_{13} & s_{13}c_{12} & s_{13}e^{-\iota\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{\iota\delta_{13}} & c_{12}c_{23} - c_{12}s_{23}s_{13}e^{\iota\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{12} - c_{12}c_{23}s_{13}e^{\iota\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{\iota\delta_{13}} & c_{23}c_{13} \end{pmatrix} \tag{1.37}$$

($c_{ab} = \cos\theta_{ab}$, $s_{ab} = \sin\theta_{ab}$). The experimental values of these parameters are given by

$s_{12} = 0.2243 \pm 0.0016$, $s_{23} = 0.0413 \pm 0.0015$, $s_{13} = 0.0037 \pm 0.0005$
$\delta_{13} = 1.05 \pm 0.24 = 60^\circ \pm 14^\circ$

Another useful parameterization was put forward by Wolfenstein showing the hierarchy among the quark mixing angles $1 \gg s_{12} \gg s_{23} \gg s_{13}$. By using the following relations in (1.37), one can get the Wolfenstein parameterization

$$s_{12} = \lambda, \quad s_{23} = A\lambda^2 \quad s_{13}e^{-\iota\delta_{13}} = A\lambda^3(\rho - \iota\eta)$$

$$V_{CKM} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - \iota\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - \iota\eta) & -A\lambda^2 & 1 \end{pmatrix} + \vartheta\left(\lambda^4\right)$$

The experimental values of the CKM matrix elements are [17]

$$V_{CKM} = \begin{pmatrix} 0.97419 \pm 0.00022 & 0.2257 \pm 0.0010 & 0.00359 \pm 0.00016 \\ 0.2256 \pm 0.0010 & 0.97334 \pm 0.00023 & 0.0415^{+0.0010}_{-0.0011} \\ 0.00874^{+0.00026}_{-0.00037} & 0.0407 \pm 0.0010 & 0.999133^{+0.000044}_{-0.000043} \end{pmatrix}$$

1.3 The Standard Model and the Neutrino Mass

In the Standard Model the left-handed neutrinos form the electroweak doublets with the charged leptons. They do not posses an electric charge and are colorless. In this theory right handed neutrinos are not included. Due to the absence of the right handed neutrinos in this theory neutrinos are massless at the tree level. Due to the presence of the exact $B - L$ symmetry in this theory, neutrinos remain massless in all orders of the perturbation theory as well as when non perturbative effects are taken into account.[19]. It is therefore natural to assume that the nonzero neutrino masses must be associated to the right-handed neutrinos and with the breaking of the $B - L$ symmetry. If the condition of explicit renormalizability of the theory is abandoned, neutrino masses can be obtained even if the SM particles are the only light degrees of freedom. The renormalizable operator generates the Majorana neutrino mass after the electroweak symmetry breaking.[20]

$$\frac{\lambda_{ij}}{M} \left(L_i H\right)^T \left(L_j H\right) \quad , \qquad i, j = e, \mu, \tau$$

$$m_{ij} = \frac{\lambda_{ij} \left\langle H \right\rangle^2}{M}$$

(λ_{ij}, H, M are the dimensionless coupling, the Higgs doublet and the cut off scale respectively). In order to get the correct masses for the neutrinos, some new physics scale below the M_{Pl} must exit.

Chapter 2

Fermion Masses Beyond the Standard Model

In the Standard Model the mixing of the quark flavors arises after diagonalizing the up and down type quark mass matrices. Both mass matrices cannot be diagonalized by unitary transformations which commute with the charged weak generators. This diagonalization-mismatch gives rise to the phenomenon of flavor mixing, whose dynamical origin is unknown. However it is implied that the mechanism responsible for the generation of the quark masses is also responsible for quark mixing [22]. In many models which go beyond the Standard electroweak model, based of flavor symmetries, the flavor mixing angles are the functions of the mass eigenvalues [23],[24]. A hierarchy exists between both the observed values of mass spectrum of quarks and the observed values of flavor mixing parameters. This hierarchical structure can be understood as a result of a specific pattern of chiral symmetries whose breaking would cause the hierarchical tower of masses to appear step by step. [25],[26]. Such a chiral evolution of the quark mass matrices leads to specific way of describing the flavor mixing. Here we describe a parameterization of flavor mixing which is unique in the sense that it incorporates the chiral evolution of the mass matrices in a natural way.

We assume that the quark mass eigenvalues are the dynamical entities whose values can be changed to study certain symmetry limits (as is done in QCD). Without loss of generality we can take the quark mass matrices as Hermitian matrices. In the limit where the masses of the u and d quarks are set to zero, the quark mass matrix $\widetilde{M}$ (both for up and down type of quarks) can be described in such a way that its $\widetilde{M}_{1i}$ and $\widetilde{M}_{i1}$ elements are zero. [25] Therefore $\widetilde{M}$ takes the form

$$\widetilde{M} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \widetilde{C} & \widetilde{B} \\ 0 & \widetilde{B}^* & \widetilde{A} \end{pmatrix} \tag{2.1}$$

The observed mass hierarchy is incorporated in this structure by taking $\widetilde{A} >> \widetilde{B}, \widetilde{C}$. The complex phases in the mass matrix (2.1) can be rotated away by subjecting both $\widetilde{M_u}$, $\widetilde{M_d}$ to the same unitary transformation. Thus in both up and down quark sectors, $\widetilde{B}$ can be taken as real. There is no CP violation at this stage. The flavor mixing matrix which diagonalizes the mass matrix (2.1), and describes the mixing between the second and the third family by the angle $\widetilde{\theta}$, is given as

$$\widetilde{V} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \widetilde{c} & \widetilde{s} \\ 0 & -\widetilde{s} & \widetilde{c} \end{pmatrix}$$

($\widetilde{c} = \cos\widetilde{\theta}$ and $\widetilde{s} = \sin\widetilde{\theta}$).

As the limit $m_u = m_d = 0$ is not far from reality, $\widetilde{\theta}$ is essentially given by $|V| = 0.039 \pm 0.002$ [27],[28] implying $\widetilde{\theta} = 2.24^\circ \pm 0.12^\circ$. At the next stage of the chiral evolution of the mass matrices, the masses of the light quarks u, d are introduced. A general hermitian mass matrix can be written as

$$M_q = \begin{pmatrix} E_q & D_q & F_q \\ D_q^* & C_q & B_q \\ F_q^* & B_q^* & A_q \end{pmatrix}$$

$(A >> C, |B| >> E, |D|, |F|)$

With a common unitary transformation of the up and the down quark fields, it is always possible to arrange the mass matrices M_u, M_d in such way that $F_q = F_u = F_d = 0$

$$M_q = \begin{pmatrix} E_q & D_q & 0 \\ D_q^* & C_q & B_q \\ 0 & B_q^* & A_q \end{pmatrix} \tag{2.2}$$

The basis in which the mass matrices take the form (2.2) is a basis in which up and down quark mass matrices exhibit two texture zero. In this basis the $(1,3)$ and $(3,1)$ elements of the mass matrices in M_u, M_d are zero. Therefore no direct mixing of the heavy quark t $(or\ b)$ and the light quark u $(or\ d)$ is present [29]. In the hierarchy limit of the quark masses the mass matrix of the type (2.2), can be diagonalized by a rotation matrix having just two angles [23]. At first the diagonal element B_q is rotated away by a rotation between the second and the third families described by the angle θ_{23}. Then the element D_q is rotated away by a transformation between the first and second families given by angle θ_{12}. There is no rotation between the first and the third families as described above. The rotation matrix for this sequence takes the form

$$R = R_{12}R_{23} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}$$

The flavor mixing matrix V is product of two such matrices one for up and the other for down type of the quark fields.

$$V = R_{12}^u R_{23}^u \left(R_{23}^d\right)^{-1} \left(R_{12}^d\right)^{-1}$$

The product of rotations $R_{23}^u \left(R_{23}^d\right)^{-1}$can be described by a single rotation matrix, given by an angle θ. The angles describing the rotations R_{12}^u and R_{12}^d are given by θ_u and θ_d respectively. Thus in the absence of the CP- violating phases the flavor mixing matrix takes the following form

$$V = \begin{pmatrix} c_u & s_u & 0 \\ -s_u & c_u & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c & s \\ 0 & -s & c \end{pmatrix} \begin{pmatrix} c_d & -s_d & 0 \\ s_d & c_d & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

($c_u = \cos\theta_u, s_u = \sin\theta_u$ etc.)

By suitably rephasing the quark fields, the flavor mixing matrix can be written in terms of a single phase φ as follows:

$$V = \begin{pmatrix} c_u & s_u & 0 \\ -s_u & c_u & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{-i\varphi} & 0 & 0 \\ 0 & c & s \\ 0 & -s & c \end{pmatrix} \begin{pmatrix} c_d & -s_d & 0 \\ s_d & c_d & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$V = \begin{pmatrix} s_u s_d c + c_u c_d e^{-i\varphi} & s_u c_d c - c_u s_d e^{-i\varphi} & s_u s \\ c_u s_d c - s_u c_d e^{-i\varphi} & c_u c_d c + s_u s_d e^{-i\varphi} & c_u s \\ -s_d s & -c_d s & c \end{pmatrix} \tag{2.3}$$

As all the three angles $\theta_u, \theta_d, \theta$ can be arranged to lie in the first quadrant, s_u, s_d, c all are positive definite. The CP violating phase φ can lie in the range $0 - 2\pi$. CP violation is present in the weak interactions if $\varphi \neq 0, \pi, 2\pi$.

The parameterization (2.3) has many advantages [22]. Some of them are as follows

1. As the mass matrix (2.3) is derived directly from the chiral expansion of the mass matrices, it naturally takes into account the hierarchy of the quark masses.

2. CP violation is linked to the first and the second families only.

3. The three mixing angles θ_u, θ_d and θ have precise physical meaning. The angle θ describes the mixing between the second and the third families in the limit $m_u << m_c << m_t$, generated by B_u and B_d in (2.2) which can be named as "heavy quark mixing". The angle θ_u describes the mixing between $u-c$ channel by term D_u in (2.2) and is called "$u-$channel mixing". Similarly the "$d-$channel mixing" is mixing given by angle θ_d which describes the mixing between $d-s$ channel by the term D_d in (2.2) in the limit $m_d << m_s << m_b$.

4. A simple relation exists between the three mixing angles and some observable quantities in the B-meson Physics. From (2.3) we can get the following simple relations

$$\sin\theta = |V_{cb}| \sqrt{1 + \left|\frac{V_{ub}}{V_{cb}}\right|^2}$$

$$\tan\theta_u = \left|\frac{V_{ub}}{V_{cb}}\right|, \ \tan\theta_d = \left|\frac{V_{td}}{V_{ts}}\right|$$

These relations make the parameterization (2.3) especially favorable for the study of B-meson Physics. An additional advantage which the parameterization (2.3) has over the standard parameterization is that the renormalization group evolution of the V is to a very high degree of accuracy associated with only angle θ . This can be verified easily if one keeps only the Yukawa coupling of t , b and neglect the possible threshold effects in renormalization group equations of the Yukawa matrices [30]. Thus if the underlying scale is changed from weak scale ($\sim 10^2$ GeV) to grand unified scale ($\sim 10^{16}$ GeV), the value of θ changes where as the values of θ_u, θ_d and φ remain independent of this variation. So heavy quark mixing is subjected to renormalization group equation effects where as $u-$and $d-$ channels and CP- violation phase are not.

2.1 Realistic Zero texture

We can predict the flavor mixing angles in terms of quark masses in the following way. We take $E_q = 0$ in the mass matrix (2.2)

$$M_q = \begin{pmatrix} 0 & D_q & 0 \\ D_q^* & C_q & B_q \\ 0 & B_q^* & A_q \end{pmatrix}$$

The physical constraints are as follows; in the flavor basis in which the (1,3) and (3,1) elements of $M_{u,d}$ vanish the (1,1) element also vanish. The vanishing of (1,1) elements can be viewed as a result of some underlying discrete or continuous flavor symmetry [31] . The prediction about the mixing angle obtained from the above texture is almost independent from renormalization-group effects. Therefore there is no need to specify any energy scale at which the above texture is realized.

2.1.1 Flavor mixing angles and masses:

For a quark mass matrix of the type (2.2) the following relations hold [31]

$$\begin{aligned} X_u &\equiv \left| \frac{|D_u|}{\lambda_1 - E_u} \frac{|D_d| \left(\lambda_3^d - A_d\right)}{|B_d| \left(\lambda_3^d - E_d\right)} + \frac{\lambda_3^d - A_d}{|B_d|} e^{i\varphi_1} + \frac{|B_u|}{\lambda_1^u - A_u} e^{i(\varphi_1+\varphi_2)} \right| \\ Y_u &\equiv \left| \frac{|D_u|}{\lambda_2 - E_u} \frac{|D_d| \left(\lambda_3^d - A_d\right)}{|B_d| \left(\lambda_3^d - E_d\right)} + \frac{\lambda_3^d - A_d}{|B_d|} e^{i\varphi_1} + \frac{|B_u|}{\lambda_2^u - A_u} e^{i(\varphi_1+\varphi_2)} \right| \end{aligned} \tag{2.4}$$

(λ_i , $i = 1, 2, 3$ represent the quark mass eigenvalues and $\varphi_{1,2}$ are related to the phases of $B_{u,d}, D_{u,d}$)

Similar relations for down type quarks can be obtained by changing the subscripts in (2.4) $u \leftrightarrow d$.

$$\begin{aligned} \tan\theta_u &= \frac{O_{21}^u}{O_{22}^u} \frac{X_u}{Y_u} \\ \tan\theta_d &= \frac{O_{21}^d}{O_{22}^d} \frac{X_d}{Y_d} \end{aligned} \tag{2.5}$$

$$\begin{aligned} \sin\theta &= \left[\left(O_{21}^u\right)^2 X_u^2 + \left(O_{22}^u\right)^2 Y_u^2 \right]^{\frac{1}{2}} O_{33}^d \\ &= \left[\left(O_{21}^d\right)^2 X_d^2 + \left(O_{22}^d\right)^2 Y_d^2 \right]^{\frac{1}{2}} O_{33}^u \end{aligned} \tag{2.6}$$

(O_{ij}^q , $ij = 21, 22$, $q = u, d$, are the elements of the matrices used to diagonalize.(2.2)).

We can define $\frac{|B_q|}{C_q} \equiv r_q$ having magnitude $\vartheta\left(1\right)$ with $C_q \neq 0$ for each quark sector. The parameters $A_q, |B_q|, C_q$ and $|D_q|$ can be expressed in terms of quark mass eigenvalues and r_q. Using relations (2.4), (2.5) and (2.6), we can find three mixing angles in terms of the quark masses given by

$$\tan\theta_u = \sqrt{\frac{m_u}{m_c}}\left(1+\triangle_u\right) \tag{2.7}$$
$$\tan\theta_d = \sqrt{\frac{m_d}{m_s}}\left(1+\triangle_d\right)$$

$$\sin\theta = \left| r_d \frac{m_s}{m_b}\left(1-\delta_d\right) - r_u \frac{m_c}{m_t}\left(1-\delta_u\right) e^{i\varphi_2} \right| \tag{2.8}$$

The next-to-leading order corrections are:

$$\triangle_u = \sqrt{\frac{m_c m_d}{m_u m_s}\frac{m_s}{m_b}} \left| \mathrm{Re}\left[e^{i\varphi_1} - \frac{r_u}{r_d}\frac{m_c m_b}{m_t m_s} e^{i(\varphi_1+\varphi_2)} \right]^{-1} \right| \tag{2.9}$$
$$\triangle_d = \sqrt{\frac{m_u m_s}{m_c m_d}\frac{m_c}{m_t}} \left| \mathrm{Re}\left[e^{i\varphi_1} - \frac{r_d}{r_u}\frac{m_t m_s}{m_c m_b} e^{i(\varphi_1+\varphi_2)} \right]^{-1} \right|$$

$$\delta_u = \frac{m_u}{m_c} + \left(1+r_u^2\right)\frac{m_c}{m_t}$$
$$\delta_d = \frac{m_d}{m_s} + \left(1+r_d^2\right)\frac{m_s}{m_b}$$

To simplify the above relations lets take $r_u = r_d = r$ as is proposed in some models with natural symmetry [31]. Then $\sin\theta$ becomes proportional to parameter r. Using the fact that $\frac{m_s}{m_b} \sim \vartheta\left(10\right)\frac{m_c}{m_t}$, the relations (2.9) can be simplified as

$$\triangle_u = \sqrt{\frac{m_c m_d}{m_u m_s}\frac{m_s}{m_b}}\cos\varphi_1 \tag{2.10}$$
$$\triangle_d = 0$$

We obtain for the mixing angles:

$$\tan\theta_u = \sqrt{\frac{m_u}{m_c}}\left(1+\sqrt{\frac{m_c m_d}{m_u m_s}\frac{m_s}{m_b}}\cos\varphi_1\right)$$
$$\tan\theta_d = \sqrt{\frac{m_d}{m_s}}$$

In leading order the above relations give

$$\tan\theta_u = \sqrt{\frac{m_u}{m_c}} \tag{2.11}$$
$$\tan\theta_d = \sqrt{\frac{m_d}{m_s}}$$

2.2 Predicting the Neutrino Mass

Our understanding of the neutrinos has changed in past few years. Now we know that neutrinos produced in a well defined flavor eigenstate, after traveling some macroscopic distance, appear as a different eigenstate. The simplest answer to this phenomenon is that neutrinos, like all other fermions are massive particles. Their mass eigenstates are different than their flavor eigenstates. This scenario opens up many of the new possibilities which does not exist for the massless neutrinos, such as, massive neutrinos can have nonzero magnetic dipole moments, they can decay to lighter neutrinos and can have a Majorana mass term [32].

At least three light physical neutrinos with left handed flavor eigenstates are required to explain the current solar and atmospheric neutrino oscillation data. These three neutrino flavor eigenstates are related to the three mass eigenstates by a mixing matrix called lepton mixing matrix or PMNS matrix V [33].

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} V_{e1} & V_{e2} & V_{e3} \\ V_{\mu 1} & V_{\mu 2} & V_{\mu 3} \\ V_{\tau 1} & V_{\tau 2} & V_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Assuming that the neutrino masses are Majorana masses V can be parameterized by the three mixing angles and three complex phases. The standard parameterization of V is similar to the parameterization of quarks and is given as

$$V_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{13}c_{12} & s_{13}e^{-\iota\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{\iota\delta_{13}} & c_{12}c_{23} - c_{12}s_{23}s_{13}e^{\iota\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{12} - c_{12}c_{23}s_{13}e^{\iota\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{\iota\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

($c_{ab} = \cos\theta_{ab}$ and $s_{ab} = \sin\theta_{ab}$) with solar angle $= \theta_{12}$, atmospheric angle $= \theta_{23}$, reactor angle $= \theta_{13}$.

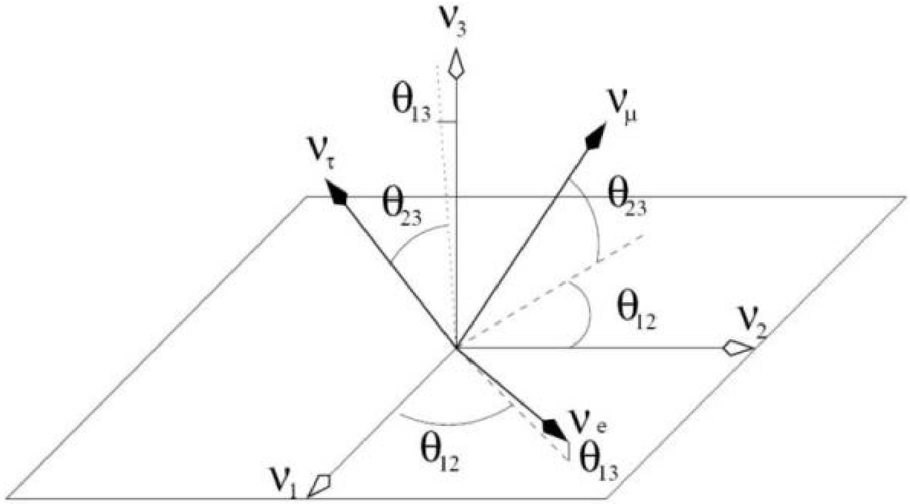

Figure 2.1: The relation between the neutrino flavor-mass eigenstates

The discussion in the previous section for quarks can be extended for the leptons in a straight forward way [35]. If neutrinos are Majorana particles, then the lepton mixing matrix is given by $V = UP$, where

$$U_{PMNS} = \begin{pmatrix} c_l & s_l & 0 \\ -s_l & c_l & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{-i\varphi} & 0 & 0 \\ 0 & c & s \\ 0 & -s & c \end{pmatrix} \begin{pmatrix} c_\nu & -s_\nu & 0 \\ s_\nu & c_\nu & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.12}$$

$$U_{PMNS} = \begin{pmatrix} s_l s_\nu c + c_l c_\nu e^{-i\varphi} & s_l c_\nu c - c_l s_\nu e^{-i\varphi} & s_l s \\ c_l s_\nu c - s_l c_\nu e^{-i\varphi} & c_l c_\nu c + s_l s_\nu e^{-i\varphi} & c_l s \\ -s_\nu s & -c_\nu s & c \end{pmatrix} \tag{2.13}$$

($c_l = \cos\theta_l$, $c_\nu = \cos\theta_\nu$, $s = \sin\theta$). The Majorana phase matrix is given as $P = \{e^{i\rho}, e^{i\sigma}, 1\}$.

These mixing angles have direct physical interpretations angle θ describes the mixing between the second and the third family, θ_l describes the mixing between charged lepton sector ($e - \mu$ mixing), and the angle θ_ν describes the solar neutrino mixing($\nu_e - \nu_\mu$ mixing). In the approximation that solar and atmospheric neutrino oscillations nearly decouple, the solar angle θ_{12}, the atmospheric angle θ_{23} and the CHOOZ angle θ_{13} can be expressed in terms of angles θ_l,θ_ν and θ.

$$\theta_{12} \approx \theta_\nu \ , \theta_{23} \approx \theta \ , \theta_{13} \approx \theta_l \sin\theta \tag{2.14}$$

Neutrino oscillation data can be described in a very convenient way by using this parameterization. In order to relate the neutrino masses with the neutrino mixing angles (similar to the case of quarks), we speculate that the neutrino has the normal mass hierarchy. By using the equation (2.11), we can write

$$\tan\theta_l = \sqrt{\frac{m_e}{m_\mu}} \quad , \quad \tan\theta_\nu = \sqrt{\frac{m_1}{m_2}} \tag{2.15}$$

Using the relations (2.15) and the fact that neutrino oscillations are associated with the mass squared differences $\triangle m_{21}^2 = m_2^2 - m_1^2$ and $\triangle m_{32}^2 = m_3^2 - m_2^2$, we arrive at the following relations:

$$m_1^2 = \frac{\sin^4\theta_\nu}{\cos 2\theta_\nu}\triangle m_{21}^2 \tag{2.16}$$

$$m_2^2 = \frac{\cos^4\theta_\nu}{\cos 2\theta_\nu}\triangle m_{21}^2 \tag{2.17}$$

$$\begin{aligned} m_3^2 &= \frac{\cos^4\theta_\nu}{\cos 2\theta_\nu}\triangle m_{21}^2 + \triangle m_{32}^2 \\ &= \frac{\cos^4\theta_\nu}{\cos 2\theta_\nu}\triangle m_{21}^2 + \left(\triangle m_{31}^2 - \triangle m_{21}^2\right) \end{aligned} \tag{2.18}$$

2.3 Tribimaximal mixing

It was conjectured independently by Cabibo [37] and Wolfenstein [39] , that the mixing matrix linking charged leptons to the neutrino could be given by

$$U_{l\nu}^{CW} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}$$

($\omega = \exp\left(\frac{2\pi i}{3}\right) = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$).
In the conventional notation this can be written as

$$\theta_{12} = \theta_{23} = 45^\circ \quad , \quad \theta_{13} = 35.3^\circ \quad , \quad \delta_{CP} = 90^\circ$$

After the discovery of the neutrino oscillations Harrison, Perkins and Scott proposed in 2002 the tribimaximal mixing matrix (or HPS) mixing matrix [40], which describes the current neutrino oscillation data very well and is given by

$$U^{TBM} = U_{l\nu}^{HPS} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \tag{2.19}$$

2.3.1 Numerical analysis of the Tribimaximal mixing:

The tribimaximal mixing leads to $\tan\theta_{12} = \frac{1}{\sqrt{2}} \Rightarrow \theta_{12} \approx 35.26°$. For the atmospheric neutrino oscillations we have $\tan\theta_{23} = 1 \Rightarrow \theta_{23} = 45°$. For the CHOOZ angle we take $\theta_{13} = 0°$.

From the matrices (2.13) and (2.19) we can calculate the numerical values of the angles θ_l, θ_ν and θ

$$\cos\theta = \frac{1}{\sqrt{2}} \Longrightarrow \theta_{23} \approx \theta = 45°$$

$$-\sin\theta_\nu \sin\theta = \frac{1}{\sqrt{6}} \Longrightarrow \theta_{12} \approx \theta_\nu = 35.26°$$

$$\cos\theta_l \sin\theta = -\frac{1}{\sqrt{2}} \Longrightarrow \theta_l = \pi \ , \quad 0 = \theta_{13} \approx \theta_l \sin\theta = 2.22°$$

Using the best fit values of the mass splitting parameters $\triangle m_{21}^2 \approx 7.65 \times 10^{-5}$ eV^2 , $\triangle m_{31}^2 \approx 2.40 \times 10^{-3}$eV^2 [36] and the mixing angle $\theta_\nu \approx 35°$, we can predict the masses of the neutrinos using relations (2.16)(2.17)(2.18)

$$\begin{aligned} m_1 &= \sqrt{\frac{\sin^4\theta_\nu}{\cos 2\theta_\nu}\triangle m_{21}^2} \\ &= \sqrt{\frac{\sin^4 35°}{\cos 2(35°)} \times 7.65 \times 10^{-5}} \\ &\approx 0.005 \quad \text{eV} \end{aligned}$$

$$\begin{aligned} m_2 &= \sqrt{\frac{\cos^4\theta_\nu}{\cos 2\theta_\nu}\triangle m_{21}^2} \\ &= \sqrt{\frac{\cos^4 35°}{\cos 2(35°)} \times 7.65 \times 10^{-5}} \\ &\approx 0.01 \quad \text{eV} \end{aligned}$$

$$\begin{aligned} m_3 &= \sqrt{\frac{\cos^4\theta_\nu}{\cos 2\theta_\nu}\triangle m_{21}^2 + (\triangle m_{31}^2 - \triangle m_{21}^2)} \\ &= \sqrt{\frac{\cos^4 35^\circ}{\cos 2(35^\circ)} \times 7.65\times 10^{-5} + 2.32\times 10^{-3}} \\ &\approx 0.05 \quad \text{eV} \end{aligned}$$

The analysis shows that the neutrinos have normal but weak hierarchy i.e. $m_1 : m_2 : m_3 = 1 : 2 : 10$. Neutrino masses obtained using tribimaximal mixing agree with those obtained by Fritzsch and Xing [35].

2.4 Near tribimaximal mixing

Although the tribimaximal mixing describes the neutrino oscillation data very well, it is not unique. A neutrino mixing matrix describing near tribimaximal mixing can be written as [41]

$$U_{NTBM} = \begin{pmatrix} \frac{2}{\sqrt{2\,(2+x^2)}} & \frac{x}{\sqrt{2+x^2}} & 0 \\ -\frac{x}{\sqrt{2\,(2+x^2)}} & \frac{1}{\sqrt{2+x^2}} & \frac{1}{\sqrt{2}} \\ \frac{x}{\sqrt{2\,(2+x^2)}} & -\frac{1}{\sqrt{2+x^2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \tag{2.20}$$

The relation $x = \sqrt{2}\tan\theta_{12}$ can be used to determine the allowed range of the parameter x. For $30^\circ < \theta_{12} < 38^\circ$ we obtain the values $0.82 \lesssim x \lesssim 1.10$. The simplest choice of $x = 1$, leading to tribimaximal mixing, is the most favored possibility, where as the value of $x = \sqrt{2}$ resulting in bimaximal neutrino mixing, is ruled out. One can determine few viable forms of V_{PMNS} , by varying x in the suitable range specified above.

2.4.1 Numerical analysis under near tribimaximal mixing:

We can determine the θ_ν in terms of the parameter x :

$$\cos 2\theta_\nu = \frac{2-x^2}{2+x^2}$$

$$\cos^4\theta_\nu = \frac{4}{x^4+4x^2+4}$$

$$\sin^4\theta_\nu = \frac{x^4}{x^4+4x^2+4}$$

With the help of the above relations and (2.16-2.18), one can obtain the following mass relations

$$\begin{aligned} m_1 &= \sqrt{\frac{x^4}{4-x^4}\times\Delta m_{21}^2} \\ &= \sqrt{\frac{x^4}{4-x^4}\times(7.65\times10^{-5})} \quad \text{eV} \end{aligned}$$

$$\begin{aligned} m_2 &= \sqrt{\frac{4}{4-x^4}\times\Delta m_{21}^2} \\ &= \sqrt{\frac{4}{4-x^4}\times(7.65\times10^{-5})} \quad \text{eV} \end{aligned}$$

$$\begin{aligned} m_3 &= \sqrt{\frac{4}{4-x^4}\times\Delta m_{21}^2+\Delta m_{32}^2} \\ &= \sqrt{\frac{4}{4-x^4}\times 7.65\times10^{-5}+(2.32\times10^{-3})} \qquad \text{eV} \end{aligned}$$

Graphs between m_1, m_2, m_3 and x are drawn given by figures 2.2 to 2.5. These graphs show that the neutrinos have a normal hierarchy. As the parameter x increases from its TMB value ($x = 1$), the neutrino masses also increase and vice versa. The mass m_3 is relatively stable against changes in the parameter x as compared to $m_1$and m_2.

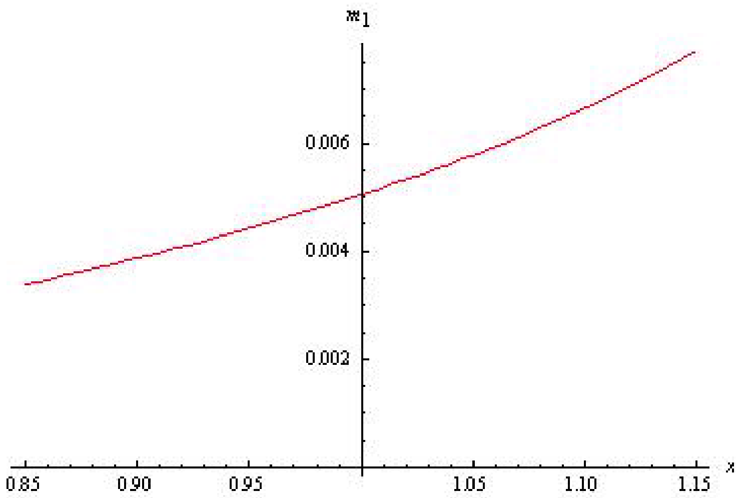

Figure 2.2: A graph between m1 and parameter x

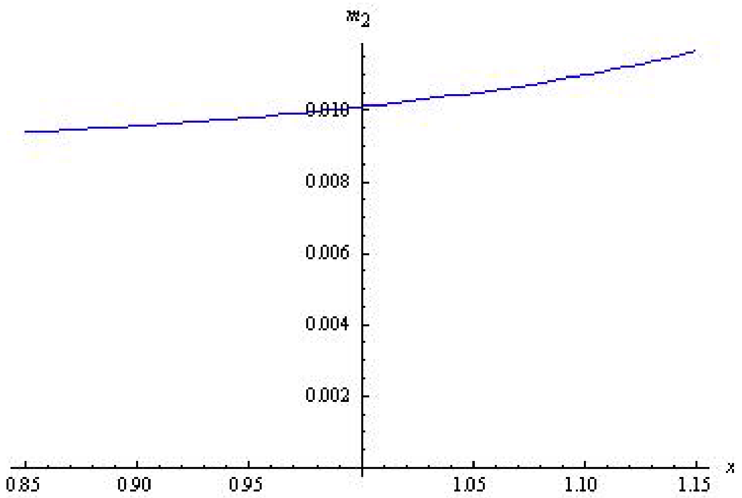

Figure 2.3: A graph between m2 and parameter x

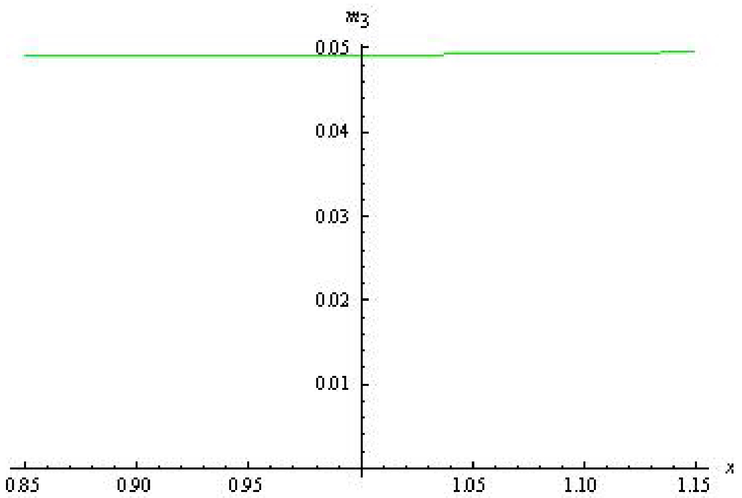

Figure 2.4: A graph between m3 and parameter x

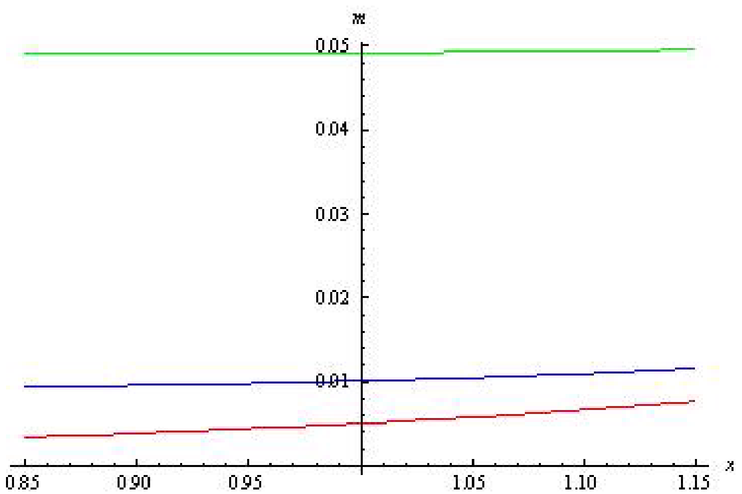

Figure 2.5: A graph between m1, m2 , m3 and parameter x

Chapter 3

Fermion Masses in Grand Unified Theories

3.1 SO(10) Grand Unified Theory

The $SO(10)$ Grand Unified Theory [42] is a candidate for a unified theory of the strong and electro-weak interactions. One of the strongest motivations for the $SO(10)$ gauge symmetry comes from the observation that all the standard model multiplets of one generation fit in a single 16-dimensional chiral spinor of $SO(10) \supset SU(3)_c \times SU(2)_L \times U(1)_Y$

$$16_F = \overbrace{(3,2,+\frac{1}{3})}^{Q_L} \oplus \overbrace{(1,2,-1)}^{L_L} \oplus \overbrace{(\overline{3},1,-\frac{4}{3})}^{u_L^c} \oplus \overbrace{(\overline{3},1,+\frac{2}{3})}^{d_L^c} \oplus \overbrace{(1,1,0)}^{\nu_L^c} \oplus \overbrace{(1,1,+2)}^{e_L^c} \tag{3.1}$$

(in terms of $SU(5)$ Grand unified theory $16 = \overline{5} \oplus 10 \oplus 1$). Hence $SO(10)$ is the minimal theory of matter unification. It has benefits

- It has a left right symmetry as a finite gauge transformation in the form of generalized charge conjugation and the Pati- Salam $SU(4)_C$ symmetry which unifies quarks and leptons.
- The right handed neutrino is automatically present which is required for the Dirac mass term for the neutrinos and for the Majorana mass term through the See-saw mechanism [43]. The Yukawa couplings, spontaneous symmetry breaking pattern and the gauge coupling unification determines the scale and structure of the Majorana mass.
- Tight connections among the Yukawa couplings exist because the gauge symmetry does not distinguish among the components of (3.1). This

reduces the number of independent parameters and the $SO(10)$ textures of the effective quark and lepton mass and mixing matrices.

- The supersymmetric version has R-parity (matter parity) as a gauge symmetry [44], a part of the center $Z4$ of $SO(10)$. In some case (tree level see-saw) it can be shown [45] that R-parity remains exact at all energies, surviving the symmetry breaking. The lightest supersymmetric particle (LSP) is then stable. This is a perfect dark matter candidate.

3.1.1 Motivation for Non-SUSY SO(10)

In the previous years most of the attention was focused on the supersymmetric version of $SO(10)$, due to the success of supersymmetric unification, and the use of supersymmetry in controlling the gauge hierarchy. However, supersymmetry may not be there [46]. It controls the Higgs gauge hierarchy, but not the cosmological constant. The long standing failure of understanding the smallness of the cosmological constant suggests that the unwelcome fine tuning may be necessary. Our fine-tuned world can be viewed, in the landscape picture simply as a selection criterion among the large number of degenerate string vacua. Or it could be that the cosmological evolution of the universe selects a light Higgs doublet [53]. If so, the main motivation behind the low-energy supersymmetry would be gone. However it is possible that the issue might be settled by the internal consistency and the predictions of a well defined grand unified theory, such as non-SUSY $SO(10)$. It is possible that simplicity and minimality at the GUT scale requires a specific low-energy theory [46].

The neutrino oscillation data strongly favors the $SO(10)$ theory. The $SO(10)$ does not require supersymmetry for a successful unification of gauge couplings. An additional Peccei-Quinn $U(1)_{PQ}$ symmetry [54] may also be introduced in order to have a dark matter candidate. This is more than sufficient motivation to carefully study ordinary non-supersymmetric $SO(10)$.

3.1.2 Symmetry Breaking in SO(10)

Unification of the gauge couplings in non-SUSY $SO(10)$ can be achieved by taking the left-right symmetry as the intermediate scale. This intermediate mass scale must be consistent with the See-Saw mechanism for the neutrino mass. The $B-L$ breaking scale which is responsible for right-handed neutrino masses cannot be too low. Because the Higgs scalar and the right-handed neutrinos, responsible for $B-L$ breaking are Standard Model singlets, they have no impact on the one loop running of renormalization group equations.

The $B-L$ breaking scale cannot be predicted. Because the couplings do not unify in the Standard Model, $B-L$ breaking scale must be below $SU(5)$ breaking. If we take M_R as the $SU(2)_R$ breaking scale, then we can write $M_{B-L} \leq M_R$. Hence M_R must be large enough. One can discriminate among many $SO(10)$ scenarios and the corresponding breaking patterns using the constraints imposed by the absolute neutrino mass scale on the position of the $B-L$ threshold and the proton decay bound on the unification scale M_U.

3.1.3 Symmetry breaking chains with two intermediate scales

With $\overline{126}_H$ one can construct a potentially realistic $SO(10)$ Yukawa sector. Together with 10_H, contractions of the the matter bilinears $16_F 16_F$ with $\overline{126}_H$ or with $\frac{\overline{16}_H \overline{16}_H}{\Lambda}$ leads to the Dirac Yukawa couplings and the Majorana mass matrices at the Standard Model level , Λ being a scale at which the effective dimension five coupling emerge.

To govern the effective Yukawa couplings up to lower energies, it is sufficient to have only two complex symmetric matrices Y_{10} and Y_{126} at renormalizable $SO(10)$ level, if 10_H , transforming under $SO(10)$, has additional quantum numbers of a complex representation of some additional symmetry ($U(1)_{PQ}$ etc.)[47].

$$16_F(10_H Y_{10} + \overline{126}_H Y_{126})16_F$$

Such models are well constrained and well motivated. D-parity, a discrete symmetry, belongs to the $SO(10)$ gauge group. Invariance under $SO(10)$ implies exact D-parity. D-parity acts as charge conjugation in left-right symmetric theories [48]. It enforces equal left and right gauge couplings. D-odd Pati-Salam (PS) singlets present in 210 may spontaneously break the D-parity, or alternatively it may be broken by a 45 Higgs representation. A left-right symmetric universe cannot lead to a baryonic asymmetry of the universe[49]. The spontaneous breaking of a discrete symmetry, such as D-parity, creates domain walls. These domain walls do not disappear, if they are massive enough for intermediate mass scales and result in over-closing the universe [50]. These problems may be avoided by confining D-parity at the GUT scale or by invoking inflation. Inflation causes the domain walls to form above the reheating temperature and enforces a lower bound on the D-parity breaking scale of 10^{12}GeV.

The symmetry breaking chain with two intermediate scales $SO(10) \rightarrow G2 \rightarrow G1 \rightarrow SM$, is Given in the table 3.1.

Chain	G2	G1
I:	$\xrightarrow[210]{} \{2_L 2_R 4_C\}$	$\xrightarrow[45]{} \{2_L 2_R 1_X 3_c\}$
II:	$\xrightarrow[54]{} \{2_L 2_R 4_C P\}$	$\xrightarrow[210]{} \{2_L 2_R 1_X 3_c P\}$
III:	$\xrightarrow[54]{} \{2_L 2_R 4_C P\}$	$\xrightarrow[45]{} \{2_L 2_R 1_X 3_c\}$
IV:	$\xrightarrow[210]{} \{2_L 2_R 1_X 3_c P\}$	$\xrightarrow[45]{} \{2_L 2_R 1_X 3_c\}$
V:	$\xrightarrow[210]{} \{2_L 2_R 4_C\}$	$\xrightarrow[45]{} \{2_L 1_R 4_C\}$
VI:	$\xrightarrow[54]{} \{2_L 2_R 4_C P\}$	$\xrightarrow[45]{} \{2_L 1_R 4_C\}$
VII:	$\xrightarrow[54]{} \{2_L 2_R 4_C P\}$	$\xrightarrow[210]{} \{2_L 2_R 4_C\}$
VIII:	$\xrightarrow[45]{} \{2_L 2_R 1_X 3_c\}$	$\xrightarrow[45]{} \{2_L 1_R 1_X 3_c\}$
IX:	$\xrightarrow[210]{} \{2_L 2_R 1_X 3_c P\}$	$\xrightarrow[45]{} \{2_L 1_R 1_X 3_c\}$
X:	$\xrightarrow[210]{} \{2_L 2_R 4_C\}$	$\xrightarrow[210]{} \{2_L 1_R 1_X 3_c\}$
XI:	$\xrightarrow[54]{} \{2_L 2_R 4_C P\}$	$\xrightarrow[210]{} \{2_L 1_R 1_X 3_c\}$
XII:	$\xrightarrow[45]{} \{2_L 1_R 4_C\}$	$\xrightarrow[45]{} \{2_L 1_R 1_X 3_c\}$

Table 3.1: SO(10) symmetry breaking chains via two intermediate gauge groups G1 and G2

3.2 Yukawa Sector of SO(10)

The fermion families are 16-dimensional spinors of $SO(10)$.

$$16_F \times 16_F = 10_H + 120_H + 126_H \tag{3.2}$$

One has three possible Yukawa coupling matrices [51], to fit all the fermion masses and mixings. If one wants a predictive theory, one should stick to the minimal one. Ideally one could use a single Higgs multiplet. A single set of Yukawas can be diagonalized. Then all the fermion mass matrices would be simultaneously diagonal. In this case we will get bad mass relations, and there will be no quark mixing and no lepton mixing in the weak currents. Therefore the minimal theory must have at least two such Higgs multiplets, hence two Yukawa matrices. As the Standard Model has at least four Yukawa matrices, it should be no surprise, that such a minimal theory is over-constrained and predictive. We have the following possibilities

(a) $\overline{126}_H + 10_H$
(b) $120_H + 10_H$
(c) $126_H + 120_H$
(d) $10_H + 10_H$
(e) $120_H + 120_H$
(f) $26_H + 126_H$

Using the Pati-Salam $SU(2)_L \times SU(2)_R \times SU(4)_C$ decomposition

$$\begin{aligned} 10_H &= (2,2,1) + (1,1,6) \\ \overline{126}_H &= (1,3,10) + (3,1,\overline{10}) + (2,2,15) + (1,1,6) \\ 120_H &= (1,3,6) + (3,1,6) + (2,2,15) + (2,2,1) + (1,1,10) + (1,1,\overline{10}) \end{aligned} \tag{3.3}$$

and the following properties of the Yukawa matrices

$$Y_{10} = Y_{10}^T \ , \ Y_{126} = Y_{126}^T \ , \ Y_{120} = -Y_{120}^T \tag{3.4}$$

we can write the M_u, M_d, M_l, M_D, $M_{\nu R}$, $M_{\nu L}$, denoting up quark, down quark, charged leptons, neutrino Dirac, right-handed neutrino and left-handed neutrino mass matrices respectively.

$$\begin{aligned}
M_u &= \langle 2,2,1\rangle^u_{10} Y_{10} + \langle 2,2,15\rangle^u_{126} Y_{126} + (\langle 2,2,1\rangle^u_{120} + \langle 2,2,15\rangle^u_{120})Y_{120} \qquad (3.5)\\
M_d &= \langle 2,2,1\rangle^d_{10} Y_{10} + \langle 2,2,15\rangle^d_{126} Y_{126} + (\langle 2,2,1\rangle^d_{120} + \langle 2,2,15\rangle^d_{120})Y_{120}\\
M_l &= \langle 2,2,1\rangle^d_{10} Y_{10} - 3\,\langle 2,2,15\rangle^d_{126} Y_{126} + (\langle 2,2,1\rangle^d_{120} - 3\,\langle 2,2,15\rangle^d_{120})Y_{120}\\
M_D &= \langle 2,2,1\rangle^u_{10} Y_{10} - 3\,\langle 2,2,15\rangle^u_{126} Y_{126} + (\langle 2,2,1\rangle^u_{120} - 3\,\langle 2,2,15\rangle^u_{120})Y_{120}\\
M_{\nu R} &= \langle 1,3,10\rangle\, Y_{126}\\
M^{II}_{\nu L} &= \langle 3,1,\overline{10}\rangle\, Y_{126}
\end{aligned}$$

Some obvious features of these relations (3.5) are

- 10 treats quarks and leptons on the same footing, because $(2,2,1)$ is a $SU(4)_C$ singlet.
- Type I and II see-saw, right handed neutrino and Georgi-Jarlskog factor [52] $m_l = -3m_d$ is given by $\overline{126}$, since $(2,2,15)$ is an adjoint of $SU(4)_C$. This works well for the second generation case.
- In the absence of $\overline{126}$, neutrinos would only have a Dirac mass, and their masses are related to the charged fermion masses. This is cured through the introduction of 16_H needed to break $B-L$, since $16_H \times 16_H = 126_H$ can simulate the direct presence of $\overline{126}$.

It is noted that (d) and (f) gives $m_d = m_e$ and $3m_d = -m_l$ for the three generations. These relations are obviously incorrect. Furthermore antisymmetry of Y_{120} in (e) implies $m_1 = 0$ and $m_2 = -m_3$, which are also wrong. Therefore we are only left with options (a),(b)and (c). We will discuss (a) and (c) only, namely $\overline{126}_H + 10_H$ and $126_H + 120_H$. A low energy supersymmetric and consistent $\overline{126}_H + 10_H$ already exists [55],[56],[57]. Here we will discuss a non-Supersymmetric version. In case of $126_H + 120_H$, only the analytic study will be done ignoring the effects of first generation. It will be shown in type I and type II See-Saw that the neutrino masses and the atmospheric mixing angle are related and the large atmospheric mixing angle fits naturally with the small V_{CB} mixing. Both (a) and (c) cases require complex Higgs fields.

The Yukawa sector of a non-supersymmetric theory does differ from the supersymmetric version in many ways. Some of them are [46]

- The running of the gauge and Yukawa couplings are changed, so are the inputs for a numerical evaluation at M_{GUT}

- Intermediate scales are necessary
- All SO(10) representations are not complex (like 16 Higgs or 126 Higgs are real), if new symmetries are not introduced.
- Radiative corrections to the Yukawa sector should be taken into account.

If we stick to the renormalizable version of the see-saw mechanism, this makes the representation $\overline{126}_H$ indispensable, since it breaks the $SU(2)_R$ group and gives a see-saw neutrino mass terms for the right handed and left handed neutrinos

$$M_{\nu R} = \langle 1,3,10\rangle \, Y_{126} \ , \ \ M_{\nu L}^{II} = \left\langle 3,1,\overline{10}\right\rangle Y_{126} \tag{3.6}$$

Thus one has both type I and type II seesaw

$$M_N = -M_{\nu D} M_{\nu R}^{-1} M_{\nu D} + M_{\nu L} \tag{3.7}$$

Right-handed neutrino acquire its mass through large vacuum expectation value (vev) of $(1,3,10)$ using type I see-saw whereas in the type II case. The left-handed triplet provides directly light neutrino masses through a small vev [58],[59]. It is very difficult to disentangle the two contributions.

3.2.1 Case 1: $\overline{126}_H + 10_H$

Schematically the Yukawa interaction in this case can be written as

$$\mathcal{L}_Y = 16_F (10_H Y_{10} + \overline{126}_H Y_{126}) 16_F + h.c. \tag{3.8}$$

where $Y_{10}$and Y_{126} are symmetric matrices in the generation space. One obtains relations for the Dirac fermion masses

$$\begin{aligned} M_D &= M_1 + M_0 \\ M_E &= -3M_1 + M_0 \\ M_U &= c_1 M_1 + c_0 M_0 \\ M_{\nu_D} &= -3c_1 M_1 + c_0 M_0 \end{aligned} \tag{3.9}$$

with the definitions

$$\begin{aligned} M_1 &= \langle 2,2,15\rangle_{126}^{d} \, Y_{126} \\ M_0 &= \langle 2,2,1\rangle_{10}^{d} \, Y_{10} \end{aligned} \tag{3.10}$$

and

$$c_0 = \frac{\langle 2,2,1\rangle_{10}^{u}}{\langle 2,2,1\rangle_{10}^{d}} \tag{3.11}$$
$$c_1 = \frac{\langle 2,2,15\rangle_{126}^{u}}{\langle 2,2,15\rangle_{126}^{d}}$$

Equations (3.6),(3.7) and (3.9) can be used to analyze [60] the fermionic spectrum. With the minimal fine-tuning the light Higgs is, in general, a mixture of, among others, $(2,2,1)$of 10_H and $(2,2,15)$ of $\overline{126}_H$. This happens at least due to the large $(1,3,10)$ vev in the term $(\overline{126}_H)^2\overline{126}_H^{\dagger}10_H$.

The mixing of $(2,2,1)$of 10_H and $(2,2,15)$ of $\overline{126}_H$ require the breaking of $SU(4)_C$ symmetry at a scale M_{PS} , and it is controlled by the ratio $\frac{M_{PS}}{M}$, where M corresponds to the mass of the heavy doublets. If $M \simeq M_{GUT}$ and $M_{PS} \ll M_{GUT}$, this would not work. Therefore one needs to tune-down M. Tuning down of the $(2,2,1)$ mass cannot have much impact on the unification constraints, but $(2,2,15)$ is a large field and could in principle cause trouble. Its contribution is tiny in this case. However should be taken into account while studying the unification constraints [46].

If real 10_H is used, then there is just one $SU(2)_L$ doublet in $(2,2,1)$ and thus $|\langle 2,2,1\rangle_{10}^{u}| = \left|\langle 2,2,1\rangle_{10}^{d}\right|$ leading to $|c_0| = 1$. The parameter space is thus smaller. Therefore using real parameters in two generations (second and third) case, the result is inconsistent with the data. A physically sensible approximation $\theta_q = V_{CB} = 0$ leads to $|c_0| \gg 1$

$$|c_0| = \frac{m_c(m_\tau - m_b) - m_t(m_\mu - m_s)}{m_s m_\tau - m_\mu m_b} \approx \frac{m_t}{m_b} \tag{3.12}$$

This conclusion is subject to the uncertainties of the full three-generation case. Although this simple model cannot be ruled out yet, there is an indication that a more complicated scenario should be considered. Introduction of a complex 10_H instead of a real 10_H introduces new Yukawa couplings in the theory, making it less predictive. However some predictions remain valid, such as the connection between $b-\tau$ unification and large atmospheric mixing angle in the type II seesaw. Because 10_H cannot distinguish down quarks from charged leptons, this connection is independently of the number of 10 dimensional Higgs representations. From $M_{\nu_L} \propto Y_{126}$, one has $M_{\nu_L} \propto M_D - M_E$. The relation among $m_b \approx m_\tau$ and θ_{ATM} can be easily established [61], [62] . In the non-supersymmetric theory, $b-\tau$ unification fails badly, $m_\tau \sim 2m_b$ [63]. The realistic theory will require a Type I seesaw, or an admixture of both possibilities.

3.2.2 Case 2: $\overline{126}_H + 120_H$

If we use 120_H instead of 10_H , we have only 3 new Yukawa couplings because Y_{126} is anti symmetric. For the charged lepton case, 120_H in addition to 10_H has already been studied by some authors [64] and it was readdressed in [65] in the case of radiative seesaw mechanism [66]. The analytic study for the second and third generation gives $b - \tau$ unification, and small quark and large leptonic mixing angle. Because of the fact that 120_H and 10_H have symmetric Yukawas, the analysis is quite similar. In case of 120_H, the Dirac mass matrices at the grand unification scale take the following form

$$\begin{aligned} M_D &= M_1 + M_2 \\ M_U &= c_1 M_1 + c_2 M_2 \\ M_E &= -3M_1 + c_3 M_2 \\ M_{\nu_D} &= -3c_1 M_1 + c_4 M_2 \end{aligned} \tag{3.13}$$

where M_1 and c_1 are given by (3.10) and (3.11) and M_2, c_2 , c_3 ,c_4 are given as

$$\begin{aligned} M_2 &= (\langle 2,2,1\rangle^d_{120} + \langle 2,2,15\rangle^d_{120})Y_{120} \\ c_2 &= \frac{\langle 2,2,1\rangle^u_{120} + \langle 2,2,15\rangle^u_{120}}{\langle 2,2,1\rangle^d_{120} + \langle 2,2,15\rangle^d_{120}} \\ c_3 &= \frac{\langle 2,2,1\rangle^d_{120} - 3\,\langle 2,2,15\rangle^d_{120}}{\langle 2,2,1\rangle^d_{120} + \langle 2,2,15\rangle^d_{120}} \\ c_4 &= \frac{\langle 2,2,1\rangle^u_{120} - 3\,\langle 2,2,15\rangle^u_{120}}{\langle 2,2,1\rangle^d_{120} + \langle 2,2,15\rangle^d_{120}} \end{aligned} \tag{3.14}$$

For the case of a real 120_H, we have similar problems as we have with real 10_H . For real bidoublets the definitions (3.14) constrain all three c_i to the same order of magnitude. But this contradicts the requirements for small second generation masses of charged leptons ($c_3 \approx 3$) and of up quarks ($c_2 \approx \frac{m_t}{m_b}$). As in previous case the remedy is to complexify the Higgs fields. This can be achieved by introducing a $U(1)_{PQ}$ global symmetry, which provides as a byproduct a dark matter candidate. The type I seesaw contribution due to right-handed neutrinos gives the light neutrino mass matrix

$$M_N^I = -M_{\nu D} M_{\nu R}^{-1} M_{\nu D} \propto 9c_1^2 M_1 - c_4^2 M_2 M_1^{-1} M_2 \tag{3.15}$$

The type II see-saw contributions is

$$M_N^{II} \propto M_1 \tag{3.16}$$

In case of only two generation (second and third) analysis, it is certainly useful to get a physical insight through analytical arguments. We assume that the effects of the first generation can be treated as a perturbation.

In the basis where M_1 is diagonal, real and nonnegative we have

$$M_1 \propto \begin{pmatrix} \sin^2\theta & 0 \\ 0 & \cos^2\theta \end{pmatrix} \tag{3.17}$$

The most general charged fermion matrix can be written as

$$M_f = \mu_f \begin{pmatrix} \sin^2\theta & i(\sin\theta\cos\theta + \epsilon_f) \\ -i(\sin\theta\cos\theta + \epsilon_f) & \cos^2\theta \end{pmatrix} \tag{3.18}$$

where $f = D$,U,E represent the charged fermions and the ϵ_f vanishes for the negligible second generation masses i.e. $\epsilon_f \propto \frac{m_2^f}{m_3^f}$. Furthermore the real parameter μ_f sets the third generation mass scale. One can determine the matrices L_f and R_f used to diagonalize the mass matrix M_f in the physically relevant approximation of small $|\epsilon_f|$. It follows:

$$M = R_f.\mathrm{Diag}\left\{-\mu_f\epsilon_f \sin 2\theta, \mu_f(1 + \epsilon_f \sin 2\theta)\right\}.L_f + \vartheta(\left|\epsilon_f^2\right|) \tag{3.19}$$

where

$$\begin{aligned} L_f &= \begin{pmatrix} 1 & -i\cos 2\theta\epsilon_f \\ -i\cos 2\theta\epsilon_f & 1 \end{pmatrix} \begin{pmatrix} \cos\theta & -i\sin\theta \\ -i\sin\theta & \cos\theta \end{pmatrix} \\ R_f &= \begin{pmatrix} 1 & i\cos 2\theta\epsilon_f \\ i\cos 2\theta\epsilon_f & 1 \end{pmatrix} \begin{pmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{pmatrix} \end{aligned} \tag{3.20}$$

Up to the leading order in $|\epsilon_f|$, using equation (3.19) we have

$$\mu_f = m_3^f \tag{3.21}$$

$$\sin 2\theta\, |\epsilon_f| = \frac{m_2^f}{m_3^f} \tag{3.22}$$

The three predictions of this theory concern (1) the neutrino mass, (2) the relation between bottom and tau masses, and (3) the quark mixing angle [46]

1. Using equations (3.15) , (3.16) and an explicit form of the 2×2 matrices one concludes that the type I and type II seesaw leads to the same structure
$$M_N^I \propto M_N^{II} \propto M_1 \tag{3.23}$$
In this basis the neutrino mass matrix is diagonal. The angle θ has to be identified with the leptonic (atmospheric) mixing angle θ_A up to terms of the order of $|\epsilon_f| \approx \frac{m_\mu}{m_\tau}$. For the neutrino masses we obtain from (3.17)
$$\frac{m_3^2 - m_2^2}{m_3^2 + m_2^2} = \frac{\cos 2\theta_A}{\frac{1-\sin^2\theta_A}{2}} + \vartheta(|\epsilon_f|) \tag{3.24}$$
In this equation the maximality of the atmospheric mixing angle measure the degeneracy of neutrino masses. Without including the effects of first generation and the running from the GUT to the weak scale one cannot make the precise determination. However some predictions can be made in a situation where this formula is approximately valid. Taking the value $\triangle m_A = |m_3^2 - m_2^2| \approx 2.5\times 10^{-3}$ eV^2 and the 99% CL limit $\theta_A = 45^\circ \pm 9^\circ$ from [67], one would get $m_2 > 30$ meV. Hence the value of m_2 could not be too small. Neutrinoless double beta decay gives an upper bound on neutrino mass, which varies from 0.14 eV to 0.5 eV, [67] . The larger the upper limit, the closer one can be to $\theta_A = 45^\circ$.

2. Contrary to the extrapolations in the Standard Model, suggesting $\frac{m_\tau}{m_b} \approx 2$, in this model at the GUT scale the ratio of tau and bottom mass is given as
$$\frac{m_\tau}{m_b} = 3 + 3\sin 2\theta_A \mathrm{Re}[\epsilon_E - \epsilon_D] + \vartheta\left(\epsilon^2\right) \tag{3.25}$$
This discrepancy can be removed by several factors, particularly by choosing suitable phases.

3. Another useful relation exists between the $|V_{CB}|$ and the atmospheric mixing angle given by
$$|V_{CB}| = |\mathrm{Re}\xi - i\cos 2\theta_A \mathrm{Im}\xi| + \vartheta\left(\epsilon^2\right) \tag{3.26}$$
where $\xi = \cos 2\theta_A(\epsilon_E - \epsilon_D)$. This equation suggest the successful coexistence of the small and large mixing angles if atmospheric mixing angle θ_A is as far away as possible from its maximal value 45°.

Chapter 4

GUTs Mass Models

In recent years many attempts have been made to understand the current mass and mixing data in the lepton sector [68] [69]. Particularly in GUT models both the lepton and quark sectors can be analyzed.

4.1 SO(10) Mass Models

The fact that all the three families of matter fields can fit into three copies of the spinor representation of $SO(10)$, 16_i $i = 1, 2, 3$, makes $SO(10)$ very attractive for unified model building. In order to break $SO(10)$ to the Standard Model, the Higgs fields 45_H , 16_H , $\overline{16}_H$ can be used. To break the electroweak symmetry, two light Higgs doublets are needed. For this purpose a single 10_H of $SO(10)$ consisting $5 + \overline{5}$ of $SU(5)$ or $(6, 1, 1) + (1, 2, 2)$ of $SU(4) \times SU(2)_R \times SU(2)_L$ can be used. The required doublet-triplet splitting of the Higgs fields can be obtained through the Dimopoulos-Wilczek mechanism [70] if the $\langle 45_H \rangle$ VEV points in the B-L direction. The electroweak breaking, $\tan\beta = \dfrac{v_u}{v_d} \sim 55$, is solely affected by 10_H .

The $SO(10)$ theory relates quarks and the leptons of the same family. However in order to avoid the bad $SU(5)$ relations among masses $m_d = m_e$ and $m_s = m_\mu$, one must invoke some horizontal symmetry. This can be done at four different levels of model building given as [68]

- Level 1: A particular texture for the mass matrices can be imposed, such as modified Fritzsch texture.

- Level 2: One can introduce an effective $\lambda \sim 0.22$ expansion for each mass matrix. However usually it is not possible in this case to determine the prefactors of the expansion parameters.

- Level 3: For each element of the mass matrix an effective operator can be assigned, possibly imposing some flavor symmetry.
- Level 4: A horizontal flavor symmetry can be introduced which assigns flavor charges to every Higgs and matter field. Renormalizable Yukawa and Higgs potentials obeying flavor symmetry are constructed. Matrix elements are obtained from the corresponding Froggatt-Nielsen diagrams.

In the literature the $SO(10)$ models differ by their choice of Higgs structure, flavor charge assignments and horizontal flavor symmetry. If $\langle 45_H \rangle$ Higgs VEV points in the B-L direction or if a $\left\langle \overline{5}\,(126_H) \right\rangle$ Higgs VEV is present, then one can easily obtain the Georgi-Jarlskog relations [71] $m_s = \frac{m_\mu}{3}$, $m_d = 3m_e$. Flavor symmetry along with $\left\langle \overline{5}\,(16_H) \right\rangle$ Higgs VEV leads to a lopsided [72] charged lepton and down quark mass matrices L and D. Such lopsided mass matrices lead to the small V_{CB} and large $U_{\mu 3}$ mixing elements [73] and enhanced the flavor violating decay rate which is within one or two orders of magnitude of the present experimental limit. Therefore this mechanism can be proven or ruled out by the future improved experiments. Generally some fine tuning is necessary to get the large mixing angle (LMA) solution in $SO(10)$ models. Those models have trouble getting LMA solution which in order to get maximal atmospheric mixing, require special features of the Dirac and right-handed Majorana mass matrices N and M_R . This can be easily achieved if the N and L are used to get maximal atmospheric mixing where as M_R is independently adjusted to yield the LMA solution.

4.2 SO(10) GUT with Zero Texture

Texture zero and some flavor symmetry is often employed to obtain the quark and lepton mass matrices. In this scheme usually quarks and leptons are treated on equal footing in accordance with observations. However there is no theoretical basis, why and how quarks and leptons are unified. A grand unified theory such as $SO(10)$ provide relations between quark and lepton mass matrices. However it is silent about the form of the matrices themselves. This suggests one to discuss the four zero texture under renormalizable $SO(10)$ GUT model. In the framework of the renormalizable $SO(10)$ GUT model, four zero texture model was analyzed by Fukuyama, Matsuda and Nishiura (FMN) [74],[75]. In this scheme only the renormalizable Higgs-fermion couplings are used. The fundamental representation of matter multiplets are 16_i , $i = 1, 2, 3$. Since $16 \times 16 = 10 + 120 + 126$, the most general form of Yukawa coupling is

$$Y_{10,ij}16_i16_j10_H + Y_{120,ij}16_i16_j120_H + Y_{126,ij}16_i16_j126_H$$

4.2.1 Embedding zero texture mass matrices in $SO(10)$

The mass term in the Lagrangian of FMN Model [75] is

$$\begin{aligned} L_M = & -\overline{q}^u_{R,i} M_{uij} q^u_{L,j} - \overline{q}^d_{R,i} M_{dij} q^d_{L,j} - \overline{l}_{R,i} M_{eij} l_{L,j} - \overline{\nu'}_{R,i} M_{Dij} \nu_{L,j} \\ & - \frac{1}{2}\overline{(\nu_{L,i})^c} M_{Lij} \nu_{L,j} - \frac{1}{2}\overline{(\nu'_{R,i})^c} M_{Rij} \nu'_{R,j} + h.c. \end{aligned} \tag{4.1}$$

and

$$q^u_{L,R} = \begin{pmatrix} u \\ c \\ t \end{pmatrix}_{L,R}, \quad q^d_{L,R} = \begin{pmatrix} d \\ s \\ b \end{pmatrix}_{L,R}, \quad l_{L,R} = \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_{L,R}$$

$$\nu_L = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_L, \quad \nu'_R = \begin{pmatrix} \nu'_e \\ \nu'_\mu \\ \nu'_\tau \end{pmatrix}_R$$

The mass matrices for the up quark, down quark, charged leptons, Dirac neutrinos, left handed Majorana neutrinos, and right handed Majorana neutrinos are given as M_u, M_d, M_e, M_D, M_L and M_R. Block- diagonalization of the neutrino mass matrix gives

$$\begin{pmatrix} M_L & M_D^T \\ M_D & M_R \end{pmatrix}$$

One gets using the See-saw mechanism, the mass matrix for the light Majorana neutrinos M_ν

$$M_\nu = M_L - M_D^T M_R^{-1} M_D \tag{4.2}$$

We consider a general renormalizable $SO(10)$ model, involving 10, 120 , $\overline{126}$ Higgs fields and hence three type of Yukawa coupling and mass matrices. These Higgs fields can be decomposed under the Pati -Salam model as follows:

$$\begin{aligned} 10 &= (6,1,1)\,(1,2,2) \\ 120 &= (15,2,2)\,(6,3,1)\,(6,1,3)\,(1,2,2)\,(10,1,1)\,\left(\overline{10},1,1\right) \\ 126 &= (10,1,3)\,\left(\overline{10},3,1\right)\,(15,2,2)\,(6,1,1) \end{aligned} \tag{4.3}$$

There are two $SU(2)$ doublets in each of 10 and 126 with Clebsch–Gordan coefficients 1 and -3. The 120 has four such doublets with no Clebsch–Gordan coefficients coefficients. Under $SU(5)$ decomposition we find

$$\begin{aligned}16 \times 16 \times 10 &\supset 5_H\,(uu^c + \nu\nu^c) + \overline{5}_H\,(dd^c + ee^c) \\ 16 \times 16 \times 120 &\supset 5_H\nu\nu^c + 45_H uu^c + \overline{5}_H\,(dd^c + ee^c) + \overline{45}_H\,(dd^c - 3ee^c) \\ 16 \times 16 \times \overline{126} &\supset 1_H\nu^c\nu^c + 15_H\nu\nu + 5_H\,(uu^c - 3\nu\nu^c) + \overline{45}_H\,(dd^c - 3ee^c)\end{aligned} \tag{4.4}$$

The right-handed and left-handed Majorana masses are given by $1_H\nu^c\nu^c$ and $15_H\nu\nu$ terms in the $\overline{126}$. The six $SO(10)$ mass matrices M_u, M_d, M_e, M_D, M_L and M_R, have the following form

$$\begin{aligned}M_u &= S + \delta' A + \epsilon S' &&\equiv S_u + A_u \\ M_d &= \alpha S + \delta A + S' &&\equiv S_d + A_d \\ M_D &= S + \delta'' A - 3\epsilon S' &&\equiv S_D + A_D \\ M_e &= \alpha S + A - 3S' &&\equiv S_e + A_e \\ M_L &= \qquad\qquad\; \beta S' &&\equiv S_L \\ M_R &= \qquad\qquad\; \gamma S' &&\equiv S_R\end{aligned} \tag{4.5}$$

Here S, S' are the common structures for mass matrices coming from 10 and 126 respectively, and A shows the common structures for mass matrices coming from 120. The relative coefficients of the vacuum expectation values (VEVs) are expressed as α,δ,δ',ϵ,β,γ. The symmetric part of M_f , ($f = u$,d,D,e,L,R) is represented by S_f, the antisymmetric part by A_f. The M_u, M_d, M_e and M_ν are hermitian. Also it is assumed that M_D, M_L and M_R have the same zero texture as M_ν [76]. In this model the $(2,2)$ components of the texture are non-zero. Quark and lepton mass matrices are treated on same footing.

In the four zero texture (FZT) model the mass matrices are given as follows:

$$\begin{aligned}M_u &= P_u \widehat{M}_u P_u^\dagger \\ &= P_u \begin{pmatrix} 0 & a_u & 0 \\ a_u & b_u & c_u \\ 0 & c_u & d_u \end{pmatrix} P_u^\dagger = \begin{pmatrix} 0 & a_u e^{i\tau_u} & 0 \\ a_u e^{-i\tau_u} & b_u & c_u e^{i\sigma_u} \\ 0 & c_u e^{-i\sigma_u} & d_u \end{pmatrix}\end{aligned} \tag{4.6}$$

$$\begin{aligned}M_d &= P_u \widehat{M}_d P_u^\dagger \\ &= P_d \begin{pmatrix} 0 & a_d & 0 \\ a_d & b_d & c_d \\ 0 & c_d & d_d \end{pmatrix} P_d^\dagger = \begin{pmatrix} 0 & a_d e^{i\tau_d} & 0 \\ a_d e^{-i\tau_d} & b_d & c_d e^{i\sigma_d} \\ 0 & c_d e^{-i\sigma_d} & d_d \end{pmatrix}\end{aligned} \tag{4.7}$$

$$
\begin{aligned}
M_D &= P_D \widehat{M}_D P_D^\dagger \\
&= P_D \begin{pmatrix} 0 & a_D & 0 \\ a_D & b_D & c_D \\ 0 & c_D & d_D \end{pmatrix} P_D^\dagger = \begin{pmatrix} 0 & a_D e^{i\tau_D} & 0 \\ a_D e^{-i\tau_D} & b_D & c_D e^{i\sigma_D} \\ 0 & c_D e^{-i\sigma_D} & d_D \end{pmatrix}
\end{aligned} \tag{4.8}
$$

$$
\begin{aligned}
M_e &= P_e \widehat{M}_e P_e^\dagger \\
&= P_e \begin{pmatrix} 0 & a_e & 0 \\ a_e & b_e & c_e \\ 0 & c_e & d_e \end{pmatrix} P_e^\dagger = \begin{pmatrix} 0 & a_e e^{i\tau_e} & 0 \\ a_e e^{-i\tau_e} & b_e & c_e e^{i\sigma_e} \\ 0 & c_e e^{-i\sigma_e} & d_e \end{pmatrix}
\end{aligned} \tag{4.9}
$$

$$
\begin{aligned}
M_\nu &= P_\nu \widehat{M}_\nu P_\nu^\dagger \\
&= P_\nu \begin{pmatrix} 0 & a_\nu & 0 \\ a_\nu & b_\nu & c_\nu \\ 0 & c_\nu & d_\nu \end{pmatrix} P_\nu^\dagger = \begin{pmatrix} 0 & a_\nu e^{i\tau_\nu} & 0 \\ a_\nu e^{-i\tau_\nu} & b_\nu & c_\nu e^{i\sigma_\nu} \\ 0 & c_\nu e^{-i\sigma_\nu} & d_\nu \end{pmatrix}
\end{aligned} \tag{4.10}
$$

$$
M_L = \begin{pmatrix} 0 & a_L & 0 \\ a_L & b_L & c_L \\ 0 & c_L & d_L \end{pmatrix} \quad , \quad M_R = \begin{pmatrix} 0 & a_R & 0 \\ a_R & b_R & c_R \\ 0 & c_R & d_R \end{pmatrix} \tag{4.11}
$$

where $P_f \equiv \mathrm{diag}\big(e^{i\gamma f1}, e^{i\gamma f1}, e^{i\gamma f1}\big)$ and $\tau_f \equiv \gamma_{f1} - \gamma_{f2}$, $\sigma_f \equiv \gamma_{f2} - \gamma_{f3}$, $f = u, d, D, e, L, R, \nu$. These matrices can be collectively described as

$$
M_f = \begin{pmatrix} 0 & a_f e^{i\tau_f} & 0 \\ a_f e^{-i\tau_f} & b_f & c_f e^{i\sigma_f} \\ 0 & c_f e^{-i\sigma_f} & d_f \end{pmatrix} \tag{4.12}
$$

Denoting the mass eigenvalues of M_f as mfi , $(i = 1, 2, 3)$, one can express a_f, b_f, c_f in terms of mfi and d_f

$$
\begin{aligned}
a_f &= \left(-\frac{mf1mf2mf3}{d_f}\right)^{\frac{1}{2}} \\
b_f &= mf1 + mf2 + mf3 - d_f \\
c_f &= \left(-\frac{(d_f - mf1)(d_f - mf2)(d_f - mf3)}{d_f}\right)^{\frac{1}{2}}
\end{aligned}
$$

	Parameters
From 10_H part of mass matrices	4
From 126_H part of mass matrices	4
From 120_H part of mass matrices	2
Coefficients of VEVs $\alpha, \delta, \delta', \epsilon$	4
Total	14

Table 4.1: Parameters in SO(10) FZT model

	Experimental Constraints
Quark masses	6
CKM mixing angles	3
The Dirac phase	1
Lepton masses	3
Total	13

Table 4.2: Experimental Constraints

here $\quad 0 < mf1 < -mf2 < mf3 \quad$ for $\quad |\ mf1| < d_f < |mf2|$

$\quad 0 < -mf1 < mf2 < mf3 \quad$ for $\quad |mf2| < d_f < |mf3|$

There are 14 parameters in this model [74] Given by the table 4.1

From the experiments we obtain 13 constraints: (table 4.2). After data fitting of the quarks and the charged leptons, only one parameter is left to be determined. Using this free parameter and δ'', β, γ we can determine the neutrino masses and mixing angles.

The four zero textures matrices can be diagonalized:

$$U_f^{\dagger} M_f U_f = \mathrm{Diag}\,(m_{f1}, m_{f2}, m_{f3})$$

$$U_f = P_f^{\dagger} O_f \ , \ U_f = \mathrm{Diag}\,(1, \tau_f, \sigma_f + \tau_f) \equiv (1, \alpha_{f2}, \alpha_{f3})$$

The orthogonal matrix O_f is

$$O_f = \begin{pmatrix} \left(\frac{\left(d_f - m_{f1}\right) m_{f2} m_{f3}}{R_{f1} d_f}\right)^{\frac{1}{2}} & \left(\frac{\left(d_f - m_{f2}\right) m_{f1} m_{f3}}{R_{f2} d_f}\right)^{\frac{1}{2}} & \left(\frac{\left(d_f - m_{f3}\right) m_{f2} m_{f1}}{R_{f3} d_f}\right)^{\frac{1}{2}} \\ -\left(-\frac{\left(d_f - m_{f1}\right) m_{f1}}{R_{f1}}\right)^{\frac{1}{2}} & \left(-\frac{\left(d_f - m_{f2}\right) m_{f2}}{R_{f2}}\right)^{\frac{1}{2}} & \left(-\frac{\left(d_f - m_{f3}\right) m_{f3}}{R_{f3}}\right)^{\frac{1}{2}} \\ \left(\frac{m_{f1}\left(d_f - m_{f2}\right)\left(d_f - m_{f3}\right)}{R_{f1} d_f}\right)^{\frac{1}{2}} & -\left(\frac{m_{f2}\left(d_f - m_{f3}\right)\left(d_f - m_{f1}\right)}{R_{f2} d_f}\right)^{\frac{1}{2}} & \left(\frac{m_{f3}\left(d_f - m_{f1}\right)\left(d_f - m_{f2}\right)}{R_{f3} d_f}\right)^{\frac{1}{2}} \end{pmatrix} \tag{4.13}$$

where

$$\begin{aligned} R_{f1} &\equiv \left(m_{f1} - m_{f2}\right)\left(m_{f1} - m_{f3}\right) \\ R_{f2} &\equiv \left(m_{f2} - m_{f3}\right)\left(m_{f2} - m_{f1}\right) \\ R_{f3} &\equiv \left(m_{f3} - m_{f1}\right)\left(m_{f3} - m_{f2}\right) \end{aligned}$$

For $f = u$,d and e, we have $\left(m_{f1}, m_{f2}, m_{f3}\right) \to \left(m_u, m_c, m_t\right), \left(m_d, m_s, m_b\right), \left(m_e, m_\mu, m_\tau\right)$, The CKM quark mixing matrix is $U_{CKM} \equiv U_u^\dagger U_d$ and can be written as [74]

$$\left(U_{CKM}\right)_{12} \approx \sqrt{\frac{|m_d|}{m_s}} - e^{i\alpha_2}\sqrt{\frac{|m_u|}{m_c}} x_u x_d - e^{i\alpha_3}\sqrt{\frac{|m_u|}{m_c}}\left(1 - x_u\right)\left(1 - x_d\right) \tag{4.14}$$

$$\left(U_{CKM}\right)_{23} \approx \sqrt{\frac{|m_d|\, m_s}{m_b^2}} - e^{i\alpha_2}\sqrt{\frac{|m_u|}{m_c}} x_u\left(1 - x_d\right) + e^{i\alpha_3}\sqrt{\frac{|m_u|}{m_c}}\left(1 - x_u\right) x_d$$

$$\left(U_{CKM}\right)_{13} \approx \sqrt{\frac{|m_u|\,|m_d|\, m_s\left(1 - x_d\right)}{m_c m_b^2 x_d}} + e^{i\alpha_2}\sqrt{x_u\left(1 - x_d\right)} - e^{i\alpha_3}\sqrt{\left(1 - x_u\right) x_d}$$

The Dirac phase is given by

$$\delta_q \approx \arg\frac{\left(e^{i\alpha_3}\sqrt{\left(1 - x_u\right)\left(1 - x_d\right)} + e^{i\alpha_2}\sqrt{x_u x_d}\right)^*}{\left(e^{i\alpha_3}\sqrt{\left(1 - x_u\right) x_d} - e^{i\alpha_2}\sqrt{x_u\left(1 - x_d\right)}\right)\left(e^{i\alpha_2}\sqrt{\left(1 - x_u\right) x_d} - e^{i\alpha_3}\sqrt{x_u\left(1 - x_d\right)}\right)^*} \tag{4.15}$$

In the limit $\left|\left(U_{CKM}\right)_{13}\right| \ll 1$ we define

$$\begin{aligned} x_f &\equiv \frac{d_f}{m_{f3}} \\ \alpha_2 &\equiv \alpha_{u2-}\alpha_{d2} = \tau_u - \tau_d \equiv \triangle\tau \\ \alpha_3 &\equiv \alpha_{u3-}\alpha_{d3} = \tau_u - \tau_d + \left(\sigma_u - \sigma_d\right) \equiv \triangle\tau + \triangle\sigma \end{aligned}$$

The CKM mixing matrix is sensitive to m_b and is not sensitive to the value of m_t. When the four zero texture model is embedded in the $SO(10)$ GUT, we have more constraints among the parameters. In the case of quarks and charged leptons, S_f and A_f obey the following relations

$$\begin{aligned} 4\alpha S_u &= (3+\alpha\epsilon)\, S_d + (1-\alpha\epsilon)\, S_e \\ \delta A_u &= \delta' A_d = \delta\delta' A_e \end{aligned}$$

Expressing these relations in a component form results in the following constraints among the parameters:

$$\begin{aligned} 4\alpha a_u \cos(\triangle\tau + \tau_d) &= (3+\alpha\epsilon)\, a_d \cos\tau_d + (1-\alpha\epsilon)\, a_e \cos\tau_e \\ 4\alpha c_u \cos(\triangle\sigma + \sigma_d) &= (3+\alpha\epsilon)\, c_d \cos\sigma_d + (1-\alpha\epsilon)\, c_e \cos\sigma_e \\ 4\alpha b_u &= (3+\alpha\epsilon)\, b_d + (1-\alpha\epsilon)\, b_e \\ 4\alpha d_u &= (3+\alpha\epsilon)\, d_d + (1-\alpha\epsilon)\, d_e \\ \delta a_u \sin(\triangle\tau + \tau_d) &= \delta' a_d \sin\tau_d = \delta\delta' a_e \sin\tau_e \\ \delta c_u \sin(\triangle\sigma + \sigma_d) &= \delta' c_d \sin\sigma_d = \delta\delta' c_e \sin\sigma_e \end{aligned} \tag{4.16}$$

Defining

$$\begin{aligned} k &\equiv \alpha\epsilon,\ r \equiv \frac{\delta'}{\delta}, \\ F(r, d_u, d_d) &\equiv \frac{c_d \sin\sigma_d}{a_d \sin\tau_d} = \frac{c_e \sin\sigma_e}{a_e \sin\tau_e} = \frac{c_e \sin\sigma_e}{a_d \sin\tau_d}\delta \end{aligned}$$

and using the restrictions

$$\begin{aligned} -1 \le \cos\tau_d &\equiv \frac{4\alpha a_u \cos(\triangle\tau + \tau_d) - (3+\alpha\epsilon)\, a_d \cos\tau_d}{(1-\alpha\epsilon)\, a_e} \le 1 \\ -1 \le \cos\sigma_e &\equiv \frac{4\alpha c_u \cos(\triangle\sigma + \sigma_d) - (3+\alpha\epsilon)\, c_d \cos\sigma_d}{(1-\alpha\epsilon)\, c_e} \le 1 \end{aligned} \tag{4.17}$$

one can express the constraints (4.16) as

$$\begin{aligned} & F(r, d_u, d_d)^2 \left[4\alpha a_u \cos(\triangle\tau + \tau_d) - (2+k)\, a_d \cos\tau_d\right]^2 \\ & - \left[4\alpha c_u \cos(\triangle\sigma + \sigma_d) - (3+k)\, c_d \cos\sigma_d\right]^2 \\ & = (1-k)^2 \left[a_e^2 F(r, d_u, d_d)^2 - c_e^2\right] \end{aligned} \tag{4.18}$$

τ_d and σ_d can be found using equations (4.16)

$$\tan\tau_d = \frac{a_u \sin\triangle\tau}{r a_d - a_u \cos\triangle\tau} \qquad (4.19)$$
$$\tan\sigma_d = \frac{c_u \sin\triangle\sigma}{r c_d - c_u \cos\triangle\sigma}$$

α and ϵ can be determined in terms of d_u , d_d and d_e

$$\alpha = \frac{(m_d + m_s + m_b)\, d_e - (m_e + m_\mu + m_\tau)\, d_d}{(m_d + m_s + m_b - m_e - m_\mu - m_\tau)\, d_u - (m_u + m_c + m_t)\,(d_d - d_e)}$$
$$k = \frac{(m_u + m_c + m_t)\,(3d_d + d_e) - [3\,(m_d + m_s + m_b) + (m_e + m_\mu + m_\tau)]\, d_u}{(m_d + m_s + m_b - m_e - m_\mu - m_\tau)\, d_u - (m_u + m_c + m_t)\,(d_d - d_e)}$$

The observed CKM values of the quark mixing matrix imposes severe constraints on the parameters d_u, d_d, $\triangle\tau$ and $\triangle\sigma$. The following values of the parameters give the best fit:

$$\triangle\tau = \frac{\pi}{2} \quad , \quad \triangle\sigma = -0.121$$
$$d_u = 0.9560\ m_t \quad , \quad d_d = 0.9477\ m_b$$

The parameters τ_d , σ_d are determined from (4.19) in terms of d_e , r. The best fit values of quarks and charged leptons at the unification scale $\mu = M_X$ are given as [77]

$$\begin{array}{rr} m_u(M_X) = 104^{+0.19}_{-0.20}\ \ \mathrm{MeV} & m_d(M_X) = 1.33^{+0.17}_{-0.19}\ \ \mathrm{MeV} \\ m_c(M_X) = 302^{+25}_{--27}\ \ \mathrm{MeV} & m_s(M_X) = 26.5^{+3.3}_{-3.7}\ \ \mathrm{MeV} \\ m_t(M_X) = 129^{+196}_{-40}\ \ \mathrm{GeV} & m_b(M_X) = 1.00 \pm 0.04\ \ \mathrm{GeV} \\ m_e(M_X) = 0.32502032 \pm 0.00000009\ \ \mathrm{MeV} & m_\mu(M_X) = 68.59813 \pm 0.00022\ \ \mathrm{MeV} \\ m_\tau(M_X) = 1171 \pm 0.2\ \ \mathrm{MeV} & \end{array} \qquad (4.20)$$

Using (4.14) , (4.15) ,and (4.20) one can determine the CKM values and the Dirac phase

$$\begin{aligned} |(U_{CKM})_{12}| &= 0.2251 \\ |(U_{CKM})_{23}| &= 0.0340 \\ |(U_{CKM})_{13}| &= 0.0032 \\ \delta_q &= 58.86^\circ \end{aligned}$$

Two remaining parameters d_e and r are determined from the equations (4.16),(4.17) and (4.18). They predict two possible solutions having $\dfrac{d_e}{m_\tau}$

values (i) 0.935883 and (ii) 0.307197. Hence the 13 parameters have been successfully and consistently fitted into the quark and lepton sectors.

4.2.2 Lepton Mixing (PMNS) Matrix

The neutrino Majorana mass matrix M_ν and charged leptons mass matrix M_e are given by (4.9 , 4.10)

$$\begin{aligned} M_\nu &= P_\nu \widehat{M}_\nu P_\nu^\dagger \\ M_e &= P_e \widehat{M}_e P_e^\dagger \end{aligned}$$

Using the unitary matrices $U_\nu = P_\nu^\dagger O_\nu Q_\nu$ and $U_{Le} = P_e^\dagger O_e$, the neutrino and charged lepton mass matrices can be diagonalized as [78]

$$\begin{aligned} U_\nu^\dagger M_\nu U_\nu &= diag\left(|m_1|, m_2, m_3\right) \\ U_{Le}^\dagger M_e U_{Le} &= diag\left(-|m_e|, m_\mu, m_\tau\right) \end{aligned}$$

In order to obtain the real positive neutrino mass matrix, an additional phase matrix $Q_\nu = diag\,(i,1,1)$ is used. Under the assumptions $|m_1| < m_2 \ll d_\nu < m_3$, $|m_e| \ll m_\mu \ll d_e < m_\tau$ for the neutrinos and the charged leptons respectively, the orthogonal matrices O_ν and O_e can be obtained using (4.13) with $f = \nu, e$ replacing m_{f1}, m_{f2}, m_{f3} by m_1, m_2, m_3 and m_e, m_μ, m_τ respectively.

$$O_\nu \simeq \begin{pmatrix} \left(\dfrac{m_2}{|m_1|+m_2}\right)^{\frac{1}{2}} & \left(\dfrac{|m_1|}{|m_1|+m_2}\right)^{\frac{1}{2}} & \left(\dfrac{|m_1|\,m_2}{m_3^2}\dfrac{1-x_\nu}{x_\nu}\right)^{\frac{1}{2}} \\ -\left(\dfrac{|m_1|}{|m_1|+m_2}x_\nu\right)^{\frac{1}{2}} & \left(\dfrac{m_2}{|m_1|+m_2}x_\nu\right)^{\frac{1}{2}} & (1-x_\nu)^{\frac{1}{2}} \\ \left(\dfrac{|m_1|}{|m_1|+m_2}(1-x_\nu)\right)^{\frac{1}{2}} & -\left(\dfrac{m_2}{|m_1|+m_2}(1-x_\nu)\right)^{\frac{1}{2}} & (x_\nu)^{\frac{1}{2}} \end{pmatrix} \tag{4.21}$$

$$O_e \simeq \begin{pmatrix} 1 & \left(\dfrac{|m_e|}{m_\mu}\right)^{\frac{1}{2}} & \left(\dfrac{|m_e|\,m_\mu}{m_\tau^2}\dfrac{1-x_e}{x_e}\right)^{\frac{1}{2}} \\ -\left(\dfrac{|m_e|}{m_\mu}x_e\right)^{\frac{1}{2}} & (x_e)^{\frac{1}{2}} & (1-x_e)^{\frac{1}{2}} \\ \left(\dfrac{|m_e|}{m_\mu}(1-x_e)\right)^{\frac{1}{2}} & -(1-x_e)^{\frac{1}{2}} & (x_e)^{\frac{1}{2}} \end{pmatrix}$$

Using the diagonal phase matrix $P_l = P_e P_\nu^\dagger = diag\left(1, e^{i\beta_2}, e^{i\beta_3}\right)$,the lepton mixing (MNS) matrix U_{MNS} can be written as

$$U_{MNS} = U_{Le}^\dagger U_\nu = O_e^T P_\nu^\dagger O_\nu Q_\nu \tag{4.22}$$

$$U_{MNS} \simeq \begin{pmatrix} i\left(\frac{m_2}{|m_1|+m_2}\right)^{\frac{1}{2}} & \left(\frac{|m_1|}{|m_1|+m_2}\right)^{\frac{1}{2}} & \left(\frac{|m_1|\, m_2}{m_3^2}\frac{1-x_\nu}{x_\nu}\right)^{\frac{1}{2}} + \xi_5\left(\frac{|m_e|}{m_\mu}\right)^{\frac{1}{2}} \\ -i\xi_1\left(\frac{|m_1|}{|m_1|+m_2}\right)^{\frac{1}{2}} & \xi_1\left(\frac{m_2}{|m_1|+m_2}\right)^{\frac{1}{2}} & \xi_3 \\ i\xi_3\left(\frac{|m_1|}{|m_1|+m_2}\right)^{\frac{1}{2}} & -\xi_3\left(\frac{m_2}{|m_1|+m_2}\right)^{\frac{1}{2}} & \xi_4 \end{pmatrix}$$

The complex quantities ξ_i , $i = 1,2,3,4,5$ are defined by

$$\begin{aligned} \xi_1 &= (x_\nu x_e)^{\frac{1}{2}}\, e^{i\beta_2} + \left[(1-x_\nu)(1-x_e)\right]^{\frac{1}{2}} e^{i\beta_3} \\ \xi_2 &= \left[(1-x_\nu)\, x_e\right]^{\frac{1}{2}} e^{i\beta_2} - \left[x_\nu(1-x_e)\right]^{\frac{1}{2}} e^{i\beta_3} \\ \xi_3 &= -\left[x_\nu(1-x_e)\right]^{\frac{1}{2}} e^{i\beta_2} + \left[(1-x_\nu)\, x_e\right]^{\frac{1}{2}} e^{i\beta_3} \\ \xi_4 &= \left[(1-x_\nu)(1-x_e)\right]^{\frac{1}{2}} e^{i\beta_2} + (x_\nu x_e)^{\frac{1}{2}}\, e^{i\beta_3} \\ \xi_5 &= \left[(1-x_\nu)\, x_e\right]^{\frac{1}{2}} e^{i\beta_2} + \left[x_\nu(1-x_e)\right]^{\frac{1}{2}} e^{i\beta_3} \end{aligned}$$

For the required maximal lepton mixing between the second and third generation we have to choose

$$|\xi_1| = |\xi_2| = |\xi_3| = |\xi_4| = \sqrt{\frac{1}{2}}$$

The above equations can be satisfied irrespective of the values of the phases β_2 and β_3 , if we choose

$$x_\nu = \frac{1}{2} \quad , \quad x_e \simeq 1$$

The explicit expressions for the components $\left|(U_{MNS})_{ij}\right|$ of the lepton mixing matrix are given as

$$|(U_{MNS})_{11}| \simeq \sqrt{\frac{m_2}{|m_1|+m_2}} \quad , \quad |(U_{MNS})_{12}| \simeq \sqrt{\frac{|m_1|}{|m_1|+m_2}}$$
$$|(U_{MNS})_{13}| \simeq \sqrt{\frac{|m_1|\,m_2}{m_3^2}} + e^{i\beta_2}\sqrt{\frac{|m_e|}{2m_\mu}} \quad , \quad |(U_{MNS})_{21}| \simeq \frac{1}{\sqrt{2}}\sqrt{\frac{|m_1|}{|m_1|+m_2}}$$
$$|(U_{MNS})_{22}| \simeq \frac{1}{\sqrt{2}}\sqrt{\frac{m_2}{|m_1|+m_2}} \quad , \quad |(U_{MNS})_{23}| \simeq \frac{1}{\sqrt{2}}$$
$$|(U_{MNS})_{31}| \simeq \frac{1}{\sqrt{2}}\sqrt{\frac{|m_1|}{|m_1|+m_2}} \quad , \quad |(U_{MNS})_{11}| \simeq \frac{1}{\sqrt{2}}\sqrt{\frac{m_2}{|m_1|+m_2}}$$
$$|(U_{MNS})_{31}| \simeq \frac{1}{\sqrt{2}}$$

The solar θ_{12}, atmospheric θ_{23}, and the CHOOZ θ_{13} angles and the phases are related to the lepton masses [78] as follows:

$$\begin{aligned}
\tan^2\theta_{12} &= \frac{|(U_{MNS})_{12}|^2}{|(U_{MNS})_{11}|^2} \simeq \frac{|m_1|}{m_2} \\
\sin^2 2\theta_{23} &= 4\left||(U_{MNS})_{23}|^2\,|(U_{MNS})_{33}|^2\right| \simeq 1 \\
|(U_{MNS})_{13}|^2 &\simeq \left|\sqrt{\frac{|m_1|\,m_2}{m_3^2}} + e^{i\beta_2}\sqrt{\frac{|m_e|}{2m_\mu}}\right|^2 \\
\delta_\nu &\simeq -\arg\left(\sqrt{\frac{|m_1|\,m_2}{m_3^2}} + e^{i\beta_2}\sqrt{\frac{|m_e|}{2m_\mu}}\right) \\
\phi_2 &\simeq \phi_3 \simeq -\frac{1}{2}
\end{aligned} \tag{4.23}$$

One can obtain the neutrino masses in terms of the mixing angles. Using the first equation $\tan\theta_{12} \simeq \frac{|m_1|}{m_2}$, one can write $\sin^2\theta_{12} = \frac{m_1}{m_1+m_2}$ and $\cos^2\theta_{12} = \frac{m_1}{m_1+m_2}$. The neutrino experiments are only sensitive to the mass squared differences $\Delta m_{21}^2 = m_2^2 - m_1^2$ and $\Delta m_{32}^2 = m_3^2 - m_2^2$. We obtain the neutrino masses:

$$m_1 = \sqrt{\frac{\sin^4\theta_{12}}{\cos 2\theta_{12}}\triangle m_{21}^2} \tag{4.24}$$

$$m_2 = \sqrt{\frac{\cos^4\theta_{12}}{\cos 2\theta_{12}} \triangle m_{21}^2} \tag{4.25}$$

$$m_3 = \sqrt{\frac{\cos^4\theta_{12}}{\cos 2\theta_{12}} \triangle m_{21}^2 + \triangle m_{32}^2} \tag{4.26}$$

4.3 Triminimal parameterization

Even if tribimaximal mixing, in the lepton sector is the result of some flavor symmetry, in general there will be deviations from this scheme. [80] [79] The triminiml parameterization is a completely general scheme of the MNSP matrix, treating Tribimaximal mixing as the zeroth order basis. Four independenent parameters of the U_{MNS} are given by ϵ_{jk}, $jk = 21, 32, 13$, which are the deviations of the θ_{jk} from their tribimaximal values and CP- violating phase δ. One can obtain the usual tribimaximal mixing by taking $\theta_{jk} = 0$. The triminimal parameterization is given by

$$U_{TMin} = R_{32}\left(\frac{\pi}{3}\right) U_\epsilon\left(\epsilon_{32}\ ,\ \epsilon_{13}\ ,\ \epsilon_{21},\ \delta\right) R_{21}\left(\sin^{-1}\frac{1}{\sqrt{3}}\right) \tag{4.27}$$

$$U_\epsilon = R_{32}\left(\epsilon_{32}\right) U_\delta^\dagger R_{13}\left(\epsilon_{13}\right) U_\delta R_{21}\left(\epsilon_{21}\right)$$

$$U_{TMin} = \begin{pmatrix} \sqrt{2} & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -1 & 1 \end{pmatrix} \frac{U_\epsilon}{\sqrt{6}} \begin{pmatrix} \sqrt{2} & 1 & 0 \\ -1 & \sqrt{2} & 0 \\ 0 & -0 & \sqrt{3} \end{pmatrix}$$

The neutrino observables up to second order in ϵ_{jk} can be expressed in terms of the triminimal parameters as

$$\begin{aligned} \sin^2\theta_{21} &= \frac{1}{3}\left(\cos\epsilon_{21} + \sqrt{2}\sin\epsilon_{21}\right)^2 \\ &\simeq \frac{1}{3} + \frac{2\sqrt{2}}{3}\epsilon_{21} + \frac{1}{3}\epsilon_{21}^2 \\ \sin^2\theta_{23} &= \frac{1}{2} + \sin\epsilon_{32}\cos\epsilon_{32} \\ &\simeq \frac{1}{2} + \epsilon_{32} \\ U_{e3} &= \sin\epsilon_{13}e^{-i\delta} \\ (\text{ This means }\ \theta_{23} &= \epsilon_{13}\ \text{ and } \delta = \delta\ ) \end{aligned} \tag{4.28}$$

4.3.1 Numerical analysis using the triminimal parameterization:

Using equations (4.28) and (4.24-4.26), one can obtain the neutrino mass relations in terms of the deviations from solar angle ϵ_{12}:

$$\begin{aligned} m_1 &\approx \sqrt{\frac{1+4\sqrt{2}\,\epsilon_{12}}{3-12\sqrt{2}\,\epsilon_{12}}\Delta m_{21}^2} \\ m_2 &\approx \sqrt{\frac{4-8\sqrt{2}\,\epsilon_{12}}{3-12\sqrt{2}\,\epsilon_{12}}\Delta m_{21}^2} \\ m_3 &\approx \sqrt{\frac{4-8\sqrt{2}\,\epsilon_{12}}{3-12\sqrt{2}\,\epsilon_{12}}\Delta m_{21}^2+\Delta m_{32}^2} \end{aligned} \tag{4.29}$$

In the limit $\epsilon_{12} \to 0$ using the best fit values of the mass squared differences $\triangle m_{21}^2 \approx 7.65 \times 10^{-5}$ eV^2 , $\triangle m_{31}^2 \approx 2.40 \times 10^{-3}$eV^2 [36], we obtain the neutrino masses for the tribimaximal mixing (TBM):

$$\begin{aligned} m_1 &\approx \sqrt{\frac{1}{3}\Delta m_{21}^2} \approx 0.00504 \quad \text{eV} \\ m_2 &\approx \sqrt{\frac{4}{3}\Delta m_{21}^2} \approx 0.010 \quad \text{eV} \\ m_3 &\approx \sqrt{\frac{4}{3}\Delta m_{21}^2+\Delta m_{32}^2} \approx 0.05 \quad \text{eV} \end{aligned}$$

The behavior of the neutrino masses in the vicinity of tribimaximal mixing (TBM) ($\epsilon_{12} = 0$) is shown by the figures 4.1 to 4.4.

The neutrino masses obtained using tribimaximal mixing agree with those obtained in chapter 2 and by Fritzsch and Xing. In the triminimal case we see that as the value of the solar angle increases from the tribimaximal value, the neutrino masses m_1, m_2, m_3 generally increase and the neutrino masses become more and more degenerate. However if the value of the solar angle decreases from the tribimaximal value, the neutrino masses tend to decrease, increasing the neutrino mass hierarchy.

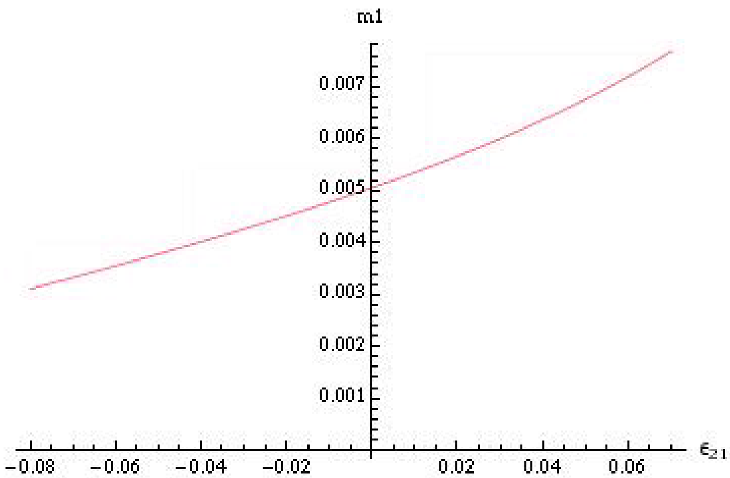

Figure 4.1: Behavior of m1 in the vicinity of TBM

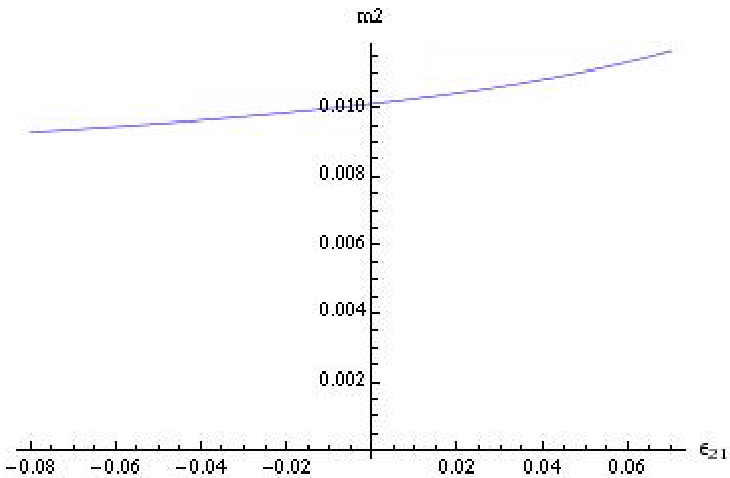

Figure 4.2: Behaviour of m2 in the vicinity of TBM

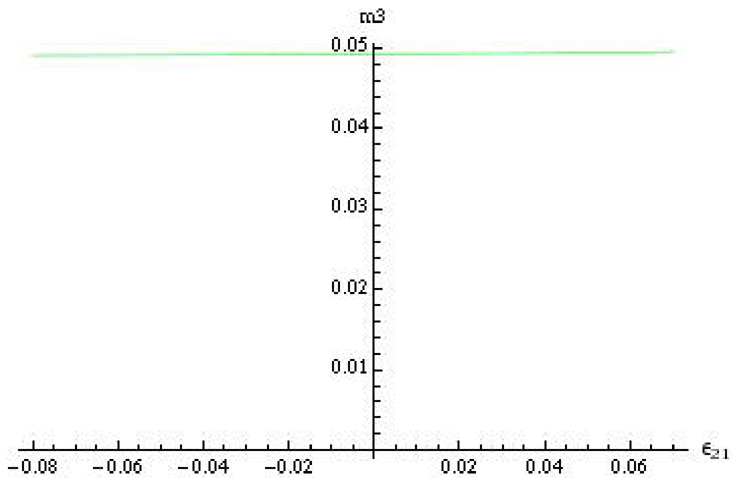

Figure 4.3: Behavior of m3 in the vicinity of TBM

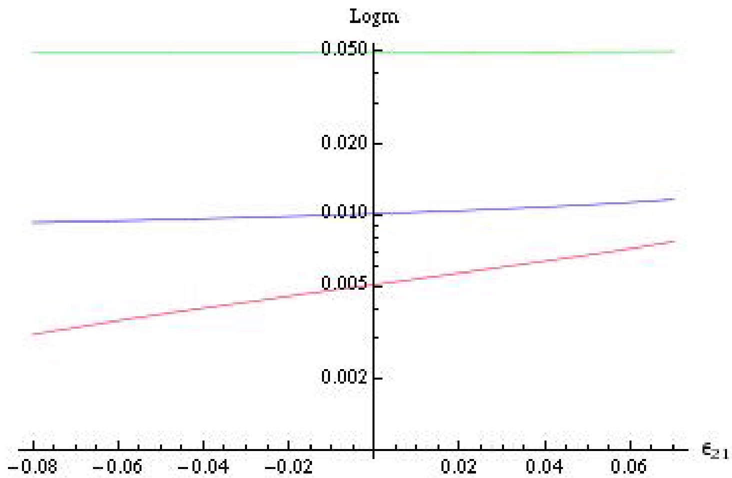

Figure 4.4: Behavior of m1, m2,m3 in the vicinity of TBM

4.4 Fermion masses and mixing in the $SO(10)\times A_4$ Model

The neutrino mixing parameters can be constrained very well, when the $SO(10)$ GUT is combined with a horizontal symmetry acting on the fermion families. We know most of the parameters in the quarks and charged lepton sectors: the masses of the quarks and the charged leptons, the quark mixing angles and the phases. The neutrino mass squared differences and the two mixing angles in the lepton sector are known. These experimental values may be the result of a discrete symmetry in the lepton sector [81]. The neutrino data can be explained by the tribimaximal mixing which one can obtain very easily using the A_4 discrete flavor symmetry. Such a discrete symmetry could be the result of some underlying high energy theory such as the super string theory or compatification of the heterotic orbifolds [82]. In the Morisi-Picariello-Torrente Lujan (MPT) non-SUSY $SO(10) \times A_4$ GUT Model [81], matter fields are treated as 16 of $SO(10)$ and triplet of A_4, in the Higgs sector a 10, a 126_s , three singlets 45 of A_4, two triplets, a 45 and a 126_t of A_4 are used. We must break the left-right symmetry at the unification scale, because the leptons and quark mass matrices cannot be symmetric. Vacuum expectation values of the Higgs A_4- triplets dynamically break the discrete symmetry A_4. In this $SO(10)$ theory the vacuum expectation values (VEVs) of the four scalar 45_s are assumed to be in the directions of T_{3R} , Y and two linear combinations of them: C and D. A contribution proportional to identity is given by 10. Higher dimensional operators contribute only to M^u, M^d, M^l and there is no contribution to M^{ν}_{Dirac} because of our choice of a particular VEV direction and the fact that 45_sappears only in a given combination. The Majorana mass matrix gets contributions from 126 only. By using the see-saw mechanism, one obtains the low energy neutrino mass matrix [83].

The Yukawa Lagrangian of the model is given by [81]

$$\mathcal{L}_Y = h_0^{ij} 16^i 10\ 16^j + h_0'^{ij} 16^i 10\ 45_{T_{3R}}\ 45_Y 16^j + h^{ijk} 16^i 10\ 45_{T_{3R}}\ 45_Y\ 45^j_C\ 45_D\ 16^k \tag{4.30}$$

$$+ \sigma^{ij} 16^i\ 45_{T_{3R}}\ 126_s\ 45_{T_{3R}} 16^j + \lambda^{ijk} 16^i 45_{T_{3R}}\ 126^j_t\ 45_{T_{3R}} 16^k$$
$$\equiv \mathcal{L}_{Dirac} + \mathcal{L}_{Majorana}$$

In the equation (4.30) i, j, k etc are the A_4 indices. The gauge indices are assumed to be summed. The three A_4 indices can be contracted in two ways in an invariant way. The 10 Higgs can not belong to triplet of A_4 as we

$SO(10)$	16	10	$45_{T_{3R}}$	45_Y	45_C	45_D	126_s	126_t
A_4	3	1	1	1	3	1	1	3

Table 4.3: Matter and Higgs field representations

want only one Higgs. The 10 Higgs can transform under A_4 in three ways, i.e. as $1, 1', 1''$. From the first term in the lagrangian the fermion masses M_f , $f = u, d, l, \nu$ can be written as

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix} \tag{4.31}$$

These mass matrices give three degenerate eigenvalues $m_u = m_c = m_\tau$. This relation is corrected by the other terms in (4.30). We assume that the A_4 triplets $45_C, 126_t$ get VeVs in the following directions

$$\langle 45_C \rangle = \upsilon_{45_C} (1, 1, 1)$$
$$\langle 126_t \rangle = \upsilon_{126_t} (1, 0, 0)$$

The quadratic part for the fermions of the lagrangian in (4.30) after symmetry breaking can be written as

$$\begin{aligned} \mathcal{L}_{Dirac} = {} & h_0 (16_1 16_1 + 16_2 16_2 + 16_3 16_3) \upsilon_{10} \\ & + h'_0 (16_1 16'_1 + 16_2 16'_2 + 16_3 16'_3) \upsilon_{10} \\ & + h_1 (16_1 16''_2 + 16_2 16''_3 + 16_3 16''_1) \upsilon_{10} \\ & + h_2 (16_1 16''_3 + 16_2 16''_1 + 16_3 16''_2) \upsilon_{10} \end{aligned} \tag{4.32}$$

$$\mathcal{L}_{Majorana} = \sigma (16'''_1 16'''_1 + 16'''_2 16'''_2 + 16'''_3 16'''_3) \upsilon_{126_s} + 16'''_2 16'''_3 \upsilon_{126_t} \tag{4.33}$$

with

$$16''_i \equiv \upsilon_{45_{T_{3R}}} \upsilon_{45_Y} \upsilon_{45_C} \upsilon_{45_D} 16_i \quad , \quad i = 1, 2, 3,$$
$$16'_i \equiv \upsilon_{45_{T_{3R}}} \upsilon_{45_Y} 16_i \quad , \quad 16'''_i \equiv \upsilon_{45_{T_{3R}}} 16_i$$

Absorbing the VEVs of the 45_s into coupling constants and representing the quantum numbers of the product of the charges T_{3R}, $Y \rightarrow x_{fL,R}$, T_{3R}, Y , C, $D \rightarrow x'_{fL,R}$ and of the charges $T_{3R} \rightarrow x''_{fL,R}$, we obtain

	X	Y	$B-L$	T_{3R}
q	1	$\frac{1}{3}$	1	0
u^c	1	$-\frac{4}{3}$	-1	$\frac{1}{2}$
d^c	-3	$\frac{2}{3}$	-1	$-\frac{1}{2}$
l	-3	-1	-3	0
e^c	1	2	3	$\frac{1}{2}$
ν^c	5	0	3	$\frac{1}{2}$

Table 4.4: Quantum numbers for the low energy matter fields

$$
\begin{aligned}
16' &= (x_{qL}\,q, x_{uR}\,u^c, x_{dR}\,d^c, x_{lL}\,l, x_{eR}\,e^c, x_{\nu R}\,\nu_R)^T\\
16'' &= \left(x'_{qL}\,q, x'_{uR}\,u^c, x'_{dR}\,d^c, x'_{lL}\,l, x'_{eR}\,e^c, x'_{\nu R}\,\nu_R\right)^T\\
16''' &= \left(x''_{qL}\,q, x''_{uR}\,u^c, x''_{dR}\,d^c, x''_{lL}\,l, x''_{eR}\,e^c, x''_{\nu R}\,\nu_R\right)^T
\end{aligned}
$$

4.4.1 Mass matrices and mixings

The terms $16_i 16''_j \upsilon_{10}$ in the Lagrangian $\mathcal{L}_{Dirac}$ do not contribute to the Dirac neutrino mass since $x_{\nu R}=0$, $x'_{lL}=0$ (Y of right handed neutrino and T_{3R} of the lepton doublet are zero). When 45_s gets a VEV, we have from the second line of equation (4.32)

$$M^{\nu}_{Dirac} = h_0 \upsilon^u I$$

I is the identity matrix and υ^u is vev of the up component of 10 Higgs. The Dirac neutrino mass matrix M^{ν}_{Dirac} is proportional to the identity. This fact is useful to realize the see-saw mechanism and the emergence of the tribimaximal mixing matrix in the lepton sector. The terms $h_1 16_1 16''_2$ and $h_2 16_2 16''_1$ from the third and fourth lines of (4.32), using the conventions $x_{uL}=x_{dL}=x_{qL}$, $x_{eL}=x_{\nu L}=x_{lL}$ and $\upsilon^e=\upsilon^u$, give the following mass terms

$$h_1\upsilon^f\left(x'_{fL}\,\overline{\psi}_{L1}\psi_{R2}+x'_{fR}\,\overline{\psi}_{L2}\psi_{R1}\right)+h_2\upsilon^f\left(x'_{fL}\,\overline{\psi}_{L2}\psi_{R1}+x'_{fR}\,\overline{\psi}_{L1}\psi_{R2}\right)+h.c.$$

We can write it as

$$\upsilon^f\begin{pmatrix} 0 & h_1x'_{fL}+h_2\,x'_{fR}\\ h_1x'_{fR}+h_2\,x'_{fL} & 0\end{pmatrix}_{12}$$

Similarly one can get other components. Suppose that

$$A_f = h_1 x'_{fL} \ + h_2 \ x'_{fR} \qquad (4.34)$$
$$B_f = h_1 x'_{fR} \ + h_2 \ x'_{fL}$$

Finally the operators proportional to the 45 representation give the following contribution to the Dirac mass matrices:

$$\upsilon^f \begin{pmatrix} 0 & A_f & B_f \\ B_f & 0 & A_f \\ A_f & B_f & 0 \end{pmatrix}$$

υ^u, υ^d represent the vevs of up and down components of 10, and h_0^f are defined by the relations

$$h_0^u = h_0 + x_{uR}\, h'_0 \quad , \, h_0^d = h_0 + x_{dR}\, h'_0 \ , \quad h_0^l = h_0 + x_{eR}\, h'_0$$

We can write the charged fermion mass matrices as

$$M^u = \upsilon^u \begin{pmatrix} h_0^u & A^u & B^u \\ B^u & h_0^u & A^u \\ A^u & B^u & h_0^u \end{pmatrix} \quad , \quad M^d = \upsilon^d \begin{pmatrix} h_0^d & A^{d,l} & B^{d,l} \\ B^{d,l} & h_0^d & A^{d,l} \\ A^{d,l} & B^{d,l} & h_0^d \end{pmatrix} \qquad (4.35)$$
$$M^l = \upsilon^d \begin{pmatrix} h_0^l & A^{d,l} & B^{d,l} \\ B^{d,l} & h_0^l & A^{d,l} \\ A^{d,l} & B^{d,l} & h_0^l \end{pmatrix}$$

The mass matrices have the same form as given in [84]. If we define $a = \sigma \upsilon_{126_s}$ and $b = \lambda \upsilon_{126_t}$ the Majorana neutrino mass matrix for the right handed neutrino can be written as

$$M_R = \begin{pmatrix} a & 0 & 0 \\ 0 & a & b \\ 0 & b & a \end{pmatrix}$$

The Dirac mass matrices (4.35) can be diagonalized by

$$U = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}$$

Using $f = u, d, l$, $\upsilon^l = \upsilon^d$ and the complex parameters h_0^f, A^f and B^f we can write the diagonalized charged fermion mass matrices

$$M^f = U \begin{pmatrix} \left(h_0^f + A^f + B^f\right) \upsilon^f & 0 & 0 \\ 0 & \left(h_0^f + \omega A^f + \omega^2 B^f\right) \upsilon^f & 0 \\ 0 & 0 & \left(h_0^f + \omega B^f + \omega^2 A^f\right) \upsilon^f \end{pmatrix} U^\dagger$$

The charged fermion mass eigenvalues can be written as

$$\begin{aligned}
m_1^f &= \left(h_0^f + A^f + B^f\right)\upsilon^f \\
m_2^f &= \left(h_0^f + \omega A^f + \omega^2 B^f\right)\upsilon^f \\
m_3^f &= \left(h_0^f + \omega B^f + \omega^2 A^f\right)\upsilon^f
\end{aligned} \tag{4.36}$$

Generally the charged fermion masses m_i^f , obtained this way are complex with unphysical phases. h_0^f, A^f, B^f are complex parameters and υ^f represent the scalar Higgs doublet in 10 with $\upsilon^l = \upsilon^d$. Using the see-saw mechanism and the fact that the Dirac neutrino mass matrix M^ν_{Dirac} is proportional to the identity, we can write the light neutrino mass matrix as

$$M^\nu = M^\nu_{Dirac}\frac{1}{M_R}\left(M^\nu_{Dirac}\right)^T$$

The low energy neutrino mass matrix $\overline{M}^\nu$, in the basis where the charged leptons are diagonal, is given by

$$\overline{M}^\nu = U^T M^\nu U = M^\nu_{Dirac}\frac{1}{U^\dagger M_R U^*}\left(M^\nu_{Dirac}\right)^T$$

For the right handed Majorana mass matrix we obtain

$$U^\dagger M_R U^* = \begin{pmatrix} a+2\frac{b}{3} & -\frac{b}{3} & -\frac{b}{3} \\ -\frac{b}{3} & 2\frac{b}{3} & a-\frac{b}{3} \\ -\frac{b}{3} & a-\frac{b}{3} & 2\frac{b}{3} \end{pmatrix}$$

A tribimaximal matrix is used to diagonalize the above mass matrix and as well as the low energy neutrino mass matrix $\overline{M}^\nu$. The eigenvalues of $\overline{M}^\nu$ are

$$\begin{aligned}
m_1 &= \frac{(h_0\upsilon^u)^2}{a+b} \\
m_2 &= \frac{(h_0\upsilon^u)^2}{a} \\
m_3 &= \frac{(h_0\upsilon^u)^2}{b-a}
\end{aligned}$$

4.4.2 Data fitting and analysis:

Using the experimental data the parameters of this model can be constrained [81]. Particularly the charged fermion mass matrices can be fitted very well. Some predictions about the absolute neutrino mass can be made.

The general solution of the equations (4.36) is

$$\begin{aligned} h_0^f &= \frac{1}{\upsilon^f}\frac{m_1^f+m_2^f+m_3^f}{3} \\ A^f &= \frac{1}{\upsilon^f}\frac{m_1^f+\omega^2 m_2^f+\omega m_3^f}{3} \\ B^f &= \frac{1}{\upsilon^f}\frac{m_1^f+\omega m_2^f+\omega^2 m_3^f}{3} \end{aligned} \tag{4.37}$$

The fermion masses fix the values of the complex parameters h_0^f, A^f, B^f up to phases. If ϕ_1 ,ϕ_2 represent the relative phases between m_1 and m_3 and between m_2 and m_3 respectively, then the absolute value of h_0^f is given by

$$\begin{aligned} \left|h_0^f\right|^2 = \left(\frac{1}{3\upsilon^f}\right)^2 &[\left(m_1^f+m_2^f+m_3^f\right)^2 \\ &-2\left\{m_1^f m_3^f\left(1-\cos\phi_1\right)+m_1^f m_2^f\left(1-\cos\left(\phi_1-\phi_2\right)\right)+m_2^f m_3^f\left(1-\cos\phi_2\right)\right\}] \end{aligned}$$

If $m_3^f > m_1^f, m_2^f$, we can write the above equation as

$$\frac{1}{3\upsilon^f}\left(m_1^f+m_2^f+m_3^f\right) \geq \left|h_0^f\right| \geq \frac{1}{3\upsilon^f}\left(m_3^f-m_1^f-m_2^f\right)$$

Similarly for $A^f$and B^fwe can have the following relations

$$\begin{aligned} \frac{1}{3\upsilon^f}\left(m_1^f+m_2^f+m_3^f\right) &\geq \left|A_0^f\right| \geq \frac{1}{3\upsilon^f}\left(m_3^f-m_1^f-m_2^f\right) \\ \frac{1}{3\upsilon^f}\left(m_1^f+m_2^f+m_3^f\right) &\geq \left|B_0^f\right| \geq \frac{1}{3\upsilon^f}\left(m_3^f-m_1^f-m_2^f\right) \end{aligned}$$

The phases of the h_0^f, A_0^f, B_0^f are very strongly constrained by the last equation in (4.36) under the assumption $m_3^f >> m_1^f, m_2^f$. From equation (4.37) we have

$$\frac{A^f}{h_0} \simeq \omega \quad , \quad \frac{B^f}{h_0} \simeq \omega^2$$

From equations (4.34) and (4.37) and using the notation $x_\pm'^f \equiv x_L'^f \pm x_R'^f$, we find

m_u (MeV)	$0.8351^{+0.1636}_{-0.1700}$
m_c (MeV)	$242.6476^{+23.5536}_{-24.7026}$
m_t (GeV)	$75.4348^{+9.9647}_{-8.5401}$
m_d (MeV)	$1.7372^{+0.4846}_{-0.2636}$
m_s (MeV)	$34.5971^{+4.8857}_{-5.1971}$
m_b (GeV)	$0.9574^{+0.0037}_{-0.0169}$
m_e (MeV)	$0.4414^{+0.0001}_{-0.0001}$
m_μ (MeV)	$93.1431^{+0.0136}_{-0.0101}$
m_τ (GeV)	$1.5834^{+10.4664}_{-13.6336}$

Table 4.5: Charged fermion masses at GUT scale

$$x'^f_+ = \frac{1}{3\upsilon^f}\frac{m^f_3+m^f_2-2m^f_1}{h_1+h_2} \quad , \quad x'^f_- = \frac{i}{\sqrt{3}\upsilon^f}\frac{m^f_3-m^f_2}{h_1-h_2}$$

The ratios $\frac{x'^f_+}{x'^{f'}_+}$ and $\frac{x'^f_-}{x'^{f'}_-}$ are independent of the parameters h_i, therefore they are experimentally determined up to the undetermined phases of the masses.

$$\begin{aligned}
\frac{x'^u_+}{x'^d_+} &= \frac{\upsilon^d}{\upsilon^u}\frac{m_t+m_c-2m_u}{m_b+m_s-2m_d} \quad , & \frac{x'^u_-}{x'^d_-} &= \frac{\upsilon^d}{\upsilon^u}\frac{m_t-m_c}{m_b-m_s} \\
\frac{x'^u_+}{x'^e_+} &= \frac{\upsilon^d}{\upsilon^u}\frac{m_t+m_c-2m_u}{m_\tau+m_\mu-2m_e} \quad , & \frac{x'^u_-}{x'^e_-} &= \frac{\upsilon^d}{\upsilon^u}\frac{m_t-m_c}{m_\tau-m_\mu} \\
\frac{x'^d_+}{x^e_+} &= \frac{m_b+m_s-2m_d}{m_\tau+m_\mu-2m_e} \quad , & \frac{x'^u_-}{x'^d_-} &= \frac{m_b-m_s}{m_\tau-m_\mu}
\end{aligned} \tag{4.38}$$

Using (non- SUSY) Standard model fermion masses at the scale 2×10^{16} GeV [81] [85], one obtains

$$\begin{aligned}
\frac{x'^u_+}{x'^d_+} &= 0.972^{+0.073}_{-0.013} \quad , & \frac{x'^u_-}{x'^d_-} &= 1.034^{+0.007}_{-0.072} \\
\frac{x'^u_+}{x'^e_+} &= 0.573^{+0.079}_{-0.011} \quad , & \frac{x'^u_-}{x'^e_-} &= 0.640^{+0.011}_{-0.077} \\
\frac{x'^d_+}{x^e_+} &= 0.590^{+0.085}_{-0.048} \quad , & \frac{x'^u_-}{x'^d_-} &= 0.619^{+0.054}_{-0.075}
\end{aligned} \tag{4.39}$$

To fit this data theoretically in the model, the values of $\frac{x_+^{\prime f}}{x_+^{\prime f'}}, \frac{x_-^{\prime f}}{x_-^{\prime f'}}$ are determined from Table 4.1 and the definition of $x_\pm^{\prime f}$. If we take, for example, the direction $C = (28X - 249Y)$ and $D = (238X - 9Y)$, we have

$$\frac{x_+^{\prime u}}{x_+^{\prime d}} = 1 \quad , \quad \frac{x_-^{\prime u}}{x_-^{\prime d}} = 1$$
$$\frac{x_+^{\prime u}}{x_+^{\prime e}} = \frac{300}{517} \quad , \quad \frac{x_-^{\prime u}}{x_-^{\prime e}} = \frac{300}{517}$$

This relation is in very good agreement with the experimental values in equation (4.39). As there are only two parameters a, b in the neutrino sector, the absolute neutrino mass scale is fixed. Using the mass squared difference values from the neutrino experiments

$$\Delta m_{12}^2 = 7.92\,(1 \pm 0.09) \times 10^{-5}\ \mathrm{eV}^2 \quad , \quad \left|\,\Delta m_{12}^2\right| = 2.2\,\left(1_{-0.26}^{+0.21}\right) \times 10^{-5}\ \mathrm{eV}^2$$

For a normal hierarchy the neutrinos masses are given by [81]:

$$m_1 = 0.0051 \pm 0.0005\ \mathrm{eV},\ m_2 = 0.0102 \pm 0.0005\ \mathrm{eV}\ ,\ m_3 = 0.049 \pm 0.004\ \mathrm{eV}$$

For the inverted hierarchy one finds

$$m_1 = 0.052 \pm 0.005\ \mathrm{eV},\ m_2 = 0.052 \pm 0.005\ \mathrm{eV}\ ,\ m_3 = 0.017 \pm 0.002\ \mathrm{eV}$$

Chapter 5

Beyond Non-SUSY $SO(10)$ GUTs

5.1 SUSY $SO(10)$ Models

The SUSY GUTs [86],[87] are well motivated theories. They are expected to clarify on many relevant problems

1. They provide a solution to the hierarchy problem by explaining, why $\upsilon_{wk} << M_{Pl}$.

2. The gauge coupling unification proceeds without intermediate energy scales.

3. As demanded by the solution to the gauge hierarchy, unification of electroweak and strong gauge couplings considering supersymmetry breaking, masses are in the TeV range.

Supersymmetric grand unified theories generally predict proton decay. Considering the current lower limit on the proton decay modes, one can rule out a number of simple SUSY GUTs [88]. The gauge coupling unification scale 10^{16} GeV and the atmospheric neutrino data fitting seesaw scale 10^{15} GeV are rather close. This suggests that the seesaw scale could be the GUT scale itself. Therefore one can hope that the supersymmetric grand unification can explain the smallness of the neutrino masses very well by the seesaw mechanism. There are many different ways to understand the large mixings in the lepton sector in the context of the SUSY $SO(10)$ GUTs. The presence of a local $B - L$ symmetry as a sub group of $SO(10)$ is the distinguishing feature of the $SO(10)$ models as compared to $SU(5)$ models.

If this $B - L$ symmetry is broken by a 16 Higgs, one necessarily obtains right handed neutrino mass arising from a nonrenormalizable coupling in a low energy MSSM with R-parity breaking such that without additional assumptions, the model cannot have a cold dark matter candidate. In the $SO(10)$ theories with 16 Higgs, one assumes that there is a further high energy theory (String theory etc.) that below the heavy scale leads to this version of $SO(10)$. In such a theories it is hard to make predictions about the fermion masses. Without additional symmetry restrictions there are less physical input than the free parameters. However in these theories the Yukawa couplings can be restricted to some extent. In some cases the predictions for the neutrino sector can be made [89]. As the first term in the type II seesaw model is negligible, neutrino masses are dictated by the second term (type I seesaw). The value of the CHOOZ angle θ_{13}, predicted by most of such theories, is very small and can not be tested in the next generation of the planned experiments.

If the $B - L$ symmetry is broken by a 126 Higgs, one gets a right handed neutrino mass from renormalizable coupling. The lightest SUSY particle as dark matter candidate results from a low energy MSSM. The neutrino masses can be obtained from first term or second term or from both terms of type II seesaw. Such models have many interesting features [90],[91]. The neutrino sector is very predictive because the flavor structures of quarks and neutrinos get unified in these models as a consequence of the 126 Higgs contribution to the fermion masses through MSSM doublets. In some cases all the mixing angles and masses can be predicted. Ignoring CP violation there are only 12 parameters, which can be determined using charged fermion masses and quark mixings. It has been shown in one such model that one gets wrong predictions of the neutrino parameters using type I seesaw [92]. However the type II seesaw model works very well, and large mixings can be obtained very easily [93]. The Model predicts $\theta_{13} \simeq 0.18^0$, which can be tested in the next generation experiments.

Here are some models with the predictions in the neutrino sector [94]

Texture Zero Models

Reference	**Hierarchy**	$\sin^2 \mathbf{2}\theta_{23}$	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$
GL1 [95]	*Normal*	1.0		≥ 0.005	

WY [96]	*Normal*	0.0006-0.0030
	Inverted	0.0006-0.0030
	Normal	<0.023
	Normal	0.017-0.14

CPP [97]	*Normal*	0.0066-0.0083
	Inverted	≥ 0.00005
	Inverted	≥ 0.032

$Le - L_\mu - L\tau$ **Models**

Reference	**Hierarchy**	$\sin^2 \mathbf{2}\theta_{23}$	$\sin^2 \theta_{12}$	$\sin^2 \theta_{13}$	$\sin^2 \theta_{23}$
BM [98]	*Inverted*			0.00029	
GMN1 [99]	*Inverted*		≥ 0.28	≤ 0.05	
PR [100]	*Inverted*		$\lesssim 0.37$	≥ 0.007	
GL2 [101]	*Inverted*		0.30	0	

$2-3$ **Symmetric Models**

Reference	**Hierarchy**	$\sin^2 \mathbf{2}\theta_{23}$	$\sin^2 \theta_{12}$	$\sin^2 \theta_{13}$	$\sin^2 \theta_{23}$
RS [102]	*Normal*	$\theta_{23} \leq 45°$		0	
	Inverted	$\theta_{23} \geq 45°$		≤ 0.02	
MN [103]	*Normal*	1.0		0.0024	
AKKL [104]	*Normal*			$0.006 - 0.016$	
	Inverted			$0.022 - 0.04$	
SRB [105]	*Inverted*	1.0	0.31	0	0.50
BY [106]	*Normal*	1.0	0.31	< 0.0025	
	Inverted	1.0	0.31	< 0.008	

S_3 Models

Reference	**Hierarchy**	$\sin^2 \mathbf{2}\theta_{23}$	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$
		1.0			
KMMR-J [107]	*Inverted*			0.000012	
CFM [108]	*Normal*			$0.00006 - 0.001$	
T [109]	*Normal*			$0.016 - 0.0036$	
TY [110]	*Inverted*	0.93	0.30	0.0025	0.37
MNY [111]	*Normal*			$0.000004 - 0.000036$	
MMP [112]	*Inverted*	1.0	0.31	0.0034	
MC [113]	*Normal*	1.0		< 0.01	

A_4 Tetrahedral Models

Reference	**Hierarchy**	$\sin^2 \mathbf{2}\theta_{23}$	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$
Ma1 [114]	*Normal*	1.0	0.31	0	0.50
	Inverted	1.0	$0.33 - 0.34$	0	0.50
ABGMP [115]	*Normal*	1.0	$0.27 - 0.30$	$0.0007 - 0.0037$	$0.51 - 0.52$
AG1 [116]	*Normal*	1.0	0.31	$0.0026 - 0.034$	$0.51 - 0.56$
HT [117]	*Normal*	1.0	$0.29 - 0.33$	< 0.0022	
AG2 [118]	*Inverted*	1.0	$0.27 - 0.34$	< 0.0012	$0.52 - 0.53$
L [119]	*Normal*	1.0	$0.29 - 0.38$	0.0025	
Ma2 [120]	*Normal*	1.0	0.32	0	0.50

S_4 Models

Reference	**Hierarchy**	$\sin^2 \mathbf{2}\theta_{23}$	$\sin^2\theta_{12}$	$\sin^2\theta_{13}$	$\sin^2\theta_{23}$
MPR [121]	*Q-deg*	0.99	$0.25 - 0.27$	$0.008 - 0.01$	0.44
HLM [122]	*Normal*	1.0	0.30	0.0044	0.50
	Normal	1.0	0.31	0.0034	0.50
Z [123]	*Normal*	$0.96 - 1.0$	0.311	< 0.030	$0.41 - 0.50$

$SO(3)$ **Models**

Reference	**Hierarchy**	$\sin^2 \mathbf{2}\theta_{23}$	$\sin^2 \theta_{12}$	$\sin^2 \theta_{13}$	$\sin^2 \theta_{23}$
M [124]	*Normal*	1.0	0.31	0.00005	
W [125]	*Normal*			$0.0027 - 0.0036$	

T' **Models**

Reference	**Hierarchy**	$\sin^2 \mathbf{2}\theta_{23}$	$\sin^2 \theta_{12}$	$\sin^2 \theta_{13}$	$\sin^2 \theta_{23}$
FM [126]	*Normal*	$0.93 - 0.95$		$0.024 - 0.036$	

5.2 Proton Decay in GUTs

Most of the grand unified theories predict an unstable proton. Therefore the proton decay can be used to determine the specific nature of the grand unified theory [19]. Since the Yukawa couplings responsible for the flavor structure of fermions arise from dimension five operators in SUSY $SO(10)$ GUTs, one can hope to learn about fermion texture from proton decay modes. Predictions about the proton decay have been studied in 16_H model [127] through mode $p \longrightarrow \overline{\nu} K^+$ and 126_H $SO(10)$ model through $n \longrightarrow \pi^0 \overline{\nu}$ [128]. However the gauge mediated proton decay $p \longrightarrow e^+ \pi^0$ being model independent, is the true test of grand unification. The lower limit of this mode is 5×10^{33} years [129]. For SUSY theories this limit is expected at about 10^{38} years and hence cannot be tested by conventional technology. The non-SUSY theories seems to be more promising in this regard. In non-SUSY $SO(10) \longrightarrow SU(2)_L \times SU(2)_R \times SU(4)_C$ theories the prediction for the proton lifetime is about $1.44 \times 10^{32.1 \pm 0.7 \pm 1.0 \pm 1.9}$ years [130]. In these theories the intermediate energy scale is at about $10^{13.6}$ GeV which can be treated as seesaw scale to predict neutrino masses

Chapter 6

Conclusion

In this thesis we have described the quark and the lepton mixing mechanism in context of hermitian mass matrices. In particular we have analyzed a realistic pattern of quark and leptons mass matrices within four zero texture (FZT). Four zero texture relates the fermions mass ratios and the mixing angles in the quark and lepton sectors [31],[35]. Using tribimaximal mixing predictions about the neutrino mass are made. Considering the mixing matrix as near tribimaximal, behavior of the neutrino masses near tribimaximal mixing is studied. The four zero texture can be imbedded in the non-SUSY $SO(10)$ [74]. In this case the model is more tightly constrained than either of FZT or $SO(10)$ alone. The model predicts the consistent values for the quarks and charged leptons. For neutrino masses we can use the triminimal mixing matrix and determine the values of neutrino masses for tribimaximal mixing given by $m_1 \approx 0.005$ eV, $m_1 \approx 0.01$ eV and $m_1 \approx 0.00504$ eV. This shows that neutrinos have weak but normal hierarchy. We have also studied the effect on neutrino masses when the value of solar angle is changed from its TBM value. Our analysis shows that the neutrinos masses m_1, m_2 in general tend to increase as the value of solar angle increases from its TBM value and vice versa. However m_3 seems to be rather stable against the change in solar angle. It appears that the neutrinos become more and more degenerate for solar angle values higher than TBM value and hierarchical for lower values of solar angle.

We speculate that in order to explain the fermion masses and mixings, the $SO(10)$ symmetry may be sufficient. No new physics beyond non-SUSY SO(10) may be needed [46]. We have described the two potentially realistic Yukawa structures based on $\overline{126}_H$ and 10_H or 120_H. It seems that both of these scenarios require $U(1)_{PQ}$. It also provides axion as a dark matter candidate. We have also reviewed the $SO(10) \times A_4$ model [81] and SUSY $SO(10)$ models.

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