Bailouts in Federations
The Role of Elections, Restrictions and Information Asymmetries

Inaugural-Dissertation
zur Erlangung des Grades
Doctor oeconomiae publicae (Dr. oec. publ.)
an der Ludwig-Maximilians-Universität München

2009

vorgelegt von
Karolina Kaiser

Referent: Prof. Dr. Andreas Hauffer
Korreferent: Prof. Dr. Thiess Büttner
Promotionsabschlussberatung: 10. Februar 2010
Acknowledgements

First of all, I would like to thank my supervisor Andreas Haufler for his invaluable advice, his challenging questions and constant encouragement during the preparation of this dissertation as well as Anja Eichhorst for drawing my interest to the topic of bailouts in federations and for inspiring me to work on bailout restrictions.

I am also indebted to Thiess Büttner who kindly agreed to become my second supervisor and gave very useful feedback on my thesis as well as to Monika Schnitzer for joining my thesis committee. Her exceptional engagement at the economics faculty, in particular her advice on identifying interesting research questions and on presenting them was precious.

I am deeply indebted to my co-author Emmanuelle Taugourdeau for kindly inviting me to the University of Paris I, for the very fruitful and interesting research collaboration, for her insightful ideas and her companionship.

During the preparation of my thesis I profited in numerous ways from the ideas and the good company of my study colleagues as well as conference and seminar participants. Especially Basak Akbel and Benno Bühlker supported me strongly both from a human and intellectual perspective. Silvia Appelt, Iris Kesternich, Nicolas Klein, Maria Lehner, Johannes Metzler, Nadine Riedel and Dominik Ruderer deserve also special mention.

The research environment that I found at the Munich Graduate School of Economics, in particular the strong focus on public economics was very valuable. The financial support I received as a member of the doctoral program „Markets, Institutions, and the Scope of Government” from the German Research Foundation (DFG) is gratefully acknowledged.

More than anything I thank my family and friends. Your love and constant support throughout my academic journey remains invaluable.
# Contents

<table>
<thead>
<tr>
<th>Preface</th>
<th>1</th>
</tr>
</thead>
</table>

1 The Timing of Elections in Federations: A Disciplining Device against Soft Budget Constraints?\(^1\)  
1.1 Introduction .................................................. 7  
1.2 The Model .................................................... 11  
1.3 Social Planner ................................................ 14  
1.4 Decentralized Setup ........................................... 15  
1.4.1 Synchronized Office Terms ................................. 15  
1.4.2 Staggered Office Terms ................................... 19  
1.5 Decentralized Setup with Political Cycles .................. 25  
1.5.1 Synchronized Office Terms ................................. 26  
1.5.2 Staggered Office Terms ................................... 29  
1.6 Welfare Analysis .............................................. 34  
1.7 Conclusion ................................................... 36

2 Restricted Bailouts and the Soft Budget Constraint Problem in Federations  
2.1 Introduction ................................................... 38

\(^1\)This chapter is joint work with Emmanuelle Taugourdeau.
2.2 Model Set-Up ......................................................... 41
2.3 Benchmark Cases ................................................. 43
  2.3.1 Centralized Decision Making ......................... 43
  2.3.2 Hard Budget Constraint Regime ..................... 44
  2.3.3 Unrestricted Bailout Regime ......................... 45
2.4 Restricted Bailout Regimes ................................. 47
  2.4.1 Fully Restricted Bailout Regime .................... 48
  2.4.2 Partially Restricted Bailout Regime ................ 48
2.5 Welfare Analysis ............................................. 51
  2.5.1 Efficiency of HBC versus SBC Regimes ............. 52
  2.5.2 Efficiency of PB vs. UB regimes .................... 54
2.6 Conclusion .................................................. 57

3 Bailouts in Federation: The Role of Asymmetric Information 59
  3.1 Introduction .............................................. 59
  3.2 Model Setup ............................................. 63
  3.3 First Best ............................................... 67
  3.4 Decentralized Setup With Partial Conditionality ........ 68
    3.4.1 The Effort Choices of the Regional Governments .... 69
    3.4.2 The Bailout Choice of the Central Government ... 71
    3.4.3 Example ........................................... 73
  3.5 Decentralized Setup with Full Conditionality ............ 76
    3.5.1 The Effort Choices of the Regional Governments ... 77
    3.5.2 The Bailout Choice of the Central Government .... 80
    3.5.3 Example continued ................................. 83
3.6 Conclusions ................................................. 85

Implications and Conclusions ........................................ 87

A Appendix .......................................................... 91

A.1 Appendix to Chapter 1 ....................................... 91
  A.1.1 Welfare analysis without Political Business Cycles ....... 91
  A.1.2 Welfare analysis with Political Business Cycles .......... 93

A.2 Appendix to Chapter 2 ....................................... 96
  A.2.1 Proof of Proposition 1 .................................... 96
  A.2.2 Proof of Proposition 2 .................................... 96
  A.2.3 Proof of Proposition 3 .................................... 96
  A.2.4 Proof of Proposition 4 .................................... 97
  A.2.5 Proof of Corollary 1 ...................................... 98
  A.2.6 Proof of Proposition 5 .................................... 99
  A.2.7 Summary of results for the logarithmic function ......... 100
  A.2.8 Proof of Proposition 6 .................................... 100

A.3 Appendix to Chapter 3 ....................................... 102
  A.3.1 Regional Government Response Functions in the Partial Conditionality Case ................. 102
  A.3.2 Bailout Responses to Parameter Changes in the Partial Conditionality Case ................. 103
  A.3.3 Mathematica Code for Comparative Statics w.r.t. Q and \( n_1 \) in the Partial Conditionality Case ................. 106
  A.3.4 Regional Government Response Functions in the Full Conditionality Case ................. 106
  A.3.5 Proof of \( b_0 > b_u \Leftrightarrow E_x[p(0,0)] > E_x[p(0,u)] \) ................. 108
A.3.6 Appendix 6: Mathematica Code for Comparative Statics w.r.t. 
Q in the Full Conditionality Case . . . . . . . . . . . . . . . . . 110

References 111
List of Figures

1.1 Timing in the SY Regime ................................. 16
1.2 Timing in the ST Regime ................................. 19
1.3 Welfare Analysis .................................. 36

2.1 Welfare Comparison HBC vs. SBC regimes ............... 53

3.1 Optimal bailouts in the partial conditionality case .......... 75
3.2 Optimal bailouts in the full conditionality case .......... 83
Preface

Over the past decades fiscal decentralization and multi-tiered government structures have experienced a growing popularity among governments around the world. Examples for countries that recently increased the level of decentralization can be found in Latin America (e.g. Brazil and Argentina)\textsuperscript{2}, Eastern Europe (e.g. Russia and Hungary)\textsuperscript{3} or Africa (e.g. South Africa)\textsuperscript{4}.

Along with this trend a growing literature on fiscal decentralization emerged, that also pointed out the dangers of fiscal decentralization, in particular the fiscal indiscipline of subnational governments (Rodden et al., 2003 or Vigneault, 2005). This danger arises because multi-tiered governmental systems allow subnational governments to build up unsustainable deficits and demand from higher level governments to assume their liabilities through transfers and bailouts financed out of the common pool of national revenues (e.g. Inman, 2001 and Rodden, 2002).

The inefficiencies associated with bailouts are the reason why higher level governments often try to establish a no bailout attitude, i.e. they announce the wish to never provide bailouts. However, experience shows that it is difficult to commit to such statements, for example because of voting considerations or because of legal requirements. The difficulty of committing to a no-bailout attitude might become obvious in the current financial crisis which starts to translate into a crisis of public budgets.

This dissertation deals with institutional safeguards helping to alleviate the inefficiencies arising from bailouts in federations. In particular, we consider the role of the timing of elections, bailout restrictions and budgetary information of comparable

\textsuperscript{2}Haldenwang (2008).
\textsuperscript{3}Bird and Wallich (1997).
\textsuperscript{4}Shah (2004).
jurisdictions. We theoretically analyze each of these institutional mechanisms in a separate chapter of this dissertation. The thesis concludes with policy implications. In the remainder of the introduction, we briefly present the basic approaches and most important results of our analyses.

The first chapter of this dissertation is joint work with Emmanuelle Taugourdeau\textsuperscript{5}. We investigate how the timing of central and regional government elections in federations affects the bailout problem. Intuitively the timing of elections should matter, because many strategic policy decisions like tax policies or expenditure cuts are decided at the beginning of an office term. These decisions have implications for the deficits arising over the course of the term and the scope for bailouts.

We analyze two regimes. On the one hand, we consider a synchronized office term regime, where regional and central governments are simultaneously elected. On the other hand, we consider a staggered office term regime where regional governments are elected in the mid-term of a central government office term.

We employ a soft budget constraint model. The underlying assumption of this generic model type is the central government’s inability to commit not to bail out lower level governments. This inability of commitment is expressed in a first mover advantage of regional governments vis-à-vis the central government. Because bailouts are financed out of the common pool of federal tax revenues, regional governments, anticipating bailouts at the second stage, have incentives to strategically raise insufficient funds to finance their public expenditures. Once the deficit arises, the central government ex-post is not able to resist giving up a part of its revenue to co-finance the regional public good.

In the synchronized office term regime this game takes place in each office period. Therefore the synchronized office term regime can be regarded as a benchmark for the standard soft budget constraint game in federations. In a staggered office term regime the game played between the central and regional governments slightly changes. In the first half of the central government’s office term, the central government is confronted with a similar dilemma like in the synchronized regime: When entering office, it has

\textsuperscript{5}Emmanuelle Taugourdeau is affiliated to the Centre national de la recherche scientifique (CNRS), the Centre d’économie de la Sorbonne (CES) and the Paris School of Economics.
to cope with the high deficits of the regional government that is still in office, and it is unable to resist providing high bailouts. However, it now has a first mover advantage vis-à-vis the regional government entering next. This changes the preferences of the central government on the allocation of its revenues over time. Anticipating the strategic behavior of the next regional government, the central government prefers to spend more funds in the first office half term. Thereby it commits to a substantial reduction of bailouts in the second half term. This forces the next regional government to finance a larger share of regional public goods than in the synchronized office term regime and increases welfare.

In the second chapter of the dissertation, we move on to a more detailed analysis of the bailout process itself. In the past, bailouts were often linked to additional obligations like savings goals, which were enforced by special monitoring boards. These boards prescribed refunding plans, they audited, inspected, approved and supervised subnational budgets during the fiscal consolidation process. Certainly one of the most famous examples is the Emergency Financial Control Board (EFCB) which controlled New York City’s government during the bailout following the 1975 debt crisis.

The public economics literature has intensely analyzed bailouts in federations, but has not paid attention to bailout restrictions. However, bailout restrictions are interesting from a theoretical perspective because they are a credible commitment device to alleviate the common pool problem of bailouts. Consider a regional government in a budgetary crisis that it is about to close schools. If just monetary bailouts were available, a central government caring for this local public service can do no better than financing the teachers’ salaries out of its own pocket. However, if additional restrictions were available, the central government could for instance enforce higher local tax rates and force the regional government into an own contribution to maintain the provision of schooling.

In this chapter, we introduce bailout restrictions in a simple soft budget constraint framework of a federation. In this framework regional governments may raise revenues through two different channels: through regional taxation and by making public good provision more efficient. We consider two degrees of bailout restrictions. In a fully
restricted bailout regime, the central government is allowed to impose restrictions on both regional revenue raising channels along with a monetary bailout. The partially restricted bailout regime is less severe because it just restricts one regional revenue raising channel. We compare both setups to the benchmark of an unrestricted regime where just a monetary bailout is provided without any further obligation.

The most striking result of our analysis is that from a welfare perspective, the partially restricted bailout regime might be worse than the unrestricted bailout regime. The intuition for this result is that the regions compensate for the restriction by distorting the unrestricted revenue instrument even more than in the case of no restrictions at all.

The analyses of the previous chapters were built on a soft budget constraint framework, which is the most frequently employed workhorse model for the analysis of bailouts in federations. As mentioned before, this model builds on the assumption that central governments are not able to commit ex-post to deny a bailout once a fiscal crisis has emerged at the regional level. One major implication of this hypothesis is that central governments are not able to condition their bailout choice on the information of why the crisis has occurred. To put it differently, the non-commitment hypothesis implies that a central government would provide the same amount of bailouts to a regional government in fiscal crisis because of fiscally imprudent behavior of local officials as to a regional government that came into a crisis because of an external shock – like a natural catastrophe.

However, cases observed in practice suggest that higher level governments are able to condition bailouts on their information of lower level governments’ negligence in the emergence of the crisis. One case in point is the denial of a bailout to the debt-ridden city of Berlin by the Federal Constitutional Court of Germany in 2006. One of the reasons brought forward by the Court was that Berlin had not sufficiently exhausted its options to raise revenue and to realize expenditure savings. The Court argued that a subnational government, failing to use all these options, cannot afterwards successfully claim financial assistance from the federation.

Even though the conditionality of bailouts on lower level governments’ negligence
promises to reduce the inefficiencies arising from bailouts, it is likely that still ineffici-
encies remain because of asymmetric information, caused by the budgetary autonomy of
multi-tiered governments in federal systems. Subnational fiscal crises often arise over
a long period of time and for many different reasons. Ex-post it seems to be diffi-
cult to disentangle the wrong decisions of politicians from forces outside the scope of
local governments. In such situations higher level governments may be interested in
obtaining information through benchmarking studies, which compare the expenditures
of the jurisdiction in fiscal crisis to the expenditures of comparable jurisdictions. This
has been done by the German Federal Constitutional Court in the denial of the Berlin
bailout.

In the third chapter of this dissertation we analyze optimal bailout schemes under
asymmetric information that condition on the observed outcomes of two regions. We
employ a model of a federation with a central government and two regional govern-
ments. The regional governments can spend costly effort to prevent a bad budgetary
outcome. The choices of both regional governments are linked to each other through a
common cost shock affecting the magnitude of the effort costs. The central government
neither observes the realization of the cost shock nor the regions’ effort, but knows the
probability and the magnitude of a high cost shock.

It is assumed that only one of the governments is poor from the outset and becomes
eligible for a bailout if it realizes a bad budgetary outcome. The magnitude of the
bailout is conditioned on both the budgetary outcome observed in the region in fiscal
crisis and the outcome in the second region that serves as a benchmark region.

We derive the result that the better the budgetary environment the lower are
bailouts. If in addition, it is ex-ante more likely to observe a uniform\textsuperscript{6} outcome than a
mixed outcome, then a higher bailout is paid to the poor region if the uniform outcome
is realized. The intuition for this result is based on the informativeness of the observed
outcome regarding the poor regional government’s negligence. If both regions are
tied to each other such that it is more likely that the regions’ budget surplus evolves
similarly, the observation of a bad budgetary outcome in the benchmark region is a

\textsuperscript{6}A uniform outcome means a bad budgetary outcome in both the poor region and the benchmark
region.
signal that the bad outcome in the poor region occurred despite sufficient effort was taken to overcome the crisis and not because of moral hazard. The opposite result and interpretation applies for the case that mixed outcomes are more likely than uniform ones. Moreover, we show that if a higher bailout is paid for the uniform outcome, the benchmarked region has an incentive to spend inefficiently high effort on the avoidance of a bad budgetary outcome because this reduces its expected contributions to the bailout. This result delivers one theoretical reason for why regions might self-impose balanced budget rules.
Chapter 1

The Timing of Elections in Federations: A Disciplining Device against Soft Budget Constraints?\(^1\)

1.1 Introduction

Between 1994 and 2003, two debt-ridden German Laender - Saarland and Bremen - received extraordinary grants to reduce their debt. Although the total funds received for that purpose amounted to more than 90 percent of the initial debt, Saarland had reduced its total debt by only 6 percent by the end of 2004 and Bremen had even increased it further by more than 20 per cent. This was probably one of the reasons why in two recent initiatives the German federal government (Bund) moved towards a no-bailout attitude. First, in 2005 a law, the German Fiscal Equalization Law allowing for discretionary grants from for Laender in budgetary crises was abolished. Secondly, in 2006 the Federal Constitutional court refused a claim from the debt ridden state of Berlin to receive discretionary grants from the Bund for debt repayment. However, in the current reform debate on fiscal federalism in Germany, the federal government agreed on financial assistance for the debt-ridden Laender of Berlin, Brenen, Saarland, Sachsen-Anhalt and Schleswig-Holstein. It is one example of a bailout resulting from

\(^{1}\)This chapter is joint work with Emmanuelle Taugourdeau.
the lack of fiscal discipline which has been studied theoretically under the generic term of ‘soft budget constraint’. Following Rodden, Eskeland and Litvack (2003), the soft budget constraint can be defined as “the situation when an entity (say, a province) can manipulate its access to funds in undesirable ways”. Although it is widely acknowledged that soft budget constraints may cause inefficiencies⁴ and should therefore be avoided, they are frequently observed (see Vigneault (2007) for an empirical survey). The recurrent emergence of soft budget constraints illustrates the difficulty of totally avoiding them and highlights the importance of better understanding the institutions which help to harden budget constraints.

To our best knowledge, very few papers have dealt with the characteristics of the political system⁵ although soft budget constraints are undoubtedly the results of interactions between two levels of government. In this context, the nature of the interactions may change depending on the electoral timetable. Since the soft budget constraint problem is a problem of commitment and therefore of timing, it might matter if regional and federal governments were elected at identical voting dates and decided at the same time on their revenues and expenditures for the upcoming term or if voting dates fall at different times. In this chapter we ask the question: does the timing of elections in federations matter for the soft budget constraint problem? In other words, in which system are budget constraints harder, with synchronized or with staggered regional and national terms of office? In practice, we observe either synchronized (Indonesia, Scotland) or staggered (Brazil, Germany, Australia) elections but the topic of concurrent vs. non-concurrent elections has received little attention for economic purposes although it has attracted the interest of political scientists⁶. The arguments in favor of one or the other system are usually based on cost considerations and on the question of voters’ motivation, but economic arguments are often missing in the

⁴There are exceptions which show that soft budget constraints may diminish inefficiencies arising from hard budget constraints (see Besançon and Lockwood, 2008).

⁵One exception is Goodspeed (2002) who considers voting in his bailout model by including exogenous re-election probabilities. However, in contrast to our paper, he only considers one (two-term) office period and therefore no interaction between current and future governments exists.

⁶In particular, the political sciences analysis has investigated how the timing of elections interacts with different political variables such as the emergence of divided governments in presidential systems (Soergel Shugart, 1995), accountability of politicians (Samuels, 2004) or voter turnout (Hanjal and Lewis, 2003).
debate. So, in contrast to the existing literature, we focus not on political but on economic variables such as the size of federal transfers, the amount of taxes raised and the magnitude of public and private consumption.

To answer our question we consider a simple model with $T$ periods. An term of office for both the federal and the local governments lasts for two periods. Local and federal office terms can either be synchronized (SY) or staggered (ST). As a synchronized office term regime we define a model setup where elections and therefore the terms of office for regional and federal governments coincide. On the contrary, a staggered office term setting is a situation where regional elections take place just in the middle of the central government office term. Therefore the terms of office of old and new federal and regional governments overlap. Finally, we consider the situation of one poor region in the federation which faces financial difficulties and therefore is eligible for a federal bailout. In our model, we consider that the soft budget constraint phenomenon is exogenously given and we focus our analysis on the influence of the electoral system on the local incentives to manipulate the access to federal bailouts.

Interestingly, we find that the staggered term of office setting clearly dominates the synchronized term of office setting. The intuition for this finding is that in the staggered term setting the central government obtains a first mover advantage vis-à-vis the regional government entering next. This advantage is not at work for synchronized elections since bailouts are always chosen once all other economic variables are decided. With staggered elections, the first mover advantage enables the federal government to anticipate the strategic behavior of the new regional government, making it relatively easier to limit the bailouts. Budget constraints are hardened by allocating less funds to the second half of the term of office and using them to improve the allocation in the first half, where the old regional government can no longer strategically respond to actions of the central government because it has already made its tax and expenditure decisions in the previous period.

When political business cycles\textsuperscript{7} at both governmental levels are introduced, it is no longer clear which system dominates. This is driven by two effects. On the one hand,

\textsuperscript{7}The existence of political business cycle implies that governments favor the private and public consumptions in periods before election in order to be reelected.
the incentive of the central government to harden budget constraints in the second half of the term is reduced by the counteracting incentive to please voters before the election. Therefore it spends less funds to improve the allocation in the first half of the term, which makes the asymmetric system less efficient. We refer to this effect as the ‘financing bias’. On the other hand, the synchronized term system also becomes less efficient because both central and regional governments are inclined to increase spending in the second period of the term in order to please voters. We refer to this effect as the ‘spending bias’. The final outcome depends on which of the two effects dominates.

Our analysis contributes to both the literatures of soft budget constraints and elections. The soft budget constraint concept was introduced by Janos Kornai (1979, 1980) in the context of socialist enterprises which got losses reimbursed by the state. Thereafter, this concept became perfectly suitable to analyze the consequences of decentralization and more precisely the interactions between several tiers of governments (for a literature overview see Kornai et al. 2003). Soft budget constraints are difficult to avoid and are associated with major incentive problems regarding the accountability of regional governments in terms of fiscal discipline. Therefore one important area of public economic research asked the question: how is the softness of budget constraints affected by different characteristics of federations such as the size of regions (Wiklasin, 1997 and Crivelli and Staal, 2006), the type of fiscal equalization system or the intensity of tax competition (Qian and Roland, 1998 and Breuillé et. al, 2006 , Koethenbuerger, 2007)? We go further in the question by introducing political economic arguments generally developed in political science. In addition, we introduce another political variable, the political business cycles, which were pointed out by the pioneering paper of Nordhaus (1975). Since his work, the theoretical economic literature dealt to an important extent with the endogenous derivation of the political business cycles and the reasons for their emergence, for instance information asymmetries between voters and incumbents (Rogoff and Sibert, 1988), preferences polarization between current and future voters (Tabellini and Alesina, 1990), the incumbent’s concern with her welfare in case of electoral defeat (Baleiras, 1997) or centralization vs. decentralization (Gonzales et al., 2006). On the other hand, the empirical literature sought to find
political business cycles in practice. The empirical results are mixed. Recent findings (Zhuravskaya, 2004) suggest that less mature economies tend to be more prone to the occurrence of political business cycles than the more mature ones. In contrast, we do not deal with the causes but rather with the consequences of political business cycles. In our analysis we take political business cycles as exogenously given and trade the inefficiencies arising from their existence against the inefficiencies arising from soft budget problems.

The remainder of the chapter is organized as follows. Section 1.2 presents the basic model setup and Section 1.3 introduces a benchmark case, where a social planner makes efficient taxation and expenditure decisions. In Section 1.4 we introduce a decentralized setup, where regional governments decide on regional taxation and expenditures and the central government is responsible for national public good provision. In addition, the central government has the opportunity to supplement regional public good provision through a bailout. In Section 1.4.1 we analyze synchronized office terms (SY) and in Section 1.4.2 staggered office terms (ST). Section 1.5 analyses the effects of the introduction of the political business cycle on the previous results. All cases are evaluated with regard to the welfare in Section 1.6 and Section 1.7 concludes.

1.2 The Model

We consider a model of a federation with one poor region, eligible for a bailout and rich regions, which are by assumption never eligible for a bailout. The population of the poor region is normalized to one, and the population of the rich regions is normalized to $N, N > 0$, such that we have a total population of $(N + 1)$.

Consumers At each date $t$, consumers in each region have an initial endowment of $w$ and derive utility from a private good $c_t$, a regional public good $g_t$ and a national public good $G_t$. The payoffs of consumers at a given date $t$ are modeled according to a log-linear utility function: $c_t + \gamma_g \ln g_t + \gamma_G \ln G_t$. 
Governments Governments are assumed to hold political power for one office term, which is divided into two sub-periods, the "post-electoral period" and the "pre-electoral period".

Regional Governments At the beginning of each office term, regional governments choose their tax and expenditure policy. For reasons of simplicity, we abstract from distortionary taxation and allow regional governments to choose a regional lump sum tax $\tau^R_t$ as a revenue raising instrument. The revenue collected can either be used for expenditures in the post-electoral period ($s_t$) or in the pre-electoral period ($s_{t+1}$). At the end of the term, the budget has to be balanced: $\tau^R_t = s_t + s_{t+1}$. While for rich regions, regional public consumption $g_t$ is solely financed by regional spending, the poor region might in addition receive a bailout $z_t > 0$ as a subsidy to regional expenditures, such that the regional budget constraint is represented by $g_t = s_t + z_t, \forall t$. The payoff of a regional official in one office term is represented by the following utility function:

$$c_t + \gamma_g \ln g_t + \gamma_G \ln G_t + c_{t+1} + \gamma_g \ln g_{t+1} + \gamma_G \ln G_{t+1}$$  \hspace{1cm} (1.1)

The separability of utility from regional, national and private consumption along with the quasi linearity assumption implies that optimal tax choices of the rich regions are completely independent from tax choices of the poor region.

Central government The central government has an own non-manipulable head tax $\tau^C_t$, which is financed from the initial endowment $w$ of consumers and is collected at the beginning of the central government’s office term from each resident of the federation. The total revenue $(1 + N) \tau^C_t$ can be used for post-electoral central government spending $S_t$ or pre-electoral spending $S_{t+1}$. In addition, in each period the central government has to decide how to split up total spending among the national public good $G_t$ and a bailout to the poor region $z_t$. The central government has to balance the budget over the whole term, i.e. $(1 + N) \tau^C_t = S_t + S_{t+1}$, as well as in each period.

\footnote{We argue that the governments decide at the beginning of their office term the level of their instruments that will enables them to implement their political program for the whole office term. In that sense, we consider that governments do not deviate from their political program overtime.}
i.e. \( S_t = G_t + z_t, \forall t. \)

In order to make the problem interesting we have to make a technical assumption regarding the exogenously given central government head tax. We assume that the revenue collected from the head tax is on the one hand large enough that a soft budget problem arises and on the other hand small enough that it is not possible that all activity of the poor region is fully financed by central government grants in all periods, i.e. \( g_t + g_{t+1} < z_t + z_{t+1} \) and \( z_t > 0, \forall t. \) This assumption is technically necessary to obtain an interior solution for the soft budget problem and to avoid corner solutions. The central government objective function is defined as:

\[
c_t + \gamma_g \ln g_t + (1 + N) \gamma_G \ln G_t + c_{t+1} + \gamma_g \ln g_{t+1} + (1 + N) \gamma_G \ln G_{t+1}
\]  \hspace{1cm} (1.2)

Since the private and the regional consumption of the rich regions is independent of the central governments actions, they are dropped from the central government’s objective function. However, this does not hold for the utility from the national public good, which is considered for all \((1 + N)\) inhabitants.

**Timing** In the synchronized office terms set-up, we face an infinitely repeated game, where at each date \( t + 2n, n \in \mathbb{Z} \) both a central and a regional government enter and stay for one office term, consisting of a post-electoral and a pre-electoral period. In the staggered office terms set-up there exists also an infinitely repeated game, where at each date \( t + 2n, n \in \mathbb{Z} \) a central government and at each date \(( t + 1) + 2n, n \in \mathbb{Z}\) a regional government enter. Regional and central governments decide when coming into office on the intertemporal distribution of spending, i.e. \( s_t, s_{t+1} \) and \( S_t, \) the level of \( \tau^R \) and \( S_{t+1} \) being derived directly from the choices of \( s_t, s_{t+1} \) and \( S_t. \) The bailouts \( z_t \) are chosen ex-post at each date \( t, \) after all other fiscal decisions have been made. The timing of each set-up is explained in more detail in the corresponding sections.

Before we move on to the construction of the synchronized and the staggered terms of office regimes, we first establish a benchmark case, where a social planner maximizes
the utility of all inhabitants over the whole office term.

1.3 Social Planner

The social planner program serves as an efficiency benchmark. The social planner solves the following problem,

$$\max_{s_t, s_{t+1}, z_t, z_{t+1}, S_t} c_t + \gamma_g \ln g_t + (1 + N) \gamma_G \ln G_t + \left( c_{t+1} + \gamma_g \ln g_{t+1} + (1 + N) \gamma_G \ln G_{t+1} \right)$$  \hspace{1cm} (1.3)

subject to the budget constraints:

$$c_t = w - \tau_t^R - \tau_t^C \hspace{1cm} c_{t+1} = w$$  \hspace{1cm} (1.4)

$$g_t = s_t + z_t \hspace{1cm} g_{t+1} = s_{t+1} + z_{t+1}$$  \hspace{1cm} (1.5)

$$G_t = S_t - z_t \hspace{1cm} G_{t+1} = S_{t+1} - z_{t+1}$$  \hspace{1cm} (1.6)

and the budget balancing constraints:

$$\tau_t^R = s_t + s_{t+1} \hspace{1cm} (1 + N) \tau_t^C = S_t + S_{t+1}$$  \hspace{1cm} (1.7)

From the first order conditions w.r.t. $s_t, s_{t+1}, z_t, z_{t+1}$ we obtain a unique solution for regional public consumption $g_t = g_{t+1} = \gamma_g$, for national public consumption: $G_t = G_{t+1} = (1 + N) \gamma_G$, for the regional tax rate $\tau_t^R = 2 \left( \gamma_g + (1 + N) \gamma_G \right) - (1 + N) \tau_t^C$ and for private consumption $c_t = w - 2 \left( \gamma_g + (1 + N) \gamma_G \right) + N \tau_t^C, c_{t+1} = w$. Through the budget balancing constraint $\tau_t^R = s_t + s_{t+1}$ the sum of optimal regional spending is determined. However, the social planner is indifferent between spending the given regional tax revenue in period $t$ or in period $t+1$, because transfers $z_t$ play as a second instrument which allows the provision of an optimal level of regional goods for each
given level of spending $0 \leq s_t \leq \tau^R_t$.

The sum of transfers over both office periods $(z_t + z_{t+1})$ equals $(1 + N) (\tau^C - 2\gamma_C)$. Because the social planner accounts for all externalities associated with the transfer, this level of transfers represents the ex-ante efficient amount. If the central government could commit, it would pay this amount no matter what tax rate the poor region chooses. However, in the subsequent set-ups we analyze a situation where the central government cannot commit to an efficient level of transfers and tax policy is decentralized. In a simple one period model, these assumptions would lead to an inefficiently low regional tax rate and to inefficiently high transfers for the poor region. The intuition for this result is that the poor region fails to account for the negative externalities associated with the transfers. In the following sections, we ask first if this result persists in the same fashion for different office term regimes (i.e. modified timings) and afterwards we analyze how the introduction of political business cycles modifies our results.

1.4 Decentralized Setup

In this section we introduce a decentralized set-up where regional governments choose the regional tax rate as well as the regional expenditures whereas the central government chooses both the level of national expenditures and the amount of transfers granted to the poor region.

1.4.1 Synchronized Office Terms

In the synchronized set-up both a new regional and a new federal government enter into office at the beginning of each term and decide simultaneously on their expenditure policy. Additionally, at the end of each period the central government decides on the level of transfers. This sequence of events is repeated infinitely.\footnote{The ex-post transfer yields an identical outcome as if an ex-ante efficient transfer is calculated which is topped up ex-post to an inefficiently high amount.} Figure 1.1 summarizes the timing of decisions. Given that new officials enter at the beginning of each term, there are no strategic interactions across terms and it is therefore sufficient to solve the
game for one office term.

\[
\begin{array}{cccc}
\text{post-electoral} & \text{pre-electoral} & \text{post-electoral} & \text{pre-electoral} \\
\text{CG} & & & \\
S_0S_{t+1} & z_t & S_{t+1} & z_{t+1} \\
t & t+1 & s_{t+2}S_{t+3} & z_{t+2} \\
t & t+1 & & t+3 \\
\end{array}
\]

Figure 1.1: Timing in the SY Regime

The basic structure of this game is identical to a standard two-stage bailout game with decentralized leadership of the regional government, which can be solved by backward induction.

At stage 2, the central government maximizes its objective function (1.2) w.r.t. \( z_t \) and \( z_{t+1} \) subject to the budget constraints (1.4) – (1.7). The solution to this problem is the following bailout scheme:

\[
z_t = \frac{\gamma_g S_t - (1 + N) \gamma_G s_t}{(1 + N) \gamma_G + \gamma_g} \quad \forall t
\]  

(1.8)

The central government is restricted in its actions in the sense that it is just allowed to increase regional revenue through bailouts, but not to reduce it, i.e. \( z_t \geq 0 \). Therefore the solution is an interior solution to a Kuhn-Tucker problem. The transfer function illustrates as well that the regional budget constraint is soft in the sense that it is optimal for the central government to compensate a reduced regional government spending with an increase of transfers \( \frac{dG}{dz_t} < 0 \). The transfer is chosen such that the preferences of the central government on the distribution of public funds across the regional and the national public good are realized. As it is typical for Cobb-Douglas type utility functions, for each type of public good a constant share of revenue is spent.
This becomes obvious when the transfer (1.8) is plugged into regional and national public consumption: 
\[ g_t = \frac{\gamma_g}{1+N\gamma_G+\gamma_g} (S_t + s_t) \] 
and 
\[ G_t = \frac{(1+N)\gamma_g}{(1+N)\gamma_G+\gamma_g} (S_t + s_t). \]

At stage 1, a Nash game between the central and the regional government is played when deciding on the expenditure policy for the following office term. While the regional government maximizes the utility of its own residents when maximizing (1.1) w.r.t. \( s_t \) and \( s_{t+1} \), the central government cares for the utility of all \((1+N)\) residents by maximizing (1.2) w.r.t. \( S_t \) (and \( S_{t+1} \) via the budget balancing constraint (1.7)), subject to all budget constraints. We obtain the following response functions for the central and the regional governments:

\[ S_t = \frac{(s_{t+1} - s_t) + (1+N) \tau_C}{2} \]
\[ s_t + s_{t+1} = (\gamma_g + \gamma_G) 2 - (1+N) \tau_C \]

The best response function of the central government (1.9) shows the intertemporal preferences of the central government. The adjustment of \( S_t \) according to (1.9) insures that a constant fraction of funds is spent in the post electoral period and in the pre-electoral period for each combination of \( s_t \) and \( s_{t+1} \) chosen by the regional government.

While the regional government’s best response function (1.10) is sensitive to the sum of central government spending \( S_t + S_{t+1} = (1+N) \tau_C \), it is insensitive to the intertemporal distribution of central government funds, \( i.e. \) to \( S_t \).

The intuition for the results can be summarized as a ‘distribution effect’ and a ‘level effect’. The distribution effect implies that the central government is decisive when it comes to the distribution of public funds through the stage 2 reaction function (1.8) as well as the stage 1 best response function (1.9). For the distribution of a given amount of funds between the regional and the national public good this is true because the central government has the final decision power by granting bailouts ex-post\(^{10}\). However, the regional government is decisive with respect to the level effect,

\[^{10}\text{Note that there is no conflict of interest between the central and the regional governments on the intertemporal distribution of revenue because both governments prefer half of the funds in period } t \text{ and half of the funds in period } t+1.\]
i.e. for the level of total funds available for public consumption. This follows directly from the assumption that the central government has an exogenously given amount of tax revenue which it cannot manipulate. The regional government clearly prefers a lower amount of spending than the central government would do if it could choose the regional lump sum tax. By reducing its tax rate, the regional government benefits because it receives in turn a bailout whose costs are borne by all the residents of the federation. This negative \textbf{interregional externality} of the bailout is characterized by a decrease of utility from national public consumption by the $N$ outsiders, which the region does not take into account.

Solving for consumption variables yields the following results for private and public consumptions:

$$c_t = w + N \tau^C - 2 \left( \gamma_g + \gamma_G \right) \quad c_{t+1} = w$$

$$g_t = \gamma_g \frac{\gamma_g + \gamma_G}{\gamma_g + (1 + N) \gamma_G} \quad g_{t+1} = \gamma_g \frac{\gamma_g + \gamma_G}{\gamma_g + (1 + N) \gamma_G}$$

$$G_t = (1 + N) \gamma_G \frac{\gamma_g + \gamma_G}{\gamma_g + (1 + N) \gamma_G} \quad G_{t+1} = (1 + N) \gamma_G \frac{\gamma_g + \gamma_G}{\gamma_g + (1 + N) \gamma_G}$$

If there were no residents outside the poor region ($N = 0$), \( \frac{\gamma_g + \gamma_G}{\gamma_g + (1 + N) \gamma_G} \) would equal one and the preferences of the regional and the central governments would fit with the social planner’s choices. $N > 0$, introduces interregional externalities that let the regional and national public goods be underprovided since the negative externalities of raising too little revenue are not taken into account by the regional government. The following proposition summarizes the implications for the level of transfers.

\textbf{Proposition 1} A decentralized leadership with synchronized office terms is associated with a higher level of transfers than the social planner set-up.

\textbf{Proof.} In the social planner solution we have:

$$(z_t + z_{t+1})^{SP} = (1 + N) \left( \tau^C - 2 \gamma_G \right)$$

whereas for the synchronized office terms solution of the decentralized leadership we
obtain:

\[(z_t + z_{t+1})^{SY} = (1 + N) \left( x^G - 2 \gamma_G \frac{(\gamma_g + \gamma_G)}{(1 + N) \gamma_G + \gamma_g} \right)\]

and clearly

\[(z_t + z_{t+1})^{SP} < (z_t + z_{t+1})^{SY} \]

This result is in line with the existing literature (e.g. Goodspeed (2002), Wildasin (1997)) and shows that the decentralized leadership, along with a lack of commitment of the federal government when deciding on the level of transfers, creates a soft budget constraint mechanism (equation (1.8)).

### 1.4.2 Staggered Office Terms

The staggered office terms set-up characterizes a federation with decentralized regional spending and taxation, where at each date \( t + 2n, n \in \mathbb{Z} \) a new central government enters office and at each date \( (t + 1) + 2n, n \in \mathbb{Z} \) new regional governments come into office. So, it differs from the synchronized elections game by the feature that regional and central election dates do not coincide, but fall on different dates (see Figure 1.2). This has first of all implications for the sequence of events and outcomes.

![Figure 1.2: Timing in the ST Regime](image)

The central government entering office at date \( t \) has to take the regional spending
decision \( s_t \) as given because it has been chosen at date \((t - 1)\). So, the central government’s position in the post-electoral period becomes weaker in the sense that its spending decision is no longer set simultaneously with the regional government. Instead, the regional government now obtains a first mover advantage. In contrast, the position of the central government at date \((t + 1)\) is strengthened \textit{vis-à-vis} the regional government as it can choose spending in the pre-electoral period \( S_{t+1} \) before the regional government chooses \( s_{t+1} \). Similarly, the position of the regional government in its post-electoral period is weakened and in its pre-electoral period strengthened. How does this affect outcomes?

We solve this game by backward induction. Given that bailouts are always granted ex-post after all other spending decisions have been made, the optimal bailout scheme, is identical to the bailout scheme in the synchronized set-up (1.8). Once the bailout scheme \( z_t(s_t, S_t), \forall t \) is determined, it can be plugged into the public consumption variables at all dates and the problem of the staggered elections set up reduces to the choice of optimal spending for all dates. Although the maximization problems of both governments remain the same, the budget constraints for private consumption (1.4) and the balanced budget constraint (1.7) change to:

\[
\begin{align*}
    c_t &= w - \tau^C_t \\
    c_{t+1} &= w - \tau^R_{t+1} \\
    \tau^R_{t+1} &= s_{t+1} + s_{t+2} \\
    (1 + N)^C &= S_t + S_{t+1}
\end{align*}
\]

This holds for all dates \( t + 2n, n \in \mathbb{Z} \) and \( (t + 1) + 2n, n \in \mathbb{Z} \) respectively. The other budget constraints stay unchanged.

We start with the calculation of the reaction functions of the regional government. The quasilinear structure of the utility function makes regional spending decisions for the post-electoral period, e.g. \( s_{t+1} \) and the pre-electoral period, e.g. \( s_{t+2} \) independent from each other. While \( s_{t+1} \) depends only on \( S_{t+1} \) (chosen by the central government which entered at date \( t \)), the choice of \( s_{t+2} \) depends on the anticipated behavior of the central government entering in the following period \( t + 2 \). Therefore, we determine in
a first step the choice of \( s_{t+1} \), then we determine central government responses \( \frac{\partial s_{t+2}}{\partial n_{t+2}} \) and in a third step, we solve for \( s_{t+2} \).

When choosing \( s_{t+1} \), the regional government maximizes the objective function:

\[
c_{t+1} + g \ln g_{t+1} + g_G \ln G_{t+1} + \left( c_{t+2} + g \ln g_{t+2} + g_G \ln G_{t+2} \right) \quad (1.11)
\]

with respect to \( s_{t+1} \) taking into account the bailout scheme (1.8) and the budget constraints (4a), (1.5), (1.7) and (7a). We obtain the following first order condition:

\[
\frac{\gamma_g}{g_{t+1}} \frac{\partial g_{t+1}}{\partial s_{t+1}} + \frac{\gamma_G}{G_{t+1}} \frac{\partial G_{t+1}}{\partial s_{t+1}} = 1 \quad \text{where} \quad \frac{\gamma_g}{g_{t+2}} \frac{\partial g_{t+2}}{\partial s_{t+1}} + \frac{\gamma_G}{G_{t+2}} \frac{\partial G_{t+2}}{\partial s_{t+1}} = \frac{\gamma_g + \gamma_G}{S_{t+1} + s_{t+1}} \quad (1.12)
\]

The left hand side shows the marginal benefits of an increase in post-electoral spending, taking the responses \( \frac{\partial s_{t+1}}{\partial s_{t+1}} \) into account, whereas the right hand side shows the marginal costs, which are constantly one due to the quasilinear consumption. Like in the synchronized elections setting, the regional government disregards the costs of forgone national public good consumption for the \( N \) citizens outside its territory. The response function derived from (1.12) : \( s_{t+1} = \left( \gamma_g + \gamma_G \right) - S_{t+1} \) illustrates that all attempts of the central government to increase public funds in period \( t + 1 \) are counteracted by a reduction of \( s_{t+1} \).

Having obtained this response function, we can move on to solve the central government problem at date \( t \), which is equal to maximizing (1.2) w.r.t. \( S_t \), taking into account response functions (1.8), (1.12) as well as the budget constraints (4a), (1.5), (1.7) and (7a). The solution to this problem is characterized by the first order condition for \( S_t \):

\[
\frac{(1+N) \gamma_G}{G_t} = 1 \quad \text{with} \quad G_t = \frac{(S_t + s_t)(1+N) \gamma_G}{\left( \gamma_g + (1+N) \gamma_G \right)} \quad \forall t + 2n, n \in \mathbb{Z} \quad (1.13)
\]

The LHS of the condition represents the marginal benefit of an increase of \( S_t \) which
equals the marginal benefit of public consumption (the benefits of regional and national public consumption are equalized through the bailout scheme). The RHS represents the marginal costs, which are equal to one. This results from the intertemporal link between $S_t$ and $S_{t+1}$ through the balanced budget constraint and the response behavior of the regional government $\frac{\partial s_{t+1}}{\partial S_{t+1}} = -1$ according to (1.12). Rearranging (1.13) to the explicit response function $S_t = (\gamma_g + (1 + N) \gamma_G) - s_t$ makes it obvious that any attempt of the regional government to increase public funds for period $t$ is fully counteracted by the central government through a decrease of $S_t$. The reaction functions (1.12) and (1.13) imply that the government entering office in a given period is decisive for the amount of public funds available in this period.

The central government response function (1.13) enables us to solve as a final step, the regional maximization problem w.r.t. $s_{t+2}$. The marginal costs of raising $s_{t+2}$ are constantly equal to one, because of the quasilinear utility structure. However, the benefits are zero because from the central government response function (1.13) it follows that $\frac{\partial b_{t+2}}{\partial s_{t+2}} = 0$ as well as $\frac{\partial G_{t+2}}{\partial s_{t+2}} = 0$. Therefore it is optimal for the region to not spend any funds in its pre-electoral period ($s_{t+2} = 0$).

Solving for consumption variables yields the following results:

$$c_t = w - \tau^C_c$$

$$c_{t+1} = w - \left(2(\gamma_g + \gamma_G) + N + \gamma_G - (1 + N)\tau^C_c\right)$$

(1.14)

$$g_t = \gamma_g$$

$$g_{t+1} = \frac{(\gamma_g + \gamma_G)\gamma_g}{\gamma_g + (1 + N)\gamma_G}$$

(1.15)

$$G_t = (1 + N)\gamma_G$$

$$G_{t+1} = \frac{(\gamma_g + \gamma_G) (1 + N)\gamma_G}{\gamma_g + (1 + N)\gamma_G}$$

(1.16)

Intuitively our findings can be explained as follows.

In periods where the central government enters, e.g. in period $t$, the central government has a weak commitment position as it moves second vis-à-vis the regional government. The latter is therefore inclined to a strong moral hazard behavior which
is illustrated by zero own contributions to regional public consumption, i.e. \( s_t = 0 \). This result seems to be extreme given that the sequence of events is similar to the synchronized office term set-up, where \( s_t \) is set first and \( z_t \) afterwards. The reason is that in addition to the interregional externality, an "intertemporal externality" is in place. This externality becomes effective via the central government budget balancing constraint which decreases the spending in period \( t + 1 \) if period \( t \) spending \( S_t \) increases. However, the regional government deciding at date \((t - 1)\) on the level of \( s_t \) is no longer in office at date \( t + 1 \) and does not consider the costs of reduced public consumption in period \( t + 1 \).

Faced with a situation of zero regional spending, what is the optimal level of central government spending at date \( t \)? Since the regional government has made all decisions for period \( t \) at date \((t - 1)\) it can no longer respond strategically to the central government’s date \( t \) choices. Therefore the benefits of period \( t \) national public spending coincide with the benefits of the social planner (who considers, like the central government, the utility of all regions). The costs of spending can be determined through the regional government response function \( \frac{ds_{t+1}}{ds_t} = 1 \). Each increase of period \( t \) national spending involves a reduction of period \((t + 1)\) national spending, which in turn forces the regional government entering at date \((t + 1)\) to finance a larger share of regional consumption by itself. This reduction is evaluated by the central government at factor 1, which also coincides with the costs faced by the social planner. Since neither the costs nor the benefits of national public spending are distorted, the central government chooses an efficient amount of regional and national public consumption in the period when it enters office (at date \( t \)). Note that, despite of the strong regional government response \( \frac{ds_{t+1}}{ds_{t+1}} = -1 \), it is still optimal for the central government to shift funds to date \((t + 1)\) because national public good provision would break down otherwise, while when the region sets \( s_t = 0 \), regional public provision would not break down because of the bailouts.

In periods where the regional government enters, e.g. at date \((t + 1)\), it has to cope with an earlier fixed amount of central government funds \( S_t+1 \). Given this amount, it chooses its level of spending such that it can implement its preferred level of public consumption in the presence of the interregional externality. Although the outcome is
identical to the post-electoral period outcome of the synchronized office terms setting, the contribution by the region to finance this outcome is generally different. Because of the first mover advantage the central government commits now to set \( S_{t+1} \) significantly lower than in the synchronized elections setting, which forces the regional government to finance a higher amount of public spending in its first period of office.

We can summarize that in every second period \((t + 2n, n \in \mathbb{Z})\) public consumption coincides with the social planner outcome and in every other period \((t + 1 + 2n, n \in \mathbb{Z})\) it coincides with the synchronized elections outcome.

We are now able to make a statement on how the timing of elections affects the softness of budget constraints:

**Proposition 2** A decentralized leadership with staggered office terms is associated with a higher level of transfers than in the social planner set-up, but with a lower level of transfers than in the synchronized elections set-up.

**Proof.** In the social planner solution we have:

\[
(z_t + z_{t+1})^{SP} = (1 + N) \left( \tau^C - 2\gamma_G \right)
\]

whereas for the synchronized office terms solution we obtain:

\[
(z_t + z_{t+1})^{SY} = (1 + N) \left( \tau^C - 2\gamma_G \frac{(\gamma_g + \gamma_G)}{(1 + N) \gamma_G + \gamma_g} \right)
\]

and for the staggered office terms solution transfers are given by:

\[
(z_t + z_{t+1})^{ST} = (1 + N) \left( \tau^C - 2\gamma_G \frac{(\gamma_g + \gamma_G + \frac{N}{2} \gamma_G)}{(1 + N) \gamma_G + \gamma_g} \right)
\]

which implies

\[
(z_t + z_{t+1})^{SP} < (z_t + z_{t+1})^{ST} < (z_t + z_{t+1})^{SY}
\]
From the clarity of this result the question arises: Why does the central government benefits more from its first mover advantage than the regional government? There are two reasons. On the one hand in the staggered regime, the regional government extracts in its first period a higher amount of transfers than it would do in the synchronized regime. It ignores that a high level of transfers today restricts the financial scope of the central government to give bailouts in the next period, because it disregards the utility of future regional governments. On the other hand, the central government has an incentive to provide in its first office period a higher bailout than it would do in the synchronized regime and commits thereby to a very low bailout in its second office period. Why? Suppose, in the synchronized regime that the regional government would choose to spend all its revenue in the second office period. Given that all regional choices are already made, the central government could do no better than using the bailouts for expenditure smoothing, i.e. to provide a large bailout of \( g^{SP} = \gamma_y \frac{\gamma_y + \gamma_G}{\gamma_y + (1 + N) \gamma_G} \) in the first period and a small bailout of \( (g^{SY} - \tau_S^{SY}) \) in the second period. In the staggered regime this is not optimal anymore. The central government can now anticipate the strategic behavior of the next regional government. It knows that each additional unit of revenues in its pre electoral period incites the future regional government to raise one unit less of revenue. Being unable, apart from providing national public consumption, to beneficially employ public funds in its pre electoral period, it becomes optimal to spend a higher amount of funds \( g^{SP}_i = \gamma_y > g^{SY}_i \) for regional public consumption and \( G^{SP}_i = (1 + N) \gamma_G > G^{SY}_i \) for national public consumption in its post electoral period. This forces the future regional government to raise an additional amount of \( (G^{SP}_i - G^{SY}_i) + (g^{SP}_i - g^{SY}_i) \) by itself. This higher regional government contribution explains the result.

1.5 Decentralized Setup with Political Cycles

In this section, we assume that governments are subject to political business cycles, i.e. they may value private and public consumption higher in the pre-electoral period. Technically, the presence of a political business cycle is taken as exogenously given and
modeled by weighting utility of the pre-electoral period with a factor $\pi > 1$\footnote{When no political business cycle exists, we have $\pi = 1$.}.

The objective of both the central and local governments writes now respectively as:

\begin{equation}
\begin{aligned}
c_t + \gamma_g \ln g_t + (1 + N) \gamma_G \ln G_t + \pi \left(c_{t+1} + \gamma_g \ln g_{t+1} + (1 + N) \gamma_G \ln G_{t+1}\right)
\end{aligned}
\end{equation}

\begin{equation}
\begin{aligned}
c_t + \gamma_g \ln g_t + \gamma_G \ln G_t + \pi \left(c_{t+1} + \gamma_g \ln g_{t+1} + \gamma_G \ln G_{t+1}\right)
\end{aligned}
\end{equation}

with $\pi > 1$.

We use this simple specification because we are not interested in endogenously deriving political business cycles, but rather interested in the impact of these cycles on the softness of budget constraints. The structure of the games and the way to solve them are the same as in section 1.4. Therefore, in this section, we only focus on the modifications of the results and their interpretation.

### 1.5.1 Synchronized Office Terms

Solving the game described in Section 1.4.1 for the new objective functions of both governments yields the following results:

\begin{equation}
\begin{aligned}
c_t = w + N \pi^C - \left(\gamma_g + \gamma_G\right) (1 + \pi) \quad c_{t+1} = w
\end{aligned}
\end{equation}

\begin{equation}
\begin{aligned}
g_t = \gamma_g \frac{\gamma_g + \gamma_G}{\gamma_g + (1 + N) \gamma_G} \quad g_{t+1} = \pi \gamma_g \frac{\gamma_g + \gamma_G}{\gamma_g + (1 + N) \gamma_G}
\end{aligned}
\end{equation}

\begin{equation}
\begin{aligned}
G_t = (1 + N) \gamma_G \frac{\gamma_g + \gamma_G}{\gamma_g + (1 + N) \gamma_G} \quad G_{t+1} = \pi (1 + N) \gamma_G \frac{\gamma_g + \gamma_G}{\gamma_g + (1 + N) \gamma_G}
\end{aligned}
\end{equation}

Compared to the setting without political cycles, we can observe that public consumption in the post-election period stays the same, whereas it increases in the pre-election period by factor $\pi$. This increase is caused by the wish of the central and
regional governments to please voters before the election. We refer to this effect in the sequel as ‘spending bias’.

Since the basic structure of this game only differs by the presence of political business cycles, the distribution effect and the level effect are still at work. Solving at stage 1, the Nash game between the central and the regional government yields the following response functions for the central and the regional governments:

\[ S_t = \frac{(s_{t+1} - \pi s_t) + (1 + N) \tau^C}{(1 + \pi)} \]  
\[ \tau^R = s_t + s_{t+1} = (\gamma_g + \gamma_G) (1 + \pi) - (S_t + S_{t+1}) \]

The best response function of the central government (1.19) shows the intertemporal preferences of the central government. Over all available public funds, it prefers a fraction \( \frac{1}{1+\pi} \) to be spent in the post-election period \( t \) and a fraction \( \frac{\pi}{1+\pi} \) in the pre-election period \( (t + 1) \). We can also observe the influence of the political business cycle in the regional government’s best response function. While it previously adjusted its expenditures such as to obtain total public expenditures of size \( 2(\gamma_g + \gamma_G) \), it now prefers a higher level of \( (1 + \pi)(\gamma_g + \gamma_G) \) in order to have more funds available for pleasing voters before the elections.

Consequently the introduction of political cycles counteracts the interregional externality in the pre-electoral period. For \( \pi > \frac{(\gamma_g + (1+N)\gamma_G)}{(\gamma_g + \gamma_G)} \), we even observe overconsumption (compared to the first best) in the pre-electoral period. If both the interregional externality and the political business cycle are in place \( (N > 0; \pi > 1) \), we always have underconsumption of public goods in the post-electoral period and depending on parameter values either under- or overconsumption in the pre-electoral period. For the particular value of \( \pi^* = \frac{(\gamma_g + (1+N)\gamma_G)}{(\gamma_g + \gamma_G)} \), both effects just cancel out and pre-electoral consumption is efficient, while post-electoral consumption is inefficiently low. In the following proposition, we turn to the implications for the amount of transfers.

**Proposition 3** In the presence of political cycles, a decentralized leadership with synchronized office terms is associated with a higher level of transfers than in the social
planner set-up for $\pi < 2\pi^* - 1$ and with a lower level of transfers for $\pi > 2\pi^* - 1$. Moreover the introduction of political cycles into the synchronized office term set-up decreases the amount of transfers.

Proof. With the social planner we have

$$(z_t + z_{t+1})^{SP} = (1 + N) \left( \tau^C - 2\gamma_G \right)$$

The synchronized office term set-up for $\pi > 1$ yields:

$$(z_t + z_{t+1})^{SY} = (1 + N) \left( \tau^C - \gamma_G \frac{(1 + \pi)(\gamma_g + \gamma_G)}{(1 + N) \gamma_G + \gamma_g} \right)$$

The synchronized office term set-up for $\pi = 1$ yields:

$$(z_t + z_{t+1})^{SY}_{\pi=1} = (1 + N) \left( \tau^C - 2\gamma_G \frac{(\gamma_g + \gamma_G)}{(1 + N) \gamma_G + \gamma_g} \right)$$

It follows:

$$(z_t + z_{t+1})^{SP} < (z_t + z_{t+1})^{SY} \iff \pi < 2\pi^* - 1$$

$$(z_t + z_{t+1})^{SY} < (z_t + z_{t+1})^{SY}_{\pi=1} \quad \forall \pi$$

The intuition of this result is that the political business cycle gives regional politicians incentives to overspend before elections. In order to please the voters, the regional government is incited to increase its own tax rate to finance more regional public goods. This counteracts the SBC mechanism which incites the regional government to lower its own spending and head tax to finance public goods. As a result, the political business cycle alleviates the SBC bias before the election and the central government is able to provide fewer transfers.
1.5.2 Staggered Office Terms

We now consider the case where office terms are staggered. As we have seen without political business cycles, staggered office terms enable the central government on the one hand to obtain a stronger position compared to the regional government in the pre-electoral period. This tends to limit the SBC phenomenon. On the other hand, the position is weakened in the period after the central government’s election. In the absence of political business cycles this has improved the outcome compared to a setting of synchronized office terms. Does this conclusion still hold when a political business cycle is introduced?

Solving the game of Section 1.4.2 for \( \pi > 1 \), yields the following results:

\[
c_t = w - \tau^C \quad c_{t+1} = w - \left( (\gamma_g + \gamma_G) - (1 + N) \tau^C + \frac{\gamma_g + (1 + N) \gamma_G}{\pi} \right)
\]

\[
g_t = \frac{\gamma_g}{\pi} \quad g_{t+1} = \frac{(\gamma_g + \gamma_G) \gamma_g}{\gamma_g + (1 + N) \gamma_G}
\]

\[
G_t = \frac{(1 + N) \gamma_G}{\pi} \quad G_{t+1} = \frac{(\gamma_g + \gamma_G) (1 + N) \gamma_G}{\gamma_g + (1 + N) \gamma_G}
\]

How do the results of staggered office terms change if political business cycles are introduced?

First of all, the bailout scheme (1.8) stays unaffected by the political business cycles because it only serves as an instrument for the central government to optimally allocate funds across public goods within periods, whereas the introduction of political business cycles changes the preferences of governments on the intertemporal allocation of public funds.

Considering regional spending in periods where the regional government enters, e.g. \( s_{t+1} \), we can summarize that the costs and benefits of regional spending are identical to the setting without political cycles (see equation 1.12). This is because the valuation
of regional spending stays unchanged for the post-electoral office term.

In contrast, the regional government’s valuation of consumption in pre-electoral office terms is increased by factor $\pi$. However, because of its first mover position with respect to date $(t + 2)$ decisions, the regional government can anticipate via the central government response function $(13a)$, holding for $t, t + 2, t + 4$, etc., that each increase of regional spending is fully counteracted by the central government by an equal reduction of central government funds. This implies that $\frac{dq_{t+2}}{ds_{t+2}} = 0$. Therefore it is still not optimal for the regional government to raise funds for the second office term, although the political bias is in place, i.e. $s_{t+2} = 0$.

For period $t$ we have:

$$S_t = \frac{\left(\gamma_g + (1 + N) \gamma_G\right)}{\pi} - s_t \quad \forall t + 2n, n \in \mathbb{Z} \quad (13a)$$

Equation $(13a)$ tells us how the central government makes its spending decisions. Compared to staggered office terms without political cycles $(1.13)$ we can observe that the preferred level of national and regional spending in periods where the central government enters is reduced to a level of $\frac{1}{\pi} \left(\gamma_g + (1 + N) \gamma_G\right)$ instead of $\left(\gamma_g + (1 + N) \gamma_G\right)$. The reason for this reduction is that the central government’s valuation of period $(t+1)$ public spending is higher in the presence of political cycles. Now, withdrawing one additional unit of funds from period $(t+1)$ by increasing period $t$ spending costs $\pi$ units $(\pi > 1)$ instead of 1. We refer to this effect in the sequel as the ‘financing bias’: because the wish to please voters at date $(t + 1)$ makes it relatively more costly to finance period $t$ spending.

Summing up, in periods where the regional government enters, consumption is distorted downwards because of the interregional externality. This is identical to the staggered office term regime without political bias. However, in periods where the central government enters, the introduction of political business cycles gives rise to a financing bias which reduces public good provision also for the remaining terms, below the efficient amount.

We are now able to compare the transfers granted to the poor region for the different
Proposition 4. In the presence of political business cycles, a decentralized leadership with staggered office terms is associated with a higher level of transfers than in the social planner set-up. The introduction of political cycles into the staggered office term set-up generally increases the level of transfers.

Proof. The total amount of transfers in the social planner setting is given by

\[(z_t + z_{t+1})^{SP} = (1 + N) \left( \tau^C - 2\gamma_G \right)\]

The total amount of transfers in the staggered office terms solution is:

\[(z_t + z_{t+1})^{ST} = (1 + N) \left( \tau^C - \gamma_G \left( 1 + \pi \right) \left( \frac{\gamma_G + \gamma_G + N\gamma_G}{\gamma_G + (1 + N)\gamma_G} \right) \right)\]

If \(\pi = 1\), this reduces to:

\[(z_t + z_{t+1})^{ST}_{\pi=1} = (1 + N) \left( \tau^C - 2\gamma_G \left( \frac{\gamma_G + \gamma_G + \frac{N}{2}\gamma_G}{(1 + N)\gamma_G + \gamma_G} \right) \right)\]

then

\[(z_t + z_{t+1})^{SP} < (z_t + z_{t+1})^{ST}\]
\[(z_t + z_{t+1})^{ST} > (z_t + z_{t+1})^{ST}_{\pi=1} \quad \forall \pi\]

At this stage we can now compare the results for staggered office terms to the results for synchronized office terms when we take the effects of political business cycles into account.

In periods when regional governments enter, both regimes yield identical results: we observe too low public consumption because of the unconsidered interregional ex-
ternality. However, in periods where the central government enters we obtain different results for both regimes. While in the staggered office terms regime public consumption is distorted downwards because of the financing bias, public consumption in the synchronized office term regime may be too low or too high depending on the level of the spending bias and the interregional externality. What does this imply for the level of transfers and the softness of budget constraints?

**Proposition 5** Compared to the synchronized election case with political cycles, staggered elections reduce the level of bailouts granted by the federal government if the political business cycle is sufficiently strong \((\pi > \sqrt{\pi^*})\).

**Proof.** In the synchronized office terms case, the level of bailouts is given by:

\[
(z_t + z_{t+1})^{SY} = (1 + N) \left( \tau^C - \gamma_g \frac{(1 + \pi) (\gamma_g + \gamma_G)}{(1 + N) \gamma_G + \gamma_g} \right)
\]

and in the staggered office terms case by:

\[
(z_t + z_{t+1})^{ST} = (1 + N) \left( \tau^C - \gamma_g \frac{(1 + \pi) (\gamma_g + \gamma_G) + N \gamma_G}{\pi (\gamma_g + (1 + N) \gamma_G)} \right)
\]

Simple algebraic manipulation reveals that:

\[
(z_t + z_{t+1})^{ST} > (z_t + z_{t+1})^{SY} \iff \pi > \sqrt{\pi^*}
\]

Remember \(\pi^* = \frac{\gamma_g + \gamma_G (1 + N)}{\gamma_g + \gamma_G} \). ■

Proposition 5 shows that for sufficiently high values of \(\pi\) the staggered regime is associated with lower transfers than the synchronized regime. At the same time we know from the previous section that for \(\pi = 1\) the staggered regime transfers are clearly lower than in the synchronized regime. The reason for this result is that for very small values of \(\pi\) the interregional externality, distorting regional spending in the synchronized regime below the level of the staggered regime, dominates. As \(\pi\) increases, the spending bias, distorting regional spending upwards, counteracts the interregional externality in the synchronized regime and moves spending towards an efficient level.
As \( \pi \) further increases regional spending becomes inefficiently high in the synchronized regime.

At the same time the financing bias created by \( \pi \) and distorting regional spending downwards in the staggered regime becomes stronger as \( \pi \) increases. At the threshold value of \( \pi^* \) the latter effect begins to dominate, in the sense that regional spending becomes lower and transfers become higher in the staggered regime compared to the synchronized regime.

**Summary of inefficiencies** Summarizing the outcomes of all cases, we can state first, that in periods where regional governments enter office public consumption equals

\[
g_t^{ST} = g_{t+1}^{ST} = \frac{\gamma_g + \gamma_G }{\gamma_g (1+N)} \gamma_g \quad \text{and} \quad G_t^{SY} = G_{t+1}^{ST} = \frac{\gamma_g + \gamma_G }{\gamma_g (1+N)} (1 + N) \gamma_G
\]

The amount of public consumption is inefficiently low due to the presence of the inter-regional externality. The level of this externality grows with the level of \( N \), i.e. if \( N \) were equal to zero public consumption would be efficient.

Second, in periods of the staggered office term regime, when the central government enters, this interregional externality is absent and public consumption amounts to

\[
g_t^{ST} = \frac{\gamma_g}{\pi} \quad \text{and} \quad G_t^{ST} = \frac{(1+N)\gamma_G}{\pi}.\]

This is as well inefficiently low but for another reason: because the costs of public good provision are distorted upwards through the financing bias. The size of this bias grows with the level of \( \pi \), and when no political business cycle is at work, public consumption is efficient.

Third, in the pre-electoral periods of the synchronized set-up, public consumption equals

\[
g_t^{SY} = \frac{\gamma_g + \gamma_G }{\gamma_g (1+N)} \gamma_g \quad \text{and} \quad G_t^{SY} = \frac{\gamma_g + \gamma_G }{\gamma_g (1+N)} (1 + N) \gamma_G
\]

and may be depending on parameters either inefficiently high or low. Two effects are driving the result. On the one hand, a downwards bias is in place, caused by the interregional externality (represented by the \( N \)). On the other hand an upwards bias is at work, caused by the wish of politicians to please voters before the elections and represented by \( \pi \) (spending bias). Depending on which of the two effects dominates, we have under- or overprovision of public goods.
1.6 Welfare Analysis

In order to facilitate comprehensibility of our analysis, we present in the following a welfare analysis for the case without political business cycles before we come to the case with political business cycles.

No Political Business Cycle For the case without political business cycles, we are able to unambiguously rank the welfare of the synchronized and staggered office term regimes.

Proposition 6 In the absence of political business cycles, the staggered office term regime clearly dominates the synchronized office term regime and we have the following ranking between outcomes:

\[
\tau_{R,SP} > \tau_{R,ST} > \tau_{R,SY}
\]

\[
\begin{align*}
g_t^{SP} & = g_t^{ST} > g_t^{SY} & g_{t+1}^{SP} & = g_{t+1}^{ST} = g_{t+1}^{SY} \\
G_t^{SP} & = G_t^{ST} > G_t^{SY} & G_{t+1}^{SP} & = G_{t+1}^{ST} = G_{t+1}^{SY}
\end{align*}
\]

Proof. See Appendix 1  ■

We drop in this section the time index for regional the tax rate because the same tax rate is set in infinite repetition.

In the absence of both the financing and the spending bias, the staggered office terms outcome coincides in periods where regional governments enter office with the synchronized office terms outcome and in periods where the central government enters with the social planner outcome. Given that the staggered office terms regime is efficient half of the time and inefficient half of the time, while the synchronized office terms regime is inefficient all of the time, the staggered office terms regime clearly dominates.

Case of Political Business Cycle When governments are subject to political cycles, all externalities and biases are at work and the welfare comparison gives:
Proposition 7 When every externality and bias is at work, we obtain the following ranking

\[(U_t + U_{t+1})^{SY} - (U_t + U_{t+1})^{ST} \begin{align*}
< 0 & \text{ for } 1 \leq \pi < \bar{\pi} \\
\geq 0 & \text{ for } \pi \leq \bar{\pi} \leq \underline{\pi} \\
< 0 & \text{ for } \pi > \underline{\pi}
\end{align*}\]

with \(\bar{\pi} = \sqrt{\pi^*}\) and \(\underline{\pi} > \bar{\pi}\) implicitly defined by \(\ln \frac{\pi^2}{\pi^*} = \frac{1}{\pi} \left( \frac{\pi^2}{\pi^*} - 1 \right)\).

where \(\pi^* = \frac{\gamma_d + (1+N)\gamma_G}{\gamma_d + \gamma_G}\).

Proof. See Appendix 2 ■

To be able to understand the mechanism we start from the special case of \(\pi = 1\) (no political business cycle). For \(\pi = 1\) the interregional externalities are the only distortive effect at work and we have shown that the ST regime dominates the SY regime.

When \(\pi\) rises, the spending and the financing bias add to the interregional distortion. Now, the spending bias counteracts for small values of \(\pi\) to the interregional externality in the SY regime. This mechanism is not at work in the ST regime since no interregional externality is in place when the central government enters. Therefore for small values of \(\pi\), an increase of \(\pi\) relatively improves the welfare of the SY regime compared to the ST regime.

Figure 1.3 illustrates for which values of \(\pi\) and \(N\) which of the regimes dominates. Let’s assume a fixed \(N > 0\). From 1 to \(\bar{\pi}\), the ST regime still dominates but the welfare difference \(U^{ST} - U^{SY}\) diminishes as \(\pi\) increases, since distortions decrease in the SY regime due to the opposite effects between the spending bias and the interregional externality. At the same time distortions increase in the ST regime because of the financing bias. At a level of \(\underline{\pi}\) the SY regime begins to dominate the ST regime. However, when \(\pi\) exceeds the level of \(\pi^*\) the welfare enhancing effect of the spending bias vanishes and further increases of \(\pi\) increase the spending bias (at a rate of \(\pi\)), whereas the financing bias of the ST regime grows at a lower rate of \(1/\pi\). In sum this leads to a relatively higher decrease of the welfare of the SY regime compared to the ST
regime and at $\pi$ the ST regime becomes again dominant. To sum up, for $\pi < \pi < \bar{\pi}$, the SY regime dominates and for the remaining values of $\pi$ the ST regime dominates.

![Figure 1.3: Welfare Analysis](image)

When $N$ varies as well, the range where the SY regime dominates shifts upwards. According to our analysis, a rise in $N$ tends to increase the interregional externality and therefore the level of $\pi$ becomes higher until the spending distortion puts a counterweight onto the interregional distortion. This increases the interval where the SY regime dominates.

### 1.7 Conclusion

The most interesting result of our analysis is that in the absence of political business cycles a staggered office terms regime always increases welfare. This is due to the fact that central government is able to harden budget constraints at least half of the time by spending most of its funds efficiently in the first half of its term of office, where “old” regional governments cannot respond strategically. In the second half of the term only a few funds are left over and the central government prefers to spend these funds for national consumption instead of bailing out sub-national jurisdictions. In the synchronized office terms regime, this commitment effect is absent and soft budget
constraints occur in all of the periods.

In the presence of political business cycles this clear-cut conclusion disappears because the central government faces a trade-off between pleasing voters before elections and committing to hard budget constraints in the second half of its office term. On the one hand, this softens budget constraints before elections and makes the staggered office terms regime less efficient. On the other hand, incentives for political business cycles might improve the welfare of the synchronized office terms regime if the negative externalities caused by soft budget constraints are sufficiently large. In this case, the wish to please voters before the elections counteracts the incentive to exploit rich regions in the federation by eliciting bailouts through undertaxation at the regional level. Therefore the staggered office terms regime becomes particularly efficient if the size of incentives for political business cycles and for extracting bailouts is intermediate. For extreme cases, i.e. for large interregional externalities or for large incentives for political business cycles the staggered office term regime still dominates.

Contrary to the political science literature which provides arguments in favor of synchronized elections, our analysis makes a case for staggered elections. Our major policy implication is that central governments should take an active role in setting up the electoral timetable. They can increase the wellbeing of their citizens by implementing a system of staggered elections, because this system improves their ability to bind themselves to spend revenues for utility enhancing national public services instead of inefficient bailouts.
Chapter 2

Restricted Bailouts and the Soft Budget Constraint Problem in Federations

2.1 Introduction

Debt crises of subnational jurisdictions in federations have been frequently observed in the past. In many cases, higher level governments contributed to overcome these crises through bailouts, which were often linked to additional obligations like savings goals. For example, in response to a series of municipal defaults during the depression in Canada, the Ontario Municipal Board (1932), the Department of Municipal Affairs (1934) and the Windsor Finance Commission (1935) were founded. These boards restricted the actions of defaulting municipalities by prescribing refunding plans, by auditing, inspecting, approving and supervising municipal budgets or even by controlling certain expenditures (Bird and Tassonyi, 2003). Another example is Brazil, where in 1997 adjustment targets were prescribed by the Law 9496 as a condition for debt relief for the Brazilian states. For instance, these targets included scheduled declines in debt-revenue ratios, limits on personnel spending or ceilings on investments (Rodden, 2003). Certainly, one of the most famous examples is the Emergency Financial Control Board (EFCB) which controlled New York City’s government during the bailout
following the 1975 debt crisis. The board could control and reject the city’s financial planning, current and capital budgets, negotiated wage contracts as well as local borrowing. In case the city had not met certain requirements, the EFCB would have had the right to control all municipal accounts and to exercise disciplinary sanctions (Eichhorst and Kaiser, 2006).

Although these examples illustrate that bailout restrictions are prevalent in practice, they have so far not found the attention of the public economics theory. In this chapter, we analyze the incentive and welfare effects of bailout restrictions in a soft budget constraint framework. At the core of the soft budget constraint framework is a lack of commitment of a higher level government in a federation to bailout a lower level government in fiscal distress. One important justification for this commitment problem is a similarity of preferences between the higher and the lower level government. Both care about the public service provision for the citizens living in the jurisdiction in fiscal distress. Such services include inter alia the provision of schooling, medical services or public transportation. Bailout restrictions are interesting from a theoretical perspective because they involve counteracting preferences between the donor and the recipient of the bailout. For instance, the higher level government providing a bailout might prefer a high tax rate in the recipient region in order to enforce a local contribution to overcome the crisis, while the region is likely to prefer low taxes and to finance the deficit out of central government funds. Therefore, once the a subnational government in crisis asks for a bailout, a higher level government that is not able to commit to no bailout at all, nevertheless might be able to commit to a restricted bailout. This might alleviate the inefficiencies arising from soft budget constraints and increase welfare.

The public economic literature has intensely analyzed soft budget constraints in federations, but has not paid attention to bailout restrictions. The idea to the generic type of soft budget constraint models has been developed by Janos Kornai (1979, 1986), who investigated the incentives of socialist firms which were bailed out by the state when a deficit occurred. Later this concept has been applied to other areas of research like bank bailouts or bailouts to lower level governments in federations. The public economic literature has intensely analyzed characteristics of federations curbing or facilitating the emergence of soft budget constraints in federations. For instance,
Wildasin (1997) as well as Crivelli and Staal (2006) investigate whether large regions are ‘too big to fail’ when contemplating the relationship between the size of regions and the softness of budget constraints. Qian and Roland (1999) as well as Breuillé et al. (2006) ask whether tax competition hardens the regional budget constraint. The relationship between the number of levels in a federation and the soft budget constraint problem is analyzed by Breuillé and Vigneault (2008). Besamille and Lockwood (2008) investigate whether soft budget constraints may be more efficient than hard budget constraints. All these papers have in common that bailouts are pure monetary transfers. The contribution of our analysis is to explicitly consider additional restrictions, e.g. mandatory regional tax rates or expenditure savings goals, which the region has to fulfill in case of accepting a monetary bailout.

We consider a simple model of a federation with two regional governments and a central government. The regional governments may finance regional public goods through two channels. On the one hand, they may collect taxes and, on the other hand, they may obtain funds through expenditure savings that result from a more efficient public good provision. The funds saved through efficiency improvements, e.g. cheaper administration, could be used for other public goods or services. The effort spent on expenditure savings could be for example the investment of money to buy software that allows administering a public service more cheaply.

In the first place, we analyze three benchmark cases. First, we consider the case of full centralization, when the central government takes all decisions on its own. Since the central government takes into account the utility of all inhabitants, this case serves as the first best benchmark (FB). Second, we investigate the case of a hard budget constraint (HBC), i.e. a case where the bailouts are restricted to be zero. This case is important because we allow the regional governments in our further analysis to reject the restricted bailouts, which requires a benchmark for the outside option of the regional governments. Third, we analyze a pure soft budget case (SBC), where no restrictions are in place as a benchmark for a pure monetary bailout.

Having defined all benchmark cases, we move on to the analysis of restricted bailouts. We consider two regimes. In a partially restricted bailout regime (PB),
the central government is just allowed to restrict one of the regional revenue channels, i.e. either to prescribe a minimum effort on expenditure savings or a minimum tax rate. The fully restricted bailout regime (FRB) takes a more comprehensive approach and allows the central government to impose restrictions on both effort on expenditure savings and regional taxation. In each of the two settings the regional governments choose their unrestricted policy variables at the first stage whereas at the second stage, the central government offers a restricted bailout scheme, which the regional governments may accept or reject. We assume that the central government makes a take-it-or-leave-it-offer of the restricted or unrestricted bailout. In particular, in case the regional government rejects the offer, it cannot hope for a more attractive offer from the central government.

As expected, we show that a fully restricted bailout clearly improves welfare. However, the most striking result of our analysis is that from a welfare perspective, the partially restricted bailout regime might be worse than the unrestricted bailout regime. The intuition for this result is that the regions compensate for the restriction by distorting the unrestricted revenue instrument even more than in the case of no restrictions.

The remainder of the chapter is organized as follows. Section 2.2 presents the basic model set-up. Section 2.3 introduces three benchmark cases: Centralized decision making as a benchmark for the first best (FB) solution, the hard budget constraint (HBC) and an unrestricted bailout (UB) regime. Section 2.4 presents the results of the fully (FRB) and the partly restricted bailout (PB) regimes. All cases are evaluated with regard to their welfare implications in section 2.5. Section 2.6 concludes.

2.2 Model Set-Up

We consider a simple model of a federation with a central government and two regional governments \((i = 1, 2)\). Each region is inhabited by representative consumers with a population size normalized to one. The total size of the population is two.

The Representative consumers derive utility from private consumption \(c_i\), regional public consumption \(g_i\) and national public consumption \(G\). The consumers' preferences
are represented by the additively separable utility function \( u(c_i) + h(g_i) + J(G) \) which exhibits standard properties, i.e. concavity \( u', h', J' > 0, u'', h'', J'' < 0 \), monotonicity, continuous differentiability in all arguments and fulfills the Inada conditions \( u'(\infty), h'(\infty), J'(\infty) = 0 \). Representative consumers are endowed (after central government taxation) with identical income \( 0 < w < \infty \). The regional governments may tax this income at a proportional rate \( t_i \geq 0 \). The private budget constraint illustrates that the income after regional taxation is used for private consumption.

\[
c_i = w(1 - t_i)
\]  

(2.1)

The regional governments obtain revenue from regional taxation, central government grants \( z_i \) and through spending effort \( a_i \geq 0 \) on providing the regional public good more efficiently. The expenditure savings realized as a consequence of the effort \( a_i \) can be used to provide other public goods. For simplicity, we assume that the governments’ effort is translated linearly into revenue for the regional public good. In sum, regional consumption is financed through three sources.

\[
g_i = wt_i + a_i + z_i
\]  

(2.2)

The effort spent on efficiency enhancements is assumed to cause convex effort costs \( k(a_i) \) with \( k'(a_i) > 0, k''(a_i) > 0 \ \forall \ a_i \geq 0 \), which diminish the payoff from public and private consumption. These costs can be interpreted as transaction costs, e.g. search costs or administration costs. The regional government maximizes the utility of the representative consumer net of effort costs \( u(c_i) + h(g_i) + J(G) - k(a_i) \).

The central government is assumed to receive a fixed amount of tax revenue \( T \) collected ex ante, which it can either spend on the national public good or on grants (transfers or bailouts) to the regional governments \( z_i \geq 0 \).

\[
T = G + z_1 + z_2
\]  

(2.3)

We abstract from the revenue raising problem of the central government in order to
focus on the bailout problem. Therefore we assume the central government taxes $T$ to be exogenously given. Since the central government cares for the utility of both regions, its payoff is the sum of regional payoffs, i.e. 

$$
\sum_{i=1}^{2} (u(c_i) + h(g_i) + J(G) - k(a_i)).
$$

In all settings the timing is such that regional governments choose in the first stage all unrestricted variables by maximizing the utility of the representative consumer in their own region. Since we assume that the soft budget problem in the federation arises because of the inability of the central government to commit to the ex-ante efficient level of transfers, the central government moves in the second stage and chooses all remaining variables by maximizing the utility of both regions.

2.3 Benchmark Cases

We consider three benchmark cases: the first best regime as a benchmark for efficiency, the hard budget regime because it defines the outside option for the region if it denies the bailout and the unrestricted bailout regime as a benchmark for a pure monetary bailout without any further restrictions.

2.3.1 Centralized Decision Making

Centralized decision making serves as a benchmark for the first best because the central government takes into account all costs and benefits of both regions. In the first best problem the central government maximizes the sum of utility over both regions with respect to all decision variables, i.e. taxes, effort choices and bailouts$^{18}$.

$$
\max_{t,a,z} \sum_{i} [u(c_i) + h(g_i) + J(G) - k(a_i)]
$$

s.t. (2.3), (2.1) and (2.2).

The solution to the central government’s optimization problem is characterized by conditions (2.5) – (2.7).

$^{18}$Where $t$ denotes $(t_1, t_2)$, $a$ denotes $(a_1, a_2)$ and $z$, $(z_1, z_2)$. 
\begin{equation}
u' (c_i) = h' (g_i) \quad \forall i \tag{2.5}\end{equation}

\begin{equation}k' (a_i) = h' (g_i) \quad \forall i \tag{2.6}\end{equation}

\begin{equation}2J' (G) = h' (g_i) \quad \forall i \tag{2.7}\end{equation}

The first two conditions express that in the efficient solution both the marginal costs of taxation (i.e. forgone private consumption) and of the effort are equalized to their marginal benefits, i.e. the additional regional public consumption. The last condition is a Samuelson type condition which shows that the benefit from increasing the regional public good by allowing one unit of transfer has to compensate for the forgone national public consumption in both regions. Because we are interested in the soft budget constraint problem, we restrict ourselves to parameter constellations for which it is optimal to have positive transfers \((z_i > 0), \forall i\) throughout the chapter. This is assured by the assumption of \(2J' (G) < h' (g_i)\) for \(z_i = 0\). Intuitively this creates a bailout motive for the central government because it makes it beneficial to reallocate public funds from the national to the regional public good. Moreover, the symmetry of the regions regarding the payoffs and the size imply that \(t_i = t_j, a_i = a_j\) and \(z_i = z_j, \forall i \neq j\). The latter result holds as well for all cases discussed below.

### 2.3.2 Hard Budget Constraint Regime

We define the hard budget constraint (HBC) regime as a regime where transfers are not available. This case is important because it defines the outside option of a region, i.e. the utility a region can obtain by denying any central government assistance and resolving the fiscal crisis on its own. We obtain the solution of the HBC problem by solving the first best problem with the additional restriction of \(z_i = 0, \forall i\). The solution is characterized by conditions (2.5) – (2.6) and is identical to the solution the regional governments would choose in the absence of transfers. Intuitively this is the case
because the only source of inefficient regional government behavior is the ignorance of the bailout costs borne by individuals outside the own region. In absence of bailouts this inefficiency is defined away.

Because, we focus on cases where it is optimal to have positive bailouts in the first best \( z_i^{FB} > 0 \), the restriction of \( z_i = 0 \) is welfare decreasing. The following proposition summarizes how tax rates and effort choices change if a hard budget regime is imposed:

**Proposition 8** Provided \( z_i^{FB} > 0 \), regions choose tax rates and effort levels too high in the HBC regime as compared to the efficient benchmark, i.e. \( t_i^{HBC} > t_i^{FB}, a_i^{HBC} > a_i^{FB} \).

Proof: see Appendix.

Technically this is the case because the reduction of bailouts from \( z_i^{FB} > 0 \) to \( z_i^{HBC} = 0 \) increases the marginal benefit of each unit of regional taxation and effort \( (h'(g_i) = h'(wt_i + a_i + z_i) > h'(wt_i + a_i + z_i)) \) while the marginal costs \( (k'(a_i) \) and \( u'(c_i) \) remain the same. Intuitively the absence of bailouts forces the regions to finance an inefficiently large share of regional public good provision with the own revenue raising instruments and to contribute to an inefficiently large extent to the resolution of the crisis.

### 2.3.3 Unrestricted Bailout Regime

The unrestricted bailout (UB) regime is the standard pure SBC case. In this setting, the central government has only the instrument of monetary grants at its disposal and cannot commit to an efficient level of grants. Non-commitment is modeled through timing. Regions move first by choosing tax rates and effort levels and the central government moves last by determining transfer policy. We solve by backward induction.

At stage two, the central government maximizes the utility of the residents in both regions by choosing grants taking tax rates and effort choices of regions as given. The solution is characterized by condition (2.7), from which we obtain through implicit differentiation the central government’s response functions.
\[
\frac{\partial z_i}{\partial h_i} = \frac{h''(z_i) + 2J''(G)}{h''(z_i) + 4J''(G)} < 0 \\
\frac{\partial z_i}{\partial a_i} = \frac{2J''(G)}{h''(z_i) + 4J''(G)} > 0 \\
\frac{\partial z_j}{\partial a_i} = \frac{2J''(G)}{h''(z_i) + 4J''(G)} > 0 \\
\frac{\partial z_j}{\partial h_i} = w \frac{2J''(G)}{h''(z_i) + 4J''(G)} > 0
\]

As in standard models, the central government responds to tax rate and effort reductions in region i with an increase of the grant to this region, whereas the grant of the second region is reduced.

At stage one, the regional governments maximize just the utility of their own residents s.t. (2.3) – (2.2) by taking the central government’s behavior into account. The solution to this problem, summarized in conditions (2.8) and (2.9), clearly differs from the efficient solution in (2.5) and (2.9).

\[
u'(c_i) = \frac{1}{2} h'(c_i) \quad \forall i \tag{2.8}\]

\[
k'(a_i) = \frac{1}{2} h'(c_i) \quad \forall i \tag{2.9}\]

The reason is that each region does not consider the effects of its own tax setting and expenditure behavior on national public consumption in other regions. Particularly in the case of two regions only half of the benefits of marginal increases of tax rates and effort levels are considered. How does this affect the regional decisions?

**Proposition 9** Provided $z_i^{FB} > 0$, the regional governments choose inefficiently low tax rates and effort levels, i.e. $t_i^{UB} < t_i^{FB}, a_i^{UB} < a_i^{FB}$.

Proof: see Appendix.

In addition, we can infer from $\frac{\partial g_i}{\partial a_i} > 0$ and $\frac{\partial g_i}{\partial h_i} > 0$\textsuperscript{19}, that regional public consumption is inefficiently low, i.e. $g_i^{UB} < g_i^{FB}$. This implies by (2.7) an inefficiently low provision of national public goods $G^{UB} < G^{FB}$ and by the central government budget constraint (2.3) inefficiently high bailouts $z_i^{UB} > z_i^{FB}$.

\textsuperscript{19} $\frac{\partial g_i}{\partial a_i} = 1 + \frac{\partial z_i}{\partial a_i} = \frac{2J''(G)}{h''(z_i) + 4J''(G)}$ and $\frac{\partial g_i}{\partial h_i} = w + w \frac{\partial z_i}{\partial h_i} = w \frac{2J''(G)}{h''(z_i) + 4J''(G)}$. 
The result not only shows that the regions reduce their effort below an efficient level as in standard models of soft budget constraints, but also that ex-ante and ex-post grants differ. The nature of these differences has been extensively discussed in Koetthenbuerger (2007) for corrective (or Pigouvian) subsidies. In our case, the difference illustrates that the ex-ante efficient grant \( z^F_i \) is not credible, because if the regions reduce their tax rates and effort to the efficient levels \( t^U_i \) and \( a^U_i \), it is optimal for the central government to increase the grant to the amount \( z^U_i \). In case of commitment the response would be zero. In the next section, we turn to the question if additional bailout restrictions can alleviate the commitment problem.

### 2.4 Restricted Bailout Regimes

In this section, we explicitly consider additional obligations or restrictions which have to be fulfilled by the regional governments if they accept bailouts from the central government. The way we model the obligations resembles the typical procedure of budgetary approval during a crisis. In practice, regions are usually allowed to prepare a budget plan, which is rejected whenever it does not meet the objectives of the monitoring agency. From a game theoretic perspective this is the same as if the agency, i.e. the central government, could choose the restricted variable directly without an initial proposal of the region. Therefore we let the regions only choose the unrestricted choice variables at stage 1 and the central government all remaining variables at stage 2.

At the same time also the central government is restricted in its actions, because regions could always reject the restricted bailout and resolve the crisis by themselves. In this case, they would obtain the utility of the no-bailout (HBC) case. In order to ensure an enforcement of the additional bailout restrictions, we rule out the possibility for regional governments to deny the restricted bailout and to wait for an unrestricted bailout. In practice, this could be assured by a law which regulates the bailout procedure. This law should allow higher level governments to take over regional decisions if regional governments fail to meet bailout restrictions as it has been done during the New York City (1975) bailout.
### 2.4.1 Fully Restricted Bailout Regime

In this regime, regions receive not only a transfer, but also they have to adhere to tax rates and effort levels prescribed by the central government, whenever they accept a bailout. We know from the benchmark case of centralized decision making (4.1) that the most preferred allocation of the central government is the first best

\[(t_i^{FB}, a_i^{FB}, z_i^{FB}).\]

Do the regions prefer the first best offer to resolving the crisis by themselves \((t_i^{HBC}, a_i^{HBC}, 0)\)? The following proposition provides the answer.

**Proposition 10** It is optimal for regional governments to accept a fully restricted bailout that implements the first best allocation.

Proof: see Appendix.

Despite the first best offer is less attractive than the allocation in an unrestricted regime from the perspective of a region, it is still more attractive than the HBC allocation and will therefore be accepted. Intuitively this is the case because accepting the first best allocation allows the regional governments to move closer to their most preferred allocation - the allocation of the unrestricted regime, where taxes and effort levels are lower and bailouts are higher than in first best. Therefore the acceptance of the first best allocation leaves the regions better off than a hard budget regime. At the same time the restrictions prevent the regional governments from further decreasing effort levels and tax rates further downwards to their most preferred level.

### 2.4.2 Partially Restricted Bailout Regime

In the partially restricted bailout (PB) regime, regions keep at least some autonomy and can choose their effort levels at stage 1, while the central government chooses tax rates and transfers at stage 2, if the bailout is accepted.\(^{20}\) Since regional governments can anticipate the outcome of this bailout game, they decide at stage 0 if they want to enter this game or if they prefer to reject the bailout and finance the regional public

\(^{20}\)The case on partial restrictions on effort, but not on taxes produces similar results, just that effort and taxation are interchanged.
good by theirselves. We first solve the two stage game and check afterwards if bailout acceptance is beneficial from the perspective of the regional governments.

At stage 2, the central government maximizes the utility of all residents subject to all budget constraints (2.3) – (2.2) and taking the regional effort choices as given. Implicit differentiation of conditions (2.5) and (2.7), which characterize the optimal tax and transfer policy of the central government, yield the following central government response functions.

$$\frac{d \tau_i}{d a_i} = \frac{d \tau_j}{d a_i} = -\frac{1}{w} h''(g_i)u''(c_i) + 4h''(g_i)J''(G) + 4u''(c_i)J''(G) < 0$$

$$\frac{d \tau_i}{d a_i} = -\frac{h''(g_i)u''(c_i) + 2h''(g_i)J''(G) + 2u''(c_i)J''(G)}{h''(g_i)u''(c_i) + 4h''(g_i)J''(G) + 4u''(c_i)J''(G)} < 0 \quad (2.10)$$

$$\frac{d \tau_i}{d a_i} = \frac{2h''(g_i)J''(G) + 2u''(c_i)J''(G)}{h''(g_i)u''(c_i) + 4h''(g_i)J''(G) + 4u''(c_i)J''(G)} > 0$$

Similar to the unrestricted bailout case, reduced effort in region $i$ elicits larger transfers to this region, but decreases the transfers to the other region. In addition, the tax rates of both regions are increased if one region reduces its effort. Summing up all effects, it can be shown that the reduced effort of region $i$, reduces regional public consumption in both regions $\left(\frac{d b_i}{d a_i} > 0, \frac{d b_j}{d a_i} > 0\right)$.

Taking this central government policy into account, the regional governments maximize the utility of their own residents subject to the budget constraints (2.3) – (2.2) at stage 1. As in the unrestricted bailout case, the optimal effort choices are characterized by condition (2.9). How does this sequence of decisions affect the equilibrium effort choices and tax rates?

**Proposition 11** In the partially restricted bailout regime, provided that $z_i^{FB} > 0$, effort levels are even lower than in the unrestricted bailout regime and tax rates are higher than in the first best, i.e. $a_i^{PB} < a_i^{UB}, t_i^{PB} > t_i^{FB}$.

Proof (including optimality of acceptance): See appendix.

As in the unrestricted bailout case, the downward distortion of the effort levels is driven by the ignorance of the bailout costs borne by the inhabitants of the other region. But why is the effort level even lower than in the unrestricted bailout case? To understand this, suppose region $i$ had chosen the effort level of the unrestricted bailout
regime \(a_i^{UB}\). As shown by the response functions (2.10), the central government compensates for effort reductions by increasing the prescribed tax rate above the first best level and hence above the level preferred by the region \(t_i^{UB}\). Ceteris paribus this reduces the regions’ marginal benefit of public consumption \(g_i\). Since tax rates and effort levels are substitutive instruments for raising revenue, the region compensates for the higher tax rates by further reducing its effort level.

Why is it optimal for the regional government to accept an restricted bailout? The intuition for this finding is that a central government that cares for the welfare of all citizens would never offer a bailout that is harmful from the perspective of a region.

**Corollary 1** In the partially restricted bailout regime, budget constraints are hardened as compared to the unrestricted regime in the sense that less transfers are paid \(z_i^{PB} < z_i^{UB}\) and more of the national and regional public goods is provided \(g_i^{PB} > g_i^{UB}, G^{PB} > G^{UB}\).

Proof: See appendix.

The reduction of transfers compared to the unrestricted bailout case is a result of the increased number of instruments available to the central government. The utilization of the second instrument, i.e., the prescription of the regional tax rate, allows the central government to reduce the bailouts \(z_i\). In effect, the central government can force the region to participate in the resolution of the crisis through taxation and therefore into the provision of a higher level of public goods before transfers, which entails a higher public good provision after transfers.

This is one central result of the chapter. However, it remains an open question, if this regime yields higher welfare than an unrestricted bailout regime. Compared to the latter, in the partially restricted bailout regime, regional as well as national public consumption are increased and effort costs are reduced, but private consumption is lower. So it is not possible to make an outright statement about the welfare effects. We move to this issue in the next section.
2.5 Welfare Analysis

Throughout the chapter, we deal with three different kinds of inefficient regimes - the unrestricted bailout regime, the partially restricted bailout regime and the hard budget constraint regime. Although the central government cannot credibly commit to a HBC regime in our two stage model, it is nevertheless important to make a welfare statement about this regime since it involves a distinctly different type of inefficiency than the UB and the PB regime. The latter two are inefficient for their soft budget constraints, i.e. the possibility for regional governments to increase the size of their budgets through their actions, which implies inefficiently low contributions of the regions to the resolution of the crisis. In contrast, the HBC regime involves no transfer at all, which renders the contributions of the central government too low and thereby enforces inefficiently high effort levels of the regions.

In the sequel, we show that depending on the wealth of the central government and on the shape of preferences, either regime may dominate the others from a welfare point of view. It is not possible to make general statements. We proceed in two steps. First, we compare the HBC regime with both SBC regimes (UB, PB) and show that the welfare evaluation depends on the tax endowment of the central government. We show that if the endowment $T$ is low, all regimes are efficient. For intermediate values, the UB and PB regimes are dominated by the HBC regime and vice versa for high values of central government endowments.

We prove in a separate section that a general statement about the welfare ranking of the PB vs. the UB regime is not possible. To show this, we use the example of a logarithmic function to show that either the UB or the PB regime may dominate the other depending on the shape of preferences. For the logarithmic function, the PB regime dominates the UB regime, if the valuation of public consumption is sufficiently high, while the UB regime dominates the PB regime if effort costs are sufficiently small.
2.5.1 Efficiency of HBC versus SBC Regimes

The evaluation of hard budget regimes as compared to soft budget regimes depends on
the tax endowment of the central government $T$ relative to the wealth endowment of
representative consumers $w$. We fix $w$ and analyze changes of $T$, because reductions of
$w$ and increases of $T$ have qualitatively similar effects.

To establish our main argument, we first define two threshold values for the central
government’s tax revenue $T$, $T$ and $\overline{T}$ with $\overline{T} > T$. For $T < T$ the central government
does not employ transfers as a financing instrument for regional public goods in first
best. For tax endowments below this threshold value ($T \leq T$), the valuation for the
national public good provided with this revenue is so high that the marginal costs of
forgiving one unit of $G$ for a transfer to the regional government exceed the benefits of
this transfer, i.e. $2J' (G) > h' (g_i)$. $T$ is the value of $T$ at which all efficiency conditions
(2.5) – (2.7) are met for $z_i = 0$.

On the other hand for all $T < \overline{T}$ the central government is so ‘rich’ that it prefers
to finance regional public consumption completely out of transfers, i.e. $g_i = z_i > 0$ and
stops to use both regional taxation as well as expenditure savings as revenue raising
instruments, i.e. $t_i = 0$ and $a_i = 0$.

Proposition 5 and figure 2.1 summarize how welfare evolves in the different regimes
across different levels of central government tax revenue.

**Proposition 12** The HBC regime is efficient for all $T \leq T$ and inefficient for all
$T > \overline{T}$. SBC regimes are efficient for all $T < \overline{T}$ as well as $T \geq \overline{T}$ and inefficient for
$\overline{T} \leq T < \overline{T}$.

Proof: see Appendix.

For $T < \overline{T}$ all regimes are efficient (fat line) because the bailout costs (marginal
costs of forgoing one unit of national public consumption) are very high. In this interval,
on the one hand the HBC regime (slim line) entails no welfare losses since a no-bailout

---

$^{21}$The existence of $T$ is assured by the Inada conditions on public and private consumption
$J' (\infty), h' (\infty), u' (\infty) = 0$, the finiteness of regional wealth $w < 0$ and hence $u' (w) > 0$ as well
as the costliness of the first unit of public effort spent, i.e. $k' (0) > 0$. 
policy is even optimal in the first best and the availability of transfers adds no value. On the other hand, the SBC regimes (dashed line) are efficient because high bailout costs allow the central government to commit to the no-bailout policy and incentives of regional governments to raise revenue stay undistorted.

At \( T = T \) the SBC regimes become inefficient because marginal deviations from the first best policy start to pay off and regions switch into the SBC equilibrium. The discontinuity in the function occurs because as long as the central government is able to commit to no bailout \( (T < T) \), the regional governments fully consider the marginal benefits of raising tax revenue and spending effort \( h'(g_i) \). This suddenly changes when budget constraints become soft \( (T > T) \). Anticipating the bailouts and ignoring the costs of the bailouts borne by individuals outside the region, the regional governments just consider half of the benefits of raising revenue for the regional public good, i.e. \( \frac{1}{2}h'(g_i) \) (see equations (2.9), (2.8)). For \( T > T \) an inefficiency of the HBC regime appears as well, but increases only gradually as \( T \) rises above \( T \). This is due to a gradual replacement of regional taxation and effort by transfers in the first best, whereas tax rates and effort are kept constant in the HBC regime.

At \( T = T \) regional taxation and effort are completely phased out as revenue raising instruments in the first best, and hence also in the SBC regimes. Given that inefficien-
cies from too low taxation or effort cannot occur anymore, the SBC regimes coincide with first best for all \( T \geq T^* \).

Besfamille and Lockwood (2007) derive similar results in a project finance framework depending on effort costs for increasing the benefit of a project and refinancing (or bailout) costs. They find that when effort costs are very low, both the HBC and the SBC regime are efficient. In contrast, the HBC regime dominates with intermediate effort and refinancing (or bailout) costs, while at high effort costs and sufficiently low refinancing costs this is true for the SBC regime.

### 2.5.2 Efficiency of PB vs. UB regimes

In this section, we show that contrary to our initial expectation, the partially restricted bailout regime does not generally dominate the unrestricted bailout regime. This finding is surprising because in the PB regime the central government has an additional instrument at its disposal and budget constraints are hardened as compared to the unrestricted regime. We employ a Cobb-Douglas type logarithmic function of the following form for the preferences of the representative consumers.

\[
U^\rho_{CG}(c_i, g_i, a_i, G) = \sum_i (\alpha \ln c_i + \beta \ln g_i + \gamma \ln (L - a_i) + \delta \ln G) \quad \rho \in (PB, UB)
\]

(2.11)

The weighting factors \( \alpha, \beta, \gamma, \delta > 0 \) add up to one: \( \alpha + \beta + \gamma + \delta = 1 \) and \( L \) is a constant. To facilitate the understanding, one could interpret \( L \) as a time endowment, \( a_i \) as time spent for making public administration more efficient and \( L - a_i \) as leisure time.

Calculating the welfare difference between the PB and the UB regimes s.t. the budget constraints (2.1) – (2.3) yields the following utility difference:\(^{22}\)

\[
\Delta (\alpha, \beta, \gamma, \delta) = U^PB_{CG} - U^UB_{CG} = \ln \frac{2\alpha + \beta}{\alpha + \beta + 2\gamma + \delta} - \alpha \ln 2
\]

(2.12)

\(^{22}\)A summary of results for this utility function can be found in the appendix.
Due to the selected functional form, this difference is independent of wealth, tax and time endowments \((w, T, L)\). We investigate for which combinations of \(\alpha, \beta, \gamma\) and \(\delta\) either the UB or the PB regime dominates. We first state our result and explain the intuition afterwards.

**Proposition 13** If the joint valuation of the regional and national public goods \((\beta + \delta)\) is sufficiently high, i.e. \(\beta + \delta \geq \left(2 - \frac{1}{\ln 2}\right)\), the PB regime at least weakly dominates the UB regime. On the other hand, the UB regime at least weakly dominates the PB Regime if the joint valuation of public and private consumption \((\alpha + \beta + \delta)\) is sufficiently small, i.e. \((\alpha + \beta + \delta) \leq \left(2 - \frac{1}{\ln 2}\right)\).

Proof: see appendix.

The interesting finding of this proposition is that the PB regime might be inferior to the UB regime although the central government has a second instrument at its disposal. Because of the multiplicity of parameters and utility components, the welfare evaluation of the result is complex. Two effects are driving the result. Firstly, a shift of welfare weight from one to another consumption variable causes a 'weighting effect', meaning that a given distortion of this variable enters more heavily into the welfare function. For instance a shift of weight from \(\alpha\) (the weight for utility from private consumption) to \(\beta\) (the weight for utility from regional public consumption), c.p. would increase the impact of a distortion of the regional public good in the welfare function. Since in the PB regime effort choices are more strongly distorted than in the UB regime, an increase of \(\gamma\) deteriorates the welfare of the PB regime relatively to the UB regime. In the same manner, an increase of the weighting factors for regional and national public consumption \((\beta, \delta)\) that are more heavily distorted in the UB regime, relatively improve the evaluation of the PB regime. The effects of \(\alpha\) are ambiguous and depend on the remaining parameters.

The second element of the welfare evaluation is an 'incentive effect', determining the the size of the distortions in the PB relative to the UB regime. In the UB regime the deviation from the first best is largest when the joint weight on forgone public consumption \((\beta + \delta)\) takes intermediate values, while for very low and very high values the
deviation is small. The intuition for this result is that a low weight on public consumption entails low provision of public consumption in the first best. Marginal reductions from initially low amounts are very costly due to sharply increasing marginal costs. Likewise for high values of \((\beta + \delta)\) marginal costs of downward deviations are virtually constant, while the benefits of additional consumption and leisure, already enjoying low valuation in the first best \((\alpha + \gamma)\), are sharply decreasing. Just for intermediate values it pays off to significantly deviate from the first best choices. Similarly, in the PB regime, incentives to deviate are maximized when \(\gamma\) and \((\alpha + \beta + \delta)\), respectively take intermediate values.

Having defined the weighting and the incentive effects, we can analyze the distortions of the PB as compared to the UB regime: For low values of \(\gamma\), both the 'incentive effect' and the 'weighting effect' in the PB regime are low. Given \(\gamma\) is low, a sufficiently high value of \((\beta + \delta)\) entails a high 'incentive effect' multiplied by a high 'weighting effect' for the UB regime, yielding a clear domination of PB.

As \(\gamma\) increases, \((\beta + \delta)\) do necessarily decrease as all weights sum up to one. At the critical value of \(\gamma \geq \frac{1}{\ln 2}\), the distortions in the PB regime become dominant. They decrease as \(\gamma\) rises further because the 'incentive effect' stops to grow and begins to fall, while the continuing rise of the 'weighting effect' dominates and maintains the dominance of the UB regime until \(\gamma\) approaches one.

Apart from the interpretation of the welfare implications for this specific type of utility function, the most important purpose of this example was to demonstrate that the PB regime, in which the central government decides on more variables than in the UB regime, may be inferior from a welfare perspective. The intuition for this result is that regional governments may create for some parameter values higher welfare damages by distorting one variable very strongly in the PB regime than distorting two variables less strongly in the UB regime.
2.6 Conclusion

In this chapter, we analyzed whether the soft budget constraint problem can be alleviated if additional obligations or restrictions are available to the higher level government. Interestingly, our analysis shows that this is not necessarily the case. Restrictions, even if they are credible, are not always welfare enhancing. If the scope of the central government is limited and in case it can require certain actions from regions just in one policy area but not in others, the outcome may be even worse than in an unrestricted regime. The intuition for this finding is that a region which is regulated in one area, e.g. by minimum tax rate requirements, may react by strongly reducing other revenue collection instruments, such as effort on cutting expenditures. In contrast, a fully restricted regime implements the first best outcome in our model.

From a policy perspective, our analysis suggests that, first; a comprehensive approach is a more promising path to reconcile the aims of helping a debt ridden jurisdiction and to enforce a sufficiently high contribution of the region to the reduction of debt. One important condition to make restrictions work in practice is that the restrictions are enforced. If a strong enforcement was absent, sub-national governments could simply ignore the restrictions and ask for further bailouts if the deficit is still present after an initial bailout.

Anecdotal evidence supports these conclusions. The quick resolution of the 1975 New York fiscal crisis under the strict surveillance of the city’s budgetary performance by the EFCB is one example for both a comprehensive approach and a strong enforcement. Moreover, the bailout of Bremen in Germany in 1994-2004 delivers anecdotal evidence that partially restricted bailouts may indeed be inefficient. During the bailout period, Bremen received in total a bailout of € 8.5 billion. During this time Bremen was not allowed to have expenditure growth above the German average. Although it adhered to this restriction, Bremen nevertheless increased its debt from € 8.8 billion to € 11.4 billion. Certainly, it is an interesting task for future research to test the hypotheses derived in this chapter on a sound empirical basis.

Finally, we want to point out that restrictions during a bailout cannot make up
for fiscally imprudent behavior before the bailout takes place. Importantly, however, such restrictions make it less attractive to enter a bailout procedure through strategic behavior and may therefore reduce the incentives to accrue unsustainable debts.
Chapter 3

Bailouts in Federation: The Role of Asymmetric Information

3.1 Introduction

The incentive problems associated with bailouts in federations, i.e. transfers from higher level governments to lower level governments with unsustainable fiscal deficits have been analyzed in the literature on soft budget constraints.\(^{42}\) Soft budget constraint models are build on the assumption that higher level governments are not able to commit ex-post to deny a bailout once a fiscal crisis has emerged at the subnational level. One implication of this hypothesis is that central governments are not able to condition bailouts on the available information of why the crisis has occurred.

Nevertheless, the incentive problems associated with bailouts, suggest that the information on the causes of the deficits should play an important role for higher level governments when deciding on whether or not to bailout lower level governments. The obvious practical problem, however, is to determine whether a fiscal crisis of a lower level government is self-inflicted and caused by negligent behavior or whether it is driven by factors outside the accountability and power of the lower level government. While this may be easy to decide in some singular cases like for example unforeseeable natural catastrophes, in the majority of cases it is difficult to disentangle wrong

\(^{42}\)For a literature overview see Kornai et al. 2003.
political decisions from external forces. Fiscal crises often arise over a long period of
time and for many different reasons, some of which are wrong decisions of politicians
and some of which are caused by forces outside the scope of local governments. One
source of information to answer this question and to determine the effort of lower level
governments on preventing a fiscal crisis are comparisons of the expenditures and rev-
ues of the jurisdiction in fiscal distress with those of other jurisdictions with similar
characteristics, e.g. of similar size or with similar economic circumstances.

Following this line of argumentation, the question of this chapter is: What is the
optimal bailout scheme that conditions the level of bailouts on the observed budgetary
situation of the jurisdiction in fiscal crisis and similarly on the budget situation of
another comparable jurisdiction not in fiscal crisis?

The relevance of this topic is underpinned by anecdotal evidence which suggests that
federal governments indeed make use of budgetary information on other jurisdictions
when deciding on fiscal bailouts. One illustrative example for this is the case of the
highly indebted City of Berlin. In 2003, Berlin advanced a claim to receive a bailout to
the Federal Constitutional court of Germany. In 2006, the claim was rejected. One of
the reasons brought forward by the Court was that Berlin has not sufficiently exhausted
its options to raise revenue and to realize expenditure savings. It was argued that a
subnational government, failing to use all these options, cannot afterwards successfully
claim financial assistance from the federation. The Court considered comparable data
on expenditures and revenues of other German states, in particular of Hamburg, which
is suitable for a comparison because it is like Berlin a City State with similar public
}

The contribution of our analysis is to introduce conditional bailouts in the theoreti-
cal bailout literature. Precisely, we account for the possibility that central governments
take into account the information about subnational governments’ strategic behavior
when deciding on the level of bailouts. In addition, we shed light on the incentives of
those lower level governments that do not receive a bailout but serve as benchmark
regions. We show that these governments might have incentives to spend inefficiently
high effort to attain a budget surplus. This result delivers one theoretical explanation of why regional governments might choose freely to consolidate their debt, for example by self-imposing balanced budget rules.

We assess this research question in a stylized model of a federation which comprises a central government and two regional governments. Both regional governments can provide effort to increase the probability of a budget surplus. While the effort itself cannot be observed by the central government, it observes the budgetary outcome in both regions. Since we presume that the effort choices in both regions are linked to each other through a common cost shock, the outcomes of both regions are informative for the central government to learn about the effort taken in the poor region. Moreover, we incorporate that a bailout is usually granted to one region only while the other serves as a source of information. More precisely, one region is assumed to be poor and therefore to be eligible for a bailout in bad budgetary situations, while the other one is assumed to be sufficiently rich to overcome a fiscal deficit by itself. The bailouts are financed through a redistributive scheme that allocates the costs of the bailout to all members of the federation according to their share of inhabitants.

We derive an optimal bailout scheme for the poor region which conditions the level of the bailout on the outcome of both regions. We show that if in equilibrium it is more probable to observe a bad outcome in both regions (uniform case) than a bad outcome in the poor region along with a good outcome in the rich region (mixed case), a higher bailout is paid in the uniform case than in the mixed case. The intuition for this outcome is that if regional outcomes are related to each other in the sense that a uniform outcome is more probable than a mixed one, observing a good outcome in the rich region along with a bad outcome in the poor region is an informative signal on the shirking of the poor region. Therefore the poor region is ‘punished’ by a lower bailout in the latter case. One interesting implication of this result is that the effort of the government in the rich region is inefficiently high. This is because the government has an additional incentive to spend effort from the purpose of avoiding a contribution to the high bailout.

The theory of public economics has comprehensively analyzed optimal redistrib-
ution schemes under asymmetric information. While considering different types of information asymmetries, e.g. on the preferences for the regional public good (Cremer et al. (1996); Bucovetsky et al. (1998); Lockwood (1999)), regional income (Cremer et al. (1996) as well as Lockwood (1999)), local input provision (Wilson and Raff, 1997), the regional tax base (Bordignon et al., 2001) and the costs of local public good provision (Lockwood (1999); Cornes, Silva (2002), Altemeyer-Bartscher (2005); Runkel and Huber (2006)), all these papers have in common that a grant scheme is offered by the central government which incentivizes regional governments to truthfully reveal their types. Our paper departs from this literature in three respects. First, we assume that the type of the regional government is known from the outset, i.e. it is ex-ante known which regions are grant recipients and which are grant contributors. Second, we do not deal with general redistribution schemes, but rather with a particular type of redistribution, which just takes place if an (already) poor region experiences a fiscal crisis. An asymmetry of information arises on the cause of the crisis in the poor region, i.e. if it emerges because of moral hazard of the regional government or because of external shocks.\textsuperscript{44} The central government is able to elicit information on the probable cause of the crisis from the observable budgetary outcomes (good or bad) of both regions. A third aspect, which differs from most of the cited literature (apart from Altemeyer-Bartscher, 2005) is that we assume that the effort choices of the regional governments are correlated through a common cost shock and that the central government conditions the grant to the poor region on all observed signals.

Our paper proceeds as follows. Section 3.2 presents the model setup and section 3.3 analyzes the optimal bailout scheme in a centralized setup without information asymmetries as a benchmark for the first best. In sections 3.4 and 3.5 we introduce a decentralized setup, in which regional governments make unobservable effort choices. In section 3.4, we first restrict our analysis to the case where the central government conditions the bailout just on the outcome in the poor region. This case serves to illustrate the inefficiencies arising from the information asymmetry between the poor

\textsuperscript{44}Conceptually, our model comprises both adverse selection with respect to the privately known cost environment and moral hazard concerning the effort choice of regional governments. However, since we do not allow for mechanisms to elicit the privately held information from regional governments, our solution technique and results build on moral hazard type models following the approach of Holmstrom (1979).
regional government and the central government. In section 3.5, we extend the decentralized setup to a case with full conditionality, i.e. we allow the central government to condition the bailout on the budgetary outcomes observed in both regions. Section 3.6 concludes.

3.2 Model Setup

We consider a federation with two regional governments $i \in \{1, 2\}$ and a central government. The population of the federation is normalized to one. Region 1 is inhabited by $n_1$ citizens and region 2 by $n_2 = (1 - n_1)$.

**Regional governments**

Regional governments differ in their eligibility for a bailout from the central government. Region 1 is a poor region and therefore receives a bailout from the central government if it realizes a fiscal deficit. In practice this could be a region which has accrued such a high level of debt and interest payments that it has difficulties to provide elementary regional public goods like schooling or hospitals. Region 2 is defined to be rich and to be able to overcome a fiscal deficit alone. Region 2 is never eligible for a bailout.

Both regional governments have equal technologies to influence their current budgetary outcome $u_i$. For simplicity, we assume that just two outcomes are possible. The outcomes are normalized such that a fiscal deficit corresponds to a budgetary outcome of $u_i = 0$ and a surplus to a positive outcome of $u_i = u > 0$. We refer in the following to $u_i = 0$ as a bad budgetary outcome and to $u_i = u$ as a good budgetary outcome. By spending effort $a_i$ regional governments can increase the probability that a good budgetary outcome arises.

For an effort level of $a_i$, the regional government has to bear effort costs of size $\rho c(a_i)$. The costs are assumed to be convex, i.e. $c'(a_i) > 0$ and $c''(a_i) > 0$ and to be zero if no effort is spent, i.e. $c(0) = 0$. The effort choices of both regional governments are related to each other through a common cost shock $\rho$ that multiplicatively increases the effort costs in both regions. With probability $\pi$ the factor $\rho$ assumes a value of
1 and with probability \((1 - \pi)\) the factor \(\rho\) assumes a value of \(r > 1\). The common shock is observed by the regional governments before they decide on their effort and represents the common environment in which the regional governments operate, like the legal framework or the economic situation. It turns out that the effort choices in a high cost environment are generally lower than in an low cost environment. Therefore we denote the choices for \(\rho = r : (a_{1}^{L}, a_{2}^{L})\) and for \(\rho = 1 : (a_{1}^{H}, a_{2}^{H})\).

Both the effort costs and the outcome \(u_{i}\) are defined in per capita terms. The total surplus \(u_{i}\) in a region amounts to \(n_{i}u_{i}\) and the total effort costs to \(n_{i}\rho \cdot c(a_{i})\). This assumption assures that a government of a small region would spend the same effort per capita as the government of a large region, other things being equal.

For a given cost shock realization \(\rho \in \{1, r\}\), the probability for an outcome combination \((u_{1}, u_{2}) \in \{(u, u), (u, 0), (0, u), (0, 0)\}\) is defined as \(p(u_{1}, u_{2}; a_{1}, a_{2})\). We refer to combinations with identical outcomes in both regions, i.e. \((u, u)\) and \((0, 0)\) as uniform outcomes and to combinations with different outcomes in both regions, i.e. \((0, u)\) and \((u, 0)\) as mixed outcomes. We assume that the probability function has the following properties:

1. Each outcome combination occurs with a positive probability: \(p(u_{1}, u_{2}; a_{1}, a_{2}) \geq 0\forall (u_{1}, u_{2})\).

2. The sum of probabilities for all outcome combinations adds up to one: 
\[
\sum_{(u_{1}, u_{2})} p(u_{1}, u_{2}; a_{1}, a_{2}) = 1.
\]

3. An increase of effort in region \(i\), \(a_{i}\), increases the probability of a good outcome in that region, i.e. \(\frac{dp(u_{1}, u_{2}; a_{1}, a_{2})}{da_{1}} > 0\) and \(\frac{dp(u_{1}, u_{2}; a_{1}, a_{2})}{da_{2}} > 0\).

4. The overall probability for a good outcome in region \(i\), \(p(u_{i} = u; a_{i}, a_{j})\) is assumed to be independent of the effort choices in region \(j\), i.e. \(\frac{dp(u_{i} = u; a_{i}, a_{j})}{da_{j}} = 0\).

This implies from the perspective of region \(i\) that for a given outcome \(u_{i}\) the government of region \(j\) is just able to reallocate probability mass from uniform to mixed outcomes and vice versa, i.e. 
\[
\frac{dp(u_{i} = u; a_{i}, a_{2})}{da_{2}} = -\frac{dp(u_{i} = u; a_{2}, a_{j})}{da_{2}}.
\]

\(^{45}\)For region 1 \(p(u_{i} = u; a_{1}, a_{2})\) is defined as \(p(u_{1} = u; a_{1}, a_{2}) = p(u, u; a_{1}, a_{2}) + p(u, 0; a_{1}, a_{2})\) and for region 2 as \(p(u_{2} = u; a_{1}, a_{2}) = p(0, u; a_{1}, a_{2}) + p(0, a_{1}, a_{2})\).
5. For simplicity it is assumed that a marginal increase of effort linearly increases the probability for a good outcome in the respective region, i.e. \[ \frac{dp(u_i = u_{i1}, a_j)}{da_i} = \text{const} \]
and \[ \frac{d^2p(u_i = u_{i1}, a_j)}{da_i^2} = 0. \] The magnitude of \[ \frac{dp(u_i = u_{i1}, a_j)}{da_i} \] is independent of the effort taken by the other regional government, i.e. \[ \frac{d^2p(u_i = u_{i1}, a_j)}{da_i da_j} = 0. \]

6. From the assumption that the regional governments have similar technologies to influence the probability for a good outcome, it follows \[ \frac{dp(u_i = u)}{da_i} = \frac{dp(u_j = u)}{da_j}. \]

One interpretation of the effort could be that a regional government implements several expenditure cuts. The uncertainty on whether the expenditure cuts translate into a budget surplus could arise from uncertain and uninfluencible tax revenues.

*The central government*

The only role of the central government is to insure the poor region’s inhabitants against a bad fiscal outcome. In order to create a bailout motive, a linear per capita benefit from a bailout of \( b(u - u_i) \) and convex per capita bailout costs of \( k(b) \) with \( k'(b) > 0, k''(b) > 0, \) implying \( b > 0, \) are assumed. The formulation of the benefit function assures that the bailout motive is absent if the poor region has a good fiscal outcome, i.e. \( b(u - u) = 0. \) For an outcome of \( u_i = 0 \) the benefit \( b(u - 0) \) is positive. The convex bailout costs are defined to be zero if there is no bailout, i.e. \( k(0) = 0. \) The per capita formulation of bailout costs and benefits assures that there are no economies of scale in the bailout technology, i.e. that the central government prefers for small regions the same bailout per capita as for large regions. Bailouts that exceed the highest outcome a regional government could achieve on its own are not efficient, that is \( k'(1) < u. \) Moreover it is assumed that the central government is allowed to differentiate the bailout \( b \) to the poor region, depending on the budgetary outcome that is observed in the rich region. We refer to the bailout that is paid if a good outcome \( u_2 = u \) in the rich region occurs as \( b_u \) and to the bailout that is paid if a bad outcome \( u_2 = 0 \) is realized as \( b_0 \)

We assume that the total bailout costs of \( n_1 k(b) \) are financed by a redistributive scheme to which each region contributes a cost share equal to its share of inhabitants. This means that region 1 pays an amount of \( n_1k(b) \) and region 2 pays an amount of
\(n_2 n_1 k (b)\) into the redistributive scheme.

**Payoffs**

To keep things simple, it is assumed that the realized budgetary outcome as well as the bailout and effort costs enter linearly into the governmental payoffs. For a given cost shock realization \(\rho \in \{1, r\}\), the payoff obtained by the government in the poor region is denoted \(U_1\) and in the rich region \(U_2\):

\[
U_1 = n_1 (p (u_1 = u) \cdot u + p (0, u) \cdot (u b_a - n_1 k (b_a)) + p (0, 0) \cdot (u b_0 - n_1 k (b_0)) - \rho c (a_1))
\]

(3.1)

\[
U_2 = n_2 (p (u_2 = u) \cdot u + p (0, u) \cdot (-n_1 k (b_a)) + p (0, 0) \cdot (-n_1 k (b_0)) - \rho c (a_2))
\]

(3.2)

The utility of the central government \(U_{CG}\) is defined as the sum of the regional governments’ payoffs.

\[
U_{CG} = U_1 + U_2
\]

(3.3)

If the central government cannot observe the cost realization, it cares for the expected utility

\[
E_\rho [U_{CG}] = E_\rho [U_1] + E_\rho [U_2]
\]

(3.4)

where\(^{46}\):

\[
E_\rho [U_1] = \pi U_1 (a_1^H, a_2^H) + (1 - \pi) U_1 (a_1^L, a_2^L)
\]

\[
E_\rho [U_2] = \pi U_2 (a_1^H, a_2^H) + (1 - \pi) U_2 (a_1^L, a_2^L)
\]

\(^{46}E_\rho [\cdot]\) refers to the expectation with respect to the cost environment \(\rho\).
3.3 First Best

In the first best setup the central government has all information available. It observes
the cost shock and chooses afterwards the effort for both regional governments as well
as the bailout \( b_u \) for the mixed outcome \((u_1, u_2) = (0, u)\) and the bailout \( b_0 \) for the
uniform outcome \((u_1, u_2) = (0, 0)\). Since the central government considers the utility
of both regions it acts like a social planner and solves the problem (3.5).

\[
\max_{a_1, a_2, b_u, b_0} U_{CG} \tag{3.5}
\]

The following optimality conditions uniquely characterize the optimal bailouts:

\[
u = k'(b_u) \quad u = k'(b_0) \tag{3.6}
\]

The optimality conditions imply that the bailouts for the uniform case and the
mixed case are identical, i.e. \( b_u = b_0 = b^* \). The intuition for this result is that the
central government can optimally insure the poor region when it has access to all
information and the possibility to choose effort efficiently by itself. In this case, it
has to equalize the marginal costs of the bailout to its marginal benefits, which are
independent of the outcome of the rich region.

For the optimal bailout \( b^* \), we obtain the optimality conditions (3.7) and (3.8)
characterizing the effort choices \( a_1 \) and \( a_2 \).

\[
\rho c' (a_1) = \frac{dp(u_1=u)}{da_1} (u (1 - b^*) + k (b^*)) \tag{3.7}
\]

\[
\rho c' (a_2) = \frac{dp(u_2=u)}{da_2} u \tag{3.8}
\]

All marginal costs and benefits are in per-capita terms. On the left hand sides
are the marginal effort costs. The factor \( \rho \) takes the value of \( r > 1 \) if the regional
governments experience a high cost shock and the value of \( \rho = 1 \) otherwise. Given the
convex effort costs, effort is lower if the cost shock occurs.
On the left hand side are the marginal net benefits of effort. The marginal net benefit of the rich region is independent of the bailout. This is because the choice of the rich region does not matter for the bailout costs which are identical for the uniform and the mixed cases. For the effort $a_1$ the bailout matters. Because of the convex bailout costs and the linear utility from the bailout it follows that the total benefit of the bailout $ub^*$ is larger than the total bailout costs $k(b^*)$ and $(u(1 - b^*) + k(b^*)) < u$. In addition, from the definition of the probability function, it is also known that both regional governments have equal opportunities to increase the probability of a good outcome, i.e. $\frac{dp(u_1 = u)}{da_1} = \frac{dp(u_2 = u)}{da_2}$. We can conclude that the poor regional government is supposed to spend less effort than the rich region. The difference arises because the effort of the poor region represents the optimal effort with insurance and the choice of the rich region without insurance. However, this distinction is not important for our further analysis because we are just interested in the change of the effort levels compared to the first best.

### 3.4 Decentralized Setup With Partial Conditionality

In this section, we consider a decentralized setup. With decentralization the regional governments have private information on the realization of the cost shock and their effort choices. The central government observes the budgetary outcomes in both regions. We consider in a first step a partial conditionality setup where the central government can condition the bailout on the outcome of the poor region, but not on the outcome of the rich region. This means that it is not possible to differentiate the bailout for the mixed case and the uniform case, i.e. $b_u = b_0 = b$. This setup is useful to show the general incentive effects that arise from the asymmetric information between the poor regional government and the central government. Moreover the setup is helpful to clearly work out the differences that occur in the bailout scheme with full conditionality where the additional information from the rich region is taken into account.

At stage 1 the central government maximizes the expected payoff (3.4) anticipating
how the regional governments respond to the bailout scheme. At stage two the regional
governments optimally choose their effort given the bailout $b$. We solve the game by
backwards induction and therefore start with the choices of the regional governments.
In section 3.4.1, we determine the optimal effort choices of the regional governments
and show how they respond to changes in the bailout $b$ as well as to parameter changes.
In section 3.4.2 we analyze the optimal bailout policy of the central government. We
illustrate our results in section 3.4.3 by a functional example which allows deriving a
graphical solution of an optimal bailout scheme.

### 3.4.1 The Effort Choices of the Regional Governments

The regional government of the poor region maximizes its objective function (3.1)
by choosing its effort level $a_1$ after having observed the cost parameter and taking
the bailout $b$ as given. The optimal effort choice is characterized by the optimality
condition (3.9).

$$\rho c'(a_1) = \frac{dp(u_1 = u)}{da_1} (u (1 - b) + n_1 k (b))$$

(3.9)

Since the capability of increasing the probability of a good budgetary outcome for
its own budget $\frac{dp(u_1 = u)}{da_1}$ is independent of the choice of the other regional government,
the choice of $a_1$ does not depend on the choice of $a_2$.

The optimality condition illustrates that the decentralized effort choice $a_1$ is lower
than the first best effort choice (3.7). The reason for this result is that the bailout is
financed out of a common pool to which the government of the rich region contributes
a share $n_2 k (b)$ and this is not taken into account by the government of the poor region.
Therefore just the share $n_1 k (b)$ instead of the total bailout costs $k (b)$ is considered as
a benefit of additional effort in (3.9).

Does the effort choice of the rich regional government also change in the decentral-
ized setup? The government of the rich region chooses its effort level so as to maximize
its objective function (3.2) for a given cost realization $\rho \in \{1, r\}$ and bailout $b$. For the
optimal choice of \( a_2 \), we obtain the following condition:

\[
\rho c' (a_2) = \frac{dp(a_2 = u)}{da_2} u
\]  

(3.10)

The choice is identical to the first best choice (3.8). The intuition for this result is that the rich regional government has no influence on the size of the bailout. Therefore the bailout costs appear as a non-manipulable fixed cost in its payoff function. This fixed cost has no influence on the marginal costs and benefits of additional effort, which do not differ from the costs and benefits a social planner would consider. In result, the optimal choice of \( a_2 \) is efficient.

**Regional Government response functions**  For our subsequent analysis of central government choices, we need the knowledge about how the regional governments’ effort responds to changes of the parameters \( b, r \) and \( n_1 \). We derive the response functions, discussed in the sequel, in appendix 1 by implicit differentiation of the regional government optimality conditions (3.9) and (3.10). We obtain the following results:

\[
\frac{da_1}{db} < 0 \quad \frac{da_1}{dn_1} > 0 \quad \frac{da_1^c}{dr} < 0 \quad \frac{da_1^l}{dr} = 0
\]

\[
\frac{da_2}{db} = 0 \quad \frac{da_2}{dn_1} = 0 \quad \frac{da_2^c}{dr} < 0 \quad \frac{da_2^l}{dr} = 0
\]

While the regional government in region 1 reduces its effort as the bailout increases \( (\frac{da_1}{db} < 0) \), the regional government in region 2 does not respond to changes of the bailout \( (\frac{da_2}{db} = 0) \). The intuition is that as illustrated by the optimality condition (3.10) the choice of region 2 is independent of the bailout \( b \). Region 1 decreases its effort in response to a marginal increase of the bailout because it obtains a higher level of bailouts if a bad outcome occurs.

Moreover region 1 increases its effort as the population \( n_1 \) increases because the increase of \( n_1 \) alleviates the common pool effect. The choice of region 2 is independent of \( n_1 \).
For the case that a high cost shock occurred ($\rho = r$) and both regional governments choose low effort $a_1^L$ and $a_2^L$, an increase of $r$ further decreases the effort variables $\left(\frac{da_1^L}{dr} < 0, \frac{da_2^L}{dr} < 0\right)$, while for $\rho = 1$ the effort choices stay unchanged $\left(\frac{da_1}{dr} = 0, \frac{da_2}{dr} = 0\right)$.

The following proposition summarizes the most important results of this section.

**Proposition 14** In a decentralized setup with partial conditionality, i.e. $b_u = b_0 = b$, the effort choice of the regional government in the poor region $a_1$ is inefficiently low and responds negatively to an increase of the bailout $b$. The effort choice of the regional government in the rich region $a_2$ is efficient and is independent of the size of the bailout $b$.

### 3.4.2 The Bailout Choice of the Central Government

Taking the regional government effort responses into account, the central government maximizes its expected utility (3.4) choosing the bailout $b$.

The bailout choice is characterized by the following optimality condition.

$$u - \frac{E_p \left[ \frac{dp(u_1=0) da_1}{db} \right]}{E_p[p(u_1=0)]} n_2 k'(b) = k'(b) \tag{3.11}$$

Since the probability for a bad outcome in region 1 decreases as the effort increases $\frac{dp(u_1=0)}{da_1} < 0$ and the effort response $\frac{da_1}{db}$ is negative, the bailout is lower than in the first best (3.6). The deduction from the first best bailout serves to incentivize the government in the poor region to spend effort and is determined by three effects, discussed in the following.

(I) **Informativeness of the observed signal**

The informativeness of the observed signal is determined by the inverse of the probability of a bad budgetary outcome in the poor region $\frac{1}{E_p[p(u_1=0)]}$.\(^{47}\) To develop

\(^{47}\)Not surprisingly, our intuition resembles that of other models of moral hazard that usually define informativeness as the ratio $\frac{f_a(x;a)}{f(x;a)}$ with $f(x;a)$ denoting the probability that outcome $x$ occurs given action $a$ and $f_a(x;a)$ the derivative of $f(x;a)$ with respect to $a$. One peculiarity of our setup is that
an intuition for this result, suppose the desired action $a_1$ is very high and therefore the probability of a bad budgetary outcome $u_1 = 0$ is close to zero. If the regional government shirks in this situation by reducing the effort by a small unit, the occurrence of a bad outcome is a very informative signal on the shirking. In this case, the central government can very effectively punish deviations from the desired effort. On the other hand, if a bad outcome is very likely under the desired action, it is difficult for the central government to distinguish if the bad outcome has occurred because of shirking or despite the right action was taken. In the latter case the deduction from the first best bailout is small.

(II) Expected Probability Response

The second effect driving the deduction is the size of the expected probability response to changes of the bailout $E_p \left[ \frac{dp(u_1=0)}{da_1} \frac{da_1}{db} \right]$. This effect depends on two factors. First, on the reduction of the regional government’s effort in response to a marginal increase of the bailout ($\frac{da_1}{db} < 0$) and second, it depends on how efficiently effort changes translate into probability changes ($\frac{dp(u_1=0)}{da_1} < 0$). The larger is the product of both effects, the higher is the deduction from the first best bailout.

(III) Common Pool Effect

The common pool effect is represented by the bailout cost share that the government of the poor region ignores in its effort decision $n_2 k(b)$. The larger the common pool effect, the larger the inefficiency of the effort choice $a_1$ and the deduction from the optimal bailout.

In appendix 2 we derive by implicit differentiation of the optimality condition (3.11) comparative static responses of the bailouts to parameter changes:

$$\frac{db}{\pi} > 0 \quad \frac{db}{a_1} > 0 \quad \frac{db}{\pi} < 0$$

We can show that the bailout positively responds to increases of the cost factor $r$ and negatively to increases of the probability for a low cost state $\pi$. Both responses have the same interpretation. As $r$ increases and (or) $\pi$ decreases, the ex-ante-probability the central government does not know the cost environment and therefore takes expectations over $p$. 
of a bad budgetary outcome in region 1 $E_P[p(u_1 = 0)]$ increases. This makes the
observation of a bad budgetary outcome in region 1 less informative about the shirking
of the regional government in the poor region. Therefore the deduction from the first
best bailout becomes smaller.

Moreover, we can state that the bailout positively responds to increases of the
population in the poor region $n_1$. The reason is that the inefficiency arising from the
common pool effect is alleviated as the population share of the poor region becomes
larger. In consequence, the need for incentivizing the regional government to spend
effort decreases and the bailout becomes larger.

### 3.4.3 Example

To illustrate the formula in (3.11) and the interpretation consider the following example.
We assume quadratic bailout costs $k(b) = b^2$ and quadratic effort costs $c(a_i) = \gamma a_i^2$, 
where $\gamma$ is a cost parameter. Since the parameter $u$ is not decisive for the qualitative
results, we assume a fixed value of $u = 1$. Moreover, we assume a very simple func-
tion, translating effort choices into probabilities. The distribution is presented in the
following matrix.

$$
\begin{bmatrix}
    p(1, 1) & p(1, 0) \\
    p(0, 1) & p(0, 0)
\end{bmatrix}
= \begin{bmatrix}
    \frac{\pi(a_1^u + a_2^u) + (1-\pi)(a_1^u + a_2^u)}{4} & \frac{\pi(a_1^u + (1-a_2^u)) + (1-\pi)(a_1^u + (1-a_2^u))}{4} \\
    \frac{\pi((1-a_1^u) + a_2^u) + (1-\pi)((1-a_1^u) + a_2^u)}{4} & \frac{\pi((1-a_1^u) + (1-a_2^u)) + (1-\pi)((1-a_1^u) + (1-a_2^u))}{4}
\end{bmatrix}
$$

This probability function meets all the criteria required for the general probability
function and is although simple, suitable for the illustration of our results. We have
to impose the restriction that the effort choices are between zero and one in order to
have positive probabilities for each outcome.

In our example, the first best bailout is $b^* = \frac{1}{2}$ and condition (3.11), that charac-
terizes
the second best bailout for the partial conditionality case reads as follows:

\[ 1 - \frac{\pi r + (1-\pi) \left(1 - n_1 2b\right)}{\left(3 - \frac{\pi r + (1-\pi) n_1^2}{2r\gamma} \right)} n_2 b^2 = 2b \]

where:

\[ E \rho \left[ \frac{dp(u_1=0)}{da_1} \frac{da_1}{db} \right] = \frac{1}{4} \frac{\pi r + (1-\pi)}{2r\gamma} (1 - n_1 2b) \]

\[ E \rho [p(u_1 = 0)] = \frac{1}{4} \left( 3 - \frac{\pi r + (1-\pi)}{2r\gamma} \right) \left( 1 - b + n_1 b^2 \right) \]

Since \( \pi, r \) and \( \gamma \) just occur in the term \( \frac{\pi r + (1-\pi)}{2r\gamma} \) we can summarize them to the factor

\[ Q \equiv \frac{\pi r + (1-\pi)}{2r\gamma} \]

representing the quality of the budgetary environment in which the regional governments operate. The quality of the budgetary environment \( Q \) is the higher, the higher the probability for a good budgetary outcome \( \pi \), the lower the cost factor \( r \) and the lower the effort cost parameter \( \gamma \). In order to assure that the effort choices are between zero and one, \( Q \) is restricted to be between zero and two.\(^{48}\) Having introduced the parameter \( Q \), the only parameters determining the optimal bailout choice are \( Q \) and \( n_1 \).

The following graph shows an optimal bailout schemes \( b \) for different levels of \( n_1 \) and \( Q \).\(^{49}\) The graph illustrates the general results obtained in the previous section. It shows that the better the budgetary environment \( Q \), the smaller is the bailout in the case that a bad budgetary outcome occurs. The intuition is the same as presented for the general solution of the partial conditionality case: The better the budgetary environment, the higher is the ex-ante probability for a good budgetary outcome in the poor region.

As \( Q \) rises, the deduction from the first best bailout \( b^* = \frac{1}{2} \) increases and \( b \) becomes

\(^{48}\)Proof: Since \( a_2^* > a_1^* \), it is sufficient to make sure that \( a_2 \leq 1 \). From the optimality condition for the effort choice \( a_2 \) (3.8) which is identical to (3.10) we can conclude that \( a_2 = \frac{1}{4r\gamma} \) in our example. This implies for the low cost state: \( a_2^H = \frac{1}{4r\gamma} \) and we can conclude: \( a_2^H \leq 1 \Leftrightarrow \gamma \geq \frac{1}{4} \). For \( r \in (1, \infty), \pi \in [0; 1] \) and \( \gamma \in \left[ \frac{1}{2}; \infty \right], Q \in (0, 2) \).

\(^{49}\)The graph was obtained with the aid of the software Mathemathica\(^{TM} \). The employed program code can be found in Appendix 3.
smaller, because observing $u_1 = 0$, although $p(u_1 = 0)$ is very small, is a very informative signal on the probability that the regional government might have shirked. This becomes obvious for one extreme case.

Consider that the desired equilibrium effort choice $a_1(b)$ is high enough in both cost environments such that a good outcome always occurs (i.e. $p(u_1 = 1) = 1$ and $p(u_1 = 0) = 0$). If the regional government shirks by decreasing its effort by a small unit, the probability $p(u_1 = 0)$ becomes positive. If $u_1 = 0$ occurs indeed, this is a perfect signal for the central government that the regional government has shirked. Since this observation does not occur in equilibrium the central government can penalize the regional government by a high deduction from the first best effort, without facing a tradeoff with its wish to insure the regional government. As the ex-ante probability for a bad outcome becomes larger, observing $u_1 = 0$ becomes less informative and therefore is penalized to a smaller extent.

A second observation from the graph is that the larger the population in the poor region $n_1$, the higher is the bailout. This is because an increase of $n_1$ increases the contribution of region 1 to the bailout costs, which alleviates the inefficiency from the common pool effect. The larger is the share of the bailout costs taken into account by the government of the poor region, the closer is its payoff function to the payoff of the social planner and the less inefficient it acts. Therefore the need for giving the regional government incentives by a deduction from the first best bailout decreases and the size
of the bailout increases as \( n_1 \) becomes larger.

In the extreme case where \( n_1 \) approaches one, the regional government considers the full bailout costs and chooses therefore the efficient level of effort. At this level of \( n_1 \) it is not necessary anymore to incentivize the regional government by a deduction from the bailout.

### 3.5 Decentralized Setup with Full Conditionality

In this section we consider bailout schemes with payments that possibly condition on both local governments’ budgetary outcomes.\(^{50}\) We refer to these bailout schemes as fully conditional. We show that as long as the outcomes of both regions are not independent, it is optimal for the central government to differentiate the bailout to the poor region depending on what outcome has occurred in the rich region. For a mixed outcome, i.e. \( u_1 = 0 \) and \( u_2 = u \) a bailout \( b_u \) is paid and for a uniform outcome, i.e. \( u_1 = u_2 = 0 \) a bailout \( b_0 \).

Similar to the the decentralized setup with partial conditionality (section 3.4.1) the realization of the regional cost shock \( \rho \) and the regional governments’ actions are unobservable to the central government. However, the central government knows the magnitude (\( r \)) and the probability of the cost shock (\( 1 - \pi \)) when it chooses the bailouts at stage one. Moreover it knows the effort cost function \( k(a_i) \). The regional governments take the bailout scheme as given and choose after having observed the realization of the cost shock their effort choices.

We solve the game by backwards induction. Therefore we start with the regional

\(^{50}\)We restrict our attention to indirect bailout schemes that depend exclusively on budgetary outcomes. In particular, we do not consider direct mechanisms with respect to cost information. Since the cost environment is perfectly correlated between both regional countries and perfectly observed by both local governments, the central government could deny any bailouts in case the announced environment of both governments does not coincide. However, a slight modification of our model so that the local governments only observed noisy signals of the cost environment, would render this mechanism less advantageous. More generally, the nonnegativity requirement of bailouts for the poor country and the convex costs of bailouts would in general impede the application of the mechanism proposed by CREMER and MCLEAN that otherwise allows to elicit correlated private information costlessly. In addition, legal restrictions might force the central government to rely exclusively on verifiable budgetary information when assigning bailouts.
government choices in section 3.5.1 and afterwards move on to the central government choices in section 3.5.2 In section 3.5.3 we illustrate our results by continuing the example introduced in section 3.4.3.

3.5.1 The Effort Choices of the Regional Governments

The regional government of the poor region maximizes its payoff (3.1) by choosing the effort \( a_1 \) knowing the realization of the cost shock and taking the bailout scheme \((b_u, b_0)\) as given. The optimal effort choice is characterized by the following optimality condition:

\[
\rho c' (a_1) = \left( \frac{dp(u_1 u)}{da_1} u + \frac{dp(0, u)}{da_1} (ub_u - n_1 k (b_u)) + \frac{dp(0, 0)}{da_1} (ub_0 - n_1 k (b_0)) \right) \tag{3.12}
\]

This optimality condition resembles that of partially conditional bailouts in many respects. On the left hand side are the marginal costs and on the right hand side are the marginal net benefits of additional effort. The marginal costs \( \rho c' (a_1) \) on the left hand side of the equation as well as the expected benefit from a higher probability of the good outcome \( \frac{dp(u_1 u)}{da_1} u \) on the right hand side are identical to condition (3.9).

However, the expected utility loss from a decrease of the probability for the bad outcome differs: When bailouts depend only on the poor country’s outcome (3.9) it amounts to \( \frac{dp(u_1 u)}{da_1} (ub - n_1 k (b)) \), where \((ub - n_1 k (b))\) constitutes the net benefit obtained with a bad budgetary outcome. If bailouts are fully conditional the expected utility loss amounts to \( \frac{dp(u_1 u)}{da_1} (ub_u - n_1 k (b_u)) + \frac{dp(0, 0)}{da_1} (ub_0 - n_1 k (b_0)) \), which results from the bailout differentiation.

Similar to the partially conditional bailout schemes, the perceived net benefit for the bad budgetary outcome is higher than in the first best solution because the regional government ignores the contribution of the rich region to the bailout costs \( n_2 k (b_0) \) and \( n_2 k (b_u) \), or to put it differently the common pool effect is still in place.

How is the effort choice of the rich region affected by the differentiated bailouts? By maximizing the utility (3.2) with respect to the effort \( a_2 \), we obtain the following
optimality condition:

$$\rho c' (a_2) = \left( \frac{dp(a_2 - u)}{da_2} u + \frac{dp(0, u)}{da_2} n_1 (k (b_0) - k (b_u)) \right)$$  \hspace{1cm} (3.13)

This optimality condition shows that in contrast to the first best and the partial conditionality cases, under fully conditional bailouts the choice of the rich region is inefficient whenever $b_u \neq b_0$. This is because the regional government of the rich region is now able to influence the contributions to the bailout. When the rich region spends more effort, it increases the probability of a mixed outcome and decreases the probability of a uniform outcome, i.e. $\frac{dp(0,u)}{da_2} = -\frac{dp(0,0)}{da_2} > 0$. If the bailout in the mixed case is lower than in the uniform case ($b_u > b_0$) then this entails an inefficiently high effort and if $b_0 < b_u$ this entails an inefficiently low effort.

**Regional Government response functions** In appendix 4 we derive by implicit differentiation of conditions (3.12) and (3.13) responses of the regional governments’ effort choices to marginal changes of the bailouts $b_u$ and $b_0$. We show that an increase of $b_0$ decreases the effort of the regional government in the poor region and increases the effort of the regional government in the rich region, provided that $b_0$ is not very large as compared to $b_u$. An increase of $b_u$ decreases both the effort of the regional government in the poor region and the effort of the regional government in the rich region, provided that $b_u$ is not much larger than $b_0$.

To understand the intuition for this result, consider first the optimal choice of the poor region’s government (3.12). An increase of the bailouts $b_0$ and $b_u$ decreases c. p. the marginal benefit of effort because a higher payoff is obtained if the bad budgetary outcome occurs. This direct effect clearly decreases the effort $a_1$. However, a counteracting indirect effect may arise from the response of the regional government in the rich region. Each positive response of $a_2$ to a change of the bailout, shifts probability mass from the uniform outcome $(0, 0)$ to the mixed outcome $(0, u)$. This increases the weight of the net benefit from the mixed outcome $(ub_u - n_1 k (b_u))$ and decreases the weight of the net benefit from the uniform outcome $(ub_0 - n_1 k (b_0))$ in the payoff function of the government in the poor region. If $b_u > b_0$ ($b_u < b_0$) this
weakens (strengthens) the incentives of the government in the poor region. Similarly, each negative response of \( a_2 \) strengthens (weakens) the incentives for \( b_u > b_0 \) (\( b_u < b_0 \)). We show in appendix 4, that the indirect effect might dominate for \( \frac{da_2}{db_u} \) (for \( \frac{da_1}{db_u} \)) if \( \frac{da_2}{db_u} < 0 \) and \( b_0 \) is significantly smaller than \( b_u \), i.e. \( b_0 << b_u \) (if \( \frac{da_2}{db_u} > 0 \) and \( b_0 \) is significantly larger than \( b_u \), i.e. \( b_0 >> b_u \)).

To understand the incentives of the government in the rich region, consider the optimality condition in (3.13). Ceteris paribus an increase of \( b_0 \) increases the marginal benefits of spending effort \( a_2 \). Intuitively this is the case because it becomes more beneficial for the government in the rich region to avoid the bailout contribution for the uniform outcome. Similarly, an increase of \( b_u \) ceteris paribus decreases the marginal benefits of effort \( a_2 \). However, a counteracting indirect effect might arise from the response of the regional government of the poor region. Since the poor regional government usually decreases its effort in response to an increase of the bailout (\( b_0 \) or \( b_u \)) this makes it more likely that a bailout occurs. Therefore, if \( (k(b_0) - k(b_u)) > 0 \), the rich region marginally saves more on contributions for bailouts when it increases its effort slightly.\(^{51}\) If \( b_u > b_0 \) this indirect effect decreases the marginal incentives to spend effort for the rich region and if \( b_0 > b_u \) this increases the incentives. We show in appendix 4 that for \( \frac{da_2}{db_u} \) the direct effect might be overcompensated by the indirect effect if \( \frac{da_1}{db_u} < 0 \) and \( b_0 >> b_u \). The same holds for \( \frac{da_2}{db_0} \) if \( \frac{da_1}{db_0} < 0 \) and \( b_u >> b_0 \).

To keep our analysis simple and since a dominance of indirect effects might only occur for extreme cases\(^{52}\), we focus in the following on cases where the direct effects dominate, i.e. we impose the following assumptions on the response functions:

\[
\frac{da_1}{db_u} < 0 \quad \frac{da_1}{db_0} < 0 \quad \frac{da_2}{db_u} < 0 \quad \frac{da_2}{db_0} > 0
\]  

(3.14)

The following proposition summarizes the most important results of this section for all cases where the direct effects of changes in bailouts on the local governments’ reactions dominate.

\(^{51}\)Likewise, if \( (k(b_0) - k(b_u)) < 0 \), the marginal losses from slightly more effort increase since a bailout is generally paid more often.

\(^{52}\)For the example analyzed in sections 4.3 and 5.3 the second order effects are even completely absent.
Proposition 15 In a decentralized setup with full conditionality, i.e. in general $b_u \neq b_0$, the effort choice of the regional government in the poor region $a_1$ is inefficiently low and responds negatively to increases of the bailouts $b_u$ and $b_0$. The effort choice of the regional government in the rich region $a_2$ is inefficiently high for $b_0 > b_u$ and inefficiently low for $b_0 < b_u$. The effort $a_2$ responds negatively to increases of the bailout $b_u$ and positively to increases of the bailout $b_0$.

3.5.2 The Bailout Choice of the Central Government

Taking the regional government effort responses into account, the central government maximizes its expected utility (3.4) by choosing the bailouts $b_u$, paid in the mixed case $(u_1 = 0, u_2 = u)$ and $b_0$, paid in the uniform case $(u_1 = 0, u_2 = 0)$. Assuming that the first order conditions characterize the global optimum, the first order condition (3.15) describes the optimal choice of $b_u$ and condition (3.16) the optimal choice of $b_0$.

\[
\begin{align*}
  u &= E_p \left[ \frac{dp(0,u)}{da_1} \frac{da_1}{dp} n_2 k(b_u) + E_p \left[ \frac{dp(0)}{da_1} \frac{da_1}{dp} n_2 k(b_0) \right] \right] - \frac{E_p \left[ \frac{-dp(0,u)}{da_2} \frac{da_2}{dp} n_2 k(b_u) \right] (NB(b_u) - NB(b_0))}{E_p[p(0,u)]} = k'(b_u) \quad (3.15)
\end{align*}
\]

\[
\begin{align*}
  u &= E_p \left[ \frac{dp(0,u)}{da_1} \frac{da_1}{dp} n_2 k(b_u) + E_p \left[ \frac{dp(0)}{da_1} \frac{da_1}{dp} n_2 k(b_0) \right] \right] + \frac{E_p \left[ \frac{-dp(0,u)}{da_2} \frac{da_2}{dp} n_2 k(b_0) \right] (NB(b_u) - NB(b_0))}{E_p[p(0,0)]} = k'(b_0) \quad (3.16)
\end{align*}
\]

Where $NB(b_u) = (ub_u - n_1 k(b_u))$ denotes the poor region’s net benefit from the bailout in the uniform case ($b_0$) and $NB(b_0) = (ub_0 - n_1 k(b_0))$ the net benefit from the bailout in the mixed case ($b_u$).

On the right hand sides of equations (3.15) and (3.16) are the marginal costs of the bailout which are positive and increasing with the size of the bailout. The left hand sides depict the marginal net benefits of the bailout.

The first term on the left hand side ($u$) represents the additional utility that the central government obtains from a marginal increase of the bailout and corresponds to
the marginal benefit in the first best. This term is followed by a second term, representing a deduction from the bailout which serves to alleviate the inefficient behavior of the government in the poor region. The third term, which may be a deduction or an increase, serves to alleviate the inefficient behavior of the government in the rich region. We refer in the sequel to the second and third terms as incentive adjustments. The size of the incentive adjustments is determined three effects that are discussed in the following.

(1) Informativeness of the Observed Signal

The informativeness of the observed signal is represented by the inverse of the probability of a bad budgetary outcome in the poor region $\frac{1}{E_p[p(0,u)]]}$ in (3.15) and $\frac{1}{E_p[p(0,0)]]}$ in (3.16), respectively. The less probable an observed outcome, the higher are the incentive adjustments from the first best level of the associated bailout.

Consider first, the actions of the government in the poor region. If a high effort $a_1$ is desired by the central government in equilibrium, i.e. $E_p[p(0,u)]$ and or $E_p[p(0,0)]$ are low, the observation of $u_1 = 0$ is a very informative signal that the regional government in the poor region might have shirked. Therefore the deductions represented by the second terms on the left hand sides of (3.15) and (3.16) are high. If to the contrary, a low level of effort $a_1$ and therefore a high probability of $(0,u)$ and or $(0,0)$ is desired by the central government, it is difficult to distinguish if e.g. $(0,u)$ has occurred because of shirking or despite the desired action has been taken. Therefore in the latter case the incentive adjustment is low.

Consider now the regional government in the rich region. For $b_0 > b_u$ it has an incentive to overprovide effort and for $b_u > b_0$ it has an incentive to underprovide effort. Moreover, consider the example that $b_0 > b_u$ and the desired effort $a_2$ is low, i.e. $E[p(0,u)]$ is low as well. Then the observation of $u_2 = u$ is a informative signal that the rich regional government has not taken the desired action and has overprovided effort. This is ‘punished’ by lowering the reward from effort overprovision through an increase of $b_u$. The impact of $\frac{1}{E_p[p(0,u)]]}$ and $\frac{1}{E_p[p(0,0)]]}$ on the remaining efficiency adjustments can be interpreted accordingly.

(II) Expected Probability Response
The expected probability response is represented in (3.15) by \( E_p \left[ \frac{dP(0,0)}{d\alpha_1} \frac{d\alpha_2}{db_0} \right] \) and \( E_p \left[ \frac{dP(0,u)}{d\alpha_1} \frac{d\alpha_2}{db_0} \right] \) and in (3.16) by the terms \( E_p \left[ \frac{dP(0,0)}{d\alpha_0} \frac{d\alpha_1}{db_0} \right] \) and \( E_p \left[ \frac{dP(0,u)}{d\alpha_0} \frac{d\alpha_1}{db_0} \right] \). Its size is determined by the effort responses of the regional governments to marginal changes in the bailout and the associated changes of the probabilities \( p(0, u) \) and \( p(0, 0) \). From the response functions (3.14) and the definition of the probability function it follows that all of these terms are positive. The efficiency adjustments to the bailouts are the higher the stronger the regional governments respond to marginal changes of the bailouts and the more sensitive the probabilities \( p(0, u) \) and \( p(0, 0) \) respond to effort changes.

(III) Externalities

The externalities ignored by the government of the poor region enter the second terms and the externalities ignored by the government of the rich region enter the third terms on the left hand sides of (3.15) and (3.16). The regional government of the poor region ignores the bailout costs borne by the residents of the rich region, \( n_2k(b_0) \) and \( n_2k(b_u) \). In section 3.4.2 we have referred to this effect as the common pool effect. The larger the common pool effect, the larger are the deductions from the first best bailouts.

The externality ignored by the rich regional government when choosing its actions is the differential net benefit from \((NB(b_u) - NB(b_0))\). This differential net benefit can be explained as follows. As the government of the rich region increases its effort, it increases the probability that the poor regional government obtains the net benefit of the mixed case \( NB(b_u) = (ub_u - n_1k(b_u)) \) instead of the net benefit of the uniform case \( NB(b_0) = (ub_0 - n_1k(b_0)) \). So, if \( b_u > b_0 \) a positive externality is ignored and if \( b_0 > b_u \), a positive externality is not taken into account. The larger are the ignored externalities the larger are the efficiency adjustments to the bailouts.

Although we were able to identify the main drivers of the incentive scheme employed by the central government, the general formulation of the problem does not allow clear-cut conclusions on which of the effects dominates. Therefore we apply in the next section the example introduced in section 3.4.3 to the full conditionality case. This allows us to graphically derive an incentive scheme. The scheme shows that both cases
$b_u > b_0$ and $b_0 > b_u$ may occur.

### 3.5.3 Example continued

Employing the specification of the bailout costs and the probability function introduced for the example presented in section 3.4.3, we obtain the following first order conditions for the optimal choice of $b_u$ and $b_0$:

$$
1 - \frac{1}{4} Q (1 - 2n_1 b_u)(b_u^2 + b_0^2) \left(1 + \frac{1}{4} Q \left(b_0 + b_u - 2n_1 b_u^2\right)\right) - \frac{1}{4} Q b_u n_1 \left((b_u - n_1 b_u^2) - (b_0 - n_1 b_0^2)\right) = 2b_u \quad \text{(FCoptimalitybu_a)}
$$

$$
1 - \frac{1}{2} \frac{1}{4} Q (1 - 2n_1 b_u)(b_u^2 + b_0^2) \left(2 + \frac{1}{4} Q \left(b_0 + b_u - 2n_1 b_u^2\right)\right) + \frac{1}{4} Q b_u n_1 \left((b_u - n_1 b_u^2) - (b_0 - n_1 b_0^2)\right) = 2b_0 \quad \text{(FCoptimalityb0_a)}
$$

The first order conditions allow deriving the optimal incentive scheme for different values of the budgetary environment $Q$ and $n_1$. Since the comparative statics with respect to $n_1$ are identical to those derived for partially conditional bailouts, we exclude $n_1$ from the discussion in this section.

![Figure 3.2: Optimal bailouts in the full conditionality case](image)

The figure shows the optimal bailouts $b_u$ and $b_0$ for different levels of the budgetary

---

$\text{(FCoptimalitybu_a)}$ where:

$$
E_{\rho} \left[ \frac{d p(0,u)}{d a_1} \frac{d a_1}{d b_u} \right] = E_{\rho} \left[ \frac{d p(0,0)}{d a_1} \frac{d a_1}{d b_u} \right] = \frac{1}{4} Q \left(1 - 2n_1 b_u\right),
$$

$$
E_{\rho} \left[ \frac{d p(0,u)}{d a_2} \frac{d a_2}{d b_u} \right] = \frac{1}{4} Q 2b_u n_1,
$$

$$
E_{\rho} \left[ p(0,u) \right] = \frac{1}{4} Q \left( b_0 + b_u - 2n_1 b_u^2 \right),
$$

$$
E_{\rho} \left[ \frac{d p(0,1)}{d a_1} \frac{d a_1}{d b_0} \right] = E_{\rho} \left[ \frac{d p(0,0)}{d a_1} \frac{d a_1}{d b_0} \right] = \frac{1}{4} \left(1 - 2n_1 b_0\right),
$$

$$
E_{\rho} \left[ \frac{d p(0,0)}{d a_2} \frac{d a_2}{d b_0} \right] = \frac{Q}{4} \frac{2b_0 n_1}{4},
$$

$$
E_{\rho} \left[ p(0,0) \right] = \frac{1}{4} \left(2 + \frac{1}{4} Q \left(b_0 + b_u - 2n_1 b_u^2\right) - Q\right), (NB(b_u) - NB(b_0)) = ((b_u - n_1 b_u^2) - (b_0 - n_1 b_0^2))
$$
environment $Q$.\footnote{The graph was obtained with the aid of the software Mathematica\textsuperscript{TM}. The employed program code can be found in Appendix 5.} In a bad budgetary environment $Q < 1$, i.e. if the probability for a high cost state $(1 - \pi)$, the effort cost parameter $\gamma$ and, or the cost factor $r$ are high, the bailout for the uniform case $b_0$ is higher than the bailout for the mixed case $b_u$. For a good budgetary environment $Q > 1$ the opposite is true. We show in appendix 6 that $Q < 1$ corresponds to the situation that the uniform outcome is more probable than the mixed one, i.e. $E_p [p (0, 0)] > E_p [p (0, u)]$.

One intuitive interpretation of a high probability for a uniform outcome is that regional budgets are positively related to each other through positively correlated tax revenues that depend on similar industries. If these industries are in a downturn, both regions are likely to experience a revenue decrease and therefore a bad budgetary outcome.

Similarly, one interpretation of a high probability of a mixed outcome is that regions generate their tax revenue from different industries. This would mean if one region’s industry (and tax revenue) is in a downturn it is likely that the other region’s industry (and tax revenue) experiences an upturn.

Having this interpretation in mind, we develop an intuition for the obtained incentive scheme. Suppose that the central government knows ex-ante that a uniform outcome is more likely than a mixed one in equilibrium. In this case, the observation of the uniform outcome is an informative signal that the bad outcome in the poor region is rather a result of the desired action than of cheating. Therefore a higher bailout is paid to the poor region, i.e. $b_0 > b_u$. For the case of $E_p [p (0, 0)] < E_p [p (0, u)]$ the interpretation is the other way around.

Moreover the downward sloping shape of the incentive scheme with respect to the quality of the budgetary environment can be similarly interpreted as in the partial conditionality case. The better is the quality of the budgetary environment, the higher is the poor regional government’s effort desired by the central government. To give the regional government the incentives to actually take this effort, the bailout is reduced.

The case of $E_p [p (0, 0)] > E_p [p (0, u)]$ seems to be more relevant in practice, because
typically similar governments are considered to be appropriate for comparisons. This is also suggested by the example of Berlin and Hamburg presented in the introduction. In this case it has been argued that Hamburg has managed to keep its expenditures at a lower level than Berlin although both cities operate under similar circumstances.

Whenever the bailout scheme bailouts satisfies \( b_0 > b_u \), condition (3.13) implies a higher effort of the rich region than in the first best. Overprovision of effort induced by fully conditional bailout schemes with \( b_0 > b_u \) is one central result of this paper. Nevertheless, our example shows that fully conditional schemes with \( b_0 > b_u \) might be optimal and therefore preferred to partial schemes that guarantee first best behavior of the rich local government. Put differently, whenever \( b_0 > b_u \) is optimal, the central government finds it optimal to use the information provided by the rich regions outcome in order to incentivize the poor regional government more effectively even though this causes inefficient behavior of the rich local government.

### 3.6 Conclusions

From a policy perspective our analysis has two important implications. First, if a higher level government in a federation is faced with the decision whether to bailout a lower level jurisdiction or not, it should use budgetary information from comparable jurisdictions. This information is helpful to uncover more efficiently if unsustainable deficits have been caused by the strategic behavior of subnational governments that are unwilling to overcome fiscal crises their selves or if the deficits emerged despite the government has taken the desired effort to resolve the crisis alone. This allows to provide more powerful incentives for a given level of insurance. If the comparable budgetary data suggests that the subnational government has not exhausted all options to overcome its fiscal crisis alone, the bailout should be lower than in the case in which the data suggests that a crisis has emerged out of an external shock like a natural catastrophe. This type of bailout conditionality is welfare enhancing because it makes strategic behavior of subnational governments less attractive.

Second, although the use of this additional information is overall welfare enhanc-
ing, it brings about a caveat. It distorts the decisions of the lower level government that serves as a benchmark. For the more relevant case that a comparison takes place between similar jurisdictions with similar expenditure and tax developments, the government serving as a benchmark has incentives to spend an inefficiently high effort for reaching a good budgetary situation. The intuition for this finding is that the bailout conditionality gives the benchmarked government the opportunity to influence the size of the bailout through its own actions. By spending more effort the benchmarked government can increase the chances to avoid a contribution to a high bailout. This observation may deliver one explanation for voluntary deficit reduction plans or self-imposed balanced budget rules of lower level governments in federations.
Implications and Conclusions

From a policy perspective the theoretical analyses of this dissertation allow several conclusions on how to alleviate the inefficiencies arising from the bailout problem and to improve the long term fiscal discipline of subnational governments in federations.

First of all the results of our analysis on the timing of elections in the first chapter of the dissertation suggest that staggered elections at the central and regional levels may alleviate the bailout problem as compared to synchronized elections of regional and central governments. The basic intuition for this result is that staggered elections make it easier for federal governments to commit to reduce the level of inefficient bailouts and to force regional governments to finance more of their expenditures by themselves and not through the common pool of federal revenues. It becomes easier for central governments to commit to a lower level of bailouts because they can decide on their political programs before future regional governments enter office. The mere anticipation of strategic regional government behavior creates incentives for central governments to avoid inefficient bailouts and instead to use national revenues for more productive purposes. On the other hand regional governments entering office and observing just a small pool of funds at the free disposal of central governments are aware of the limited scope for bailouts and account for that by raising more revenue through own regional revenue channels.

When are these policy recommendations relevant in practice? Rodden (2002) empirically analyzes which characteristics of federal systems facilitate unsustainable lower level government debt. He identifies two important drivers of subnational deficits: First, a high level of transfer dependency and second, weak subnational governments’ borrowing restrictions. In the light of these empirical findings, staggered elections
might improve the allocation of public funds in federations where, e.g. for historical reasons, borrowing restrictions at subnational levels are difficult to implement and where the level of transfer dependency is difficult to reduce.

While these results of the first chapter concern the interactions of two government levels as a whole, the policy implications derived from the analyses of the second and third chapter of this dissertation derive more detailed conclusions on the bailout process. Our results suggest that an elaborate design of a bailout procedure may substantially reduce the payoffs of bailouts and therefore make it less attractive for subnational governments to elicit bailouts through strategic behavior.

Our analysis in the second chapter of the dissertation shows that restrictions, such as savings goals for subnational governments during a bailout procedure, may reduce the inefficiencies arising from bailouts. One important condition for the usefulness of bailout restrictions is that they fully restrict regional revenue raising channels. In practice, this means that either both expenditure savings goals and revenue raising objectives should be defined for subnational jurisdictions or that there is a regulation on the residual of revenues and expenditures, i.e. on the size of an acceptable budget surpluses (or deficits). Surprisingly, we find that restrictions on just one revenue raising channel might produce larger inefficiencies than bailouts with no restrictions at all. Anecdotal evidence suggests that this result indeed might hold in practice. The Laender of Saarland and Bremen received from 1993 to 2003 bailouts for the reduction of their unsustainable deficits from the German federal government. During this period both Laender were not allowed to have expenditure growth rates above the German average. Although it was adhered to this rule and despite of substantial bailouts, Saarland just insignificantly reduced its debt and Bremen even further increased it. If a balanced budget rule had been imposed, Saarland would have reduced its initial debt by 90% and Bremen by 97%.

A second important condition for efficient bailout restrictions is a strong enforcement. If a strong enforcement was absent, subnational governments could simply ignore the restrictions and ask for further bailouts if the deficit still is present after an initial bailout. One way to put this recommendation into practice is to endow central
governments with the right to curtail regional governments’ budgetary autonomy if requirements are not met, i.e. to allow the central government to execute expenditure cuts and revenue raising actions itself. This has been done by the Emergency Financial Control Board (EFCB) which successfully monitored New York City’s government during the bailout following the 1975 debt crisis. The board could control and reject the city’s financial decisions as well as local borrowing. If the city had not met the requirements, the EFCB would have had the right to control all municipal accounts and to exercise disciplinary sanctions.

Our results suggest that an elaborate design of a bailout procedure affects the incentives of subnational governments to actually accrue unsustainable debts and to call for bailouts. Another implication along these lines is derived in the third chapter of the dissertation. Central governments should use, when deciding on the level of bailouts, the available information of why a subnational crisis has occurred, i.e. if the crisis is self-inflicted and caused by negligent behavior or whether it is driven by factors outside the accountability and power of the lower level government.

One obstacle to put such conditionality into practice is the information asymmetry that is introduced by fiscal decentralization and the associated separation of local, regional and central budgets. The incomplete knowledge of subnational budgets opens up the possibility for lower level governments to shift the blame of a fiscal crisis onto developments outside their scope. For instance in the case of Berlin it was argued that revenue downturns and expenditure increases resulting from the reunification created such a high burden for the city’s budget that it was not possible for Berlin’s government to overcome the deficit without federal assistance.\footnote{Wieland (2002).}

Nevertheless benchmarking studies can help to obtain information on the behavior of subnational governments in eliciting a fiscal crisis by comparing the level of the expenditure and revenue categories of several jurisdictions to each other. The comparisons may uncover, e.g. if subnational governments in fiscal distress have spent more than their colleagues in comparable jurisdictions. In this regard they can shed light on the question if a government indeed has exhausted all options to overcome
the crisis alone before asking for a bailout. When designing bailout schemes that de-
pend on the relative performance of regions, it should be taken into account that this
might distort incentives of countries that are usually not eligible for bailout. They may
have incentives to inefficiently high effort to obtain a good budgetary outcome. The
reason is that it becomes beneficial to suggest to the central government that it was
easier to realize savings than it actually was to avoid a contribution to a high bailout.
This observation may deliver one explanation for voluntary deficit reduction plans or
self-imposed balanced budget rules of lower level governments in federations.
Appendix A

A.1 Appendix to Chapter 1

A.1.1 Welfare analysis without Political Business Cycles

The following table provides an overview of the results in the SP, SY and ST regimes.

<table>
<thead>
<tr>
<th></th>
<th>SP</th>
<th>SY</th>
<th>ST</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_t$</td>
<td>$\gamma_g$</td>
<td>$\gamma_g \frac{\gamma_g + \gamma_G}{\gamma_g + (1+N)\gamma_G}$</td>
<td>$\gamma_g$</td>
</tr>
<tr>
<td>$g_{t+1}$</td>
<td>$\gamma_g$</td>
<td>$\gamma_g \frac{\gamma_g + \gamma_G}{\gamma_g + (1+N)\gamma_G}$</td>
<td>$\gamma_g \frac{\gamma_g + \gamma_G}{\gamma_g + (1+N)\gamma_G}$</td>
</tr>
<tr>
<td>$G_t$</td>
<td>$(1 + N) \gamma_G$</td>
<td>$(1 + N) \gamma_G \frac{\gamma_g + \gamma_G}{\gamma_g + (1+N)\gamma_G}$</td>
<td>$(1 + N) \gamma_G$</td>
</tr>
<tr>
<td>$G_{t+1}$</td>
<td>$(1 + N) \gamma_G$</td>
<td>$(1 + N) \gamma_G \frac{\gamma_g + \gamma_G}{\gamma_g + (1+N)\gamma_G}$</td>
<td>$(1 + N) \gamma_G \frac{\gamma_g + \gamma_G}{\gamma_g + (1+N)\gamma_G}$</td>
</tr>
<tr>
<td>$\tau^R$</td>
<td>$2 \left( \gamma_g + \gamma_G (1+N) \right) - (1+N) \tau^C$</td>
<td>$\left( \gamma_g + \gamma_G \right) - (1+N) \tau^C$</td>
<td>$2 \left( \gamma_g + \gamma_G (1+N) \right) - N\gamma_G$ - \left(1+N\right) \tau^C$</td>
</tr>
</tbody>
</table>

From the comparison of the results we obtain:

$$\tau^{R,SP} > \tau^{R,ST} > \tau^{R,SY}$$

$$g^{SP}_t = g^{ST}_t > g^{SY}_t \quad g^{SP}_{t+1} > g^{ST}_{t+1} = g^{SY}_{t+1}$$

$$G^{SP}_t = G^{ST}_t > G^{SY}_t \quad G^{SP}_{t+1} > G^{ST}_{t+1} = G^{SY}_{t+1}$$

The payoffs of the social planner are given by:
\[ U^{SP} = 2w + N \tau^C - 2(\gamma_g + (1 + N) \gamma_G) + 2\gamma_g \ln \gamma_g + 2(1 + N) \gamma_G \ln \gamma_G (1 + N) \]

\[ U^{SY} = 2w + N \tau^C - 2(\gamma_g + \gamma_G) + 2(\gamma_g + (1 + N) \gamma_G) \ln \frac{(\gamma_g + \gamma_G)}{\gamma_g + (1 + N) \gamma_G} + 2\gamma_g \ln \gamma_g + 2\gamma_G (1 + N) \ln (1 + N) \gamma_G \]

\[ U^{ST} = 2w + N \tau^C + 2\gamma_g \ln \gamma_g + 2(1 + N) \gamma_G \ln (1 + N) \gamma_G - (\gamma_g + \gamma_G) - ((1 + N) \gamma_G + \gamma_g) + (\gamma_g + (1 + N) \gamma_G) \ln \frac{(\gamma_g + \gamma_G)}{\gamma_g + (1 + N) \gamma_G} \]

The difference between the ST welfare and the SY welfare is given by:

\[ U^{SY} - U^{ST} = (\gamma_g + (1 + N) \gamma_G) \left( \ln \frac{(\gamma_g + \gamma_G)}{\gamma_g + (1 + N) \gamma_G} + 1 - \frac{(\gamma_g + \gamma_G)}{(\gamma_g + (1 + N) \gamma_G)} \right) < 0 \]

if we set \( x = \frac{(\gamma_g + \gamma_G)}{\gamma_g + (1 + N) \gamma_G} \in (0; 1) \) the function \( f(x) = 1 - x + \ln x < 0 \forall \, x \)

Obviously, the welfare difference between the social planner and the staggered office terms solutions has to be positive:

\[ U^{SP} - U^{ST} = -(\gamma_g + (1 + N) \gamma_G) + (\gamma_g + \gamma_G) - \left( (\gamma_g + (1 + N) \gamma_G) \ln \frac{(\gamma_g + \gamma_G)}{\gamma_g + (1 + N) \gamma_G} \right) \]

\[ = (\gamma_g + (1 + N) \gamma_G) \left( -1 + \frac{\gamma_g + \gamma_G}{\gamma_g + (1 + N) \gamma_G} - \ln \frac{(\gamma_g + \gamma_G)}{\gamma_g + (1 + N) \gamma_G} \right) > 0 \]

if we set \( x = \frac{(\gamma_g + \gamma_G)}{\gamma_g + (1 + N) \gamma_G} \in (0; 1) \) the function \( f(x) = -1 + x - \ln x > 0 \forall \, x \)

Result:
\( U^{SP} > U^{ST} > U^{SY} \)

### A.1.2 Welfare analysis with Political Business Cycles

The following table provides an overview of the results in the SP, SY and ST regimes.

<table>
<thead>
<tr>
<th></th>
<th>SP</th>
<th>SY</th>
<th>ST</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g_t )</td>
<td>( \gamma_g )</td>
<td>( \gamma_g \frac{\gamma_g + \gamma_G}{\gamma_g + (1+N)\gamma_G} )</td>
<td>( \frac{\gamma_g}{\pi} )</td>
</tr>
<tr>
<td>( g_{t+1} )</td>
<td>( \gamma_g )</td>
<td>( \pi \gamma_g \frac{\gamma_g + \gamma_G}{\gamma_g + (1+N)\gamma_G} )</td>
<td>( \frac{\gamma_g + \gamma_G}{\gamma_g + (1+N)\gamma_G} )</td>
</tr>
<tr>
<td>( G_t )</td>
<td>( (1+N) \gamma_G )</td>
<td>( (1+N) \gamma_G \frac{\gamma_g + \gamma_G}{\gamma_g + (1+N)\gamma_G} )</td>
<td>( \frac{(1+N)\gamma_G}{\gamma_g + (1+N)\gamma_G} )</td>
</tr>
<tr>
<td>( G_{t+1} )</td>
<td>( (1+N) \gamma_G )</td>
<td>( \pi (1+N) \gamma_G \frac{\gamma_g + \gamma_G}{\gamma_g + (1+N)\gamma_G} )</td>
<td>( \frac{(1+N)\gamma_G}{\gamma_g + (1+N)\gamma_G} )</td>
</tr>
<tr>
<td>( \tau^R )</td>
<td>( 2 \left( \gamma_g + \gamma_G (1+N) \right) - (1+N) \tau^C )</td>
<td>( \left( \gamma_g + \gamma_G \right) (1 + \pi) ) - (1+N) \tau^C )</td>
<td>( \left( \gamma_g + \gamma_G \right) \frac{\gamma_g + (1+N)\gamma_G}{\gamma_g + \gamma_G} ) - (1+N) \tau^C )</td>
</tr>
</tbody>
</table>

The welfare of the different regimes is given as:

\[
U^{SP} = 2w + N\tau^C - 2 \left( \gamma_g + (1+N) \gamma_G \right) + 2\gamma_g \ln \gamma_g + 2(1+N) \gamma_G \ln \gamma_G (1+N)
\]

\[
U^{SY} = 2w + N\tau^C - (1+\pi) \left( \gamma_g + \gamma_G \right) + \left( \gamma_g + (1+N) \gamma_G \right) \ln \pi
\]

\[
+ 2 \left( \gamma_g + (1+N) \gamma_G \right) \ln \frac{\gamma_g + \gamma_G}{\gamma_g + (1+N) \gamma_G} + 2\gamma_g \ln \gamma_g + 2\gamma_G (1+N) \ln (1+N) \gamma_G
\]

\[
U^{ST} = 2w + N\tau^C + 2\gamma_g \ln \gamma_g + 2(1+N) \gamma_G \ln (1+N) \gamma_G - \left( \gamma_g + (1+N) \gamma_G \right) \ln \pi
\]

\[
- \left( \gamma_g + \gamma_G \right) - \frac{(1+N) \gamma_G}{\pi} + \left( \gamma_g + (1+N) \gamma_G \right) \ln \frac{\gamma_g + \gamma_G}{\gamma_g + (1+N) \gamma_G}
\]

The difference between the ST welfare function and the SY welfare function is given by:

\[
U^{SY} - U^{ST} = \left( \gamma_g + (1+N) \gamma_G \right) \left( \ln \frac{\pi^2 \left( \gamma_g + \gamma_G \right)}{\gamma_g + (1+N) \gamma_G} + \frac{1}{\pi} \left( 1 - \frac{\pi^2 \left( \gamma_g + \gamma_G \right)}{\gamma_g + (1+N) \gamma_G} \right) \right)
\]
We analyze the welfare difference for different values of $\pi$:

Using $x = \frac{(\gamma_g + \gamma_G)}{(\gamma_g + (1 + N) \gamma_G)}$, we obtain

$$U^{SY} - U^{ST} = (\gamma_g + (1 + N) \gamma_G) \left( \ln \pi^2 x + \frac{1}{\pi} - \pi x \right)$$

$$U^{SY} - U^{ST} > 0 \iff f(\pi) = \left( \ln \pi^2 x + \frac{1}{\pi} - \pi x \right) > 0$$

The analysis of function $f(\pi)$ gives:

$$f(1) = (\ln x + 1 - x) < 0$$

$$f\left(\frac{1}{\sqrt{x}}\right) = \left( \ln \frac{1}{\sqrt{x^2}} x + \frac{x}{\sqrt{x}} - \frac{x}{\sqrt{x}} \right) = 0$$

$$f\left(\frac{1}{x}\right) = \left( \ln \pi^2 x + \frac{1}{\pi} - \pi x \right) > 0$$

$$\lim_{\pi \to \infty} f(\pi) = \lim_{\pi \to \infty} \pi \left( \frac{2 \ln \pi}{\pi} + \frac{\ln x}{\pi} + \frac{2}{\pi^2} - x \right) \to -\infty$$

This analysis shows that there is at least one root between $\pi = \frac{1}{x}$ and $\pi \to \infty$. In order to prove that there is only one root, we analyze the derivative of $f(\pi)$.

$$f'(\pi) = \frac{1}{\pi} \left( 2 - \frac{1}{\pi} \right) - x$$

and

$$\text{sign} f'(\pi) = \text{sign} g(\pi) \quad \text{with} \quad g(\pi) = -x \pi^2 + 2\pi - 1$$

The analysis of $g(\pi)$ shows that:

$$g(\pi) \begin{cases} > 0 & \text{for} \ 1 < \pi < \frac{1 + \sqrt{1-x}}{x} \\ < 0 & \text{for} \ \pi > \frac{1 + \sqrt{1-x}}{x} \end{cases} \quad \text{with} \quad \frac{1 + \sqrt{1-x}}{x} > \frac{1}{x}$$

$$f''(\pi) = -\frac{2}{\pi^2} \left( 1 - \frac{1}{\pi} \right) < 0 \forall \pi > 1$$
The monotonicity of the function \( g(\pi) \) shows that there can only be one root between \( \pi = \frac{1}{x} \) and \( \pi \to \infty \). We refer to the value of \( \pi \) defining the root: \( f(\pi) = (\ln \pi^2 x + \frac{1}{x} - \pi x) = 0 \) with \( \pi \in \left( \frac{1}{x}, \infty \right) \) as \( \bar{\pi} \). An analogous definition of \( \bar{\pi} \) can be obtained by the equation:

\[
\ln \frac{\pi^2}{\pi^*} = \frac{1}{\bar{\pi}} \left( \frac{\pi^2}{\pi^*} - 1 \right)
\]

where \( \pi^* = \frac{1}{x} = \frac{(\gamma + (1+N)\gamma_G)}{(\gamma + \gamma_G)} \) and \( \bar{\pi} > \bar{\pi} \).

To sum up, we have shown that there exists a \( \bar{\pi} \) and a \( \bar{\pi} \) with \( \bar{\pi} < \bar{\pi} \) such that:

\[
U^{SY} - U^{ST} \begin{cases} 
< 0 & \text{for } 1 \leq \pi \leq \bar{\pi} \\
\geq 0 & \text{for } \bar{\pi} \leq \pi \leq \bar{\pi} \\
< & \text{for } \pi > \bar{\pi}
\end{cases}
\]
A.2 Appendix to Chapter 2

A.2.1 Proof of Proposition 1

We next show that if \( z_i^F > 0 \), then assumptions \( h'(g_i) > 0, h''(g_i) < 0 \) imply that \( t_i^HBC > t_i^F \) and \( a_i^HBC > a_i^F \).

Proof: Suppose \( t_i^F, a_i^F \) is optimal in the hard budget regime.

Then \( wt_i^F + a_i^F = g_i^HBC < g_i^F = wt_i^F + a_i^F + z_i^F \) imply

\[
\frac{\partial U_i^{HBC}}{\partial t_i} = -wu'(c_i) + wh'(g_i) > 0 \quad \text{and} \quad \frac{\partial U_i^{HBC}}{\partial a_i} = h'(g_i) - k'(a_i) > 0,
\]

meaning that it pays to deviate upwards in both the tax and the effort dimensions, which provides the contradiction.

In addition equalization of marginal benefits of consumption and marginal effort costs \( (u'(c_i) = k'(a_i)) \) in both the HBC and the FB regimes insures that upwards deviation occurs in both dimensions.

A.2.2 Proof of Proposition 2

Next we show \( t_i^U < t_i^F, a_i^U < a_i^F \) whenever \( z_i^F > 0 \).

Proof: Suppose \( t_i^U = t_i^F, a_i^U = a_i^F \), then \( u'(c_i^F) = h'(g_i^F) \) and \( k'(a_i^F) = h'(g_i^{F}) \) imply

\[
\frac{\partial U_i^{HBC}}{\partial t_i} = -wu'(c_i) + w\frac{1}{2}h'(g_i) < 0 \quad \frac{\partial U_i^{HBC}}{\partial a_i} = \frac{1}{2}h'(g_i) - k'(a_i) < 0,
\]

downward deviation increases regional utility. Again the equality of marginal effort costs and marginal utility from private consumption \( (u'(c_i) = k'(a_i)) \), both in the first best and in the soft budget regimes excludes solutions where one variable deviates upwards, while the other deviates downwards.

A.2.3 Proof of Proposition 3

We show that given \( z_i^F > 0 \ \forall i \), the central government offer is more valuable to the region than the maximal utility which the region can obtain under a hard bud-
get regime:  \( U_i(a_i^{FB}, t_i^{FB}, z_i^{FB}) > U_i(a_i^{HBC}, t_i^{HBC}, 0) \), where \( a \) denotes the vector \((a_1, a_2)\), \( t = (t_1, t_2) \) and \( z = (z_1, z_2) \). We proceed in two steps:

(1) Part I of the proof shows, if transfers are paid in first best \( z_i^{FB} > 0 \), then it is as well optimal to pay transfers at the higher tax rate \( t_i^{HBC} \) and the higher effort level \( a_i^{HBC} : z_i^{HBC} \equiv z_i^* \left( t_i^{HBC}, a_i^{HBC} \right) > 0 \), where the * indicates the optimal central government policy for a given vector of regional choices. To see this, suppose to the contrary \( z_i^* \left( t_i^{HBC}, a_i^{HBC} \right) = 0 \). Then the HBC solution would be characterized by the following conditions: \( h' (g_i^{HBC}) = w' \left( t_i^{HBC} \right) \), \( h' (g_i^{HBC}) = k' (a_i^{HBC}) \), \( h' (g_i^{HBC}) \leq 2J' (w_C) \). But if this was true, then \( z_i^{FB} = 0 \). The central government condition for transfers: \( h' (g_i) > 2J' (G) \) assures in addition that regional utility would be raised if the region could obtain the grant \( z_i^{HBC} \).

(2) Part II of the proof shows that if the region could choose freely, it would prefer \((a_i^{FB}, t_i^{FB}, z_i^{FB})\) over \((a_i^{HBC}, t_i^{HBC}, z_i \left( t_i^{HBC}, a_i^{HBC} \right))\). The first and second order conditions of the SBC problem imply \( \forall \ t_i > t_i^{UB} \land a_i > a_i^{UB} \):

\[
\frac{\partial U_i}{\partial t_i} = -wu'(c_i) + w \frac{1}{2} h'(g_i) < 0 \quad \text{and} \quad \frac{\partial U_i}{\partial a_i} = \frac{1}{2} h'(g_i) - k'(a_i) < 0.
\]

Therefore it is profitable to deviate downwards from \((t_i^{HBC}, a_i^{HBC})\) to \((t_i^{FB}, a_i^{FB})\).

**A.2.4 Proof of Proposition 4**

Proposition \( t_i^{PB} > t_i^{FB}, a_i^{PB} < a_i^{UB} \). We proof the proposition in two steps and in a third step that acceptance is optimal. Define \( t_i^* (a) \), \( z_i^* (a) \) as optimal central government policy for a given set of effort choices \((a_1, a_2)\) in the PB regime.

(1) \( t_i^{PB} > t_i^{FB}, a_i^{PB} < a_i^{FB} \):

Suppose to the contrary \( a^{PB} = a^{FB} \) and hence \( t_i^* (a^{FB}) = t_i^{FB} \) and \( z_i^* (a^{FB}) = z_i^{FB} \). The central government optimality conditions imply

\[
\frac{\partial^2 U_i}{\partial a_i^2} = -\frac{\partial (wu'(c_i) + w \frac{1}{2} k'(g_i))}{\partial a_i} = w^2 \left( u'' (c_i) + \frac{1}{2} h'' (g_i) \right) < 0, \quad \frac{\partial^2 U_i}{\partial a_i \partial t_i} = \frac{\partial (wu'(c_i) + w \frac{1}{2} k'(g_i))}{\partial a_i} = \frac{1}{2} w h'' (g_i) < 0,
\]

\[
\frac{\partial^2 U_i}{\partial t_i^2} = \frac{\partial (\frac{1}{2} h'(g_i) - k'(a_i))}{\partial a_i} = \frac{1}{2} h'' (g_i) - k'' (a_i) < 0 \quad \text{and hence} \quad D > 0.
\]
(2) Now we show in addition: $a^P_B < a^U_B < a^F_B$.

Suppose to the contrary: $a^F_B > a^P_B \geq a^U_B$.

$t^*_i (a^F_B) = t^F_B$ and $\frac{\partial t^*_i}{\partial a_i} < 0 \Rightarrow t^*_i (a^P_B) > t^*_i (a^F_B) > t^*_i (a^U_B)$

It follows from $a^P_B \geq a^U_B$ and $t^*_i (a^P_B) > t^*_i (a^U_B) \Rightarrow h' (g^P_B) > h' (g^U_B)$ and by condition (2.9), $k' (a^P_B) < k' (a^U_B)$, which in turn implies $a^P_B < a^U_B$, contradicting our initial statement.

(3) Acceptance of the partially restricted bailout is more valuable from the perspective of the region than resolving the crisis alone.

We have already shown in the proof to proposition 3 that $z^*_i (a^{HBC}, t^{HBC}) > 0$, i.e. the central government would give a transfer if the regions chose $(a^{HBC}, t^{HBC})$ in the unrestricted regime and the acceptance of this grant would increase the utility of the regions $U_i (a^{HBC}, t^{HBC}, z^*_i (a^{HBC}, t^{HBC})) > U_i (a^{HBC}, t^{HBC}, 0)$. Given, the central government could choose $t_i$ and $z_i$ freely, it would prefer by the central government response functions $(\frac{\partial t_i}{\partial a_i} < 0, \frac{\partial z_i}{\partial a_i} < 0), t^*_i (a^{HBC}) < t^*_i (a^{HBC})$ and $z_i (t^* (a^{HBC}), a^{HBC}) > z_i (t^{HBC}, a^{HBC})$. A move to this $(z_i, t_i)$ combination would be again utility enhancing from the perspective of the regions, because the central government would reduce $t_i$ only as long as $u' (c_i) > h' (g_i)$ and increase $z_i$ only as long as $h' (g_i) \geq 2J' (G)$. Therefore $U_i (a^{HBC}, t^*_i (a^{HBC}), z_i (t^* (a^{HBC}), a^{HBC})) > U_i (a^{HBC}, t^{HBC}, z_i (t^{HBC}, a^{HBC}))$ and by revealed preference:

$U_i (a^{P_B}, t^*_i (a^{P_B}), z^*_i (t^* (a^{P_B}), a^{P_B})) > U_i (a^{HBC}, t^*_i (a^{HBC}), z_i (t^* (a^{HBC}), a^{HBC}))$,

which establishes the result that acceptance is indeed optimal.

### A.2.5 Proof of Corollary 1

$z^*_i < z^*_i < g^*_i > g^*_i$:

From $a^P_B < a^P_B \Rightarrow h' (g^P_B) < h' (g^P_B) \Rightarrow g^P_B > g^P_B$

$(2.7) \Rightarrow G^P_B > G^P_B \Rightarrow z^*_i < z^*_i$.
A.2.6 Proof of Proposition 5

We denote in the following $U^\rho_{CG}(z,t,a)$ as the utility of the central government in regime $\rho \in (FB, UB, PB, HBC)$. We proceed in four steps.

(1) For $T \leq T_c$, the first order conditions of the HBC regime coincide with the FB regime.

(2) Differentiation of the utility difference between the first best and the HBC regime for $T > T_c$ shows that the HBC regime constantly deteriorates compared to the efficient solution as $T$ increases. This can be illustrated by the derivative of the utility difference:

$$\frac{dU_i^{FB}}{dT} - \frac{dU_i^{HBC}}{dT} = 2J'(G^{FB}) - 2J'(G^{HBC}) > 0$$

(note that $G^{HBC} = T$ and $G^{FB} = T - z_i^{FB} - z_j^{FB} < T$).

(3) The SBC regimes (UB, PB) are efficient for $T < T_c$ because over this range $\frac{d\alpha_i}{dT} = \frac{d\alpha_j}{d\alpha_i} = 0$ and marginal deviations from the first best policy do not pay off for regions. At $T = T_c$ central government responses ($\frac{d\alpha_i}{dT}$ and/or $\frac{d\alpha_j}{d\alpha_i}$) change from zero to positive values and marginal deviations from first best become beneficial. Furthermore, because regions distort their choices until their optimality conditions ((2.8) and/or (2.9)) are met, there is a discrete utility difference between SBC and FB at $T = T_c$.

(4) As $T$ rises, in the HBC regime tax rates and effort levels stay constant, i.e. $\frac{d\alpha_i}{dT} = \frac{d\alpha_j}{d\alpha_i} = 0$, whereas they depend negatively on $T$ in all the remaining regimes (see table), i.e. $\frac{d\alpha_i}{dT}, \frac{d\alpha_j}{d\alpha_i} < 0$, entailing $\frac{d\alpha_i}{dT} > 0, \rho \in (UB, PB, FB)$.

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>FB</th>
<th>PB</th>
<th>UB</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{d\alpha_i}{dT}$</td>
<td>$\frac{2hjk-2hju+2jku}{4hjk-4hju+hku+4jku}$</td>
<td>$\frac{2hjk-2hju+2jku}{4hjk-4hju+hku+4jku}$</td>
<td>$\frac{hjk-hju+2jku}{2hjk-2hju+hku+4jku}$</td>
</tr>
<tr>
<td>$\frac{d\alpha_j}{dT}$</td>
<td>$\frac{1}{4hjk-4hju+hku+4jku}$</td>
<td>$\frac{1}{4hjk-4hju+hku+4jku}$</td>
<td>$\frac{1}{2hjk-2hju+hku+4jku}$</td>
</tr>
<tr>
<td>$\frac{d\alpha_j}{d\alpha_i}$</td>
<td>$\frac{2hju}{4hjk-4hju+hku+4jku}$</td>
<td>$\frac{hju}{4hjk-4hju+hku+4jku}$</td>
<td>$\frac{hju}{2hjk-2hju+hku+4jku}$</td>
</tr>
</tbody>
</table>

Notation: $u \equiv u''(\alpha_i^\rho)$, $h \equiv h''(g_i^\rho)$, $k \equiv k''(a_i^\rho)$, $j \equiv J''(G^\rho)$, $\rho \in (FB, PB, UB)$.

By proposition 2 it follows from $a_i^{FB} = 0$ and $t_i^{FB} = 0$, that $a_i^{UB} = 0$ and $t_i^{UB} = 0$.

---

2We abstract from possible SBC equilibria below $T_c$ because they just add complexity but qualitatively do not change results.
and by proposition 4 $a^{PB}_i = 0$, which proofs that PB and UB coincide with FB if $T \geq \bar{T}$.

### A.2.7 Summary of results for the logarithmic function

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>$FB$</th>
<th>$UB$</th>
<th>$PB$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_i^p$</td>
<td>$\frac{2L + 2w + T}{2 \alpha + \beta + \gamma + \delta}$</td>
<td>$\frac{2L + 2w + T}{2 \alpha + \beta + \gamma + \delta}$</td>
<td>$\frac{2L + 2w + T}{2 \alpha + \beta + \gamma + \delta}$</td>
</tr>
<tr>
<td>$g_i^p$</td>
<td>$\frac{2L + 2w + T}{2 \alpha + \beta + \gamma + \delta}$</td>
<td>$\frac{2L + 2w + T}{2 \alpha + \beta + \gamma + \delta}$</td>
<td>$\frac{2L + 2w + T}{2 \alpha + \beta + \gamma + \delta}$</td>
</tr>
<tr>
<td>$l_i^p$</td>
<td>$\frac{2L + 2w + T}{2 \alpha + \beta + \gamma + \delta}$</td>
<td>$\frac{2L + 2w + T}{2 \alpha + \beta + \gamma + \delta}$</td>
<td>$\frac{2L + 2w + T}{2 \alpha + \beta + \gamma + \delta}$</td>
</tr>
<tr>
<td>$G^p$</td>
<td>$\frac{2L + 2w + T}{2 \alpha + \beta + \gamma + \delta}$</td>
<td>$\frac{2L + 2w + T}{2 \alpha + \beta + \gamma + \delta}$</td>
<td>$\frac{2L + 2w + T}{2 \alpha + \beta + \gamma + \delta}$</td>
</tr>
</tbody>
</table>

### A.2.8 Proof of Proposition 6

Consider the utility difference: $\Delta(\alpha, \beta, \gamma, \delta) = U_{CG}^{PB} - U_{CG}^{UB} = \ln \frac{2^{\alpha + \beta + \gamma + \delta}}{2^{\alpha + \beta + \gamma + \delta}} - \alpha \ln 2$.

1. We substitute $(\beta + \delta)$ by $x = (\beta + \delta)$. In order to cover all possible cases, we use two specifications of $\Delta(\alpha, \beta, \gamma, \delta)$.

2. First we replace $x$, using the restriction of $\alpha + \gamma + x = 1$, by $1 - (\alpha + \gamma)$ to obtain a utility difference, which only depends on $\alpha$ and $\gamma$:

   $\Delta(\alpha, \gamma) = U_{CG}^{PB} - U_{CG}^{UB} = \ln \frac{1^{\alpha + \beta + \gamma}}{1^{\alpha + \gamma}} - \alpha \ln 2$. When is $\Delta(\alpha, \gamma)$ negative, i.e. the UB regime dominates?

   $$
   \ln \left(\frac{1^{\alpha + \beta + \gamma}}{1^{\alpha + \gamma}}\right) - \alpha \ln 2 < 0 \quad \Leftrightarrow \quad \gamma > \frac{1 + \alpha - 2^{\alpha}}{(2^{\alpha} - 1)}
   $$

   $$
   \rightarrow \min \left(\frac{1 + \alpha - 2^{\alpha}}{(2^{\alpha} - 1)}\right) = \lim_{\alpha \to 0} \frac{1 + \alpha - 2^{\alpha}}{(2^{\alpha} - 1)} = \lim_{\alpha \to 0} \frac{1 - (\ln 2)^{2^{\alpha}}}{(\ln 2)^{2^{\alpha}}} = \frac{1 - \ln 2}{\ln 2} \approx 0.44.
   $$

   Hence if $\gamma > \frac{1 - \ln 2}{\ln 2}$, the UB regime always dominates.

3. Second, we use again the restriction $\alpha + \gamma + x = 1$ to replace $\gamma$ and to obtain another specification of the utility difference, only depending on $a$ and $x$: $\Delta(\alpha, x) = \ln \frac{2^{x-a}}{2^{x-a} - \alpha \ln 2}$. When is $\Delta(\alpha, x)$ positive, i.e. PB dominates?

   $$
   \ln \frac{2^{x-a}}{2^{x-a} - \alpha \ln 2} > 0 \quad \Leftrightarrow \quad x > \frac{(2 - \alpha)2^{\alpha} - 2^{\alpha} - \alpha}{(2^{\alpha} - 1)}
   $$

   $$
   \min \left(\frac{(2 - \alpha)2^{\alpha} - 2^{\alpha} - \alpha}{(2^{\alpha} - 1)}\right) = \lim_{\alpha \to 0} \frac{(2 - \alpha)2^{\alpha} - 2^{\alpha}}{(2^{\alpha} - 1)} = \lim_{\alpha \to 0} \frac{2 - \frac{2^{\alpha}(1 + \alpha \ln(2))}{(\ln 2)^{2^{\alpha}}}}{2 - \frac{1}{\ln 2}} = 2 - \frac{1}{\ln 2} \approx 0.56.
   $$

   Hence the PB regime dominates for all parameter constellations fulfilling $(\beta + \delta) > \frac{1 - \ln 2}{\ln 2}$.
2 - \frac{1}{m^2}.

4. The last possible specification of the utility difference, i.e. $\Delta(x, \gamma)$ does not yield any additional insights and is therefore not considered.

5. To demonstrate, that there are solutions for both cases fulfilling the parameter constraints, we calculate two examples. As endowment parameters we choose:

$L = 0.5 \quad w = 0.1 \quad w_G = 0.4.$

Example 1 (dominance of UB regime):

$\gamma > 0.44 : \quad \alpha = 0.13 \quad \beta = 0.2 \quad \gamma = 0.47 \quad \delta = 0.2$

$U_{CG}^{PB} - U_{CG}^{UB} \approx -5.4 \times 10^{-3} < 0 \quad t_i^{UB} \approx 0.11 > 0 \quad a_i^{PB} \approx 0.15 > 0.$

Example 2 (dominance of PB regime):

$\beta + \delta > 0.56 : \quad \alpha = 0.1 \quad \beta = 0.3 \quad \gamma = 0.3 \quad \delta = 0.3$

$U_{CG}^{PB} - U_{CG}^{UB} \approx 4.8 \times 10^{-3} > 0 \quad t_i^{UB} \approx 0.21 > 0 \quad a_i^{PB} \approx 0.25 > 0.$
A.3 Appendix to Chapter 3

A.3.1 Regional Government Response Functions in the Partial Conditionality Case

The regional government response functions are calculated using implicit differentiation.

1. Responses of the Regional Regional Governments to Changes in \( r \)

\[
\frac{da_1}{dr} = -\frac{\delta \text{FOC}(a_1^T)}{\delta a_1} = \frac{\partial}{\partial a_1} \left( \frac{dp(u_1=u)}{\delta a_1} (u(1-b)+n_1 k(b)) - r c'(a_1^T) \right) = -\frac{c'(a_1^T)}{r c''(a_1^T)} < 0
\]

\[
\frac{da_1}{dr} = -\frac{\delta \text{FOC}(a_1^H)}{\delta a_1} = \frac{\partial}{\partial a_1} \left( \frac{dp(u_1=u)}{\delta a_1} (u(1-b)+n_1 k(b)) - r c'(a_1^H) \right) = -\frac{c'(a_1^H)}{r c''(a_1^H)} = 0
\]

2. Responses of the Regional Regional Governments to Changes in \( n_1 \)

\[
\frac{dp(u_1=u)}{da_1} \frac{k(b)}{pc''(a_1)} > 0
\]

\[
\frac{dp(u_1=u)}{da_1} \frac{k(b)}{pc''(a_1)} = \frac{\partial}{\partial a_1} \left( \frac{dp(u_1=u)}{\delta a_1} (u(1-b)+n_1 k(b)) - r c'(a_1^T) \right) = -\frac{c'(a_1^T)}{r c''(a_1^T)} < 0
\]

\[
\frac{dp(u_1=u)}{da_1} \frac{k(b)}{pc''(a_1)} = \frac{\partial}{\partial a_1} \left( \frac{dp(u_1=u)}{\delta a_1} (u(1-b)+n_1 k(b)) - r c'(a_1^H) \right) = -\frac{c'(a_1^H)}{r c''(a_1^H)} = 0
\]

3. Responses of the Regional Regional Governments to Changes in \( b \)

\[
\frac{dp(u_1=u)}{db} \frac{n_1 k''(b)}{pc''(a_1)} > 0
\]

\[
\frac{dp(u_1=u)}{db} \frac{n_1 k''(b)}{pc''(a_1)} = \frac{\partial}{\partial b} \left( \frac{dp(u_1=u)}{\delta b} (u(1-b)+n_1 k(b)) - r c'(a_1^T) \right) = -\frac{c'(a_1^T)}{r c''(a_1^T)} < 0
\]

We calculate also the derivatives of the response functions because they are needed for calculating the comparative statics of the bailouts.
\[
\frac{d a_{n1}}{d n_1} = \frac{d \left( \frac{d p(u_1 = u)}{d a_{n1}} \right)}{d r}\left( -\frac{p u_1}{\rho u_1} \right) = \frac{d p(u_1 = u)}{d a_{n1}} \frac{k'(b)}{\rho u_1} = \frac{da_{n1}}{db} \frac{u - n_{1} k'(b)}{u - n_{1} k'(b)} > 0
\]

A.3.2 Bailout Responses to Parameter Changes in the Partial Conditionality Case

For the following calculations, we denote the first order condition of the central government objective function with respect to the bailout: \( FOC \ (b) \) and the second order derivative \( SOD \ (b) \).

1. Response function w.r.t. \( r \)

\[
\frac{d b}{d r} = - \frac{\frac{d FOC(b)}{d r}}{SOD(b) < 0}
\]

\[
= - \frac{d \left( (u - k' (b)) E_{F} [p(u_1 = 0)] + E_{F} \left[ \frac{dp(u_1 = 0)}{da_{n1}} \frac{d a_{n1}}{d r} \frac{-n_{2} k(b)}{1} \right] \right)}{SOD(b) < 0}
\]

\[
= - \frac{E_{F} \left[ \frac{dp(u_1 = 0)}{da_{n1}} \frac{d a_{n1}}{d r} \right] + E_{F} \left[ \frac{dp(u_1 = 0)}{da_{n1}} \frac{d a_{n1}}{d r} \frac{-n_{2} k(b)}{1} \right]}{SOD(b) < 0} > 0
\]

Proof:

From appendix 1 we know: \( \frac{d a_{n1}}{d r} < 0 \) and \( \frac{dp(u_1 = 0)}{da_{n1}} < 0 \) by assumption. From FOC \( (b) \) we know that \( b < b^* \) and consequently \( k'(b) < k'(b^*) = u \), which implies that: \( (u - k'(b)) > 0 \).

Using the response functions from appendix 1, it is possible to calculate:

\[
E_{F} \left[ \frac{d a_{n1}}{d r} \right] = \pi \frac{d a_{n1}}{d r} + (1 - \pi) \frac{d a_{n1}}{d r} = \pi \cdot 0 + (1 - \pi) \left( -\frac{da_{n1}}{db} \right) > 0
\]

2. Response function w.r.t. \( n_1 \)

\[
\frac{d b}{d n_1} = - \frac{\frac{d FOC(b)}{d n_1}}{SOD(b) < 0}
\]

\[
= - \frac{d \left( (u - k' (b)) E_{F} [p(u_1 = 0)] + E_{F} \left[ \frac{dp(u_1 = 0)}{da_{n1}} \frac{d a_{n1}}{d n_1} \frac{-n_{2} k(b)}{1} \right] \right)}{SOD(b) < 0}
\]

\[
= \frac{E_{F} \left[ \frac{dp(u_1 = 0)}{da_{n1}} \frac{d a_{n1}}{d n_1} \frac{n_{2} k(b)}{1} \right] + E_{F} \left[ \frac{dp(u_1 = 0)}{da_{n1}} \frac{d a_{n1}}{d n_1} \frac{-k(b)}{1} \right]}{-SOD(b)} > 0
\]

Proof:

From FOC \( (b) \) we know that \( b < b^* \) and consequently \( k'(b) < k'(b^*) = u \), which
implies that: \((u - k'(b)) > 0\). \(\frac{dp(u_1=0)}{da_1}\) is negative by assumption. From appendix 1, we know \(\frac{da_1}{dn_1} > 0\) and \(\frac{da_1}{db} < 0\) and \(\frac{dn_1}{db} > 0\).

Since the terms in the FOC are ambiguous, we have to summarize the terms using the response functions derived in appendix 1:

\[
\frac{da_1}{dn_1} = \frac{dp(u_1=0)}{da_1} \frac{k(b)}{\rho c'(a_1)} - \frac{dp(u_1=0)}{da_1} \frac{k'(b)}{\rho c'(a_1)} = \frac{da_1}{dn_1} - \frac{dn_1}{db} \frac{k'(b)}{\rho c'(a_1)}
\]

\[
\frac{d^2a_1}{dn_1} = \frac{dp(u_1=0)}{da_1} \frac{k''(b)}{\rho c''(a_1)} - \frac{dp(u_1=0)}{da_1} \frac{k''(b)}{\rho c''(a_1)} = \frac{da_1}{dn_1} - \frac{dn_1}{db} \frac{k''(b)}{\rho c''(a_1)}
\]

plug this in:

\[
\frac{\partial FOC(b)}{\partial n_1} = E_p \left[ \frac{dp(u_1=0)}{da_1} \frac{k(b)}{\rho c'(a_1)} + \frac{dp(u_1=0)}{da_1} \frac{k'(b)}{\rho c'(a_1)} \right]
\]

\[
\frac{\partial FOC(b)}{\partial n_1} = E_p \left[ \frac{dp(u_1=0)}{da_1} \frac{k(b)}{\rho c'(a_1)} - \frac{dp(u_1=0)}{da_1} \frac{k'(b)}{\rho c'(a_1)} \right]
\]

\[
\frac{\partial FOC(b)}{\partial n_1} = E_p \left[ \frac{dp(u_1=0)}{da_1} \frac{k(b)}{\rho c'(a_1)} - \frac{dp(u_1=0)}{da_1} \frac{k'(b)}{\rho c'(a_1)} \right]
\]

We can conclude:

\[
\frac{db}{dn_1} = -\frac{\partial FOC(b)}{\partial n_1} \frac{SOD(b) < 0}{SOD(b) < 0} = \frac{E_p \left[ \frac{dp(u_1=0)}{da_1} \frac{k(b)}{\rho c'(a_1)} - \frac{dp(u_1=0)}{da_1} \frac{k'(b)}{\rho c'(a_1)} \right] > 0}{SOD(b) < 0} > 0
\]

3. Response function w.r.t. \(\pi\)

For notational convenience, we denote the probabilities that hold for the high cost shock, and consequently with the low effort choices by \(p^L\) and for the high effort choices \((\rho = 1)\) with \(p^H\).

\[
\frac{db}{d\pi} = -\frac{\partial FOC(b)}{\partial \pi} \frac{SOD(b) < 0}{SOD(b) < 0} = \frac{\partial \left( (u - k'(b))E_p[p(u_1=0)] + E_p \left[ \frac{dp(u_1=0)}{da_1} \frac{k(b)}{\rho c'(a_1)} \right] \right)}{SOD(b) < 0}
\]

\[
\frac{\partial FOC(b)}{\partial \pi} = \frac{(u - k'(b))}{1} dE_p[p(u_1=0)] \frac{d\pi}{dE_p[p(u_1=0)]} + \frac{dE_p \left[ \frac{dp(u_1=0)}{da_1} \frac{k(b)}{\rho c'(a_1)} \right]}{d\pi} \frac{d\pi}{SOD(b) < 0}
\]

\[
\frac{\partial FOC(b)}{\partial \pi} = \frac{(u - k'(b))}{1} d\pi \left[ p^H(u_1=0) - p^L(u_1=0) \right] + \frac{d\pi \left[ \frac{dp^H(u_1=0)}{da_1} \frac{k(b)}{\rho c'(a_1)} \right]}{d\pi} \frac{d\pi}{SOD(b) < 0}
\]

from \(\frac{dp^H(u_1=0)}{da_1} \frac{da_1^H}{db} = \frac{dp^L(u_1=0)}{da_1} \frac{da_1^L}{db}\)

\[
\frac{\partial FOC(b)}{\partial \pi} = \frac{(u - k'(b))}{1} \frac{dp^H(u_1=0)}{da_1} \frac{da_1^H}{db} - \frac{dp^L(u_1=0)}{da_1} \frac{da_1^L}{db} \frac{d\pi}{SOD(b) < 0}
\]


\[
\frac{\partial FOC(b)}{\partial \pi} = \frac{(u-k'(b))}{1} p^H(u_1=0) - p^L(u_1=0)
\]
\[
+ \frac{dp(u_1=0)}{da_1} \left( \left( - \frac{dp(u_1=0)}{da_1} u-n_1 k'(b) \right) \frac{c''(a_1)}{r} \right) \frac{(-n_2 k(b))}{1} \]
\[
\frac{\partial FOC(b)}{\partial \pi} = \frac{(u-k'(b))}{1} p^H(u_1=0) - p^L(u_1=0)
\]
\[
+ \frac{dp(u_1=0)}{da_1} \left( \left( - \frac{dp(u_1=0)}{da_1} u-n_1 k'(b) \right) \frac{r}{r} \right) \frac{(-n_2 k(b))}{1} \]
\[
\frac{\partial FOC(b)}{\partial \pi} = \frac{(u-k'(b))}{1} p^H(u_1=0) - p^L(u_1=0)
\]
\[
\frac{dp(u_1=0)}{da_1} \begin{cases} \frac{2}{1} & \text{if } r > 0 \\ \frac{1}{1} & \text{if } r < 0 \end{cases} \left( \frac{r - 1}{r} \right) \frac{u-n_1 k'(b)}{c''(a_1)} \frac{(-n_2 k(b))}{1} < 0
\]
\[
\frac{db}{d\pi} = -\frac{\frac{\partial FOC(b)}{\partial \pi}}{SOD(b)<0} < 0
\]

4. Second Order Condition:

\[
SOD(b) = \frac{d^2F(b)}{db^2} = \frac{\partial}{\partial b} \left( \frac{\partial}{\partial b} (u-k'(b)) E_p[p(u_1=0)] + E_p \left[ \frac{dp(u_1=0)}{da_1} \frac{da_1}{db} \right] \frac{(-n_2 k(b))}{1} \right)
\]
\[
= \frac{d(u-k'(b))}{db} E_p[p(u_1=0)] + \frac{d(u-k'(b))}{db} E_p \left[ \frac{dp(u_1=0)}{da_1} \frac{da_1}{db} \right] + E_p \left[ \frac{dp(u_1=0)}{da_1} \frac{da_1}{db} \right] \frac{(-n_2 k(b))}{1}
\]
\[
+ E_p \left[ \frac{dp(u_1=0)}{da_1} \frac{da_1}{db} \right] \frac{(-n_2 k(b))}{1}
\]

from Appendix 1, 3.

\[
\frac{d^3a}{db^3} = \frac{da_1}{db} \frac{-n_1 k''(b)}{(u-n_k'(b))} > 0
\]
\[
= -k''(b) E_p[p(u_1=0)] + \frac{d(u-k'(b))}{db} E_p \left[ \frac{dp(u_1=0)}{da_1} \frac{da_1}{db} \right] + E_p \left[ \frac{dp(u_1=0)}{da_1} \frac{da_1}{db} \right] \frac{(-n_2 k(b))}{1}
\]
\[
+ E_p \left[ \frac{dp(u_1=0)}{da_1} \frac{da_1}{db} \right] \frac{(-n_2 k(b))}{1}
\]
\[
= \left( \frac{d^2a}{db^2} \right) \frac{d^2a}{db^2} \frac{(-n_2 k(b))}{1} \frac{(-n_2 k(b))}{1}
\]
\[
= -k''(b) E_p[p(u_1=0)] \left( 1 - \frac{E_p \left[ \frac{dp(u_1=0)}{da_1} \frac{da_1}{db} \right] \frac{(-n_2 k(b))}{1} \frac{(-n_2 k(b))}{1} \right)
\]
\[
+ E_p \left[ \frac{dp(u_1=0)}{da_1} \frac{da_1}{db} \right] \frac{(-n_2 k(b))}{1}
\]

from FOC (b):

\[
E_p \left[ \frac{dp(u_1=0)}{da_1} \frac{da_1}{db} \right] = \frac{u-k'(b)}{n_k(b)}
\]
\[
= \left( \frac{d^2a}{db^2} \right) \frac{d^2a}{db^2} \frac{(-n_2 k(b))}{1} \frac{(-n_2 k(b))}{1}
\]
\[
= E_p[p(u_1=0)] \left( \frac{k''(b)(1-n_1)u}{(u-n_k'(b)^2)} \right) + E_p \left[ \frac{dp(u_1=0)}{da_1} \frac{da_1}{db} \right] \left( \frac{(u-k'(b))}{1} - \frac{n_2 k(b)}{1} \right)
\]
\[
= E_p[p(u_1=0)] \left( \frac{k''(b)(1-n_1)u}{(u-n_k'(b)^2)} \right) - E_p \left[ \frac{dp(u_1=0)}{da_1} \frac{da_1}{db} \right] \left( \frac{(2-n_1)k'(b)^2}{1} \right) < 0
\]
A.3.3 Mathematica Code for Comparative Statics w.r.t. $Q$ and $n_1$ in the Partial Conditionality Case

\[
\begin{align*}
&\text{ContourPlot}\left[\left\{\begin{array}{l}
(1-2\ b) - \frac{Q (1-2\times0.1\ b)}{3-Q (1-0.1\ b^2 + 1)} = 0, \\
(1-2\ b) - \frac{Q (1-2\times0.5\ b)}{4-Q (1-0.5\ b^2 + 1)} = 0,
\end{array}\right\}, (0, 0, 2), (b, 0.42, 0.5)\right]\end{align*}
\]

A.3.4 Regional Government Response Functions in the Full Conditionality Case

We use implicit differentiation of the regional governments’ first order derivatives w.r.t. their effort choices for calculating the response of effort choices to changes of the bailouts $b_u$ and $b_0$.

The first order conditions for $a_1$ and $a_2$ are denoted $FOC\ (a_1)$ and $FOC\ (a_2)$, respectively.

\[FOC\ (a_1):\]
\[0 = \left(\frac{dp(u_1=u)}{da_1}u + \frac{dp(0,u)}{da_1}(ub_u - n_1k(b_u)) + \frac{dp(0,0)}{da_1}(ub_0 - n_1k(b_0))\right) - \rho c'\ (a_1)\]

\[FOC\ (a_2):\]
\[0 = \frac{dp(u_2=u)}{da_2}u + \frac{dp(0,u)}{da_2}(n_1k(b_0) - k(b_u)) - \rho c'\ (a_2)\]

Total differentiation of the first order condition w.r.t. $b_u$ and $b_0$ yields the following results:

\[
\begin{align*}
\frac{da_1}{db_u} &= \frac{\partial FOC(a_1)}{\partial a_1} \frac{\partial FOC(a_2)}{\partial a_2} - \frac{\partial FOC(a_1)}{\partial a_1} \frac{\partial FOC(a_2)}{\partial a_2}
\frac{da_1}{db_0} &= \frac{\partial FOC(a_1)}{\partial a_1} \frac{\partial FOC(a_2)}{\partial a_2} - \frac{\partial FOC(a_1)}{\partial a_1} \frac{\partial FOC(a_2)}{\partial a_2}
\frac{da_2}{db_u} &= \frac{\partial FOC(a_2)}{\partial a_2} \frac{\partial FOC(a_1)}{\partial a_1} - \frac{\partial FOC(a_2)}{\partial a_2} \frac{\partial FOC(a_1)}{\partial a_1}
\frac{da_2}{db_0} &= \frac{\partial FOC(a_2)}{\partial a_2} \frac{\partial FOC(a_1)}{\partial a_1} - \frac{\partial FOC(a_2)}{\partial a_2} \frac{\partial FOC(a_1)}{\partial a_1}
\end{align*}
\]

We are interested in the sign of the responses. At first we determine the sign of the denominator, which is equal for all response functions:

\[
\begin{align*}
\frac{\partial FOC(a_1)}{\partial a_1} \frac{\partial FOC(a_2)}{\partial a_2} - \frac{\partial FOC(a_1)}{\partial a_2} \frac{\partial FOC(a_2)}{\partial a_1} = \\
\left(-\rho c''\ (a_1)\right) - \left(-\rho c''\ (a_2)\right) - \left(\frac{d}{da_1} \left(\frac{dp(0,0)}{da_1} (ub_0 - n_1k(b_0)) + (ub_u - n_1k(b_u))\right)\right) - \left(\frac{d}{da_1} \left(\frac{n_1k(b_0) - k(b_u)}{1}\right)\right)
\end{align*}
\]
Note that from \( \frac{dp(u_1=0)}{da_2} = 0 \) follows: \( \frac{dp(0,0)}{da_2} + \frac{dp(0,u)}{da_2} = 0 \) and \( \frac{dp(0,u)}{da_2} = -\frac{dp(0,0)}{da_2} \).

Using this condition and from Young’s Theorem \( f_{xy} = f_{yx} \) we can conclude:

\[
\frac{d}{da_1} \frac{dp(u)}{da_2} = \frac{d}{da_1} \frac{dp(0)}{da_2} = -\frac{d}{da_2} \frac{dp(0)}{da_1}
\]

It follows:

\[
\frac{\partial FOC(a_1)}{\partial a_1} \frac{\partial FOC(a_2)}{\partial a_2} - \frac{\partial FOC(a_1)}{\partial u_1} \frac{\partial FOC(a_2)}{\partial u_2} = \rho^2 c'' (a_1) c'' (a_2) + \\
\left( \frac{d}{da_2} \frac{dp(0)}{da_2} \right)^2 \frac{1}{(ub_0 - n_1 k (b_0)) - (ub_n - n_1 k (b_n))} \frac{1}{n_1 (k (b_0) - k (b_n))} > 0
\]

Since the denominator is positive, the sign of the response function is equal to the sign of the numerator.

**Numerator of \( \frac{da_1}{db_0} \):**

\[
um \left( \frac{da_1}{db_0} \right) = \frac{\partial FOC(a_1)}{\partial a_1} \frac{\partial FOC(a_2)}{\partial a_2} - \frac{\partial FOC(a_1)}{\partial u_1} \frac{\partial FOC(a_2)}{\partial u_2}
\]

\[
= \left( \frac{d}{da_2} \frac{dp(0)}{da_2} \frac{1}{(ub_0 - n_1 k (b_0)) - (ub_n - n_1 k (b_n))} \right) \left( \frac{dp(0,u)}{da_2} \frac{n_1 k' (b_0)}{1} - \left( \frac{dp(0,u)}{da_1} \frac{u-n_1 k' (b_0)}{1} \right) \left( \frac{dp(0,u)}{da_2} \frac{n_1 k' (b_n)}{1} \right) \right)
\]

The numerator is unambiguously negative if \( b_u > b_0 \) and might be positive if \( b_0 >> b_u \).

**Numerator of \( \frac{da_1}{db_u} \):**

\[
um \left( \frac{da_1}{db_u} \right) = \frac{\partial FOC(a_1)}{\partial a_1} \frac{\partial FOC(a_2)}{\partial a_2} - \frac{\partial FOC(a_1)}{\partial u_1} \frac{\partial FOC(a_2)}{\partial u_2}
\]

\[
= \left( \frac{d}{da_2} \frac{dp(0)}{da_2} \frac{1}{(ub_0 - n_1 k (b_0)) - (ub_n - n_1 k (b_n))} \right) \left( \frac{dp(0,u)}{da_2} \frac{n_1 k' (b_0)}{1} - \left( \frac{dp(0,u)}{da_1} \frac{u-n_1 k' (b_0)}{1} \right) \left( \frac{dp(0,u)}{da_2} \frac{n_1 k' (b_n)}{1} \right) \right)
\]

The numerator is unambiguously negative if \( b_0 > b_u \) and might be positive if \( b_u >> b_0 \).

**Numerator of \( \frac{da_2}{db_0} \):**
The numerator is unambiguously positive if \( b_0 > b_u \) and might be negative if \( b_u >> b_0 \).

**Numerator of \( \frac{d\alpha_2}{d\theta} \):**

\[
num \left( \frac{d\alpha_2}{d\theta} \right) = \frac{\partial \alpha_2}{\partial \theta} \left( \frac{\partial \alpha_1}{\partial u} \right) - \frac{\partial \alpha_2}{\partial \theta} \left( \frac{\partial \alpha_1}{\partial u} \right) \\
= \left( \frac{d\alpha_2}{d\theta} \right) \left( \frac{\partial \alpha_1}{\partial u} \right) \left( \frac{\partial \alpha_1}{\partial u} \right) - \left( \frac{d\alpha_2}{d\theta} \right) \left( \frac{\partial \alpha_1}{\partial u} \right) \left( \frac{\partial \alpha_1}{\partial u} \right)
\]

The numerator is unambiguously negative if \( b_u > b_0 \) and might be positive if \( b_0 >> b_u \).

### A.3.5 Proof of \( b_0 > b_u \Leftrightarrow E_\rho \left[ p(0,0) \right] > E_\rho \left[ p(0, u) \right] \)

To show our result, we first undertake a variable transformation of our problem. We set \( b_u = \frac{u\beta(1-\alpha)}{2} \) and \( b_0 = \frac{u\beta(1+\alpha)}{2} \). The variable \( \alpha \) represents the relative bailout difference between the bailouts \( b_0 \) and \( b_u \). The variable \( \beta \) denotes the size of the second best optimal bailout relative to the first best bailout. We have \( b_0 > b_u \Leftrightarrow \alpha > 0 \).

The optimality condition derived for \( \alpha \) is the following:

\[
0 = \frac{1}{4} n_1 \beta \left( \frac{1-\beta}{1-Q} \right) - \frac{\alpha \beta}{4} \left( \frac{1-2(1+Q)\beta n_1}{1-2(1+Q)\beta n_1} \right) - \frac{1}{4} \left( \frac{1-2(1+Q)\beta n_1}{1-2(1+Q)\beta n_1} \right)
\]

\( \Leftrightarrow \alpha = \frac{1-\beta}{\beta} \left( \frac{1-2(1+Q)\beta n_1}{1-2(1+Q)\beta n_1} \right) \)

We show in a first step that the denominator \( (3 - Q) + \frac{R \beta}{4} \left( \frac{2(1+2n_1)}{1-2(1+Q)\beta n_1} \right) \) is positive:
\[
\frac{d}{dn_1} \left( \frac{2(1+2n_1) - \beta n_1}{1} \cdot \frac{2(a^2+2) + n_1(1-a^2)}{1} \right) = \frac{4}{1} \left( \frac{1-\beta}{1} - \frac{\beta n_1 + a^2(1-n_1)}{1} \right) \\
\frac{d^2}{dn_1^2} \left( \frac{\beta n_1}{1} \cdot \frac{n_1(1-a^2) + 2a^2 + 4 - 2(1+2n_1)}{1} \right) = \frac{d}{dn_1} \left( \frac{1-\beta}{1} - \frac{\beta n_1 + a^2(1-n_1)}{1} \right) = -2\beta \frac{1-a^2}{1} < 0
\]

⇒ we have a minimum either at \( n_1 = 0 \) or \( n_1 = 1 \):
for \( n_1 = 0 \):
\[
\frac{2(1+2n_1) - \beta n_1}{1} \cdot \frac{2(a^2+2) + n_1(1-a^2)}{1} = 2 \text{ is true}
\]
for \( n_1 = 1 \):
\[
\frac{2(1+2n_1) - \beta n_1}{1} \cdot \frac{2(a^2+2) + n_1(1-a^2)}{1} = 6 - \beta (5 + a^2)
\]

From the assumption \( k'(1) < u \), i.e. that it is not efficient to pay bailouts that are larger than the maximal outcome a regional government can achieve on its own, it follows that \( \beta < 1 \). Moreover, \( b > 0 \) prohibits that \( \alpha > 1 \). This implies that \( 6 - \beta (5 + a^2) > 0 \).

In addition, we need the parameter restrictions that result from \( a_2^{FB} \leq 1 \) and from the maximal and minimal values for the parameter restrictions, we can derive an maximal value for \( Q = \frac{\pi r + (1-\pi)}{2\gamma} \):

The values of \( \pi \) and \( r \) maximizing \( Q \):
\[
\max_{\pi=1, r=1} \frac{\pi r + (1-\pi)}{2\gamma} = \frac{1}{2\gamma}
\]

Furthermore, from: \( a_2^{FB} \leq 1 \) follows: \( \frac{1}{4\gamma} \leq 1 \rightarrow \gamma \geq \frac{1}{4\gamma} \), for \( r = 1 : \gamma \geq \frac{1}{4} \).

This implies an maximal value of \( Q \) equal to:
\[
\max_{\pi=1, r=1, \gamma=0.25} \frac{\pi r + (1-\pi)}{2\gamma} = 2.
\]

From \( \frac{2(1+2n_1) - \beta n_1}{1} \cdot \frac{2(a^2+2) + n_1(1-a^2)}{1} > 0 \) and \( Q < 2 \), we conclude \( (3-Q) + \frac{Q}{4} \left( \frac{2(1+2n_1) - \beta n_1}{1} \cdot \frac{2(a^2+2) + n_1(1-a^2)}{1} \right) > 0 \).

This implies that \( \alpha > 0 \), i.e. \( b_0 > b_u \iff Q < 1 \).

In a second step, we establish the relationship of \( \alpha \) with the expected probabilites \( E_\rho [p(0,0)] > E_\rho [p(0, u)] \). For the transformed problem, we obtain in the full conditionality equilibrium the following values for the probabilities:
\[
E_\rho [p(0,0)] = \frac{1}{4} \left( 2 + \frac{Q}{4} \beta \frac{2n_1 \beta (1-\alpha)^2}{1} - Q \right) \\
E_\rho [p(0, u)] = \frac{1}{4} \left( 1 + \frac{Q}{4} \beta \frac{2n_1 \beta (1-\alpha)^2}{1} \right) \\
E_\rho [p(0,0)] - E_\rho [p(0, u)] = \frac{1}{4} \left( 1 - Q - \frac{Q}{4} \beta \frac{2n_1 \alpha}{1} \right)
\]
For $Q = 1$ and $\alpha = 0$ : $E_\rho [p (0, 0)] - E_\rho [p (0, u)] = 0$

Since $\beta < 1$, the unique root of $\alpha$ is at $Q = 1$. Moreover from

$(E_\rho [p (0, 0)] - E_\rho [p (0, u)]) > 0$ at $Q = 0$ and $(E_\rho [p (0, 0)] - E_\rho [p (0, u)]) < 0$ at $Q = 2$, we can conclude that $\alpha > 0$, i.e. $b_0 > b_u \Leftrightarrow (E_\rho [p (0, 0)] - E_\rho [p (0, u)]) > 0$.

Note:

For $Q = 0$ we have:

$E_\rho [p (0, 0)] - E_\rho [p (0, u)] = \frac{1}{4} > 0$

For $Q = 2$ we have:

$E_\rho [p (0, 0)] - E_\rho [p (0, u)] = -\frac{1}{4} \frac{1+2 \beta}{1+2 \beta} \frac{2(1+n_1)}{1- \beta(1+n_1) \left(1 - \frac{2(\alpha^2+1)n_1(1-\alpha^2)}{1} \right)} < 0$

where:

$\left( \frac{2(1+n_1)}{1} - \frac{\beta n_1}{1} \right) > 0$

A.3.6 Appendix 6: Mathematica Code for Comparative Statics w.r.t. $Q$ in the Full Conditionality Case

```mathematica
Clear[a, b]
u = 1
n = 2
Solve[[u, n] :=
{b (1 - a) ^ 2, b (1 - a) ^ 2} /.
ToRules[[
Reduce\\n[[
1 - b (1 - a) ^ 2, 3 - 0 * u , 8 * (2 b - 1) (1 - 2 u), 0 * b u ^ 2, b u ^ 2 (2 b - 1) (1 - a) ^ 2, 6 (a u) ^ 2 (2 b - 1), 2 - 3 b (1 - a) ^ 2) \rightarrow 0, 
(1 - 0 * u) (1 - b) - a (1 b (3 - 2 * u) - 1 ^ 4 b u * b - (2 (b + b (1 - a) ^ 2) + 2 a ^ 2 b) - 2 (1 + 2 a)) \rightarrow 0.66 \frac{1}{2} \leq a \leq \frac{1}{2} 6 \leq \beta < 1], 
{a, b}, Reals, Backsubstitution -> True]]]]
Plot[Solve[[u, n] (0, 2), PlotPoints -> 20, MaxRecursion -> 0]
```
References


Curriculum Vitae

November 2007 - Februar 2008
Research Stay at the Centre d’Economie de la Sorbonne Université Paris I

October 2005 - September 2009
Doctoral Studies, Munich Graduate School of Economics

May 2003 - September 2005
Research Assistant, Institute of Finance, University of Leipzig

August 2002 - January 2003
Economics (Erasmus), University of Lund, Sweden

October 1998 - April 2003
Economics, University of Leipzig

July 1998
Abitur, Wilhelm-Ostwald Gymnasium Leipzig