Frameworks for the Theoretical and Empirical Analysis of Monetary Policy

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CHAPTER 1

Introduction and summary

Monetary policy has been very successful in most countries in recent years. Average inflation rates have declined considerably since the 1980s. Furthermore, a number of authors such as Stock and Watson (2002) and Martin and Rowthorn (2005) also attribute the observed decline in macroeconomic volatility, i.e. in the variance of inflation and output, at least partly to better monetary policy.

But the last decade has not passed without new challenges for central banking in theory and practice. In theory, the New Keynesian or New Neoclassical Synthesis model became the standard workhorse for monetary macroeconomics and some of its most prominent proponents such as Woodford (2003) argued for a new 'timeless-perspective' approach to policy as the allegedly optimal monetary policy. In practice, the creation of the European Monetary Union, with the European Central Bank (ECB) being responsible for monetary policy since 1999, represented an enormous challenge for policy-makers in 'unchartered waters' (Duisenberg, 1998). Furthermore, the world faced several severe liquidity crises on financial markets that threatened the stability of the financial system. This thesis develops and applies three different frameworks to analyse these challenges in detail within three self-contained chapters.

Besides the focus on frameworks for monetary policy analysis, the special role of rules represents another unifying theme for all three essays. Chapter 2 investigates the optimality of the timeless perspective rule in the New Keynesian model and chapter 3 uses Taylor rules to examine if the ECB conducted a stabilising monetary policy with respect to inflation and output. Finally, chapter 4 studies the role of the liquidity provision principle as an optimal response to liquidity crises on financial markets.

The debate about rules in monetary policy dates at least back to the beginning of the 19th century as reported in Flandreau (2006). Wicksell (1898) wrote a comprehensive treatise of monetary policy emphasising an interest rate rule that provides the basis for the modern analysis in Woodford (2003). After the Great Depression, Simons (1936, p. 30) argued in a similar vein as Wicksell (1898, p. 4) that

> [a] monetary rule of maintaining the constancy of some price-index, preferably an index of prices of competitively produced commodities, appears to afford the only promising escape from present monetary chaos and uncertainties.

While this proposal comes already very close to current mandates of most central banks, the thinking about rules versus discretion after the rational expectations revolution in macroeconomics in the 1970s has been mainly shaped by Kydland and Prescott (1977): Since private agents include expectations about future policies in their current actions, discretionary monetary policy that follows optimal control theory results in suboptimal economic outcomes. Hence, rule-based policy-making can increase welfare.

The timeless perspective proposed by Woodford (1999, 2003) represents a prominent modern form of such a rule in monetary policy analysis. It helps to overcome not only the traditional inflation bias in the sense of Barro and Gordon (1983), but also the stabilisation bias, a dynamic loss stemming from cost-push shocks in the New Keynesian model as described in Clarida, Galí and Gertler (1999). These represent the long-run gains from rule-based policy-making in the New Keynesian model.

Chapter 2 shows, however, that the timeless perspective is associated with short-run costs because the monetary authority demonstrates its commitment to the timeless perspective by not exploiting given inflation expectations in the initial period. Instead, it follows a policy 'to which it would have been optimal to commit to at a date far in the past,' i.e. it behaves as if the policy rule had been in place already for a long time. This policy is strategically coherent because it avoids any initial period effects that are one reason for the time inconsistency of standard commitment solutions, but it is initially suboptimal. These short-run costs from the timeless perspective are the price to pay to make the commitment to it arguably more credible than an overall optimal commitment solution that exploits given inflation expectations. Using this framework, chapter 2 analyses under which circumstances these short-run costs exceed the long-run gains from commitment.

After deriving a formal condition for the superiority of discretion over the timeless perspective rule, I investigate the influence of structural and preference parameters on the performance of monetary policy both under discretion and the timeless perspective. Discretion gains relatively to the timeless perspective rule, i.e. the short-run losses become relatively more important, if the private sector behaves less forward-looking or if the monetary authority puts a greater weight on output gap stabilisation. For empirically reasonable values of price stickiness, the relative gain from discretion rises with stickier prices. A fourth parameter which influences the relative gains is the persistence of shocks: The introduction of serial correlation into the model only strengthens the respective relative performance of policies in the situation without serial correlation in shocks. In particular, I show conditions for each parameter under which discretion performs strictly better than the timeless perspective rule.

Furthermore, the framework of short-run losses and long-run gains also allows explaining why an economy that is sufficiently far away from its steady-state suffers rather than gains from implementing the timeless perspective rule. In general, chapter 2 uses unconditional expectations of the loss function as welfare criterion, in line with most of the literature. The analysis of initial conditions, however, requires reverting to expected losses conditional on the initial state of the economy because unconditional expectations of the loss function implicitly treat the economy's initial conditions as stochastic. Altogether, in the normal New Keynesian model all conditions for the superiority of discretion need not be as adverse as one might suspect.

Finally, I introduce an 'optimal' timeless policy rule based on Blake (2001) and Jensen and McCallum (2002). While the general influence of structural and preferences parameters on the performance of monetary policy under this rule is not affected, discretion is never better than this rule when evaluated with unconditional expectations as it is common in the literature on monetary policy rules. The reason is that this allegedly optimal rule optimally accounts for the use of unconditional expectations as the welfare criterion. For *any* timeless rule, however, initial conditions can be sufficiently adverse to make the rule inferior to discretion.

As a policy conclusion of chapter 2, the timeless perspective in its standard formulation is not optimal for all economies at all times. In particular, if an economy is characterised by rigid prices, a low discount factor, a high preference for output stabilisation or a sufficiently large deviation from its steady state, it should prefer discretionary monetary policy over the timeless perspective. The critical parameter values obtained in this paper with the simplest version of the New Keynesian model suggest that – for a number of empirically reasonable combinations of parameters – the long-run losses from discretion may be less relevant than previously thought. Furthermore, the short-run costs in this paper can be interpreted as a lower bound for the actual costs because they are derived under the assumption of full credibility of the monetary authority. Incomplete credibility would raise the costs from commitment even further, since it takes some time until the central bank can reap the full gains from commitment.

Another important theoretical result of the New Keynesian literature model is that monetary policy can and should stabilise the inflation rate around its target rate and real output around its 'natural' level, i.e. the level in the absence of nominal rigidities. For example Woodford (2003) shows that the Taylor-rule developed by Taylor (1993) fulfills both stabilisation objectives as it implies countercyclical real interest rates in response to deviations of inflation and output from their respective target values. In particular, the so-called 'Taylor-principle' states that the central bank should increase the nominal interest rate by more than one for one in response to an increase of inflation in order to raise the real interest rate. A specific advantage of the Taylor principle is its robustness in a wide range of different theoretical models.

Over the last decade, this simple instrument policy rule has become a popular framework for evaluating monetary policy of the Federal Reserve and other central banks. Chapter 3, which is joint work with Jan-Egbert Sturm, presents one of the first empirical studies of actual monetary policy in the euro area. By estimating several instrument policy reaction functions for the ECB, we look back over the 'Duisenberg-era' and explore what role the output gap has played in actual ECB policy and how actively the ECB has really responded to changes in inflation. We compare these results with those for the Bundesbank in order to get a clearer picture of the new institutional monetary setting in Europe.[1]

Looking at contemporaneous Taylor rules, the presented evidence clearly confirms previous research and suggests that the ECB is accommodating changes in inflation and hence follows a destabilising policy. The differences between the Bundesbank and the ECB are significant. Such an interpretation gives rise to the conjecture that the ECB follows a policy quite similar to the pre-Volcker era of US monetary policy, a time also known as the 'Great Inflation' (Taylor, 1999).

One focus of chapter 3 refers to data uncertainties faced by policy-makers. They base their decisions upon data which will most likely be revised in the future. Yet most studies on central bank behaviour neglect this issue and use so-called 'current' or 'ex-post' data, i.e. data published in the latest release, to estimate monetary policy rules. In reality, central bankers can only use so-called 'real-time' data, i.e. data available when taking the decision. In his influential paper, Orphanides (2001)

[1]Since the ECB is a supranational institution and can set only one interest rate for the whole euro area, it is a 'natural consequence' that the ECB defined its mandate of price stability in terms of overall inflation in the euro area (ECB, 2004, p. 51). Hence, its policy can only be reasonably assessed in chapter 3 with data for the euro area aggregate. The consequences of nationally heterogenous inflation rates for the economic development of member states are discussed in Henzel and Sauer (2006), for example.

shows that estimated policy reaction functions obtained by using the ex-post revised data can yield misleading descriptions of historical policy in the case of the US. We explore whether data revisions contain similar problems for the euro area. In this line of argument, the use of survey data which are rarely being revised in the course of time, readily available, and timely (as opposed to most official data) can be very helpful.

A second important aspect of survey data is its prevalent forward-looking perspective. It is well known that central banks not only respond to past information, but use a broad range of information. In particular, they consider forecasts of inflation and output in their decision process. The theoretical justification for such a forward-looking approach is given by, e.g., Clarida et al. (1999) and Woodford (2003) within a New Keynesian model. In addition to investigating policy reaction functions based on survey data, we follow Clarida, Galí and Gertler (1998,1999,2000) and estimate forward-looking Taylor rules in order to compare the relevance of real-time versus forward-looking aspects.

The impression of a destabilising monetary policy by the ECB, which is based on contemporaneous Taylor rules, seems to be largely due to the lack of a forward-looking perspective. Either assuming rational expectations and using a forward-looking specification as suggested by Clarida, Galí and Gertler (1998), or using expectations as derived from surveys result in Taylor rules which do imply a stabilising role of the ECB. In such forward-looking cases, the weights attached to the inflation rate by the Bundesbank and the ECB do no longer significantly differ. Furthermore, the ECB appears to have responded to real economic developments at least as strongly as the Bundesbank.

The use of real-time industrial production data, as suggested by Orphanides (2004), hardly influences the results. Estimations for an extended sample until the end of 2006 confirm the results obtained for the Duisenberg-era; contemporaneous specifications find an insufficient response to inflation developments in the euro area, but forward-looking rules indicate a stabilising role of the ECB.

In the low-inflation environment of recent years, a lot of central banks have begun to add concerns about financial stability in addition to the maintenance of price stability and limited output and employment volatility to the top of their agenda. The increased tendency of major central banks such as the ECB, the Bank of England or the Swedish Riksbank to publish 'Financial Stability Reports' represents a widely visible evidence for this conjecture. The prevention of financial crises is an important reason for this behaviour.

The model in chapter 4 helps to provide guidance for central banks in the event of such crises. In particular, it offers a framework to analyse emergency liquidity assistance of central banks on financial markets in response to aggregate and idiosyncratic liquidity shocks.

Liquidity is an important concept in finance and macroeconomics. The microeconomic literature in finance views liquidity roughly as the ability to sell assets quickly and costlessly. In macroeconomics, liquidity refers to a generally accepted medium of exchange or, in brief, money. Money is the most liquid asset due to the fact that it does not need to be converted into anything else in order to make purchases of real goods or other assets. This feature makes money valuable in both perspectives.

Chapter 4 uses this common perspective of money and links liquidity risk on an asset market with aggregate demand and aggregate supply on a goods market. Spillover effects from the asset market to the goods market can justify a central bank intervention on the asset market even if the central bank does not take the welfare of investors on the asset market into account. Hence, the model provides a framework to analyse the perceived insurance against severe financial turmoil by the Federal Reserve under Alan Greenspan, which has been termed the 'Greenspan put' in the popular press and 'liquidity provision principle' by Taylor (2005).

The chapter begins with a survey of empirical and historical evidence for the relevance of liquidity for asset prices, in particular during financial crises. The stock market crash in October 1987 or the LTCM-crisis in September 1998 represent 'flight to quality' or 'flight to liquidity' episodes in which investors wanted to shift from relatively illiquid medium to long-term assets such as shares into safe and liquid government bonds or cash. While liquidity provision has been studied in the literature with a focus on the role of financial intermediaries within 'real' models, chapter 4 develops a model in nominal units in order to look at optimal monetary interventions on financial markets.

In the model, investors can invest on an asset market in liquid money and potentially illiquid, but productive assets, in order to optimally satisfy their uncertain consumption needs on a separated goods market over two periods. Two channels link the goods market to the asset market: First, the amount of money held by investors determines together with the size of a liquidity shock the aggregate demand of investors on the goods market which is subject to a cash-in-advance constraint. Second, a dramatic decrease of the asset price negatively influences the goods supply in the final period because it forces investors to costly liquidate their asset. Confronted with a liquidity crisis, the central bank faces a trade-off between injecting liquidity and thus incurring risks to price stability and negative supply effects in the future. The size of the optimal intervention increases in the size of the liquidity shock, the weight on output relative to inflation and the extent of negative supply effects of the crisis. It decreases in the size of the associated inflation in goods prices which is linked to the possibility to sterilise the intervention and the amount of liquidity initially held by investors.

Furthermore, the anticipation of central bank interventions by private investors leads to a moral hazard effect in the form of less private liquidity provision and thus an increase in the likelihood of financial crises. At the same time, less liquidity provision means more productive investment and thus greater aggregate supply in the absence of a financial crisis. If the central bank is able to credibly commit to some future policy, the optimal liquidity provision rule has to take these additional effects into account.

After the analysis of idiosyncratic liquidity shocks within this framework, chapter 4 offers a thorough discussion of mechanisms that can turn small shocks into large ones. Finally, I review the related literature on the Greenspan put, market segmentation, market microstructure theory and the public supply of liquidity.

This summary shows that the different chapters of this thesis apply a wide range of economic methodologies to the analysis of monetary policy. Chapter 2 looks at optimal monetary policy in the modern micro-founded New Keynesian macroeconomic model, while chapter 3 offers an empirical investigation of monetary policy in the euro area. The final chapter 4 combines a microeconomic model of liquidity shocks on an asset market that includes features of market microstructure theory with a model of the goods market inspired by nominal rigidities as common in macroeconomic models. All three chapters are connected by the prominent role of different rules and the objective to develop and apply frameworks for the analysis of monetary policy from a theoretical and empirical perspective.

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CHAPTER 2

Discretion rather than rules? When is discretionary policy-making better than the timeless perspective?

Abstract

Discretionary monetary policy produces a dynamic loss in the New Keynesian model in the presence of cost-push shocks. The possibility to commit to a specific policy rule can increase welfare. A number of authors since Woodford (1999) have argued in favour of a timeless perspective rule as an optimal policy. The short-run costs associated with the timeless perspective are neglected in general, however. Rigid prices, relatively impatient households, a high preference of policy makers for output stabilisation and a deviation from the steady state all worsen the performance of the timeless perspective rule and can make it inferior to discretion.

2.1 Introduction

Kydland and Prescott (1977) showed that rule-based policy-making can increase welfare. The timeless perspective proposed by Woodford (1999) represents a prominent modern form of such a rule in monetary policy analysis. It helps to overcome not only the traditional inflation bias in the sense of Barro and Gordon (1983), but also the stabilisation bias, a dynamic loss stemming from cost-push shocks in the New Keynesian model as described in Clarida, Galí and Gertler (1999). It is, however, associated with short-run costs that may be larger than the long-run gains from commitment.

After deriving a formal condition for the superiority of discretion over the timeless perspective rule, this paper investigates the influence of structural and preference parameters on the performance of monetary policy both under discretion and the timeless perspective in the sense of Woodford (1999). Discretion gains relatively to the timeless perspective rule, i.e. the short-run losses become relatively more important, if the private sector behaves less forward-looking or if the monetary authority puts a greater weight on output gap stabilisation. For empirically reasonable values of price stickiness, the relative gain from discretion rises with stickier prices. A fourth parameter which influences the relative gains is the persistence of shocks: Introducing serial correlation into the model only strengthens the respective relative performance of policies in the situation without serial correlation in shocks. In particular, we show conditions for each parameter, under which discretion performs strictly better than the timeless perspective rule.

Furthermore, the framework of short-run losses and long-run gains also allows explaining why an economy that is sufficiently far away from its steady-state suffers rather than gains from implementing the timeless perspective rule. In general, this paper uses unconditional expectations of the loss function as welfare criterion, in line with most of the literature. The analysis of initial conditions, however, requires reverting to expected losses conditional on the initial state of the economy because unconditional expectations of the loss function implicitly treat the economy's initial conditions as stochastic. Altogether, in the normal New Keynesian model all conditions for the superiority of discretion need not be as adverse as one might suspect.

We also introduce an 'optimal' timeless policy rule based on Blake (2001), Jensen and McCallum (2002) and Jensen (2003). While the general influence of structural and preferences parameters on the performance of monetary policy under this rule is not affected, discretion is never better than this rule when evaluated with unconditional expectations as it is common in the literature on monetary policy rules. The reason is that this allegedly optimal rule optimally accounts for the use of unconditional expectations as the welfare criterion. For *any* timeless rule, however, initial conditions can be sufficiently adverse to make the rule inferior to discretion.

The following section 2.2 presents the canonical New Keynesian Model. Section 2.3.1 explains the relevant welfare criteria. The analytical solution in section 2.3.2 is followed by simulation results and a thorough economic interpretation of the performance of policies under discretion and the timeless perspective, while section 2.3.4 concludes the discussion of Woodford's timeless perspective by looking at the effects of initial conditions. Section 2.4 introduces the optimal timeless policy rule and repeats the analysis from section 2.3.3, whereas section 2.5 concludes.

2.2 New Keynesian Model

The New Keynesian or New Neoclassical Synthesis model has become the standard toolbox for modern macroeconomics. While there is some debate about the exact functional forms, the standard setup consists of a forward-looking Phillips curve, an intertemporal IS-curve and a welfare function.[1] Following, e.g., Walsh (2003), the New Keynesian Phillips curve based on Calvo (1983) pricing is given by

$$\pi_t = \beta E_t \pi_{t+1} + \alpha y_t + u_t \tag{2.1}$$

with

$$\alpha \equiv \frac{(1-\zeta)(1-\beta\zeta)}{\zeta}. \tag{2.2}$$

π_t denotes inflation, E_t the expectations operator conditional on information in period t, y_t the output gap, and u_t a stochastic shock term that is assumed to follow a stationary AR(1) process with AR-parameter ρ and innovation variance σ^2. While the output gap refers to the deviation of actual output from natural or flexible-price output, u_t is often interpreted as a cost-push shock term that captures time-varying distortions from consumption or wage taxation or mark-ups in firms' prices or wages. It is the source of the stabilisation bias. $0 < \beta < 1$ denotes the (private sector's) discount factor and $0 \leq \zeta < 1$ is the constant probability that a firm is not able to reset its price in period t. A firm's optimal price depends on current and (for $\zeta > 0$) future real marginal costs, which are assumed to be proportional to the respective output gap.[2] Hence, ζ and α reflect the degree of price rigidity in this model which is increasing in ζ and decreasing in α.

The policy-maker's objective at an arbitrary time $t = 0$ is to minimise

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t L_t \qquad \text{with} \quad L_t = \pi_t^2 + \omega y_t^2, \tag{2.3}$$

where $\omega \geq 0$ reflects the relative importance of output-gap variability in policymaker preferences. We assume zero to be the target values of inflation and the output gap, respectively. While the former assumption is included only for notational simplicity and without loss of generality, the latter is crucial for the absence of a traditional

[1] Depending on the purpose of their paper, some authors directly use an instrument rule or a targeting rule without explicitly maximising some welfare function.

[2] In (2.1), the proportionality factor is set equal to 1.

inflation bias in the sense of Barro and Gordon (1983).

The New Keynesian model also includes an aggregate demand relationship based on consumers' intertemporal optimisation in the form of

$$y_t = E_t y_{t+1} - b(R_t - E_t \pi_{t+1}) + v_t, \tag{2.4}$$

where R_t is the central bank's interest rate instrument and v_t is a shock to preferences, government spending or the exogenous natural-rate value of output, for example.[3] The parameter $b > 0$ captures the output gap elasticity with respect to the real interest rate. Yet, for distinguishing between the timeless-perspective and the discretionary solution, it is sufficient to assume that the central bank can directly control π_t as an instrument. Hence, the aggregate demand relationship can be neglected below.[4]

2.2.1 Model Solutions

If the monetary authority neglects the impact of its policies on inflation expectations and reoptimises in each period, it conducts monetary policy under *discretion*. This creates both the Barro and Gordon (1983) inflation bias for positive output gap targets and the Clarida et al. (1999) stabilisation bias caused by cost-push shocks. To concentrate on the second source of dynamic losses in this model, a positive inflation bias is ruled out by assuming an output gap target of zero in the loss function (2.3). Minimising (2.3) subject to (2.1) and to given inflation expectations $E_t \pi_{t+1}$ results in the Lagrangian

$$\Lambda_t = \pi_t^2 + \omega y_t^2 - \lambda_t(\pi_t - \beta E_t \pi_{t+1} - \alpha y_t - u_t) \qquad \forall\, t = 0, 1, 2, \ldots \quad . \tag{2.5}$$

The first order conditions

$$\begin{aligned} \frac{\partial \Lambda_t}{\partial y_t} &= \quad 2\omega y_t + \alpha \lambda_t \quad = 0 \\ \frac{\partial \Lambda_t}{\partial \pi_t} &= \quad 2\pi_t - \lambda_t \quad = 0 \end{aligned}$$

imply

$$\pi_t = -\frac{\omega}{\alpha} y_t. \tag{2.6}$$

If instead the monetary authority takes the impact of its actions on expectations

[3] v_t is generally referred to as a *demand* shock. But in this model, y_t reflects the output *gap* and not output alone. Hence, shocks to the flexible-price level of output are also included in v_t. See, e.g., Woodford (2003, p. 246).

[4] Formally, adding (2.4) as a constraint to the optimisation problems below gives a value of zero to the respective Lagrangian multiplier.

into account and possesses an exogenous possibility to credibly commit itself to some future policy, it can minimise the loss function (2.3) over an enhanced opportunity set. Hence, the resulting *commitment solution* must be at least as good as the one under discretion. The single-period Lagrangian (2.5) changes to

$$\Lambda = E_0 \sum_{t=0}^{\infty} \beta^t \left[(\pi_t^2 + \omega y_t^2) - \lambda_t (\pi_t - \beta \pi_{t+1} - \alpha y_t - u_t) \right]. \tag{2.7}$$

This yields as first order conditions

$$\begin{aligned}
\frac{\partial \Lambda}{\partial y_t} &= 2\omega y_t + \alpha \lambda_t = 0, & t &= 0, 1, 2, \ldots, \\
\frac{\partial \Lambda}{\partial \pi_t} &= 2\pi_t - \lambda_t = 0, & t &= 0, \\
\frac{\partial \Lambda}{\partial \pi_t} &= 2\pi_t - \lambda_t + \lambda_{t-1} = 0, & t &= 1, 2, \ldots,
\end{aligned}$$

implying

$$\pi_t = -\frac{\omega}{\alpha} y_t, \qquad t = 0 \quad \text{and} \tag{2.8}$$

$$\pi_t = -\frac{\omega}{\alpha} y_t + \frac{\omega}{\alpha} y_{t-1}, \qquad t = 1, 2, \ldots \quad . \tag{2.9}$$

The commitment solution improves the short-run output/inflation trade-off faced by the monetary authority because short-run price dynamics depend on expectations about the future. Since the authority commits to a history-dependent policy in the future, it is able to optimally spread the effects of shocks over several periods. The commitment solution also enables the policy maker to reap the benefits of discretionary policy in the initial period without paying the price in terms of higher inflation expectations, since these are assumed to depend on the future commitment to (2.9). Indeed, optimal policy is identical under commitment and discretion in the initial period. In two recent paper, Dennis and Söderström (2006) and Levine, McAdam and Pearlman (2007) compare the welfare gains from commitment over discretion under different scenarios.

However, the commitment solution suffers from time inconsistency in two ways: First, by switching from (2.9) to (2.6) in any future period, the monetary authority can exploit given inflationary expectations and gain in the respective period. Second, the monetary authority knows at $t = 0$ that applying the same optimisation procedure (2.7) in the future implies a departure from today's optimal plan, a feature McCallum (2003, p. 4) calls 'strategic incoherence'.

To overcome the second form of time inconsistency and thus gain true credibility, many authors since Woodford (1999) have proposed the concept of policy-making under the *timeless perspective*: The optimal policy in the initial period should be chosen such that it would have been optimal to commit to this policy at a date far in the past, not exploiting given inflationary expectations in the initial period.[5] This implies neglecting (2.8) and applying (2.9) in all periods, not just in $t = 1, 2, \ldots$:

$$\pi_t = -\frac{\omega}{\alpha} y_t + \frac{\omega}{\alpha} y_{t-1}, \qquad t = 0, 1, \ldots \quad . \tag{2.10}$$

Hence, the only difference to the commitment solution lies in the different policy in the initial period, unless the economy starts from its steady-state with $y_{-1} = 0$.[6] But since the commitment solution is by definition optimal for (2.7), this difference causes a loss of the timeless perspective policy compared to the commitment solution. If this loss is greater than the gain from the commitment solution (COM) over discretion, rule-based policy making under the timeless perspective (TP) causes larger losses than policy under discretion (DIS):

$$L_{TP} - L_{COM} > L_{DIS} - L_{COM} \qquad \Leftrightarrow \qquad L_{TP} > L_{DIS}. \tag{2.11}$$

The central aim of the rest of this paper is to compare the losses from TP and DIS.

2.2.2 Minimal state variable (MSV) solutions

Before we are able to calculate the losses under the different policy rules, we need to determine the particular equilibrium behaviour of the economy, which is given by the New Keynesian Phillips curve (2.1)[7] and the respective policy rule, i.e. DIS (2.6) or TP (2.10). Following McCallum (1999), the minimal state variable (MSV) solution to each model represents the rational expectations solution that excludes bubbles and sunspots.

Under *discretion*, u_t is the only relevant state variable in (2.1) and (2.6)

$$\begin{aligned} \pi_t &= \beta E_t \pi_{t+1} + \alpha y_t + u_t \\ \pi_t &= -\frac{\omega}{\alpha} y_t, \end{aligned}$$

[5] Woodford (1999) compares this ‘commitment’ to the ‘contract’ under John Rawls’ veil of uncertainty.

[6] Due to the history-dependence of (2.10), the different initial policy has some influence on the losses in subsequent periods, too.

[7] Without loss of generality but to simplify the notation, the MSV solutions are derived based on (2.1) without reference to (2.2). The definition of α in (2.2) is substituted into the MSV solutions for the simulation results in section 2.3.3.

so the conjectured solution is of the form

$$\begin{aligned}\pi_{t,DIS} &= \phi_1 u_t \\ y_{t,DIS} &= \phi_2 u_t.\end{aligned}$$

Since $E_t\pi_{t+1} = \phi_1\rho u_t$ in this case, the MSV solution is given by

$$\pi_{t,DIS} = \frac{\omega}{\omega(1-\beta\rho)+\alpha^2}u_t \tag{2.12}$$

$$y_{t,DIS} = \frac{-\alpha}{\omega(1-\beta\rho)+\alpha^2}u_t. \tag{2.13}$$

Under the *timeless perspective*, y_{t-1} and u_t are the relevant state variables from (2.1) and (2.10):

$$\begin{aligned}\pi_t &= \beta E_t\pi_{t+1} + \alpha y_t + u_t \\ \pi_t &= -\frac{\omega}{\alpha}y_t + \frac{\omega}{\alpha}y_{t-1}.\end{aligned}$$

Hence, the conjectured solution becomes

$$\pi_{t,TP} = \phi_{11}y_{t-1} + \phi_{12}u_t \tag{2.14}$$

$$y_{t,TP} = \phi_{21}y_{t-1} + \phi_{22}u_t. \tag{2.15}$$

After some calculations,[8] the resulting MSV solution is described by

$$\pi_{t,TP} = \frac{\omega(1-\delta)}{\alpha}y_{t-1} + \frac{1}{\gamma-\beta(\rho+\delta)}u_t \tag{2.16}$$

$$y_{t,TP} = \delta y_{t-1} - \frac{\alpha}{\omega(\gamma-\beta(\rho+\delta))}u_t. \tag{2.17}$$

with $\gamma \equiv 1+\beta+\frac{\alpha^2}{\omega}$ and $\delta \equiv \frac{\gamma-\sqrt{\gamma^2-4\beta}}{2\beta}$. Given these MSV solutions, we are now able to evaluate the relative performance of monetary policy under discretion and the timeless perspective rule.

2.3 Policy Evaluation

2.3.1 Welfare criteria

Unconditional expectations: The standard approach to evaluate monetary policy performance is to compare average values for the period loss function, i.e. values

[8] These calculations include a quadratic equation in ϕ_{21}, of which only one root, $0 < \delta < 1$, is relevant according to both the stability and MSV criteria.

of the unconditional expectations of the period loss function in (2.3), denoted as $E[L]$.[9] We follow this approach for the analysis of the influence of preference and structural parameters mainly because it is very common in the literature[10] and allows an analytical solution. However, it includes several implicit assumptions.

First, π_t and y_t need to be covariance-stationary. This is not a problem in our setup since u_t is stationary by assumption and $0 < \delta < 1$ is chosen according to the stability criterion, see footnote 8. Second, using unconditional expectations of (2.3) implies treating the initial conditions as stochastic (see, e.g., King and Wolman, 1999, p.377) and thus averages over all possible initial conditions. Third, Rudebusch and Svensson (1999) and Dennis (2004, Appendix A) show that the standard approach is formally correct only for $\lim \beta \to 1$, the central bank's discount factor being close to 1. This may influence the precise parameter values for which DIS performs better than TP in section 2.3.3, but it only strengthens the general argument with respect to the influence of β as will be shown below.

Conditional expectations: At the same time, using unconditional expectations impedes an investigation of the effects of specific initial conditions and transitional dynamics to the steady state on the relative performance of policy rules. For this reason and to be consistent with the microfoundations of the New Keynesian model, Kim and Levin (2005), Kim, Kim, Schaumburg and Sims (2005) and Schmitt-Grohé and Uribe (2004) argue in favour of conditional expectations as the relevant welfare criterion. If future outcomes are discounted, i.e. $\beta < 1$, the use of conditional expectations, i.e. $\mathcal{L}$ in (2.3) as welfare criterion, implies that short-run losses from TP become relatively more important to the long-run gains compared to the evaluation with unconditional expectations.

Both concepts can be used to evaluate the performance of monetary policy under varying parameter values and the results are qualitatively equivalent. Besides its popularity and analytical tractability, the choice of unconditional expectations as the general welfare measure has a third advantage: by implicitly averaging over all possible initial conditions and treating all periods the same, we can evaluate policies for all current and future periods and thus consider the policy problem from a 'truly timeless' perspective in the sense of Jensen (2003), that does not bias our results in favour of discretionary policy-making. Only the analysis of the effects of different initial conditions requires reverting to conditional expectations.

[9]The unconditional expectations of the period loss function L_t are equal to the unconditional expectations of the total loss function $\mathcal{L}$ in (2.3), scaled down by the factor $(1 - \beta)$.

[10]See, e.g., various articles in the conference volume by Taylor (1999) and Clarida et al. (1999), Woodford (1999), Jensen and McCallum (2002).

2.3.2 Analytical solution

In principle, the relative performance of DIS and TP can be solved analytically if closed form solutions for the unconditional expectations of the period loss function are available. This is possible, since

$$L_i = E[L_{t,i}] = E[\pi_{t,i}^2] + \omega E[y_{t,i}^2], \quad i \in \{DIS,\, TP\} \tag{2.18}$$

from (2.3) and the MSV solutions in section 2.2.2 determine the unconditional variances $E[\pi_{t,i}^2]$ and $E[y_{t,i}^2]$. The MSV solution under *discretion*, (2.12) and (2.13) with u_t as the only state variable and $E[u_t^2] = \frac{1}{1-\rho^2}\sigma^2$, give the relevant welfare criterion

$$\begin{aligned} L_{DIS} &= \left[\frac{\omega}{\omega(1-\beta\rho)+\alpha^2}\right]^2 \frac{1}{1-\rho^2}\,\sigma^2 + \omega\left[\frac{-\alpha}{\omega(1-\beta\rho)+\alpha^2}\right]^2 \frac{1}{1-\rho^2}\,\sigma^2 \\ &= \frac{\omega(\omega+\alpha^2)}{[\omega(1-\beta\rho)+\alpha^2]^2}\cdot\frac{1}{1-\rho^2}\,\sigma^2. \end{aligned} \tag{2.19}$$

For the *timeless perspective*, the MSV solution (2.16) and (2.17) depends on two state variables, y_{t-1} and u_t. From the conjectured solution in (2.14) and (2.15), we have

$$\begin{aligned} E[\pi_{t,TP}^2] &= \phi_{11}^2 E[y_{t-1}^2] + \phi_{12}^2 E[u_t^2] + 2\phi_{11}\phi_{12}E[y_{t-1}u_t] \\ E[y_{t,TP}^2] &= \phi_{21}^2 E[y_{t-1}^2] + \phi_{22}^2 E[u_t^2] + 2\phi_{21}\phi_{22}E[y_{t-1}u_t]. \end{aligned} \tag{2.20}$$

These two equations are solved and plugged into (2.18) in Appendix 2.A. The result is

$$L_{TP} = \frac{2\omega(1-\delta)(1-\rho)+\alpha^2(1+\delta\rho)}{\omega(1-\delta^2)(1-\delta\rho)[\gamma-\beta(\delta+\rho)]^2}\cdot\frac{1}{1-\rho^2}\,\sigma^2. \tag{2.21}$$

Hence, discretion is superior to the timeless perspective rule, if

$$\begin{aligned} L_{DIS} < L_{TP} \quad &\Leftrightarrow \quad \frac{\omega(\omega+\alpha^2)}{[\omega(1-\beta\rho)+\alpha^2]^2} < \frac{2\omega(1-\delta)(1-\rho)+\alpha^2(1+\delta\rho)}{\omega(1-\delta^2)(1-\delta\rho)[\gamma-\beta(\delta+\rho)]^2} \\ &\Leftrightarrow \quad RL \equiv L_{TP}/L_{DIS} - 1 > 0. \end{aligned} \tag{2.22}$$

(2.22) allows analytical proofs of several intuitive arguments: First, the variance of cost-push shocks $\frac{1}{1-\rho^2}\,\sigma^2$ affects the magnitude of absolute losses in (2.19) and (2.21), but has no effect on the relative loss RL because it cancels out in (2.22). Second, economic theory states that with perfectly flexible prices, i.e. $\zeta = 0$ and $\alpha \to \infty$, respectively, the short-run Phillips curve is vertical at $y_t = 0$. In this case, the short-run output/inflation trade-off and hence the source of the stabilisation bias disappears completely and no difference between DIS, COM and TP can exist.

Third, if the society behaves as an 'inflation nutter' (King, 1997) and only cares about inflation stabilisation, i.e. $\omega = 0$, inflation deviates from the target value neither under discretion nor under rule-based policy-making. This behaviour eliminates the stabilisation bias because the effect of shocks cannot be spread over several periods. Shocks always enter the contemporaneous output gap completely. Furthermore, the initial conditions do not matter, since y_{-1} receives a weight of 0 in (2.10) and no short-run loss arises. The last two statements are summarised in the following proposition.

Proposition 2.1 *Discretion and Woodford's timeless perspective are equivalent for*

1. *perfectly flexible prices or*
2. *inflation nutter - preferences.*

Proof. 1. $\lim_{\alpha\to\infty} RL = 0$. 2. $\lim_{\omega\to 0} RL = 0$. ■

Finally, proposition 2.2 states that discretion is not always inferior to Woodford's timeless perspective. If the private sector discounts future developments at a larger rate, i.e. β decreases, firms care less about optimal prices in the future, when they set their optimal price today. Hence, the potential to use future policies to spread the effects of a current shock via the expectations channel decreases. Therefore, the loss from the stabilisation bias under DIS, where this potential is not exploited, i.e. the long-run gains $L_{DIS} - L_{COM}$, also decreases with smaller β, while the short-run costs from TP, $L_{TP} - L_{COM}$, remain unaffected under rule (2.10). In the extreme case of $\beta = 0$, expectations are irrelevant in the Phillips curve (2.1) and the source of the stabilisation bias disappears. If the reduction in the long-run gain is sufficiently large, conditions (2.11) and (2.22) are fulfilled.

Proposition 2.2 *There exists a discount factor β small enough such that discretion is superior to Woodford's timeless perspective as long as some weight is given to output stabilisation and prices are not perfectly flexible.*

Proof. RL is continuous in β because stability requires $0 \le \delta, \rho < 1$. Furthermore, $\lim_{\beta\to 0} RL = \frac{[\alpha^2+2(1-\rho)\omega+(1+\rho)\omega](\alpha^2+\omega)}{(\alpha^2+2\omega)[\alpha^2+(1-\rho)\omega]} - 1 > 0$ for $\omega > 0 \wedge \alpha < \infty$. ■

In principle, (2.22) could be used to look at the influence of structural (ζ, ρ) and preference (β, ω) parameters on the relative performance of monetary policy under discretion and the timeless perspective rule more generally.[11] Unfortunately, (2.22) is too complex to be analytically tractable. Hence, we have to turn to results from simulations.

[11]Please note that it would be conceptually nonsense to compare one policy over several values of a preference parameter. Here, however, we always compare two policies (DIS and TP) holding all preference and structural parameters constant.

2.3.3 Simulation results

Preference (β, ω) and structural (ζ, ρ) parameters influence the relative performance of monetary policy under discretion and the timeless perspective rule. To evaluate each effect separately, we start from a benchmark model with parameter values presented in table 2.1 and then vary each parameter successively.

Table 2.1: *Parameter values for the benchmark model.*

Parameter	β	ω	ζ	α	ρ
Value	0.99	0.0625	0.8722	0.02	0

If one period in the model reflects one quarter, the discount factor of $\beta = 0.99$ corresponds to an annual real interest rate of 4%. Setting $\omega = 1/16$ implies an equal weight on the quarterly variances of annualised inflation and the output gap. For $\beta = 0.99$, $\zeta = 0.8722$ corresponds to $\alpha = 0.02$, the value used in Jensen and McCallum (2002) based on empirical estimates in Galí and Gertler (1999).[12]

Discount factor β: Figure 2.1 presents the results for the variation of the discount factor β as the loss from the timeless perspective relative to discretionary policy, RL. A positive (negative) value of RL means that the loss from the timeless perspective rule is greater (smaller) than the loss under discretion, while an increase (decrease) in RL implies a relative gain (loss) from discretion.

The simulation shows that RL increases with decreasing β, i.e. DIS gains relative to TP, if the private sector puts less weight on the future. This pattern reflects proposition 2.2 in the previous section. Since the expectations channel becomes less relevant with smaller β, the stabilisation bias and thus the long-run gains from commitment also decrease in β, whereas short-run losses remain unaffected.

In particular, DIS becomes superior to TP in the benchmark model for $\beta < 0.839$, but with $\omega = 1$ already for $\beta < 0.975$. Differentiating between the central bank's and the private sector's discount factor β as in section 2.4, when the optimal timeless policy rule is derived analytically, shows that the latter drives RL because it enters the Phillips curve, while the former is irrelevant due to the use of unconditional expectations as the welfare criterion as discussed in section 2.3.1. But since using the unconditional expectations of the loss function gives equal weight to all periods and hence greater weight to future periods than actually valid for $\beta < 1$, this effect only strengthens the general argument.

[12] ζ and α are linked through the definition of α in (2.2).

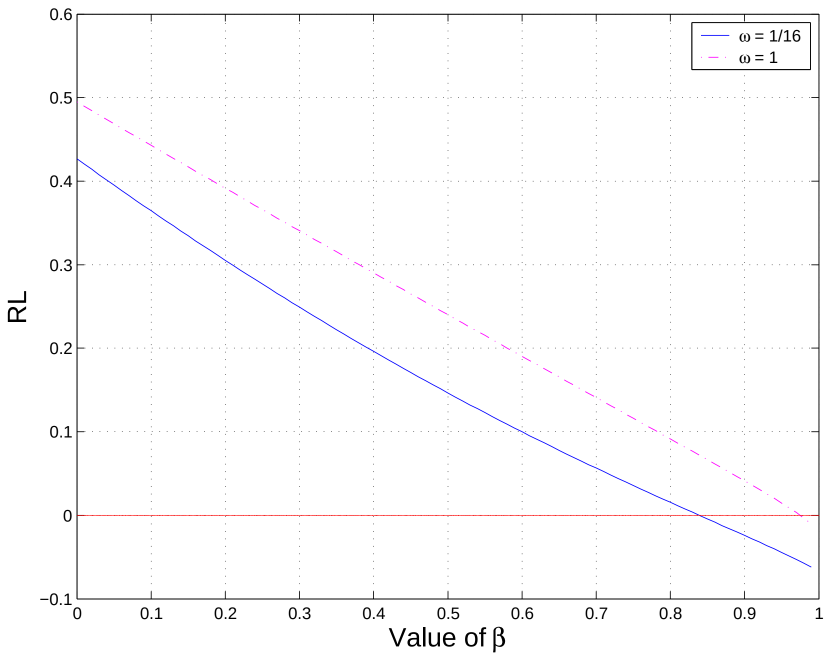

Figure 2.1: *Variation of discount factor β, TP vs. DIS.*

This can be shown with the value of the loss function (2.3), $\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t L_t$, *conditional* on expectations at $t = 0$ instead of the *unconditional* expectations $E[L]$. As figure 2.2 demonstrates, the general impact of β on RL is similar to figure 2.1.[13] The notable difference is the absolute superiority of DIS over TP in our benchmark model, independently of β. In order to get a critical value of β for which DIS and TP produce equal losses, other parameters of the benchmark model have to be adjusted such that they favour TP, e.g. by reducing ω as explained below. Hence, figure 2.2 provides evidence that the use of unconditional expectations does not bias the results towards lower losses for discretionary policy. For reasons presented in section 2.3.1, we focus only on unconditional expectations from now on.

Output gap weight ω: In Barro and Gordon (1983), the traditional inflation bias increases in the weight on the output gap, while the optimal stabilisation policies are identical both under discretion and under commitment.[14] In my intertemporal model without structural inefficiences, however, the optimal stabilisation policies are different under DIS and COM/TP. The history-dependence of TP in (2.10) improves the monetary authority's short-run output/inflation trade-off in each period because it makes today's output gap enter tomorrow's optimal policy with the opposite sign,

[13] The use of conditional expectations requires setting the initial conditions, i.e. y_{-1} and u_0, to specific values. In figure 2.2, $y_{-1} = -0.01$ and $u_0 = 0$.

[14] In Barro and Gordon (1983), a larger ω increases the marginal utility of higher inflation. Under discretion, the marginal utility of higher inflation must equal its marginal cost such that the ex ante expected policy is also ex post optimal *on average*, which leaves the optimal stabilisation policy unaffected.

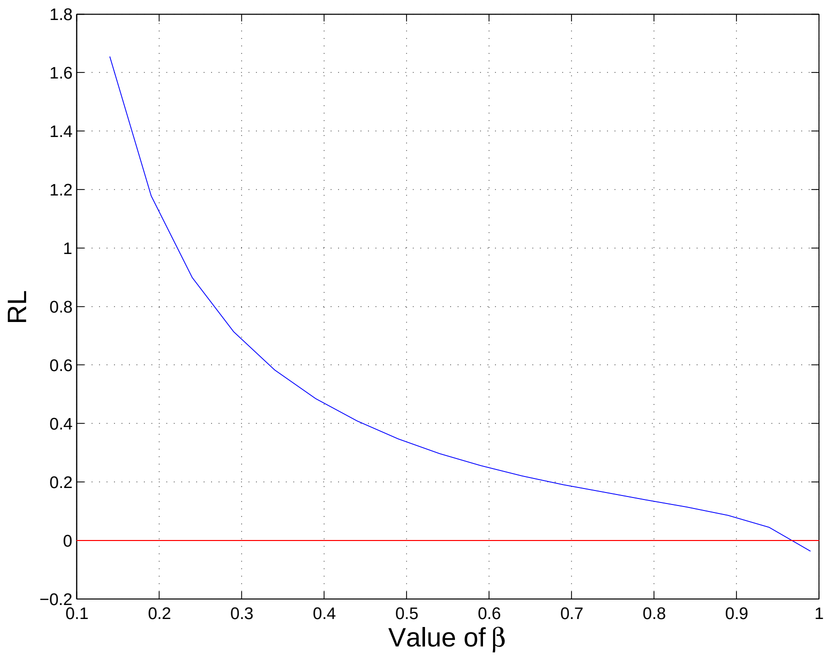

Figure 2.2: *Variation of discount factor β using conditional expectations of loss function, TP vs. DIS.*

but the same weight ω/α in both periods. Hence, optimal current inflation depends on the *change* in the output gap under TP, but only on the contemporaneous output gap under DIS. This way, rule-based policy-making eliminates the stabilisation bias and reduces the relative variance of inflation and output gap, which is a prominent result in the literature.[15]

The short-run costs from TP arise because the monetary authority must be tough on inflation already in the initial period. These short-run costs increase with the weight on the output gap ω.[16] The long-run gains from TP are caused by the size of the stabilisation bias and the importance of its elimination given by the preferences in the loss function. Equation (2.10) shows that increasing ω implies a softer policy on inflation today, but is followed by a tougher policy tomorrow. Although the effect of tomorrow's policy is discounted by the private sector with β, the size of the stabilisation bias, i.e. the neglection of the possibility to spread shocks over several periods, appears to be largely independent from ω. However, the reduction in the relative variance of inflation due to TP becomes less important the larger the weight on the variance of the output gap in the loss function, i.e. the long-run gains from TP decrease in ω. Since short-run costs increase and long-run gains decrease in the weight on the output gap ($\omega \uparrow$), a larger preference for output gap stabilisation favours DIS relative to TP for reasonable ranges of parameters.

[15]See, e.g., Woodford (1999) and Dennis and Söderström (2006).
[16]The optimal output gap y_t under DIS is decreasing in ω, see equation (2.6).

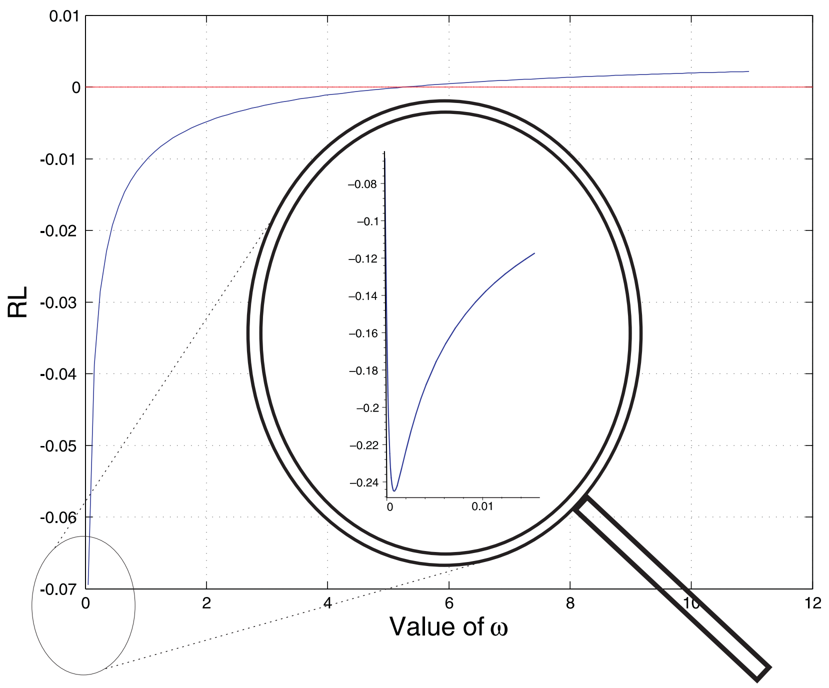

Figure 2.3: *Variation of weight on the output gap ω, TP vs. DIS.*

In the benchmark model of figure 2.3, RL initially decreases from 0 for $\omega = 0$ with $\omega \uparrow$.[17] But for reasonable values of ω, i.e. $\omega > 0.0009$ in the benchmark model, RL increases in the preference for output stabilisation and becomes even positive for $\omega > 5.28$.[18]

Price rigidity $\boldsymbol{\zeta}$: Proposition 2.1 states that DIS and TP are equivalent for perfectly flexible prices, i.e. $\zeta = 0$ or $\alpha \to \infty$, respectively. Increasing price rigidity, i.e. increasing ζ, has two effects: First, firms' price-setting becomes more forward-looking because they have less opportunities to adjust their prices. This effect favours TP over DIS for $\zeta \uparrow$ because TP optimally incorporates forward-looking expectations. Second, more rigid prices imply a flatter Phillips curve and thus the requirement of TP to be tough on inflation already in the initial period becomes more costly. Hence, the left-handside of (2.11), the short-run losses from TP over DIS, increases. Figure 2.4 demonstrates that for $\zeta > 0.436$, the second effect becomes more important, and for $\zeta > 0.915$, the second effect even dominates the first.[19]

Galí and Gertler (1999) provide evidence that empirically reasonable estimates for price rigidity lie within $\alpha \in [0.01; 0.05]$, i.e. $\zeta \in [0.909; 0.804]$. In this range, figure 2.5 shows that RL increases with the firms' probability of not being able to

[17]Note the magnifying glass in figure 2.3.

[18]RL may approach 0 again for $\omega \to \infty$, the (unreasonable) case of an 'employment nutter'.

[19]Since the relationship between ζ and α given by equation (2.2) also depends on β, there is a qualitatively irrelevant and quantitatively negligible difference between varying the probability of no change in a firm's price, ζ, and directly varying the output gap coefficient in the Phillips curve, α.

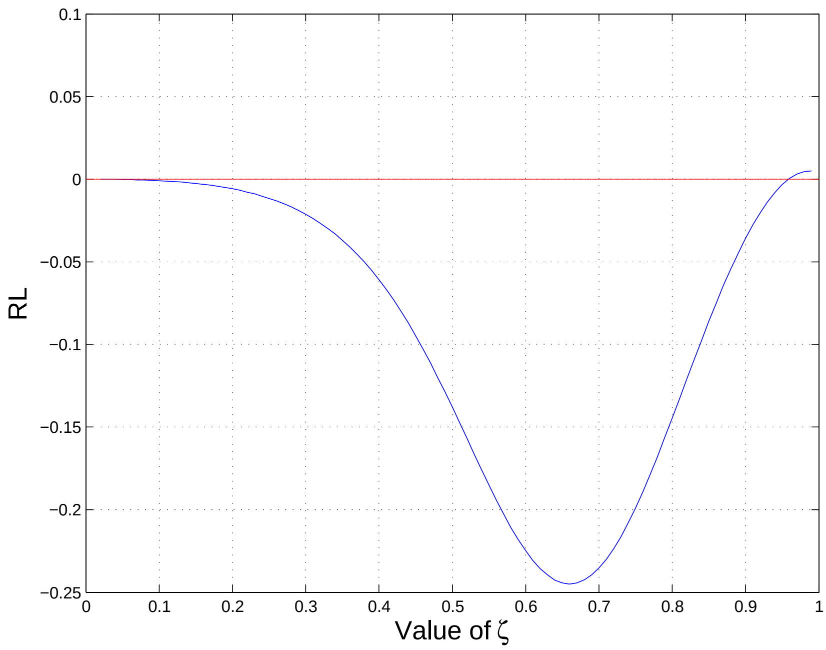

Figure 2.4: *Variation of degree of price rigidity ζ, TP vs. DIS.*

reset their price, ζ, and exceeds 0 for $\zeta > 0.915$ or $\alpha < 0.009$.

Correlation of shocks ρ: The analysis of the influence of serial correlation in cost push shocks, ρ, is more complex. L_{DIS} exceeds L_{TP} in the benchmark model with $\rho = 0$ and raising ρ ceteris paribus strengthens the advantage of TP as demonstrated in figure 2.6. If shocks become more persistent, their impact on

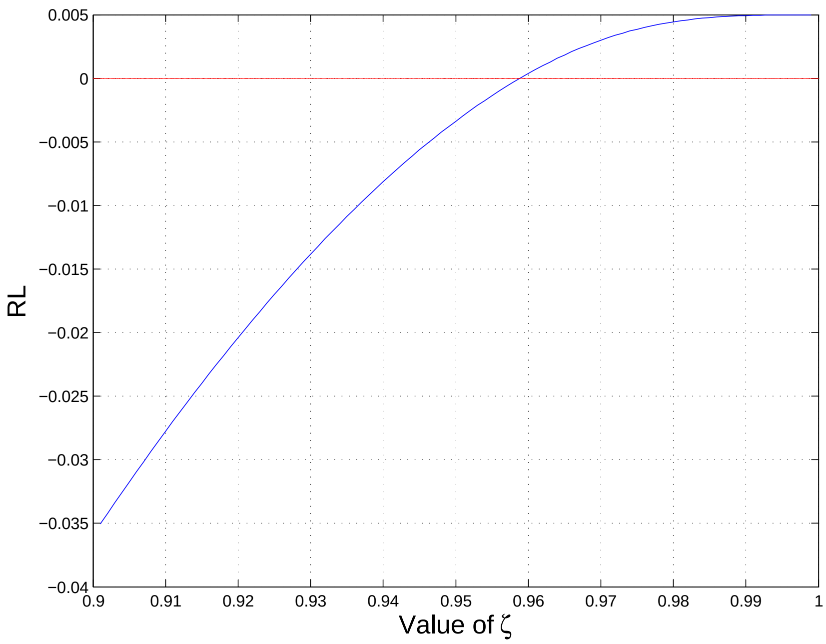

Figure 2.5: *Variation of degree of price rigidity ζ for $\zeta > 0.9$, TP vs. DIS.*

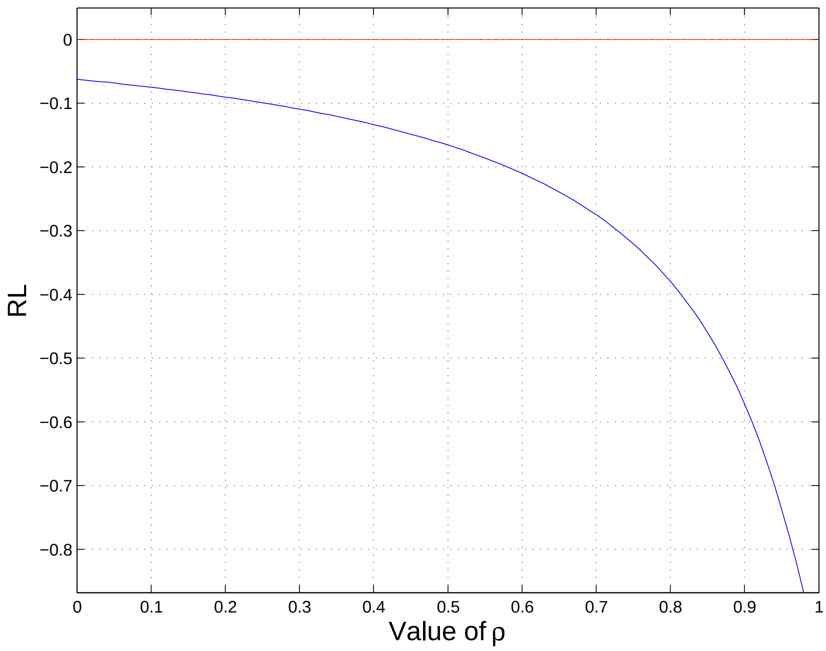

Figure 2.6: *Variation of degree of serial correlation ρ in the benchmark model, TP vs. DIS.*

future outcomes increases and thus TP gains relative to DIS because it accounts for these effects in a superior way. The long-run gains from TP dominate its short-run losses and RL decreases with ρ.

However, the relationship between ρ and RL is not independent of the other parameters in the model, while the relationships between RL and β, ζ and ω, respectively, appear to be robust to alternative specifications of other parameters. Broadly speaking, as long as $L_{DIS} > L_{TP}$ for $\rho = 0$, varying ρ results in a diagram similar to figure 2.6, i.e. $L_{DIS} > L_{TP}$ for all $\rho \in [0; 1)$ and RL decreases in ρ. If, however, due to an appropriate combination of β, ζ and ω, $L_{DIS} \leq L_{TP}$ for $\rho = 0$, a picture symmetric to the horizontal axis in figure 2.6 emerges, as shown in figure 2.7.[20] That means that a higher degree of serial correlation only strengthens the dominance of either TP or DIS already present without serial correlation. Hence, serial correlation on its own seems not to be able to overcome the result of the trade-off between short-run losses and long-run gains from TP implied by the other parameter values.[21]

[20]For parameter combinations that result in L_{DIS} in the neighbourhood of L_{TP} for $\rho = 0$, increasing ρ has hardly any influence on RL, but for high degrees of serial correlation from about $\rho > 0.8$, RL increases rapidly.

[21]This shows that the results in McCallum and Nelson (2004, p. 48), who only report the relationship visible in figure 2.6, do not hold in general.

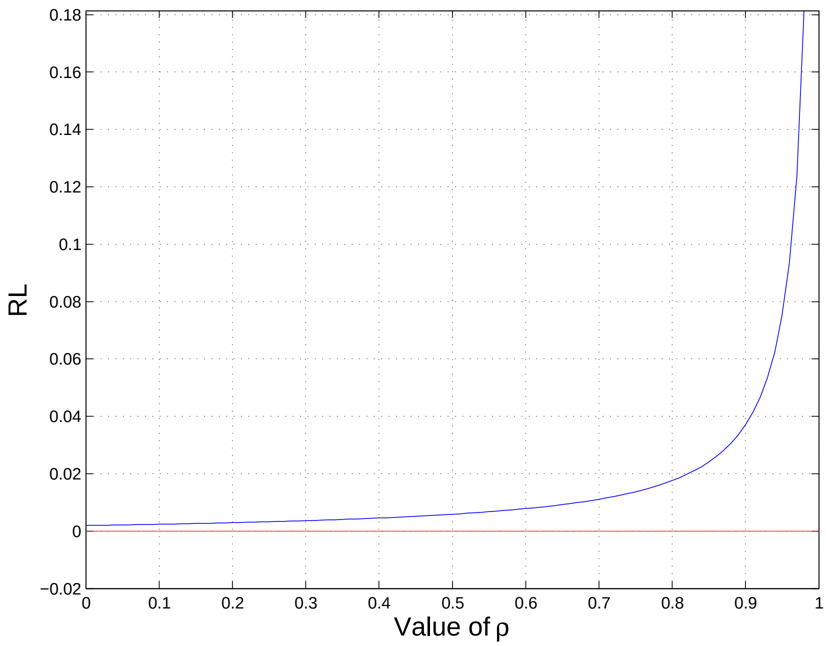

Figure 2.7: *Variation of degree of serial correlation ρ with $\omega = 10$, TP vs. DIS.*

2.3.4 Effects of initial conditions

As argued in section 2.3.1, we have to use conditional expectations of $\mathcal{L}$ in 2.3 in order to investigate the effects of the initial conditions, i.e. the previous output gap y_{-1} and the current cost-push shock u_0 on the relative performance of policy rules. Figure 2.8 presents the relative loss $\widehat{RL} = \mathcal{L}_{TP}/\mathcal{L}_{DIS} - 1$ conditional on y_{-1} and u_0.

Starting from the steady state with $y_{-1} = u_0 = 0$ where $\widehat{RL} = -0.0666$ in the benchmark model,increasing the absolute value of the initial lagged output gap $|y_{-1}|$ increases the short-run cost from following TP instead of DIS and leaves long-run gains unaffected: While $\pi_{0,DIS} = y_{0,DIS} = 0$ from (2.12) and (2.13), $\pi_{0,TP}$ and $y_{0,TP}$ deviate from their target values as can be seen from the history-dependence of (2.10) or the MSV solution (2.16) and (2.17). Hence, TP becomes suboptimal under conditional expectations for sufficiently large $|y_{-1}|$. Note also that this short-run cost is of course symmetric to the steady-state value $y_{-1} = 0$.

If in addition to $|y_{-1}| > 0$ a cost-push shock $|u_0| > 0$ hits the economy, the absolute losses both under DIS and TP increase. Since TP allows an optimal combination of the short-run cost from TP, the inclusion of $|y_{-1}| > 0$ in (2.10), with the possibility to spread the impact of the initial shock $|u_0| > 0$ over several periods, a larger shock u_0 alleviates the short-run cost from TP. Hence, the relative loss $\widehat{RL}$ from TP decreases in $|u_0|$ for any given $|y_{-1}| > 0$.

However, this effect is the weaker the closer $|y_{-1}|$ is to 0, as can be seen from the less bent contour lines in figure 2.8. If $y_{-1} = 0$, the size of $|u_0|$ has no influence

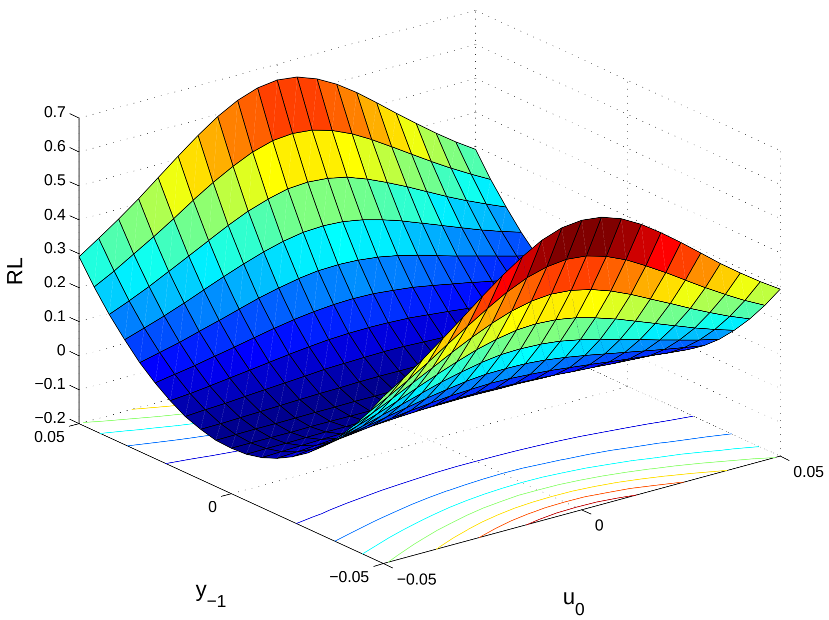

Figure 2.8: $\widehat{RL}$ *depending on* y_{-1} *and* u_0.

on $\widehat{RL}$ any more since DIS and TP do not differ in $t = 0$.[22] In this case, $\widehat{RL}$ is parallel to the u_0-axis. While u_0 still influences the *absolute* loss-values $\mathcal{L}$ under both policies and how these losses are spread over time under TP, it has no influence on the *relative* gain from TP as measured by $\widehat{RL}$, which is solely determined by the long-run gains from TP for $y_{-1} = 0$.

Note that RL is symmetric both to $y_{-1} = 0$ for any given u_0 and to $u_0 = 0$ for any given y_{-1}. Under DIS, y_{-1} has no impact because (2.6) is not history-dependent and u_0 only influences the respective period loss L_0, which is the weighted sum of the *variances* π_0^2 and y_0^2. Hence, $\mathcal{L}_{DIS}$ is independent of y_{-1} and symmetric to $u_0 = 0$.

Under TP, however, the history-dependence of (2.9) makes y_{-1} and u_0 influence current and future losses. While the transitional dynamics differ with the relative sign of u_0 and y_{-1}, the total absolute loss $\mathcal{L}_{TP}$ does not for any given combination of $|y_{-1}|$ and $|u_0|$. If the economy was in a recession ($y_{-1} < 0$), for example,[23] the price to pay under TP is to decrease π_0 through dampening y_0. In figure 2.9, the shift of the steady-state aggregate demand curve AD^* to AD_0 reflects this policy response.

Scenario 1: If additionally a negative cost-push shock $u_0 < 0$ hits the economy, i.e. with the *same* sign as $y_{-1} < 0$, this shock lowers π_0 further as the Phillips curve (2.1) is shifted downwards from its steady-state locus AS^* to AS_0' in figure 2.9. At

[22] To be precise, the policy 'rules' (2.6) and (2.10) do not differ in $t = 0$, but the losses differ because of the more favourable output-inflation trade-off through the impact of TP on $E_0\pi_1$ in (2.1). This benefit of TP is part of the long-run gains, however, because it is also present under COM.

[23] The following arguments run in a completely analogous manner for $y_{-1} > 0$.

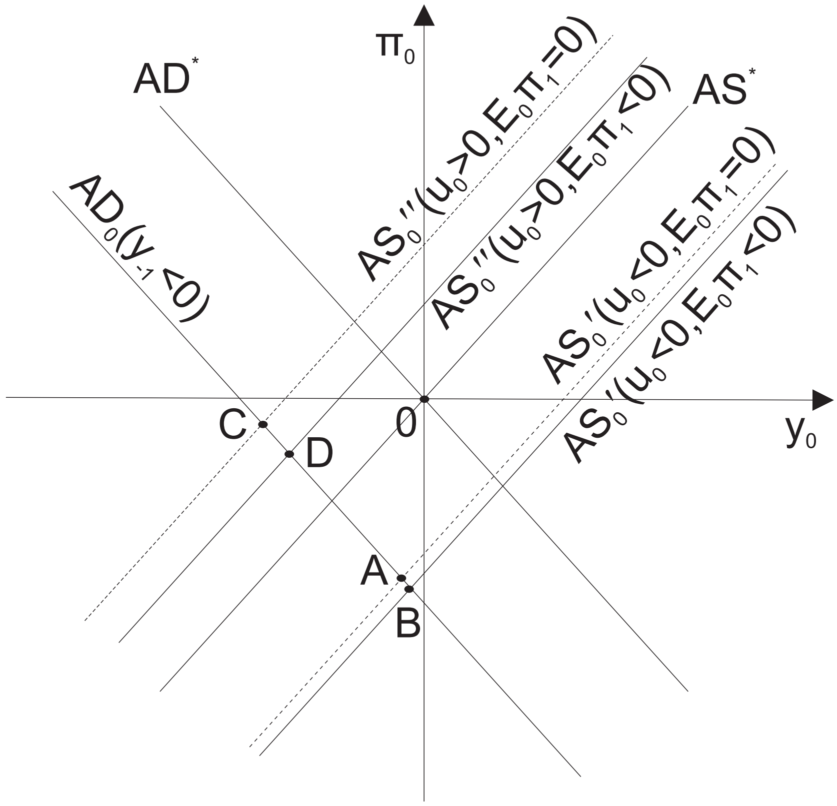

Figure 2.9: *AS-AD-Diagram in $t = 0$ for two symmetric cost-push shocks u_0.*

the same time, $u_0 < 0$ increases y_0 ceteris paribus,[24] brings y_0 closer to the target of 0 and thus reduces the price to pay for TP in the next periods $t = 1, \ldots$. The anticipation of this policy in turn lowers inflation expectations $E_0\pi_1$ compared to the steady-state and thus shifts AS_0' even further down. B denotes the resulting equilibrium in figure 2.9 and is always closer to the π_0-axis than A.

Scenario 2: If, however, the initial cost-push shock u_0 is positive, i.e. of *opposite* sign to $y_{-1} < 0$, the transitional dynamics are reversed. The Phillips curve (2.1) is shifted upwards to AS_0'' in figure 2.9. In contrast to scenario 1 with $u_0 < 0$, this reduces the negative impact of y_{-1} on π_0 but increases y_0 to point C. Hence, the price to pay under TP in $t = 1$ is larger than in scenario 1, which in turn also lowers inflation expectations $E_0\pi_1$ by more. The additional shift of AS_0'' downwards is thus larger than for $u_0 < 0$ and the new equilibrium is at point D.

Figure 2.10 presents the discounted period losses under TP for both cases in the benchmark model. The behaviour of the economy as described above causes a larger loss in the initial period for the first scenario with $\text{sign}(y_{-1}) = \text{sign}(u_0)$ compared to the case with $\text{sign}(y_{-1}) = -\text{sign}(u_0)$ because the expectations channel has a smaller

[24]Formally, partial derivatives of (2.16) and (2.17) with respect to both state variables (y_{t-1}, u_t) show that both have the same qualitative effect on π_t and an opposing effect on y_t: $\partial\pi_t/\partial y_{t-1} = \frac{\omega(1-\delta)}{\alpha} > 0$ and $\partial\pi_t/\partial u_t = \frac{1}{\gamma-\beta(\rho+\delta)} > 0$ while $\partial y_t/\partial y_{t-1} = \delta > 0$ and $\partial y_t/\partial u_t = \frac{-\alpha}{\omega(\gamma-\beta(\rho+\delta))} < 0$.

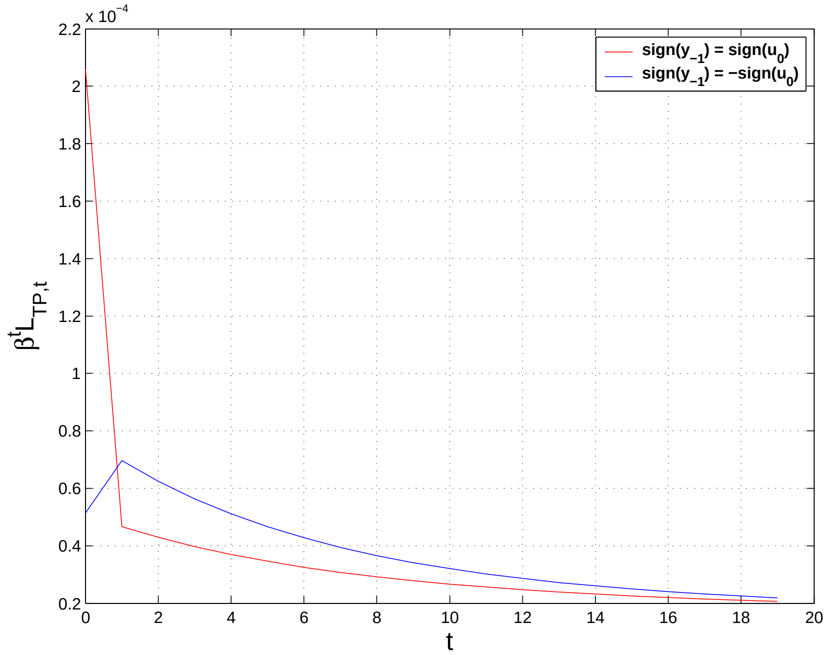

Figure 2.10: *Discounted per-period loss values $L_{TP,t}$ for $|y_{-1}| = 0.02$ and $|u_0| = 0.01$.*

impact, but a reversal of the magnitude of losses for $t \geq 1$ because the price to pay for TP then is larger until the period loss converges to its unconditional value. Since the sum of the discounted losses, however, is equal in both scenarios, $\mathcal{L}_{TP}$ is symmetric to $u_0 = 0$ given y_{-1} and to $y_{-1} = 0$ given u_0.

To summarise, Figure 2.8 presents the influences of the initial conditions on the relative performance of TP and DIS and the rest of this section provides intuitive explanations of the effects present in the model. $\widehat{RL}$ becomes positive, i.e. DIS performs better than TP, in the benchmark model for quite realistic values of the initial conditions, e.g. $\widehat{RL} > 0$ for $|y_{-1}| = 0.015$ and $|u_0| = 0.01$. Hence, it may not be welfare increasing for an economy to switch from DIS to TP if it is not close to its steady state.

2.4 Optimal timeless policy rule

So far, we have compared policy under discretion and under the timeless perspective rule in the sense of Woodford (1999). The latter appears to be the most common 'optimal' rule in the recent literature on monetary policy. However, as noted in the introduction, several authors have already mentioned that TP is not always an optimal rule - without providing an analysis of the influence of different parameters on the performance of TP and without an intuitive interpretation of their result, the main objectives of this chapter. In particular, Blake (2001) and Jensen (2003) derive the optimal timeless policy (OP) based on the unconditional expectations of the timeless perspective's MSV solution, i.e. equations (2.16) and (2.17) in section 2.2.2, as

$$\pi_t = -\frac{\omega}{\alpha} y_t + \beta \frac{\omega}{\alpha} y_{t-1} \qquad \forall t. \tag{2.23}$$

Starting from the root of the problem, however, and differentiating between the monetary authority's discount factor β_{MA}, at which the intertemporal losses in (2.3) are discounted, and the private sector's discount factor β_{PS}, that enters the New Keynesian Phillips curve (2.1), allow further insights. The intertemporal Lagrangian (2.7) changes to

$$\Lambda = E_0 \sum_{t=0}^{\infty} \beta_{MA}^t \left[(\pi_t^2 + \omega y_t^2) - \lambda_t (\pi_t - \beta_{PS} \pi_{t+1} - \alpha y_t - u_t) \right]. \tag{2.24}$$

This yields as first order conditions

$$\begin{aligned}
\frac{\partial \Lambda}{\partial y_t} &= & 2\omega y_t + \alpha \lambda_t = 0, & \qquad t = 0, 1, 2, \ldots, \\
\frac{\partial \Lambda}{\partial \pi_t} &= & 2\pi_t - \lambda_t = 0, & \qquad t = 0, \\
\frac{\partial \Lambda}{\partial \pi_t} &= & 2\beta_{MA}\pi_t - \beta_{MA}\lambda_t + \beta_{PS}\lambda_{t-1} = 0, & \qquad t = 1, 2, \ldots,
\end{aligned}$$

implying

$$\pi_t = -\frac{\omega}{\alpha} y_t, \qquad t = 0 \quad \text{and} \tag{2.25}$$

$$\pi_t = -\frac{\omega}{\alpha} y_t + \frac{\beta_{PS}}{\beta_{MA}} \frac{\omega}{\alpha} y_{t-1}, \qquad t = 1, 2, \ldots \quad . \tag{2.26}$$

Again, the timeless perspective requires neglecting (2.25) and applying (2.26) in all periods. We know from the discussion in section 2.3.1 and Dennis (2004, Appendix A) that using the 'truly timeless' perspective with unconditional expectations implicitly sets $\beta_{MA} = 1$. Hence, the optimal timeless rule by Blake (2001) and Jensen (2003) is in fact[25]

$$\pi_t = -\frac{\omega}{\alpha} y_t + \beta_{PS} \frac{\omega}{\alpha} y_{t-1}, \qquad \forall t. \tag{2.27}$$

This rule causes a loss under unconditional expectations of

$$L_{OP} = \frac{\omega[1 - \eta^2 + (\beta_{PS} - \eta)^2] + \alpha^2}{\omega(1 - \eta^2)(\xi - \beta_{PS}\eta)^2} \cdot \sigma^2, \tag{2.28}$$

[25] Recall that Blake (2001) and Jensen (2003) cannot account for the difference between β_{PS} and β_{MA} because they optimise over unconditional expectations of the loss function.

where $\xi \equiv 1 + \beta_{PS}^2 + \frac{\alpha^2}{\omega}$, $\eta \equiv \frac{\xi - \sqrt{\xi^2 - 4\beta_{PS}^2}}{2\beta_{PS}}$ and $\rho = 0$ for simplicity.

Performing simulations analogous to the ones in section 2.3.3, but with the optimal timeless rule (2.27) instead of (2.10) and $\widetilde{RL} \equiv L_{OP}/L_{DIS} - 1$, gives graphs with similar patterns to the respective figures in section 2.3.3. The critical difference is that $\widetilde{RL}$ never becomes positive for *any* parameter combinations (see figures 2.12 to 2.16 in Appendix 2.B), even for figure 2.7, where RL is positive, but $\widetilde{RL}$ negative for all ρ.[26] This suggests that as long as the private sector is not completely myopic[27] and some weight is given to output stabilisation and prices are not perfectly flexible, the inclusion of β_{PS} in the optimal policy rule (2.27) is superior to DIS from a truly timeless perspective.[28] The optimal policy rule reduces its reaction to the lagged output gap in all periods and thus optimally accounts for the decreasing potential to use future policies to spread the effects of a current shock both in the initial and future periods, given that the future is not discounted in the welfare function ($\beta_{MA} = 1$). The reason is that (2.27) reduces the weight on y_{t-1} by β_{PS} whereby today's output gap receives exactly the same weight in tomorrow's policy with which the private sector discounts tomorrow's policy today.[29]

Hence, the inclusion of β_{PS} optimally accounts for the use of unconditional expectations as the welfare criterion. But the general argument, that the relative performance of policy-making under the timeless perspective and discretion reflects the trade-off between short-run losses and long-run gains in (2.11), remains valid for two reasons: First, the general pattern of the parameter influences is not affected by OP. Second, the influence of initial conditions on the relative performance is alleviated, but still present in the benchmark model with $\beta = 0.99$ as can be seen in figure 2.11, which plots $\widehat{RL}$ as in figure 2.8, but with OP instead of TP compared to DIS.

2.5 Conclusion

This paper explores the theoretical implications of different policy rules and discretionary policy under varying parameters in the New Keynesian model. With the comparison of short-run gains from discretion over rule-based policy and long-run

[26] Here, $L_{DIS} \geq L_{OP}$ for $\rho = 0$ with any combination of parameters, and increasing ρ only aggravates this situation.

[27] For a completely myopic private sector, i.e. $\beta_{PS} = 0$, the optimal timeless rule causes a loss equivalent to the one under discretion because equations (2.6) and (2.23) are identical for $\beta_{PS} = 0$. Hence, there is no equivalent to Proposition 2.2 for OP.

[28] An analytical proof of this result could be given as follows: Since $\lim_{\beta_{PS} \to 0} L_{OP} = L_{DIS}$ and $\frac{dL_{OP}}{d\beta} < 0$ for $0 < \beta \leq 1$, while $\frac{dL_{DIS}}{d\beta} = 0$, $L_{OP} < L_{DIS}$ for $0 < \beta \leq 1$. But $\frac{dL_{OP}}{d\beta}$ is too complex to allow an analytical determination of sign $\left(\frac{dL_{OP}}{d\beta}\right)$.

[29] Recall also the discussion of the influence of β and ω in sections 2.3.2 and 2.3.3.

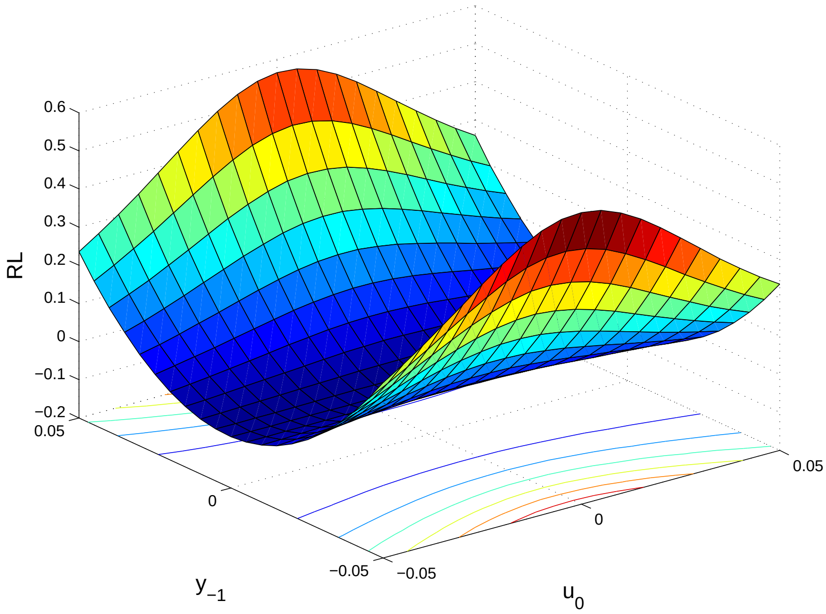

Figure 2.11: $\widehat{RL} = \frac{\mathcal{L}_{OP}}{\mathcal{L}_{DIS}}$ *depending on* y_{-1} *and* u_0.

losses from discretion, we have provided a framework in which to think about the impact of different parameters on monetary policy rules versus discretion. This framework allows intuitive economic explanations of the effects at work.

Already Blake (2001), Jensen and McCallum (2002) and Jensen (2003) provide evidence that a policy rule following the timeless perspective can cause larger losses than purely discretionary modes of monetary policy making in special circumstances. But none of these contributions considers an economic explanation for this rather unfamiliar result let alone analyses the relevant parameters as rigorously as this chapter.

What recommendations for economic policy making can be derived? Most importantly, the timeless perspective in its standard formulation is not optimal for all economies at all times. In particular, if an economy is characterised by rigid prices, a low discount factor, a high preference for output stabilisation or a sufficiently large deviation from its steady state, it should prefer discretionary monetary policy over the timeless perspective. The critical parameter values obtained in this chapter suggest that – for a number of empirically reasonable combinations of parameters – the long-run losses from discretion may be less relevant than previously thought.

In an overall laudatory review of Woodford (2003), Walsh (2005) argues that Woodford's book 'will be widely recognized as the definitive treatise on the new Keynesian approach to monetary policy.' He critisises the book, however, for its lack of an analysis of the potential short-run costs of adopting the timeless perspective rule. Walsh (2005) sees these short-run costs arising from incomplete credibility of the central bank. Our analysis has completely abstracted from such credibility effects and still found potentially significant short-run costs from the timeless perspective. Obviously, if the private sector does not fully believe in the monetary authority's commitment, the losses from sticking to a rule relative to discretionary policy are even greater than in the model used in this chapter. One way to incorporate such issues is to assume that the private sector has to learn the monetary policy rule. Evans and Honkapohja (2001) provide a convenient framework to analyse this question in more detail.

Appendix

2.A Derivation of L_{TP}

The unconditional loss for the timeless perspective, equation (2.21), can be derived in several steps. The MSV solution (2.16) and (2.17) depends on two state variables, y_{t-1} and u_t. From the conjectured solution in (2.15), we have

$$E[y_t^2] = \phi_{21}^2 E[y_{t-1}^2] + \phi_{22}^2 E[u_t^2] + 2\phi_{21}\phi_{22}E[y_{t-1}u_t]. \tag{2.29}$$

$E[y_{t-1}u_t]$ can be calculated from (2.15) with $u_t = \rho u_{t-1} + \epsilon$ as

$$\begin{aligned} E[y_{t-1}u_t] &= E[(\phi_{21}y_{t-2} + \phi_{22}(\rho u_{t-2} + \epsilon_{t-1}))(\rho u_{t-1} + \epsilon_t)] \\ &= E[\phi_{21}\rho \underbrace{y_{t-2}u_{t-1}}_{=E[y_{t-1}u_t]} + \phi_{22}(\rho^2 \underbrace{u_{t-1}u_{t-2}}_{=\rho\sigma_u^2} + \rho \underbrace{u_{t-1}\epsilon_{t-1}}_{=\sigma^2})] + 3 \cdot 0, \end{aligned} \tag{2.30}$$

since the white noise shock ϵ_t is uncorrelated with anything from the past. Solving for $E[y_{t-1}u_t]$ with $\sigma_u^2 = \frac{1}{1-\rho^2}\,\sigma^2$ gives

$$E[y_{t-1}u_t] = \frac{\phi_{22}\rho}{1-\phi_{21}\rho} \cdot \frac{1}{1-\rho^2}\,\sigma^2. \tag{2.31}$$

Plugging this into (2.29), using $E[y_t^2] = E[y_{t-1}^2] = E[y^2]$ and ϕ_{21}, ϕ_{22} from the MSV solution (2.17) leaves

$$\begin{aligned} E[y^2] &= \frac{1}{1-\phi_{21}^2}\left(\phi_{22}^2 + \frac{2\phi_{21}\phi_{22}^2\rho}{1-\phi_{21}\rho}\right)\frac{1}{1-\rho^2}\,\sigma^2 \\ &= \frac{\alpha^2(1+\delta\rho)}{\omega^2(1-\delta^2)(1-\delta\rho)[\gamma-\beta(\delta+\rho)]^2} \cdot \frac{1}{1-\rho^2}\,\sigma^2. \end{aligned} \tag{2.32}$$

From the conjectured solution in (2.14), we have

$$E[\pi_t^2] = \phi_{11}^2 E[y_{t-1}^2] + \phi_{12}^2 E[u_t^2] + 2\phi_{11}\phi_{12}E[y_{t-1}u_t]. \tag{2.33}$$

Combining this with the previous results and the MSV solution (2.16) results in

$$E[\pi^2] = \frac{2(1-\rho)}{(1+\delta)(1-\delta\rho)[\gamma-\beta(\delta+\rho)]^2} \cdot \frac{1}{1-\rho^2}\,\sigma^2. \tag{2.34}$$

Hence, L_{TP} as the weighted sum of $E[\pi^2]$ and $E[y^2]$ is given by

$$L_{TP} = \frac{2\omega(1-\delta)(1-\rho) + \alpha^2(1+\delta\rho)}{\omega(1-\delta^2)(1-\delta\rho)[\gamma-\beta(\delta+\rho)]^2} \cdot \frac{1}{1-\rho^2}\,\sigma^2. \tag{2.35}$$

2.B Influence of parameters on $\widetilde{RL}$

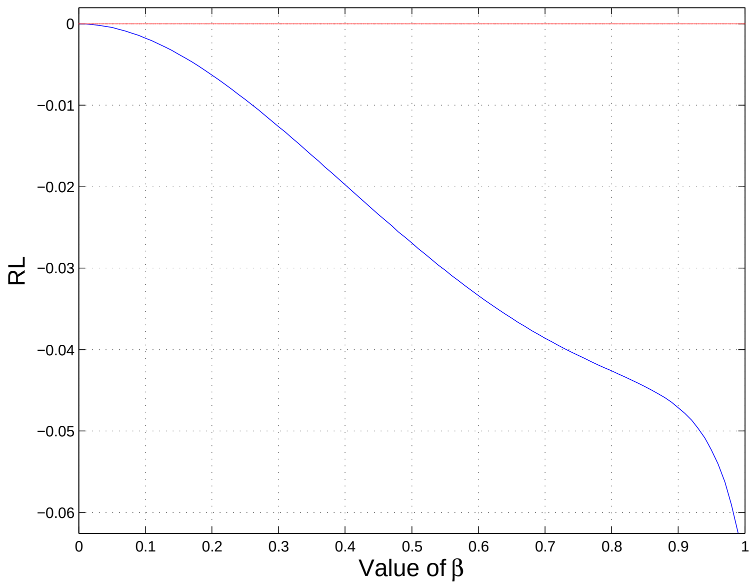

Figure 2.12: *Variation of discount factor β, OP vs. DIS.*

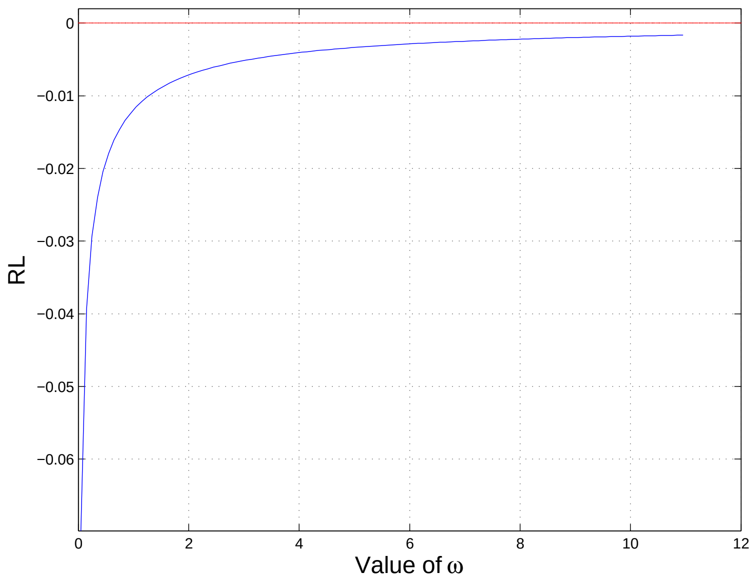

Figure 2.13: *Variation of weight on the output gap ω, OP vs. DIS.*

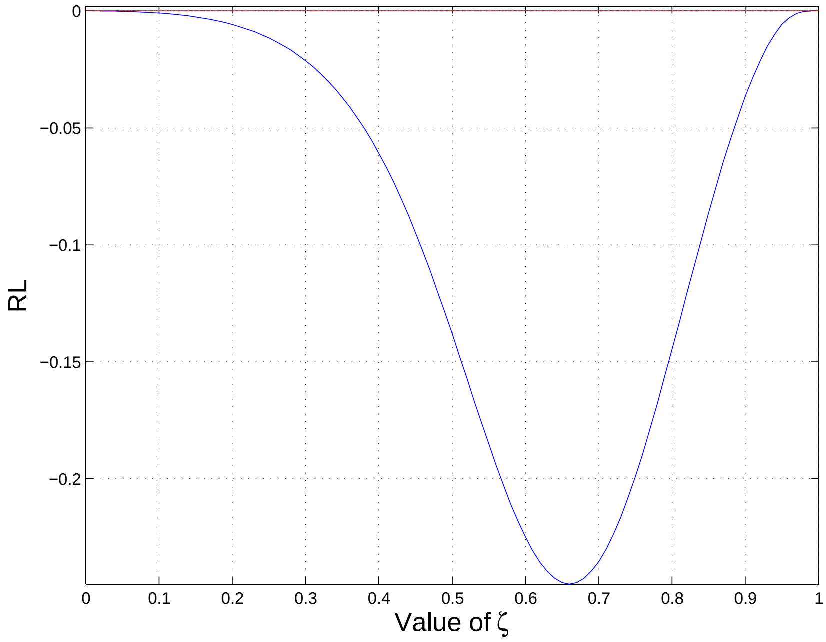

Figure 2.14: *Variation of degree of price rigidity ζ, OP vs. DIS.*

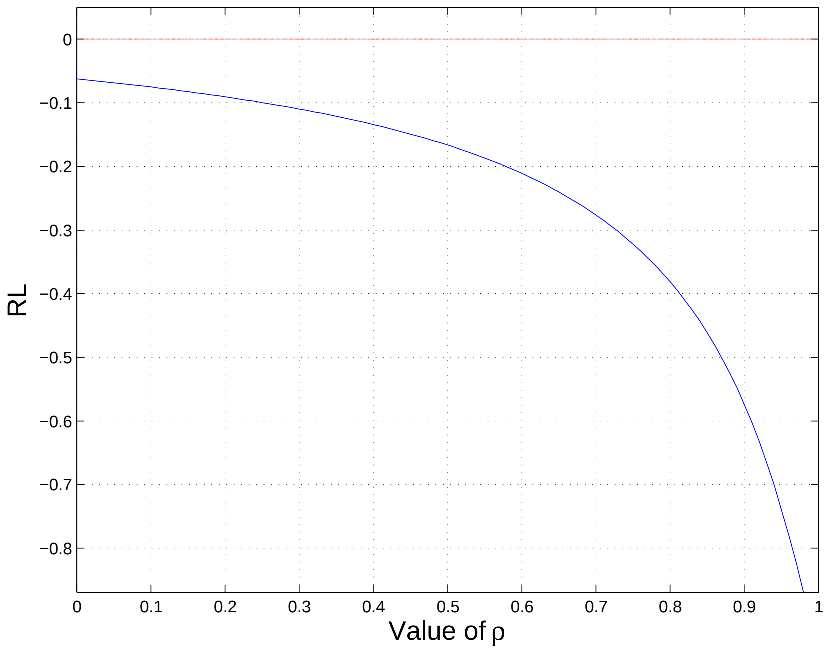

Figure 2.15: *Variation of degree of serial correlation ρ in the benchmark model, OP vs. DIS.*

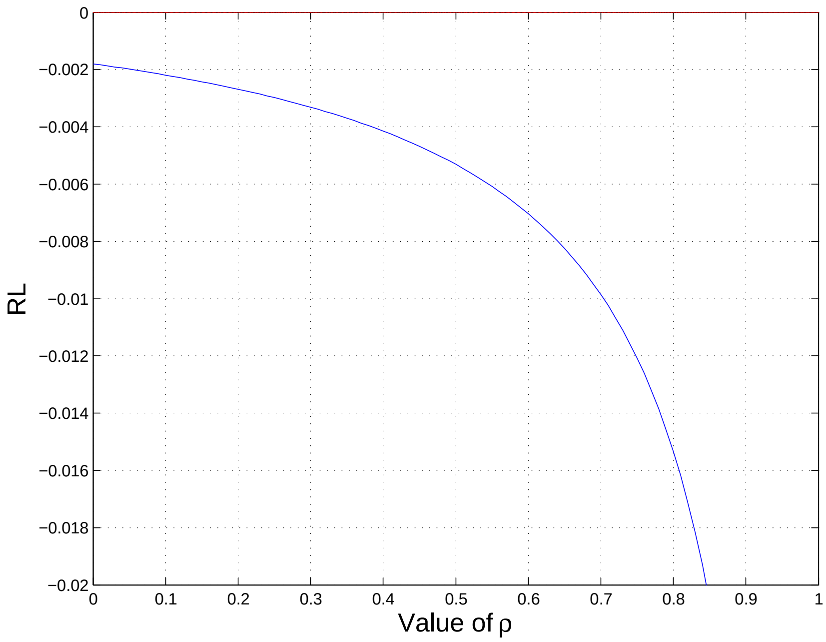

Figure 2.16: *Variation of degree of serial correlation ρ with $\omega = 10$, OP vs. DIS.*

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CHAPTER 3

Using Taylor rules to understand ECB monetary policy*

Abstract

Over the last decade, the simple instrument policy rule developed by Taylor (1993) has become a popular tool for evaluating monetary policy of central banks. As an extensive empirical analysis of the ECB's past behaviour still seems to be in its infancy, we estimate several instrument policy reaction functions for the ECB to shed some light on actual monetary policy in the euro area under the presidency of Wim Duisenberg and answer questions like whether the ECB has actually followed a stabilising or a destabilising rule so far.

Looking at contemporaneous Taylor rules, the presented evidence suggests that the ECB is accommodating changes in inflation and hence follows a destabilising policy. However, this impression seems to be largely due to the lack of a forward-looking perspective in such specifications. Either assuming rational expectations and using a forward-looking specification, or using expectations as derived from surveys result in Taylor rules which do imply a stabilising role of the ECB. The use of real-time industrial production data does not seem to play such a significant role as in the case of the US.

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*This chapter is based on joint work with Jan-Egbert Sturm and provides an extended and updated version of Sauer and Sturm (2007).

3.1 Introduction

Over the last decade, the simple instrument policy rule developed by Taylor (1993) has become a popular tool for evaluating monetary policy of central banks. Besides numerous papers on the behaviour of the Federal Reserve and other central banks,[2] some authors have applied this rule as a policy guide for the European Central Bank (ECB) in advance of the introduction of the euro in 1999.[3] Since then, the Taylor rule has been used mainly as a rough guide for the evaluation of the ECB policy by many ECB watchers in several periodicals such as 'Monitoring the ECB' by the CEPR. In contrast to that evidence and despite the end of term of the ECB's first president, Mr. Duisenberg, an extensive empirical analysis of the ECB's past behaviour still seems to be in its infancy. Referring to its short history, most papers on ECB monetary policy have estimated a Bundesbank or a hypothetical ECB reaction function prior to 1999 and then, e.g., by testing for out-of-sample stability, compared the implied interest rates with the actual ECB policy.[4] Only few researchers, such as Fourçans and Vranceanu (2002), Gerdesmeier and Roffia (2003) and Ullrich (2003), have actually estimated an ECB reaction function.

We add to this latter literature by estimating several instrument policy reaction functions for the ECB. In this way we intend to shed some light on actual monetary policy in the euro area. Looking back over the 'Duisenberg-era', we explore what role the output gap has played in the actual ECB policy and how actively the ECB has really responded to changes in inflation. By comparing these results with those for the Bundesbank, we hope to get a clearer picture of the new institutional monetary setting in Europe.

In describing actual monetary policy of the ECB by so-called Taylor rules, we will focus on data uncertainties faced by policy-makers. They base their decisions upon data which will most likely be revised in the future. Still most studies on central bank behaviour neglect this issue and use so-called 'current' or 'ex-post' data, i.e. data published in the latest release, to estimate monetary policy rules. In reality, central bankers can only use so-called 'real-time' data, i.e. data available when taking the decision. Croushore and Stark (2001) and Swanson, Ghysels and Callan (1999) show that data revisions in the case of the US affect policy analysis and economic forecasts to a substantial degree. In his influential paper, Orphanides (2001) shows that estimated policy reaction functions obtained using the ex-post revised data can yield misleading descriptions of historical policy in the case of the

[2]See, e.g., Clarida, Galí and Gertler (1998), Clarida, Galí and Gertler (2000), Judd and Rudebush (1998), Kozicki (1999), Orphanides (2001), Rudebusch (2002) and Taylor (1999).

[3]See, e.g., Gerlach and Schnabel (2000) and Peersman and Smets (1998).

[4]See, e.g., Clausen and Hayo (2002), Faust, Rogers and Wright (2001) and Smant (2002) for the first approach and, e.g., Clausen and Hayo (2002) and Gerlach-Kristen (2003) for the latter.

US. We explore whether data revisions contain similar problems for the euro area. In this line of argument, the use of survey data which are rarely being revised in the course of time, readily available, and timely (as opposed to most official data) can be very helpful.

A second important aspect of survey data is its prevalent forward-looking perspective. It is well known that central banks not only respond to past information, but use a broad range of information. In particular, they consider forecasts of inflation and output in their decision process. The theoretical justification for such a forward-looking approach is given by, e.g., Clarida, Galí and Gertler (1999) within a New Keynesian model. In addition to investigating policy reaction functions based on survey data, we follow Clarida et al. (1998, 1999, 2000) and estimate forward-looking Taylor rules in order to compare the relevance of real-time versus forward-looking aspects.

We conclude that, without assuming a forward-looking attitude of ECB policymakers, past policy rate changes are identified as having been too small with respect to changes in inflation and the ECB's policy reaction function does clearly differ from that of the Bundesbank. However, once forward-looking behaviour of the ECB is taken into account, it has followed a stabilising course, i.e. nominal policy rate changes were large enough to actually influence real short term interest rates. In that case, it becomes more difficult to statistically distinguish between the way the Bundesbank has carried out its mandate of achieving price stability in the nineties and the way the ECB has done it since. Specifications using survey information, and therefore combining a forward-looking aspect with the use of real-time data, result in by far the best fit. Unlike for the US, the use of real-time – instead of ex-post data – does not make such a clear difference for any of our conclusions for the euro area.

The next section introduces the Taylor rule. Section 3.3 offers a short overview of the relevant empirical literature. The following two sections present our own results. Amongst others, we exemplify the use of real-time as well as forward-looking data in estimating Taylor rules for the ECB. We end with some concluding remarks.

3.2 The Taylor rule

The Maastricht Treaty has made the ECB very independent. Nowadays, it is widely believed that a high level of central bank independence and an explicit mandate for the bank to restrain inflation are important institutional devices to assure price stability. It is thought that an independent central bank can give full priority to low levels of inflation. In case of the ECB, its statutes define its primary objective to be price stability, which according to the Governing Council of the ECB is measured by a year-on-year increase of the harmonised index of consumer prices (HICP) for the euro area of below, but close to 2 per cent over the medium term. In countries with a more dependent central bank, other considerations (notably, re-election perspectives of politicians and a low level of unemployment) may interfere with the objective of price stability.

The monetary policy strategy of the ECB rests on two 'pillars'.[5] One pillar, the monetary analysis, gives a prominent role to money. As inflation in the long run is considered to be a monetary phenomenon, the ECB Governing Council has announced a quantitative reference value for the annual growth rate of a broad monetary aggregate (M3). The other pillar, the economic analysis, is a broadly based assessment both of the outlook regarding price developments and of the risks to price stability in the euro area as a whole. As noted by Issing, Caspar, Angeloni and Tristani (2001), a wide range of economic and financial indicator variables – like output gap measures (i.e. measures of the discrepancy between output, or its factors of production, and their equilibrium values) – is used for this purpose.

The above suggests that, like for the US, it might be possible to describe monetary policy in the euro area by a rule depending upon both inflation and output gap developments. A natural starting point is the rule as advocated by Taylor (1993) to describe the monetary policy of the Federal Reserve in the US:[6]

$$i_t = r^* + \pi_t + 0.5(\pi_t - \pi^*) + 0.5y_t = (r^* - 0.5\pi^*) + 1.5\pi_t + 0.5y_t. \tag{3.1}$$

i_t represents the policy interest rate, r^* the equilibrium or natural real rate, π_t the rate of inflation (as a proxy for expected inflation), π^* the inflation target and y_t the output gap in period t.

From a theoretical point of view, Svensson (1999) shows that such a rule is the optimal reaction function for a central bank pursuing an inflation target in a simple backward-looking model (using an IS and a Phillips curve).[7] In line with the economic analysis of the ECB's policy strategy, the output gap is useful in forecasting future inflation and therefore enters the reaction function of the central bank even when it has a strict inflation target.

An important question relates to the weight on inflation. Since it is the real

[5] The announced changes by the ECB Governing Council on May 8th 2003 are primarily intended to improve communication. For instance, the two pillars have been interchanged and relabelled to stress the way in which information under the two pillars are cross-checked.

[6] As common in this line of literature, the nominal short-term interest rate on the money market is considered to reflect the stance of monetary policy.

[7] For other examples which motivate such a specification theoretically, we refer to Svensson (1997), Bernanke and Woodford (1997), Ball (1999) and Woodford (2001, 2003a).

interest rate which actually drives private decisions, the size of this weight needs to assure that – as a response to a rise in inflation – the nominal interest rate is raised enough to actually increase the real interest rate. This so-called 'Taylor principle' implies that this coefficient has to be greater than 1. Appendix 3.A derives the Taylor principle using the model of Svensson (1999) and the New Keynesian model of chapter 2.

The idea that an 'active' monetary policy that reacts strongly to inflation determines the equilibrium of an economy goes at least back to Leeper (1991). If the central bank does not follow such a 'leaning against the wind' policy, self-fulfilling bursts of inflation may be possible (see, e.g., Bernanke and Woodford, 1997; Clarida et al., 1998, 2000; Woodford, 2001, 2003a).[8]

In order to compare the original Taylor rule (3.1) with actual monetary policy, we need to set the equilibrium real interest rate and the inflation target and find proxies for the actual stance of monetary policy, the rate of inflation and the output gap.[9] With the ECB's inflation target of (close to, but) under 2 per cent and a mean ex-post real interest rate of roughly 1.5 per cent over the Duisenberg era, Taylor's (1993) original values of $\pi^* = 2$ and $r^* = 2$ for the US should also do reasonably well for the euro area. We measure actual monetary policy with the Euro Overnight Index Average (EONIA) lending rate on the money market.[10] Inflation is measured by the year-on-year percentage change in the harmonised index of consumer prices for the euro area, i.e. the price index used by the ECB to measure price stability.[11]

The most difficult variable to quantify in this context is the output gap. Given the relatively short time span since the introduction of the euro and the monthly frequency in which the governing council of the ECB meets and discusses the stance of monetary policy, we follow, e.g., Clarida et al. (1998) and Faust et al. (2001) and use monthly data. This restricts our option with respect to an output gap measure. In line with, e.g., Clarida et al. (1998), we take the industrial production index

[8]Within the literature on adaptive learning, Bullard and Mitra (2002) show that the Taylor principle completely characterises learnability of the fundamental (minimum state variable) rational expectations equilibrium. Honkapohja and Mitra (2004) demonstrate that policies violating the Taylor principle lead to indeterminacy and also non-fundamental rational expectations equilibria are then unlearnable.

[9]Appendix 3.B contains a list of all time series used and their sources.

[10]There is some discussion about what is the correct short-term interest rate for the euro area. We focus on the EONIA as it is the European equivalent of the Federal Funds rate for the US. Nevertheless, Pérez Quirós and Sicilia (2002) challenge its relevance because of the relatively high volatility when looking at a daily frequency due to short-term liquidity needs. As monthly averages smooth out such movements, this does not appear to be relevant for our study; all results are robust to using the 3-month EURIBOR instead.

[11]We use ex-post available data with respect to the inflation rate, i.e. the major revision of the German CPI as published in March 2003 is included. This revision has reduced inflation rates in the euro area up to 0.5 percentage points mainly in the year 2000. Taking older releases, however, does not change any of our qualitative conclusions (not shown).

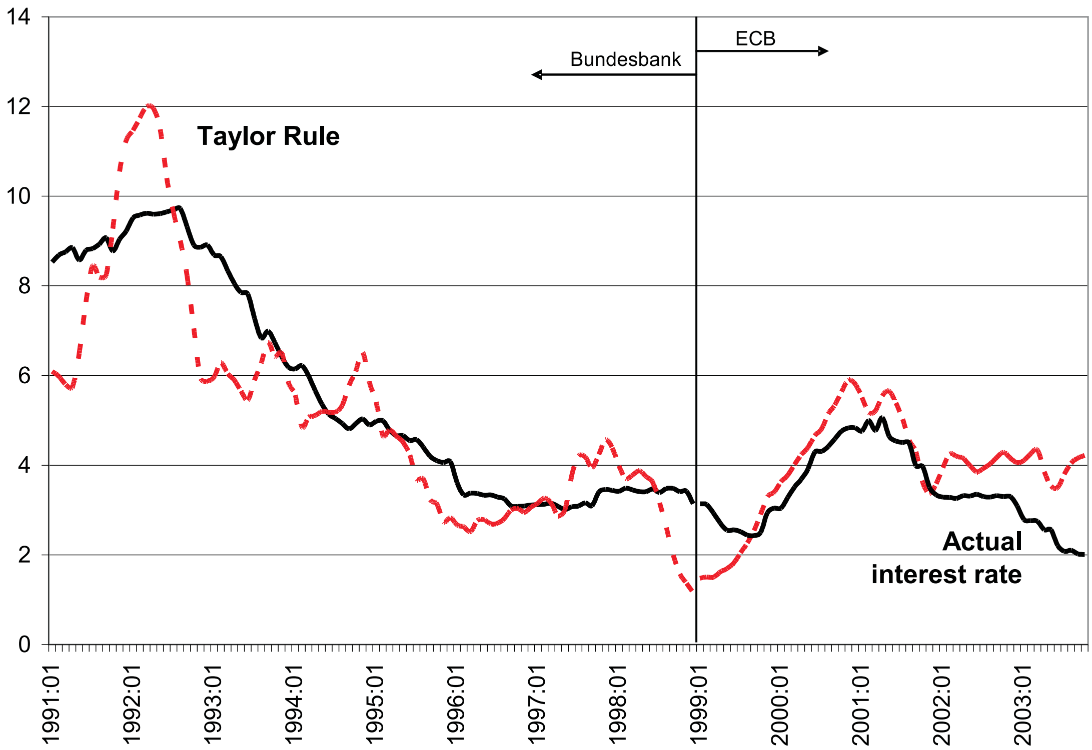

Figure 3.1: *The nominal interest rate and the Taylor rule in Germany and the euro area.*

Notes: The data before 1999 refer to Germany and monetary policy as conducted by the German Bundesbank. From 1999 onwards, the data refer to the euro area and the ECB. The solid line equals the Frankfurt overnight interest rate / EONIA, whereas the dotted line shows the three months moving average Taylor rule, in which the inflation rate is measured as the year-to-year percentage change in the Harmonised Index of Consumer Prices (for respectively Germany and the euro area) and the output gap is measured as the deviation of (German / euro area) industrial production from a Hodrick-Prescott filtered trend.

for the euro area, apply a standard Hodrick-Prescott filter (with the smoothing parameter set at $\lambda = 14,400$ and calculate our measure of the output gap as the deviation of the logarithm of actual industrial production from its trend.[12] Despite the increasing share of services in the overall economy, it is still generally believed that the industrial sector is the 'cycle maker' in the sense that it leads and influences large parts of the economy.[13]

Using these measures, figure 3.1 depicts actual monetary policy together with the Taylor rule as given by equation (3.1).[14] To enhance comparison with the Bun-

[12]To calculate a reliable measure of the output gap, we use data for euro area industrial production from 1985 onwards.

[13]As will be discussed later, industrial production data are frequently revised. For that reason, we will also look at real-time industrial production and at the European Sentiment Indicator (ESIN) as measures of the output gap.

[14]Since our measure of the output gap based on industrial production is more volatile than Taylor's (1993) original GDP-based output gap, it might be argued that it is more appropriate to

desbank era, the same graph also shows both time series for Germany using the Frankfurt overnight interest rate and other German counterparts for the remaining series.[15] In general, the coincidence of the actual nominal interest rate and the Taylor rule is quite striking especially given the sometimes volatile movements in industrial production.[16] Only during three time periods, the discrepancy between the two series appears to be relatively persistent: First, in the aftermath of German unification and the following crisis of the European Exchange Rate Mechanism (ERM) until mid-1993. Second, during the second half of 1998 and the first half of 1999. Hence, the change towards the euro seems to have had its effect on actual monetary policy. Finally, the gap since 11 September 2001 appears to be rather widening.

3.3 An overview of the empirical literature

Using such a simple rule for monetary policy and building on the experience of Taylor (1993), several authors have tried to estimate the weights given to deviations of inflation and output from their optimum by central bankers rather than choosing a symmetric weight of 0.5 as in equation (3.1). The general idea of such work is to estimate:

$$i_t = \alpha + g_\pi \pi_t + g_y y_t + \varepsilon_t \tag{3.2}$$

where the constant α captures the term $(r^* - 0.5\pi^*)$ in equation (3.1), g_π and g_y represent the estimated weights on inflation and the output gap, respectively, and ε_t is an i.i.d. error term.

In practice, it is commonly observed that, especially since the early 1990s, central banks worldwide tend to move policy interest rates in small steps without reversing direction quickly.[17] To capture this so-called interest rate smoothing, equation (3.2) is viewed as the mechanism by which the *target* interest rate i_t^* is determined. The *actual* interest rate i_t partially adjusts to this target according to $i_t = (1-\rho)i_t^* + \rho i_{t-1}$, where ρ is the smoothing parameter. This results in the following equation to be

use a lower weight on y_t than 0.5. Adjusting this weight by the ratio of the standard deviations of the output gaps based on GDP and industrial production (=0.68 for euro area since 1999) does not alter figure 3.1 in any relevant way.

[15]For the period before 1999, we have also experimented with using industrial production and inflation for the euro area. The data, however, suggest that actual policy of the German Bundesbank has been more concerned with inner German developments.

[16]Using quarterly data and taking deviations of GDP from its trend to measure the output gap results in an even better fit. This explains why, for example, the fit as shown in figure 3.1 is not as perfect as in Taylor (1993) for the federal funds rate.

[17]See, e.g., Amato and Laubach (1999), Rudebusch (2002).

estimated:

$$i_t = (1-\rho)\alpha + (1-\rho)(g_\pi \pi_t + g_y y_t) + \rho i_{t-1} + \varepsilon_t \tag{3.3}$$

Table 3.1 presents a review of different Taylor rule estimates for the euro area and the Bundesbank using monthly or quarterly data.[18] All regressions show that monetary policy prior to 1999 followed the Taylor principle as g_π exceeds 1 consistently. This holds for both Germany and the hypothetical euro area.[19] One reason for the small differences between the Bundesbank and the hypothetical euro area might be the fact that Germany possesses a very large weight in the calculation of the hypothetical euro area interest rate due to its economic size and some authors included merely a subset of all euro member countries in their studies.[20]

Furthermore, note that studies which allow the central banks to behave in a forward-looking manner do not seem to differ significantly from those which do not. This result can be interpreted in different ways. One possibility is that the period of estimation has been relatively stable which would make actual measures of the business cycle and the inflation differential good indicators of (short-term) future developments. In less stable environments – as arguably encountered by the ECB in the last couple of years – this convenient attribute of contemporaneous measures might fail.

With respect to actual ECB policy the story looks rather different; the results of Gerdesmeier and Roffia (2003) and Ullrich (2003) – who use standard output gap measures based on Hodrick-Prescott-filtered industrial production as described above – contradict those of Fourçans and Vranceanu (2002) – who take annual growth rate of industrial production as business cycle measure – and the literature on Taylor rules for both Germany and the hypothetical euro area. While Fourçans and Vranceanu (2002) find the ECB to react strongly to variations in the inflation rate and much less to output variations, Gerdesmeier and Roffia (2003), Surico (2003), as well as Ullrich (2003) estimate small reactions to inflation movements – suggesting a destabilising role of the ECB – and (both in relative and in absolute terms) strong replies to output deviations. Furthermore, Ullrich (2003) observes a structural break between pre-1999 and post-1999 monetary policy in the euro area.

[18] Recently, some authors have used ordered probit models to estimate the probability of discrete policy interest rate *changes* rather than to explain interest rate *levels* with inflation and output gap measures (see, e.g., Carstensen, 2006; Gerlach, 2005; Ullrich, 2005). Since the estimation technique and the interpretation of the parameters are very different to the standard approach used in this chapter, such papers are not included in table 3.1.

[19] The way the hypothetical euro area is being defined slightly varies across the cited papers. However, any measure is dominated by the three largest economies in the euro area, i.e. Germany, France and Italy.

[20] The striking difference of Clausen and Hayo's (2002) value for euro area g_y in comparison with all other papers and their own value for the Bundesbank might be due to their special estimation technique; they estimate a simultaneous equation model using full information maximum likelihood.

Table 3.1: *Review of Taylor rule estimations for the euro area and the Bundesbank.*

Study	Type of rule	Sample period	α	g_π	g_y	ρ
Germany						
Clarida et al. (1998)	Fwd.	1979:3-1993:12	3.14	1.31	0.25	0.91
			(0.28)	(0.09)	(0.04)	(0.01)
Peersman and Smets (1998)	Fwd.	1979:1-1997:12	2.52	1.30	0.28	0.93
			(0.32)	(0.10)	(0.05)	(0.01)
Faust et al. (2001)	Fwd.	1985:1-1998:12	2.85	1.31	0.18	0.91
			(0.85)	(0.35)	(0.16)	(0.03)
Clausen and Hayo (2002)	Cont.	1979:I-1996:IV	3.83	2.89	0.49	0.88
Smant (2002)	Fwd.	1979:3-1998:12	3.32	1.73	0.45	0.91
			(0.26)	(0.25)	(0.17)	(0.02)
Bohl and Siklos (2007)	Fwd.	1982:1-1998:12		1.20	0.28	0.92
				(0.20)	(0.59)	(0.03)
Hypothetical euro area						
Peersman and Smets (1998)	Fwd.	1980:I-1997:IV	3.87	1.20	0.76	0.76
			(0.44)	(0.09)	(0.13)	(0.13)
Gerlach and Schnabel (2000)	Cont.	1990:I-1998:IV	2.40	1.58	0.45	
			(0.30)	(0.09)	(0.06)	
	Cont.	1990:I-1998:IV	2.65	1.51	0.49	0.32
	Fwd.	1990:I-1998:IV	1.95	1.51	0.28	0.18
Clausen and Hayo (2002)	Cont.	1979:I-1996:IV	4.07	2.15	2.12	0.86
Gerlach-Kristen (2002)	Cont.	1988:I-1999:IV		1.72	0.91	0.75
				(0.33)	(0.28)	(0.05)
Ullrich (2003)	Cont.	1995:1-1998:12	1.97	1.25	0.29	0.23
Castelnuovo (2007)*	Cont.	1980:I-2003:IV		1.15	0.98	
				(0.28)	(0.22)	
	Fwd.	1980:I-2003:IV		1.00	1.80	
				(0.60)	(0.85)	

[Table continued on the next page]

To summarise, in contrast to the evidence of the Bundesbank and the hypothetical euro area, the actual ECB policy since 1999 does not necessarily seem to comply with the Taylor principle. In the rest of the chapter, we intend to shed some more light on this issue by estimating several reaction functions of the ECB and elaborating on the relevance of the output gap measures. Furthermore, we will go into the forward-looking behaviour of actual monetary policy in recent years. Table 3.1 already reveals that all papers published after the working paper version of this chapter (Sauer and Sturm, 2003) confirm the Taylor principle with a forward-looking rule, but reject it with a contemporaneous one.

Table 3.1: (continued)

Study	Type of rule	Sample period	α	g_π	g_y	ρ
Actual euro area						
Fourçans and Vranceanu	Cont./	1999:4-2002:2	1.22	1.16	0.18	0.73
(2002)[§]	Fwd.			(0.15)	(0.04)	(0.06)
Gerdesmeier and Roffia	Cont.	1999:1-2002:1	2.60	0.45	0.30	0.72
(2003)			(0.20)	(0.10)	(0.03)	(0.04)
Surico (2003)	Cont.	1997:7-2002:10	3.77	0.77	0.47	0.77
			(0.06)	(0.11)	(0.07)	(0.03)
Ullrich (2003)	Cont.	1999:1-2002:8	2.96	0.25	0.63	0.19
Fourçans and Vranceanu	Cont.	1999:4-2003:10	1.80	0.84	0.32	0.90
(2004)	Fwd.	1999:4-2003:10	-2.53	2.80	0.19	0.84
Belke et al. (2005)	Cont.	1999:I-2005:II	0.02	0.49	1.94	0.75
			(0.00)	(0.19)	(0.08)	(0.02)
Fendel and Frenkel (2005)	Fwd.	1999:1-2005:6	-1.26	1.67	0.57	0.86
			(1.08)	(0.48)	(0.05)	(0.02)
Fendel and Frenkel (2006)	Fwd.	1999:1-2003:12	-0.68	2.00	0.53	0.84
			(0.65)	(0.30)	(0.03)	(0.01)
Hayo and Hofmann (2006)	Fwd.	1999:1-2004:5	0.32	1.48	0.60	0.85
			(1.77)	(0.79)	(0.10)	(0.05)
Fourçans and Vranceanu	Cont.	1999:1-2005:10	0.07	0.63	1.08	0.94
(2006)	Fwd.	1999:1-2005:10	-0.34	4.88	0.66	0.96

Notes: Contemporaneous (Cont.) Taylor rules refer to equation (3.3), forward-looking (Fwd.) Taylor rules to equation (3.5). If reported, standard errors are within parentheses.

[*]This author employs a procedure developed by English, Nelson and Sack (2003) in order to test for active interest rate smoothing (see also section 3.6). This test requires to estimate equations (3.3) and (3.5) in first differences, i.e. using $\Delta i_t = i_t - i_{t-1}$ as a dependant variable instead of i_t, and these results are reported here.

[§]These authors use so-called growth-rate cycles (instead of growth cycles) of industrial production. As shown by, e.g., Nierhaus and Sturm (2003), a property of growth rate cycles is that business cycle turning point usually show up sooner than in case of growth cycles. Relative to growth cycles, the use of growth rate cycles therefore introduces a limited form of forward-looking behaviour. Sauer and Sturm (2003, 2005) show that this actually explains the difference in results between these and other authors using actual euro area data for 1999-2003.

3.4 Contemporaneous rules for the ECB

3.4.1 Using ex-post data

Columns (1) and (4) of table 3.2 report the results of estimating equations (3.2) and (3.3) using ex-post data. In order to get a clearer impression of the institutional changes related to the ECB taking up monetary policy in the euro area, the regressions have been conducted for the period 1991:1-2003:10, the end of Wim Duisenberg's presidentship. However, all parameters are estimated separately for the Bundesbank (1991:1-1998:12) and the ECB (1999:1-2003:10) period.[21] In this way, we can test whether significant changes have occurred. Without for the time being going into the details of the different regressions, the last two rows of the table – presenting the results of this Chow test – clearly reject the assumption of identical monetary policy reaction functions. Figure 3.1 suggests that this might be mainly due to the transition period, i.e. the second half of 1998 and the first half of 1999. To test this, Columns (2) and (5) do not take data from 1998:7-1999:6 into account. The results of the Chow test are hardly influenced by this. Hence, Bundesbank policy for Germany during the 1990s clearly differs from ECB policy under Duisenberg.

To explain in what way policies diverge, we look at the individual parameter estimates. Column (1) shows the outcomes when estimating equation (3.2). The inflation parameter for the ECB period (g_π^{ECB}) is higher than the output parameter (g_y^{ECB}), but does, however, not exceed one. Hence, the ECB moves to accommodate changes in inflation, but does not increase it sufficiently to keep the real interest rate from declining. This is confirmed by one of the last rows of table 3.2 labelled Prob $\left(g_\pi^{ECB} > 1\right)$, which reports the probability of the ECB inflation parameter to exceed one.

The middle half of table 3.2, reporting the extent to which ECB and Bundesbank coefficients differ, shows that the Bundesbank did not pursue such an accommodative strategy. The point estimate for g_π^{BuBa} equals $(0.47 + 0.90 =)1.37$. The difference between the two point estimates is highly significant. Hence, the Bundesbank more clearly followed a policy stabilising inflation as compared to the ECB. This finding is quite robust in the sense that the difference between the inflation parameters is significantly positive across almost all specifications tested.

Furthermore, the row labelled $g_y^{BuBa} - g_y^{ECB}$ reports highly significant differences between the Bundesbank and the ECB with respect to the output variable when estimating equation (3.2). The ECB seems to respond much more to changes in the business cycle than the Bundesbank has during the last years in which it determined monetary policy.

A consistent feature of OLS estimates of such simple rules as equation (3.2) is a high degree of serial correlation in the error term. Both the low Durbin-Watson statistic and the high maximum gap reported by the Durbin Cumulated Periodogram test clearly indicate severe problems with respect to serial correlation in the error term.[22] Furthermore, the Engle and Granger (1987) cointegration tests indicate

[21] Changing the sample period for the Bundesbank to 1994:1-1998:12, i.e. excluding the aftermath of German unification and the ERM crisis, does not alter our qualitative results.

[22] As we report Newey and West (1987) standard errors this should – in principle – not affect our ability to interpret the reported standard errors.

Table 3.2: *Estimated contemporaneous Taylor rules based on Hodrick-Prescott filtered output gap, 1991:1-2003:10 (with and without transition period).*

	(1) $y^{ex-post}$	(2) $y^{ex-post}$	(3) $y^{real-time}$	(4) $y^{ex-post}$	(5) $y^{ex-post}$	(6) $y^{real-time}$
	ECB-coefficients					
α^{ECB}	2.49	1.54	1.93	4.81	5.75	5.27
	(5.55)	(4.44)	(4.56)	(2.82)	(1.60)	(0.81)
g_{π}^{ECB}	0.47	0.89	0.83	-0.84	-1.28	-0.27
	(2.04)	(5.24)	(3.64)	(-0.89)	(-0.71)	(-0.10)
g_{y}^{ECB}	0.38	0.43	0.19	1.45	1.52	3.01
	(4.83)	(5.40)	(2.13)	(1.99)	(1.66)	(0.65)
ρ^{ECB}				0.94	0.95	0.98
				(25.50)	(22.68)	(29.81)
	Difference BuBa-ECB coefficients					
$\alpha^{BuBa} - \alpha^{ECB}$	-0.72	0.01		-5.77	-6.78	
	(-1.44)	(0.02)		(-1.70)	(-1.48)	
$g_{\pi}^{BuBa} - g_{\pi}^{ECB}$	0.90	0.54		2.29	2.78	
	(3.74)	(2.83)		(2.19)	(1.49)	
$g_{y}^{BuBa} - g_{y}^{ECB}$	-0.39	-0.44		0.08	-0.06	
	(-4.14)	(-4.55)		(0.04)	(-0.04)	
$\rho^{BuBa} - \rho^{ECB}$				0.04	0.03	
				(0.93)	(0.68)	
# Obs.	154	142	58	153	142	57
adj. R^2	0.85	0.86	0.32	1.00	1.00	0.96
DW/Durbin's h	0.27	0.32	0.13	0.13	0.13	0.13
Cum. Per. Test	0.64	0.61	0.69	0.12	0.12	0.15
Engle-Granger	-3.69	-3.91	-0.77	-11.36	-11.40	-7.15
Prob ($g_{\pi}^{ECB} > 1$)	0.01	0.27	0.22	0.03	0.11	0.32
Chow-test	39.15	37.55		20.10	18.32	
p-value	0.00	0.00		0.00	0.00	

Notes: Columns (1), (2) and (3) show the results for equation (3.2) using OLS with Newey and West (1987) standard errors allowing for serial correlation up to order 3. Columns (4), (5) and (6) present non-linear least squares estimates of equation (3.3) again using Newey and West (1987) standard errors. Columns (2) and (5) exclude the transition period to EMU, i.e. 1998:7-1999:6. The output gap is measured by the Hodrick-Prescott filtered industrial production. Columns (3) and (6) take detrended real-time industrial production, shifted back by two months, for the output gap variable. The row identified as DW/Durbin's h presents the Durbin-Watson test statistic for Columns (1), (2) and (3) and Durbin's h for Columns (4), (5) and (6). The Durbin Cumulated Periodogram Test (Cum. Per. Test) – a test for general serial correlation using frequency frequency domain techniques – shows the maximum gap between the theoretical spectral distribution of a white noise process and the actual residuals. In case it is significantly different from zero, we cannot reject the null of general serial correlation. For Columns (1), (2), (4) and (5) the approximate rejection limits for this test are 0.17 (1%), 0.14 (5%) and 0.12 (10%). In case of Columns (3) and (6) these rejection limits are 0.24 (1%), 0.20 (5%) and 0.18 (10%). The row labelled Engle-Granger denotes the t-statistics of the Engle and Granger (1987) cointegration test. The MacKinnon (1991) critical value using 154 observations and 6 explanatory variables equals -5.41 at the 1 per cent level. The MacKinnon (1991) critical value using 58 (57) observations and 3 (4) explanatory variables equals -4.55 (-4.73) at the 1 per cent level. The next row shows the probability of the coefficient g_{π}^{ECB} being larger than 1. The null hypothesis of the Chow test is that the coefficients for the Bundesbank and ECB period are the same (i.e. no break). t-statistics are within parentheses.

that the residuals are non-stationary, which implies that at least some variables are non-stationary and indicates that it might be problematic to interpret the estimated coefficients the way we did.[23] While interest rates and inflation are likely to be stationary in large samples, augmented Dickey and Fuller (1979, 1981) tests nevertheless indicate the presence of a unit root in our sample (not shown).[24]

To cope with the non-stationarity of some of our series and to take a possible cointegration relationship into account, we have also applied the fully modified estimator of Phillips and Hansen (1990).[25] This method provides an alternative to the use of error correction models (ECM) that are of growing popularity in empirical research.[26] As shown in Phillips (1988), the semi-parametric fully modified method and the parametric ECM approach are asymptotically equivalent in some cases. In other cases (characterised by feedback among the innovations) the fully modified method is preferable in terms of asymptotic behaviour. The fully modified estimation results (not shown) do not differ much from the results presented in the first columns. The point estimate for the ECB inflation parameter is even nearly identical. The ECB output parameter, and the differences between the Bundesbank and ECB period are generally found to be larger, albeit less significant.[27]

The other more conventional answer to the reported high serial correlation in the residuals of equation (3.2) is to include a lagged interest rate as an additional explanatory variable and hence turn to empirical estimates of equation (3.3). Column (4) of table 3.2 reports the results. The inclusion of the lagged interest rate both improves the fit of the regression and lowers the degree of serial correlation

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[23] We prefer the use of the Engle-Granger cointegration test, instead of the Durbin-Watson test on cointegration, because '[t]he use of [the Durbin-Watson] statistic is problematic in the present setting. First, the test statistic for co-integration depends upon the number of regressors in the co-integrating equation and, more generally, on the data-generation process and hence on the precise data matrix. Second, the bounds diverge as the number of regressors is increased, and eventually cease to have any practical value for the purpose of inference. Finally, the statistic assumes the null where [the residual vector] is a random walk, and the alternative where [the residual vector] is a stationary first-order autoregressive process (...). However, the tabulated bounds are not correct if there is higher-order residual autocorrelation, as will commonly occur.' (Banerjee, Dolado, Galbraith and Hendry, 1993, p. 207).

[24] By using the Hodrick-Prescott filter to calculate our measure of the output gap, this variable is by construction stationary. This is confirmed by augmented Dickey-Fuller tests. However, according to, e.g., Nelson and Plosser (1982) or Harvey and Jaeger (1993), the use of the Hodrick-Prescott filter might create artificial business cycles in the output gap variable (if the underlying industrial production series is non-stationary). A solution to this potential problem is the use of (stationary) survey data.

[25] The underlying idea of cointegration is that non-stationary time series (such as interest and inflation rates) can move apart in the short run, but will be brought back to an equilibrium relation in the long run.

[26] See in the present context Gerlach-Kristen (2003), for example.

[27] As the fully modified OLS method continues to produce similar outcomes to other methods, we will in the remaining of the chapter neither report nor discuss them; these results are available upon request.

in the errors. Both the Durbin-h statistic and the Durbin Cumulated Periodogram test cannot reject the hypothesis that the residuals behave normal. Furthermore, the Engle and Granger cointegration test clearly rejects non-stationarity of the residuals.

As compared to the first column, the ECB inflation parameter reduces in value and becomes even negative. Hence, its difference to that of the Bundesbank further increases albeit becomes less significant. For the output gap parameter, the point estimate for the ECB becomes larger. However, the difference between the Bundesbank and the ECB is no longer significant. Column (5) shows that these conclusions are hardly driven by the inclusion of the period 1998:7 until 1999:6 in which the transition towards a single currency took place and appears to have affected monetary policy (see figure 3.1).

In general, these results confirm Gerdesmeier and Roffia (2003) as well as Ullrich (2003) and suggest that the ECB reacts to a rise in inflation by raising nominal short-term interest rates by a relatively small amount and thus letting real short-term interest rates decline. As argued before, such accommodating behaviour constitutes a destabilising policy with respect to inflation. Hence, instead of continuing the inflation stabilising policy line as conducted by the Bundesbank, the ECB appears to have followed a policy rather comparable to the pre-Volcker era of the Federal reserve, for which, e.g., Taylor (1999) and Clarida et al. (2000) have found values for g_π well below one.

3.4.2 Using real-time data

A general critique to estimated policy rules such as (3.2) and (3.2) has been proposed in a sequence of articles by Orphanides (2001, 2002, 2004). He suggests that appropriateness of the Taylor rule requires the use of 'real-time' data, i.e. data actually available to the central bank at the time of its decision making. The first step to acknowledge this argument is by referring to expectations and the use of an available information set to form these expectations. Often then – to get rid of the problem of real-time data – rational expectations with unbiased forecast errors with respect to the final data are assumed.[28] However, as shown by Orphanides (2001, 2002, 2004), the actual use of real-time data in the case of the Federal Reserve for the US can cause important differences. While he uses information provided by the Greenbook for Federal Reserve Board meetings, we have to rely on publicly available data for the euro area.

In accordance with Coenen, Levin and Wieland (2005), one way to solve this problem is to take real-time data from the ECB's Monthly Bulletin statistics for the

[28] See, e.g., Clarida et al. (1998, 1999, 2000).

HICP and industrial production. The time lag of publication varies between one and two months for the inflation rate[29] and three to four months for the industrial production index.[30] Coenen et al. (2005, Table 1), document the extent of revisions of these figures, which can be summarised as being negligible for the inflation rate,[31] but substantial and frequent for the industrial production index. For this reason, we focus on the consequences of using real-time data for our measure of the output gap.

Converting our business cycle measure into real time not only involves the use of real-time industrial production data. In the previous section – and as usual in this line of literature –, we have estimated potential output in one run using all ex-post data available. However, policymakers do not have access to future information necessary to properly calculate potential output. Our monthly measure of the real-time output gap is therefore based only on data available up to two months before the month in question, i.e. potential output is calculated using the Hodrick-Prescott (HP) filter for each month separately using each time 10 preceding years of data.[32] In each run, we use the first release of industrial production for the six most recent monthly observations; ex-post data are used for older observations. Hence, we assume that the major revisions will take place within the first half year after release.[33]

Figure 3.2 shows, amongst others, the output gap measures as calculated using ex-post data (IP) and the version based on real-time data (real-time IP) since 1999:1, i.e. the ECB period. Especially during the period between the second half of 2000 and the first half of 2002, the use of real-time data clearly underestimates the expansionary phase in which the European industrial sector was situated. This might explain the relatively low interest rate during that period as compared to the Taylor rule shown in figure 3.1.

[29] Since November 2001, Eurostat base their first estimate on only a selected number of countries. This allows the first estimate to be published one month earlier than before.

[30] In fact, Eurostat releases its figures already one month before they are published in the ECB Monthly Bulletin. Therefore, we will assume that data for month $t-2$ is the latest information available on industrial production in month t.

[31] The only noticeable exception is the major revision in March 2003 as mentioned in footnote 11. Nevertheless, using real-time inflation rates does not affect any of our results in any notable way.

[32] To circumvent the end-point problem in calculating potential output using the Hodrick-Prescott filter, we also experimented with taking an autoregressive method to forecast several additional months which are then added to the series before applying the Hodrick-Prescott filter. This does not affect the outcomes in a substantial way. To not already introduce some form of forward-looking behaviour, we decided to refrain from doing so at this stage of the analysis. When estimating a forward-looking rule in section 3.5.2, the real-time output gap is based on 12-months forecasts using an AR(3) process.

[33] We experimented with slightly different procedures to construct the real time output gap. The point estimates from the different procedures do not differ much and focusing on the method proposed in the text does not affect any of the qualitative conclusions.

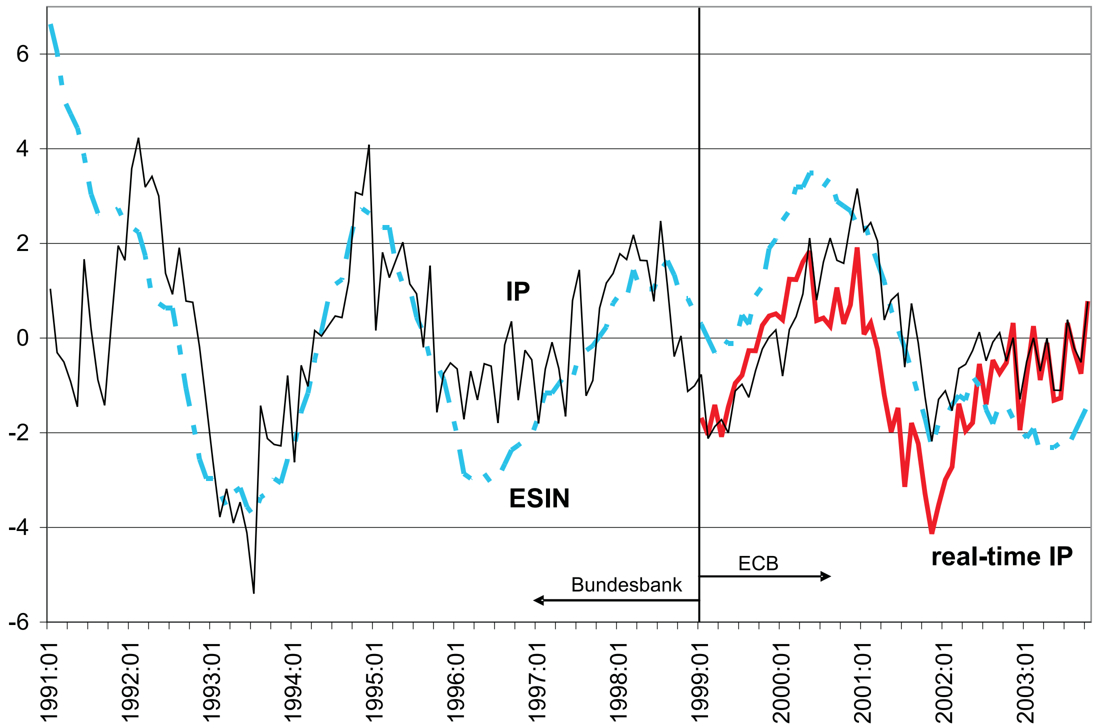

Figure 3.2: *Different indicators for the output gap of Germany and the euro area.* Notes: The data before 1999 refer to Germany, after 1999 to the euro area. The thin solid line labelled IP stands for the detrended industrial production index. The Hodrick-Prescott filter (with $\lambda = 14,400$) has been used to detrend the series. The thick solid real-time IP line shows the detrended European industrial production index this time only using real-time data as explained in the text. The dashed line labelled ESIN depicts the European Economic Sentiment Indicator from which its average over the relevant time period has been subtracted.

To investigate the consequences of this 'under-estimation' in real time, Columns (3) and (6) of table 3.2 show results when using a real-time HP measure of the output gap instead of using ex-post data. In the specification of equation (3.2), the use of real-time data results in the size of the inflation parameter to increase somewhat, without, however, exceeding one. Nevertheless, the last row of table 3.3 shows that, instead of having a probability of (nearly) zero of having the inflation parameter to exceed 1, this probability increases to 22 and 32 per cent, respectively.

Albeit the likelihood of the ECB to conduct a stabilising monetary policy has increased to more than 20%, overall we have to conclude that the use of real-time data does not lead to significantly different results. The explanatory power – as denoted by the adjusted R^2 – even declines (somewhat).

3.5 Forward-looking rules for the ECB

The ECB Governing Council has on several occasions explicitly announced that price stability is to be maintained over the medium term. Since monetary policy operates with a lag, successful stabilisation policy therefore needs to be forward-looking. Hence, an explicitly forward-looking version of the Taylor rule – with inflation and output forecasts as arguments – might be more appropriate than contemporaneous versions as estimated above.

3.5.1 Using survey data

One way to include forward-looking elements into the analysis is to use survey information to proxy business cycle movements. As survey information not only becomes available much sooner than statistical information and in general includes questions regarding future developments, it is nowadays widely believed that the former is a good leading indicator for the latter.

Since 1962 – the year in which the first harmonised business survey in industry for the EU was launched – there has been a spectacular growth of business and consumer surveys. This allowed the scope and sectors covered by such surveys to expand over time. Since 1985, the European Commission publishes the composite EU Economic Sentiment Indicator (ESIN) on a monthly basis.[34] The ESIN provides a picture of economic activity one to two months before industrial production statistics become available.[35]

Figure 3.2 shows the deviations of the ESIN from its average together with our other two indicators for the output gap. In general, the patterns of these three indicators are rather similar. The only clear difference is in volatility: the ESIN is by far the least volatile measure. A somewhat less pronounced difference is that – for the ECB period – the ESIN appears to lead the other two indicators.

By taking the ESIN as our output gap measure into the regressions, the inflation parameter gets close to – or even slightly larger than – one (Columns (1) and (3) of table 3.3). The probability of the ECB stabilising inflation increases to respectively 42 and 62 per cent, respectively (coming from close to zero in Columns (1) and (4) of table 3.2). The output parameter reduces slightly in size without losing significance. This suggests that ECB's apparent accommodative behaviour can be explained by differences between contemporaneous and forward-looking data.

[34]The EU ESIN comprises of an industrial confidence indicator, a consumer confidence indicator, a construction confidence indicator and a retail trade confidence indicator. In May 2004, i.e. after the time period considered, a service sector indicator was added to ESIN.

[35]For the relevance of ESIN as a business cycle indicator for the EU, see, e.g., Goldrian, Lindlbauer and Nerb (2001).

Table 3.3: *Estimated forward-looking Taylor rules using survey data, 1991:1-2003:10.*

	(1) y^{ESIN}	(2) $+\pi^{forecast}$	(3) y^{ESIN}	(4) $+\pi^{forecast}$
	ECB-coefficients			
α^{ECB}	1.50	0.12	1.09	0.13
	(3.61)	(0.17)	(2.16)	(0.19)
g_π^{ECB}	0.96	1.95	1.08	1.85
	(4.93)	(4.73)	(4.28)	(4.63)
g_y^{ECB}	0.28	0.23	0.66	0.59
	(4.71)	(4.54)	(6.89)	(5.81)
ρ^{ECB}			0.88	0.87
			(35.14)	(28.58)
	Difference BuBa-ECB coefficients			
$\alpha^{BuBa} - \alpha^{ECB}$	0.33	0.34	-0.20	0.69
	(0.72)	(0.37)	(-0.26)	(0.71)
$g_\pi^{BuBa} - g_\pi^{ECB}$	0.39	-0.22	0.25	-0.45
	(1.89)	(-0.46)	(0.81)	(-0.85)
$g_y^{BuBa} - g_y^{ECB}$	-0.13	-0.12	0.05	-0.42
	(-1.31)	(-1.75)	(0.20)	(-3.46)
$\rho^{BuBa} - \rho^{ECB}$			0.08	-0.01
			(2.42)	(-0.12)
# Obs.	154	118	153	117
adj. R^2	0.87	0.76	1.00	0.98
DW/Durbin's h	0.28	0.20	0.26	0.26
Cum. Per. Test	0.65	0.67	0.10	0.08
Engle-Granger	-3.51	-2.20	-12.72	-11.72
Prob ($g_\pi^{ECB} > 1$)	0.42	0.99	0.62	0.98
Chow-test	37.92	4.02	12.21	17.71
p-value	0.00	0.26	0.02	0.00

Notes: Columns (1) and (3) take the European Sentiment Indicator (ESIN) as output gap measure. Columns (2) and (4) also use forecasted inflation as published in *The Economist* instead of actual inflation. All parameters are estimated separately for the Bundesbank (1991:1-1998:12 in case of (1) and (3), 1994:1-1998:12 in case of (2) and (4)) and the ECB (1999:1-2003:10) period, based on data for the respective region. See notes of table 3.2.

Instead of relying on statistical releases of the (contemporaneous) inflation rate, we can also use (forward-looking) survey results to get an idea of inflation developments. The newspaper *The Economist* has published inflation forecasts based on a poll of a group of forecasters every month since 1994.[36] Figure 3.3 shows these survey forecasts together with our regular inflation measure. The inflation forecast measure is less volatile.

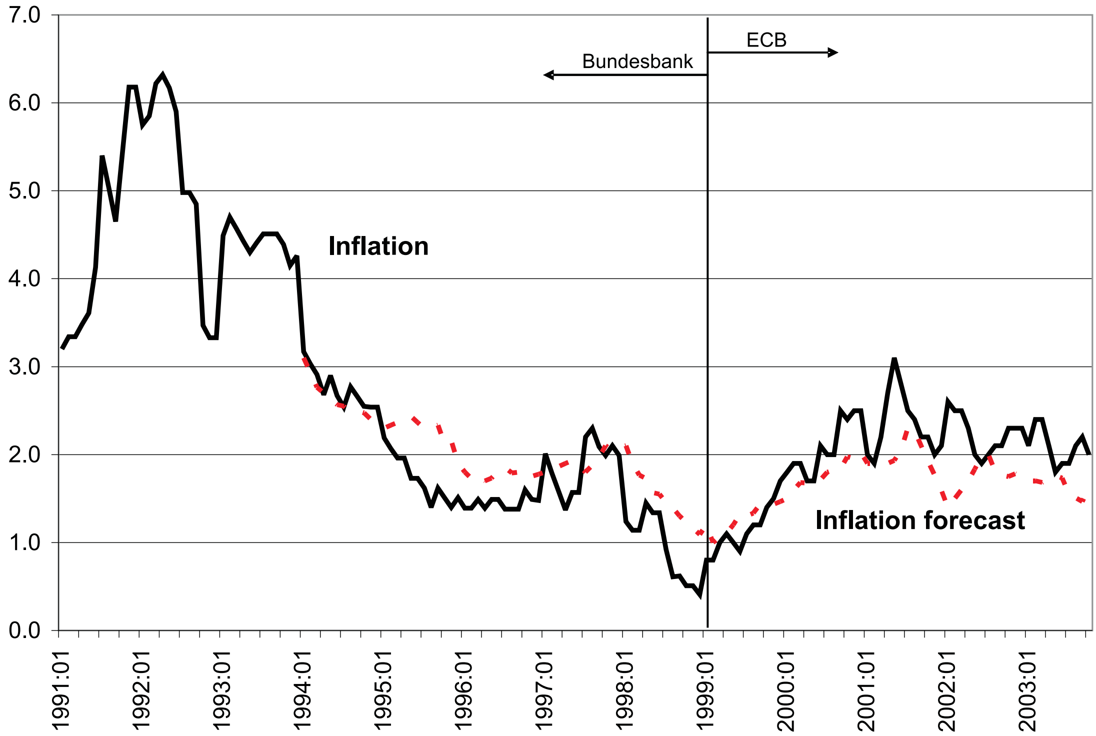

Figure 3.3: *Different indicators for inflation in Germany and the euro area.*
Notes: The solid line labelled 'Inflation' shows the year-on-year percentage change of the Harmonised Index of Consumer Prices (HICP) for Germany and the euro area, respectively. The dotted line shows the inflation forecasts taken from *The Economist* for the respective regions.

Columns (2) and (4) of table 3.3 show the results in case we combine both forward-looking survey measures, i.e. replace actual inflation in Columns (1) and (3) by this inflation forecast measure. In both specifications, the inflation parameter both for the Bundesbank and the ECB is – with a probability of close to 100 per cent – larger than one without significantly affecting the output parameter. Without the interest rate smoothing term, the structural break between the Bundesbank and the ECB disappears, while in column (4) the break is driven solely by the significantly

[36] Unfortunately, these figures are only annual average inflation rates, not true 12-month inflation forecasts. To convert these into monthly moving figures, we take as the 12-month forecast of inflation the weighted average of the forecast for the current and the following year, where the weights are $x/12$ for the x remaining months in the current year and $(12 - x)/12$ for the following year's forecast. See also Smant (2002, p. 7).

smaller reaction of the Bundesbank to the German ESIN-output gap rather than different reactions to inflation forecasts. Hence, taking these survey measures as proxies for our theoretical output gap and inflation variables shows that the ECB has appeared to have followed a stabilising policy rule with respect to both.

3.5.2 Using HP-filtered industrial production

As survey measures also bear real-time aspects – they are usually available without long time lags and without (substantial) revisions – it could be argued that the improved results in table 3.3 (as compared to table 3.2) should be attributed to the use of real-time data instead of taking a forward-looking perspective. However, note that the use of real-time data in so-called contemporaneous rules (Columns (3) and (6) of table 3.2) seems to reject that hypothesis. To nevertheless shed some additional light on this, we will now estimate explicitly forward-looking models in which ex-post and real-time data on industrial production are used.

As an enhancement of the standard Taylor-rule framework, many economists follow Clarida et al. (1998) and use a forward-looking rule, where the target interest rate i_t^* is set in response to expected inflation and output. Expectations are based on the available information set Ω at time t and reach k and l periods into the future, respectively.

$$i_t = \alpha + g_\pi E\left(\pi_{t+k}|\Omega_t\right) + g_y E\left(y_{t+l}|\Omega_t\right) + \varepsilon_t \tag{3.4}$$

$$i_t = (1-\rho)\,\alpha + (1-\rho)\left(g_\pi E\left(\pi_{t+k}|\Omega_t\right) + g_y E\left(y_{t+l}|\Omega_t\right)\right) + \rho i_{t-1} + \varepsilon_t \tag{3.5}$$

Assuming rational expectations, these equations are estimated using the generalised method of moments (GMM). Table 3.4 reports results for $k = 6$ and $l = 3$.[37]

Independent of whether we use ex-post or real-time data to measure the output gap, the inflation parameter is with high probability larger than one. Hence, by explicitly including forward-looking behaviour on account of the ECB, monetary policy in recent years has – at least ex ante – been sufficiently aggressive to stabilise inflation in the euro area. The use of real-time data as compared to ex-post data does not seem to make much of a difference. From table 3.2, however, we know that it is not sufficient to use real-time data in a contemporaneous set-up. Without taking a forward-looking perspective, ECB's monetary policy cannot be considered to have stabilised inflation.

Comparing the results in table 3.4 with those in table 3.3 reveals that the use of

[37] The set of instruments are up to six lags of the inflation and output gap corresponding to data employed in the regression, and – in case we model interest rate smoothing – the money market rate.

Table 3.4: *Estimated forward-looking Taylor rules using GMM, 1991:1-2003:10.*

	(1) $y^{ex-post}$	(2) $y^{real-time}$	(3) $y^{ex-post}$	(4) $y^{real-time}$
	ECB-coefficients			
α^{ECB}	1.15	0.15	-1.36	-6.23
	(1.45)	(0.18)	(-1.10)	(-0.61)
g_{π}^{ECB}	1.05	1.54	2.15	6.62
	(2.88)	(4.06)	(3.83)	(0.90)
g_{y}^{ECB}	0.22	0.01	1.10	9.24
	(1.60)	(0.05)	(3.22)	(0.64)
ρ^{ECB}			0.91	0.98
			(31.20)	(35.57)
	Difference BuBa-ECB coefficients			
$\alpha^{BuBa} - \alpha^{ECB}$	0.66		-2.93	
	(0.78)		(-0.16)	
$g_{\pi}^{BuBa} - g_{\pi}^{ECB}$	0.49		-1.59	
	(1.30)		(-0.50)	
$g_{y}^{BuBa} - g_{y}^{ECB}$	-0.15		8.71	
	(-1.07)		(0.29)	
$\rho^{BuBa} - \rho^{ECB}$			0.08	
			(2.44)	
# Obs.	145	55	141	51
adj. R^2	0.80	0.27	1.00	0.96
DW/Durbin's h	0.20	0.24	0.24	0.24
Cum. Per. Test	0.69	0.61	0.07	0.17
Engle-Granger	-2.93	-2.04	-11.36	-9.01
Prob ($g_{\pi}^{ECB} > 1$)	0.55	0.92	0.98	0.77
Chow-test	93.65		8.62	
p-value	0.00		0.07	

Notes: Under rational expectations, forward-looking estimates of inflation and the output gap are used, i.e. we set $k = 6$, $l = 3$ in equation (3.4) and (3.5). The results are estimated by the Generalised Method of Moments (GMM) with Newey-West heteroscedasticity and serial correlation robust estimators. As instruments, we use up to six months lagged inflation and output gaps corresponding to data employed in the regression, and – in case we model interest rate smoothing – interest rates. Columns (1) and (3) use ex-post data. Columns (2) and (4) take detrended real-time industrial production, shifted back by two months, for the output gap variable. For Columns (1) and (3), the approximate rejection limits for the Durbin Cumulated Periodogram Test (Cum. Per. Test) are 0.17 (1%), 0.14 (5%) and 0.12 (10%). In case of Columns (2) and (4) these rejection limits are 0.24 (1%), 0.20 (5%) and 0.18 (10%). The MacKinnon (1991) critical value using 145 observations and 6 explanatory variables equals -5.43 at the 1 per cent level, or $-4.57(-4.99)$ when using 55 (51) observations and 3 (4) explanatory variables. See notes of table 3.2.

survey data results in a better fit than does the use of industrial production data in forward-looking specifications like equation (3.4) or (3.5).

3.6 Concluding remarks

In this chapter, we have explored different ECB Taylor rules for the euro area. We have asked ourselves, whether or not the ECB has in its first years of existence under the presidency of Mr. Duisenberg been following a stabilising or a destabilising rule. Already Faust et al. (2001) argue that the ECB puts too high a weight on the output gap relative to inflation and in comparison to the Bundesbank.

Looking at contemporaneous Taylor rules, the presented evidence clearly confirms previous research and suggests that the ECB is accommodating changes in inflation and hence follows a destabilising policy. The differences between the Bundesbank and the ECB are significant. Such an interpretation gives rise to the conjecture that the ECB follows a policy quite similar to the pre-Volcker era of US monetary policy, a time also known as the 'Great Inflation' (Taylor, 1999).[38]

However, this impression seems to be largely due to the lack of a forward-looking perspective. Either assuming rational expectations and using a forward-looking specification as suggested by Clarida et al. (1998), or using expectations as derived from surveys result in Taylor rules which do imply a stabilising role of the ECB. In such forward-looking cases, at least the weights attached to the inflation rate by the Bundesbank and the ECB do no longer significantly differ. The use of real-time industrial production data, as suggested by Orphanides (2004), hardly helps in this respect.

Our preferred specification involves the use of survey data; their real-time character combined with their forward-looking nature seems to produce the best results, in the sense that its explanatory power is the largest and the parameters do confirm a stabilising role for the ECB. Furthermore, an important advantage of survey data is that one does not have to rely upon (artificial) decomposition methods like the Hodrick-Prescott filter introducing several additional problems – problems which we barely touched upon in this chapter.

The chapter so far concentrated on the empirical analysis of the 'Duisenberg-era' from January 1999 to October 2003. In November 2003, Jean-Claude Trichet succeeded Wim Duisenberg as the president of the ECB. Table 3.5 provides a comparison of a contemporaneous Taylor rule with our preferred specification based on

[38] Taylor (1999) finds values of $g_\pi = 0.81$ and $g_y = 0.25$ with ex-post data for the US for that period, while Orphanides (2004) estimates a forward-looking rule with real-time data and reports $g_\pi = 1.64$ and $g_y = 0.57$.

survey data for the full ECB sample for which data are available, i.e. 1999:1-2006:12. The results confirm the evidence for the shorter sample period: Based on the contemporaneous specification, the ECB appears to have followed a destabilising policy with respect to inflation; the estimated weight on inflation is even significantly negative. Using survey data, however, provides similar results to Column (4) in table 3.3 and thus corroborates the view that the ECB follows a forward-looking, stabilising policy with respect to inflation expectations and output developments.

Table 3.5: *Estimated Taylor rules for the full ECB period, 1999:1-2006:12.*

	(1) Contemporaneous	(2) Forward-looking
α^{ECB}	5.06	-0.26
	(3.55)	(-0.16)
g_π^{ECB}	-1.00	1.87
	(-1.60)	(2.03)
g_y^{ECB}	1.50	0.21
	(3.48)	(4.95)
ρ^{ECB}	0.95	0.94
	(54.84)	(61.24)
# Obs.	93	95
adj. R^2	0.98	0.98
DW/Durbin's h	0.05	0.06
Cum. Per. Test	0.12	0.14
Engle-Granger	-9.97	-11.39
Prob ($g_\pi^{ECB} > 1$)	0.00	0.83

Notes: Columns (1) and (2) present non-linear least squares estimates of equations (3.3) and (3.5), respectively, using Newey and West (1987) standard errors. Column (1) repeats the estimation of Column (4) in table 3.2, but for the sample 1999:1-2006:10. Column (2) reflects Column (4) in table 3.3 for the sample 1999:1-2006:12. See notes of table 3.2.

We have also largely abstracted from the second pillar of the ECB's monetary policy strategy, the monetary analysis. Formally, results from the monetary analysis serve to 'cross-check' the shorter-term inflationary risks emerging from the economic analysis and the ECB has emphasised its relevance on numerous occasions. Empirically, however, Fourçans and Vranceanu (2004) and Fendel and Frenkel (2006) find that simply adding money growth as an additional explanatory variable to equations (3.2) to (3.5) has no statistically significant impact on the estimation.[39] In a recent paper, Hofmann, Sauer and Strauch (2007) report a positive, systematic role of monetary aggregates on interest rates only for different empirical specifications reflecting the idea of 'cross-checking' in a more elaborate way.[40]

A final result of this chapter is that the data show a large degree of partial adjustment in the interest rate, i.e. short-term interest rates tend to be changed in several sequential steps in one direction. In principle, this could imply that policy responds too little and too late to changes in the economic environment. Rudebusch (2002, 2006) reports comparable outcomes for the US. In contrast to the conventional wisdom that the Federal Reserve smoothes adjustments in the interest rate, Rudebusch argues – based on quarterly data – that this view is an illusion and the apparent inertia rather reflect persistent shocks to the economy.[41] Castelnuovo (2007) tests for Rudebusch's hypothesis using data for the hypothetical euro area from 1980 to 2003. His results[42] suggest that the observed gradualism in the interest rate is to a significant extent endogenous, i.e. stemming from the systematic component of monetary policy in the hypothetical euro area. Whether this is also true for the ECB since 1999 is a question that is left for future research.

[39]Berger, de Haan and Sturm (2006) construct indices measuring the different aspects of the ECB's strategy in its monthly press statements explaining interest rate decisions. They obtain no significant impact of the index related to monetary developments on actual interest rate decisions.

[40]Inter alia, they include inflation projections based on information from monetary aggregates rather than the monetary aggregates themselves in empirical reaction functions as additional variables.

[41]Sack and Wieland (2000) offer three explanations of interest-rate smoothing: forward-looking behaviour by market participants, measurement error associated with key macroeconomic variables and uncertainty regarding relevant structural parameters. Goodfriend (1991) stresses the financial instability associated with potential market overreactions in response to volatile policy interest rates. Ellis and Lowe (1997) emphasise that repeated changes in the direction of interest rate adjustments may be perceived by the public as policy 'mistakes' and weaken the announcement effect of interest rate changes in the transmission mechanism of monetary policy. Further arguments in favour of interest rate smoothing involve the zero lower bound on nominal interest rates (Reifschneider and Williams, 2000) and the history dependence of optimal monetary policy as advocated by Woodford (2003a,b) and analysed in chapter 2 of this thesis.

[42]The results of the estimated reaction functions are reported in table 3.1.

Appendices

3.A Theoretical foundations of the Taylor principle

The Taylor principle, i.e. the increase of the nominal interest rate i_t by more than one-for-one in response to an increase in inflation π_t or inflation expectations $E_t\pi_{t+1}$ in order to raise the real interest rate, has proven to be a robust guideline for prudent monetary policy in a wide range of macroeconomic models. In this appendix, we derive the Taylor principle in two models that have a non-vertical short-run aggregate supply curve, an aggregate demand relationship that depends on the real interest rate and a loss function or an explicit interest rate rule for the central bank.

3.A.1 Backward-looking model

Svensson (1997) uses a model of the economy, where the transmission lag of interest rate changes to real activity is one period and to inflation two periods:[43]

$$\pi_{t+1} = \pi_t + \gamma y_t + \varepsilon_{t+1} \tag{3.6}$$

$$y_{t+1} = \delta y_t - \varphi(i_t - E_t\pi_{t+1} - r^*) + \eta_{t+1}, \tag{3.7}$$

where E_t denotes expectations conditional upon information available at t. γ, δ, φ are positively defined parameters and ε_t and η_t i.i.d. are shocks with mean zero. Equation (3.6) represents a backward-looking, accelerationist Phillips curve, (3.7) an aggregate demand relationship. The central bank controls the nominal interest rate $\{i_t\}_{t=0}^{\infty}$ and minimises

$$E_0 \sum_{t=0}^{\infty} \beta^t (\pi_t - \pi^*)^2. \tag{3.8}$$

Plugging (3.7) in (3.6) shifted forward by one period yields

$$\pi_{t+2} = \pi_t + \gamma y_t + \varepsilon_{t+1} + \gamma\{\delta y_t - \varphi[i_t - (\pi_t + \gamma y_t) - r^*] + \eta_{t+1}\}$$

and the expected inflation rate

$$E_t\pi_{t+2} = (1 + \gamma\varphi)\pi_t + \gamma(1 + \delta + \gamma\varphi)y_t - \gamma\varphi(i_t - r^*). \tag{3.9}$$

[43] The timing of the model is consistent with results from a number of VAR-studies, if one interprets one period as roughly one year (see, e.g., Christiano, Eichenbaum and Evans, 1996).

Since the central bank can influence inflation with its instrument i_t only with a two-period lag, the first-order condition for optimal policy in $t \geq 0$ is

$$\begin{aligned}\frac{\partial E_t \beta^2(\pi_{t+2}-\pi^*)^2}{\partial i_t} &= E_t\left[2\beta^2(\pi_{t+2}-\pi^*)\frac{\partial \pi_{t+2}}{\partial i_t}\right] \\ &= -2\beta^2\gamma\varphi(E_t\pi_{t+2}-\pi^*) = 0. \\ \Leftrightarrow \qquad & E_t\pi_{t+2} = \pi^* \qquad (3.10)\end{aligned}$$

Combining the expected inflation rate (3.9) and the first-order condition (3.10) gives the optimal interest rate rule

$$i_t = r^* + \pi_t + \frac{1}{\gamma\varphi}(\pi_t - \pi^*) + \left(\gamma + \frac{1+\delta}{\varphi}\right) y_t. \qquad (3.11)$$

Equation (3.11) corresponds to the Taylor rule (3.1) with general weights instead of 0.5 as initially suggested by Taylor (1993). In particular, the rule (3.11) fulfills the Taylor principle as $\frac{1}{\gamma\varphi} > 0$. In line with the second pillar of the ECB's monetary policy strategy, the output gap is useful in forecasting future inflation and therefore enters the reaction function of the central bank even when it has a strict inflation target.[44]

3.A.2 New Keynesian model

Using the forward-looking New Keynesian model of chapter 2, Woodford (2003a) shows that the Taylor principle must hold in order to determine the price level with an interest rate rule. Let the forward-looking New Keynesian Phillips curve (3.12) and the aggregate demand relationship (3.13) based on intertemporal optimisation be given by

$$\pi_t = \beta E_t \pi_{t+1} + a y_t + u_t \qquad (3.12)$$

$$y_t = E_t y_{t+1} - b(i_t - E_t\pi_{t+1}) + v_t \qquad (3.13)$$

with a, b as positively defined parameters, u_t, v_t i.i.d. shocks with mean zero and the natural real interest rate $r^* = 0$. The model is closed with a general interest rate rule in which the central bank reacts only to the inflation rate and not to the output gap:

$$i_t = \phi_\pi \pi_t. \qquad (3.14)$$

[44]Svensson (1997) shows that the Taylor principle also holds in the optimal interest rate rule if the loss function explicitly includes an output gap term, i.e. the period loss function is $L_t = (\pi_t - \pi^*)^2 + \omega y_t^2$. The loss function (3.8) reflects the special case with a weight $\omega = 0$ on the output gap term.

The rule is not explicitly derived from a loss function and the inflation target $\pi^* = 0$ for simplicity. The system can be rewritten as

$$\begin{bmatrix} E_t\pi_{t+1} \\ E_t y_{t+1} \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{1}{\beta} & -\frac{a}{\beta} \\ bg_\pi - \frac{b}{\beta} & 1 + \frac{ab}{\beta} \end{bmatrix}}_{\equiv \mathbf{A}} \underbrace{\begin{bmatrix} \pi_t \\ y_t \end{bmatrix}}_{\equiv \mathbf{z}_t} + \underbrace{\begin{bmatrix} -\frac{1}{\beta} & 0 \\ \frac{b}{\beta} & 1 \end{bmatrix}}_{\equiv \mathbf{B}} \underbrace{\begin{bmatrix} u_t \\ v_t \end{bmatrix}}_{\equiv \mathbf{e}_t}$$

which can be summarised as

$$E_t\mathbf{z}_{t+1} = \mathbf{A}\mathbf{z}_t + \mathbf{B}\mathbf{e}_t.$$

Since $\mathbf{e}_t$ is stationary by assumption, the rational expectations equilibrium is determinate if and only if the matrix $\mathbf{A}$ has both eigenvalues outside the unit circle. Given that the trace $\mathrm{tr}\mathbf{A} = 1 + \beta^{-1}(1 + ab) > 1$ and the determinant $\det \mathbf{A} = \beta^{-1}(1 + abg_\pi) > 1$, Woodford (2003a) shows that the eigenvalues of $\mathbf{A}$ fulfill this condition if and only if $\det \mathbf{A} - \mathrm{tr}\mathbf{A} > -1$, i.e.

$$\beta^{-1}(1 + abg_\pi) - 1 - \beta^{-1}(1 + ab) > -1,$$

which simplifies to the Taylor principle

$$g_\pi > 1.$$

In a recent working paper, Cochrane (2006) challenges the conventional wisdom and argues that 1) the Taylor principle would not determine the price level or the inflation rate in the New Keynesian model and that 2) the Taylor rule coefficients could not be identified in a Taylor rule regression. The first conjecture is based on the observation that the Taylor principle guarantees only a unique *local* equilibrium as it is derived from a log-linear approximation of the true non-linear model. Cochrane relates this to the fiscal theory of the price level which claims that the government satisfies its budget constraint only in equilibrium and only this equilibrium condition could determine the price level. For example, Buiter (2002) provides a thorough critique of the fiscal theory of the price level.

Cochrane's second conjecture crucially depends on the assumption that the interest rate shock x_{it} in the interest rate rule

$$i_t = g_\pi\pi_t + x_{it}$$

represents the *only* state variable in the system. If there are other state variables such as cost-push shocks u_t, demand shocks v_t or lagged inflation rates and output gaps which could be due to habit formation, for example, Cochrane's strong conclusions break down.[45]

3.B Data

3.B.1 Interest rates

For the nominal interest rate of the euro area, we take the Euro Overnight Index Average (EONIA). In case of Germany, we use the Frankfurt Interbank Offered Rate Overnight. Both interest rates are provided as monthly averages by the Bundesbank's time series data base: `http://www.bundesbank.de/stat/zeitreihen/index.htm`

3.B.2 Inflation rates

Annual inflation for the euro area is measured by the harmonised index of consumer prices (HICP). This series is not adjusted for seasonally effects and is taken from the ECB website: `http://www.ecb.int/stats/mb/eastats.htm.`

For Germany, we take the annual inflation rate based on the consumer price index (CPI) (not seasonally adjusted) as published by the Federal Statistical Office Germany.

Real-time inflation for the euro area is based on first published figures for the respective month as available in the ECB Monthly Bulletins. The inflation forecasts are based on data published by the newspaper *The Economist*. In that case, the calculation of each monthly data point is described in footnote 36.

3.B.3 Output gap measures

As first measure for the output gap, we take the European industrial production index starting in 1985, apply a standard Hodrick-Prescott filter with the smoothing parameter of $\lambda = 14,400$ and calculate the output gap as the deviation of the logarithm of actual industrial production from trend. Our measure of the euro area industrial production index excludes construction, is seasonally and working day adjusted, and is taken from the ECB website.

Alternative estimates of the output gap include a 'real-time' industrial production index and the European Sentiment Indicator (ESIN). The former consists of first published figures for the respective months and is collected from the ECB

[45] I have developed this argument in joint research with Agostino Consolo.

Monthly Bulletins. The latter, which is a weighted combination of an industrial confidence indicator, a consumer confidence indicator, a construction confidence indicator, and a retail trade confidence indicator, is taken from the European Commission website: http://europa.eu.int/comm/economy_finance/indicators/business_consumer_surveys/bcsseries_en.htm

German industrial production is seasonally adjusted and taken from Eurostat.

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CHAPTER 4

Liquidity risk and monetary policy

Abstract

This chapter provides a framework to analyse emergency liquidity assistance of central banks on financial markets in response to aggregate and idiosyncratic liquidity shocks. The model combines the microeconomic view of liquidity as the ability to sell assets quickly and at low costs and the macroeconomic view of liquidity as a medium of exchange that influences the aggregate price level of goods. The central bank faces a trade-off between limiting the negative output effects of dramatic asset price declines and more inflation. Furthermore, the anticipation of central bank intervention causes a moral hazard effect with investors. This gives rise to the possibility of an optimal monetary policy under commitment.

4.1 Introduction

Liquidity is an important concept in finance and macroeconomics. The microeconomic literature in finance views liquidity roughly as the ability to sell assets quickly and costlessly. In macroeconomics, liquidity refers to a generally accepted medium of exchange or, in brief, money. Money is the most liquid asset due to the fact that it does not need to be converted into anything else in order to make purchases of real goods or other assets. This feature makes money valuable in both perspectives.

This chapter uses this common perspective of money and links liquidity risk on an asset market with aggregate demand and aggregate supply on a goods market. Spillover effects from the asset market to the goods market can justify a central bank intervention on the asset market even if the central bank does not take the welfare of investors on the asset market into account. Hence, the model provides a framework to analyse the perceived insurance against severe financial turmoil by the Federal Reserve under Alan Greenspan, which has been termed the 'Greenspan put' in the popular press and 'liquidity provision principle' by Taylor (2005).

Liquidity provision has been studied in the literature with a focus on the role of financial intermediaries (see, e.g., Allen and Gale, 1998; Diamond and Dybvig, 1983; Diamond and Rajan, 2001, 2005; Goodhart and Illing, 2002). Considerably less research looked at liquidity provision by financial markets (see, e.g., Allen and Gale, 1994; Holmström and Tirole, 1998). Furthermore, all of these papers use models with real assets and claims. If the aim is to analyse optimal monetary interventions on financial markets, however, it seems to be natural that one has to use a model in nominal units, since modern central banks provide nominal fiat money but not real goods. Only recently, Gale (2005) and Diamond and Rajan (2006) have made first steps in that direction and developed models with nominal assets.[1] Contributing to this literature, I develop an analytical framework based on the cash-in-the-market pricing model of Allen and Gale (1994, 2005) that directly links monetary policy and liquidity on financial markets.

Before I turn to the details of the model, the following two sections provide empirical and historical evidence of the role of liquidity on asset prices and in financial crises.

4.1.1 Empirical evidence for the role of liquidity on asset prices

One of the first studies that empirically links asset pricing and liquidity is Amihud and Mendelson (1986), who show that shares' excess returns increase in the size of the average bid-ask spread, a well-known measure of an asset's level of liquidity. Recent research has provided further important empirical evidence on the relevance of time-varying market-wide liquidity on asset pricing and of the effects of monetary expansions on liquidity during crisis periods.

Pastor and Stambaugh (2003) measure market liquidity as the equally weighted average of individual shares' expected return reversal. The authors start from the idea that a sell (buy) order should be accompanied by a negative (positive) price

[1]Allen and Gale (1998) contains discussions about both monetary policy to limit some inefficiencies of bank runs and the effects of an asset market. Gale (2005, p. 2) himself, however, argues that this and more recent papers by Allen and Gale that use the same methodology are 'essentially real (non-monetary) models' and 'focus on banks and banking, to the exclusion of other parts of the financial system.'

impact that one expects to be partially reversed in the future if the share is not perfectly liquid. Sharp declines in this measure coincide with market declines and 'flight to quality' or 'flight to liquidity' episodes in which investors want to shift from relatively illiquid medium to long-term assets such as shares into safe and liquid government bonds or cash. Examples of such incidents are discussed in the following section 4.1.2. Market-wide liquidity as measured by Pastor and Stambaugh (2003) appears to be a state variable that is important for share prices. Shares whose returns are more sensitive to aggregate liquidity have substantially higher expected returns, even as the authors control for exposures to the market, size and value factors of Fama and French (1993) and a momentum factor.

Acharya and Pedersen (2005) derive and estimate a liquidity-adjusted capital asset pricing model. In addition to the standard market beta, their model has three betas representing different forms of liquidity risk. One beta resembles the analysis in Pastor and Stambaugh (2003): Investors are willing to accept a lower expected return on an asset with a high return in times of market illiquidity. Futhermore, Acharya and Pedersen (2005) show that investors require a higher expected return for a security that becomes illiquid when the market in general becomes illiquid. Finally, investors require a lower expected return for an asset that is liquid if the market return is low. In the authors' estimations, the last effect appears to have the strongest impact on expected returns.

Most importantly for this chapter, Chordia, Sarkar and Subrahmanyam (2005) establish an empirical link between the macro- and the micro-perspective of liquidity. The authors find that 'money flows (...) account for part of the commonality in stock and bond market liquidity.' Furthermore, they use vector autoregressions to provide evidence that a loose monetary policy, measured as a decrease in net borrowed reserves or a negative interest rate surprise,[2] is associated with lower bid-ask spreads, i.e. increased liquidity, in times of crises.

4.1.2 Historical liquidity crises and central banks' reactions

Besides these empirical studies, there is also a lot of anecdotal evidence how central banks reacted to liquidity crises, since the last decades have shown a number of such crises on financial markets. For example, Davis (1994) describes five severe liquidity crises in international markets: The Penn Central Bankruptcy in 1970, the crisis in the floating-rate notes market in the UK in 1986, the failure of the US-High Yield

[2]Net borrowed reserves represent the difference between the amount of reserves banks need to have to satisfy their reserve requirements and the amount which the Fed is willing to supply. A negative interest rate surprise is defined as a drop of the federal funds target rate below market expectations (Chordia et al., 2005, pp. 112-113).

bond market in 1989, the Swedish Commercial Paper crisis in 1990 and the collapse of the ECU bond market in 1992. Greenspan (2004) highlights three crises during his chairmanship at the Federal Reserve (Fed), in which market participants wanted to convert illiquid medium to long-term assets into cash because they favoured safety and liquidity over uncertainty: The stock market crash in 1987, the LTCM-crisis 1998 and the terrorist attacks of September 11, 2001. This section provides a brief review of these three events and the central banks', in particular the Fed's, reactions to them.

On 19 October 1987 ('Black Monday'), the Dow Jonex Index dropped by 22.6%. Many commentators blamed institutional investors that followed a portfolio insurance investment strategy for the dramatic crash in prices.[3] Similar to stop-loss-orders, portfolio insurance implies automatic sell orders when the value of a portfolio or single shares falls below a certain threshold. If the absorption capacity of the market is limited, portfolio insurance can cause a vicious circle of price falls and further sell orders (see also section 4.4.3).

Grossman and Miller (1988) describe the events on 19 and 20 October against the background of their model in which market liquidity is determined by the demand and supply of immediacy, i.e. the willingness to trade immediately rather than to wait some time for a possibly better price. They argue that order imbalances were so great[4] that market makers became incapable of supplying further immediacy. Market illiquidity materialised as delays in the execution and confirmation of trades and as the virtual impossibility of executing market sell orders at the quoted prices at the time of order entry.

As chairman of the Fed, Alan Greenspan managed to improve the confidence of investors and the liquidity of the market by issuing the following statement at 9am on 20 October 1987:

> The Federal Reserve, consistent with its responsibilities as the Nation's central bank, affirmed today its readiness to serve as a source of liquidity to support the economic and financial system (Greenspan, 1987).

The Dow Jones regained 5.9% and 10.1% on this and the following day, respectively. Garcia (1989) discusses the different tools the Fed used to limit the extent of the stock market crash. These included, besides communication via the quoted statement, mainly open market operations and the use of the discount window to

[3] For example, Gammill and Marsh (1988) report official statistics that show that institutional investors who followed a portfolio insurance investment strategy were the heaviest net sellers on the New York Stock Exchange and in the S&P 500 index futures market.

[4] After a more than 10% decline of the Dow Jones between Wednesday, 14 October, and Friday, 16 October, Gammill and Marsh (1988) note an 'overhang of incomplete portfolio selling' by portfolio insurers which caused additional selling pressure on the morning of Black Monday.

provide liquidity in the form of additional money to the market. The handling of the crisis by Alan Greenspan, who had been appointed as Fed Chairman only two months earlier, laid the foundations for the belief in an insurance against stock market losses, the alleged 'Greenspan put' (see also section 4.5.1).

In September 1998, the near-collapse of the hedge fund Long-term Capital Management (LTCM) caused severe turmoil on financial markets.[5] After years of extraodinary performance, LTCM experienced below-average returns in 1997 and even losses in the first half of 1998. In response, LTCM increased its leverage, i.e. its debt/equity ratio, and focused even more on investments in relatively illiquid assets. The Russian default in August 1998 caused a flight to quality into liquid government bonds, while the prices of more illiquid assets fell dramatically. Margin calls forced LTCM to sell its assets into the falling market, which exacerbated the crisis. Other market participants could not (and some did not want to, see Brunnermeier and Pedersen, 2005) step in and buy assets, not least because they had copied LTCM's trading strategies and were constrained in their available funds. LTCM's supposedly sophisticated risk management system had not taken this endogeneity of risk sufficiently into account and its imminent collapse threatened the functioning of the Treasury bond market because of LTCM's large short-positions on this market.

On 23 September, the New York Fed organised a private bailout of LTCM by 14 banks that had lent to the fund. In the following weeks, the Fed lowered its policy rate three times by 25 basis points in order to provide sufficient liquidity for financial markets. Both Greenspan (2004) and Meyer (2004), who was on the Fed's Board of Governors at that time, admit that the purpose of these rate cuts was to calm financial markets rather than to stimulate the still expanding real economy. Indeed, the second cut boosted financial markets[6] and, for example, considerably lowered spreads on repos, swaps, corporate bonds and off-the-run treasuries, which all had increased dramatically after the Russian default (IMF, 1998, p. 39). Nevertheless, the Fed still feared the downside risks and lowered its policy rate a third time on 17 November despite lingering positive GDP data. Given the subsequent rise in inflation and equity prices until 2000, Meyer (2004, p. 121) later regretted this last cut.

The terrorist attacks in the morning of 11 September 2001 represented a very different form of a liquidity shock to financial markets. Liquidity evaporated from the financial system not because of margin calls, portfolio insurance strategies or a preference shock, but rather because large parts of the communication system and

[5] For a more detailed analysis of the LTCM-crisis, see e.g. IMF (1998), Jorion (2000) or Sauer (2002).

[6] The cut was implemented between two scheduled meetings of the Federal Open Market Committee on 15 October 1998, a very rare step by the Fed under Alan Greenspan.

a lot of back offices in lower Manhattan were physically destroyed. One immediate response of the authorities was to leave the New York Stock Exchange, the American Stock Exchange and NASDAQ closed until 17 September. Hence, liquidity problems concentrated in the payment and settlement system and did not affect the stock market immediately. In that sense, the effects were limited and the Fed could quickly withdraw the additional 108 billion US-$ in discount window credits, overnight repos and check floats it had supplied to banks until 13 September already by 20 September (Lacker, 2004, table 1).

In Europe, the European Central Bank (ECB) immediately issued the following press statement on 11 September:

> After the unprecedented and tragic events in the United States today, the Eurosystem stands ready to support the normal functioning of the markets. In particular, the Eurosystem will provide liquidity to the markets, if need be. (ECB, 2001a)

Furthermore, the ECB conducted two one-day fine-tuning operations on 12 and 13 September with a volume of 69.3 and 40.5 billion Euro, respectively, in which all bids were satisfied. It also entered into a swap agreement with the Fed over 50 billion US-$ to provide dollar liquidity to European banks on 12 September (ECB, 2001b). However, the ECB left its key interest rates unchanged on its regular meeting on 13 September.

Just before U.S. stock markets reopened on the morning of Monday 17 September, the Fed cut its target rate by 50 basis points. The ECB followed suit and also lowered its key interest rates by the same amount. The Fed continued to cut rates on 2 October, 6 November and 11 December, while the ECB reduced its rates only on 9 November. Although Lacker (2004, p. 961) argues that 'the [Fed] interest rate cuts following September 11 are probably best viewed as addressing the medium- and longer-term macroeconomic consequences' rather than a necessary response to disruptions in the payment system, the contemporaneous action of central banks worldwide on 17 September[7] hints that this move was also aimed at rebuilding confidence and signalling that central banks would continue to provide liquidity if necessary. Indeed, on 17 September the Dow Jones opened only 3.2% below the closing value on 10 September. Until 21 September, the Dow lost 14.3% compared to 10 September, but regained quickly in the following weeks and reached the pre-terrorist attacks level already in October.

A common feature of these crises is that the Fed lowered its interest rate to provide emergency liquidity to the market, although the mandate of the Fed in

[7]Besides the Fed and the ECB, also the Bank of England, the Swedish Riksbank, the Bank of Canada and other central banks worldwide lowered their policy rates on the same day.

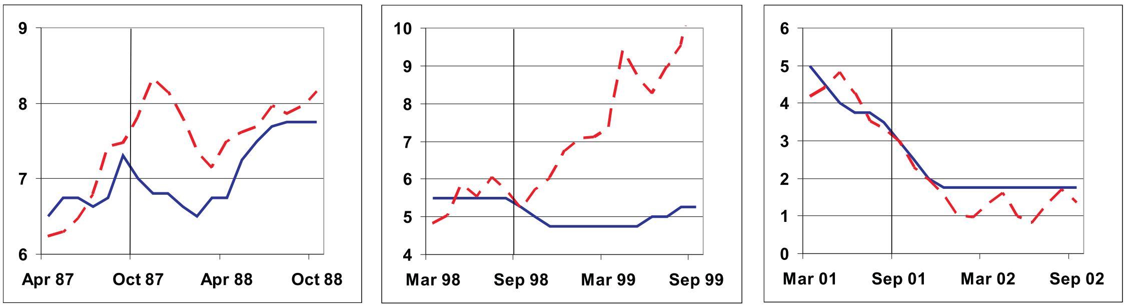

Figure 4.1: *Federal funds target rate (solid line) and Taylor rule rate (dashed line) in the U.S. during the crises in 1987, 1998 and 2001.*
Notes: The Taylor rule rate is based on equation (4.1) with π_t measured as the annual growth rate of the consumer price index and y_t measured as the quarterly OECD-output gap transformed into monthly data with a cubic spline. The Taylor rate is adjusted for time-varying r_t^* and π_t^* by matching the average Taylor rate in the six months prior to the respective crisis with the average Federal funds target rate over this period. Data source: Thomson Financial Datastream.

the Humphrey-Hawkins Act of 1978 focuses on price stability and full employment. Taylor (1993) suggested a simple interest rate rule to capture these two goals:

$$i_t = r_t^* + \pi_t + 0.5(\pi_t - \pi_t^*) + 0.5y_t. \tag{4.1}$$

The nominal interest rate i_t should rise with the natural real rate r_t^*, inflation π_t relative to its target rate π_t^* and the output gap y_t. The comparison of the actual Fed funds target rate with the recommendation from this Taylor rule provides a simple test for the liquidity provision principle, i.e. a temporary departure of interest rates from the Taylor rule during financial crises (Taylor, 2005) in order to avoid negative spillover effects from the asset to the goods market. Figure 4.1 shows that the Fed decreased its policy rate in the months following all three crises as noted above. The Taylor rule, however, recommended a rise of the interest rate after the crises of 1987 and 1998. Therefore, monetary policy appears expansionary for about six months until April 1988 and even more so after the LTCM-crisis 1998. In contrast, the Taylor rate matches the actual Fed funds rate after the terrorist attacks in 2001 quite closely. From the beginning of 2002, actual monetary policy looks even restrictive compared to the Taylor rule.

Figure 4.2 reveals considerable differences in the development of inflation in the aftermath of the crises. For comparison, inflation is measured as the annual growth rate of both the consumer price index (CPI) and the personal consumption expenditure index (PCE), but the differences appear to be negligible. The average inflation rate one and a half to two years after the crises compared to average inflation in the six months up to the crises increased by 0.8 percentage points after

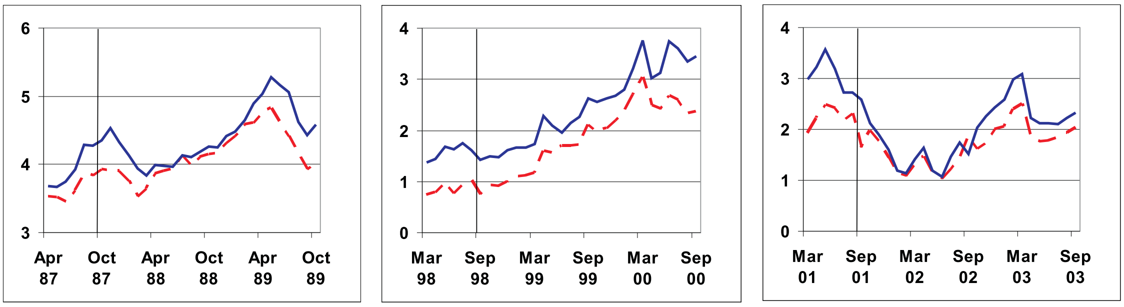

Figure 4.2: *CPI (solid line) and PCE (dashed line) inflation rates in the U.S. after the crises in 1987, 1998 and 2001.*

Notes: Inflation is measured as the annual growth rate of the consumer price index (CPI) and the personal consumption expenditure index (PCE). Data source: Thomson Financial Datastream.

1987 and 1.7 points after 1998.[8] In contrast, inflation decreased by 0.4 (PCE) or 0.9 (CPI) points after 2001. Therefore, expansionary monetary policy via the liquidity provision principle appears to have contributed to price increases after 1987 and 1998, while a normal or even restrictive stance of monetary policy added to a decline of inflation after 2001.

All three historical episodes of liquidity crises demonstrate that central banks, and in particular the Fed under Alan Greenspan, stood ready to provide liquidity in times of financial crises. Greenspan (2004, p. 38) states that the 'immediate response on the part of the central bank to such financial implosions must be to inject large quantities of liquidity,' roughly in line with the traditional Bagehot (1873) principle for a lender of last resort activity to 'lend freely at a high rate against good collateral.' But the events also indicate that not all financial crises are alike and central banks face a difficult task to decide on the optimal policy, which depends on the associated costs and benefits. The rest of this chapter develops a stylised model of an asset market and a goods market which provides a framework to analyse the relevant trade-offs for the central bank.

4.1.3 The model in a nutshell

The model consists of two separate markets, an asset market and a goods market. The main focus is on developments on the asset market, but these developments have important implications for the goods market. Although the monetary authority only cares about deviations of goods prices and quantities from the optimal values, the spillover effects from the asset market may require a central bank intervention on this market.

[8]Besides the rise in consumer prices, expansionary monetary policy may also have contributed to the boom and bust period of equity prices in the five years following the LTCM-crisis.

In the model, investors can invest on an asset market in liquid money and potentially illiquid, but productive assets, called shares, in order to optimally satisfy their uncertain consumption needs on the goods market over two periods. Two channels link the goods market to the asset market: First, the amount of money held by investors determines together with the size of a liquidity shock the aggregate demand of investors on the goods market which is subject to a cash-in-advance constraint. Second, a dramatic decrease of the asset price negatively influences the goods supply in the final period because it forces investors to costly liquidate their asset. Hence, the central bank faces a trade-off between inflating a demand shock today, which causes higher losses today, and limiting a negative supply shock tomorrow, which will cause higher losses tomorrow. Expectations of central bank intervention give rise to a moral hazard effect with additional investment in less liquid, but productive shares. If the central bank has the possibility to commit to some future policy, it should optimally weight these productivity gains against the expected intervention costs.

Section 4.2 analyses the basic model under certainty and aggregate risk. Section 4.3 provides further insights into the trade-off the central bank faces and derives the optimal central bank intervention before section 4.4 discusses the impact of idiosyncratic risk. After a review of the related literature in section 4.5, section 4.6 concludes.

4.2 The model

4.2.1 Framework

A continuum of ex ante identical investors i is uniformally distributed on an intervall $I = [0; 1]$. They can invest on an asset market and buy goods for consumption on a separate goods market. An investor i derives utility from consumption c_t in periods $t = 1$ and 2 according to the utility function

$$U_i(c_1, c_2) = \gamma\zeta_i \ln c_1 + \beta \ln c_2. \tag{4.2}$$

$\gamma\zeta_i$ represents a liquidity shock that consists of an aggregate liquidity shock γ and an individual liquidity shock ζ_i. The distribution functions of both shocks are assumed to be uncorrelated, symmetric, having a positive support and an expected value of 1 in $t = 0$, i.e. $E_0 [\gamma\zeta_i] = \int_{-\infty}^{\infty} \gamma f(\gamma) d\gamma \cdot \int_{-\infty}^{\infty} \zeta_i f(\zeta_i) d\zeta_i = 1$.

Every investor is endowed with nominal wealth w that can be invested in $t = 0$ in nominal money m and a real asset s, called shares, on a primary market with price $q_0 = 1$ fixed and s endogenous. The asset pays a fixed nominal return R in $t = 2$

Table 4.1: *Payoffs of money and shares in* $t = 0, 1, 2$.

	0	1	2
$m = w - s$	-1	1	0
		0	1
s	-1	q	0
		0	R
		ρ	0

and can be traded at the nominal price q on a secondary asset market in $t = 1$ after the realisation of the liquidity shock $\gamma\zeta_i$, but before goods are traded on the goods market. Besides, investors have access to a costly real liquidation technique, which transforms z units of the asset s into ρz units of additional consumption goods in period 1 with $\rho < 1$. The individual cost of liquidation is the missed nominal return Rz in $t = 2$ and the social cost is a reduction of aggregate supply in $t = 2$ by $\Delta(z)$.[9] The asset s can also be interpreted as a nominal bond with a fixed interest rate R and a real put option with a strike price of ρ. Table 4.1 summarises the payoffs of m and s in $t = 0, 1, 2$.

At the beginning of $t = 1$ and 2, homogenous, infinitely divisible and non-storable consumption goods are produced with capital and labour input from workers who can participate only on the goods market and receive a nominal wage ψ_t that is determined at $t - 1$.[10] These goods must be bought by investors and workers with money, i.e. they are subject to a cash-in-advance constraint. The price of consumption goods p_t is determined by demand for goods from workers and investors and the aggregate supply of goods. Markets are competitive but incomplete. Figure 4.3 summarises the timing of the model.

[9]For example, Shleifer and Vishny (1992) and Allen and Gale (1998) contain a discussion of the costs of premature liquidation of assets. The costly liquidation technology shall represent investors possibility to a) partly liquidate their capital, b) sell their capital to less productive owners or c) cut down replacement investments because firms' refinancing possibilities depend on their share price as in the financial accelerator model by Bernanke, Gertler and Gilchrist (1999). In this model, the assumption $\rho < 1$ guarantees that money is not fully dominated by the asset given the price determination on the goods market as explained in section 4.2.2 and the absence of central bank interventions. For the corresponding condition with central bank intervention, see Corollary 4.2 on page 105.

[10]Section 4.5.2 discusses the literature on market segmentation.

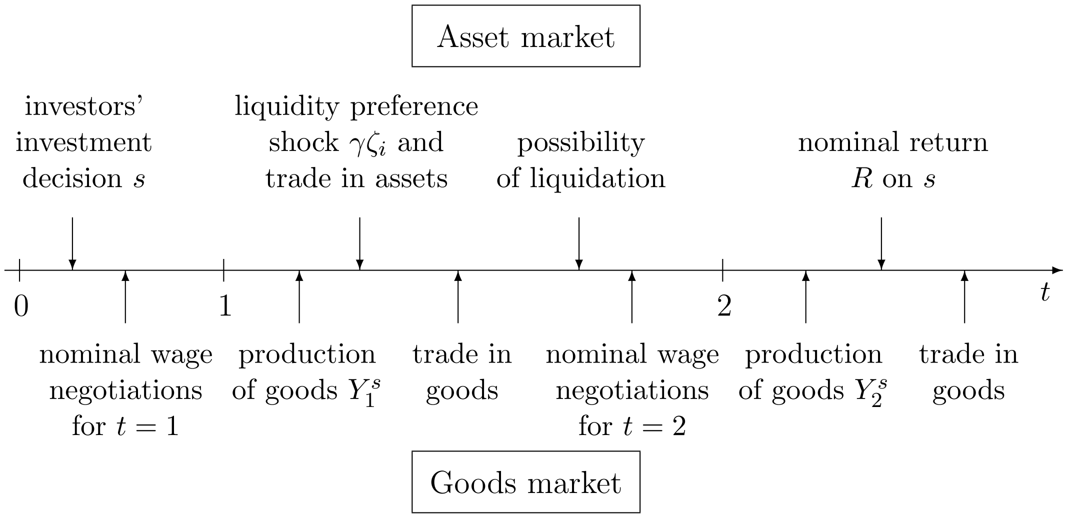

Figure 4.3: *Time structure of the model.*

4.2.2 Under certainty

Investors' problem and asset market

Before I analyse the effects of liquidity shocks $\gamma\zeta_i$, I solve the model under certainty, i.e. $\gamma = \zeta_i = 1$. The individual investor maximises her utility function (4.2) subject to her budget constraint and her cash-in-advance constraint (CIA) in $t = 1$.[11] She controls her initial investment in the asset s, her consumption c_t in $t = 1$ and 2 bought on the goods market with cash, her demand for additional assets in $t = 1$, $\hat{s}$, and the extent of costly liquidation z, which is subject to a non-negativity constraint:[12]

$$\max_{s,c_1,c_2,\hat{s},z} U(c_1', c_2) = \ln(c_1 + \rho z) + \beta \ln c_2 \quad \text{s.t.} \tag{4.3}$$

$$p_1 c_1 + p_2 c_2 \leq w - s + Rs + (R - q)\hat{s} - Rz$$

$$p_1 c_1 + q\hat{s} \leq w - s$$

$$0 \leq z \leq s$$

Note that an investors' total consumption in $t = 1$, c_1', is the sum of the consumption purchased via the goods market, c_1, and the real return from the possible liquidation

[11] The budget constraint implicitly includes the CIA for $t = 2$ as the investor holds only cash when she enters the goods market in $t = 2$.

[12] The Cobb-Douglas utility function (4.2) makes $c_t > 0$ as long as $w > 0$.

of assets, ρz. Solving the maximisation problem with the Lagrangian

$$\begin{aligned}\max_{s,c_1,c_2,\hat{s},z} \Lambda &= \ln(c_1+\rho z)+\beta\ln c_2\\ &-\lambda\left[p_1c_1+p_2c_2-(w-s)-Rs-(R-q)\hat{s}+Rz\right]\\ &-\mu\left[p_1c_1+q\hat{s}-(w-s)\right]\end{aligned}$$

yields as first-order conditions

$$\frac{d\Lambda}{dc_1}=\frac{1}{c_1+\rho z}-\lambda p_1-\mu p_1=0 \quad\longrightarrow\quad \mu+\lambda=\frac{1}{p_1\left(c_1+\rho z\right)} \tag{4.4a}$$
$$\frac{d\Lambda}{dc_2}=\frac{\beta}{c_2}-\lambda p_2=0 \quad\longrightarrow\quad \lambda=\frac{\beta}{p_2c_2} \tag{4.4b}$$
$$\frac{d\Lambda}{ds}=-\lambda+\lambda R-\mu=0 \quad\longrightarrow\quad \mu=\lambda\left(R-1\right) \tag{4.4c}$$
$$\frac{d\Lambda}{d\hat{s}}=\lambda\left(R-q\right)-\mu q=0 \quad\longrightarrow\quad \mu=\lambda\left(\frac{R}{q}-1\right) \tag{4.4d}$$
$$\frac{d\Lambda}{dz}=\frac{1}{c_1+\rho z}\rho-\lambda R\le 0 \tag{4.4e}$$
$$\frac{d\Lambda}{d\lambda}=-p_1c_1-p_2c_2+(w-s)+Rs+(R-q)\hat{s}\ge 0 \tag{4.4f}$$
$$\frac{d\Lambda}{d\mu}=-p_1c_1-q\hat{s}+(w-s)\ge 0 \tag{4.4g}$$

and $\frac{dL}{dz}z=0$, $\frac{dL}{d\lambda}\lambda=0$ and $\frac{dL}{d\mu}\mu=0$ as complementary slackness conditions.[13]

Since the costly liquidation is inefficient for $p_1\rho<1$, investors will not use it under certainty, and $z=0$.[14] As will become clear from the discussion of the goods market in the next section, the price of goods p_1 equals its expected value, i.e. $p_1=1$, under certainty, so $\rho<1$ is a necessary and sufficient condition for $z=0$.

(4.4c) and (4.4d) show that $q=1$ in the equilibrium under certainty because holding money would be dominated from $t=0$ to $t=1$ for $q>1$ such that $s=w$, while holding shares would be dominated from $t=0$ to $t=1$ for $q<1$ such that $s=0$. For $q=1$, money and shares are equivalent assets from $t=0$ to $t=1$. Since money is dominated by shares over the long run, the CIA is binding in $t=1$.[15] The only possible symmetric equilibrium is $\hat{s}=0$, i.e. there is no trade on the asset market in $t=1$, and money is only held for consumption in $t=1$: (4.4g) reduces to $p_1c_1=w-s$. The combination of (4.4a) and (4.4b) shows that a binding CIA drives

[13]The second-order conditions for a maximum are fulfilled, since (4.3) maximises a strictly concave utility function under linear constraints and the optimum is an interior solution.

[14]By plugging μ from (4.4c) in (4.4a), solving for λ and then plugging λ in the inequality (4.4e), it can be shown that $d\Lambda/dz$ is negative and thus $z=0$ as long as $p_1\rho<1$.

[15]Since $R>1$ by assumption and $\lambda>0$ from (4.4b), the first-order condition for optimal investment in s yields $\mu>0$.

a wedge μ, the marginal utility of cash's liquidity services, between the marginal utilities of consumption in $t = 1$ and $t = 2$:[16]

$$\mu + \frac{\beta}{p_2 c_2} = \frac{1}{p_1 c_1}.$$

According to (4.4c), the wedge μ equals the marginal utility of wealth, λ, times the excess return of shares over money, $R - 1$, such that the marginal rate of intertemporal substitution equals the price ratio times the return on shares:

$$\frac{c_2}{\beta c_1} = \frac{p_1}{p_2} R.$$

Given the optimal consumption in $t = 1$ and 2, the budget constraint (4.4f) and the CIA (4.4g), the optimal investment decision in $t = 0$ is

$$\begin{aligned} s &= \frac{\beta}{1+\beta} w \quad \text{and} \\ m &= \frac{1}{1+\beta} w. \end{aligned}$$

An individual investor has consumption demands of[17]

$$\begin{aligned} c_1 &= \frac{w}{(1+\beta)\, p_1} \quad \text{and} \\ c_2 &= \frac{\beta R w}{(1+\beta)\, p_2}. \end{aligned}$$

Finally, the investment and consumption decisions of individual investors i can be aggregated to aggregate investment and consumption. Let capital letters denote aggregate values of the respective variable, i.e. $W \equiv \int_{i \in I} w di$, $M \equiv \int_{i \in I} m di$, $S \equiv \int_{i \in I} s di$, $C_1 \equiv \int_{i \in I} c_1 di$ and $C_2 \equiv \int_{i \in I} c_2 di$. Given $I = [0; 1]$, the following Proposition 4.1 summarises the situation under certainty:

Proposition 4.1 *In the symmetric equilibrium under certainty, investors split their wealth in money* $\left(M = \frac{1}{1+\beta} W\right)$ *and shares* $\left(S = \frac{\beta}{1+\beta} W\right)$ *and consume* $C_1 = \frac{1}{p_1(1+\beta)} W$ *and* $C_2 = \frac{\beta R}{p_2(1+\beta)} W$. *The asset price* $q = 1$ *and no assets are traded in the symmetric equilibrium.*

Plugging $R = 1/\beta$ into the results of Proposition 4.1 yields a special result:

[16]Note that $\mu \geq 0$ represents the standard complementary slackness condition: If the CIA is not binding ($\mu = 0$), the marginal utility of money's liquidity services is zero; but if the marginal utility of money's liquidity services is positive, the liquidity constraint becomes binding ($\mu > 0$).

[17]For completeness, the Lagrangian parameters are $\lambda = \frac{1+\beta}{Rw}$ and $\mu = \lambda (R - 1)$.

Corollary 4.1 *If the interest rate R equals the discount rate $1/\beta$, investors spend the same amount of money in both periods, i.e. $p_1 C_1 = p_2 C_2$, and consume the same amount of goods, i.e. $C_1 = C_2$, if prices remain constant.*

To concentrate on the intertemporal substitution effects of liquidity preference shocks, I start from the situation in Corollary 4.1 with perfect consumption smoothing and thus assume $\beta R = 1$ where useful below.

Goods production and goods market

Because I want to focus on events on the asset market, in particular on the effects of emergency liquidity provision by the central bank in section 4.3, and the direct spillover effects to the goods market, the model includes a very stylised version of a goods market. Non-storable goods are produced by a mass of 1 of identical competitive firms at the beginning of periods $t = 1, 2$ with total labour input $N_t = \bar{N}$ from identical workers who cannot participate on the asset market and capital input K_t according to a Cobb-Douglas production function

$$Y_t = K_t^{\alpha} \bar{N}^{1-\alpha} \tag{4.5}$$

with $0 < \alpha < 1$. Trade on the goods market takes place after the realisation of the liquidity shock for investors and after trade on the asset market. While aggregate supply is already produced and thus fixed at Y_t, aggregate demand consists of demand from workers based on their nominal labour income ψ_t and from investors as derived in the previous subsection.

Given a Cobb-Douglas production function with constant returns to scale and perfect competition, the Euler theorem states that production factors are paid their marginal product times the respective factor input. With the production function (4.5), workers should receive the share of total output Y_t that reflects their relative importance in production as captured by $1 - \alpha$, while capital owners should receive αY_t. Furthermore, I assume that investors' demand C_t represents the whole factor income of capital, such that $C_t = \alpha Y_t$ and that the real investment S determines the constant producible aggregate real supply $\bar{Y}$ with $\partial \bar{Y}/\partial K \cdot dK/dS > 0$.[18]

Since I have a model in nominal units, labour income for period t is determined in nominal wage negotiations between workers and firms[19] at the end of period $t-1$ such that their expected real income is $\Psi_t = (1-\alpha)Y_t$. Hence, the agreed nominal

[18]Although this is an obvious departure from a full general equilibrium model where the income from capital is directly linked to the marginal product of capital, the crucial effects of the model should still hold in general equilibrium under the assumption of a cash-in-advance constraint for investors and limited asset market participation.

[19]Firms only produce consumption goods and negotiate wages in the model.

wage is $\psi_t = \Psi_t E_{t-1}[p_t] = (1-\alpha) Y_t E_{t-1}[p_t]$ given the expected price level $E_{t-1}[p_t]$. $E_0[p_1]$ is normalised to 1.[20] For simplicity, I assume that workers build their price expectations based on the quantity equation, i.e. they expect that investors use all their available nominal funds for the purchase of consumption goods in the respective period.[21] Hence, money holdings $M = W - S = E_0[p_1 C_1]$ and the supply of goods $\bar{Y}$ represent the information set for the wage negotiations in $t = 0$. The nominal return from the investment RS plus any unused M from $t = 1$ equal $E_1[p_2 C_2]$. Together with Y_2^s, this provides the information for the negotiations in $t = 1$. The expected nominal demand $E_{t-1}[p_t C_t]$ in turn has to be equal to the expected income share of capital, $E_{t-1}[p_t] \alpha Y_t$. Due to the normalisation $E_0[p_1] = 1$, $C_1 = \alpha \bar{Y}$.[22]

Under certainty, this also means that $E_1[p_2] = p_2 = 1$ as well if $\beta R = 1$ because the CIA binds ($\mu > 0$) and investors transfer no money to $t = 2$. Hence, investors' nominal funds are thus identical in $t = 1$ and 2. If $\beta R \neq 1$, investors' nominal funds differ in both periods under certainty. The nominal wage negotiations in $t = 1$ determine ψ_2 such that the price p_2 adjusts such that workers receive $1 - \alpha$ and investors α of the constant aggregate supply $\bar{Y}$ in $t = 2$. Hence, aggregate demand Y_t^d and aggregate supply Y_t^s are

$$Y_t^d = \frac{\psi_t}{p_t} + C_t = \Psi_t + C_t \quad \text{and} \tag{4.6}$$

$$Y_t^s = \bar{Y}. \tag{4.7}$$

To summarise the equilibrium on the goods market under certainty for $\beta R = 1$, the expected price of goods $E_{t-1}[p_t]$ equals the actual price $p_t = 1$ for $t = 1, 2$. Investors consume $C_1 = C_2 = W/(1+\beta)$, while total production equals $Y_1 = Y_2 = W/[\alpha(1+\beta)]$ and workers consume $\frac{1-\alpha}{\alpha}$ times investors' consumption, i.e. $\Psi_1 = \Psi_2 = \frac{1-\alpha}{\alpha} W/(1+\beta)$.

4.2.3 Aggregate risk

What is the efficient response to a positive aggregate demand shock in $t = 1$? If the supply of goods can be adjusted to the increased demand, it will be increased until the marginal costs of doing so equal the marginal benefit. In this model, production takes place before the shock, so the liquidation technology offers the only way to increase supply in $t = 1$. Since the liquidation costs are very high, investors will

[20] This assumption avoids any problems with a possible indeterminacy of the price level.

[21] For example, Illing (1997) and Walsh (2003) model aggregate demand with a quantity equation.

[22] Actually, C_1 is determined by W, S and p_1 (see table 4.2). With nominal W and real $C_1 = \alpha Y$ fixed, p_1 is no free parameter any more. But the link between S and K and thus $\bar{Y}$ could be normalised such that $E_0[p_1] = 1$.

use it only for large shocks. In an intermediate range, prices adjust such that the marginal rate of intertemporal substitution equals the relative prices.

Since the optimal investment strategy in $t = 0$ depends on expectations about developments on the asset and the goods market in $t = 1$ and 2, the model has to be solved by backward induction. Hence, the allocations on the goods market in $t = 2$ and $t = 1$ as well as the influence of the shocks on the optimal behaviour of investors on the asset market in $t = 1$ have to be taken into account when one solves the utility maximisation problem of investors in $t = 0$. For illustrative purposes, however, it will be easier to begin with the description of the asset market, turn to the goods market afterwards and then solve the initial investment problem given the behaviour in $t = 1, 2$.

Asset market

The optimal investment decision problem for an individual investor under aggregate risk becomes

$$\max_{s,c_1,c_2,\hat{s},z} E\left[U(c_1', c_2)\right] = \int_{-\infty}^{\infty} \left(\gamma \ln\left(c_1 + \rho z\right) + \beta \ln c_2\right) f(\gamma) d\gamma \quad \text{s.t.} \tag{4.8}$$

$$\begin{aligned} p_1 c_1 + p_2 c_2 &\leq w - s + Rs + (R - q)\hat{s} - Rz \\ p_1 c_1 + q\hat{s} &\leq w - s \\ 0 \leq z &\leq s. \end{aligned}$$

The solution to this maximisation problem in section 4.A of the appendix uses the Leibniz-Rule and yields as first order conditions

$$\frac{\partial \Lambda}{\partial c_1} = \frac{\gamma}{c_1 + \rho z} - \lambda p_1 - \mu p_1 = 0 \tag{4.9a}$$

$$\frac{\partial \Lambda}{\partial c_2} = \frac{\beta}{c_2} - \lambda p_2 = 0 \tag{4.9b}$$

$$\frac{\partial \Lambda}{\partial \hat{s}} = \lambda\left(R - q\right) - \mu q = 0 \tag{4.9c}$$

$$\frac{\partial \Lambda}{\partial z} = \frac{\gamma}{c_1 + \rho z}\rho - \lambda R \leq 0 \tag{4.9d}$$

$$\frac{\partial \Lambda}{\partial \lambda} = -p_1 c_1 - p_2 c_2 + w + \left(R - 1\right)s + (R - q)\hat{s} - Rz \geq 0 \tag{4.9e}$$

$$\frac{\partial \Lambda}{\partial \mu} = -p_1 c_1 - q\hat{s} + w - s \geq 0 \tag{4.9f}$$

$$\frac{d\Lambda}{ds} = \int_{-\infty}^{\infty} \left[\lambda\left(R - 1\right) - \mu\right] f\left(\gamma\right) d\gamma = 0. \tag{4.9g}$$

and $\frac{\partial\Lambda}{\partial z}z = 0$, $\frac{\partial\Lambda}{\partial\lambda}\lambda = 0$, $\frac{\partial\Lambda}{\partial\mu}\mu = 0$ and $\frac{d\Lambda}{ds}E_0[s] = 0$ as complementary slackness conditions.[23]

Since all investors are identical without idiosyncratic risk, they all want to sell or buy assets in response to an aggregate liquidity shock γ at the same time in $t = 1$ in order to adjust their money holdings optimally to their desired consumption which is subject to the CIA. As the aggregate stock of assets is determined in $t = 0$, however, they cannot sell or buy in the aggregate. Hence, the asset price q has to adjust to exclude any excess demand or supply of assets, i.e. market clearing in $t = 1$ requires that $\hat{S} = \int_{i\in I}\hat{s}di = 0$.

Depending on the realisation of the liquidity shock γ, the asset price q, the Lagrangian parameters λ and μ and the choice variables c_1, c_2 and z lie in three different ranges. For $\gamma < \frac{\beta(W-S)}{RS} \equiv CIA$, investors want to transfer wealth into the next period. This drives up the asset price q, which is bounded by R: Nobody would be willing to pay more for the asset than the asset's fixed payoff in the next period. In this case, the CIA becomes non-binding ($\mu = 0$).

For greater values of γ, however, the CIA is binding and the asset price depends on the cash in the market as in Allen and Gale (1994, 2005). As long as investors do not liquidate their assets, the asset price captures the full effect of $\gamma \in \left[\frac{\beta(W-S)}{RS}; \frac{\beta(W-S)}{p_1\rho S}\right]$. For sufficiently large liquidity shocks $\gamma > \frac{\beta(W-S)}{p_1\rho S} \equiv LIQ$, the asset price q falls to a level where the investors become indifferent between liquidating the asset and selling the asset. Since they cannot sell in the aggregate, they costly liquidate part of their assets ($z > 0$). Table 4.2 summarises the equilibrium values of the relevant variables in the three ranges of γ.[24] Figure 4.4 illustrates the asset price q and the two Lagrangian parameters on the budget constraint and the CIA as a function of γ for $R = 1/\beta = 1.1, W = 1, S = \frac{\beta}{1+\beta}W$ and $\rho = 0.7$. The possibility of a severe drop in q captures the microeconomic view of liquidity, as an illiquid asset cannot be sold quickly without costs.

Turning to the optimal investment decision in $t = 0$, the first-order condition for optimal investment in the asset is given by equation (4.9g). Using the results for λ and μ from table 4.2 and the definitions of the cumulative distribution function $F(x) \equiv \int_{-\infty}^{x} f(\gamma)\, d\gamma$ of the liquidity shock γ and the function $G(x) \equiv \int_{-\infty}^{x} \gamma f(\gamma)\, d\gamma$, section 4.A in the appendix shows that the determination of the optimal investment s requires an explicit parameterisation of the shock's den-

[23] As for the maximisation problem (4.3) under certainty, the second-order conditions for a maximum are fulfilled since (4.8) maximises a strictly concave utility function under linear constraints and the optimum is an interior solution.

[24] Note that the Cobb-Douglas preferences (4.2) determine the relative expenditures p_1c_1 to p_2c_2 such that c_1 is independent from p_2 and c_2 is independent from p_1 in general. Only for $\gamma > LIQ$ and thus $z > 0$, c_2 depends on $p_1\rho$ because this is the nominal value of liquidation in $t = 1$. Without central bank intervention, $p_1 = 1$ in this case as demonstrated in the next section 4.2.3.

Table 4.2: *Summary of the values of the asset price q, the Lagrangian parameters λ and μ and the choice variables c_1, c_2 and z after the realisation of γ in $t = 1$.*

	$\gamma < \frac{\beta(W-S)}{RS} \equiv CIA$	$\frac{\beta(W-S)}{RS} \leq \gamma \leq \frac{\beta(W-S)}{p_1\rho S}$	$\gamma > \frac{\beta(W-S)}{p_1\rho S} \equiv LIQ$
q	R	$\frac{\beta(W-S)}{\gamma S}$	$p_1\rho$
λ	$\frac{\beta+\gamma}{w-s+Rs}$	$\frac{\beta(W-S+RS)}{RS(w-s+Rs)}$	$\frac{p_1\rho(\beta+\gamma)}{R(w-s+p_1\rho s)}$
μ	0	$\lambda\left(\frac{\gamma RS}{\beta(W-S)} - 1\right)$	$\lambda\left(\frac{R}{p_1\rho} - 1\right)$
z	0	0	$\frac{\gamma p_1\rho s-\beta(w-s)}{p_1\rho(\beta+\gamma)}$
c_1	$\frac{\gamma}{p_1(\beta+\gamma)}(w - s + Rs)$	$\frac{w-s}{p_1}$	$\frac{w-s}{p_1}$
c_2	$\frac{\beta}{p_2(\beta+\gamma)}(w - s + Rs)$	$\frac{Rs}{p_2}$	$\frac{\beta R(w-s+p_1\rho s)}{p_2p_1\rho(\beta+\gamma)}$

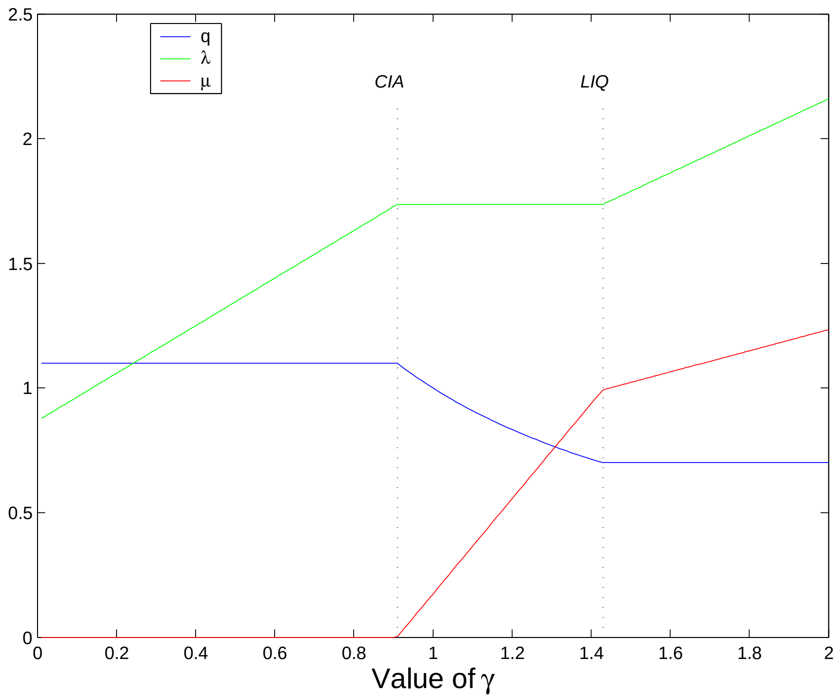

Figure 4.4: *q, λ, μ as a function of γ and given different parameter values.*

Table 4.3: *Summary of $f(\gamma), F(\gamma), G(\gamma)$ in $t=1$.*

	$\gamma \in [a;b]$	$\gamma \notin [a;b]$	$\gamma = CIA$	$\gamma = LIQ$
$f(\gamma)$	$\frac{1}{b-a}$	0	$\frac{1}{b-a}$	$\frac{1}{b-a}$
$F(\gamma)$	$\frac{\gamma-a}{b-a}$	0	$\frac{\frac{\beta(W-S)}{RS}-a}{b-a}$	$\frac{\frac{\beta(W-S)}{p_1\rho S}-a}{b-a}$
$G(\gamma)$	$\frac{\gamma-a}{b-a}\cdot\frac{1}{2}(\gamma+a)$	0	$F(CIA)\cdot \frac{1}{2}\left(\frac{\beta(W-S)}{RS}+a\right)$	$F(LIQ)\cdot \frac{1}{2}\left(\frac{\beta(W-S)}{p_1\rho S}+a\right)$

sity function $f(\gamma)$. I assume γ to be uniformly distributed between a and b with $0 < a < b$. Table 4.3 provides a summary of $f(\gamma), F(\gamma)$ and $G(\gamma)$ in $t = 1$ which is derived in the appendix.

There is only one variable left that depends on the realisation of γ, namely the goods price p_1, which is determined on the goods market as described in the following subsection. As noted above, however, the utility function (4.2) implies that p_1 only matters for λ, μ, C_t in the range $\gamma \geq \frac{\beta(W-S)}{p_1\rho S}$. Table 4.2 shows that in this range investors use all their nominal funds $w - s$ to buy consumption goods on the goods market. The detailed description of the goods market in the next section 4.2.3 shows that $p_1 = 1$ in this case. Given this information, one can now solve for the optimal investment in the asset s.

Figure 4.5 illustrates that the optimal investment is decreasing in the standard deviation of γ, $\sigma(\gamma) = \frac{b-a}{2\sqrt{3}}$, while this effect is more pronounced for a lower real payoff of the liquidation technology ρ. Without aggregate risk, Proposition 4.1 states that investors hold $S = \frac{\beta}{1+\beta}W \approx 0.4762$ for $R = 1/\beta = 1.1$ and $W = 1$. Initially, introducing aggregate risk does not affect S because the asset price q absorbs the full impact of the liquidity shock for the chosen parameter values, i.e. the CIA always binds ($F(CIA) = 0$) and no assets are liquidated ($F(LIQ) = 1$) given the equilibrium S. Further increasing $\sigma(\gamma)$ makes the risk-averse investors reduce their investment S. As the real payoff of liquidations Z increases in ρ and the liquidation threshold LIQ decreases in S, the reduction in S caused by increased aggregate risk is dampened by a greater ρ and the solid line ($\rho = 0.9$) lies above the dashed line ($\rho = 0.5$) in figure 4.5.

This is the solution of the model with aggregate risk and access to a costly real liquidation technology for investors. The analysis of an emergency liquidity assistance by the central bank requires at first a deeper discussion of the goods market in the next subsection. Furthermore, the costs and benefits of such an intervention need to be based on an explicit welfare function for the central bank. I

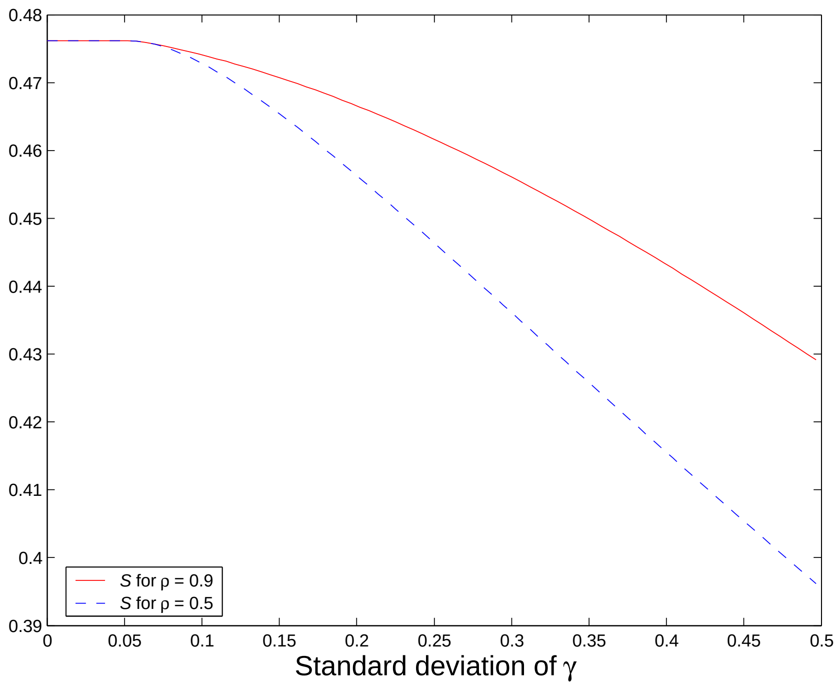

Figure 4.5: *Optimal investment S for $R = 1/\beta = 1.1$ and $W = 1$.*

turn to this issue in section 4.3.

Goods market

Investors' liquidity shocks in $t = 1$ can spill over to the goods market via a demand effect in $t = 1$ and a supply effect in $t = 2$. Let η denote the first channel that links the asset market and the goods market: For small realisations of the liquidity shock $\gamma < CIA$, the CIA of investors becomes non-binding and they do not use all their money for consumption in $t = 1$. This represents a negative nominal aggregate demand shock on the goods market, represented by $\eta < 0$. If the liquidity shock γ is in the range of $CIA \leq \gamma \leq LIQ$, the asset price q absorbs the full effect of the liquidity shock as noted in the previous section and investors' nominal demand $p_1 C_1 = W - S$. For large liquidity shocks $\gamma > LIQ$, investors liquidate part of their assets and thus increase the total resources available for consumption in $t = 1$ beyond $\bar{Y}$. Since investors satisfy ρZ of their desired consumption goods with the liquidation technology, they still demand $p_1 C_1 = W - S$ on the goods market. If, however, the central bank intervenes on the asset market and injects additional money in case of large realisations of γ as will be shown in the following section 4.3, investors' nominal demand rises above the level expected in the wage negotiations. This is represented by a positive aggregate demand shock $\eta > 0$.

The rest of aggregate demand depends on nominal labour income ψ_t, which is determined in nominal wage negotiations at $t-1$ as explained in section 4.2.2: Perfect competition and the Cobb-Douglas production function (4.5) require that workers can consume $(1-\alpha)\,Y_t$ in t given the expected price level $E_{t-1}[p_t]$ which is normalised to 1 for $t=1$. Workers build their price expectations based on the quantity equation, i.e. they expect that the total amount of money held by investors at the time of the wage negotiations is spent in $t=1$. Hence, the expected nominal demand $E_0[p_1 C_1] = W - S$ has to be equal to the expected capitalists' income share $E_0[p_1]\,\alpha Y_t = \alpha \bar{Y}$ as $E_0[p_1] = 1$.[25] Therefore, the aggregate demand relationship from equation (4.6) becomes

$$Y_1^d = \frac{\psi_1 + W - S + \eta}{p_1}, \tag{4.10}$$

while aggregate supply is again fixed to[26]

$$Y_1^s = \bar{Y}.$$

Note that the price impact of nominal demand shocks η originating from the asset market is less than 1 as ψ_1 is constant. Hence, the first channel that links the asset with the goods market, η, causes a redistribution effect from investors' consumption share at $p_1 = 1$ towards workers for $\eta < 0$ and from workers towards investors for $\eta > 0$. Given the determination of $E_0[p_1]$ described above, positive price shocks can only occur with additional money from the central bank which will be discussed extensively in the following section 4.3.

The exercise of the real put option acquired with the asset s, i.e. the application of the costly liquidation technique, in response to large liquidity shocks $\gamma > LIQ$ with no or insufficient emergency liquidity assistance by the central bank causes the second link between the asset market and the goods market: Without costly liquidations, the capital stock K_t is fixed over the time horizon of this model and aggregate output is $\bar{Y}$, given the initial investment S. If investors choose to liquidate part of their shares, i.e. $Z > 0$, this liquidation takes place after production in $t=1$ and increases the real resources available for consumption in $t=1$, but reduces K_2.

[25]This assumption is a short-cut from the rational $E_0^{rat}[p_1 C_1]$ because investors will spend all their money in $t=1$ only as long as their CIA binds, i.e. $\gamma \geq CIA$, and less for $\gamma < CIA$. This implies $E_0^{rat}[C_1] < C_1\,(\gamma \geq CIA)$ and $E_0[\eta] < 0$ without central bank intervention. Hence, workers get more than their expected share of aggregate supply $\bar{Y}$ in $t=1$ on average and are thus implicitly compensated for their real income risk in $t=1$. To summarise, the way workers form their expectations and the normalisation of $E_0[p_1]$ determine the size of the redistribution effect of investors' nominal demand on workers after the realisation of γ, but not the possibility of such redistributions.

[26]$\bar{Y}$ may be different from the one under certainty, however, since it depends on S which may decrease with the extent of aggregate risk as demonstrated in figure 4.5.

The lower capital input in $t = 2$ lowers $\bar{Y}$ by $\Delta(Z)$, with $\frac{d\Delta}{dZ} > 0$, and aggregate supply becomes

$$Y_2^s = \bar{Y} - \Delta(Z). \tag{4.11}$$

Any risk has disappeared from the model at the time of the nominal wage negotiations for $t = 2$. Workers build their price expectations $E_1[p_2]$ based on investors' safe nominal revenues $R(S - Z)$, potentially unused money holdings $W - S - p_1C_1$ and the known Y_2^s. Again, perfect competition allows them to consume $\Psi_2 = (1 - \alpha) Y_2$, which implies a nominal wage of $\psi_2 = E_1[p_2](1 - \alpha) Y_2$.[27] The aggregate demand equation for $t = 2$ then is

$$Y_2^d = \frac{\psi_2 + R(S - Z) + W - S - p_1C_1}{p_2}. \tag{4.12}$$

and equals aggregate supply at $p_2 = E_1[p_2]$ in equilibrium:

$$\Psi_2 + C_2 = \bar{Y} - \Delta(Z).$$

p_2 and its expected value adjust relative to p_1 such that investors' real consumption $C_2 = \alpha\left[\bar{Y} - \Delta(Z)\right]$. For example, if investors' liquidity shock γ is within the intermediate range $CIA \leq \gamma \leq LIQ$, the CIA is binding and $W - S = p_1C_1$, but no assets are liquidated, i.e. $Z = 0$. Equation (4.12) reduces to

$$Y_2^d = \frac{\psi_2 + RS}{p_2}$$

and $p_2 = p_1 = 1$ for $\beta R = 1$ and a sufficiently small variance of γ that leaves $S = \beta/(1 + \beta) W$ from the certainty case unaffected (see also figure 4.5). Since investors' Cobb-Douglas-preferences smooth nominal expenditures over $t = 1$ and 2, p_2 has no effect on investors' behaviour in $t = 1$ given S.

To summarise, the two direct channels that link the asset market to the goods market in this model are the aggregate demand shock η in period 1 and the aggregate supply shock Δ in period 2, which both depend on the realisation of the liquidity shock γ in $t = 1$.

[27] Note again that investors do not react to possible changes of p_2 relative to a constant p_1 because the Cobb-Douglas-preferences determine the expenditure share rather than real consumption in each period. Hence, given the constant produceable aggregate supply $\bar{Y}$ and workers' desired income of $(1 - \alpha) Y_2$, p_2 would have to deviate from p_1 even if $\gamma = E_0[\gamma] = 1$ for example if $\beta R \neq 1$ in order to equate investors' intertemporal rate of substitution to the relative price p_1/p_2 for constant real consumption $C_2 = C_1 = \alpha\bar{Y}$.

4.3 Central bank intervention

4.3.1 Welfare function

The direct spillover effects from the asset market to the goods market mean that the central bank may intervene on the asset market even if it does not take investors' welfare into account. The loss function $\mathcal{L}$ of the central bank consists of the weighted sum of two parts: The increase of p_1 above the desired price level p_1^* because of the associated real income loss of workers and the deviation of aggregate supply Y_2^s from $\bar{Y}$ caused by liquidations Z, $\Delta(Z)$:[28]

$$\mathcal{L} = (p_1 - p_1^*) - \omega \left(Y_2^s - \bar{Y} \right). \tag{4.13}$$

ω reflects the weight on the real income loss of workers in $t = 2$ relative to the weight on the real income loss of workers in $t = 1$ caused by a rise in p_1 and thus implicitly includes the central bank's time discount factor. Let p_1^* be normalised to 1 and thus equal the expected price level $E_0[p_1]$. It is sufficient to concentrate on p_1 and Y_2^s in this stylised model because Y_1^s is produced before any shocks occur and thus not directly influencable by monetary policy under discretion[29] and nominal wage negotiations for $t = 2$ take place after any shocks and determine p_2 such that workers receive $\Psi_t = (1 - \alpha) Y_2^s$.

The concentration on goods markets can be justified with several arguments: From a positive perspective because price stability and – differently accentuated – output stability are the mandate of most central banks in the world, where price stability is generally interpreted as a low but positive growth rate of some form of a consumer price index. From a political economy perspective, since people living mainly from their nominal labour income represent the majority of voters in a society and, as I show in this chapter, this focus may even improve the welfare of investors as well. Finally, also from a normative perspective within the New Keynesian framework as argued by Woodford (2003) because asset prices are in general a lot more flexible than goods prices and the monetary authority should focus on a measure of relatively sticky core inflation to limit the distortions caused by nominal rigidities.[30]

[28]The discussion below shows that the central bank cannot intervene symmetrically in this model. Hence, the linear loss function represents a useful simplification. The results of the model are robust to a loss function that is quadratic in inflation and output deviations from their respective targets, but the comparative static analysis and the restrictions on some parameter values become more complex (see section 4.B in the appendix).

[29]The indirect effect of central bank intervention on aggregate supply will be analysed in section 4.3.5. Section 4.3.6 discusses how optimal monetary policy can take the indirect effect into account.

[30]Note, however, that the normative argument has been subject of a long discussion in macroeconomics that goes far beyond the scope of this chapter. For example, Woodford's argument

4.3.2 Asset market

The central bank has the possibility to prevent the costly liquidation of shares if it acts as a lender or rather liquidity provider of last resort to the financial market. That means it can enter repurchasing agreements with investors at a price (just) high enough to prevent liquidations and thus provide extra liquidity to the market. In such an emergency repurchasing agreement, the central bank buys l assets at a nominal price $\underline{q}$ and sells them to the same investor in $t = 2$ for the asset's nominal payoff R. The total amount $L \equiv \int_{i \in I} ldi$ of assets bought, their buying price $\underline{q}$ and thus the liquidity costs for investors $\left(R - \underline{q}\right) L$ all depend on the preferences of the central bank in (4.13).[31] As in section 4.2.3, I begin with the asset market and an investors' optimal behaviour.

The possibility of a central bank intervention alters the optimal investment decision problem for an individual investor. The maximisation problem (4.8) under aggregate risk becomes

$$\max_{s,c_1,c_2,\hat{s},z,l} E\left[U(c_1', c_2)\right] = \int_{-\infty}^{\infty} \left(\gamma \ln\left(c_1 + \rho z\right) + \beta \ln c_2\right) f(\gamma) d\gamma \quad \text{s.t.} \tag{4.14}$$

$$p_1 c_1 + p_2 c_2 \leq w - s + Rs + (R - q)\hat{s} - Rz - \left(R - \underline{q}\right) l$$

$$p_1 c_1 + q\hat{s} \leq w - s + \underline{q} l$$

$$0 \leq z \leq s; \; 0 \leq l \leq s; \; l + z \leq s.$$

The problem (4.14) is solved as in section 4.2.3. While the first-order conditions (4.9a) to (4.9d) and (4.9g) remain unchanged, the derivatives with respect to the

neglects the information content of asset prices about future consumer price inflation that was emphasised by Alchian and Klein (1973). These authors concluded that asset prices should receive a very high weight in the price index that the central bank tries to stabilise; their argument was rejected mostly for practical reasons (see Cecchetti, Genberg, Lipsky and Wadhwani (2000), for example).

The modern discussion rather ranges between, e.g., Bernanke and Gertler (1999), who argue that asset price changes are only relevant for monetary policy insofar as they change the forecasts of consumer price inflation and output, while, e.g., Cecchetti et al. (2000) favour a more direct response to asset prices because this should limit the extent of asset price bubbles and thus dampen the volatility of output and inflation. The discussion below shows that the spillover effects from the asset to the goods market justify a direct monetary policy response to asset prices even if the central bank neglects the welfare of asset holders and there are no bubbles.

[31] The individual costs of emergency liquidity provision $\left(R - \underline{q}\right) l$ represent a deadweight loss in the model. Actually, these costs equal the nominal seigniorage income for the central bank. Section 4.3.6 includes a discussion of the optimal use of this seigniorage income.

Lagrangian parameters (4.9e) and (4.9f) become

$$\frac{\partial \Lambda}{\partial \lambda} = -p_1 c_1 - p_2 c_2 + w + (R-1)\, s + (R-q)\hat{s} - Rz - \left(R - \underline{q}\right) l \geq 0 \qquad (4.15a)$$

$$\frac{\partial \Lambda}{\partial \mu} = -p_1 c_1 - q\hat{s} + w - s + \underline{q} l \geq 0 \qquad (4.15b)$$

and the new first-order condition

$$\frac{\partial \Lambda}{\partial l} = -\lambda \left(R - \underline{q}\right) + \mu \underline{q} \leq 0, \quad l \geq 0 \qquad (4.16)$$

is added to the system.

In order to limit the increase of the price level on the goods market p_1 caused by the extra liquidity in the market, the central bank will provide this liquidity at the highest cost for investors that still prevents the costly liquidation, i.e. $\underline{q}$ is as low as possible. Since (4.16) implies that $\lambda \left(\frac{R}{\underline{q}} - 1\right) = \mu$ for $l > 0$, it is obvious from (4.9c) that $q = \underline{q}$ in equilibrium in this case. At the same time, the discussion in section 4.2.3 shows that $\lambda \left(\frac{R}{p_1 \rho} - 1\right) = \mu$ for $z > 0$, i.e. $\gamma > LIQ$. $\underline{q} = p_1 \rho = q$ causes investors' indifference between consuming by liquidating assets ($z > 0$) or by buying c_1 for p_1 on the goods market with cash from selling the asset at $\underline{q}$ to the central bank or at q on the asset market. Hence, $\underline{q} = p_1 \rho$ is the lowest price at which the central bank can prevent costly liquidations in response to large liquidity shocks γ.

4.3.3 Goods market

A closer look at the goods market in $t = 1$ and 2 illuminates the mechanism of the model and the trade-off the central bank faces. In particular, the central bank needs to quantify the costs and benefits of additional liquidity to determine the optimal amount of nominal aggregate liquidity provision.

As in section 4.2.3, the aggregate demand shock η in (4.10) can be negative in $t = 1$, as investors transfer money into $t = 2$ for $\gamma < CIA$. Due to the central bank intervention, however, η can also be positive. For $\gamma > LIQ$, the central bank increases the amount of money available for consumption purchases in the economy by $\underline{q}L$. Since aggregate supply is already produced at the beginning of $t = 1$, the additional nominal funds $\underline{q}L$ cause a rise in the price of goods p_1 by $\tau \underline{q} L$.[32] Given workers' fixed nominal wage ψ_1, this price increase reduces workers' real consumption Ψ_1 and increases the amount of goods investors can buy on the goods market with money. Investors' total consumption C_1' is then the sum of goods bought

[32] Using the parameters and variables of the model, the price impact can be expressed as $\tau = \alpha p_1 \rho / (W - S)$. To simplify the exposition of the arguments, I continue to use τ for the price impact of L.

on the goods market C_1 with initial money holdings plus the liquidity provision $\underline{q}L$ and the proceeds from the real liquidation ρZ that the central bank optimally admits. Crucially, once the nominal wage ψ is fixed based on the expected nominal demand such that workers expect to receive $(1-\alpha)\bar{Y}$, a liquidity provision by the central bank that exceeds workers' expectations, independently of their expectation formation mechanism, will always induce this redistribution effect and increase the amount of real funds available for investors' consumption in $t=1$.

At the same time, the real liquidation of Z assets causes a reduction of aggregate supply in $t=2$ by $\Delta(Z)=\kappa Z$.[33] As the central bank intervention reduces the amount of liquidations by L, it increases Y_2^s proportionately by κL. Hence, aggregate supply $Y_2^s=\bar{Y}-\kappa Z$, where Z denotes the amount of optimally admitted liquidations, and the aggregate demand equation (4.12) becomes

$$Y_2^d=\frac{\psi_2+R(S-Z-L)+W-S-p_1C_1}{p_2}. \tag{4.17}$$

Since any risk in the model is dissolved by the time of the wage negotiations for $t=2$, the nominal wage ψ_2 guarantees a real consumption of $\Psi_2=(1-\alpha)\left(\bar{Y}-\kappa Z\right)$ and $E_1[p_2]=p_2$. As in section 4.2.3, $p_2=p_1=1$, if $\gamma\in[CIA,LIQ]$, $\beta R=1$ and $S=\beta/(1+\beta)W$, for example.

4.3.4 Optimal central bank intervention

The trade-off between the price impact τ and the output effect κ determines the optimal amount of liquidity L^* provided by the central bank. I define Z^* as the maximal aggregate amount of desired asset liquidations in response to a shock γ, i.e. $Z^*\equiv\int_{i\in I}zdi=\frac{\gamma p_1\rho S-\beta(W-S)}{p_1\rho(\beta+\gamma)}$ with $z=\frac{\gamma p_1\rho s-\beta(w-s)}{p_1\rho(\beta+\gamma)}$ taken from table 4.2 in section 4.2.3. The liquidation of Z^* produces an output loss of κZ^* in $t=2$ with $\kappa>0$. An intervention of L causes an increase in p_1 of $\tau\underline{q}L$ with $\tau>0$ above the expected price level $E_0[p_1]=1$. At the same time, it reduces the extent of actual liquidations Z by L, which increases aggregate supply in $t=2$ by κL. I assume $\omega\kappa>\rho\tau$ such that the value of the output gain is sufficiently high for a positive level of L in response to large shocks γ. The endogeneity of the lowest intervention

[33] The linearity of the output loss serves again the purpose of expositional ease. Given the way the size of the economy $\bar{Y}$ is linked to the amount of assets S, $\kappa=R/(\alpha p_2)$.

price $\underline{q} = p_1\rho$ implies that

$$\begin{aligned} & p_1 = 1 + \tau\underline{q}L \\ \Leftrightarrow \quad & p_1 = 1 + \tau p_1 \rho L \\ \Leftrightarrow \quad & p_1 = \frac{1}{1-\tau\rho L} \end{aligned} \tag{4.18}$$

and requires $\tau\rho L < 1$ for an equilibrium. Given this information about the price and output impacts of its intervention, the central bank optimises

$$\begin{aligned} \min_L \mathcal{L} &= (p_1 - p_1^*) - \omega\left(Y_2^s - \bar{Y}\right) \qquad (4.19) \\ &= \left(\frac{1}{1-\tau\rho L} - 1\right) - \omega\left(\bar{Y} - \kappa\left(Z^* - L\right) - \bar{Y}\right) \\ &= \frac{\tau\rho L}{1-\tau\rho L} + \omega\kappa\left(\frac{\gamma S - \frac{\beta(W-S)(1-\tau\rho L)}{\rho}}{\beta+\gamma} - L\right). \end{aligned}$$

In the optimum, the marginal costs of higher prices p_1 just equal the marginal benefit of greater output Y_2^s,

$$\begin{aligned} & \frac{d\mathcal{L}}{dL} = \frac{\tau\rho}{(1-\tau\rho L)^2} + \omega\kappa\left(\frac{\beta\left(W-S\right)\tau}{\beta+\gamma} - 1\right) \stackrel{!}{=} 0 \\ \Leftrightarrow \quad & \underbrace{\frac{\tau\rho}{(1-\tau\rho L)^2}}_{\text{direct marginal cost of } \frac{dp_1}{dL}} + \underbrace{\omega\kappa\frac{\beta\left(W-S\right)\tau}{\beta+\gamma}}_{\text{indirect marginal cost of } \frac{\partial Y_2^s}{\partial Z^*}\cdot\frac{\partial Z^*}{\partial p_1}\cdot\frac{dp_1}{dL}} = \underbrace{\omega\kappa}_{\text{marginal benefit of } \frac{\partial Y_2^s}{\partial L}}. \end{aligned} \tag{4.20}$$

Note that $\frac{dZ^*}{dL} > 0$ since the goods price increase associated with $L > 0$ makes the real liquidation technology more attractive. The optimal liquidity provision L^* that fulfills the stability criterion $\tau\rho L < 1$ is

$$L^* = \frac{1}{\rho\tau} - \sqrt{\frac{\beta+\gamma}{\omega\kappa\tau\rho\left[\beta+\gamma-\beta\left(W-S\right)\tau\right]}}. \tag{4.21}$$

Proposition 4.2 *The optimal amount of assets purchased by the central bank L^* increases in the size of the liquidity shock γ, the weight on the output gap ω and its marginal reduction of output losses κ. L^* decreases in its marginal price impact τ, the real payoff of the liquidation technology ρ and the amount of money $W-S$ initially held by investors if $\omega\kappa > \rho\tau$ and $\gamma > LIQ$.*

Proof. The derivatives of L^* in equation (4.21) are positive with respect to γ, ω, κ and negative with respect to ρ, τ given the assumptions about the parameters. ■

Proposition 4.2 shows that the central bank will provide more liquidity in response to a greater shock γ because it reduces the indirect marginal costs of intervening.[34] The opposite is true for larger money holdings $W-S$ and more investment in the illiquid asset S: More initial liquidity increases the marginal costs of L as the same endogenous rise of p_1 raises the desired liquidations Z^* by more. Furthermore, L^* increases with the weight on output gap stabilisation relative to price stabilisation, ω, because this makes an output loss due to liquidation more costly relative to a price increase due to central bank intervention. A greater output impact κ of an intervention or a smaller price impact τ also improve the benefits of intervening relative to its costs and thus raise L^*. Finally, a greater ρ amplifies the price impact of the necessary intervention ceteris paribus and thus lowers L^*.

A special situation emerges if the central bank provides so much liquidity that p_1 rises until $p_1\rho = \underline{q} = R$. A further increase of $\underline{q}$ means that the central bank actually *pays* investors not to liquidate their assets and $\mu < 0$ from (4.9c). But $\underline{q} > R$ may become necessary as it is the nominal value of the asset's real put option in $t = 1$, $p_1\rho$, that determines Z, not the asset's final payoff R (see table 4.2). This situation will not occur, however, as long as

$$\begin{aligned} p_1 &< \frac{R}{\rho} \\ \Leftrightarrow \quad \frac{1}{1-\tau\rho L^*} &< \frac{R}{\rho}. \end{aligned}$$

Taking L^* from (4.21) and neglecting the indirect marginal costs of L that only reduce L^* shows that

$$\begin{aligned} \frac{1}{1-\tau\rho\left(\frac{1}{\rho\tau}-\sqrt{\frac{1}{\omega\kappa\tau\rho}}\right)} &< \frac{R}{\rho} \\ \Leftrightarrow \quad R\sqrt{\frac{\tau}{\omega\kappa\rho}} &> 1 \end{aligned}$$

is a sufficient condition for $p_1 < \frac{R}{\rho}$ and thus $\underline{q} < R$.

4.3.5 Welfare implications and the moral hazard effect

How is the utility and the behaviour of investors affected by the central bank intervention? First, the central bank chooses $\underline{q}$ such that it can prevent the real liquidation

[34] If the loss function (4.13) was quadratic in the output gap in $t = 2$, also the marginal benefit of L would increase with Z^* and thus with γ (see section 4.B in the appendix).

of L^* assets at the lowest price impact, i.e. $\underline{q} = p_1\rho$. At this price, the individual costs of liquidating the asset $(R - p_1\rho)$ and the costs of selling it to the central bank in exchange for cash $(R - \underline{q})$ are identical. (4.9c), (4.9d) and (4.16) show that individual investors are indifferent between liquidating, selling to the central bank and selling on the market as $\underline{q} = p_1\rho = q$. Nevertheless, the central bank intervention raises the welfare of investors ceteris paribus because it lessens the cash-in-advance constraint via the endogenous rise of p_1 and the corresponding increase in the value of the asset in $t = 1$, $p_1\rho = \underline{q} = q$.[35] Since the nominal income of workers and the supply of goods $Y_1^s = \bar{Y}$ are fixed, the price increase causes a redistribution from workers to investors in $t = 1$.

The anticipation of central bank intervention also affects the initial investment decision of investors. The first-order condition for optimal investment in the asset is (4.9g),

$$\frac{dL}{ds} = \int_{-\infty}^{\infty} \left[\lambda\left(R-1\right) - \mu\right] f\left(\gamma\right) d\gamma = 0. \tag{4.22}$$

In the optimum, the excess return of the asset over money $(R - 1)$ evaluated with the expected marginal utility of wealth λ equals the expected marginal utility of money's liquidity services μ. Investors anticipate that the central bank will provide extra liquidity for some realisations of γ. These interventions raise the rationally expected price of goods $E_0\left[p_1\right]$ relative to the one without expectations of interventions. The higher expected price level lowers the value of money's nominal payoff relative to the liquidated asset's real payoff of ρ in $t = 1$, or, in nominal terms, raises the nominal value of a liquidated asset $p_1\rho$ relative to the constant nominal payoff of money of 1. Since the asset becomes more valuable relative to money, investors will increase their investment s. This represents the so-called *moral hazard effect* of central bank intervention because investors increase their holdings of the asset whose value is possibly subject to liquidity risk as they anticipate the liquidity provision by the central bank.

Proposition 4.3 *The anticipation of a central bank intervention in $t = 1$ to limit the extent of real liquidations of the asset S causes an increase in the investment in S in $t = 0$ relative to the case without the possibility of a central bank intervention.*

Proof. The moral hazard effect arises for two reasons. Taking the aggregate investment level S as given, the higher goods price p_1 first raises the optimal amount of assets liquidated or sold to the central bank because $\frac{\partial z}{\partial p_1}|_{\gamma > LIQ} > 0$ (for z, λ, μ, see the last column of table 4.2). This is reflected in (4.22) in a lower expected marginal utility of money, $\frac{\partial \mu}{\partial p_1}|_{\gamma > LIQ} < 0$, and a greater marginal utility of wealth,

[35] See also figure 4.6 and the discussion of the moral hazard effect below.

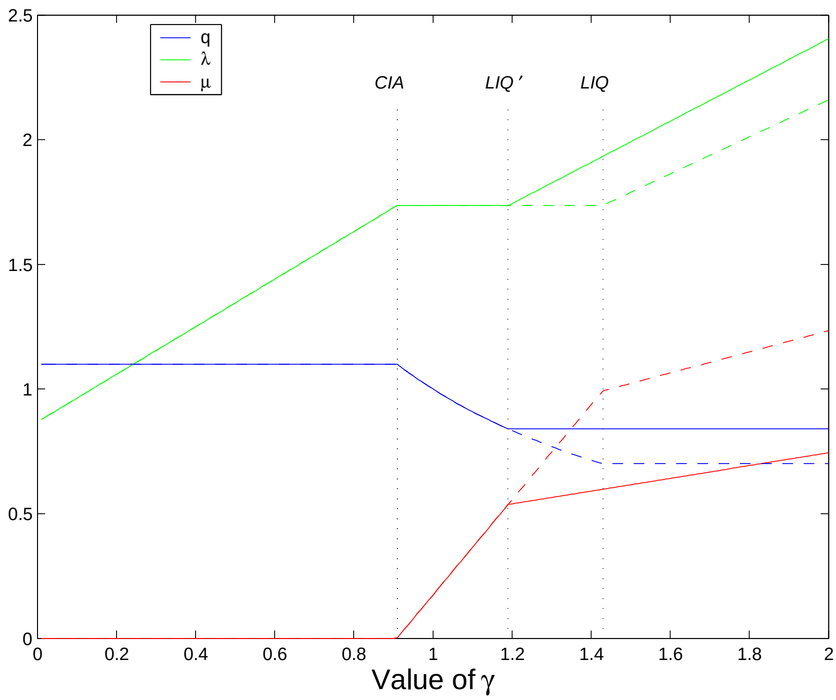

Figure 4.6: *q, λ, μ as a function of γ and different parameter values for $p_1 = 1.2$ (solid line) and $p_1 = 1$ (dashed line).*

$\frac{\partial \lambda}{\partial p_1}|_{\gamma > LIQ} > 0$. Second, the increase in p_1 lowers the threshold of the realisation of γ, $LIQ = \frac{\beta(W-S)}{p_1 \rho S}$, for which Z and L become positive. Since $CIA = \frac{\beta(W-S)}{RS}$ remains unchanged for a given S, the lower LIQ reduces the intermediate range $CIA \leq \gamma \leq LIQ$ for which the effect of the liquidity shock is fully absorbed by the asset price and consumption remains unchanged (see table 4.2). The constant consumption levels imply that the marginal utility of wealth, λ, is also constant in this range, while the cash-in-advance constraint becomes very costly, i.e. μ rises rapidly with γ. Equation (4.22) shows that a greater expected marginal utility of wealth and a smaller expected marginal utility of money increase the optimal individual investment s. This raises also the aggregate investment $S = \int_{i \in I} sdi$ in equilibrium.
■

The two effects can be seen in figure 4.6 which replicates figure 4.4 for the case of no central bank intervention. It shows the shift to the left of the threshold LIQ, i.e. the right kink in the three curves, and the higher values of λ and the lower values of μ in the range of $\gamma > LIQ$ for a greater price p_1 due to a central bank intervention.

The moral hazard effect of Proposition 4.3 can be so severe that investors stop holding money as stated in the following Corollary 4.2:

Corollary 4.2 *Holding no money from $t = 0$ to $t = 1$ represents an equilibrium if investors expect the central bank to intervene at a price $\underline{q}$ greater than 1.*

Proof. Assume all investors except i hold only the asset, i.e. $S = W$. Then, $CIA = LIQ = 0$ and the central bank has to intervene with certainty. Equation (4.22) simplifies to $\frac{dL}{ds} = \int_{-\infty}^{\infty} \left[\frac{(\beta+\gamma)(\underline{q}-1)}{w-s+\underline{q}s} \right] f(y)\, d\gamma \geq 0$ which will be strictly positive for $E_0 \left[\underline{q} \right] > 1$ as γ has a positive support, $cov(\gamma, \underline{q}) > 0$ and the denominator $w + (\underline{q} - 1)s > 0$. Hence, investing the full endowment w in the asset will be optimal for i, i.e. $s = w$, and $S = W$ represents an equilibrium. ■

Corollary 4.2 implies that the parameters of the model, for example the real payoff of liquidation ρ or the weight on output stabilisation ω, have to be chosen such that the liquidity provision is sufficiently costly and $E_0 \left[\underline{q} \right]$ sufficiently smaller than 1 in order to prevent the possibility of a complete moral hazard scenario caused by full insurance against liquidity shocks provided by the central bank.

What happens to the welfare of workers? Given the investment S, their welfare clearly rises if the central bank's relative weight on output in the loss function (4.13), ω, represents their own preferences. The central bank sets L^* and the corresponding price $\underline{q}$ such that the marginal cost of the price increase equals the marginal benefit of less liquidated assets in equation (4.20). The increase in S due to the moral hazard effect is double-edged, however: The higher real investment causes a rise in producible output $\bar{Y}$ as $\partial \bar{Y} / \partial K \cdot dK/dS > 0$. At the same time, it increases the extent of desired liquidations Z^* and central bank intervention L^* ceteris paribus. In general, the overall welfare effect for workers depends on the gain from greater output $\bar{Y}$ due to the increase in S relative to the associated costs in $t = 1, 2$. The following section discusses the optimal monetary policy when the central bank takes this additional trade-off into account. That section also examines what happens if not only investors, but also workers anticipate the central bank intervention.

4.3.6 Monetary policy under commitment and further model extensions

In section 4.3.4, the optimal central bank intervention in $t = 1$ was calculated based on the central bank loss function (4.13) after the realisation of the liquidity shock and given aggregate investment S. This reflects the absence of a commitment possibility of the central bank in this model. In other words, the solutions presented so far represent optimal monetary policy under discretion. The optimal second-best solution given the cash-in-advance constraint, however, could be achieved by a central bank with the possibility to commit to a specific intervention policy in $t = 1$ at $t = 0$.

In that case, the central bank has to optimally weight the increased aggregate supply $\bar{Y}$ associated with the moral hazard-effect against the costs of liquidations and interventions in $t = 1$ and 2. More generally, if private investors anticipate a liquidity insurance by the central bank, they hold less liquidity and invest their funds more productively. A lower level of aggregate liquidity, however, makes the financial sector less resilient, such that financial crises and central bank interventions become more likely.

Hence, the loss function (4.13) has to be extended to take the productivity gain from the moral hazard effect into account. As before, the loss increases in $p_1 - p_1^*$ and the output costs of liquidations $\Delta(Z)$. Additionally, the loss decreases with aggregate output $\bar{Y}$, such that optimal monetary policy under commitment solves

$$\min_{S,L} E_0 \left[\mathcal{L}(\bar{Y}, p_1 - p_1^*, \Delta(Z)) \right] \tag{4.23}$$

to find the optimal level of private investments S and the optimal liquidity provision L conditional on the realised liquidity shock.

So far in this chapter, workers build their price expectations based on the money holdings $W - S$ of investors (see sections 4.2.2 and 4.2.3). The question what happens if not only investors, but also workers anticipate the central bank intervention, is related to the brief discussion in footnote 25 of the effects if workers' formed their price expectations in the wage negotiations with rational expectations rather than the quantity equation. For a given level of S, the central bank will provide extra liquidity if $\gamma > LIQ$. This increases the expected amount of cash available for purchases of consumption goods relative to the situation without central bank intervention and thus raises the expected price of consumption goods or – in a repeated version of the model – the expected inflation rate. Since rational workers want to be compensated for the higher expected price with higher nominal wages, this leads to an 'inflation bias' which the central bank should consider in the optimal monetary policy under commitment.[36] But once wages are determined, the central bank can always provide more liquidity than expected. Hence, the trade-off in $t = 1$ between redistribution losses for workers today versus less supply tomorrow continues to exist, independently of the way workers form their price expectations.

Another important feature of the central bank intervention is the possibility of a sterilisation of its intervention before the additional money causes price increases on the goods market. The example of September 11 in section 4.1.2 shows that the Fed was indeed able to quickly sterilise the emergency liquidity issued directly after the terrorist attacks. But this liquidity crisis was mostly limited to the payments

[36]Note that although the moral hazard effect lowers private money holdings $W - S$, the central bank intervention still raises the expected overall nominal demand from investors on the goods market. The reason is that investors reduce their money holdings precisely because they expect an easing of their CIA on average relative to the situation without central bank intervention.

and settlement system. In the other two examples of section 4.1.2, the crises in 1987 and 1998, the Fed had to lower interest rates despite buoyant GDP growth and rising inflation and provide liquidity for a much longer time to calm the markets (see figure 4.1 and the discussion in section 4.1.2). In these cases, the trade-off analysed in this chapter increases in relevance for the optimal policy response to the crises as demonstrated by the different developments of inflation after 1987, 1998 and 2001, illustrated in figure 4.2.[37] Nevertheless, a sterilisation-possibility of interventions could be easily included into the model by making the nominal aggregate demand shock η that spills over from the asset to the goods market a function of the sterilisation possibilities of the central bank.

In the model so far, the individual costs of emergency liquidity provision $\left(R - \underline{q}\right) l$ represented a deadweight loss. Actually, these costs for investors correspond to seigniorage income for the central bank. If the central bank or the government used this seigniorage to buy consumption goods in $t = 2$, the aggregate demand equation (4.12) included the additional term $\left(R - \underline{q}\right) L$ in the numerator. The welfare effects depend on the use of the real seigniorage income and should be taken into account accordingly when the central bank provides liquidity in $t = 1$. The inclusion of seignorage does not change the general trade-offs in the model, but it reduces the costs of liquidity provision if the seigniorage income is distributed to workers.

Finally, the traditional Bagehot (1873) principles suggest that the central bank should provide liquidity only to an illiquid, but solvent bank. The judgement between illiquidity and insolvency requires a lot of information about banks' assets and liabilities on behalf of the lender of last resort, the central bank. This identification problem transfers to financial markets, where the central bank faces the question if asset price declines are caused by illiquidity or by deteriorating fundamentals. In contrast to the case of financial intermediaries, this judgement seems to be less difficult on financial markets since a number of illiquidity measures exist and are easily observable: For example, bid-ask spreads, the quoted depth, i.e. the number of shares available at the bid/ask price, respectively, the volatility of returns and the size of order flows (see, e.g., Chordia et al., 2005). If all of these criteria signal liquidity problems, the central bank most probably faces a liquidity crisis. It should then act as a 'liquidity provider of last resort' and judge its actions according to the framework developed in this chapter.[38]

[37] Taylor (2005) supports the liquidity provision principle and thus a temporary departure of interest rates from the recommendations of a Taylor rule that includes only inflation and output. But he argues that policy should have returned to a standard rule more quickly after the crises in 1987 and 1998, i.e. sterilised the liquidity provision (Taylor, 2005, p. 114).

[38] Besides, my model could also easily capture worsening fundamentals by a lower real value of the liquidation technology, ρ.

4.4 Idiosyncratic risk

Having analysed the asset market, the goods market and central bank interventions under aggregate risk in the previous sections, I now focus on the question under which circumstances idiosyncratic shocks can influence asset prices. If liquidations are optimal for individual investors, the optimal central bank intervention and the spillover effects to the goods market are identical to the case under aggregate risk. Hence, this section concentrates on the asset market.

4.4.1 Standard model

The optimal investment decision problem for an individual investor under idiosyncratic risk resembles the one under aggregate risk with central bank intervention in (4.14)[39]

$$\max_{s,c_1,c_2,\hat{s},z,l} E\left[U(c_1',c_2)\right] = \int_{-\infty}^{\infty} \left(\zeta_i \ln\left(c_1+\rho z\right) + \beta \ln c_2\right) f(\gamma) d\gamma \quad \text{s.t.} \tag{4.24}$$
$$p_1 c_1 + p_2 c_2 \leq w - s + Rs + (R-q)\hat{s} - Rz - \left(R - \underline{q}\right) l$$
$$p_1 c_1 + q\hat{s} \leq w - s + \underline{q} l$$
$$0 \leq z \leq s;\ 0 \leq l \leq s;\ l + z \leq s$$

and is solved analogically. The first-order conditions are the same as in section 4.3.2 with ζ_i replacing γ.[40] Again, market clearing in $t=1$ requires that $\hat{S} = \int_{i\in I} \hat{s} di = 0$. Hence, the asset price q has to adjust to equalise excess demand and supply of assets by individual investors. The crucial difference to the case with only aggregate risk is that the market clearing condition does *not* imply that $\hat{s} = 0$ and no assets are traded.

The demand for shares in $t=1$ by investor i, $\hat{s}_i$, is determined by

$$\hat{s}_i = \frac{\beta\left(w-s\right) - q\zeta_i s}{q\left(\beta + \zeta_i\right)}. \tag{4.25}$$

Note that that the Cobb-Douglas utility function (4.2) implies that $\hat{s}_i$ is a convex function of ζ_i for a given asset price q as $\partial^2 \hat{s}_i / \left(\partial \zeta_i\right)^2 > 0$.[41]

Assume that each investor has an ex-ante probability of one half of belonging to group A who receive a shock ζ_A and to group B with shock ζ_B, respectively, and $\zeta_A \geq \zeta_B$ without loss of generality. The condition $E[\zeta_i] = 1$ and the positive support

[39] In order to explicitly exclude short sales, the constraint $\hat{s} \geq -s$ had to be added to (4.24). Footnote 41 shows that this is redundant given the specification of the model.

[40] That means equations (4.9a) to (4.9d), (4.9g), (4.15a), (4.15b) and (4.16).

[41] Furthermore, $\hat{s}_i > -s$ as $\beta(w-s) > -q\beta s$ such that the short sale constraint is redundant.

of ζ_i imply $\zeta_A \in [1;2)$, $(\zeta_A + \zeta_B)/2 = 1$ and the absence of an aggregate shock. As usual, market clearing requires

$$\int_{i \in A} \frac{\beta(w-s) - q\zeta_A s}{q(\beta + \zeta_A)} di + \int_{i \in B} \frac{\beta(w-s) - q\zeta_B s}{q(\beta + \zeta_B)} di = 0.$$

The pricing kernel for q becomes

$$q = \min\left[\frac{\beta(1+\beta)(W-S)}{[\zeta_A(2-\zeta_A)+\beta]S}; R\right] \qquad \text{for} \quad \frac{\beta(W-S)}{S} \geq p_1\rho \tag{4.26}$$

with W and S defined as before. Without idiosyncratic shocks, i.e. $\zeta_i = 1$, equation (4.26) simplifies to $q = \beta(W-S)/S$, the same asset price as for $\gamma = 1$ in section 4.2.3. The condition $\beta(W-S)/S \geq p_1\rho$ excludes liquidations $z > 0$ for $\zeta_i = 1$.[42]

Note that q in (4.26) increases in the heterogeneity of A and B, i.e. in the absolute value $|\zeta_i - 1|$. The reason is the convexity of $\hat{s}_i$ in (4.25) mentioned above. The convexity implies that the additional demand of the agents with the low liquidity shock ζ_B is always sufficiently large such that agents with the high shock ζ_A do not need to liquidate their asset. Figure 4.7 illustrates the convexity of $\hat{s}_i$ for $R = 1/\beta = 1.1$ and $S = \beta/(1+\beta)W$, the investment in the case of certainty. The solid line represents the excess demand for the asset which is 0 for $\zeta_A = 1$, given an asset price of $q = 1$ in the left panel. For $\zeta_A > 1$ (and thus $\zeta_B = 2 - \zeta_A < 1$), $\hat{s}_B$ rises faster than $\hat{s}_A$ falls, the excess demand becomes positive and $q > 1$ for $\zeta_A > 1$. For $\zeta_A \approx 1.413$, the asset price increases to $q = R$, since the excess demand is 0 at this combination of q and ζ_A in the right panel. For $\zeta_A > 1.413$, investors hit by the low shock ζ_B transfer money into $t = 2$ as their CIA becomes unbinding.

To summarise the effects, the structure of the model, in particular the Cobb-Douglas utility function (4.2) that causes the convexity of $\hat{s}_i$ and the dissolution of risk in $t = 1$, imply that idiosyncratic shocks alleviate the CIA given a fixed initial investment S. In general, however, idiosyncratic shocks can have a negative impact on asset prices if the absorption capacity of the market is limited. This happens in reality and in other models for example if investors are risk-averse and future returns are risky (see, e.g., Huang and Wang, 2006). A further feature of reality is the presence of brokers and market-makers on financial markets rather than a Walrasian auctioneer. As they smooth price fluctuations by providing liquidity to financial markets, they earn income in the form of bid-ask spreads. Models that analyse the microstructure of financial markets explain the behaviour of these market participants and the implications for transaction prices. The following subsection presents an extension to the standard model of this section that includes transaction

[42]The more general form of (4.26) is $q = \min\left[\max\left(\frac{\beta(1+\beta)(W-S)}{[\zeta_A(2-\zeta_A)+\beta]S}; p_1\rho\right); R\right]$.

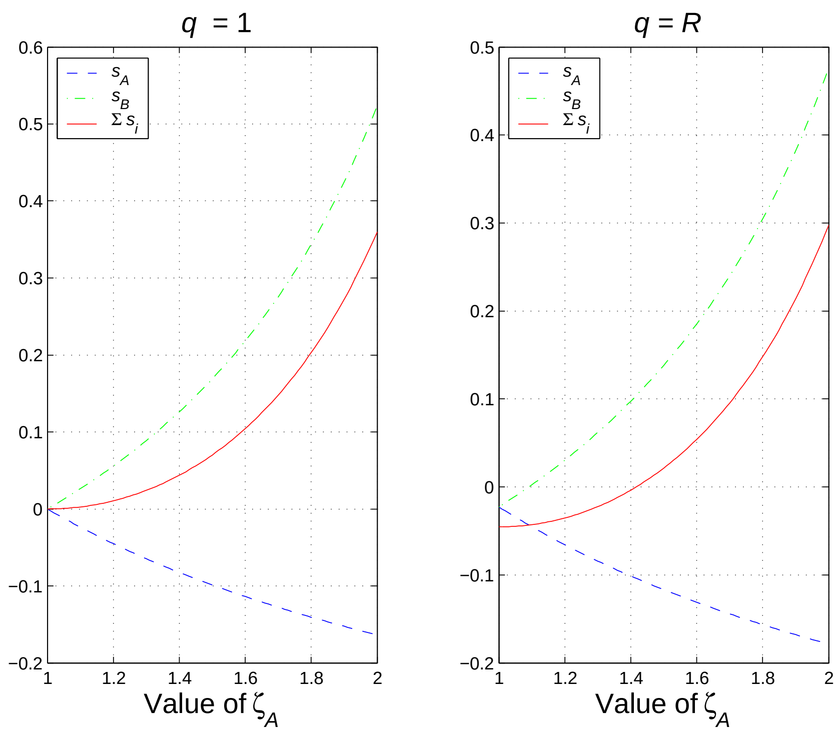

Figure 4.7: *Convexity of* $\hat{s}_i$*:* $\hat{s}_A$*,* $\hat{s}_B$ *and* $\Sigma\widehat{s_i} = \hat{s}_A + \hat{s}_B$ *as a function of* ζ_A *for* $q = 1$ *and* $q = 2$*.*

costs in the form of bid-ask spreads, and section 4.4.3 discusses different mechanisms how small shocks can have large impacts on asset prices.

4.4.2 Model with transaction costs

The market microstructure literature has developed models based on order-handling costs, asymmetric information or strategic behaviour, where idiosyncratic shocks can have (severe) impacts on asset prices. As Biais, Glosten and Spatt (2005, p. 218) formulate it:

> In perfect markets, Walrasian equilibrium prices reflect the competitive demand curves of all potential investors. While the determination of these fundamental equilibrium valuations is the focus of (most) asset pricing, market microstructure studies how, in the short term, transaction prices converge to (or deviate from) long-term equilibrium values.

A full market microstructure model is beyond the scope of this chapter, but the most important literature in this field is discussed in section 4.5.3. A simple way to summarise the relevant issues of market microstructure as developed, e.g., in O'Hara (1995) and Biais et al. (2005), is to assume transaction costs in the form of a bid-ask spread Ξ that decreases in total liquidity $M = \int_{i \in I} mdi$ available and increases in the order size $\hat{s}_i$, i.e. $\Xi = \Xi\left(\underset{-}{M}, \underset{+}{\hat{s}_i}\right)$.[43] Market-makers buy the asset in $t = 1$ at a bid price of q^{bid} from investors with the high shock $\zeta_A > 1$ and sell it to the low-shock types B with $\zeta_B = 2 - \zeta_A$ at an ask price $q^{\text{ask}} = q^{\text{bid}} + \Xi$. They earn the spread Ξ, with which they buy consumption goods on the goods market in $t = 1$ such that aggregate demand for goods is not directly affected by the presence of market-makers in the model.

The transaction cost Ξ is a measure of an asset's liquidity from the micro-perspective[44] and has a number of interpretations beyond completely exogenous transaction costs as, e.g., in Vayanos (2004) and Favero, Pagano and von Thadden (2006): It represents the time-varying illiquidity cost of shares in Acharya and Pedersen (2005) as $\hat{s}$ varies with the size of the shock.[45] It also captures search costs from search and matching models of financial markets as developed by, e.g., Duffie, Gârleanu and Pedersen (2005) because more available liquidity increases the probability of quickly finding a buyer, but larger orders decrease it. A further interpretation of Ξ are the random order-execution delays in Weill (2007). They are low in normal times but can become severe in times of liquidity crises such as October 1987 or September 1998 as described in section 4.1.2. These are precisely the times when order sizes tend to be large and aggregate liquidity tends to be low, at least until a central bank intervention calms markets.

Adding the transaction cost Ξ to the standard model (4.24) for idiosyncratic risk results in

$$\begin{aligned}
\max_{s,c_1,c_2,\hat{s},z} E\left[U(c_1', c_2)\right] = \int_{-\infty}^{\infty} \left(\zeta_i \ln\left(c_1 + \rho z\right) + \beta \ln c_2\right) f(\gamma) d\gamma \quad \text{s.t.} \qquad (4.27) \\
p_1 c_1 + p_2 c_2 \leq w - s + Rs + (R - q^j)\hat{s} - Rz - \left(R - \underline{q}\right) l \\
p_1 c_1 + q^j \hat{s} \leq w - s + \underline{q} l \\
0 \leq z \leq s;\ 0 \leq l \leq s;\ l + z \leq s \\
q^j = \begin{cases} q^{\text{bid}} & \text{for} \quad \zeta_i = \zeta_A \\ q^{\text{ask}} = q^{\text{bid}} + \Xi(M, \hat{s}) & \text{for} \quad \zeta_i = \zeta_B. \end{cases}
\end{aligned}$$

---

[43] For the positive relation between order size and bid-ask spreads, see chapters 3 and 6 in O'Hara (1995), for example. M represents a proxy for the size of the market making sector, which has a negative impact on the size of the spread as demonstrated in different models in O'Hara (1995). It also captures the public good character of liquidity as discussed below. Amihud and Mendelson (1986) provide empirical evidence for the role of bid-ask spreads in asset pricing.

[44] Other measures of liquidity such as the size of order flows were listed in section 4.3.6.

[45] Furthermore, M may be time-varying in a dynamic model in which this three-period game is repeatedly played.

In the equilibrium with $\zeta_A > \zeta_B$, investors of group A cannot be buyers of s in $t = 1$, i.e. $\hat{s}_A \leq 0$, such that their constraints are based on q^{bid}, while investors of group B cannot be sellers of s, i.e. $\hat{s}_A \geq 0$, and their constraints include q^{ask}.

The spread drives an additional wedge between the assets final payoff R and the achievable price for sellers, q^{bid}. Hence, costly liquidation ($z > 0$) is optimal for a wider range of parameters and shocks, which in turn leads to an extension of central bank intervention as the central bank optimally weights the output costs of intervention against the price increase associated with additional money.

Finally, the negative dependence of Ξ on aggregate liquidity $M = W - S$ introduces the public good character of liquidity and financial stability into the model.[46] While a decrease in S would lower the expected bid-ask spread and thus decrease the probability of costly liquidation in $t = 1$, the individual investor does not take this external effect into account in $t = 0$ since she is a price taker, i.e. $dM/ds = 0$.

4.4.3 From small shocks to large impacts: Propagation mechanisms

It may be questionable if transaction costs Ξ can become so large that idiosyncratic shocks can cause financial crises. But modern financial systems exhibit a number of feedback mechanisms that can amplify small shocks once the price impact exceeds a certain threshold.[47] These propagation mechanisms include margin calls, capital adequacy ratios, marking to market accounting rules and modern risk management.

Margins serve as collateral on markets for derivatives and for credit-financed investments. Combined with some form of a financing constraint, they can generate negative feedback mechanisms. In Morris and Shin (2004), 'liquidity black holes' arise because of exogenous loss limits for traders. Extending the market microstructure model of Grossman and Miller (1988), Brunnermeier and Pedersen (2007) use the concepts of market liquidity and funding liquidity: In normal times, capital constrained traders use external funds to smooth price fluctuations and provide market liquidity. If traders' outside financiers cannot distinguish illiquidity shocks from fundamental ones[48] and increase the required margins in response to an increase in price volatility, they can create a vicious circle: A negative liquidity shock causes losses and higher margins for traders, which reduces their ability to provide market

[46] Other papers that model liquidity as a public good include Holmström and Tirole (1998), Huang and Wang (2006) and Illing (2007). For a practitioner's view, see Geithner (2006).

[47] In the model of this chapter, this may be particularly relevant if idiosyncratic shocks are combined with positive aggregate shocks.

[48] Brunnermeier and Pedersen (2007) borrow this idea from the performance-based arbitrage argument in Shleifer and Vishny (1997), which is also applied for example in Gromb and Vayanos (2002).

liquidity and thus further increases traders' losses and required margins. Schnabel and Shin (2004) use the financial crisis in northern Europe in 1763 as a historic example of such negative feedback effects. Sauer (2002) adopts the stylised model of Schnabel and Shin (2004) to explain the LTCM-crisis in 1998.

Closely related to margin calls for leveraged investors are capital adequacy ratios for banks. Shin (2005a,b) and Illing (2007) show that capital adequacy requirements for banks can set off a vicious circle of asset sales similar to the one triggered by the funding constraints in Brunnermeier and Pedersen (2007).[49] Recent international reform proposals of accounting rules suggest to extend the use of market prices in accounting of financial firms instead of valuations based on historical costs, an approach already common among hedge funds, for example. While such marking to market gives a clearer picture of the true value of firms in general, it may cause excessive price volatility, i.e. volatility not reflecting fundamentals, and exacerbate or even trigger a financial crisis. In the model by Shin (2005a,b), marking to market is not necessary (Illing, 2007, p. 10), but accelerates the feedback effects via banks' balance sheets. Plantin, Sapra and Shin (2005) describe in a global games setup, how marking to market accounting rules can cause large losses in less liquid markets because asset sales are strategic complements under this accounting regime. They find that the damage done by marking to market is greatest when claims are long-lived, illiquid and senior. Cifuentes, Ferrucci and Shin (2007) combine marking to market accounting with regulatory solvency requirements to show that balance sheet interlinkages among financial institutions and contagion via changes in asset prices can cause contagious failures of financial institutions as a result of small shocks.

Financial risk management is a core competence of modern financial institutions and continuously evolving, not least in response to financial crises. In the 1980's, portfolio insurance became a popular form of risk management for investment funds. The discussion of the 1987-crash in section 4.1.2 highlights the negative impact of portfolio insurance during the crash. Today, value at risk (VaR) has become the standard risk measure used by financial institutions. Banks' capital requirements in the Basel-I accords have been linked to market risk based on VaR-calculations since 1998. Yet, VaR is no panacea, either. For example, Gârleanu and Pedersen (2007) show that a feedback effect can arise between tighter risk management and a reduction in liquidity. The former reduces the amount of liquidity provided to the market and the latter increases the effective risk of positions because it takes

[49] In Shin (2005b), financial intermediaries also want to maintain a minimum level of leverage. This creates a 'virtuous circle' of rising asset prices and increased lending. Thus, 'booms can be understood as a mirror image of liquidity drains.' While Shin (2005a) just mentions possible asymmetries due to default or inefficient liquidations, Illing (2007) extends Shin's model with a kinked net supply curve of assets, in this case property, due to information asymmetries. These market imperfections cause asymmetries between boom and bust periods in asset prices.

longer to sell them. The heart of the problem is the endogeneity of risk as described by Danielsson (2001) and Danielsson and Shin (2003). Financial market risk is not given exogenously by nature, but depends on the actions of market participants. This property becomes particularly important if financial institutions follow very similar investment strategies[50] and use the same standardised methods for their risk management (IMF, 1998).

Stress testing or liquidity-adjusted VaR measures are ways to incorporate liquidity risk into risk management. Nevertheless, the fundamental problem of financial risk's endogeneity remains unsolved.[51] In particular, individuals neglect the external effect of their decisions on aggregate liquidity. The public good character of liquidity, however, becomes most relevant during financial crises.

4.5 Related theoretical literature

Besides the different propagation mechanisms discussed in the previous section, the model developed in this chapter is linked to a number of theoretical contributions in the literature. This section reviews papers that analyse the Greenspan put option, segmented asset and goods markets as well as market microstructure theory and papers on the public supply of liquidity.

4.5.1 Greenspan put option

The 'Greenspan Put', i.e. the supposed insurance against severe financial turmoil by the Federal Reserve under Alan Greenspan, has become a well-known argument in the popular financial press. To my knowledge, only Miller, Weller and Zhang (2002) and Illing (2004) have developed an explicit theoretical analysis of the Greenspan put, focusing on the situation in which the central bank insures against asset price declines caused by a deterioration of the fundamental value of the asset.

In Miller et al. (2002), the expected present value of all current and future dividends determines the fundamental value of shares. Better management of financial crises by the Fed under Alan Greenspan, as indicated by the examples of 1987 and 1998 in section 4.1.2, may have fundamentally reduced the risk of shares and thus increased their fundamental value. But Miller et al. (2002) argue that investors additionally hold the erroneous belief that the Fed could insure them against *any* fall in asset prices, i.e. not only against price drops that are due to a financial crisis

[50] Many proprietary traders of investment banks copied the until then highly profitable strategies of LTCM in 1998 (see, e.g., Morris and Shin, 1999, p. 52).

[51] In Gârleanu and Pedersen (2007), the feedback effects are even stronger for a liquidity-adjusted VaR than a standard one.

but also against ‘normal’ declines that are due to a decrease in current and future dividends. For example, lower productivity growth than expected may reduce dividends and thus justify and require a revaluation of shares. Hence, investors appear to have overconfidence in the ability of the Fed to put a lower bound, a put option, on the share price and this leads to an overvaluation of shares which represents a bubble. The bubble is, however, completely independent of *actual* monetary policy, which is the focus of the present chapter.

Illing (2004) uses a model that concentrates on nominal debt and financial intermediation. Equity owners have a residual claim on the risky payoff of firms that are leveraged with nominal bank debt. A severely negative aggregate shock to the fundamental value of firms limits their ability to repay their debt, which in turn leads to the threat of bank runs by depositors. The collapse of banks would extinguish the knowledge capital from relationship lending in the economy and prompts the central bank to provide liquidity to the banks. The additional money reduces the real value of firms’ nominal debt, which represents a capital gain for equity owners. The anticipation of the monetary injection causes a bubble ex ante, as it raises the firms’ share price above its fundamental value. Illing (2004) notes that the central bank in his model faces a trade-off between the bubble created by moral hazard, i.e. the expected capital gain, and the risk associated with the disruption of financial intermediation.

The model by Illing (2004) can be easily classified in terms of the framework provided in this chapter, although I focus on financial markets rather than intermediation and leverage is not crucial in my model. After the realisation of the aggregate shock, the central bank in Illing’s model does not face any trade-off because ex post it is always optimal to prevent bank runs and keep banks’ knowledge capital.[52] This is different in my model where the central bank optimally chooses between inflation today and an output loss tomorrow after the shock. The trade-off emphasised by Illing only arises if the central bank has the possibility to commit to a specific intervention policy in response to a negative shock. This situation is akin to the optimal commitment solution discussed in section 4.3.6 in my model.

Both Miller et al. (2002) and Illing (2004) analyse the central bank insurance against asset price declines caused by a deterioration of the fundamental value of the asset, either erroneously expected or actually conducted. This represents probably a part of the public perception of the alleged ‘Greenspan put’. But the analysis in my model is more closely related to a ‘liquidity provision principle’, i.e. a temporary deviation of central bank policy from a standard Taylor rule that responds only

[52] A similar effect occurs in Illing (2007) within a different model with propagation effects as discussed in section 4.4.3. There, the central bank always lowers interest rates to prevent fire sales of assets by distressed banks which have to restore their capital adequacy requirements.

to output and inflation in order to inject large quantities of liquidity in a financial crisis (Taylor, 2005). It is the dramatic increase in μ, the marginal utility of cash's liquidity services, that induces investors to partly liquidate their assets and triggers the central bank's response, not a change in the fundamental pay-off of the asset R. Hence, it is the microeconomic view of liquidity, the missing ability to sell assets quickly and costlessly, that causes a monetary injection in my model.

4.5.2 Market segmentation

A crucial assumption in the model relates to the limited participation on the asset market, as workers cannot buy assets. If they could do so and their liquidity needs were not perfectly correlated with investors liquidity shocks, they might provide the extra liquidity needed to smooth investors liquidity shocks. The assumption of segmented markets follows the models in Allen and Gale (1994, 2005) and Huang and Wang (2006), where limited market participation emerges from participation costs.[53] The same impact has the assumption of separate cash-in-advance constraints on the asset and the goods market for all agents in Gale (2005). Gale (2005) uses his model to show that liquidity must be costly in order to guarantee the determinacy of the price level. Furthermore, the asset price fluctuates without affecting the goods price as the central bank stabilises the goods prices via its real seigniorage income. Both features are present in my model as the cash-in-advance constraint never binds ($\mu = 0$) if the central bank provides liquidity for free, i.e. $\underline{q} = R$. Asset price volatility without spillover effects to the goods market occurs in my model in the intermediate range of $CIA \leq \gamma \leq LIQ$. The main contributions of my chapter are the analysis of financial crises and the focus on emergency liquidity provision rather than on seigniorage income as in Gale (2005).

In reality, participation in asset markets is limited because economic agents lack the required expertise, have limited attention, institutional barriers or other costs of entry (Gale, 2005). Empirically, Landon-Lane and Occhino (2006) use Bayesian techniques to estimate the fraction of households participating in financial markets to be approximately 22%, while Campbell and Mankiw (1989) find that about 40% to 50% of the population in the U.S. consume only their current income rather than smooth their consumption via savings and dissavings. Statistics from the Survey of Consumer Finances 2004 reported in Bucks, Kennickell and Moore (2006) show that merely 20.7% of U.S. households hold publicly traded shares directly and only

[53] In Allen and Gale (1994), private agents decide about their participation on the asset market before liquidity shocks occur, in Huang and Wang (2006) after the realisation of idiosyncratic shocks which can thus have aggregate effects. Other papers that use models with limited market participation include Alvarez, Atkeson and Kehoe (2002) and Williamson (1994), for example.

48.6% hold some shares either directly or indirectly, e.g. via retirement accounts.

4.5.3 Market microstructure theory

The literature on market microstructure analyses the trading mechanism for financial securities and its impact on short-term asset price behaviour. O'Hara (1995) provides an excellent summary of the earlier literature, Biais et al. (2005) survey more recent developments. Amihud, Mendelson and Pedersen (2005) review the connection between liquidity as derived from the theoretical and empirical microstructure literature and asset pricing.

A common feature of this literature is that it does not distinguish between nominal and real assets and payoffs. Technically, most models maximise agents' expected utility of terminal wealth and thus abstract from real goods (see the models in O'Hara, 1995). For example, Grossman and Miller (1988) model liquidity as the price of immediacy. Market makers are willing to smooth temporary order imbalances for an asset with a risky final payoff if they can expect a positive excess return compared to the investment in a riskless asset.

An alternative way to model asset trading and possible illiquidity is the search and matching literature that has been inspired by Duffie et al. (2005). Again, these models do not differentiate between nominal and real payoffs as an asset pays one unit of a consumption good per period that serves as numéraire. One application of the model by Duffie et al. (2005) are endogenous feedback effects between risk-management and liquidity in Gârleanu and Pedersen (2007) as discussed in section 4.4.3.

By providing a framework that links asset price developments caused by liquidity shocks to the real sector of the economy via two spillover effects, this chapter makes one of the first steps to link the findings from the market microstructure literature with the analysis of optimal monetary policy in the macroeconomic literature.

4.5.4 Public supply of liquidity

A prominent paper that investigates the public provision of liquidity is Holmström and Tirole (1998), but it differs from the model in this chapter in important respects. First, liquidity is defined as the availability of instruments to transfer wealth across periods rather than to sell assets quickly and costlessly. Furthermore, the paper looks at the production side of the economy as firms may have a demand for liquidity to refinance their investment projects. Firms financing is subject to an agency problem such that firms cannot pledge the full value of the firm as collateral for credit lines or marketable assets. While this is not problematic in their model without aggregate uncertainty given the right private institutions such as banks, private 'liquidity' is insufficient in the presence of aggregate uncertainty. The government can overcome the agency problem and issue government bonds that are not subject to the agency problem because it can enforce tax payments. The social optimum in the model can be achieved with state-contingent government bonds, i.e. an active management of public liquidity, as their existence averts any private excess liquidity. Hence, Holmström and Tirole (1998) is not a paper about financial crises but rather about the involvement of the state in the financial system in normal times.

More generally, however, public provision of liquidity refers to the lender of last resort activity of a public authority, usually the central bank, as emergency liquidity assistance to the financial system. Most of the literature on the lender of last resort concentrates on banks and the interbank market. The collection of a wide range of papers on the lender of last resort in Goodhart and Illing (2002) includes only one paper by Kaufman (2002) that discusses the response to fire sales on asset markets in an informal way. More recent treatments like Freixas, Parigi and Rochet (2004) also neglect liquidity crises on asset markets, which are the focus of my chapter.[54] Given the substantial growth of financial markets relative to traditional banking in continental Europe and the continuous introduction of new financial instruments like credit derivatives, an appreciation of the effects of liquidity provision in response to liquidity crises on financial markets appears to be necessary.

4.6 Conclusion

The different specifications of the general model in this chapter help to provide guidance for central banks in the event of liquidity crises. Confronted with a liquidity crisis, the central bank faces a trade-off between injecting liquidity and thus incurring risks to price stability and negative supply effects in the future. The size of the optimal intervention increases in the size of the liquidity shock, the weight on output relative to inflation and the extent of negative supply effects of the crisis. It decreases in the size of the associated inflation in goods prices which is linked to the possibility to sterilise the intervention and the amount of liquidity initially held by investors.

Furthermore, the anticipation of central bank interventions by private investors leads to a moral hazard effect in the form of less private liquidity provision and

[54]One exception is Caballero and Krishnamurthy (2007) who develop a model of financial crises based on liquidity shortages and Knightian uncertainty aversion in which public and private liquidity serve as complements: The promise by the central bank to provide liquidity in extreme events, i.e. a 'double wave of liquidity shocks' in the model, but not for intermediate events, i.e. only 'one wave' of liquidity shocks, makes private agents provide their own liquidity for intermediate events as they are insured against an uncertain second wave of shocks.

thus an increase in the likelihood of financial crises. At the same time, less liquidity provision means more productive investment and thus greater aggregate supply in the absence of a financial crisis. Optimal monetary policy under commitment has to take these additional effects into account.

Motivated by the actual behaviour of the Fed under Alan Greenspan, the chapter has concentrated on the optimal *monetary* policy response to liquidity crises. However, this does not exclude the possibility that other policy tools exist to limit the probability and the extent of such crises. Regulatory measures represent an obvious candidate for appropriate ex ante action, in particular in the light of the external effects of private liquidity provision. A promising proposal seems to be the introduction of procyclical liquidity requirements for financial institutions. Such requirements could help to prevent the buildup of excessive positions in illiquid assets during boom periods via balance sheet feedback effects converse to the ones described in section 4.4.3 and at the same time limit vicious circles during market downturns (see, e.g., Illing, 2007). But even with an appropriate regulatory environment, liquidity crises may emerge and the trade-offs emphasised in this chapter remain relevant.

Finally, in view of the substantial growth of financial markets relative to traditional banking in particular in continental Europe and the introduction of new financial instruments like credit derivatives, the concentration on the banking system for financial stability as common in the literature appears to be inadequate. Instead, the understanding of the interlinkages between money, liquidity on financial markets, financial crises, inflation and real production is very important for financial stability and the continuation of successful monetary policy in the future. The increased tendency of major central banks such as the ECB, the Bank of England or the Swedish Riksbank to publish 'Financial Stability Reports' that take a very broad perspective on risks to the stability of the financial system represents a widely visible evidence that central bankers acknowledge this development. This chapter has provided a theoretical contribution to a better understanding of the relevant arguments. The obvious next step is to transfer this model into a stochastic dynamic general equilibrium framework and thus gain additional insights, in particular about the optimal monetary policy under commitment.

Appendices

4.A Solution to investors' problem under aggregate risk

The Lagrangian for the optimal investment decision problem for an individual investor under aggregate risk reads as

$$\begin{aligned}\Lambda = \int_{-\infty}^{\infty} &\{\gamma \ln (c_1 + \rho z) + \beta \ln c_2 \\ &- \lambda \left[p_1 c_1 + p_2 c_2 - (w - s) - Rs - (R - q)\hat{s} + Rz\right] \\ &- \mu \left[p_1 c_1 + q\hat{s} - (w - s)\right]\} f(\gamma)\, d\gamma.\end{aligned}$$

Using the Leibniz-Rule $\frac{d}{dx}\int_a^b f(x,z)\,dz = \int_a^b \frac{\partial}{\partial x} f(x,z)\,dz$, i.e. pointwise differentiation, the first-order conditions become

$$\frac{d\Lambda}{dc_1} = \int_{-\infty}^{\infty} \left(\frac{\gamma}{c_1 + \rho z} - \lambda p_1 - \mu p_1\right) f(\gamma)\, d\gamma = 0 \tag{4.28a}$$

$$\frac{d\Lambda}{dc_2} = \int_{-\infty}^{\infty} \left(\frac{\beta}{c_2} - \lambda p_2\right) f(\gamma)\, d\gamma = 0 \tag{4.28b}$$

$$\frac{d\Lambda}{d\hat{s}} = \int_{-\infty}^{\infty} \left(\lambda (R - q) - \mu q\right) f(\gamma)\, d\gamma = 0 \tag{4.28c}$$

$$\frac{d\Lambda}{dz} = \int_{-\infty}^{\infty} \left(\frac{\gamma}{c_1 + \rho z}\rho - \lambda R\right) f(\gamma)\, d\gamma \leq 0 \tag{4.28d}$$

$$\frac{d\Lambda}{d\lambda} = \int_{-\infty}^{\infty} \left(-p_1 c_1 - p_2 c_2 + (w - s) + Rs + (R - q)\hat{s} + Rz\right) f(\gamma)\, d\gamma \geq 0 \tag{4.28e}$$

$$\frac{d\Lambda}{d\mu} = \int_{-\infty}^{\infty} \left(-p_1 c_1 - q\hat{s} + w - s\right) f(\gamma)\, d\gamma \geq 0 \tag{4.28f}$$

$$\frac{d\Lambda}{ds} = \int_{-\infty}^{\infty} \left(\lambda (R - 1) - \mu\right) f(\gamma)\, d\gamma \leq 0 \tag{4.28g}$$

and $\frac{d\Lambda}{dz} \cdot \int_{-\infty}^{\infty} z f(\gamma)\, d\gamma = 0$, $\frac{d\Lambda}{d\lambda} \cdot \int_{-\infty}^{\infty} \lambda f(\gamma)\, d\gamma = 0$, $\frac{d\Lambda}{d\mu} \cdot \int_{-\infty}^{\infty} \mu f(\gamma)\, d\gamma = 0$ and $\frac{d\Lambda}{ds} \cdot \int_{-\infty}^{\infty} s f(\gamma)\, d\gamma = 0$ as complementary slackness conditions.

To derive the expected values of the Lagrangian parameters λ and μ in $t = 0$, it is easier to use the optimal values of $c_1, c_2, \hat{s}, z$ given a realisation of γ in $t = 1$ and then to integrate over all possible values of γ afterwards. This is equivalent to solving for the optimal values of $c_1, c_2, \hat{s}, z$ given the partial derivatives of the integrands in the first-order conditions above.

$$\frac{\partial \Lambda}{\partial c_1} = \frac{\gamma}{c_1 + \rho z} - \lambda p_1 - \mu p_1 = 0 \tag{4.29a}$$
$$\frac{\partial \Lambda}{\partial c_2} = \frac{\beta}{c_2} - \lambda p_2 = 0 \tag{4.29b}$$
$$\frac{\partial \Lambda}{\partial \hat{s}} = \lambda (R - q) - \mu q = 0 \tag{4.29c}$$
$$\frac{\partial \Lambda}{\partial z} = \frac{\gamma}{c_1 + \rho z} \rho - \lambda R \leq 0 \tag{4.29d}$$
$$\frac{\partial \Lambda}{\partial \lambda} = -p_1 c_1 - p_2 c_2 + w + (R - 1) s + (R - q) \hat{s} - R z \geq 0 \tag{4.29e}$$
$$\frac{\partial \Lambda}{\partial \mu} = -p_1 c_1 - q \hat{s} + w - s \geq 0 \tag{4.29f}$$

and $\frac{\partial \Lambda}{\partial z} z = 0$, $\frac{\partial \Lambda}{\partial \lambda} \lambda = 0$ and $\frac{\partial \Lambda}{\partial \mu} \mu = 0$ as complementary slackness conditions. Equations (4.29a) to (4.29f) and equation (4.28g) are equations (4.9a) to (4.9g) in section 4.2.3.

The first-order condition for optimal investment in the asset is given by equation (4.28g). Using the results for λ and μ from table 4.2 produces

$$\begin{aligned}
\frac{d\Lambda}{ds} = & \int_{-\infty}^{\frac{\beta(W-S)}{RS}} \left[\frac{\beta + \gamma}{w + (R-1) s} (R - 1) \right] f(\gamma) \, d\gamma \\
& + \int_{\frac{\beta(W-S)}{RS}}^{\frac{\beta(W-S)}{p_1 \rho S}} \left[\frac{\beta (W + (R-1) S)}{RS (w + (R-1) s)} (R - 1) - \frac{\gamma (W + (R-1) S)}{(w + (R-1) s) (W - S)} + \lambda \right] f(\gamma) \, d\gamma \\
& + \int_{\frac{\beta(W-S)}{p_1 \rho S}}^{\infty} \left[\frac{p_1 \rho (\beta + \gamma)}{R (w + (p_1 \rho - 1) s)} (R - 1) - \frac{\beta + \gamma}{w + (p_1 \rho - 1) s} + \lambda \right] f(\gamma) \, d\gamma \\
& = 0.
\end{aligned}$$

Solving this using $G(x) \equiv \int_{-\infty}^{x} \gamma f(\gamma) \, d\gamma$ and $F(x) \equiv \int_{-\infty}^{x} f(\gamma) \, d\gamma$ with $CIA \equiv \frac{\beta(W-S)}{RS}$ and $LIQ \equiv \frac{\beta(W-S)}{p_1 \rho S}$ gives

$$\begin{aligned}
& \frac{\beta (R-1)}{w + (R-1) s} F(CIA) + \frac{(R-1)}{w + (R-1) s} G(CIA) \\
& + \frac{\beta (W + (R-1) S)}{S (w + (R-1) s)} [F(LIQ) - F(CIA)] \\
& - \frac{(W + (R-1) S)}{(w + (R-1) s)(W - S)} [G(LIQ) - G(CIA)] \\
& + \frac{\beta (p_1 \rho - 1)}{w + (p_1 \rho - 1) s} [1 - F(LIQ)] + \frac{(p_1 \rho - 1)}{w + (p_1 \rho - 1) s} [1 - G(LIQ)] \\
& = 0.
\end{aligned}$$

In equilibrium, all investors follow the same investment strategy and the assumption of a mass 1 of ex-ante identical investors makes $s = S$. The definition of conditional expectations

$$E[\gamma|\gamma < x] = \frac{\int_{-\infty}^{x} \gamma f(\gamma)\, d\gamma}{F(x)}$$

leads to

$$G(x) = \int_{-\infty}^{x} \gamma f(\gamma)\, d\gamma = F(x)\, E[\gamma|\gamma < x],$$

but this does not allow to solve for s without explicitly parameterising the density function of the liquidity shock $f(\gamma)$. Assuming a uniform distribution for γ, i.e. $F(x) = \frac{x-a}{b-a}$ for $a \leq x \leq b$, gives the conditional expected value of $E[\gamma|\gamma \leq x] = \frac{1}{2}(x+a)$ for $0 < a < b$. The definition of the thresholds as $CIA = \frac{\beta(W-S)}{RS}$ and $LIQ = \frac{\beta(W-S)}{p_1 \rho S}$ results in the conditional expected values $E[\gamma|\gamma < CIA] = \frac{1}{2}\left(\frac{\beta(W-S)}{RS} + a\right)$ and $E[\gamma|\gamma < LIQ] = \frac{1}{2}\left(\frac{\beta(W-S)}{p_1 \rho S} + a\right)$. Table 4.3 in section 4.2.3 summarises this information.

4.B Optimal central bank intervention with a quadratic loss function

In section 4.3, the loss function (4.13) of the central bank is linear in the increase of p_1 above the desired price level p_1^* and the deviation of aggregate supply Y_2^s from $\bar{Y}$ caused by liquidations Z, $\Delta(Z)$. This section shows that the results of the model are robust to the loss function (4.30) that is quadratic in inflation and output deviations from their respective targets, but the first-order condition and thus the comparative static analysis become more complex:

$$\mathcal{L}_2 = (p_1 - p_1^*)^2 + \omega\left(Y_2^s - \bar{Y}\right)^2. \tag{4.30}$$

The optimisation problem (4.19) for the central bank becomes

$$\begin{aligned}
\min_L \mathcal{L}_2 &= (p_1 - p_1^*)^2 + \omega\left(Y_2^s - \bar{Y}\right)^2 \\
&= \left(\frac{1}{1-\tau\rho L} - 1\right)^2 + \omega\left(\bar{Y} - \kappa\left(Z^* - L\right) - \bar{Y}\right)^2 \\
&= \left(\frac{\tau\rho L}{1-\tau\rho L}\right)^2 + \omega\kappa^2\left(\frac{\gamma S - \frac{\beta(W-S)(1-\tau\rho L)}{\rho}}{\beta+\gamma} - L\right)^2.
\end{aligned} \tag{4.31}$$

The first-order condition turns out to be

$$\frac{d\mathcal{L}_2}{dL} = \frac{\tau^2\rho^2 L}{(1-\tau\rho L)^3} + \omega\kappa^2 (Z^* - L)\left(\frac{\beta(W-S)\tau}{\beta+\gamma} - 1\right) \stackrel{!}{=} 0 \tag{4.32}$$

$$\Leftrightarrow \underbrace{\frac{\tau^2\rho^2 L}{(1-\tau\rho L)^3}}_{\text{direct marginal cost of } \frac{dp_1}{dL}} + \underbrace{\omega\kappa^2 (Z^* - L)\frac{\beta(W-S)\tau}{\beta+\gamma}}_{\text{indirect marginal cost of } \frac{\partial Y_2^s}{\partial Z^*}\cdot\frac{\partial Z^*}{\partial p_1}\cdot\frac{dp_1}{dL}} = \underbrace{\omega\kappa^2 (Z^* - L)}_{\text{marginal benefit of } \frac{\partial Y_2^s}{\partial L}}.$$

Overall, the quadratic loss function has an impact on the relative size of direct and indirect marginal costs and benefits, but it does not change the general structure of the first-order condition. In particular, the direct marginal cost continues to increase in L, while the indirect marginal cost and the marginal benefit decrease with L as $d(Z^* - L)/dL < 0$ given the assumptions about the parameters. The optimal L^* becomes the solution to a fourth-degree polynomial. With a linear loss function, the last two effects are constant, instead, and L^* is the solution to the quadratic equation (4.20).

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