Optimal Policies in the Presence of Tax Evasion

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Preface

Overview

All essays that make up this dissertation formulate economic models where some economic agents – either individuals or firms – may try to pay only part or even nothing at all of their tax bill. Tax evasion and tax avoidance are phenomena present in all societies that use taxes to finance government expenditures. Sometimes every possible effort is made to reduce the tax burden on the side of taxpayers. Effort is also present on the side of the government and the (tax) law enforcement agency: they try to deter evasion and avoidance or at least to detect and convict tax evaders. Each activity, evasion, avoidance and deterrence, uses resources and adds to the deadweight loss of a tax system.

Tax evasion, in particular, can take on different forms: non-declaration or underreporting of the tax base (which may be income, sales, wealth etc.), overreporting of deductible expenses, moonlighting, smuggling and others. This dissertation abstracts from the particular mode that is used to save on the tax bill – with the exception of chapter 3 – and I have to disappoint the reader that tries to find hints on how he could best reduce his own tax payments.[1] Rather the models discuss especially how a benevolent government (the so-called social planner) could and should react to the presence of tax evasion activities.

I present a short overview of research in the field in chapter 2. The remaining chapters present new models and results. Chapters 3 and 4 investigate the impact of tax evasion on firm behavior from a microeconomic perspective. Notably, I discuss the relationship between the tax evasion and the production decision in a monopolistic and duopoly market and characterize optimal tax and enforcement policies. In particular, I discuss the implications of profit tax evasion by a monopolist for his production decision (chapter 3). I show that there are conditions under which even a benevolent government should not deter evasion activities completely even if deterrence does not entail any resource costs in a duopoly model with sales tax evasion (chapter 4). Chapters 5 and 6 deal with macroeconomic issues of tax eva-

[1] The Economist reports some frequently used ways of tax evasion and tax avoidance in The Economist (2001).

sion with a special emphasis on the relationship between tax evasion and economic growth. I show that under some conditions tax evasion has no consequences for long-run economic growth if the government adjusts its tax policy appropriately (chapter 5). Nevertheless, tax evasion is an important factor that determines economic growth in the real world as an empirical investigation shows that I provide (chapter 6).

Structure

In detail, the structure of this dissertation is as follows:

Chapter 1 introduces and discusses some main issues in the economic theory of tax evasion.

Chapter 2 selectively summarizes existing results on positive and normative issues in the economic theory of tax evasion. It can be viewed as a short – and by far not complete – survey of the literature on tax evasion that is related to the issues of the following chapters. Therefore, it also serves as a reference point for the models that are developed and used later in the text. Contrary to other surveys like Cowell (1985a), Pyle (1991), Andreoni *et al.* (1998), Franzoni (2000), Slemrod & Yitzhaki (2002), it includes an overview on the literature of tax evasion of firms. In particular, it emphasizes the importance of the evasion or concealment technology and possible government policies. It also includes an overview on macroeconomic literature in the field. Additionally, it hints at the general direction that the literature on tax evasion takes. The chapter allows to compare and criticize some of the results presented later in the text.

Chapters 3 and 4 discuss models of imperfect competition that allow for tax evasion on the side of firms. Both chapters analyze the interdependency of the production and the tax evasion decision and characterize optimal policy in the case that the production decision may be influenced by the possibility to evade taxes.

In particular, chapter 3 discusses the relationship between the decision to evade a profit tax and the production decision in a monopolistic market. It provides a simple condition that generates a one-sided separability: the evasion decision is separable from the production decision but not vice versa if the firm chooses to overreport production costs and the concealment cost function is strictly convex. This condition also implies that if the monopolist chooses to evade taxes, it has an incentive to increase production beyond the monopoly output with full enforcement of taxes. The normative analysis shows how the government designs its tax and tax enforcement policy in the light of tax evasion. The optimality condition implies that both instruments should be used to the extent that their marginal excess burden per marginal revenue unit is equal.

Chapter 4 puts forward a similar tax evasion model as chapter 3 but it covers sales (and unit) tax evasion in a duopoly market. Additionally, compared to chapter 3, this model allows for firm heterogeneity. Again, a one-sided separability of the evasion and the production decision holds. In equilibrium, prices and market shares are (in part) determined by the technology that each firm uses to produce and to conceal its evasion activities. Stricter enforcement leads to higher tax costs and is shifted into higher prices as are increases in production costs. If firms differ in the concealment or the production technology, policy may also influence market shares which are relevant for social welfare. The chapter characterizes the socially optimal tax and tax enforcement policy. A central result is that it may be optimal not to enforce taxes completely even if enforcement does not entail any resource costs. The chapter also characterizes tax and enforcement policies that are determined by a majority vote. In this respect, it provides a positive theory of actual tax enforcement in a democracy. Earlier versions of this chapter have been presented at the 2005 Public Economic Theory (PET) conference in Marseille, France, and faculty seminars at the Universities of Baton Rouge, USA, Pisa and Brescia, Italy, and Saarbrücken, Germany. The chapter has been accepted for upcoming presentation at the 2006 Annual Congress of the Swiss Society for Economics and Statistics (SGVS) in Lugano, Switzerland, and the 2006 Annual Conference of the Scottish Economic Society.

Chapters 5 and 6 discuss implications of tax evasion from a macroeconomic perspective.

Chapter 5 studies the macroeconomic impact of tax evasion with a special emphasis on the rate of growth. The impact of tax evasion on economic growth is examined in an endogenous growth model which emphasizes the role of the government that takes tax evasion into account when designing the tax structure. It is shown that even if public goods are nonproductive, tax evasion does not lead to higher growth rates (as claimed in the previous literature) if the government adjusts its tax rate upwards to ensure the efficient provision of public goods. Earlier versions of this chapter have been presented at a PhD-student conference at the Royal Institute of Technology (KTH) in Stockholm, Sweden, in 2004, the 2005 Annual Meeting of the Public Choice Society (PCS) in New Orleans, USA, the 2005 Annual Congress of the Swiss Society for Economics and Statistics (SGVS) in Zurich, Switzerland, and the 2005 Annual Conference of the Royal Economic Society (RES) in Nottingham, UK.

Chapter 6 is devoted to the empirical relationship between tax evasion and economic growth. In order to quantify the impact of tax evasion on growth, a tax evasion measure based on estimates of the shadow economy is developed and discussed. It is used to investigate the impact of tax evasion on economic growth empirically in a cross-section data set of about 70 countries. It is found that tax evasion and growth are negatively related. This is true irrespectively of the development stage of a country. The relationship is economically important: if a country could improve tax compliance by one standard deviation, it may improve growth by about 0.8 percentage points.

Reading

All chapters are self-contained and may be read independently from another. However, as they are all concerned with issues in tax evasion the dissertation is presented in the form of a monograph with a general introduction (chapter 1) and a separate overview on related literature (chapter 2). Technical details are relegated to appendices if reading would otherwise be disturbed.

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Chapter 1

Introduction

Tax evasion (and tax avoidance)[1] is a phenomenon present in all societies that use taxes to finance government expenditures. It can take on different forms: non-declaration or underreporting of the tax base (which may be income, sales, wealth etc.), overreporting of deductible expenses, moonlighting, smuggling and others.[2] Sometimes, every possible effort is made (and a substantial criminal resolve is being shown) to reduce the tax burden, for example in the evasion of value-added taxes (see U.K. Government. HM Customs & Excise (2004) and Caplan *et al.* (2003) for a description of value-added tax (VAT) evasion with a particular emphasis on the VAT missing trader fraud and its carousel variant). On the other hand, the tax administration tries to identify possible evaders and a considerable amount of resources is used during this detection process.[3]

[1]The distinction between tax evasion, which is illegal, and tax avoidance, which is not, may not always be clear in practice. In particular, it is determined by the interpretation of the tax law by the courts. In this respect, one may argue that the distinction is ambiguous only ex ante that is before a court decision established the legal status. According to German tax law, there is even a threefold distinction: additionally to tax evasion (Steuerhinterziehung) and tax avoidance (Steuervermeidung) §42 AO defines activities that in general are not illegal but against the spirit of the law: "Durch Missbrauch von Gestaltungsmöglichkeiten des Rechts kann das Steuergesetz nicht umgangen werden." (Steuerumgehung). Steuerumgehung is only illegal if the taxpayer did not comply with his reporting duties (unehrliche Steuerumgehung). As this is a characteristic of the German tax code – similar provisions exist in Israel, Sweden and Finland (see Tipke & Kruse (2004, §42 AO Tz. 23))– and its implications have not been discussed in the literature, the twofold distinction is used here throughout. Nevertheless, it is acknowledged that this is a simplification. The important conceptual difference between tax evasion and tax avoidance is considered to be the consequence that the first introduces uncertainty of future consumption prospects whereas the latter does not (Cowell (1985a)). Both share the similarity to be motivated by tax rate differentials and the desire to minimize tax payments.

[2]Tax avoidance typically involves the restructuring of a transaction to minimize its associated tax liability. The Economist mentions several more peculiar ways by which taxes are avoided. For example, people change their citizenship (The Economist (2000, 2001)) or get a divorce (The Economist (1997b)).

[3]For estimates of administrative costs of taxation for a number of countries see Sandford *et al.* (1989).

To give an impression of its magnitude – empirical methods and results are discussed in chapter 6 in more detail: the average size of the shadow economy – where no taxes are paid by definition – in OECD countries is 17% of GDP with the highest relative extent in Greece (28%) and the lowest in the U.S. and Switzerland (9% each). In transition and developing countries it amounts to 38% respectively 41% of GDP during the period 1999 – 2000 (Schneider (2005)).[4] In particular, the Internal Revenue Service (IRS) estimates that about 17% of U.S. income tax liability is not paid (U.S. Department of the Treasury. Internal Revenue Service (1996)).[5] For 1998 the tax gap has been estimated at $280 billion (U.S. Department of the Treasury. Internal Revenue Service (2002)). As far as the VAT is concerned, for example, Nam *et al.* (2003) estimate that about 10% of it was not paid in Germany in 2001. These numbers already illustrate why tax evasion and avoidance is not only a topic in developing countries but also in high developed ones.[6]

Although tax evasion is illegal it arouses the interest of public finance economists more than other criminal activities because of its practical relevance and its implications for the (optimal) tax system. It raises a number of positive and normative questions which are dealt with in this dissertation:[7]

- Which factors determine the extent of tax evasion?
- How does and how should the tax and enforcement structure reflect the presence of tax evasion and how does tax evasion affect the optimal provision of public goods?
- What are the implications of tax evasion for the economy as a whole?

The determining factors of tax evasion are varied and emphasis can be placed on economic as well as sociological influences. In this economic discussion the latter

[4] Note that the size of the shadow economy gives only a first hint on the amount of tax evasion. For example, the evasion of capital income taxes is not captured. Compared to other indices it is a relatively broad indicator that includes the base of taxes (and social security contributions) on the side of the firm and of individuals.

[5] More recent estimates of a Taxpayer Compliance Measurement Program are not available except for compliance for the Earned Income Tax Credit; see U.S. Department of the Treasury. Internal Revenue Service (2000b, 2002).

[6] Bird (1983) and Mansfield (1988) emphasize the importance of evasion activities especially in countries of the former group. Issues of administration are of particular importance there. See Busse (2000) for a description of tax evasion in Russia.

[7] According to Aigner *et al.* (1988, p.297), tax evasion has three consequences: it leads to a loss in tax revenue, macroeconomic indicators like GDP or the rate of unemployment are unreliable, the distribution of after-tax income cannot be inferred from the distribution of pre-tax income and the tax schedule alone (and vice versa). Additionally, tax evasion may lead to efficiency losses.

only play a minor role.[8]

Politicians (governments) are highly concerned with the effects of tax evasion and underground economic activity in particular on tax receipts, fearing an erosion of tax revenue. However, the impact of tax evasion on tax receipts is not clear because of at least two reasons. First, the tax base itself may be sensitive to the effective tax rate it is subjected to – including the possibility of tax evasion. The so called *tax gap*, defined as the difference between the taxes actually paid and the taxes due according to law, gives only a naive expression of the impact of tax evasion on tax revenue (an upper bound if one emphasizes the discouraging effect of taxes: at least part of the taxable activities would not have been undertaken under a stricter system of tax enforcement).

Second, although the tax base may evade taxation on one occasion it is possible that it is taxed in a different form. Taxes on capital income can be evaded easily in many countries. If the additional net income is spent at least partly, value-added tax revenues increase.

It is important for policy makers – also apart from concerns on tax revenue – to consider that taxes are evaded and to interpret information and act accordingly. The problem of distorted official statistics is most eminent if government policy is based solely on these official figures, disregarding estimates of tax evasion or the underground economy.[9]

Normative questions are of very high relevance in the context of tax evasion and the underground economy. On the one hand, tax evasion and the shadow economy might lead to the loss of public funds and a lower provision of public goods. On the other hand, the underground economy is seen as an efficient entrepreneurial sector that circumvents inefficiencies brought on by taxes and regulations.

There is also an issue of redistribution associated with tax evasion. Tax evasion redistributes income from the honest to the dishonest. As such, studying tax evasion is concerned with horizontal equity: people with similar incomes pay different taxes because not everybody pays the amount due according to law and not every evader is detected. Possibilities to evade may also vary across occupations (and across different income structures even if total income is identical). Vertically, tax evasion possibilities may vary across income brackets. Again, it is important for policy makers to take into account that tax evasion may alter the redistributive pattern of the statutory tax system.[10]

[8] The sociological and psychological literature on tax evasion is enormous and economic and psychological advances are compared and combined in many recent publications, see e.g. Elffers *et al.* (1987), Porcano (1988), Falkinger (1995), Kim (2002), with a special focus on fairness.

[9] Feige (1989) goes even so far to argue that stagflation in the UK and elsewhere in the 1970s was only a phenomenon of *official* statistics.

[10] Persson & Wissén (1984) formulate an economic model for this issue and Bishop *et al.* (1997)

The established tax system can (and does)[11] reflect the fact that taxes are evaded. If policy makers design the tax system with regard to the possibility of tax evasion, statutory tax rates give only a blurred picture of the effective tax structure (and *intended* tax revenue).

A question of central focus in the economic theory of tax evasion is how optimal government policy can be characterized if tax evasion occurs. The general task is how to strike the right balance between a strict enforcement system that hinders economic activity or where some convictions may be incorrect and a lenient system where public funds are scarce and tax morale deteriorates.

Tax evasion led economists to think more generally about optimal tax systems and gave rise to a number of new instruments that the government may use to control evasion and avoidance activities. Answers to the question of how an optimal tax system looks are still an active field of research. So far, the answers are quite sensitive to the model's assumptions, in particular to the restrictions that are imposed exogenously on available instruments. This issue also arises in this dissertation.

find empirical support for the redistributive effects of tax evasion for the U.S.

[11] For example, Wallschutzky (1991) provides a detailed description of the changes in the Australian tax system in the 1980s, which "have been designed, at least in part, to reduce evasion of income tax"(Wallschutzky (1991, p.166)). He also quotes that compliance rates for capital gains taxes in the U.S. increased by 27% from 1981 to 1987 due to the introduction of information reporting (Wallschutzky (1991, p.166)). See also Kesselman (1993) for a related discussion concerning the question of the optimal tax mix in Australia.

Chapter 2

A Short Overview on Related Literature

This chapter provides a selective overview on positive and normative issues in the economic theory of tax evasion. It is by far not complete.[1] Rather, in addition to introducing the basic model of individual income tax evasion it presents and discusses results that are closer related to the following chapters than by the general topic of tax evasion.[2] In particular, it provides an overview on results if tax evasion by firms instead of private individuals is considered. Although a number of reviews of the tax evasion and avoidance literature exist (Cowell (1985a, 2004), Pyle (1991), Andreoni *et al.* (1998), Franzoni (2000), Slemrod & Yitzhaki (2002)) they all more or less neglect this strand of literature. Different implications (concerning comparative statics or policy recommendations) may arise and may be traced back to differences in the underlying evasion technology. Furthermore, results of some macroeconomic papers on tax evasion are stated and discussed. These are especially relevant for the models concerned with tax evasion and economic growth. On the whole, the chapter allows to compare and criticize some of the results presented later in the text.

2.1 The Basic Model

The basic model of tax evasion by Allingham & Sandmo (1972) assumes that an individual is endowed with an amount of income, $\bar{y}$, which is known to him but

[1] Some streams of economic literature are completely neglected. To read about amnesties the reader is referred to Lerman (1986), Uchitelle (1989), Andreoni (1991a), Malik & Schwab (1991), Stella (1991), Alm & Beck (1993), Hasseldine (1998), Christian *et al.* (2002). Apart from the adverse long-run effects on compliance which counterbalance increased revenues in the short-run, amnesties may be seen as an insurance instrument.

[2] If results are stated in a different form than in the original paper, the calculations are provided in the Appendix.

not to the tax collection agency and subjected to a tax.[3] By disregarding the labor supply decision for the moment it is possible to focus on the decision of how much of a given income to report to the tax authorities and how much to evade. Theoretically, there is no conceptual difference between a household's problem of how much personal (labor, capital etc.) income, wealth or any other given tax base to declare. All tax bases are subsumed as income (for the analysis of the evasion and avoidance of particular taxes see, for example, Eller *et al.* (2001) (estate tax) or Feinstein & Ho (2001) (gift tax)). Practically, different parameter values may apply. For example, the chances of being detected underreporting labor income in the U.S. or Germany (and many other countries) exceed the ones evading capital income due to information reporting from the side of the employer. Additionally, there is no conceptual difference between a firm's decision of how much sales or profits to report if the tax base already exists. However, it is less common to consider only the evasion decision leaving the output decision aside within a theory of the firm because the output decision is a central focus of firm market behavior.

Then, the following definition is appropriate.[4]

Definition 2.1 (Tax evasion). *Tax evasion* is the deliberate failure to disclose all or part of one's income to the tax authority. ▲

The decision of how much income to declare is a decision that involves uncertainty if detection is assumed to be imperfect. Its description constitutes an application of the theory of crime by Becker (1968) in this respect. A penalty is to be paid if and only if the taxpayer has evaded taxes and is discovered by the tax authority.[5] To be more precise, assume that there are only two states of the world: either the tax authority investigates the taxpayer's return, then it detects all evaded income and the individual is fined (call it state 1), or it does not and the tax cheater gets away with it (state 2). Clearly, this is a simplification. As far as the decision on actual audits are concerned it is also unlikely that the enforcement agency audits at random. In the U.S., for example, the Internal Revenue Service (IRS) employs a range of methods to detect evaders. In particular, it uses the results of its program

[3]Note that the concept of *fiscal income* defined by legislative tax statutes is relevant here rather than the broader concept of *economic income*; for a further discussion of this difference see Feige (1989).

[4]The U.S. Department of the Treasury. Internal Revenue Service (2000a, p.3) distinguishes between payment compliance, filing compliance and reporting compliance. The present definition only covers the last aspect. Payment and filing compliance have been addressed by Andreoni (1992) respectively Lee (2001) among others.

[5]Usually, this fine depends on the amount of evaded income or the amount of evaded taxes (see, for example, the German personal income tax code: §238(1) AO); details are provided below. The problem of the administration to have sufficient evidence such that detected evasion also leads to a court ruling is neglected. Andreoni (1991b) formalizes a model where a juror decides on conviction; then the fine and the probability of conviction are not independent.

of intensive audits: the Taxpayer Compliance Measurement Program (TCMP). On the basis of its results it assigns to each tax report a likelihood that it is incorrect. Andreoni *et al.* (1998) state that over 50% of audits are based on this score. According to their calculations, the yield of a random audit is \$289 compared to \$5.500 for non-random ones.[6]

A tradeoff arises if an individual can reduce the obligation to pay taxes by underreporting income but risks to be detected and fined. The solution to this tradeoff for given enforcement parameters depends on the characteristics of the individual's preferences.

Assumption 2.1. *Assume that preferences are represented by a twice differentiable utility function defined on net income $u : \mathbb{R} \to \mathbb{R}$, $y \mapsto u(y)$ with $u''(y) \leqslant 0 < u'(y)$. Assume, furthermore, that the von Neumann-Morgenstern axioms of expected utility hold, i.e. individuals are expected utility maximizers.* ▼

If the case of a linear tax schedule $T(y_d) = \tau y_d$, where y_d denotes declared income, τ, $0 \leqslant \tau \leqslant 1$, a constant tax rate, and a fine schedule that is linear in evaded income $F(y_e) = \zeta y_e$, where y_e denotes unreported income (and $\bar{y} = y_d + y_e$), ζ, $0 < \zeta$, a constant fine rate, net income in the two states is:[7]

$$y^1 = \bar{y} - \tau y_d - \zeta y_e, \tag{2.1}$$
$$y^2 = \bar{y} - \tau y_d. \tag{2.2}$$

With net income in case of full tax payment of $(1-\tau)\bar{y}$ the individual can raise his income in state of the world 2 by underreporting his income by an amount y_e, which leads to an increase of y^2 by τy_e (the tax savings); in case state 1 occurs the individual's evasion is detected and he is punished with the fine payment of ζy_e leaving him with a change of income of $(\tau - \zeta)y_e$. The slope of the budget line that denotes the ratio at which one unit of income in case of detection can be substituted for income in case of nondetection is therefore $\frac{\tau}{\tau-\zeta} < 0$. It is assumed

[6] Nevertheless, the model uses random audits for simplicity. This simplification makes it possible to illustrate the model in a two states of the world-diagram and build some intuition later. See Slemrod & Yitzhaki (2002, section 4) or Yitzhaki (1987) for slight extensions of this assumption.

[7] Note that in this formulation repayment of evaded taxes in case of detection is included in the fine function. An alternative formulation is to define $y^1 = (1-\tau)\bar{y} - \hat{\zeta} y_e$, for $0 < \hat{\zeta}$. This does not alter the problem as the fine rates are related by $\zeta = 1 + \hat{\zeta}$. The question of the appropriate base for the fine rate is relegated to footnote 14. The assumption of a constant fine rate is a simplification. Typically, civil or criminal penalties may apply. Andreoni *et al.* (1998) state that in the U.S. "...civil penalties are applied at a rate of 20 percent of the portion of the underpayment of tax resulting from a specified misconduct [...]. However, in cases of fraud [...] a civil penalty may be applied at a rate of 75 percent." Serious evasion cases may even be punished with criminal penalties (fines of not more than \$100.000 or imprisonment of not more than 5 years). Similar provisions exist for Germany.

that the tax rate is strictly below the fine rate, $\tau < \zeta$, which is satisfied if $1 < \zeta$ for any tax rate $0 \leqslant \tau \leqslant 1$ because otherwise the tradeoff vanishes as tax evasion is always profitable even in case of detection (regardless of the audit probability). The maximum amount of y^2 that can be attained for $0 \leqslant y_e$ is $\bar{y}$ where no income is declared. Then the individual risks to be left with $(1-\zeta)\bar{y} < 0$ in case of an audit. Although the budget line allows the individual to report no income at all, the model does not specify how the possibly resulting large fine could be paid. Formally, the utility function is not defined on negative values of income. It is reasonable to consider only parameter constellations where the individual evades income only to such an extent that he can pay the fine in case of audit or to assume $\lim_{y \searrow 0} u'(y) = +\infty$. In the latter case the individual does not find it optimal to evade income to an extent that he may not be able to pay the fine in case of detection. Alternatively, an additional source of income that is not evadable may be assumed. If the individual may even claim deductible credits, he could even increase his income beyond the pre-tax income (if he is not audited). This case is neglected in the following.

In order to recognize the similarity of the tax evasion problem and the problem of optimal portfolio choice, reformulate the respective incomes as first Christiansen (1980) did as

$$y^1 = (1-\tau)\bar{y} + (\tau - \zeta)y_e, \tag{2.3}$$
$$y^2 = (1-\tau)\bar{y} + \tau y_e. \tag{2.4}$$

Now, starting with certain income net of taxes $(1-\tau)\bar{y}$ in case no taxes are evaded, underreporting income is equivalent to investment in a security (or gambling) with uncertain return of either $\tau - \zeta$ or τ per unit of evaded income. This equivalence allows to apply a result from the theory of *investment under uncertainty* (Arrow (1970)).

Remark 2.1. *An individual engages in tax evasion if and only if its expected return is positive: $\rho(\tau-\zeta) + (1-\rho)\tau > 0$, i.e. the tax rate must be larger than the expected fine $\rho\zeta < \tau$.* △

The algebraic problem of expected utility maximization can be formulated as[8]

$$\max_{0 \leqslant y_e \leqslant \bar{y}} \Phi(y_e) := \mathbb{E}[u(\tilde{y})] = \rho u(y^1) + (1-\rho)u(y^2). \tag{2.5}$$

[8] A ˜ means that the associated variable is random and $\mathbb{E}$ denotes the expectation operator.

The necessary condition for an interior maximum of problem (2.5) is

$$\begin{aligned}\Phi_{y_e}(y_e) := \frac{\mathrm{d}\Phi}{\mathrm{d}y_e}(y_e) &= \rho u'(y^1)(\tau-\zeta) + (1-\rho)u'(y^2)\tau = 0 \\ \Leftrightarrow -\frac{\rho u'(y^1)}{(1-\rho)u'(y^2)} &= \frac{\tau}{\tau-\zeta} \end{aligned} \tag{2.6}$$

which is also sufficient if the individual is risk-averse ($u'' < 0$) because the second derivative

$$\Phi_{y_e y_e}(y_e) := \frac{\mathrm{d}^2\Phi}{\mathrm{d}y_e^2}(y_e) = \rho u''(y^1)(\tau-\zeta)^2 + (1-\rho)u''(y^2)\tau^2 \tag{2.7}$$

is strictly negative in this case. In an interior optimum the marginal rate of substitution of income in state 1 for income in state 2 (left-hand side of equation (2.6)) and the respective relative price (right-hand side) coincide.[9]

The first-order condition (2.6) implicitly defines the optimum amount of evaded income as a continuous function of the parameters $\bar{y}$, ρ, τ and ζ, $y_e = y_e(\bar{y}, \rho, \tau, \zeta)$,[10] with the following comparative statics results.

Proposition 2.1 (Allingham & Sandmo (1972)). *The signs of the comparative statics of the optimal amount of evaded income with respect to changes in the parameters are as follows:*

$$\frac{\partial y_e}{\partial \rho} < 0, \ \frac{\partial y_e}{\partial \zeta} < 0, \tag{2.8}$$

$$\frac{\partial y_e}{\partial \bar{y}} \begin{cases} > 0, & \textit{for} \quad r_A(u, y^1) > r_A(u, y^2), \\ = 0, & \textit{for} \quad r_A(u, y^1) = r_A(u, y^2), \\ < 0, & \textit{for} \quad r_A(u, y^1) < r_A(u, y^2), \end{cases} \tag{2.9}$$

$$\frac{\partial y_e}{\partial \tau} \gtreqless 0, \tag{2.10}$$

where $r_A(u, y)$ denotes the coefficient of absolute risk aversion for utility function u at income level y as defined and discussed by Pratt (1964) and Arrow (1970). □

CALCULATIONS: See the Appendix. ■

[9] To ensure an interior solution $0 < y_e < \bar{y}$ the parameters must be such that $\Phi_{y_e}(\bar{y}) < 0 < \Phi_{y_e}(0)$ holds. In particular, evaluating the first derivative of Φ at the left boundary $y_e = 0$ proves the sufficiency of Arrow's condition (Remark 2.1). Altogether, $\tau\left(\rho + (1-\rho)\frac{u'(\bar{y})}{u'((1-\zeta)\bar{y})}\right) < \rho\zeta < \tau$ is sufficient.

[10] This is assured by Berge's *theorem of the maximum* as an extremum of a continuous function on a bounded and closed, i.e. compact, set is obtained. Differentiability is also implied by Assumption 2.1 (u twice differentiable). Only interior solutions are discussed in the following.

The amount of evaded income falls in the audit rate because the marginal rate of substitution increases (in absolute value), i.e. the willingness to pay for an additional unit of income in state 1 in terms of income in state 2 rises, therefore the individual substitutes income in state 2 for income in state 1 and evades a lower amount.

The amount of evaded income also falls in the fine rate. This is the case because an increase in the fine rate increases the relative price of income in state 1 in terms of income in state 2. Therefore, the individual substitutes income in state 2 for income in state 1, i.e. he evades less. The reaction of the individual is said to be determined by a *substitution effect*.

If the amount of (pre tax) income changes, the change in the marginal rate of substitution is determined by the behavior of the individual's utility function – in particular on his risk preference. If the individual exhibits decreasing absolute risk aversion (DARA), he is more willing to take a given risk if he is richer and, therefore, chooses to evade a larger amount of income. The reaction of the individual is said to be determined by an *income effect*.[11,12]

A change in the statutory tax rate changes the relative price and the individual's income. It therefore leads to a substitution and an income effect which may lead into opposite directions. On the one hand, an increase in the tax rate leads to a decrease in the relative price of income in state 1 in terms of income in state 2. Therefore, the individual substitutes income in state 2 for income in state 1 and evades more. On the other hand, an increase in the tax rate makes the individual poorer and, therefore, if he has DARA preferences, he will increase his exposure to risk and evade a larger amount of income.[13]

There is an alternative for a linear fine schedule, which can be used to sign the comparative static of the amount of evaded income with respect to the tax rate unambiguously for DARA utility functions:[14] a fine proportional to the amount of

[11] How the share of income that is not reported reacts to changes in income is determined by the coefficient of relative risk aversion. It is increasing (constant, decreasing) if the coefficient of relative risk aversion is decreasing (constant, increasing).

[12] Allingham & Sandmo (1972) stated the ambiguity of $\frac{\partial y_d}{\partial \bar{y}}$. This does not contradict the present formulation because $\frac{\partial y_e}{\partial \bar{y}} = 1 - \frac{\partial y_d}{\partial \bar{y}}$. See the Appendix for the precise calculations.

[13] Note that for CARA or IARA utility functions $\frac{\partial y_e}{\partial \tau} > 0$ unambiguously as the income effect is absent or moves in the same direction as the substitution effect. Balassone & Jones (1998) argue that the empirical literature failed to establish a positive relationship between tax rates and the amount of evaded income and that IARA utility is not an assumption as strong as it is considered to be because it merely says that demand for tax evasion is an inferior good.

[14] This alternative is also considered to be more realistic. Yitzhaki (1974) states that fines are proportional to the amount of evaded tax under American and Israeli tax laws. Wallschutzky (1991, p.171) quotes Section 223 of the Australian tax law which suggests this as well (with $\zeta = 2$): "...the taxpayer is liable to pay, by way of penalty, additional tax equal to double the amount of the excess." There is no specification on how the fine is to be determined in German tax law besides the provision that interest is to be paid on the evaded taxes (§238(1) AO) and

evaded tax. It was first introduced by Yitzhaki (1974).

Assume that the fine is linear in the amount of evaded tax, that is to say that if an individual underreports true income by an amount of y_e and is detected he has to pay a fine of $\zeta\tau y_e$.

In this case, the income in state of the world 1 changes to

$$y^1 = \bar{y} - \tau y_d - \zeta\tau y_e = (1-\tau)\bar{y} + \tau(1-\zeta)y_e \tag{2.11}$$

and the necessary condition for an interior maximum of the tax evasion problem is given by

$$\begin{aligned} \Phi_{y_e}(y_e) &= \rho u'(y^1)\tau(1-\zeta) + (1-\rho)u'(y^2)\tau = 0 \\ \Leftrightarrow -\frac{\rho u'(y^1)}{(1-\rho)u'(y^2)} &= \frac{1}{1-\zeta}, \end{aligned} \tag{2.12}$$

again defining implicitly the optimal amount of evaded income y_e.[15] The comparative statics of y_e with respect to $\bar{y}$, ρ, and ζ are signed as in Allingham & Sandmo (1972). Yet, now it is possible to sign unambiguously the comparative statics of y_e with respect to changes in the tax rate for DARA utility functions.

Proposition 2.2 (Yitzhaki (1974)). *If the fine to be paid in case of detection is proportional to the amount of evaded tax, then*

$$\frac{\partial y_e}{\partial \tau} < 0 \tag{2.13}$$

for DARA utility functions. □

Calculations: See the Appendix. ■

The intuitive reason for this clear-cut result is the absence of the substitution effect: a change in the tax rate does not change the relative price of income across the two states of the world. The tax rate change encourages evasion by the same rate as it deters it (the tax savings from evasion as well as the penalty increase proportionally with an increase of the tax rate). The income effect for DARA utility functions is negative, again.[16]

that the fine should reflect the economic situation of the felon (§40 StGB) and the extent of evasion. If *Steuerumgehung* is detected, it is taxed as if the activity in question were done in the appropriate way (§42(2) AO).

[15] Analogously to the Allingham/Sandmo-case, evaluating the first-order condition at $y_e = 0$ and $y_e = \bar{y}$ respectively, $\rho\zeta < 1$ and $\frac{u'(\bar{y})}{u'((1-\tau\zeta)\bar{y})} < \frac{\rho(1-\zeta)}{1-\rho}$ have to hold to obtain an interior solution.

[16] The comparative static of the amount of evaded income with respect to changes in the tax rate

These comparative statics results can be illustrated diagrammatically and it is useful to do so for some of them to build some further intuition. The diagrammatic exposition uses a two states of the world diagram that is a diagram, where income in case of detection is shown on the horizontal axis and income in case of no audit is depicted on the vertical axis (see Figure 2.1).

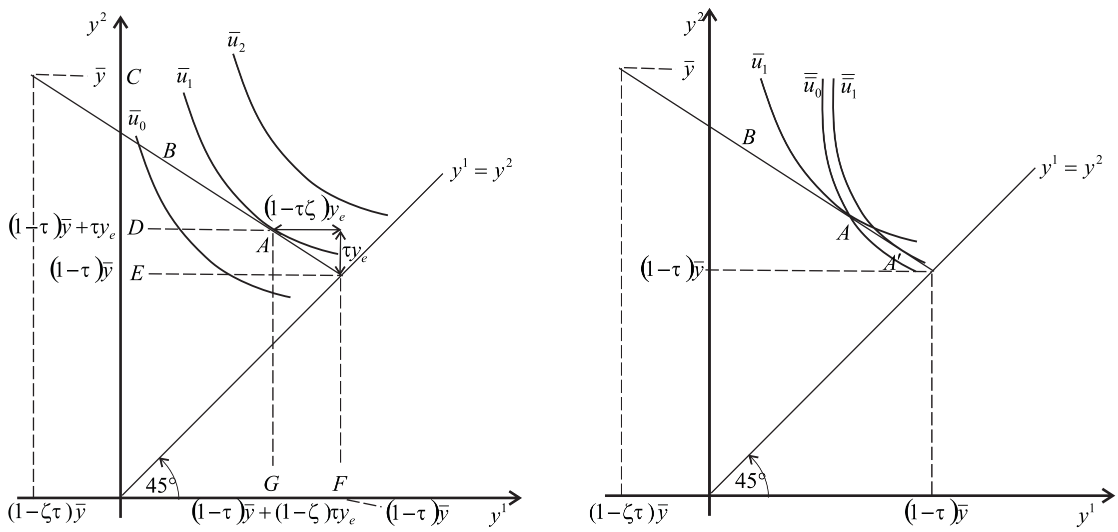
First of all, note that the slope of the budget line (labelled as B) is $\frac{1}{1-\zeta}$ independent of τ in the Yitzhaki-case. Indifference curves for risk-averse individuals are convex to the origin. They are indexed in increasing order of utility $\bar{u}_0 < \bar{u}_1 < \dots$ (respectively $\bar{\bar{u}}_0 < \bar{\bar{u}}_1 < \dots$). At the certainty line (the diagonal where $y^1 = y^2$) the marginal rate of substitution of income between the two states is $\frac{\rho}{1-\rho}$ (in absolute value). An interior optimum is characterized by the tangency of the highest achievable indifference curve for a given budget line. Such an optimum is depicted by point A in Figure 2.1(a).

The tax payment of the individual with optimum at point A can be read from the diagram as the distance from point C to point D. Consequently, it evades taxes of amount of distance from D to E. If the tax evader is audited (and evaded income is detected and the evader is fined), the fine in excess of the regular tax can be read from the distance of point F to G.

Consider now the illustration of the impact of a change in the probability of audit on the amount of evaded income. With an increase in the probability of detection the indifference curves become steeper.[17] The new indifference curve that passes through point A is labelled $\bar{\bar{u}}_0$. Optimally, this leads to an increase in the amount of declared income (in the figure the new solution is depicted as point A' with the associated utility level $\bar{\bar{u}}_1$). Behavior changes in the same direction for an increase in the fine rate. Graphically, the budget line instead of the indifference curves

has been at the focus of these early contributions and stimulated subsequent research. One reason might have been that experimental studies, for example, Baldry (1987), Friedland *et al.* (1987) and econometric studies, for example, Clotfelter (1983), Crane & Nourzad (1990), Joulfaian & Rider (1996, 1998) have found positive correlations (Dubin & Wilde (1988), Alm *et al.* (1990) find negative Feinstein (1991) mixed correlations). For a theoretical model that implies this positive relationship (at least under some circumstances), see, for example, Gordon (1989), Yaniv (1999), Al-Nowaihi & Pyle (2000), Lee (2001) or Bernasconi & Zanardi (2004) (depending on the reference income). However, none of their modifications (adding psychic or stigma costs or using prospect theory) is necessary to obtain the presumably intuitive relationship between the nominal tax rate and the amount of evaded income. According to Yaniv (1994), it is sufficient to assume that the highest fine is the expropriation of all income and that relative risk aversion is constant and bounded from above by the inverse of the penalty rate.

[17] Note that the indifference curve for utility level $\bar{u}$ is algebraically given by the equation $\bar{u} = \rho u(y^1) + (1-\rho)u(y^2)$ with the slope, i.e. marginal rate of substitution, of $\mathrm{mrs}(\bar{u}) := \left.\frac{\mathrm{d}y^2}{\mathrm{d}y^1}\right|_{\mathrm{d}\bar{u}=0} = -\frac{\rho u'(y^1)}{(1-\rho)u'(y^2)} < 0$. The derivative of the marginal rate of substitution is strictly negative, $\frac{\partial\,\mathrm{mrs}}{\partial\rho} = -\frac{u'(y^1)}{(1-\rho)^2 u'(y^2)} < 0$, at any given point (y^1, y^2). Thus, the slope of the indifference curve at any given point strictly decreases (increases in absolute value) in the audit probability.



(a) Optimal amount of evaded income. (b) Comparative statics of y_e w.r.t. ρ.

Figure 2.1: Individual income tax evasion.

becomes steeper (see Figure 2.2). A change in the tax rate does not turn the budget line, but only shifts it in a parallel manner (no substitution effect).[18]

The comparative statics suggest first policy implications. For example, if tax evasion is considered as socially harmful a priori and should be eliminated, it can be reduced by either increasing the audit probability or increasing the fine rate. The tax enforcement parameters are substitutes in the sense that both lead to higher compliance. They are also substitutes in the sense that both raise additional net revenue if additional audits are not too costly.[19] Kolm (1973) is the first to model the comparative efficiency of increases in the probability of audit which uses resources and in the fine rate which does not. The limit case suggests the use of maximal penalties with a negligible audit probability to minimize enforcement costs.[20]

Proposition 2.3 (Kolm (1973)). *If the cost of detecting evasion uses resources (audit costs) while subjecting an individual to a higher fine does not, then the cost minimizing policy to raise a given revenue is to use low audit and high fine rates.*[21,22] □

[18]Several other graphical illustrations of the comparative static effects, including mean-preserving spreads, are discussed in Cowell (1985a).

[19]Increasing the fine rate is commonly assumed to be costless.

[20]Even if there are no direct resource costs associated with increasing the fine rate there may well be other reasons why penalties should be limited, see, for example, Cowell (1989).

[21]Although Kolm (1973) uses the Allingham & Sandmo (1972) specification of the fine schedule, the result holds true for the Yitzhaki (1974) specification, too.

[22]Note that the result of Kolm (1973) is mathematically imprecise as the function that captures

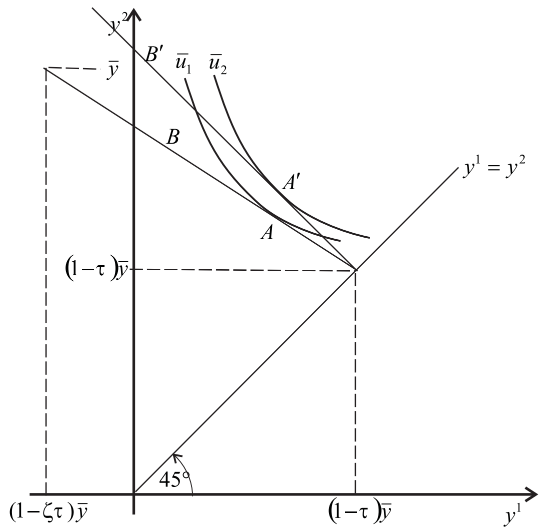


Figure 2.2: Comparative statics of y_e w.r.t. ζ.

CALCULATIONS: The result follows immediately from the comparative statics. Calculations are unnecessary. ■

Some other early papers are also concerned with the optimal design of the enforcement system if the optimal policy is to deter all evasion activities. Singh (1973) investigates the probability of detection that induces taxpayers to declare income fully. Fishburn (1979) investigates analogously the *prohibitive penalty*.

Similarly, compliance could be increased by increasing the extent of complexity or ambiguity of the tax code (Scotchmer (1989), Scotchmer & Slemrod (1989)). However, it is counterintuitive that such a more ambiguous tax code is desirable from a social point of view.

These policy conclusions already highlight a limitation of the basic model. It is only a descriptive model and it does not answer, for example, the question to which extent taxes should be enforced from the perspective of social welfare. In particular, if taxes cannot be enforced completely (because the costs of full enforcement are prohibitively high), such a model must specify how to balance the ex post horizontal inequity arising because only those evaders get punished that are caught and convicted – especially if many evaders escape detection.[23] Some normative issues are discussed below in greater detail.

the total resource costs does not have a minimum. His result depicts an infimum.

[23] Actually, the Bundesverfassungsgericht even declared a tax on realized profits from share holdings void because evasion was so wide spread; see BVerfG, 2 BvL 17/02 as at 9.3.2004 – Bundesverfassungsgericht (2004).

Additionally, there are other limitations.[24] For example, only monetary punishments are considered. In reality, a court might also – alternatively or additionally – sentence a tax evader to prison. The model also lacks a form of intrinsic motivation to be honest or social interactions as, for example, the individual's reputation might be damaged if he is known to be a tax cheater.[25] It is debatable whether these other forms of punishment are equivalent to monetary penalties.[26]

Allingham & Sandmo (1972) assume that the taxpayers know precisely the probability of being caught and the penalties that they will receive so that they can make the cost-benefit calculation. In reality, this information is kept confidential by the tax administration.[27]

2.2 Tax Evasion with an Endogenous Tax Base

If the labor supply decision is endogenous, there are two ways to evade taxes: the first one is to underreport income obtained through legal work, the other is to divide total labor time between work in the official and work in the informal sector, where no taxes are paid in the latter by definition.[28]

[24] The influence of progressive taxation (Srinivasan (1973), Koskela (1983a,b), Witte & Woodbury (1985)), an audit probability increasing in the amount of evasion (Yitzhaki (1987)) and other portfolio decisions (Landskroner *et al.* (1990)) have been included in related extensions of the basic model. A number of papers discusses implications if the basic model is extended to more than a single mode of tax evasion (Klepper & Nagin (1989), Martinez-Vazquez & Rider (2005)). Substitution effects among these modes of evasion arise. Overall, comparative statics become ambiguous even if the fine rate is to be paid on the amount of evaded tax. These extensions imply that increased enforcement may even decrease tax compliance. Empirical studies support the existence of such effects (Feinstein (1991), Joulfaian & Rider (1998), Martinez-Vazquez & Rider (2005)). Cross & Shaw (1982), Alm (1988), Cowell (1990) discuss combined evasion and avoidance models.

[25] Allingham & Sandmo (1972) already make this remark.

[26] Other non-monetary incentives to comply with the tax law have been included in subsequent research (disutility to dishonesty: Skinner & Slemrod (1985); psychic or stigma costs: Benjamini & Maital (1985), Gordon (1989), tax morale: Schmölders (1970), Feld & Frey (2002), Feld & Tyran (2002); guilt and shame: Erard & Feinstein (1994); satisfaction of taxpayers with government: Spicer & Lundstedt (1976); fairness and satisfaction with government: Bordignon (1993), Pommerehne *et al.* (1994); trust in government: Kucher & Götte (1998); political involvement: Torgler *et al.* (2003)). Some of these issues have been found to be a predictor of compliance (Spicer & Becker (1980), Erard & Feinstein (1994), Alm *et al.* (1992)).

[27] This also implies that the government has an additional instrument at hand to influence compliance: the degree of uncertainty about the actual tax payment; see Scotchmer & Slemrod (1989).

[28] If occupations differ by the possibilities to evade taxes, this may influence the choice of occupation, see Pestieau & Possen (1991), Parker (2003) and for an empirical analysis Lemieux *et al.* (1994). Empirical evidence on differences in the effective tax rate across sectors is provided by Alm *et al.* (1991a,b) for Jamaica. It also bears some surprising implications for the impact of

Models that include the labor supply decision into a tax evasion model, however, do not yield clear-cut comparative statics results unless the utility function is restricted to particular forms (Andersen (1977), Baldry (1979), Pencavel (1979), Isachsen & Strom (1980), Cowell (1985b)).[29] The reason is again an income effect. An increase in tax enforcement for a given income leads to a decrease in the relative price of income in state 1 in terms of income in state 2 and the individual substitutes to greater honesty in the formulation of the fine due to Allingham & Sandmo (1972). It may also increase the average tax rate of income and the individual may choose to work additional hours if the labor supply curve is backward bending.[30] The total effect on compliance is therefore ambiguous. Similar explanations apply for other parameter changes.

The analysis of tax evasion by firms differs in the objective function, different applicable taxes and other ways of evading them. Firms may evade (sales, profit, payroll) taxes by underreporting of revenue/sales or overreporting of costs.[31] The relation between the production and the evasion decision is an often addressed issue in models with firm tax evasion.

So far, this literature has discussed implications of tax evasion for efficiency in a market with perfect competition (Virmani (1989)), analyzed oligopolistic (Cournot) competition with risk-averse firms (Marrelli & Martina (1988)) and characterized optimal tax and audit policy in a Ramsey-type model with perfect competition (Cremer & Gahvari (1992, 1993)). A series of articles has discussed the relationship between the compliance and the production decision of a monopolist (Kreutzer & Lee (1986), Wang & Conant (1988), Kreutzer & Lee (1988), Yaniv (1995, 1996), Lee (1998), Panteghini (2000), Goerke & Runkel (2005)) with the particular emphasis of providing sufficient conditions such that a profit tax is still neutral if it may be evaded (and does not affect the production decision). An example for a non-neutrality is provided in chapter 3. Most interestingly, this literature lacks the discussion of normative implications.[32]

The previously discussed result that the amount of evaded income may be negatively related to the tax rate may also change. In order to see this, assume that a

policies; see Das-Gupta (1994).

[29] Cowell (1981, 1985b) assumes a separability of the labor supply function. To derive stronger results the utility function may be structured such that the decision about how much labor to supply overall is separated from that of how to divide the labor between legal and evasion activities. Sandmo (1981) characterizes optimal policy in such an environment.

[30] There is also a theory of labor supply in the presence of a tax avoidance, see Mayshar (1991), Slemrod (1994), Feldstein (1995) or Agell & Persson (2000) for empirical results.

[31] The first model that investigates tax evasion of firms is Marrelli (1984), who discusses evasion of sales taxes by underreporting revenue. Evasion of profit taxes by overreporting of production costs is first discussed by Kreutzer & Lee (1986) and underreporting of wage payments to evade payroll taxes is discussed by Yaniv (1996). VAT fraud is a growing concern.

[32] Cremer & Gahvari (1992, 1993) are notable exceptions.

company is active in an industry that produces a homogenous good under perfect competition. The firm uses a linear production technology and c, $0 \leqslant c$, denotes the constant average and marginal cost parameter. The output q is subjected to a unit tax at rate t, $0 \leqslant t \leqslant 1$. The firm takes the price of the good, p, as given and maximizes its expected profit π^e by deciding on its tax declaration. To evade a fraction e, $0 \leqslant e \leqslant 1$, of its output the firm has to incur resource costs of $g(e)$ per unit of output concealed to shield its evasion activity from cursory examination. It may nevertheless be detected. This is the case if and only if it is subjected to a tax audit which happens with probability ρ, $0 \leqslant \rho \leqslant 1$. Therefore, if the firm is not audited (state of the world 1; this happens with probability $1 - \rho$), it earns profits of $\pi^1 = [p - c - t(1 - e) - g(e)e]q$. If the firm is audited (state of the world 2; happens with probability ρ), it earns profits of $\pi^2 = \pi^1 - t\zeta eq$, where the last term denotes the fine the firm has to pay proportionally at rate ζ to the amount of evaded output (Yitzhaki (1974)). Expected profit, $\pi^e = (1-\rho)\pi^1 + \rho\pi^2$, is therefore given by

$$\pi^e = [p - (1 - \bar{r}e)t - g(e)e - c]q, \tag{2.14}$$

where $\bar{r} = (1 - \rho\zeta)$ denotes the marginal gain of evasion per unit of evaded tax. In the optimum the firm evades taxes to the point where the marginal tax savings equal the marginal concealment costs

$$\bar{r}t = g(e) + g'(e)e. \tag{2.15}$$

The zero profit condition then establishes the market equilibrium

$$p = c + g(e)e + t^e, \tag{2.16}$$

where $t^e := (1 - \bar{r}e)t$ denotes the effective tax rate that the firm pays (and e is evaluated at the optimum).

Proposition 2.4 (Cremer & Gahvari (1993), comparative statics). *The comparative statics are signed as follows:*

$$\frac{\partial p}{\partial t} > 0, \; \frac{\partial p}{\partial \rho} > 0, \; \frac{\partial p}{\partial \zeta} > 0, \tag{2.17}$$

$$\frac{\partial e}{\partial t} > 0, \; \frac{\partial e}{\partial \rho} < 0, \; \frac{\partial e}{\partial \zeta} < 0. \tag{2.18}$$

□

CALCULATIONS: See the Appendix. ■

An increase in the statutory tax rate increases the *tax costs* (the sum $g(e)e + t^e$) and therefore the equilibrium price. The same argument explains why stricter enforcement leads to higher prices.

The share of evaded output increases in the tax rate because the marginal tax savings per unit of evaded output increase. On the contrary, if tax enforcement is intensified the marginal gain of evasion decreases leading to a lower share of evaded output.

2.3 Optimal Policy

The problem of characterizing optimal policy is at the center of public finance. It has also been a major line of research in the economic theory of tax evasion. It has even been hinted at already in the first formal treatment of tax evasion by Allingham & Sandmo (1972, section 6). Even in this simple model tax evasion gives rise to an additional welfare loss.

If income is given, the excess burden of a tax is the loss in (expected) utility beyond that which would be incurred if a lump-sum tax of equal amount were collected. It is a consequence of the induced uncertainty.[33]

Definition 2.2 (Excess burden of tax evasion, Yitzhaki (1987)). The *excess burden* of tax evasion is the loss in expected utility due to uncertainty. ▲

Unlike a cursory argument might suggest Slemrod & Yitzhaki (1987) show that the optimal extent of enforcement is characterized by the equality of the associated marginal reduction in the excess burden of tax evasion and the marginal increase in resource costs (administrative costs, for example, audit costs) and not the equality of marginal (tax and fine) revenue and marginal resource costs because the latter only represents a transfer from individuals to the government.[34]

If it is not optimal to enforce taxation fully (for example because there are administrative costs), the possibility of tax evasion always leads to horizontal inequity (ex post) because even if two individuals have the same pre tax income and the same risk preferences if only one of them is audited (and caught) their incomes available for consumption are different. Therefore, the more the government is concerned about this inequity the more the optimal policy relies on audits instead of fines (Cowell (1989)).

Kaplow (1990), Cremer & Gahvari (1993) and Boadway *et al.* (1994) investigate the question of optimal audit and taxation in a framework extending the model of optimal commodity taxation by Ramsey (1927) to the case with tax evasion and

[33] In models with risk-neutral agents like firms, for example, a alternative welfare loss may be introduced, see Virmani (1989), Cremer & Gahvari (1993).

[34] See also Scotchmer & Slemrod (1989). Nevertheless, a series of paper has discussed optimal policy if the government maximizes revenue, see Reinganum & Wilde (1985) and the related discussion below.

costly administration and enforcement. The results provide modifications of the Ramsey-formula for optimal commodity taxation in a model with perfect competition.[35]

Proposition 2.5 (Cremer & Gahvari (1993), optimal policy). *Optimal commodity tax rates satisfy*

$$\frac{\sum_{i=1}^{n} t_i^e \frac{\partial q_k^c}{\partial p_i}}{q_k} = \sum_{i=1}^{n} t_i^e \frac{\partial q_i}{\partial \bar{y}} + \frac{\mu}{\lambda} - \frac{\partial t_k^e}{\partial t_k} \Big/ \frac{\partial p_k}{\partial t_k} \tag{2.19}$$

$\forall\, k = 1, \ldots, n$, where n is the number of commodities, q_k^c denotes the (compensated) demand for good k; $t_k^e = (1 - \bar{r}e)t$ denotes the effective tax rate in market k; λ is the Lagrange multiplier for the government budget constraint and μ denotes the marginal utility of income.[36] □

Proof: See Cremer & Gahvari (1993). ■

Compared to equation (2.19) the original Ramsey-formula is[37]

$$\frac{\sum_{i=1}^{n} t_i \frac{\partial q_k^c}{\partial p_i}}{q_k} = \sum_{i=1}^{n} t_i \frac{\partial q_i}{\partial \bar{y}} + \frac{\mu}{\lambda}. \tag{2.20}$$

Therefore, the optimal tax rate of a commodity is higher the larger is $\frac{\partial t_k^e}{\partial t_k} \Big/ \frac{\partial p_k}{\partial t_k}$, i.e. the smaller the distortion of taxation by evasion. This is intuitive because the government tries to minimize the excess burden of taxation.

Falkinger (1991) discusses the problem of optimal public good supply in the presence of tax evasion. He shows that for general utility functions it is indeterminate whether the optimal amount of a public good supplied is lower or larger if tax evasion is allowed because the marginal rate of substitution and the marginal rate of transformation change simultaneously.[38]

---

[35] In another modification of the standard taxation problems, optimal income taxation, Cremer & Gahvari (1994) show that the effect of tax evasion on the marginal tax rate cannot be determined in general.

[36] Cremer & Gahvari (1992) also discuss the special case of the inverse elasticity rule where no cross-price effects are present.

[37] Ramsey assumed quasi-linear utility; Samuelson (1986) allowed for income effects.

[38] The optimal amount of the public good remains unaltered if a public good exhibits zero income effects, i.e. the utility function is of the form $u(y, g) = f(y + v(g))$, where f, v are twice continuously differential functions with $f'', v'' < 0 < f', v'$. A modification including the objective to redistribute income is provided by Balestrino & Galmarini (2003).

A methodologically different strand of research has investigated more sophisticated investigation schemes to maximize expected government receipts or to enforce truthful reporting at low costs (Graetz *et al.* (1984), Graetz & Reinganum (1986), Greenberg (1984), Reinganum & Wilde (1984, 1985), Scotchmer (1987), Mookherjee & P'ng (1989), Sánchez & Sobel (1993), Cremer & Gahvari (1996)). The results critically depend on whether the tax enforcement agency can commit to an announced audit rule, or whether it cannot commit. Interestingly, in the former case, some papers find that the optimal policy is characterized by an income threshold below which each individual is subjected to an audit with a constant probability and above which individuals are never audited. The details, however, are beyond the scope of this survey.[39]

Even more importantly, the study of tax evasion initiated the enlargement of the set of policy instruments available to the government beyond the determination of enforcement policies. In this more general sense the *theory of optimal tax systems* (Slemrod (1990)) explicitly acknowledges the fact that results may critically rely on the restrictions imposed on the policy set and includes the choice of tax bases (Stern (1982), Yitzhaki (1989)). For example, tax evasion is a possibility to explain the simultaneous existence of direct and indirect taxes: Boadway *et al.* (1994).[40] Similarly, Barreto & Alm (2003) investigate the impact of *corruption* on the optimal tax mix in a neoclassical growth model with consumptive and productive public goods. They find that the optimal tax mix in a corrupt economy relies more heavily on consumption taxes than on income taxes.[41,42] Gordon & Nielsen (1997), Emran & Stiglitz (2005) investigate the welfare effects of changes towards a higher reliance on the VAT if an informal sector is present.

2.4 Further Extensions

The results presented so far have been generalized in several directions. For example, there are other possible ways to evade taxes in a dynamic context: untimely filing of the tax return or payment of amounts due. The previous definition there-

[39] Recent extensions include the optimal remuneration scheme of the tax inspectors; see Hindricks *et al.* (1999).

[40] It is interesting to note that Allingham & Sandmo (1972, p.338) already speculated that in the presence of an evadable income tax, employing solely an income tax might not be optimal.

[41] Kesselman (1993) reaches a contrary conclusion in a two-sector general equilibrium model. He shows that increases in the commodity tax rate of the informal sector are shifted into prices of goods in the formal sector.

[42] For discussions of economic issues of corruption, see Shleifer & Vishny (1993), Mauro (1995), Bardhan (1997), Rose-Ackerman (1999), Tanzi & Davoodi (2000), Barreto & Alm (2003), Méndez & Sepúlveda (2006). It is also frequently discussed in the popular press (The Economist (2004a)).

fore might be extended to include timely payments.

Definition 2.3 (Tax evasion). *Tax evasion* is the deliberate failure to disclose all or part of one's income on time to the tax authority or to delay payments associated with taxation. ▲

Additionally, a time-lag between the filing of the tax report and possible investigations may arise. Andreoni (1992) shows that this time-lag might be used by individuals to borrow funds from the tax administration in case the shadow value of income is high because of low income realizations, for example, in bad overall economic conditions.

On the other hand, the tax administration might also be able to investigate earlier reports. Allingham & Sandmo (1972, section 5) also investigate an individual's problem of making a sequence of declaration decisions of a given (exogenous) income stream. The decisions are interrelated because they assume that a detected tax evader's past tax reports are also investigated. They show that under some circumstances an individual might increase its declarations over time.[43]

The bulk of the literature on tax evasion is microeconomic and neglects macroeconomic questions. However, two different approaches describe the influence of tax evasion on the macroeconomy – a short term approach that concentrates on the aggregate demand effects of tax evasion in static Keynesian models and a long term approach that emphasizes the implications of tax evasion for private savings/investment and economic growth.

The initial work of the first approach is Peacock & Shaw (1982). They recognize that estimates of the revenue loss through tax evasion may not set national income ceteris paribus, that is to say that if national income as the total tax base can be decomposed into declared and evaded income $Y = Y_d + Y_e = (1-e)Y + eY$, where $e := \frac{Y_e}{Y}$, revenue loss is not equal to τeY due to the fact that consumption may increase with evasion which increases national income and tax payments. Therefore, they develop a Keynesian model of income determination in the presence of tax evasion, later extended by Ricketts (1984), Lai & Chang (1988), von Zameck (1989), Lai *et al.* (1995), Chang & Lai (1996). Counteracting factors may be also present (for example government demand may decline) and the total effect of increased evasion on national income depends on the specification of the model.

As an example of the second approach, Roubini & Sala-i-Martin (1995) develop a macroeconomic model, where the government reacts to evasion: in countries with tax evasion the government increases seignorage by repressing the financial sector and increasing inflation rates. This government policy tends to reduce the amount

[43] Engel & Hines Jr. (1998) show how such retrospective audits may explain aggregate compliance behavior in the U.S.

of services that the financial sector provides to the economy, therefore the result is lower growth.

Some other papers also discuss the relationship between tax evasion and growth. Caballé & Panadés (1997) study in particular how tax compliance policy in the form of auditing and fining affects the rate of economic growth in a (discrete time) overlapping generations model, where tax financed public goods are productive. They find that the effect of stricter enforcement on growth is in general ambiguous and depends on the importance of public inputs in the production process because (if compliance is not perfect) stricter enforcement increases compliance, leading to two effects in opposite directions. On the one hand, private savings fall with falling expected disposable income. On the other hand, the rise of public inputs leads to higher investment because of the increased productivity of private capital.

Similar effects drive the results of Chen (2003). In his model the government optimizes the tax rate, auditing probability and fine rate taking as given the consumer's evasion decision. In general, these policies have ambiguous effects, but for realistic parameter constellations he finds that growth declines with tax evasion.

2.5 Conclusions

This chapter has summarized some main findings in the economic literature on tax evasion. Some fundamental elements of the tax evasion model of Allingham & Sandmo (1972) and its extension of Yitzhaki (1974) have been discussed as well as the model of firm tax evasion by Cremer & Gahvari (1992, 1993). These models are varied in the following chapters. The basic intuition for some implications of these models has also been discussed. Some normative implications have been highlighted.

One preliminary result from the literature review is that the results (for example the comparative statics) are sensitive to the particular specification of the problem as a problem of individual or firm tax evasion. These differences may arise because of differing evasion technologies that are used – sometimes related to the objective function at hand. Even if tax evasion is discussed from the perspective of firms the particular specification of the evasion or concealment technology plays an important role. This issue is discussed in the following chapter. A particular concealment technology is assumed that implies that the government may influence the production decision via the possibility to evade taxes. Its implications for optimal policy are discussed.

Results from macroeconomic models are discussed again in chapters 5 and 6.

2.A Appendix

Calculations for Proposition 2.1

CALCULATIONS: Applying the *theorem for implicit functions* on equation (2.6), one obtains:

$$\frac{\partial y_e}{\partial \bar{y}} = -\frac{\Phi_{y_e \bar{y}}}{\Phi_{y_e y_e}} = -\frac{\rho u''(y^1)(\tau-\zeta)(1-\tau) + (1-\rho)u''(y^2)\tau(1-\tau)}{\Phi_{y_e y_e}}, \tag{A.21}$$

$$\frac{\partial y_e}{\partial \rho} = -\frac{\Phi_{y_e \rho}}{\Phi_{y_e y_e}} = -\frac{u'(y^1)(\tau-\zeta) - \rho u'(y^2)\tau}{\Phi_{y_e y_e}}, \tag{A.22}$$

$$\begin{aligned}\frac{\partial y_e}{\partial \tau} = -\frac{\Phi_{y_e \tau}}{\Phi_{y_e y_e}} &= -\frac{\rho u''(y^1)(\tau-\zeta)(y_e-\bar{y}) + \rho u'(y^1)}{\Phi_{y_e y_e}} \\ &\quad -\frac{(1-\rho)u''(y^2)\tau(y_e-\bar{y}) + (1-\rho)u'(y^2)}{\Phi_{y_e y_e}},\end{aligned} \tag{A.23}$$

$$\frac{\partial y_e}{\partial \zeta} = -\frac{\Phi_{y_e \zeta}}{\Phi_{y_e y_e}} = -\frac{\rho u''(y^1)(\tau-\zeta)(-y_e) - \rho u'(y^2)}{\Phi_{y_e y_e}}. \tag{A.24}$$

In order to sign these derivatives, remember that $u''(y) < 0 < u'(y), \forall y \in \mathbb{R}_0^+$, $\tau - \zeta < 0$, $0 \leqslant \tau \leqslant 1$ and $\Phi_{y_e y_e} < 0$ if the individual is risk-averse. Therefore, unambiguously, $\frac{\partial y_e}{\partial \rho}, \frac{\partial y_e}{\partial \zeta} < 0$; an increase in the probability of detection or in the fine rate decreases the optimal amount of evaded income.

In order to sign $\frac{\partial y_e}{\partial \bar{y}}$, the coefficient of absolute risk aversion is used.

Definition A.4 (Coefficient of absolute/relative risk aversion). The coefficient of absolute risk aversion, r_A, is defined as:

$$r_A(u, y) := -\frac{u''(y)}{u'(y)}, \text{ for } u'(y) \neq 0. \tag{A.25}$$

The higher this coefficient the more risk-averse is an individual in the sense that his risk premium for a given lottery is larger.

The coefficient of relative risk aversion, r_R, is defined as:

$$r_R(u, y) := r_A \cdot y. \tag{A.26}$$

▲

From equation (2.6) it follows that $-\rho u'(y^1)(\tau - \zeta) = (1-\rho)u'(y^2)\tau \ (> 0)$.

Therefore,[44]

$$\frac{\partial y_e}{\partial \bar{y}} \stackrel{s}{=} -\frac{u''(y^1)}{u'(y^1)} + \frac{u''(y^2)}{u'(y^2)} \stackrel{s}{=} r_A(u, y^1) - r_A(u, y^2)$$
$$\stackrel{s}{=} \begin{cases} 1 \Leftrightarrow r_A(u, y^1) > r_A(u, y^2), \\ 0 \Leftrightarrow r_A(u, y^1) = r_A(u, y^2), \\ -1 \Leftrightarrow r_A(u, y^1) < r_A(u, y^2). \end{cases} \tag{A.27}$$

The optimal amount of evaded income increases with income if the individual has a DARA utility function.

$\frac{\partial y_e}{\partial \tau}$ cannot be signed for DARA utility functions:

$$\frac{\partial y_e}{\partial \tau} \stackrel{s}{=} [r_A(u, y^2) - r_A(u, y^1)]y_d - \frac{\zeta}{\tau(\tau - \zeta)}. \tag{A.28}$$

For DARA utility the first term is negative (the second is always positive). ∎

Calculations for Proposition 2.2

Calculations: The application of the theorem for implicit functions on equation (2.12) yields

$$\frac{\partial y_e}{\partial \tau} = -\frac{\Phi_{y_e\tau}}{\Phi_{y_e y_e}} = -\frac{\rho u''(y^1)\tau(1-\zeta)([1-\zeta]y_e - \bar{y}) + \rho u'(y^1)(1-\zeta)}{\Phi_{y_e y_e}} - \frac{(1-\rho)u''(y^2)\tau(y_e - \bar{y}) + (1-\rho)u'(y^2)}{\Phi_{y_e y_e}} \tag{A.29}$$

which using the first-order condition (2.12) implies

$$\frac{\partial y_e}{\partial \tau} \stackrel{s}{=} -\frac{u''(y^1)}{u'(y^1)}[(1-\zeta)y_e - \bar{y}] + \frac{u''(y^2)}{u'(y^2)}(y_e - \bar{y}). \tag{A.30}$$

For DARA utility function (in fact, non-increasing absolute risk aversion suffices) it follows: $r_A(u, y^1) > r_A(u, y^2) \Rightarrow r_A(u, y^1)(y_e - \bar{y}) < r_A(u, y^2)(y_e - \bar{y}) \Rightarrow r_A(u, y^1)([1-\zeta]y_e - \bar{y}) < r_A(u, y^2)(y_e - \bar{y})$. For utility functions of the DARA class the amount of evaded income decreases unambiguously as the tax rate increases. ∎

[44] The sign $\stackrel{s}{=}$ has to be read *is of the same sign as*.

Calculations for Proposition 2.4

Calculations: Differentiate the first-order condition for the optimal share of evaded output, equation (2.15), to obtain

$$\mathrm{d}\frac{\partial \pi^e}{\partial e} = -(2g' + g'')\mathrm{d}e + \bar{r}\mathrm{d}t - (\zeta \mathrm{d}\rho + \rho \mathrm{d}\zeta)t = 0. \tag{A.31}$$

It follows that

$$\frac{\partial e}{\partial t} = \frac{\bar{r}}{2g' + g''} > 0, \tag{A.32}$$

$$\frac{\partial e}{\partial \rho} = \frac{-\zeta t}{2g' + g''} < 0, \tag{A.33}$$

$$\frac{\partial e}{\partial \zeta} = \frac{-\rho t}{2g' + g''} < 0. \tag{A.34}$$

From the differentiation of the market equilibrium condition, equation (2.16) one obtains (where equation (2.15) has been used to simplify)

$$\mathrm{d}p = (1 - \bar{r}e)\mathrm{d}t + (\zeta \mathrm{d}\rho + \rho \mathrm{d}\zeta)et. \tag{A.35}$$

The comparative statics of the equilibrium price with respect to policy changes therefore are

$$\frac{\partial p}{\partial t} = 1 - \bar{r}e > 0, \tag{A.36}$$

$$\frac{\partial p}{\partial \rho} = et\zeta > 0, \tag{A.37}$$

$$\frac{\partial p}{\partial \zeta} = et\rho > 0. \tag{A.38}$$

This completes the calculations. ■

Chapter 3

The Non-neutrality of a Profit Tax With Cost Overreporting*

This chapter discusses the relationship between the decision to evade a profit tax and the production decision in a monopolistic market. It provides a simple condition that generates a one-sided separability: the evasion decision is separable from the production decision but not vice versa if the firm chooses to overreport production costs and the concealment cost function is strictly convex. This condition also implies that if the monopolist chooses to evade taxes it has an incentive to increase production beyond the monopoly output with full payment of tax. The normative analysis shows how the government designs its tax and tax enforcement policy in the light of tax evasion. The optimality condition implies that both instruments should be used to the extent that their marginal excess burden per marginal revenue unit is equal.

3.1 Introduction

Profit tax evasion (and tax avoidance) is a problem that arouses public interest in particular (see The Economist (1999, 2004b)). The present chapter discusses how optimal policy can be characterized if profit tax evasion occurs in a monopolistic market. A particular emphasis is laid on the case where the neutrality of taxation is invalid. In this latter sense it stresses the importance of the particular technology that is used for evasion or avoidance activities (Slemrod & Yitzhaki (2002)).

Suppose that a monopolist sells quantity q of a good of which (inverse) market demand is given by a strictly downward sloping (twice differentiable) function $p(q)$. The monopolist's revenue for any quantity produced is therefore given by the product pq.[1] Suppose, furthermore, that the monopolist can produce additional units at

*Helpful hints by and discussions with Felix Bierbrauer and Mark Wipprich are gratefully acknowledged.

[1]Function arguments are suppressed where no confusion can arise.

constant marginal costs $c > 0$ such that total costs amount to cq. The government raises a profit tax at constant rate τ, $0 \leqslant \tau \leqslant 1$, such that net profit after tax is given by

$$\pi = (1-\tau)(p-c)q \tag{3.1}$$

if no taxes may be evaded.

Taxation does not affect the production decision in such a setting in the sense that the monopolist produces a quantity q^m (the standard monopoly quantity) that equates marginal revenue to marginal costs, i.e. that satisfies

$$p(q^m) + p'(q^m)q^m = c, \tag{3.2}$$

irrespectively of the tax rate because it affects marginal revenue and marginal costs at the same proportional rate.[2]

However, if the monopolist may choose to evade (some) taxes, this neutrality result is sensitive to the particular specification of the problem as the discussion of Kreutzer & Lee (1986, 1988), Wang & Conant (1988), Wang (1990), Yaniv (1995, 1996), Lee (1998), Panteghini (2000) and Goerke & Runkel (2005) has shown. All models that derive an independence of the production decision from the evasion decision assume that apart from risk no other inefficiency arises. Additionally, risk is only inefficient because these authors assume risk-averse firms.[3]

The subsequent discussion shows that the production decision is not independent of the evasion decision if the monopolist evades taxes by *overreporting* his costs (as in Kreutzer & Lee (1986, 1988), Wang & Conant (1988)) and if it has to cover its tax evasion activities from cursory examination by spending resources (and the associated concealment cost function is strictly convex as in Cremer & Gahvari (1993)).

The chapter proceeds as follows. The model is developed in section 3.2. First, the monopolist's optimization problem is described in section 3.2.1. It includes the derivation and interpretation of the optimality conditions and the analysis of the non-neutrality property. A graphical illustration of the monopolist's optimum decision is also provided. Furthermore, section 3.2.1 includes the derivation of some comparative statics results. The possibility of influencing the monopolist's production decision through tax and tax enforcement policy is investigated in section 3.2.2. The welfare maximization problem for a benevolent government is set out and solved. Among other results an intuitive equation that a welfare optimal policy satisfies is derived. A summary and conclusion follow in section 3.3.

[2] Note that equation (3.2) necessarily holds only for an interior solution, but corner solutions for production choice are not discussed. Assume therefore that $p(0) > c$ and $\lim_{q\to+\infty} p(q) < c$ hold which is sufficient to guarantee the existence of such an interior solution for q.

[3] A detailed discussion of the appropriate risk attitude of firms follows below.

3.2 The Model

3.2.1 Production and Evasion Decision

The model is developed in a standard two states of the world setup frequently used in models of tax evasion and goes back to the seminal paper of Allingham & Sandmo (1972).[4] It is adapted to the situation of a monopolistic firm along the lines of the models in Kreutzer & Lee (1986, 1988), Cremer & Gahvari (1992, 1993).

If the monopolist evades taxes by overreporting a fraction δ of its costs, he takes a gamble with uncertain return. Suppose that the company is audited with some constant probability ρ, $0 \leqslant \rho \leqslant 1$.[5] The auditor detects any amount of evaded tax in case of an audit if the firm did not report its true tax liability, and it has to pay a fine proportional to the amount of evaded tax at rate ζ, $1 < \zeta$ (Yitzhaki (1974)). The firm either earns profit $\pi^1 = (1-\tau)(p-c)q + (\tau - g)\delta cq$ if it is not audited or $\pi^2 = \pi^1 - \zeta\tau\delta cq$ in case of an audit. Here, $g\delta$ denotes the concealment costs of overreporting production costs of δcq, and g itself is assumed to be strictly increasing in δ (in particular, assume $g(0) = g'(0) = 0$, $g' > 0$, for $\delta > 0$) such that the concealment technology is strictly convex.[6] Firms can be assumed to be risk-neutral if well-diversified portfolios exist.[7]

[4] Early contributions also include Srinivasan (1973) and Kolm (1973). Cowell (1985a, 2004), Pyle (1991), Andreoni *et al.* (1998), Franzoni (2000), Slemrod & Yitzhaki (2002) review the literature.

[5] Clearly, this is a simplification. As far as the decision on actual audits is concerned it is also unlikely that the enforcement agency audits at random. In the U.S., for example, the IRS employs a range of methods to detect evaders. In particular, it uses the results of its program of intensive audits: the Taxpayer Compliance Measurement Program (TCMP). On the basis of its results it assigns to each tax report a likelihood that it is incorrect. Andreoni *et al.* (1998) state that over 50% of audits are based on this score. According to their calculations, the yield of a random audit is \$289 compared to \$5.500 for non-random ones. Nevertheless, the model uses random audits for simplicity. On theoretical grounds this may be defended by the assumption of the concealment cost function. It is assumed that if an individual incurred the concealment costs there is no hint left that he has evaded taxes. He can only be distinguished from an honest taxpayer by audit.

[6] Several interpretations of this tax evasion model are possible. One neglected aspect that is captured by it is that the cost of concealment function is a possibility to deal with an important difference of tax evasion compared to other crimes. In many cases the taxpayer files a report to the agency and the agency looks for hints on evasion that may lead it to start a more thorough investigation. The cost of concealment function captures the costs that an agent incurs to cover the tracks of his evasion activities. These costs may consist of payments to unscrupulous tax advisers or taxpayer time. An alternative explanation may be costs of corrupting public officials as in Hibbs & Piculescu (2005) where additionally to taxation the circumvention of other government regulations is addressed. Niepelt (2005) develops a dynamic model where convex concealment cost function arises endogenously.

[7] Some work on tax evasion by firms treats firms as risk-averse, see Wang & Conant (1988), Wang (1990), Yaniv (1995, 1996), Lee (1998). The assumption of risk aversion of firms may only be rationalized satisfactorily in two environments. First, where it is a manager that decides under

The first result states that a profit tax is not neutral in this model.

Proposition 3.1 (Non-neutrality of a profit tax). *If the economic environment is as specified above where a risk-neutral monopolist in particular incurs convex resource costs to overreport a fraction of its production costs, then a profit tax is not production neutral if it is optimal to evade taxes. The monopolist produces a larger quantity than with full tax enforcement if the marginal revenue is strictly decreasing in the quantity sold.* □

PROOF: The algebraic profit maximization problem of the monopolist is:

$$\max_{\{q,\delta\}} \pi^e = (1-\tau)(p(q)-c)q + ((1-\rho\zeta)\tau - g(\delta))\delta c q, \qquad (3.3)$$
$$\text{s.t. } 0 \leqslant q,\ 0 \leqslant \delta.$$

The equation system

$$\frac{\partial \pi^e}{\partial q} = (1-\tau)(p+p'q-c) + ((1-\rho\zeta)\tau - g)\delta c = 0$$
$$\Leftrightarrow (1-\tau)(p+p'q) + (1-\rho\zeta)\tau\delta c = (1-\tau)c + g\delta c, \qquad (3.4)$$
$$\frac{\partial \pi^e}{\partial \delta} = [(1-\rho\zeta)\tau - g - g'\delta]cq = 0$$
$$\Leftrightarrow (1-\rho\zeta)\tau = g + g'\delta, \qquad (3.5)$$

implicitly describes an interior optimum $(q^*, \delta^*) \in (0, +\infty) \times (0, +\infty)$. A solution to (3.4), (3.5) exists if $\rho\zeta < 1$, $g(0) = g'(0) = 0$, $\lim_{\delta \to \infty} g(\delta) = 1$, $(1-\tau)p(0) > c$, $\lim_{q \to +\infty} p(q) < c$. The first two conditions guarantee that some evasion is profitable. The second condition guarantees that the optimal production decision leads to a positive finite quantity. The sufficient conditions for a maximum boil down to the one that both second derivatives are strictly negative, i.e. $\frac{\partial^2 \pi^e}{\partial q^2} = (1-\tau)(2p' + p''q) < 0$, and $\frac{\partial^2 \pi^e}{\partial \delta^2} = -(2g' + g''\delta) < 0$ which is guaranteed if $2p' + p''q < 0$ and $g'' = 0$. The latter assumption is not important. The first, however, rules out that quantities are strategic complements (Bulow *et al.* (1985)). It is widely made and defended by Hahn (1962), for example.

In order to see that the quantity produced in such an optimum exceeds the monopoly

some discretion of the owners. And second, when capital markets are imperfect. In both cases, other important issues should be included in the analysis, either the agency problem between management and owners (Chen & Chu (2002), Crocker & Slemrod (2005)) or the source of the capital market imperfection and its implications (Andreoni (1992)). If both discussions are absent, the assumption of risk aversion remains purely technical to guarantee an interior solution of the evasion decision, see Marrelli & Martina (1988, eq. 2).

quantity without tax evasion, substitute equation (3.5) into equation (3.4) to obtain

$$(1-\tau)(p+p'q-c)+g'\delta^2 c=0. \tag{3.6}$$

Equation (3.6) cannot hold at q^m because $0<g'\delta^2$ for $0<\delta$ and the first term $(1-\tau)(p+p'q-c)$ is zero at q^m. As the marginal revenue is assumed to be strictly decreasing the optimal quantity q^* is larger than the monopoly quantity without the possibility of tax evasion $q^m < q^*$.[8] ■

The reason for this result is that the possibility to evade taxes operates like a random production subsidy administered through tax evasion. The monopolist can raise profit through official sales (the first term in equation (3.3)) and through tax evasion (second term). The marginal profit of production through tax evasion is positive at q^m because the associated concealment costs enter on average which is always below the marginal concealment costs for a convex concealment cost function. Therefore, it is profitable for the monopolist to increase output beyond the production optimum in case of full enforcement. The special mode by which tax evasion occurs (and the convexity of the concealment technology) ensures that the effective costs to produce additional units of output $(1-\tau)c-((1-\rho\zeta)\tau-g)\delta c=(1-\tau)c-g'\delta^2 c$ are lowered by the possibility to evade taxes $((1-\tau)c-g'\delta^2 c<(1-\tau)c)$.

The same result has been achieved by Kreutzer & Lee (1986, 1988). The reason why it has been criticized in strong terms may be that their model lacks a thorough microeconomic foundation. Especially the way they introduce risk by multiplying profits by a strictly decreasing function $\phi(\delta)$, with $\phi(0)=1$ (Kreutzer & Lee (1988, eq.1)) comes as an ad hoc assumption. Actually, there is no ϕ such that their model is equivalent to the one developed above.[9]

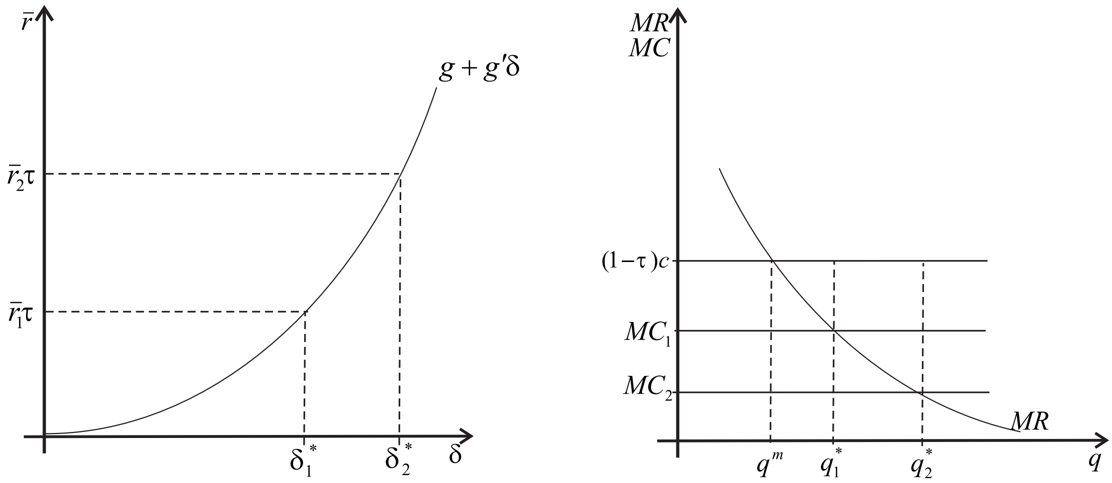
In the following it is assumed that some evasion is profitable and the sufficient conditions for an interior maximum hold. In this case, the relationship of production and evasion decision is as follows.

Remark 3.1 (Separation result). *The evasion decision is separable from the production decision (but not vice versa).* △

The separation is only one-sided. The overreport of the costs is independent of the production (q does not appear in equation (3.5)). It is only necessary to equate the marginal expected return of one unit of evaded tax that is solely determined by policy parameters to its marginal costs because they are both proportionally

[8] Optimal values are only indicated by an * if confusion arises otherwise.

[9] Equation (1) of Kreutzer & Lee (1988) reads in the notation of the present chapter $\pi^e=\phi(\delta)[(p-c)q-\tau(p-(1+\delta)c)q]$ which may be written as $\pi^e=\phi(\delta)(1-\tau)(p-c)q+\phi(\delta)\tau\delta cq$ which is not equivalent to equation (3.3) for any choice of ϕ.



(a) Evasion decision. (b) Production decision.

Figure 3.1: Profit maximum.

affected by the total costs.[10] The production decision depends on the marginal return to production which consists of the marginal revenue from sales and from tax evasion. The latter in turn is affected by the marginal costs of evasion and the extent to which the monopolist overreports his costs. And whenever some evasion is profitable, the marginal return to production at the standard monopoly quantity q^m is positive.

This separation implies that changes in the government's tax enforcement policy influence the production decision only directly through their impact on the marginal return to production and not through their impact on the marginal return to evasion and the associated changes in tax evasion.

The solution of the profit maximization problem can be illustrated by Figure 3.1. Figure 3.1(a) shows the determination of the optimal rate of cost overreport δ^* for two possible scenarios, one where the expected marginal gain of one unit of evaded tax, $\bar{r} := 1 - \rho\zeta$, is low, denoted by $\bar{r}_1$, and the other where it is high, $\bar{r}_2$. The respective rates of cost overstatement are ordered $\delta_1^* < \delta_2^*$.

Figure 3.1(b) depicts the marginal revenue curve (net of tax), $\mathrm{MR} = (1-\tau)(p+p'q)$, as strictly falling (at a decreasing rate) as the sufficient conditions for a maximum of (3.3) demand. The intersection of the marginal revenue and the marginal cost curve (again net of tax) defines the optimum q^m if no tax evasion exists. If costs are overstated at rate $0 < \delta^*$, this shifts the marginal cost curve down by $(\bar{r}\tau - g^*)\delta^* c = g^{*\prime}\delta^{*2}c$ units. For $\bar{r}_i$ the relevant marginal cost curve is denoted by MC_i, $i \in \{1, 2\}$. Consequently, the intersections of the marginal revenue and the new marginal cost

[10]This separation generalizes to the case of a strictly convex production cost function.

curves at points q_1^* and q_2^* are to the right of q^m with $q_1^* < q_2^*$.

It must be noted that the result depends crucially on the mechanism through which taxes are evaded, that is cost overreporting (and the associated form of the concealment function). If evasion is done in such a way that the concealment costs are of the form $g(e)e(p-c)q$, where e denotes the fraction of the tax base that is evaded, i.e. the concealment costs are proportional to the amount of evaded profit, the neutrality result prevails because evasion gains and concealment costs are proportionally affected by quantity or profit changes. In effect, it is an empirical question which form of the concealment technology is more appropriate and this issue deserves further examination.

To investigate the impact of the policy and market parameters formally, consider the following result concerning the comparative statics in an interior optimum (q^*, δ^*).

Proposition 3.2 (Comparative statics). *The comparative statics in the model described above have the following signs:*

$$\frac{\partial q}{\partial \tau} > 0,\ \frac{\partial q}{\partial \rho} < 0,\ \frac{\partial q}{\partial \zeta} < 0,\ \frac{\partial q}{\partial c}\begin{cases} < 0 \text{ if } \tau < \tau^c \\ = 0 \text{ if } \tau = \tau^c \\ > 0 \text{ if } \tau > \tau^c \end{cases} \tag{3.7}$$

$$\frac{\partial \delta}{\partial \tau} > 0,\ \frac{\partial \delta}{\partial \rho} < 0,\ \frac{\partial \delta}{\partial \zeta} < 0,\ \frac{\partial \delta}{\partial c} = 0, \tag{3.8}$$

□

where $0 < \tau^c < 1$.

PROOF: See the Appendix. ■

The reason why the monopolist produces larger quantities at higher tax rates is that an increase in the tax rate is equivalent to an increase in a production subsidy. The same result has been obtained by Kreutzer & Lee (1986). The production and the shifting decision have also been analyzed in other papers mentioned above. The present result contrasts, for example, with the one by Marrelli & Martina (1988). In their model a profit tax is not shifted (production decisions are not affected by tax declarations). In particular, the shifting result in previous models seems to depend on the audit strategy (Marrelli (1984)).

Increases in tax enforcement parameters increase the effective production costs and, therefore, diminish the return to production at the margin. Higher tax rates increase the marginal return to evasion and lead to higher overreporting (as in Cremer & Gahvari (1992, 1993)). As in the basic model of individual income taxation (Allingham & Sandmo (1972)) stronger enforcement reduces evasion. The reason in the present model is that the expected gain of tax evasion, $\bar{r}$, decreases in enforcement.

The monopolist produces smaller quantities if the marginal production cost rises if the tax rate is relatively low. Vice versa it produces higher quantities if τ is relatively large. The intuition is that for low τ the negative production incentive through the increase in the marginal production costs outweighs the positive incentive through the higher return to production by evasion and vice versa for high τ.[11] A nonsymmetric response to increases in tax compared to production costs has also been found by Slemrod (2001). In the present model even the sign of the relationship between production and marginal costs is ambiguous.

The derived comparative statics imply that not only the fraction of costs that is overreported increases in the tax rate but also the total amount of tax evasion is positively related to τ.

Corollary 3.1 (Amount of evasion). *The amount of evaded profit tax is strictly increasing in the tax rate.* □

PROOF: The proof is obvious and is therefore left out. ■

In addition to the direct effect of a tax increase on the amount of evaded taxes (holding δ and q constant) the production quantity and the share of cost overreport increase.

In the model studied so far, tax and tax enforcement policy have been assumed to be arbitrary and unaware of the existence of tax evasion. This assumption is abolished in the following where a particular form of optimal policy is discussed.

3.2.2 Optimal Tax and Audit Policy

Suppose that the government can choose the tax rate τ and the audit probability ρ to influence the monopolist's production and evasion decision in a socially desirable way. The monopolist's first-order conditions determine the optimal production and evasion decision (q^*, δ^*) for a given tax and audit policy, $q^* = q^*(\tau, \rho)$, $\delta^* = \delta^*(\tau, \rho)$, and the government takes the monopolist's production and evasion decision into account when it sets its policy. Note that the set of possible tax rates is restricted to constant ones between 0 and 1 and the set of possible audit strategies is restricted to random policies where the firm is audited with a constant probability. The fine rate is assumed to be fixed at rate ζ. It is assumed that the *commitment* problem is solved, that is to say that the government sets its policy and agents react to it without the possibility that the policy is changed again. This is the approach also followed by Cremer & Gahvari (1992, 1993, 1994).[12]

[11] If $1 - g'\delta^2 < \tau$, the effective costs are negative and an increase in c is equivalent to an increase in the production (cost) subsidy.

[12] A methodologically different strand of research has investigated more sophisticated investigation schemes to maximize expected government receipts or to enforce truthful reporting at low costs (Graetz *et al.* (1984), Graetz & Reinganum (1986), Greenberg (1984), Reinganum & Wilde

A particularly interesting question is whether it is possible for the government to influence the production decision of the monopolist such that it sets price like in the competitive environment equal to the marginal costs.

Remark 3.2 (Marginal cost pricing). *If the government allows tax evasion, it can use its tax policy to induce the monopolist to set price equal marginal cost, i.e.* $\forall\ \rho, \zeta : \rho\zeta < 1\ \exists\ \hat{\tau},\ 0 < \hat{\tau} < 1 : p = c$ *is optimal.* △

Proof: Suppose that the audit and fine rates are such that the monopolist evades taxes, i.e. $\rho\zeta < 1$. If the monopolist has to find it optimal to set $p = c$, the corresponding first-order condition has to hold for some τ, $0 \leqslant \tau \leqslant 1$. Mathematically, the question therefore is whether some $\hat{\tau}$ exists such that (combining equations (3.5) and (3.4), again)

$$h(\tau, \rho) := \left.\frac{\partial \pi^e}{\partial q}\right|_{p=c} = (1-\tau)p'q + g'\delta^2 c \tag{3.9}$$

is equal to zero. However, this is always guaranteed because $h(0, \rho) = p'(0, \rho)q(0, \rho) < 0$, $h(1, \rho) = g'(\delta(1, \rho))\delta^2(1, \rho)c > 0$ and h is continuous in τ. Thus, actually $0 < \hat{\tau} < 1$. ■

Note that an additional condition has to be satisfied such that an analogous result holds for the audit rate.[13]

Note also that this argument neglects the revenue effect of different tax rates so far. Clearly, as the firm makes no actual profits at $\hat{\tau}$ and overreports its costs the expected tax revenue is negative $R^e(\hat{\tau}, \rho) = -\bar{r}\tau\delta c q < 0$. The monopolist earns a subsidy on production that is administered through the possibility to evade taxes.

An interesting question concerns the second-best policy where the government is restricted to a balanced budget or some other predefined revenue goal $\bar{R}$. After the impact of different policies on the expected revenue is investigated this second-best policy is characterized.

Analogously to taxation in a model with perfect tax compliance if tax evasion occurs, i.e. if $\rho\zeta < 1$ there may be a Laffer curve effect.

(1984, 1985), Scotchmer (1987), Mookherjee & P'ng (1989), Sánchez & Sobel (1993), Cremer & Gahvari (1996)). The results critically depend on whether the tax enforcement agency can commit to an announced audit rule, or whether it cannot commit. Interestingly, in the former case, some papers find that the optimal policy is characterized by an income threshold below which each individual is subjected to an audit with a constant probability and above which individuals are never audited.

[13] In particular, it has to be true that $\left.\frac{\partial \pi^e}{\partial q}\right|_{p=c}(\tau, 0) = (1-\tau)p'(q(\tau, 0))q(\tau, 0) + g'(\delta(\tau, 0))\delta^2(\tau, 0)c < 0$ which is satisfied if the concealment costs are low enough in the optimum.

Lemma 3.1 (Laffer curve). *The expected tax and fine revenue R^e may be a non-monotonous function of the nominal tax rate with an expected revenue maximizing rate $\tau^{\max}$, $0 < \tau^{\max} < 1$. It is increasing in τ for low tax rates.* □

PROOF: The expected tax and fine revenue is given by

$$R^e = \tau[(p-c)q - \bar{r}\delta cq] - \chi(\rho), \tag{3.10}$$

where it is assumed that auditing is costly according to the increasing and strictly convex audit cost function $\chi : [0,1] \to \mathbb{R}_0^+$, $\rho \mapsto \chi(\rho)$ with $\chi(0) = 0$ and if an interior solution of the government's optimization problem should be assured $\lim_{\rho \to 1} \chi(\rho) = +\infty$. An increase in the fine rate may be considered as costless.

Therefore, the derivative of R^e with respect to τ is given by

$$\begin{aligned} \frac{\partial R^e}{\partial \tau} &= (p-c)q - \bar{r}\delta cq + \tau \left[(p + p'q - c)\frac{\partial q}{\partial \tau} - \bar{r}c \left(\frac{\partial \delta}{\partial \tau} q + \delta \frac{\partial q}{\partial \tau} \right) \right] \\ &= (p - (1+\bar{r}\delta)c)\,(1 + \eta_{q,\tau})\,q + \left(p' \frac{\partial q}{\partial \tau} - \bar{r}c \frac{\partial \delta}{\partial \tau} \right) \tau q, \end{aligned} \tag{3.11}$$

with $\eta_{q,\tau} := \frac{\tau}{q}\frac{\partial q}{\partial \tau}$ denoting the elasticity of production with respect to the nominal tax rate. The first term of equation (3.11) is of the same sign as the revenue requirement and the second term is always negative.

Like in Cremer & Gahvari (1999) it can not be concluded that the marginal revenue is lower in case that tax evasion is possible because the elasticity also changes. This question is not investigated further here.

Using the comparative statics results equation (3.11) can be rewritten as

$$\begin{aligned} \frac{\partial R^e}{\partial \tau} = {} & (p - (1+\bar{r}\delta)c) \left(q + \frac{p + p'q - (1+\bar{r}\delta)c}{(1-\tau)(2p' + p''q)} \tau \right) \\ & + \left(\frac{p + p'q - (1+\bar{r}\delta)c}{(1-\tau)(2p' + p''q)} p' - \frac{\bar{r}^2 c}{2g' + g''\delta} \right) \tau q. \end{aligned} \tag{3.12}$$

It is not possible to read off immediately a sufficient condition for the marginal expected revenue to be negative. However, this case is more likely if $g'' = 0$.[14]

At small tax rates the marginal expected revenue is positive

$$\left. \frac{\partial R^e}{\partial \tau} \right|_{\tau=0} = [p(q(0,\rho)) - (1 + \bar{r}\delta(0,\rho))c]q(0,\rho) > 0. \tag{3.13}$$

■

[14]The Appendix provides an example where a Laffer curve arises.

A similar result holds with respect to the tax enforcement parameters ρ and ζ. However, the associated costs are decisive for the sign of the partial derivatives of the expected revenue function. If additional audits are sufficiently costly, R^e is decreasing in ρ at the margin.

Lemma 3.2 (Laffer curve). *The expected tax and fine revenue R^e is increasing in the tax enforcement parameters ρ respectively ζ if the marginal audit costs are negligible initially, i.e. $\chi'(0) = 0$. R^e is decreasing in ρ at some point if the marginal audit costs χ' are sufficiently high.* □

PROOF: The partial derivatives of R^e with respect to ρ and ζ are given by

$$\frac{\partial R^e}{\partial \rho} = \tau \left[(p + p'q - c)\frac{\partial q}{\partial \rho} + \delta\zeta cq - \bar{r}c\left(\frac{\partial \delta}{\partial \rho}q + \delta\frac{\partial q}{\partial \rho}\right)\right] - \chi', \tag{3.14}$$

$$\frac{\partial R^e}{\partial \zeta} = \tau \left[(p + p'q - c)\frac{\partial q}{\partial \zeta} + \delta\rho cq - \bar{r}c\left(\frac{\partial \delta}{\partial \zeta}q + \delta\frac{\partial q}{\partial \zeta}\right)\right]. \tag{3.15}$$

The term in the square brackets is positive. The marginal revenue is increasing in ρ and ζ due to the deterrent effect on production (which is subsidized at the margin) and on tax evasion activities. Therefore, in particular, expected revenue is increasing for low levels of auditing if $\chi'(0) = 0$.[15]

$$\begin{aligned}\left.\frac{\partial R^e}{\partial \rho}\right|_{\rho=0} = \tau \Bigg[&(p(q(0,\tau)) + p'(q(0,\tau))q(0,\tau) - c)\frac{\partial q}{\partial \rho}(0,\tau) \\ &+ \delta\zeta cq(0,\tau) - \bar{r}c\left(\frac{\partial \delta}{\partial \rho}(0,\tau)q(0,\tau) + \delta(0,\tau)\frac{\partial q}{\partial \rho}(0,\tau)\right)\Bigg] > 0,\end{aligned} \tag{3.16}$$

$\forall\ \tau > 0$.

The expected revenue is decreasing

$$\frac{\partial R^e}{\partial \rho} = \tau \left[(p + p'q - c)\frac{\partial q}{\partial \rho} + \delta\zeta cq - \bar{r}c\left(\frac{\partial \delta}{\partial \rho}q + \delta\frac{\partial q}{\partial \rho}\right)\right] - \chi' < 0 \tag{3.17}$$

if the marginal auditing costs are sufficiently high.

If the limit condition $\lim_{\rho \to 1/\zeta} \chi'(\rho) = +\infty$ holds, $\frac{\partial R^e}{\partial \rho}$ is negative for some $\rho < \frac{1}{\zeta}$ (the right-hand side denotes the value of ρ where tax evasion is deterred completely).■

Therefore, at least for a range of small values of tax and audit rate both instruments may be used as substitutes in raising revenue.

[15] With a slight abuse of notation the arguments of q, δ and their derivatives relate to the audit probability and the tax rate. The fine rate is left out for simplicity.

A welfare maximizing government could now try to maximize the total surplus S in the market. This consists of consumer surplus

$$\mathrm{CS} := \int_0^q p(x)\mathrm{d}x - pq \tag{3.18}$$

and producer surplus

$$\mathrm{PS} := \pi^e. \tag{3.19}$$

The total surplus – holding total expected revenue constant such that the utility of public goods can be neglected[16] – can therefore be calculated to be given by

$$S := \int_0^q p(x)\mathrm{d}x - (1+g\delta)cq. \tag{3.20}$$

Equation (3.20) shows that besides the benefits that consumers derive from consumption and the production costs the social planer takes the costs that are associated with tax evasion into account.[17]

Proposition 3.3 (Optimal tax and audit policy). *The second-best (interior) tax and audit policy in the environment described above satisfies*

$$\frac{\partial R^e}{\partial \tau}\Big/\frac{\partial R^e}{\partial \rho} = [p-(1+g\delta)c]\frac{\partial q}{\partial \tau} - c\bar{r}\tau\frac{\partial \delta}{\partial \tau}q\Big/[p-(1+g\delta)c]\frac{\partial q}{\partial \rho} - c\bar{r}\tau\frac{\partial \delta}{\partial \rho}q. \tag{3.22}$$

□

Proof: A welfare maximizing government that maximizes total surplus

$$S = \int_0^q p(x)\mathrm{d}x - (1+g\delta)cq$$

[16]To be more precise, expected revenue has to be constant and it has to be equal to the actual revenue. By the law of large numbers this is guaranteed if there is a large number of markets like the one discussed here. Otherwise, individuals may be worse off even if expected revenue is constant if the level of the public good provision is stochastic and individuals are risk-averse with respect to consumption of the public good.

[17]Therefore, if tax evasion cannot be eliminated, the first-best optimal quantity satisfies

$$p(q) = (1+g\delta)c. \tag{3.21}$$

As it has been shown that even the standard monopoly quantity can be induced by the appropriate choice of a tax rate this also holds true for any smaller quantity like the one that satisfies equation (3.21).

subject to the revenue constraint $R^e = \tau[p - (1 + \bar{r}\delta)c]q \geqslant \bar{R}$ solves the problem

$$\max_{\{\tau,\rho\}} S = \int_0^q p(x)\mathrm{d}x - (1 + g\delta)cq \tag{3.24}$$
$$s.t.\ R^e \geqslant \bar{R},\ 0 \leqslant \tau \leqslant 1,\ 0 \leqslant \rho \leqslant \frac{1}{\zeta}.$$

A positive amount of revenue can only be collected if $0 < \tau$ and $0 < \rho$. It is also assumed that setting $\rho = \frac{1}{\zeta}$ is too costly such that it is sufficient only to include the revenue constraint and the upper bound on the tax rate in the algebraic formulation of the problem.

The associated Lagrangian that allows the optimal tax rate to be at the upper bound (with λ and μ as Lagrange parameters) is given by

$$\mathcal{L} = \int_0^q p(x)\mathrm{d}x - (1 + g\delta)cq - \lambda(\bar{R} - R^e) - \mu(\tau - 1). \tag{3.25}$$

The first-order Kuhn-Tucker-conditions are given by

$$\frac{\partial \mathcal{L}}{\partial \tau} = 0, \tag{3.26}$$
$$\frac{\partial \mathcal{L}}{\partial \rho} = 0, \tag{3.27}$$
$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0, \tag{3.28}$$
$$\frac{\partial \mathcal{L}}{\partial \mu} \geqslant 0,\ \mu(\tau - 1) = 0, \tag{3.29}$$

and the partial derivatives of the Lagrangian can be calculated to be

$$\frac{\partial \mathcal{L}}{\partial \tau} = [p - (1 + g\delta)c]\frac{\partial q}{\partial \tau} - (g + g'\delta)\frac{\partial \delta}{\partial \tau}cq + \lambda\frac{\partial R^e}{\partial \tau} - \mu, \tag{3.30}$$
$$\frac{\partial \mathcal{L}}{\partial \rho} = [p - (1 + g\delta)c]\frac{\partial q}{\partial \rho} - (g + g'\delta)\frac{\partial \delta}{\partial \rho}cq + \lambda\frac{\partial R^e}{\partial \rho}, \tag{3.31}$$
$$\frac{\partial \mathcal{L}}{\partial \lambda} = R^e - \bar{R}, \tag{3.32}$$
$$\frac{\partial \mathcal{L}}{\partial \mu} = 1 - \tau. \tag{3.33}$$

If one assumes that the solution entails an interior tax rate such that $\mu = 0$, one eliminates the Lagrange parameter λ by combining equations (3.30) and (3.31) and one takes the first-order condition of equation (3.5) into account, one obtains the result. ■

The right-hand side of equation (3.22) is the ratio at which both instruments can be substituted for each other holding expected revenue constant. The left-hand side denotes the ratio of the associated welfare effects.

The result can easily be interpreted if it is rearranged to

$$\frac{[p-(1-g\delta)c]\frac{\partial q}{\partial \tau}-\bar{r}\tau\frac{\partial \delta}{\partial \tau}cq}{\frac{\partial R^e}{\partial \tau}}=\frac{[p-(1+g\delta)c]\frac{\partial q}{\partial \rho}-\bar{r}\tau\frac{\partial \delta}{\partial \rho}cq}{\frac{\partial R^e}{\partial \rho}}. \tag{3.34}$$

It says that the marginal welfare loss of the last unit of revenue must be equal for both instruments.

The result needs some further clarifications. Equation (3.22) only holds for an interior solution. Nevertheless, it is also possible that it is optimal to set the highest possible tax rate $\tau = 1$. In order to see this, the first-order condition for τ needs to be investigated in greater detail.

By using the results on the comparative statics (and the first-order condition for the evasion choice, equation (3.5)) equation (3.30) can be rewritten as

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial \tau} &= (p-(1+g\delta)c)\frac{p+p'q-(1+\bar{r}\delta)c}{(1-\tau)(2p'+p''q)}-\frac{\bar{r}^2\tau cq}{2g'+g''\delta} \\ &+\lambda\left[(p-(1+\bar{r}\delta)c)\left(q+\frac{p+p'q-(1+\bar{r}\delta)c}{(1-\tau)(2p'+p''q)}\tau\right)\right. \\ &+\left.\left(\frac{p+p'q-(1+\bar{r}\delta)c}{(1-\tau)(2p'+p''q)}p'-\frac{\bar{r}^2c}{2g'+g''\delta}\right)\tau q\right]-\mu=0\end{aligned} \tag{3.35}$$

This equation is satisfied if no Laffer curve effect is present (and the marginal tax revenue is even positive at $\tau = 1$) and the marginal welfare gain through higher production (first term) is larger than the welfare loss through higher evasion (second term).

Nevertheless, $\tau < 1$ is also a possible solution. In this case, an increase in the tax rate leads to a welfare loss at the margin. This is particularly likely if the gain from tax evasion $\bar{r}$ is relatively high (for example because enforcement is difficult and costly).

This result qualifies the one by Slemrod & Yitzhaki (1987). They show for the case of direct tax evasion that it is not optimal to set the audit rate at the revenue maximizing rate if individual income taxes may be evaded.

Intuitively, the model may also imply that the government should not enforce taxes strictly because this has a detrimental effect on the production of the monopolist.

3.3 Conclusions

The model has shown that the production decision of a monopolist is not independent of the evasion decision if the monopolist evades taxes by *overreporting* his costs (as in Kreutzer & Lee (1986, 1988), Wang & Conant (1988)) and if he tries to cover its tax evasion activities by spending resources (and the associated concealment cost function is strictly convex as in Cremer & Gahvari (1993)). The reason for this is that the possibility to evade taxes operates like a random production subsidy administered through tax evasion. The special mode by which tax evasion occurs ensures that the effective costs to produce additional units of output are decreased providing an additional production incentive.

This result shows, in particular, that the mode by which taxes are evaded is a critical determinant in the discussion of the neutrality profit taxes if they may be evaded. The model shows similarities with Slemrod (2001) in this respect. He also emphasizes the role of the avoidance technology in a framework of individual income tax avoidance with endogenous labor supply. He shows that for particular forms of this avoidance technology, working is subsidized because the associated higher income makes a given income tax avoidance more plausible (Slemrod (2001, p.121) terms this the *avoidance facilitating* effect). This effect does not exist in the present model. To include it the concealment technology could be altered to be of the form $g = g(\delta, q)$ with $\frac{\partial g}{\partial q} < 0$. In the end, the actual form of the evasion or avoidance technology is an empirical question and the relevancy of the presented mode of operation in practical examples is left open for future research.

The paper also derived an equation that characterizes the optimal tax and tax enforcement policy assuming that the government can commit to its policy. In the optimum the marginal welfare loss of the last unit of revenue must be equal for both instruments, the tax and the audit rate. It may even imply that tax evasion may be used deliberately to decrease input costs and, therefore, increase production. This may even be welfare increasing if the total amount of government revenue can be held constant (for example by an appropriate increase in the tax rate). In this respect tax evasion could be seen as a regulatory instrument.

Apart from the empirical aspect of the particular tax evasion mode that has been investigated in this paper the question arises under which circumstances the government should not subsidize production directly but use the possibility to evade taxes. This question has to be left open in the present model. It might be argued that the latter mode gives the government some possibilities to differentiate among firms and, therefore, to discriminate which might not necessarily be true for general production subsidies. This issue is discussed further in the following chapter.

3.A Appendix

Formal Proofs

Proof of Proposition 3.2

Proof: Total differentiation of equations (3.4) and (3.5) leads to the linear equation system:[18]

$$\underbrace{\begin{pmatrix} (1-\tau)(2p'+p''q) & 0 \\ 0 & -(2g'+g''\delta) \end{pmatrix}}_{=:A} \begin{pmatrix} \mathrm{d}q \\ \mathrm{d}\delta \end{pmatrix}$$
$$= \begin{pmatrix} (p+p'q-c-\bar{r}\delta c)\mathrm{d}\tau + \delta c\tau(\zeta \mathrm{d}\rho + \rho \mathrm{d}\zeta) + (1-\tau-(\bar{r}\tau-g)\delta)\mathrm{d}c \\ -\bar{r}\mathrm{d}\tau + \tau(\zeta\mathrm{d}\rho + \rho\mathrm{d}\zeta) \end{pmatrix}$$

where it was used that $\bar{r}\tau = g + g'\delta$ in an optimum.

From

$$|A| = -(1-\tau)(2p'+p''q)(2g'+g''\delta) > 0 \tag{A.36}$$

(if $p'' \leqslant 0$, $g'' \geqslant 0$) one can conclude that[19]

$$\frac{\partial q}{\partial \tau} \stackrel{s}{=} -(p+p'q-c-\bar{r}\delta c)(2g'+g''\delta) \stackrel{s}{=} +1$$

because $p+p'q-c<0$ and $\rho\zeta<1$ in an interior optimum. Furthermore,

$$\begin{aligned} \frac{\partial q}{\partial \rho} &\stackrel{s}{=} -\delta c\tau\zeta(2g'+g''\delta) \stackrel{s}{=} -1, \\ \frac{\partial q}{\partial \zeta} &\stackrel{s}{=} -\delta c\rho\tau(2g'+g''\delta) \stackrel{s}{=} -1, \\ \frac{\partial q}{\partial c} &\stackrel{s}{=} -(1-\tau-(\bar{r}\tau-g)\delta)(2g'+g''\delta) \begin{cases} <0, & \text{if} \quad \tau<\tau^c, \\ =0, & \text{if} \quad \tau=\tau^c, \\ >0, & \text{if} \quad \tau>\tau^c. \end{cases} \end{aligned}$$

The monopolist produces smaller quantities if the marginal production costs rise if the tax rate is low enough. Then $1-\tau$ is large and $g'(\delta)\delta$ is low. Vice versa it produces higher quantities if τ is relatively large because then $1-\tau-g'(\delta)\delta^{*2}<0$. The critical tax rate where $\frac{\partial q}{\partial c}$ switches sign is defined to be τ^c (and $0<\tau^c<1$).

[18] Because of the one-sided separation one could also start with the first-order condition for the evasion decision and then calculate the comparative statics of the production decision subsequently.

[19] The sign $\stackrel{s}{=}$ has to be read *is of the same sign as.*

And for the cost overreporting decision the following comparative statics hold:

$$\frac{\partial\delta}{\partial\tau} \stackrel{s}{=} -(1-\tau)(2p' + p''q)(1-\rho\zeta) \stackrel{s}{=} +1,$$
$$\frac{\partial\delta}{\partial\rho} \stackrel{s}{=} (1-\tau)(2p' + p''q)\delta c\tau\zeta \stackrel{s}{=} -1,$$
$$\frac{\partial\delta}{\partial\zeta} \stackrel{s}{=} (1-\tau)(2p' + p''q)\delta c\rho\tau \stackrel{s}{=} -1.$$

Clearly, in addition $\frac{\partial\delta}{\partial c} = 0$. ■

Example

Suppose that the demand function is given by the linear function

$$p(q) := 1 - q. \tag{A.38}$$

Then a monopolist that cannot (or does not find it optimal to) evade taxes solves the profit maximization problem

$$\max_{q} \pi = (1-\tau)(1-q-c)q \tag{A.39}$$

and produces

$$q^m = \frac{1-c}{2} \tag{A.40}$$

units of output.

If he may evade part of its profit tax liability by overreporting costs and the concealment cost function is given by

$$g(\delta) := \frac{\delta}{2}, \tag{A.41}$$

it solves the optimization problem

$$\max_{q,\delta} \pi = (1-\tau)(1-q-c)q + \left(\bar{r}\tau - \frac{\delta}{2}\right)\delta cq \tag{A.42}$$

and sets

$$q^* = \frac{2(1-c)(1-\tau) + r^2\tau^2 c}{4(1-\tau)} = q^m + \frac{r^2\tau^2 c}{4(1-\tau)}, \tag{A.43}$$
$$\delta^* = \bar{r}\tau. \tag{A.44}$$

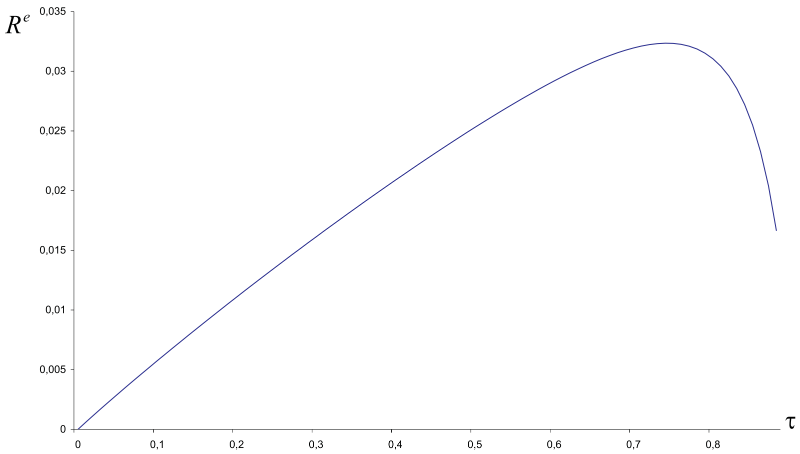


Figure A.2: Laffer curve.

The optimal price is therefore

$$p^* = 1 - \frac{2(1-c)(1-\tau) + r^2\tau^2 c}{4(1-\tau)} = \frac{2(1-\tau) + 2(1-c)\tau - \bar{r}^2\tau^2 c}{4(1-\tau)}. \tag{A.45}$$

The total expected revenue can be calculated to be

$$\begin{aligned} R^e &= \tau\left(\frac{2(1-\tau) + 2(1-c)\tau - \bar{r}^2\tau^2 c}{4(1-\tau)} - c - \bar{r}^2\tau c\right)\frac{2(1-c)(1-\tau) + r^2\tau^2 c}{4(1-\tau)} \\ &= \tau\frac{2(1-2c) + 2c\tau + \bar{r}^2(3\tau-4)\tau c}{4(1-\tau)}\frac{2(1-c)(1-\tau) + r^2\tau^2 c}{4(1-\tau)}. \end{aligned} \tag{A.46}$$

The expected revenue function may be non-monotonous. For example, Figure A.2 depicts it for the parameter constellation $c = 0.5$, $r = 0.25$.

Chapter 4

Tax Evasion and Tax Enforcement in a Duopoly Model[*]

This chapter discusses sales (and unit) tax evasion in a duopoly market. It allows for firm heterogeneity and investigates the impact of tax and enforcement policy as regulatory instruments on the market outcome. In equilibrium, prices and market shares are (in part) determined by the technology that each firm uses to produce and to conceal its evasion activities. Stricter enforcement leads to higher tax costs and is shifted into higher prices as are increases in production costs. If firms differ in the concealment or the production technology, policy may also influence market shares which are relevant for social welfare. The chapter characterizes the socially optimal tax and tax enforcement policy.

4.1 Introduction

Since the seminal paper by Allingham & Sandmo (1972) positive and normative aspects of tax evasion have been discussed extensively (see Cowell (1985a, 2004), Andreoni *et al.* (1998), Franzoni (2000), Slemrod & Yitzhaki (2002) for overviews of the literature). The focus has mainly been on the evasion of individual income taxes and its determinants. The model most widely used incorporates tax evasion into a model of decision making under risk. The main objection to this model concerns the empirical validity of its predictions. In particular, the model seems to

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predict a larger extent of evasion than actually observed.[1]

An obvious question in the theory of *optimal (tax) law enforcement* is why punishment is limited (Becker (1968)). If the punishment for a crime is only harsh enough, people are deterred from violating the law and the punishment does not even have to be carried out. Tax evasion provides a primary example for this line of reasoning. Kolm (1973) has studied the comparative efficiency of costly increases in the probability of audit and costless increases in the fine rate in a model with income tax evasion. The limit case suggests the use of 'infinitely high' penalties with a 'negligible' audit probability: hanging tax evaders with probability zero; thereby deterring all evasion activities.[2,3] Thus, some subsequent papers have assumed that punishment is limited in order to investigate the optimal policy in the presence of tax evasion, such that tax evasion occurs in equilibrium. They have focused on optimal (random) audits (Cremer *et al.* (1990), Cremer & Gahvari (1992, 1993), Richter & Boadway (2005)). An upper bound on punishment has been rationalized by bankruptcy, equity and political considerations and possible imperfections in the auditing process.[4]

However, it is not self-evident that complete compliance to the law is desirable under all possible circumstances. The present chapter shows that optimal government policy may accept some extent of tax evasion even if tax enforcement does not entail any resource costs. The reason is that the possibility to evade taxes also provides an incentive for firms to increase production. Thus, the efficiency gain from the production side in a market with imperfect competition may be larger than the efficiency loss from higher evasion activities.[5] Moreover, the chapter answers the

[1]Note that this argument typically neglects that the probability of detection may vary significantly by income source due to information reporting and possible cross-controls. Bernasconi (1998) provides an alternative theoretical explanation relying on a distinction between orders of risk aversion applied to similar phenomena in financial and insurance markets.

[2]Note that the result of Kolm (1973) is mathematically imprecise as the function that captures the total resource costs does not have a minimum. His result depicts an infimum.

[3]This has also been found to be the optimal policy if the government maximizes expected utility of individuals (Cowell (1989)) or maximizes the rate of economic growth under the restriction of complete honesty (Caballé & Panadés (1997)).

[4]See, among others, Border & Sobel (1987), Cowell (1989), Andreoni (1991b), Pestieau *et al.* (1998), Boadway & Sato (2000), Pestieau *et al.* (2004). Additionally, Polinsky & Shavell (2000) have argued that corrupt bureaucrats may abuse the entailed threat of harsh punishments to collect bribes from someone who makes an honest mistake. A concern for equity may also underlie the argument of Slemrod & Yitzhaki (2002) who argue that high fines mandate higher accuracy in the detection process. Under special circumstances, tax evasion may also increase with higher sanctions (Boadway *et al.* (2002), Borck (2004)).

[5]Stiglitz (1982) provides a normative reason for stochastic taxation acting on the supply-side with similar implications, as well. His paper argues for positive work incentives and is not explicitly concerned with tax evasion. Weiss (1976) also suggests that the possibility to evade may increase an individual's labor supply and may mitigate the distortive effect of taxation on the labor market. However, Yitzhaki (1987) shows that the examples he uses also imply that

question to what degree taxes should be enforced if full enforcement is not optimal.

The chapter also makes contributions to the theory of tax evasion by firms. So far, this literature has discussed implications of tax evasion for efficiency in a market with perfect competition (Virmani (1989)), analyzed oligopolistic (Cournot) competition with risk-averse firms (Marrelli & Martina (1988)) and characterized optimal tax and audit policy in a Ramsey-type model with perfect competition (Cremer & Gahvari (1992, 1993)). A series of articles has discussed the relationship between the compliance and the production decision of a monopolist (Kreutzer & Lee (1986), Wang & Conant (1988), Kreutzer & Lee (1988), Yaniv (1995, 1996), Lee (1998), Panteghini (2000), Goerke & Runkel (2005)) with the particular emphasis of providing sufficient conditions such that a profit tax is still neutral if it may be evaded (and does not affect the production decision).

The present chapter adds to this literature in two respects. First, it discusses the implications of tax evasion for competition in a duopoly market with a horizontally differentiated good and price-setting firms (Bertrand competition). It is shown that tax evasion can be interpreted as a technology that increases expected (marginal) profit for any level of output. It leads to stronger competition in the product market and lower prices even if not all firms find it optimal to evade taxes.[6] As prices are strategic complements even if only one firm can (or finds it optimal to) evade taxes this puts competitive pressure on the competitor to lower his price, too.

Second, the chapter jointly characterizes both optimal tax and optimal enforcement policy in such a market. An explicit model of social welfare is derived that allows to examine the desirability of various combinations of the available policy instruments. Welfare effects arise from the amount of resources spent to conceal tax evasion and from the production structure on the side of firms. On the buyer side it is necessary to include the impact that government policy has on market prices and the ensuing purchasing decisions of consumers in the welfare analysis. The optimal tax and

the initial policy is on the declining side of the Laffer curve and that the welfare improvement is therefore not caused by allowing evasion. Slemrod & Yitzhaki (2002, p.1451) conclude on this issue that "[...] neither the practical nor hypothetical relevance of this point has yet been demonstrated." The conclusion that complete honesty is not necessarily implied by an optimal policy has also been obtained by Cowell (1989). His social welfare function is non-standard and includes the government's concern for inequality. For an expected utility maximizing government and a fixed probability of audit his model also yields the result that full enforcement is optimal if it is costless, see Cowell (1989, p.610). See Andreoni (1992) for an alternative second-best argument.

[6] The chapter, in particular, discusses sales tax evasion. Recently, this has captured increasing attention of economists (Matthews & Lloyd-Williams (2001)). Value-added tax (VAT) fraud is a particular concern (Caplan *et al.* (2003), U.K. Government. HM Customs & Excise (2004)). Nam *et al.* (2003) estimate that about 10% of VAT in Germany was not paid in 2001. Webley *et al.* (2002) investigate the determinants of VAT compliance in the UK and find similar results as for income tax compliance.

enforcement policy takes these welfare effects into account at the margin. Note again that the optimal enforcement policy implies, in particular, that it may be below the full enforcement level even if it is costless.[7]

Lastly, the chapter discusses a political economy model of tax evasion. In this respect, it provides a positive theory of actual tax enforcement in a democracy. Consumers are aware of the impact that different policies have on the prices that they pay and vote on the one that minimizes their expenditures. In the optimum a marginal change in the tax rate per unit of marginal revenue has the same effect on the price as a marginal change in enforcement per unit of marginal revenue.

The chapter is organized as follows: the model is presented in section 4.2. Section 4.2.1 lays out the general assumptions of the model and derives and discusses the demand side of the model. Section 4.2.2 discusses the firms' objective functions, calculates and interprets the first-order conditions for profit maximization. The existence of a Nash equilibrium in prices and evasion is established in section 4.2.3. The comparative statics of the tax, tax enforcement and other parameters are derived and interpreted. Optimal tax and optimal tax enforcement policy is then characterized and discussed in section 4.2.4. Section 4.2.5 derives a condition that holds if the electorate decides on tax and enforcement policy. Finally, section 4.3 concludes and scope for further research is discussed.

4.2 The Model

A duopoly model with price competition and a given degree of product differentiation is considered such that it is possible for one firm to comply with the tax laws even if the other firm does not, stay in the market and sustain competition in equilibrium.[8] The implications of heterogeneity among firms is also investigated. It is the first duopoly model that allows for tax evasion of firms and discusses its impact on competition if these firms are risk-neutral and compete in prices.[9]

[7] The models that include tax evasion of firms so far have neglected normative policy issues; Cremer & Gahvari (1992, 1993) are notable exceptions.

[8] It is not possible to have a compliant and an evading firm in Bertrand competition with a homogenous good (except in the special case where one firm can exactly make up for a production cost disadvantage by a concealment cost advantage) because an evading firm could undercut a compliant firm and serve the whole market.

[9] The paper of Marrelli & Martina (1988) addresses similar positive questions as the present model in the framework of Cournot competition but uses a rather special model of risk-averse firms where the only inefficiency that arises is the risk to be detected by the tax enforcement agency that firms bear if they are involved in tax evasion activities. Firms do not incur additional concealment efforts or produce inefficiently (underground). The assumption of risk aversion of firms may only be rationalized satisfactorily in two environments. First, where it is a manager that decides under some discretion of the owners. And second, when capital markets are imperfect. In

4.2.1 Market Demand and Profits

In order to allow for price competition with the possibility of heterogeneity among firms and profits in equilibrium, a model of spatial competition is used: the linear city. The city consists of a street of length 1. One firm is located at either end of the street at $q = 0$ (call it firm 0) and at $q = 1$ (firm 1).[10] They sell their output on a single market and compete in prices (Bertrand competition).

Assume that the consumers are uniformly distributed along the street with mass 1. They all buy one unit of the good or nothing at all if prices exceed their maximum willingness to pay of $\bar{v} \in \mathbb{R}^+$. Individuals only differ in taste specified by the spot, where the individual is situated which is labelled by q, $0 \leqslant q \leqslant 1$. They are therefore identified by their location. Individual q buys at store 0 if the total costs he incurs are lower than if buying at store 1 and total expenses do not exceed his valuation for the good. The individual that is just indifferent (the marginal consumer) is denoted by $\hat{q} = \hat{q}(p_0, p_1)$. He is located at a point where his total costs that consist of the price and the transportation costs are equal irrespectively of where he buys the good such that

$$\theta T(\hat{q}) + p_0 = \theta T(1 - \hat{q}) + p_1 \tag{4.1}$$

holds, where $T : [0, 1] \to \mathbb{R}_0^+$, $q \mapsto T(q)$, $T(0) = 0$, $0 < T'$ for $q \neq 0$, $0 \leqslant T''$, is the (common) transportation cost function; θ denotes a positive parameter that captures the extent of differentiation in the market and p_j is the price to be paid for the good at store j, $j \in \{0, 1\}$. The location of the marginal consumer is important for the derivation of the demand for each firm: all consumers to the left of $\hat{q}$ buy at firm 0, all to the right buy at firm 1. These demands are labelled q_0 and q_1 respectively ($q_0 := \hat{q}$, $q_1 := 1 - \hat{q}$). It is assumed that the maximum willingness to pay is high enough that every individual buys in equilibrium.

Figure 4.1 illustrates the demand side of the model. The interval $[0, 1]$ represents the city and $\hat{q}$ is determined by the intersection of the total cost curves of $\hat{q}$ depicted at each end of the city for a given combination of prices (p_0, p_1) (note that the transportation cost curves are drawn linearly in Figure 4.1 for simplicity).

both cases, other important issues should be included in the analysis, either the agency problem between management and owners (Chen & Chu (2002), Crocker & Slemrod (2005)) or the source of the capital market imperfection and its implications (Andreoni (1992)). If both discussions are absent, the assumption of risk aversion remains purely technical to guarantee an interior solution of the evasion decision, see Marrelli & Martina (1988, eq. 2). Additionally, Marrelli & Martina (1988) do not discuss optimal policy.

[10] One may consider this model as the second stage of a Hotelling model (Hotelling (1929)) with endogenous product differentiation choice that leads to maximal differentiation, see, for example, d'Aspremont *et al.* (1979). The first model of spatial competition is attributed to Launhardt (1885).

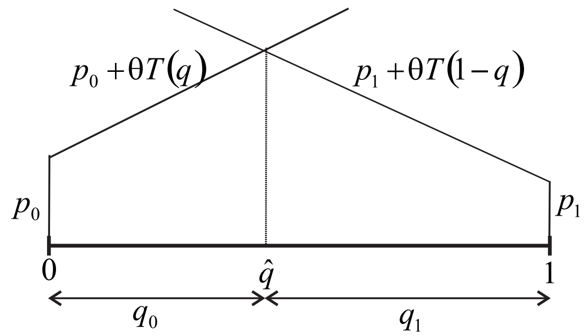


Figure 4.1: Marginal consumer and demand.

Total demand is fixed at one unit. The prices that firms set determine the respective market shares. The following result shows that the model captures the idea that a firm may increase its market share by lowering its own price.

Remark 4.1 (Shape of demand functions). *If both companies serve some customers, the demand functions are strictly decreasing in the own and strictly increasing in the competitor's price but nothing can be said a priori about the sign of their second derivatives. It is determined by the second and third derivative of the transportation cost function. For interesting implications of the model at hand it is sufficient to restrict attention to linear and quadratic transportation cost functions. Note that a linear or quadratic transportation cost function implies that the demand functions are linearly decreasing (increasing) in the own (competitor's) price and consequently the second derivatives of the demand functions vanish:* $\frac{\partial^2 q_j}{\partial p_k \partial p_l} = 0$, $j, k, l \in \{0, 1\}$.

PROOF: An increase in the own price leads to a lower market share (and vice versa for a price increase of the competitor) if both firms serve some customers as the partial derivatives of the demand functions may be calculated to be[11]

$$\frac{\partial q_0}{\partial p_0} = \frac{-1}{\theta[T'(q_0) + T'(q_1)]} = -\frac{\partial q_1}{\partial p_0} < 0, \tag{4.2}$$

$$\frac{\partial q_0}{\partial p_1} = \frac{1}{\theta[T'(q_0) + T'(q_1)]} = -\frac{\partial q_1}{\partial p_1} > 0. \tag{4.3}$$

[11]Function arguments are suppressed where no confusion can arise in the following.

The second derivatives follow from the differentiation of equation (4.2):

$$\frac{\partial^2 q_0}{\partial p_0^2} = \frac{T''(q_0) - T''(q_1)}{\theta[T'(q_0) + T'(q_1)]^2} \frac{\partial q_0}{\partial p_0}, \tag{4.4}$$

$$\frac{\partial^2 q_0}{\partial p_0 \partial p_1} = \frac{T''(q_0) - T''(q_1)}{\theta[T'(q_0) + T'(q_1)]^2} \frac{\partial q_0}{\partial p_1}, \tag{4.5}$$

which implies $\frac{\partial^2 q_0}{\partial p_k \partial p_l} = 0$, for $k, l \in \{0, 1\}$ if T'' is constant (which is the case if T is linear or quadratic). The same holds for firm 1. ■

The model that is developed below does not rely on the assumption that demand functions are linear. However, some sufficient conditions naturally hold for this case. Therefore, although the results may carry over to more general demand forms, linear demands will be used frequently to prove the sufficiency of first-order conditions under simple circumstances.

Two examples are used frequently to illustrate the model and prove the *existence* of several results.

Example 4.1 (Quadratic T) Assume that the transportation cost function is quadratic, $T(q) = A + Bq + Cq^2$, with $0 \leqslant A, B$, $0 < C$. For given prices p_0, p_1 the marginal consumer is located at

$$\hat{q} = \frac{p_1 - p_0 + \theta d}{2\theta d} \text{ with } d := B + C, \tag{4.6}$$

and, therefore, the demand of each firm is given by

$$q_0 = \frac{p_1 - p_0 + \theta d}{2\theta d}, \tag{4.7}$$

respectively

$$q_1 = \frac{p_0 - p_1 + \theta d}{2\theta d}. \tag{4.8}$$

In this special case

$$\frac{\partial q_0}{\partial p_0} = -\frac{1}{2\theta d}. \tag{4.9}$$

Note that $\lim\limits_{\theta \to \infty} \frac{\partial q_0}{\partial p_0} = \lim\limits_{d \to \infty} \frac{\partial q_0}{\partial p_0} = 0.$ *

Firms are assumed to be expected profit maximizers. Profits before tax are given by the difference of sales revenue and production costs $p_j q_j - \gamma_j c(q_j)$, where c denotes the convex production cost function (γ_j a positive parameter that allows variations of the marginal production costs). A firm is subjected to a sales tax at constant rate

τ and may evade some of its tax liability (for example by underreporting sales or overreporting deductible expenses).[12,13] If it does so, it has to shield its evasion from immediate detection through costless cursory examination by spending resources of $\beta_j g(e_j)e_j p_j q_j$,[14] where $g(e_j)$ denotes the marginal concealment cost function, β_j, $\beta_j > 0$, a parameter that allows variations of the marginal concealment costs and e_j, $0 \leqslant e_j \leqslant 1$, the share of the tax base, i.e. here of sales, that is not reported by firm j. Therefore, $e_j p_j q_j$ denotes the amount of evaded sales. Here, g denotes the concealment costs per unit and g itself is assumed to be strictly increasing in e. Thus, the concealment cost function is assumed to be strictly convex. In case the firm is not audited (state of the world 1), which happens with probability $1-\rho$, it earns profits $\pi_j^1 = p_j q_j - \gamma_j c(q_j) - \tau(1-e_j)p_j q_j - \beta_j g(e_j)e_j p_j q_j$.[15] If the firm is audited (state of the world 2), which occurs with probability ρ, it earns profits of $\pi_j^2 = \pi_j^1 - \tau\zeta e_j p_j q_j$, where the last term denotes the fine that the firm has to pay proportionally at rate ζ to the amount of evaded sales (Yitzhaki (1974)).[16]

[12] All results stated here are also valid for the case of a unit tax if price-effects are not decisive, see the Appendix.

[13] Several detailed descriptions and analyses of ways to evade sales taxes exist in the literature (Caplan *et al.* (2003), U.K. Government. HM Customs & Excise (2004)). For example, Fedeli & Forte (1999) discuss joint income-tax and VAT-chain evasion. The particular mode of tax evasion is not at the focus of the discussion here.

[14] This evasion model is developed in Cremer & Gahvari (1993). Several interpretations of this tax evasion model are possible. One neglected aspect that is captured by it is that the cost of concealment function is a possibility to deal with an important difference of tax evasion compared to other crimes. In many cases the taxpayer files a report to the agency and the agency looks for hints on evasion that may lead it to start a more thorough investigation. The cost of concealment function captures the costs that an agent incurs to cover the tracks of his evasion activities. These costs may consist of payments to unscrupulous tax advisers or taxpayer time. An alternative explanation may be costs of corrupting public officials as in Hibbs & Piculescu (2005) where additionally to taxation the circumvention of other government regulations is addressed. Niepelt (2005) develops a dynamic model where convex concealment cost function arises endogenously.

[15] The probability of audit is assumed to be constant. Clearly, this is a simplification. As far as the decision on actual audits is concerned it is also unlikely that the enforcement agency audits at random. In the U.S., for example, the IRS employs a range of methods to detect evaders. In particular, it uses the results of its program of intensive audits: the Taxpayer Compliance Measurement Program (TCMP). On the basis of its results it assigns to each tax report a likelihood that it is incorrect. Andreoni *et al.* (1998) state that over 50% of audits are based on this score. According to their calculations, the yield of a random audit is \$289 compared to \$5.500 for non-random ones. Nevertheless, the model uses random audits for simplicity. On theoretical grounds this may be defended by the assumption of the concealment cost function. It is assumed that if an individual incurred the concealment costs there is no hint left that he has evaded taxes. He can only be distinguished from an honest taxpayer by audit.

[16] The model only considers monetary sanctions of law violations. Additionally, there may be other non-monetary incentives to comply with the tax law, see, for example, Cowell & Gordon (1988), Gordon (1989) and Myles & Naylor (1996).

Expected profit, $\pi_j^e = (1-\rho)\pi_j^1 + \rho\pi_j^2$, of firm j is therefore given by:[17]

$$\pi_j^e = [1 - \tau(1-(1-\rho\zeta)e_j) - \beta_j g(e_j)e_j]p_j q_j - \gamma_j c(q_j),\ j \in \{0,1\}. \tag{4.10}$$

The formulation implies that tax evasion leads to an effective tax rate per unit of sales $\tau_j^e := \tau(1-(1-\rho\zeta)e_j)$ that is lower than the statutory rate whenever the expected fine per unit of evaded tax $\rho\zeta$ is lower than 1 (the gain per unit of tax concealed) which is a necessary condition for tax evasion to occur (see below).

Example 4.1 continued. Assume, furthermore, that the marginal concealment cost function is linear $g(e) = \frac{e}{2}$ and the production cost function quadratic $c(q) = \frac{q^2}{2}$. Expected profit of firm 0 and firm 1 respectively then amounts to

$$\pi_0^e = \left(1 - \tau(1-\bar{r}e_0) - \frac{\beta_0 e_0^2}{2}\right)\frac{p_1 - p_0 + \theta d}{2\theta d}p_0 - \frac{\gamma_0}{2}\left(\frac{p_1 - p_0 + \theta d}{2\theta d}\right)^2, \tag{4.11}$$

$$\pi_1^e = \left(1 - \tau(1-\bar{r}e_1) - \frac{\beta_1 e_1^2}{2}\right)\frac{p_0 - p_1 + \theta d}{2\theta d}p_1 - \frac{\gamma_1}{2}\left(\frac{p_0 - p_1 + \theta d}{2\theta d}\right)^2, \tag{4.12}$$

where $\bar{r} := 1 - \rho\zeta$. $\ast$

4.2.2 Tax Evasion and Optimal Prices

The first-order Kuhn-Tucker-conditions for profit maximization that allow for evasion optima at the full compliance boundary are

$$\frac{\partial \pi_j^e}{\partial p_j} = 0, \tag{4.13}$$

$$\frac{\partial \pi_j^e}{\partial e_j} \leqslant 0,\ e_j \geqslant 0,\ e_j\frac{\partial \pi_j^e}{\partial e_j} = 0, \tag{4.14}$$

and the partial derivatives of the expected profit function may be calculated to be

$$\frac{\partial \pi_j^e}{\partial p_j} = (1 - \tau_j^e - \beta_j g(e_j)e_j)\left(q_j + p_j\frac{\partial q_j}{\partial p_j}\right) - \gamma_j c'(q_j)\frac{\partial q_j}{\partial p_j}, \tag{4.15}$$

$$\frac{\partial \pi_j^e}{\partial e_j} = [(1-\rho\zeta)\tau - \beta_j(g(e_j) + g'(e_j)e_j)]p_j q_j. \tag{4.16}$$

[17] A conceptual advantage of a tax evasion model with imperfect competition is that it does not imply that actual profits are positive if and only if the firm is not audited. Models of perfect competition do not specify how fines should be paid in case of an audit. This may still be possible out of profits in the present model if the fines are not too high.

In the optimum, prices are set such that the expected marginal revenue after tax is equal to the marginal costs (consisting of production and concealment costs). Each firm takes into account that raising its own price decreases its market share.

The evasion decision implies that the firm minimizes the expected tax payments net of concealment costs. Firms evade taxes until the marginal expected tax saving is equal to the marginal costs of tax evasion in an interior optimum. If the concealment costs on the first unit exceed the marginal tax saving, the firm reports truthfully.

Note that the optimal choice of firm j only depends on the pricing decision of its competitor and on the evasion decision only indirectly via its impact on its competitor's pricing decision and vice versa.

Remark 4.2 (Market power). *Firms charge a price above marginal production costs if* $0 < \tau_j^e + \beta_j g(e_j)e_j < 1$*. Sufficient conditions for this are that the marginal concealment costs are sufficiently large and* $\tau < 1$*.* △

PROOF: The first-order condition for p_j is given as before

$$(1 - \tau_j^e - \beta_j g(e_j)e_j)\left(q_j + p_j \frac{\partial q_j}{\partial p_j}\right) = \gamma_j c'(q_j)\frac{\partial q_j}{\partial p_j}. \tag{4.17}$$

Therefore, the optimal price p_j satisfies

$$(1 - \tau_j^e - \beta_j g(e_j)e_j)q_j = -[(1 - \tau_j^e - \beta_j g(e_j)e_j)p_j - \gamma_j c'(q_j)]\frac{\partial q_j}{\partial p_j} \tag{4.18}$$

and if $0 < \tau_j^e + \beta_j g(e_j)e_j < 1$ the left-hand side of equation (4.17) is positive. Therefore, the right-hand side has to be positive, too, i.e. $(1 - \tau_j^e - \beta_j g(e_j)e_j)p_j > \gamma_j c'(q_j)$ which implies $p_j > \gamma_j c'(q_j)$. $\tau < 1$ is sufficient to guarantee that $\tau_j^e + \beta_j g(e_j)e_j < 1$ because the firm evades taxes to reduce its effective tax payment net of concealment costs. Thus, $\tau_j^e + \beta_j g(e_j)e_j \leqslant \tau$ always holds. If $\frac{1-\tau}{g(1)} < \beta_j$ the following inequalities hold $\tau + \beta_j g(1) < 1 \Rightarrow \tau + \beta_j g(e_j)e_j < 1 \Rightarrow 0 < 1 - \tau^e - \beta_j g(e_j)e_j$. ■

Note that the general formulation allows that the effective tax payment of a firm (even net of concealment costs) may be negative, i.e. $\tau_j^e + \beta_j g(e_j)e_j < 0$. A necessary condition for this to be the case is that the firm may reduce its expected tax payments to be negative $\tau_j^e < 0$. If firms differ in the evasion possibilities, this may be the case even with positive tax revenue overall if not all firms can reduce their expected tax payments in such a radical way. Nevertheless, a sufficient condition to exclude $\tau_j^e + \beta_j g(e_j)e_j < 0$ is that the concealment cost function is sufficiently convex. The following discussion focuses on the latter case.

Note that the evasion decision is separable from the production decision (pq may be cancelled from equation (4.16)) but not the other way round. It is sufficient to equate the marginal expected gain of one unit of evaded sales that is solely determined by policy parameters to its marginal costs because they are both proportionally affected by the amount of sales (note that concealment costs are linear in the tax base). The production decision depends on the marginal return to production which consists of the marginal revenue from sales and from tax evasion (or put differently: the marginal revenue net of tax including the expected reduction of the statutory tax rate due to evasion). The latter in turn is affected by the marginal costs of concealment and the extent to which the firm incurs in tax evasion.

Example 4.1 continued. Because of the (one-sided) separability between evasion and production choices each firm's profit maximization problem can be solved in succession starting with the tax evasion decision. The first-order condition for e_j equates the marginal return of tax evasion to its marginal costs

$$\frac{\partial \pi_0^e}{\partial e_0} = (\tau \bar{r} - \beta_0 e_0) \frac{p_1 - p_0 + \theta d}{2\theta d} p_0 = 0, \tag{4.19}$$

$$\frac{\partial \pi_1^e}{\partial e_1} = (\tau \bar{r} - \beta_1 e_1) \frac{p_0 - p_1 + \theta d}{2\theta d} p_1 = 0, \tag{4.20}$$

and can be solved for the shares of evaded sales in the optimum: if $\bar{r} \geqslant 0$ and appropriate lower bounds on β_j guarantee that $e_j \leqslant 1$, then

$$e_0 = \frac{\tau \bar{r}}{\beta_0}, \tag{4.21}$$

$$e_1 = \frac{\tau \bar{r}}{\beta_1}. \tag{4.22}$$

If $\bar{r} \leqslant 0$ or $\tau = 0$ or in the limit case $\beta_j = +\infty$, then $e_j = 0$.

Substituting these optimum evasion values into the profit functions (4.11) and (4.12) respectively yields after some manipulation

$$\pi_0^e = \left(1 - \tau + \frac{(\bar{r}\tau)^2}{2\beta_0}\right) \frac{p_1 - p_0 + \theta d}{2\theta d} p_0 - \frac{\gamma_0}{2} \left(\frac{p_1 - p_0 + \theta d}{2\theta d}\right)^2, \tag{4.23}$$

$$\pi_1^e = \left(1 - \tau + \frac{(\bar{r}\tau)^2}{2\beta_1}\right) \frac{p_0 - p_1 + \theta d}{2\theta d} p_1 - \frac{\gamma_1}{2} \left(\frac{p_0 - p_1 + \theta d}{2\theta d}\right)^2, \tag{4.24}$$

where the term $\frac{(\bar{r}\tau)^2}{2\beta_j}$ results from the fact that the marginal concealment costs are higher than the average concealment costs (the concealment cost function is assumed to be strictly convex) and is therefore the expected reduction in the statutory tax rate due to evasion. It may also be interpreted as the marginal return from increasing sales by one unit that results from evading part of its tax liability. Although $0 < \frac{(\bar{r}\tau)^2}{2\beta_j}$ it may well be assumed that the government collects a positive amount of taxes from any tax evading firm. This is the case if the marginal concealment cost parameter is above the lower bound $\beta_j > \frac{\bar{r}^2\tau}{2} =: \underline{\beta}$ such that $\tau - \frac{(\bar{r}\tau)^2}{2\beta_j}$ is still positive.

The first-order conditions for optimal prices equate the marginal revenue from sales with its marginal production costs:

$$\left(1 - \tau + \frac{(\bar{r}\tau)^2}{2\beta_0}\right)\frac{2p_0 - p_1 - \theta d}{2\theta d} = \gamma_0 \frac{p_1 - p_0 + \theta d}{4(\theta d)^2}, \tag{4.25}$$

$$\left(1 - \tau + \frac{(\bar{r}\tau)^2}{2\beta_1}\right)\frac{2p_1 - p_0 - \theta d}{2\theta d} = \gamma_1 \frac{p_0 - p_1 + \theta d}{4(\theta d)^2}. \tag{4.26}$$

The concealment costs determine the effective tax costs of each firm. $*$

Example 4.2 (One evading firm) Assume that T, g and c are given as in Example 4.1. If it is assumed additionally that $\beta_1 = +\infty$ such that firm 1 reports truthfully, expected profit of firm 0 and firm 1 respectively then amount to (after substituting the optimum evasion values)

$$\pi_0^e = \left(1 - \tau + \frac{(\bar{r}\tau)^2}{2\beta_0}\right)\frac{p_1 - p_0 + \theta d}{2\theta d}p_0 - \frac{\gamma_0}{2}\left(\frac{p_1 - p_0 + \theta d}{2\theta d}\right)^2, \tag{4.23}$$

$$\pi_1 = (1 - \tau)\frac{p_0 - p_1 + \theta d}{2\theta d}p_1 - \frac{\gamma_1}{2}\left(\frac{p_0 - p_1 + \theta d}{2\theta d}\right)^2, \tag{4.27}$$

with interpretations as in Example 4.1.

The first-order conditions for optimal prices in this case are

$$\left(1 - \tau + \frac{(\bar{r}\tau)^2}{2\beta_0}\right)\frac{2p_0 - p_1 - \theta d}{2\theta d} = \gamma_0 \frac{p_1 - p_0 + d}{4(\theta d)^2}, \tag{4.25}$$

$$(1 - \tau)\frac{2p_1 - p_0 - \theta d}{2\theta d} = \gamma_1 \frac{p_0 - p_1 + \theta d}{4(\theta d)^2}. \tag{4.28}$$

$*$

In order to build an intuition on the relationship between the evasion and the production decision, the strategic interaction is neglected for the moment.

Remark 4.3 (Evasion and pricing). *If tax evasion is profitable and firm j's marginal profit is decreasing in the own price (in particular* $\left.\frac{\partial^2 \pi_j^e}{\partial p_j^2}\right|_{e_j=0} < 0$ *has to hold), a firm's optimal price is lower than in the case of full enforcement.* $\triangle$

PROOF: If $0 < e_j$, then $\frac{\partial \pi_j^e}{\partial e_j} = 0$ which using equation (4.16) can be rewritten as

$$(1 - \rho\zeta)\tau - \beta_j g(e_j) = \beta_j g'(e_j)e_j. \tag{4.29}$$

Substituting the right-hand side of equation (4.29) into equation (4.15) and setting the resulting condition for the optimal price equal to zero, one obtains

$$(1-\tau+\beta_j g'(e_j)e_j^2)\left(q_j+p_j\frac{\partial q_j}{\partial p_j}\right)=\gamma_j c'(q_j)\frac{\partial q_j}{\partial p_j}, \tag{4.30}$$

which (if $g' > 0$) cannot hold at the same price that equates marginal revenue to marginal costs in the case of full enforcement. Rearranging this equation to

$$(1-\tau)\left(q_j+p_j\frac{\partial q_j}{\partial p_j}\right)-\gamma_j c'(q_j)\frac{\partial q_j}{\partial p_j}=-\beta_j g'(e_j)e_j^2\left(q_j+p_j\frac{\partial q_j}{\partial p_j}\right) \tag{4.31}$$

shows that the left-hand side which equals zero in the case of full enforcement now is positive in the optimum. Therefore, if the marginal profit is falling in p_j (a sufficient condition is that c is linear and T linear or quadratic), p_j is lower than in the case of full enforcement. ■

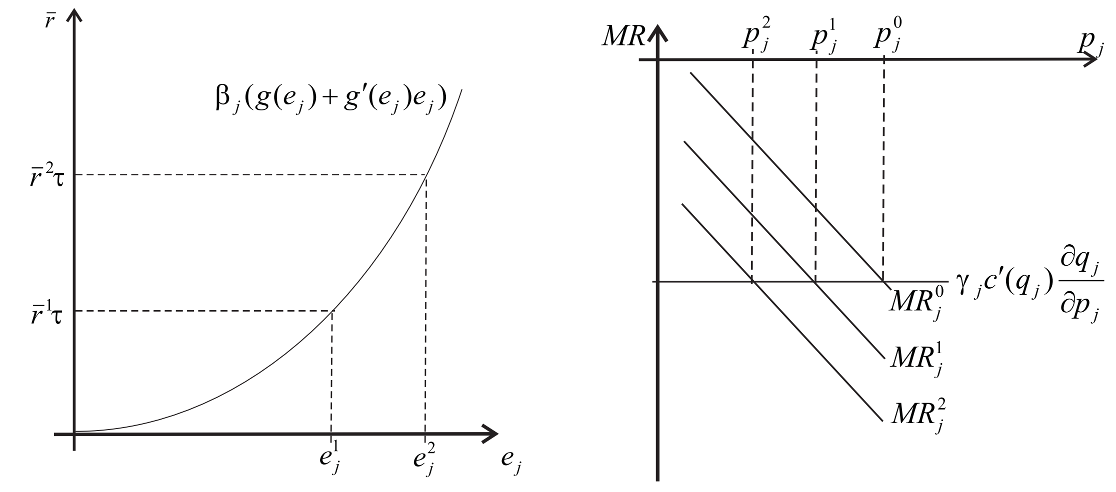
Although the argument in the preceding remark neglects the strategic interaction and is actually valid only for the monopolistic case, the result and its intuition carry over to the duopoly case for marginal changes in enforcement policy (the proof is formulated in section 4.2.3).

The reason for this result is that a firm can lower its effective tax payment net of concealment costs if evasion is profitable and, therefore, increases the after-tax marginal revenue. This advantage is translated into higher supply and lower prices.

Note that the possibility of tax evasion also leads to lower prices if the concealment cost function is linear, i.e. g is constant. Then the firm either evades all of its tax liability ($e = 1$) or nothing at all ($e = 0$). The evasion function which gives the optimal share of evaded sales depending on exogenous parameters has a discontinuity where $1 = \rho\zeta$. Nevertheless, the optimal price in case of tax evasion (excluding the knife-edge case where exactly $1 = \rho\zeta$) is lower also in the case where $g' = 0$ because the effective tax rate is lower. To avoid the discontinuity of the evasion function in the following $g' > 0$, for $e_j > 0$, is assumed throughout.

The solution of the profit maximization problem of a firm is illustrated by Figure 4.2. Figure 4.2(a) shows the determination of the optimal share of evaded sales for firm j, e_j, for two possible scenarios; one where the expected marginal gain of one unit of evaded tax, $\bar{r} := 1 - \rho\zeta$, is low, denoted by $\bar{r}^1$, and the other where it is high, $\bar{r}^2$. The respective shares of evaded sales are ordered $e_j^1 < e_j^2$.

Figure 4.2(b) depicts the marginal revenue curve (net of the full nominal tax payment), $\mathrm{MR}_j^0 = (1-\tau)\left(q_j + p_j\frac{\partial q_j}{\partial p_j}\right)$, as strictly falling (in fact, linearly falling as in the case of a linear or quadratic transportation cost function). The intersection of the marginal revenue and the marginal cost curve (which is depicted constant as it



(a) Evasion decision.

(b) Production decision.

Figure 4.2: Profit maximum.

is for a linear production and linear or quadratic transportation cost function) defines the optimal price p_j^0 if no tax evasion exists. If $0 < e_j$, this shifts the marginal revenue curve down by $\beta_j g'(e_j)e_j^2 p_j q_j$ units.[18] For $\bar{r}^i$ the relevant marginal revenue curve is denoted by MR_j^i, $i \in \{1, 2\}$. Consequently, the intersections of the new marginal revenue and cost curve at point p_j^1 and p_j^2 are to the left of p_j^0 with $p_j^2 < p_j^1$.

Lemma 4.1 (Unique Optimum). *Equations (4.13) and (4.14) are sufficient to guarantee a unique solution p_j, $0 < p_j$, e_j, $0 \leqslant e_j < 1$, (for any given price of the other firm), if*

1. $c'(0) = 0$, $\lim_{q \to 1} c'(q) = +\infty$ *and*

2. *the profit function π_j^e is strictly concave, i.e. if the Hessian matrix for π_j^e*

$$H_j(p_0, p_1, e_j) := \begin{pmatrix} \frac{\partial^2 \pi_j^e}{\partial p_j^2}(p_0, p_1, e_j) & \frac{\partial^2 \pi_j^e}{\partial e_j \partial p_j}(p_0, p_1, e_j) \\ \frac{\partial^2 \pi_j^e}{\partial p_j \partial e_j}(p_0, p_1, e_j) & \frac{\partial^2 \pi_j^e}{\partial e_j^2}(p_0, p_1, e_j) \end{pmatrix} \tag{4.32}$$

is negative definite $\forall (p_0, p_1, e_j) \in (\mathbb{R}_0^+)^2 \times [0, 1]$. *If* $\frac{\partial^2 \pi_j^e}{\partial p_j^2}(p_0, p_1, e_j) < 0$, *this is*

[18] Note that the price is the endogenous variable in this model. It may be reformulated in terms of quantities such that as usual the marginal revenue (as well as the marginal cost) curve lies in the positive quadrant of the analogous diagram and is shifted up.

the case if and only if:

$$0 < \frac{\partial^2 \pi_j^e}{\partial p_j^2}(p_0,p_1,e_j)\frac{\partial^2 \pi_j^e}{\partial e_j^2}(p_0,p_1,e_j) - \left[\frac{\partial^2 \pi_j^e}{\partial p_j \partial e_j}(p_0,p_1,e_j)\right]^2$$
$$\Leftrightarrow \Bigg[(1-\tau_j^e - \beta_j g(e_j)e_j)\left(2\frac{\partial q_j}{\partial p_j} + p_j\frac{\partial^2 q_j}{\partial p_j^2}\right) - \gamma_j\left(c''(q_j)\left(\frac{\partial q_j}{\partial p_j}\right)^2 + c'(q_j)\frac{\partial^2 q_j}{\partial p_j^2}\right)\Bigg]\beta_j(2g'(e_j) + g''(e_j)e_j) < 0. \tag{4.33}$$

In particular, if $g'' \geqslant 0$ the second bracket is positive and the first bracket has to be negative. Sufficient conditions are that the transportation cost function is linear or quadratic and $\frac{1-\tau}{g(1)} < \beta_j$, $\tau < 1$ and $c'' \geqslant 0$. □

Assume that for each firm the conditions for a unique interior optimum (given the competitor's choice) are satisfied. If the sufficient conditions for an optimum hold, a higher production of one firm triggers a higher production of its competitor.

Lemma 4.2 (Increasing reaction function). *The reaction correspondence of firm 0, $\Phi_0 : \mathbb{R}_0^+ \times [0,1] \to \mathbb{R}_0^+ \times [0,1]$, $(p_1, e_1) \mapsto \Phi_0(p_1, e_1)$, is a continuous* function *that satisfies $0 < \Phi_0(0,\cdot)$ and is increasing in p_1.* □

Proof: See the Appendix. ■

The analogous result holds for firm 1. Note that this is the standard strategic relationship in a Bertrand model.

Define the maximal optimum price of firm j by $\bar{p}_j := \Phi_j(\bar{v}, \cdot)$. Now, it is possible to proof the existence of a Nash equilibrium and derive some properties that every Nash equilibrium satisfies.

4.2.3 Nash Equilibrium

The following result holds:

Proposition 4.1 (Existence of a pure strategy Nash equilibrium). *If each firm's profit maximization problem has a unique solution and the marginal concealment cost and production cost functions satisfy $c'(0) = 0$, $\lim_{q \to 1} c'(q) = +\infty$, $\lim_{e \to 1} g(e) = 1$, there exists a Nash equilibrium in pure strategies $(p_0, p_1, e_0, e_1) \in (\mathbb{R}^+)^2 \times [0,1)^2$.* □

PROOF: A Nash equilibrium has to satisfy: $(p_0, e_0) = \Phi_0(p_1, e_1)$ and $(p_1, e_1) = \Phi_1(p_0, e_0)$, i.e.

$$(p_1, e_1) = \Phi_1(\Phi_0(p_1, e_1)). \tag{4.34}$$

Brouwer's fixed point theorem can be applied to the problem at hand as follows: define $\mathbb{D} := [0, \bar{p}] \times [0, 1] \subset \mathbb{R}^2$, where $\bar{p} := \max\{\bar{p}_0, \bar{p}_1\}$. Then $\mathbb{D}$ is compact and convex. Define $f : \mathbb{D} \to \mathbb{D}$, $(p_1, e_1) \mapsto f(p_1, e_1) := \Phi_1(\Phi_0(p_1, e_1)) - (p_1, e_1)$. f is a continuous function (for any e_0). Therefore, f has a fixed point in $\mathbb{D}$.

If the transportation cost function is linear or quadratic, every Nash equilibrium is in *pure* strategies because it is a strong equilibrium (Harsanyi (1973)) (see Remark 4.1).

There is no Nash equilibrium where $p_0 = 0 = p_1$ because $0 < \Phi_0(0, \cdot), \Phi_1(0, \cdot)$ if $c'(0) = 0$. At every $0 < p_0, p_1$: $0 < e_0$, $0 < e_1$ if $0 = g'(0)$. The limit conditions guarantee that (p_j, e_j) are finite respectively strictly below 1. ■

The Nash equilibrium is stable if the slope of the reaction functions is below one.

Example 4.1 continued. The equation system (4.25), (4.26) can be solved for the following prices in the Nash equilibrium (sums and products are always over $j = 0, 1$, $\hat{\tau} := (1 - \tau)$):

$$p_0 = \frac{(\bar{r}^2\tau^2\theta d + \beta_0\gamma_0 + 2\hat{\tau}\beta_0\theta d)(3\bar{r}^2\tau^2\theta d + 2\beta_1\gamma_1 + 6\hat{\tau}\beta_1\theta d)}{3\bar{r}^4\tau^4\theta d + 2\hat{\tau}\prod\beta_j(\sum\gamma_j + 6\hat{\tau}\theta d) + \bar{r}^2\tau^2(\sum\beta_j\gamma_j + 6\hat{\tau}\theta d\sum\beta_j)}, \tag{4.35}$$

$$p_1 = \frac{(\bar{r}^2\tau^2\theta d + \beta_1\gamma_1 + 2\hat{\tau}\beta_1\theta d)(3\bar{r}^2\tau^2\theta d + 2\beta_0\gamma_0 + 6\hat{\tau}\beta_0\theta d)}{3\bar{r}^4\tau^4\theta d + 2\hat{\tau}\prod\beta_j(\sum\gamma_j + 6\hat{\tau}\theta d) + \bar{r}^2\tau^2(\sum\beta_j\gamma_j + 6\hat{\tau}\theta d\sum\beta_j)}. \tag{4.36}$$

The resulting equilibrium quantity for firm 0 is

$$q_0 = \frac{(\bar{r}^2\tau^2 + 2\hat{\tau}\beta_0)(3\bar{r}^2\tau^2 + 2\beta_1\gamma_1 + 6\hat{\tau}\beta_1\theta d)}{2(3\bar{r}^4\tau^4\theta d + 2\hat{\tau}\prod\beta_j(\sum\gamma_j + 6\hat{\tau}\theta d) + \bar{r}^2\tau^2(\sum\beta_j\gamma_j + 6\hat{\tau}\theta d\sum\beta_j))}. \tag{4.37}$$

Note that $q_0 = \frac{1}{2}$ if the firms are identical, i.e. $\beta_0 = \beta_1$ and $\gamma_0 = \gamma_1$. *

Proposition 4.2 (Comparative statics). *If the transportation cost function is linear or quadratic, $0 \leqslant g''$, $0 \leqslant c''$ the comparative statics in a pure strategy Nash equilibrium (p_0, p_1, e_0, e_1) with respect to the policy parameters are of the following signs:*

$$\frac{\partial p_j}{\partial \tau} > 0,\ \frac{\partial p_j}{\partial \rho} > 0,\ \frac{\partial p_j}{\partial \zeta} > 0, \tag{4.38}$$

$$\frac{\partial e_j}{\partial \tau} > 0,\ \frac{\partial e_j}{\partial \rho} < 0,\ \frac{\partial e_j}{\partial \zeta} < 0. \tag{4.39}$$

Under the same conditions the comparative statics with respect to market and firm parameters are of the following signs:

$$\frac{\partial p_j}{\partial \theta} > 0,\ \frac{\partial p_j}{\partial \beta_j} > 0,\ \frac{\partial p_j}{\partial \beta_{-j}} > 0,\ \frac{\partial p_j}{\partial \gamma_j} > 0,\ \frac{\partial p_j}{\partial \gamma_{-j}} > 0, \tag{4.40}$$

$$\frac{\partial e_j}{\partial \theta} = 0,\ \frac{\partial e_j}{\partial \beta_j} < 0,\ \frac{\partial e_j}{\partial \beta_{-j}} = 0,\ \frac{\partial e_j}{\partial \gamma_j} = 0,\ \frac{\partial e_j}{\partial \gamma_{-j}} = 0, \tag{4.41}$$

where $-j = 1$ if $j = 0$ and $-j = 0$ if $j = 1$. □

Proof: See the Appendix. ■

Like in Cremer & Gahvari (1992, 1993), Anderson *et al.* (2001) taxes are shifted. An increase in the statutory tax rate decreases the effective marginal revenue. The firm increases its price to match its marginal costs.[19] The same argument explains why stricter enforcement leads to higher prices.

The share of evaded sales increases in the tax rate because the marginal tax savings per unit of evaded sales increase. On the contrary, if tax enforcement is intensified the marginal gain of evasion decreases leading to a lower share of evaded sales.

An increase in the differentiation parameter leads to higher prices. This is the case because consumers are more reluctant to switch to the other producer and firms are left with higher market power (the responsiveness of demand decreases). Increases in the marginal production or concealment costs lead to higher effective production costs. This cost increase is translated into higher prices.

The share of evaded sales falls in the marginal concealment costs because it is increasingly difficult not to be an obvious evader. This effect also explains why this increases the optimal price: the effective rate of taxation increases.[20] Similarly to an increase in the marginal production costs it leads to a fall in marginal profit which is matched by a price increase.

The strategic relationship between prices (prices are strategic complements) explains why the same effects are present for changes on the side of the competitor. An increase in the marginal (production or concealment) costs leads to higher prices independently of the side of the market that is subject to this change (nevertheless,

[19] A detailed discussion of the shifting decision in a market with imperfect competition and quantity (Cournot) competition without tax evasion is contained in Katz & Rosen (1985), Myles (1987), Stern (1987), Hamilton (1999). Policy incidence in models with product differentiation is discussed by Gruenspecht (1984) for export subsidies and by Cremer & Thisse (1994) for commodity taxes. Empirical results are provided by Fershtman *et al.* (1999). The latter models allow for endogenous determination of quality.

[20] This result on *tax incidence* is similar to the one obtained by Kesselman (1989) in a model with two sectors – formal and informal – with two distinct goods.

the change may affect both firms to a different degree).[21]

Changes in the parameters that do not affect the marginal return of evasion for a firm do not change the share of evaded sales because of the one-sided separability between production and evasion activity.

Note that a firm is subjected to additional competitive pressure if tax evasion is possible and lowers its price even if it does not evade taxes itself in case enforcement policies are relaxed. Even for $e_j = 0$ (for example because $\beta_j = +\infty$) $\frac{\partial p_j}{\partial \rho}, \frac{\partial p_j}{\partial \zeta} > 0$ if $e_{-j} > 0$. The underlying reason is again that prices are strategic complements in the Bertrand (or Hotelling) model. A laxer enforcement policy allows the evading firm to lower its price. This puts competitive pressure on its competitor to lower its price, too.

The comparative statics of Proposition 4.2 do not imply that the total amount of evaded sales falls in the enforcement parameters (or the tax rate). In fact, the total amount of evaded sales may as well increase in enforcement.

Corollary 4.1 (Total amount of evasion). *The total amount of evaded sales may in- as well as decrease with stricter enforcement.* □

Proof: The total amount of evaded sales is given by

$$E := \sum_{j=0}^{1} e_j p_j q_j = q_0(e_0 p_0 - e_1 p_1) + e_1 p_1 \tag{4.42}$$

with partial derivative with respect to increases in the probability of audit (analogous results are obtained for increases in the fine rate)

$$\frac{\partial E}{\partial \rho} = \sum_{j=0}^{1} \left(\frac{\partial e_j}{\partial \rho} p_j q_j + e_j \frac{\partial p_j}{\partial \rho} q_j + e_j p_j \frac{\partial q_j}{\partial \rho} \right). \tag{4.43}$$

The first term in the sum captures the reduction in evasion due to the effect that the share of evaded sales decreases in the probability of audit. The next term leads in the opposite direction. Increases in enforcement lead to higher prices. The last term is indeterminate in sign and may therefore add to any of the previous two effects depending on how stricter enforcement influences the market shares (a detailed discussion of the influence of policy parameters on market shares follows below).

[21] An interesting special case occurs if the firms are identical. In this case, the comparative statics of the equilibrium price with respect to a change in the marginal production costs takes the form $\frac{\partial p}{\partial \gamma} = \frac{2}{3} c'(\frac{1}{2})$. Thus, $\frac{2}{3}$ of the cost increase are shifted into prices the remaining $\frac{1}{3}$ reduces profits.

Nevertheless, even for identical firms where there is no quantity effect, i.e. the market shares are identical for any choice of policy, the increase in the price may overweigh the decrease in the share of evaded sales.

The existence of this case is proved using Example 4.1.

Example 4.1 continued. Assume that the two firms are identical, i.e. assume that $\gamma_0 = \gamma_1 = \gamma$ and $\beta_0 = \beta_1 = \beta$. Then, $e_j = e$, $p_j = p$, for $j \in \{0, 1\}$, and the partial derivative of the total amount of evaded sales (in this case $E = ep = \frac{(\theta d[\bar{r}^2\tau^2+2(1-\tau)\beta]+\gamma\beta)\bar{r}\tau}{(\bar{r}^2\tau^2+2(1-\tau)\beta)\beta}$) can be calculated to be

$$\begin{aligned}\frac{\partial E}{\partial \rho} &= \frac{\partial e}{\partial \rho}p + e\frac{\partial p}{\partial \rho} \\ &= \frac{2(\theta d[\bar{r}^2\tau^2 + 2(1-\tau)\beta] + \gamma\beta)\bar{r}^2\tau^3\zeta}{(\bar{r}^2\tau^2 + 2(1-\tau)\beta)^2\beta} \\ &\quad - \frac{2\theta d\bar{r}^2\tau^3\zeta + (\theta d[\bar{r}^2\tau^2 + 2(1-\tau)\beta] + \gamma\beta)\zeta\tau}{(\bar{r}^2\tau^2 + 2(1-\tau)\beta)\beta}\end{aligned} \tag{4.44}$$

which is zero for

$$\hat{\theta} := \frac{\theta d}{\gamma} = \frac{\beta(\bar{r}^2\tau^2 - 2(1-\tau)\beta)}{4(1-\tau)\beta[\bar{r}^2\tau^2 + (1-\tau)\beta] + \bar{r}^4\tau^4}. \tag{4.45}$$

The denominator is always positive; the numerator switches sign at $\hat{\beta} = \frac{\bar{r}^2\tau^2}{2(1-\tau)}$ and is positive for $\beta < \hat{\beta}$. Furthermore, $\frac{\partial^2 E}{\partial\rho\partial\theta} = -\frac{\tau\zeta d}{\beta} < 0$. Therefore, if $0 < \theta < \hat{\theta}$ and $0 < \beta < \hat{\beta}$ the total amount of evaded sales increases in the audit probability. This case may occur even if firms face a positive effective tax rate. A sufficient condition for this case is that $\frac{1}{2} < \tau$ because then $\frac{\bar{r}^2\tau}{2} < \beta < \frac{\bar{r}^2\tau^2}{2(1-\tau)}$ always holds. *

This proves finally Corollary 4.1. ■

An analogous result for increases in the tax rate may also occur. E has partial derivative with respect to τ

$$\frac{\partial E}{\partial \tau} = \sum_{j=0}^{1} \left(\frac{\partial e_j}{\partial \tau}p_j q_j + e_j\frac{\partial p_j}{\partial \tau}q_j + e_j p_j\frac{\partial q_j}{\partial \tau} \right). \tag{4.46}$$

Although the first two terms in the sum are positive the overall effect may in principle be negative due to a reallocation of market shares (last term in the sum). The existence of a case where $\frac{\partial E}{\partial \tau} < 0$ and the firms face a positive effective tax rate could not be proved neither for the general case nor for the two examples. For Example 4.2 $\frac{\partial E}{\partial \tau} > 0$ always holds.

The possibility to evade taxes has implications for competition in the market. A competitive advantage can arise out of different production and evasion possibilities. For example, if the production cost functions of both firms are identical, the concealment cost function is decisive for relative prices and market shares.

Corollary 4.2 (Competitive advantage). *If the production cost functions are identical, i.e. $\gamma_0 = \gamma_1$, and if one company (without loss of generality firm 0) has a competitive advantage in the evasion of taxes in the sense that for a given share of evaded sales its marginal concealment costs are always lower than the ones of its competitor, i.e. $\beta_0 < \beta_1$, and the marginal concealment costs of the first unit are lower than the expected tax savings, this firm evades a larger share of sales, sets a lower price and serves a larger share of the market in equilibrium:*

$$\text{if } \gamma_0 = \gamma_1,\ \beta_0 < \beta_1 \text{ then } e_0 > e_1,\ p_0 < p_1 \text{ and } q_1 < \frac{1}{2} < q_0. \tag{4.47}$$

□

Proof: If $\beta_0 < \beta_1$, then $e_0 > e_1$ follows from the comparative statics. The rest of the proof is by contradiction. Suppose both firms set the same price $p_0 = p_1 = p$. Then they share the market equally, $\hat{q} = \frac{1}{2}$. But if the first-order conditions for firm 1 are satisfied at (p, e_1), the marginal profit of firm 0 is positive because it optimally evades a larger share of sales and pays a lower effective tax rate net of concealment costs. In effect, firm 0 has an incentive to lower its price: $p_0 < p_1$. This implies $q_1 < \frac{1}{2} < q_0$. ■

Better possibilities to evade taxes may be interpreted as a cost or technology advantage. They allow to reduce the tax costs for any given policy. This cost reduction is (partly) translated into lower prices to increase market share.

A related result has been derived by Trandel (1992) who shows that market power declines if consumers have an incentive to shop across borders because they can evade paying taxes there. Consumers benefit indirectly from tax evasion and find it optimal to enforce taxes only laxly. A related empirical result is obtained by Goolsbee (2000) who shows that different sales tax rates explains the propensity to buy online where sales tax is difficult to enforce.

Also interesting for the normative implications is to note that if one firm has a production cost advantage this may be (partly) compensated by the competitor if he has an advantage in the tax concealment technology.

Example 4.1 continued. Market shares are equal ($q_0 = \frac{1}{2}$) if a firm can make up for a concealment cost disadvantage by a superior production cost technology and vice versa. This is the case if the production and concealment cost parameters satisfy

$$\frac{\gamma_0}{\gamma_1} = \frac{(\bar{r}^2\tau^2 + 2(1-\tau)\beta_0)\beta_1}{(\bar{r}^2\tau^2 + 2(1-\tau)\beta_1)\beta_0}. \tag{4.48}$$

Note that this condition is independent of θ.

The following limit results hold $\lim_{\theta\to\infty} q_0 = \lim_{d\to\infty} q_0 = \frac{1}{2}$, $\lim_{\gamma_j\to\infty} q_j = 0$. In the first case the firm's technology is irrelevant for market shares, each firm serves its own market. In the latter case company j does not even produce because of its bad production technology. No concealment technology can make up for it.

Similarly, if $\gamma_1 < \gamma_0 < +\infty$, firm 0 can always make up for this production cost disadvantage if its concealment technology is sufficiently good. Market shares are equal if

$$\beta_0 = \frac{\beta_1\gamma_1\bar{r}^2\tau^2}{2(1-\tau)(\gamma_0-\gamma_1)\beta_1+\gamma_0\bar{r}^2\tau^2}. \tag{4.49}$$

An analogous result is obtained in the example with a single evading company.

Example 4.2 continued. If only firm 0 evades taxes, market shares are equal if

$$\frac{\gamma_0}{\gamma_1} = \frac{\bar{r}^2\tau^2+2(1-\tau)\beta_0}{2(1-\tau)\beta_0}, \tag{4.50}$$

$$\beta_0 = \frac{\bar{r}^2\tau^2\gamma_1}{2(1-\tau)(\gamma_0-\gamma_1)} \tag{4.51}$$

respectively. *

Of particular interest are the changes in the market shares with respect to policy changes *at the margin.*

Corollary 4.3 (Market shares and policy). *If the transportation cost function is linear or quadratic, the impact of a policy change on the equilibrium market shares depends on the relative impact of this policy change on each firm's optimal price:*[22]

$$\frac{\partial q_0}{\partial\kappa} \overset{s}{=} \frac{\partial p_1}{\partial\kappa} - \frac{\partial p_0}{\partial\kappa}, \text{ for } \kappa\in\{\tau,\rho,\zeta\}. \tag{4.52}$$

In particular,

1. *if in addition $c'' \geqslant 0$ and $\beta_0 = \beta_1$, then*

$$\frac{\partial q_0}{\partial\kappa} \overset{s}{=} \gamma_0-\gamma_1, \text{ for } \kappa\in\{\tau,\rho,\zeta\}; \tag{4.53}$$

[22]The sign $\overset{s}{=}$ has to be read *is of the same sign as.*

2. *if in addition* $c'' \geqslant 0$, g *is linear and* $0 < \beta_0 < \beta_1 = +\infty$, *then* $\exists\ \hat{\gamma}_1$, $0 < \hat{\gamma}_1$ *such that*

$$\frac{\partial q_0}{\partial \tau} \begin{cases} < 0 \text{ if } \gamma_1 < \hat{\gamma}_1, \\ = 0 \text{ if } \gamma_1 = \hat{\gamma}_1, \\ > 0 \text{ if } \hat{\gamma}_1 < \gamma_1, \end{cases} \tag{4.54}$$

3. *if in addition* $c'' \geqslant 0$, g *is linear and* $0 = \gamma_0 < \gamma_1 < +\infty$, *then* $\exists\ \hat{\beta}_1$, $0 < \hat{\beta}_1$, *such that*

$$\frac{\partial q_0}{\partial \tau} \begin{cases} < 0 \text{ if } \beta_1 < \hat{\beta}_1, \\ = 0 \text{ if } \beta_1 = \hat{\beta}_1, \\ > 0 \text{ if } \hat{\beta}_1 < \beta_1. \end{cases} \tag{4.55}$$

4. *if in addition* $c'' \geqslant 0$, g *is linear and* $0 < \beta_0 < \beta_1 = +\infty$, *then*

$$\frac{\partial q_0}{\partial \nu} < 0, \ \text{ for } \nu \in \{\rho, \zeta\}, \tag{4.56}$$

5. *if in addition* $c'' \geqslant 0$, g *is linear and* $0 = \gamma_0 < \gamma_1 < +\infty$, *then*

$$\frac{\partial q_0}{\partial \nu} > 0, \ \text{ for } \nu \in \{\rho, \zeta\}. \tag{4.57}$$

□

PROOF: The impact of a change in parameter κ on q_0 can be reformulated as $\frac{\partial q_0}{\partial \kappa} = \frac{\partial q_0}{\partial p_0}\frac{\partial p_0}{\partial \kappa} + \frac{\partial q_0}{\partial p_1}\frac{\partial p_1}{\partial \kappa} = -\frac{\partial q_0}{\partial p_0}\left(\frac{\partial p_1}{\partial \kappa} - \frac{\partial p_0}{\partial \kappa}\right)$, for $\kappa \in \{\tau, \rho, \zeta\}$, where the last step is valid for linear and quadratic transportation cost functions because $\frac{\partial q_0}{\partial p_0} = -\frac{\partial q_0}{\partial p_1}$ holds in this case (see equations (4.2),(4.3) above). This proves equation (4.52).

From the comparative statics follows that

$$\begin{aligned} \frac{\partial p_1}{\partial \nu} - \frac{\partial p_0}{\partial \nu} &= \frac{\nu^- \tau}{a_{11}a_{33} - a_{13}a_{31}} \left[e_1 \left(q_1 + p_1 \frac{\partial q_1}{\partial p_1} \right) (a_{11} + a_{13}) \right. \qquad (4.58) \\ &\qquad \left. - e_0 \left(q_0 + p_0 \frac{\partial q_0}{\partial p_0} \right) (a_{31} + a_{33}) \right] \\ &\stackrel{s}{=} e_0 \left(q_0 + p_0 \frac{\partial q_0}{\partial p_0} \right) (1 - \tau_1^e - \beta_1 g(e_1) e_1) \qquad (4.59) \\ &\quad - e_1 \left(q_1 + p_1 \frac{\partial q_1}{\partial p_1} \right) (1 - \tau_0^e - \beta_0 g(e_0) e_0). \end{aligned}$$

For $\beta_0 = \beta_1 = \beta \Rightarrow e_0 = e_1 = e$ and $\tau_0^e = \tau_1^e = \tau^e$. This term simplifies to

$$\begin{aligned} \frac{\partial p_1}{\partial \nu} - \frac{\partial p_0}{\partial \nu} &= \frac{(1 - \bar{r}e)\nu^- \tau e (1 - \tau^e - \beta g(e) e)}{a_{11}a_{33} - a_{13}a_{31}} \left[q_0 + p_0 \frac{\partial q_0}{\partial p_0} - q_1 - p_1 \frac{\partial q_1}{\partial p_1} \right] \\ &\stackrel{s}{=} \gamma_0 - \gamma_1, \end{aligned}$$

for $\nu \in \{\rho, \zeta\}$ and $\nu^- = \zeta$ if $\nu = \rho$ and $\nu^- = \rho$ if $\nu = \zeta$. The last $\overset{s}{=}$ follows from the first-order condition (4.15) for the case of a linear production cost function.

Analogously,

$$\frac{\partial p_1}{\partial \tau} - \frac{\partial p_0}{\partial \tau} = \frac{1}{a_{11}a_{33} - a_{13}a_{31}} \left[(1 - \bar{r}e_1) \left(q_1 + p_1 \frac{\partial q_1}{\partial p_1} \right) (a_{11} + a_{13}) \right. \tag{4.60}$$
$$\left. -(1 - \bar{r}e_0) \left(q_0 + p_0 \frac{\partial q_0}{\partial p_0} \right) (a_{31} + a_{33}) \right]$$
$$\overset{s}{=} (1 - \bar{r}e_0) \left(q_0 + p_0 \frac{\partial q_0}{\partial p_0} \right) (1 - \tau_1^e - \beta_1 g(e_1)e_1) \tag{4.61}$$
$$- (1 - \bar{r}e_1) \left(q_1 + p_1 \frac{\partial q_1}{\partial p_1} \right) (1 - \tau_0^e - \beta_0 g(e_0)e_0).$$

For $\beta_0 = \beta_1 =: \beta \Rightarrow e_0 = e_1 =: e$ and $\tau_0^e = \tau_1^e =: \tau^e$ this simplifies to

$$\frac{\partial p_1}{\partial \tau} - \frac{\partial p_0}{\partial \tau} = \frac{(1 - \bar{r}e)(1 - \tau^e - \beta g(e)e)}{a_{11}a_{33} - a_{13}a_{31}} \left[q_0 + p_0 \frac{\partial q_0}{\partial p_0} - q_1 - p_1 \frac{\partial q_1}{\partial p_1} \right]$$
$$\overset{s}{=} \gamma_0 - \gamma_1.$$

This proves equation (4.53).

Items 2.-5. are proved using Example 4.1 and 4.2.

Example 4.1 continued. Assume that $0 = \gamma_0 < \gamma_1 < +\infty$. Then it follows that

$$\frac{\partial q_0}{\partial \tau} = \frac{3\beta_1\gamma_1\theta d(\beta_1 - \bar{r}^2\tau)}{(3\bar{r}^2\tau^2\theta d + 6(1-\tau)\beta_1\theta d + \beta_1\gamma_1)^2}. \tag{4.62}$$

Consequently,

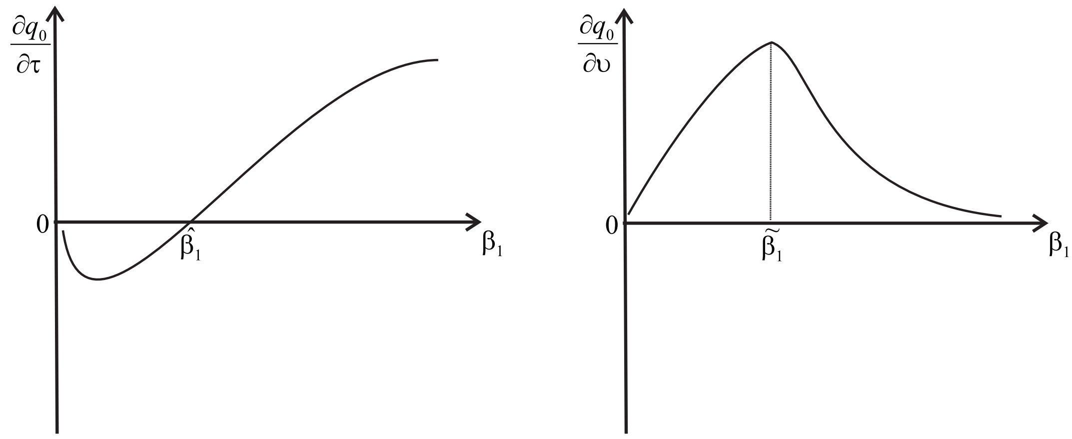
$$\frac{\partial q_0}{\partial \tau} \begin{cases} < 0 \text{ if } \beta_1 < \hat{\beta}_1, \\ = 0 \text{ if } \beta_1 = \hat{\beta}_1, \\ > 0 \text{ if } \hat{\beta}_1 < \beta_1, \end{cases}$$

where $\hat{\beta}_1 = \bar{r}^2\tau$.

The behavior of $\frac{\partial q_0}{\partial \tau}$ is qualitatively depicted in Figure 4.3(a) in this case ($\lim_{\beta_1 \to \infty} \frac{\partial q_0}{\partial \tau} = \frac{3\theta d\gamma_1}{(6(1-\tau)\theta d + \gamma_1)^2} > \frac{\partial q_0}{\partial \tau}$ for any finite β_1 and $\lim_{\beta_1 \to 0} \frac{\partial q_0}{\partial \tau} = 0$ hold).
If $0 = \gamma_0 < \gamma_1 < +\infty$,

$$\frac{\partial q_0}{\partial \nu} = \frac{3\theta d\beta_1\gamma_1\tau^2\bar{r}\nu^-}{(3\theta d(\bar{r}^2\tau^2 + 2(1-\tau)\beta_1) + \beta_1\gamma_1)^2} > 0, \tag{4.64}$$

where $\nu \in \{\rho, \zeta\}$ and $\nu^- = \zeta$ if $\nu = \rho$ and $\nu^- = \rho$ if $\nu = \zeta$. It is qualitatively depicted in Figure 4.3(b) ($\lim_{\beta_1 \to 0} \frac{\partial q_0}{\partial \nu} = 0 = \lim_{\beta_1 \to \infty} \frac{\partial q_0}{\partial \nu}$ hold such that the



(a) Case 3. (b) Case 5.

Figure 4.3: Market shares and policy if $0 = \gamma_0 < \gamma_1 < +\infty$.

existence of a $\tilde{\beta}_1$ where $\frac{\partial q_0}{\partial \nu} = 0$ is assured). *

Example 4.2 continued. If $\beta_0 < \beta_1 = +\infty$,

$$\frac{\partial q_0}{\partial \tau} \begin{cases} < 0 \text{ if } \gamma_1 < \hat{\gamma}_1, \\ = 0 \text{ if } \gamma_1 = \hat{\gamma}_1, \\ > 0 \text{ if } \hat{\gamma}_1 < \gamma_1, \end{cases} \tag{4.65}$$

where

$$\hat{\gamma}_1 = \frac{12\theta d \beta_0 \gamma_0 (1-\tau)^2 (\beta_0 - \bar{r}^2 \tau)}{12(1-\tau)^2 \beta_0^2 \theta d + 2(2-\tau)\beta_0 \gamma_0 \bar{r}^2 \tau + 12(1-\tau)\beta_0 \theta d \bar{r}^2 \tau^2 + 3\theta d \bar{r}^4 \tau^4} \tag{4.66}$$

and it can be shown that $\frac{\partial^2 q_0}{\partial \tau \partial \gamma_1} > 0$ at $\hat{\gamma}_1$ such that to the left of $\hat{\gamma}_1$ $\frac{\partial q_0}{\partial \tau}$ is negative and to the right of $\hat{\gamma}_1$ $\frac{\partial q_0}{\partial \tau}$ is positive.

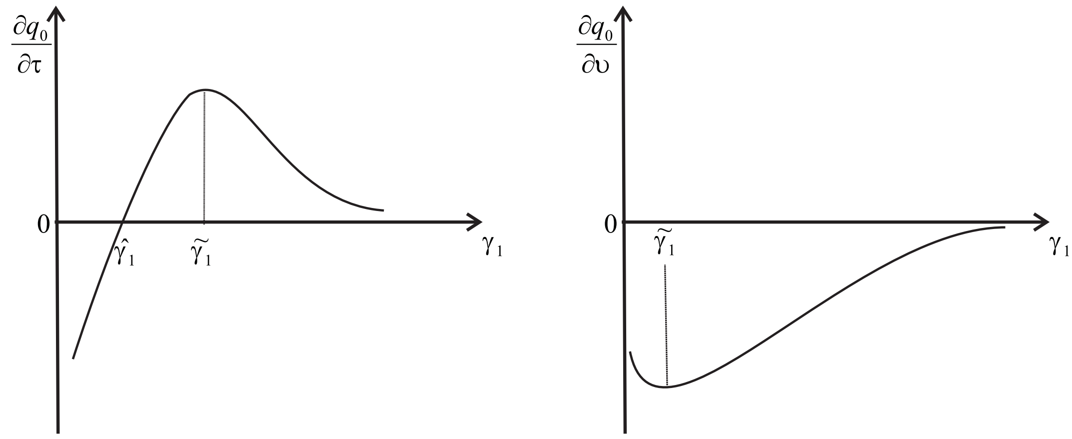
The behavior of $\frac{\partial q_0}{\partial \tau}$ is qualitatively depicted in Figure 4.4(a) in this case ($\lim\limits_{\gamma_1 \to 0} \frac{\partial q_0}{\partial \tau} < 0$, $\lim\limits_{\gamma_1 \to \infty} \frac{\partial q_0}{\partial \tau} = 0$ holds).

Additionally,

$$\frac{\partial q_0}{\partial \nu} = -\frac{4\bar{r}\tau\nu^-(1-\tau)(3(1-\tau)\theta d + \gamma_1)\beta_0\gamma_0}{(6\theta d(1-\tau)(\bar{r}^2\tau^2 + 2\beta_0(1-\tau)) + 2(1-\tau)(\gamma_0+\gamma_1)\beta_0 + \gamma_1 \bar{r}^2 \tau^2)^2} < 0 \tag{4.67}$$

and $\lim_{\gamma_1 \to 0} \frac{\partial q_0}{\partial \nu} = -\frac{3\tau^2 \bar{r}\nu^- \theta d \beta_0 \gamma_0}{(3\theta d \tau^2 \bar{r}^2 + 6(1-\tau)\theta d + \beta_0 \gamma_0)^2} < 0$, $\lim_{\gamma_1 \to \infty} \frac{\partial q_0}{\partial \nu} = 0$. See Figure 4.4(b) for a qualitative illustration of this case. *

This finishes the proof of Corollary 4.3. ■



(a) Case 2. (b) Case 4.

Figure 4.4: Market shares and policy if $0 < \beta_0 < \beta_1 = +\infty$.

If a change in the parameter κ increases prices, equation (4.52) says that the market share of firm 0 increases if and only if the parameter change increases the price of firm 0 less than the price of firm 1. If both firms have identical marginal concealment costs, the change in the equilibrium market share is solely determined by differences in the marginal production costs. If both firms also have identical marginal production costs, they always serve the same market share irrespectively of policy.

However, an advantage in either aspect of technology can be made up by a disadvantage in the other (if $\gamma_j < +\infty$) as has been shown above. The discussed cases show that this is also the case for changes at the margin. Note that $\frac{\partial q_0}{\partial \tau}$, $\frac{\partial q_0}{\partial \rho}$ and $\frac{\partial q_0}{\partial \zeta}$ are bounded in both examples.

4.2.4 Social Optimum

An interesting and important question now concerns optimal government policy. As has been shown by the comparative statics results, the government may be able to influence the market outcome by its tax and tax enforcement policy. The question that is answered in this section is what tax, audit and fine rate the government should choose in order to reach a social optimum. It is assumed that the government does not have any other regulatory instruments available. Therefore, it might use the available tax and enforcement instruments to influence the market outcome.[23]

[23] This analysis does neither suggest that there are no other instruments available for regulating a duopoly nor that regulation should be down via the tax evasion channel. But it neglects these instruments to show that tax and enforcement policy may also be used to some extent in this respect if other instruments are imperfect. The broader question of the role of tax evasion in a

From the outset the optimization problem has therefore a second-best character. The possible tax schedules are restricted to linear ones, the possible audit rates are restricted to constant ones such that any firm is audited with equal probability independent of its tax report or some other (outside) information. Additionally, the fine rate may only be constant.

It is assumed that the government chooses its policy and can commit to it. This is the approach also followed by Cremer & Gahvari (1992, 1993, 1994).[24] As the government sets its policy it takes any reactions of the firms and the consumers into account.

The government minimizes the welfare losses in the economy. These welfare losses occur because

- tax evasion causes inefficiencies. If tax evasion is part of the optimum (for example because full enforcement is infinitely costly), firms spend resources to conceal their evasion activities;
- production might be inefficient. If the marginal production costs of the firms differ in equilibrium, production should be carried out to a larger extent by the more efficient firm, i.e. the firm with the lower marginal production costs; and
- consumption leads to inefficiencies. Only consumers located at $q = 0$ and at $q = 1$ obtain their preferred good. If the market is not equally split and the transportation costs of the marginal consumer of buying from firm 0 are lower (higher) than of buying from firm 1, the total transportation costs can be reduced if firm 0 serves more (less) customers.

There may be even other welfare losses that are not investigated. Most prominently, it is assumed that the government can raise sufficient funds to finance a predetermined level of public services. Therefore, no welfare loss from underprovision of public goods arises.

larger set of regulatory instruments is not investigated.

[24] A methodologically different strand of research has analyzed more sophisticated investigation schemes to maximize expected government receipts or to enforce truthful reporting at low costs (Graetz *et al.* (1984), Graetz & Reinganum (1986), Greenberg (1984), Reinganum & Wilde (1984, 1985), Scotchmer (1987), Mookherjee & P'ng (1989), Sánchez & Sobel (1993), Cremer & Gahvari (1996)). The results critically depend on whether the tax enforcement agency can commit to an announced audit rule, or whether it cannot commit. Interestingly, in the former case, some papers find that the optimal policy is characterized by an income threshold below which each individual is subjected to an audit with a constant probability and above which individuals are never audited.

All these social costs affect the sum of consumer and producer surplus.[25] The following discussion shows how these welfare losses are balanced at the margin in the optimum.

The Government's Objective Function

An exogenous revenue requirement is imposed later. Therefore, the following surplus discussion neglects utility derived from consumption of public goods.

An individual that is situated at spot q and buys at firm 0 obtains a consumer surplus of $\bar{v} - \theta T(q) - p_0$ from consumption of the private good in question. The consumer surplus of all customers of firm 0 from consumption of the good therefore is given by

$$\mathrm{CS}_0 = (\bar{v} - p_0)q_0 - \theta \int_0^{q_0} T(x)\mathrm{d}x. \tag{4.68}$$

Analogously, a consumer at spot q that buys from firm 1 obtains consumer surplus of $\bar{v} - \theta T(1-q) - p_1$. Aggregation leads to the consumer surplus of consumers that buy at firm 1 of

$$\begin{aligned}\mathrm{CS}_1 &= (\bar{v} - p_1)q_1 - \theta \int_{q_0}^{1} T(1-x)\mathrm{d}x \\ &= (\bar{v} - p_1)(1-q_0) - \theta \int_{q_0}^{1} T(1-x)\mathrm{d}x. \end{aligned} \tag{4.69}$$

Total consumer surplus is therefore given by

$$\begin{aligned}\mathrm{CS} &= \mathrm{CS}_0 + \mathrm{CS}_1 \\ &= \bar{v} - p_0 q_0 - p_1(1-q_0) - \theta \left[\int_0^{q_0} T(x)\mathrm{d}x + \int_{q_0}^{1} T(1-x)\mathrm{d}x \right]. \end{aligned} \tag{4.70}$$

Producer surplus is equal to the sum of expected profits net of tax as given by

[25] The approach adopted here to model the objective function is the standard one of total rent (surplus) maximization. This approach neglects possible objectives concerned with horizontal and vertical equity. See Cowell (1989) for an alternative.

equation (4.10)

$$\begin{aligned}\mathrm{PS} =\ & \pi_0^e + \pi_1^e \\ =\ & \sum_{j=0}^{1} \left([1 - \tau(1 - \bar{r}e_j) - \beta_j g(e_j) e_j] p_j q_j - \gamma_j c(q_j) \right). \end{aligned} \tag{4.71}$$

The total surplus given by the sum of consumer and producer surplus $S = \mathrm{CS} + \mathrm{PS}$ therefore is given by

$$\begin{aligned} S =\ & \bar{v} - p_0 q_0 - p_1(1 - q_0) - \theta \left[\int_0^{q_0} T(x)\mathrm{d}x + \int_{q_0}^{1} T(1-x)\mathrm{d}x \right] \\ & + \sum_{j=0}^{1} \left([1 - \tau(1 - \bar{r}e_j) - \beta_j g(e_j) e_j] p_j q_j - \gamma_j c(q_j) \right) \\ =\ & \bar{v} - \theta \left[\int_0^{q_0} T(x)\mathrm{d}x + \int_{q_0}^{1} T(1-x)\mathrm{d}x \right] - \sum_{j=0}^{1} \left(\beta_j g(e_j) e_j p_j q_j + \gamma_j c(q_j) \right) \\ & - \tau \sum_{j=0}^{1} (1 - \bar{r}e_j) p_j q_j . \end{aligned} \tag{4.72}$$

The maximization of S holding total expected tax and fine revenue constant[26] is equivalent to the minimization of L, the total costs associated with production and consumption of a unit of output, where

$$L = \theta \left[\int_0^{q_0} T(x)\mathrm{d}x + \int_{q_0}^{1} T(1-x)\mathrm{d}x \right] + \sum_{j=0}^{1} \left(\beta_j g(e_j) e_j p_j q_j + \gamma_j c(q_j) \right). \tag{4.73}$$

Therefore, if the government has to finance a given expected revenue it does so to minimize the welfare losses that are associated with the consumption of output (the first term of equation (4.73)), the total concealment costs (first term in the sum) and total production costs (second term in the sum).

The next section discusses the government's budget constraint and derives the

[26] To be more precise, expected revenue has to be constant and it has to be equal to the actual revenue. By the law of large numbers this is guaranteed if there is a large number of markets like the one discussed here. Otherwise individuals may be worse off even if expected revenue is constant if the level of the public good provision is stochastic and individuals are risk-averse with respect to consumption of the public good.

impact of the policy variables on expected revenue.

The Government's Budget Constraint

The expected tax and fine revenue is given by

$$R^e = \tau \sum_{j=0}^{1} (1 - \bar{r} e_j) p_j q_j - \chi(\rho), \tag{4.74}$$

where it is assumed that auditing is costly according to the increasing and strictly convex audit cost function $\chi : [0,1] \to \mathbb{R}_0^+$, $\rho \mapsto \chi(\rho)$ with $\chi(0) = 0$ and if an interior solution of the government's optimization problem should be assured $\lim\limits_{\rho \to 1} \chi(\rho) = +\infty$. An increase in the fine rate may be considered costless.

Before the government's problem is posed, the impact of tax and tax enforcement instruments on expected revenue is discussed to determine under which conditions they are substitutes in raising revenue.

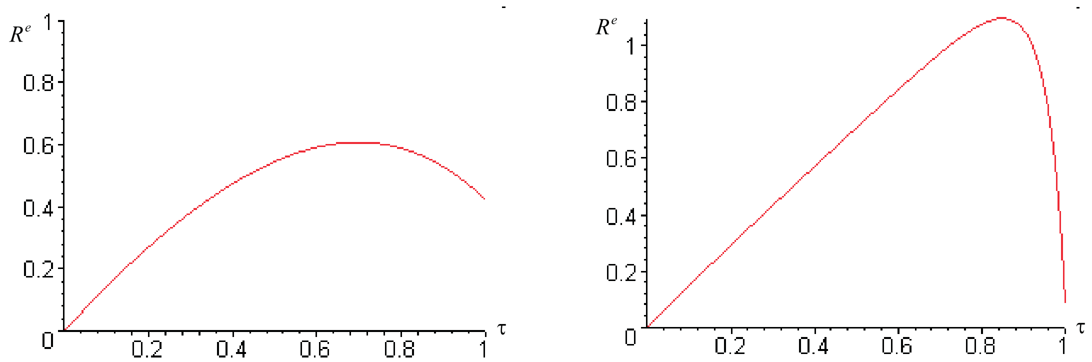
Lemma 4.3 (Effect of tax and enforcement policy on revenue). *In general, total (expected) tax revenue (including fine payments) may be increasing as well as decreasing in the policy parameters. It is increasing in the tax rate if firms are identical and the share of evaded sales does not increase too dramatically if the tax rate rises. It is increasing in enforcement (the audit or the fine rate) if firms are identical and the marginal enforcement costs are not too high.* □

PROOF: The government's expected tax and fine revenue, $R^e = \tau \sum\limits_{j=0}^{1} (1 - \bar{r} e_j) p_j q_j - \chi(\rho)$ has the first derivative with respect to τ

$$\begin{aligned} \frac{\partial R^e}{\partial \tau} &= \sum_{j=0}^{1} \left[(1 - \bar{r} e_j) p_j q_j + \tau \left((1 - \bar{r} e_j) \left(\frac{\partial p_j}{\partial \tau} q_j + p_j \frac{\partial q_j}{\partial \tau} \right) - \bar{r} \frac{\partial e_j}{\partial \tau} p_j q_j \right) \right] \\ &= \sum_{j=0}^{1} [1 - \bar{r} e_j (1 + \eta_{e_j,\tau})] p_j q_j + \tau ((1 - \bar{r} e_j) \left(\frac{\partial p_j}{\partial \tau} q_j + p_j \frac{\partial q_j}{\partial \tau} \right), \end{aligned} \tag{4.75}$$

where $\eta_{e_j,\tau} := \frac{\tau}{e_j} \frac{\partial e_j}{\partial \tau}$ denotes the elasticity of the evaded share of sales with respect to the nominal tax rate.

The direct revenue effect of an increase in the tax rate (holding the tax base constant) is always positive and the effect that higher tax rates are shifted into higher prices adds to this effect. An opposing effect is also present: the term $-\bar{r} \sum_{j=0}^{1} e_j \eta_{e_j,\tau} p_j q_j$ captures the loss in marginal revenue that is brought about by



(a) Laffer curve with identical firms.

(b) Laffer curve with heterogenous firms.

Figure 4.5: Laffer curves.

a higher share of evaded sales (holding prices constant). Additionally, there may be a revenue effect from the reallocation of market shares. This effect is captured by the term $\tau \sum_{j=0}^{1}(1-\bar{r}e_j)p_j\frac{\partial q_j}{\partial \tau}$. As has been shown above some form of heterogeneity is necessary for $\frac{\partial q_j}{\partial \tau}$ to be nonzero (the effect is not present if the firms are identical: $\beta_0 = \beta_1$ and $\gamma_0 = \gamma_1$). This revenue effect is positive if the policy change increases (decreases) the market share of firm 0 and firm 0 initially demands a higher (lower) price than firm 1. In order to see this, rearrange $p_0\frac{\partial q_0}{\partial \tau} + p_1\frac{\partial q_1}{\partial \tau} = (p_0 - p_1)\frac{\partial q_0}{\partial \tau}$. The following conclusions arise.

1. If firms are identical, $\frac{\partial R^e}{\partial \tau}$ is positive if and only if the loss in marginal revenue due to increased evasion activities is not too large.

2. If firms are heterogenous, $\frac{\partial R^e}{\partial \tau}$ may be positive as well as negative. The Laffer curve may arise from a reallocation of market shares or from a large impact on the share of evaded sales.

The proof of the existence of Laffer curves uses Examples 4.1 and 4.2.

Example 4.1 continued. For identical firms and the parameter constellation $\beta_0 = \beta_1 = \gamma_0 = \gamma_1 = 1$, $\bar{r} = 0.9$, the relationship of the tax rate and the total revenue is illustrated by Figure 4.5(a). $\ast$

Example 4.2 continued. For heterogenous firms and the parameter constellations $\beta_0 = \gamma_0 = \gamma_0 = 1$, $\beta_1 = +\infty$, $\bar{r} = 0.99$, the relationship of the tax rate and the total revenue is illustrated by Figure 4.5(b). $\ast$

Similar results hold for the first derivative of R^e with respect to the enforcement parameters:

$$\frac{\partial R^e}{\partial \rho} = \tau \sum_{j=0}^{1} \left[\zeta e_j p_j q_j - \bar{r} \frac{\partial e_j}{\partial \rho} p_j q_j + (1 - \bar{r} e_j) \left(\frac{\partial p_j}{\partial \rho} q_j + p_j \frac{\partial q_j}{\partial \rho} \right) \right] - \chi'(\rho) \quad (4.76)$$

$$\frac{\partial R^e}{\partial \zeta} = \tau \sum_{j=0}^{1} \left[\rho e_j p_j q_j - \bar{r} \frac{\partial e_j}{\partial \zeta} p_j q_j + (1 - \bar{r} e_j) \left(\frac{\partial p_j}{\partial \zeta} q_j + p_j \frac{\partial q_j}{\partial \zeta} \right) \right]. \quad (4.77)$$

Positive revenue effects result from the higher payments of detected tax evaders and from the deterrent effect on evasion. Additionally, the price increases add to the positive revenue effect. If the marginal enforcement costs are not too high and in case the effect from possible reshuffling of market shares is negligible, $\frac{\partial R^e}{\partial \rho}$ is positive. The same applies for $\frac{\partial R^e}{\partial \zeta}$. It is positive at least in the case of identical firms. ■

The previous result has shown that, although expected government revenue may be increasing as well as decreasing, there is a range of suitably small parameter values where tax and tax enforcement policies are substitutes on the left side of the Laffer curves. Let the government's budget constraint be reflected by a desired tax and fine revenue $\bar{R}$. The government now tries to minimize the costs associated with financing $\bar{R}$. This minimization does not have a solution if it is possible to vary all three policy parameters, especially both tax enforcement parameters ρ and ζ simultaneously. In this latter case any gain from evasion $\bar{r}$, $0 < \bar{r}$, should always be obtained at least costs using a 'large' fine rate and 'negligible' audit probability. Mathematically, a minimum does not exist; the least-cost solution is given by an infimum. To exclude this case the discussion is limited to the case where only the tax and a single enforcement parameter may be varied simultaneously.

The government's problem can therefore be formulated as follows[27]

$$\min_{\tau,\nu} L = \theta \left[\int_0^{q_0} T(x) \mathrm{d}x + \int_{q_0}^{1} T(1-x) \mathrm{d}x \right] + \sum_{j=0}^{1} (\beta_j g(e_j) e_j p_j q_j + \gamma_j c(q_j)) \quad (4.78)$$
$$\text{s.t. } R^e = \bar{R},\ \rho\zeta \leqslant 1,$$

for $\nu \in \{\rho, \zeta\}$.

[27] The restriction $\rho\zeta \leqslant 1$ is due to the fact that any policy where $\rho\zeta > 1$ holds leads to full compliance. For a fixed fine rate full compliance can therefore be reached at minimal costs for $\rho = \frac{1}{\zeta}$. For a fixed audit probability all policies where $\rho\zeta > 1$ can be substituted with a policy where $\zeta = \frac{1}{\rho}$ without loss of generality.

The associated Lagrangian (with λ, μ as the Lagrange multipliers) is

$$\begin{aligned}\mathcal{L} = \theta \left[\int_0^{q_0} T(x)\mathrm{d}x + \int_{q_0}^{1} T(1-x)\mathrm{d}x \right] + \sum_{j=0}^{1} (\beta_j g(e_j) e_j p_j q_j + \gamma_j c(q_j)) \\ + \lambda(\bar{R} - R^e) + \mu(\rho\zeta - 1)\end{aligned} \tag{4.79}$$

with first-order Kuhn-Tucker conditions that allow for the case of an optimum at the full enforcement level

$$\frac{\partial \mathcal{L}}{\partial \tau} = 0, \tag{4.80}$$

$$\frac{\partial \mathcal{L}}{\partial \nu} \geqslant 0, \tag{4.81}$$

$$\frac{\partial \mathcal{L}}{\partial \mu} \leqslant 0 = \frac{\partial \mathcal{L}}{\partial \lambda}, \ \mu(1 - \rho\zeta) = 0, \tag{4.82}$$

where the partial derivatives of the Lagrangian function can be calculated to be

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial \tau} = \theta[T(q_0) - T(1-q_0)]\frac{\partial q_0}{\partial \tau} + \sum_{j=0}^{1} \Bigg[\beta_j \Bigg((g(e_j) + g'(e_j)e_j)\frac{\partial e_j}{\partial \tau} p_j q_j \\ + g(e_j)e_j \left(\frac{\partial p_j}{\partial \tau} q_j + p_j \frac{\partial q_j}{\partial \tau} \right) \Bigg) + \gamma_j c'(q_j)\frac{\partial q_j}{\partial \tau} \Bigg] - \lambda \frac{\partial R^e}{\partial \tau},\end{aligned} \tag{4.83}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial \nu} = \theta[T(q_0) - T(1-q_0)]\frac{\partial q_0}{\partial \nu} + \sum_{j=0}^{1} \Bigg[\beta_j \Bigg((g(e_j) + g'(e_j)e_j)\frac{\partial e_j}{\partial \nu} p_j q_j \\ + g(e_j)e_j \left(\frac{\partial p_j}{\partial \nu} q_j + p_j \frac{\partial q_j}{\partial \nu} \right) \Bigg) + \gamma_j c'(q_j)\frac{\partial q_j}{\partial \nu} \Bigg] - \lambda \frac{\partial R^e}{\partial \nu} + \mu\nu^-,\end{aligned} \tag{4.84}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \bar{R} - R^e, \tag{4.85}$$

$$\frac{\partial \mathcal{L}}{\partial \mu} = \rho\zeta - 1. \tag{4.86}$$

Note first, that the term for the minimization of the total transportation costs drops out if the firms are identical. Policy cannot change the equilibrium market shares and, therefore, cannot change the total transportation costs. In this case, the first-order conditions for the tax and tax enforcement parameters, equations (4.83) and (4.84) simplify to (equation (4.16) has been used to simplify and firm indices are finally dropped)

$$\frac{\partial \mathcal{L}}{\partial \tau} = \sum_{j=0}^{1} \beta_j \left[(g(e_j) + g'(e_j)e_j) \frac{\partial e_j}{\partial \tau} p_j q_j + g(e_j)e_j \frac{\partial p_j}{\partial \tau} q_j \right] - \lambda \frac{\partial R^e}{\partial \tau}$$

$$= \begin{cases} \bar{r}\tau \frac{\partial e}{\partial \tau} p + \beta g(e) e \frac{\partial p}{\partial \tau} - \lambda \frac{\partial R^e}{\partial \tau} & \text{if} \quad \rho\zeta < 1, \\ -\lambda \frac{\partial R^e}{\partial \tau} + \mu\nu^- & \text{if} \quad \rho\zeta = 1, \end{cases} \tag{4.87}$$

$$\frac{\partial \mathcal{L}}{\partial \nu} = \sum_{j=0}^{1} \beta_j \left[(g(e_j) + g'(e_j)e_j) \frac{\partial e_j}{\partial \nu} p_j q_j + g(e_j)e_j \frac{\partial p_j}{\partial \nu} q_j \right] - \lambda \frac{\partial R^e}{\partial \nu}$$

$$= \begin{cases} \bar{r}\tau \frac{\partial e}{\partial \nu} p + \beta g(e) e \frac{\partial p}{\partial \nu} & \text{if} \quad \rho\zeta < 1, \\ \mu\nu^- & \text{if} \quad \rho\zeta = 1. \end{cases} \tag{4.88}$$

Although the sum of the first two terms of the right-hand side of equation (4.87) may be negative this case coincides with the case that the marginal revenue is negative. Equation (4.87) shows that the optimal tax rate is on the increasing side of the Laffer curve if evasion occurs in equilibrium. The audit rate may in principle be on either side of the Laffer curve because increases in the audit rate may have an ambiguous effect on welfare (because it is not clear whether the total concealment costs increase or decrease). Note that tax evasion leads to social costs as far as it is associated with concealment costs. These concealment costs may in principle de- as well as increase in the audit rate. They decrease if the impact of an increase in the audit rate on the extent of evasion is larger than the associated price effect.[28] Both terms together determine the impact of additional audits on the total concealment costs at the margin.

It shows that $\frac{\partial R^e}{\partial \nu}$ is positive in the optimum, i.e. in particular optimal auditing is below the point at which marginal revenue from auditing is equal to marginal costs. This corresponds to the result of Slemrod & Yitzhaki (1987) for optimal auditing in a model of individual income tax evasion.

If firms are not identical, the sign of the first term depends on the market shares in the initial Nash equilibrium. It is positive if and only if $q_0 > \frac{1}{2}$. If stricter enforcement increases q_0, it leads to a welfare loss at the margin due to a more inefficient allocation of customers. Note that comparative statics are difficult at this level of generality and cannot be deduced immediately from the first-order conditions.

Proposition 4.3 (Optimal taxation and enforcement). *The optimal tax and*

[28] Obviously, a price effect does not occur in the case of a unit tax. In this case, the audit rate is also on the increasing side of the Laffer curve.

enforcement policy to finance expected revenue of R^e in an interior optimum obeys:

$$\frac{\partial R^e}{\partial \tau} \bigg/ \frac{\partial R^e}{\partial \nu} = [T(q_0) - T(1-q_0)]\frac{\partial q_0}{\partial \tau} + \sum_{j=0}^{1}\left[\bar{r}\tau\frac{\partial e_j}{\partial \tau}p_j q_j \right. \\ \left. +\beta_j g(e_j)e_j\left(\frac{\partial p_j}{\partial \tau}q_j + p_j\frac{\partial q_j}{\partial \tau}\right) + \gamma_j c'(q_j)\frac{\partial q_j}{\partial \tau}\right] \bigg/ \\ [T(q_0) - T(1-q_0)]\frac{\partial q_0}{\partial \nu} + \sum_{j=0}^{1}\left[\bar{r}\tau\frac{\partial e_j}{\partial \nu}p_j q_j \right. \\ \left. +\beta_j g(e_j)e_j\left(\frac{\partial p_j}{\partial \nu}q_j + p_j\frac{\partial q_j}{\partial \nu}\right) + \gamma_j c'(q_j)\frac{\partial q_j}{\partial \nu}\right], \tag{4.89}$$

for $\nu \in \{\rho, \zeta\}$. □

Proof: The result follows from eliminating the Lagrange parameter from equations (4.83), (4.84) and using equation (4.16). ■

Proposition 4.3 has an intuitive interpretation. The left-hand side is the ratio at which both instruments can be substituted for each other such that the expected revenue stays unchanged. The right-hand side is the ratio of the associated marginal welfare effects. In the optimum both have to be equal. If, for example, the left-hand side were larger than the right-hand side then the tax rate could be reduced, tax enforcement raised and welfare would increase without effects on total expected tax revenue (see also Cremer & Gahvari (1993)).

An important implication of equation (4.89) is that if firms are heterogenous full enforcement may not be optimal even if it does not entail any resource costs.

Corollary 4.4 (Full enforcement). *If firms differ in marginal production and concealment costs such that one firm has a comparative advantage in either activity (one firm has a comparative advantage in production having lower marginal production costs, the other firm has a comparative advantage in evasion having lower marginal concealment costs) tax enforcement below the full compliance level may be optimal even if it does not entail any resource costs.* □

Proof: The proof uses Example 4.2.

Example 4.2 continued. Suppose that the government wants to finance a revenue of 0.126 units. In order to do this in the case without tax evasion, where the government, for example, sets the fine rate such that $\bar{r} = 1 - \rho\zeta = 0$, it needs to set a tax rate of 10%. Now, assume that it does not enforce taxes strictly but allows for tax evasion and sets $\bar{r} = 0.4$. Leaving the tax rate at 10% is not sufficient to finance the desired revenue: it falls to 0.117 units. In order to make up for this decrease

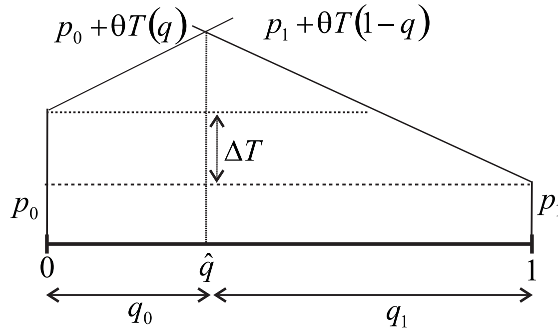


Figure 4.6: Welfare improvement through reallocation of market shares.

in revenue, it is necessary to increase the statutory tax rate to 11%. Although in the latter case the equilibrium entails tax evasion the total surplus is higher than in the equilibrium without tax evasion ($S = .126$ compared to $S = 0.141$). $*$

This proofs Corollary 4.4. ■

The intuition for the result is as follows: the government can use its enforcement policy to influence competition and may allow for tax evasion if the efficiency loss that it induces is lower than the efficiency loss otherwise arising out of (highly) different market shares.[29] It is illustrated in Figure 4.6. The welfare gain of a reallocation of the market shares due to a decrease in the total transportation costs is depicted by ΔT. Although other inefficiencies arise (in the example provided above the firm with lower marginal concealment costs is also less efficient and, additionally, tax evasion occurs which implies welfare loss in terms of concealment costs) these welfare losses may be outweighed by the welfare gain of consumers.

That avoidance behavior may be the result of conscious tax policy, has also been noted by Stiglitz (1985) in the context of individual retirement accounts (IRAs). The goal here is to induce individuals to increase savings. Boadway & Keen (1998) also provide an argument for lax tax enforcement. The role of tax evasion in their model is that it is used to lower the tax burden in case the tax rate itself is set too high because the government cannot commit to the optimal lower rate. A similar result is also obtained by Polinsky & Shavell (1979). They show that a penalty

[29]Note that some form of heterogeneity is necessary to obtain this result. Otherwise, if firms are identical $q_0 = q_1 = \frac{1}{2}$ and $R^e = 2\tau(1-\bar{r}e)pq$. In order to obtain the same revenue as in the case with full enforcement $\tilde{R} = 2\tilde{\tau}\tilde{p}q$, the government has to increase its tax rate to $\tau = \frac{\tilde{\tau}\tilde{p}}{2(1-\bar{r}e)p}$. As this even increases the evasion costs and does not entail any benefits, full enforcement is optimal if it is costless.

below the maximal penalty is optimal if there are crimes in which the private benefit to the criminal exceed the social cost of the criminal activity.

An important question is why firms should differ in their concealment costs if they operate in the same market. There are several possibilities that explain these differences that are in line with the present model. For example, different marginal concealment costs may arise from different reporting requirements. Information reporting is extensive in the U.S. (see U.S. Department of the Treasury. Internal Revenue Service (1990)), however, different requirements apply conditional on the legal status of the firm leaving larger possibilities to evade for sources where information requirements are low like for sole proprietorships. Hibbs & Piculescu (2005) argue for firm-specific thresholds of tax toleration determined by firm-specific institutional benefits available when producing officially and the costs of corruption required to produce unofficially. Their model may also explain the variation of tax evasion (and underground production) across firms in the same institutional and regulatory environment.

In a sense the presented model also explains and provides a normative reason for differences in reporting requirements. If one argues that the concealment cost function also captures compliance costs even in case of truthful reporting and that the marginal concealment costs are an instrument of the government because it may set reporting requirements, it may explain the optimal degree of information reporting that is imposed on firms.[30] In order to explain this in more detail, assume that $G(e, w)pq$ denotes the compliance cost function. Here, w denotes the degree of information reporting that the government imposes (with higher w being associated with higher reporting requirements). With respect to evasion this function may be assumed to behave like the strictly convex concealment cost function ge discussed above except that $G(0, w)$ should now be assumed to be positive to capture the idea that even honest firms have to bear compliance costs in addition to the statutory tax payment. Additionally, $0 < \frac{\partial G}{\partial w}$ should hold to capture the welfare loss that is implied by the additional administrative effort involved with higher reporting requirements. Analogously to the discussion above, in the optimum firms should be subjected to information reporting, for example, required to keep business records, to an extent that the marginal compliance costs of the last unit of revenue are equal to the marginal reduction in concealment costs per unit of revenue.

[30] On a more abstract level, the economic theory of tax evasion has lead to a broader discussion of instruments available to the government; see Slemrod (1990, 2001), Slemrod & Yitzhaki (2002).

4.2.5 Determination of Tax and Enforcement Policy by Majority Vote

Another question – especially if one regards the social planner as an idealization not existing in the real world – concerns how the actual policy can be characterized if a majority decides on the tax enforcement level. It is not surprising that it votes for lax tax enforcement if it evades taxes itself. However, this seems not to be the case – at least for the U.S.[31] Nevertheless, the present model where the majority does *not* evade taxes (in fact, individuals do not evade taxes at all, only companies may evade sales taxes) may be suitable to answer this question. The benefit of reduced tax payment by tax evasion not only accrues to shareholders of a tax evading company (in this case one might consider the firm as an instrument or an intermediary to evade taxes) but to all its customers through higher competition in the product market.[32]

It is assumed in this section that tax enforcement is determined by a majority vote. There are only a few models of tax evasion and voting. Fuest & Huber (2001) explain the difficulty of tax coordination in a model of tax competition with tax evasion and Borck (2004) establishes a positive relationship between the fine rate and tax evasion in a median voter model. An empirical contribution is Hunter & Nelson (1995) who investigate the determinants of tax enforcement levels among different states and find that enforcement is lower in states represented by legislators with oversight responsibility for the tax agency. The following discussion constitutes the first political economy model of tax enforcement, where enforcement and taxation are determined simultaneously by majority voting. It is a very special model of spatial voting and further research is obviously necessary.

First of all, the set of policies that individuals may vote on is restricted. As in the preceding section it is assumed that the government has to finance an exogenous amount of revenue, $\bar{R}$. The voters are not allowed to vote on $\bar{R}$, but only on the way that it is financed. Therefore, they may only vote on *budget-neutral* (joint) changes of tax and tax enforcement policy.

As proved before certain revenues can be financed by different combinations of tax

[31] The IRS's Taxpayer Compliance Measurement Program (TCMP), a program for intensive audits on a stratified random sample of tax returns, has revealed that about 40% of U.S. households have evaded taxes in 1988 (among which some may even have underpaid their taxes unintentionally as the TCMP revealed that 7% have overpaid their tax obligations).

[32] It should be easy to show that shareholders of a company that evades taxes favor a weak tax enforcement policy because it lowers the effective tax payment and raises profits available for distribution as dividends or higher profits increase the value of their share holdings. However, this is not convincing in a majority voting model (at least for Germany) because only a small percentage of Germans actually own shares: in 2004 about 7% of the total German population owned shares, 17% owned shares and mutual funds, Deutsches Aktieninstitut (DAI) (2004).

rates and enforcement effort. The purpose of majority voting in this context is to determine the individually preferred pair of enforcement effort and tax rate that leads to the same predetermined revenue. Each individual determines his optimum by maximizing the (indirect) utility function. An individual situated at spot q that buys at firm j solves[33]

$$\max_{\tau,\rho} \bar{v} - p_j - \theta T(\tilde{q}_j) + w(\bar{R}), \quad \text{s.t. } R^e \geqslant \bar{R},\ 0 \leqslant \tau \leqslant 1,\ 0 \leqslant \rho \leqslant \frac{1}{\zeta}, \tag{4.90}$$

where

$$\tilde{q}_j = \begin{cases} q & \text{if} \quad j = 0, \\ 1 - q & \text{if} \quad j = 1. \end{cases} \tag{4.91}$$

Here, w denotes the utility derived from a public good. All revenue is assumed to be spent on this public good. As it is assumed that the individual may only vote on budget-neutral policies the utility of the public good as well as the fixed reservation value can be dropped from the maximization and problem (4.90) is equivalent to the minimization of the total costs associated with the purchasing decision and may therefore be written as

$$\min_{\tau,\rho} p_j + \theta T(\tilde{q}_j), \quad \text{s.t. } R^e = \bar{R},\ 0 \leqslant \tau \leqslant 1,\ 0 \leqslant \rho \leqslant \frac{1}{\zeta}. \tag{4.92}$$

Especially for individuals away from the ends of the Hotelling street it is not clear a priori at which firm they buy and different policies may lead to different decisions. Therefore, on the whole each individual solves

$$\min_{j\in\{0,1\}} \{\min_{\tau,\rho} p_j + \theta T(\tilde{q}_j)\}, \quad \text{s.t. } R^e = \bar{R},\ 0 \leqslant \tau \leqslant 1,\ 0 \leqslant \rho \leqslant \frac{1}{\zeta}. \tag{4.93}$$

Nevertheless, under the condition that the decisive consumer at $\hat{q}$ does not change the decision of where to buy the following conclusion can be drawn.[34]

Proposition 4.4 (Policy determined by majority). *The optimal tax and tax*

[33] Contrary to the preceding section it is assumed that individuals vote on the tax rate and the extent to which firms are audited for concreteness. The fine rate is assumed to be fixed exogenously.

[34] A sufficient condition for this to be the case is that one firm has a competitive advantage in concealment and its production cost disadvantage is not too large (see Corollary 4.3).

enforcement policy that is determined via a majority vote and leads to expected revenue $\bar{R}$ satisfies

$$\frac{\partial R^e}{\partial \tau} \bigg/ \frac{\partial R^e}{\partial \rho} = \frac{\partial p_j}{\partial \tau} \bigg/ \frac{\partial p_j}{\partial \rho}. \tag{4.94}$$

□

Proof: The associated Lagrangian of problem (4.92) is given by

$$\mathcal{L} = p_j + \theta T(\tilde{q}_j) + \lambda(\bar{R} - R^e) \tag{4.95}$$

with the first-order conditions:

$$\frac{\partial \mathcal{L}}{\partial \tau} = \frac{\partial p_j}{\partial \tau} - \lambda \frac{\partial R^e}{\partial \tau} = 0, \tag{4.96}$$

$$\frac{\partial \mathcal{L}}{\partial \nu} = \frac{\partial p_j}{\partial \rho} - \lambda \frac{\partial R^e}{\partial \rho} = 0, \tag{4.97}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \bar{R} - R^e = 0. \tag{4.98}$$

Eliminating the Lagrange parameter from equations (4.96),(4.97) leads to the result. ■

Equation (4.94) has again an intuitive interpretation along the same lines as before. The left-hand side denotes the rate at which the two tax instruments may be substituted for each other such that the expected tax revenue is held constant. The right-hand side denotes the ratio of marginal changes of the price at the store where the consumer buys which are induced by these policy changes. In the optimum both sides have to be equal. If, for example, the left-hand side where (in absolute value) larger than the right-hand side the tax rate could be increased and simultaneously the audit rate decreased in a way that expected revenue can be held constant and the price that the consumer pays is smaller than before. The case of an inequality of equation (4.94) can therefore not denote an optimum.

Tanzi (2000, p.172) provides a nice example that might be explained by the model. He quotes a case where salaried workers have demonstrated in large numbers in the streets to call for reduction in tax evasion by independent professionals and other groups.[35] The model sketched above implies that these workers may have been concerned of paying too high prices in addition to other – more obvious – disadvantages associated with tax evasion.

[35]The Economist (1997a) also reports that corruption has become an important issue in elections in Argentina.

4.3 Conclusions

The model integrates the possibility of tax evasion into a duopoly model with a horizontally differentiated product (Hotelling model) and price setting firms. It has been shown that a Nash equilibrium exists and that allowing higher tax evasion increases competition and leads to lower prices at the margin.

The competitive advantage of a firm with a superior concealment technology suggests that different tax evasion opportunities (or differential tax treatments) may be a source of a competitive advantage – presumably *unfair* competition. This competitive advantage is present even if there is no advantage on the side of the production technology. Nevertheless, it has been shown that the government may intentionally allow such competition to improve the efficiency of the market equilibrium. It is clear from that the restriction of the available policy instruments is a critical input of the model and the result has a second-best character.

The assumption that the concealment technology takes the given form is crucial for the results. A more thorough investigation of its determinants seems mandatory. Across industry sectors it may vary according to the nature of a good (for example its visibility) or the size of the firms (Cowell (2004)). In particular, if the evasion advantage stems from some learning activity in the market it might be a significant barrier to entry because the established firms can sustain low prices even if they do not have any cost advantage or own superior production technologies through R&D or experience. Nevertheless, differences in production and evasion possibilities do not have to be identical even in the same market. For example, different legal organizations may entail different reporting duties – to the tax administration as well as to other stake holders. This is true in particular for partnerships versus joint-stock companies (or a C corporation compared to an S corporation in the U.S.). In this sense, the reporting duties might be associated different evasion possibilities (different β's). Empirical evidence on tax compliance of firms is very limited. Using micro data of U.S. corporations, Rice (1992) finds that firms evade less if they are publicly traded or belong to a highly regulated industry.[36] Thus, the differences that the present chapter discussed could arise from a different legal organization of the firm with a concealment cost disadvantage for a publicly traded corporation. One could also allow the extent of information reporting required by the tax agency to be a choice variable of the government and some intuition on this extension has been discussed above.

Similar results should be obtained for other forms of competition, for example, for the case of quantity competition (Cournot) or monopolistic competition. The only difference is that the welfare function takes total quantity effects rather than how a given quantity is allocated among consumers into account.

[36] He also finds that tax evasion arises if a firm earns lower profits relative to the industry median.

Naturally, the results are not limited to tax evasion but carry over to other situations with possible law violations (for example environmental pollution).

4.A Appendix

Formal Proofs

Reaction Function of Firm 0

Proof: Totally differentiate the first-order condition (4.15) for firm 0 to obtain:

$$\begin{aligned}
\mathrm{d}\frac{\partial \pi_0^e}{\partial p_0} &= (1-\tau_0^e-\beta_0 g e_0)\left[\left(2\frac{\partial q_0}{\partial p_0}+p_0\frac{\partial^2 q_0}{\partial p_0^2}\right)\mathrm{d}p_0+\left(\frac{\partial q_0}{\partial p_1}+p_0\frac{\partial^2 q_0}{\partial p_0 \partial p_1}\right)\mathrm{d}p_1\right] \\
&\quad -\gamma_0\left[\left(c''\left(\frac{\partial q_0}{\partial p_0}\right)^2+c'\frac{\partial^2 q_0}{\partial p_0^2}\right)\mathrm{d}p_0+\left(c''\frac{\partial q_0}{\partial p_0}\frac{\partial q_0}{\partial p_1}+c'\frac{\partial^2 q_0}{\partial p_0^2 \partial p_1}\right)\mathrm{d}p_1\right]=0
\end{aligned}$$

The result of Lemma 4.2 follows from rearranging to

$$\frac{\partial p_0}{\partial p_1}=-\frac{(1-\tau_0^e-\beta_0 g e_0)\left(2\frac{\partial q_0}{\partial p_0}+p_0\frac{\partial^2 q_0}{\partial p_0^2}\right)-\gamma_0\left(c''\left(\frac{\partial q_0}{\partial p_0}\right)^2+c'\frac{\partial^2 q_0}{\partial p_0^2}\right)}{(1-\tau_0^e-\beta_0 g e_0)\left(\frac{\partial q_0}{\partial p_1}+p_0\frac{\partial^2 q_0}{\partial p_0 \partial p_1}\right)-\gamma_0\left(c''\frac{\partial q_0}{\partial p_0}\frac{\partial q_0}{\partial p_1}+c'\frac{\partial^2 q_0}{\partial p_0^2 \partial p_1}\right)} \tag{A.99}$$

which is positive if the transportation cost function is linear or quadratic and $0 \leqslant c''$. ■

Proof of the Comparative Statics

The first-order conditions for profit maximization lead to the following (4×4) equation system that is satisfied in any (interior) Nash equilibrium (p_0, p_1, e_0, e_1):

$$\frac{\partial \pi_j^e}{\partial p_j}(p_0,p_1,e_j)=(1-\tau_j^e-\beta_j g(e_j)e_j)\left(q_j+p_j\frac{\partial q_j}{\partial p_j}\right)-\gamma_j c_j'(q_j)\frac{\partial q_j}{\partial p_j}=0, \tag{4.15}$$

$$\frac{\partial \pi_j^e}{\partial e_j}(p_0,p_1,e_j)=((1-\rho\zeta)\tau-\beta_j(g(e_j)+g'(e_j)e_j))p_j q_j=0, \tag{4.16}$$

for $j \in \{0,1\}$.

In order to derive the comparative statics, totally differentiate (4.15), (4.16) and remember that $\frac{\partial^2 q_j}{\partial p_k \partial p_l}=0$, for $j,k,l \in \{0,1\}$ for a linear or quadratic transportation cost function. One obtains (all endogenous variables take optimum values):

$$
\begin{aligned}
\mathrm{d}\frac{\partial \pi_0^e}{\partial p_0} =& \left(2(1-\tau_0^e-\beta_0 g(e_0)e_0)-\gamma_0 c''(q_0)\frac{\partial q_0}{\partial p_0}\right)\frac{\partial q_0}{\partial p_0}\mathrm{d}p_0+\Bigg(1-\tau_0^e-\beta_0 g(e_0)e_0\\
&-\gamma_0 c''(q_0)\frac{\partial q_0}{\partial p_0}\Bigg)\frac{\partial q_0}{\partial p_1}\mathrm{d}p_1-[(1-\bar{r}e_0)\mathrm{d}\tau+(\zeta\mathrm{d}\rho+\rho\mathrm{d}\zeta)\tau e_0]\left(q_0+p_0\frac{\partial q_0}{\partial p_0}\right)\\
&-g(e_0)e_0\left(q_0+p_0\frac{\partial q_0}{\partial p_0}\right)\mathrm{d}\beta_0-c'(q_0)\frac{\partial q_0}{\partial p_0}\mathrm{d}\gamma_0\\
&+\frac{(1-\tau_0^e-\beta_0 g(e_0)e_0)(p_0-\gamma_0 c'(q_0))}{\theta^2(T'(q_0)+T'(q_1))}\mathrm{d}\theta=0\\
\mathrm{d}\frac{\partial \pi_0^e}{\partial e_0} =& -\beta_0(2g'(e_0)+g''(e_0)e_0)\mathrm{d}e_0+\bar{r}\mathrm{d}\tau-\tau(\zeta\mathrm{d}\rho+\rho\mathrm{d}\zeta)\\
&-(g(e_0)+g'(e_0)e_0)\mathrm{d}\beta_0=0\\
\mathrm{d}\frac{\partial \pi_1^e}{\partial p_1} =& \left(1-\tau_1^e-\beta_1 g(e_1)e_1-\gamma_1 c''(q_1)\frac{\partial q_1}{\partial p_1}\right)\frac{\partial q_1}{\partial p_0}\mathrm{d}p_0+\Bigg(2(1-\tau_1^e-\beta_1 g(e_1)e_1)\\
&-\gamma_1 c''(q_1)\frac{\partial q_1}{\partial p_1}\Bigg)\frac{\partial q_1}{\partial p_1}\mathrm{d}p_1-[(1-\bar{r}e_1)\mathrm{d}\tau+(\zeta\mathrm{d}\rho+\rho\mathrm{d}\zeta)\tau e_1]\left(q_1+p_1\frac{\partial q_1}{\partial p_1}\right)\\
&-g(e_1)e_1\left(q_1+p_1\frac{\partial q_1}{\partial p_1}\right)\mathrm{d}\beta_1-c'(q_1)\frac{\partial q_1}{\partial p_1}\mathrm{d}\gamma_1\\
&+\frac{(1-\tau_1^e-\beta_1 g(e_1)e_1)(p_1-\gamma_1 c'(q_1))}{\theta^2(T'(q_0)+T'(q_1))}\mathrm{d}\theta=0\\
\mathrm{d}\frac{\partial \pi_1^e}{\partial e_1} =& -\beta_1(2g'(e_1)+g''(e_1)e_1)\mathrm{d}e_1+\bar{r}\mathrm{d}\tau-\tau(\zeta\mathrm{d}\rho+\rho\mathrm{d}\zeta)\\
&-(g(e_1)+g'(e_1)e_1)\mathrm{d}\beta_1=0
\end{aligned}
$$

or in Matrix form (to abbreviate set $(2g'(e_j)+g''(e_j)e_j):=\tilde{g}_j$):

$$\underbrace{\begin{pmatrix} \left[2(1-\tau_0^e-\beta_0 g(e_0)e_0)-\gamma_0 c''(q_0)\frac{\partial q_0}{\partial p_0}\right]\frac{\partial q_0}{\partial p_0} & 0 & \left[1-\tau_0^e-\beta_0 g(e_0)e_0-\gamma_0 c''(q_0)\frac{\partial q_0}{\partial p_0}\right]\frac{\partial q_0}{\partial p_1} & 0 \\ 0 & -\tilde{g}_0 & 0 & 0 \\ \left[1-\tau_1^e-\beta_1 g(e_1)e_1-\gamma_1 c''(q_1)\frac{\partial q_1}{\partial p_1}\right]\frac{\partial q_1}{\partial p_0} & 0 & \left[2(1-\tau_1^e-\beta_1 g(e_1)e_1)-\gamma_1 c''(q_1)\frac{\partial q_1}{\partial p_1}\right]\frac{\partial q_1}{\partial p_1} & 0 \\ 0 & 0 & 0 & -\tilde{g}_1 \end{pmatrix}}_{(a_{ij})_{i,j=1}^4 =: A =:}$$

$$\times \begin{pmatrix} \mathrm{d}p_0 \\ \mathrm{d}e_0 \\ \mathrm{d}p_1 \\ \mathrm{d}e_1 \end{pmatrix} = \begin{pmatrix} [(1-\bar{r}e_0)\mathrm{d}\tau+(\zeta\mathrm{d}\rho+\rho\mathrm{d}\zeta)\tau e_0+g(e_0)e_0\mathrm{d}\beta_0]\left[q_0+p_0\frac{\partial q_0}{\partial p_0}\right]+c'(q_0)\frac{\partial q_0}{\partial p_0}\mathrm{d}\gamma_0-\frac{(1-\tau_0^e-\beta_0 g(e_0)e_0)(p_0-\gamma_0 c'(q_0))}{\theta^2(T'(q_0)+T'(q_1))}\mathrm{d}\theta \\ -\bar{r}\mathrm{d}\tau+\tau(\zeta\mathrm{d}\rho+\rho\mathrm{d}\zeta)+(g(e_0)+g'(e_0)e_0)\mathrm{d}\beta_0 \\ [(1-\bar{r}e_1)\mathrm{d}\tau+(\zeta\mathrm{d}\rho+\rho\mathrm{d}\zeta)\tau e_1+g(e_1)e_1\mathrm{d}\beta_1]\left[q_1+p_1\frac{\partial q_1}{\partial p_1}\right]+c'(q_1)\frac{\partial q_1}{\partial p_1}\mathrm{d}\gamma_1-\frac{(1-\tau_1^e-\beta_1 g(e_1)e_1)(p_1-\gamma_1 c'(q_1))}{\theta^2(T'(q_0)+T'(q_1))}\mathrm{d}\theta \\ -\bar{r}\mathrm{d}\tau+\tau(\zeta\mathrm{d}\rho+\rho\mathrm{d}\zeta)+(g(e_1)+g'(e_1)e_1)\mathrm{d}\beta_1 \end{pmatrix}$$

with

$$\det(A) = a_{22}a_{44}(a_{11}a_{33}-a_{13}a_{31}) > 0, \tag{A.100}$$

as the bracketed term is positive because $|a_{11}| > a_{13}$ and $|a_{33}| > a_{31}$.

Consequently,[37]

$$
\begin{aligned}
\frac{\partial p_0}{\partial \tau} &\overset{s}{=} \det \begin{pmatrix} (1-\bar{r}e_0)\left(q_0 + p_0\frac{\partial q_0}{\partial p_0}\right) & 0 & a_{13} & 0 \\ -\bar{r} & a_{22} & 0 & 0 \\ (1-\bar{r}e_1)\left(q_1 + p_1\frac{\partial q_1}{\partial p_1}\right) & 0 & a_{33} & 0 \\ -\bar{r} & 0 & 0 & a_{44} \end{pmatrix} \\
&= a_{22}a_{44}\left[(1-\bar{r}e_0)\left(q_0 + p_0\frac{\partial q_0}{\partial p_0}\right)a_{33} - (1-\bar{r}e_1)\left(q_1 + p_1\frac{\partial q_1}{\partial p_1}\right)a_{13}\right] \\
&\overset{s}{=} +1,
\end{aligned} \tag{A.101}
$$

$$
\begin{aligned}
\frac{\partial p_0}{\partial \nu} &\overset{s}{=} \det \begin{pmatrix} \nu^-\tau e_0\left(q_0 + p_0\frac{\partial q_0}{\partial p_0}\right) & 0 & a_{13} & 0 \\ \nu^-\tau & a_{22} & 0 & 0 \\ \nu^-\tau e_1\left(q_1 + p_1\frac{\partial q_1}{\partial p_1}\right) & 0 & a_{33} & 0 \\ \nu^-\tau & 0 & 0 & a_{44} \end{pmatrix} \\
&= a_{22}a_{44}\nu^-\tau\left[e_0\left(q_0 + p_0\frac{\partial q_0}{\partial p_0}\right)a_{33} - e_1\left(q_1 + p_1\frac{\partial q_1}{\partial p_1}\right)a_{13}\right] \\
&\overset{s}{=} +1,
\end{aligned} \tag{A.102}
$$

where $\nu \in \{\rho, \zeta\}$ and $\nu^- = \zeta$ if $\nu = \rho$ and $\nu^- = \rho$ if $\nu = \zeta$.

Symmetrically,

$$
\begin{aligned}
\frac{\partial p_1}{\partial \tau} &\overset{s}{=} \det \begin{pmatrix} a_{11} & 0 & (1-\bar{r}e_0)\left(q_0 + p_0\frac{\partial q_0}{\partial p_0}\right) & 0 \\ 0 & a_{22} & -\bar{r} & 0 \\ a_{31} & 0 & (1-\bar{r}e_1)\left(q_1 + p_1\frac{\partial q_1}{\partial p_1}\right) & 0 \\ 0 & 0 & -\bar{r} & a_{44} \end{pmatrix} \\
&= a_{22}a_{44}\left[(1-\bar{r}e_1)\left(q_1 + p_1\frac{\partial q_1}{\partial p_1}\right)a_{11} - (1-\bar{r}e_0)\left(q_0 + p_0\frac{\partial q_0}{\partial p_0}\right)a_{31}\right] \\
&\overset{s}{=} +1,
\end{aligned} \tag{A.103}
$$

[37] The sign $\overset{s}{=}$ has to be read *is of the same sign as.*

$$
\begin{aligned}
\frac{\partial p_1}{\partial \nu} &\overset{s}{=} \det \begin{pmatrix} a_{11} & 0 & \nu^{-}\tau e_0\left(q_0 + p_0\frac{\partial q_0}{\partial p_0}\right) & 0 \\ 0 & a_{22} & \nu^{-}\tau & 0 \\ a_{31} & 0 & \nu^{-}\tau e_1\left(q_1 + p_1\frac{\partial q_1}{\partial p_1}\right) & 0 \\ 0 & 0 & \nu^{-}\tau & a_{44} \end{pmatrix} \\
&= a_{22}a_{44}\nu^{-}\tau\left[e_1\left(q_1 + p_1\frac{\partial q_1}{\partial p_1}\right)a_{11} - e_0\left(q_0 + p_0\frac{\partial q_0}{\partial p_0}\right)a_{31}\right] \\
&\overset{s}{=} +1, \qquad \text{(A.104)}
\end{aligned}
$$

where, again, $\nu \in \{\rho, \zeta\}$ and $\nu^{-} = \zeta$ if $\nu = \rho$ and $\nu^{-} = \rho$ if $\nu = \zeta$. Additionally,

$$
\begin{aligned}
\frac{\partial p_0}{\partial \beta_0} &\overset{s}{=} \det \begin{pmatrix} g(e_0)e_0\left(q_0 + p_0\frac{\partial q_0}{\partial p_0}\right) & 0 & a_{13} & 0 \\ g(e_0) + g'(e_0)e_0 & a_{22} & 0 & 0 \\ 0 & 0 & a_{33} & 0 \\ 0 & 0 & 0 & a_{44} \end{pmatrix} \\
&= g(e_0)e_0\left(q_0 + p_0\frac{\partial q_0}{\partial p_0}\right)a_{22}a_{33}a_{44} \overset{s}{=} +1. \qquad \text{(A.105)}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial p_0}{\partial \beta_1} &\overset{s}{=} \det \begin{pmatrix} 0 & 0 & a_{13} & 0 \\ 0 & a_{22} & 0 & 0 \\ g(e_1)e_1\left(q_1 + p_1\frac{\partial q_1}{\partial p_1}\right) & 0 & a_{33} & 0 \\ g(e_1) + g'(e_1)e_1 & 0 & 0 & a_{44} \end{pmatrix} \\
&= -g(e_1)e_1\left(q_1 + p_1\frac{\partial q_1}{\partial p_1}\right)a_{13}a_{22}a_{44} \overset{s}{=} +1, \qquad \text{(A.106)}
\end{aligned}
$$

$$
\frac{\partial p_0}{\partial \gamma_0} \overset{s}{=} \det \begin{pmatrix} c'(q_0)\frac{\partial q_0}{\partial p_0} & 0 & a_{13} & 0 \\ 0 & a_{22} & 0 & 0 \\ 0 & 0 & a_{33} & 0 \\ 0 & 0 & 0 & a_{44} \end{pmatrix} = c'(q_0)\frac{\partial q_0}{\partial p_0}a_{22}a_{33}a_{44} \overset{s}{=} +1, \qquad \text{(A.107)}
$$

$$
\frac{\partial p_0}{\partial \gamma_1} \overset{s}{=} \det \begin{pmatrix} 0 & 0 & a_{13} & 0 \\ 0 & a_{22} & 0 & 0 \\ c'(q_1)\frac{\partial q_1}{\partial p_1} & 0 & a_{33} & 0 \\ 0 & 0 & 0 & a_{44} \end{pmatrix} = -c'(q_1)\frac{\partial q_1}{\partial p_1}a_{13}a_{22}a_{44} \overset{s}{=} +1. \qquad \text{(A.108)}
$$

With respect to the differentiation parameter the comparative statics can be calculated to be given as follows:

$$
\begin{aligned}
\frac{\partial p_0}{\partial \theta} &\overset{s}{=} \det \begin{pmatrix} -\frac{(1-\tau_0^e-\beta_0 g(e_0)e_0)(p_0-\gamma_0 c'(q_0))}{\theta^2(T'(q_0)+T'(q_1))} & 0 & a_{13} & 0 \\ -\bar{r} & a_{22} & 0 & 0 \\ -\frac{(1-\tau_1^e-\beta_1 g(e_1)e_1)(p_1-\gamma_1 c'(q_1))}{\theta^2(T'(q_0)+T'(q_1))} & 0 & a_{33} & 0 \\ -\bar{r} & 0 & 0 & a_{44} \end{pmatrix} \\
&= \Bigg[\frac{(1+\tau_1^e-\beta_1 g(e_1)e_1)(p_1-\gamma_1 c'(q_1))}{\theta^2(T'(q_0)+T'(q_1))} a_{13} \\
&\quad - \frac{(1+\tau_0^e-\beta_0 g(e_0)e_0)(p_0-\gamma_0 c'(q_0))}{\theta^2(T'(q_0)+T'(q_1))} a_{33}\Bigg] a_{22}a_{44} \overset{s}{=} +1, \qquad \text{(A.109)} \\
\frac{\partial p_1}{\partial \theta} &\overset{s}{=} \det \begin{pmatrix} a_{11} & 0 & -\frac{(1-\tau_0^e-\beta_0 g(e_0)e_0)(p_0-\gamma_0 c'(q_0))}{\theta^2(T'(q_0)+T'(q_1))} & 0 \\ 0 & a_{22} & -\bar{r} & 0 \\ a_{31} & 0 & -\frac{(1+\tau_1^e-\beta_1 g(e_1)e_1)(p_1-\gamma_1 c'(q_1))}{\theta^2(T'(q_0)+T'(q_1))} & 0 \\ 0 & 0 & -\bar{r} & a_{44} \end{pmatrix} \\
&= \Bigg[\frac{(1-\tau_1^e-\beta_0 g(e_0)e_0)(p_0-\gamma_0 c'(q_0))}{\theta^2(T'(q_0)+T'(q_1))} a_{31} \\
&\quad - \frac{(1+\tau_1^e-\beta_1 g(e_1)e_1)(p_1-\gamma_1 c'(q_1))}{\theta^2(T'(q_0)+T'(q_1))} a_{11}\Bigg] a_{22}a_{44} \overset{s}{=} +1. \qquad \text{(A.110)}
\end{aligned}
$$

Furthermore,

$$
\begin{aligned}
\frac{\partial e_0}{\partial \tau} &\overset{s}{=} \det \begin{pmatrix} a_{11} & (1-\bar{r}e_0)\left(q_0 + p_0\frac{\partial q_0}{\partial p_0}\right) & a_{13} & 0 \\ 0 & -\bar{r} & 0 & 0 \\ a_{31} & (1-\bar{r}e_0)\left(q_1 + p_1\frac{\partial q_1}{\partial p_1}\right) & a_{33} & 0 \\ 0 & -\bar{r} & 0 & a_{44} \end{pmatrix} \\
&= \bar{r}a_{44}(a_{13}a_{31} - a_{11}a_{33}) \overset{s}{=} +1, \qquad \text{(A.111)} \\
\frac{\partial e_0}{\partial \nu} &\overset{s}{=} \det \begin{pmatrix} a_{11} & \nu^- \tau e_0\left(q_0 + p_0\frac{\partial q_0}{\partial p_0}\right) & a_{13} & 0 \\ 0 & \nu^- \tau & 0 & 0 \\ a_{31} & \nu^- \tau e_1\left(q_1 + p_1\frac{\partial q_1}{\partial p_1}\right) & a_{33} & 0 \\ 0 & \nu^- \tau & 0 & a_{44} \end{pmatrix} \\
&= \nu^- \tau a_{44}(a_{11}a_{33} - a_{13}a_{31}) \overset{s}{=} -1, \qquad \text{(A.112)}
\end{aligned}
$$

$$\frac{\partial e_0}{\partial \beta_0} \stackrel{s}{=} \det \begin{pmatrix} a_{11} & g(e_0)e_0\left(q_0 + p_0 \frac{\partial q_0}{\partial p_0}\right) & a_{13} & 0 \\ 0 & g(e_0) + g'(e_0)e_0 & 0 & 0 \\ a_{31} & 0 & a_{33} & 0 \\ 0 & 0 & 0 & a_{44} \end{pmatrix}$$
$$= (g(e_0) + g'(e_0)e_0)a_{44}(a_{11}a_{33} - a_{13}a_{31}) \stackrel{s}{=} -1, \tag{A.113}$$

where $\nu \in \{\rho, \zeta\}$ and $\nu^- = \zeta$ if $\nu = \rho$ and $\nu^- = \rho$ if $\nu = \zeta$.

The symmetric comparative statics are obtained for firm 1:

$$\frac{\partial e_1}{\partial \tau} \stackrel{s}{=} \det \begin{pmatrix} a_{11} & 0 & a_{13} & (1 - \bar{r}e_0)\left(q_0 + p_0 \frac{\partial q_0}{\partial p_0}\right) \\ 0 & a_{22} & 0 & -\bar{r} \\ a_{31} & 0 & a_{33} & (1 - \bar{r}e_0)\left(q_1 + p_1 \frac{\partial q_1}{\partial p_1}\right) \\ 0 & 0 & 0 & -\bar{r} \end{pmatrix}$$
$$= \bar{r}a_{22}(a_{13}a_{31} - a_{11}a_{33}) \stackrel{s}{=} +1, \tag{A.114}$$

$$\frac{\partial e_1}{\partial \nu} \stackrel{s}{=} \det \begin{pmatrix} a_{11} & 0 & a_{13} & \nu^- \tau e_0\left(q_0 + p_0 \frac{\partial q_0}{\partial p_0}\right) \\ 0 & a_{22} & 0 & \nu^- \tau \\ a_{31} & 0 & a_{33} & \nu^- \tau e_1\left(q_1 + p_1 \frac{\partial q_1}{\partial p_1}\right) \\ 0 & 0 & 0 & \nu^- \tau \end{pmatrix}$$
$$= \nu^- \tau a_{22}(a_{11}a_{33} - a_{13}a_{31}) \stackrel{s}{=} -1, \tag{A.115}$$

$$\frac{\partial e_1}{\partial \beta_1} \stackrel{s}{=} \det \begin{pmatrix} a_{11} & 0 & a_{13} & g(e_0)e_0\left(q_0 + p_0 \frac{\partial q_0}{\partial p_0}\right) \\ 0 & a_{22} & 0 & g(e_0) + g'(e_0)e_0 \\ a_{31} & 0 & a_{33} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$
$$= (g(e_0) + g'(e_0)e_0)a_{44}(a_{11}a_{33} - a_{13}a_{31}) \stackrel{s}{=} -1, \tag{A.116}$$

where, again, $\nu \in \{\rho, \zeta\}$ and $\nu^- = \zeta$ if $\nu = \rho$ and $\nu^- = \rho$ if $\nu = \zeta$.

This finishes the proof of Proposition 4.2.

The Case of a Unit Tax

If the firms are subjected to a unit tax at rate t, $0 \leqslant t \leqslant 1$, the expected profits are given by

$$\pi_j^e = (p_j - (1 - \bar{r}e_j)t - \beta_j g(e_j)e_j)q_j - \gamma_j c(q_j), \text{ for } j \in \{0, 1\}, \tag{A.117}$$

and the first-order conditions for interior solutions may be calculated to be

$$\frac{\partial \pi_j^e}{\partial p_j} = q_j + (p_j - (1 - \bar{r}e_j)t - \beta_j g(e_j)e_j)\frac{\partial q_j}{\partial p_j} - \gamma_j c'(q_j)\frac{\partial q_j}{\partial p_j} = 0, \tag{A.118}$$

$$\frac{\partial \pi_j^e}{\partial e_j} = [\bar{r}t - \beta_j(g(e_j) + g'(e_j)e_j)]q_j = 0. \tag{A.119}$$

The production decision is separable from the tax evasion decision (but not vice versa).

This separation implies that changes in the government's tax enforcement policy influence the production decision only directly through their impact on the marginal return to production and not through their impact on the marginal return to evasion and the associated changes in tax evasion.

With the analogous reasoning as in the case of a sales tax it can be concluded that tax evasion leads to lower prices if the marginal revenue is strictly decreasing.

Comparative Statics

A Nash equilibrium exists and the comparative statics can be derived to be signed as follows:

$$\frac{\partial p_j}{\partial t} > 0, \ \frac{\partial p_j}{\partial \rho} > 0, \ \frac{\partial p_j}{\partial \zeta} > 0, \tag{A.120}$$

$$\frac{\partial e_j}{\partial t} > 0, \ \frac{\partial e_j}{\partial \rho} < 0, \ \frac{\partial e_j}{\partial \zeta} < 0, \ \frac{\partial e_j}{\partial \beta_j} < 0. \tag{A.121}$$

Proof: The production decision is separable from the evasion decision. Therefore, the comparative statics of the latter decision can be derived first. The results are the same as in the case of sales tax evasion because gains and costs are linear in the tax base.

Differentiate the first-order condition for optimal evasion

$$\frac{\partial \pi_j^e}{\partial e_j} = \bar{r}t - \beta_j(g(e_j) + g'(e_j)e_j) = 0 \tag{A.122}$$

to obtain

$$\begin{aligned}\mathrm{d}\frac{\partial \pi_j^e}{\partial e_j} = & -\beta_j(2g'(e_j) + g''(e_j)e_j)\mathrm{d}e_j + \bar{r}\mathrm{d}t - (\rho \mathrm{d}\zeta + \zeta \mathrm{d}\rho)t \\ & -(g(e_j) + g'(e_j)e_j)\mathrm{d}\beta_j = 0.\end{aligned}$$

Therefore,

$$\frac{\partial e_j}{\partial t} = \frac{\bar{r}}{\beta_j(2g'(e_j) + g''(e_j)e_j)}, \tag{A.123}$$

$$\frac{\partial e_j}{\partial \rho} = \frac{\zeta t}{\beta_j(2g'(e_j) + g''(e_j)e_j)}, \tag{A.124}$$

$$\frac{\partial e_j}{\partial \zeta} = \frac{\rho t}{\beta_j(2g'(e_j) + g''(e_j)e_j)}, \tag{A.125}$$

$$\frac{\partial e_j}{\partial \beta_j} = \frac{-(g(e_j) + g'(e_j)e_j)}{\beta_j(2g'(e_j) + g''(e_j)e_j)}, \tag{A.126}$$

with the given signs if $g'' \geqslant 0$.

The comparative statics for the optimal prices are obtained from the differentiation of equation (A.118) for $j = 0, 1$ (for simplicity it is assumed that T is linear or quadratic such that $\frac{\partial^2 q_j}{\partial p_k \partial p_l} = 0$, for $j, k, l \in \{0, 1\}$)

$$\begin{aligned}\mathrm{d}\frac{\partial \pi_0^e}{\partial p_0} = & \left[2\frac{\partial q_0}{\partial p_0} - \gamma_0 c''(q_0)\left(\frac{\partial q_0}{\partial p_0}\right)^2\right]\mathrm{d}p_0 + \left[\frac{\partial q_0}{\partial p_1} - \gamma_0 c''(q_0)\frac{\partial q_0}{\partial p_0}\frac{\partial q_0}{\partial p_1}\right]\mathrm{d}p_1 \\ & -(1-\bar{r}e_0)\frac{\partial q_0}{\partial p_0}\mathrm{d}t - (\rho \mathrm{d}\zeta + \zeta \mathrm{d}\rho)\, e_0 t\frac{\partial q_0}{\partial p_0} = 0, \\ \mathrm{d}\frac{\partial \pi_1^e}{\partial p_1} = & \left[\frac{\partial q_1}{\partial p_0} - \gamma_1 c''(q_1)\frac{\partial q_1}{\partial p_1}\frac{\partial q_1}{\partial p_0}\right]\mathrm{d}p_0 + \left[2\frac{\partial q_1}{\partial p_1} - \gamma_1 c''(q_1)\left(\frac{\partial q_1}{\partial p_1}\right)^2\right]\mathrm{d}p_1 \\ & -(1-\bar{r}e_1)\frac{\partial q_1}{\partial p_1}\mathrm{d}t - (\rho \mathrm{d}\zeta + \zeta \mathrm{d}\rho)\, e_1 t\frac{\partial q_1}{\partial p_1} = 0.\end{aligned}$$

Arrange this equation system into a matrix form

$$\begin{aligned}&\underbrace{\begin{pmatrix} \left(2 - \gamma_0 c''(q_0)\frac{\partial q_0}{\partial p_0}\right)\frac{\partial q_0}{\partial p_0} & \left(1 - \gamma_0 c''(q_0)\frac{\partial q_0}{\partial p_0}\right)\frac{\partial q_0}{\partial p_1} \\ \left(1 - \gamma_1 c''(q_1)\frac{\partial q_1}{\partial p_1}\right)\frac{\partial q_1}{\partial p_0} & \left(2 - \gamma_1 c''(q_1)\frac{\partial q_1}{\partial p_1}\right)\frac{\partial q_1}{\partial p_1}\end{pmatrix}}_{B=:}\begin{pmatrix}\mathrm{d}p_0 \\ \mathrm{d}p_1\end{pmatrix} \\ &\qquad = \begin{pmatrix}(1-\bar{r}e_0)\frac{\partial q_0}{\partial p_0}\mathrm{d}t + (\rho\mathrm{d}\zeta + \zeta\mathrm{d}\rho)e_0 t\frac{\partial q_0}{\partial p_0} \\ (1-\bar{r}e_1)\frac{\partial q_1}{\partial p_1}\mathrm{d}t + (\rho\mathrm{d}\zeta + \zeta\mathrm{d}\rho)e_1 t\frac{\partial q_1}{\partial p_1}\end{pmatrix}\end{aligned}$$

It follows that

$$\begin{aligned}\det(B) = &\left(2-\gamma_0 c''(q_0)\frac{\partial q_0}{\partial p_0}\right)\left(2-\gamma_1 c''(q_1)\frac{\partial q_1}{\partial p_1}\right)\left(\frac{\partial q_0}{\partial p_0}\right)^2 \\ &-\left(1-\gamma_0 c''(q_0)\frac{\partial q_0}{\partial p_0}\right)\left(1-\gamma_1 c''(q_1)\frac{\partial q_1}{\partial p_1}\right)\left(\frac{\partial q_0}{\partial p_1}\right)^2 > 0. \quad \text{(A.127)}\end{aligned}$$

Therefore,

$$\begin{aligned}\frac{\partial p_0}{\partial t} &= \frac{1}{\det(B)}\left[(1-\bar{r}e_0)\left(2-\gamma_1 c''(q_1)\frac{\partial q_1}{\partial p_1}\right)\left(\frac{\partial q_0}{\partial p_0}\right)^2\right. \\ &\left.\quad -(1-\bar{r}e_1)\left(1-\gamma_0 c''(q_0)\frac{\partial q_0}{\partial p_0}\right)\frac{\partial q_0}{\partial p_1}\frac{\partial q_1}{\partial p_1}\right] > 0, \quad \text{(A.128)} \\ \frac{\partial p_1}{\partial t} &= \frac{1}{\det(B)}\left[(1-\bar{r}e_1)\left(2-\gamma_0 c''(q_0)\frac{\partial q_0}{\partial p_0}\right)\left(\frac{\partial q_1}{\partial p_1}\right)^2\right. \\ &\left.\quad -(1-\bar{r}e_0)\left(1-\gamma_1 c''(q_1)\frac{\partial q_1}{\partial p_1}\right)\frac{\partial q_0}{\partial p_0}\frac{\partial q_1}{\partial p_0}\right] > 0, \quad \text{(A.129)} \\ \frac{\partial p_0}{\partial \rho} &= \frac{1}{\det(B)}\left[e_0 t\zeta\left(2-\gamma_1 c''(q_1)\frac{\partial q_1}{\partial p_1}\right)\left(\frac{\partial q_0}{\partial p_0}\right)^2\right. \\ &\left.\quad -e_1 t\zeta\left(1-\gamma_0 c''(q_0)\frac{\partial q_0}{\partial p_0}\right)\frac{\partial q_0}{\partial p_1}\frac{\partial q_1}{\partial p_1}\right] > 0, \quad \text{(A.130)} \\ \frac{\partial p_1}{\partial \rho} &= \frac{1}{\det(B)}\left[e_1 t\zeta\left(2-\gamma_0 c''(q_0)\frac{\partial q_0}{\partial p_0}\right)\left(\frac{\partial q_1}{\partial p_1}\right)^2\right. \\ &\left.\quad -e_0 t\zeta\left(1-\gamma_1 c''(q_1)\frac{\partial q_1}{\partial p_1}\right)\frac{\partial q_0}{\partial p_0}\frac{\partial q_1}{\partial p_0}\right] > 0, \quad \text{(A.131)}\end{aligned}$$

$$\begin{aligned}\frac{\partial p_0}{\partial \zeta} &= \frac{1}{\det(B)}\left[e_0 t\rho\left(2-\gamma_1 c''(q_1)\frac{\partial q_1}{\partial p_1}\right)\left(\frac{\partial q_0}{\partial p_0}\right)^2\right. \\ &\left.\quad -e_1 t\rho\left(1-\gamma_0 c''(q_0)\frac{\partial q_0}{\partial p_0}\right)\frac{\partial q_0}{\partial p_1}\frac{\partial q_1}{\partial p_1}\right] > 0, \quad \text{(A.132)} \\ \frac{\partial p_1}{\partial \zeta} &= \frac{1}{\det(B)}\left[e_1 t\rho\left(2-\gamma_0 c''(q_0)\frac{\partial q_0}{\partial p_0}\right)\left(\frac{\partial q_1}{\partial p_1}\right)^2\right. \\ &\left.\quad -e_0 t\rho\left(1-\gamma_1 c''(q_1)\frac{\partial q_1}{\partial p_1}\right)\frac{\partial q_0}{\partial p_0}\frac{\partial q_1}{\partial p_0}\right] > 0. \quad \text{(A.133)}\end{aligned}$$

This completes the proof. ■

Social Welfare

As in the case of sales tax evasion a suitable welfare measure is total surplus,

$$S = \bar{v} + w(R^e) - \theta \left[\int_0^{q_0} T(x)\mathrm{d}x + \int_{q_0}^1 T(1-x)\mathrm{d}x \right] - \sum_{j=0}^{1} \left(\beta_j g(e_j) e_j q_j + \gamma_j c(q_j) \right) \\ - \tau \sum_{j=0}^{1} (1 - \bar{r} e_j) q_j \tag{A.134}$$

the maximization of which holding total (expected) revenue constant is equivalent to the minimization of

$$L = \theta \left[\int_0^{q_0} T(x)\mathrm{d}x + \int_{q_0}^1 T(1-x)\mathrm{d}x \right] + \sum_{j=0}^{1} \left(\beta_j g(e_j) e_j q_j + \gamma_j c(q_j) \right). \tag{A.135}$$

Writing down the associated Lagrangian (with λ, μ as the Lagrange multipliers)

$$\mathcal{L} = \theta \left[\int_0^{q_0} T(x)\mathrm{d}x + \int_{q_0}^1 T(1-x)\mathrm{d}x \right] + \sum_{j=0}^{1} \left(\beta_j g(e_j) e_j q_j + \gamma_j c(q_j) \right) \\ + \lambda(\bar{R} - R^e) + \mu(\rho\zeta - 1) \tag{A.136}$$

one obtains the first-order conditions

$$\frac{\partial \mathcal{L}}{\partial t} = \theta[T(q_0) - T(1-q_0)] \frac{\partial q_0}{\partial t} + \sum_{j=0}^{1} \left[\beta_j \left((g(e_j) + g'(e_j)e_j) \frac{\partial e_j}{\partial t} q_j \right.\right. \\ \left.\left. + g(e_j) e_j \frac{\partial q_j}{\partial t} \right) + \gamma_j c'(q_j) \frac{\partial q_j}{\partial t} \right] - \lambda \frac{\partial R^e}{\partial t} = 0, \tag{A.137}$$

$$\frac{\partial \mathcal{L}}{\partial \nu} = \theta[T(q_0) - T(1-q_0)] \frac{\partial q_0}{\partial \nu} + \sum_{j=0}^{1} \left[\beta_j \left((g(e_j) + g'(e_j)e_j) \frac{\partial e_j}{\partial \nu} q_j \right.\right. \\ \left.\left. + g(e_j) e_j \frac{\partial q_j}{\partial \nu} \right) + \gamma_j c'(q_j) \frac{\partial q_j}{\partial \nu} \right] - \lambda \frac{\partial R^e}{\partial \nu} + \mu \nu^- = 0 \tag{A.138}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = R^e - \bar{R} = 0, \tag{A.139}$$

$$\frac{\partial \mathcal{L}}{\partial \mu} = \rho\zeta - 1 = 0, \tag{A.140}$$

where $\nu \in \{\rho, \zeta\}$ and $\nu^- = \zeta$ if $\nu = \rho$ and $\nu^- = \rho$ if $\nu = \zeta$.

Using the first-order condition for the evasion decision and eliminating the Lagrange multiplier yields the equation that any optimal tax and enforcement policy to finance expected revenue of R^e obeys:

$$\begin{aligned} \frac{\partial R^e}{\partial t} \Bigg/ \frac{\partial R^e}{\partial \nu} = {} & [T(q_0) - T(1-q_0)]\frac{\partial q_0}{\partial t} + \sum_{j=0}^{1}\left[\bar{r}t\frac{\partial e_j}{\partial t}q_j \right. \\ & \left. + \beta_j g(e_j)e_j\frac{\partial q_j}{\partial t} + \gamma_j c'(q_j)\frac{\partial q_j}{\partial t}\right] \Bigg/ \\ & [T(q_0) - T(1-q_0)]\frac{\partial q_0}{\partial \nu} + \sum_{j=0}^{1}\left[\bar{r}t\frac{\partial e_j}{\partial \nu}q_j \right. \\ & \left. + \beta_j g(e_j)e_j\frac{\partial q_j}{\partial \nu} + \gamma_j c'(q_j)\frac{\partial q_j}{\partial \nu}\right], \end{aligned} \tag{A.141}$$

for $\nu \in \{\rho, \zeta\}$.

Also in the case of a unit tax it is possible that in the optimum $\bar{r} > 0$ is chosen even if enforcement is not associated with resource costs. The proof uses Example 4.2.

PROOF: Suppose that the government wants to finance a revenue of 0.2 units. To do this in the case without tax evasion, where the government, for example, sets the fine rate such that $\bar{r} = 1 - \rho\zeta = 0$ it needs to set a tax rate of 20%. Now, assume that it does not enforce taxes strictly but allows for tax evasion and sets $\bar{r} = 0.5$. Leaving the tax rate at 20% is not sufficient to finance the desired revenue: it falls to 0.18 units. To make up for this decrease in revenue it is necessary to increase the statutory tax rate to 23.5%. Although in the latter case the equilibrium entails tax evasion the total surplus is higher than in the equilibrium without tax evasion. ■

Chapter 5

The Implications of Tax Evasion for Economic Growth*

Although growth theory is a very active field of economic research, models that explicitly take the possibility of tax evasion into account are rare. The notable exceptions are discussed below in greater detail. It is quite surprising that the literature does not provide an unambiguous prediction on the relationship between tax evasion and economic growth. One purpose of this chapter is to stress the role of a benevolent government in such models and to show that under the standard assumption of a welfare maximizing government tax evasion cannot be beneficial for growth (or welfare) even in the case of a purely consumptive public good.

5.1 Introduction

Tax evasion may affect the allocation of productive factors in several ways. On the one hand, all effort to evade taxation on the side of taxpayers and equally all effort to detect evaders on the side of the tax authority is a deadweight loss. On the other hand, the after-tax return of production in the shadow economy – where no taxes are paid by definition – is higher than in the official sector and some factors may be employed underground. Tax evasion may also affect the incentives to invest and, therefore, long-run economic growth. This is also true for the redistribution associated with tax evasion. Tax evasion redistributes income from the honest to the dishonest and from the evaders caught to the ones not detected. If the marginal propensity to save and invest are different for both groups (maybe because both

*Earlier versions of this chapter have been presented at a PhD-student conference at the Royal Institute of Technology (KTH) in Stockholm, Sweden, in 2004, the 2005 Annual Meeting of the Public Choice Society (PCS) in New Orleans, USA, the 2005 Annual Congress of the Swiss Society for Economics and Statistics (SGVS) in Zurich, Switzerland, and the 2005 Annual Conference of the Royal Economic Society (RES) in Nottingham, UK. Particularly helpful were comments by Cecilia Garcia-Peñalosa, Marco Sahm and Ulrich Woitek.

are determined by an underlying difference in the propensity to take risk), this redistribution likely affects the overall amount of investment.

Some papers that deal with one or the other aspect are discussed shortly in the following.

Wrede (1995) formulates a (discrete time) overlapping generations model of endogenous growth with a completely rival productive public good, where saving and tax evasion (of interest income) decisions are endogenous. With individuals having logarithmic preferences and a production function with increasing returns to scale he shows that tax evasion has a negative impact on growth because the loss in tax revenue leads to lower levels of the public good and income (and savings) falls. With respect to the tax enforcement parameters his results are ambiguous and depend on the intertemporal elasticity of substitution.

Roubini & Sala-i-Martin (1995) develop a model, where the government reacts to evasion: in countries with tax evasion the government increases seignorage by repressing the financial sector and increasing inflation rates. This government policy tends to reduce the amount of services that the financial sector provides to the economy, therefore the result is lower growth.

Caballé & Panadés (1997) study in particular how tax compliance policy in the form of auditing and fine rate affects the rate of economic growth in a (discrete time) overlapping generations model with identical individuals with logarithmic utility, where tax financed public goods are productive. They find that this effect is in general ambiguous, and depends on the importance of public inputs in the production process because (if compliance is not perfect) stricter enforcement increases compliance, leading to two effects in opposite directions. On the one hand, private savings fall with falling disposable income. On the other hand, the rise of public inputs leads to higher investment because of the increased productivity of private capital.

Lin & Yang (2001) adapt part of the growth model of Barro (1990) in a continuous time endogenous growth model with tax evasion. If public goods have consumptive character only, Barro (1990) finds that the growth rate is strictly decreasing in the tax rate. Lin & Yang (2001) show that for individuals with logarithmic preferences economic growth is increasing in tax evasion because resources are diverted from the unproductive government sector to the productive private sector.

Chen (2003) investigates an endogenous growth model in continuous time with a Cobb-Douglas production function with public capital financed by an income tax which can be evaded. He investigates the optimum decision of saving and evasion in an environment without uncertainty assuming that individuals hold assets of enough firms so that auditing for a fraction of income is certain by the law or large numbers. The government optimizes the tax rate, the auditing probability and the fine rate given the consumer's evasion decision. In general, these policies have ambiguous effects, but for realistic parameter constellations he finds that growth declines with tax evasion.

A main result that is challenged in the following chapter is the presumption that tax evasion is growth enhancing if the productivity of the public good that is provided is sufficiently low.

The structure of this chapter is as follows. A dynamic tax evasion model is developed along the lines of the model of Lin & Yang (2001) in section 5.2. The structure of the economy, household preferences, production possibilities, and the role of the government are defined subsequently in section 5.2.1. Closed-form solutions for the household's intertemporal optimization problem are derived (section 5.2.2). In Section 5.2.3 it is shown that if public goods are purely consumptive and utility is separable in the private and the public good tax evasion spurs growth because it leads to higher (expected) income and, therefore, higher savings (and investment). If the interaction between taxpayers and government is modelled explicitly, this conclusion need not hold because a welfare maximizing government adjusts its tax rate upwards to ensure the efficient provision of a public good (section 5.2.4). In case evasion involves no other costs than risk a neutrality result is derived: tax evasion has no impact on the growth rate.[1] Section 5.3 concludes.

5.2 The Model

5.2.1 Assumptions

Consider a continuum of identical individuals of mass 1 with no population growth. Each individual is identified by a utility function defined on a private good c (consumption) and a pure public consumption good g, both considered as flows.

Assumption 5.1 (Utility). *Assume that for each point in time the utility function u, $u : \mathbb{R}_0^+ \times \mathbb{R}_0^+ \to \mathbb{R}, (c, g) \mapsto u(c, g)$, exhibits a constant coefficient of relative risk aversion in the private good and is separable in c and g, i.e. u is of the form*

$$u(c,g) := \begin{cases} (1-\theta)\frac{c^{1-\alpha}-1}{1-\alpha}, +\theta v(g) & \text{for} \quad \alpha \neq 1, \\ (1-\theta)\ln(c) + \theta v(g) & \text{for} \quad \alpha = 1, \end{cases} \tag{5.1}$$

where α denotes the coefficient of relative risk aversion with respect to consumption defined and discussed by Pratt (1964) and Arrow (1970); θ denotes the weight attached to consumption of the public good; $v : \mathbb{R}_0^+ \to \mathbb{R}_0^+$, $g \mapsto v(g)$, is a strictly

[1] Clearly, this is a knife-edge case and if tax evasion involves resource costs or other inefficiencies, this does not hold and tax evasion goes along with lower growth rates.

increasing, strictly concave and continuously differentiable function.[2] *Assume, furthermore, that the von Neumann-Morgenstern axioms hold, i.e. individuals are expected-utility maximizers. Intertemporally, assume that utility is additive separable and state and time independent. Individuals discount future utility at a constant rate* κ, $0 < \kappa < 1$. ▼

Note that α also denotes the inverse of the elasticity of substitution of consumption between two points in time that is to say for high levels of α an individual is less willing to depart from a smooth consumption path.

Consumers optimize their consumption and evasion stream over an infinite planning horizon $T := [0, +\infty)$, where time is continuous, taking as given the restrictions imposed by the production possibility set and the tax and penalty system.[3]

In order to derive closed-form solutions with ongoing growth, a model of endogenous growth is used assuming that the production function is of the AK-type, i.e. exhibits constant returns to capital.[4]

Assumption 5.2 (Production possibilities). *Let output per capita, y, be produced by the constant returns to capital production function:*

$$y = Ak, \tag{5.2}$$

where k denotes the per capita capital stock net of depreciation and A, $0 < A$, a technological parameter. Across time, the production function is assumed to be stationary. ▼

Taxes are levied on income and tax revenues are used to finance the provision of the public good. For simplicity, a linear tax and penalty framework is considered.

[2]The CRRA specification of utility ($\alpha \neq 1$) is, for example, used by Barro (1990). It is widely argued that CRRA-utility is a reasonable assumption, see, for example, Pratt (1964) and Arrow (1970, Chapter 3). It is assumed that utility is separable in the private and the public good. A more general form of u could take account of complementarity or substitutability but makes the problem more difficult because the individual would have to form a believe on g (taking possible tax evasion of others into account). The logarithmic specification was used by Lin & Yang (2001).

[3]This modifies the first growth models with consumer optimization of Ramsey (1928), Cass (1965) and Koopmans (1965).

[4]According to Barro & Sala-i-Martin (1995, p.39), this production function was first used by von Neumann (1937). Possible explanations for constant returns to capital are technological progress and a broad interpretation of capital including human capital, see e.g. Romer (1986), Rebelo (1991). Note that, technically, this assumption is crucial for the continuous-time model because only then drift and shocks in the stochastic differential equation (5.9) are proportional to the current state.

Assumption 5.3 (Tax and penalty system). *The tax system is fully specified by a constant income tax rate τ for positive declarations of income, $0 < \tau < 1$. The penalty system is described by a constant fine rate ζ, $1 < \zeta$, to be paid on the amount of evaded tax.*[5] *To be more precise, the tax schedule is given by the function $T : \mathbb{R} \to \mathbb{R}_0^+$, $y_d \mapsto T(y_d)$:*

$$T(y_d) := \begin{cases} 0, y_d < 0, \\ \tau y_d, 0 \leqslant y_d, \end{cases} \tag{5.3}$$

where y_d denotes declared income. The fine schedule (including tax repayment) is given by the random function $\tilde{\zeta} : \mathbb{R} \times \Omega \to \mathbb{R}_0^+$, $(y_e, \omega) \mapsto \tilde{\zeta}(y_e, \omega)$:

$$\tilde{\zeta}(y_e, \omega) := \begin{cases} \zeta \tau y_e, \ 0 \leqslant y_e \wedge \omega = y_e, \\ -\tau y_e, \ y_e < 0 \wedge \omega = y_e, \\ 0, \ \omega = 0, \end{cases} \tag{5.4}$$

where $(\Omega, \mathcal{F}, \mu)$ is the probability space with event set $\Omega := \{y_e, 0\}$, σ-algebra $\mathcal{F} := \mathcal{P}(\Omega)$ and probability measure $\mu(y_e) := \rho$, $\mu(0) := 1 - \rho$, for some detection probability ρ, $0 < \rho < 1$. ▼

Remark 5.1. *As there is* no loss offset *and overpayment of taxes is only repaid in case of an audit, this specification of the tax and penalty structure ensures that a risk-averse individual always chooses $y_d \in [0, y]$.* △

With identical individuals this setup leads to a representative consumer-producer economy. Per capita output accrues to the consumer as income. At each point in time t, an individual with given income $y(t)$ has to decide simultaneously how much income to declare $y_d(t)$, $0 \leqslant y_d(t) \leqslant y(t)$, and, respectively, how much to evade, $y_e(t)$, where $y_d(t) + y_e(t) = y(t)$ always holds. Tax evasion is therefore possible by underreporting income. This approach is common in the tax evasion literature since the seminal paper by Allingham & Sandmo (1972).

The government does not know initial capital per capita k_0 (therefore cannot infer the true income stream), investigates a fraction ρ of all individuals and detects evasion if and only if the tax cheater is subjected to such a *random* audit.[6] The

[5]This specification of the fine follows Yitzhaki (1974) and is the more realistic alternative to the one proposed first by Allingham & Sandmo (1972), where the penalty is imposed on evaded income.

[6]Clearly, this is a simplification. As far as the decision on actual audits is concerned it is also unlikely that the enforcement agency audits at random. In the U.S., for example, the IRS employs a range of methods to detect evaders. In particular, it uses the results of its program of intensive audits: the Taxpayer Compliance Measurement Program (TCMP). On the basis of its results it assigns to each tax report a likelihood that it is incorrect. Andreoni *et al.* (1998) state

government's actions are exogenous for the moment, i.e. τ, ρ and ζ are given.[7] Individuals are assumed to be fully informed about the penalty and audit rate. Thus, from an individual's point of view, auditing is random and disposable income after taxes and fines is a binary random variable:

$$\tilde{y}(t,\omega) = \begin{cases} y(t, y_e) & = (1-\tau)y(t) + (1-\zeta)\tau y_e(t) \quad \text{with probability} \quad \rho, \\ y(t, 0) & = (1-\tau)y(t) + \tau y_e(t) \quad \text{with probability} \quad 1-\rho. \end{cases} \tag{5.5}$$

At each point in time the individual trades off a lower tax payment by evading income with the risk of a fine in case of an audit.

Tax evasion resembles the portfolio decision with a safe and a risky asset (evaded income). Denote by $\bar{r} := 1 - \rho\zeta$ the expected return of one unit of evaded tax. Then a well-known result due to Arrow (1970) can be applied.

Remark 5.2. *In a static setup a risk-averse individual takes risk, i.e. in the present context evades taxes if and only if the expected return on the first unit is positive, i.e.* $0 < \bar{r}$. △

Tax evasion implies that the effective tax rate differs from the statutory rate.

Lemma 5.1 (Expected tax rate). *For given income y, statutory tax rate τ and enforcement parameters ρ and ζ, and (endogenously determined) share of evaded income $e := \frac{y_e}{y}$, the expected tax rate is:*

$$\tau^e = (1 - \bar{r}e)\tau. \tag{5.6}$$

□

PROOF: Fix t, y, τ, ρ, ζ, e and remember $\bar{r} = 1 - \rho\zeta$. Then the expected tax and penalty payment is:

$$\begin{aligned} \mathbb{E}[T(y_d) + \tilde{\zeta}(y_e,\omega)] &= (1-\rho)\tau(1-e)y + \rho((1-e)\tau y + \zeta\tau e y) \\ &= \tau(1-\bar{r}e)y. \end{aligned} \tag{5.7}$$

Therefore, income is taxed at the expected rate $\tau(1-\bar{r}e) =: \tau^e$. ■

that over 50% of audits are based on this score. According to their calculations, the yield of a random audit is \$289 compared to \$5.500 for non-random ones. Nevertheless, the model uses random audits for simplicity.

[7] Technically, the probability space is endowed with a constant filtration (especially, the possibility that the probability of audit may depend on the results of prior audits is excluded). To keep notation as simple as possible the probability space on T is identified with the probability space $(\Omega, \mathcal{F}, \mu)$.

As a continuum of individuals is assumed this expected rate is equal to the effective rate of taxation in the whole economy by the law of large numbers.

It is assumed that all revenue is spent for a public good.

Assumption 5.4 (Balanced government budget). *Contemporaneous tax revenues and penalties are used to finance the public good. Thus,*

$$g(t) = \tau^e(t)y(t), \forall t \in T. \tag{5.8}$$

▼

5.2.2 Individual Optimum

The first step of the analysis is the optimal allocation of capital across time in an uncertain environment from an individual's perspective. As disposable income is random, consumption and savings depend on whether an individual has been audited or not. Savings, s, equal investment in this closed economy and augment the capital stock. They are therefore the driving force of growth.

Taking the uncertainty of an audit into account, the capital stock per capita follows a linear stochastic Itô differential equation[8]

$$\mathrm{d}k(t) = [(1 - \tau(t) + \bar{r}(t)\tau(t)e(t))y(t) - \delta k(t) - c(t)]\,\mathrm{d}t + (\sigma(t)e(t)y(t))^2\mathrm{d}z, \tag{5.9}$$

where δ, $0 \leqslant \delta$, is a constant depreciation rate, $\sigma^2 := \rho(1-\rho)(\zeta\tau)^2$ is the instantaneous variance of the process and $\{z(t), t \in T\}$ is assumed to follow a one-dimensional standard Wiener process on the probability space $(\Omega, \mathcal{F}, \mu)$.

Thus, the state of the economy is described by a stochastic differential equation involving variables, which can be adjusted by the individual so that his objective of maximal (expected discounted) utility is achieved. $(1 - \tau + \bar{r}\tau e)Ak - \delta k - c$ denotes the average drift of the capital process, which is perturbed by a noisy term depending on whether an audit has occurred or not.

At any instant t the individual chooses $\psi(t) := (c(t), e(t))$ in order to control (the moments of) the process. To ensure that the individual's objective functional is well-defined $\psi(t)$ must at least be measurable. In the following, only Markov controls are considered.[9]

[8] See Dixit & Pindyck (1994) for an introduction to stochastic processes and its applications and Lin & Yang (2001) for the application to tax evasion. The derivation of this process is shown in Lin & Yang (2001) and the present formulation takes the correction first hinted at by Caballé & Panadés (2001) into account, that is the fact that the instantaneous variance is quadratic in the tax rate.

[9] If the value function of problem (5.10) below is maximized using some other control, there is also a Markov control that does not perform worse under certain mild extra conditions (Oksendal (1998, Theorem 11.2.3, p.232)).

Assumption 5.5 (Markov controls). *Assume that the individual chooses a plan among feasible Markov controls, i.e. controls that only depend on the current state.* ▼

Consider the decision of the taxpayer of how much to consume and save respectively and how much to evade. The individual's problem is[10]

$$\begin{aligned}&\max_{\psi} \mathbb{E}_0 \left\{ \int_0^\infty \left[(1-\theta)\frac{[c(t)]^{1-\alpha}-1}{1-\alpha} + \theta v(g(t)) \right] \exp(-\kappa t)\mathrm{d}t \right\}, \\ &\text{s.t. } (5.8), (5.9), \\ &\qquad 0 \leqslant c(t) \leqslant y(t),\ 0 \leqslant e(t) \leqslant 1, \\ &\qquad 0 \leqslant k(t),\ \text{all } t \in T, k(0) = k_0 > 0,\end{aligned} \tag{5.10}$$

where $\mathbb{E}_t\{\cdot\} := \mathbb{E}\{\cdot|k(t)\}$ states that in choosing the plan ψ, the individual takes the available information about the state of the system at time t into account. The utility derived from the public good may be dropped from the optimization problem because every individual takes its provision as given. As θ is a constant it may also be dropped from the maximization.

The solution of problem (5.10) can be found using the approach of *stochastic dynamic programming.* The Bellman equation for problem (5.10) is[11]

$$\begin{aligned}\kappa I(k) = \max_{\psi} &\left\{ \frac{c^{1-\alpha}-1}{1-\alpha} + I'(k)\left(\left[1 - \tau + \bar{r}\tau e - \frac{\delta}{A}\right] Ak - c \right) \right. \\ &\left. + \frac{1}{2} I''(k)(\sigma e A k)^2 \right\},\end{aligned} \tag{5.11}$$

where $I(k) := \max_{\psi} \mathbb{E}_0 \left\{ \int_0^\infty \frac{[c(t)]^{1-\alpha}-1}{1-\alpha} \exp(-\kappa t)\mathrm{d}t \right\}$ denotes the value function and the right-hand side yields the necessary conditions for an interior optimum:

$$c^{-\alpha} - I'(k) = 0, \tag{5.12}$$

$$I'(k)\bar{r}\tau Ak + I''(k)(\sigma y)^2 e = 0, \tag{5.13}$$

such that the marginal value of capital equals the marginal utility of an additional unit of consumption and the marginal utility of an additional unit of income evaded equals the marginal disutility of increased risk.

[10] The solutions are continuous in α, therefore, results for the case of logarithmic utility can be obtained from the solution to the general CRRA utility case.

[11] Time indices are suppressed in the following calculations.

It follows:

$$c(t) = [I'(k(t))]^{-\frac{1}{\alpha}}, \tag{5.14}$$

$$e(t) = -\frac{I'(k(t))\bar{r}(t)\tau(t)}{I''(k(t))\sigma^2(t)Ak(t)}. \tag{5.15}$$

Substituting equations (5.14) and (5.15) into equation (5.11) one obtains

$$\begin{aligned}
\kappa I(k) &= \frac{[I'(k)]^{-\frac{1-\alpha}{\alpha}} - 1}{1-\alpha} + I'(k)\left(1 - \tau - \frac{\delta}{A}\right)Ak - \frac{[I'(k)\bar{r}\tau]^2}{I''(k)\sigma^2} \\
&\quad - [I'(k)]^{-\frac{1-\alpha}{\alpha}} + \frac{1}{2}I''(k)\left[\frac{I'(k)\bar{r}\tau}{I''(k)\sigma}\right]^2 \\
&= \frac{\alpha}{1-\alpha}[I'(k)]^{-\frac{1-\alpha}{\alpha}} - \frac{1}{1-\alpha} + I'(k)\left(1 - \tau - \frac{\delta}{A}\right)Ak - \frac{[I'(k)\bar{r}\tau]^2}{2I''(k)\sigma^2},
\end{aligned} \tag{5.16}$$

an implicit nonlinear differential equation of order 2, of which the solution can be written as $I(k) = \frac{Ck^{1-\alpha}-1}{\kappa(1-\alpha)}$, where C is a constant to be determined as follows:

$$\frac{Ck^{1-\alpha}}{1-\alpha} = \frac{\alpha}{1-\alpha}\left(\frac{C}{\kappa}\right)^{-\frac{1-\alpha}{\alpha}} k^{1-\alpha} + [(1-\tau)A - \delta]\frac{C}{\kappa}k^{1-\alpha} + \frac{1}{2\alpha}\left[\frac{\bar{r}\tau}{\sigma}\right]^2\frac{C}{\kappa}k^{1-\alpha} \tag{5.17}$$

$$\begin{aligned}
&\Leftrightarrow C = \alpha\left(\frac{C}{\kappa}\right)^{-\frac{1-\alpha}{\alpha}} + (1-\alpha)[(1-\tau)A - \delta]\frac{C}{\kappa} + \frac{1-\alpha}{2\alpha}\left[\frac{\bar{r}\tau}{\sigma}\right]^2\frac{C}{\kappa} \\
&\Leftrightarrow \kappa = \alpha C^{-\frac{1}{\alpha}}\kappa^{\frac{1}{\alpha}} + (1-\alpha)[(1-\tau)A - \delta] + \frac{1-\alpha}{2\alpha}\left[\frac{\bar{r}\tau}{\sigma}\right]^2 \\
&\Leftrightarrow C = \kappa\left(\frac{\kappa}{\alpha} - \frac{1-\alpha}{\alpha}\left((1-\tau)A - \delta + \frac{1}{2\alpha}\left[\frac{\bar{r}\tau}{\sigma}\right]^2\right)\right)^{-\alpha}.
\end{aligned} \tag{5.18}$$

Now the optimal consumption and evasion plans can be determined.

Proposition 5.1 (Individual optimum for CRRA utility). *The optimal consumption and evasion plan are given by:*

$$c(t) = Dk(t), \tag{5.19}$$

$$y_e(t) = \frac{\bar{r}\tau}{\alpha\sigma^2}k(t), \tag{5.20}$$

where $D := \frac{\kappa}{\alpha} - \frac{1-\alpha}{\alpha}\left((1-\tau)A - \delta + \frac{1}{2\alpha}\left[\frac{\bar{r}\tau}{\sigma}\right]^2\right)$.

At each point in time it is optimal to consume a constant share of the current capital stock per capita. Furthermore, evaded income is a constant share of the *current capital stock per capita. In analogy to the static model:*

$$0 < y_e \Leftrightarrow 0 < \bar{r}. \tag{5.21}$$

A risk-averse expected utility maximizer evades taxes if and only if the expected return of doing so is positive. □

The result of separability of the optimal tax evasion decision from the intertemporal consumption decision is analogous to the one found by Samuelson (1969) and Merton (1969, 1971) for the consumption-based capital asset pricing model.

In order to ensure that the restrictions of problem (5.10) are satisfied, the following inequalities have to hold:

$$\begin{aligned}(1-\alpha)((1-\tau)A-\delta) + \frac{1-\alpha}{2\alpha}\left[\frac{\bar{r}\tau}{\sigma}\right]^2 &\leqslant \kappa \\ &\leqslant \alpha A + ((1-\tau)A-\delta) + \frac{1-\alpha}{2\alpha}\left[\frac{\bar{r}\tau}{\sigma}\right]^2.\end{aligned} \tag{5.22}$$

The time preference rate may not be too small, otherwise the future would matter too little and may not be too large, otherwise the individual would like to postpone all consumption into the future.

For the evasion decision

$$0 \leqslant \bar{r} \leqslant \alpha\rho(1-\rho)\zeta^2 A \tag{5.23}$$

has to hold.

It is now interesting to investigate the comparative dynamics.

Proposition 5.2 (Comparative Dynamics). *For the optimal consumption plan the following comparative dynamics hold (time arguments are left out for simplicity):*

$$\frac{\partial c}{\partial \kappa} = \frac{1}{\alpha}k > 0. \tag{5.24}$$

For an increase of the instantaneous rate of time preference, the individual becomes less patient and it is optimal to substitute future for present consumption.

$$\frac{\partial c}{\partial \tau} = \frac{1-\alpha}{\alpha}Ak \gtreqless 0. \tag{5.25}$$

The effect of an increase in the tax rate on consumption is ambiguous in general. Income and substitution effects may go in opposite directions depending on the value of α.[12] *For $\alpha = 1$, $\frac{\partial c}{\partial \tau} = 0$ because income and substitution effects exactly*

[12] Sialm (2006) discusses a general equilibrium model of portfolio choice where asset prices also adjust to tax changes.

cancel in this case.

$$\frac{\partial c}{\partial \alpha} = \frac{1}{\alpha^2}\left((1-\tau)A - \delta + \frac{2-\alpha}{2\alpha}\left[\frac{\bar{r}\tau}{\sigma}\right]^2 - \kappa\right) k \gtreqless 0. \tag{5.26}$$

On the one hand, an increase in α means an increase in risk aversion and, therefore, leads to higher savings (lower consumption); on the other hand, $-\frac{1}{\alpha}$, the instantaneous elasticity of substitution of consumption over time, falls leading to higher consumption. A priori, it is unclear which effect dominates.

The comparative dynamics for the amount of evaded income are the following:

$$\frac{\partial y_e}{\partial \tau} = -\frac{\bar{r}}{\rho(1-\rho)\zeta^2\tau^2} k < 0, \tag{5.27}$$

$$\frac{\partial y_e}{\partial \bar{r}} = \frac{1}{\rho(1-\rho)\zeta^2\tau} k > 0, \tag{5.28}$$

Tax evasion decreases in the tax rate. There is no substitution effect of higher tax rates (see Yitzhaki (1974)). However, the variance increases in the tax rate. In continuous time all individuals behave locally according to the $\mu - \sigma$ criterion. Therefore, a risk-averse individual evades a smaller amount of his income. An increase in the expected return clearly makes tax evasion more attractive, leading to higher evasion. A mean-preserving spread decreases evasion – again in analogy to portfolio models, see, for example, Rothschild & Stiglitz (1970, 1971).

Additionally:

$$\frac{\partial y_e}{\partial \alpha} = -\frac{\bar{r}}{\alpha^2\rho(1-\rho)\zeta^2\tau} k < 0. \tag{5.29}$$

An increase in the coefficient of relative risk aversion leads to a reduction in the amount of evaded income reflecting the lower willingness to take risk. □

5.2.3 The Impact on Growth

The growth rate in the whole economy is certain because of the law of large numbers.

Proposition 5.3 (Growth Rate). *The expected growth rate of capital per capita is given by:*

$$\bar{\gamma}_k = \frac{1}{\alpha}\left((1-\tau)A + \frac{1+\alpha}{2\alpha}\left[\frac{\bar{r}\tau}{\sigma}\right]^2 - \kappa\right). \tag{5.30}$$

□

Proof: According to equation (5.9), it follows that the expected value of an incremental change in k is given by

$$\mathbb{E}[\mathrm{d}k] = \left(\left(1 - \tau - \frac{\delta}{A} + \bar{r}\tau e \right) Ak - c \right) \mathrm{d}t. \tag{5.31}$$

It follows with the optimal plans given by equations (5.19) and (5.20) and the definition $\mathbb{E}\left[\frac{\mathrm{d}k}{\mathrm{d}t}\right] = \mathbb{E}[\dot{k}]$, that the growth rate $\frac{\mathbb{E}[\dot{k}]}{k} = \bar{\gamma}_k$ can be written as

$$\begin{aligned}
\bar{\gamma}_k &= (1-\tau)A - \delta + \frac{1}{\alpha}\left[\frac{\bar{r}\tau}{\sigma}\right]^2 - D \\
&= (1-\tau)A - \delta + \frac{1}{\alpha}\left[\frac{\bar{r}\tau}{\sigma}\right]^2 - \frac{1}{\alpha}\left(\kappa - (1-\alpha)[(1-\tau)A - \delta] - \frac{1-\alpha}{2\alpha}\left[\frac{\bar{r}\tau}{\sigma}\right]^2\right) \\
&= \frac{1}{\alpha}\left((1-\tau)A - \delta + \frac{1+\alpha}{2\alpha}\left[\frac{\bar{r}\tau}{\sigma}\right]^2 - \kappa\right).
\end{aligned} \tag{5.32}$$

■

Although the calculations are not provided there are intuitive relationships between the underlying parameters and the growth rate. For example, a higher rate of technological progress increases growth, a higher rate of depreciation as well as a higher discount factor decreases it. A higher gain of tax evasion leads to increased evasion therefore to higher expected income after taxes and higher savings overall spurring to growth. The impact of changes in α and τ are ambiguous because of the ambiguous impact on consumption respectively on countervailing effects on consumption and evasion.

Now the growth rate of the above economy is compared with an otherwise identical economy without tax evasion. This is a special case of the model above if one assumes that $\bar{r} = 0$ such that the individual does not find it optimal to evade taxes. The only relevant decision to investigate is therefore the savings-consumption decision. Mathematically, the problem may be formed as[13]

$$\max_{c(t)} u := \int_0^\infty \frac{c(t)^{1-\alpha} - 1}{1-\alpha} exp(-\kappa t)\mathrm{d}t \tag{5.33}$$

$$\begin{aligned}
\text{s.t. } & \dot{k}(t) = (1-\tau)Ak(t) - \delta k(t) - c(t), \\
& 0 \leqslant c(t) \leqslant (1-\tau)Ak(t), \\
& 0 \leqslant k(t),\ k(0) = k_0.
\end{aligned}$$

[13] Again, the utility from the consumption of the public good is dropped immediately as it is independent of any individual's decision.

In terms of the present value Hamiltonian $\mathcal{H} := \frac{c(t)^{1-\alpha}-1}{1-\alpha}\exp(-\kappa t) - \lambda(t)[(1-\tau)Ak(t) - \delta k(t) - c(t)]$ the necessary conditions for an interior optimum are

$$\frac{\partial \mathcal{H}}{\partial c} = c(t)^{-\alpha}\exp(-\kappa t) + \lambda(t) = 0, \tag{5.34}$$

$$\dot{\lambda}(t) = -\frac{\partial \mathcal{H}}{\partial k} = -\lambda(t)((1-\tau)A - \delta), \tag{5.35}$$

a system of first-order differential equations and the transversality condition. The solution of the second equation is given by:

$$\lambda(t) = E \cdot \exp(-((1-\tau)A - \delta)t) \tag{5.36}$$

for some constant E. Inserting this into equation (5.34) one obtains:

$$\begin{aligned} c(t)^{-\alpha} &= E\exp([\kappa - ((1-\tau)A - \delta)]t) \\ \Leftrightarrow c(t) &= \frac{1}{E}\exp\left(\frac{1}{\alpha}[(1-\tau)A - \delta - \kappa]t\right). \end{aligned} \tag{5.37}$$

Therefore, the growth rate of consumption is $\frac{1}{\alpha}[(1-\tau)A - \delta - \kappa]$. The capital stock grows at the same rate: $\gamma_k(t) = \frac{1}{\alpha}[(1-\tau)A - \delta - \kappa]$.

If no evasion is possible (or optimal), the growth rate of the capital stock per capita is given by[14]

$$\gamma_k = \frac{1}{\alpha}((1-\tau)A - \delta - \kappa). \tag{5.38}$$

Immediately on obtains:

Proposition 5.4 (Growth comparison). *An economy where tax evasion is possible grows at a larger rate than if enforcement is strict. In the limit for $\alpha \to \infty$ both grow at the same rate of zero.*

$$\bar{\gamma}_k > \gamma_k; \tag{5.39}$$

$$\lim_{\alpha \to +\infty} \bar{\gamma}_k = \gamma_k = 0. \tag{5.40}$$

□

An economy with tax evasion grows at a higher rate because the increased expected income is partly saved and contributes to the accumulation of productive capital while it is used as a pure consumption good in government spending. Note that the assumption that the government does not change its tax rate (or more general: its policy) is crucial for this result. The next section shows that this result must be qualified if the government adjusts its policy as a reaction to tax evasion. In

[14]Note that one obtains this growth rate by setting $\bar{r} = 0$ in equation (5.32).

particular, it is investigated how the government should adjust its tax rate to ensure the efficient provision of a public good.

5.2.4 A Welfare Maximizing Government

Assume that the government maximizes welfare W in the economy without tax evasion. As all individuals are equal, welfare maximization is equivalent to maximization of the utility of a representative individual, i.e. the government solves

$$\max_{\tau} W(\tau) := \int_0^\infty u(c,g)\exp(-\kappa t)\mathrm{d}t, \tag{5.41}$$

taking the effect of taxation on the individual's consumption and evasion decision given by (5.19), (5.20) respectively (and the growth rate) into account. All tax revenues are spent for a public good, therefore $g(t) = \tau y(t) = \tau A k(t)$.

This problem is solved in the following for the special case discussed above assuming that u is additively separable, i.e. may be written as $u = f(c) + v(g)$ and that f and v are logarithmic.

By choosing the optimal tax rate, the government trades off a higher tax revenue from given income with a lower growth rate because of reduced savings. For the logarithmic utility function, in particular, the government's maximization problem is

$$\max_{\tau} W(\tau) = \int_0^\infty [(1-\theta)\ln(\kappa k(t)) + \theta\ln(\tau A k(t))]\exp(-\kappa t)\mathrm{d}t, \tag{5.42}$$

where capital per capita follows the differential equation

$$\dot{k} = ((1-\tau)A - \delta - \kappa)k, \tag{5.43}$$

with initial condition $k(0) = k_0$.

At time $t \in T$ it is therefore given by

$$k(t) = k_0\exp([(1-\tau)A - \delta - \kappa]t). \tag{5.44}$$

The optimization problem becomes

$$\begin{aligned}\max_{\tau} W(\tau) = \int_0^\infty &[(1-\theta)\ln(\kappa k_0\exp([(1-\tau)A - \delta - \kappa]t)) \\ &+ \theta\ln(\tau A k_0\exp([(1-\tau)A - \delta - \kappa]t))]\exp(-\kappa t)\mathrm{d}t\end{aligned} \tag{5.45}$$

The maximization of W can be equivalently be formulated as the maximization of

$$\begin{aligned}\hat{W} =& \int_0^\infty [((1-\tau)A-\delta-\kappa)t + \theta\ln(\tau)]\exp(-\kappa t)\mathrm{d}t \\ =&((1-\tau)A-\delta-\kappa)\int_0^\infty t\exp(-\kappa t)\mathrm{d}t + \theta\ln(\tau) \\ =&\frac{(1-\tau)A-\delta-\kappa}{\kappa} + \theta\ln(\tau),\end{aligned} \tag{5.46}$$

where constant terms have been dropped with first-order condition

$$\frac{\mathrm{d}\hat{W}}{\mathrm{d}\tau} = -\frac{A}{\kappa} + \frac{\theta}{\tau} = 0 \tag{5.47}$$

so in the optimum:

$$\tau^* = \frac{\kappa\theta}{A}. \tag{5.48}$$

The comparative statics of this optimal tax rate are straightforward: if the weight attached to consumption of the public good increases, the tax rate should be higher because consumption of the public good is more important to individuals; if the rate of time preference increases, future consumption (and, therefore, the growth rate) is less important and taxation should be higher to supply a larger quantity of the public good immediately; if the return of capital increases, it allows the government to lower the tax rate to obtain a sufficient level of public goods today and even achieve a higher growth rate.

The corresponding growth rate is:

$$\begin{aligned}\gamma_k^* &= (1-\tau^*)A-\delta-\kappa = \left(1-\frac{\kappa\theta}{A}\right)A-\kappa \\ &= A-\delta-\kappa(1+\theta).\end{aligned} \tag{5.49}$$

Consider now again the economy with tax evasion. If the government sets a tax rate of

$$\hat{\tau} = \tau^* + \frac{\bar{r}^2}{\rho\,(1-\rho)\,\zeta^2 A}, \tag{5.50}$$

it can induce the same growth rate. But $\hat{\tau}$ might not be optimal because also current income is taxed at this higher rate. The optimization in this case is[15]

$$\max_{\tau} \bar{W}(\tau) := \int_0^\infty [(1-\theta)\ln(\kappa k(t)) + \theta \ln(\tau(1-\bar{r}e(t))Ak(t))] \exp(-\kappa t)\mathrm{d}t \quad (5.51)$$
$$\text{s.t. } 0 \leqslant \tau \leqslant 1,\ \dot{k}(t) = [(1-\tau+\bar{r}\tau e(t))A - c(t)]k(t), \forall t \in T,\ k(0) = k_0,$$

where $c(t) = \kappa k(t)$, $y_e(t) = \frac{\bar{r}}{\rho(1-\rho)\zeta^2\tau}k(t)$ from individual optimization. The problem

$$\begin{aligned}\max_{\tau} \bar{W}(\tau) =& \int_0^\infty [(1-\theta)\ln(\kappa k_0 \exp([(1-\tau+\bar{r}\tau e)A - \delta - \kappa]t))] \exp(-\kappa t)\mathrm{d}t \\ &+ \int_0^\infty \theta \ln\left(\tau - \frac{\bar{r}^2}{\rho(1-\rho)\zeta^2 A}\right) \exp([(1-\tau+\bar{r}\tau e)A - \delta - c]t)) \exp(-\kappa t)\mathrm{d}t\end{aligned}$$

is equivalent to the maximization of

$$\begin{aligned}\tilde{W} =& [(1-\tau+\bar{r}\tau e)A - \delta - \kappa]\int_0^\infty t\exp(-\kappa t)\mathrm{d}t + \theta\ln\left(\tau - \frac{\bar{r}^2}{\rho(1-\rho)\zeta^2 A}\right)\int_0^\infty \exp(-\kappa t)\mathrm{d}t \\ =& \frac{1}{\kappa}\left[\left(1-\tau+\frac{\bar{r}^2}{\rho(1-\rho)\zeta^2}\right)A - \delta - \kappa\right] + \theta\ln\left(\tau - \frac{\bar{r}^2}{\rho(1-\rho)\zeta^2 A}\right) \quad (5.52)\end{aligned}$$

with first-order conditions:

$$\begin{aligned}-\frac{A}{\kappa} + \frac{\theta}{\tau - \frac{\bar{r}^2}{\rho(1-\rho)\zeta^2 A}} &= 0 \\ \Leftrightarrow \hat{\tau} = \frac{\theta\kappa}{A} + \frac{\bar{r}^2}{\rho(1-\rho)\zeta^2 A}&, \quad (5.53)\end{aligned}$$

[15] It has been debated whether the government should take the utility of an individual that evades taxes (Cowell (1989)) into account. In the present model it is difficult to take the risk preferences of the individuals into account because – as time passes – individuals become heterogenous as a consequence of different individual audit histories. It is not clear which probability measure should be used to capture an objective of expected welfare. Therefore, the risk attitude is left out of the analysis by the social planner.

and growth rate

$$\begin{aligned}\hat{\hat{\gamma}}_k &:= (1 - \hat{\tau} + \bar{r}\hat{\tau}e)A - \delta - \kappa \\ &= \left(1 - \frac{\theta\kappa}{A} - \frac{\bar{r}^2}{\rho(1-\rho)\zeta^2 A} + \frac{\bar{r}^2}{\rho(1-\rho)\zeta^2 A}\right) A - \delta - \kappa \\ &= A - \delta - \kappa(1+\theta) = \gamma_k^*. \end{aligned} \tag{5.54}$$

Thus, the following result:

Proposition 5.5 (Neutrality). *If a welfare maximizing government takes the consumption and evasion decision of individuals into account, it raises the tax rate to ensure the efficient provision of public goods. Overall, tax evasion has no impact on the growth rate of the economy.* □

Note that in order to establish optimal tax policy, it is not necessary for the government to possess any information on the capital stock.

5.3 Conclusions

Ceteris paribus, tax evasion increases private savings and is therefore growth enhancing if government revenues are used for a good that is unproductive. The assumption that policy is not changed if taxes are evaded is crucial for this result. If the benevolent government adjusts its tax policy to meet an efficiency objective, it raises statutory tax rates and at best tax evasion does not affect the growth rate (if enforcement costs are negligible).

Note that a crucial assumption for the growth effect is, that the possibility that the public good is productive has been excluded; see, for example, Barro (1990) or Turnovsky (1997). Therefore, there is no trade-off between the disincentiving role of taxation on private saving and public spending raising the private return to capital and, therefore, encouraging saving. It is justified by a tractability argument and the objective of the present chapter to show that evasion does not lead to higher growth even if the public good has low productivity. Nevertheless, a more complete model of the effects of tax evasion on growth should incorporate more realistic values for this productivity. Additionally, it may include an underground economy that captures activity that would otherwise not be undertaken and not only underreporting of existing income.

The auditing strategy of the government is particularly restrictive in this dynamic setup. It is unlikely that the given policy of random audits at every point in time is optimal. First of all, the tax authority may choose to audit only at discrete times. Second, as the only risk comes from auditing a single audit is sufficient to learn

k_0 and to infer the income of the agent for the whole future. Another source of asymmetric information should be added to make the audit strategy more plausible.

Another issue of the model is that one cannot separate effects of risk and time preferences in expected-utility models; see Epstein & Zin (1989) for an alternative. Chatterjee *et al.* (2004) showed that the effects of tax changes on the equilibrium growth rate, its volatility, and welfare are sensitive to independent variations of the rate of time preference and the coefficient of risk aversion using a numerical analysis with recursive preferences. It is therefore of interest to investigate how the results change when such preferences are used.[16]

[16]The methodology employed above cannot be used because the stochastic Bellman equation cannot be solved in closed-form for these preferences. Nevertheless, Campbell & Viceira (2002) show how to obtain an approximate a solution.

5.A Appendix

The Discrete-time Problem

In discrete time $T := \{t_0, t_1, \dots\}$ the capital process follows a random difference equation:

$$\tilde{k}\left(t_{j+1}\right) = (1-\delta)\, k\left(t_j\right) + \tilde{s}\left(t_j, \omega\right), \tag{A.55}$$

where

$$\tilde{s}\left(t_j, \omega\right) = \begin{cases} (1-\tau+\tau e(t_j))y(t_j) - c(t_j), & \text{for} \quad \omega = 0, \\ (1-\tau-\zeta\tau e(t_j))y(t_j) - c(t_j) & \text{for} \quad \omega = y_e. \end{cases}$$

Here, $\tilde{s}(t_j)$ denotes the savings in period t_j. It is random because it may be paid after an audit which may reveal some evasion.

For $0 < \alpha, \alpha \neq 1$ and discrete time the individual's problem is

$$\begin{aligned} &\max_{\psi} \mathbb{E}_0 \left\{ \sum_{j=0}^{\infty} \left[(1-\theta) \frac{[c(t_j)]^{1-\alpha} - 1}{1-\alpha} + \theta v(g(t_j)) \right] \exp(-\kappa t_j) \right\} \qquad\qquad (A.57) \\ &\text{s.t. } \tilde{k}(t_{j+1}) = (1-\delta)k(t_j) + \tilde{s}(t_j) \\ &\qquad 0 \leqslant c(t_j) \leqslant y(t_j),\ 0 \leqslant e(t_j) \leqslant 1,\ 0 \leqslant k(t_j), \text{ all } t_j \in T,\ k(0) = k_0 > 0, \end{aligned}$$

as he takes the level of the public good as given.

As the solution to (A.57) is very tedious it is only shown for the special case of logarithmic utility ($\alpha = 1$). Then the problem is (where constant parameters have been dropped)

$$\max_{\psi} \mathbb{E}_0 \left\{ \sum_{j=0}^{\infty} (1-\theta) \ln(c(t_j)) \exp(-\kappa t_j) \right\}. \tag{A.58}$$

The Bellman principle states that the value function satisfies the functional equation (for $j \in \mathbb{N}_0$)[17]

$$\begin{aligned} I(k(t_j)) &= \max_{c(t_j), e(t_j)} \left\{ (1-\theta) \ln c(t_j) + \beta \mathbb{E}_{t_j}[I(\tilde{k}(t_{j+1}))] \right\} \\ &= \max_{c(t_j), e(t_j)} \{ (1-\theta) \ln c(t_j) + \beta[\rho I[(1-\delta)k(t_j) \qquad\qquad (A.59) \\ &\qquad + (1-\tau+(1-\zeta)\tau e(t_j))Ak(t_j) - c(t_j)] \\ &\qquad + (1-\rho) I((1-\delta)k(t_j) + [1-\tau+\tau e(t_j)]Ak(t_j) - c(t_j))]\}, \end{aligned}$$

[17]With equidistant points in time of interval length Δt ($t_j := j\Delta t$, $j \in \mathbb{N}_0$), define the one-period discount factor $\beta := \exp(-\Delta t_j)$.

with first-order conditions of the right-hand side (for an interior maximum):

$$\begin{aligned}
&\frac{1-\theta}{c(t_j)} + \beta[\rho I'(k(t_{j+1})|\omega = y_e) + (1-\rho)I'(k(t_{j+1})|\omega = 0)](-1) = 0 \\
\Leftrightarrow\ &\frac{1-\theta}{c(t_j)} = \beta[\rho I'(k(t_{j+1})|\omega = y_e) + (1-\rho)I'(k(t_{j+1})|\omega = 0)], && \text{(A.60)} \\
&\rho I'(k(t_{j+1})|\omega = y_e)(1-\zeta)\tau + (1-\rho)I'(k(t_{j+1})|\omega = 0)\tau = 0 \\
\Leftrightarrow\ &-\frac{\rho I'(k(t_{j+1})|\omega = y_e)}{(1-\rho)I'(k(t_{j+1})|\omega = 0)} = \frac{1}{1-\zeta}. && \text{(A.61)}
\end{aligned}$$

At the optimum, the marginal utility of present consumption is equal to the discounted expected value of capital and the marginal rate of substitution between income in the state of the world where evasion is detected and the state where no audit takes place is equal to the price ratio.

The value function takes the form $I(k(t_j)) = C_1 \ln(k(t_j)) + C_2$, where C_1, C_2 are constants to be determined below, so equations $(A.60)$ and $(A.61)$ become (with $n(t_j) := (1-\delta+(1-\tau)A)k(t_j)$):[18]

$$\begin{aligned}
\frac{1-\theta}{c} &= \beta C_1 \left[\frac{\rho}{n+(1-\zeta)\tau eAk - c} + \frac{1-\rho}{n+\tau eAk - c}\right], && \text{(A.62)} \\
\frac{\rho(1-\zeta)\tau}{n+(1-\zeta)\tau eAk - c} &= \frac{(1-\rho)\,\tau}{n+\tau eAk - c}. && \text{(A.63)}
\end{aligned}$$

The (2×2)-system (A.62), (A.63) yields the solutions:

$$\begin{aligned}
c &= \frac{n(1-\theta)}{1-\theta+\beta C_1} = \frac{(1-\theta)(1-\delta+(1-\tau)A)}{1-\theta+\beta C_1}k, && \text{(A.64)} \\
e &= \frac{n\beta C_1 \bar{r}}{(1-\theta+\beta C_1)(1-\zeta)\tau Ak} = \frac{1-\delta+(1-\tau)A)\beta C_1 \bar{r}}{(1-\theta+\beta C_1)(1-\zeta)\tau A}. && \text{(A.65)}
\end{aligned}$$

The constants C_1 and C_2 are determined such that the Bellman equation (A.59) is

[18] Time indices are suppressed in the following calculations.

satisfied. Substituting equations (A.64) and (A.65) into equation (A.59) yields

$$
\begin{aligned}
C_1 \ln(k) + C_2 = (1-\theta)\ln&\left(\frac{(1-\theta)(1-\delta+(1-\tau)A)}{1-\theta+\beta C_1}k\right) \\
&+ \beta\left[\rho C_1 \ln\left((1-\delta+(1-\tau)A)\left[1-\frac{1-\theta+\beta C_1\bar{r}}{1-\theta+\beta C_1}\right]\right)\right. \\
&+ (1-\rho)C_1 \ln\left((1-\delta+(1-\tau)A)\left[1+\frac{\beta C_1 \bar{r}}{(1-\theta+\beta C_1)(1-\zeta)}\right.\right. \\
&\left.\left.\left.-\frac{1-\theta}{1-\theta+\beta C_1}\right]\right) + \beta C_1 \ln k + \beta C_2.
\end{aligned}
$$

Collecting terms:

$$
\begin{aligned}
[(1-\beta)C_1 - (1-\theta)]\ln(k) + (1-\beta)C_2 = \ & (1-\theta)\ln\left(\frac{(1-\theta)(1-\delta+(1-\tau)A)}{1-\theta+\beta C_1}\right) \\
+ \beta\Bigg[& C_1 \ln(1-\delta+(1-\tau)A) + \rho C_1 \ln\left(\frac{\beta C_1(1-\bar{r})}{1-\theta+\beta C_1}\right) \\
& + (1-\rho)C_1 \ln\left(\frac{\beta C_1((1-\zeta)+\bar{r})}{(1-\theta+\beta C_1)(1-\zeta)}\right)\Bigg]
\end{aligned}
$$

which can only be satisfied if

$$
\begin{aligned}
0 &= [(1-\beta)C_1 - (1-\theta)]\ln(k) \\
\Leftrightarrow C_1 &= \frac{1-\theta}{1-\beta} \qquad \text{(A.66)}
\end{aligned}
$$

and

$$
\begin{aligned}
(1-\beta)C_2 = \ & \ln((1-\beta)(1-\delta+(1-\tau)A)) + \frac{\beta}{1-\beta}\Bigg[\ln(1-\delta+(1-\tau)A) \\
& + \rho\ln(\beta(1-\bar{r})) + (1-\rho)\ln\left(\frac{\beta\,(1-\zeta+\bar{r})}{1-\zeta}\right)\Bigg] \\
\Leftrightarrow C_2 = \ & \frac{1-\theta}{1-\beta}\left(\ln(1-\beta) + \frac{1-\beta\theta}{1-\beta}\ln(1-\delta+(1-\tau)A)\right. \qquad \text{(A.67)} \\
& \left. + (1-\theta)\frac{\beta}{1-\beta}\left(\ln(\beta) + \rho\ln(1-\bar{r}) + (1-\rho)\ln\left(1+\frac{\bar{r}}{1-\zeta}\right)\right)\right).
\end{aligned}
$$

Thus, optimal plans are given by

$$c(t_j) = (1-\beta)(1-\delta+(1-\tau)A)k(t_j), \tag{A.68}$$

$$y_e(t_j) = \frac{(1-\delta+(1-\tau)A)\beta\bar{r}}{(1-\zeta)\tau}k(t_j). \tag{A.69}$$

As in continuous time the optimal amount consumed and evaded each period is proportional to the capital stock.

The expected rate of growth of per capita capital can be derived using equation (A.55):

$$\begin{aligned}
\mathbb{E}[k(t_{j+1})] &= \rho[(1-\delta)k(t_j) + (1-\tau+(1-\zeta)\tau e)Ak(t_j) - c(t_j)] \\
&\quad + (1-\rho)[(1-\delta)k(t_j) + (1-\tau+\tau e(t_j))Ak(t_j) - c(t_j)] \\
&= (1-\delta)k(t_j) + (1-\tau)Ak(t_j) + (1-\delta+(1-\tau)A)\beta\frac{\bar{r}^2}{1-\zeta} \\
&\quad - (1-\beta)(1-\delta+(1-\tau)A)k(t_j) \\
\Rightarrow \bar{\gamma}_k(t_j) &= \beta(1-\delta+(1-\tau)A)\frac{1-\zeta+\bar{r}^2}{1-\zeta} - 1,
\end{aligned} \tag{A.70}$$

independent of time.

One obtains an analogous result as in continuous time:

$$0 < \bar{r} \Rightarrow \bar{\gamma}_k = \beta(1-\delta+(1-\tau)A)\frac{\zeta+\bar{r}^2}{\zeta} > \beta(1-\delta+(1-\tau)A) = \gamma_k. \tag{A.71}$$

However, some of the comparative dynamics differ from the case of continuous time, in particular, because consumption is affected by taxation in the present case even for $\alpha = 1$.

Chapter 6

Tax Evasion and Economic Growth - An Empirical Investigation*

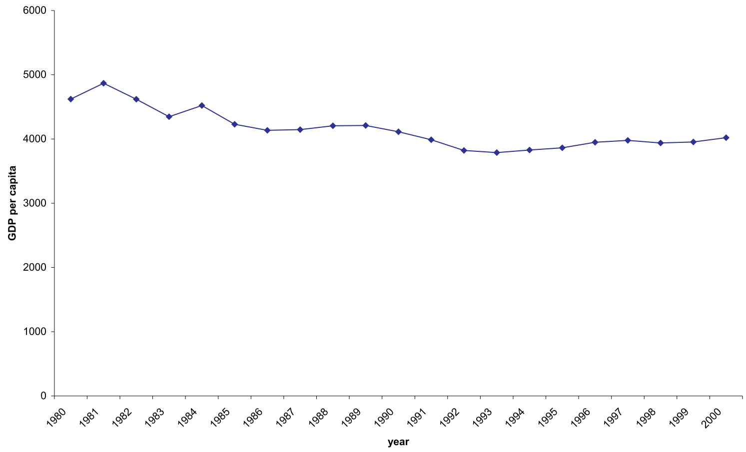
6.1 Introduction

Economic growth is an important welfare indicator because it determines available consumption opportunities of a country in the long run. A central issue in economics is to identify determinants of growth. The question is of particular importance because growth rates differ widely across countries. For example, during the period 1980 to 2000, South Africa's income per capita decreased by about 0.7% annually (on average), a decline of 13% over the whole 20 years period. On the other hand, Botswana's income per capita grew at an annual rate of 4.6% during the same time period, which amounts to a total increase of GDP per capita of about 146% (see Figure 6.1).

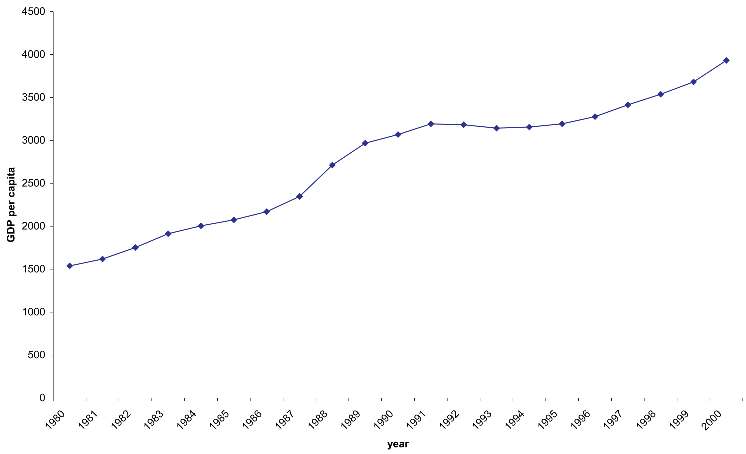
Figure 6.1 also shows how the absolute GDP per head differs in those two countries. In 1980 Botswana's GDP per head was 1,538 USD, while South Africa's amounted to 4,620 USD – about 3 times as much as Botswana's. By the year 2000, this gap has nearly vanished.

There is a huge body of economic literature which tries to identify decisive factors for economic growth. Neoclassical growth models dating back to Solow (1956) and Swan (1956) imply that growth is solely determined by population growth in the long run (steady state). At a per capita base, these models imply the absence of growth except for exogenous technological progress. In the short run, differences in growth rates are explained by differences in the currently accumulated capital per capita and in saving and depreciation rates. This view has been extended to include externalities from learning by doing (Romer (1986)), human capital formation (Lucas (1988), Rebelo (1991)), government expenditure (Barro (1990)), or endogenous technological development (Grossman & Helpman (1991), Aghion

*Earlier versions of this chapter have been presented at seminars at the University of Munich. Particularly helpful were hints by and discussions with Frank Westermann and Ulrich Woitek.



(a) GDP per capita of South Africa.



(b) GDP per capita of Botswana.

Figure 6.1: Comparison of the development of GDP per capita of South Africa and Botswana.

& Howitt (1992)) which allow ongoing growth even in the absence of exogenous technological progress. Levine & Renelt (1992), Quah & Durlauf (1999) provide a broad overview on the results of the empirical literature.

The present chapter discusses whether the extent of tax evasion helps to explain differences in growth rates across countries. To the best of the author's knowledge no paper so far exists that links tax evasion to economic growth. The goal of the present chapter is to provide a discussion of this relationship and to complement the analysis of Schneider (2005) on the relationship between the shadow economy and economic growth.

Theoretically, the implications of tax evasion for the development of a country are ambiguous. As Caballé & Panadés (1997) pointed out the increase in (expected) private income after tax increases private savings at the expense of government savings. Although tax evasion leads to an erosion of tax revenue and a lower provision of public goods, the overall effect of tax evasion on capital formation and economic growth depends on the relative productivity of public and private capital goods (and the magnitude of auditing costs).[1] As the preceding chapter argued, even in the case of purely consumptive public goods this may not be the case if the government adjusts its tax policy optimally. Even if the arguments for a negative relationship between tax evasion and growth are more convincing, the question remains to what extent growth is affected by evasion activities.

In order to answer this question, a tax evasion measure is developed and discussed. It is used to investigate the impact of tax evasion on economic growth empirically in a cross-section data set. The main finding is that tax evasion and growth are indeed negatively related. This is true irrespectively of the development stage of a country. The relationship is economically important: a country which could improve tax compliance by one standard deviation may improve growth by about 0.8 percentage points. The chapter also discusses possible underlying determinants of this relationship.

The chapter is organized as follows: section 6.2 contains the empirical analysis. Section 6.2.1 provides an overview on the concept of the shadow economy and the methods that have been employed to estimate its extent. The size of the shadow economy is used to create an indicator of tax evasion. Its construction is discussed in section 6.2.2 and section 6.2.3 describes the data set under analysis. The results of the estimation are presented and discussed in section 6.2.4. Scope for further research is discussed in section 6.3.

[1] The same result is implied by Lin & Yang (2001). Such an effect may also be generated using a model with credit constraints; see Andreoni (1992).

6.2 Concepts, Data and Estimations

6.2.1 Size Estimates of the Shadow Economy

An important part of the literature is devoted to the problem of estimating the extent of the shadow economy in different countries and over time.[2] This section provides a short overview on the methods employed and the results obtained.[3] The growth implications of the size of the underground economy have not been investigated until very recently.[4]

There is no consensus among economists about how to define the shadow economy. The following definition is widely used.[5]

Definition 6.1 (Shadow Economy). The *shadow economy* comprises all economic activities that escape detection in the official estimates of GDP and that would be taxed if it were known. ▲

All illegal activities are excluded by this definition.

For the purpose at hand it is important to note that the underground economy and tax evasion cannot be separated. In particular, no taxes are paid in the underground economy by definition.

Empirical research of the shadow economy or tax evasion naturally has to deal with measurement problems. The hidden economy does not appear in official statistics and is difficult to quantify. Potential approaches may rely on direct estimates or trace its hints indirectly. Tax evasion of individuals and firms is estimated based on official reports, where a sample of taxpayers is audited and the result is extrapolated for the entire population. An example for this method is the American Taxpayer Compliance Measurement Program (TCMP) of the Internal Revenue Service (IRS) in the U.S.; another approach uses self-reports in interviews (see Mork (1975)).[6] Indirect methods rely on (presumed) effects that the shadow economy has on observable variables, for example, on the amount of large denomination bank notes (similarly: Gutmann (1977)), the cash/deposit ratio or the ratio of high denomina-

[2] To the best of the author's knowledge estimates for the extent of tax evasion do not exist for a large number of countries.

[3] See Schneider & Enste (2000), Schneider (2003, 2004, 2005), Chaudhuri *et al.* (2006) for details.

[4] Schneider (2005) estimates the relationship between the shadow economy and economic growth in a panel data set for the time 1990–2000. No similar study exists for tax evasion.

[5] A broader definition is provided by Smith (1985, p.18): 'Market based production of goods and services, whether legal or illegal, that escape detection in the official estimates of GDP.' For a discussion of the controversy, see Schneider & Enste (2000, p.78).

[6] See, for example, Song (1978), Isachsen *et al.* (1982), Isachsen & Strom (1985). A related alternative are experimental studies, see Friedland *et al.* (1987), Spicer & Becker (1980), Baldry (1987), Becker *et al.* (1987).

tion to low denomination notes, the average life of paper currency or the difference between income and expenditure based gross domestic product data.[7,8]

6.2.2 A Proxy of Tax Evasion

Estimating the amount of evaded taxes is a difficult task because people try to conceal it from authorities. A first proxy may be the size of the shadow economy. As this chapter is mainly concerned with the revenue effect associated with the existence of the underground sector this proxy neglects that the shadow economy is only the tax base that is not declared. Across countries tax rates differ widely and the loss in government revenue can therefore not be proxied by the extent of the shadow economy alone. In order to obtain a (crude) measure of evaded tax payments one has to include the tax rate that this tax base would have been subjected to if it were known. For example, the (estimated) size of the shadow economy in India in 1999/2000 was 23% of GDP, in Ecuador it was estimated at 34.5% of GDP. These numbers suggest at first sight that the revenue loss as a fraction of GDP in Ecuador is much higher than in India. If one also notes that, for example, the corporate tax rate in India was at 41% and in Ecuador only at 25% each unit of evaded GDP in India amounted to a larger revenue loss compared to the full compliance case than in Ecuador (if it is assumed that it would have been taxed at the corporate tax rate). In effect, if the whole activity in the shadow economy were taxed (at the corporate tax rate), the tax gap per unit of GDP in both countries does not differ to the extent that the estimates of the shadow economy suggest, their relative extent is even reversed – in India it amounts to 9.5%, in Ecuador to about 9% of GDP. Therefore, the estimates of the size of the shadow economy are only used as a starting point.

A measure of the extent of tax evasion does not exist for a large number of countries. However, estimates exist for the shadow economy. Denote this measure by s_j for the (relative) size of the underground sector in country j. The activities in the shadow economy would be subject to tax and social security payments if they were known. For example, if a construction work remains undeclared, the payment to the company escapes sales taxation. There may be even more tax payments involved. The company itself pays its employees a salary subject to a wage tax and

[7]See Cagan (1958), MacAfee (1980), Contini (1981), Del Boca (1981), Petersen (1982), Smith (1985), O'Higgins (1989), Thomas (1992), Williams & Windebank (1995), Johnson *et al.* (1994), Giles (1999a,b), Bajada & Schneider (2003).

[8]It should be clear from the outset that different empirical procedures measure different components of underground activity and results cannot be directly compared across methods. The methodology determines the aspect of the shadow economy that is measured. The currency demand approach described above does not detect transactions where bartering is used and, therefore, only describes the cash economy part of the shadow economy.

social security contributions. Finally, a profit tax may apply. Denote by m the number of taxes that the shadow economy were subjected to if it were known. If ξ_{lj} denotes the part of the underground activities s_j that is subject to the tax (or social security) rate τ_{lj}, the total amount of tax that were collected in the underground sector amounts to

$$T = \sum_{l=1}^{m} \xi_{lj} \tau_{lj}, \tag{6.1}$$

where $\sum_{l=1}^{m} \xi_{lj} = s_j$, for all countries j.

This assumes that the elasticity of the activities in the underground sector with respect to taxation is zero.

Practically, it is difficult to assess the structure of the underground economy and to apply a weighted average tax rate to its overall extent. The present chapter therefore makes the simplifying assumption that either all activities in the shadow economy are subject to the corporate tax rate (specification 1) or the individual income tax rate (specification 2).

6.2.3 Description of Data

The data set includes countries from all over the world (except the former socialist countries in Eastern Europe for which the appropriate tax data is not available). Table A.5 of the appendix lists all investigated countries. The actual number of countries analyzed depends in particular on whether all variables are available and may differ across the estimations.

Table A.5 also shows the respective size of the underground sector as a percentage of official GDP as estimated by Schneider (2005). This size ranges from 8.7% (USA), 8.8% (Switzerland) to 64.1% (Panama) and 67.1% (Bolivia) in the 1999/2000 average. The average size of the shadow economy relative to GDP in the OECD countries of his sample is 16.9%. In the rest of the sample it amounts to 35.9%. A detailed description of the distribution of the used sample can be found in Table 6.1. The size of the underground sector multiplied either by the corporate (specification 1) or an individual income tax rate (specification 2) is used as an estimate of the extent of tax evasion in a country. The tax rates are also listed in Table A.5.

Table 6.1: Descriptive statistics of regression variables.

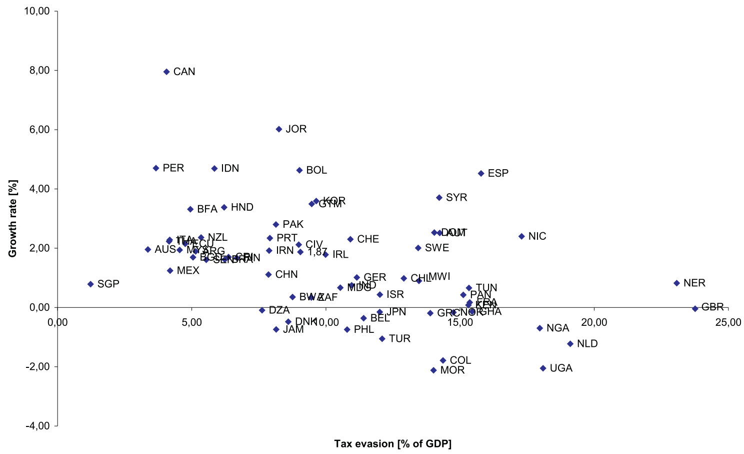
Variable	Mean	Stand.dev.	Minimum	Maximum
Growth Rate [%]	1.34	1.94	-2.64	7.95
Initial GDP per capita [USD]	8,624	11,313	145	45,951
Schooling [%]	60.0	30.6	6.6	119.5
Government share [% of GDP]	15.5	6.6	4.2	43.5
Inflation rate [%]	10.6	12.5	0.6	71.4
Life expectancy [years]	64.4	9.7	42.5	76.9
Population growth rate [%]	1.8	1.1	0	3.7
Openness [index]	.23	2.04	-6.12	15.67
Shadow economy [% of GDP]	31.0	14.4	8.7	67.1
Tax evasion (spec. 1) [% of GDP]	10.4	4.9	1.2	23.8
Tax evasion (spec. 2) [% of GDP]	11.3	5.5	1.1	29.1

Notes:
Data from Devereux *et al.* (2002), Tornell *et al.* (2005), World Bank (2003); see also Table A.6.

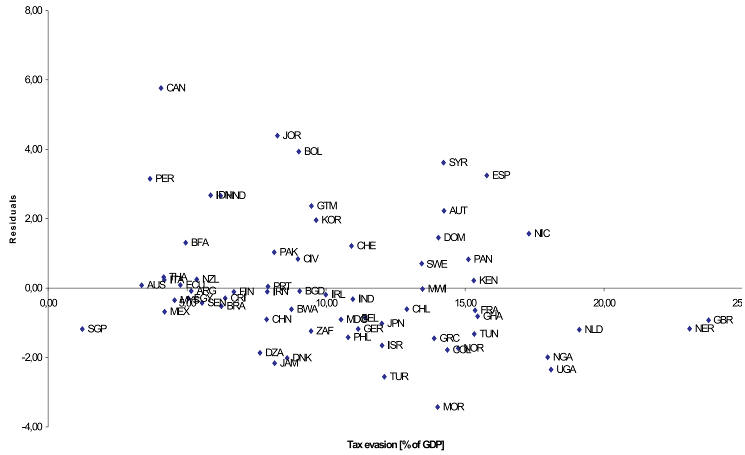
Figure 6.2(a) shows a scatter diagram suggesting a negative relationship between tax evasion (specification 1) and economic growth. This relationship remains present even if the residuals of a standard cross-section growth regression that corrects for (the logarithm of) initial income per capita, educational attainment and life expectancy are plotted against the estimated extent of tax evasion (Figure 6.2(b)).[9] The same is true for the analogous scatter diagrams for specification 2 (Figures 6.3(a) and 6.3(b)). It is not clear from the visual inspection of a scatter plot, whether this relationship also holds in high-income countries only (see Figure 6.4).

The next section investigates whether the hypothesis that such a negative link does not exist can be rejected on the basis of results from standard statistical procedures.

[9] The regression output is displayed in Table A.8 of the appendix.

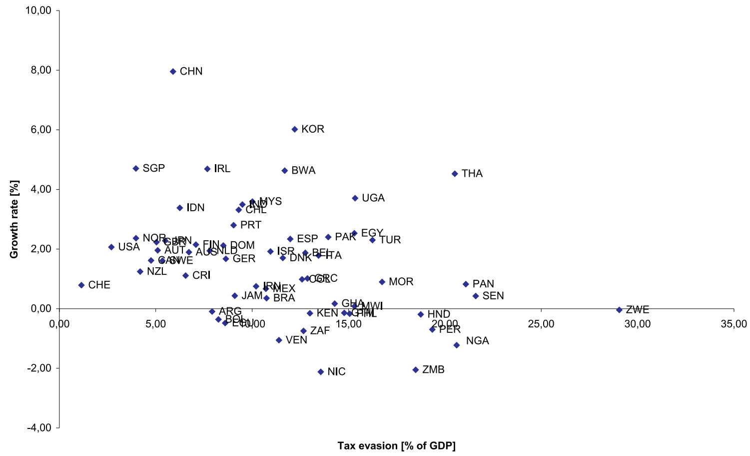


(a) Scatter diagram: tax evasion vs. growth (specification 1).

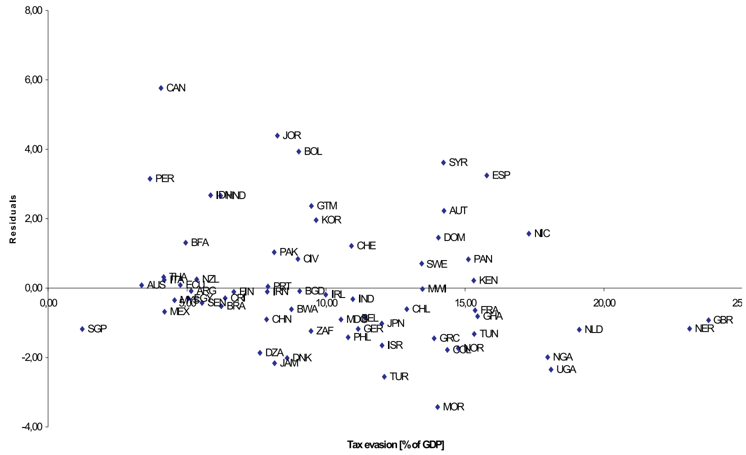


(b) Scatter diagram: tax evasion vs. residuals of cross-section growth regression with (logarithm of) initial income, education and life expectancy as explaining variables (specification 1).

Figure 6.2: Scatter diagrams for specification 1.



(a) Scatter diagram: tax evasion vs. growth (specification 2).



(b) Scatter diagram: tax evasion vs. residuals of cross-section growth regression with (logarithm of) initial income, education and life expectancy as explaining variables (specification 2).

Figure 6.3: Scatter diagrams for specification 2.

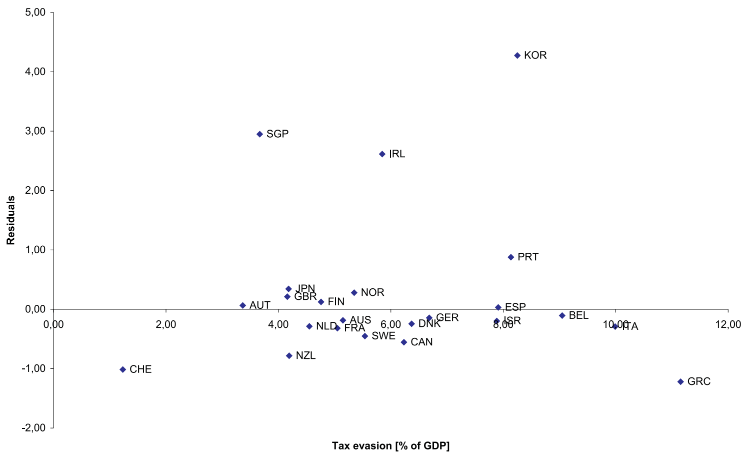


Figure 6.4: Scatter diagram: tax evasion in high income countries vs. residuals of cross-section growth regression with (logarithm of) initial income, education and life expectancy as explaining variables (specification 1).

6.2.4 Regression Analysis

The impact of tax evasion on growth is estimated in a standard cross-section growth regression. The estimated model is given by the following linear growth equation

$$\Delta y_i = \lambda_0 y_{i0} + \lambda_1 s_i + \lambda_2 l_i + \gamma^T X_i + \beta e_i + \epsilon_i, \tag{6.2}$$

where Δy_i is the average growth rate of per-capita GDP of country i; y_{i0} is the initial level of per capita GDP, s_i an education variable that proxies for the amount of human capital of a country together with the life expectancy variable l_i. X_i is a vector of other control variables that have been found important in the literature and e_i denotes the share of taxes relative to GDP per capita that is not collected due to tax evasion.[10]

In the OLS regressions 1980 is the initial year, the average growth rate is computed over the period $1981 - 2000$. The size estimate of the shadow economy reflects the average over the years 1999/2000. Table 6.2 reports the regression results.

Column (1) seems to imply that there is little evidence on absolute convergence (a feature also present in Barro & Sala-i-Martin (1995)). The parameters for the usual proxies for physical and human capital are not statistically different from zero. This also holds for several alternative functional forms and interaction terms (like using the logarithm of the life or schooling variable and interaction terms; not shown); it is also robust to the inclusion of current investment (not shown). Note that the possibility of endogeneity exists and the parameters may be biased (see below).

The parameter of the tax evasion variable is nonzero across all estimations for usual significance measures except in the last one. In estimation (5) it is only weakly significantly different from zero. If other parameters are estimated to be statistically different from zero, they have the usual signs.

As growth may also affect the extent of tax evasion the estimated coefficients may be biased. An instrumental variable (IV) regression is used to overcome the problem of endogeneity. Lagged values of the included variables are used as instruments. The hypothesis behind this is that the lagged variables do not have any influence on the mean growth rate except for the one that is given by the current right-hand side variables. The results of the instrumental variable estimation are reported in Table 6.3.

The IV estimates show in particular that there is some evidence on convergence in the sample. Also the life expectancy variable is significantly different from zero and

[10] Mauro (1995), Méndez & Sepúlveda (2006) use a similar specification to investigate the impact of corruption on economic growth.

Table 6.2: OLS regressions (specification 1).

Dependent variable: Mean growth rate	(1)	(2)	(3)	(4)	(5)
Constant	3.475	4.687**	2.472	2.542**	2.608**
	(2.830)	(1.929)	(2.782)	(1.081)	(1.062)
$\ln(y_0)$	-.554$^{\hat{*}}$	-.139			
	(.363)	(.180)			
Secondary schooling	.008		-.005		
	(.016)		(.014)		
Life expectancy	.060		.016		
	(.054)		(.046)		
Tax evasion	-.193***	-.205***	-.172***	-.085*	-.078$^{\hat{*}}$
	(.060)	(.060)	(.059)	(.050)	(.050)
Inflation				-.032**	-.033**
				(.016)	(.015)
Investment				.052$^{\hat{*}}$	.055*
				(.032)	(.032)
Government share				-.072**	-.073**
				(.029)	(.029)
Population growth				9.163	
				(17.173)	
Openness				.096	
				(.088)	
# of observations	66	66	66	64	64
R^2	.228	.205	.199	.317	.297
F-statistic	4.516**	8.101***	5.133***	4.416***	6.235***

Notes:
Regression results of standard OLS regressions of variables commonly associated with economic growth and a tax evasion variable (specification 1) on the average growth rate in the period 1981–2000.
The variables are taken from World Bank (2003); see Table A.6 for details. Standard errors are reported in parentheses. $^{\hat{*}}$ denotes significance at fifteen, * ten, ** five, *** one percent.

has a positive sign. This may show that human capital is one positive determinant of growth – in line with the literature. Although the size of the effects varies across equations, the sign does not: the estimates show that tax evasion has a negative impact on growth. For example, the results in column (5) show that an increase in compliance by one standard deviation would lead to an increase in the growth rate of $5 \times 0.155 \doteq 0.8$ percentage points.[11]

The following regressions test the robustness of the results to the presence of outliers and the occurrence of wars. As statistical outliers have been excluded the three countries with the lowest and highest residuals in the IV (5) regression (China, Ecuador, Jordan, Korea, Nicaragua, Thailand). The following countries have been excluded because of the occurrence of wars: Guatemala, Iran, Nicaragua, Peru, Philippines, South Africa, Uganda.[12]

The reasons for the negative relationship cannot be identified by the regressions. The reasons may stem from different institutional efficiency or underlying attitudes towards taxation or the government in general.

6.3 Conclusions

The empirical analysis establishes a significant negative relationship between tax evasion and economic growth. If a country with could improve tax compliance by one standard deviation, it may improve growth by about 0.8 percentage points.

Further research is clearly necessary and could address several questions. First, further examination should be undertaken to answer the question whether the employed proxy for tax evasion is appropriate. If data on the actual structure of the shadow economy across countries is available, one could weigh parts of it with the appropriate tax rate and thereby capture the revenue loss in a more appropriate way.

[11] Schneider (2005) has a similar goal to the one in the present chapter. As in the economic literature of tax evasion and growth the relation between the shadow economy and economic growth is ambiguous (Loayza (1996)). Even the underlying reason for this result is similar to the result discussed previously. A reduction in the shadow economy leads to an increase in tax revenues and greater quantity or quality of public goods which might enhance growth in the official sector (see also Adam & Ginsburgh (1985) for estimates for Belgium). Schneider (2005) therefore tries to sign the correlation and finds that "[i]f the shadow economy in industrialized countries increases by 1 percentage point of GDP, official growth increases by 7.7%; in contrast, for developing countries, an increase in the shadow economy by 1 percentage point of official GDP is associated with a decrease in the official growth rate by 4.9%."

[12] The results are not as robust for specification 2 where the individual income tax is used. The regression results for this case are shown in Table A.9 in the appendix. The present estimates are preferred because of their higher R^2.

Table 6.3: Cross section IV regressions (specification 1).

Dependent variable: Mean growth rate

	(1)	(2)	(3)	(4)	(5)
Constant	4.204	7.972***	2.430	3.603***	2.577
	(3.293)	(2.280)	(3.229)	(1.106)	(2.625)
$\ln(y_0)$	-1.457***	-.444**			-.952**
	(0.407)	(.206)			(.408)
Secondary schooling	.011		-.015		
	(.020)		(.017)		
Life expectancy	.170***		.033		.134**
	(.060)		(.055)		(.061)
Tax evasion	-.263***	-.277***	-.205***	-.096*	-.155**
	(.072)	(.072)	(.071)	(.053)	(.072)
Inflation				-.055*	-.048*
				(.028)	(.025)
Government share				-.056	
				(.051)	
Population growth				15.591	
				(22.539)	
Openness				.087	
				(.200)	
# of observations	60	60	60	57	57
R^2	.266	.227	.229	.242	.184
F-statistic	7.540***	8.080***	5.083***	3.154**	4.449**

Notes:
Regression results of instrumental variable regressions of variables commonly associated with economic growth and a tax evasion variable (specification 1) on the average growth rate in the period 1981–2000.
The variables are taken from World Bank (2003); see also Table A.6 for details.
Standard errors are reported in parentheses. $\hat{*}$ denotes significance at fifteen, * ten, ** five, *** one percent.

Table 6.4: Outliers.

Dependent variable: Mean growth rate

	Statistical outliers	War cases
Constant	2.310	2.704
	(2.195)	(2.356)
$\ln(y_0)$	-1.051***	-.683*
	(.340)	(.383)
Life expectancy	.150***	.094$^{\hat{*}}$
	(.052)	(.058)
Tax evasion	-.164***	-.140**
	(.057)	(.068)
Inflation	-.044**	-.032$^{\hat{*}}$
	(.021)	(.019)
# of observations	51	48
R^2	.348	.191
F-statistic	6.675***	3.236**

Notes:
Regression results of instrumental variable regressions (specification 1).
Standard errors are reported in parentheses.
$^{\hat{*}}$ denotes significance at fifteen, * ten, ** five, *** one percent.
War cases: Algeria, Guatemala, Iran, Nicaragua, Peru, Philippines, South Africa, Uganda
Outliers: China, Ecuador, Jordan, Korea, Nicaragua,

It is also interesting to investigate whether the results are robust to panel estimations.

Another interesting point might be to investigate the relationship between corruption and tax evasion and to include variables for both in a cross-section estimation. This may clarify whether and how close both phenomena are related and whether one of both has a greater impact on growth.

6.A Appendix

Data Set

Table A.5 provides a list of the investigated countries and their respective average growth rate and size of the shadow economy. It also includes the corporate and individual income tax rates that are used to calculate the estimate of tax evasion in a given country.

Table A.5: List of investigated countries.

Country	Country abbrev.	Mean growth rate 1981-2000	Size of the shadow economy (% of GDP)		Tax rate			
					Corporate		Individual	
			1990/91 avg.	1999/2000 avg.	1981-90	1991-2000	1981-90	1991-2000
Argentina	ARG	-0.09	22.1	25.4	.27	.29	.43	.31
Australia	AUS	1.89	10.1	14.3	.39	.36	.56	.47
Austria	AUT	1.96	6.9	9.0	.30	.33	.60	.50
Bangladesh	BGD	2.51	28.4	35.6	NA	.40	NA	NA
Belgium	BEL	1.87	19.3	22.2	.42	.39	.55	.55
Bolivia	BOL	-0.36	55.4	67.1	.22	.17	.22	.12
Botswana	BWA	4.63	27.6	33.4	.40	.28	.58	.35
Brazil	BRA	0.35	32.5	39.8	.30	.23	.49	.27
Canada	CAN	1.62	12.8	16.2	.38	.38	.32	.29
Chile	CHL	3.31	13.6	19.8	.33	.26	.55	.47
China	CHN	7.95	10.5	13.1	.40	.32	.45	.45
Colombia	COL	0.98	33.4	39.1	.30	.32	.42	.32
Costa Rica	CRI	1.11	22.0	26.2	.30	.30	.45	.25
Cote d'Ivoire	CIV	-1.79	33.4	39.9	.40	.36	NA	NA
Denmark	DNK	1.70	10.8	18.0	.45	.35	.61	.64
Dominican Rep.	DOM	2.12	28.4	.46	.32	.30	.72	.27
Ecuador	ECU	-0.48	28.9	34.4	.23	.25	.40	.25
Egypt, Ar.Rep.	EGY	2.53	30.5	35.1	.40	.40	.65	.44
Finland	FIN	2.15	13.4	18.1	.29	.26	.50	.39
France	FRA	1.69	9.0	15.2	.38	.34	NA	NA
Germany	DEU	1.67	11.8	14.9	.53	.41	.56	.53
Ghana	GHA	0.17	32.9	38.4	.50	.41	NA	.37
Greece	GRC	1.01	22.6	28.6	.46	.39	.58	.45
Guatemala	GTM	-0.14	41.4	51.5	.34	.30	.44	.29

Table A.5: (continued)

Country	Country abbrev.	Mean growth rate 1981-2000	Size of the shadow economy (% of GDP) 1990/91 avg.	1999/2000 avg.	Tax rate: Corporate 1981-90	Corporate 1991-2000	Individual 1981-90	Individual 1991-2000
Honduras	HND	-0.19	40.7	49.6	.38	.29	.40	.38
India	IND	3.49	20.6	23.1	.50	.41	.55	.41
Indonesia	IDN	3.38	15.4	19.4	.35	.32	.38	.32
Iran	IRN	0.75	13.7	18.9	.65	.60	.83	.54
Ireland, Rep. of	IRL	4.69	11.0	15.9	.45	.37	.60	.49
Israel	ISR	1.92	16.3	21.9	NA	.36	NA	.50
Italy	ITA	1.78	22.8	27.3	.36	.36	.62	.50
Jamaica	JAM	0.43	31.4	36.4	.33	.33	NA	.25
Japan	JPN	2.28	8.2	11.3	.40	.37	.66	.49
Jordan	JOR	-0.74	15.4	19.4	NA	.42	NA	NA
Kenya	KEN	-0.15	28.4	34.3	.44	.36	.60	.38
Korea, Rep. of	KOR	6.02	22.3	27.5	.30	.30	.54	.44
Malawi	MWI	0.08	33.5	40.3	.50	.39	.50	.38
Malaysia	MYS	3.59	25.1	31.1	.35	.31	.51	.32
Mexico	MEX	0.67	24.1	30.1	.37	.35	.51	.36
Morocco	MAR	0.90	29.8	36.4	.40	.37	.58	.46
Netherlands	NLD	1.94	11.9	13.5	.35	.35	.71	.60
New Zealand	NZL	1.24	9.2	12.8	.31	.33	.57	.33
Nicaragua	NIC	-2.12	40.1	45.2	NA	.31	.50	.30
Nigeria	NGA	-1.23	46.7	57.9	.40	.34	.64	.36
Norway	NOR	2.36	14.8	19.1	.28	.28	.29	.21
Pakistan	PAK	2.40	28.2	36.8	.53	.47	.53	.38
Panama	PAN	0.82	51.4	64.1	.50	.37	.56	.33
Peru	PER	-0.70	47.1	59.9	.35	.30	.54	.32

Table A.5: (continued)

Country	Country abbrev.	Mean growth rate 1981-2000	Size of the shadow economy (% of GDP) 1990/91 avg.	1999/2000 avg.	Tax rate Corporate 1981-90	Corporate 1991-2000	Individual 1981-90	Individual 1991-2000
Philippines	PHL	-0.17	37.2	43.4	.35	.34	.39	.35
Portugal	PRT	2.80	15.9	23.1	.37	.38	.53	.40
Senegal	SEN	0.42	35.1	43.2	.34	.35	.59	.50
Singapore	SGP	4.70	9.8	13.1	.33	.28	.39	.30
South Africa	ZAF	-0.74	22.1	28.4	.50	.39	.46	.45
Spain	ESP	2.34	16.1	22.7	.35	.35	.62	.53
Sweden	SWE	1.61	15.8	19.2	.46	.30	.78	.28
Switzerland	CHE	0.79	6.7	8.6	.11	.14	.19	.13
Syrian Ar.Rep.	SYR	0.34	12.8	19.3	NA	.49	NA	NA
Thailand	THA	4.53	43.2	52.6	.30	.30	.60	.39
Tunisia	TUN	2.01	30.9	38.4	NA	.35	NA	NA
Turkey	TUR	2.30	26.3	32.1	.46	.34	.54	.51
Uganda	UGA	3.71	37.2	43.1	NA	.33	NA	.36
United Kingdom	GBR	2.23	9.6	13.0	.35	.33	.58	.40
United States	USA	2.06	6.7	8.7	NA	NA	.42	.31
Uruguay	URY	0.66	41.3	51.1	.30	.30	NA	NA
Venezuela, RB	VEN	-1.06	27.4	33.6	.50	.36	.45	.34
Zambia	ZMB	-2.05	40.7	48.9	.45	.38	.78	.38
Zimbabwe	ZWE	-0.04	47.3	59.4	.50	.41	.48	.49

Notes:
Mean growth rate is the geometric average of the growth rates of GDP in the period 1981–2000.
For OECD countries the average size of the shadow economy is listed for the years 1989/1990 instead of 1990/1991.
A detailed description of the variables can be found in Table A.6.

Table A.6: Definitions and sources of variables.

Variable	Definition	Source
GDP per capita	Ratio of total GDP to total population. GDP is in 1985 PPP-adjusted USD	World Development Indicators (WDI) of World Bank (2003)
GDP per capita growth	Log difference of real GDP per capita	WDI of World Bank (2003)
Initial GDP per capita	Initial value of ratio of total GDP to total population. GDP is in 1985 PPP-adjusted USD	WDI of World Bank (2003)
Education (Schooling)	Ratio of total secondary enrollment, regardless of age, to the population of the age group that officially corresponds to that level of education.	WDI of World Bank (2003)
Government share	Ratio of government consumption to GDP	WDI of World Bank (2003)
CPI	Consumer price index at the end of year (1995 = 100)	Tornell *et al.* (2005)
Inflation rate	Annual % change in CPI	Tornell *et al.* (2005)
Life expectancy (Life)	Life expectancy at birth	World Bank (2003)
Shadow economy	GDP produced in the shadow economy in % of officially measured GDP. The estimates have been made using the currency demand approach of Cagan (1958), Tanzi (1983) and the dynamic multiple indicators multiple causes method by Frey & Weck-Hannemann (1984).	Schneider (2005), Tables 3.4, 3.5, 3.6, 3.8.
Corporate tax rate	Statutory profit tax rate (inclusive of local taxes)	Devereux *et al.* (2002)

Table A.6: (continued)

Variable	Definition	Source
Individual income tax rate	Top marginal tax rate on earned income at the federal or national level	OECD statistical compendium 2003

Table A.7: Country groupings.

Country	Development stage				Severe war period
	High inc.	Upper-middle-inc.	Lower-middle-inc.	Low inc.	
Argentina		x			
Australia	x				
Austria	x				
Bangladesh				x	
Belgium	x				
Bolivia			x		
Botswana		x			
Brazil			x		
Canada	x				
Chile		x			
China			x		
Colombia			x		
Costa Rica		x			
Cote d'Ivoire				x	
Denmark	x				
Dominican Rep.			x		
Ecuador			x		
Egypt, Arab Rep.			x		
Finland	x				
France	x				
Germany	x				
Ghana				x	
Greece	x				
Guatemala			x		x
Honduras			x		

Table A.7: (continued)

Country	Development stage				Severe war period
	High inc.	Upper-middle-inc.	Lower-middle-inc.	Low inc.	
India				x	
Indonesia			x		
Iran			x		x
Ireland, Rep. of	x				
Israel	x				
Italy	x				
Jamaica			x		
Japan	x				
Jordan			x		
Kenya				x	
Korea, Rep. of	x				
Malawi				x	
Malaysia		x			
Mexico		x			
Morocco			x		
Netherlands	x				
New Zealand	x				
Nicaragua				x	
Nigeria				x	x
Norway	x				
Pakistan				x	
Panama		x			
Peru			x		x
Philippines			x		x
Portugal	x				

Table A.7: (continued)

Country	Development stage				Severe war period
	High inc.	Upper-middle-inc.	Lower-middle-inc.	Low inc.	
Senegal				x	
Singapore	x				
South Africa			x		x
Spain	x				
Sweden	x				
Switzerland	x				
Syrian Arab Rep.			x		
Thailand			x		
Tunisia			x		
Turkey			x		
Uganda				x	x
United Kingdom	x				
United States	x				
Uruguay		x			
Venezuela, RB		x			
Zambia				x	
Zimbabwe				x	

Notes:
Classification of countries by development stage according to the World Bank.
Countries that have an (estimated average number of violent deaths/average population) *100 above 0,005 for two consecutive years are classified as having a severe war period; Heidelberg Institute of International Conflict Research (HIIK).

Regression Results

Table A.8: Cross section OLS regression.

Dependent variable: Mean growth rate	
Constant	-2.955
	(2.240)
$\ln(y_0)$	-.032
	(.369)
Schooling	.009
	(.017)
Life expectancy	.097*
	(.054)
# of observations	71
R^2	.144
F-statistic	3.747**

Notes:
Regression result of ordinary least squares regression of variables that proxy for physical and human capital endowment on the average growth rate in the period 1981–2000. The variables are taken from World Bank (2003); see also Table A.6 for details.
Standard errors are reported in parentheses.
* denotes significance at ten, ** five, *** one percent.

Table A.9: Cross section IV regressions (specification 2).

Dependent variable: Mean growth rate	(1)	(2)	(3)	(4)
Constant	2.430	7.297***	2.028	3.360***
	(3.871)	(2.629)	(3.744)	(1.030)
$\ln(y_0)$	-1.314***	-.358$^{\hat{*}}$		
	(.486)	(.235)		
Secondary schooling	.014		-.012	
	(.025)		(.020)	
Life expectancy	.017		.035	
	(.075)		(.063)	
Tax evasion	-.199**	-.249***	-.177**	-.063
	(.081)	(.077)	(.079)	(.052)
Inflation				-.084**
				(.032)
Government share				-.056
				(.043)
Population growth				36.737
				(26.207)
Openness				.044
				(.110)
# of observations	56	56	56	54
R^2	.060	.009	.095	.147
F-statistic	4.705**	5.706***	3.556**	3.048**

Notes:
Regression results of instrumental variable regressions of variables commonly associated with economic growth and a tax evasion variable (specification 2) on the average growth rate in the period 1981–2000.
The variables are taken from World Bank (2003); see also Table A.6 for details.
Standard errors are reported in parentheses. $\hat{*}$ denotes significance at fifteen, * ten, ** five, *** one percent.

Bibliography

Adam, Markus C., & Ginsburgh, Victor C. 1985. The Effects of Irregular Markets on Macroeconomic Policy: Some Estimates for Belgium. *European Economic Review*, **29**(1), 15–33.

Agell, Jonas, & Persson, Mats. 2000. Tax Arbitrage and Labor Supply. *Journal of Public Economics*, **78**(1), 3–24.

Aghion, Philippe, & Howitt, Peter. 1992. A Model of Growth through Creative Destruction. *Econometrica*, **60**(2), 323–351.

Aigner, Dennis J., Schneider, Friedrich, & Ghosh, Damayanti. 1988. Me and My Shadow: Estimating the Size of the U.S. Hidden Economy From Time Series Data. *Pages 297–334 of:* Barnett, William A., & Berndt, Ernst R. (eds), *Dynamic Econometric Modeling: Proceedings of the Third International Symposium in Economic Theory and Econometrics*. Boston: Cambridge University Press.

Al-Nowaihi, Ali, & Pyle, David J. 2000. Income Tax Evasion: A Theoretical Analysis. *Pages 249–266 of:* MacDonald, Ziggy, & Pyle, David J. (eds), *Illicit Activity: The Economics of Crime, Drugs and Tax Fraud*. Ashgate: Aldershot.

Allingham, Michael G., & Sandmo, Agnar. 1972. Income Tax Evasion: A Theoretical Analysis. *Journal of Public Economics*, **1**(3/4), 323–338.

Alm, James. 1988. Compliance Costs and the Tax Avoidance–Tax Evasion Decision. *Public Finance Quarterly*, **16**(1), 31–66.

Alm, James, & Beck, William. 1993. Tax Amnesties and Compliance in the Long Run: A Time Series Analysis. *National Tax Journal*, **46**(1), 53–60.

Alm, James, Bahl, Roy, & Murray, Matthew N. 1990. Tax Structure and Tax Compliance. *The Review of Economics and Statistics*, **72**(4), 603–613.

Alm, James, Bahl, Roy, & Murray, Matthew N. 1991a. Income Tax Evasion. *Chap. 5, pages 181–214 of:* Bahl, Roy (ed), *The Jamaican Tax Reform*. Cambridge: Lincoln Institute of Land Policy.

Alm, James, Bahl, Roy, & Murray, Matthew N. 1991b. Tax Base Erosion in Developing Countries. *Economic Development and Cultural Change*, **39**(4), 849–872.

Alm, James, Jackson, Betty R., & McKee, Michael. 1992. Estimating the Determinants of Taxpayer Compliance with Experimental Data. *National Tax Journal*, **65**(1), 107–114.

Andersen, Per. 1977. Tax Evasion and Labor Supply. *Scandinavian Journal of Economics*, **79**(3), 375–383.

Anderson, Simon P., de Palma, Andre, & Kreider, Brent. 2001. Tax Incidence in Differentiated Product Oligopoly. *Journal of Public Economics*, **81**(2), 173–192.

Andreoni, James. 1991a. The Desirability of a Permanent Tax Amnesty. *Journal of Public Economics*, **45**(2), 143–159.

Andreoni, James. 1991b. Reasonable Doubt and the Optimal Magnitude of Fines: Should the Penalty Fit the Crime? *RAND Journal of Economics*, **2**(3), 385–395.

Andreoni, James. 1992. IRS as Loan Shark. *Journal of Public Economics*, **49**(3), 35–46.

Andreoni, James, Erard, Brian, & Feinstein, Jonathan S. 1998. Tax Compliance. *Journal of Economic Literature*, **36**(2), 818–860.

Arrow, Kenneth J. 1970. *Essays in the Theory of Risk-Bearing*. Amsterdam and London: North-Holland.

Bajada, Christopher, & Schneider, Friedrich. 2003. *The Size and Development of the Shadow Economies in the Asia-Pacific*. Working Paper No. 2003-01, Johannes Kepler Universität Linz.

Balassone, Fabrizio, & Jones, Philip. 1998. Tax Evasion and Tax Rates: Properties of a Penalty Structure. *Public Finance Review*, **26**(3), 270–285.

Baldry, Jonathan C. 1979. Tax Evasion and Labour Supply. *Economics Letters*, **3**(1), 53–56.

Baldry, Jonathan C. 1987. Income Tax Evasion and the Tax Schedule: Some Experimental Results. *Public Finance/Finances Publiques*, **42**(3), 357–383.

Balestrino, Alessandro, & Galmarini, Umberto. 2003. Imperfect Tax Compliance and the Optimal Provision of Public Goods. *Bulletin of Economic Research*, **55**(1), 37–52.

Bardhan, Pranab. 1997. Corruption and Development: A Review of Issues. *Journal of Economic Literature*, **35**(3), 1320–1346.

Barreto, Raul A., & Alm, James. 2003. Corruption, Optimal Taxation and Growth. *Public Finance Review*, **31**(10), 1–34.

Barro, Robert J. 1990. Government Spending in a Simple Model of Endogenous Growth. *The Journal of Political Economy*, **98**(5), S103–S125.

Barro, Robert J., & Sala-i-Martin, Xavier. 1995. *Economic Growth*. New York et al.: McGraw-Hill.

Becker, Gary S. 1968. Crime and Punishment: An Economic Approach. *The Journal of Political Economy*, **76**(2), 169–217.

Becker, Winfried, Büchner, Heinz-Jürgen, & Sleeking, Simon. 1987. The Impact of Public Transfer Expenditures on Tax Evasion. *Journal of Public Economics*, **34**(2), 243–252.

Benjamini, Yael, & Maital, Shlomo. 1985. Optimal Tax Evasion and Optimal Tax Evasion Policy: Behavioral Aspects. *Pages 245–264 of:* Gärtner, Wulf, & Wenig, Alois (eds), *The Economics of the Shadow Economy: Proceedings of the International Conference on the Economics of the Shadow Economy Held at the University of Bielefeld, West Germany, October 10–14, 1983.* Studies in Contemporary Economics. Berlin et al.: Springer.

Bernasconi, Michele. 1998. Tax Evasion and Orders of Risk Aversion. *Journal of Public Economics*, **67**(1), 123–134.

Bernasconi, Michele, & Zanardi, Alberto. 2004. Tax Evasion, Tax Rates, and Reference Dependence. *FinanzArchiv*, **60**(3), 422–445.

Bird, Richard M. 1983. Income Tax Reform in Developing Countries – The Administrative Dimension. *Bulletin for International Fiscal Documentation*, **37**(1), 3–14.

Bishop, John A., Chow, K. Victor, Formby, John P., & Ho, Chih-

Chin. 1997. The Redistributive Effects of Non-Compliance and Tax Evasion in the US. *Pages 28–47 of:* Creedy, John (ed), *Taxation, Poverty and Income Distribution*. Aldershot: Edward Elgar.

Boadway, Robin, & Keen, Michael. 1998. Evasion and Time Consistency in the Taxation of Capital Income. *International Economic Review*, **39**(2), 461–476.

Boadway, Robin, & Sato, Motohiro. 2000. The Optimality of Punishing Only the Innocent: The Case of Tax Evasion. *International Tax and Public Finance*, **7**(6), 641–664.

Boadway, Robin, Marchand, Maurice, & Pestieau, Pierre. 1994. Towards a Theory of the Direct-Indirect Tax Mix. *Journal of Public Economics*, **55**(1), 71–88.

Boadway, Robin, Marceau, Nicolas, & Mongrain, Steeve. 2002. Joint Tax Evasion. *Canadian Journal of Economics/Revue canadienne d'Economique*, **35**(3), 418–435.

Borck, Rainald. 2004. Stricter Enforcement May Increase Tax Evasion. *European Journal of Political Economy*, **20**(3), 725–737.

Border, Kim C., & Sobel, Joel. 1987. Samurai Accountant: A Theory of Auditing and Plunder. *Review of Economic Studies*, **54**(4), 525–540.

Bordignon, Massimo. 1993. A Fairness Approach to Income Tax Evasion. *Journal of Public Economics*, **52**(3), 345–362.

Bulow, Jeremy I., Geanakoplos, John D., & Klemperer, Paul D. 1985. Multimarket Oligopoly: Strategic Substitutes and Complements. *The Journal of Political Economy*, **93**(3), 488–511.

Bundesverfassungsgericht. 2004. Verfassungswidrigkeit der Besteuerung Privater Spekulationsgewinne 1997 und 1998 mit Wertpapieren. *Neue Juristische Wochenschrift*, **57**(14), 1022–1030. 9.3.2004 - 2 BvL 17/02.

Busse, Eva. 2000. The Embeddedness of Tax Evasion in Russia. *Pages 129–143 of:* Ledeneva, Alena V., & Kurkchiyan, Marina (eds), *Economic Crime in Russia*. The Hague: Kluwer Law International.

Caballé, Jordi, & Panadés, Judith. 1997. Tax Evasion and Economic Growth. *Public Finance/Finances Publiques*, **52**(3/4), 318–340.

Caballé, Jordi, & Panadés, Judith. 2001. *On the Relation Between Tax*

Rates and Evasion in a Multi-period Economy. Working Paper No. 500.01, Unitat de Fonaments de l'Análisi Económia (UAB) and Institute d'Análisi Económica (CSIC).

Cagan, Phillip. 1958. The Demand for Currency Relative to the Total Money Supply. *The Journal of Political Economy*, **66**(4), 303–328.

Campbell, John Y., & Viceira, Luis M. 2002. *Strategic Asset Allocation.* Oxford: Oxford University Press.

Caplan, David, Ruffles, David, Tily, Geoff, & Tudor, Sandra. 2003. VAT Missing Trader Intra-Community Fraud: the Effect on Balance of Payment Statistics and UK National Accounts. *Economic Trends*, **597**, 58–70.

Cass, David. 1965. Optimum Growth in an Aggregative Model of Capital Accumulation. *Review of Economic Studies*, **32**(3), 233–240.

Chang, Wen-Ya, & Lai, Ching-Chong. 1996. The Implication of Efficiency Wages on Tax Evasion and Tax Collections. *Public Finance Quarterly*, **24**(2), 163–172.

Chatterjee, Santanu, Giuliano, Paola, & Turnovsky, Stephen J. 2004. Capital Income Taxes and Growth in a Stochastic Economy: A Numerical Analysis of the Role of Risk Aversion and Intertemporal Substitution. *Journal of Public Economic Theory*, **6**(2), 277–310.

Chaudhuri, Kausik, Schneider, Friedrich, & Chattopadhyay, Sumana. 2006. The Size and Development of the Shadow Economy: An Empirical Investigation from States of India. *Journal of Development Economics*, **79**(forthcoming).

Chen, Been-Lon. 2003. Tax Evasion in a Model of Endogenous Growth. *Review of Economic Dynamics*, **6**(2), 381–403.

Chen, Kong-Pin, & Chu, C.Y. Cyrus. 2002. Internal Control vs. External Manipulation: a Model of Corporate Income Tax Evasion. *RAND Journal of Economics*, **36**(1), 151–164.

Christian, Charles W., Gupta, Sanjay, & Young, James C. 2002. Evidence on Subsequent Filing from the State of Michigan's Income Tax Amnesty. *National Tax Journal*, **55**(4), 703–721.

Christiansen, Vidar. 1980. Two Comments on Tax Evasion. *Journal of Public Economics*, **13**(3), 389–393.

Clotfelter, Charles T. 1983. Tax Evasion and Tax Rates: An Analysis of Individual Returns. *The Review of Economics and Statistics*, **65**(3), 363–373.

Contini, Bruno B. 1981. Labor Market Segmentation and the Development of the Parallel Economy: the Italian Experience. *Oxford Economic Papers*, **33**, 401–412.

Cowell, Frank A. 1981. Taxation and Labour Supply With Risky Activities. *Economica*, **48**(192), 365–379.

Cowell, Frank A. 1985a. The Economic Analysis of Tax Evasion. *Bulletin of Economic Research*, **37**(3), 163–193.

Cowell, Frank A. 1985b. Tax Evasion with Labour Income. *Journal of Public Economics*, **26**(1), 19–34.

Cowell, Frank A. 1989. Honesty Is Sometimes the Best Policy. *European Economic Review*, **33**(2/3), 605–617.

Cowell, Frank A. 1990. *Cheating the Government: The Economics of Evasion.* Cambridge: MIT Press.

Cowell, Frank A. 2004. Carrots and Sticks in Enforcement. *Pages 230–275 of:* Aaron, Henry J., & Slemrod, Joel (eds), *The Crisis in Tax Administration.* Washington, D.C.: The Brookings Instiution Press.

Cowell, Frank A., & Gordon, James P.F. 1988. Unwillingness to Pay: Tax Evasion and Public Good Provision. *Journal of Public Economics*, **36**(3), 305–321.

Crane, Steven E., & Nourzad, Farrokh. 1990. Tax Rates and Tax Evasion: Evidence from Californian Amnesty Data. *National Tax Journal*, **43**(2), 189–199.

Cremer, Helmuth, & Gahvari, Firouz. 1992. Tax Evasion and the Structure of Indirect Taxes and Audit Probabilities. *Public Finance/Finances Publiques*, **47(Suppl)**, 351–365.

Cremer, Helmuth, & Gahvari, Firouz. 1993. Tax Evasion and Optimal Commodity Taxation. *Journal of Public Economics*, **50**(2), 261–275.

Cremer, Helmuth, & Gahvari, Firouz. 1994. Tax Evasion, Concealment and the Optimal Linear Income Tax. *Scandinavian Journal of Economics*, **96**(2), 219–239.

CREMER, HELMUTH, & GAHVARI, FIROUZ. 1996. Tax Evasion and the Optimum General Income Tax. *Journal of Public Economics*, **60**(2), 235–249.

CREMER, HELMUTH, & GAHVARI, FIROUZ. 1999. Excise Tax Evasion, Tax Revenue, and Welfare. *Public Finance Review*, **27**(1), 77–95.

CREMER, HELMUTH, & THISSE, JACQUES-FRANÇOIS. 1994. Commodity Taxation in Differentiated Oligopoly. *International Economic Review*, **35**(1), 613–633.

CREMER, HELMUTH, MARCHAND, MAURICE, & PESTIEAU, PIERRE. 1990. Evading, Auditing and Taxing: The Equity-Compliance Tradeoff. *Journal of Public Economics*, **43**(1), 67–92.

CROCKER, KEITH J., & SLEMROD, JOEL. 2005. Corporate Tax Evasion with Agency Costs. *Journal of Public Economics*, **89**(9/10), 1593–1610.

CROSS, RODNEY, & SHAW, G. KEITH. 1982. On the Economics of Tax Aversion. *Public Finance/Finances Publiques*, **37**(1), 36–47.

DAS-GUPTA, ARINDAM. 1994. A Theory of Hard-to-Tax Groups. *Public Finance/Finances Publiques*, **49(Suppl)**, 28–39.

D'ASPREMONT, CLAUDE, GABSZEWICZ, JEAN J., & THISSE, JACQUES-FRANÇOIS. 1979. On Hotelling's 'Stability in Competition'. *Econometrica*, **47**(5), 1145–1150.

DEL BOCA, DANIELA. 1981. Parallel Economy and Allocation of Time. *Micros (Quarterly Journal of Microeconomics)*, **4**(2), 13–18.

DEUTSCHES AKTIENINSTITUT (DAI). 2004. *DAI-Factbook*. Tech. rept. Deutsches Aktieninstitut (DAI), Frankfurt a.M.

DEVEREUX, MICHAEL P., GRIFFITH, RACHEL, & KLEMM, ALEXANDER. 2002. Corporate Income Tax Reforms and International Tax Competition. *Economic Policy*, **17**(35), 449–495.

DIXIT, AVINASH K., & PINDYCK, ROBERT S. 1994. *Investment Under Uncertainty*. Princeton: Princeton University Press.

DUBIN, JEFFREY A., & WILDE, LOUIS L. 1988. An Empirical Analysis of Federal Income Tax Auditing and Compliance. *National Tax Journal*, **41**(1), 61–74.

THE ECONOMIST. 1997a. Smelling Salts. *The Economist Newspaper*, **345**(8039), 36–36.

THE ECONOMIST. 1997b. Wedded to Complications. *The Economist Newspaper*, **345**(8039), 28–28.

THE ECONOMIST. 1999. Rupert Laid Bare. *The Economist Newspaper*, **350**(8111), 63–64.

THE ECONOMIST. 2000. Getting Personal. *The Economist Newspaper*, **354**(8155), 20–21.

THE ECONOMIST. 2001. A Taxing Question. *The Economist Newspaper*, **359**(8226), 9–12.

THE ECONOMIST. 2004a. Fighting Corruption. *The Economist Online Edition*, 29 April. www.economist.com.

THE ECONOMIST. 2004b. A Taxing Battle. *The Economist Newspaper*, **370**(8360), 71–72.

EICHHORN, CHRISTOPH, & SAHM, MARCO. 2005a. Billige WM-Tickets dank Sponsoring. *Wirtschaftsdienst*, **85**(4), 255–257.

EICHHORN, CHRISTOPH, & SAHM, MARCO. 2005b. *Monopoly Pricing, Demand Quality and Two-sided Markets*. Mimeo.

EICHHORN, CHRISTOPH, & SAHM, MARCO. 2005c. Warum Sind die WM-Tickets so Billig? *WiSt – Wirtschaftswissenschaftliches Studium*, **34**(7), 395–398.

EICHHORN, CHRISTOPH, SAHM, MARCO, & WESTERMANN, FRANK. 2005. Eine Ökonomische Nachbetrachtung der Münchner Hochhausabstimmung. *ifo Schnelldienst*, **58**(1), 40–40.

ELFFERS, HENK, WEIGEL, RUSSELL H., & HESSING, DICK J. 1987. The Consequences of Different Strategies for Measuring Tax Evasion Behavior. *Journal of Economic Psychology*, **8**(3), 311–337.

ELLER, MARTHA BRITTON, ERARD, BRIAN, & HO, CHIH-CHIN. 2001. The Magnitude and Determinants of Federal Estate Tax Noncompliance. *Pages 375–410 of:* GALE, WILLIAM G., HINES, JR., JAMES R., & SLEMROD, JOEL (eds), *Rethinking Estate and Gift Taxation*, vol. 8. Washington, D.C.: The Brookings Institution Press.

EMRAN, M. SHAHE, & STIGLITZ, JOSEPH E. 2005. On Selective Indirect Tax Reform in Developing Countries. *Journal of Public Economics*, **89**(4), 599–623.

ENGEL, EDUARDO M.R.A., & HINES JR., JAMES R. 1998. *Understanding Tax Evasion Dynamics*. Centro de Economía Aplicada Departamento de Ingeniería Industrial Facultad de Ciencias Físicas y Matemáticas Universidad de Chile Serie Economie No. 47.

EPSTEIN, LARRY G., & ZIN, STANLEY E. 1989. Substitution, Risk Aversion and the Temporal Behavior of Consumption and Asset Returns: a Theoretical Framework. *Econometrica*, **57**(4), 937–969.

ERARD, BRIAN, & FEINSTEIN, JONATHAN S. 1994. Honesty and Evasion in the Tax Compliance Game. *RAND Journal of Economics*, **25**(1), 3–19.

FALKINGER, JOSEF. 1991. On Optimal Public Good Provision With Tax Evasion. *Journal of Public Economics*, **45**(1), 127–133.

FALKINGER, JOSEF. 1995. Tax Evasion, Consumption of Public Goods and Fairness. *Journal of Economic Psychology*, **16**(1), 63–72.

FEDELI, SILVIA, & FORTE, FRANCESCO. 1999. Joint Income-Tax and VAT-Chain Evasion. *European Journal of Political Economy*, **15**(3), 391–415.

FEIGE, EDGAR L. 1989. The Meaning and Measurement of the Underground Economy. *Chap. 1, pages 13–56 of:* FEIGE, EDGAR L. (ed), *The Underground Economies - Tax Evasion and Information Distortion*. Cambridge et al.: Cambridge University Press.

FEINSTEIN, JONATHAN S. 1991. An Econometric Analysis of Income Tax Evasion and its Detection. *RAND Journal of Economics*, **22**(1), 14–35.

FEINSTEIN, JONATHAN S., & HO, CHIH-CHIN. 2001. Elderly Asset Management and Health. vol. 8. Washington, D.C.: The Brookings Institution Press.

FELD, LARS P., & FREY, BRUNO S. 2002. Trust Breeds Trust: How Taxpayers are Treated. *Economics of Governance*, **3**(3), 87–99.

FELD, LARS P., & TYRAN, JEAN-ROBERT. 2002. Tax Evasion and Voting: An Experimental Analysis. *Kyklos*, **55**(2), 197–222.

FELDSTEIN, MARTIN. 1995. The Effect of Marginal Tax Rates on Taxable Income: A Panel Study of the 1986 Tax Reform Act. *The Journal of Political Economy*, **103**(3), 551–572.

FERSHTMAN, CHAIM, GANDAL, NEIL, & MARKOVICH, SARIT. 1999. Estimating the Effect of Tax Reform in Differentiated Product Oligopolistic Markets. *Journal* *of Public Economics*, **74**(2), 151–170.

Fishburn, Geoffrey. 1979. On How to Keep Taxpayers Honest (or Almost so). *The Economic Record*, **55**(150), 267–270.

Franzoni, Luigi A. 2000. Tax Evasion and Tax Compliance. *Pages 52-–94 of:* Bouckaert, Boudewijn, & Geest, Gerrit De (eds), *Encyclopedia of Law and Economics*, vol. 4. Cheltenham: Edward Elgar.

Frey, Bruno S., & Weck-Hannemann, Hannelore. 1984. The Hidden Economy as an 'Unobserved' Variable. *European Economic Review*, **26**(1), 33–53.

Friedland, Nehemiah, Maital, Shlomo, & Rutenberg, Aryeh. 1987. A Simulation Study of Income Tax Evasion. *Journal of Public Economics*, **10**(1), 107–116.

Fuest, Clemens, & Huber, Bernd. 2001. Why is There so Little Tax Coordination? The Role of Majority Voting and International Tax Evasion. *Regional Science and Urban Economics*, **31**, 299–317.

Giles, David E.A. 1999a. Measuring the Hidden Economy: Implications for Econometric Modelling. *The Economic Journal*, **109**(456), F370–F380.

Giles, David E.A. 1999b. Modelling the Hidden Economy and the Tax-Gap in New Zealand. *Empirical Economics*, **24**(4), 621–640.

Goerke, Laszlo, & Runkel, Marco. 2005. *Profit Tax Evasion Under Oligopoly with Endogenous Market Structure*. Mimeo.

Goolsbee, Austan. 2000. In a World Without Borders: The Impact of Taxes on Internet Commerce. *The Quarterly Journal of Economics*, **115**(2), 561–575.

Gordon, James P.F. 1989. Individual Morality and Reputation Costs as Deterrents to Tax Evasion. *European Economic Review*, **33**(4), 797–805.

Gordon, Roger H., & Nielsen, Soren Bo. 1997. Tax Evasion in an Open Economy: Value-Added Vs. Income Taxation. *Journal of Public Economics*, **66**(2), 173–197.

Graetz, Michael J., & Reinganum, Jennifer F. 1986. The Tax Compliance Game: Toward an Interactive Theory of Tax Enforcement. *Journal of Law, Economics and Organization*, **2**(1), 1–32.

Graetz, Michael J., Reinganum, Jennifer F., & Wilde, Louis L. 1984

(November). *A Model of Tax Compliance With Budget-constrained Auditors.* Social Science Working Paper 520, California Institute of Technology.

Greenberg, Joseph. 1984. Avoiding Tax Avoidance: A (Repeated) Game Theoretic Approach. *Journal of Economic Theory*, **32**(1), 1–13.

Grossman, Gene, & Helpman, Elhanan. 1991. Quality Ladders in the Theory of Growth. *Review of Economic Studies*, **58**(1), 43–61.

Gruenspecht, Howard K. 1984. Export Subsidies for Differentiated Products. *Journal of International Economics*, **24**(3/4), 331–344.

Gutmann, Pierre M. 1977. The Subterranean Economy. *Financial Analysts Journal*, **34**(1), 26–28.

Hahn, Frank H. 1962. The Stability of the Cournot Oligopoly Solution. *Review of Economic Studies*, **29**, 329–331.

Hamilton, Stephen F. 1999. Tax Incidence Under Oligopoly: a Comparison of Policy Approaches. *Journal of Public Economics*, **71**(2), 233–245.

Harsanyi, John. 1973. Oddness of the Number of Equilibrium Points: A New Proof. *International Journal of Game Theory*, **2**, 235–250.

Hasseldine, John. 1998. Tax Amnesties: An International Review. *Bulletin for International Fiscal Documentation*, **52**(7), 303–310.

Hibbs, Douglas A., & Piculescu, Violeta. 2005. *Institutions, Corruption and Tax Evasion in the Unofficial Economy.* Working Papers in Economics No. 173, University of Göteborg.

Hindricks, Jean, Keen, Michael, & Muthoo, Abhinay. 1999. Corruption, Extortion and Evasion. *Journal of Public Economics*, **74**(3), 395–430.

Hotelling, Harold. 1929. Stability in Competition. *The Economic Journal*, **39**(1), 41–57.

Hunter, William J., & Nelson, Michael A. 1995. Tax Enforcement: A Public Choice Perspective. *Public Choice*, **82**(1/2), 53–67.

Isachsen, Arne, & Strom, Steiner. 1985. The Size and Growth of the Hidden Economy in Norway. *The Review of Income and Wealth*, **31**(1), 21–38.

Isachsen, Arne J., & Strom, Steiner. 1980. The Hidden Economy: The

Labor Market and Tax Evasion. *Scandinavian Journal of Economics*, **82**(2), 304–311.

Isachsen, Arne J., Klovland, Jan, & Strom, Steiner. 1982. The Hidden Economy in Norway. *Pages 209–231 of:* Tanzi, Vito (ed), *The Underground Economy in the United States and Abroad*. Heath: Lexington.

Johnson, Simon, Kaufmann, Daniel, & Zoido-Lobatón, Pablo. 1994. Regulatory Discretion and the Unofficial Economy. *The American Economic Review*, **88**(2), 387–392.

Joulfaian, David, & Rider, Mark. 1996. Tax Evasion in the Presence of Negative Income Tax Rates. *National Tax Journal*, **49**(4), 553–570.

Joulfaian, David, & Rider, Mark. 1998. Differential Taxation and Tax Evasion by Small Business. *National Tax Journal*, **51**(4), 675–687.

Kaplow, Louis. 1990. Optimal Taxation With Costly Enforcement and Evasion. *Journal of Public Economics*, **43**(2), 221–236.

Katz, Michael L., & Rosen, Harvey. 1985. Tax Analysis in an Oligopoly Model. *Public Finance Quarterly*, **13**(1), 3–19.

Kesselman, Jonathan R. 1989. Income Tax Evasion - An Intersectoral Analysis. *Journal of Public Economics*, **38**(2), 137–182.

Kesselman, Jonathan R. 1993. Evasion Effects of Changing the Tax Mix. *The Economic Record*, **69**(205), 131–148.

Kim, Chung Kweon. 2002. Does Fairness Matter in Tax Reporting Behavior? *Journal of Economic Psychology*, **23**(6), 771–785.

Klepper, Steven, & Nagin, Daniel. 1989. The Anatomy of Tax Evasion. *Journal of Law, Economics and Organization*, **5**(1), 1–24.

Kolm, Serge-Christophe. 1973. A Note on Optimum Tax Evasion. *Journal of Public Economics*, **2**(3), 265–270.

Koopmans, Tjalling C. 1965. On the Concept of Optimal Economic Growth. North Holland: Amsterdam.

Koskela, Erkki. 1983a. A Note on Progression, Penalty Schemes and Tax Evasion. *Journal of Public Economics*, **22**(1), 127–133.

Koskela, Erkki. 1983b. On the Shape of Tax Schedule, the Probability of Detection, and the Penalty Schemes as Deterrents to Tax Evasion. *Public Finance/Finances Publiques*, **38**(1), 71–80.

Kreutzer, David, & Lee, Dwight R. 1986. On Taxation and Understated Monopoly Profits. *National Tax Journal*, **39**(2), 241–243.

Kreutzer, David, & Lee, Dwight R. 1988. Tax Evasion and Monopoly Output Decisions: A Reply. *National Tax Journal*, **41**(4), 583–584.

Kucher, Marcel, & Götte, Lorenz. 1998. Trust Me: An Empirical Analysis of Taxpayer Honesty. *FinanzArchiv*, **55**(3), 429–444.

Lai, Ching-Chong, & Chang, Wen-Ya. 1988. Tax Evasion and Tax Collections: An Aggregate Demand - Aggregate Supply Analysis. *Public Finance/Finances Publiques*, **43**(1), 138–146.

Lai, Ching-Chong, Chang, Juin-Jen, & Chang, Wen-Ya. 1995. Tax Evasion and Efficient Bargains. *Public Finance/Finances Publiques*, **50**(1), 96–105.

Landskroner, Yoram, Paroush, Jacob, & Swary, Itzhak. 1990. Tax Evasion and Portfolio Decisions. *Public Finance/Finances Publiques*, **45**(3), 409–422.

Launhardt, C.F. Wilhelm. 1885. *Mathematische Begründung der Volkswirtschaftslehre*. Aalen: Scintia. Neudruck 1963.

Lee, Kangoh. 1998. Tax Evasion, Monopoly, and Nonneutral Profit Taxes. *National Tax Journal*, **51**(2), 333–338.

Lee, Kangoh. 2001. Tax Evasion and Self-Insurance. *Journal of Public Economics*, **81**(1), 73–81.

Lemieux, Thomas, Fortin, Bernard, & Fréchette, Pierre. 1994. An Empirical Model of Labor Supply in the Underground Economy. *The American Economic Review*, **84**(1), 231–254.

Lerman, Allen H. 1986. Tax Amnesty: The Federal Perspective. *National Tax Journal*, **39**(3), 325–332.

Levine, Ross, & Renelt, David. 1992. A Sensitivity Analysis of Cross-Country Growth Regressions. *The American Economic Review*, **82**(4), 942–963.

Lin, Wen-Zhung, & Yang, Cheng-Chen. 2001. A Dynamic Portfolio Choice Model of Tax Evasion: Comparative Statics of Tax Rates and Its Implication for

Economic Growth. *Journal of Economic Dynamics and Control*, **25**(11), 1827–1840.

LOAYZA, NORMAN V. 1996. *The Economics of the Informal Sector: a Simple Model and Some Empirical Evidence from Latin America.* Carnegie-Rochester Conference Series on Public Policy 45.

LUCAS, ROBERT E. 1988. On the Mechanics of Economic Development. *Journal of Monetary Economics*, **22**(1), 3–42.

MACAFEE, KERRICK. 1980. A Glimpse of the Hidden Economy in the National Accounts. *Economic Trends*, **316**, 81–87.

MALIK, ARUN S., & SCHWAB, ROBERT M. 1991. The Economics of Tax Amnesties. *Journal of Public Economics*, **46**(1), 29–49.

MANSFIELD, CHARLES. 1988. Tax Administration in Developing Countries. *International Monetary Fund Staff Papers*, **35**(1), 181–197.

MARRELLI, MASSIMO. 1984. On Indirect Tax Evasion. *Journal of Public Economics*, **25**(1/2), 181–196.

MARRELLI, MASSIMO, & MARTINA, RICCARDO. 1988. Tax Evasion and Strategic Behaviour of the Firms. *Journal of Public Economics*, **37**(1), 55–69.

MARTINEZ-VAZQUEZ, JORGE, & RIDER, MARK. 2005. Multiple Modes of Tax Evasion: Theory and Evidence. *National Tax Journal*, **58**(1), 51–76.

MATTHEWS, KENT, & LLOYD-WILLIAMS, JEAN. 2001. The VAT-Evading Firm and VAT Evasion: An Empirical Analysis. *International Journal of the Economics of Business*, **8**(1), 39–49.

MAURO, PAOLO. 1995. Corruption and Growth. *The Quarterly Journal of Economics*, **110**(3), 681–712.

MAYSHAR, JORAM. 1991. Taxation with Costly Administration. *Scandinavian Journal of Economics*, **93**(1), 75–88.

MÉNDEZ, FABIO, & SEPÚLVEDA, FACUNDO. 2006. Corruption, Growth and Political Regimes: Cross Country Evidence. *European Journal of Political Economy*, **22**(1), 82–98.

MERTON, ROBERT C. 1969. Lifetime Portfolio Selection Under Uncertainty: The Continuous-Time Case. *The Review of Economics and Statistics*, **51**(3), 247–257.

MERTON, ROBERT C. 1971. Optimum Consumption and Portfolio Rules in a Continuous-Time Model. *Journal of Economic Theory*, **3**(4), 373–413.

MOOKHERJEE, DILIP, & P'NG, IVAN P.L. 1989. Optimal Auditing, Insurance, and Redistribution. *The Quarterly Journal of Economics*, **104**(2), 399–415.

MORK, KNUT ANTON. 1975. Income Tax Evasion: Some Empirical Evidence. *Public Finance/Finances Publiques*, **30**, 70–76.

MYLES, GARETH D. 1987. Tax Design in the Presence of Imperfect Competition: An Example. *Journal of Public Economics*, **34**(3), 367–378.

MYLES, GARETH D., & NAYLOR, ROBIN A. 1996. A Model of Tax Evasion with Group Conformity and Social Customs. *European Journal of Political Economy*, **12**(1), 49–66.

NAM, CHANG WOON, GEBAUER, ANDREA, & PARSCHE, RÜDIGER. 2003. *Is the Completion of the EU Single Market Hindered by VAT Evasion*. CESifo Working Paper Nr. 974.

VON NEUMANN, JOHN. 1937. Über ein Ökonomisches Gleichungssystem und eine Verallgemeinerung des Brouwerschen Fixpunktsatzes. *Pages 73–83 of:* MENGER, KARL (ed), *Ergebnisse eines Mathematischen Colloquiums*, vol. 8.

NIEPELT, DIRK. 2005. Timing Tax Evasion. *Journal of Public Economics*, **89**(9/10), 1611–1637.

O'HIGGINS, MICHAEL. 1989. Assessing the Underground Economy in the United Kingdom. *Pages 175–195 of:* FEIGE, EDGAR L. (ed), *The Underground Economies - Tax Evasion and Information Distortion*. Cambridge et al.: Cambridge University Press.

OKSENDAL, BERNT. 1998. *Stochastic Differential Equations*. 5th edn. Universitext. New York et al.: Springer.

PANTEGHINI, PAOLO. 2000. Tax Evasion and Entrepreneurial Flexibility. *Public Finance Review*, **28**(3), 199–209.

PARKER, SIMON C. 2003. Does Tax Evasion Affect Occupational Choice? *Oxford Bulletin of Economics and Statistics*, **65**(3), 379–394.

PEACOCK, ALAN, & SHAW, G. KEITH. 1982. Tax Evasion and Tax Revenue Loss. *Public Finance/Finances Publiques*, **37**(2), 269–278.

PENCAVEL, JOHN H. 1979. A Note on Income Tax Evasion, Labor Supply, and Nonlinear Tax Schedules. *Journal of Public Economics*, **12**(1), 115–124.

PERSSON, MATS, & WISSÉN, PEHR. 1984. Redistributional Aspects of Tax Evasion. *Scandinavian Journal of Economics*, **86**(2), 131–149.

PESTIEAU, PIERRE, & POSSEN, URI M. 1991. Tax Evasion and Occupational Choice. *Journal of Public Economics*, **45**(1), 107–125.

PESTIEAU, PIERRE, POSSEN, URI M., & SLUTSKY, STEVEN M. 1998. The Value of Explicit Randomization in the Tax Code. *Journal of Public Economics*, **67**(1), 87–103.

PESTIEAU, PIERRE, POSSEN, URI M., & SLUTSKY, STEVEN M. 2004. Jointly Optimal Taxes and Enforcement Policies in Response to Tax Evasion. *Journal of Public Economic Theory*, **6**(2), 337–374.

PETERSEN, HANS-GEORG. 1982. Size of the Public Sector, Economic Growth and the Informal Economy: Development Trends in the Federal Republic of Germany. *Review of Income and Wealth*, **28**(2), 191–215.

POLINSKY, A. MITCHELL, & SHAVELL, STEVEN. 1979. The Optimal Tradeoff Between the Probability and Magnitude of Fines. *The American Economic Review*, **69**(5), 880–891.

POLINSKY, A. MITCHELL, & SHAVELL, STEVEN. 2000. The Economic Theory of Public Enforcement of Law. *Journal of Economic Literature*, **38**(1), 45–76.

POMMEREHNE, WERNER W., HART, ALBERT, & FREY, BRUNO S. 1994. Tax Morale, Tax Evasion and the Choice of Policy Instruments in Different Political Systems. *Public Finance/Finances Publiques*, **49(Suppl)**, 52–69.

PORCANO, THOMAS M. 1988. Correlates of Tax Evasion. *Journal of Economic Psychology*, **9**(1), 47–67.

PRATT, JOHN W. 1964. Risk Aversion in the Small and in the Large. *Econometrica*, **32**(1/2), 122–136.

PYLE, DAVID J. 1991. The Economics of Taxpayer Compliance. *Journal of Economic Surveys*, **5**(2), 163–198.

QUAH, DANNY T., & DURLAUF, STEVEN N. 1999. The New Empirics of Economic Growth. *Chap. 4, pages 235–308 of:* TAYLOR, JOHN B., & WOODFORD, MICHAEL (eds), *Handbook of Macroeconomics*, vol. 1. Amsterdam, New York and Oxford: North-Holland.

Ramsey, Frank P. 1927. A Contribution to the Theory of Taxation. *The Economic Journal*, **37**(145), 47–61.

Ramsey, Frank P. 1928. A Mathematical Theory of Saving. *The Economic Journal*, **38**(152), 543–559.

Rebelo, Sergio M. 1991. Long-Run Policy Analysis and Long-Run Growth. *The Journal of Political Economy*, **99**(3), 500–521.

Reinganum, Jennifer F., & Wilde, Louis L. 1984 (April). *Sequential Equilibrium Detection and Reporting Policies in a Model of Tax Evasion.* Social Science Working Paper 525, California Institute of Technology.

Reinganum, Jennifer F., & Wilde, Louis L. 1985. Income Tax Compliance in a Principal-Agent Framework. *Journal of Public Economics*, **26**(1), 1–18.

Rice, Eric M. 1992. The Corporate Tax Gap: Evidence on Tax Compliance by Small Corporations. *Pages 125–161 of:* Slemrod, Joel (ed), *Why People Pay Taxes: Tax Compliance and Enforcement.* Ann Arbor: University of Michigan Press.

Richter, Wolfram F., & Boadway, Robin W. 2005. Trading Off Tax Distortion and Tax Evasion. *Journal of Public Economic Theory*, **7**(3), 361–381.

Ricketts, Martin. 1984. On the Simple Macroeconomics of Tax Evasion: An Elaboration of the Peacock-Shaw Approach. *Public Finance/Finances Publiques*, **39**(3), 420–424.

Romer, Paul M. 1986. Increasing Returns and Long-Run Growth. *The Journal of Political Economy*, **94**(5), 1002–1037.

Rose-Ackerman, Susan. 1999. *Corruption and Government. Causes, Consequences and Reform.* Cambridge: Cambridge University Press.

Rothschild, M., & Stiglitz, Joseph E. 1970. Increasing Risk I: A Definition. *Journal of Economic Theory*, **2**(3), 225–243.

Rothschild, M., & Stiglitz, Joseph E. 1971. Increasing Risk II: Its Economic Consequences. *Journal of Economic Theory*, **3**(1), 66–84.

Roubini, Nouriel, & Sala-i-Martin, Xavier. 1995. A Growth Model of Inflation, Tax Evasion, and Financial Repression. *Journal of Monetary Economics*,

35(2), 275–301.

Samuelson, Paul A. 1969. Lifetime Portfolio Selection by Dynamic Stochastic Programming. *The Review of Economics and Statistics*, **51**(3), 239–246.

Samuelson, Paul A. 1986. Theory of Optimal Taxation. *Journal of Public Economics*, **30**(2), 137–143.

Sánchez, Isabel, & Sobel, Joel. 1993. Hierarchical Design and Enforcement of Income Tax Policies. *Journal of Public Economics*, **50**(3), 345–369.

Sandford, Cedric, Goodwin, Michael, & Hardwick, Peter. 1989. *Administrative and Compliance Costs of Taxation*. Bath: Fiscal Publications.

Sandmo, Agnar. 1981. Income Tax Evasion, Labour Supply, and the Equity-Evasion Tradeoff. *Journal of Public Economics*, **16**(3), 265–288.

Schmölders, Günter. 1970. Survey Research in Public Finance: A Behavioral Approach to Fiscal Theory. *Public Finance/Finances Publiques*, **25**(2), 300–306.

Schneider, Friedrich. 2003. Shadow Economy. *Pages 286–296 of:* Rowley, Charles K., & Schneider, Friedrich (eds), *The Encyclopedia of Public Choice*, vol. II. Dordrecht, Boston and London: Kluwer Academic Publishers.

Schneider, Friedrich. 2004. *The Size of the Shadow Economies of 145 Countries all over the World: First Results over the Period 1999 to 2003*. Discussion Paper No. 1431, Institute for the Study of Labor (IZA).

Schneider, Friedrich. 2005. Shadow Economies Around the World: What do We Really Know? *European Journal of Political Economy*, **21**(3), 598–642.

Schneider, Friedrich, & Enste, Dominik H. 2000. Shadow Economies: Size, Causes, and Consequences. *Journal of Economic Literature*, **38**(1), 77–114.

Scotchmer, Suzanne. 1987. Audit Classes and Tax Enforcement Policy. *The American Economic Review Papers and Proceedings*, **77**(2), 229–233.

Scotchmer, Suzanne. 1989. Who Profits from Taxpayer Confusion? *Economic Letters*, **29**(1), 49–55.

Scotchmer, Suzanne, & Slemrod, Joel. 1989. Randomness in Tax Enforcement. *Journal of Public Economics*, **38**(1), 17–32.

Shleifer, Andrei, & Vishny, Robert W. 1993. Corruption. *The Quarterly*

Journal of Economics, **108**(3), 599–617.

Sialm, Clemens. 2006. Stochastic Taxation and Asset Pricing in Dynamic General Equilibrium. *Journal of Economic Dynamics and Control*, **30**(2), 511–540.

Singh, Balbir. 1973. Making Honesty the Best Policy. *Journal of Public Economics*, **2**(3), 257–263.

Skinner, Jonathan, & Slemrod, Joel. 1985. An Economic Perspective on Tax Evasion. *National Tax Journal*, **38**(3), 345–353.

Slemrod, Joel. 1990. Optimal Taxation and Optimal Tax Systems. *Journal of Economic Perspectives*, **4**(1), 157–178.

Slemrod, Joel. 1994. Fixing the Leak in Okun's Bucket: Optimal Tax Progressivity When Avoidance Can be Controlled. *Journal of Public Economics*, **55**(1), 41–51.

Slemrod, Joel. 2001. A General Model of the Behavioral Response to Taxation. *International Tax and Public Finance*, **8**(2), 119–128.

Slemrod, Joel, & Yitzhaki, Shlomo. 1987. The Optimal Size of a Tax Collection Agency. *Scandinavian Journal of Economics*, **89**(2), 183–192.

Slemrod, Joel, & Yitzhaki, Shlomo. 2002. Tax Avoidance, Evasion, and Administration. *Chap. 22, pages 1423–1470 of:* Auerbach, Alan J., & Feldstein, Martin (eds), *Handbook of Public Economics*, vol. 3. Amsterdam et al.: Elsevier.

Smith, James D. 1985. Market Motives in the Informal Economy. *Pages 161–177 of:* Gärtner, Wulf, & Wenig, Alois (eds), *The Economics of the Shadow Economy: Proceedings of the International Conference on the Economics of the Shadow Economy Held at the University of Bielefeld, West Germany, October 10–14, 1983*. Studies in Contemporary Economics. Berlin et al.: Springer.

Solow, Robert M. 1956. A Contribution to the Theory of Economic Growth. *The Quarterly Journal of Economics*, **70**(1), 30–53.

Song, Yarbrough. 1978. Tax Ethics and Taxpayers Attitudes: A Survey. *Public Administration Review*, **38**(5), 442–457.

Spicer, Michael W., & Becker, Lee A. 1980. Fiscal Inequity and Tax Evasion: An Experimental Approach. *National Tax Journal*, **33**(2), 171–175.

Spicer, Michael W., & Lundstedt, Sven B. 1976. Understanding Tax Evasion. *Public Finance/Finances Publiques*, **31**(2), 295–305.

Srinivasan, T.N. 1973. Tax Evasion: A Model. *Journal of Public Economics*, **2**(4), 339–346.

Stella, Peter. 1991. An Economic Analysis of Tax Amnesties. *Journal of Public Economics*, **46**(3), 383–400.

Stern, Nicholas H. 1982. Optimum Taxation with Errors in Administration. *Journal of Public Economics*, **17**(2), 181–211.

Stern, Nicholas H. 1987. The Effects of Taxation, Price Control and Government Contracts in Oligopoly and Monopolistic Competition. *Journal of Public Economics*, **32**(2), 133–158.

Stiglitz, Joseph E. 1982. Self-Selection and Pareto Efficient Taxation. *Journal of Public Economics*, **17**(2), 213–240.

Stiglitz, Joseph E. 1985. The General Theory of Tax Avoidance. *National Tax Journal*, **38**(3), 325–337.

Swan, Trevor W. 1956. Economic Growth and Capital Accumulation. *The Economic Record*, **32**, 334–361.

Tanzi, Vito. 1983. *The Underground Economy in the United States: Annual Estimates, 1930-1980.* IMF-Staff Papers 33/2.

Tanzi, Vito. 2000. A Primer on Tax Evasion. *Chap. 10, pages 171–185 of:* Tanzi, Vito (ed), *Policies, Institutions and the Dark Side of Economics.* Cheltenham: Edward Elgar.

Tanzi, Vito, & Davoodi, Hamid R. 2000. *Corruption, Growth, and Public Finances.* IMF Working Paper No. WP/00/182.

Thomas, Jim J. 1992. *Informal Economic Activity.* LSE Handbooks in Economics. London: Harvester Wheatsheaf.

Tipke, Klaus, & Kruse, Heinrich W. 2004. *Abgabenordnung – Finanzgerichtsordnung.* Köln: Dr. Otto Schmidt.

Torgler, Benno, Schaltegger, Christoph A., & Schaffner, Markus. 2003. Is Forgiveness Divine? A Cross-Culture Comparison of Tax Amnesties. *Schweizer Zeitschrift für Nationalökonomie und Statistik*, **139**(3), 375–396.

Tornell, Aaron, Ranciere, Romain, & Westermann, Frank. 2005 (January). *Systemic Crises and Growth.* NBER Working Paper No. 11076.

Trandel, Gregory. 1992. Evading the Use Tax on Cross-Border Sales – Pricing and Welfare Effects. *Journal of Public Economics*, **49**(3), 313–331.

Turnovsky, Stephen J. 1997. Public and Private Capital in an Endogenously Growing Economy. *Macroeconomic Dynamics*, **1**(3), 615–639.

Uchitelle, Elliot. 1989. The Effectiveness of Tax Amnesty Programs in Selected Countries. *Federal Reserve Bank of New York Quarterly Review*, **14**(3), 48–53.

U.K. Government. HM Customs & Excise. 2004 (December). *Measuring and Tackling Indirect Tax Losses.* www.hmce.gov.uk.

U.S. Department of the Treasury. Internal Revenue Service. 1990. *Income Tax Compliance Research: Net Tax Gap and Remittance Gap Estimates.* Tech. rept. 1415 (4-90). Internal Revenue Service, Washington, D.C.

U.S. Department of the Treasury. Internal Revenue Service. 1996. *Federal Tax Compliance Research: Individual Income Tax Estimates for 1985, 1988, and 1992.* Tech. rept. 1415 (Rev. 4-96). Internal Revenue Service, Washington, D.C.

U.S. Department of the Treasury. Internal Revenue Service. 2000a. *Compliance Estimates for Earned Income Tax Credit Claimed on 1997 Returns.* Tech. rept. Internal Revenue Service, Washington, D.C.

U.S. Department of the Treasury. Internal Revenue Service. 2000b. *Compliance Estimates for Earned Income Tax Credit Claimed on 1999 Returns.* Tech. rept. Internal Revenue Service, Washington, D.C.

U.S. Department of the Treasury. Internal Revenue Service. 2002. *Compliance Estimates for Earned Income Tax Credit Claimed on 1999 Returns.* Tech. rept. Internal Revenue Service, Washington, D.C.

Virmani, Arvind. 1989. Indirect Tax Evasion and Production Efficiency. *Journal of Public Economics*, **39**(2), 223–237.

Wallschutzky, Ian. 1991. Australia: Reforming a Tax System to Reduce Opportunities for Tax Evasion. *Bulletin for International Fiscal Documentation*, **45**(4), 165–175.

Wang, Leonard F.S. 1990. Tax Evasion and Monopoly Output Decisions with

Endogenous Probability of Detection. *Public Finance Quarterly*, **18**, 480–487.

Wang, Leonard F.S., & Conant, John L. 1988. Corporate Tax Evasion and Output Decisions of the Uncertain Monopolist. *National Tax Journal*, **41**(4), 579–581.

Webley, Paul, Adams, Caroline, & Elffers, Henk. 2002. *VAT Compliance in the United Kingdom.* Working Paper No. 41, The Australian National University.

Weiss, Laurence. 1976. The Desirability of Cheating Incentives and Randomness in the Optimal Income Tax. *The Journal of Political Economy*, **86**(6), 1343–1352.

Williams, Colin C., & Windebank, Jan. 1995. Black Market Work in the European Community: Peripheral Work for Peripheral Localities? *International Journal of Urban and Regional Research*, **19**(1), 23–39.

Witte, Ann D., & Woodbury, Diane F. 1985. The Effect of Tax Laws and Tax Administration on Tax Compliance: The Case of the U.S. Individual Income Tax. *National Tax Journal*, **38**(1), 1–13.

World Bank. 2003. *World Development Indicators.* Washington, D.C.: International Bank for Reconstruction and Development.

Wrede, Matthias. 1995. Tax Evasion and Growth. *Finnish Economic Papers*, **8**, 82–90.

Yaniv, Gideon. 1994. Tax Evasion and the Income Tax Rate: A Theoretical Reexamination. *Public Finance/Finances Publiques*, **49**(1), 107–112.

Yaniv, Gideon. 1995. A Note on the Tax-Evading Firm. *National Tax Journal*, **48**(1), 113–120.

Yaniv, Gideon. 1996. Tax Evasion and Monopoly Output Decisions: Note. *Public Finance Quarterly*, **24**(4), 501–505.

Yaniv, Gideon. 1999. Tax Compliance and Advance Tax Payments: A Prospect Theory Analysis. *National Tax Journal*, **52**(4), 753–764.

Yitzhaki, Shlomo. 1974. Income Tax Evasion: A Theoretical Analysis. *Journal of Public Economics*, **3**(2), 201–202.

Yitzhaki, Shlomo. 1987. On the Excess Burden of Tax Evasion. *Public Finance Quarterly*, **15**(2), 123–137.

Yitzhaki, Shlomo. 1989. A Note on Optimal Taxation and Administrative Costs. *The American Economic Review*, **69**(2), 475–480.

von Zameck, Walburga. 1989. Tax Evasion and Tax Revenue Loss: Another Elaboration of the Peacock-Shaw Approach. *Public Finance/Finances Publiques*, **44**(2), 308–315.

Eidesstattliche Versicherung

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