Structural Estimation of Search Equilibrium Models with Wage Posting

by

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Preface

The search equilibrium environment is designed to model an equilibrium in the labour market in the presence of search frictions. Following Pissarides (2002), under search friction we understand any time delay in getting a job by unemployed worker or, similarly, filling an open vacancy by a firm. Historically, search equilibrium models originate from one-sided job search models that described exclusively the behaviour of the labour supply side under incomplete information about offered wages. Early contributions to the economics of job search include McCall (1970) and Gronau (1971) among many others. In relatively similar environments they derive an optimal stopping rule for an unemployed agent who has a limited access to job offers, drawing only one such offer per period of time. Later on this framework was extended to include the possibilities of quitting and dismissals . Other influential contributions to the development of one-sided job search models set the following benchmarks. Jovanovic (1979) introduces job matching with noise and uses this to explain age-earnings profile and the negative correlation between quits and tenure. Mortensen (1977) studies time-limited unemployment insurance and its effects on exit from unemployment. Van den Berg (1990) generalizes a job search model to allow non-stationary wage offer distribution and duration dependence in both benefit payments and arrival rates of job offers.

None of the papers on one-sided job search, however, was able to reproduce the empirically observed wage dispersion. Moreover, agents behaviour represented by these models may not be consistent with equilibrium behaviour, since the labour demand side was completely ignored. These two facts brought to life extensions of one-sided models that resulted in the creation of a search equilibrium framework.

There exist two main ways of modelling search equilibrium on the labour market, which mostly depend on the view of the nature of search frictions and the nature of equilibrium wage setting. The first approach is to view search frictions as incomplete information about the location of the vacancy, which generates a time delay until the unemployed worker and firm with the vacancy are matched. This approach was taken by Diamond (1982), Mortensen (1982) and Pissarides (1985). In this setting the wage is determined through a decentralized Nash bargaining process as long as the application of the Nash solution to the equilibrium wage determination is justified (see Binmore et al., 1986). The second approach to modelling a search equilibrium is to assume that search frictions are the result of workers' incomplete information about offered wages. In this case workers sequentially draw wage offers (one per period) and then accept or reject it before each new draw. This view of search frictions is taken form the early job search models and is integrated into the search equilibrium framework by Diamond (1971), Albrecht and Axell (1984) and Burdett and Mortensen (1998). In view of the "take it or leave it" nature of the match formation, wage setting in this framework is modelled as a result of wage posting game among employers.

Both approaches have their comparative advantages. As shown by Pissarides (1990), the first one has a richer potential for describing equilibrium flows into and out of unemployment, since the relevant hazards can be functions of labour market tightness, workers' search intensity, etc. At the same time the approach is less informative about on-the-job search and wage offer distributions. Namely no endogenous wage offer distribution can be obtained using this approach, which implies limited possibilities for the empirical applications. In contrast the model with wage posting and on-the-job search, solves for the unique wage offer distribution which is a key feature that facilitates the estimation of the model. Moreover, this environment is more suitable for studying heterogeneous workers and firms and therefore interconnections between individual qualities, labour market institutions and market equilibrium outcomes. Since the work presented in this thesis is of the empirical, rather than economic-theoretical nature, we concentrate on the second class of models.

In the framework of a search equilibrium with job search and wage-posting utility-maximizing workers search for a better offer while profit-maximizing employers set wages given the acceptance criteria of workers (e.g. reservation wage) and wages that are posted by competitors. These two assumptions together imply that from the very outset search frictions in the form of uncertainty about a better offer gives the employer relatively more power in wage setting. However, as Mortensen and Pissarides (1999) indicate, this relative monopsony advantage is constrained by competition with other employers in wage posting.

The first wage-posting model of search equilibrium was constructed after Diamond (1971) . In this model information available to workers was limited to one offer per period of time and no offers after job acceptance, i.e. no search on the job. Workers were assumed to be homogeneous with respect to their opportunity costs of employment and employers were assumed to be homogeneous with respect to their productivity. The main result implied by Diamond's (1971) analysis was that there exists only one wage offer in equilibrium from all competing employers and it is exactly the reservation wage.

To overcome the problem of a unique equilibrium wage offer Albrecht and Axell (1984) suggest that searching agents may have different values of leisure when unemployed. This assumption triggers the heterogeneity of the labour supply side with respect to reservation wages. As a result the equilibrium solution for the model is a solution with a dispersed wage offer. Albrecht and Axell (1984) solve their model for two different levels of leisure value and end up with a discrete wage offer distribution with two points of support and an endogenously determined probability of offering each of these wages. Mortensen and Pissarides (1999) also show that without loss of generality the assumption of different leisure value can be treated as an assumption of differential costs of search. In both cases, the most remarkable result is the endogeneity of the wage offer distribution. However, the intrinsic feature of this distribution is such, that all points of its support are necessarily reservation wages of one or another group of workers.

To address the inconsistency of a reservation wage based support with empirical facts, Burdett and Mortensen (1998) extend information assumptions, suggesting that after any accepted offer worker can still search on the job for a better offer. They show that with allowing for on-the-job search it becomes possible to derive an equilibrium solution with endogenously determined dispersed wage offers even with purely homogeneous workers and employers. Furthermore in their solution, the wage offer is characterized by a continuous offer distribution. The fact that in this solution wages greater than the reservation wage are also paid can be explained by efficiency wage considerations and the threat of losing part of the match surplus by employers. It is also worth noticing that by setting the arrival rate of job offers to employed workers to zero (i.e. eliminating on the job search) the solution of Burdett and Mortensen (1998) converges to that of Diamond (1971).

In the case of a purely homogeneous model, however, Burdett and Mortensen (1998) face a strictly increasing wage offer density, which theoretically implies strictly increasing earnings density. Since in reality empirical earnings densities have a decreasing right tail, two further extensions of the model were attempted. These were the introduction of heterogeneity of workers with respect to opportunity costs of employment and the introduction of heterogeneity of employers with respect to productivity. Burdett and Mortensen (1998) show that in the second case the solution implies acceptable (however still locally increasing) shapes for the earnings density. Moreover, if the productivity dispersion is discrete the resulting equilibrium wage offer density has discontinuous jumps. Continuity of the productivity distribution, to the contrary, implies continuity of the offer distribution.

Bontemps et al. (1999) attempt to solve the Burdett-Mortensen model with simultaneous heterogeneity of both supply and demand side. However, because of the theoretical complexity, an analytical solution in their case could be derived only under the assumption of equal arrival rates of job offers to unemployed and employed agents, which is a strong assumption. To overcome the restriction to arrival rates equality and at the same time keep the model tractable Postel-Vinay and Robin (2002) suggest somewhat different assumptions on the wage setting process. They rather assume that firms can vary wage offers according to some particular characteristics of the worker and firms can counter the offers from outside firms. These two amendments generate a new mechanism of wage setting that implies tenure-increasing wage trajectories for any worker within a firm. Moreover, job to job transitions with an initial wage cut but the expectation higher payment in future become possible.

Along with the development of the theoretical models of search equilibrium with wage posting, with key contributions just listed above, there has also been emerging a literature on the empirical implementation of these models. Structural econometric models considered in this literature aim at connecting the parameters of the theoretical model and the derived solution for the wage offer distribution to the observed wage and duration data. Similarly to the evolution of the theory, empirical search equilibrium models originate from conventional duration models and one-sided empirical job search models.[1] Pioneering work in the estimation of search equilibrium models with wage-posting was accomplished by Eckstein and Wolpin (1990) who estimate Albrecht and Axell's (1984) model extended to more than two types of workers. Unlike all the successors who were building their estimation methodology on the basis of hazard rate

[1]Extensive literature review on the one-sided empirical job search models can be found in Devine and Kiefer (1991).

models, Eckstein and Wolpin (1990) viewed unemployment duration as a period by period probability of staying in this given state of the labour market. Such an approach results in a likelihood function that is a mixture of negative binomial distributions with parameters restricted by economic-theoretical implications of the model. In their analysis Eckstein and Wolpin (1990) find that the model fails to fit the data and that almost all wage variation is due to the introduced measurement error. This finding along with absence of on-the-job search (and hence implications about employment durations) and the support problem of the wage offer distribution inherent to Albrecht and Axell (1984) model called for the use of a better specification, which turned out to be a model of Burdett and Mortensen (1998).

A homogeneous version of the empirical Burdett-Mortensen equilibrium search model was first estimated by Kiefer and Neumann (1993) and Ridder and van den Berg (1993), (1998). In contrast to Eckstein and Wolpin (1990) the likelihood function for this model was already relying on the observed wage and duration data. Apart from the improvements in the estimated offer distribution, an important feature of this model is the inclusion of employment durations. Using the hazard rates implied by theory and appropriately specified transition probabilities the model offers a possibility to study job to job transitions and outflows into unemployment. The relative simplicity of the econometric model allows to follow workers for more than one change of the state, as Ridder and van den Berg (1998) do. They also introduce measurement error and parameterize search intensities, making them dependent on the segment of the supply side, which every worker belongs to. This segmentation approach to heterogeneity is particularly helpful, since empirically it allows to study equilibrium behaviour of heterogeneous groups of workers even in the case when the underlying theoretical model is purely homogeneous.

Still, the shortcoming of the homogeneous model lies in the fact that the predicted theoretical earnings distribution has an increasing density. To overcome this complication and attain a decreasing right tail, Koning et al. (1995) and Bowlus et al. (1995), (2001) estimate the model with heterogeneous productivity of the demand side. Koning et al. (1995) assume the lognormal productivity distribution. Bowlus et al. (1995), (2001), to the contrary do not make any assumptions on the parametric form productivity distribution. It is rather assumed that this distribution is discrete. Using Mortensen's (1990) findings on the endogeneity of the productivity distribution, the latter group of authors estimates its support and point mass probability values from the data. Their approach is more advantageous in the sense of minimizing the impact of distributional assumptions on productivity levels. However, the estimated wage offer and earnings densities are discontinuous with jump points at the wages that correspond to estimated productivity points. Wage offer and earnings densities estimated by Koning et al. (1995) do not suffer from discontinuity. Though, Bontemps et al. (2000) argue that the true productivity distribution cannot be approximated by lognormal or any other density. As a remedy to the danger of misspecification Bontemps et al. (2000) assume that the productivity distribution is continuous and suggest a nonparametric procedure for estimation of the structural parameters of the model. The procedure turns out to be both easy in computation and robust to specification errors for the underlying productivity distribution.

The most comprehensive version of the model with heterogeneity of both supply and demand sides represented by continuous distributions of opportunity costs of employment and productivity of employers was estimated by Bontemps et al. (1999). However, as already mentioned in the discussion of the theory, unemployed and employed workers in this model are assumed to have the same arrival rates of job offer. This assumption is too restrictive at least in view of the results of Bontemps et al. (2000), who reject the formal hypothesis of arrival rates equality. Consequently, Postel-Vinay and Robin (2002) assume a different wage setting mechanism that already allows estimating the structural model with heterogeneity on both sides of the market with unrestricted arrival rates. Their estimation procedure is a further extension of the nonparametric method offered by Bontemps et al. (2000). The advantage of the empirical model developed by Postel-Vinay and Robin (2002) is that, in a unified search equilibrium framework, it both represents workers' heterogeneity with respect to opportunity costs of employment and offers a possibility to study the dependence of workers' search behaviour on their observed characteristics. The disadvantage is, however, a higher degree of computational complexity compared to Bontemps et al. (2000).

Reviewing the literature on the estimation of the equilibrium search model with wage posting one can notice that the assumption of the continuous productivity dispersion and subsequent nonparametric procedures become standard because of their relative simplicity and a "perfect" fit (the nonparametric estimate of the observed earnings distribution directly substitutes the theoretical earnings distribution in the likelihood function). At the same time all the existing methods of this type assume that the earnings data are completely observed. In the first chapter of this thesis we address the case in which wages are top coded (right-censored) and suggest a method of estimating the the specification of Bontemps et al. (2000) even in presence of incomplete earnings information. The suggested econometric model is easy to implement and shares all the features of the original specification of Bontemps et al. (2000) model, taking into account the bias due to the right censoring.

The next chapter further investigates the application of the nonparametric methods of estimating the search equilibrium models. Here we look into the properties of the productivity distribution implied by the specification of Bontemps et al. (2000). The authors state that the model provides consistent estimates of search frictions only if the predicted productivity density is a proper density that does not take negative values. However it is not possible to exclude the improper densities without an additional constraint that is also developed by Bontemps et al. (2000). In our analysis we find that this constraint cannot always be fulfilled and there may exist situations in which no positive Poisson arrival rate of job offer can guarantee the nonnegative values of the productivity density. Such situations, dubbed "constraint inconsistency", occur whenever there appear clusters of rich individuals far at the right tail of the earnings density. We quantify this phenomenon and derive a simple condition that tells beforehand about the applicability of the nonparametric method of Bontemps et al. (2000). At the same time we also find that the alternative model of Bowlus et al. (2001) with discrete productivity dispersion is robust to the above described clustering. The fully parametric approach, which in this particular case outperforms the nonparametric one, is applied to learn about the changes in the search behaviour of unskilled and elder workers induced by the reform that prolonged the entitlement to unemployment insurance benefits in West Germany. We find that the low-skilled workers were adversely affected by the reform which made the increase in their unemployment rate disproportionately high. The reform has turned out to have no effect on the unemployment rates of the old workers. However it has significantly increased their incentives to go into early retirement.

The third chapter provides further elaborations on the specification of the econometric search equilibrium model. From the review of the theoretical and empirical literature above we can see that essentially the only way to induce the decreasing right tail of the theoretical earnings density predicted by the model is to introduce productivity differentials (either discrete or continuous). In the third chapter we consider the extension of the original Burdett-Mortensen model that assumes explicit skill differences and a general production function with complementary skill inputs and the degree of homogeneity greater than one. The theoretical extension is developed by Christian Holzner and is adapted from Holzner and Launov (2005). Within this extension it is possible to show that whenever the homogeneity degree of the production function is sufficiently high we get the theoretical earnings density with a falling right tail even in the absence of employer heterogeneity. Subsequent introduction of different productivity types is essential only for a better fit to the data. The econometric counterpart of the theoretical extension draws on the ideas of Bowlus et al. (2001). However in our setting we have such new features as identification problem for the structural parameters and specification restrictions that insure continuous offer distributions (in the theoretical part continuity must be assumed since with skill multiplicity it is impossible to rule out the mass points in the offer distribution). Our estimated parameters for the degree of homogeneity clearly indicate the increasing returns and are broadly consistent with the results reported in the productivity analysis literature. We apply the model to estimate the effect of the marginal shift in the skill structure towards more high skilled workers in West Germany. The question of interest here is whether this marginal shift can generate a positive excess value after covering private costs of further education of a marginal individual. Evidence of such excess value would point at the underinvestment into education and, as a consequence, at the sub-optimality of the existing skill structure. Our findings indicate a strong underinvestment into education at the low-to-medium skill level.

To summarize, the thesis deals with extending the existing structural econometric models of search equilibrium in different directions (incomplete earnings information in Chapter 1, skill multiplicity with increasing returns technology in Chapter 3) and further elaborating on the properties of these models (limitations of nonparametric approach in Chapter 2).

Chapter 1

The Model With Continuous Productivity Dispersion

1.1 Introduction

A considerable part of the empirical work on the estimation of search equilibrium models is connected to the estimation of a Burdett-Mortensen model, in which the equilibrium wage offer is determined as a solution to a wage-posting game among competing employers. In pioneering contributions Kiefer and Neumann (1993), van den Berg and Ridder (1993), (1998) consider the specification with identical workers and employers. This specification, however, has a drawback. Namely, the theory implies that the earnings density has an increasing right tail, which contradicts general empirical evidence. Taking this into account Burdett and Mortensen (1998) and Bontemps et al. (2000) demonstrate that assuming productivity differentials among employers can lead to a decreasing right tail of the theoretical earnings density. Using this result Bontemps et al. (2000) and Bowlus et al. (2001) formulate the econometric models with a heterogenous labour demand side. The two latter contributions differ from each other in their assumptions about the productivity distribution of the firms. Bontemps et al. (2000) assume that the productivity distribution is continuous and get a strictly decreasing right tail. Bowlus et al. (2001) consider the discrete productivity distribution and get a locally increasing right tail. Different assumptions about the productivity distribution also lead to the conceptually different estimation strategies. The procedure of Bowlus et al. (2001) is less attractive computationally because along with the structural parameters of the model one needs to estimate the kink points of the wage offer distribution and the likelihood function is discontinuous at these kink points. The estimation procedure of Bontemps et al. (2000) does not have this feature and maximum likelihood estimates of the structural parameters can be found using standard gradient-based methods.

However, despite the estimation method of Bontemps et al. (2000) is preferable to the one of Bowlus et al. (2001), its applicability may not always meet the data requirements. One such case occurs when the earnings data are top-coded (i.e. censored from above). In this case the former method cannot be used, because it relies on the nonparametric estimates of the earnings density and distribution functions and the direct application of kernel estimators to the sample with top-coded earnings will necessarily lead to inconsistent estimates of the structural parameters. The procedure of Bowlus et al. (2001) to the contrary, uses the theoretically derived functional forms for the earnings density and distribution. This allows treating the right-censored wage information in the standard way and thereby estimate the structural parameters of the model consistently. Application of the latter strategy, however, comes at great computational cost once we seek to provide a successful approximation to the continuous productivity distribution by a discrete one.

In this chapter we suggest a simple amendment to the nonparametric procedure of Bontemps et al. (2000) which will make it robust to the top-coded wages. The idea is to provide an approximation of the right tail of the earnings density on the top-coded subsample by a certain parametric form. Knowledge of the functional form will allow specifying censored individual contributions to the likelihood function in the correct way. Combined with the nonparametric specification for the rest of the observations we obtain the model that consistently estimates the parameters and still shares all the advantages of the Bontemps et al. (2000) specification. It turns out that when the amount of censoring is not too large the sought approximation of the right tail is readily available. Fichtenbaum and Shahidi (1988) discuss a similar problem in application to inequality measurement with incompletely observed data. Their main result is that on its rightmost end any earnings density can be successfully approximated by the Pareto density. The result of Fichtenbaum and Shahidi (1988) is nothing but additional evidence that supports the original finding of Vilfredo Pareto, who has discovered that after crossing a certain threshold the plot of the log-number of income-earning individuals is linear in the log-income scale. This phenomenon is also known as “Pareto Power law” (see Reed, 2001, for the extensive discussion). In this chapter we exploit this feature of the earnings distribution to formulate the likelihood function that correctly accounts for the top-coded wages.

Our results can be particularly important to those who try estimating the parameters of the search equilibrium models using the data from the Social Security records etc., since wage information in this kind of administrative data is typically top-coded. The reason for using administrative data instead of household surveys is, usually, their higher quality. As a rule, such data contain daily information about the job/unemployment durations, which both minimizes measurement error and increases the precision of the estimates. Furthermore, for certain countries the appropriate and long enough panel data surveys may simply not exist.

Apart from discussing the methodology we also estimate the model with the data from the Austrian Social Security records and describe the equilibrium outcomes in the Austrian labour market at the two different points of time. In this application we consider three indicators of the labour market performance. First of all we analyze changes in the expected unemployment durations predicted by the model. Then we focus on the changes in career advancement under which we understand job-to-job transitions and speed of climbing up the earnings ladder. Finally we analyze the changes in firms' profitability and their monopsony power in wage setting. It is worth noticing that with respect to job-to-job changes Mayrhuber and Url (1999) find that job mobility in Austria is becoming surprisingly high. Using our search equilibrium setting in which expected job duration depends not only on the arrival rate of a job offer, but also on the complete wage offer distribution, we can explicitly represent the expected job duration as a function of the prospects of getting a better offer. This allows disentangling job mobility patterns of the workers who belong to different income groups and thereby getting a more detailed picture of the speed of job change.

Finally, we also consider the robustness of the model to parametric assumptions on equilibrium offer and productivity distributions.

The chapter is structured as follows. The second section provides brief overview of the main theoretical results of the model. The third section describes the data. In the fourth section we discuss the nonparametric method of Bontemps et al. (2000). Here we emphasize the problem of incompletely observed earnings and suggest the solution. The fifth section contains the estimation results and the results of the sensitivity analysis. Summary of our main findings is presented in the conclusion.

1.2 Main Theoretical Results

The model incorporates both labour supply side [workers] and labour demand side [employers] who meet on the market. Workers search for jobs and employers offer job opportunities. Both types of agents are rational. Workers maximize their utility of being employed and employers maximize their profits.

Workers are risk-neutral and homogeneous with respect to their opportunity cost of employment b. Equal opportunity costs of employment leads to a common reservation wage R.

There are two states in which workers can be, namely, "employment" and "unemployment", and workers are allowed to search whenever both employed and unemployed. Change of states is assumed to follow Poisson process. Transition from current to a better paid job is also qualified as a change of state, so there are three Poisson arrival rates that govern all transitions in the working history. We define arrival rates of a job offer to an unemployed and employed worker as λ_0 and λ_1 respectively. Arrival rate of a layoff is δ. The search process of an individual is formalized as a repeated drawing of the offers from a certain [known to worker] distribution $F(w)$ and acceptance or rejection of the offer after each draw. It is important to notice that rejected wage offers are unobserved. Available earnings data are just current salaries of employed individuals and so are necessarily the accepted wages. Therefore instead of offer distribution $F(w)$ only earnings distribution $G(w)$ can be observed. Searching workers face an optimal stopping problem. If the agent is unemployed, Mortensen and Neumann (1988) show that the solution for this problem is a reservation wage

$$R = b + (\lambda_0 - \lambda_1) \int_R^{\overline{w}} \frac{\bar{F}(x)}{\delta + r + \lambda_1 \bar{F}(x)} dx \tag{1.1}$$

where $\bar{F}(x) = 1 - F(x)$, $supp(F) = [R, \overline{w}]$, b stands for benefits and r is the real interest rate. If the agent is employed, the solution is to accept any wage greater than the currently earned one. This constitutes workers' prescription for utility maximizing behavior. Following Mortensen and Neumann (1988) without loss of generality we can associate λ_0 and λ_1 that satisfy (1.1) with agents optimal search intensities.

To formulate the employers' problem we start with two important findings, both due to Burdett and Mortensen (1998). Let U be a steady state number of unemployed workers, M the total number of supplying agents and N the steady state number of active firms. Equating equilibrium flow into and out of unemployment Burdett and Mortensen (1998) demonstrate that the equilibrium rate of unemployment is

$$U/M = \delta\left(\delta + \lambda_0\right)^{-1} \tag{1.2}$$

and there exists a unique dependence between the unobserved offer and the observed earnings distribution (density) functions, that can be written down as

$$\bar{F}(w) = \frac{\delta}{\delta + \lambda_1 G(w)} \bar{G}(w) \tag{1.3a}$$

$$f(w) = \frac{\delta\left(\delta + \lambda_1\right)}{\left[\delta + \lambda_1 G(w)\right]^2} g(w) \tag{1.3b}$$

where $\bar{G}(w) = 1 - G(w)$. Moreover Burdett and Mortensen (1998) derive the amount of workers l attracted in the steady state by a firm that offers wage w

$$l(w) = \frac{M-U}{N} \frac{\delta\left(\delta + \lambda_1\right)}{\left[\delta + \lambda_1 \bar{F}(w)\right]^2} \tag{1.4}$$

where $l(w)$ is an increasing function of the wage offered.

Returning to the employers' problem, every firm maximizes once own profit with respect to wage paid

$$\pi = \max_w (p-w) l(w) \tag{1.5}$$

with l given by (1.4).

In this chapter we consider a version of the model in which employers are heterogeneous with respect to their productivity and the probability distribution of the productivity across active firms $\Gamma(p)$, is continuous, $supp(\Gamma) = [\underline{p}, \overline{p}]$. Solving (1.5) Bontemps et al. (1997), (2000) firstly show that whenever $\Gamma(p)$ is continuous there exists a unique single valued, monotone and continuous function $w = K(p)$, which maps the support of the productivity distribution Γ into the support of the wage offer distribution F. Secondly they demonstrate that more productive firms offer higher wages. These two facts imply that

$$F(w) = \Gamma(K^{-1}(w)) \tag{1.6}$$

which is a generalization of the well-known result of Mortensen (1990) for the discrete productivity distributions (see also Chapter 2, p.46). Using (1.6) Bontemps et al. (2000) find the solution to the optimal wage setting problem (1.5) of a p-type firm

$$K(p) = p - \left[\delta + \lambda_1 \bar{\Gamma}(p)\right]^2 \int_R^p \frac{dx}{\left[\delta + \lambda_1 \bar{\Gamma}(x)\right]^2} \tag{1.7}$$

which completes the steady-state solution of the model. Eventually, Bontemps et al. (2000) show that whenever the upper bound of the support of the productivity distribution is finite there exists at least one equilibrium on the market. A formal definition of market equilibrium follows.

Definition 1.1: A market search equilibrium is a triple $\{F(w), \mathcal{W}, \mathcal{K}_p\}$ such that:

1. The distribution of wage offers is $F(w) = \int F(w|p) d\Gamma(p)$, where $\Gamma(p)$ is a productivity distribution of firms, active in the market

2. $\mathcal{W} = \max\{R, w_{current}\}$ is the workers' best response to firms' wage-posting behavior; R defined in (1.1)

3. $\mathcal{K}_p = \arg\max\{\pi(p, w)|R \leq w \leq \overline{w}\}$ is a profit-maximizing wage posted in equilibrium by each p-type firm; $\pi(p, w)$ defined in (1.5) and $\mathcal{K}_p$ defined in (1.7)

Unlike in the case with discrete $\Gamma(p)$ considered by Mortensen (1990), the theoretical solution of the model with continuous productivity dispersion does not provide the closed form for the equilibrium wage offer distribution $F(w)$. It is easy to see that it is impossible to solve (1.6) given (1.7) analytically. This creates a potential problem for the conventional parametric estimation of the model. However, Bontemps et al. (2000) overcome this drawback by suggesting a very simple yet powerful nonparametric alternative. Their "nonparametric three-step" method is reviewed in Section 1.4.1.

1.3 The Data

In the present survey we use the data from the Austrian Social Security records. They represent working history of individuals who were followed through a fifteen year period from 1984 to 1998. All observations are made on 30.05 of each year. Each year-specific block of individual data includes gender, age, earnings before tax, professional affiliation, employment status and the dates and types of employment history events

Table 1.1: Duration Data for the Austrian Labour Market

	Sample 1988	Sample 1994
Number of Individuals	3404 [1.000]	3726 [1.000]
Employed:	3110 [0.914]	3361 [0.902]
Unemployed:	294 [0.086]	365 [0.098]
Employed Agents	3110 [1.000]	3361 [1.000]
Uncensored observations with:		
job-to-job transition:	646 [0.208]	658 [0.196]
job-to-unemployment transition:	500 [0.161]	630 [0.187]
mean time spell between the two states:	43.189	42.324
(std. deviation):	(34.884)	(33.008)
Censored observations:		
a) left-censored durations only:		
with job-to-job transition:	365 [0.117]	73 [0.022]
with job-to-unemployment transition:	257 [0.083]	77 [0.023]
b) right-censored durations only:	257 [0.083]	1085 [0.323]
c) both left- and right-censored durations:	1085 [0.349]	838 [0.249]
mean duration [both censored and uncens.]:	91.793	88.019
(std. deviation):	(59.159)	(56.026)
Unemployed Agents	294 [1.000]	365 [1.000]
Uncensored observations (u $\rightarrow$ j transition):	34 [0.116]	29 [0.079]
mean time spell between the two states:	7.962	5.638
(std. deviation):	(8.256)	(3.226)
Censored observations		
a) left-censored (u $\rightarrow$ j transition) only:	–	–
b) right-censored durations only:	3 [0.010]	48 [0.132]
c) both left- and right-censored durations:	257 [0.874]	288 [0.789]
mean duration [both censored and uncens.]:	18.978	23.630
(std. deviation):	(26.496)	(27.461)

that have happened during the previous year. Using the guidelines of van den Berg and Ridder (1998) we draw two samples at 1988 and 1994 and restore labour market histories of all sampled individuals. To restore the employment history we track every individual backward(forward) until the date of his/her entry to(exit from) the current state or until we reach the lower(upper) end of the observation period.

All individuals in the data set are divided into the four employment status categories: "employed", "unemployed", "on study" and "other". In what follows we consider only "employed" and "unemployed" ones. The reason is that the theoretical model is restricted to only these two states of the labour market. It is believed that individuals who fall into the rest of the groups have incentives different from the agents described by the model. Therefore common practice leaves them out (see van den Berg and Ridder, 1993, 1998).

For the same reason we exclude part-time workers. Unfortunately, there is no direct indication of part-time employment in the data. To draw our samples with the least possible noise we argue that if an agent works on a full-time basis, his/her income is at least as high as the legal minimum wage before tax. Thus individuals, with income below the minimum should be left out of consideration. Here we should also notice that in fact there is no uniform legal minimum wage in Austria. Instead, every year unions in all industries bargain with employers for minimum wages that should be paid in respective industries throughout the whole year. As an approximation for a single minimum wage we take an average of the agreed within-industry wages, which are available from the annual reports of the Austrian Central Statistical Office. Such way of leaving out potential non-participants also provides a more reliable estimate of the lower bound of support of the unknown offer distribution, which otherwise would have been $\hat{R} = \min\{w\}$ as suggested by Kiefer and Neumann (1993). This commonly used minimum sample estimator is sensitive to measurement error, which in view of no full-time employment indication would be definitely present in our data.

Proceeding as above we end up with two data samples with 3404(294) employed(unemployed) at 30.05.88 and 3726(365) employed(unemployed) at 30.05.94. Summary statistics of the duration lengths is given in Table 1.1. Durations are measured in months; fraction of the sample is given in square brackets. From Table 1.1 it is easy to see that the average job duration was remaining roughly the same over the observed period. So, if there were changes in the job mobility, as Mayrhuber and Url (1999) indicate, they may be detected only after considering heterogenous groups of agents.

Figure 1.1: Earnings Density 1988

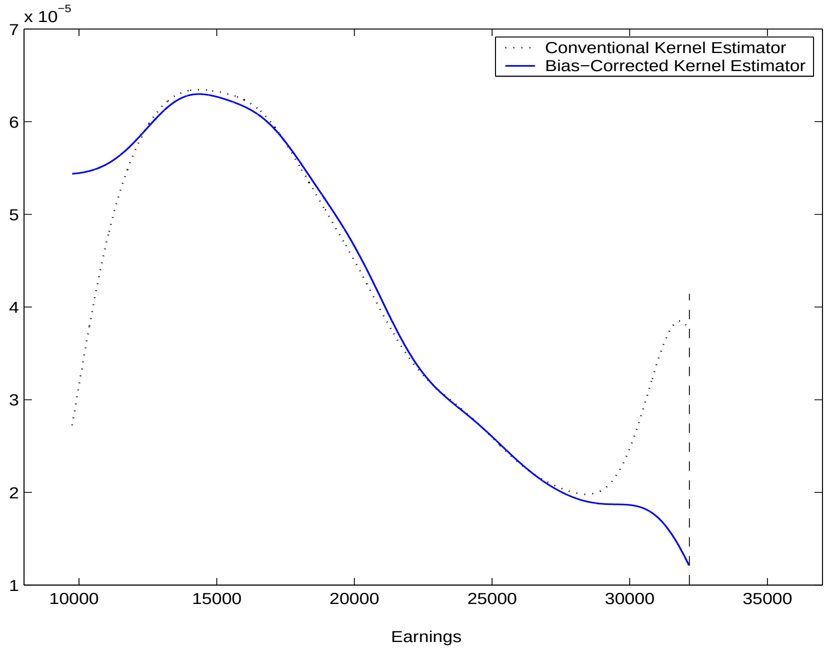
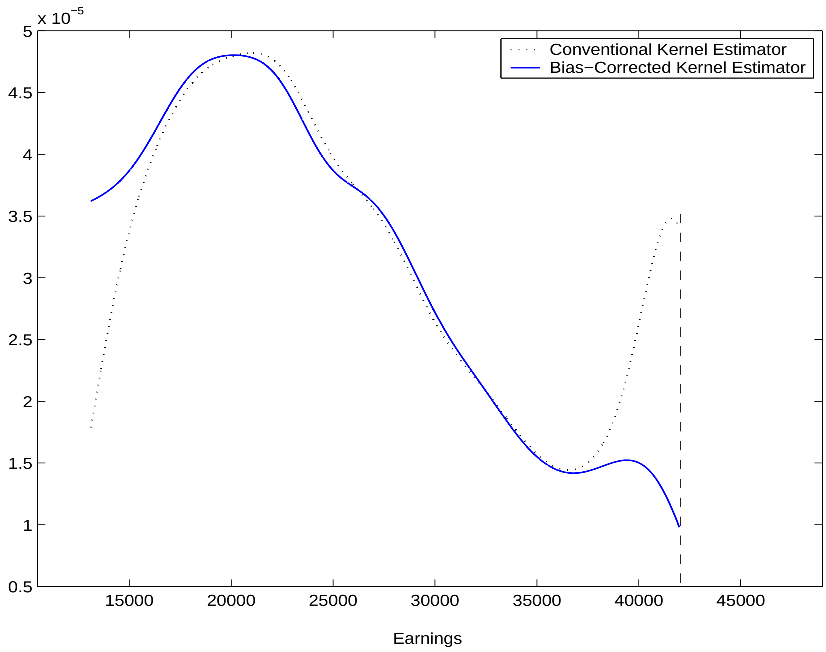


Figure 1.2: Earnings Density 1994



One can also notice the considerable discrepancy between the average duration of the completed employment spells and the average duration of both completed and incomplete employment spells. This is a sign of relatively high degree of duration dependence in the data. The same can be said about the unemployment spells. Additionally the expected length of unemployment in the later period has become somewhat longer than in the earlier one.[1] Finally, there was a slight increase in the unemployment rate from 8.64% to 9.80%.

Summary of the earnings information is presented by the two kernel density plots in Figures 1.1-2 for years 1988 and 1994 respectively. We can see that the conventional kernel density estimator is severely biased at the right tail. This bias occurs because kernel function tries to smooth out a point mass generated by top-coding. For year 1988 this point mass, i.e. the share of right-censored wages in the whole sample, makes 8,03%. In the sample of 1994 as much as 9.61% of the wages are top-coded. Figures 1.1-2 also plot the bias corrected estimator which not only accounts for top-coding but also provides the left tail correction (see Section 1.4.2 for details). However, despite being able to consistently estimate the density on the observed part of the support of the earnings distribution the bias-corrected estimator tells us nothing about the shape of the right tail above the censoring threshold (dashed vertical line on both plots). Facilitating the parameter estimation under no information about the shape of the right tail behind the censoring threshold is the primary goal of this chapter.

In comparable prices, mean earnings in years 1998 and 1994 have made ATS 19678 and 21105 respectively. Standard deviations for both statistics are ATS 6797.4 and ATS 7033.5 (note that due to right censoring the reported means are downward biased).

1.4 Estimation Methodology

Structural parameters $\{\lambda_0, \lambda_1, \delta\}$ are estimated from the econometric model that relies on wage and duration data. The general approach to the construction of the likelihood function is based on Lancaster (1990) and van den Berg and Ridder (1998). Additionally we discuss the modifications, which are necessary to account for the top-coded earnings data.

[1] One should keep in mind, however, that all the measures of duration lengths in Table 1.1 are downward biased because of incomplete spells. The specification of the econometric model which will be discussed in the next section will take this into account.

1.4.1 The Likelihood Function

The backbone process of the model is Poisson, so the waiting time between any two adjacent events is exponentially distributed with parameter θ. However, this property can not be applied directly, because the sampling scheme used to retrieve the job histories is not random. Ridder (1984) demonstrates that under the described above sampling the spells with the longer length have higher probability of being included into the resulting data set. To overcome this problem Ridder (1984) suggests analyzing a joint distribution of elapsed (t_e) and residual (t_r) durations of a spell.

About the distribution of the elapsed duration it is known that certain time t_e ago there was a renewal of states and since then an individual spent at least t_e in a new state. Renewal probability for Poi(θ) is shown to be equal to θ (see Lancaster, 1990). On the distribution of the residual duration our knowledge is that given a certain elapsed time t_e an individual spends in this state additional time t_r ($t_r > 0$). Therefore the appropriate densities are:

$$\begin{array}{ll} \text{Elapsed:} & f(t_e) = \theta e^{-\theta t_e} \\ \text{Residual:} & f(t_r|t_e) = \theta e^{-\theta t_r},\ t_r > 0 \\ \text{Joint:} & f(t_e, t_r) = \theta^2 e^{-\theta(t_e+t_r)},\ t_r > 0 \end{array}$$

For unemployed agents the corresponding Poisson rate is just λ_0. For unemployed ones the correct Poisson rate is a sum of transition intensities to either unemployment δ or a better-paid job $\lambda_1 \bar{F}(w)$, i.e. $\theta = \delta + \lambda_1 \bar{F}(w)$.

To complete the formulation of the individual contributions to the likelihood we notice that:

- *for Unemployed:* Equilibrium probability of sampling an unemployed agent is given by (1.2). In case the subsequent job transition is observed we know the offered wage and can record the value of the wage offer density $f(w)$.
- *for Employed:* Equilibrium probability of sampling an agent who earns wage w is $g(w)\lambda_0/(\delta + \lambda_0)$. In case the agents' transition to the next state is observed we record the destination state. The probabilities of exit to unemployment and to next job are $\pi_{j\to u} = \delta/\left(\delta + \lambda_1 \bar{F}(w)\right)$ and $\pi_{j\to j} = \lambda_1 \bar{F}(w)/\left(\delta + \lambda_1 \bar{F}(w)\right)$ respectively.

Taking an account of incompletely observed elapsed/residual durations is relatively straightforward. In case of left censoring we drop the renewal probability and in case of right censoring we drop exit probability and change the residual density with the survivor function. With these results $\mathcal{L}_u$ and $\mathcal{L}_e$ individuals become

$$\mathcal{L}_u = \frac{\delta}{\delta + \lambda_0} \lambda_0^{2-d_r-d_l} \exp\{-\lambda_0(t_e + t_r)\} f(w)^{1-d_r} \tag{1.8}$$

$$\mathcal{L}_e = \frac{\lambda_0 g(w)}{\delta + \lambda_0} \left[\delta + \lambda_1 \bar{F}(w)\right]^{1-d_l} \exp\{-\left[\delta + \lambda_1 \bar{F}(w)\right](t_e + t_r)\} \\ \times \left[\left[\lambda_1 \bar{F}(w)\right]^{d_t} \delta^{1-d_t}\right]^{1-d_r} \tag{1.9}$$

where $d_l = 1$, if a spell is left-censored, 0 otherwise, $d_r = 1$, if a spell is right-censored, 0 otherwise and $d_t = 1$ if there is a job-to-job transition, 0 otherwise.

One can see that both (1.8) and (1.9) involve the unknown theoretical wage offer distribution and density functions. As it was mentioned in Section 1.2 no analytical solution for $F(w)$ is available. In view of this Bontemps et al. (2000) suggest the following "nonparametric three-step procedure" for the estimation of the structural parameters:

1. On the first step compute the non-parametric estimates of $g(w)$ and $G(w)$
2. On the second step use (1.3a)-(1.3b) and $\hat{g}(w)$ and $\hat{G}(w)$ to substitute the unknown offer density and distribution functions in (1.8)-(1.9) with the expressions that contain structural parameters only. Maximize the likelihood function with respect to $\{\lambda_0, \lambda_1, \delta\}$.
3. Use $\left\{\hat{\lambda}_0, \hat{\lambda}_1, \hat{\delta}\right\}$ and $\hat{g}(w)$ and $\hat{G}(w)$ to calculate the unknown productivity levels p and productivity density $\gamma(p)$.

For the third step Bontemps et al. (2000) show that

$$p = w + \frac{\delta + \lambda_1 G(w)}{2\lambda_1 g(w)} \tag{1.10}$$

and

$$\gamma(p) = \frac{2\delta\lambda_1(\delta + \lambda_1) g(w)^3}{3\lambda_1 g(w)^2[\delta + \lambda_1 G(w)]^2 - g'(w)[\delta + \lambda_1 G(w)]^3} \tag{1.11}$$

Two points worth mentioning here. First of all the model is well-specified only if the denominator in (1.11) is positive. In case it is not, Bontemps et al. (2000) suggest the constrained maximum likelihood estimation. Though, as we will show in Chapter 2 there may exist the cases in which the constrained optimization may not always be feasible and the whole nonparametric approach of Bontemps et al. (2000) breaks down. Secondly, the formulation above relies on the completely observed earnings data. As we have discussed in the overview of the data this is not the case in the present application. In Section 1.4.2. we suggest the way out.

Finally we comment on the estimation of the support bounds of earnings distribution. Usual practice (see Kiefer and Neumann, 1993, or Bowlus et al., 1995) is to use $R = \min(w)$ and $\overline{w} = \max(w)$. In the present application we rather use average of agreed within-industry minimum wages as an estimator for R. For $\overline{w}$ we keep $\overline{w} = \max(w)$.

1.4.2 The Problem of Top-Coded Wages

As it was mentioned in Section 1.3, about 10% of top earnings observations in both samples are censored. Absence of information on the wages from the upper decile of the earnings distribution makes the nonparametric estimation of its right tail unfeasible. To solve this problem we follow the suggestion of Fichtenbaum and Shahidi (1988) and Reed (2001) and approximate the top-coded right tail by the right tail of the Pareto distribution. Informally, we split the support of $G(w)$ in two intervals. On the first interval we use nonparametric estimates of $g(w)$ and $G(w)$, on the second one we take $g(w)$ and $G(w)$ to be of a Pareto form.

Consider the first interval. To consistently estimate the distribution function we use the Product-Limit estimator which is known to be robust to the right censoring of the data (see Lancaster, 1990, for the derivation and the overview of basic properties).

To estimate the density we follow Padgett (1988), who suggests a version of a kernel estimator suited to the case when right censoring is not random and the censoring threshold is the same constant. The estimator has a simple form

$$\tilde{g}(w) = \left[nh^{-1}\right] \sum_{j=1}^{n} K\left(\frac{w - w_j}{h}\right) [I_j : w_j < w^c] \tag{1.12}$$

where I_j is an indicator function that takes value 1 if w_j is less then the value of the censoring threshold w^c and zero otherwise. Padgett (1988) also justifies an application of Gaussian kernel for (1.12).

Additionally, Vuong et al. (2000) show that whenever the distribution is defined on a compact set any kernel density estimator is asymptotically downward biased towards tails. Bontemps et al. (2000) state that this bias is precisely $E\left[\hat{g}(w)\right] \rightarrow \frac{g(w)}{2}$ and suggest the following bias-corrected kernel estimator:

$$\hat{g}(w) = \tilde{g}(w)\left[\Phi\left(\frac{x-w}{h}\right)\right]^{-1} \tag{1.13}$$

Note that in view of right censoring and subsequent Pareto approximation the expression in (1.13) presents a version of the estimator with only left tail correction making it suitable for our study.

Now consider the approximation of the earnings distribution on the second interval. Let $\rho\left(w|\alpha,\beta\right)$ and $\mathcal{P}\left(w|\alpha,\beta\right)$ denote the Pareto density and distribution functions. The survivor function for the Pareto distribution can be written down as

$$1-\mathcal{P}\left(w|\alpha,\beta\right) = \alpha^{\beta}w^{-\beta}$$

One of the key properties of this survivor function is that the plot of $\ln(1-\mathcal{P}\left(w\right))$ against $\ln\left(w\right)$ is linear. Thus the approximation of the earnings distribution of the free form $G(w)$ by the Pareto distribution will be justified only on the segment where $\ln(1-\hat{G}(w))$ is linear against $\ln(w)$.

With our data linearity of the log-log plot of $1-\hat{G}(w)$ amounts to the top 10% of the *observed* wages in both samples. This corresponds to broad evidences that Pareto law of incomes is a property of the two upper deciles of the earnings distribution (see Reed, 2001). By substitution of $\ln\left(1-\hat{G}(w)\right)$ into the l.h.s. of

$$\ln(1-\mathcal{P}(w_i)) = \beta\ln\alpha - \beta\ln(w_i) + \varepsilon_i \tag{1.14}$$

both scale parameter α and shape parameter β of the Pareto distribution are estimated by least squares using the *observed* part of the second interval.[2]

Knowing α and β, hence the exact form of the Pareto tail , we specify the individual contributions to the likelihood function in a standard way. If the wage is top-coded

[2]Roughly, this will be the ninth decile of the earnings distribution.

the individual contribution to the likelihood (1.8)-(1.9) becomes

$$\tilde{\mathcal{L}}_u = \frac{\delta}{\delta + \lambda_0} \lambda_0^{2-d_r-d_l} \exp\left\{-\lambda_0(t_e + t_r)\right\} \left[\left(\delta \bar{\mathcal{P}}(w^c)\right) / \left(\delta + \lambda_1 \mathcal{P}(w^c)\right)\right]^{1-d_r} \tag{1.15}$$

$$\tilde{\mathcal{L}}_e = \frac{\lambda_0 \bar{\mathcal{P}}(w^c)}{\delta + \lambda_0} \left[\delta + \lambda_1 \bar{F}(w^c)\right]^{1-d_l} \exp\{-\left[\delta + \lambda_1 \bar{F}(w^c)\right](t_e + t_r)\} \\ \times \left[\left[\lambda_1 \bar{F}(w^c)\right]^{d_t} \delta^{1-d_t}\right]^{1-d_r} \tag{1.16}$$

where $\bar{\mathcal{P}}(w) = 1 - \mathcal{P}(w)$. If, the wage is observed but still falls into the interval where Pareto form is valid, the likelihood function remains the same as in (1.9) with the only difference that now $\rho(w)$ and $\mathcal{P}(w)$ will be substituting $g(w)$ and $G(w)$ in (1.8)-(1.9).

Statistical properties of the MLE from (1.15)-(1.16) will be the same as of those obtained from the fully observed earnings sample, provided that (1.14) consistently estimates the parametric form of the right tail of the earnings distribution. Asymptotic covariance matrix of the estimated structural parameters will be, however, unknown at least because of the presence of the nonparametric estimates $\hat{g}$ and $\hat{G}$ in the likelihood function. Therefore, as in the original paper of Bontemps et al. (2000), we bootstrap the confidence intervals for $\left\{\hat{\lambda}_0, \hat{\lambda}_1, \hat{\delta}\right\}$.

Before concluding we also need to notice that yet another way of estimating Pareto parameters can be suggested. In the alternative formulation the break point between non-parametric and parametric forms could be explicitly introduced as a parameter of the likelihood function. This, however, would lead to the fact that the support of the likelihood contributions for certain individuals would become a function of the unknown β, which would lead to MLEs with non-standard properties. Moreover, in this situation the application of extreme order statistics (see Kiefer and Neumann, 1993, and Donald and Paarsch, 2002) is not possible because the lower bound of the support of the Pareto distribution for the tail does not need to coincide with the observed sample minimum wage. However, this alternative formulation a promising way of research.

1.5 Estimation Results and Discussion

In this section we consider the implementation of the proposed method. The main purpose of this part of the chapter is not to make a particular statement about the economy but rather to consider the performance of the suggested econometric procedure.

1.5.1 Pareto Parameters

We start with discussing the fit of Pareto distribution to the right tail of the earnings distribution. As already mentioned before, in both samples the linearity of $1 - \hat{G}(w)$ in the log-log plot against wages amounts to the top 10% of the observed earnings data. Using this sub-sample we estimate the parameters of the Pareto distribution by non-linear least squares applied to (1.14). Table 1.2 contains the results.

From this Table it is easy to see that both regressions provide quite reliable estimates of the Pareto parameters. First of all the number of observations is large enough to secure high precision of the estimates. Secondly, in both cases the range of the regression interval and its ratio to the range of the whole sample is sufficiently big to make sure that the inference is not made on a cluster. Finally the coefficients of determination indicate a very good fit of Pareto tail to the actually observed tail.

Altogether, the results presented in Table 1.2 show that on the rightmost of $G(w)$ Pareto cdf closely predicts the true cdf. Knowledge about the functional form of the right tail of $G(w)$ enables us to estimate the model even with the top-coded wage sample. Notice though, that the precision of the above prediction (hence the performance

Table 1.2: Estimated Parameters for the Pareto Tails

	Sample 1988		Sample 1994	
	Coeff.	(Std.Error)	Coeff.	(Std.Error)
α	19978.81	(106.35)	26638.20	(125.20)
β	5.0336	(0.0658)	4.9645	(0.0588)
R^2	0.9740		0.9793	
Obs.	236		241	
Range	[28424, 32159]		[37620, 41995]	
$\frac{\text{Range}}{\text{Total Range}}$	0.167		0.151	

of the proposed method) depends on the degree of right-censoring in the data: the higher it is the less accurate will be the resulting estimates.

1.5.2 Structural Parameters

Next we estimate the model specified in (1.8)-(1.9). Whenever wage observations are top-coded we use (1.15)-(1.16) where $\mathcal{P}(w)$ is determined by the Pareto parameters from Table 1.2. If the wages are observed but still belong to the interval on which Pareto distribution consistently estimates earnings distribution we use (1.8)-(1.9) inserting $\rho\left(w|\hat{\alpha},\hat{\beta}\right)$ and $\mathcal{P}\left(w|\hat{\alpha},\hat{\beta}\right)$ instead of $\hat{g}(w)$ and $\hat{G}(w)$.

For convenience of inference in Table 1.3 we rather report the reciprocals of the estimated parameters. Consider first the unemployed agents. Following Mortensen and Neumann (1988) mean unemployment duration is just a reciprocal hazard of exit from unemployment, i.e. λ_0^{-1}. From Table 1.3 we see that the expected length of unemployment did not significantly change during the analyzed period of time. For the individuals sampled in 1998 it was at the level of about two years. In the sample drawn at 1994 it has kept remaining to be so. The expected length of the unemployment duration is quite in line with the sample measures (see Table 1.1; keep in mind that sample information is downward biased because of incomplete durations). At the same time the unemployment rates predicted by the model overestimate the observed ones. Inserting $\hat{\lambda}_0$ and $\hat{\delta}$ into (1.2) one obtains the unemployment rates of 15.5% for 1988 and 17.3% for 1994 which is too high in comparison to the sample counterparts of 8.6% and 9.8% respectively. This misfit is an evidence of either underestimated λ_0 or overestimated δ. Though, as we have shown above the estimated λ_0 quite accurately predicts the mean unemployment duration. So the most likely reason is a not rich enough specification of quit behaviour. Going back to the formulation of the likelihood function one can recall that we have only two states in the model, i.e. only employed and unemployed individuals are sampled. However, there also exist the working-age non-participants. Even though we do not include them into the sample we can later observe the transitions from job and unemployment to non-participation. Since we don't have "non-participation" as state such spells are censored at the exit date. If we had had one, however, the spell would have been complete and we would have also known the quit probability to non-participation. This probability, like both $\pi_{j\to j}$ and $\pi_{j\to u}$ obviously depends on the quit rate δ, which would enrich specification of this parameter and most likely improve the fit of the model. To our knowledge, so far there were almost no applications that would move in this direction. The only known to us paper with non-participation as an additional state is that of Bowlus and Grogan (2001). Their estimation method, however, is not the nonparametric one. Furthermore, even if it was, there would be no straightforward way to compare the estimates because the model with two states is not nested in the three-state specification, but rather is a degenerate case of the latter with zero probability mass put on the non-participation. Our intuition is that certain selection bias would still exist.

Inference about expected employment durations predicted by the model is less straightforward. By the same argument of Mortensen and Neumann (1988) expected job duration equals to the reciprocal hazard of exit to a better job, which is $[\lambda_1 \bar{F}(w)]^{-1}$. In other words calculation of the expected job duration predicted by the model presumes the knowledge of the wage offer distribution. Even though it is possible to estimate the offer distribution in this setting (see Figures A.1-2 of the Appendix), so far all the inference in the existing literature was based only on the reciprocal employed search intensity λ_1^{-1}. In the present application we rather consider $[\lambda_1 \bar{F}(w_i)]^{-1}$. In this way we provide more comprehensive description of the expected employment duration linking it to the promotion prospects of the agent. To make the estimated expected durations comparable across time we evaluate $\bar{F}(w_i)$ at the average wages of the respective deciles of the earnings distribution. Visual presentation of our results is given in Figure A.3 of the Appendix.

First we notice that expected job duration of agents, whose earnings belong to the upper two deciles, exceeds potential job tenure (more than 50 years). This result is, however, quite natural, since it tells that people who earn very high income lose pecuniary incentives to search for a better job and are happy to stick to their current job forever. Considering the results for the rest of the workers we see that, like in the unemployment case, no significant changes occurred to the predicted durations. In Figure A.3, be it sample of 1988 or 1994, the expected job durations of one year do not fall out of the confidence bounds for another one. Thus we do not find support to Mayrhuber and Url (1998), who indicate significant upward shifts in job mobility. On one hand our analysis is more comprehensive, since we explicitly take the promotion prospects into account. On the other hand, as we have noticed above, the two-state model estimated in this chapter may be too restrictive to provide the precise fit to the available data.

Table 1.3: Estimated Frictional Parameters*

	Sample 1988	Sample 1994
$1/\lambda_0$	25.8792 (1.1305) [22.9610, 28.4108]	26.7293 (1.0003) [24.3911, 29.4752]
$1/\lambda_1$	32.9377 (1.5851) [29.2327, 36.1881]	35.7950 (1.7150) [32.3762, 40.0264]
$1/\delta$	138.6262 (3.1019) [130.7908, 145.0097]	127.8525 (2.6710) [122.6721, 134.7831]
log(Likelihood)	−43386.396	−51654.443

* Standard errors in parenthesis; bootstrap 95% confidence intervals in square brackets (200 replications)

To describe firms' behaviour we use the following tools. Firstly, it is the index of monopsony power, which is defined by Bontemps et al. (2000) as

$$MPI = (p - K(p))\,/p$$

Secondly, using (1.4), (1.5) and some algebra, we derive a profit ratio

$$\frac{\pi^*}{\pi} = \frac{M^*/N^*}{M/N}\frac{p^* - w^*}{I_w\,(p - w)}\left[\frac{\lambda_0^*(\delta_0 + \kappa_0)}{\lambda_0(\delta^* + \kappa_0^*)}\frac{(\delta^* + \lambda_1^*)(\delta + \lambda_1 \bar{F}(w))}{(\delta + \lambda_1)(\delta^* + \lambda_1^* \bar{F}^*(w^*))}\right]$$

in which π^* and π are the profits at the two different periods of time. To make the productivity-wage differences comparable the denominator is scaled by the index of agreed minimum wages I_w; M/N stands for total labour force over the number of active firms.[3]

Figure A.4 presents the plot of the monopsony power indices for 1988 and 1994 (in comparable scale). In Figure A.5 one can see that plot of the ratio of profits in 1994

[3]Henceforward we assume that $[M^*/N^*]\,/\,[M/N] = 1$.

to profits in 1988 evaluated for the average productivity value of every decile of the productivity distribution. An interesting observation can be made here. First of all we see that the monopsony power has slightly gone down. This would imply that the firms became weaker in exploiting search frictions and their profits should therefore reduce. The same fact would be told by the graphs of the offer densities and distributions. As one can see from Figures A.1-2 the shape of the wage offer has changed towards posting higher wages more frequently, which *ceteris paribus* should also imply a reduction in the equilibrium profits of the firms. However, inspecting Figure A.5 we find out that for the majority of the firms (loosely speaking the core from the 2nd to 9th deciles of the productivity distribution) profits did not alter considerably. Provided that M/N did not change this fact can be be explained by the increase in the productivity of the firms which was large enough to cover the partial loss of monopsony power and offer higher wages keeping the profits relatively intact.

It is also worth noticing that the profits of the lowest-productive firms went up and the profits of the highest productive firms went down considerably. This has also quite an intuitive explanation once we look at Figure A.2 and recall (1.6), which tells that in the model with continuous productivity dispersion the shapes of $F(w)$ and $\Gamma(p)$ are identical. The rightward shift of $F(w)$ hence $\Gamma(p)$ implies that the number of low-productive firms has reduced and the number of high-productive firms has increased. This has reduced competition among employers at the bottom of $\Gamma(p)$ and intensified competition on its' top. Exactly this fact is conveyed by the profit ratio plot at the leftmost and the rightmost of the support of $\Gamma(p)$.

1.5.3 Sensitivity Analysis

Some earlier contributions to the estimation of search equilibrium models with heterogeneous demand side were attempting to specify the productivity distribution as a member of a certain parametric family. Koning et al. (1995) estimate the basic Burdett-Mortensen model assuming that productivity parameter in the equilibrium wage offer distribution is distributed log-normally. Christensen et al. (2000) use the same framework with the only difference that the productivity parameter in their specification is multiplied by a dispersion factor exponent of which has normal distribution with zero mean and unknown variance. However, from the Botnemps et al. (2000) discussion of the properties of the equilibrium productivity distribution it turns out that

Table 1.4: Goodness of Fit Tests for the Wage Offer Distribution

Distribution under H_0	Test	Minimum Test Statistic		5% Cr.Value
		Sample 1988	Sample 1994	
Pareto:	Kolmogorov-D	1.4635	2.5273	1.2240
	Kuiper	2.6864	5.0382	1.7470
Log-Normal:	Kolmogorov-D	5.0417	4.5024	1.2240
	Kuiper	7.2385	6.1587	1.7470

only a limited class of known parametric families can meet the conditions determined by the equilibrium solution of the model (see Bontemps et al. 2000, Proposition 8). Namely, the authors state that the right tail of the appropriate productivity density should converge to zero slower then p^{-3}, which invalidates application of the log-normal distribution for p. At the same time Bontemps et al. (2000) notice that the family of Pareto distributions meets this convergence criterion. The purpose of this concluding subsection is to perform the analysis of the robustness of the estimation results to parametric assumptions on the productivity distribution.

For the sensitivity analysis it will be more convenient to consider $F(w)$ rather than $\Gamma(p)$. In what follows we take the estimates of $\hat{F}(w)$ obtained from (1.3a) and compare them with the distribution function from a known parametric family. Specifically, we test the equivalence of $\hat{F}(w)$ to log-normal and Pareto distribution.

In our analysis we use Kolmogorov-D

$$D(x) = \max_{\{x\}} \left(|F^*(x) - F_0(x|\boldsymbol{\theta})| \right) \tag{1.17}$$

and Kuiper

$$V(x) = \max_{\{x\}} \left(F^*(x) - F_0(x|\boldsymbol{\theta}) \right) + \max_{\{x\}} \left(F_0(x|\boldsymbol{\theta}) - F^*(x) \right) \tag{1.18}$$

tests for equivalence of two distributions. In both above statistics $F^*(x)$ stands for the nonparametric estimate of the distribution of x and $F_0(x|\boldsymbol{\theta})$ stands for the parametric form of this distribution under null (see Stephens, 1974, for the overview and tabulated critical values).

With both (1.17) and (1.18) we minimize the test statistics with respect to $\boldsymbol{\theta}$ and check whether the obtained minimum is less then the critical value for the test. In case it is we conclude that there exists a subset of the parameter space of $F_0(w)$ on which this distribution and the predicted $\hat{F}(w)$ are statistically equivalent. This would imply that imposing functional form assumptions on the offer(productivity) distribution should not result into a misspecified model. In case the minimum exceeds the critical value, we conclude that imposing parametric assumptions on the offer(productivity) distribution must lead to the inconsistent estimates of the parameters of the model.

The results of the analysis are presented in Table 1.4. We find that in both samples Pareto and log-normal specifications are rejected in favor of the nonparametric alternative. Firstly this finding supports the tail behavior result of Bontemps et al (2000) that rules out the log-normal family. Secondly, we demonstrate that even assuming the theoretically admissible Pareto family we cannot guarantee satisfactory performance of the parametric estimation of the model. Interesting enough, Bontemps et al (2000) also mention this fact with their data.

To cut it short, our analysis demonstrates that arbitrary parametric assumptions on the productivity distribution lead to inconsistent estimates. So the nonparametric estimation method should rather be applied. From this perspective the suggested in this chapter method of dealing with top-coded wages using the nonparametric 3-step method can be especially important.

1.6 Concluding Remarks

In this chapter we consider a problem of non-parametric estimation of the search equilibrium model with continuous productivity dispersion in the situation when individual earnings information is top-coded. To facilitate estimation of the structural parameters from the right-censored earnings samples we use the Pareto power law property of the earnings distribution and formulate a simple amendment to the specification of the model. Our formulation also preserves such attractive features of the original specification as fast convergence and simplicity of implementation.

To analyze the performance of the suggested method we estimate the model from the two samples of employment history data drawn from the Austrian Social Security records. It turns out that in moderately censored samples (top coded earnings make about 10% of the total number of observations) local linear regression indeed provides a very close fit of the Pareto functional form to the partially observed right tail of the earnings distribution.

At the same time the estimates of the separation rate imply that the standard formulation of the model with only two states of the market might be too restrictive.

Additionally we investigate whether non-parametric inference could be successfully substituted by the one with *ad hoc* parametric assumptions. In our application it turns out that neither log-normal nor Pareto distribution is a good enough proxy for the unobserved productivity distribution. This implies that arbitrary parametric assumptions may most likely lead to the inconsistent estimates of the structural parameters.

Chapter 2

Discrete vs. Continuous Productivity Dispersion

2.1 Introduction

As discussed in the previous chapter, nonparametric estimation of the Burdett-Mortensen type of search equilibrium models has a number of advantages over the parametric one. However, once we considering endogenous productivity distributions, in order to perform the nonparametric estimation the continuous productivity distribution must be assumed. In contrast, a fully parametric approach relies on the finite number of productivity types in the economy. This difference becomes important once we address the specification of the econometric model.

Bontemps et al. (2000), when developing their nonparametric method, notice that it assures the consistent estimates of the parameters of interest only when the model predicts the proper productivity distribution. Since theoretically it was not possible to exclude the cases in which the implied productivity density can take negative values, Bontemps et al. (2000) derive the appropriate restriction that would provide a properly estimated density. In this chapter we demonstrate that there are situations in which there exist no positive arrival rate of wage offer to the employed worker, which can satisfy mentioned restriction and avoid the nonnegativity of the productivity density. This implies that the procedure of Bontemps et al. (2000) breaks down and one needs to search for an alternative approach to estimate the model. We also develop a simple data-driven condition which can in advance tell about the applicability of the nonparametric method. Reviewing the alternative method of Bowlus et (2001) we find that assuming the discrete productivity dispersion does not suffer from the specification failures of the considered art, which makes it eventually a substitute for the inapplicable first best. Our analysis is applied to learning about the influence of the extension of the entitlement to the UI benefits on the subsequent dynamics of unemployment rates among old and unskilled workers in West Germany.

Generous unemployment insurance benefit is one potential reason for the high level of unemployment in European economies. Nickell (1997) and Siebert (1997) provide evidence for this hypothesis. Furthermore, Nickell (1997) and Nickell and Layard (1999) demonstrate that there exists a positive dependence between long-lasting entitlements to unemployment benefit and long-term unemployment. The German labour market is a typical representative of the above pattern. Evidence for this is presented for instance in Hunt (1995) or Steiner (1997) who, in a reduced form estimation of a duration model, show that the increasing length of unemployment duration is associated with an increase in the entitlement.

The time profile of the west German unemployment rates shows some well-known and interesting features: from the mid-1980s to the mid-1990s the unemployment rate of low-skilled and old-age workers was rising faster than that of the other skill or age groups. Relatively high unemployment rates of low-skilled workers are not only a German phenomenon, but they are particularly high in Germany. Nickell and Layard (1999) present figures of the unemployment rates of low and highly educated male workers for ten OECD countries from the 1970s to the early 1990s: from 1983 to 1986, the unemployment rate of low skilled workers relative to the total unemployment rate was 2.2 in West Germany For the other countries the ratio ranged from 0.6 to about 1.8, and the average was about 1.4. Moreover, until 1991 to 1993, for Germany this ratio rose by about 18%, while in the other countries the increase was lower. For elder workers, figures from the OECD Employment Outlook (1996) on standardized unemployment rates show that the West German situation substantially differs from that of many large economies. For instance, in 1983 in France, Italy, Spain, and the US the ratio of the unemployment rate of workers from 55 to 64 years old to the total unemployment rate was below one. Until the year 1990 it rose only for Spain. In contrast, for West Germany this ratio was about 1.16 in 1983 and more than doubled by 1990 demonstrating again the highest value and the sharpest increase.

There may exist quite a number of reasons why by the mid 90s unemployment of unskilled and elder workers in West Germany was increasing. In this chapter we concentrate on the entitlement length, which we consider especially important. In the mid 80s the government has introduced a series of reforms aimed at raising the length of entitlement to unemployment insurance benefits. Additionally, the increase in the entitlement length was the highest for elder unemployed individuals. We expect that as a result of these reforms reemployment incentives among elder unemployed workers have significantly gone down. Furthermore, the reforms may have had particularly strong adverse effect on the searching incentives of low-skilled unemployed workers.

Considerations of this type are not unfamiliar in the literature that documents the German labour market. For instance, Sinn (2002) argues that changes in the unemployment benefit system can potentially have an adverse effect on the incentives of low skilled workers, because typically such workers work for low wages. For elder workers, longer entitlement to unemployment benefit could be interpreted as a de facto reduction of the (early) retirement age.

In the present application we try to investigate empirically the impact of the increased length of entitlement to unemployment benefits on the unemployment rates of low-skilled and elder workers in West Germany. To do so we study the arrival rates of job offers and exit rates from full-time employment into unemployment in the mid 1980s and mid 1990s for different skill and age groups. We choose as a framework for the analysis the Burdett-Mortensen model, because through the adjustment of individual search behavior it enables us establishing the link between the increase in the entitlement duration and the dynamics of unemployment rates.

The setting of the theoretical model estimated in this chapter is similar to the one briefly reviewed in Chapter 1. So here we will not make any additional introduction to the theory (for an extensive treatment of the theory interested readers are referred to Burdett and Mortensen, 1998, Mortensen, 1990, and Bontemps et al., 2000). At the same time this chapter will provide the detailed analysis of the two existing structural estimation methods and discuss the instances when the more attractive nonparametric procedure of Bontemps et al. (2000) cannot not be applicable.

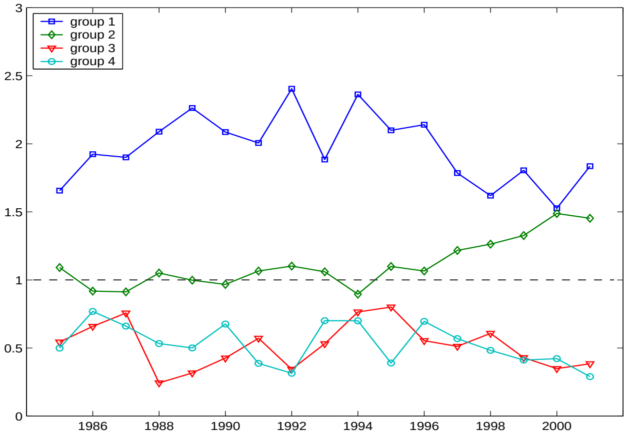
The second chapter is organized as follows. Section 2.2 motivates our study. Here we describe the evolution of unemployment rates in West Germany for different skill and age groups. We also provide a number of potential explanations of such a pattern of unemployment outcomes. In Section 2.3 we present an overview of the theoretical results from the search equilibrium modelling and develop an argument that links the entitlement reforms with the unemployment rate dynamics. Section 2.4 discusses the data. Methodological questions on the estimation of empirical search equilibrium models are discussed in Section 2.5. Here we demonstrate the limitation of the non-parametric method of Bontemps at al. (2000) and review the alternative procedure. We also discuss some further inference-related issues. Section 2.6 presents our estimation results and discusses their main economic implications. Summary and conclusions are given in Section 2.7.

2.2 Unemployment Rates in West Germany

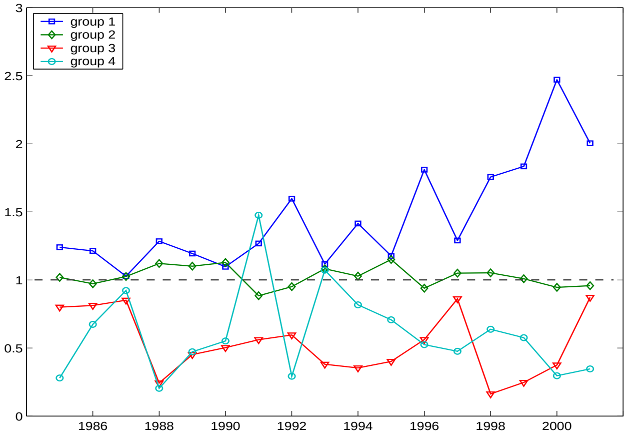
Already in the 1980s unemployment rates of unskilled and elder workers were particularly high relative to the overall unemployment rate. Moreover they kept rising considerably starting from the early 1990s. The figures below illustrate this phenomenon. Figure 2.1 shows the unemployment rates of four different skill-groups relative to the overall unemployment rate for each gender. Figure 2.2 repeats this exercise for different age groups. These figures were computed using data from the German Socio-economic Panel (GSOEP). The samples are limited to workers who are 16 to 64 years old. As to qualification, the GSOEP categorizes workers according to "International Standard Classification of Education" (ISCED) code, which takes into account both general schooling and occupational qualifications. We discern four such groups: 1 - inadequately trained or with general elementary schooling, 2 - middle vocational training, 3 - vocational training and college entrance exam or higher vocational training and 4 - higher education.

Figure 2.1(a) demonstrates that for skill-group 1 male unemployment rate is far above the average unemployment rate in the economy. Since 1988 it is most of the time about twice as high as the average male unemployment rate. Figure 2.1(b) displays relative unemployment rates for women. Its striking feature is again the unemployment rate in the lowest skill group. From 1985 to 1991 it exceeds the overall unemployment rate by roughly 13 up to 34%. After 1991 the differences start increasing and eventually become much higher then in the earlier period, ranging from 22 to about 130 %. In the skill group 2 there are most of the time no remarkable difference between the group-specific and economy wide unemployment rate for both males and females. The

Figure 2.1: Relative Unemployment Rates by Skill Groups

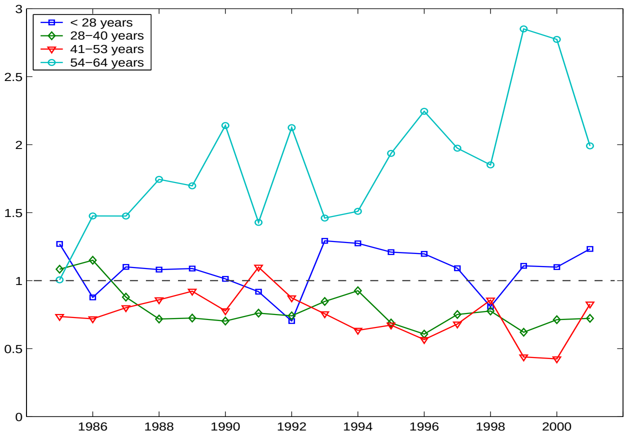


(a) Males

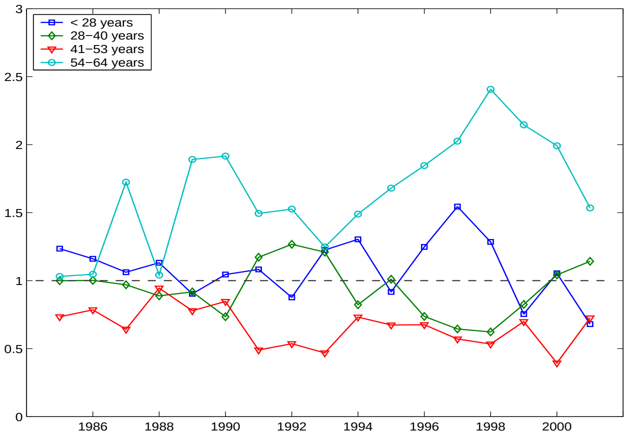


(b) Females

Figure 2.2: Relative Unemployment Rates by Age Groups



(a) Males



(b) Females

unemployment rates of the two highest skill groups are usually somewhat lower (and occasionally considerably lower) than those of the entire economy. To sum up, Figure 2.1 demonstrates that unemployment rates of the unskilled workers are the highest among all other skill groups and for women their relative deviation from the economy-wide unemployment rate has become particularly high in the 1990s.

The evolution of such skill-based differences in the West German unemployment rates is also highlighted by Sinn (2002) who points out that high unemployment rates of the unskilled could reflect adverse effects of changes in benefits. Indeed, the standard argument that increased benefit levels may raise the reservation wage and/or decrease job search intensity and therefore induce a higher level of unemployment, may apply. And this can be especially important for the low-skilled unemployed workers, whose potential earnings are relatively close to the benefits that they receive. However, in the period under review the replacement rates of the German unemployment benefit system were not increased. So this can hardly explain why unemployment rates of the low-skilled rose considerably from the 1980s to the mid 1990s. At the same time, as we will discuss in more detail below, in the mid 1990s for some groups of workers unemployment insurance benefits were paid for a much longer period of time. So it might have rather been an increase in the entitlement period that could have adversely affected the unemployment rates of low-skilled workers.

Let us consider now the age dimension. Figure 2.2 tracks the unemployment rates for several age groups of workers relative to the total unemployment rate. The most important detail of this plot is the evolution of the unemployment rate of the eldest group. Looking at Figure 2.2(a) in the year 1985 unemployment rate of the eldest males is still relatively close to the aggregate male unemployment rate. But from 1986 to 1989 it exceeds the aggregate unemployment rate by about 46 to 74%. From 1995 to 2001, this relative difference ranged even from 79 and 167%. The unemployment rates of all other age groups deviate much less from the aggregate rate. The corresponding relative unemployment rates for women are shown in Figure 2.2(b). The evidence on the eldest workers is not exactly the same as for men. Still, the figure shows that the unemployment rate of those who are 54 to 64 years old tends to exceed the aggregate unemployment rate in the second half of the 1980s and the first half the 1990s. And its deviation from the overall unemployment rate becomes remarkable since about 1992. Sometimes the unemployment rate in this group is even more than twice as high as the overall unemployment rate of all females.

In Germany two important institutional changes may have contributed to the increase in the relative unemployment rates of the eldest workers. First of all over the 1980s several benefit reforms tended to raise the potential length of the unemployment insurance (UI) benefits. Table 2.1 shows the length of UI benefit receipt over several time periods.

We start with the year 1985, as we will analyze the period from the mid 80s until the year 2000. The length of UI receipt depends positively on work-history in insured employment in the seven years prior to the benefit claim. The first column of Table 2.1 shows the relevant work-history intervals in months. The effect of additional work-history on the UI entitlement length also depends on age-limits.[1] These age-limits are shown in brackets next to the entitlement lengths in the other columns of the Table 2.1. The table shows the rules for the entitlement lengths, which are measured in months, that were in force in the year 1985 (second column), from January 1986 to March 1987 (third column), from April 1987 to March 1997 (fourth column) and from April 1997 to December 2003 (fifth column). We also notice that due to some special exemptions the rules displayed by the last column fully affected unemployed workers only two years after their introduction (see Wolff, 2003, for details).

Table 2.1 demonstrates that except of the last reform, all benefit reforms were raising the length of UI entitlement. One can also see that this increase was usually limited to certain age-groups and that the reforms were making the benefit system more and more generous for elder workers. With a sufficient work-history, unemployed workers older than 54 from July 1987 to March 1997 could be entitled to UI benefits for up to 32 months, while it was only 24 months from January 1986 to June 1987 and 18 months in the year 1985. For workers younger than 42 years to the contrary, the maximum length of UI entitlement was never raised in the 1980s and they could never receive UI for more than 12 months. However, the amount of work-history to achieve this maximum was reduced from 36 in 1985 to 24 by March 1987.

Along with the eldest workers, the reforms of the late eighties have stretched out the

[1]Note that unemployed people who run out of their UI benefit may still receive unemployment assistance benefit (UA). UA is generally lower than UI benefit and is not time limited. It can be paid until people reach the regular retirement age. Before 1994 the formal replacement rates of the UA benefit were 58 % for parents and 56 % for childless people, while for UI they were 68 % and 63 %, respectively. In 1994 these replacement rates were cut for UA benefit to 57 % and 53 % and for UI benefit to 67 % and 60 %. However, the UA benefit is means-tested and the benefit level may hence by far lower than the formal replacement rates suggest.

Table 2.1: Entitlement Length of Unemployment Insurance Benefit

Work History (months)	Length of UI entitlement during specific periods: January 1985 to December 1985	January 1986 to June 1987	July 1987 to March 1997	April 1997 to December 2003
$12-15$	4	4	6	6
$16-17$	4	4	8	8
$18-19$	6	6	8	8
$20-23$	6	6	10	10
$24-27$	8	8	12	12
$28-29$	8	8	14 (age $\geq$ 42)	14 (age $\geq$ 45)
$30-31$	10	10	14 (age $\geq$ 42)	14 (age $\geq$ 45)
$32-35$	10	10	16 (age $\geq$ 42)	16 (age $\geq$ 45)
$36-39$	12	12	18 (age $\geq$ 42)	18 (age $\geq$ 45)
$40-41$	12	12	20 (age $\geq$ 44)	20 (age $\geq$ 47)
$42-43$	14 (age $\geq$ 49)	14 (age $\geq$ 44)	20 (age $\geq$ 44)	20 (age $\geq$ 47)
$44-47$	14 (age $\geq$ 49)	14 (age $\geq$ 44)	22 (age $\geq$ 44)	22 (age $\geq$ 47)
$48-51$	16 (age $\geq$ 49)	16 (age $\geq$ 44)	24 (age $\geq$ 49)	24 (age $\geq$ 52)
$52-53$	16 (age $\geq$ 49)	16 (age $\geq$ 44)	26 (age $\geq$ 49)	26 (age $\geq$ 52)
$54-55$	18 (age $\geq$ 49)	18 (age $\geq$ 49)	26 (age $\geq$ 49)	26 (age $\geq$ 52)
$56-59$	18 (age $\geq$ 49)	18 (age $\geq$ 49)	28 (age $\geq$ 54)	28 (age $\geq$ 57)
$60-63$	18 (age $\geq$ 49)	20 (age $\geq$ 49)	30 (age $\geq$ 54)	30 (age $\geq$ 57)
$64-65$	18 (age $\geq$ 49)	20 (age $\geq$ 49)	32 (age $\geq$ 54)	32 (age $\geq$ 57)
$66-71$	18 (age $\geq$ 49)	22 (age $\geq$ 54)	32 (age $\geq$ 54)	32 (age $\geq$ 57)
≥ 72	18 (age $\geq$ 49)	24 (age $\geq$ 54)	32 (age $\geq$ 54)	32 (age $\geq$ 57)

maximum entitlement lengths for the 42 to 53 years old individuals. But the increase for those older then 54 was still higher. As a result the incentives to actively search for a job have gone down particularly for the eldest workers.

On top of that UI recipients older than 54 were always facing additional disincentive to search for a job. Namely, this is an option of exit into early retirement at the age of 60. To qualify for early retirement one must have at least 12 months of unemployment in the 18 months prior to reaching this age limit (see Lampert, 1996, p.267). For workers near sixty, this type of early retirement together with the high length of UI entitlement could be an additional disincentive to search actively for a job. Finally, since the reform of the Employment Promotion Act in 1986, unemployed workers older than 57 could agree with the labour offices to enter early retirement at the earliest possible date (see Steffen, 2003). In turn they need not be (fully) available for suitable job offers. This has pushed the disincentives for elder workers further up and paved the way into early retirement within the two years prior to reaching the age limit of 60 years. Even though elder workers are highly protected against dismissal, in practice these rules made their dismissal for both the employer and the employee more attractive. Arnds and Bonin (2002) argue that these reforms enabled employers to change the structure of their staff towards younger workers. And apart from the unemployment benefit, dismissed elder workers could even receive some additional financial support from their last employer.

Based on the overview above, for both unskilled and elder workers we ask one and the same question. We are interested in how far the increase in the UI entitlement length and the introduction of less strict requirements for getting the UI have influenced the magnitude of the equilibrium unemployment rate of this workers. To answer this question we need a theory that would link UI entitlement reform with equilibrium unemployment rates. We consider such theory in the next section.

2.3 Necessary Theoretical Results

The Burdett and Mortensen (1998) model of search equilibrium formalizes strategic interactions between supply and demand sides of the labour market. Representatives of the supply side, i.e. workers, search for better jobs while representatives of the demand side (employers) offer job opportunities. Workers maximize their utility of being employed and employers maximize their profits. The model describes equilibrium flows between the two states of the labour market, namely "employment" and "unemployment" by means of three key parameters: the arrival rate of a job offer to unemployed worker, λ_0, the arrival rate of a job offer to employed worker, λ_1, and the arrival rate of a match dissolution and return to unemployment, δ. The individual search process in any of these two states is viewed as a repeated drawing of job offers from a certain probability distribution, $F(w)$, and acceptance or rejection of the offer after each draw.

Three components of the equilibrium solution of the Burdett-Mortensen are central to the application of this chapter. First of all it is the steady state level of unemployment:

$$u = \frac{\delta}{\delta + \lambda_0}. \tag{2.1}$$

Secondly, it is the reservation wage of any unemployed agent who has an opportunity cost of employment b

$$R = b + (\lambda_0 - \lambda_1) \int_R^{\overline{w}} \frac{1 - F(w)}{\delta + \lambda_1 (1 - F(w))} dw. \tag{2.2}$$

In the economy with no black market work the parameter b in (2.2) reflects the level of potential unemployment benefits. Additionally, Mortensen and Neumann (1988) argue that in (2.2) the arrival rates of job offers, λ_0 and λ_1 can, without loss of generality, be interpreted as search intensities of the participating workers. This interpretation will be useful later on. Finally, Burdett and Mortensen (1998) show that whenever all the employers are homogeneous with respect to their productivity, p, the equilibrium wage offer distribution takes a form

$$F(w) = \frac{\delta + \lambda_1}{\lambda_1} \left[1 - \sqrt{\frac{p - w}{p - R}} \right] \tag{2.3}$$

One can further relax the assumption of employer homogeneity which will lead to the wage offer distribution of a form $F(w) = \int F(w|p) d\Gamma(p)$ where $\Gamma(p)$ is a certain productivity distribution that can be also derived endogenously. In his earlier contribution Mortensen (1990) derives the theoretical wage offer distribution assuming that $\Gamma(p)$ is discrete. Bontemps et al. (2000) study the case where the productivity distribution is continuous. In this chapter we estimate the model with both discrete and continuous productivity distributions. Since we find that under the continuity assumption the econometric model can be misspecified, we reserve the discussion of the issues related to the functional form of the wage offer distribution for Section 2.5, where we treat the structural econometric estimation of the model in detail.

We use (2.1)-(2.3) to link equilibrium unemployment rate, search intensity, reservation wage and the extension of the entitlement to UI with each other.

Consider first (2.1). Differentiating u with respect to λ_0 one can see that u is a decreasing function of λ_0. *Ceteris paribus* a reduction in search intensity of unemployed workers leads to an increase in the equilibrium unemployment rate. The opposite is true with respect to δ: A higher incidence of exit into unemployment raises the equilibrium unemployment rate.

Next consider (2.2). After some algebra (2.2) can be represented as an implicit function: $\mathcal{G}(R, \lambda_0, \lambda_1, \delta, b) = 0$. Simple application of the Implicit Function Theorem to $\mathcal{G}$ leads to a number of interesting results. Firstly one finds that for $\lambda_1 \in [\delta, \lambda_0)$ an increase in b has positive impact on R and δ, negative for λ_0 and ambiguous for λ_1, changing sign from positive to negative as λ_1 goes from δ to λ_0:

$$\frac{\partial R}{\partial b} > 0, \quad \frac{\partial \lambda_0}{\partial b} < 0, \quad \frac{\partial \lambda_1}{\partial b} \gtrless 0, \quad \frac{\partial \delta}{\partial b} > 0, \tag{2.4}$$

Secondly, adjustment behavior of workers to the entitlement reforms further influences the reservation wage level. Again for $\lambda_1 \in [\delta, \lambda_0)$ it can be shown that:[2]

$$\frac{\partial R}{\partial \lambda_0} > 0, \quad \frac{\partial R}{\partial \lambda_1} \gtrless 0, \quad \frac{\partial R}{\partial \delta} < 0 \tag{2.5}$$

The partial derivatives $\partial R/\partial \lambda_0$ and $\partial R/\partial \lambda_1$ have quite an intuitive interpretation. They show that unemployed workers who search more actively, i.e have higher λ_0, must have higher reservation wages. Better prospects of promotion on the job reflected by a high λ_1 reduce the reservation wage and create an incentive to accept lower wages in order to get out of unemployment faster (note that similarly to the previous chapter each promotion on the job is treated as a job change). Poor promotion possibilities, i.e. low λ_1, increase R creating thus an additional incentive to stay longer in unemployment and wait for better times.

The results above make it easy to show how increasing entitlement length would influence the dynamics of unemployment rates. We would suggest the following argument. Even though it is not explicitly stated in (2.2) that considers only the current benefit level and not its discounted present value, it is reasonable to suggest that an increase in the duration of UI benefit payments induces an increase in the net present value of unemployment. At the same time it is true that for any agent the search process is associated with certain disutility generated by search efforts. Therefore, facing the exogenous increase in the value of unemployment, unemployed agents will tend to substitute certain degree of search intensity that brings disutility for some other activities, i.e. search less. Considering (2.1) we conclude that this will unambiguously shift the equilibrium unemployment rate up. This establishes the direct effect of the

[2] Even though the condition $\lambda_1 \in [\delta, \lambda_0)$ might seem to bee too restrictive, indeed it is not so. The reason is that $\lambda_1 \leq \lambda_0$ implies that expected job duration is at least as high as expected unemployment duration. Furthermore $\lambda_1 \geq \delta$ implies that for employed workers with no job-to-job changes so far the probability of finding the next job is at least as high as the probability of being fired. Thus, the values of λ_1 will typically lie in the interval $[\delta, \lambda_0]$.

extension on the unemployment rates.[3]

Along with a direct effect there can also exist an indirect one. One can expect that the exogenous increase of the value of unemployment can make unemployed workers choosy with respect to the arriving wage offers (van den Berg, 1990). This will be reflected by the increasing reservation wage. At the same time, from (2.4)-(2.5) follows that the just described reduction of unemployment search intensity will drive the reservation wage down. This will counteract the initial exogenous increase in R. As a result of the initial exogenous shock and subsequent unemployed search behavior adjustment we will receive a new equilibrium level of the reservation wage. An interesting case arises whenever this new level is higher then the one before the entitlement extension. In this situation the low-productivity firms with limited capacities for productivity enhancement may offer too low a wage to attract any worker. This will result in a higher degree of structural unemployment among lower-skilled workers.

Finally, the contribution to an increase in equilibrium unemployment rates may come from the side of the match dissolution parameter δ. Even though in the model this parameter is exogenous to the worker it still reflects some effects induced by a longer duration of UI receipt. In particular, the increased generosity of the UI system may increase the incentives to shirk and as a result drive the incidence of match breaks up. From (2.1) we know that an increase in the frequency of match dissolution incidents leads to the increase in the equilibrium unemployment rate.

The arguments presented above imply that by analyzing empirically the key parameters of the model before and after the reform we will be able to tell whether the rise in UI entitlement length indeed contributed to the increase in unemployment rates of unskilled and elder workers as discussed in the previous section. Even though the reservation wage equation in the contemporary formulation of the model does not explicitly include the timing of UI payments, the available econometric procedures are robust to this theoretical shortcoming (see Section 2.5, page 46). So we will be able to avoid possible specification error in our structural estimation and find the estimates that are consistent with the most general formalization of UI payment schedules that

[3] The logic of this argument also goes in line with the results of van den Berg (1990) who in a simple job search model with time-limited UI payments demonstrates that " ... any future shift in the time path of exogenous variables that benefits the expected discounted lifetime income induces job searchers to be more selective in their search process" (see van den Berg, 1990, p. 262). Even though the model of van den Berg (1990) does not incorporate on-the-job search and quits it should be expected that the discovered regularity would hold in the extended model as well.

would consider the duration of benefit payments.

To conclude we notice that the effect of benefit reforms on the search behavior of employed workers λ_1 is unclear theoretically. In addition, empirical studies by Belzil (1995), (2001) demonstrate that changes in the duration of benefit payments do not significantly alter the length of subsequent reemployment spells. For these reasons, our discussion will concentrate exclusively on the arrival rate of job offers for unemployed individuals (λ_0) and on the match dissolution parameter δ.

2.4 The Data

We use data from the German Socio-economic Panel – an annual longitudinal survey of German households started at 1984. The information comes from the waves of years 1984 to 2001. The analysis is restricted to samples A and B of the GSOEP. Sample A represents households with a household head being a native West German. Sample B represents households whose head belongs to main groups of foreigners in west Germany. Additionally, we only include respondents from 16 to 64 years old.

2.4.1 Classification of Workers in the Stock Samples

Estimation of the empirical model of search equilibrium relies on stock sampling. We analyze the stocks of employed and unemployed people from two specific waves: the wave of the year 1986 and the wave of the year 1995. As the effect of the increase of the potential length of UI benefit receipt started manifesting itself in-between, such a sampling scheme should be the appropriate one for investigating the reaction in the search behavior. The choice of years is also influenced by the fact that macroeconomic conditions in these two years were rather similar, i.e. the economy was in roughly the same phase of the cycle. Finally, this choice minimizes the amount of censored job and unemployment durations in the samples under study.

For the reasons already spelled out in Section 1.3 we analyze the agents who are "unemployed" and "full-time employed". We classify workers as "unemployed" if for the modal interview month of the chosen year they reported to be registered as unemployed. For this classification, we use information from the subsequent wave's retrospective labour force status calendarium. In contrast, we classify people as "full-time employed" on the basis of their current labour force status reported at the interview.

Unlike with the social security records data used in the previous chapter, GSOEP always provides an information of whether an individual is a full-time or part-time worker. This minimizes measurement error when estimating the endpoints of the support of the offer/earnings distribution (see Section 1.3 for the discussion).

2.4.2 Unemployment and Job Durations, Exit States

To construct the likelihood function for the model we need to use both wage and duration data. Whenever we observe a change of states, we need to record information about the new state. In the setting of the model, job-to-job changes are also considered as a "change of state".

Unemployment duration is calculated from the retrospective labour force status calendarium of the GSOEP, in which respondents have to provide their labour force status for every month of the previous calendar year. Apart form completed spells, unemployment spells can also be left-censored, right-censored or both. In our sample, unemployment spells are left-censored mainly because a respondent was already unemployed before he/she first filled in the labour force status calendarium. The main reasons for right-censoring is either that a respondent temporarily did not respond to the GSOEP or due to the fact that the respondent completely dropped out of the panel study. Finally, some of the spells did not terminate before the end of our observation period.

The information on the beginning and end of a job spell is more difficult to obtain. There are various pieces of information on the job history of individuals that the GSOEP collects retrospectively. First of all respondents who state that they are currently employed provide the calendar year and the calendar month of the start of the job. Secondly, provided that there is a job change, employed respondents have to state in which calendar month this job event took place and indicate the type of job event: first job, new employer, self-employment, change within the firm, company takeover, or return to work. Combining these two sources of information we restore the calendar beginning and end of the jobs for all employed individuals.

Similar to unemployment spells, job spells can be left-censored, right-censored or both and we proceed in similar fashion to the treatment of unemployment spells. Here it would be insightful to point out one substantial difference between the household

Table 2.2: Descriptive Statistics of Event History Data for the Two Stock Samples *

	1986			1995		
	Full Sample	Elder	Low-Skilled	Full Sample	Elder	Low-Skilled
Number of individuals:	4873 [1.000]	571 [1.000]	1401 [1.000]	4030 [1.000]	637 [1.000]	933 [1.000]
Employed:	4551 [0.934]	518 [0.907]	1272 [0.908]	3681 [0.913]	533 [0.837]	780 [0.836]
Unemployed:	322 [0.066]	53 [0.093]	129 [0.092]	349 [0.087]	104 [0.163]	153 [0.164]
Employed Agents:	4551 [1.000]	518 [1.000]	1401 [1.000]	3681 [1.000]	533 [1.000]	780 [1.000]
Uncensored observations with:						
job → job transition:	706 [0.155]	6 [0.012]	138 [0.108]	423 [0.114]	7 [0.013]	49 [0.063]
job → unemployment transition:	385 [0.085]	42 [0.081]	157 [0.123]	277 [0.075]	68 [0.128]	101 [0.129]
mean time spell between two states [job duration]:	139.95	248.94	150.53	106.82	248.33	129.35
(std. deviation):	(115.44)	(138.18)	(113.66)	(101.08)	(141.28)	(115.29)
Censored observations						
a) left-censored durations only						
with job → job transition:	97 [0.021]	5 [0.010]	24 [0.019]	22 [0.006]	1 [0.002]	3 [0.004]
with job → unemployment transition:	74 [0.016]	16 [0.031]	28 [0.022]	16 [0.004]	2 [0.004]	1 [0.001]
b) right-censored durations only:	2898 [0.637]	361 [0.697]	784 [0.616]	2857 [0.776]	445 [0.835]	603 [0.773]
c) both left- and right-censored durations:	391 [0.086]	88 [0.170]	141 [0.111]	86 [0.023]	10 [0.019]	23 [0.029]
Mean time spell [both uncensored and censored]:	168.85	236.69	158.99	155.05	263.87	161.88
(std. deviation):	(136.41)	(167.16)	(123.92)	(118.89)	(143.26)	(117.13)
Unemployed Agents:	322 [1.000]	53 [1.000]	129 [1.000]	349 [1.000]	104 [1.000]	153 [1.000]
Uncensored observations (u → j transition):	116 [0.360]	3 [0.057]	42 [0.326]	105 [0.301]	4 [0.038]	38 [0.248]
mean time spell between two states [job duration]:	14.18	11.67	14.91	20.81	14.50	19.92
(std. deviation):	(18.94)	(4.16)	(12.57)	(22.95)	(8.66)	(14.30)
Censored observations						
a) left-censored durations (u → j transition) only:	14 [0.043]	-	11 [0.085]	3 [0.009]	-	1 [0.007]
b) right-censored durations only:	160 [0.497]	33 [0.623]	58 [0.450]	226 [0.648]	96 [0.923]	106 [0.693]
c) both left- and right-censored durations:	32 [0.099]	17 [0.321]	18 [0.140]	15 [0.043]	4 [0.038]	8 [0.052]
Mean time spell [both uncensored and censored]:	29.20	45.51	34.95	35.43	47.25	40.92
(std. deviation):	(33.02)	(37.02)	(36.07)	(33.35)	(36.75)	(36.25)

* Duration data in Months. Share of the sample in square brackets.

survey data of this chapter and social security records data of the previous one. With household surveys it happens that we may not observe the exact start or end of the job spell even if it does not exceed the observation period. We may see only a year of the job change, but not know the month. In this cases the elapsed(residual) duration is taken to be the the smallest observed one and the spell is qualified as censored. Social security records to the contrary always provide the exact timing of the spell.

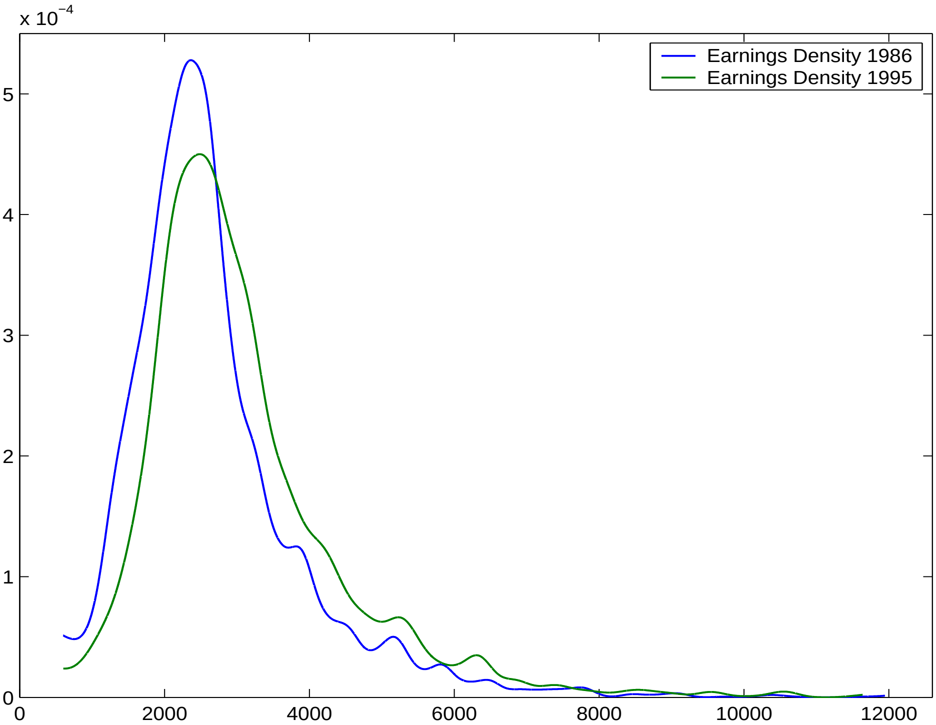
For all spells where we could observe the calendar end, we determine the exit state. In case of the unemployment spells, using the retrospective labour force status calendarium information, we check whether they ended in full-time employment or in any other labour force state. In case of the job spells, we use the labour force status calendarium and job events information to see whether a job has ended by the transition to unemployment, another job or non-participation.

Table 2.2 provides summary statistics for employment and unemployment spells in the resulting stock samples. Additionally it shows the percentage of the spells that are completed, left-, right- and both left- and right-censored. From Table 2.2 one can again see how big the difference between the economy-wide unemployment rate and unemployment rates of the unskilled and elder workers is. Moreover we see the increasing difference: in 1986 the rates for these both groups were about 1.5 times bigger than the total rate; in 1995 this gap widens and unemployment rates of the unskilled and old workers become twice as big as the total rate. This fact was extensively discussed in Section 2.2. Inspecting the data set it may also be interesting to compare the West-German unemployment rates with the Austrian ones. From Table 1.1 in Section 1.3 of the first Chapter we realize that the dynamics of unemployment rates in a country with no entitlement reforms was by far not as drastic. Similar observation can be made about the unemployment duration in Austria and Germany. Comparing the duration lengths (both censored and uncensored spells) we see that generally in Germany unemployment lasts longer.

2.4.3 Wages and Benefits

Wages: The final piece of information necessary for the estimation of the model is earnings. We use the data on net wages provided by the GSOEP. Individuals who are employed at their interview provide the monthly net wage in the month prior to the interview. For the stock sample of job spells we use the wage information that the

Figure 2.3: Kernel Estimates of Earnings Densities in 1986 and 1995



respondents stated at the year for which the sample is drawn. For the stock sample of unemployment spells we use the first reported wage after the end of unemployment, provided that the unemployment spell is not right-censored. All wages are deflated by the west German consumer price index to prices of 1998. Kernel estimates of earnings densities are presented in Figure 2.3. (kernel plots account for tail correction of Vuong et al., 2000, on both left and right tails).

Benefit Levels: Having once estimated the model we compute the reservation wages predicted by the theory. To do this we need to know either the true benefit receipt or a potential benefit level of our sample members. We consider three types of benefits: unemployment insurance benefits (UI), unemployment assistance benefits (UA) and welfare benefits (WB). UI and UA benefits are determined by formal replacement rates. Though the UA and WB are means-tested and therefore may eventually be lower than the formal replacement rates.

For unemployed people, we set the UI or UA benefit to the level that they received at the date when the stock sample was drawn. These benefit levels are reported retrospectively in the subsequent wave. The respondents provide the monthly average benefit level for the months in which they received the benefit during the previous calendar year. There are also a few unemployed individuals in our sample who receive a training benefit but no unemployment benefit. For all full-time employed individuals, we set their unemployment benefit level to the value of the replacement rate of the UI benefit multiplied by their net wage.

We did not attempt to simulate the means-test for the households in our sample in order to compute a welfare benefit level. However, we used information on social benefits provided by the household heads for the households in which the respondents live. We took into account receipt of rent subsidy payments, continuous aid for living expenses as well as social welfare assistance to meet special contingencies in life.

2.5 Structural Econometric Model

2.5.1 The Likelihood Function

The construction of the likelihood function is thoroughly considered in Chapter 1, Section 1.4.1. Here we only show the result, rewriting it a bit differently for convenience of the subsequent argument and estimation. Define $\kappa_0 = \lambda_0/\delta$, $\kappa_1 = \lambda_1/\delta$. Keeping the conventional notation of $\bar{F}(w) = 1 - F(w)$ we get the following likelihood contributions of unemployed ($\mathcal{L}_u$) and employed ($\mathcal{L}_e$) individuals:

$$\mathcal{L}_u = \frac{1}{1+\kappa_0} \left[\delta\kappa_0\right]^{2-d_r-d_l} \exp\left\{-\delta\kappa_0 \left[t_e + t_r\right]\right\} \left[f(w)\right]^{1-d_r}, \tag{2.6}$$

$$\begin{aligned}\mathcal{L}_e = \frac{\kappa_0 g(w)}{1+\kappa_0} \left[\delta\left(1+\kappa_1\bar{F}(w)\right)\right]^{1-d_l} \exp\left\{-\delta\left(1+\kappa_1\bar{F}(w)\right)\left[t_e+t_r\right]\right\} \times \\ \left[\left[\delta\kappa_1\bar{F}(w)\right]^{d_t}\delta^{1-d_t}\right]^{1-d_r}\end{aligned} \tag{2.7}$$

In (2.6) and (2.7) $d_l = 1$, if a spell is left-censored, 0 otherwise; $d_r = 1$, if a spell is right-censored, 0 otherwise; $d_t = 1$ if there is a job-to-job transition, 0 otherwise. Since all labor suppliers are assumed to act independently, the total likelihood is a product of all individual contributions.

2.5.2 Nonparametric Estimation and Its Limitations

Define the observed earnings density and distribution as $g(w)$ and $G(w)$ respectively. Then using the steady state identities of the theoretical Burdett-Mortensen model

$$\bar{F}(w) = \frac{1 - G(w)}{1 + \kappa_1 G(w)} \quad \text{and} \quad f(w) = \frac{(1 + \kappa_1)\, g(w)}{[1 + \kappa_1 G(w)]^2} \tag{2.8}$$

Bontemps et al. (2000) propose the following 3-step estimation procedure. On the first step $g(w)$ and $G(w)$ in (2.8) are estimated nonparametrically. On the second step the expressions in (2.8) are substituted into (2.6) and (2.7) and the likelihood function is maximized with respect to $\{\kappa_0, \kappa_1, \delta\}$. On the third step the equilibrium productivity levels

$$p = K^{-1}(w) = w + \frac{1 + \kappa_1 G(w)}{2\kappa_1 g(w)} \tag{2.9}$$

and productivity density

$$\gamma(p) = \frac{2\kappa_1(1 + \kappa_1) g(w)^3}{3\kappa_1 g(w)^2 [1 + \kappa_1 G(w)]^2 - g'(w)[1 + \kappa_1 G(w)]^3} \tag{2.10}$$

are calculated. Bontemps et al. (2000) notice that the third step is possible only if the model is well specified with respect to the equilibrium productivity distribution, i.e., if $3\kappa_1 g(w)^2 - g'(w)[1 + \kappa_1 G(w)] > 0$. In case this condition is not satisfied they suggest to perform the second step of the procedure under this theoretically implied constraint, which can be conveniently written down as

$$\kappa_1 \left[3g(w)^2 - g'(w)G(w)\right] > g'(w) \quad \{w : g'(w) \geq 0\}\,.^4 \tag{2.11}$$

In the applications of the proposed methodology so far (see, for instance, Bontemps et al., 2000) the constraint in (2.11) was never violated. The present application, to the contrary, faces the opposite case. Therefore, we follow the suggestion of Bontemps et al. (2000) and on the second step maximize the likelihood with respect to (2.11). It turns out, however, that the constrained optimization may not always be feasible. To see this notice that for some values of w the term $3g(w)^2 - g'(w)G(w)$ on the l.h.s. of (2.11) can be negative. This is exactly the case when we observe clusters of those who earn very high wages. Such clustering is represented by a bump far on the right

[4]Notice that if $g'(w) < 0$ productivity density $\gamma(p)$ is always positive.

tail of the estimated earnings density. Whenever such a bump occurs, $g'(w)$ is greater than zero and at the same time $G(w) \to 1$ and $g(w) \to 0$. So the value of $g(w)$ may be too small to make the whole term $3g(w)^2 - g'(w)G(w)$ positive. In this situation the constraint yields

$$\kappa_1 < \min_{\{w\}} \frac{g'(w)}{3g(w)^2 - g'(w)G(w)} < 0 \quad \{w : g'(w) \geq 0\} \tag{2.12}$$

As a result there is no κ_1 that can satisfy (2.11), since κ_1 is always greater than zero. We will refer to this case as *"constraint inconsistency"*.

In the opposite situation when $3g(w)^2 - g'(w)G(w) > 0$ the constraint becomes

$$\kappa_1 > \max_{\{w\}} \frac{g'(w)}{3g(w)^2 - g'(w)G(w)} > 0 \quad \{w : g'(w) \geq 0\} \tag{2.13}$$

and constrained maximization on the second step indeed returns an appropriate estimate of κ_1. A typical example for this case will be the left tail of earnings distribution, where $g(w)$ increases, but its values are high enough to ensure that $3g(w)^2 - g'(w)G(w) > 0$ holds $\forall w : g'(w) \geq 0$.

As we find that constraint inconsistency is purely the earnings data property, the

$$\texttt{sign}\left[3g(w)^2 - g'(w)G(w)\right] \tag{2.14}$$

on $\{w : g'(w) \geq 0\}$ becomes a simple criterion that would allow checking in advance whether the nonparametric 3-step procedure is applicable.

In our application it turns out that for both data samples (2.14) is not uniformly positive, i.e. we face the case of an inconsistent constraint. This implies that we cannot apply the nonparametric estimation procedure directly. The regions of the tail where (2.14) is negative can be actually seen on Figure 2.5. These are the peaks of the "waves" to the right of about DM 7000.

Finally, we also warn from using oversmoothing of the kernel density estimator in order to achieve the "consistent" constraint. By oversmoothing one can indeed get a strictly decreasing right tail with minor changes of the curvature of the rest of estimated density. However, from (2.13) it can be seen that by manipulating the magnitude of the bandwidth one arbitrarily fixes the value of the lower bound of the constraint. This inevitably biases the estimated $\hat{\kappa}_1$.

2.5.3 Parametric Estimation of the Model

Facing the situation of constraint inconsistency we cannot perform the nonparametric estimation of the model any longer. So we need to use the alternative parametric procedures. In other words we have to impose certain assumptions concerning the form of either the earnings or the productivity distribution.

Parametric Assumptions on the Earnings Distribution

The easiest way to avoid an inconsistent constraint is to assume some parametric form for $g(w)$ instead of using its nonparametric estimate in (2.6)-(2.7). Inspecting the shape of the kernel estimate of the earnings distribution the most natural suggestion would be that $g(w)$ is log-normal. We estimate the model under this assumption and find that indeed (2.11) is always satisfied. However, calculating (2.9) we discover that it violates the requirement that the offered wage is a monotone increasing function of productivity.[5] This generates an improper estimated productivity density and implies the necessity of imposing parametric assumptions on the productivity distribution directly.

Parametric Assumptions on the Productivity Distribution

This approach differs from the one in Section 2.5.2 by the fact that now the productivity parameter p appears in the likelihood function explicitly. p emerges because instead of nonparametric estimates of $g(w)$, $f(w)$ and $F(w)$ we take the theoretical expressions of these same functions. The theoretical offer and earnings distributions constitute a part of the equilibrium solution of the model and depend on both search intensity parameters and the productivity parameter p (see Mortensen, 1990, and Burdett and Mortensen, 1998, for derivations).

If employers were identical, the theoretical offer distribution would be given in (2.3). However, to get a decreasing right tail of the theoretical earnings density, we must assume productivity differentials among employers. This would imply a certain distribution for p.

In the literature there exist two approaches to estimation of the model with heterogeneous employers. The first one is taken by Koning et al. (1995) and Christensen

[5]Monotonicity of offered wages as a function of productivity follows from *Proposition 10* of Bontemps et al. (1997), which is a generalization of Burdett and Mortensen (1998) finding that more productive firms pay higher wages.

et al. (2000). It is based on *ad hoc* assumptions about the shape of the productivity distribution. Namely, Koning et al. (1995) assume that firms' productivity is distributed lognormally with parameters μ and σ and consider the marginal likelihood, where marginalization is made with respect to productivity. The likelihood function is maximized with respect to $\{\kappa_0, \kappa_1, \delta, \mu, \sigma\}$. Christensen et al. (2000) rather suggest that the unknown productivity parameter p is multiplied by the term $\exp\{\eta\}$, where $\eta \sim N(0, \sigma^2)$. The likelihood function in their application is maximized with respect to $\{\kappa_0, \kappa_1, \delta, p, \sigma\}$. However, despite assuming the continuous productivity distribution both these contributions ignore the result

$$F(w) = \Gamma(p)$$

stated later by Bontemps et al. (2000). In addition to that, in Chapter 1 we have shown that arbitrary assumptions on the productivity distribution may lead to inconsistently estimated structural parameters (see Section 1.5.3). Therefore this approach can hardly be attractive.

The second approach to the specification of the productivity distribution is due to Bowlus et al. (1995), (2001). It assumes that the productivity distribution is discrete rather then continuous. The exact form of the distribution is *a priori* unknown and the support points and corresponding probability mass values are estimated together with the structural parameters of the model. Furthermore Bowlus et al. (1995), (2001) take into account the theoretical dependence between the continuous offer distribution and the discrete productivity distribution (see 2.16 below) and, unlike Koning et al. (1995) and Christensen et al. (2000), consider a completely specified structural model. These positive features of the second approach makes it an attractive choice for our present application.

The specification relies on the closed form solution for the wage offer distribution derived by Mortensen (1990). Mortensen (1990) shows that whenever the productivity distribution is discrete and has Q points of support the theoretical wage offer distribution has Q kinks, each of them corresponding to the highest wage paid by a p_j-type employer ($j = 1, ..., Q$). Moreover, firms with higher productivity pay higher wages, which implies that the lowest wage paid by p_j-type employer (w_{L_j}) is equal to the highest wage of p_{j-1} -type employer ($w_{H_{j-1}}$). Consequently the ranking $w_{H_{j-1}} < w_{H_j}$ $\forall j = 1, ..., Q$ applies. This leads to the theoretical wage offer distribution with Q

distinct productivity types

$$F(w) = \frac{1+\kappa_1}{\kappa_1}\left[1 - \frac{1+\kappa_1\left(1-\gamma_{j-1}\right)}{1+\kappa_1}\sqrt{\frac{p_j - w}{p_j - w_{H_{j-1}}}}\right], \quad w \in (w_{L_j}, w_{H_j}] \tag{2.15}$$

$j = 1, ..., Q$, ($w_{L_1} = R$, $w_{H_Q} = \overline{w}$). In the expression above γ_j indicates the fraction of employers with productivity level p_j or less ($\gamma_0 = 0$, $\gamma_Q = 1$). From the fact that more productive firms pay higher wages Mortensen (1990) concludes that

$$F(w_{H_j}) = \gamma_j \tag{2.16}$$

Bowlus et al. (1995) use (2.15) and (2.16) to formulate their estimation procedure. First, differentiating (2.15) with respect to w and using (2.8) Bowlus et al. (1995) get the theoretical wage offer and earnings densities

$$f(w) = \frac{1+\kappa_1\left(1-\gamma_{j-1}\right)}{2\kappa_1}\frac{1}{\sqrt{p_j - w}\sqrt{p_j - w_{H_{j-1}}}}, \tag{2.17}$$

$$g(w) = \frac{1+\kappa_1}{2\kappa_1\left(1+\kappa_1\left(1-\gamma_{j-1}\right)\right)}\frac{1}{p_j - w}\sqrt{\frac{p_j - w_{H_{j-1}}}{p_j - w}} \tag{2.18}$$

$w \in (w_{L_j}, w_{H_j}]$, $j = 1, ..., Q$. Substitution of (2.15)-(2.18) into (2.6)-(2.7) gives the likelihood function with unknown parameters $\left\{\kappa_0, \kappa_1, \delta, \gamma_1, ..., \gamma_{Q-1}, p_1, ..., p_Q, R, w_{H_1}, ..., w_{H_{Q-1}}, \overline{w}\right\}$. Next, following Mortensen (1990) the productivity level is represented as a function of cutoff wages w_{H_j}, probability mass points γ_j and structural parameters

$$p_j = \frac{w_{H_j} - B_j w_{H_{j-1}}}{1 - B_j}, \tag{2.19}$$

where $B_j = \left[\frac{1+\kappa_1(1-\gamma_j)}{1+\kappa_1(1-\gamma_{j-1})}\right]^2$ and (2.19) is substituted into (2.15), (2.17) and (2.18). After this Bowlus et al. (1995) consider the subsets $\theta_1 = \{R, \overline{w}\}$, $\theta_2 = \left\{w_{H_1}, ..., w_{H_{Q-1}}\right\}$ and $\theta_3 = \{\kappa_0, \kappa_1, \delta\}$ of the parameter space. Following Kiefer and Neumann (1993) as an estimator of θ_1 they suggest using the extreme order statistics of the observed wage sample: $\hat{\theta}_1 = \{w_{\min}, w_{\max}\}$.[6] Then the estimation procedure is stepwise:

[6] This fact is especially useful for the present application, because the estimator $\hat{R} = w_{\min}$ allows getting the consistent estimate of R even when the timing of UI benefit payments is not explicitly introduced in the model

1. On the first step given $\hat{\theta}_1$ and starting values for $\{\kappa_0, \kappa_1, \delta\}$ and $\left\{\gamma_1, ..., \gamma_{Q-1}\right\}$ the set of cutoff wages θ_2 is estimated;
2. On the second step given $\hat{\theta}_1$, $\hat{\theta}_2$ and starting values for $\{\kappa_0, \kappa_1, \delta\}$ and $\left\{\gamma_1, ..., \gamma_{Q-1}\right\}$ the likelihood function (2.6)-(2.7) is maximized with respect to $\theta_3 = \{\kappa_0, \kappa_1, \delta\}$;
3. Using $\hat{\theta}_3$ and (2.8) and (2.16) the point mass probabilities γ_j are found and thy cycle repeats again.

Since (2.15) has Q kinks in w_{H_j} the likelihood function is discontinuous in $\theta_2 = \left\{w_{H_1}, ..., w_{H_{Q-1}}\right\}$. To estimate the discontinuity points on the first step of the above procedure Bowlus et al. (1995), (2001) suggest using the Simulated Annealing algorithm as introduced by Kirkpatrick et al., 1983, (directions for implementation of the algorithm can be found in Goffe et al., 1994). On the smooth second step the likelihood is maximized by standard methods.

The number of elements in θ_2, hence the number of mass points in $\Gamma(p_j)$, is treated as unknown. We start from the homogeneous case ($Q = 1$) and add productivity levels one by one. The exact distribution of the likelihood ratio in this particular case is also not known. Bowlus et al. (2001) propose a quasi-LRT test $V = -2\left(\log L_{j-1} - \log L_j\right) < \chi^2(1)$. Performing a simulation study they notice, however, that this criterion can be applied for small Q only, as the critical region increases with Q. Therefore we make our choice of the number of mass points on the basis of information criteria (consistent AIC, SBC).

Finally, we conclude with a short note on the covariance matrix estimator for the structural parameters. Chernozhukov and Hong (2004) demonstrate that for a general class of structural models in which discontinuity points of the likelihood function are not determined by the parameter subset $\hat{\theta}_3$ asymptotic distribution of $\hat{\theta}_3$ is $\sqrt{n}\left(\hat{\theta}_3 - \theta_{3.n}\right) \rightarrow N(0, \mathbf{I}^{-1})$, where $\mathbf{I}$ is the outer product of first derivatives of the total likelihood with respect to θ_3.

2.6 Estimation Results and Discussion

2.6.1 Preliminary Discussion

In Section 2.3 we argue that entitlement extension should negatively effect the search intensity of the unemployed λ_0 and increase the exit rate to unemployment δ. According to the theory both these changes must result in an upward shift of the equilibrium unemployment rate. We also stipulate that the adjustment dynamics of search intensity induced by the reforms may change the reservation wage level, which can contribute to an increase in structural unemployment.

With respect to skills we expect that the increased generosity of the benefit system will affect the arrival rates of job offers and reservation wages of the low skilled workers more than those of the high skilled ones. One line of argument to support this hypothesis is that the value of household production of skilled and unskilled workers is about the same. At the same time the ratio of benefits plus value of household production while unemployed to the potential wages plus the value of household production while employed is much higher for the low-skilled than for the skilled workers. Thus extending the entitlement length may affect the job search behavior of the low-skilled workers more than that of skilled workers.[7]

Considering age groups, the increase in the potential duration of UI benefit receipt is higher the older the workers are. So we expect that the search intensity of elder workers must fall faster than that of younger workers, which will result into higher group-specific unemployment rate. The opposite should be true for the reservations wages.

We also have to notice that even though the comparison of reservation wages predicted by the model using (1.1) may provide us with rather useful results we have only limited a possibility to interpret it. The reason is that the reservation wage calculation relies on a formula that contains the opportunity cost of employment b. We set this quantity equal to the benefit level received by the agents. By doing so we do not explicitly take into account other possible contributions to b such as household production, black market work etc. In this way we may underestimate the magnitude of the reservation wage. Furthermore, reservation wages predicted by means of (1.1) will also ignore the actual change in the entitlement period. This shortcoming hampers the inference about the possible contribution to the unemployment dynamics. Limited possibilities to interpret the predicted reservation wages also prevents us from inferring much from the changes in employed search intensity λ_1. However, the latter fact

[7]One should note, though, that such differences may also be caused by other influences in the labour market. For instance it could be a skill-biased technological change that could decrease the relative demand for low-skilled workers. As a result we may expect a reduction in the arrival rate of job offers to unskilled relative to skilled workers as well as an increase in their relative rate of job loss.

can hardly be important for the inference, since we know that the dynamics λ_1 is not determined by the entitlement extension reforms (see Section 2.3).

2.6.2 Overall Fit of the Model

Let us first consider the estimation results for the whole economy.[8] The model for the whole economy is estimated primarily for analyzing its' fit to the data. The estimation results are presented in Table B.1. The procedure terminates at $Q = 8$ for the sample of 1986 and $Q = 10$ for the data of 1995. The most appealing criterion of the goodness of fit is the discrepancy between the predicted theoretical earnings distribution and the nonparametric estimate of earnings distribution obtained from wage data. From Figures B.1-2 we can see that for both 1986 and 1995 samples this fit is very close, which should assure sound inference from the obtained estimation results. Furthermore, the fit can be improved to an arbitrarily high degree by simply adding points to the support of the productivity distribution. This, however, does not significantly change the estimated parameters and does not improve the model any further in terms of information criteria.

We can also compare the equilibrium unemployment rates predicted by the model with actual unemployment rates reported in Table 2.2. Using (2.1) and the results in Table B.1 we find that the model predicts unemployment rates of 7.3% and 9.7% for 1986 and 1995 samples, respectively. From Table 2.2 we see that the share of unemployed workers in 1986 and 1995 was 6.6% and 8.7% respectively. Again, this reflects a fairly good fit of the estimated model to the data. So again we conclude that the chosen model can provide reliable information for our subsequent analysis.

In what follows we estimate the model for different skill and age groups. As above, the number of points of increase in the productivity distribution is treated as unknown. We start with the homogeneous ($Q = 1$) model and, adding the support points one by one, use information criteria to find the best specification. Estimation results for skill groups are reported in Table B.2 and for age groups in Table B.3. Using the fact that $\lambda_i = \kappa_i \delta$ $(i = 0, 1)$ in Tables B.2-3 we report the results already in the form of arrival rates of job offers. Our attention will be mainly focused on the results for the least-skilled workers (Table B.2, group 1) and elder workers (Table B.3, group 4).

[8] We do not carry out the estimation separately for men and women. The reason is that when we estimate the models for different skill or age groups, the gender-specific samples become too small.

Table 2.3: Test Results for Search Intensities

	H_0 :	$\chi^2_{(1)}$	p-Value
Skills	$\lambda_0^{(86)} = \lambda_0^{(95)}$	19.5996	0.0000
(Group 1)	$\lambda_1^{(86)} = \lambda_1^{(95)}$	16.4417	0.0000
	$\delta^{(86)} = \delta^{(95)}$	16.9996	0.0000
Age	$\lambda_0^{(86)} = \lambda_0^{(95)}$	0.9781	0.3227
(Group 4)	$\lambda_1^{(86)} = \lambda_1^{(95)}$	0.1451	0.7033
	$\delta^{(86)} = \delta^{(95)}$	15.1216	0.0001

2.6.3 Estimation Results by Skills

Table B.2 of the Appendix presents the estimation results for different skill groups in both the year 1986 and 1994. The least skilled workers of group 1 are those who went through an inadequate training or only general elementary education. We find that the predicted equilibrium rate of unemployment in this group of workers goes up from 10.3% in 1986 to 15.1% in 1995. The results somewhat underpredict the true shift from 9.2% to 16.4% observed in the data (see Table 2.2). However, this underprediction is minor. These results also demonstrate a considerable increase of the unemployment rate of the low-skilled relative to those of all other skill groups, which matches the empirical regularity presented in Figures 2.1-2.

Remembering that the equilibrium unemployment rate is found to be $\delta/(\delta + \lambda_0)$, let us have a look at how λ_0 and δ were changing over the observation period. For the unemployed unskilled workers the arrival rate of job offers fell from 0.0373 to 0.0273, i.e., by roughly 27%. This change was significant, as the Wald test for the constancy of this parameter in the first row of Table 2.3 demonstrates. So we may conclude that by significantly slowing down search intensity of the unskilled unemployed individuals the entitlement extension reform has indeed contributed to the increase in their equilibrium unemployment rate.

Moreover, the search intensity of the unskilled has fallen much more than search intensity of the higher skill groups. For both skill group 2 (middle vocational training) and group 3 (vocational training and college entrance exam or higher vocational training) it has decreased by only about 12%. This finding goes in line with the argument that the benefit reform may potentially have an adverse negative effect on the unemployment rates of the unskilled.

A remarkable result displayed in Table B.2 is that for workers with the highest skills (group 4) λ_0 has gone up from 0.0659 in 1986 to 0.0864 in 1995 which amounts to more than 30%. Moreover, their reservation wage rose by about 1,200 D-Mark. In contrast, for skill groups 1 and 3 it nearly did not change and for group 2 it rose by only about 200 D-Mark. A potential explanation for these results can be a skill-biased technological change that raised the productivity and therefore the demand for qualified workers. This hypothesis is supported by the fact that over the observation period, the wage offer density for the high-skilled became less skewed and has considerably shifted to the right (see Figure B.3). Such a change of curvature is a reflection of the relative increase in firms' productivity which is in line with the acceleration of technological progress.

Consider now the separation rate δ. For the unskilled workers of group 1, it rose from 0.0043 in 1986 to 0.0049 in 1995; i.e., by about 14%. Remarkable enough is that for group 2 the upward shift of the separation rate was about the same, whereas for group 3 it became about 26% and for group 4 even 33%. So the unemployment rates of the least qualified workers were actually subject to a weaker pressure of match break incidence. Still, taking a look at the third row of Table 2.3 we see that the observed 14% increase of the exit rate to unemployment among the least qualified workers is statistically significant. This implies that the increase of δ has also significantly contributed to the upward shift of the unemployment rate. However, as long as in the model δ is exogenous to the worker and theoretically absorbs all other possible reasons for match dissolution, it is an open question what share in the observed eventual 14% increase is due to the entitlement extension.

Summarizing the findings above we conclude that the entitlement extension reforms of the late 1980s have led to a significant slowdown in the search intensity among the unemployed low-skilled workers. Moreover it is likely that they have also increased shirking incentives among the employed low-skilled workers, which has lead to higher separation rates. Taken together these two effects have led to the leap of unemployment rates of least qualified workers. Furthermore, as it could be generally expected (see for instance Nickell and Layard, 1999), this reform has brought about a significantly longer duration of unemployment for the unskilled.[9]

We also observe the similar influence for the second and third qualification groups. However, the magnitude is much lower. For the highest skill group, skill biased technological change may have counteracted this effect over the period under review since as a result no changes in unemployment rates were predicted.

Now let us proceed with the parameter estimates for λ_1, the arrival rate of job offers while employed. They are also displayed in Table B.2. From 1986 to 1995 for skill groups 1, 2 and 4, the estimates reveal a decrease of this arrival rate of about 20%, 12% and 34%, respectively. For group 3 instead, it rose by roughly 20%. These results are somewhat puzzling. With skill-biased technological change, one would have expected, that the higher is the skill level, the bigger should be the percentage change in the arrival rate of job offers. Instead, however, we observe that after having increased for the third group the arrival rate of job offers has immensely decreased for group 4, workers with the highest skills. As a possible explanation to this phenomenon one may suggest that firms post too high wages for workers of the skill group 4 because the highly skilled personnel becomes increasingly important. However, we regard this interpretation as rather speculative.

Finally, considering the changes in λ_1 for the first two qualification groups we may think that the skill-biased technological progress obscures the promotion prospects of the low-skilled. We observe that the less qualified the worker is, the fewer chances of finding a better paid job he/she has.

Concluding the discussion of this subsection it would be natural to go over the policy measures that our results imply. As we have discovered, the extension of entitlement to UI has significantly affected the search intensity of unemployed low-skilled workers and through this contributed to the increase of equilibrium unemployment rate in this group. Moreover it has also raised the expected duration of being unemployed. Therefore if one pursues the goal of reducing the unemployment rate and tries to enhance incentives to return to work faster, changing the entitlement length could be a valuable instrument.

[9]This conclusion follows automatically, since the expected unemployment duration within the theoretical model is just a reciprocal search intensity parameter of unemployed workers.

2.6.4 Estimation Results by Age

Let us consider the results for different age groups (see Table B.3 of the Appendix). Here we would expect that the reforms should induce a significant increase in the unemployment rate of the eldest workers. Furthermore, the absolute value of this increase is also expected to be the biggest among all the other groups. Our estimation results indeed reflect such kind of dynamics. Table B.3 shows that from 1986 to 1995 the expected unemployment rate of workers over 54 went up by 28% (from 11.4% to 14.6%). However we significantly underpredict the magnitude of the actual change because the sample fraction of unemployed individuals in this age group has risen from 9.3% to 16.3%, i.e. by more than 70% (see Table 2.2). Percentage changes of the predicted unemployment rates for the other age groups 1 (16-27 years), 2 (28-40 years), and 3 (41-53 years) are about 15%, 44% and 22% respectively. But again, for the second age group the percentage increase turns out to be higher than the corresponding increase in the eldest group. And this is in odds with the entitlement extension scheme, since for the workers younger than 42 years maximum duration of UI receipt has remained intact. So, unlike in the case with skill groups, for the eldest workers the model fails to provide a good fit to the data.

Let us analyze the results for λ_0 and δ in more detail. From Table B.3 one can see that for the oldest group search intensity in unemployment has reduced by roughly 10%. *Ceteris paribus*, this change would have lifted the equilibrium unemployment rate of the elder workers up to 12.6%. Yet according to the Wald test in Table 3 for these workers we cannot reject the hypothesis that the parameter λ_0 is the same in 1986 and 1995. This means that our model does not support the argument that the entitlement extension affects the unemployment rates of the elder workers through inhibiting their search intensity.

As to the other age groups, we could have expected that the arrival rate of job offers would have fallen more for the elder workers than for the younger ones. If compared with the group of workers that are 16 to 27 years old, the results are indeed in line with our expectation: the arrival rate of job offers of the youngest fell by only about 2% (see Table B.3). Though in the other two groups the group-specific percentage change is quite similar to that of the eldest workers.

Consider now the arrival rate of employer-employee match break. From 1986 to 1995 the incidence of job loss among the eldest individuals rose by about 18%, whereas for the three younger groups, it rose by 14%, 30% and 13% respectively. So again we cannot even state that the exit rate positively depends on age, which could have been otherwise expected.

To see whether the observed 18% increase in δ has significantly contributed to the increase of unemployment rates among the oldest workers we again test the hypothesis of the constancy of δ over time. The results in Table 2.3 indicate rejection. This establishes the fact that changes in unemployment profiles of elder workers are mainly explained by the increased match break incidence. Partly this result is in line with our expectations. Due to the Employment Promotion Act of 1986 unemployed people of at least 58 years old were granted a possibility to be no longer available for mediation into jobs, provided that they would retire early (at the age of 60). This could have made a job loss for elder workers more acceptable and therefore could have increased their incentives to shirk. Consequently the likelihood that firms terminate the employment of elder workers would go up, since such termination could be done amicably given the generous (essentially, infinite) benefit entitlements that the elder workers have become able to get. As a result, match dissolution incidence should go up. And that is exactly what we find.

Table B.3 also displays the predicted reservation wages of the four age groups for the years 1986 and 1995. Reservation wages of those 54 to 64 years old have hardly changed over this period indicating no impact of the UI reforms.

To summarize, we find that the chosen theoretical model is not rich enough to shed light on the precise mechanisms that shifted up the unemployment rate of the oldest workers. For this group the predicted change of the equilibrium unemployment rate is considerably lower than the actual change in the sample. Hence we would also expect that the changes in the parameter estimates are biased. This may be the reason why our results for elder workers do not generally reflect our expectations about changes of their search intensities, job loss rate and reservation wages. We discover that the dynamics of unemployment rates of the eldest workers is mostly determined by the changes in their group-specific separation rate. But still we see that the institutional influence in this group is more complex, because now it consists of not only extended entitlement to UI benefits but also of the possibility of earlier retirement. As long as under the assumptions of the model the rate of job loss is exogenous we are not able to say definitely which of the two stands behind it. Though in view of statistical insignificance of the changes in search behavior of the oldest group, we would tend to think that the suggested early retirement argument may be an explanation.

2.7 Summary and Conclusions

In this chapter we ask whether the reforms that extended the entitlement length to UI benefit payments in West Germany had a significant contribution to the increase in unemployment rates among unskilled and elder workers. We try to answer this question by estimating the parameters of the theoretical search equilibrium model of Burdett and Mortensen (1998) with heterogenous employers. Our choice of the theoretical framework is determined by the fact that through the individual search behavior the model makes it possible to link the increased UI entitlement length with the subsequent dynamics of unemployment rates.

To estimate the model we firstly use the structural nonparametric approach suggested by Bontemps et al. (2000). However, we discover that this procedure cannot be always applicable and find a data-driven condition, which demonstrates the limitations of this estimation techniques. As long as the applicability condition which we refer to as "constraint inconsistency" is not satisfied in our case, we proceed with the structural estimation method suggested by Bowlus et al. (1995), (2001).

We find that for unskilled workers the extension of the entitlement period has significantly influenced search behavior. Both arrival rates of job offers to unemployed and employed workers went down. Moreover the arrival rate of employee-employer match went up considerably. A slowdown in unemployment search intensity along with increased incentives to shirk induced by the UI system after the reforms has led to the increase of predicted unemployment rates in this group. The unemployment rate for the unskilled predicted by the model shifts from 10.3% to 15.1% which almost completely matches the 9.2% to 16.4% increase of the same rate observed in the data.

As to the elder workers, a pure search intensity adjustment argument turns out to be insufficient to present a satisfactory explanation of unemployment rate dynamics. However, the model mirrors the phenomenon of increased unemployment rates between 1986 and 1995 predicting a higher exit from jobs into unemployment. We know that for this group of workers the entitlement to unemployment benefit payments became particularly long in the second half of the 1980s. Additionally, whenever out of job, under certain conditions elder workers were granted a possibility to retire earlier. Taken together this may have made a job loss more acceptable and gave employers an incentive to dismiss old workers rather than the others. So it looks like the benefit and retirement reforms have affected the exit rates into unemployment rather than search behavior of the elder. Still our model is not rich enough to separate these two institutional effects.

In this context, it is interesting to note that recent labour market reforms instituted in Germany are likely to reverse some of the phenomena discussed in this chapter. In particular the duration of entitlement to UI has been shortened twice (in 1997 and in 2003) and the levels of UA benefits are being adjusted downwards to the level of social assistance (starting from 2005). Furthermore reforms in the job referral system of employment agencies aimed at lowering the costs of job search were undertaken in 2003. Given the analysis of this chapter, such reforms should eventually induce a significant reduction in unemployment rates.

Chapter 3

Extension to Non-Linear Technology with Increasing Returns

3.1 Introduction

It is generally agreed that the shape of the wage earnings distribution is determined by the skill distribution of the work force, the production technology employed by the economy and the search and matching frictions that govern the allocation of workers to jobs. The aim of this chapter is to provide a theoretical and still empirically tractable model that takes all three factors and its interactions into account. For doing so we extend the search equilibrium model of Burdett and Mortensen (1998) and derive an explicit functional form for the wage offer and earnings distributions. Our extension explicitly introduces different skill groups that are linked via a production function, which permits increasing returns to scale. Introduction of skill differences allows for the analysis of firms' wage posting behavior, where firms simultaneously compete for heterogeneous workers. The theoretical model presented in this chapter is developed by Christian Holzner and adopted from Holzner and Launov (2005).

Since the endogenous wage distribution generated by the original Burdett-Mortensen model has an upward-sloping density, which is at odds with the empirical observation of a flat right tail, there has been a lot of effort to extend the original model in order to generate a more realistic shape of the wage distribution. Mortensen (1990) introduced differences in firm productivity and Bowlus et al. (1995) showed that this greatly improves the fit to the empirical wage distribution. Bontemps et al. (2000) and Burdett and Mortensen (1998) formulate a closed-form solution for a continuous productivity distribution, which translates into a right-tailed wage earnings distribution, depending on the assumed productivity dispersion. Postel-Vinay and Robin (2002) extend this for both employer and worker heterogeneity.

In this chapter our extension demonstrates that with skill multiplicity and a production function that permits increasing returns to scale we get a unimodal right-skewed wage offer and earnings densities with a decreasing right tail. Even though we also introduce productivity dispersion the result about the shape of offer and earnings densities is true even with identical employers. While the structural estimates of models with continuous productivity dispersion as suggested by Bontemps et al. (2000) and Postel-Vinay and Robin (2002) improve the fit to the empirical wage earnings distribution and the estimates of the labor market transition rates, they tell us nothing about the production parameters governing the productivity dispersion (see Manning, 2003, p.106f). In this chapter different production technologies are explicitly introduced. As a result this allows us estimating the parameters of the production functions even without using firms' data.

We use the estimated parameters of our model to analyze whether there is over- or underinvestment in human capital from a social welfare point of view, i.e. whether the increase in output coming from educating the marginal individual is larger than the marginal private costs of the shift of the skill structure. Underinvestment in (undirected) search or matching models are analyzed by Acemoglu (1996) and Masters (1998). Following Grout (1984) they provide models where underinvestment results from the fact that search or matching frictions make it impossible for workers to capture the whole return on their investment. The same mechanism is at work in the theoretical model of the present chapter (however, underinvestment cannot be attributed to rent sharing exclusively). Furthermore, allowing for segmented labor markets, where unskilled workers do not search for the same jobs as skilled workers do (and vise versa), makes both over- or underinvestment into education possible. The simple idea is that a lower unemployment rate among high skilled workers can increase the return to human capital investment as shown by Saint-Paul (1996).[1]

The estimation methodology applied in this chapter is based on the one considered in Bowlus et al. (1995), (2001). However, skill-multiplicity and Cobb-Douglas produc-

[1]Acemoglu and Shimer (1999) show that the hold-up problem can be overcome if workers are able to direct their search to potentially different markets.

tion function used in the econometric model impose additional restrictions that must be taken into account when applying the original method. First, these are the restrictions that allow representing the subset of production parameters as a function of search frictions parameters and the homogeneity degree of the Cobb-Douglas technology. Second, these are the identification restrictions that appear with an introduction of employer heterogeneity. Our estimation problem can be also related to that of Bowlus and Eckstein (2002). Within the simple Burdett-Mortensen model Bowlus and Eckstein (2002) analyze discrimination and skill differences by allowing for different productivity and different transition parameters across races as well as incorporating discrimination of employers. However, unlike in Bowlus and Eckstein (2002), we estimate the parameters of interest by maximum likelihood.

Our empirical investigation tries to answer whether over- or underinvestment into skills is present in the German economy. The main result of the empirical application of the model is that there is a strong underinvestment into education at the low-to-medium skill level.

The chapter proceeds as follows. The theory is summarized in Section 2. Here we discuss the extension the original Burdett-Mortensen framework. The empirical implementation of the model is treated in Section 3. We formulate the appropriate likelihood function and discuss the relevant estimation method and identification issues. Thereafter we provide a brief description of the data set and in detail discuss the results of the structural estimation of the model. Section 4 concludes.

3.2 Theory

In this section we outline the extension of the original BM model by introducing different skill groups and technologies that link the skill groups in the production process.

3.2.1 Framework

The model has an infinite horizon, is set in continuous time and concentrates on steady states. Workers are assumed to be risk neutral and to discount at rate r. Each worker belongs to a skill group $i = 1, 2, ..., I$ whose measures are defined as q_i, satisfying $\sum q_i = m$. The measure u_i of workers is unemployed and the measure $q_i - u_i$ is employed. Before choosing a skill-group workers incur a cost c_i for skill-specific education. By assuming perfect capital market workers are able to borrow the cost of education. Once incurred, the education cost is sunk.

Workers search for a job in the skill-segmented labor markets. With probability λ_i unemployed workers of skill group i encounter a firm that makes them a wage offer corresponding to their education, and with probability λ_e employed workers encounter a firm (common for all skills). Then workers decide whether to accept or reject the job offer. Job-worker match is destroyed at an exogenous rate $\delta > 0$.

We assume that there exist J distinct production technologies $Y_j(\mathbf{l}(\mathbf{w} \mid \mathbf{R}, F(\mathbf{w})))$ indexed by j, where $\mathbf{l}(\mathbf{w} \mid \mathbf{R}, F(\mathbf{w}))$ is the vector of skill groups $l_i(w \mid R_i, F_i(w))$ employed by a firm with technology j. The size $l_i(w \mid R_i, F_i(w))$ of the skill group depends on the firm's wage offer w_i, the workers' reservation wage R_i and the skill specific wage offer distribution $F_i(w)$. We further assume that skill inputs in the production function are complementary.

3.2.2 Workers' Search Strategy

The optimal search strategy for a worker of occupation i is characterized by a reservation wage R_i, where an unemployed worker is indifferent between accepting or rejecting a wage offer, i.e. $U_i = V_i(R_i)$, where U_i is the value of being unemployed and $V_i(R_i)$ the value of being employed at the reservation wage R_i. Flow values of being unemployed and employed

$$rU_i = \lambda_i \int_{R_i}^{\bar{w}_i} (V_i(x_i) - U_i)\, dF_i(x_i) - c_i, \tag{3.1a}$$

$$rV_i(w_i) = w_i + \lambda_e \int_{w_i}^{\bar{w}_i} (V_i(x_i) - V_i(w_i))\, dF_i(x_i) + \delta\,(U_i - V_i(w_i)) - c_i \tag{3.1b}$$

respectively, can be solved for a reservation wage[2]

$$R_i = (\lambda_i - \lambda_e) \int_{R_i}^{\bar{w}_i} \left(\frac{1 - F_i(x)}{r + \delta + \lambda_e(1 - F_i(x^-))} \right) dx. \tag{3.2}$$

It can be readily seen that for the workers' optimal behavior there is no conceptual difference between single skill model and skill multiplicity. The only new feature here

[2]The solution completely carries over from Mortensen and Neumann (1988).

is just skill-segmented wage offer. The wage offer distribution is given by $F_i(w) = F_i(w^-) + \upsilon_i(w)$, where $\upsilon_i(w)$ is the mass of firms offering wage w to skill group i.

3.2.3 Employers' Wage Posting

Equating the flows in and out of unemployment gives the steady state measure of unemployed per skill group, i.e.

$$u_i = \frac{\delta}{\delta + \lambda_i} q_i. \tag{3.3}$$

Given the assumptions of constant Poisson arrival rates λ_i, λ_e and the constant separation rate δ Mortensen (1999) has shown that skill group size evolves according to a special Markov-chain known as stochastic birth-death process. Expected value of the skill-specific labor force size is given by

$$E\left[l_i\left(w \mid R_i, F_i\left(w\right)\right)\right] = \frac{\delta\lambda_i\left(\delta + \lambda_e\right) / \left(\delta + \lambda_i\right)}{\left[\delta + \lambda_e \overline{F}_i(w)\right]\left[\delta + \lambda_e \overline{F}_i(w^-)\right]} q_i, \tag{3.4}$$

where it can be seen that the expected skill group size is an increasing function of the offered wage w. Furthermore steady-state measure of employed workers earning a wage less than w becomes

$$G_i(w^-)(q_i - u_i) = \frac{\lambda_i F_i(w^-) u_i}{\delta + \lambda_e \overline{F}_i(w^-)}. \tag{3.5}$$

which is analog of (1.3a) in Chapter 1.

Each firm posts a wage schedule $\mathbf{w}$ in order to maximize its profit, taking as given the workers' search strategy, i.e. the reservation wage vector $\mathbf{R}$, and the other firms' wage posting behavior, i.e. $F\left(\mathbf{w}\right)$. For tractability it is assumed that a firm can specify its wage policy $\mathbf{w}$ only once (see Holzner and Launov, 2005, for discussion). Thus the profit maximization problem of a type j firm as

$$\pi_j = \max_{\mathbf{w}} \left[Y_j\left(E\left[\mathbf{l}\left(\mathbf{w}\right)\right]\right) - \mathbf{w}^T E\left[\mathbf{l}\left(\mathbf{w}\right)\right]\right]. \tag{3.6}$$

Denote by $\mathbf{W}_j$ the set of wage offers that maximize equation (3.6), i.e. $\mathbf{W}_j = \arg\max_{\mathbf{w}} \pi_j$, and the corresponding I-dimensional wage offer distribution for each firm type j by $F_j\left(\mathbf{w}\right) = (F_{1j}(w), F_{2j}(w), ..., F_{Ij}(w))$, where $F_{ij}(w)$ denotes the wage offer distribution of type j firms for skill group i. Then the following definition of equilibrium can be stated.

Definition 3.1: A *steady state wage posting equilibrium* is a wage offer distribution $F_j(\mathbf{w})$ with $\mathbf{w} \in \mathbf{W}_j$ for each firm type $j \in J$ such that

$$\begin{aligned} \pi_j &= Y_j(E[\mathbf{l}(\mathbf{w})]) - \mathbf{w}^T E[\mathbf{l}(\mathbf{w})] \text{ for all } \mathbf{w} \text{ on the support of } F_j(\mathbf{w}), \\ \pi_j &\geq Y_j(E[\mathbf{l}(\mathbf{w})]) - \mathbf{w}^T E[\mathbf{l}(\mathbf{w})] \text{ otherwise,} \end{aligned} \tag{3.7}$$

given the reservation wage R_i for each skill group $i = 1, 2, ..., I$ and a corresponding skill group wage offer distribution $F_i(w)$ such that the reservation wage R_i satisfies equation (3.2) given $F_i(w)$.

Equilibrium concept in the extended model is the same as in the basic Burdett-Mortensen model. Given the solution for the optimal workers' behavior derivation of the equilibrium wage offer distribution is performed in three steps. First, it is shown that complementarity of skill inputs in the production technology implies positive wage correlation within the firm. Whenever skill-specific offer distributions are continuous positive wage correlation implies that for all $\mathbf{w} \in \mathbf{W}_j$

$$F_{ij}^k(w) = F_{lj}^k(w) \text{ for all } i, l = 1, 2, ..., I. \tag{3.8}$$

Second, it is demonstrated that also in the multiple-skill framework more productive employers offer higher wages, which leads to the fact that

$$F_i(\overline{w}_{ij}) = \gamma_j \tag{3.9}$$

Sufficient condition for continuity of $F_{ij}(w)$ is given by an inequality in which marginal increase in the offered wage generates the increase in output that is strictly greater than the corresponding marginal increase in labour costs.

Assuming that the sufficient condition for continuity holds let the j-th type technology $Y_j(\mathbf{l}(\mathbf{w}_j))$ have a degree of homogeneity ξ_j and be twice differentiable with respect to production factors, which leads to the Taylor Expansion

$$Y_j(\mathbf{l}(\mathbf{w}_j)) = Y_j(\mathbf{r}_j) + \sum\nolimits_i Y_j'(\mathbf{r}_j)[r_{ij} h_j(w) - r_{ij}] + \frac{1}{2} \sum\nolimits_i \sigma_{ij} [h_j(w) - 1]^2,$$

where

$$
\begin{aligned}
h_j(w) &= \frac{\left[\delta+\lambda_e\left(1-\gamma_{j-1}\right)\right]^2}{\left[\delta+\lambda_e\overline{F}_j(w)\right]^2}, \qquad r_{ij} = \frac{\delta\left(\delta+\lambda_e\right)\lambda_i/\left(\delta+\lambda_i\right)}{\left[\delta+\lambda_e\left(1-\gamma_{j-1}\right)\right]^2}q_i, \\
Y_j'\left(\mathbf{r}_j\right) &= \frac{\partial Y_j\left(\mathbf{r}_j\right)}{\partial l_i} \quad \text{and} \quad \sigma_{ij} = \sum\nolimits_l \frac{\partial^2 Y_j\left(\mathbf{r}_j\right)}{\partial l_i \partial l_l} r_{lj} r_{ij} = \left(\xi_j - 1\right) Y_j'\left(\mathbf{r}_j\right) r_{ij}.
\end{aligned}
$$

Using the above listed results, invoking the equal profit condition $\pi_j = \pi_j^{\mathbf{R}}$ and applying the first order condition on (3.6) the following solution for $F_i(w)$ as a function of w obtains.

Proposition 3.1 *Given that production functions $Y_j\left(E\left[\mathbf{l}\left(\mathbf{w}\right)\right]\right) \forall j = 1,2,...,J$ are homogeneous of degree $\xi_j \geq 1$, and if no mass point exists a unique equilibrium wage offer distribution $F_{ij}(w)$ for each skill group $i = 1,2,...,I$ exists that has the form*
(i) for $\xi_j = 1$

$$
F_{ij}(w) = \frac{\delta+\lambda_e}{\lambda_e} - \frac{\delta+\lambda_e(1-\gamma_{j-1})}{\lambda_e}\sqrt{\frac{Y_j'\left(\mathbf{r}_j\right)-w}{Y_j'\left(\mathbf{r}_j\right)-\underline{w}_{ij}}}, \tag{3.10}
$$

(ii) for $\xi_j > 1$

$$
\begin{aligned}
F_{ij}(w) = {} & \frac{\delta+\lambda_e}{\lambda_e} \\
& - \frac{\delta+\lambda_e\left(1-\gamma_{j-1}\right)}{\lambda_e\sqrt{\frac{\left(Y_j'(\mathbf{r}_j)-w_i\right)r_{ij}-\sigma_{ij}-\sqrt{\left(\left(Y_j'(\mathbf{r}_j)-w_i\right)r_{ij}-\sigma_{ij}\right)^2+4\left(\sigma_{ij}-\mu_{ij}\right)\left(\left(Y_j'(\mathbf{r}_j)-\underline{w}_{ij}\right)r_{ij}-\mu_{ij}\right)}}{-2\left(\sigma_{ij}-\mu_{ij}\right)}}}
\end{aligned} \tag{3.11}
$$

for any $w \in [\underline{w}_{ij}, \overline{w}_{ij}]$, where

$$
\mu_{ij} = \frac{r_{ij}}{\sum_i r_{ij}}\frac{1}{2}\sum\nolimits_i \sigma_{ij},
$$

Proof. See Holzner and Launov (2005). ■

The aggregate wage offer distribution is given by

$$
F(w) = \sum_{i=1}^{I}\frac{q_i}{m}F_i(w) = \sum_{i=1}^{I}\frac{q_i}{m}\sum_{j=1}^{J}\left(\gamma_j-\gamma_{j-1}\right)F_{ij}(w).
$$

There also exists special case for $F_{ij}(w)$ when $\left(Y_j'(\mathbf{r}_j) - \underline{w}_{ij}\right) r_{ij} = \mu_{ij}$. This case, however, implies artificial restrictions on ξ_j considering this case here is neither interesting nor useful.

3.2.4 Properties of the Equilibrium Offer and Earnings Distributions

For a production function with homogeneity of degree one the explicit wage offer distribution resembles the distribution derived in Burdett and Mortensen (1998) and has its typical increasing density. However, the functional form of (3.11) also permits the densities with falling tail. From proof of Proposition 3.1 (see Holzner and Launov, 2005) it immediately follows that the necessary condition for an upward sloping wage offer density $\partial F_i(w)/\partial w$ is

$$\left(2 - \xi_j\right) \frac{\partial Y_j(\mathbf{r}_j)}{\partial r_{ij}} - w > 0. \tag{3.12}$$

This result implies $\xi_j^* > 2$ as a sufficient degree of homogeneity that guaranties a density function with the decreasing tail. Furthermore from (3.12) follows that as the wage w increases the inequality becomes more likely to be violated implying that the wage offer density can have an upward sloping part for small wages and an downward sloping part for large wages.

Consider now the equilibrium earnings distribution $G_i(w)$. Decreasing tail of $f_{ij}(w)$ implies the same for the earnings density $g_{ij}(w)$. It is well known that the actual earnings data have a heavy right tail that decreases slower then that of the densities from the exponential family (see, for instance, Singh and Maddala, 1976, or Reed, 2001). To see if the model with increasing returns can predict the appropriate densities we seek for the limiting behaviour of the right tail of $g_{ij}(w)$.

The tail of the density function defined on $[\underline{w}_{i1}, \overline{w}_{iJ}]$ converges at the highest possible rate. However letting $\{\underline{w}_{iJ}, \overline{w}_{iJ}\}$ go to infinity we get the following result.

Proposition 3.2 *Let $\underline{w}_{iJ} \to \infty$ and $\overline{w}_{iJ} \to \infty$. Under the sufficient condition for a decreasing right tail of $f_{iJ}(w_i)$ the right tail of the equilibrium earnings density $g_{iJ}(w_i)$ converges at the rate faster then w^{-2}. Speed of convergence is a power law that positively depends on the degree of homogeneity of the production function.*

Proof. Using (3.5) and (3.11) one obtains the closed form solution for the equilibrium earnings density

$$
\begin{aligned}
g_{iJ}(w_i) &= \frac{(\delta+\lambda_e)r_{iJ}}{2\lambda_e\left(\delta+\lambda_e\left(1-\gamma_{J-1}\right)\right)} \\
&\times \frac{\sqrt{-\frac{\left(Y_J'(\mathbf{r}_J)-w_i\right)r_{iJ}-\sigma_{iJ}}{2(\sigma_{iJ}-\mu_{iJ})}+\frac{\sqrt{\left(\left(Y_J'(\mathbf{r}_J)-w_i\right)r_{iJ}-\sigma_{iJ}\right)^2+4(\sigma_{iJ}-\mu_{iJ})\left(\left(Y_J'(\mathbf{r}_J)-\underline{w}_{iJ}\right)r_{iJ}-\mu_{iJ}\right)}}{2(\sigma_{iJ}-\mu_{iJ})}}}{\sqrt{\left(\left(Y_J'\left(\mathbf{r}_J\right)-w_i\right)r_{iJ}-\sigma_{iJ}\right)^2+4\left(\sigma_{iJ}-\mu_{iJ}\right)\left(\left(Y_J'\left(\mathbf{r}_J\right)-\underline{w}_{iJ}\right)r_{iJ}-\mu_{iJ}\right)}}.
\end{aligned}
$$

Define

$$
\begin{aligned}
A(w_i) &\equiv \frac{\left(Y_J'(\mathbf{r}_J)-w_i\right)r_{iJ}-\sigma_{iJ}-\sqrt{\left(\left(Y_J'(\mathbf{r}_J)-w_i\right)r_{iJ}-\sigma_{iJ}\right)^2+4(\sigma_{iJ}-\mu_{iJ})\left(\left(Y_J'(\mathbf{r}_J)-\underline{w}_{iJ}\right)r_{iJ}-\mu_{iJ}\right)}}{-2(\sigma_{iJ}-\mu_{iJ})} \text{ and} \\
B(w_i) &\equiv \left(\left(Y_J'\left(\mathbf{r}_J\right)-w_i\right)r_{iJ}-\sigma_{iJ}\right)^2+4\left(\sigma_{iJ}-\mu_{iJ}\right)\left(\left(Y_J'\left(\mathbf{r}_J\right)-\underline{w}_{iJ}\right)r_{iJ}-\mu_{iJ}\right).
\end{aligned}
$$

Then the first derivative of $g_{iJ}(w_i)$ can be written down as

$$
g_{iJ}'(w_i) = -\frac{(\delta+\lambda_e)r_{iJ}^2}{2\lambda_e\left(\delta+\lambda_e\left(1-\gamma_{J-1}\right)\right)}A^{\frac{1}{2}}(w_i)\left[\frac{A(w_i)}{B^{\frac{3}{2}}(w_i)}-\frac{3}{2}\frac{1}{B(w_i)}\right].
$$

For $\underline{w}_{iJ} \to \infty$ and $\overline{w}_{iJ} \to \infty$ $A(w_i) = O(1)$ and $B(w_i) = O\left(w_i^{2(\xi_J-1)}\right)$, which leads to

$$
g_{iJ}'(w_i) = O\left(w_i^{-2(\xi_J-1)}\right).
$$

Finally, under the sufficient condition for the decreasing right tail of the $f_{iJ}(w_i)$ we get $g_{iJ}'(w_i) = O\left(w_i^{-2-\delta}\right)$, where $\delta > 0$. ■

The result of Proposition 3.2 tells us that the equilibrium earnings density encompasses the family of Pareto and Singh-Maddala densities, right tail of which is acknowledged to have the best fit to the observed high-earnings data (see Singh and Maddala, 1976). Similarly to the equilibrium densities of Bontemps et al. (2000), tail behaviour of $g_{iJ}(w_i)$ excludes the distributions with the exponential speed of convergence (e.g. lognormal) form the set of possible functional form candidates for the equilibrium earnings distribution. Furthermore, increasing returns of the production function extend the result of Proposition 8 in Bontemps et al. (2000) allowing earnings density to converge both slower and faster then w^{-3}.

Finally, we have to note that the assumption of the continuity of $F_{ij}(w)$ made to facilitate the derivation of (3.11) must be respected when the model is implemented empirically (this issue is discussed in detail in Section 3.3.4).

3.3 Econometric Model

Here we consider in detail the structural econometric model based on the theory presented above. We assume a Cobb-Douglas production technology which allows for constant and increasing returns to scale, i.e.

$$Y_j(l(\mathbf{w}_j)) = p_j \prod_{i=1}^{I} l_i(w_j)^{\alpha_{ij}} \tag{3.13}$$

with $\sum_i \alpha_{ij} = \xi_j \geq 1$, $\alpha_{ij} > 0$.

In general, we build upon the model developed by Bowlus et al. (1995), (2001). In the discussion to follow we put special emphasis on such new features as parameter identification and related modification of the estimation procedure.

3.3.1 The Likelihood Function

Let us start from the formulation of the likelihood function. The formulation goes along the lines plotted in Section 1.4.1, taking only an account of skill multiplicity.

For Poisson process with rate θ the joint distribution of the elapsed (t_e) and residual (t_r) duration of time spent by an individual in a certain state of the labour market is $f(t_e, t_r) = \theta^2 e^{-\theta(t_e+t_r)}$. For an individual that belongs to i-th skill group the appropriate Poisson rates are λ_i if the person is unemployed and $\delta + \lambda_e [1 - F_i(w)]$ if the person is employed at wage w. Furthermore:

- *For Unemployed:* Equilibrium probability of sampling an unemployed agent who belongs to i-th skill group is $m^{-1}q_i\delta/(\delta + \lambda_i)$. In case the subsequent job transition is observed we know the offered wage and can record the value of the wage offer density $f_i(w)$.
- *For Employed:* Equilibrium probability of sampling an agent who belongs to i-th skill group and earns wage w is $m^{-1}q_i g_i(w)\lambda_i/(\delta + \lambda_i)$. In case the transition to the next state is observed we record the destination state. The probabilities of

exit to unemployment and to next job are $\rho_{j \to u} = \delta / \left(\delta + \lambda_e \overline{F}_i\left(w\right)\right)$ and $\rho_{j \to j} = \lambda_e \overline{F}_i\left(w\right) / \left(\delta + \lambda_e \overline{F}_i\left(w\right)\right)$ respectively.

For convenience of estimation, define $\kappa_i = \lambda_i / \delta$, $\kappa_e = \lambda_e / \delta$. Then the likelihood contributions of unemployed ($\mathcal{L}_{(i)\,u}$) and employed ($\mathcal{L}_{(i)\,e}$) individuals affiliated with i-th skill group is:

$$\mathcal{L}_{(i)\,u} = \frac{q_i}{m\left(1+\kappa_i\right)} \left[\delta \kappa_i\right]^{2-d_r-d_l} e^{-\delta \kappa_i \left[t_e+t_r\right]} \left[f_i(w)\right]^{1-d_r}, \tag{3.14}$$

$$\mathcal{L}_{(i)\,e} = g_i(w) \frac{q_i}{m} \frac{\kappa_i}{1+\kappa_i} \left[\delta\left(1+\kappa_e \overline{F}_i\left(w\right)\right)\right]^{1-d_l} e^{-\delta\left(1+\kappa_e \overline{F}_i(w)\right)\left[t_e+t_r\right]} \times \left[\left[\delta \kappa_e \overline{F}_i\left(w\right)\right]^{d_t} \delta^{1-d_t}\right]^{1-d_r}. \tag{3.15}$$

In (3.14) and (3.15) $d_l = 1$, if a spell is left-censored, 0 otherwise; $d_r = 1$, if a spell is right-censored, 0 otherwise; $d_t = 1$ if there is a job-to-job transition, 0 otherwise. Substitution of the appropriate $g_i\left(w\right)$, $f_i(w)$ and $F_i(w)$ into (3.14) and (3.15) completes the formulation of the likelihood function ($g_i(w)$ is obtained from $F_i(w)$ using (3.5)).

Notice that except of probability terms $m^{-1} q_i / (1+\kappa_i)$ and $m^{-1} q_i \kappa_i / (1+\kappa_i)$ (3.14) and (3.15) are the same as in the previous two Chapters. The main differences are rather driven by the functional forms of the offer and earnings distributions.

3.3.2 Homogeneous Firms

It is instructive to start with the model with no productivity dispersion, since the theory allows obtaining an earnings density with a decreasing right tail even with homogeneous employers. This density will have $I-1$ jumps at infimum wages and $I-1$ spike at supremum wages of each skill group.

Under employer homogeneity the assumed production function modifies to $Y(l(\mathbf{w})) = p \prod_{l=1}^{I} l_l(w)^{\alpha_l}$. Functional form of the wage offer distribution with homogeneous employers is $F(w) = \sum_{i=1}^{I} \frac{q_i}{m} F_i(w)$, where $F_i(w)$ is given in Proposition 3.1 with $J = 1$. Rewritten in terms of $\kappa_{i,e}$ the skill-specific offer distribution becomes

$$F_i(w_i) = \frac{1+\kappa_e}{\kappa_e} - \frac{1+\kappa_e}{\kappa_e \sqrt{\frac{\left(Y_i'(\mathbf{r})-w\right) r_i - \sigma_i - \sqrt{\left(\left(Y_i'(\mathbf{r})-w\right) r_i - \sigma_i\right)^2 + 4(\sigma_i - \mu_i)\left(\left(Y_i'(\mathbf{r}) - \underline{w}_i\right) r_i - \mu_i\right)}}{-2(\sigma_i - \mu_i)}}}, \tag{3.16}$$

where

$$r_i = \frac{\kappa_i}{(1+\kappa_e)(1+\kappa_i)} q_i, \qquad Y_i'(\mathbf{r}) = \frac{\alpha_i}{r_i} p \prod_{i=1}^{I} r_i^{\alpha_i},$$
$$\sigma_i = \alpha_i (\xi - 1) Y(\mathbf{r}), \quad \text{and} \quad \mu_i = \frac{r_i}{\sum_i r_i} \frac{1}{2} \sum_i \sigma_i.$$

Recognizing that $F_i(\overline{w}_i) = 1$ we use $Y(l(\mathbf{w}))$ to get the following solution for the common productivity parameter

$$p = \frac{r_i}{\prod_{i=1}^{I} r_i^{\alpha_i}} \left[\alpha_i - \frac{\xi - 1}{\eta} \left(\frac{\xi (1+\eta) r_i}{2 \sum_i r_i} - \alpha_i \right) \right]^{-1} \left(\frac{\overline{w}_i - \eta \underline{w}_i}{1-\eta} \right). \tag{3.17}$$

where $\eta = (1+\kappa_e)^{-2}$.

Consider the unknowns of the econometric model. The skill measures $\{q_i\}_{i=1}^{I}$ are known from the data and they are nothing else but sample sizes of each skill group. Furthermore, to avoid bounds of the likelihood function depending on the parameters, Kiefer and Neumann (1993) suggest extreme order statistics $\{\min(w_i), \max(w_i)\}$ as the consistent estimates for $\underline{w}_i$ and $\overline{w}_i$ respectively. Finally, from the fact that (3.17) holds for any i one can represent any α_i as a function of ξ and the rest of structural parameters. Namely (3.17) implies that for any $i, l = 1, .., I$ there holds

$$\alpha_i \frac{(\overline{w}_l - \eta \underline{w}_l) r_l}{(\overline{w}_i - \eta \underline{w}_i) r_i} - \alpha_l = \frac{\xi (\xi - 1)(1+\eta) r_l}{2 (\xi + \eta - 1) \sum_{k=1}^{I} r_k} \left[\frac{\overline{w}_l - \eta \underline{w}_l}{\overline{w}_i - \eta \underline{w}_i} - 1 \right],$$

Without loss of generality setting $i = 1$, $l = 2, ..., I$ and recognizing that $\alpha_1 = \xi - \sum_{k=2}^{I} \alpha_k$, we get a system of $I - 1$ linear equations that is easily verified to provide a unique solution for $\boldsymbol{\alpha}$ in terms of $\left\{ \{\kappa_i\}_{i=1}^{I}, \kappa_e, \delta, \xi \right\}$. To see this it is sufficient to rewrite the system in the matrix form. The matrix to be inverted will have a particular structure that never allows one row to be a linear combination of the others since $\frac{\overline{w}_l - \eta \underline{w}_l}{\overline{w}_i - \eta \underline{w}_i} > 0 \; \forall i, l$.

To demonstrate that the model with the parameter space that eventually reduces to ξ and search frictions is identifiable it is enough to notice that frictions parameters $\left\{ \{\kappa_i\}_{i=1}^{I}, \kappa_e, \delta \right\}$ are uniquely identified from the duration data irrespective of the functional form of the offer distribution (e.g. Koning et al., 1995). From this follows that production size ξ is uniquely identified from the earnings data.

3.3.3 Heterogeneous Firms

For heterogeneous employers the production functions are given in (3.13). The relevant occupation-specific wage offer distribution $F_i(w)$ is provided in Proposition 3.1. Rewritten in $\kappa_{i,e}$ terms it becomes

$$F_i(w_i) = \frac{1+\kappa_e}{\kappa_e} - \frac{1+\kappa_e\left(1-\gamma_{j-1}\right)}{\kappa_e\sqrt{\frac{\left(Y_j'(\mathbf{r}_j)-w_i\right)r_{ij}-\sigma_{ij}-\sqrt{\left(\left(Y_j'(\mathbf{r}_j)-w_i\right)r_{ij}-\sigma_{ij}\right)^2+4\left(\sigma_{ij}-\mu_{ij}\right)\left(\left(Y_j'(\mathbf{r}_j)-\underline{w}_{ij}\right)r_{ij}-\mu_{ij}\right)}}{-2\left(\sigma_{ij}-\mu_{ij}\right)}}}, \quad (3.18)$$

where

$$\begin{aligned} r_{ij} &= \frac{\kappa_i/\left(1+\kappa_i\right)\left(1+\kappa_e\right)}{\left[1+\kappa_e\left(1-\gamma_{j-1}\right)\right]^2}q_i, \qquad Y_j'\left(\mathbf{r}_j\right) = \frac{\alpha_{ij}}{r_{ij}}p_j\prod\nolimits_{i=1}^{I} r_{ij}^{\alpha_{ij}}, \\ \sigma_{ij} &= \alpha_{ij}\left(\xi_j-1\right)Y_j\left(\mathbf{r}_j\right), \quad \text{and} \quad \mu_{ij} = \frac{r_{ij}}{\sum_i r_{ij}}\frac{1}{2}\sum\nolimits_i \sigma_{ij} \end{aligned}$$

for all $w_i \in [\underline{w}_{ij}, \overline{w}_{ij}]$, $i = 1, ..., I$ and $j = 1, ..., J$. Additionally we assume that for any i and j none of α_{ij} is equal to each other.

Remembering that $\gamma_j = F_i(\overline{w}_{ij})$ we use (3.13) and (3.18) to derive the productivity level of the firm

$$p_j = \frac{r_{ij}}{\prod_{i=1}^{I} r_{ij}^{\alpha_{ij}}}\left[\alpha_{ij} - \frac{\xi_j-1}{\eta_j}\left(\frac{\xi_j\left(1+\eta_j\right)r_{ij}}{2\sum_i r_{ij}} - \alpha_{ij}\right)\right]^{-1}\left(\frac{\overline{w}_{ij}-\eta_j\underline{w}_{ij}}{1-\eta_j}\right), \quad (3.19)$$

where $\eta_j = \left[\left(1+\kappa_e[1-\gamma_j]\right)/\left(1+\kappa_e[1-\gamma_{j-1}]\right)\right]^2$.

Consider the unknowns of the econometric model with heterogeneous firms. As before, skill group size and group-specific bounds for the offer distributions are available from the data. At the same time there appears an additional set of unknown cutoff wages $\{\overline{w}_{ij}\}_{i,j=1}^{I,J-1}$ for the firm-specific wage offer. Unlike in the homogeneous model, existence of the unknown cutoff wages does not allow using (3.19) to write down α_{ij} as a function of exclusively ξ_j and frictional parameters. However, knowing that $\overline{w}_{ij} = \underline{w}_{ij-1}$ provides us with additional cross-restrictions on p_{j-1} and p_j. Using these cross-restrictions together with the fact that (3.19) is the same for all i and noticing that the parameter subsets $\{\alpha_{ij}\}_{i,j=1}^{I-1,J}$ and $\{\overline{w}_{ij}\}_{i,j=1}^{I,J-1}$ are completely determined by (3.19) two representations of the model are possible:

1. cutoff wages $\{\overline{w}_{ij}\}_{i,j=1}^{I,J-1}$ are expressed as a function of production parameters $\{\alpha_{ij}\}_{i,j=1}^{I-1,J}$, search frictions and $\boldsymbol{\xi}$,
2. production parameters $\{\alpha_{ij}\}_{i,j=1}^{I-1,J}$ are expressed as a function of cutoff wages $\{\overline{w}_{ij}\}_{i,j=1}^{I,J-1}$, search frictions and $\boldsymbol{\xi}$.

First of all, irrespective of the choice of the parameter subset to be substituted out, (3.19) implies that there exist $J(I-1)$ independent equations that completely determine cutoff wages and production parameters.[3] Moreover, for I skill groups there exist $(J-1)I$ unknown production parameters and $J(I-1)$ unknown cutoff wages. Since both above representations must be equivalent to each other we conclude that the parameters cannot be identified whenever $J(I-1) \neq (J-1)I$. From this follows that $I = J$ symmetry is a necessary condition for identification of the model with employer heterogeneity.

Next, we notice that despite both specifications are equally possible, expressing cutoff wages as a function of the rest of the parameters, is the strictly dominated one. The reason is that cutoff wages are the discontinuity points of the likelihood function, so substituting them with known functions of the rest of the parameters means that no gradient-based methods can be used when estimating the model. Even though derivative-free methods are available a serious problem may appear when the assumption of no mass points in the offer distribution becomes violated at the solution. This case will imply constrained derivative-free optimization subject to the no mass point condition (for detailed discussion see p.72 later on), which is already a very difficult task.

Choosing the second way to represent the model one can show that (3.19) implies that for any $i, l = 1, .., I$ there holds an identity

$$\alpha_{ij} \frac{\left(\overline{w}_{lj} - \eta_j \underline{w}_{lj}\right) r_{lj}}{\left(\overline{w}_{ij} - \eta_j \underline{w}_{ij}\right) r_{ij}} - \alpha_{lj} = \frac{\xi_j \left(\xi_j - 1\right) \left(1 + \eta_j\right) r_{lj}}{2 \left(\xi_j + \eta_j - 1\right) \sum_{k=1}^{I} r_{kj}} \left[\frac{\overline{w}_{lj} - \eta_j \underline{w}_{lj}}{\overline{w}_{ij} - \eta_j \underline{w}_{ij}} - 1 \right],$$

which gives rise to a system of $J(I-1)$ linear equations with $J(I-1)$ unknown cutoff wages. It is also easy to see that for $J = 1$ the above identity reduces to the one

[3] i.e. neither $\{\overline{w}_{ij}\}_{i,j=1}^{I,J-1}$ nor $\{\alpha_{ij}\}_{i,j=1}^{I-1,J}$ appear outside the system of these equations.

described in the previous subsection. Rewriting the implied system in a matrix form one can find that the matrix to be inverted is block-diagonal. Each and every block in it has the same structure as the matrix of a corresponding problem in Section 3.3.2, out of which invertability follows.

Unique solution for $\{\alpha_{ij}\}_{i,j=1}^{I-1,J}$ reduces the parameter space to the subset of the location parameters of the discontinuity points of the likelihood function $\{\overline{w}_{ij}\}_{i,j=1}^{I,J-1}$ and the subset of shape parameters $\boldsymbol{\theta} \equiv \left\{ \{\kappa_i\}_{i=1}^{I}, \delta, \kappa_e, \left\{\xi_j\right\}_{j=1}^{J} \right\}$. Chernozhukov and Hong (2004) demonstrate that in the considered class of models shape and location parameters are independent of each other. Thus conditional identifiability will imply joint identifiability of the both. Within the subset of shape parameters search frictions are uniquely identified using the duration data. From this follows that production sizes are uniquely identified from the earnings data.

The above representation of the model fits into a convenient stepwise estimation strategy developed by Bowlus et al. (1995). At the first step, given the starting values for the structural parameters, cutoff wages are estimated by simulated annealing. At the second step, given the estimates of the cutoff wages, the likelihood function is maximized with respect to $\boldsymbol{\theta}$. The second step is a "smooth" optimization and can be efficiently executed using the gradient-based methods. Given the estimates from both steps into (3.5) and (3.9) we calculate the new point mass values γ_j

$$\gamma_j = 1 - \sum_{i=1}^{I} \frac{q_i}{m} \frac{1 - \hat{G}_i(\overline{w}_{ij})}{1 + \kappa_e \hat{G}_i(\overline{w}_{ij})}, \tag{3.20}$$

where $\hat{G}_i$ is a nonparametric estimate of the skill-specific earnings distribution, and the cycle repeats.

Provided that the maximum likelihood estimates are consistent with the assumption of the continuity of $F_{ij}(w)$ we can apply the result of Chernozhukov and Hong (2004) who show that the asymptotic distribution of the subset of shape parameters is $N(0, \mathbf{I}^{-1})$, where

$$\mathbf{I} = n^{-1} \sum_{l=1}^{n} \frac{\partial}{\partial \boldsymbol{\theta}} \mathcal{L}_l(\boldsymbol{\theta}) \frac{\partial}{\partial \boldsymbol{\theta}} \mathcal{L}_l(\boldsymbol{\theta}'). \tag{3.21}$$

Furthermore Chernozhukov and Hong (2004) validate bootstrap methods for the estimation of the asymptotic covariance matrix above.

3.3.4 Specification Check

We have derived the wage offer distribution (3.11) under the assumption that all skill specific wage offer distributions $F_i(w)$ are continuous. As argued in Section 3.2.3 a mass point can only exist, if increasing the wage further would imply that the additional wage cost outweighs the additional output produced with the additionally recruited workers. Consider an arbitrary skill group h. Then the distribution function $F_h(w)$ is continuous, if for a type j firm $\lim_{\varepsilon \to 0} \pi_j(w_h + \varepsilon, \mathbf{w}_{-h}) > \pi_j(\mathbf{w})$, i.e.

$$p_j \left(\frac{\kappa_h(1+\kappa_e)/(1+\kappa_h)}{\left[1+\kappa_e \overline{F}_h(w)\right]^2} q_h \right)^{\alpha_{hj}} \prod_{\substack{i=1 \\ i \neq h}}^{I} l_i(w)^{\alpha_{ij}} - \frac{\kappa_h(1+\kappa_e)/(1+\kappa_h)}{\left[1+\kappa_e \overline{F}_1(w)\right]^2} w_h q_h >$$

$$> \left(\frac{\kappa_h(1+\kappa_e)/(1+\kappa_h)}{\left[1+\kappa_e \overline{F}_h(w)\right]\left[1+\kappa_e \overline{F}_h(w^-)\right]} q_h \right)^{\alpha_{hj}} p_j \prod_{\substack{i=1 \\ i \neq h}}^{I} l_i(w)^{\alpha_{ij}} - \frac{\kappa_h(1+\kappa_e)/(1+\kappa_h)}{\left[1+\kappa_e \overline{F}_h(w)\right]\left[1+\kappa_e \overline{F}_h(w^-)\right]} w_h q_h.$$

First, note that this condition is satisfied for $\alpha_{hj} \geq 1$. For $\alpha_{hj} < 1$ the concavity of the production function implies that if a mass point exists at $w_h \in [\underline{w}_{hj}, \overline{w}_{hj}]$, then increasing the wage by ε still implies that the additional wage cost outweighs the additional output produced. Thus, if a mass point exists, then it exists at the upper bound of the support of F_{hj} : $supp(F_{hj}) = [\underline{w}_{hj}, \overline{w}_{hj}]$. Together with the fact that $F_h(\overline{w}_{hj}) = \gamma_j$ this implies that $F_h(\overline{w}_{hj}^-) = \gamma_j - \upsilon_h(\overline{w}_{hj})$. Substituting $\overline{w}_{hj}$ for w_h in the equation above and rearranging gives the following inequality:

$$\frac{1 - \left(\frac{1+\kappa_e\left(1-\gamma_j\right)}{1+\kappa_e\left(1-\gamma_j+\upsilon_1(\overline{w}_{hj})\right)} \right)^{\alpha_{hj}}}{1 - \frac{1+\kappa_e\left(1-\gamma_j\right)}{1+\kappa_e\left(1-\gamma_j+\upsilon_1(\overline{w}_{hj})\right)}} > \frac{\overline{w}_{hj} l_h(\overline{w}_{hj})^{1-\alpha_{hj}}}{p_j \prod_{\substack{i=1 \\ i \neq h}}^{I} l_i(w)^{\alpha_{ij}}}. \tag{3.22}$$

From (3.22) a necessary condition for continuity follows whenever $\lim_{\upsilon_h(\overline{w}_{hj}) \to 0}(lhs) > (rhs)$. Taking limit of the lhs and applying (3.8) to the rhs we get

$$a_{hj} > \frac{\overline{w}_{hj} l_h(\overline{w}_{hj})}{p_j \prod_{i=1}^{I} l_i(\overline{w}_{ij})^{\alpha_{ij}}} \tag{3.23}$$

The estimated parameters are consistent only when the model is properly specified, i.e. when (3.23) holds.

It is also easy to see that in a special case with no skill differentiation, constant returns and unique productivity type firms, which is the original Burdett-Mortensen model, (3.23) gives us $1 > \overline{w}/p$, which is always true, implying continuous offer distribution in the basic BM model.

Furthermore the estimated parameters must be consistent with the assumption that profits of the firms with different technologies are ranked, i.e.

$$0 \leq \pi_{j-1} < \pi_j. \tag{3.24}$$

In terms of the Burdett-Mortensen model with discrete employer heterogeneity the above condition will imply the ranking of productivity levels. Possibility of violation of productivity ranking in applied work is discussed by Bowlus et al. (1995), p.S127.

One should also keep in mind that whenever any of the above restrictions is binding at the maximum the asymptotic covariance matrix of the ML estimator is no longer given by (3.21) and the exact form of it is unknown. Moreover even in the simpler models with inequality constraints it is shown that bootstrap fails to consistently estimate the covariance matrix when the true parameter is on the boundary of the parameter space (see Andrews, 2000, for discussion).

Finally we notice that in the extended model with distinct productivity types another (weaker) way to see whether (3.8) holds is to consider $\hat{G}_i\left(\overline{w}_{ij}|\arg\max_{\{\boldsymbol{\theta},\gamma_j\}}(\mathcal{L})\right)$. Both (3.8) and (3.5) imply that $\hat{G}_i = \hat{G}_l \ \forall i,l \in [2,I]$. At the same time (3.20) does not restrict $\hat{G}_i$ to be equal to each other. Thus, if $\{\boldsymbol{\theta},\gamma_j\} \ \forall j \in [2, J-1]$ is a consistent estimate of the true parameters the values of the empirical earnings distribution at the skill-specific cutoff wages must not be significantly different from each other.

3.4 Empirical Application

3.4.1 The Data

To estimate the model we use the data from the German Socio-Economic Panel. The theoretical formulation of the model provides enough restrictions to estimate the parameters of the production function even without firm data. Thus, as before, we apply duration and wage information only. In the present chapter we use the sample of 1995.

Table 3.1: Descriptive Statistics of Event History Data

	Skills			
	Low	Medium	High	Full Sample
Number of individuals:	898	1931	1062	3891
Employed:	746	1786	1025	3557
Unemployed:	152	145	37	334
Employed Agents:				
Uncensored observations with:				
job → job transition:	49	187	178	414
job → unemployment transition:	98	126	41	256
mean time spell between two states [job duration]:	129.639	109.815	89.566	107.576
(std. deviation):	(114.92)	(102.14)	(85.42)	(101.01)
Censored observations				
a) left-censored durations only				
with job → job transition:	3	12	6	21
with job → unemployment transition:	1	13	1	15
b) right-censored durations only:	575	1407	781	2763
c) both left- and right-censored durations:	20	41	18	79
Mean time spell [both uncensored and censored]:	163.637	153.259	154.096	155.677
(std. deviation):	(116.23)	(118.84)	(120.30)	(118.76)
Unemployed Agents:				
Uncensored observations (u → j transition):	37	49	13	99
mean time spell between two states [job duration]:	19.595	22.429	10.538	19.808
(std. deviation):	(14.35)	(26.72)	(12.22)	(21.41)
Censored observations				
a) left-censored durations (u → j transition) only:	1	2	-	3
b) right-censored durations only:	106	89	24	219
c) both left- and right-censored durations:	8	5	-	13
Mean time spell [both uncensored and censored]:	40.974	32.310	24.270	35.362
(std. deviation):	(36.37)	(31.90)	(23.07)	(33.61)

Duration data in Months

Target groups, sampling scheme and retrieving duration data is in detail described in Section 2.4 so we do not repeat it here once again. Collecting earnings information relevant for the estimation of the extended Burdett-Mortensen model needs, however, some further explanations. In this chapter we differentiate between net wage received by the worker and labour costs to the firm. In the theoretical model we have two sets of parameters, namely workers' search intensities and production parameters. Since the theory states that reservation wage and labour size depend on just the position of the firm in the wage offer distribution, frictional parameters can be estimated using any of the two types of earnings data, provided that the ordering of the firms does not change when we pass from net wages to labour costs. For identification of the production parameters, to the contrary, labour costs are crucial because they enter the employers' profit maximization problem explicitly.

GSOEP provides the data on both net and gross wages. Individuals who are employed at their interview provide the earnings information of one month prior to the interview. For the sample of job spells we use wage information provided by respondents at the year for which the sample is drawn. For the sample of unemployment spells we use the first reported wage after the end of unemployment, given that the transition to the job is observed. All wages are deflated by the West German consumer price index at prices of 1998. Labour costs are defined as a sum of gross wage and firms' contributions to the employees' social security payments. Information on the latter is available form the Social Security Office.

In our application we estimate the model with three different productivity levels and three different skill groups. Skill stratification of the sample is performed on the basis of the International Standard Classification of Education (ISCED). We identify as "low- skilled" all individuals who have inadequate or general elementary training. To "medium-skilled" group belong those who have got middle vocational training. Finally, as "high-skilled" we qualify all the rest, i.e. those with higher vocational training, university education etc.

Summary of duration and wage data is presented in Table 3.1 and Table 3.2 respectively. Along with the information about the full sample we present summary statistics for the three skill groups. The data on skills reflect such basic facts about less skilled in comparison to higher skilled as higher level of unemployment, higher rate of job loss and longer unemployment duration. Additionally net wages and labour costs are summarized by kernel density plots (see Figures C.1-2 in the Appendix). As expected,

Table 3.2: Descriptive Statistics of Earnings Data

	Skills			
	Low	Medium	High	Full Sample
Labour Costs:				
Sample Minimum:	734.	1038.	1646.	734.
Mean Cost:	4431 (1417)	5245 (1903)	6950 (2642)	5554 (2258)
Sample Maximum:	12057.	17348.	20523.	20523.
Net Wages:				
Sample Minimum:	604.	635.	952.	604.
Mean Wage:	2472 (809)	2880 (1083)	3967 (1667)	3101 (1356)
Sample Maximum:	6878.	9524.	11534.	11534.

density of both net earnings and labour costs of the low-skilled are more peaked at its' leftmost part of the support than those of the higher skills. Also mean net wage of high-skilled workers amounts to DM 3967 which exceeds that of medium-skilled by 27% and of low-skilled by more then 37%. Labour costs are roughly the same across the skills and almost double the net wage.[4]

3.4.2 Estimation Results: Fit of the Model

First we estimate the model with identical employers setting off with the constant returns assumption (see Table C.1 in the Appendix). When doing so we also fit the original Burdett-Mortensen model with no productivity dispersion to compare it with the results provided by our extension.[5] It turns out that the structural parameters estimated with both original and extended constant-returns specifications do not significantly differ from each other, which implies that from the empirical perspective sole

[4]Note the difference between the skill group definitions in this Chapter and Chapter 2. Here we use higher level of aggregation uniting groups 3 and 4 into "high" category. Also because for certain individuals gross wages were unavailable the sample used in this Chapter has about 4% fewer observations than the one of Chapter 2.

[5]We do not report the estimates from the original model here.

introduction of skill differences does not improve the estimates of search frictions. Predicted theoretical offer and labour costs densities (Figures C.3-4 respectively) for the extended theoretical model with constant returns have two jumps at the reservation wages of the medium- and high-skilled workers and two spikes at the maximum wages of the low- and medium-skilled workers. This generates a quasi-"falling" right tail of the aggregate density despite that skill-specific ones are strictly increasing. However, even with large I the model with constant returns has limited potential of fitting the data.

The results change when we switch to the increasing returns technology specification (the second column in Table C.1). First, when inserted into (3.3), the estimates of κ_i fit the observed skill-specific unemployment rates closer. Second, and more important, the model with increasing returns provides much more realistic estimates for κ_e and δ. Though the most interesting result is displayed in Figures C.3-4 where we see that increasing returns imply offer and labour costs densities with strictly decreasing right tail even in absence of productivity dispersion. Even though the predicted labour costs density is still too flat pointing towards existence of heterogeneous production technologies in the data, this result alone is already remarkable.

The initial unrestricted estimates of the model with increasing returns to scale do not meet the "no mass point condition" of Section 3.3.4. Therefore in Table C.1 we report the estimates which are obtained by maximizing the likelihood function subject to (3.23). Furthermore we restrict profits to be non-negative. It turns out that at the constrained maximum the condition in (3.23) becomes inactive. However, the non-negativity of profits is violated on the upper end of the offer distribution and the non-negativity constraint on profits remains binding at the maximum. As a consequence the asymptotic covariance matrix of the estimated parameters becomes unknown.[6]

Next we estimate the model with employer heterogeneity. As before, we also fit the original Burdett-Mortensen model with $J = 3$.[7] Again, the results of the original Burdett-Mortensen model and our extension with constant returns almost do not differ from each other. Even though two jumps at the left tail and two spikes at the right one improve the fit of the aggregate labour costs density (see Figures C.5-6),

[6]We report confidence intervals based on (3.21). However, since the true parameters lie on the boundary of the parameter space, (3.21) underestimates the true covariance matrix (see also Section 3.3.4).

[7]Again we do not report the estimates from the basic model.

locally increasing right tail of individual-specific densities still keeps the fit from being satisfactory.

Relaxing the assumption of constant returns again changes the picture. Though, similarly to the case with identical firms, the unrestricted MLEs still violate profit ranking requirement. Therefore we perform the estimation of the model given (3.23) and (3.24). Remarkable enough, in the restricted maximum the "no mass point condition" of Section 3.3.4 is again inactive which empirically supports the k-percent rule (3.8). However $\pi(\overline{w}_{ij-1}) < \pi(\underline{w}_{ij})$ turn out to be binding. On one hand this may be simply a consequence of the insufficient heterogeneity of the production side. On the other, this can also be interpreted as an empirical indication of the restrictiveness of the equal-profit condition among the firms with the same technology. While the first interpretation opens a purely empirical issue that can be amended by just increasing the number of distinct skill levels, resolution of the alternative case would require a more refined theoretical model.

The estimates of the model with increasing returns and three-point productivity dispersion are presented in the second column of Table C.2. Comparing them with the estimates from the specification with identical firms and increasing returns technology two further improvements can be noticed. First, we manage to obtain a better fit for the degree of returns to scale in the whole economy. According to our estimates the homogeneity degrees are 1.04 for the "low-productive" technology, 1.40 for the "medium-productive" technology and 4.92 for the "high-productive" one. Given the estimated fraction of each technology $\left[\gamma_j - \gamma_{j-1}\right]$ in the economy these estimates imply the economy-wide returns to scale at the level of 1.20. This goes in line with numerous evidences from the literature on the estimation of the returns to scale using different types of production functions. Typical estimates in this literature support the increasing returns hypothesis and range from about 1.1 to about 1.35 (see Färe at al., 1985, Kim, 1992, and Zellner and Ryu, 1998, and references therein). Second, and even more important, productivity dispersion along with increasing returns technologies also leads to a better fitting offer and labour costs densities. In Figures C.5-6 one can easily see the dominance of increasing over constant returns specification in terms of both shape of the right tail and smoothed out spikes around the mean.

3.4.3 Estimation Results: Social Returns to Education

We use our estimation results to investigate whether the education level in the economy is efficient, i.e. whether the increase in output coming from educating the marginal individual equals the individual's and the government's investment costs.

In our model the increase in education is reflected by the marginal shift of the skill structure towards the higher fraction of more skilled workers. From the point of view of the social welfare planner positive externality will exist if the expected output increase induced by this marginal shift of the skill structure will be big enough to cover private costs of educating a marginal worker to the next level and will generate a positive excess value.[8]

Denote the measure of any adjacent skill groups by n so that $n = q_i + q_{i+1}$. It is easy to show that for a j-type firm the marginal change in output due to educating a marginal i-skilled worker one level up (i.e. due to the marginal increase of the measure of $i+1$-skilled workers) is

$$\begin{aligned} \frac{\partial Y_j(\mathbf{l}(\mathbf{w}))}{\partial q_{i+1}} &= Y_j(\mathbf{l}(\mathbf{w})) \left[\sum_{k=1}^{I} \frac{\alpha_{kj}}{l_k(w)} \frac{\partial l_k(w)}{\partial q_k} \right] \\ &= Y_j(\mathbf{l}(\mathbf{w})) \left[\frac{\alpha_{i+1j}}{q_{i+1}} - \frac{\alpha_{ij}}{n - q_{i+1}} + \frac{2\kappa_e \left(\alpha_{ij} + \alpha_{i+1j}\right)}{1 + \kappa_e \left[1 - F\right]} \left(\frac{\partial F}{\partial q_i} \right) \right] \end{aligned}$$

which implies an expected change in the total output

$$E(\Delta Y) = \int_0^1 \frac{\partial Y_j(\mathbf{l}(\mathbf{w}))}{\partial q_{i+1}} dF = \sum_{j=1}^{J} \int_{\gamma_{j-1}}^{\gamma_j} \frac{\partial Y_j(\mathbf{l}(\mathbf{w}))}{\partial q_{i+1}} dF. \tag{3.25}$$

In order to learn whether the social returns from educating an agent to a higher skill level exceed the private returns of doing so, we proceed in comparing the marginal increase in output caused by a change in the skill structure with the cost the marginal individual incurs to acquire this skill level. It has to be true that in equilibrium the marginal worker is exactly indifferent between the two skill groups, i.e. $U_i = U_{i-1}$. Thus, using (3.1a), the private cost of educating oneself one level up can be written as

[8]For more rigorous theoretical treatment of the social welfare planner's problem see Holzner and Launov, 2005, Section 4.3

$$
\begin{aligned}
(c_i - c_{i-1})\, a_i^I &= rU_i - rU_{i-1} \\
&= \kappa_i \int_{w_i^r}^{\bar{w}_i} \frac{\bar{F}_i(w)}{1 + r/\delta + \kappa_e \bar{F}_i(w)} dw - \kappa_{i-1} \int_{w_{i-1}^r}^{\bar{w}_{i-1}} \frac{\bar{F}_{i-1}(w)}{1 + r/\delta + \kappa_e \bar{F}_{i-1}(w)} dw.
\end{aligned} \tag{3.26}
$$

Note, that (3.26) refers to the optimal decision of the searching *individual*, which implies that the net wages w_i^r and $\overline{w}_i$ – not the wage costs – are the bounds of the distribution of the net offer. Therefore in order to compute the correct difference in private costs we have to translate the estimated cutoff wages expressed in terms of labour costs into the cutoff wages expressed in terms of net earnings (which is possible since we know nonparametric estimates of both labour costs and net earnings cdfs). Finally, drawing on the OECD statistics, the average real interest rate over the considered period of 1984-2001 is equal to 3.6%.[9]

We use the estimates of the structural parameters to evaluate (3.25)-(3.26) and see whether present skill structure is efficient. In doing so we consider two cases, namely:

1. Marginal shift from Medium to High skills (the fraction of low-skilled is constant),
2. Marginal shift from Low to Medium skills (the fraction of high-skilled is constant).

Our key finding is that indeed a marginal change of the skill structure towards a larger share of skilled workers uniformly generates an increase in output.

Taking the first case, the marginal increase of the fraction of high-skilled workers by educating the medium-skilled ones induces the expected output increase of DM 2269.88. At the same time the period private cost of this increase amount to DM 2277.60, from which follows that the fraction of high-skilled workers almost precisely matches its' socially optimal level.

For the next case, however, the situation is different. The output effect of the marginal change of the skill structure towards increasing the share of medium-skilled workers in the economy is again positive and, although somewhat smaller in absolute value, amounts to DM 2057.27. But the individual costs that trigger this effect lie at the level of DM 821.67, which is more than twice as low. Thus we obtain a strong

[9]Source: OECD Economic Outlook, No.77. Price base for the calculation is set to 1998, as that of the earnings data.

evidence of underinvestment into skills at the low-to-medium level and conclude that subsidizing the education of the low-skilled must be welfare improving from the social prospective. Going back to the definition of skills this means that it would be socially optimal to reduce the fraction of workers with inadequate or general elementary training and increase the fraction of those with middle-vocational training.

To conclude, the present paper offers a new approach to measuring social returns to education within an equilibrium framework which takes the skill specific unemployment risk explicitly into account. As a result we are able to provide insights of whether there is over- or underinvestment in an economy. At the same time our framework does not allow determining the source of the inefficiency. The detected underinvestment could either be caused by the hold-up problem that workers face when making their investment decision or by a positive human capital externality due to an education spillover.

Abstracting from the application to social returns, our results also appear to be in line with those of Falk and Koebel (1999) who show that output is a positive and increasing function of skills and that output effect dominates in explaining the shift away from unskilled labour in Germany.

3.5 Conclusion

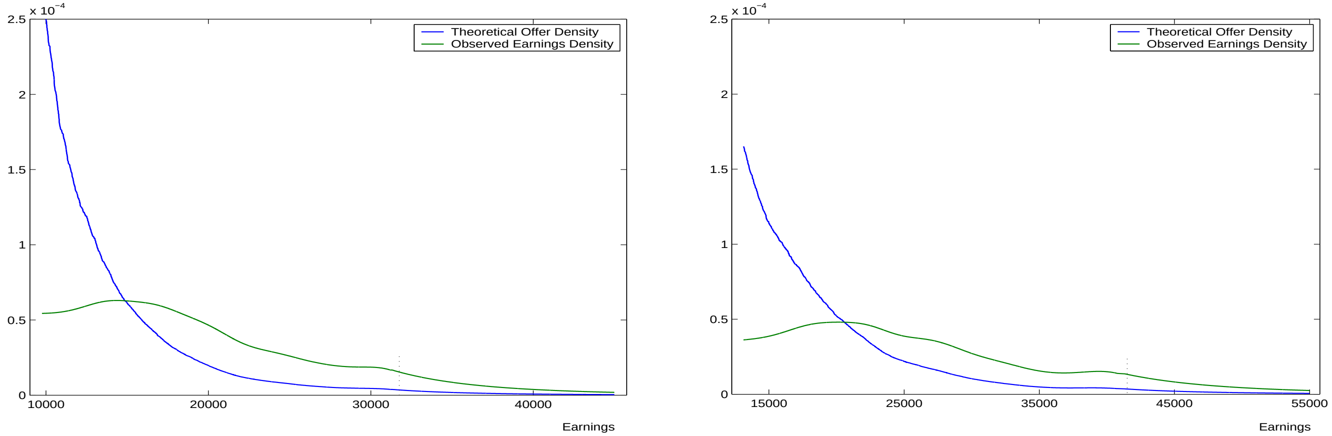
In this chapter we consider the extension of the search equilibrium model of Burdett and Mortensen (1998). The extension of the original model introduces different skill groups and links them via a production function that permits both constant and increasing returns to scale. The main theoretical contribution of the extension is that whenever the production function exhibits increasing returns to scale a decreasing wage offer density obtains. Subsequent introduction of employer heterogeneity leads to a further improvement of the shape of wage offer and earnings distributions predicted by the model.

The theoretical solution of our extension suggests a structural econometric model that allows estimating not only search frictions inherent to the labour market but also the parameters of the production function. The richness of the theoretical model enables us to estimate all parameters of interest using wage and duration data only, which requires no additional information on output.

We apply our model to learn whether there is over- or underinvestment into human capital in Germany. Our results suggest that the cost of a marginal shift of the skill composition of the workforce away from the low-skilled and towards a larger share of medium-skilled workers is expected to be lower than the expected increase in output, induced by this shift. This suggests that social returns to education exceed private returns and that a policy designed to promote education at lower levels would be welfare improving.

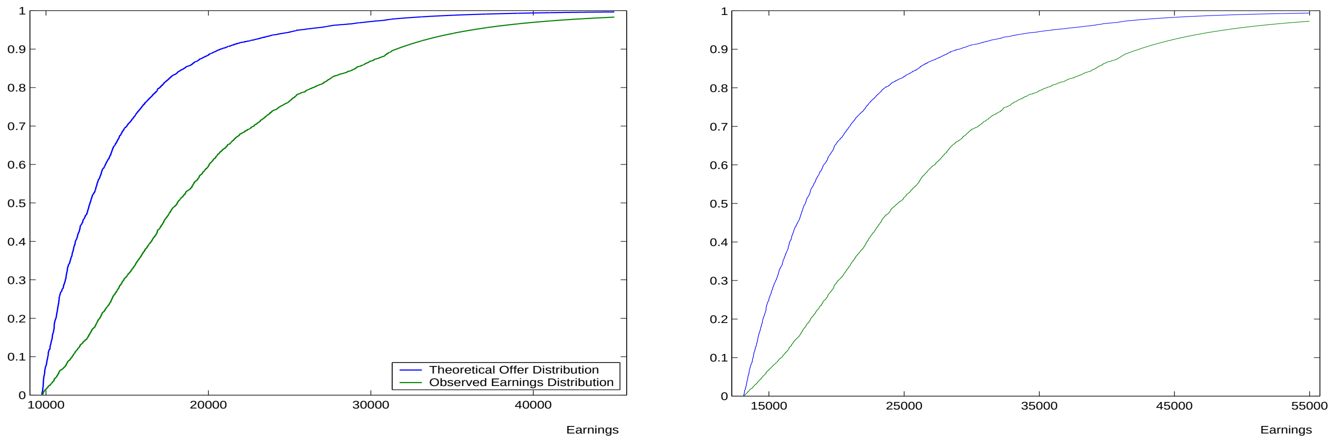
Appendix A

Figure A.1: Wage Offer and Earnings Densities for the Two Samples



(a) 1988 (b) 1994

Figure A.2: Wage Offer and Earnings Distributions for the Two Samples



(a) 1988 (b) 1994

Figure A.3: Expected Job Duration (years)

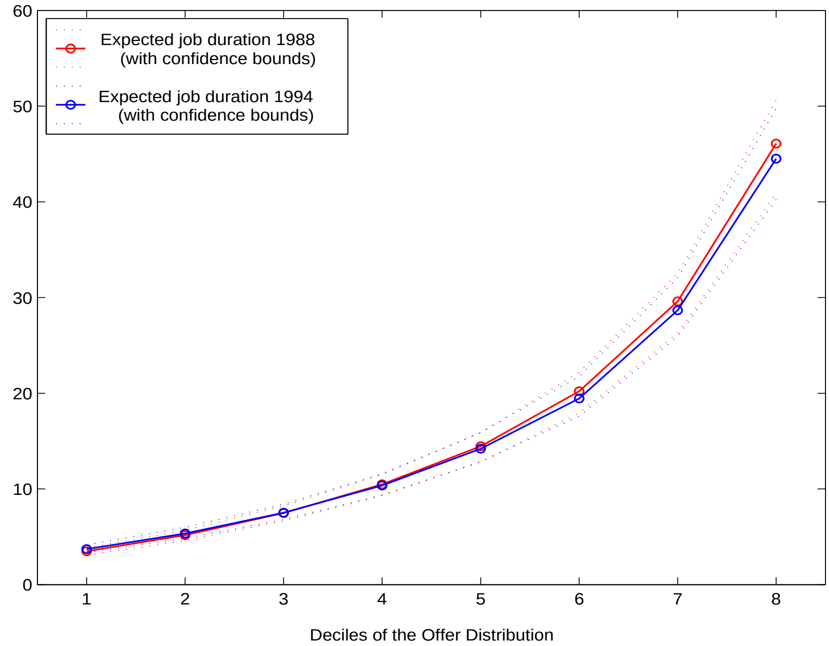


Figure A.4: Index of Monopsony Power

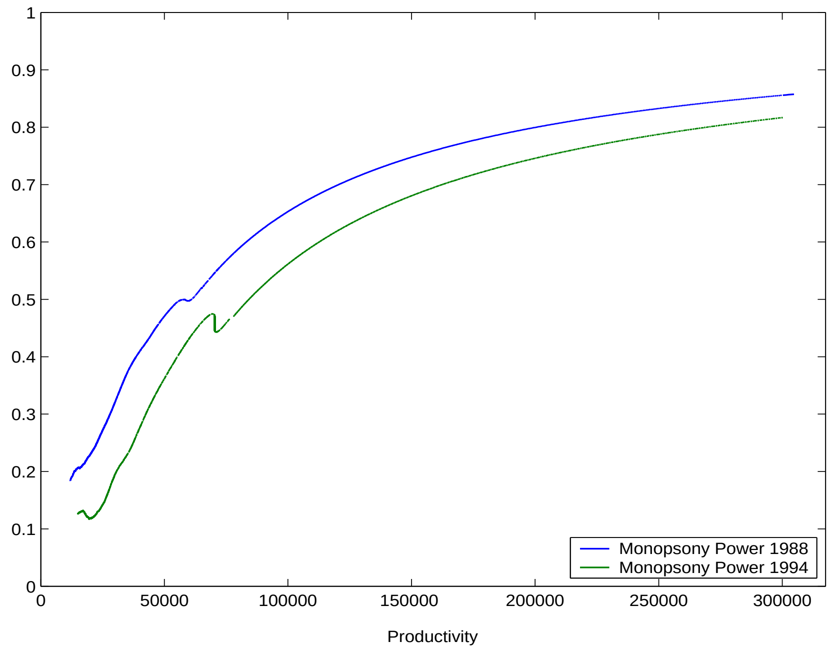
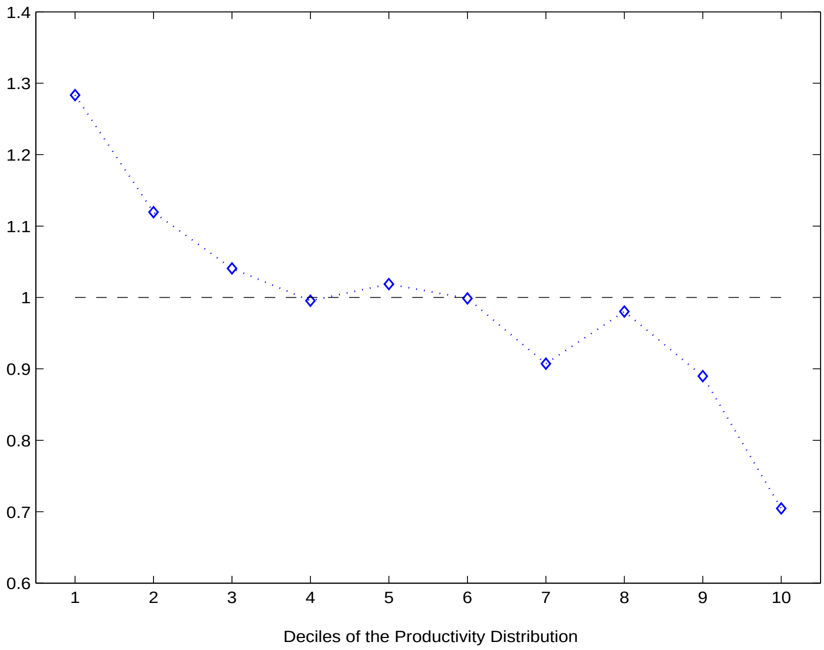


Figure A.5: Profit Ratio Plot



Appendix B

Table B.1: Estimated Model for the Whole Economy

Sample: 1986			Sample: 1995		
	Coefficients	(Std.Errors)		Coefficients	(Std.Errors)
κ_0	12.7914	(0.4767)	κ_0	9.3459	(0.3428)
κ_1	4.8014	(0.1157)	κ_1	4.0133	(0.1126)
δ	0.0036	($6.3 \cdot 10^{-5}$)	δ	0.0041	($8.0 \cdot 10^{-5}$)
Estimated Productivity Distribution:			Estimated Productivity Distribution:		
i :	P_i	γ_i	i :	P_i	γ_i
1	2304.6	0.65561	1	2758.0	0.62421
2	2726.6	0.81784	2	3120.4	0.79455
3	3289.8	0.90804	3	3845.8	0.88384
4	4601.5	0.95306	4	4738.5	0.92208
5	7997.2	0.98269	5	6147.2	0.94792
6	18630.5	0.99529	6	8673.3	0.97320
7	62728.1	0.99897	7	13906.5	0.98731
8	437143.1	1	8	24442.1	0.99331
			9	53593.8	0.99769
			10	232585.7	1
Log(Likelihood):		-74245.072	Log(Likelihood):		-61075.378

Table B.2: Estimated Model for Different Qualification Groups

Qualification Groups	1986 Coefficients (Std.Errors)			1986 u	1986 R	1995 Coefficients (Std.Errors)			1995 u	1995 R
Group I:	λ_0	0.0373	(0.0023)			λ_0	0.0273	(0.0017)		
(inadequately or general	λ_1	0.0110	(0.0006)	0.103	1768.9	λ_1	0.0088	(0.0006)	0.151	1800.4
elementary)	δ	0.0043	(1.4×10^{-4})			δ	0.0049	(1.9×10^{-4})		
	Number of mass points in $\gamma(p)$:			7		Number of mass points in $\gamma(p)$:			7	
	Number of observations:			1401		Number of observations:			933	
	Log(Likelihood):			−20331.83		Log(Likelihood):			−13319.29	
Group II:	λ_0	0.0513	(0.0028)			λ_0	0.0449	(0.0024)		
(middle vocational)	λ_1	0.0179	(0.0006)	0.067	2036.0	λ_1	0.0157	(0.0006)	0.087	2238.6
	δ	0.0037	(9.1×10^{-5})			δ	0.0043	(1.2×10^{-4})		
	Number of mass points in $\gamma(p)$:			8		Number of mass points in $\gamma(p)$:			7	
	Number of observations:			2381		Number of observations:			1973	
	Log(Likelihood):			−35969.58		Log(Likelihood):			−29618.12	
Group III:	λ_0	0.0746	(0.0104)			λ_0	0.0659	(0.0080)		
(vocational + "Abitur"	λ_1	0.0184	(0.0012)	0.040	2852.1	λ_1	0.0221	(0.0015)	0.055	2858.3
or higher vocational)	δ	0.0031	(1.5×10^{-4})			δ	0.0039	(1.9×10^{-4})		
	Number of mass points in $\gamma(p)$:			8		Number of mass points in $\gamma(p)$:			7	
	Number of observations:			616		Number of observations:			587	
	Log(Likelihood):			−9643.23		Log(Likelihood):			−9078.60	
Group IV:	λ_0	0.0659	(0.0103)			λ_0	0.0864	(0.0131)		
(higher education)	λ_1	0.0434	(0.0031)	0.043	2488.2	λ_1	0.0285	(0.0021)	0.044	3687.5
	δ	0.0030	(1.8×10^{-4})			δ	0.0040	(2.2×10^{-4})		
	Number of mass points in $\gamma(p)$:			9		Number of mass points in $\gamma(p)$:			7	
	Number of observations:			450		Number of observations:			488	
	Log(Likelihood):			−7146.4097		Log(Likelihood):			−7828.00	

Table B.3: Estimated Model for Different Age Groups

Qualification Groups	1986 Coefficients (Std.Errors)			1986 u	1986 R	1995 Coefficients (Std.Errors)			1995 u	1995 R
Group I:	λ_0	0.0781	(0.0053)			λ_0	0.0766	(0.0060)		
(16-27 years old)	λ_1	0.0533	(0.0026)	0.096	1016.1	λ_1	0.0357	(0.0022)	0.111	1479.2
	δ	0.0083	(3.1×10^{-4})			δ	0.0095	(4.2×10^{-4})		
	Number of mass points in $\gamma(p)$:			7		Number of mass points in $\gamma(p)$:			6	
	Number of observations:			1110		Number of observations:			745	
	Log(Likelihood):			-15799.39		Log(Likelihood):			-10334.69	
Group II:	λ_0	0.0664	(0.0047)			λ_0	0.0584	(0.0037)		
(28-40 years old)	λ_1	0.0164	(0.0007)	0.057	2437.3	λ_1	0.0238	(0.0010)	0.081	2244.5
	δ	0.0040	(1.2×10^{-4})			δ	0.0052	(1.6×10^{-4})		
	Number of mass points in $\gamma(p)$:			7		Number of mass points in $\gamma(p)$:			8	
	Number of observations:			1609		Number of observations:			1527	
	Log(Likelihood):			-25096.58		Log(Likelihood):			-23409.19	
Group III:	λ_0	0.0408	(0.0028)			λ_0	0.0372	(0.0027)		
(41-53 years old)	λ_1	0.0068	(0.0003)	0.070	2983.0	λ_1	0.0056	(0.0004)	0.087	3328.2
	δ	0.0031	(9.1×10^{-5})			δ	0.0035	(1.3×10^{-4})		
	Number of mass points in $\gamma(p)$:			7		Number of mass points in $\gamma(p)$:			4	
	Number of observations:			1583		Number of observations:			1121	
	Log(Likelihood):			-24712.09		Log(Likelihood):			-17411.64	
Group IV:	λ_0	0.0215	(0.0021)			λ_0	0.0194	(0.0015)		
(54-64 years old)	λ_1	0.0032	(0.0003)	0.114	3165.2	λ_1	0.0030	(0.0003)	0.146	3114.4
	δ	0.0028	(1.5×10^{-4})			δ	0.0033	(1.5×10^{-4})		
	Number of mass points in $\gamma(p)$:			5		Number of mass points in $\gamma(p)$:			7	
	Number of observations:			575		Number of observations:			637	
	Log(Likelihood):			-8021.41		Log(Likelihood):			-9344.46	

Figure B.1: Estimated Theoretical Offer and Earnings Distributions for the whole Economy: Sample 1986

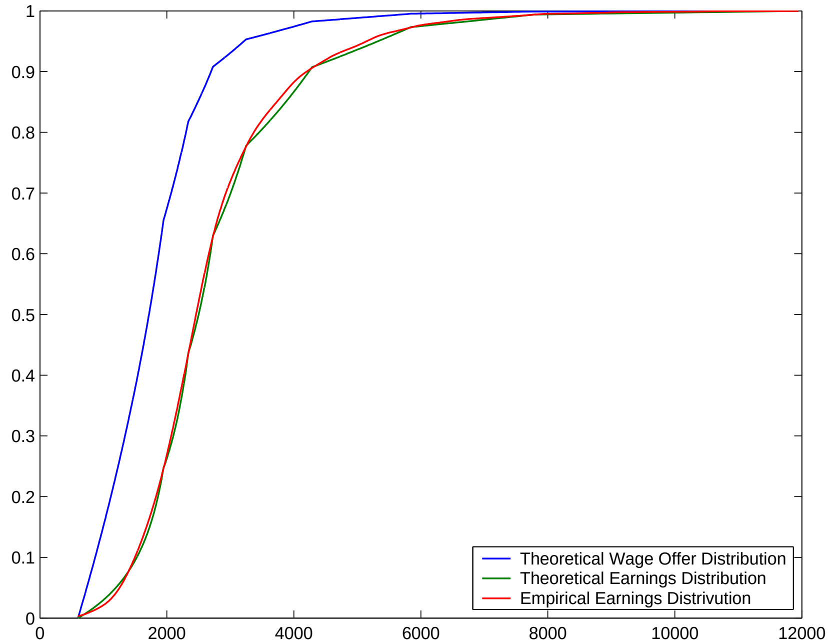


Figure B.2: Estimated Theoretical Offer and Earnings Distributions for the whole Economy: Sample 1995

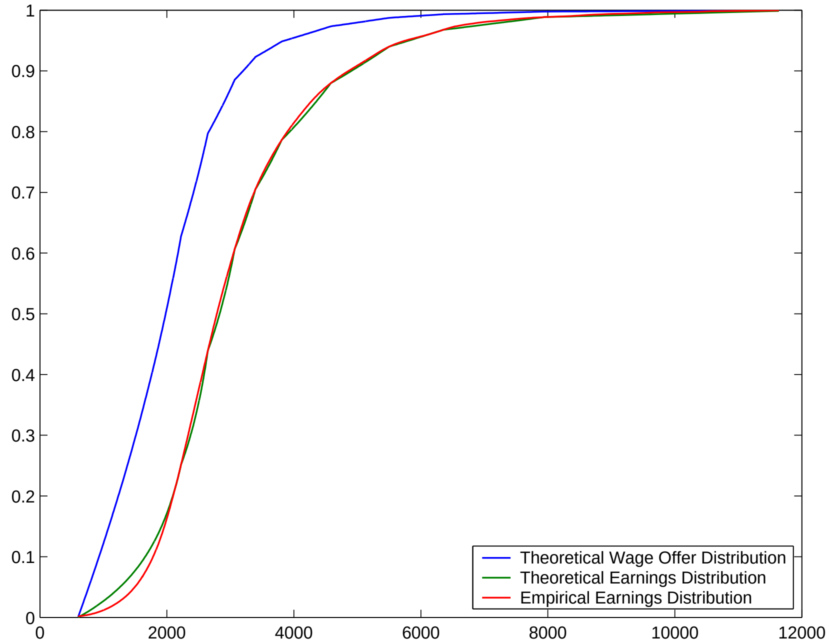
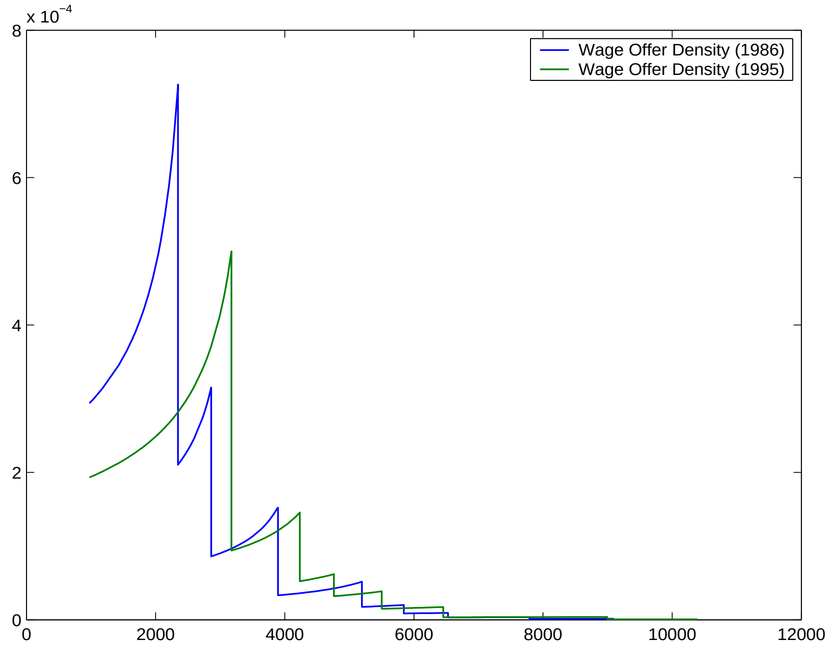


Figure B.3: Estimated Theoretical Wage Offer Densities for the High Skilled Group



Appendix C

Figure C.1: Kernel Estimates of Earnings Densities

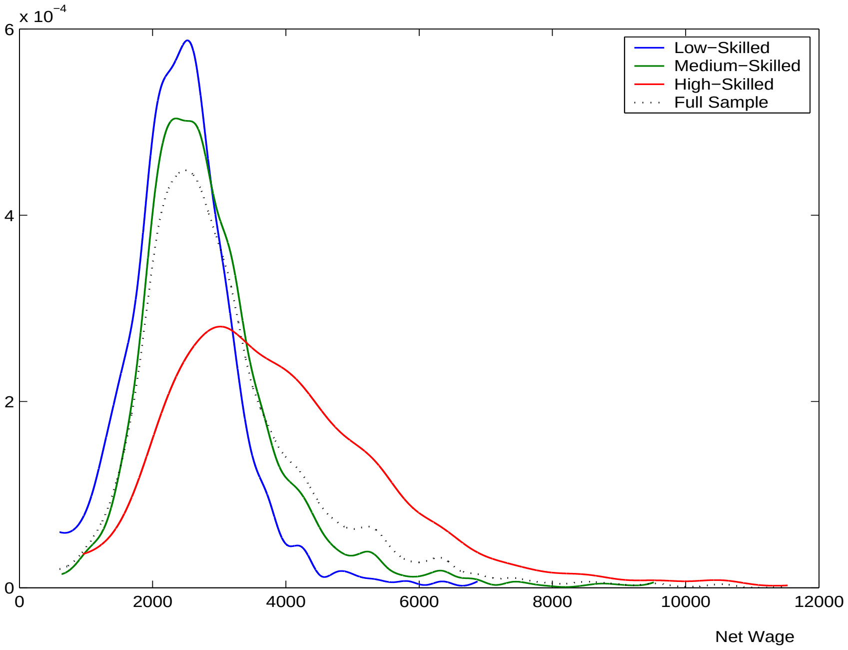


Figure C.2: Kernel Estimates of Labour Cost Densities

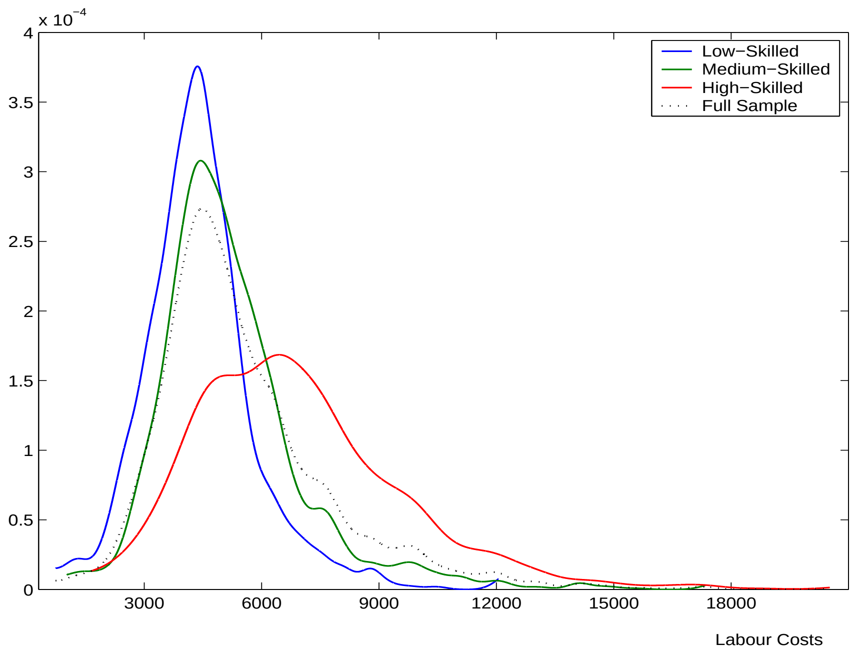


Table C.1: Estimation Results: Homogeneous Firms

	Specification			
	Constant Returns*		Increasing Returns	
κ_{u1}	4.6182	[4.1640, 5.0725]	5.9115	[5.2372, 6.5858]
κ_{u2}	8.2312	[7.6093, 8.8531]	10.4875	[9.5566, 11.4183]
κ_{u3}	14.1192	[12.5421, 15.6963]	17.8712	[15.4814, 20.2611]
κ_e	0.1605	[0.1421, 0.1789]	2.0963	[1.7342, 2.4585]
δ	0.0066	[0.063, 0.0068]	0.0043	[0.0041, 0.0045]
ξ			2.0000	[1.7945, 2.2053]
α_1	0.1513		0.3704	
α_2	0.5080		1.0044	
$\ln(\mathcal{L})$	-68248.06		-66758.10	

* *Here and henceforward 95% confidence intervals in square brackets*

Figure C.3: Aggregate Wage Offer Densities: Homogeneous Firms

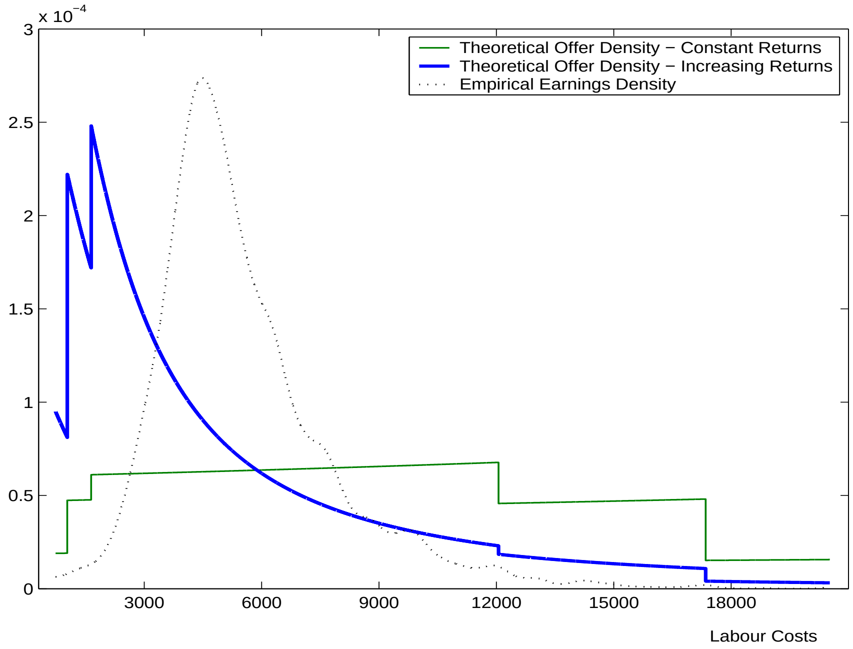


Figure C.4: Theoretical Earnings Densities: Homogeneous Firms

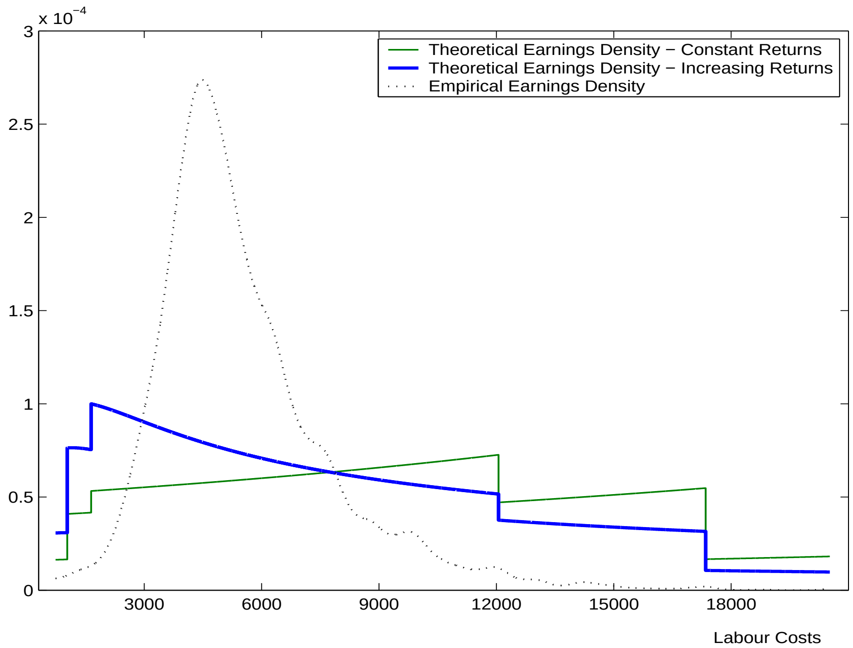


Table C.2: Estimation Results: 3-Point Employer Heterogeneity

	Specification						
	Constant Returns				Increasing Returns		
κ_{u1}	5.6156	[4.9973, 6.2339]		κ_{u1}	5.9612	[5.2742, 6.6481]	
κ_{u2}	9.9702	[9.1169, 10.8234]		κ_{u2}	10.6176	[9.6662, 11.5691]	
κ_{u3}	17.0121	[14.8258, 19.1985]		κ_{u3}	18.0656	[15.6320, 20.4991]	
κ_e	2.1277	[1.9869, 2.2684]		κ_e	3.6432	[3.3926, 3.8939]	
δ	0.0047	[0.0045, 0.0049]		δ	0.0042	[0.0040, 0.0044]	
				ξ_1	1.0381	[1.0324, 1.0437]	
				ξ_2	1.3961	[1.2977, 1.4945]	
				ξ_3	4.9201	[3.1342, 6.7060]	
$\{\alpha_{ij}\}$	$j=1$	$j=2$	$j=3$	$\{\alpha_{ij}\}$	$j=1$	$j=2$	$j=3$
$i=1$	0.1772	0.1449	0.1499	$i=1$	0.1896	0.2466	0.9822
$i=2$	0.4622	0.4939	0.5212	$i=2$	0.4850	0.6586	2.4929
$\{\overline{w}_{ij}\}$		$j=1$	$j=2$	$\{\overline{w}_{ij}\}$		$j=1$	$j=2$
$i=1$		4431	5698	$i=1$		4431	5698
$i=2$		5065	7597	$i=2$		5065	6964
$i=3$		6964	9992	$i=3$		6964	9992
		$j=1$	$j=2$			$j=1$	$j=2$
γ_j		0.7905	0.9610	γ_j		0.8485	0.9685
$\ln(\mathcal{L})$	-65059.96			$\ln(\mathcal{L})$	-64843.50		

Figure C.5: Aggregate Wage Offer Densities: 3-Point Employer Heterogeneity

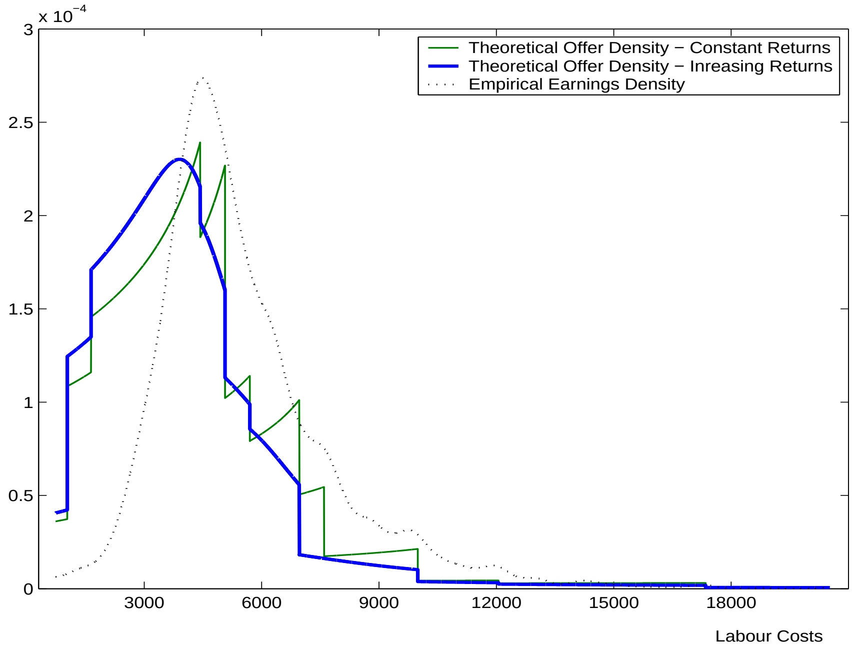
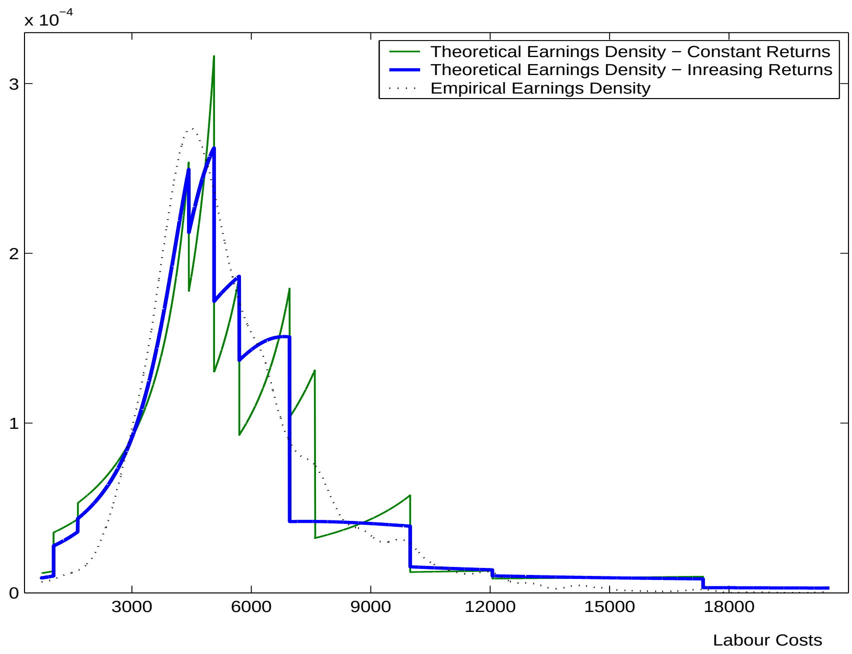


Figure C.6: Theoretical Earnings Densities: 3-Point Employer Heterogeneity



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Eidesstattliche Versicherung

Ich versichere hiermit eidesstattlich, dass ich die vorliegende Arbeit selbständig und ohne fremde Hilfe verfasst habe. Die aus fremden Quellen direkt oder indirekt übernommenen Gedanken sowie mir gegebene Anregungen sind als solche kenntlich gemacht.
Die Arbeit wurde bisher keiner anderen Prüfungsbehörde vorgelegt und auch noch nicht veröffentlicht.

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