Education and Asymmetric Information in the Labor Market
Three Essays in Search Theory

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Preface

Observable worker characteristics that are supposed to account for differences between worker productivities explain only about 30% of the variation in wages. The remaining 70% could be accounted for by unobservable worker characteristics. Systematic differences in wages like the persistence of inter-industry wage differentials over time and their similarity across countries (Katz and Summers, 1988), the firm-size wage effect (Davis and Haltiwager, 1996), and the change of the wage distribution over time after accounting for observable worker characteristics (Katz and Autor, 1999) suggest that similar workers are paid differently.

Using matched employer employee data sets Abowd, Kramarz and Margolis (1999) and Abowd, Finer and Kramarz (1999) decompose industry and firm size differentials into average employer and worker components. The importance of the average employer effect in explaining these systematic differences indicate that firms have some scope to set a particular wage policy. As manifold as worker pay policies can be, so are the different wage formation assumptions used in different theories. The use of different wage policies also suggests that workers cannot easily move between employers and compare wage offers. These mobility frictions have many different sources. In the following, I briefly discuss the different driving forces governing the wage formation process and the matching of workers and jobs.

Matching function

Pissarides (2000), in his book "Equilibrium Unemployment Theory", argues that decentralized trade in the labor market is characterized by coordination failure and involves costly information acquisition about the productivity of workers and jobs as well as their location. Backed by a lot of empirical evidence he suggests that the various frictions can best be captured by a general matching function with constant returns to scale. Similar to a production function, the inputs into a matching function, i.e. the number of vacancies v and searching workers s, result in a specific number of matches $m(v,s)$. The number of matches divided by the number of vacancies gives the matching probability for a vacancy and the number of matches divided by the number of searching workers gives the matching probability of a worker. Using the constant returns to scale assumption implies that the matching probability can be written as a function of the labor market tightness $\theta = v/s$ defined as the ratio of vacancies to searching workers. The matching probability of a worker increases in θ and the matching probability of a vacancy decreases in θ.

Hall (1979), Pissarides (1979), Blanchard and Diamond (1994), and many other authors including Acemolgu and Shimer (1999) have used the exponential form derived from the urn-ball game as matching function. The urn-ball game assumes that workers' arrival at vacancies follows a Poisson process. Thus, with probability $1 - e^{-1/\theta}$ at least one worker applies for a vacancy and with the ex-ante probability $\left(1 - e^{-1/\theta}\right)/\theta$ a worker is offered a job. While the derivation based on the urn-ball game is mechanical Burdett, Shi, and Wright (2001) derive the matching function as the most stable mixed strategy equilibrium in a model where sellers post prices and buyers decide whom to visit given the observed prices. Other models deriving matching functions from primative assumptions include Montgomery (1991) who allows for strategic price setting of firms that take the outside option of workers as given, Lagos (2000) who endogenizes the moving decision of agents, Coles and Smith (1998) who propose an alternative matching game where the stock of unemployed workers can only match with the flow of incoming vacancies and vice versa.

Wage formation

The early search literature assumed an exogenously given wage dispersion to motivate the searching process and they derived a reservation wage rule. In these models unemployment arises because workers rejected low wage offers. This led Rothschild (1973) to his criticism that in equilibrium firms should not offer wages that are not accepted and he asked for wage dispersion to be derived as an equilibrium outcome.

Diamond (1971) presented the first equilibrium search model where firms make publicly not observable take-it-or-leave-it wage offers to workers who have to decide immediately whether to accept the wage offer before sampling another offer. This sequential search assumption implies that workers cannot induce firms to compete with their wage offers. Furthermore, since wage offers are not publicly observable workers cannot chose which firm to contact. Thus, firms have all the bargaining power and offer a monopsony wage which takes away the incentive for workers to search. This is called the Diamond paradox. By introducing bargaining over the match rent Diamond (1982), Mortensen (1982) and Pissarides (2000) ensured that workers get more than their reservation wage. Albrecht and Axell (1984), Salop and Stiglitz (1976) and others introduced ex-ante heterogeneity in search cost, information acquisition cost or production cost to generate a price or wage dispersion. However, the problem of whether wage dispersion can exist in models with identical workers and firms remained.

In order to solve the Diamond paradox one strand of the literature introduced some competition between firms by assuming that some workers get multiple offers. They maintained, however, the assumption that wage offers are private information. The non-sequential search models by Wilde (1977) and Burdett and Judd (1983) allow workers to sample more than one job offer before deciding. Given the non-sequential search behavior of workers Burdett and Judd (1983) prove that a wage dispersion exists if one proportion of workers sample only one job offer and the rest more than one job offer. Acemoglu and Shimer (2000) extend their finding and show that workers who sample only one job offer free-ride on workers who sample two job offers. This externality implies that in equilibrium there are too few workers sampling two job offers. Noisy search models (see also Burdett and Judd, 1983) assume that a worker incurs a fixed cost and receives several job offers with corresponding wages. The job offers are randomly distributed, where the distribution mechanism is assumed to be exogenous. Lately, Gautier and Moraga-González (2004) endogenize this distribution by allowing workers to make multiple applications. The job offer distribution results from the coordination failure between firms, since they do not know which worker was already contacted by a competing firm.

The Diamond paradox can also be overcome if workers are assumed to observe wage offers before they start to apply for a job. Models based on that assumption are called directed search models or competitive search models. This literature started with Montgomery (1991), Moen (1997), and Acemoglu and Shimer (1999). The fact that wages are publicly observable does not imply that workers are paid their marginal product or that wages are unique. The reason is that the coordination failure among workers implies that several workers can apply for the same job. If a firm offers a high wage, then the likelihood is high that it attracts many workers. At the same time each single worker has a low probability of getting the job. In addition firms cannot expand the demand for labor without bounds, since they face some cost for opening a vacancy, which implies that matching frictions prevail. The exact probability of being matched depends on the formulation of the matching function (see below). In equilibrium workers might be indifferent between joining a long queue at a job offering a high wage or applying for a job offering a low wage, where the probability of getting that job is high. Regardless of whether multiple equilibria and wage dispersion exist or not, the fact that wages are publicly observable implies that firms offer a wage that maximizes the workers life-time utility. Since firms make zero profit, maximizing the workers' utility is equivalent to maximizing social welfare.

Burdett and Mortensen (1998) kept the assumption of sequential search but introduced on-the-job search. This also implies competition between firms, because a potential future employer has to offer a higher wage than the worker is currently earning in order to poach him. The model is able to explain the firm-size wage effect, since firms offering a high wage have a high steady-state labor force. The underlying intuition is that a firm offering high wages attracts more workers from firms offering lower wages and loses fewer workers to employers paying higher wages. Firms are, however, indifferent between offering a low or a high wage, since a high wage leads not only to a large firm but also implies a low profit per worker. The reverse is true for firms paying a low wage. Thus, the model generates a continuous wage distribution. Unfortunately, the predicted wage offer distribution has an increasing convex density which is not compatible with the data.

In order to generate a more realistically shaped wage distribution, researchers extended the simple model by Burdett and Mortensen (1998). Mortensen (1990) and Burdett and Mortensen (1998) combine their simple model with atomless ex-ante worker heterogeneity as in the model of Albrecht and Axell (1984). This does, however, generally not generate a right tailed wage distribution. Mortensen (1990) introduced differences in firm productivity and showed that more productive employers pay higher wages. Bontemps, Robin and van den Berg (2000) and Burdett and Mortensen (1998) formulate a closed form solution for a continuous productivity distribution. While the structural estimates of the models with continuous productivity dispersion as suggested by Bontemps, Robin and van den Berg (2000) and Postel-Vinay and Robin (2002) improve the fit to the empirical wage earnings distribution, they tell us nothing about the production parameters governing the assumed productivity dispersion.

Burdett and Mortensen (1998) assume a constant offer arrival rate, because they concentrate on the steady state wage structure and abstract from productivity or other shocks affecting the labor market tightness over the business cycle. As Mortensen (2000) shows the restriction to a constant Poisson arrival rate of offers can be relaxed to allow for a general matching function.

Content of the dissertation

The extension of the Burdett-Mortensen model presented in chapter 1 allows for a flexible production function that incorporates and links different skill groups. Complementarity between different skill groups in the production process is shown to generate a positive intra-firm wage correlation across different skill groups, a result backed by empirical findings by Barth and Dale-Olsen (2003). The flexibility of the production function allows for decreasing and increasing returns to scale, where the later can generate a unimodal wage distribution with a long right tail without any ex-ante productivity dispersion.

The extension in chapter 1, therefore, provides a method of estimating the production parameters governing the productivity dispersion that generates a unimodal wage distribution. Holzner and Launov (2005) fit the wage distribution for low, medium and high skilled workers in Germany and find that increasing returns to scale estimates fit the wage distribution quite well.

In chapter 2 I focus on the asymmetric information that exists between the current employer and the outside market regarding a worker's productivity. The model shows that for severe enough search frictions, a market for employed workers with wage gains emerges despite the presence of adverse selection. Asymmetric information about a worker's productivity between the worker's current employer and the outside market enables the current employer to keep its best employees from joining the outside market by promoting them or by making them counter offers. Since outside wage offers are uncertain, firms promote or make counter offers only to their best workers. The resulting adverse selection, though, leads to an initial breakdown of the market for employed workers. As low-productivity workers are laid off over time, tenure serves as a positive signal about a worker's productivity. After enough badly performing workers were laid off, the signal is strong enough to counteract the negative effect of adverse selection and a market for employed workers emerges.

Within this framework it is possible to explain several empirical findings about the job mobility and wage dynamics of young workers. The model presented in chapter 2 can explain that movers outperform stayers in the short run but in the long run the on average higher productivity of stayers implies that they earn more in the long run. The model also explains that the hazard of job termination increases initially but is decreasing in the long run. The learning aspect dealt with in chapter 2 is related to Jovanovic (1979, 1984) and leads for the same reasons to a wage tenure effect. Jovanovic, however, analyzes a framework where workers and their employers learn about the productivity of a match, which implies that there is no asymmetric information between the current employer and the outside market about the worker's productivity. In chapter 2 it is also shown that a market for employed workers paying higher wages than the current employer might not exist. One reason for non-existence could be that a high matching rate for employed workers induces current employers to promote their best workers in order to prevent them from starting to search. The resulting adverse selection can lead to a breakdown of the market.

In chapter 3 I use a general matching function to model the frictions in the labor market and assume, as first done by Pissarides (1979), that firms open vacancies as long as the value of a vacancy is positive. This drives the profits of opening a new position down to zero. The fact that firms' profits are driven down by vacancy creation is central to the result in chapter 3 that search frictions per se do not imply that firms invest inefficiently in general training. The zero profit condition resulting from the creation of vacancies implies that future employers of trained workers do not profit from the training in other firms. This allows the training firm and its trainee to enter into a long-term contract that guarantees that the training level will be efficient if workers are not credit constrained.

Chapter 3 also shows that it is more expensive for firms to hire high skilled workers than low skilled workers, which provides an incentive for firms to provide some training for unskilled workers. In addition firms want to reduce turnover of the newly trained worker and promote workers as soon as possible to take away their incentive to search for an outside job. Thus, training firms demand a lump-sum payment equivalent to the value of the promotion from workers that are not credit constrained and promote them immediately. Credit constrained workers are paid a trainee wage of zero and promoted only with some probability making them indifferent between staying unskilled and being trained. Only if labour mobility is very high and workers are credit constrained, then workers will gain from training.

Chapter 1

The Role of Skill Groups and the Production Function for the Shape of the Wage Distribution

This chapter extends the Burdett-Mortensen model of on-the-job search by introducing different skill groups and links them via a general production function. Depending on the degree of homogeneity of the production function the model generates a log-normal-shaped wage earnings distribution. The extended model provides a method of estimating technological substitution elasticities between the different skill groups and of estimating the returns to education taking skill specific unemployment into account.

1.1 Introduction

It is generally believed that the shape of the wage earnings distribution is determined by the skill distribution of the work force, the production technology employed by the economy, the search and matching frictions that govern the allocation of workers to jobs and by institutions like a legally bounded minimum wage. The aim of this chapter is to provide a theoretical and yet empirically tractable model that takes all these factors and its interactions into account. In order to do so I extend the on-the-job search equilibrium model by Burdett and Mortensen (1998) since this model generates an explicit functional form for the wage offer and earnings distribution, which allows to estimate the parameters that influence the wage distribution. I extend it by introducing different skill groups and link them via a general production function with constant or increasing returns to scale. The extension to different skill groups allows for the analysis of firms' wage posting behavior, where firms simultaneously compete for workers of different skill groups. It is shown that supermodularity in production implies a positive correlation between the wages of workers in different skill groups within firms. The extension to a production function with increasing returns to scale can generate a unimodal wage offer and earnings distributions with a long right tail and provides a method of deriving the technological substitution elasticities between skill groups from the wage earnings distribution.

The original Burdett-Mortensen model introduces on-the-job search into the search literature and is thus able to generate a wage offer distribution. The reason for the existence of a wage distribution is that firms offering higher wages attracted more workers from firms offering lower wages and lose less workers to employers paying higher wages. This leads to a higher steady-state labor force for firms offering higher wages. In equilibrium firms are indifferent between paying low or high wages since high wages imply not only a large labor force but also a low profit per worker. The reverse is true for firms offering low wages.

Since the endogenous wage distribution generated by the original Burdett-Mortensen model has an upward-sloping density, which is at odds with the empirical observation of a flat right tail, there has been a lot of effort to extend the original model in order to generate a more realistic-shaped wage distribution. Mortensen (1990) introduced differences in firm productivity and Bowlus et al. (1995) showed that this greatly improves the fit to the empirical wage distribution. Bontemps et al. (2000) and Burdett and Mortensen (1998) formulated a closed-form solution for a continuous atomless productivity distribution, which translates into a right-tailed wage earnings distribution, depending on the assumed productivity dispersion. A model with endogenous employer heterogeneity is given by Mortensen (2000) who shows that wage dispersion gives rise to different investment incentives for employers such that higher-paying employers invest more in match-specific capital.

This extension is the first one to allow for the possibility that the production function incorporates several skill groups and exhibits increasing returns to scale, which can generate a log-normal shaped wage offer and earnings density given one production technology. While the structural estimates of the models with continuous productivity dispersion as suggested by Bontemps et al. (2000) and Postel-Vinay and Robin (2002) improve the fit to the empirical wage earnings distribution and thus the estimates of the labor market transition rates, they tell us nothing about the production parameters governing the assumed productivity dispersion – a critique already mentioned by Manning (2003, p. 106f).

The advantage of having one or few production technologies is to enable empirical researchers to derive structural estimates of the production technology from the wage earnings distribution and the data on durations of employment or unemployment.[1] So far the literature has derived technology parameters from estimates of labor demand functions (for a summary of the literature see Hamermesh, 1996). This literature assumed a competitive labor market with (or without) adjustment costs. Thus, firms react to current or expected changes in relative input prices and output by adjusting their labor force accordingly. Manning (2003) and Barth and Dale-Olsen (2002) found,

[1] I also allow for employer heterogeneity, i.e. that different technologies are used by different firms, in order to make the model more attractable for empirical research (see Holzner and Launov, 2005).

however, that firms face a locally upward sloping labor supply curve which gives them the monopsony power to set wages. Thus, firms are able to adjust their labor force by either offering higher or lower wages. If this is the case, then the estimates of production parameters in the traditional studies should be biased.

Holzner and Launov (2005) show that the extension of the Burdett-Mortensen model to three skill groups and three Cobb-Douglas production technologies fits the data quite well. They use panel data of German full-time, male workers to estimate the values of being a new market entrant as low and high skilled workers.

Finally, I demonstrate in this chapter that whenever skills are complementary in the production process, a positive within-firm correlation between wages of workers with different skills exists. Positive wage correlation within firms is a well established fact, empirical evidence of which are presented in Katz and Summers (1989) and Barth and Dale-Olsen (2003) among many others. Theoretical consideration of the issue is performed by Kremer (1993). In his O-ring theory Kremer also uses a production function that exhibits complementarity of the working colleagues' abilities not to make a mistake when performing a sequence of tasks in order to complete the final good. Exploiting this property, Kremer shows that firms recruit equal ability types for each task. Since higher ability types are more productive, they are paid higher wages in a competitive market. Thus wage differentials in the O-ring theory are attributable to workers' ability, whereas wage differentials in this chapter are explained by the monopsony power of firms to set wages. The O-ring theory also suggests that wages increase with the number of tasks performed in a firm, since only high ability workers are able to produce goods that require many tasks. Thus, the O-ring theory implies a positive correlation between wages and the number of tasks and therefore the overall size of the workforce, whereas this model predicts a positive correlation between skill-group size and wages. However, Barth and Dale-Olsen (2002) find empirically that the employer-size wage effect vanishes once they look at the skill-group size. This provides some evidence in favor of the labor market frictions approach of this model rather then O-ring theory of Kremer (1993).

This chapter proceeds as follows. The theory is presented in Section 2, where

I extend the existing Burdett-Mortensen framework, solve for optimal strategies of workers and firms and define the labor market equilibrium. The discussion of the properties of the resulting equilibrium wage offer distribution is treated in Section 3. Section 4 presents the results of Holzner and Launov (2005) who take this theoretical model to the data. Section 5 concludes.

1.2 The Model

In this section I extend the original Burdett-Mortensen model of on-the-job search by introducing different skill groups and different technologies that link the skill groups via the production function.

Framework

The model has an infinite horizon, is set in continuous time and concentrates on steady states. Workers are assumed to be risk neutral and to discount at rate r. Each worker belongs to a skill group $i = 1, 2, ..., I$ whose measures are defined as q_i, satisfying $\sum q_i = m$. The measure u_i of workers is unemployed and the measure $q_i - u_i$ is employed. Before choosing a skill-group workers incur a one-off cost c_i for skill-specific education. By assuming perfect capital market workers are able to borrow the cost of education.

Workers search for a job in the skill-segmented labor markets. With probability λ_i unemployed workers of skill group i encounter a firm that makes them a wage offer corresponding to their education, and with probability λ_e employed workers encounter a firm.[2] Then workers decide whether to accept or reject the job offer. Job-worker match is destroyed at an exogenous rate $\delta > 0$. Laid off workers start again as unemployed.

I assume that there exist J distinct production technologies $Y_j\left(\mathbf{l}\left(\mathbf{w} \mid \mathbf{w^r}, F\left(\mathbf{w}\right)\right)\right)$ indexed by j, where $\mathbf{l}\left(\mathbf{w} \mid \mathbf{w^r}, F\left(\mathbf{w}\right)\right)$ is the vector of skill groups $l_i\left(w \mid w_i^r, F_i\left(w\right)\right)$ employed by a firm with technology j. The size $l_i\left(w \mid w_i^r, F_i\left(w\right)\right)$ of the skill group depends on the firm's wage offer w_i, the workers' reservation wage w_i^r and the skill specific wage offer distribution $F_i(w)$. I further assume that the production function

[2] λ_e is not skill group specific, since we would otherwise not be able to derive an explicit wage offer distribution function.

$Y_j(\mathbf{l}(\mathbf{w} \mid \mathbf{w^r}, F(\mathbf{w})))$ is supermodular in $\mathbf{l}(\mathbf{w} \mid \mathbf{w^r}, F(\mathbf{w}))$, i.e. has increasing differences in $\mathbf{l}(\mathbf{w} \mid \mathbf{w^r}, F(\mathbf{w}))$ as defined below, and is twice continuously differentiable in $l_i(w \mid w_i^r, F_i(w))$.

Definition 1: For any $\mathbf{l} \equiv \mathbf{l}(\mathbf{w} \mid \mathbf{w^r}, F(\mathbf{w}))$ and $\mathbf{l'} \equiv \mathbf{l'}(\mathbf{w} \mid \mathbf{w^r}, F(\mathbf{w}))$, $Y_j(\mathbf{l})$ is *supermodular* in $\mathbf{l}$, if

$$Y_j(\mathbf{l} \wedge \mathbf{l'}) + Y_j(\mathbf{l} \vee \mathbf{l'}) \geq Y_j(\mathbf{l}) + Y_j(\mathbf{l'}),$$

where $l \vee l' \equiv (\max(l_1, l'_1), ..., \max(l_I, l'_I))$ and $l \wedge l' \equiv (\min(l_1, l'_1), ..., \min(l_I, l'_I))$. Supermodularity in l_i implies *increasing differences* in l_i, i.e. for $\mathbf{l} \geq \mathbf{l'}$ it follows that

$$Y_j(l_i, \mathbf{l}_{-i}) + Y_j\left(l'_i, \mathbf{l'}_{-i}\right) \geq Y_j\left(l_i, \mathbf{l'}_{-i}\right) + Y_j(l'_i, \mathbf{l}_{-i}),$$

where $-i$ denotes the vector of all skill groups except i.

Firms maximize profits by offering a wage schedule $\mathbf{w} = (w_1, w_2, ..., w_I) = (w_i, \mathbf{w}_{-i})$.

Workers' Search Strategy

The optimal search strategy for a worker of occupation i is characterized by a reservation wage w_i^r, where an unemployed worker is indifferent between accepting or rejecting a wage offer, i.e. $U_i = V_i(w_i^r)$, where U_i is the value of being unemployed and $V_i(w_i^r)$ the value of being employed at the reservation wage w_i^r. Flow values of being unemployed and employed

$$rU_i = \lambda_i \int_{w_i^r}^{\bar{w}_i} (V_i(x_i) - U_i)\, dF_i(x_i) - c_i, \tag{1.1a}$$

$$rV_i(w_i) = w_i + \lambda_e \int_{w_i}^{\bar{w}_i} (V_i(x_i) - V_i(w_i))\, dF_i(x_i) + \delta(U_i - V_i(w_i)) - c_i \tag{1.1b}$$

respectively, can be solved for a reservation wage[3]

$$w_i^r = (\lambda_i - \lambda_e) \int_{w_i^r}^{\bar{w}_i} \left(\frac{1 - F_i(x)}{r + \delta + \lambda_e(1 - F_i(x^-))} \right) dx. \tag{1.2}$$

In order to keep the analysis simple, for the remainder of the paper I assume that $r/\lambda_i \to 0$ as done in the original model by Burdett and Mortensen (1998). The wage

[3]The details of the derivation can be found in Mortensen and Neumann (1988).

offer distribution is given by $F_i(w) = F_i(w^-) + \upsilon_i(w)$, where $\upsilon_i(w)$ is the mass of firms offering wage w to skill group i. Since offering a wage lower than the reservation wage does not attract any worker, I assume with out loss of generality that no firm offers a wage below the reservation wage, i.e. $F_i(w) = 0$ for $w < w_i^r$.

Steady State Flows and Skill Group Size

Equating the flows in and out of unemployment gives the steady state measure of unemployed per skill group, i.e.

$$u_i = \frac{\delta}{\delta + \lambda_i} q_i. \tag{1.3}$$

Given the assumptions of constant Poisson arrival rates λ_i, λ_e and the constant separation rate δ Mortensen (1999) has shown that skill group size evolves according to a special Markov-chain known as stochastic birth-death process.

The birth rate of a job offered by a firm posting a wage w is given by the average rate at which a job is filled. There are u_i unemployed who leave unemployment at rate λ_i and $(q_i - u_i)$ employed workers who leave their current employer at rate $\lambda_e G_i(w^-)$ to join the firm offering a wage w, where $G_i(w) = G_i(w^-) + \vartheta_i(w)$ denotes the cumulative wage earnings distribution for skill group i. A worker-employer pair split at rate δ or a worker receives a higher wage offer from another firm, which occurs at rate λ_e, and accepts it, which happens with probability $\overline{F}_i(w) \equiv (1 - F_i(w))$. The death rate of a job is, therefore, given by $\delta + \lambda_e \overline{F}_i(w)$. Mortensen (1999) shows that the skill group size is Poisson distributed with mean

$$E\left[l_i\left(w \mid w_i^r, F_i\left(w\right)\right)\right] = \frac{\lambda_i u_i + \lambda_e G_i(w^-)(q_i - u_i)}{\delta + \lambda_e \overline{F}_i(w)}.$$

Equating the inflow and outflow gives the steady-state measure of employed workers earning a wage less than w

$$G_i(w^-)(q_i - u_i) = \frac{\lambda_i F_i(w^-) u_i}{\delta + \lambda_e \overline{F}_i(w^-)}. \tag{1.4}$$

Substituting gives

$$E\left[l_i\left(w \mid w_i^r, F_i\left(w\right)\right)\right] = \frac{\delta \lambda_i \left(\delta + \lambda_e\right) / \left(\delta + \lambda_i\right)}{\left[\delta + \lambda_e \overline{F}_i(w)\right]\left[\delta + \lambda_e \overline{F}_i(w^-)\right]} q_i, \tag{1.5}$$

From (1.5) it follows that the expected skill group size $E\left[l_i\left(w \mid w_i^r, F_i\left(w\right)\right)\right]$ is (i) increasing in w, if $w \geq w_i^r$, (ii) continuous except where $F_i\left(w\right)$ has a mass point and is (iii) strictly increasing on the support of $F_i\left(w\right)$ and constant on any connected interval off the support of $F_i\left(w\right)$. The intuition behind this result is that on-the-job search implies that the higher the wage offered by a firm the more employed workers are attracted from firms offering lower wages and the less workers quit to employers paying higher wages. This leads to a higher steady-state skill group size for firms offering higher wages. For notational simplicity from now on I use $l_i\left(w\right)$ instead of $l_i\left(w \mid w_i^r, F_i\left(w\right)\right)$.

Wage Posting

Each firm posts a wage schedule $\mathbf{w}$ in order to maximize its profit, taking as given the workers' search strategy, i.e. the reservation wage vector $\mathbf{w^r}$, and the other firms' wage posting behavior, i.e. $F\left(\mathbf{w}\right)$.

$$\pi_j = \max_{\mathbf{w}} E\left[Y_j\left(\mathbf{l}\left(\mathbf{w}\right)\right) - \mathbf{w}^T \mathbf{l}\left(\mathbf{w}\right)\right].$$

The expectation operator in the equation above is over all possible realizations of the different skill group sizes $l_i\left(w \mid w_i^r, F_i\left(w\right)\right)$ a firm can realize given its choice of the wage schedule and the birth-death process characterized above. Hence, in the steady state a firm might choose to adjust its wage policy according to the realizations of the different skill group sizes $l_i\left(w \mid w_i^r, F_i\left(w\right)\right)$. Since this problem is intractable, I assume that a firm can specify its wage policy $\mathbf{w}$ only once. This implies that I can write the maximization problem of a type j firm as

$$\pi_j = \max_{\mathbf{w}} \left[Y_j\left(E\left[\mathbf{l}\left(\mathbf{w}\right)\right]\right) - \mathbf{w}^T E\left[\mathbf{l}\left(\mathbf{w}\right)\right]\right]. \tag{1.6}$$

Denote by $\mathbf{W}_j$ the set of wage offers that maximize equation (1.6), i.e. $\mathbf{W}_j = \arg\max_{\mathbf{w}} \pi_j$, and the corresponding I-dimensional wage offer distribution for each firm type j by $F_j\left(\mathbf{w}\right) = \left(F_{1j}(w), F_{2j}(w), ..., F_{Ij}(w)\right)$, where $F_{ij}(w)$ denotes the wage offer distribution of type j firms for skill group i.

Definition 2: A *steady state wage posting equilibrium* is a wage offer distribution

$F_j(\mathbf{w})$ with $\mathbf{w} \in \mathbf{W}_j$ for each firm type $j \in J$ such that

$$\begin{aligned}
\pi_j &= Y_j\left(E\left[\mathbf{l}(\mathbf{w})\right]\right) - \mathbf{w}^T E\left[\mathbf{l}(\mathbf{w})\right] \text{ for all } \mathbf{w} \text{ on the support of } F_j(\mathbf{w}), \quad (1.7)\\
\pi_j &\geq Y_j\left(E\left[\mathbf{l}(\mathbf{w})\right]\right) - \mathbf{w}^T E\left[\mathbf{l}(\mathbf{w})\right] \text{ otherwise,}
\end{aligned}$$

given the reservation wage w_i^r for each skill group $i = 1, 2, ..., I$ and a corresponding skill group wage offer distribution $F_i(w)$ such that the reservation wage w_i^r satisfies equation (1.2) given $F_i(w)$.

1.3 Properties of the Wage Offer Distribution

Following Mortensen (1990) I next describe the properties of the aggregate and the skill specific wage offer distributions.

Given the supermodularity property of the production function and the fact that the expected skill group size given in equation (1.5) is increasing in w and upper semi-continuous implies that profits π_j are supermodular in w_i. Thus, a firm paying higher wages for one skill group also pays higher wages for another skill group.

Proposition 1: *Take a firm of type $j \in [1, J]$ offering $w \in W_j$ and another firm of type j offering $w' \in W_j$, where w and $w' \geq w^{\mathbf{r}}$, then either $w \geq w'$ or $w \leq w'$.*

Proof: For any $\mathbf{w}$ and $\mathbf{w}' \geq \mathbf{w}^{\mathbf{r}}$, $\pi_j(w_i, \mathbf{w}_{-i})$ is supermodular, i.e.

$$\pi_j\left(w_i \wedge w_i', \mathbf{w}_{-i} \wedge \mathbf{w}'_{-i}\right) + \pi_j\left(w_i \vee w_i', \mathbf{w}_{-i} \vee \mathbf{w}'_{-i}\right) \geq \pi_j(w_i, \mathbf{w}_{-i}) + \pi_j\left(w_i', \mathbf{w}'_{-i}\right),$$

because the same inequality holds for output $Y_j\left(E\left[\mathbf{l}(w_i, \mathbf{w}_{-i})\right]\right)$ and the wage cost cancel out.

Now, I prove $\mathbf{w} \geq \mathbf{w}'$ by contradiction. For any $\mathbf{w}$ and $\mathbf{w}' \in \mathbf{W}_j$ with $w_i > w_i'$, suppose $\mathbf{w}_{-i} < \mathbf{w}'_{-i}$. The following chain of inequalities results in the desired contradiction.

$$\begin{aligned}
0 &< \pi_j(w_i, \mathbf{w}_{-i}) - \pi_j\left(w_i \vee w_i', \mathbf{w}_{-i} \vee \mathbf{w}'_{-i}\right)\\
&\leq \pi_j\left(w_i \wedge w_i', \mathbf{w}_{-i} \wedge \mathbf{w}'_{-i}\right) - \pi_j\left(w_i', \mathbf{w}'_{-i}\right) < 0
\end{aligned}$$

The first and the last inequality result from optimality of $\mathbf{w}$ and $\mathbf{w}'$, the second inequality comes from the supermodularity shown above. $\square$

This positive correlation between the wages of workers in different skill groups within firms is a well established fact. Katz and Summers (1989) show evidence that secretaries earn more in firms where average wages are higher. More recently, Barth and Dale-Olsen (2003) find that "[h]igh-wage establishments for workers with higher education are high-wage establishments for workers with lower education as well". The explanation provided for this empirical observation in this paper rests on two pillars. Firstly, labor market frictions lead to an upward sloping labor supply curve for each skill group which can be seen from equation (1.5). Secondly, I need the complementarity of the skill groups in the production process. This guarantees that increasing both labor inputs simultaneously is optimal. The empirical regularity mentioned above justifies my choice of the production function, where labor inputs are complements.

Note, that Proposition 1 does not guarantee that a firm occupies the same position in the wage offer distribution of all skill groups, because it is possible that there is a mass point in the wage offer distribution of skill group i but not in the wage offer distribution in the other $-i$ skill groups.

Given that the skill group size is increasing in the wage w_i, it would be a waist of money, if the support of the wage offer distributions was not a compact set.

Lemma 1: *The support of each skill specific wage offer distribution $F_i(w)$ is a compact set, i.e. $supp(F_i) = [w_i^r, \overline{w}_i]$.*

Proof: Suppose not, i.e. no firms offer a wage $w_i \in (w_i^*, w_i^{**}) \subset [w_i^r, \overline{w}_i]$. This cannot be profit maximizing, since the firm offering w_i^{**} can offer $\lim_{\varepsilon \to 0} (w_i^* + \varepsilon)$, have the same skill group size, i.e. $l_i (w_i^{**} \mid w_i^r, F_i(w_i^{**})) = \lim_{\varepsilon \to 0} l_i ((w_i^* + \varepsilon) \mid w_i^r, F_i(w_i^* + \varepsilon))$, since $\lim_{\varepsilon \to 0} F_i(w_i^* + \varepsilon) = F_i(w_i^{**})$, and can thus make higher profit. Thus, the support of the wage offer distribution is connected. By the same argument w_i^r is part of the support. The equal profit condition (1.7) together with the equation for the skill group size (1.5) implies that the support is also closed at the upper end. $\square$

Firms with different technologies j make potentially different profits π_j in equilibrium, compare equation (1.7). I index the technologies according to their profitability, i.e. $\pi_j \geq \pi_{j-1} \forall j = 1, 2, ..., J$. The next proposition shows that for any skill group i more profitable firms pay higher wages.

Proposition 2: *Let $F_j : supp(F_j) = [\underline{\mathbf{w}}_j, \overline{\mathbf{w}}_j]$ and $F_{j-1} : supp(F_{j-1}) = [\underline{\mathbf{w}}_{j-1}, \overline{\mathbf{w}}_{j-1}]$ be the I-dimensional wage offer distributions of j and $j-1$-type firms respectively. Then, for any wage schedule $w_j \in [\mathbf{w}^{\mathbf{r}}, \overline{\mathbf{w}}]$ and $w_{j-1} \in [\mathbf{w}^{\mathbf{r}}, \overline{\mathbf{w}}]$ it is true that $w_j \geq w_{j-1}$.*

Proof: From the steady state equilibrium condition (1.7) it follows that:

$$\begin{aligned} \pi_j &= Y_j\left(E\left[\mathbf{l}\left(\mathbf{w}_j\right)\right]\right) - \mathbf{w}_j^T E\left[\mathbf{l}\left(\mathbf{w}_j\right)\right] \quad \forall \mathbf{w}_j \in supp(F_j) \\ \pi_j &\geq Y_j\left(E\left[\mathbf{l}\left(\mathbf{w}_{j-1}\right)\right]\right) - \mathbf{w}_{j-1}^T E\left[\mathbf{l}\left(\mathbf{w}_{j-1}\right)\right] \quad \forall \mathbf{w}_{j-1} \notin supp(F_j) \end{aligned}$$

Using the result above I can write

$$\begin{aligned} \pi_j &= Y_j(E\left[\mathbf{l}\left(\mathbf{w}_j\right)\right]) - \mathbf{w}_j^T E\left[\mathbf{l}\left(\mathbf{w}_j\right)\right] \geq Y_j(E\left[\mathbf{l}\left(\mathbf{w}_{j-1}\right)\right]) - \mathbf{w}_{j-1}^T E\left[\mathbf{l}\left(\mathbf{w}_{j-1}\right)\right] \\ &\geq Y_{j-1}(E\left[\mathbf{l}\left(\mathbf{w}_{j-1}\right)\right]) - \mathbf{w}_{j-1}^T E\left[\mathbf{l}\left(\mathbf{w}_{j-1}\right)\right] = \pi_{j-1} \geq Y_{j-1}(E\left[\mathbf{l}\left(\mathbf{w}_j\right)\right]) - \mathbf{w}_j^T E\left[\mathbf{l}\left(\mathbf{w}_j\right)\right], \end{aligned}$$

where the second inequality results from the fact that $\pi_j \geq \pi_{j-1}$.

The difference of the first and the last terms in this inequality is greater than or equal to the difference of its middle terms, i.e $Y_j(E\left[\mathbf{l}\left(\mathbf{w}_j\right)\right]) - Y_{j-1}(E\left[\mathbf{l}\left(\mathbf{w}_j\right)\right]) \geq Y_j(E\left[\mathbf{l}\left(\mathbf{w}_{j-1}\right)\right]) - Y_{j-1}(E\left[\mathbf{l}\left(\mathbf{w}_{j-1}\right)\right])$. Since $\mathbf{l}\left(\mathbf{w}\right)$ is an increasing function of wages $\mathbf{w}$, the claim follows. $\square$

In order to be able to identify a particular technology in the empirical estimation, I assume that technologies strictly dominate each other by profits, i.e. $\pi_j > \pi_{j-1}$. Since Proposition 2 holds true for any wage pair $\mathbf{w}_j, \mathbf{w}_{j-1}$ and thus also for $\underline{\mathbf{w}}_j = \inf \mathbf{w}_j$ and $\overline{\mathbf{w}}_{j-1} = \sup \mathbf{w}_{j-1}$, it follows that $\underline{\mathbf{w}}_j \geq \overline{\mathbf{w}}_{j-1}$. Thus, the more productive firms with technology j pay higher wages for all skill groups.

Furthermore, let γ_j denote the cumulative measure of technology j with $\gamma_j > \gamma_{j-1} > 0 \ \forall j = 1, 2, ..., J$ and $\gamma_J = 1$. Thus, Proposition 3 implies that the fraction of firms with technologies earning profit π_j or less post wages $\overline{\mathbf{w}}_j$ or below. Thus, for every skill group i the wage offer distribution at $\overline{w}_{ij}$is given by γ_j, i.e.

$$F_i\left(\overline{w}_{ij}\right) = \gamma_j \tag{1.8}$$

The next proposition shows under which condition it is not optimal for a type j firm to offer the same wage w_i as a mass of other type j firms does.

Proposition 3: *The wage offer distributions $F_i(w_i)$ of type j firms for skill group i is continuous, if*

$$\begin{aligned} & Y_j\left[E\left[l_i\left(w_i \mid w_i^r, F_i\left(w_i\right)\right)\right], E\left[\mathbf{l}\left(\mathbf{w}_{-i}\right)\right]\right] - Y_j\left[E\left[l_i\left(w_i \mid w_i^r, F_i\left(w_i^-\right)\right)\right], E\left[\mathbf{l}\left(\mathbf{w}_{-i}\right)\right]\right] \\ > \; & w_{ij}\left(E\left[l_i\left(w_i \mid w_i^r, F_i\left(w_i\right)\right)\right] - E\left[l_i\left(w_i \mid w_i^r, F_i\left(w_i^-\right)\right)\right]\right). \end{aligned} \tag{1.9}$$

If a mass point exists, then it can only exist at the upper bound of the support of $F_i(w_i)$, i.e. $F_i\left(w_i^-\right) = \gamma_j - \upsilon_i\left(\overline{w}_{ij}\right)$.
If the marginal product at the upper bound of the support of $F_i(w_i)$ exceeds $\overline{w}_{ij}$, then mass points can be ruled out, i.e.

$$\frac{\partial Y_j\left[E\left[\mathbf{l}\left(\overline{\mathbf{w}}\right)\right]\right]}{\partial E\left[l_i\left(\overline{w}_{ij} \mid w_i^r, \gamma_j\right)\right]} > \overline{w}_{ij}. \tag{1.10}$$

Proof: uppose a mass point exists at $w_i \in \left[\underline{w}_{ij}, \overline{w}_{ij}\right]$. Equation (1.6), and the fact that the cdf $F_i(w_i)$ is right continuous implies

$$\begin{aligned} & \lim_{\varepsilon \to 0} \pi_j\left(w_i + \varepsilon, \mathbf{w}_{-i}\right) + \mathbf{w}_{-i}^T E\left[\mathbf{l}\left(\mathbf{w}_{-i}\right)\right] \\ = \; & Y_j\left[E\left[l_i\left(w_i \mid w_i^r, F_i\left(w_i\right)\right)\right], E\left[\mathbf{l}\left(\mathbf{w}_{-i}\right)\right]\right] - w_i E\left[l_i\left(w_i \mid w_i^r, F_i\left(w_i\right)\right)\right] \qquad (1.11) \\ > \; & Y_j\left[E\left[l_i\left(w_i \mid w_i^r, F_i\left(w_i^-\right)\right)\right], E\left[\mathbf{l}\left(\mathbf{w}_{-i}\right)\right]\right] - w_i E\left[l_i\left(w_i \mid w_i^r, F_i\left(w_i^-\right)\right)\right] \\ = \; & \pi_j\left(\mathbf{w}\right) + \mathbf{w}_{-i}^T E\left[\mathbf{l}\left(\mathbf{w}_{-i}\right)\right] \end{aligned}$$

since $F_i(w_i) - F_i(w_i^-) = \upsilon_i(w_i) > 0$. If the above inequality holds, when a mass point exists at w_i.
To show that mass points can only exist at the upper bound of the support of $F_i(w_i)$ note that equation (1.5) together with Proposition 2 implies that $E\left[l_i\left(w_i \mid w_i^r, F_i\left(w_i\right)\right)\right]$ is strictly increasing in w_i on its support $\left[\underline{w}_{ij}, \overline{w}_{ij}\right]$, i.e. $\Delta E\left[l_i\left(w_i \mid w_i^r, F_i\left(w_i\right)\right)\right] / \Delta w_i >$

0. Using the equal profit condition (1.7) implies

$$\frac{\Delta E\left[l_i\left(w_i\right)\right]}{\Delta w_i} =$$

$$\frac{E\left[l_i\left(w_i\right)\right]}{Y_j\left[E\left[l_i\left(w_i\right)\right], E\left[\mathbf{l}\left(\mathbf{w}_{-i}\right)\right]\right] - Y_j\left[E\left[l_i\left(w_i^-\right)\right], E\left[\mathbf{l}\left(\mathbf{w}_{-i}\right)\right]\right] - w_i\left(E\left[l_i\left(w_i\right)\right] - E\left[l_i\left(w_i^-\right)\right]\right)},$$

where $E\left[l_i\left(w_i^-\right)\right] = E\left[l_i\left(w_i \mid w_i^r, F_i\left(w_i^-\right)\right)\right]$. This expression is only positive if and only if inequality (1.11) holds, i.e. only if no mass point exists. Thus, a mass point cannot exist in the interior of the support of $F_i\left(w_i\right)$ but only at the upper bound, i.e. $F_i\left(w_i^-\right) = \gamma_j - \upsilon_i\left(\overline{w}_{ij}\right)$.

Rewriting inequality (1.11) and using the fact that $F_i\left(w_i^-\right) = \gamma_j - \upsilon_i\left(\overline{w}_{ij}\right)$ gives

$$\frac{Y_j\left[E\left[l_i\left(w_i\right)\right], E\left[\mathbf{l}\left(\mathbf{w}_{-i}\right)\right]\right] - Y_j\left[E\left[l_i\left(w_i^-\right)\right], E\left[\mathbf{l}\left(\mathbf{w}_{-i}\right)\right]\right]}{E\left[l_i\left(w_i\right)\right] - E\left[l_i\left(w_i^-\right)\right]} > \overline{w}_{ij}.$$

A necessary condition for no mass point to exist obtains by letting $\upsilon_i\left(\overline{w}_{ij}\right) \to 0$, i.e.

$$\lim_{\upsilon_i\left(\overline{w}_{ij}\right)\to 0} \frac{Y_j\left[E\left[l_i\left(w_i\right)\right], E\left[\mathbf{l}\left(\mathbf{w}_{-i}\right)\right]\right] - Y_j\left[E\left[l_i\left(w_i^-\right)\right], E\left[\mathbf{l}\left(\mathbf{w}_{-i}\right)\right]\right]}{E\left[l_i\left(w_i\right)\right] - E\left[l_i\left(w_i^-\right)\right]} = \frac{\partial Y_j\left[E\left[\mathbf{l}\left(\overline{\mathbf{w}}\right)\right]\right]}{\partial E\left[l_i\left(\overline{w}_{ij} \mid w_i^r, \gamma_j\right)\right]}.$$

□

The basic argument as to why the wage offer distributions can be continuous is given by Burdett and Mortensen (1998). If all firms offer the same wage for one skill group, then individual firms could attract a significantly larger expected skill group size by offering a slightly higher wage. This wage increase can be arbitrarily small, whereas the resulting increase in the skill group size is significant, since all workers currently working for the "mass-point" wage will change to the new employer as soon as they get this higher wage offer. The deviation from a mass point is, thus, profitable if the increase in total output is higher than the increase in total wage cost induced by a slight wage increase. This is stated by the condition (1.9) in Proposition 4.

In order to be able to derive an explicit solution for the wage offer distribution, I continue under assumption that no mass points exist. If all wage offer distributions are continuous, then an immediate result of Proposition 1 is that a firm occupies the same position in the wage offer distribution of every skill group. To formalize this let us introduce an index k, which orders the firms of type j as they increase their wage offer for skill group 1 (i.e. firm $k = 1$ offers $\underline{w}_{1j}$), then Proposition 1 implies that for all $\mathbf{w} \in \mathbf{W}_j$

$$F_{ij}^k(w) = F_{lj}^k(w) \text{ for all } i, l = 1, 2, ..., I. \tag{1.12}$$

In order to be able to us the above property I introduce the following separation of a skill group size, where I rewrite the skill group size as

$$E\left[l_i\left(w \mid w_i^r, F_i(w)\right)\right] = r_{ij} h_j(w),$$

where

$$h_j(w) = \frac{\left[\delta + \lambda_e\left(1 - \gamma_{j-1}\right)\right]^2}{\left[\delta + \lambda_e \overline{F}_j(w)\right]\left[\delta + \lambda_e \overline{F}_j(w^-)\right]}, \qquad r_{ij} = \frac{\delta\left(\delta + \lambda_e\right)\lambda_i / \left(\delta + \lambda_i\right)}{\left[\delta + \lambda_e\left(1 - \gamma_{j-1}\right)\right]^2} q_i.$$

The fact that $h_j(w)$ depends only on the position the firm takes in the wage offer distribution, i.e. $F_j(w)$, implies that $h_j(w)$ does not depend on any skill specific parameter. Since I want to derive an explicit functional form for the wage offer distribution for each skill group i I additionally have to approximate the production technology j by using a second order Taylor Expansion around the minimum wage $\underline{w}_{ij}$ that firms with technology j post. Given a technology $Y_j(\mathbf{r}_j)$ is homogeneous of degree ξ_j the Taylor Expansion is given by

$$Y_j\left(\mathbf{l}\left(\mathbf{w}_j\right)\right) = Y_j\left(\mathbf{r}_j\right) + \sum\nolimits_i Y_j'\left(\mathbf{r}_j\right)\left[r_{ij} h_j(w) - r_{ij}\right] + \frac{1}{2}\sum\nolimits_i \sigma_{ij}\left[h_j(w) - 1\right]^2,$$

where

$$Y_j'\left(\mathbf{r}_j\right) = \frac{\partial Y_j\left(\mathbf{r}_j\right)}{\partial l_i} \quad \text{and} \quad \sigma_{ij} = \sum\nolimits_l \frac{\partial^2 Y_j\left(\mathbf{r}_j\right)}{\partial l_i \partial l_l} r_{lj} r_{ij} = \left(\xi_j - 1\right) Y_j'\left(\mathbf{r}_j\right) r_{ij}.$$

Using the results of Propositions 1-3 I invoke the equal profit condition $\pi_j = \pi_j^r$ and apply the Taylor Expansion and the first order condition to derive the skill-specific wage offer distribution. Proposition 4 provides the solution for $F_i(w_i)$ as a function of w_i.

Proposition 4: *Given that production functions $Y_j\left(E\left[\mathbf{l}(\mathbf{w})\right]\right)$ $\forall j = 1, 2, ..., J$ are supermodular and given that no mass point exists, then a unique equilibrium wage offer* *distribution $F_{ij}(w_i)$ for each skill group $i = 1, 2, ..., I$ exists that has the following form (i) for $\xi_j = 1$*

$$F_{ij}(w_i) = \frac{\delta + \lambda_e}{\lambda_e} - \frac{\delta + \lambda_e(1 - \gamma_{j-1})}{\lambda_e} \sqrt{\frac{Y_j'(\mathbf{r}_j) - w_i}{Y_j'(\mathbf{r}_j) - \underline{w}_{ij}}}, \tag{1.13}$$

(ii) for $\xi_j \neq 1$

$$F_{ij}(w_i) = \frac{\delta + \lambda_e}{\lambda_e} \tag{1.14}$$

$$- \frac{\delta + \lambda_e \left(1 - \gamma_{j-1}\right)}{\lambda_e \sqrt{\frac{\left(Y_j'(\mathbf{r}_j) - w_i\right) r_{ij} - \sigma_{ij} - \sqrt{\left(\left(Y_j'(\mathbf{r}_j) - w_i\right) r_{ij} - \sigma_{ij}\right)^2 + 4\left(\sigma_{ij} - \mu_{ij}\right)\left(\left(Y_j'(\mathbf{r}_j) - \underline{w}_{ij}\right) r_{ij} - \mu_{ij}\right)}}{-2\left(\sigma_{ij} - \mu_{ij}\right)}}}$$

for any $w_i \in [\underline{w}_{ij}, \overline{w}_{ij}]$, where

$$\mu_{ij} = \frac{r_{ij}}{\sum_i r_{ij}} \frac{1}{2} \sum\nolimits_i \sigma_{ij},$$

A necessary condition for an upward sloping wage offer density $f_{ij}(w_i)$ is

$$\left(2 - \xi_j\right) \frac{\partial Y_j(\mathbf{r}_j)}{\partial r_{ij}} - w_i > 0. \tag{1.15}$$

Proof: See Appendix. □

The aggregate wage offer distribution is given by

$$F(w) = \sum_{i=1}^{I} \frac{q_i}{m} F_i(w_i) = \sum_{i=1}^{I} \frac{q_i}{m} \sum_{j=1}^{J} \left(\gamma_j - \gamma_{j-1}\right) F_{ij}(w_i).$$

A special case for $F_{ij}(w_i)$ when $\left(Y_j'(\mathbf{r}_j) - \underline{w}_{ij}\right) r_{ij} = \mu_{ij}$ is shown in the proof of Proposition 5. Since it implies artificial restrictions on ξ_j considering this case here is neither interesting nor useful.

For a production function with homogeneity of degree one the explicit wage offer distribution resembles the distribution derived in Burdett and Mortensen (1998) and has its typical increasing density. Since an upward-sloping earnings density is at odds with the empirical observation of a flat right tail, Mortensen (1990) introduces differences in firm productivity by allowing for different productivity levels in order to improve the fit to the empirical wage earnings distribution. Bowlus et al. (1995) demonstrate that this greatly improves the fit to the empirical earnings distribution. Bontemps et al. (2000) and Burdett and Mortensen (1998) formulate a closed-form solution for a continuous atomless productivity distribution, which translates into a right-tailed wage earnings distribution, depending on the assumed productivity dispersion.[4]

The novelty is that the wage offer distribution given in Proposition 5 can have an increasing and a decreasing density for a given production technology. Although I allow for the possibility that heterogeneous production technologies are used, I do not need any technology dispersion to get a hump-shaped density. As stated in condition (1.15) only technologies with homogeneity of degree $2 > \xi_j$ can have an increasing density. Notice further that as the wage w increases condition (1.15) is more likely to be violated implying that the wage offer density can have an upward sloping part for small wages and an downward sloping part for large wages. A production technology with decreasing returns to scale would result in a negative wage offer density for at least one skill group, hence violate the first order condition and result in a violation of the continuity condition.

The reason for why increasing returns to scale can bend the wage offer density in such a way that is depicts a long right tail has its cause in the equal profit condition. Let us focus on the case with a homogenous production function with increasing returns to scale and compare it to an economy with constant returns to scale, where the marginal product of firms offering the reservation wage schedule are equivalent in both environments. First note that the skill group size is determined solely by the firm's position in the wage offer distribution. Thus, the shape of the wage offer distribution does not matter for the output generated. Due to increasing returns to scale the output of firms at the top of the wage offer distribution increases more than compared to an

[4]However, tail behavior of the productivity density, hence offer and earnings densities, in this case is subject to additional restrictions (see Bontemps et al., 2000; Proposition 8).

economy with constant returns to scale. In order for firms on the top of the wage offer distribution to make the same profits as firms at the lower end, the firms in an environment with increasing returns to scale have to pay higher wage in order to satisfy the equal profit condition as compared to firms in an environment with constant returns to scale who are at the same position of the wage offer distribution (except of course the firm offering the reservation wage schedule). Thus, the wage offer distribution in an economy with increasing returns to scale is more dispersed. If the returns to scale are large enough, the wage difference paid by "neighboring" firms at the upper end of the wage offer distribution increases generating a decreasing wage offer density.

Mortensen (2000) makes implicitly a similar restriction to production functions with increasing returns to scale when deriving endogenously the employer heterogeneity based on match specific capital. He assumes that the production technology has constant returns with respect to labor but on increasing economies of scale due to the capital k employed by the firm, i.e. $Y(l(w)) = k^{\alpha} l(w)$. By simulation he shows that for positive α the wage offer distribution has a flat right tail.

Finally, the comparative statics results of the original Burdett-Mortensen model are still valid for the general wage offer distribution function. If the arrival rate of on-the-job offers, i.e. λ_e, goes to zero, then the wage offer distribution $F_i(w)$ collapses to a mass point at the reservation wage w_i^r, which equals the Diamond (1971) monopsony solution. If moving from one job to another becomes very easy, i.e. λ_e goes to infinity, the competition among firms drives wages up and the wage earnings distribution $G_i(w)$ converges to a mass point at the marginal product of the skill group.

1.4 Empirical Evidence

Holzner and Launov (2005) use monthly data on duration and wages for full-time, male workers from the German Socio-Economic Panel – a longitudinal survey of German households – and estimate the model using three distinct Cobb-Douglas production function with low, medium and high skilled workers. The first remarkable result is that unconstrained estimates of the increasing returns specification meets the no-mass-point condition derived in Proposition 3, indicating that the estimates are consistent with the model.

The constant returns to scale specification has two jumps at the left tail and eight spikes at the right, however the locally increasing right tail of individual-specific densities renders the fit of the aggregate earnings density far from satisfactory. Relaxing the assumption of constant returns improves the picture (see figure 2). The predicted wage offer density is shown in figure 1.

Along with frictional parameters they also estimate the size of the returns to scale. According to their estimates the homogeneity degrees are 1.04 for the "low-productive" technology, 1.40 for the "medium-productive" technology and 4.92 for the "high-productive" one. Given the estimated fraction of each technology $[\gamma_j - \gamma_{j-1}]$ in the economy these estimates imply the economy-wide returns to scale at the level of 1.20. This goes in line with numerous evidences from the literature on the estimation of the returns to scale using different types of production functions. Typical estimates in this literature support the increasing returns hypothesis and range from about 1.1 to about 1.35 (see Färe at al., 1985, Kim, 1992, and Zellner and Ryu, 1998, and references therein). Second, and even more important, productivity dispersion along with increasing returns technologies also leads to a better fitting offer and labour costs densities.

Figure 1: "Aggregate Wage Offer Densities: 3-Point Employer Heterogeneity"

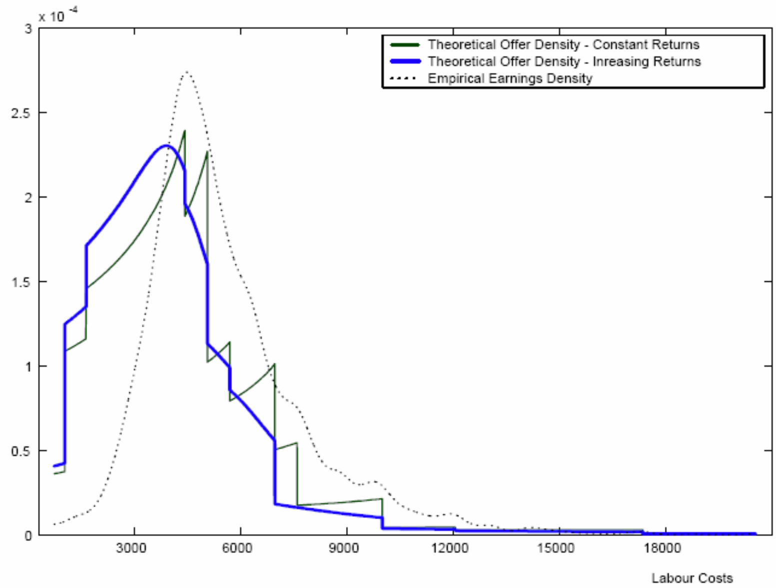
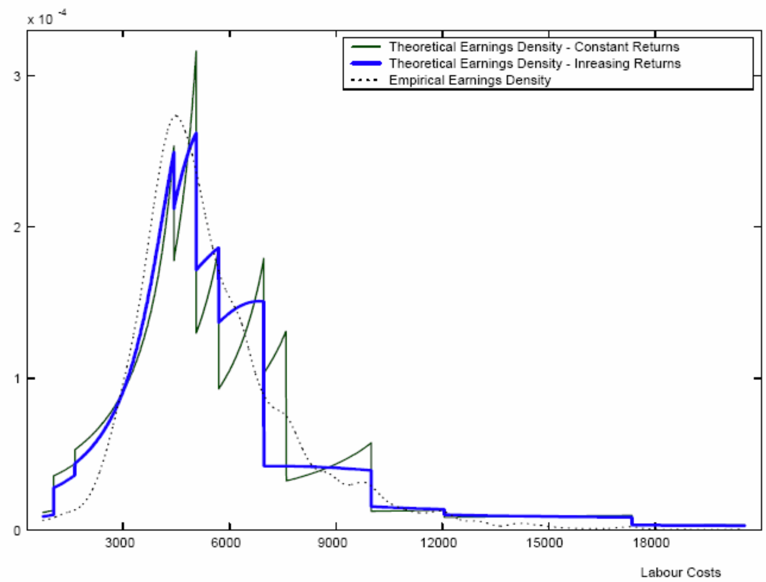


Figure 2: "Theoretical Earnings Densities Predicted by the Model: 3-Point Employer Heterogeneity"



Source: Holzner and Launov (2005), Figure A4 and A5.

1.5 Conclusion

This chapter extends the on-the-job search equilibrium model by Burdett and Mortensen (1998) by introducing different skill groups and linking them via a production function which allows for constant and increasing returns to scale. Whenever skills are complementary in the production process, a positive within-firm correlation between wages of workers with different skills arises. Depending on the degree of homogeneity of the production function the extension is able to generate a unimodal wage offer and earnings distributions with a long right tail given one production technology.

Many empirical researchers have estimated on-the-job search models assuming a continuous productivity dispersion across firms. Although these models fit the data very well, they provide no insight into the technology generating the continuous distribution. The focus on only few production technologies that are flexible enough to fit the data as suggested in this model enables empirical researchers to estimate the structural parameters underpinning the production dispersion. This extension also allows one to estimate the returns to education taking skill specific unemployment into account and to calculate the cost for acquiring additional skill.

1.6 Appendix: Proof of Proposition 4

Define

$$\begin{aligned} h_j(w) &= \frac{\left[\delta+\lambda_e\left(1-\gamma_{j-1}\right)\right]^2}{\left[\delta+\lambda_e\overline{F}_j(w)\right]^2},\ r_{ij}=\frac{\delta\lambda_i(\delta+\lambda_e)}{(\delta+\lambda_i)\left[\delta+\lambda_e\left(1-\gamma_{j-1}\right)\right]^2}q_i \\ Y_j'(\mathbf{r}_j) &= \frac{\partial Y_j(\mathbf{r}_j)}{\partial l_i},\text{ and } \sigma_{ij}=\sum\nolimits_l \frac{\partial^2 Y_j(\mathbf{r}_j)}{\partial l_i\partial l_l} r_{lj}r_{ij}. \end{aligned}$$

The second order Taylor-Expansion of the production function around r_j is given by

$$Y_j(\mathbf{l}(\mathbf{w}_j)) = Y_j(\mathbf{r}_j)+\sum\nolimits_i Y_j'(\mathbf{r}_j)\left[r_{ij}h_j(w)-r_{ij}\right]+\frac{1}{2}\sum\nolimits_i \sigma_{ij}\left[h_j(w)-1\right]^2.$$

Note, that $h_j(w)$ is independent of the skill group i, because of equation (1.12). Using the equal profit condition for the equilibrium, i.e. $\pi_j(\mathbf{w}_j)=\pi_j\left(\underline{\mathbf{w}}_j\right)$, and substituting gives

$$\begin{aligned} D = \sum\nolimits_i \left(Y_j'(\mathbf{r}_j)-w_i\right) r_{ij}h_j(w)+\frac{1}{2}\sum\nolimits_i \sigma_{ij}\left(h_j(w)-1\right)^2 \\ -\sum\nolimits_i \left(Y_j'(\mathbf{r}_j)-\underline{w}_{ij}\right) r_{ij}=0 \end{aligned} \tag{A.1}$$

The first order condition for wage w_i satisfies,

$$\left(\frac{\partial Y_j(\mathbf{l}(\mathbf{w}))}{\partial l_i(w_i)}-w_i\right) l_i(w_i) = l_i(w_i)^2\left[\frac{dl_i(w_i)}{dw_i}\right]^{-1}, \tag{A.2}$$

where rhs can be written as

$$l_i(w_i)^2\left[\frac{dl_i(w_i)}{dw_i}\right]^{-1} = \left[r_{ij}h_j(w)\right]^2\left[r_{ij}\frac{dh_j(w)}{dw_i}\right]^{-1}$$

According to the result that all firms occupy the same position in all wage offer distribution, changing the wage for one skill group implies a change of all other wages in the same direction, i.e. according to equation (A.1)

$$\begin{aligned} &\left[r_{ij}h_j(w)\right]^2\left[r_{ij}\frac{dh_j(w)}{dw_i}\right]^{-1} = r_{ij}h_j(w)^2\left(\frac{-\partial D/\partial h_j(w)}{-\sum_i \partial D/\partial w_i}\right) \\ &= \frac{r_{ij}}{\sum_i r_{ij}}\left(\sum\nolimits_i \left(Y_j'(\mathbf{r}_j)-w_i\right) r_{ij}h_j(w)+\sum\nolimits_i \sigma_{ij}\left(h_j(w)^2-h_j(w)\right)\right). \end{aligned}$$

Using a Taylor-Expansion for the first derivative of the production function and substituting $l_l(w_l)$ out gives

$$Y_j'(\mathbf{l}(\mathbf{w})) = Y_j'(\mathbf{r}_j)+\sum\nolimits_l \frac{\partial^2 Y_j(\mathbf{r}_j)}{\partial l_i\partial l_l}\left(r_{lj}h_j(w)-r_{lj}\right).$$

The first order condition can therefore be written as

$$\begin{aligned} & \left(Y_j'(\mathbf{r}_j) - w_i\right) r_{ij} h_j(w) + \sigma_{ij}\left(h_j(w)^2 - h_j(w)\right) \\ = \; & \frac{r_{ij}}{\sum_i r_{ij}} \left(\sum\nolimits_i \left(Y_j'(\mathbf{r}_j) - w_i\right) r_{ij} h_j(w) + \sum\nolimits_i \sigma_{ij}\left(h_j(w)^2 - h_j(w)\right)\right). \end{aligned}$$

Substituting $\sum_i \left(Y_j'(\mathbf{r}_j) - w_i\right) r_{ij} h_j(w)$ from equation (A.1) gives

$$\begin{aligned} & \left(Y_j'(\mathbf{r}_j) - w_i\right) r_{ij} h_j(w) + \sigma_{ij}\left(h_j(w)^2 - h_j(w)\right) \\ = \; & \frac{r_{ij}}{\sum_i r_{ij}} \sum\nolimits_i \left(Y_j'(\mathbf{r}_j) - \underline{w}_{ij}\right) r_{ij} + \frac{r_{ij}}{\sum_i r_{ij}} \frac{1}{2} \sum\nolimits_i \sigma_{ij}\left[h_j(w)^2 - 1\right]. \end{aligned}$$

Evaluating this equation at $\underline{w}_{ij}$ and substituting $\frac{r_{ij}}{\sum_i r_{ij}} \sum_i \left(Y_j'(\mathbf{r}_j) - \underline{w}_{ij}\right) r_{ij}$ gives

$$\begin{aligned} & \left(Y_j'(\mathbf{r}_j) - w_i\right) r_{ij} h_j(w) + \sigma_{ij}\left(h_j(w)^2 - h_j(w)\right) \\ = \; & \left(Y_j'(\mathbf{r}_j) - \underline{w}_{ij}\right) r_{ij} + \frac{r_{ij}}{\sum_i r_{ij}} \frac{1}{2} \sum\nolimits_i \sigma_{ij}\left[h_j(w)^2 - 1\right]. \end{aligned}$$

Rearranging gives

$$\left(\sigma_{ij} - \mu_{ij}\right) h_j(w)^2 + \left(\left(Y_j'(\mathbf{r}_j) - w_i\right) r_{ij} - \sigma_{ij}\right) h_j(w) = \left(Y_j'(\mathbf{r}_j) - \underline{w}_{ij}\right) r_{ij} - \mu_{ij}, \quad \text{(A.3)}$$

where $\mu_{ij} = \frac{r_{ij}}{\sum_i r_{ij}} \frac{1}{2} \sum_i \sigma_{ij}$.

For a production function with homogeneity of degree one $\sigma_{ij} = 0$ for all i I get

$$F_{ij}(w_i) = \frac{\delta + \lambda_e}{\lambda_e} - \frac{\delta + \lambda_e(1 - \gamma_{j-1})}{\lambda_e} \sqrt{\frac{Y_j'(\mathbf{r}_j) - w_i}{Y_j'(\mathbf{r}_j) - \underline{w}_{ij}}}.$$

Apart from this a special cases appear if $\left(Y_j'(\mathbf{r}_j) - \underline{w}_{ij}\right) r_{ij} - \mu_{ij} = 0$. In this case I get

$$F_{ij}(w_i) = \frac{\delta + \lambda_e}{\lambda_e} - \frac{\delta + \lambda_e(1 - \gamma_{j-1})}{\lambda_e} \sqrt{\frac{\left(Y_j'(\mathbf{r}_j) - \underline{w}_{ij}\right) r_{ij} - \sigma_{ij}}{\left(Y_j'(\mathbf{r}_j) - w\right) r_{ij} - \sigma_{ij}}}.$$

Otherwise, I get the following solution for the quadratic function, i.e.

$$\begin{aligned} h_j(w) = & -\frac{\left(Y_j'(\mathbf{r}_j) - w_i\right) r_{ij} - \sigma_{ij}}{2\left(\sigma_{ij} - \mu_{ij}\right)} \\ & \pm \frac{\sqrt{\left(\left(Y_j'(\mathbf{r}_j) - w_i\right) r_{ij} - \sigma_{ij}\right)^2 + 4\left(\sigma_{ij} - \mu_{ij}\right)\left(\left(Y_j'(\mathbf{r}_j) - \underline{w}_{ij}\right) r_{ij} - \mu_{ij}\right)}}{2\left(\sigma_{ij} - \mu_{ij}\right)}. \quad \text{(A.4)} \end{aligned}$$

The wage offer density implied by the quadratic function (A.3) has to be positive, i.e.

$$\frac{dF_{ij}(w)}{dw_i} = -\frac{-r_{ij}h_j\left(w\right)}{\left(2\left(\sigma_{ij}-\mu_{ij}\right)h_j\left(w\right)+\left(\left(Y_j'\left(\mathbf{r}_j\right)-w_i\right)r_{ij}-\sigma_{ij}\right)\right)\frac{\partial h_j(w)}{\partial F_{ij}(w)}} > 0$$

Since $\frac{\partial h_j(w)}{\partial F_{ij}(w)} > 0$, it follows that $2\left(\sigma_{ij}-\mu_{ij}\right)h_j\left(w\right)+\left(\left(Y_j'\left(\mathbf{r}_j\right)-w_i\right)r_{ij}-\sigma_{ij}\right)$ has to be greater than zero. Rewriting equation (A.4) implies that only the positive solution is valid, i.e.

$$\begin{aligned} &+\sqrt{\left(\left(Y_j'\left(\mathbf{r}_j\right)-w_i\right)r_{ij}-\sigma_{ij}\right)^2+4\left(\sigma_{ij}-\mu_{ij}\right)\left(\left(Y_j'\left(\mathbf{r}_j\right)-\underline{w}_{ij}\right)r_{ij}-\mu_{ij}\right)} \\ &= 2\left(\sigma_{ij}-\mu_{ij}\right)h_j\left(w\right)+\left(Y_j'\left(\mathbf{r}_j\right)-w_i\right)r_{ij}-\sigma_{ij} > 0. \qquad \text{(A.5)} \end{aligned}$$

Hence the cumulative wage offer distribution is given by

$$F_{ij}(w_i) = \frac{\delta+\lambda_e}{\lambda_e} - \frac{\delta+\lambda_e\left(1-\gamma_{j-1}\right)}{\lambda_e\sqrt{\frac{\left(Y_j'(\mathbf{r}_j)-w_i\right)r_{ij}-\sigma_{ij}-\sqrt{\left(\left(Y_j'(\mathbf{r}_j)-w_i\right)r_{ij}-\sigma_{ij}\right)^2+4\left(\sigma_{ij}-\mu_{ij}\right)\left(\left(Y_j'(\mathbf{r}_j)-\underline{w}_{ij}\right)r_{ij}-\mu_{ij}\right)}}{-2\left(\sigma_{ij}-\mu_{ij}\right)}}}.$$

In order to see that the wage offer density can be increasing and decreasing consider the explicit solution to the wage offer density

$$\begin{aligned} f_{ij}(w_i) &= \frac{\left(\delta+\lambda_e\left(1-\gamma_{j-1}\right)\right)r_{ij}}{2\lambda_e\sqrt{\left(\left(Y_j'(\mathbf{r}_j)-w_i\right)r_{ij}-\sigma_{ij}\right)^2+4\left(\sigma_{ij}-\mu_{ij}\right)\left(\left(Y_j'(\mathbf{r}_j)-\underline{w}_{ij}\right)r_{ij}-\mu_{ij}\right)}} \\ &\times \frac{1}{\sqrt{\frac{\left(Y_j'(\mathbf{r}_j)-w_i\right)r_{ij}-\sigma_{ij}-\sqrt{\left(\left(Y_j'(\mathbf{r}_j)-w_i\right)r_{ij}-\sigma_{ij}\right)^2+4\left(\sigma_{ij}-\mu_{ij}\right)\left(\left(Y_j'(\mathbf{r}_j)-\underline{w}_{ij}\right)r_{ij}-\mu_{ij}\right)}}{-2\left(\sigma_{ij}-\mu_{ij}\right)}}}. \end{aligned}$$

The slope of the wage offer density is given by

$$\begin{aligned} \frac{\partial f_{ij}(w)}{\partial w} &= -\frac{\left(\left(Y_j'(\mathbf{r}_j)-w_i\right)r_{ij}-\sigma_{ij}\right)^2+4\left(\sigma_{ij}-\mu_{ij}\right)\left(\left(Y_j'(\mathbf{r}_j)-\underline{w}_{ij}\right)r_{ij}-\mu_{ij}\right)-2r_{ij}\left(\left(Y_j'(\mathbf{r}_j)-w_i\right)r_{ij}-\sigma_{ij}\right)}{\left(\left(Y_j'(\mathbf{r}_j)-w_i\right)r_{ij}-\sigma_{ij}\right)^2+4\left(\sigma_{ij}-\mu_{ij}\right)\left(\left(Y_j'(\mathbf{r}_j)-\underline{w}_{ij}\right)r_{ij}-\mu_{ij}\right)} \\ &\times \frac{\frac{\left(\delta+\lambda_e\left(1-\gamma_{j-1}\right)\right)r_{ij}^2}{4\lambda_e\sqrt{\left(\left(Y_j'(\mathbf{r}_j)-w_i\right)r_{ij}-\sigma_{ij}\right)^2+4\left(\sigma_{ij}-\mu_{ij}\right)\left(\left(Y_j'(\mathbf{r}_j)-\underline{w}_{ij}\right)r_{ij}-\mu_{ij}\right)}}}{\sqrt{\frac{\left(Y_j'(\mathbf{r}_j)-w_i\right)r_{ij}-\sigma_{ij}-\sqrt{\left(\left(Y_j'(\mathbf{r}_j)-w_i\right)r_{ij}-\sigma_{ij}\right)^2+4\left(\sigma_{ij}-\mu_{ij}\right)\left(\left(Y_j'(\mathbf{r}_j)-\underline{w}_{ij}\right)r_{ij}-\mu_{ij}\right)}}{-2\left(\sigma_{ij}-\mu_{ij}\right)}}} \end{aligned}$$

Thus, a necessary condition for the wage offer density to be upward sloping is that $\left(Y_j'\left(\mathbf{r}_j\right)-w_i\right)r_{ij}-\sigma_{ij} > 0$. Substituting σ_{ij}, and using the Euler Theorem gives the stated condition.

Chapter 2

On-the-Job Search and Asymmetric Information in the Labor Market

This chapter focuses on the asymmetric information that exists between the current employer and the outside market regarding a worker's productivity. The model shows that for severe enough search frictions, a market for employed workers with wage gains emerges despite the presence of adverse selection. Asymmetric information about a worker's productivity between the worker's current employer and the outside market enables the current employer to keep its best employees from joining the outside market by promoting them or by making them counter offers. Since outside wage offers are uncertain, firms promote or make counter offers only to their best workers. The resulting adverse selection, though, leads to an initial breakdown of the market for employed workers. As low-productivity workers are laid off over time, tenure serves as a positive signal about a worker's productivity. After enough badly performing workers were laid off, the signal is strong enough to counteract the negative effect of adverse selection and a market for employed workers emerges.

2.1 Introduction

This chapter considers the implications the interaction of asymmetric information and search have on lay-off and promotion decisions and on the corresponding wage dynamics. As the worker's productivity is revealed to the current employer and the worker over time the information advantage of the current employer compared to the outside market increases. The outside market makes non-observable job offers to employed workers in order to prevent their current employers from using their information to make counter offers. The current employer tries to exploit this information advantage by promoting his best performing workers in order to prevent them from searching. At the same time the outside market tries to separate well performing workers from their "average" productive colleagues by inducing them to search in different markets.

I present a learning model with asymmetric information in a labor market where search frictions are the same for all firms and where workers are passive agents. This basic model can already explain several empirical findings about the job mobility of young workers, the wage tenure effect, and about the wage gains and wage losses of "stayers" and "movers" over time. While part of these findings are already explained by other models, the model presented in this chapter provides an explanation for several common features of the labor market.

The model explains an empirical finding by Farber (1994), namely that the hazard rate of job termination increases up to a maximum and declines thereafter. The explanation given in this chapter is as follows. Initially, the probability is zero, because the cost of recruitment implies that a firm will wait some time before laying off a worker. During that time tenure will not carry any information about a worker's productivity, implying that the market for employed workers is likely to break down. The hazard becomes positive as the first workers are laid off. After the first layoffs, a market with wage increases for employed workers need not resume immediately, since it takes some time to ensure that enough badly performing workers are laid off such that the pool of searching workers has a sufficiently high productivity to enable outside firms to offer wages above the worker's current wage. At some point the hazard declines, because promoted workers quit at a lower rate and more and more workers are promoted over time. In addition the likelihood that an employed worker is revealed to be of a bad type and hence is laid off declines over time. This is similar to Jovanovic (1979) who shows in a model with symmetric information where match-specific productivity is revealed to the worker and the firm over time. He shows that the hazard can increase in the first periods and declines thereafter. Pissarides (1994) explains in an on-the-job model with firm-specific human capital a declining hazard.

As mentioned above the best-performing workers might be promoted by their current employer in order to prevent them from searching or quitting. Thus, for high productivity workers the option to get an outside wage offer is enough to induce their current firm to increase their wage. The consequent promotion of the best-performing workers then implies together with the assumed downward wage rigidity that wages increase with tenure. Jovanovic (1979) already showed that the revelation of match-specific productivity over time can lead to a wage-tenure effect. Since in his model wages adjust to the expected productivity at the match, bad matches, which imply decreasing wages, are terminated voluntarily while good matches, which imply wage increases, persist. Even if identical workers search on-the-job it is optimal for the current employer to increase wages over tenure in order to reduce the worker's quit rate, as shown by Burdett and Coles (2003). In the model presented here both mechanisms are at work to generate a wage-tenure effect.

While the explanation based on the theory of firm-specific human capital developed by Becker (1993) and Hashimoto (1981) – where a worker and a firm share the investment and the returns on the investment into firm-specific human capital to reduce turnover – is seen by many empirical researchers as the key source for why wages increase with tenure, there are also empirical studies by Abraham and Farber (1987), Altonji and Shakotko (1987) and more recently Altonji and Williams (1997) supporting the explanation given by Jovanovic and in this paper based on the revelation of productivity over time. They show that the measured positive cross-sectional return to seniority found in studies of Mincer and Jovanovic (1981) and Bartel and Borjas (1981) among others is largely a statistical artifact due to the correlation of high seniority with an omitted variable representing the quality of the worker, job or worker-employer match.

On-the-job search as in Burdett and Mortensen (1998) explains that wages increase with experience since workers search for better paying jobs while being employed. On-the-job search models also predict wage cuts for workers that are laid off. Such wage cuts are also explained by the adverse selection models of Greenwald (1986) and Gibbons and Katz (1991) among others. The predicted wage dynamics following a layoff are different, however. The on-the-job search models predict a closing wage difference between laid off workers and employed workers, since individuals are identical and unemployment is seen as a random shock that will in the long-run affect everybody equally. The adverse selection models predict a persisting wage gap since stayers are more productive than movers. Combining adverse selection with on-the-job search implies a declining wage gap with some persistent difference in the long-run.

Furthermore, I provide conditions under which it is optimal for the current employer to ignore outside offers. This is important because the early on-the-job search models by Burdett (1978), Jovanovic (1984), Pissarides (1994) and Burdett and Mortensen (1998) assume that the current employer does not react to outside wage offers. Thus, workers receiving higher outside wage offers climb up the wage ladder. Postel-Vinay and Robin (2002) assume the opposite, namely that the current employer observes and reacts to an outside wage offer by making a counter offer. The resulting Bertand competition between firms drives the wage up so that the firm with the highest productivity wins the bidding game. Although it is quite obvious that making a counter offer is optimal if outside wage offers are observable, it turns out that if outside wage offers are not observable, it is optimal not to make counter offers only if search frictions are large enough.

The outline of this chapter is as follows. The next section I presents the framework of the model. In section 3 I analyze the learning model with asymmetric information and derive the results on the job mobility of young workers, the wage tenure correlation and about the wage gains and wage losses of stayers and movers over time.

2.2 The Framework

Driving forces in this model are three core assumptions. Firstly, I assume that an outside wage offer is not observed by the worker's current employer until it is accepted. This assumption is most easily justified by the fact that it is in the interest of outside firms to conceal their outside offers, since making an observable outside wage offer would trigger a counter offer of the current employer and lead to a breakdown of the on-the-job market in an economy with homogenous employers.[1]

The second core assumption is that wages cannot be negotiated downward.[2] If wages could be negotiated downward, then nobody would be laid off and tenure could not be used as a positive production signal. Adverse selection caused by the promotion strategy of the current firm then implies that only a market for the least productive workers exists.

Thirdly, I assume that outside firms can condition their wage offers on the employment status of a worker and on the periods t an applicant has been employed for. This is necessary in order for tenure to serve as a signal about productivity.[3] I further assume that future employers know the tenure at the previous employer.

[1]Lazear (1986) and Waldman (1990) present a model where learing takes place in a diffuse fashion, i.e. not only the current employer but also the raiding firm learns with some probability the worker's type. Both authors show that outside firms only make an observable wage offer, if it is informed about the worker's type. Since in the model presented here, the current employer is always better informed about the worker's type than the outside market, the assumption that outside offers are not observable by the current employer is without loss of generality.

[2]A good literature review justifying downward wage rigidity is given in Weiss (1991) and Bewley (1999).

[3]The idea that the firm's retention decision serves as a signal about a worker's productivity goes back to Waldman (1990) and Gibbons and Katz (1991). Waldman's model about up-or-out contracts analyses the incentives to invest in human capital in a model where the current and the outside firm learn about a worker's ability in a diffuse fashion. Gibbons and Katz focus in their two period model on the wage differential of workers that are laid off compared to those displaced.

The model is set in discrete time and has an infinite horizon. A measure n of new market entrants enter the labor market every period and each worker survives with some probability until the next period. The analysis focuses on one cohort. Workers differ in the probability p to produce output of unit 1, where the distribution of types is given by a cumulative distribution function $F(p)$ with support $p \in [\underline{p}, \overline{p}]$. This chapter focuses on the asymmetric information which exists between the current employer and the outside market regarding a worker's productivity. There is symmetric information between the worker and the current employer about the worker's productivity. Workers are, however, better informed than their second employers. In the basic model I, therefore, concentrate on what happens during a worker's first job and assume that new market entrants have the same prior belief about their type as their first employers have.

The prior belief about a worker's expected productivity equals the average production probability of all new market entrants, i.e.

$$b_0 = \int p dF(p). \tag{2.1}$$

The beliefs are updated by Bayes' rule. The worker's and his first employer's belief about the worker's productivity is, therefore, given by

$$b_t^y = b_t^{y(h_{t,s})} = \frac{\int p^{y(h_{t,s})+1} (1-p)^{t-y(h_{t,s})} dF(p)}{\int p^{y(h_{t,s})} (1-p)^{t-y(h_{t,s})} dF(p)}, \tag{2.2}$$

where $y(h_{t,s})$ denotes the aggregate output of a worker with working history $h_{t,s}$ at tenure t and last employment status $s \in \{u, e\}$, where u stands for unemployed and e for employed workers. The subscript s is used as long as it is unknown whether unemployed and employed workers are searching in the same or in separate markets. The development of the expected productivity b_t^y is shown in figure 1, where firms will update their belief at every t after production has taken place between $t-1$ and t.

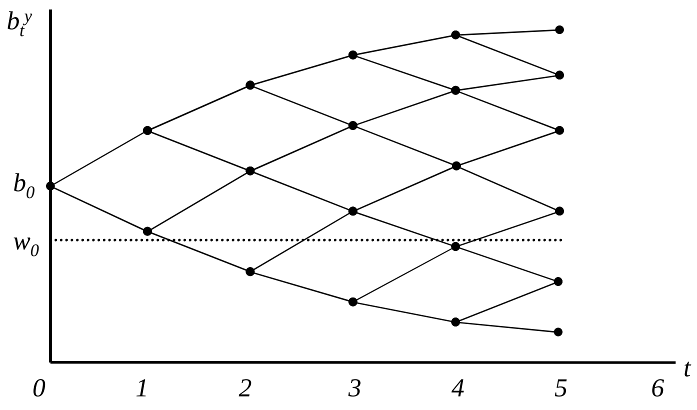


Figure 1: Development of the expected productivity b_t^y.

The outside market does not observe the aggregate output of a worker $y(h_{t,s})$. Outside firms only observe the tenure t of a worker, whether a worker is unemployed or employed and in which market he is currently searching or the firm he works for. Let $H_{t,s}^m$ be the set of working histories $h_{t,s}$ of workers with the last labor market status s who are believed to search in market $m \in M$, where M denotes the set of existing markets at tenure t. The believed market productivity $\mu_{t,s}^m$ is therefore given by

$$\mu_{t,s}^m = \sum_{h_{t,s} \in H_{t,s}^m} \frac{\int p^{y(h_{t,s})+1} (1-p)^{t-y(h_{t,s})} dF(p)}{\sum_{h_{t,s} \in H_{t,s}^m} \int p^{y(h_{t,s})} (1-p)^{t-y(h_{t,s})} dF(p)}. \tag{2.3}$$

The second employer's belief about a worker recruited from a market with average productivity $\mu_{t,s}^m$ is also updated by Bayes' rule. After a worker produced aggregate output x in d periods the second employer's belief about his productivity is given by

$$b_d^x|_m = \sum_{h_{t,s} \in H_{t,s}^m} \left(\frac{\displaystyle\int p^{y(h_{t,s})+x+1} (1-p)^{t-y(h_{t,s})+d-x} dF(p)}{\displaystyle\int p^{y(h_{t,s})+x} (1-p)^{t-y(h_{t,s})+d-x} dF(p)} \times \frac{\int p^{y(h_{t,s})} (1-p)^{t-y(h_{t,s})} dF(p)}{\sum_{h_{t,s} \in H_{t,s}^m} \int p^{y(h_{t,s})} (1-p)^{t-y(h_{t,s})} dF(p)} \right).$$

I assume that searching workers receive a wage offer with the given probability $(1-s) \in (0,1)$ and that firms pay a fixed cost R to recruit a worker. In order to simplify the worker's decision problem, I assume that workers are myopic. Since workers are myopic they need not be insured against future layoffs, i.e. it is optimal for firms to write one period contracts. Furthermore, the myopic behavior of workers rules out that workers with the same observable characteristics but different unobserved productivity might be induced to search in separate markets since workers accept a higher wage regardless of the future risk of being laid off.

Firms exist forever and discount future payments with a factor $\beta \in (0,1)$, which includes the exit probability of a worker. Free entry of firms ensures that firms make zero profit, i.e. $\Pi\left(w^m_{t,s}|\mu^m_{t,s}\right)$ the expected value of employing a worker with believed market productivity $\mu^m_{t,s}$ and last labor market status s at the market wage $w^m_{t,s}$ equals the recruitment cost R, i.e.

$$\Pi\left(w^m_{t,s}|\mu^m_{t,s}\right) = R. \tag{2.4}$$

The value of employing a worker with believed productivity b^y_t is given by his expected productivity b^y_t minus his current wage w_t plus the expected discounted continuation payoff $\beta E_t\left[\Pi\left(w_t|b^y_{t+1}\right)\right]$, i.e.

$$\Pi\left(w_t|b^y_t\right) = \max\left[\max_{w_t\in[w_{t-1},\infty)}\left[b^y_t - w_t + \beta E_t\left[\Pi\left(w_t|b^y_{t+1}\right)\right]\right], 0\right] \tag{2.5}$$

By the assumption of downward wage rigidity, the firm can not lower wages, i.e. pay $w_t \in [w_{t-1}, \infty)$. It can, of course, lay off a worker. Clearly,

$\Pi\left(w_t|b^y_t\right)$ is strictly increasing in per period profit $b^y_t - w_t$.

To ensure that the worst performing workers will eventually be laid off, I assume that the productivity of a new market entrant minus the cost of recruiting him exceeds the productivity of the least productive workers, i.e.

$$b_0 - R > \underline{p}, \tag{2.6}$$

Let me now turn to the game between the worker and his current firm after each production period.

Step 1: A firm decides whether or not to lay off a worker. The firm cannot recall a laid-off worker.

Step 2: If a worker was not laid off, the current firm decides whether or not to promote him and whether to prolong the contract.

Step 3: Outside firms observe the labor market status (unemployed versus employed) and the tenure of all searching workers. Given that information they decide on the wage offers $w^m_{t,s}$ for those laid off and for those still employed. The promotion decision is assumed not to be observable.

Step 4: Workers observe all wage offers. Since search is costless, I assume that workers start to search only if a market wage offer $w^m_{t,s}$ exceeding his current wage w_t exists. If more such outside wage offer exists, workers decide in which market $m \in M$ to search in.

Step 5: Matched firms of market m decide whether to employ a matched worker given his observed characteristics.

Before solving the game, it seems useful to comment on the assumptions about the information that is transmitted from the current employer's actions to the market and compare them to the assumptions underlying other adverse selection models.

In Greenwald's (1986) model of "Adverse selection in the labour market" workers move either because of an exogenous reason or because the current employer induces the worker to quit by offering them a lower wage than the market wage. Gibbons and Katz (1991) show that this is only a corner equilibrium of a continuum of equilibria, because the current employers can use other layoff rules, e.g. laying off the worst workers and inducing the rest of the workers it want to get rid off to quit. Since the current employer is indifferent between laying off workers or inducing them to leave, it depends on the equilibrium belief of all firms which layoff policy is implemented. In both models the retention decision is a positive signal about the workers' productivity either in the same period as in Gibbons and Katz or one period later as in the Greenwald.

Promotion is assumed not to be observable by the outside market, thus promotion should rather be seen as a pay increase not as an assignment to a different task. If I assumed contrary the current employers have to worry that this positive signal enables outside firms to raid their best workers as it is common in assignment models (see Waldman, 1984, or Ricart i Costa, 1988). The assignment literature assumes that jobs differ in productivity such that more able workers are more productive in the assigned job. If the jobs do not differ in productivity, then the assignment literature has shown that nobody should be promoted.

2.3 The Model

The stage game is solved by backward induction:

Step 5: Matched firms employ a matched worker, if the expected profit is positive, i.e.

$$\mu_{t,s}^m - w_{t,s}^m + \beta E_t \left[\Pi \left(w_{t,s}^m | \mu_{t,s}^m \right) \right] \geq 0.$$

This is always the case as long as $\mu_{t,s}^m \geq w_{t,s}^m$, since $\mu_{t,s}^m \geq w_{t,s}^m \Longrightarrow E_t \left[\Pi \left(w_{t,s}^m | \mu_{t,s}^m \right) \right] \geq 0$.

Step 4: Since workers are myopic, they search if and only if the market wage exceeds the current wage, i.e.

$$w_{t,s}^m > w_t, \tag{2.7}$$

where $w_t \in [w_{t-1}, \infty)$.

Step 3: Outside firms observe the labor market status s and the tenure t of all workers and form a belief about the workers' productivity $\mu_{t,s}^m$ of workers searching in market m. For the laid off workers the market wage is given by

$$w_{t,s}^m = \mu_{t,s}^m + \beta E_t \left[\Pi \left(w_{t,s}^m | \mu_{t,s}^m \right) \right] - R,$$

For employed workers the corresponding market belief – as given in equation (2.3) – has to exceed the initial belief, i.e. $\mu_{t,s}^m > b_0$, in order for outside firms to be able to offer a market wage $w_{t,s}^m$ that satisfies the free entry condition and exceeds the initial wage, i.e.

$$w_{t,s}^m = \mu_{t,s}^m + \beta E_t \left[\Pi \left(w_{t,s}^m | \mu_{t,s}^m \right) \right] - R > w_0, \tag{2.8}$$

where $b_1^x|_m$ denotes the second employer's belief after the worker stayed with him one period.

Step 2: Given that the current firm's belief about the average productivity of workers searching is $\mu^m_{t,s}$, it promotes, i.e. pays a worker $w_t = w^m_{t,s}$, where $w^m_{t,s}$ is given by equation (2.8) if and only if the payoff of the promoted worker is higher than the payoff of paying him the last period's wage w_{t-1} and taking into account that he will get and accept an outside offer with probability $1 - s$, i.e. stay with probability s,

$$b^y_t - w^m_{t,s} + \beta E_t \left[\Pi\left(w^m_{t,s}|b^y_{t+1}\right)\right] > s\left(b^y_t - w_{t-1} + \beta E_t \left[\Pi\left(w_{t-1}|b^y_{t+1}\right)\right]\right). \tag{2.9}$$

The promotion-threshold productivity b^p, if a worker earned wage w_{t-1} last period and the market offers a wage $w^m_{t,s}$, is given by

$$b^p - w^m_{t,s} + \beta E_t \left[\Pi\left(w^m_{t,s}|b^p\right)\right] = s\left(b^p - w_{t-1} + \beta E_t \left[\Pi\left(w_{t-1}|b^p\right)\right]\right). \tag{2.10}$$

Promotion can lead to adverse selection and multiple equilibria on the market for employed workers, since the very best workers are promoted and kept off the labor market, whereas the average productive workers are not promoted and therefore search.

Step 1: Finally, the least productive workers are laid off, if and only if their expected current payoff plus their continuation payoff is negative. The timing of the game implies that a worker is not promoted and laid off at the same time, hence the layoff decision is based on the wage of the last period w_{t-1}, i.e.

$$b^y_t - w_{t-1} + \beta E_t \left[\Pi\left(w_{t-1}|b^y_{t+1}\right)\right] < 0. \tag{2.11}$$

The layoff-threshold belief $b^l(w_{t-1})$, given the worker earned wage w_{t-1} last period, is defined such that the worker's believed productivity makes the current firm just indifferent between continuing to employ this worker or laying him off and searching for a new worker, which has a value of zero. Given that the continuation payoff at the threshold is zero, i.e. $E_t\left[\Pi\left(w_{t-1}|b^l(w_{t-1})\right)\right] = 0$, if follows that,

$$b^l(w_{t-1}) = w_{t-1}. \tag{2.12}$$

As shown in figure 2 depending on the current employer's belief about the worker's productivity b^y_t the worker will either be laid off if his expected productivity is below the last period's wage, he will be promoted if his productivity is high enough, i.e.

$b_t^y > b^p$, or he will start searching if he is not promoted and a market offering a wage higher than his last periods wage exists.

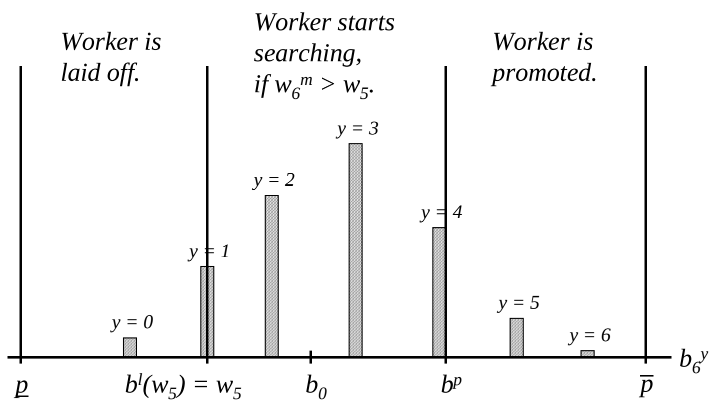


Figure 2: Action taken by the current employer or the worker depending on b_6^y.

The core of the forthcoming analysis will be to look at the development of the distribution of the workers' believed productivities (shaded columns) and the layoff and the promotion thresholds as time unfolds. At $t = 0$ all new market entrants have a believed productivity of b_0 and the mass is concentrated at b_0. The situation depicted in figure 2 corresponds to $t = 6$ since workers' aggregate output y ranges from zero up to six units and so there are seven mass points corresponding to the current firm's belief b_6^y.

The next step will be to look under which condition an outside market with wage increases exists and how workers' mobility and wages develop over time. Consider first the following lemmas.

Lemma 1: *Employed workers do not search and outside firms are inactive as long as nobody is laid off.*

Proof: If nobody has been laid off so far, then the belief about the employed workers' productivity equals the initial belief b_0. A potential market wage offer $w_{t,s}^m$ could therefore be no higher than the initial wage w_0. Since due to the downward wage rigidity the employed workers earn at least their initial wage, they would not start to search for an outside job, i.e. $H^m_{t,s}$ is empty. Thus, outside firms are inactive as long as nobody is laid off. $\square$

A direct consequence of Lemma 1 is that the current employer will not promote anyone as long as no market exists, since workers have no other option as to stay with their current employer as long as no market exists. An immediate, testable prediction following from Lemma 1 is that newly recruited workers should not be promoted as long as nobody from their cohort is laid off.

Note further that even if I assumed that some workers quit because of exogenous reasons as is frequently done in the adverse selection literature in order to ensure that an on-the-job market exists (see Greenwald, 1986, Gibbons and Katz, 1991 or Acemoglu and Pischke, 1998 among others), a market with a wage higher than the initial wage would not exist as long as nobody is laid off. The reason is that badly performing workers cannot be induced to quit by offering them a lower than the current wage as done by Greenwald or Gibbons and Katz, since I assumed that wages cannot be negotiated downward.

Lemma 2: *From some point $0 < \tau < \infty$ onward, badly performing workers are laid off.*

Proof: Lemma 1 implies that the layoff decision at τ is based on the initial wage w_0. The recruitment cost R implies that there is an option value for the current employer to wait before laying off a worker, i.e. the free entry condition (2.4) and equation (2.12) imply $b_0 > b^l(w_0)$ which gives $0 < \tau$.
$b_0 - R > \underline{p}$ and the free entry condition (2.4) imply $w_0 - \beta E_0 [\Pi(b_1, w_0)] > \underline{p}$. Substituting the initial wage w_0 using equation (2.12) implies $b^l(w_0) > \underline{p}$.
Furthermore, there are some unlucky workers who never produce. The believed productivity of these workers approaches $\underline{p}$ as $t \to \infty$, i.e. $\lim_{t\to\infty} b^0_t \to \underline{p}$. Given $b^l(w_0) > \underline{p}$, it follows that as $t \to \infty$ a point in time $\tau < \infty$ exists where $b^0_\tau < b^l(w_0)$. $\square$

After the first workers have been laid off, the average productivity of the workers still employed is higher than the average productivity of the new market entrant. Tenure is, thus, a positive signal about a worker's productivity. Since outside firms can observe a worker's tenure and employment status, they can use these as a positive signal about a worker's productivity.

Outside firms can, however, not forbid unemployed workers to apply for the same jobs as employed workers do. They can only enforce separation of workers with different characteristics, i.e. different labor market status s and different tenure t, if they can commit to turn down a matched worker who does not posses the right characteristics.

Lemma 3:

(i) Unemployed workers will not search in a market offering a wage $w_{t,e}^m \geq w_0$.

(ii) Employed workers of one cohort cannot be separated.

Proof:

(i) According to the layoff decision (2.11) a worker is not employed if his value of being employed is negative. For unemployed workers this is the case if and only if the market offers a wage $w_{t,e}^m \geq w_0$, since $b_t^y < b^l(w_0)$. With no chance of being employed in a market offering a wage $w_{t,e}^m \geq w_0$ unemployed will not search in that market.

(ii) Employed workers of one cohort cannot be separated since they are indistinguishable for outside firms and accept higher wages regardless of their believed productivity. □

Lemma 3 implies that after the first workers have been laid off outside firms offering a wage above the initial wage can be sure that the average productivity of the workers searching in that market is above the layoff-threshold belief, since offering a wage higher than the initial wage induces unemployed and employed workers to search in separate markets. The second property of Lemma 3 implies that employed workers search in a common market. Thus, for employed workers of one cohort only one market can exist.

Lemma 3 does, however, not guarantee that an on-the-job market with wage increases emerges, since outside firms know that not all employed workers necessarily search, since the best workers might have been promoted by the current employer in order to keep them away from the market. Looking at the promotion condition (2.9) reveals that the believed productivity of the searching employed workers $\mu_{t,e}^m$ exceeds the initial productivity b_0, if the probability s of staying is large enough. This is stated in the following proposition characterizing the equilibrium.

Proposition 1:

(i) At all $t < \tau$ all workers earn w_0, they do not search on-the-job, current employers do not promote and outside firms are inactive.

(ii) At all $t \geqslant \tau$ workers with an expected productivity below their current wage $b_t^y < w_t$ are laid off,

a) if $s \geqslant \underline{s} = \left(\overline{p} - w_{\tau,e}^m\right) / \left(\overline{p} - w_0\right)$, all workers with $b_t^y \geq w_t$ search on-the-job, current employers do not promote and outside firms offer $w_{\tau,e}^m > w_0$ according to equation (2.8),

b) if $s < \underline{s}$ either (i) holds or workers with $b^p > b_t^y \geq w_t$ search on-the-job, current employers promote workers with $b_t^y \geq b^p$ and outside firms offer $w_{\tau,e}^m > w_t$ according to equation (2.8).

Proof: (i) Follows from Lemma 1.

(ii) a) Since all employers observe that some workers have been laid off at τ, they believe that for employed workers $H_{\tau,e}^m$ is such that $\mu_{\tau,e}^m > b_0$ if nobody is promoted. Given $\mu_{\tau,e}^m$ the outside wage offer is according to equation (2.8) greater than the initial wage, i.e. $w_{\tau,e}^m > w_0$. Form the promotion condition (2.9) it follows that nobody is promoted if and only if

$$s\left[\overline{p} - w_0\right] \geqslant \overline{p} - w_{\tau,e}^m. \tag{2.13}$$

Since $w_{\tau,e}^m > w_0$, there exists an $\underline{s} = \frac{\overline{p} - w_{\tau,e}^m}{\overline{p} - w_0} \in (0,1)$ such that for all $s \geqslant \underline{s}$ nobody is promoted.

b) Suppose for some staying probability $s < \underline{s}$ the outside wage offer $w_{\tau,e}^m$ is such that the best workers are promoted according to the promotion condition (2.9). If $\mu_{\tau,e}^m \leq b_0$ then situation (i) holds, if $\mu_{\tau,e}^m > b_0$ outside firms offer wages according to equation (2.8) greater than the initial wage, i.e. $w_{\tau,e}^m > w_0$. For worker promoted at time t_p the current wage is given by $w_t = w_{t_p,e}^m$ and the argument follows again (ii) b). □

The intuition behind Proposition 1 is simply that large search frictions decrease the firm's willingness to promote, because it is harder for non-promoted workers to get an outside wage offer. Thus, fewer high-performance workers are promoted and start to search. The fact that more high-performance workers search increases the average productivity of the searching workers and enables outside firms to offer a wage higher than the initial wage.

On the other hand, a lower staying probability or equivalently lower search frictions induce firms to promote highly productive workers in order to take away their incentive to search for an outside job. This decreases the average productivity $\mu_{t,e}^{m}$ of those workers searching and subsequently the market wage $w_{t,e}^{m}$ outside firms can offer. A lower market wage again implies that firms are willing to promote not only the highly productive workers but also less productive workers. This adverse selection effect might cause the average productivity of searching workers to be so low, i.e. $\mu_{t,e}^{m} \leq b_0$, that no on-the-job market with wage increases ever emerges.

The explanation for Farber's (1994) finding – that the hazard of job termination increases up to a maximum and declines thereafter – can be described as follows. Initially, the hazard is zero, because the cost of recruitment implies that a firm will wait some time before laying off a worker. During that time tenure will not carry any information about a worker's productivity, implying that the market for employed workers breaks down. The hazard becomes positive as the first workers are laid off. After the first layoffs, the market for employed workers generally does not resume immediately, since it takes some time to ensure that enough badly performing workers are laid off such that the pool of searching workers has a sufficiently high productivity to enable outside firms to offer wages above the worker's current wage. After some point the hazard declines, not because the probability of finding a job declines, but because fewer workers are searching because the probability to be promoted increases as the outside wage offers increase over time. In addition, the likelihood that an employed worker is of the bad type and hence is laid off declines as well.

As mentioned above, the best-performing workers are promoted by their current employer in order to prevent them from searching or quitting. Thus, for high productivity workers the option to get an outside wage offer is enough to induce their current firm to increase their wage. Furthermore, since more and more badly performing workers are laid off over time, it follows that the productivity of the pool of employed workers and of the pool of searching workers increases over time. With an increase in the productivity of searching workers over time the outside wage offer increases, too. The subsequent promotion of the best-performing workers together with the assumption of downward wage rigidity implies that wages increase with tenure.

Corollary 2: *Wages weakly increase with tenure.*

Greenwald (1986) emphasizes that adverse selection inhibits turnover since workers that are induced to change jobs are marked to be part of the inferior group. This statement is still true but has to be qualified, since only laid off workers are stigmatized. Turnover is, however, still inhibited since only a low turnover rate can prevent the current employers from using their superior information to promote the best workers. Greenwald also predicts that stayers outperform movers in the long-run since the retention decision at t is a positive signal about the workers' productivity one period later. By the same argument the basic model implies that "stayers" outperform laid off workers. At the same time voluntary "movers" outperform "stayers" in the short run, since "stayers" are composed of the promoted workers who earn a wage equal to the "movers" and of workers who are searching but are not matched and, therefore, earn their current wage. Later on promoted workers might also start to search and earn even higher wages such that the initial "stayers" eventually outperform the initial "movers".

2.4 Conclusion

The model explains several empirical findings about the job mobility of young workers, the wage tenure effect and about the wage gains and wage losses of stayers and movers over time. The finding that the hazard of job termination increases up to a maximum and declines thereafter is also consistent with the model.

Chapter 3

Credit Constraints, Promotion and Firm Financed General Training

This chapter investigates firm-financed general training in a Pissarides matching framework. It assumes that workers of different skills search in skill-segmented markets. It shows that it is more expensive for firms to hire high skilled workers than low skilled workers, which provides an incentive for firms to provide some training for unskilled workers. In addition firms want to reduce turnover of the newly trained worker and promote workers as soon as possible to take away their incentive to search for an outside job. Thus, training firms demand a lump-sum payment equivalent to the value of the promotion from workers that are not credit constrained and promote them immediately. Credit constrained workers are paid a trainee wage of zero and promoted only with some probability making them indifferent between staying unskilled and being trained. Only if labour mobility is very high and workers are credit constrained, then workers will gain from training.

3.1 Introduction

Becker (1993) shows that in a competitive labor market workers should pay for general training since they receive the full return to training. In a search model with bargaining, I show that search frictions per se do not necessarily cause underinvestment into general training and thus confirm Becker's result that investment into general training can be efficient if workers are not credit constrained. The underlying reason is that future employers need not profit from the training in other firms, if their profits are driven down to zero as positive profits trigger vacancy creation. Since future employers of trained workers do not benefit from the training in other firms, the training firm and the worker can, if workers are not credit constrained, enter into a long-term contract that guarantees that the training level will be efficient. The training firm will offer a contract that demands a lump-sum payment equivalent to the value of the promotion from its trainee and promotes him immediately. Thus, the skilled worker will not search and training will be efficient. If workers are credit constrained firms want to reduce turnover of the newly trained worker and promote them as soon as possible to take away their incentive to search for an outside job. Credit constrained workers are paid a trainee wage of zero and are promoted with some probability making them indifferent between staying unskilled and being trained.

If workers are credit constrained, then the training firm will still provide some training, because when deciding whether to train an unskilled worker or not, the firm faces the trade off between training an unskilled worker at its own expense or recruiting a skilled worker from the market. The difference in recruitment costs between unskilled workers and skilled workers can be used to pay for the general training, a point already mentioned by Oatey (1970) and Stevens (1994, 2001). While Stevens (1994, 2001) assumed different recruitment cost, the model presented here endogenizes the recruitment cost.[1]

Even if workers are credit constrained, they need not benefit from training, because when posting the trainee contract the training firm can set the trainee wage to zero

[1] Oatey (1970) presents no formal model.

and commit to a promotion rate such that it takes away the trained worker's expected gain from searching for another job. Trainees gain from training only if workers are credit constrained and the matching probability for a trained worker is high enough.

The model presented in this chapter follows the line of other research showing that labor market frictions provide an incentive for firms to invest in general training. Acemoglu and Pischke (1999) show that a compressed wage structure is sufficient for firms to pay at least partly for general training and that credit constraints, which are mentioned by Becker (1993) as a reason why firms may pay for general training, are not necessary. This compressed wage structure may be the result of an information asymmetry between training firms and not-training firms about the ability of individual workers as Katz and Ziderman (1990), Chang and Wang (1996) and Acemoglu and Pischke (1998) show. Acemoglu (1997) or Acemoglu and Pischke (1999) model firm financed general training with search frictions. They find that firms can only extract part of the return since future employer benefit from the training in other firms. Hence they are not willing to finance general training up to the efficient level. The contribution of this paper is, that it shows that a promotion commitment can increase the training firm's return to training by reducing turnover. Thus, promotion can increase training activities. A third strand of the literature explains general training in combination of firm-specific training.

The plan of the chapter is as follows. Section 2 presents the framework. Section 3 derives the workers' behavior followed by the analysis of labor turnover in the steady state in Section 4. Section 5 derives the firms' vacancy creation decision, the general training condition in a situation where workers are credit constrained and where they are not. Section 6 establishes that multiple labor market equilibria exist if workers are credit constrained. Section 7 presents extensions concerning the wage formation. The chapter concludes by summarizing the main results.

3.2 The Framework

Firms

The model considers an infinite-horizon, stationary labor market in continuous time. The measure of firms is normalized to unity. Firms are assumed to be risk neutral and to discount future payments by the rate of interest r. All firms live infinitely. Firms search for workers by creating vacancies v_i for the respective labor markets, where $i \in \{s, u\}$. s stands for the labor market of skilled workers and u for the labor market of unskilled workers. The fact that workers of different skill are assumed to search in different markets implies that firms opening a vacancy for one type of worker have no use for another type of worker and can therefore commit not to employ a worker of another type. The advertising cost for a vacancy per time unit is given by adt.

The bargaining wages w_i for skilled and unskilled workers are taken as given by the firm when it chooses the training γ and the promotion rate ρ. The firm offers with probability γdt an employed, unskilled workers a training contract specifying a trainee-wage w_t and the commitment by the firm to pay the education cost c. The general training contract is a take-it-or-leave-it offer by the firm. The large number of unskilled workers per firm implies that the firm has effectively all market power and can therefore offer a contract that makes an unskilled worker exactly indifferent between accepting and rejecting the offer.

Firms produce according to a constant return to scale production function. The output produced over the period dt is given by an strictly increasing, concave and twice continuously differentiable function

$$ydt = F\left(l_u, l_s + l_t\right) dt.$$

Since training is instantaneous, trainees are able to work as skilled labor. Therefore, the skilled labor force $(l_s + l_t)$ is given by the sum of skilled workers and trainees. The unskilled labor force is given by l_u.

Firms promote trainees to a full skilled job with a respective market wage at rate ρdt. Furthermore, I assume that the firm is able to commit to its promotion promise.

Workers

New market entrants start their working career as unskilled workers, whose measure is defined by m. If they are trained by the firm, they become skilled worker. Workers are assumed to be risk neutral and to discount future payments at rate r. If workers are credit constrained, they cannot make any payment to the firm. A worker's stay in the labor market is exponentially distributed with parameter $\delta > 0$. If a worker exits the labor market, he is replaced by a new individual.

All unskilled workers start searching as unemployed in the labor market for unskilled. During that period they receive unemployment income normalized to zero. Individuals only search if the expected gain is strictly positive. Thus, only trained workers that are not promoted start to search for a skilled job. The labor markets for skilled and unskilled are separated. For simplicity, I assume that employed workers cannot become unemployed. Employment ends with a positive probability per period (here δdt) because of workers exiting the labor market.

Matching

Define s_i as the measure of workers searching in a particular labor market. The labor market tightness is defined as the ratio of vacancies to searching workers, $\theta_i \equiv v_i/s_i$. Define $M(v_i, s_i)$ as a Pissarides-type matching function, where $M(0, s_i) = M(v_i, 0) = 0$. It is assumed to be increasing, twice continuously differentiable, concave and linearly homogeneous. It hence has constant returns to matching and can be written in terms of the labor market tightness $M(v_i, s_i) \equiv s_i q(\theta_i)$. The properties of $M(v_i, s_i)$ imply that $q(\theta_i)$ is an increasing function of θ_i and satisfies the Inada conditions:

$$\text{i) } q(0) = 0, \quad \text{ii) } \lim_{\theta_i \to +0} q(\theta_i)' = \infty, \quad \text{iii) } \lim_{\theta_i \to +\infty} q(\theta_i)' = 0.$$

A searching worker meets a vacancy at the Poisson rate $M(v_i, s_i)/s_i = q(\theta_i)$. A vacancy is in turn contacted by a worker at the Poisson rate $M(v_i, s_i)/v_i = q(\theta_i)/\theta_i$. For notational reasons I define:

$$\lambda_i \equiv q(\theta_i), \qquad \text{and} \qquad \eta_i \equiv q(\theta_i)/\theta_i.$$

Bargaining

Wages are negotiated by unions and an employers' association. The unions' bargaining power is given by β. Thus, for each skill level $i \in \{s, u\}$ the agreed wage is given by

$$w_i = \beta F'_{l_i}\left(l_u, l_s + l_t\right), \tag{3.1}$$

where $F'_{l_i}\left(l_u, l_s + l_t\right)$ denotes the marginal product of a worker with skill level i. Firms and workers take these wages as given when they make their decisions. In Section 7, I allow for individual bargaining.

3.3 Individuals' Behavior

As new market entrants are unemployed, they start to search for a job. Once employed the individual can be offered training. This enables him to search for a skilled job afterwards if he is not promoted by his current employer. The flow value of being unemployed as unskilled worker is given by $(r + \delta) U$. At the rate λ_u he meets an unskilled job vacancy and gets the wage w_u.

$$(r + \delta) U = \lambda_u \max\left[V_u(w_u) - U, 0\right]. \tag{3.2}$$

The value of being employed as an unskilled worker at wage w_u is given by $V_u(w_u)$,

$$(r + \delta) V_u(w_u) = w_u + \gamma \max\left[V_t(w_t) - V_u(w_u), 0\right], \tag{3.3}$$

where the current employer offers the worker a training contract at rate γ. A trainee is promoted with probability ρ by the current employer. At the same time he can search for a skilled job vacancy at another firm (and matches with probability λ_s). The implicit assumption that the firm matches the outside wage when promoting its trainee is without loss of generality. Promoting and paying a wage less than w_s cannot be optimal since the trainees would still search and leave at the same rate λ_s as before. Paying a higher wage would reduce the firms profit. The value of being employed as trainee at wage w_t is thus given by

$$(r + \delta) V_t(w_t) = w_t + (\lambda_s + \rho) \max\left[V_{ts}(w_s) - V_t(w_t), 0\right]. \tag{3.4}$$

The value for a former trainee to be employed as skilled worker at wage w_s is given by:

$$(r+\delta)\, V_{ts}(w_s) = w_s. \tag{3.5}$$

The four Bellman equations (3.2), (3.3), (3.4), and (3.5) can be used to derive the conditions under which it is profitable for a worker to change status and hence to start actively searching for a vacancy in the corresponding labor market.

For a trainee to search for a skilled job vacancy, it has to be true that the wage for a skilled worker has to exceed the wage earned as trainee:

$$V_{ts}(w_s) > V_t(w_t) \quad \Leftrightarrow \quad w_s > w_t. \tag{3.6}$$

For an employed unskilled worker to accept the training contract, the value of being employed as trainee $V_t(w_t)$ must be at least as great as the value of being employed as unskilled worker $V_u(w_u)$[2]

$$V_t(w_t) \geq V_u(w_u) \quad \Leftrightarrow \quad \frac{(r+\delta)w_t + (\lambda_s+\rho)\, w_s}{r+\delta+\lambda_s+\rho} \geq w_u. \tag{3.7}$$

In other words, the expected wage income from starting as a trainee and later being employed (with probability $\lambda_s + \rho$) as a skilled worker has to exceed or be equal to the current wage earned as an unskilled worker.

Since it will be optimal for the firm to offer a wage w_t such that the worker is indifferent between accepting and rejecting condition (3.7) will hold with equality if workers are not credit constrained. If workers are credit constrained, then the training wage is bounded below by zero, i.e. $w_t \geq 0$. Furthermore, the firm can choose its promotion strategy ρ, which allows the firm to reduce the turnover of trained workers. By increasing the promotion rate the firm is able to lower the wage w_t acceptable to a trainee to make the worker indifferent between staying unskilled or being trained.

Returning to the individual's behavior, it follows from condition (3.7) that condition (3.6) is satisfied as long as $w_s > w_u$.

[2] This condition does not require a strict inequality, since workers are offered training contracts without the necessity to participate in search.

3.4 Steady State Turnover

Unemployment Measures

For every individual who leaves the labor market, a new individual enters unemployment as an unskilled worker. Thus, the measure δm of individuals enter the unemployment pool as unskilled workers. The measure $\lambda_u u_u$ of unemployed exit into employment. In addition, there are some individuals, i.e. δu_u, that exit the labor market before finding a job. The steady state unemployment measures of unskilled workers is

$$u_u = \frac{\delta}{\lambda_u + \delta} m. \tag{3.8}$$

Employment Measures

Since only one wage prevails in each labor market, workers cannot improve their situation by searching for an identical job. Consequently, only unemployed and trainees search.

The inflow into employment out of unemployment is given by $\lambda_u u_u$. Workers of every type exit employment at the rate δl_i. From the unskilled labor force γl_u become trainees, so that the measure of employed unskilled workers is given by

$$l_u = \frac{\lambda_u}{\gamma + \delta} u_u = \frac{\lambda_u}{\lambda_u + \delta} \frac{\delta}{\gamma + \delta} m. \tag{3.9}$$

The outflow from unskilled labor γl_u equals the inflow into the measure of trainees. The outflow from the trainee status is made up by the sum of individuals who exit the labor market altogether (i.e. δl_t), and by the individuals who find a skilled job vacancy at another firm or are promoted by their current firm (i.e. $(\lambda_s + \rho) l_t$). The measure of trainees is hence given by

$$l_t = \frac{\gamma}{\delta + \lambda_s + \rho} l_u = \frac{\delta}{\delta + \lambda_s + \rho} \frac{\lambda_u}{\lambda_u + \delta} \frac{\gamma}{\gamma + \delta} m. \tag{3.10}$$

Skilled workers are recruited internally and externally. From the pool of employed trainees $\lambda_s l_t$ are recruited externally and ρl_t internally. Given the outflow of δl_s from skilled labor the total measure of skilled labor is

$$l_s = \frac{\lambda_s + \rho}{\delta} l_t = \frac{\lambda_s + \rho}{\delta + \lambda_s + \rho} \frac{\lambda_u}{\lambda_u + \delta} \frac{\gamma}{\gamma + \delta} m. \tag{3.11}$$

Note, that the sum of trainees and skilled workers is independent of ρ, since promotion alters the status of the workers but not their role in production

$$l_t + l_s = \frac{\lambda_u}{\lambda_u + \delta} \frac{\gamma}{\gamma + \delta} m. \tag{3.12}$$

For later analysis, let us briefly focus on the ratio of skilled to unskilled labor, which determines the marginal product of the respective labor forces and hence their wages in equilibrium

$$\frac{l_t + l_s}{l_u} = \frac{\gamma}{\delta}. \tag{3.13}$$

The ratio increases with γ, the rate at which unskilled workers are recruited as trainees, but is independent of the promotion strategy ρ and the labor market frictions of either market. If a firm does not train while all other firms do but recruits skilled workers from the external market, it is able to achieve a labor ratio of

$$\frac{l_s}{l_u} = \frac{\lambda_s + \rho}{\delta + \lambda_s + \rho} \frac{\gamma}{\delta}, \tag{3.14}$$

which depends on the other firms training γ and promotion rate ρ.

Measure of Searching Individuals

The measure of individuals searching for unskilled job vacancies are the unskilled unemployed, i.e. $s_u = u_u$. Employed unskilled workers have no incentive to search for an identical job at another firm, since they would just earn the same wage.

The measure of workers searching for skilled job vacancies are the trainees, i.e.

$$s_s = l_t = \frac{\delta}{\delta + \lambda_s + \rho} \frac{\lambda_u}{\lambda_u + \delta} \frac{\gamma}{\gamma + \delta} m. \tag{3.15}$$

Since firms are small, they influence s_s through γ and ρ without taking it into account. By granting more unskilled workers general training, firms increase the pool of people searching for skilled job vacancies. This makes it easier for other firms to recruit skilled labor. The resulting externality does not automatically lead to inefficient investment into training, as shown in the next section.

3.5 Firms' Behavior

Firms maximize their present value. The instruments at hand are to create vacancies v_i for unskilled and skilled workers, to offer unskilled workers general training contracts at rate γ, to determine the trainee-wage w_t and to decide how many trainees ρ are promoted and given a full skilled worker's contract. The firm takes the wages for skilled and unskilled workers as given. Formally

$$\max_{v_i,\gamma,\rho} \pi = \int_0^\infty \left(F(l_u, l_s + l_t) - \sum_{i\in\{s,u\}} [w_i l_i + a v_i] - w_t l_t - \gamma l_u c \right) e^{-rt} dt \tag{3.16}$$

$$\begin{aligned} \text{s.t.} \quad \dot{l}_u &= \eta_u v_u - (\gamma + \delta) l_u \\ \dot{l}_t &= \gamma l_u - (\delta + \lambda_s + \rho) l_t \\ \dot{l}_s &= \eta_s v_s + \rho l_t - \delta l_s \\ w_t &= \begin{cases} w_u - (\lambda_s + \rho) \dfrac{w_s - w_u}{r+\delta} \geq 0, \\ \text{if workers are credit constrained and} \\ w_u - (\lambda_s + \rho) \dfrac{w_s - w_u}{r+\delta}, \ \text{if not.} \end{cases} . \end{aligned}$$

The total training costs for a firm is $\gamma l_u c$, which equals the inflow of new trainees multiplied by the cost of education. The firm contacts a worker with probability η_i per vacancy, so that the inflow out of unemployment into the skilled and unskilled labor force is given by $\eta_i v_i$.

Note that the marginal product of a trainee is the same as the marginal product of a skilled worker, since I assume that training is instantaneous. Denote x_i as the co-state variable associated with (3.16). Then the resulting Euler-conditions are:

$$\begin{aligned}
\frac{\partial H}{\partial v_u} &= a - \eta_u x_u \overset{!}{=} 0 \qquad\qquad \frac{\partial H}{\partial v_s} = a - \eta_s x_s \overset{!}{=} 0 \\
\frac{dx_u}{dt} &= x_u r - F'_{l_u}(l_u, l_s + l_t) + w_u + c\gamma + x_u(\delta + \gamma) - x_t \gamma \\
\frac{dx_t}{dt} &= x_t r - F'_{l_s}(l_u, l_s + l_t) + w_t + x_t(\delta + \lambda_s + \rho) - x_s \rho \\
\frac{dx_s}{dt} &= x_s r - F'_{l_s}(l_u, l_s + l_t) + w_s + x_s \delta \\
\frac{\partial H}{\partial \gamma} &= -c + x_t - x_u \overset{!}{=} 0 \qquad\qquad \frac{\partial H}{\partial \rho} = -x_t + x_s \overset{!}{=} 0 \\
w_t &= \begin{cases} w_u - (\lambda_s + \rho)\dfrac{w_s - w_u}{r + \delta} \geq 0, \\ \text{if workers are credit constrained and} \\ w_u - (\lambda_s + \rho)\dfrac{w_s - w_u}{r + \delta}, \text{ if not.} \end{cases}
\end{aligned}$$

Recruitment Cost

The steady state solution to this problem gives the vacancy creation condition for each labor market, i.e.

$$a = \eta_i^* \frac{F'_{l_i}(l_u, l_s + l_t) - w_i}{r + \delta} \quad \text{for } i \in \{s, u\}. \tag{3.17}$$

The vacancy creation condition requires that the cost of creating a vacancy a equals the expected return of a match. In the simple Pissarides (2000) model the vacancy creation condition determines together with the zero profit condition the number of firms (vacancies) in equilibrium. Here, the measure of firms is fixed to unity, so that the vacancy creation condition determines the size of a firm. This also guarantees that the value of creating a vacancy is equal to zero.

Proposition 1: *Given all other firms engage in training, the recruitment cost for skilled labor is higher than for unskilled labor.*

Proof: Define $\Phi(\theta_i) \equiv a\theta_i / q(\theta_i)$. Given the properties of the matching function, it follows that

$$\Phi'(\theta_i) > 0, \;\; \Phi''(\theta_i) < 0, \;\; \lim_{\theta_i \to +0} \Phi(\theta_i) = 0 \;\text{ and }\; \lim_{\theta_i \to +\infty} \Phi(\theta_i) = \infty.$$

Hence, $\Phi(\theta_i)$ is a strictly increasing and concave function of θ_i, with domain $[0, \infty)$ and range $[0, \infty)$.

Denote $\widetilde{F}'_{l_i}(\theta_s, \gamma) \equiv F'_{l_i}(l_u, l_t + l_s)$, where γ is the training rate of all other firms. From (3.14) and the properties of the production function, it follows that the marginal product of an unskilled worker is increasing in θ_s and the marginal product of a skilled worker is decreasing in θ_s if the firm does not train. If it trains the marginal product for each skill level is independent of search frictions, see equation (3.13). Hence, a strictly positive and unique θ_i^* exists, where $\Phi(\theta_i^*) = \widetilde{F}'_{l_i}(\theta_s^*, \gamma)$.

Since all other firms train, the training rate γ is such that $\widetilde{F}'_{l_s}(\theta_s, \gamma) > \widetilde{F}'_{l_u}(\theta_s, \gamma)$. Given equation (3.1) it follows that $F'_{l_s}(l_u, l_s + l_t) - w_s > F'_{l_u}(l_u, l_s + l_t) - w_u$ whether the firm trains or not. Thus, according to equation (3.17) the recruitment cost of a skilled worker is higher than for an unskilled worker, i.e.

$$\frac{av_s}{M(v_s, s_s)} > \frac{av_u}{M(v_u, s_u)}.$$

□

Rearranging equation (3.17) shows that the recruitment cost per match equals the discounted marginal revenue of a matched worker.

$$\frac{av_i}{M(v_i, s_i)} = \frac{F'_{l_i}(l_u, l_s + l_t) - w_i}{r + \delta}$$

In equilibrium the cash flow (i.e. $F'_{l_i}(l_u, l_s + l_t) - w_i$) of a skilled worker is greater than the cash flow of an unskilled worker. The firm will therefore pay more for the recruitment of a skilled worker than for an unskilled worker. While for a firm it is harder to find skilled workers than unskilled workers (i.e. $\eta_s^* < \eta_u^*$), the matching technology implies that it is easier for searching skilled individuals to find a vacancy than for unskilled individuals (i.e. $\lambda_s^* > \lambda_u^*$).

Firms make zero profit, since they pay one part of the marginal product for recruitment and the other part in wages to workers themselves. Thus, firms that recruit trained workers pay them their effective marginal product and hence do not profit from recruiting trained workers. The fact that the future employer of a trained worker does not benefit from the training in other firms implies that search frictions per se need not cause underinvestment in training.

Promotion Decision

The firm can use promotion to prevent trainees from searching for a skilled job vacancy at another employer. The promotion condition requires that the shadow value of a trainee equals the shadow value of a skilled worker

$$\frac{\partial H}{\partial \rho} = -x_t + x_s \stackrel{!}{=} 0. \tag{3.18}$$

The value of a trainee x_t after substituting the trainee wage w_t out is given by

$$x_t = \frac{F'_{l_s}(l_u, l_s + l_t) - w_t + \rho x_s}{r + \delta + \lambda_s + \rho}, \tag{3.19}$$

which implies that $x_t < x_s$ for all ρ, except for $\rho \to \infty$, where $x_t = x_s$. Hence, it is optimal for the firm to promote at the highest rate possible. If workers are not credit constrained, then they can be promoted immediately, i.e. $\rho^{NC} \to \infty$, in turn for a lump-sum payment that is equivalent to the value of the promotion, i.e. $(w_s - w_u)/(r + \delta)$, because the firm can extract all rent from an unskilled worker when posting the training contract. If workers are credit constrained, then it is optimal to chose the promotion probability ρ^C such that the trainee wage is set to zero, i.e. $w_t = 0$,

$$\rho^C = \max\left[(r + \delta)\frac{w_u}{w_s - w_u} - \lambda_s, 0\right]. \tag{3.20}$$

If the offer arrival rate for skilled workers λ_s exceeds $\overline{\lambda_s} = (r + \delta) w_u/(w_s - w_u)$, then nobody is promoted, since workers will accept a trainee contract even without any promotion promise by the training firm, as can be seen by substituting $\overline{\lambda_s}$ into condition (3.7).

Training Decision

Firms promote all trainees if workers are not credit constrained, and thereby keep them off the skilled labor market. This implies that workers do not benefit from training, since they pay for their promotion up front. At the same time future employers do not benefit from the training of other firms either, since all skilled workers stay with their training firm. Thus, if workers are not credit constrained the training level will be equal to the level of training in a competitive market, where workers pay for training.

Proposition 2: *If workers are not credit constrained, firms will provide training up to the first best level, i.e.* $(r+\delta)\,c = \widehat{F}'_{l_s}(\gamma^*) - \widehat{F}'_{l_u}(\gamma^*)$, *where* $\widehat{F}'_{l_i}(\gamma) \equiv F'_{l_i}(l_u, l_t + l_s)$.

Proof: The rent extracted from the difference in recruitment cost is

$$\frac{\widehat{F}'_{l_s}(\gamma^*) - w_s}{r+\delta} - \frac{\widehat{F}'_{l_u}(\gamma^*) - w_u}{r+\delta}.$$

The lump-sum payment equivalent to the value of the promotion is given by

$$\frac{w_s - w_u}{r+\delta}.$$

Adding up gives the same training condition as in a competitive market, where worker pay for training.

$$(r+\delta)\,c = \widehat{F}'_{l_s}(\gamma^*) - \widehat{F}'_{l_u}(\gamma^*). \tag{3.21}$$

Note that the difference in the marginal products between skilled and unskilled workers is higher for firms that do not train than for training firms, since $(l_t + l_s)/l_u > l_s/l_u$ according to equation (3.13) and (3.14). The return to training will therefore exceed the cost of training so that training is optimal. □

If workers are credit constrained, then firms cannot extract all the rent from workers by promoting them immediately. Firms will therefore pay a trainee wage $w_t = 0$ and promote at rate ρ^C given by equation (3.20).

The difference in recruitment costs can still be used to pay for the general training of some unskilled workers. This can be seen by looking at the Euler equation, which implies that the difference in the shadow value of a trainee and the shadow value of employing an unskilled worker has to equal the cost of training (i.e. $c = x_t - x_u$). In other words, the cost of general training has to equal the discounted cash flows between trainees and unskilled workers. Using the first order conditions to subsitute the value of being a trainee and an unskilled worker and rearranging gives

$$(r+\delta)\,c = \frac{\left(r+\delta+\rho^C\right) F'_{l_s}\left(l_u, l_s + l_t\right) - \rho^C w_s}{r+\delta+\lambda_s+\rho^C} - F'_{l_u}\left(l_u, l_s + l_t\right) + w_u. \tag{3.22}$$

Proposition 3: *If workers are credit constrained and* $0 < \lambda_s \leq \overline{\lambda_s}$, *where* $\overline{\lambda_s} = (r+\delta)\,w_u/\left(w_s - w_u\right)$, *the training level* γ^1 *is below the first best level* γ^* *(since trainees* *leave their training firm). For $\lambda_s > \overline{\lambda_s}$ the training level $\gamma^2 < \gamma^1 < \gamma^*$ is even lower, since unskilled workers receive part of the return to training.*

Proof: For $0 < \lambda_s \leq \overline{\lambda_s}$ and after substituting the promotion rate ρ^C for credit constrained workers given in equation (3.20) and the wage for skilled workers out, the training condition is according to equation (3.22) given by

$$(r+\delta)\, c = \frac{r+\delta-(1-\beta)\,\lambda_s}{r+\delta}\left(\widehat{F}'_{l_s}(\gamma^1) - \widehat{F}'_{l_u}(\gamma^1)\right). \tag{3.23}$$

Comparing this condition to the competitive level

$$(r+\delta)\, c = \widehat{F}'_{l_s}(\gamma^*) - \widehat{F}'_{l_u}(\gamma^*),$$

and noting that the marginal product of a skilled worker is decreasing in γ and the marginal product of a unskilled worker is increasing in γ as well as noting that $\beta \in (0,1)$, it follows that $\gamma^1 < \gamma^*$.
For $\lambda_s > \overline{\lambda_s}$ the training condition is according to equation (3.22) given by

$$(r+\delta)\, c = \frac{r+\delta}{r+\delta+\lambda_s}\widehat{F}'_{l_s}(\gamma^2) - (1-\beta)\,\widehat{F}'_{l_u}(\gamma^2).$$

Substituting $\overline{\lambda_s}$ for λ_s implies

$$(r+\delta)\, c < \widehat{F}'_{l_s}(\gamma^2) - (2-\beta)\,\widehat{F}'_{l_u}(\gamma^2). \tag{3.24}$$

Substituting $\overline{\lambda_s}$ in equation (3.23) gives

$$(r+\delta)\, c \geq \widehat{F}'_{l_s}(\gamma^1) - (2-\beta)\,\widehat{F}'_{l_u}(\gamma^1). \tag{3.25}$$

Comparing equation (3.24) and (3.25) gives $\gamma^2 < \gamma^1$.
The fact that workers get part of the return to training can be seen by looking at equations (3.2) to (3.5) and comparing (i) $w_t = 0$ and $\rho^C = (r+\delta)\,\frac{w_u}{w_s-w_u} - \lambda_s$ with (ii) $w_t = 0$ and $\rho^C = 0$. In the case of (i) it follows according to condition (3.7) that $V_t(w_t) = V_u(w_u)$. In case of (ii) condition (3.7) holds with equality, i.e.

$$\frac{\lambda_s}{r+\delta+\lambda_s}w_s > \frac{\overline{\lambda_s}}{r+\delta+\overline{\lambda_s}}w_s = w_u,$$

which implies $V_t(w_t) > V_u(w_u)$. Hence unskilled workers gain from being trained. $\square$

Although future employers do not benefit from employing trained workers, training will be inefficient as Proposition 3 shows, because workers are credit constrained. The reason is that for the training firm to recover its training expenses fully, all trained workers would have to stay with their training firm for their entire working life and receive the wage of an unskilled worker. Outside firms are, however, willing to pay them the wage of a skilled worker. This induces trained workers to search for another employer.

The training firm can prevent workers from starting to search by promoting them immediately and paying them the market wage of a high skilled worker. If workers are not credit constrained, then the firm can make the worker indifferent between being unskilled or becoming a trainee. The reason is that the firm temporarily possesses all the bargaining power when offering the trainee contract. It can therefore demand the value of the promotion as a lump-sum payment up-front. This guarantees that a training firm gets all the return from training and will therefore invest efficiently.

If workers are credit constrained, then the training firm will try to extract as much rent as possible from its trainees by setting the trainee wage equal to zero. In order to get the worker to accept the trainee contract, the training firm commits to a certain promotion rate, that makes the unskilled worker exactly indifferent between becoming a trainee or staying unskilled. Since workers are searching for a skilled job as long as they are not promoted, training will be inefficient due to worker turnover.

If the probability for trainees to get an outside offer is high enough then the training firm does not need to promise a promotion in order to get the worker to accept the trainee contract. This in turn implies that with a high matching rate for skilled workers, trainees profit from general training, since the training firm is not able to extract the whole surplus from its trainee. Thus, the level of training provided by the firm will be even lower.

3.6 Labor Market Equilibrium

The aim of this section is to show that in an economy with credit constrained workers there may be multiple training equilibria. If workers are not credit constrained, promotion in turn for an equivalent lump-sum payment from the trainee to the training firm prevents trainees from quitting and leads to a unique labor market equilibrium.

Definition: Labor Market Equilibrium

A labor market equilibrium satisfies the following conditions: Firms create vacancies according to (3.17), offer general training at rate γ satisfying (3.22) if workers are credit constrained and (3.21) if workers are not credit constrained and are promoted immediately. Workers follow an optimal search strategy according to (3.2) - (3.5) and bargaining wages are formed according to (3.1).

Proposition 4: *If workers are not credit constrained, then a unique labor market equilibrium exists.*
If workers are credit constrained, multiple equilibria with inefficient training can exist, where a high training equilibrium is sustained by a low matching rate for trainees and vice versa, i.e. for any two equilibria a and b, I have $\lambda_s^a < \lambda_s^b$ and $\gamma^* > \gamma^a > \gamma^b$.

Proof: Part 1: Existence and uniqueness if workers are not credit constrained.
Since $\widehat{F}'_{l_s}(\gamma) - \widehat{F}'_{l_u}(\gamma)$ goes to infinity for $\gamma \to 0$ and to zero for $\gamma \to \infty$, a unique $\gamma^* > 0$ for the training rate in equation (3.21) exists. The wages are w_u^*, w_s^* and the market tightness θ_u^*, θ_s^* are functions of γ^* via the marginal product of a worker but not vice versa. Thus, the vacancy creation condition (3.17) implies a unique market tightness θ_i^* for each market. Wages are uniquely determined by equation (3.1) via the marginal product.
Part 2: Existence and multiplicity, if workers are credit constrained.
Again the property of the production function implies that $\gamma^j > 0$ for $j = 1, 2$ for the training rate in equation (3.22) exists. To establish the possibility of multiplicity it is sufficient to show that there are multiple (θ_s^*, γ^j) that satisfy the vacancy creation condition (3.17) for skilled workers and the training equation (3.22). The training

condition can be written as

$$(r+\delta)\,c = f_j\left(\theta_s^*\right)\overline{F}'_{l_s}(\gamma^j) - h_j\overline{F}'_{l_u}(\gamma^j), \tag{3.26}$$

where

$$\begin{aligned} f_1\left(\theta_s^*\right) &= \frac{r+\delta-(1-\beta)\,\lambda_s}{r+\delta} \text{ and } h_1 = 1 \text{ for } j=1 \text{ and} \\ f_2\left(\theta_s^*\right) &= \frac{r+\delta}{r+\delta+\lambda_s} \text{ and } h_2 = 1-\beta \text{ for } j=2. \end{aligned}$$

Note that $f_j\left(\theta_s^*\right)$ is decreasing in θ_s^* and the rhs of equation (3.26) is decreasing in γ^j. In the vacancy creation condition (3.17) the rhs is decreasing in θ_s^* and in γ^j. Thus, multiple equilibria a and b can exist for $\gamma^a > \gamma^b$ and $\theta_s^{*a} < \theta_s^{*b}$. □

If workers are credit constrained, firms are deprived of the promotion instrument, and general training generates a search externality since firms do not take into account that by training they increase the pool s_s of people searching for skilled job vacancies – compare equation (3.15) – and that by doing so it becomes harder for other trainees to find a job. This lower separation rate increases the firm's return to general training, which sustains a high training level and a low market tightness for skilled labor. On the other side, a low training equilibrium can exist where the probability for trainees to find a job at another firm is high. This decreases the return to general training such that firms train less, which sustains a high matching rate for trainees.

Only if unskilled workers are not credit constrained can the current firm extract the whole rent from general training and prevent its trainees from searching. This eliminates this externality and leads to an efficient investment in general training.

3.7 Extensions

Individual Bargaining

Assume that wages are negotiated after a worker contacted a firm. Firms take these wages as given then; they choose the number of vacancies, the training rate and the promotion rate. Nature chooses with probability β the worker to make an offer and with probability $1-\beta$ the firm. Workers and firms are assumed to have some bargaining power (i.e. $0 > \beta > 1$). If the other party accepts the offer, a wage contract is written and production starts immediately thereafter. If the offer is rejected, the respondent can leave the negotiation table and continue searching (both parties), or he can wait for the bargaining game to start again next period.

During this period the worker receives the flow-utility of leisure normalized to zero, since an employed worker has to take a day leave while bargaining with a different firm. The firm makes no loss or gain, since it does not advertise the job vacancy during negotiations.

At the same time there is a positive probability δdt that the worker exits the labor market. This could result in a breakdown of the negotiations, where the worker receives a flow utility of zero and the firm continues searching with the unfilled vacancy, which has a value of zero due to free entry. The firm's payoff while negotiations are postponed is also zero, as mentioned above.

The outside options of the workers are to take another day leave which gives him zero utility. The outside option for a firm is to walk away and to search for another worker. Since the value of a vacancy (i.e. searching) is zero in equilibrium, the outside option of the firm has a value of zero.

In case of a breakdown, payoffs are zero. The outside and the inside options are not binding so that the bargaining model simplifies to a random proposer Rubinstein model. Furthermore, the fact that the discount rates for firms and workers are identical implies that the bargaining power is equivalent to the probability of being chosen by nature to make an offer. Muthoo (1999, ch. 3.2 and 7.2.4) shows that the solution to the bargaining scenario - as $dt \to 0$ - is given by

$$w_i^* = \beta F'_{l_i}\left(l_u, l_s + l_t\right).$$

The assumption that an employed worker receives only the value of leisure and not his wage while negotiations are postponed ensures a single wage for each type of labor. This implies that employed workers do not gain by searching for an identical job at another firm. Therefore, only the unemployed and trainees will search. This assumption is relaxed below.

On-the-job Search and Search Intensity

In the preceding analysis the bargaining game was chosen such that only unemployed and trainees searched but not the skilled and unskilled workers. If one assumes that the inside option of a worker is his current wage and not the value of leisure, then on-the-job search will arise since workers can increase their wage every time they meet a new employer, i.e.

$$w_{i,e} = (1-\beta)w_{i,e-1} + \beta F'_{l_i}(l_u, l_s + l_t), \tag{3.27}$$

where e is an index for the number of employers the worker was/is employed with and $w_{i,e-1}$ indicates the wage at the last employer or in the case of the first employer the value of leisure normalized to zero. Thus, employed workers will continue searching as long as they earn less than their marginal product.

Promotion would keep trainees away from the skilled labor market and lead to efficient investment in general training if workers are not credit constrained, since the training firm can recover the promotion cost up-front via a lump-sum payment for training equivalent to the cost of promotion. If workers are not credit constrained, then the training firm will not promote the trained workers. It can, however, reduce the trainee wage in order to capture the future wage increases the worker expects to get from searching on-the-job.

The result that training firms do not promote, or demand the lowest possible trainee wage, only changes if the search intensity is no longer fixed and costless for workers. To introduce search intensity I follow Pissarides (2000). The matching rate depends not only on the market tightness θ_i, but also on a worker's search intensity $\sigma_{i,e}$, which will vary with his wage and thus with the number of jobs he already occupied, and it will depend on the average search intensity σ_i of all workers from his skill group. The transition rate for a worker is therefore given by

$$\sigma_{i,e}\phi_i \equiv \sigma_{i,e}\frac{q(\sigma_i,\theta_i)}{\sigma_i} = \sigma_{i,e}\frac{M(v_i,\sigma_i s_i)}{\sigma_i s_i}.$$

Assume that the search cost function $k(\sigma_{i,e})$ is convex and $k(0) = 0$, then the Bellman equation for a trainee is given by:

$$(r+\delta)\,V(w_{t,e})=\max_{\sigma_{t,e}}\left[w_{t,e}-k(\sigma_{t,e})+\sigma_{t,e}\phi_t\left(V(w_{t,e+1})-V(w_{t,e})\right)\right].$$

It follows that the optimal search intensity equates the marginal cost of searching with the marginal expected gain from being employed at the new employer at wage $w_{t,e+1}$, i.e.

$$\frac{\partial k(\sigma_{t,e})}{\partial\sigma_{t,e}}=\phi_t\left(V(w_{t,e+1})-V(w_{t,e})\right).$$

The convex search cost function and the fact that the expected utility gain of changing employer, i.e. $V(w_{t,e+1})-V(w_{t,e})$, decreases[3] with a higher current wage guarantees that each trainee will search less if his current wage is higher. However, trainees will continue to search as long as they earn less than their marginal product. Nevertheless, firms might be able to extract some rent from their trainees by promoting them immediately after training since the promotion saves the trainees search costs and reduces their incentive to search more intensively. A firm will promote a trainee, i.e. pay him a wage $w_{s,e}>w_{t,e}$, if and only if the lower matching probability compensates the firm for the cost of promotion, i.e.

$$\begin{aligned}&\max_{w_{s,e}\in\left(w_{t,e},F'_{l_s}(l_u,l_s+l_t)\right)}\left[F'_{l_s}(l_u,l_s+l_t)-w_{s,e}+\sigma_{s,e}\phi_s\left[0-J(w_{s,e})\right]\right]\\ >\;&F'_{l_i}(l_u,l_s+l_t)-w_{t,e}+\sigma_{t,e}\phi_t\left[0-J(w_{t,e})\right],\end{aligned}$$

where $J(w_{i,e})$ is the value of employing a worker at wage $w_{i,e}$. If the worker leaves, then the value to the firm is zero. Provided the convexity of the search cost function is severe enough, then the training firm will promote its trained workers.

3.8 Conclusion

The model presented in this chapter shows that in a search model where vacancy creation drives profits down to zero such that future employers of trained workers do

[3] This can easily be seen from equation (3.27) and the fact that $V(w_{t,e})$ is bounded above by the discounted sum of the workers marginal product.

not benefit from the training in other firms, then firm's investment into general training will only be below the competitive level if workers are credit constrained. The reason is that unskilled workers have to pay their expected gain from training to the training firm in exchange for being trained.

If workers are credit constrained, then the training firm cannot recover the cost of training, since trained workers will search for a better paid job. This, however, does not imply that trainees will benefit from training, since the firm can extract the worker's expected gain from searching for another employer by paying him a low trainee wage as long as the worker stays with the training firm. Only if the trainee wage is bounded by the workers' credit constraints do trainees gain from training.

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