A Network Analysis of Contagion Risk in the Interbank Market

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Acronyms

AUR	Area Under the ROC-curve
BAKIS	Bankaufsichtliches Informationssystem
BIS	Bank for International Settlements
CDF	Cumulative Distribution Function
CDS	Credit Default Swap
CoVaR	Conditional Value at Risk
CR	Concentration Ratio
EBA	European Banking Authority
ECB	European Central Bank
EUR	Euro
FEDS	Finance and Economics Discussion Series
HHI	Herfindahl-Hirschman Index
IMF	International Monetary Fund
LGD	Loss Given Default
MiMiK	Mikrodatenbank Millionenkredite
OLS	Ordinary Least Squares
ROC	Receiver Operating Characteristic
RWA	Risk Weighted Assets
VaR	Value at Risk

Chapter 1

Introduction

1.1 Research on financial stability

The continuing financial crisis starting in 2007 forces policy makers as well as academics around the world to think about adequate actions to guarantee the proper functioning of the global financial system. At the latest the collapse of Lehman brothers in 2008 showed that a globally interconnected financial network can transmit shocks to financial centers all over the world. And recently the emerging European sovereign debt crisis shows that there is an additional danger of spillover effects from a sovereign default to the banking system which could serve again (e.g. through the interbank market) as a shock transmitter to financial institutions all over the world. As a response to these events happening during the financial crisis, policy makers as well as academics are working on solutions to make the financial system more resilient to shocks. Thus, more and more models and methods are developed to evaluate and improve the stability of the financial system. Additionally, a great effort is made to develop and implement a new regulatory framework that mitigates systemic risk.

A comprehensive summary of research on this topic has been provided by the Global Financial Stability Report of the International Monetary Fund in April 2009.[1] In this report, different approaches to measure systemic risk due to the existence of financial interlinkages are presented: One of these approaches is the Co-Risk analysis developed by Adrian and Brunnermeier (2008). This approach uses publicly available market equity data as well as balance sheet data of financial institutions to measure the Value at Risk (VaR) of one institution in distress conditional on another institution (or the whole financial system) being in distress. This "Conditional Value at Risk" (or CoVaR) can be calculated by using quantile regressions. Another approach introduced by the International Monetary Fund (2009) is the distress dependence matrix developed by Segoviano and Goodhart (2009). They generate a

[1] See International Monetary Fund (2009), Chapter 2

multivariate distribution describing implied asset price movements of different institutions. From this multivariate distribution, pairwise conditional probabilities of distress can be derived. Thus, it is possible to calculate the probability of one institution falling into distress conditional on the probability of another institution being in distress.

These approaches use market data of financial institutions. This makes it possible to assess systemic risk due to direct and indirect exposures between financial institutions, as co-movements of risk should be, under the assumption of market efficiency, captured by co-movements of the respective indicators (like equity returns or CDS spreads). An additional advantage of market data is that they are publicly available, usually at a high frequency. As a result, various studies have been developed recently that exploit the information inherent in market data to evaluate systemic risk in financial systems. Another paper that uses market data for the analysis of financial stability is Acharya et al. (2010). In this study, the authors use equity return as well as CDS data to estimate the relationship between banks' losses in times of severe distress compared to moderately bad days. The European Central Bank provides a summary and brief discussion of these and other studies that try to quantify systemic risk in a special feature of its Financial Stability Review of December 2010.[2] However, one drawback of the approach using market data is that markets are not efficient - especially not in times of distress. Market participants tend to overreact in crisis times and underestimate risk in tranquil times. Additionally, the anticipation of a government bail-out for institutions that are considered as too big to fail may lead to an incorrect evaluation of the true risk inherent in certain financial institutions.

Another approach mentioned in the Global Financial Stability Report of the International Monetary Fund (2009) considers the financial system as an interconnected network of financial institutions. This approach usually does not rely on

[2] See European Central Bank (2010a), pp. 147-153

market data, but on detailed information about mutual interbank exposures. Of course, this approach faces some problems of data availability, especially on an international level. Additionally, there is usually only a focus on direct interlinkages between banks. However, an advantage of this approach is, that results can be clearly assigned to one specific channel of shock transmission and that it normally uses quite reliable data. In this modeling approach, the financial system can be seen as a directed graph with financial institutions being the nodes (or vertices) and exposures between these institutions being the edges (or arcs). Within this so-called network approach techniques originated within the theory of complex networks can be used. These techniques are already widely applied to other disciplines like physics, computer science or sociology. In this context, a special feature in the June 2010 Financial Stability Review of the European Central Bank[3] as well as Haldane and May (2011) provide an introduction how the theory of complex networks can be applied to analyze the stability of financial systems.

1.2 Summary and contribution

Chapter 2 of this thesis is a contribution to the growing literature on financial networks. It is based on the paper "Completeness, interconnectedness and distribution of interbank exposures - a parameterized analysis of the stability of financial networks".[4] In this chapter, the stability of a stylized financial system dependent on certain characteristics is evaluated. Whereas existing empirical literature on this topic has its focus on one particular network, theoretical (simulation as well as model-based) literature on this topic concentrates on the effect of the completeness and interconnectedness of the network on financial stability. This chapter extends the existing literature on theoretical network analysis by one parameter: *the degree*

[3] See European Central Bank (2010b), pp. 155-160

[4] See Sachs (2010)

of equality of the distribution of interbank exposures (measured by entropy).

In this chapter, a financial system is characterized by the total number of banks, the total number of assets in the system, the share of interbank assets to total assets and the banks' equity ratio. Furthermore, three network structures are investigated: a *complete network*, where a directed link between each financial institution exists, a *random graph*, which denotes a usually incomplete network, where (in this case homogeneous) banks form their links randomly, and a so-called *money center system*, which consists of large and strongly interconnected core banks as well as smaller banks in the periphery that are linked to exactly one core bank. The crucial component for the simulations is the matrix of interbank exposures. For given row and column sums of the matrix, which are exogenously given by the parameters mentioned above, a large number of valid matrices of interbank exposures is generated by simulation. These matrices are then characterized by the degree of equality how the exposures are distributed.

After creating these matrices for a given financial system, domino effects are simulated. Thus, it is investigated what happens if one bank fails for some exogenous reason. If losses on the exposures to the failed bank exceed the creditor banks' capital, the creditor banks also fail.[5] Several rounds of this contagion mechanism could occur leading to a whole cascade of bank failures. One important component in the contagion analysis is the loss given default (LGD), i.e. the share of the total exposure to the failing bank that is actually lost and leads to write-downs on the creditor banks' equity. To keep the contagion analysis as simple as possible and to be able to exclusively focus on the impact of the structure of the matrix of interbank exposures, I assume a constant LGD within this chapter.[6]

[5] Note that Chapter 2 does not deal with the existence of any regulatory minimum capital ratios. Thus, I assume that a bank can operate properly as long as it has positive capital. The following Chapters 3 and 4 will take a minimum capital ratio into account.

[6] In Chapter 3 and 4 that contain an empirical analysis of the stability of the German banking system, this assumption is not applied any more. Instead, a whole distribution of loss given default is used for simulations.

The first simulation exercise deals with a complete network and not too extreme parameter values (i.e. very low equity ratios or very high LGDs). The first result is that a more equal distribution of exposures leads to a more stable system as exposures are better diversified among counterparties. In a next step, this analysis is extended to a random graph. Looking at the average number of bank defaults for a given connectivity (i.e. a given probability that a certain link exists), the results of the theoretical model of Allen and Gale (2000), i.e. the non-monotonic relationship between the completeness of the network and its stability, can be confirmed by simulation. If, however, the distribution of interbank exposures is additionally considered, some deviant results can be shown. Thus, the second result of this chapter is that financial stability does not only depend on the completeness and interconnectedness of the network, but also on the degree of equality how the exposures are distributed. Furthermore, two key parameters, the equity ratio and the loss given default, are varied. It turns out that for parameter values that yield a very unstable system (i.e. a very low equity ratio and a very high LGD) a more unequal distribution of interbank exposures leads to a more stable system. Thus, in this case, an equal distribution of exposures helps to spread the initial shock all over the system. The next simulation exercise deals with money center models. Not surprisingly, the more concentrated assets are in the core of the system, the more unstable it is. The last simulation exercise in this chapter deals with a comparison of the stability of money center systems to the stability of random graphs. On the reasonable assumption that exposures among core banks are at least as large as exposures from core to periphery banks, a money center system is less stable compared to a network of homogeneous banks that form their links randomly.

To be able to analyze the influence of specific parameters like the structure of the matrix of interbank exposures, banks' capitalization or the loss given default, the theoretical simulations in Chapter 2 use a highly simplified structure of a banking system. Real world banking systems are, of course, much more complex, in particular in terms of the system size and the heterogeneity of the banks. Chapter 3 and 4 of this thesis focus on a real-world banking system and investigate its stability using actually realized supervisory data. To be more precise, we analyze the stability of the German banking system using detailed information on interbank exposures as well as data on actually realized loss given default in the interbank market.

Chapter 3 is based on joint work with Christoph Memmel and Ingrid Stein (Deutsche Bundesbank) and provides a revised version of the paper "Contagion at the Interbank Market with Stochastic LGD".[7] The main emphasis of this chapter is on modeling the loss given default. Our dataset shows that the empirical frequency distribution of the loss given default is markedly u-shaped, i.e. in most of the cases the LGD is either very low (e.g. due to good collateralization) or very high. This u-shaped pattern is found for different subsamples concerning the type and the size of the banks. A suitable approximation for our u-shaped LGD distribution can be derived by using a beta distribution with parameters being less than one. Existing empirical literature on interbank contagion mostly uses an exogenously given constant LGD for simulations and then derives results dependent on the specification of the LGD. However, as we have a whole distribution of interbank loss given defaults available, it is possible to run our simulations on the assumption of a stochastic LGD. Thus, we repeatedly simulate the failure of one particular bank, each time drawing a set of LGD values from the estimated beta distribution. This exercise is repeated for each bank in our sample. Contrary to the case of a constant LGD, where only one number of bank defaults is obtained, our simulation method yields a whole distribution of bank failures and therefore makes it possible to distinguish between different scenarios. In our simulations that use on- and off-balance sheet exposure data of German banks in the fourth quarter of 2010, we find that contagion in the German interbank market may happen. Furthermore, we find that off-balance sheet

[7] See Memmel et al. (2011)

exposures considerably contribute to systemic risk and that netting (if enforceable) could be a potential solution to the problem of direct interbank contagion.

Chapter 4 builds on Chapter 3 and is based on the paper "Contagion in the Interbank Market and its Determinants", which is a joint work with Christoph Memmel.[8] In this chapter, the analysis of contagion within the German banking system is extended to a whole time *period* (from the first quarter of 2008 to the second quarter of 2011). Thus, we run the contagion analysis already applied in Chapter 3 for each quarter within the time period under consideration. The result of this exercise is that the system becomes less vulnerable to direct interbank contagion over time. To investigate the impact of our assumption of a stochastic LGD, we run the same simulations again, but this time by assuming a constant LGD which equals the mean of our LGD-dataset. We find that the effect of our assumption of a stochastic LGD depends on the overall stability of the financial system. The assumption of a constant LGD leads to an overestimation of the number of bank failures if the system is rather unstable, as it is not taken into account that parts of the interbank exposures have a high recovery rate (i.e. a low loss given default). In contrast, if the system is rather stable, the assumption of a constant LGD leads to an underestimation of the number of bank failures, as it is possible that some key exposures have a very low recovery rate (i.e. a high loss given default). Thus, we conclude that it is important to take into account the *distribution* of the LGD when running a contagion analysis. Simulating by averaging out the LGD can lead to an over- or underestimation of the stability of the financial system, respectively.

As we run stochastic simulations, we obtain a whole distribution of bank failures as a result. To be able to compare the distributions of bank failures for different points in time, we use the concept of stochastic dominance. However, to implement further analysis it is desirable to condense the information of the whole distribution into

[8] See Memmel and Sachs (2011)

one indicator. By estimating a logit model, we show that it is possible to predict the probability of a dominance relationship by the absolute difference in the expected number of bank defaults. Thus, most of the information can be condensed into one indicator, which is the expected number of bank defaults. This result simplifies the investigation of the main determinants of financial stability. Following the theoretical literature on interbank contagion (e.g. the simulation analysis in Chapter 2), we estimate the impact of the following determinants: the banks' capitalization, their interbank lending, the mean of the beta distribution of the loss given default and as a really systemic measure, the degree of equality how banks spread their exposures (measured by entropy). Thus, we quantify the impact of the different determinants of system stability. Additionally, we can confirm the results of the theoretical simulations in Chapter 2 that a higher equity ratio, a lower amount of interbank lending, a lower average loss given default and a more equal distribution of interbank exposures leads to a more stable system.

Hence, some implications for the optimal design of a stable financial network can be derived out of the following chapters. First, a rather equal distribution of interbank exposures and thus a careful risk diversification makes the system more resilient to an exogenous shock as long as banks are not too weak to absorb shocks (e.g. due to a very low capitalization). Additionally, a centralized banking system with few large banks in the core and many small banks that are linked to these few large banks bears the risk that the failure of one large bank is a threat to all remaining banks in the system. A decentralized system with banks of rather equal size naturally allows more opportunities of risk diversification leading to a more stable system. Not surprisingly, a better capitalization of banks (which is already included in the Basel III framework) and a lower amount of interbank lending decrease the danger of direct interbank contagion. This leads to the conclusion that a higher reliance on wholesale funding by banks reduces the stability of the financial system. Another important implication can be drawn for the detection of systemic risk. In the following chapters, it is shown that simulations of direct contagion on the interbank market are remarkably influenced by assumptions concerning parameters like the loss given default. Thus, it is desirable to take into account a more realistic modeling approach of the loss given default by using a u-shaped frequency distribution.

During recent years, a great effort has already been made to enhance the resilience of the financial system to certain shocks. Part of these efforts comprise the development of the Basel III regulatory framework that is, among other things, designed to improve banks' capital and liquidity endowment. Furthermore, institutions like the European Systemic Risk Board (responsible for the European Union) or the Financial Stability Oversight Council (responsible for the United States) were created to improve macroprudential surveillance and detect systemic risk.[9] But, as often stated among policy makers and academics, a lot of research still has to be done in order to sufficiently understand various channels that influence financial stability. This thesis constitutes a quantitative contribution to this research partially using real data.

[9] For a description of their tasks and possibilities of action see International Monetary Fund (2011), Chapter 3, pp. 3-5

Chapter 2

A parameterized analysis of the stability of financial networks

2.1 Introduction

Research on financial stability is an important topic in order to assess certain risks and dangers within financial systems that potentially lead to huge losses for the overall economy. Especially the investigation of various channels of interbank contagion has been in the focus of recent research. This is also the aim of this chapter. In this context, interbank contagion means that the failure of one financial institution leads to the failure of other financial institutions. To be more precise, this chapter examines pure domino effects between banks. Thus, it is investigated what happens if one bank fails and therefore a part of other banks' claims to that bank also fail. It is then possible that creditor banks lose all their capital and therefore fail as well. In the worst case, there are subsequent rounds of failures until the whole system defaults. Of course, this is just one channel through which interbank contagion can occur. Further channels can be contagion due to liquidity problems because of correlated asset portfolios among banks, contagion due to refinancing problems affecting banks or contagion due to information spillovers. As a starting point, however, to be able to exclusively focus on the effect of the structure of the liability matrix on the stability of the financial network, only domino effects are considered.

The main contributions of this chapter are, first, that there is an explicit investigation of the impact of the structure of the matrix of interbank liabilities on the stability of the interbank network. In this context, for given balance sheets of a hypothetical banking system, a large number of valid matrices of interbank exposures is created and characterized by the degree of equality of the distribution of exposures (measured by entropy). Second, this chapter examines, how the impact of the structure of the matrix of interbank exposures on the stability of the financial system interacts with other parameters like banks' capitalization or the loss given default. Third, this chapter provides a comparison of the stability of the financial system between different network topologies like a complete network, a random graph and a money center system with a core-periphery structure.

The main results are, first, that not only the topology of the network (e.g. its completeness and interconnectedness) determines its stability but also how equally interbank exposures are distributed. The second result is that the sign of the correlation between the degree of equality of the distribution of interbank exposures and the average number of bank failures depends on the number of interbank links within the financial system as well as on banks' equity ratio and the loss given default. Additionally, by assuming reasonable parameter values concerning the amount of bilateral interbank exposures, money center systems with asset concentration among core banks are more unstable than networks with banks of homogeneous size that form their links randomly.

This chapter is organized as follows. Section 2.2 gives an overview on the related literature as well as this chapters' main contributions to this literature. In Section 2.3 the basic structure of the financial system is defined. Section 2.4 explains in more detail how interbank liability matrices are created and characterized. Simulations of domino effects are run and results are presented in Section 2.5 which is divided into the investigation of complete networks (Section 2.5.1), random graphs (Section 2.5.2) and money center models (Section 2.5.3). Section 2.6 summarizes the main findings.

2.2 Literature

Various fields of studies have been developed to capture the numerous facets of this comprehensive topic (for a literature survey, see Allen and Babus (2009)). From the theoretical point of view, Allen and Gale (2000) show that interbank connections can be useful in order to provide an insurance against liquidity shocks. Because of these interbank linkages, however, the bankruptcy of one bank can lead to the bankruptcy of other banks. In this context, Allen and Gale show that a financial system with a complete network structure is less prone to contagion than a financial system with an incomplete network structure. In addition, they state that a disconnected system is useful to limit contagion. Freixas et al. (2000) implement, among other things, a theoretical analysis of contagion within a “money center system”, where banks in the “periphery” are linked to one “core bank” but not to each other. They show that there are parameter constellations under which the failure of a periphery bank does not lead to contagion, whereas the failure of the “core bank” does.

Another part of the literature that investigates financial stability are empirical studies that use supervisory data to analyze the danger of domino effects within a banking system (for example van Lelyveld and Liedorp (2006), Upper and Worms (2004), Wells (2004), Furfine (2003), Sheldon and Maurer (1998)). As a lot of detailed data on interbank exposures are necessary but often not available, assumptions such as maximum entropy are made concerning the structure of these exposures. This means that banks are assumed to spread their interbank claims as equally as possible among their counterparties. However, it is likely that, under the maximum entropy assumption, results are biased. In his summary of the analysis of interbank contagion, Upper (2007) states that maximum entropy assumptions tend to underestimate the incidence but overestimate the severity of contagion. Mistrulli (2011) investigates interbank contagion using actual Italian interbank data and compares his findings with an analysis using the maximum entropy assumption. He finds that, for most parameter constellations, the maximum entropy assumption tends to underestimate the extent of contagion. There are, however, also some parameter constellations (in particular a high loss given default) where the maximum entropy assumption overestimates the scope of contagion.

Cifuentes et al. (2005) extend a contagion model of domino effects by simulations that include contagion due to liquidity problems. Within their simulations they use a clearing algorithm developed by Eisenberg and Noe (2001).[10] Liquidity effects are considered in a similar way in the network model of Bluhm and Krahnen (2011). Furthermore, the analysis of pure domino effects is extended by Chan-Lau (2010) and Espinosa-Vega and Solé (2010) by additionally considering contagion due to banks' refinancing problems and due to risk transfers stemming from off-balance sheet exposures. Also, the impact of market and funding liquidity risk on the stability of a financial network is investigated by Aikman et al. (2009).

In recent years there has been a growing literature which uses theory of complex networks to describe real-world financial systems and simulate the effects of potentially dangerous events. For example, Boss et al. (2004) analyze the network topology of the Austrian interbank market. Iori et al. (2008) apply network theory to describe the Italian overnight money market. Haldane (2009) provides a characterization of the world's financial network. Georg (2011) models a financial system including liquidity provision by the central bank and investigates the effect of the structure of the network (e.g. a small-world network and a scale-free network) on its stability. Additionally, Gai and Kapadia (2010), as well as Nier et al. (2007), use random graphs to analyze the danger of contagion dependent on certain characteristics of the financial system by simulation. Gai and Kapadia (2010) find that for a high connectivity of the network, the probability that contagion occurs is low but the impact if contagion occurs can be high. Nier et al. (2007) find out by parameterized simulation some non-linearities between certain parameter values and financial stability. Contrary to most of the empirical literature, where one special financial system is considered to test the danger of contagion, simulation-based work instead tries to find out the main characteristics that make a financial system especially vulnerable to contagion.

As, up to now, only few studies exist about the detailed structure of real-world finan-

[10] Eisenberg and Noe showed that under mild regularity conditions (strong connectivity and at least one node has positive equity value or all nodes have positive operating cash flows) there exists a unique fixed point that describes the clearing payment vector of the financial system.

cial networks, this chapter considers several stylized structures and investigates their impact on financial stability by simulation. This chapter builds on the empirical literature that uses entropy methods to construct and characterize interbank linkages as well as on literature that tries to simulate the danger of contagion according to certain characteristics of the financial system, in particular the matrix of interbank exposures. For example, it extends the work of Mistrulli (2011) in such a way that not only one matrix is compared to the maximum entropy solution but a great variety of randomly generated matrices with different network structures. A new approach in this chapter is that a large set of valid interbank matrices is constructed by a random generator and then characterized according to certain properties, such as entropy, relative entropy to the maximum entropy solution or connectivity.

Additionally, this chapter differs from Nier et al. (2007) in the sense that, first, balance sheets are constructed and, as a second step, the liability matrix is generated, which is, besides row and column sums of the matrix, independent from banks' balance sheets. Thus, stability results obtained in this chapter can be attributed purely to changes in the liability matrix.

Furthermore, results can be interpreted as an extension to the theoretical literature about the impact of certain network patterns on contagion. Up to now the focus has been exclusively on the completeness and interconnectedness of interbank networks (see Allen and Gale (2000)). In this context, banks are modeled as completely homogeneous, especially with all interbank exposures being the same size. This work, however, investigates a large number of possible matrices with various possible specifications of interbank exposures and can thus have an additional focus on the *distribution* of claims within the network for given completeness and interconnectedness. Results of this chapter show that the distribution of interbank claims within the network is an important parameter affecting the stability of the network.

2.3 Structure of the financial system

The financial system is modeled as a network of N nodes where nodes 1 to $N-1$ are financial institutions (referred to as banks in the following) and node N constitutes the external (non-banking) sector (such as households or non-financial companies). These nodes are linked by directed edges that depict direct claims/obligations between the financial institutions and the external sector. For some of the subsequent financial networks modeled, it is assumed that there are two different types of banks, core banks and periphery banks, that are equal within their groups but differ across groups with regard to their connectivity and size, respectively. The distribution of assets among the two types of banks is given by a concentration ratio CR, that denotes the share of total bank assets that core banks hold.
Bank i's balance sheet has the following structure:

$$A_i^{IB} + A_i^E = L_i^{IB} + L_i^E + E_i \tag{2.1}$$

$\forall\ i \ \epsilon\ \{1, ..., N-1\}$.
Interbank assets A_i^{IB} (liabilities L_i^{IB}) are claims (obligations) between banks. External assets A_i^E are interpreted as credit to the external sector. External liabilities L_i^E denote obligations of banks to the external sector such as customer deposits. The balance sheet is completed by equity E_i that is given by the difference of bank i's total assets A_i $(= A_i^{IB} + A_i^E)$ and liabilities L_i $(= L_i^{IB} + L_i^E)$.
The (risk unweighted) equity ratio, which is presumed to be equal across banks, is given by:[11]

$$r = \frac{E_i}{A_i^{IB} + A_i^E} \tag{2.2}$$

[11] More general models including banks with heterogeneous equity ratios can be implemented in a straightforward way.

$\forall\ i\ \epsilon\ \{1, ..., N-1\}$.

The financial system is characterized by the total amount of banks' assets A^{banks}, as well as the total amount of banks' interbank assets A^{IB}. The ratio of interbank to total assets in the financial system is defined as:

$$\phi = \frac{A^{IB}}{A^{banks}} \tag{2.3}$$

Bank i's total assets, total liabilities and equity of this stylized financial system can be perfectly described by the total amount of banks' assets A^{banks}, the total number of banks $N-1$, the number of core banks n_{core}, the concentration ratio CR and banks' equity ratio r.

The direct connections between the nodes can be illustrated by a liability matrix:

$$\mathbf{L} = \begin{array}{c} \\ L_1 \\ L_2 \\ \vdots \\ L_{N-1} \\ L_N \end{array} \begin{array}{c} \begin{array}{ccccc} A_1 & A_2 & \dots & A_{N-1} & A_N \end{array} \\ \begin{pmatrix} 0 & L_{1,2} & \dots & L_{1,N-1} & L_{1,N} \\ L_{2,1} & 0 & \dots & L_{2,N-1} & L_{2,N} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ L_{N-1,1} & L_{N-1,2} & \dots & 0 & L_{N-1,N} \\ L_{N,1} & L_{N,2} & \dots & L_{N,N-1} & 0 \end{pmatrix} \end{array}$$

with $L_{i,j}$ being the obligation of bank i to bank j ($i,\ j\ \epsilon\ \{1, ..., N-1\}$). Because banks and the external sector do not lend to themselves, $L_{i,i} = 0\ \forall\ i\ \epsilon\ \{1, .., N\}$. Additionally, as banks are linked on both sides of their balance sheets, it is easy to interpret row sums (= total liabilities) and column sums (= total assets) of the matrix. The elements of the last row, $L_{N,i}$ ($\forall\ i\ \epsilon\ \{1, .., N-1\}$), are equal to banks' external assets A_i^E. Thus, the sum of the elements in the last row of the matrix is equivalent to banks' total external assets, which are given by $(1-\phi)\cdot A^{banks}$. The elements of the last column, $L_{i,N}$ ($\forall\ i\ \epsilon\ \{1,..,N-1\}$), are equal to banks' external liabilities L_i^E. Hence, the sum of the elements in the last column of the matrix (A_N) is equivalent to the total amount of external liabilities of banks. Furthermore, it is assumed that the system is closed, i.e. there is no lending / borrowing to somewhere outside the network. Technically, this means that the sum of row sums of the liability matrix has to be equal to the sum of column sums. Thus, total external liabilities of banks (or total assets of the external sector A_N) can be calculated by the difference between total liabilities in the system (the external sector included) and total assets of banks.[12]

2.4 Creation and characterization of liability matrices

Regulators often face the problem of limited data. Sometimes only the row sums and column sums of the liability matrix are observable. At least it is quite common for some elements of the liability matrix to be missing. As already mentioned, this problem is often surrounded by using the assumption that banks spread their exposures as evenly as possible, which is equivalent to maximizing the entropy of the (normalized) liability matrix.[13] However, using matrices under the maximum entropy assumption tends to bias the results.

The approach of this chapter is to abstract from generating only one matrix using the maximum entropy assumption but to create, for given row and column sums, a large number of valid liability matrices by a random generator. This is done in

[12] In the aggregate A_N, it is not considered that the external sector might be the owner of the banks. Thus, A_N only comprises the amount of banks' liabilities that is provided by the external sector.

[13] For the calculation of the maximum entropy solution of a matrix with given row and column sums, see Appendix 1.

two steps: first, a random number L_{ij}^{rand}, that does not exceed the number of total liabilities in the system L^{total} (or total assets in the system A^{total}, respectively), is assigned to each off-diagonal element. This random number is drawn from a uniform distribution with $L_{ij}^{rand} \ \epsilon \ \left[0, L^{total}\right], \forall \ i \ \neq \ j$, where $RSgoal(i)$ is the aspired row sum and $CSgoal(j)$ is the aspired column sum associated to this element. The interval the random number is drawn from seems at first more restrictive than it is. A reduction/expansion of the interval of the uniform distribution to some smaller/higher upper bound does not change the simulation results. To make the matrix fit exactly, the RAS algorithm is applied:[14] In a first step, each element of the matrix is multiplied by the ratio of the aspired row sum ($RSgoal(i)$) and the actual row sum ($rs(i)$).

$$L_{ij} = L_{ij} \cdot \frac{RSgoal(i)}{rs(i)} \tag{2.4}$$

In a second step, each element of the matrix is multiplied by the ratio of the aspired column sum ($CSgoal(j)$) and the actual column sum ($cs(j)$).

$$L_{ij} = L_{ij} \cdot \frac{CSgoal(j)}{cs(j)} \tag{2.5}$$

By repeating these two steps sufficiently often, a matrix with elements that fit to the aspired row and column sums will be generated.

The RAS algorithm shows some interesting features. First, restrictions to connectivity can be imposed by setting certain elements equal to zero. These elements will remain zero after running the algorithm. Second, given certain random starting values within the matrix, the RAS algorithm yields a unique solution, independent of the “position” of a certain bank within the matrix. The algorithm is also robust to a transposition of the matrix. Third, the randomly generated starting values determine a certain correlation structure within the matrix. The RAS algorithm determines a unique solution that matches the given correlation structure as well as

[14] For a detailed description of the RAS algorithm, see Blien and Graef (1991).

possible and that fulfills row and column sum restrictions. However, there are cases where the RAS algorithm does not provide a valid solution. This happens especially when too many zero restrictions are imposed. Within the simulations, randomly generated matrices that do not fit are dropped.

After liability matrices are generated, they have to be characterized. As the aim of this chapter is to investigate the stability of the financial system dependent on the matrix of *interbank exposures*, the focus is, for the following characterizations, on the $(N-1 \times N-1)$ matrix that covers the interbank market. It is created by deleting the last row and the last column of the $(N \times N)$ liability matrix L.[15] As a next step, there has to be some normalization of matrices because entropy measures have to be applied on probability fields. This is done by dividing all elements by the total amount of interbank liabilities or interbank assets, respectively. As a result, the elements of the normalized matrices add up to 1 and thus can be treated as probabilities. In the following, all normalized elements are marked with a superscript p and are written in lower case letters.

After normalization, the next step is to characterize matrices according to the following measures:

- **Entropy:** In information theory, entropy is a measure for information and can, for example, be explained in the context of search problems. To be more precise, entropy is a lower bound of the average path length from the root to the leaves of a binary search tree. Thus, entropy is a lower bound to the average number of yes/no questions that is needed to obtain full information. The more equal the probability distribution of the elements in the search space, the more questions are on average needed to obtain the desired element and, hence, the higher entropy is. The more unequal the probability distribution,

[15] As the sum of banks' interbank assets has to be equal to banks' interbank liabilities, the sum of row sums of the $(N-1 \times N-1)$ matrix is still equal to the sum of column sums.

the lower entropy is. The lowest entropy (equal to zero) can be obtained when one element in the search space occurs with probability 1 and the other elements with probability 0, i.e. the most unequal distribution of elements occurs.

This entropy measure can be reinterpreted to quantify the inequality of the distribution of claims of a liability matrix. Using the normalization mentioned above, the elements of the matrix can be seen as realizations of a probability distribution of elements within a search space that need not be defined more specifically. Entropy measures the amount of information inherent in these realizations and is maximal if banks spread their claims / obligations as equally as possible. The higher the entropy, the more equally interbank claims are distributed for given row and column sums. The entropy is calculated by:[16]

$$ENT = -\sum_{i=1}^{N-1}\sum_{j=1}^{N-1} l_{ij}^p \cdot ln\left(l_{ij}^p\right) \tag{2.6}$$

with $0 \cdot ln(0) := 0$.

- **Relative entropy (Kullback-Leibler divergence)** to maximum entropy solution: The relative entropy is a measure for the difference between two probability distributions. Given two normalized liability matrices X^p (in this case the maximum entropy solution X^*, see Appendix 1, with last row and last column deleted and normalized by the total amount of interbank liabilities) and L^p (in this case a valid normalized liability matrix generated by random generator), the relative entropy is given by:

$$RE = \sum_{i=1}^{N-1}\sum_{j=1}^{N-1} l_{ij}^p ln\left(\frac{l_{ij}^p}{x_{ij}^p}\right) \tag{2.7}$$

[16] When applying the entropy measure in the context of binary search trees, $log2$ is used. However, in economics literature it is more common to use the natural logarithm. This is equivalent to multiplying a constant factor.

with $0 \cdot ln(0) := 0$ and $0 \cdot ln(\frac{0}{0}) := 0$.

A higher value of RE denotes a greater difference between the two distributions. In the financial system modeled here, a higher relative entropy means a greater distance to the probability distribution of the maximum entropy solution and thus a more unequal distribution of claims among banks. As long as the relative entropy to the *maximum entropy solution* is considered and banks are assumed to be of equal size, there is a negative linear relationship between the entropy of a matrix and its relative entropy to the maximum entropy solution.[17]

- **Connectivity:** The connectivity of the financial system can be described by the probability that a directed link between two banks exists. While constructing the liability matrix of a random graph, each off-diagonal interbank element is (independently) given a certain positive real number with probability p and 0 with probability $1-p$. This probability p is called Erdös-Rényi probability. However, during implementation one has to be careful that, for given starting values (including zeros with a certain probability), the RAS algorithm is able to find a valid solution of matrix entries. This problem increases with decreasing connectivity. The algorithm used in this chapter simply drops matrices that are not valid.

[17] The general derivation of this linear relationship is provided in Appendix 2. Simulation results confirm this theoretical finding.

2.5 Simulation of domino effects

Within these simulations, pure domino effects are modeled dependent on characteristics of the interbank liability structure.[18] As a trigger event, one bank fails.[19] Assuming a certain loss given default (LGD), creditor banks lose a share of their claims to the defaulting bank.[20] If this lost share is larger than the creditor bank's equity, the creditor bank also fails. If one or more banks fail due to the first failure, the next round starts with banks losing additional shares of their claims to failing banks. Thus, a bank fails if:

$$\sum \text{Interbank exposures to failed banks} * \text{LGD} > \text{Equity}$$

For a large number of randomly generated matrices, it is investigated how many banks fail on average, after the failure of one bank, dependent on the characteristics of the liability matrix mentioned in Section 2.4. To be more precise, it is calculated which percentage of total assets of the banking system belongs to failing banks, i.e. which percentage of total bank assets is affected by bank failure.[21] Note, however, that this does not mean that all assets affected by bank failure actually default. The amount of assets that actually default depends on the value of the loss given default.

To depict the results graphically, value intervals of characteristics have to be defined. One possibility to do this is to adjust interval size according to the number of observations. After the random generation of matrices, they are sorted according

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18 For advantages and disadvantages of modeling domino effects, see Upper (2007).

19 This is a rather simple way to model a shock on the financial system. A more sophisticated approach is, for example, used by Elsinger et al. (2006), who apply aggregate macroeconomic shocks to test for the resilience of the Austrian interbank market.

20 In this chapter, a constant, exogenously given LGD is assumed. An obvious extension is to endogenize the LGD as, for example, in Degryse and Nguyen (2007).

21 An alternative target value to measure the harm of interbank contagion is the loss of the external sector, which can be computed easily within this model.

to their characterization values, and then intervals are defined with each interval having the same number of observations.

The network simulations are run several times and for different banks failing first to check how robust these results are with respect to sample changes and to changes in the trigger event.

2.5.1 Complete networks

To begin with, simulations are run for complete networks, i.e. it is assumed that there exists a directed link from each node to all other nodes. The parameter values used for subsequent simulations are $A^{banks} = 1.000$, $N = 11$, $n_{core} = 10$, $CR = 1$ (i.e. all banks are the same size), $\phi = 0.3$, $r = 0.06$ and $LGD = 0.5$. The following figures show, for $50,000$ randomly generated matrices of interbank exposures, the average percentage of total assets of the banking system affected by bank failures in a network with 11 nodes (10 banks and the external sector) dependent on entropy (Figure 2.1) and relative entropy to the maximum entropy solution (Figure 2.2), each color representing a randomly generated sample.

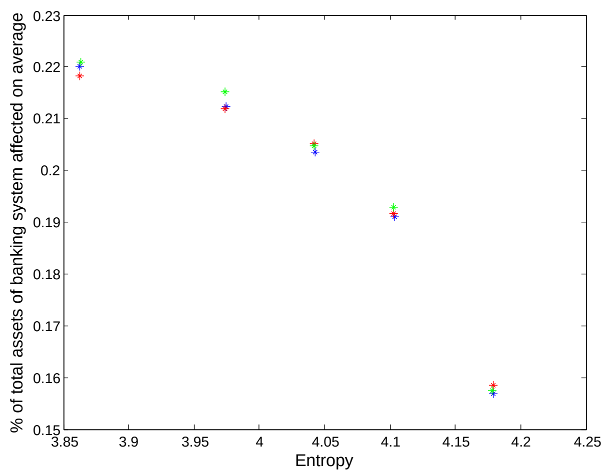

Figure 2.1: Stability of a complete network dependent on entropy

From Figure 2.1 it can be seen that an increase in entropy leads to a lower average percentage of banks' assets affected. On the assumption that all banks are of equal size, the average number of bank defaults dependent on entropy can be derived easily by multiplying the average percentage of banks' assets affected by failure by the total number of banks (in this case, 10). Hence, the more equally banks spread their claims, the fewer institutions default on average. These results suggest that, within a complete network and for the parameter values given above, shocks are absorbed best if banks diversify their (credit) risk exposures well. The results, as well as all subsequent simulation results, are robust to changes in the sample and to changes in the bank that fails first.

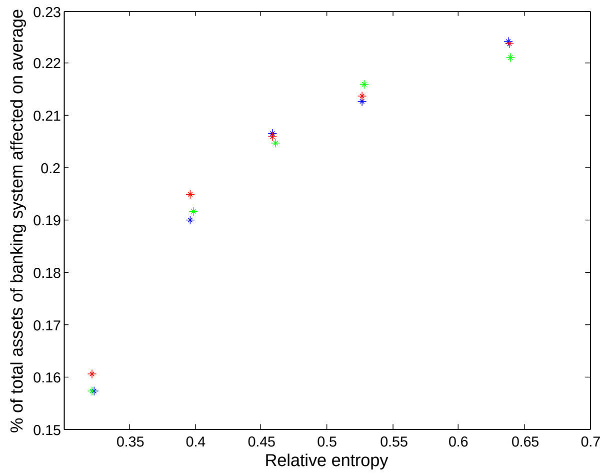

Figure 2.2: Stability of a complete network dependent on relative entropy

Figure 2.2, which shows the relation between relative entropy to the maximum entropy solution and average percentage of banks' assets affected, yields the inverse result compared to Figure 2.1: The higher the relative entropy, the higher the percentage of banks' assets affected and therefore the higher the average number of bank failures. Thus, because of the negative linear relationship between entropy and relative entropy shown in Appendix 2, Figure 2.2 also confirms that a more equal distribution of claims leads to a more stable system.

Up to now, the impact of the distribution of claims on financial stability can be summed up as follows:

Result 1: In a complete network, for the parameter values given above,[22] a liability matrix with an equal distribution of interbank exposures (a high entropy or a low relative entropy to the maximum entropy solution, respectively) leads to a more stable system than a liability matrix with an unequal distribution of interbank exposures.

2.5.2 Random graphs

A connectivity of 100% is rather unrealistic. Thus, some network has to be designed that omits some directed links within the financial system. One option in this context is to model *random graphs*.

Concerning completeness and interconnectedness of the network only and assuming that banks are completely homogeneous, especially with a completely homogeneous asset / liability structure, subsequent results should be expected according to the theoretical findings of Allen and Gale (2000). They examine three types of networks that are displayed in Figure 2.3. The complete and perfectly interconnected network (Figure 1 in Allen and Gale) is equivalent to a random graph with an Erdös-Rényi probability of 100%. In this case, the possibility that contagion occurs is rather low because the more complete a financial system, the greater is the potential for risk diversification. With decreasing connectivity, the network structure moves towards systems that are still highly interconnected but also incomplete (equivalent to Figure 2 in Allen and Gale). Allen and Gale show that these systems are more vulnerable to contagion. With connectivity decreasing further, the network structure becomes equivalent to the disconnected system in Figure 3 in Allen and Gale. This

[22] In the sections below, it is specified in more detail for which parameter values these results hold.

disconnection can limit the extent of contagion. Hence there is a non-monotonic relationship between completeness of the network and financial stability.

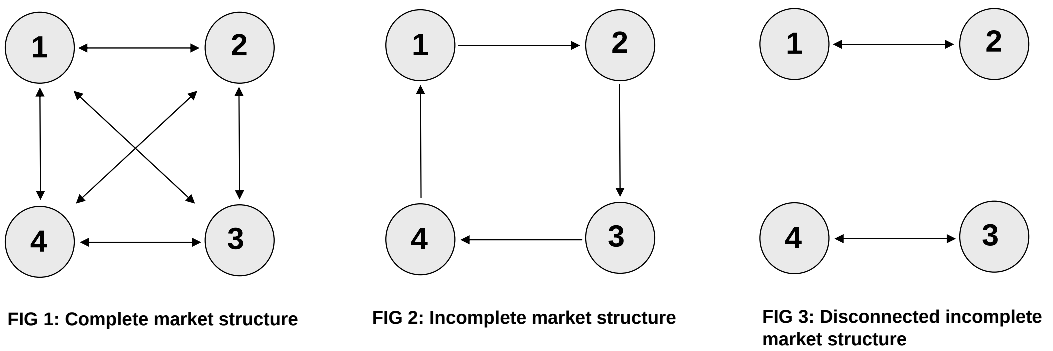

Figure 2.3: Types of networks investigated by Allen and Gale (2000)

In the following simulations, the degree of disconnection is measured by the average number of strongly connected components across all matrices in a sample. Within a strongly connected component, every bank can be reached by every other bank. This does not mean that there are direct links between all banks as in a complete network. It is sufficient that there exists a directed path between all nodes. If the graph contains only one strongly connected component, the failure of one bank can (potentially after several rounds of contagion) cause the failure of all other banks. If there is more than one strongly connected component, however, it is possible that the failure of one bank cannot cause the failure of all other banks because not all banks can be reached by the failing bank. Hence, the higher the average number of strongly connected components for a given Erdös-Rényi probability, the more disconnected is the system.

As mentioned above, the analysis of Allen and Gale is based on a banking system with completely homogeneous banks with a completely equal asset / liability structure. Within the simulations of this chapter, in addition to completeness and interconnectedness, a *third* aspect is introduced into the analysis: the distribution of claims within the system.

2.5.2.1 Varying connectivity

In the following, the default algorithm is run for different values of the Erdös-Rényi probability p, all other parameter values kept equal to those in Section 2.5.1. Each off-diagonal element of the liability matrix that denotes an *interbank* claim / obligation is set equal to zero with probability $1 - p$.[23] As for certain zero constellations, the RAS algorithm is not able to find a valid solution; matrices that do not fit are dropped. Furthermore, matrices where the actual share of existing links to total possible links deviates more than 0.02 from the desired connectivity are also dropped. Thus, only matrices that fit exactly to desired row and column sums and (almost) exactly to desired connectivity are used for the analysis.

To capture the degree of disconnection of the randomly generated network, the number of strongly connected components is computed for each graph. After generating a large number of matrices, the average number of strongly connected components for a given Erdös-Rényi probability is calculated. It turns out that the system starts to become disconnected for $p = 0.5$ with an average number of strongly connected components of around 1.03. For $p = 0.3$, more randomly generated graphs are not perfectly interconnected any more, which yields an average number of strongly connected components of about 1.80. The degree of disconnection jumps up for $p = 0.1$, where the average number of strongly connected components is around 9.07.

All the following simulations are implemented by generating $50,000$ matrices for p = 100%, 90%, 70%, 50%, 30% and 10%, respectively. Table 2.1 shows overall correlations between entropy ($= ENT$), relative entropy ($= RE$) and connectivity ($= p$) for an Erdös-Rényi probability of 10% to 100% (with $25,000$ matrices generated in 10%-steps, respectively) using the same parameter values as in the previous

[23] Alternatively, the whole graph (including the node that denotes the external sector) can be modeled as a random graph. The aim of this chapter is, however, to investigate the impact of the network topology of the interbank market on financial stability. Furthermore, it is certainly more realistic to assume that all banks have connections to the external sector.

section.

characteristic	p	ENT	RE
p	1	0.94	-0.94
ENT		1	-1
RE			1

Table 2.1: Correlation coefficients between characteristics in a random graph

As a first step, to capture the effect of completeness and interconnectedness on financial stability, the average percentage of bank assets affected by failure dependent on the connectivity of the financial system is calculated. Figure 2.4 shows the average percentage of banks' assets affected by failure dependent on the median entropy for a given Erdös-Rényi probability. It can be seen that a complete network (i.e. with $p = 100\%$) leads on average to matrices that are characterized by high entropy. With decreasing connectivity, entropy also decreases, meaning that claims are distributed more unequally (according to Table 2.1, there is a high correlation of 0.94 between entropy and the Erdös-Rényi probability). Furthermore, Figure 2.4 shows that with decreasing completeness (i.e. a decreasing Erdös-Rényi probability) the average percentage of assets affected by bank failure rises. This is in line with the finding of Allen and Gale that an incomplete but perfectly interconnected network leads to a less stable financial system than a complete network. The effect appearing in Allen and Gale's disconnected network can be observed for $p = 10\%$. For $p = 10\%$, the average percentage of assets affected (and therefore the average number of bank failures) is much lower than for $p = 30\%$ which can be explained by the large rise in the average number of strongly connected components from around 1.80 to around 9.07.

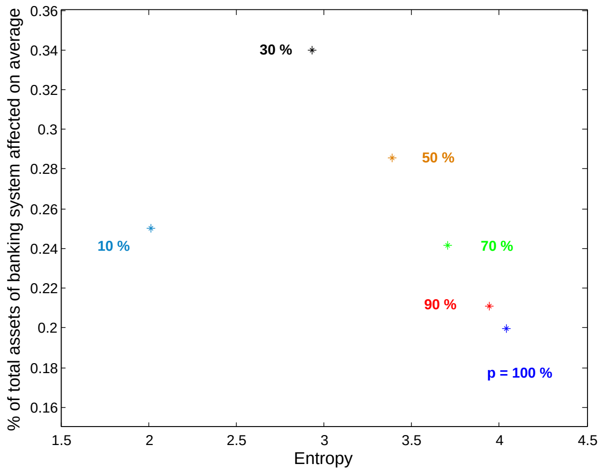

Figure 2.4: Average stability of the network dependent on connectivity and entropy

These results shown in Figure 2.4 can be obtained *on average* if networks are exclusively characterized by their completeness and interconnectedness. However, the effect of the structure of the financial system on financial stability can be analyzed in more detail by additionally considering the effect of the distribution of claims for a given connectivity. Intervals of characteristics are, as in Section 2.5.1, defined in a way that the number of observations is the same within all intervals. Simulations show that results are still not dependent on which bank failed first and the sample generated.

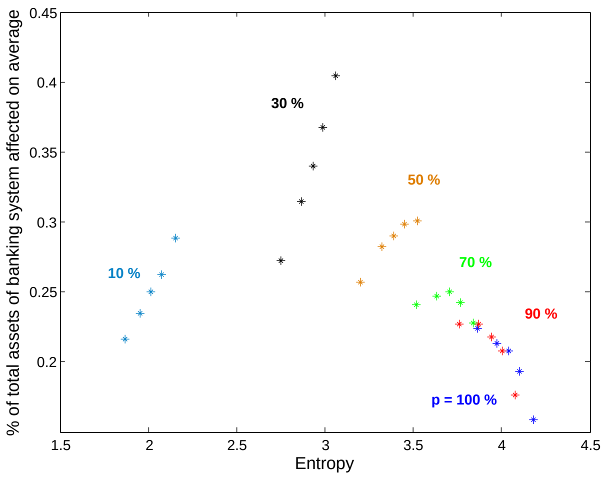

Figure 2.5: Stability of the network dependent on connectivity and distribution of exposures (measured by entropy)

In Figure 2.5 it can be seen that for $p = 100\%$ the same result is obtained as in Figure 2.1. A liability matrix with a higher entropy leads to a lower average percentage of assets affected by bank failure and therefore a lower average number of bank failures. However, with the 100% connectivity assumption, only rather high values for entropy can be generated. Inserting zero off-diagonal "interbank" elements into the matrix with 10% probability, which is equivalent to $p = 90\%$, generates matrices with a lower entropy. The negative correlation between bank failures and entropy still holds for $p = 90\%$. With decreasing connectivity, matrices with an even lower entropy can be created. The negative correlation between entropy and average bank failures, however, becomes weaker and turns into a positive correlation. This means that for a given low Erdös-Rényi probability (for example $p = 50\%$, $p = 30\%$ or $p = 10\%$), a comparatively high entropy leads on average to more banks defaulting than a comparatively low entropy.

An interpretation for this observation is that, for a high connectivity, an equal distribution of interbank claims is the best shock absorber due to credit risk diversification, whereas an unequal distribution of claims increases the probability that there is a second-round effect after the failure of one bank. On the contrary, when connectivity is low, the failure of one bank is very likely to cause second-round effects because the average amount of interbank exposures to the few connected banks is very high. Hence, the more equal the distribution to the few other banks, the higher the probability that all these banks fail because there are not enough counterparties to diversify the losses induced by the shock. On the other hand, the more unequal the distribution to the few other banks, the higher the probability that not all of these banks fail and therefore the average number of failures is smaller in this case. Thus, a change in the average percentage of assets in the banking sector affected by failure (i.e. a change in the average number of bank failures) is not just due to a change in connectivity but can also be due to a change in the distribution of interbank claims. For example, though the overall average number of banks failing is higher for a connectivity of 30% compared to a connectivity of 50% (see Figure 2.4), a system with a very unequal distribution of interbank claims (low entropy) and 30% connectivity is more stable than a system with a rather equal distribution of interbank claims (high entropy) and 50% connectivity (see Figure 2.5). Also, a system with a very equal distribution of interbank claims (high entropy) and 90% connectivity is more stable than a network with 100% connectivity and a very unequal distribution of interbank claims (low entropy).

Thus, by additionally considering the distribution of claims within the system, it can be seen that a complete network can be more unstable than an incomplete but perfectly interconnected network. This finding extends the work of Allen and Gale in a way that results could change if interbank claims are allowed to be heterogeneous.

Figure 2.6 and 2.7 show the average number of bank failures dependent on the relative entropy to the maximum entropy solution. As entropy and relative entropy are exactly negatively correlated (see Appendix 2 and Table 2.1), these two figures can be regarded as the mirror image of Figures 2.4 and 2.5.

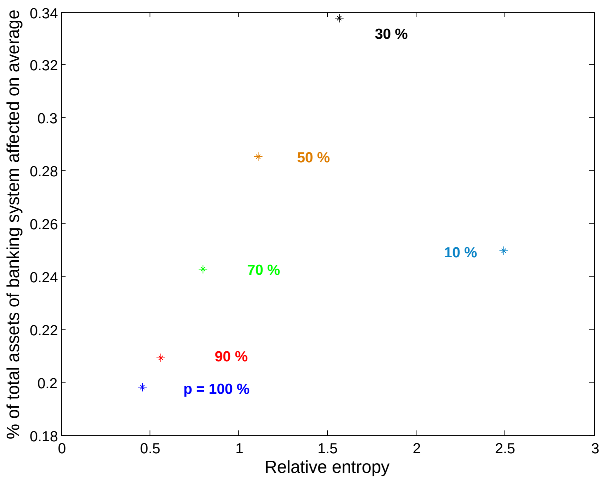

Figure 2.6: Average stability of the network dependent on connectivity and relative entropy

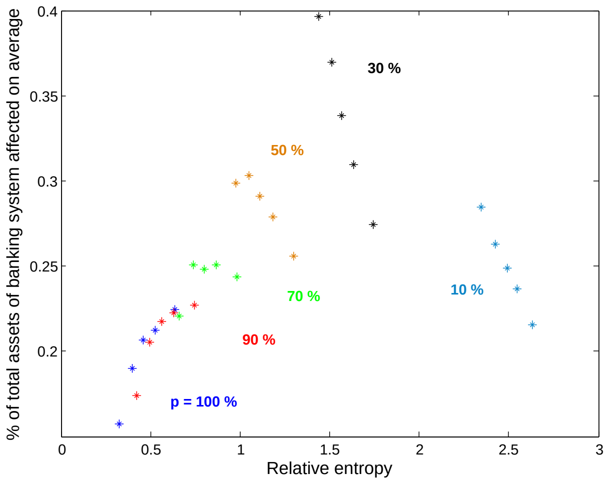

Figure 2.7: Stability of the network dependent on connectivity and distribution of exposures (measured by relative entropy)

The results of this paragraph can be summarized as follows:

Result 2: Financial stability does not only depend on the completeness and interconnectedness of the network but also on the *distribution of claims* within the system.

Result 3: For the parameter values given above,[24] the sign of the correlation between the equality of the distribution of claims (measured by entropy and relative entropy) and financial stability changes with decreasing completeness of the network. For high completeness (and high interconnectedness) an equal distribution of claims leads to the most stable system. For lower completeness (but still high interconnectedness) the positive correlation between entropy and number of banks failing weakens. For very low completeness (and low interconnectedness) a more unequal distribution of interbank claims leads to a more stable system.

As long as banks are assumed to be of equal size, the characterization of matri-

[24] In the sections below, it is specified in more detail for which parameter values these results hold.

ces by entropy and relative entropy yields exactly the same results. Thus, all the following investigations are only made dependent on entropy. As a next step, some sensitivity analysis is done by varying one parameter (LGD, equity ratio or ratio of interbank assets to total assets in the banking system), as well as connectivity, and fixing all other parameters at their benchmark value set in Section 2.5.1.

2.5.2.2 Varying loss given default and connectivity

Figure 2.8 shows the not very surprising result that, for a given connectivity, the average percentage of assets affected by bank failure (and thus also the average number of bank failures) increases with an increasing loss given default. An interesting observation is that for a high LGD (= 100%) the effect of a disconnected system (equivalent to Figure 3 in Allen and Gale) is already visible between $p = 50\%$ and $p = 30\%$. Starting from $p = 50\%$ the average number of bank defaults decreases with decreasing connectivity. Hence, the impact of the disconnection of the financial system becomes more important for high rates of LGD where the average number of bank defaults tends to be very high.

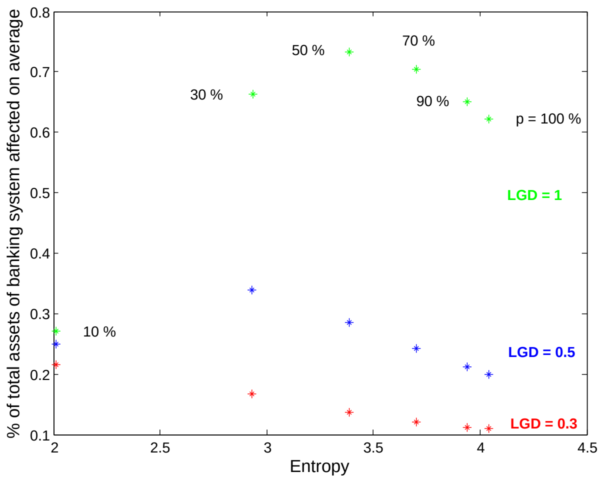

Figure 2.8: Average stability of the network dependent on the loss given default

Figure 2.9 shows that a LGD of 100% leads to a positive correlation between assets affected by bank failure and entropy, even for a high connectivity of the network. A LGD of 30% leads to a negative correlation between assets affected by bank failure and entropy, even for a low connectivity (up to $p = 30\%$) of the network. An explanation for this observation is that a high LGD makes a system vulnerable to interbank contagion as a high share of claims to the failing banks defaults. Thus, it can be assumed that contagion occurs with certainty. In this case, it is better to have a relatively unequal distribution of interbank exposures so that only few banks are hit by a second-round effect of contagion. For a low LGD, the system is rather resilient to interbank contagion. In this case, second-round effects only occur if interbank exposures are distributed very unequally, i.e. interbank claims are not well diversified among counterparties. This result is exactly in line with the findings of Mistrulli (2011), who shows that for low and medium LGDs, the maximum entropy assumption tends to underestimate the severity of contagion. For high LGDs, however, using the maximum entropy assumption leads to an overestimation of the severity of contagion.

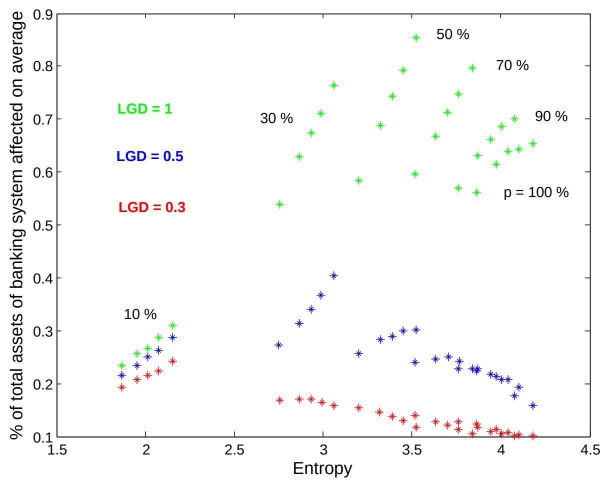

Figure 2.9: Stability of the network dependent on the loss given default and the distribution of interbank exposures

2.5.2.3 Varying equity ratio and connectivity

Figure 2.10 shows, not surprisingly, that a lower equity ratio leads, for a given Erdös-Rényi probability, to a higher average number of bank assets affected by failure. Again, the impact of the disconnection of the financial system becomes more important for parameter constellations where the average number of bank defaults tends to be very high, i.e. for low equity ratios.

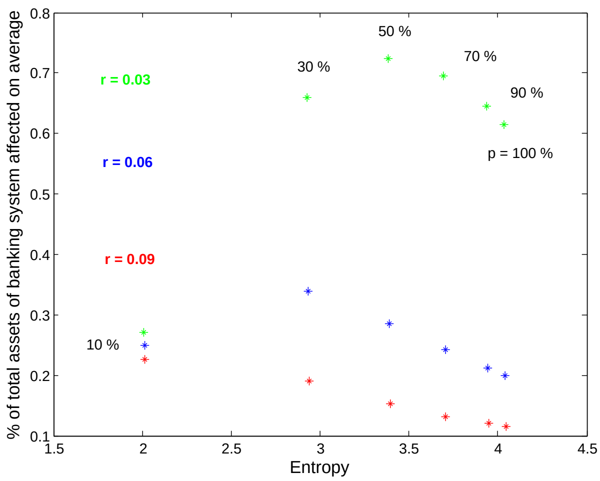

Figure 2.10: Average stability of the network dependent on the equity ratio

Looking at the effect of the distribution of claims for given connectivity in Figure 2.11, it can be seen that for an equity ratio of 9% the correlation of entropy and percentage of assets affected by bank failure is negative for all values of p (except $p = 10\%$). For an equity ratio of 3% the correlation is always positive. Similar to the variation of LGD, systems with high equity ratios are more stable with an equal distribution of claims because second-round effects only occur when interbank claims are not well diversified. Systems with low equity ratios are, for a given Erdös-Rényi probability, more stable with an unequal distribution of claims so that second-round effects (that occur almost with certainty) only hit few counterparties in the financial system.

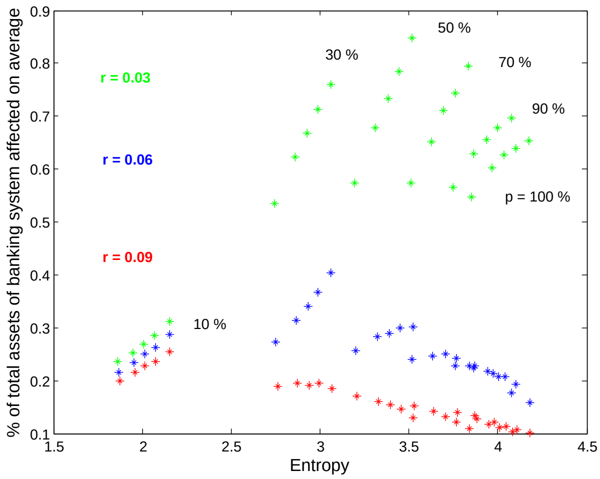

Figure 2.11: Stability of the network dependent on the equity ratio and the distribution of interbank exposures

2.5.2.4 Varying ratio of interbank assets to total assets and connectivity

Additionally, the ratio of interbank assets to total assets ϕ can be varied. Figure 2.12 shows that a higher ratio of interbank assets to total assets leads, for a given Erdös-Rényi probability, to a higher average percentage of assets affected by bank failure. This is not surprising because, for a given equity ratio and LGD, banks become more vulnerable to the default of a neighboring bank. The reason is that, on average, the bilateral exposure per counterparty increases with an increasing ratio of interbank assets to total assets. Furthermore, for high numbers of bank failures (i.e. high numbers of ϕ) the effect of a disconnected system is again already visible for an Erdös-Rényi probability below 50%.

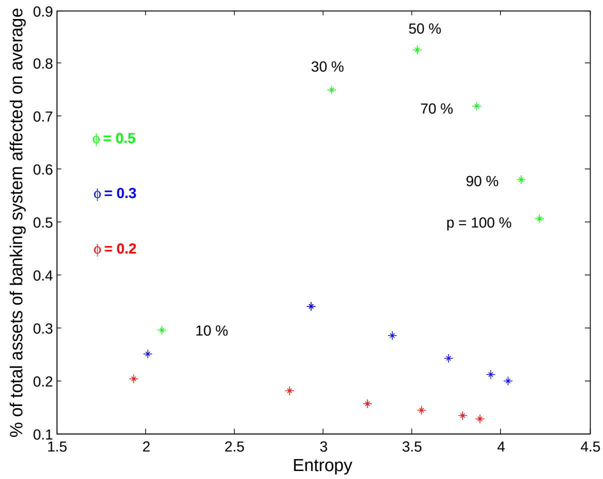

Figure 2.12: Average stability of the network dependent on the share of interbank assets

Figure 2.13 shows that the correlation between entropy and average percentage of assets affected by failure changes from positive to negative at an Erdös-Rényi probability of around 70%, independent of the value of ϕ. Thus, contrary to the variation of LGD and r, there is no influence of ϕ on the correlation between entropy and average number of banks defaults.[25] However, one has to be careful when comparing samples of matrices with different ϕ. Looking at the range of entropy values created for a given Erdös-Rényi probability in Figure 2.13 it can be seen that, for higher values of ϕ, the random generator on average creates matrices with a higher entropy. Thus, not the same entropy intervals can be compared when investigating the average percentage of bank assets affected by failure for different values of ϕ.

[25] In additional simulations the LGD and the equity ratio were varied for different values of ϕ. The result is that high values of LGD or low values of r lead to a positive correlation between entropy and the average percentage of assets affected, and low values of LGD or high values of r lead to a negative correlation, independent of ϕ. Thus, a variation of LGD and r changes the correlation between entropy and average percentage of assets affected by failure; a variation of ϕ, however, does not.

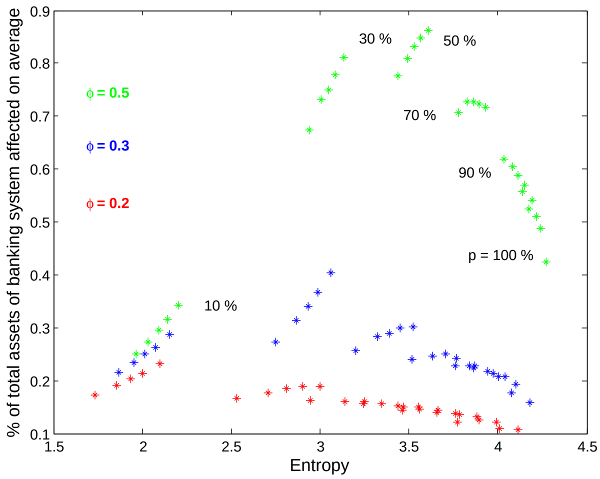

Figure 2.13: Stability of the network dependent on the share of interbank assets and the distribution of interbank exposures

Paragraphs 2.5.2.2 to 2.5.2.4 can be summarized by the following result:

Result 4: For a given total amount of interbank assets, the sign of the correlation between the equality of the distribution of claims (entropy) and the average percentage of bank assets affected by failure (or average number of banks defaulting, respectively) tends to be positive for parameters that make a system vulnerable to interbank contagion (i.e. a high LGD and low equity ratios). On the other hand, the sign of the correlation tends to be negative for parameters that make the system resilient to interbank contagion (i.e. a low LGD and high equity ratios). For intermediate parameter values, the sign of the correlation between the equality of the distribution of claims and average number of banks defaulting changes from negative to positive with decreasing connectivity (see results in Section 2.5.1 and 2.5.2.1).

In further simulations, additional parameters are varied. At first, the total number of assets was changed. This, however, does not alter the results as relative numbers (for example the equity ratio or the ratio of interbank assets to total assets) do not change. Increasing the number of banks in the system makes it (all other variables kept equal) less vulnerable to the failure of one bank as the average amount of bilateral exposures per counterparty decreases with an increasing number of counterparties. Furthermore, these simulations were run for “extreme” parameter values, i.e. parameters that all make a financial system very unstable (a high LGD, a low equity ratio and a high share of interbank assets to total assets) or stable, respectively. The results confirmed the main findings summarized in Result 4.

2.5.3 Money center systems

The analysis of the impact of the distribution of claims on financial stability in random graphs leads to some interesting results. It is, however, questionable, whether a random graph is a good description of real world financial networks. In the literature, it is sometimes stated that a more adequate model of a financial system is a scale-free network (see, for example, Boss et al. (2004) and Soramäki et al. (2007)). Craig and von Peter (2010) prefer to model financial systems as *(multiple) money center systems*, where few large core banks are strongly interconnected (i.e. they form a complete network) and a larger number of small banks in the periphery are only connected to core banks but not to other banks in the periphery. Figure 2.14 shows an example of a money center model (without the external sector).

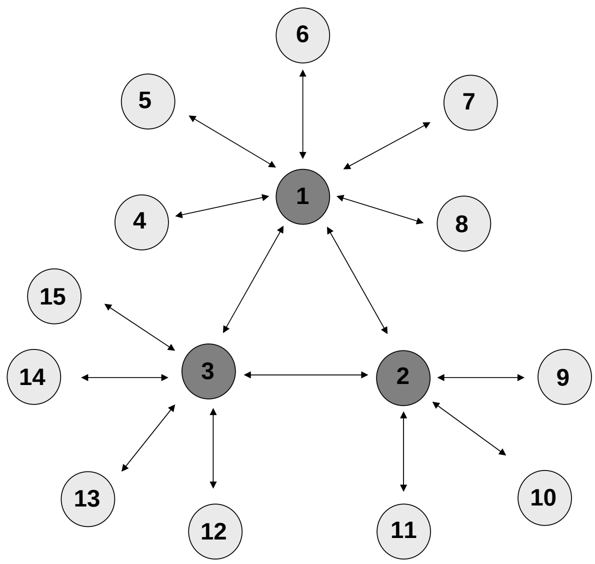

Figure 2.14: Example of a money center system with 15 banks (3 core banks and 12 banks in the periphery)

The financial network in Figure 2.14, including the external sector, can be described by an adjacency matrix:

$$\mathbf{L} = \begin{array}{cc} & \begin{array}{ccccccccc} A_{1,core} & A_{2,core} & A_{3,core} & & A_{4,per} & A_{5,per} & \dots & A_{15,per} & A_N \end{array} \\ \begin{array}{c} L_{1,core} \\ L_{2,core} \\ L_{3,core} \\ \\ L_{4,per} \\ L_{5,per} \\ \vdots \\ L_{15,per} \\ L_N \end{array} & \left(\begin{array}{ccccccccc} 0 & 1 & 1 & | & 1 & 1 & \dots & 0 & 1 \\ 1 & 0 & 1 & | & 0 & 0 & \dots & 0 & 1 \\ 1 & 1 & 0 & | & 0 & 0 & \dots & 1 & 1 \\ - & - & - & - & - & - & - & - & \\ 1 & 0 & 0 & | & 0 & 0 & \dots & 0 & 1 \\ 1 & 0 & 0 & | & 0 & 0 & \dots & 0 & 1 \\ \vdots & \vdots & \vdots & | & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 1 & | & 0 & 0 & \dots & 0 & 1 \\ 1 & 1 & 1 & & 1 & 1 & \dots & 1 & 0 \end{array} \right) \end{array}$$

The interbank part of this adjacency matrix, i.e. the $(N-1) \times (N-1)$ matrix that is obtained when deleting the last row and the last column of $\mathbf{L}$, can be written in block matrix form:[26]

$$\mathbf{L^{IB}} = \begin{pmatrix} \Lambda_1 & | & \Lambda_2 \\ - & - & - \\ \Lambda_3 & | & \Lambda_4 \end{pmatrix}$$

By assumption, the money center model applied in this chapter has to follow cer-

[26] The way this money center system is constructed is very similar to the "block-model approach" of Craig and von Peter (2010). The conditions a money center system has to fulfill in this chapter are, however, slightly different.

tain patterns. First, all core banks are strongly connected to each other. Thus, all off-diagonal elements of the top left corner of the adjacency matrix $\mathbf{L}^{IB}$ (Λ_1) have to be equal to one. Second, each bank in the periphery is linked to exactly one money center bank in both directions. Thus, the top right corner (Λ_2) has exactly one non-zero element per column and the bottom left corner of the adjacency matrix (Λ_3) is the exactly transposed version of the top right corner. Third, banks in the periphery are not linked to each other. Hence, the bottom right corner (Λ_4) contains only zeros. Thus, by construction, the strongly connected component always includes all financial institutions in the system.

After constructing the adjacency matrix, the edges of the graph obtain weights that are, similar to the case of the random graph, created by a random generator. For given row and column sums, elements are again adjusted by using the RAS algorithm.

2.5.3.1 Varying the number of core banks and the concentration ratio

At first, the stability of a money center model is investigated by varying two main parameters that characterize its pattern: the number of core banks n_{core} and the concentration ratio CR. To create a more reasonable ratio of core banks to periphery banks, the number of banks in the financial system is increased to 15, i.e. $N = 16$. Remaining parameters are set at their benchmark values, i.e. $LGD = 0.5$, $r = 0.06$ and $\phi = 0.3$. Total assets in the banking system are again set at $A^{banks} = 1,000$ and each simulation is run for a sample of $50,000$ randomly generated matrices. While for the random graph it does not matter which bank fails first, for the money center model it is assumed that a core bank fails.[27]

Figures 2.15 to 2.18 show the average percentage of total assets that are affected by bank failure for $n_{core} = 5$ to $n_{core} = 2$. For each number of core banks the con-

[27] Additionally, simulations with a periphery bank failing were run. Not surprisingly, in this case, the financial system is more stable than in the case of a core bank failing.

centration ratio is varied from $CR = 0.99$ (core banks hold almost all assets in the banking system) to $CR = 0.4$ (core banks hold 40% of total assets in the banking system). The lower bound of the concentration ratio is set in such a way that, for all values of n_{core} investigated, it cannot happen that core banks have smaller balance sheet totals than periphery banks.

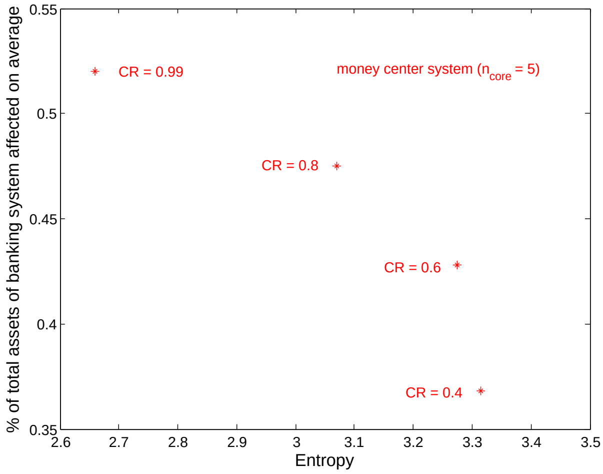

Figure 2.15: Stability of the network with five core banks

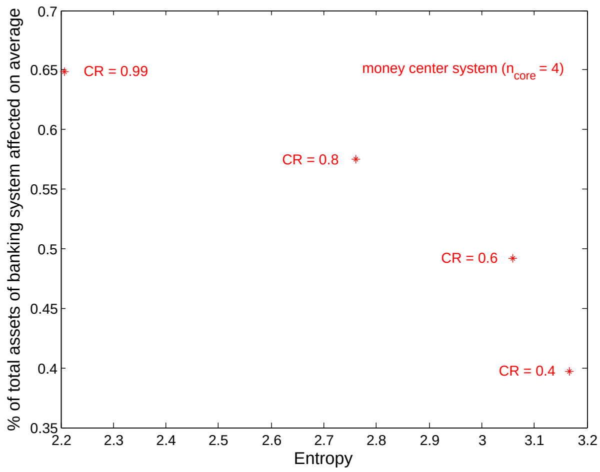

Figure 2.16: Stability of the network with four core banks

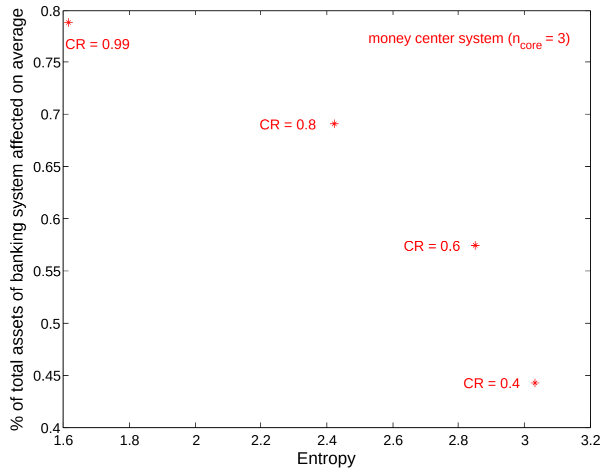

Figure 2.17: Stability of the network with three core banks

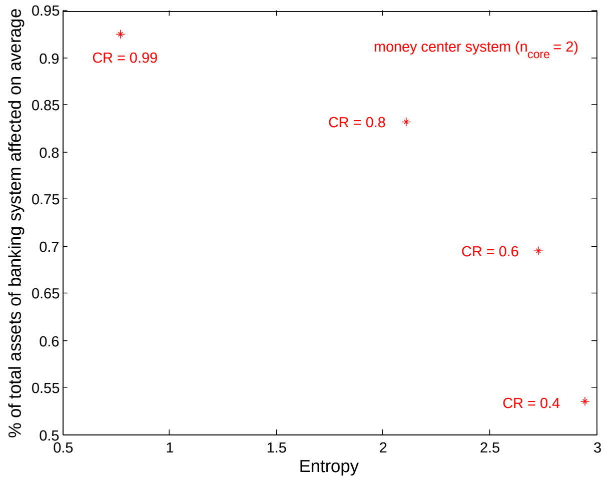

Figure 2.18: Stability of the network with two core banks

Not surprisingly, entropy decreases with an increasing concentration ratio as a higher concentration of assets among few core banks implies a more unequal distribution of claims within the financial system. Furthermore, for a given number of core banks, the average percentage of assets affected by bank failure increases with the concentration ratio. One reason is that the total amount of assets of the bank that fails first, and therefore the initial percentage of assets affected by bank failure, is higher. But this is not the only effect. With increasing size of core banks compared to banks in the periphery (i.e. a higher CR), and all other variables kept equal, the average weights of the links between core banks increase. Thus, the amount of interbank

assets between core banks becomes larger on average, which makes core banks more vulnerable to interbank contagion. And the more core banks fail, the more banks in the periphery are on average affected by domino effects.
Comparing the stability of the financial system by varying the number of core banks n_{core} it can be seen that, all other parameters kept equal, the stability of the financial system increases with an increasing number of core banks. This effect is again due to the size of the core banks. For a given concentration ratio, the size of the core banks decreases with an increasing number of core banks. Thus, the average amount of interbank assets between core banks also becomes smaller and the probability that domino effects between core banks occur, is reduced.

Hence, the main result of this paragraph is:

Result 5**:** Increasing asset concentration (a higher concentration ratio for a given number of core banks or a lower number of core banks for a given concentration ratio) within a money center system makes it more unstable on average.

2.5.3.2 Comparison to random graphs

Additionally, the stability of a money center system is compared to the stability of a random graph. As a benchmark, the investigation of the stability of random graphs with the same system size (in terms of total assets) and the same number of banks as in the money center system is included in each subsequent figure. When modeling the random graph, it is assumed that all banks have homogeneous balance sheet totals and are linked randomly with a certain probability. The money center system is interlinked according to the description in Section 2.5.3.
However, one has to be careful when setting the parameter values for the money center system. Within a random graph, the average amount of bilateral interbank

exposures (over the whole sample of generated matrices) is the same between all banks. In a money center model, though, if the concentration ratio is chosen too low (or the number of core banks is chosen too high), i.e. core banks do not have a balance sheet total that is large enough compared to periphery banks, the assumed network topology of the money center model leads to interbank connections where core banks have less exposures to each other than to banks in the periphery. The reason is that periphery banks have all their interbank exposures to one core bank by assumption. To obtain a valid liability matrix for given balance sheet totals, the result is a very low weight on interbank exposures among core banks. This does not fit to realistic banking systems. Thus, in the following simulations, it is assumed that the average amount of bilateral interbank assets each core bank holds against another core bank is at least as high as the average amount of bilateral interbank assets a core bank holds against a periphery bank. This amounts to a minimum concentration ratio of $CR = 0.25$ for $n_{core} = 2$, $CR = 0.35$ for $n_{core} = 3$, $CR = 0.4$ for $n_{core} = 4$ and $CR = 0.45$ for $n_{core} = 5$.[28]

Figures 2.19 to 2.22 show that (for the same values of N, A^{banks}, ϕ, LGD and r as in Section 2.5.3.1) the random graph is always more stable on average than the money center system with the minimum concentration ratio derived above.[29]

[28] These results are obtained by calculating the average amount of bilateral interbank assets between two core banks and between a core and a periphery bank by simulation for different values of CR (in steps of 0.05). The value of CR, where the discrepancy between the average amount of interbank assets each core bank holds against another core bank and the average amount of interbank assets a core bank holds against a periphery bank is minimal, is then chosen as the minimum concentration ratio for further simulations.

[29] In additional simulations the same result was obtained for extreme values of LGD and r.

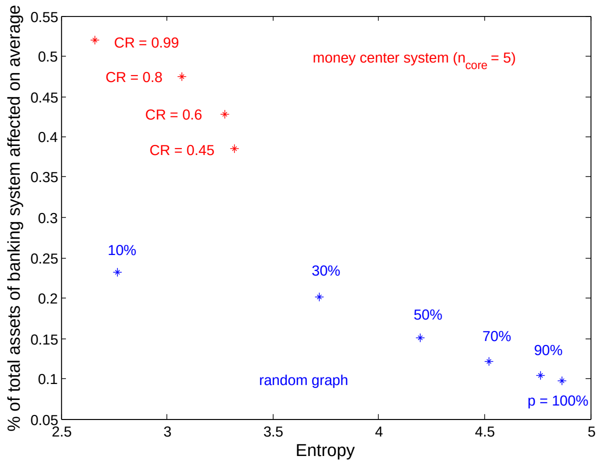

Figure 2.19: Stability of money center system (five core banks) and random graph

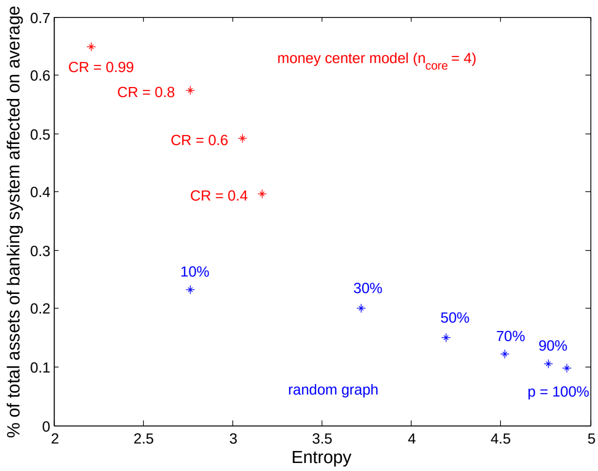

Figure 2.20: Stability of money center system (four core banks) and random graph

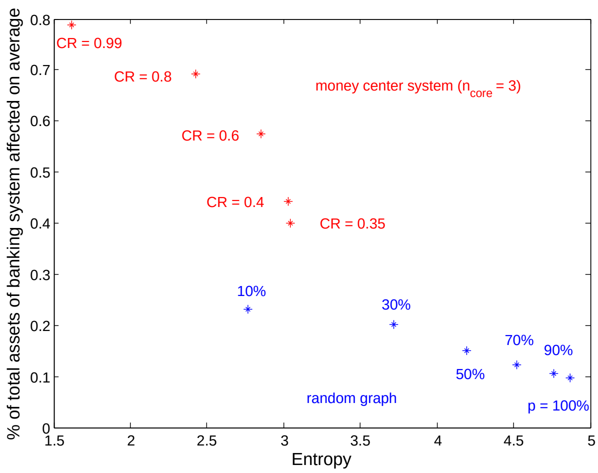

Figure 2.21: Stability of money center system (three core banks) and random graph

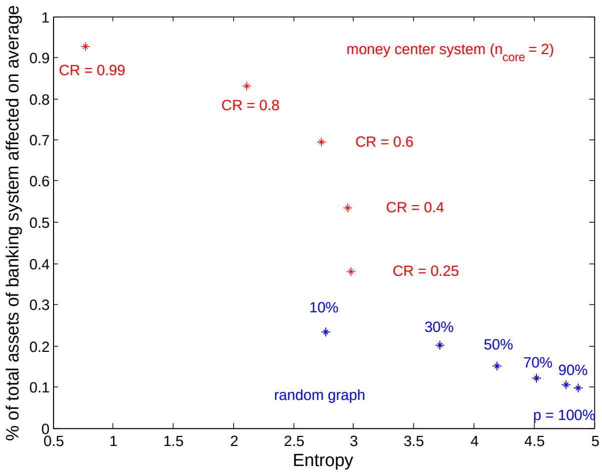

Figure 2.22: Stability of money center system (two core banks) and random graph

Several reasons for this result can be mentioned. First, in the money center model, the initial percentage of assets affected by bank failure is larger than in the random graph. The reason is that, for the given minimum concentration ratio, core banks in the money center model are always larger than banks in a random graph. Second, a large balance sheet total of the failing core bank implies on average a high amount of interbank claims defaulting in the first round. Thus, the initial shock is larger compared to a random graph. Third, among core banks there are only limited possibilities of risk diversification, as they are only linked to the other core banks and to few periphery banks. Additionally, there is no sufficient risk diversification possible for periphery banks that hold all their claims against one core bank. Fourth, money center models are always strongly connected by assumption, i.e. they have only one strongly connected component. Hence, domino effects can never be curtailed by disconnection of the financial system.

Thus, to summarize the main findings of this paragraph:

Result 6: On the assumption that the average amount of bilateral interbank assets between two core banks is at least as high as between a core and a periphery bank, a money center system with asset concentration among core banks is (in all previous simulations) less stable on average than a system of banks with homogeneous size that follows a random graph.

2.6 Conclusion

In this chapter, the impact of the structure of the matrix of interbank liabilities on financial stability is analyzed. After characterizing the financial system according to the number of banks, total assets in the banking system, equity ratio, the ratio of interbank to total assets and loss given default, a large number of valid interbank liability matrices is created by a random generator for given row and column sums. Thus, a new approach of this chapter is that interbank contagion is investigated for a large sample of interbank matrices. These matrices are then characterized by entropy and relative entropy to the maximum entropy solution, which constitute measures of the equality of the distribution of interbank exposures. As a next step, domino effects resulting from the default of one bank are modeled. As long as banks are assumed to be of equal size (to be able to only focus on the effects of the structure of the liability matrix), it does not matter which bank fails first. Additionally, as a large number of valid matrices is generated, results do not depend on the sample of matrices.

The first simulations are run for complete networks and "intermediate" parameter values. The main result is that a more equal distribution of interbank claims leads to a more stable financial system. These results, however, change if an incomplete network is considered. Starting with a random graph and, again, "intermediate" parameter values, it can be seen that the sign of the correlation between equality of distribution of claims and percentage of assets affected by bank failure changes with decreasing connectivity. Furthermore, a crucial result of these simulations is that not only completeness and interconnectedness of a financial network, as investigated theoretically in Allen and Gale (2000), matters, but also the *distribution of claims* within the financial network. In this chapter, cases can be shown where, contrary to the findings of Allen and Gale, a complete network (with an unequal distribution of claims) is less stable than an incomplete but perfectly interconnected network (with an equal distribution of claims).

As a next step, further sensitivity analysis is implemented by varying loss given default, banks' equity ratio and the ratio of interbank assets to total assets. The main result in this context is that the sign of the correlation between entropy and the average percentage of assets affected by bank failure depends on connectivity, loss given default and equity ratio. For high values of LGD and low values of r (i.e. parameters that make a financial system vulnerable to interbank contagion) the sign of the correlation between entropy and the average percentage of assets affected tends to be positive, while for low values of LGD and high values of r (i.e. parameters that make a financial system resilient to interbank contagion) the sign of the correlation tends to be negative. For "intermediate" parameter values the sign of the correlation changes from negative to positive with decreasing connectivity.

A second, probably more realistic, approach to modeling incomplete networks is to consider money center systems. The main idea of money center models is to distinguish between large core banks that are strongly connected to each other and small banks in the periphery that are only linked to one core bank. Not surprisingly, the more concentrated assets are within a money center system, the less stable it is if a core bank fails. Additionally, using reasonable parameters for the number of core banks and the concentration of assets among core banks, it turns out that, for all simulations run, the money center system is less stable than a random graph with homogeneous bank size.

As a conclusion, this chapter extends the existing literature on interbank contagion within a financial network by explicitly considering the distribution of claims within the financial system, and therefore gives a variety of insights into the determinants of financial stability. This approach can be widened to aspects of interbank contagion extending domino effects (for example contagion due to liquidity problems). Therefore, this approach leaves a lot of new topics for future research.

2.7 Appendix

2.7.1 Appendix 1: Generation of the maximum entropy solution of an interbank liability matrix

The starting point is a matrix X with given row sums L_i, $i \in \{1, ..., N\}$ and column sums A_j, $j \in \{1, ..., N\}$.[30]

$$\mathbf{X} = \begin{array}{c} \\ L_1 \\ L_2 \\ \vdots \\ L_N \end{array} \begin{array}{c} \begin{array}{cccc} A_1 & A_2 & \cdots & A_N \end{array} \\ \begin{pmatrix} \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \vdots \\ \cdots & \cdots & \cdots & \cdots \end{pmatrix} \end{array}$$

with $\sum_{i=1}^{N} L_i = L$, $\sum_{j=1}^{N} A_j = A$ and $A = L$.

As entropy methods must be applied on probability fields, some normalization of row and column sums is necessary:

$$\mathbf{X^p} = \begin{array}{c} \\ l_1^p \\ l_2^p \\ \vdots \\ l_N^p \end{array} \begin{array}{c} \begin{array}{cccc} a_1^p & a_2^p & \cdots & a_N^p \end{array} \\ \begin{pmatrix} \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \vdots \\ \cdots & \cdots & \cdots & \cdots \end{pmatrix} \end{array}$$

with $l_i^p = \frac{L_i}{L}$ and $a_j^p = \frac{A_j}{A}$.

[30] See Upper and Worms (2004)

Furthermore $\sum_{i=1}^{N} l_i^p = \sum_{j=1}^{N} a_j^p = 1$.

The a^p's and l^p's are interpreted as realizations of the marginal distributions $f(a)$ and $f(l)$, the elements of the liability matrix $X^p \left(= x_{ij}^p\right)$ as realizations of their joint distribution $f(a,l)$. If $f(a)$ and $f(l)$ are independent, the elements x_{ij}^p of the normalized matrix are given by $x_{ij}^p := l_i^p \; a_j^p$. This results in maximizing the entropy of X^p.

$$
\mathbf{X^p} = \begin{array}{c} \\ l_1^p \\ l_2^p \\ \vdots \\ l_N^p \end{array}
\begin{array}{c} \begin{array}{cccc} a_1^p & a_2^p & \cdots & a_N^p \end{array} \\
\begin{pmatrix} x_{11}^p & x_{12}^p & \cdots & x_{1N}^p \\ x_{21}^p & x_{22}^p & \cdots & x_{2N}^p \\ \vdots & \vdots & \vdots & \vdots \\ x_{N1}^p & x_{N2}^p & \cdots & x_{NN}^p \end{pmatrix} \end{array}
$$

with $\sum_{j=1}^{N} x_{ij}^p = l_i^p$ and $\sum_{i=1}^{N} x_{ij}^p = a_j^p$.

The problem is that the matrix X^p has non-zero elements on the main diagonal which means that banks lend to themselves. To avoid this phenomenon, a new matrix X_0^p with zero elements on the diagonal has to be created, i.e. x_{ij}^p is set equal to zero for $i = j$.

$$
\mathbf{X_0^p} = \begin{array}{c} \\ l_1^p \\ l_2^p \\ \vdots \\ l_N^p \end{array}
\begin{array}{c} \begin{array}{cccc} a_1^p & a_2^p & \cdots & a_N^p \end{array} \\
\begin{pmatrix} 0 & x_{12}^p & \cdots & x_{1N}^p \\ x_{21}^p & 0 & \cdots & x_{2N}^p \\ \vdots & \vdots & \vdots & \vdots \\ x_{N1}^p & x_{N2}^p & \cdots & 0 \end{pmatrix} \end{array}
$$

The new matrix should deviate from the maximum entropy solution as little as possible. Thus, out of all possible normalized matrices $\overline{X^p}$, a matrix X^* has to be created that minimizes the relative entropy with respect to the matrix X_0^p.

$$x^* = argmin \ \overline{x^{p'}} \cdot ln\frac{\overline{x^p}}{x_0^p} \tag{2.8}$$

s.t. $x^* \geq 0$ and $Ax^* = \left[a^{p'}, l^p\right]'$,

where x^* and x_0^p are $(N^2 - N) \times 1$ vectors containing the off-diagonal elements of X^* and X_0^p, a^p and l^p are the row and column sums of X^p ($\left[a^{p'}, l^p\right]'$ has the size $2N \times 1$), and A is a $2N \times (N^2 - N)$ matrix containing zeros and ones so that the restrictions concerning row and column sums are fulfilled.

This minimization problem can be either solved using the RAS algorithm (see Blien and Graef (1991)) or using the "fmincon-command" of MATLAB's optimization toolbox. Both approaches lead to the same results.[31]

After solving the minimization problem, a matrix X^* is obtained that deviates from the assumption of independence as little as possible.

$$\mathbf{X}^* = \begin{array}{c c} & \begin{array}{cccc} a_1^p & a_2^p & \cdots & a_N^p \end{array} \\ \begin{array}{c} l_1^p \\ l_2^p \\ \vdots \\ l_N^p \end{array} & \begin{pmatrix} 0 & x_{12}^* & \cdots & x_{1N}^* \\ x_{21}^* & 0 & \cdots & x_{2N}^* \\ \vdots & \vdots & \vdots & \vdots \\ x_{N1}^* & x_{N2}^* & \cdots & 0 \end{pmatrix} \end{array}$$

As a last step, X^* can be transformed back into a "real" liability matrix X by multiplying each element x_{ij}^* as well as the row and column sums l_i^p and a_j^p with L or A.

[31] However, using the "fmincon-command" can become computationally intensive if the liability matrices are large.

2.7.2 Appendix 2: Specification of the linear relationship between entropy and relative entropy to the maximum entropy solution

Consider a network of N nodes ($N-1$ banks and one external sector) and, in particular, the $(N-1)\times(N-1)$ interbank liability matrix that is normalized by the total amount of interbank assets / liabilities. Normalization implies that the sum of row as well as the sum of column sums has to be equal to 1. The characteristic of the maximum entropy solution of the interbank matrix is that claims are distributed as equally as possible for given row and column sums. Thus, on the assumption that all banks are of equal size, the general result of the maximum entropy solution is a matrix with row and column sums of $\frac{1}{N-1}$ and off-diagonal elements of $\frac{1}{(N-1)(N-2)}$, respectively:

$$
\mathbf{X}^* = \begin{array}{cc}
 & \begin{array}{cccc} \frac{1}{N-1} & \frac{1}{N-1} & \cdots & \frac{1}{N-1} \end{array} \\
\begin{array}{c} \frac{1}{N-1} \\ \frac{1}{N-1} \\ \vdots \\ \frac{1}{N-1} \end{array} &
\begin{pmatrix}
0 & \frac{1}{(N-1)(N-2)} & \cdots & \frac{1}{(N-1)(N-2)} \\
\frac{1}{(N-1)(N-2)} & 0 & \cdots & \frac{1}{(N-1)(N-2)} \\
\vdots & \vdots & \vdots & \vdots \\
\frac{1}{(N-1)(N-2)} & \frac{1}{(N-1)(N-2)} & \cdots & 0
\end{pmatrix}
\end{array}
$$

The relative entropy is given by:
$RE = \sum_{i=1}^{N-1}\sum_{j=1}^{N-1} l_{ij}^p ln\left(\frac{l_{ij}^p}{x_{ij}^p}\right)$
with $0 \cdot ln(0) := 0$ and $0 \cdot ln(\frac{0}{0}) := 0$.

This equation can be rearranged:
$RE = \sum_{i=1}^{N-1}\sum_{j=1}^{N-1} l_{ij}^p ln\left(l_{ij}^p\right) - \sum_{i=1}^{N-1}\sum_{j=1}^{N-1} l_{ij}^p ln\left(x_{ij}^p\right)$

Assuming that x_{ij}^p constitutes an element of the matrix of the maximum entropy solution, yields the following equation:

$RE = \sum_{i=1}^{N-1}\sum_{j=1}^{N-1} l_{ij}^p ln\left(l_{ij}^p\right) - \sum_{i=1}^{N-1}\sum_{j=1}^{N-1} l_{ij}^p ln\left(\frac{1}{(N-1)(N-2)}\right)$

$= \sum_{i=1}^{N-1}\sum_{j=1}^{N-1} l_{ij}^p ln\left(l_{ij}^p\right) - ln\left(\frac{1}{(N-1)(N-2)}\right)\left(\sum_{i=1}^{N-1}\sum_{j=1}^{N-1} l_{ij}^p\right)$

$= \sum_{i=1}^{N-1}\sum_{j=1}^{N-1} l_{ij}^p ln\left(l_{ij}^p\right) - ln\left(\frac{1}{(N-1)(N-2)}\right)\cdot 1$

Entropy is given by:

$ENT = -\sum_{i=1}^{N-1}\sum_{j=1}^{N-1} l_{ij}^p \cdot ln\left(l_{ij}^p\right)$

with $0 \cdot ln(0) := 0$.

Inserting the equation for entropy yields the following result:

$RE = -ENT - ln\left(\frac{1}{(N-1)(N-2)}\right)$

or

$RE = ln\left((N-1)(N-2)\right) - ENT$

As an example, consider 11 nodes in the system (10 banks and one external sector). The relationship between entropy and relative entropy to the maximum entropy solution is thus given by:

$RE = 4.4998 - ENT$

This equation can be confirmed by simulation (see Figure 2.23).

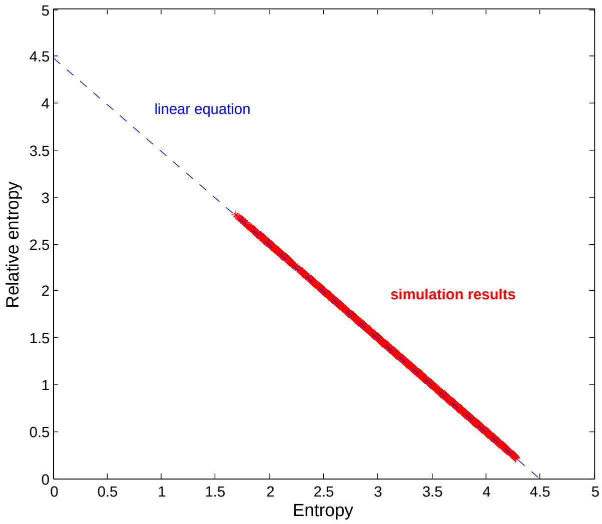

Figure 2.23: Negative linear relationship between entropy and relative entropy to the maximum entropy solution for $N = 11$

Chapter 3

Contagion in the interbank market with stochastic loss given default*

* This chapter is based on joint work with Christoph Memmel and Ingrid Stein. Opinions expressed are those of the authors and do not necessarily reflect the views of the Deutsche Bundesbank.

3.1 Introduction

The collapse of Lehman Brothers turned the 2007/2008 turmoil into a deep global financial crisis. But even before the Lehman default interbank markets ceased to function properly. In particular, the fear of contagion via interbank markets played a crucial role. While banks could gauge their direct losses from exposure to so-called toxic assets, they could not assess their counterparties' losses and creditworthiness and were therefore not willing to lend money to other banks, causing the breakdown of interbank markets. This led to an unprecedented liquidity extension of central banks and government rescue packages (see Stolz and Wedow (2010)) which, however, could not avoid deep recessions in many countries of the world. From an economic perspective, it is therefore essential to have a tool allowing to assess potential contagion risks via interbank markets.

Creating such a tool is the aim of this chapter. We study contagion in the German interbank market, one of the largest interbank markets in Europe. We carry out a simulation exercise where we assume that a certain bank fails and examine how this failure affects other banks' solvency via direct effects and chain reactions in the banking system. Throughout this (and also the next) chapter, our focus is on 14 large and internationally active German banks and the sectors of savings and cooperative banks.

In our contagion analysis, we only investigate the direct, mechanic contagion effects in the interbank market, which means that we analyze the direct (on and off-balance sheet) exposure between the banks. What we do not consider are effects like a general loss in confidence among banks which could lead to a drying up of the interbank market and thereby to a liquidity shortage, contagion due to market perception, i.e. that all banks with a similar business model are subject to distrust when such a bank runs into distress, or herding behavior, where massive sales can drive the price of an asset below its fundamental value and banks using fair-value accounting have to adjust asset values. Hence, our analysis covers only part of the possible contagion effects. However, this analysis is relatively precise because it is based on hard data and not so much on estimated economic relationships.

We investigate in particular the role of the loss given default (LGD) in the contagion process which is a key factor for the extent of contagion in analyses like this. The LGD, multiplied by the total exposure of a creditor bank to a debtor bank, gives the actual loss of the creditor bank in the event that the debtor bank fails. The LGD can vary between 0% (e.g. in the event that the defaulted loan is fully collateralized) and 100% (which is equivalent to a zero recovery rate of the defaulted loan). As there is usually only sparse information about recovery rates in the case of bank defaults, the standard approach in the literature on interbank contagion is to assume a fixed value of the LGD and repeat the simulation exercise with different values of this LGD. The literature generally finds that losses in the total banking system crucially depend on the LGD value. Below a certain threshold of LGD potential losses are minor. However, as soon as the LGD exceeds a certain threshold, there are considerable risks of large parts of the banking system being affected and heavy losses in the banking system occurring (see e.g. Upper and Worms (2004) and van Lelyveld and Liedorp (2006)). Therefore, the standard approach has the considerable drawback that an assessment of contagion risks in the real world is difficult and associated with great uncertainties. We, however, overcome this shortcoming by using a unique dataset of realized LGDs of defaulted interbank exposures.

Our contributions to the literature are as follows. First, by using this dataset of realized LGDs on the interbank market, we are able to investigate the empirical patterns of actual LGDs of bank loans. Second, unlike the vast majority of papers in the literature, we dispose of detailed data about the pairwise interbank exposures and do not need to estimate them. Instead, we are able to precisely quantify interbank exposures (including off-balance sheet and derivative positions) within the national market. Third, in contrast to most papers in the literature, we conduct the simulation exercise with a *stochastic* LGD derived from the observed distribution of LGDs (instead of a stepwise increase of *constant* values). We thereby obtain a distribution of the number of contagious bank defaults which allows a more realistic assessment of contagion risks.

Our main findings are, first of all, that LGDs follow a markedly u-shaped distribution, which can be reasonably well approximated by a beta distribution. Second of all, using the precise information about interbank exposures and the distribution of LGD, we find that contagion in the German interbank market may happen. Third, for the point in time under consideration, we find that the number of bank defaults increases on average when we assume a stochastic LGD instead of a constant one.

This chapter is structured in the following way: In Section 3.2, we give a brief overview of the literature on interbank contagion as well as LGD modeling and state our contribution to the literature. Section 3.3 deals with the description of the contagion exercise and the structure of the interbank network. Section 3.4 summarizes how we model the LGD. In Section 3.5, we show the results of the contagion exercise and in Section 3.6 the conclusion is presented.

3.2 Literature

This chapter relates to three strands of the literature. The first strand is about empirical simulation studies of interbank contagion (see Upper (2011) for an overview). Especially national European interbank markets have been the focus of empirical studies (see, for instance, van Lelyveld and Liedorp (2006) for the Netherlands, Sheldon and Maurer (1998) for Switzerland or Mistrulli (2011) for Italy). In addition to studies based on national interbank markets, there are cross-border contagion simulations. These studies are either based on BIS data on consolidated banking statistics (see Espinosa-Vega and Solé (2010) and Degryse et al. (2010)) or analyze international sector interlinkages (see Castrén and Kavonius (2009)). Most papers in this strand do not have direct access to information on interbank exposures but apply either statistical methods to derive the bilateral exposures or rely on data which cover only part of the interbank exposures. We have a certain advantage compared to these studies since we are able to precisely quantify the amount of bilateral exposures for a system of 14 large and internationally active German banks as well as the sectors of the savings and the cooperative banks. Our dataset is based on the German credit register and includes off-balance sheet and derivative positions. It contains all bilateral exposures of the 14 banks and two sectors above a threshold of EUR 1.5m. This threshold is not relevant for the purpose of our study since interbank exposures are typically large.

The second strand of literature we contribute to deals with extensions of the usual contagion exercises. Cifuentes et al. (2005) introduce additional stress due to declining asset prices as a result of fire sales; Elsinger et al. (2006) integrate the interbank contagion model in a stress testing setting that includes macroeconomic shocks. Espinosa-Vega and Solé (2010) and Chan-Lau (2010) do not only consider credit risk, but funding risk as well. They argue that the banks' funding is hindered when the interbank market does not function properly. Aikman et al. (2009) incorporate various of these aspects into one quantitative model of systemic stability. Degryse and Nguyen (2007) explicitly model the LGDs, deriving them endogenously from the banks' balance sheet composition. Our extension, too, is about LGD modeling. However, we model the LGDs as stochastic.

The third strand of literature deals with the distribution of LGDs. Huang et al. (2009) and Tarashev and Zhu (2008) choose a stochastic setting for the LGD. They assume a triangular distribution with the probability mass concentrated in the center of the distribution (more precisely at 55% and 50%, respectively). Crouhy et al. (2000) model a stochastic LGD with the help of a beta distribution. They estimate the parameters by using bond market data. Their estimations yield the result that the LGD follows a unimodal beta distribution. Our contribution consists in estimating the distribution of the LGDs of *interbank exposures.* We have a unique dataset of realized interbank LGDs at our disposal. This data suggests a markedly u-shaped density for the LGD, i.e. a distribution with a vast probability mass at zero and 100 per cent. This finding is in line with Dermine and de Carvalho (2006) and Bastos (2010) who use a dataset of defaulted loans provided by a large Portuguese bank and find a u-shaped LGD distribution for non-financial firms.

3.3 Round-by-round algorithm

In the event that a bank fails, the banks that have granted loans to this bank suffer losses from their exposures. The contagion process in the interbank market may stop after the first round, but may also propagate further through the system. Banks, which fell into distress as a consequence of the initial distress, may now themselves become a source of contagion. This process will continue round-by-round until no new banks are affected (possibly leading to a large number of failures in total) or until the supervisory authorities manage to put an end to this process.

In this section we describe a simulation exercise so as to study the extent to which the German banking system may be prone to such a contagious process. We apply the round-by-round algorithm as described in Upper (2011).

1. Initially, bank i fails exogenously.
2. As a result, banks whose exposure to bank i multiplied by the loss given default (LGD) exceeds their buffer of tier-1 capital, also fail. We define a bank to be in default in the event that its tier-1 capital ratio is below 6 per cent of its risk weighted assets. This default definition is in line with the new Basel accord where the minimum capital requirement is also set at 6%.[32] We do not take into account potential reactions of the lender banks. For example,

[32] See Basel Committee on Banking Supervision (2010), paragraph 50.

the lender banks may have hidden reserves which they release to raise their tier-1 capital. Instead, we assume that write-offs on interbank loans decrease the lender's tier-1 capital by the same amount.

3. Further banks may fail if their combined exposure to the banks that have failed so far (times the LGD) is greater than their capital buffer.

4. The contagious process stops when there is a round with no new failures.

Thus, bank j is in distress, if

$$\frac{E_j - \sum_k (LGD_{jk} \cdot x_{jk} \cdot 1_{k \in D})}{RWA_j - 0.2 \cdot \sum_k (x_{jk} \cdot 1_{k \in D})} < 0.06 \tag{3.1}$$

In this context, E_j is the tier-1 capital of bank j, x_{jk} is the exposure of bank j to bank k, $1_{k \in D}$ is an indicator variable that takes on the value 1 in the event that bank k is in distress (and 0 otherwise), LGD_{jk} is the loss given default associated with the exposure of bank j to bank k and RWA_j are the risk weighted assets of bank j. We assume that interbank claims receive a weight of 0.2 in banks' risk weighted assets.[33] When calculating the tier-1 capital ratio, we also take into account that every claim to a bank that failed completely disappears from the creditor bank's risk weighted assets.

One can argue that a bank can serve its debt unless the capital of this bank is totally consumed and becomes negative. However, we use the stricter criterion of a minimum level of 6% tier-1 capital. We do this for the following reasons: (i) When a bank is shut down and liquidated (because, for instance, it no longer meets the minimum regulatory capital requirements), it is questionable whether one receives the bank assets' book value, especially the book value of its illiquid positions may be far higher than the proceeds from a hasty fire sale. (ii) A bank with a sharp drop in its capital ratio will no longer have any access to short term funding at sustainable

[33] The risk weight of 0.2 stems from the Basel I and Basel II framework applied to German banks, see e.g. Deutsche Bundesbank (2004), p.77.

rates. Soon afterwards, the unfavorable funding conditions will have consumed what is left of the capital.

We carry out this simulation exercise for each of 14 large and internationally active banks in Germany (the biggest private commercial banks and the central institutes of the savings and cooperative banks) and the two sectors of the savings and cooperative banks, which we treat as aggregate sectors. Thus, we consider 16 units in total which potentially have bilateral exposures to each other. We treat the savings and cooperative banks in an aggregate way because single banks that belong to this group are usually very small.[34] Hence, it is almost sure that the default of one of these small banks would not trigger contagious reactions. If, however, the whole sector of savings or cooperative banks were to be hit by an aggregate shock (which is not completely unlikely because of similar business models), a contagious effect on the rest of the banking system is quite possible. To sum up, we cover about 67% of the total assets of the German banking system in our analysis.

To run the round-by-round algorithm, information is needed on (i) the pairwise exposures between the banks and (ii) the appropriate loss, given a bank fails. Concerning the pairwise exposures, we have detailed information on exposures within the German interbank market. This leaves the question of determining the loss given default. From the literature we know that this is crucial for the contagion exercises (see e.g. Upper and Worms (2004)). Different solutions are possible.

1. Constant *LGD*. The loss given default is exogenously set to a constant value, say 40% or 45%.[35] To account for the fact that the LGD crucially drives the

[34] Craig and von Peter (2010) show that only a small number of banks form the so-called core of the German interbank market and that these core banks act as an intermediary for numerous small banks (like savings and cooperative banks).

[35] Kaufman (1994) gives an overview of loss given default estimates for bank failures; the estimates vary considerably. James (1991) finds that the average loss of failed US banks during the period of 1985 to 1988 was about 30%. In addition, there were direct costs associated with the bank closures of 10% of the assets. In our dataset, the mean LGD is about 45%.

results, one can vary the constant loss given default over a wide range of values. The contagion exercise is then run for each different value of the LGD.

2. Endogenous *LGD*. If information on the actual over-indebtedness of the distressed bank, the bankruptcy cost and the degree of collateralization were available, it would be possible to endogenously calculate the loss given default.

3. Stochastic *LGD*. Our supervisory data concerning the write-offs of interbank loans show that the loss given default varies considerably, with a large portion of the probability mass at 0% and at 100%. A possible explanation for this quasi-dichotomy may be that the loans are either fully collateralized (as in the Repo-market) or completely unsecured. This finding is not in line with the assumption of a constant LGD (solution 1). Solution 1 would rather be in line with a distribution of the LGDs concentrated in one point.

In this study, we use the third solution. In contrast to the existing literature that exogenously assumes some constant LGD value, we have a unique dataset of actually realized LGDs on the interbank market. This dataset provides an empirical frequency distribution of LGDs. The exact properties of the LGD distribution are investigated in Section 3.4.

As outlined above, the first step for running the round-by-round algorithm consists of establishing the matrix of mutual interbank exposures. We use Bundesbank data from the German credit register (MiMiK) to obtain the necessary information.[36] Unlike credit registers in most other countries, the German credit register also includes interbank loans and is not confined to non-financials. This data base offers us a certain data advantage compared to other studies since we are able to determine the complete matrix of interbank exposures. By contrast, balance sheet data only show (for each bank) the aggregate amount lent to or borrowed from all banks.

[36] See Schmieder (2006) for more details about this database.

Moreover, payment data or large exposure data are generally less comprehensive than credit register data and include, for example in the case of payment data, information about short-term lending only.

The German credit register contains quarterly data on large exposures of banks to individual borrowers or single borrower units (e.g. groups). Banking institutions located in Germany are required to report if their exposures to an individual borrower or the sum of exposures to borrowers belonging to one borrower unit exceeds the threshold of EUR 1.5m at least once in the respective quarter. We think that the threshold of EUR 1.5m does not cause a serious bias since the typical interbank loan is relatively large and exceeds the threshold of EUR 1.5m.

The credit register applies a broad definition of a loan. Loans in this sense include traditional loans, bonds, off-balance sheet positions and exposures from derivative positions. However, trading book positions are excluded. We start by analyzing gross on- and off-balance sheet exposures as a benchmark case. In Section 3.5.2, we run simulations considering on-balance sheet exposures only. Furthermore, we investigate the case of netting. It is, however, by far not clear whether netting can be enforced in case of a bank failure.[37]

For the simulation exercise, we use data from the fourth quarter of 2010. The resulting matrix of interbank exposures gives some interesting insight into the German interbank market. As we consider 14 large and internationally active German banks as well as the savings and cooperative sector, we obtain a 16×16 matrix of interbank exposures. Not surprisingly, we almost have a complete network, i.e. most of the off-diagonal elements of this matrix are non-zero. To be precise, only two off-diagonal elements are zero, i.e. only two of the 240 possible interbank relations do not exist.

Table 3.1 in Appendix 1 shows some summary statistics of the interbank network we consider in this chapter. To capture the inequality of how banks spread their inter-

[37] See Mistrulli (2011) for this and other arguments concerning the simulation method.

bank assets/liabilities among their counterparties and thus to evaluate how banks differ in terms of their connections to other banks, we calculated for each bank the *normalized* Herfindahl-Hirschman index (HHI) of the share of single interbank assets/-liabilities to the bank's total interbank assets/-liabilities (that are included in our analysis).[38] The maximum HHI of 1 would indicate that a bank has all its interbank assets/-liabilities towards one single counterparty. The minimum HHI of 0 would indicate that a bank spreads its interbank assets/-liabilities as equally as possible. The large difference between the 25% quantile (with an HHI of 0.05 and 0.11, respectively) and the 75% quantile (with an HHI of 0.28 and 0.38, respectively) implies that results differ substantially among banks. The reason is that the central institutions of the savings and cooperative banks concentrate their exposures on the savings and cooperative sector, respectively, large private banks, however, do not. It is also remarkable that banks in our network tend to spread their interbank assets more equally than their interbank liabilities. The relative size of interbank exposures (i.e. interbank assets/liabilities over tier-1 capital) is already quite large at the 25% quantile, with 4% and 5%, respectively. The median of the relative size is at 12% and 10%, respectively. This confirms that interbank exposures are a considerable source of contagion.[39]

3.4 Loss given default (LGD)

As stated in the previous section, another key component for the contagion exercise is the loss given default (LGD). We have some information about the loss rate banks face in the event that a debtor bank defaults. More precisely, our LGD data are

[38] Another method to measure the inequality would be to calculate for each bank the entropy of the shares of single interbank exposures.

[39] Generally, a single loan must not exceed 25% of a bank's liable capital. Exceptions are, however, exposures between banks within the associations of savings and cooperative banks, respectively.

assigned to the respective lender bank and not - as usually - to the debtor bank. Although we do not know the LGD of the lender bank for a default of a *specific* debtor bank, we know for each lender bank the average LGD of interbank exposures (at an annual frequency). We have data on the volume of non-performing interbank loans and on the corresponding write-downs. For each bank and each year, two figures are provided: the amount (in euro) of interbank loans for which provisions have been made and the amount of these provisions. We interpret the ratio of these two figures as a realization of the stochastic LGD of a single interbank relationship, not of an average of two or more LGDs. It is important to have realizations of LGDs of single interbank relationships because realizations of average LGDs tend to be biased towards unimodal distributions; the average of, let's say 50 LGDs, is by virtue of the central limit theorem approximately normally distributed, even if the distribution of single LGDs is markedly u-shaped.[40] The data are taken from the quantitative supervisory reports collected by the Bundesbank on banks in Germany.[41] Based on this data, we can estimate the distribution of LGDs.[42]

Looking at Figure 3.1, we see that the empirical distribution of the LGDs is markedly u-shaped. This characteristic and the nature of the LGDs, especially its range between 0 and 1, suggest modeling the LGD distribution with the beta distribution. Figure 3.1 also displays the probability density function of a beta distribution with the estimated parameters. Compared to the empirical frequency distribution, only small deviations can be observed. Statistical tests confirm this observation. The null hypothesis of a χ^2 goodness-of-fit test on whether our data follow a beta distribution with the estimated parameters $\hat{\alpha}$ and $\hat{\beta}$, cannot be rejected on a 5% significance level. Choosing ten equidistant intervals and comparing the observed frequency to

[40] We will discuss this topic in more detail later in this section.

[41] For more details on these data see Memmel and Stein (2008).

[42] Note that the LGD data and the exposure data are not fully compatible: Whereas the LGD data refers to unconsolidated accounts and includes both the trading and the banking book, the exposure data refers to consolidated accounts and does not include the trading book. We believe, however, that these lacks in compatibility do not call in question the use of the data.

the expected frequency within the intervals yields a p-value of ≈ 0.075.[43]

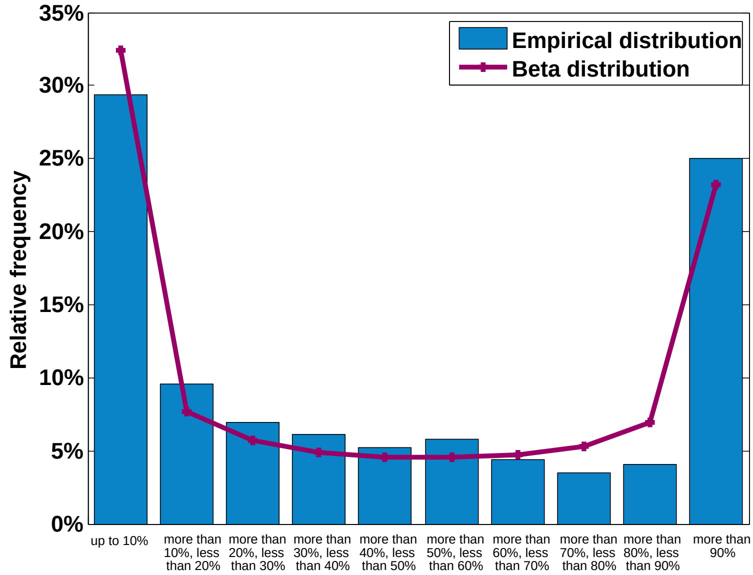

Figure 3.1: Relative frequency of the loss given default for interbank loans, derived from data on German private commercial banks and the central institutions of the savings and cooperative banks. 344 observations for the period 1998-2008

Therefore, we use the beta distribution for further analysis. The density of the beta distribution is given by

$$f(x) = \frac{1}{B(\alpha, \beta)} x^{\alpha-1}(1-x)^{\beta-1} \quad x \in (0, 1) \tag{3.2}$$

with

$$B(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}, \tag{3.3}$$

43 The result of this test gives strong evidence that the assumed distribution is very close to the observed distribution, as the test is very sensitive due to the large number of observations. To illustrate the correlation between the number of observations and the sensitiveness of the test, we run simulations with a sample randomly drawn from a beta distribution. Drawing 344 observations from a beta(0.28,0.35)-distribution $10,000$ times and testing each sample against a beta(0.18,0.25)-distribution yields a probability of making a type II error (i.e. the error of falsely accepting the null hypothesis) of around 18%. Repeating this exercise for only half of the sample (i.e. drawing 172 observations each time) leads to a probability of making a type II error of 62%. Thus, the larger the sample, the more sensitive the test becomes to only small deviations from the distribution tested.

where $\Gamma(\cdot)$ is the Gamma-function. The parameters $\alpha > 0$ and $\beta > 0$ determine the shape of this distribution.[44] The beta distribution is especially suited to model the LGD because (i) the domain is confined to the economic sensible interval from 0 to 1, (ii) it is highly flexible and (iii) nests other distributions.[45] For instance, when both parameters equal one, then the beta distribution becomes a uniform distribution. When both of the parameters are smaller than one, the probability density function is u-shaped with a large portion of the probability mass close to zero and one. For parameter values close to zero, this distribution converges to the binomial distribution. By contrast, the density is unimodal in the case of both parameters α and β being greater than one. For very large parameter values, it converges to the degenerate distribution, where the entire probability mass is concentrated on one point. The expectation and the variance of a random variable X following a beta distribution are functions of the parameters α and β:

$$E(X) =: \mu = \frac{\alpha}{\alpha + \beta} \tag{3.4}$$

and

$$var(X) =: \sigma^2 = \frac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)} \tag{3.5}$$

Given estimates for the expectation and the variance, estimators for the parameters α and β are obtained by solving the equations (3.4) and (3.5) for α and β, respectively:[46]

$$\hat{\alpha} = \hat{\mu}\left(\frac{\hat{\mu}(1 - \hat{\mu})}{\hat{\sigma}^2} - 1\right) \tag{3.6}$$

[44] Figure 3.6 in Appendix 2 summarizes the possible shapes of the probability density function dependent on the parameter values.

[45] See e.g. Hahn and Shapiro (1967), p. 91.

[46] This procedure is called method of matching moments, see e.g. Hahn and Shapiro (1967), p.95. We do not use maximum likelihood-estimation because there is a considerable amount of observations which equal exactly 0 and 1 and for which, therefore, the likelihood function is not defined.

$$\hat{\beta} = (1 - \hat{\mu}) \left(\frac{\hat{\mu}(1 - \hat{\mu})}{\hat{\sigma}^2} - 1 \right) \tag{3.7}$$

We calculate the sample mean and variance of the distribution of the LGD for the whole sample and different subsamples and then estimate the parameters α and β (see Tables 3.2 and 3.3 in Appendix 4). It is noteworthy that the average LGD of savings banks being the creditors (= 58%) is well above the average LGD of the total sample (= 38%). Cooperative banks, whose business model is comparable, however, suffer only from a rather low LGD of 24% on average in the event they incur losses on the interbank market. The average LGD incurred by large and internationally active banks is in between (= 45%). Furthermore, we see that the LGD tends to be higher the larger the lender bank (measured as the lender bank's total assets). Irrespective of the subsample under consideration, we observe a u-shaped distribution. We explicitly test the null hypothesis that the beta distribution is not u-shaped, i.e. that $\alpha \geq 1$ or $\beta \geq 1$ (see Figure 3.6). We do this by applying the delta method.[47] The result is that we can reject the null hypothesis on a 1% and 5% significance level, respectively, in all cases. Thus, we can conclude that, irrespective of the banking group and size of the lender banks, we can assume a u-shaped distribution of the LGD.[48]

As our analysis focuses mostly on large and internationally active banks in Germany, it would be obvious to use this subsample to estimate the parameters of the LGD distribution. This would give us 101 observations of realized LGDs. However, the problem is that these LGDs are probably not only due to one single credit but due to several credit relationships in distress. If we assume that the number of defaulted credit relationships per bank and year follows a Poisson distribution, we obtain for the case that exactly one debtor bank defaulted, given the bank reports

47 The details on the delta method are described in Appendix 3.

48 In the literature, however, the LGD is often modeled by using a unimodal distribution (which implies that $\alpha > 1$ and $\beta > 1$) or as a constant. Hence, these results may also have further implications for this literature.

non-zero write-downs, a probability of only 71%. Thus, we have the problem that the reported LGD value of that bank is often (in about 29% of the cases) just an average of several LGD values and the LGD distribution is therefore biased towards unimodal distributions.[49]

We can mitigate this problem by including further banks in our sample for which it is reasonable to assume that exactly one credit relationship is in distress, given the bank reports non-zero write-downs. Thus, we include all private commercial banks in our sample which yields a probability that exactly one single credit defaulted (given the bank reported non-zero write-downs) of 93%. Regional savings banks and cooperative banks, which are generally small and medium-sized, are not included in our sample. The reason is that we consider their position in the German interbank market as less representative for our stability analysis because these banks' interbank market activities are very much characterized by relationships to their central institutes. This is not the case for the smaller private banks. In addition, the mean LGD, which is not affected by the aforementioned problem, is quite similar (around 45%) in the sample we chose and in the sample of the large and internationally active banks (see Table 3.2). We therefore believe that our sample is a balanced compromise between statistical properties (a high share of single default events) and economic fit (similarity of the banks in the contagion exercise and estimation of the LGD distribution).

As our LGD-data are applied to situations of severe stress in the interbank market, an important point to investigate is how LGDs change in crisis time compared to normal times. Our data enable this as they contain the period from 1998 to 2008 and thus include the crisis year 2008. By comparing the mean LGD of the pre-crisis years (1998-2007) to the crisis year 2008, we obtain the surprising result that the LGDs in 2008 are, on average, lower compared to the period before. For the sample

[49] The higher standard deviation of the sample used for the contagion analysis compared to the sample of large and internationally active banks confirms our assumption, see Table 3.2.

including all banks in 2008, we obtain 251 observations with a mean LGD of 0.27 (compared to 0.44 in pre-crisis years). The subsample of all private commercial banks and the central institutions of the savings and cooperative banks (28 observations) yields a mean LGD of only 0.22 (compared to 0.47 in pre-crisis years). A possible explanation for this fact is that banks become more cautious in times of stress and e.g. demand more collateral for interbank lending. Thus, we conclude that the potential rise of LGDs in times of severe stress (e.g. due to reduced asset values in banks' balance sheets) is counteracted by more precautious lending by banks.

Our final sample of LGD observations consists of 344 observations in the period from 1998 to 2008. Figure 3.1 shows the frequency distribution of the LGDs. Using the sample mean $\hat{\mu}$ and variance $\hat{\sigma}^2$ as an estimator for the population mean and variance, we obtain $\hat{\mu} = 0.45$ and $\hat{\sigma}^2 = 0.15$. Inserting $\hat{\mu}$ and $\hat{\sigma}^2$ into equation (3.6) and (3.7) yields $\hat{\alpha} = 0.28$ and $\hat{\beta} = 0.35$. These parameter values indicate a u-shaped distribution (see Figure 3.6).

As stated above, LGDs of banks can in theory be derived endogenously from their balance sheet composition. However, we do not apply this solution because we would have to make a lot of additional assumptions in our contagion exercise.[50] For example, we would have to make assumptions about who has to bear the losses that arise from the bank failures. A standard assumption in a case like this is that losses are distributed pro rata among creditors (see the clearing algorithm of Eisenberg and Noe (2001)), which is definitely a strong assumption. For instance we find that the mean LGD for totally unsecured interbank exposures is 64%, whereas it is only 24% for the at least partly collateralized ones. Additionally, it would be necessary to model losses due to fire sales of assets of distressed banks. A detailed contagion analysis with an endogenous LGD is thus not feasible since we lack the necessary data. Besides, our data on realized LGDs suggest that the borrower banks' balance

[50] See Upper (2011) for an overview of these assumptions.

sheet composition and other bank specific variables only explain a small fraction of the LGD variation. We carried out a variance decomposition of the LGDs and we find that most of the variation is due to the lender bank and due to the nature of the relationship, i.e. the variation owing to the balance sheet composition of the borrower bank is less important. Furthermore, endogenizing the LGD disregards the time dimension. Upper (2011) cites the default of Bankhaus Herstatt as an example for the observation that the LGD varies across the time horizon, i.e. the LGD decreases when the recovery horizon becomes longer. This observation is backed up by Bastos (2010) who shows that the recovery rate ($= 1 - LGD$) (though for defaulted loans to non-financials) increases steadily with the recovery horizon. Thus, in our opinion, the best approach is to use the u-shaped frequency distribution of the LGD data that are derived from actual write-downs following the default of a bank.

3.5 Results

3.5.1 Benchmark case

The initial assumption for our simulations is that one of the 16 banks/banking sectors[51] fails. This could trigger a cascade of failures if the ratio of tier-1 capital to risk weighted assets of one of the creditor banks falls below 6%. The simulations (based on a stochastic LGD) are run by drawing from a beta distribution with parameters $\alpha = 0.28$ and $\beta = 0.35$. This means that, for each exposure of a creditor bank to a bank in distress, we randomly draw an LGD value from the beta distribution estimated in Section 3.4. We repeat this exercise by varying the bank that fails first from bank number 1 to 16. In contrast to simulations based on a constant LGD, the approach with a stochastic LGD yields for each of the 16 banks a *distribution* of the number of banks in distress (and not only one single number of

[51] For notational convenience we will call the sectors in the following just banks.

subsequent failures). We repeat the contagion exercise $100,000$ times for each bank, with a different of the 16 banks starting the contagious process each time.

Figure 3.2 indicates the relative frequency of the number of bank failures, assuming that the probability of the initial failure is the same for all of the 16 banks. The figure shows that in 51% of the 1,600,000 simulation runs, no further failure occurs. In 8% of the cases, however, 11 subsequent bank failures occur. On average, we observe 3.06 subsequent bank failures (i.e. 4.06 bank failures in total) in our simulations. Figure 3.3 shows, among others, that in almost 18% of the cases more than ten banks fail. These results indicate that there is a considerable risk of interbank contagion.

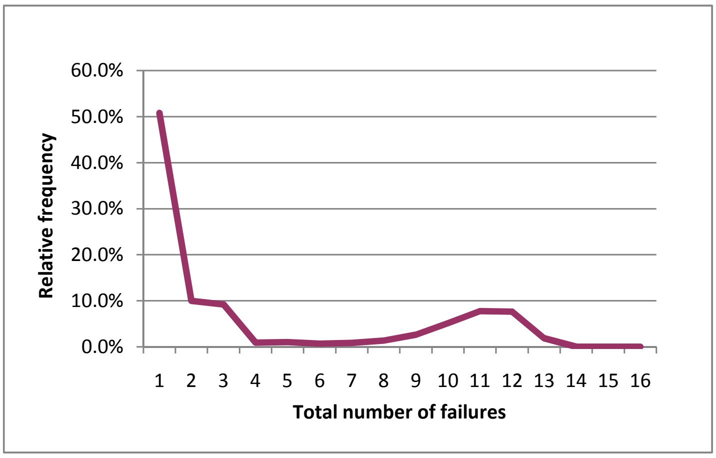

Figure 3.2: Frequency distribution of bank failures

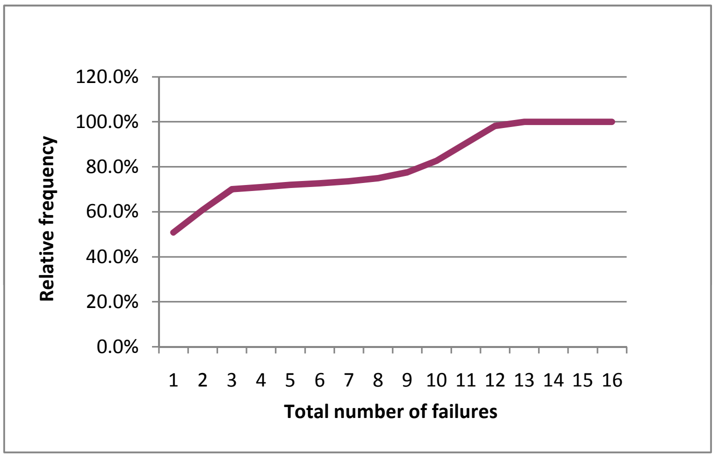

Figure 3.3: Distribution function of bank failures

3.5.2 Robustness checks

We carry out robustness checks concerning six issues. The first three checks consider the robustness of the results to different LGD specifications. First, we investigate if drawing from the beta distribution estimated from our dataset is a good approximation for the empirical distribution. Second, we examine if results change significantly when using different LGD distributions for the savings and cooperative sector. Third, we compare the simulation results of a stochastic LGD with the results under the assumption of a constant LGD that is equal to the mean of our dataset. The next two checks consider the specification of the matrix of interbank exposures. With one special feature of our dataset (compared to most of the existing literature) being the inclusion of off-balance sheet exposures, we thus additionally run simulations excluding off-balance sheet exposures and compare the results. In our fifth robustness check, we examine if netting of interbank assets and liabilities between counterparties can solve the problem of contagion. Finally, we check whether the number of bank failures is a good indicator for the stability of the system as it is, of course, not only important how many banks fail but also how many assets are affected by failure. Thus, as a last robustness check, we take the balance sheet total of failing banks into account when judging the severity of contagion.

Drawing from the empirical LGD distribution: To investigate the sensitivity of our results with respect to the assumed distribution, we draw from the discrete distribution observed by the data instead of the beta distribution. For this purpose, one observed LGD value is randomly allocated to each exposure of a creditor bank to a bank in distress. Compared to drawing the LGD from a beta distribution, the results of this exercise do not differ much. The average amount of bank failures is 4.11 (compared to 4.06 in Section 3.5.1). Furthermore, if we look at the relative frequency distribution as well as the cumulative distribution function of the total number of bank failures, there are virtually no differences to the results of the simulations with the beta-distributed LGD. We can therefore conclude that drawing from the beta distribution is a good approximation for our observed LGD values.

Different LGD distribution for savings and cooperative banks: As our interbank network consists not only of private commercial banks but also of savings and cooperative banks, an obvious question is how results are driven by the parameters of the beta distribution the LGDs are drawn from. As Table 3.2 shows, LGDs corresponding to write-downs of savings and cooperative banks rather resemble a beta(0.42,0.30) and beta(0.08,0.24) distribution, respectively. Thus, we run our simulations by drawing from the respective distributions for exposures of the savings and cooperative sector and from the "standard" beta(0.28,0.35) distribution for the exposures of the remaining banks.

The results of the contagion analysis differ only slightly from our benchmark results. The overall expectation of bank failures is now at 4.24 (compared to 4.06 in the benchmark case). On the bank level, it is not clear whether the system is more stable than in the benchmark case. The initial default of 5 of the 16 banks triggers more failures in the benchmark case; for the initial default of 9 of the 16 banks, less failures occur in the benchmark case and 2 of the 16 banks do not trigger any further bank failure in any case. The deviations of the expected number of bank defaults, given that one specific bank fails are, however, rather small regarding the benchmark case and do not exceed 0.95.

Stochastic versus constant LGD: As the standard assumption in the existing literature is a constant LGD, we compare our simulation results generated under the assumption of a stochastic LGD with results under the assumption of a constant LGD. We set the constant LGD equal to the mean of our LGD-dataset (= 0.45, see Table 3.2). Contrary to the case of the stochastic LGD, where we receive for each trigger bank a whole distribution of results, we obtain for each trigger bank one single number of failures under the assumption of a constant LGD. For only 4 of 16 initial bank failures, a constant LGD yields a more unstable system (compared to the average number of bank failures under the assumption of a stochastic LGD). In total, we obtain on average 2.69 bank failures under the assumption of a constant LGD (compared to the average of 4.06 bank failures under the assumption of a stochastic LGD). Thus, we conclude that there is a certain risk of underestimating the effects of a bank failure on financial stability if the distribution of the LGD is not considered.

On-balance sheet exposures only: Additionally, we examine the impact of including off-balance sheet positions in our simulations. Most literature on interbank contagion ignores off-balance sheet exposures due to data restrictions, while we have considered them in our above simulations. We therefore repeat the simulation exercise by excluding off-balance sheet positions. According to our dataset, the ratio of off-balance sheet exposures to total exposures varies considerably between banks. Table 3.1 shows that 25% of the banks hold less than 6% of total interbank assets (3% of total interbank liabilities) off-balance sheet. There are, however, also 25% of the banks that have a share of more than 24% (10%) of off-balance sheet interbank assets (liabilities).

Not surprisingly, banks with a high amount of off-balance sheet positions on their liability side trigger much less bank failures when ignoring these exposures. In total, the average amount of bank failures is only 3.47 (compared to 4.06 when considering all exposures).

To elaborate the differences between the simulation results with and without off-balance sheet exposures, we calculate the difference between the two relative frequency distributions of bank failures (see Figure 3.4). Figure 3.4 shows, for example, that the overall relative frequency of observing only one bank failure (i.e. contagion effects not occurring) is five percentage points higher when only considering balance sheet exposures. For high numbers of bank defaults, the result is reversed. For instance, the overall relative frequency of observing 12 bank failures is more than five percentage points higher when off-balance sheet exposures are considered. Thus, Figure 3.4 shows that the inclusion of off-balance sheet exposures leads to a higher frequency of observing extreme events and therefore captures tail risk in a more adequate way. Therefore, we can conclude that off-balance sheet exposures considerably contribute to the interdependence of banks and possibly change the results of the stability analysis in a remarkable way.

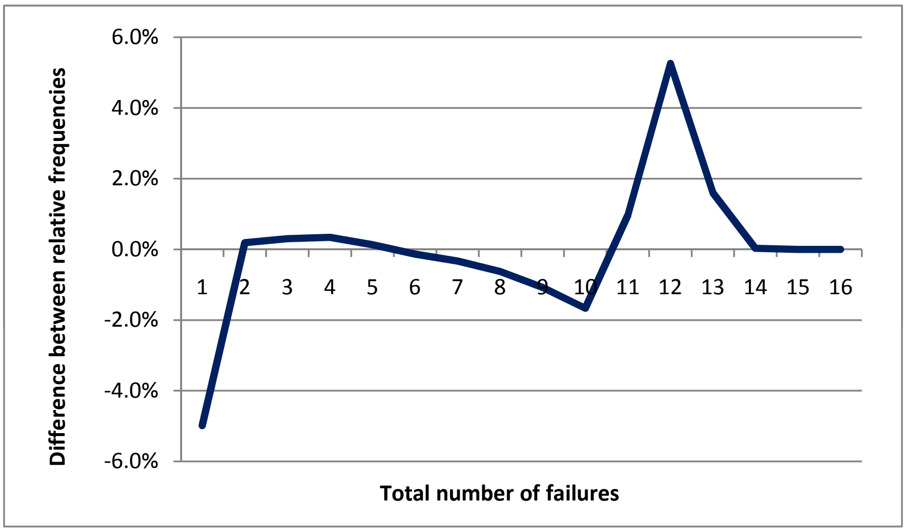

Figure 3.4: Difference between the relative frequency distributions of bank failures considering total exposures and on-balance sheet exposures only

Netting: As a next robustness check we examine how netting affects our results. Thus, we assume that banks net their exposures to each other. Technically, this means that we calculate the difference of element (i,j) and (j,i) of the matrix of interbank exposures and change all negative entries to zero. The outcome is a matrix of net interbank exposures.

The result is that, of course, far fewer bank failures occur. This could be seen easily by looking at our simulation method. A significant reduction of interbank exposures necessarily induces less contagious bank failures. What is surprising, however, is that contagion could still occur. Our simulations show that in almost 13% of the 1,600,000 simulation runs, a second round effect occurs. On average, 1.16 banks fail, which is naturally much less than in our benchmark case (= 4.06 bank failures on average).

Balance sheet vs. number of banks Of course one could argue that the number of bank failures is not a good indicator for financial stability as also the size of the defaulted bank matters. Hence, as an additional robustness check, we use the ratio of assets that belong to banks that fail in reaction to the trigger event to total assets that could theoretically fail as an indicator for the contagious effects. A value of zero would thus mean that only the trigger bank fails and no subsequent bank failures occur. A value of one means that all banks in the system default.

In Figure 3.5 we compare the distribution function of the total number of bank failures (which is the same as in Figure 3.3) with the distribution function of the share of assets that belong to failing banks (without the trigger bank). To make these two functions comparable, we divided the share of assets that belong to failed banks into 16 intervals of the same size and counted the frequency of results being in a particular interval. It is now easy to see that it is e.g. more likely to observe 50% or less of the total assets of the banking system (without trigger bank) failing compared to observe 8 or less bank failures.

Furthermore, our simulations show that on average 14% of assets in the remaining banking system (without the trigger bank) are affected by bank failure. By comparing this result to the average share of banks that fail subsequently ($= 3.05/15 \approx 20\%$) we can conclude that the banks that fail in our simulations belong on average to the smaller banks of our sample.

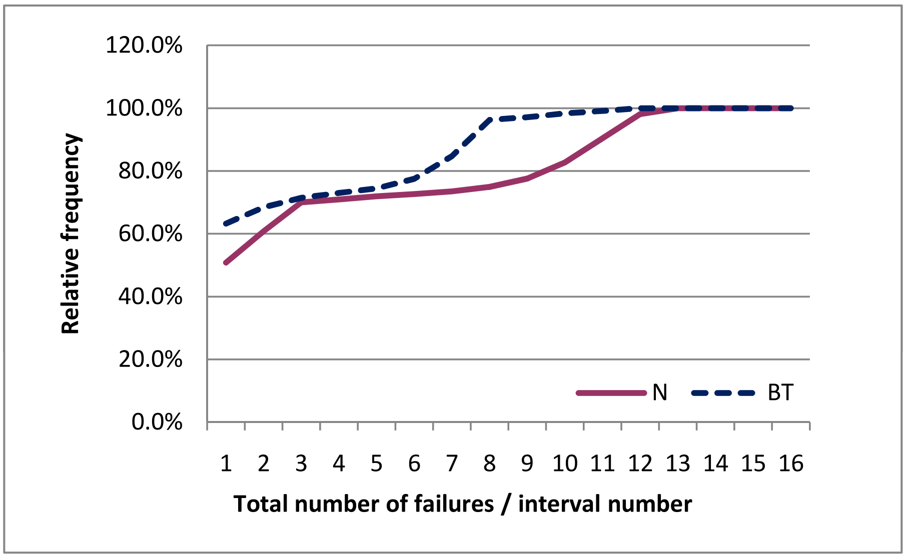

Figure 3.5: Distribution function of bank failures (N) and share of assets that belong to failing banks (BT)

3.6 Conclusion

In this chapter, we investigate contagion risk in the German interbank market. We have access to a unique dataset on loss given defaults (LGDs) of interbank exposures. Our data reveal that the frequency distribution of the LGD is markedly u-shaped, i.e. defaults of interbank loans often imply either a low or a high loss. This markedly u-shaped distribution stands in contrast to the assumption of a unimodal LGD distribution in the literature.

Next, we run simulations investigating the extent of potential contagion in the German interbank market. For this purpose, we focus on 14 systematically relevant German banks and the sectors of the savings and cooperative banks. We run simulations under the assumption of a stochastic LGD by drawing from a beta distribution. The shape of the beta distribution is derived from our LGD dataset.

The result of our simulations is that contagion in the German interbank market may happen. For the period of time under review (end 2010), we find that the contagion exercise under the assumption of a stochastic LGD yields on average a more vulnerable system than under the assumption of a constant LGD. Furthermore, banks' off-balance sheet exposures considerably contribute to the interdependence of banks and change the results of the stability analysis in a remarkable way.

An open question for research is to compare the loss distribution at different points in time and to develop an indicator showing by how far the interbank market is prone to contagious processes.

3.7 Appendix

3.7.1 Appendix 1: Summary statistics of the interbank network

	p25	median	p75	N
HHI(AIB)	0.05	0.10	0.28	16
HHI(LIB)	0.11	0.26	0.38	16
AIB/E	0.04	0.12	0.24	240
LIB/E	0.05	0.10	0.26	240
off-bs(AIB)	0.06	0.09	0.24	16
off-bs(LIB)	0.03	0.05	0.10	16

Table 3.1: HHI = normalized Herfindahl-Hirschman index; HHI(AIB)/HHI(LIB) = bank-specific normalized HHI of the share of single interbank exposures to total exposures on the asset/liability side of the bank's balance sheet; AIB/E (LIB/E) = ratio of single interbank exposures on the asset (liability) side of the bank's balance sheet to its tier-1 capital; off-bs(AIB)/off-bs(LIB) = bank-specific ratio of off-balance sheet exposures to total exposures on the asset/liability side of the bank's balance sheet; p25 = 0.25-percentile; p75 = 0.75-percentile; N = number of observations

3.7.2 Appendix 2: Beta distribution

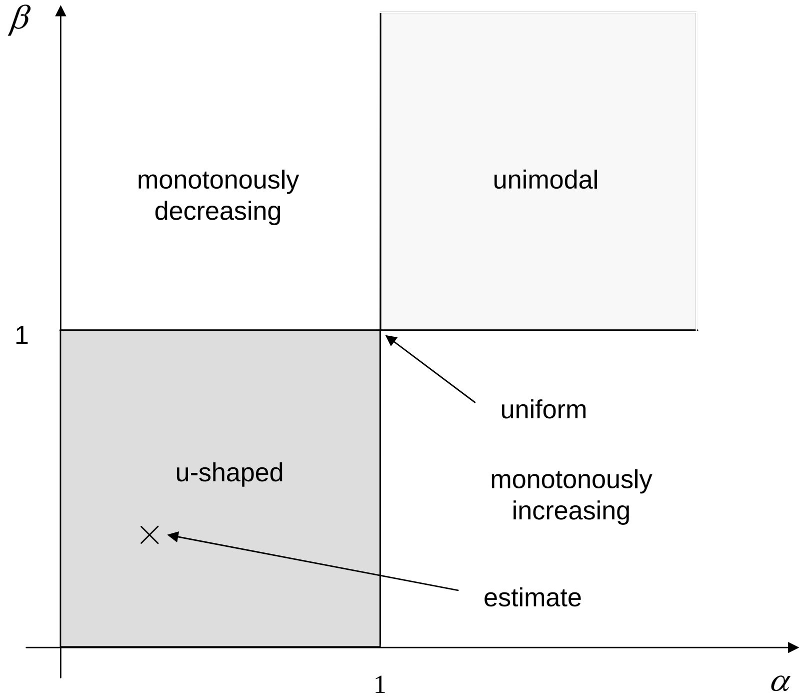

Figure 3.6: Shapes of the probability density function of the beta distribution dependent on the value of the parameters α and β

3.7.3 Appendix 3: Delta method to test for the u-shape of the beta distribution

Our goal is to explicitly test whether the observed LGD distribution is significantly u-shaped, i.e. we test the null hypothesis that $\alpha \geq 1$ or $\beta \geq 1$. We carry out a sequence of two t-tests with the two null hypotheses $\alpha \geq 1$ and $\beta \geq 1$, respectively. In the event that we can reject both null hypotheses, we accept the hypothesis $\alpha < 1$ and $\beta < 1$. Given the same significance level in both t-tests, the significance level of the joint hypothesis $\alpha < 1$ and $\beta < 1$ is at least as strong (see Frahm et al. (2010)). Using the delta method and the relations given in Equations (3.6) and (3.7), we derive the asymptotic distribution of the estimates for $\hat{\alpha}$ and $\hat{\beta}$, respectively. Using a first-order Taylor expansion, the delta method gives us a relation between the variance-covariance matrix of the estimators $\hat{\mu}$ and $\hat{\sigma}^2$, and the variance-covariance matrix of $\hat{\alpha}$ and $\hat{\beta}$:

$$Var\begin{pmatrix}\hat{\alpha}\\ \hat{\beta}\end{pmatrix} \approx \nabla\begin{pmatrix}f_1\left(\hat{\mu},\hat{\sigma}^2\right)\\ f_2\left(\hat{\mu},\hat{\sigma}^2\right)\end{pmatrix}^T \cdot Var\begin{pmatrix}\hat{\mu}\\ \hat{\sigma}^2\end{pmatrix} \cdot \nabla\begin{pmatrix}f_1\left(\hat{\mu},\hat{\sigma}^2\right)\\ f_2\left(\hat{\mu},\hat{\sigma}^2\right)\end{pmatrix} \tag{3.8}$$

with $f_1\left(\hat{\mu},\hat{\sigma}^2\right) = \hat{\alpha} = \hat{\mu}\left(\frac{\hat{\mu}(1-\hat{\mu})}{\hat{\sigma}^2} - 1\right)$ and $f_2\left(\hat{\mu},\hat{\sigma}^2\right) = \hat{\beta} = (1-\hat{\mu})\left(\frac{\hat{\mu}(1-\hat{\mu})}{\hat{\sigma}^2} - 1\right)$.
The variance-covariance matrix of $\hat{\mu}$ and $\hat{\sigma}^2$ is given by:

$$Var\begin{pmatrix}\hat{\mu}\\ \hat{\sigma}^2\end{pmatrix} = \begin{pmatrix}\sigma^2_{\hat{\mu}} & \sigma_{\hat{\mu},\hat{\sigma}^2}\\ \sigma_{\hat{\mu},\hat{\sigma}^2} & \sigma^2_{\hat{\sigma}^2}\end{pmatrix} = \begin{pmatrix}\frac{1}{N}\sigma^2 & \frac{1}{N}\mu_3\\ \frac{1}{N}\mu_3 & \frac{1}{N}\left(\mu_4 - \frac{N-3}{N-1}\sigma^4\right)\end{pmatrix} \tag{3.9}$$

where μ_3 and μ_4 denote the third and fourth central moments, respectively.[52] For implementation purposes, we replace the true moments by their estimators, i.e. $\hat{\sigma}^2$, $\hat{\mu}_3$ and $\hat{\mu}_4$ are given by $\frac{1}{N-1}\sum_{i=1}^{N}\left(x_i-\hat{\mu}\right)^2$, $\frac{1}{N}\sum_{i=1}^{N}\left(x_i-\hat{\mu}\right)^3$ and $\frac{1}{N}\sum_{i=1}^{N}\left(x_i-\hat{\mu}\right)^4$,

[52] See, for example, Mood et al. (1974), p. 228, and Zhang (2007) for the variances and covariances of the estimators $\hat{\mu}$ and $\hat{\sigma}^2$.

respectively.[53] From the Equations (3.8) and (3.9), we see that the variances of $\hat{\alpha}$ and $\hat{\beta}$ are linear combinations of $\sigma^2_{\hat{\mu}}$, $\sigma_{\hat{\mu},\hat{\sigma}^2}$ and $\sigma^2_{\hat{\sigma}^2}$:

$$Var\left(\hat{\alpha}\right) = \left(\frac{\partial f_1}{\partial \hat{\mu}}\right)^2 \cdot \sigma^2_{\hat{\mu}} + 2 \cdot \left(\frac{\partial f_1}{\partial \hat{\mu}}\right) \cdot \left(\frac{\partial f_1}{\partial \hat{\sigma}^2}\right) \cdot \sigma_{\hat{\mu},\hat{\sigma}^2} + \left(\frac{\partial f_1}{\partial \hat{\sigma}^2}\right)^2 \cdot \sigma^2_{\hat{\sigma}^2} \tag{3.10}$$

$$Var\left(\hat{\beta}\right) = \left(\frac{\partial f_2}{\partial \hat{\mu}}\right)^2 \cdot \sigma^2_{\hat{\mu}} + 2 \cdot \left(\frac{\partial f_2}{\partial \hat{\mu}}\right) \cdot \left(\frac{\partial f_2}{\partial \hat{\sigma}^2}\right) \cdot \sigma_{\hat{\mu},\hat{\sigma}^2} + \left(\frac{\partial f_2}{\partial \hat{\sigma}^2}\right)^2 \cdot \sigma^2_{\hat{\sigma}^2} \tag{3.11}$$

Calculations based on our sample (i.e. all private commercial banks and the central institutions of the savings and cooperative banks) yield $Var\left(\hat{\alpha}\right) = 0.0007$ and $Var\left(\hat{\beta}\right) = 0.0013$. As a next step, we use these values to calculate the test statistics T for the t-test with the null hypothesis that $\alpha \geq 1$ and $\beta \geq 1$. The results $T_\alpha \approx -27$ and $T_\beta \approx -18$ clearly show that the null hypothesis can be rejected. Thus, we can conclude that, contrary to the common assumption of a unimodal LGD distribution in the literature, our dataset of the LGD follows a u-shaped distribution.

[53] See Hahn and Shapiro (1967), p. 48.

3.7.4 Appendix 4: Characteristics of the LGD distribution for different subsamples of lender banks

Sample	N	Loss given default		Beta distribution	
		Mean	Standard dev.	α	β
All banks	667	0.38	0.39	0.20***	0.33***
Sample used for simulations	344	0.45	0.39	0.28***	0.35***
Large and internationally active banks	101	0.45	0.32	0.62***	0.76**
Savings banks	50	0.58	0.38	0.42***	0.30***
Cooperative banks	222	0.24	0.37	0.08***	0.24***

Table 3.2: Mean and standard deviation of the empirical frequency distribution of the LGD and estimated parameters of the respective beta distribution dependent on different samples of lender banks; N = number of observations; "Large and internationally active" includes large private commercial banks and central institutions of the savings and cooperative banks, "sample used for simulations" includes large and internationally active banks and all private commercial banks; **/ *** means significantly < 1 on the 5%/1%-level.

Size group	Loss given default		Beta distribution	
	Mean	Standard dev.	α	β
Smallest 20%	0.26	0.40	0.05***	0.14***
2nd quintile	0.35	0.41	0.12***	0.22***
3rd quintile	0.38	0.40	0.17***	0.28***
4th quintile	0.48	0.38	0.33***	0.36***
Largest 20%	0.42	0.33	0.53***	0.73**

Table 3.3: Mean and standard deviation of the empirical frequency distribution of the LGD and estimated parameters of the respective beta distribution dependent on the lender banks' size (= lender banks' balance sheet total). Whole sample of 667 observations; **/ *** means significantly < 1 on the 5%/1%-level.

Chapter 4

Contagion in the interbank market and its determinants*

* This chapter is based on joint work with Christoph Memmel. Opinions expressed are those of the authors and do not necessarily reflect the views of the Deutsche Bundesbank.

4.1 Introduction

The ongoing financial crisis shows the importance of stress testing exercises in testing the resilience of financial systems given the occurrence of shocks. These results are important for regulatory purposes as a more unstable system has to be regulated more strictly. Furthermore, stress testing is important for bailout decisions: If there is a danger of one financial institution failing, some careful analysis has to be made on the issue of what this would mean for the rest of the financial system. To create meaningful stress testing exercises, one has to think about various channels through which financial distress could spread from one financial institution to another.

In many studies, the interbank market has been identified as one of these channels. To be more precise, the failure of one bank can trigger the failure of its creditor banks due to their direct exposures. This is the case if the write-downs on the exposures to the failed bank cannot be absorbed by the creditor banks' capital buffers. If one of these creditor banks also fails, there could be another round of bank failures. This procedure can lead to several rounds of bank failures and is therefore often denoted as "domino effects". Thus, one obvious stress testing exercise is to investigate how many subsequent bank failures occur as a consequence of direct exposures in the event that one bank fails for some exogenous reason.

Of course, there are other transmission channels of contagion, e.g. due to liquidity problems that result out of asset fire sales, refinancing problems because of dried up interbank markets or information contagion. Here, however, we exclusively deal with contagion effects due to direct interbank exposures. We concentrate on this channel because we have detailed data about German banks' mutual credit exposures at our disposal. This enables us to simulate the failure of one of the large and internationally active German banks and to investigate the effects on other German banks that arise from direct interbank linkages.

This analysis can be carried out for all banks in a banking system for a certain point in time. Repeating this exercise for different points in time makes it possible to judge how the stability of the financial system (in terms of the danger of a domino effect) evolves over time. This could give regulators important information on how e.g. certain regulatory actions affect the stability of the financial system.

Our aim is to condense the results of the contagion exercises into one indicator for each point in time and then to investigate its determinants. Investigating the determinants of this indicator can help in two ways: First, determinants derived from theoretical considerations can be empirically validated and their importance can be assessed. Second, on the assumption that all interbank markets are similar, one can transfer the results obtained here to interbank markets for which there is no detailed data available.

Our analysis consists of three steps. First, we investigate the danger and the extent of contagion for each point in time from the first quarter of 2008 to the second quarter of 2011. Besides mutual exposures, a very important input variable for the simulations is the loss given default (LGD), i.e. the percentage of the interbank exposure that actually has to be written off in case of default. Thus, a LGD of 0% means that there are no write-downs (e.g. because of good collateral), a LGD of 100% means a complete write-down of the exposures in the event of failure. In most existing studies of contagion in the interbank market, an exogenously given and constant LGD is used. Thus, the outcome of these contagion studies crucially depends on the value of the LGD. We have, however, a unique dataset of actually realized LGD available. Thus, following Chapter 3 we use a different approach, i.e. we draw randomly from a beta distribution that is fitted to the empirical frequency distribution of our dataset. Hence, our simulations are based on a stochastic instead of a constant LGD. As a robustness check, we then compare these results with results under the assumption of a constant LGD that equals the mean of our dataset. It turns out that for rather stable systems, the assumption of a constant LGD systematically yields a lower number of bank failures than the assumption of a stochastic LGD (and vice versa). We use the distribution functions of bank failures for each point in time (which can be compared by using the concept of stochastic dominance) as well as the expectation of bank failures as an indicator to investigate how financial stability evolves over time. It turns out that the system becomes less vulnerable to direct domino effects over the time span considered.

Second, we empirically check whether the information of a whole loss distribution can be sufficiently summarized in a single indicator. Our metric is by how far an indicator can predict whether or not the loss distribution of a given quarter dominates the loss distribution of another quarter, i.e. the comparison of a whole distribution (by using the concept of stochastic dominance) is condensed into a single indicator. In this context, we use the expected number of failures as the indicator. The discriminatory power of this indicator proves to be sufficiently high.

Third, having chosen this indicator, we investigate its determinants. Following the literature on interbank contagion, we suggest four determinants: the capital in the system, the percentage of interbank assets relative to total assets, the loss given default and – as the really systemic measure – the degree of equality in the distribution of bilateral interbank exposures (measured by the entropy of the matrix). We find that the coefficients for the four determinants have the expected sign and are all significant. More important, they can explain more than 80% of the variation of the indicator.

This chapter is structured as follows: In Section 4.2, we provide a short overview of the literature in this field and point out our contribution. Then, in Section 4.3, we describe the data, explain the contagion algorithm and show our results under the assumption of a constant and a stochastic LGD. In Section 4.4, we investigate if the expected number of bank failures is a suitable indicator for the stability of the interbank market and, in Section 4.5, we explore the indicator's determinants. Section 4.6 concludes.

4.2 Literature

This chapter contributes to three strands of the literature. First, our method for simulating domino effects is similar to the empirical contagion analysis already applied to many countries (see e.g. Upper and Worms (2004) for Germany, Mistrulli (2011) for Italy or van Lelyveld and Liedorp (2006) for the Netherlands). Upper (2011) provides a comprehensive overview of this topic. Our approach, however, differs from this "standard approach" as we do not model the LGD as constant but as stochastic (see Chapter 3). To be able to evaluate how the vulnerability of the system to interbank contagion evolves over time, we use a time series of 14 quarters. A similar approach has been used by Degryse and Nguyen (2007). They investigate contagion in the Belgian interbank market over a ten years period ending in 2002. Another related paper in this context is Cont et al. (2010). They use a detailed dataset on exposures in the Brazilian interbank market and investigate by using a contagion exercise how the stability of the Brazilian banking system evolves from mid 2007 to the end of 2008. Though the basic simulation mechanism of these two papers is similar to ours, there are various differences to our approach (e.g. the design of the shock, the way the loss given default is modeled and the way the stability of the system is evaluated).

Second, we develop an indicator of the interbank market's resilience. Cont et al. (2010) summarize their simulation results by developing an indicator of the systemic importance of financial institutions for different points in time. Like these authors, we have detailed information on direct interbank exposures. Additionally, we use a dataset on actually realized loss given default (LGD) on the interbank market. Thus, contrary to market-based indicators that are, for example, developed by Acharya et al. (2010), Adrian and Brunnermeier (2008) and Huang et al. (2011), our stability indicator relies on detailed supervisory data.

Third, the aim of this chapter is to find out which simple indicators of a financial system help to explain our (more sophisticated) stress testing results. Simple indicators would be much more convenient for regulators to calculate and interpret compared to more sophisticated ones. In this context, Drehmann and Tarashev (2011) study the effects of simple indicators (such as bank size and interbank lending / borrowing) on the systemic importance of banks. They find that these simple indicators contribute well to the explanation of the more sophisticated systemic risk measures of banks. Degryse and Nguyen (2007) find that a move from a complete structure of claims towards a multiple money center structure within the Belgian banking sector (measured by the share of domestic interbank exposures of large banks to total domestic interbank exposures) as well as its increasing internationalization (measured by the share of total domestic interbank exposures to total interbank exposures) reduced the danger of contagion in the domestic interbank market. Additionally, the banks' capitalization is identified as a crucial determinant of interbank contagion. Cont et al. (2010) find that the size of interbank liabilities as well as some structural features of the interbank network (measured by newly created indicators) have an impact on financial stability.

The selection of the main determinants of our financial stability indicator is based on literature that focuses on theoretical simulations of interbank contagion. In this context, Nier et al. (2007) investigate, among other things, how the variation of banks' capital ratio, the size of banks' interbank exposures as well as banks' connectivity affects the stability of the system. Gai and Kapadia (2010) show, among other things, the impact of banks' connectivity and capital ratio on financial stability. The theoretical simulations in Chapter 2 examine the impact of banks' equity ratio, the amount of interbank lending, the loss given default and the degree of equality in how banks spread their claims on the stability of the network. In addition to theoretical simulations, the model of Allen and Gale (2000) also shows that it is important to consider the network structure of the banking system for the stability analysis. We test for four determinants of the vulnerability to interbank contagion: banks' capitalization, interbank lending, the loss given default and how equally banks spread their claims among counterparties. The empirical investigations in this chapter confirm, among others, the theoretical simulations of Chapter 2, which show that a higher capital ratio, less interbank lending within the system, a lower loss given default and a more equal distribution of interbank claims (given not too extreme parameter values) in a complete network yield a more stable system.

4.3 Simulation exercise

4.3.1 Data

Our simulation exercise starts with the exogenous failure of one bank within our sample.[54] Given the matrix of mutual interbank exposures and a loss given default (LGD) assigned to each of these exposures, we calculate the losses (i.e. the write-downs) of the creditor banks. If the tier-1 capital ratio of one of the creditor banks falls below 6%, which is the critical threshold according to the Basel III capital requirements as well as the EBA stress tests in 2010 (and implicitly in 2009), this bank will also become distressed and fail. If at least one bank fails after the failure of the trigger bank, there will be a next round in which the losses of the creditor banks are calculated. This contagious process comes to an end if there is a round with no new bank failures.

Thus, the required data for this analysis are, first, information on banks' capital as well as their risk weighted assets, second, data on banks' mutual exposures and third, data on the LGD. Our sample consists of 14 large and internationally active German banks as well as the aggregate sectors of the savings and cooperative banks, for which we have data from the first quarter of 2008 to the second quarter of 2011.[55]

[54] For a general discussion of the round-by-round algorithm applied in this chapter see Upper (2011). For a detailed description of the contagion algorithm see Chapter 3

[55] For simplicity, these 16 entities are just called banks in the following.

The banks' equity (tier-1 capital) and their risk weighted assets (RWA) are taken from the supervisory data storage system BAKIS.[56] Data on the bilateral exposures are taken from the German credit register, where all bilateral exposures are collected provided that they exceed (or are equal to) a threshold of EUR 1.5m.[57]

To get a first overview of the data, Figures 4.1 to 4.3 show how key characteristics of the banking system under consideration evolve over time. Figure 4.1 shows that the capitalization of the banking system increased substantially over time from an average of about 8.5% tier-1 capital relative to risk weighted assets in the first quarter of 2008 to more than 12% in the second quarter of 2011. This is due to an increase in banks' tier-1 capital on the one hand and a reduction in risk weighted assets on the other hand. Thus, banks raised their capital buffers during this time span to improve their resilience to potential shocks. Additionally, the weighted share of interbank assets (and thus the size of interbank linkages) relative to the sum of banks' balance sheet totals tends to decrease over time, as Figure 4.2 shows.[58] Following an average of more than 13.5% of interbank assets in the third quarter of 2009, the ratio decreased to 11.5% in the second quarter of 2011. The decreasing ratio of interbank assets to total assets shows a decreasing amount of interbank assets rather than an increase in banks' balance sheet totals. To see how the degree of equality in the distribution of interbank exposures evolves over time, we calculate the entropy of the matrix of interbank linkages. Entropy methods have been used in the literature on interbank contagion mostly to fill in missing data into the matrix of bilateral interbank exposures.[59] The underlying assumption of this method is that banks spread these exposures as equally as possible among

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56 For more information about the supervisory data in Germany see Memmel and Stein (2008).

57 For more information on the German credit register see Schmieder (2006).

58 One has to bear in mind that we only consider interbank assets within the system. As we consider large and internationally active banks, it is quite likely that some banks will have most of their interbank exposures abroad. However, looking at aggregate interbank lending (of domestic banks) to all banks (including foreign banks) over the time span considered yields the same result, i.e. a decrease in the share of interbank assets to total assets.

59 See e.g. Upper and Worms (2004)

their counterparties, which is equivalent to maximizing the entropy of the matrix of interbank exposures. In this chapter, we use this approach the other way round. As the whole matrix of bilateral exposures is available, we calculate the entropy of the matrix as a measure of how equally/unequally exposures are distributed. Figure 4.3 shows that the entropy of the matrix of mutual exposures has steadily decreased, which means that interbank exposures tend to be distributed more unequally over time. The network we consider is almost complete, i.e. there are no more than two off-diagonal zero entries in the 16×16 matrix of interbank exposures for each point in time.

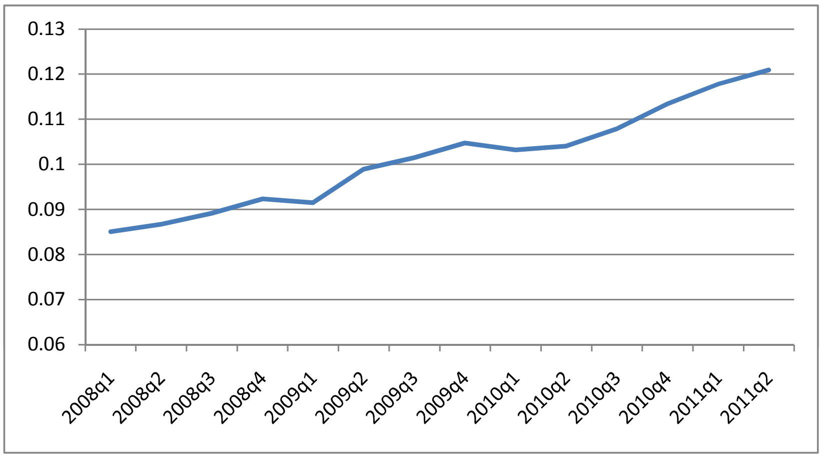

Figure 4.1: Development of the weighted tier-1 capital ratio of all 16 entities

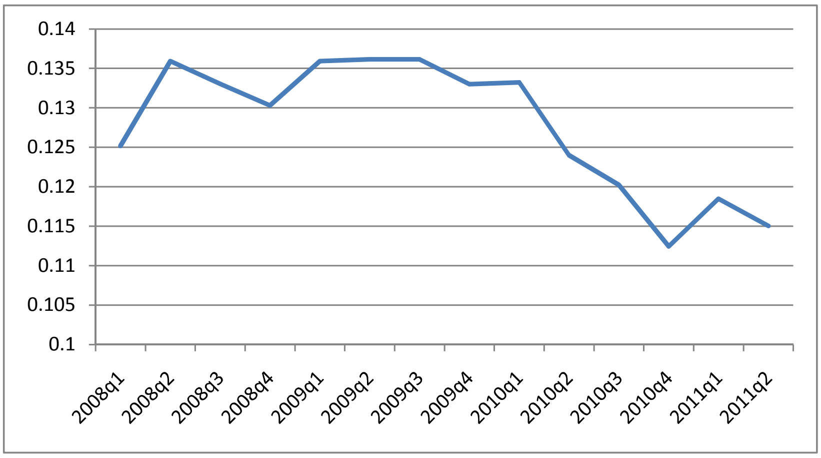

Figure 4.2: Development of the ratio of interbank assets within the system to total assets of all 16 entities

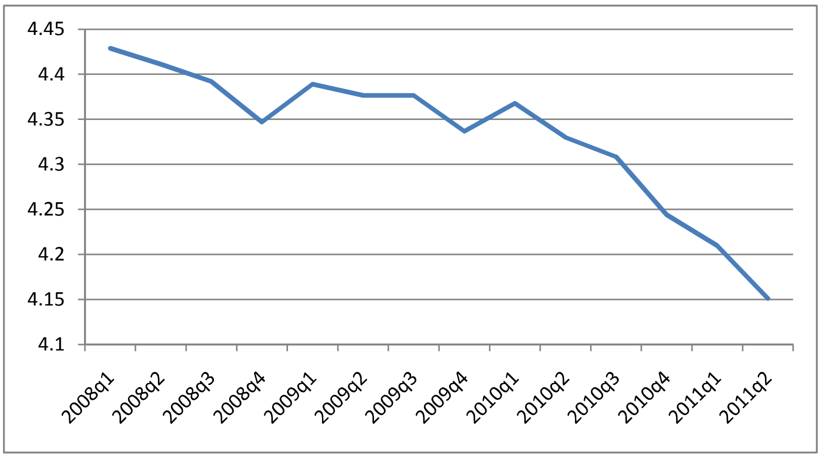

Figure 4.3: Development of the entropy of the matrix of interbank exposures

Furthermore, we need data on the loss given default for our contagion exercise. In this context, we use LGD-data from the quantitative supervisory reports for banks in Germany, where once a year each bank had to report the actual provisions on interbank loans as well as the total volume of the loan for which provisions have been made. As in Chapter 3, we use the subsample of all German private commercial banks plus the central institutions of the savings and cooperative banks. This gives us an empirical frequency distribution with a mean of 0.45 and a standard deviation of 0.39. Using this information and equations (3.6) and (3.7), we can approximate the empirical frequency distribution by a (markedly u-shaped) beta distribution with parameters $\alpha = 0.28$ and $\beta = 0.35$. Figure 4.4 shows the empirical frequency distribution of the actually observed LGD-data as well as the fitted beta distribution. To incorporate the LGD as an explanatory variable into our analysis of the main determinants of financial stability (see Section 4.5), we carry out the contagion exercises for different LGD distributions: We change the parameters α and β of the beta distribution (which can be easily calculated dependent on the mean and variance of the distribution) so as to have expected LGDs of 25%, 35%, 55% and

65%, respectively.[60] The different beta distributions we use for our simulations are shown in Figure 4.7 in Appendix 1.

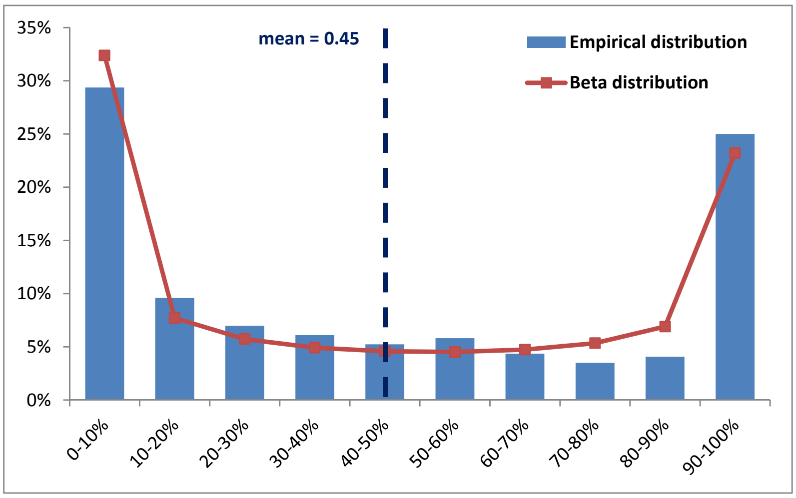

Figure 4.4: Empirical frequency distribution of the LGD data as well as the fitted beta distribution

4.3.2 Stochastic loss given default

The simulation exercise is carried out using the round-by-round algorithm described in Section 4.3.1. At first, all simulations are run by assuming that the LGD is stochastic and follows the distribution shown in Figure 4.4, i.e. for each exposure to a failing bank, we randomly draw a LGD from the estimated beta distribution with parameters $\alpha = 0.28$ and $\beta = 0.35$. To be more precise, we let one bank (e.g. bank i) at a particular time (e.g. time t) fail, assign a randomly drawn LGD to each interbank exposure and calculate how many banks fail in total due to domino effects. We repeat that exercise (i.e. calculating the consequences of the failure of bank i at time t) 100,000 times in total, each time randomly drawing a new set of

60 It is straightforward to adjust the variance of the beta distribution: In order to preserve the structure of its density function, the ratio of the variance of a binomially distributed random variable and a beta distributed random variable with the same mean should be constant, i.e. $\frac{\mu_1(1-\mu_1)}{\sigma_1^2} = \frac{\mu_2(1-\mu_2)}{\sigma_2^2}$ with $\mu_i(1-\mu_i)$ being the variance of a binomially distributed variable with mean μ_i and σ_i^2 being the variance of a beta distributed variable with mean μ_i. With μ being the expected value of the LGD distribution, we can thus calculate the parameters $\alpha = \mu \cdot 0.65$ and $\beta = (1-\mu) \cdot 0.65$.

LGDs from the beta distribution with estimated parameters. As a next step, we let another bank in the system fail (e.g. bank j) and calculate, again 100,000 times, the number of bank failures. By repeating this exercise for each of the 16 banks in the sample, we obtain a total of 1,600,000 results of bank failures for time t. We aggregate these results in order to receive an empirical frequency distribution and the respective cumulative distribution function of bank failures. As we have data for 14 points in time, we can generate 14 cumulative distribution functions that indicate the stability of the banking system in each respective quarter. Additionally, we calculate the overall mean of bank failures for each quarter. As we also investigate the impact of the loss given default on the expected number of bank failures, we repeat this contagion exercise for each point in time four times, each time drawing from another LGD distribution shown in Figure 4.7.

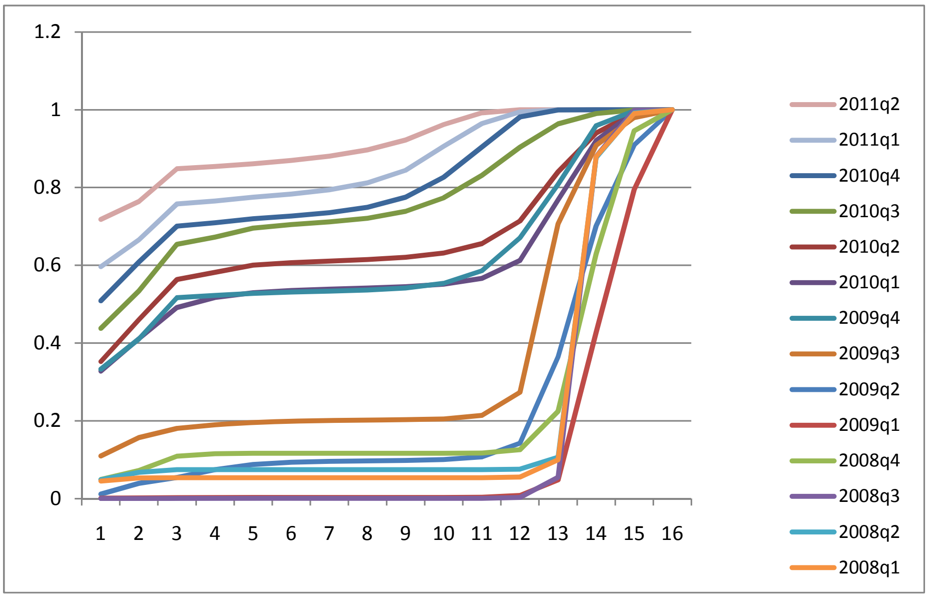

Figure 4.5: Distribution function of bank failures for the first quarter of 2008 to the second quarter of 2011 (stochastic LGD)

Figure 4.5 shows the simulation results for the first quarter of 2008 to the second quarter of 2011 under the assumption that the LGD follows the beta distribution shown in Figure 4.4 (i.e. the distribution that is derived from our LGD-dataset). The cumulative distribution function of the first quarter of 2008 (2008q1), for example, indicates that, under the assumption that each of the 16 entities fails with equal probability, the probability of observing 13 or fewer bank failures is about 10%. The probability of observing exactly 14 bank failures is around 80%. Thus, in the vast majority of cases in the first quarter of 2008, more than 13 banks fail (including the bank that fails first). This yields a rather unstable system. Looking at the cumulative distribution function of the second quarter of 2011 (2011q2), we find a different result. Here, the probability of observing just one bank failure (which is the bank that failed exogenously) is almost 72%. In only 15% of the cases more than 3 banks fail in total. Thus, in 2011q2, our results yield a considerably more stable system compared to previous quarters. One interesting result we obtain is that there is a substantial increase in system stability after the third quarter of 2009 (2009q3).

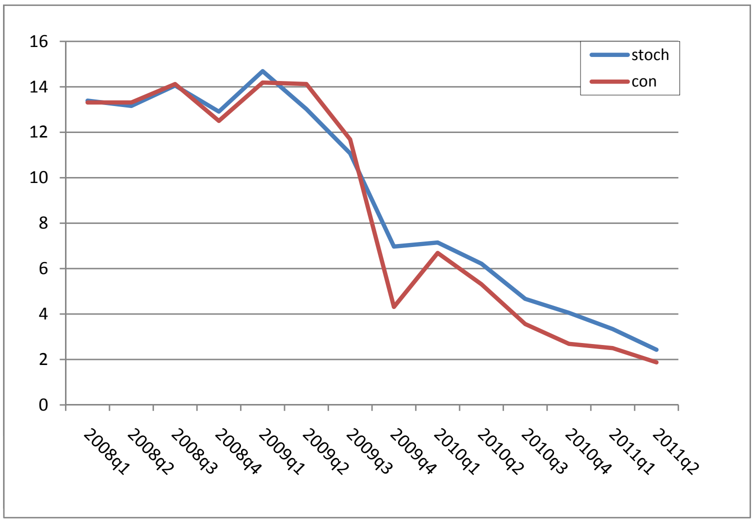

Figure 4.6: Overall expectation of the number of bank failures for the period from the first quarter of 2008 to the second quarter of 2011 (stochastic and constant LGD)

In addition to the cumulative distribution function, we characterize the stability of the system for each point in time by one single number: the expectation of the total number of bank defaults if one of the 16 entities fails. Thus, we calculate the average number of bank defaults of all 1,600,000 simulation runs (again under the assumption that the loss given default follows the beta distribution shown in Figure 4.4) for each point in time. Figure 4.6 shows the development of this expectation over time. The highest value is reached in the first quarter of 2009 with an expectation of more than 14 bank defaults. In the following quarters the expectation continuously decreases to fewer than 3 bank defaults in the second quarter of 2011. However, one has to bear in mind that our simulations only consider direct contagion via domino effects. Our simulations do not consider shocks on banks' assets other than direct interbank exposures. Thus, our simulations do not take into account, for example, risks due to sovereign default and therefore show a very stable system in the first and second quarter of 2011.

4.3.3 Stochastic versus constant loss given default

As a robustness check, we repeat the contagion exercise by assuming a constant LGD which equals the mean of our empirical distribution (= 45%). Thus, we assign the same LGD to each interbank exposure. This procedure yields, in contrast to the stochastic case, only one number of bank failures given that bank i fails at time t. Again, for each point in time we let each of the 16 entities fail and derive a number of subsequent bank failures. And similarly to the case of the stochastic LGD, we can summarize our results for each point in time by a cumulative distribution function as well as the overall average number of bank failures.[61]

Figure 4.6 shows that there is, for most points in time, not very much difference in the overall expectation of the number of bank failures between simulations with a stochastic LGD and a constant LGD. On the bank level, however, there can be a considerable variation in the results. For each of the 16 entities, we calculate the results of the 14 points in time, which yields 224 observations. In 16% of these 224

[61] The cumulative distribution functions (CDFs) under the assumption of a constant LGD look, in terms of the relative positions of the different CDFs, very similar to the CDFs in Figure 4.5.

observations, there is a deviation of more than 4 bank failures, in more than 40% of the cases there is a deviation of more than one bank failure. The direction of the deviation, however, varies. In 52% of the cases, a constant LGD yields a more unstable system, in 39% of the cases, a constant LGD yields a more stable system and in the remaining 9% of the cases there is no difference (this only happens when there are no further bank failures in both cases).

A straightforward question in this context is what drives the result regarding whether a constant LGD yields a more stable or unstable system. Visual inspection suggests that the total number of bank failures, given that bank i fails at time t, is a crucial factor. Thus, let $D_{i,t}$ be a dummy variable that takes on the value one if the failure of bank i at time t yields a less stable system under the assumption of a constant LGD and zero otherwise.[62] In addition, let $AV_{i,t}$ be the average of the expected number of bank failures (following the failure of bank i at time t) under the assumption of a stochastic LGD and the respective number of bank failures under the assumption of a constant LGD. We model the probability that the assumption of a constant LGD will lead to a less stable system with a logit model, using $AV_{i,t}$ as the explanatory variable.

$$Pr\left(D_{i,t}=1\right)=\frac{1}{1+\exp\left[-(\beta_0+\beta_1 AV_{i,t})\right]} \tag{4.1}$$

Table 4.1 shows that we obtain the highly significant result that a higher average number of bank failures increases the probability that a constant LGD will yield a higher number of bank failures compared to a stochastic LGD.

As a robustness check we use a standard OLS regression to investigate the relationship between the discrepancy of the results under a constant and a stochastic LGD and the average number of bank failures. Let $CS_{i,t}$ be the difference between the number of failures under the assumption of a constant LGD and the expected

62 In this estimation, the cases where the constant and the stochastic LGD yield the same results are included ($D_{i,t}$ takes on the value zero in these cases). As a robustness check we estimate the model without these data. However, there is hardly any change in the results.

number of failures under the assumption of a stochastic LGD (following the failure of bank i at time t). This yields the following equation:

$$CS_{i,t} = \beta_0 + \beta_1 AV_{i,t} + \varepsilon_{i,t} \tag{4.2}$$

The last column of Table 4.1 shows that a higher average number of bank failures indicates a higher value of $CS_{i,t}$. The interpretation depends on the sign of $CS_{i,t}$. For a low average number of bank failures, the difference is negative, i.e. a stochastic LGD yields a more unstable system and an increase in $AV_{i,t}$ moves $CS_{i,t}$ towards zero. For a high average number of bank failures, the difference is positive, i.e. a constant LGD yields a more unstable system and an increase in $AV_{i,t}$ also increases the difference between the results of a constant and a stochastic LGD. Again, all results are highly significant.

Variable	Logit D	CS
$AV_{i,t}$	0.28***	0.23***
	(0.03)	(0.02)
constant	-2.38***	-2.50***
	(0.34)	(0.33)
Nobs	224	224
(Pseudo) R^2	0.30	0.20
AUR	0.76	-

Table 4.1: Logit regression with $D_{i,t}$ being a dummy variable indicating that the failure of bank i at time t yields more bank failures under the assumption of a constant LGD compared to a stochastic LGD, and $AV_{i,t}$ corresponding to the average number of bank failures with a constant and a stochastic LGD. OLS regression with $CS_{i,t}$ being the difference of the (expected) number of failures under the assumption of a constant and a stochastic LGD. Robust standard errors. AUR gives the area under the ROC-curve; *** denotes significance at 1%-level.

Intuitively, if the system is rather unstable (e.g. due to a low tier-1 capital ratio of banks), a constant LGD leads to a higher average number of bank defaults than a stochastic LGD as it is not possible in the constant case to randomly draw a very low LGD that avoids contagion from one bank to another. In contrast, if the system is rather stable, a constant LGD leads to a lower average number of bank defaults compared to a stochastic LGD as it is not possible in the constant case to randomly draw a very high LGD.

Empirically, we find that the LGD is not rather constant, but markedly u-shaped (see Figure 4.4), i.e. the LGD is often low or high, but little probability mass is centered around the expectation of the distribution in the middle. Thus, the simplifying assumption of a constant LGD cannot be justified by empirical data, which has important implications for our contagion exercise. Under the assumption of a constant LGD, one tends to overestimate the extent of contagion in unstable systems and to underestimate it in rather stable systems. In Chapter 3, we investigate the extent of contagion for one point in time (the fourth quarter of 2010). In this context, we also compare the assumptions of a constant LGD and a stochastic LGD and find that the assumption of a constant LGD underestimates the extent of contagion. This is in line with the results of this section as Figure 4.5 shows a rather stable system in 2010q4.

4.4 Development of an indicator

4.4.1 Stochastic dominance

As a next step, to evaluate our results from Section 4.3.2 in more detail, we have to find a measure that allows us to compare the different distributions (and not only numbers) of bank failures over time. One concept that makes this possible without many assumptions is stochastic dominance.[63] This measure can be e.g. used in decision theory if a preference relation between two assets with stochastic returns has to be found. In our case, we can also form preference relations by assuming that fewer bank failures are preferred to more bank failures. In this context, assume that there are two cumulative distribution functions $F(\cdot)$ and $G(\cdot)$. The distribution $F(\cdot)$ is said to have first-order stochastic dominance over the function $G(\cdot)$ if

$$F(x) \geq G(x) \tag{4.3}$$

for all x and strict inequality for at least one x.[64] If there is first-order stochastic dominance, every individual preferring less bank failures to more bank failures and having the choice between two distributions, prefers the distribution that dominates the other one according to the definition given by Equation (4.3).

There are two main drawbacks of the concept of first-order stochastic dominance: First, there is no statement possible by how far one distribution is preferred to another (dominated) distribution and, second, the comparison is not complete in a mathematical sense, i.e. there is not always a dominance relationship between two distributions.

The results of the analysis of first-order stochastic dominance are shown in Appendix 2. The matrix that describes the results confirms our findings in Section 4.3.2. The most favorable distribution function of bank defaults is given in the second quarter of 2011. This distribution dominates the distribution function of all other points in time. The distribution function of the first quarter of 2009 does not dominate any

[63] See Bawa (1975) and Schmid and Trede (2006), chapter 8, for more information about the concept of stochastic dominance.

[64] Note, however, that the definition of stochastic dominance in this paper is not exactly the same as the standard definition which is based on the assumption that the respective utility function is increasing (and not decreasing as in this paper). However, we can redefine the utility function in a way that it is dependent on the number of solvent banks $y := 16 - x$ (with x being the number of failed banks). Thus, it follows that $\frac{\partial u}{\partial y} > 0$. Using this assumption, the condition for first-order stochastic dominance is, that $F(\cdot)$ dominates $G(\cdot)$ if $F(y) \leq G(y)$ for all y and strict inequality for at least one y. Redefining each CDF by making it dependent on y and using this standard condition for stochastic dominance yields the same results.

other quarter but is dominated by 12 other quarters. This result indicates that in 2009q1 the distribution function of bank defaults was quite unfavorable. However, there is a dominance relationship between different points in time in only 70 of the 91 cases.[65]

Using a higher order stochastic dominance, one can mitigate the problem of completeness – at the expense of imposing additional assumptions.[66]

4.4.2 Whole distribution versus expectation

It is now possible (for most points in time) to compare the distribution functions of bank failures. To analyze the main determinants for the stability of the financial system it would, however, be much easier to use single numbers as an indicator of financial stability. In the end, the question of whether a single number is suitable to condense the information of a whole distribution needs to be answered with empirical data. Our aim is to show that a statement based on the comparison of two distribution functions is more or less equivalent to the comparison of the expectations. To do so, we proceed as follows: Having 14 different distribution functions (one for each quarter), we can make 91 (=14*13/2) bilateral comparisons, i.e. we exclude comparisons with itself and double counts. Let F and G be the cumulative distribution functions of time t_1 and t_2, respectively. Whenever there is a (first-order) dominance relationship between F and G (irrespective of the direction),

65 We also calculated the relationships of first-order stochastic dominance for the distributions of bank failures under the assumption of a constant LGD. In this case, we can even compare 87 of the 91 cases using the concept of first-order stochastic dominance.

66 We additionally investigated the second-order dominance relationships. To be able to do this, we have to redefine each CDF to make it dependent on the number of solvent banks y. Under the assumption that individuals prefer more solvent banks to less solvent banks and are risk averse ($\frac{\partial u}{\partial y} > 0$ and $\frac{\partial^2 u}{\partial y^2} < 0$), $F(\cdot)$ is preferred to $G(\cdot)$ if $\int\limits_{-\infty}^{y} F(t)\,dt \leq \int\limits_{-\infty}^{y} G(t)\,dt$ for all y and strict inequality for at least one y. However, the number of dominance relationships ($= 72$) only slightly increases compared to first-order stochastic dominance.

the indicator variable D_{t_1,t_2} takes on the value one. The variable $\Delta F_{t_1,t_2} := \text{abs}(F_{t_1} - F_{t_2})$ is the corresponding absolute difference in the expected number of bank failures. We model the probability of an existing dominance relationship with a logit model and explain this probability with the absolute difference $\Delta F_{t_1,t_2}$.

$$Pr\left(D_{t_1,t_2} = 1\right) = \frac{1}{1 + \exp\left[-(\beta_0 + \beta_1 \Delta F_{t_1,t_2})\right]} \tag{4.4}$$

We expect a positive coefficient for β_1: the larger the absolute difference in the expected number of bank failures, the more likely the existence of a dominance relationship. As a robustness check and to account for possible non-linearities, we also include the squared term ΔF^2 in the model. The results are displayed in Table 4.2.

Variable	Logit D	Logit D
ΔF	0.45***	0.99***
	(0.13)	(0.35)
ΔF^2		-0.06**
		(0.03)
constant	-0.50	-1.14**
	(0.40)	(0.55)
Nobs	91	91
Pseudo R^2	0.25	0.27
AUR	0.85	0.82

Table 4.2: Logit regression where D_{t_1,t_2} is a dummy variable indicating (first-order) stochastic dominance between the distributions in t_1 and t_2 or vice versa, and ΔF is the corresponding absolute difference in the expected number of bank failures. Robust standard errors. AUR gives the area under the ROC-curve; ** and *** denote significance at the 5% and 1%-level, respectively.

As expected, the coefficient of the variable ΔF is positive and highly significant. Additionally, to evaluate the discriminatory power of the model, we calculate the area under the ROC-curve (AUR). The ROC (receiver operating characteristic)-curve plots the type 1 error rate of the model against one minus the type 2 error rate for different thresholds. The better the predictive power of the model, the lower the type 2 error rate for a given type 1 error rate and the higher the area under the ROC-curve. In the case of our model, the AUR is high with 0.85. Therefore, we conclude that, in this case, the comparison of two distribution functions on the one hand and the comparison of the two expectations on the other hand yields rather similar results, i.e. the expected number of failures is a suitable indicator for measuring the vulnerability of a banking system to contagion. There is no use in including a quadratic term as done in the robustness check: The AUR is then even lower than in the case without this quadratic term.

4.5 Determinants

As shown in the previous section, the information included in the whole distribution can be summarized in one number without much loss of information. This single number is the expectation of bank failures F and will be our indicator for the vulnerability of the (German) interbank market. Drehmann and Tarashev (2011) show that highly sophisticated measures of systemic risk contribution can be well approximated by more objective figures like size and interbank lending. Following this idea, we try to replicate our indicator using relatively easily available measures. Following the results of the theoretical simulations in Chapter 2, we look at four different determinants: the capital ratio CR, the extent of interbank lending IBL, the average loss given default LGD and the structure of the interbank market ENT, measured by the entropy of the matrix of bilateral interbank exposures. As outlined in Section 4.3.1, the entropy is a statistical tool that measures the degree of equality;

the higher the entropy, the higher the degree of equality. Theoretical considerations as laid down in Chapter 2 argue that a rather complete system ceteris paribus becomes (for not too extreme parameter values) more stable the more equalized its linkages are, i.e. the higher the entropy is.

We run the following linear regression:

$$F_{t,i,j} = \beta_0 + \beta_1 CR_{t,i,j} + \beta_2 IBL_{t,i,j} + \beta_3 LGD_{t,i,j} + \beta_4 ENT_{t,i,j} + \varepsilon_{t,i,j} \tag{4.5}$$

where $CR_{t,i,j}$ is the tier-1 capital ratio of the banking system at time t excluding bank i, which is originally and exogenously in distress. Accordingly, $IBL_{t,i,j}$ is the cumulated interbank lending of the banking system over the aggregate total assets of the system at time t excluding bank i. $ENT_{t,i,j}$ is the entropy of the matrix of bilateral interbank exposures at time t, excluding bank i. Figures 4.1 to 4.3 show the evolution over time for the three determinants (in contrast to our regressors, these figures show the determinants for the whole financial system). On the one hand, the capital ratio in the system has increased significantly and interbank lending tends to decrease; these two developments are believed to make the system more stable. On the other hand, the exposures have become less equally distributed, which should lead to a less stable system.

By creating variables in the way described above, we not only have variation in the time dimension (as shown in Figures 4.1 to 4.3) but also in the cross-section (i.e. between banks). The descriptive statistics shown in Table 4.3 indicate that the endogenous variable $F_{t,i,j}$ and two exogenous variables are characterized by a substantial part of cross-sectional variation. This enables us to apply a panel analysis. The index $j = 1, ..., 5$ denotes the different average LGDs, ranging from 25% to 65% in steps of 10 percentage points. As the variable LGD is set exogenously, we do not report its mean and standard deviation in Table 4.3.

Variable	Mean	Stand. dev.	Between variation (Percentage)
F	9.08	5.00	15%
CR (%)	10.12	1.11	1%
IBL (%)	12.83	1.51	74%
LGD	-	-	
ENT	4.21	0.10	39%

Table 4.3: Descriptive Statistics (for the case of an average LGD of 45%). The column "Between variation" gives the between variance of the given variable as a share of the total variance.

Table 4.4 shows the results of the linear regression (4.5) with bank dummies to account for possible bank-specific effects. The results are in line with expectations. We find that an increase in the capital ratio of the whole system by one percentage point reduces the expected number of failing banks by more than four. This result is highly significant. The exposure to the interbank market is also of great importance. When interbank lending (relative to total assets) increases by one percentage point, the number of expected failures will go up by 1.8 banks. When the LGD increases by one percentage point, then the number of expected bank failures will go up by 0.14. The capital ratio and interbank lending – although calculated for the whole system – are, after all, the (weighted) average of the single ratios, i.e. these ratios are bank-specific measures by nature. The LGD is even specific to each borrower-lender-relationship. By contrast, the entropy of the system is a truly systemic measure. As mentioned above, it shows how equally interbank lending is distributed in the banking system. A higher entropy means that banks spread their interbank assets / liabilities more equally among other banks in the system. The simulation results in Chapter 2 indicate that for a rather complete network a more equal distribution of interbank lending, i.e. a higher entropy of the matrix of interbank lending, leads (for not too extreme parameter values like an extremely low capitalization of banks) to a more resilient system.

Variable	Coefficient	Stand. Dev.
CR	-4.02***	0.145
IBL	1.81***	0.127
LGD	0.14***	0.005
ENT	-24.13***	2.289
constant	130.07***	10.618
Adj R^2	0.813	
Nobs	1120	

Table 4.4: Results of the regression $F_{t,i,j} = \beta_0 + \beta_1 CR_{t,i,j} + \beta_2 IBL_{t,i,j} + \beta_3 LGD_{t,i,j} + \beta_4 ENT_{t,i,j} + \varepsilon_{t,i,j}$, where $F_{t,i,j}$ is the expected number of failing banks in quarter t given that bank i fails exogenously and the LGD is drawn from beta distribution j. Pooled OLS regression with dummies for each bank and robust standard errors; *** denotes significance at the 1%-level

Indeed, we find that an increase in entropy (= a more equal distribution of interbank lending) leads to a reduction in the expected number of bank failures.[67] As the network we consider is almost complete (for each point in time there are no more than two off-diagonal zero-entries in the 16×16 matrix of interbank liabilities) and parameter values are not too extreme (banks' capitalization is not extremely low and the LGD is not extremely high), our empirical results confirm the theoretical simulation results in Chapter 2. These results are also in line with the theoretical findings of Allen and Gale (2000), who show that a complete network (with maximum entropy of the matrix of mutual exposures) is more stable than an incomplete but perfectly interconnected network (with a lower entropy of the matrix of mutual

[67] This finding is in contrast to Figure 4.3, which shows a more unequal distribution of claims over time although the stability of the system increased. However, it is quite likely that the negative effect of a decreasing entropy is outweighed by the effect of banks' capitalization and interbank lending.

exposures). The bank dummies also have a high explanatory power. All of them are highly significant as well. In total, the four determinants as well as the bank dummies can explain more than 80% of the variation in the indicator.

The relative importance of three of the four determinants can be assessed by assuming a change in each determinant by one standard deviation. For the LGD, we do not have a meaningful standard deviation because its variation is exogenously set by us. Using the standard deviations reported in Table 4.3 and the estimated coefficients in Table 4.4, we see a decrease of 4.5 in the number of expected failures when the capital ratio in the system increases by one standard deviation (here: 1.11 percentage points). The corresponding numbers for interbank lending and entropy are 2.7 and 2.4, respectively. Hence, the capital ratio is the most important determinant. However, some of the determinants show a rather high correlation among themselves, which has to be kept in mind when trying to quantify their exact contribution to system stability.

4.6 Conclusion

This chapter investigates interbank contagion due to direct exposures for different points in time. We have data on mutual interbank exposures from the first quarter of 2008 to the second quarter of 2011. At first, following Chapter 3, we run contagion simulations by drawing the loss given default from a beta distribution that is fitted to a distribution of actually realized data of loss given default on the interbank market. As a result, we obtain for each point in time a whole distribution as well as the expected number of bank failures. We find that the system has become less vulnerable to domino effects over time. As a robustness check, we compare these results with the results obtained assuming a constant LGD and find that for a rather stable system, the assumption of a constant LGD tends to underestimate the extent of contagion, whereas for a rather unstable system the assumption of a constant

LGD tends to overestimate the extent of contagion.

As a next step, we analyze whether the information of the whole distribution of bank failures can be summarized in a single indicator like the expected number of bank failures. Using the concept of stochastic dominance to compare the distributions for different points in time, we find that the discriminatory power of the single indicator is sufficiently high.

Finally, we investigate the main determinants of this indicator. We find that the banks' capital ratio, the share of interbank assets in the system in relation to total assets, the loss given default and the degree of equality in the distribution of interbank exposures (measured by entropy) are important determinants for financial stability. We are thus able to confirm the importance of these determinants derived from theoretical considerations.

4.7 Appendix

4.7.1 Appendix 1: Beta distribution

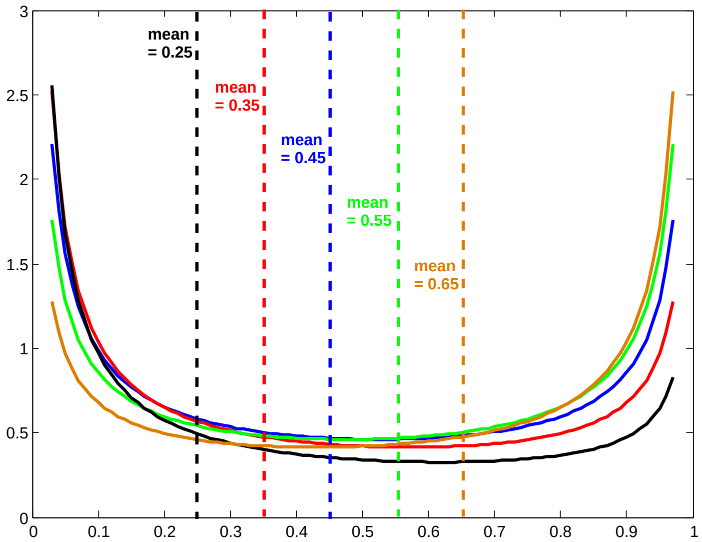

Figure 4.7: Different beta distributions of the loss given default used for the contagion simulations

4.7.2 Appendix 2: Dominance relationships

Analysis of dominance relationships from the first quarter of 2008 ($08q1$) to the second quarter of 2011 ($11q2$). If the element in row x and column y of the matrix is equal to one, the distribution function of time y (first-order) stochastically dominates the distribution function of time x.

$$
\begin{array}{rl}
 & \begin{array}{cccccccccccccc} 08q1 & 08q2 & 08q3 & 08q4 & 09q1 & 09q2 & 09q3 & 09q4 & 10q1 & 10q2 & 10q3 & 10q4 & 11q1 & 11q2 \end{array} \\
\begin{array}{r} 08q1 \\ 08q2 \\ 08q3 \\ 08q4 \\ 09q1 \\ 09q2 \\ 09q3 \\ 09q4 \\ 10q1 \\ 10q2 \\ 10q3 \\ 10q4 \\ 11q1 \\ 11q2 \end{array} &
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\end{array}
$$

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