# Early Universe Cosmology

## In Supersymmetric Extensions of the Standard Model

Jochen Peter Baumann

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Dissertation

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[1] S. Antusch, M. Bastero-Gil, J. P. Baumann, K. Dutta, S. F. King, and P. M. Kostka, "Gauge Non-Singlet Inflation in SUSY GUTs," *JHEP* **08** (2010) 100 , `arXiv:1003.3233 [hep-ph]`

[2] S. Antusch, J. P. Baumann, V. F. Domcke, and P. M. Kostka, "Sneutrino Hybrid Inflation and Nonthermal Leptogenesis," *JCAP* **1010** (2010) 006 , `arXiv:1007.0708 [hep-ph]`

[3] J. P. Baumann, "Gauge Non-Singlet (GNS) Inflation in SUSY GUTs," *J. Phys. Conf. Ser.* **259** (2010) 012046


Betreut von Prof. Dr. Stefan Antusch
am Max-Planck-Institut für Physik, München.

Erstgutachter: PD Dr. Georg G. Raffelt
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# Zusammenfassung

In dieser Dissertation untersuchen wir mögliche Verbindungen zwischen kosmologischer Inflation und Leptogenese auf der einen und teilchenphysikalischen Modellen auf der anderen Seite. Wir arbeiten mit supersymmetrischen Erweiterungen des Standard Modells. Eine zentrale Rolle nimmt hierbei das rechtshändige Sneutrino ein, der Superpartner des am Typ I Seesaw Mechanismus beteiligten rechtshändigen Neutrinos.

Wir untersuchen ein Modell für Inflation und nicht-thermale Leptogenese, welches eine einfache Erweiterung des Minimalen Supersymmetrischen Standard Modells (MSSM) mit erhaltener R-Parität darstellt und dem wir drei rechtshändige Neutrino-Superfelder hinzufügen. Die Inflaton-Richtung ist durch die imaginären Komponenten der jeweiligen skalaren Felder gegeben, die durch eine Shift-Symmetrie im Kählerpotential vor dem $\eta$-Problem in Supergravitation (SUGRA) geschützt werden. Wir diskutieren das Modell zunächst in einem global supersymmetrischen (SUSY) Kontext und danach im Kontext von Supergravitations-Theorien und berechnen die inflationären Vorhersagen des Modells.

Des Weiteren untersuchen wir Reheating und nicht-thermale Leptogenese in diesem Modell. Eine numerische Simulation zeigt, dass kurz nach dem Wasserfall-Phasenübergang, der Inflation beendet, das Universum von rechtshändigen Sneutrinos dominiert wird, deren Nicht-Gleichgewichts-Zerfälle die erwünschte Materie-Antimaterie-Asymmetrie erzeugen können. Unter Benutzung einer vereinfachten, zeitlich gemittelten Beschreibung leiten wir analytische Ausdrücke für die Vorhersagen des Modells her. Durch eine Kombination der Resultate bezüglich Inflation und Leptogenese gelingt es uns den erlaubten Parameterraum des Modells von zwei Seiten einzuschränken, was Implikationen für die Niederenergie-Neutrinophysik mit sich bringt.

Eine weitere Richtung, in der sich unsere Untersuchungen bewegen, ist die Verallgemeinerung des obigen Inflationsmodells zu dem Falle, dass das Inflaton unter einer Eichsymmetrie geladen ist. Dies ist durch die Tatsache motiviert, dass das rechtshändige (S)Neutrino ein unverzichtbarer Bestandteil von links-rechts symmetrischen, supersymmetrischen Grossen Vereinheitlichten Theorien (SUSY GUTs) wie z.B. SUSY Pati-Salam Modellen oder SUSY $SO(10)$ Modellen ist. In solchen Modellen muss das rechtshändige (S)Neutrino also nicht von Hand eingefügt werden wie im Falle des MSSM.

Wir diskutieren die neu entstehenden Probleme für die Umsetzung von Slow-Roll Inflation im Zusammenhang mit einem geladenen Inflaton und illustrieren die grundlegenden Ideen unseres Ansatzes am Beispiel eines Inflatons mit einer $U(1)$-Ladung. Danach betrachten wir ein realistisches Modell für Nicht-Singlet Inflation im Rahmen von SUSY Pati-Salam Vereinheitlichung. Für den speziellen Fall von Sneutrino Inflation in SUSY Pati-Salam Vereinheitlichung führen wir eine detailierte Untersuchung der Inflationsdynamik durch und berechnen potentiell gefährliche Ein- und Zwei-Loop-Beiträge zum Inflaton-Potential. Wir zeigen, dass diese keine Gefahr für unser Modell darstellen. Zum Abschluss verallgemeinern wir dieses Modell zu SUSY $SO(10)$ Modellen und diskutieren eine mögliche Einbettung in SUGRA-Theorien.

# Abstract

In this thesis we investigate possible connections between cosmological inflation and leptogenesis on the one side and particle physics on the other side. We work in supersymmetric extensions of the Standard Model. A key role is played by the right-handed sneutrino, the superpartner of the right-handed neutrino involved in the type I seesaw mechanism.

We study a combined model of inflation and non-thermal leptogenesis that is a simple extension of the Minimal Supersymmetric Standard Model (MSSM) with conserved R-parity, where we add three right-handed neutrino superfields. The inflaton direction is given by the imaginary components of the corresponding scalar component fields, which are protected from the supergravity (SUGRA) $\eta$-problem by a shift symmetry in the Kähler potential. We discuss the model first in a globally supersymmetric (SUSY) and then in a supergravity context and compute the inflationary predictions of the model.

We also study reheating and non-thermal leptogenesis in this model. A numerical simulation shows that shortly after the waterfall phase transition that ends inflation, the universe is dominated by right-handed sneutrinos and their out-of-equilibrium decay can produce the desired matter-antimatter asymmetry. Using a simplified time-averaged description, we derive analytical expressions for the model predictions. Combining the results from inflation and leptogenesis allows us to constrain the allowed parameter space from two different directions, with implications for low energy neutrino physics.

As a second thread of investigation, we discuss a generalisation of the inflationary model discussed above to include gauge non-singlet fields as inflatons. This is motivated by the fact that in left-right symmetric, supersymmetric Grand Unified Theories (SUSY GUTs), like SUSY Pati-Salam unification or SUSY $SO(10)$ GUTs, the right-handed (s)neutrino is an indispensable ingredient and does not have to be put in by hand as in the MSSM.

We discuss the new problems that arise in connection with realising slow-roll inflation with a charged inflaton and illustrate our basic ideas with the example of an inflaton charged under $U(1)$. We then move on to discuss a realistic model of gauge non-singlet inflation in SUSY Pati-Salam unification. For the special case of sneutrino inflation in SUSY Pati-Salam unification, we discuss in detail the inflationary dynamics as well as potentially dangerous one- and two-loop contributions to the inflaton potential and we show that these contributions do not spoil slow-roll inflation. We then generalise this model to SUSY $SO(10)$ GUTs and conclude with a possible embedding into a SUGRA framework.

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# Introduction

One of the truly remarkable achievements of modern day science has to be the fact that we can talk with considerable confidence about what happened in the *early universe* some 13.7 billion years ago. Detailed knowledge of such distant events could only be obtained by combining results from many different domains of experimental and theoretical research, spanning from the smallest sub-atomic scales of high energy particle physics to the largest scales of galaxy clusters and beyond. The challenge, in particular for theoretical cosmology, is to weave together all these different strands of knowledge to form a single coherent picture. In this thesis we try to contribute to this endeavour.

As a matter of fact, there has already been enormous progress in this undertaking over the last decades, leading to what is often called the *Standard Model of Cosmology.* In its most prominent manifestation, the *Lambda Cold Dark Matter model* ($\Lambda$CDM), it can successfully describe the evolution of our universe in accordance with recent experimental data [4, 5, 6]. The $\Lambda$CDM model has only 6 free parameters [1] and it can, for example, account for the production of the light elements during Big Bang Nucleosynthesis (BBN), the emission of the *Cosmic Microwave Background Radiation* (CMBR) during recombination, the formation of the large scale structure due to the attractive nature of the gravitational interaction and the current accelerated expansion of our universe. Despite this success, however, the $\Lambda$CDM model still does not provide an answer to some very fundamental questions. In particular, it neither explains the particle physics mechanism that lies behind *cosmological inflation* nor the generation of the observed *matter-antimatter asymmetry.*

Cosmological inflation [7], an early epoch of accelerated expansion of the universe, is a key ingredient of the $\Lambda$CDM model as it provides a solution to the otherwise unexplained horizon, flatness, and monopole problems. It also predicts the generation of tiny energy density fluctuations [8] that eventually act as seeds for the formation of the large scale structure in our universe. These density fluctuations also manifest themselves as temperature anisotropies in the CMB radiation as observed by the COBE [9] and WMAP [10] satellite, as well as the currently operating Planck satellite [11]. The CMB temperature anisotropies are therefore often said to be the "fingerprint of inflation".

It is usually assumed that inflation is driven by a scalar field called the *inflaton*, whose large potential energy dominates over its kinetic energy, leading to an exponential expansion of the universe. In a semi-classical approximation we can picture the inflaton to be slowly rolling down its potential until the potential becomes too steep and the inflaton starts to pick up speed. Once the kinetic energy of the inflaton becomes comparable to its potential energy, the exponential expansion stops and the universe enters a stage of decelerated expansion. This setup is called *slow-roll inflation* [12, 13] and it is the scenario most commonly considered.

The big question now is: Who is the inflaton? In many models, the inflaton is simply added to the theory "by hand" without any connection to particle physics models such as the Standard Model. This is clearly an unsatisfactory situation. One of the main goals of this thesis is to establish such a possible connection between cosmological inflation and particle physics.

[1] The 6 parameters are: the physical baryon density, the physical dark matter density, the dark energy density, the scalar spectral index, the curvature fluctuation amplitude and the reionisation optical depth.

Our current knowledge about the world on subatomic scales is summarised in the *Standard Model of particle physics* (SM) [14], describing the electroweak and strong interactions of the known elementary particles based on the principle of local gauge invariance. It has been tested to very high precision up to energies of the order of a few hundred GeV [15]. Like its cousin, the Standard Model of Cosmology, however, it leaves a number of fundamental questions unanswered. Some of these questions are currently under scrutiny at the *Large Hadron Collider* (LHC), for example the question of the origin of the mass of the elementary particles and the related problem of the stability of the electroweak scale, also called the hierarchy problem [16]. A solution to the first question is implemented in the SM through the Higgs mechanism [17, 18] and the search for the Higgs boson is one of the main missions of the LHC. A solution to the hierarchy problem, on the other side, can be provided by a new symmetry called *supersymmetry* (SUSY) [19, 20], and the search for supersymmetric partner particles to the well known particles of the Standard Model is another major aspect on the agenda of the LHC. Supersymmetry is especially interesting from a cosmologist's point of view since SUSY introduces many additional scalar particles, which make them possible inflaton candidates.

Another problem left unexplained in the Standard Model is the origin and size of the neutrino masses. Even if the Higgs boson is found, the SM does not provide a mechanism to explain these neutrino masses since, by definition, it does not include any right-handed neutrinos. Adding two or more heavy right-handed neutrinos, on the other hand, provides a very elegant way of explaining the small SM neutrino masses through the *type I seesaw mechanism* [21]. The superpartner of such a heavy right-handed neutrino, the *right-handed sneutrino* turns out to be an excellent inflaton candidate.

While the introduction of the right-handed (s)neutrinos is therefore very well motivated, so far they are still put in "by hand". The situation changes, however, if we consider the possibility of (partial) unification of the gauge interactions. In particular, in left-right symmetric *supersymmetric Grand Unified Theories* (SUSY GUTs), like SUSY Pati-Salam unification [22] or SUSY $SO(10)$ GUTs [23], the right-handed (s)neutrinos are members of some *matter representation* that also contains Standard Model particles, and they are therefore an indispensable ingredient of the theory. In this sense, we get the right-handed (s)neutrinos "for free" in such left-right symmetric models.

The price we have to pay is that the right-handed sneutrinos now carry a gauge charge. This seems to be problematic for inflationary model building since conventional wisdom dictates that the inflaton must be a gauge singlet. In particular in supersymmetric models of inflation, scalar component fields that carry a gauge charge have quartic terms in their potential (due to D-terms) that induce a slope of the inflaton potential that is too large. Also, as is commonly believed, radiative corrections at the one- and two-loop level [24] spoil the flatness of the potential for a non-singlet inflaton field. A major result of this thesis is that these problems can indeed be overcome and that it is possible to construct viable models of inflation using gauge non-singlet (GNS) fields as inflatons.

If inflation is to last long enough to solve the horizon, flatness, and monopole problems, any particles present before inflation are diluted to practically zero density after inflation. This brings us back to the problem of the generation of the matter-antimatter asymmetry – or *baryogenesis* – mentioned in the beginning. Here again, the right-handed (s)neutrino can act as a link between early universe cosmology and particle physics: the out-of-equilibrium decays of the right-handed (s)neutrinos satisfy all the necessary conditions [25] for the generation of a matter-antimatter asymmetry via baryogenesis through *non-thermal leptogenesis* [26, 27]. If we furthermore assume that inflation is also driven by the right-handed sneutrino, this mechanism can operate very efficiently and combining both inflation and leptogenesis within the same model makes the model very economical and predictive. Such a combined model of inflation and leptogenesis is another major result of this thesis.

Sneutrino inflation and subsequent leptogenesis has previously been studied in the context of chaotic inflation [27, 28] and hybrid inflation [29]. The inflationary models we consider in this thesis belong to a class of models called *tribrid inflation* [30, 31, 32], which is a modification of SUSY F-term hybrid inflation [33, 34, 35]. The advantage of tribrid inflation is that it is perfectly suited to use GNS fields as inflatons. It also facilitates the embedding of our models into a *supergravity* (SUGRA) framework [30, 31, 36] with a symmetry solution to the $\eta$-problem of inflation in SUGRA [34, 37].

This work is organised in three parts

- **Part I** introduces the theoretical tools and methods used in this thesis:

  In chapter 1 we introduce the concepts of supersymmetry and supergravity and state the results relevant for this thesis. In chapter 2 we introduce the Minimal Supersymmetric Standard Model (MSSM) and then move on to discuss SUSY Pati-Salam unification based on the gauge group $G_{\text{PS}} = SU(4)_C \times SU(2)_L \times SU(2)_R$ and SUSY $SO(10)$ GUTs. Chapter 3 contains an introduction to inflation. We discuss slow-roll inflation and the generation of the density perturbations in some detail and give an overview of the classes of models discussed in the literature. Finally, chapter 4 provides a discussion of reheating after inflation and baryogenesis, focusing on baryogenesis through non-thermal leptogenesis.

- **Part II** contains a discussion of the inflationary phase of the early universe and its relation to particle physics:

  In chapter 5 we consider sneutrino tribrid inflation in a simple extension of the MSSM with conserved R-parity, where we add three heavy right-handed neutrino superfields that contain the inflaton. We discuss the inflationary dynamics in both global and local supersymmetry and compute the inflationary predictions of the model. Motivated by the discussion above, we then move on to consider inflation with a GNS inflaton field, starting with an explanation of the basic ideas in chapter 6. For simplicity, we constrain ourselves to an inflaton charged under a $U(1)$ gauge symmetry in this chapter. The more complicated case of matter inflation in SUSY GUTs is discussed in chapter 7. We start with a discussion of matter inflation in SUSY Pati-Salam models, discussing the special case of sneutrino inflation in some detail. After this we address the problem of the radiative corrections to the inflaton potential and show that these corrections do not pose a threat to our model. Then we move on to discuss the generalisation of our model to SUSY $SO(10)$ GUTs and the embedding into a SUGRA framework.

- **Part III** discusses baryogenesis through non-thermal leptogenesis after sneutrino tribrid inflation:

  In chapter 8 we come back to the model of chapter 5 and discuss reheating and non-thermal leptogenesis after inflation in this model. Using a time-averaged description, we derive analytical expressions for the model predictions. Combining the results from both chapters allows us to constrain the parameter space of this model from two different directions. We find an allowed region in parameter space where both inflation and baryogenesis through non-thermal leptogenesis in accordance with the latest experimental data is possible. The bounds we derive also have implications for low-energy neutrino physics, thus providing another link between early universe cosmology and particle physics.

After this we summarise the main results and draw our conclusions.

# PART I

# Theoretical Foundations

In this part we start out with an introduction to the theoretical tools and methods that are of importance for this thesis. We start with an introduction to *supersymmetry* (SUSY) in chapter 1. After a quick motivation for SUSY, we begin by discussing the $\mathcal{N} = 1$ supersymmetry algebra and linear representations in form of differential operators that act on superspace in section 1.2. This leads us to the discussion of chiral and vector superfields in section 1.3. These two kinds of superfields are then used in section 1.4 to build interaction Lagrangians that are invariant under SUSY transformations. We start with a collection of interacting chiral superfields in section 1.4.1 and then move on to include supersymmetric gauge interactions in section 1.4.2. After this we quickly discuss spontaneous breaking of supersymmetry in section 1.5 and we finish the chapter with a short overview of the basic ideas and most important results of local supersymmetry or *supergravity* (SUGRA) in section 1.6.

Following that, we review some of the important aspects of the *Minimal Supersymmetric Standard Model* (MSSM) and of *supersymmetric Grand Unification* in chapter 2. We begin with a discussion of the field content and the interactions in the MSSM in section 2.1 and then move on to discuss supersymmetric Pati-Salam models in section 2.2. We show how the field content of the MSSM can be embedded into representations of $G_{\mathrm{PS}} = SU(4)_C \times SU(2)_L \times SU(2)_R$ and discuss spontaneous symmetry breaking of $G_{\mathrm{PS}} \to G_{\mathrm{SM}}$ as well as interactions between the Pati-Salam supermultiplets. Finally, we consider supersymmetric $SO(10)$ unification in section 2.3.

After that we turn our attention to the early universe and discuss the fundamentals of *inflation* and then *reheating* and *leptogenesis* in subsequent chapters 3 and 4.

Chapter 3 starts with a motivation for inflation by discussing the horizon and flatness problems in section 3.1. We also discuss topological defects and the question of the origin of the large scale structure in the universe. We then move on to introduce the important concept of *slow-roll inflation* in section 3.2. The next section deals with the quantum fluctuations of such a slowly-rolling inflaton field and we derive expressions describing the most important statistical properties of these fluctuations in terms of the slow-roll parameters. After that we give an overview of the different classes of models discussed in the literature, both in non-supersymmetric theories (section 3.4), as well as in supersymmetric theories (section 3.5). We conclude the chapter by discussing inflationary model building in SUGRA theories, in particular the $\eta$-problem, in section 3.6. This also leads us to the introduction of a special class of supersymmetric models of inflation called *tribrid inflation*, which is of paramount importance throughout this thesis.

Finally, chapter 4 begins with a discussion of particle production after a phase of inflation, or *reheating*, in section 4.1. We then move on to discuss under what conditions a matter-antimatter asymmetry can be generated in the universe in section 4.2 and finish the chapter with a discussion of *non-thermal leptogenesis*. This concludes part I of this thesis.

CHAPTER 1

# Supersymmetry

Supersymmetry (SUSY) [19, 20], a spacetime symmetry relating bosonic and fermionic degrees of freedom, remains one of the most commonly considered extensions of the Standard Model. The reason is that SUSY provides some very interesting and welcome features, both from a theoretical and a phenomenological point of view.

On the phenomenological side, it can e.g. provide a very compelling dark matter candidate [38] and it predicts a light Higgs boson, which is favoured by currently available electroweak precision observables, cf. for example [39] and references therein. Furthermore, it allows for gauge coupling unification at high energies [40, 41], which does not quite work out in the (non-supersymmetric) Standard Model. (For a very readable review article covering these points and many more in connection with beyond-the-Standard-Model (BSM) physics, we suggest e.g. [42] where also a lot of additional references can be found.)

From a theoretical viewpoint, SUSY provides a very elegant solution [43, 40, 44] to the gauge hierarchy problem [16]. On an even more fundamental level, SUSY seems to be indispensable for a consistent formulation of string theories (see e.g. [45] for a textbook discussion), which are at the moment the most popular candidate for a quantum theory of gravitation. Finally, SUSY allows to extend the Poincaré group in a non-trivial way [46], circumventing the Coleman-Mandula theorem [47] by introducing fermionic generators of the symmetry transformations.

We take the last of these arguments as our starting point for the discussion of supersymmetry. After a quick detour to discuss the notion of invariance of a physical system under a group of symmetry transformations, we introduce the SUSY algebra as a non-trivial extension of the Poincaré algebra. This leads us to the notion of superspace and this in turn to superfields, which provide a clean and concise way to formulate interacting theories that are invariant under supersymmetry transformations. We then proceed to construct such SUSY invariant theories, beginning with a set of interacting chiral superfields. Subsequently, we also include vector superfields which automatically leads us to supersymmetric gauge invariance. Following this discussion we take a quick look at the basics of spontaneous supersymmetry breaking and end this chapter with a short introduction to supergravity theories.

The main reference we have consulted in writing the sections on global supersymmetry is the excellent introductory article by Signer [48] and we largely follow the discussion there. We also use the notations and conventions established in that article. Section 1.6 on supergravity is mainly based on [45, 49]. Further details on supersymmetry, both in its global and local form, can e.g. be found in [50, 51, 52, 53, 54] and references therein.

## 1.1 Symmetries in Field Theory

A physical system is said to be invariant under a group of symmetry transformations if these transformations leave the equations of motion of the physical system unchanged. If the symmetry transformations depend continuously on one or more parameters the Noether theorem tells us that associated to every such symmetry there is a conserved quantity of the physical system. In this section we quickly sketch how such symmetry transformations are implemented in classical and quantum field theories, respectively.

### 1.1.1 Classical Field Theory

A continuous symmetry transformation can be written in terms of the transformation parameters $\lambda^a$ and the generators $\mathcal{T}^a$ of the group as

$$S(\lambda) = \mathrm{e}^{-\mathrm{i}\,\lambda^a\,\mathcal{T}^a}\,. \tag{1.1}$$

It acts on a classical field $\Phi$ as

$$\Phi \to \Phi' = \mathrm{e}^{-\mathrm{i}\,\lambda^a\,\mathcal{T}^a}\,\Phi\,, \tag{1.2}$$

or for an infinitesimal transformation

$$\Phi \to \Phi' = \Phi - \mathrm{i}\,\lambda^a\,(\mathcal{T}^a\,\Phi)\,. \tag{1.3}$$

The explicit form of the generators $\mathcal{T}^a$ depends on the nature of the field the symmetry transformation acts on (in a more mathematical language it depends on the representation under which the field transforms).

The structure of the symmetry group close to the identity element, however, is encoded in the commutation relations between the generators of the group

$$[\mathcal{T}^a, \mathcal{T}^b] = f^{abc}\,\mathcal{T}^c\,, \tag{1.4}$$

which do not depend on the explicit form of the generators but are the same for all representations. This is called the algebra of the symmetry group and the $f^{abc}$ are called the structure constants.

For example, under a Poincaré transformation, $x^\mu$ transforms as

$$x^\mu \to x'^\mu = x^\mu + \omega^{\mu\nu} x_\nu + a^\mu\,. \tag{1.5}$$

To describe such a transformation, we need 10 parameters: 6 Lorentz parameters (3 boosts and 3 rotation angles) written as an antisymmetric rank 2 tensor $\omega^{\mu\nu} = -\omega^{\nu\mu}$ and 4 spacetime translation parameters $a^\mu$. Correspondingly, there are 10 generators of the Poincaré group: 6 generators $\mathcal{M}^{\mu\nu}$ (antisymmetric in $\mu$ and $\nu$) that generate the Lorentz transformations and the four spacetime translation operators $\mathcal{P}^\mu$.

For a spin 0 field the generators are given by

$$\mathcal{P}^\mu_{(0)} = \mathrm{i}\,\partial^\mu\,, \tag{1.6}$$
$$\mathcal{M}^{\mu\nu}_{(0)} = \mathrm{i}\,(x^\mu\partial^\nu - x^\nu\partial^\mu)\,, \tag{1.7}$$

whereas for a spin 1/2 field they take the following form

$$\mathcal{P}^\mu_{(1/2)} = \mathrm{i}\,\partial^\mu\,, \tag{1.8}$$
$$\mathcal{M}^{\mu\nu}_{(1/2)} = \mathrm{i}\,(x^\mu\partial^\nu - x^\nu\partial^\mu) + \frac{\mathrm{i}}{4}\,[\gamma^\mu,\gamma^\nu]\;. \tag{1.9}$$

The last part in the expression for $\mathcal{M}^{\mu\nu}_{(1/2)}$ corresponds to the spin of the particle.

Independent of the explicit form of the generators, they always satisfy the same commutation relations

$$[\mathcal{P}^\mu,\mathcal{P}^\nu] = 0\,, \tag{1.10}$$
$$[\mathcal{P}^\mu,\mathcal{M}^{\nu\sigma}] = \mathrm{i}\,(\eta^{\mu\nu}\mathcal{P}^\sigma - \eta^{\mu\sigma}\mathcal{P}^\nu)\,, \tag{1.11}$$
$$[\mathcal{M}^{\mu\nu},\mathcal{M}^{\sigma\tau}] = -\,\mathrm{i}\,(\eta^{\mu\sigma}\mathcal{M}^{\nu\tau} + \eta^{\nu\tau}\mathcal{M}^{\mu\sigma} - \eta^{\mu\tau}\mathcal{M}^{\nu\sigma} - \eta^{\nu\sigma}\mathcal{M}^{\mu\tau})\,. \tag{1.12}$$

This is called the Poincaré algebra and it will be important when introducing the notion of supersymmetry and superspace in section 1.2.

### 1.1.2 Quantum Field Theory

In a quantum field theory the fields get promoted to operators $\hat{\Phi}$ on a Hilbert space. Symmetry transformations are now implemented as unitary operators $\hat{\mathsf{U}}$ acting on the physical states as

$$|\psi\rangle \to |\psi'\rangle = \hat{\mathsf{U}}\,|\psi\rangle\,. \tag{1.13}$$

The corresponding transformation for the field operators reads

$$\hat{\Phi} \to \hat{\Phi}' = \hat{\mathsf{U}}\,\hat{\Phi}\,\hat{\mathsf{U}}^\dagger\,. \tag{1.14}$$

We can again write $\hat{\mathsf{U}}$ as

$$\hat{\mathsf{U}} = \mathrm{e}^{\mathrm{i}\,\lambda^a\hat{\mathsf{T}}^a}\,, \tag{1.15}$$

where $\lambda^a$ are the transformation parameters and the $\hat{\mathsf{T}}^a$ are now the Hermitian generators of the symmetry group on the Hilbert space. This means they are themselves constructed out of field operators and are thus to be distinguished from the generators $\mathcal{T}^a$ used in the previous section. They do, however, satisfy the same commutation relations as the $\mathcal{T}^a$'s since they furnish a representation of the same algebra.

Expanding Eqn. (1.14) to first order in the transformation parameters $\lambda^a$, we find that for infinitesimal symmetry transformations

$$\hat{\Phi} \to \hat{\Phi} + \mathrm{i}\,\lambda^a[\hat{\mathsf{T}}^a, \hat{\Phi}]\,, \tag{1.16}$$

which is to be compared with Eqn. (1.3) for the classical case.

## 1.2 Supersymmetry Algebra and Superspace

Let us now go back to the Poincaré algebra Eqns. (1.10) - (1.12). Since the search for more and more symmetry is not only aesthetically pleasing but has also been a very successful guiding principle in high energy particle physics, it is natural to ask whether the Poincaré group can be extended to an even greater symmetry group. This can, of course, be done in the form of gauge symmetries, which are internal symmetries that do not act on spacetime but rather on internal degrees of freedom of the fields involved in the theory. In other words, the generators $\mathcal{G}^a$ of any gauge group commute with all the generators of the Poincaré group

$$[\mathcal{G}^a, \mathcal{P}^\mu] = [\mathcal{G}^a, \mathcal{M}^{\sigma\tau}] = 0\,. \tag{1.17}$$

As it turns out, this is not a coincidence, but is in fact a consequence of the aforementioned Coleman-Mandula theorem [47] which in essence states that any symmetry group compatible with an interacting relativistic quantum field theory is of the form of a direct product of the Poincaré group with an internal symmetry.

There is, however, a way around this no-go-theorem, because in the Coleman-Mandula theorem it is assumed that the generators of all the symmetries in the theory do not alter the spin of the state they act on. Let us therefore introduce a set of *fermionic generators* $\mathcal{Q}_\alpha$ and $\bar{\mathcal{Q}}_{\dot{\beta}}$ [1], which change the spin of the state they act on by 1/2

$$\mathcal{Q}_\alpha|_{s=0}\rangle = |_{s=1/2}\rangle_\alpha\,, \qquad \mathcal{Q}_\alpha|_{s=1/2}\rangle^\alpha = |_{s=0}\rangle\,. \tag{1.18}$$

According to the Haag-Lopuszanski-Sohnius theorem [46], the introduction of one such set of fermionic generators [2] indeed allows us to extend the Poincaré algebra in a non-

[1] Details on Weyl spinors, Grassmann numbers, $\sigma$-matrices and other notations and conventions we use can be found in Appendix A.

[2] In this thesis we solely concern ourselves with $\mathcal{N} = 1$ supersymmetry, corresponding to one set of fermionic generators only.

trivial way and end up with a consistent interacting quantum field theory. The resulting structure is the $\mathcal{N}=1$ super Poincaré algebra

$$[\mathcal{Q}_\alpha, \mathcal{P}^\mu] = [\bar{\mathcal{Q}}_{\dot\alpha}, \mathcal{P}^\mu] = 0\,, \tag{1.19}$$
$$[\mathcal{Q}_\alpha, \mathcal{M}^{\mu\nu}] = \mathrm{i}\,(\sigma^{\mu\nu})_\alpha^{\ \beta}\mathcal{Q}_\beta\,, \tag{1.20}$$
$$[\bar{\mathcal{Q}}_{\dot\alpha}, \mathcal{M}^{\mu\nu}] = \mathrm{i}\,(\bar\sigma^{\mu\nu})_{\dot\alpha}^{\ \dot\beta}\bar{\mathcal{Q}}_{\dot\beta}\,, \tag{1.21}$$
$$\{\mathcal{Q}_\alpha, \bar{\mathcal{Q}}_{\dot\beta}\} = 2\,(\sigma^\mu)_{\alpha\dot\beta}\,\mathcal{P}_\mu\,, \tag{1.22}$$
$$\{\mathcal{Q}_\alpha, \mathcal{Q}_\beta\} = \{\bar{\mathcal{Q}}_{\dot\alpha}, \bar{\mathcal{Q}}_{\dot\beta}\} = 0\,. \tag{1.23}$$

In extending the Poincaré group to include fermionic generators, we also have to formally enlarge spacetime to include fermionic coordinates $\theta^\alpha$ and $\bar\theta^{\dot\alpha}$. These are Grassmann valued quantities, i.e.

$$\{\theta^\alpha, \theta^\beta\} \;=\; \{\theta^\alpha, \bar\theta^{\dot\beta}\} \;=\; \{\bar\theta^{\dot\alpha}, \bar\theta^{\dot\beta}\} \;=\; 0\,. \tag{1.24}$$

In contrast to the normal spacetime coordinates with mass dimension $[x^\mu] = -1$, the Grassmann coordinates have mass dimension $[\theta^\alpha] = [\bar\theta^{\dot\alpha}] = -1/2$. The resulting space with coordinates $\boldsymbol{X} = (x^\mu, \theta^\alpha, \bar\theta^{\dot\alpha})$ is called superspace and fields $\boldsymbol{\Omega}(x^\mu, \theta^\alpha, \bar\theta^{\dot\alpha})$ defined on this superspace are called superfields [55]. Before turning our attention to them in more detail, we first derive a linear representation of the SUSY algebra Eqns. (1.19) - (1.23) in terms of differential operators. This representation enables us to define chiral superfields, which turn out to be a fundamental building block for supersymmetric model building.

### 1.2.1 Linear Representation of the SUSY Algebra

In order to find the linear representation we are after, let us first consider the action of two subsequent SUSY transformations

$$\hat{\mathsf{S}}(x,\theta,\bar\theta) \equiv \mathrm{e}^{\mathrm{i}(\theta^\alpha\hat{\mathsf{Q}}_\alpha + \bar\theta_{\dot\alpha}\hat{\bar{\mathsf{Q}}}^{\dot\alpha} + x^\mu\hat{\mathsf{P}}_\mu)}\,, \tag{1.25}$$
$$\hat{\mathsf{S}}(a,\xi,\bar\xi) \equiv \mathrm{e}^{\mathrm{i}(\xi^\alpha\hat{\mathsf{Q}}_\alpha + \bar\xi_{\dot\alpha}\hat{\bar{\mathsf{Q}}}^{\dot\alpha} + a^\mu\hat{\mathsf{P}}_\mu)}\,. \tag{1.26}$$

Using the Baker-Campbell-Hausdorff formula and the fact that the only relevant non-vanishing commutators are [3]

$$[\xi\hat{\mathsf{Q}}, \bar\theta\hat{\bar{\mathsf{Q}}}] = 2\,\xi\sigma^\mu\bar\theta\,\hat{\mathsf{P}}_\mu\,, \tag{1.27}$$
$$[\bar\xi\hat{\bar{\mathsf{Q}}}, \theta\hat{\mathsf{Q}}] = -\,2\,\theta\sigma^\mu\bar\xi\,\hat{\mathsf{P}}_\mu\,, \tag{1.28}$$

we find that

$$\hat{\mathsf{S}}(a,\xi,\bar\xi)\,\hat{\mathsf{S}}(x,\theta,\bar\theta) = \hat{\mathsf{S}}(x^\mu + a^\mu + \mathrm{i}\,\xi\sigma^\mu\bar\theta - \mathrm{i}\,\theta\sigma^\mu\bar\xi\,,\,\theta+\xi\,,\,\bar\theta+\bar\xi)\,. \tag{1.29}$$

[3] Which can be derived from Eqn. (1.22).

This tells us that under the SUSY transformation $\hat{\mathsf{S}}(a,\xi,\bar{\xi})$ the point $\boldsymbol{X}$ is mapped to $\boldsymbol{X'}$

$$\boldsymbol{X} = (x^\mu, \theta^\alpha, \bar{\theta}^{\dot{\alpha}}) \to \boldsymbol{X'} = (x^\mu + a^\mu + \mathrm{i}\,\xi\sigma^\mu\bar{\theta} - \mathrm{i}\,\theta\sigma^\mu\bar{\xi}\,,\, \theta + \xi\,,\, \bar{\theta} + \bar{\xi})\,, \tag{1.30}$$

while a superfield $\boldsymbol{\Omega}(x^\mu, \theta^\alpha, \bar{\theta}^{\dot{\alpha}})$ transforms as

$$\begin{aligned}\boldsymbol{\Omega}(x,\theta,\bar{\theta}) \to\, & \hat{\mathsf{S}}(a,\xi,\bar{\xi})\,\boldsymbol{\Omega}(x,\theta,\bar{\theta})\,\hat{\mathsf{S}}^\dagger(a,\xi,\bar{\xi}) \\ &= \boldsymbol{\Omega}(x^\mu + a^\mu + \mathrm{i}\,\xi\sigma^\mu\bar{\theta} - \mathrm{i}\,\theta\sigma^\mu\bar{\xi}\,,\, \theta + \xi\,,\, \bar{\theta} + \bar{\xi})\,.\end{aligned} \tag{1.31}$$

What we are after are *differential operators* $\mathcal{P}_\mu$, $\mathcal{Q}_\alpha$ and $\bar{\mathcal{Q}}^{\dot{\alpha}}$ that allow us to express the transformation properties Eqn. (1.31) as

$$\begin{aligned}&\boldsymbol{\Omega}(x^\mu + a^\mu + \mathrm{i}\,\xi\sigma^\mu\bar{\theta} - \mathrm{i}\,\theta\sigma^\mu\bar{\xi}\,,\, \theta + \xi\,,\, \bar{\theta} + \bar{\xi}) = \\ &\qquad \mathrm{e}^{-\mathrm{i}(\xi^\alpha\mathcal{Q}_\alpha + \bar{\xi}_{\dot{\alpha}}\bar{\mathcal{Q}}^{\dot{\alpha}} + a^\mu\mathcal{P}_\mu)}\,\boldsymbol{\Omega}(x,\theta,\bar{\theta})\,.\end{aligned} \tag{1.32}$$

Taking $a, \xi, \bar{\xi}$ to be small parameters and Taylor expanding both sides of Eqn. (1.32) to first order, we can match coefficients to find

$$\mathcal{P}_\mu = \mathrm{i}\,\partial_\mu\,, \tag{1.33}$$
$$\mathcal{Q}_\alpha = \mathrm{i}\,\partial_\alpha - (\sigma^\mu)_{\alpha\dot{\alpha}}\,\bar{\theta}^{\dot{\alpha}}\,\partial_\mu\,, \tag{1.34}$$
$$\bar{\mathcal{Q}}_{\dot{\alpha}} = -\,\mathrm{i}\,\bar{\partial}_{\dot{\alpha}} + \theta^\alpha(\sigma^\mu)_{\alpha\dot{\alpha}}\,\partial_\mu\,, \tag{1.35}$$

which is the desired result. It is easy to check that these operators indeed satisfy the SUSY algebra Eqns. (1.19) - (1.23).

We can now define SUSY covariant derivatives $\mathcal{D}_\alpha$ and $\bar{\mathcal{D}}_{\dot{\alpha}}$ that satisfy

$$\{\mathcal{D}_\alpha, \mathcal{Q}_\beta\} = \{\mathcal{D}_\alpha, \bar{\mathcal{Q}}_{\dot{\beta}}\} = \{\bar{\mathcal{D}}_{\dot{\alpha}}, \mathcal{Q}_\beta\} = \{\bar{\mathcal{D}}_{\dot{\alpha}}, \bar{\mathcal{Q}}_{\dot{\beta}}\} = 0\,, \tag{1.36}$$
$$\{\mathcal{D}_\alpha, \mathcal{D}_\beta\} = \{\bar{\mathcal{D}}_{\dot{\alpha}}, \bar{\mathcal{D}}_{\dot{\beta}}\} = 0\,, \tag{1.37}$$
$$\{\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot{\alpha}}\} = -2\,\mathrm{i}\,(\sigma^\mu)_{\alpha\dot{\alpha}}\,\partial_\mu\,. \tag{1.38}$$

With our conventions, their explicit form is given by [48]

$$\mathcal{D}_\alpha \equiv \partial_\alpha - \mathrm{i}\,(\sigma^\mu)_{\alpha\dot{\alpha}}\,\bar{\theta}^{\dot{\alpha}}\,\partial_\mu\,, \tag{1.39}$$
$$\bar{\mathcal{D}}_{\dot{\alpha}} \equiv \bar{\partial}_{\dot{\alpha}} - \mathrm{i}\,\theta^\alpha(\sigma^\mu)_{\alpha\dot{\alpha}}\,\partial_\mu\,. \tag{1.40}$$

They get their name from the fact that their action is covariant with respect to SUSY transformation, i.e. $\mathcal{D}_\alpha\boldsymbol{\Omega}$ and $\bar{\mathcal{D}}_{\dot{\alpha}}\boldsymbol{\Omega}$ transform in the same way under SUSY transformations as $\boldsymbol{\Omega}$ does. We use this fact in section 1.3.1 to define chiral superfields which are a crucial ingredient in building SUSY invariant theories.

## 1.3 Superfields

Since the fermionic coordinates $\theta$ and $\bar{\theta}$ are Grassmann valued, the most general expansion of any Lorentz invariant superfield in these coordinates terminates after a finite number of terms:

$$\begin{aligned}\mathbf{\Omega}(x,\theta,\bar{\theta}) = c(x) \,&+\, \theta\psi(x) \,+\, \bar{\theta}\bar{\delta}(x) \,+\, \theta\theta\, F(x) \,+\, \bar{\theta}\bar{\theta}\, G(x) \\ &+\, \theta\sigma^{\mu}\bar{\theta}\, v_{\mu}(x) \,+\, \theta\theta\,\bar{\theta}\bar{\zeta}(x) \,+\, \bar{\theta}\bar{\theta}\,\theta\lambda(x) \\ &+\, \bar{\theta}\bar{\theta}\,\theta\theta\, D(x)\,. \end{aligned} \tag{1.41}$$

In this expression, $c(x), F(x), G(x)$ and $D(x)$ are complex scalar fields, $v_{\mu}(x)$ is a complex four-vector field and $\psi(x), \lambda(x), \bar{\delta}(x), \bar{\zeta}(x)$ are Weyl spinor fields. Together they are called the component fields of the superfield $\mathbf{\Omega}(x,\theta,\bar{\theta})$.

We see that the superfield $\mathbf{\Omega}$ contains 8 complex bosonic and 8 complex fermionic degrees of freedom. The fact that the number of bosonic and fermionic degrees of freedom exactly match is a consequence of supersymmetry and is true for all superfields. However, by imposing SUSY covariant constraints on the superfields we can define superfields with a smaller number of degrees of freedom. The two kinds of such constraint superfields we are interested in are the chiral superfield discussed in section 1.3.1 and the vector superfield discussed in section 1.3.2.

### 1.3.1 Chiral Superfields

Let us start our discussion of the different types of superfields with the chiral superfield. In particular, we are interested in left-handed chiral superfields $\mathbf{\Phi}(x,\theta,\bar{\theta})$ which are defined via the constraint [4]

$$\bar{\mathcal{D}}_{\dot{\alpha}}\mathbf{\Phi} = 0\,. \tag{1.42}$$

This constraint is preserved under SUSY transformations, because the SUSY covariant derivative $\bar{\mathcal{D}}_{\dot{\alpha}}$ commutes with the generators of SUSY transformations $\mathcal{Q}_{\alpha}$ and $\bar{\mathcal{Q}}_{\dot{\alpha}}$, cf. Eqn. (1.36). The constraint Eqn. (1.42) on the superfield reduces the number of both bosonic and fermionic degrees of freedom in the superfield. As turns out, the left-handed chiral superfield only contains left-handed Weyl spinors as fermionic component fields, which gives the superfield its name. These left-handed chiral superfields will later on contain the matter fields, i.e. all the quarks and leptons, as well as the Higgs scalars, along with their superpartners as component fields, while the gauge fields and their superpartners, the gauginos, will be assigned to vector superfields, which are discussed in section 1.3.2.

To study the properties of a left-handed chiral superfield in more detail, let us first introduce the new variable $y^{\mu}$ defined as

$$y^{\mu} \equiv x^{\mu} - \mathrm{i}\,\theta\sigma^{\mu}\bar{\theta}\,. \tag{1.43}$$

[4] Right-handed chiral superfields are defined through the equation $\mathcal{D}_{\alpha}\mathbf{\Phi}^{\dagger} = 0$.

Because $\bar{\mathcal{D}}_{\dot\alpha}\theta^\alpha = 0$ and $\bar{\mathcal{D}}_{\dot\alpha} y^\mu = 0$, the general solution to Eqn. (1.42) can be written as

$$\mathbf{\Phi}(y,\theta) = \phi(y) + \sqrt{2}\,\theta\psi(y) - \theta\theta\,F(y)\,. \tag{1.44}$$

Expanding back in $(x^\mu, \theta, \bar\theta)$ yields

$$\begin{aligned}\mathbf{\Phi}(x,\theta,\bar\theta) = \phi(x) + \sqrt{2}\,\theta\psi(x) - \mathrm{i}\,\theta\sigma^\mu\bar\theta\,\partial_\mu\phi(x) + \frac{\mathrm{i}}{\sqrt{2}}\,\theta\theta\,\partial_\mu\psi(x)\,\sigma^\mu\bar\theta \\ - \frac{1}{4}\theta\theta\,\bar\theta\bar\theta\,\partial^\mu\partial_\mu\phi(x) - \theta\theta\,F(x)\,.\end{aligned} \tag{1.45}$$

We find that a left-handed chiral superfield contains 2 complex scalar fields $\phi(x), F(x)$ and one left-handed complex Weyl spinor $\psi(x)$ as component fields, i.e. 4 real bosonic and fermionic degrees of freedom. The spinor $\psi$ of such a superfield e.g. describes the left-handed quarks and leptons of a SUSY extension of the Standard Model [5] and the complex scalar field $\phi$ represents their supersymmetric partners, the squarks and sleptons. The Higgs bosons and their SUSY partners, the Higgsinos, also form chiral superfields. As it turns out, the scalar field $F$ does not contain any physical degrees of freedom and can be eliminated through its equation of motion. It is needed in order for the SUSY algebra to close off-shell and can thus be looked upon as a kind of "book-keeping device". It plays, however, an important role when we construct interaction Lagrangians that are invariant under SUSY transformations. To understand why, we must find the transformation properties of the component fields under a SUSY transformation.

Acting with the generators $\mathcal{Q}_\alpha, \bar{\mathcal{Q}}_{\dot\alpha}$ defined in Eqns. (1.34), (1.35) on the left-handed chiral superfield Eqn. (1.45) yields

$$\delta\phi = \sqrt{2}\,\xi\psi(x)\,, \tag{1.46}$$

$$\delta\psi_\alpha = -\sqrt{2}\,F(x)\,\xi_\alpha - \mathrm{i}\sqrt{2}\,(\sigma^\mu)_{\alpha\dot\alpha}\,\bar\xi^{\dot\alpha}\,\partial_\mu\phi(x)\,, \tag{1.47}$$

$$\delta F = \partial_\mu\big(-\mathrm{i}\sqrt{2}\;\psi(x)\sigma^\mu\bar\xi\,\big)\,, \tag{1.48}$$

where $\xi$ and $\bar\xi$ are the SUSY transformation parameters. The important point to note here is that the auxiliary field $F$ only changes by a total derivative under SUSY transformations, a fact which greatly facilitates the construction of SUSY invariant Lagrangians.

### 1.3.2 Vector Superfields

In order to also include gauge fields in our theories, we need a type of superfield that contains a vector field as one of its component fields. The proper choice is the vector superfield $\boldsymbol{V}(x,\theta,\bar\theta)$ which is defined to satisfy the constraint

$$\boldsymbol{V}(x,\theta,\bar\theta) = \boldsymbol{V}^\dagger(x,\theta,\bar\theta)\,. \tag{1.49}$$

[5] The right-handed quarks and leptons are usually introduced through their charge conjugate fields, which are again left-handed.

Again, this constraint is preserved under SUSY transformations.

Taking the general expansion of a superfield in its component fields Eqn. (1.41) and enforcing the constraint Eqn. (1.49), we can write a general vector superfield as [6]

$$\begin{aligned} \boldsymbol{V}(x,\theta,\bar{\theta}) = {} & s(x) + \theta\delta(x) + \bar{\theta}\bar{\delta}(x) + \theta\theta\, N(x) + \bar{\theta}\bar{\theta}\, N^{\dagger}(x) \\ & + \theta\sigma^{\mu}\bar{\theta}\, v_{\mu}(x) + \theta\theta\,\bar{\theta}\bar{\lambda}(x) + \bar{\theta}\bar{\theta}\,\theta\lambda(x) \\ & + \theta\theta\,\bar{\theta}\bar{\theta}\, D(x)\,, \end{aligned} \tag{1.50}$$

where $s(x)$ and $D(x)$ are now real scalar fields, $N(x)$ is a complex scalar field, $v_\mu(x)$ is a real vector field and $\delta(x), \lambda(x)$ are complex Weyl spinors. Thus the vector superfield contains both 8 bosonic and 8 fermionic real degrees of freedom. It turns out, however, that some of these degrees of freedom are unphysical and can be eliminated. In particular, the real scalar field $D(x)$ is unphysical. It plays a role similar to the component field $F(x)$ of the chiral superfield and it also changes only by a total derivative under SUSY transformations

$$\delta D \propto \partial_\mu\big(\xi\sigma^\mu\bar{\lambda}(x) + \lambda(x)\sigma^\mu\bar{\xi}\,\big)\,. \tag{1.51}$$

This fact is again exploited in the construction of SUSY invariant Lagrangians.

## 1.4 Interacting Superfields

Having defined the main ingredients we need to build models that are invariant under supersymmetry transformation, we now commence to explicitly write down such SUSY invariant actions. We start with interacting chiral superfields and subsequently also include vector superfields. This automatically leads us to supersymmetric gauge theories.

### 1.4.1 Chiral Superfields: The Wess-Zumino Model

Let us start by considering SUSY invariant interactions between a set of left-handed chiral superfields $\boldsymbol{\Phi}_i$ $(i = 1, \ldots, n)$ and their conjugates $\boldsymbol{\Phi}_i^\dagger$, which transform as right-handed chiral superfields.

Any product $\boldsymbol{\Phi}_i\boldsymbol{\Phi}_j ... \boldsymbol{\Phi}_k$ of left-handed chiral superfields is again a left-handed chiral superfield, because the SUSY covariant derivative obeys the product rule. Thus, for example, for the product of two such left-handed chiral superfields,

$$\bar{\mathcal{D}}_{\dot{\alpha}}(\boldsymbol{\Phi}_i\boldsymbol{\Phi}_j) = (\bar{\mathcal{D}}_{\dot{\alpha}}\boldsymbol{\Phi}_i)\boldsymbol{\Phi}_j + \boldsymbol{\Phi}_i(\bar{\mathcal{D}}_{\dot{\alpha}}\boldsymbol{\Phi}_j) = 0 \tag{1.52}$$

and similarly for higher products. Also, a product of right-handed chiral superfields is again a right-handed chiral superfield.

[6] The vector superfield is w.l.o.g. often written in the following form:
$\boldsymbol{V} = s(x) + \theta\sigma^\mu\bar{\theta}v_\mu(x) + \frac{1}{2}\theta\theta\,\bar{\theta}\bar{\theta}\big(D(x) - \frac{1}{2}\partial^\mu\partial_\mu s(x)\big) + \big\{\mathrm{i}\,\theta\delta(x) + \mathrm{i}\,\theta\theta\, N(x) + \mathrm{i}\,\theta\theta\,\bar{\theta}\big(\bar{\lambda}(x) + \frac{\mathrm{i}}{2}\partial_\mu\delta(x)\sigma^\mu\big) + H.c.\big\}$
The advantage of this form is that it is easier to see how the unphysical degrees of freedom can be gauged away in what is commonly referred to as the Wess-Zumino gauge [56], cf. section 1.4.2.

Furthermore, the product of a left-handed chiral superfield with its conjugate is a vector superfield since (no summation here)

$$(\mathbf{\Phi}_i^\dagger \mathbf{\Phi}_i)^\dagger = \mathbf{\Phi}_i^\dagger \mathbf{\Phi}_i\,. \tag{1.53}$$

We saw in the last section that the F-term of a chiral superfield (i.e. the $\theta\theta$ component of a left-handed chiral superfield and the $\bar{\theta}\bar{\theta}$ component of a right-handed chiral superfield) as well as the D-term of a vector superfield (i.e. the $\theta\theta\,\bar{\theta}\bar{\theta}$ component) only change by a total derivative under supersymmetry transformation.

Let us therefore write [7]

$$\mathcal{L}_{\mathrm{WZ}} = \left[\mathbf{\Phi}_i^\dagger \mathbf{\Phi}_i\right]_{\theta\theta\,\bar{\theta}\bar{\theta}} + \left[W(\mathbf{\Phi}_i)\right]_{\theta\theta} + \left[W^\dagger(\mathbf{\Phi}_i^\dagger)\right]_{\bar{\theta}\bar{\theta}} \tag{1.54}$$

where the superpotential $W(\mathbf{\Phi}_i)$ is a holomorphic function of the left-handed chiral superfields $\mathbf{\Phi}_i$. Furthermore, $W^\dagger(\mathbf{\Phi}_i^\dagger)$ is the Hermitian conjugate of $W(\mathbf{\Phi}_i)$ and the notation $\left[\,...\,\right]_{\theta\theta}$ indicates to take only the $\theta\theta$ component of the expression in brackets, etc. [8] Since this Lagrangian consequently only changes by a total derivative under a supersymmetry transformation, the action

$$S = \int \mathrm{d}^4x\, \mathcal{L}_{\mathrm{WZ}} \tag{1.55}$$

is SUSY invariant. This is called the Wess-Zumino model [20, 57] of interacting chiral superfields.

In order to analyse this model further we have to specify our superpotential. First, we want to confine ourselves to renormalisable interactions in this introductory chapter. Thus we only allow products of chiral superfields with up to three superfields, since higher order products lead to non-renormalisable operators in the Lagrangian [9]. Secondly, we set a possible term linear in the superfields to zero [10]. Then our superpotential is of the following form

$$W = \frac{1}{2} m_{ij} \mathbf{\Phi}_i \mathbf{\Phi}_j + \frac{1}{3!} y_{ijk} \mathbf{\Phi}_i \mathbf{\Phi}_j \mathbf{\Phi}_k\,. \tag{1.56}$$

In this expression, $m_{ij}$ is symmetric in $i, j$ and has dimension $[m_{ij}] = 1$ while $y_{ijk}$ is symmetric in $i, j, k$ and is dimensionless, $[y_{ijk}] = 0$.

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[7] More generally, we can write $\mathcal{L}_{\mathrm{WZ}} = \left[K(\mathbf{\Phi}_i, \mathbf{\Phi}_j^\dagger)\right]_{\theta\theta\,\bar{\theta}\bar{\theta}} + \left[W(\mathbf{\Phi}_i)\right]_{\theta\theta} + \left[W^\dagger(\mathbf{\Phi}_i^\dagger)\right]_{\bar{\theta}\bar{\theta}}$ , where $K(\mathbf{\Phi}_i, \mathbf{\Phi}_j^\dagger)$ with $i, j = 1, ..., n$ is a real function of the superfields. This becomes important later on when we discuss supergravity in section 1.6.

[8] This can also be written as an integral over the fermionic coordinates as explained in section 1.4.3 and Appendix A. For example, $\int \mathrm{d}^2\theta\, W(\mathbf{\Phi}_i) = \left[W(\mathbf{\Phi}_i)\right]_{\theta\theta}$.

[9] In this thesis we also encounter superpotentials that contain non-renormalisable operators. These operators can, however, be treated with the same formalism that is developed in this chapter such that we can confine ourselves to renormalisable interactions in this section for simplicity.

[10] Such a term becomes important when we discuss F-term SUSY breaking in section 1.5.1, but for now we choose to set it to zero to keep things as simple as possible.

Expanding the Lagrangian Eqn. (1.54) with the superpotential Eqn. (1.56) into the component fields yields

$$\begin{aligned}\mathcal{L}_{\mathrm{WZ}} = F_i^\dagger F_i + (\partial_\mu \phi_i^\dagger)(\partial^\mu \phi_i) + \frac{\mathrm{i}}{2}\psi_i \sigma^\mu (\partial_\mu \bar{\psi}_i) - \frac{\mathrm{i}}{2}(\partial_\mu \psi_i)\sigma^\mu \bar{\psi} \\ - \left\{ m_{ij}\phi_i F_j + \frac{m_{ij}}{2}\psi_i\psi_j + \frac{y_{ijk}}{2}\phi_i\phi_j F_k + \frac{y_{ijk}}{2}\phi_i\psi_j\psi_k + H.c. \right\} .\end{aligned} \tag{1.57}$$

Since this expression does not contain kinetic terms for the auxiliary fields $F_i$, we can use their equations of motion

$$0 = \partial_\mu \frac{\delta \mathcal{L}_{\mathrm{WZ}}}{\delta(\partial_\mu F_i)} - \frac{\delta \mathcal{L}_{\mathrm{WZ}}}{\delta F_i} = -\frac{\delta \mathcal{L}_{\mathrm{WZ}}}{\delta F_i} = -F_i^\dagger + m_{ij}\phi_j + \frac{y_{ijk}}{2}\phi_j\phi_k \tag{1.58}$$

to eliminate them from the Lagrangian. The resulting Lagrangian can be written as

$$\begin{aligned}\mathcal{L}_{\mathrm{WZ}} = (\partial_\mu \phi_i^\dagger)(\partial^\mu \phi_i) + \frac{\mathrm{i}}{2}\psi_i \sigma^\mu (\partial_\mu \bar{\psi}_i) - \frac{\mathrm{i}}{2}(\partial_\mu \psi_i)\sigma^\mu \bar{\psi} \\ - \sum_i \left| \frac{\partial W(\phi_i)}{\partial \phi_i} \right|^2 - \frac{1}{2}\left[ \left( \frac{\partial^2 W(\phi_i)}{\partial \phi_i \partial \phi_j} \right) \psi_i \psi_j + H.c. \right] ,\end{aligned} \tag{1.59}$$

where in the last line the superpotential is considered as a function of the scalar component fields $\phi_i$ only. We see that, in particular, the effective scalar potential is given by

$$V(\phi_i) = \sum_i \left| \frac{\partial W(\phi_i)}{\partial \phi_i} \right|^2 . \tag{1.60}$$

This is true for any superpotential (not just the renormalisable form considered here), provided the "kinetic terms" in the Lagrangian are of the minimal form $[\mathbf{\Phi}_i^\dagger \mathbf{\Phi}_i]_{\theta\theta\,\bar{\theta}\bar{\theta}}$ and the chiral superfields are not coupled to any other fields.

The fermionic mass matrix, on the other hand, reads

$$\mathcal{M}_{ij}^{\mathrm{ferm}} = \frac{\partial^2 W(\phi_i)}{\partial \phi_i \, \partial \phi_j} = m_{ij} . \tag{1.61}$$

### 1.4.2 Vector Superfields and SUSY Gauge Invariance

Given a vector superfield $\boldsymbol{V} = \boldsymbol{V}^\dagger$, any product $\boldsymbol{V}^n$ of $\boldsymbol{V}$ with itself is again a vector superfield. Thus, we can use the D-term (i.e. the $\theta\theta\,\bar{\theta}\bar{\theta}$ component) of such products to build SUSY invariant actions. It turns out, however, that such terms do not provide the necessary kinetic terms for the corresponding vector fields $v_\mu$. Furthermore, since we want these vector fields to eventually describe gauge fields we also have to define how gauge transformations act on the superfields of our theory and how the vector superfields can be coupled to the chiral superfields (which contain the matter fields) in a gauge invariant way. We start our discussion with the case of a local $U(1)$ gauge symmetry and then generalise this discussion to the case of non-Abelian gauge symmetries.

### Supersymmetric, Abelian Gauge Symmetry

In this section we want to generalise the notion of local gauge invariance to a theory containing superfields. For simplicity we start out with the case of an Abelian $U(1)$ symmetry.

In a non-supersymmetric theory, as for example the Standard Model, the vector potential $A_\mu(x)$ changes as

$$A_\mu(x) \to A'_\mu(x) = A_\mu(x) + \partial_\mu \upsilon(x) \tag{1.62}$$

under a local $U(1)$ gauge transformation. Here, $\upsilon(x)$ is a spacetime-dependent transformation parameter. The corresponding transformation of the matter fields reads

$$\phi(x) \to \phi'(x) = \mathrm{e}^{-\mathrm{i}q\upsilon(x)}\phi(x)\,. \tag{1.63}$$

We now want to generalise this to supersymmetric theories.

From the discussion of the vector superfield in section 1.3.2, we know that such a vector superfield $\boldsymbol{V}$ contains a four-vector field $v_\mu(x)$ as the $\theta\sigma^\mu\bar{\theta}$ component field (the reader is kindly reminded not to confuse $\upsilon(x)$ and $v_\mu(x)$ in this paragraph). This we want to identify with the $U(1)$ vector potential $A_\mu(x)$ in Eqn. (1.62). Furthermore, given a left-handed chiral superfield $\boldsymbol{\Upsilon}$ with scalar component field $\upsilon(x)$ one can show that

$$\left[\mathrm{i}\,(\boldsymbol{\Upsilon} - \boldsymbol{\Upsilon}^\dagger)\right]_{\theta\sigma^\mu\bar{\theta}} \;=\; -\partial_\mu(\upsilon(x) + \upsilon^\dagger(x))\,. \tag{1.64}$$

As suggested by Wess and Zumino [56], let us therefore generalise Eqn. (1.62) to

$$\boldsymbol{V} \to \boldsymbol{V'} = \boldsymbol{V} + \mathrm{i}\,(\boldsymbol{\Upsilon} - \boldsymbol{\Upsilon}^\dagger)\,. \tag{1.65}$$

In anticipation of extending this discussion to non-Abelian gauge-theories we can also write this as

$$\mathrm{e}^{\boldsymbol{V}} \to \mathrm{e}^{-\mathrm{i}\boldsymbol{\Upsilon}^\dagger}\mathrm{e}^{\boldsymbol{V}}\mathrm{e}^{\mathrm{i}\boldsymbol{\Upsilon}}\,. \tag{1.66}$$

Eqn. (1.63) on the other hand is generalised to

$$\boldsymbol{\Phi} \to \boldsymbol{\Phi'} = \mathrm{e}^{-2\mathrm{i}g\boldsymbol{\Upsilon}}\boldsymbol{\Phi} \tag{1.67}$$

for a left-handed chiral superfield $\boldsymbol{\Phi}$. Here, $g$ is the gauge coupling and the factor of 2 has been introduced to end up with the right normalisation for the component fields in the end. With these transformation laws, the term

$$\left[\boldsymbol{\Phi}^\dagger \mathrm{e}^{2g\boldsymbol{V}}\boldsymbol{\Phi}\right]_{\theta\theta\,\bar{\theta}\bar{\theta}} \tag{1.68}$$

is now invariant under supersymmetry transformations as well as local gauge transformations.

Using the gauge transformation Eqn. (1.66), it is possible to gauge away many non-physical degrees of freedom of the vector superfield $\boldsymbol{V}$. In the resulting gauge, called the Wess-Zumino gauge [56], $\boldsymbol{V}$ has the simple form

$$\boldsymbol{V}_{\mathrm{WZ}}(x,\theta,\bar{\theta}) = \theta\sigma^{\mu}\bar{\theta}\, v_{\mu}(x) + \mathrm{i}\,\theta\theta\,\bar{\theta}\bar{\lambda}(x) - \mathrm{i}\,\bar{\theta}\bar{\theta}\,\theta\lambda(x) + \frac{1}{2}\,\theta\theta\,\bar{\theta}\bar{\theta}\,D(x)\,. \tag{1.69}$$

What is still missing are supersymmetric, gauge invariant kinetic terms for the vector superfield $\boldsymbol{V}$. Defining

$$\boldsymbol{U}_{\alpha} \equiv -\,\frac{1}{4}\,(\bar{\mathcal{D}}_{\dot{\alpha}}\bar{\mathcal{D}}^{\dot{\alpha}})\,\mathcal{D}_{\alpha}\,\boldsymbol{V}\,, \tag{1.70}$$

$$\bar{\boldsymbol{U}}_{\dot{\alpha}} \equiv -\,\frac{1}{4}\,(\mathcal{D}^{\alpha}\mathcal{D}_{\alpha})\,\bar{\mathcal{D}}_{\dot{\alpha}}\,\boldsymbol{V}\,, \tag{1.71}$$

we first note that $\boldsymbol{U}_{\alpha}$ is a left-handed chiral superfield $\bar{\mathcal{D}}_{\dot{\alpha}}\boldsymbol{U}_{\alpha} = 0$ and similarly $\bar{\boldsymbol{U}}_{\dot{\alpha}}$ is a right-handed chiral superfield. Furthermore, using the transformation law Eqn. (1.66), it can be shown that for an Abelian symmetry, $\boldsymbol{U}_{\alpha}$ and $\bar{\boldsymbol{U}}_{\dot{\alpha}}$ are gauge invariant. We can therefore include the following supersymmetric, gauge invariant terms in the Lagrangian

$$\mathcal{L} \supset \frac{1}{4}\big[\boldsymbol{U}^{\alpha}\boldsymbol{U}_{\alpha}\big]_{\theta\theta} + \frac{1}{4}\big[\bar{\boldsymbol{U}}_{\dot{\alpha}}\bar{\boldsymbol{U}}^{\dot{\alpha}}\big]_{\bar{\theta}\bar{\theta}} \tag{1.72}$$

and these terms indeed yield the desired kinetic terms for the vector superfield $\boldsymbol{V}$.

Putting everything together and assuming the superpotential $W$ only contains gauge invariant products of chiral superfields [11], we end up with the following supersymmetric, gauge invariant Lagrangian for a local Abelian $U(1)$ gauge symmetry, coupling a set of chiral superfields $\boldsymbol{\Phi}_i$ to the vector superfield $\boldsymbol{V}$

$$\begin{aligned}\mathcal{L} = {} & \frac{1}{4}\big[\boldsymbol{U}^{\alpha}\boldsymbol{U}_{\alpha}\big]_{\theta\theta} + \frac{1}{4}\big[\bar{\boldsymbol{U}}_{\dot{\alpha}}\bar{\boldsymbol{U}}^{\dot{\alpha}}\big]_{\bar{\theta}\bar{\theta}} + \big[\boldsymbol{\Phi}_i^{\dagger}\,\mathrm{e}^{2g\boldsymbol{V}}\boldsymbol{\Phi}_i\big]_{\theta\theta\,\bar{\theta}\bar{\theta}} \\ & + \big[W(\boldsymbol{\Phi}_i)\big]_{\theta\theta} + \big[W^{\dagger}(\boldsymbol{\Phi}_i^{\dagger})\big]_{\bar{\theta}\bar{\theta}} + 2\big[\xi\boldsymbol{V}\big]_{\theta\theta\,\bar{\theta}\bar{\theta}}\,.\end{aligned} \tag{1.73}$$

Here, we have added a Fayet-Iliopoulos term $2\big[\xi\boldsymbol{V}\big]_{\theta\theta\,\bar{\theta}\bar{\theta}} = 2\,\xi\,D(x)$ [58], which will become important when we discuss D-term supersymmetry breaking in section 1.5.2. It is important to note, however, that such a term is gauge invariant only for an Abelian gauge superfield.

We could now continue to write out the Lagrangian in component fields, which would show that the field $D(x)$ is non-dynamical and can be eliminated via its equation of motion much in the same way as the field $F(x)$ was eliminated in the Wess-Zumino model in section 1.4.1. However, we postpone this task until we have generalised our discussion to non-Abelian gauge symmetries in the next section.

[11] In particular, linear terms of the form $a_i\boldsymbol{\Phi}_i$ are only allowed for gauge singlet superfields.

### Supersymmetric, Non-Abelian Gauge Symmetry

Let us now generalise the discussion of the previous section to a general non-Abelian gauge group $\mathcal{G}$ with generators $\mathcal{T}^a$. The generalisation of Eqn. (1.67) for a set of left-handed chiral superfields $\boldsymbol{\Phi}_i$ transforming under some representation of $\mathcal{G}$ is given by

$$\boldsymbol{\Phi}_i \rightarrow \boldsymbol{\Phi}_i' = \left(\mathrm{e}^{-2\mathrm{i}g\tilde{\boldsymbol{\Upsilon}}}\right)_{ij}\boldsymbol{\Phi}_j\,, \tag{1.74}$$

where we have defined

$$\tilde{\boldsymbol{\Upsilon}} \equiv \boldsymbol{\Upsilon}^a\mathcal{T}^a \tag{1.75}$$

with a collection of left-handed chiral superfields $\boldsymbol{\Upsilon}^a$ and where the form of the generators $\mathcal{T}^a$ depends on the representation under which the $\boldsymbol{\Phi}_i$ transform.

For each generator $\mathcal{T}^a$ we need a corresponding vector superfield $\boldsymbol{V}^a$. Defining

$$\tilde{\boldsymbol{V}} \equiv \boldsymbol{V}^a\mathcal{T}^a\,, \tag{1.76}$$

the transformation law for the vector superfields now reads

$$\mathrm{e}^{2g\tilde{\boldsymbol{V}}} \rightarrow \mathrm{e}^{-2\mathrm{i}g\tilde{\boldsymbol{\Upsilon}}^\dagger}\mathrm{e}^{2g\tilde{\boldsymbol{V}}}\mathrm{e}^{2\mathrm{i}g\tilde{\boldsymbol{\Upsilon}}}\,, \tag{1.77}$$

and it can be shown that

$$\left[\boldsymbol{\Phi}_i^\dagger\left(\mathrm{e}^{2g\tilde{\boldsymbol{V}}}\right)_{ij}\boldsymbol{\Phi}_j\right]_{\theta\theta\,\bar{\theta}\bar{\theta}} \tag{1.78}$$

is supersymmetric and gauge invariant.

Last, the definition of $\boldsymbol{U}_\alpha$ and $\bar{\boldsymbol{U}}_{\dot{\alpha}}$ has to be modified. Writing

$$\tilde{\boldsymbol{U}}_\alpha \equiv (\boldsymbol{U}^a)_\alpha\,\mathcal{T}^a\,, \tag{1.79}$$

$$\tilde{\bar{\boldsymbol{U}}}_{\dot{\alpha}} \equiv (\bar{\boldsymbol{U}}^a)_{\dot{\alpha}}\,\mathcal{T}^a\,, \tag{1.80}$$

their definition now reads

$$\tilde{\boldsymbol{U}}_\alpha \equiv -\,\frac{1}{8g}\,(\bar{\mathcal{D}}_{\dot{\alpha}}\bar{\mathcal{D}}^{\dot{\alpha}})\,\mathrm{e}^{-2g\tilde{\boldsymbol{V}}}\,\mathcal{D}_\alpha\,\mathrm{e}^{2g\tilde{\boldsymbol{V}}}\,, \tag{1.81}$$

$$\tilde{\bar{\boldsymbol{U}}}_{\dot{\alpha}} \equiv -\,\frac{1}{8g}\,(\mathcal{D}^\alpha\mathcal{D}_\alpha)\,\mathrm{e}^{-2g\tilde{\boldsymbol{V}}}\,\bar{\mathcal{D}}_{\dot{\alpha}}\,\mathrm{e}^{2g\tilde{\boldsymbol{V}}} \tag{1.82}$$

and it can be shown that the trace over the group indices

$$\mathrm{Tr}\,\tilde{\boldsymbol{U}}^\alpha\tilde{\boldsymbol{U}}_\alpha = \frac{1}{2}\,(\boldsymbol{U}^a)^\alpha\,(\boldsymbol{U}^a)_\alpha \;\equiv\; \tilde{\boldsymbol{U}}\cdot\tilde{\boldsymbol{U}}\,, \tag{1.83}$$

$$\mathrm{Tr}\,\tilde{\bar{\boldsymbol{U}}}_{\dot{\alpha}}\tilde{\bar{\boldsymbol{U}}}^{\dot{\alpha}} = \frac{1}{2}\,(\bar{\boldsymbol{U}}^a)_{\dot{\alpha}}\,(\bar{\boldsymbol{U}}^a)^{\dot{\alpha}} \;\equiv\; \tilde{\bar{\boldsymbol{U}}}\cdot\tilde{\bar{\boldsymbol{U}}} \tag{1.84}$$

is indeed gauge invariant and gives the right kinetic terms for the gauge fields.

Collecting all terms we end up with the following supersymmetric, gauge invariant Lagrangian

$$\begin{aligned}\mathcal{L} = {} & \frac{1}{4}\big[\tilde{\boldsymbol{U}}\cdot\tilde{\boldsymbol{U}}\big]_{\theta\theta} + \frac{1}{4}\big[\tilde{\bar{\boldsymbol{U}}}\cdot\tilde{\bar{\boldsymbol{U}}}\big]_{\bar{\theta}\bar{\theta}} + \left[\boldsymbol{\Phi}_i^\dagger\left(\mathrm{e}^{2g\tilde{\boldsymbol{V}}}\right)_{ij}\boldsymbol{\Phi}_j\right]_{\theta\theta\,\bar{\theta}\bar{\theta}} \\ & + \big[W(\boldsymbol{\Phi}_i)\big]_{\theta\theta} + \big[W^\dagger(\boldsymbol{\Phi}_i^\dagger)\big]_{\bar{\theta}\bar{\theta}}\,,\end{aligned} \tag{1.85}$$

plus a possible Fayet-Iliopoulos term

$$\mathcal{L}_{\mathrm{FI}} = 2\sum_a \xi^a \big[\boldsymbol{V}^a\big]_{\theta\theta\,\bar\theta\bar\theta} \tag{1.86}$$

for $U(1)$ gauge superfields.

It is now possible to write out this Lagrangian in the component fields. Upon doing so it turns out that the auxiliary fields $D^a$ of the gauge superfields $\boldsymbol{V}^a$ are non-dynamical (i.e. their equations of motion do not contain any derivatives) and can thus be eliminated from the theory. This yields the final result of this section, the supersymmetric Lagrangian in the Wess-Zumino gauge for chiral superfields $\boldsymbol{\Phi}_i$ (with component fields $\phi_i, \psi_i$) and vector superfields $\boldsymbol{V}^a$ (with component fields $v^a_\mu, \lambda^a$) for a general non-Abelian gauge group $\mathcal{G}$

$$\begin{aligned}\mathcal{L} = {} & (\mathrm{D}_\mu\phi)^\dagger_i(\mathrm{D}^\mu\phi)_i + \frac{\mathrm{i}}{2}\psi_i\,\sigma^\mu(\mathrm{D}_\mu\bar\psi)_i - \frac{\mathrm{i}}{2}(\mathrm{D}_\mu\psi)_i\,\sigma^\mu\bar\psi_i \\ & - \frac{1}{4}(F^a)_{\mu\nu}\,(F^a)^{\mu\nu} + \frac{\mathrm{i}}{2}\lambda^a\sigma^\mu(\mathrm{D}_\mu\bar\lambda)^a - \frac{\mathrm{i}}{2}(\mathrm{D}_\mu\lambda)^a\sigma^\mu\bar\lambda^a \\ & - \mathrm{i}\sqrt{2}\,g\,\bar\psi_i\bar\lambda^a\,(\mathcal{T}^a)_{ij}\,\phi_j + \mathrm{i}\sqrt{2}\,g\,\phi^\dagger_i\,(\mathcal{T}^a)_{ij}\,\psi_j\lambda^a \\ & - \frac{1}{2}\left[\frac{\partial^2 W(\phi_i)}{\partial\phi_i\,\partial\phi_j}\psi_i\psi_j + H.c.\right] - V(\phi_i,\phi^\dagger_j)\,,\end{aligned} \tag{1.87}$$

with $i,j = 1,\ldots,n$. The covariant derivative $\mathrm{D}_\mu$ acts as follows on the scalars $\phi_i$, chiral fermions $\psi_i$ and gauginos $\lambda^a$, respectively,

$$(\mathrm{D}_\mu\phi)_i = \partial_\mu\phi_i + \mathrm{i}\,g\,v^a_\mu\,(\mathcal{T}^a)_{ij}\,\phi_j\,, \tag{1.88}$$

$$(\mathrm{D}_\mu\psi)_i = \partial_\mu\psi_i + \mathrm{i}\,g\,v^a_\mu\,(\mathcal{T}^a)_{ij}\,\psi_j\,, \tag{1.89}$$

$$(\mathrm{D}_\mu\lambda)^a = \partial_\mu\lambda^a - gf^{abc}\,v^b_\mu\,\lambda^c\,. \tag{1.90}$$

The field-strength tensor is given by

$$(F^a)_{\mu\nu} = \partial_\mu\,v^a_\nu - \partial_\mu\,v^a_\nu - gf^{abc}\,v^b_\mu\,v^c_\nu\,. \tag{1.91}$$

Finally, the scalar potential is given by

$$\boxed{\begin{aligned}V(\phi_i,\phi^\dagger_j) &= \sum_i F^\dagger_i F_i + \frac{1}{2}\sum_a (D^a)^2 \\ &= \sum_i \left|\frac{\partial W}{\partial\phi_i}\right|^2 + \frac{1}{2}\sum_a \big(\sum_x g_a\,\overrightarrow{\phi_x}^\dagger\!\cdot\mathcal{T}^a_x\cdot\overrightarrow{\phi_x}\;+\;\xi^a\big)^2.\end{aligned}} \tag{1.92}$$

In this formula, $a$ labels the generators of the group and the $g_a$ are the corresponding coupling constants of the possibly different factors of the gauge group. We have also included the case here that there are more than one representation involved in the model: the index $x$ runs over all representations and

$$\overrightarrow{\phi_x} = (\phi_x)_k \quad , \quad k = 1,\ldots,d \tag{1.93}$$

denotes a $d$-dimensional field-multiplet transforming under the corresponding $d$-dimensional representation of the group $\mathcal{G}$ with generators

$$(\mathcal{T}_x^a)_{kl} \quad , \quad k,l = 1,\ldots,d\,. \tag{1.94}$$

Finally, the Fayet-Iliopoulos term $\xi^a$ is only allowed for Abelian gauge fields. This formula is of great importance in inflationary model building.

### 1.4.3 R-Symmetries

The form of the Lagrangians Eqns. (1.54), (1.85) motivates the introduction of a further global, continuous symmetry one can impose on the theory. To understand this, let us first notice that e.g. Eqn. (1.54) can also be written in the following form

$$\mathcal{L}_{\mathrm{WZ}} = \int \mathrm{d}^2\theta\,\mathrm{d}^2\bar\theta\; K(\mathbf{\Phi}_i, \mathbf{\Phi}_j^\dagger) + \int \mathrm{d}^2\theta\; W(\mathbf{\Phi}_i) + \int \mathrm{d}^2\bar\theta\; W^\dagger(\mathbf{\Phi}_i^\dagger)\,, \tag{1.95}$$

with $i,j = 1,\ldots,n$ (cf. Appendix A).

Let us define a global $U(1)_R$ symmetry, under which the fermionic coordinates transform as

$$\theta_\alpha \to \mathrm{e}^{\mathrm{i}\delta}\theta_\alpha \quad , \quad \bar\theta^{\dot\alpha} \to \mathrm{e}^{-\mathrm{i}\delta}\bar\theta^{\dot\alpha}\,, \tag{1.96}$$

that is, they carry charge 1 and $-1$ under this symmetry, respectively. This also implies

$$\mathcal{Q}_\alpha \to \mathrm{e}^{-\mathrm{i}\delta}\mathcal{Q}_\alpha \quad , \quad \bar{\mathcal{Q}}^{\dot\alpha} \to \mathrm{e}^{\mathrm{i}\delta}\bar{\mathcal{Q}}^{\dot\alpha} \tag{1.97}$$

for the generators of supersymmetry transformations. Obviously, they do not commute with the generator $\mathcal{R}$ of the $U(1)_R$ symmetry, but rather

$$\big[\mathcal{R},\mathcal{Q}\big] = -\mathcal{Q} \quad , \quad \big[\mathcal{R},\bar{\mathcal{Q}}\big] = \bar{\mathcal{Q}}\,. \tag{1.98}$$

Since the different component fields of a given superfield, however, are related by the action of $\mathcal{Q}, \bar{\mathcal{Q}}$ , this means that the component fields have different charges under the $U(1)_R$ symmetry. Let's look at this in slightly more detail.

A generic superfield $\boldsymbol{S}(x,\theta,\bar\theta)$ with $U(1)_R$ charge $r_S$ by definition transforms as

$$\boldsymbol{S}(x,\theta,\bar\theta) \to \boldsymbol{S'}(x',\theta',\bar\theta') = \mathrm{e}^{\mathrm{i}\,r_S\,\delta}\boldsymbol{S}(x,\theta,\bar\theta) \tag{1.99}$$

under a $U(1)_R$ transformation.

In case of a chiral superfield $\mathbf{\Phi} = (\phi,\psi,F)$ with $U(1)_R$-charge $r_\Phi$ we find for the component fields [12]

$$\phi \to \mathrm{e}^{\mathrm{i}\,r_\Phi\delta}\,\phi \tag{1.100}$$

$$\psi \to \mathrm{e}^{\mathrm{i}\,(r_\Phi-1)\delta}\,\psi \tag{1.101}$$

$$F \to \mathrm{e}^{\mathrm{i}\,(r_\Phi-2)\delta}\,F\,. \tag{1.102}$$

[12] A more detailed derivation can be found in Appendix B.

Thus, they carry charge $r_\Phi, r_\Phi - 1$ and $r_\Phi - 2$, respectively. The component fields of $\boldsymbol{\Phi}^\dagger$ carry the opposite charges.

A vector superfield (in the Wess-Zumino gauge) $\boldsymbol{V} = (v_\mu, \lambda, D) = \boldsymbol{V}^\dagger$, on the other hand, necessarily has charge $r_V = 0$ and its component fields transform as

$$v_\mu \to v_\mu \tag{1.103}$$

$$\lambda \to \mathrm{e}^{\mathrm{i}\delta}\,\lambda \tag{1.104}$$

$$D \to D\,, \tag{1.105}$$

i.e. they carry charge $0, 1$ and $0$, respectively. For a product of superfields, the resulting $U(1)_R$ charge is simply the sum of the individual charges.

Looking at Eqn. (1.95) we find that the superpotential $W(\boldsymbol{\Phi}_i)$ has to carry $U(1)_R$ charge $r_W = 2$ whereas $K(\boldsymbol{\Phi}_i, \boldsymbol{\Phi}_j^\dagger)$ has to have charge $r_K = 0$ if our Lagrangian is to be invariant under the $U(1)_R$ symmetry [13]. This can be used in model building to put constraints on the form of the superpotential. All terms in the Lagrangian Eqn. (1.85) containing vector superfields, on the other hand, are automatically $U(1)_R$ invariant. Let us conclude this section by remarking that it is also possible to restrict the symmetry to a discrete subgroup $\mathbb{Z}_n$ of the $U(1)_R$ group. The most important example of such a discrete symmetry is *R-parity*, which is discussed in section 2.1 in the context of the MSSM.

## 1.5 Spontaneous Breaking Of Supersymmetry

If supersymmetry were unbroken, the superpartners of the Standard Model particles would have the same mass as the Standard Model particles themselves [14]. Since no such particles have been observed, it is clear that supersymmetry has to be broken at low energies.

Since we do not want to break supersymmetry by brute force, we have to investigate if and how supersymmetry can be spontaneously broken. A symmetry is spontaneously broken if the Lagrangian is invariant under that particular symmetry, but the ground state of the theory is not. For the case of supersymmetry this means that we must have

$$\mathcal{Q}_\alpha|0\rangle \neq 0 \quad , \quad \bar{\mathcal{Q}}_{\dot\alpha}|0\rangle \neq 0\,. \tag{1.106}$$

Furthermore, in the case of global supersymmetry the Hamiltonian can be related to the generators of supersymmetry transformations using Eqn. (1.22) and $\bar{\mathcal{Q}}_{\dot\alpha} = (\mathcal{Q}_\alpha)^\dagger$

$$4H = 4\mathcal{P}^0 = \mathcal{Q}_1\mathcal{Q}_1^\dagger + \mathcal{Q}_1^\dagger\mathcal{Q}_1 + \mathcal{Q}_2\mathcal{Q}_2^\dagger + \mathcal{Q}_2^\dagger\mathcal{Q}_2\,. \tag{1.107}$$

Thus, for unbroken global supersymmetry with $\mathcal{Q}_\alpha|0\rangle = \bar{\mathcal{Q}}_{\dot\alpha}|0\rangle = 0$, the vacuum has zero energy $\langle 0|H|0\rangle = 0$, whereas for spontaneously broken SUSY the vacuum must have positive energy

$$\langle 0|H|0\rangle = \frac{1}{4}\Big(\big\|\mathcal{Q}_1^\dagger|0\rangle\big\|^2 + \big\|\mathcal{Q}_1|0\rangle\big\|^2 + \big\|\mathcal{Q}_2^\dagger|0\rangle\big\|^2 + \big\|\mathcal{Q}_2|0\rangle\big\|^2\Big) > 0\,. \tag{1.108}$$

[13] Note that $\mathrm{d}\theta$ has $U(1)_R$ charge $-1$, since by definition $\int \mathrm{d}\theta\,\theta = 1$ etc. Once again, one has to be careful when dealing with Grassmann valued expressions.

[14] The reason for this is that the mass-squared operator $\mathcal{P}_\mu\mathcal{P}^\mu$ commutes with the generators $\mathcal{Q}_\alpha, \bar{\mathcal{Q}}_{\dot\alpha}$ of supersymmetry transformations.

In the absence of spacetime-dependent effects and fermion condensates we have $\langle 0|H|0\rangle = \langle 0|V|0\rangle$, with the scalar potential given by Eqn. (1.92). Therefore, supersymmetry is spontaneously broken if either $F_i$ or $D^a$ do not vanish in the ground state [15]. We discuss both cases, referred to as F-term breaking and D-term breaking respectively, in the next sections.

### 1.5.1 F-Term SUSY Breaking

Let us start with a discussion of breaking supersymmetry by a non-vanishing F-term. The canonical example of F-term SUSY breaking is given by the O'Raifeartaigh model [59] with the following superpotential

$$W_{\mathrm{OR}} = -a\mathbf{\Phi}_1 + m\mathbf{\Phi}_2\mathbf{\Phi}_3 + \frac{y}{2}\mathbf{\Phi}_1\mathbf{\Phi}_3^2\,. \tag{1.109}$$

Here $\mathbf{\Phi}_1, \mathbf{\Phi}_2$ and $\mathbf{\Phi}_3$ are gauge-singlet chiral superfields, such that $V_D = 0$. (A possible Fayet-Iliopoulos term is set to 0 in this model.) The scalar potential arising from this superpotential is given by

$$\begin{aligned} V_{\mathrm{OR}} = V_F = \sum_i F_i^\dagger F_i = \sum_i \left|\frac{\partial W_{\mathrm{OR}}(\phi_i)}{\partial\phi_i}\right|^2 \\ = \left|\frac{y}{2}\phi_3^2 - a\right|^2 + |m\,\phi_3|^2 + |m\,\phi_2 + y\,\phi_1\phi_3|^2\,. \end{aligned} \tag{1.110}$$

Looking at the first two terms we see that $V_{\mathrm{OR}} > 0$, which is what we wanted. Assuming $a < m^2/y$, the absolute minimum of the potential is located at $\langle\phi_2\rangle = \langle\phi_3\rangle = 0$ with $\langle\phi_1\rangle$ undetermined at tree-level [16]. Such a direction in field-space along which the scalar potential is constant (at tree-level) is called a flat direction. These flat directions play an important role in the construction of viable supersymmetric inflationary models as we will see later on.

Coming back to the O'Raifeartaigh model and additionally assuming $\langle\phi_1\rangle = 0$ we can compute the mass spectrum to explicitly see the breaking of supersymmetry as a mass splitting between the masses of some of the bosonic and fermionic component fields. The resulting mass spectrum is summarised in Table 1.1.

As we can see, SUSY breaking is manifest in the mass splitting between the real and imaginary parts of the complex scalar field $\phi_3$ and the third fermionic mass eigenstate $\mathrm{i}\,(\psi_2 - \psi_3)/\sqrt{2}$. In the SUSY conserving limit $a \to 0$ this mass splitting vanishes, as it should be the case [17].

Another interesting observation is that one Weyl spinor $\psi_1$ remains massless. This is to be expected since we are breaking a global *fermionic* symmetry. Thus, this massless fermion, called the Goldstino, can be seen as the analogon of the massless Goldstone boson we get from the breaking of a global *bosonic* symmetry.

[15] It is also possible that SUSY breaking is realised as a combination of both F-term and D-term breaking.
[16] A mass term for the $\phi_1$ direction is generated at the loop-level. We say that the flat direction is lifted by quantum corrections.
[17] In this model the order parameter of SUSY breaking is given by $\langle 0|F_1|0\rangle = -a$.

| Mass Eigenstate | Squared Mass |
|---|---|
| $\mathrm{Re}(\phi_1), \mathrm{Im}(\phi_1)$ | $0$ |
| $\mathrm{Re}(\phi_2), \mathrm{Im}(\phi_2)$ | $m^2$ |
| $\mathrm{Re}(\phi_3)$ | $m^2 - ay$ |
| $\mathrm{Im}(\phi_3)$ | $m^2 + ay$ |
| $\psi_1$ | $0$ |
| $(\psi_2 + \psi_3)/\sqrt{2}$ | $m^2$ |
| $\mathrm{i}\,(\psi_2 - \psi_3)/\sqrt{2}$ | $m^2$ |

**Table 1.1:** Mass spectrum of the O'Raifeartaigh model [59] of F-term SUSY breaking in the minimum $\langle\phi_i\rangle = 0$ with $\phi_i = \frac{1}{\sqrt{2}}(\mathrm{Re}(\phi_i) + \mathrm{i}\,\mathrm{Im}(\phi_i))$.

Unfortunately it turns out that although the O'Raifeartaigh model seems quite appealing, it does not break SUSY in the way we want from a phenomenological point of view: we can see from Table 1.1 that we have one scalar field $\mathrm{Re}(\phi_3)$ with a mass that is smaller than the mass of the corresponding fermionic mass eigenstate. If these fermionic mass eigenstates are to represent the Standard Model particles then we should already have seen some scalar particles with masses smaller than their corresponding Standard Model partners. This is clearly not the case. Also, this is no coincidence but follows from the general tree-level result for theories that contain only chiral superfields

$$\mathrm{STr}\,\mathcal{M}^2 = \sum_j (-1)^{2j}(2j+1)m_j^2 = 0\,, \tag{1.111}$$

where the supertrace STr denotes the trace of the mass-squared matrix over the real fields of the theory. In the presence of SUSY breaking, however, this result gets modified at loop-level [18]. One can therefore try to break SUSY via the O'Raifeartaigh mechanism in what is called the *hidden sector*, which then communicates the SUSY breaking via loop effects to the visible sector in such a way that all SUSY partners end up heavier than the corresponding Standard Model particles.

### 1.5.2 D-Term SUSY Breaking

Let us now turn to the second possibility, which is breaking SUSY spontaneously via non vanishing D-terms [58]. Let us start by considering the case of a $U(1)$ vector superfield $\boldsymbol{V}$ coupled to a chiral superfield $\boldsymbol{\Phi}$ with $U(1)$-charge $q$ and vanishing superpotential $W(\boldsymbol{\Phi}) = 0$. In this case the scalar potential looks like the following

$$V = V_D = \frac{1}{2}(g\,q\,\phi^\dagger\phi + \xi)^2\,, \tag{1.112}$$

[18] It can also get modified in theories that contain vector superfields with some non-vanishing D-term.

where we have included a non-zero Fayet-Iliopoulos term $\xi \neq 0$. We want $V_D > 0$. However, what can happen is that the scalar component field $\phi$ develops a non-zero vacuum expectation value. In particular, if

$$\langle 0|\phi^\dagger \phi|0\rangle = -\frac{\xi}{g\,q}\,, \tag{1.113}$$

then $\langle 0|V|0\rangle = 0$, i.e. supersymmetry is unbroken. The $U(1)$ gauge symmetry, on the other hand, is broken by the non-zero VEV of $\phi$. This is not what we want. What we have to do in order to ensure $V > 0$ is to prevent the scalar field from acquiring a VEV. This can be done by introducing a large mass for the scalar component field $\phi$ through suitable terms in the superpotential. However, since $\phi$ is charged under the $U(1)$ gauge symmetry and the superpotential has to be a holomorphic, gauge invariant function of the chiral superfields, we need at least two such chiral superfields with opposite charges $q_1 = -q_2$. Then we can write

$$W = m\,\mathbf{\Phi}_1 \mathbf{\Phi}_2\,, \tag{1.114}$$

where we take $m$ to be real for simplicity. The resulting scalar potential is given by

$$V = V_F + V_D = m^2 \big(\sum_{i=1}^{2} \phi_i^\dagger \phi_i\big) + \frac{1}{2}\big(\xi + g \sum_{i=1}^{2} q_i\, \phi_i^\dagger \phi_i\big)^2\,. \tag{1.115}$$

If we choose $m^2 > g\,|q_i|\,\xi$, the minimum of the potential is indeed given by $\langle 0|\phi_i|0\rangle = 0$, which gives

$$\langle 0|V|0\rangle = \xi^2/2 \neq 0\,. \tag{1.116}$$

We find that the order parameter of D-term SUSY breaking is given by the Fayet-Iliopoulos term $\xi$. The mass spectrum of this model of D-term SUSY breaking is summarised in Table 1.2.

| Mass Eigenstate | Squared Mass |
|---|---|
| Gauge field $v_\mu$ | $0$ |
| Gaugino $\lambda$ | $0$ |
| $\mathrm{Re}(\phi_1), \mathrm{Im}(\phi_1)$ | $m^2 + g\,q\,\xi$ |
| $\mathrm{Re}(\phi_2), \mathrm{Im}(\phi_2)$ | $m^2 - g\,q\,\xi$ |
| $(\psi_1 + \psi_2)/\sqrt{2}$ | $m^2$ |
| $\mathrm{i}\,(\psi_1 - \psi_2)/\sqrt{2}$ | $m^2$ |

**Table 1.2:** Mass spectrum of the Fayet-Iliopoulos model [58] of D-term SUSY breaking in the minimum $\langle \phi_i \rangle = 0$ with $\phi_i = \frac{1}{\sqrt{2}}(\mathrm{Re}(\phi_i) + \mathrm{i}\,\mathrm{Im}(\phi_i))$.

Again, we find that some of the scalar particles ends up lighter than the corresponding fermion which means that this type of SUSY breaking, too, can only be applied indirectly to a supersymmetric extension of the Standard Model. Also, since the gauge symmetry remains unbroken, the gauge field remains massless. Its superpartner, the gaugino, also remains massless in this scenario and can be identified as the Goldstino arising from the breaking of global SUSY.

## 1.6 Supergravity

So far the transformation parameters of supersymmetry transformations have been constants (cf. for example Eqn. (1.32) or Eqns. (1.46) - (1.48)), i.e. we have been dealing with supersymmetry as a global symmetry. Since most symmetries in particle physics are, however, realised as local symmetries it is natural to ask whether supersymmetry can also be generalised to a local symmetry including transformation parameters $(a^\mu(x), \xi^\alpha(x), \bar{\xi}_{\dot{\alpha}}(x))$ that depend on the spacetime coordinates [60, 61, 62, 63].

Similar to the case of promoting a global gauge symmetry to a local one, the generalisation of a globally supersymmetric theory to one containing spacetime dependent transformation parameters requires the introduction of an additional field, the "gauge field of local supersymmetry transformations", in order to sustain invariance of the action under such transformations. For the case of local supersymmetry transformations, it turns out that this "gauge field" has spin 3/2, in contrast to the spin 1 vector bosons for the case of ordinary local gauge invariance. Furthermore, since we are dealing with supersymmetric theories, it is accompanied by its superpartner, a symmetric rank two tensor field with spin 2. It turns out that this rank two tensor can be identified with the spacetime metric $g_{\mu\nu}$, whose quantum fluctuations are the mediators of the gravitational interaction. This tensor field is therefore called the *graviton field* and its spin 3/2 superpartner, the gauge field of local supersymmetry transformations, is called the *gravitino field*.

As we can see, a locally supersymmetric theory necessarily includes a theory of general relativity. This is also borne out by the fact that the supersymmetry algebra, when generalised to include spacetime dependent transformation parameters, includes generators that induce spacetime dependent coordinate transformations, which lie at the heart of general relativity. Therefore local supersymmetry is often referred to as *supergravity* or SUGRA in short.

In the next section we sketch how one can obtain a locally supersymmetric theory starting from a globally supersymmetric one using the Noether method and then move on to list the results that are most important for this thesis.

### 1.6.1 The Noether Method

In this section we sketch how to use the Noether procedure to obtain a Lagrangian for a massless chiral supermultiplet that is invariant under local supersymmetry transformations. For simplicity, we will be working directly with the component fields rather then the superfields in this section.

Let us start with the globally supersymmetric Lagrangian for one free, massless chiral supermultiplet [19]

$$\mathcal{L}_0 = (\partial_\mu \phi^*)(\partial^\mu \phi) + \mathrm{i}\,\bar{\psi}\bar{\sigma}^\mu(\partial_\mu \psi)\,. \tag{1.117}$$

The resulting action is invariant under the global SUSY transformations

$$\delta\phi = \sqrt{2}\,\xi^\alpha \psi_\alpha \tag{1.118}$$

$$\delta\psi_\alpha = -\,\mathrm{i}\sqrt{2}\,(\sigma^\mu)_{\alpha\dot{\alpha}}\,\bar{\xi}^{\dot{\alpha}}\,\partial_\mu\phi\,, \tag{1.119}$$

cf. Eqns. (1.46), (1.47).

We can also write these relations in terms of two Majorana spinors

$$\psi_\mathsf{m} = \begin{pmatrix} \psi_\alpha \\ \bar{\psi}^{\dot{\alpha}} \end{pmatrix} \quad , \quad \xi_\mathsf{m} = \begin{pmatrix} \xi_\alpha \\ \bar{\xi}^{\dot{\alpha}} \end{pmatrix} \tag{1.120}$$

and two real scalar fields

$$\phi = \frac{1}{\sqrt{2}}(a + \mathrm{i}\,b) \tag{1.121}$$

as

$$\mathcal{L}_0 = \frac{1}{2}(\partial_\mu a)(\partial^\mu a) + \frac{1}{2}(\partial_\mu b)(\partial^\mu b) + \frac{\mathrm{i}}{2}\,\bar{\psi}_\mathsf{m}\gamma^\mu\,(\partial_\mu \psi_\mathsf{m}) \tag{1.122}$$

with the transformations of the fields now given by

$$\delta a = \bar{\xi}_\mathsf{m}\psi_\mathsf{m}\,, \tag{1.123}$$

$$\delta b = \mathrm{i}\,\bar{\xi}_\mathsf{m}\gamma_5\,\psi_\mathsf{m}\,, \tag{1.124}$$

$$\delta\psi_\mathsf{m} = -\,\mathrm{i}\,\gamma^\mu \xi_\mathsf{m}\,\partial_\mu(a + \mathrm{i}\gamma_5 b)\,. \tag{1.125}$$

If we now allow the transformation parameter $\xi_\mathsf{m}$ to depend on spacetime

$$\xi_\mathsf{m} \to \xi_\mathsf{m}(x)\,, \tag{1.126}$$

the action is no longer invariant under such local SUSY transformations. Rather, we get, up to a total divergence,

$$\delta\mathcal{L}_0 = (\partial_\mu \bar{\xi}_\mathsf{m})\,j^\mu\,, \tag{1.127}$$

with

$$j^\mu = \gamma^\nu \partial_\nu (a - \mathrm{i}\gamma_5 b)\,\gamma^\mu \psi_\mathsf{m}\,. \tag{1.128}$$

To compensate this extra term, we have to introduce a term in the Lagrangian which, by analogy with theories of local gauge invariance, couples the current $j^\mu$ to the "gauge field of local supersymmetry transformations", $\Psi_\mu$. Since the supersymmetry generators are spinorial quantities, this gauge field $\Psi_\mu$ turns out to be a vector spinor and it can

[19] This Lagrangian can be derived from Eqn. (1.59) by setting $W = 0$ and using integration by parts as well as Eqn. (A.30) .

be identified with the gravitino field as discussed above. Its coupling to the current $j^\mu$ is given by

$$\mathcal{L}_1 = -\frac{\kappa}{2}\bar{\Psi}_\mu j^\mu \tag{1.129}$$

with

$$\kappa = \sqrt{8\pi G} = 1/M_{\mathrm{P}} \tag{1.130}$$

and to leading order it transforms as

$$\Psi_\mu \to \Psi_\mu + \frac{2}{\kappa}\partial_\mu \xi_{\mathsf{m}} + ... \tag{1.131}$$

under local SUSY transformations. This cancels the variations $\delta\mathcal{L}_0$ and $\delta\mathcal{L}_1$ at zeroth order in $\kappa$. At next-to-leading order in $\kappa$, however, our Lagrangian is still not invariant under local SUSY transformations, but rather we have

$$\delta\mathcal{L}_1 = \mathrm{i}\,\kappa\big(\bar{\Psi}^\mu\gamma^\nu\xi_{\mathsf{m}}\,T_{\mu\nu} + ...\big) + \mathcal{O}(\kappa^2)\,, \tag{1.132}$$

where $T_{\mu\nu}$ is the energy-momentum tensor of the scalar fields $a$ and $b$. This term can now be cancelled at $\mathcal{O}(\kappa)$ by adding a coupling

$$\mathcal{L}_2 = -g_{\mu\nu}T^{\mu\nu} \tag{1.133}$$

between the graviton and the energy momentum tensor to the Lagrangian.

Continuing in this fashion and including appropriate terms for the supergravity multiplet, one finds that at $\mathcal{O}(\kappa^2)$ this procedure terminates and one ends up with a locally supersymmetric Lagrangian, coupling a chiral multiplet $(\phi, \psi_{\mathsf{m}})$ to the supergravity multiplet $(\Psi_\mu, g_{\mu\nu})$.

Since this has only been a schematic sketch of how the Noether method can be used to work out a locally supersymmetric Lagrangian, the reader is referred to the literature for further details (we can suggest e.g. [45, 49] for readable expositions on the subject).

One could now proceed with the same strategy to also include vector supermultiplets in the theory. However, since this turns out to be very tedious indeed, in practise one usually uses the more efficient local tensor calculus [64] (an introduction on the subject can e.g. be found in [53]) or the superfield formalism. We refrain from a discussion of these topics in this thesis and only state the results most relevant for us in the next sections.

### 1.6.2 SUGRA Lagrangian for Chiral and Vector Supermultiplets

In this section we summarise the most important results for the supergravity Lagrangian. For simplicity, we again work directly with component fields rather than superfields and, for example, the notation $W(\phi_i)$ indicates that we treat the superpotential as a function of the scalar fields only. That is to say, $W(\phi_i)$ is obtained from $W(\mathbf{\Phi}_i)$ by replacing all superfields by their corresponding scalar component fields

$$W(\phi_i) = W(\mathbf{\Phi}_i)\Big|_{\mathbf{\Phi}_i\to\phi_i}\,, \tag{1.134}$$

and so forth.

### SUGRA Lagrangian for Chiral Supermultiplets

Let us start with a collection of chiral superfields $\mathbf{\Phi}_i$ and their conjugates $\mathbf{\Phi}_j^\dagger$ with $i, j = 1, \dots, n$ , coupled to the supergravity multiplet in a locally supersymmetric way. It turns out that the resulting Lagrangian is fully determined by a single real function $G(\phi_i, \phi_j^*)$ of the scalar fields $\phi_i$ and their complex conjugates $\phi_j^*$, called the *Kähler function*. It is given by

$$G(\phi_i, \phi_j^*) \equiv \frac{K(\phi_i, \phi_j^*)}{M_\mathrm{P}^2} + \ln\left[\frac{\left|W(\phi_i)\right|^2}{M_\mathrm{P}^6}\right], \tag{1.135}$$

where $W(\phi_i)$ is the superpotential and the *Kähler potential* is an arbitrary real function that generalises the "kinetic terms" $K(\mathbf{\Phi}_i, \mathbf{\Phi}_j^\dagger) = \delta^{ij}\,\mathbf{\Phi}_j^\dagger \mathbf{\Phi}_i = \mathbf{\Phi}_i^\dagger \mathbf{\Phi}_i$ of the Wess-Zumino model, c.f. the footnote on page 20. This is important since SUGRA has to be regarded as an effective theory, such that non-canonical kinetic terms as well as non-renormalisable operators have in general to be included, in the Kähler potential and also in the superpotential.

The Kähler function (1.135) is invariant under

$$W \longrightarrow \mathrm{e}^{-h(\phi_i)} W\,, \tag{1.136}$$

$$M_\mathrm{P}^{-2}\, K \longrightarrow M_\mathrm{P}^{-2}\, K + h(\phi_i) + h^*(\phi_i^*)\,, \tag{1.137}$$

where $h(\phi_i)$ is an arbitrary holomorphic function of the scalar fields. This is called Kähler invariance.

In supergravity, the scalar fields can be regarded as coordinates of a special kind of complex manifold, called a Kähler manifold. The metric on this manifold, called the *Kähler metric*, is given by

$$K_{i\bar{j}} = M_\mathrm{P}^2\, G_{i\bar{j}} = \frac{\partial^2 K}{\partial\phi_i\,\partial\phi_j^*}\,, \tag{1.138}$$

where we have defined

$$\partial_i \equiv \frac{\partial}{\partial\phi_i}\,, \tag{1.139}$$

$$\partial_{\bar{j}} \equiv \frac{\partial}{\partial\phi_j^*}\,. \tag{1.140}$$

The Kähler metric is a Hermitian matrix and it determines the kinetic terms of the scalar fields

$$\alpha^{-1}\mathcal{L}_\mathrm{kin}^{(\mathrm{scalar})} = K_{i\bar{j}}\,(\partial_\mu\phi_j^*)\,(\partial^\mu\phi_i)\,, \tag{1.141}$$

with $\alpha = \sqrt{-\det g_{\mu\nu}}$.
The inverse Kähler metric $K^{i\bar{j}}$ is defined to satisfy

$$K^{i\bar{j}} K_{\bar{j}k} = \delta^i_k\,. \tag{1.142}$$

With these definitions we are now in a position to write down the scalar F-term potential of a SUGRA theory. It is given by

$$\boxed{\begin{aligned} V_F(\phi_i, \phi_j^*) &= M_{\mathrm{P}}^4 \, \mathrm{e}^G \left[ G_i \, G^{i\bar{j}} \, G_{\bar{j}} - 3 \right] \\ &= \mathrm{e}^{K/M_{\mathrm{P}}^2} \left[ \mathcal{D}_i W \, K^{i\bar{j}} \, \mathcal{D}_{\bar{j}} W^* - 3 \, M_{\mathrm{P}}^{-2} \, |W|^2 \right] \end{aligned}} \tag{1.143}$$

Here, we have defined

$$\mathcal{D}_i W \equiv W_i + M_{\mathrm{P}}^{-2} \, W \, K_i \, , \tag{1.144}$$

$$\mathcal{D}_{\bar{j}} W^* \equiv \left( \mathcal{D}_j W \right)^* . \tag{1.145}$$

Another important result we will use later on is the fermionic mass matrix for a collection of chiral superfields coupled to the SUGRA multiplet. It is given by

$$\begin{aligned} \left( \mathcal{M}_F \right)_{ij} = \mathrm{e}^{K/2M_{\mathrm{P}}^2} \Big( & W_{ij} - K^{k\bar{l}} \, K_{ij\bar{l}} \, \mathcal{D}_k W \\ & + M_{\mathrm{P}}^{-2} \, (K_{ij} \, W + K_i \, W_j + K_j \, W_i) + M_{\mathrm{P}}^{-4} \, K_i \, K_j \, W \Big) . \end{aligned} \tag{1.146}$$

### SUGRA Gauge Invariance

Let's now move on to also include vector supermultiplets and a non-trivial gauge group. In this situation there is one more holomorphic function $f_{ab}(\phi_i)$ we have to specify in order to determine the final form of the SUGRA Lagrangian. This function is called the *gauge kinetic function* since it determines the form of the kinetic terms for the gauge fields

$$\alpha^{-1} \mathcal{L}_{\mathrm{kin}}^{(\mathrm{vector})} = -\frac{1}{4} \, \mathrm{Re}(f_{ab}) \, (F^a)_{\mu\nu} (F^b)^{\mu\nu} + \frac{\mathrm{i}}{4} \, \mathrm{Im}(f_{ab}) \, (F^a)_{\mu\nu} (\tilde{F}^b)^{\mu\nu} \, , \tag{1.147}$$

where $(\tilde{F}^b)^{\mu\nu} \equiv \epsilon^{\mu\nu\sigma\tau} \, (F^b)_{\sigma\tau}$ is the dual field strength tensor and $a, b$ are group indices.

For a non-trivial gauge group, we also have D-term contributions to the scalar potential. With a gauge coupling $g$ these are given by

$$\boxed{V_D(\phi_i, \phi_j^*) = M_{\mathrm{P}}^4 \, \frac{g^2}{2} \, \mathrm{Re}(f_{ab})^{-1} \big( G_i \, (\mathcal{T}^a)_{ij} \, \phi_j \big) \big( G_k \, (\mathcal{T}^b)_{kl} \, \phi_l \big)} \tag{1.148}$$

with $G_i = \partial G / \partial \phi_i$ as above and where $\mathcal{T}^a$ are the generators of the gauge group under consideration.

This concludes our discussion of the SUGRA Lagrangian. For more details and results the reader is referred to the literature, e.g. [52, 53, 54, 45, 49] and references therein. Concerning the main part of this thesis, Eqns. (1.143) and (1.148) are the main results of this section.

### 1.6.3 Spontaneous Breaking of SUGRA

Let us conclude this section on supergravity issues by a short discussion of the possibilities to spontaneously break supergravity. Again, not considering fermionic condensates or spacetime dependent effects, what we have to do is to give VEVs to one or more scalar fields in such a way that the resulting ground state is not invariant under local supersymmetry transformations. As in global supersymmetry, there are essentially two ways of doing so, called F-term SUGRA breaking and D-term SUGRA breaking, respectively (a combination of both mechanisms is also possible). There are, however, some important differences between the spontaneous breaking of global SUSY and SUGRA, the former being a global symmetry while the latter is a local one.

First of all, whereas in the case of breaking global SUSY the resulting spectrum contains the massless Goldstino, this is not the case for breaking SUGRA spontaneously. Instead, the Goldstino gets "eaten" by the massless gravitino to give rise to a massive spin 3/2 particle. This mechanism is analogous to the case of spontaneously breaking a local gauge theory, where the vector bosons get their mass from "swallowing" some of the Goldstone bosons. It is therefore often referred to as the *super-Higgs mechanism* [65, 61, 63].

Second, in SUGRA theories, the vacuum energy is no longer positive semi-definite. In particular, there may exist ground states that spontaneously break SUGRA with a cosmological constant equal to zero $\Lambda = \langle 0|V|0\rangle = 0$. This is phenomenologically very appealing, since experiments prefer a cosmological constant very close but not equal to 0, which drives the current accelerated expansion of our universe (assuming the standard 6 parameter $\Lambda$CDM model, cf. [66]). The lack of a particle physics explanation for such a small, non-vanishing cosmological constant is usually called the *cosmological constant problem*. Reviews on this problem can e.g. be found in [67].

Third, the sum rule Eqn. (1.111) gets modified in the context of spontaneously broken local supersymmetry in such a way that it is easier to reconcile it with experimental bounds, cf. Eqn. (1.158).

Let us now turn in more detail to the two possibilities mentioned above to break SUGRA spontaneously. The first one is called F-term SUGRA breaking. It comprises giving a non-zero vacuum expectation value to (at least) one of the auxiliary component fields $F_i$ of the chiral supermultiplets of the theory [20]

$$\langle F_i\rangle = M_{\mathrm{P}}\,\langle \mathrm{e}^{G/2}\,G^{i\bar{j}}\,G_{\bar{j}}\rangle \neq 0\,. \tag{1.149}$$

In this case, the Goldstino $\eta$ is given by the linear combination

$$\eta = M_{\mathrm{P}}\langle G_i\rangle\psi_i\,. \tag{1.150}$$

Assuming a minimal Kähler potential $K = \phi_i^*\phi_i$ and defining the Majorana spinor

$$\eta_{\mathsf{m}} = \begin{pmatrix}\eta_\alpha \\ \bar{\eta}^{\dot{\alpha}}\end{pmatrix}\,, \tag{1.151}$$

[20] In SUGRA, the equation of motion for the auxiliary field $F_i$ actually yields the more complicated result $F_i = M_{\mathrm{P}}\,\mathrm{e}^{G/2}G^{i\bar{j}}G_{\bar{j}} - G^{i\bar{k}}G_{jl\bar{k}}\,\psi_j\psi_l + \frac{1}{2}\psi_i G_{\bar{j}}\psi_j +$ *gaugino terms*, but the only part that can get a VEV is the one displayed in Eqn. (1.149), cf. [49] for example.

the Goldstino can be eliminated from the theory by a redefinition of the gravitino field

$$\Psi'_\mu = \Psi_\mu - \frac{\mathrm{i}}{3\sqrt{2}}\,\gamma_\mu \eta_{\mathsf{m}} - \frac{\sqrt{2}}{3\,M_{\mathrm{P}}} \mathrm{e}^{\langle -G/2 \rangle}\, \partial_\mu \eta_{\mathsf{m}}\,. \tag{1.152}$$

This leaves us with a mass term for the redefined gravitino field (for a definition of the matrices $\Sigma^{\mu\nu}$, cf. Eqn. (A.7))

$$\mathcal{L}_{m_{3/2}} = \frac{\mathrm{i}}{2} M_{\mathrm{P}}\, \mathrm{e}^{\langle G/2 \rangle}\, \bar{\Psi}'_\mu\, \Sigma^{\mu\nu}\, \Psi'_\nu\,, \tag{1.153}$$

from which we read of

$$m_{3/2} = M_{\mathrm{P}}\, \mathrm{e}^{\langle G/2 \rangle} = M_{\mathrm{P}}^{-2}\, \langle \mathrm{e}^{K/2M_{\mathrm{P}}^2}\, |W| \rangle\,. \tag{1.154}$$

Turning to D-term SUGRA breaking, this is realised by giving a non-zero vacuum expectation value to (at least) one of the auxiliary component fields $D^a$ of the vector supermultiplets of the theory [21]

$$\langle D^a \rangle = \mathrm{i}\, g\, M_{\mathrm{P}}^2\, \langle\, \mathrm{Re}(f_{ab})^{-1}\, G_i\, (\mathcal{T}^a)_{ij}\, \phi_j\, \rangle \neq 0\,. \tag{1.155}$$

The Goldstino is now given by the linear combination

$$\eta = M_{\mathrm{P}} \langle G_i \rangle \psi_i - \frac{g}{\sqrt{2}}\, \langle \mathrm{e}^{-G/2} G_i\, (\mathcal{T}^a)_{ij}\, \phi_j \rangle \lambda_a \tag{1.156}$$

and one ends up with essentially the same results as above.

Comparing F-term SUGRA breaking and D-term SUGRA breaking we see that in both cases a necessary condition for breaking local supersymmetry reads

$$\langle G_i \rangle \neq 0 \tag{1.157}$$

for at least one $i$.

Coming back to F-term SUGRA breaking, another interesting result can be noted for the case of $N$ chiral supermultiplets coupled to the supergravity multiplet. The sum rule Eqn. (1.111) gets modified, as mentioned above. It now reads [22]

$$\mathrm{STr}\mathcal{M}^2 = 2(N-1) m_{3/2}^2\,, \tag{1.158}$$

which tells us that on average the scalar particles have to be heavier than their fermionic superpartners. This is exactly what we want from a phenomenological point of view. Combining this with the fact that in spontaneously broken supergravity the SUGRA breaking ground state can have zero cosmological constant, as opposed to global supersymmetry

[21] Again, the actual equation of motion for the auxiliary field $D^a$ is more complicated, but the part displayed is the only one that can acquire a non-zero VEV.

[22] Note that this formula can again get modified e.g. in the presence of vector supermultiplets with non-vanishing VEVs for some of the auxiliary D fields $\langle D^a \rangle \neq 0$.

where the SUSY breaking ground state always has positive energy, this scenario appears quite promising to construct a viable supersymmetric model of particle physics. It turns out, however, that the most successful applications of F-term SUGRA breaking to particle physics models are theories in which the breaking of SUGRA occurs in a hidden sector, where the breaking effects are communicated to the visible sector only through gravitational interactions. To conclude this section on supergravity let us therefore have a short look at a simple toy model of such hidden sector supergravity breaking, called the Polonyi model.

### Polonyi Model

The Polonyi superpotential is given by [68]

$$W_{\text{Polonyi}} = m^2(\mathbf{\Phi} + \beta)\,, \tag{1.159}$$

where $\mathbf{\Phi}$ is a gauge singlet chiral superfield and $m, \beta$ are real parameters with dimensions of mass. For simplicity, we assume a canonical Kähler potential $K = \mathbf{\Phi}^\dagger \mathbf{\Phi}$. Since $\mathbf{\Phi}$ is a gauge singlet superfield, the scalar potential is given by the F-term contribution alone. It reads

$$\begin{aligned} V &= M_P^4\, \mathrm{e}^G \big(G_\phi\, G^{\phi\bar\phi}\, G_{\bar\phi} - 3\big) \\ &= m^4\, \mathrm{e}^{\phi^*\phi/M_P^2} \left( \left| \frac{\phi^*(\phi+\beta)}{M_P^2} + 1 \right|^2 - \frac{3|\phi+\beta|^2}{M_P^2} \right) . \end{aligned} \tag{1.160}$$

Choosing

$$\beta = (2-\sqrt{3})M_P\,, \tag{1.161}$$

the global minimum of the potential is located at

$$\langle\phi\rangle = (\sqrt{3}-1)M_P \tag{1.162}$$

with $\langle V\rangle = 0$. In this minimum supergravity is broken since

$$\langle G_\phi\rangle = \sqrt{3}/M_P \tag{1.163}$$

and we end up with a massive gravitino with mass

$$m_{3/2} = \mathrm{e}^{(\sqrt{3}-1)^2/2}\, \frac{m^2}{M_P^2}\, M_P\,. \tag{1.164}$$

If we don't want to spoil supersymmetry as a solution to the hierarchy problem, we need the gravitino mass to lie somewhere around the electroweak scale $M_W$ since it turns out that the soft SUSY breaking parameters are of the order of the gravitino mass in this class of models. If this is the case, however, the bosonic component field of the Polonyi superfield also has a mass of the order of the gravitino mass, i.e. of the order $M_W$. The problem with such a light Polonyi field is that it typically dominates the energy density of the universe after inflation and is very long-lived, which leads to serious cosmological problems. It can for example spoil successful Big Bang Nucleosynthesis (BBN) if it decays after the onset of BBN. This is often referred to as the "Polonyi problem" [69]. Thus, the Polonyi model can only be regarded as a simple toy model and we refrain from a further discussion.

CHAPTER 2

# Supersymmetric Grand Unified Theories

After having dealt with supersymmetry in the last chapter, the next topic we are going to cover are supersymmetric grand unified theories, in short SUSY GUTs. The idea of a grand unified theory, unifying the electroweak and the strong force has been around for a long time and it still is a very active field of high energy physics.

Since one of the main goals of this thesis is to establish a possible connection between early universe cosmology (in particular inflation and leptogenesis) and particle physics and since the right-handed (s)neutrino will play a paramount role in this endeavour, we focus our discussion on SUSY GUTs that naturally include the right-handed (s)neutrino field. In particular, we discuss left-right symmetric Pati-Salam models with gauge group $G_{\text{PS}} = SU(4)_C \times SU(2)_L \times SU(2)_R$ [22] and models with gauge group $SO(10)$ [23], which are also left-right symmetric. On the other hand, we do not discuss the "classic" model with gauge group $SU(5)$ [70], since here the right-handed (s)neutrino is still a singlet and has to be introduced by hand.

We start our discussion of SUSY GUTs with the Minimal Supersymmetric Standard Model (MSSM), which does not in itself classify as a grand unified theory. It does, however, set, in a sense, the framework for supersymmetric Pati-Salam unification and $SO(10)$ GUTs, which we discuss afterwards. The discussion in this introductory chapter roughly follows the lines of [71] and further details on group theory and GUTs in general can e.g. be found in [72, 73, 74, 42] and references therein.

## 2.1 The Minimal Supersymmetric Standard Model

The MSSM [52, 75] is defined as being the globally supersymmetric extension of the Standard Model (SM) [76, 77] with the smallest number of new particles. It is obtained from the SM by promoting all of the SM fields to chiral or vector superfields [1], such that each SM particle is accompanied by its supersymmetric partner particle with the same quantum numbers under the SM gauge group $G_{\text{SM}} = SU(3)_C \times SU(2)_L \times U(1)_Y$.

[1] The SM matter fields – the quarks and leptons – belong to chiral superfields and the gauge bosons are part of vector superfields.

Additionally, we have to introduce a new up-type Higgs $\boldsymbol{h_u}$ supermultiplet that is a doublet under $SU(2)_L$. The necessity for introducing this new Higgs doublet is two-fold.

First, in the MSSM the Yukawa interactions are encoded in the superpotential $W(\Phi)$. However, since this is a holomorphic function of left-handed superfields only, we cannot, like in the SM, use the conjugate of the down-type Higgs doublet $\boldsymbol{h_d^\dagger}$ to introduce the Yukawa interactions with the up-type quarks. Instead, we have to use another left-handed chiral superfield $\boldsymbol{h_u}$ that has the opposite hypercharge as $\boldsymbol{h_d}$.

Secondly, we need this second Higgs doublet in order to guarantee anomaly cancellation: in the SM the gauge anomalies [78] (for a textbook review cf. e.g. [79]) are automatically cancelled by the fermionic field content within one family. The SM Higgs bosons on the other hand, being scalar particles, do not contribute to these anomalies. In the MSSM, the situation is different. The cancellation between the leptons and quarks within one family is still intact since their superpartners are all scalar particles. But now we also have the *fermionic* superpartners of the down-type Higgs doublet $\boldsymbol{h_d}$. Their contribution to the gauge anomalies can only be cancelled by a second set of Higgsinos with opposite hypercharge. As it turns out, the fermionic component fields of $\boldsymbol{h_u}$ have exactly the right quantum numbers to do the job.

Taking these considerations into account, the field content of the MSSM is given by Table 2.1.

| L$\chi$ Superfield | $G_{\text{SM}}$ | Spin 0 / Spin 1 | Spin 1/2 | Name |
|---|---|---|---|---|
| $\boldsymbol{q}$ | $(\mathbf{3},\mathbf{2},+1/6)$ | $\tilde{q}_L = (\tilde{u}_L, \tilde{d}_L)$ | $q_L = (u_L, d_L)$ | (s)quarks |
| $\boldsymbol{u^c}$ | $(\bar{\mathbf{3}},\mathbf{1},-2/3)$ | $\tilde{u}_R^*$ | $u_R^\dagger$ | |
| $\boldsymbol{d^c}$ | $(\bar{\mathbf{3}},\mathbf{1},+1/3)$ | $\tilde{d}_R^*$ | $d_R^\dagger$ | |
| $\boldsymbol{l}$ | $(\mathbf{1},\mathbf{2},-1/2)$ | $\tilde{l}_L = (\tilde{\nu}_L, \tilde{e}_L)$ | $l_L = (\nu_L, e_L)$ | (s)leptons |
| $\boldsymbol{e^c}$ | $(\mathbf{1},\mathbf{1},+1)$ | $\tilde{e}_R^*$ | $e_R^\dagger$ | |
| $\boldsymbol{h_u}$ | $(\mathbf{1},\mathbf{2},+1/2)$ | $h_u = (h_u^+, h_u^0)$ | $\tilde{h}_u = (\tilde{h}_u^+, \tilde{h}_u^0)$ | Higgs(inos) |
| $\boldsymbol{h_d}$ | $(\mathbf{1},\mathbf{2},-1/2)$ | $h_d = (h_d^0, h_d^-)$ | $\tilde{h}_d = (\tilde{h}_d^0, \tilde{h}_d^-)$ | |
| $\boldsymbol{G^a}$ | $(\mathbf{8},\mathbf{1},0)$ | $G^a$ | $\tilde{G}^a$ | gauge bosons |
| $\boldsymbol{W^i}$ | $(\mathbf{1},\mathbf{3},0)$ | $W^i$ | $\tilde{W}^i$ | and |
| $\boldsymbol{B^0}$ | $(\mathbf{1},\mathbf{1},0)$ | $B^0$ | $\tilde{B}^0$ | gauginos |

**Table 2.1:** Field content of the MSSM. $G_{\text{SM}} = SU(3)_C \times SU(2)_L \times U(1)_Y$ is the SM gauge group. For simplicity we only show the first family of (s)quarks and (s)leptons and colour indices are suppressed. The right-handed electrons and quarks have been introduced through their charge conjugates which are also left-handed, such that the whole theory can be described in terms of left-handed chiral superfields (L$\chi$ superfields).

The field content of the MSSM and the requirement of invariance under global supersymmetry transformations and local gauge transformations determines the form of the allowed interactions. On top of that, the MSSM is very often augmented by an additional discrete symmetry, called *R-parity* [77]. For a specific component field it is defined as

$$P_R \equiv (-1)^{3(B-L)+2s}\,, \tag{2.1}$$

where $B$ and $L$ are the baryon and lepton number of the particle under consideration, while $s$ is the particle's spin. On the superfield level, it is equivalent to another discrete symmetry called *matter parity* [80, 40, 81], defined for a given superfield as

$$P_M \equiv (-1)^{3(B-L)}\,. \tag{2.2}$$

Invariance with respect to matter-parity requires all terms in the superpotential to be $P_M$-even. This reproduces invariance under R-parity at the level of the component fields in the Lagrangian.

The introduction of this additional symmetry has two main virtues: first, it can forbid $B$- and $L$-violating operators that mediate proton decay. It can therefore only be very weakly broken [2] in order to be consistent with the latest experimental bounds on the proton lifetime [83]. Secondly, it makes the lightest supersymmetric particle – the LSP – stable. If the LSP turns out to be neutral, it is a very promising dark matter candidate.

Requiring renormalisability, the allowed terms in the superpotential of the MSSM with conserved R-parity now read

$$W_{\text{MSSM}} = y_u\, \boldsymbol{h_u}.\boldsymbol{q}.\boldsymbol{u^c} + y_d\, \boldsymbol{h_d}.\boldsymbol{q}.\boldsymbol{d^c} + y_e\, \boldsymbol{h_d}.\boldsymbol{l}.\boldsymbol{e^c} + \mu\, \boldsymbol{h_u}.\boldsymbol{h_d}\,. \tag{2.3}$$

Here, $y_u, y_d$ and $y_e$ are the up-type quark, down-type quark and charged lepton Yukawa coupling matrices and contraction of all gauge and family indices, indicated by a dot, is assumed [3]. $\mu$ is a parameter with mass dimension 1. In order to obtain electroweak symmetry breaking (s. below) at the right scale, $\mu$ should be of the order of 100 GeV. There is, however, no a priori reason why the value of $\mu$ should lie in this range. In the literature, this is referred to as the *$\mu$-problem*. A further discussion of the $\mu$-problem and possible solutions (like e.g. the NMSSM, the Kim-Nilles mechanism [84] or the Giudice-Masiero mechanism [85]) can for example be found in [51].

Up to now we have assumed that global supersymmetry is an exact symmetry of our model. However, this is clearly not realised in nature, at least not at low energies. Thus, if supersymmetry is at all a symmetry of nature, it has to be (spontaneously) broken. The exact mechanism of supersymmetry breaking is still not known. It is usually assumed that the spontaneous breaking of SUSY happens in a *hidden sector* and that the effects of this breaking are then transferred to the visible sector e.g. through gravity or gauge interactions. In the MSSM, the effects of such hidden sector SUSY breaking are parametrised

[2] A detailed account of the effects of R-parity violating couplings within the MSSM and the different bounds on these couplings can be found in [82].

[3] For example, writing out the $SU(2)_L$-indices explicitly, we have $y_u\, \boldsymbol{h_u}.\boldsymbol{q}.\boldsymbol{u^c} = y_u\, (\boldsymbol{h_u})^a\, \epsilon_{ab}\, (\boldsymbol{q})^b\, \boldsymbol{u^c}$ and so forth, where $\epsilon_{ab}$ is the Levi-Civita tensor.

via *soft supersymmetry breaking terms*. These are terms that have coupling parameters with positive mass dimension and such terms are said to be super-renormalisable since they do not introduce any new divergences. They have to be gauge invariant and respect R-parity but they break supersymmetry explicitly. In order not to spoil supersymmetry as a solution to the hierarchy problem, however, the mass scale of these soft SUSY breaking terms must not be larger than $m_{\text{soft}} \simeq \mathcal{O}(\text{TeV})$ [4]. We do not discuss the issue of supersymmetry breaking within the MSSM further. Additional information on hidden sector and soft supersymmetry breaking can for example be found in [51] and references therein.

Finally, electroweak symmetry breaking within the MSSM occurs when the neutral scalar component fields of the up- and down-type Higgs supermultiplets develop vacuum expectation values

$$\langle h_u \rangle = \begin{pmatrix} 0 \\ \langle h_u^0 \rangle \end{pmatrix} \quad , \quad \langle h_d \rangle = \begin{pmatrix} \langle h_d^0 \rangle \\ 0 \end{pmatrix} , \tag{2.4}$$

with

$$v^2 = (174\,\text{GeV})^2 = \langle h_u^0 \rangle^2 + \langle h_d^0 \rangle^2 \,. \tag{2.5}$$

The parameter $\tan\beta$ is defined as the ratio of the two VEVs

$$\tan\beta \equiv \frac{\langle h_u^0 \rangle}{\langle h_d^0 \rangle} \,. \tag{2.6}$$

The potential necessary to generate the electroweak phase transition is given by a combination of SUSY conserving F- and D-term contributions as well as contributions stemming from the soft SUSY breaking terms discussed above. The reader is again referred to the literature for further details, for example [51] and references therein.

Last, requiring the VEVs of the Higgses to be electrically neutral under the remaining $U(1)_Q$-symmetry fixes the relation between the generators of the 3$^{\text{rd}}$ weak isospin component $I_3$, the hypercharge $Y$ and the electric charge $Q$ to be

$$Q = I_3 + Y \,. \tag{2.7}$$

This concludes our discussion of the MSSM.

## 2.2 Supersymmetric Pati-Salam Models

As the next step towards unification of all known gauge interactions within one simple gauge group we discuss supersymmetric Pati-Salam unification [22] based on the gauge group

$$G_{\text{PS}} = SU(4)_C \times SU(2)_L \times SU(2)_R \,. \tag{2.8}$$

[4] It has been pointed out that strictly speaking this has only to be true for a certain part of the SUSY spectrum, namely two stops and one sbottom, two higgsinos as well as the gluino. The other particles of the SUSY spectrum can be somewhat heavier. For a much more detailed discussion we recommend [86], which also contains an extensive list of further references.

In this model, leptons and quarks are united in the same representations by treating lepton number as a fourth colour. However, the right- and left-handed quark and lepton fields still belong to different representations of $G_{\rm PS}$. A unification of all these fields within one single representation is only achieved in the next section, when we consider the gauge group $SO(10)$.

Nonetheless, the Pati-Salam model has some very attractive features, the main one for us being the fact that the right-handed (s)neutrino is automatically present in the spectrum once we decompose the representations of $G_{\rm PS}$ with respect to the SM gauge group. This is of special importance to the main line of investigation of this thesis, which tries to establish a connection between early universe cosmology and particle physics. Since the right-handed (s)neutrino will play an outstanding role in the models proposed, its automatic inclusion makes left-right symmetric theories [5] a promising and economical choice for our purpose.

### 2.2.1 Decomposition into SM Representations

In order to see how the particles of the MSSM can be embedded into representations of the Pati-Salam gauge group $G_{\rm PS}$, we have to decompose the relevant representations with respect to the SM gauge group $G_{\rm SM}$. Details of the underlying group theory can for example be found in [73]. The results are summarised in Table 2.2.

| $G_{\rm PS}$ | $G_{\rm SM}$ | Sector |
|---|---|---|
| $(\mathbf{1},\bar{\mathbf{2}},\mathbf{2})$ | $(\mathbf{1},\bar{\mathbf{2}},+1/2)\oplus(\mathbf{1},\bar{\mathbf{2}},-1/2)$ | Higgs |
| $(\mathbf{4},\mathbf{2},\mathbf{1})$ | $(\mathbf{3},\mathbf{2},+1/6)\oplus(\mathbf{1},\mathbf{2},-1/2)$ | L$\chi$ matter fields |
| $(\bar{\mathbf{4}},\mathbf{1},\bar{\mathbf{2}})$ | $(\bar{\mathbf{3}},\mathbf{1},-2/3)\oplus(\bar{\mathbf{3}},\mathbf{1},+1/3)\oplus(\mathbf{1},\mathbf{1},+1)\oplus(\mathbf{1},\mathbf{1},0)$ | R$\chi$ matter fields |
| $(\mathbf{15},\mathbf{1},\mathbf{1})$ | $(\mathbf{8},\mathbf{1},0)\oplus(\mathbf{1},\mathbf{1},0)\oplus(\mathbf{3},\mathbf{1},+2/3)\oplus(\bar{\mathbf{3}},\mathbf{1},-2/3)$ | |
| $(\mathbf{1},\mathbf{1},\mathbf{3})$ | $(\mathbf{1},\mathbf{1},0)\oplus(\mathbf{1},\mathbf{1},+1)\oplus(\mathbf{1},\mathbf{1},-1)$ | gauge fields |
| $(\mathbf{1},\mathbf{3},\mathbf{1})$ | $(\mathbf{1},\mathbf{3},0)$ | |

**Table 2.2:** Decomposition of the (low-dimensional) representations of the Pati-Salam gauge group $G_{\rm PS} = SU(4)_C \times SU(2)_L \times SU(2)_R$ with respect to the SM gauge group $G_{\rm SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$.

Comparing this with the field content of the MSSM displayed in Table 2.1 we can make the following identifications:

[5] Another example of a left-right symmetric theory that automatically includes the right-handed (s)neutrino is based on the gauge group $G_{\rm LR} = SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ [87]. Since, apart from issues concerning the breaking of the unified gauge group to the SM gauge group, the discussion regarding the main results of this thesis would not much differ from the discussion within the Pati-Salam model, we focus on the latter and do not discuss the minimal left-right symmetric model with gauge group $G_{\rm LR}$ in more detail.

- **Higgs sector**
  The two MSSM Higgs doublets can be assigned to the following representation

$$\boldsymbol{h} \sim (\mathbf{1}, \bar{\mathbf{2}}, \mathbf{2}) = \underbrace{(\mathbf{1}, \bar{\mathbf{2}}, +1/2)}_{-\epsilon\, \boldsymbol{h_u}} \oplus \underbrace{(\mathbf{1}, \bar{\mathbf{2}}, -1/2)}_{-\epsilon\, \boldsymbol{h_d}} \,. \tag{2.9}$$

  Here, we have defined $\boldsymbol{h}$ to transform as the anti-fundamental representation rather than the fundamental representation for the $SU(2)_L$-factor in order to facilitate the contraction of the corresponding gauge indices later on. We are free to make this choice, since for the group $SU(2)$ both representations are equivalent and are related by $\bar{\mathbf{2}}_a = \epsilon_{ab}\, \mathbf{2}^b$.

- **Left-handed matter superfields**
  We find that the left-handed (s)quark and (s)lepton doublets can be nicely unified into one representation as

$$\boldsymbol{L} \sim (\mathbf{4}, \mathbf{2}, \mathbf{1}) = \underbrace{(\mathbf{3}, \mathbf{2}, +1/6)}_{\boldsymbol{q}} \oplus \underbrace{(\mathbf{1}, \mathbf{2}, -1/2)}_{\boldsymbol{l}} \,. \tag{2.10}$$

- **Right-handed matter superfields**
  The right-handed (s)quarks and (s)leptons, which are singlets under the $SU(2)_L$ group, on the other hand, reside in

$$\boldsymbol{R^c} \sim (\bar{\mathbf{4}}, \mathbf{1}, \bar{\mathbf{2}}) = \underbrace{(\bar{\mathbf{3}}, \mathbf{1}, -2/3)}_{\boldsymbol{u^c}} \oplus \underbrace{(\bar{\mathbf{3}}, \mathbf{1}, +1/3)}_{\boldsymbol{d^c}} \oplus \underbrace{(\mathbf{1}, \mathbf{1}, +1)}_{\boldsymbol{e^c}} \oplus (\mathbf{1}, \mathbf{1}, 0) \,. \tag{2.11}$$

  As we can see, we are left with one SM singlet. This we identify with the aforementioned right-handed neutrino superfield

$$\boldsymbol{\nu^c} \sim (\mathbf{1}, \mathbf{1}, 0) \tag{2.12}$$

  that contains the right-handed sneutrino $\nu^c$ and the right-handed neutrino $\psi_{\nu^c}$ as component fields [6]. It can provide a solution to the problem of the small physical neutrino masses within the SM via the type I seesaw mechanism [21] (for the type II and type III seesaw mechanism, cf. [88], [89]) and it will play a crucial role in the rest of this work.

Summarising our findings so far, we have made the following assignments

$$(\boldsymbol{L})^{\beta a} = \begin{pmatrix} \boldsymbol{u_1} & \boldsymbol{u_2} & \boldsymbol{u_3} & \boldsymbol{\nu_L} \\ \boldsymbol{d_1} & \boldsymbol{d_2} & \boldsymbol{d_3} & \boldsymbol{e_L} \end{pmatrix} , \tag{2.13}$$

$$(\boldsymbol{R^c})_{\gamma k} = \begin{pmatrix} \boldsymbol{u_1^c} & \boldsymbol{u_2^c} & \boldsymbol{u_3^c} & \boldsymbol{\nu^c} \\ \boldsymbol{d_1^c} & \boldsymbol{d_2^c} & \boldsymbol{d_3^c} & \boldsymbol{e^c} \end{pmatrix} , \tag{2.14}$$

$$(\boldsymbol{h})_a^k = \begin{pmatrix} -\boldsymbol{h_u^0} & -\boldsymbol{h_d^-} \\ \boldsymbol{h_u^+} & \boldsymbol{h_d^0} \end{pmatrix} . \tag{2.15}$$

[6] In chapters 4, 5 and 8 we denote the right-handed sneutrino by $N$ and the right-handed neutrino by $\psi_N$.

Here, $\beta, \gamma = 1, ..., 4$ denote $SU(4)_C$ gauge indices, $a$ is an $SU(2)_L$-index and $k$ belongs to the group $SU(2)_R$. Upper indices denote the fundamental representation, lower ones the anti-fundamental representation. In our notation, the Higgs matrix transforms vertically as $\bar{\mathbf{2}}$ of $SU(2)_L$ and horizontally as $\mathbf{2}$ of $SU(2)_R$.

- **Gauge bosons**
  The gauge bosons can be found in the adjoint representations of the respective gauge factors. For $SU(4)_C$ this representation is 15-dimensional, whereas for $SU(2)_L$ and $SU(2)_R$ it is 3-dimensional. Looking at the decomposition of the representations in question in terms of the SM gauge group (cf. Table 2.2), we can make the following identifications

$$(\mathbf{15}, \mathbf{1}, \mathbf{1}) = \underbrace{(\mathbf{8}, \mathbf{1}, 0)}_{\boldsymbol{G^a}} \oplus \underbrace{(\mathbf{1}, \mathbf{1}, 0)}_{\boldsymbol{A^{15}}} \oplus (\mathbf{3}, \mathbf{1}, +2/3) \oplus (\bar{\mathbf{3}}, \mathbf{1}, -2/3)\,, \tag{2.16}$$

$$(\mathbf{1}, \mathbf{1}, \mathbf{3}) = \underbrace{(\mathbf{1}, \mathbf{1}, 0)}_{\boldsymbol{A^{18}}} \oplus (\mathbf{1}, \mathbf{1}, +1) \oplus (\mathbf{1}, \mathbf{1}, -1)\,, \tag{2.17}$$

$$(\mathbf{1}, \mathbf{3}, \mathbf{1}) = \underbrace{(\mathbf{1}, \mathbf{3}, 0)}_{\boldsymbol{W^i}}\,. \tag{2.18}$$

  The 8 gluons $\boldsymbol{G^a}$ and the three $SU(2)_L$ gauge bosons $\boldsymbol{W^i}$, as well as the linear combination

$$\boldsymbol{B^0} = \sqrt{\frac{2}{5}} \left( \boldsymbol{A^{15}} + \sqrt{\frac{3}{2}} \boldsymbol{A^{18}} \right)\,, \tag{2.19}$$

  corresponding to the gauge boson of $U(1)_Y$, remain massless in the breaking of $G_{\mathrm{PS}}$ to $G_{\mathrm{SM}}$. The linear combination orthogonal to $\boldsymbol{B^0}$ as well as the remaining gauge bosons, on the other hand, become massive in this phase transition.

As we can see, we have been able to nicely fit all of the MSSM particles into (low dimensional) representations of the Pati-Salam gauge group. Additionally, we have seen that the right-handed (s)neutrino field naturally appears in the spectrum if we go from $G_{\mathrm{SM}}$ to $G_{\mathrm{PS}}$. What remains to be done is to specify the superpotential and describe in more detail how $G_{\mathrm{PS}}$ is broken down to $G_{\mathrm{SM}}$. This is done in the following two paragraphs.

### 2.2.2 Spontaneous Symmetry Breaking of $G_{\mathrm{PS}}$

In this section we want to have a closer look at how the Pati-Salam gauge group is spontaneously broken down to the SM gauge group

$$G_{\mathrm{PS}} = SU(4)_C \times SU(2)_L \times SU(2)_R \longrightarrow G_{\mathrm{SM}} = SU(3)_C \times SU(2)_L \times U(1)_Y\,. \tag{2.20}$$

Since the $SU(2)_L$ factor stays intact in this breaking channel, the responsible Higgs supermultiplet(s) have to transform non-trivially only under $SU(4)_C$ and $SU(2)_R$. The lowest dimensional representations that fulfil this requirement are

$$\boldsymbol{H^c} = \begin{pmatrix} \boldsymbol{u^c_H} & \boldsymbol{u^c_H} & \boldsymbol{u^c_H} & \boldsymbol{\nu^c_H} \\ \boldsymbol{d^c_H} & \boldsymbol{d^c_H} & \boldsymbol{d^c_H} & \boldsymbol{e^c_H} \end{pmatrix} \sim (\bar{\mathbf{4}}, \mathbf{1}, \bar{\mathbf{2}})\,, \tag{2.21}$$

$$\bar{\boldsymbol{H}}^{\boldsymbol{c}} = \begin{pmatrix} \bar{\boldsymbol{u}}^{\boldsymbol{c}}_{\boldsymbol{H}} & \bar{\boldsymbol{u}}^{\boldsymbol{c}}_{\boldsymbol{H}} & \bar{\boldsymbol{u}}^{\boldsymbol{c}}_{\boldsymbol{H}} & \bar{\boldsymbol{\nu}}^{\boldsymbol{c}}_{\boldsymbol{H}} \\ \bar{\boldsymbol{d}}^{\boldsymbol{c}}_{\boldsymbol{H}} & \bar{\boldsymbol{d}}^{\boldsymbol{c}}_{\boldsymbol{H}} & \bar{\boldsymbol{d}}^{\boldsymbol{c}}_{\boldsymbol{H}} & \bar{\boldsymbol{e}}^{\boldsymbol{c}}_{\boldsymbol{H}} \end{pmatrix} \sim (\mathbf{4}, \mathbf{1}, \mathbf{2})\,, \tag{2.22}$$

and once the scalar component fields develop vacuum expectation values in the right-handed neutrino direction

$$\langle H^c \rangle = \begin{pmatrix} 0 & 0 & 0 & \langle \nu^c_H \rangle \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \langle \bar{H}^c \rangle = \begin{pmatrix} 0 & 0 & 0 & \langle \bar{\nu}^c_H \rangle \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{2.23}$$

the Pati-Salam gauge group is indeed broken down to the SM gauge group. In chapter 7 we construct an explicit model that realises this breaking scenario by forcing the fields $H^c$ and $\bar{H}^c$ to develop their VEVs in the desired direction in the waterfall phase transition that ends inflation.

Once the symmetry is broken from $G_{\rm PS}$ to $G_{\rm SM}$, only the gauge bosons corresponding to the remaining unbroken generators remain massless. These are the 8 gluons $\boldsymbol{G^a}$, the three gauge bosons $\boldsymbol{W^i}$ corresponding to the unbroken $SU(2)_L$ symmetry, and the one gauge boson for the unbroken generator of $U(1)_Y$-hypercharge. The latter one is required to give zero hypercharge when acting on the vacuum state Eqn. (2.23)

$$Y \langle H^c \rangle = Y \langle \bar{H}^c \rangle = 0\,. \tag{2.24}$$

This fixes its form and it is given by

$$Y = \sqrt{\frac{2}{3}}\,\mathcal{T}^{15} + \mathcal{T}^{18}\,. \tag{2.25}$$

Explicit calculations of the masses of the gauge bosons and the conventions for the generators we use can be found in Appendix B.

### 2.2.3 Pati-Salam Superpotential

Let us conclude our discussion of supersymmetric Pati-Salam models with a few comments about the possible form of the superpotential. We do not address the issue of supersymmetry breaking here, so only supersymmetric interactions are allowed. If we furthermore limit ourselves to renormalisable interactions for simplicity, the allowed terms for the chiral supermultiplets introduced so far read

$$W_{\rm PS} = y\, \boldsymbol{L}.\boldsymbol{h}.\boldsymbol{R^c} + \frac{1}{2}\,\mu\, \boldsymbol{h}.\boldsymbol{h}\,, \tag{2.26}$$

plus possible terms containing the Pati-Salam symmetry breaking Higgs fields $\boldsymbol{H^c}, \bar{\boldsymbol{H}}^{\boldsymbol{c}}$.

After breaking $G_{\mathrm{PS}}$ to $G_{\mathrm{SM}}$, the superpotential (2.26) reproduces the MSSM superpotential (2.3) plus one additional term, which reads

$$W_\nu = y_\nu \, \boldsymbol{h_u}.\boldsymbol{l}.\boldsymbol{\nu^c} \,. \tag{2.27}$$

This term gives rise to Dirac mass terms for the SM neutrinos of the order

$$m_D \simeq y_\nu \langle h_u \rangle \simeq y_\nu \cdot \mathcal{O}(100\,\mathrm{GeV}) \,. \tag{2.28}$$

We see that we would need very small neutrino Yukawa couplings to get the masses for the active neutrinos right if this were the full story. However, as already mentioned before, the MSSM is typically regarded only as an effective low energy approximation to some more fundamental theory at higher energies. In such a case one also has to consider non-renormalisable operators in the superpotential. These operators typically are suppressed by a high energy scale, which roughly corresponds to the mass of the heavy particles that are integrated out to obtain the operators.

In particular, in the class of models we are going to study, we encounter operators of the form [7]

$$W_\nu = \frac{\lambda}{\Lambda} \langle H \rangle^2 \boldsymbol{\nu^c} \boldsymbol{\nu^c} \,, \tag{2.29}$$

with, for example (cf. e.g. our model in chapter 5),

$$\frac{\lambda}{\Lambda} \approx \frac{0.01}{M_{\mathrm{P}}} \quad , \quad \langle H \rangle \approx 3 \cdot 10^{-3} M_{\mathrm{P}} \,. \tag{2.30}$$

This yields a Majorana mass for the right-handed neutrino [8] roughly of the order

$$m_M \approx \mathcal{O}(10^{11}\,\mathrm{GeV}) \,. \tag{2.31}$$

In the presence of such a Majorana mass term, the complete neutrino mass matrix now reads

$$m_\nu = \begin{pmatrix} 0 & m_D \\ m_D & m_M \end{pmatrix} \tag{2.32}$$

with $m_M$ much larger that $m_D$. Upon diagonalisation we end up with one heavy neutrino with mass

$$m_{\mathrm{heavy}} \approx m_M \,, \tag{2.33}$$

which is predominantly made up of the right-handed neutrino $\psi_{\nu^c}$ and one very light neutrino with mass

$$m_{\mathrm{light}} \approx \frac{m_D^2}{m_M} \,, \tag{2.34}$$

[7] Operators of this form also quite naturally appear in other classes of models not necessarily connected to early universe cosmology.

[8] For the sake of simplicity, we discuss the type I Seesaw mechanism [21] with one left-handed and one right-handed neutrino only. A generalisation to more than one family is straightforward. Pedagogical review articles on the subject of neutrino masses include for example [90].

which is predominantly made up of the active left-handed SM neutrino $\nu_L$. This is called the *type I seesaw mechanism* (cf. above) and it is now possible to get the right masses for the SM neutrinos with moderate values of the neutrino Yukawa couplings (of the same order of magnitude as, say, the electron Yukawa coupling).

## 2.3 Supersymmetric $SO(10)$ Unification

Let us now go even one step further and unify all of the MSSM matter superfields as well as the right-handed (s)neutrino in one single representation, the **16** spinor representation of $SO(10)$ [23]. Furthermore, all the gauge bosons and gauginos are unified in another single $SO(10)$ representation, the adjoint **45** of $SO(10)$, and we have a gauge group that consists of just one factor, with one gauge coupling. In this sense the supersymmetric $SO(10)$ model truly deserves its name as a grand unified theory.

Before looking in more detail at the embedding of the MSSM fields within representations of $SO(10)$ let us quickly say a few words about the breaking of the $SO(10)$ symmetry. Since we encounter $SO(10)$ unification only in the context of inflationary model building in section 7.3 and since in that model the $SO(10)$ group is assumed to be already broken down to $G_{\text{PS}}$ during inflation, we assume this scenario here. In other words we assume the following breaking pattern

$$SO(10) \xrightarrow{\text{e.g. } \mathbf{54}} G_{\text{PS}} \xrightarrow{\boldsymbol{H^c},\bar{\boldsymbol{H}}^{\boldsymbol{c}}} G_{\text{SM}}\,, \tag{2.35}$$

where the first stage of spontaneous symmetry breaking (e.g. via the VEV of a **54** of $SO(10)$) has already happened before inflation and we ignore possible effects of this breaking on the subsequent phase of inflation. We also assume that the *doublet-triplet splitting problem* (cf. below) is resolved in this stage of the breaking chain.

Assuming this breaking scenario, we now proceed to show how the representations of the Pati-Salam gauge group containing the superfields we are interested in can be embedded into representations of the gauge group $SO(10)$. These representations of $G_{\text{PS}}$ in turn contain the well-known MSSM superfields, cf. Table 2.1.

- **Matter fields**
  The Pati-Salam supermultiplets $\boldsymbol{L}$ and $\boldsymbol{R^c}$ containing all of the MSSM matter fields of one family can be nicely unified into one single representation of SUSY $SO(10)$

$$\boldsymbol{F} \sim \mathbf{16} = \underbrace{(\mathbf{4},\mathbf{2},\mathbf{1})}_{\boldsymbol{L}} \oplus \underbrace{(\bar{\mathbf{4}},\mathbf{1},\bar{\mathbf{2}})}_{\boldsymbol{R^c}}\,. \tag{2.36}$$

- **Gauge Bosons**
  All of the Pati-Salam gauge bosons, on the other hand, fit into the adjoint representation of $SO(10)$, which decomposes as

$$\boldsymbol{A} \sim \mathbf{45} = \underbrace{(\mathbf{15},\mathbf{1},\mathbf{1})}_{SU(4)_C} \oplus \underbrace{(\mathbf{1},\mathbf{3},\mathbf{1})}_{SU(2)_L} \oplus \underbrace{(\mathbf{1},\mathbf{1},\mathbf{3})}_{SU(2)_R} \oplus (\mathbf{6},\mathbf{2},\mathbf{2})\,. \tag{2.37}$$

Here, we have indicated to which factor of $G_{\text{PS}}$ the corresponding representations belong. The Pati-Salam gauge bosons in turn contain the MSSM gauge bosons as described in section 2.2.2. The gauge bosons residing in the $(\mathbf{6},\mathbf{2},\mathbf{2})$, on the other hand, get GUT-scale masses in the breaking of $SO(10)$ to $G_{\text{PS}}$.

- **MSSM Higgses**
  The $(\mathbf{1},\mathbf{2},\mathbf{2})$-representation of $G_{\text{PS}}$, which contains both the MSSM up-type Higgs and down-type Higgs doublets, fits into the following representation of $SO(10)$ [9]

$$\boldsymbol{h_{10}} \sim \mathbf{10} = \underbrace{(\mathbf{1},\mathbf{2},\mathbf{2})}_{\boldsymbol{h}} \oplus (\mathbf{6},\mathbf{1},\mathbf{1})\,. \tag{2.38}$$

  Here a potential problem of $SO(10)$ unification emerges: the additional degrees of freedom contained in the $(\mathbf{6},\mathbf{1},\mathbf{1})$ include colour triplets under $G_{\text{SM}}$, which would lead to rapid proton decay. This can only be prevented by making these colour-triplets very heavy (which suppresses the dangerous operators responsible for proton decay), while keeping the MSSM Higgses sufficiently light. This problem is referred to as the *doublet-triplet splitting problem* in the literature and as already mentioned above we assume that a solution to this problem is provided in the first stage of the breaking chain Eqn. (2.35). A more detailed discussion of the doublet-triplet splitting problem within supersymmetric GUTs and possible solutions (like e.g. the *Dimopoulos-Wilczek mechanism* [91], cf. also [92]) can for example be found in [74, 93].

With these definitions and sticking to renormalisable operators, the allowed terms in the superpotential read

$$W = y\,\boldsymbol{F}.\boldsymbol{h_{10}}.\boldsymbol{F} + \frac{1}{2}\,\mu\,\boldsymbol{h_{10}}.\boldsymbol{h_{10}}\,. \tag{2.39}$$

After the breaking to $G_{\text{PS}}$ and then to $G_{\text{SM}}$ this superpotential reproduces the Yukawa couplings and the $\mu$-term, cf. Eqns. (2.26) and (2.3).

We can also embed the Higgs fields responsible for the breaking of $G_{\text{PS}}$ to $G_{\text{SM}}$ into two additional spinor representations $\mathbf{16}$ and $\overline{\mathbf{16}}$. We call these representations $\boldsymbol{H_{16}}$ and $\bar{\boldsymbol{H}}_{\boldsymbol{16}}$, respectively. Given that they have to get VEVs in the right-handed neutrino direction in order to break to the right SM vacuum, operators of the form

$$\frac{\lambda}{\Lambda}\,\boldsymbol{F}.\bar{\boldsymbol{H}}_{\boldsymbol{16}}\,\boldsymbol{F}.\bar{\boldsymbol{H}}_{\boldsymbol{16}}\,, \tag{2.40}$$

again yield a Majorana mass term for the right-handed neutrinos, which together with the superpotential Eqn. (2.39) gives small masses to the active SM neutrinos through the type I Seesaw mechanism. Since these same operators also play a crucial role in the models of inflation we are going to study in chapter 7, they can provide a link between particle

[9] Remember that for $SU(2)$, $\mathbf{2}$ and $\bar{\mathbf{2}}$ are equivalent. Here, we use the fundamental representation $\mathbf{2}$ for the $SU(2)_L$ factor for convenience, in contrast to section 2.2.1, where we used the anti-fundamental representation $\bar{\mathbf{2}}$.

| Name | $SO(10)$ | $G_{\text{PS}}$ Decomp. | Contains MSSM Fields | Sector |
|---|---|---|---|---|
| $\boldsymbol{F}$ | **16** | $(\mathbf{4},\mathbf{2},\mathbf{1})$ | $\boldsymbol{q},\boldsymbol{l}$ | Matter - |
| | | $(\bar{\mathbf{4}},\mathbf{1},\bar{\mathbf{2}})$ | $\boldsymbol{u^c},\boldsymbol{d^c},\boldsymbol{l^c},\boldsymbol{\nu^c}$ | (Super)Fields |
| $\boldsymbol{A}$ | **45** | $(\mathbf{15},\mathbf{1},\mathbf{1})$ | $\boldsymbol{G^a},\boldsymbol{A^{15}}$ | |
| | | $(\mathbf{1},\mathbf{1},\mathbf{3})$ | $\boldsymbol{A^{18}}$ | Gauge Bosons & |
| | | $(\mathbf{1},\mathbf{3},\mathbf{1})$ | $\boldsymbol{W^+},\boldsymbol{W^0},\boldsymbol{W^-}$ | Gauginos |
| | | $(\mathbf{6},\mathbf{2},\mathbf{2})$ | | |
| $\boldsymbol{h_{10}}$ | **10** | $(\mathbf{1},\mathbf{2},\mathbf{2})$ | $\boldsymbol{h_u},\boldsymbol{h_d}$ | Higgs Bosons & |
| | | $(\mathbf{6},\mathbf{1},\mathbf{1})$ | | Higgsinos |

**Table 2.3:** Decomposition of some $SO(10)$ representations with respect to $G_{\text{PS}}$ and the MSSM fields that are embedded within these representations. For simplicity we only show the first family of matter fields. Colour indices are suppressed.

physics on the one hand and early universe cosmology on the other hand. The study of such possible connections is one of the main subjects of this thesis.

In section 7.1.1, we explicitly show how such operators can be generated by integrating out heavy messenger particles.

# CHAPTER 3
# Cosmological Inflation

After having introduced the necessary particle physics tools and models in the previous two chapters we now turn our attention to cosmology. In particular, we are going to discuss two very important events in the history of our universe: cosmological inflation and the generation of the baryon asymmetry. The first, inflation, was originally introduced by Starobinsky and Guth [7] and later in a modified version (called slow-roll inflation) also by Linde, Albrecht and Steinhardt [12, 13] to solve what is commonly referred to as the horizon and flatness problems of the Standard Hot Big Bang Model (SBB). It has since developed into an integral part of our understanding of the early universe, as inflation can, for example, also account for the origin of the large-scale structure we observe in the universe today. The second, baryogenesis, addresses the question of how we can live in a universe filled with matter but no appreciable amount of antimatter given that our universe started out in a state that is symmetric between the two.

In this chapter we focus on inflation, starting with a discussion of the issues that motivated its introduction and then discussing how inflation works in its most popular manifestation, slow-roll inflation. After that we survey the zoology of inflationary models and address some of the problems connected to the embedding of inflationary models within supergravity.

The motivation for inflation, the basics of slow-roll inflation and the different types of inflationary models are discussed in a large number of review articles and text books. The ones we have mainly consulted include [94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104].

Baryogenesis and in particular baryogenesis through leptogenesis are discussed in the next chapter.

## 3.1 The Problem of Initial Conditions

First, we want to discuss in more detail the problems that led to the idea of inflation and sketch the way inflation actually solves these problems. When talking about the Standard Hot Big Bang Model in this section, we always mean a matter or radiation dominated universe, that expands and cools as it evolves, but does *not* include a stage of accelerated expansion as part of its history.

### The Horizon Problem

The first of these problems is called the *horizon problem* and it is connected to the homogeneity and isotropy of the *Cosmic Microwave Background Radiation* (CMBR) that we can observe today and that fills up the entire sky.

This radiation, which has a temperature of [105]

$$T_{\mathrm{CMB}} = 2.72548 \pm 0.00057\,\mathrm{K} \tag{3.1}$$

and exhibits a nearly perfect black body spectrum, is incredibly homogeneous and isotropic [106, 6]. This poses a serious problem for the SBB model, since it turns out that in a universe with a normal thermal history without a phase of accelerated expansion, the radiation reaching us today from different directions in the sky must have originated from regions of spacetime that could never have been in causal contact at the time the radiation was emitted. How, then, can it be so extremely homogeneous and isotropic? Let us quantify this problem to show its severity.

The CMB radiation was created everywhere in the universe in an event commonly termed *last scattering* at a redshift of

$$1 + z_{\mathrm{ls}} \equiv \frac{a(t_0)}{a(t_{\mathrm{ls}})} \approx 1090\,, \tag{3.2}$$

approximately 378000 years after the Big Bang [6, 66]. However, the radiation we receive at earth today originated from a spherical surface around the earth, called the *surface of last scattering*. The radius $r_{\mathrm{ls}}$ of this surface increased from the time of last scattering until today, following the expansion of the universe

$$r_{\mathrm{ls}} \propto a(t)\,. \tag{3.3}$$

Since the CMBR is visible to us today, the current radius of this surface must be smaller that the radius of the currently visible universe, roughly given by the Hubble distance

$$r_{\mathrm{visible}} \simeq \mathcal{H}^{-1}\,. \tag{3.4}$$

However, for a universe filled with "ordinary matter" described by an equation of state with $1 + 3\omega > 0$ (cf. Appendix C for more details), the horizon distance $r_{\mathrm{visible}}$ expands faster than the distance between two comoving points following the expansion of the universe. Thus, more and more parts of the universe "fall inside our horizon" as the universe evolves. In particular

$$r_{\mathrm{visible}} \propto t\,, \tag{3.5}$$

$$r_{\mathrm{ls}} \propto t^f\,, \tag{3.6}$$

with $0 < f < 1$ for matter or radiation dominance. Going back in time, on the other hand, this means that what is visible to us today, and thus is in causal contact, must not necessarily have been in causal contact in the past. This is in particular true for different regions on the surface of last scattering. Extrapolating back what we know about our universe – its age, its energy content and the time of last scattering – we find that at the creation of the CMB radiation, the surface of last scattering consisted of a huge number of causally disconnected regions. How many? Viewed from earth, the horizon size at the time of last scattering subtended only an angle of about 1 degree on the surface of last scattering [102].

The solution inflation provides to this problem is that inflation can stretch a causal patch that initially lay well inside the horizon to a size much large than the horizon, large enough to encompass the whole surface of last scattering and thereby establishing causal contact on it [1].

### The Flatness Problem

The second problem is connected to the geometry of our universe. We know from observation that today our universe is almost perfectly spatially flat [6]

$$\Omega_0 = 1.02 \pm 0.02\,, \tag{3.7}$$

cf. Eqn. (C.19). However, from the Friedmann equation (C.18) we get

$$\frac{\mathrm{d}|\Omega - 1|}{\mathrm{d}\ln a} > 0 \quad \text{for} \quad 1 + 3\omega > 0\,, \tag{3.8}$$

which holds, in particular, for a matter or radiation dominated universe. Extrapolating back e.g. to the time of last scattering or Big Bang Nucleosynthesis (BBN), respectively, the geometry of our universe must have been incredibly fine-tuned [102],

$$|\Omega_{\mathrm{ls}} - 1| < 0.0004\,, \tag{3.9}$$

$$|\Omega_{\mathrm{BBN}} - 1| < 10^{-12}\,, \tag{3.10}$$

to end up with the universe we see today. Why should the universe have started out in such a very special state?

An answer to this *flatness problem* is again given by a phase of accelerated expansion of the universe, because during such a phase the geometry of our universe is actually driven *towards* flatness, not away from it. This can again be seen directly from the Friedmann equation (C.18), because during inflation the comoving Hubble radius $(a\mathcal{H})^{-1}$ decreases [2], such that any "curvature term" on the right hand side of Eqn. (C.18) becomes negligible.

### Topological Defects

Another problem that inflation can solve is the problem of topological defects (also called the monopole problem on occasion, because monopoles turn out to be particularly dangerous). These topological defects can be generated e.g. during phase transitions in the early

[1] As we will see shortly, during inflation the physical horizon size stays almost constant whereas the physical distance between comoving observers grows exponentially, cf. Eqns. (3.20).

[2] During inflation $\ddot{a} > 0$ by definition. Thus, as $\dot{a} > 0$ increases, $(a\mathcal{H})^{-1} = \dot{a}^{-1}$ decreases.

universe when a GUT symmetry is broken down spontaneously to a smaller symmetry group [18, 107].

Whether or not and what kinds of topological defects are generated in such a symmetry breaking phase transition depends on group-theoretical issues [3] as well as the exact mechanism that governs the phase transition [4].

In most of the standard GUT scenarios, however, these topological defects are quite copiously produced and an estimate shows that they would give rise to a catastrophically large contribution to the energy density of the universe $\Omega_{\rm top} \gg 1$ (cf. for example [96] for a detailed discussion). This would dramatically over-close the universe and make it re-collapse onto itself, which seems to rule out most of the GUT scenarios.

If, however, these phase transitions take place before inflation, then during inflation the energy density contribution of the produced defects can be diluted to an acceptably low value $\Omega_{\rm top} \ll 1$. For GUT model building, on the other hand, this means that any phase transition that happens after inflation must not produce additional topological defects [5].

For a more detailed discussion of topological defects in the early universe we refer the reader to [111, 96, 100] and references therein.

### Initial Perturbations

The last point we want to discuss as a motivation for inflation is the question what set the initial conditions for the structure formation in our universe. Whereas it might seem that after inflation the universe is a pretty boring place, void of any structure, this is actually not the case. While it is true that inflation tends to wash out any localised structure or energy density perturbation present before the onset of inflation, at the same time it provides its own mechanism to generate such energy density perturbations: during inflation the quantum fluctuations of the inflaton field can get stretched across the horizon and become classical quantities [8]. These field perturbations are coupled to the spacetime metric, leading to gravitational perturbations of the homogeneous background metric, which in turn lead to temperature anisotropies in the CMB radiation and later on to the formation of the large scale structure in our universe.

The fact that these perturbations can be computed for a given inflationary model and be compared to experimental data on the temperature anisotropies of the CMB, showing an astonishing agreement between theory and experiment for a wide class of inflationary models, firmly established inflation as part of our understanding of early universe cosmology. As a matter of fact, inflation is the only known mechanism that can naturally reproduce the peak structure of the CMB power spectrum (and simultaneously

[3] To be more precise, it depends on the fact whether or not one of the homotopy groups $\pi_i(\mathcal{M})$ of the vacuum manifold $\mathcal{M}$ is non-trivial. The vacuum manifold is given by the quotient group $\mathcal{M} = G/H$, where G the group that is spontaneously broken down to $H \subset G$.

[4] In section 7.1 we construct a model that avoids the production of topological defects in such a phase transition, notwithstanding the fact that they would be allowed from a group-theoretical perspective.

[5] Cosmic strings might be allowed to a certain extend and it was even thought that they might help to push the preferred value of the spectral index $n_s$ towards (or even above) $n_s = 1$, cf. e.g. [108, 109]. According to a more recent analysis [110], however, a scale invariant initial perturbation spectrum with $n_s = 1$ is now disfavoured at $2.4\sigma$ even if strings are present.

solve the horizon, flatness, and monopole problems). This comes about because once the fluctuation modes have been stretched across the horizon and become classical quantities, the all re-enter the horizon in phase and their interference pattern produces the peak structure in the CMB power spectrum. A more thorough discussion of this very interesting topic can be found in [112, 104].

In the remainder of this chapter, we discuss these points in more detail.

## 3.2 Slow-Roll Inflation

In this section we want to discuss in more detail the mechanism that lies behind inflation. By inflation we mean a stage of accelerated expansion of the universe with $\ddot{a} > 0$. The most common way inflation is implemented in a quantum field theoretical framework is called *slow-roll inflation* [12, 13], where a scalar field whose potential energy dominates over its kinetic energy drives inflation. The name slow-roll stems from the fact that in a semi-classical approximation, the field is slowly rolling down its potential.

To see how this works in detail let us recall from the discussion in Appendix C that the evolution of the scale factor in a homogeneous, isotropic universe described by the Friedmann-Robertson-Walker metric (C.1), filled with a perfect fluid is governed by the Friedmann Eqns. (C.14), (C.15).

For a scalar field $\phi$ coupled to the FRW metric $g_{\mu\nu}$ through the action

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{1}{2}\, g^{\mu\nu}\, \partial_\mu\phi\, \partial_\nu\phi - V(\phi)\right) , \tag{3.11}$$

the energy momentum tensor is given by

$$T^\mu{}_\nu = \partial^\mu\phi\, \partial_\nu\phi - \mathcal{L}(\phi, \partial_\mu\phi)\, \delta^\mu{}_\nu\,. \tag{3.12}$$

Focusing on the zero mode of the scalar field with $\nabla_i\,\phi = 0$ in a semi-classical approximation, the energy-momentum tensor takes on the form of a perfect fluid

$$T^\mu{}_\nu = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & -p \end{pmatrix} \tag{3.13}$$

with

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)\,, \tag{3.14}$$

$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi)\,. \tag{3.15}$$

Assuming a spatially flat universe with $k = 0$ (which is very well justified after the first few e-folds of inflation), the evolution of the scalar field is described by its equation of motion

$$\ddot{\phi} + 3\,\mathcal{H}\,\dot{\phi} + \frac{\mathrm{d}V(\phi)}{\mathrm{d}\phi} = 0\,, \tag{3.16}$$

with

$$\mathcal{H}^2 = \frac{1}{3M_{\mathrm{P}}^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right) . \tag{3.17}$$

In the slow-roll regime with

$$V(\phi) \gg \frac{1}{2}\dot{\phi}^2 , \tag{3.18}$$

the scalar field resembles a perfect fluid with equation of state

$$\omega_\phi \approx -1 , \tag{3.19}$$

leading to an (quasi)exponential expansion of the universe with

$$a(t) \simeq a(t_0)\, \mathrm{e}^{\mathcal{H}t} . \tag{3.20}$$

Here, the Hubble parameter is nearly constant and related to the potential energy of the scalar field via

$$\mathcal{H} \simeq \sqrt{\frac{V(\phi)}{3M_{\mathrm{P}}^2}} . \tag{3.21}$$

In this approximation, and furthermore requiring that

$$|\ddot{\phi}| \ll |3\,\mathcal{H}\,\dot{\phi}| \tag{3.22}$$

in order to fulfil the slow-roll condition Eqn. (3.18) for a sufficiently long period of time, the equation of motion of the scalar field simplifies to

$$3\,\mathcal{H}\,\dot{\phi} + V'(\phi) \simeq 0 , \tag{3.23}$$

where a prime denotes a derivative with respect to the scalar field $\phi$.

Defining the slow-roll parameters

$$\epsilon \equiv \frac{M_{\mathrm{P}}^2}{2}\left(\frac{V'}{V}\right)^2 \tag{3.24}$$

$$\eta \equiv M_{\mathrm{P}}^2\left(\frac{V''}{V}\right) , \tag{3.25}$$

the slow-roll conditions Eqns. (3.18), (3.22) imply

$$\epsilon \ll 1 \quad , \quad |\eta| \ll 1 . \tag{3.26}$$

We also need the parameter

$$\xi^2 \equiv M_{\mathrm{P}}^4\left(\frac{V'V'''}{V^2}\right) \tag{3.27}$$

when we compute the predictions for the running of the spectral index in section 3.3.2.

As already indicated, the second slow-roll condition Eqn. (3.22) is needed to sustain the exponential expansion of the universe for a long enough period of time. Long enough in this case means long enough to solve the problems discussed at the beginning of this chapter. In order to quantify this statement one usually defines the *number of e-folds* between the onset and the end of inflation to be

$$\mathcal{N} \equiv \ln \frac{a(t_e)}{a(t_i)} , \tag{3.28}$$

which, in the slow-roll approximation, can be rewritten as

$$\mathcal{N} = \int_{t_i}^{t_e} \mathcal{H}\, \mathrm{d}t \simeq M_{\mathrm{P}}^{-2} \int_{\phi_e}^{\phi_i} \frac{V(\phi)}{V'(\phi)}\, \mathrm{d}\phi \, = M_{\mathrm{P}}^{-1} \int_{\phi_e}^{\phi_i} \frac{1}{\sqrt{2\epsilon}}\, \mathrm{d}\phi \, , \tag{3.29}$$

where $\phi_i$ and $\phi_e$ are defined to be the field values of $\phi$ at the onset and the end of inflation, respectively. The end of inflation occurs when one of the slow-roll conditions gets violated, which is equivalent to one of the slow-roll parameters becoming $\mathcal{O}(1)$ [6].

According to [94], around 70 e-folds are enough to solve all the problems mentioned in section 3.1.

## 3.3 Quantum Fluctuations of the Inflaton Field

So far we have only studied the dynamics of the homogeneous part of the inflaton field $\phi$ and the corresponding evolution of the homogeneous and isotropic background spacetime. In this section we want to investigate the behaviour of the *quantum fluctuations* of the inflaton field during and after slow-roll inflation. Since these fluctuations of the inflaton field lead to fluctuations in the energy-momentum tensor, which are themselves coupled to the spacetime metric through Einstein's equations, we also have to include metric fluctuations to be self-consistent. In our discussion we mainly follow [104], where additional information and more detailed computations can be found.

We will find that the most important quantity in our discussion is the *comoving curvature perturbation* $\mathcal{R}$ [113]. This is a gauge-invariant quantity which for a given scale is practically constant once this scale leaves the horizon. It measures the gravitational potential on comoving hypersurfaces where $\delta\phi = 0$ (cf. e.g. the discussion in [98]). $\mathcal{R}$ can be related to the temperature fluctuations in the CMBR via what is called a transfer function and is thus a direct link between theoretical models of inflation and experimental data [7]. The main results of this section are a number of expressions giving the amplitude of $\mathcal{R}$ and its most important statistical properties in terms of the inflaton potential and the slow-roll parameters Eqns. (3.24), (3.25) and (3.27).

[6] More strictly speaking, inflation ends when the kinetic energy becomes comparable to the potential energy, which happens when $\epsilon_{\mathcal{H}} = -\dot{\mathcal{H}}/\mathcal{H}^2$ becomes $\mathcal{O}(1)$. Within the slow-roll regime, however, this is equivalent to the conditions stated above.

[7] More on transfer functions and how theoretical predictions can be related to measurements of the CMB radiation can e.g. be found in [114].

Let us begin our study of cosmological perturbations for single field inflationary models [8] by decomposing the full inflaton field $\phi(t,\boldsymbol{x})$ into a homogeneous background field $\phi(t)$, whose dynamics we studied in the previous section 3.2, and spacetime-dependent fluctuations $\delta\phi(t,\boldsymbol{x})$

$$\phi(t,\boldsymbol{x}) = \phi(t) + \delta\phi(t,\boldsymbol{x})\,. \tag{3.30}$$

The fluctuations $\delta\phi(t,\boldsymbol{x})$ lead to fluctuations $\delta T_{\mu\nu}$ in the energy-momentum tensor, which are coupled to fluctuations in the spacetime metric via Einstein's equations

$$\delta G_{\mu\nu} = M_{\mathrm{P}}^{-2}\,\delta T_{\mu\nu}\,, \tag{3.31}$$

where for small fluctuations it is enough to consider the linearised equations.

At this point we have to make a few comments about gauge invariance. Working with the perturbed spacetime metric we are now in a situation where there isn't a preferred coordinate system with respect to which we can define the perturbation of, let's say, the energy density of the scalar field. At any rate, we are always allowed to make a general coordinate transformation. (We could also have done this in our discussion of the homogeneous field in the previous section to make things more complicated.) Such general coordinate transformations, however, can lead to *unphysical* effects. To avoid such effects, one best works with quantities that are invariant under such coordinate transformations [115, 116]. The comoving curvature perturbation $\mathcal{R}$ we are interested in is one such gauge invariant quantity.

To proceed, let us write the perturbed metric as [115]

$$\begin{aligned}\mathrm{d}s^2 &= g_{\mu\nu}\,\mathrm{d}x^\mu \mathrm{d}x^\nu \\ &= (1+2\Phi)\,\mathrm{d}t^2 - 2\,a(t)\,B_i\,\mathrm{d}x^i\mathrm{d}t - a(t)^2\left[(1-2\Psi)\,\delta_{ij} + E_{ij}\right]\mathrm{d}x^i\mathrm{d}x^j\,,\end{aligned} \tag{3.32}$$

which is a spatially flat FRW background metric with metric fluctuations superimposed. These fluctuations can be decomposed into scalar, vector, and tensor components as

$$\begin{aligned} B_i &= \partial_i B - S_i \quad \text{with} \\ &\qquad \partial^i S_i = 0\,, \end{aligned} \tag{3.33}$$

$$\begin{aligned} E_{ij} &= 2\,\partial_i\partial_j E + 2\,\partial_{(i}F_{j)} + h_{ij} \quad \text{with} \\ &\qquad \partial^i F_i = 0 \quad \text{and} \quad h^i_i = 0\,,\ \partial^i h_{ij} = 0\,. \end{aligned} \tag{3.34}$$

Since the vector perturbations $S_i$ and $F_i$ are not sourced by the models of inflation we will consider, we are only interested in the scalar perturbations $\Phi, \Psi, B$, and $E$ as well as the tensor perturbations $h_{ij}$.

Whereas the tensor perturbations are gauge invariant, the scalar perturbations are not. As it turns out, there are actually only two gauge invariant quantities describing the scalar perturbations, namely $\zeta$ and $\mathcal{R}$ (in the notation used in [104]). They are equal

[8] Since all the models we are concerned with in this thesis can effectively be described as a single field model during the inflationary phase, we restrict ourselves to single field inflation in this section.

during slow-roll inflation and for superhorizon scales $k < a\mathcal{H}$. Furthermore, for such scales outside the horizon, they are nearly constant.

During inflation, the comoving curvature perturbation $\mathcal{R}$ can be directly related to fluctuations of the inflaton field (in any gauge) via the equation

$$\mathcal{R} = \Psi + \frac{\mathcal{H}}{\dot{\phi}(t)}\,\delta\phi(t, \boldsymbol{x})\,. \tag{3.35}$$

With this we are now in a position to compute the comoving curvature perturbation and its statistical properties for a single field inflationary model.

Since $\mathcal{R}$ is a gauge invariant quantity, we are free to choose whatever gauge we want to do computations. Here, we choose a gauge in which

$$\delta\phi = 0\,, \tag{3.36}$$

$$g_{ij} = a(t)^2\big[(1-2\mathcal{R})\delta_{ij} + h_{ij}\big]\,. \tag{3.37}$$

We start with the following action

$$S = \int \mathrm{d}^4x\,\sqrt{-g}\,\left[\frac{1}{2}\,g^{\mu\nu}\,\partial_\mu\phi\,\partial_\nu\phi - V(\phi) - \frac{1}{2}M_\mathrm{P}^2\,R\right], \tag{3.38}$$

which describes a scalar field $\phi$ minimally coupled to gravity. Since, in particular, we want to compute two-point correlation functions, we have to expand this action up to second order in the linear perturbations. We only sketch how to proceed from here for the scalar perturbations. For the tensor perturbations we only state the main results. A very good discussion of all these points with many more computational details can e.g. be found in [104].

In the gauge we have chosen, all scalar degrees of freedom can be computed from the comoving curvature perturbation $\mathcal{R}$. Expanding the action Eqn. (3.38) to second order in $\mathcal{R}$ yields, after some tedious calculation [117]

$$S_{(2)} = \int \mathrm{d}^4x\,\frac{a^3}{2}\,\frac{\dot{\phi}^2}{\mathcal{H}^2}\left[\dot{\mathcal{R}}^2 - a^{-2}\left(\partial_i\mathcal{R}\right)^2\right]. \tag{3.39}$$

Changing to conformal time $\tau$ defined by

$$\mathrm{d}\tau = \frac{\mathrm{d}t}{a(t)} \tag{3.40}$$

and defining the Mukhanov variable $v$ [118]

$$v = \mathcal{R}z \quad , \quad z = a\phi'/\mathcal{H}\,, \tag{3.41}$$

we can rewrite this as

$$S_{(2)} = \frac{1}{2}\int \mathrm{d}\tau\,\mathrm{d}^3x\,\left[\left(v'\right)^2 + \left(\partial_i v\right)^2 + \frac{z''}{z}v^2\right], \tag{3.42}$$

where prime denotes derivative with respect to conformal time in this case. Fourier expanding the spatial part of $v$ as

$$v(\tau, \boldsymbol{x}) = \int \frac{\mathrm{d}^3 k}{(2\pi)^3}\, v_{\boldsymbol{k}}(\tau)\, \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}} \tag{3.43}$$

yields the following equation for the mode functions

$$v_k'' + \left(k^2 + \frac{z''}{z}\right) v_k = 0 \tag{3.44}$$

with $k = |\boldsymbol{k}|$. To quantise the perturbations we now promote the Fourier components $v_{\boldsymbol{k}}$ to operators as

$$v_{\boldsymbol{k}} \;\to\; \hat{v}_{\boldsymbol{k}} = v_k(\tau)\, \hat{a}_{\boldsymbol{k}} + v_{-k}^*(\tau)\, \hat{a}_{-\boldsymbol{k}}^\dagger\,. \tag{3.45}$$

The creation and annihilation operators satisfy the usual commutation relations

$$\left[\hat{a}_{\boldsymbol{k}}, \hat{a}_{\boldsymbol{p}}^\dagger\right] = (2\pi)^3\, \delta(\boldsymbol{k} - \boldsymbol{p}) \tag{3.46}$$

with the mode function normalisation given by

$$\mathrm{i}\left(v_k^* v_k' - v_k^{*\prime} v_k\right) = 1\,. \tag{3.47}$$

This fixes one of the boundary conditions for the solution of Eqn. (3.44).

As the second boundary condition we take the requirement that for modes well inside the horizon, where the modes do not feel the curvature of spacetime, the vacuum state of the fluctuations defined by

$$\hat{a}_{\boldsymbol{k}}|0\rangle = 0 \tag{3.48}$$

coincides with the usual Minkowski vacuum. This is called the *Bunch-Davies vacuum* [119]. Assuming de Sitter space with a constant Hubble radius, each mode once lay inside the horizon if we go far enough back in time [9]. Therefore, the above statement can be translated into the following asymptotic behaviour of the mode functions

$$\lim_{\tau\to-\infty} v_k(\tau) = \frac{\mathrm{e}^{-\mathrm{i}\tau k}}{\sqrt{2k}}\,. \tag{3.49}$$

This second boundary condition now completely fixes the solutions of Eqn. (3.44).

[9] A mode with comoving wave number $k$ is well inside the horizon if $k \gg a\mathcal{H}$. In de Sitter space, $\mathcal{H}$ is constant and one can show that in this case the conformal time is given by $\tau = -(a\mathcal{H})^{-1}$. Thus, the mode is well inside the horizon if $|k\tau| \gg 1$. This is true for every mode if we go far enough back in conformal time $\tau \to -\infty$.

### 3.3.1 Solution in (Quasi) De Sitter Spacetime

We are now in a position to solve Eqn. (3.44), subject to the boundary conditions Eqns. (3.47) and (3.49), in the limit of de Sitter spacetime with constant Hubble parameter $\mathcal{H} \to$ const. In this limit we have

$$\frac{z''}{z} = \frac{a''}{a} = \frac{2}{\tau^2} \tag{3.50}$$

and the unique solution for the mode functions is given by

$$v_k(\tau) = \frac{\mathrm{e}^{-\mathrm{i}\tau k}}{\sqrt{2k}} \left(1 - \frac{\mathrm{i}}{\tau k}\right) . \tag{3.51}$$

Defining

$$\hat{\psi}_{\boldsymbol{k}} = a^{-1} \hat{v}_{\boldsymbol{k}} \, , \tag{3.52}$$

we can now compute the two point correlation function

$$\begin{aligned} \langle \hat{\psi}_{\boldsymbol{k}}(\tau)\, \hat{\psi}_{\boldsymbol{p}}(\tau) \rangle &\equiv (2\pi)^3\, \delta(\boldsymbol{k}+\boldsymbol{p})\, \mathcal{P}_\psi(k) \\ &\equiv (2\pi)^3\, \delta(\boldsymbol{k}+\boldsymbol{p})\, \frac{2\pi^2}{k^3}\, \Delta_\psi^2(k) \, . \end{aligned} \tag{3.53}$$

The result is given by

$$\langle \hat{\psi}_{\boldsymbol{k}}(\tau)\, \hat{\psi}_{\boldsymbol{p}}(\tau) \rangle = (2\pi)^3\, \delta(\boldsymbol{k}+\boldsymbol{p})\, \frac{\mathcal{H}^2}{2k^3}\, (1 + k^2\tau^2) \, . \tag{3.54}$$

On superhorizon scales we have $|k\tau| \ll 1$ and this result approaches a constant, as claimed in the beginning of this section:

$$\langle \hat{\psi}_{\boldsymbol{k}}(\tau)\, \hat{\psi}_{\boldsymbol{p}}(\tau) \rangle \;\to\; (2\pi)^3\, \delta(\boldsymbol{k}+\boldsymbol{p})\, \frac{\mathcal{H}^2}{2k^3} \, . \tag{3.55}$$

That is

$$\mathcal{P}_\psi = \frac{\mathcal{H}^2}{2k^3} \quad , \quad \Delta_\psi^2(k) = \left(\frac{\mathcal{H}}{2\pi}\right)^2 . \tag{3.56}$$

Using $\mathcal{R} = v/z = \psi \mathcal{H}/\dot{\phi}$, we finally arrive at the result of the two point correlator for the Fourier modes of the comoving curvature perturbation

$$\langle \mathcal{R}_{\boldsymbol{k}}(\tau)\, \mathcal{R}_{\boldsymbol{p}}(\tau) \rangle = (2\pi)^3\, \delta(\boldsymbol{k}+\boldsymbol{p})\, \frac{\mathcal{H}_\star^2}{2k^3}\, \frac{\mathcal{H}_\star^2}{\dot{\phi}_\star^2} \, . \tag{3.57}$$

For a given mode $k$ this expression stays constant once this scale leaves the horizon, which happens when $k = a\mathcal{H}$. Thus, for each separate mode we have to evaluate Eqn. (3.57) at the time $t_\star$ when $a(t_\star)\mathcal{H}(t_\star) = k$. This is indicated by the star-notation $(...)_\star$ in Eqn. (3.57).

The virtue of this treatment is that it also extends from a pure de Sitter spacetime to a slowly changing quasi de Sitter spacetime as for example given during a phase of slow-roll inflation: since we evaluate the correlation function separately for each mode at horizon crossing we implicitly track the evolution of the (slowly) changing background spacetime. It can indeed be show that within the slow-roll approximation the result obtained in this way agrees with the result one gets from a more rigorous treatment (cf. e.g. [104]).

This leaves us with the main result for the scalar perturbations, which is called the *power spectrum*

$$\Delta^2_{\mathcal{R}}(k) \equiv \Delta^2_s(k) = \frac{\mathcal{H}^2_\star}{(2\pi)^2}\,\frac{\mathcal{H}^2_\star}{\dot{\phi}^2_\star}\,. \tag{3.58}$$

Finally, the scale dependence of the power spectrum is encoded in the *spectral index* $n_s$, defined as

$$n_s - 1 \equiv \frac{\mathrm{d}\ln\Delta^2_{\mathcal{R}}}{\mathrm{d}\ln k}\,. \tag{3.59}$$

A similar treatment can now be done for the tensor fluctuations $h_{ij}$. We do not discuss this here in any detail but just quote the most important results. Further details can e.g. be found in [104]. The main result is the power spectrum of the tensor fluctuations, which is given by

$$\Delta^2_t(k) = \frac{2}{\pi^2}\,\frac{\mathcal{H}^2_\star}{M^2_{\mathrm{P}}}\,, \tag{3.60}$$

again to be evaluated when the mode in question leaves the horizon.

Finally, the tensor-to-scalar ratio is defined as

$$r \equiv \frac{\Delta^2_t(k)}{\Delta^2_s(k)}\,. \tag{3.61}$$

Since $\Delta^2_s$ is determined from experiment to be of the order $\mathcal{O}(10^{-9})$ [106, 6] and since $\Delta^2_t \propto \mathcal{H}^2 \simeq V/(3M^2_{\mathrm{P}})$ during inflation, we can derive the following interesting relation between the tensor-to-scalar ratio $r$ and the energy scale of inflation $V^{1/4}$

$$V^{1/4} \simeq \left(\frac{r}{0.01}\right)^{1/4} 10^{16}\,\mathrm{GeV}\,. \tag{3.62}$$

Among other things, we will try to connect inflation to particle physics in the main part of this thesis by identifying the waterfall phase transition that ends hybrid inflation with a GUT symmetry breaking phase transition. Since the GUT scale is roughly $\mathcal{O}(10^{16}\,\mathrm{GeV})$, this relation already tells us that such a model should have a rather small tensor-to-scalar ratio $r \approx 0.01$. This indeed turns out to be the case.

Another interesting result regarding the tensor-to-scalar ratio is what is usually referred to as the *Lyth bound*, which states that the variation of the inflaton field value during inflation $\Delta\phi$ is connected to $r$ in the following way [120]

$$\Delta\phi \simeq \mathcal{O}(1) \times \left(\frac{r}{0.01}\right)^{1/2} M_{\mathrm{P}}\,. \tag{3.63}$$

Hence, for a measurable tensor-to-scalar ratio $r > 0.01$ we need $\Delta\phi > M_P$. Such models are called *large field models*, cf. section 3.4.1.

### 3.3.2 Results in the Slow-Roll Approximation and Experimental Values

To conclude this section, let us state the main results in terms of the slow-roll parameters $\epsilon, \eta$ and $\xi^2$ as introduced in Eqns. (3.24), (3.25) and (3.27), as well as the currently available experimental values/bounds.

To leading order in the slow-roll parameters, the power spectra of the scalar and tensor perturbations are given by

$$\Delta_s^2 = \frac{1}{24\pi^2 M_\mathrm{P}^4}\frac{V}{\epsilon} = \frac{1}{12\pi^2 M_\mathrm{P}^6}\frac{V^3}{(V')^2}\,, \tag{3.64}$$

$$\Delta_t^2 = \frac{2}{3\pi^2 M_\mathrm{P}^4} V\,. \tag{3.65}$$

The spectral index and the tensor-to-scalar ratio are given by

$$n_s - 1 = 2\eta - 6\epsilon\,, \tag{3.66}$$

$$r = 16\epsilon\,. \tag{3.67}$$

Finally, the running of the spectral index is given by

$$\alpha_s \equiv \frac{\mathrm{d}n_s}{\mathrm{d}\ln k} = 16\epsilon\eta - 24\epsilon^2 - 2\xi^2\,. \tag{3.68}$$

When comparing these values with data obtained from measurements of the CMB, all of the above expressions have to be evaluated at the time when the CMB fluctuations exited the horizon. This time is approximately 60 e-folds before the end of inflation.

Turning to experiment, the extraction of these parameters from experimental data depends somewhat on the underlying model for the late-time evolution of the universe one uses to fit the data. Using the ΛCDM concordance model, for example, a combination of measurements of the baryon acoustic oscillations [4], of the present value of the Hubble parameter $\mathcal{H}_0$ [5], and of the 7 year WMAP data [6] gives [10]

$$0.951 < n_s < 0.975 \qquad (68\%\ \mathrm{CL})\,, \tag{3.69}$$

$$0.939 < n_s < 0.987 \qquad (95\%\ \mathrm{CL})\,, \tag{3.70}$$

$$-0.061 < \alpha_s < 0.017 \qquad (95\%\ \mathrm{CL})\,, \tag{3.71}$$

as well as

$$\Delta_s^2(k_0) = (2.441^{+0.088}_{-0.092}) \cdot 10^{-9} \tag{3.72}$$

with $k_0 = 0.002\,\mathrm{Mpc}^{-1}$. In the ΛCDM model all density perturbations are adiabatic by definition and, in particular, no tensor fluctuations are included. Considering an extended

[10] The values quoted here are the values recommended by the WMAP collaboration after the 7-year WMAP data release [66]. These values are based on a computation of the recombination history of the universe using RECFAST version 1.4.2. In [6] the computation has been re-done with the updated version RECFAST 1.5. This shifts the central value of the spectral index slightly upwards to $0.956 < n_s < 0.980$ at 68% CL such that the significance with which $n_s = 1$ is excluded is no longer more than $3\sigma$. In this thesis, however, we use the recommended values from [66].

model that also allows for tensor fluctuations, experimental data yield the following upper bound on the tensor-to-scalar ratio

$$r < 0.24 \qquad (95\%\ \mathrm{CL})\,. \tag{3.73}$$

The hope is that the currently operating Planck satellite [11] will improve this bound, allowing to make a more profound statement about whether small or large field models are preferred by experiment.

## 3.4 Model Overview

Since the predictions for a model of inflation depend mainly on the number of fields involved, the form of the inflaton potential and the inflaton field values during inflation, it is possible to classify the existing inflationary models in terms of these categories. In this section we provide a short overview of the main classes of models that are currently under discussion. For more details the reader is referred to the review articles and textbooks mentioned in the beginning of this chapter and references therein.

### 3.4.1 Single Field Models

In single field models only one field is relevant for the dynamics during and after inflation. Single field models can be further compartmentalised according to whether the inflaton evolution during inflation involves super-Planckian values $\Delta\phi > M_P$ or not.

#### Large Field Models

Large field models are characterised by a super-Planckian field evolution during inflation $\Delta\phi > M_P$, leading to a rather large tensor-to-scalar ratio and an appreciable amount of gravitational waves produced during inflation (s. below). The detection of such primordial gravitational waves would therefore strongly hint towards large field inflationary models.

The simplest form of large field models are *chaotic inflation* models [13] with monomial inflaton potentials

$$V(\phi) \propto \phi^p\,. \tag{3.74}$$

The name chaotic stems from the fact that these models rely on chaotic initial conditions with inflaton field values larger than the Planck scale at the Planck time. From there the inflaton rolls down towards the potential minimum as shown in Fig. 3.1.

The predictions for chaotic models depend on the power $p$ of the monomial in the potential. Experimental data rule out chaotic inflation models with minimal coupling to gravity and $p > 2$ (odd powers of $\phi$ are usually forbidden by symmetry arguments) as can be seen in Fig. 3.2.

Since the simplest model of chaotic inflation with

$$V(\phi) = \frac{1}{2}m^2\phi^2 \tag{3.75}$$

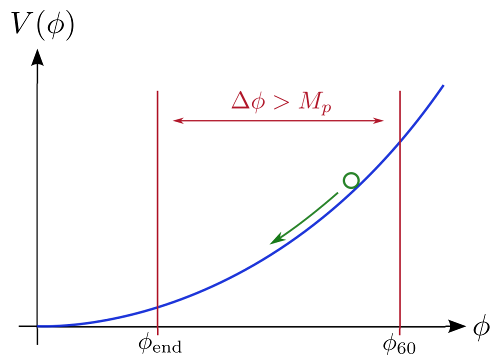



**Figure 3.1:** Typical form of the potential for *chaotic inflation* models [13].

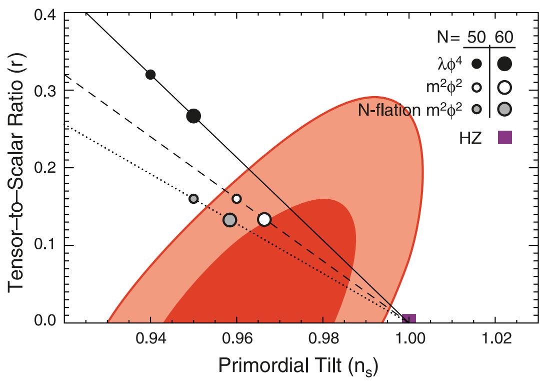



**Figure 3.2:** Predictions for chaotic inflation models with quadratic (white circles) and quartic (black circles) potential as well as for N-flation (grey circles). The small circles indicate evaluation of the spectral index and tensor-to-scalar ratio 50 e-folds before the end of inflation, the large circles 60 e-folds before the end of inflation. The different red shadings indicate the 68% and 95% confidence level of the 7-year WMAP data. The figure is taken from [6].

is still allowed experimentally and it is one of the few models where the predictions can be computed analytically, we take the opportunity here to illustrate how such computations are done (more on the strategies and methods to compute predictions from inflationary models and compare them to CMB measurements can e.g. be found in [121]).

Starting with the potential Eqn. (3.75), the slow-roll parameters $\epsilon$ and $\eta$ from Eqns. (3.24) and (3.25) are given by

$$\epsilon = \eta = \frac{2}{\phi^2} M_{\mathrm{P}}^2 \,. \tag{3.76}$$

Inflation ends when $\epsilon \approx 1$, which in this case means

$$\phi_{\text{end}} = \sqrt{2}\, M_{\mathrm{P}} \,. \tag{3.77}$$

To compute the inflationary predictions we have to integrate the equation of motion for $\phi$ back to the point where the relevant scales leave the horizon, which we take to be 60 e-folds before the end of inflation. We use Eqn. (3.29) to get

$$\mathcal{N} = M_{\mathrm{P}}^{-1} \int_{\phi_{\mathrm{end}}}^{\phi_{60}} \frac{1}{\sqrt{2\epsilon}} \, \mathrm{d}\phi = M_{\mathrm{P}}^{-2} \int_{\sqrt{2}M_{\mathrm{P}}}^{\phi_{60}} \frac{\phi}{2} \, \mathrm{d}\phi = \frac{1}{4M_{\mathrm{P}}^2} \left(\phi_{60}^2 - 2M_{\mathrm{P}}^2\right) \stackrel{!}{=} 60 \,, \tag{3.78}$$

that is

$$\phi_{60} \approx 15.55 \, M_P \,. \tag{3.79}$$

The inflaton field value at the time the CMB fluctuations exited the horizon was approximately 15.5 times the Planck scale!

With this value we can now use the measured amplitude of the CMB fluctuations to determine the mass scale $m$. We have

$$\sqrt{\Delta_s^2} = \frac{1}{2\pi\sqrt{3}M_{\mathrm{P}}^3} \frac{V^{3/2}}{|V'|} = \frac{1}{4\pi\sqrt{6}} \frac{(\phi_{60})^2}{M_{\mathrm{P}}^2} \frac{m}{M_{\mathrm{P}}} \stackrel{!}{=} 4.94 \cdot 10^{-5} \,. \tag{3.80}$$

Hence

$$m = 6.29 \cdot 10^{-6} \, M_{\mathrm{P}} = 1.53 \cdot 10^{13} \, \mathrm{GeV} \,. \tag{3.81}$$

Finally, we can compute the spectral index and the tensor-to-scalar ratio. We find

$$n_s = 1 + 2\eta - 6\epsilon = 1 - \frac{8M_{\mathrm{P}}^2}{(\phi_{60})^2} \approx 0.967 \,, \tag{3.82}$$

$$r = 16\epsilon = \frac{32M_{\mathrm{P}}^2}{(\phi_{60})^2} \approx 0.132 \,. \tag{3.83}$$

This concludes the discussion of chaotic inflation with quadratic potential.

An interesting variant of chaotic inflation models that has received a lot of attention recently is *Standard Model Higgs Inflation* with modified coupling to gravity. The idea to use the Standard Model Higgs boson as the inflaton particle is of course very tempting since its presence is very strongly motivated by particle physics considerations and thus such a model would be very economical. However, it turns out that the requirement of successful electroweak symmetry breaking forces the Higgs potential to be such that it is impossible to realise slow-roll inflation with predictions in accordance to experiment in it.

This problem can be overcome, either by coupling the SM Higgs Boson $H$ non-minimally to the curvature scalar $R$ through a term like $\mathcal{L} \supset -\xi H^\dagger H R$ as was done in [122], or by modifying the Higgs kinetic terms to contain a coupling to the Einstein tensor $G^{\mu\nu}$ of the form $\mathcal{L} \supset -\omega^2 G^{\mu\nu} D_\mu H^\dagger D_\nu H$ as was done in [123]. This last model was called *New Higgs Inflation* by the authors.

Both theories are compatible with the latest WMAP data. For further details on these models and also a discussion of possible difficulties with them, e.g. the UV completion of the first model or the value of the parameter $\xi$ therein, the reader is referred to the literature.

### Small Field Models

Small field models are characterised by a sub-Planckian field evolution during inflation $\Delta\phi < M_P$. Looking at the Lyth-bound Eqn. (3.63) this immediately tells us that such models lead to a small tensor-to-scalar ratio, i.e. a very small amplitude for the gravitational waves produced during inflation, which will be nearly impossible to detect experimentally.

The typical form of the potential for such models is given by

$$V(\phi) = V_0 \left[1 - \left(\frac{\phi}{\mu}\right)^p\right] + ... \tag{3.84}$$

(the dots denote operators that become important at the end and after inflation) and the inflaton rolls from an unstable local maximum in the potential towards a displaced vacuum configuration as shown in Fig. 3.3. Such potentials are e.g. encountered in spontaneous symmetry breaking phase transitions.

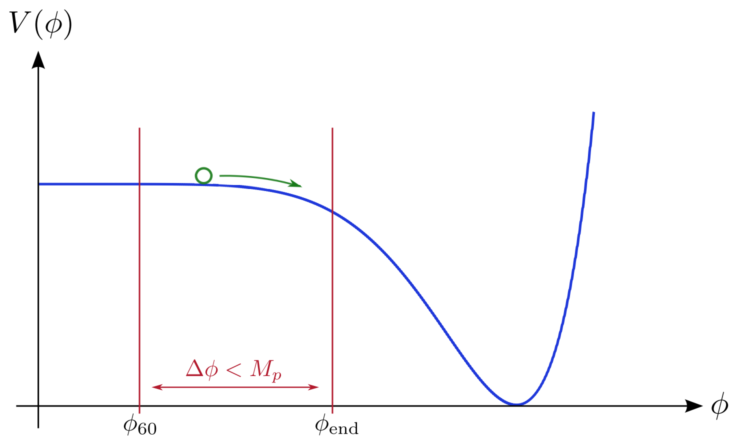



**Figure 3.3:** Typical form of the potential for *new inflation* models [12].

Models of this type are also called *new inflation* models and they were first introduced in [12].

### Natural Inflation

Natural inflation [124] can be a small field model or a large field model, depending on whether the periodicity $2\pi f$ of the potential

$$V(\phi) = V_0 \left[1 + \cos\left(\frac{\phi}{f}\right)\right] \tag{3.85}$$

is smaller or larger than $M_P$ (cf. also the discussion in [125]). The form of the potential is depicted in Fig. 3.4. Such a potential arises e.g. when the inflaton is taken to be an axion-like particle.

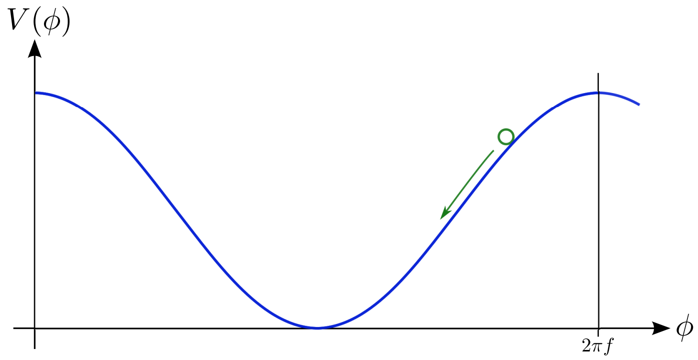



**Figure 3.4:** Typical form of the potential for *natural inflation* models [124].

### 3.4.2 Multi Field Models

We now want to consider models with more than one field involved in the dynamics during and after inflation.

#### N-flation

Motivated by attempts to realise slow-roll inflation in string theory, N-flation tries to avoid the super-Planckian field values that arise in chaotic inflation with quadratic potential while retaining the good agreement with experimental data of that model.

The main idea behind the model [126] is to combine $N \gg 1$ axion-like fields (which naturally arise in many string compactifications) with sub-Planckian field values into one effective inflaton particle that reproduces chaotic inflation with a quadratic potential. In particular, the potential for the $N$ axion fields is of the form

$$V(\phi_i) = \sum_i \Lambda_i^4 \left[1 - \cos\left(\frac{\phi_i}{f_i}\right)\right] , \tag{3.86}$$

similar to the natural inflation model presented above. The potential is independent for each of the different axion fields. Taylor expanding around the minimum leads to the Lagrangian

$$\mathcal{L} \simeq \frac{1}{2}\partial_\mu \phi_i \partial^\mu \phi_i - \frac{1}{2} m_i^2 \phi_i^2 \,. \tag{3.87}$$

Assuming the masses $m_i$ to be nearly equal and a common initial displacement $\bar{\phi}$ from the origin, all fields $\phi_i$ move in unison towards the origin, mimicking one single field $\phi$ with initial displacement $\sqrt{N}\bar{\phi}$. Thus, even for small displacements $\phi_i \ll M_P$ the effective field $\phi$ can satisfy the slow-roll conditions for quadratic chaotic inflation. The predictions for this model are also shown in Fig. 3.2

### Hybrid Inflation

Let us now discuss the class of models we are mainly concerned with in this thesis: *hybrid inflation*. In its original form [33], the potential for hybrid inflation is given by

$$V(\sigma, \phi) = \frac{1}{4\lambda}(M^2 - \lambda\sigma^2)^2 + \frac{1}{2}m^2\phi^2 + \frac{g^2}{2}\phi^2\sigma^2\,, \tag{3.88}$$

and one typically assumes $m^2 \ll \mathcal{H}^2 \ll M^2$ (cf. below). As we can see, two fields are relevant for the dynamics in this model: the field $\phi$, which acts as inflaton and slowly rolls down the potential, and the *waterfall field* $\sigma$. This field has an inflaton dependent mass given by

$$m_\sigma^2 = g^2\phi^2 - M^2\,, \tag{3.89}$$

which is positive for $\phi > \phi_c = M/g$ and in this range the waterfall field is stabilised at zero. This defines the slow-roll trajectory

$$V(\phi)_{\text{slow-roll}} = V(\phi, \sigma = 0) = \frac{M^4}{4\lambda} + \frac{1}{2}m^2\phi^2 \tag{3.90}$$

and inflation is mainly driven by the large vacuum energy

$$\mathcal{H}^2 \simeq \frac{V}{3M_{\text{P}}^2} \simeq \frac{M^4}{12M_{\text{P}}^2\lambda}\,. \tag{3.91}$$

Once the inflaton field approaches the critical values $\phi_c$, the mass of $\sigma$ becomes tachyonic and the field destabilises. Around this point the slow-roll conditions are violated, slow-roll inflation ends and in the subsequent *waterfall phase transition* both fields roll down to their true minima located at $\phi = 0$ and $\sigma = \pm M/\sqrt{\lambda}$. The situation is depicted in Fig. 3.5.

The nice feature of this model is that it allows to establish a connection between inflation and particle physics by identifying the waterfall phase transition with e.g. the spontaneous breaking of a GUT or flavour symmetry. This idea is one of the main motivations for the studies carried out in the main part of this thesis.

In its simplest form presented here, however, hybrid inflation is almost ruled out by experiment, since it predicts a spectral index $n_s \geq 1$ [34], which is clearly not preferred. This problem can be overcome by embedding hybrid inflation within a supersymmetric framework, as discussed in the next section. In any case, belonging to the class of small-field models, hybrid inflation models typically predict a very small tensor-to-scalar ratio.

## 3.5 Inflation in Supersymmetry

We now want to extend our discussion of models of inflation to supersymmetric models. As already stated above this is quite important, at least for hybrid-type inflationary models, since without SUSY there is a big tension between theory and experiment.

It should be clear by now that the most important piece of information about a model of inflation is the scalar potential. Looking at Eqn. (1.92), we see that in a supersymmetric

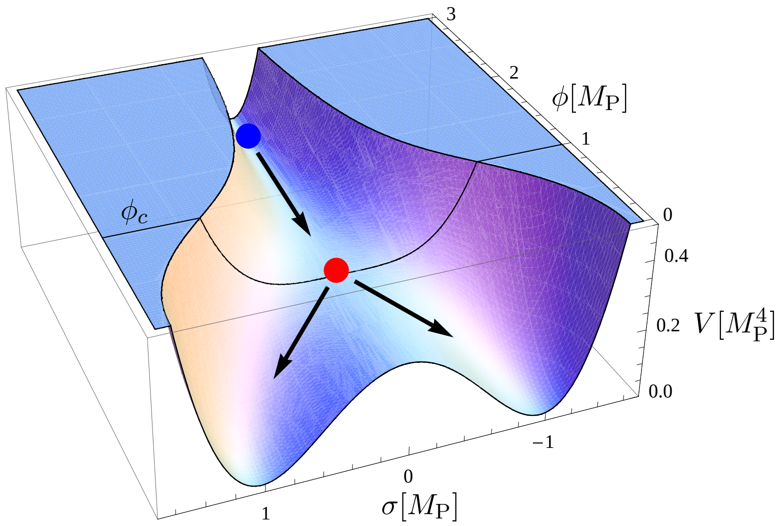



**Figure 3.5:** Typical form of the potential for *hybrid inflation* models [33]. During the inflationary phase (blue ball), the waterfall field is trapped at zero and the inflaton field slowly rolls towards the critical value $\phi_c$. Once this value is reached, the waterfall direction becomes tachyonic and both fields roll down to their true minima (red ball) located at $\phi = 0$ and $\sigma = \pm M/\sqrt{\lambda}$. Here, we have plotted the potential for $\lambda = g = 1, m = 0.1\, M_\mathrm{P}$ and $M = 1\, M_\mathrm{P}$.

theory there are two contributions to it. Accordingly, there are two main classes of supersymmetric inflationary models, called *D-term inflation* and *F-term inflation*, respectively. In this section we present a prototype model for each class, with special emphasis on the SUSY F-term hybrid model, since this acts as a kind of blueprint for the models we will investigate later on in the main part of this thesis.

### 3.5.1 D-Term Inflation

The simplest model for D-term inflation introduced in [36, 127] is defined by the superpotential

$$W = \lambda\, \boldsymbol{\Phi}\, \boldsymbol{H}_+\, \boldsymbol{H}_- \,, \tag{3.92}$$

where we have assumed a $U(1)$ gauge symmetry under which $\boldsymbol{H}_+$ and $\boldsymbol{H}_-$ carry opposite charges. According to Eqn. (1.92) the resulting scalar potential reads

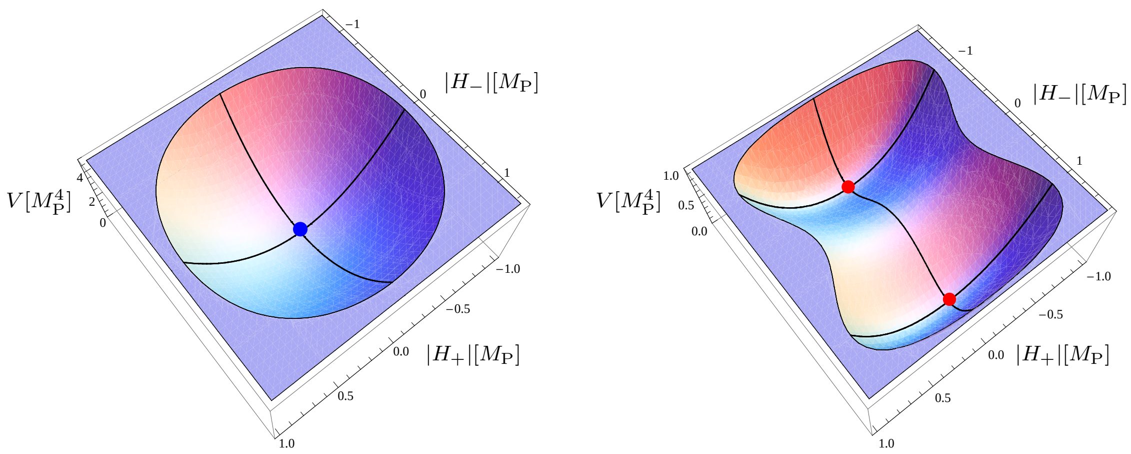
$$V = \lambda^2|\phi|^2\left(|H_-|^2 + |H_+|^2\right) + \lambda^2|H_-H_+|^2 + \frac{g^2}{2}\left(|H_+|^2 - |H_-|^2 + \xi\right)^2 \tag{3.93}$$

with a (positive) Fayet-Iliopoulos term $\xi$ (hence the $U(1)$ gauge symmetry).

For $|\phi| > \phi_c = g/\lambda \cdot \sqrt{\xi}$ the potential has a minimum for $|H_-| = |H_+| = 0$ and along this trajectory the potential is flat in the $|\phi|$ direction. Since it can be shown that the field corresponding to the phase of $\phi$ quickly settles to a constant value in this regime, we can identify the canonically normalised scalar field $\varphi = \sqrt{2}|\phi|$ as the inflaton and inflation is driven by the large vacuum energy

$$V_{\text{tree}} = \frac{g^2}{2}\xi^2 \,. \tag{3.94}$$

Since this vacuum energy comes from the D-term contribution and all F-term contributions vanish along the inflationary trajectory, the name D-term inflation is justified.




(a) $\varphi > \varphi_c$ (b) $\varphi = 0$

**Figure 3.6:** Scalar potential of the D-term inflation model [36, 127] in the $|H_-|$–$|H_+|$ plane for $\lambda = g = 1$ and $\xi = M_P^2$.

(a) $\varphi = 2M_P > \varphi_c$. We can see that here both fields are trapped at $|H_-| = |H_+| = 0$. In this case the potential is flat at tree-level in the $\varphi$-direction. This flatness is lifted by quantum corrections, which drive $\varphi$ towards $\varphi_c$.

(b) $\varphi = 0$. The SUSY conserving, $U(1)$-breaking global minimum of the potential is located at $\varphi = |H_+| = 0$ and $|H_-| = \pm\sqrt{\xi}$.

So far the potential is exactly flat in the inflaton direction such that inflation would go on forever. However, during inflation supersymmetry is broken, which is manifest in a mass splitting between the scalar component fields of the superfields $\boldsymbol{H}_+$ and $\boldsymbol{H}_-$ and the fermionic component fields

$$m^2_{\text{Re}(H_+)} = m^2_{\text{Im}(H_+)} = \frac{\varphi^2\lambda^2}{2} + g^2\xi \,, \tag{3.95}$$

$$m^2_{\text{Re}(H_-)} = m^2_{\text{Im}(H_-)} = \frac{\varphi^2\lambda^2}{2} - g^2\xi \,, \tag{3.96}$$

$$m_{\chi_+} = m_{\chi_-} = \frac{\varphi\lambda}{\sqrt{2}} \,. \tag{3.97}$$

These masses provide a slope for the inflaton potential through inflaton-dependent contributions to the *Coleman-Weinberg effective one-loop potential* [128]

$$V_{\text{loop}} = \frac{1}{64\pi^2}\,\text{STr}\left[\mathcal{M}^4\left(\ln\frac{\mathcal{M}^2}{Q^2} - \frac{3}{2}\right)\right] . \tag{3.98}$$

Here, $Q$ is the renormalisation scale, $\mathcal{M}$ is the mass matrix of the theory and the supertrace STr runs over all bosonic and fermionic degrees of freedom (cf. Eqn. (1.111)). In our case it is sufficient, however, to consider the sum over such superfields that contain component fields with inflaton-dependent masses since only these fields contribute to the slope of the potential (the remaining terms only lead to a renormalisation of $V_0$). In the case at hand we get

$$V = V_{\text{tree}} + V_{\text{loop}} = \frac{g^2}{2}\xi^2\left[1 + \frac{g^2}{8\pi^2}\ln\left(\frac{\lambda^2\varphi^2}{2Q^2}\right)\right] . \tag{3.99}$$

The slope hereby generated drives the inflaton towards the critical value where the slow-roll conditions get violated and the scalar component fields of $\boldsymbol{H}_-$ become tachyonic. Subsequently all fields roll down to the global minimum of the potential.

According to [129], however, this model with a field-independent FI-term is only consistent in global SUSY, not in a SUGRA framework. For this reason, and since we are exclusively dealing with F-term inflationary models in the main part of this thesis, we refrain from a further discussion of D-term inflation.

### 3.5.2 F-Term Hybrid Inflation

Let us now come to a discussion of a supersymmetric version of the hybrid inflation model introduced in section 3.4. Since the potential solely stems from F-term contributions in this model, it is an example of F-term inflation.

The superpotential of the model is given by [34, 35]

$$W = \kappa\,\boldsymbol{\Phi}\left(\boldsymbol{H}^2 - M^2\right) , \tag{3.100}$$

where both $\boldsymbol{\Phi}$ and $\boldsymbol{H}$ are assumed to be gauge singlets for simplicity. The more complicated case involving gauge non-singlet fields is one of the main results of this work and is discussed in chapter 7.

Since all the fields are singlets, there are only F-term contributions to the scalar potential, which reads

$$V = \kappa^2\left|\left(H^2 - M^2\right)\right|^2 + 4\kappa^2\,|\phi|^2\,|H|^2 . \tag{3.101}$$

For $|\phi| > \phi_c = M/\sqrt{2}$, the minimum of the potential is given by $H = 0$ and along this trajectory the potential for the inflaton, which we again take to be $\varphi = \sqrt{2}\,|\phi|$, is flat at tree-level by construction and given by

$$V_{\text{tree}} = \kappa^2 M^4 . \tag{3.102}$$

This vacuum energy breaks SUSY, which results in a mass splitting for the component fields of $\boldsymbol{H}$

$$m^2_{\mathrm{Re}(H)} = 2\,\kappa^2\left(\varphi^2 - M^2\right)\,, \tag{3.103}$$
$$m^2_{\mathrm{Im}(H)} = 2\,\kappa^2\left(\varphi^2 + M^2\right)\,, \tag{3.104}$$
$$m_{\chi_H} = \sqrt{2}\,\kappa\,\varphi\,. \tag{3.105}$$

This provides the necessary slope for the inflaton potential through the Coleman-Weinberg potential Eqn. (3.98) and the complete potential at one-loop level now reads

$$V = V_{\mathrm{tree}} + V_{\mathrm{loop}} = \kappa^2 M^4\left[1 + \frac{\kappa^2}{8\pi^2}\ln\left(\frac{2\kappa^2\varphi^2}{Q^2}\right)\right]\,. \tag{3.106}$$

Using this potential and the slow-roll approximation, the field value $\varphi_{\mathcal{N}}$ at the time $\mathcal{N}$ e-folds before the end of inflation can now be approximated as (cf. Appendix B for more details on the computation)

$$\varphi^2_{\mathcal{N}} \simeq \varphi_c^2 + \frac{\kappa^2 M_{\mathrm{P}}^2}{2\pi^2}\mathcal{N} \simeq \frac{\kappa^2 M_{\mathrm{P}}^2}{2\pi^2}\mathcal{N}\,, \tag{3.107}$$

where we have anticipated

$$\varphi_c^2 = M^2 \approx 10^{-5} M_{\mathrm{P}}^2 \ll \frac{\kappa^2 M_{\mathrm{P}}^2}{2\pi^2}\mathcal{N}\,. \tag{3.108}$$

With these approximations and taking $\mathcal{N} = 60$ we now get

$$n_s \simeq 1 - \frac{1}{\mathcal{N}} \quad\longrightarrow\quad n_s \approx 0.98\,, \tag{3.109}$$
$$r \simeq \frac{\kappa^2}{\pi^2\mathcal{N}} \quad\longrightarrow\quad r \approx \kappa^2\cdot\mathcal{O}(10^{-3})\,, \tag{3.110}$$
$$\Delta_s^2 \simeq \frac{2}{3}\frac{M^4}{M_{\mathrm{P}}^4}\mathcal{N} \quad\longrightarrow\quad M \approx 3\cdot 10^{-3} M_{\mathrm{P}}\,. \tag{3.111}$$

We see that the predicted value for $n_s$ lies somewhat above the central value and slightly outside the 68% CL of the WMAP data but well within the 95% CL. As anticipated for a small field model, the tensor-to-scalar ratio is very small [11]. Note that the energy scale of inflation set by the mass-scale $M \approx 3\cdot 10^{-3} M_{\mathrm{P}}$ lies roughly around the GUT scale. This suggests that there might be a relation between the waterfall phase transition ending inflation and the spontaneous breaking of a GUT symmetry. We explore this idea further in chapter 7.

[11] It has been pointed out that for waterfall fields that are charged under some gauge group which allows for the production of cosmic strings in the waterfall phase transition, $\kappa$ is bounded to be $\lesssim 10^{-2}$ due to the effects of the produced strings on e.g. the spectral index [108].

## 3.6 Inflation in SUGRA and the $\eta$-Problem

To conclude this chapter, let us go one final step further and discuss the embedding of F-term inflationary models within a supergravity framework. Naively one could expect that such an embedding only leads to small corrections to the scalar potential since e.g. derivations from a canonical Kähler potential are suppressed by inverse powers of the Planck scale $M_{\mathrm{P}}$. However, this turns out not to be the case. On the contrary, as we will shortly discuss, inflationary models within any effective field theory (EFT) with cut-off scale $M_{\mathrm{P}}$ quite generically suffer from a serious problem called the *$\eta$-problem* [34, 37]. In essence, this describes the fact that in such a case, corrections to the scalar potential tend to make the slow-roll parameter $\eta$ become at least $\mathcal{O}(1)$, which immediately spoils slow-roll inflation. In this section we discuss how the $\eta$-problem arises in a generic EFT with cut-off $M_{\mathrm{P}}$ and in SUGRA models of F-term inflation in particular and then discuss how it can be avoided in the latter case. In doing so we introduce a new class of F-term hybrid inflationary models, called *tribrid inflation*, which is particularly well suited to apply a symmetry solution to the $\eta$-problem.

### 3.6.1 The $\eta$-Problem

As already mentioned in section 1.6.2, supergravity has to be regarded as an effective theory approximation to a more fundamental, UV-complete theory like e.g. string theory. As a consequence, non-renormalisable operators have to be taken into account.

Looking at the scalar potential in the presence of a large vacuum energy $V_0$ in a generic effective field theory with natural cut-off scale $M_{\mathrm{P}}$, non-renormalisable terms like

$$V \supset V_0 \left( \frac{\phi^* \phi}{M_{\mathrm{P}}^2} + \frac{(\phi^* \phi)^2}{M_{\mathrm{P}}^4} + ... \right) \tag{3.112}$$

for any gauge singlet or non-singlet field $\phi$ are a priori not forbidden. Again, one might expect that for small field models with $\phi < M_P$ such terms are negligible, however this is not the case.

Indeed, from the first term

$$V \supset V_0 \frac{\phi^* \phi}{M_{\mathrm{P}}^2} \tag{3.113}$$

we find

$$\eta = M_{\mathrm{P}}^2 \frac{V''}{V} \simeq \frac{V_0}{V_0} = 1\,, \tag{3.114}$$

where we have assumed that the vacuum energy $V_0$ dominates during inflation. This immediately spoils slow-roll inflation and is called the *$\eta$-problem* [34, 37].

For large field models, on the other side, the situation is even more severe, because then it is not even clear in what sense an expansion of e.g. the Kähler potential or the superpotential over powers of the Planck scale is still meaningful. This is one of the reasons why we only consider small fields models in this thesis.

Let us now discuss how this problem can be avoided in the case of SUGRA. First we have to note that a SUGRA model is usually not defined in terms of the scalar potential directly, but in terms of the Kähler potential $K$ and the superpotential $W$. Thus, we should look at expansions of these more fundamental objects rather than an expansion of the scalar potential itself.

Looking, for example, at a F-term inflation model with canonical Kähler potential $K = \phi^*\phi$, we immediately see how the $\eta$-problem arises in this case by Taylor-expanding the SUGRA scalar F-term potential Eqn. (1.143) to find

$$V \simeq V_0 \left(1 + \frac{\phi^*\phi}{M_\mathrm{P}^2} + ...\right) , \tag{3.115}$$

which again gives $\eta \approx 1$ [12].

Since in this case the problem arises from the expansion of the exponential factor $\mathrm{e}^{K/M_\mathrm{P}^2}$ in the formula Eqn. (1.143) for the scalar F-term potential, this suggests that a possible solution to the $\eta$-problem might lie in resorting to D-term inflationary models [36, 127]. While it is true that such models indeed do not suffer from the $\eta$-problem, their embedding into a supergravity context presents difficulties of its own (cf. section 3.5.1) and we do not discuss this approach further.

Returning to F-term models, there are basically two ways to avoid the $\eta$-problem. One is fine-tuning the coefficients in a general expansion of the Kähler potential

$$K = \phi_i^*\phi_i + \sum_{n=2} \kappa_i^{(n)} \frac{(\phi_i^*\phi_i)^n}{M_\mathrm{P}^{2n-2}} + \sum_{k=l+m} \kappa_{ij}^{(k)} \frac{(\phi_i^*\phi_i)^l (\phi_j^*\phi_j)^m}{M_\mathrm{P}^{2k-2}} + ... \tag{3.116}$$

in such a way to keep the curvature of the potential small, i.e. $|\eta| \ll 1$ . This does work (cf. for example [71]), but it involves a lot of fine-tuning.

A more appealing option is the application of a symmetry solution to the $\eta$-problem. Such a solution restricts the Kähler potential to be a function of the field combination $\rho$ only, where $\rho$ is different from the inflaton direction and invariant under the imposed symmetry. This protects the inflaton direction from obtaining SUGRA corrections that spoil slow-roll inflation.

Consider, for example, a Nambu-Goldstone like *shift symmetry* [130], under which the complex scalar $\phi$ is defined to transform as

$$\phi \,\to\, \phi + \mathrm{i}\,\mu\,, \tag{3.117}$$

where $\mu$ is a real transformation parameter. In this case the invariant field combination $\rho$ is given by

$$\rho = \frac{\phi + \phi^*}{\sqrt{2}} \tag{3.118}$$

[12] For the standard hybrid inflation model with $W = \kappa\, \mathbf{\Phi} \left(\boldsymbol{H}^2 - M^2\right)$ and canonical Kähler potential there is actually *no* $\eta$-problem because of a rather miraculous cancellation in the F-term scalar potential, cf. for example the detailed discussion in [71]. Some people take this as an argument in favour of that model. However, we regard this rather as a coincidence for a very special case and concentrate on more general approaches to solve the $\eta$-problem in our discussion.

and if we restrict the Kähler potential to be a function only of $\rho$ but not of $\phi$ directly, then the direction perpendicular to $\rho$,

$$\varphi = \frac{\phi - \phi^*}{\mathrm{i}\sqrt{2}}\,, \tag{3.119}$$

is protected from SUGRA corrections and thus a viable direction for inflation. Note, however, that such a shift symmetry can only be applied to a gauge singlet field.

Another symmetry that has been discussed in the literature is called *Heisenberg symmetry* [131] and in addition to the complex scalar containing the inflaton direction it involves another complex scalar field $T$, called a modulus field. The combined transformation properties of the scalar fields under such a Heisenberg transformation are given by

$$T \to T + \mathrm{i}\,\nu\,, \tag{3.120}$$
$$T \to T + M_{\mathrm{P}}^{-1}\big(\alpha^*\phi + \alpha^*\alpha/2\big)\,, \tag{3.121}$$
$$\phi \to \phi + \alpha\,, \tag{3.122}$$

with $\nu$ being a real and $\alpha$ a complex transformation parameter. The invariant field combination in this case is given by

$$\rho = T + T^* - M_{\mathrm{P}}^{-1}\phi^*\phi \tag{3.123}$$

and it is possible to show that if the Kähler potential respects the Heisenberg symmetry and only depends on $\rho/M_{\mathrm{P}}$, the modulus of $\phi$ is a viable inflaton direction. It is interesting to note that the Heisenberg symmetry transformation can be generalised to include gauge non-singlet fields, too.

### 3.6.2 Tribrid Inflation

To apply such a symmetry solution, a slightly modified version of F-term hybrid inflation has turned out to be particularly well suited. This version is called *tribrid inflation* [30, 31, 32] and it is defined to have a superpotential of the following form

$$W = \kappa\,\boldsymbol{S}\left(\boldsymbol{H}^2 - M^2\right) + g\left(\boldsymbol{\Phi}, \boldsymbol{H}\right)\,, \tag{3.124}$$

with the additional requirement that during inflation

$$\begin{aligned} &W = W_\phi = W_H = 0\,,\\ &W_S \neq 0\,,\\ &S = H = 0\,. \end{aligned} \tag{3.125}$$

These conditions have been shown to be very desirable when using a symmetry solution to solve the $\eta$-problem [36]. They further restrict the allowed form of the function $g\left(\boldsymbol{\Phi}, \boldsymbol{H}\right)$ and we exclusively use the following form throughout this thesis

$$g = \left(\boldsymbol{\Phi}, \boldsymbol{H}\right) = \frac{\lambda}{M_*}\boldsymbol{\Phi}^2\boldsymbol{H}^2\,. \tag{3.126}$$

The roles of the fields are the following. The scalar component field $\phi$ of the superfield $\boldsymbol{\Phi}$ contains the inflaton direction $\varphi$, which is protected from SUGRA corrections either by a shift symmetry or a Heisenberg symmetry. The scalar component field $H$ of the superfield $\boldsymbol{H}$ acts as waterfall field. Finally, the role of the scalar component field $S$ is to provide the large vacuum energy that drives inflation via its F-term $F_S$. It is therefore called the *driving field*. As we can see, the main difference to the standard hybrid scenario lies in the fact that the role of the field that provides that large vacuum energy and the field that acts as inflaton are separated in the case of tribrid inflation. This is crucial for the application of a symmetry solution to solve the $\eta$-problem [36].

A more detailed discussion of how inflation and the subsequent waterfall phase transition work in tribrid inflation is given in the context of the two specific models in chapters 5 and 7.

Finally, for more extensive discussions of symmetry solutions to the $\eta$-problem in F-term inflationary models in general and in tribrid inflation in particular, the reader is referred to [71] and references therein.

CHAPTER 4

# Reheating and Leptogenesis

The final piece of information that is missing before we can plunge into a detailed investigation of inflation and subsequent leptogenesis in the framework of particle physics models is a discussion of baryogenesis, in particular baryogenesis through non-thermal leptogenesis and its relation to the field dynamics after inflation.

We know from observation that there is no substantial amount of antimatter in our universe. Put quantitatively, the number density of baryons $n_B$ minus the number density of antibaryons $n_{\bar{B}}$, normalised to the number density of photons $n_\gamma$ at the present time is given by

$$\eta = \frac{n_B - n_{\bar{B}}}{n_\gamma} = (6.19^{+0.15}_{-0.15}) \cdot 10^{-10} \quad (68\% \text{ CL})\,, \tag{4.1}$$

which can be inferred from the WMAP 7 year data [6]: the ratio of the first-to-second acoustic peaks in the CMB power spectrum is sensitive to the baryon density (for a very nice discussion of the various features of the CMB radiation and their dependence on cosmological parameters, cf. [132, 114]). The photon density can be calculated using $n_\gamma \propto T^3_{\text{CMB}}$. From this the value Eqn. (4.1) can be deduced. This immediately raises two questions.

Since inflation depletes the universe of any particles that might have been present before, the first question is how we can end up in a universe with non-zero net baryon number having started out with zero net baryon number after inflation. There are three necessary conditions for this, called Sakharov's conditions [25], and we will discuss them in due course.

The second question is, of course, how any particles are created after inflation in the first place. This goes under the name of reheating [133] and it is the first issue we discuss in this chapter. For a textbook discussion of reheating after inflation, cf. e.g. [95].

## 4.1 Reheating after Inflation

At the end of inflation the inflaton field (and in hybrid models also the waterfall field) starts to accelerate down the potential. Eventually, the field reaches its minimum, overshoots it because of its kinetic energy and starts oscillating around its minimum. Assuming the potential around the minimum can be approximated as that of a simple harmonic oscillator and using

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)\,, \tag{4.2}$$

the virial theorem gives

$$\langle \dot{\phi}^2/2 \rangle = \langle V(\phi) \rangle = \rho_\phi/2\,. \tag{4.3}$$

Multiplying the equation of motion for the inflaton field with $\dot{\phi}$, time-averaging over oscillations and using the above result we get

$$\dot{\rho_\phi} + 3\,\mathcal{H}\rho_\phi = 0\,. \tag{4.4}$$

Thus, the oscillating inflaton field can be viewed as a collection of massive, non-relativistic particles.

Assuming that the inflaton field is coupled to other fields, the oscillations of the inflaton around its minimum are further damped by the emission of (ultra-relativistic) particles. Let $\Gamma_\phi$ denote the decay rate of the inflaton. If preheating effects are negligible (cf. [134, 135] for parametric preheating and [136, 137] for tachyonic preheating), as is the case in the model we are going to discuss (cf. section 8.3.1), the equation of motion describing such a decaying inflaton field during the oscillatory phase gets modified to [135]

$$\ddot{\phi} + 3\,\mathcal{H}\,\dot{\phi} + \Gamma_\phi\dot{\phi} + V'(\phi) = 0\,, \tag{4.5}$$

and we end up with

$$\dot{\rho_\phi} + (3\,\mathcal{H} + \Gamma_\phi)\,\rho_\phi = 0\,. \tag{4.6}$$

The energy-density $\rho_{\rm rad}$ of the ultra-relativistic particles produced by the inflaton decays, on the other hand, satisfies the following equation

$$\dot{\rho}_{\rm rad} + 4\,\mathcal{H}\,\rho_{\rm rad} - \Gamma_\phi\,\rho_\phi = 0\,. \tag{4.7}$$

The different numeric prefactor of the second term stems from the different scaling behaviour of the energy-density of ultra-relativistic particles, $\rho_{\rm rad} \propto a^{-4}$, as compared to the energy-density of massive particles, $\rho_{\rm m} \propto a^{-3}$, cf. Tab. C.1.

As long as $\Gamma_\phi \ll \mathcal{H}$, the inflaton decays can be neglected, the oscillations are predominantly damped by the Hubble expansion and the universe is in a matter-dominated era. However, as soon as $\Gamma_\phi \approx \mathcal{H}$, the production of ultra-relativistic particles becomes dominant and their interaction rates become large enough for the universe to thermalise. This is the first time in the evolution of the universe that there exists a thermal bath and that it is possible to define a temperature. Roughly around the same time, the universe changes from matter to radiation dominance. The temperature the universe has at this point in time is called the *reheat temperature* $T_R$. The situation is depicted in Fig. 4.1

If $\rho_{\text{rad}}$ is the energy density of the ultra-relativistic particles at the time the universe thermalises, then

$$T_R^4 = \frac{30}{g_* \pi^2}\, \rho_{\text{rad}}(T_R)\,, \tag{4.8}$$

where $g_*$ denotes the effective number of relativistic degrees of freedom. One can show that the reheat temperature can be related to the inflaton decay rate via (cf. e.g. [95])

$$T_R \propto g_*^{-1/4} \left(\Gamma_\phi M_P\right)^{1/2} . \tag{4.9}$$

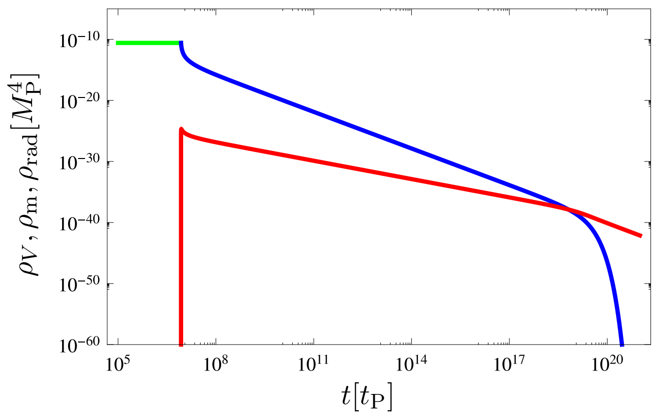



**Figure 4.1:** Evolution of the energy densities during and after inflation. During inflation the universe is dominated by the large vacuum energy $V_0$ (green). After slow-roll inflation ends this potential energy is converted into energy of the oscillating inflaton field, which can be regarded as non-relativistic matter (blue). With the onset of these oscillations, ultra-relativistic particles (red) are also produced from inflaton decays. However, for $\Gamma_\phi \ll \mathcal{H}$ they get quickly diluted and only contribute subdominantly to the energy budget of the universe. Only when $\Gamma_\phi \approx \mathcal{H}$, the universe thermalises and becomes radiation dominated. This plot is based on the model discussed in chapters 5 and 8.

## 4.2 Conditions for Baryogenesis

Having found a possibility to populate the universe with particles after inflation, let us now return to the question how we can produce an excess of particles over antiparticles. Assuming that we start out with a universe with baryon number equal to zero after inflation, Sakharov [25] showed that there are three necessary conditions, called *Sakharov conditions*, that have to be fulfilled in the universe in order to generate a baryon asymmetry:

1. Baryon number violation.
2. $\mathcal{C}$ and $\mathcal{CP}$ violation.
3. Departure from thermal equilibrium.

Only if these three conditions are simultaneously satisfied, an excess of baryons over antibaryons can be created. Let us discuss them in more detail.

### Baryon Number Violation

Since we assume the net baryon number to be equal to zero after inflation, it is clear that baryon number has to be violated in some process taking place after inflation if we want to end up with a universe with more baryons than antibaryons.

### $\mathcal{C}$ and $\mathcal{CP}$ Violation

To understand why both $\mathcal{C}$ and $\mathcal{CP}$ symmetry have to be violated let us first assume that $\mathcal{C}$ symmetry were intact. Assume furthermore that there is a baryon number violating process $i \longrightarrow f$ that produces net baryon number

$$\Delta B = B_f - B_i \ , \tag{4.10}$$

where $B_f$ and $B_i$ are the sum of the baryon numbers of all particles in the final and initial state, respectively. Now, $\mathcal{C}$ symmetry replaces all particles by their corresponding antiparticles, so the $\mathcal{C}$ conjugate process $\bar{i} \longrightarrow \bar{f}$ produces net baryon number

$$\Delta B_{\mathcal{C}} = B_{\bar{f}} - B_{\bar{i}} = -B_f + B_i = -\Delta B \ . \tag{4.11}$$

If $\mathcal{C}$ symmetry is intact, both processes operate at the same rate and the produced baryon asymmetries of both processes exactly cancel each other. Thus $\mathcal{C}$ symmetry has to be violated.

Regarding $\mathcal{CP}$ violation, the argument is similar to the one above, only that in addition the momenta of all particles involved in the $\mathcal{CP}$ conjugate process are reversed. However, assuming $\mathcal{CP}$ is conserved and integrating both the original and the $\mathcal{CP}$ conjugate process over the allowed phase spaces, which are identical, the produced net baryon number again cancels to zero. Thus both $\mathcal{C}$ and $\mathcal{CP}$ symmetry have to be violated.

### Departure from Thermal Equilibrium

Finally, if the universe is in thermal equilibrium, then every $\mathcal{C}$, $\mathcal{CP}$, and baryon number violating process $i \longrightarrow f$ that produces net baryon number $\Delta B$ is accompanied by the inverse reaction $f \longrightarrow i$ at the same rate, which immediately washes out any produced baryon asymmetry. Thus, there has to be a departure from thermal equilibrium to produce a sustainable baryon asymmetry in the universe.

In principle there is a source for $\mathcal{C}$, $\mathcal{CP}$, and baryon number violation as well as for the departure from thermal equilibrium already to be found in the Standard Model. However, one finds that the $\mathcal{CP}$ violation and the departure from thermal equilibrium within the SM are not sufficient to explain the measured value for $\eta$ (cf. for example the discussion in [138, 139, 95] for more details). Thus, we have to go beyond the Standard Model to explain the excess of baryons over antibaryons that we see in our universe.

There are a number of models and mechanisms on the market, such as GUT baryogenesis, electroweak baryogenesis [140, 141], Affleck-Dine baryogenesis [142] and baryogenesis through (thermal) leptogenesis [143]. All of them have their vices and virtues but we do not discuss them here since in this thesis we are only concerned with *non-thermal leptogenesis* [26, 27], which we discuss in the next section. For more details on the subject the reader is referred to the review articles and textbooks [144, 138, 145, 139, 146, 147, 95, 96] and references therein.

## 4.3 Non-Thermal Leptogenesis

The idea behind baryogenesis through leptogenesis – thermal or non-thermal – is to generate a lepton asymmetry that is transferred to the baryon sector through SM sphaleron processes [148, 140]. These are non-perturbative processes that violate $(B+L)$ while conserving $(B-L)$ [149] and that are due to the non-trivial topology of the vacuum manifold of the SM. Contrary to instanton effects, which correspond to tunnelling processes through the potential barrier separating the different vacuum configurations, and which are very much suppressed, sphaleron processes occur via thermal fluctuations over the barrier and can be much more efficient at finite temperature (cf. e.g. [95]). The produced baryon asymmetry $n_B$ is related to the lepton asymmetry $n_L$ via

$$n_B = \frac{C}{C-1}\, n_L = C\, n_{B-L}\ , \tag{4.12}$$

where $C$ is a numerical factor of $\mathcal{O}(1)$, which depends on the precise field content of the model in question and the temperature $T_s$ at which the sphalerons leave thermal equilibrium (typically, $T_s \approx T_{\mathrm{EW}}$). In the MSSM, for example [147]

$$C_{\mathrm{MSSM}} \approx \frac{1}{3}\ , \tag{4.13}$$

which is the number we will be working with.

In the simplest non-supersymmetric model of leptogenesis [143], the SM is augmented by (at least two) heavy right-handed Majorana neutrinos $\psi_{N^i}$ and the lepton asymmetry is generated through the decays of these neutrinos. They are the same right-handed neutrinos that can be used to explain the lightness of the SM neutrino masses in the type I seesaw mechanism. Hence, their introduction is very well motivated from a particle physics point of view. Also, the same neutrino Yukawa couplings that are employed in the type I seesaw mechanism

$$\mathcal{L} = -(y_\nu)_{ij}\, \bar{\psi}_{N^i}\, \Big((h)_a\, \epsilon^{ab}\, (l^j)_b\Big) + \mathrm{H.c.}\,, \tag{4.14}$$

where $h$ and $l^j$ are the SM Higgs and lepton doublets, mediate the lepton number violating decays of the right-handed neutrinos

$$\psi_{N^i} \longrightarrow l^j\, h\,, \tag{4.15}$$

$$\psi_{N^i} \longrightarrow \bar{l}^j\, \bar{h}\,. \tag{4.16}$$

These decays also violate $\mathcal{C}$ symmetry. However, at tree-level, $\mathcal{CP}$ symmetry is conserved in the SM augmented by right-handed neutrinos, such that the parameter

$$\epsilon = \frac{\Gamma(\psi_{N^i} \to l^j\, h) - \Gamma(\psi_{N^i} \to \bar{l}^j\, \bar{h})}{\Gamma(\psi_{N^i} \to l^j\, h) + \Gamma(\psi_{N^i} \to \bar{l}^j\, \bar{h})}\,, \tag{4.17}$$

measuring the $\mathcal{CP}$ violation per neutrino decay, vanishes at tree-level. Including at least two right-handed neutrinos in the theory and taking into account the interference with the one-loop diagrams shown in Fig. 4.2, on the other hand, yields a non-vanishing value for $\epsilon$ [150].

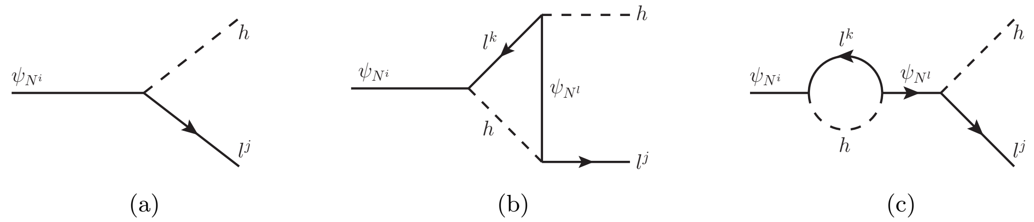



**Figure 4.2:** Decays of a right-handed neutrino into lepton and Higgs doublet. The interference between the tree-level (a) and the two one-loop diagrams (b),(c) generates the $\mathcal{CP}$ violation necessary for leptogenesis [150].

The scenario described above can be generalised in a straightforward way to include supersymmetry [151, 27]. In this case the right-handed sneutrino, the superpartner of the right-handed neutrino, can take the place of the particle responsible for the generation of the lepton asymmetry. Considering for example a simple extension of the MSSM with only three right-handed neutrino superfields added, the Yukawa couplings introduced in Eqn. (4.14) are generalised to the following couplings in the superpotential

$$W = (y_\nu)_{ij}\, \boldsymbol{N}^i \left( (\boldsymbol{h_u})_a\, \epsilon^{ab}\, (\boldsymbol{l}^j)_b \right)\,. \tag{4.18}$$

Assuming the right handed (s)neutrinos to be strongly hierarchical $m_{N^1} \ll m_{N^2} \ll m_{N^3}$ (which implies that the dominant contribution to the produced lepton asymmetry comes from decays of the lightest sneutrino), the $\mathcal{CP}$ violation per $N^i$- decay is bounded to be [150, 152]

$$\epsilon_i < \frac{3}{8\pi} \frac{\sqrt{\Delta m^2_{\mathrm{atm}}}\, m_{N^i}}{\langle h_u \rangle^2}\,. \tag{4.19}$$

This is the situation we are going to study in chapter 8 in the context of the model introduced in chapter 5. It turns out that in this model the lightest right-handed sneutrino *is the inflaton* and the universe is dominated by sneutrinos after inflation. The decays of these sneutrinos produce a lepton asymmetry which in the end is converted to a baryon asymmetry. Since the sneutrino is the inflaton in this scenario, leptogenesis is very efficient.

This also answers a question regarding the third Sakharov condition, namely how the departure from thermal equilibrium can be achieved. Since in our model the sneutrinos are "created" non-thermally at a point in time when the universe is far from thermal equilibrium, this condition is automatically satisfied. A model where the (s)neutrinos responsible for leptogenesis are produced in such a non-thermal way is consequently called *non-thermal* leptogenesis. If the heavy right-handed (s)neutrinos are produced in a thermal bath, on the other hand, one speaks of *thermal* leptogenesis.

# PART II

# Inflation

We have already seen in chapter 3 that cosmological inflation can dynamically resolve the flatness and horizon problems of the early universe and also explain the absence of relics from early phase transitions. Furthermore, it provides a way to generate the seeds for the formation of the large scale structure in the universe. As a matter of fact, inflation is the only known mechanism that can naturally reproduce the peak structure of the CMB power spectrum (and simultaneously solve the horizon, flatness and monopole problems), due to the way the fluctuations are imprinted into spacetime by inflation. All this reinforces our conviction that inflation is indeed the right mechanism to solve the aforementioned problems. However, the connection to particle physics is still unclear.

In this part of this thesis we try to establish such a possible connection. A key role in this endeavour is played by the right-handed sneutrinos, the superpartners of the right-handed neutrinos involved in the type I Seesaw mechanism.

We start the discussion in chapter 5 with a model that is a simple extension of the MSSM with conserved R-parity and that uses the scalar component fields of the right-handed neutrino superfields as inflatons. This model also contains Yukawa couplings between the right-handed neutrino superfields and the MSSM $SU(2)_L$ lepton supermultiplets. They become important when we discuss non-thermal leptogenesis within this model in chapter 8. In chapter 5, however, we concentrate on the inflationary dynamics of the model. In section 5.1 we describe the framework we are working in. In section 5.2 we then show that under the assumption of a hierarchical spectrum for the right-handed (s)neutrinos, we can treat our model as an effective one generation model, which greatly simplifies the discussion. Following that we discuss an explicit realisation of our model in global supersymmetry in section 5.3 and derive analytical expressions for the inflationary predictions. In section 5.4 we refine our discussion to include supergravity effects and again compute the inflationary predictions for our model. These predictions are then used in section 5.5 to constrain the parameters of the model that are relevant during inflation.

In the model discussed in chapter 5, the right-handed neutrino superfield containing the inflaton field is still a singlet and "put in by hand". In chapters 6 and 7 we investigate the possibility to use a gauge non-singlet (GNS) field as inflaton. In particular, we try to use fields associated to the *matter sector* of SUSY GUTs as inflatons. By matter sector we mean GUT representations that also contain the SM leptons and quarks. Before realising this idea in an explicit model of inflation in SUSY Pati-Salam unification and SUSY $SO(10)$ in chapter 7, we discuss our general setup in 6, using an inflaton charged under an Abelian $U(1)$ group. We also point out several additional complications that arise because of the gauge charge the inflaton now carries. In particular, we introduce the *gauge $\eta$-problem* in section 6.2 and discuss the production of topological defects in the waterfall phase transition after inflation in section 6.3.

Then we move on to an explicit realisation of GNS inflation in supersymmetric GUT models in chapter 7. We start with a discussion of the field content and superpotential of the model we want to consider, which is based on supersymmetric Pati-Salam unification. We show how inflation can proceed along a D-flat direction in field space in section 7.1.3 and then work out the specific example of *sneutrino inflation in SUSY Pati-Salam unification* in quite some detail in section 7.1.4. In section 7.2 we discuss one- and two-loop contributions to the inflaton potential and we show, in particular, how the gauge $\eta$-problem is resolved in our class of models. Finally, we generalise the model to supersymmetric $SO(10)$ GUTs in section 7.3 and discuss a possible embedding into supergravity in section 7.4. This then concludes our investigation of the inflationary epoch. In part III we come back to the model of chapter 5 and investigate non-thermal leptogenesis within that model.

CHAPTER 5

# Singlet Sneutrino Tribrid Inflation

In this chapter we discuss sneutrino tribrid inflation in an extension of the MSSM with conserved R-parity, where we add three additional gauge-singlet right-handed (s)neutrinos, which acquire large masses after the waterfall phase transition ending inflation. The Higgs-like waterfall field is also taken to be a singlet under the SM gauge group for simplicity in this chapter. The model is of the *tribrid inflation* type as introduced in section 3.6, which facilitates its embedding into a SUGRA framework.

Furthermore, the model also contains all the necessary ingredients to describe baryogenesis through non-thermal leptogenesis after inflation. This allows us to constrain the allowed parameter space of this model from two different directions – requiring successful inflation in accordance with the latest experimental bounds on the one side and the production of the right matter-antimatter asymmetry on the other side – which makes this framework quite predictive.

In this chapter, however, we concentrate on the inflationary dynamics within this model and move on to leptogenesis and a combined analysis of the allowed parameter space in chapter 8. We start with a general discussion of the framework we are working with in this chapter, then discuss a realisation of our model first within global supersymmetry and then in the context of supergravity. In both cases we compute the inflationary predictions of the model. These predictions are then used to put bounds on the model parameters that govern the inflationary dynamics.

## 5.1 Framework

The superpotential we are working with in this chapter is given by

$$W = W_{\mathrm{MSSM}} + (y_\nu)_{ij}\, \boldsymbol{N}^i\, (\boldsymbol{h_u})_a\, \epsilon^{ab} (\boldsymbol{l}^j)_b + \frac{\lambda_{ii}}{M_{\mathrm{P}}}\, (\boldsymbol{N}^i)^2 \boldsymbol{H}^2 + \kappa\, \boldsymbol{S}\left(\boldsymbol{H}^2 - M^2\right) + \dots \ . \quad (5.1)$$

Here, the superfields $\boldsymbol{N}^i$ are gauge singlet superfields describing the heavy right-handed (s)neutrinos and $i = 1, 2, 3$ is a flavour index. The canonically normalised imaginary parts $n_I^i$ of the respective scalar component fields $N^i = (n_R^i + \mathrm{i}\, n_I^i)/\sqrt{2}$ act as inflatons, since they are protected from the SUGRA $\eta$-problem by a shift symmetry in the Kähler potential as discussed in section 5.4.

$\boldsymbol{H}$ and $\boldsymbol{S}$ are two additional gauge singlet superfields. The canonically normalised real part $h_R$ of the scalar component field $H = (h_R + \mathrm{i}\, h_I)/\sqrt{2}$ constitutes the "waterfall" field responsible for ending inflation. The F-term of $\boldsymbol{S}$, the "driving superfield", on the other hand, provides the large vacuum energy density that drives inflation. As shown in section 5.4, the scalar component field of $\boldsymbol{S}$ is fixed at zero during inflation by SUGRA corrections, $S = 0$ [1], and does therefore not affect the inflationary dynamics. We work in a basis in which $\lambda_{ij}$ is diagonal with $\lambda_{ii}$ real and positive for $i = 1, 2, 3$. For simplicity, we also take $\kappa$ to be real and positive.

Finally, $\boldsymbol{h_u}$ and $\boldsymbol{l}^j$ are the Standard Model up-type Higgs and lepton doublet superfields charged under $SU(2)_L$, where $j = 1, 2, 3$ is again a flavour index.

A word on the notation we use for the superfields and their component fields: for all MSSM superfields, the SM component fields are denoted by letters in normal print while their MSSM-superpartners are denoted by a tilde, cf. Table 2.1. Thus, for example, $l^j$ denotes the SM lepton doublet while $\tilde{l}^j$ denotes the corresponding slepton doublet. For all non-MSSM superfields $\boldsymbol{R}$, the scalar component field is denoted in normal print as $R$, while the fermionic component field is denoted as $\psi_R$. We trust this is no cause for confusion.

Let us now discuss the roles of the different operators in the superpotential Eqn. (5.1). The operators that are relevant during inflation are the latter two. In the false vacuum with the $n_I^i$ acting as inflatons and the waterfall field $h_R$ stabilised at zero, the vacuum energy $V_0 = \kappa^2 M^4$ stemming from the $F_S$ term drives the quasi-exponential growth of the scale factor in inflation. Driven by the small slope of the effective one-loop Coleman-Weinberg potential as discussed in sections 3.5.1 and 3.5.2, the inflaton fields $n_I^i$ roll towards smaller field values. Once they fall below a critical value $n_{\mathrm{crit}}$, the negative contribution to the squared mass of $h_R$ from the term $\kappa\, \boldsymbol{S}\, (\boldsymbol{H}^2 - M^2)$ starts dominating over the positive contribution from the terms $\lambda_{ii}/M_{\mathrm{P}}\, (\boldsymbol{N}^i)^2 \boldsymbol{H}^2$. Hence, $h_R$ becomes tachyonic, which triggers the waterfall phase transition. Around this point the slow-roll conditions are violated and inflation ends.

During inflation, all the MSSM fields are stabilised at zero by large SUGRA mass corrections, such that $W_{\mathrm{MSSM}}$ in the superpotential Eqn. (5.1) is irrelevant during inflation. The scalar potential resulting from the scenario described here is of the typical form for hybrid-like inflation models as plotted in Fig. 3.5.

After inflation, when the large vacuum energy contribution vanishes and SUSY is approximately restored, the fields $n_I^i$ and $h_R$ perform damped oscillations around their global minima $n_I^i \approx 0$ and $h_R \approx \pm\sqrt{2} M$ and account for a matter dominated universe as discussed in section 4.1. At this stage, due to the large VEV of $h_R$, the second to last operator $\lambda_{ii}/M_{\mathrm{P}}\, (\boldsymbol{N}^i)^2 \boldsymbol{H}^2$ generates large masses for the right-handed (s)neutrinos.

[1] Technically speaking we should write $\langle S \rangle = 0$. However, if not stated otherwise, we will always be talking about the homogeneous $k = 0$ modes of the different scalar fields throughout the rest of this thesis and we will omit the angle brackets in order not to clutter up notation too much. This is also common practise in the literature and we have already used this convention in chapter 3. Keep in mind, however, that the full quantum field is given by $S = \langle S \rangle + \delta S$, cf. also Eqn. (3.30).

Taking into account their Yukawa couplings to the lepton and up-type Higgs doublets, $(y_\nu)_{ij}\, \boldsymbol{N}^i \boldsymbol{h_u} \,.\, \boldsymbol{l}^j$, their decays can generate the desired lepton asymmetry. These issues are discussed in much more detail in chapter 8. Finally, the dots in Eqn. (5.1) represent higher dimensional operators.

As a further remark we note that discrete symmetries can be used in an explicit model to restrict the superpotential to the form we use here and to distinguish between the different fields [31, 1].

During the inflationary phase the following parameters are of relevance in our model

- The **phase transition scale** $M$ is equal to the VEV of the scalar field $H$ in the true vacuum and it determines the energy scale of the waterfall phase transition.
- The parameter $\lambda_{11} > 0$ determines the **seesaw scale**, which corresponds to the mass of the lightest right-handed neutrino $m_{N^1} = 2\lambda_{11} M^2/M_{\mathrm{P}}$ in the true vacuum of the theory.
- The **vacuum energy parameter** $\kappa > 0$ fixes the the vacuum energy density $V_0$ that drives inflation with respect to the phase transition scale $M$, $V_0 = \kappa^2 M^4$.
- The **effective first generation Yukawa coupling** $\tilde{y}_1 \equiv \sqrt{(y_\nu y_\nu^\dagger)_{11}}$, which is related to the mass scale of the active Standard Model neutrinos.

Eventually, we will be working in a SUGRA framework with SUGRA corrections stabilising the scalar component fields of $\boldsymbol{S}$, $\boldsymbol{l}$, $\boldsymbol{h_u}$ and all other MSSM scalar fields at 0 during inflation. Furthermore, we will use a shift symmetry in the Kähler potential that keeps the $n_I^i$ directions flat. This solves the $\eta$-problem of SUGRA inflation for this model. The details of such a SUGRA framework are discussed in section 5.4. There we also find that another additional parameter becomes important, namely

- The **SUGRA correction parameter** $\delta$, which controls the SUGRA corrections to the loop potential.

We introduce this parameter here for completeness. However, to illustrate the underlying physics more clearly we first focus on a globally supersymmetric (SUSY) model and take the features mentioned above for granted.

Before doing so, however, we first have to discuss another important simplification, namely that assuming a hierarchical spectrum for the right-handed neutrinos we can treat this model as an effective one generation model for all practical purposes.

## 5.2 Simplification to an Effective One Generation Model

In the superpotential Eqn. (5.1) we have introduced three right-handed neutrino superfields $\boldsymbol{N}^i$. We choose to work with three (s)neutrinos for two reasons. First, we want to employ the type I seesaw mechanism to explain the smallness of the Standard Model neutrino masses. Since we know that at least two of the three Standard Model neutrinos are massive with different masses, we need at least two heavy right-handed neutrinos that are involved in the seesaw mechanism.

Second, we want to generate a lepton asymmetry through decays of the right-handed sneutrinos after inflation in order to produce the observed baryon asymmetry in our universe via leptogenesis. Consequently, Sakharov's conditions have to be satisfied. In particular, as already mentioned in section 4.3, we need more than one such right-handed (s)neutrino in order to have a non-vanishing $\mathcal{CP}$ violation in such decays. For reasons of symmetry we choose to work with three right-handed neutrino superfields.

Assuming that the right-handed (s)neutrinos are strongly hierarchical, i.e. one of them is significantly lighter than the other two, it turns out that for reasonable values of the model parameters, the scalar components of the two heavier superfields can be stabilised at their minima before the final 60 e-folds of inflation begin, cf. Fig. 5.1. Thus the time evolution of the lightest right-handed sneutrino controls the relevant slow-roll dynamics and it can therefore be identified as the inflaton.

Furthermore, as discussed more thoroughly in chapter 8, the outcome of leptogenesis is governed by the sneutrino with the smallest decay rate, which also implies a comparatively small mass and small Yukawa couplings.

Thus, in this chapter and in chapter 8 we concentrate on the case described above, where the lightest sneutrino drives both inflation and leptogenesis and the three generation model can be simplified to an effective one generation model in the right-handed (s)neutrino sector. The only remaining effect of the other two generations is a non-vanishing $\mathcal{CP}$ violation for leptogenesis.

With these considerations in mind we concentrate on $i = 1$ in Eqn. (5.1) and denote the relevant inflaton direction by $n \equiv n_I^1$ and the respective coupling constant by $\lambda \equiv \lambda_{11}$.

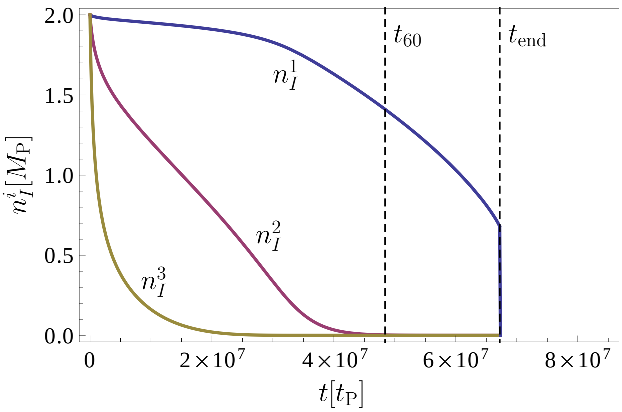



**Figure 5.1:** Inflationary dynamics for the case of three strongly hierarchical right-handed sneutrinos $N^i$. The inflaton direction is given by the imaginary direction $n_I^i$ of the corresponding scalar field $N^i$. These directions are protected from the SUGRA $\eta$-problem by a shift symmetry as discussed in section 5.4. The parameters used for this plot are $\kappa = 0.5, M = 0.0032\,M_\mathrm{P}, \lambda_{11} = 0.005, \lambda_{22} = 0.1, \lambda_{33} = 0.5$ and $\delta = 1$. The time when the CMB fluctuations exited the horizon is marked as $t_{60}$, inflation ends at $t_\mathrm{end}$. During the last 60 e-folds of inflation, which are relevant for the predictions of the model, we can treat our model as an effective one generation model.

## 5.3 Realisation in Global Supersymmetry

In this section we investigate the inflationary dynamics in a globally supersymmetric realisation of our model and derive (approximate) analytical expressions for the inflationary predictions.

In section 5.4 we then refine our discussion by including SUGRA effects and we use the predictions from our model to derive restrictions on the model parameters governing the inflationary dynamics, requiring successful inflation in accordance with the latest observational data.

We finally come back to the very same model in chapter 8, where we discuss non-thermal leptogenesis after inflation within this model. This allows us to considerably narrow down the allowed parameter space for the model by combining the constraints from inflation on the one side and successful leptogenesis on the other side.

Since the discussion of the model at hand is very typical for the investigation of hybrid/tribrid inflation models in general, we are quite explicit in this section in order to familiarise the reader with the steps that are necessary to derive predictions from such a model. This also allows us to be a bit more terse in the chapters on gauge non-singlet inflation later on.

The first step in the investigation is to derive the scalar potential from the superpotential (and the Kähler potential if we are working in supergravity). For a globally supersymmetric model, the scalar potential is given by Eqn. (1.92). Taking into account that the MSSM scalar fields are kept at zero during inflation by SUGRA corrections, it turns out that the D-term potential vanishes during inflation, $V_D = 0$. Also, all possible mass contributions from the D-term potential to the masses of the fields relevant for the one-loop corrections to the inflaton potential vanish for the same reason. Thus, it suffices to consider the F-term contributions to the scalar potential. Recall that we are treating our model as an effective one generation model with $N^1 \equiv N$ and $\lambda_{11} \equiv \lambda$. We have

$$F_S = \kappa \left(H^2 - M^2\right) , \tag{5.2}$$

$$F_H = 2\,\lambda\, H\, N^2/M_{\mathrm{P}} \, + 2\,\kappa\, S\, H\,, \tag{5.3}$$

$$F_N = 2\,\lambda\, N\, H^2/M_{\mathrm{P}} \, + \sum\nolimits_{j,a,b} (y_\nu)_{1j}\, (h_u)_a\, \epsilon^{ab}\, (\tilde{l}^j)_b\,, \tag{5.4}$$

$$F_{(h_u)_a} = \sum\nolimits_{j,b} (y_\nu)_{1j}\, N\, \epsilon^{ab}\, (\tilde{l}^j)_b\,, \tag{5.5}$$

$$F_{(\tilde{l}^j)_b} = \sum\nolimits_{a} (y_\nu)_{1j}\, N\, (h_u)_a\, \epsilon^{ab}\,. \tag{5.6}$$

Setting $S = 0 = \tilde{l}^j = h_u$ during the inflationary epoch, the resulting scalar potential is given by

$$V_{\mathrm{inf}} = \kappa^2 \left|H^2 - M^2\right|^2 + 4\frac{\lambda^2}{M_{\mathrm{P}}^2}\,|H|^2\,|N|^4 + 4\frac{\lambda^2}{M_{\mathrm{P}}^2}\,|N|^2\,|H|^4\ . \tag{5.7}$$

It is of the typical hybrid form, as depicted in Fig. 3.5.

During inflation, for inflaton values larger than the critical value $n > n_{\mathrm{crit}}$, both canonically normalised component fields $h_R$ and $h_I$ of $H = (h_R + \mathrm{i}\, h_I)/\sqrt{2}$ have masses larger than the Hubble scale (see below) and are therefore stabilised at zero. Along this trajectory the inflaton potential is tree-level flat and given by the large vacuum energy

$$V_0 = V_{\text{inf}}(n > n_{\text{crit}}, H = 0) = \kappa^2 M^4 \tag{5.8}$$

that drives inflation. A slope for the potential, which drives the inflaton towards its critical value, is generated by the Coleman-Weinberg one-loop corrections to the tree-level potential, which we discuss below.

Once the inflaton approaches the critical value, the field $h_R$ becomes tachyonic and triggers the waterfall phase transition. The critical value can be computed from the requirement $m^2_{h_R}(n_{\text{crit}}) = 0$. With the help of equation (5.11) this yields for our case

$$n^2_{\text{crit}} = \sqrt{2}\,\frac{\kappa}{\lambda} M M_{\text{P}}\,. \tag{5.9}$$

After inflation and the waterfall phase transition, both fields $n$ and $h_R$ roll down to their global minimum located at $n = 0$ and $h_R = \pm\sqrt{2}M$, around which they perform damped oscillation, cf. chapter 8. In this true vacuum, the large vacuum energy $V_0$ vanishes and SUSY is restored. (We assume a different mechanism to be responsible for the soft SUSY breaking within the MSSM but we do not discuss this issues further in this thesis.)

To continue our discussion we must now compute the Coleman-Weinberg one-loop corrections (cf. Eqn. (3.98))

$$V_{\text{loop}} = \frac{1}{64\pi^2}\,\text{STr}\left[\mathcal{M}^4\left(\ln\frac{\mathcal{M}^2}{Q^2} - \frac{3}{2}\right)\right]\,, \tag{5.10}$$

which generate the necessary slope of the inflaton potential. Relevant for the effective inflaton potential are only the $n$-dependent bosonic and fermionic mass terms, which can be calculated from the superpotential Eqn. (5.1) with $H = 0 = n_R = S = h_u = \tilde{l}^j$ along the inflationary trajectory.

We end up with the following mass terms [2]

$$\big[m_H^{(s)}\big]^2 = 2\,\kappa^2 M^2\,(x-1)\,, \tag{5.11}$$
$$\big[m_H^{(p)}\big]^2 = 2\,\kappa^2 M^2\,(x+1)\,, \tag{5.12}$$
$$\big[m_H^{(f)}\big]^2 = 2\,\kappa^2 M^2 x\,, \tag{5.13}$$
$$\big[m_{(h_u)_a}^{(s)}\big]^2 = \big[m_{(h_u)_a}^{(p)}\big]^2 = \big[m_{(h_u)_a}^{(f)}\big]^2 = n^2\,\tilde{y_1}^2/2\,, \tag{5.14}$$
$$\big[m_{(l^j)_b}^{(s)}\big]^2 = \big[m_{(l^j)_b}^{(p)}\big]^2 = \big[m_{(l^j)_b}^{(f)}\big]^2 = n^2\,|(y_\nu)_{1j}|^2/2\,, \tag{5.15}$$

with

$$x \equiv \left(\frac{n}{n_{\text{crit}}}\right)^4 = \frac{n^4\lambda^2}{2\,\kappa^2 M^2 M_{\text{P}}^2}\,. \tag{5.16}$$

[2] For a chiral superfield $\boldsymbol{\phi} = (\phi, \psi_\phi, F_\phi)$ with complex spin-0 component field $\phi = (\varphi_R + \mathrm{i}\,\varphi_I)/\sqrt{2}$, the index (s) (for scalar) denotes the mass term of the real part $\varphi_R$ of $\phi$ whereas the index (p) (for pseudo-scalar) marks the mass term of the purely imaginary part $\varphi_I$. The index (f) marks the mass term of the corresponding fermionic component field $\psi_\phi$.

Note that the $\boldsymbol{l}^j$ and $\boldsymbol{h_u}$ terms in the supertrace vanish since the degeneracy in the respective fermionic and bosonic masses leads to a cancellation of these contributions. This situation changes once we include SUGRA effects in the next section.

To continue, we fix the renormalisation scale to $Q = \sqrt{2}\,\kappa\, M$, which is the order of magnitude of the SUSY breaking scale.

To compute the one-loop contributions, note that during inflation

$$n > n_{\text{crit}} \quad \Rightarrow \quad \frac{1}{x} < 1\,. \tag{5.17}$$

We can use this to expand the logarithms as

$$\begin{aligned}\ln(x \pm 1) &= \ln\left[x \cdot \left(1 \pm \frac{1}{x}\right)\right] = \ln x + \ln\left(1 \pm \frac{1}{x}\right) \\ &= \ln x \pm \frac{1}{x} - \frac{1}{2x^2} + \mathcal{O}\left(\frac{1}{x^3}\right)\,,\end{aligned} \tag{5.18}$$

with

$$\ln(1 \pm a) = \pm a - \frac{a^2}{2} + \mathcal{O}(a^3) \quad \text{for} \quad a < 1\,. \tag{5.19}$$

With this we obtain

$$\begin{aligned}V_{\text{loop}} &= \frac{1}{64\pi^2}\Bigg[(m_H^{(s)})^4\left(\ln\frac{(m_H^{(s)})^2}{Q^2} - \frac{3}{2}\right) + (m_H^{(p)})^4\left(\ln\frac{(m_H^{(p)})^2}{Q^2} - \frac{3}{2}\right) \\ &\qquad - 2(m_H^{(f)})^4\left(\ln\frac{(m_H^{(f)})^2}{Q^2} - \frac{3}{2}\right)\Bigg] \\ &= \frac{4\kappa^4 M^4}{64\pi^2}\Bigg[(x-1)^2\left(\ln(x-1) - \frac{3}{2}\right) + (x+1)^2\left(\ln(x+1) - \frac{3}{2}\right) \\ &\qquad - 2x^2\left(\ln x - \frac{3}{2}\right)\Bigg] \\ &= \frac{\kappa^4 M^4}{8\pi^2}\left(\ln x + \mathcal{O}\left(\frac{1}{x^2}\right)\right)\,,\end{aligned} \tag{5.20}$$

and the effective potential along the inflationary trajectory at one-loop level is given by

$$V = V_0 + V_{\text{loop}} = \kappa^2 M^4 + \frac{\kappa^4 M^4}{8\pi^2}\ln x + \mathcal{O}\left(\frac{1}{x^2}\right)\,. \tag{5.21}$$

Similar to the discussion of SUSY F-term hybrid inflation in section 3.5.2 and Appendix B, we can now plug this into the equation of motion for the inflaton field

$$3\,\mathcal{H}\,\dot{n} \simeq -\frac{\partial V}{\partial n} = -\frac{\kappa^4 M^4}{2\pi^2}\frac{1}{n}\,, \tag{5.22}$$

and then solve for the inflaton field value $n_\mathcal{N}$ at the time $\mathcal{N}$ e-folds before the end of inflation. We get

$$n_\mathcal{N}^2 \simeq n_\mathrm{crit}^2 + \frac{\kappa^2 M_\mathrm{P}^2}{\pi^2}\mathcal{N}\,. \tag{5.23}$$

Neglecting the one-loop contribution compared to the tree-level contribution in the parameter range of interest, $V_0 = \kappa^2 M^4 \gg V_\mathrm{loop}$, we can derive the following analytical approximations for the slow-roll parameters

$$\epsilon \simeq \frac{M_\mathrm{P}^2}{2}\left(\frac{V'}{V_0}\right)^2 \simeq \frac{\kappa^4 M_\mathrm{P}^2}{8\pi^4\, n_\mathcal{N}^2}\,, \tag{5.24}$$

$$\eta \simeq M_\mathrm{P}^2 \frac{V''}{V_0} \simeq -\frac{\kappa^2 M_\mathrm{P}^2}{2\pi^2\, n_\mathcal{N}^2}\,, \tag{5.25}$$

$$\xi^2 \simeq M_\mathrm{P}^4 \frac{V'V'''}{V_0^2} \simeq \frac{\kappa^4 M_\mathrm{P}^4}{2\pi^4\, n_\mathcal{N}^4}\,, \tag{5.26}$$

to be evaluated at $n_\mathcal{N} = n_{60}$.

With these expressions and to leading order in the slow-roll approximation, the inflationary predictions for our model are given by

$$n_s = 1 - 6\epsilon + 2\eta \simeq 1 - \frac{\kappa^2 M_\mathrm{P}^2}{\pi^2\, n_\mathcal{N}^2}\left(1 + \frac{3\kappa^2}{4\pi^2}\right)\,, \tag{5.27}$$

$$r = 16\epsilon \simeq \frac{2\,\kappa^4 M_\mathrm{P}^2}{\pi^4\, n_\mathcal{N}^2}\,, \tag{5.28}$$

$$\alpha_s = 16\epsilon\eta - 24\epsilon^2 - 2\xi^2 \simeq -\frac{\kappa^4 M_\mathrm{P}^4}{\pi^4\, n_\mathcal{N}^4}\left(1 + \frac{\kappa^2}{\pi^2} + \frac{3\kappa^4}{8\pi^4}\right)\,, \tag{5.29}$$

$$\Delta_s^2 = \frac{1}{12\,\pi^2 M_\mathrm{P}^6}\frac{V_0^3}{(V')^2} \simeq \frac{\pi^2 M^4}{3\,\kappa^2 M_\mathrm{P}^6}\, n_\mathcal{N}^2\,. \tag{5.30}$$

Assuming

$$\kappa\,\lambda\,\frac{\mathcal{N}}{\pi^2} \gg \sqrt{2}\,\frac{M}{M_\mathrm{P}} \iff n_\mathrm{crit}^2 \ll \frac{\kappa^2 M_\mathrm{P}^2}{\pi^2}\mathcal{N} \tag{5.31}$$

and $\kappa^2/\pi^2 \ll 1$, which holds in a wide range of the parameter space we are interested in (cf. section 5.5), we arrive at the following analytical approximations, to be evaluated at $\mathcal{N} \approx 60$ :

$$n_s \simeq 1 - \frac{1}{\mathcal{N}} \quad \longrightarrow \quad n_s \approx 0.98\,, \tag{5.32}$$

$$r \simeq \frac{2\kappa^2}{\pi^2 \mathcal{N}} \quad \longrightarrow \quad r \approx \kappa^2 \cdot \mathcal{O}(10^{-3})\,, \tag{5.33}$$

$$\alpha_s \simeq -\frac{1}{\mathcal{N}^2} \quad \longrightarrow \quad \alpha_s \approx \mathcal{O}(10^{-4})\,, \tag{5.34}$$

$$\Delta_s^2 \simeq \frac{M^4}{3M_\mathrm{P}^4}\mathcal{N} \quad \longrightarrow \quad \Delta_s^2 \approx 20\, M^4/M_\mathrm{P}^4\,. \tag{5.35}$$

Note, in particular, the last expression: to leading order the amplitude of the CMB fluctuations depends only on the phase transition scale $M$. Together with the measured value of $\Delta_s^2$ this allows us to unambiguously fix the value for $M$. This in turn means that we can now use the parameter $\lambda$ and the mass of the lightest sneutrino $m_N \equiv m_{N^1}$ after inflation interchangeably, notwithstanding the fact that the inflaton is, of course, nearly massless throughout the inflationary epoch. Once $M$ is fixed the two parameters are completely equivalent and related by $m_N = 2\,\lambda\, M^2/M_{\mathrm{P}}$. We make use of this fact e.g. in Fig. 5.3, where the inflationary results are plotted over $m_N$ rather than $\lambda$ for later convenience (cf. chapter 8).

## 5.4 Realisation in Supergravity

Let us now turn to the embedding of our model within a supergravity context. We consider the following Kähler potential together with the superpotential given in Eqn. (5.1)

$$K = K_{\mathrm{MSSM}} + |S|^2 + |H|^2 + \sum\nolimits_i \frac{1}{2}\big(N^i + N^{i*}\big)^2 + \frac{\kappa_{\mathrm{S}}}{M_{\mathrm{P}}^2}|S|^4 + \frac{\kappa_{\mathrm{SH}}}{M_{\mathrm{P}}^2}|S|^2|H|^2 + \ldots \;. \tag{5.36}$$

$K_{\mathrm{MSSM}}$ describes a canonical Kähler potential for all MSSM scalar fields, including the up-type Higgs doublet $h_u$ and the slepton doublets $\tilde{l}^j$. These canonical terms lead to a stabilisation of all these fields during inflation due to SUGRA masses larger than the Hubble scale $m^2 > \mathcal{H}^2$ as described in section 3.6 and in particular in Eqn. (3.113).

The remaining terms are the ones relevant during inflation. As already advertised, the Kähler potential utilises a shift symmetry to protect the imaginary parts $n_I^i$ of the scalar fields $N^i$ from large SUGRA mass corrections to solve the $\eta$-problem. Consequently, the Kähler potential is a function of the real combinations $N^i + N^{i*}$ only.

We have also explicitly shown two non-minimal couplings with coupling constants $\kappa_{\mathrm{S}}$ and $\kappa_{\mathrm{SH}}$. The first one is necessary in order to stabilise the driving field $S$ at zero, as we will shortly see. The second one gives the leading contribution to the SUGRA corrections to the one-loop potential and thus the inflationary predictions of our model. Finally, the dots denote additional non-minimal couplings, which turn out to be negligible for our discussion.

Given the superpotential Eqn. (5.1) and the Kähler potential Eqn. (5.36) we can now proceed to calculate the mass spectrum of the model using Eqns. (1.143) and (1.146) along the inflationary trajectory. From now on we again treat our model as an effective one generation model and ignore the fields $N^2$ and $N^3$.

For the driving superfield $\boldsymbol{S}$ we find

$$\big[m_S^{(s)}\big]^2 = \big[m_S^{(p)}\big]^2 = -4\,\kappa^2 M^4\,\frac{\kappa_{\mathrm{S}}}{M_{\mathrm{P}}^2}\,, \tag{5.37}$$

$$\big[m_S^{(f)}\big]^2 = 0\,. \tag{5.38}$$

The Hubble scale during inflation, on the other hand, is given by

$$\mathcal{H}^2 \simeq \frac{V}{3M_{\mathrm{P}}^2} \simeq \frac{\kappa^2 M^4}{3M_{\mathrm{P}}^2}\,. \tag{5.39}$$

If the scalar component fields $s_R$ and $s_I$ are to be stabilised at zero with a mass larger than the Hubble scale, we therefore need

$$-4\,\kappa^2 M^4\,\frac{\kappa_{\rm S}}{M_{\rm P}^2} > \frac{\kappa^2 M^4}{3M_{\rm P}^2} \quad\Rightarrow\quad \kappa_{\rm S} < -\frac{1}{12}\,. \tag{5.40}$$

We assume this situation in the following and consequently set $S = 0$ during inflation, as anticipated above. Moreover, since these masses are independent of the inflaton field value $n$ we do not have to include them in the loop corrections to the inflaton potential.

Turning to the up-type Higgs and lepton doublets $\boldsymbol{h_u}$ and $\boldsymbol{l^j}$, their masses are given by

$$\big[m^{(s)}_{(h_u)_a}\big]^2 = \big[m^{(p)}_{(h_u)_a}\big]^2 = \frac{\kappa^2 M^4}{M_{\rm P}^2} + n^2\,\tilde{y}_1^2/2\,, \tag{5.41}$$

$$\big[m^{(f)}_{(h_u)_a}\big]^2 = n^2\,\tilde{y}_1^2/2\,, \tag{5.42}$$

and

$$\big[m^{(s)}_{(l^j)_b}\big]^2 = \big[m^{(p)}_{(l^j)_b}\big]^2 = \frac{\kappa^2 M^4}{M_{\rm P}^2} + n^2\,|(y_\nu)_{1j}|^2/2\,, \tag{5.43}$$

$$\big[m^{(f)}_{(l^j)_b}\big]^2 = n^2\,|(y_\nu)_{1j}|^2/2\,. \tag{5.44}$$

As we can see, the masses for the scalar fields are always larger than the Hubble scale and $h_u = 0 = \tilde{l}^j$ is justified during inflation. We can also see that the degeneracy between the masses of the scalar and fermionic component fields we encountered in the SUSY case is lifted. There appears now a mass splitting, characterised by the *SUGRA mass splitting scale*

$$Q_{\rm SUGRA} = \kappa M^2/M_{\rm P} \ll \sqrt{2}\kappa M = Q_{\rm SUSY}\,, \tag{5.45}$$

which is much smaller than the SUSY breaking scale $\sqrt{2}\kappa M$. Consequently, even though there *is* now a contribution from the superfields $\boldsymbol{h_u}$ and $\boldsymbol{l^j}$ to the Coleman-Weinberg potential, for the parameter range of interest, which will turn out to be $\tilde{y}_1 < 10^{-2}$, this contribution is completely negligible compared to the contribution from the waterfall superfield $\boldsymbol{H}$.

Speaking of which, the waterfall field masses are

$$\big[m_H^{(s)}\big]^2 = 2\,\kappa^2 M^2\left[x - 1 + \left(\frac{M}{M_{\rm P}}\right)^2\left(\frac{1-\kappa_{\rm SH}}{2}\right)\right], \tag{5.46}$$

$$\big[m_H^{(p)}\big]^2 = 2\,\kappa^2 M^2\left[x + 1 + \left(\frac{M}{M_{\rm P}}\right)^2\left(\frac{1-\kappa_{\rm SH}}{2}\right)\right], \tag{5.47}$$

$$\big[m_H^{(f)}\big]^2 = 2\,\kappa^2 M^2\,x\,, \tag{5.48}$$

with $x$ given by

$$x = \frac{n^4\lambda^2}{2\,\kappa^2 M^2 M_{\rm P}^2}\,, \tag{5.49}$$

as before. They give the dominant contributions to the loop potential. From the expression for the mass of the real waterfall component field we can compute the critical value in the case of SUGRA by setting $[m_H^{(s)}(n_{\rm crit})]^2 = 0$. We get

$$n_{\rm crit}^2 = \sqrt{2}\,\frac{\kappa\, M M_{\rm P}}{\lambda}\left[1 - \left(\frac{M}{M_{\rm P}}\right)^2\left(\frac{1-\kappa_{\rm SH}}{2}\right)\right]^{1/2}. \tag{5.50}$$

Finally, as a cross check we find

$$\left[m_N^{(p)}\right]^2 = m_n^2 = 0\,, \tag{5.51}$$

$$\left[m_N^{(s)}\right]^2 = 2\,\frac{\kappa^2}{M_{\rm P}^2} M^4\,, \tag{5.52}$$

$$\left[m_N^{(f)}\right]^2 = 0\,. \tag{5.53}$$

By construction, the inflaton $n \equiv n_I^1$ is massless at tree-level because it is protected by the shift symmetry, whereas the real part, which is not protected, receives a SUGRA mass correction larger than the Hubble scale, which stabilises it at zero during inflation.

We are now in a position to compute the loop corrections to the effective potential. Taylor-expanding the dominant contribution stemming from the inflaton-dependent waterfall masses as discussed above, the result is given by

$$V_{\rm loop} = \frac{\kappa^4 M^4}{8\pi^2}\left[\ln x + 2\,x\,y\,(\ln x - 1) + \mathcal{O}\left(\frac{1}{x^2}, y^2\right)\right], \tag{5.54}$$

with y defined as

$$y \equiv \left(\frac{M}{M_{\rm P}}\right)^2\left(\frac{1-\kappa_{\rm SH}}{2}\right) \ll 1\,, \tag{5.55}$$

parametrising the magnitude of the SUGRA corrections to the effective potential. Since $M$ will be fixed by the value of $\Delta_s^2$, the decisive quantity turns out to be the *SUGRA correction parameter*

$$\delta \equiv 1 - \kappa_{\rm SH}\,, \tag{5.56}$$

and $\delta = 0$ recovers the results from global SUSY [3]. Figure 5.2 shows the quality of the Taylor expansion in Eqn. (5.54) for realistic values of the model parameters compared to a full numerical computation.

## 5.5 Predictions

With an expression for the SUGRA one-loop effective potential, we are finally in a position to derive the inflationary predictions from our model in the locally supersymmetric case.

[3] We expect this to hold only up to the order we have expanded the Kähler potential to. Higher dimensional operators again induce SUGRA corrections to the observables, even for $\delta = 0$. These effects, however, are very much suppressed and can therefore be safely neglected.

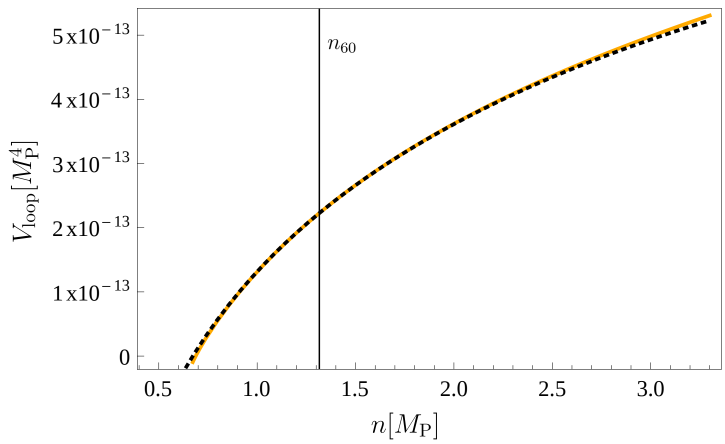



**Figure 5.2:** Quality of the Taylor approximation given in Eqn. (5.54) (dashed, black) to the SUGRA one-loop effective potential. The solid orange line shows a full numerical evaluation of the Coleman-Weinberg one-loop potential. The parameters for this plot are $\kappa = 0.5, M = 0.0032\, M_{\text{P}}, \lambda = 5 \cdot 10^{-3}$ and $\delta = 1$.

To do so we must again integrate back the equations of motion for the inflaton field to find the field value $n_{60}$ at the time 60 e-folds before the end of inflation. This value can then be plugged into the slow-roll expressions for the amplitude of the scalar fluctuations $\Delta_s^2$, the spectral index $n_s$, the tensor-to-scalar ratio $r$ and the running of the spectral index $\alpha_s$.

This has been done in a full numeric simulation for the SUGRA case at hand. The results thus obtained allow us to constrain the allowed parameter space of our model by comparing them with the latest bounds from the WMAP collaboration [6]. The results are show in Fig. 5.3

Let us now discuss these results and the implied restrictions on the model parameters in some more detail.

- The **phase transition scale** $M$ is fixed to $M \approx 0.0032\, M_{\text{P}} \approx 8 \cdot 10^{15}$ GeV with a slight deviation in the region of large SUGRA corrections. This is consistent with the global SUSY calculation which gives $M^4 \simeq 3\Delta_s^2/\mathcal{N} \cdot M_{\text{P}}^4$ for $n_{\mathcal{N}} \gg n_{\text{crit}}$ (cf. Eqn. (5.35)).

- The width of the bands in Fig. 5.3 is given by the variation of the **vacuum energy parameter** $\kappa$. A priori we would expect $\kappa$ to be an $\mathcal{O}(1)$ parameter, thus we shall assume $0.5 < \kappa < 2$. In Fig. 5.3, larger values of $\kappa$ are associated with SUGRA corrections becoming relevant at smaller values of $m_N$. In particular the tensor-to-scalar ratio $r$ is quite sensitive to $\kappa$, with $\kappa = 2$ leading to a comparatively large $r \sim \mathcal{O}(10^{-2})$.

- Fig. 5.3 also demonstrates the effect of the **SUGRA correction parameter** $\delta$. The respective quantities are marked in lavender for $\delta = 0$, which corresponds to the globally supersymmetric limit, and in green (blue) for $\delta = +1\,(-1)$, which corresponds to turning on the SUGRA corrections in the $\boldsymbol{H}$ mass terms with positive (negative) sign. In the considered SUGRA context the value of $\delta$ is a priori undetermined. Thus we would in general not expect to find global SUSY restored, which would correspond to $\delta$ exactly equal to zero.

- The second parameter controlling the effect of the SUGRA corrections is the coupling constant $\lambda$, which for fixed $M$ is directly proportional to the **seesaw scale** $m_N = 2\,\lambda\,M^2/M_{\mathrm{P}}$. Fig. 5.3 shows that the observables are independent of $\delta$ for small values of $\lambda$. In this range, and with $M$ fixed to $M = 0.0032 M_{\mathrm{P}}$, the model predicts

$$0.98 < n_s < 1\,, \tag{5.57}$$
$$3 \cdot 10^{-4} < \alpha_s < 0\,, \tag{5.58}$$
$$r < 0.013\,, \tag{5.59}$$

which again corresponds to the case of global SUSY. For very small values of $\lambda$, the spectral index $n_s$ approaches 1, which is not preferred by the latest WMAP data.
On the other hand, all solutions with $\delta \neq 0$ [4] leave the experimentally preferred region (at the 95% CL) for the spectral index at large values of $\lambda$. In combination, we find the preferred regions

$$5 \cdot 10^{-4} < \lambda < 0.13 \qquad \text{for} \qquad \delta = -1\,, \tag{5.60}$$
$$5 \cdot 10^{-4} < \lambda < 0.043 \qquad \text{for} \qquad \delta = +1\,, \tag{5.61}$$

respectively, which translates to

$$2 \cdot 10^{10}\,\mathrm{GeV} \lesssim m_N \lesssim 7 \cdot 10^{12}\,\mathrm{GeV} \qquad \text{for} \qquad \delta = -1\,, \tag{5.62}$$
$$2 \cdot 10^{10}\,\mathrm{GeV} \lesssim m_N \lesssim 2 \cdot 10^{12}\,\mathrm{GeV} \qquad \text{for} \qquad \delta = +1\,. \tag{5.63}$$

Summarising our findings so far, we take the phase transition scale to be fixed at

$$M \approx 0.0032\,M_{\mathrm{P}} \approx 8 \cdot 10^{15}\,\mathrm{GeV}\,, \tag{5.64}$$

and we concentrate on the following parameter space in the further discussion of this model in chapter 8:

$$2 \cdot 10^{10}\,\mathrm{GeV} < m_N < 7 \cdot 10^{12}\,\mathrm{GeV}\,, \tag{5.65}$$
$$0.5 < \kappa < 2\,, \tag{5.66}$$
$$|\delta| > 0.1\,. \tag{5.67}$$

This yields

$$-0.0004 \lesssim \alpha_s \lesssim 0.0002\,, \tag{5.68}$$
$$r \lesssim 0.015\,, \tag{5.69}$$

for the running of the spectral index and the tensor-to-scalar ratio.

[4] As mentioned above, we do not in general expect to find global SUSY restored and we therefore concentrate our discussion on the case $\delta \neq 0$, cf. Eqn. (5.67).

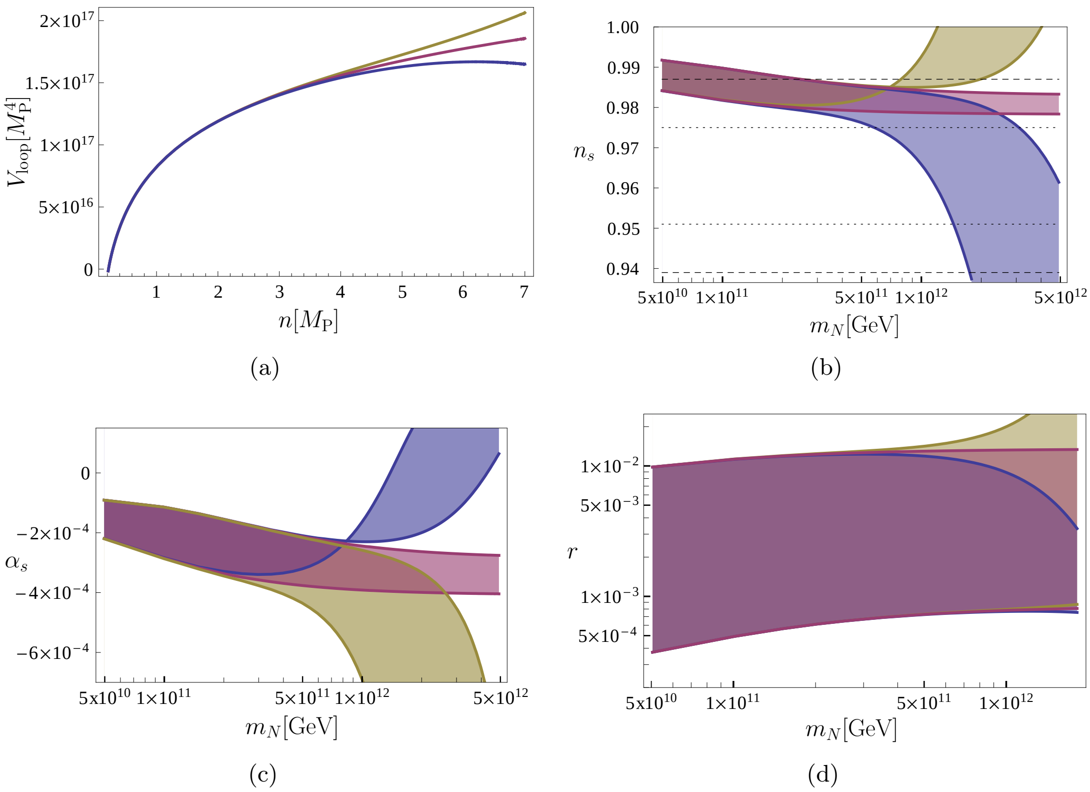



**Figure 5.3:** Loop potential and predictions for CMB observables for values of the SUGRA correction parameter $\delta = +1$ (green), $\delta = 0$ (lavender) and $\delta = -1$ (blue) from a full numerical study with $\mathcal{N} = 60$.

(a) Loop potential for $\kappa = 0.5$, $M = 0.0032\, M_{\rm P}$, $m_N = 2.5 \cdot 10^{11}$ GeV.

(b) - (d): Spectral index $n_s$, running of the spectral index $\alpha_s$ and tensor-to-scalar ratio $r$. The width of the bands is given by the variation of the vacuum energy parameter $\kappa = 0.5 - 2$. On the left border of plots (b) and (c), $\kappa = 0.5$ corresponds to the upper set of lines, and in plot (d) to the lower set of lines. For labelling the $x$-axis, the phase transition scale was set to $M = 0.0032\, M_{\rm P}$. The 95% and 68% CL experimental bounds from Eqns. (3.70) and (3.69) are marked by dashed and dotted lines in (b), respectively.

CHAPTER 6

# Inflation with a Charged Inflaton

In this chapter and the next, we want to discuss tribrid inflation with an inflaton that carries a gauge charge. This leads to a number of complications when we want to realise slow-roll inflation and the discussion in this chapter is meant to familiarise the reader with the basic setup and strategies we use to circumvent these problems. To keep things simple we consider an inflaton field charged under a $U(1)$ gauge symmetry in this chapter.

We move on to the more complicated case that the inflaton resides in a SUSY GUT multiplet in the next chapter, where we discuss GNS inflation in SUSY Pati-Salam unification and SUSY $SO(10)$ GUTs.

## 6.1 SUSY Tribrid Inflation with a Charged Inflaton

To start the discussion, let us remind ourselves of the main differences between SUSY hybrid inflation and SUSY tribrid inflation (cf. also section 3.6.2) under the premise of a charged inflaton. Standard SUSY hybrid inflation is based on the superpotential [35]

$$W_0 = \kappa \boldsymbol{S} \left( \boldsymbol{H}\bar{\boldsymbol{H}} - M^2 \right) , \tag{6.1}$$

where the superfield $\boldsymbol{S}$ has to be a singlet under the gauge group $G$, while the superfields $\boldsymbol{H}$ and $\bar{\boldsymbol{H}}$ reside in conjugate representations of $G$. (For simplicity, we have so far assumed that $\boldsymbol{H}$ is also a gauge singlet, meaning we were able to write $\kappa\, \boldsymbol{S}(\boldsymbol{H}^2 - M^2)$, cf. Eqn. (5.1), for example.) The role of $\boldsymbol{S}$ in this model is twofold: its F-term $F_S$ provides the vacuum energy to drive inflation and the scalar component field $S$ contains the slowly rolling inflaton. The scalar component fields $H$ and $\bar{H}$ act as waterfall fields, which take on zero values during inflation but are switched on when the inflaton reaches some critical value, ending inflation in the waterfall phase transition and breaking the gauge group $G$ at their global minimum $H = \bar{H} = M$. In a realistic model, $G$ can e.g. be identified with a GUT group and $H$, $\bar{H}$ are the Higgs which break that group [35]. Since $S$ has to be a singlet in this case, SUSY hybrid inflation is clearly not a good candidate to achieve our eventual goal of embedding the inflaton in a SUSY GUT representation.

Tribrid inflation, on the other side, splits the role of the superfield $\boldsymbol{S}$ that provides the large vacuum energy that drives inflation, and the superfield $\boldsymbol{\phi}$ that contains the inflaton. Along the lines discussed in section 3.6.2 let us therefore consider the following simple extension of the superpotential Eqn. (6.1),

$$W = W_0 + \frac{\zeta}{\Lambda} \left(\boldsymbol{\phi}\, \bar{\boldsymbol{\phi}}\right) \left(\boldsymbol{H} \bar{\boldsymbol{H}}\right) . \tag{6.2}$$

Here, we have included a pair of charged superfields $\boldsymbol{\phi}$ and $\bar{\boldsymbol{\phi}}$ in conjugate representations of $G$. As we want to consider the case $G = U(1)$ in this chapter this means that they have respective $U(1)$-charges $q$ and $-q$. They couple to the Higgs superfields via a non-renormalisable coupling controlled by a dimensionless coupling constant $\zeta$ and a scale $\Lambda$. This modification now allows for the new possibility that inflation is realised via slowly rolling scalar fields contained in the superfields $\boldsymbol{\phi}$ and $\bar{\boldsymbol{\phi}}$, with the singlet field $S$ staying fixed at zero during and after inflation. This is again ensured by SUGRA effects, similar to the discussion in section 5.4. On the other hand, large SUGRA mass contributions can be avoided for $\boldsymbol{\phi}$ and $\bar{\boldsymbol{\phi}}$ using a Heisenberg symmetry [30] as discussed in section 7.4. Note that a shift symmetry cannot be used to solve the $\eta$-problem here, since the inflaton is now charged. It is important to realise that the same setup can also be used for SUSY GUT multiplets $\boldsymbol{R^c}$ and $\bar{\boldsymbol{R}}^{\boldsymbol{c}}$ in place of the charged superfields $\boldsymbol{\phi}$ and $\bar{\boldsymbol{\phi}}$. This case is discussed in the next chapter.

To work out the important basic concepts more clearly, however, let us for now go back to the simplest case $G = U(1)$, where $\boldsymbol{\phi}$ and $\bar{\boldsymbol{\phi}}$ are two superfields with $U(1)$ charge $q$ and $-q$, respectively, and $\boldsymbol{H}$ and $\bar{\boldsymbol{H}}$ are also charged under the group $U(1)$ with opposite charges. Let us now discuss the inflationary dynamics in more detail.

To do so we need to calculate the full global SUSY potential. As usual, it is given by

$$V = V_F + V_D = F_i^\dagger F_i + \frac{1}{2}\, D^a D^a . \tag{6.3}$$

Let us assume, for simplicity, that the charges of $\boldsymbol{\phi}$ and $\boldsymbol{H}$ are equal. Then we find (setting a possible Fayet-Iliopoulos term to zero)

$$V_D = \frac{q^2}{2} \left(|\phi|^2 - |\bar{\phi}|^2 + |H|^2 - |\bar{H}|^2\right)^2 . \tag{6.4}$$

On the inflationary trajectory with $H = \bar{H}^* = 0$ this obviously has a D-flat direction given by the condition

$$\phi = \bar{\phi}^* . \tag{6.5}$$

Under the assumption that the D-term potential Eqn. (6.4) has already stabilised the fields in the D-flat valley, we only need to consider the F-term part

$$\begin{aligned} V_F = \; & \left|\kappa \left(H\bar{H} - M^2\right)\right|^2 + \left|\frac{\zeta}{\Lambda}\, \bar{\phi}\, (H\bar{H})\right|^2 + \left|\frac{\zeta}{\Lambda}\, \phi\, (H\bar{H})\right|^2 \\ & + \left|\kappa\, S\, \bar{H} + \frac{\zeta}{\Lambda}\, (\phi\, \bar{\phi})\, \bar{H}\right|^2 + \left|\kappa\, S\, H + \frac{\zeta}{\Lambda}\, (\phi\, \bar{\phi})\, H\right|^2 . \end{aligned} \tag{6.6}$$

Plugging the D-flatness condition $\phi = \bar{\phi}^*$ into Eqn. (6.6) and setting $S = 0$, the F-term potential reduces to

$$V_F = \left|\kappa^2 \left(M^2 - H\bar{H}\right)\right|^2 + 2\frac{|\zeta|^2}{\Lambda^2}|\phi|^2|H|^2|\bar{H}|^2 + \frac{|\zeta|^2}{\Lambda^2}|\phi|^4|H|^2 + \frac{|\zeta|^2}{\Lambda^2}|\phi|^4|\bar{H}|^2\,. \tag{6.7}$$

The situation within the D-flat valley is depicted in Fig. 6.1(a) for all model parameters set to unity.

As we can see, in the inflationary valley $S = 0 = H = \bar{H}$ the potential has a flat inflaton direction $|\phi|$ and a tachyonic waterfall direction below some critical value $\phi_{\mathrm{crit}}$, which is indicated in Fig. 6.1(a).

## 6.2 Radiative Corrections and Inflationary Predictions

As in the models discussed before, the tree-level flat inflaton direction is now lifted radiatively due to inflaton-dependent, SUSY breaking waterfall masses. On the inflationary trajectory they can be computed from Eqns. (6.1) and (6.2). Writing

$$H = \frac{h_R + \mathrm{i}\,h_I}{\sqrt{2}}\,, \quad \bar{H} = \frac{\bar{h}_R + \mathrm{i}\,\bar{h}_I}{\sqrt{2}}\,, \tag{6.8}$$

we find the following mass-squared matrices in the scalar and pseudo-scalar sector, respectively

$$\mathcal{M}_s^2 = \begin{pmatrix} \frac{|\phi^4|\,\zeta^2}{\Lambda^2} & -\kappa^2 M^2 \\ -\kappa^2 M^2 & \frac{|\phi^4|\,\zeta^2}{\Lambda^2} \end{pmatrix}\,, \quad \mathcal{M}_p^2 = \begin{pmatrix} \frac{|\phi^4|\,\zeta^2}{\Lambda^2} & \kappa^2 M^2 \\ \kappa^2 M^2 & \frac{|\phi^4|\,\zeta^2}{\Lambda^2} \end{pmatrix}\,. \tag{6.9}$$

Diagonalising these mass matrices, we find the following real mass eigenstates

$$\left.\begin{matrix} \bar{h}_R - h_R \\ \bar{h}_I + h_I \end{matrix}\right\} \longrightarrow m_+^2 = \frac{|\phi^4|\,\zeta^2}{\Lambda^2} + \kappa^2 M^2 \tag{6.10}$$

and

$$\left.\begin{matrix} \bar{h}_R + h_R \\ \bar{h}_I - h_I \end{matrix}\right\} \longrightarrow m_-^2 = \frac{|\phi^4|\,\zeta^2}{\Lambda^2} - \kappa^2 M^2\,. \tag{6.11}$$

Additionally, the spectrum contains two Weyl spinors with squared masses

$$m_f^2 = |\phi^4|\,\zeta^2/\Lambda^2\,. \tag{6.12}$$

One possible problem that arises when the inflaton is not a gauge singlet is that two-loop corrections to the inflaton potential can induce a mass for the inflaton that is larger than the Hubble scale during inflation and would thus spoil slow-roll inflation [24]. We call this issue the *gauge $\eta$-problem*, and it is discussed in more detail in section 7.2.2. For now, let us only quote the result, which tells us that the two-loop corrections turn out to be negligible in the setup we are discussing. This is essentially true due to the fact that the large VEV of the inflaton already breaks the gauge symmetry during inflation in the inflaton direction. Therefore, the inflaton effectively behaves like a singlet during inflation and decouples from the gauge interactions mediating the dangerous two-loop processes.

It is therefore enough to consider the effective potential up to one-loop level when calculating predictions for the observables. Moreover, since inflation proceeds along a predefined, straight trajectory in field space, characterised by the D-flatness condition Eqn. (6.5), the relevant inflationary predictions for the model discussed here are the number of e-folds $\mathcal{N}$ of inflation, the amplitude $\Delta_s^2$, spectral index $n_s$ and running of the spectral index $\alpha_s$ of the power spectrum for the scalar metric perturbations as well as the tensor-to-scalar ratio $r$, giving the amplitude of the tensor metric perturbations. These quantities can all be calculated from the potential as usual.

This concludes the discussion of the basic setup and strategies we use. Let us quickly summarise what we have discussed so far. Since the superfield $\boldsymbol{S}$ of standard SUSY hybrid inflation by construction has to be a gauge singlet, we have to modify the superpotential in order to include a charged inflaton. A very promising choice is a superpotential of the tribrid form

$$W = \kappa\, \boldsymbol{S} \left(\boldsymbol{H}\bar{\boldsymbol{H}} - M^2\right) + \frac{\zeta}{\Lambda} \left(\boldsymbol{\phi}\,\bar{\boldsymbol{\phi}}\right) \left(\boldsymbol{H}\bar{\boldsymbol{H}}\right) \,, \tag{6.13}$$

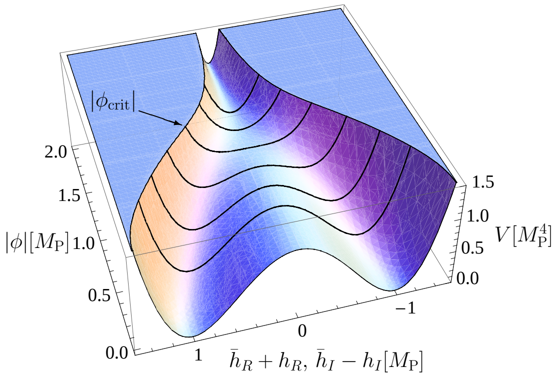
which is also tailor-made for a symmetry solution to the $\eta$-problem via a Heisenberg symmetry as discussed in sections 3.6 and 7.4. Here, $\boldsymbol{H}$ and $\bar{\boldsymbol{H}}$ are two Higgs superfields in conjugate representations of the gauge group $G$, which contain the waterfall field(s) as scalar component fields. The form of the superpotential allows to include a charged inflaton direction through the two additional superfields $\boldsymbol{\phi}$ and $\bar{\boldsymbol{\phi}}$, which also lie in conjugate representations of $G$. During inflation the scalar component fields $S$ as well as $H$ and $\bar{H}$ are stabilised at zero and inflation proceeds along a direction in field space along which the D-term contributions to the scalar potential vanish. This leads to a constraint on the VEVs of the scalar fields $\phi$ and $\bar{\phi}$, which defines the inflaton direction. During inflation, the gauge symmetry is broken in this direction by the large inflaton VEV and as a result the inflaton effectively behaves like a gauge singlet. This protects the inflaton from possibly dangerous two-loop mass corrections that would otherwise threaten to spoil slow-roll inflation. Once the inflaton reaches a critical value, inflation ends through the waterfall phase transition and subsequently the gauge group $G$ is broken down to some subgroup $G' \subset G$ through the VEV of the Higgs fields $H = \bar{H} = M$ in the global minimum of the potential.

Before we finally move on to a realisation of these ideas within an explicit SUSY GUT scenario, let us comment on one further complication that arises when the waterfall fields break some gauge symmetry in the waterfall phase transition.

## 6.3 Topological Defects

The potential problem that arises if the waterfall phase transition is associated with the breaking of a gauge symmetry $G$ is the possibility of copiously producing topological defects [18, 107]. In particular, if the gauge group $G$ is non-Abelian, the possibility of the production of magnetic monopoles arises. The abundance of such objects is very severely constraint from observations, which poses a serious thread to any model that predicts their production (cf. the discussion of the monopole problem in section 3.1).

For such topological defects to form it is necessary that at the critical value when the waterfall occurs, several different vacuum directions have degenerate masses and none is favoured over the other. If the same vacuum is chosen everywhere in space, no topological defects can form. This can be ensured by effective operators containing terms (e.g. in the superpotential) like $\boldsymbol{H}^n \bar{\boldsymbol{H}}^m \boldsymbol{\phi}^p \bar{\boldsymbol{\phi}}^q$ that can lead to a deformation of the potential, which can force the waterfall to happen in a particular field direction everywhere in space, thus avoiding the production of potentially problematic topological defects. This is illustrated in Fig. 6.1(b) for the example of the Abelian $U(1)$ symmetry discussed in this chapter. In this case, domain walls can be generated in the waterfall phase transition as should be clear from Fig. 6.1(a). A more detailed discussion for the case of a non-Abelian gauge group will ensue in section 7.1.




(a) Original potential (b) Deformed potential

**Figure 6.1:** Plot of the F-term tribrid inflation potential in the D-flat valleys $\phi = \bar{\phi}^*$, $H = \bar{H}^*$. For this plot we have chosen $\kappa = \zeta = 1$ and $\Lambda = M = M_{\rm P}$.

(a): Without deformations by higher-dimensional effective operators.

(b): Deformed potential, where an effective superpotential term $W_{\rm eff} = \tau\, \boldsymbol{H} \bar{\boldsymbol{\phi}}$ with $\tau = -0.4$ has been switched on. This term gives rise to a slope at $H = \bar{H} = 0$ that forces the field into the global minimum at positive $M$.

CHAPTER 7

# Inflation with a GUT Multiplet

In this chapter we discuss a realistic example of SUSY tribrid inflation with a gauge non-singlet (GNS) inflaton, where $G$ is identified with the SUSY Pati-Salam gauge group. Following the general ideas presented in the previous chapter, in the model under construction inflation proceeds along a trajectory in field space where the D-term contributions to the scalar potential vanish and the F-term contributions provide the vacuum energy that drives inflation. In addition to that we want to associate the inflaton field to the "matter sector" of the theory such that the model is closely related to low energy particle physics.

However, if we *only* include matter superfields fields $\boldsymbol{R^c_i} \sim (\bar{\mathbf{4}}, \mathbf{1}, \bar{\mathbf{2}})$ in our model (cf. section 2.2 for details on the representations of $G_{\mathrm{PS}}$), this typically leads to large D-term contributions incompatible with inflation. Therefore, in addition to the matter superfields $\boldsymbol{R^c_i}$, we also introduce another superfield $\boldsymbol{\bar{R}^c} \sim (\mathbf{4}, \mathbf{1}, \mathbf{2})$ in the conjugate representation of the gauge group. For simplicity, we discuss here the case where $i = 1, \ldots, 4$ and where there is only one $\boldsymbol{\bar{R}^c}$. As we will see, the introduction of $\boldsymbol{\bar{R}^c}$ is also necessary in order to keep all the waterfall directions stabilised during inflation. After inflation, one linear combination of the superfields $\boldsymbol{R^c_i}$ pairs up with $\boldsymbol{\bar{R}^c}$ and becomes heavy, while three other combinations remain light (and contain the three generations of SM fields), apart from the superfields containing the right-handed neutrinos of the type I seesaw mechanism, which also obtain large masses after inflation.

In addition to the introduction of the model in this section, we also work out an example in full detail, where the inflaton moves along a flat direction such that both $\bar{R}^c$ and one of the $R^c_i$ get a VEV in the sneutrino direction. Following that, we discuss consistency of the model with respect to one- and two-loop quantum corrections. We then discuss the embedding of the model into SUSY $SO(10)$ GUTs: starting with the Pati-Salam model, we first make it explicitly left-right symmetric and then describe how its field content can be embedded in $SO(10)$ representations. Finally, we sketch how the model can be embedded into a SUGRA framework.

## 7.1 GNS Inflation in SUSY Pati-Salam

Let us start now with the discussion of a realistic model of GNS inflation, where the inflaton resides in the matter sector of a supersymmetric GUT based on the Pati-Salam gauge group

$$G_{\mathrm{PS}} = SU(4)_C \times SU(2)_L \times SU(2)_R \,. \tag{7.1}$$

Details of $G_{\mathrm{PS}}$ and its representations have been discussed in section 2.2. For simplicity, we focus on the right sector of the theory only, i.e. fields that are charged under $SU(2)_R$. From the point of view of the Higgs sector breaking Pati-Salam to the SM this is sufficient, since VEVs of one $(\mathbf{4},\mathbf{1},\mathbf{2})$ and one $(\bar{\mathbf{4}},\mathbf{1},\bar{\mathbf{2}})$ are enough for this purpose. Therefore, let us first introduce the left-chiral $SU(2)_R$-doublet leptoquark superfields and their conjugate representation as discussed above

$$\boldsymbol{R}^{\boldsymbol{c}}_i = \begin{pmatrix} \boldsymbol{u}^{\boldsymbol{c}}_i & \boldsymbol{u}^{\boldsymbol{c}}_i & \boldsymbol{u}^{\boldsymbol{c}}_i & \boldsymbol{\nu}^{\boldsymbol{c}}_i \\ \boldsymbol{d}^{\boldsymbol{c}}_i & \boldsymbol{d}^{\boldsymbol{c}}_i & \boldsymbol{d}^{\boldsymbol{c}}_i & \boldsymbol{e}^{\boldsymbol{c}}_i \end{pmatrix} \sim (\bar{\mathbf{4}},\mathbf{1},\bar{\mathbf{2}}) \,, \tag{7.2}$$

$$\bar{\boldsymbol{R}}^{\boldsymbol{c}} = \begin{pmatrix} \bar{\boldsymbol{u}}^{\boldsymbol{c}} & \bar{\boldsymbol{u}}^{\boldsymbol{c}} & \bar{\boldsymbol{u}}^{\boldsymbol{c}} & \bar{\boldsymbol{\nu}}^{\boldsymbol{c}} \\ \bar{\boldsymbol{d}}^{\boldsymbol{c}} & \bar{\boldsymbol{d}}^{\boldsymbol{c}} & \bar{\boldsymbol{d}}^{\boldsymbol{c}} & \bar{\boldsymbol{e}}^{\boldsymbol{c}} \end{pmatrix} \sim (\mathbf{4},\mathbf{1},\mathbf{2}) \,, \tag{7.3}$$

where we have omitted colour indices for convenience and $i$ denotes a generation index. The $\boldsymbol{R}^{\boldsymbol{c}}_i$ multiplets contain the right-handed neutrino superfields, which are singlets under the SM gauge group. The waterfall Higgs superfields breaking Pati-Salam to the SM after inflation reside in the multiplets

$$\boldsymbol{H}^{\boldsymbol{c}} = \begin{pmatrix} \boldsymbol{u}^{\boldsymbol{c}}_{\boldsymbol{H}} & \boldsymbol{u}^{\boldsymbol{c}}_{\boldsymbol{H}} & \boldsymbol{u}^{\boldsymbol{c}}_{\boldsymbol{H}} & \boldsymbol{\nu}^{\boldsymbol{c}}_{\boldsymbol{H}} \\ \boldsymbol{d}^{\boldsymbol{c}}_{\boldsymbol{H}} & \boldsymbol{d}^{\boldsymbol{c}}_{\boldsymbol{H}} & \boldsymbol{d}^{\boldsymbol{c}}_{\boldsymbol{H}} & \boldsymbol{e}^{\boldsymbol{c}}_{\boldsymbol{H}} \end{pmatrix} \sim (\bar{\mathbf{4}},\mathbf{1},\bar{\mathbf{2}}) \,, \tag{7.4}$$

$$\bar{\boldsymbol{H}}^{\boldsymbol{c}} = \begin{pmatrix} \bar{\boldsymbol{u}}^{\boldsymbol{c}}_{\boldsymbol{H}} & \bar{\boldsymbol{u}}^{\boldsymbol{c}}_{\boldsymbol{H}} & \bar{\boldsymbol{u}}^{\boldsymbol{c}}_{\boldsymbol{H}} & \bar{\boldsymbol{\nu}}^{\boldsymbol{c}}_{\boldsymbol{H}} \\ \bar{\boldsymbol{d}}^{\boldsymbol{c}}_{\boldsymbol{H}} & \bar{\boldsymbol{d}}^{\boldsymbol{c}}_{\boldsymbol{H}} & \bar{\boldsymbol{d}}^{\boldsymbol{c}}_{\boldsymbol{H}} & \bar{\boldsymbol{e}}^{\boldsymbol{c}}_{\boldsymbol{H}} \end{pmatrix} \sim (\mathbf{4},\mathbf{1},\mathbf{2}) \,. \tag{7.5}$$

In addition, we introduce two further gauge singlet superfields, namely $\boldsymbol{S}$ and $\boldsymbol{X}$. The symmetry assignments to all the fields are given in the upper half of Tab. 7.1. As we can see, we have introduced two additional symmetries: a R-symmetry and a discrete $\mathbb{Z}_{10}$-symmetry. The lower half of Tab. 7.1 can be ignored until we introduce the left doublets in a more general framework in section 7.3.1. We would also like to remark at this point that the symmetries and charge assignments of Tab. 7.1 are not unique and should mainly illustrate that it is possible to obtain the desired form of the superpotential by symmetry arguments.

### 7.1.1 Effective Dimension 5 Operators in Pati-Salam

In addition to the restrictions imposed on the superpotential by these symmetries, we want to limit our discussion to the case where we consider effective operators up to dimension five that are generated by the exchange of singlet messenger fields only.

| Superfield | $G_{\text{PS}}$ | R | $\mathbb{Z}_{10}$ |
|---|---|---|---|
| $\boldsymbol{S}$ | $(\mathbf{1},\mathbf{1},\mathbf{1})$ | 1 | 0 |
| $\boldsymbol{X}$ | $(\mathbf{1},\mathbf{1},\mathbf{1})$ | 0 | 7 |
| $\boldsymbol{H^c}$ | $(\bar{\mathbf{4}},\mathbf{1},\bar{\mathbf{2}})$ | 0 | 1 |
| $\bar{\boldsymbol{H}}^{\boldsymbol{c}}$ | $(\mathbf{4},\mathbf{1},\mathbf{2})$ | 0 | 2 |
| $\boldsymbol{R}^{\boldsymbol{c}}_i$ | $(\bar{\mathbf{4}},\mathbf{1},\bar{\mathbf{2}})$ | 1/2 | 3 |
| $\bar{\boldsymbol{R}}^{\boldsymbol{c}}$ | $(\mathbf{4},\mathbf{1},\mathbf{2})$ | 1/2 | 4 |
| $\boldsymbol{H}$ | $(\mathbf{4},\mathbf{2},\mathbf{1})$ | 0 | 1 |
| $\bar{\boldsymbol{H}}$ | $(\bar{\mathbf{4}},\bar{\mathbf{2}},\mathbf{1})$ | 0 | 2 |
| $\boldsymbol{L}_i$ | $(\mathbf{4},\mathbf{2},\mathbf{1})$ | 1/2 | 3 |
| $\bar{\boldsymbol{L}}$ | $(\bar{\mathbf{4}},\bar{\mathbf{2}},\mathbf{1})$ | 1/2 | 4 |

**Table 7.1:** Superfield content of the $G_{\text{PS}}$ model and associated symmetries.

To find all such operators, let us first focus on the $SU(4)_C$ gauge structure. Under $SU(4)_C$, $\bar{\boldsymbol{R}}^{\boldsymbol{c}}, \bar{\boldsymbol{H}}^{\boldsymbol{c}} \sim \mathbf{4}$, whereas $\boldsymbol{R^c}, \boldsymbol{H^c} \sim \bar{\mathbf{4}}$. Furthermore, we know that

$$\mathbf{4} \otimes \bar{\mathbf{4}} = \mathbf{1} \oplus \mathbf{15}\,, \tag{7.6}$$
$$\mathbf{4} \otimes \mathbf{4} = \mathbf{10} \oplus \bar{\mathbf{6}}\,, \tag{7.7}$$
$$\bar{\mathbf{4}} \otimes \bar{\mathbf{4}} = \bar{\mathbf{10}} \oplus \mathbf{6}\,. \tag{7.8}$$

In order to form a singlet messenger we therefore have to couple one field transforming as a $\mathbf{4}$ to one transforming as a $\bar{\mathbf{4}}$. Note that coupling two such fields also yields a singlet under $SU(2)_R$, since in our model such fields transform as $\mathbf{2}$ and $\bar{\mathbf{2}}$ under $SU(2)_R$, respectively. The allowed fundamental vertices are shown in figure 7.1.

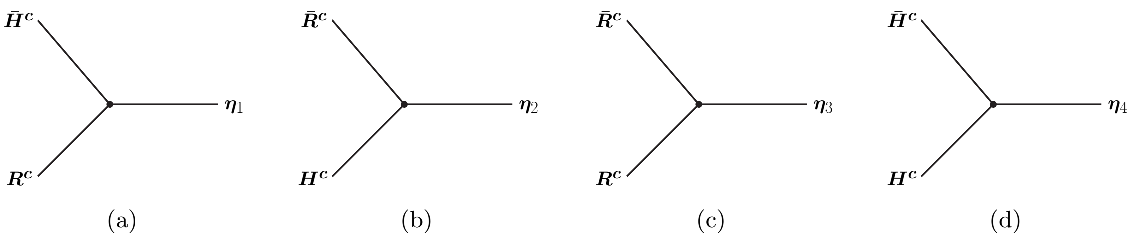



**Figure 7.1:** Allowed fundamental interaction vertices yielding singlet messenger fields.

When combining two of these fundamental vertices to form an effective $d = 5$ operator, we have to introduce a mass insertion, cf. Fig. 7.2. The corresponding term in the superpotential reads

$$W \supset \Lambda\, \boldsymbol{\eta}_i \boldsymbol{\eta}_j\ . \tag{7.9}$$

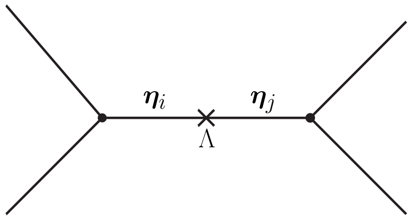



**Figure 7.2:** Feynman diagram generating the effective $d = 5$ operators.

From this we see that the R- and $\mathbb{Z}_{10}$-quantum numbers of the messenger fields involved have to add up to 1 and a multiple of 10, respectively. The quantum numbers of the messenger fields resulting from the vertices in Fig. 7.1 can be found in Tab. 7.2.

| Messenger | R | $\mathbb{Z}_{10}$ |
|---|---|---|
| $\boldsymbol{\eta}_1$ | 1/2 | 5 |
| $\boldsymbol{\eta}_2$ | 1/2 | 5 |
| $\boldsymbol{\eta}_3$ | 0 | 3 |
| $\boldsymbol{\eta}_4$ | 1 | 7 |

**Table 7.2:** Quantum numbers of the singlet messenger fields.

Thus, we can couple $\boldsymbol{\eta}_1$ and $\boldsymbol{\eta}_2$ to themselves, $\boldsymbol{\eta}_1$ to $\boldsymbol{\eta}_2$ and finally $\boldsymbol{\eta}_3$ to $\boldsymbol{\eta}_4$. After integrating out the heavy messengers, the following effective operators are generated, where the dot notation denotes contraction of the $SU(4)_C$ and $SU(2)_R$ indices as before

$$\mathcal{O}_1^{d=5} = \frac{\lambda}{\Lambda}\left(\boldsymbol{R^c}.\boldsymbol{\bar{H}^c}\right)\left(\boldsymbol{R^c}.\boldsymbol{\bar{H}^c}\right) , \tag{7.10}$$

$$\mathcal{O}_2^{d=5} = \frac{\gamma}{\Lambda}\left(\boldsymbol{\bar{R}^c}.\boldsymbol{H^c}\right)\left(\boldsymbol{\bar{R}^c}.\boldsymbol{H^c}\right) , \tag{7.11}$$

$$\mathcal{O}_3^{d=5} = \frac{\zeta}{\Lambda}\left(\boldsymbol{R^c}.\boldsymbol{\bar{R}^c}\right)\left(\boldsymbol{H^c}.\boldsymbol{\bar{H}^c}\right) , \tag{7.12}$$

$$\mathcal{O}_4^{d=5} = \frac{\xi}{\Lambda}\left(\boldsymbol{R^c}.\boldsymbol{\bar{H}^c}\right)\left(\boldsymbol{\bar{R}^c}.\boldsymbol{H^c}\right) . \tag{7.13}$$

### 7.1.2 GNS Superpotential in Pati-Salam

With these considerations taken into account and the symmetry assignments of Tab. 7.1, the allowed terms in the superpotential are

$$W = \kappa\, \boldsymbol{S} \left( \frac{\langle \boldsymbol{X} \rangle}{\Lambda}\, \boldsymbol{H^c}.\bar{\boldsymbol{H}}^{\boldsymbol{c}} - M^2 \right) + \frac{\lambda_{ij}}{\Lambda} (\boldsymbol{R}_i^{\boldsymbol{c}}.\bar{\boldsymbol{H}}^{\boldsymbol{c}}) (\boldsymbol{R}_j^{\boldsymbol{c}}.\bar{\boldsymbol{H}}^{\boldsymbol{c}}) + \frac{\gamma}{\Lambda} (\bar{\boldsymbol{R}}^{\boldsymbol{c}}.\boldsymbol{H^c}) (\bar{\boldsymbol{R}}^{\boldsymbol{c}}.\boldsymbol{H^c}) + \frac{\zeta_i}{\Lambda} (\boldsymbol{R}_i^{\boldsymbol{c}}.\bar{\boldsymbol{R}}^{\boldsymbol{c}}) (\boldsymbol{H^c}.\bar{\boldsymbol{H}}^{\boldsymbol{c}}) + \frac{\xi_i}{\Lambda} (\boldsymbol{R}_i^{\boldsymbol{c}}.\bar{\boldsymbol{H}}^{\boldsymbol{c}}) (\bar{\boldsymbol{R}}^{\boldsymbol{c}}.\boldsymbol{H^c})\,. \tag{7.14}$$

The roles of the superfields in this model are the following. $\boldsymbol{S}$ is the gauge singlet superfield contributing the large vacuum energy during inflation by its F-term, i.e $F_S \neq 0$. Its scalar component field $S$ stays at zero both during and after inflation. A large mass for $S$ that keeps it at zero can be generated by SUGRA effects due to non-canonical terms in the Kähler potential, as discussed before. The $SU(2)_R$-doublet superfields $\boldsymbol{H^c}$ and $\bar{\boldsymbol{H}}^{\boldsymbol{c}}$ contain as scalar component fields the waterfall fields, which are zero during inflation and become tachyonic subsequently, ending inflation in the waterfall phase transition and breaking $SU(4)_C \times SU(2)_L \times SU(2)_R$ to the MSSM by their VEVs. The $SU(2)_R$-charged leptoquark superfields $\boldsymbol{R}_i^{\boldsymbol{c}}$ together with $\bar{\boldsymbol{R}}^{\boldsymbol{c}}$ contain the slowly rolling inflaton directions as scalar component fields.

After the end of inflation we want all component fields of three generations of $\boldsymbol{R}_i^{\boldsymbol{c}}$, except for their right-handed (s)neutrinos, to be light. All component fields of $\bar{\boldsymbol{R}}^{\boldsymbol{c}}$, on the other hand, need to be heavy. With the number of generations of $\boldsymbol{R}_i^{\boldsymbol{c}}$ larger than the one of $\bar{\boldsymbol{R}}^{\boldsymbol{c}}$ by three, e.g. $i = 1, ..., 4$, all the $\bar{\boldsymbol{R}}^{\boldsymbol{c}}$ component fields pair up with some $\boldsymbol{R}_i^{\boldsymbol{c}}$ component field and form Dirac-type mass terms at the GUT scale and decouple from the theory. Only three $\boldsymbol{R}_i^{\boldsymbol{c}}$ generations remain light, apart from their right-handed (s)neutrinos, which also become heavy.

Let us now discuss the superpotential given in Eqn. (7.14) in more detail. The term proportional to $\zeta_i$ provides masses to all the components of the scalar multiplets $H^c$ and $\bar{H}^c$ during inflation when $R_i^c$ and $\bar{R}^c$ get VEVs. Looking at the superpotential we can easily convince ourselves that without the presence of the $\bar{R}^c$ multiplet, not all of the squared masses of the waterfall fields were positive during inflation and their immediate destabilisation would not allow for slow-roll inflationary dynamics. The introduction of the superfield $\boldsymbol{X}$ is motivated as follows: we have imposed the discrete $\mathbb{Z}_{10}$-symmetry to forbid a direct mass term for the $\boldsymbol{R}_i^{\boldsymbol{c}}$ and $\bar{\boldsymbol{R}}^{\boldsymbol{c}}$ fields, therefore charging $\boldsymbol{R}_i^{\boldsymbol{c}}.\bar{\boldsymbol{R}}^{\boldsymbol{c}}$ under the symmetry. On the other hand, we have allowed the operator $\boldsymbol{R}_i^{\boldsymbol{c}}.\bar{\boldsymbol{R}}^{\boldsymbol{c}}\, \boldsymbol{H^c}.\bar{\boldsymbol{H}}^{\boldsymbol{c}}$ in Eqn. (7.14), thus $\boldsymbol{H^c}.\bar{\boldsymbol{H}}^{\boldsymbol{c}}$ cannot be invariant under this discrete symmetry. Therefore, a superpotential term of the form $\boldsymbol{S}.\boldsymbol{H^c}.\bar{\boldsymbol{H}}^{\boldsymbol{c}}$ is forbidden. However, in the presence of the gauge singlet superfield $\boldsymbol{X}$ that gets a VEV around the Planck scale and breaks the discrete symmetry spontaneously, a similar term, $\frac{1}{\Lambda}\,\boldsymbol{S}.\boldsymbol{X}.\boldsymbol{H^c}.\bar{\boldsymbol{H}}^{\boldsymbol{c}}$, is allowed and it effectively generates the desired term after $X$ gets its VEV. To allow the term $\frac{1}{\Lambda}\,\boldsymbol{S}.\boldsymbol{X}.\boldsymbol{H^c}.\bar{\boldsymbol{H}}^{\boldsymbol{c}}$, the $\boldsymbol{X}$ field carries a charge equal to the charge of the product $\boldsymbol{R}_i^{\boldsymbol{c}}.\bar{\boldsymbol{R}}^{\boldsymbol{c}}$ under the discrete symmetry, as can be seen in Tab. 7.1.

### 7.1.3 D-Flat Inflaton Directions

The inflationary epoch is determined by the scalar potential given by both F-term and D-term contributions. For the sake of simplicity, in this section we investigate only the global SUSY limit. A possible embedding in supergravity is sketched in section 7.4.

Tribrid inflation requires a large vacuum energy density to drive the exponential expansion of the scale factor and a nearly flat inflaton direction. As explained above, in our model inflation proceeds along a trajectory in the (scalar) field space of $R_i^c$ and $\bar{R}^c$ along which the D-term contributions vanish. In such a D-flat valley, the F-term contribution from the $S$ field provides the necessary vacuum energy. Both the $R_i^c$ and $\bar{R}^c$ fields do not have any tree-level F-term mass contributions. On the other hand, due to the large F-term contributions to their masses, the waterfall fields remain at zero during inflation. Therefore, in our Pati-Salam framework, the tree-level F-term inflaton potential becomes $V_F = \kappa^2 M^4$, whereas the D-term potential reduces to

$$V_D = \frac{g^2}{2} \sum_{a=1}^{18} \Big( \sum_{i=1}^{4} -R_i^{c\dagger}\, \mathcal{T}^{a*}\, R_i^c \;+\; \bar{R}^{c\dagger}\, \mathcal{T}^a \bar{R}^c \Big)^2 . \tag{7.15}$$

Here, $\mathcal{T}^a$ with $a = 1, \dots, 18$ denote the 18 generators of the Pati-Salam gauge group, cf. Eqns. (B.22) and (B.23), and we assume gauge coupling unification around the GUT scale. From Eqn. (7.15) we can read of the following D-flatness conditions

$$\sum_{i=1}^{4} R_i^{c\dagger}\, \mathcal{T}^{a*}\, R_i^c = \bar{R}^{c\dagger}\, \mathcal{T}^a\, \bar{R}^c \,, \quad a = 1, \dots, 18 \,. \tag{7.16}$$

During inflation, these constraint equations have to be imposed on the F-term scalar potential.

Using Eqn. (7.16), it can be shown that several flat directions exist in this model. All these directions can in principle be valid trajectories for inflation to occur. During inflation $R_i^c$ and $\bar{R}^c$ acquire VEVs along one of these directions and break the Pati-Salam symmetry. The gauge fields coupled to this particular direction in field space become massive. This direction is classically flat and lifted only by radiative corrections such that it is suitable for inflation. On the other hand, other flat directions in field space along which the gauge symmetry is not broken and the gauge fields are still massless, acquire large two-loop mass contributions as will be clarified in section 7.2.2. Such large mass contributions lift these other flat directions strongly and drive their VEVs to zero.

After inflation, the breaking of the Pati-Salam gauge group is realised by the VEVs of $H^c$ and $\bar{H}^c$. In the next subsection we consider inflation along the right-handed sneutrino directions $\nu^c$ and $\bar{\nu}^c$, which provides one possible D-flat direction in field space. We show explicitly that in this case the VEVs of $H^c$ and $\bar{H}^c$ will be aligned in the right-handed sneutrino direction as well. We thus have an example model of "sneutrino inflation", realised with the inflaton residing in a non-singlet representation of $G_{\mathrm{PS}}$. It is important to emphasise that although the inflaton belongs to a non-singlet representation, it effectively behaves like a singlet since $G_{\mathrm{PS}}$ is already broken to $G_{\mathrm{SM}}$ during inflation. This also turns out to be very important with respect to quantum corrections to the inflaton potential.

### 7.1.4 An Example: Sneutrino Inflation

As we have mentioned in the last section, the model has several tree-level flat directions in the scalar field space of $R_i^c$ and $\bar{R}^c$, and in principle inflation can proceed along any of them. In this section we would like to discuss the inflationary dynamics when the matter fields acquire VEVs along the sneutrino direction. In addition we also discuss the waterfall mechanism in more detail. It turns out to be an interesting feature of this particular flat direction that at the end of inflation, and for generic choices of parameters, the waterfall fields $H^c$ and $\bar{H}^c$ acquire VEVs along the corresponding right-handed sneutrino directions $\nu_H^c$ and $\bar{\nu}_H^c$ as well. We also discuss how this preferred waterfall direction helps to avoid the production of stable monopoles after inflation.

Let us consider the explicit example where only one of the $R^c \equiv R_1^c \neq 0$ and $\bar{R}^c \neq 0$ are slow-rolling, while all the others remain at zero $R_{i\neq 1}^c = 0$. In addition, we want to realise inflation along the sneutrino direction

$$R^c = \begin{pmatrix} 0 & 0 & 0 & \nu^c \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad \bar{R}^c = \begin{pmatrix} 0 & 0 & 0 & \bar{\nu}^c \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{7.17}$$

This reduces our inflationary superpotential in Eqn. (7.14) to the effective form

$$\begin{aligned} W_{\text{inf}} = {} & \kappa\, \boldsymbol{S} \left(\boldsymbol{H^c}.\bar{\boldsymbol{H}}^{\boldsymbol{c}} - M^2\right) \\ & + \lambda\, (\boldsymbol{\nu^c}\, \bar{\boldsymbol{\nu}}^{\boldsymbol{c}}_{\boldsymbol{H}})^2 + \gamma\, (\bar{\boldsymbol{\nu}}^{\boldsymbol{c}}\, \boldsymbol{\nu}^{\boldsymbol{c}}_{\boldsymbol{H}})^2 + \xi\, (\boldsymbol{\nu^c}\, \bar{\boldsymbol{\nu}}^{\boldsymbol{c}})\, (\boldsymbol{\nu}^{\boldsymbol{c}}_{\boldsymbol{H}}\, \bar{\boldsymbol{\nu}}^{\boldsymbol{c}}_{\boldsymbol{H}}) + \zeta\, (\boldsymbol{\nu^c}\, \bar{\boldsymbol{\nu}}^{\boldsymbol{c}})\, (\boldsymbol{H^c}.\bar{\boldsymbol{H}}^{\boldsymbol{c}})\,, \end{aligned} \tag{7.18}$$

where we have absorbed all factors of $\Lambda^{-1}$ (and also the VEV of $X$) into the coupling constants. Due to the VEVs in Eqn. (7.17), $G_{\text{PS}}$ is already broken to $G_{\text{SM}}$ during inflation. If we can also ensure that the waterfall is forced into the $\nu_H^c$ and $\bar{\nu}_H^c$ directions in field space, no monopoles are produced after inflation.

With $R^c$ and $\bar{R}^c$ pointing in the right-handed sneutrino direction and the generators of $G_{\text{PS}}$ given by Eqns. (B.22) and (B.23), the global SUSY D-term potential reads

$$V_D = \frac{5}{16}\, g^2 \left(|\nu^c|^2 - |\bar{\nu}^c|^2\right)^2 . \tag{7.19}$$

This vanishes for $|\nu^c| = |\bar{\nu}^c|$. From now on, we assume that inflation occurs in this D-flat valley. Therefore, the scalar potential during inflation has to be calculated in the inflationary trajectory $S = 0 = H^c = \bar{H}^c$ with the D-flatness condition $|\nu^c| = |\bar{\nu}^c|$ imposed.

For simplicity, let us take the VEVs of $\nu^c$ and $\bar{\nu}^c$ to be real. Then, for the D-flat direction $\nu^c = \bar{\nu}^c$, the field combination $\text{Re}(\bar{\nu}^c - \nu^c)$ with mass $m^2 = 5\, g^2\, (\nu^c)^2/2$ is orthogonal to the flat, massless direction

$$\nu_+^c \equiv \text{Re}(\bar{\nu}^c + \nu^c)\,, \tag{7.20}$$

which is the inflaton direction in this case.

On the other hand, for the other D-flat direction $\nu^c = -\bar{\nu}^c$, the field combination $\mathrm{Re}(\bar{\nu}^c + \nu^c)$ becomes massive with mass $m^2 = 5\, g^2\, (\nu^c)^2/2$, while the massless inflaton direction is given by [1]

$$\nu^c_- \equiv \mathrm{Re}(\bar{\nu}^c - \nu^c)\,. \tag{7.22}$$

Next, we want to discuss how the waterfall mechanism works in our particular example. For definiteness, we take the inflaton direction to be $\nu^c_+$ with the VEVs satisfying $\nu^c = \bar{\nu}^c$. Then, the full F-term potential contains the following terms

$$\begin{aligned} V_F = \; & \left|\kappa \left(H^c.\bar{H}^c - M^2\right)\right|^2 + \left|2\,\lambda\,(\nu^c)^2 \bar{\nu}^c_H + \xi\,(\nu^c\,\bar{\nu}^c)\,\nu^c_H + \zeta\,(\nu^c\,\bar{\nu}^c)\,\nu^c_H\right|^2 \\ & + \left|\kappa\, S\, \bar{H}^c + \zeta\,(\nu^c\,\bar{\nu}^c)\,\bar{H}^c\right|^2 + \left|2\,\gamma\,(\bar{\nu}^c)^2 \nu^c_H + \xi\,(\nu^c\,\bar{\nu}^c)\,\bar{\nu}^c_H + \zeta\,(\nu^c\,\bar{\nu}^c)\,\bar{\nu}^c_H\right|^2 \\ & + \left|\kappa\, S\, H^c + \zeta\,(\nu^c\,\bar{\nu}^c)\,H^c\right|^2 + \left|2\,\gamma\,\bar{\nu}^c\,(\nu^c_H)^2 + \xi\,\nu^c\,(\nu^c_H \bar{\nu}^c_H) + \zeta\,\nu^c\,(H^c.\bar{H}^c)\right|^2 \\ & + \left|2\,\lambda\,\nu^c\,(\bar{\nu}^c_H)^2 + \xi\,\bar{\nu}^c\,(\nu^c_H \bar{\nu}^c_H) + \zeta\,\bar{\nu}^c\,(H^c.\bar{H}^c)\right|^2 . \end{aligned} \tag{7.23}$$

Terms containing uncontracted $H^c$ and $\bar{H}^c$ multiplets have to be summed over all components. We now decompose all complex scalar fields into real and imaginary components. Thus, for example,

$$\nu^c_H = \frac{\nu^c_{H_R} + \mathrm{i}\,\nu^c_{H_I}}{\sqrt{2}}\,, \tag{7.24}$$

$$\bar{\nu}^c_H = \frac{\bar{\nu}^c_{H_R} + \mathrm{i}\,\bar{\nu}^c_{H_I}}{\sqrt{2}}\,, \tag{7.25}$$

and analogous for all the other waterfall fields. Furthermore, we can parametrise the inflaton direction $\nu^c_+$ via a canonically normalised, real scalar field $\tilde{\nu}^c$ by defining

$$\nu^c = \tilde{\nu}^c/\sqrt{2}\,, \tag{7.26}$$

$$\bar{\nu}^c = \tilde{\nu}^c/\sqrt{2}\,. \tag{7.27}$$

In the scenario we are discussing here, all other scalar degrees of freedom from the $R^c_i$ and $\bar{R}^c$ multiplets obtain large masses, either directly from the D-term potential or through two-loop corrections as discussed in section 7.2.2, and are thus stabilised at zero during inflation. Therefore, we can ignore them for the time being.

Due to large F-term contributions to their masses from the VEVs of the inflaton fields, the waterfall fields are also fixed at zero during inflation. However, as the inflaton fields slowly roll towards smaller values, the masses of the waterfall fields decrease and finally one

[1] We remind the reader that we are usually working in a semi-classical approximation where we only talk about the homogeneous $k = 0$ mode of a given scalar field and we have agreed to omit the angle brackets denoting the VEV of such a field, cf. the footnote on page 94. However, the full quantum field is always given by $\phi = \langle\phi\rangle + \delta\phi$ (where $\delta\phi$ denote the quantum fluctuations around the VEV) and the effective mass of such a scalar field is given by

$$m^2 = \frac{\partial^2\, V\big(\langle\phi\rangle + \delta\phi\big)}{\partial\, \delta\phi^2}\Big|_{\delta\phi=0} = \frac{\partial^2\, V(\phi)}{\partial\, \phi^2}\Big|_{\phi=\langle\phi\rangle}\,. \tag{7.21}$$

or more directions in field space becomes tachyonic. The corresponding waterfall component fields now quickly roll to their true minima and inflation ends by the "waterfall". We now discuss in which direction in field space the waterfall is initiated, i.e. which direction becomes tachyonic first.

To start with, the masses of $(u^c_H, \bar{u}^c_H)$, $(d^c_H, \bar{d}^c_H)$ and $(e^c_H, \bar{e}^c_H)$ obtain universal contributions from the terms with couplings $\kappa$ and $\zeta$ and are block-diagonal within this basis (i.e. there is no mixing between e.g. the "up-type" fields $(u^c_H, \bar{u}^c_H)$ and the "down-type" fields $(d^c_H, \bar{d}^c_H)$, etc.). Concentrating, for example, on the $(u^c_H, \bar{u}^c_H)$ fields, the mass squared matrices for the scalar and pseudo-scalar sector are given by

$$\mathcal{M}^2_{(u^c_{H_R}, \bar{u}^c_{H_R})} = \begin{pmatrix} \frac{|\zeta|^2|\tilde{\nu}^c|^4}{4} & -|\kappa|^2 M^2 \\ -|\kappa|^2 M^2 & \frac{|\zeta|^2|\tilde{\nu}^c|^4}{4} \end{pmatrix} , \tag{7.28}$$

$$\mathcal{M}^2_{(u^c_{H_I}, \bar{u}^c_{H_I})} = \begin{pmatrix} \frac{|\zeta|^2|\tilde{\nu}^c|^4}{4} & |\kappa|^2 M^2 \\ |\kappa|^2 M^2 & \frac{|\zeta|^2|\tilde{\nu}^c|^4}{4} \end{pmatrix} , \tag{7.29}$$

respectively. Analogous to section 6.2, we can now diagonalise these matrices by defining the following mass eigenstates

$$\left.\begin{matrix} \bar{u}^c_{H_R} - u^c_{H_R} \\ \bar{u}^c_{H_I} + u^c_{H_I} \end{matrix}\right\} \longrightarrow m^2_{+\,(u)} = \frac{|\zeta|^2|\tilde{\nu}^c|^4}{4} + |\kappa|^2 M^2 \tag{7.30}$$

and

$$\left.\begin{matrix} \bar{u}^c_{H_R} + u^c_{H_R} \\ \bar{u}^c_{H_I} - u^c_{H_I} \end{matrix}\right\} \longrightarrow m^2_{-\,(u)} = \frac{|\zeta|^2|\tilde{\nu}^c|^4}{4} - |\kappa|^2 M^2 \,. \tag{7.31}$$

The same expressions also hold for the $(d^c_H, \bar{d}^c_H)$ and $(e^c_H, \bar{e}^c_H)$ fields.

For the SM singlet directions $(\nu^c_H, \bar{\nu}^c_H)$, however, the situation is different. Due to the additional contributions from the non-universal couplings $\lambda$, $\gamma$, and $\xi$, the masses in the scalar and pseudo-scalar sector become

$$\left(\mathcal{M}^2_s\right)_{11} = \left(\mathcal{M}^2_s\right)_{22} = \frac{\left(|\zeta+\xi|^2 + 4|\gamma|^2\right)|\tilde{\nu}^c|^4}{4} \,, \tag{7.32}$$

$$\left(\mathcal{M}^2_s\right)_{12} = \left(\mathcal{M}^2_s\right)_{21} = \frac{\mathrm{Re}\left((\gamma+\lambda)^*(\zeta+\xi)\right)|\tilde{\nu}^c|^4}{2} - |\kappa|^2 M^2 \,, \tag{7.33}$$

and

$$\left(\mathcal{M}^2_p\right)_{11} = \left(\mathcal{M}^2_s\right)_{22} = \frac{\left(|\zeta+\xi|^2 + 4|\gamma|^2\right)|\tilde{\nu}^c|^4}{4} \,, \tag{7.34}$$

$$\left(\mathcal{M}^2_p\right)_{12} = \left(\mathcal{M}^2_s\right)_{21} = \frac{\mathrm{Re}\left((\gamma+\lambda)^*(\zeta+\xi)\right)|\tilde{\nu}^c|^4}{2} + |\kappa|^2 M^2 \,. \tag{7.35}$$

To simplify the further discussion, we consider the case $\gamma = \lambda$ from now on. Then, the resulting mass eigenstates are given by

$$\bar{\nu}^c_{H_R} + \nu^c_{H_R} \longrightarrow m^2_{+\,(\nu,R)} = \frac{|\zeta + \xi - 2\gamma|^2}{4}\,|\tilde{\nu}^c|^4 + |\kappa|^2 M^2\,, \tag{7.36}$$

$$\bar{\nu}^c_{H_I} + \nu^c_{H_I} \longrightarrow m^2_{+\,(\nu,I)} = \frac{|\zeta + \xi + 2\gamma|^2}{4}\,|\tilde{\nu}^c|^4 + |\kappa|^2 M^2\,, \tag{7.37}$$

$$\bar{\nu}^c_{H_R} - \nu^c_{H_R} \longrightarrow m^2_{-\,(\nu,R)} = \frac{|\zeta + \xi + 2\gamma|^2}{4}\,|\tilde{\nu}^c|^4 - |\kappa|^2 M^2\,, \tag{7.38}$$

$$\bar{\nu}^c_{H_I} - \nu^c_{H_I} \longrightarrow m^2_{-\,(\nu,I)} = \frac{|\zeta + \xi - 2\gamma|^2}{4}\,|\tilde{\nu}^c|^4 - |\kappa|^2 M^2\,. \tag{7.39}$$

With these expressions, one can now easily calculate the critical values at which the system gets destabilised by setting the dynamical masses to zero. Setting, for example, $m^2_{-\,(u)} = m^2_{-\,(d)} = m^2_{-\,(e)} = 0$, we find the critical value

$$|\tilde{\nu}^{c\,(u,d,e)}_{\text{crit}}| = \sqrt{\frac{2\,|\kappa|\,M}{|\zeta|}}\,, \tag{7.40}$$

for the $(\bar{u}^c_{H_R} + u^c_{H_R})$, $(\bar{u}^c_{H_I} - u^c_{H_I})$, ... directions (cf. Eqn. (7.31)).

From Eqns. (7.38), (7.39), on the other hand, we get

$$m^2_{-\,(\nu,R)} = 0 \longrightarrow |\tilde{\nu}^{c\,(\nu,R)}_{\text{crit}}| = \sqrt{\frac{2\,|\kappa|\,M}{|\zeta + \xi + 2\gamma|}} \tag{7.41}$$

$$m^2_{-\,(\nu,I)} = 0 \longrightarrow |\tilde{\nu}^{c\,(\nu,I)}_{\text{crit}}| = \sqrt{\frac{2\,|\kappa|\,M}{|\zeta + \xi - 2\gamma|}}\,, \tag{7.42}$$

for the $(\bar{\nu}^c_{H_R} - \nu^c_{H_R})$ and $(\bar{\nu}^c_{H_I} - \nu^c_{H_I})$ directions, respectively.

The important observation here is, that for generic non-zero values of $\gamma$ (and for example small values of $\xi$), either the $(\bar{\nu}^c_{H_R} - \nu^c_{H_R})$ direction or the $(\bar{\nu}^c_{H_I} - \nu^c_{H_I})$ direction becomes tachyonic for larger values of the inflaton VEV than the $(\bar{u}^c_{H_R} + u^c_{H_R})$, $(\bar{u}^c_{H_I} - u^c_{H_I})$, ... directions. Consequently, it destabilises first and the waterfall is initiated in this unique direction in field space. This avoids the production of magnetic monopoles in our model.

We note that with the effective operators in Eqn. (7.14) included in this discussion, there is still the possibility of domain wall formation associated with the $\mathbb{Z}_2$-symmetry $\nu^c_H \to -\nu^c_H$ and $\bar{\nu}^c_H \to -\bar{\nu}^c_H$. However, additional effective operators at higher order which contain odd powers of $\boldsymbol{H^c}$ and $\bar{\boldsymbol{H}}^{\boldsymbol{c}}$ (in particular terms linear in $\boldsymbol{H^c}$ and $\bar{\boldsymbol{H}}^{\boldsymbol{c}}$) can lift this degeneracy and force the waterfall to occur in one unique direction. An example for such a deformed inflationary potential is shown in Fig. 6.1(b). For other possibilities to evade the cosmological domain wall problem, the reader is referred to [153].

In summary, since the gauge symmetry is already broken by the inflaton VEV during inflation, higher-dimensional operators allow to force the waterfall into one single direction in field space such that a particular vacuum is chosen everywhere and the production of topological defects such as monopoles can be avoided.

## 7.2 The Effective Potential and Radiative Corrections

As in the models of SUSY F-term inflation discussed before, our Pati-Salam model discussed here also has a tree-level flat inflaton potential. Hence, we need to consider radiative corrections to the effective potential in order to generate a small slope that drives the inflaton field(s) towards the critical value. Unlike the case with a singlet inflaton discussed before, however, for the case of GNS inflation also two-loop corrections have to be considered, as it was pointed out that they might even be dominant and lead to an instant violation of the slow-roll conditions in this case [24]. We call this the *gauge $\eta$-problem*.

In subsection 7.2.1 we summarise the full mass spectrum during Pati-Salam sneutrino inflation as calculated in detail in Appendix B and section 7.1.4 and compute the resulting one-loop Coleman-Weinberg potential. As it turns out, in the absence of SUGRA masses for the gauginos, only the fields of the waterfall sector show a splitting between the masses of the scalar and fermionic component fields and hence contribute to the lifting of the flat direction at one-loop level.

In subsection 7.2.2, we give estimates for the potentially dangerous two-loop corrections pointed out in [24] and show that they are small and can be neglected in our model.

### 7.2.1 One-Loop Corrections

In our previous studies we have already shown how to calculate the one-loop contributions to the inflaton potential due to the inflaton-dependent masses of the scalar and fermionic components of the waterfall sector superfields. The calculation here can be performed analogously. In addition to the waterfall sector, we have to consider the gauge sector of the theory for the one-loop contributions here, i.e. the loop contributions from inflaton-dependent masses of gauge bosons and gauginos.

Let us start with the gauge sector masses of our model, since we will see that under certain assumptions, SUSY-breaking does not directly affect this sector. These assumptions include the absence of a direct SUGRA gaugino mass term

$$\mathcal{L}_{\text{gaugino}} = \frac{1}{4}\, M_{\text{P}}\, \mathrm{e}^{-\langle G/2\rangle} \left\langle G^{i\bar{j}}\, G_i\, \left(\partial_{\bar{j}}\, f^*_{ab}\right)\right\rangle \lambda^a\, \lambda^b \,+\, \text{H.c.}\,, \tag{7.43}$$

where $G$ denotes the Kähler function defined in Eqn. (1.135), and $f_{ab}(\phi_i)$ is the gauge kinetic function, cf. Eqn. (1.147). The presence or absence of this contribution to the gaugino masses depends on the details of the SUGRA model. In the following we assume that the gauge kinetic function is diagonal and constant $f_{ab} = \delta_{ab}$ such that the contribution Eqn. (7.43) vanishes [2].

The resulting gauge sector mass spectrum is summarised in Tab. 7.3. As can easily be seen, the supertrace over these contributions vanishes and they do not contribute to the Coleman-Weinberg potential.

[2] More generally, the SUGRA contribution to the gaugino masses Eqn. (7.43) vanishes if the gauge kinetic function does not depend on fields that obtain a non-zero F-term such as the driving field $S$ in our model.

| Fields | Squared Masses $m^2$ |
|---|---|
| 8 gauge bosons | $g^2\,(\tilde{\nu}^c)^2/2$ |
| 1 gauge boson | $5\,g^2\,(\tilde{\nu}^c)^2/4$ |
| 8 Dirac fermions | $g^2\,(\tilde{\nu}^c)^2/2$ |
| 1 Dirac fermion | $5\,g^2\,(\tilde{\nu}^c)^2/4$ |
| 8 real scalars | $g^2\,(\tilde{\nu}^c)^2/2$ |
| 1 real scalar | $5\,g^2\,(\tilde{\nu}^c)^2/4$ |

**Table 7.3:** Mass spectrum of the gauge sector.

The $\tilde{\nu}^c$-dependent, SUSY-breaking contributions relevant for the Coleman-Weinberg potential arise from the waterfall sector masses only. Their squared masses are displayed in Tab. 7.4. These masses carry the SUSY-mass splittings $\mu = \kappa\,M$ and thus contribute to the one-loop Coleman-Weinberg inflaton potential, lifting the tree-level flat inflaton direction and driving the inflaton field(s) towards the critical value.

| Fields | Squared Masses $m^2$ |
|---|---|
| 7 Dirac fermions | $\lvert\zeta\rvert^2\,(\tilde{\nu}^c)^4/4$ |
| 1 Majorana fermion | $\lvert 2\,\gamma - \zeta - \xi\rvert^2\,(\tilde{\nu}^c)^4/4$ |
| 1 Majorana fermion | $\lvert 2\,\gamma + \zeta + \xi\rvert^2\,(\tilde{\nu}^c)^4/4$ |
| 7 complex scalars | $\lvert\zeta\rvert^2\,(\tilde{\nu}^c)^4/4 - \lvert\kappa\rvert^2 M^2$ |
| 7 complex scalars | $\lvert\zeta\rvert^2\,(\tilde{\nu}^c)^4/4 + \lvert\kappa\rvert^2 M^2$ |
| 1 real scalar | $\lvert\zeta + \xi - 2\gamma\rvert^2\,(\tilde{\nu}^c)^4/4 + \lvert\kappa\rvert^2 M^2$ |
| 1 real scalar | $\lvert\zeta + \xi - 2\gamma\rvert^2\,(\tilde{\nu}^c)^4/4 - \lvert\kappa\rvert^2 M^2$ |
| 1 real scalar | $\lvert\zeta + \xi + 2\gamma\rvert^2\,(\tilde{\nu}^c)^4/4 + \lvert\kappa\rvert^2 M^2$ |
| 1 real scalar | $\lvert\zeta + \xi + 2\gamma\rvert^2\,(\tilde{\nu}^c)^4/4 - \lvert\kappa\rvert^2 M^2$ |

**Table 7.4:** Mass spectrum of the waterfall sector.

For an example set of parameters $\kappa = 0.1$, $\xi = 0.1\,M_{\mathrm{P}}^{-1}$, $\gamma = -0.1\,M_{\mathrm{P}}^{-1}$, $\zeta = 0.2\,M_{\mathrm{P}}^{-1}$, and $M = 0.003\,M_{\mathrm{P}}$ [3] we have plotted the one-loop effective potential in Fig. 7.3. It has the typical shape of the Coleman-Weinberg potential in hybrid/tribrid inflation. Since in the case considered here the inflationary trajectory is a straight line in field space (cf. Eqns. (7.26) and (7.27)), we are effectively dealing with a single field model and the

[3] Remember that we have absorbed factors of the inverse mass scale $\Lambda^{-1}$ into the coupling constants $\xi, \gamma, \zeta$, whereas $\kappa$ is dimensionless since it has absorbed a factor $\langle X\rangle/\Lambda$, cf. Eqn (7.18).

inflationary predictions can be directly calculated from the slow-roll parameters. The negative curvature of the potential gives rise to a spectral index below one (typically $n_{\rm s} \approx 0.98$), as well as a small tensor-to-scalar ratio $r \lesssim 10^{-2}$. The COBE normalisation $\Delta_s^2 \approx 2.441 \cdot 10^{-9}$ fixes the scale $M$ and we have assumed $\mathcal{N} = 60$. Furthermore, we do not expect large non-gaussianities since as mentioned above the inflationary trajectory is not curved in field space [4].

We also note that the prediction for $n_s$ can be further lowered and thus brought even closer to the best fit value of the latest WMAP results [6] via the inclusion of a non-canonical coupling of the waterfall multiplets to the driving field $S$ in the Kähler potential of a SUGRA embedding of our model (cf. section 7.4) [5].

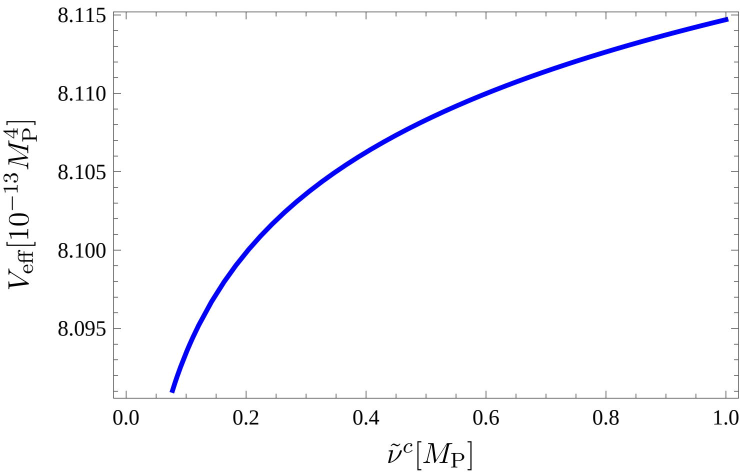



**Figure 7.3:** Effective one-loop Coleman-Weinberg potential. The negative curvature of the potential gives rise to a red tilted spectral index. The parameters chosen for this plot are $\kappa = 0.1$, $\xi = 0.1\, M_{\rm P}^{-1}$, $\gamma = -0.1\, M_{\rm P}^{-1}$, $\zeta = 0.2\, M_{\rm P}^{-1}$, and $M = 0.003\, M_{\rm P}$.

### 7.2.2 Two-Loop Corrections

Let us now move on to discuss what we call the *gauge $\eta$-problem*. This was first pointed out in [24]. There, it was stated that for a GNS inflaton two-loop corrections to the inflaton mass lead to a violation of the slow-roll conditions. However, we will show that the discussion in [24] cannot be applied directly to our model and we explain why such two-loop corrections are suppressed in our case.

Let us start, however, with a general discussion of the problem. For a GNS inflaton to suffer from the gauge $\eta$-problem, there are two basic conditions that have to be fulfilled. First of all, there has to be one superfield $\boldsymbol{S}$, which contributes the large vacuum energy

[4] We note that for more complicated trajectories, non-gaussianities may arise.

[5] We note that a lower spectral index in SUSY hybrid inflation models can also be achieved by different means [154].

by its F-term $F_S \neq 0$. Secondly, this superfield has to be coupled to some non-singlet superfields, in our case $\boldsymbol{H^c}$, $\boldsymbol{\bar{H}^c}$. A relevant superpotential term reads, for example,

$$W \supset \kappa\, \boldsymbol{S} \left( \boldsymbol{H^c}.\boldsymbol{\bar{H}^c} - M^2 \right) . \tag{7.44}$$

If these premises are given, any gauge non-singlet direction $\phi$ receives two-loop contributions to its effective mass of the order

$$\delta m^2 \simeq \frac{g^4}{(4\pi)^4} \frac{|F_S|^2}{m_f^2} , \tag{7.45}$$

where $g$ is the gauge coupling constant and $m_f$ refers to the SUSY conserving mass of the $\boldsymbol{H^c}$ and $\boldsymbol{\bar{H}^c}$ superfields. In Fig. 7.4, we have displayed the diagrams contributing to the mass corrections.

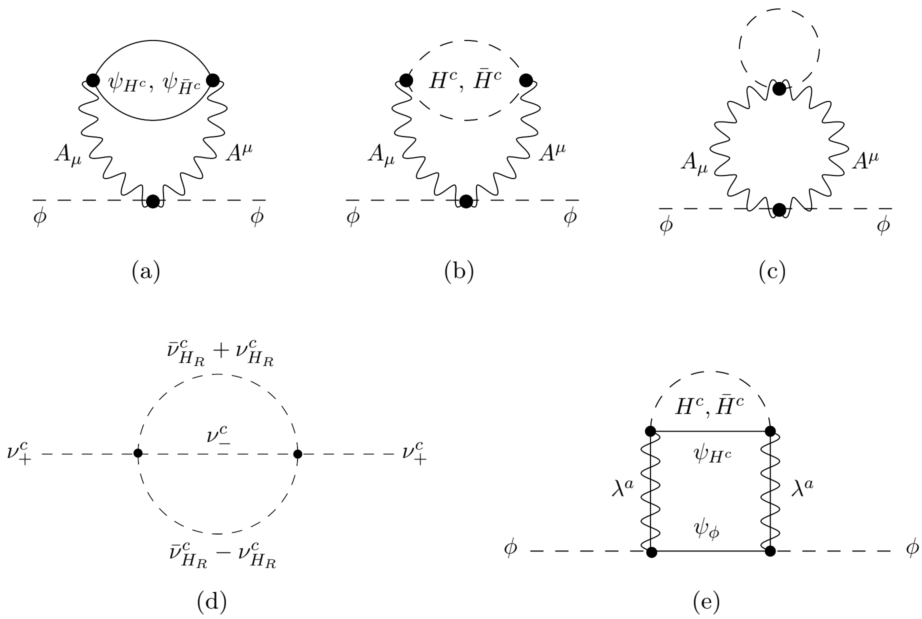



**Figure 7.4:** Two-loop diagrams contributing to the gauge $\eta$-problem that was pointed out in [24].

Typically, a contribution as in Eqn. (7.45) is large enough to provide a mass larger than the Hubble scale during inflation $\delta m > \mathcal{H}$. If we apply this to the inflaton, it violates the second slow-roll condition $|\eta| < 1$. We encountered the same situation when we discussed the $\eta$-problem of inflation in supergravity in section 3.6. Since here, however, the problem arises because of gauge interactions, we call it the gauge $\eta$-problem. In the further discussion we are interested in $\phi \in \{R^c, \bar{R}^c\}$.

As already mentioned in the beginning of this section it turns out, however, that Eqn. (7.45) cannot be applied to our model. Let us explain why. Eqn. (7.45) is calculated under the assumption that the gauge bosons $A_\mu$ mediating the loops are massless. Since the inflaton VEV already breaks the gauge symmetry $G_{\rm PS}$ during inflation, however, this is not the case in our model. Indeed, the broken gauge symmetry during inflation corresponds to large gauge boson masses that suppress the two-loop contributions of Fig. 7.4 for the inflaton: taking $\phi \in \{\nu^c, \bar{\nu}^c\}$ we find that the gauge bosons in Fig. 7.4 are contained in the coset $G_{\rm PS}/G_{\rm SM}$, which contains the massive gauge bosons after spontaneous symmetry breaking. Thus, their contributions get suppressed compared to massless gauge bosons, which were assumed in the derivation of Eqn. (7.45). Another way to say this is that the effective gauge symmetry during inflation is $G_{\rm SM}$ under which the inflaton direction is a singlet. All other directions $\phi \in \{u^c, d^c, e^c, \bar{u}^c, \bar{d}^c, \bar{e}^c\}$ couple to gauge bosons that are still massless, which allows the use of Eqn. (7.45). As a consequence, they obtain additional mass contributions helping to keep them at zero during the inflationary epoch.

We now want to estimate the typical size of the two-loop corrections in our model in the large gauge boson mass limit $M_g \gg p$ ($p$ is the momentum of the gauge boson) to justify the discussion above. For the SUSY-split waterfall masses, we take

$$m_+^2 = m_f^2 + \mu^2\,, \qquad m_-^2 = m_f^2 - \mu^2\,, \tag{7.46}$$

where

$$m_f^2 = \zeta^2\,(\tilde{\nu}^c)^4/4 \tag{7.47}$$

is the mass of the waterfall chiral fermion and $\mu = \kappa\, M$ is the SUSY-breaking scale. Due to the non-renormalisation theorem, all contributions not proportional to powers of $\mu$ must cancel such that in the SUSY-limit $\mu \to 0$ the total two-loop contribution vanishes. Thus, we expand the final loop integrals in terms of $\mu$.

In analogy to the calculations in [155] we find that in the large gauge boson mass limit the diagrams in Fig. 7.4 lead to the following two-loop mass contributions

$$\delta m^2 \simeq \frac{g^4}{(4\pi)^4}\,\frac{m_f^2\,\mu^4}{M_g^4}\,, \tag{7.48}$$

$$\delta m^2 \simeq \frac{g^4}{(4\pi)^4}\,\frac{\mu^4}{M_g^2}\,, \tag{7.49}$$

$$\delta m^2 \simeq \frac{g^4}{(4\pi)^4}\,\frac{m_f\,\mu^4}{M_g^3}\,. \tag{7.50}$$

The Hubble scale during inflation is given by $\mathcal{H}^2 \simeq \kappa^2 M^4/(3M_{\rm P}^2)$. This yields

$$\frac{\delta m^2}{\mathcal{H}^2} \simeq \frac{3\,\kappa^2}{(4\pi)^4}\,\zeta^2 M_{\rm P}^2\,, \tag{7.51}$$

$$\frac{\delta m^2}{\mathcal{H}^2} \simeq \frac{6\,g^2\kappa^2}{(4\pi)^4}\,\frac{M_{\rm P}^2}{(\tilde{\nu}^c)^2}\,, \tag{7.52}$$

$$\frac{\delta m^2}{\mathcal{H}^2} \simeq \frac{\sqrt{2}\,3\,g\,\kappa^2}{(4\pi)^4}\,\frac{\zeta\,M_{\rm P}^2}{\tilde{\nu}^c}\,. \tag{7.53}$$

Using the values $\kappa = 0.05$, $\zeta = 0.2\, M_{\mathrm{P}}^{-1}$, $g = 0.5$, $M = 3.4 \cdot 10^{-3}\, M_{\mathrm{P}}$, and $\tilde{\nu}^c = 0.36\, M_{\mathrm{P}}$ at about 50 e-folds before the end of inflation, taken from Ref. [30] where a similar effective superpotential has been analysed, we can finally estimate

$$\frac{\delta m^2}{\mathcal{H}^2} \approx \mathcal{O}(10^{-8})\,, \tag{7.54}$$
$$\frac{\delta m^2}{\mathcal{H}^2} \approx \mathcal{O}(10^{-6})\,, \tag{7.55}$$
$$\frac{\delta m^2}{\mathcal{H}^2} \approx \mathcal{O}(10^{-7})\,. \tag{7.56}$$

Hence, we can conclude that the two-loop contributions can be neglected in our model.

## 7.3 $SO(10)$ SUSY GUTs

We now turn to the embedding of the model into $SO(10)$ GUTs. Starting with the model of the previous section, we first make it explicitly left-right symmetric and then describe how its field content can be embedded in $SO(10)$ representations.

### 7.3.1 Left-Right Extension

In order to make our example model of the previous section explicitly left-right-symmetric, we need to add supermultiplets charged under $SU(2)_L$ to the theory. In addition to the right-charged matter fields and their conjugates, defined in Eqns. (7.2) and (7.3), we therefore introduce left-doublet leptoquarks contained in the following supermultiplets

$$\boldsymbol{L}_i = \begin{pmatrix} \boldsymbol{u}_i & \boldsymbol{u}_i & \boldsymbol{u}_i & \boldsymbol{\nu}_i \\ \boldsymbol{d}_i & \boldsymbol{d}_i & \boldsymbol{d}_i & \boldsymbol{e}_i \end{pmatrix} \sim (\mathbf{4}, \mathbf{2}, \mathbf{1})\,, \tag{7.57}$$

$$\bar{\boldsymbol{L}} = \begin{pmatrix} \bar{\boldsymbol{u}} & \bar{\boldsymbol{u}} & \bar{\boldsymbol{u}} & \bar{\boldsymbol{\nu}} \\ \bar{\boldsymbol{d}} & \bar{\boldsymbol{d}} & \bar{\boldsymbol{d}} & \bar{\boldsymbol{e}} \end{pmatrix} \sim (\bar{\mathbf{4}}, \bar{\mathbf{2}}, \mathbf{1})\,, \tag{7.58}$$

where we omitted the colour indices for convenience and $i$ denotes a generation index as before. The waterfall Higgs superfields breaking $G_{\mathrm{PS}}$ to the $G_{\mathrm{SM}}$ by VEVs of their scalar component fields are given in Eqns. (7.4) and (7.5). Making the field content left-right symmetric, we now have their left-charged counterparts as well, which read

$$\boldsymbol{H} = \begin{pmatrix} \boldsymbol{u_H} & \boldsymbol{u_H} & \boldsymbol{u_H} & \boldsymbol{\nu_H} \\ \boldsymbol{d_H} & \boldsymbol{d_H} & \boldsymbol{d_H} & \boldsymbol{e_H} \end{pmatrix} \sim (\mathbf{4}, \mathbf{2}, \mathbf{1})\,, \tag{7.59}$$

$$\bar{\boldsymbol{H}} = \begin{pmatrix} \bar{\boldsymbol{u}}_{\boldsymbol{H}} & \bar{\boldsymbol{u}}_{\boldsymbol{H}} & \bar{\boldsymbol{u}}_{\boldsymbol{H}} & \bar{\boldsymbol{\nu}}_{\boldsymbol{H}} \\ \bar{\boldsymbol{d}}_{\boldsymbol{H}} & \bar{\boldsymbol{d}}_{\boldsymbol{H}} & \bar{\boldsymbol{d}}_{\boldsymbol{H}} & \bar{\boldsymbol{e}}_{\boldsymbol{H}} \end{pmatrix} \sim (\bar{\mathbf{4}}, \bar{\mathbf{2}}, \mathbf{1})\,. \tag{7.60}$$

The symmetry assignments are given in Tab. 7.1. We note that at this stage the model contains two copies of the inflaton sector discussed in the previous section, one charged under $SU(2)_R$ and one charged under $SU(2)_L$, as well as additional couplings between the two sectors. In the absence of a discrete left-right symmetry we would not expect the couplings in the left and right sector to be exactly equal. With two potential sectors for inflation, inflation may happen in both of them with the respective sneutrinos playing the role of the inflaton. Thus we might have an "inflaton race" between the two sectors. Once the waterfall occurs in one of the two sectors (with different couplings in each sector we do not expect this to happen simultaneously), inflation ends since the vacuum energy given by the $F_S$-term vanishes. At the same time, the masses of the matter fields get fixed by the VEVs of the waterfall fields and the couplings between the left and the right sector. When this happens, we *(re)name* the sector in which the waterfall has occurred as the right sector under the SM gauge group. Before the breaking of $G_{\text{PS}}$ to $G_{\text{SM}}$, the names right-charged and left-charged were arbitrary (referring with right-charged and left-charged to $SU(2)_R$ and $SU(2)_L$, respectively) and a renaming is always possible at this stage. Thus, without loss of generality, we can assume that $G_{\text{PS}}$ is broken to $G_{\text{SM}}$ by the VEV of a right-charged Pati-Salam Higgs field.

### 7.3.2 Embedding into $SO(10)$

One attractive feature of $SO(10)$ GUTs is that all matter fields of a family, including the right-handed neutrinos, are contained in one **16** representation of $SO(10)$. If we furthermore consider a SUSY GUT, these fields are accompanied by their scalar superpartners. It is then tempting to try to realise inflation by one (or more) of the scalar fields belonging to such a **16** superfield.

A detailed discussion of $SO(10)$ SUSY GUTs and in particular of the decomposition of representations of $SO(10)$ with respect to $G_{\text{PS}}$ and $G_{\text{SM}}$ can be found in section 2.3. Let us quickly recapitulate the important results. In terms of the Pati-Salam framework considered in the preceding sections, the left- and right-charged leptoquark superfields are unified into **16** representations, while their conjugate counterparts are unified into $\bar{\mathbf{16}}$ representations as

$$\mathbf{16} = (\mathbf{4},\mathbf{2},\mathbf{1}) \oplus (\bar{\mathbf{4}},\mathbf{1},\bar{\mathbf{2}})\,, \tag{7.61}$$

$$\bar{\mathbf{16}} = (\bar{\mathbf{4}},\bar{\mathbf{2}},\mathbf{1}) \oplus (\mathbf{4},\mathbf{1},\mathbf{2})\,. \tag{7.62}$$

In addition, the MSSM Higgs doublets can be embedded into a **10** multiplet, which decomposes under $G_{\text{PS}}$ as

$$\mathbf{10} = (\mathbf{1},\mathbf{2},\mathbf{2}) \oplus (\mathbf{6},\mathbf{1},\mathbf{1})\,. \tag{7.63}$$

The breaking of $SO(10)$ can take place via various hierarchies of intermediate subgroups [156]. One possibility, corresponding in some sense to the strategy followed so far in this paper, is via the intermediate Pati-Salam group $G_{\text{PS}}$

$$\begin{aligned} SO(10) \;\longrightarrow\; & G_{\text{PS}} = SU(4)_C \times SU(2)_L \times SU(2)_R \\ & \longrightarrow\; G_{\text{SM}} = SU(3)_C \times SU(2)_L \times U(1)_Y\,. \end{aligned} \tag{7.64}$$

In this breaking pattern, monopoles can in principle get produced in the first and in the second stage of the breaking. In section 7.1.4 we have already discussed how the monopole production in the second stage can be avoided in our model of sneutrino inflation. If we assume that $SO(10)$ is broken to $G_{\rm PS}$ before inflation, the monopoles produced in this stage of the breaking are diluted and thus unproblematic.

We would also like to note that the breaking via $G_{\rm PS}$ is not the only possible breaking pattern compatible with GNS sneutrino inflation. For example, one could break $SO(10)$ to the minimal left-right symmetric model and then to the SM, avoiding monopole production completely at the second stage. Since, apart from this, the discussion would be analogous to the breaking via $G_{\rm PS}$, we will not dwell on this in any more detail.

Let us now turn to the formulation of the model in the $SO(10)$ framework. As described above, we unify the left- and right-charged multiplets into $\mathbf{16}$'s and $\bar{\mathbf{16}}$'s. The matter fields containing the SM fermions and their superpartners are denoted as $\boldsymbol{F}_i \sim \mathbf{16}$ and $\bar{\boldsymbol{F}} \sim \bar{\mathbf{16}}$. The waterfall Higgs fields are unified into the $SO(10)$ representations $\boldsymbol{H}_{\mathbf{16}} \sim \mathbf{16}$ and $\bar{\boldsymbol{H}}_{\mathbf{16}} \sim \bar{\mathbf{16}}$. The symmetry assignments are basically chosen as in the previous sections, with the exception of an additional $\mathbb{Z}_2$-symmetry that we introduce to constrain the allowed form of the superpotential, cf. below. An example superfield content with associated symmetry assignments is displayed in Tab. 7.5.

| Superfield | $SO(10)$ | R | $\mathbb{Z}_{10}$ | $\mathbb{Z}_2$ |
|---|---|---|---|---|
| $\boldsymbol{S}$ | $\mathbf{1}$ | 1 | 0 | + |
| $\boldsymbol{X}$ | $\mathbf{1}$ | 0 | 7 | + |
| $\boldsymbol{H}_{\mathbf{16}}$ | $\mathbf{16}$ | 0 | 1 | + |
| $\bar{\boldsymbol{H}}_{\mathbf{16}}$ | $\bar{\mathbf{16}}$ | 0 | 2 | + |
| $\boldsymbol{F}_i$ | $\mathbf{16}$ | 1/2 | 3 | + |
| $\bar{\boldsymbol{F}}$ | $\bar{\mathbf{16}}$ | 1/2 | 4 | + |
| $\boldsymbol{h}_{\mathbf{10}}$ | $\mathbf{10}$ | 0 | 4 | − |
| $\boldsymbol{\theta}$ | $\mathbf{1}$ | 0 | 0 | − |

**Table 7.5:** Example $SO(10)$ superfield content and associated symmetries.

With this superfield content and symmetry assignments, however, one immediately encounters a potential problem for realising inflation, connected to the Yukawa couplings of the matter representations to the $\boldsymbol{h}_{\mathbf{10}}$ Higgs representation. If the theory contains renormalisable Yukawa interactions, i.e. terms of the form

$$y\, \boldsymbol{F}.\boldsymbol{h}_{\mathbf{10}}.\boldsymbol{F}\,, \tag{7.65}$$

then the F-term of the $\boldsymbol{h}_{\mathbf{10}}$ yields a contribution to the scalar potential

$$V \supset |y\, F.F|^2 \tag{7.66}$$

that would represent quartic couplings of the inflaton field(s). Such a quartic term in the inflaton potential is, unless $y$ is extremely small, strongly disfavoured by the WMAP data [6]. On the other hand, in many flavour models based on GUTs combined with family symmetries, the Yukawa couplings, especially the ones for the first two families, do not arise from renormalisable couplings but rather from higher-dimensional operators. The suppression of the higher-dimensional operators allows to explain the hierarchical structure of the charged fermion masses. The Yukawa couplings are then generated after some family symmetry breaking Higgs field $\theta$, called flavon in the following, gets its VEV. Such Yukawa couplings can schematically be written as

$$y\,\frac{\langle\boldsymbol{\theta}\rangle}{\Lambda}\,\boldsymbol{F}.\boldsymbol{h_{10}}.\boldsymbol{F}\,, \tag{7.67}$$

where $\langle\boldsymbol{\theta}\rangle/\Lambda$ stands for the suppression of the Yukawa couplings by an effective operator and $\Lambda$ is the family symmetry breaking scale. It represents, in a simplified notation, the typically more complicated flavour sector of the theory. As long as the flavon field $\theta$ obtains its VEV after inflation (and has zero VEV during inflation), the potentially problematic coupling in Eqn. (7.65) is not appearing during inflation. We assume this situation in the following.

Keeping these points in mind, the allowed terms in the superpotential up to dimension seven read

$$\begin{aligned} W = \kappa\,\boldsymbol{S}\left(\frac{\langle\boldsymbol{X}\rangle}{\Lambda}\,\boldsymbol{H_{16}}.\bar{\boldsymbol{H}}_{\boldsymbol{16}} - M^2\right) & \\ + \frac{\lambda_{ij}}{\Lambda}\,\boldsymbol{F}_i.\boldsymbol{F}_j.\bar{\boldsymbol{H}}_{\boldsymbol{16}}.\bar{\boldsymbol{H}}_{\boldsymbol{16}} + \frac{\gamma}{\Lambda}\,\bar{\boldsymbol{F}}.\bar{\boldsymbol{F}}.\boldsymbol{H_{16}}.\boldsymbol{H_{16}} + \frac{\zeta_i}{\Lambda}\,\boldsymbol{F}_i.\bar{\boldsymbol{F}}.\boldsymbol{H_{16}}.\bar{\boldsymbol{H}}_{\boldsymbol{16}} & \\ + y_{ij}\,\frac{\langle\boldsymbol{\theta}\rangle}{\Lambda}\,\boldsymbol{F}_i.\boldsymbol{h_{10}}.\boldsymbol{F}_j + \tilde{y}\,\frac{\langle\boldsymbol{\theta}\rangle}{\Lambda^3}\,\boldsymbol{h}^2_{\boldsymbol{10}}.\bar{\boldsymbol{F}}.\boldsymbol{h_{10}}.\bar{\boldsymbol{F}} + \ldots\ , & \end{aligned} \tag{7.68}$$

where the dot notation again indicates contraction of all gauge indices. Like in the Pati-Salam version of the model, we assume that $X$ has already acquired its large VEV $\langle X\rangle \approx \Lambda$ before inflation has started. Furthermore we assume $\langle\theta\rangle = 0$ during inflation as explained above.

Then, the part of the superpotential of our model relevant for inflation has the form

$$\begin{aligned} W_{\text{inf}} = \kappa\,\boldsymbol{S}\left(\boldsymbol{H_{16}}.\bar{\boldsymbol{H}}_{\boldsymbol{16}} - M^2\right) & \\ + \frac{\lambda_{ij}}{\Lambda}\,\boldsymbol{F}_i.\boldsymbol{F}_j.\bar{\boldsymbol{H}}_{\boldsymbol{16}}.\bar{\boldsymbol{H}}_{\boldsymbol{16}} + \frac{\gamma}{\Lambda}\,\bar{\boldsymbol{F}}.\bar{\boldsymbol{F}}.\boldsymbol{H_{16}}.\boldsymbol{H_{16}} + \frac{\zeta_i}{\Lambda}\,\boldsymbol{F}_i.\bar{\boldsymbol{F}}.\boldsymbol{H_{16}}.\bar{\boldsymbol{H}}_{\boldsymbol{16}} + \ldots\ . & \end{aligned} \tag{7.69}$$

We assume that $SO(10)$ is broken to $G_{\text{PS}}$ before inflation and then inflation as well as the waterfall after inflation are realised as discussed in section 7.1.

We would like to emphasise at this point that the minimalist field content and the choice of symmetries mainly serve the purpose of giving a proof of existence that GNS inflation can be realised in $SO(10)$ GUTs. In a fully realistic model, which e.g. may also contain a full flavour sector, different symmetries may have to be chosen and the field content may have to be extended.

## 7.4 Generalisation to Supergravity

So far, we have investigated the proposed model within a global SUSY framework only. The purpose of this section is to outline how GNS inflation can be generalised to supergravity. As already explained in section 3.6, a typical problem that arises when dealing with inflationary model building in SUGRA is the $\eta$-problem. In this section we want to give a brief summary of how the Heisenberg symmetry approach can be applied to our type of model.

In order to embed our model into a SUGRA framework and solve the $\eta$-problem we introduce a Kähler potential as proposed in [30] that is invariant under a Heisenberg symmetry [131]. As discussed in section 3.6.1, in this approach an additional modulus field $T$ is introduced. For our $SO(10)$ model, the Heisenberg group transformations

$$T \to T + \mathrm{i}\,\nu \tag{7.70}$$

$$T \to T + M_{\mathrm{P}}^{-1}\big(\alpha^{*I} F_I + |\alpha_I|^2/2\big) \tag{7.71}$$

$$F_I \to F_I + \alpha_I\,, \tag{7.72}$$

give rise to the following invariant combination

$$\rho = T + T^* - M_{\mathrm{P}}^{-1}\big(F_i^\dagger F_i - \bar{F}^\dagger \bar{F}\big)\,, \tag{7.73}$$

where the $\alpha_I$ and $\nu$ are infinitesimal transformation parameters. Note that the index $I$ runs over all generation indices, gauge indices and representations (i.e. also $\bar{F}$).

Following [30], a suitable Heisenberg symmetry invariant Kähler potential is given by

$$\begin{aligned} K = {} & M_{\mathrm{P}}^2\, k(\rho/M_{\mathrm{P}}) + \big(1 + M_{\mathrm{P}}^{-2}\kappa_S\,|S|^2 + M_{\mathrm{P}}^{-1}\kappa_\rho\,\rho\,\big)\,|S|^2 \\ & + H_{16}^\dagger H_{16} + \bar{H}_{16}^\dagger \bar{H}_{16} + h_{10}^\dagger h_{10}\,, \end{aligned} \tag{7.74}$$

where the dagger indicates complex conjugation and summation over all gauge indices. Note that the function $k(\rho/M_{\mathrm{P}})$ can be an arbitrary function, which is only constrained by the requirement that the resulting potential has a stable minimum at $\rho_{\mathrm{min}}$ in which $\rho$ can settle and that $k'(\rho_{\mathrm{min}}/M_{\mathrm{P}}) < 0$ to obtain positive kinetic terms for the inflaton fields. More details on this can be found in [30]. An important feature of Eqn. (7.74) is the term $M_{\mathrm{P}}^{-2}\kappa_S|S|^4$. For negative $\kappa_S$, this gives a large mass to the $S$ field, which stabilises it at zero during inflation (cf. also the discussion in section 5.4).

We would like to note at this point that the Heisenberg symmetry is not meant to be an exact symmetry of the theory, but rather an approximate one. It is even necessary to break the Heisenberg symmetry at some level, since otherwise the inflaton potential would be exactly flat and inflation could not end. In our model, the Heisenberg symmetry is broken by effective operators in the superpotential (which conserve tree-level flatness but induce a slope of the inflaton potential at loop level) as well as by the gauge interactions. At tree-level, the latter effects vanish in the D-flat valley (the gauge loop effects have been discussed in detail in section 7.2.2). Thus, the breaking of the Heisenberg symmetry in our scenario is capable of generating the desired slope of the inflaton potential but does not endanger the solution to the $\eta$-problem.

If we choose $\rho$ and the components of $F_i$ and $\bar{F}$ to be the independent degrees of freedom and eliminate the $T$-degrees of freedom, the F-term potential in the inflationary minimum $S = H_{16} = \bar{H}_{16} = h_{10} = 0$ is of the form

$$V_F \simeq \kappa^2 M^4 \, \frac{\mathrm{e}^{k(\rho/M_\mathrm{P})}}{(1 + M_\mathrm{P}^{-1} \kappa_\rho \, \rho)} \tag{7.75}$$

and thus flat at tree-level in the direction of the $F_i$ and $\bar{F}$ components. As can be seen from Eqn. (7.75), the additional coupling $\kappa_\rho$ in the Kähler potential is essential to stabilise the modulus field $\rho$. This is possible for negative $\kappa_\rho$. Again, more details can be found in [30].

Furthermore, in a SUGRA framework, under the assumption of a constant diagonal gauge kinetic function $f_{ab} = \delta_{ab}$, the D-term potential for the matter fields is also $\rho$-dependent and of the form

$$\begin{aligned} V_D(F_i, \bar{F}) &\simeq M_\mathrm{P}^4 \, \frac{g^2}{2} \sum_a \bigg( \sum_i \frac{k'(\rho/M_\mathrm{P})}{M_\mathrm{P}} \Big( F_i^\dagger \, \mathcal{T}^a F_i - \bar{F}^\dagger \, \mathcal{T}^{a*} \bar{F} \Big) \bigg)^2 \\ &\propto \frac{g^2}{2} \, k'(\rho/M_\mathrm{P})^2 \sum_a \Big( \sum_i F_i^\dagger \, \mathcal{T}^a F_i - \bar{F}^\dagger \, \mathcal{T}^{a*} \bar{F} \Big)^2 . \end{aligned} \tag{7.76}$$

The basic difference to the global SUSY D-term contribution Eqn. (7.15) is the global factor of $k'(\rho/M_\mathrm{P})^2$. Due to the fact that the modulus quickly settles to its minimum at the very beginning of inflation from Eqn. (7.75), $k'(\rho_\mathrm{min}/M_\mathrm{P})^2$ soon approaches a constant value and the D-flatness conditions basically do not change with respect to the global SUSY ones.

At this point we would once again like to emphasise the special properties of the superpotential of our model, Eqn. (7.14). In our setup the inflationary superpotential vanishes during inflation and the vacuum energy originates from the F-term of some field different from the inflaton. It has been pointed out in [36, 32] that, due to this property, the class of models considered here is generically very well suited for the generalisation from global SUSY to SUGRA (cf. also the discussion in section 3.6).

We furthermore emphasise that the Heisenberg symmetry approach is especially suitable for solving the $\eta$-problem for GNS inflation in SUGRA, in contrast to e.g. a shift symmetry $\phi \rightarrow \phi + \mathrm{i}\,\mu$, which cannot be applied to a GNS inflation, since it does not respect gauge symmetry.

# PART III

# Reheating and Leptogenesis

After having discussed inflation in part II, we now come to another outstanding event in the history of the early universe, leptogenesis. In particular we discuss non-thermal leptogenesis within the model discussed in chapter 5. We start by generalising the discussion of section 4.1 to also include the waterfall field in section 8.1. To analyse the dynamics in our model we then compute the necessary decay rates in section 8.1.1 and show the results of a full numerical simulation of the field dynamics after inflation in section 8.1.2. These results enable us to further analyse our model using a simplified analytical treatment in section 8.2. This finally allows us to compute predictions and compare them to experimental data, which is done in section 8.3. Finally, we combine the results from this chapter and chapter 5 to constrain the parameter space of the model from two different directions in section 8.4.

CHAPTER 8

# Non-Thermal Leptogenesis after Sneutrino Tribrid Inflation

Let us now resume the discussion of the singlet sneutrino model from chapter 5. We have already sketched what happens after inflation in section 4.1 where we discussed reheating after inflation. However, there we only considered the case of a single field inflation model.

Here the situation is slightly more complicated, because in hybrid-type inflationary models two fields are relevant in the reheating process after inflation: the inflaton field $n$ and the waterfall field $h \equiv h_R$ (cf. chapter 5 for the exact definitions of the fields and the nomenclature we are using). Depending on the form of the potential and the initial conditions at the onset of the waterfall phase transition as well as the decay rates of the inflaton and waterfall field, one or the other (or both) dominates the dynamics of reheating.

Our first task is therefore to compute the decay rates of the inflaton and the waterfall field and investigate how the two fields evolve after the waterfall phase transition.

## 8.1 Field Dynamics after Sneutrino Tribrid Inflation

To start with, let us generalise the pair of equations Eqns. (4.5) and (4.7) describing the reheating process for a single field model to our multi field case. Adding the equation of motion for the decaying waterfall field with a decay rate $\Gamma_H$ to ultra-relativistic particles, we get

$$\ddot{n} + 3\,\mathcal{H}\,\dot{n} + \frac{\partial V}{\partial n} + \Gamma_N\,\dot{n} = 0\,, \tag{8.1}$$

$$\ddot{h} + 3\,\mathcal{H}\,\dot{h} + \frac{\partial V}{\partial h} + \Gamma_H\,\dot{h} = 0\,, \tag{8.2}$$

$$\dot{\rho}_{\text{rad}} + 4\,\mathcal{H}\,\rho_{\text{rad}} - \Gamma_N\,\rho_N - \Gamma_H\,\rho_H = 0\,. \tag{8.3}$$

Here, the energy density of the inflaton and the waterfall field is given by

$$\rho_{\text{m}} \equiv \rho_N + \rho_H = \frac{1}{2}\dot{n}^2 + \frac{1}{2}\dot{h}^2 + V(n,h)\,. \tag{8.4}$$

To close the system of equations, we have to add the Friedmann equation

$$\mathcal{H}^2 = \frac{1}{3M_{\mathrm{P}}} \left(\rho_N + \rho_H + \rho_{\mathrm{rad}}\right) . \tag{8.5}$$

Having solved this system of equations, and neglecting back-reactions and $2 \leftrightarrow 2$-scattering processes, the produced lepton asymmetry can be calculated from the following Boltzmann equation [157]

$$\dot{n}_L + 3\,\mathcal{H}\,n_L = \epsilon_1\,\Gamma_N\,\frac{\rho_N}{m_N} + \epsilon_3\,\Gamma_H\,\frac{\rho_H}{m_{N^3}}\,, \tag{8.6}$$

with $\epsilon_i$ bounded by Eqn. (4.19) and $m_N \equiv m_{N^1}$.

To proceed any further, we now have to compute the decay rates of the inflaton and the waterfall field into ultra-relativistic particles, $\Gamma_N$ and $\Gamma_H$.

### 8.1.1 Decay Rates

Relevant for us is the decay rate of a particle $A$ decaying into two particles $b, c$ which is given by

$$\Gamma = \iint \frac{1}{32\pi^2} |\mathcal{M}|^2 \frac{|\,p\,|}{m_A^2}\,\mathrm{d}\cos\theta\,\mathrm{d}\phi\,. \tag{8.7}$$

The matrix element $\mathcal{M}$ can be evaluated using the standard Feynman rules and $|\,p\,|$ is given by

$$|\,p\,| = \frac{1}{2m_A}\Big[\left(m_A^2 - (m_b+m_c)^2\right)\left(m_A^2 - (m_b-m_c)^2\right)\Big]^{1/2}. \tag{8.8}$$

Since we are interested e.g. in the decay of the inflaton into up-type Higgs(ino) and (s)lepton doublet, we need to know the masses of the corresponding fields after the waterfall phase transition in the true vacuum of the theory. Neglecting SUGRA effects, which are now irrelevant because the large vacuum energy $V_0$ has vanished, a straightforward computation yields

$$m_S = m_H = 2\,\kappa\,M\,, \tag{8.9}$$

$$m_{N^i} = 2\,\frac{\lambda_{ii}}{M_{\mathrm{P}}}\,M^2\,, \tag{8.10}$$

$$m_{l^j} = m_{h_u} = 0\,. \tag{8.11}$$

Here a comment is in place: in the treatment that follows we assume that $V = 0$ and that SUSY is approximately restored after the waterfall phase transition, such that the above expressions hold for every component field of the corresponding superfields. As long as $n$ and $h$ are strongly oscillating, this is of course not true. Since it turns out, however, that the outcome of reheating in our model is mainly determined by decays occurring towards the end of the reheating phase where $h$ has already settled to its minimum and the amplitude of the inflaton oscillations are very small, such a treatment is justified.

Using these masses we can now compute the relevant decay rates. Let us begin with inflaton decays. More details on the computation of these decay rates can be found in [157].

### Inflaton Decays

The inflaton (being the pseudo-scalar component of the lightest right-handed sneutrino) can decay into lepton and up-type Higgsino doublet

$$n \longrightarrow l^j \, \tilde{h}_u \tag{8.12}$$

via the Yukawa coupling in Eqn. (5.1) or into slepton and up-type Higgs doublet

$$n \longrightarrow \tilde{l}^j \, h_u \,. \tag{8.13}$$

via the $F_N$ contribution to the scalar potential. The corresponding partial decay rates are given by (summing over all generations of leptons)

$$\Gamma_{n \to l \, \tilde{h}_u} = \Gamma_{n \to \tilde{l} \, h_u} = \frac{1}{4\pi} \tilde{y_1}^2 \frac{\lambda}{M_{\mathrm{P}}} M^2 \,. \tag{8.14}$$

The decay into the heavier (s)neutrinos or into $H$ and $S$ particles, on the other hand, is kinematically forbidden in the parameter space of interest.

Therefore, the total decay rate of the inflaton particle reads

$$\Gamma_N = \frac{1}{2\pi} \tilde{y_1}^2 \frac{\lambda}{M_{\mathrm{P}}} M^2 \,. \tag{8.15}$$

### Waterfall Field Decays

Having no direct coupling to the MSSM fields [1] and taking kinematics into account, the relevant decay channels for the waterfall field $h \equiv h_R$ are the decay into two right-handed neutrinos

$$h \longrightarrow \psi_{N^i} \, \psi_{N^i} \tag{8.16}$$

or two right-handed sneutrinos

$$h \longrightarrow N^i \, N^i \,. \tag{8.17}$$

Since we work in a basis where $\lambda_{ij}$ is diagonal, only decays into two (s)neutrinos of the same generation occur. The dominant decay channel is the decay into the heaviest right-handed neutrinos, $h \to \psi_{N^3} \, \psi_{N^3}$ and the corresponding decay rate equates to

$$\Gamma_{h \to \psi_{N^3} \, \psi_{N^3}} = \frac{\kappa \, M^3 \, \lambda_{33}^2}{8\pi M_{\mathrm{P}}^2} \,. \tag{8.18}$$

The heavy neutrinos hereby produced decay further into lepton and up-type Higgs doublet or slepton and up-type Higgsino doublet

$$\psi_{N^3} \longrightarrow l^j \, h_u \qquad , \qquad \psi_{N^3} \longrightarrow \tilde{l}^j \, \tilde{h}_u \,. \tag{8.19}$$

[1] There are Planck scale suppressed couplings of $h$ to the MSSM fields from SUGRA effects that are, however, negligible here.

The decay widths for these processes are given by

$$\Gamma_{\psi_{N^3}\to l\, h_u} = \Gamma_{\psi_{N^3}\to \tilde{l}\, \tilde{h}_u} = \frac{1}{\pi}\tilde{y_3}^2 \frac{\lambda_{33}}{M_{\mathrm{P}}} M^2\,, \tag{8.20}$$

and the total decay width of the heaviest right-handed neutrino $\psi_{N^3}$ is given by

$$\Gamma_{\psi_{N^3}} = \frac{2}{\pi}\tilde{y_3}^2 \frac{\lambda_{33}}{M_{\mathrm{P}}} M^2\,. \tag{8.21}$$

Combining these results we find for the dominant decay chain of the waterfall field

$$\begin{aligned}
\Gamma_H &\simeq \left(\Gamma^{-1}_{h\to\psi_{N^3}\,\psi_{N^3}} + \Gamma^{-1}_{\psi_{N^3}}\right)^{-1} \\
&\simeq \min\left\{\Gamma_{h\to\psi_{N^3}\,\psi_{N^3}}\,,\ \Gamma_{\psi_{N^3}}\right\} \\
&\simeq \frac{2}{\pi}\frac{M^2\lambda_{33}}{M_{\mathrm{P}}}\min\left\{\tilde{y_3}^2\,,\ \frac{\kappa M\lambda_{33}}{16 M_{\mathrm{P}}}\right\}.
\end{aligned} \tag{8.22}$$

### 8.1.2 Numerical Results

Using these expressions, we did a full numerical simulation of the system of equations (8.1) - (8.6). The results for the evolution of the inflaton and the waterfall field, starting in the phase of slow-roll inflation and extending into the oscillatory phase after the waterfall phase transition, are shown in Fig. 8.1.

The simulation exhibits the following preeminent features: at the beginning of the reheating phase $\mathcal{H} \gg \Gamma_N, \Gamma_H$ holds which implies that the decaying particles are damped predominantly by Hubble expansion, not by decays, and the produced ultra-relativistic particles are strongly diluted. This fact is also illustrated in Fig. 4.1, which is based on the model discussed here and shows the time evolution of the respective energy densities during and after inflation. The decays into ultra-relativistic particles become significant for $t = \mathcal{H}^{-1} \approx \min\{\Gamma_N^{-1}, \Gamma_H^{-1}\}$. At this stage it is safe to assume that the waterfall field has completely settled to its true minimum $h = \sqrt{2}\,M$ and the inflaton oscillations are very small $n \ll 1$.

Summarising the results found so far, the regime of reheating is characterised by oscillating scalar fields and can be divided into three distinct phases: After the end of inflation both $n$ and $h$ fall to their true minimum and begin to oscillate. After only a few oscillations the waterfall field $h$ settles at its minimum and the dynamics of the system is governed by the oscillations of the inflaton (e.g. right-handed sneutrino) field $n$. The further evolution of the $n$ oscillations is governed by Hubble damping: as long as $\mathcal{H} \gg \Gamma_N$, the universe is dominated by the oscillating sneutrino field $n$, which can be interpreted as (decaying) heavy sneutrino particles. This implies a matter dominated universe out of thermal equilibrium. Ultra-relativistic particles are produced through the decays of these heavy particles, however they are diluted by the expansion of the universe. Only when $\mathcal{H} \approx \Gamma_N$, the radiation energy density becomes dominant and the light particles begin to thermalise. This marks the end of reheating and determines the reheat temperature and also the asymmetry $n_L/s$.

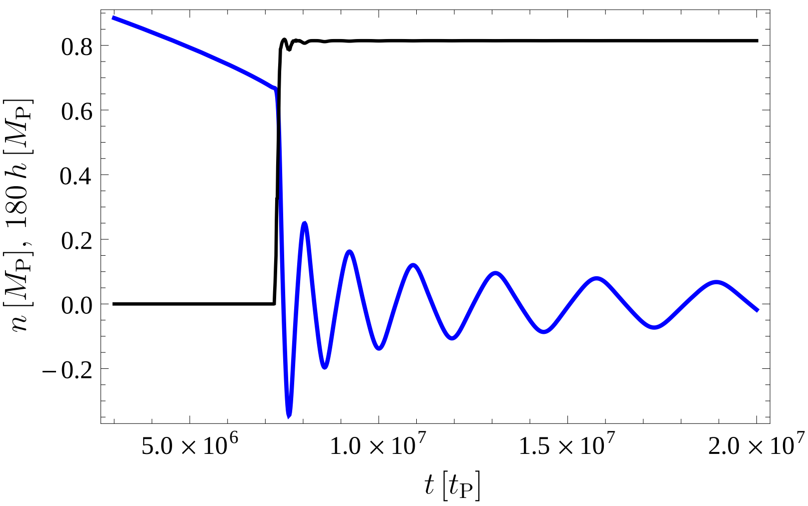



**Figure 8.1:** Time evolution of the inflaton field $n$ (blue) and the waterfall field $h$ (black, rescaled). The simulation was for this plot was run with the following parameter values : $\kappa = 0.5$, $M = 0.0032\,M_\mathrm{P}$, $m_N = 2.5\cdot 10^{11}\,\mathrm{GeV}$, and $\kappa_\mathrm{SH} = 0.5$ . The initial values at $t = 0$ were chosen to be $n(0) = 1$, $\dot{n}(0) = 0$, $h(0) = 0$ and $\dot{h}(0) = 10^{-9}$.

## 8.2 Simplified Analytic Treatment

In principle, we could now proceed to determine the finally produced baryon asymmetry from the numerical simulation and compare it to the experimental bounds in order to constrain the model parameters. Using the insight into the reheating phase gained so, however, allows us to simplify the system of equations (8.1) - (8.6) to a simpler set of equations that allows for approximate analytic solutions.

Note, in particular, that the Boltzmann equations (8.3) and (8.6) imply a splitting of the total matter energy density $\rho_\mathrm{m}$ into $\rho_N$ and $\rho_H$, which is not straightforward if the respective degrees of freedom are highly coupled. However, since $\Gamma_H \gg \Gamma_N$ in our setting in the preferred region of parameter space [2], any radiation energy density produced by $h$ decays is strongly diluted during the following matter dominated phase governed by oscillations of the sneutrino. With $\rho_\mathrm{m} \approx \rho_N$ shortly after the end of inflation due to the strong damping of the $h$ field we can thus substitute (8.3) and (8.6) by the simpler equations

$$\dot{\rho}_\mathrm{rad} + 4\,\mathcal{H}\,\rho_\mathrm{rad} - \Gamma_N \rho_\mathrm{m} \simeq 0\,, \tag{8.23}$$

$$\dot{n}_L + 3\,\mathcal{H}\,n_L \simeq \epsilon_1\,\Gamma_N\,\frac{\rho_\mathrm{m}}{m_N}\,, \tag{8.24}$$

[2] The assumption of a hierarchical spectrum of right-handed neutrinos implies $\lambda_{33} \gg \lambda_{11}$ and the assumption $\Gamma_{N^3} > \Gamma_{N^1}$ (cf. section 5.2) implies $\lambda_{33}\,\tilde{y}_3^2 > \lambda_{11}\,\tilde{y}_1^2$. This gives $\Gamma_H \gg \Gamma_N$.

without introducing a significant error for the finally produced radiation density. The effect of this approximation on the Hubble expansion rate is negligible since the Friedmann equation is predominantly governed by $\rho_{\rm m}$ for $t < \Gamma_N^{-1}$.

A further simplification can be achieved by time-averaging the equations of motion of the scalar fields. This is quite a common strategy (e.g. [158]) and it results in a set of Boltzmann equations for the matter energy density $\rho_{\rm m} = \rho_N + \rho_H$ and the radiation energy density $\rho_{\rm rad}$ completed by the Friedmann equation:

$$\dot{\rho}_{\rm m} + 3\,\mathcal{H}\,\rho_{\rm m} = -\Gamma_N \rho_{\rm m}\,, \tag{8.25}$$

$$\dot{\rho}_{\rm rad} + 4\,\mathcal{H}\,\rho_{\rm rad} = \Gamma_N \rho_{\rm m}\,, \tag{8.26}$$

$$\frac{1}{3\,M_{\rm P}^2}\,(\rho_{\rm m} + \rho_{\rm rad}) = \mathcal{H}^2\,. \tag{8.27}$$

The lepton asymmetry is determined by Eqn. (8.24).

While this set of equations has the big advantage that it allows for approximate analytical solutions, it requires one further important assumption concerning the scalar potential $V(n,h)$ (cf. e.g. [158]): in order to rewrite the time-averaged kinetic energy density in terms of the total energy density by exploiting the virial theorem we must assume that we can write the scalar potential as $V(n,h) = V_n(n) + V_h(h)$ with $V_n \propto n^r$ and $V_h \propto h^r$. Eqns. (8.25) - (8.27) are obtained with $r = 2$.

Numerical simulations of the full system of equations (8.1), (8.2), (8.5), (8.23) and (8.24) show that this assumption is not justified in the early oscillatory phase in our model since the large oscillations of the $n$ field result in a highly coupled system with higher order terms in the scalar potential playing a non-negligible role. This effect can e.g. be observed in the asymmetric shape of the $h$ oscillations and the change of frequency of the $n$ oscillations, cf. Fig. 8.1. However, the simulations also show that for $t = \mathcal{H}^{-1} \approx \Gamma_N^{-1}$, the simpler system of differential equations (8.25) - (8.27) does give a good approximation. This is the point of time relevant for the predictions of the reheating phase.

Having seen that the results of the numerical solutions to the full field equations for $t \approx \Gamma_N^{-1}$ can be approximated reasonably well by the simpler set of Boltzmann differential equations (8.25) - (8.27), we can now derive approximate analytical solutions to the latter and use these expressions to find estimates for the reheat temperature and the produced baryon asymmetry.

After the waterfall phase transition, while the universe is in a matter dominated phase, we have $\mathcal{H} > \Gamma_N$ and the Hubble parameter evolves as $\mathcal{H} \simeq 2/3t$. The initial conditions for the description of this matter dominated phase can be given in a good approximation to be

$$\rho_{\rm m}(t_i) = V_0 = \kappa^2 M^4\,, \qquad \rho_{\rm rad}(t_i) = 0\,, \tag{8.28}$$

where $V_0$ is the vacuum energy that drives inflation. From Eqn. (8.27) we therefore get

$$\frac{1}{3M_{\rm P}^2}\rho_{\rm m}(t_i) \simeq \mathcal{H}(t_i)^2 \simeq \frac{4}{9\,t_i^2} \quad \Rightarrow \quad t_i^2 \simeq \frac{4M_{\rm P}^2}{3V_0}\,. \tag{8.29}$$

Moreover, integrating Eqn. (8.25) and neglecting $\Gamma_N$ with respect to $3\,\mathcal{H}$ yields

$$\frac{\mathrm{d}\,\rho_{\mathrm{m}}}{\mathrm{d}\,t} - \frac{2}{t}\,\rho_{\mathrm{m}} = 0 \quad \Rightarrow \quad \rho_{\mathrm{m}} \simeq V_0 \left(\frac{t_i}{t}\right)^2 . \tag{8.30}$$

Plugging these results into Eqn. (8.26) leaves us with

$$\frac{\mathrm{d}\,\rho_{\mathrm{rad}}}{\mathrm{d}\,t} + \frac{8}{3\,t}\rho_{\mathrm{rad}} = \Gamma_N\,V_0\,\frac{t_i^2}{t^2} \quad \Rightarrow \quad \rho_{\mathrm{rad}}(t) \simeq \frac{4}{5}\frac{\Gamma_N\,M_{\mathrm{P}}^2}{t^{8/3}}\left(t^{5/3} - t_i^{5/3}\right) . \tag{8.31}$$

The universe thermalises when $\mathcal{H} \approx \Gamma_N$, that is

$$t_R \simeq \frac{2}{3\,\Gamma_N}\,, \tag{8.32}$$

and at this time the energy density of the ultra-relativistic particles is given by

$$\rho_{\mathrm{rad}}(t_R) \simeq \frac{6}{5}\,\Gamma_N^2\,M_{\mathrm{P}}^2 \simeq \frac{6}{5}\,M_{\mathrm{P}}^2\,\frac{\tilde{y}_1^4}{(2\pi^2)^2}\left(\frac{m_N}{2}\right)^2 , \tag{8.33}$$

where we have used Eqns. (8.15) and (8.10). Plugging this into the formula for the reheat temperature Eqn. (4.8) finally yields

$$T_R \simeq \left(\frac{9}{4\,\pi^4\,g_*}\right)^{1/4} \tilde{y}_1\,\sqrt{m_N\,M_{\mathrm{P}}}\,, \tag{8.34}$$

where $g_*$ is the effective number of degrees of freedom, which equals 914/4 in the MSSM. We see that for fixed $m_N$, the reheat temperature is directly proportional to the effective first generation Yukawa coupling $\tilde{y}_1$.

With these results, we can now proceed to derive an expression for the lepton asymmetry. From Eqn. (8.24) we get

$$\frac{\mathrm{d}\,n_L}{\mathrm{d}\,t} + \frac{2}{t}\,n_L = \epsilon_1\,\frac{\Gamma_N}{m_N}\,V_0\left(\frac{t_i}{t}\right)^2 \quad \Rightarrow \quad n_L(t) \simeq \frac{4}{3}\,\epsilon_1\,\frac{\Gamma_N\,M_{\mathrm{P}}^2}{m_N}\,\frac{t - t_i}{t^2} . \tag{8.35}$$

With the entropy density for ultra-relativistic particles given by

$$s = \frac{2\pi^2 g_* T^3}{45} \tag{8.36}$$

we can derive the following relation

$$\frac{n_L}{s}(t_R) \simeq \frac{5}{4}\frac{\epsilon_1 T_R}{m_N} . \tag{8.37}$$

Finally, using $n_B = \frac{C}{C-1} n_L$ due to sphaleron processes with $C \approx 1/3$ in the MSSM and $s = \alpha\,n_\gamma$ with today's conversion factor given by $\alpha = 7.04$ [95], we obtain the following prediction for the value of today's baryon asymmetry $\eta(t_0)$ from our model

$$\eta(t_0) = \frac{n_B}{n_\gamma}(t_0) \simeq 3.45\,\frac{C}{C-1}\,g_*^{-1/4}\,\epsilon_1 \tilde{y}_1 \left(\frac{M_{\mathrm{P}}}{m_N}\right)^{1/2} . \tag{8.38}$$

For fixed $m_N$ this yields

$$\eta(t_0) \propto \epsilon_1 \, \tilde{y}_1 \propto \epsilon_1 \, T_R \,. \tag{8.39}$$

Note also that our result reproduces the familiar relation $n_B/s \sim \epsilon \, T_R/m_N$ (see e.g. [95, 158]).

## 8.3 Results from Leptogenesis

The prediction from our model Eqn. (8.38) has now to be compared with the experimental value Eqn. (4.1). This yields a lower bound on the reheat temperature (cf. below), which in our model is given by

$$T_R > 1.4 \cdot 10^6 \, \mathrm{GeV} \,. \tag{8.40}$$

Additionally, the reheat temperature is bounded from above by the so called gravitino problem [159]: a high reheat temperature would result in an overproduction of gravitinos. If these are stable, then the fact that their energy density can not be larger than the present total energy density of the universe leads to a bound on the reheat temperature in terms of the gravitino mass $m_{3/2}$. On the other hand, if gravitinos are not stable, they can either decay before or during and after Big Bang Nucleosynthesis (BBN).

In the former case (i.e. heavy gravitinos), with R-parity conserved, the gravitinos will decay into the lightest supersymmetric particle (LSP) and their production is thus constrained by the dark matter abundance. This yields a fairly model independent bound of $T_R < 2 \cdot 10^{10}$ GeV for an LSP mass of about 100 GeV to 150 GeV.

In the latter case (i.e. light gravitinos), the decay of the gravitinos would alter the outcome of BBN and create a conflict between BBN predictions and observations. This yields even stronger, however model dependent, constraints on the reheat temperature. Combining these arguments yields a constraint on the reheat temperature of typically

$$T_R < 10^7 - 10^{10} \, \mathrm{GeV} \,, \tag{8.41}$$

depending mainly on the model under consideration and on the value of $m_{3/2}$.

The resulting preferred region in $(m_N, \tilde{y}_1)$-parameter space is depicted in blue in Fig. 8.2 with the different shadings corresponding to different upper bounds on the reheat temperature. Going to lower values of the reheat temperature for fixed values of $m_N$ (i.e. lower values of $\tilde{y}_1$, cf. Eqn. (8.34)) requires larger values of the $\mathcal{CP}$ violation parameter $\epsilon_1$ in order to produce a sufficient amount of baryon asymmetry in accordance with the experimental value, cf. Eqn. (8.39). Since $\epsilon_1$ is bounded from above by Eqn. (4.19), however, reheat temperatures below $1.4 \cdot 10^6$ GeV do not allow for the production of a sufficient baryon asymmetry, hence the lower bound on the reheat temperature, Eqn. (8.40). The red dashed line corresponds to $T_R = m_N$, which marks the boundary between thermal leptogenesis (above) and non-thermal leptogenesis (below). Our calculations only hold beneath this line, since we have negelcted wash-out effects and other complications that can arise if leptogenesis is predominantly of the thermal type.

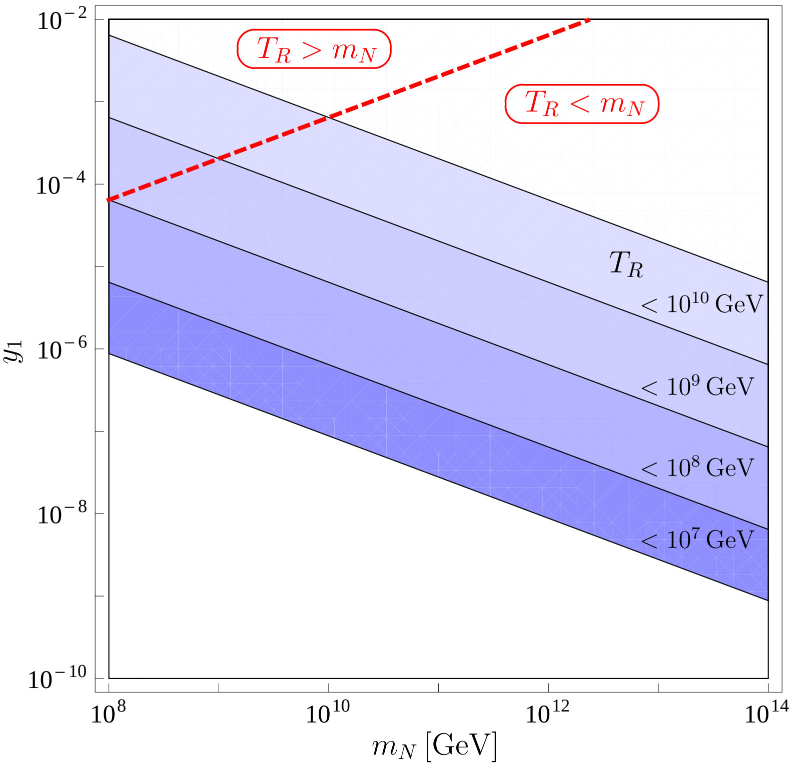



**Figure 8.2:** Preferred region in $(m_N, \tilde{y}_1)$-parameter space at 95% CL from the requirement of successful leptogenesis in accordance with the experimental value, Eqn. (4.1). The different shadings of blue indicate different upper bounds on the reheat temperature from the gravitino problem. The lower bound at $T_R > 1.4 \cdot 10^6$ GeV is due to the upper bound on $\epsilon_1$, Eqn. (4.19), and the dependence of $\eta$ on $\epsilon_1$ and $T_R$ for fixed $m_N$, Eqn. (8.39). The red dashed line, corresponding to $T_R = m_N$, indicates the boundary between predominantly thermal leptogenesis (above) and predominantly non-thermal leptogenesis (below).

### 8.3.1 Remarks on Preheating

Note that throughout this chapter we have focused on the evolution of the homogeneous fields $n$ and $h$. It has been pointed out that under certain circumstances this might not be sufficient, since modes with $k \neq 0$ of all fields in the model can be strongly excited at the end of inflation and before the beginning of reheating in a process referred to as preheating. There are two types of preheating worth mentioning in the context of hybrid inflation, namely preheating via parametric resonance [134, 135, 160] and tachyonic preheating [136, 137, 161]. In the former case, the coupling of fermions and bosons to the oscillating inflaton field results in oscillating mass terms for these particles. Solving the respective equations of motion (roughly the equation of an harmonic oscillator with an oscillating mass as described, e.g., by the Mathieu equation) can yield explosive particle production. However, in the region of parameter space of interest to us, any heavy particles that are produced by this mechanism decay back into heavy (s)neutrinos or into radiation. The radiation produced directly or indirectly through this process at the beginning of the reheating phase, however, is strongly diluted during the ongoing matter dominated phase and is thus insignificant for the outcome of the reheating phase. Thus, in our model, parametric resonance will not affect the results discussed above, mainly due to the structure of the mass spectrum and the very small effective Yukawa coupling $\tilde{y}_1$.

Tachyonic preheating occurs when the squared mass of the $h$ field becomes negative, triggering the waterfall ending inflation. Modes of the $h$ field with $k < |m_h|$ grow exponentially [3], causing particle production of bosonic and fermionic fields coupled to the waterfall field [161] and creating an inhomogeneous field $H(x,t)$ which can cause the formation of topological defects when the waterfall occurs [136]. It was stated in [136] that the production of fermions and bosons coupling to the waterfall field with a coupling strength $g$ is suppressed by $\rho_f/\rho_V \approx \rho_b/\rho_V \sim 10^{-3}\, g$ with $\rho_V$ denoting the energy density during inflation. Thus in the parameter region of interest, this is negligible in our model. On the other hand, the production of topological defects could indeed dominate the evolution of the universe in an early stage. However, since we have not observed any topological defects yet, a mechanism to prevent or dilute these objects as for example discussed in sections 6.3 and 7.1.4 is typically implemented. We assume that the higher dimensional operators denoted by dots in Eqn. (5.1) provide such a solution so that at some time after the waterfall, the universe is dominated by the lightest right-handed sneutrino. The evolution from this point on is correctly described by the classical theory of reheating, as discussed above. Other possible scenarios in which the evolution of the universe may not be dominated by the homogeneous component of the inflaton field remain to be explored in this context.

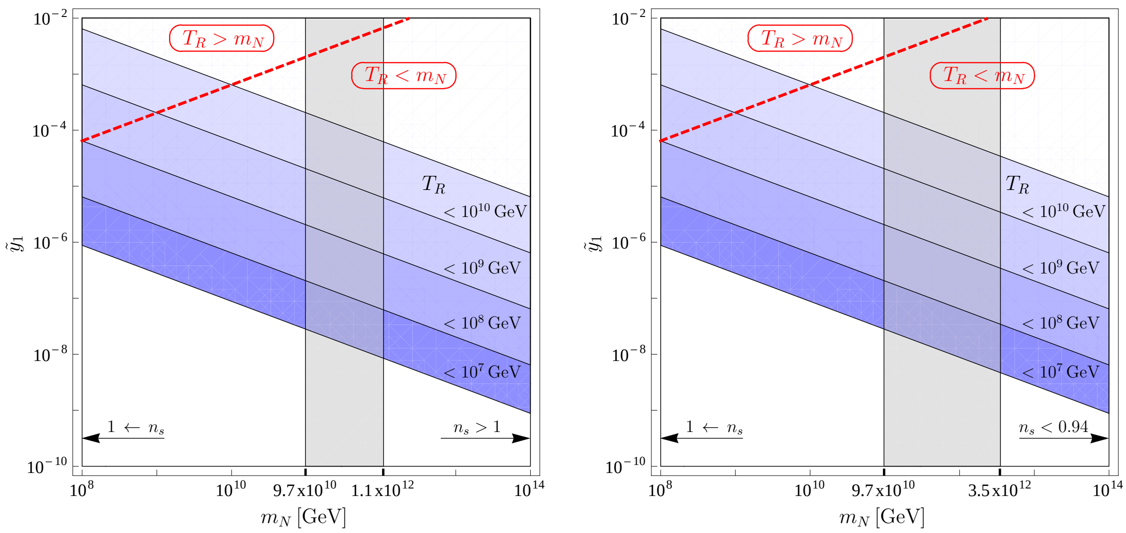
## 8.4 Combined Results from Inflation and Leptogenesis

In this chapter and chapter 5 we have investigated the conditions under which inflation, with primordial perturbations in accordance with the latest WMAP results, as well as successful leptogenesis can be realised simultaneously in a model of sneutrino tribrid inflation. The model is defined through its superpotential, Eqn. (5.1), and Kähler potential, Eqn. (5.36). In this section we want to combine the results from both chapters, in particular sections 5.5 and 8.3, to narrow down the allowed parameter space for the model from two different directions. The results thus obtained are summarised in Fig. 8.3.

We finish this chapter with an overview over the most salient features of the model and a discussion of the constraints we found from comparing the predictions with experimental data.

The dynamics of inflation in our model is governed by the scale $M$ of the phase transition ending tribrid inflation, the mass of the lightest right-handed (s)neutrino $m_N$, the

[3] It was pointed out in [162] that in some hybrid inflation models a fragmentation of the inflaton condensate can occur, causing the evolution of the universe to be dominated by these 'lumps' instead of by the homogeneous component of the inflaton field. However this 'lump' formation requires a flatter than $\phi^2$ potential (with $\phi \in \{n,\ h\}$), which does not appear in our model as can easily be checked from Eqn. (5.1).




(a) case: $\delta = +1$ (b) case: $\delta = -1$

**Figure 8.3:** Preferred region in $(m_N, \tilde{y}_1)$-parameter space at 95% CL from a combined analysis of inflation and leptogenesis. The preferred region derived from inflation, in particular from the WMAP constraints on the spectral index $n_s$, is marked in grey. The favoured region obtained from reheating and leptogenesis is depicted in blue. The meaning of the different blue shadings and the dashed red line is explained in Fig. 8.2. For the plots we have chosen $\kappa = 1$ and the phase transition scale is fixed at $M = 0.0032\, M_{\mathrm{P}}$.

(a) shows the situation for $\delta \equiv 1 - \kappa_{SH} = +1$. Note that after reaching a minimum at $n_s \approx 0.984$, the spectral index acquires large values for increasing $m_N$.

(b) depicts the situation for $\delta \equiv 1 - \kappa_{SH} = -1$. In this case, the sign of the SUGRA corrections flips and $n_s$ decreases for large values of $m_N$.

vacuum energy parameter $\kappa$ (= waterfall field self coupling) and the parameter $\delta \equiv 1-\kappa_{SH}$ controlling the size of the SUGRA corrections. In principle, terms depending on the neutrino Yukawa coupling matrix could contribute, too. However, in our case the comparatively small first generation Yukawa couplings make these contributions negligible. With $M$ fixed by the amplitude of the scalar CMB fluctuations and $\kappa \sim \mathcal{O}(1)$, the spectrum of the CMB fluctuations is primarily dependent on the lightest right-handed (s)neutrino mass $m_N$. For large values of $m_N$, SUGRA corrections controlled by $\delta$ become important, with the sign of these contributions depending on the sign of $\delta$. For the spectral index, its running and the tensor-to-scalar ratio the predictions are shown in Fig. 5.3. Recent WMAP observations constrain the preferred region for the spectral index $n_s$, thus imposing a constraint on the preferred region for $m_N$. For $\kappa = 1$ and $\delta = \pm 1$ this is marked in grey in Fig. 8.3.

On the other hand, the decisive quantities of reheating and leptogenesis, namely the reheat temperature $T_R$ and the baryon asymmetry $n_B/n_\gamma$ depend on the effective first generation neutrino Yukawa coupling $\tilde{y}_1$, the $\mathcal{CP}$ asymmetry $\epsilon_1$ and the mass of the lightest right-handed (s)neutrino $m_N$ (cf. Eqns. (8.34) and (8.38)). The latter parameter is thus the link between inflation and leptogenesis. The preferred region of parameter space resulting from bounds on these quantities is marked in blue in Fig. 8.3. It is bounded from below by the experimental value of the baryon asymmetry measured by WMAP and by an upper bound on the $\mathcal{CP}$ violation per (s)neutrino decay Eqn. (4.19). From above it is bounded by constraints imposed on the reheat temperature from the gravitino problem. Since these are model dependent, we have plotted the regions satisfying $T_R < 10^{10}$, $10^9$, $10^8$, $10^7$ GeV in different shadings. Note that a higher reheat temperature at a fixed value for $m_N$ automatically corresponds to a smaller value of $\epsilon_1$ in order to match the measured baryon asymmetry. The resulting preferred region in parameter space implies an effective first generation Yukawa coupling $\tilde{y}_1 = \mathcal{O}(10^{-9} - 10^{-4})$. The upper part of this range is of the same order as the first family quark and charged lepton Yukawa couplings, which in the MSSM with moderate $\tan\beta$ are of the order $10^{-4}$ to $10^{-6}$.

Throughout the discussion we have assumed non-thermal leptogenesis. Assuming that the light neutrinos obtain masses via the type I seesaw mechanism [4], this assumptions depends on the value of the effective light neutrino mass parameter (also dubbed washout parameter)

$$\tilde{m}_1 \equiv \tilde{y}_1^2 \langle v \rangle^2 / m_N \, . \tag{8.42}$$

More explicitly, one can easily see from Eqn. (8.34) that

$$\left( \frac{T_R}{m_N} \right)^2 \approx 4.0 \cdot 10^2 \, \frac{\tilde{m}_1}{\text{eV}} \, . \tag{8.43}$$

Thus, constant values of $\tilde{m}_1$ correspond to a fixed relation between $T_R$ and $m_N$. They also give the order of magnitude for the mass of the left-handed neutrino $m_{\nu_1} \approx \tilde{m}_1$. In the

[4] This implies the following mass matrix for the left-handed neutrinos $(m_\nu)_{ij} = -(y_\nu^T M^{-1} y_\nu)_{ij} \langle v \rangle^2 / 2$.

preferred region of parameter space we find $\tilde{m}_1 < 3.4 \cdot 10^{-5}$ eV, thus implying non-thermal leptogenesis with $T_R \ll m_N$ and

$$m_{\nu_1} \ll \sqrt{\Delta m^2_{\text{atm, sol}}}\,, \tag{8.44}$$

implying a hierarchical spectrum for the SM neutrinos.

Finally, we want to comment on possible extensions of this scenario. In the results shown in Fig. 8.3 we have fixed $\kappa = 1$. Allowing for $0.5 < \kappa < 2$ gives qualitatively the same picture with a somewhat shifted grey region. For example, for $\kappa = 2$ the grey region is extended to the left to $m_N = 2 \cdot 10^{10}$ GeV whereas for $\kappa = 0.5$ it is extended to the right to $m_N = 7 \cdot 10^{12}$ GeV for $\delta = -1$. The maximally allowed region in $(m_N, \tilde{y}_1)$-parameter space at 95% CL, obtained from varying $0.5 < \kappa < 2$ with $\delta = \pm 1$, is shown in Fig. 8.4.

Another interesting possibility would arise if the experimentally preferred region for the spectral index was raised, favouring a spectral index closer to 1. This would lower the preferred range for the lightest (s)neutrino mass $m_N$ significantly and thus open up the region of thermal leptogenesis and allow for

$$m_{\nu_1} \approx \mathcal{O}(\sqrt{\Delta m^2_{\text{atm, sol}}})\,. \tag{8.45}$$

The forthcoming results of the Planck satellite will make the requirements for $m_N$ more accurate.

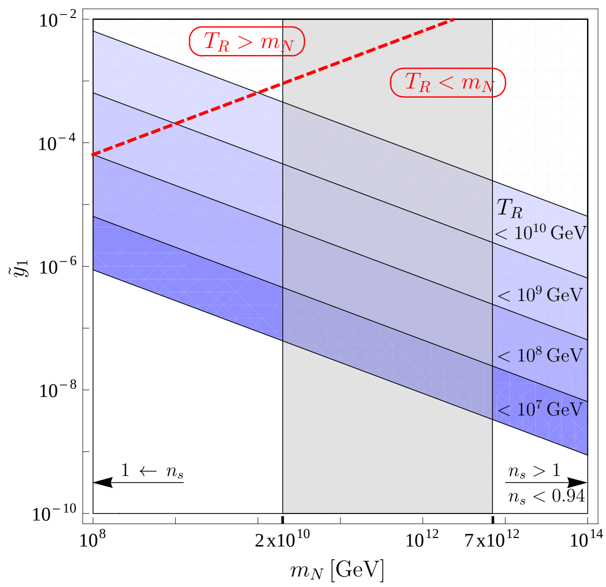



**Figure 8.4:** Maximally allowed region in $(m_N, \tilde{y}_1)$-parameter space at 95% CL, obtained from varying $0.5 < \kappa < 2$ with $\delta = \pm 1$. The preferred region derived from inflation, in particular from the WMAP constraints on the spectral index $n_s$, is marked in grey. The favoured region obtained from reheating and leptogenesis is depicted in blue. The meaning of the different blue shadings and the dashed red line is explained in Fig. 8.2.

# Summary and Conclusions

We have constructed a combined model of sneutrino tribrid inflation and subsequent baryogenesis through non-thermal leptogenesis in chapters 5 and 8. The model is a simple extension of the MSSM with conserved R-parity, where we have added three right-handed neutrino superfields. It is defined through the superpotential Eqn. (5.1) and the Kähler potential Eqn. (5.36). We have used a shift symmetry in the Kähler potential to protect the imaginary directions of the right-handed sneutrino fields from the SUGRA $\eta$-problem. These directions are tree-level flat and act as inflatons. Assuming a hierarchical spectrum for the right-handed (s)neutrinos, the inflationary dynamics during the last 60 e-folds of inflation as well as the outcome of leptogenesis after inflation are governed by the dynamics of the lightest right-handed (s)neutrino. This has allowed us to treat our model as an effective one-generation model. The tree-level flatness of the inflaton potential is lifted by one-loop Coleman-Weinberg corrections, which generate the slope necessary to drive the inflaton towards its critical value. We have computed these corrections and the resulting inflationary predictions for the model both in a SUSY and a SUGRA framework. Requiring successful inflation with the amplitude and spectral index of the scalar fluctuations in accordance with the latest experimental values [4, 5, 6] (within the 95% CL), we have derived the following bound on the mass of the lightest right-handed (s)neutrino

- Mass of the lightest right-handed (s)neutrino

$$m_N = \mathcal{O}(10^{10} - 10^{13})\ \mathrm{GeV}\,. \tag{8.46}$$

Further inflationary predictions in the allowed parameter range are

- A small tensor-to-scalar ratio

$$r \lesssim 0.015\,. \tag{8.47}$$

- A running of the spectral index

$$-0.0004 \lesssim \alpha_s \lesssim 0.0002\,. \tag{8.48}$$

These results have been discussed in more detail in section 5.5 and are summarised in Fig. 5.3.

After inflation, the lightest sneutrino field and the waterfall field perform damped oscillations around their respective global minima. Using a full numerical simulation we have shown that for the parameter values allowed from the requirement of successful inflation in accordance with the latest data from the WMAP satellite, the waterfall field quickly settles down to its true minimum and the universe is dominated by the oscillating right-handed sneutrino field as shown in Figs. 4.1 and 8.1. The $\mathcal{CP}$ violating, out-of-equilibrium decays of these sneutrinos produce the desired matter-antimatter asymmetry through non-thermal leptogenesis. Since the inflaton *is* the right-handed sneutrino in our model, non-thermal leptogenesis is very efficient and allows for a comparably small reheat temperature. Using a time-averaged description of the dynamics, which is a very good approximation concerning the outcome of leptogenesis, we have derived analytical expressions for the produced baryon asymmetry and the reheat temperature in our model. The results of this analysis have been discussed in detail in section 8.3 and are shown in Fig. 8.2

Combining the results from both inflation and non-thermal leptogenesis and comparing these results with the latest experimental data [4, 5, 6] we have constrained the allowed parameter space of our model from two different directions. The final results are summarised in Figs. 8.3 and 8.4 and have been discussed in detail in section 8.4. From the combined analysis we have found the following additional constraints:

- An effective first generation Yukawa coupling

$$\tilde{y}_1 = \mathcal{O}(10^{-9} - 10^{-4})\,. \tag{8.49}$$

- A mass for the lightest SM neutrino $\nu_1$

$$m_{\nu_1} < \mathcal{O}(10^{-4})\ \mathrm{eV}\,. \tag{8.50}$$

Currently running high precision experiments like the Planck satellite [11] or the KATRIN experiment [163] can help to clarify the status of this model.

The second major result of this thesis is the construction of a viable model of inflation using gauge-non singlet (GNS) fields as inflatons. In particular, we have constructed a model of inflation using fields from the matter sector of SUSY Pati-Salam unification and SUSY $SO(10)$ GUTs as inflatons. This is motivated by the observation that in such left-right symmetric SUSY GUTs, the right-handed neutrino superfield is an indispensable ingredient of any phenomenologically interesting model and has thus not to be put in "by hand" as in the model discussed in chapter 5, for example.

However, for a charged inflaton, new challenges arise in the form of D-term as well as one- and two-loop contributions to the inflaton potential, all of which threaten to spoil slow-roll inflation. We have found that by combining fields from some matter representation with fields from the conjugate representation it is possible to avoid the D-term contributions if inflation proceeds along a D-flat direction in field space. Along such a trajectory, the GUT symmetry is *broken in the inflaton direction* during inflation, such that the inflaton effectively decouples from all gauge interactions and behaves like a singlet. This is the crucial observation that helps to avoid the problematic one- and two-loop contributions to the effective inflaton potential. It also helps to avoid the production of topological defects in the waterfall phase transition ending inflation, because the broken symmetry is "taken over" by the waterfall fields which are thereby forced to settle in a unique vacuum state after inflation.

We have first discussed these ideas for an inflaton charged under a $U(1)$ symmetry in chapter 6 to illustrate the main features of our approach in as simple a form as possible. The basic form of the superpotential for this toy model has been introduced in Eqns. (6.1) and (6.2). We have then moved on to consider matter inflation in SUSY Pati-Salam unification in chapter 7. Combining fields from the matter representations $\boldsymbol{R_i^c} \sim (\bar{\mathbf{4}}, \mathbf{1}, \bar{\mathbf{2}})$ with fields from the conjugate representation $\bar{\boldsymbol{R}}^{\boldsymbol{c}} \sim (\mathbf{4}, \mathbf{1}, \mathbf{2})$ we have shown how inflation can proceed along a D-flat trajectory. Furthermore, we have argued that for $i = 4$ one linear combination of the $\boldsymbol{R_i^c}$ will pair up with $\bar{\boldsymbol{R}}^{\boldsymbol{c}}$ to form Dirac-type mass terms after inflation (i.e. around the GUT scale) and become heavy. We therefore end up with three light generations containing the SM quarks and leptons. We have then investigated the special case of *sneutrino inflation in Pati-Salam unification* in some detail in section 7.1.4. For this case we have explicitly calculated the mass spectrum during inflation (cf. Tabs. 7.3 and 7.4) and we have shown how the waterfall is triggered into a preferred direction. Using these results we have discussed how the fact that the gauge symmetry is broken in the inflaton direction helps to avoid potentially dangerous one- and two-loop contributions to the effective inflaton potential in section 7.2. Finally, in sections 7.3 and 7.4, we have elaborated on possible generalisations to SUSY $SO(10)$ GUTs and to SUGRA and shown that it is possible to realise inflation with the 16-dimensional spinor representations $\mathbf{16}$ and $\bar{\mathbf{16}}$ of SUSY $SO(10)$ while avoiding the SUGRA $\eta$-problem with the help of a Heisenberg symmetry.

In conclusion, the problem to forge a connection between early universe cosmology on the one side and particle physics models on the other side is a very challenging one, not least because of the limited amount of experimental data available on the former. Only further high precision experiments will help to clarify the situation. In this thesis we have shown how progress can be made using inflation and leptogenesis as complementary probes for the parameters of a model that implements both in a single setup, and by using the concept of (partial) unification of gauge interactions. In both approaches the right-handed neutrino superfield has played a paramount role.

# Appendix

APPENDIX A

# Notations and Conventions

In this chapter we summarise the notations and conventions used throughout this thesis.

Spacetime indices are denoted by $\mu, \nu, \sigma, \tau, ... \in \{0, 1, 2, 3\}$.
Spinor indices are denoted by $\alpha, \beta, \dot{\alpha}, \dot{\beta}, ... \in \{1, 2\}$.
If not stated otherwise the summation convention is always used.
Throughout this thesis we use units in which $c = \hbar = 1$.

## Spinor Conventions

The Minkowski spacetime metric we use is given by

$$\eta_{\mu\nu} = \eta^{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)\,. \tag{A.1}$$

The Pauli matrices are given by

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\,, \quad \sigma^2 = \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}\,, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\,. \tag{A.2}$$

In the chiral representation, the Dirac matrices are given by

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}\,, \tag{A.3}$$

where we have defined

$$\sigma^\mu \equiv (\mathbb{1}_2, \boldsymbol{\sigma})\,, \tag{A.4}$$

$$\bar{\sigma}^\mu \equiv (\mathbb{1}_2, -\boldsymbol{\sigma}) = \sigma_\mu\,. \tag{A.5}$$

They satisfy the Clifford-algebra

$$\{\gamma^\mu, \gamma^\nu\} = 2\,\eta^{\mu\nu}\mathbb{1}_4\,. \tag{A.6}$$

In the section on supergravity, we make use of the matrices

$$\Sigma^{\mu\nu} = \frac{i}{2}\big[\gamma^\mu, \gamma^\nu\big]\,. \tag{A.7}$$

Working in the chiral representation, we find

$$\gamma_5 \equiv \mathrm{i}\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -\mathbb{1}_2 & 0 \\ 0 & \mathbb{1}_2 \end{pmatrix} \tag{A.8}$$

and the chirality projection operators are given by

$$P_L \equiv \frac{1}{2}(1-\gamma_5) = \begin{pmatrix} \mathbb{1}_2 & 0 \\ 0 & 0 \end{pmatrix}\,, \tag{A.9}$$

$$P_R \equiv \frac{1}{2}(1+\gamma_5) = \begin{pmatrix} 0 & 0 \\ 0 & \mathbb{1}_2 \end{pmatrix}\,. \tag{A.10}$$

With these expressions, we can decompose a Dirac spinor as

$$\Psi = \begin{pmatrix} \psi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix}\,, \tag{A.11}$$

where $\psi_\alpha$ is a left-handed Weyl spinor and $\bar{\chi}^{\dot{\alpha}}$ is a right-handed Weyl spinor. Defining

$$\sigma^{\mu\nu} \equiv \tfrac{1}{4}(\sigma^\mu\bar{\sigma}^\nu - \sigma^\nu\bar{\sigma}^\mu)\,, \tag{A.12}$$

$$\bar{\sigma}^{\mu\nu} \equiv \tfrac{1}{4}(\bar{\sigma}^\mu\sigma^\nu - \bar{\sigma}^\nu\sigma^\mu)\,, \tag{A.13}$$

and

$$S_1(\Lambda) \equiv \exp\left(\tfrac{1}{2}\,\omega_{\mu\nu}\,\sigma^{\mu\nu}\right)\,, \tag{A.14}$$

$$S_2(\Lambda) \equiv \exp\left(\tfrac{1}{2}\,\omega_{\mu\nu}\,\bar{\sigma}^{\mu\nu}\right)\,, \tag{A.15}$$

they transform in the following way under Lorentz transformations

$$\psi_\alpha \to S_1(\Lambda)_\alpha{}^\beta\,\psi_\beta\,, \tag{A.16}$$

$$\bar{\chi}^{\dot{\alpha}} \to S_2(\Lambda)^{\dot{\alpha}}{}_{\dot{\beta}}\,\bar{\chi}^{\dot{\beta}}\,, \tag{A.17}$$

with $S_2^{-1} = S_1^\dagger$.

The two kinds of Weyl spinors are related via

$$\bar{\psi}_{\dot{\alpha}} = (\psi_\alpha)^\dagger\,, \quad \chi^\alpha = (\bar{\chi}^{\dot{\alpha}})^\dagger\,, \tag{A.18}$$

and indices can be raised or lowered as

$$\psi_\alpha = \epsilon_{\alpha\beta}\,\psi^\beta\,, \quad \psi^\alpha = \epsilon^{\alpha\beta}\,\psi_\beta\,, \tag{A.19}$$

$$\bar{\chi}_{\dot{\alpha}} = \epsilon_{\dot{\alpha}\dot{\beta}}\,\bar{\chi}^{\dot{\beta}}\,, \quad \bar{\chi}^{\dot{\alpha}} = \epsilon^{\dot{\alpha}\dot{\beta}}\,\bar{\chi}_{\dot{\beta}}\,, \tag{A.20}$$

where

$$\epsilon_{\alpha\beta} = \epsilon_{\dot\alpha\dot\beta} = -\mathrm{i}\sigma^2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \tag{A.21}$$

$$\epsilon^{\alpha\beta} = \epsilon^{\dot\alpha\dot\beta} = \mathrm{i}\sigma^2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{A.22}$$

The "charge-conjugate" $\Psi^c$ of a Dirac spinor $\Psi$ is defined as

$$\Psi = \begin{pmatrix} \psi_\alpha \\ \bar\chi^{\dot\alpha} \end{pmatrix} \quad \longleftrightarrow \quad \Psi^c = \begin{pmatrix} \chi_\alpha \\ \bar\psi^{\dot\alpha} \end{pmatrix}, \tag{A.23}$$

and a (4 component) Majorana spinor $\Psi_{\mathsf{m}}^c = \Psi_{\mathsf{m}}$ can be written as

$$\Psi_{\mathsf{m}} = \begin{pmatrix} \psi_\alpha \\ \bar\psi^{\dot\alpha} \end{pmatrix}. \tag{A.24}$$

The Lorentz invariant scalar product between two left- respectively right handed Weyl spinors is defined as

$$\chi\psi \equiv \chi^\alpha\psi_\alpha = \psi^\alpha\chi_\alpha = \psi\chi, \tag{A.25}$$

$$\bar\chi\bar\psi \equiv \bar\chi_{\dot\alpha}\bar\psi^{\dot\alpha} = \bar\psi_{\dot\alpha}\bar\chi^{\dot\alpha} = \bar\psi\bar\chi. \tag{A.26}$$

When manipulating expressions like these it is important to be careful about the placement of the indices and to remember that spinors are Grassmann valued variables (cf. below), because e.g.

$$\chi^\alpha\psi_\alpha = -\,\chi_\alpha\psi^\alpha, \tag{A.27}$$

$$\bar\chi_{\dot\alpha}\bar\psi^{\dot\alpha} = -\,\bar\chi^{\dot\alpha}\bar\psi_{\dot\alpha}. \tag{A.28}$$

The Hermitian conjugate of the scalar product of two Weyl spinors is defined as

$$(\chi\psi)^\dagger \equiv \psi^\dagger\chi^\dagger = \bar\psi\bar\chi. \tag{A.29}$$

Furthermore,

$$\chi^\alpha(\sigma^\mu)_{\alpha\dot\beta}\,\bar\psi^{\dot\beta} = -\bar\psi_{\dot\beta}(\bar\sigma^\mu)^{\dot\beta\alpha}\,\chi_\alpha, \tag{A.30}$$

$$\bar\chi_{\dot\alpha}(\bar\sigma^\mu)^{\dot\alpha\beta}\,\psi_\beta = -\psi^\beta(\sigma^\mu)_{\beta\dot\alpha}\,\bar\chi^{\dot\alpha} \tag{A.31}$$

both transform as 4-vectors under Lorentz transformation and we have

$$(\chi\sigma^\mu\bar\psi)^\dagger = \psi\sigma^\mu\bar\chi = -\bar\chi\bar\sigma^\mu\psi = -(\bar\psi\bar\sigma^\mu\chi)^\dagger. \tag{A.32}$$

With this we are now in position to write the usual covariant Dirac bilinears in terms of two component Weyl spinors. Writing

$$\Psi = \begin{pmatrix} \psi_\alpha \\ \bar\chi^{\dot\alpha} \end{pmatrix}, \quad \Phi = \begin{pmatrix} \varphi_\alpha \\ \bar\eta^{\dot\alpha} \end{pmatrix}, \tag{A.33}$$

we get

$$\bar{\Psi}\Phi = \bar{\psi}\bar{\eta} + \chi\varphi = (\bar{\Phi}\Psi)^\dagger\,, \tag{A.34}$$

$$\bar{\Psi}\gamma^5\Phi = \bar{\psi}\bar{\eta} - \chi\varphi = -(\bar{\Phi}\gamma^5\Psi)^\dagger\,, \tag{A.35}$$

$$\bar{\Psi}\gamma^\mu\Phi = \chi\sigma^\mu\bar{\eta} + \bar{\psi}\bar{\sigma}^\mu\varphi = (\bar{\Phi}\gamma^\mu\Psi)^\dagger\,. \tag{A.36}$$

In particular, the standard Dirac mass term is given by

$$m\,\bar{\Psi}\Psi = m\,(\bar{\psi}\bar{\chi} + \chi\psi)\,, \tag{A.37}$$

whereas for a Majorana spinor it is given by

$$\frac{1}{2}m\,\bar{\Psi}_{\mathsf{m}}\Psi_{\mathsf{m}} = \frac{1}{2}m\,(\bar{\psi}\bar{\psi} + \psi\psi)\,. \tag{A.38}$$

## Grassmann Variables

When manipulating expressions containing (Weyl) spinors or fermionic coordinates / transformation parameters, it is important to remember that these are Grassmann valued variables

$$\{\theta_\alpha, \theta_\beta\} = \{\theta_\alpha, \bar{\xi}_{\dot{\alpha}}\} = \{\bar{\xi}_{\dot{\alpha}}, \bar{\xi}_{\dot{\beta}}\} = 0 \quad , \quad \alpha, \beta, \dot{\alpha}, \dot{\beta} \in \{1, 2\}\,. \tag{A.39}$$

In particular, any product $\theta^n$ with $n > 2$ of such a two-component Grassmann spinor with itself vanishes. As a consequence, the Taylor expansion of any function $\Phi(\theta)$ terminates after a finite number of terms

$$\Phi(\theta) = a + \zeta^\alpha\theta_\alpha + b\,\theta\theta\,, \tag{A.40}$$

where $a, b$ are complex numbers and $\zeta$ is a constant Grassmann spinor. Similar expressions hold for functions $\Phi^\dagger(\bar{\theta})$ and $\Omega(\theta, \bar{\theta})$, where in the last case the expansion contains terms up to $\theta\theta\,\bar{\theta}\bar{\theta}$.

Integration with respect to Grassmann variables is defined in such a way that it always picks out the highest part in such an expansion, i.e.

$$\int \Phi(\theta)\,\mathrm{d}^2\theta = \big[\Phi(\theta)\big]_{\theta\theta}\,, \tag{A.41}$$

$$\int \Omega(\theta, \bar{\theta})\,\mathrm{d}^2\theta\,\mathrm{d}^2\bar{\theta} = \big[\Omega(\theta, \bar{\theta})\big]_{\theta\theta\,\bar{\theta}\bar{\theta}}\,. \tag{A.42}$$

Finally, derivatives with respect to Grassmann variables are defined as

$$\partial_\alpha = \frac{\partial}{\partial\theta^\alpha}\,, \quad \partial^\alpha = \epsilon^{\alpha\beta}\partial_\beta\,, \tag{A.43}$$

$$\bar{\partial}_{\dot{\alpha}} = \frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}\,, \quad \bar{\partial}^{\dot{\alpha}} = \epsilon^{\dot{\alpha}\dot{\beta}}\bar{\partial}_{\dot{\beta}}\,. \tag{A.44}$$

This entails the somewhat complicated results

$$\partial_\alpha \theta^\beta = \delta_\alpha^\beta\,, \qquad \partial^\alpha \theta_\beta = -\delta_\beta^\alpha\,, \qquad \partial^\alpha \theta^\beta = \epsilon^{\alpha\beta}\,, \qquad \partial_\alpha \theta_\beta = -\epsilon_{\alpha\beta}\,, \tag{A.45}$$

$$\bar\partial_{\dot\alpha} \bar\theta^{\dot\beta} = \delta_{\dot\alpha}^{\dot\beta}\,, \qquad \bar\partial^{\dot\alpha} \bar\theta_{\dot\beta} = -\delta_{\dot\beta}^{\dot\alpha}\,, \qquad \bar\partial^{\dot\alpha} \bar\theta^{\dot\beta} = \epsilon^{\dot\alpha\dot\beta}\,, \qquad \bar\partial_{\dot\alpha} \bar\theta_{\dot\beta} = -\epsilon_{\dot\alpha\dot\beta}\,, \tag{A.46}$$

which are necessary for consistency, however. Furthermore, the fermionic derivatives anticommute with other Grassmann variables. Thus, for example,

$$\partial_\alpha(\theta\theta) = (\partial_\alpha\theta^\beta)\theta_\beta - \theta^\beta(\partial_\alpha\theta_\beta) = \delta_\alpha^\beta\theta_\beta + \epsilon_{\alpha\beta}\theta^\beta = 2\theta_\alpha\,, \tag{A.47}$$

$$\partial^{\dot\alpha}(\bar\theta\bar\theta) = (\partial^{\dot\alpha}\bar\theta_{\dot\beta})\bar\theta^{\dot\beta} - \bar\theta_{\dot\beta}(\partial^{\dot\alpha}\bar\theta^{\dot\beta}) = -\delta_{\dot\beta}^{\dot\alpha}\bar\theta^{\dot\beta} - \bar\theta_{\dot\beta}\epsilon^{\dot\alpha\dot\beta} = -2\bar\theta^{\dot\alpha}\,. \tag{A.48}$$

As one can see, one has to be very careful of all the minus signs as the above results entail, for example,

$$\Phi(\theta+\zeta) = \Phi(\theta) + \zeta^\alpha\,\partial_\alpha\Phi(\theta) + \mathcal{O}(\zeta\zeta)\,, \tag{A.49}$$

$$\Phi^\dagger(\bar\theta+\bar\zeta) = \Phi^\dagger(\bar\theta) - \bar\zeta_{\dot\alpha}\,\partial^{\dot\alpha}\Phi^\dagger(\bar\theta) + \mathcal{O}(\bar\zeta\bar\zeta)\,. \tag{A.50}$$

# APPENDIX B

# Sample Calculations

In this chapter we derive some of the results used in this thesis in more detail.

## R-Symmetry Charges for a Chiral Superfield

A chiral superfield $\mathbf{\Phi}(x,\theta)$ with $U(1)_R$-charge $r_\Phi$ is defined to transform in the following way under a $U(1)_R$ transformation

$$\mathbf{\Phi}(x,\theta) \to \mathbf{\Phi'}(x',\theta') = e^{ir_\Phi\delta}\,\mathbf{\Phi}(x,\theta)\,. \tag{B.1}$$

This induces the following transformations on the component fields $(\phi,\psi,F)$

$$\phi(x) \to \phi'(x') = e^{ir_\phi\delta}\,\phi(x)\,, \tag{B.2}$$

$$\psi(x) \to \psi'(x') = e^{ir_\psi\delta}\,\psi(x)\,, \tag{B.3}$$

$$F(x) \to F'(x') = e^{ir_F\delta}\,F(x)\,. \tag{B.4}$$

Let us use this to write out some of the terms of Eqn. (B.1) in more detail. We find

$$\begin{aligned}\mathbf{\Phi'}(x',\theta') &= \phi'(x') + \sqrt{2}\theta'\psi'(x') + \theta'\theta' F'(x') + \,...\\ &= e^{ir_\Phi\delta}\left(\phi(x) + \sqrt{2}\theta\psi(x) + \theta\theta F(x) + \,...\right)\\ &= e^{ir_\Phi\delta}\Big(e^{-ir_\phi\delta}\,\phi'(x') + \sqrt{2}\,e^{-i\delta}\theta'\cdot e^{-ir_\psi\delta}\,\psi'(x')\\ &\qquad + e^{-2i\delta}\theta'\theta'\cdot e^{-ir_F\delta}\,F'(x') + \,...\Big)\,,\end{aligned} \tag{B.5}$$

with

$$x \to x' = x\,, \quad \theta \to \theta' = e^{i\delta}\theta\,. \tag{B.6}$$

We find that

$$e^{i\delta(r_\Phi - r_\phi)}\,\phi'(x') = \phi'(x')\,, \tag{B.7}$$

$$e^{i\delta(r_\Phi - 1 - r_\psi)}\,\psi'(x') = \psi'(x')\,, \tag{B.8}$$

$$e^{i\delta(r_\Phi - 2 - r_F)}\,F'(x') = F'(x')\,, \tag{B.9}$$

from which we can read of the corresponding charges

$$r_\phi = r_\Phi \,, \tag{B.10}$$
$$r_\psi = r_\Phi - 1 \,, \tag{B.11}$$
$$r_F = r_\Phi - 2 \,. \tag{B.12}$$

## Derivation of Equation (3.107)

To compute the field value $\varphi_\mathcal{N}$ at the time $\mathcal{N}$ e-folds before the end of inflation in the slow-roll approximation we use the two equations

$$3\,\mathcal{H}\,\dot{\varphi} \simeq -V'(\varphi) \,, \tag{B.13}$$
$$\mathcal{H} \simeq M_\mathrm{P}^{-1}\,\sqrt{V/3} \,. \tag{B.14}$$

For the case of the F-term hybrid inflation model we have

$$\mathcal{H} \simeq \frac{\kappa M^2}{\sqrt{3} M_\mathrm{P}} \,, \tag{B.15}$$
$$V' \simeq \frac{\kappa^4 M^4}{4\pi^2 \varphi} \,. \tag{B.16}$$

Thus, we can write

$$\int_{t_\mathcal{N}}^{t_c} -\frac{\kappa^4 M^4}{4\pi^2} \frac{M_\mathrm{P}}{\sqrt{3}\,\kappa\, M^2}\,\mathrm{d}t = \int_{\varphi_\mathcal{N}}^{\varphi_c} \varphi\,\mathrm{d}\varphi \,. \tag{B.17}$$

This yields

$$\frac{1}{2}\left(\varphi_c^2 - \varphi_\mathcal{N}^2\right) = -\frac{\kappa^3 M^2 M_\mathrm{P}}{\sqrt{48}\pi^2}\left(t_c - t_\mathcal{N}\right) \,. \tag{B.18}$$

Now we use

$$\mathcal{N} = \int_{t_\mathcal{N}}^{t_c} \mathcal{H}\,\mathrm{d}t = \mathcal{H}\int_{t_\mathcal{N}}^{t_c} \mathrm{d}t = \mathcal{H}\left(t_c - t_\mathcal{N}\right) \,, \tag{B.19}$$

that is

$$t_c - t_\mathcal{N} = \frac{\mathcal{N}}{\mathcal{H}} = \frac{\sqrt{3} M_\mathrm{P}}{\kappa M^2}\mathcal{N} \,. \tag{B.20}$$

Plugging this into the equation above yields

$$\varphi_\mathcal{N}^2 - \varphi_c^2 = \frac{\kappa^2 M_\mathrm{P}^2}{2\pi^2}\mathcal{N} \,. \tag{B.21}$$

## Mass Spectrum during Inflation

Here, we calculate the masses of the relevant fields during inflation for the model of section 7.1.4. In particular, we calculate the gauge boson masses, the fermion masses corresponding to the chiral supermultiplets $\boldsymbol{H^c}$ and $\bar{\boldsymbol{H}}^{\boldsymbol{c}}$ and the fermion masses arising from the mixing between the chiral and gauge multiplets. The results are summarised in the main text in Tab. 7.3 and Tab. 7.4 and they have been used in calculating the one-loop radiative corrections in section 7.2.1. The scalar masses for the waterfall sector have been calculated in the main text, section 7.1.4.

### Gauge Boson Masses

We now calculate the gauge boson masses corresponding to the gauge factors $SU(2)_R$ and $SU(4)_C$ of the Pati-Salam gauge group $G_{\text{PS}}$. As we will see, some of the gauge fields become massive when the inflaton fields acquire VEVs during inflation.

In our calculation, we set the coupling constants $g_R = g_C \equiv g$ close to the GUT scale and we use the following generators for the fundamental representation $(\mathbf{4}, \mathbf{1}, \mathbf{2})$ (ignoring the trivial $SU(2)_L$ part)

$$\mathcal{T}^a = T^a \otimes \mathbb{1}_{2\times 2}\,, \quad a = 1, \,\dots\,, 15\,, \tag{B.22}$$

$$\mathcal{T}^{16} = \mathbb{1}_{4\times 4} \otimes \tfrac{1}{2}\sigma^1\,, \quad \mathcal{T}^{17} = \mathbb{1}_{4\times 4} \otimes \tfrac{1}{2}\sigma^2\,, \quad \mathcal{T}^{18} = \mathbb{1}_{4\times 4} \otimes \tfrac{1}{2}\sigma^3\,. \tag{B.23}$$

Here, $\sigma^b$ are the Pauli matrices Eqn. (A.2) and $T^a$ are the 15 generators of $SU(4)_C$ displayed in Tab. B.1.

The masses for the gauge bosons are given by the following term in the Lagrangian

$$\mathcal{L}_{GB} \;= \Big| \sum_{a=1}^{18} g\, A_\mu^a\, \mathcal{T}^a \bar{R}^c \Big|^2 + \text{ terms for } R^c\,, \tag{B.24}$$

where $R^c$ and $\bar{R}^c$ contain the VEVs of the sneutrinos acting as inflatons, cf. Eqn. (7.17).

We can easily see that the gauge fields corresponding to the generators $\mathcal{T}^1, \,\dots\,, \mathcal{T}^8$ remain massless. On the other hand, e.g. for the gauge fields corresponding to the generators $\mathcal{T}^9$ and $\mathcal{T}^{10}$ we find

$$\begin{aligned} \mathcal{L}_{GB} \supset \frac{1}{8}\, g^2\, (\tilde{\nu}^c)^2 \Big| A_\mu^9 - i\, A_\mu^{10} \Big|^2 + \text{ terms for } R^c \\ = \frac{1}{4}\, g^2\, (\tilde{\nu}^c)^2 \Big[ (A_\mu^9)^2 + (A_\mu^{10})^2 \Big]. \end{aligned} \tag{B.25}$$

This yields

$$m_9^2 = m_{10}^2 = g^2\, (\tilde{\nu}^c)^2/2\,. \tag{B.26}$$

Similarly, the gauge bosons corresponding to the generators $\mathcal{T}^{11}, \,\dots\,, \mathcal{T}^{14}$ as well as $\mathcal{T}^{16}$ and $\mathcal{T}^{17}$ acquire the same mass. The generators $\mathcal{T}^{18}$ and $\mathcal{T}^{15}$ are diagonal and the corresponding gauge bosons mix. We find

$$\mathcal{L}_{GB} \supset\; g^2 \frac{(\tilde{\nu}^c)^2}{8} \left( A_\mu^{18} - \sqrt{\frac{3}{2}}\, A_\mu^{15} \right)^2 + \text{ terms for } R^c\,. \tag{B.27}$$

$$T^1 = \tfrac{1}{2}\begin{pmatrix}0&1&0&0\\1&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix} \quad T^2 = \tfrac{1}{2}\begin{pmatrix}0&-\mathrm{i}&0&0\\\mathrm{i}&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix} \quad T^3 = \tfrac{1}{2}\begin{pmatrix}1&0&0&0\\0&-1&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}$$

$$T^4 = \tfrac{1}{2}\begin{pmatrix}0&0&1&0\\0&0&0&0\\1&0&0&0\\0&0&0&0\end{pmatrix} \quad T^5 = \tfrac{1}{2}\begin{pmatrix}0&0&-\mathrm{i}&0\\0&0&0&0\\\mathrm{i}&0&0&0\\0&0&0&0\end{pmatrix} \quad T^6 = \tfrac{1}{2}\begin{pmatrix}0&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&0\end{pmatrix}$$

$$T^7 = \tfrac{1}{2}\begin{pmatrix}0&0&0&0\\0&0&-\mathrm{i}&0\\0&\mathrm{i}&0&0\\0&0&0&0\end{pmatrix} \quad T^8 = \tfrac{1}{2\sqrt{3}}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-2&0\\0&0&0&0\end{pmatrix} \quad T^9 = \tfrac{1}{2}\begin{pmatrix}0&0&0&1\\0&0&0&0\\0&0&0&0\\1&0&0&0\end{pmatrix}$$

$$T^{10} = \tfrac{1}{2}\begin{pmatrix}0&0&0&-\mathrm{i}\\0&0&0&0\\0&0&0&0\\\mathrm{i}&0&0&0\end{pmatrix} \quad T^{11} = \tfrac{1}{2}\begin{pmatrix}0&0&0&0\\0&0&0&1\\0&0&0&0\\0&1&0&0\end{pmatrix} \quad T^{12} = \tfrac{1}{2}\begin{pmatrix}0&0&0&0\\0&0&0&-\mathrm{i}\\0&0&0&0\\0&\mathrm{i}&0&0\end{pmatrix}$$

$$T^{13} = \tfrac{1}{2}\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix} \quad T^{14} = \tfrac{1}{2}\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&-\mathrm{i}\\0&0&\mathrm{i}&0\end{pmatrix} \quad T^{15} = \tfrac{1}{2\sqrt{6}}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&-3\end{pmatrix}$$

**Table B.1:** Generators of $SU(4)_C$ in the fundamental representation **4**.

Defining the new normalised field

$$Z_\mu^\parallel \equiv \sqrt{\frac{2}{5}}\left(A_\mu^{18} - \sqrt{\frac{3}{2}}A_\mu^{15}\right) \tag{B.28}$$

this becomes

$$\mathcal{L}_{GB} \supset \frac{5}{8}\, g^2\, (\tilde{\nu}^c)^2\, \left(Z_\mu^\parallel\right)^2. \tag{B.29}$$

The combination orthogonal to $Z_\mu^\parallel$, i.e

$$Z_\mu^\perp \equiv \sqrt{\frac{2}{5}}\left(A_\mu^{15} + \sqrt{\frac{3}{2}}A_\mu^{18}\right) \tag{B.30}$$

remains massless. The gauge boson masses are summarised in Tab. 7.3.

### Fermion Mass Spectrum

In a SUSY theory there are two contributions to the fermion masses, one coming directly from the superpotential and another one from the mixing between the chiral and the gauge multiplets.

The contribution from the superpotential is given by

$$\mathcal{L}_1 = -\frac{1}{2}\,\frac{\partial^2 W}{\partial\phi_i\,\partial\phi_j}\left(\psi_i\,\psi_j + \bar{\psi}_i\,\bar{\psi}_j\right)\,. \tag{B.31}$$

Here, $\phi_i$ and $\psi_i$ are the scalar boson and chiral fermion contained in the chiral superfield $\boldsymbol{\phi_i} \ni \phi_i, \psi_i$ and $W$ is the superpotential regarded as a function of the scalar fields only.

Using the form of the superpotential in Eqn. (7.14) and keeping in mind that the VEVs of the scalar component fields of $\boldsymbol{H^c}$ and $\boldsymbol{\bar{H}^c}$ remain at zero during inflation, we conclude that Eqn. (B.31) does not contribute to the fermion masses for the chiral supermultiplets $\boldsymbol{R^c}$ and $\boldsymbol{\bar{R}^c}$. It does, however, contribute to the fermion masses for the $\boldsymbol{H^c}$ and $\boldsymbol{\bar{H}^c}$ supermultiplets:

$$\begin{aligned}\mathcal{L}_1 = &-\frac{1}{2}\,\zeta\,(\tilde{\nu}^c)^2\Big[\psi_{u^c_{1H}}\psi_{\bar{u}^c_{1H}} + \,...\, + \psi_{d^c_{3H}}\psi_{\bar{d}^c_{3H}} + \psi_{e^c_H}\psi_{\bar{e}^c_H} + \text{H.c.}\Big]\\ &-\frac{1}{4}\,(\tilde{\nu}^c)^2\Big[2\,\gamma\,\psi_{\nu^c_H}\psi_{\nu^c_H} + 2\,(\zeta+\xi)\,\psi_{\nu^c_H}\psi_{\bar{\nu}^c_H} + 2\,\lambda\,\psi_{\bar{\nu}^c_H}\psi_{\bar{\nu}^c_H} + \text{H.c.}\Big]\,.\end{aligned} \tag{B.32}$$

Combining two chiral spinors to a Dirac spinor

$$\Psi_{u^c_{1H}} = \begin{pmatrix}\psi_{u^c_{1H}}\\ \bar{\psi}_{\bar{u}^c_{1H}}\end{pmatrix}\,,\quad ...\,, \tag{B.33}$$

the first part becomes

$$\mathcal{L}_1 \supset -\frac{1}{2}\,\zeta\,(\tilde{\nu}^c)^2\Big[\bar{\Psi}_{u^c_{1H}}\Psi_{u^c_{1H}} + \,...\, + \bar{\Psi}_{d^c_{3H}}\Psi_{d^c_{3H}} + \bar{\Psi}_{e^c_H}\Psi_{e^c_H}\Big]\,. \tag{B.34}$$

Diagonalising the mass matrix of the second part, we find

$$\mathcal{L}_1 \supset -\frac{1}{4}\,(\tilde{\nu}^c)^2\Big[(2\,\gamma-\zeta-\xi)\,\psi_a\,\psi_a \;+\; (2\,\gamma+\zeta+\xi)\,\psi_b\,\psi_b + \text{H.c.}\Big]\,, \tag{B.35}$$

where

$$\begin{pmatrix}\psi_a\\ \psi_b\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}\psi_{\bar{\nu}^c_H} - \psi_{\nu^c_H}\\ \psi_{\bar{\nu}^c_H} + \psi_{\nu^c_H}\end{pmatrix} \tag{B.36}$$

and we have set $\gamma = \lambda$ for simplicity.

Finally, defining the two Majorana spinors

$$\Psi_a = \begin{pmatrix}\psi_a\\ \bar{\psi}_a\end{pmatrix}\,,\qquad \Psi_b = \begin{pmatrix}\psi_b\\ \bar{\psi}_b\end{pmatrix}\,, \tag{B.37}$$

this becomes

$$\mathcal{L}_1 \supset -\frac{1}{4}\,(\tilde{\nu}^c)^2 \Big[\; (2\,\gamma - \zeta - \xi)\;\bar{\Psi}_a \Psi_a \;+\; (2\,\gamma + \zeta + \xi)\;\bar{\Psi}_b \Psi_b \Big]\,. \tag{B.38}$$

The resulting masses are summarised in Tab. 7.4.

Next, we turn to the second contribution due to the mixing between the chiral fermions $\psi_i$ of the chiral superfields and the gauginos. It is given by

$$\mathcal{L}_2 = -\sqrt{2}\,g \sum_a \left(\phi^*_{\bar{R}^c}\,\mathcal{T}^a\,\psi_{\bar{R}^c}\right)\,\lambda^a - \sqrt{2}\,g \sum_a \bar{\lambda}^a \left(\bar{\psi}_{\bar{R}^c}\,\mathcal{T}^a\,\phi_{\bar{R}^c}\right) + \text{terms for } R^c, \tag{B.39}$$

where $\phi_{\bar{R}^c}$ and $\psi_{\bar{R}^c}$ are the scalar and fermionic fields contained in the chiral supermultiplet $\boldsymbol{\bar{R}^c}$, etc.

Plugging in the VEVs of the $R^c$ and $\bar{R}^c$ fields we end up with

$$\begin{aligned}\mathcal{L}_2 = -\frac{g}{2}\,\tilde{\nu}^c \Big[\, & \psi_{u^c_1}\left(-\lambda^9 + \mathrm{i}\,\lambda^{10}\right) \,+\, \psi_{\bar{u}^c_1}\left(\lambda^9 + \mathrm{i}\,\lambda^{10}\right) + \\ & \psi_{u^c_2}\left(-\lambda^{11} + \mathrm{i}\,\lambda^{12}\right) \,+\, \psi_{\bar{u}^c_2}\left(\lambda^{11} + \mathrm{i}\,\lambda^{12}\right) + \\ & \psi_{u^c_3}\left(-\lambda^{13} + \mathrm{i}\,\lambda^{14}\right) \,+\, \psi_{\bar{u}^c_3}\left(\lambda^{13} + \mathrm{i}\,\lambda^{14}\right) + \\ & \psi_{e^c}\left(-\lambda^{16} - \mathrm{i}\,\lambda^{17}\right) \,+\, \psi_{\bar{e}^c}\left(\lambda^{16} - \mathrm{i}\,\lambda^{17}\right) + \\ & \psi_{\nu^c}\Big(\sqrt{\tfrac{3}{2}}\,\lambda^{15} - \lambda^{18}\Big) \,+\, \psi_{\bar{\nu}^c}\Big(-\sqrt{\tfrac{3}{2}}\,\lambda^{15} + \lambda^{18}\Big) + \text{H.c.}\Big]\,.\end{aligned} \tag{B.40}$$

Defining the following normalised left-chiral fields

$$\begin{aligned}\chi_1 &= \frac{1}{\sqrt{2}}\left(-\lambda^9 + \mathrm{i}\,\lambda^{10}\right)\,, & \chi_2 &= \frac{1}{\sqrt{2}}\left(\lambda^9 + \mathrm{i}\,\lambda^{10}\right)\,, \\ \chi_3 &= \frac{1}{\sqrt{2}}\left(-\lambda^{11} + \mathrm{i}\,\lambda^{12}\right)\,, & \chi_4 &= \frac{1}{\sqrt{2}}\left(\lambda^{11} + \mathrm{i}\,\lambda^{12}\right)\,, \\ \chi_5 &= \frac{1}{\sqrt{2}}\left(-\lambda^{13} + \mathrm{i}\,\lambda^{14}\right)\,, & \chi_6 &= \frac{1}{\sqrt{2}}\left(\lambda^{13} + \mathrm{i}\,\lambda^{15}\right)\,, \\ \chi_{e^c} &= -\frac{1}{\sqrt{2}}\left(\lambda^{16} + \mathrm{i}\,\lambda^{17}\right)\,, & \chi_{\bar{e}^c} &= \frac{1}{\sqrt{2}}\left(\lambda^{16} - \mathrm{i}\,\lambda^{17}\right)\,, \end{aligned} \tag{B.41}$$

$$\begin{aligned}\psi^{\parallel}_{\nu} &= \frac{1}{\sqrt{2}}\left(\psi_{\nu^c} - \psi_{\bar{\nu}^c}\right)\,, & \psi^{\perp}_{\nu} &= \frac{1}{\sqrt{2}}\left(\psi_{\nu^c} + \psi_{\bar{\nu}^c}\right)\,, \\ \chi^{\parallel}_{\nu^c} &= \sqrt{\tfrac{2}{5}}\Big(\sqrt{\tfrac{3}{2}}\lambda^{15} - \lambda^{18}\Big)\,, & \chi^{\perp}_{\nu^c} &= \sqrt{\tfrac{2}{5}}\Big(\sqrt{\tfrac{3}{2}}\lambda^{18} + \lambda^{15}\Big)\,, \end{aligned} \tag{B.42}$$

we can combine these with the chiral fermion fields from the $\boldsymbol{R^c}$ and $\boldsymbol{\bar{R}^c}$ superfields to form the following Dirac spinors

$$\Psi_1 = \begin{pmatrix}\psi_{u^c_1}\\ \bar{\chi}_1\end{pmatrix}\,,\quad \Psi_2 = \begin{pmatrix}\psi_{\bar{u}^c_1}\\ \bar{\chi}_2\end{pmatrix}\quad,\;\;\dots\;\;,\quad \Psi^{\parallel}_{\nu^c} = \begin{pmatrix}\psi^{\parallel}_{\nu^c}\\ \bar{\chi}^{\parallel}_{\nu^c}\end{pmatrix}\,,\quad \Psi^{\perp}_{\nu^c} = \begin{pmatrix}\psi^{\perp}_{\nu^c}\\ \bar{\chi}^{\perp}_{\nu^c}\end{pmatrix}\,. \tag{B.43}$$

With these, we can now write

$$\mathcal{L}_2 = -\frac{g}{\sqrt{2}}\,\tilde{\nu}^c\Big[\bar{\Psi}_1\Psi_1 + \ldots + \bar{\Psi}_6\Psi_6 + \bar{\Psi}_{e^c}\Psi_{e^c} + \bar{\Psi}_{\bar{e}^c}\,\Psi_{\bar{e}^c}\Big] \tag{B.44}$$

$$-\sqrt{\frac{5}{4}}\,g\,\tilde{\nu}^c\;\bar{\Psi}^{\parallel}_{\nu^c}\Psi^{\parallel}_{\nu^c}. \tag{B.45}$$

The resulting mass spectrum is listed in Tab. 7.3.

APPENDIX C

# Hot Big Bang Cosmology

In this appendix we summarise some important facts about the Standard Hot Big Bang (SSB) model. More details on the subject can, for example, be found in [158, 95, 96, 100, 164] and references therein.

We know from observation that the visible universe is homogeneous and isotropic on scales larger than 100 Mpc and it expands. Such a spacetime is described by the Friedmann-Robertson-Walker (FRW) metric

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a(t)^2 \left( \frac{\mathrm{d}r^2}{1 - kr^2} + r^2 \Big( \mathrm{d}\theta^2 + \sin^2\theta \, \mathrm{d}\phi^2 \Big) \right) . \tag{C.1}$$

Here, the curvature parameter $k$ is $+1$ for positively curved spatial geometry, 0 for flat geometry and $-1$ for negatively curved geometry of space. The coordinates $\boldsymbol{r} = (r, \theta, \phi)$ are called comoving coordinates, i.e. coordinates that follow the expansion of the universe. The time coordinate $t$ is the proper time measured by a comoving observer. The physical distance between two comoving observers at a given point in time is given by

$$R = a(t)\Delta\boldsymbol{r} , \tag{C.2}$$

and $a(t)$ is called the scale factor. The expansion rate of the universe is measured by the Hubble parameter

$$\mathcal{H}(t) \equiv \frac{\dot{a}(t)}{a(t)} . \tag{C.3}$$

It is an important quantity, because it sets the fundamental time and length scale of the FRW spacetime

$$t_{\mathcal{H}} = d_{\mathcal{H}} \simeq \mathcal{H}^{-1} , \tag{C.4}$$

called Hubble time and Hubble length. The first provides an estimate for the age of the universe, the second for the distance light can travel while the universe expands by an appreciable amount. On distances much smaller than the Hubble length and time scales much smaller than the Hubble time, the expansion of the universe can be neglected. In particular, causal processes can only operate on length scales smaller than the Hubble length. Applied to a wave with comoving wavenumber $k$, i.e. physical wavelength $\lambda_k \simeq a/k$, this means that for

$$\frac{d_{\mathcal{H}}}{\lambda_k} \simeq \frac{k}{a\mathcal{H}} < 1 \tag{C.5}$$

this mode cannot be affected by causal processes and is thus said to be outside the horizon.

On large scales, matter in a FRW universe can approximately be described as a perfect fluid (in order to sustain homogeneity and isotropy) with energy-momentum tensor

$$T_{\mu\nu} = (\rho + p)U_\mu U_\nu - p\, g_{\mu\nu}\ , \tag{C.6}$$

where $\rho$ and $p$ are the energy density and pressure in the fluid rest frame and $U^\mu$ is the 4-velocity of the fluid. Different types of matter are described via their equation of state parameter $\omega$, which is defined as

$$\omega \equiv p/\rho\ . \tag{C.7}$$

For pressure-less dust, radiation and a constant vacuum energy we have, respectively,

$$\omega_{\text{dust}} = 0\,, \tag{C.8}$$
$$\omega_{\text{rad}} = 1/3\,, \tag{C.9}$$
$$\omega_{\text{vac}} = -1\ . \tag{C.10}$$

The dynamics of the scale factor $a(t)$ in the presence of matter described by a perfect fluid as discussed above is governed by Einstein's Equations

$$G_{\mu\nu} - \Lambda\, g_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}\, R\, g_{\mu\nu} - \Lambda\, g_{\mu\nu} = 8\pi G\, T_{\mu\nu}\ , \tag{C.11}$$

where $G$ is Newton's constant, $R_{\mu\nu}$ is the Ricci tensor and $R$ is the Ricci scalar. $G$ is related to the reduced Planck mass $M_{\text{P}}$ via

$$M_{\text{P}} = \big(8\pi G\big)^{-1/2} \approx 2.43 \cdot 10^{18}\,\text{GeV}\,. \tag{C.12}$$

Since a non-vanishing cosmological constant $\Lambda \neq 0$ can always be accounted for by a contribution

$$T^{(\Lambda)}_{\mu\nu} = \frac{\Lambda}{8\pi G}\, g_{\mu\nu} \equiv \rho^{(\Lambda)}\, g_{\mu\nu} \tag{C.13}$$

to the energy-momentum tensor, we include it as a contribution to the matter content of the universe from now on and set the explicit cosmological term in Einstein's Equations to zero.

For $\Lambda = 0$, the 00-component of Einstein's Equations (C.11) yields, using the FRW metric Eqn. (C.1) and the energy-momentum tensor Eqn. (C.6),

$$\mathcal{H}^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}\ . \tag{C.14}$$

This is called the Friedmann equation. From the $ij$-components of Einstein's Equations we arrive at the following equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \; . \tag{C.15}$$

From this equation we find that for a matter or radiation dominated universe, in which $\rho + 3p > 0$ holds, $\ddot{a}$ is always negative, i.e. the expansion of the universe is slowed down by gravity. Also, since $\dot{a} > 0$ today, we conclude that $\dot{a}$ has always been positive, i.e. $a(t)$ was always increasing. Extrapolating backwards in time, we therefore arrive at a point in the history of our universe when $a = 0$ – the Big Bang.

Turning our attention back to the Friedmann equation (C.14), we find that

$$\rho = \rho_{\rm cr} \equiv \frac{3\mathcal{H}^2}{8\pi G} \tag{C.16}$$

implies $k = 0$, i.e. a spatially flat universe. $\rho_{\rm cr}$ is therefore called the critical density. Note, however, that since $\mathcal{H}$ is a time-dependent quantity, so is the critical density.

The density parameter $\Omega$, defined as

$$\Omega \equiv \frac{\rho}{\rho_{\rm cr}} \; , \tag{C.17}$$

measures the energy density of the universe as a fraction of the critical density. With it we can rewrite the Friedmann equation (C.14) as

$$\Omega - 1 = \frac{k}{(a\mathcal{H})^2} \tag{C.18}$$

and we see that $\Omega = 1$ corresponds to a flat universe. In general, $\Omega$ is again a time-dependent quantity. The present value of $\Omega$, denoted by $\Omega_0$, is [6]

$$\Omega_0 = 1.02 \pm 0.02 \; , \tag{C.19}$$

compatible with a flat universe.

| Type of Matter | $\omega = p/\rho$ | $\rho(a)$ | $a(t)$ |
|---|---|---|---|
| Dust | $0$ | $\rho \sim a^{-3}$ | $a \sim t^{2/3}$ |
| Radiation | $1/3$ | $\rho \sim a^{-4}$ | $a \sim t^{1/2}$ |
| Cosm. Constant | $-1$ | $\rho = \text{const}$ | $a \sim e^{\mathcal{H}t}$ |

**Table C.1:** Behaviour of different types of matter in a flat FRW universe.

Taking the time derivative of the Friedman equation (C.14) and using Eqn. (C.15) yields the continuity equation

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} + 3\mathcal{H}(\rho + p) = 0 \,, \tag{C.20}$$

which can also be derived from energy-momentum conservation $\nabla_\mu T^{\mu\nu} = 0$.

This equation can be integrated to obtain the dependence $\rho(a)$ of the energy density on the scale factor for different types of matter with equation of state $p = \omega\rho$. For a flat universe with $k = 0$, $\rho(a)$ can then be plugged into the Friedman equation Eqn. (C.14) to compute the time dependence $a(t)$ of the scale factor. The results are summarised in Table C.1.

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