Three Essays on Commitment and Information Problems

Inaugural-Dissertation
zur Erlangung des Grades
Doctor oeconomiae publicae (Dr. oec. publ.)
an der Ludwig-Maximilians-Universität München

2011

vorgelegt von

Matthias Fahn

Referent:	Prof. Ray Rees
Korreferent:	Prof. Niko Matouschek
Promotionsabschlussberatung:	16. November 2011

Acknowledgements

During the work on this dissertation, I received support from many people to whom I am extremely grateful. First and foremost I want to thank my supervisor and co-author Ray Rees for many inspiring discussions, various reference letters, and for encouraging me to merge the fields of relational contracts and household economics, which finally led to our joint paper. I am also very greatful to Niko Matouschek. He not only agreed to be my second supervisor, but also made my second stay at Northwestern University possible, connected me to the Management and Strategy Department, and repeatedly discussed economics and many other issues with me. Furthermore, I want to thank Carsten Eckel for being my third examiner, and Peter Egger, Valeria Merlo, and Georg Wamser, for the work on our joint paper.

I am grateful to Florian Englmaier who gave me a lot of helpful advice and took me to Konstanz. Furthermore, I very much benefited from being part of the Munich Graduate School of Economics and I would like to thank all professors, administrative staff and colleagues I was allowed to meet. Financial support from the Deutsche Forschungsgemeinschaft is gratefully acknowledged.

I am indebted to Matthias Doepke, who made my first stay at Northwestern University possible. There, I benefited a lot from discussions with many people and am very grateful for that. Especially, I want to thank Jin Li for all the encouragement and time, and Mark Satterthwaite for officially inviting me. Furthermore, I gratefully acknowledge financial support from the DAAD (Deutscher Akademischer Austauschdienst) during my second stay.

Many people gave me advice on specific parts of this dissertation. I am grateful to Asnia Asim, Robert Ulbricht, and Mauricio Varela for helpful comments. Furthermore, the papers benefited from seminar audiences in Munich, Konstanz, the 2009 Annual Conference of the European Association of Law and Economics (Rome), the 2009 Conference of the German Economics Association (Magdeburg), and the 2010 Meeting of the American Law and Economics Association (Princeton).

Very importantly, I want to thank all my friends in Munich, Evanston and everywhere else for always supporting me and just being there. You know who you are.

Finally, I want to thank my parents and family for all their support and patience during the last years.

Contents

Preface

Economic agents often face a lack of commitment power. Even when best intentions to honor promises are present *ex ante*, agents might have incentives to renounce *ex post*. If it is not feasible to pin down all contingencies in a formal contract, cooperation might not be possible. However, long-term relationships can help to solve commitment problems. If partners interact over and over again, they might be able sustain cooperation even if this is costly for them in the short run. Cooperation today is then rewarded by future collaboration, whereas reneging is followed by a termination of the present relationship. Thus, a self-enforcing system of collaboration can be created, with the use of so-called relational contracts. This explains why (costly) cooperation can even be sustained if economic agents only pursue their own interest.

This dissertation presents three chapters, where two of them deal with commitment problems and the impact of different kinds of laws on the ability to sustain cooperation, whereas one chapter considers an information problem.

In the first chapter, the behavior of couples within a household is considered. Agreements there are to a large extent implicit, and partners must trust each other to honor promises. Since punishment in a relationship might assume the form of a separation, divorce laws can affect cooperation within a household. The second chapter further divides the reputational aspects of an ongoing relationship. If a relationship is potentially long-lasting and the current partner might be replaced, it is also important whether a player is trusted by potential new partners. However, if it is not possible to build up an external reputation – for example due to a lack of market transparency – cooperation within a relationship can only be enforced if sufficient turnover costs are present. This, however, leads to efficiency losses, which can be mitigated by a minimum wage. The third chapter addresses information problem that a multinational enterprise faces if it wants to explore new investment opportunities. Before market entry, it is uncertain whether it will be successful there, and learns about that after entry. Since conditions in different market are correlated, the firm can also use information gathered in one market to learn about the terms in other markets. This correlated learning can explain why firms pursue a gradual expansion strategy over time rather than exploring all profitable opportunities simultaneously.

All three chapters capture different kinds of problems and can be read independently of each other.

The first chapter, "Household Relational Contracts for Marriage, Fertility, and Divorce" (joint with Ray Rees) derives conditions for cooperation within a relationship when both partners are solely driven by their self-interest and not able to make formally binding commitments. Thus, all promises must be self enforcing, i.e., part of an equilibrium of the dynamic game. More precisely, we analyze the decision of a couple whether to get married, how many children to have, and whether to remain together or break up. Generally, children not only provide utility, but are also associated with costs. One partner (denoted the secondary earner) will usually stay at home for some time to raise the children. Thus, she faces opportunity costs in the form of current income losses as well as future earning reductions – the latter induced by a reduced accumulation of human capital. Therefore, the secondary earner will usually be less inclined to have children than her partner. However, funds within the household can be reallocated in a way to compensate the secondary earner for the associated opportunity costs. Since it is not possible to write a formal contract, all related promises have to be self-enforcing and part of equilibrium strategies. Cooperative behavior is only individually rational if reneging is followed by sufficient and credible punishment. Since the outside option in a relationship is to a large extent captured by separation, spouses use the possibility to leave as a punishment threat. Then, payoffs in equilibrium as well as off equilibrium determine the feasible amount of cooperation. Furthermore, we explore the impact of several divorce laws on the enforceability of transfers, and thus on fertility. Divorce costs, for example, generally have a positive impact on fertility by increasing relationship stability and decreasing players' reservation utilities. Thus, a marriage can serve as a commitment device to enforce cooperation within a relationship. However, higher divorce costs do not necessarily increase welfare. Making a divorce more difficult also induces couples to stay together when their match quality has become relatively bad and they would rather prefer to break up (absent divorce costs). If the gains from an increased commitment are lower than this welfare loss, a couple might abstain from getting married ex ante and rather choose to cohabit.

The second chapter, "Minimum Wages and Relational Contracts", develops a tractable model that shows that if agents must be motived to exert effort, various – empirically observed – consequences of a minimum wage can be explained. Furthermore, if relational contracts, i.e., contracts based on observable but non-verifiable measures, are used and agents can be replaced, an appropriate minimum wage increases the total surplus created within an employment relationship. The driving factor behind these results is a firm's optimal choice of incentives. If firms are forced to pay a higher wage than actually intended, they will also require their employees to work harder. More precisely, a labor market with many homogenous firms and employees exists, with more employees than firms. The market is frictionless, and no (exogenous) turnover costs exist, why it is always possible for a firm to costlessly replace an agent. Furthermore, the market is not fully transparent in a sense that if turnover occurs, it is not possible to detect the reason, i.e., if an agent is fired or leaves voluntarily. Thus, a firm cannot build up an external – or market – reputation for honoring its promises. This creates a commitment problem: Instead of making promised payments as a reward for previous effort, firms might have an incentive to renounce and replace employees. Therefore, the only way to induce agents to work is the existence of endogenous turnover costs. However, firms are also exposed to these turnover costs whenever their employees leave for exogenous reasons. Although they have all bargaining power, firms are thus not able to capture the whole surplus of an employment relationship. Then, they face a tradeoff between giving high incentives (induced by high wages) and reducing turnover costs (which also increase with equilibrium wages). Even if maximum incentives are possible, employers voluntarily decrease them and enforce an effort level which is inefficiently low. Forcing firms to pay a minimum wage will make it optimal for firms to let agents work harder, inducing a surplus increase. To capture employment effects as well, the model is extended accordingly. In one specification it is assumed that profits are positive. Furthermore, a firm can employ many agents. Then, employment is chosen efficiently for a given level of equilibrium effort. However, since firms voluntarily decrease incentives to reduce endogenous turnover costs, effort and consequently also employment will be inefficiently low. By increasing effort, a minimum wage thus also induces a firm to employ more agents than before.

The third chapter, "On the Genesis of Multinational Networks" (joint with Peter Egger, Valeria Merlo, and Georg Wamser) deals with information problems. Specifically, this part explores how multinational enterprises (MNEs) develop their network of foreign affiliates. It is commonly observed that MNEs tend to pursue a gradual expansion strategy of their network of foreign affiliates over time rather than exploring all profitable opportunities simultaneously. They typically establish themselves in their home countries and then enter new foreign markets step by step. We propose a model where MNEs face uncertainty concerning their success in new markets and learn about that after entry. Conditions in different markets are not independent, and the information gathered in one country can also be used to learn about conditions in other, in particular, similar countries. This so-called correlated learning can explain why firms expand step by step: market entry is associated with considerable costs, and sequential investments help to economize on these costs by reducing uncertainty. The learning model developed in this paper serves to derive a number of testable hypotheses regarding market entry in general and simultaneous versus sequential market entry in specific. These hypotheses are assessed in a data-set of the universe of German MNEs and their foreign affiliates. The results provide empirical evidence for correlated learning as a main driver behind international expansion strategies.

Chapter 1

Household Relational Contracts for Marriage, Fertility, and Divorce[1]

1.1 Introduction

Countries worldwide have observed a substantial decline in fertility rates over the past 50 years.[2] Besides various other aspects,[3] the strong increase in female labour force participation is very likely to be a driving factor of this development (Michael, 1985, Ahn and Miro, 2002). While women still have to take major parts of the responsibilities of raising children, female education has considerably improved over time and reached or even exceeded male levels in many countries. This has substantially increased women's opportunity costs for having children. Current income losses when staying at home have become higher, as well as future earning reductions – the latter induced by a reduced accumulation of human capital. In many discussions on how to deal with low fertility, there is a large focus on how to reduce these costs by reconciling work and family life – for example with the help of child-care facilities or part-time occupations. However, spouses can also compensate each other for their income losses by means of an adequate allocation of funds within the household. No *ex ante* commitment to any such allocation is possible,

[1]This chapter is joint work with Ray Rees.

[2]For example, total fertility rates (live births per woman) in Europe have gone down from 2.56 (1960-1965) to 1.53 (2005-2010). During the same time period, the decrease in the US was from 3.31 to 2.07 and in Canada from 3.68 to 1.65. Source: United Nations Department of Economic and Social Affairs (2011).

[3]For example the birth control pill, an easier access to abortion, or the decline in infant mortality (see Doepke, 2005).

though, and there is always the possibility that a partner breaks a given promise.

This paper derives conditions for cooperation within a relationship. Funds within the household are reallocated in a way to compensate the partner responsible for raising the couple's joint children for the associated opportunity costs. However, both partners are solely driven by their self-interest and furthermore not able to make formally binding commitments. Thus, the corresponding transfers must be self enforcing, i.e., part of an equilibrium of the dynamic game. An important aspect of feasible cooperation are the implications of a divorce. After a divorce, implicit agreements are replaced by formal rules, and having separation as an option – as a possibility happening in equilibrium or as a threat to punish a partner for not cooperating – is important for the enforceability of a specific within-household allocation. Generally, more restrictive divorce laws increase commitment and the scope for cooperation. Therefore, they consequently raise fertility within a marriage. On the other hand, if the match quality has become low and partners would prefer to break up, complications associated with a divorce decrease welfare. Couples contemplating a marriage thus face a trade-off when a potential later separation is more difficult. Increased commitment allows for more redistribution, whereas ending a bad match becomes more difficult.

More precisely, this paper analyzes the interrelationships among decisions on whether to marry or cohabit, how many children to have, and whether to stay in the relationship or not. The underlying model consists of two risk-neutral players, the primary (he) and the secondary earner (she), who form a – potentially – long-lasting relationship. First, they decide whether to marry or cohabit and then how many children they want to have. When children are present, the secondary earner abstains from work for some time. This causes current income losses as well as a reduced accumulation of human capital, thus inducing lower future wages. In later periods, the couple decides whether to remain together or to separate. Such a decision will also be affected by factors that are usually not part of an economic model, like love and caring for the present as well as a potential new partner. To capture these aspects, we introduce a stochastic utility component that is added to players' payoffs once the broke up. Realizations of these outside utilities are drawn at the beginning of every period and revealed to the spouses. Then, remaining together is efficient if the sum of players' payoff levels within the relationship is higher than outside.[4] However, reallocations of funds can also be needed to maintain an efficient marriage. One partner might prefer a separation, while the other might want to stay together. Then, the former has to be sufficiently compensated. Such a transfer might or might not always be enforceable, depending on the underlying assumptions concerning the timing of payments. We analyze both cases, namely that the couple only breaks up if it is efficient, and the situation that also inefficient separations can occur.

Since she bears the associated opportunity costs, the secondary earner will usually be less inclined to have children than her partner. However, this decision has to be made unanimously, why individual utility maximization will lead to an inefficiently low fertility level. Then, a Pareto improvement is possible. The secondary earner might agree to having more children in exchange for a more preferable allocation of funds within the households. More precisely, if utilities from staying together are higher than those from breaking up, the resulting surplus can be redistributed and used to give incentives for cooperation.

However, it is not possible to formally commit to a certain contingent allocation of this surplus *ex ante*. Therefore, all related promises have to be self-enforcing and part of equilibrium strategies.[5] Cooperative behavior is only individually rational if reneging is followed by sufficient and credible punishment. Formally, the maximum feasible amount of cooperation can be pinned down into one enforceability constraint which states that the total future equilibrium surplus (net of reservation utilities, i.e., their payoffs absent any cooperation) must be higher than the reneging temptations today. Whether the respective constraint binds (and then reduces feasible cooperation) depends on the relationship stability in equilibrium, the “steepness” of the functions determining the secondary earner’s opportunity costs of having children, and the severity of punishment. Since the outside option in a relationship is to a large extent captured by separation, spouses use the possibility to leave as a punishment threat. However, low separation payoffs alone are not sufficient to foster cooperation, primarily because separation must actually be a punishment and the respective threat has to be credible. Concerning the first, as established earlier, the couple might also break up in equilibrium. If this is very

[4]The common assumption that the couple receives utility just by being together is captured by making the values of these outside utilities net of any internal "relationship utilities".

[5]Although bargining models might implicitly assume a dynamic setting to support efficient decisions, they do not make precise the conditions necessary for cooperation.

likely to happen soon anyway, payoff differences between on- and off-equilibrium outcomes are small. Thus, a separation only is a severe punishment if the relationship is relatively stable in equilibrium.[6] Furthermore, a separation must be credible in a sense that it has be optimal for players to actually terminate the relationship absent any cooperation. If a separation off equilibrium is very unattractive for both players, it will probably take some time until sufficiently high values of the stochastic outside utilities are realized and the punishment is actually carried out. Then, the scope for cooperation in equilibrium is reduced. However, we take the possibility into account that the value of the relationship will automatically go down after a spouse reneges on their promises. Consequently, if the relationship is sufficiently unpleasant, a separation will always occur immediately after a deviation, and low separation payoffs always help to sustain cooperation.

After identifying the Pareto-efficient equilibrium, we analyze the situation following a divorce in more detail. While agreements during a relationship are to a large extent implicit, this changes after a divorce. When all goodwill is lost, issues like financial support or access to children are mainly governed by law. Thus, we take an institutional perspective and analyze the impact of different policy changes on fertility, marriage stability, and the propensity to get married (versus cohabiting). In doing so, we want to contribute to the discussion concerning low birth rates in many countries. We argue that – besides reasons that deal with the above mentioned increased opportunity costs – a change in the enforceability of transfers induced by legal amendments can be an important factor determing fertility levels.

A major part of this analysis deals with divorce costs. Our model predicts that divorce costs generally have a positive impact on fertility by increasing relationship stability and decreasing players' reservation utilities. Only if they make an off-equilibrium divorce threat too uncredible, they might create adverse effects. Thus, a marriage can serve as a commitment device to enforce cooperation within a relationship. This idea has already been discussed by Becker (1991) and Rawthorn (1999), formally derived by Matouschek and Rasul (2008), and empirically tested by various authors (see Rasul, 2003, Stevenson, 2007, Matouschek and Rasul, 2008, or Bellido and Marcen, 2011). The latter use the move

[6]The result that a high probability of getting divorced reduces cooperation within a relationship goes back to Lommerud (1989), who assumes that cooperation is driven by "voice enforcement" rather then players' self interest within a repeated setup. It is supported empirically by Lundberg and Rose (1999), where a higher divorce risk is associated with lower levels of specialization.

from mutual-consent to unilateral divorce regimes that occured in US and Europe in the second half of the 20th century.[7] Assuming that a separation is easier and thus less costly (also taking non-monetary aspects into account) under unilateral divorce, they present evidence that lower divorce costs imply lower fertility levels. However, higher divorce costs do not necessarily increase welfare in our model. Making a divorce more difficult also induces couples to stay together when their match quality has become relatively bad and they would rather prefer to break up (absent divorce costs). If the gains from an increased commitment are lower than this welfare loss, a couple might abstain from getting married ex ante and rather choose to cohabit. Putting it differently, higher divorce costs will increase fertility given a couple is married. If, however, they induce the partners to cohabit (where separation costs are substantially lower), higher divorce costs might ultimately decrease fertility. The claim that total welfare might not necessarily increase in divorce costs is in line with the empirical results presented by Alesina and Giuliano (2007) who – different from Rasul (2003) or Matouschek and Rasul (2008) – find that unilateral divorce does not imply a decrease but rather an increase in the number of marriages. Concerning fertility, Alesina and Guiliano (2007) also find that in wedlock fertility basically remains unaffected by the adoption of unilateral divorce laws, while out of wedlock fertility decreases significantly and fertility rates for newly wedded couples go up. This supports our view that the impact of divorce costs on marriage and total fertility is not as obvious as it might seem and captures more aspects than just an increased degree of commitment.

In addition to divorce costs, we analyze the impact of wealth division rules (more precisely, we consider alimony payments that are solely based on income differences – as players are risk neutral, however, there is no accumulation of wealth in addition to human capital). Although having no (direct) impact on relationship stability, they can help to increase fertility. Since raising children is associated with a decrease in future income for one spouse, alimony payments can serve as an insurance against this human capital loss. Both effects together – no direct impact on relationship stability in equilibrium combined with an increased slackness of the enforceability constraint – increase the relative benefits of being married compared to cohabiting for higher alimony payments.

[7] Whereas a mutual-consent divorce regime either requires both partners to agree on a divorce or alternatively a proof of one partner's misbehaving, just one partner is sufficient to induce a separation under unilateral divorce.

Finally, we allow for a reduction of the primary earner's access to his children following a separation. By fostering marriage stability, reducing reservation utility and therefore increasing the punishment following no cooperation, such restrictions can help to increase fertility as well. However, their impact on a couple's propensity to marry should be limited, since the primary earner's access to his children is usually not only reduced after a divorce but also after breaking up a cohabitation.

Related Literature

This paper relates to the vast literature on (theoretical) household economics, where a large part of this literature is underlied by the assumption that family members act cooperatively and necessarily achieve Pareto efficient allocations.[8] For example, the Nash bargaining models of household behavior originating with Manser and Brown (1980) and McElroy and Horney (1981) assume that household allocations are Pareto efficient and can somehow be enforced as binding agreements even in a one-shot game. Also the collective models of the houshold (for instance, see Browning and Chiappori, 1998) assume that the household maximizes a weighted sum of individual utilities. Using cooperative game theory, all previous papers consider individual utilities but take cooperative behavior as given. Early challenges to this assumption were made by Ulph (1988), Woolley (1988) and, within the Nash bargaining framework, by Ott (1992), Konrad and Lommerud (1995), and Lundberg and Pollak (2003), among others. Applying non-cooperative game theory to household decision making in a static environment, they identify sources of inefficient behavior of household members.

Instead of *assuming* either cooperative or non-cooperative behavior, we derive conditions for cooperation in a dynamic setup where players are solely driven by their self-interest and not able to write exogenously enforceable agreements. Thus, the present paper directly relates to the theory of relational contracts, which provides an appropriate tool to gain new insights into decision-making within households. Relational contracts are dynamic games based on actions or outcomes that are observable but not verifiable, i.e., the associated contracts are not legally enforceable. As agreements in household relationships are to a large extent implicit and extend over quite long periods of time, they

[8]See Apps and Rees (2009), Chapter 3 for an extensive survey and list of references.

present a good subject for an analysis with a relational contracts model. Starting with Bull (1987), relational contract were initially developed to analyze labour markets and agency situations. MacLeod/Malcomson (1989) provide a complete analysis for perfect information, while Levin (2003) explores the case of imperfect public monitoring. Furthermore, Baker, Gibbons, and Murphy (1994) or Schmidt and Schnitzer (1995) study the interaction between relational and formal contracts. This is linked to our model since after a divorce, implicit agreements between partners are basically replaced by formal arrangements.

Most closely related to the present analysis is the aforementioned paper by Matouschek and Rasul (2008). They develop a model where ongoing cooperation within the household creates an exogenously given benefit and has to be enforced by sufficient punishment threats. Divorce costs serve as a commitment device and thus increase cooperation. Our paper differs by making precise a couple's fertility decision – which only happens in relatively early stages of a relationship – and the consequences of having children, i.e., the associated opportunity costs.

1.2 The Model

Two individuals decide whether to form a household and, if so, whether to marry or cohabit. In each case a household consists of a primary ("he") and a secondary earner ("she")[9], denoted $i = 1, 2$. The time horizone is infinite, $t = 0, 1, 2, \ldots$, and players discount the future with the factor $\delta \in (0, 1)$[10].

In the first period of the game, i.e., in $t = 0$, the couple has $n \geq 0$ children (how n is determined is analyzed below). For convenience, we assume n is a real number. This makes the secondary earner spend a share $g(n)$ of her total time allocation (normalized to 1) for raising children in $t = 0$, with $g(0) = 0$, $g' > 0$, $g'' \geq 0$ and $g(n) \leq 1$ for relevant fertility levels. The remaining time she is working and earns $(1 - g(n))\, w_{20}$, where w_{20} is her wage in period $t = 0$. In all future periods, the secondary earner supplies her total time allocation to the labour market, earning $w_{2t}(n) \geq 0$. Because of work-related human

[9] The pronouns reflect the fact that 70-90% of secondary earners in North America and Europe are female. See Immervoll et al., 2009, Table 1, for country specific numbers.

[10] In addition to discounting, $1 - \delta$ can also represent an exogenous probability of separation.

capital acquisition, the wage is an increasing concave function of her period 0 labor supply and therefore a decreasing convex function of fertility, i.e., $w'_{2t} < 0$ and $w''_{2t} \geq 0$.[11] It is assumed that human capital accumulation only occurs in the first period[12], and the wage $w_{2t}(n) \equiv \overline{w}_2(n)$ is constant for $t \geq 1$.

The primary earner works full time in every period and earns w_{1t} in period t. As his human capital accumulation is of no interest to our analysis, w_{1t} is constant over time and equals $\overline{w}_1$. Furthermore, we assume that $\overline{w}_1 > \overline{w}_2(n) \geq w_{20}$.

Per period utility functions if the household is formed are $u_{it} = x_{it} + \varphi_i(n)$, with $\varphi_i(0) = 0$, $\varphi'_i > 0$ and $\varphi''_i < 0$ for $i = 1, 2$, where x is a private consumption good. The individual consumptions are defined by:

$$x_{1t} = \overline{w}_1 - p_t \quad t = 0, 1, ... \tag{1.1}$$

$$x_{20} = w_{20}[1 - g(n)] + p_0 \tag{1.2}$$

$$x_{2t} = \overline{w}_2(n) + p_t \quad t = 1, 2, ... \tag{1.3}$$

where $p_t \gtreqless 0$ is a payment made from one partner to the other. If $p_t > 0$, the primary earner makes the payment, whereas $p_t < 0$ implies a transfer from the secondary to the primary earner. The payment p_t need not be explicit - its value is *implied by any choice* of n *and the* x_{it}, given $\overline{w}_1, w_{20}$ and $\overline{w}_2(n)$. For analytical purposes however it is useful to treat this *as if* it were an explicit payment. On the other hand, no explicit contract on the p_t is feasible, it has to be part of an equilibrium supported by the household relational contract (HRC), defined below.

In periods $t = 1, 2, ...$, the couple makes the decision whether to separate or remain together. A separation has the following consequences

- Each receives an exogenously given outside net utility $\tilde{v}_i$ in every period which reflects possibilities outside the relationship, such as potential new partners, as well as

[11] Note that the reduced wage can also reflect difficulties of re-entering the labor market after abstaining from it for some time.

[12] This assumption has no qualitative impact on our results.

those within, like love or caring for the existing partner. The common assumption that the couple receives utility just by being together is captured by making the values of these outside utilities net of any internal "relationship utilities". Thus, outside utilities also determine the match quality of the couple in a given period. $\tilde{v}_i$ is a random variable, drawn in each period with distribution $F_i(\tilde{v}_i)$ and continuous density $f_i(\tilde{v}_i)$, strictly positive everywhere on the support $[v_i^0, v_i^1]$. Furthermore, we assume $v_i^0 \leq 0 < v_i^1$ and denote the unconditional expectation $\mathrm{E}[\tilde{v}_i] \equiv \overline{v}_i$. For now, we do not impose further restrictions on the distributions. We will implicitly do so later to have second order conditions satisfied. Note that $\tilde{v}_1$ is independently distributed from $\tilde{v}_2$ (and vice versa) and that the $\tilde{v}_i$ are independently and identically distributed over time. We also assume that both outside utility realizations, v_1 and v_2, are observed by each partner.[13]

After a player broke a promise (what this means will be made precise below), the general quality of living together can be negatively affected. Thus, the reneging partner's outside utility is increasing by the amount Δv_i in every subsequent period.

- The utility derived from children by the primary earner after a separation is $\theta\varphi_1(n)$, $\theta \in [0, 1]$. Here, we want to allow for differences in legislation determining the access of the primary earner to his children, given the assumption that custody is granted to the second earner.[14]

If the couple had chosen to marry, as opposed to cohabiting, a separation is a divorce and has two further effects:

- The partners bear possibly unequal divorce costs $k_i > 0$.
- The secondary earner receives a monetary transfer $\phi\{w_{1t} - w_{2t}(n)\}$ from 1. We will refer to this transfer as a wealth division rule or alimony payments. Since we only consider risk neutral actors and savings are irrelevant, both terms mean the same in our setup (especially as the only wealth players accumulate is human capital).

[13]The assumption that spouses know their partners' outside option fairly well is supported by Peters (1986)

[14]Note, we do not take account of any perceived disutility to the children arising from divorce. This could be treated as a factor, say $\rho_i \in (0, 1]$, applied to *both* utilities. Nothing in the following discussion would change qualitatively as a result, as long as the value of ρ for the secondary earner was not so much smaller than that for the primary earner as to outweigh the effects of θ as analyzed here.

> Although the transfer does not directly depend on the number of children, n enters via its impact on 2's wages. The factor ϕ is assumed to be known ex ante and is determined by divorce law. Note that we are assuming that this law takes into account the effects of the second earner's withdrawal from the labour market on her human capital, in assessing the value of the alimony payment. Then, ϕ measures the weight given to this effect.

The values for k_i, θ and ϕ remain constant over time. The separation decision is irreversible, and we assume that a divorced couple is never getting together again. After a separation, no voluntary transfer is made anymore. We assume that then, all trust between (former) spouses is lost, implying that transfers can not be self enforcing (i.e., part of an equilibrium – this concept is further specified below) anymore.

As a result of this, the separation utilities in periods $t = 1, 2, ..$ of the partners are (as all parameters or expectations are constant over time, the time subscript is omitted), if married

$$\tilde{U}_1(v_1) = \frac{1}{1-\delta}\left(\overline{w}_1 + \theta\varphi_1(n) - \phi[\overline{w}_1 - \overline{w}_2(n)]\right) - k_1 + v_1 + \frac{\delta}{1-\delta}\overline{v}_1 \tag{1.4}$$

and

$$\tilde{U}_2(v_2) = \frac{1}{1-\delta}\left(\overline{w}_2(n) + \varphi_2(n) + \phi[\overline{w}_1 - \overline{w}_2(n)]\right) - k_2 + v_2 + \frac{\delta}{1-\delta}\overline{v}_2 \tag{1.5}$$

Furthermore, we denote the expectation of separation utilities $\mathrm{E}[\tilde{U}_i(\tilde{v}_i)] = \tilde{U}_1(\overline{v}_i) \equiv \tilde{U}_i$.

If the partners are not married, we simply set $k_i = \phi = 0$. Thus we model cohabitation as essentially the decision to avoid divorce costs and dispense with legal regulation of alimony payments and child custody/access arrangements.[15]

To complete the model setup we specify the timing of events within one period. At the beginning of $t = 0$, the couple decides whether to get married or live together in cohabitation.[16] Formally, each player announces a value $m_i \in \{0, 1\}$. $m_i = 1$ indicates that player i wants to get married, whereas $m_i = 0$ implies that the player prefers to cohabit. The spouses marry if and only if both agree, i.e., if $m \equiv m_1 m_2 = 1$. Otherwise, the couple cohabits. Then, they unanimously decide on n, the number of children. If the

[15] As well as the costs or utility of the act of getting married in itself.

[16] The matching process, i.e., how the spouses meet, is beyond the scope of our analysis and taken as exogenously given.

individually desired levels differ, the lower of both is realized.

Afterwards, the primary earner works, while the secondary earner allocates her time between work and raising children, as specified above. After players received their income, a transfer p_0 is made.

At the beginning of each subsequent period and if the couple is still together, both observe the realizations of this period's outside utilities, $v_{1,t}$ and $v_{2,t}$. Taking these into account, the spouses then decide whether to remain together or not. Formally, players announce $d_{it} \in \{0,1\}$, where $d_{it} = 1$ indicates that player i wants to remain together for period t. If at least one of them chooses $d_{it} = 0$, they irrevocably break up.[17] The variable $d_t \in \{0,1\}$ indicates whether the relationship is still active in period t. It is defined recursively by $d_t = d_{t-1}d_{1t}d_{2t}$, with $d_0 = 1$. Once they separate, spouses receive their previously specified separation utilities $\tilde{U}_i$. Otherwise, both work and receive their income, followed by the transfer p_t. We do not impose any exogenous bound on the transfer levels. This implicitly assumes that players can save or borrow, an issue not modelled here due to the lack of additional insight when players are risk neutral.

Note that we abstract from monetary aspects of having children. If we included such costs and assumed a given allocation among partners, our results would not be affected qualitatively. The same would be true for laws supporting parents financially. Furthermore, our alimony payments or wealth division rules do not take the utility of children into account and are only supposed to compensate the secondary earner for her human capital loss. Thus, we do not consider child support laws. These are beyond the scope of our analysis, especially as one problem associated with them is that fathers often do not pay despite the existence of a legal title (see Allen and Brinig, 2010).

Our assumptions with regard to $g(n)$, i.e., the time needed to raise children and the fact that only the secondary earner participates, require some attention. If child-care facilities were available, $g(n)$ and the associated human capital loss could be reduced. However, this would not affect our results, as long as a substantial amount of time still has to be spent by parents. Since especially newborns can not be given away immediately after birth, $g(n)$ must be positive even in the presence of child-care facilities. Another aspect not covered in our model is that parents actually derive utility from *spending* time

[17] If they are married, we thus assume a unilateral divorce regime.

with their children (instead of just having them). Thus, parents might decide to stay at home for a longer time than is actually necessary.

The assumption that only the secondary earner stays at home is quite restrictive and causes inefficiently low fertility levels, generating the need for redistribution within the relationship. This will become clear below, just note that if the partners were able to commit to an arbitrary allocation of $g(n)$, they could choose it in a way to obtain efficient fertility. Taking a certain allocation of $g(n)$ among players as exogenously given (which is the crucial aspect here; the assumption that only the secondary earner partially abstains from the labour market is just made for convience) thus has to be justified by issues outside our model. We argue that this restriction is sensible and that especially cultural reasons often prevent fathers from assuming their share of responsibilities “efficiently”.[18]

For example, many (especially Western) countries regard it as a problem that women still are mainly responsible for raising children – especially since men are not better educated anymore. The human capital loss induced by women’s difficulties to re-enter the labor market after a pregnancy is seen as one the mean reasons for low fertility rates. Many suggestions have been made how these responsibilities could be distributed more equally. As an example, Sweden and Germany offer financial assistance to men staying at home for some time with their newborns (these programmes are not gender-specific – yet, participation of men is either a prerequesite or extends their duration).

Furthermore, many jobs are designed in ways that doing them part-time is not possible. Then, the couple is just not able to share the time needed to take care of children more equally and has to make a decision which of both partners completely stays at home. Finally, ex-ante promises made by the primary earner to stay at home for some time might not be credible. Taking this into account, the secondary earner will be more reluctant to have children.

[18] One has to be careful using the term efficient here, as many aspect beyond a purely economic view may enter.

1.3 Household Relational Contracts

1.3.1 The game

Players have to decide on whether to form a household, and if they do so the legal form of their relationship, the number of children they want to have, payments p_t, and whether they separate or stay together in later periods. We assume that - while together - they formulate a Household Relational Contract (HRC) which is a subgame perfect equilibrium of the game and specifies all actions players will take conditional on all possible histories. However, this can not be a legally binding contract contingent on actions or outcomes, because of the non-verifiability of the payments p_t.[19]

Let us briefly give a formal characterization of actions, strategies and conditions for a subgame perfect equilibrium, i.e., a Household Relational Contract. There, instead of just referring to the net transfer p_t, we split it into the individual contributions of players 1 and 2, with $p_t = p_{1,t} + p_{2,t}$, where $p_{1,t} \geq 0$ and $p_{2,t} \leq 0$. Obviously, only the net transfer p_t is relevant and thus used in all other sections. However, splitting it into two components simplifies a characterization of strategies.

Furthermore, the number of children is realized as follows. Each player announces their preferred preferred fertility level n_i, $i = 1, 2$. Since this decision has to be made unanimously, we assume for convenience that realized fertility n is the smaller of both players' announcements, i.e., $n = \min\{n_1, n_2\}$.

Then, the history h_t specifies all events that occur at time t. For $t = 0$, we have

$h_0 = \{m_1, m_2, n_1, n_2, d_{1,0}, d_{2,0}, p_{1,0}, p_{2,0}\}$ (note that $d_{1,0} = d_{2,0} = d_0 = 1$ by assumption). For all $t \geq 1$, the history is $h_t = \{d_{1,t}, d_{2,t}, p_{1,t}, p_{2,t}\}$. Then, $h^t = \{h_\tau\}_{\tau=0}^{t-1}$ is the history path at beginning of period t, with $h^0 = \emptyset$. $H^t = \{h^t\}$ characterizes the set of history paths until time t, while $H = \cup_t H^t$ is the set of histories.

A strategy σ_i for player i, $i = 1, 2$, is a sequence of functions $M_i \cup N_i \cup \{P_{i,t}, D_{i,t}\}_{t=0}^{\infty}$, where $M_i : H^0 \to \{0, 1\}$ specifies whether the couple gets married (if $m = m_1 m_2 = 1$) or cohabits ($m = 0$). $N_i : H^0 \cup \{m_1, m_2\} \to [0, \infty)$ describes the process determining

[19] This is supported by the argument that individual consumptions within a household cannot be verifiably measured. In reality of course there is a far richer set of reasons for the impossibility of complete marital contracts than this.

fertility at the beginning of period $t = 0$, and $n = \min\{n_1, n_2\}$ is the realized fertility level. $D_{i,t} : H^t \cup \{v_{1,t}, v_{2,t}\} \rightarrow \{0, 1\}$, $t \geq 1$, characterizes players' decisions whether they want to remain together ($d_{i,t} = 1$) or separate, with $d_t = d_{t-1}d_{1,t}d_{2,t}$ and $d_0 = 1$. Finally, transfers are determined by $P_{1,t} : H^t \cup \{v_{1,t}, v_{2,t}, d_{1,t}, d_{2,t}\} \rightarrow [0, \infty)$ and $P_{2,t} : H^t \cup \{v_{1,t}, v_{2,t}, d_{1,t}, d_{2,t}\} \rightarrow (-\infty, 0]$ for periods $t \geq 1$, where the net transfer p_t equals $p_{1,t} + p_{2,t}$. In $t = 0$, the functions are $P_{1,0} : H^0 \cup \{m_1, m_2, n_1, n_2\} \rightarrow [0, \infty)$ and $P_{2,0} : H^0 \cup \{m_1, m_2, n_1, n_2\} \rightarrow (-\infty, 0]$.

Denoting a player's payoffs following history h_t $U_i(\sigma_1, \sigma_2 \mid h_t)$, a strategy profile (σ_1, σ_2) is a subgame perfect equilibrium if and only if following any history h_t,

$$\sigma_1 \in \underset{\tilde{\sigma}_1}{\operatorname{argmax}}\, U_1(\tilde{\sigma}_1, \sigma_2 \mid h_t)$$

$$\sigma_2 \in \underset{\tilde{\sigma}_2}{\operatorname{argmax}}\, U_1(\sigma_1, \tilde{\sigma}_2 \mid h_t)$$

1.3.2 Fertility, Transfers, and Constraints

The spouses will use the payments p_t as an incentive tool to either raise fertility or maintain the relationship.[20] This increases efficiency as – absent any transfers – fertility is too low and separation probabilities are too high. Note that we refer to efficiency as the outcome players would choose if they were able to fully commit.

Too low fertility is induced by the exogenously given distribution of the costs and benefits of having children, i.e., the fact that only the secondary earner (partially) refrains from the labor market to raise them in period $t = 0$. Thus - and since the decision about n has to be made unanimously - it is very likely that the individually optimal levels of n differ between spouses. Then, gains from cooperation exist which the partners can try to exploit. The partner bearing relatively higher costs (in relation to the benefits) might be willing to agree on having more children than individually optimal if compensation is (credibly) promised.

Inefficient separation has to be prevented if staying together is efficient, i.e, outside utility realizations (v_1, v_2) in a period are low enough that the sum of utility streams when remaining together is higher than $\sum_i \tilde{U}(v_i)$, but if one player would find it individually

[20]Transfers always can always contain a purely redistributive component as well.

optimal to split. Then, a transfer exists that makes it optimal for both to remain together.

However, transfers to increase efficiency by giving incentives to raise fertility and prevent inefficient divorce must be self-enforcing, i.e., part of a subgame perfect equilibrium as specified above. For example, assume a transfer is supposed to be positive. Then, it must be in the interest of the primary earner to actually make it *ex post*, i.e., after the secondary earner has delivered, either by agreeing on a higher fertility level or abstaining from inducing a separation. Thus, although he might be willing to make that transfer *ex ante*, the limits of commitment - recall that no explicit contingent contracts are feasible - might make him renounce this promise ex post. As this is anticipated by the secondary earner, her ex-ante willingness to cooperate would be limited by his credibility.

More precisely, a payment will only be made if reneging triggers sufficient punishment. We use the standard dynamic games/relational contracts approach[21] and assume that after someone reneged, the relationship has become unpleasant, and any trust between the partners is lost. Thus, the harshest possible punishment is used (Abreu, 1986), implying that the equilibrium with the lowest payoff for the player that reneged (pushing that player down to their reservation utility) is played. As the only decision players can make in periods $t \geq 1$ determines whether they want to remain together, punishment here must take the form of a separation.[22] Still, this punishment threat has to be credible. Assume that a player did not act as intended and is supposed to be punished by a separation. If staying together is in the interest of both in the following period, the punishment will be postponed until a sufficiently high draw of one of the outside utilities is realized.[23] Furthermore, a separation only effectively penalizes a player if it does not occur in equilibrium anyway. Thus, transfers to reward cooperation can be enforced more easily if separation is less likely in equilibrium and if the probability that – absent transfers – one partner breaks up is higher. However, both issues might contradict each other to some extent. As an example, take 2's divorce costs k_2. It is straightforward and will be

[21]For example, see MacLeod and Malcomson (1989).

[22]In the household Nash bargaining literature, considerable discussion has taken place over whether separation is too drastic a punishment for failure to disagree, and this has led to models which take as threat points non-cooperative Nash equilibria within an ongoing household. It is said, for example, that one would not threaten divorce over a failure to agree on the colour of a sofa. While we agree with that viewpoint, the class of household decisions being analyzed in this paper is we believe sufficiently fundamental that threats based on separation are the appropriate ones to assume.

[23]Formally, a player can always make sure to receive his/her minmax-payoff. Furthermore, note that as reneging triggers all cooperation to cease, it does not matter whose outside utility is sufficiently high.

shown below that higher divorce costs generally make divorce less likely. But they also increase the probability that the secondary earner is willing to stay within the marriage anyway and does not need a compensation. Therefore, it is not clear whether higher divorce costs k_2 have a positive or negative impact on the enforceability of transfers.

However, a broken promise can also have an impact on the general quality of living together as well. Thus we assume that after a player reneged, the partner's outside utility increases by Δv_i in every period in each state. The size of Δv_i has no qualitative impact on our results, unless it is so large that a punishment is always credible and divorce immediately occurs after a deviation. Both cases will be analyzed separately below. Concluding, a spouse who wants to reward one's partner for cooperation has to make a payment now but is punished in the future for not acting accordingly. Thus, inefficiency can still pertain in equilibrium, if either a punishment cannot be enforced immediately after a deviation and/or if the future is not valuable enough.

While having children is a "discrete" act, the same is not necessarily true for making transfers and inducing a breakup. The period between which payments are feasible could be made arbitrarily small (Wickelgren, 2007, among others, is making this argument), and there are good reasons why an artificial division into fixed periods does not reflect the real life of a couple. Thus, the main part of our paper will impose the assumption that the decision whether to separate or remaining together is always made efficiently in equilibrium. In the Appendix, we show that we approach this outcome arbitrarily close by assuming that time is continuous and each period of a given (discrete) length can be divided into subperiods. If transfers can be made and a separation be induced in each of these subperiods,[24] making the latter arbitrarily small lets the couple getting separated almost only when this is actually efficient. The reason is that any reneging is almost immediately followed by a punishment. Afterwards, we take the initially assumed discrete nature of the game literal and show what happens if inefficient breakups can happen on the equilibrium path.

Before continuing with the formal analysis, we briefly want to discuss a separation always is necessary to punish a devation from cooperative behavior, as it destroys surplus (off equilibrium) and thus is not renegotiation proof. However, it is possible to construct an off-equilibrium outcome that actually is renegotiation proof. Instead of breaking up

[24] Still, the interval between new draws of outside utilities remain fixed.

after a deviation, the couple can continue to play a cooperative equilibrium, but where the reneging player is subsequently pushed down to their reservation utility. This is possible because of one important feature of relational contracts – namely that any surplus distribution can be induced as long as both players at least receive their reservation utilities. However, both approaches give the same equilibrium outcomes, and our focus on a non-renegotiation-proof equilibrium is without loss of generalitiy.

In the following, we derive necessary (and sufficient) conditions to induce allocations that increase efficiency. These results do not depend on how the resulting surplus is shared among players. Assuming a transfer to maintain the relationship can be enforced, actually any surplus distribution is feasible - as long as both players at least receive their reservation utilities. Thus, our objective is to characterize the set of subgame perfect equilibria that are Pareto optimal and maximize the sum of players' utilities.

1.4 Constraints in $t = 0$

In this paragraph, we derive a general condition that specifies to what extent utility transfers in period $t = 0$ are enforceable. If it binds, this condition determines (Pareto optimal) equilibrium fertility. If it does not bind, the efficient fertility level can be attained. Note that all results derived here hold independent of whether we assume that the separation decision is always made efficiently or not.

It will further become clear that only the (promised) allocation of utility streams that matters for players' willingness to cooperate, and using an explicit formulation in form of the transfer p_0 is just a useful tool to obtain that objective. This also implies that players are indifferent between receiving/giving current funds (via p_0) or expected future payoffs as a reward for cooperative behavior - both can be substituted arbitrarily, and we do not have to specifiy how exactly funds are redistributed.

We start with the definition of the relevant payoff streams. Define

$\overline{U}_i^* \equiv i$**'s expected discounted continuation utility *in* equilibrium path in** $t = 1$**,** taking into account both non-divorce and divorce states.[25]

Furthermore, define

$$\overline{U}_i \equiv i\text{'s expected discounted utility } \textit{off} \text{ equilibrium in } t = 1.$$

Note that off-equilibrium or reservation utilities $\overline{U}_i$ do not necessarily coincide with expected separation utilities $\tilde{U}_i$ (defined above) because they might cover states where divorce does not occur. To what extent they differ depends on the credibility of punishment threats. Furthermore, we omit the time subscript. In this section, this is done for convenience (only continuation utilities in period $t = 1$ are relevant). However, also in later periods we do not have to make payoffs depend on calendar time without loss of generality.[26]

At the beginning of period $t = 0$, both spouses unanimously decide on equilibrium fertility n^* and an associated transfer p_0, taking future utility allocations into account (which might be a function of fertility as well).[27] If they fail to reach an agreement, they have n^{**} children and play the non-cooperative equilibrium from then on. n^{**} is defined as $n^{**} = \min\{n_1^{**}, n_2^{**}\}$, where n_i^{**} is player i's individually preferred non-cooperative fertility level, i.e., if $p_t = 0$ for all $t \geq 0$.

Knowledge of n_i^{**} also tells us who needs to be compensated in equilibrium, namely the one with a lower level. We will generally assume that $n_1^{**} > n_2^{**}$, which seems natural as player 2's human capital reduction is substantially reducing her future earnings. Then, the players can agree on the following deal at the beginning of period 0. Player 2 is willing to accept $n^* > n_2^{**}$. After the children are born, she receives a transfer $p_0(n^*)$ at the end of period 0 and/or the promise of higher continuation payoffs in the future. If she insists on any smaller number of children, there will be no transfer in period $t = 0$ as well as in any other subsequent period. Thus, if she insists on a smaller n, she will always choose n^{**}.

The opposite is true for $n_1^{**} < n_2^{**}$. Then, the primary earner needs be compensated

[25]Furthermore, corresponding to the $\overline{U}_i^*$ will be (possibly implicit) side payments p which in general also vary across states

[26]The reason is that current and future payoffs are perfect substitutes, and we can thus focus on stationary contracts.

[27]For a formal description see section 3.1 above.

for agreeing on a higher fertility level, either with the (now negative) transfer $p_0(n^*)$ or a higher expected continuation utility.

Two kinds of conditions have to be satisfied that n^* can actually be part of an equilibrium. First of all, given players believe the transfers are made, it has to be optimal for both to choose n^* rather than any other level. Furthermore, transfers have to be credible, i.e., making them has to give players a higher utility than not making them.

Making n^*optimal for both players giving that promised transfers are made is captured by incentive compatibility (IC) constraints, which are

(IC1)

$$u_{11}(n^*) - p_0(n^*) + \delta\overline{U}_1^*(n^*) \geq u_{11}(n^{**}) + \delta\overline{U}_1^*(n^{**})$$

for player 1, and

(IC2)

$$u_{21}(n^*) + p_0(n^*) + \delta\overline{U}_2^*(n^*) \geq u_{21}(n^{**}) + \delta\overline{U}_2^*(n^{**})$$

for the second player.

There, $u_{i1}(n)$ is player i's period-0 utility and $\overline{U}_i^*(n^*)$ player i's expected discounted equilibrium payoff stream in period 1.

Note that both constraints have to hold for $n_1^{**} > n_2^{**}$ and $n_1^{**} < n_2^{**}$. Which of both cases is actually true only determines whether transfers are negative or positive. In the first case, i.e., when $n_1^{**} > n_2^{**}$, $p_0(n^*)$ is positive. Otherwise, (when $n_1^{**} < n_2^{**}$) $p_0(n^*)$ is negative.[28]

Furthermore, it has to be in the interest of players to make a promised transfer. This is only the case if their utility is higher than otherwise. Thus, reneging requires a punishment. As discussed above, this punishment takes the form of pushing a player down to one‘s reservation utility. The dynamic enforcement (DE) constraints make these arguments precise. If a transfer is positive, the primary earner has to decide whether to make it or renege. He will only keep his promise, if

[28]If continuation utilities alone give sufficient incentives for first-best equilibrium fertility, it would even be possible that we observe $n_1^{**} > n_2^{**}$ and a negative transfer. However, $p_0(n^*)$ then is solely used for redistributive purposes in period $t = 0$ and not to give incentives. We are not further interested in this possibility, as it would imply that constraints in period $t = 0$ do not bind in equilibrium and thus are not relevant.

(DE1)

$$p_0(n^*) \leq \delta[\overline{U}_1^*(n^*) - \overline{U}_1(n^*)]$$

is satisfied. If the payment is supposed to be negative, the secondary earner makes the relevant decision. She will only cooperate if her utility after making it is at least as high as if not, or if

(DE2)

$$-p_0(n) \leq \delta[\overline{U}_2^*(n^*) - \overline{U}_2(n^*)]$$

is satisfied.

Note that the (DE) constraints require players to believe that future equilibrium transfers are made as well.

Combining (IC) and (DE) constraints then gives just one constraint which is both necessary and sufficient for an equilibrium fertility level n^* to be enforceable.

Proposition 1: If $n_1^{**} > n_2^{**}$ a fertility level n^* can be enforced if and only if it satisfies the condition

$$u_{21}(n^*) - u_{21}(n^{**}) + \delta\left[\overline{U}_1^*(n^*) + \overline{U}_2^*(n^*) - \left(\overline{U}_1(n^*) + \overline{U}_2(n^{**})\right)\right] \geq 0 \tag{1.6}$$

.

If $n_1^{**} < n_2^{**}$, the necessary and sufficient condition for equilibrium fertility n^* is

$$u_{11}(n^*) - u_{11}(n^{**}) + \delta\left[\overline{U}_1^*(n^*) + \overline{U}_2^*(n^*) - \left(\overline{U}_1(n^{**}) + \overline{U}_2(n^*)\right)\right] \geq 0 \tag{1.7}$$

The proof for Proposition 1 can be found in the Appendix.

The (IC-DE) constraint states that the gains from deviating today, i.e., $u_{21}(n^{**}) - u_{21}(n^*)$ in (1.6), must not exceed the future surplus, i.e., equilibrium payoffs net of reservation utilities.

The chosen fertility level has a direct impact on the enforceability of transfers, a feature usually not observed in relational contracting models, where the production process tends to be independent across periods. This aspect becomes especially important when inefficient separation occurs in equilibrium, and higher fertility can help to increase relationship stability. Furthermore, although the kind of production process usually used in the literature is independent over time, it still remains identical. This implies a further dimension where our setup differs, as incentives to increase fertility are provided by using the surplus from remaining together.

The fact that satisfying the (IC-DE) constraint is also sufficient for enforcing a fertility level n^* (Proposition 1) allows us to separate surplus distribution from incentive giving. This implies that any surplus distribution that gives players at least their reservation utilities is enforceable. Thus, we can confine our interest to the (constrained) Pareto optimal equilibrium, without having to worry about who gets what.

Denoting players' expected payoffs at the beginning of period $t = 0$ but after the marriage decision has been made U_i, equilibrium fertility n^* solves

$$\underset{n}{\mathrm{Max}}\, U = U_1 + U_2 \tag{1.8}$$

subject to (IC-DE).

Depending on the realizations of outside utilities, the couple might break up in equilibrium. Thus, the enforceability of transfers crucially depends on the perceived relationship stability. If partners are convinced that they will end up getting separated anyway, they are less willing to find ways to cooperate early on. In the following, we will thus further analyze the determinants of relationship stability in and off equilibrium.

1.5 Efficient Separation

In this section, we derive conditions for when breaking up is optimal for the couple. As already pointed out, we also assume that a separation in equilibrium only occurs if it is efficient (i.e., what the partners would choose if they were able to fully commit) during most of our analysis and allow for inefficient separations later.

This assumption somehow neglects the discreteness of the model. As we will see later, taking the discreteness seriously will always induce to situations where remaining together is efficient but not not possible, as the necessary transfer is not enforceable. Yet, it is not obvious why it should not be possible to make the separation decision - as well as corresponding transfers - at any point in time. In Appendix II, we show that if time is continuous and each original period is subdivided into very small subperiods, we can get arbitrarily close to the outcome that the couple breaks up if and only if that is actually efficient.[29] Thus - even when this assumption is imposed - players still act within the framework of a relational contract and not within a bargaining game. If the latter were true and a bargaining structure like in MacLeod and Malcomson (1995) or Shaked and Sutton (1984) would be imposed, the surplus distribution on and off equilibrium would be the same - as well as the decision whether to remain together or break up. Then, no punishment would be feasible, making it impossible to enforce any transfer. However, as the game continues with positive probability after a transfer has been made and since trust in the partner's ongoing willingness to cooperate is necessary to sustain cooperation at any point in time, remaining within the relational contracts framework even with the assumption regarding efficient separations seems sensible. This implies two further issues. If all trust between the players is lost after one reneged, the couple will break up off equilibrium even if remaining together would be optimal. Furthermore, any surplus distribution is feasible.[30]

Take periods $t \geq 1$ (note the couple gets together at the beginning of the period $t = 0$; thus, the first time it can break up is $t = 1$) and assume that the couple is married.[31] Define

$$u_1^0 = \overline{w}_1 + \varphi_1(n) \tag{1.9}$$

$$u_2^0 = \overline{w}_2(n) + \varphi_2(n) \tag{1.10}$$

[29]Note that we do not just want to assume that $\delta \to 1$, although this would yield a related outcome as well. But this could cause problems when relating non-recurring factors - like divorce costs - to ongoing effects. Furthermore, we would have to specify different discount factors between period 0 and subsequent periods.

[30]Again, this would allow us to get an outcome that is renegotiation proof - even off equilibrium, the separation decision could be made efficiently, yet pushing the player who deviated down to his/her reservation utility.

[31]The issue marriage versus cohabitation is considered below.

as the per-period utilities the partners would have within the relationship with $p_t = 0$ for $t \geq 1$ and U_i^0 as the respective infinite discounted payoff streams (taking into account that a divorce might occur in future periods). If $\sum_{i=1}^{2}(U_i^0 - \tilde{U}_i) < 0$, a separation is optimal and will occur.

As all utility components are fixed and constant over time except the realizations of $\tilde{v}_i$, the latter determine whether the couple should break up. More precisely, this is specified by the sum of outside utility realizations, i.e., $v_1 + v_2$, independent of the respective individual values. Thus, define

$$\tilde{v} \equiv \tilde{v}_1 + \tilde{v}_2.$$

$\tilde{v}$ has distribution $F(\tilde{v})$ and continous density $f(\tilde{v})$ (specified below) and is strictly positive everywhere on the support $[v_1^0 + v_2^0, v_1^1 + v_2^1]$.

Lemma 1: Assume the separation decision is made efficiently. Then, a divorce takes place if and only if $\tilde{v} > \hat{v}$, where $\hat{v}$ is defined by

$$\varphi_1(n)(1-\theta) + (1-\delta)(k_1 + k_2) + \delta \int_{v_1^0+v_2^0}^{\hat{v}} f(\tilde{v})(\hat{v} - \tilde{v})d\tilde{v} - \hat{v} = 0 \tag{1.11}$$

Proof:

The assumption that the couple chooses to get separated if and only if $\sum_{i=1}^{2}(U_i^0 - \tilde{U}_i) <$ 0 is the first component needed to establish the existence of the threshold $\hat{v}$. In addition, we need that given the threshold setting $\sum_{i=1}^{2}(U_i^0 - \tilde{U}_i) = 0$ exists, $\sum_{i=1}^{2}(U_i^0 - \tilde{U}_i)$ is decreasing in $\hat{v}$.

Finding a value $\hat{v}$ that satisfies $\sum_{i=1}^{2}(U_i^0 - \tilde{U}_i) = 0$ is done recursively. First, we assume this threshold exists and that a divorce takes place if and only if $v > \hat{v}$ for any value of $\hat{v}$. Then, we derive the conditions for this behavior actually being optimal, i.e., specify $\hat{v}$.

Given the threshold $\hat{v}$, the partners‘ expected discounted payoff streams within the relationship when $p_t = 0$ for an arbitrary period $t \geq 1$ (which also allows as to omit time subscripts) are

$$U_1^0 = \overline{w}_1 + \varphi_1(n) + \delta \left[F(\hat{v}) U_1^0 + (1 - F(\hat{v})) \, \mathrm{E}[\tilde{U}_1 \mid v \geq \hat{v}] \right] \tag{1.12}$$

$$U_2^0 = \overline{w}_2(n) + \varphi_2(n) + \delta \left[F(\hat{v}) U_2^0 + (1 - F(\hat{v})) \, \mathrm{E}[\tilde{U}_2 \mid v \geq \hat{v}] \right] \tag{1.13}$$

Furthermore, recall that the payoff streams in a period where a divorce happens equal

$$\begin{aligned} \tilde{U}_1(v_1) &= \frac{1}{1-\delta} \left(\overline{w}_1 + \theta\varphi_1(n) - \phi[\overline{w}_1 - \overline{w}_2(n)] \right) - k_1 + v_1 + \frac{\delta}{1-\delta} \overline{v}_1 \\ \tilde{U}_2(v_2) &= \frac{1}{1-\delta} \left(\overline{w}_2(n) + \varphi_2(n) + \phi[\overline{w}_1 - \overline{w}_2(n)] \right) - k_2 + v_2 + \frac{\delta}{1-\delta} \overline{v}_2 \end{aligned}$$

where we take into account the assumption that once a couple breaks up, it will not get together again in the future. To obtain a characterization of $\mathrm{E}[\tilde{U}_i \mid v \geq \hat{v}]$, the realizations of v_i in $\tilde{U}_i(v_i)$ only have to be replaced by $\mathrm{E}[v_i \mid v \geq \hat{v}]$.

There, note that (as v_1 and v_2 are independently distributed)

$$f(\tilde{v}) = (f_1 * f_2)(\tilde{v}) = \int_{v_1^0}^{v_1^1} f_1(v_1) f_2(\tilde{v} - v_1) dv_1 = \int_{v_2^0}^{v_2^1} f_1(\tilde{v} - v_2) f_2(v_2) dv_2$$

and

$$F(\tilde{v}) = \int_{v_1^0 + v_2^0}^{\hat{v}} f(\tilde{v}) d\tilde{v} = \int_{v_1^0 + v_2^0}^{\hat{v}} \left(\int_{v_1^0}^{v_1^1} f_1(v_1) f_2(\tilde{v} - v_1) dv_1 \right) d\tilde{v}$$

Thus,

$\mathrm{E}[v_1 \mid v \geq \hat{v}] = \frac{1}{1-F(\hat{v})} \int_{\hat{v}}^{v_1^1+v_2^1} \left(\int_{v_1^0}^{v_1^1} f_1(v_1) f_2(\tilde{v} - v_1) v_1 dv_1 \right) d\tilde{v}$ and

$\mathrm{E}[v_2 \mid v \geq \hat{v}] = \frac{1}{1-F(\hat{v})} \int_{\hat{v}}^{v_1^1+v_2^1} \left(\int_{v_2^0}^{v_2^1} f_1(\tilde{v} - v_2) f_2(v_2) v_2 dv_2 \right) d\tilde{v}$

Plugging all expressions into $U_1^0 + U_2^0 = \tilde{U}_1(v_1) + \tilde{U}_2(v_2)$, applying Bayes' rule (i.e., $\overline{v}_i = \mathrm{E}[v_i \mid v > \hat{v}](1 - F(\hat{v})) + \mathrm{E}[v_i \mid v \leq \hat{v}] F(\hat{v})$) and rearranging gives (1.11).

Finally, it remains to show that (1.11) is decreasing in $\hat{v}$. Differentiating 1.11 with respect to $\hat{v}$ gives $-(1-\delta F(\hat{v}))<0$, which completes the proof.

Note that this proof does not require $\hat{v} \leq v_1^1 + v_2^1$, i.e., that the threshold is below the upper bound of the support of $\tilde{v}$. Thus, we also cover the case that divorce never occurs in equilibrium. It is then easy to prove

Proposition 2: Given that the separation decision is efficient, divorce in a period is less likely - for a given distribution of outside options - the higher are divorce costs, the lower is the primary earner's post-separation right of access to the children, θ, and the higher the number of children, while it is independent of the wealth division parameter ϕ, the wage gap $\overline{w}_1 - \overline{w}_2$ and the second earner's labour supply $1-g(n)$.

Proof: These follow straightforwardly from implicitly differentiating (1.11), which gives $\frac{d\hat{v}}{dk_1} = \frac{d\hat{v}}{dk_2} = \frac{(1-\delta)}{(1-\delta F(\hat{v}))} > 0$, $\frac{d\hat{v}}{d\theta} = \frac{-\varphi_1(n)}{(1-\delta F(\hat{v}))} < 0$ and $\frac{d\hat{v}}{dn} = \frac{\varphi_1'(1-\theta)}{(1-\delta F(\hat{v}))} > 0$

These results are perfectly intuitive: wealth division simply represents a transfer between the partners. Although it makes the primary earner less prone to file for a divorce, the opposite is true for the secondary earner, with a net effect of zero. Loss of the primary earner's access to the children is a form of deadweight loss to the couple, as are divorce costs. This suggests that there is a tradeoff from society's point of view between the primary earner's post-divorce right of access to the children and the divorce rate, since increasing the former also raises the latter, other things equal. In the restricted context of the separation decision, higher fertility leads to a lower divorce rate, since the deadweight loss from divorce increases with n, given $\varphi_1'(n) > 0$ and $\theta < 1$. Since fertility is endogenous, however, there is still much more to be said on the relationship between fertility and divorce.

Note that the results for k_i are valid for couples married at the time when the law changes. They do not imply that divorce rates have to go up in the long run (if costs are reduced and is θ increased). Instead, a new institutional setting also changes incentives to actually become married, thus affecting subsequent divorce propensities. We further explore this issue in section 9 below, just note that short-run indeed appear to differ from long-run effects. As an example, take the change to unilateral divorce laws in many US states some decades ago, which could be regarded as a reduction of divorce costs. In the short run, divorce rates went up, confirming our predictions; however, they basically returned to their initial levels after some time (see Wolfers, 2006, or Matouschek and Rasul, 2008).

1.6 Fertility

We will now proceed with the characterization of equilibrium fertility and derive comparative statics with respect to several divorce laws. This contributes to the public discussion on why couples in (especially) Western countries have less and less children. As already pointed out, this discussion usually restricts attention to a simple benefit-cost analysis and discusses the effectivity of various policies to reduce various costs (also including parents' human capital loss). All these issues could be incorporated into our model as well, yielding the predicted results.[32] Here, we take a different approach and show that legislation that is not directly aimed towards influencing peoples' propensity to have children might have a substantial impact as well. Since costs and benefits are at least partially exogenously given and fixed (for reasons explained above), redistribution within the household is needed to equalize the burden among spouses. However, no formal contract determining within-household allocation can be written, and all transfers have to be self-enforcing.

We assume that all cooperation ceases after a separation, and the implicit agreement is replaced by formal rules.[33] Different divorce laws have an impact on relationship stability and/or the absolute and relative welfare levels of spouses after a separation. Thus, these rules will directly affect each partner's utility as well as the enforceability of transfers by having an impact on the credibility of punishment threats as well as the risk of being left alone. Just note that the following results are true for a couple given it becomes married. It does not necessarily imply that divorce laws have the predicted consequences on an

[32] For example, providing child-care facilities could to some extent reduce $g(n)$ and thus increase fertility.

[33] We will further specify below how our setup relates to the general matter of interactions between explicit and implicit contracts, as for example analyzed by Baker, Gibbons and Murphy (1994).

aggregate level. Instead, couples might also adjust their marriage-versus-cohabitation decision. We will further explore this issue in section 9 below.

Absent any transfers, individually optimal fertility levels n_1^{**} and n_2^{**} will generally differ. To what extent the spouses' interests can be aligned depends on the enforceability of transfers.

Recall that the couples solves

$$\max_n U = U_1 + U_2$$

subject to the (IC-DE) constraint derived above, where U_i are the expected utility streams at the beginning of period $t = 0$, i.e., when the household has just been formed. The objective in period 0 is set on lifetime utility streams, taking into account the utilities that will actually be chosen in each state (including divorce utilities in the corresponding states). The distributional variables are those relevant in period 0, when the allocation is being chosen. The decision must take into account the effect of the current fertility choice on all future utilities along the equilibrium path.

If the (IC-DE) constraint does not bind, we get the efficient outcome (in a sense that it would be chosen by the couple under full commitment), and equilibrium fertility is described by

Proposition 3: Assuming (IC-DE) does not bind, optimal fertility n^* satisfies

$$\alpha\varphi_1'(n^*) + \varphi_2'(n^*) = (1-\delta)w_{20}g_0'(n^*) - \delta\overline{w}_2'(n^*) \tag{1.14}$$

where

$$\alpha \equiv \frac{1-\delta+\delta(1-F(\hat{v}))\theta}{1-\delta F(\hat{v})} \leq 1 \tag{1.15}$$

The proof for Proposition 3 can be found in the Appendix.

This leads to the conclusion that in the presence of a positive probability of divorce $(1 - F(\hat{V}_2) > 0)$ and less than complete access to the children after divorce for the primary earner $(\theta < 1)$ there will be a lower fertility rate than is socially optimal, since this would require $\alpha = 1$. The marginal social benefit of fertility is $\frac{1}{1-\delta}\left(\varphi_1'(n^*) + \varphi_2'(n^*)\right)$, and the marginal social cost (recall that child consumption costs have been set to zero) is the marginal value of the time the second earner spends in child rearing in both periods, taking into account also the value in period 1 of the loss of human capital in period 0.

This immediately allows us to obtain some comparative statics with respect to divorce laws when the efficient fertility level is feasible. As we think it is more relevant given only the secondary earner stays at home when having children, we only discuss it for the case of $n_1^{**} > n_2^{**}$.

Proposition 4: Assume that $n_1^{**} > n_2^{**}$ and the respective (IC-DE) constraint does not bind. Then, higher divorce costs increase equilibrium fertility, a lower access of the primary earner to his children might or might not increase fertility, while wealth division laws have no impact.

Proof: Efficient equilibrium fertility is characterized by (1.14). Note that the second order condition is satisfied by construction, and we must have $\frac{\partial^2 U}{\partial n^2} < 0$ at the optimumal n^*.

Thus, we have

$$\frac{dn^*}{dk_1} = \frac{dn^*}{dk_2} = -\frac{f(\hat{v})\delta(1-\delta)^2}{(1-\delta F(\hat{v}))^3}\frac{\varphi_1'(n)(1-\theta)}{\frac{\partial^2 U}{\partial n^2}} > 0$$

$$\frac{dn^*}{d\phi} = 0$$

$$\frac{dn^*}{d\theta} = -\frac{\frac{f(\hat{v})(1-\delta)}{(1-\delta F(\hat{v}))^2}\delta\frac{d\hat{v}}{d\theta}\varphi_1'(n)(1-\theta) + \frac{\delta(1-F(\hat{v}))\theta}{1-\delta F(\hat{v})}\varphi_1'(n)}{\frac{\partial^2 U}{\partial n^2}} \lesseqgtr 0$$

As wealth division rules after a divorces only redistribute funds between spouses after a divorce, they cancel out when the constraint does not bind and thus have no impact on equilibrium fertility.

Condition 1.14 in Proposition 3 gives some intuition on whether increasing divorce costs or reducing θ could be expected to raise or lower fertility. Clearly for $\theta = 1$ we have $\alpha = 1$ and so fertility will be at its first best level, since the probability of divorce no longer plays a role in determining fertility. However, realistically we must have $\theta < 1$ if a couple ceases to cohabit after divorce and the children remain with the secondary earner.[34] Then, higher divorce costs always increase fertility by making divorce less likely. The probability of the efficiency loss induced by a separation is reduced, inducing the couple to have more children. Thus, divorce costs serve as a commitment device, an outcome supported empirically by Rasul (2005), Stevenson (2007), Matouschek and Rasul (2008), or Bellido and Marcï£¡n (2011).

The results of reduing θ are ambiguous because there are two opposing effects. On the one hand, the marginal return to fertility across divorce states goes down. Yet. the probability of no divorce increases, and the net effect depends on parameter values and the form of the distribution function $F(.)$. If θ is close to 1, the latter effect is negligible, and a reduction of a father's access to his children after a separation always leads to a fertility reduction.

Finally, reducing the marginal cost on the right hand side of (1.14) would also increase fertility, and this could be achieved by reducing the rate at which increased fertility reduces the second earner loss of human capital, clearly strengthening the argument for policies that allow second earners to combine raising a family with pursuing a career.

The question arises whether higher divorce costs or a decrease in θ are beneficiary - especially when they increase fertility. Although low birth rates affect a society as a whole (just take the discussions on the financing of the welfare state), we restrict attention to the impact of divorce laws on the utitlities of the involved partners. Furthermore, in the long run only the couple's welfare is relevant. If their utilities are lower in the presence of divorce laws, they will simply abstain from getting married and instead cohabit. See section 9 for an analysis of this matter.

Then, as long as the (IC-DE) constraint is not binding, restrictions like higher costs or a decreased access to children after a divorce reduce total utility.

[34] Even if there are no legal restriction to a primary earner's access, the pure fact that the parents do not live together anymore will reduce the time he can spend with his children.

Lemma 3: Given the efficient fertility level can be enforced, higher divorce costs and a lower θ decrease total equilibrium surplus U.

Proof: Applying the envelope theorem gives

$\frac{dU}{dk_i} = \frac{\partial U}{\partial k_i} = -\delta \frac{(1-F(\hat{v}))}{1-\delta F(\hat{v})} < 0$

$\frac{dU}{d\phi} = 0$

$\frac{dU}{d\theta} = \frac{\partial U}{\partial \theta} = \frac{\delta}{1-\delta}\varphi_1(n)\frac{1-F(\hat{v})}{1-\delta F(\hat{v})} > 0$

The reason is that although higher costs or a lower access reduce the probability of divorce, this destroys surplus as players can not consume outside utilities v_i where it would otherwise be optimal (note that marriage stability has no value per se). Thus, although divorce costs and a lower value of θ serve as a commitment device to increase fertility, the increased commitment is harmful if the (IC-DE) does not bind.

Binding (IC-DE) constraint

If the relationship is relatively unstable or the difference between n_1^{**} and n_2^{**} large (for example because 2's marginal human capital loss is high), it is likely that the (IC-DE) constraint binds and equilibrium fertility is smaller than the efficient level.[35]

Then, n^* is determined by the binding constraint, or, in case of $n_1^{**} > n_2^{**}$,

$$u_{21}(n^*) - u_{21}(n^{**}) + \delta\left[\overline{U}_1^*(n^*) + \overline{U}_2^*(n^*) - \left(\overline{U}_1(n^*) + \overline{U}_2(n^{**})\right)\right] = 0$$

There, it is worth to further specify off-equilibrium utilities $\overline{U}_1(n^*)$ and $\overline{U}_2(n^{**})$. Recall that after a player deviated, no more transfers are made in the future. The couple breaks up if inducing a divorce is optimal for at least one player, which depends on the

[35] To see that equilibrium fertility is lower than the level specified by (1.14), take the Lagrange function $L = U^0 + \lambda[(IC-DE)]$. The first order condition equals $\frac{dU^0}{dn} + \lambda\frac{d(IC-DE)}{dn}$. If (IC-DE) does not bind, $\lambda = 0$ and we are in the unconstrained case. If it binds, $\frac{d(IC-DE)}{dn}$ has to be negative. The reason is that otherwise, increasing fertility would relax the constraint, contradicting that we are at an optimum. Thus, $\frac{dU^0}{dn}$ has to be positive in an equilibrium with the (IC-DE) constraint binding. As $\frac{d^2U^0}{dn^2}$ must has to be negative as well, equilibrium fertility is lower when (IC-DE) binds. Furthermore, it decreases with λ.

realizations of outside utilities. If those are not sufficiently high, both just receive their relationship-utilities $w_i + \varphi_i(n)$ in the respective period and wait for the next draw of $\tilde{v}_i$. In addition, the partner's failure to make an agreed payment has a negative impact on the overall quality of the relationship and thus increases one's own outside utility by $\Delta v_i \geq 0$ in every subsequent period. This has an impact on the couple's ability to redistribute funds, but only as it affects the likelihood of a separation off equilibrium.

In the Appendix we derive thresholds v_i^* and v_i^{**} (depending on who deviated) for individual outside utilities. Only if either one of these thresholds is exceeded, a separation occurs in the respective period. Otherwise, i.e., if both realizations of outside utilities are below these thresholds, both players prefer to stay together for at least one more period even though the partnership is not working properly anymore. Then, they just wait until at least one's outside utility is sufficiently high to get out of the relationship.

The actual levels of Δv_i have no qualitative impact on comparative statics unless they are that high that both thresholds are below the lower bound of the support of outside utilities, i.e., if $v_i^*/v_i^{**} \leq v_i^0$. Then, a deviation is immediately followed by a separation in any case. This changes the impact of divorce laws on fertility if (IC-DE) binds, as increasing marriage stability then does not make a divorce threat less credible, and thus has an unambiously positive effect. Therefore, we treat the cases where Δv_i are high enough to always induce a punishment and where this is not the case separately, still focusing on the situation with $n_1^{**} > n_2^{**}$.

In the first case, we have

Proposition 5 : Assume $n_1^{**} > n_2^{**}$, the (IC-DE) constraint binds and that both values of Δv_i are sufficiently high that any deviation is immedialety followed by a separation for all realizations of $\tilde{v}_i$. Then, higher divorce costs and higher alimony payments increase fertility, as well as a lower access of the primary earner to his children following a divorce.

Proof: As $v_i^*/v_i^{**} \leq v_i^0$, we have $\overline{U}_i = \tilde{U}_i$. Plugging all values into the binding (IC-DE) constraint gives

(IC-DE)

$$w_{20}\left(g_0(n^{**})-g_0(n^*)\right)+\varphi_2(n^*)-\varphi_2(n^{**})$$
$$+\tfrac{\delta}{1-\delta}\left(\overline{w}_2(n^*)+\varphi_2(n^*)-\overline{w}_2(n^{**})-\varphi_2(n^{**})\right)$$
$$+\delta\int_{v_1^0+v_2^0}^{\hat{v}} f(\tilde{v})(\hat{v}-\tilde{v})d\tilde{v}+\tfrac{\delta}{1-\delta}\phi\left(\overline{w}_2(n^{**})-\overline{w}_2(n^*)\right)\geq 0.$$

Taking into account that if it binds in equilibrium, $\frac{\partial(IC-DE)}{\partial n}<0$ (otherwise, a higher fertility would relax the constraint, contradicting that it binds and fertility is too low at the same time) we have $\frac{dn^*}{dk_i}=-\frac{d\hat{v}}{dk_i}\frac{\delta F(\hat{v})}{\frac{\partial(IC-DE)}{\partial n}}>0$, $\frac{dn^*}{d\theta}=-\frac{d\hat{v}}{d\theta}\frac{\delta F(\hat{v})}{\frac{\partial(IC-DE)}{\partial n}}\leq 0$, and $\frac{dn^*}{d\phi}=-\frac{\delta}{1-\delta}\frac{\overline{w}_2(n^{**})-\overline{w}_2(n^*)}{\frac{\partial(IC-DE)}{\partial n}}\geq 0$ (since $n^{**}\leq n^*$).

As higher divorce costs reduce the likelihood of a divorce without decreasing the severity of punishment, the range of states where surplus can be redistributed and used to provide incentives becomes larger. As empirically established, divorce costs thus also serve as a commitment device when the (IC-DE) constraint binds.

Higher alimony payments partially compensate the secondary earner for her human capital loss and thus reduce her marginal costs of having children. For a given fertility level the difference between her on- and off-equilibrium fertility gets higher as $\overline{w}_2(n^{**})>\overline{w}_2(n^*)$. Thus, more redistribution between the spouses can be enforced, allowing them to increase n^*. Note that the impact of higher alimony is not driven by reducing 1's reservation equilibrium utility, as this cancels out against 2's increased reservation utility. Although having no direct impact on relationship stability, alimony payments thus make a separation less likely in equilibrium, namely as the probability of a divorce decreases in equilibrium fertility n^*.

A reduction of θ now has an unambiguosly positive impact on fertility. As fertility is too low, the utility reduction in case of a separation as a factor reducing fertility is obviously not taken into account. Thus, a lower access of the primary earner increases fertility by relaxing the (IC-DE) constraint.

As fertility is inefficiently low when (IC-DE) binds, a divorce that is more regulated can even increase the total relationship surplus. This is always the case for alimony payments, which have no impact the surplus if the efficient fertility level is enforceable.

Lemma 3: Assume $n_1^{**}>n_2^{**}$, the (IC-DE) constraint is binds and that both values of Δv_i are sufficiently high that any deviation is immedialety followed by a separation for all realizations of $\tilde{v}_i$. Then, higher divorce costs and a lower θ might or might not increase the relationship surplus. Higher alimony payments always increase the surplus

Proof: As fertility is inefficiently low $\frac{\partial U(n^*)}{\partial n} > 0$. Thus,

$$\frac{dU(n^*)}{dk_i} = \frac{\partial U(n^*)}{\partial n}\frac{\partial n^*}{\partial k_i} + \frac{\partial U(n^*)}{\partial k_i} = \frac{\partial U(n^*)}{\partial n}\frac{\partial n^*}{\partial k_i} - \delta(1 - F(\hat{v})) \lesseqgtr 0$$

$$\frac{dU(n^*)}{d\phi} = \frac{\partial U(n^*)}{\partial n}\frac{\partial n^*}{\partial \phi} + \frac{\partial U(n^*)}{\partial \phi} = \frac{\partial U(n^*)}{\partial n}\frac{\partial n^*}{\partial \phi} > 0$$

$$\frac{dU(n^*)}{d\theta} = \frac{\partial U(n^*)}{\partial n}\frac{\partial n^*}{\partial \theta} + \frac{\partial U(n^*)}{\partial \theta} = \frac{\partial U(n^*)}{\partial n}\frac{\partial n^*}{\partial \theta} + \frac{\delta}{1-\delta}\varphi_1(n^*)\frac{1-F(\hat{v})}{1-\delta F(\hat{v})} \lesseqgtr 0$$

If a player's deviation is not triggered by a sufficiently high increase of the partners outside utilities and thus not immediately followed by a separation (i.e., $v_i^*/v_i^{**} > v_i^0$), the impact of divorce laws is less obvious. Now, they do not only affect divorce utilities but also the likelihood that punishment can actually be carried out. Thus, we have

Proposition 6: Assume $n_1^{**} > n_2^{**}$ and $v_i^*/v_i^{**} > v_i^0$. Then, the impact of divorce laws on fertility is ambiguous.

The proof for Proposition 6 can be found in the Appendix.

The effect of higher divorce costs and a lower level of θ is ambiguous as these policies not only lower utilities in case of a divorce (which helps to enforce transfers) but also make it less likely that a punishment is actually carried - as it becomes more attractive for players to remain within a marriage even if partners do not cooperate anymore. Concerning wealth division rules, we have the same effect as before increasing equilibrium fertility. Furthermore, 2's willingness to actually induce a divorce in each period off equilibrium increases, while the primary earner is less likely to do that. Which effect dominates depends on the distributions of outside utilities. If those are for example uniformly distributed on the same support, both effects cancel out, and the impact of alimony on fertility is unambiguously positive.

The effect of divorce laws on total efficiency is ambiguous as well, and we omit a formal analysis. When they increase fertility, they might increase total surplus for reason that are the same as above.

1.7 Inefficient Separation

In the preceding parts, we assumed that the separation decision is always made efficiently in equilibrium. We further showed in Appendix II that this outcome can be approximated arbitrarily closely in a continuous-time setting where the period length between decisions gets smaller and smaller. Now, we take a different approach and the discreteness of the game literally. The truth will probably lie somewhere in between, although we think assuming an efficient decision is more convincing. For example, the fact that initiating a divorce takes time does not play a role here. This creates a lag, namely between the decision to break up and the time from which on the institutional changes - costs, wealth division, and a reduced access - come into force. However, this just makes us discount the relevant parameters accordingly and thus also affect the threshold $\hat{v}$, giving the same qualitative results as above.

The main difference to above is that a separation can occur even if it is efficient to remain together. The reason is that not all necessary transfers are enforceable anymore - since their enforceability only depends on expectations about future payoffs. Still, the results are not too different from above; the possibility of inefficient divorce is just always taken into account. Furthermore, fertility is affected, as children do not just provide utility per se but can also have an impact on the likelihood of a separation. If this likelihood is reduced, fertility can even be above the efficient level.

To start with the formal analysis, we briefy recall the timing within a period $t \geq 1$ (if the couple is still together):

- The realizations of players‘ outside utilities $\tilde{v}_i$ are revealed to both. Then, they continue the relationship or break up. After a separation, players immediately receive their reservation utilities $\tilde{U}_i(v_i)$.

- If they remain together, they work and receive their wages. Then, the transfer p_t (also allowed to be negative) is made from 1 to 2. Finally, they consume the private consumption good x_t and enjoy utility from their kids.

Furthermore, once the couple separates, it is assumed that the partners never come together again. Note that the timing of the transfer is not important - if it can already be made after outside utilities are revealed but before players have to make the separation decision, no additional stability is created.

Obtaining the states in which a separation occurs is slightly more involved now, as there does not exist just one value for the outside utilities above which the couple breaks up. Instead we have three thresholds, one for $\tilde{v}_1$, one for $\tilde{v}_2$ and one for the sum $\tilde{v}$. If any one of these threshold is exceeded, a separation will occur. The reason is that we do not only have to worry about efficiency, but also about the enforceability of transfers to maintain the relationship. As the latter only depends on expectations about future payoffs, thresholds for enforceability and efficiency generally will not coincide.

For concreteness, again assume that the couple chooses to marry. Recall the definitions $u_i^0 = \overline{w}_i + \varphi_i(n)$ as the per-period utilities within the relationship with $p_t = 0$ for any $t \geq 1$. Then, player i must be compensated to be willing to stay in the relationship if $u_i^0 + \delta\overline{U}_i^* - \tilde{U}_i(v_i) < 0$, where $\overline{U}_i^*$ is the expected equilibrium utility stream of player i (including transfers)[36] and $\tilde{U}_i(\tilde{v}_i)$ player i's utility if filing for divorce in the respective period. Any transfer $p(v_i)$ must make it optimal for both players to stay within the relationship. Furthermore, it has to be in the interest of player 1 to provide a positive payment and for 2 to provide a negative payment. Concerning off-equilibrium payoffs, we assume for concreteness that if any player does not keep a promise, the increase in the other's outside utility, i.e., Δv_i, is large enough to immediately induce a subsequent divorce.

To derive the relevant constraints, let us first assume that $u_2^0 + \delta\overline{U}_2^* - \tilde{U}_2(v_2) < 0$, i.e., the secondary earner needs a transfer to remain within the relationship. This transfer, denoted it $p(v_2)$, must be large enough to satisfy 2's individual rationality (IR) constraint

$$u_2^0 + p(v_2) + \delta\overline{U}_2^* - \tilde{U}_2(v_2) \geq 0 \tag{1.16}$$

[36]Note that we omit a time subscript and thus do not allow expected equilibrium payoffs to change over time. This is just done for convenience and without loss of generality, as - as we will see when computing the relevant constraints - the enforceability of any transfer as well as the efficiency of a separation always depends on the sum of players' utilities in equilibrium and not on the surplus distribution.

Furthermore, 1's (IR) constraint must hold, which is obviously the case if breaking up would be inefficient, i.e., if $v_1 + v_2 \leq \hat{v}$[37] If this is the case, the primary earner must actually be willing to make the transfer $p(v_2)$. This is captured by 1's dynamic enforcement (DE) constraint,

$$p(v_2) \leq \delta[\overline{U}_1^* - \overline{U}_1] \tag{1.17}$$

Obviously, the right hand side of (1.17) is independent of current realizations of outside utilities. Thus, the maximum feasible value of $p(v_2)$ is the same in each period. Denoting this maximum feasible transfer $\max p \equiv \delta[\overline{U}_1^* - \overline{U}_1]$, a transfer that keeps 2 in the relationship and satisfies 1's (DE) constraint exists if $u_2^0 + \max p + \delta\overline{U}_2^* - \tilde{U}_2(v_2) \geq 0$. Whether it is actually made also depends on 1's (IR) constraint, i.e., whether v_1 is sufficiently small such that $v \leq \hat{v}$.

Concluding the previous arguments, a transfer that keeps 2 in the relationship and satisfies 1's (DE) constraint exists if $v_2 \leq v_2^{max}$, where v_2^{max} is defined by

$$u_2^0 + \delta\overline{U}_2^* - \tilde{U}_2(v_2^{max}) + \delta[\overline{U}_1^* - \overline{U}_1] = 0$$

Equivalent considerations help us to define the threshold v_1^{max}, stating when a negative transfer exists that keeps player 1 within the relationship and satisfies 2's (DE) constraint as long as $v_1 \leq v_1^{max}$:

$$u_1^0 + \delta\overline{U}_1^* - \tilde{U}_1(v_2^{max}) + \delta[\overline{U}_2^* - \overline{U}_2] = 0$$

Therefore, a separation can never be prevented if either $v_1 > v_1^{max}$ or $v_2 > v_2^{max}$. As already pointed out, this does not imply that if $v_1 \leq v_1^{max}$ and $v_2 \leq v_2^{max}$ are satisfied, the couple remains together. It still has to be in the interest of a player to make a transfer. More precisely, the other's (IR) constraint must be satisfied as well, which will be the case if staying together is efficient. Concluding, a couple will not break up in any period t, if at the same time $v_1 \leq v_1^{max}$, $v_2 \leq v_2^{max}$ and $v_1 + v_2 = v \leq \hat{v}$, where $\hat{v}$ is characterized by all combinations of (v_1, v_2) that satisfy $u_1^0 + u_2^0 + \delta\overline{U}_2^* + \delta\overline{U}_1^* = \tilde{U}_1(v_1) + \tilde{U}_2(v_2)$.

In any period, the likelihood of remaining together thus is $\text{Prob}(\tilde{v}_1 \leq v_1^{max} \cap \tilde{v}_2 \leq$

[37] Note that $\hat{v}$ here does not coincide with its value above, as separation probabilities will be different. Just the meaning is identical, namely that in a given period a separation is efficient if $v > \hat{v}$.

$v_2^{max} \cap v \leq \hat{v}$). In the Appendix, we give an explicit formulation of this probability and also prove

Proposition 7: The divorce probability is higher than when this decision is made efficiently

The proof for Proposition 7 can be found in the Appendix.

Proposition 7 is very intuitive and does not require further explanation - because a transfer necessary to maintain a relationship might not be enforceable, the couple can also break up in states where this is not efficient. What we also show in the proof to proposition 7 is that the divorce probability is strictly lower (unless $k_i = \phi = (1-\theta) = 0$) than in the case of efficient divorce, i.e., that $v_1^{max} \geq \hat{v}$ and $v_2^{max} \geq \hat{v}$ cannot be satisfied at the same time.

In the following proposition, we state the impact of divorce laws and fertility on marriage stability.

Proposition 8: The probality of a divorce decreases with higher divorce costs and a lower value of θ. The impact of alimony payments and more children on marriage stability is ambiguous.

The proof for Proposition 8 can be found in the Appendix.

As before, divorce becomes more likely for lower divorce costs and a higher θ. They increase the efficient threshold $\hat{v}$ but also make transfers to maintain the relationship easier to enforce (recall that we focus ont the case with sufficiently high Δv_i). Higher alimony payments can have a positive or negative impact on marriage stability. On the one hand, they make it more difficult to enforce a positive transfer, as the secondary earner already gets alimony in the period of divorce, while the primary earner only takes the future into account when considering whether to make the transfer. The opposite is true for negative payments. Which effect dominates depends on the exact specifications of the distribution functions of players' outside utilities. More children, on the one hand, make a marriage more stable by reducing utility after a separation if $\theta < 1$. Still, children also affect stability via alimony payments. Thus, if those make a divorce more likely (recall that their impact on stability is ambiguous), the total effect of a higher fertility level on stability might be negative. Yet, the net impact of alimony payments on stability should not be too high as it consists of two countervailing effects which might cancel out depending on distributation. Thus, it seems more convincing that children generally have a positive impact on relationship stability, by increasing the gap between the primary earner's utility within and outside the relationship. Some characterstics of equilibrium fertility are given in

Proposition 9: Assume the respective (IC-DE) constraint does not bind. If the impact of children on marriage stability is positive, equilibrium fertility might be higher than under full commitment. Otherwise it is lower. The impact of divorce laws on equilibrium fertility is ambiguous.

The proof for Proposition 9 can be found in the Appendix.

Two issues are different compared to before, when the divorce decision was always made efficiently. As the marriage is less stable, the couple's propensity to have children is lower, because the likelihood of the utility loss induced by $\theta < 1$ is higher. Yet, if more children increase marriage stability, a countervailing effect exists, and each of them can dominate. In addition to providing utility, children might thus be "used" as a commitment device that makes a separation less likely.

Concerning comparative statics, higher divorce costs and a lower value of θ still might increase fertility by increasing relationship stability. Yet, if children also make a separatin less likely, some substitution between these two instruments takes place. As the commitment role of children is needed less due to a higher k_i and lower θ, a countervailing effect decreasing fertility exists, and it is not clear which one dominates.

When the constraint binds, the situation changes, and the impact of a change in costs and θ becomes unambiguous again.

Proposition 10: Assume $n_1^{**} > n_2^{**}$ and that the respective (IC-DE) constraint binds. Then, higher divorce costs and a lower level of θ increase equilibrium fertility. The impact of alimony payments is ambiguous.

The proof for Proposition 10 can be found in the Appendix.

As fertility is too low anyway, potential substitution effects between fertility and higher costs and a lower access play no role. Thus, these forms of increased regulation unambiguously increase fertility as the increase maximum enforceable transfers by reducing off equilibrium utilities. Higher alimony payments have a positive impact on fertility if their effect on marriage stability is not too large. Then, a higher ϕ increases the secondary earner's compensation for her human capital loss in divorce states, induced by the wage difference $\overline{w}(n^{**}) - \overline{w}(n^*)$.

1.8 The Interaction of Formal and Informal Arrangements

This paper explores interactions between the institutional settings following a divorce and the ability to informally enforce cooperation within a relationship. From a theoretical point of view, this relates to the literature on the interaction between explicit and implicit contracts (see Baker, Gibbons, Murphy, 1994, or Schmidt and Schnitzer, 1995). Generally, the two contractual arrangements are substitutes, and the existence of a profitable explicit contract reduces the efficiency of the implicit relationship. To see that point, assume that absent any formal contract, an implicit agreement yields the efficient outcome. Now, the same matter can also be governed within an explicit framework. However, the latter is less inefficient - for example because the verifiable signal is less precise than the non-verifiable one. Thus, players still prefer the informal agreement. But their situation might be worse now. Since they cannot commit to not use the formal agreement off equilibrium, i.e., after a player reneged, reservation utilities go up. This makes it more difficult to enforce cooperation under the implicit contract, and efficiency goes down. If players there were able to reduce the attractiveness of the explicit contract, they would do so.

To some extent, divorce laws assume an identical role in our setup. Higher k_i or a lower θ decrease the utilities after a separation and thus reservation utilities. However – as a divorce happens in equilibrium as well – divorce laws not only have an impact on off-equilibrium but also on on-equilibrium payoffs. Thus, formal and informal arrangements can be complements or substitutes.

More precisely, the impact of lower payoffs after a divorce on total welfare is threefold. First of all, equilibrium utilities are reduced as divorce occurs in equilibrium. Furthermore - if Δv_i is not sufficiently large after one's partner reneged - it gets more difficult to enforce a punishment, also reducing welfare. Finally, lower reservation utilities improves the enforceability of transfer, potentially fostering efficiency.

The latter point is not relevant if the (IC-DE) constraint does not bind. Then, only the reduction of on-equilibrium payoffs is relevant. If the (IC-DE) constraint binds, however, reducing players' payoffs under the formal arrangement of a divorce can help as it slackens the constraint. This point becomes more important in relation to the utility reduction in equibrium if the welfare loss associated with the binding constraint is high.

Another issue is interesting when focussing on the primary earner's utility reduction after a divorce, captured by θ. There, the choice of n also has an impact on the relative attractiveness of formal compared to informal cooperation, since a higher n increase this difference for a given $\theta < 1$.

Thus, our setup includes a nice application and extension of the interaction between explicit and implicit arrangements. It can help to reduce the attractiveness of formal contracts to foster informal cooperation. However – as all kinds of relationships can break up – one has to be aware that a formal contract might also be used in equilibrium at some point.

The case is slightly different for alimony payments/wealth division rules. These have no impact on efficiency and thus on the "quality" of the explicit relative to the implicit contract. Since funds are redistributed to the weaker side and since these funds are increasing with the amount of previous cooperation, i.e., the fertility level, implicit cooperation is enhanced by this formal redistribution rule.

1.9 Marriage Versus Cohabitation

Until now, we remained silent about the couple's decision whether to get married or live in cohabitation or to link it the discussion of the previous section, when the couple is willing to choose the more regulated outside reservation utility, always taking into account that it will also have to deal with it in equilibrium. The major part of our analysis - especially when considering the impact of divorce laws - rather assumed that the couple is married. In this section, we briefly explore conditions for the optimality of marriage. If $k = \phi = 0$ (1's access to his children is also reduced after the termination of a relationship without marriage), both settings are identical. Then, the couple will only become married if the regulation induced after a divorce increases their utilities ex ante. For convenience, we assume that it is sufficient that the sum of utilities is higher under marriage, i.e., we do not have to focus on individual utility levels. This can be rationalized by letting partners make ex-ante transfers, which have to be self-enforcing as well (if a partner receives a transfer and then refuses to marry, this is regarded as a deviation.). We do not explore this issue further and make the assumption that the couple gets married if and only if

$$U(m = 1) \geq U(m = 0),$$

where $m = m_1 m_2 \in \{0, 1\}$ denotes the marriage decision and $U(m) = U_1(m) + U_2(m)$ is the sum of discounted payoff streams.

If the separation decision is always made efficiently in equilibrium, we can state a first result, namely that a marriage will not occur if the (IC-DE) constraint does not bind under cohabition.

Proposition 11: Assume the separation decision is always made efficiently in equilibrium and that the relevant (IC-DE) does not bind for $m = 0$. Then, the couple will not become married if $k_i > 0$, $i = 1, 2$. The spouses are indifferent if $k_i = 0$ and $\phi \geq 0$.

Proof: Follows from Lemma 2, which states that U is monotonically decreasing in k and independent of ϕ if (IC-DE) does not bind.

Since fertility is at its efficient level, divorce costs are not needed as a commitment device. Furthermore, they decrease utility in case of a divorce and unnecessarily increase marriage stability (note that stability does not have a value per se - it can prevent the spouses from consuming their outside utilities). This can change if the separation decision is not always made efficiently and if the associated efficiency loss is sufficiently large. We do not further explore the case of inefficient separation in this paragraph, just note that an increased relationship stability induced by a marriage can then be beneficial.

Thus, a marriage can only be a useful institution in our setting if under cohabitation, fertility is too low because of commitment problems. Before further exploring this issue, let us make some general remarks. Of course, the present analysis only captures a limited part of potential benefits of a marriage. For some couples, marriage might have a value per se,[38] and young adults still face more or less pressure to get married in some societies. Furthermore, the tax system can induce a marriage, especially if joint instead of individual taxation is applied. Finally, relationship stability can have a value per se. Compensation for ongoing household production (like cooking, cleaning...) might have to be self enforcing, and a higher stability increase the scope for cooperation[39]. Also the welfare of children - which is left aside in the current analysis - might be negatively affected by a separation.[40]

In our setting, the institutional framework a marriage provides can only be beneficial if the (IC-DE) constraint binds for a cohabiting couple. To simplify the analysis, we now focus on the case where Δv_i, i.e., the increase in outside utilities after a partner reneged, is sufficiently high for a divorce to occur immediately after someone deviated.[41] Then, if divorce costs are negligible and a wealth division rule $\phi > 0$ is in place, a marriage will always be optimal if $n_1^{**} > n_2^{**}$,[42] captured in

[38]Which could be captured in our model by assuming different distributions of outside utilities for spouses that are cohabiting and those that are married.

[39]It is not too convincing, though, that aspects of household production that can more easily be substituted by the market than raising children are a driving force making a couple entering the unflexible institution of a marriage.

[40]However, if spouses care about their children's welfare, they will take the negative impact of a separation into account and should not need commitment induced by divorce costs.

[41]If this is not the case, potential benefits of divorce costs and thus a marriage are even lower, since they do not only increase marriage stability in but also off equilibrium.

[42]Which we again assume for reasons stated above. To be fully precise, we would have to take into account the possibility that for example under cohabitation, $n_1^{**} > n_2^{**}$, while $n_1^{**} < n_2^{**}$ after a marriage. This could be induced by substantial alimony payments, which increase the secondary player's benefits

Proposition 12: Assume the separation decision is always made efficiently in equilibrium, that $n_1^{**} > n_2^{**}$, and that (IC-DE) binds for $m = 0$. Then, the couple will marry if $k_i = 0$, $i = 1, 2$ and $\phi > 0$. If $k_i > 0$, the couple might or might not get married

Proof: Follows from Lemma 3, which establishes $\frac{dU(n^*)}{d\phi} > 0$ for $n_1^{**} > n_2^{**}$, and a binding (IC-DE) constraint. Furthermore, Lemma 3 establishes that $\frac{dU(n^*)}{dk_i} = \frac{\partial U(n^*)}{\partial n}\frac{\partial n^*}{\partial k_i} - \delta(1 - F(\hat{v})) \lesseqgtr 0$.

A generous wealth division rule combined with laws making a divorce very easy will thus induce the couple to marry, since ϕ increases the enforceability of transfers without having a negative impact on efficiency. However, we have to be careful since we only consider the impact of divorce laws on the sum of payoffs. If an ex-ante redistribution is not feasible, things might be different. Despite an inefficiently low fertility level, it can then be in the interest of the primary earner to abstain from a marriage. To our knowledge, the impact of wealth division rules on marriage rates has only found limited attention in the empirical literature. One exception is Rasul (2003), who finds a negative effect on marriage rates of a change to an equal division of property after a divorce. However, his results have to be treated with care when comparing them to our analysis. Table 14 of his paper shows that the explained negative impact is strongly and significantly negative only for spouses contemplating a second marriage. For those not previously married, the coefficient is not significant at the 10% level, and positive for men and negative for women. Since our prediction of a positive impact of an equitable wealth division after divorce is only due to a subsequent increase of fertility rates, his results might be driven by spouses who do not consider having any more children. Thus, wealth division rules should be analyzed empirically in more detail, especially in connection to fertility levels.

The impact of divorce costs on marriage rates has received more attention in the empirical literature, mainly due to the replacement of consent with unilateral divorce laws in many US states some decades ago. Regarding this change as a reduction of divorce costs,[43] results are ambiguous. Whereas Rasul (2003) or Matouschek and Rasul

from children in case of a divorce. However, we restrict our attention to the case where $n_1^{**} > n_2^{**}$, no matter whether the couple is cohabiting or married.

[43]Which is supported by the fact divorce rates immediately went up after the introduction of unilateral

(2008) observe a decline in marriage rates, Alesina and Giuliano (2007) do not find this effect but rather an increase. This supports our claim that more commitment by higher divorce costs is not automatically preferred by couples, since divorce utilities are reduced as well as the option to utilize relatively high realizations of outside utilities. Only if the utility loss induced by the fact that the (IC-DE) constraint binds is sufficiently high, the existence of divorce costs can make a marriage optimal. If divorce costs are relatively high, their reduction might make more couples willing to use them as a commitment device to increase fertility. Alesina and Guiliano's (2007) results are perfectly in line with this interpretation. In wedlock fertility basically remains unaffected by the adoption of unilateral divorce laws, while out of wedlock fertility decreases significantly and fertility rates for newly wedded couples go up.

In the remainder of the section, we consider conditions that actually make the (IC-DE) constraint bind and thus increase a couple's propensity to get married when substantial divorce costs are present. For $n_1^{**} > n_2^{**}$ and with Δv_1 sufficiently large, the (IC-DE) constraint for a cohabiting couple equals

$$w_{20}\left(g_0(n^{**}) - g_0(n^*)\right) + \varphi_2(n^*) - \varphi_2(n^{**}) + \frac{\delta}{1-\delta}\left(\overline{w}_2(n^*) - \overline{w}_2(n^{**})\right)$$
$$+\frac{\delta}{1-\delta}\left(\varphi_2(n^*) - \varphi_2(n^{**})\right) + \delta \int_{v_1^0+v_2^0}^{\hat{v}} f(\tilde{v})(\hat{v}-\tilde{v})d\tilde{v} \geq 0,$$

where $\hat{v}$ is defined by $\varphi_1(n)(1-\theta) + \delta \int_{v_1^0+v_2^0}^{\hat{v}} f(\tilde{v})(\hat{v}-\tilde{v})d\tilde{v} - \hat{v} = 0.$

Note that the first line of the expression above is negative because of $n_1^{**} > n_2^{**}$. Furthermore - since $n^* \geq n_2^{**}$ - it is decreasing in n^*.

Then, the (IC-DE) constraint is more likely to bind if more additional time is needed to raise more children ($g_0(n)$ is steeper) and if more children imply a higher human capital loss for the secondary earner ($\overline{w}_2(n^*)$ is steeper). This will also make it more likely that the desired transfer is larger, implied by a bigger difference between n_1^{**} and n_2^{**}. Furthermore, a lower relationship stability (captured by a smaller value of $\int_{v_1^0+v_2^0}^{\hat{v}} f(\tilde{v})(\hat{v}-\tilde{v})d\tilde{v}$) decreases

divorce, see Friedberg (1998) or Matouschek and Rasul (2008).

the value of the left hand side of the condition for given amounts of n^* and n^{**}.

Although the first components are exogenously given within our model, they deserve some attention here. For example, $g_0(n)$ could be steeper if less child-care facilities were available at reasonable costs. Thus, couples without the option to give their children away for some time might be more inclined to marry. This point is also important for the second aspect, a higher human capital loss associated with children. There, recall our assumption that the secondary earner alone is responsible for raising the couple's children. As argued above, it seems unlikely that the fact that women still assume major parts of the responsibilities associated with having children is purely driven by an optimal (in an economic sense) allocation of tasks, but should rather be given by issues outside our model, for example cultural reasons.

If men were willing to substantially participate in child-rearing and if jobs were sufficiently flexible, i.e., if the couple were able to commit to any allocation of $g(n)$, it would be possible to obtain efficient fertility without the need of additional transfers. Thus, if the couple is closer to an optimal time allocation, the (IC-DE) is less likely to bind. Therefore, couples with more traditional views should be more likely to get married, a claim that is supporated by empirical evidence[44].

1.10 Conclusion

This paper analyzed interactions between divorce laws and fertility. Making precise the conditions under which cooperation within a relationship can be enforced, we showed how the institutional setting following a separation can make it easier or more difficult to allocate resources within a household and thus compensate a partner for the human capital loss associated with having children. However, our approach can only be a first step to analyze (often unintended) consequences legislative changes. Future research could evolve along three lines.

Further empirical research is needed to test some predictions. Whereas the impact of divorce laws on divorce rates, the propensity to marriage, and on fertility has been extensively tested – especially using the natural experiment of a switch from consensual

[44] Kaufmann (2004) for example finds out that men with egalitarian counterparts are more likely to cohabit than those with more traditional views

or no-fault to unilateral divorce – the same remains to be done for alimony payments or wealth division rules and a reduced access of one partner to his children after a separation.

Furthermore, our model is general enough to incorporate further laws that are important for a marriage. The impact of different forms of income taxation – for example joint versus individual taxation – could be analyzed. The model is also precise enough to look at the question consent versus unilateral divorce in more detail. Of course, a unilateral divorce is very likely to be associated with lower divorce costs, what is shown by the vast empirical literature that found an immediate increase in divorce rates following its introduction. However, taking this matter literally and noting that under a consent divorce regime, one partner alone can not easily induce a divorce anymore might allow us to explain some empirical results that can not be explained by a change in commitment power alone. For example, Alesina and Giuliano (2007) find that after the introduction of unilateral laws, fertility rates for newly wedded couples went up, while out-of-wedlock fertility decreased. They claim that lower divorce costs make couples enter marriage more easily, however without further specifying this explanation. Applying our model can also give an explanation, since a consent divorce regime taken literally makes it much more difficult to enforce payments between spouses. The reason is that no more transfers are needed to keep the partner within a marriage, and allocations in and off equilibrium do not differ.[45]

Finally, the model setup itself can be extended, for example by taking children's welfare into account or assuming risk-averse players. In the latter case, it will be necessary to analyze alimony payments and wealth division laws separately, since savings will become an important aspect. In addition, these laws might not only compensate the secondary earner for her human capital loss, but also help to equalize income across states. Furthermore, relationship stability might have a value per se. Then, laws reducing the attractiveness of a divorce could rather be welfare enhancing in comparison to the present setup. Also, the role of children as a device to increase stability (via the access parameter θ) would have to be reassessed.

We plan to pursue some of these extensions in our future research and hope they will lead to further interesting results.

[45] This is even the case if transfers to prevent the inefficient continuation of a marriage are feasible, since such transfers go in hand with the end of the relationship.

Appendix to Chapter 1

Appendix I – The Fertility Choice

Omitted proofs

Proposition 1:

If $n_1^{**} > n_2^{**}$ a fertility level n^* can be enforced if and only if it satisfies the condition

$$u_{21}(n^*) - u_{21}(n^{**}) + \delta\left[\overline{U}_1^*(n^*) + \overline{U}_2^*(n^*) - \left(\overline{U}_1(n^*) + \overline{U}_2(n^{**})\right)\right] \geq 0$$

.

If $n_1^{**} < n_2^{**}$, the necessary and sufficient condition for equilibrium fertility n^* is

$$u_{11}(n^*) - u_{11}(n^{**}) + \delta\left[\overline{U}_1^*(n^*) + \overline{U}_2^*(n^*) - \left(\overline{U}_1(n^{**}) + \overline{U}_2(n^*)\right)\right] \geq 0$$

Proof:

Assume that $n_1^{**} > n_2^{**}$. Then, we do not have to consider (IC1), since it is automatically satisfied if equilibrium fertility is smaller or equal than the efficient level. Obviously, we do not want to induce fertility that is higher than efficient, where efficiency is defined as the level players would choose if they were able to commit.

Necessity:

Rewriting (IC2) gives

$p_0(n^*) \geq u_{21}(n^{**}) + \delta\overline{U}_2^*(n^{**}) - [u_{21}(n^*) + \delta\overline{U}_2^*(n^*)]$. Plugging this into (DE1) which is the relevant constraint as $p_0(n^*)$ is supposed to be positive[46] yields condition 1.6.

[46]Note that in the case where continuation utilities alone are sufficient to yield incentives for first-best fertility, a negative transfer p_0 would be feasible in this case as well. Yet, then this transfer is solely used

Sufficiency:

Assume that 1.6 is satisfied.

Set $p_0^+ \equiv \delta[\overline{U}_1^*(n^*) - \overline{U}_1(n^*)] \geq 0$ and plug it into (IC-DE), which becomes $p_0^+ + u_{21}(n^*) - u_{21}(n^{**}) + \delta\left[\overline{U}_2^*(n^*) - \overline{U}_2(n^{**})\right] \geq 0$. Thus, (IC2) is satisfied. Furthermore, (DE1) is satisfied by construction of p_0^+.

Necessity and sufficiency for $n_1^{**} < n_2^{**}$ is proven accordingly.

Proposition 3: Assuming (IC-DE) does not bind, optimal fertility n^* satisfies

$$\alpha\varphi_1'(n) + \varphi_2'(n) = (1-\delta)w_{20}g_0'(n) - \delta\overline{w}_2'(n)$$

where

$$\alpha \equiv \frac{1-\delta+\delta(1-F(\hat{v}))\theta}{1-\delta F(\hat{v})} \leq 1$$

Proof: If the respective (IC-DE) constraint does not bind, players maximize the sum of players' expected utility streams, which equals

$U^0 = w_{10} + w_{20}(1-g_0(n)) + \varphi_1(n) + \varphi_2(n)$

$+\delta\left(F(\hat{v})[U_1^0 + U_2^0] + (1-F(\hat{v}))[\mathrm{E}[\tilde{U}_1 \mid v > \hat{v}] + \mathrm{E}[\tilde{U}_2 \mid v > \hat{v}]]\right)$

$= w_{10} + w_{20}(1-g_0(n)) + \varphi_1(n) + \varphi_2(n)$

$+\frac{\delta}{1-\delta}\left(\overline{w}_1 + \overline{w}_2(n) + \varphi_2(n)\right) + \delta\varphi_1(n)\frac{F(\hat{v})+(1-F(\hat{v}))\frac{1}{1-\delta}\theta}{1-\delta F(\hat{v})}$

$+\frac{\delta(1-F(\hat{v}))}{1-\delta F(\hat{v})}\left(-k_1 - k_2 + \frac{\delta}{1-\delta}\mathrm{E}[\tilde{v}_1] + \frac{\delta}{1-\delta}\mathrm{E}[\tilde{v}_2]\right) + \frac{\delta}{1-\delta F(\hat{v})}\int_{\hat{v}}^{v_1^1+v_2^1} f(\tilde{v})\tilde{v}d\tilde{v}$

As $\frac{\partial U^0}{\partial \hat{v}} = 0$

the first order conditions becomes

for redistributive reasons, making the (IC) constraints featuring the fertility decision irrelevant.

$$\alpha\varphi_1'(n) + \varphi_2'(n) = (1-\delta)w_{20}g_0'(n) - \delta\overline{w}_2'(n)$$

$$\alpha \equiv \frac{1-\delta+\delta(1-F(\hat{v}))\theta}{1-\delta F(\hat{v})}$$

We assume that the second order condition, is satisfied by construction.

Binding (IC-DE) constraint

We first characterize the maximum feasible punishment off equilibrium. After any player reneged, any trust in the relationship is lost, and both know there will be no transfer anymore. Thus, a divorce occurs in any period if it is in the interest of at least one player to file for a divorce, i.e. if any $\tilde{v}_i$ exceeds a threshold of outside utilities such that divorce is associated with a higher utility stream than waiting for at least one further period.

The thresholds will depend on who reneged, as this determines the off-equilibrium fertility level and further whose outside utility increases by Δv_i in each subsequent period.

First, assume that player 1 reneged, i.e., he refused to make a positive transfer after 2 agreed on equilibrium fertility n^*. This implies that 2‘s outside utility increases by Δv_2 in each subsequent period. This gives

Lemma A1: Assume that 1 refused to make a promised transfer $p_0(n^*)$. Then, a divorce off equilibrium occurs in the first period with either $v_1 \geq v_1^*$ or $v_2 \geq v_2^*$, where v_1^* and v_2^* are characterized by

$$\varphi_1(n^*)(1-\theta) + \phi[\overline{w}_1 - \overline{w}_2(n^*)] + (1-\delta)k_1 + \delta F(v_2^*)\int_{v_1^0}^{v_1^*} f_1(v_1)(v_1^* - v_1)dv_1 - v_1^* = 0 \quad (1.18)$$

$$-\phi[\overline{w}_1 - \overline{w}_2(n^*)] + (1-\delta)k_2 + \delta F(v_1^*)\int_{v_2^0}^{v_2^*} f_2(v_2)(v_2^* - v_2)dv_2 - \Delta v_2 - v_2^* = 0 \tag{1.19}$$

Proof: We have off equilibrium utilities

$\overline{w}_i + \varphi_i(n) + \delta\left[F_1(v_1^*)F_2(v_2^*)U_i^0 + (1 - F_1(v_1^*)F_2(v_2^*))\,\mathrm{E}[\tilde{U}_i \mid v_1 > v_1^* \cup v_2 > v_2^*]\right]$ and

$$\mathrm{Prob}[v_1 > v_1^* \cup v_2 > v_2^*] = \mathrm{Prob}[v_1 > v_1^*] + \mathrm{Prob}[v_2 > v_2^*] - \mathrm{Prob}[v_1 > v_1^*]\mathrm{Prob}[v_2 > v_2^*]$$
$$= \int_{v_1^*}^{v_1^1} f_1(v_1)dv_1 + \int_{v_2^*}^{v_2^1} f_2(v_2)dv_2 - \int_{v_1^*}^{v_1^1} f_1(v_1)dv_1 \int_{v_2^*}^{v_2^1} f_2(v_2)dv_2$$
$$= \int_{v_1^*}^{v_1^1} f_1(v_1)dv_1 + \int_{v_2^*}^{v_2^1} f_2(v_2)dv_2 \left(1 - \int_{v_1^*}^{v_1^1} f_1(v_1)dv_1\right)$$
$$= \int_{v_1^*}^{v_1^1} f_1(v_1)dv_1 + \int_{v_2^*}^{v_2^1} f_2(v_2)dv_2 \left(\int_{v_1^0}^{v_1^*} f_1(v_1)dv_1\right)$$

Then,

$$\mathrm{E}[v_1 \mid v_1 > v_1^* \cup v_2 > v_2^*] = \tfrac{1}{1-F(v_1^*)F(v_2^*)}\left(\int_{v_1^*}^{v_1^1} f_1(v_1)v_1dv_1 + \int_{v_2^*}^{v_2^1} f_2(v_2)dv_2 \int_{v_1^0}^{v_1^*} f_1(v_1)v_1dv_1\right)$$

$$\mathrm{E}[v_2 \mid v_1 > v_1^* \cup v_2 > v_2^*] = \tfrac{1}{1-F(v_1^*)F(v_2^*)}\left(\int_{v_2^*}^{v_2^1} f_2(v_2)v_2dv_2 + \int_{v_1^*}^{v_1^1} f_1(v_1)dv_1 \int_{v_2^0}^{v_2^*} f_2(v_2)v_2dv_2\right)$$

Substituting the respective expressions and rearranging gives (1.18) and (1.19).

Equivalently, we obtain

Lemma A2: Assume that 2 refused on equilibrium fertility n^*. Then, a divorce off equilibrium occurs in the first period with either $v_1 \geq v_1^{**}$ or $v_2 \geq v_2^{**}$, where v_1^{**} and v_2^{**} are characterized by

$$\varphi_1(n^{**})(1-\theta) + \phi[\overline{w}_1 - \overline{w}_2(n^{**})] + (1-\delta)k_1 - \delta F(v_2^{**})\int_{v_1^0}^{v_1^{**}} f_1(v_1)(v_1^{**} - v_1)dv_1 - \Delta v_1 - v_1^{**} = 0 \tag{1.20}$$

$$-\phi[\overline{w}_1 - \overline{w}_2(n^{**})] + (1-\delta)k_2 + \delta F(v_1^{**}) \int_{v_2^0}^{v_2^{**}} f_2(v_2)(v_2^{**} - v_2)dv_2 - v_2^{**} = 0 \qquad (1.21)$$

Proof: Now, we have $U_1 = \tilde{U}_1(v_1^{**} + \Delta v_1)$ and $U_2 = \tilde{U}_2(v_2^{**})$, and the fertility level is n^{**}. All other steps are as in the proof to Lemma A2.

Comparative Statics

For later use, we will now derive some comparative statics for v_2^* and v_1^{**}. It will become clear below that we do not need the ones for v_1^* and v_2^*:

Lemma A3: Comparative statics for v_2^* are $\frac{dv_2^*}{dk_1} \geq 0$, $\frac{dv_2^*}{dk_2} \geq 0$, $\frac{dv_2^*}{d\phi} \leq 0$ and $\frac{dv_2^*}{d\theta} \leq 0$, while those for v_1^{**} satisfy $\frac{dv_1^{**}}{dk_1} \geq 0$, $\frac{dv_1^{**}}{dk_2} \geq 0$, $\frac{dv_1^{**}}{d\phi} \geq 0$, $\frac{dv_1^{**}}{d\theta} \leq 0$.

Proof:

Denote equation (1.18) F_1^*, equation (1.19) F_2^*, equation (1.20) F_1^{**} and equation (1.21) F_2^{**}.

Then, the implicit function theorem gives

$$\frac{dv_2^*}{dk_1} = \frac{\begin{vmatrix} \frac{\partial F_1^*}{\partial v_1^*} & -\frac{\partial F_1^*}{\partial k_1} \\ \frac{\partial F_2^*}{\partial v_1^*} & -\frac{\partial F_2^*}{\partial k_1} \end{vmatrix}}{\begin{vmatrix} \frac{\partial F_1^*}{\partial v_1^*} & \frac{\partial F_1^*}{\partial v_2^*} \\ \frac{\partial F_2^*}{\partial v_1^*} & \frac{\partial F_2^*}{\partial v_2^*} \end{vmatrix}} = \frac{\begin{vmatrix} -\left(1-\delta F(v_1^*)F(v_2^*)\right) & -(1-\delta) \\ \delta f(v_1^*) \int_{v_2^0}^{v_2^*} f_2(v_2)\left(v_2^* - v_2\right) dv_2 & 0 \end{vmatrix}}{\begin{vmatrix} -\left(1-\delta F(v_1^*)F(v_2^*)\right) & \delta f(v_2^*) \int_{v_1^0}^{v_1^*} f_1(v_1)\left(v_1^* - v_1\right) dv_1 \\ \delta f(v_1^*) \int_{v_2^0}^{v_2^*} f_2(v_2)\left(v_2^* - v_2\right) dv_2 & -\left(1-\delta F(v_1^*)F(v_2^*)\right) \end{vmatrix}}$$

$$= \frac{\delta(1-\delta) f(v_1^*) \int_{v_2^0}^{v_2^*} f_2(v_2)\left(v_2^* - v_2\right) dv_2}{\begin{vmatrix} \frac{\partial F_1^*}{\partial v_1^*} & \frac{\partial F_1^*}{\partial v_2^*} \\ \frac{\partial F_2^*}{\partial v_1^*} & \frac{\partial F_2^*}{\partial v_2^*} \end{vmatrix}} > 0$$

Note that the denominator must be positive as the matrix of first derivatives with respect to v_i^* must be negative definite. The reason is that otherwise, a higher outside utility than v_i^* would make a separation optimal, implying that the game is not in an equilibrium.

Equivalently, we have

$$\frac{dv_2^*}{dk_2} = \frac{(1-\delta)\left(1-\delta F(v_1^*)F(v_2^*)\right)}{\begin{vmatrix} \frac{\partial F_1^*}{\partial v_1^*} & \frac{\partial F_1^*}{\partial v_2^*} \\ \frac{\partial F_2^*}{\partial v_1^*} & \frac{\partial F_2^*}{\partial v_2^*} \end{vmatrix}} > 0,$$

$$\frac{dv_2^*}{d\phi} = \frac{[\overline{w}_1 - \overline{w}_2(n^*)]\left(\delta f(v_1^*)\int_{v_2^0}^{v_2^*} f_2(v_2)\left(v_2^* - v_2\right)dv_2 - \left(1-\delta F(v_1^*)F(v_2^*)\right)\right)}{\begin{vmatrix} \frac{\partial F_1^*}{\partial v_1^*} & \frac{\partial F_1^*}{\partial v_2^*} \\ \frac{\partial F_2^*}{\partial v_1^*} & \frac{\partial F_2^*}{\partial v_2^*} \end{vmatrix}} < 0,$$ where the nominator is negative due to the requirements for a stable equilibrium as well (which also requires that $\frac{\partial F_1^*}{\partial v_1^*} + \frac{\partial F_1^*}{\partial v_2^*} < 0$), and

$$\frac{dv_2^*}{d\theta} = \frac{-\varphi_1(n^*)\delta f(v_1^*)\int_{v_2^0}^{v_2^*} f_2(v_2)\left(v_2^* - v_2\right)dv_2}{\begin{vmatrix} \frac{\partial F_1^*}{\partial v_1^*} & \frac{\partial F_1^*}{\partial v_2^*} \\ \frac{\partial F_2^*}{\partial v_1^*} & \frac{\partial F_2^*}{\partial v_2^*} \end{vmatrix}} < 0$$

Furthermore,

$$\frac{dv_1^{**}}{dk_1} = \frac{\begin{vmatrix} -\frac{\partial F_1^{**}}{\partial k_1} & \frac{\partial F_1^{**}}{\partial v_2^{**}} \\ -\frac{\partial F_2^{**}}{\partial k_1} & \frac{\partial F_2^{**}}{\partial v_2^{**}} \end{vmatrix}}{\begin{vmatrix} \frac{\partial F_1^{**}}{\partial v_1^{**}} & \frac{\partial F_1^{**}}{\partial v_2^{**}} \\ \frac{\partial F_2^{**}}{\partial v_1^{**}} & \frac{\partial F_2^{**}}{\partial v_2^{**}} \end{vmatrix}} = \frac{\begin{vmatrix} -(1-\delta) & \delta f(v_2^{**})\int_{v_1^0}^{v_1^{**}} f_1(v_1)\left(v_1^{**} - v_1\right)dv_1 \\ 0 & -\left(1-\delta F(v_1^{**})F(v_2^{**})\right) \end{vmatrix}}{\begin{vmatrix} -\left(1-\delta F(v_1^{**})F(v_2^{**})\right) & \delta f(v_2^{**})\int_{v_1^0}^{v_1^{**}} f_1(v_1)\left(v_1^{**} - v_1\right)dv_1 \\ \delta f(v_1^{**})\int_{v_2^0}^{v_2^{**}} f_2(v_2)\left(v_2^{**} - v_2\right)dv_2 & -\left(1-\delta F(v_1^{**})F(v_2^{**})\right) \end{vmatrix}}$$

$$= \frac{(1-\delta)\left(1-\delta F(v_1^{**})F(v_2^{**})\right)}{\begin{vmatrix} \frac{\partial F_1^{**}}{\partial v_1^{**}} & \frac{\partial F_1^{**}}{\partial v_2^{**}} \\ \frac{\partial F_2^{**}}{\partial v_1^{**}} & \frac{\partial F_2^{**}}{\partial v_2^{**}} \end{vmatrix}} > 0,$$

$$\frac{dv_1^{**}}{dk_2} = \frac{(1-\delta)\delta f(v_2^{**})\int_{v_1^0}^{v_1^{**}} f_1(v_1)\left(v_1^{**} - v_1\right)dv_1}{\begin{vmatrix} \frac{\partial F_1^{**}}{\partial v_1^{**}} & \frac{\partial F_1^{**}}{\partial v_2^{**}} \\ \frac{\partial F_2^{**}}{\partial v_1^{**}} & \frac{\partial F_2^{**}}{\partial v_2^{**}} \end{vmatrix}} > 0,$$

$$\frac{dv_1^{**}}{d\phi} = \frac{[\overline{w}_1 - \overline{w}_2(n^{**})]\left[\left(1-\delta F(v_1^{**})F(v_2^{**})\right) - \delta f(v_2^{**}) \int_{v_1^0}^{v_1^{**}} f_1(v_1)\left(v_1^{**}-v_1\right)dv_1\right]}{\begin{vmatrix} \frac{\partial F_1^{**}}{\partial v_1^{**}} & \frac{\partial F_1^{**}}{\partial v_2^{**}} \\ \frac{\partial F_2^{**}}{\partial v_1^{**}} & \frac{\partial F_2^{**}}{\partial v_2^{**}} \end{vmatrix}} > 0 \text{ and}$$

$$\frac{dv_1^{**}}{d\theta} = \frac{-\varphi_1(n^{**})\left(1-\delta F(v_1^{**})F(v_2^{**})\right)}{\begin{vmatrix} \frac{\partial F_1^{**}}{\partial v_1^{**}} & \frac{\partial F_1^{**}}{\partial v_2^{**}} \\ \frac{\partial F_2^{**}}{\partial v_1^{**}} & \frac{\partial F_2^{**}}{\partial v_2^{**}} \end{vmatrix}} < 0$$

This helps us to prove

Proposition 6: Assume $n_1^{**} > n_2^{**}$ and $v_i^*/v_i^{**} > v_i^0$. Then, the impact of divorce laws on fertility is ambiguous.

Proof:

The binding (IC-DE) constraint now equals

$$w_{20}(1-g_0(n^*)) + \varphi_2(n^*) - (w_{20}(1-g_0(n^{**})) + \varphi_2(n^{**}))$$
$$+\tfrac{\delta}{1-\delta}\left(\overline{w}_2(n^*) + \varphi_2(n^*)\right) - \tfrac{\delta}{1-\delta}\left(\overline{w}_2(n^{**}) + \varphi_2(n^{**})\right) + \delta\varphi_1(n^*)\frac{F(\hat{v})+(1-F(\hat{v}))\frac{1}{1-\delta}\theta}{1-\delta F(\hat{v})}$$
$$+\tfrac{\delta(1-F(\hat{v}))}{1-\delta F(\hat{v})}\left(-k_1 - k_2 + \tfrac{\delta}{1-\delta}\mathrm{E}[\tilde{v}_1] + \tfrac{\delta}{1-\delta}\mathrm{E}[\tilde{v}_2]\right) + \tfrac{\delta}{1-\delta F(\hat{v})}\int_{\hat{v}}^{v_1^1+v_2^1} f(\tilde{v})\tilde{v}d\tilde{v}$$
$$-\delta\frac{\varphi_1(n^*)\left[F_1(v_1^*)F_2(v_2^*) + \frac{\theta}{1-\delta}\left(1-F_1(v_1^*)F_2(v_2^*)\right)\right]}{\left(1-\delta F_1(v_1^*)F_2(v_2^*)\right)}$$
$$+\delta\tfrac{1-F_1(v_1^*)F_2(v_2^*)}{1-\delta F_1(v_1^*)F_2(v_2^*)}\left(\tfrac{1}{1-\delta}\phi[\overline{w}_1 - \overline{w}_2(n^*)] + k_1 - \tfrac{\delta}{1-\delta}\mathrm{E}[\tilde{v}_1]\right)$$
$$-\delta\tfrac{1}{1-\delta F_1(v_1^*)F_2(v_2^*)}\left(\int_{v_1^*}^{v_1^1} f_1(v_1)v_1dv_1 + \int_{v_2^*}^{v_2^1} f_2(v_2)dv_2\int_{v_1^0}^{v_1^*} f_1(v_1)v_1dv_1\right)$$
$$-\delta\tfrac{1-F_1(v_1^{**})F_2(v_2^{**})}{1-\delta F_1(v_1^{**})F_2(v_2^{**})}\left(\tfrac{1}{1-\delta}\phi[\overline{w}_1 - \overline{w}_2(n^{**})] - k_2 + \tfrac{\delta}{1-\delta}\mathrm{E}[\tilde{v}_2]\right)$$
$$-\delta\tfrac{1}{1-\delta F_1(v_1^{**})F_2(v_2^{**})}\left(\int_{v_2^{**}}^{v_2^1} f_2(v_2)v_2dv_2 + \int_{v_1^{**}}^{v_1^1} f_1(v_1)dv_1\int_{v_2^0}^{v_2^{**}} f_2(v_2)v_2dv_2\right) = 0$$

There, it can be shown that

$$\frac{\partial(IC-DE)}{\partial v_1^*} = 0$$

Furthermore,

$$\frac{\partial(IC-DE)}{\partial v_2^*} = -\delta f_2(v_2^*)\frac{\left(F_1(v_1^*)(\varphi_1(n^*)(1-\theta)+\phi[\overline{w}_1-\overline{w}_2(n^*)]+(1-\delta)k_1)-\int\limits_{v_1^0}^{v_1^*} f_1(v_1)v_1 dv_1\right)}{\left(1-\delta F_1(v_1^*)F_2(v_2^*)\right)^2} < 0$$

$$\frac{\partial(IC-DE)}{\partial v_1^{**}} = -\delta f_1(v_1^{**})\frac{F_2(v_2^{**})((1-\delta)k_2-\phi[\overline{w}_1-\overline{w}_2(n^{**})])-\int\limits_{v_2^0}^{v_2^{**}} f_2(v_2)v_2 dv_2}{\left(1-\delta F_1(v_1^{**})F_2(v_2^{**})\right)^2} < 0$$

and

$$\frac{\partial(IC-DE)}{\partial v_2^{**}} = 0$$

Using these results, it follows that

$$\frac{d(ICDE)}{dk_1} \lesseqgtr 0$$

$$\frac{d(ICDE)}{dk_2} \lesseqgtr 0$$

$$\frac{d(ICDE)}{d\phi} \lesseqgtr 0$$

Appendix II – Relational Contracts in Periods $t \geq 1$

Continuous Time Approximation

Here, we derive how the assumption that a separation occurs if and only if it is efficient (in equilibrium) could be generated endogenously, by slightly adjusting the model setup. Note that the analysis of decisions in period $t = 0$ remains unaffected as long as this period remains sufficiently large (i.e., discrete).

Now, assume that time $t \in [1, \infty)$ is continous and the discount rate equals$r > 0$. A new realization of outside options v_i is drawn for each interval of length $\Delta t \gg 0$ and immediately observed by both players. Furthermore, Δt is divided into K subintervals of equal length, $K \in \mathbb{N}$, where the length of each subinterval equals $\frac{\Delta t}{K}$.

At the beginning of each subintervall $k = 1, ..., K$, both players first decide whether they want to stay together or break up. Subsequently, a transfer $p_k(v)$ is made.

This allows us to state

Proposition A1: Assume all transfers $p_k(v)$ must be self enforcing. Then, the probability of an inefficient divorce goes to zero as K approaches infinity.

Proof:

To get a better idea of the impact of continuous time, let us first characterize the relevant continuation utilities.

If a divorce occurs at the beginning of date t, the players receive the following utility levels.

$$\tilde{U}_{1,t}(v_1) = \int_t^{t+\Delta t} e^{-r(\tau-t)} v_1 d\tau + e^{-r\Delta t} \int_t^{\infty} e^{-r(\tau-t)} \overline{v}_1 d\tau - k_1$$
$$+ \int_t^{\infty} e^{-r(\tau-t)} \left(\overline{w}_1 + \theta\varphi_1(n) - \phi[\overline{w}_1 - \overline{w}_2(n)]\right) d\tau$$
$$= [1 - e^{-r\Delta t}]\tfrac{1}{r} v_1 + e^{-r\Delta t}\tfrac{1}{r}\overline{v}_1 - k_1 + \tfrac{1}{r}\left(\overline{w}_1 + \theta\varphi_1(n) - \phi[\overline{w}_1 - \overline{w}_2(n)]\right)$$
$$\tilde{U}_{2,t}(v_2) = \int_t^{t+\Delta t} e^{-r(\tau-t)} v_2 d\tau + e^{-r\Delta t} \int_t^{\infty} e^{-r(\tau-t)} \overline{v}_2 d\tau - k_2$$
$$+ \int_t^{\infty} e^{-r(\tau-t)} \left(\overline{w}_2(n) + \varphi_2(n) + \phi[\overline{w}_1 - \overline{w}_2(n)]\right) d\tau$$
$$= [1 - e^{-r\Delta t}]\tfrac{1}{r} v_2 + e^{-r\Delta t}\tfrac{1}{r}\overline{v}_2 - k_2 + \tfrac{1}{r}\left(\overline{w}_2(n) + \varphi_2(n) + \phi[\overline{w}_1 - \overline{w}_2(n)]\right)$$

Note that although new realizations of $\tilde{v}_i$ are only drawn every Δt "periods", the expectation $\overline{v}_i$ is the same throughout. After the interval of length Δt with the realization of v_i is over, expected discounted utility streams of the outside options thus equal $\int_t^{\infty} e^{-r(\tau-t)} \overline{v}_i d\tau$ from then on.

If a divorce is initiated at the beginning of any subinterval k, we have

$$\tilde{U}_{1,t+(k-1)\frac{\Delta t}{K}}(v_1) = \int_{t+(k-1)\frac{\Delta t}{K}}^{t+\Delta t} e^{-r\left(\tau-(t+(k-1)\frac{\Delta t}{K})\right)} v_1 d\tau + e^{-r(\Delta t-(k-1)\frac{\Delta t}{K})}\tfrac{1}{r}\overline{v}_1 - k_1$$
$$+\tfrac{1}{r}\left(\overline{w}_1 + \theta\varphi_1(n) - \phi[\overline{w}_1 - \overline{w}_2(n)]\right)$$
$$= [1 - e^{-r\Delta t\frac{K-(k-1)}{K}}]\tfrac{1}{r} v_1 + e^{-r(\Delta t-(k-1)\frac{\Delta t}{K})}\tfrac{1}{r}\overline{v}_1 - k_1 + \tfrac{1}{r}\left(\overline{w}_1 + \theta\varphi_1(n) - \phi[\overline{w}_1 - \overline{w}_2(n)]\right)$$

and

$$\tilde{U}_{2,t+(k-1)\frac{\Delta t}{K}}(v_2) = [1 - e^{-r\Delta t\frac{K-(k-1)}{K}}]\tfrac{1}{r} v_2 + e^{-r(\Delta t-(k-1)\frac{\Delta t}{K})}\tfrac{1}{r}\overline{v}_2 - k_2$$
$$+\tfrac{1}{r}\left(\overline{w}_2(n) + \varphi_2(n) + \phi[\overline{w}_1 - \overline{w}_2(n)]\right)$$

Accordingly, when the couple is married at the beginning of any subinterval $k \geq 1$ within the period Δt, continuation utilities are

$$U_{1,t+(k-1)\frac{\Delta t}{K}}(v) = \int_{t+(k-1)\frac{\Delta t}{K}}^{t+k\frac{\Delta t}{K}} e^{-r\left(\tau-(t+(k-1)\frac{\Delta t}{K})\right)} \left(\overline{w}_1 - p_k(v) + \varphi_1(n)\right) d\tau$$
$$+ \int_{t+k\frac{\Delta t}{K}}^{t+\Delta t} e^{-r\left(\tau-(t+(k-1)\frac{\Delta t}{K})\right)} \left(\overline{w}_1 + \varphi_1(n)\right) d\tau - \sum_{\kappa=k+1}^{K} \int_{t+(\kappa-1)\frac{\Delta t}{K}}^{t+\kappa\frac{\Delta t}{K}} e^{-r(\tau-(t+(k-1)\frac{\Delta t}{K}))} p_\kappa(v) d\tau$$
$$+ e^{-r(\Delta t-(k-1)\frac{\Delta t}{K})} \overline{U}^*_{1,t+\Delta t}$$
$$= [1-e^{-r\frac{\Delta t}{K}}]\tfrac{1}{r}\left(\overline{w}_1 - p_k(v) + \varphi_1(n)\right) + \tfrac{1}{r}[e^{-r\frac{\Delta t}{K}} - e^{-r\Delta t\frac{K-(k-1)}{K}}]\left(\overline{w}_1 + \varphi_1(n)\right)$$
$$-\tfrac{1}{r}\sum_{\kappa=k+1}^{K}[e^{-r\frac{\Delta t}{K}(\kappa-k)} - e^{-r\frac{\Delta t}{K}(\kappa-(k-1))}]p_\kappa(v) + e^{-r(\Delta t-(k-1)\frac{\Delta t}{K})}\overline{U}^*_{1,t+\Delta t} \text{ and}$$
$$U_{2,t+(k-1)\frac{\Delta t}{K}}(v) = [1-e^{-r\frac{\Delta t}{K}}]\tfrac{1}{r}\left(\overline{w}_2(n) + p_k(v) + \varphi_2(n)\right)$$
$$+\tfrac{1}{r}[e^{-r\frac{\Delta t}{K}} - e^{-r\Delta t\frac{K-(k-1)}{K}}]\left(\overline{w}_2(n) + \varphi_2(n)\right)$$
$$+\tfrac{1}{r}\sum_{\kappa=k+1}^{K}[e^{-r\frac{\Delta t}{K}(\kappa-k)} - e^{-r\frac{\Delta t}{K}(\kappa-(k-1))}]p_\kappa(v) + e^{-r(\Delta t-(k-1)\frac{\Delta t}{K})}\overline{U}^*_{2,t+\Delta t}$$

Now, assume a positive transfer is needed in subinterval $k \geq 1$ to maintain the marriage.[47] Assuming all subsequent transfers are made, the payment in subinterval k must satisfy the secondary earner's (IC) constraint:

(IC2)

$$[1-e^{-r\frac{\Delta t}{K}}]\frac{1}{r}p_k(v) \geq \tilde{U}_{2,t+(k-1)\frac{\Delta t}{K}}(v_2) - \left(U_{2,t+(k-1)\frac{\Delta t}{K}}(v) - [1-e^{-r\frac{\Delta t}{K}}]\frac{1}{r}p_k(v)\right),$$

where we just added and substracted the transfer-term to the condition $U_{2,t+(k-1)\frac{\Delta t}{K}}(v) \geq \tilde{U}_{2,t+(k-1)\frac{\Delta t}{K}}(v_2)$.

Furthermore, given all subsequent transfers are made, the primary earner is willing to make $p(v)$ in the first subinterval (i.e., if he reneges, divorce will be initiated at the beginning of the second subinterval), if $U_{1,t+(k-1)\frac{\Delta t}{K}}(v) \geq [1-e^{-r\frac{\Delta t}{K}}]\frac{1}{r}\left(\overline{w}_1 + \varphi_1(n)\right) + e^{-r\frac{\Delta t}{K}}\tilde{U}_{1,t+k\frac{\Delta t}{K}}(v_1)$ is satisfied. Using $\tilde{U}_{1,t+(k-1)\frac{\Delta t}{K}}(v_1) - e^{-r\frac{\Delta t}{K}}\tilde{U}_{1,t+k\frac{\Delta t}{K}}(v_1)$

[47] Note that if divorce occurs at any point within the a time interval $[t, t+\Delta t]$, it will happen at the beginning (if it happened later, no earlier transfer would be enforceable.
Still, transfer for any subinterval k must be self enforcing.

$= \left(1 - e^{-r\frac{\Delta t}{K}}\right) \frac{1}{r} \left[v_1 - rk_1 + \overline{w}_1 + \theta\varphi_1(n) - \phi[\overline{w}_1 - \overline{w}_2(n)]\right]$ and adding and substracting the term $[1 - e^{-r\frac{\Delta t}{K}}]\frac{1}{r}\left(\overline{w}_1 - p_k(v) + \varphi_1(n)\right)$, we obtain

(DE1)

$$\begin{aligned}
&[1 - e^{-r\frac{\Delta t}{K}}]\frac{1}{r}p_k(v) \\
&\leq U_{1,t+(k-1)\frac{\Delta t}{K}}(v) - \tilde{U}_{1,t+(k-1)\frac{\Delta t}{K}}(v_1) + [1 - e^{-r\frac{\Delta t}{K}}]\frac{1}{r}p_k(v) \\
&+ \left(1 - e^{-r\frac{\Delta t}{K}}\right)\frac{1}{r}\left[v_1 - rk_1 - \varphi_1(n)(1-\theta) - \phi[\overline{w}_1 - \overline{w}_2(n)]\right]
\end{aligned}$$

Note that the last term on the right hand side must be negative, as otherwise (IC1) would not be satisfied.

Merging (IC2) and (DE1) gives

(IC-DE)

$$\begin{aligned}
&U_{1,t+(k-1)\frac{\Delta t}{K}}(v) + U_{2,t+(k-1)\frac{\Delta t}{K}}(v) - \tilde{U}_{1,t+(k-1)\frac{\Delta t}{K}}(v_1) - \tilde{U}_{2,t+(k-1)\frac{\Delta t}{K}}(v_2) \\
&\quad + \left(1 - e^{-r\frac{\Delta t}{K}}\right)\frac{1}{r}\left[v_1 - rk_1 - \varphi_1(n)(1-\theta) - \phi[\overline{w}_1 - \overline{w}_2(n)]\right] \geq 0
\end{aligned}$$

For $\lim\limits_{K\to\infty}\left(1 - e^{-r\frac{\Delta t}{K}}\right) = 0$, the last term approaches zero, and (IC-DE) and the condition for an efficient divorce coincide.

Note that the assumption of a sufficiently high Δv to make an immediate divorce a credible punishment threat is not necessary for this argument. If at any point in time within the interval Δt, the divorce threat is not credible, this also implies that a transfer to maintain the marriage is not required anymore.

Inefficient Divorce Feasible

A divorce can only be prevented if at the same time $v_1 \leq v_1^{max}$, $v_2 \leq v_2^{max}$ and $v \leq \hat{v}$. Thus, the probability of remaining together equals $\mathrm{P} \equiv \mathrm{Prob}(\tilde{v}_1 \leq v_1^{max} \cap \tilde{v}_2 \leq v_2^{max} \cap v \leq \hat{v})$. Recall that v_1^{max}, v_2^{max} and $\hat{v}$ are characterized by

$$\overline{w}_1 + \varphi_1(n) + \delta\overline{U}_1^* + \delta\overline{U}_2^* = \delta\tilde{U}_2 + \tilde{U}_1(v_1^{max})$$

$$\overline{w}_2(n) + \varphi_2(n) + \delta\overline{U}_1^* + \delta\overline{U}_2^* = \delta\tilde{U}_1 + \tilde{U}_2(v_2^{max})$$

$$U_1 + U_2 = \tilde{U}_1(v_1) + \tilde{U}_2(v_2), \text{ with } v_1 + v_2 = \hat{v}$$

where $\tilde{U}_i = \mathrm{E}[\tilde{U}(\tilde{v}_i)]$ and we use make use of the assumption that Δv_i is large enough for $\overline{U}_i = \tilde{U}_i$.

In the following we will denote

$$\mathrm{P} \equiv \text{Probability of no divorce in a given period}$$

and the conditional expectation

$$\mathrm{E}[\tilde{v} \mid \mathrm{NODIV}] \equiv \text{Expected value of } \tilde{v} \text{ conditional on no divorce}$$

Then, plugging in values and using Bayes' rule gives

$$\varphi_1(n)(1-\theta) + (1-\delta\mathrm{P})\left(\phi[\overline{w}_1 - \overline{w}_2(n)] - v_1^{max}\right) + (1-\delta)(k_1 + \delta\mathrm{P}k_2) - \delta\mathrm{P}\mathrm{E}[\tilde{v} \mid \mathrm{NODIV}] = 0 \tag{1.22}$$

$$\delta\mathrm{P}\varphi_1(n)(1-\theta) - (1-\delta\mathrm{P})\left(\phi[\overline{w}_1 - \overline{w}_2(n)] + v_2^{max}\right) + (1-\delta)(\delta\mathrm{P}k_1 + k_2) - \delta\mathrm{P}\mathrm{E}[\tilde{v} \mid \mathrm{NODIV}] = 0 \tag{1.23}$$

$$\varphi_1(n)(1-\theta) + (1-\delta)(k_1 + k_2) - \delta\mathrm{P}\mathrm{E}[\tilde{v} \mid \mathrm{NODIV}] - (1-\delta\mathrm{P})\,\hat{v} = 0 \tag{1.24}$$

Proposition 7: The divorce probability is higher than when this decision is made efficiently.

Proof:

It is only possible that the couple separates efficiently if $v_1^{max} \geq \hat{v}$ and $v_2^{max} \geq \hat{v}$ are satisfied simultaneously. Assume this is the case.

Then, the proof to Lemma A4 below shows that we can use equations (1.22) to (1.24) to obtain $\hat{v} = \frac{\delta \mathrm{PE}[\tilde{v}|\mathrm{NODIV}] + (v_1^{max} + v_2^{max})}{(1+\delta \mathrm{P})}$.

For $v_1^{max} \geq \hat{v}$, we need $v_1^{max} \geq \frac{\delta \mathrm{PE}[\tilde{v}|\mathrm{NODIV}] + (v_1^{max} + v_2^{max})}{(1+\delta \mathrm{P})}$ or $\delta \mathrm{P} v_1^{max} \geq \delta \mathrm{PE}[\tilde{v} \mid \mathrm{NODIV}] + v_2^{max}$, whereas $v_2^{max} \geq \hat{v}$ requires $\delta \mathrm{P} v_2^{max} \geq \delta \mathrm{PE}[\tilde{v} \mid \mathrm{NODIV}] + v_1^{max}$. Combining both gives us a necessary conditions that has to be satisfied, namely $v_1^{max}(1 - \delta \mathrm{P}) \leq -\delta \mathrm{PE}[\tilde{v} \mid \mathrm{NODIV}]$. Merging this with equation (1.22) yields the condition $\varphi_1(n)(1-\theta) + (1-\delta \mathrm{P})\, \phi[\overline{w}_1 - \overline{w}_2(n)] + (1-\delta)k_1 + \delta \mathrm{P}(1-\delta)k_2 \leq 0$, which is never satisfied unless $(1-\theta) = \phi = k_1 = k_2 = 0$.

Lemma A4: The probability of remaining together equals

$$\mathrm{P} = \int\limits_{v_1^0+v_2^0}^{\hat{v}} f(\tilde{v}) d\tilde{v} - \int\limits_{v_1^0+v_2^0}^{\hat{v}} \left(\int\limits_{v_1^{max}}^{v_1^1} f_1(v_1) f_2(\tilde{v} - v_1) dv_1 \right) d\tilde{v}$$
$$- \int\limits_{v_1^0+v_2^0}^{\hat{v}} \left(\int\limits_{v_2^{max}}^{v_2^1} f_1(\tilde{v} - v_2) f_2(v_2) dv_2 \right) d\tilde{v}$$

Proof:

The term above states that the probability of remaining together equals the probility that remaining together is efficient minus the probability that it is efficient but a necessary transfer cannot be enforced. We have to show that $v_1^{max} + v_2^{max} \geq \hat{v}$, as otherwise the term above would double-count the state where we are below $\hat{v}$ and at the same time above v_1^{max} and v_2^{max}.

Thus, assume that $v_1^{max} + v_2^{max} < \hat{v}$ is possible and use equations (1.22) and (1.23) to obtain

$(1-\delta)(k_1+k_2) = \frac{2\delta \text{PE}[\tilde{v}|\text{NODIV}]+(1-\delta\text{P})(v_1^{max}+v_2^{max})}{(1+\delta\text{P})} - \varphi_1(n)(1-\theta)$. Plugging this into (1.24) (which still holds although it is not necessary to determine the probability of divorce when $v_1^{max}+v_2^{max} < \hat{v}$) gives

$$\hat{v} = \frac{\delta \text{PE}[\tilde{v} \mid \text{NODIV}] + (v_1^{max}+v_2^{max})}{(1+\delta\text{P})}$$

Then, $\hat{v} > v_1^{max}+v_2^{max}$ implies $\frac{\delta \text{PE}[\tilde{v}|\text{NODIV}]+(v_1^{max}+v_2^{max})}{(1+\delta\text{P})} > v_1^{max}+v_2^{max}$ or $\text{PE}[\tilde{v} \mid \text{NODIV}] > \text{P}\,(v_1^{max}+v_2^{max})$.

Furthermore, we use that $\text{P} = F_1(v_1^{max})F_2(v_2^{max})$ and

$\text{PE}[\tilde{v} \mid \text{NODIV}] = \text{P}\left(\text{E}[\tilde{v}_1 \mid \text{NODIV}] + \text{E}[\tilde{v}_2 \mid \text{NODIV}]\right)$
$= \int_{v_1^0}^{v_1^{max}} f_1(v_1)v_1dv_1F_2(v_2^{max}) + \int_{v_2^0}^{v_2^{max}} f_2(v_2)v_2dv_2F_1(v_1^{max})$.

Thus, $\text{PE}[\tilde{v} \mid \text{NODIV}] > \text{P}\,(v_1^{max}+v_2^{max})$ becomes

$F_2(v_2^{max}) \int_{v_1^0}^{v_1^{max}} f_1(v_1)v_1dv_1 + F_1(v_1^{max}) \int_{v_2^0}^{v_2^{max}} f_2(v_2)v_2dv_2$

$> F_2(v_2^{max}) \int_{v_1^0}^{v_1^{max}} f_1(v_1)dv_1v_1^{max} + F_1(v_1^{max}) \int_{v_2^0}^{v_2^{max}} f_2(v_2)dv_2v_2^{max}$or

$F_2(v_2^{max}) \int_{v_1^0}^{v_1^{max}} f_1(v_1)(v_1 - v_1^{max})dv_1 + F_1(v_1^{max}) \int_{v_2^0}^{v_2^{max}} f_2(v_2)(v_2 - v_2^{max})dv_2 > 0$, which is not possible as the term within the integral, $(v_1 - v_1^{max}) \leq 0$. Thus, $\hat{v} \leq v_1^{max}+v_2^{max}$, proving the Lemma.

Now, we can use the characterization of P to prove

Proposition 8: The probality of a divorce decreases with higher divorce costs and a lower value of θ. The impact of alimony payments and more children on marriage stability is ambiguous.

Proof:

Let us denote equation (1.22) g^1, (1.23) g^2 and (1.24) g^3 and define the matrix $D \equiv$

$\begin{pmatrix} \frac{\partial g^1}{\partial v_1^{max}} & \frac{\partial g^1}{\partial v_2^{max}} & \frac{\partial g^1}{\partial \hat{v}} \\ \frac{\partial g^2}{\partial v_1^{max}} & \frac{\partial g^2}{\partial v_2^{max}} & \frac{\partial g^2}{\partial \hat{v}} \\ \frac{\partial g^3}{\partial v_1^{max}} & \frac{\partial g^3}{\partial v_2^{max}} & \frac{\partial g^3}{\partial \hat{v}} \end{pmatrix}$. D must be negative definite as otherwise the situation would not be stable in a sense that values of the outside utilities above the thresholds would not necessarily lead to a termination. Thus, we assume negative definitess of D from now on . To obtain the impact of divorce laws on the probability P, we first find the impact of divorce laws on each of the three thresholds and then combine all outcomes. For example, note that $\frac{d\mathrm{P}}{dk_i} = \frac{\partial \mathrm{P}}{\partial v_1^{max}}\frac{\partial v_1^{max}}{\partial k_i} + \frac{\partial \mathrm{P}}{\partial v_2^{max}}\frac{\partial v_2^{max}}{\partial k_i} + \frac{\partial \mathrm{P}}{\partial \hat{v}}\frac{\partial \hat{v}}{\partial k_i}$.

To get tractable characterizations for the comparative statics, we first derive some helpful expression:

Using above expression for P and PE[$\tilde{v}$ | NODIV], we obtain

$$\frac{\partial \mathrm{P}}{\partial v_1^{max}} = \int\limits_{v_1^0+v_2^0}^{\hat{v}} f_1(v_1^{max}) f_2(\tilde{v} - v_1^{max}) d\tilde{v} \geq 0$$

$$\frac{\partial \mathrm{P}}{\partial v_2^{max}} = \int\limits_{v_1^0+v_2^0}^{\hat{v}} f_1(\tilde{v} - v_2^{max}) f_2(v_2^{max}) d\tilde{v} \geq 0$$

$$\frac{\partial \mathrm{P}}{\partial \hat{v}} = f(\hat{v}) - \int\limits_{v_1^{max}}^{v_1^1} f_1(v_1) f_2(\hat{v} - v_1) dv_1 - \int\limits_{v_2^{max}}^{v_2^1} f_1(\hat{v} - v_2) f_2(v_2) dv_2 \geq 0 \text{ as } v_1^{max} + v_2^{max} \geq \hat{v}.$$

$$\frac{\partial \mathrm{PE}[\tilde{v}|\mathrm{NODIV}]}{\partial v_1^{max}} = \int\limits_{v_1^0+v_2^0}^{\hat{v}} f_1(v_1^{max}) f_2(\tilde{v} - v_1^{max}) \tilde{v} d\tilde{v}$$

$$\frac{\partial \mathrm{PE}[\tilde{v}|\mathrm{NODIV}]}{\partial v_2^{max}} = \int\limits_{v_1^0+v_2^0}^{\hat{v}} f_1(\tilde{v} - v_2^{max}) f_2(v_2^{max}) \tilde{v} d\tilde{v}$$

$$\frac{\partial \mathrm{PE}[\tilde{v}|\mathrm{NODIV}]}{\partial \hat{v}} = f(\hat{v})\hat{v} - \int\limits_{v_1^{max}}^{v_1^1} f_1(v_1) f_2(\hat{v} - v_1) dv_1 \hat{v} - \int\limits_{v_2^{max}}^{v_2^1} f_1(\hat{v} - v_2) f_2(v_2) dv_2 \hat{v}$$

Using the definitions of g^1, g^2 and g^3 and the previous results then gives

$$\frac{\partial g^1}{\partial v_1^{max}} = \delta \frac{\partial \mathrm{P}}{\partial v_1^{max}} \hat{v} - \delta \frac{\partial \mathrm{PE}[\tilde{v}|\mathrm{NODIV}]}{\partial v_1^{max}} - (1 - \delta \mathrm{P})$$

$$= \delta \int\limits_{v_1^0+v_2^0}^{\hat{v}} f_1(v_1^{max}) f_2(\tilde{v} - v_1^{max})(\hat{v} - \tilde{v}) d\tilde{v} - (1 - \delta \mathrm{P})$$

$$\frac{\partial g^1}{\partial v_2^{max}} = \delta \frac{\partial \mathrm{P}}{\partial v_2^{max}} \hat{v} - \delta \frac{\partial \mathrm{PE}[\tilde{v}|\mathrm{NODIV}]}{\partial v_2^{max}} = \delta \int\limits_{v_1^0+v_2^0}^{\hat{v}} f_1(\tilde{v} - v_2^{max}) f_2(v_2^{max})(\hat{v} - \tilde{v}) d\tilde{v}$$

$$\frac{\partial g^1}{\partial \hat{v}} = \delta \frac{\partial \mathrm{P}}{\partial \hat{v}} \hat{v} - \delta \frac{\partial \mathrm{PE}[\tilde{v}|\mathrm{NODIV}]}{\partial \hat{v}} = 0$$

$$\frac{\partial g^2}{\partial v_1^{max}} = \delta \frac{\partial \mathrm{P}}{\partial v_1^{max}} \hat{v} - \delta \frac{\partial \mathrm{PE}[\tilde{v}|\mathrm{NODIV}]}{\partial v_1^{max}} = \delta \int\limits_{v_1^0+v_2^0}^{\hat{v}} f_1(v_1^{max}) f_2(\tilde{v} - v_1^{max})(\hat{v} - \tilde{v}) d\tilde{v}$$

$$\frac{\partial g^2}{\partial v_2^{max}} = \delta \frac{\partial \mathrm{P}}{\partial v_2^{max}} \hat{v} - \delta \frac{\partial \mathrm{PE}[\tilde{v}|\mathrm{NODIV}]}{\partial v_2^{max}} - (1 - \delta \mathrm{P})$$

$$= \delta \int_{v_1^0+v_2^0}^{\hat{v}} f_1(\tilde{v} - v_2^{max}) f_2(v_2^{max})(\hat{v} - \tilde{v}) d\tilde{v} - (1 - \delta \mathrm{P})$$

$$\frac{\partial g^2}{\partial \hat{v}} = \delta \frac{\partial \mathrm{P}}{\partial \hat{v}} \hat{v} - \delta \frac{\partial \mathrm{PE}[\tilde{v}|\mathrm{NODIV}]}{\partial \hat{v}} = 0$$

$$\frac{\partial g^3}{\partial v_1^{max}} = \delta \frac{\partial \mathrm{P}}{\partial v_1^{max}} \hat{v} - \delta \frac{\partial \mathrm{PE}[\tilde{v}|\mathrm{NODIV}]}{\partial v_1^{max}} = \delta \int_{v_1^0+v_2^0}^{\hat{v}} f_1(v_1^{max}) f_2(\tilde{v} - v_1^{max})(\hat{v} - \tilde{v}) d\tilde{v}$$

$$\frac{\partial g^3}{\partial v_2^{max}} = \delta \frac{\partial \mathrm{P}}{\partial v_2^{max}} \hat{v} - \delta \frac{\partial \mathrm{PE}[\tilde{v}|\mathrm{NODIV}]}{\partial v_2^{max}} = \delta \int_{v_1^0+v_2^0}^{\hat{v}} f_1(\tilde{v} - v_2^{max}) f_2(v_2^{max})(\hat{v} - \tilde{v}) d\tilde{v}$$

$$\frac{\partial g^3}{\partial \hat{v}} = \delta \frac{\partial \mathrm{P}}{\partial \hat{v}} \hat{v} - \delta \frac{\partial \mathrm{PE}[\tilde{v}|\mathrm{NODIV}]}{\partial \hat{v}} - (1 - \delta \mathrm{P}) = -(1 - \delta \mathrm{P})$$

Now, we show that $\frac{d\mathrm{P}}{dk_i} = \frac{\partial \mathrm{P}}{\partial v_1^{max}} \frac{dv_1^{max}}{dk_i} + \frac{\partial \mathrm{P}}{\partial v_2^{max}} \frac{dv_2^{max}}{dk_i} + \frac{\partial \mathrm{P}}{\partial \hat{v}} \frac{d\hat{v}}{dk_i} \geq 0$ for $i = 1, 2$. As $\frac{\partial \mathrm{P}}{\partial v_1^{max}}$, $\frac{\partial \mathrm{P}}{\partial v_2^{max}}$ and $\frac{\partial \mathrm{P}}{\partial \hat{v}}$ are positive, it remains to show that $\frac{dv_1^{max}}{dk_i}$, $\frac{dv_2^{max}}{dk_i}$ and $\frac{\partial \hat{v}}{\partial k_i}$ are positive as well.

$$\frac{dv_1^{max}}{dk_1} = \frac{\begin{vmatrix} -(1-\delta) & \frac{\partial g^1}{\partial v_2^{max}} & \frac{\partial g^1}{\partial \hat{v}} \\ -\delta \mathrm{P}(1-\delta) & \frac{\partial g^2}{\partial v_2^{max}} & \frac{\partial g^2}{\partial \hat{v}} \\ -(1-\delta) & \frac{\partial g^3}{\partial v_2^{max}} & \frac{\partial g^3}{\partial \hat{v}} \end{vmatrix}}{|D|} = (1-\delta)(1 - \delta \mathrm{P})^2 \frac{\left(\delta \int_{v_1^0+v_2^0}^{\hat{v}} f_1(\tilde{v}-v_2^{max}) f_2(v_2^{max})(\hat{v}-\tilde{v}) d\tilde{v} - 1\right)}{|D|} \geq$$

0, as $|D| \leq 0$ and the negative definites of D further requires that

$$\frac{\partial g^1}{\partial v_1^{max}} = \delta \int_{v_1^0+v_2^0}^{\hat{v}} f_1(v_1^{max}) f_2(\tilde{v} - v_1^{max})(\hat{v} - \tilde{v}) d\tilde{v} - (1 - \delta \mathrm{P}) \leq 0.$$

$$\frac{dv_2^{max}}{dk_1} = \frac{\begin{vmatrix} \frac{\partial g^1}{\partial v_1^{max}} & -(1-\delta) & \frac{\partial g^1}{\partial \hat{v}} \\ \frac{\partial g^2}{\partial v_1^{max}} & -\delta \mathrm{P}(1-\delta) & \frac{\partial g^2}{\partial \hat{v}} \\ \frac{\partial g^3}{\partial v_1^{max}} & -(1-\delta) & \frac{\partial g^3}{\partial \hat{v}} \end{vmatrix}}{|D|} = -(1-\delta)(1 - \delta \mathrm{P})^2 \frac{\left(\delta \int_{v_1^0+v_2^0}^{\hat{v}} f_1(v_1^{max}) f_2(\tilde{v}-v_1^{max})(\hat{v}-\tilde{v}) d\tilde{v} + \delta \mathrm{P}\right)}{|D|} \geq$$

0

$$\frac{d\hat{v}}{dk_1} = \frac{\begin{vmatrix} \frac{\partial g^1}{\partial v_1^{max}} & \frac{\partial g^1}{\partial v_2^{max}} & -(1-\delta) \\ \frac{\partial g^2}{\partial v_1^{max}} & \frac{\partial g^2}{\partial v_2^{max}} & -\delta \mathrm{P}(1-\delta) \\ \frac{\partial g^3}{\partial v_1^{max}} & \frac{\partial g^3}{\partial v_2^{max}} & -(1-\delta) \end{vmatrix}}{|D|} = (1-\delta)(1 - \delta \mathrm{P})^2 \frac{\left[\delta \int_{v_1^0+v_2^0}^{\hat{v}} f_1(\tilde{v}-v_2^{max}) f_2(v_2^{max})(\hat{v}-\tilde{v}) d\tilde{v} - 1\right]}{|D|} \geq$$

0

$$\Rightarrow \frac{d\mathrm{P}}{dk_1} \geq 0$$

$$\frac{dv_1^{max}}{dk_2} = -(1-\delta)\left(1-\delta\mathrm{P}\right)^2 \frac{\left(\delta \int\limits_{v_1^0+v_2^0}^{\hat{v}} f_1(\tilde{v}-v_2^{max})f_2(v_2^{max})(\hat{v}-\tilde{v})d\tilde{v}+\delta\mathrm{P}\right)}{|D|} \geq 0$$

$$\frac{dv_2^{max}}{dk_2} = (1-\delta)\left(1-\delta\mathrm{P}\right)^2 \frac{\left(\delta \int\limits_{v_1^0+v_2^0}^{\hat{v}} f_1(v_1^{max})f_2(\tilde{v}-v_1^{max})(\hat{v}-\tilde{v})d\tilde{v}-1\right)}{|D|} \geq 0$$

$$\frac{d\hat{v}}{dk_2} = (1-\delta)\left(1-\delta\mathrm{P}\right)^2 \frac{\left[\delta \int\limits_{v_1^0+v_2^0}^{\hat{v}} f_1(v_1^{max})f_2(\tilde{v}-v_1^{max})(\hat{v}-\tilde{v})d\tilde{v}-1\right]}{|D|} \geq 0$$

$$\Rightarrow \frac{d\mathrm{P}}{dk_2} \geq 0$$

Concerning $\frac{d\mathrm{P}}{d\phi} = \frac{\partial \mathrm{P}}{\partial v_1^{max}}\frac{dv_1^{max}}{d\phi} + \frac{\partial \mathrm{P}}{\partial v_2^{max}}\frac{dv_2^{max}}{d\phi} + \frac{\partial \mathrm{P}}{\partial \hat{v}}\frac{d\hat{v}}{d\phi}$, we have

$$\frac{dv_1^{max}}{d\phi} = \left(1-\delta\mathrm{P}\right)^2\left[\overline{w}_1-\overline{w}_2(n)\right]\frac{2\delta \int\limits_{v_1^0+v_2^0}^{\hat{v}} f_1(\tilde{v}-v_2^{max})f_2(v_2^{max})(\hat{v}-\tilde{v})d\tilde{v}-\left(1-\delta\mathrm{P}\right)}{|D|} \gtreqless 0$$

$$\frac{dv_2^{max}}{d\phi} = -\left(1-\delta\mathrm{P}\right)^2\left[\overline{w}_1-\overline{w}_2(n)\right]\frac{2\delta \int\limits_{v_1^0+v_2^0}^{\hat{v}} f_1(v_1^{max})f_2(\tilde{v}-v_1^{max})(\hat{v}-\tilde{v})d\tilde{v}-\left(1-\delta\mathrm{P}\right)}{|D|} \gtreqless 0$$

$$\frac{d\hat{v}}{d\phi} = \left(1-\delta\mathrm{P}\right)^2\left[\overline{w}_1-\overline{w}_2(n)\right]\frac{\delta \int\limits_{v_1^0+v_2^0}^{\hat{v}} f_1(\tilde{v}-v_2^{max})f_2(v_2^{max})(\hat{v}-\tilde{v})d\tilde{v}-\delta \int\limits_{v_1^0+v_2^0}^{\hat{v}} f_1(v_1^{max})f_2(\tilde{v}-v_1^{max})(\hat{v}-\tilde{v})d\tilde{v}}{|D|} \gtreqless 0$$

$$\Rightarrow \frac{d\mathrm{P}}{d\phi} \gtreqless 0$$

Finally,

$\frac{d\mathrm{P}}{d\theta} = \frac{\partial \mathrm{P}}{\partial v_1^{max}}\frac{dv_1^{max}}{d\theta} + \frac{\partial \mathrm{P}}{\partial v_2^{max}}\frac{dv_2^{max}}{d\theta} + \frac{\partial \mathrm{P}}{\partial \hat{v}}\frac{d\hat{v}}{d\theta} \leq 0$, since

$$\frac{dv_1^{max}}{d\theta} = -\varphi_1(n)\left(1-\delta\mathrm{P}\right)^2 \frac{\left(\delta \int\limits_{v_1^0+v_2^0}^{\hat{v}} f_1(\tilde{v}-v_2^{max})f_2(v_2^{max})(\hat{v}-\tilde{v})d\tilde{v}-1\right)}{|D|} \leq 0$$

$$\frac{dv_2^{max}}{d\theta} = \varphi_1(n)\left(1-\delta\mathrm{P}\right)^2 \frac{\left(\delta \int\limits_{v_1^0+v_2^0}^{\hat{v}} f_1(v_1^{max})f_2(\tilde{v}-v_1^{max})(\hat{v}-\tilde{v})d\tilde{v}+\delta\mathrm{P}\right)}{|D|} \leq 0$$

$$\frac{d\hat{v}}{d\theta} = -\varphi_1(n)\left(1-\delta\mathrm{P}\right)^2 \frac{\left[\delta \int\limits_{v_1^0+v_2^0}^{\hat{v}} f_1(\tilde{v}-v_2^{max}) f_2(v_2^{max})(\hat{v}-\tilde{v})d\tilde{v} - 1\right]}{|D|} \leq 0$$

$$\Rightarrow \frac{d\mathrm{P}}{d\theta} \leq 0$$

The impact of children on marriage stability is ambiguous. On the one hand, a higher n decreases stability due to the efficiency loss after a divorce induced by $\theta < 1$. On the other hand, a potentially negative effect of alimony payments on marriage stability might induce a countereffect.

Fertility

Proposition 9: Assume the respective (IC-DE) constraint does not bind. If the impact of children on marriage stability is positive, equilibrium fertility might be higher than under full commitment. Otherwise it is lower. The impact of divorce laws on equilibrium fertility is ambiguous.

Proof: As the (IC-DE) constraint does not bind, players choose fertility to maximize

$U = w_{10} + w_{20}(1 - g_0(n)) + \varphi_1(n) + \varphi_2(n)$

$+\delta\left(\mathrm{P}[U_1^* + U_2^*] + (1-\mathrm{P})[\mathrm{E}[\tilde{U}_1 \mid \mathrm{DIV}] + \mathrm{E}[\tilde{U}_2 \mid \mathrm{DIV}]]\right)$

$= w_{10} + w_{20}(1 - g_0(n)) + \varphi_1(n) + \varphi_2(n)$

$+\frac{\delta}{1-\delta}\left(\overline{w}_1 + \overline{w}_2(n) + \varphi_2(n) + \mathrm{E}[\tilde{v}_1] + \mathrm{E}[\tilde{v}_2]\right) + \delta\varphi_1(n)\frac{\mathrm{P}+(1-\mathrm{P})\frac{1}{1-\delta}\theta}{1-\delta\mathrm{P}}$

$-\frac{\delta(1-\mathrm{P})}{1-\delta\mathrm{P}}\left(k_1 + k_2\right) - \frac{\delta}{1-\delta\mathrm{P}}\mathrm{P}\mathrm{E}[\tilde{v} \mid \mathrm{NODIV}]$

Making use of $\frac{\partial U}{\partial \hat{v}} = \frac{\delta}{(1-\delta\mathrm{P})}\left(\frac{d\mathrm{P}}{d\hat{v}}\hat{v} - \frac{d\mathrm{P}(\mathrm{E}[\tilde{v}|\mathrm{NODIV}])}{d\hat{v}}\right) = 0$ gives

$\frac{\partial U}{\partial n} = \frac{\partial U}{\partial v_1^{max}}\frac{dv_1^{max}}{dn} + \frac{\partial U}{\partial v_2^{max}}\frac{dv_2^{max}}{dn} - g' w_{20} + \frac{\delta}{1-\delta}\overline{w}_2' + \frac{1}{1-\delta}\varphi_2' + \varphi_1'\left(\frac{1+(1-\mathrm{P})\frac{\delta}{1-\delta}\theta}{1-\delta\mathrm{P}}\right) = 0$ or

$$\alpha^*\varphi_1' + \varphi_2' + (1-\delta)\left(\frac{\partial U^0}{\partial v_1^{max}}\frac{dv_1^{max}}{dn} + \frac{\partial U^0}{\partial v_2^{max}}\frac{dv_2^{max}}{dn}\right) = (1-\delta)g' w_{20} - \delta\overline{w}_2'$$

where

$$\alpha^* \equiv \frac{1-\delta+\delta(1-\mathrm{P})\theta}{1-\delta\mathrm{P}}$$

As $F(\hat{v}) \geq \mathrm{P}$and $\frac{d\alpha^*}{d\mathrm{P}} = \frac{\delta(1-\theta)(1-\delta)}{\left(1-\delta\mathrm{P}\right)^2} \geq 0$, we have $\alpha \geq \alpha^*$, where α was defined above for the case where the divorce decision is always made efficiently. On the one hand, having $\alpha^* \leq \alpha$ decreases equilibrium fertility as long as $\theta < 1$. Yet, if the term in brackets is positive (which is the case if $\frac{dv_i^{max}}{dn} > 0$ as $\frac{\partial U^0}{\partial v_1^{max}} = \frac{\delta}{\left(1-\delta\mathrm{P}\right)} \int\limits_{v_1^0+v_2^0}^{\hat{v}} f_1(v_1^{max}) f_2(\tilde{v}-v_1^{max})(\hat{v}-\tilde{v}) d\tilde{v} \geq 0$ and $\frac{\partial U^0}{\partial v_2^{max}} = \frac{\delta}{\left(1-\delta\mathrm{P}\right)} \int\limits_{v_1^0+v_2^0}^{\hat{v}} f_1(\tilde{v}-v_2^{max}) f_2(v_2^{max})(\hat{v}-\tilde{v}) d\tilde{v} \geq 0$), a countervailing effect exists, and we can not say which one dominates.

Comparative statics

Here we subsume all effects of for example k_i on *relationship stability* under $\frac{\partial^2 U}{\partial \mathrm{P}^2} \frac{\partial \mathrm{P}}{\partial k_i} \frac{\partial \mathrm{P}}{\partial n} + \frac{\partial U}{\partial \mathrm{P}} \frac{\partial^2 \mathrm{P}}{\partial n \partial k_i}$, where the sign is ambiguous for all divorce laws, since it depends on exact characteristics of the outside utility's distribution functions (which we did not specify further).

Thus,

$$\frac{dn^*}{dk_i} = -\frac{\frac{\partial^2 U^0}{\partial \mathrm{P}^2} \frac{\partial \mathrm{P}}{\partial k_i} \frac{\partial \mathrm{P}}{\partial n} + \frac{\partial U^0}{\partial \mathrm{P}} \frac{\partial^2 \mathrm{P}}{\partial n \partial k_i} + \delta \varphi_1' \left(\frac{1-\theta}{\left(1-\delta\mathrm{P}\right)^2} \right) \frac{\partial \mathrm{P}}{\partial k_i}}{\frac{d^2 U^0}{dn^2}} \gtrless 0$$

$$\frac{dn^*}{d\phi} = -\frac{\frac{\partial^2 U^0}{\partial \mathrm{P}^2} \frac{\partial \mathrm{P}}{\partial \phi} \frac{\partial \mathrm{P}}{\partial n} + \frac{\partial U^0}{\partial \mathrm{P}} \frac{\partial^2 \mathrm{P}}{\partial n \partial \phi} + \delta \varphi_1' \left(\frac{1-\theta}{\left(1-\delta\mathrm{P}\right)^2} \right) \frac{\partial \mathrm{P}}{\partial \phi}}{\frac{d^2 U^0}{dn^2}} \gtrless 0$$

$$\frac{dn^*}{d\theta} = -\frac{\frac{\partial^2 U^0}{\partial \mathrm{P}^2} \frac{\partial \mathrm{P}}{\partial \theta} \frac{\partial \mathrm{P}}{\partial n} + \frac{\partial U^0}{\partial \mathrm{P}} \frac{\partial^2 \mathrm{P}}{\partial n \partial \theta} + \delta \varphi_1' \left(\frac{1-\theta}{\left(1-\delta\mathrm{P}\right)^2} \right) \frac{\partial \mathrm{P}}{\partial \theta}}{\frac{d^2 U^0}{dn^2}} \gtrless 0$$

Proposition 10: Assume $n_1^{**} > n_2^{**}$ and that the respective (IC-DE) constraint binds. Then, higher divorce costs and a lower level of θ increase equilibrium fertility. The impact of alimony payments is ambiguous.

Proof:

Recall that equilibrium fertility when $n_1^{**} > n_2^{**}$ is characterized by the binding (IC-DE) constraint, taking into account that Δv_i is sufficiently big to make an immediate divorce optimal off equilibrium:

$$u_{21}(n^*) - u_{21}(n^{**}) + \delta\left[\overline{U}_1^*(n^*) + \overline{U}_2^*(n^*) - \left(\overline{U}_1(n^*) + \overline{U}_2(n^{**})\right)\right] = 0 \text{ or}$$

$$u_{21}(n^*) - u_{21}(n^{**}) + \tfrac{\delta}{1-\delta}\left(\overline{w}_2(n^*) + \varphi_2(n^*)\right) - \tfrac{\delta}{1-\delta}\left(\overline{w}_2(n^{**}) + \varphi_2(n^{**})\right)$$
$$+\frac{\delta \mathrm{P}_{\varphi_1(n^*)(1-\theta)}}{(1-\delta\mathrm{P})} + \frac{\delta\mathrm{P}}{(1-\delta\mathrm{P})}(1-\delta)(k_1+k_2)$$
$$-\frac{\delta}{(1-\delta\mathrm{P})}\mathrm{PE}[\tilde{v} \mid \mathrm{NODIV}] + \tfrac{\delta}{1-\delta}\phi[\overline{w}_2(n^{**}) - \overline{w}_2(n^*)] = 0$$

As

$$\frac{\partial(IC-DE)}{\partial v_1^{max}} = \frac{\delta}{(1-\delta\mathrm{P})}\int_{v_1^0+v_2^0}^{\hat{v}} f_1(v_1^{max})f_2(\tilde{v} - v_1^{max})(\hat{v} - \tilde{v})d\tilde{v} \geq 0$$

$$\frac{\partial(IC-DE)}{\partial v_2^{max}} = \frac{\delta}{(1-\delta\mathrm{P})}\int_{v_1^0+v_2^0}^{\hat{v}} f_1(\tilde{v} - v_2^{max})f_2(v_2^{max})(\hat{v} - \tilde{v})d\tilde{v} \geq 0 \text{ and}$$

$\frac{\partial(IC-DE)}{\partial \hat{v}} = 0$, we have

$$\frac{dn}{dk_i} = -\frac{\frac{\partial(IC-DE)}{\partial v_1^{max}}\frac{dv_1^{max}}{dk_i} + \frac{\partial(IC-DE)}{\partial v_2^{max}}\frac{dv_2^{max}}{dk_i} + \frac{\partial(IC-DE)}{\partial k_i}}{\frac{\partial(IC-DE)}{\partial n}} \geq 0$$

$$\frac{dn}{d\phi} = -\frac{\frac{\partial(IC-DE)}{\partial v_1^{max}}\frac{\partial v_1^{max}}{\partial \phi} + \frac{\partial(IC-DE)}{\partial v_2^{max}}\frac{\partial v_2^{max}}{\partial \phi} + \frac{\partial(IC-DE)}{\partial \phi}}{\frac{\partial(IC-DE)}{\partial n}} \gtreqless 0$$

$$\frac{dn}{d\theta} = -\frac{\frac{\partial(IC-DE)}{\partial v_1^{max}}\frac{\partial v_1^{max}}{\partial \theta} + \frac{\partial(IC-DE)}{\partial v_2^{max}}\frac{\partial v_2^{max}}{\partial \theta} + \frac{\partial(IC-DE)}{\partial \theta}}{\frac{\partial(IC-DE)}{\partial n}} \leq 0$$

where we make use of the fact that $\frac{\partial(IC-DE)}{\partial n}$ must be negative.

Chapter 2

Minimum Wages and Relational Contracts

2.1 Introduction

Minimum wage laws and associated positive or negative effects are one of the most controversially debated issues in economics. Many discussions deal with distributional aspects or employment effects, but also the impact on employees not directly affected by the minimum wage is analyzed. When trying to understand its consequences, however, only limited attention has been paid to how a minimum wage might affect incentives. Especially aspects like the hold-up problem, relationship specific investments, or the efficiency of employment relationships have basically been neglected, and we are just beginning to understand potential interactions (see MacLeod, 2010).

The following paper shows that if agents must be motived to exert effort, various – empirically observed – consequences of a minimum wage can be explained. Then, a higher minimum wage can increase employment, reduce turnover of employees, have spillover effects on higher wages, and reduce wage dispersion. The driving factor behind these results is a firm's optimal choice of incentives. If firms are forced to pay a higher wage than actually intended, they will also require their employees to work harder. Furthermore, if only relational contracts, i.e., contracts based on observable but non-verifiable measures, can be used, an appropriate a minimum wage can even be efficiency-enhancing and increase the surplus within an employment relationship.

The need to give incentives has largely been neglected in the (theoretical) literature on minimum wages.[1] For example, this aspect is absent in search-and-matching models, which assume a prominent part in explaining a lot of empirical results (see Flinn, 2006, or Dube, Lester, and Reich, 2011). However, many employees receiving a minimum wage are subject to performance pay, like waiters or retail employees (Kadan and Swinkels, 2010). In addition, it might be impossible or too costly to verify certain aspects of an occupation within these industries. For example, employees of a fast food restaurant are supposed to be friendly to customers and careful when preparing their products. A barber will have to exert effort to provide sufficient quality, and a nightwatchman can be more or less attentive. While all these jobs have standardized components – for example the time an employee must be present – the aspects pointed out generally can not be verified. Then, incentives can only be given in dynamic games, with the use of relational contracts.[2] More precisely, this implies that employees need incentives to perform a desired task but that neither effort nor a signal induced by the employee's effort are verifiable[3]. A court-enforceable contract that induces the agent to work is thus not feasible, and all contingent compensation must be enforceable within an equilibrium of the dynamic game. Then, cooperation can only be sustained if the discounted future value of the employment relationship is sufficiently high.

The present paper uses relational contracts to analyze the impact of a minimum wage. In addition, a labor market with many homogeneous firms (also denoted as principals) and employees (or agents) exists, with more employees than firms.[4] The market is frictionless, and no (exogenous) turnover costs exist, why it is always possible for a firm to costlessly replace an agent. Furthermore, the market is not fully transparent in a sense that if turnover occurs, it is not possible to detect the reason, i.e., if an agent is fired or leaves voluntarily. Thus, a firm cannot build up an external – or market – reputation for honoring its promises. This creates a commitment problem: Instead of making promised payments as a reward for previous effort, firms might have an incentive to renounce and replace employees. Therefore, the only way to induce agents to work is the existence

[1] Exceptions are Rebitzer and Taylor (1995) who apply an efficiency wage model, or Kadan and Swinkels (2010), who use a standard moral hazard setting.

[2] For an elaborate and complete analysis of relational contracts see MacLeod & Malcomson (1989) and Levin (2003).

[3] Alternatively, they might be verifiable, but actually verifying them is too costly.

[4] See MacLeod & Malcomson (1998) for a relational contracts model within a market setting.

of endogenous turnover costs, i.e., as part of an equilibrium. A natural way to induce these costs (and preventing surplus destruction) is an equilibrium where all new agents receive a rent (as in MacLeod and Malcomson, 1998) which is at least as high as the payment promised to agents as a reward for their effort. However, firms are also exposed to these turnover costs whenever their employees leave for exogenous reasons. Although they have all bargaining power, firms are thus not able to capture the whole surplus of an employment relationship. Then, they face a tradeoff between giving high incentives (induced by high wages) and reducing turnover costs (which also increase with equilibrium wages). Even if maximum incentives are possible, employers voluntarily decrease them and enforce an effort level which is inefficiently low. Forcing firms to pay higher wages – for example induced by a minimum wage – will make it optimal for firms to let agents work harder, inducing a surplus increase.

Although the minimum wage increases efficiency, profits of the firms go down, which should have an impact on employment. However, the previous results were derived in a setup where the number of firms is fixed and each of them employs exactly one agent. Then, a mininum wage does not affect employment, at least as long as firms' profits are still positive. Instead, it redistributes surplus from firms to employees, a result empirically confirmed by Holzer, Katz and Krueger (1991).

To capture employment effects as well, the model is extended accordingly. A minimum wage can have positive or negative employment effects, which crucially depends on whether firms make positive profits or not. Firms make positive profits if they have considerable market power or had to face sunk investment costs at the time of entry. Profits are certainly made in some industries where a minimum wage applies (take the fast-food industry and some of its major players), but probably not for all. Thus, we analyze both cases separately. First, firms have no market power and can freely enter the market, implying that employment is characterized by a zero-profit condition. As a minimum wage decreases firms' profits, it also lowers total employment. However, the surplus of each remaining individual employment relationship increases. Thus, the impact of a minimum wage on total (or market) efficiency might or might not be positive.

Subsequently, we assume that profits are positive. Furthermore, a firm can employ many agents. Then, employment is chosen efficiently for a given level of equilibrium effort. However, since firms voluntarily decrease incentives to reduce endogenous turnover costs, effort and consequently also employment will be inefficiently low. By increasing effort, a minimum wage thus also induces a firm to employ more agents than before.

The impact of a minimum wage on employment thus depends on the affected firms' profitability. This might explain why positive employment effects can be observed in the fast-food industry (for the most prominent example see Card & Krueger, 1994), where several large players have a substantial degree of market power.

Furthermore, we test the robustness of our main result – that a minimum wage increases induced effort levels – in an almost identical setup, but where the principal is not able to observe an agent's effort. Instead, he can just use the resulting output as an imperfect signal. If a minimum wage is sufficiently high, it will still cause higher effort and efficiency levels. In addition, asymmetric information can make it optimal to use a termination threat to give incentives. The reason is that if only contingent payments are used, compensation for a good will be higher than for a low outcome. Since the low wage must not exceed the minimum wage, compensation after observing the good outcome must be adjusted accordingly to maintain incentives (giving rise to spillover effects caused by the minimum wage). However, it is also possible to fire the agent after a low outcome. This raises incentives but also increases turnover costs (which are still required to keep the principal from reneging). If the minimum wage is sufficiently high, it is optimal to always fire an agent after a observing the low outcome. Then, only one wage is paid to all agents, implying that compensation is more compressed than before.

Summarizing, turnover levels are generally higher when a minimum wage is present. However, a marginal increase of the minimum wage will at some point induce a decrease of employment turnover. The latter is implied by the positive impact of the wage floor on effort. When agents work harder, the likelihood of a low output – and correspondingly the probability of termination – decreases.

All of these results have been observed empirically. Industries facing a minimum wage are usually characterized by high turnover levels (see Brown et al., 1982). We show that this does not have to be an exogenuously given property but can also be driven by a firm's consideration to give incentives optimally. Furthermore, the negative marginal impact of a higher minimum wage on turnover has been found by Portugal and Cardoso (2006), Dube, Naid and Reich (2007) and Dube, Lester, and Reich (2011). Evidence for a negative impact of minimum wages on wage dispersion is provided by Grossman (1983), Katz and Krueger (1998) or Lee (1999). Finally, spillover effects of a minimum wage have been found as well (see Card and Krueger, 1995, or Neumark and Wascher, 2008, for summaries).

In the last part of this paper, we discuss the assumption of non-verifiability of effort and output. This aspect is necessary for the normative component of our arguments, namely that a minimum wage can increase efficiency. The positive component, i.e., the explanation of many observed consequences of the minimum wage, does not require this assumption. It is also possible to get these results if the agent's effort is verifiable, only the need to give incentives is crucial. Therefore, we show that for a binding minimum wage, a further decrease also raises the induced effort level if effort can be verified. However, higher effort now is not associated with more efficiency, since the principal would induce first-best effort absent a minimum wage. Furthermore, the minimum wage has a spillover effect on higher wages and – if a firm making positive profits can employ more than one agent – also might raise employment.

Related Literature

An important and considerable amount of research deals with employment effects of minimum wages. The hypothesis derived from the standard textbook model of a labor market – that a binding minimum wage leads to job losses – is now seriously questioned. Empirical studies like Card & Krueger (1994), Katz and Krueger (1992), Machin and Manning (1994) and most recently Dube, Lester and Reich (2010) suggest that the employment effect of a minimum wage is not necessarily negative and might even be slightly positive. Other papers (for a good overview see Brown (1982) or Neumark and Wascher (2007)) still claim that a minimum wage destroys jobs. Many theoretical models have been developed to explain the observed patterns. Bhashkar and To (1999) develop a model of monopolistic competition where a minimum wage raises employment per firm but causes firms to exit the market. However, they do not take the need to give incentives into account, and require job characteristics and agents' reservation wages to be heterogenous.

Other models stress the importance of match specific human capital (see Miller, 1984 or Flinn, 1986). Generally, most theoretical papers assume that labor markets are characterized by rent-creating frictions (for a recent approach see Flinn, 2006). The present paper does not need labor-market frictions to get positive employment effects of a minimum wage. Furthermore, none of our results depends on heterogeneity with respect to job and/or worker characteristics. Even if all principals and agents are identical and the labor market is frictionless, a minimum wage can increase employment. We only require that firms make positive profits ex post, no matter whether this is induced by fixed entry costs or reduced competition on the product market.

Less focus has been put on the interaction of a minimum wage with individual incentives and effort choices. Exceptions are Kadan and Swinkels (2009, 2010), where the impact of a wage floor in the standard moral hazard setting is analyzed. They claim that a minimum wage generally has a negative impact on induced effort levels. Different from our setting, they assume that agents are risk averse, effort can not be observed and an explicit contract is feasible. Then, a higher wage floor (i.e. payments that have to be made for the lowest output realization) generally increase the marginal costs of inducing effort, reducing total incentives given to employees. Total employment might or might not fall, depending on whether incentives are redistributed among agents. Unfortunately - as also noted by Kadan and Swinkels (2010) - there seems to be no empirical paper so far studying the impact of minimum compensation on productivity.

To some extent, we can use results from the literature on the impact of unionization on firm productivity. At least in the USA, this interaction appears to be positive, and unionized firms are more productive (for example, see Brown and Medoff, 1978, Clark, 1980 or Allen, 1984). Furthermore, although their impact on productivity seems positive, the effect of unions on employment is small. This outcome has been regarded as unintuitive by Wessels (1985), but perfectly fits with our model as long as firms make positive profits.

However, our results depend on the ability of firms to costlessly fire agents. This is not the case for countries with rigid employment protection laws. There, a minimum wage is likely to have only a little or no positive impact on efficiency. This might explain why unionized firms tend to be more productive in the US, whereas this relation is more ambiguous in countries like the UK, Germany or Japan (Doucouliagos and Laroche, 2003).

2.2 Model Setup

The economy consists of a mass M of small, identical firms ("principal", "he") and a mass of $N > M$ identical employees ("agent", "she"). Principals and agents are risk neutral. The time horizon is infinite, time is discrete (periods are denoted $t = 1, 2, ...$), and all players share a common discount factor δ. As principals and agents are identical, they are not further indexed.

At the beginning of each period t, we distinguish between whether a player currently is part of a match or not (in $t = 1$, everyone is obviously unmatched). Each unmatched principals can offer a contract to exactly one unmatched agent (we assume that this process is completely arbitrary and contains no frictions).[5] This offer consists of a legally enforceable wage payment w_t which is not restricted to positive values. A principal who does not make an offer gets an outside utility $\overline{\pi}$ in the respective period, where we make the normalization $\overline{\pi} = 0$. If an unmatched agent receives no offer from any principal or receives one and rejects it, she has to consume her exogenuous outside utility, which is normalized to zero as well. If she gets an offer and accepts it, she consumes w_t and then chooses an effort level $e_t \in [0, 1]$. This leads to output $y_t = \theta$ with probabability e_t, and to $y_t = 0$ with probability $(1 - e_t)$. While the output is directly consumed by the principal, the agent suffers effort costs $c(e_t)$, with c', c'', $c''' > 0$, $c(0)$, $c(0)' = 0$ and $c(1)$ sufficiently large to never be optimal. After output realization, each agent - no matter whether employed or not - leaves the market for exogenuous reasons – for example because the partner found a job somewhere else – with probability $(1 - \gamma)$ and remains for another period with probability γ. If an agent exits the market for exogenous reasons, she also receives a payoff equalized to zero.[6] Furthermore, she does not return in any subsequent period. To keep the number of employees fixed over time, $(1 - \gamma)N$ new agents enter the market in every period. At the end of the period, the principal can make a new offer consisting of the wage payment w_{t+1} to a remaining agent. If the agent accepts it, the above procedure is repeated in the next period: the agent receives w_{t+1} and chooses effort e_{t+1}, after which the output y_{t+1} is realized. In each other case, i.e. if the principal does not make an offer or the agent does not accept, both enter the matching market in the

[5] The case where a principal can employ more than 1 agents is considered below.

[6] Note that this assumption is without loss of generality even when the agent expects a positive utility while being on the market.

next period.

Using $d_t^P \in \{0,1\}$ to describe whether a principal is in a relationship, the payoff stream of an arbitrary principal at the beginning of a period t equals

$$\Pi_t = \mathrm{E}\left[\sum_{\tau=t}^{\infty} \delta^{\tau-t} d_\tau^P \left(e_\tau \theta - w_\tau\right)\right]$$

where the expectation is over effort and wage levels, which might depend on whether the principals enters a new relationship in a period or keeps his past employee.

Using $d_t^A \in \{0,1\}$ to describe whether an agent is in a relationship, an arbitrary agent receives

$$U_t = \mathrm{E}\left[\sum_{\tau=t}^{\infty} \delta^{\tau-t} d_t^A \left(w_t - c(e_t)\right)\right]$$

Here, expectation is over d_t^A – an agent can leave the market for exogenuous reasons, while if she remains, she might or might not receive an offer from a principal.

We assume the following information structure. Within a match, there is no asymmetric information. This implies that effort and output can be observed by both players. Still, neither effort nor output are verifiable, i.e., no explicit contract using them can be written.

All players outside a match ("the market") can not observe anything that happens within.[7] However, the market can see whether an agent leaves a relationship. It cannot detect the reason, i.e., whether she left for exogenous reasons, did not receive a new offer from her previous employer, or decided to not accept an offer herself. Finally, firms can not distinguish new agents from those that already have been on the market for a longer time.

Although information within a match is perfectly symmetric, we thus have an infinitely repeated game of imperfect public monitoring. Any employee has no information concerning the offers a principal made in the past but can use each firm's turnover history as an imperfect signal. Therefore, we follow the literature and use the solution concept of a public perfect equilibrium (PPE) in a sense that each player's actions only depend

[7] Whether or not the only verifiable component - the wage payment - can be observed by the market is of no relevance here.

on the public history they share with the respective partner. Put differently, the offer a principal makes to a new agent only depends on his turnover history; the same is true for the agent's acceptance- and first-period effort-decision. When a match has been active for a while, players' actions in addition depend on the events that were observed in the relationship.[8] We will later impose some additional restrictions on strategies used by players. These restrictions simplify the analysis with having a substantial impact on the results.

Note that no additional bonus is used, and payment contingent of the agent's effort level only occurs wia the wage w_t. However, this assumption is without loss of generality given our model assumptions (for an equivalent argument see Fong and Li, 2010), since bonus payments could be postponed until the next period (taking the possibility of exogenous termination into account), then becoming part of the fixed wage.

2.3 Stationary Contracts and the Commitment Problem

Contracts can be stationary without loss of generality in a sense that for any new match, effort and contingent compensation are constant as long as the relationship remains.[9] The reason is that it does not matter for an agent whether she is compensated for her effort immediately (i.e., by the wage paid in the subsequent period) or by a (credible) promise for some future payments (always taking into account the possible termination of the relationship). Each non-stationary contract could thus be replaced by a stationary one, namely by averaging out changes in promised continuation payoffs.

However, focusing on individually stationary contracts does not automatically imply that stationarity of the whole game can be assumed without loss of generality. Each new contract can depend on the whole turnover history of a principal. This is relevant because market intransparency does not allow a principal to build up an external reputation for keeping his promises, and thus creates an additional commitment problem. The market

[8] We analyze the case of asymmetric information - the principal cannot observe the agent's effort choice - later; then, the equilibrium concepts remains the same in a sense that within a relationship, actions only depend on what was observed by both players.

[9] See Levin (2003) or the Appendix of MacLeod and Malcomson (1998), where the latter considers the case with replaceable agents.

only receives an imperfect signal, namely a firm's total turnover, but cannot detect the reason. Thus, principals are not able to extract the whole feasible surplus, although they are endowed with all the bargaining power. To better understand this point, assume that the maximum feasible surplus is realized (feasible in a sense that it satisfies all relevant constraints derived below). Furthermore, assume that the principal completely gets this surplus. Then, he would always renege, fire an agent at the end of the period instead of offering her a new contract, and employ a new one. Therefore, turnover has to be costly for the principal, where these costs could for example assume some kind of surplus destruction or an upfront payment to new agents (i.e., a payment that cannot be used to give incentives). For convenience, we assume that the principals use the latter tool, and pay new agents a rent.

To simplify the following analysis, we restrict attention to *contract-specific* strategies in the sense of Board and Meyer-ter-Vehn (2011). This implies that actions of the firm and workers do not depend on the identity of the worker, calendar time, or history outside the current relationship. However, this assumption has only a slight impact on generality in our setup. Not using information outside a current relationship implies that the firm's total turnover history is not part of strategies. Instead, the punishment imposed on a principal when starting a new employment relationship (i.e., the rent going to a new agent) is always the same. Thus, we only have to distinguish between any period in an ongoing and the first period of a new relationship. In Appendix I, we prove that giving new agents the same rent independent of the whole turnover history is without loss of generality.

Furthermore, conditioning strategies on calendar time could increase a firm's rent. The reason is that future turnover costs are needed to persuade agents today that promises are honored. Thus, turnover costs are only necessary for all periods except for the first period of the whole game. However, relaxing the assumption of contract-specific strategies and allowing for a different countract in the first period of the game would have no qualitative impact on our results and just slightly increase profits and equilibrium effort levels.

Concluding, the punishment, i.e., the rent going to new agents, will keep the principal from reneging and firing agents in equilibrium. However, he cannot completely prevent this punishment, but will be exposed to it all the time an agent leaves for exogenuous reasons and has to be replaced. As these costs must increase with equilibrium compensation, the principal faces a trade-off between giving optimal incentives and reducing turnover costs, a problem inducing him to voluntarily reduce the feasible effort level and thus the relationship surplus.

2.4 Payoffs and Constraints

Let us denote the stationary equilibrium effort level e^*, the wage in ongoing relationships w, and U the agent's payoff in such an ongoing relationship. Then, U can be defined recursively as

$$U = w - c(e^*) + \delta\gamma U \tag{2.1}$$

The discounted future stream δU only enters with probability γ, since there is a chance that the agent leaves the market for exogenous reasons (happening with probability $(1-\gamma)$), then receiving a payoff of zero.

If an agent is on the market and gets a job, her expected payoff in the first period of employment equals

$$U^0 = w^0 - c(e^*) + \delta\gamma U$$

where w^0 denotes the wage an agent receives in this first period of employment.

Finally, an agent currently unemployed but on the market at the beginning of a period receives a job offer with probability $\mu \equiv \frac{(1-\gamma)M}{(N-M)+(1-\gamma)M} = \frac{(1-\gamma)M}{N-\gamma M}$. Thus, an agent's endogenous reservation utility $\overline{U}$ equals

$$\overline{U} = \mu U^0 + \delta(1-\mu)\gamma\overline{U}$$

There, note that an agent remains in the market with probability γ, no matter whether she currently is employed or not.

In equilibrium, some constraints have to be satisfied for each agent who is part of a match. First of all, an agent must prefer to be employed rather than not. This implies that the utilities U and U^0 must exceed the endogenuous reservation utility $\overline{U}$. This is captured by an agent's individual rationality (IRA) constraints,

$$\begin{aligned} U &\geq \overline{U} \\ \text{and} \quad U^0 &\geq \overline{U} \end{aligned}$$

Furthermore, given U, U^0 and $\overline{U}$, it must be in the interest of an employed agent to actually choose equilibrium effort e^*, i.e., her incentive compatibility (IC) constraint must be satisfied. Here, we assume that an agent who does not exert e^* is fired[10] and re-enters the job market in the subsequent period. If an agent deviates and chooses effort $e \neq e^*$, she will obviously set $e = 0$ (or put differently: if satisfied for $e = 0$, (IC) also holds for any other effort level). Thus, (IC) equals

$$c(e^*) \leq \delta\gamma \left(U - \overline{U}\right)$$

A principal's payoff starting a new relationship is denoted Π^0, while he gets Π in an ongoing match. These payoffs can be characterized recursively as well, giving

$$\begin{aligned} \Pi &= e^*\theta - w + \delta[\gamma\Pi + (1-\gamma)\Pi^0] \\ \text{and} \quad \Pi^0 &= e^*\theta - w^0 + \delta[\gamma\Pi + (1-\gamma)\Pi^0] \end{aligned}$$

Each principal faces some constraints as well. First of all, starting a new employment relationship should be better than shutting down completely, giving the individual rationality (IRP) constraint

$$\Pi^0 \geq 0.$$

Furthermore, each principal must have an incentive to honor his promises. If he reneges and offers a wage different from the one specified in the relational contract, we make the standard assumption that all trust is lost in the specific relationship, the employed agent does not believe the firm‘s promises anymore, and thus is not willing to exert positive effort from then on. After a deviation, the principal thus has the choice to either shut down and receive his exogenous reservation utility with a value of zero, or to

[10]Which is optimal for the principal due to the observability of effort and will not occur in equilibrium.

employ a new agent. Since either choice must not be optimal, we have two constraints, where the incentive compatibility (ICP) constraint covers the first case and equals

$$\Pi \geq 0$$

The second one is denoted the non-reneging (NR) constraint, and characterized by

$$\Pi \geq \Pi^0$$

Obviously, (ICP) is automatically satisfied given (IRP) and (NR), and we can omit it from now on.

2.5 Basic Results

As firms have all bargaining power, we focus on outcomes that maximize a principal's profits (subject to the relevant constraints). This can be motivated by assuming that each firm with a vacant employment opportunity makes a take-it-or-leave-it offer to an arbitrary agent. Considering profit maximizing equilibria is without loss of generality in a standard relational contracts setting with one principal and one agent. There, giving incentives can be separated from surplus distribution. However, since the market is not transparent, the profit maximizing equilibrium yields a different outcome with respect to effort and efficiency than other equilibria, where either agents' rents or total surplus is maximized. The missing transparency creates a commitment problem and forces each principal to induce endogenous turnover costs. This creates a tradeoff between surplus maximization and the minimization of these turnover costs. Therefore, firms decide to enforce an effort level lower than the one that (possibly) maximizes total surplus.

Solving for the equilibrium that maximizes a firm's profits, we assume players only use the aforementioned contract-specific strategies. Since we do not consider the possibility of collusion among firms, every principal takes $\overline{U}$ as given. Recall that the equilibrium wage level in an ongoing relationship is denoted w, while the wage paid at the beginning of a new employment relationship equals w^0.

Then, a principal's (NR) constraint – which is supposed to keep him from renouncing and replacing an employed agent – boils down to

$$w^0 \geq w. \tag{2.2}$$

As this constraint obviously binds, an agent always receives the same wage, no matter whether she is a new employee or not.

Proposition 1: Given $\Pi > 0$, the induced per period effort level e^* is characterized by $c' = \delta\gamma\theta$.

Proof: First, note that (NR) and (IC) constraints are satisfied as equalities in equilibrium. Since w^0 can not be used to give incentives and thus just redistributes funds to the agent, the principal will set it as low as possible. Similarily, for a given effort level e^*, there is no point in offering the agent a higher wage than necessary to satisfy her (IC) constraint.

This allows us to compute wage levels, $w^0 = w = \frac{c(e^*)}{\delta\gamma} + \overline{U}(1-\delta\gamma)$, and substitute them into the principal's profit function,

$\Pi = \frac{1}{1-\delta}\left(e\theta - w\right) = \frac{1}{1-\delta}(e\theta - \frac{c(e^*)}{\delta\gamma} - \overline{U}(1-\delta\gamma))$.

Then, the first order condition (which also is sufficient given the modelling assumptions) with respect to induced effort gives $\theta - \frac{c'}{\delta\gamma} = 0$, proving the proposition.

The induced effort level is lower than the efficient one, which is characterized by

$$c' = \theta$$

Note that - unlike in standard relational contracts models - the inefficiency is not induced by the future surplus being too low (which we assume it is not) but by the fact that when giving stronger incentives and consequently increasing the surplus, a principal also raises the rent he has to give to new agents. The bigger the probability that agents leave for exogenous reasons, the stronger are the tendencies to limit these losses by reducing incentives. As minimum wage workers tend to have high turnover rates (Brown et al, 1982), the efficiency loss induced by limited commitment should be considerable in these markets.

2.6 The Minimum Wage and Efficiency

The introduction of a minimum wage above market compensation has the following impact on production. Given that θ is sufficiently high (such that principals' profits remain non-negative), a binding minimum wage increases effort and total surplus of each employment relationship.

Proposition 2: A mandatory wage increase (for example induced by the introduction of a minimum wage) increases agents' effort and can raise the surplus created within each occupation.

Proof: Since $w = w^0$, and since the (IC) constraint binds, effort as a function of the wage level is implicitly defined by $w = \frac{c(e^*)}{\delta\gamma} + (1-\delta\gamma)\overline{U}$. Since a minimum wage holds for all firms, we also must take its impact on $\overline{U}$ into account, and have $\frac{de}{dw} = \frac{\delta\gamma(1-\mu)}{c'} > 0$.

The per period surplus created by an employed agent equals $s \equiv e\theta - c(e)$. Therefore, we have,

$$\frac{ds}{dw} = \frac{de}{dw}(\theta - c')$$

which is positive as long as $\theta \geq c'$.

The introduction of a (binding) minimum wage has two kinds of consequences. First of all, resources are distributed from firms to agents, which is in line with empirical results found by Holzer, Katz, and Krueger (1991). Furthermore, the total surplus created in the relationship increases.

Firms' profits, however, go down. For relationship starting after the introduction of the minimum wage, the impact on a firm's profits is given by

$$\frac{d\Pi^0}{dw} = \frac{1}{1-\delta}\frac{de}{dw}(\theta - \frac{c'}{\gamma}\frac{1}{1-\mu}) < 0$$

All our results are only valid as long as the principals' (IR) constraints do not bind and each firm makes a strictly positive profit. If the constraint binds, a mandatory wage increase would lead the firms to leave the market and not produce anymore, since otherwise they would make losses. We will analyze the case of (partial) market exit in more detail below .

Finally, note that using a bonus paid at the end of the period would not improve the situation for principals, as long as the total compensation in a period must not be below the miniumg wage.

2.7 Employment Effects of the Minimum Wage

The previous sections contain the assumption that each firm can employ exactly one agent and that – recall that M is fixed – total employment remains constant. This further requires each firm to make positive profits, since otherwise a minimum wage would have made it optimal to not employ an agent and receive the outside option of zero instead.

If we relax the assumption that M is fixed and analyze potential employment effects of a minimum wage, the results depend on whether firms make positive profits or not. If not, a (binding) minimum wage will in any case cause employment losses. If profits are made, a minimum wage can even increase employment and efficiency. In the following, we analyze both cases in isolation.

First, we assume that firms can enter the market at no costs and that the productivity parameter θ decreases with M. Since each firm only employs one agent, and since the number of firms is determined by a zero-profit condition, a minimum wage – reducing a firm's profits – will obviously induce an employment reduction.

Then, we allow firms to make positive profits. This might be a consequence of positive entry costs (i.e., although ex-ante profits are zero, they are positive after entry) or market power on the product market (which is certainly the case for big fast food chains). Now, a firm can hire more than one agents, but faces decreasing returns to scale. In this case, the profit maximizing employment level is efficient given the induced effort. As the commitment problem makes it optimal for firms to voluntarily reduce effort, however, employment is inefficiently low. Since a minimum wage increases effort, it will raise employment as well. Note that the mechanism here has nothing to do with arguments using monopsony power, which probably is the classical example for positive employment effects of a minimum wage. The latter requires agents to be heterogeneous with respect to their outside options. Here, the employment effect is solely driven by the minimum wage's positive impact on effort, and homogeneous agents make labor supply infinitely elastic.

In the first part, firms neither have market power on the product market, nor do they face substantial entry costs. Instead, they can freely enter and leave the market. Thus, M is endogenous and determined by a zero-profit condition. Each output depends on M via the differentiable and (strictly) decreasing function $\theta(M)$.[11] Furthermore, N is assumed to be sufficiently large for $N > M$ always to be satisfied in equilibrium.

Since we focus on contract-specific strategies, firms entering the market are treated equally to those already present for a longer time.[12] Thus, equilibrium employment M^* – as well as equilibrium effort – is characterized by the zero- profit-condition $\Pi(1-\delta) = e^*\theta(M^*) - w = 0$. Wage payments are given by an agent's (IC) constraint, which is identical to above. Thus, we can further specify equilibrium employment M^*, namely by

$$e^*\theta(M^*) - \frac{N - \gamma M^*}{N - M^*}\frac{c(e^*)}{\delta\gamma} = 0. \tag{2.3}$$

This immediately gives

Proposition 3: Assume equilibrium employment is characterized by 3.9. Then, the introduction of a (binding) minimum wage reduces employment.

Proof: Implicitly differentiating 3.9 gives $\frac{dM^*}{dw} = -\frac{\frac{de^*}{dw}(\theta(M^*) - \frac{c'}{\delta\gamma(1-\mu^*)})}{e^*\theta' - \frac{N(1-\gamma)}{(N-M^*)^2}\frac{c(e^*)}{\delta\gamma}}$. As equilibrium effort is still defined by $\theta(M^*) = \frac{c'}{\delta\gamma}$, the nominator – as well as the denominator – are

[11] For simplicity, we assume that this relation is independent of the number of realized total successes but that - given the output is high - the level depends on M always in the same way.

[12] If this were not the case new firms did not have to pay a rent to their first employed agent, they could immediately exit the market after their first agent left for exogenuous reasons. Since this would allow them to make strictly positive profits, establishing an equilibrium could be difficult. If $\theta(M)$ were strictly positive for all M, the number of firms would approach infinity (at some point, however, it would exceed the number of agents N). If $\theta(M)$ would be zero for $M \geq \tilde{M}$, at least $\tilde{M}$ firms, all inducing zero effort, would be on the market.

negative.

However, there will be a transition period. After the minimum wage is introduced, firms remain in the market until their agent leaves for exogenuous reasons (note that the payoff decrease firms face is mainly driven by employment relationships starting in the future. The existing agent only receives a slightly higher rent). Therefore, it will take some time until employment actually falls.

The impact of a minimum wage on total efficiency now is ambiguous. While individual matches become more productive, some output is lost due to the employment reduction. However, combining the minimum wage with a subsidy S can unambiguously increase efficiency. Just assume that each firm that employs an agent for the minimum wage receives S. If this subsidy is just high enough to offset the profit loss induced by the minimum wage, total employment will be identical to before. To finance the subsidy, an income tax $T = S$ could be collected. Then, agents are still better off than in the situation without a minimum wage, since the surplus increase more than offsets the tax T. An appropriate subsidy and tax policy combined with a minimum wage can therefore lead to a Pareto improvement in the respective market.

If firms make positive profits, because of market power or since they face sufficiently high entry costs,[13] a minimum wage rather increases employment. Now, assume that the number of firms is fixed but that each of them can employ an arbitrary number of agents. Furthermore, the production function of one firm only depends on the number of agents employed by this firm, and not on the total number of principals or employees in the market. Then, it is without loss of generality to that the market consists of just one principal (this implies that $\overline{U} = 0$). This firm's employment level is given by J, where we maintain the assumption that $J < N$ is satsified in equilibrium. The productivity of each employed agent is characterized by the differentiable and decreasing function $\theta(J)$.[14] Furthermore, $\theta'' < 0$ is assumed as a sufficient condition for a maximum.

[13]Furthermore, it is required that, upon entry, the firm did not expect the minimum wage to come.

[14]For convenience, we assume that an additional agent does not have a lower productivity than those employed previously, but that this new agents uniformly lowers the productivities of all employees. Assuming a production function where an additional agent has no impact on others' productivities would not give different results.

Here, we have to further specify the assumption concerning contract-specific strategies in a sense that the strategies of employed agents do not depend on the firm's behavior towards their colleagues. Otherwise, multilateral contracts in the sense of Levin (2002) could be feasible.

Then, the constraints that have to be satisfied are equivalent to above. As long as profits are positive, only an agent's (IC) and the firm's (NR) constraint – stating that all new agents have to receive at least the same wage as those with longer tenure – are relevant. As both will obviously bind, the firm's payoff stream in an arbitrary period $t > 1$ equals

$$\Pi(J) = J\frac{e\theta(J) - w}{1 - \delta}$$

Maximizing total profits, an agent's effort again is characterized by $\delta\gamma\theta(J^*) = c'$ as long as no binding minimum wage is present. Otherwise, i.e., with a minimum wage sufficiently high, effort is given by an agent's (IC) constraint, namely $\overline{w} = \frac{c(e^*)}{\delta\gamma}$.

Then, equilibrium employment J^* is characterized by $e\theta(J^*) + Je\theta' - w = 0$. For given effort and wage levels, employment is efficient. The reason is that labor supply is infinitely elastic. If effort remained constant, a minimum wage increase (given the minimum wage binds) would thus always be associated with an employment reduction, i.e., $\frac{dJ}{d\overline{w}} \mid_e$ constant$= \frac{1}{2e\theta' + Je\theta''} < 0$. However, a higher effort countervails the direct negative of a minimum wage on employment.

Proposition 4: Assume equilibrium effort and employment are characterized by $w = \frac{c(e^*)}{\delta\gamma}$ and $e\theta(J^*) + Je\theta' - w = 0$. Then, a higher minimum wage increases an agent's effort and total employment J.

Proof:

Note that we are only interested in the case that the minimum wage binds. Then, effort and employment are characterized by

$\overline{w} - \frac{c(e^*)}{\delta\gamma} = 0$ and $e\theta(J) + Je\theta' - \overline{w} = 0$.

Thus, we have $\frac{de}{dw} = \frac{\begin{vmatrix} -1 & 0 \\ 1 & 2e\theta' + Je\theta'' \end{vmatrix}}{\begin{vmatrix} -\frac{c'}{\delta\gamma} & 0 \\ \theta(J) + J\theta' & 2e\theta' + Je\theta'' \end{vmatrix}} = \frac{-(2e\theta'+Je\theta'')}{\begin{vmatrix} -\frac{c'}{\delta\gamma} & 0 \\ \theta(J) + J\theta' & 2e\theta' + Je\theta'' \end{vmatrix}} > 0,$ since $2e\theta' + Je\theta''$ is negative. Furthermore

$\frac{dJ^*}{dw} = \frac{\begin{vmatrix} -\frac{c'}{\delta\gamma} & -1 \\ \theta(J) + J\theta' & 1 \end{vmatrix}}{\begin{vmatrix} -\frac{c'}{\delta\gamma} & 0 \\ \theta(J) + J\theta' & 2e\theta' + Je\theta'' \end{vmatrix}} = \frac{-\frac{c'}{\delta\gamma}+\theta(J)+J\theta'}{\begin{vmatrix} -\frac{c'}{\delta\gamma} & 0 \\ \theta(J) + J\theta' & 2e\theta' + Je\theta'' \end{vmatrix}}.$ Substituting $\theta(J^*) + J^*\theta' = \frac{w}{e}$ gives $\frac{-\frac{c'}{\delta\gamma}+\frac{w}{e}}{\begin{vmatrix} -\frac{c'}{\delta\gamma} & 0 \\ \theta(J) + J\theta' & 2e\theta' + Je\theta'' \end{vmatrix}}$, which is positive as $-\frac{c'}{\delta\gamma} + w = 0$ and $e < 1$.

Thus, a minimum wage initially also increases efficiency. If it becomes too high, however, effort and employment turn out to be inefficiently high as well.

Summarizing, a minimum wage can only have a positive impact on employment if firms make positive profits. In many real life situations, intermediate cases might exist. Firms have some market power, but it is some extent bounded by entry. Then, a minimum wage might or might not increase employment.

2.8 Asymmetric Information

In this section, we show that our results continue to hold if the agent's effort is her private information and if the principal can only observe the resulting output. This result is important, since in many cases it will not be possible or simply too expensive for a firm to continuously monitor an employee's actions. For example, McDonald's does not have managers steadily taking care of whether its employees treat their customers nicely, and the owner of a barber shop will not always have an eye on all their staff.

We will thus check the robustness of our main result – that a minimum wage can increase the surplus of an employment relationship – in an identical setup but where the principal is not able to observe an agent's effort. Instead, he just can use the resulting output as an imperfect signal for an employee's cooperation. Then, the impossibility to create an external reputation again leads to an imposed effort level that is inefficiently small (even if first best effort were enforceable). If a minimum wage is sufficiently high, it will also lead to effort and surplus that is higher than before. However, the impact of a higher minimum wage on effort is not monotone. When it just becomes binding, the minimum wage first decreases effort and needs a certain level to impose the positive impact derived above. We do not explicitly analyze employment effects in this section. Those should not be too different from before as – in the case where firms make positive profits – the employment increase was solely driven by higher effort and not by effort being observable.

Furthermore, the case of asymmetric information raises some additional issues absent before. If the minimum wage becomes binding, it can be optimal for the principal to fire the agent with positive probability after a low outcome. The reason is that if an agent is never fired in equilbrium, incentives are solely given by two wage levels - a high wage following high outcome, and a low wage when $y = 0$. Since the low wage can not be below $\overline{w}$, the principal must also increase the high wage (for a given effort level). However, the inability to build up an external reputation forces the principal to pay the high wage to any new agent – otherwise, the principal would always renege after a good outcome. Instead of only using wages to give incentives, the principal could fire the agent after a low outcome with positive probability. Then, expected turnover costs increase for a given high wage, since agents do not only leave the firm for exogenuous reasons anymore. On the other hand, it becomes cheaper to give incentives, as the agent does not necessarily receive the minimum wage after a failure. If a minimum wage is sufficiently high, it becomes optimal for a firm to solely use a termination threat to induce effort. If agents are always fired after a low outcome, only one wage level for all agents – whether new or remaining – is paid.

Thus, a minimum wage not only has an impact on chosen effort levels, but also on turnover and wage compression within an industry. With respect to turnover, our model thus makes the following predictions. If a minimum wage exists, turnover should generally be higher than otherwise. However, if the minimum wage is sufficiently high, a further increase will lead to less turnover. The latter is implied by the minimum wage's positive impact on effort, which makes a realization of the low output – and thus a layoff – less likely. Furthermore, a minimum wage implies a compression of wages. When the minimum wage is low and employees are also kept after a low outcome, wages paid after a low and those after a high output realization are different. When the minimum wage is high and only termination is used to give incentives, all workers receive the same wage in every period.

Finally, spillover effects exist, i.e., minimum wages also have an impact on higher wages. As long as different wage levels exist (i.e, the termination probability after a low output is smaller than 1), the lower one will be equal to the minimum wage. However, the high wage will be affected as well – if incentives are supposed to remain at a constant level, for example, it might increase.[15]

There exists a considerable amount of empirical evidence that our predictions with respect to turnover, wage compression, and spillover effects are indeed observed when a minimum wage is present.

Generally, industries where a minimum wage is relevant – like the fast food industry – are characterized by high turnover levels (see Brown et al., 1982). Although low wage industries are generally considered to generally face high turnover, some of it might be driven by the a firm's consideration to give incentives optimally. Furthermore, early empirical suggest that industries where a minimum wage applies are actually associated with higher turnover rates (for example, see Wessels, 1980).

The negative marginal impact of a minimum wage increase on turnover is well established empirically. Portugal and Cardoso (2006) find that separations of teenage workers in Portugal decreased after a minimum wage increase, while Dube, Naid and Reich (2007) observe that average tenure rose substantially in restaurants in San Francisco. The most recent contribution is Dube, Lester, and Reich (2011), who also find strong evidence that turnover rates for teens and restaurant workers fall after a minimum wage increase.

An early contribution exploring wage compression is Grossman (1983), later followed by Katz and Krueger (1998), who find that the minimum wage has greatly compressed wages in the Texas fast food industry. Furthermore, Lee (1999) provides evidence that

[15]Below, we find that spillover effects also exist when effort is verifiable.

a substantial decline of the real minimum wage in the US was mainly responsible for a sharp increase of the wage dispersion among low income workers in the eighties.

Finally, for evidence on spillover effects see Card and Krueger (1995), or Neumark and Wascher (2008). Several reasons have been provided, for example that firms substitute low wage with high workers or that an adjustment of wages is necessary to maintain differentials between high and low skilled workers (Grossman (1983)). In our setting, spillover effects occur even with homogenous workers, and we neither need fairness perceptions (Falk et al., 2006) or a more advanced bargaining concept (Dittrich, Knabe, 2010) to show that an increase in reservation utilities induced by non-binding minimum wages has this impact.

We will now briefly present the main results formally, a full formal analysis – as well as the proofs to the associated propositions – is delegated to Appendix II. In each period t, the principal can observe y_t but not e_t (still, the output remains non-verifiable). Thus, the agent's compensation can only be based on past output levels. Let us denote the wage an agent receives after a success in the previous period w_t^+, while w_t^- equals the compensation after $y_{t-1} = 0$. In the latter case, i.e. after observing a low outcome, the principal might also terminate the relationship. We denote the probability of a continuation of the relationship after a low output was observed (conditional on the agent not leaving for exogenuous reasons) α_t.[16]

We derive the equilibrium in contract-specific strategies that maximizes each principal's profits at the beginning of a new employment relationship.[17] The most relevant constraints are an agent's incentive compatibility (IC) constraint and a principal's non-reneging (NR) constraint. The latter states that a principal must not have an incentive to

[16]Note that α can only adopt an intermediate value, i.e., strictly lie between 0 and 1, if a public randomization device exists. If an intermediate value was supposed to just be supported by a mixed strategy, the principal would always keep an agent after a low output. The reason is that turnover is associated with real costs. A new agents has to receive w^+ and is thus more expensive than keeping an agent whose output was low and who is supposed to receive w^-. Without a randomization device, only equilibria where an agent either gets never or always replaced after a low output can be supported. We will assume that this randomization device exists and thus allow for intermediate values of α. Still, it might be that $\alpha \in \{0, 1\}$, with the conditions made more precise in the Appendix.

[17]Note that the restriction on stationary contracts is not without loss of generality here. Instead of terminating the relationship with the same probability in every period, the principal could make α_t contingent on the whole respective output history. Fong and Li (2010) provide a complete characterization of optimal relational contracts in non-market relationships (i.e., with just one principal and one agent), where the agent faces a limited liability constraint and effort is binary.

replace an agent when this is not part of an equilibrium. Since $w^+ \geq w^-$, this constraint is only relevant after a success. Thus, a new agent's wage, w^0, must not be below w^+. Finally, a principal's profits must remain positive, also when a minimum wage is present. We subsequently assume that this is the case.

We solve this problem for all different levels of a minimum wage, and have the following development. If the minimum wage does not bind for w^- (note that we do not impose any further limited liability constraint), $\alpha = 1$, i.e. no termination is used.

Proposition 5: Assume the minimum wage is lower than the optimal level of w^-. Then, $\alpha = 1$.

Here, a failure allows the principal to set w^- low enough to extract the whole future surplus the agent is expects from this relationship. Since a new agent always gets a rent, it is never optimal to fire an employee.

However, if the minimum wage is sufficiently large, an agent always is fired after a low output.

Proposition 6: There exists a $\overline{w}^{\#}$ such that $\alpha = 0$ for all $\overline{w} \geq \overline{w}^{\#}$.

Firing an agent with positive probability after a low output increases her incentives to exert effort, since remaining in a relationship is strictly better than getting fired and receiving $\overline{U}$. However, termination is costly for the principal who has to pay new agents the wage w^+. If an agent is always fired after a low outcome, only one wage level exists, and turnover costs are reduced again. Thus, if the minimum wage is sufficiently high, it becomes optimal to only use a termination threat to provide incentives.

Concerning the development of α (given changes of the minimum wage), it is either possible that α gradually falls from $\alpha = 1$ to $\alpha = 0$, or that it only adopts corner solutions, i.e., $\alpha \in \{0, 1\}$. In the latter case, $\alpha = 1$ for $\overline{w} < \overline{w}^{\#}$ and $\alpha = 0$ for $\overline{w} \geq \overline{w}^{\#}$, where $\overline{w}^{\#}$ is specificied in Proposition 6.

Turning to equilibrium effort, we first show that, absent a minimum wage, the commitment problem makes the principal induce an inefficiently low effort level.

Proposition 7: If the minimum wage does not bind (i.e., $w^{-} > \overline{w}$), equilibrium effort is characterized by $\theta - c' - c'' \frac{1-\delta\gamma}{\delta\gamma} = 0$. Then, U^{-} – the payoff an agent receives after a low output – is equal to $\overline{U}$. When $\overline{w}$ increases and becomes binding, effort continuously decreases to the level specified by $\theta - c' - c'' \frac{(1-\delta\gamma(1-e))}{\delta\gamma} = 0$, where it remains as long as $\alpha = 1$.

As long as $\alpha = 1$, a higher minimum wage thus has no impact on the induced effort level. Only if α adopts a level strictly smaller than 1, $\overline{w}$ has an actual impact on equilibrium effort. When α can also assume intermediate values, $\frac{de}{d\overline{w}}$ can be positive or negative, depending on parameter values. If $\overline{w}$ is large enough such that $\alpha = 0$, a higher minimum wage always leads to a higher equilibrium effort level.

Proposition 8: Assume $\alpha = 0$. Then, $\frac{de}{d\overline{w}} \geq 0$.

As long as the minimum wage does not bind when $\alpha = 0$ (which might or might not be possible), we have $\frac{de}{d\overline{w}} = 0$. When it is binding, the impact of the minimum wage on equilibrium effort becomes strictly positive.

Finally, the impact of a minimum wage increase on turnover is negative if $\alpha = 0$, captured in

Proposition 9: Assume $\alpha = 0$. Then, a higher minimum wage reduces total turnover.

This result immediately follows from the previous discussion. Since a higher minimum wage increases effort, the probability of $y = 0$ is reduced as well.

2.9 The Minimum Wage and Non-Verifiability of Effort and Output

Our main results consist of a normative and a positive components. Since the commitment problem makes it optimal for firms to induce inefficiently low effort levels, a minimum wage can increase efficiency. On the other hand, various empirically observed consequences of a minimum wage are explained. However, the latter would still hold if markets were fully transparent in a sense that everyone could detect deviations (i.e., the principal would be able to build up an external reputation), and even if an agent's effort or output were verifiable. Therefore, our positive results are robust to changes in firms' commitment power. Only the need to give agents incentives remains crucial.

In this section, we briefly validate that this claim is true in the case of verifiable effort. More precisely, we first show that the minimum wage increases agents' effort levels if it binds (however, the level where the minimum wage becomes binding is higher than before). This outcome continues to hold since an agent's (IC) constraint when effort is verifiable is identical to above. But higher effort now is not associated with more efficiency, since the principal would induce first-best effort absent a minimum wage. Furthermore, the minimum wage has a spillover effect on higher wages and – if a firm making positive profits can employ more than one agent – also can raise employment. The latter is only true if the minimum wage binds. As long as it does not bind, it reduces employment. These results are implied by the fact that the condition determining a firm's optimal employment level is identical to above. Thus, higher individual effort levels increase employment.

We do not formally analyze the case where a formal contract can only be written on an agent's output. However, the impact of a minimum wage on turnover well basically be the same as above, since an agent's (IC) constraint is unaffected by the firm's commitment power. For a sufficiently high minimum wage, it will then be optimal to solely use termination threats to provide incentives, and increased effort levels again reduce turnover rates.

Verifiable Effort

If effort is verifiable and firms are not forced to pay a minimum wage, they will choose first-best effort (assuming that production is efficient per se). Furthermore, they are able to keep agents at their reservation utilities. In this section, we slightly adjust the contracts firms write. Since effort is verifiable, contingent payments are made at the end of a period, before agents might leave the market for exogenous reasons. However, if a minimum wage has to be paid, this affects the total wage paid in a period. Note that if we used the same contract as before (payment occurs at the beginning of the next period, but only if the agent has not left the market), the following results would not be affected. Throughout, we assume that a firm's profits are always positive.

Thus, we have

Proposition 10: Assume the agent's effort is verfiable. As long as no positive minimum wage exists, equilibrium effort is characterized by $\theta = c'$, i.e. the efficient effort level e^{eff}. Furthermore, the firm can capture the full rent of each relationship.

Proof: First, note that $\overline{U} = 0$, which follows from the fact that firms are just required to compensate agents for their effort. The latter also determines the (IC) constraint, which will bind in equilibrium, and thus gives $w = c(e^*)$. Plugging into the principal's profit function, $\Pi(1-\delta) = e^*\theta - c(e^*)$, and solving for e^* proves the proposition.

Now, assume that each firm is forced to pay a strictly positive minimum wage $\overline{w}$ in every period. Then, the introduction of a minimum wage will initially have no impact on the efficiency of the relationship (which is equal to the first best), but only lead to a redistribution to employees. When $\overline{w}$ is further increased, though, effort will become inefficiently high.

Furthermore, even if a minimum wage does not bind, it has spillover effect on higher wages. More precisely, assume the minimum wage $\overline{w}$ is set at a level between the original values of w^0 and w. This immediately increases U^0, but also has a positive impact on agents already employed. Since reservation utilities $\overline{U}$ increase after the introduction of the minimum wage, firms either have to reduce induced effort levels or increase w as well. It turns out that they choose the latter. Furthermore, even if the minimum wage is set at a level above the original value of w, it might not bind ex post, and firms might adjust wages to a level even above $\overline{w}$.

Proposition 11: Assume a strictly positive minimum wage $\overline{w}$ is introduced. Then, a firm wil induce equilibrium effort $e^* \geq e^{eff}$. As long as $e^* = e^{eff}$, the wage in an ongoing relationship, w, remains higher than the minimum wage. Only if $e^* > e^{eff}$, the minimum wage will bind for ongoing relationships.

Proof: An employed agent's (IC) constraint equals $w^* - c(e^*) + \delta\gamma U \geq \tilde{w} + \delta\gamma\overline{U}$, where w^* denotes the agent's wage after exerting equilibrium effort, and $\tilde{w}$ the corresponding wage for no effort. Obviously, a firm will set $\tilde{w} = \overline{w}$.

Using $U = \frac{w^* - c(e^*)}{1-\delta\gamma}$ and the fact that (IC) will bind in equilibrium, we have

$w^* = c(e^*) + (1-\delta\gamma)\left(\overline{w} + \delta\gamma\overline{U}\right)$.

Thus, the principal solves

$\max\limits_{e} \Pi = \frac{1}{1-\delta}(e^*\theta - w^*)$

s.t.

(IC) $w^* = c(e^*) + (1-\delta\gamma)\left(\overline{w} + \delta\gamma\overline{U}\right)$

(MW) $w^* \geq \overline{w}$

Substituting gives the Lagrange function

$L = \frac{1}{1-\delta}\left[e^*\theta - c(e^*) - (1-\delta\gamma)\left(\overline{w} + \delta\gamma\overline{U}\right)\right] + \lambda_{MW}\left[c(e^*) + (1-\delta\gamma)\left(\overline{w} + \delta\gamma\overline{U}\right) - \overline{w}\right]$, and the first order condition

$$\frac{1}{1-\delta}\left[\theta - c'\right] + \lambda_{MW}c' = 0$$

If $\lambda_{MW} = 0$, $\theta = c'$; if $\lambda_{MW} > 0$, $\theta < c'$, proving the proposition.

Thus, the minimum wage induces upward pressure on all wage levels, i.e., we observe spillover effects. Furthermore, wage dispersion is decreased since all agents – independent of their tenure – receive the same wage at some point.

Furthermore, we can establish

Proposition 12: As long as $\overline{w}(1-\mu) < \frac{c(e^{eff})}{\delta\gamma}$, where μ denotes the probability an unemployed agent finds a job in any period, a marginal increase of the minimum wage has no impact on equilibrium effort. If $\overline{w}(1-\mu) \geq \frac{c(e^{eff})}{\delta\gamma}$, a further increase of $\overline{w}$ increases effort.

Proof: Note that an agent's payoff equals $U = \frac{w^* - c(e^*)}{1-\delta\gamma}$. Substituting the binding (IC) constraint, i.e., $w^* = c(e^*) + (1-\delta\gamma)\left(\overline{w} + \delta\gamma\overline{U}\right)$, we have $U = \overline{w} + \delta\gamma\overline{U}$. This further gives $\overline{U} = \mu U + \delta(1-\mu)\gamma\overline{U}$, and $\overline{U} = \frac{\mu}{1-\delta\gamma}\overline{w}$. The latter expression is again plugged into (IC) to obtain

$$w^* - \overline{w}\left(1 - \delta\gamma(1-\mu)\right) = c(e^*)$$

Now, assume that $\overline{w}$ is gradually increased starting from $\overline{w} = 0$, and recall that $e^* \geq e^{eff}$. Then, w^* must rise as well to keep effort at e^{eff}, but relatively less than $\overline{w}$. When $\overline{w}$ reaches the level $\frac{c(e^{eff})}{\delta\gamma(1-\mu)}$, the minimum wage thus just becomes binding. Therefore, $\frac{de^*}{d\overline{w}} = 0$ for $\overline{w}(1-\mu) < \frac{c(e^{eff})}{\delta\gamma}$. If $\overline{w}(1-\mu) \geq \frac{c(e^{eff})}{\delta\gamma}$, equilibrium effort is determined by $\overline{w}(1-\mu) = \frac{c(e^*)}{\delta\gamma}$, and $\frac{de^*}{d\overline{w}} = \frac{(1-\mu)\delta\gamma}{c'} > 0$.

Finally, we show that positive employment effects of a minimum wage can also be observed if an agent's effort is verifiable. Thus, we return to the case where only one firm making positive profits is on the market and can employ several agents.

This gives

Proposition 13: Assume that effort is verifiable, a firm can employ more than one agent, and that each agent's productivity is characterized by the decreasing and convex function $\theta(J)$, where J denotes firm employment. Then, equilibrium effort $e^* = e^{eff}$ if $\overline{w} \leq \frac{c(e^{eff})}{\delta\gamma}$, and $e^* > e^{eff}$ if $\overline{w} > \frac{c(e^{eff})}{\delta\gamma}$. Furthermore, increasing the minimum wage $\overline{w}$ has no impact on equilibrium effort and decreases employment J^* as long as $\overline{w} \leq \frac{c(e^{eff})}{\delta\gamma}$. When $\overline{w} > \frac{c(e^{eff})}{\delta\gamma}$, further raising $\overline{w}$ increases equilibrium effort as well as equilibrium employment.

Proof: Since we only consider one firm, $\overline{U} = 0$. Thus, an agent's (IC) gives $w^* = c(e) + (1-\delta\gamma)\overline{w}$. As the firm maximizes $\Pi = J\frac{e\theta(N)-w}{1-\delta}$, the Lagrange function equals $L = J\frac{e\theta(J)-c(e)-(1-\delta\gamma)\overline{w}}{1-\delta} + \lambda_{MW}\left(c(e) - \delta\gamma\overline{w}\right)$, which gives first order conditions

$$\theta(J^*) - c' + \frac{1-\delta}{J^*}\lambda_{MW}c' = 0$$

$$e^*\theta(J^*) - c(e^*) - (1-\delta\gamma)\overline{w} + J^*e^*\theta' = 0$$

If the minimum wage constraint is not binding (which will be the case if $\overline{w} < \frac{c(e^{eff})}{\delta\gamma}$), $\lambda_{MW} =$ and $e^* = e^{eff}$. Furthermore, $\frac{de^*}{d\overline{w}} = 0$, and

$\frac{dJ^*}{d\overline{w}} = \frac{(1-\delta\gamma)}{2e^*\theta'+N^*e^*\theta''} < 0.$

If $\overline{w} \geq \frac{c(e^{eff})}{\delta\gamma}$, the minimum wage constraint binds, $\lambda_{MW} > 0$ and thus equilibrium effort $e^* > e^{eff}$. Furthermore, this allows us to characterize equilibrium effort by $\overline{w} - \frac{c(e^*)}{\delta\gamma} = 0$ and equilibrium employment by $e^*\theta(J^*) - \overline{w} + J^*e^*\theta' = 0$. Thus,

$$\frac{de^*}{d\overline{w}} = \frac{\begin{vmatrix} -1 & 0 \\ 1 & 2e^*\theta' + J^*e^*\theta'' \end{vmatrix}}{\begin{vmatrix} -\frac{c'}{\delta\gamma} & 0 \\ \theta(J^*) + J*\theta' & 2e^*\theta' + J^*e^*\theta'' \end{vmatrix}} = \frac{-2e^*\theta' - J^*e^*\theta''}{\begin{vmatrix} -\frac{c'}{\delta\gamma} & 0 \\ \theta(J^*) + J*\theta' & 2e^*\theta' + J^*e^*\theta'' \end{vmatrix}} > 0 \text{ and}$$

$$\frac{dJ^*}{d\overline{w}} = \frac{\begin{vmatrix} -\frac{c'}{\delta\gamma} & -1 \\ \theta(J^*) + N^*\theta' & 1 \end{vmatrix}}{\begin{vmatrix} -\frac{c'}{\delta\gamma} & 0 \\ \theta(J^*) + J*\theta' & 2e^*\theta' + J^*e^*\theta'' \end{vmatrix}} = \frac{-\frac{c'}{\delta\gamma} + \theta(J^*) + J^*\theta'}{\begin{vmatrix} -\frac{c'}{\delta\gamma} & 0 \\ \theta(J^*) + J*\theta' & 2e^*\theta' + J^*e^*\theta'' \end{vmatrix}}.$$

Substituting $\theta(J^*) + J^*\theta' = \frac{\overline{w}}{e^*}$, the nominator becomes $-\frac{c'}{\delta\gamma} + \frac{\overline{w}}{e^*}$. This expression is positive, since $\overline{w} - \frac{c(e^*)}{\delta\gamma} = 0$ and $e^* < 1$.

Thus, an increase of a non-binding minimum wage destroys employment, while further raising a binding minimum wage increases employment. However, a minimum wage is always associated with an efficiency loss if an agent's effort is verifiable.

2.10 Conclusion

We showed how a minimum wage can increase effort and employment, reduce turnover, have spillover effects, and reduce wage diversion if incentives are taken into account. Moreover, if effort and output are not verifiable, and if a firm can not build up an external reputation to honor its promises, the minimum wage has a positive impact on efficiency.

Several further aspects deserve are worth pursuing. First of all, a minimum wage can not be analyzed in isolation from other labor market institutions. Especially in many European countries, many labor markets are highly regulated, implying that a minimum wage might have different consequences on employment or production than in a less regulated country like the US. Take employment protection, which effectively induces costs on a firm which wants to fire employees. These costs can be used as a commitment device as well, allowing the principal to capture the whole rent (see Yang, 2008). A combination of a minimum wage with employment protection might therefore either increase or decrease employment and efficiency.

Furthermore, we do not take the possibility of collustion into account. Firms do not consider the impact of their choices on agents' outside options and thus on rents other firms have to give away. By colluding and choosing the wage that maximizes joint profits, the induced effort levels (then characterized by $c' = \delta\gamma(1-\mu)\theta$, compared to $c' = \delta\gamma\theta$ absent collusion) would even be lower relative to the first best, increasing potential benefits of a minimum wage. This might be difficult to sustain in a market with many small firms. However, our results continue to hold in a market with a few large players. It is just necessary that the different principals have a substantial amount of freedom when making their decisions and that the information concerning (non-verifiable) performance of agents is not shared between restaurants. This might hold for the fast food industry, where many restaurants are run by franchisees who are relatively independent when running their outlets. Coordinating on wages (and thus indirectly on effort) would increase their profits. Then, a minimum has an even bigger potential to increase the productivity of employment relations.

Appendix to Chapter 2

Appendix I:

Here, we show that the restriction to contract-specific strategies is without loss of generality in a sense that allowing strategies to depend on the whole turnover history of a firm would not increase attainable profits. Thus, we make

Proposition A1: Denote the maximum payoff a principal can get in any period t if equilibrium strategies by any player are only contingent on the turnover in period $t-1$ Π^{max}. Then, Π^{max} will be the maximum payoff of the firm in any equilibrium.

Proof:

Let us use the replacement variable $r_t \in \{0,1\}$, where $r_t = 1$ if a firm starts an new employment relationship in period t and $r_t = 0$ otherwise. Then, $R^t = \{r_1, r_2, ..., r_t\}$ denotes the full replacement history of the respective principal until the beginning of period $t+1$. R^t is observable by the whole market. We want to show that $\Pi^0(R^{t-1}) = \Pi^0(\overline{R}^{t-1})$, all R^{t-1}, $\overline{R}^{t-1}$, in the class of contracts that maximize the principal‘s profits.

Note that contracts can be assumed to be stationary with the exception that they might depend on the full turnover history. Thus, each firm faces the constraints

(IC)

$$c(e^*(R^{t-1})) \leq \delta\gamma(U(R^{t-1}, r_t = 0) - \overline{U})$$

(IRA)

$$U(R^t) \geq \overline{U}$$

(IRP)

$$\Pi(R^t) \geq 0$$

(NR)

$$\Pi(R^{t-1}, r_t = 0) \geq \Pi(R^{t-1}, r_t = 1)$$

for all $t \geq 1$.

To satisfy (NR), the principal must get punished after a turnover. We already pointed out that we use a rent going to new agents as the means of punishment; here, we still keep it general and thus refer to the punishment firms face after turnover. This punishment is denoted $P(R^t)$.

First of all, note that (ICA) must bind, as the market cannot observe payments made between the principal and the agent. If it were not binding, the principal could increase profits by decreasing U without violating any constraint. Therefore, agents who remain in an employment relationship get no rent, and the surplus stream in each period, S_t, consists of expected profits Π_t and expected punishments, P_t. Obviously, a punishment never occurs if no replacement has taken place.

As a next step, we establish that equilibrium effort can without loss be independent of the replacement history R^t. To see that, first note that the replacement history has no impact on *enforceable* effort levels. Thus, if it is optimal to have effort differ in equilibrium based on the turnover history, this is solely used to punish the principal. But then we can just play a payoff equivalent equilibrium where effort is independent of R^t and the respective punishment is carried out via other means, for example a rent going to new agents.

Furthermore, the principal's non-reneging constraint $\Pi(R^{t-1}, r_t = 0) \geq \Pi(R^{t-1}, r_t = 1)$ has to bind for every history. Assume this is not the case. Then, we show that by increasing $\Pi_t^0(R^{t-1})$, the principal can increase profits Π_1 without violating any constraints in any period after and before t.

First, take periods later than t and note that all future effort levels are independent of the replacement literature. Furthermore, it can be assumed without loss that a necessary punishment occurs immediately, i.e., in the period of turnover. Then, the punishment in period t can obviously be reduced without violating any further incentives.

Concerning incentives for periods before t, again assume that $\Pi_t(R^{t-1}) > \Pi_t^0(R^{t-1})$

and that all constraints in previous periods are satisfied. Now, consider the following change. Decrease $P(R^{t-1})$ until (NR) as satisfied as an equality in period t and denote the difference between the old and the new amount of punishment ΔP. As $\Pi_{t-1}(R^{t-1}) = e - w(R^{t-1}) - p(R^{t-1}) + \delta[\gamma\Pi(R^{t-1}, r = 0) + (1+\gamma)\Pi(R^{t-1}, r = 1)]$ and since $\Pi(R^{t-1}, r = 1)$ is reached with positive probability starting at every period along this replacement history together with the assumption that everything else remains unchanged, this increases the expected profit stream in each period along the history path R^t. For all periods τ along this history where $r_\tau = 0$, this does not impose a problem for (IRP). If $r_\tau = 1$, (IRP) might be violated now. If this is the case, increase p_τ such that (IRP) is just not violated anymore. Still, profits in any period prior to τ are not lower than originally. To see that, take an arbitrary period $\tau < t$ with $r_\tau = 1$. Assume the history R^t requires k replacements between τ and t (the exact order is not important for the argument used here). Then, Π^0_τ increases by the discounted value of Δp_t times the probability that the principal actually gets there, namely by $\delta^{t-\tau}\Delta p_t \gamma^{t-\tau-k}(1-\gamma)^k$. If (IRP) was binding in period τ (otherwise, our argument is satisfied even easier), p_τ has to be increased by exactly this amount. Now, it is obvious that profits before period τ are not smaller than in the original situation.

Having derived that equilibrium effort is independent of the replacement history R^t and that $\Pi(R^{t-1}, r = 0) = \Pi(R^{t-1}, r = 1)$ for each period t and any replacement history, we can now show that in each period t, $\Pi^0(R^{t-1}) = \Pi^0(\overline{R}^{t-1})$ for all replacement histories R^{t-1} and $\overline{R}^{t-1}$. First of all, $\Pi^0(R^{t-1}) = \Pi^0(\overline{R}^{t-1})$ implies $\Pi(R^{t-1}) = \Pi(\overline{R}^{t-1})$. Note that $\Pi(R^{t-1}) = e\theta - w(R^{t-1}) + \delta[\gamma\Pi(R^t, r = 0) + (1-\gamma)\Pi(R^t, r = 1)]$. Furthermore, from $\Pi(R^t, r = 0) = \Pi(R^t, r = 1)$ follows $\Pi(R^{t-1}) = e\theta - w(R^{t-1}) + \delta\Pi(R^t, r = 0)$; thus, by induction we can say that if equality is true at period t, this also has to be the case for all future periods for all possible histories.

Finally, it may never be optimal to fire the agent in equilibrium: If an agent was fired with positive probability, payoff needed to incentivize him would increase. But this also increases necessary punishment in the case where no replacement occurs.

Appendix II: The Optimal Stationary Relational Contract Under Asymmetric Information

Here, we are interested in the optimal (i.e., profit maximizing) stationary contract when the principal is not able to observe the agent's effort but only the respective output. Then, it might be optimal to use a termination of the relationship to provide incentives when a binding minimum wage is present. Thus, the principal fires the agent with probability $(1-\alpha)$ when $y=0$ is observed.

This implies the following timing. At the beginning of the period, the agent receives her wage. Then, effort is exerted and the output realized. After that, the agent leaves for exogenuous reasons with probability $(1-\gamma)$. If this does not happen, the principal offers her to remain for an additional period, receiving the wage w^+, if $y=\theta$ was observed. If the output was low, the agent gets fired with probability $(1-\alpha)$. Otherwise, she can stay and gets offered to receive w^- in the following period.

Note again that $\alpha \in [0,1]$ is only feasible if a public randomization exists (what we assume from now on). Otherwise, α can only adopt the values 0 or 1. If only a mixed strategy was used, the principal would always keep the agent after a low output - as new agents have to receive w^+ and $w^+ \geq w^-$.

In the following, we assume that such a randomization device exists.

As already mentioned, we derive the contract that maximizes each principal's payoff stream at the beginning of the whole game. Here, it is identical to the payoff he receives after a failure is observed, i.e. Π^-.

The following constraints have to be satisfied: It may never be optimal to replace an agent instead of compensating her after a high output. Thus, the wage a new agent receives, w^0, has to be at least as high as w^+. Obviously, $w^0=w^+$, which we already substitute in the following. After a success or when starting a new relationship, i.e., when the wage w^+ has to be paid, the principal's payoff must be larger than his outside option, namely $\Pi^+ \geq 0$. Furthermore, the agent's utility after a failure must not lie below her outside option, implying $U^- \geq \overline{U}$. The agent's effort is determined by her incentive compatibility (IC) constraint, claiming that $e \in \arg\max -c(e)+\delta\gamma[eU^+ + (1-e)\left(\alpha U^- + (1-\alpha)\overline{U}\right)]$. For us to be able to use the first order approach, some conditions concerning the agents' functions have to be satisfied. We will make them precise below.

Finally, α must lie between 0 and 1, and the wages paid by the principal must exceed a potential minimum wage.

For convenience, we also set $\overline{U} = 0$ in this section, i.e., assume that there is only one firm present. However, changing this has no impact on our results.

Then, the objective is to maximize profits from starting a new relationship, i.e.,

$$\max_{e,\alpha,w^+,w^-} \Pi^+ = e\theta - w^+ + \delta\left[\gamma\left(e\Pi^+ + (1-e)\left(\alpha\Pi^- + (1-\alpha)\Pi^0\right)\right) + (1-\gamma)\Pi^0\right]$$

$$= \frac{e\theta - w^+ + \delta\alpha\gamma(1-e)(w^+ - w^-)}{1-\delta}$$

s.t.

(IRP) $\Pi^+ = \frac{e\theta - w^+ + \delta\alpha\gamma(1-e)(w^+ - w^-)}{1-\delta} \geq 0$

(NR) $w^0 \geq w^+$

(IRA) $U^- = w^- - c(e) + \delta\gamma[eU^+ + (1-e)\alpha U^-] \geq 0$

(IC) $e \in \text{argmax} - c(e) + \delta\gamma[eU^+ + (1-e)\alpha U^-]$

(MW+) $w^+ \geq \overline{w}$

(MW-) $w^- \geq \overline{w}$

$0 \leq \alpha \leq 1$

First of all, note that (MW+) can only bind when $\alpha = 0$, as otherwise $w^+ > w^- \geq \overline{w}$.

Then, we can use the first order approach to rewrite the agent's (IC) constraint. Note that some additional conditions have to be imposed on the agent's effort cost function to make the first order approach valid in our case. We will make these conditions precise below.

Thus, an agent's effort is characterized by

$$-c'\left(1 - \delta\gamma\left(e + \alpha(1-e)\right)\right) + \delta\gamma\left(w^+ - \alpha w^- - c(e)(1-\alpha) - \delta\gamma\alpha\left(w^+ - w^-\right)\right) = 0$$

This allows us to substitute w^+ in the principal's problem by

$$w^{+} = \frac{\frac{c'}{\delta\gamma}\left(1-\delta\gamma\left(e+\alpha(1-e)\right)\right)+c(e)(1-\alpha)+(1-\delta\gamma)\alpha w^{-}}{(1-\delta\gamma\alpha)}$$

Taken together, we get the Lagrange function, which equals

$$L = \frac{e\theta - w^{+} + \delta\alpha\gamma(1-e)(w^{+}-w^{-})}{1-\delta} + \lambda_{IRP}\Pi^{+} + \lambda_{IRA}\frac{w^{-}-c(e)+\delta\gamma e\left(w^{+}-w^{-}\right)}{1-\delta\gamma(e+\alpha(1-e))} + \lambda_{MW+}(w^{+}-\overline{w})$$
$$+\lambda_{MW-}(w^{-}-\overline{w}) + \lambda_{\alpha\geq 0}\alpha + \lambda_{\alpha\leq 1}(1-\alpha)$$

and gives the first order conditions

$$\frac{\partial L}{\partial \alpha} = \frac{-\frac{\partial w^{+}}{\partial\alpha}+\delta\alpha\gamma(1-e)(\frac{\partial w^{+}}{\partial\alpha}-\frac{\partial w^{-}}{\partial\alpha})+\delta\gamma(1-e)(w^{+}-w^{-})}{1-\delta} + \lambda_{IRP}\frac{\partial\Pi^{+}}{\partial\alpha}$$
$$+\lambda_{IRA}\left(\frac{\left(\frac{\partial w^{-}}{\partial\alpha}+\delta\gamma e\left(\frac{\partial w^{+}}{\partial\alpha}-\frac{\partial w^{-}}{\partial\alpha}\right)\right)(1-\delta\gamma(e+\alpha(1-e)))+\delta\gamma(1-e)\left(w^{-}-c(e)+\delta\gamma e\left(w^{+}-w^{-}\right)\right)}{(1-\delta\gamma(e+\alpha(1-e)))^{2}}\right)$$
$$+\lambda_{MW+}\frac{\partial w^{+}}{\partial\alpha} + \lambda_{MW-}\frac{\partial w^{-}}{\partial\alpha} + \lambda_{\alpha\geq 0} - \lambda_{\alpha\leq 1} = 0$$

$$\frac{\partial L}{\partial e} = \frac{\theta-\frac{\partial w^{+}}{\partial e}+\delta\alpha\gamma(1-e)(\frac{\partial w^{+}}{\partial e}-\frac{\partial w^{-}}{\partial e})-\delta\alpha\gamma(w^{+}-w^{-})}{1-\delta} + \lambda_{IRP}\frac{\partial\Pi^{+}}{\partial e}$$
$$+\lambda_{IRA}\frac{\left(\frac{\partial w^{-}}{\partial e}-c'+\delta\gamma\left(w^{+}-w^{-}\right)+\delta\gamma e\left(\frac{\partial w^{+}}{\partial e}-\frac{\partial w^{-}}{\partial e}\right)\right)(1-\delta\gamma(e+\alpha(1-e)))+\delta\gamma(1-\alpha)\left(w^{-}-c(e)+\delta\gamma e\left(w^{+}-w^{-}\right)\right)}{(1-\delta\gamma(e+\alpha(1-e)))^{2}}$$
$$+\lambda_{MW+}\frac{\partial w^{+}}{\partial e} + \lambda_{MW-}\frac{\partial w^{-}}{\partial e} = 0$$

$$\frac{\partial L}{\partial w^{-}} = \frac{-\frac{\partial w^{+}}{\partial w^{-}}-\delta(\frac{\partial w^{+}}{\partial w^{-}}-1)(1-\alpha\gamma(1-e))}{1-\delta} + \lambda_{IRP}\frac{\partial\Pi^{+}}{\partial w^{-}}$$
$$+\lambda_{IRA}\left(\frac{1+\delta\gamma e(\frac{\partial w^{+}}{\partial w^{-}}-1)}{1-\delta\gamma(e+\alpha(1-e))}\right) + \lambda_{MW+}\frac{\partial w^{+}}{\partial w^{-}} + \lambda_{MW-} = 0$$

From now on, assume that $\Pi^{+} > 0$. This implies that the principal makes positive profits after a success.

Now we can prove

Proposition 5: Assume the minimum wage is lower than the optimal w^{-}. Then, $\alpha = 1$.

and establish

Lemma A1: Assume the minimum wage is lower than w^-. Then $U^- = \overline{U} = 0$, and equilibrium effort is determined by $\theta - c' - c''\frac{1-\delta\gamma}{\delta\gamma}$

The assumption means that $\lambda_{MW-} = 0$ ($\Rightarrow \lambda_{MW+} = 0$ as well)

This is the case if there is either no minimum wage or that it is that small to have no impact on the principal's decisions.

Then, $\lambda_{IRA} > 0$, implying $U^- = 0$ and thus

$$w^- = c(e) - ec'$$

$\Rightarrow w^+ = c'\left(\frac{1}{\delta\gamma} - e\right) + c(e)$ and $w^+ - w^- = \frac{c'}{\delta\gamma}$

Thus, $\frac{\partial w^+}{\partial e} = c''\left(\frac{1}{\delta\gamma} - e\right)$ and $\frac{\partial w^-}{\partial e} = -ec''$

This allows us to rewrite the conditions above as

$\frac{\partial L}{\partial \alpha} = \frac{(1-e)c'}{1-\delta} + \lambda_{IRA}\frac{-\delta\gamma(1-e)+\delta\gamma(1-e)}{(1-\delta\gamma(e+\alpha(1-e)))} + \lambda_{\alpha\geq 0} - \lambda_{\alpha\leq 1} = 0$ and

$\frac{(1-e)c'}{1-\delta} + \lambda_{\alpha\geq 0} - \lambda_{\alpha\leq 1} = 0 \Rightarrow \lambda_{\alpha\leq 1} > 0$, and no termination occurs in equilibrium

$\frac{\partial L}{\partial e} = \frac{\theta - \alpha c' - c''\left(\frac{1}{\delta\gamma} - e\right) + \alpha(1-e)c''}{1-\delta} = 0$ and - as $\alpha = 1$ -

$$\theta - c' - c''\frac{1-\delta\gamma}{\delta\gamma}$$

Lemma A2: Assume the minimum wage binds for w^-, i.e., $w^- = \overline{w}$. As long as $\alpha = 1$, equilibrium effort is determined by $\theta - c' - c''\frac{(1-\delta\gamma(1-e))}{\delta\gamma} + (1-\delta)\lambda_{IRA}\frac{e(c'')^2}{(1-\delta\gamma)} = 0$, where $\lambda_{IRA} > 0$ for a sufficiently low minimum wage and then becomes zero as $\overline{w}$ increases.

Proof:

Now, we have $\lambda_{MW-} > 0$ and thus $w^- = \overline{w}$.

If $\lambda_{IRA} > 0$, effort is determined by $\overline{w} = c(e) - ec'$. Thus,

$w^+ = c'\left(\frac{1}{\delta\gamma} - e\right) + c(e)$ and

$w^+ - \overline{w} = \frac{c'}{\delta\gamma}$

Furthermore,

$\frac{\partial L}{\partial \alpha} = \frac{(1-e)c'}{1-\delta} + \lambda_{MW+}\frac{\partial w^+}{\partial \alpha} + \lambda_{\alpha\geq 0} - \lambda_{\alpha\leq 1} = 0 \Rightarrow \lambda_{\alpha\leq 1} > 0$ and $\alpha = 1$, implying $\lambda_{MW+} = 0$, and

$\frac{\partial L}{\partial e} = \frac{\theta - c' - c''\frac{(1-\delta\gamma(1-e))}{\delta\gamma}}{1-\delta} + \lambda_{IRA}\frac{\delta\gamma e c''\frac{\partial w^+}{\partial e}}{(1-\delta\gamma)} = 0$

Thus, we have a further characterization of e, namely

$$\theta - c' - c''\frac{(1-\delta\gamma(1-e))}{\delta\gamma} + (1-\delta)\lambda_{IRA}\frac{e\,(c'')^2}{(1-\delta\gamma)} = 0$$

This allows us to prove

Proposition 7: If the minimum wage does not bind (i.e., $w^- > \overline{w}$), equilibrium effort is characterized by $\theta - c' - c''\frac{1-\delta\gamma}{\delta\gamma} = 0$. Then, U^- - the payoff an agent receives after a low output - is equal to $\overline{U}$. When $\overline{w}$ becomess binding, effort continuously decreases to the level specified by $\theta - c' - c''\frac{(1-\delta\gamma(1-e))}{\delta\gamma} = 0$, where it remains as long as $\alpha = 1$.

Proof:

Follows from Lemmas and and the fact that $e(\overline{w})$ has to be continuous as long as $\alpha = 1$. Therefore, $\lambda_{IRA} > 0$ when the minimum wage just becomes binding.

$$-\frac{\partial w^+}{\partial \alpha}(1-\delta\alpha\gamma(1-e)) + \delta\gamma(1-e)(w^+ - w^-) + (1-\delta)\left(\lambda_{MW+}\frac{\partial w^+}{\partial \alpha} + \lambda_{\alpha\geq 0} - \lambda_{\alpha\leq 1}\right) = 0 \tag{2.4}$$

$$\theta - \frac{\partial w^+}{\partial e}(1-\delta\alpha\gamma(1-e)) - \alpha\gamma\delta(w^+ - w^-) + (1-\delta)\lambda_{MW+}\frac{\partial w^+}{\partial e} = 0 \tag{2.5}$$

Now we want to show that for a sufficiently high minimum wage, the constraint MW+ must bind. Our approach is to assume that if $\lambda_{MW+} = 0$, the left hand side of equation (2.5) becomes strictly positive at some level of $\overline{w}$.

Lemma A3: There exists a $\overline{w}^*$ such that if $\overline{w} \geq \overline{w}^*$, $\lambda_{MW+} > 0$.

Proof:

Assume that is not the case, then there are levels $\overline{w} \geq \overline{w}^*$ where

$$\theta - \frac{\partial w^+}{\partial e}\left(1 - \delta\alpha\gamma(1-e)\right) - \alpha\gamma\delta(w^+ - w^-) = 0 \tag{2.6}$$

is satisfied.

Adding and substracting the terms $\frac{1}{e}\alpha\delta\gamma(w^+ - \overline{w})$ and $\frac{1}{e}w^+$ in (3.8) gives

$\theta - \alpha\gamma\delta(w^+ - w^-) + \frac{1}{e}\alpha\delta\gamma(w^+ - \overline{w}) - \frac{1}{e}w^+ + \frac{1}{e}w^+ - \frac{1}{e}\alpha\delta\gamma(w^+ - \overline{w}) - \frac{\partial w^+}{\partial e}\left(1 - \delta\alpha\gamma(1-e)\right)$

$= \frac{1}{e}\left[e\theta - w^+ + \alpha\delta\gamma(w^+ - \overline{w})(1-e) + w^+ - \alpha\delta\gamma(w^+ - \overline{w}) - e\frac{\partial w^+}{\partial e}\left(1 - \delta\alpha\gamma(1-e)\right)\right]$

$= \frac{1}{e}\left[(1-\delta)\Pi^+ + w^+ - \alpha\delta\gamma(w^+ - \overline{w}) - e\frac{\partial w^+}{\partial e}\left(1 - \delta\alpha\gamma(1-e)\right)\right]$

$\geq \frac{1}{e}\left[w^+ - \alpha\delta\gamma(w^+ - \overline{w}) - e\frac{\partial w^+}{\partial e}\left(1 - \delta\alpha\gamma(1-e)\right)\right]$ as $\Pi^+ \geq 0$

Now, we use (3.8) to get $\alpha\gamma\delta(w^+ - w^-) = \theta - \frac{\partial w^+}{\partial e}\left(1 - \delta\alpha\gamma(1-e)\right)$ and substitute it into the previous term, which becomes

$\frac{1}{e}\left[w^+ - \theta + \frac{\partial w^+}{\partial e}\left(1 - \delta\alpha\gamma(1-e)\right)(1-e)\right]$

As long as $MW+$ does not bind, $w^+ > \overline{w}$, and thus (furthermore using that $\frac{\partial w^+}{\partial e} \geq 0$)

$\frac{1}{e}\left[w^+ - \theta + \frac{\partial w^+}{\partial e}\left(1 - \delta\alpha\gamma(1-e)\right)(1-e)\right]$

$\geq \frac{1}{e}\left[w^+ - \theta\right]$

$> \frac{1}{e}\left[\overline{w} - \theta\right]$, which is positive for $\overline{w}$ sufficiently large, contradicting the assumption

This immediately allows us to prove

Proposition 6: There exists a $\overline{w}^\#$ such that $\alpha = 0$ for all $\overline{w} \geq \overline{w}^\#$.

Proof: Follows from Lemma A3: If $w^+ = \overline{w}$, α has to be equal to zero.

Corner versus interior solution

Note that for an interior solution of α,

$$-\frac{\partial w^+}{\partial \alpha}(1-\delta\alpha\gamma(1-e))+\delta\gamma(1-e)(w^+-w^-)+(1-\delta)\left(\lambda_{MW+}\frac{\partial w^+}{\partial \alpha}+\lambda_{\alpha\geq 0}-\lambda_{\alpha\leq 1}\right)=0$$

and

$$\theta-\frac{\partial w^+}{\partial e}(1-\delta\alpha\gamma(1-e))-\alpha\gamma\delta(w^+-w^-)+(1-\delta)\lambda_{MW+}\frac{\partial w^+}{\partial e}=0$$

must be satisfied. Furthermore, the matrix of second derivatives, i.e., $\begin{vmatrix} \Pi^-_{\alpha\alpha} & \Pi^-_{\alpha e} \\ \Pi^-_{e\alpha} & \Pi^-_{ee} \end{vmatrix}$, has to be negative definite, implying that we need $\frac{\partial^2\Pi^-}{\partial\alpha^2}<0$, $\frac{\partial^2\Pi^-}{\partial e^2}<0$ and $\frac{\partial^2\Pi^-}{\partial\alpha^2}\frac{\partial^2\Pi^-}{\partial e^2}-\left(\frac{\partial^2\Pi^-}{\partial\alpha\partial e}\right)^2>0$.

Since $\frac{\partial^2\Pi^+}{\partial\alpha^2}=-2\delta\gamma\left[1-\delta\gamma\right]\frac{c'e-c(e)+\overline{w}}{(1-\delta\gamma\alpha)^3(1-\delta)}\left[\delta+e\right]=-2\delta\gamma\frac{[1-\delta\gamma]}{(1-\delta)}\frac{U^-}{(1-\delta\gamma\alpha)^2}e<0$, an interior solution is feasible.

Furthermore, $\frac{\partial^2\Pi^+}{\partial e^2}=\frac{1}{1-\delta}\left(-\frac{\partial^2 w^+}{\partial e^2}(1-\delta\alpha\gamma(1-e))-2\alpha\gamma\delta\frac{\partial w^+}{\partial e}\right)<0$. Note that this always has to be the case, even if α only assumes corner solutions, i.e., if $\alpha\in\{0,1\}$. Thus, if $\alpha=0$, $\frac{\partial^2 w^+}{\partial e^2}$ has to be positive. If $\alpha=1$, $\frac{\partial^2 w^+}{\partial e^2}=\frac{\frac{c'''}{\delta\gamma}(1-\delta\gamma(e+\alpha(1-e)))-c''(1-\alpha)}{(1-\delta\gamma\alpha)}$ is positive for sure.

There might be a case where each agent's cost function is such that $\frac{\partial^2 w^+}{\partial e^2}$ is positive for $\alpha=0$ and $\alpha=1$ but negative for intermediate values. Still, it will not have a too big impact on our results if we assume that $\frac{\partial^2 w^+}{\partial e^2}>0$, which we do from now on.

When we can have an interior solution, i.e., α does not fall from 1 to 0 but moves smoothly, this generally happens monotonically. It might only be the case that somewhere α increases again. We already know that for a sufficiently high minimum wage, $\alpha=0$. Furthermore, we know that effort decreases initially, i.e., when the minimum wage just becomes binding, and remains at a certain level as long as $\alpha=1$ (which still is the case when IRA just gets non-binding). However, the exact pattern is of no interest for us. Thus, we omit comparative statics for the range when α assumes an interior solution and immediately move on to levels of $\overline{w}$ where $\alpha = 0$, and analyze comparative statics with respect to effort in this range.

$\alpha \in \{0, 1\}$

Now, assume an interior solution for α does not exist.

Proposition: If $\alpha \in \{0, 1\}$, $\alpha = 1$ for $\overline{w} < \overline{w}^{\#}$ and $\alpha = 0$ for $\overline{w} \geq \overline{w}^{\#}$. When $\alpha = 0$, $\frac{de}{d\overline{w}} = 0$ if the constraint MW+ does not bind and $\frac{de}{d\overline{w}} > 0$ if it binds. When $\alpha = 1$ and $w^- = \overline{w}$, $\frac{de}{d\overline{w}} < 0$ as long as (IRA) binds. If it does not bind, $\frac{de}{d\overline{w}} = 0$.

The result that $\frac{de}{d\overline{w}} < 0$ as long as the (IRA) constraint binds has already proven above, as well as the fact that $\frac{de}{d\overline{w}} = 0$ as long as $\alpha = 1$ if (IRA) does not bind.

The rest of the proof proceeds as follows. We determine equilibrium profits for $\alpha = 1$ and $\alpha = 0$ separately and show that the former decrease stronger with $\overline{w}$ than the latter, because if $\alpha = 0$, then $\frac{de}{d\overline{w}} \geq 0$. Then, we determine $\overline{w}^{\#}$ by setting $\Pi^-(\alpha = 1) = \Pi^-(\alpha = 0)$.

If $\alpha = 1$ and (IRA) does not bind anymore, equilibrium effort is characterized by $\theta - c' - c'' \left(\frac{1-\delta\gamma(1-e)}{\delta\gamma}\right) = 0$, implying that $\frac{de}{d\overline{w}} = 0$ in that range.

Thus, $\Pi^+(\alpha = 1) = \frac{e\theta - w^+ + \delta\gamma(1-e)(w^+ - \overline{w})}{1-\delta} = \frac{e\theta - \overline{w} - \frac{(1-\delta\gamma(e+(1-e)))}{(1-\delta\gamma)} \frac{c'}{\delta\gamma}\delta(1-\gamma(1-e))}{1-\delta}$, and

$\frac{d\Pi^-(\alpha=1)}{d\overline{w}} = -\frac{1}{1-\delta}$

If $\alpha = 0$ and the (MW+) constraint does not bind, equilibrium effort is characterized by $\theta - c''\frac{(1-\delta\gamma e)}{\delta\gamma}$, implying that $\frac{de}{d\overline{w}} = 0$.

Then, we have $\Pi^+(\alpha = 0, w^+ > \overline{w}) = \frac{e\theta - \frac{c'}{\delta\gamma}(1-\delta\gamma e) + c(e)}{1-\delta}$ and

$\frac{d\Pi^-(\alpha=0, w^+>\overline{w})}{d\overline{w}} = 0$

If $\alpha = 0$ and $w^+ = \overline{w}$, equilibrium effort is determined by the minimum wage, namely by $\overline{w} = \frac{c'}{\delta\gamma}(1 - \delta\gamma e) + c(e)$.

Then, $\frac{de}{d\overline{w}} = -\frac{1}{-\frac{c''}{\delta\gamma}(1-\delta\gamma e)} > 0$.

Then, we have $\Pi^-(\alpha = 0, w^+ = \overline{w}) = \frac{e\theta - \overline{w}}{1-\delta}$, with

$\frac{d\Pi^-(\alpha=0,w^+=\overline{w})}{d\overline{w}} = \frac{\frac{de}{d\overline{w}}\theta-1}{1-\delta} > -\frac{1}{1-\delta}$

Thus, setting profits with $\alpha = 1$ equal to $\alpha = 0$, we get the desired threshold. As $\frac{d\Pi^-(\alpha=0)}{d\overline{w}} > \frac{d\Pi^-(\alpha=1)}{d\overline{w}}$, $\alpha = 0$ remains optimal for all minimum wage levels to the right of $\overline{w}^{\#}$. As we already figured out that $\alpha = 1$ is optimal for relatively low levels of $\overline{w}$, we are done with the proof.

The proof to Proposition then is sufficient for

Proposition 8: Assume $\alpha = 0$. Then, $\frac{de}{d\overline{w}} \geq 0$.

Finally, it could be of interest whether (MW+) binds at $\overline{w}^{\#}$. There, both cases are possible, depending on respective effort levels, why we do not explore this issue further.

Proposition 9: Assume $\alpha = 0$. Then, a higher minimum wage reduces total turnover.

This follows from previous results and the production function.

Chapter 3

On the Genesis of Multinational Networks[1]

3.1 Introduction

Multinational enterprises (MNEs) tend to pursue a *gradual expansion strategy* of their network of foreign affiliates over time rather than exploring all profitable opportunities *simultaneously*. They typically establish themselves in their home countries and then enter new foreign markets step by step. This paper studies the optimal dynamic behavior of MNEs to explore international growth opportunities. It contributes to the literature on the international organization of firms by investigating sequential location decisions.

We propose a model where MNEs face uncertainty concerning their success in new markets and learn about that after entry. Conditions in different markets are not independent, and the information gathered in one country can also be used to learn about conditions in other, in particular, similar countries. This so-called correlated learning can explain why firms expand step by step: market entry is associated with considerable costs, and sequential investments help to economize on these costs by reducing uncertainty. The learning model developed in this paper serves to derive a number of testable hypotheses regarding market entry in general and *simultaneous* versus *sequential* market entry in specific. These hypotheses are assessed in a data-set of the universe of German MNEs and their foreign affiliates. The results provide empirical evidence for correlated

[1] This chapter is joint work with Peter Egger, Valeria Merlo, and Georg Wamser.

learning as a main driver behind international expansion strategies.

Our paper is related to recent work on sequential exporting. For instance, Evenett and Venables (2002) point out that initial exports to one market are typically followed by exports to adjacent markets at the product level. Eaton, Eslava, Kugler, and Tybout (2007) find that Columbian firms start exporting in a single foreign market and gradually enter additional destinations. They also show that further expansions crucially depend on the export market served initially. A similar pattern was found by Schmeiser (2009) who, using Russian firm-level data, demonstrates that export experience determines export dynamics: a typical firm first enters one destination and then slowly expands. More recently, Albornoz, Pardo, Corcos, and Ornelas (2011) explore how firms learn about their export profitability. They illustrate that firms use their first export market as a "testing ground" to learn about their export profitability and, subsequently, exit, continue to export, or enter further markets. Hence, the first export decision not only provides information about the export market, it also reveals information about the firm itself (in a given market). We argue that learning is particularly crucial for foreign direct investment (FDI; as an alternative to exporting), which definitely involves discrete real investments while exporting does not necessarily.

We also relate to the literature on the mode and depth of firms' international activities. Models of heterogenous firms describe how firms make decisions depending on the associated costs and their productivity levels. Assuming that fixed costs are higher for exporting than for domestic sales only and that they are even higher for foreign plant set-up and running a multinational network than for exporting, the most productive firms engage in FDI, less productive companies export, and the least productive firms stay in the domestic market only (see Helpman, Melitz, and Yeaple, 2004). This theoretically predicted pattern has been supported by a number of empirical studies (for the difference between exporters and non-exporters, see Arnold and Hussinger, 2005a; Bernard and Jensen, 1999; Clerides, Lach, and Tybout, 1998; Girma, Greenaway, and Kneller, 2002; for the difference between exporters and MNEs, see Girma, Kneller, and Pisu, 2005; Arnold and Hussinger, 2005b). Recently, Conconi, Sapir, and Zanardi (2010) illustrate that learning through exporting matters for the decision of how to serve a market, via exports or FDI. Empirically, many MNEs are multi-plant units which are established gradually. It appears that no research on the *genesis of multinational networks* exists to this date, and it is this paper's purpose to fill this gap.

Given that establishing a multinational network is profitable per se beyond other options, further choices are available to the firm. For example, it has to decide on *where* to locate the first foreign entity (location choice). This choice among several alternative first locations may depend on local factor costs, on the accessibility of production factors, or on various measures of proximity to the home market (for empirical investigations on the location choice of MNEs, see Devereux and Griffith, 1998; Becker, Egger, and Merlo, 2009; Chen and Moore, 2010). Managers of the firm then have to answer related questions of the following kind. Should the first investment involve high or low capacity levels? Is the first investment the basis for other investments in the region? Given the location choice of previous investments, where should subsequent affiliates be located?

In this context, the present paper analyzes foreign location decisions of MNEs, why sequential entry patterns can be optimal, and how decisions depend on earlier location choices. Our theoretical approach is related to the theoretical learning (or bandit) literature. Early contributions to this literature include Bellman (1956) and Berry and Fristedt (1979), while a learning process similar to ours has been applied recently by Bergemann and Hege (1998, 2005) and Keller, Rady, and Cripps (2005). Specific to our model is the possibility that entry decisions in different countries depend on each other, since market conditions exhibit similarities. How consumers preferences or attitudes of employees differ across countries depends on issues such as geographical or cultural distance. If the correlation between market features is sufficiently high, a firm can make use of the knowledge it gains in one market to learn about conditions elsewhere. Then, a firm may want to enter a second country if it was sufficiently successful in the first one. This leads to one of our main results: even if expected profits in a market are positive, it can be optimal to delay or later on even abandon subsequent entry. The reason is that market entry is costly, and sequential investments can increase expected profits by using information gathered elsewhere. On the other hand, the reduced uncertainty through delayed entry comes at the cost of foregone profits. This result is related to the vast literature on investment under uncertainty (starting with McDonald and Siegel, 1986; see Dixit and Pindyck, 1994, for an overview). If the value of an irreversible investment project follows a stochastic process, the option to wait for a better realization is valuable even if immediate entry would be profitable. Our result follows a similar logic. Uncertainty combined with correlated learning creates an option value of waiting, and a sufficient amount of uncertainty must exist to make sequential entry potentially optimal. The main difference is that learning is not induced exogenously but by a firm's activities elsewhere. Thus, the firm can also influence the degree of learning by adjusting its investment levels in other markets.

Based on the proposed theoretical model, we derive several testable hypotheses. First, entry should be more likely in foreign markets where expected profits are higher. Expected profits do not only increase only with market size and productivity but also with proximity to the domestic market. The reason for the latter is the following. Firms for which FDI is possibly attractive are successful at home. Such firms will enter closer foreign markets more likely, since their positive experience at home is ceteris paribus more valuable there. Second, sequential entry rather than simultaneous entry abroad can be optimal with sufficient uncertainty about returns on FDI and high-enough success at markets entered first. Then, with sequential entry more proximate countries should be entered first on average. The reason is that uncertainty creates a value of waiting, rendering it worthwhile to stagger FDI decisions across markets in an order which declines in expected profits. Third, subsequent foreign entry is more likely in markets which are proximate to previous investments for the same correlated learning reasons as before.

We assess these hypotheses empirically using a unique micro-level panel data-set provided by Deutsche Bundesbank (the German Central Bank) that allows us to track German MNEs' sequential location decisions over time. We are able to identify the first, the second, etc., up to the eleventh location decision of firms across large-enough samples. Using a conditional logit model for the empirical analysis, we find that first, foreign entry is more likely for countries that are closer to the MNE's home base and where higher profits may be expected in general terms. This finding is supported by variables measuring the proximity of markets at large, e.g., whether the same language is spoken or if the target country used to be a colony of the home country. Moreover, proximate countries tend to be entered first as a multinational network evolves. Third, subsequent entry in later expansion phases is generally more likely in markets that are closer to the ones entered previously.

To analyze whether it is actually correlated learning that drives the observed expansion patterns or not, we conduct a number of tests. An important result of our theoretical model is that the average capacity of investments should ceteris paribus be higher in case of a sequential entry pattern compared to the average capacity of investments when entities are established simultaneously. The reason is that in the former case a higher capacity in a country not only raises expected revenues there, but also the amount of learning about other markets.

Furthermore, the reason for not observing sequential investments could either be that *isolated entry* or *simultaneous entry* was intended from the beginning or that a firm initially planned *sequential entry* but was not sufficiently successful in the first market. There, we can use the result that the average capacity in sequentially entered markets is higher than in simultaneously entered ones. This allows us to hypothesize that, if a firm only enters one market in one phase and does not establish any subsequent affiliates, the more successful ones should have a lower capacity than the others. The latter is consistent with the notion that affiliates with an above-average capacity were intended to be followed by sequential investments elsewhere. Then, one reason for a lack of subsequent investments to high-capacity first investments should be that first investments were not sufficiently successful.

Finally, correlated learning makes the firm ceteris paribus more optimistic about the prospects in a market. Thus, it will lower its requirements for later entry with respect to market size or entry costs over time.

We find support for all of these hypotheses in our empirical analysis, leading us to the conclusion that the proposed correlated learning mechanism is indeed an important factor determining international expansion strategies of multinational networks. To investigate whether other alternative mechanisms can explain the results of our model as reported above, we analyze alternative mechanisms such as stochastic shocks across markets, diseconomies of scale (i.e., constrained resources available to firms), or learning by doing. All of these mechanisms may be used to derive dynamic expansion strategies of MNEs. But, as we illustrate, none of these models fits the data as well as correlated learning does.

The remainder of the paper is organized as follows. We present a theoretical model and main results in Sections 2 and 3. Section 4 derive testable hypotheses, introduces the data and empirical model, and summarizes the benchmark estimates. Section 5 provides extensions and robustness tests, while Section 6 develops alternative models that might also explain the observed firm behavior. Section 7 offers concluding remarks.

3.2 Model Setup

The following model portrays the international expansion pattern of a firm. This firm (or "multinational", or "MNE") is active for two periods, $t = 1, 2$, and considers establishing affiliates in two countries, $j = \{A, B\}$. Entry in country j is possible in either period. Upon entry, the firm chooses a capacity level X_j which can not be adjusted subsequently.[2] In the period of entry, the investment level X_j is associated with costs $K_j(X_j) = F_j + k_j \frac{(X_j)^2}{2}$, where $F_j \geq 0$ are fixed entry costs and $k_j > 0$ captures marginal investment costs.

Each investment may be profitable or not. More precisely, the firm possesses an exogenously given type θ_j in country j, with $\theta_j \in \{0, \theta^h\}$ and $0 < \theta^h \leq 1$. The type θ_j covers firm- as well as market-specific characteristics and is related to the idea of a matching quality between the firm and the market in the spirit of Jovanovic (1979).[3] If $\theta_j = \theta^h$, the affiliate generates a constant return $R_j > 0$ with probability $X_j\theta^h$ in each period.[4] Future profits are discounted with the factor $\delta \in [0, 1]$. If $\theta_j = 0$, the project does not yield any profits. Formally, per-period returns are denoted by $Y_{jt} \in \{0, R_j\}$, with $\text{Prob}(Y_{jt} = R_j \mid X_j, \theta_j) = X_j\theta_j \in [0, 1]$. The latter requires sufficiently high marginal investment costs, $k_j \geq \theta^h R_j(1 + \delta)$, $j = A, B$, which we assume subsequently. Finally, the firm is not financially constrained.

Note that, for the sake of simplicity, we restrict the firm's success to two states – an affiliate is either profitable or not. Allowing for several or even continuous degrees of success would have no qualitative impact on our results but substantially complicate the

[2] This assumption has no qualitative impact on our results; see Appendix III for a characterization of capacity investments when X_j can be adjusted later on.

[3] For instance, θ_j captures the success of a marketing campaign and other specific characteristics of demand for a firm's products in country j. It could also reflect the ability to make use of natural resources and other local factors in country j and, in general, the efficiency of the firm's production process there.

[4] We interpret R_j quite broadly. It may include revenues attributable to the investment in j but also general efficiency gains to the firm through a foreign investment. Furthermore, we choose the probability function to be linear in X_j for convenience. Generally, any probability function which is monotonic in X_j and less convex than the cost function $K_j(X_j)$ would serve our purpose.

analysis.

A Firm's Belief About Its Type

One crucial element of the proposed model is that, before market entry, an MNE does not know whether its type in country j is high or low. Instead, it assigns the (subjective) probability ρ_j to being the high type. In the following, we call this probability the firm's *belief*. The belief is given before period $t = 1$ and may have been formed by previous activities in this market such as market research. A firm's type is specific to a market so that being of the high type at home does not guarantee to be the high type also in a foreign market.[5]

For a firm in our model, there are three relevant markets in each of which the firm is either of the high or the low type: home, A, and B. All firms initially are active at home and learn about their type there over time. We confine our interest to firms of the high type at home. This is consistent with results in Helpman, Melitz, and Yeaple (2004) suggesting that only the most productive firms become MNEs. While decisions about home do not feature in our analysis, the type there is relevant because conditions in different markets – and thus the realizations of types – are not independent. For example, geographical or cultural neighborhood across markets is a source of such correlation.

In our model, the type at home is drawn first. Recall that we focus on firms of θ^h at home. Then, the type in country A is realized. The type is determined by two different components. The first component relates to the type at home, the second one is idiosyncratic. To be precise, with weight $r_A \in [0, 1]$ the type in A is high with the same probability – namely unity – as at home. r_A captures the proximity between home and country A and is larger if markets are geographically and culturally close to each other. With weight $(1 - r_A)$, the type in A is high with an idiosyncratic probability of ρ_A^0. The latter is formed by generally available information, market research, or other previous activities. Thus, the firm's subjective ex-ante belief of being a high type in country A

[5] Prior to foreign market entry, a firm faces substantial uncertainty concerning its profitability there. It may be argued that firms considering FDI in some market have already gathered information about local market conditions. This does not contradict the maintained assumptions, as long as there is still some uncertainty left.

equals

$$\rho_A = r_A + (1 - r_A)\rho_A^0.$$

Finally, the type in country B is realized. It is identical to home with weight r_B. With weight $(1 - r_B)$, the type is determined by B's idiosyncratic component and its proximity to country A. Formally, the belief in B is characterized by

$$\rho_B = r_B + (1 - r_B)[r_{AB}\rho_A^0 + (1 - r_{AB})\rho_B^0].$$

The parameter r_{AB} captures potential correlations between A and B that are not already covered by the proximity to home, r_A and r_B, respectively. Hence, we introduce different dimensions of proximity. For example, assume that home is Germany and that the MNE considers investments in Austria, Switzerland, and Denmark. The (geographical and cultural) distance of each of these countries to Germany is quite low. However, while Austria and Switzerland share a common language and other cultural aspects with each other so that they are quite proximate in general terms (high r_{AB}), the geographical and cultural distance between Austria and Denmark is much bigger (lower r_{AB}) than the one between Austria and Switzerland.

The beliefs are increasing in the respective proximity parameters r_j, so that the firm is more optimistic about a country closer to home. The parameters ρ_j^0, r_j and r_{AB} are known ex ante and determine the subjective beliefs ρ_j. After the first period, when respective output values are observed, these observations are used to update beliefs using Bayes' rule. We will explore the updating process in more detail below when analyzing respective entry patterns.

3.3 Optimal Behavior

3.3.1 Entry Patterns

Conditional on being active abroad, the MNE will choose one of the three following options.

- **Isolated Entry**: Entry into one country in period 1, no further entry in period 2.

- **Simultaneous Entry**: Entry into both countries in period 1.
- **Sequential Entry**: Entry into one country in period 1 and into the other one in period 2, conditional on a success of the first foreign investment.

All other possibilities are dominated by one of the options mentioned above. Isolated or simultaneous entry in the second period would come at the cost of foregone profits in the first period. Entering a second market sequentially after a failure in the first one would be dominated ex-ante by simultaneous entry. The reason for the latter lies in the correlated updating of beliefs, as will become clear below.

The MNE will choose isolated entry into one country if beliefs in the other country are too low to ever justify an investment there. Otherwise, the firm will consider simultaneous or sequential entry, facing the following tradeoff. Under sequential entry, the firm looses potential profits in the first period. On the other hand, the risk of wasting investment costs is reduced, i.e., there is a value of waiting. The reason is that the firm learns something about the conditions in the second market because of the correlation of the firm's types in the two markets. We will analyze this tradeoff in more detail below.

3.3.2 Isolated Entry

In this section, the MNE only considers entry into one country, which allows us to omit the country subscript. If entry is optimal, it will already occur in period 1. Then, the firm chooses a capacity X to maximize the expected discounted profit stream

$$\Pi^{iso} = X\left(\rho_1 R\theta^h + \delta \mathrm{E}[\rho_2] R\theta^h\right) - K(X), \tag{3.1}$$

where expectation is taken concerning ρ_2, the belief in period 2. To be able to characterize the optimal level of X, we have to consider the updating process. ρ_2 is derived using Bayes' rule, given the initial belief and capacity X. To simplify issues, we omit time subscripts and denote the initial belief ρ. After a success ($Y_1 = R$), the period-2 belief equals ρ^+, while after a failure ($Y_1 = 0$), that belief equals ρ^-.

As the bad type always fails, a success immediately reveals a good type, and

$$\rho^+ = 1.$$

After a failure, we have[6]

$$\rho(X)^- = \frac{(1 - X\theta^h)\rho}{1 - \rho X\theta^h}.$$

As $\frac{\partial\rho(X)^-}{\partial X} < 0$, a larger investment is generally associated with more learning. More updating occurs for intermediate values of the initial belief, while there is less updating if the belief is close to zero or unity.[7]

Note that, as running the affiliate requires no further costs once the capacity is set, it is never optimal to exit a market, no matter how low the belief may be.[8] The assumption of no operating costs has no substantial impact on our results. In a more general model with operating costs and a longer time horizon, the possibility of exit in the future would affect the decisions associated with entry. However, this impact is lower the further ahead a potential exit lies in the future. In our analysis, firms are sufficiently productive to engage in FDI and face substantial entry costs. This implies that the belief necessary to make entry optimal has to be high enough that an immediate exit after only a few failures will not occur. The optimal activities of the MNE in case of isolated entry may be described as follows.

Proposition 1 (Isolated Entry): Given market entry, the optimal capacity under isolated entry equals

$$X^{iso} = \frac{\rho\theta^h R(1+\delta)}{k}, \tag{3.2}$$

and entry is only optimal if

$$\Pi^{iso} = \frac{(\rho\theta^h R)^2(1+\delta)^2}{2k} - F \geq 0. \tag{3.3}$$

Proof: See Appendix II for the proof.

[6]Recall that $\rho(X)^- = \text{Prob}[\theta = \theta^h \mid Y_1 = 0] = \frac{\text{Prob}[\theta=\theta^h \cap Y_1=0]}{\text{Prob}[Y_1=0]}$
$= \frac{\text{Prob}[Y_1=0|\theta=\theta^h]\text{Prob}[\theta=\theta^h]}{\text{Prob}[Y_1=0|\theta=\theta^h]\text{Prob}[\theta=\theta^h]+\text{Prob}[Y_1=0|\theta=0]\text{Prob}[\theta=0]} = \frac{(1-X\theta^h)\rho}{(1-X\theta^h)\rho+1(1-\rho)}.$

[7]This is the case since $\rho - \rho(X)^- = \rho X\theta^h \frac{1-\rho}{1-\rho X\theta^h}$.

[8]Thus, we do not consider the "standard" value of learning, namely the option to stop the project. If running the affiliate was costly, this option value would make the firm willing to accept some expected short-term losses in the first period.

Throughout, we assume that condition 3.3 is satisfied for each country if $\rho_j = 1$, i.e., market entry is profitable at profit-maximizing capacity levels for sufficiently high beliefs. Comparative statics can easily be derived. The capacity is increasing in R (which could reflect fundamentals such as market size). It is decreasing in the distance to home (i.e., a larger value of r) and in investment costs k. Higher R and r as well as lower k also render entry more likely, whereas larger fixed costs (which in a market perspective could reflect fundamentals such as corruption or investment freedom) make entry less likely.

3.3.3 Simultaneous Entry

When choosing simultaneous entry, the firm enters both countries A and B at the beginning of period 1. Now, beliefs in country A are also affected by outcomes in B (and vice versa). But, after capacities are set, events in A have no impact on decisions in B and vice versa. Thus, correlated learning does not provide an additional benefit under simultaneous entry, and we postpone the analysis of the correlated updating process to the case of sequential entry. Total expected profits of the firm just equal the sum of profits under isolated entry:

$$\Pi^{sim} = \left(X_A^{sim}\rho_A\theta^h R_A + X_B^{sim}\rho_B\theta^h R_B\right)(1+\delta) - K_A(X_A^{sim}) - K_B(X_B^{sim}).$$

Therefore, the chosen capacity levels are identical to above and we get

$$X_j^{sim} = \frac{\rho_j\theta^h R_j(1+\delta)}{k_j},$$

yielding total profits

$$\Pi^{sim} = \frac{(\rho_A\theta^h R_A)^2(1+\delta)^2}{2k_A} + \frac{(\rho_B\theta^h R_B)^2(1+\delta)^2}{2k_B} - F_A - F_B.$$

The non-negativity condition is identical to the one under isolated entry and has to be satisfied here as well. Comparative statics for capacity and the likelihood of entry are also the same as under isolated entry.

3.3.4 Sequential Entry

When choosing sequential entry, the firm uses information gathered in one country, say A, to update its beliefs about B. At the beginning of period 1, it enters A. Observing a success, it subsequently invests in B in period 2. Otherwise, it just remains in A without any further investments. Note that entry in B after a failure in A can not be optimal since this would be dominated by simultaneous entry.

The (relative) profitability of sequential entry depends on several aspects. As already mentioned, the firm faces a tradeoff when comparing sequential and simultaneous entry. Under the former regime, it can reduce its risk and only has to bear investment costs for relatively high beliefs. On the other hand, it looses potential profits from the second country in period 1. Crucial for the aspect of risk reduction is the actual amount of correlated learning, which determines the option value of waiting. This depends on the distance between A and B, captured by the parameter r_{AB}. Furthermore, observing a success in A has to be a sufficiently strong signal. In case that r_A is very close to unity, a success in A does not contain much new information, as the firm already is quite optimistic to face a high type there ex ante. This limits updating in B, rendering the gains of sequential entry negligible. Therefore, a considerable amount of uncertainty in A has to prevail for sequential entry to be optimal.

Beliefs and Correlated Learning

Considering correlated learning, the updating process is slightly different from above, as the outcome in one country also affects beliefs in the other one.

Recall that ex-ante beliefs (or *priors*) about markets A and B equal $\rho_A = r_A + (1-r_A)\rho_A^0$ and $\rho_B = r_B + (1-r_B)[r_{AB}\rho_A^0 + (1-r_{AB})\rho_B^0]$, respectively, where r_j is a proxy for the (cultural or geographical) distance of country $j \in \{A, B\}$ to the MNE's home market, while r_{AB} considers the proximity between the two foreign target countries. With sequential entry, we can not analyze both countries' beliefs in isolation anymore and have to consider four possible states for the set of types (θ_A, θ_B), namely (θ^h, θ^h), $(\theta^h, 0)$, $(0, \theta^h)$, and $(0, 0)$. Updating occurs conditional on observing the outcome (Y_A, Y_B), which takes one of the realizations (R_A, R_B), $(R_A, 0)$, $(0, R_B)$, or $(0, 0)$. As before, we use superscripts to denote updated beliefs. For instance, ρ_j^{+-} denotes the period-2 belief in country j after a success in A and a failure in B were observed. Note that not entering B in period 1 automatically implies that a failure was observed there. In Appendix I, we derive general characterizations of the updated beliefs.[9] Note that, after a success, the belief in the respective country still jumps to unity. What we need for the analysis of sequential entry – i.e., entry in the second country after a success in the first one – is the updated belief in B after a success was observed in A, and vice versa.

Under sequential entry starting in A and with $Y_A = R_A$ (which implies that the type in A must be high), the belief in B becomes

$$\rho_B^{+-} = \left(r_B + (1 - r_B)[r_{AB}\frac{\rho_A^0}{\rho_A} + (1 - r_{AB})\rho_B^0] \right).$$

Obviously, $\rho_B^{+-} > \rho_B$. Starting out by investing in B and observing a success there yields

$$\rho_A^{-+} = \left(r_B + (1 - r_B)[r_{AB}\frac{\rho_A^0}{\rho_A} + (1 - r_{AB})\rho_B^0] \right) \frac{\rho_A}{\rho_B} = \rho_B^{+-}\frac{\rho_A}{\rho_B}.$$

Profits and Capacities

In this section we derive profits and capacities under sequential entry. For convenience, we continue to assume that the MNE enters country A first, and B subsequently.

Sequential entry yields total expected profits of

$$\Pi^{seq} = X_A^{seq}\rho_A\theta^h R_A(1+\delta) - k_A\frac{(X_A^{seq})^2}{2} - F_A + \delta X_A^{seq}\rho_A\theta^h \left(X_B^{seq}\rho_B^{+-}\theta^h R_B - k_B\frac{(X_B^{seq})^2}{2} - F_B \right). \tag{3.4}$$

$X_A^{seq}\rho_A\theta^h R_A(1+\delta) - k_A\frac{(X_A^{seq})^2}{2} - F_A$ collects profits generated in A. The term $X_A^{seq}\rho_A\theta^h \left(X_B^{seq}\rho_B^{+-}\theta^h R_B - k_B\frac{(X_B^{seq})^2}{2} - F_B \right) = X_A^{seq}\rho_A\theta^h\Pi_B^{seq}$ describes expected profits from entering country B valued in period 2. It is the product of the probability that this actually happens, i.e., the probability of success in A, and the expected profits in B given

[9] We also show that under correlated learning beliefs follow a martingale – implying that they do not change in expectation – just as before when we considered the updating process for one country in isolation

entry there. Then, the capacity in B equals

$$X_B^{seq} = \frac{\rho_B^{+-} \theta^h R_B}{k_B},$$

yielding $\Pi_B^{seq} = \frac{\left(\rho_B^{+-} \theta^h R_B\right)^2}{2k_B} - F_B$.

When determining X_A^{seq}, the potential profits in B are taken into account, and we obtain

$$X_A^{seq} = \frac{\rho_A \theta^h R_A (1+\delta) + \delta \rho_A \theta^h \Pi_B^{seq}}{k_A}. \tag{3.5}$$

Proposition 2 (Sequential Entry): The capacity chosen in the first country under sequential entry is larger than the capacity in this country under simultaneous entry.

Proof: See Appendix II for the proof.

Capacity in A is higher under sequential than under simultaneous entry because it not only raises expected revenues there but also the probability for entry in B for a given belief ρ_B^{+-}. As the expected profits in B are positive by construction (otherwise, isolated entry would be better), a larger capacity in A increases the likelihood of a realization of these profits. This implies that expected total profits (net of investment costs) in A are lower than under simultaneous entry. Whether the capacity in B is lower or higher depends on the size of the discount factor.

Different from the standard literature on investment under uncertainty (see McDonald and Siegel, 1986; Dixit and Pindyck, 1994), the degree of learning is not exogenously given but implied by the capacity choice in country A. Thus, the firm balances costs of learning (higher capacity in A than individually optimal) with its benefits (higher probability of realizing profits in B).

Proposition 2 is very important for empirical robustness tests as conducted in Section 3.5 below. If sequential entry were unobserved in reality, we would not not know whether simultaneous or isolated entry were intended from the beginning or whether the firm had planned sequential entry originally but did not observe a success in the first country.[10] If simultaneous or isolated entry were observed, we could use capacity differences between simultaneous or isolated entry and sequential entry as a proxy for the intended entry pattern.

Finally, sequential entry upon entry in A gives expected profits

$$\Pi^{seq} = \frac{1}{2k_A}\left[\rho_A\theta^h R_A(1+\delta) + \delta\rho_A\theta^h\left(\frac{\left(\rho_B^{+-}\theta^h R_B\right)^2}{2k_B} - F_B\right)\right]^2 - F_A. \qquad (3.6)$$

Similar to the cases of simultaneous and isolated entry, the probability of sequential entry to be profitable (which does not mean that it is actually optimal) increases with expected profits, i.e., ρ_j, r_j, θ^h, and R_j, and it decreases with costs, i.e., with k_j and F_j. The next proposition contains our first main result.

Proposition 3: Sequential entry or simultaneous entry can be optimal, depending on parameter values. Sequential entry is even possible if individual expected profits in both countries are positive at the beginning.

Proof: See Appendix II for the proof.

This proposition links our results to the one proposed by optimal investment decisions under uncertainty (McDonald and Siegel, 1986; Dixit and Pindyck, 1994). If expected profits in both countries are strictly positive ex ante, delaying entry for one of them might still be optimal. Despite the positive net-present-value of an investment, the option value of waiting may be higher.

However, sequential entry will only be optimal if sufficient correlated learning occurs, which requires two elements. The distance between countries A and B must not be too high (r_{AB} must be high enough). In addition, some uncertainty has to prevail, as otherwise no substantial updating can occur. If a firm is already very optimistic about its type in one country (ρ_j is close to unity), beliefs will only be updated marginally.

[10] A further option would be that sequential entry is still planned and only was not realized yet. We will come back to that point later on.

Let us use these results and attend to the tradeoff the firm faces when considering sequential entry. It can reduce the total risk of investment (investment costs in markets with a low type) by using information gained in country A for activities in B.[11] The information gathered in A is only valuable for B, if the difference between ρ_B^{+-} and ρ_B is sufficiently large. To see this, consider the extreme case where $\rho_B = \rho_B^{+-}$ (which will be the case if either $\rho_A = 1$ or $\rho_B = 1$). Then, simultaneous or isolated entry always dominates sequential entry (for a formal analysis see the proof with regard to Hypothesis 2 in Appendix II). This remains the case as long as $\Delta\rho_B^{seq} \equiv \rho_B^{+-} - \rho_B$ is relatively small. Since this is an important aspect, let us take a closer look at $\Delta\rho_B^{seq} = (1 - r_B) r_{AB} \frac{\rho_A^0}{\rho_A} (1 - \rho_A)$. As $\rho_A = r_A + (1 - r_A)\rho_A^0$, we arrive at the following comparative static results:

$$\frac{\partial \Delta\rho_B^{seq}}{\partial r_{AB}} = (1 - r_B) \left(\frac{\rho_A^0}{\rho_A} - \rho_A^0 \right) > 0, \tag{3.7}$$

$$\frac{\partial \Delta\rho_B^{seq}}{\partial r_A} = -(1 - r_B) r_{AB} \frac{\rho_A^0 (1 - \rho_A^0)}{\rho_A^2} < 0, \tag{3.8}$$

and $\frac{\partial \Delta\rho_B^{seq}}{\partial r_B} = -r_{AB} \left(\frac{\rho_A^0}{\rho_A} - \rho_A^0 \right) < 0$, $\frac{\partial^2 \Delta\rho_B^{seq}}{\partial r_A \partial r_B} = r_{AB} \frac{\rho_A^0 (1 - \rho_A^0)}{\rho_A^2}$. Expression (3.7) implies that if conditions in countries A and B are more similar to each other, the correlation in learning is higher, and a positive outcome in A is a stronger signal concerning the profitability in B. Expression (3.8) states that if A is closer to home, entry there makes the firm learn less about the conditions in B. In the extreme case, if $r_A = 1$, the type in A can not be distinguished from the type at home. Then, $\Delta\rho_B^{seq} = 0$, and it is not possible to learn something from A about the conditions in B. Again, this part relates to the question of optimal investment under uncertainty. A higher degree of uncertainty increases the option value of waiting and thus raises the threshold of required profits to make entry actually optimal. Here, a sufficient degree of learning as characterized by $\Delta\rho_B^{seq}$ is required to render sequential entry an optimal choice.[12]

To sum up, for sequential entry to be optimal, a success must reveal sufficient information about the first country that is entered as well as the second one.

[11] For example, a high level of F_B will make the risk reduction through waiting and learning more valuable.

[12] Note that we can not establish a simple monotone rule claiming that a higher level $\Delta\rho_B^{seq}$ increases the profitability of sequential relative to simultaneous entry. The reason is that it is not possible to analyze a change of $\Delta\rho_B^{seq}$ in isolation, as all its components have an impact on other issues than just the degree of learning or uncertainty.

3.4 Empirical Analysis

3.4.1 Testable Hypotheses

In this section, we use the theoretical model to derive predictions and formulate them in a way that allows us to test them empirically (all predictions are proven in Appendix II). We will refer to these *predictions* as testable *hypotheses.*

Hypothesis 1:
Foreign market entry should be more likely for larger levels of R_j and θ^h, for lower costs k_j and F_j, and for a larger value of general proximity r_j.

R_j or θ^h capture a firm's profitability in a market provided that it is generally successful there (its type is high). Profitability may be affected by market size, which can be measured by a country's GDP, and other aspects that have a direct impact on profits, such as a country's profit tax rate. Concerning the costs of market entry, we consider measures such as corruption, investment freedom, or the general costs of starting a business in a country (see Chen and Moore, 2010). Finally, a high proximity to the home country should render entry into a market more likely. Geographical distance is obviously a good proxy for the parameter r_j. But also cultural factors such as a common language are expected to positively affect a firm's propensity to enter a foreign market.

Hypothesis 2:
Sequential entry can be the optimal entry mode. If it is chosen, the country with a higher level of proximity r_j should generally be entered first. For levels of r_j close to 1*, simultaneous entry would be optimal.*

Hypothesis 2 states that the marginal effect of the proximity parameters should be larger in absolute value for earlier compared to later entry. The reason is that the first foreign investment of a firm may relate to sequential or simultaneous entry and should thus be close to home on average. In contrast, all subsequent investments must be part of a sequential entry strategy.

With sequential entry, the closer country should generally be entered first as long as two foreign markets do not differ too much in size (and, hence, profitability). If r_j is close to unity, almost no correlated learning occurs; see equations (3.7) and (3.8) and the related discussion. Then, simultaneous entry is optimal.

Hypothesis 3:
Provided that market A is entered in period 1 but B is not, a higher value of proximity between A and B, r_{AB}, should increase the probability that the MNE enters B in period 2.

Hypothesis 3 predicts that a greater (geographical) distance between countries of different expansion phases will reduce the probability to enter a country at a later stage. In this sense, later expansion phases depend on all previous investments.

The above hypotheses suggest that a firm should rather enter more promising markets in terms of market size and costs than others. Furthermore, an expansion of a multinational network should, on average, follow a certain pattern – starting in closer countries, then gradually increasing in distance from home but remaining close to markets entered previously. In the following, we show that such entry patterns are indeed observed in our data. For this empirical analysis, we use a unique micro-level data-set provided by Deutsche Bundesbank (the German Central Bank) that allows us to track the universe of German MNEs' sequential location decisions over time. We will see that the patterns observed in the data are largely in line with those hypotheses.[13]

3.4.2 Empirical Model Specification

Let us index German MNEs by $i = 1, ..., N$ and focus on the location choice of their affiliates among $j = 0, 1, ..., J$ foreign host countries. In any phase $p = 1, ..., P$

[13] At this point, the patterns described by Hypotheses 1-3 and found in the data could still be generated by other mechanisms than the proposed correlated learning channel. However, we analyze this issue theoretically below and provide evidence supporting the proposed learning process for the genesis of multinational networks.

(corresponding to periods in our theoretical model), MNE i can choose among the J host markets with regard to location of its foreign entities.[14] Since we are interested in the genesis of MNEs' networks, we associate expansion phases of the network with p. While MNEs typically set up one foreign affliate per phase p, in some cases they locate in several markets simultaneously in p. Each of these decisions will be treated individually below.[15] There is a number of options for modeling such a multinomial choice problem by means of nonlinear multinomial probability models. Examples thereof are the classes of multinomial probit-type models and multinomial logit-type models. A great advantage of the latter is that they follow from utility maximization of households or, as in our case, profit maximization of firms (see Wooldridge, 2002, p. 500f.). The same would be true for multinomial probit-type models, but with a huge number of $N = 15,171$ firms choosing among as many as $J = 104$ host countries as in our case,[16] it is natural to resort to multinomial logit-type models due to their tractability and numerical stability.[17] In the class of logit-type models, the conditional logit is a natural candidate for modeling the problem at stake, since it allows for regressors which are indexed by alternative j (and possibly also by firm i).[18]

We postulate that firm i would receive latent net profits Π^*_{ijp} from locating an affiliate at market j in phase p consistent with our theoretical model according to the process

$$\Pi^*_{ijp} = Z_{ijp}\beta_p + \alpha_{ijp}, \quad i = 1, ..., N, \ j = 0, 1, ..., J, \ p = 1, ..., P \tag{3.9}$$

[14]Notice that investment phases are unequally spaced in real time across firms. Hence, *phases* should not be confused with *years*. For instance, the first foreign investment of firm i may take place in any year covered by our sample period. Hence, a phase is associated with a vintage of foreign investments per firm.

Furthermore, note that the restrictions to two firms, two host countries, and two periods in our theoretical model has no qualitative impact on the derived hypotheses.

[15]Accordingly, index i in fact denotes the choice of an MNE about a specific affiliate. However, for the ease of presentation, it is sufficient to refer to i as a firm.

[16]In principle, MNEs may enter as many as 162 countries, but in 58 of them not a single investment occurs so that those choices are dropped in the analysis.

[17]Multivariate probit-type models require integrating numerically a multivariate normal whose dimensions are determined by the number of choices taken. In spite of the efficient simulation algorithms available nowadays, for a choice problem as large as ours and a data-set which is not accessible locally so that computers can not be employed over extended time spans, it is virtually impossible to run multinomial probit-type or nested logit-type models.

[18]What is referred to as the multinomial logit model in a narrow sense assumes that the regressors only vary across firms i but not alternatives j in any phase p, while the parameters on those regressors vary across alternatives. It is well known that this model can be represented by the conditional logit model, where regressors rather than parameters are specific to the alternatives. Again, for as many alternatives as in our case, it appears unnatural to estimate J parameter vectors.

where the $1 \times L_p$ vector Z_{ijp} contains determinants of the profits which depend on the alternative and, eventually, on firm i in any phase p. The $L_p \times 1$ vector of weights β_p on Z_{ijp} are unknown and will be estimated by maximum likelihood estimation. α_{ijp} represents unobservable variables affecting the choice. The actual choice $C_{ip} \in 0, 1, ..., J$ is based on the maximum attainable profit, $\arg\max(\Pi^*_{i0p}, ..., \Pi^*_{iJp})$. Following McFadden (1974) in assuming that the α_{ijp} are independently distributed across alternatives with a type I extreme value distribution and using the notation $Z_{ijp} = (Z_{i0p}, ..., Z_{iJp})$,

$$P_{ijp} \equiv Pr(C_{ip} = j | Z_{ijp}) = \frac{\exp(Z_{ijp}\beta)}{\sum_{j=0}^{J} \exp(Z_{ijp}\beta)}, \quad \text{for all } i, j, p \tag{3.10}$$

for which the marginal effect of the kth variable Z_{ijp} is $\partial P_{ijp}/\partial Z_{ijpk} = P_{ijp}(1-P_{ijp})\beta_{pk}$ for all i, j, p, k and $\partial P_{ijp}/\partial Z_{i\ell pk} = -P_{ijp}P_{i\ell p}\beta_{pk}$ for all $i, \ell \neq j, p, k$. A well-known assumption taken by this approach is the one of independence from irrelevant alternatives (i.e., that the choices taken with regard to alternatives j versus ℓ are not affected when adding further alternatives).[19]

3.4.3 Data

We use data on the universe of German MNEs' foreign entities according to the classification taken by Deutsche Bundesbank[20] and as collected in and made available through the Bank's MiDi (Microdatabase Direct Investment) database (see Lipponer, 2009, for details). Individual MNEs and their affiliates can be tracked annually in MiDi since 1996. Since the database contains the universe of German MNEs' foreign affiliates, it is particularly suited for an analysis of the genesis of multinational networks of foreign affiliates.

The vector of determinants of location decisions of firm i in phase p, Z_{ijp} in (3.9),

[19] Alternative modeling choices such as multivariate probit or nested logit models do not assume an irrelevance of the relative odds between choices j and ℓ from irrelevant alternatives. However, as said before, these models are computationally demanding and, with a choice and firm data-set as large as ours and the conditions imposed on empirical analysis through computing at the site of the data source, even infeasible to estimate.

[20] All German firms and households which hold 10% or more of the shares or voting rights in a foreign enterprise with a balance-sheet total of more than 3 million Euros are required by German law to report balance-sheet information to Deutsche Bundesbank. Indirect participating interests have to be reported whenever foreign affiliates hold 10% or more of the shares or voting rights in other foreign enterprises. These reporting requirements are set by the Foreign Trade and Payments Regulation.

contains the following regressors. The statutory corporate tax rate of the host country, $Tax_{jp} \in [0,1]$, reduces a firm's profitability ceteris paribus. The log of real GDP at constant U.S. dollars of the year 2000, $\log GDP_{jp}$, is a measure of j's market size. A number of variables are supposed to reflect the fixed investment costs F in terms of our theoretical model, namely an investment freedom index, $InvestFree_{jp} \in [0,100]$, and a corruption perception index $CPI_{jp} \in [0,10]$,[21] as inverse measures of investment costs, as well as $InvestCost_{jp} \in [0,\infty)$ that reflects cost of starting a business relative to income per capita. The stock of German investments prior to firm i's investment in j and phase p, $StockInv_{jp} \in [0,\infty)$, is included as a general measure of market j's attractiveness for German investors beyond the aforementioned measures thereof. Furthermore, a number of variables determine the correlation between markets entered in p and $1 \leq \ell < p$ in terms of economic, cultural, and geographical proximity: host market j's geographical distance to Germany, $\log Distance_{jp}\ to\ \ell - th \in (-\infty,\infty)$, a common border indicator between Germany and host market j, $Border_{jp}\ to\ \ell - th \in \{0,1\}$, a common language indicator between Germany and host market j, $Language_{jp}\ to\ \ell - th \in \{0,1\}$, a former colony indicator between Germany and host market j, $Colony_{jp}\ to\ \ell - th \in \{0,1\}$, and a preferential trade agreement indicator between Germany and host country j, $GTA_{jp}\ to\ \ell - th \in \{0,1\}$.

When analyzing subsequent investment decisions (see below) for $p \geq 2$, we will also control for the indicator variable $Same_{jp} \in \{0,1\}$, which is unity if host country j and the country of the previous investment i are the same. Since $Same_{j1} = 0$ for all host countries j in the sample, $Same_{jp}$ is included only in the specifications for the second and subsequent investment phases. The sources for the data on the control variables are the World Bank's World Development Indicators 2009 ($\log GDP_{jp}$, $InvestCost_{jp}$), International Bureau of Fiscal Documentation (Tax_{jp}), Ernst&Young (Tax_{jp}), Price Waterhouse Coopers (Tax_{jp}), Transparency International (CPI_{jp}), Deutsche Bundesbank's MiDi ($StockInv_{jp}$), the Centre d'ï£¡tudes Prospectives et d'Informations Internationales ($\log\ Distance_{jp}\ to\ \ell - th$, $Border_{jp}\ to\ \ell - th$, $Language_{jp}\ to\ \ell - th$, $Colony_{jp}\ to\ \ell - th$, $Same_{jp}\ to\ \ell - th$), and the World Trade Organization as well as individual preferential trade agreement secretariates' webpages ($GTA_{jp}\ to\ \ell - th$).

Since the purpose of our analysis is to shed light on the determinants of an establish-

[21] Higher values of that index measure lower levels of perceived corruption.

ment of foreign affiliates per phase of investment, we restrict our interest to those firms for which we know that they did not operate any foreign affiliates in the first available year of the data, 1996. Hence, phase $p = 1$ with the first foreign investment of firm i may correspond to 1997 or any later year. Our data-set covers all first or subsequent investments of firms that became MNEs in 1997 or thereafter until 2008. Moreover, there are as many as $P - 1 = 11$ subsequent expansion phases possible in the data between 1998 and 2007. All of a firm's new affiliates which are founded across different years are associated with specific phases p and dubbed *sequential* investments, while a number of new affiliates founded within the same year are associated with the same expansion phase p and dubbed *simultaneous* investments. The design is such that $p = 1$ refers to the first set-up of one or more affiliates of firm i abroad, no matter in which year between 1997 and 2007 it occurred, and similarly for subsequent phases $p \geq 2$.

First Foreign Investments (p=1):

For first investments, $Z_{ij1} = Z_{j1}$ in (3.9) includes only determinants which pertain to the host country the first affiliate may be or is located in. First foreign investments may in principle occur in more than a single host market as investments in any phase p. We will relate subsequent investments to the biggest investment in phase $p - 1$ in terms of *fixed assets* for any phase $p \geq 2$. Using *total assets* as an alternative criterion does not lead to alternative conclusions. See also Section 3.4.5 for further sensitivity checks on this issue.

Second and Subsequent Foreign Investments (p>1):

According to our theoretical model, firm-specific decisions about first investments matter for subsequent foreign investments. Therefore, the determinants for subsequent expansions of the MNE network will be collected in the matrix Z_{ijp} for $p \geq 2$ in equation (3.9), which is indexed by i as well as j apart from p. In phases $p \geq 2$, Z_{ijp} includes regressors which are specific to host market j for the p-th investment, but it also includes ones that are firm-and-host-market specific in the sense that they relate to previous investments for firm i in phases $\ell < p$. By design, the number of regressors is $L_p > L_\ell > L_1$ for all phases $\ell < p$ with $p \geq 2$.

Covariates which relate foreign investments in phase p to previous ones are the following: the log distance of an affiliate set up in market j and phase p to the investments in earlier phases, log $Distance_{jp}\ to\ \ell - th$; a common border indicator between an affiliate set up in market j and phase p with the investments in earlier phases, $Border_{jp}\ to\ \ell - th$; a common language indicator between country j entered in phase p and countries entered in previous phases, $Language_{jp}\ to\ \ell - th$; and similarly with colonial relationships ($Colony_{jp}\ to\ \ell - th$), same country relationships ($Same_{jp}\ to\ \ - th$), and membership in a common goods trade agreement ($GTA_{jp}\ to\ \ell - th$). Table 1 presents descriptive statistics of all variables.

– Insert Table 1 about here –

3.4.4 Estimation Results

Table 2 summarizes results for sequential location decisions of MNEs. In every phase p, firms choose among approximately 100 host countries.

– Insert Table 2 about here –

We observe 15,165 *first* location decisions of MNEs in our sample analyzed in Column 1 of Table 2. As expected by Hypothesis 1, a bigger market size ($\log GDP$) raises the probability of an investment. A higher tax burden measured by the statutory tax rate of a country (Tax) implies a lower probability to choose a location. This is consistent with the impact of R_j on the location choice in the theoretical model. Moreover, as stated by Hypothesis 1, lower costs of entry as captured by more investment freedom ($InvestFree$), lower fixed costs ($InvestCost$), and less corruption perception (CPI) are associated with a higher probability to locate in a country. This is consistent with the impact of F_j and k_j on the location choice in the theoretical model. Finally, the included measures of proximity suggest that the probability of choosing a location increases with r_j as stated in Hypothesis 1. For instance, a larger distance between Germany and a potential host country ($\log Distance\ to\ parent$) reduces the probability of a first investment there. Similarly, if a potential host country shares a border with Germany ($\log Border\ to\ parent$), the location probability of a first investment increases. The variables *Language same as parent* and *Colony of parent* measure proximity in terms of cultural similarity and historic ties, respectively. In both cases, the impact on the location probability of a first investment is positive. Finally, if Germany has signed a goods trade agreement with a host country (*GTA with parent*), this affects the *first* foreign investment decision significantly.

Columns 2 to 5 of Table 2 summarize the results for the *second* up to the *fifth* location decision (phase). The findings with respect to the (unilateral) host-country variables are qualitatively very similar and all coefficient point estimates have the expected signs. Note that a positive *fifth* location decision is observed for only 958 affiliates but the number of (columns in Z_{ij5} and) parameters to be estimated is largest among all models in Table 2. Hence, the coefficients in the last column of Table 2 are estimated with less precision than the ones pertaining to the *first* to the *fourth* investments. All of that is also broadly consistent with Hypothesis 1.

The results for the *second*, *third*, and *fourth* location decisions reveal an interesting pattern, confirming our theoretical considerations as stated in Hypotheses 2 and 3. While the distance effect between foreign investments in phase p to ones in phase $p-1$ is always negative (Hypothesis 3), it becomes less important in terms of magnitude over the expansion path of a multinational network (Hypothesis 2). This pattern clearly confirms some form of regional development of MNE networks, similar to the development of export networks identified in the literature on sequential exporting (see Evenett and Venables, 2002; Albornoz et al., 2011). This feature does not accrue to the sample composition but is also reflected in the marginal effects (see Table 3).

– Insert Table 3 about here –

In addition to the proximity variables, Table 3 presents further marginal effects. Lines 1 and 2 imply that the marginal impact of parameters referring to potential profitability in a market declines over time, wheres lines 3-5 suggest that the marginal impact of lower fixed costs gets broadly less important for later stages. We use this result as a robustness test below, addressed in Hypotheses R4 and R5.

Let us particularly emphasize two results in Table 2. First, whether or not a host country was a former colony seems relatively important in expansion phases $p \geq 2$. One reason for this result may be that the variable Colony captures many different aspects of *proximity*. Second, while we find that having a goods trade agreement (GTA = 1) with the parent makes it less likely to locate in a country, trade agreements between countries of subsequent location decisions increase the probability of establishing affiliates there. This stays in contrast to the literature on tariff-jumping FDI which stipulates that trade agreements may lead to a consolidation of foreign affiliates in response to preferential tariff liberalization (see Raff, 2004).

3.4.5 Sensitivity Analysis

In contrast to the models estimated in Table 2, the ones in Table 4 include the total stock of German investments in market j and phase p (StockInv_{jp}) prior to a firm's location decision there while otherwise including the same regressors as in Table 2. This modification aims at checking whether or not the estimated coefficients are mainly driven by agglomeration effects – such as a general tendency of German firms to locate in just a few countries. StockInv should be a good measure of a market's general attractiveness for German investors beyond the dimensions captured by the covariates included in the regressions of Tables 1 and 2. The results in Table 4 suggest that the earlier findings are robust against the inclusion of StockInv. In fact, most of the coefficients are hardly affected by the additional control variable and, hence, are not biased due to omitted determinants of location choice.

– Insert Table 4 about here –

Recall that MNEs may establish more than one affiliates in an expansion phase p. If two or more investments are conducted in different countries in phase $\ell < p$, the reference of investments in p to ones in phase ℓ through Distance, Border, Language, Colony, etc., is no longer clear. We solved this problem in Table 2 by using the country of the bigger previous investment in terms of *fixed assets* as the reference country in phase p. In Table 5 we use the biggest previous investment in terms of *total assets* as alternative criterion to determine the reference country in phase p. The results displayed in Table 5 show that using an alternative criterion does not lead to alternative conclusions.

– Insert Table 5 about here –

Table 6 presents results for a subsample of firms and affiliates where all investments of any previous phase $p-1$ occurred in only one country (the firms might have established several affiliates in this country, though). Then, the *bilateral* variables Distance, Border, Language, Colony, etc., refer to a unique reference country throughout. The findings in Table 6 confirm our previous results in broad terms. However, we should note that the strategy applied in Table 6 leads to a significant loss of degrees of freedom along the expansion path of MNE networks. The reason is that many MNEs set up foreign entities simultaneously in several countries at some point of the genesis of their multinational network. Therefore, from the *third* location decision onwards, the coefficients can not be estimated precisely any more, due to the reduction in sample size as compared to the findings in Tables 2, 4, and 5.

– Insert Table 6 about here –

3.5 Further Hypothesis Tests and Robustness

Although our results concerning the genesis of a multinational network appear to be robust regarding some general features, it is not per se obvious that learning under uncertainty is the main factor driving the observed patterns of investments. In what follows, we will derive further – more specific – hypotheses, referred to as R1-R5, arising from our theoretical model.

Let us first address the point that sequential entry is only observable *ex post*. More precisely, when there is foreign market entry in one period but no subsequent expansion of the network, we do not know whether this was intended from the beginning or not. Instead, an MNE could have planned to enter markets sequentially, but it could have turned out that it was not sufficiently successful in the first-entered markets to undertake subsequent investments. By this reasoning, firms that actually take the second step and make a sequential investment should have been relatively more successful in their first market(s). On the other hand, firms that only remained in their initial markets will include those that chose isolated or simultaneous foreign investments and were either successful or not; it will also include those that had planned sequential entry but were not successful in the first markets. Furthermore, an investment in a country entered as second under sequential entry will on average be more successful than an otherwise identical investment in a country entered under simultaneous or isolated entry. The reason is that correlated learning together made its belief increase, implying higher expected profits. Therefore, the pool of firms where sequential entry is observed should on average be more successful than the pool of MNEs where only simultaneous or isolated entry is observed. This gives

Hypothesis R1:
Firms where sequential entry is observed are on average more successful than firms where isolated or simultaneous entry is observed in otherwise identical markets.

Proof: The proof associated with Hypothesis R1 follows from the discussion.

The following hypothesis uses the result that if a country is chosen as the first market of a planned sequential entry path, the MNE will have a larger capacity there than otherwise. Although the pool of observed isolated or simultaneous entries also contains planned but not realized sequential patterns, the capacity there should on average be smaller. Thus, we state

Hypothesis R2:
Firms where sequential entry is observed have, on average, a larger capacity than firms where only isolated or simultaneous entry is observed in otherwise identical markets.

Proof: Follows from the definition of sequential entry and Proposition 2. Furthermore, note the belief in markets entered in later stages is ceteris paribus higher and that these markets also serve as "first" countries for later stages. Under sequential entry, capacities should thus be higher along the whole investment path.

Hypotheses R1 and R2 relate to the size and profitability of MNEs. In particular, MNEs may differ in these dimensions depending on whether they enter markets simultaneously or sequentially. Table 7 presents regression results, where we use the indicator variable *Sequential entry* to distinguish between sequential and simultaneous entries. To be precise, the variable *Sequential entry* is unity if we identify an observation as a sequential entry and zero else.

We analyze three different dependent variables in Table 7: the *fixed assets*, the *total assets*, and the *sales-to-total-assets ratio* of the *average* investment, respectively. The different columns refer to the maximum number of entities a firm consists of. For example, the column denoted by (3) includes firms that have established 2 or 3 entities. By focusing on firms that are always 2-plant, 3-plant, etc., MNEs, we can distinguish between simultaneous and sequential market entry.

All results support Hypotheses R1 and R2: if sequential entry is observed ex post, the previously established affiliates are on average more successful and larger than those where only simultaneous or isolated entry is observed.

– Insert Table 7 about here –

Now, let us only consider investments where no sequential entry is observed (yet). As pointed out above, such investments may include ones where sequential entry was intended but not (yet) exercised.[22] The corresponding firms should, on average, be less successful than other MNEs. As their capacity is higher as well, we formulate

Hypothesis R3:

For firms where simultaneous or isolated entry is observed, the more successful ones should, on average, exhibit a higher capacity.

Proof: The proof associated with Hypothesis R3 follows from the discussion.

Table 8 presents a test of Hypothesis R3, focusing on one-plant MNEs. The dependent variable is fixed assets of a foreign affiliate. Consistent with Hypothesis R3, we observe that for firms where only isolated entry is observed ex post, more profitable ones (measured by the sales-to-fixed-assets ratio or the sales-to-total assets ratio) have lower amounts of fixed assets. We conduct the same test in column 2, but additionally include all simultaneous-plant units in the estimation sample. The findings are very similar.

[22]This also contains investments where sequential entry is still planned. However, by selecting on firms that did not exercise sequential entry ex post within a given time span, there is an intended bias towards firms which will not exercise sequential entry in the future. The latter should be sufficient for the proposed inference.

– Insert Table 8 about here –

Furthermore, a crucial component of learning is that if a market is entered at a later phase, the belief about that market is higher compared with an earlier entry. This can have interesting implications on the (marginal) propensity to enter a market. Take a country that can either be entered using simultaneous entry or, as second investment, under sequential entry. To make entry optimal, the associated R_j must exceed (and equivalently fixed costs F_j must be lower than) a certain threshold, for a given belief. Since the belief is higher if the market is entered under sequential entry (and if a success in the first country was observed), the relevant threshold making entry optimal for R_j should be lower and the one for F_j higher than when this country is entered under simultaneous entry. Although this result is less straightforward when the country is entered first under sequential entry, if countries are not too different, we can confirm the above finding. Thus, we propose

Hypothesis R4:
If a country is entered at later expansion phases, the minimum market size necessary to enter should be smaller. Moreover, the maximum fixed costs making entry just profitable should be higher.

Proof: The proof associated with Hypothesis R4 can be found in Appendix II.

Lines 1-5 of Table 3 give marginal effects of variables characterizing market size (lines 1 and 2) and fixed costs of market entry. Until the third investment, the marginal effects have the predicted patterns. Marginal effects of *Tax* and *log GDP* decrease in absolute terms, indicating that the correlated learning is important for entry decisions. Furthermore, while the marginal effect of *InvestCost* does not seem to differ much for different entry stages, the development of the marginal effects of *InvestFree* and *CPI* are largely as predicted.

Finally, let us establish another hypothesis that makes use of correlated learning. For higher fixed entry costs, learning is more valuable, i.e., the option value of waiting is larger.

Hypothesis R5:

If fixed entry costs in one country are ceteris paribus higher, it is more likely that this country is chosen as second under sequential entry. Furthermore, the relative profitability of sequential compared to simultaneous entry increases if the fixed entry costs in the second target country are larger. Thus, fixed costs should, on average, be higher for countries entered at later stages.

Proof: The proof associated with Hypothesis R5 can be found in Appendix II.

Hypothesis R5 is supported by Table 1, where lines 3-5 give average values for parameters capturing fixed entry costs. There, especially *InvestFree* and *InvestCost* are as predicted, where the former decreases and the later increases along expansion phases.

Furthermore, Hypothesis R5 can help to explain the seemingly counterintuitive impact of trade agreements (GTA) with the parent with countries entered in the second and later phases. Whereas the first investment is positively affected by such an agreement, the impact is negative for later ones. If GTAs are associated with fixed cost, this contradicts the (otherwise empirically supported) hypothesis that higher fixed costs should generally be associated with a lower probability of entry. However, if firms enter countries with high fixed cost, this will rather happen at later stages.

3.6 Alternative Explanations for the Genesis of Multinational Networks

In this section, we present alternative explanations for the dynamic pattern of foreign market entry of MNEs. These alternative explanations fail to predict one or more substantial aspects of the observed investment pattern. In fact, we find conclusive evidence that supports the suggested correlated learning mechanism rather than any of the considered alternatives.

Although the observed sequential entry and expansion patterns of MNE networks can not be explained by static models of market entry, there is a number of alternative dynamic models which could lead to predictions that are qualitatively similar to the ones derived from our model. Here, we briefly analyze three prime candidates of alternative models, namely stochastic shocks, diseconomies of scale, and learning by doing. The main difference between those models and ours is that, in each period, the MNE would face uncertainty concerning its type in the respective market in the alternative modeling environments. While we would agree that any one of the three models may be consistent with some empirical findings concerning the gradual expansion of MNE networks, we will show that they fail to explain important features of the data. The reason is that the learning model renders decisions in later periods contingent on the outcome in earlier periods, while the three alternative explanations do not. In the absence of uncertainty concerning success in a market, second-period actions are generally independent of success in the first period.

Alternative 1: Stochastic Shocks

One reason for why a firm might not want to enter all markets simultaneously is that exogenous factors affect its profitability there. Then, it will not invest unless market conditions turn out to be sufficiently good.

The setup for such a model is identical to the one derived above, with two exceptions. The firm's type in each country is not identical over time, but a new realization is drawn at the beginning of each period. The probability that the type in country j is high in a given period equals q_j, $j = A, B$. We impose no further structure on q_j, however it could depend on the realization in the previous period as well as the distance to home (r_j) or to the other potential host country (r_{AB}). Furthermore, the firm can observe the realizations of θ_j, $j = A, B$, in each period, so that the only uncertainty it faces concerns next period's value of θ_j.

The MNE's entry decision with respect to country A is now independent of its entry decision for B (and vice versa). The reason is that past decisions have no impact on the likelihood of having a high type in the future. Thus, we can focus on optimal actions for just one market.

In the first period, the firm will not enter country j if $\theta_j = 0$, since this would yield negative profits in period 1 (without a positive impact on future profits). If $\theta_j = \theta^h$ in period 1, expected profits for a given capacity X_j are

$$X_j \theta^h R_j (1 + \delta q_j).$$

Conditional on entry, it will choose a capacity level $X_j = \frac{\theta^h R_j (1+\delta q_j)}{k_j}$ and finally enter the market if $\Pi_j = \frac{(\theta^h R_j)^2 (1+\delta q_j)^2}{2k_j} - F_j \geq 0$.

If $\theta_j = 0$ in the first period, the MNE will enter the market in period 2 if the type is then high and if expected profits are positive, i.e., whenever $\Pi = \frac{(\theta^h R)^2}{2k_j} - F_j \geq 0$.

If these conditions are satisfied, entry in periods 1 and 2 occurs with probability q_j and $(1 - q_j)q_j$, respectively. The total likelihood of entry thus equals $2q_j - q_j^2$.

A bigger market size and lower entry costs are also associated with a higher likelihood of entry. We might even construct a sequential entry pattern as observed in the data, with closer countries entered first, followed by a gradual expansion to markets farther away. This would require the assumption $\frac{\partial q_j}{\partial r_j} > 0$. It would already be less straightforward to construct assumptions – such that the role of r_{AB} would be similar to our benchmark model – however not impossible. But Hypotheses R1-R5 will definitely not hold. Take Hypothesis R1, where we claim that firms that enter sequentially are on average more successful. Assume A is entered in the first period but B not. Then, the decision whether to enter B in period 2 is independent of what happened in A. Thus, expected profits in A are always the same, no matter whether B is entered in period 1, 2, or not at all. Similar arguments can be used to reject Hypothesis R2 (sequential entry is associated with higher capacity levels), and Hypothesis R3 (when countries are entered simultaneously, the ones with a lower capacity should be less successful). Furthermore, Hypothesis R4 is not supported, as the thresholds do depend on whether a country is entered earlier or later, taking aside time horizon effects. Finally, the average fixed costs will not be higher if a country is entered in period 2, dismissing Hypothesis R4. Hence, we can reject the first alternative model as an explanation for the observed empirical patterns.

Alternative 2: Diseconomies of Scale

Here, we take into account that an MNE's resources in one period might be constrained. For simplicity, let us focus on financial resources and assume that investment costs in one period may not exceed the value D. Also, let us assume that it is known that the MNE's type is high in both markets. All else is identical to the original model setup. Thus, without financial constraint, the MNE would enter both countries at the beginning of period 1. Then, the firm would choose capacities $X_j = \frac{(\theta^h R_j)(1+\delta)}{k_j}$ and obtain expected profits $\Pi_j = \frac{(\theta^h R_j)^2(1+\delta)^2}{2k_j} - F_j$. For $D \geq k_A \frac{(X_A)^2}{2} + k_B \frac{(X_B)^2}{2} + F_A + F_B$, the budget constraint does not bind and simultaneous entry occurs. To simplify issues, we assume that $k_j \frac{(X_j)^2}{2} + F_j \leq D \leq k_j \frac{(X_j)^2}{2} + F_A + F_B$. Accordingly, it is feasible to enter one country with the first-best capacity, but not possible to enter the second one at all. Hence, higher revenues or lower costs render first entry more likely again, and Hypothesis 1 would hold. If we further assume that R_j decreases with the distance to home, the geographically closer country would more likely be entered first, which is in line with Hypothesis 2. It would be more difficult to justify why the distance between two host markets A and B should matter for the sequential entry pattern, and Hypothesis R1 could only be obtained for this model under the assumption that the budget constraint in the second period is relaxed after the realization of a success in period 1. But Hypotheses R2 and R3 would definitely not flow from the diseconomies of scale model, since chosen capacities are independent of other entries. Note that Hypothesis R2 (first country of sequential entry has larger capacity) holds for otherwise identical markets and we can not use the argument that affiliates at initially entered markets are more successful. Finally, while Hypothesis R5 is in line with a model of diseconomies of scale (countries with higher fixed costs are on average entered later), this is not true for Hypothesis R4, since the relative thresholds above which entry is profitable do not change along entry phases.

Although the diseconomies of scale model does a relatively better job in explaining the pattern of MNE network formation observed in the data than the stochastic shocks model, neither of them provides an explanation for the different capacity levels and their correlation with observed success. Thus, the second alternative model can be rejected on those grounds as an explanation for the observed empirical features as well.

Alternative 3: Learning by Doing

Finally, let us assume that second-period returns in both countries depend on first-period production, i.e., $R_j(X_A, X_B)$. We use the explicit linear expression for the second-period returns, which equal $R_A(X_A, X_B) = rX_A + \alpha r X_B + T_A$ and $R_B(X_A, X_B) = \alpha r X_A + rX_B + T_B$, with $\alpha, r \geq 0$ and $\alpha \leq 1$. First-period returns thus equal T_j. Now, sequential entry might also be used to save investment costs in the first period and use learning benefits from A about B. It is easy to derive formal results, which we omit here; just note that Hypothesis 1 is satisfied as in the benchmark learning model. If T_j is larger for the closer country and α is larger if the two host countries are closer to each other, Hypothesis 2 can be met if countries closer to home are more profitable, while Hypothesis 3 is supported by learning by doing if the learning parameter α is larger for host countries that are close to each other.

Since entry into the second country in period 2 occurs for sure, Hypotheses R1-R5 are not generally true in the case of learning by doing. Hypotheses R2 (sequential entry is associated with a larger capacity), R4 (entry thresholds differ across phases) and R5 (fixed costs are higher in later entry phases) may or may not be true, depending on parameter values. But Hypotheses R1 (observed sequential entry associated with higher success) and R3 (larger capacity correlated with lower success for simultaneous or isolated entry) will not be satisfied here, because outcomes in the first period have no impact on later decisions. Hence, the third alternative model can also not explain all empirical results.

3.7 Conclusion

This paper has shown that multinational enterprises develop their networks of foreign affiliates gradually over time. Instead of exploring all profitable opportunities immediately, they first establish themselves in their home countries and then enter new markets stepwise. We explain this gradual expansion pattern by proposing a model where MNEs face uncertainty concerning their success in new markets and learn about that after entry. Conditions in different markets are not independent, and the information gathered in one country can also be used to learn about conditions in other, in particular, similar countries.

This so-called correlated learning mechanism serves us to derive a number of testable hypotheses regarding market entry in general and *simultaneous* versus *sequential* market entry in specific. These hypotheses are assessed in a data-set of the universe of German

MNEs and their foreign affiliates provided by the Deutsche Bundesbank. Using a conditional logit model for the empirical analysis, we find that first foreign entry is more likely for countries that are closer to the MNE's home base and where higher profits may be expected in general terms. This finding is supported by variables measuring the proximity of markets at large, e.g., whether the same language is spoken or if the target country used to be a colony of the home country. Moreover, proximate countries tend to be entered first as a multinational network evolves. Third, subsequent entry in later expansion phases is generally more likely in markets that are closer to the ones entered previously.

Although other reasons like diseconomies of scale, stochastic shocks, or learning by doing certainly also influence a multinational firm's expansion, we show in a number of additional tests that correlated learning plays a substantial role in explaining the *genesis of multinational networks*.

The way how MNEs expand their network of foreign affiliates over time and, in particular, correlated learning as identified in this paper may have important policy implications. Understanding whether, how, and where firms grow is crucial to anticipate for policy makers – not only with respect to domestic policies (such as tax policy) but also with regard to international policies (such as bilateral or multilateral preferential agreements). This is especially important since different country variables do not only affect location decisions of MNEs, it also affects entry patterns on an overall scale. Since different entry patterns are associated with different investment levels, countries may pursue policy strategies that take into account such effects.

In our future research, we aim at analyzing learning processes in more detail. For example, the role of learning might be different contingent on whether a firm acquires an existing affiliate or establishes a new plant. Correlated learning can also have an impact on market exit, which we have neglected so far. In the latter case, policies of neighboring countries may be even more important. We also aim at quantifying the gains and costs of learning in comparison to the impact of other fundamentals on firm behavior.

Appendix to Chapter 3

Appendix I – Correlated Learning

The ex-ante joint beliefs for being in one of the four potential states (θ^h, θ^h), $(\theta^h, 0)$, $(0, \theta^h)$ or $(0, 0)$ are characterized in the following Corollary.

Lemma A1: The ex-ante probabilities of being in state (θ_A, θ_B) equal

$\text{Prob}(\theta^h, \theta^h) \equiv p^{hh} = \left(r_B + (1 - r_B)[r_{AB}\frac{\rho_A^0}{\rho_A} + (1 - r_{AB})\rho_B^0]\right)\rho_A$

$\text{Prob}(\theta^h, 0) \equiv p^{hl} = (1 - r_B)\left(1 - [r_{AB}\frac{\rho_A^0}{\rho_A} + (1 - r_{AB})\rho_B^0]\right)\rho_A$

$\text{Prob}(0, \theta^h) \equiv p^{lh} = (r_B + (1 - r_B)(1 - r_{AB})\rho_B^0)(1 - \rho_A)$

$\text{Prob}(0, 0) \equiv p^{ll} = (1 - r_B)(1 - (1 - r_{AB})\rho_B^0)(1 - \rho_A)$

Proof:

As $\text{Prob}(\theta^h, \theta^h) = \text{Prob}(\theta_B^h \mid \theta_A^h)\text{Prob}(\theta_A^h)$, we need $\text{Prob}(\theta_B^h \mid \theta_A^h)$. Taking $\rho_B = r_B + (1 - r_B)[r_{AB}\rho_A^0 + (1 - r_{AB})\rho_B^0]$, we obtain

$\text{Prob}(\theta_B^h \mid \theta_A^h) = r_B + (1 - r_B)[r_{AB}\text{E}[\rho_A^0 \mid \theta_A^h] + (1 - r_{AB})\rho_B^0]$. Bayes' rule can be used to compute $\text{E}[\rho_A^0 \mid \theta_A^h]$, and we get

$\text{E}[\rho_A^0 \mid \theta_A^h] = \frac{\rho_A^0(r_A+(1-r_A)\cdot 1)}{\rho_A^0(r_A+(1-r_A)\cdot 1)+(1-\rho_A^0)(r_A+(1-r_A)\cdot 0)} = \frac{\rho_A^0}{\rho_A}$. Equivalently,

$\text{E}[\rho_A^0 \mid \theta_A^l] = \frac{\rho_A^0[1-(r_A+(1-r_A))]}{\rho_A^0[1-(r_A+(1-r_A))]+(1-\rho_A^0)[1-(r_A+(1-r_A)\cdot 0)]} = 0$, proving the Corollary.

Updating occurs for each of the potential outcome realizations
$(Y_A, Y_B) \in \{(R_A, R_B), (R_A, 0), (0, R_B), (0, 0)\}$:

1. $(Y_A, Y_B) = (R_A, R_B)$:

   $\rho_A^{++} = \rho_B^{++} = 1$

2. $(Y_A, Y_B) = (R_A, 0)$

   $\rho_A^{+-} = 1$

   $\rho_B^{+-} = (p^{hh})^{+-} + (p^{lh})^{+-} = \frac{p^{hh}X_A\theta^h(1-X_B\theta^h)}{p^{hh}X_A\theta^h(1-X_B\theta^h)+p^{hl}X_A\theta^h} + 0 = \frac{p^{hh}(1-X_B\theta^h)}{(1-X_B\theta_B^h)p^{hh}+p^{hl}}$

3. $(Y_A, Y_B) = (0, R_B)$

$\rho_A^{-+} = \frac{p^{hh}(1-X_A\theta^h)}{(1-X_A\theta^h)p^{hh}+p^{lh}}$

$\rho_B^{-+} = 1$

4. $(Y_A, Y_B) = (0, 0)$

$\rho_A^{--} = \frac{p^{hh}(1-X_A\theta^h)(1-X_B\theta^h)+p^{hl}(1-X_A\theta^h)}{(1-X_A\theta^h)\left(p^{hh}(1-X_B\theta^h)+p^{hl}\right)+\left(p^{lh}(1-X_B\theta^h)+p^{ll}\right)}$

$\rho_B^{--} = \frac{p^{hh}(1-X_A\theta^h)(1-X_B\theta^h)+p^{lh}(1-X_B\theta^h)}{(1-X_A\theta^h)\left(p^{hh}(1-X_B\theta^h)+p^{hl}\right)+\left(p^{lh}(1-X_B\theta^h)+p^{ll}\right)}$

Observing (R_A, R_B), both beliefs jump to 1, i.e. $\rho_A^{++} = \rho_B^{++} = 1$. If a success is only realized in country A but not in B, implying $(R_A, 0)$, $\rho_A^{+-} = 1$, while $\rho_B^{+-} = (p^{hh})^{+-} + (p^{lh})^{+-} = \frac{p^{hh}X_A\theta^h(1-X_B\theta^h)}{p^{hh}X_A\theta^h(1-X_B\theta^h)+p^{hl}X_A\theta^h} + 0 = \frac{p^{hh}(1-X_B\theta^h)}{(1-X_B\theta_B^h)p^{hh}+p^{hl}}$. Conversely, the realization $(0, R_B)$ gives $\rho_A^{-+} = \frac{p^{hh}(1-X_A\theta^h)}{(1-X_A\theta^h)p^{hh}+p^{lh}}$ and $\rho_B^{-+} = 1$.

Finally, after a double failure, beliefs fall to $\rho_A^{--} = \frac{p^{hh}(1-X_A\theta^h)(1-X_B\theta^h)+p^{hl}(1-X_A\theta^h)}{(1-X_A\theta^h)\left(p^{hh}(1-X_B\theta^h)+p^{hl}\right)+\left(p^{lh}(1-X_B\theta^h)+p^{ll}\right)}$ and $\rho_B^{--} = \frac{p^{hh}(1-X_A\theta^h)(1-X_B\theta^h)+p^{lh}(1-X_B\theta^h)}{(1-X_A\theta^h)\left(p^{hh}(1-X_B\theta^h)+p^{hl}\right)+\left(p^{lh}(1-X_B\theta^h)+p^{ll}\right)}$.

The case we are interested in is where entry initially occurs only in one country, say A. This is covered by setting $X_B = 0$ and taking into account that a "failure" there occurs with probability 1. If the MNE only enters A and observes a success, the belief in B becomes

$\rho_B^{+-} = \left(r_B + (1-r_B)[r_{AB}\frac{\rho_A^0}{\rho_A} + (1-r_{AB})\rho_B^0]\right) > \rho_B$. Recall that ρ_B^{-+} is not of interest as B is never entered after a failure in A.

Starting out by investing in B and observing a success there yields

$\rho_A^{-+} = \left(r_B + (1-r_B)[r_{AB}\frac{\rho_A^0}{\rho_A} + (1-r_{AB})\rho_B^0]\right)\frac{\rho_A}{\rho_B} = \rho_B^{+-}\frac{\rho_A}{\rho_B}$.

Finally, beliefs also follow a martingale here; to see this, take the expected belief change in A for arbitrary investment levels X_A and X_B. Keeping in mind that $\rho_A = p^{hh} + p^{hl}$, we have $\mathrm{E}[\rho_{At+1} \mid \rho_{At}] = p^{hh}X_AX_B\theta^h\theta^h\rho_A^{++} + \left[p^{hh}X_A\theta^h(1-X_B\theta^h) + p^{hl}X_A\theta^h\right]\rho_A^{+-}$

$+ \left[p^{hh}(1-X_A\theta^h)X_B\theta^h + p^{lh}X_B\theta^h\right]\rho_A^{-+}$

$+ \left[p^{hh}(1-X_A\theta^h)(1-X_B\theta^h) + p^{hl}(1-X_A\theta^h) + p^{lh}(1-X_B\theta^h) + p^{ll}\right]\rho_A^{--}$

$= X_A\theta^h\rho_A + (1-X_A\theta^h)(p^{hh}+p^{hl}) = \rho_A$

Appendix II – Proofs

Proof of Proposition 1

Note that beliefs follow a martingale, i.e., they do not change in expectation:

$\mathrm{E}[\rho_{t+1} \mid \rho_t, X_t] = X_t\rho_t\theta^h\rho_t^+ + (1 - X_t\rho_t\theta^h)\rho_t^-(X_t) = \rho_t$. Thus, (3.1) can be rewritten as

$\Pi^{iso} = \max_{X\geq 0} \left[\{1\}_{X=0}0 + \{1\}_{X>0}\left(X\rho R\theta^h(1+\delta) - [F + k\frac{(X)^2}{2}]\right)\right].$

Since there is no market exit after a failure and beliefs assume the martingale feature, expected profits in period 1 and 2 are identical (from the perspective of period 1). The first-order condition yields (3.2). The second-order condition is satisfied by the assumption of convexity of the investment cost function. As entry will only occur for non-negative profits, fixed investment costs have to be covered as well in expectation, i.e., $X(\rho\theta^h R - c)(1+\delta) - [F + k\frac{(X)^2}{2}] \geq 0$, yielding (3.3).

Proof of Proposition 2

This immediately follows from comparing $X_A^{seq} = \frac{\rho_A\theta^h R_A(1+\delta) + \delta\rho_A\theta^h\left(\frac{\left(\rho_B^{+-}\theta^h R_B\right)^2}{2k_B} - F_B\right)}{k_A}$ with $X_A^{sim} = \frac{\rho_A\theta^h R_A(1+\delta)}{k_A}$. The term $\left(\frac{\left(\rho_B^{+-}\theta^h R_B\right)^2}{2k_B} - F_B\right)$ has to be positive as otherwise entry into B would not occur.

Proof of Proposition 3

Assume $\Pi_B^{sim} = \frac{(\rho_B\theta^h R_B)^2(1+\delta)^2}{2k_B} - F_B^* = 0$, implying that isolated entry (only in A) and simultaneous entry yield identical profits.

$\Pi^{seq} > \Pi^{sim}$, if

$\frac{1}{2k_A}\left[\rho_A\theta^h R_A(1+\delta) + \delta\rho_A\theta^h\left(\frac{\left(\rho_B^{+-}\theta^h R_B\right)^2}{2k_B} - F_B^*\right)\right]^2 - F_A > \frac{(\rho_A\theta^h R_A)^2(1+\delta)^2}{2k_A} - F_A$ or

$\frac{\left(\rho_B^{+-}\theta^h R_B\right)^2}{2k_B} - F_B^* > 0.$

Thus, we need

$\rho_B^{+-} > \rho_B(1+\delta).$

As $\rho_B^{+-} > \rho_B$, there is always a δ such that this is satisfied.

For the part that sequential entry can be optimal even if ex ante profits in country B are strictly positive, assume that $F_B = F_B^* - \left(\rho_A \theta^h\right)^2 \delta\varepsilon \frac{(2R_A+\delta\varepsilon)(1+\delta)^2}{2k_A}$, $\varepsilon > 0$, and entry into B already in the first period would yield a profit $\left(\rho_A \theta^h\right)^2 \delta\varepsilon \frac{(2R_A+\delta\varepsilon)(1+\delta)^2}{2k_A}$.

For $\Pi^{seq} > \Pi^{sim}$, we need

$$\frac{1}{2k_A}\left[\rho_A\theta^h R_A(1+\delta) + \delta\rho_A\theta^h\left(\frac{\left(\rho_B^{+-}\theta^h R_B\right)^2}{2k_B} - F_B^* + \left(\rho_A\theta^h\right)^2 \delta\varepsilon \frac{(2R_A+\delta\varepsilon)(1+\delta)^2}{2k_A}\right)\right]^2 - F_A$$

$$> \frac{(\rho_A\theta^h R_A)^2(1+\delta)^2}{2k_A} - F_A + \frac{(2\delta\rho_A\theta^h\rho_A\theta^h R_A\varepsilon + \left(\delta\rho_A\theta^h\varepsilon\right)^2)(1+\delta)^2}{2k_A} \text{ or}$$

$$\frac{\left(\rho_B^{+-}\theta^h R_B\right)^2}{2k_B} - F_B^* > \varepsilon\left((1+\delta) - \delta\left(\rho_A\theta^h\right)^2 \frac{(2R_A+\delta\varepsilon)(1+\delta)^2}{2k_A}\right).$$

This possibly for a ε sufficiently small.

Finally, we have to make sure that entry into B after a failure in A is not optimal, which requires

$$\frac{\left(\rho_B^{--}\theta^h R_B\right)^2}{2k_B} - F_B^* + \left(\rho_A\theta^h\right)^2 \delta\varepsilon \frac{(2R_A+\delta\varepsilon)(1+\delta)^2}{2k_A} < 0.$$

We know that $\frac{\left(\rho_B\theta^h R_B\right)^2(1+\delta)^2}{2k_B} - F_B^* = 0$ and that $\rho_B^{--} < \rho_B$ for $X_A^{seq} > 0$. Thus, the above condition is satisfied for ε sufficiently small.

For the potential optimality of simultaneous entry, see Lemma 3 below, which states that there exists a value r_A^* such that for $r_A \geq r_A^*$, sequential entry is never chosen. Then, there are always values for F_A and F_B making simultaneous (and not isolated) entry optimal.

Proofs of Propositions underlying Hypotheses 1-3, R4 and R5

Hypothesis 1

We aim at showing that the marginal impact on respective profits of R_i, r_i and θ^h is positive, while it should be negative for k_i and F_i. The claim is obvious for isolated and simultaneous entry, where individual profits equal $\frac{(\rho_i\theta^h R_i)^2(1+\delta)^2}{2k_i} - F_i$ and comparative statics yield the predicted signs. Total profits under sequential entry are

$$\Pi^{seq} = X_A^{Seq}\rho_A\theta^h R_A(1+\delta) - k_A\frac{(X_A^{Seq})^2}{2} - F_A + \delta X_A^{Seq}\rho_A\theta^h\left(X_B^{Seq}\rho_B^{+-}\theta^h R_B - k_B\frac{(X_B^{Seq})^2}{2} - F_B\right)$$

$$= \frac{1}{2k_A}\left[\rho_A\theta^h R_A(1+\delta) + \delta\rho_A\theta^h\left(\frac{\left(\rho_B^{+-}\theta^h R_B\right)^2}{2k_B} - F_B\right)\right]^2 - F_A.$$

The Hypothesis is easily satisfied for entry into B, where profits, given a success in A was observed, equal $\frac{\left(\rho_B^{+-}\theta^h R_B\right)^2}{2k_B} - F_B$, and $\frac{\partial\rho_B^{+-}}{\partial r_B} > 0$. Concerning entry in period 1, comparative statics with respect to R_A, θ^h, k_A and F_A are unambiguous. This is different for r_A, as $\frac{\partial\rho_B^{+-}}{\partial r_A} < 0$, and we can not exclude $\frac{\partial\Pi^{seq}}{\partial r_A} < 0$. Still, to determine the likelihood of entry, we focus on the margin, i.e., where $\Pi^{seq} = 0$. But if $\frac{\partial\Pi^{Seq}}{\partial r_A}\mid_{\Pi^{seq}=0} < 0$, the MNE would choose isolated or simultaneous instead of sequential entry. As derived above, this becomes more likely for a larger value of r_A.

Hypothesis 2

For the part that sequential entry might be optimal, see Proposition 2.

When choosing sequential entry, we first show that for two countries which are identical and only differ in their distance to home, the MNE will enter the closer country first. Afterwards, we compare the profits under sequential entry when A is entered first with those when B is entered first. We look at the impact of r_A on the difference between these two measures and show that – as long as the countries are not too different – this impact works in favor of first entering A.

Here, we are mainly interested on the impact of distance, i.e., if we expect to observe entry into closer countries first. Although a larger r_A decreases the updating in B, it generally makes it more likely that A is entered first. Let us first derive the result for the most stylized case where both countries are identical except for their distance to home. Then, it can be shown that entry first occurs into the country with the higher r_i.

Lemma A2: Assume $R_A = R_B \equiv R$, $\rho_A^0 = \rho_B^0 \equiv \rho^0$, $k_A = k_B \equiv k$, $F_A = F_B \equiv F$, and that sequential entry is chosen. Then, the MNE will first enter A if (and only if) $r_A \geq r_B$.

Proof:

Define $\Delta\Pi^{seq} \equiv \Pi^{seq}(AB) - \Pi^{seq}(BA)$. Then,

$$\Delta\Pi^{seq} = \frac{1}{2k}\left[\rho_A\theta^h R(1+\delta) + \delta\rho_A\theta^h\left(\frac{\left(\rho_B^{+-}\theta^h R\right)^2}{2k} - F\right)\right]^2 - F$$

$-\frac{1}{2k}\left[\rho_B\theta^h R(1+\delta)+\delta\rho_B\theta^h\left(\frac{\left(\rho_A^{-+}\theta^h R\right)^2}{2k}-F\right)\right]^2+F\geq 0$ or

$\left(\rho_A\theta^h\left[R(1+\delta)+\delta\left(\frac{\left(\rho_B^{+-}\theta^h R\right)^2}{2k}-F\right)\right]+\rho_B\theta^h\left[R(1+\delta)+\delta\left(\frac{\left(\rho_A^{-+}\theta^h R\right)^2}{2k}-F\right)\right]\right)\cdot$
$\left(\rho_A\theta^h\left[R(1+\delta)+\delta\left(\frac{\left(\rho_B^{+-}\theta^h R\right)^2}{2k}-F\right)\right]-\rho_B\theta^h\left[R(1+\delta)+\delta\left(\frac{\left(\rho_A^{-+}\theta^h R\right)^2}{2k}-F\right)\right]\right)\geq$
0.

As the first term is always positive, the sign of $\Delta\Pi^{seq}$ is determined by

$$\left(\theta^h R(1+\delta)(\rho_A-\rho_B)+\delta\rho_A\theta^h\left(\frac{\left(\rho_B^{+-}\theta^h R\right)^2}{2k}-F\right)-\delta\rho_B\theta^h\left(\frac{\left(\rho_A^{-+}\theta^h R\right)^2}{2k}-F\right)\right)$$

which – as $\rho_A^{-+}=\frac{\rho_A}{\rho_B}\rho_B^{+-}$ – can be rewritten as

$$\theta^h(\rho_A-\rho_B)\left[R+\delta\left(R-\frac{\frac{\rho_A}{\rho_B}\left(\rho_B^{+-}\theta^h R\right)^2}{2k}-F\right)\right]. \tag{3.11}$$

If we can show that the squared bracket of 3.11 is always positive, then $\text{sgn}\Delta\Pi^{seq}=\text{sgn}(\rho_A-\rho_B)$.

As we assume that $k\geq\theta^h R(1+\delta)$ ($X\leq 1$ even if a type is known to be high) and $\frac{(\theta^h R)^2(1+\delta)^2}{2k}>F$ (entry is optimal for the high type),

$\left[R+\delta\left(R-\frac{\frac{\rho_A}{\rho_B}\left(\rho_B^{+-}\theta^h R\right)^2}{2k}-F\right)\right]\geq\left[R+\delta\left(R-\frac{\frac{\rho_A}{\rho_B}\left(\rho_B^{+-}\theta^h R\right)^2}{2k}-\frac{(\theta^h R)^2(1+\delta)^2}{2k}\right)\right]$

$\geq\left[R+\delta\left(R-\frac{\frac{\rho_A}{\rho_B}\left(\rho_B^{+-}\theta^h R\right)^2}{2\theta^h R(1+\delta)}-\frac{(\theta^h R)^2(1+\delta)^2}{2\theta^h R(1+\delta)}\right)\right]=R\left[1+\delta\left(1-\theta^h\frac{\frac{\rho_A}{\rho_B}\left(\rho_B^{+-}\right)^2}{2(1+\delta)}-\frac{\theta^h(1+\delta)}{2}\right)\right]$

$\geq R\left[1-\frac{\delta}{1+\delta}\theta^h\frac{\frac{\rho_A}{\rho_B}\left(\rho_B^{+-}\right)^2}{2}\right]$, as $1-\frac{\theta^h(1+\delta)}{2}\geq 0$

(A) $\rho_A-\rho_B\geq 0$

Now,

$\frac{\rho_A}{\rho_B}\left(\rho_B^{+-}\right)^2=\frac{\rho_A}{\rho_B}\left(\rho_B+(1-r_B)r_{AB}\frac{\rho^0}{\rho_A}(1-\rho_A)\right)^2$

$=\rho_A\rho_B+2(1-r_B)r_{AB}\rho^0(1-\rho_A)+\frac{1}{\rho_A\rho_B}\left((1-r_B)r_{AB}\rho^0(1-\rho_A)\right)^2$

$=\rho_A\rho_B+2(1-r_B)r_{AB}\rho^0(1-\rho_A)+\frac{1}{\rho_B\rho_A}\left((1-r_B)r_{AB}\rho^0(1-\rho_A)\right)^2$

and note that, $\rho_A\rho_B \leq 1$, $2(1-r_B)r_{AB}\rho^0\left(1-\rho_A\right) \leq 2$ and

$\frac{1}{\rho_B\rho_A}\left((1-r_B)r_{AB}\rho^0\left(1-\rho_A\right)\right)^2 = \frac{(\rho^0)^2}{(r_B+(1-r_B)\rho^0)(r_A+(1-r_A)\rho^0)}\left((1-r_B)r_{AB}\left(1-\rho_A\right)\right)^2$

$\leq \frac{(\rho^0)^2}{(1-r_B)\rho^0(1-r_A)\rho^0}\left((1-r_B)r_{AB}\left(1-\rho_A\right)\right)^2$

$= (1-r_B)(1-\rho^0)r_{AB}^2\left(1-\rho_A\right) \leq 1$

Thus, $\frac{\rho_A}{\rho_B}\left(\rho_B^{+-}\right)^2 \leq 4$ and

$\left[1-\frac{\delta}{1+\delta}\theta^h\frac{\frac{\rho_A}{\rho_B}\left(\rho_B^{+-}\right)^2}{2}\right] \geq \left[1-\frac{\delta}{1+\delta}2\theta^h\right]$

$= \frac{1}{1+\delta}\left[1-\delta\theta^h+\delta-\delta\theta^h\right] \geq 0.$

(B)$\rho_A - \rho_B < 0$

$\left[1-\frac{\delta}{1+\delta}\theta^h\frac{\frac{\rho_A}{\rho_B}\left(\rho_B^{+-}\right)^2}{2}\right] \geq \left[1-\frac{\delta}{1+\delta}\theta^h\frac{\left(\rho_B^{+-}\right)^2}{2}\right] \geq \left[1-\frac{\delta}{1+\delta}\theta^h\frac{1}{2}\right] \geq 0.$

To get a better idea, we now allow for general parameter values and analyze $\frac{d\Delta\Pi^{seq}}{dr_A}$ (recall that $\Delta\Pi^{seq} = \Pi^{seq}(AB) - \Pi^{seq}(BA)$):

$\frac{d\Delta\Pi^{seq}}{dr_A} = X_A^{AB}\frac{d\rho_A}{dr_A}\theta^h R_A(1+\delta) + \delta X_A^{AB}\frac{d\rho_A}{dr_A}\theta^h\left(\frac{(\rho_B^{+-}\theta^h R_B)^2}{2k_B} - F_B\right)$

$+\delta X_A^{AB}\rho_A\theta^h X_B^{AB}\frac{d\rho_B^{+-}}{dr_A}\theta^h R_B - \delta X_B^{BA}\rho_B\theta^h X_A^{BA}\frac{d\rho_A^{-+}}{dr_A}\theta^h R_A$, where X_A^{AB} is the capacity chosen in A under sequential entry starting in A. The first term describes increased profits in A, while the second term covers the increased likelihood of entry into B. The third term is negative, as ρ_B^{+-} decreases with r_A. Finally, the fourth term captures foregone profits when A is entered as the second country.

The expression can be rewritten as

$\delta\theta^h R_A(1-\rho_A^0)\left[X_A^{AB} - \delta X_B^{BA}\theta^h X_A^{BA}(r_B+(1-r_B)(1-r_{AB})\rho_B^0)\right]$

$+\theta^h(1-\rho_A^0)X_A^{AB}\left[R_A+\delta\left(\frac{(\rho_B^{+-}\theta^h R_B)^2}{2k_B} - F_B - (1-r_B)r_{AB}\frac{\rho_A^0}{\rho_A}\frac{\rho_B^{+-}\theta^h R_B}{k_B}\theta^h R_B\right)\right].$

Taking the term in squared brackets of the first line gives

$\left(X_A^{AB} - X_B^{BA}\delta\theta^h X_A^{BA}(r_B+(1-r_B)(1-r_{AB})\rho_B^0)\right)$

$= \frac{\rho_A\theta^h R_A(1+\delta)+\delta\rho_A\theta^h\left(\frac{\left(\rho_B^{+-}\theta^h R_B\right)^2}{2k_B}-F_B\right)}{k_A}$

$-\frac{\rho_B\theta^h R_B(1+\delta)+\delta\rho_B\theta^h\left(\frac{\left(\rho_A^{+-}\theta^h R_A\right)^2}{2k_A}-F_A\right)}{k_B}\delta\theta^h\frac{\rho_A^{+-}\theta^h R_A}{k_A}(r_B+(1-r_B)(1-r_{AB})\rho_B^0)$

$$\geq \frac{\rho_A\theta^h R_A(1+\delta)}{k_A} - \frac{\rho_B\theta^h R_B(1+\delta)+\delta\rho_B\theta^h\left(\frac{\left(\rho_A^{+-}\theta^h R_A\right)^2}{2k_A}-F_A\right)}{k_B}\delta\theta^h\frac{\rho_A^{+-}\theta^h R_A}{k_A}(r_B+(1-r_B)(1-r_{AB})\rho_B^0)$$

$$\geq \left(\frac{\rho_A\theta^h R_A(1+\delta)}{k_A} - \delta\theta^h\frac{\rho_A^{+-}\theta^h R_A}{k_A}(r_B+(1-r_B)(1-r_{AB})\rho_B^0)\right)$$

(since $k \geq \theta^h R(1+\delta)$ and $\frac{(\theta^h R)^2(1+\delta)^2}{2k} > F$)

$$= \frac{1}{k_A}\theta^h R_A\rho_A\left((1+\delta) - \delta\theta^h\frac{\rho_B^{+-}}{\rho_B}\left(\rho_B-(1-r_B)r_{AB}\rho_A^0\right)\right)$$

$$= \frac{1}{k_A}\theta^h R_A\rho_A\left((1+\delta) - \delta\theta^h\rho_B^{+-} + \delta\theta^h\frac{\rho_B^{+-}}{\rho_B}(1-r_B)r_{AB}\rho_A^0\right) \geq 0$$

The term in squared brackets of the second line equals

$$\left[R_A + \delta\frac{(\theta^h R_B)^2}{k_B}\rho_B^{+-}\left(\frac{\rho_B^{+-}}{2}-(1-r_B)r_{AB}\frac{\rho_A^0}{\rho_A}\right)-\delta F_B\right]$$

$$\geq \left[R_A - \delta(1-r_B)r_{AB}\frac{\rho_A^0}{\rho_A}\frac{\rho_B^{+-}\theta^h R_B}{k_B}\theta^h R_B\right] \geq \left[R_A - \frac{\delta}{(1+\delta)}(1-r_B)r_{AB}\frac{\rho_A^0}{\rho_A}\rho_B^{+-}\theta^h R_B\right].$$

As R_B is multiplied with terms that are all smaller than 1, the last term can only be negative if R_B is much larger than R_A.

Therefore, $\frac{d\Delta\Pi^{seq}}{dr_A}$ will generally be positive.

Considering simultaneous entry, we can establish the following Lemma.

Lemma A3: There exists a value r_A^* such that for $r_A \geq r_A^*$, sequential entry is never chosen.

Proof:

Now, assume without loss of generality that if sequential entry is chosen, the MNE starts in A and define $\Delta\Pi = \Pi^{seq} - \Pi^{sim}$.

Rewriting gives

$\Delta\Pi = \Pi_A^{seq} - \Pi_A^{sim} + \delta X_A^{seq}\rho_A\theta^h\Pi_B^{seq} - \Pi_B^{sim}$

Furthermore, $\Delta\Pi_A \equiv \Pi_A^{seq}-\Pi_A^{sim} = -\left(\delta\rho_A\theta^h\right)^2\frac{\left(\frac{\left(\rho_B^{+-}\theta^h R_B\right)^2}{2k_B}-F_B\right)^2}{2k_A} \leq 0$, as the capacity in A under sequential entry is too high if just profits in A are considered.

Furthermore, $\lim\limits_{r_A\to 1}\rho_B^{+-} = \rho_B$. For $r_A \to 1$, Π_B^{seq} approaches a value smaller or equal than Π_B^{Sim}.

As $\delta X_A^{seq}\rho_A\theta^h < 1$, $\delta X_A^{seq}\rho_A\theta^h\Pi_B^{seq} - \Pi_B^{sim}$ is negative for $r_A = 1$. By continuity, the desired value r_A^* exists.

Note that this Lemma does not imply that for $r_A < r_A^*$, sequential entry is always optimal. This might or might not be the case, depending on parameter values.

Hypothesis 3

The profits in B given sequential entry is chosen equal $\left(\frac{\left(\rho_B^{+-}\theta^h R_B\right)^2}{2k_B} - F_B\right)^2$. They are increasing in ρ_B^{+-}, which itself increases in r_{AB}.

Hypothesis R4

Note that we leave aside the modeling restrictions imposed by having only two periods here. The reason is that this restriction decreases expected profit streams for countries entered later per se. Since the expected time horizon should not automatically differ for different entry phases, effects induced by the reduced time horizon should not be emphasized too much.

Concerning the minimum requirements for market size R_j, compare profits when a country is entered under isolated or simultaneous and when it is entered, as second investment, under sequential entry. Without loss of generality, assume that this country is A.

In the first case, the requirement for entry is

$$R_A \geq \frac{\sqrt{2k_AF_A}}{\rho_A\theta^h(1+\delta)}. \tag{3.12}$$

.

In the second case, entry occurs if and only if

$$R_A \geq \frac{\sqrt{2k_AF_A}}{\rho_A^{-+}\theta^h}. \tag{3.13}$$

Since $\rho_A^{-+} \geq \rho_A$, the right hand side of (3.12) is larger than the right hand side of

(3.13), abstracting from the longer time horizon in the first case.

Equivalently, we show that the threshold with respect to F_j is larger in the second than in the first case.

If A is entered first under sequential entry, the condition for entry equals

$$\Pi^{seq} = \frac{1}{2k_A}\left[\rho_A\theta^h R_A(1+\delta) + \delta\rho_A\theta^h\left(\frac{\left(\rho_B^{+-}\theta^h R_B\right)^2}{2k_B} - F_B\right)\right]^2 - F_A \geq 0.$$

Since the thresholds now also depend on characteristics in B, it is not possible to make a general statement. However, let us assume that both countries are identical and only differ in R_j, giving respective thresholds $R_A \geq \frac{\sqrt{2kF}}{\rho^{-+}\theta^h}$ (if entered as second under sequential entry) and $R_A \geq \frac{\sqrt{2kF}-\delta\rho\theta^h\left(\frac{\left(\rho^{+-}\theta^h R_B\right)^2}{2k}-F\right)}{\rho\theta^h(1+\delta)}$ (if entered as first under sequential entry). R_B still plays a role in determining the relevant thresholds. However, we can claim that if both countries are identical except their values of R_j and sequential entry is chosen, the one with a higher R_j is always entered first, completing the argument. To see this point take $\Delta\Pi^{seq}$ defined as the difference when A is entered first and when B is entered first under sequential entry. It equals

$$\Delta\Pi^{seq} = \frac{1}{2k}\left[\rho\theta^h R_A(1+\delta) + \delta\rho\theta^h\left(\frac{\left(\rho^{+-}\theta^h R_B\right)^2}{2k} - F\right)\right]^2$$

$$-\frac{1}{2k}\left[\rho\theta^h R_B(1+\delta) + \delta\rho\theta^h\left(\frac{\left(\rho^{+-}\theta^h R_A\right)^2}{2k} - F\right)\right]^2$$

$$= \frac{1}{2k}\left\{\left[\rho\theta^h R_A(1+\delta) + \delta\rho\theta^h\left(\frac{\left(\rho^{+-}\theta^h R_B\right)^2}{2k} - F\right)\right] + \left[\rho\theta^h R_B(1+\delta) + \delta\rho\theta^h\left(\frac{\left(\rho^{+-}\theta^h R_A\right)^2}{2k} - F\right)\right]\right\}$$

$$\cdot\left\{\left[\rho\theta^h R_A(1+\delta) + \delta\rho\theta^h\left(\frac{\left(\rho^{+-}\theta^h R_B\right)^2}{2k} - F\right)\right] - \left[\rho\theta^h R_B(1+\delta) + \delta\rho\theta^h\left(\frac{\left(\rho^{+-}\theta^h R_A\right)^2}{2k} - F\right)\right]\right\}.$$

As the first line of the previous expression is always positive, it is sufficient to look at the last line. It equals

$(R_A - R_B)\rho\theta^h\left[(1+\delta) - \delta\left(\frac{\left(\rho^{+-}\theta^h\right)^2}{2k}\right)(R_A + R_B)\right]$. Thus, it remains to show that the term in squared brackets is always positive. Then, the sign of $\Delta\Pi^{seq}$ is only determined by the sign of $(R_A - R_B)$.

Thus,

$$\left[(1+\delta) - \delta\left(\rho^{+-}\theta^h\right)^2\frac{(R_A+R_B)}{2k}\right] \geq \left[(1+\delta) - \delta\left(\rho^{+-}\theta^h\right)^2\frac{2\max\{R_A,R_B\}}{2k}\right]$$

$$\geq \left[(1+\delta) - \delta\left(\rho^{+-}\theta^h\right)^2\frac{2\max\{R_A,R_B\}}{2\theta^h\max\{R_A,R_B\}(1+\delta)}\right] \text{ as } k_j \geq \theta^h R_j(1+\delta)$$

$$= \left[(1+\delta) - \delta\left(\rho^{+-}\right)^2 \theta^h \tfrac{1}{(1+\delta)}\right] \geq 0$$

Equivalently, we can show that if both countries are identical but only differ with respect to their fixed entry costs, the one with a higher level of F_j should be entered later.

Hypothesis R5

First, we derive $\Delta\Pi^{Seq}$, the difference between profits under sequential entry when A and when B is chosen first.

Here, $\frac{\partial \Delta \Pi^{seq}}{dF_B} = -\delta X_A^{AB} \rho_A \theta^h + 1 > 0$ and $\frac{\partial \Delta \Pi^{seq}}{dF_A} = -1 + \delta X_B^{BA} \rho_B \theta^h < 0$. Second, we derive the difference between profits under sequential and simultaneous entry and get $\frac{d\Delta\Pi}{dF_B} = -\delta X_A^{Seq} \rho_A \theta^h + 1 > 0$.

Finally, it helps to establish that if fixed costs are very small, sequential entry can never be optimal. Note that when sequential entry is chosen, entry into B after a failure in A can not be optimal (otherwise, the firm could increase expected profits by choosing simultaneous entry). Thus, the belief, ρ_B^{--}, i.e., the belief in B after a failure in A must satisfy $\frac{\left(\rho_B^{--}\theta^h R_B\right)^2}{2k_B} - F_B \leq 0$. As $\rho_B^{--} > 0$, F_B needs to be sufficiently large to make this condition hold.

Appendix III – Adjustable Capacity

Assume that the capacity can be adjusted upwards in the second period. We assume that the cost function is a function of the total capacity, i.e. the marginal investment cost for the first capacity unit in period 2 equals the marginal cost for the last capacity unit in the first period. Generally, the option to adjust the capacity later will allow the firms to increase investments in period 2 if a success was observed in $t = 1$. After a failure, nothing changes. Obviously, the capacity in the first period will be smaller than without the adjustment option. What we shoe here is that sequential entry is still associated with a higher investment level in country A. All other main results will obviously hold as well.

Isolated Entry

As the MNE only considers entry into one country, we can omit the country subscript. Define X_1 as the first period and $X_2 = X_1 + \Delta X$ as the total second-period-capacity following a success. Furthermore, define $\Delta K(X_2) = K(X_2) - K(X_1)$ as the costs of the capacity increase.

We first have to determine the capacity adjustment in the second period after a success has been observed. Generally, expected profits then equal $X_2 R\theta^h - \Delta K(X_2) = X_2 R\theta^h - \left(k\frac{X_2^2}{2} - k\frac{X_1^2}{2}\right)$. The gives an optimal capacity level $X_2 = \frac{R\theta^h}{k}$ and implies second-period profits $\Pi_2^+ = \frac{\left(R\theta^h\right)^2}{2k} + k\frac{X_1^2}{2}$. As $\rho^- < \rho$, the capacity does not get adjusted after a failure, yielding expected second-period profits $\Pi_2^- = \rho_2^- X_1\theta^h R = \frac{\rho(1-X_1\theta^h)}{\rho(1-X_1\theta^h)+(1-\rho)}X_1\theta^h R$.

This allows us to state

Lemma A4: Assume the capacity can be adjusted in the second period. Then, the first-period investment level under isolated entry equals $X_1 = \frac{\left(k+2\delta\rho\theta^h R\theta^h\right) - \sqrt{\left(k+2\delta\rho\theta^h R\theta^h\right)^2 - 3\delta\rho\theta^h\rho\theta^h R\left(\delta R\left(\theta^h\right)^2 + 2k(1+\delta)\right)}}{3\delta\rho\theta^h k}$.

Proof:

Total expected profits are equal to

$$\Pi^{iso} = X_1\rho R\theta^h - K(X_1) + \delta\left(\rho X_1\theta^h\Pi_2^+ + (1-\rho X_1\theta^h)\Pi_2^-\right)$$

Substituting allows us to state the first-order condition:

$$\rho R\theta^h - kX_1 + \delta\left(\rho\theta^h\frac{\left(R\theta^h\right)^2}{2k} + 3\rho\theta^h k\frac{X_1^2}{2} + \rho\theta^h R - 2\rho X_1\theta^h R\theta^h\right) = 0, \text{ with}$$

$$X_1 = \frac{\left(k+2\delta\rho\theta^h R\theta^h\right) \pm \sqrt{\left(k+2\delta\rho\theta^h R\theta^h\right)^2 - 3\delta\rho\theta^h\rho\theta^h R\left(\delta R\left(\theta^h\right)^2 + 2k(1+\delta)\right)}}{3\delta\rho\theta^h k}$$

The second order condition then guarantees that the stated level is a maximum, while the other level constitutes a minimum.

Sequential Entry

We first proceed with sequential entry and show that the resulting capacity in A is higher than under isolated entry. Under sequential entry, the situation in country B is identical to the case without the option to adjust one's capacity; thus $X_B^{Seq} = \frac{\rho_B^{+-}\theta^h R_B}{k_B}$, yields expected profits in B, $\Pi_B^{seq} = \frac{\left(\rho_B^{+-}\theta^h R_B\right)^2}{2k_B} - F_B$.

Furthermore, the considerations in A in the second period are equivalent to isolated entry. A success yields a second-period capacity $X_{2A} = \frac{R_A\theta^h}{k_A}$ associated with profits $\Pi_{2A}^+ = \frac{\left(R_A\theta^h\right)^2}{2k_A} + k_A\frac{X_{1A}^2}{2}$. A failure leaves the capacity unchanged and gives second-period profits $\Pi_2^- = \rho_{2A}^- X_{1A}\theta^h R_A = \frac{\rho_A(1-X_{1A}\theta^h)}{\rho_A(1-X_{1A}\theta^h)+(1-\rho_A)} X_{1A}\theta^h R_A$.

Now we can state

Lemma A5: Assume the capacity can be adjusted in the second period. Then, the first-period investment level in the first country entered under sequential entry equals

$$X_{1A}^{Seq} = \frac{2\delta\rho_A\theta^h\theta^h R_A + k_A - \sqrt{\left(2\delta\rho_A\theta^h\theta^h R_A + k_A\right)^2 - 4\frac{3}{2}\delta\rho_A\theta^h k_A\rho_A\theta^h\left(R_A(1+\delta) + \delta\frac{\left(R_A\theta^h\right)^2}{2k_A} + \delta\Pi_B^{seq}\right)}}{3\delta\rho_A\theta^h k_A}.$$

Proof:

Total profits equal

$\Pi^{Seq} = X_{1A}^{Seq}\rho_A\theta^h R_A - k_A\frac{(X_{1A}^{Seq})^2}{2} - F_A + \delta X_{A1}^{Seq}\rho_A\theta^h\left(\frac{\left(R_A\theta^h\right)^2}{2k_A} + k_A\frac{X_{1A}^2}{2} + \Pi_B^{seq}\right)$

$+\delta\rho_A(1 - X_{1A}\theta^h)X_{1A}\theta^h R_A$, which implies the first order condition

$(X_{A1}^{Seq})^2\frac{3}{2}\delta\rho_A\theta^h k_A - X_{1A}^{Seq}\left(2\delta\rho_A\theta^h\theta^h R_A + k_A\right) + \rho_A\theta^h\left(R_A(1+\delta) + \delta\frac{\left(R_A\theta^h\right)^2}{2k_A} + \delta\Pi_B^{seq}\right) =$ 0 and potential capacity levels

$$X_{1A}^{Seq} = \frac{2\delta\rho_A\theta^h\theta^h R_A + k_A \pm \sqrt{\left(2\delta\rho_A\theta^h\theta^h R_A + k_A\right)^2 - 4\frac{3}{2}\delta\rho_A\theta^h k_A\rho_A\theta^h\left(R_A(1+\delta) + \delta\frac{\left(R_A\theta^h\right)^2}{2k_A} + \delta\Pi_B^{seq}\right)}}{3\delta\rho_A\theta^h k_A}.$$

The second order condition guarantees that the stated level is a maximum.

This allows us to constitute

Lemma A6: Assume the capacity can be adjusted in the second period. Then, the first-period investment level in the first country entered under sequential entry is higher there than when this country was entered in isolation.

Proof:

$X_{1A}^{seq} \geq X_{1A}^{iso}$ is satisfied as long as $2k_A \delta \Pi_B^{seq} > 0$, which obviously is the case.

Simultaneous Entry

We can not always state that simultaneous and isolated entry lead to identical outcomes. Now, a success in A could induce a capacity adjustment in B even after a failure in B was observed. Thus, the capacity under simultaneous might be higher than under isolated entry. Yet, it will never be as high as under sequential entry. The reason is that sequential entry constitutes the most extreme case of the possibility stated above. As $X_{1B} = 0$, $\rho_B^{+-} > \rho_B$, and a success in A leads to an increase of the capacity in B even though a failure occured there (which had to happen with probability 1). As the higher capacity in A under sequential entry is induced by the extra profits expected in B, it is obvious that sequential entry is always associated with a higher capacity than simultaneous entry. The reason is that these expected extra profits are highest when no previous investments in B occured (otherwise, the updated belief would be lower).

Appendix IV – Tables

Table 1: Descriptive Statistics (Mean Values)

	Foreign Investment of the MNE: 1st	2nd	3rd	4th	5th
Dependent Variable:					
Location Decision	0.013	0.012	0.011	0.012	0.013
Host-country Variables:					
Tax	0.301	0.292	0.291	0.282	0.276
log GDP	25.750	25.572	25.575	25.762	25.882
InvestFree	60.860	59.292	58.100	57.850	57.518
InvestCost	26.782	28.383	31.163	28.539	21.810
CPI	4.756	4.676	4.562	4.695	4.895
Bilateral Variables:					
log Distance to parent	8.103	8.131	8.158	8.106	8.025
log Distance to 1st		8.407	8.427	8.373	8.273
log Distance to 2nd			8.461	8.418	8.312
log Distance to 3rd				8.441	8.400
log Distance to 4th					8.498
Border to parent	0.104	0.093	0.090	0.097	0.102
Border to 1st		0.032	0.031	0.033	0.037
Border to 2nd			0.030	0.032	0.039
Border to 3rd				0.034	0.034
Border to 4th					0.041
Language same as parent	0.026	0.023	0.023	0.024	0.026
Language same as 1st		0.137	0.132	0.129	0.111
Language same as 2nd			0.137	0.129	0.095
Language same as 3rd				0.124	0.137
Language same as 4th					0.133
Colony of parent	0.025	0.023	0.023	0.024	0.026
Colony of 1st		0.052	0.048	0.048	0.223
Colony of 2nd			0.049	0.043	0.198
Colony of 3rd				0.048	0.194
Colony of 4th					0.210
Same country as 1st		0.012	0.011	0.014	0.017
Same country as 2nd			0.010	0.013	0.016
Same country as 3rd				0.011	0.007
Same country as 4th					0.013
GTA with parent	0.221	0.199	0.206	0.231	0.261
GTA with 1st		0.173	0.181	0.202	0.242
GTA with 2nd			0.192	0.201	0.262
GTA with 3rd				0.196	0.241
GTA with 4th					0.179
Observations	*1,164,529*	*402,359*	*199,168*	*90,716*	*74,876*

Table 2: Sequential Location Decision (Basic Results)

	Foreign Investment of the MNE:				
	1st	*2nd*	*3rd*	*4th*	*5th*
Host-country Variables:					
Tax	-1.626***	-1.619***	-1.613***	-3.346***	0.742
	(0.191)	(0.343)	(0.468)	(0.698)	(0.704)
log GDP	0.836***	0.757***	0.664***	0.749***	0.524***
	(0.010)	(0.016)	(0.022)	(0.032)	(0.031)
InvestFree	0.017***	0.010***	0.011***	0.009***	0.018***
	(0.001)	(0.001)	(0.002)	(0.003)	(0.003)
InvestCost	-0.008***	-0.010***	-0.012***	-0.010***	-0.009***
	(0.001)	(0.002)	(0.002)	(0.003)	(0.003)
CPI	0.042***	0.003	-0.002	-0.038	-0.021
	(0.006)	(0.012)	(0.017)	(0.026)	(0.028)
Bilateral Variables:					
log Distance to parent	-0.522***	-0.251***	-0.033	0.011	0.315***
	(0.016)	(0.030)	(0.043)	(0.069)	(0.076)
log Distance to 1st		-0.538***	-0.358***	-0.217***	-0.267***
		(0.021)	(0.030)	(0.050)	(0.063)
log Distance to 2nd			-0.347***	-0.216***	-0.148***
			(0.030)	(0.047)	(0.057)
log Distance to 3rd				-0.367***	-0.455***
				(0.048)	(0.058)
log Distance to 4th					-0.272***
					(0.044)
Border to parent	0.535***	0.371***	0.318***	0.474***	0.272**
	(0.028)	(0.051)	(0.074)	(0.108)	(0.114)
Border to 1st		-0.156**	0.016	-0.130	-0.568***
		(0.059)	(0.084)	(0.152)	(0.180)
Border to 2nd			0.199**	-0.064	0.352**
			(0.083)	(0.142)	(0.146)
Border to 3rd				-0.078	0.630***
				(0.141)	(0.148)
Border to 4th					-0.258*
					(0.150)

continued

Table 2: Sequential Location Decision (Basic Results)
(continued)

	Foreign Investment of the MNE:				
	1st	*2nd*	*3rd*	*4th*	*5th*
Language same as parent	0.378***	0.251***	0.297***	0.031	-0.486***
	(0.033)	(0.062)	(0.092)	(0.150)	(0.167)
Language same as 1st		0.087	-0.321***	-0.129	0.424***
		(0.057)	(0.087)	(0.132)	(0.154)
Language same as 2nd			-0.276***	-0.059	-0.438***
			(0.087)	(0.130)	(0.166)
Language same as 3rd				-0.188	-0.554***
				(0.132)	(0.149)
Language same as 4th					0.163
					(0.127)
Colony of parent	0.361***	0.339***	0.624***	-0.208	0.565***
	(0.047)	(0.085)	(0.118)	(0.196)	(0.204)
Colony of 1st		0.429***	0.611***	0.273*	-0.113
		(0.066)	(0.096)	(0.157)	(0.198)
Colony of 2nd			0.653***	0.247*	0.132
			(0.095)	(0.150)	(0.205)
Colony of 3rd				0.525***	-0.076
				(0.140)	(0.193)
Colony of 4th					0.019
					(0.146)
Same country as 1st		-0.226**	-0.282*	0.105	-0.415
		(0.100)	(0.162)	(0.256)	(0.340)
Same country as 2nd			-0.551***	-0.006	-0.061
			(0.170)	(0.223)	(0.251)
Same country as 3rd				-0.107	-0.680*
				(0.249)	(0.370)
Same country as 4th					-0.214
					(0.273)

continued

TABLE 2: SEQUENTIAL LOCATION DECISION (BASIC RESULTS)
(concluded)

	Foreign Investment of the MNE:				
	1st	*2nd*	*3rd*	*4th*	*5th*
GTA with parent	0.073**	-0.221***	-0.188**	-0.543***	-0.088
	(0.029)	(0.055)	(0.081)	(0.128)	(0.149)
GTA with 1st		0.535***	0.333***	0.330***	0.153
		(0.051)	(0.078)	(0.119)	(0.148)
GTA with 2nd			0.410***	0.350***	0.223*
			(0.078)	(0.121)	(0.126)
GTA with 3rd				0.316***	-0.310**
				(0.116)	(0.136)
GTA with 4th					0.471***
					(0.120)
Pseudo R2	0.2258	0.2819	0.2706	0.2553	0.2255
Observations	1,164,529	402,359	199,168	90,716	74,876
Location decisions	15,165	4,694	2,249	1,099	958
Years between decisions		1.999	1.611	1.478	1.326

Notes: Conditional logit model. If the MNE has chosen two (or more) locations in phase $p-1$, we use the greater investment (measured in fixed assets) as reference for the investment in phase p. Robust standard errors reported in parentheses. *, **, and *** indicate significance at 10%, 5%, and 1%, respectively. *Location decisions* reports the actual number of location decisions made (Location decision = 1). *Years between decisions* are the average years between the respective (sequential) location decisions made by the multinationals in the sample. Control variables are take from different sources. *Tax* is the statutory tax rate of a host country. The tax data is collected from databases provided by the International Bureau of Fiscal Documentation (IBFD) and tax surveys provided by Ernst&Young, PwC, and KPMG. *log GDP* measures the real GDP at constant U.S. dollars of the year 2000 and is taken from the World Bank's World Development Indicators 2009. The investment freedom index *InvestFree* is taken from the Heritage Indicators database. The index can take on values between 0 and 100; higher values are associated with more investment freedom. *InvestCost* is from World Bank's Doing Business Database and measures the cost of starting a business relative to income per capita. *CPI* (Corruption Perception Index) is published annually by Transparency International. It ranks countries in terms of perceived levels of corruption, as determined by expert assessments and opinion surveys. The scores range from 10 (country perceived as virtually corruption free) to 0 (country perceived as almost totally corrupt). *log Distance* is the log of the distance (in kilometer) between the most populated cities in the host country and the country of the previous investment. As to the bilateral variables for the first investment, we use Germany as the reference country. *Border* is a common border indicator, *Language* a common language indicator, *Colony* a former colony indicator, *Same country* a dummy indicating whether the host country and the country of the previous investment are the same. *GTA* is an indicator for the existence of a general trade agreement indicator between the host country and the country of the previous investment. The bilateral variables are either taken from the Centre d'Études Prospectives et d'Informations Internationales (*log Distance, Border, Language, Colony, Same country*), or from the World Trade Organization (*GTA*).

Table 3: Marginal Effects of Continuous Variables

	Foreign Investment of the MNE:				
	1st	*2nd*	*3rd*	*4th*	*5th*
Host-country Variables:					
Tax	-.0200	-.0173	-.0168	-.0372	.0088
log GDP	.0103	.0081	.0069	.0083	.0062
InvestFree	.0002	.0001	.0001	.0001	.0002
InvestCost	-.0001	-.0001	-.0001	-.0001	-.0002
CPI	.0005	.0000	-.0000	-.0004	-.0001
Bilateral Variables:					
log Distance to parent	-.0064	-.0027	-.0003	.0001	.0037
log Distance to 1st		-.0057	-.0037	-.0024	-.0032
log Distance to 2nd			-.0036	-.0024	-.0017
log Distance to 3rd				-.0041	-.0054
log Distance to 4th					-.0032

Notes: Marginal effects correspond to Table 2 (Basic Results). The values shown are the average marginal effects. The latter are obtained as $p_j(x)/\partial x_{jk} = p_j(x)[1 - p_j(x)]\beta_k$, where p_j is the response probability given by Equation 3.10.

Table 4: Sequential Location Decision (Sensitivity I)

	Foreign Investment of the MNE:				
	1st	*2nd*	*3rd*	*4th*	*5th*
Host-country Variables:					
Tax	-1.279***	-1.599***	-1.637***	-3.331***	0.909
	(0.193)	(0.343)	(0.469)	(0.698)	(0.708)
log GDP	0.584***	0.667***	0.606***	0.764***	0.617***
	(0.012)	(0.020)	(0.027)	(0.042)	(0.041)
InvestFree	0.016***	0.010***	0.011***	0.008***	0.020***
	(0.001)	(0.001)	(0.002)	(0.002)	(0.003)
InvestCost	-0.006***	-0.009***	-0.011***	-0.010***	-0.009***
	(0.001)	(0.001)	(0.002)	(0.003)	(0.003)
CPI	0.011*	-0.009	-0.010	-0.035	-0.017
	(0.006)	(0.012)	(0.017)	(0.026)	(0.027)
Bilateral Variables:					
log Distance to parent	-0.469***	-0.232***	-0.021	0.008	0.299***
	(0.016)	(0.029)	(0.043)	(0.069)	(0.075)
log Distance to 1st		-0.530***	-0.354***	-0.219***	-0.283***
		(0.020)	(0.031)	(0.050)	(0.061)
log Distance to 2nd			-0.345***	-0.217***	-0.151***
			(0.029)	(0.046)	(0.056)
log Distance to 3rd				-0.368***	-0.461***
				(0.047)	(0.057)
log Distance to 4th					-0.271***
					(0.043)
Border to parent	0.313***	0.293***	0.268***	0.485***	0.343***
	(0.028)	(0.052)	(0.076)	(0.109)	(0.115)
Border to 1st		-0.148**	0.031	-0.133	-0.564***
		(0.059)	(0.084)	(0.151)	(0.179)
Border to 2nd			0.194**	-0.061	0.341**
			(0.083)	(0.142)	(0.146)
Border to 3rd				-0.078	0.628***
				(0.141)	(0.146)
Border to 4th					-0.275*
					(0.150)

continued

Table 4: Sequential Location Decision (Sensitivity I)
(continued)

	Foreign Investment of the MNE:				
	1st	*2nd*	*3rd*	*4th*	*5th*
Language same as parent	0.326***	0.254***	0.304***	0.027	-0.515***
	(0.032)	(0.061)	(0.092)	(0.150)	(0.169)
Language same as 1st		0.092	-0.321***	-0.132	0.361**
		(0.056)	(0.087)	(0.132)	(0.157)
Language same as 2nd			-0.281***	-0.059	-0.419**
			(0.086)	(0.130)	(0.166)
Language same as 3rd				-0.186	-0.557***
				(0.132)	(0.148)
Language same as 4th					0.157
					(0.126)
Colony of parent	0.441***	0.355***	0.627***	-0.205	0.601***
	(0.046)	(0.084)	(0.117)	(0.196)	(0.206)
Colony of 1st		0.378***	0.587***	0.282*	0.017
		(0.067)	(0.097)	(0.157)	(0.202)
Colony of 2nd			0.636***	0.252*	0.192
			(0.095)	(0.150)	(0.202)
Colony of 3rd				0.531***	-0.041
				(0.139)	(0.191)
Colony of 4th					0.055
					(0.147)
Same country as 1st		-0.213**	-0.281*	0.105	-0.450
		(0.101)	(0.162)	(0.255)	(0.340)
Same country as 2nd			-0.549***	-0.008	-0.063
			(0.171)	(0.222)	(0.249)
Same country as 3rd				-0.108	-0.693*
				(0.248)	(0.368)
Same country as 4th					-0.218
					(0.271)

continued

Table 4: Sequential Location Decision (Sensitivity I)
(concluded)

	Foreign Investment of the MNE:				
	1st	*2nd*	*3rd*	*4th*	*5th*
GTA with parent	0.063**	-0.214***	-0.179**	-0.546***	-0.135
	(0.028)	(0.054)	(0.080)	(0.128)	(0.148)
GTA with 1st		0.519	0.325***	0.332***	0.183
		(0.051)	(0.078)	(0.119)	(0.146)
GTA with 2nd			0.401***	0.351***	0.222*
			(0.078)	(0.121)	(0.124)
GTA with 3rd				0.317***	-0.284**
				(0.115)	(0.134)
GTA with 4th					0.472***
					(0.118)
StockInv	0.428***	0.173***	0.117***	-0.034	-0.229***
	(0.015)	(0.028)	(0.041)	(0.060)	(0.077)
Pseudo R2	0.2307	0.2827	0.2710	0.2553	0.2265
Observations	1,164,529	402,359	199,168	90,716	74,876
Location decisions	15,165	4,694	2,249	1,099	958
Years between decisions		1.999	1.611	1.478	1.326

Notes: Conditional logit model. *Sensitivity I*: All estimations additionally include the stock of all German investments in country j prior to firm i's investment, *StockInv*. If the MNE has chosen two (or more) locations in phase $p-1$, we use the greater investment (measured in fixed assets) as reference for the investment in phase p. Robust standard errors reported in parentheses. *, **, and *** indicate significance at 10%, 5%, and 1%, respectively. *Location decisions* reports the actual number of location decisions made (Location decision = 1). *Years between decisions* are the average years between the respective (sequential) location decisions made by the multinationals in the sample. Control variables are take from different sources. *Tax* is the statutory tax rate of a host country. The tax data is collected from databases provided by the International Bureau of Fiscal Documentation (IBFD) and tax surveys provided by Ernst&Young, PwC, and KPMG. *log GDP* measures the real GDP at constant U.S. dollars of the year 2000 and is taken from the World Bank's World Development Indicators 2009. The investment freedom index *InvestFree* is taken from the Heritage Indicators database. The index can take on values between 0 and 100; higher values are associated with more investment freedom. *InvestCost* is from World Bank's Doing Business Database and measures the cost of starting a business relative to income per capita. *CPI* (Corruption Perception Index) is published annually by Transparency International. It ranks countries in terms of perceived levels of corruption, as determined by expert assessments and opinion surveys. The scores range from 10 (country perceived as virtually corruption free) to 0 (country perceived as almost totally corrupt). *log Distance* is the log of the distance (in kilometer) between the most populated cities in the host country and the country of the previous investment. As to the bilateral variables for the first investment, we use Germany as the reference country. *Border* is a common border indicator, *Language* a common language indicator, *Colony* a former colony indicator, *Same country* a dummy indicating whether the host country and the country of the previous investment are the same. *GTA* is an indicator for the existence of a general trade agreement indicator between the host country and the country of the previous investment. The bilateral variables are either taken from the Centre d'Études Prospectives et d'Informations Internationales (*log Distance, Border, Language, Colony, Same country*), or from the World Trade Organization (*GTA*).

TABLE 5: SEQUENTIAL LOCATION DECISION (SENSITIVITY II)

	Foreign Investment of the MNE:				
	1st	*2nd*	*3rd*	*4th*	*5th*
Host-country Variables:					
Tax	-1.626***	-1.622***	-1.498***	-3.418***	0.688
	(0.190)	(0.343)	(0.469)	(0.689)	(0.725)
log GDP	0.836***	0.757***	0.663***	0.754***	0.536***
	(0.009)	(0.016)	(0.022)	(0.032)	(0.031)
InvestFree	0.017***	0.010***	0.009***	0.008***	0.015***
	(0.001)	(0.001)	(0.002)	(0.002)	(0.003)
InvestCost	-0.008***	-0.010***	-0.012***	-0.011***	-0.008***
	(0.001)	(0.001)	(0.002)	(0.003)	(0.003)
CPI	0.042***	0.003	-0.007	-0.042	-0.023
	(0.006)	(0.011)	(0.016)	(0.026)	(0.028)
Bilateral Variables:					
log Distance to parent	-0.521***	-0.251***	-0.042	0.047	0.352***
	(0.016)	(0.029)	(0.044)	(0.069)	(0.078)
log Distance to 1st		-0.538***	-0.375***	-0.228***	-0.249***
		(0.020)	(0.030)	(0.051)	(0.066)
log Distance to 2nd			-0.296***	-0.219***	-0.246***
			(0.032)	(0.048)	(0.057)
log Distance to 3rd				-0.358***	-0.363***
				(0.046)	(0.058)
log Distance to 4th					-0.297***
					(0.048)
Border to parent	0.535***	0.371***	0.281***	0.462***	0.315***
	(0.027)	(0.051)	(0.074)	(0.108)	(0.114)
Border to 1st		-0.161***	0.143*	-0.139	-0.509***
		(0.059)	(0.086)	(0.151)	(0.176)
Border to 2nd			-0.083	-0.025	0.208
			(0.098)	(0.142)	(0.155)
Border to 3rd				0.006	0.641***
				(0.137)	(0.145)
Border to 4th					-0.302**
					(0.147)

continued

Table 5: Sequential Location Decision (Sensitivity II)
(continued)

	Foreign Investment of the MNE:				
	1st	*2nd*	*3rd*	*4th*	*5th*
Language same as parent	0.377***	0.251***	0.339***	0.052	-0.429***
	(0.033)	(0.062)	(0.093)	(0.148)	(0.166)
Language same as 1st		0.092	-0.373***	-0.113	0.389***
		(0.056)	(0.089)	(0.134)	(0.149)
Language same as 2nd			-0.202**	-0.188	-0.605***
			(0.084)	(0.130)	(0.164)
Language same as 3rd				-0.191	-0.483***
				(0.138)	(0.146)
Language same as 4th					0.395***
					(0.118)
Colony of parent	0.361***	0.339***	0.659***	-0.146	0.509**
	(0.046)	(0.085)	(0.119)	(0.195)	(0.204)
Colony of 1st		0.427***	0.716***	0.346**	-0.194
		(0.066)	(0.101)	(0.160)	(0.186)
Colony of 2nd			0.443***	0.224	-0.007
			(0.096)	(0.150)	(0.201)
Colony of 3rd				0.382	0.234
				(0.146)	(0.190)
Colony of 4th					-0.248*
					(0.149)
Same country as 1st		-0.225**	-0.452***	0.046	-0.448
		(0.100)	(0.165)	(0.252)	(0.341)
Same country as 2nd			-0.084	0.347	0.417
			(0.162)	(0.222)	(0.255)
Same country as 3rd				-0.220	-0.606**
				(0.249)	(0.315)
Same country as 4th					-0.104
					(0.275)

continued

TABLE 5: SEQUENTIAL LOCATION DECISION (SENSITIVITY II)
(concluded)

	Foreign Investment of the MNE:				
	1st	*2nd*	*3rd*	*4th*	*5th*
GTA with parent	0.073***	-0.220***	-0.186**	-0.574***	-0.072
	(0.028)	(0.054)	(0.082)	(0.130)	(0.155)
GTA with 1st		0.534***	0.309***	0.333***	0.206
		(0.051)	(0.078)	(0.122)	(0.151)
GTA with 2nd			0.418***	0.299**	0.209
			(0.081)	(0.122)	(0.133)
GTA with 3rd				0.364***	-0.107
				(0.115)	(0.137)
GTA with 4th					0.152
					(0.132)
Pseudo R2	0.2258	0.2819	0.2646	0.2564	0.2271
Observations	1,164,529	402,240	198,531	91,212	75,348
Location decisions	15,165	4,693	2,242	1,105	964
Years between decisions		1.998	1.613	1.485	1.332

Notes: Conditional logit model. *Sensitivity II*: If the MNE has chosen two (or more) locations in phase $p-1$, we use the greater investment (measured in *total assets* rather than in fixed assets) as reference for the investment in phase p. Robust standard errors reported in parentheses. *, **, and *** indicate significance at 10%, 5%, and 1%, respectively. *Location decisions* reports the actual number of location decisions made (Location decision = 1). *Years between decisions* are the average years between the respective (sequential) location decisions made by the multinationals in the sample. Control variables are take from different sources. *Tax* is the statutory tax rate of a host country. The tax data is collected from databases provided by the International Bureau of Fiscal Documentation (IBFD) and tax surveys provided by Ernst&Young, PwC, and KPMG. *log GDP* measures the real GDP at constant U.S. dollars of the year 2000 and is taken from the World Bank's World Development Indicators 2009. The investment freedom index *InvestFree* is taken from the Heritage Indicators database. The index can take on values between 0 and 100; higher values are associated with more investment freedom. *InvestCost* is from World Bank's Doing Business Database and measures the cost of starting a business relative to income per capita. *CPI* (Corruption Perception Index) is published annually by Transparency International. It ranks countries in terms of perceived levels of corruption, as determined by expert assessments and opinion surveys. The scores range from 10 (country perceived as virtually corruption free) to 0 (country perceived as almost totally corrupt). *log Distance* is the log of the distance (in kilometer) between the most populated cities in the host country and the country of the previous investment. As to the bilateral variables for the first investment, we use Germany as the reference country. *Border* is a common border indicator, *Language* a common language indicator, *Colony* a former colony indicator, *Same country* a dummy indicating whether the host country and the country of the previous investment are the same. *GTA* is an indicator for the existence of a general trade agreement indicator between the host country and the country of the previous investment. The bilateral variables are either taken from the Centre d'Études Prospectives et d'Informations Internationales (*log Distance, Border, Language, Colony, Same country*), or from the World Trade Organization (*GTA*).

Table 6: Sequential Location Decision (Sensitivity III)

	Foreign Investment of the MNE:				
	1st	*2nd*	*3rd*	*4th*	*5th*
Host-country Variables:					
Tax	-1.626***	-1.623***	-2.346***	-3.411**	1.710
	(0.191)	(0.343)	(0.795)	(1.678)	(2.936)
log GDP	0.836***	0.758***	0.760***	0.794***	0.525***
	(0.010)	(0.016)	(0.036)	(0.085)	(0.112)
InvestFree	0.017***	0.010***	0.010***	0.009	0.027**
	(0.001)	(0.001)	(0.003)	(0.006)	(0.013)
InvestCost	-0.008***	-0.010***	-0.012***	-.012*	-0.007
	(0.001)	(0.001)	(0.004)	(0.007)	(0.012)
CPI	0.042***	0.003	-0.046*	-0.067	-0.160
	(0.006)	(0.011)	(0.027)	(0.056)	(0.117)
Bilateral Variables:					
log Distance to parent	-0.521***	-0.252***	-0.099	0.144	0.634*
	(0.016)	(0.030)	(0.069)	(0.167)	(.337)
log Distance to 1st		-0.538***	-0.429***	-0.280**	-1.064***
		(0.020)	(0.045)	(.114)	(0.227)
log Distance to 2nd			-0.348***	-0.368***	0.243
			(0.049)	(0.112)	(0.315)
log Distance to 3rd				-0.248**	-0.732***
				(0.125)	(0.150)
log Distance to 4th					-0.271
					(0.222)
Border to parent	0.535***	0.372***	0.345***	0.240	0.739
	(0.027)	(0.051)	(0.122)	(0.284)	(0.534)
Border to 1st		-0.160***	-0.109	0.517	-0.645
		(0.059)	(0.145)	(0.349)	(0.846)
Border to 2nd			-0.163	-0.749*	0.896
			(0.156)	(0.400)	(0.637)
Border to 3rd				0.320	0.383
				(0.316)	(0.571)
Border to 4th					0.292
					(0.638)

continued

Table 6: Sequential Location Decision (Sensitivity III)
(continued)

	Foreign Investment of the MNE:				
	1st	*2nd*	*3rd*	*4th*	*5th*
Language same as parent	0.377***	0.251***	0.225	0.601	-0.974
	(0.033)	(0.061)	(0.155)	(0.380)	(0.808)
Language same as 1st		0.091	-0.278**	-1.018***	-0.215
		(0.056)	(0.137)	(0.335)	(0.673)
Language same as 2nd			0.163	-.023	0.122
			(0.130)	(0.340)	(0.623)
Language same as 3rd				-0.109	0.337
				(0.340)	(0.583)
Language same as 4th					-1.121
					(0.719)
Colony of parent	0.361***	0.339***	0.650***	0.686*	0.434
	(0.046)	(0.085)	(.181)	(0.409)	(0.947)
Colony of 1st		0.427***	0.437***	0.844**	0.851
		(0.066)	(0.162)	(0.350)	(0.670)
Colony of 2nd			0.320**	0.314	-1.620
			(0.154)	(0.389)	(1.223)
Colony of 3rd				-0.136	-1.049
				(0.381)	(1.187)
Colony of 4th					-0.365
					(0.773)
Same country as 1st		-0.224**	-0.296	-2.006**	-0.292
		(0.100)	(0.267)	(0.875)	(0.930)
Same country as 2nd			0.033	-0.009	-1.211
			(0.245)	(0.578)	(1.216)
Same country as 3rd				1.030**	-1.531
				(0.439)	(1.360)
Same country as 4th					1.958*
					(1.056)

continued

Table 6: Sequential Location Decision (Sensitivity III)
(concluded)

	Foreign Investment of the MNE:				
	1st	*2nd*	*3rd*	*4th*	*5th*
GTA with parent	0.073***	-0.220***	-0.278**	-0.157	-0.004
	(0.028)	(0.054)	(0.127)	(0.296)	(0.697)
GTA with 1st		0.534***	0.256**	-0.290	-0.821
		(0.051)	(0.125)	(0.278)	(0.551)
GTA with 2nd			0.305**	0.293	-0.055
			(0.128)	(0.308)	(0.600)
GTA with 3rd				0.367	-0.396
				(0.281)	(0.565)
GTA with 4th					0.440
					(0.606)
Pseudo R2	0.2258	0.2819	0.2845	0.2695	0.3331
Observations	1,164,529	402,256	79,677	15,999	4,688
Location decisions	15,165	4,693	885	190	60
Years between decisions		1.999	1.821	2.034	1.90

Notes: Conditional logit model. *Sensitivity III*: We only include sequential investments if the MNE has chosen only one location in phase $p-1$. In such cases, we have precise information on the reference investments in phase p. Robust standard errors reported in parentheses. *, **, and *** indicate significance at 10%, 5%, and 1%, respectively. *Location decisions* reports the actual number of location decisions made (Location decision = 1). *Years between decisions* are the average years between the respective (sequential) location decisions made by the multinationals in the sample. Control variables are take from different sources. *Tax* is the statutory tax rate of a host country. The tax data is collected from databases provided by the International Bureau of Fiscal Documentation (IBFD) and tax surveys provided by Ernst&Young, PwC, and KPMG. *log GDP* measures the real GDP at constant U.S. dollars of the year 2000 and is taken from the World Bank's World Development Indicators 2009. The investment freedom index *InvestFree* is taken from the Heritage Indicators database. The index can take on values between 0 and 100; higher values are associated with more investment freedom. *InvestCost* is from World Bank's Doing Business Database and measures the cost of starting a business relative to income per capita. *CPI* (Corruption Perception Index) is published annually by Transparency International. It ranks countries in terms of perceived levels of corruption, as determined by expert assessments and opinion surveys. The scores range from 10 (country perceived as virtually corruption free) to 0 (country perceived as almost totally corrupt). *log Distance* is the log of the distance (in kilometer) between the most populated cities in the host country and the country of the previous investment. As to the bilateral variables for the first investment, we use Germany as the reference country. *Border* is a common border indicator, *Language* a common language indicator, *Colony* a former colony indicator, *Same country* a dummy indicating whether the host country and the country of the previous investment are the same. *GTA* is an indicator for the existence of a general trade agreement indicator between the host country and the country of the previous investment. The bilateral variables are either taken from the Centre d'Études Prospectives et d'Informations Internationales (*log Distance, Border, Language, Colony, Same country*), or from the World Trade Organization (*GTA*).

TABLE 7: SIMULTANEOUS VS. SEQUENTIAL ENTRY

Maximum # of investments:	**(2)**	**(3)**	**(4)**
	Dependent variable is mean investment size measured as fixed assets		
Sequential entry	2307.46***	1843.77***	948.83*
	(827.86)	(635.99)	(586.86)
R-sq	0.9279	0.9167	0.8444
Observations	*1,812*	*2,543*	*2,954*
	Dependent variable is mean investment size measured as total assets		
Sequential entry	23596.19***	18705.9**	14582.75*
	(6560.25)	(7739.50)	(8228.59)
R-sq	0.9409	0.8581	0.7853
Observations	*1,812*	*2,543*	*2,954*
	Dependent variable is mean sales-to-total-asset ratio		
Sequential entry	.253***	.216***	.208***
	(.069)	(.056)	(.052)
R-sq	0.0425	0.0386	0.0392
Observations	*1,812*	*2,543*	*2,954*
Share of sequential entries	.582	.629	.651

Notes: Maximum # of investments is the maximum number of foreign entities per firm which have been established (one-plant firms are not considered). For example, the column denoted by (3) indicates that firms have established 3 or 2 investments. The dummy variable *Sequential entry* indicates whether the investments have been established simultaneously (Sequential entry = 0) or sequentially (Sequential entry = 1). Robust standard errors reported in parentheses. *, **, and *** indicate significance at 10%, 5%, and 1%, respectively. For reasons of comparability of firms, the above coefficients are obtained from cross-section OLS regressions which also control for a firm's total sum of investments and averages of all country controls used in the conditional logit regressions above.

TABLE 8: SIGNALS FOR ONE-PLANT & SIMULTANEOUS-PLANT UNITS

	Dependent variable is fixed assets of an affiliate	
	one-plant units	*one-plant & all simultaneous-plant units*
Sales/Fixed Assets	-2.510*	-0.234*
	(1.417)	(0.127)
Observations	*6,130*	*7,477*
Sales/Total Assets	-957.711**	-686.600**
	(494.686)	(329.551)
Observations	*6,765*	*8,357*

Notes: OLS estimation. Robust standard errors reported in parentheses. *, **, and *** indicate significance at 10%, 5%, and 1%, respectively. For reasons of comparability, the above coefficients are obtained from cross-section OLS regressions which also control for averages of all country controls used in the conditional logit regressions above.

Bibliography

Abreu, D., (1986), “Extremal Equilibria of Oligopolistic Supergames” , *Journal of Economic Theory*, 39, 191-235.

Abreu, D., Pearce, D., and E. Staccetti (1990), “Towards a Theory of Discounted Repeated Games with Imperfect Monitoring”, *Econometrica*, 58, 1041-1063.

Ahn, N., and P. Mira (2002), “A Note on the Changing Relationship Between Fertility and Female Employment Rates in Developed Countries”, Journal of Population Economics, 15, 4, 667-682.

Albornoz, F., Pardo, H.F.C., Corcos, G., and E. Ornelas (2010), “Sequential Exporting,” CEPR Discussion Paper No. DP8103.

Allen, S.G. (1984), “Unionized Construction Workers are more Productive”, *Quarterly Journal of Economics*, 99(2), 251-274.

Allen, D.W., and M. Brinig (2010), “Child Support Guidelines and Divorce Incentives”, mimeo, Simon Fraser University and University of Notre Dame.

Apps, P., and R. Rees (2009), *Public Economics and the Household*, Cambridge University Press, 2009.

Arnold, J.M., and K. Hussinger (2005a), “Export Behavior and Firm Productivity in German Manufacturing,” *Review of World Economics/Weltwirtschaftliches Archiv*, 141, 219-243.

Arnold, J.M., and K. Hussinger (2005b), “Export versus FDI in German Manufacturing: Firm Performance and Participation in International Markets,” ZEW Discussion Paper No. 05-73.

Baker, G., Gibbons, R., and K.J. Murphy (1994), "Subjective Performance Measures in Optimal Incentive Contracts" , *Quarterly Journal of Economics*, 109, 1125-1156.

Becker, G. S., (1991), *A Treatise on the Family*, Harvard University Press, 1991.

Becker, S.O., Egger P., and V. Merlo (2009), "How Low Business Tax Rates Attract Multinational Headquarters: Municipality-Level Evidence from Germany," CESifo Working Paper No. 2517.

Bellido, H., and M. Marcï£¡n, (2011), "Divorce Laws and Fertility Decisions", MPRA Paper No. 30243.

Bellman , R. (1956), "A Problem in the Sequential Design of Experiments", *Sankhyā: The Indian Journal of Statistics*, 16, 221-229.

Bergemann, D., and U. Hege (1998), "Dynamic Venture Capital Financing, Learning and Moral Hazard", *Journal of Banking and Finance* 22, 703-735.

Bergemann, D., and U. Hege (2005), "The Financing of Innovation: Learning and Stopping", *RAND Journal of Economics,* 36, 4, 719-752.

Bernard, A., and B. Jensen (1999), "Exceptional Exporter Performance: Cause, Effect, or Both?", *Journal of International Economics*, 47, 1-25.

Berry and Fristedt (1979), "Bernoulli One-Armed Bandits - Arbitrary Discount Sequences", *The Annals of Statistics*, 7, 5, 1086-1105.

Bhashkar, T., and T. To (1999), "Minimum Wages for Ronald McDonald Monopsonies: A Theory of Monopsonistic Competition", *The Economic Journal*, 109, 190-203.

Board, S., and M. Meyer-ter-Vehn (2011), "Relational Contracts in Competitive Labor Markets", mimeo, UCLA.

Bradshaw, J., and N. Finch, (2002), "*A Comparison of Child Benefit Packages in 22 Countries*" , Research Report No. 174, London: Department for Work and Pensions.

Brown, C., and J.L. Medoff (1978), "Trade Unions in the Production Process", *Journal of Political Economy*, 86(3), 355-378.

Brown, C., Gilroy, C., and A. Kohen (1982), "The Effect of the Minimum Wage on Employment and Unemployment", *Journal of Economic Literature*, 20(2), 487-528.

Brown, C. (1999), "Minimum Wages, Employment, and the Distribution of Income", *Handbook of Labor Economics*, Vol. 3, Part 2, 2101-2163.

Browning, M., and P.-A. Chiappori (1998), "Efficient Intra-Household Allocation: a Characterisation and Tests", *Econometrica*, 66, 1241-1278.

Bull, C. (1987), "The Existence of Self-Enforcing Implicit Contracts", *Quarterly Journal of Economics*, 102(1), 147-159.

Card, D., and A.B. Krueger (1994), "Minimum Wages and Employment: A Case Study of the Fast-Food Industry in New Jersey and Pennsylvania", *American Economic Review*, 84, 772-793.

Card, D., and A.B. Krueger (1995), "Myth and Measurement. The New Economics of the Minimum Wage", Princeton: Princeton University Press.

Carruth, A., Dickerson, A., and A. Henley (2000), "What do we know about Investment under Uncertainty?", Journal of Economic Surveys, 14, 2, 119-154.

Chen, M.X., and M.O. Moore (2010), "Location Decision of Heterogeneous Multinational Firms", *Journal of International Economics*, 80, 188-199.

Clark, K.B. (1980), "Unionization and Productivity: Micro-Econometric Evidence", *Quarterly Journal of Economics,* 95(4), 613-639.

Clerides, S. K., S. Lach, and J.R. Tybout (1998), "Is Learning by Exporting Important? Micro- Dynamic Evidence from Colombia, Mexico, and Morocco", *Quarterly Journal of Economics*, 113, 903-947.

Conconi, P., Sapir, A., and M. Zanardi (2010), "The Internationalization Process of Firms from Exports to FDI?", mimeo.

Devereux, M., and Griffith, R. (1998), "Taxes and the location of production-evidence from a panel of US multinationals", *Journal of Public Economics.* 68, 335-367.

Dittrich, M., and A. Knabe (2010), "Wage and Employment Effects of Non-Binding Minimum Wages", CESifo Working Paper No. 3149, Muenchen.

Dixit, A.K., and R.S. Pindyck (1994), "Investment under Uncertainty", Princeton University Press.

Doepke, M. (2005), "Child Mortality and Fertility Decline: Does the Barro-Becker Model Fit the Facts?", *Journal of Population Economics*, 18, 2, 337-366.

Doucouliagos, C., and P. Laroche (2003), "What Do Unions Do To Productivity? A Meta-Analysis", *Industrial Relations: A Journal of Economy and Society,* 42(4), 650-691.

Dube, A., Lester, T.W., and M. Reich (2010), "Minimum Wage Effects Across State Borders: Estimates Using Contiguous Counties", *The Review of Economics and Statistics*, 92, 945-964.

Dube, A., Lester, T.W., and M. Reich (2011), "Do Frictions Matter in the Labor Market? Accessions, Separations and Minimum Wage Effects", Working Paper, Institute for Labor and Employment, UC Berkeley.

Dube, A., Lester, S., and M. Reich (2007), "The Economic Effects of Citywide Minimum Wages.", *Industrial and Labor Relations Review,* 60(4), 522-543.

Eaton, J., Eslava, M., Kugler, M., and Tybout, J. (2007), "Export Dynamics in Colombia: Firm-Level Evidence", NBER Working Paper No. W13531.

Evenett, S.J., and A.J. Venables (2002), "Export Growth by Developing Economies: Market Entry and Bilateral Trade',' unpublished manuscript, University of St. Gallen.

Falk, A., Fehr, E., and C. Zehnder (2006), "Fariness Perceptions and Reservation Wages - The Behavioral Effects of Minimum Wage Laws", *Quarterly Journal of Economics*, 121, 1347-1381.

Flinn, C. (1986), "Wage and Job Mobility of Young Workers", *Journal of Political Economy*, 94, 88-110.

Freeman, Richard B. (1996), "The Minimum Wage as a Redistributive Tool", *The Economic Journal*, 436, 639-649.

Fong, Y. and J. Li (2010), "Relational Contracts, Efficiency Wages, and Employment Dynamics", mimeo, Northwestern University.

Friedberg, L., (1998), "Did Unilateral Divorce Raise Divorce Rates? Evidence from Panel Data" , *American Economic Review*, 88, 608-627.

Girma, S., Greenaway, D. , and R. Kneller (2002), "Does Exporting Lead to Better Performance? A Microeconometric Analysis of Matched Firms", University of Nottingham Research Paper Series 09.

Girma, S., Kneller, R., and M. Pisu (2005), "Exports versus FDI: An Empirical Test", *Review of World Economics,* 141, 193-218.

Grossberg, A.J., and P. Sicilian (1993), "Do Legal Minimum Wages Create Rents? A Re-examination of the Evidence", *Southern Economic Journal*, 69(1), 201-209.

Grossman, J.B. (1983), "The Impact of the Minimum Wage on Other Wages", *Journal of Human Resources*, 18, 359-378.

Helpman, E., M. J. Melitz, and S. R. Yeaple (2004), "Export versus FDI with Heterogeneous Firms", *American Economic Review,* 94, 300-316.

Holzer, H.J., Katz, L.K., and A.B. Krueger (1991), "Job Queues and Wages", *Quarterly Journal of Economics*, 106(3), 739-768.

Immervoll, H., Kleven, H.J., Kreiner, C.T., and N. Verdelin (2009), "An Evaluation of the Tax-Transfer Treatment of Married Couples in European Countries", IZA Discussion Paper No. 3965.

Jovanovic, B. (1979), "Job Matching and the Theory of Turnover", *The Journal of Political Economy*, 87, 5, 972-990.

Kadan, O., and J. M. Swinkels (2009), "Minimum Payments and Induced Effort in Moral Hazard Problems", mimeo.

Kadan, O., and J. M. Swinkels (2010), "Minimum Payments, Incentives, and Markets", mimeo, Northwestern University.

Katz, L., and A.B. Krueger (1998), "The Effect of the Minimum Wage on the Fast Food Industry", *Industrial and Labor Relations Review*, 46, 6-21.

Kaufman, Gayle (2004), "Do Gender Role Attitudes Matter? Family Formation And Dissolution Among Traditional And Egalitarian Men And Women", in *Readings In Family Theory*, SAGE Publications, 225-236.

Keller, G., S. Rady, and M. Cripps (2005), "Strategic Experimentation with Exponential Bandits", *Econometrica,* 73, 39-68.

Konrad, K. A., and K. E. Lommerud (2000), "The Bargaining Family Revisited" , *Canadian Journal of Economics*, 33, 471-487.

Kraft, K., and P. Stebler, (2006), "An Economic Analysis of Financial Support after Divorce" , *International Game Theory Review*, 04, 561-574.

Lee, D.S. (1999), "Wage Inequality in the United States During the 1980s: Rising Dispersion or Falling Minimum Wage?", *Quarterly Journal of Economics*, 114(3), 977-1023.

Levin, J. (2002), "Multilateral Contracting and the Employment Relationship", *Quarterly Journal of Economics*, 117(3), 1075-1103.

Levin, J. (2003), "Relational Incentive Contracts", *American Economic Review*, 93(3), 835-857.

Lipponer, A. (2007). "Microdatabase Direct Investment – MiDi. A brief guide", Bundesbank Discussion Paper, Frankfurt.

Lommerud, K. E. (1989), "Marital Division of Labor with Risk of Divorce: The Role of "Voice" Enforcement of Contracts", *Journal of Labor Economics*, 7, 113-136.

Lundberg, S., and E. Rose, (1999), "The Determinants of Specialization Within Marriage", mimeo.

Lundberg, S., and R. A. Pollak (2003), "Efficiency in Marriage" , *Review of Economics of the Household*, 1, 153-168.

Machin, S., and A. Manning (1994), "The Effects of Minimum Wages on Wage Dispersion and Employment: Evidence from the U.K. Wage Councils", *Industrial and Labor Relations Review*, 47, 319-329.

MacLeod, W. Bentley and James M. Malcomson (1989), "Implicit Contracts, Incentive Compatibility, and Involuntary Unemployment", *Econometrica*, 57(2), 447-480.

MacLeod, B. W., and J. M. Malcomson (1995), "Contract Bargaining with Symmetric Information" , The Canadian Journal of Economics, 28, 336-367.

MacLeod, W. Bentley and James M. Malcomson (1998), "Motivation and Markets", *American Economic Review*, 88(3), 388-411

MacLeod, W. Bentley (2010), "Great Expectations: Law, Employment Contracts, and Labor Market Performance", NBER Working Papers 16048.

Manser, M., and M. Brown (1980), "Marriage and Household Decision Making: a Bargaining Analysis" , *International Economic Review*, 21, 31-44.

Matouschek, N., and I. Rasul, (2008), "The Economics of the Marriage Contract: Theories and Evidence" , *Journal of Law and Economics*, 51, 59-110.

McDonald, R., and D.R. Siegel (1986), "The Value of Waiting to Invest", Quarterly Journal of Economics, 101, 4, 707-727.

McElroy, B., and M.J. Horney (1981), "Nash-Bargained Household Decisions Making: Toward a Generalization of the Theory of Demand" , *International Economic Review*, 22, 333-350.

McFadden, D. (1974), "Conditional Logit Analysis of Qualitative Choice Behavior". In P. Zarembka (Ed.), Frontiers in Econometrics (pp. 105-142). New York: Academic.

Michael, R. (1985), "Consequences of the Rise in Female Labor Force Participation Rates: Question and Probes", Journal of Labor Economics, 3, 1, Part 2, S117-S146.

Miller, R. (1984), "Job Matching and Occupational Choice", *Journal of Political Economy*, 92, 1086-1120.

Neumark, D., and W.L. Wascher (2007), "Minimum Wages and Employment", IZA Discussion Paper 2570.

Neumark, D., and W.L. Wascher (2008), "Minimum Wages", Cambridge: MIT Press.

Ott, N., (1992), *Intrafamily Bargaining and Household Decisions.* Berlin: Springer-Verlag.

Peters, H.E. (1986), “Marriage and Divorce: Informational Constraints and Private Contracting” , *The American Economic Review*, 76, 437-454.

Portugal, P., and A.R. Cardoso (2006), “Disentangling the Minimum Wage Puzzle: An Analysis of Worker Accessions and Separations”, *Journal of the European Economic Association,* 4(5), 988-1013.

Raff, H., (2004), “Preferential Trade Agreements and Tax Competition for Foreign Direct Investment”, *Journal of Public Economics,* 88, 2745-2763.

Rasul, I. (2003), “The Impact of Divorce Laws on Marriage”, mimeo, University of Chicago and CEPR.

Rasul, I., (2008), “Household Bargaining over Fertility: Theory and Evidence from Malaysia” , *Journal of Development Economics*, 86, 215-241.

Rebitzer, J.B., and L.J. Taylor (1995), “The Consequences of Minimum Wage Laws - Some New Theoretical Ideas”, *Journal of Public Economics*, 56, 245-255.

Riano, A., (2010), “Exports, Investments and Firm-level Sales Productivity”, CESifo Working Paper 3319.

Rowthorn, R., (1999), “Marriage and trust: some lessons from economics” , *Cambridge Journal of Economics*, 23, 661-691.

Rubinstein, A. (1982), “Perfect Equilibrium in a Bargaining Model” , *Econometrica*, 50, 97-109.

Schmeiser (2009), “Learning to Export: Export Growth and the Destination Decision of Firms”, Working Paper.

Schmidt, K.M., and M. Schnitzer (1995), “The Interaction of Explicit and Implicit Contracts”, Economics Letters, 48, 2, 193-199.

Shaked, A., and J. Sutton (1984), “Involuntary Unemployment as a Perfect Equilibrium in a Bargaining Model” , *Econometrica*, 52, 1351-1364.

Stevenson, B. (2007), “The Impact of Divorce Laws on Marriage-Specific Capital” , *Journal of Labor Economics*, 25, 75-94.

Stevenson, B., and J. Wolfers, (2007), "Marriage and Divorce: Changes and their Driving Forces" , *Journal of Economic Perspectives*, 21, 27-52.

Ulph, D. (1988). "A General Non-Cooperative Nash Model of Household Consumption Behaviour". Working paper 88-205, Department of Economics, University of Bristol.

United Nations, Department of Economic and Social Affairs (2011), Population Division, Population Estimates and Project Selection, http://esa.un.org/unpd/wpp/Excel-Data/fertility.htm

Wessels, W.J. (1980), "The Effect of Minimum Wages in the Presence of Fringe Benefits: An Expanded Model", *Economic Inquiry*, 18(2), 293-313.

Wickelgren, A., (2009), "Why Divorce Laws Matter: Incentives for Non-Contractible Marital Investments under Unilateral and Consent Divorce", *Journal of Law, Economics & Organization,*25, 80-106.

Wolfers, J., (2006), "Did Unilateral Divorce Raise Divorce Rates? A Reconciliation and New Results", *American Economic Review*, 96, 1802-1820.

Wooldridge, J. M. (2002), "Econometric Analysis of Cross Section and Panel Data", MIT Press, Cambridge, MA.

Woolley, F. (1988), "A Non-Cooperative Model of Family Decision Making". DP TIDI 125, London School of Economics.

Yang, H. (2008), "Efficiency Wages and Subjective Performance Pay", *Economic Inquiry*, 46, 179-196.

Eidesstattliche Versicherung

Ich versichere hiermit eidesstattlich, dass ich die vorliegende Arbeit selbständig und ohne fremde Hilfe verfasst habe. Die aus fremden Quellen direkt oder indirekt übernommenen Gedanken sowie mir gegebene Anregungen sind als solche kenntlich gemacht. Die Arbeit wurde bisher keiner anderen Prüfungsbehörde vorgelegt und auch noch nicht veröffentlicht.

Datum: 01.07.2011

Matthias Fahn