Particle Physics Models of Inflation in Supergravity and Grand Unification

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Zusammenfassung

Im ersten Teil dieser Dissertation untersuchen wir Klassen von Hybrid- und Chaotischen Inflationsmodellen in vierdimensionaler $\mathcal{N} = 1$ Supergravitationstheorie. Darin kann das η-Problem durch fundamentale Symmetrien im Kählerpotential behoben werden. Konkret untersuchen wir explizite Realisierungen von Superpotentialen, in welchen die Flachheit des Inflatonpotentials in Bornscher Näherung durch eine Shiftsymmetrie oder eine Heisenbergsymmetrie im Kählerpotential geschützt wird. Im letzteren Fall kann das zugehörige Modulusfeld während der Inflation durch Supergravitationseffekte stabilisiert werden.

Im Rahmen der Hybridinflation erweist sich eine neuartige Klasse von Modellen, welche wir als „Tribridinflation“ bezeichnen, als besonders verträglich mit solchen Symmetrielösungen des η-Problems. Strahlungskorrekturen infolge von Operatoren im Superpotential, welche die betreffende Symmetrie brechen, erzeugen die nötige kleine Steigung des Inflatonpotentials. Zusätzliche effektive Operatoren im Kählerpotential können den vorhergesagten Spektralindex senken, sodass er mit den neuesten Beobachtungen übereinstimmt.

Innerhalb eines Modells der Chaotischen Inflation in Supergravitation mit quadratischem Potential verwenden wir die Heisenbergsymmetrie, um Inflation bei Feldwerten oberhalb der Planckskala zu ermöglichen, wobei der zugehörige Modulus stabilisiert ist. Wir zeigen, dass Strahlungskorrekturen in diesem Zusammenhang vernachlässigbar sind.

Im zweiten Teil verwenden wir die Tribridinflationsmodelle dazu, Inflation in nichttrivialen Darstellungen einer Eichgruppe zu realisieren. Dies wird auf den Materiesektor in supersymmetrischen großen vereinheitlichten Theorien basierend auf der Pati–Salam Eichgruppe angewandt.

Für das spezielle Szenario, in welchem das rechtshändige Sneutrino das Inflaton ist, untersuchen wir das Skalarpotential in einem D-flachen Tal. Wir zeigen, dass trotz potenziell gefährlicher Zweischleifen-Korrekturen die notwendige Flachheit des Potentials beibehalten werden kann. Der Grund dafür ist die starke Unterdrückung von Eichwechselwirkungen des Inflatonfeldes aufgrund seines symmetriebrechenden Vakuumerwartungswertes. Zusätzlich kann die Erzeugung stabiler magnetischer Monopole am Ende der Inflationsphase vermieden werden.

Am Ende skizzieren wir, wie die in den beiden Teilen diskutierten Konzepte in Tribridinflationsmodellen verbunden werden können, um Inflation mittels Heisenbergsymmetrie in lokal supersymmetrischer großer Vereinheitlichung basierend auf der $SO(10)$ Eichgruppe zu verwirklichen.

Abstract

In the first part of this thesis, we study classes of hybrid and chaotic inflation models in four-dimensional $\mathcal{N} = 1$ supergravity. Therein, the η-problem can be resolved relying on fundamental symmetries in the Kähler potential. Concretely, we investigate explicit realizations of superpotentials, in which the flatness of the inflaton potential is protected at tree level by a shift symmetry or a Heisenberg symmetry in the Kähler potential. In the latter case, the associated modulus field can be stabilized during inflation by supergravity effects.

In the context of hybrid inflation, a novel class of models, to which we refer as "tribrid inflation," turns out to be particularly compatible with such symmetry solutions to the η-problem. Radiative corrections due to operators in the superpotential, which break the respective symmetry, generate the required small slope of the inflaton potential. Additional effective operators in the Kähler potential can reduce the predicted spectral index so that it agrees with latest observational data.

Within a model of chaotic inflation in supergravity with a quadratic potential, we apply the Heisenberg symmetry to allow for viable inflation with super-Planckian field values, while the associated modulus is stabilized. We show that radiative corrections are negligible in this context.

In the second part, the tribrid inflation models are extended to realize gauge non-singlet inflation. This is applied to the matter sector of supersymmetric Grand Unified Theories based on the Pati–Salam gauge group.

For the specific scenario in which the right-handed sneutrino is the inflaton, we study the scalar potential in a D-flat valley. We show that despite potentially dangerous two-loop corrections, the required flatness of the potential can be maintained. The reason for this is the strong suppression of gauge interactions of the inflaton field due to its symmetry breaking vacuum expectation value. In addition, the production of stable magnetic monopoles at the end of the stage of inflation can be avoided.

Finally, we sketch how in tribrid inflation models the concepts discussed in the two parts can be combined to realize inflation via Heisenberg symmetry in local supersymmetric $SO(10)$ grand unification.

Contents

Part I

Introduction

Chapter 1

Introduction

Curiosity incites us to find out more about our most distant history as well as the smallest building blocks of nature. From the viewpoint of physics, these two aspects are closely related. Looking back further into the past of the universe leads us to higher energy densities which is also the case when trying to resolve structure on smaller scales at collider experiments. Hence, it is mandatory that any sensible theory of the early universe cosmology has to be consistent with a particle physics description at very high energies. Consequently, a connection between cosmology and high energy particle physics is desirable. This is the guiding principle of this dissertation.

The Standard Cosmology (SC) provides a theoretical description of phenomena throughout most of the evolution of our universe. Most prominently, the Lambda Cold Dark Matter (ΛCDM) model is in good agreement with recent observations [5, 6, 7]. With only six parameters, it accounts for the production of the light elements during Big Bang nucleosynthesis (BBN) and for the decoupling of the Cosmic Microwave Background Radiation (CMBR) during the time of recombination, when electrons and protons formed hydrogen atoms. In addition, it is able to provide an appropriate description of structure formation via gravitational collapse as well as of the present accelerated expansion.

The Standard Model (SM) of particle physics is another important development in theoretical physics. It successfully describes the electroweak (EW) and strong interactions of all known elementary particles [8] and it has been experimentally tested to very high precision up to the EW scale. Thus, the SC and the SM are two mutually independent, successful theories of nature.

Despite the success of both theoretical frameworks, each of them has got its shortcomings. Important in the context of this work, the SC fails to explain the observation that the universe is spatially flat, homogeneous and isotropic on cosmological scales with tiny density perturbations which act as seeds for structure formation on smaller scales. The latter are observed as anisotropies in the CMBR by experiments such as the Wilkinson Microwave Anisotropy Probe (WMAP) [5] or the Planck satellite [9]. A simultaneous explanation of these otherwise extremely fine-tuned initial conditions can be given by *cosmic inflation* [10, 11] which describes a phase of accelerated expansion in the very early universe.

Likewise, the SM does not seem to be the fundamental theory of nature, lacking for example a solution to the hierarchy problem and an explanation of the unknown source of dark matter (DM). As a simple extension of the spacetime symmetries, low energy *supersymmetry* (SUSY) [12, 13, 14] offers a theory beyond the SM which is able to resolve these issues. Furthermore, the SM can neither explain charge quantization nor anomaly cancellation, nor the origin of the light neutrino masses. The concept of a *Grand Unified Theory* (GUT) [15, 16, 17, 18], i.e., a unification of the fundamental forces of the SM within one simple gauge group at the so-called GUT scale, addresses these problems. Moreover, the almost exact gauge coupling unification [19, 20] in a SUSY extension of the SM makes a combination of SUSY with GUTs very appealing.

Following our aspiration to establish connections between cosmology and particle physics, the motivation for the work presented in this thesis is to merge the aforementioned extensions of the SC on the one hand and of the SM on the other hand. In more specific terms, our goal is to identify the inflaton particle which drives cosmic inflation within the particle content provided by SUSY GUTs.

According to the standard paradigm of *slow-roll inflation* [21, 22, 23], quasi-exponential expansion in the early universe is driven by the vacuum energy density of some scalar condensate which is characterized by an equation of state with negative pressure. This requires a strong domination of the classical scalar field's potential energy over its kinetic energy and thus a sufficiently flat region in the potential. In other words, the mass of the corresponding inflaton particle is bound to be much smaller than the Hubble scale during inflation. Such a light scalar field can slowly roll down its potential, thus accounting for accelerated expansion. This sets the required initial conditions of a spatially flat, homogeneous and isotropic universe void of any relic particle species.

During the phase of accelerated expansion, the quantum fluctuations of the inflaton field about its classical value quickly get stretched to scales larger than the causal horizon size. At this point, their amplitude freezes and they imprint themselves as classical perturbations on our spacetime [24, 25, 26, 27]. When they re-enter the causal patch later in the post-inflationary epoch, they give rise to curvature perturbations which serve as seeds for structure formation, thus readily explaining the anisotropies observed in the CMBR. The statistical properties of the CMBR can be directly related to parameters of the underlying inflaton potential, enabling us in principle to discriminate between different models of inflation in future high-precision experiments.

At first glance, the introduction of SUSY opens up a multitude of possibilities to realize inflation, because of the many additional scalar fields it involves. However, since inflation can be operative at very high energies close to the fundamental scale of gravity, the Planck scale, threshold effects from the yet unknown theory of quantum gravity cannot be neglected. $\mathcal{N} = 1$ supergravity (SUGRA), i.e., local SUSY in four spacetime dimensions [28, 29, 30], offers an effective description of quantum gravity.

Within the theoretical framework of SUGRA, it turns out that viable inflation models are subject to severe constraints. This is mainly due to the fact that SUGRA comprises a theory of gravity which couples to everything. Correspondingly, this means that gravitational interactions of the inflaton may violate the requirement for a small inflaton mass. In particular, the inflaton field can couple to the large vacuum energy density. This typically induces scalar masses of the order of the Hubble scale inconsistent with slow-roll inflation, referred to as the *η-problem of SUGRA inflation* [31, 32]. Another problem in SUGRA is caused by the presence of additional scalar fields, so-called moduli fields. These can give rise to severe problems for cosmology [33, 34, 35]. Especially if the moduli potentials are of the runaway-type, their dominant kinetic energy spoils the vacuum energy equation of state that drives inflation. Therefore, the moduli have to be fixed in stable minima of the potential during inflation, which we refer to as the *moduli stabilization problem*. The first objective of this thesis is to find solutions to the problems of inflation in SUGRA within different classes of models, with an emphasis on symmetry solutions to the η-problem.

Among the several mechanisms proposed for inflation, hybrid inflation [36, 37] offers a particularly interesting possibility to link a phase of inflation to high energy particle physics. This originates from the fact that the waterfall field ending hybrid inflation can be identified with some symmetry breaking Higgs field. Applied to GUTs, the end of inflation may thus be related to the phase transition in which the unified gauge group is broken to the SM gauge group. From the viewpoint of inflation model building in SUSY GUTs, a particularly well motivated scenario is therefore SUSY hybrid inflation [31, 38].

Moreover, an intriguing property of left-right symmetric SUSY GUTs, important for this work, is the inevitable presence of a right-handed neutrino and its scalar superpartner, the right-handed sneutrino. With a very heavy Majorana mass for the right-handed neutrino, obtained in the breaking of the GUT, the small physical neutrino masses can be explained via the seesaw mechanism [39, 40, 41, 42, 43]. A promising model that can naturally combine the above advantages is sneutrino hybrid inflation [44].

In viable variants of hybrid inflation in SUSY GUTs, such as smooth hybrid inflation [45, 46] and shifted hybrid inflation [47], the inflaton typically remains a gauge singlet. Nevertheless, we attempt to identify the inflaton with one of the scalar superpartners of the matter fermions, which transform as non-singlet representations (reps) under the unified gauge group. Thus, we have to allow for charged inflaton fields, bringing along new problems related to the fact that gauge interactions tend to endanger the flatness of the inflaton potential [48]. Following the above nomenclature, we shall refer to these issues as *gauge η-problems*. In addition, in GUT phase transitions, copious production of stable topological defects, in particular magnetic monopoles, can take place after the end of inflation leading to a disastrously high energy density. This is known as the *monopole problem* [49, 50]. The second objective of this thesis is to propose a novel class of hybrid inflation models in SUSY GUTs, inspired by sneutrino hybrid inflation, with gauge non-singlet (GNS) inflaton fields in which the above problems are under control.

A specific class of models turns out to be appropriate for combining the two objectives of this work and realize inflation in the matter sector of SUGRA GUTs. This new class of models, denoted *tribrid inflation*, appears to open up new possibilities to connect cosmology and particle physics.

The present thesis is structured as follows. Part II deals with the theoretical background, including inflationary cosmology, the derivation of relevant SUSY and SUGRA Lagrangians as well as a SUSY GUT embedding of the SM field content into a Pati–Salam and $SO(10)$ framework.

Part III is dedicated to possible realizations of inflation models in SUGRA. To this goal, in Ch. 5, the specific problems inflation has to face in SUGRA are reviewed. In Ch. 6, viable solutions within standard hybrid inflation are reviewed and the new class of tribrid inflation models is presented. By examining explicit realizations of this new setup, we show that it is particularly suitable for a joint resolution of the aforementioned problems by imposing symmetries on the Kähler potential. In Ch. 7, we present two solutions within the context of chaotic inflation, one based on a shift symmetry and a new one based on the Heisenberg symmetry. In the latter case we introduce a mechanism to stabilize the associated modulus field during inflation.

Part IV addresses the question of how inflation can arise in the matter sector of SUSY GUTs. In Ch. 8, we show that the class of tribrid inflation models is also a good candidate for GNS inflation by looking at a simple toy model based on a $U(1)$ gauge symmetry. In Ch. 9, a more realistic model of matter inflation in the Pati–Salam gauge group is presented and a full study of the inflaton potential at tree level, one-loop and two-loop level is performed for a right-handed sneutrino inflaton direction. We sketch how these ideas can be generalized to $SO(10)$ in Ch. 10. As a highlight, we combine the concepts presented in Part III and IV to realize tribrid inflation in SUGRA $SO(10)$ via Heisenberg symmetry.

We summarize the results and draw our conclusions in Part V. More in-depth supplements have been collected in the appendix Part VI, as sources of information for the interested reader.

Part II

Theoretical Foundations

Chapter 2

Inflationary Cosmology

This chapter is dedicated to giving a concise overview over the basic ideas of the slow-roll inflationary paradigm and over the common inflation models. In Sec. 2.1, we start with a description of the shortcomings within the SC, that can collectively be solved by a phase of inflation. For textbook reviews on the SC and inflationary cosmology, the reader is referred to, e.g., [51, 52, 53, 54]. Next, the basic toolkit for constructing such a phase of accelerated expansion using canonical scalar fields is presented in Sec. 2.2. Sec. 2.3 is dedicated to the connection between the quantum fluctuations of the inflaton field on the smallest scales and the spectra of curvature and tensor perturbations on very large scales, making contact to observables in the CMBR [5, 55]. Finally, Sec. 2.4 gives a short overview over the most prominent models of slow-roll inflation and their generic predictions.

2.1 Motivation for Inflation

The SC with a Friedmann–Lemaître–Robertson–Walker (FLRW) metric[1] describing the evolution of the universe by perfect fluids supplemented by a cosmological constant Λ and a cold DM component, namely the ΛCDM model, is very successful in accounting for most observations of the late time cosmology.

Nevertheless, it has some very severe initial condition problems when extrapolating back towards the big bang. In the following, we will very briefly summarize these problems.

Horizon Problem: A very fundamental problem within the SC is related to the fact that, in a radiation- or matter-dominated universe, there can be many causally disconnected regions at the time of last scattering. This is badly in conflict with observations in the CMBR, suggesting that the universe at the time of last scattering was homogeneous to a high degree.

Let us state this argument a little more quantitatively. According to the SC where gravity is always attractive, the scale factor has a time-dependence $a(t) \sim t^q$ with some $q < 1$ depending on the exact equation of state. This implies that the Hubble expansion

[1] A definition of the corresponding line element is given by Eq. (2.10).

rate is given by

$$\mathcal{H} \equiv \left(\frac{\dot{a}}{a}\right) \sim t^{-1} \,. \tag{2.1}$$

Hence, the particle horizon scale which encompasses the observable part of the universe can be approximated as $d_{\mathrm{H}}(t) \simeq \mathcal{H}^{-1} \sim t$. This patch is known to be homogeneous and isotropic today at t_0. At some initial time t_{i}, this region was scaled down to $d_{\mathrm{H}}(t_{\mathrm{i}}) \sim t_0\,[a(t_{\mathrm{i}})/a(t_0)]$. At this early time, the typical distance scale of a domain in causal contact is $d_{\mathrm{C}}(t_{\mathrm{i}}) \simeq t_{\mathrm{i}}$. Upon comparing the ratio of the homogeneity scale and the causality scale, we find

$$\frac{d_{\mathrm{H}}(t_{\mathrm{i}})}{d_{\mathrm{C}}(t_{\mathrm{i}})} \sim \frac{\dot{a}(t_{\mathrm{i}})}{\dot{a}(t_0)} \,, \tag{2.2}$$

where we have used Eq. (2.1).

In a universe where gravity always decelerates the expansion $\ddot{a} < 0$ and thus $\dot{a}(t_{\mathrm{i}}) > \dot{a}(t_0)$ for any initial time $t_{\mathrm{i}} < t_0$, Eq. (2.2) implies that $d_{\mathrm{H}}(t_{\mathrm{i}}) > d_{\mathrm{C}}(t_{\mathrm{i}})$. In turn, this means that the scale containing our observable patch of the universe today contained many causally disconnected regions at early times. Thus at the time of last scattering when the photons of the CMBR stopped interacting with matter, they would have started to travel to us from different regions which have never been in causal contact. This contradicts the high degree of homogeneity of this radiation observed on the largest scales.

A solution to this problem is to invert the relation above to $d_{\mathrm{H}}(t_{\mathrm{i}}) < d_{\mathrm{C}}(t_{\mathrm{i}})$. Looking at Eq. (2.2), this is possible if there has been some epoch in the evolution of our early universe where

$$\ddot{a} > 0 \,, \tag{2.3}$$

or in other words, a phase of accelerated expansion in which gravity acted as a repulsive force. Such a phase is called inflation and we explain how to realize it within the context of scalar field dynamics in Sec. 2.2.

Flatness Problem: Another problem is the missing explanation for why the spatial part of our metric is flat. Rewriting the Friedmann equation (2.11) in terms of the critical energy density $\varrho_{\mathrm{crit}} = 3\,\mathcal{H}^2/(8\pi G)$, one obtains

$$\Omega(t) - 1 = \frac{k}{(a\mathcal{H})^2} \,, \tag{2.4}$$

where the ratio $\Omega = \varrho/\varrho_{\mathrm{crit}}$ has been defined.

Since in SC, the comoving Hubble scale $(a\mathcal{H})^{-1}$ increases with time, even a very small spatial curvature term on the right-hand side of Eq. (2.4) at early times can give rise to a huge contribution today. Therefore, within the SC evolution, only extremely fine-tuned initial conditions $\Omega(t_{\mathrm{i}}) = 1$ can account for $\Omega(t_0) = 1$ today with a general spatial curvature k. Another way of saying this is that for $t \to 0$, the velocity $\dot{a} = a\mathcal{H}$ has to diverge in order to satisfy for a spatially flat universe today, which seems rather unnatural.

Just as for the horizon problem, an accelerated phase of expansion with $\ddot{a} > 0$ can also provide a solution in this case. With a comoving Hubble scale $(a\mathcal{H})^{-1}$ that decreases sufficiently long during a phase of inflation, the initial conditions are in some sense set "dynamically" by this period of rapid growth of the scale factor as the physical curvature radius of our three-dimensional hypersurface $R^{(3)}_{\rm curv}(t) \equiv a(t)\,|k|^{-1/2}$ becomes very large.

Monopole Problem: The third problem within the SC is related to the possible presence of superheavy stable particles. Such relic species are commonly produced as topological defects, e.g., in the breaking of GUTs. These non-trivial vacuum configurations can give rise to catastrophic contributions to the energy density $\Omega(t_0) \gg 1$.

Inflation also offers a solution to this problem if the phase transition producing the defects has taken place before the end of inflation. The quasi-exponential evolution of the scale factor dilutes any form of energy density such that at the end of inflation nothing besides a constant vacuum energy density is left. In Part IV, we will discuss ways of avoiding defect production after inflation in certain classes of GUT inflation models.

Initial Perturbation Problem: In order for structure to form in the first place, an exactly homogeneous, isotropic universe is not enough. The large-scale structure that we observe can only be accounted for by gravitational collapse if the initial density contrast had perturbations $\delta\varrho/\varrho = \mathcal{O}(10^{-5})$ on galactic scales.

Unlike the other problems mentioned in this section, the initial metric perturbations required to solve this problem cannot be explained purely in terms of accelerated expansion. However, in a quantum field theoretical description of inflation using a scalar inflaton field, the homogeneous background dynamics of the scalar condensate can account for accelerated expansion, see Sec. 2.2, while the inflaton quantum fluctuations can source inhomogeneities through their couplings to metric perturbations.

In Sec. 2.3 we introduce the basic ideas behind a connection between quantum fluctuations and cosmological perturbations. It is the success of this theory that made it possible to predict the power spectra of scalar and tensor perturbations from the underlying model of inflation and test them against observations from CMBR measurements such as WMAP [5, 55] or the currently operating Planck satellite [9].

2.2 Slow-Roll Inflation and Scalar Fields

In order to describe a phase of exponential expansion of the spatial components of a FLRW universe as scalar field dynamics, we only need two basic ingredients. First of all we need the action of our spacetime invariant under general coordinate transformations, namely the Einstein–Hilbert action

$$S_{\rm EH} = -\frac{1}{16\,\pi\,G} \int \mathrm{d}^4x\,\sqrt{-g}\,\left(R + 2\,\Lambda\right)\,, \tag{2.5}$$

where g is the determinant of the metric tensor $g_{\mu\nu}$ and R denotes the Ricci curvature scalar. In addition, assuming the only form of "particles" present in the early universe is described by a real scalar field, we need the action giving rise to a scalar field minimally coupled to gravity

$$S_{\mathrm{M}} = \int \mathrm{d}^4x \,\sqrt{-g}\,\left(\frac{1}{2}\,g^{\mu\nu}\,\partial_\mu\phi\,\partial_\nu\phi - V(\phi)\right)\,, \tag{2.6}$$

which is also invariant under general coordinate transformations. Therefore, in the presence of only this scalar degree of freedom (DOF) the full action of such a simple early universe cosmology is just given by $S = S_{\mathrm{EH}} + S_{\mathrm{M}}$.

Minimizing the action with respect to (w.r.t.) the metric tensor $g_{\mu\nu}$, we obtain the Einstein equations

$$R_{\mu\nu} - \frac{1}{2}\,R\,g_{\mu\nu} = 8\,\pi\,G\,T_{\mu\nu} + \Lambda\,g_{\mu\nu}\,, \tag{2.7}$$

with the curvature of the spacetime manifold on the left-hand side and the particle physics or "matter" input in form of the stress-energy tensor of some relativistic fluid on the right-hand side. For the primordial scalar field, the stress-energy tensor reads

$$T^{\mu\nu} = \partial^\mu\phi\,\partial^\nu\phi - \mathcal{L}(\phi,\partial_\mu\phi)\,g^{\mu\nu}\,. \tag{2.8}$$

In the following, we will assume a vanishing cosmological term by setting $\Lambda = 0$. Minimizing (2.6) w.r.t. the scalar field and its derivatives, we end up with the Klein–Gordon equation of a scalar field in curved spacetime

$$\frac{1}{\sqrt{-g}}\,\partial_\mu\left(\sqrt{-g}\,g^{\mu\nu}\,\partial_\nu\phi\right) + \frac{\partial V(\phi)}{\partial\phi} = 0\,. \tag{2.9}$$

Basically, the system of differential Eqs. (2.7) and (2.9) is sufficient to fully depict the early universe dynamics in the presence of a simple classical scalar field theory.

The line element of the explicit FLRW metric is given by

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)\left[\frac{\mathrm{d}r^2}{1-k\,r^2} + r^2\left(\mathrm{d}\theta^2 + \mathrm{d}\varphi^2\,\sin^2\theta\right)\right]\,, \tag{2.10}$$

where $k = -1, 0, +1$ parametrizes the spatial curvature and the growth of physical scales is determined by the time-dependence of the scale factor $a(t)$.

Assuming homogeneity and isotropy of the classical scalar field due to vanishing spatial derivatives $\nabla_i\,\phi = 0$, the evolution equations simplify a great bit. The scalar condensate resembles a perfect fluid with stress-energy tensor $T_{\mu\nu} = \mathrm{diag}(\varrho, -p, -p, -p)$ having an energy density ϱ and pressure $p = \mathcal{L}$, cf. (2.13). From Eq. (2.7), one can derive the Friedmann-Lemaître equations

$$\mathcal{H}^2 = \frac{8\,\pi\,G}{3}\,\varrho - \frac{k}{a^2}\,, \qquad \left(\frac{\ddot{a}}{a}\right) = -\frac{4\,\pi\,G}{3}\,(\varrho + 3\,p)\,, \tag{2.11}$$

where we have used the Hubble scale $\mathcal{H}$ as defined in (2.1). The evolution of the classical scalar field living in the curved background reduces to the ordinary differential equation

$$\ddot{\phi} + 3\,\mathcal{H}\,\dot{\phi} + V'(\phi) = 0\,. \tag{2.12}$$

Here, a dot denotes the derivative w.r.t. cosmic time whereas a prime denotes the derivative w.r.t. the scalar field.

An accelerated expansion with $\ddot{a} > 0$, cf. Eq. (2.11), is only possible with a negative pressure fluid and hence, an equation of state $p = w\,\varrho$ with $w < -1/3$. For our simple system with only one homogeneous, isotropic scalar DOF carrying an energy and pressure density

$$\varrho = \mathcal{L}_{\text{kin}} + V(\phi)\,, \qquad p = \mathcal{L}_{\text{kin}} - V(\phi)\,, \tag{2.13}$$

an equation of state with $p = -\varrho$ can approximately be achieved in an epoch where the potential energy dominates strongly over the kinetic terms

$$V(\phi) \gg \mathcal{L}_{\text{kin}}\,. \tag{2.14}$$

Thus, the scalar field is effectively "frozen" to its potential due to a big cosmic viscosity[2] $\mathcal{H} \simeq \sqrt{V/3}$ which damps the field, cf. Eq. (2.12), and gives rise to negligible acceleration $\ddot{\phi} \ll \mathcal{H}\,\dot{\phi}$. This is the so-called slow-roll regime. With a nearly constant Hubble scale during inflation, the large vacuum energy density V_0, i.e., the height of the potential, drives the accelerated expansion or inflation and Eq. (2.11) are consistent with an approximate solution

$$a(t) \simeq a(t_0)\,\exp\left(\mathcal{H}\,t\right)\,. \tag{2.15}$$

Next, we want to parametrize the slow-roll regime in terms of simple constraints on the scalar potential and its derivatives. Therefore, we just rewrite the equation of motion (EOM) (2.12) of the scalar field in the slow-roll approximation, which reads

$$3\,\mathcal{H}\,\dot{\phi} \simeq -V'(\phi)\,. \tag{2.16}$$

One can define three slow-roll parameters which parametrize the slope, curvature and a higher order derivative respectively

$$\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2, \qquad \eta = \left(\frac{V''}{V}\right), \qquad \xi^2 = \left(\frac{V'V'''}{V^2}\right)\,. \tag{2.17}$$

If we use Eq. (2.16) together with the claim $\dot{\phi}^2/2 \ll V$, the first slow-roll condition on the slope is given by $\epsilon \ll 1$. The time derivative of Eq. (2.16) in combination with $\ddot{\phi} \ll \mathcal{H}\,\dot{\phi}$ leads to the second slow-roll condition on the curvature $|\eta| \ll 1$.

In order to resolve the initial condition problems with the SC mentioned in Sec. 2.1, a phase of inflation needs to be sustained for a sufficiently long time, i.e., over a sufficient range of the field space from the initial value ϕ_{i} to the value at the end of inflation ϕ_{e}. Typically, for this purpose, one defines the number of e-folds before the end of inflation as

$$N_{\text{e}} \equiv \ln\frac{a(t_{\text{e}})}{a(t_{\text{i}})}\,, \tag{2.18}$$

[2]From now on, we work in units where we set the reduced Planck scale $M_{\text{P}} = 1/\sqrt{8\,\pi\,G}$ to one.

which can be translated into an integral over the field configuration exploiting Eq. (2.16)

$$N_{\mathrm{e}} = \int_{t_{\mathrm{i}}}^{t_{\mathrm{e}}} \mathrm{d}t\, \mathcal{H}(t) = \int_{\phi_{\mathrm{e}}}^{\phi_{\mathrm{i}}} \mathrm{d}\phi\, \frac{V(\phi)}{V'(\phi)}\,. \tag{2.19}$$

60 e-folds of slow-roll inflation can solve the aforementioned problems. In most explicit models, this is easy to achieve.

2.3 Perturbations from Inflation

In the previous section we have discussed the evolution of a classical homogeneous scalar field. Here, we consider the quantum fluctuations of our almost massless scalar field in a de Sitter universe. Such a universe with a positive constant four-curvature is a very good approximation to describe inflation in most scenarios. It turns out that the accelerated expansion stretches the quantum modes to macroscopic scales giving rise to anisotropies on galactic scales today when they have re-entered the horizon [24, 25, 26, 27]. There are many good reviews on the issue, e.g. [56, 57].

For the sake of simplicity, in this section, we work with conformal time η which is negative in de Sitter space $-\infty < \eta < 0$ and defined by $a(\eta)\,\mathrm{d}\eta \equiv \mathrm{d}t$. We consider a line element with a conformal scale factor

$$\mathrm{d}s^2 = a^2(\eta)\left[-(1+2A)\,\mathrm{d}\eta^2 + 2B_i\,\mathrm{d}x^i\,\mathrm{d}\eta + (\delta_{ij}+h_{ij})\,\mathrm{d}x^i\,\mathrm{d}x^j\right]\,, \tag{2.20}$$

which is a spatially flat metric with linear perturbations superimposed. In Eq. (2.20), A, B_i and h_{ij} parametrize the perturbations about the flat background. These perturbations are typically decomposed into scalar, vector and tensor modes by

$$\begin{aligned} B_i &= \nabla_i\, B + \bar{B}_i\,, \\ h_{ij} &= 2\,C\,\delta_{ij} + 2\,\nabla_i\nabla_j\,E + (\nabla_i E_j + \nabla_j E_i) + \bar{E}_{ij}\,, \end{aligned} \tag{2.21}$$

with transverse vector modes $\nabla_i \bar{B}^i = \nabla_i \bar{E}^i = 0$ and transverse and traceless tensor modes $\nabla_i \bar{E}^{ij} = \bar{E}^{ij}\delta_{ij} = 0$. Since the scalar modes are the most relevant ones in cosmology, we restrict ourselves to them in the following and obtain a perturbed metric of the form

$$g_{\mu\nu} + \delta g_{\mu\nu} = a^2(\eta)\begin{pmatrix} -(1+2A) & \nabla_i B \\ \nabla_j B & (1+2C)\,\delta_{ij} + 2\nabla_i\nabla_j E \end{pmatrix}\,. \tag{2.22}$$

In the same manner, the fluctuation of the scalar inflaton field is written as an expansion around the classical, homogeneous background as

$$\phi(\eta,\vec{x}) = \phi(\eta) + \delta\phi(\eta,\vec{x})\,. \tag{2.23}$$

We now start by expanding the action up to second order in the linear perturbations

$$S\left[\phi+\delta\phi, g_{\mu\nu}+\delta g_{\mu\nu}\right] = S^{(0)}\left[\phi, g_{\mu\nu}\right] + S^{(1)}\left[\delta\phi, \delta g_{\mu\nu}; \phi, g_{\mu\nu}\right] + S^{(2)}\left[\delta\phi, \delta g_{\mu\nu}; \phi, g_{\mu\nu}\right]\,, \tag{2.24}$$

where the insertion of the EOMs eliminates $S^{(1)}$, and $S^{(0)}$ contains only the homogeneous part. The second order term $S^{(2)}$ is what gives us the EOMs for the linear perturbations via the Euler–Lagrange equation. In addition it allows us to quantize the perturbations. Especially, for the gauge-invariant combination

$$v = a\left(\delta\phi - \frac{\phi'}{\mathcal{H}}C\right), \tag{2.25}$$

which is a linear combination of metric and scalar field perturbations, the second order action in Eq. (2.24) is simply given by

$$S^{(2)}[v] = \frac{1}{2}\int \mathrm{d}\eta\,\mathrm{d}^3x\left(v'^2 + \partial_i v\,\partial^i v + \frac{z''}{z}v^2\right). \tag{2.26}$$

Eq. (2.26) is equivalent to one of a scalar field v in flat Minkowski spacetime $\eta_{\mu\nu}$ with an effective time-dependent mass squared $m^2_{\mathrm{eff}} = -z''/z$ where we have defined $z \equiv a\,\phi'/\mathcal{H}$. Note that we use a prime to denote derivatives w.r.t. conformal time in this section.[3]

As a next step, we want to quantize our true DOF v. This can be done in the canonical way by Fourier expanding

$$\hat{v}(\eta,\vec{x}) = \frac{1}{(2\pi)^{3/2}}\int \mathrm{d}^3k\left(\hat{a}_{\vec{k}}\,v_k(\eta)\,\mathrm{e}^{\mathrm{i}\vec{k}\vec{x}} + \hat{a}^\dagger_{\vec{k}}\,v^*_k(\eta)\,\mathrm{e}^{-\mathrm{i}\vec{k}\vec{x}}\right), \tag{2.27}$$

where the creation and annihilation operators $\hat{a}^\dagger_{\vec{k}}$ and $\hat{a}_{\vec{k}}$ satisfy the equal time commutation relations

$$[\hat{a}_{\vec{k}},\hat{a}^\dagger_{\vec{k}'}] = \delta^{(3)}(\vec{k}-\vec{k}')\,, \qquad [\hat{a}_{\vec{k}},\hat{a}_{\vec{k}'}] = [\hat{a}^\dagger_{\vec{k}},\hat{a}^\dagger_{\vec{k}'}] = 0\,, \tag{2.28}$$

and the complex mode functions $v_k(\eta)$ fulfill the classical EOM in momentum space

$$v''_k + \left(k^2 - \frac{z''}{z}\right)v_k = 0\,. \tag{2.29}$$

Under the assumption of equal time canonical commutation relations for the field and its conjugate momentum, the normalization is given by the Wronskian

$$v_k\,{v'_k}^* - v^*_k\,v'_k = \mathrm{i}\,. \tag{2.30}$$

Since during inflation the curvature scale $\mathcal{H}^{-1}$ is nearly constant, for every mode there is a time in the very early past $\eta \to -\infty$ at which it did not feel the curvature of spacetime. Thus it makes sense to use the well-known Minkowski vacuum $v_k \sim \mathrm{e}^{-\mathrm{i}k\eta}$ in the limit $k|\eta| \gg 1$. From Eq. (2.11), one finds that the scale factor during de Sitter expansion behaves as $a(\eta) = -1/\mathcal{H}\,\eta$. With this knowledge, one can easily check that in the slow-roll approximation $z''/z \sim a''/a \sim 1/\eta^2$. If we plug the Ansatz $v_k = \alpha\,u_k\,\mathrm{e}^{-\mathrm{i}k\eta}$ in Eq. (2.29)

[3] Despite an ambiguity in our notation, it should be obvious to the educated reader if the prime refers to a derivative w.r.t. a scalar field or w.r.t. conformal time.

and use the Wronskian in Eq. (2.30) to find the correct normalization factor α, we end up with the solution, the so-called Bunch–Davies vacuum

$$v_k \simeq \sqrt{\frac{1}{2k}}\,\mathrm{e}^{-\mathrm{i}k\eta}\left(1-\frac{\mathrm{i}}{k\eta}\right)\,. \tag{2.31}$$

Using the definition of the correlation function of a scalar field ϕ in terms of its power spectrum $\mathcal{P}_\phi(k)$ given by

$$\langle 0|\hat{\phi}(\vec{x})\,\hat{\phi}(\vec{y})|0\rangle = \int \mathrm{d}^3k\,\mathrm{e}^{\mathrm{i}\,\vec{k}\,(\vec{x}-\vec{y})}\,\frac{\mathcal{P}_\phi(k)}{4\pi k^3}\,, \tag{2.32}$$

and applying it to the true scalar DOF $\hat{v}$ as defined in Eq. (2.27), we obtain the two power spectra

$$2\,\pi^2\,k^{-3}\,\mathcal{P}_v(k) = |v_k|^2\,, \qquad 2\,\pi^2\,k^{-3}\,\mathcal{P}_\mathcal{R}(k) = \frac{|v_k|^2}{z^2}\,, \tag{2.33}$$

where we have used the fact that the comoving curvature perturbation is related to v through $\mathcal{R} = -v/z$. In terms of the number of e-folds, the curvature perturbation can more conveniently be written as

$$\mathcal{R} = \delta N_\mathrm{e} = \frac{\partial N_\mathrm{e}}{\partial\phi}\,\delta\phi\,. \tag{2.34}$$

Plugging the Bunch-Davies vacuum (2.31) in the super-horizon limit $k|\eta| \ll 1$ into Eq. (2.33), we can find the infamous result for the spectrum of scalar cosmological perturbations generated from quantum fluctuations during slow-roll inflation

$$\mathcal{P}_\mathcal{R}(k) \simeq \frac{1}{4\pi^2}\left(\frac{\mathcal{H}^4}{\dot{\phi}^2}\right)_{k=a\mathcal{H}}\,, \tag{2.35}$$

which is given in terms of cosmic time. This is the generic prediction of a nearly scale-invariant power spectrum since both $\mathcal{H}$ and $\dot{\phi}$ do not significantly evolve during slow-roll and there is no explicit k-dependence. Note however, that $\mathcal{P}_\mathcal{R}$ is evaluated when a relevant scale exits the horizon, i.e., when $k = a\mathcal{H}$. For each scale, this occurs at a different moment, hence a slight scale-dependence of the spectrum remains. With the use of $\mathcal{H}^2 \approx V/3$ and the slow-roll EOM (2.16), one can connect the prediction for the amplitude of the spectrum to functional properties of the fundamental inflaton potential

$$\mathcal{P}_\mathcal{R}^{1/2}(k) \simeq \frac{1}{2\sqrt{3}\,\pi}\left(\frac{V^{3/2}}{|V'|}\right)_{k=a\mathcal{H}}\,. \tag{2.36}$$

Under the simple assumption of a spectrum merely proportional to some power of k, which is parametrized by the spectral index defined as

$$n_\mathrm{s} - 1 \equiv \frac{\mathrm{d}\ln\mathcal{P}_\mathcal{R}(k)}{\mathrm{d}\ln k} = \frac{\mathrm{d}\ln\mathcal{P}_\mathcal{R}(k)}{\mathrm{d}\phi}\,\frac{\dot{\phi}\,\mathrm{d}t}{\mathrm{d}\ln(a\mathcal{H})}\,, \tag{2.37}$$

and using again the slow-roll approximation together with $\mathcal{H} \approx$ const, one can derive the spectral index in terms of the slow-roll parameters (2.17), which reads

$$n_{\mathrm{s}} - 1 \simeq 2\,\eta - 6\,\epsilon\,. \tag{2.38}$$

A similar calculation for the mode decomposition of the tensor perturbations $\bar{E}_{ij}$ in Eq. (2.21) leads to the power spectrum of primordial gravitational waves which has a relative amplitude

$$r = \frac{\mathcal{P}_{\mathcal{T}}(k)}{\mathcal{P}_{\mathcal{R}}(k)} = 16\,\epsilon\,, \tag{2.39}$$

and is dubbed tensor-to-scalar ratio. Since the spectral index of the tensor power spectrum is extremely close to a scale-invariant one in the common models, we will not consider it in our predictions. In addition, its measurement in experiments to a sufficient accuracy is far from accomplishable at the time of writing.

Furthermore, we can also relate the running of the spectral index to the the slow-roll parameters and obtain

$$\frac{\mathrm{d}\,n_{\mathrm{s}}}{\mathrm{d}\ln k} \simeq 16\,\epsilon\,\eta - 24\,\epsilon^2 - 2\,\xi^2\,. \tag{2.40}$$

All of the above predictions have to be evaluated when scales corresponding to the present particle horizon size crossed the curvature scale during inflation, i.e., about 60 e-folds before the end of inflation at ϕ_{i}. In order not to be ruled out, a model of inflation has to predict these quantities in agreement with observations. Let us briefly give the latest results of combined data from WMAP [5], distance measurements from the Baryon Acoustic Oscillations (BAO) in the distribution of galaxies [6] and measurements of the present Hubble parameter ($\mathcal{H}_0$) [7]. The observables are determined to be

$$\begin{aligned} \mathcal{P}_{\mathcal{R}}(k_0) &= \left(2.441^{+0.088}_{-0.092}\right)\cdot 10^{-9}\,,\\ n_{\mathrm{s}} &= 0.963 \pm 0.012\,,\\ r &< 0.24\,, \end{aligned} \tag{2.41}$$

where the amplitude of the curvature power spectrum as well as the spectral index are given at the 68% confidence level (CL) for $k_0 = 0.002\,\mathrm{Mpc}^{-1}$. The upper bound on the tensor-to-scalar ratio however is determined at 95% CL. In addition, upper and lower bounds on the deviation from a power law spectrum are given in terms of the running of the spectral index $-0.061 < \mathrm{d}\,n_{\mathrm{s}}/\mathrm{d}\ln k < 0.017$ at 95% CL. Hopefully the Planck satellite [9] will soon deliver refined measurements of the observables in Eq. (2.41).

2.4 Model Overview

From the derivation of the observable power spectra in the previous section, it should be clear that a model of slow-roll inflation is basically defined through its scalar potential. In the following, we introduce the three established categories of models discussed in the literature.

Small-Field Inflation: The first class of models which has been proposed is known as *new inflation* [21, 22]. These models have their slow-roll trajectory at small field values $\phi_{\mathrm{i}} \ll 1$, close to an unstable maximum of the potential as encountered for the tachyonic directions in spontaneous symmetry breaking (SSB). Generally, the form of the potential is given by $V(\phi) \sim V_0 \cdot (1 - \phi^p)$ and can be interpreted as the lowest order term in a Taylor expansion about the origin. We skip any details of new inflation models, since they are irrelevant for the remainder of this thesis.

Large-Field Inflation: This class of models is called large field models, since they involve classical field values or initial displacements from the potential minimum which are larger than the Planck scale $\phi_{\mathrm{i}} > 1$. The prototype of these models are the so-called *chaotic inflation* models as first introduced in [23]. The name chaotic inflation has been introduced, since in this class of models, the universe is assumed to be subject to chaotic initial conditions

$$\dot{\phi}^2 \simeq \left(\vec{\nabla}\phi\right)^2 \simeq V(\phi) \simeq 1\,, \tag{2.42}$$

around the Planck time.

Generally, the chaotic models have monomial potentials $V(\phi) \sim \phi^p$. Since potentials with $p > 3$ are inconsistent with the observational bounds on r and typically discrete symmetries such as $\mathbb{Z}_2$ forbid odd p, in this work we restrict our focus on quadratic potentials

$$V(\phi) = \frac{1}{2}\, m^2\, \phi^2\,. \tag{2.43}$$

At times, we use the name chaotic inflation interchangeably for the quadratic potentials.

If we plug the potential (2.43) in Eq. (2.19), we can integrate back from ϕ_{e} at which the slow-roll conditions are violated to the field value ϕ_{60}, 60 e-folds before the end of inflation. With the simple quadratic potential at hand, this turns out to be at $\phi_{60} \approx 16$. Thus, with the WMAP normalization on the amplitude in (2.41) together with Eq. (2.36), we can calculate the inflaton mass $m \approx 6 \cdot 10^{-6}\, M_{\mathrm{P}}$. The corresponding scalar potential is displayed in Fig. 2.1.

The generic prediction for the spectral tilt and the tensor-to-scalar ratio calculated from Eqs. (2.38) and (2.39) are

$$n_{\mathrm{s}} \approx 0.97\,, \qquad r \approx 0.13\,. \tag{2.44}$$

As one can see from Eq. (2.41), they are in very good agreement with observational data. A nice feature, which distinguishes the large field models from the small field ones is the prediction of a large gravitational wave contribution r which will be tested by experiments in the near future.

In Ch. 7, we describe chaotic inflation models in a SUGRA context. We show that it is possible to circumvent typical problems of chaotic inflation in SUGRA using symmetry arguments and reproduce the generic results above.

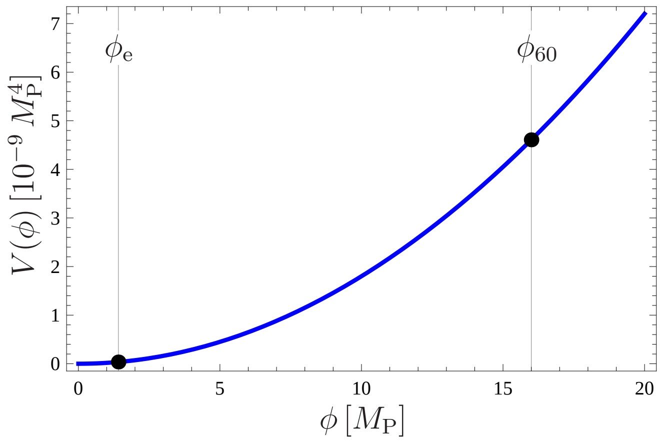

Figure 2.1: Quadratic inflaton potential. ϕ_{60} denotes the field value 60 e-folds before the end of inflation and at ϕ_e, the slow-roll conditions are violated.

Hybrid Inflation: Hybrid inflation is the most recent class of models [36, 37]. The fundamental difference to the aforementioned models is the fact that hybrid inflation involves two scalar fields. In its original realization, the model is given by the effective potential

$$V(\sigma, \phi) = \frac{1}{4\lambda}\left(M^2 - \lambda\,\sigma^2\right)^2 + \frac{1}{2}\,m^2\,\phi^2 + \frac{g^2}{2}\,\phi^2\,\sigma^2\,, \tag{2.45}$$

where ϕ is the slow-rolling inflaton scalar field and σ the so-called waterfall field which provides the mechanism to end inflation. Both M and m are dimensionful parameters, while λ and g are dimensionless coupling constants.

The waterfall field σ has an effective mass squared $m_\sigma^2 = -M^2 + g^2\phi^2$. Due to this fact, it gets stabilized at zero for inflaton field values $\phi > \phi_c$ above some critical value $\phi_c = M/g$. Once the critical field value is reached, the negative mass squared dominates, inducing SSB in which the waterfall field acts as a Higgs field and quickly acquires its global minimum at $\sigma_{\rm min} = \pm M/\sqrt{\lambda}$. The vacuum energy density $V(0,0) = M^4/4\lambda$ that drives inflation disappears in the global minimum $\sigma_{\rm min}$, ending inflation.

Hybrid inflation owes its name to the fact that the slow-rolling inflaton field and the field contributing the large vacuum energy density during inflation are two distinct scalar DOFs, hence a hybrid of two scalar fields. Typically, one chooses $m^2 \ll \mathcal{H}^2 \ll M^2$ in this class of models. We have plotted the hybrid inflation potential of Eq. (2.45) in Fig. 2.2 for $\lambda = g = 1$ and $m = 0.1\,M$. A very intriguing feature of hybrid inflation is the possibility to make a close connection to particle physics models, since the waterfall field can be identified with the Higgs field of some phase transition in the early universe.

Ch. 6 is dedicated to hybrid inflation models in a SUGRA framework. In the global SUSY context [31, 38], hybrid inflation in its most common realizations basically looks like Eq. (2.45) with $m = 0$ and thus an exactly flat inflaton direction at tree-level. The slope driving the inflaton towards its critical value appears through one-loop radiative

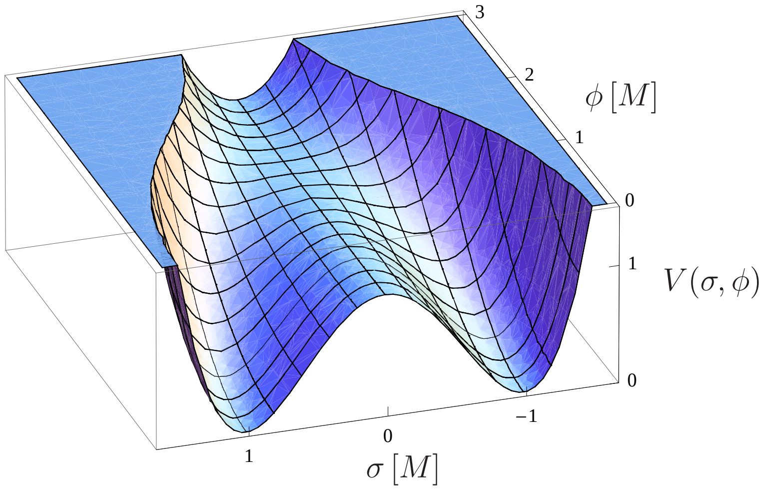

Figure 2.2: Original hybrid inflation potential for parameters $m = 0.1\,M$ and dimensionless couplings $\lambda = g = 1$.

corrections given by the Coleman–Weinberg (CW) [58, 59, 60] effective potential

$$V_{\text{eff}}(\phi) = V_{\text{tree}} + V_{\text{loop}}(\phi)\,, \tag{2.46}$$

where the one-loop correction reads

$$V_{\text{loop}}(\phi) = \frac{1}{64\,\pi^2}\sum_i (-1)^{2s_i}(2s_i+1)\,m_i^4(\phi)\left[\ln\left(\frac{m_i^2(\phi)}{Q^2}\right) - \frac{3}{2}\right]. \tag{2.47}$$

The sum in Eq. (2.47) is taken over all bosonic as well as fermionic DOFs i with spins s_i and mass eigenvalues m_i. Q denotes the renormalization scale.

In Part IV we discuss a variant of hybrid inflation, where the waterfall field is the Higgs field breaking Pati–Salam to the SM gauge group.

Chapter 3

Supersymmetry and Supergravity

Supersymmetry is a symmetry between bosonic and fermionic DOFs. In the context of particle physics, it proves very useful since it can cure [61, 62] problematic quadratic dependencies of the Higgs mass on any scale of new physics. This problem of the SM is usually referred to as the *hierarchy problem* [63, 64, 65, 66]. Also on the cosmology side, SUSY is a welcome stool. From the perspective of an inflation model builder, SUSY has the appealing property of inevitably providing us with a sufficient amount of fundamental scalar fields, some of which can play the role of the inflaton particle. In addition, SUSY extensions of the SM can account for the large amount of DM in our universe [67] which has no explanation within the SM.

Since cosmology inherently has to invoke a theory of gravity, local SUSY or SUGRA becomes very interesting. In addition to the desirable features mentioned above, the spin-3/2 gravitino as the gauge field of SUGRA transformations has a superpartner spin-2 tensor field that can be identified with the metric tensor. Thus local SUSY automatically engages a theory of gravity and is therefore the perfect playground for people seeking for connections between particle physics and cosmology. Especially for models of inflation which can typically involve energy scales close to the fundamental gravity scale M_{P}, SUGRA effects cannot be neglected.

In this chapter, we want to introduce the main concepts of SUSY and SUGRA and highlight their important aspects with regard to model building. Most of this chapter is based on Refs. [68, 69, 70, 71, 72]. Sec. 3.1 is dedicated to the construction of globally supersymmetric Lagrangians using the concept of superpotentials. The following Sec. 3.2 then generalizes to local SUSY with a focus on the relevant parts, for our purposes, of the rather lengthy SUGRA Lagrangian. Due to the fact that for an understanding of the calculations within this thesis, the elegant but rather involved superfield and superspace formalism [73, 74] is not necessary, we have decided to keep matters as simple as possible and build up the theory from simple SUSY invariant Lagrangians in four dimensions as in Ref. [68]. For a pedagogical introduction to the superfield formalism, the ambitious reader is referred to [69].

3.1 Globally Supersymmetric Lagrangians

As mentioned in the introduction to this chapter, the hierarchy problem and its solution within SUSY theories is one of the main motivations for extending the SM by SUSY. Consider a Lagrangian which contains a Yukawa coupling of some complex scalar field h to some Dirac fermion F and a coupling to some other complex scalar degree of freedom S given by

$$\mathcal{L} = \lambda_F \, h \, \bar{F} \, F + \lambda_S \, |h|^2 |S|^2 \,. \tag{3.1}$$

Here, λ_F and λ_S denote dimensionless coupling parameters. The terms in the Lagrangian (3.1) induce squared masses for the fermion F and the scalar S

$$m_F^2 = |\lambda_F|^2 \, |h|^2, \qquad m_S^2 = \lambda_S \, |h|^2 \,, \tag{3.2}$$

where h is assumed to be a complex scalar field. This induces one-loop radiative corrections to the scalar potential $V_{\text{1-loop}} \sim \Lambda^2(m_S^2 - 2\, m_F^2)/32\, \pi^2$ which are quadratically dependent on the cutoff scale Λ. In terms of mass corrections of the h field, this reads

$$\Delta m_h^2 = \frac{\Lambda^2}{16\, \pi^2} \left(\lambda_S - 2\, |\lambda_F|^2 \right) \,. \tag{3.3}$$

For any new physics entering at a high scale Λ, the hierarchy problem immediately becomes obvious. If you think of h as being the SM Higgs field and F as being the SM fermions, a correction as in Eq. (3.3) results in a very strong sensitivity of the Higgs mass to physics at any scale Λ. This spoils the hierarchy between the electroweak scale Higgs mass and the high scale Λ and is thus dubbed hierarchy problem.

An obvious solution to this problem can be a relation between fermionic and bosonic degrees of freedom in such a way that for every Dirac fermion F there are two complex scalars S with exactly the same masses $m_F = m_S$, or in other words $\lambda_S = |\lambda_F|^2$. In this case, the corrections from fermions and bosons in Eq. (3.3) cancel and the problem is resolved. SUSY is a spacetime symmetry which transforms bosonic states into fermionic states and vice versa with equivalent masses and thus gives the aforementioned relations. Due to the fact that the SUSY transformation changes the spin of the state, its generator $\hat{Q}$ has to be a spinor operator and schematically, it transforms the states as

$$\hat{Q}\, |\text{Boson}\rangle = |\text{Fermion}\rangle \,, \qquad \hat{Q}\, |\text{Fermion}\rangle = |\text{Boson}\rangle \,. \tag{3.4}$$

Being spin-1/2 operators, $\hat{Q}$ and its conjugate $\hat{Q}^\dagger$ generate a spacetime symmetry and this is the reason for them extending the Poincaré algebra in the following anticommuting way

$$\begin{aligned} \{\hat{Q}_\alpha, \hat{Q}_\beta\} &= \{\hat{Q}^\dagger_{\dot{\alpha}}, \hat{Q}^\dagger_{\dot{\beta}}\} = 0 \,, \\ \{\hat{Q}_\alpha, \hat{Q}^\dagger_{\dot{\alpha}}\} &= 2\, \sigma^\mu_{\alpha\dot{\alpha}}\, \hat{P}_\mu \,. \end{aligned} \tag{3.5}$$

Note that here $\hat{Q}$ and $\hat{Q}^\dagger$ are assumed to be anticommuting two component Weyl spinor operators with $\alpha, \dot{\alpha} = 1, 2$, cf. App. A. Their commutation relations with the generators of the Poincaré algebra are given by

$$\begin{aligned}
[\hat{Q}_\alpha, \hat{P}_\mu] &= [\hat{Q}^\dagger_{\dot{\alpha}}, \hat{P}_\mu] = 0\,, \\
[\hat{Q}_\alpha, \hat{S}_{\mu\nu}] &= -\frac{1}{2}\, \left(\sigma_{\mu\nu}\right)_\alpha^{\ \beta}\, \hat{Q}_\beta\,, \\
[\hat{Q}^\dagger_{\dot{\alpha}}, \hat{S}_{\mu\nu}] &= -\frac{1}{2}\, \left(\bar{\sigma}_{\mu\nu}\right)_{\dot{\alpha}}^{\ \dot{\beta}}\, \hat{Q}^\dagger_{\dot{\beta}}\,,
\end{aligned} \tag{3.6}$$

where $\hat{P}_\mu$ denotes the four-momentum generator of spacetime translations and $\hat{S}_{\mu\nu}$ the generators of Lorentz transformations defined in App. A. The mass degeneracy of the single-particle states within one such supermultiplet is a simple consequence of the commutation of the SUSY generators with the four-momentum generator as given in Eq. (3.6), since it immediately implies $\hat{M}^2\,|\text{Boson}\rangle = \hat{P}^\mu \hat{P}_\mu\,|\text{Boson}\rangle = \hat{M}^2\,|\text{Fermion}\rangle$. In addition to that, the SUSY generators also commute with the generators of gauge transformations. Therefore, the so-called superpartners within one supermultiplet also carry the same quantum numbers under the gauge group.

It can be proven that within each supermultiplet, the number of bosonic DOFs identically matches the number of fermionic DOFs. For unextended SUSY[1] this gives rise to several phenomenologically interesting supermultiplets.

The simplest possibility is the *chiral supermultiplet*. It contains one single complex scalar and a corresponding Weyl fermion, combining spin-0 and spin-1/2 states. To make the DOFs match, an auxiliary complex scalar field is added. Such chiral supermultiplets are suitable e.g. for accounting for the "matter" fermions and their superpartners within the minimal supersymmetric SM (MSSM).

Furthermore, it is important to include spin-1 vector bosons in the theory if one wants to describe gauge interactions. It turns out that the only renormalizable way to do this is a supermultiplet containing a spin-1/2 Weyl fermion called the gaugino and a spin-1 gauge boson. This is called a *vector supermultiplet* and it includes a real scalar auxiliary field to match the bosonic and fermionic DOFs.[2]

Another possibility is to combine a spin-3/2 fermion and a massless spin-2 boson within the same supermultiplet. As we will explain in Sec. 3.2, such a multiplet arises automatically in SUGRA theories, where the so-called spin-3/2 gravitino is the gauge field of local SUSY transformations and has a spin-2 superpartner, the graviton. Since in this section we are concerned with global SUSY only, we postpone SUGRA to the following section.

In Sec. 3.1.1 we first discuss the simple non-interacting chiral supermultiplet and subsequently generalize to the phenomenologically more interesting interacting case. Sec. 3.1.2 is dedicated to the vector multiplet and the possible interactions in SUSY gauge theories. Since SUSY breaking is especially important in the context of SUSY inflation models, we discuss spontaneous SUSY breaking in Sec. 3.1.3.

[1]In unextended or $\mathcal{N} = 1$ SUSY, there is only one set of SUSY generators $\hat{Q}_\alpha$ and $\hat{Q}^\dagger_{\dot{\alpha}}$. Throughout this work, we will only consider unextended SUSY and SUGRA in $D = 4$ spacetime dimensions.

[2]At least in the massless case where the gauge symmetry remains unbroken.

3.1.1 Chiral Supermultiplets

The simplest possibility of a supermultiplet is one two-component Weyl fermion χ in combination with one complex scalar ϕ which transform into each others under SUSY transformations. Throughout the course of this chapter, we stick to the spinor conventions introduced in App. A.

First of all, let us look at a free field theory which contains only the kinetic terms

$$\mathcal{L}_\mathrm{S} = \partial^\mu \phi^* \, \partial_\mu \phi \,, \qquad \mathcal{L}_\mathrm{F} = \mathrm{i}\, \chi^\dagger \bar{\sigma}^\mu \partial_\mu \chi \,. \tag{3.7}$$

This is called the massless, non-interacting Wess–Zumino model [14]. Its action is given by

$$S = \int \mathrm{d}^4 x \left(\mathcal{L}_\mathrm{S} + \mathcal{L}_\mathrm{F} \right) , \tag{3.8}$$

and, as we show below, a simple set of SUSY transformations under which $\delta S = 0$ reads

$$\delta\phi = \varepsilon^\alpha \chi_\alpha \,, \qquad \delta\chi_\alpha = \mathrm{i} \left(\sigma^\mu \varepsilon^\dagger \right)_\alpha \partial_\mu \phi \,. \tag{3.9}$$

The parameter ε in Eq. (3.9) is a constant, infinitesimal Weyl fermion object which parametrizes the SUSY transformation. Using the above SUSY transformations on the fields, the variations of the scalar and fermionic part of the Lagrangian density read respectively

$$\begin{aligned} \delta\mathcal{L}_\mathrm{S} =& \, \varepsilon\, (\partial^\mu \chi)(\partial_\mu \phi^*) + \varepsilon^\dagger (\partial^\mu \chi^\dagger)(\partial_\mu \phi) \,, \\ \delta\mathcal{L}_\mathrm{F} =& - \varepsilon\, (\partial^\mu \chi)(\partial_\mu \phi^*) - \varepsilon^\dagger \, (\partial^\mu \chi^\dagger)(\partial_\mu \phi) \\ & + \partial_\mu \left[\varepsilon\, \sigma^\nu \bar{\sigma}^\mu \chi \, (\partial_\nu \phi^*) + \varepsilon\, \chi \, (\partial^\mu \phi^*) + \varepsilon^\dagger \chi^\dagger \, (\partial^\mu \phi) \right] \,, \end{aligned} \tag{3.10}$$

where to avoid cluttering up the notation, we have suppressed spinor indices. To arrive at the form for the fermionic Lagrangian variation in Eq. (3.10) one has to apply some of the Pauli matrix identities given in App. A. The second line of the fermionic variation is a total derivative which is just a boundary term in the variation of the action which can be set to zero. The remaining terms cancel against the ones from the scalar part and we have confirmed that the action is invariant under the SUSY transformations Eq. (3.9) such that $\delta S = 0$.

However, so far the Poincaré algebra extended by SUSY closes on-shell only. This can be seen by looking at the commutator of two SUSY transformations on the fields. For the scalar field, it is given by

$$\left[\delta_{\varepsilon_2}, \delta_{\varepsilon_1} \right] \phi = \mathrm{i} \left(\varepsilon_1 \, \sigma^\mu \, \varepsilon_2^\dagger - \varepsilon_2 \, \sigma^\mu \, \varepsilon_1^\dagger \right) \partial_\mu \phi \,, \tag{3.11}$$

and here indeed, we obtain a result proportional to the generator of spacetime translations ∂_μ which is of a similar structure as the anti-commutator (3.5). For the Weyl fermion field, the situation is a little less satisfying. The commutator of two SUSY transformations after a little algebraic gymnastics reads

$$\left[\delta_{\varepsilon_2}, \delta_{\varepsilon_1} \right] \chi_\alpha = \mathrm{i} \left(\varepsilon_1 \, \sigma^\mu \, \varepsilon_2^\dagger - \varepsilon_2 \, \sigma^\mu \, \varepsilon_1^\dagger \right) \partial_\mu \chi_\alpha - \mathrm{i}\, \varepsilon_{1\alpha} \left(\varepsilon_2 \, \bar{\sigma}^\mu \, \partial_\mu \chi \right) + \mathrm{i}\, \varepsilon_{2\alpha} \left(\varepsilon_1 \, \bar{\sigma}^\mu \, \partial_\mu \chi \right) . \tag{3.12}$$

In Eq. (3.12) the last two terms, which spoil resulting in the same spacetime translation as in the scalar case, are problematic. On-shell however, if the classical EOM $\bar{\sigma}^\mu\,\partial_\mu\chi = 0$ holds for the fermionic DOFs, these terms vanish identically and closure of the algebra is ensured.

Dealing with quantum phenomena, we have to require that the algebra closes off-shell. Indeed this is possible by a simple trick. The reason for why our simplistic approach failed is due to the fact that in our supermultiplet (ϕ, χ) so far, the numbers of scalar and fermionic DOFs are not the same. We have two scalar DOFs from the complex scalar but all in all four DOFs from the two complex Weyl spinor components. The trick to make things work off-shell is to add the missing scalar DOFs to the supermultiplet in form of a so-called auxiliary complex scalar field F which is non-propagating. Hence, the supermultiplet now consists of (ϕ, χ, F) and the Lagrangian density of the auxiliary field is just given by

$$\mathcal{L}_{\mathrm{aux}} = F^* F\,, \tag{3.13}$$

implying the trivial EOM $F = F^* = 0$.

In order to maintain SUSY invariance of the action, the field transformations have to be adjusted in an adequate way, giving rise to the new SUSY transformations

$$\delta\phi = \varepsilon^\alpha \chi_\alpha\,, \qquad \delta\chi_\alpha = \mathrm{i}\left(\sigma^\mu \varepsilon^\dagger\right)_\alpha \partial_\mu\phi + \varepsilon_\alpha\, F\,, \qquad \delta F = \mathrm{i}\,\varepsilon^\dagger\,\bar{\sigma}^\mu\,\partial_\mu\chi\,. \tag{3.14}$$

In Eq. (3.14), the additional term in $\delta\chi_\alpha$ is now responsible for an exact cancellation between the new terms in $\delta\mathcal{L}_{\mathrm{F}}$ and

$$\delta\mathcal{L}_{\mathrm{aux}} = \mathrm{i}\,\varepsilon^\dagger\,\bar{\sigma}^\mu\,(\partial_\mu\chi)\,F^* - \mathrm{i}\,(\partial_\mu\chi^\dagger)\,\bar{\sigma}^\mu\,\varepsilon\,F\,. \tag{3.15}$$

In fact, it can be shown that SUSY invariance of the full action holds according to

$$\delta S = \int \mathrm{d}^4x\,(\delta\mathcal{L}_{\mathrm{S}} + \delta\mathcal{L}_{\mathrm{F}} + \delta\mathcal{L}_{\mathrm{aux}}) = 0\,, \tag{3.16}$$

and in addition, the algebra closes even without imposing the classical EOMs, such that for each of the supermultiplet components $\Phi = (\phi, \chi, F)$, the commutator of two SUSY transformations gives

$$[\delta_{\varepsilon_2}, \delta_{\varepsilon_1}]\,\Phi = \mathrm{i}\left(\varepsilon_1\,\sigma^\mu\,\varepsilon_2^\dagger - \varepsilon_2\,\sigma^\mu\,\varepsilon_1^\dagger\right)\partial_\mu\Phi\,, \tag{3.17}$$

and similarly for their conjugates $\Phi^\dagger = (\phi^*, \chi^\dagger, F^*)$.

In conclusion, we have shown that the Lagrangian density of a free theory containing the chiral supermultiplet $\Phi = (\phi, \chi, F)$

$$\mathcal{L}_{\mathrm{chiral}}^{(\mathrm{free})} = \partial^\mu\phi^*\,\partial_\mu\phi + \mathrm{i}\,\chi^\dagger\bar{\sigma}^\mu\partial_\mu\chi + F^*F\,, \tag{3.18}$$

is invariant under the SUSY transformations Eq. (3.14) and the SUSY extended Poincaré algebra closes classically as well as for quantum fields.

Let us now take the logical next step and try to extend this rather unspectacular non-interacting theory to a SUSY theory which accounts for Yukawa couplings and self-interactions generated by interactions between different chiral supermultiplets. As it will turn out, all these interactions in a SUSY theory can be written very conveniently just in terms of one simple holomorphic function $W(\phi_i)$ of the scalar components of the different chiral supermultiplets $\Phi_i = (\phi_i, \chi_i, F_i)$ labeled by i. This function is called the superpotential and plays an essential role in the further course of the thesis.

The most general Lagrangian which is renormalizable and compatible with the SUSY transformations (3.14) for each supermultiplet separately has one free part as before

$$\mathcal{L}_{\text{chiral}}^{(\text{free})} = \partial^\mu \phi^{*i}\, \partial_\mu \phi_i + \mathrm{i}\, \chi^{\dagger i} \bar{\sigma}^\mu \partial_\mu \chi_i + F^{*i} F_i\,, \tag{3.19}$$

and an interaction part given by

$$\mathcal{L}_{\text{chiral}}^{(\text{int})} = \left(-\frac{1}{2}\, W^{ij}\, \chi_i\, \chi_j + W^i\, F_i \right) + \text{h.c.}\,. \tag{3.20}$$

Up to this point, W^i and W^{ij} are merely coefficients. From the SUSY invariance of the part in $\delta\mathcal{L}_{\text{chiral}}^{(\text{int})}$ that contains four spinors, one can conclude that $W^{ij} = M^{ij} + y^{ijk}\phi_k$ is analytic and contains the symmetric fermion mass matrix M^{ij} and the Yukawa couplings y^{ijk} which are totally symmetric under interchange of i, j, k. Thus, we can write it as a second derivative

$$W^{ij} = \frac{\delta W}{\delta\phi_i \delta\phi_j}\,, \tag{3.21}$$

where the function W is our foreshadowed superpotential and is usually given by

$$W = \frac{1}{2}\, M^{ij}\, \phi_i\, \phi_j + \frac{1}{6}\, y^{ijk}\, \phi_i\, \phi_j\, \phi_k\,. \tag{3.22}$$

Looking at the the part of $\delta\mathcal{L}_{\text{chiral}}^{(\text{int})}$ containing spacetime derivatives

$$\delta\mathcal{L}_{\text{chiral}}^{(\text{int})}|_{\partial_\mu} = -\mathrm{i}\left[W^{ij}\,(\partial_\mu \phi_j)\, \chi_i + W^i \partial_\mu \chi_i\right]_\alpha (\sigma^\mu \varepsilon^\dagger)^\alpha - \text{h.c.}\,, \tag{3.23}$$

and plugging in (3.21), the expression in squared brackets can only be a total derivative, contributing a boundary term in the variation of the action, if

$$W^i = \frac{\delta W}{\delta \phi_i}\,. \tag{3.24}$$

All remaining terms in $\delta\mathcal{L}_{\text{chiral}}^{(\text{int})}$ cancel given the previous results for W^i and W^{ij}. It is possible to eliminate the auxiliary DOFs by imposing their simple EOMs, taken from the full Lagrangian $\mathcal{L}_{\text{chiral}}^{(\text{free})} + \mathcal{L}_{\text{chiral}}^{(\text{int})}$, which are given by

$$F_i = -W_i^*\,, \qquad F^{*i} = -W^i\,. \tag{3.25}$$

This allows us to write down the full Lagrangian for a theory of interacting chiral supermultiplets in terms of the derivatives of the superpotential only:

$$\mathcal{L}_{\text{chiral}} = \partial^\mu \phi^{*i} \partial_\mu \phi + \mathrm{i}\, \chi^{\dagger i} \bar{\sigma}^\mu \partial_\mu \chi_i - \frac{1}{2} \left(W^{ij} \chi_i \chi_j + W^*_{ij} \chi^{\dagger i} \chi^{\dagger j} \right) - W^i W^*_i \,. \tag{3.26}$$

In summary, all matter-like interactions between chiral supermultiplets $\Phi_i = (\phi_i, \chi_i, F_i)$ in a global SUSY theory can be accounted for by the analytic superpotential which gives rise to the SUSY Lagrangian Eq. (3.26).

The most general renormalizable superpotential is actually not necessarily of the form as in Eq. (3.22) but in addition can contain a linear and a constant term and thus reads

$$W = W_0 + L^i \, \Phi_i + \frac{1}{2} M^{ij} \, \Phi_i \, \Phi_j + \frac{1}{6} y^{ijk} \, \Phi_i \, \Phi_j \, \Phi_k \,. \tag{3.27}$$

The nature of the linear term with the dimensionful coupling L_i in Eq. (3.27) can give rise to spontaneous SUSY breaking which is discussed in Sec. 3.1.3. The constant contribution W_0 has mass dimension three. Note that we have written W as a function of the supermultiplets or *superfields* Φ_i, which is justified by the superfield formalism [73, 74]. However, it can just be treated as a function of the complex scalar components ϕ_i contained in the respective multiplet.

3.1.2 Vector Supermultiplets

A vector supermultiplet contains the propagating DOFs of a gauge boson A^a_μ and the superpartner Weyl fermion which is called gaugino λ^a. Here, the index a runs over the adjoint representation of the gauge group under consideration. Similar as for the chiral supermultiplet, this is not enough for an off-shell formulation since the three real DOFs of the gauge boson field do not match the four real DOFs of the gaugino field. Hence, we introduce the additional real scalar auxiliary field D^a to make the theory manifestly SUSY invariant on the quantum level.

Using this knowledge, we can write down the Lagrangian density for the non-interacting vector supermultiplet $V^a = (A^a_\mu, \lambda^a, D^a)$, given by

$$\mathcal{L}_{\text{vector}} = -\frac{1}{4} F^a_{\mu\nu} \, F^{a\mu\nu} + \mathrm{i}\, \lambda^{\dagger a} \, \bar{\sigma}^\mu \, D_\mu \lambda^a + \frac{1}{2} D^a D^a \,, \tag{3.28}$$

where we have introduced the Yang-Mills field strength tensor $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\, f^{abc} A^b_\mu A^c_\nu$ and the covariant derivative of the gaugino field $D_\mu \lambda^a = \partial_\mu \lambda^a + g\, f^{abc} A^b_\mu \, \lambda^c$. The gauge coupling is denoted by[3] g and the totally antisymmetric f^{abc} are the structure constants of the gauge group. With the gaugino included, we obtain extended gauge transformations in the SUSY case given by

$$\begin{aligned} \delta_{\text{gauge}}\, A^a_\mu &= \partial_\mu \Lambda^a + g\, f^{abc}\, A^b_\mu \, \Lambda^c \,, \\ \delta_{\text{gauge}}\, \lambda^a &= g\, f^{abc} \lambda^b \, \Lambda^c \,, \end{aligned} \tag{3.29}$$

[3]Note, that the definition of g is ambiguous since in the context of particle physics it denotes the gauge coupling constants whereas in the cosmology context, we have introduced it as the determinant of the metric tensor. However, if not noted explicitly the distinction should always be clear.

where Λ^a is the infinitesimal gauge transformation parameter.

Analogous to the treatment in Sec. 3.1.1, one can show that the Lagrangian density (3.28) is invariant under the following SUSY transformations of the fields:

$$\begin{aligned} \delta A^a_\mu &= \frac{1}{\sqrt{2}} \left(\varepsilon^\dagger \, \bar{\sigma}_\mu \, \lambda^a + \lambda^{\dagger a} \, \bar{\sigma}_\mu \, \varepsilon \right) , \\ \delta \lambda^a_\alpha &= \frac{\mathrm{i}}{2\sqrt{2}} \left(\sigma^\mu \bar{\sigma}^\nu \, \varepsilon \right)_\alpha + \frac{1}{\sqrt{2}} \, \varepsilon_\alpha \, D^a , \\ \delta D^a &= \frac{\mathrm{i}}{\sqrt{2}} \left(\varepsilon^\dagger \, \bar{\sigma}^\mu \, D_\mu \lambda^a - D_\mu \lambda^{\dagger a} \, \bar{\sigma}^\mu \, \varepsilon \right) . \end{aligned} \tag{3.30}$$

Furthermore, with the use of Eq. (3.28), we can also check that the SUSY algebra closes off-shell such that for any component $V^a = (A^a_\mu, \lambda^a, D^a)$ within the vector supermultiplet

$$[\delta_{\varepsilon_2}, \delta_{\varepsilon_1}] \, V^a = \mathrm{i} \left(\varepsilon_1 \, \sigma^\mu \, \varepsilon_2^\dagger - \varepsilon_2 \, \sigma^\mu \, \varepsilon_1^\dagger \right) \, D_\mu V^a \, . \tag{3.31}$$

Again, this is only possible by introducing the auxiliary field D^a which has a trivial EOM $D^a = 0$ in the non-interacting case at hand.

Turning on couplings between chiral supermultiplets Φ_i describing matter fields with their superpartners and vector supermultiplets V^a describing gauge bosons and their according superpartners is our next step. Since SUSY and gauge transformations commute, all component fields within the chiral supermultiplet Φ_i must be contained in the same representation of the gauge group and therefore transform in the same way

$$\delta_{\text{gauge}} \, \Phi_i = \mathrm{i} \, g \, \Lambda^a \left(\mathcal{T}^a \, \Phi \right)_i \, , \tag{3.32}$$

under gauge transformations. The $\mathcal{T}^a$ are the generators of the gauge group.

In order to obtain a Lagrangian which is invariant under gauge transformations, as usual we replace all spacetime derivatives by their gauge-covariant counterpart and hence

$$D_\mu \phi_i = \partial_\mu \phi_i - \mathrm{i} \, g \, A^a_\mu \left(\mathcal{T}^a \, \phi \right)_i \, , \qquad D_\mu \chi_i = \partial_\mu \chi_i - \mathrm{i} \, g \, A^a_\mu \left(\mathcal{T}^a \, \chi \right)_i \, . \tag{3.33}$$

Within the Lagrangian density, Eq. (3.33) induces couplings between the vector bosons and the superpartners of the chiral supermultiplets.

But this is not the whole story. Since there are additional component fields in the vector supermultiplet, we also have to include all other terms allowed by gauge-invariance. As it turns out, the three possibilities that are renormalizable read

$$\mathcal{L}^{\text{chiral}}_{\text{vector}} = -\sqrt{2} \, g \left(\phi^* \, \mathcal{T}^a \, \chi \right) \lambda^a - \sqrt{2} \, g \, \lambda^{\dagger a} \left(\chi^\dagger \, \mathcal{T}^a \, \phi \right) + g \left(\phi^* \, \mathcal{T}^a \, \phi \right) D^a \, , \tag{3.34}$$

and contain interactions between D^a or the gauginos λ^a with the scalars ϕ_i and the Weyl fermions χ_i. Due to the fact that they are gauge interactions, all of them couple with strength g. The coefficients are chosen for convenience, such that the SUSY field transformations of the chiral supermultiplets change in the following way

$$\begin{aligned} \delta \phi_i &= \varepsilon \, \chi_i \, , \\ \delta \chi_{i\alpha} &= \mathrm{i} \left(\sigma^\mu \, \varepsilon^\dagger \right)_\alpha D_\mu \phi_i + \varepsilon_\alpha F_i \, , \\ \delta F_i &= \mathrm{i} \, \varepsilon^\dagger \, \bar{\sigma}^\mu \, D_\mu \chi_i + \sqrt{2} \, g \left(\mathcal{T}^a \, \phi \right)_i \varepsilon^\dagger \lambda^{\dagger a} \, . \end{aligned} \tag{3.35}$$

Substituting the modified EOM for the auxiliary field $D^a = -g\,(\phi^*\,\mathcal{T}^a\,\phi)$, we obtain the complete Lagrangian density of a global SUSY gauge theory containing chiral as well as vector supermultiplets

$$\begin{aligned}\mathcal{L}_{\text{SUSY}} = & (D^\mu\phi^i)^*(D_\mu\phi_i) + \mathrm{i}\,\chi^{\dagger i}\bar{\sigma}^\mu D_\mu\chi_i - \frac{1}{4}\,F^a_{\mu\nu}F^{a\mu\nu} + \mathrm{i}\,\lambda^{\dagger a}\bar{\sigma}^\mu D_\mu\lambda^a \\ & - \sqrt{2}\,g\left[\,(\phi^*\mathcal{T}^a\chi)\,\lambda^a + \lambda^{\dagger a}\left(\chi^\dagger\mathcal{T}^a\phi\right)\right] \\ & - \frac{1}{2}\left(W^{ij}\,\chi_i\,\chi_j + W^*_{ij}\,\chi^{\dagger i}\chi^{\dagger j}\right) \\ & - W^i W^*_i - \frac{1}{2}\sum_a g_a^2\,(\phi^*\mathcal{T}^a\phi)^2\ ,\end{aligned} \tag{3.36}$$

where the first line contains kinetic terms and gauge interactions from the covariant derivatives, the second line is responsible for couplings of gauginos to the scalars and fermions within the chiral supermultiplets, line three gives rise to direct fermion masses and Yukawa couplings and last but not least the tree-level SUSY scalar potential is given in line four.

The SUSY and gauge invariant Lagrangian (3.36) is the main result of this section. Concerning SUSY breaking and inflationary model building, the scalar potential in the last line plays an essential role. In fact, inflation and SUSY breaking in the early universe are closely related since the vacuum energy density driving inflation necessarily breaks SUSY spontaneously. We discuss the concept and different mechanisms of spontaneous SUSY breaking in some detail in the next section.

3.1.3 Spontaneous Breaking of Supersymmetry

The SM describes particle interactions at energies up to the electroweak scale very accurately. Within this energy range, no superpartners of the SM particles have been experimentally observed. Therefore, if our spacetime obeys SUSY at high scales, it has to be broken at the low scales observed so far. However, also during a phase of inflation which requires a large vacuum energy density, SUSY breaking is inevitable as we explain in this section.

By definition, broken SUSY means that the true vacuum state $|0\rangle$ is not invariant under the action of the SUSY generators, i.e. $\hat{Q}\,|0\rangle \neq 0$ and $\hat{Q}^\dagger\,|0\rangle \neq 0$. Exploiting Eq. (3.5), one can easily see that this gives rise to a non-vanishing vacuum energy

$$\langle 0|V|0\rangle = \langle 0|P_0|0\rangle = \frac{1}{4}\,\mathrm{Tr}\,\langle 0|\{\hat{Q}_\alpha, \hat{Q}^\dagger_{\dot{\alpha}}\}|0\rangle > 0\,, \tag{3.37}$$

where we have assumed that spacetime-dependent effects and fermion condensates are negligible w.r.t. the scalar potential V and that the Hilbert space has a positive norm.

From Eq. (3.36), we know that the scalar potential of a SUSY gauge theory is completely given in terms of

$$V(\phi_i, \phi_i^*) = V_F + V_D = W_i^* W^i + \frac{1}{2}\sum_a g_a^2\,(\phi^*\mathcal{T}^a\phi)^2\ . \tag{3.38}$$

The first term in Eq. (3.38) is called the *F-term* contribution to the scalar potential since it arises from the EOMs of the auxiliary fields F_i of the chiral supermultiplets. Originating from the auxiliary fields D^a of the vector supermultiplets, the second term is the so-called *D-term* contribution which vanishes in the absence of gauge interactions. Obviously, in order to break SUSY either $F_i \neq 0$ or $D^a \neq 0$ or both have to be satisfied.

As for the spontaneous breaking of any global symmetry, there is a massless Nambu–Goldstone mode related to the spontaneous breaking of global SUSY. It carries the same quantum numbers as the broken symmetry generator. In the case of global SUSY, this is the fermionic charge generator $\hat{Q}_\alpha$. Therefore, the Nambu–Goldstone particle is a neutral Weyl fermion referred to as *goldstino*.

First of all, let us consider D-term SUSY breaking. The mechanism responsible for this is the *Fayet–Iliopoulos* mechanism [75, 76]. If the gauge group under consideration includes a $U(1)$ factor, the symmetries allow for an additional term in the Lagrangian linear in the auxiliary field belonging to the $U(1)$, namely $\mathcal{L}_{\mathrm{FI}} = -\xi D$, which gives rise to the D-term scalar potential

$$V_D = \xi\, D - \frac{1}{2}\, D^2 - g\, D\, q_i |\phi_i|^2 \,. \tag{3.39}$$

Here, the q_i are the $U(1)$ charges of the fields ϕ_i. When one solves Eq. (3.39) for the EOMs, the auxiliary field can be eliminated by

$$D = \xi - g\, q_i |\phi_i|^2 \,. \tag{3.40}$$

Assuming SUSY conserving F-term masses m_i for the chiral supermultiplets Φ_i, the full scalar potential reads

$$V = |m_i|^2 |\phi_i|^2 + \frac{1}{2}\left(\xi - g\, q_i |\phi_i|^2\right)^2 \,, \tag{3.41}$$

which cannot identically vanish. In the simplest case $\langle \phi_i \rangle = 0$ where the gauge symmetry is unbroken, we obtain a non-zero D-term $D = \xi$ and the vacuum energy density that breaks SUSY is given by

$$V_D = \frac{1}{2}\, \xi^2 \,. \tag{3.42}$$

Therefore, the SUSY breaking scale $\sqrt{\xi}$ determines the mass splittings within the chiral supermultiplets. This is obvious when looking at the effective squared masses from Eq. (3.41) of the ϕ_i scalars given by $|m_i|^2 - g\, \xi\, q_i$, while the fermionic superpartner squared masses remain $|m_i|^2$. In the simple case just described, the massless goldstino can be identified with the gaugino.

The other prominent mechanism of SUSY breaking is realized by non-vanishing F-term VEVs. It is referred to as *O'Raifeartaigh* models [77]. Basically, these models are realized by a set of chiral superfields Φ_i with a superpotential that does not allow for all auxiliary field EOMs giving $F^i = 0$ simultaneously. In Eq. (3.27) we have introduced the most general renormalizable superpotential including a linear term in the superfields. As it turns out, such a linear term is necessary to realize F-term SUSY breaking in a renormalizable theory to obtain a non-vanishing vacuum energy at the minimum. The O'Raifeartaigh model in its most simple form reads

$$W = \kappa\, \Phi_1 \left(\Phi_3^2 - \mu^2\right) + m\, \Phi_2 \Phi_3 \,, \tag{3.43}$$

from which we can calculate the F-terms of the three chiral supermultiplets

$$F_1^* = -\kappa \left(\phi_3^2 - \mu^2\right) \,, \qquad F_2^* = -m\, \phi_3 \,, \qquad F_3^* = -m\, \phi_2 - 2\,\kappa\, \phi_1 \phi_3 \,. \tag{3.44}$$

The first two F-terms in Eq. (3.44) are incompatible with $F_1^* = F_2^* = 0$ and hence, they break SUSY. Looking at the F-term contribution to the scalar potential

$$V_F = \left|\kappa \left(\phi_3^2 - \mu^2\right)\right|^2 + \left|m\, \phi_3\right|^2 + \left|m\, \phi_2 + 2\,\kappa\, \phi_1 \phi_3\right|^2 \,, \tag{3.45}$$

and assuming μ, m and κ are real and positive, for $m^2 > 2\,\kappa^2 \mu^2$ the global minimum of the potential lies at $\langle \phi_2 \rangle = \langle \phi_3 \rangle = 0$ with $\langle \phi_1 \rangle$ undetermined at tree-level. Note that this is an example of a flat direction common in SUSY theories. For such a flat direction, there is no tree-level mass in the global minimum of the potential.

A mass is only generated at the loop-level, the flat direction is said to be lifted by quantum corrections. Indeed, as we discuss in later chapters, such flat directions are especially suitable for realizing the slowly rolling directions giving rise to inflation and the model in Eq. (3.43) is very similar to some of the common SUSY models of hybrid inflation.

Coming back to the O'Raifeartaigh model, the vacuum energy which breaks SUSY due to the F-terms in the true minimum is given by

$$V_F = \kappa^2 \mu^4 \,, \tag{3.46}$$

where μ is the SUSY breaking parameter. If we assume $\langle \phi_1 \rangle = 0$ in addition, the SUSY breaking becomes obvious from the mass spectrum again. The flat direction ϕ_1 decouples from the other two scalars in the mass matrix and is massless, the same is true for its fermionic superpartner. For the remaining mass matrices in (Φ_2, Φ_3) basis, we then obtain the mass eigenvalues by diagonalization. The Weyl fermion mass matrix has the two eigenvalues (m^2, m^2), while the scalar eigenvalues are given by $(m^2, m^2 - \kappa^2 \mu^2)$ and the pseudoscalar ones by $(m^2, m^2 + \kappa^2 \mu^2)$. Hence, one can see that in the SUSY conserving limit $\mu \to 0$, the mass degeneracy within the supermultiplets is restored. In the remainder of this thesis, we will mainly be concerned with F-term SUSY breaking during inflation, however in the presence of SUSY gauge interactions, one has to consider the full scalar potential Eq. (3.38).

As a concluding remark to this section, let us mention the fact that in any theory with spontaneously broken SUSY and non-anomalous gauge symmetry, there exists a sum rule for the masses. In terms of the supertrace of all mass matrices, it reads

$$\mathrm{STr}\, \mathcal{M}^2 = \sum_j (-1)^j \, (2\, j + 1) \; \mathrm{Tr}\, \mathcal{M}_j^2 = 0 \,, \tag{3.47}$$

where the sum is taken over all particles with spin j. For the above example, Eq. (3.47) obviously holds.

3.2 Locally Supersymmetric Lagrangians

The main difference between a global and a local symmetry is the fact that the symmetry transformation parameter explicitly depends on spacetime. In the context of SUSY this implies that the SUSY transformation parameter is promoted to a field, hence $\varepsilon \to \varepsilon(x)$. One obvious consequence when comparing this to the global SUSY transformations, say, e.g., in Eq. (3.17), is that the required closure of the local SUSY algebra generates spacetime translations

$$\{\delta_{\varepsilon_2}, \delta_{\varepsilon_1}\}\, \Phi \sim \mathrm{i}\, a^\mu(x)\, \partial_\mu \Phi\,,^{4} \tag{3.48}$$

which depend on x^μ and are hence *general coordinate transformations*. This implies that local SUSY inherently contains a theory of general relativity (GR), which is the reason for calling it supergravity.

As for any gauge theory, also in local SUSY one has to introduce a gauge field in order to sustain invariance of the Lagrangian. This gauge field has to be a spin-3/2 Rarita–Schwinger field and is called the *gravitino*. Its name is chosen for the fact that its spin-2 superpartner tensor field turns out to be the metric tensor $g_{\mu\nu}$ whose quantum fluctuation is the mediator of gravity, the *graviton*.

In the remainder of this section, we shall first sketch in a very schematic fashion how gravity arises in a local SUSY theory. Then, we describe the different terms contained in the SUGRA Lagrangian and how they are fully determined by three functions of the scalar components in the chiral supermultiplets. The full derivation of the Lagrangian is very lengthy and goes beyond the scope of this thesis. The reader is referred to the extensive literature on the topic, e.g. [69, 70, 71, 78, 79].

Let us now demonstrate by the use of the Noether method how SUGRA implies the presence of a spin-3/2 gravitino gauge field as well as the spin-2 graviton in form of the metric tensor. Assume again a free theory with a complex scalar field ϕ and a spin-1/2 Weyl fermion superpartner χ given by

$$\mathcal{L} = \partial^\mu \phi^*\, \partial_\mu \phi + \mathrm{i}\, \chi^\dagger \bar{\sigma}^\mu \partial_\mu \chi\,, \tag{3.49}$$

as in (3.7). However, instead of the global SUSY transformations (3.9) we require invariance under spacetime dependent transformations

$$\delta\phi = \varepsilon(x)\chi\,, \qquad \delta\chi_\alpha = \mathrm{i}\left[\sigma^\mu \varepsilon^\dagger(x)\right]_\alpha \partial_\mu \phi\,. \tag{3.50}$$

Due to the local field transformations, in addition to the variations given in Eq. (3.10), we now obtain terms proportional to the spacetime derivatives $\partial_\mu \varepsilon^\alpha$ of the transformation parameter summarized in

$$\delta\mathcal{L}|_{\partial\varepsilon} = \partial_\mu \varepsilon^\alpha \cdot K^\mu_\alpha + \text{h.c.}\,, \tag{3.51}$$

where the new object carrying both a spinor and a Lorentz index is given by

$$K^\mu_\alpha = \left[\, \chi\, (\partial^\mu \phi^*) - \sigma^\mu \bar{\sigma}^\nu \chi\, (\partial_\nu \phi^*)\, \right]_\alpha\,. \tag{3.52}$$

[4] The vector field $a^\mu(x) \sim \varepsilon_2(x)\, \sigma^\mu\, \varepsilon_1^\dagger(x)$.

Being a tensor product of spin $(1 \otimes 1/2) = 3/2 \oplus 1/2$ under the Lorentz group this basically is an object in an irreducible representation with spin-3/2.[5] Thus the term in the Lagrangian which has to cancel Eq. (3.51) should couple the above spin-3/2 object to the corresponding gauge field. In turn, this means that the gauge field of local SUSY transformations has to be a spin-3/2 gravitino Ψ_α^μ in order to sustain Lorentz invariance in the Lagrangian with the Noether coupling

$$\mathcal{L}_\mathrm{N} = k\, K_\mu^\alpha\, \Psi_\alpha^\mu + \text{h.c.}\,. \tag{3.53}$$

In order to give the correct mass dimension, the parameter k is dimensionful. Given the Majorana vector spinor field Ψ_α^μ transforms as

$$\Psi_\alpha^\mu \;\to\; \Psi_\alpha^\mu + \frac{1}{k}\,\partial^\mu \varepsilon_\alpha\,, \tag{3.54}$$

the Lagrangian variation Eq. (3.51) can be cancelled by the first term in

$$\delta\mathcal{L}_\mathrm{N} = k\left(K_\mu^\alpha\, \delta\Psi_\alpha^\mu + \Psi_\alpha^\mu\, \delta K_\mu^\alpha\right) + \text{h.c.}\,. \tag{3.55}$$

However, due to the second term in Eq. (3.55) our theory is still not fully invariant. To cure this flaw, the strategy is to add yet another term to the Lagrangian which cancels the remaining piece. This additional term can be rewritten as

$$\delta\mathcal{L}_\mathrm{N} \supset k\, T^{\mu\nu}\left(\Psi_\mu^\dagger\, \bar\sigma_\nu\, \varepsilon + \Psi_\mu\, \sigma_\nu\, \varepsilon^\dagger\right)\,. \tag{3.56}$$

Here, $T^{\mu\nu}$ is the energy-momentum tensor of our quantum fields ϕ and χ. The only term that can cancel the remaining part of (3.56) is given by

$$\mathcal{L}_g = -g_{\mu\nu}\, T^{\mu\nu}\,, \tag{3.57}$$

and obviously has to contain another rank two tensor field $g_{\mu\nu}$ whose SUGRA transformation reads

$$\delta g_{\mu\nu} = k\left(\Psi_\mu^\dagger\, \bar\sigma_\nu\, \varepsilon + \Psi_\mu\, \sigma_\nu\, \varepsilon^\dagger\right)\,. \tag{3.58}$$

This spin-2 field is the metric tensor and thus represents the graviton as the bosonic mediator of gravity and the superpartner of the gravitino. Hence, in some sense SUGRA can be viewed as a quantum theory of gravity, however lacking to be ultraviolet finite due to its non-renormalizability. In conclusion, if we want to study SUSY as a local symmetry, we have to include the pure gravity supermultiplet $(g_{\mu\nu}, \Psi_\mu^\alpha)$ containing the graviton and gravitino respectively.

Having shown how the gravity supermultiplet arises in SUGRA, the next step is to go to an interacting theory of SUGRA including chiral multiplets, vector multiplets and all possible interactions, gauge as well as non-gauge. In the previous Sec. 3.1 for the global SUSY Lagrangian, this could easily be achieved using the Noether method, i.e. just arranging for additional terms iteratively such that in the end the action stays invariant under the symmetry transformations. In principle, one could apply this same method for

[5] The spin-1/2 irreducible representation can be gauged away by the parameter ε.

a SUGRA theory. However, this is extremely tedious and in practice the more efficient *tensor calculus* method[6] or the superspace formulation turn out to be more convenient.

In full analogy to the last section dealing with the global SUSY case, the remainder of this section is organized as follows. First of all, we present the interacting chiral field content of a SUGRA theory and the resulting Lagrangian in Sec. 3.2.1. Sec. 3.2.2 then generalizes the Lagrangian to include vector multiplets and thus gauge interactions. Finally, we discuss the spontaneous breaking of SUGRA within Sec. 3.2.3. Except for the signature of the metric, we stick to the conventions of Ref. [69].

3.2.1 Chiral Supergravity Multiplets

First of all, we summarize the results for the chiral SUGRA Lagrangian for $i, j = 1, \ldots, n$ chiral superfields $\Phi^i = (\phi^i, \chi^i, F^i)$ and the corresponding conjugates $\Phi^{\dagger\bar{i}} = (\phi^{*\bar{i}}, \chi^{\dagger\bar{i}}, F^{*\bar{i}})$. All interactions can be very conveniently condensed into one arbitrary real function called the *Kähler function*[7]

$$G(\phi^i, \phi^{*i}) = K(\phi^i, \phi^{*\bar{i}}) + \ln |W(\phi^i)|^2 \,, \tag{3.59}$$

where as for global SUSY, W denotes the analytic superpotential that only depends on the scalar components ϕ^i of the chiral superfields. The new part is the *Kähler potential* K which is an arbitrary real function depending on the fields ϕ^i as well as their conjugates $\phi^{*\bar{i}}$. One property of the Kähler function Eq. (3.59) is the Kähler invariance, i.e. invariance under the transformations

$$W \to \mathrm{e}^{-f(\phi^i)}\, W \,, \qquad K \to K + f(\phi^i) + f^*(\phi^{*\bar{i}}) \,, \tag{3.60}$$

with an arbitrary holomorphic function $f(\phi^i)$. The scalar fields are the coordinates of a manifold called the Kähler manifold. The metric on this manifold is referred to as the *Kähler metric* and can be calculated in terms of the second derivatives[8]

$$K_{i\bar{j}} = G_{i\bar{j}} \equiv \frac{\partial^2 K}{\partial\phi^i\, \partial\phi^{*\bar{j}}} \,. \tag{3.61}$$

It is a Hermitian matrix which determines the kinetic terms. Note that in our notation, an index with a bar refers to derivatives w.r.t. the complex conjugate fields. The inverse Kähler metric can be written $K^{i\bar{j}}$ such that $K^{i\bar{j}} K_{\bar{j}l} = \delta^i_l$.

All Lagrangian terms that can be derived for the chiral multiplet can be split up in three parts

$$\mathcal{L}_{\text{chiral}} = \mathcal{L}^{\text{B}}_{\text{chiral}} + \mathcal{L}^{\text{F}}_{\text{chiral}} + \mathcal{L}^{\text{FK}}_{\text{chiral}} \,, \tag{3.62}$$

where the first bosonic part $\mathcal{L}^{\text{B}}_{\text{chiral}}$ contains scalar fields only, the second piece $\mathcal{L}^{\text{F}}_{\text{chiral}}$ contains fermion field couplings without any dynamics and $\mathcal{L}^{\text{FK}}_{\text{chiral}}$ contains fermion kinetic terms involving covariant derivatives w.r.t. gravity.

[6] For a review see, e.g., [78] and references therein.

[7] Here and in the following, we use units in which the reduced Planck scale $M_{\text{P}} \approx 2.4 \cdot 10^{18}$ GeV has been set to unity.

[8] As in the last section, indices on the superpotential, Kähler potential and Kähler function refer to derivatives w.r.t. the respective scalar field component.

The first scalar part of the Lagrangian is the most important one for our purposes since for an inflaton scalar field it gives rise to kinetic terms as well as to a potential. It can be expressed in a very compact form

$$e^{-1}\mathcal{L}^{\mathrm{B}}_{\mathrm{chiral}} = -\frac{1}{2}\,R + G_{i\bar{j}}\,\partial_\mu\phi^i\,\partial^\mu\phi^{*\bar{j}} - \mathrm{e}^G\left[G_i\,G^{i\bar{j}}G_{\bar{j}} - 3\right], \tag{3.63}$$

where the first term is the Einstein–Hilbert piece with the Ricci scalar familiar from GR. As mentioned before, the Kähler metric determines the kinetic terms which can be seen from the second term in Eq. (3.63), while the last term determines the F-term scalar potential in SUGRA. In addition, we have introduced $e = \det e^\mu_m = \sqrt{-g}$ where e^μ_m represent the *Vierbein* carrying a local Lorentz index μ and an index m in tangent space. The Vierbein has to be introduced in order to consistently treat spinors in a curved background and its relation to the metric is given by $g_{mn} = e^\mu_m\,e^\nu_n\,\eta_{\mu\nu}$. Note that this is somewhat opposite to the notation usually applied in the literature, where local Lorentz indices are typically referred to by latin letters. However, in order to avoid confusion with the indices labeling the different chiral superfield components, we have opted for this notation.

The second Lagrangian piece in Eq. (3.62) contains fermion mass and interaction terms

$$\begin{aligned} e^{-1}\mathcal{L}^{\mathrm{F}}_{\mathrm{chiral}} = &- \mathrm{e}^{G/2}\left(\Psi^\mu\sigma_{\mu\nu}\Psi^\nu + \Psi^{\dagger\mu}\bar{\sigma}_{\mu\nu}\Psi^{\dagger\nu}\right)\\ &- \frac{1}{2}\left[\mathrm{e}^{G/2}\left(G_{ij} + G_iG_j - G_{ij\bar{k}}G^{\bar{k}l}G_l\right)\chi^i\,\chi^j + \mathrm{h.c.}\right]\\ &- \mathrm{i}\,\mathrm{e}^{G/2}\left[G_i\,\chi^i\sigma^\mu\,\Psi^\dagger_\mu + G_{\bar{i}}\,\chi^{\dagger\bar{i}}\bar{\sigma}^\mu\,\Psi_\mu\right]/\sqrt{2}\\ &+ \text{four fermion interactions}\,, \end{aligned} \tag{3.64}$$

in which the terms in the first line give a mass to the gravitino. In the presence of non-vanishing scalar field VEVs such that $\mathrm{e}^{G/2} \neq 0$ this mass term occurs if local SUSY is broken. We discuss these issues in more detail in Sec. 3.2.3 . From the second line of Eq. (3.64), one obtains the direct mass terms for all chiral fermions in the theory. Line three of Eq. (3.64) are interaction terms coupling chiral fermions to the gravitino and the scalar fields, while the omitted terms contain non-renormalizable four fermion vertices.

Third and last, let us write down the Lagrangian terms which contain the fermion kinetic terms, i.e. for the chiral fermions as well as the gravitino. It is given by

$$\begin{aligned} \mathcal{L}^{\mathrm{FK}}_{\mathrm{chiral}} = &\,\epsilon^{\mu\nu\lambda\rho}\Psi^\dagger_\mu\bar{\sigma}_\nu\tilde{\mathcal{D}}_\lambda\Psi_\rho + \mathrm{i}\,G_{i\bar{j}}\,\chi^{\dagger\bar{j}}\bar{\sigma}^\mu\mathcal{D}_\mu\chi^i\\ &- \frac{1}{\sqrt{2}}\,G_{i\bar{j}}\,\partial_\mu\phi^{*\bar{j}}\chi^i\sigma^\nu\bar{\sigma}^\mu\Psi_\nu - \frac{1}{\sqrt{2}}\,G_{i\bar{j}}\,\partial_\mu\phi^i\chi^{\dagger\bar{j}}\bar{\sigma}^\nu\sigma^\mu\Psi^\dagger_\nu\,, \end{aligned} \tag{3.65}$$

and the kinetic terms for the gravitino and the chiral fermions are contained in the first line. The rest of Eq. (3.65) is made up of non-renormalizable derivative interaction terms. In order to allow for such a compact notation, we have introduced the covariant derivatives for the gravitino and the chiral fermions respectively

$$\begin{aligned} \tilde{\mathcal{D}}_\mu\Psi^\alpha_\nu &= \partial_\mu\Psi^\alpha_\nu + \Psi^\beta_\nu\,\omega^\alpha_{\mu\beta} + \frac{1}{4}\left(G_{\bar{i}}\,\partial_\mu\phi^{*\bar{i}} - G_i\,\partial_\mu\phi^i\right)\Psi^\alpha_\nu\,,\\ \mathcal{D}_\mu\chi^{i\alpha} &= \partial_\mu\chi^{i\alpha} + \chi^{i\beta}\,\omega^\alpha_{\mu\beta} - \frac{1}{4}\left(G_{\bar{i}}\,\partial_\mu\phi^{*\bar{i}} - G_i\,\partial_\mu\phi^i\right)\chi^{i\alpha} + \Gamma^i_{jl}\,\partial_\mu\phi^j\,\chi^{l\alpha}\,, \end{aligned} \tag{3.66}$$

with the spin connection $\omega^{\alpha}_{\mu\beta}$ and the connection of the Kähler geometry $\Gamma^i_{jl} = G^{i\bar{k}} G_{jl\bar{k}}$.

Let us briefly summarize the results mainly relevant for our considerations in this thesis. Since we are interested in implementing models of inflation, the most interesting part of the aforementioned SUGRA Lagrangian is without any doubt the F-term scalar potential. In the absence of gauge interactions, i.e. in an interacting SUGRA theory of chiral superfields only, the F-term scalar potential as taken from Eq. (3.63) reads

$$V_F = \mathrm{e}^K \left[K^{i\bar{j}}\, \mathcal{D}_i W\, \mathcal{D}_{\bar{j}} W^* - 3\,|W|^2 \right] , \tag{3.67}$$

which was derived from the form in Eq. (3.63) using the definition of the Kähler function Eq. (3.59). We are concerned with this scalar potential a lot in Part III of this work, where we investigate SUGRA inflation without including gauge interactions. Note that we have introduced the covariant derivative

$$\mathcal{D}_i W \equiv W_i + W K_i \,, \tag{3.68}$$

which should not be confused with the covariant derivatives carrying local Lorentz indices.

For our purpose in this thesis, another important result is the fermion mass matrix, since we need it to calculate the one-loop radiative corrections to the above tree-level scalar potential. We can directly extract it from the Lagrangian and using the definition of the Kähler function (3.59), it can be rewritten as

$$\left(\mathcal{M}_{\mathrm{F}}\right)_{ij} = \, \mathrm{e}^{K/2} \big(W_{ij} + K_{ij}\, W + K_i\, W_j + K_j\, W_i + K_i\, K_j\, W - K^{k\bar{l}}\, K_{ij\bar{l}}\, \mathcal{D}_k W \big) \,. \tag{3.69}$$

In order to solve for the EOMs of the background fields giving the starting point for simulations of any dynamical processes, the kinetic terms are indispensable ingredients. For the scalar fields, these are all contained in the second term of Eq. (3.63) as mentioned before.

3.2.2 Vector Supergravity Multiplets

Now we turn to the more realistic case of a full SUGRA gauge theory. Therefore, the field content of the previous section is once more extended by the vector supermultiplets $V^a = (A^a_\mu, \lambda^a, D^a)$ carrying a gauge index a. In addition to the superpotential and Kähler potential, there is one further arbitrary analytic function determining the theory. This is the *gauge kinetic function* and it carries two gauge indices and can depend on the scalar chiral superfield components

$$f_{ab} = f_{ab}(\phi^i) \,. \tag{3.70}$$

As indicated by its name, Eq. (3.70) determines the kinetic terms for gauge fields and gauginos. Note that constant $f_{ab} = \delta_{ab}$ correspond to canonical kinetic terms in the gauge sector.

The full SUGRA Lagrangian containing both chiral as well as vector multiplets and their most general interactions can thus be written in schematic form as

$$\mathcal{L}_{\mathrm{SUGRA}} = \tilde{\mathcal{L}}_{\mathrm{chiral}} + \mathcal{L}_{\mathrm{vector}} \,, \tag{3.71}$$

where $\tilde{\mathcal{L}}_{\text{chiral}}$ is the Lagrangian given in Eq. (3.62), however, in gauge covariant form. Due to the fact that the full SUGRA Lagrangian is extremely lengthy and not very illuminating, we focus on the most important terms for our phenomenological purposes in this work. For all possible terms, the reader is referred to [69]. We split up the remaining Lagrangian density into three parts

$$\mathcal{L}_{\text{vector}} = \mathcal{L}^{\text{B}}_{\text{vector}} + \mathcal{L}^{\text{F}}_{\text{vector}} + \mathcal{L}^{\text{FK}}_{\text{vector}}\,, \tag{3.72}$$

where the gauge boson kinetic terms and the D-term scalar potential are contained in

$$e^{-1}\mathcal{L}^{\text{B}}_{\text{vector}} = -\frac{1}{4}\,[\text{Re}f_{ab}]\,F^a_{\mu\nu}F^{b\mu\nu} + \frac{1}{8}\,[\text{Im}f_{ab}]\,F^a_{\mu\nu}\tilde{F}^{b\mu\nu} - \frac{1}{2}\left[\text{Re}f^{-1}_{ab}\right]D^aD^b\,, \tag{3.73}$$

and the SUGRA D-term EOMs depending on the derivative of the Kähler function are given by $D^a = -g\,G_i\,(\mathcal{T}^a)^i_j\,\phi^j$. The tilde on the field strength tensor denotes its dual $\tilde{F}^{\mu\nu} = \epsilon^{\mu\nu\lambda\rho}F_{\lambda\rho}$.

Next, let us write down the terms which describe fermion masses and additional fermion-fermion mixings. The Lagrangian density reads

$$\begin{aligned} e^{-1}\mathcal{L}^{\text{F}}_{\text{vector}} =& \frac{1}{4}\,\mathrm{e}^{G/2}\,G^{i\bar{j}}G_i\,\frac{\partial f^*_{ab}}{\partial\phi^{*\bar{j}}}\,\lambda^a\lambda^b + \frac{1}{4}\,\mathrm{e}^{G/2}\,G^{i\bar{j}}G_{\bar{j}}\,\frac{\partial f_{ab}}{\partial\phi^i}\,\lambda^{\dagger a}\lambda^{\dagger b} \\ &+ \frac{1}{2}\,D^a\,\Psi_\mu\,\sigma^\mu\lambda^{\dagger a} - \frac{1}{2}\,D^a\,\Psi^\dagger_\mu\bar{\sigma}^\mu\lambda^a \\ &+ \mathrm{i}\,\sqrt{2}\left(\frac{\partial D^a}{\partial\phi^i}\right)\chi^i\lambda^a - \mathrm{i}\,\sqrt{2}\left(\frac{\partial D^a}{\partial\phi^{*\bar{i}}}\right)\chi^{\dagger\bar{i}}\lambda^{\dagger a} \\ &+ \frac{\mathrm{i}}{4}\sqrt{2}\left(\frac{\partial f^*_{ab}}{\partial\phi^{*\bar{i}}}\right)D^a\,\chi^{\dagger\bar{i}}\lambda^{\dagger b} - \frac{\mathrm{i}}{4}\sqrt{2}\left(\frac{\partial f_{ab}}{\partial\phi^i}\right)D^a\,\chi^i\lambda^b \\ &+ \text{four fermion interactions}\,, \end{aligned} \tag{3.74}$$

where the first line determines the gaugino masses. The second line gives rise to gravitino-gaugino mixings which couple to the auxiliary fields D^a and the terms in line three and four induce scalar-gaugino-chiral fermion couplings.

The last missing piece in the Lagrangian density for the vector supermultiplet Eq. (3.72) are the fermion-kinetic terms $\mathcal{L}^{\text{FK}}_{\text{vector}}$, summarized in

$$\begin{aligned} e^{-1}\mathcal{L}^{\text{FK}}_{\text{vector}} =& \frac{\mathrm{i}}{2}\,\text{Re}f_{ab}\left(\lambda^a\sigma^\mu\,\tilde{\mathcal{D}}_\mu\lambda^{\dagger b} + \lambda^{\dagger a}\bar{\sigma}^\mu\,\tilde{\mathcal{D}}_\mu\lambda^b\right) - \frac{1}{2}\,\text{Im}f_{ab}\,\tilde{\mathcal{D}}_\mu\left[e\,\lambda^a\sigma^\mu\lambda^{\dagger b}\right] \\ &-\frac{1}{4}\,\sqrt{2}\left(\frac{\partial f_{ab}}{\partial\phi^i}\right)\chi^i\sigma^{\mu\nu}\lambda^aF^b_{\mu\nu} - \frac{1}{4}\,\sqrt{2}\left(\frac{\partial f^*_{ab}}{\partial\phi^{*\bar{i}}}\right)\chi^{\dagger\bar{i}}\bar{\sigma}^{\mu\nu}\lambda^{\dagger a}F^b_{\mu\nu} \\ &-\frac{\mathrm{i}}{4}\,\text{Re}f_{ab}\left(\Psi_\mu\,\sigma^{\rho\nu}\sigma^\mu\lambda^{\dagger a} + \Psi^\dagger_\mu\,\bar{\sigma}^{\rho\nu}\bar{\sigma}^\mu\lambda^a\right)\left[F^b_{\rho\nu} + \hat{F}^b_{\rho\nu}\right]\,, \end{aligned} \tag{3.75}$$

which account for the gaugino kinetic terms, given by the first line. Also, couplings between the gauge field strength tensor, the gauginos and the chiral fermions are introduced by the second line of Eq. (3.75). Line three contains couplings of the gauge field strength tensor to the gauginos and the gravitino. A detailed list of the relevant supercovariant quantities such as the field strength tensor $\hat{F}_{\mu\nu}$ and new derivatives $\tilde{\mathcal{D}}_\mu$, as well as the SUSY transformations of all component fields can be found in [69].

As in the previous section, we now summarize the most important pieces of the complete gauge-invariant SUGRA Lagrangian which are relevant for the main objectives of this thesis. Since we typically choose a constant, diagonal gauge kinetic function $f_{ab} = \delta_{ab}$, the Lagrangian simplifies quite a bit, due to the fact that all terms proportional to the derivatives $\partial f_{ab}/\partial\phi^i$ vanish. From Eq. (3.74), we thus obtain vanishing direct gaugino masses. The only remaining contributions are gaugino-chiral fermion and gaugino-gravitino mixings for which one has to calculate the eigenvalues to end up with the physical fermion masses. This is relevant for calculations of the one-loop effective potential in the presence of gauge interactions, which we need to consider once we introduce GNS inflatons in Part IV of this work.

The most crucial new piece of information in the context of inflation is the classical D-term contribution to the scalar potential

$$V_D = \frac{1}{2}\left[\mathrm{Re}f^{-1}_{ab}\right] D^a D^b = \frac{g^2}{2}\left[\mathrm{Re}f^{-1}_{ab}\right]\left[G_i\,(\mathcal{T}^a)^i_j\,\phi^j\right]\left[G_i\,(\mathcal{T}^b)^i_j\,\phi^j\right] , \tag{3.76}$$

which possibly induces large masses for an inflaton charged w.r.t. the gauge group under consideration. One crucial result of this thesis is that in SUGRA GUTs, it is indeed possible to find so-called D-flat directions where $V_D = 0$ and there are no D-term masses for the inflaton, such that inflation is purely due to the F-term potential Eq. (3.67), while at the same time large SUGRA corrections to the inflaton mass from the latter can be forbidden by symmetry arguments.

Summarizing Secs. 3.2.1 and 3.2.2, a phase of slow-roll inflation in a SUGRA invariant gauge theory is governed by the scalar potential

$$V_{\mathrm{SUGRA}} = \mathrm{e}^G\left[G_i\,G^{i\bar{j}}G_{\bar{j}} - 3\right] + \frac{g^2}{2}\left[\mathrm{Re}f^{-1}_{ab}\right]\left[G_i\,(\mathcal{T}^a)^i_j\,\phi^j\right]\left[G_i\,(\mathcal{T}^b)^i_j\,\phi^j\right] , \tag{3.77}$$

with both F-term and D-term contributions. Eq. (3.77) is the main object to be studied in the course of this work. In Part III, we focus on the F-term part only, in the absence of gauge interactions. Part IV is dedicated to the study of GNS inflaton directions, where also the D-term contribution plays a key role.

3.2.3 Spontaneous Breaking of Supergravity

As in the process of spontaneous symmetry breaking (SSB) of any gauge symmetry, there should also be an analog of the Higgs mechanism in a SUGRA theory. Due to the fact that in this particular case the broken symmetry generators are SUSY generators, the breaking process is referred to as the *super-Higgs mechanism* in which some scalar fields obtain vacuum expectation values (VEVs). In fact, with the gravitino as the gauge field of local SUSY, it has to acquire a mass in the SSB by swallowing the Goldstone DOFs. Since the

Goldstone field is a combination of the fermionic superpartners of the scalar field directions developing their VEVs, it is denoted *goldstino*.

Similar to the discussion in Sec. 3.1.3, we shall describe the two possibilities of F-term and D-term SUGRA breaking. Again, this occurs by non-vanishing auxiliary field F^i and D^a VEVs respectively

$$\langle F^i \rangle \neq 0 \qquad \text{or} \qquad \langle D^a \rangle \neq 0\,. \tag{3.78}$$

Note that there are other ways to break local SUSY such as by *gaugino condensation*. The details of SUSY breaking by gaugino condensates are irrelevant for this thesis and we thus do not discuss it here. For more details, see [80] and references therein.

Let us first consider local SUSY breaking by some non-vanishing F-terms which, in SUGRA, are given by

$$\langle F^i \rangle = \langle \mathrm{e}^{G/2}\, G^{i\bar{j}}\, G_{\bar{j}} \rangle \neq 0\,. \tag{3.79}$$

Provided that all D-terms vanish, in the unbroken phase $\langle F^i \rangle = 0$, the F-term scalar potential contained in (3.77) reduces to $V_F = -3\,\mathrm{e}^K |W|^2$. This is a crucial difference between global and local SUSY. The vacuum state conserving SUGRA can account for negative vacuum energy. In turn, in the SUGRA breaking minimum (3.79), the vacuum energy can vanish which is not possible in a theory with broken global SUSY. This feature of SUGRA is particularly appealing with respect to the *cosmological constant problem*,[9] which is the lack of an explanation for the extremely small, yet non-vanishing cosmological constant accounting for the present day acceleration of our universe. The fact that V_F can vanish with SUSY simultaneously being broken in SUGRA theories does not make them a solution to the cosmological constant problem. Nevertheless, if one is willing to admit for some fine-tuning, at least the problem seems to become manageable within SUGRA which is not the case in global SUSY.

In the above F-term SUSY breaking scenario, the super-Higgs mechanism works in the following way. Looking at the fermion mass terms, the goldstino corresponding to the non-zero $\langle G_i \rangle$ is given by

$$\eta = G_i\, \chi^i\,, \tag{3.80}$$

mixes with the gravitino via the third line in Eq. (3.64). Under the assumption of a minimal Kähler potential $K = \phi^i \phi^{*\bar{i}}$, the goldstino DOFs get swallowed by the redefined gravitino field

$$\Psi'_\mu = \Psi_\mu - \frac{\mathrm{i}}{3\sqrt{2}}\, \sigma_\mu\, \eta - \frac{\sqrt{2}}{3}\, \mathrm{e}^{-G/2}\, \partial_\mu \eta\,, \tag{3.81}$$

and we end up with the simple fermion mass terms

$$e^{-1}\mathcal{L}^{\mathrm{F}}_{\mathrm{chiral}} = -\mathrm{e}^{G/2}\Psi'^{\mu}\sigma_{\mu\nu}\Psi'^{\nu} - \frac{1}{2}\,\mathrm{e}^{G/2}\left(G_{ij} + \frac{1}{3}\, G_i G_j\right)\chi^i\,\chi^j + \text{h.c.}\,, \tag{3.82}$$

where the chiral fermions and the gravitino are decoupled. By swallowing the goldstino DOFs, the gravitino obtains its mass

$$m_{3/2} = \mathrm{e}^{G/2} = \mathrm{e}^{K/2}|W|\,. \tag{3.83}$$

[9]For reviews on the cosmological constant problem, see, e.g., [81, 82].

Next, we shall consider the SSB of SUGRA by non-vanishing D-terms given by

$$\langle D^a \rangle = \mathrm{i}\, g \, \langle \left[f_{ab}^{-1} \right] G_i \, (\mathcal{T}^b)^i_j \, \phi^j \rangle \neq 0 \,. \tag{3.84}$$

Therefore, in addition to the gravitino-chiral fermion mixing terms in Eq. (3.64), we also have to take into account the mixing between the gravitino and the gaugino in the second line of Eq. (3.74). This has the simple consequence that the goldstino direction is now a linear combination of the chiral fermions and the gauginos which reads

$$\eta = G_i \, \chi^i - \frac{g}{\sqrt{2}} \, \mathrm{e}^{-G/2} \, G_i \, (\mathcal{T}^a)^i_j \, \phi^j \lambda^a \,. \tag{3.85}$$

The super-Higgs mechanism works essentially in the same way as in the F-term breaking example above.

Note that from comparing F-term breaking Eq. (3.79) with D-term breaking Eq. (3.84) one can deduce a necessary condition for SUSY breaking, namely that

$$\langle G_i \rangle \neq 0 \,. \tag{3.86}$$

In the inflationary scenarios discussed in this thesis, we limit ourselves to the case where SUGRA is broken by non-vanishing F-terms only. The sum rule Eq. (3.47) which holds in broken global SUSY gets modified in the context of SUGRA. One finds that

$$\mathrm{STr}\, \mathcal{M}^2 = 2\,(N-1)\, m_{3/2}^2 \,, \tag{3.87}$$

where N is the number of chiral supermultiplets present in the theory. It is interesting to see that Eq. (3.87) does not vanish in the general case. We encounter this feature of SUGRA in our models in the following parts on a regular basis. In addition, this is desirable since it states that on average, the scalar particles within the chiral superfields are heavier than their fermionic counterparts. Within the MSSM such a statement should certainly be true, since lacking their observation up to the date of writing, the sleptons and squarks have to be heavier than the observed leptons and quarks.

Chapter 4

Supersymmetric Grand Unification

The idea of a GUT, i.e., the unification of the fundamental forces in nature (except for gravity) above some high energy scale, is very intriguing for different reasons. From the top-down perspective there are some compelling features of having a larger simple gauge group with only one independent gauge coupling above the scale of grand unification M_{GUT}, the so-called GUT scale. Besides unifying the gauge interactions, it can also explain charge quantization in a simple way. In addition, some of the many free parameters of the SM can be related in GUTs, a desirable feature of a more fundamental theory of nature. Furthermore, left-right symmetric GUTs have a built-in explanation for anomaly cancellation and the origin of the light neutrino masses.

From the bottom-up perspective there is also some hint for unification. This is the renormalization group (RG) evolution of the gauge couplings which hints at their unification at a high energy scale. In this respect, GUTs and light SUSY fit together especially well because taking into account the SUSY partners in addition to the SM particle content, the gauge couplings meet almost exactly at $M_{\mathrm{GUT}} \approx 2 \cdot 10^{16}$ GeV [19, 20].

Historically, the Pati–Salam (PS) model [83, 84] was the first step towards unification, where the authors have unified quarks and leptons in multiplets of the gauge group $G_{\mathrm{PS}} = SU(4)_C \times SU(2)_L \times SU(2)_R$. Complete unification within a simple group and hence one single gauge coupling has been proposed subsequently in the context of $SU(5)$ [15], $SO(10)$ [16, 17] and E_6 [18] gauge symmetries.

Within this work we are concerned with identifying the inflaton particle with some of the scalar superpartners of *matter fermions* within reps of a SUSY GUT, in particular the right-handed sneutrino. Therefore, we focus on the G_{PS} gauge group first, which automatically contains the right-handed neutrino superfield in the antifundamental rep. We then extend our considerations to $SO(10)$ which goes even further. It is the smallest simple symmetry group which can accommodate a complete generation of SM fermions and the corresponding right-handed neutrino within one single **16** dimensional spinor rep.

We briefly introduce the particle content of the minimal supersymmetric standard model (MSSM) [85, 86, 87, 88] in Sec. 4.1. Subsequently Secs. 4.2 and 4.3 are dedicated to embedding the MSSM particle content within reps of G_{PS} and $SO(10)$ respectively.

4.1 Particle Content of the MSSM

In order to be able to embed the particles within the SM into SUSY GUTs in the following sections, let us first introduce the particle content of the simplest global SUSY extension of the SM, namely the MSSM. Therefore, all the fields introduced are not only multiplets under the gauge transformations but also multiplets under SUSY transformations as defined in Ch. 3. Most of this section is based on [68, 89]. Note that gauge symmetry and SUSY generators commute which implies that all members of a supermultiplet have the same gauge quantum numbers.

All interactions within the SM obey the principle of gauge invariance under the product group[1] $G_{\rm SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$. This determines the strong interactions of quantum chromodynamics (QCD) given in terms of invariance under non-abelian $SU(3)_C$ as well as the electroweak (EW) interactions invariant under $SU(2)_L \times U(1)_Y$. All matter fields as well as force carriers and Higgs fields transform as irreducible unitary reps under the group $G_{\rm SM}$.

While the matter and Higgs fields typically lie in the *fundamental* (and its conjugate) rep,[2] the gauge fields belong to the *adjoint* rep. On the SUSY side, spin-0 Higgs scalars and spin-1/2 matter fermions should live in chiral supermultiplets, see Sec. 3.1.1. Being spin-1 particles, the gauge bosons belong to vector supermultiplets under SUSY, see Sec. 3.1.2.

Let us start with the chiral superfield content as summarized in Tab. 4.1. For the sake of simplicity, we write down the first generation only and suppress generation indices. However, one should keep in mind that there are actually three copies of this generation. The left-handed quark superfields are color triplets and $SU(2)_L$ doublets

$$q = \begin{pmatrix} u_1 & u_2 & u_3 \\ d_1 & d_2 & d_3 \end{pmatrix} , \tag{4.1}$$

while their right-handed color anti-triplet, $SU(2)_L$ singlet counterparts are given by[3]

$$u^c = \begin{pmatrix} u^c_1 & u^c_2 & u^c_3 \end{pmatrix} , \qquad d^c = \begin{pmatrix} d^c_1 & d^c_2 & d^c_3 \end{pmatrix} . \tag{4.2}$$

Note that in horizontal direction, the multiplets transform under $SU(3)_C$ and the indices refer to the three colors. In vertical direction, the multiplet transforms under $SU(2)_L$.

The lepton sector which is uncharged under $SU(3)_C$ contains one left-handed $SU(2)_L$ doublet and a singlet superfield, respectively

$$l = \begin{pmatrix} \nu \\ e \end{pmatrix} , \qquad e^c . \tag{4.3}$$

In the MSSM, one single $SU(2)_L$ Higgs doublet is not enough. On the one hand, having only one, not all the Yukawa couplings are allowed which are mandatory in order to

[1]For an introduction to Lie algebras and Lie groups, see Ref. [90].
[2]This is not necessarily true when the fields are embedded into reps of some unifying symmetry group.
[3]The ^{c} indicates that they correspond to the charge conjugated, right-handed counterparts.

Label	$G_{\rm SM}$	Boson Component	Fermion Component	Sector
q	$(\mathbf{3},\mathbf{2},+1/6)$	$\tilde{q}_L=(\tilde{u}_L,\tilde{d}_L)^T$	$q_L=(u_L,d_L)^T$	(s)quarks
u^c	$(\bar{\mathbf{3}},\mathbf{1},-2/3)$	$\tilde{u}^*_R$	$u^\dagger_R$	
d^c	$(\bar{\mathbf{3}},\mathbf{1},+1/3)$	$\tilde{d}^*_R$	$d^\dagger_R$	
l	$(\mathbf{1},\mathbf{2},-1/2)$	$\tilde{l}_L=(\tilde{\nu}_L,\tilde{e}_L)^T$	$l_L=(\nu_L,e_L)^T$	(s)leptons
e^c	$(\mathbf{1},\mathbf{1},+1)$	$\tilde{e}^*_R$	$\tilde{e}^\dagger_R$	
h_u	$(\mathbf{1},\mathbf{2},+1/2)$	$h_u=(h_u^+,h_u^0)^T$	$\tilde{h}_u=(\tilde{h}_u^+,\tilde{h}_u^0)^T$	Higgs(inos)
h_d	$(\mathbf{1},\mathbf{2},-1/2)$	$h_d=(h_d^0,h_d^-)^T$	$\tilde{h}_d=(\tilde{h}_d^0,\tilde{h}_d^-)^T$	
G^a	$(\mathbf{8},\mathbf{1},0)$	G^a	$\tilde{G}^a$	gauge bosons
W^i	$(\mathbf{1},\mathbf{3},0)$	W^i	$\tilde{W}^i$	(gauginos)
B^0	$(\mathbf{1},\mathbf{1},0)$	B^0	$\tilde{B}^0$	

Table 4.1: Superfield content of the MSSM. To streamline notation we have suppressed color and generation indices for the chiral supermultiplets. For the vector supermultiplets, the index $a=1,\dots,8$ runs over the adjoint rep of $SU(3)_C$ and $i=1,2,3$ over the adjoint rep of $SU(2)_L$. We follow the standard convention that all chiral supermultiplets are defined in terms of left-chiral Weyl spinors. A tilde denotes the SUSY partner of SM fields.

generate the fermion masses in the process of EW symmetry breaking. On the other hand, having only one doublet would let the EW gauge symmetry suffer from a gauge anomaly. This can be seen from the condition for anomaly cancellation

$$\mathcal{A}^{abc}=\mathrm{Tr}\left[\mathcal{T}^a\{\mathcal{T}^b,\mathcal{T}^c\}\right]=0\,, \tag{4.4}$$

where the trace is taken over all fermionic DOFs and the $\mathcal{T}^a$ denote the generators contributing to external currents in the triangle diagrams which generate the anomaly. For the MSSM, the $SU(3)_C$ part of the condition is readily fulfilled by the above particle content, however the remaining EW part of Eq. (4.4) implies the condition $\mathrm{Tr}[I_3^2 Y]=\mathrm{Tr}[Y^3]$.[4] In the SM, this condition is also satisfied by the known quarks and leptons. However, if we add a chiral Higgs supermultiplet with a fermionic higgsino component it must be a $SU(2)_L$ doublet with either hypercharge $Y=\pm 1/2$. Thus one must add yet another Higgs doublet with the opposite hypercharge Y to account for Eq. (4.4). The two Higgs doublet superfields are given by

$$h_u=\begin{pmatrix} h_u^+\\ h_u^0\end{pmatrix}\,,\qquad h_d=\begin{pmatrix} h_d^0\\ h_d^-\end{pmatrix}\,, \tag{4.5}$$

which complete the chiral supermultiplets of the MSSM. All of the MSSM chiral and vector supermultiplets are summarized in Tab. 4.1.

[4]The quantum number of the third component of weak isospin is dubbed I_3. Y refers to the weak hypercharge.

As noted, the gauge bosons are contained in vector supermultiplets. The eight gluons G^a responsible for the strong interactions come in supermultiplets with their spin-1/2 Weyl fermion superpartners, the gluinos. EW gauge symmetry is associated with the three W^i gauge bosons of $SU(2)_L$ and the B^0 gauge boson of $U(1)_Y$. Their Weyl fermion superpartners are referred to as winos and bino. After EW symmetry breaking (EWSB), the gauge eigenstates W^3 and B^0 mix to form one massive mass eigenstate Z^0 and the photon γ, a zero mass eigenstate corresponding to the remaining unbroken electromagnetic $U(1)_Q$. EWSB occurs when the neutral components of the Higgs doublets develop their VEVs

$$\langle h_u \rangle = \begin{pmatrix} 0 \\ \langle h_u^0 \rangle \end{pmatrix} , \qquad \langle h_d \rangle = \begin{pmatrix} \langle h_d^0 \rangle \\ 0 \end{pmatrix} , \tag{4.6}$$

where $v^2 = \langle h_u^0 \rangle^2 + \langle h_d^0 \rangle^2 = (174\,\text{GeV})^2$ and the ratio is defined by $\tan\beta \equiv \langle h_u^0 \rangle / \langle h_d^0 \rangle$. With the gauge quantum numbers as displayed in Tab. 4.1 and the requirement that the new vacuum Eq. (4.6) has eigenvalues zero under the Abelian electromagnetic charge generator $Q\langle h_u \rangle = Q\langle h_d \rangle = 0$, one can relate the generator of $U(1)_Q$ to weak isospin and hypercharge by

$$Q = I_3 + Y \, . \tag{4.7}$$

The potential required to induce EWSB of $SU(2)_L \times U(1)_Y \to U(1)_Q$ arises from a combination of SUSY conserving F- and D-term contributions as described in Sec. 3.1.3 and from soft SUSY breaking terms.

The soft breaking terms have to be of order $m_{\text{soft}} \simeq \mathcal{O}(\text{TeV})$, otherwise the solution to the hierarchy problem is endangered. One expects that SUSY is broken in some *hidden sector*[5] and mediated to the visible sector either via the superconformal anomaly in theories with extra dimensions, via gauge interactions or via gravity. In the latter case SUSY breaking is induced by SUGRA as discussed in Sec. 3.2.3. Since soft SUSY breaking is not of relevance for our considerations in this thesis, we do not further discuss it.

Apart from the soft terms, all possible interactions in the MSSM can simply be obtained by requiring invariance under the gauge symmetry G_{SM}, under *R-parity* and under SUSY. R-parity [87] is defined as

$$P_R = (-1)^{3(B-L)+2s} \; , \tag{4.8}$$

where B and L refer to baryon and lepton number respectively, while s is the spin. It is introduced to make the lightest supersymmetric particle (LSP) stable and hence a promising cold dark matter candidate. Thus, the Lagrangian has to be P_R-even. In addition, R-parity can forbid B and L violating operators in the superpotential since it reproduces *matter parity* $P_M = (-1)^{3(B-L)}$ for the superfields. Therefore, viable superpotential terms (3.27) should combine the gauge quantum numbers of the chiral supermultiplets in Tab. 4.1 to form gauge singlets and carry even matter parity. The allowed superpotential reads

$$W_{\text{MSSM}} = y_u \, h_u \, q \, u^c + y_d \, h_d \, q \, d^c + y_e \, h_d \, l \, e^c + \mu \, h_u h_d \, , \tag{4.9}$$

[5]Which entirely consists of SM gauge singlet superfields.

where contraction over all gauge indices is assumed and y_u, y_d, y_e are up-type, down-type and charged lepton 3×3 Yukawa coupling matrices when taking into account all three families. To streamline notation, we have suppressed family indices. μ is a parameter of positive mass dimension which should be of the order of 100 GeV such that the Higgs potential has its VEV at the right scale. The lack of an a priori explanation for the smallness of μ is called the *μ-problem* [91]. All interactions of the MSSM particles can now be derived from Eq. (3.36).

4.2 Supersymmetric Pati–Salam Unification

This section is dedicated to a unification of the MSSM particles within reps of the gauge group $G_{\rm PS} = SU(4)_C \times SU(2)_L \times SU(2)_R$. From now on, we work with supermultiplets only and do not explicitly mention the superpartners as in Sec. 4.1. In order to assign MSSM particles to reps of $G_{\rm PS}$, let us first list some of the lower dimensional PS reps and their decompositions under $G_{\rm SM}$. Extensive lists of $SO(10)$ reps and their various decompositions under the most important intermediate symmetry groups down to $G_{\rm SM}$ are given in [92].

For the chiral superfield content of the MSSM it is sufficient to work with the following three PS multiplets whose $G_{\rm SM}$ decompositions read

$$\begin{aligned}
(\mathbf{1},\mathbf{2},\mathbf{2}) &= (\mathbf{1},\mathbf{2},+1/2) \oplus (\mathbf{1},\mathbf{2},-1/2)\,,\\
(\mathbf{4},\mathbf{2},\mathbf{1}) &= (\mathbf{3},\mathbf{2},+1/6) \oplus (\mathbf{1},\mathbf{2},-1/2)\,,\\
(\bar{\mathbf{4}},\mathbf{1},\mathbf{2}) &= (\bar{\mathbf{3}},\mathbf{1},-2/3) \oplus (\bar{\mathbf{3}},\mathbf{1},+1/3) \oplus (\mathbf{1},\mathbf{1},+1) \oplus (\mathbf{1},\mathbf{1},0)\,.
\end{aligned} \tag{4.10}$$

To simplify the task of contracting $SU(2)$ indices, in the following we define the multiplets with the antifundamental $\bar{\mathbf{2}}$ where appropriate. Upon comparison of Eq. (4.10) to the MSSM chiral superfields given in Tab. 4.1, one can obviously accommodate the two MSSM Higgs doublets within the PS bi-doublet which we dub

$$h = (\mathbf{1},\bar{\mathbf{2}},\mathbf{2}) = \underbrace{(\mathbf{1},\bar{\mathbf{2}},+1/2)}_{\epsilon\, h_u} \oplus \underbrace{(\mathbf{1},\bar{\mathbf{2}},-1/2)}_{\epsilon\, h_d}\,. \tag{4.11}$$

The ϵ symbolically stands for the Levi–Civita symbol with two indices which is used to obtain the antifundamental rep of $SU(2)_L$ according to $\bar{\mathbf{2}}_a = \epsilon_{ab}\,\mathbf{2}^b$. The PS multiplet containing the fundamental rep of $SU(4)_C$ which is charged under $SU(2)_L$ has exactly the right SM decomposition to unify the doublet quark and doublet lepton superfields of one generation as

$$L = (\mathbf{4},\mathbf{2},\mathbf{1}) = \underbrace{(\mathbf{3},\mathbf{2},+1/6)}_{q} \oplus \underbrace{(\mathbf{1},\mathbf{2},-1/2)}_{l}\,, \tag{4.12}$$

which we denote L because it is the common rep of the left-doublet SM fermion supermultiplets. Furthermore, the PS multiplet which is charged under $SU(2)_R$ must be able to account for the $SU(2)_L$ singlet SM fields and indeed we find

$$R^c = (\bar{\mathbf{4}}, \mathbf{1}, \bar{\mathbf{2}}) = \underbrace{(\bar{\mathbf{3}}, \mathbf{1}, -2/3)}_{u^c} \oplus \underbrace{(\bar{\mathbf{3}}, \mathbf{1}, +1/3)}_{d^c} \oplus \underbrace{(\mathbf{1}, \mathbf{1}, +1)}_{e^c} \oplus (\mathbf{1}, \mathbf{1}, 0)\,, \tag{4.13}$$

where the name R^c refers to the fact that they are charged under $SU(2)_R$. Notice that in the decomposition Eq. (4.13) there is one G_{SM} singlet field left over which cannot be identified with any of the MSSM chiral superfields. Rather than being a problem, this is quite desirable because it can account for the right-handed neutrino which is needed to generate the small physical neutrino masses via the seesaw mechanism [39, 40, 41, 42, 43]. Thus by simply assigning MSSM particles to PS multiplets we have already found an explanation for a missing piece within the SM and we define

$$\nu^c = (\mathbf{1}, \mathbf{1}, 0)\,. \tag{4.14}$$

To summarize, the full chiral superfield content of the MSSM plus a right-handed neutrino superfield can fit very economically into three PS multiplets which in our basis are given by

$$L^{\beta a} = \begin{pmatrix} u_1 & u_2 & u_3 & \nu \\ d_1 & d_2 & d_3 & e \end{pmatrix}\,, \qquad (R^c)_{\beta x} = \begin{pmatrix} u^c_1 & u^c_2 & u^c_3 & \nu^c \\ d^c_1 & d^c_2 & d^c_3 & e^c \end{pmatrix}\,, \qquad h^x_a = \begin{pmatrix} h^0_u & h^-_d \\ h^+_u & h^0_d \end{pmatrix}\,, \tag{4.15}$$

where $\beta = 1, \ldots, 4$ is an $SU(4)_C$ index and $a, x = 1, 2$ are $SU(2)_{L,R}$ indices. Note that one is basically free in choosing the upper and lower component of $SU(2)_R$ in R^c but one has to adjust the entries of the Higgs matrix rep h accordingly. In our notation it transforms vertically under $SU(2)_L$ and horizontally under $SU(2)_R$. At the renormalizable level, the only superpotential terms allowed for the three PS multiplets are

$$W_{\mathrm{PS}} = y\, h\, L\, R^c + \mu\, h\, h\,, \tag{4.16}$$

which after PS symmetry breaking to the MSSM can be identified with all the terms in Eq. (4.9) plus one additional Yukawa term

$$W_\nu \supset y_\nu\, h_u\, l\, \nu^c\,, \tag{4.17}$$

giving rise to the Dirac neutrino mass.

At first glance this might seem as a problem since it generates a Dirac mass matrix $\hat{m}_{\mathrm{D}}$ for the neutrinos with a typical mass scale determined by the EW Higgs VEV hence $\langle h^0_u \rangle \simeq 100\,\mathrm{GeV}$. Without extremely fine-tuned Yukawa couplings y_ν, there is a priori no explanation for the small neutrino masses $m_\nu \lesssim \mathcal{O}(\mathrm{eV})$. However, as stated above there is help in form of the seesaw mechanism. As we will see in Part IV of this work, if one invokes higher dimensional effective operators, one typically obtains a Majorana-type mass matrix $\hat{M}$ for the right-handed neutrinos in the breaking of PS with a typical mass scale of the order of M_{GUT}. Therefore, we obtain an effective low energy superpotential for the neutrino sector given by

$$W_\nu \simeq \hat{m}_{\mathrm{D}}\, \nu\, \nu^c + \hat{M}\, \nu^c\, \nu^c\,, \tag{4.18}$$

with a mixing mass matrix

$$\hat{\mathcal{M}}_\nu = \begin{pmatrix} 0 & \hat{m}_\mathrm{D}^T \\ \hat{m}_\mathrm{D} & \hat{M} \end{pmatrix} . \tag{4.19}$$

Under the legitimate assumption that the entries of $\hat{M}$ are much larger than the ones of $\hat{m}_\mathrm{D}$ we can bring the mass matrix into block-diagonal form. We obtain one mass matrix with large eigenvalues of the mostly right-handed fields and one mass matrix with small eigenvalues of the mostly left-handed fields

$$\hat{M}_{\nu^c} \simeq \hat{M} , \qquad \hat{m}_\nu \simeq -\hat{m}_\mathrm{D}^T \hat{M}^{-1} \hat{m}_\mathrm{D} . \tag{4.20}$$

Plugging in the EW scale and the GUT scale as typical values, we obtain small physical neutrino mass eigenvalues m_ν which are exactly in the right ballpark. What we have just described is the simplest type-I seesaw mechanism.

Next, we discuss the gauge bosons of the PS model. They can be found in the adjoint rep of each factor group. In the case of $SU(4)_C$ this is a **15** and both $SU(2)$ factors have a three dimensional adjoint rep each with the G_SM decompositions

$$\begin{aligned} (\mathbf{15},\mathbf{1},\mathbf{1}) &= \underbrace{(\mathbf{8},\mathbf{1},0)}_{G^a} \oplus \underbrace{(\mathbf{1},\mathbf{1},0)}_{A^{15}} \oplus (\mathbf{3},\mathbf{1},+2/3) \oplus (\bar{\mathbf{3}},\mathbf{1},-2/3) , \\ (\mathbf{1},\mathbf{1},\mathbf{3}) &= \underbrace{(\mathbf{1},\mathbf{1},0)}_{A^{18}} \oplus (\mathbf{1},\mathbf{1},+1) \oplus (\mathbf{1},\mathbf{1},-1) , \\ (\mathbf{1},\mathbf{3},\mathbf{1}) &= \underbrace{(\mathbf{1},\mathbf{3},0)}_{W^i} . \end{aligned} \tag{4.21}$$

We can already identify the eight gluons G^a and the three W^i gauge bosons which must remain massless in the breaking of PS to obtain the MSSM. In addition there are two DOFs denoted A^{15} and A^{18}. In the breaking of PS, these gauge bosons form one massless linear combination giving rise to the B^0 gauge boson of $U(1)_Y$, cf. Eq. (D.8), and one orthogonal linear combination which becomes heavy by consuming a Goldstone boson. Also, all the remaining gauge bosons become massive.

This brings us directly to the spontaneous symmetry breakdown of PS to the MSSM, with $SU(2)_L$ remaining intact, given by

$$SU(4)_C \times SU(2)_R \to SU(3)_C \times U(1)_Y . \tag{4.22}$$

Therefore, we need the VEVs of some additional Higgs fields transforming non-trivially under both $SU(4)_C$ and $SU(2)_R$ in order to break them. The lowest dimensional reps that can achieve this are given by the fundamental and antifundamental of both factor groups and we define them by

$$\begin{aligned} H^c &= (\bar{\mathbf{4}},\mathbf{1},\bar{\mathbf{2}}) = \begin{pmatrix} u_H^c & u_H^c & u_H^c & \nu_H^c \\ d_H^c & d_H^c & d_H^c & e_H^c \end{pmatrix} , \\ \bar{H}^c &= (\mathbf{4},\mathbf{1},\mathbf{2}) = \begin{pmatrix} \bar{u}_H^c & \bar{u}_H^c & \bar{u}_H^c & \bar{\nu}_H^c \\ \bar{d}_H^c & \bar{d}_H^c & \bar{d}_H^c & \bar{e}_H^c \end{pmatrix} , \end{aligned} \tag{4.23}$$

where from now on we suppress gauge indices if the notation is unambiguous. With VEVs in the right-handed neutrino direction

$$\langle H^c \rangle = \begin{pmatrix} 0 & 0 & 0 & \langle \nu^c_H \rangle \\ 0 & 0 & 0 & 0 \end{pmatrix} , \qquad \langle \bar{H}^c \rangle = \begin{pmatrix} 0 & 0 & 0 & \langle \bar{\nu}^c_H \rangle \\ 0 & 0 & 0 & 0 \end{pmatrix} , \tag{4.24}$$

the PS symmetry is broken exactly in the right direction to make all gauge bosons in the coset $G_{\text{PS}}/G_{\text{SM}}$ heavy which corresponds to the breaking channel Eq. (4.22).

The details of how the gauge bosons corresponding to certain generators become massive are discussed in App. D.1. Via the gauge covariant derivatives in the kinetic terms of H^c and $\bar{H}^c$ the VEV in Eq. (4.24) makes 9 out of the 21 gauge bosons of G_{PS} massive. These correspond to the generators $\mathcal{T}^9, \ldots, \mathcal{T}^{14}, \mathcal{T}^{16}, \mathcal{T}^{17}$ and one linear combination of $\mathcal{T}^{15}$ and $\mathcal{T}^{18}$. Being a diagonal generator, the orthogonal linear combination can give rise to $U(1)_Y$. And indeed requiring $Y\langle H^c \rangle = Y\langle \bar{H}^c \rangle = 0$, the hypercharge generator acting on the fundamental rep with correct normalization factor is given by

$$Y = \sqrt{\frac{2}{3}}\, \mathcal{T}^{15} + \mathcal{T}^{18} \,. \tag{4.25}$$

This corresponds to the aforementioned linear combination of the gauge bosons A^{15} and A^{18} where $\mathcal{T}^{18}$ is the diagonal generator of $SU(2)_R$ isospin. Apart from these, the gauge bosons which belong to the generators $\mathcal{T}^1, \ldots, \mathcal{T}^8$ are the eight gluons G^a and the W^i are not affected since the Higgs VEVs in Eq. (4.23) are $SU(2)_L$ singlets. Hence we end up in the MSSM vacuum with a total of 12 massless gauge bosons which correspond to the eight gluons, three W-bosons and the B^0-boson.

To summarize, we have discussed how the MSSM field content can be embedded in reps of G_{PS}. The presence of a right-handed neutrino as an implication of the left-right symmetry in PS makes it possible to explain the light neutrino masses via the seesaw mechanism. Furthermore, we have discussed the SSB of PS by the VEVs of Higgs reps $(\bar{\mathbf{4}}, \mathbf{1}, \bar{\mathbf{2}})$ and $(\mathbf{4}, \mathbf{1}, \mathbf{2})$. In Part IV we construct scalar potentials in which the mechanism ending inflation induces symmetry breakdown in exactly these directions, cf. Eq. (4.24). In this scenario the right-handed neutrino superfield (4.14) plays a crucial role since its scalar component, the right-handed sneutrino, is the inflaton particle.

4.3 Supersymmetric $SO(10)$ Grand Unification

Probably one of the most appealing properties of an $SO(10)$ SUSY GUT is the fact that all MSSM matter superfields of one family including the right-handed neutrino can reside most economically in one single rep, the **16** spinor rep of $SO(10)$. Following the bottom-up fashion of Sec. 4.2 we proceed to embed the MSSM field content in $SO(10)$ reps in this section. We limit ourselves to give a concise presentation of the $SO(10)$ embeddings of the PS multiplets needed for this work only, cutting down on any details of the breaking of $SO(10)$ to G_{PS} or any group theoretical aspects. For details of the underlying group theory, the reader is referred to Ref. [90].

The strategy we follow throughout the course of this thesis is to consider a breaking hierarchy of the $SO(10)$ GUT via an intermediate $G_{\rm PS}$ symmetry down to $G_{\rm SM}$ as

$$SO(10) \rightarrow SU(4)_C \times SU(2)_L \times SU(2)_R \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y\,, \tag{4.26}$$

however dealing with the field content on PS-level. Therefore, we assume that $SO(10)$ is already broken down to $G_{\rm PS}$ during inflation which is possible, e.g., by the VEV of a **54** Higgs rep.

Since $SO(10)$ respects left-right symmetry the matter **16** must contain both the left- and right-handed fields of one MSSM family and indeed it decomposes under $G_{\rm PS}$ as

$$F = \mathbf{16} = \underbrace{(\mathbf{4},\mathbf{2},\mathbf{1})}_{L} \oplus \underbrace{(\bar{\mathbf{4}},\mathbf{1},\bar{\mathbf{2}})}_{R^c}\,, \tag{4.27}$$

where the label F indicates that it accommodates the SM fermions.

The two MSSM Higgs doublets which form a bi-doublet under $G_{\rm PS}$ lie in the **10** vector rep

$$h = \mathbf{10} = \underbrace{(\mathbf{1},\mathbf{2},\mathbf{2})}_{h} \oplus (\mathbf{6},\mathbf{1},\mathbf{1})\,, \tag{4.28}$$

where the additional DOFs contain $G_{\rm SM}$ color triplets which can mediate rapid proton decay.[6] To suppress these decay channels sufficiently in agreement with lower bounds on the proton lifetime, one has to make sure that there is some mechanism at work which makes these color triplet Higgs fields sufficiently massive while keeping the $SU(2)_L$ doublets light. This goes under the name of *doublet-triplet splitting problem* [61, 94, 95, 96] and is typically solved during one of the stages of SSB. For our purposes, we assume that the antisymmetric **6** of $G_{\rm PS}$ obtains an effective mass term in the breaking of $SO(10)$ and can subsequently be integrated out, thus decoupling from the theory.

The gauge bosons of $SO(10)$ reside in the **45** adjoint rep which we refer to as A. Its PS-decomposition is given by

$$A = \mathbf{45} = \underbrace{(\mathbf{15},\mathbf{1},\mathbf{1})}_{SU(4)_C} \oplus \underbrace{(\mathbf{1},\mathbf{3},\mathbf{1})}_{SU(2)_L} \oplus \underbrace{(\mathbf{1},\mathbf{1},\mathbf{3})}_{SU(2)_R} \oplus (\mathbf{6},\mathbf{2},\mathbf{2})\,, \tag{4.29}$$

where we have indicated to which reps the different PS factor groups belong to. Notice that in the breaking $SO(10) \rightarrow G_{\rm PS}$ the gauge bosons in the $(\mathbf{6},\mathbf{2},\mathbf{2})$ obtain GUT-scale masses $M_{\rm GUT}$.

Now we can see why $SO(10)$ is so attractive. With only three reps, one for each sector (matter, Higgs, gauge bosons), we have all we need to describe the complete set of MSSM fields. This is very economical and we have summarized the reps with their decompositions under $G_{\rm PS}$ and how the MSSM fields are embedded in Tab. 4.2.

The only renormalizable operators constructed from the **10** and **16** that can enter the superpotential as gauge singlets are

$$y\,\mathbf{16}.\mathbf{10}.\mathbf{16} + \mu\,\mathbf{10}.\mathbf{10}\,, \tag{4.30}$$

[6] Rapid proton decay clearly contradicts experimental constraints [93].

	$SO(10)$	G_{PS}	MSSM Fields
F_i	$\mathbf{16}$	$(\mathbf{4},\mathbf{2},\mathbf{1})$	q_i, l_i
		$(\bar{\mathbf{4}},\mathbf{1},\bar{\mathbf{2}})$	$u_i^c, d_i^c, l_i^c, \nu_i^c$
h	$\mathbf{10}$	$(\mathbf{1},\bar{\mathbf{2}},\mathbf{2})$	h_u, h_d
		$(\mathbf{6},\mathbf{1},\mathbf{1})$	
A	$\mathbf{45}$	$(\mathbf{15},\mathbf{1},\mathbf{1})$	g^a, A^{15}
		$(\mathbf{1},\mathbf{1},\mathbf{3})$	A^{18}
		$(\mathbf{1},\mathbf{3},\mathbf{1})$	W^+, W^0, W^-
		$(\mathbf{6},\mathbf{2},\mathbf{2})$	

Table 4.2: Superfield content of the MSSM unified in PS multiplets as well as $SO(10)$ reps. Color indices are suppressed for convenience. The index $i = 1, 2, 3$ runs over the three different families while $a = 1, \ldots, 8$ runs over the adjoint rep of $SU(3)_C$.

which can reproduce the Yukawa couplings and μ-term in the superpotential (4.16) in the broken phase of $SO(10)$. If we furthermore want to include the Higgs fields defined in Eq. (4.23) which break PS, we have to add one $\mathbf{16}_H$ and one $\overline{\mathbf{16}}_H$ dubbed H and $\bar{H}$. Since their VEVs have to point in the direction of the right-handed neutrinos,[7] the latter can obtain their large Majorana masses quite naturally via effective dimension five operators

$$\gamma \, \frac{\mathbf{16}.\mathbf{16}.\overline{\mathbf{16}}_H.\overline{\mathbf{16}}_H}{\Lambda} \,, \tag{4.31}$$

where γ is a dimensionless coupling parameter and Λ is a suppression scale. In App. D.2 we give an example of singlet messenger exchange to demonstrate how such dimension five operators arise. These same operators turn out to be an essential ingredient to realize hybrid inflation in the matter sector as proposed in Part IV of this work.

[7]This is mandatory in order to end up in G_{SM}.

Part III

Inflation in Supergravity

Chapter 5

Cosmological Problems in SUGRA

This chapter is dedicated to motivating the work presented in this part of the thesis.

On the one hand, it is a generic feature of SUGRA theories to induce mass corrections of the order of the Hubble scale for any scalar DOF. Such mass corrections spoil viable slow-roll inflation by violating the slow-roll condition for η. This is the η-problem of SUGRA inflation [31, 32, 97].

On the other hand, especially in SUGRA arising from higher dimensional theories, there are typically additional scalar fields present. These scalar DOFs have their origin in the compactification process of the spacetime manifold to $D = 4$ dimensions and are denoted moduli fields. The latter can generate severe problems for cosmology [33, 34, 35]. First of all, moduli typically obtain a mass of the order of the gravitino mass $m_{3/2}$ and couple only weakly through gravitational interactions to particles in the observable sector. Hence, they can easily survive until after BBN. If they do not decay until after BBN, their large matter-like energy density overcloses the universe. Therefore, they must decay before BBN, which is referred to as the *cosmological moduli problem* [32, 98, 99]. Even if they decay before BBN, but later than about $10^{-2}\,$s after inflation, their decay products inject additional photons, hadrons and leptons during BBN, ruining its successful predictions. This puts a lower bound of about $30\,$TeV on the moduli masses [100, 101, 102]. Furthermore, the potentials of the moduli can have a runaway behavior which, lacking a mechanism that stabilizes them, quickly leads to a dominance of their kinetic energy. This endangers any stage of inflation and we refer to it as the moduli stabilization problem.

Our motivation is to look for viable simultaneous solutions to both the η-problem as well as the moduli stabilization problem during inflation. In some sense, this is basically a scaling problem because we want to have all scalar DOFs besides the inflaton stable in their minima with masses larger than the Hubble scale. At the same time we require the inflaton mass to obey the opposite condition. This induces a hierarchy of scales $m_{\mathrm{mod}} > \mathcal{H} > m_{\mathrm{inf}}$.

The chapter consists of two sections. Sec. 5.1 gives a short description of what the η-problem is in general and how it arises in SUGRA theories. In addition, we present some possible solutions common in the literature. The moduli stabilization problem is introduced in Sec. 5.2.

5.1 The η - Problem

From an effective field theory viewpoint, for any gauge singlet or non-singlet field ϕ, non-renormalizable higher dimensional operators like

$$V_{\mathrm{eff}}(\phi) = V_0 \sum_n \frac{(\phi^*\phi)^n}{M_{\mathrm{P}}^{2n}}\,, \tag{5.1}$$

cannot be forbidden in the potential. Here V_0 denotes the vacuum energy. Already the first term in such an expansion ($n = 1$) induces a large contribution to the inflaton mass proportional to the vacuum energy density, i.e., $V'' \sim V_0$. Plugged in the formula for the slow-roll parameter η defined in (2.17), this generically spoils inflation due to a leading contribution $\eta \approx 1$.

Within SUGRA theories, this so-called η-problem typically appears, since gravity couples to everything and thus also induces a coupling of all the fields to the vacuum energy density V_0. Especially in the F-term contribution to the scalar potential given by Eq. (3.67) this is obvious, since for a minimal Kähler potential $K = \phi^*\phi$ giving rise to canonical kinetic terms, an expansion of the exponential in the tree level potential leads to the form

$$V_F \sim \left(1 + \frac{\phi^*\phi}{M_{\mathrm{P}}^2} + \ldots\right) V_0\,. \tag{5.2}$$

Comparing (5.2) to (5.1), we note that it is exactly these dangerous terms that reappear in the F-term potential of a SUGRA theory. This states the η-problem of SUGRA inflation [31, 32].

Note that for small field inflation models[1] with $\phi < M_{\mathrm{P}}$, the $n = 1$ contribution of Eq. (5.1) is bad enough to spoil inflation. The η-problem becomes a lot more severe when dealing with large field models with $\phi > M_{\mathrm{P}}$. This is due to the fact that then, an expansion as in Eq. (5.1) breaks down, since every higher order contributes an even larger mass making the η-problem a catastrophe.

In order to control or even solve the η-problem altogether, there are several different approaches proposed in the literature. First of all, if the scalar potential is not dominated by the F-term contribution, but instead arises mainly from the D-term part (3.76), the η-problem is simply not present. The reason is that in this scenario of *D-term inflation* [103], the exponential factor e^K does not appear. Nevertheless, we are interested in building F-term models of inflation in SUGRA and thus do not further elaborate on the idea of D-term inflation.

With the focus on the F-term part of the scalar potential Eq. (3.67), we are basically left with two possible ways to solve the η-problem. Lacking an ultraviolet complete description, the Kähler potential can be an arbitrary real function of the scalar components of the chiral superfields. One proposal is to stick to the most general non-minimal Kähler potential as

[1] Which include the hybrid inflation models discussed in this work.

an expansion in terms of all scalar fields present [44, 104, 105]

$$K = \phi_i^* \phi^i + \sum_{n=2} \kappa_i^{(n)} \frac{(\phi_i^* \phi^i)^n}{M_{\rm P}^{2n-2}} + \sum_{k=m+n} \kappa_{ij}^{(k)} \frac{(\phi_i^* \phi^i)^n (\phi_j^* \phi^j)^m}{M_{\rm P}^{2k-2}} + \dots\,, \tag{5.3}$$

where the leading terms give rise to canonical kinetic terms and $\phi_i^* = \delta_{i\bar{j}}\, \phi^{*\bar{j}}$. For the higher order terms with expansion parameters $\kappa_i^{(n)}, \kappa_{ij}^{(k)}, \dots$, one obtains off-diagonal contributions to the Kähler metric. This Ansatz solves the η-problem if the expansion parameters are tuned in such a way that the scalar potential is flat enough for slow-roll inflation.

The second possibility is to apply some fundamental symmetry on the Kähler potential to forbid operators as in Eq. (5.2) for the inflaton direction which give rise to the η-problem. One common feature of a solution by symmetry arguments is that only the invariant field combination ρ under the symmetry appears explicitly in the Kähler potential

$$K = k(\rho)\,, \tag{5.4}$$

where ρ is a DOF different from the inflaton direction which protects the latter from obtaining large SUGRA mass corrections. Within the remainder of the thesis, we extensively study such symmetry solutions. If one applies these, one should make sure that the symmetry allows for canonical normalization of the kinetic terms in a not too complicated manner.

The simplest candidate symmetry which can account for the aforementioned properties is a Nambu–Goldstone-like *shift symmetry* [106, 107, 108] under which the complex scalar containing the inflaton direction transforms as

$$\phi \to \phi + {\rm i}\,\mu\,. \tag{5.5}$$

Here, μ denotes a real transformation parameter. The invariant combination in the Kähler potential is then given by

$$\phi + \phi^* = 2\,{\rm Re}(\phi)\,. \tag{5.6}$$

Therefore, the imaginary part ${\rm Im}(\phi)$ is a good inflaton direction since it gets protected by the shift symmetry Eq. (5.5).

Another more involved symmetry is called *Heisenberg symmetry* and is based on non-compact Heisenberg group transformations of two or more complex scalar fields as

$$\begin{aligned} T &\to T + {\rm i}\,\mu\,, \\ T &\to T + \alpha_i^* \phi^i + \frac{\alpha_i^* \alpha^i}{2}\,, \\ \phi^i &\to \phi^i + \alpha^i\,, \end{aligned} \tag{5.7}$$

where μ is again a real transformation parameter and the α^i are complex transformation parameters. The complex scalar field T, belonging to a chiral supermultiplet, is a modulus field associated with the Heisenberg symmetry. It was first discussed in the context of string-inspired models in Ref. [109] and its use for inflation model building has been studied in Refs. [110, 111], however, lacking an explicit model and its predictions. The invariant DOF under the transformations in Eq. (5.7) is given by

$$\rho = T + T^* - \phi_i^* \phi^i \,. \tag{5.8}$$

If the Kähler potential satisfies the symmetry and thus depends on ρ only, the $|\phi^i|$ are viable inflaton directions as we show in the next two chapters. Also, as we shall discuss in more detail, the Heisenberg symmetry has another advantage. Going to a basis where instead of the fundamental DOFs $\{\phi, \mathrm{Re}(T)\}$, we treat the DOFs $\{\phi, \rho\}$ as independent, the Kähler metric and thus the kinetic terms are diagonal. This facilitates the task of canonical normalization.

5.2 The Moduli Stabilization Problem

As mentioned in the introduction to this chapter, we are also concerned with the problem of stabilizing any additional modulus field during inflation which is present in the theory. Our objective in this section is to outline the problem in a simple example.

Therefore, we assume the modulus sector chiral superfield T to have a vanishing superpotential and a *no-scale* Kähler potential [112, 113] given by

$$K_{\mathrm{mod}} = -3 \ln(T + T^*) = -3 \ln \tau \,. \tag{5.9}$$

Note that this Kähler potential includes a shift symmetry in the modulus sector according to Eq. (5.5) with an invariant combination $\tau = T + T^*$. Without any other sector involved, Eq. (5.9) gives rise to a vanishing cosmological constant due to $V_F = 0$ and a gravitino mass $m_{3/2}$ which is undetermined at tree level. The only input mass scale is the Planck scale M_{P}. It has been shown in Refs. [114, 115, 116, 117, 118] that such no-scale Kähler potentials typically arise in the compactification of ten-dimensional supergravity on Calabi–Yau manifolds.

In addition, let us assume that we have SUSY breaking realized by some hidden sector chiral superfields F-term. We use the simple linear *Polonyi superpotential* [119] given by a singlet S under all symmetries and a minimal Kähler potential giving rise to canonically normalized kinetic terms reading

$$W = S\,\mu^2 \,, \qquad K_{\mathrm{hid}} = |S|^2 \,, \tag{5.10}$$

where μ determines the SUSY breaking scale. As we will see below, the SUSY breaking minimum of the F-term scalar potential in our simple setup is given by $S = 0$. The F-term that breaks SUSY obviously reads $\mathcal{D}_S W|_{S=0} = \mu^2$. This also induces a non-vanishing F-term scalar potential as calculated below.

Putting together Eqs. (5.9) and (5.10), the full example toy model is given by

$$W = S\,\mu^2 \,, \qquad K = K_{\mathrm{mod}} + K_{\mathrm{hid}} \,, \tag{5.11}$$

which, using Eq. (3.61), results in a diagonal Kähler metric in the (S, T)-basis

$$K_{i\bar{j}} = \begin{pmatrix} 1 & 0 \\ 0 & 3/\tau^2 \end{pmatrix} . \tag{5.12}$$

Therefore, kinetic mixing between the two sectors is absent. In addition, the F-term scalar potential as taken from Eq. (3.67) simplifies to[2]

$$V_F = \mathrm{e}^{-3\ln\tau}\, \mathrm{e}^{|S|^2} \left(K^{S\bar{S}}\, \mathcal{D}_S W\, \mathcal{D}_{\bar{S}} W^* + K^{T\bar{T}}\, \mathcal{D}_T W\, \mathcal{D}_{\bar{T}} W^* - 3\,|W|^2 \right) , \tag{5.13}$$

where due to the no-scale form of the modulus Kähler potential, the second and the last term in the brackets cancel exactly. The first term in Eq. (5.13) is only dependent on the Polonyi field S and thus a coupling between S and T is merely induced gravitationally via the exponential pre-factor e^K. Thus the F-term scalar potential reduces to the very simple form

$$V_F = \frac{\mu^4}{\tau^3} \left(1 + |S|^2\right)^2 \mathrm{e}^{|S|^2} . \tag{5.14}$$

Since the potential factorizes, the minimum in S-direction is independent of the value of τ. Minimizing Eq. (5.14) w.r.t. the $|S|$-direction, we find that the global minimum of the potential is located at $S = S^* = 0$ as noted above.

Assuming that S is stabilized in its minimum for field values of $\tau \neq 0$, the remaining scalar potential in τ-direction is of the runaway-type

$$V_F|_{S=0} = \mu^4/\tau^3 . \tag{5.15}$$

If any such modulus field direction with a scalar potential as in Eq. (5.15) is present in addition to the slow-roll inflaton potential, we encounter a cosmological problem. For any initial field value of τ, the modulus field immediately starts to accelerate and due to the runaway potential quickly $\partial_0\tau \to \infty$ as $\tau \to \infty$. This implies that the condition Eq. (2.14) gets violated and hence, slow-roll inflation ends.

The above argument highlights the importance of the condition that all scalar field directions which are present in the theory in addition to the inflaton field should not have a runaway potential. Therefore, we need some mechanism at work which generates a stable potential minimum for the moduli fields (at least during the phase of inflation) giving them a mass $m_\tau > \mathcal{H}$. Simultaneously, this mechanism must not affect the flatness of the inflaton direction. This is what we refer to as the moduli stabilization problem during inflation.

In the literature, moduli stabilization has mostly been realized invoking nonperturbative effects such as gaugino condensation or instantons. In the seminal work referred to as the KKLT model [35], an exponential superpotential induced by non-perturbative effects helps to stabilize the volume modulus. This model has been modified by an extra non-perturbative exponential term to allow for a SUSY Minkowski vacuum in Refs. [120, 121],

[2]Note that here and in the following, we denote the chiral superfields and their complex scalar component fields by the same symbols if the distinction is obvious.

which we refer to as the KL model. In the following chapters, we demonstrate how in our setup moduli stabilization during inflation can be achieved by different means. We use perturbative effects by higher dimensional effective operators in the Kähler potential. These allow for a large modulus mass due to the coupling of the modulus to the large vacuum energy density during inflation.

Chapter 6

Hybrid vs. Tribrid Inflation Models

In this chapter, we discuss possible realizations of hybrid inflation within the context of SUGRA. We focus on solutions to the η-problem of SUGRA inflation and the moduli stabilization problem, which we have explained in Ch. 5.

First of all, we review the standard SUSY hybrid inflation model introduced in [31, 38] and discuss the possibilities of its SUGRA embeddings [31, 105] and generic problems that arise when imposing symmetries on the Kähler potential [122, 123].

Furthermore, we introduce the new class of tribrid inflation models, based mainly on Refs. [1, 2, 124] which is motivated by sneutrino inflation [44]. The basic features of these models within the global SUSY framework are discussed before we show explicit SUGRA realizations using three different Kähler potentials.

One of our main objectives in this thesis is to explain the absence of operators in the Kähler potential, giving rise to the η-problem, by fundamental symmetries. Important terms which can break such symmetries shall leave the model natural in 't Hooft's sense [125]. For the hybrid and for the tribrid inflation models, we study such symmetry solutions as well as their counterparts, where the Kähler potential is just taken to be a general expansion in terms of all the fields present in the theory.

Since we want to focus mainly on the issue of realizing such models in SUGRA and postpone the problems related to realizing inflation in non-singlet reps under some gauge group to Part IV, we work with gauge singlet chiral superfields only. Within the chapters of this part, we do not give any justification by symmetries but rather investigate the typical features of the superpotentials.

This chapter is structured as follows. Sec. 6.1 is dedicated to reviewing the standard SUSY hybrid inflation superpotential and its properties. In Sec. 6.1.1, we describe its combination with a general Kähler potential expansion. In Sec. 6.1.2, the problems of hybrid inflation in combination with a shift symmetry imposed on the Kähler potential are described. Sec. 6.2 is dedicated to presenting the tribrid inflation superpotential and its defining properties. Possible combinations of the tribrid-inflation-type superpotential with a general Kähler potential expansion in Sec. 6.2.1, with a shift symmetric Kähler potential in Sec. 6.2.2 and with a Heisenberg symmetric Kähler potential in Sec. 6.2.3 are subsequently introduced.

6.1 Supersymmetric Hybrid Inflation

To begin with, let us summarize the basic features of standard F-term SUSY hybrid inflation as presented in [31, 38]. The superpotential of global SUSY hybrid inflation is given by the overall form[1]

$$W = \kappa\,\Phi\left(H^2 - M^2\right)\,, \tag{6.1}$$

where the superfield content consists of the inflaton Φ and the waterfall field H. The parameters are a dimensionless coupling κ and the symmetry breaking scale M which basically sets the scale of inflation, namely the SUSY breaking scale during inflation and the VEV of H after inflation.

Note that during inflation, when $\Phi \neq 0$ and $H = 0$, the superpotential, as well as its first derivative are non-zero

$$W|_{H=0} = -\kappa\,M^2\Phi\,, \qquad W_\Phi|_{H=0} = -\kappa\,M^2\,. \tag{6.2}$$

At this point, this may not seem very important but it is in fact crucial to distinguish the superpotential above from other model classes considered in Sec. 6.2.

Due to the absence of a D-term part for the scalar potential without gauge interactions, we only need to consider the F-term part which, plugging (6.1) into (3.38), can be calculated to[2]

$$V_F = \kappa^2\left|\left(H^2 - M^2\right)\right|^2 + 4\,\kappa^2\,|\Phi|^2|H|^2\,. \tag{6.3}$$

Basically, the potential (6.3) has two minima. The SUSY conserving vacuum with $V_F = 0$ is given by the VEVs $\langle\Phi\rangle = 0$ and $\langle H\rangle = \pm M$ and a SUSY breaking inflationary vacuum $V_F = \kappa^2 M^4$ at $\langle H\rangle = \langle H^*\rangle = 0$ stabilized by a large positive mass squared contribution for $|\Phi| > |\Phi_\mathrm{c}|$ with the critical value $|\Phi_\mathrm{c}| = M/\sqrt{2}$. In this false vacuum, the large vacuum energy density drives quasi-exponential expansion of the scale factor. Note that for $|\Phi| > |\Phi_\mathrm{c}|$, the tree level potential is exactly flat in the canonically normalized inflaton direction $\phi \equiv \sqrt{2}\,|\Phi|$.

Thus, in order to drive the inflaton towards its critical value, one needs to take quantum corrections into account. At the one-loop level, the correction to the potential is of the CW type [58], cf. Eq. (2.47). The CW one-loop radiative correction to the effective potential in a supersymmetric theory [58, 59, 60] is given by

$$V_\mathrm{loop}(\phi) = \frac{1}{64\,\pi^2}\,\mathrm{STr}\,\mathcal{M}^4(\phi)\left(\ln\frac{\mathcal{M}^2(\phi)}{Q^2} - \frac{3}{2}\right)\,, \tag{6.4}$$

where $\mathcal{M}$ is the mass matrix and Q is the renormalization scale. Since SUSY is broken in the inflationary minimum by the F-term of Φ, the supertrace over all bosonic and fermionic DOFs entering the loop potential does not vanish and we obtain the effective potential

$$V_\mathrm{eff}(\phi) = V_F + V_\mathrm{loop}(\phi)\,. \tag{6.5}$$

[1] As mentioned in the introduction to this chapter, since we only work with gauge singlet chiral superfields, instead of a superfield H and its conjugate $\bar{H}$, we simply replace invariant operators such that $H\bar{H} \to H^2$.

[2] Assuming that κ is real.

This comes about due to ϕ-dependent masses of the components of the chiral superfield H. To be explicit, these are given by the Weyl fermion χ_H, whose mass calculated from Eq. (6.1) is given by

$$m_{\rm F} = \sqrt{2}\,\kappa\,\phi\,, \tag{6.6}$$

and the squared masses of the real and pseudoscalar part of the scalar component of H, which read

$$m_{\rm R}^2 = 2\,\kappa^2\phi^2 - 2\,\kappa^2 M^2\,, \qquad m_{\rm P}^2 = 2\,\kappa^2\phi^2 + 2\,\kappa^2 M^2\,. \tag{6.7}$$

Thus, the radiative correction to the effective potential along the inflationary trajectory, $\phi > \phi_{\rm c}$ and $|H| = 0$, from Eq. (6.4) reads

$$V_{\rm loop}(\phi) = \frac{1}{64\,\pi^2}\left[m_{\rm R}^4\left(\ln\frac{m_{\rm R}^2}{Q^2} - \frac{3}{2}\right) + m_{\rm P}^4\left(\ln\frac{m_{\rm P}^2}{Q^2} - \frac{3}{2}\right) - 2\,m_{\rm F}^4\left(\ln\frac{m_{\rm F}^2}{Q^2} - \frac{3}{2}\right)\right]\,, \tag{6.8}$$

and it drives the inflaton towards the critical value $\phi_{\rm c}$ close to which slow-roll inflation should end. For $\phi \gg \phi_{\rm c}$ this can be Taylor expanded to

$$V_{\rm loop}(\phi) \simeq \frac{\kappa^4 M^4}{8\,\pi^2}\left(\ln\frac{2\,\kappa^2\phi^2}{Q^2} - \frac{3}{2}\right)\,. \tag{6.9}$$

The inflationary predictions of this model can be calculated from the slow-roll parameters defined in Eq. (2.17). Assuming that the end of inflation[3] occurs at $\phi_{\rm c}$, and that scales corresponding to the present horizon size exited the de Sitter horizon 60 e-folds before inflation ended, we can use the slow-roll EOMs (2.16) to determine the corresponding field value. Exploiting Eq. (6.9), it can be approximated to $\phi^2(N_{\rm e}) \simeq \phi_{\rm c}^2 + \kappa^2 N_{\rm e}/2\,\pi^2 \simeq \kappa^2 N_{\rm e}/2\,\pi^2$. In addition, the WMAP normalization on the amplitude of the scalar metric perturbation $\mathcal{P}_{\mathcal{R}}^{1/2} \sim \sqrt{N_{\rm e}}\,M^2 \approx 5\cdot 10^{-5}$ taken from Eq. (2.41) basically fixes the scale $M \approx 10^{-3}$. Using the above approximations, the scalar spectral index and the tensor-to-scalar ratio respectively can be estimated to

$$n_{\rm s} \simeq 1 - \frac{1}{N_{\rm e}} \simeq 0.98\,, \qquad r \simeq \frac{\kappa^2}{N_{\rm e}\,\pi^2}\,. \tag{6.10}$$

The tensor-to-scalar ratio is generically very small, e.g., for $\kappa = \mathcal{O}(1)$, it has an upper bound $r \lesssim 10^{-3}$. The predictions in Eq. (6.10) are very robust and typical for global SUSY models with a superpotential of the form (6.1).

However, since we are interested in an embedding of inflation in SUGRA, we also need to specify a Kähler potential for the aforementioned model. In the following, we combine the superpotential of Eq. (6.1) with different Kähler potentials. Viable possibilities as well as problematic issues are discussed.

[3]Note that the slow-roll conditions can be violated at field values slightly different from the critical value. Since this gives rise to a difference of a few e-folds only, which does not affect the predictions significantly, we assume that inflation ends at $\phi_{\rm c}$ for all practical purposes.

6.1.1 Hybrid Inflation with Kähler Expansion

In this section, we review how the hybrid inflation superpotential (6.1) can be combined with a minimal Kähler potential and a non-minimal Kähler potential expansion in terms of higher dimensional effective operators. The section is based on [31, 105].

Let us consider the SUSY hybrid inflation superpotential (6.1) combined with a Kähler potential as an expansion up to mass dimension four in the scalar component fields given by

$$K = |\Phi|^2 + |H|^2 + \frac{\kappa_\Phi}{4\,\Lambda^2}\,|\Phi|^4 + \frac{\kappa_H}{4\,\Lambda^2}\,|H|^4 + \frac{\kappa_{\Phi H}}{\Lambda^2}\,|\Phi|^2|H|^2 + \ldots\,, \tag{6.11}$$

where in the following, we set the suppression scale of the effective operators to $\Lambda = 1$.

The minimal Kähler potential, which gives rise to canonical kinetic terms without kinetic mixing is just given by a sum of the absolute values squared of the scalar components in all chiral supermultiplets. Thus, as the simplest SUGRA extension of the standard SUSY hybrid inflation, let us first study a minimal Kähler potential by switching off all higher dimensional operators setting $\kappa_\Phi = \kappa_H = \kappa_{\Phi H} = \ldots = 0$.

As stated in Ch. 5, we want to study the viability of inflation models in SUGRA regarding the η-problem and the problem of moduli stabilization. In the simple case at hand, there are no additional moduli fields involved, so we only have to study the SUGRA corrections to the tree level inflaton mass. Using Eq. (3.67), the F-term scalar potential obtained from Eq. (6.11) reads

$$V_F = \mathrm{e}^{|\Phi|^2+|H|^2}\left[|(W_\Phi + W K_\Phi)|^2 + |(W_H + W K_H)|^2 - 3|W|^2\right]\,, \tag{6.12}$$

since $K^{i\bar{j}} = \delta^{i\bar{j}}$.

As explained in Sec. 5.1, the exponential factor in Eq. (6.12) contributes the most serious potential source of the η-problem in SUGRA inflation. It turns out that with a minimal Kähler potential, to leading order in the SUGRA expansion, the mass squared terms for the inflaton exactly cancel. This is a very special and desirable feature of minimal SUSY hybrid inflation since mysteriously, the η-problem is not present in this simple case, as has been pointed out in [97, 105]. Let us push on to see how this comes about. Expanding Eq. (6.12), one obtains to leading order

$$V_F \simeq 4\,\kappa^2|\Phi|^2|H|^2 + \kappa^2\left(|H|^2 - M^2\right)^2\left(1 + |H|^2 + \frac{|\Phi|^4}{2} + |\Phi|^2|H|^2\right) + \ldots\,, \tag{6.13}$$

where the positive mass squared contribution for $|\Phi|$ comes from the exponential in (6.12) and the negative one from the sum of terms within the the squared brackets. They cancel and there is no dangerous tree level mass term for the inflaton at leading order.

Just as in the case of global SUSY hybrid inflation, the waterfall direction obtains its large mass squared contribution for $|\Phi| > |\Phi_\mathrm{c}|$. This stabilizes it at zero during the inflationary epoch. With a canonically normalized inflaton direction $\phi = \sqrt{2}\,|\Phi|$, the resulting one-loop effective potential in the inflationary minimum $|H| = 0$ is approximately given by

$$V_\mathrm{eff}^\mathrm{min}(\phi) \simeq \kappa^2 M^4\left(1 + \frac{\phi^4}{8} + \ldots\right) + V_\mathrm{loop}^\mathrm{min}(\phi)\,. \tag{6.14}$$

The one-loop correction $V_{\rm loop}^{\rm min}(\phi)$ is again given by Eq. (6.4), however, with the mass matrices calculated using the SUGRA formulae displayed in Sec. 3.2. Note that the difference to the global SUSY potential is the ϕ^4-dependence of the tree level potential in Eq. (6.14) generated by SUGRA corrections. Since typically in the hybrid models we are interested in $\phi \ll 1$, these corrections are subdominant in the relevant inflationary field space. As κ is increased, the field values at a fixed number of e-folds increases and the SUGRA effects become more important. This can be seen, e.g., in Fig. 6.1, where the spectral index is driven to larger values for increasing κ.

Next, let us explore the effect of the higher dimensional operators in the Kähler potential of Eq. (6.11). Due to the resulting non-diagonal Kähler metric, the F-term potential contains many additional terms. Since this is not very enlightening, we only give the expansion

$$V_F \simeq 4\,\kappa^2|\Phi|^2|H|^2 + \kappa^2\left(|H|^2 - M^2\right)^2\left(1 - \kappa_\Phi\,|\Phi|^2 + \gamma_H\,|H|^2 + \gamma_\Phi\,\frac{|\Phi|^4}{2}\right) + \ldots\,, \tag{6.15}$$

where the new parameters have been defined as

$$\begin{aligned}\gamma_H &= 1 - \kappa_{\Phi H}\,,\\ \gamma_\Phi &= 1 - \frac{7}{2}\,\kappa_\Phi + 2\,\kappa_\Phi^2\,.\end{aligned} \tag{6.16}$$

Given the tree level F-term potential, we can again calculate the one-loop effective potential in the inflationary trajectory, $\phi \gg \phi_{\rm c}$ and $|H| = 0$, given by

$$V_{\rm eff}^{\rm non\text{-}min}(\phi) \simeq \kappa^2 M^4\left(1 - \kappa_\Phi\,\frac{\phi^2}{2} + \gamma_\Phi\,\frac{\phi^4}{8} + \ldots\right) + V_{\rm loop}^{\rm non\text{-}min}(\phi)\,. \tag{6.17}$$

Note, that the effective operator corresponding to κ_Φ induces a (negative) mass squared term at tree level. Thus, fulfilling the slow-roll conditions requires a tuning to make these parameters somewhat small. A parameter $\kappa_\Phi = \mathcal{O}(1)$ would be unacceptable because it is inconsistent with slow-roll inflation. As it should be, switching off all higher dimensional operators by setting $\kappa_\Phi = \kappa_H = \kappa_{\Phi H} = \ldots = 0$, Eq. (6.17) just reproduces the potential resulting from the minimal Kähler potential in Eq. (6.14). Since the Kähler potential is in principle arbitrary, we consider the minimal Kähler potential to be a specific version of the more general expansion of Eq. (6.11) where all higher order expansion parameters have been set to zero.

In order to study the effect of the crucial model parameter κ_Φ, we fix the number of e-folds of observable inflation to $N_{\rm e} = 60$. From the critical value $\phi_{\rm c}$, we use the slow-roll EOM Eq. (2.16) applied to the potential Eq. (6.17) to calculate back to the field value at which scales corresponding to the present horizon size crossed the de Sitter horizon during inflation. As before, the WMAP normalization on $P_{\mathcal{R}}$ in Eq. (2.41) then fixes the scale $M = \mathcal{O}(10^{-3})$ for the relevant parameters. We have implemented this calculation numerically.

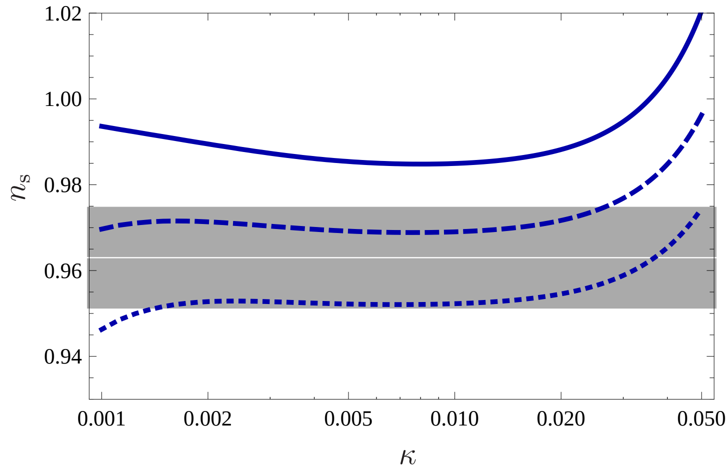

Figure 6.1: Predicted spectral index depending on the fundamental coupling parameter κ for different values of κ_Φ. The solution for minimal Kähler potential is represented by the solid line ($\kappa_\Phi = 0$), while higher dimensional operators are taken into account in the dashed ($\kappa_\Phi = 5 \cdot 10^{-3}$) and dotted ($\kappa_\Phi = 10^{-2}$) lines. The gray shaded region corresponds to the region preferred by the latest WMAP data at 68% CL.

The resulting spectral index for different values of κ_Φ depending on the model parameter κ is displayed in Fig. 6.1, where for convenience we have set all other model parameters to zero. The case of a minimal Kähler potential is then just obtained by setting $\kappa_\Phi = 0$, which corresponds to the solid line in the plot. This reproduces the results of Ref. [105]. For $\kappa_\Phi \simeq 10^{-3} - 10^{-2}$, the predicted value can lie well inside the region preferred by the seven-year WMAP data at 68% CL which is highlighted in grey. Successively larger values of κ_Φ can effectively reduce the spectral tilt which is due to the fact that it contributes to a negative squared mass pushing η to more negative values. Looking at Eq. (2.38), this leads to a smaller spectral tilt n_s. As in the global SUSY result, the predicted tensor-to-scalar ratio is again very small.

6.1.2 Hybrid Inflation with Shift Symmetry

As we have seen in the last section, a general Kähler potential which is not restricted to any particular form typically gives rise to the η-problem. The latter comes about by contributing operators as in Eq. (5.1) to the scalar potential. One can now either tune the expansion parameters of such higher order operators to be small or one can try to apply a symmetry that forbids them.

In this section, we impose a shift symmetry[4] as in Eq. (5.5) to protect the inflaton direction from large SUGRA mass contributions. This is based on Refs. [122, 123] which have pointed out severe problems for realizing F-term SUGRA hybrid inflation in this way. As it turns out, without further tuning or strong restrictions on the couplings between the

[4] A shift symmetry can also arise in the context of string cosmology, see, e.g., Refs. [126, 127, 128].

inflaton and additional moduli sectors, F-term hybrid inflation does not work.

Again, we consider the superpotential of F-term SUSY hybrid inflation (6.1), however this time using a Kähler potential of the form

$$K = \frac{1}{2} (\Phi + \Phi^*)^2 + |H|^2 \,, \tag{6.18}$$

which is invariant under the shift symmetry transformation (5.5) in the scalar component of the inflaton chiral superfield Φ. The waterfall superfield H is just subject to a minimal Kähler potential. A nice feature of the Kähler potential (5.5) is that the shift symmetric form does not destroy canonical normalization of the kinetic terms, since just as for the minimal Kähler potential $K_{i\bar{j}} = \delta_{i\bar{j}}$.

We can decompose the inflaton scalar component into canonically normalized real and imaginary DOFs according to

$$\Phi = \frac{1}{\sqrt{2}} (\phi_{\mathrm{R}} + \mathrm{i}\, \phi) \ . \tag{6.19}$$

In Eq. (6.18), the shift symmetry protects the canonically normalized imaginary part $\phi = \sqrt{2}\,\mathrm{Im}(\Phi)$ from obtaining large SUGRA corrections while the latter stabilize the real part $\phi_{\mathrm{R}} = \sqrt{2}\,\mathrm{Re}(\Phi)$ at zero. Thus, the imaginary part ϕ would be a promising inflaton candidate. The problem is however, that in the F-term scalar potential Eq. (3.67) the term $-3\,|W|^2$, which compensated positive mass squared terms in the case of a minimal Kähler potential, does not get cancelled anymore. In the inflationary trajectory, the resulting F-term potential reads

$$V_F|_{\phi_{\mathrm{R}}=|H|=0} \simeq \kappa^2 M^4 \left(1 - \frac{3}{2}\,\phi^2\right) \ . \tag{6.20}$$

This induces a tree level mass squared

$$m_\phi^2 = -3\,\kappa^2 M^4 \,, \tag{6.21}$$

which renders the inflaton direction tachyonic with a large negative curvature $\eta \approx -3$ that violates the slow-roll conditions.

It has been pointed out in [123] that the tachyonic inflaton mass can be cured by invoking a modulus field T with a Kähler potential of no-scale form (5.9) that should be added to Eq. (6.18). This would work under the condition that the modulus field gets stabilized by some mechanism. The authors' conclusion when adding either a KKLT- or KL-type superpotential [35, 121] for the modulus field is, that F-term hybrid inflation does not work, independent of the form of the Kähler potential. This comes from the fact that small variations in the moduli minima reintroduce a large negative curvature in the inflaton direction with $\eta \approx -3$. Therefore, any modulus dynamics brings along a new η-problem and slow-roll inflation can be ruled out.

It is only in very special setups that the F-term hybrid inflation superpotential can be combined with a shift symmetric Kähler potential to give rise to viable inflation. Either one accepts a lot of fine-tuning or one multiplies the inflaton and the modulus superpotentials.

Even though in the latter case viable inflation with simultaneous modulus stabilization can successfully work, constraints on the moduli sector parameters remain.

In the following section, we introduce an alternative class of superpotentials which can give rise to hybrid inflation in SUGRA. One promising feature of this new class is the absence of any additional inflaton mass contributions from inside the square brackets in the F-term scalar potential Eq. (3.67). The presence of such a contribution was the decisive factor in ruling out the model above. Therefore, the new class of models turns out to have the right properties suitable for solving the η-problem via symmetries in the Kähler potential.

6.2 Supersymmetric Tribrid Inflation

In this section, we want to discuss in detail a new class of SUSY hybrid inflation models referred to as *tribrid inflation*. The content of the section is mainly based on the publications [1, 2, 124]. First of all, the name tribrid is used because instead of two chiral superfields, we have an extended field content of three superfields $\{H, S, \Phi\}$ instead of two. Again, Φ denotes the superfield which contains the slowly rolling inflaton direction as its scalar component and H is the waterfall superfield whose scalar component gets destabilized and triggers the end of inflation below some critical value of Φ. Furthermore, we introduce the superfield S whose task is to contribute the SUSY breaking vacuum energy that drives inflation by its F-term. Also, the F-term drives the VEV of the waterfall H and can thus be called a driving field. In this sense, the two important tasks of a slowly rolling inflaton field and a field that induces SUSY breaking by a linear term in the superpotential are accounted for by two separate fields. Remember that in the hybrid scenario described in Sec. 6.1, both of these tasks where simultaneously taken care of by Φ.

Let us start by considering the following general framework where the superpotential is of the form

$$W = \kappa\, S \left(H^2 - M^2\right) + g(\Phi, H)\,, \tag{6.22}$$

where M fixes the scale of inflation and the VEV of H after inflation just as in the SUSY hybrid inflation case. As before, κ is a dimensionless parameter assumed to be real. The above superpotential is very general and we will have to further specify our model by choosing a particular functional dependence of $g(\Phi, H)$. As main features of the general framework we require that

$$W = 0\,, \qquad W_\Phi = 0\,, \qquad W_H = 0\,, \qquad W_S \neq 0\,, \tag{6.23}$$

while S and H both stay at zero during inflation. It has been emphasized in [129] that these criteria are desirable for solving the η-problem using some specific symmetries of the Kähler potential.

The conditions Eqs. (6.23) also restrict the possible analytical functions $g(\Phi, H)$. For all practical purposes in this thesis, we use the non-renormalizable effective dimension five operator

$$g(\Phi, H) = \frac{\lambda}{M_*}\,\Phi^2 H^2\,, \tag{6.24}$$

which is inspired by sneutrino hybrid inflation [44]. A further motivation for using the effective operator (6.24) is postponed to Part IV of this work where it proves to be very useful for realizing viable gauge non-singlet inflation. λ is a dimensionless parameter assumed to be real and M_* is a suppression mass scale at which the above operator is effectively generated. The purpose of the operator (6.24) is to provide a positive Φ-dependent mass squared for H during inflation as long as $|\Phi| \gg |\Phi_c|$. When the waterfall field H finally gets destabilized below the critical value, inflation ends and H acquires a large VEV $\langle |H| \rangle = M$. This generates a large mass for the inflaton superfield Φ after inflation.

One can easily check that Eq. (6.24) fulfills the conditions (6.23) as long as[5] $S = H = 0$. At the global SUSY level, there is a priori no justification for assuming $S = 0$ since it is a massless mode, however, in the SUGRA context discussed below, SUGRA corrections can easily account for a large S-mass which stabilizes it at zero. For pedagogical reasons however, we derive the tribrid scalar potential in global SUSY first, keeping in mind that $S = 0$ can be realized when embedding the model in SUGRA.

To summarize, the tribrid inflation superpotential we study in this thesis is given by[6]

$$W = \kappa\, S\left(H^2 - M^2\right) + \frac{\lambda}{M_*}\,\Phi^2 H^2\,, \tag{6.25}$$

and using Eq. (3.38) we can derive the F-term scalar potential

$$V_F = \kappa^2\left|\left(H^2 - M^2\right)\right|^2 + 4\left|\left(\kappa\, S\, H + \frac{\lambda}{M_*}\,\Phi^2 H\right)\right|^2 + 4\left|\frac{\lambda}{M_*}\,\Phi\, H^2\right|^2 . \tag{6.26}$$

Assuming that $S = 0$, the scalar potential simplifies to

$$V_F = \kappa^2\left|\left(H^2 - M^2\right)\right|^2 + 4\left(\frac{\lambda}{M_*}\right)^2\left[|\Phi|^4|H|^2 + |\Phi|^2|H|^4\right], \tag{6.27}$$

where the negative mass squared contribution for H is exactly the same as in Eq. (6.3), arising however from the F-term of the driving field S. During inflation, it gets overcompensated by the positive contribution $\sim |\Phi|^4|H|^2$ from the effective operator thus stabilizing H at zero.

The inflaton-dependent waterfall sector mass spectrum during inflation can be calculated in a straightforward way. Again, canonically normalized scalar fields shall be used. The inflaton direction is given by $\phi = \sqrt{2}\,|\Phi|$, whereas the waterfall field decomposes into

[5] Note that also different operators such as the renormalizable $g(\Phi, H) = \lambda\,\Phi\,H^2$ fulfill the conditions Eqs. (6.23) and can give rise to viable inflation. The predictions derived from the resulting potentials are very similar to the ones obtained for Eq. (6.24) in the remainder of this section.

[6] We note that the minimal superpotential of Eq. (6.25) may be justified by introducing additional symmetries and spurion fields, as outlined in App. B.2.

normalized real and imaginary parts as $H = (h_\mathrm{R} + \mathrm{i}\, h_\mathrm{I})/\sqrt{2}$. For convenience, we introduce the new dimensionless variable

$$x \equiv \left(\frac{\lambda}{\kappa}\right)^2 \frac{\phi^4}{2\,(M M_*)^2}\,. \tag{6.28}$$

For the Weyl fermion component field, according to $m_\mathrm{F} = W_{HH}$, the mass then reads

$$m_\mathrm{F} = \kappa\, M \sqrt{2\, x}\,, \tag{6.29}$$

while the real scalar h_R and pseudoscalar h_I masses squared derived from Eq. (6.27) respectively are given by

$$m_\mathrm{R}^2 = 2\,\kappa^2 M^2\,(x-1)\ , \qquad m_\mathrm{P}^2 = 2\,\kappa^2 M^2\,(x+1)\ . \tag{6.30}$$

Once m_R^2 changes its sign from positive to negative, the waterfall phase transition takes place ending inflation. Thus the critical value ϕ_c can be calculated by setting $m_\mathrm{R}^2 = 0$ (corresponding to $x = 1$) and we obtain

$$\phi_\mathrm{c} = \pm\, 2^{1/4} \sqrt{\left(\frac{\kappa}{\lambda}\right)\, M_* M}\,. \tag{6.31}$$

As in Sec. 6.1, we use the CW radiative correction (6.4) with the masses from Eqs. (6.29) and (6.30) plugged in to lift the tree level flat potential $V_\mathrm{tree} \simeq \kappa^2 M^4$ by the quantum correction

$$V_\mathrm{loop}(\phi) = \frac{1}{64\,\pi^2}\left[m_\mathrm{R}^4\left(\ln\frac{m_\mathrm{R}^2}{Q^2} - \frac{3}{2}\right) + m_\mathrm{P}^4\left(\ln\frac{m_\mathrm{P}^2}{Q^2} - \frac{3}{2}\right) - 2\, m_\mathrm{F}^4\left(\ln\frac{m_\mathrm{F}^2}{Q^2} - \frac{3}{2}\right)\right]\,. \tag{6.32}$$

For $\phi \gg \phi_\mathrm{c}$, the loop-correction can be approximated as

$$V_\mathrm{loop}(\phi) \simeq \frac{\kappa^4 M^4}{16\,\pi^2}\left[2\,\ln\left(\frac{\lambda^2\phi^4}{Q^2 M_*^2}\right) - \frac{3}{2}\right]\,. \tag{6.33}$$

Applying the slow-roll EOMs (2.16) to Eq. (6.33), we can calculate the field values in terms of the number of e-folds to be $\phi^2(N_\mathrm{e}) \simeq \phi_\mathrm{c}^2 + \kappa^2 N_\mathrm{e}/\pi^2 \simeq \kappa^2 N_\mathrm{e}/\pi^2$. Similar as in SUSY hybrid inflation, the estimated amplitude of the spectrum $P_\mathcal{R}^{1/2} \sim \sqrt{N_\mathrm{e}}\, M^2$. For $N_\mathrm{e} = 60$, the WMAP normalization on the amplitude in Eq. (2.41) fixes the scale $M \approx 3 \cdot 10^{-3}$ and the predictions of the scalar spectral index and the tensor-to-scalar ratio respectively read

$$n_\mathrm{s} \simeq 1 - \frac{1}{N_\mathrm{e}} \approx 0.98\,, \qquad r \simeq \frac{2\,\kappa^2}{N_\mathrm{e}\,\pi^2}\,. \tag{6.34}$$

In this simple approximation, the prediction for the spectral index n_s is the same as for the hybrid inflation superpotential, however r is predicted to be twice as large. Thus, for $\kappa = \mathcal{O}(1)$, we obtain the upper bound $r \lesssim 10^{-2}$.

The above estimation is simplified, but it gives a pretty robust prediction of the typical values of $n_{\rm s}$ and r. Nevertheless, as mentioned before, SUGRA corrections have to be taken into account inevitably, since on the global SUSY level, there is no justification for the stabilization of the driving field S at zero. Such SUGRA corrections can induce altered mass splittings for the waterfall component fields which contribute dominantly to the loop-correction (6.33). In turn, this alters the predictions in SUGRA tribrid inflation scenarios. In the following sections, SUGRA effects from different Kähler potentials combined with the tribrid inflation superpotential (6.24) are studied in detail.

6.2.1 Tribrid Inflation with Kähler Expansion

In this section, the possibility to combine the tribrid inflation superpotential defined in Eq. (6.25) with a non-minimal Kähler potential shall be investigated. We want to emphasize that the results presented are new and have not been published before. Similar as in Sec. 6.1.1, we choose a Kähler potential of the form

$$\begin{aligned} K = |\Phi|^2 + |H|^2 + |S|^2 + \frac{\kappa_\Phi}{4\,\Lambda^2}\,|\Phi|^4 + \frac{\kappa_H}{4\,\Lambda^2}\,|H|^4 + \frac{\kappa_S}{4\,\Lambda^2}\,|S|^4 \\ + \frac{\kappa_{H\Phi}}{\Lambda^2}\,|\Phi|^2|H|^2 + \frac{\kappa_{S\Phi}}{\Lambda^2}\,|\Phi|^2|S|^2 + \frac{\kappa_{SH}}{\Lambda^2}\,|S|^2|H|^2 + \dots\,, \end{aligned} \tag{6.35}$$

where Λ is the suppression scale of the higher dimensional operators. Having field values well below Λ, we obtain approximately canonically normalized kinetic terms for $\phi = \sqrt{2}\,|\Phi|$, $H = (h_{\rm R} + {\rm i}\,h_{\rm I})/\sqrt{2}$ and $s = \sqrt{2}\,|S|$.

This section is meant to be a proof of existence that it is possible to make inflation work in the tribrid superpotential combined with a Kähler potential (6.35) rather than a detailed study. To keep matters simple and also, since there has to be quite some tuning involved in order to avoid running into the η-problem, we fix many of the parameters. Due to the fact that the scalar potential and masses are rather lengthy, we use the SuperCosmology code [130] to calculate them and expand up to $\mathcal{O}(\phi^4)$.

Using Eq. (3.67) to calculate the tree level scalar potential, it turns out that it has a minimum at $s = h_{\rm R} = h_{\rm I} = 0$. Within this minimum, the inflaton mass squared up to order $\mathcal{O}(\phi^2)$ reads

$$m_\phi^2 = (1 - \tilde\kappa_{S\Phi})\,\kappa^2 M^4 + \frac{3}{4}\,\kappa^2 M^4\left(2 + \tilde\kappa_\Phi + 4\,\tilde\kappa_{S\Phi}^2 - 4\,\tilde\kappa_{S\Phi}\right)\phi^2 + \mathcal{O}(\phi^4)\,, \tag{6.36}$$

where we have introduced the effective dimensionless parameters $\tilde\kappa_i \equiv \kappa_i\,(M_{\rm P}/\Lambda)^2$ and $\tilde\kappa_{ij} \equiv \kappa_{ij}\,(M_{\rm P}/\Lambda)^2$ in which i, j run over all superfields.

Since we want a flat inflaton direction in order to circumvent the η-problem, we require a vanishing inflaton mass up to the order displayed in Eq. (6.36). The aforementioned tuning thus includes that $\tilde\kappa_{S\Phi} = 1$ and $\tilde{\kappa_\Phi} = -2$. Under this assumption, it turns out that even up to $\mathcal{O}(\phi^4)$ the inflaton mass m_ϕ is zero. For the other mass eigenvalues, the parameter $\tilde\kappa_H$ is irrelevant and does not affect any predictions during inflation, since $H = 0$. The effects of $\tilde\kappa_{SH}$ and $\tilde\kappa_{H\Phi}$ are of similar importance, thus to further simplify our calculation, we assume $\tilde{\kappa}_{SH} = \tilde{\kappa}_{H\Phi}$. Now, we are left with four dimensionless parameters, κ, λ, $\tilde{\kappa}_S$ and $\tilde{\kappa}_{SH}$.

The potential energy density that drives inflation in the minimum $s = h_\mathrm{R} = h_\mathrm{I} = 0$ for our above parameter choice is constant up to

$$V_\mathrm{tree} = V_F = \kappa^2 M^4 + \mathcal{O}(\phi^6)\,, \tag{6.37}$$

which gives rise to a Hubble scale $\mathcal{H} \approx \sqrt{\kappa^2 M^4/3}$. Furthermore, the mass of the scalar field s is given by

$$m_s^2 = \kappa^2 M^4 \left(-\tilde{\kappa}_S + \frac{1}{2}\,\tilde{\kappa}_S\,\phi^2 - \frac{1}{4}\,\tilde{\kappa}_S\,\phi^4\right) + \mathcal{O}(\phi^6)\,, \tag{6.38}$$

and for $\phi \ll 1$, the first term dominates accounting for a positive mass squared with a minimum at $s = 0$ if $\tilde{\kappa}_S$ is negative. To be stabilized sufficiently quick before any mentionable slow-roll dynamics occurs, $m_s^2/\mathcal{H}^2 > 1$ which leads to the constraint that $\tilde{\kappa}_S < -1/3$. We therefore choose $\tilde{\kappa}_S = -1$ in the following.

At the order of expansion considered, the tree level flat inflaton direction is lifted by CW one-loop corrections. For this reason, we have to take into account all ϕ-dependent masses in the spectrum. The scalar component of the waterfall supermultiplet H contains the following real scalar and pseudoscalar masses up to order $\mathcal{O}(\phi^4)$ respectively

$$\begin{aligned} m_\mathrm{R}^2 &= \kappa^2 M^4\,(1-\tilde{\kappa}_{SH}) - 2\,\kappa^2 M^2 + \tilde{\kappa}_{SH}\,\kappa^2 M^4 \phi^2 + \left(\frac{\lambda^2}{M_*^2} - \frac{1}{4}\,\tilde{\kappa}_{SH}\,\kappa^2 M^4\right)\phi^4\,, \\ m_\mathrm{P}^2 &= \kappa^2 M^4\,(1-\tilde{\kappa}_{SH}) + 2\,\kappa^2 M^2 + \tilde{\kappa}_{SH}\,\kappa^2 M^4 \phi^2 + \left(\frac{\lambda^2}{M_*^2} - \frac{1}{4}\,\tilde{\kappa}_{SH}\,\kappa^2 M^4\right)\phi^4\,. \end{aligned} \tag{6.39}$$

In addition, we need to calculate the fermion mass matrix squared, which gives rise to only one non-zero eigenvalue

$$m_\mathrm{F}^2 = \frac{\lambda^2}{M_*^2}\,\phi^4\,. \tag{6.40}$$

Using Eq. (6.4), we find that the ϕ-dependent one-loop correction to the effective potential is given by

$$\begin{aligned} V_\mathrm{loop}(\phi) = \frac{1}{64\,\pi^2}\Bigg[& m_\mathrm{R}^4\left(\ln\frac{m_\mathrm{R}^2}{Q^2} - \frac{3}{2}\right) + m_\mathrm{P}^4\left(\ln\frac{m_\mathrm{P}^2}{Q^2} - \frac{3}{2}\right) \\ & + 2\,m_s^4\left(\ln\frac{m_s^2}{Q^2} - \frac{3}{2}\right) - 2\,m_\mathrm{F}^4\left(\ln\frac{m_\mathrm{F}^2}{Q^2} - \frac{3}{2}\right)\Bigg]\,, \end{aligned} \tag{6.41}$$

where the factor of two in front of the term containing m_s^2 accounts for the scalar and pseudoscalar DOFs and the masses calculated above have to be plugged in. Adding the tree level potential (6.37) and the loop-correction (6.41) we obtain the effective inflaton potential

$$V_\mathrm{eff} \simeq \kappa^2 M^4 + V_\mathrm{loop}(\phi)\,. \tag{6.42}$$

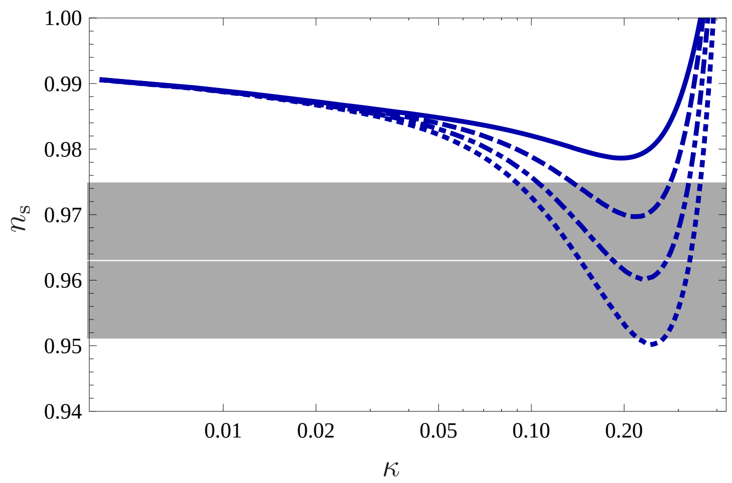

Figure 6.2: Predicted spectral index n_s depending on κ for different values of $\tilde{\kappa}_{SH} \equiv \kappa_{SH}(M_\mathrm{P}/\Lambda)^2$ and fixed $\lambda = 0.1$. The prediction is plotted for $\tilde{\kappa}_{SH} = 5$ (solid line), $\tilde{\kappa}_{SH} = 10$ (dashed line), $\tilde{\kappa}_{SH} = 15$ (dotted dashed line) and $\tilde{\kappa}_{SH} = 20$ (dotted line).

The predictions of our model given by Eq. (6.42) are now straightforward to obtain. As before, we fix the number of e-folds to $N_\mathrm{e} = 60$ and for the renormalization scale, we choose $Q = \sqrt{2}\,\kappa\, M$. Numerically, we calculate the field value 60 e-folds before the end of inflation at ϕ_c which can be obtained by solving $m_\mathrm{R}^2 = 0$ with respect to ϕ. Furthermore, we fix $\lambda = 0.1$ and the scale $M_* = 1$. Again, the WMAP normalization fixes the scale $M \approx 3 \cdot 10^{-3}$. For different $\tilde{\kappa}_{SH}$, we plot the predicted spectral index depending on κ in Fig. 6.2 and it can lie within the seven-year WMAP 1σ range for large enough $\tilde{\kappa}_{SH}$. The generic prediction of a very small tensor-to-scalar ratio $r \lesssim 10^{-3}$ is unaffected by the Kähler potential expansion (6.35).

To summarize, in this section we have given a proof of existence that combining the tribrid inflation superpotential (6.25) with a Kähler potential expansion (6.35) can account for viable inflation. The expansion parameters in the Kähler potential can be tuned in such a way that the η-problem is under control. Certain parameters controlling higher dimensional operators in the Kähler potential can reduce the predicted spectral index via loop-corrections to be in full agreement with latest observations.

6.2.2 Tribrid Inflation with Shift Symmetry

As emphasized before, the tribrid inflation superpotential is especially suitable for solving the η-problem by fundamental symmetries imposed on the Kähler potential without fine-tuning. In order to study these interesting possibilities, let us start in this section with the simplest case of a Nambu–Goldstone-like shift symmetry in the inflaton direction as introduced in Eq. (5.5). In Ref. [2] we have proposed a minimal setup described by the superpotential (6.25) and the Kähler potential[7]

$$
\begin{aligned}
K = |H|^2 + |S|^2 + \frac{1}{2}(\Phi + \Phi^*)^2 + \frac{\kappa_H}{\Lambda^2}|H|^4 + \frac{\kappa_S}{\Lambda^2}|S|^4 + \frac{\kappa_\Phi}{4\,\Lambda^2}(\Phi + \Phi^*)^4 \\
+ \frac{\kappa_{SH}}{\Lambda^2}|S|^2|H|^2 + \frac{\kappa_{S\Phi}}{2\,\Lambda^2}|S|^2(\Phi + \Phi^*)^2 + \frac{\kappa_{H\Phi}}{2\,\Lambda^2}|H|^2(\Phi + \Phi^*)^2 + \ldots .
\end{aligned} \tag{6.43}
$$

The Kähler potential (6.43) is symmetric under the shift of Φ by (5.5) which requires it to be a function $K(\Phi, \Phi^*) = K(\Phi + \Phi^*)$.[8] For generality, and since all field values are well below the Planck scale, we use an expansion in the absolute values squared of the fields H and S and in even powers of $(\Phi + \Phi^*)/\sqrt{2}$, where higher order operators are again suppressed by suitable powers of some scale Λ. As in the previous Sec. 6.2.1, we use the tilde notation for effective dimensionless parameters $\tilde{\kappa}_i \equiv \kappa_i\,(M_{\mathrm{P}}/\Lambda)^2$ and $\tilde{\kappa}_{ij} \equiv \kappa_{ij}(M_{\mathrm{P}}/\Lambda)^2$.

The Kähler metric is given by the second derivatives w.r.t. the fields and their conjugates. In the limit $S, H \to 0$, it diagonalizes and in the (H, S, Φ)-basis is given by

$$
K_{i\bar{j}} = \delta_{i\bar{j}} + \mathrm{diag}\left(\tilde{\kappa}_{H\Phi}\,\phi_{\mathrm{R}}^2\,,\,\tilde{\kappa}_{S\Phi}\,\phi_{\mathrm{R}}^2\,,\,6\,\tilde{\kappa}_\Phi\,\phi_{\mathrm{R}}^2\right)\,. \tag{6.44}
$$

With $\phi_{\mathrm{R}} = 0$, according to Eq. (3.63) we recover canonically normalized kinetic terms for $\Phi = (\phi_{\mathrm{R}} + \mathrm{i}\,\phi)/\sqrt{2}$, $H = (h_{\mathrm{R}} + \mathrm{i}\,h_{\mathrm{I}})/\sqrt{2}$ and $S = s/\sqrt{2}$. The shift symmetry, together with the tribrid inflation superpotential (6.25), protects the imaginary part of Φ from obtaining any mass term at tree level.

The fact that the shift symmetry in the Kähler potential provides a solution to the η-problem in SUGRA inflation can be seen by looking at the full F-term scalar potential Eq. (3.67). As described in Sec. 5.1, the η-problem is the tendency that in SUGRA an inflaton mass of the order of the Hubble scale is typically generated, which spoils slow-roll conditions. Because of the shift symmetry, the exponential SUGRA factor e^K in Eq. (3.67) is independent of the imaginary part ϕ, identified as the inflaton direction. This solves the usual η-problem in SUGRA inflation.

The problem of conventional SUGRA hybrid inflation with a shift symmetric Kähler potential described in Sec. 6.1.2, namely that a large tachyonic inflaton mass is induced by the $-3\,|W|^2$ term in Eq. (3.67), can be avoided in the tribrid setup. This is due to the special property of having $W = 0$ during inflation, cf. Eq. (6.23). Furthermore, the properties (6.23) also eliminate certain couplings between additional moduli sectors and the inflaton which have turned out to be problematic for realizing inflation [122, 123]. Recently, it has been shown in [132] that combining our tribrid setup presented in this section with a KL-type moduli sector [121], it is possible to simultaneously realize viable inflation and moduli stabilization while the scale of SUSY breaking can be low. Finally, we note that a specific shift symmetric Kähler potential may be chosen instead of the expansion we have used here for generality. The main point we would like to emphasize is that in our tribrid class of models satisfying conditions (6.23) during inflation, a shift symmetry naturally solves the η-problem.

[7] A proof of existence by symmetry assignments is given in App B.2.

[8] Shift symmetry in the context of natural inflation has been mentioned in [131].

The F-term scalar potential is given by Eq. (3.67). In the following, we assume that the fields $s, h_{\rm R}, h_{\rm I}$ and $\phi_{\rm R}$ have already settled to their minima at $s = h_{\rm R} = h_{\rm I} = \phi_{\rm R} = 0$. This is justified because, as we show below, the fields can easily have masses larger than the Hubble scale $\mathcal{H}$. From Eq. (6.44) we see that, in the minimum $K_{i\bar{j}} = \delta_{i\bar{j}}$, such that all fields are already canonically normalized. The vacuum energy density is then given by the tree level potential which reads[9]

$$V_{\rm tree} = V_F \simeq {\rm e}^K K^{S\bar{S}} \left|W_S\right|^2 \simeq \kappa^2 M^4 \simeq 3\,\mathcal{H}^2\,, \tag{6.45}$$

and is flat in the ϕ-direction. Note that the shift symmetry is slightly broken by the effective operator $\frac{\lambda}{M_*}\Phi^2 H^2$ in the superpotential (6.25). While the inflaton potential is flat at tree level due to the symmetry, this term gives rise to a slope of the inflaton potential at loop level, as will be discussed below.

Using the SuperCosmology code [130], the scalar and pseudoscalar mass matrices are calculated to be diagonal and the mass spectrum is given by

$$m_\phi^2 = 0\,, \qquad m_{\phi_{\rm R}}^2 = 2\,\kappa^2 M^4 \left(1 - \tilde{\kappa}_{S\Phi}\right)\,, \qquad m_s^2 = -\,4\,\tilde{\kappa}_S\,\kappa^2 M^4\,. \tag{6.46}$$

The directions in field space different from the inflaton are stable provided that their masses are larger than the Hubble scale. This requirement leads to the constraints $\tilde{\kappa}_{S\Phi} < 5/6$ and $\tilde{\kappa}_S < -1/12$.

For the fermions, we can calculate the SUGRA mass matrix using Eq. (3.69). It is also diagonal in the inflationary minimum and has only one non-vanishing eigenvalue during inflation. The fermion mass matrix squared in the (H, S, Φ)-basis is obtained to be

$$\left(\mathcal{M}_{\rm F}^\dagger \mathcal{M}_{\rm F}\right)_{ij} = {\rm diag}\left(\frac{\lambda^2}{M_*^2}\,\phi^4\,,\,0\,,\,0\right)\,. \tag{6.47}$$

Since for both the scalars and pseudoscalars, as well as for the fermions, only the components of the waterfall superfield H contribute ϕ-dependent masses, it is enough to take into account their contributions to the one-loop potential. We denote their squared masses by

$$\begin{aligned} m_{\rm R}^2 &= 2\,\kappa^2 M^2\left[x - 1 + \frac{M^2}{2}\left(1 - \tilde{\kappa}_{SH}\right)\right]\,,\\ m_{\rm P}^2 &= 2\,\kappa^2 M^2\left[x + 1 + \frac{M^2}{2}\left(1 - \tilde{\kappa}_{SH}\right)\right]\,,\\ m_{\rm F}^2 &= 2\,\kappa^2 M^2\,x\,, \end{aligned} \tag{6.48}$$

where we have used the same definition for x as in Eq. (6.28). Note that additional mass splittings from SUGRA effects in Eq. (6.48) are suppressed stronger by a factor M^2 when

[9]This result can be reproduced by making use of the conditions (6.23).

compared to the global SUSY mass squared formulae (6.30). As usual, the critical value of ϕ at which the waterfall field destabilizes can be calculated from $m_{\rm R}^2 = 0$ and is given by

$$\phi_{\rm c}^2 = \frac{\kappa}{\lambda}\,\sqrt{2}\,(M M_*)\,\sqrt{1 - \frac{M^2}{2}\,(1 - \tilde{\kappa}_{SH})}\,. \tag{6.49}$$

The CW one-loop contribution to the effective potential (6.4) using the squared masses in Eq. (6.48) reads

$$V_{\rm loop} = \frac{1}{64\,\pi^2}\left[m_{\rm R}^4\left(\ln\frac{m_{\rm R}^2}{Q^2} - \frac{3}{2}\right) + m_{\rm P}^4\left(\ln\frac{m_{\rm P}^2}{Q^2} - \frac{3}{2}\right) - 2\,m_{\rm F}^4\left(\ln\frac{m_{\rm F}^2}{Q^2} - \frac{3}{2}\right)\right]\,, \tag{6.50}$$

where we fix the renormalization scale $Q = m_{\rm F}/\sqrt{x} = \sqrt{2}\,\kappa\,M$.

The complete effective scalar potential up to one-loop thus reads

$$V_{\rm eff}(\phi) = V_{\rm tree} + V_{\rm loop}(\phi)\,. \tag{6.51}$$

For a particular choice of parameters, we have plotted the potential in Fig. 6.3. Depending on the value of the parameter $\tilde{\kappa}_{SH}$, the shape of the potential can be changed. The red curve represents $\tilde{\kappa}_{SH} < 1$, in which case the SUGRA correction adds a positive curvature to the potential resulting in a less negative slow-roll parameter η and hence, due to Eq. (2.38) a larger spectral index. Having $\tilde{\kappa}_{SH} = 1$ (displayed by the dotted red curve) is a very special case, since then we recover exactly the global SUSY limit and SUGRA corrections cancel identically. This can be seen explicitly by comparing the waterfall sector masses (6.48) to the ones of global SUSY tribrid inflation given in Eqs. (6.29) and (6.30). In the third case of $\tilde{\kappa}_{SH} > 1$ which is plotted in blue, the potential obtains an even larger negative curvature η and hence, a reduced spectral index.

Note, that in the latter case, the shape of the potential is of hilltop-type, generated purely by radiative corrections.[10] In order to achieve at least 60 e-folds of inflation, the initial condition for the inflaton field is that it has to start between ϕ_{60}, about 60 e-folds before the end of inflation, and the top of the hill, i.e., the local maximum of the potential.

Fixing the parameters $M_* = 1$ and Q as before, we vary $\tilde{\kappa}_{SH}$ for different values of κ while keeping $\lambda = 0.1$ fixed. The predicted values of the observables calculated in this way are displayed in Fig. 6.4. Note, that for a relatively wide and generic range of the parameter $\tilde{\kappa}_{SH}$, the predicted spectral index $n_{\rm s}$ lies within the 1σ-range as taken from the seven-year WMAP data [5]. The tensor-to-scalar ratio is very small, $r = \mathcal{O}(10^{-5})$, as typical for hybrid/tribrid and small-field models of inflation and easily satisfies the observational upper bound. For a running of the spectral index, the current observational evidence is still not considered as significant. Our prediction ${\rm d}n_{\rm s}/{\rm d}\ln k = \mathcal{O}(\pm 10^{-4})$ is in full agreement with this range. We note that consistent with our earlier estimations, the scale M obtained from the WMAP normalization for the amplitude of the power spectrum predicts $\langle H\rangle = \mathcal{O}(10^{-3})$, close to the GUT scale. In this simplest type of effective single-field model, we do not expect large non-Gaussianities.

[10] It has recently been argued in Ref. [133] that such hilltop-type potentials can also arise in non-supersymmetric hybrid inflation when the inflaton is coupled to the right-handed neutrinos.

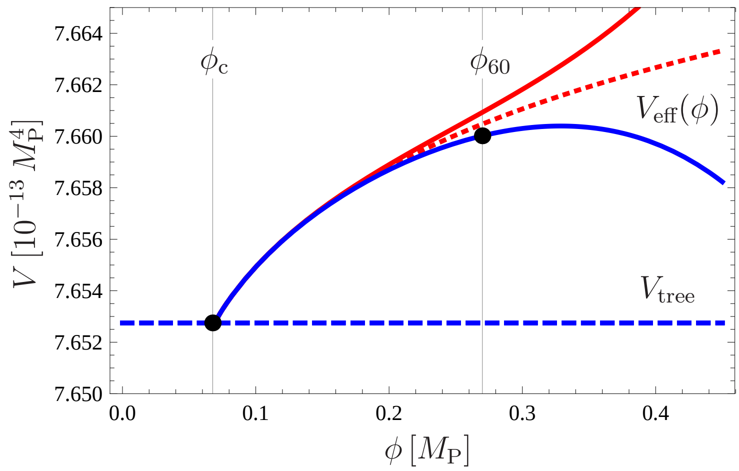

Figure 6.3: Graphical illustration of the one-loop effective potential for ϕ with typical values of the field $N_{\mathrm{e}} = 60$ before the end of inflation ϕ_{60} and at the critical value ϕ_{c} where inflation ends. The parameter choice in this example was $\kappa = 0.12$, $\lambda = 0.1$ and $M = 0.0027\, M_{\mathrm{P}}$. For different values of $\tilde{\kappa}_{SH}$ the potential is displayed in solid blue ($\tilde{\kappa}_{SH} = 40$), dotted red ($\tilde{\kappa}_{SH} = 1$) and solid red ($\tilde{\kappa}_{SH} = -38$). The dashed blue line represents the constant tree level vacuum energy density $V_0 \approx \kappa^2 M^4$.

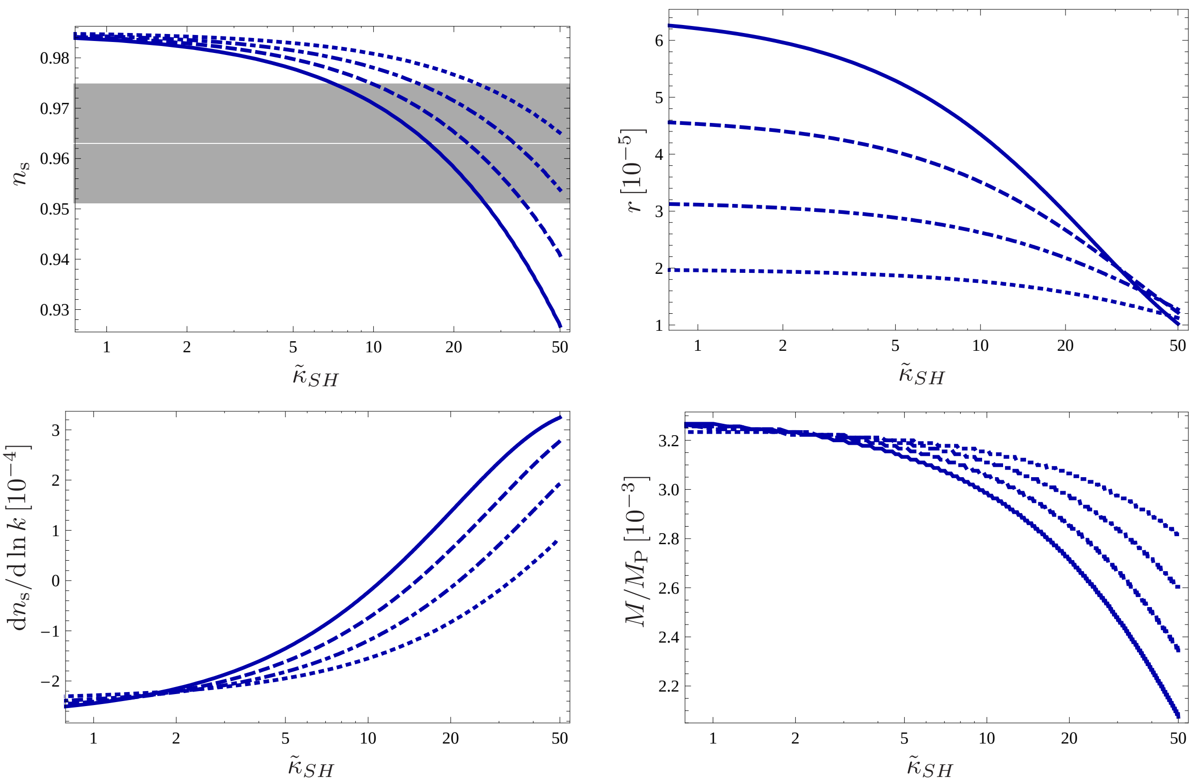

Figure 6.4: Predicted values of the spectral index n_{s}, the tensor-to-scalar ratio r, the running of the spectral index $\mathrm{d}n_{\mathrm{s}}/\mathrm{d}\ln k$ and the scale M for $\lambda = 0.1$ and $\kappa = 0.14$ (solid line), $\kappa = 0.12$ (dashed line), $\kappa = 0.10$ (dotted dashed line), $\kappa = 0.08$ (dotted line) depending on $\tilde{\kappa}_{SH} = \kappa_{SH}\,(M_{\mathrm{P}}/\Lambda)^2$. The shaded region in the first plot highlights the seven-year WMAP 1σ-range of the spectral index.

6.2.3 Tribrid Inflation with Heisenberg Symmetry

Motivated by the possibility to generalize to inflation in non-singlet reps of some symmetry group, in this section, we impose a Heisenberg symmetry (5.7) on the Kähler potential combined with the tribrid inflation superpotential (6.25). With only one single inflaton field Φ, the invariant field combination under the Heisenberg symmetry is given by

$$\rho = T + T^* - |\Phi|^2 \,. \tag{6.52}$$

Since with the Heisenberg symmetry, we also introduce new DOFs in the complex modulus field T, we have to be concerned about their stabilization during slow-roll inflation as outlined in Sec. 5.2. In the following, we show in detail how both successful inflation and simultaneous modulus stabilization can be achieved using a Heisenberg symmetric Kähler potential.

The explicit example model which we will investigate in the remainder of this section is again defined by the superpotential Eq. (6.25) and a new Kähler potential

$$K = |H|^2 + \big(1 + \kappa_S \,|S|^2 + \kappa_\rho \,\rho\big)\,|S|^2 + f(\rho)\,, \tag{6.53}$$

where κ_S and κ_ρ are dimensionless parameters. Note that here, we do not use the tilde notation as in the previous sections. This is justified because throughout this section, we assume that the suppression scale of higher dimensional effective operators in the Kähler potential is $\Lambda = 1$ and thus $\tilde{\kappa}_i = \kappa_i$ and $\tilde{\kappa}_{ij} = \kappa_{ij}$.

The purpose of the coupling κ_S is to give a large mass to the S-field, which keeps it at zero both during and after inflation. We have not included a term proportional to κ_{SH}, since its effect has already been studied in the previous section. As in the context of shift symmetry, $\kappa_{SH} \approx \mathcal{O}(10)$ allows to lower the predictions for the spectral index. Finally, the additional coupling constant κ_ρ which admits a coupling between the modulus ρ and S is needed in order to generate a stabilizing minimum in the scalar potential for ρ. After transforming to the basis in which ρ and Φ are treated as independent DOFs, the potential for Φ is flat at tree level due to the Heisenberg symmetry.

Next, let us derive the background EOMs of all relevant fields and calculate the tree level potential. In addition, we describe how to transform to the (Φ, ρ)-basis and why this basis is convenient. We assume that $S = H = 0$ during inflation, such that the conditions (6.23) are satisfied. Below, we will explicitly show from the full scalar potential that these assumptions are justified.

The Kähler metric can be calculated as the second derivative of the Kähler potential (6.53) with respect to the superfields and their conjugates which in the (H, S, Φ, T)-basis reads

$$\big(K_{i\bar{j}}\big) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 + \kappa_\rho\,\rho + 4\,\kappa_S\,|S|^2 & -\kappa_\rho\,\Phi\,S^* & \kappa_\rho\,S^* \\ 0 & -\kappa_\rho\,\Phi^*\,S & f''(\rho)\,|\Phi|^2 - f'(\rho) - \kappa_\rho\,|S|^2 & -f''(\rho)\,\Phi^* \\ 0 & \kappa_\rho\,S & -f''(\rho)\,\Phi & f''(\rho) \end{pmatrix} . \tag{6.54}$$

With $S = 0$ during inflation, this reduces to the block-diagonal form

$$\left(K_{i\bar{j}}\right) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1+\kappa_\rho\,\rho & 0 & 0 \\ 0 & 0 & f''(\rho)\,|\Phi|^2 - f'(\rho) & -f''(\rho)\,\Phi^* \\ 0 & 0 & -f''(\rho)\,\Phi & f''(\rho) \end{pmatrix}, \tag{6.55}$$

which suggests that the (Φ, T)-sub-block can be treated independently. Since S basically remains static during and after inflation, we do not take its EOM into account. The kinetic sector of the waterfall field H decouples from (Φ, T) and its kinetic term is canonical.

Since the phases of the scalar fields S, H and Φ as well as $\mathrm{Im}(T)$ very quickly approach a constant value in an expanding universe and subsequently decouple from the absolute values and $\mathrm{Re}(T)$ in the EOMs, as we discuss in detail in App. B.1, we only consider the absolute values and $\mathrm{Re}(T)$ in what follows and denote them by lowercase letters $s = \sqrt{2}\,|S|$, $h = \sqrt{2}\,|H|$, $\phi = \sqrt{2}\,|\Phi|$ and $t/2 = \sqrt{2}\,\mathrm{Re}(T)$. We set the phases and $\mathrm{Im}(T)$ to a constant, or without loss of generality to zero. The kinetic terms for t and ϕ are then obtained to be

$$\mathcal{L}_{\mathrm{kin}} = \frac{f''(\rho)}{4}\,\phi^2\,(\partial_\mu\phi)^2 - \frac{f'(\rho)}{2}\,(\partial_\mu\phi)^2 - \frac{f''(\rho)}{2\sqrt{2}}\,\phi\,\partial_\mu\phi\,\partial^\mu t + \frac{f''(\rho)}{8}\,(\partial_\mu t)^2\,. \tag{6.56}$$

In order to transform to the independent DOFs ρ and ϕ, we use the definition (6.52) and end up with the kinetic Lagrangian terms

$$\mathcal{L}_{\mathrm{kin}} = \frac{f''(\rho)}{4}\,(\partial_\mu\rho)^2 - \frac{f'(\rho)}{2}\,(\partial_\mu\phi)^2\,, \tag{6.57}$$

which are diagonal in the field derivatives $\partial_\mu\rho$ and $\partial_\mu\phi$. In addition, the spatial derivatives $\nabla\phi_i$ vanish in a homogeneous, isotropic universe.

Upon application of the Euler–Lagrange equation and introduction of the Hubble scale $\mathcal{H}$, we obtain the EOMs[11] for the classical scalar fields

$$\begin{aligned} \ddot{\phi} + 3\,\mathcal{H}\,\dot{\phi} + \frac{f''(\rho)}{f'(\rho)}\,\dot{\rho}\,\dot{\phi} - \frac{1}{f'(\rho)}\frac{\partial V}{\partial\phi} &= 0\,, \\ \ddot{\rho} + 3\,\mathcal{H}\,\dot{\rho} + \frac{f^{(3)}(\rho)}{2f''(\rho)}\,\dot{\rho}^2 + \dot{\phi}^2 + \frac{2}{f''(\rho)}\frac{\partial V}{\partial\rho} &= 0\,. \end{aligned} \tag{6.58}$$

For the simulation of the evolution of the scale factor during inflation, we add the Friedmann Eq. (2.11) assuming vanishing spatial curvature $k = 0$, where the energy density is given by Eq. (2.13). In our case, the potential is only determined by the F-terms $V = V_F$.

With $S = H = 0$ during inflation, the tree level F-term scalar potential in Eq. (3.67) reduces to the simple form

$$V_{\mathrm{tree}} = V_F = \mathrm{e}^{f(\rho)}\,K^{-1}_{SS^*}\left|\frac{\partial W}{\partial S}\right|^2 = \kappa^2 M^4\cdot\frac{\mathrm{e}^{f(\rho)}}{(1+\kappa_\rho\,\rho)}\,. \tag{6.59}$$

[11] Note from the EOMs (6.58) that for $f'(\rho_0) = 0$ there is a divergence in the acceleration of ϕ. This can be avoided for non-vanishing κ_ρ, such that the minimum of the potential ρ_{min} is shifted away from the minimum of $f(\rho)$ and thus $\rho_{\mathrm{min}} \neq \rho_0$. As we will see below, $f(\rho)$ does not even have to have a minimum in order to stabilize ρ.

From Eq. (6.57) we can see that in order to have a positive kinetic term for the inflaton field in the minimum of the potential in ρ-direction, the function $f(\rho)$ should fulfill the requirement that $f'(\rho_{\min})$ is negative. During inflation, the value $\rho_{\min} = \langle\rho\rangle$ determines the VEV of ρ.

As mentioned in the beginning of this chapter, we want to summarize why the new (ϕ, ρ)-basis is more convenient. We have shown in this section that in this basis, the Kähler metric diagonalizes during inflation, which is a great simplification. In addition, the tree level potential is exactly flat in ϕ-direction. This is due to the Heisenberg symmetry which protects ϕ from obtaining large mass corrections in the SUGRA expansion. Hence, the η-problem of SUGRA inflation has a simple solution. Moreover, as we will see in the next section, not only the Kähler metric, but also the mass matrices are simultaneously diagonal in this basis. Due to the diagonal Kähler metric, the kinetic terms are diagonal in the (ϕ, ρ)-basis and thus the CW formalism of calculating the effective potential from radiative corrections applies,[12] which is well known in the literature [58]. Therefore, the one-loop radiative corrections are easy to calculate. Having everything diagonalized in the independent fields (ϕ, ρ), we consider this basis to be the physically relevant one. It is important to note that even though the kinetic energies of the fields are diagonal, they are not yet canonically normalized except for the field h. We execute the normalization procedure at $\rho = \langle\rho\rangle$ later in this section in order to obtain the physical mass spectrum.

As a next step, the assumptions $S = H = 0$, used above, accounting for Eq. (6.23) must be proven from the full scalar potential. These assumptions are justified, if the potential has minima in all relevant directions at $s = h = 0$ with masses of the fields larger than the Hubble scale $m_i^2 > \mathcal{H}^2$. Therefore, using the SuperCosmology code [130] we calculate the F-term scalar potential from Eqs. (6.25) and (6.53) making use of (3.67), and since the potential is very lengthy, we do not write it down explicitly. But we show all results derived from the potential.

Before proceeding further we need to specify the function $f(\rho)$. As explained in Sec. 5.2, one well motivated example of the general function is the following no-scale form

$$f(\rho) = -\,3\,\ln(\rho). \tag{6.60}$$

We emphasize that this is only one specific choice within the class of models where the Kähler potential has the form of Eq. (6.53). Making the curvature of the potential along the ρ-direction larger than the Hubble scale helps to stabilize the modulus very quickly. In our case, the term proportional to κ_ρ generates the minimum of the potential w.r.t. ρ by switching on the coupling between S and ρ.

Let us now verify our assumptions. First, we have to check that both s and h have an extremum at $s = h = 0$, i.e.,

$$\left.\frac{\partial V}{\partial s}\right|_{s=h=0} = \left.\frac{\partial V}{\partial h}\right|_{s=h=0} = 0\,. \tag{6.61}$$

[12] Apart from a normalization factor.

After transforming the potential to the (ϕ, ρ)-basis by the substitution $t \to \rho + \phi^2/2$, the curvatures of the potential along the ϕ-, s-, ρ- and h-direction respectively at $s = h = 0$ are given by

$$\begin{aligned}
\left.\frac{\partial^2 V}{\partial \phi^2}\right|_{s=h=0} &= 0\,, \\
\left.\frac{\partial^2 V}{\partial s^2}\right|_{s=h=0} &= \frac{\kappa^2 M^4\, \mathrm{e}^{f(\rho)}}{3\,(1+\kappa_\rho \rho)^2}\left[-12\,\kappa_S + (3+4\kappa_\rho\rho)^2\right]\,, \\
\left.\frac{\partial^2 V}{\partial \rho^2}\right|_{s=h=0} &= \frac{2\,\kappa^2 M^4\, \mathrm{e}^{f(\rho)}}{\rho^2\,(1+\kappa_\rho \rho)^3}\left[6 + 15\,\kappa_\rho\,\rho + 10\,\kappa_\rho^2\,\rho^2\right]\,, \\
\left.\frac{\partial^2 V}{\partial h^2}\right|_{s=h=0} &= \mathrm{e}^{f(\rho)}\left[\frac{\lambda^2}{M_*^2}\,\phi^4 + \frac{2\,(\kappa M)^2}{(1+\kappa_\rho\rho)}\left(\frac{M^2}{2}-1\right)\right]\,.
\end{aligned} \tag{6.62}$$

Strictly speaking, these values of the curvatures cannot be interpreted as the squared masses m_i^2 of the respective fields, since the fields are not yet canonically normalized, except for the waterfall field h. From Eq. (6.57), we know that the normalization depends on the ρ-modulus only, and as we will see soon, the latter settles to its minimum at the very beginning of inflation.

We will justify this by both comparing the mass of the ρ-modulus at the minimum of V to the Hubble scale and also by looking at the full evolution of the fields solving Eq. (6.58). After the ρ-modulus has settled to its minimum, we can easily canonically normalize the fields and this normalization typically induces changes of $\mathcal{O}(1)$.

Note that of all the curvatures (6.62), only the one of the h field depends on the field value of the inflaton ϕ. Therefore it will be the only relevant contribution to the one-loop effective potential. In addition, we have verified that all the cross terms vanish. Hence, the full mass matrix is diagonal

$$\left.\mathcal{M}^2\right|_{s=h=0} = \mathrm{diag}\big(m_{\tilde{h}}^2\,,\, m_{\tilde{s}}^2\,,\, 0\,,\, m_{\tilde{\rho}}^2\big)\,, \tag{6.63}$$

with a flat ϕ-direction $m_{\tilde{\phi}}^2 = 0$, as expected. From now on we indicate all canonically normalized fields by a tilde.

Depending on the choice of κ_ρ and hence the VEV $\langle\rho\rangle$, the other masses can be fairly large in the inflationary trajectory. The potential at $s = h = 0$ is given by Eq. (6.59) together with the no-scale form (6.60) of $f(\rho)$ and is depicted in Fig. 6.5. As we can see from Eq. (6.59), for the modulus to be stable during inflation, the initial field value of ρ must be less than $-\kappa_\rho^{-1}$, where the pole is located. The potential then gets minimized at

$$\langle\rho\rangle = -\frac{3}{4\,\kappa_\rho}\,. \tag{6.64}$$

At the minimum, the canonically normalized fields in terms of the non-canonically normalized ones are given by the following relations: $\tilde{s} = \frac{s}{2}$, $\tilde{\rho} = \sqrt{8/3}\,\rho$, $\tilde{\phi} = 2\,\phi$, and $\tilde{h} = h$. The

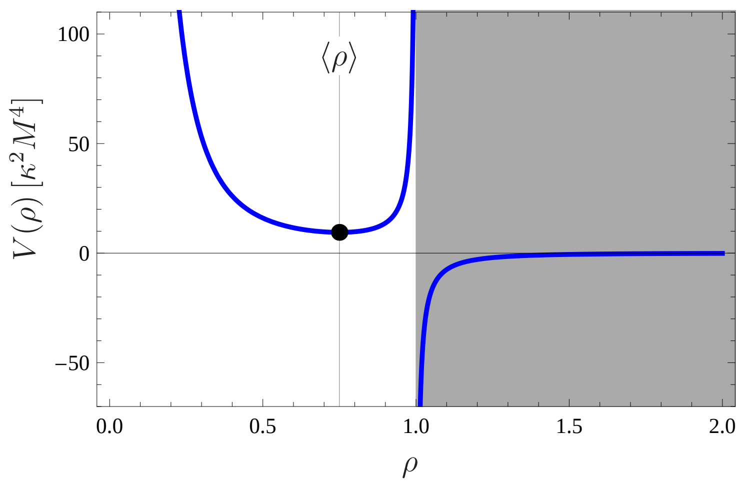

Figure 6.5: Tree level scalar potential depending on ρ for $\kappa_\rho = -1$. The modulus ρ is given in units of the reduced Planck mass $M_{\rm P}$. Note that there is a pole at $\rho = 1$ and the minimum at $\rho_{\rm min} = 3/4$ is highlighted by a vertical line and a black dot. In order to be stabilized within the minimum, the initial conditions should not lie above the pole in the grey shaded region.

normalization factors can be calculated with the kinetic terms from Eqs. (6.55) and (6.57). In the minimum at $\langle\rho\rangle$ during inflation, the squared masses (6.62) of the scalars read

$$\begin{aligned} m^2_{\tilde\phi} &= 0\,, \\ m^2_{\tilde s} &= \frac{4096}{27}\,\kappa_\rho^3\,\kappa_S\,\kappa^2 M^4\,, \\ m^2_{\tilde\rho} &= -\frac{16384}{81}\,\kappa_\rho^5\,\kappa^2 M^4\,, \\ m^2_{\tilde h} &= -\frac{64}{27}\,\kappa_\rho^3\left[\frac{\lambda^2}{16\,M_*^2}\,\tilde\phi^4 + 8\,\kappa^2 M^2\left(\frac{M^2}{2} - 1\right)\right]. \end{aligned} \tag{6.65}$$

To see that the last three are stable during inflation, we need to compare them to the Hubble scale squared in the same patch, given by

$$\mathcal{H}^2 = \left.\frac{V(\rho_{\rm min})}{3}\right|_{s=h=0} = -\frac{256}{81}\,\kappa_\rho^3\,\kappa^2 M^4\ . \tag{6.66}$$

For the modulus squared mass, the requirement $(m_{\tilde\rho}/\mathcal{H})^2 > 1$ is easily fulfilled, since $(m_{\tilde\rho}/\mathcal{H})^2 = 64\,\kappa_\rho^2$ and the condition is thus satisfied if $|\kappa_\rho| > 1/8$. Since only the case of a negative sign generates a minimum in the potential, we can even require $\kappa_\rho < -1/8$. The $\tilde s$ field can be heavier than the Hubble scale if the condition $\kappa_S < -1/48$ holds.

In the model at hand, the waterfall mechanism works in the usual way. From Eq. (6.65), it is clear that the mass of the waterfall field can be arbitrarily high if the field value of $\tilde\phi$ is large enough. Once $\tilde\phi$ drops below its critical value $\tilde\phi_{\rm c}$ at which $m^2_{\tilde h} = 0$, the waterfall field gets destabilized and slow-roll inflation ends. From Eq. (6.65) the critical value of the waterfall field is found to be

$$\tilde\phi_{\rm c}^2 = 8\,\frac{\kappa}{\lambda}\,(M M_*)\,\sqrt{2 - M^2}\,. \tag{6.67}$$

Having shown that all fields are stabilized during inflation in the inflationary trajectory $s = h = 0$ and that the inflaton direction $\tilde{\phi}$ is exactly flat at the classical level, we now calculate the one-loop radiative corrections to the effective potential. These corrections are induced by Heisenberg symmetry breaking superpotential couplings given by Eq. (6.24) in combination with SUSY breaking during inflation. They will serve to generate a slope for the inflaton field driving it towards the critical value where inflation can end.

The CW one-loop radiative correction is taken from Eq. (6.4). It is important to note that we are evaluating the effective potential in the approximation that the ρ field has stabilized to its minimum at $\langle\rho\rangle$. There, as we have shown above, only the masses of the component fields within the H supermultiplet contribute $\tilde{\phi}$-dependent mass terms to the effective potential. Upon introduction of the new dimensionless variable

$$y \equiv (1 + \kappa_\rho \, \rho) \; x \, , \tag{6.68}$$

where x was defined in Eq. (6.28), the squared masses are of a simple form.

The bosonic contribution comes from the scalar and pseudoscalar masses of the H field. In order to calculate the latter, instead of looking at the absolute value h only, we have to expand the complex waterfall quantum field around its VEV into its real and imaginary part $H = (h_\mathrm{R} + \mathrm{i}\, h_\mathrm{I}) / \sqrt{2}$. Taking the second derivatives of the F-term scalar potential w.r.t. h_R and h_I evaluated at the inflationary minimum, their masses are given by

$$m^2_{\mathrm{R/P}} = 2 \, \frac{(\kappa \, M)^2}{(1 + \kappa_\rho \, \rho)} \, \mathrm{e}^{f(\rho)} \left[y \mp 1 + \frac{M^2}{2} \right] , \tag{6.69}$$

where the minus refers to the real scalars and the plus to the pseudoscalars.

With the considered tribrid inflation superpotential satisfying the conditions (6.23), the mass of the fermionic superpartner according to Eq. (3.69) reduces to $m_\mathrm{F} = \mathrm{e}^{K/2} \, W_{HH}$. Hence, the fermion mass squared is obtained to be

$$m^2_\mathrm{F} = 2 \, \frac{(\kappa \, M)^2}{(1 + \kappa_\rho \, \rho)} \, \mathrm{e}^{f(\rho)} \, y \, . \tag{6.70}$$

Taking into account the spin-multiplicity for the fermions, the resulting one-loop correction is given by

$$\begin{aligned} V_\mathrm{loop}(y) = \frac{(\kappa M)^4}{64 \, (1 + \kappa_\rho \, \rho)^2 \, \pi^2} \Bigg(& \\ 4 \, \mathrm{e}^{2f(\rho)} \left(y - 1 + \frac{M^2}{2} \right)^2 & \left[\ln \left(\frac{2 \, \kappa^2 M^2 \, \mathrm{e}^{f(\rho)}}{(1 + \kappa_\rho \, \rho) \; Q^2} \right) + \ln \left(y - 1 + \frac{M^2}{2} \right) - 3/2 \right] \\ + 4 \, \mathrm{e}^{2f(\rho)} \left(y + 1 + \frac{M^2}{2} \right)^2 & \left[\ln \left(\frac{2 \, \kappa^2 M^2 \, \mathrm{e}^{f(\rho)}}{(1 + \kappa_\rho \, \rho) \; Q^2} \right) + \ln \left(y + 1 + \frac{M^2}{2} \right) - 3/2 \right] \\ - 8 \, \mathrm{e}^{2f(\rho)} \, y^2 & \left[\ln \left(\frac{2 \, \kappa^2 M^2 \, \mathrm{e}^{f(\rho)}}{(1 + \kappa_\rho \, \rho) \; Q^2} \right) + \ln (y) - 3/2 \right] \Bigg) \, . \end{aligned} \tag{6.71}$$

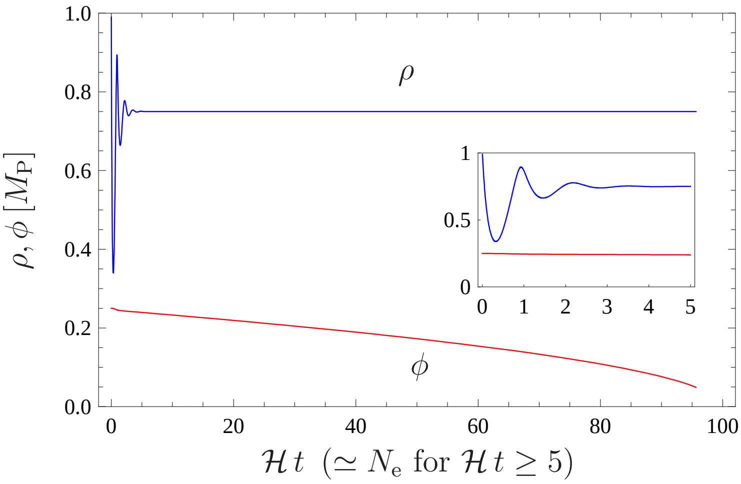

Figure 6.6: Evolution of the modulus field ρ (blue) and the inflaton field ϕ (red) as a function of $\mathcal{H}\,t$. The inlay shows the behavior of the fields for the period $\mathcal{H}\,t \leq 5$ during which ρ settles to its minimum. ρ and ϕ are given in units of the reduced Planck mass M_{P}.

Now, we make a few clarifying remarks concerning the calculation of the one-loop effective potential. First of all, neglecting all mass eigenvalues besides the ones for H is justified, since under the assumption that ρ has settled to its VEV, all other terms are field-independent and therefore just contribute a constant energy density which adds to V_{tree}. Fixing the renormalization scale $Q = m_{\mathrm{F}}/\sqrt{y}$ as we do for calculating the predictions below, it turns out that all these contributions can be safely neglected w.r.t. the tree level potential (6.59). Furthermore, we are aware of the fact that there is a remaining Q-dependence in the observables. Using sensible values of Q around the scale of inflation, a change of Q only results in a shift of the model parameters.[13] The predictions for the observable quantities do not change by a noteworthy amount. Moreover, as all observables are calculated at horizon exit, i.e., around 50–60 e-folds before the end of inflation, for all practical purposes we substitute $\rho = \langle\rho\rangle$ taken from Eq. (6.64) in the above expression. Strictly speaking, to calculate the one-loop potential for a dynamical ρ, one would have to canonically normalize both ϕ and ρ at every moment in time.

In order to show that the assumption $\rho = \langle\rho\rangle$ is a legitimate one, we numerically simulate the full evolution of the non-canonically normalized fields from the EOMs of Eq. (6.58) using

$$V_{\mathrm{eff}}(\phi, \rho) = V_{\mathrm{tree}}(\rho) + V_{\mathrm{loop}}(\phi, \rho)\,, \tag{6.72}$$

with rather generic initial field values. One example of such a numerical solution is shown in Fig. 6.6. We can see that ρ indeed settles to its minimum very quickly and we can achieve a large enough number of e-folds of inflation with ϕ moving to smaller values while ρ remains stabilized at the minimum of its potential. For the plot we have chosen example model parameters $\kappa = 0.05$ and $\lambda/M_* = 0.2$, which are compatible with the observational constraints. For a complete simulation including also the DOFs of the imaginary part of

[13] Due to the renormalization group flow.

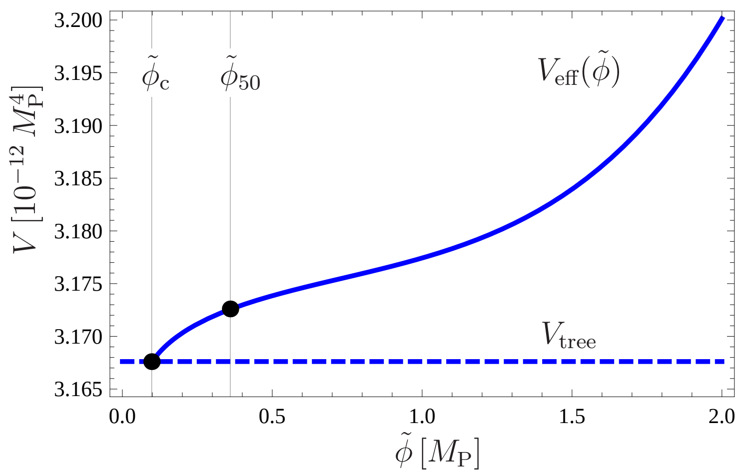

Figure 6.7: Graphical illustration of the one-loop effective potential for $\tilde{\phi}$ with typical values of the field $N_{\rm e} \approx 50$ e-folds before the end of inflation $\tilde{\phi}_{50}$ and at the critical value $\tilde{\phi}_{\rm c}$ where inflation ends. $\tilde{\phi}$ is given in units of the reduced Planck mass.

T and the phase of the complex inflaton Φ, have a look at App. B.1.

To obtain the predictions of our model, we have calculated the observables from the full loop-corrected potential. We want to stress that all fields besides the inflaton direction ϕ acquire a constant value very quickly such that the model can effectively be treated as a single-field model of inflation. Hence, Eqs. (2.36), (2.38), (2.39) and (2.40) apply and there is no curving of the trajectory in field space and thus no isocurvature mode. Therefore, we have fixed $\langle\rho\rangle = 3/4$ to its minimum for $\kappa_\rho = -1$, cf. Eq. (6.64). Since only the combination λ/M_* is relevant, we can also fix $M_* = 1$ without loss of generality. For each point in parameter space, the scale M has been calculated numerically at horizon exit such that the amplitude of the curvature perturbations $P_{\mathcal{R}}^{1/2}$ resembles the observed value given in Eq. (2.41) to one sigma. In addition, the renormalization scale is taken to be $Q = m_{\rm F}/\sqrt{y}$ which makes the constant logarithmic contribution vanish in the loop-potential (6.71) and thereby makes $(1/y)$ a good expansion parameter.

In order to investigate the parameter space and give the predictions for the spectral index and the tensor-to-scalar ratio in this model, we scan this two-dimensional space. Therefore, we fix the other parameters and the renormalization scale as above. The results are displayed in Fig. 6.8. In the upper left plot, the contour lines of the spectral index $n_{\rm s}$ are plotted over a wider range of the parameter space, where both λ and κ have been varied from 0 to 0.2. The other three plots show contour lines of $n_{\rm s}$, r and the scale M in regions in which λ has been varied from 0 to 0.04 and κ from 0.2 to 0.8. There, a minimum of the spectral index has been found. In the ranges depicted, the spectral index is found to be below $n_{\rm s} < 1$, but above $n_{\rm s} \gtrsim 0.98$. The tensor-to-scalar ratio is $r \lesssim 10^{-2}$ and $M = \mathcal{O}(10^{-3})$. As typical for an effective single-field inflation model, the non-Gaussianity parameter $f_{\rm NL}$ is negligible.

We stress that the above results have been calculated using the minimal model defined in Eqs. (6.25) and (6.53) with $f(\rho)$ of no-scale form (6.60). Although the no-scale form is particularly well motivated, in general this assumption might be relaxed and a different function $f(\rho)$ may be chosen. The main requirement for $f(\rho)$ is that the potential for ρ has a minimum around the Planck scale and that the shape of the potential forces ρ to settle rapidly at its minimum. After ρ has settled in the minimum, the values of $f(\rho_{\min})$ and its derivatives affect the normalization of the inflaton field and also the field-dependent masses which finally enter the CW one-loop potential. We have analyzed some examples with generalized functions $f(\rho)$ and found that in the considered cases the shape of the potential was not affected and the effects on the observables were negligible. To give one explicit example, for $f(\rho) = 1/\rho$ and $\kappa_\rho = -1$, we find a minimum at $\langle\rho\rangle = (\sqrt{5} - 1)/2$ where the modulus stabilizes quickly such that inflation can occur. In this scenario, the minimal value of the spectral index also lies around $n_{\mathrm{s}} \approx 0.98$ and the tensor-to-scalar ratio, the scale of inflation as well as the running of the spectral index are only slightly changed.

On the other hand, as noted already in the beginning of this section, we find that the inclusion of additional couplings, for instance of $\kappa_{SH} \neq 0$ as in Eq. (6.35) or (6.43), could lower the spectral index at loop-level. The reason is that, with such modifications, the form of the potential changes qualitatively as we have shown in the context of shift symmetry in Fig. 6.3. For example, adding to our Kähler potential (6.53) a term $\kappa_{SH}|S|^2|H|^2$, with the model parameters $(\kappa, \lambda, \kappa_{SH}) = (0.05, 0.2, 10)$ we find a spectral tilt $n_{\mathrm{s}} \approx 0.953$ at horizon exit where the WMAP normalization fixes $M \approx 2.9 \cdot 10^{-3}$. In contrast, without the additional term for $(\kappa, \lambda, \kappa_{SH}) = (0.05, 0.2, 0)$ we obtain $n_{\mathrm{s}} \approx 0.982$ and $M \approx 3.4 \cdot 10^{-3}$.

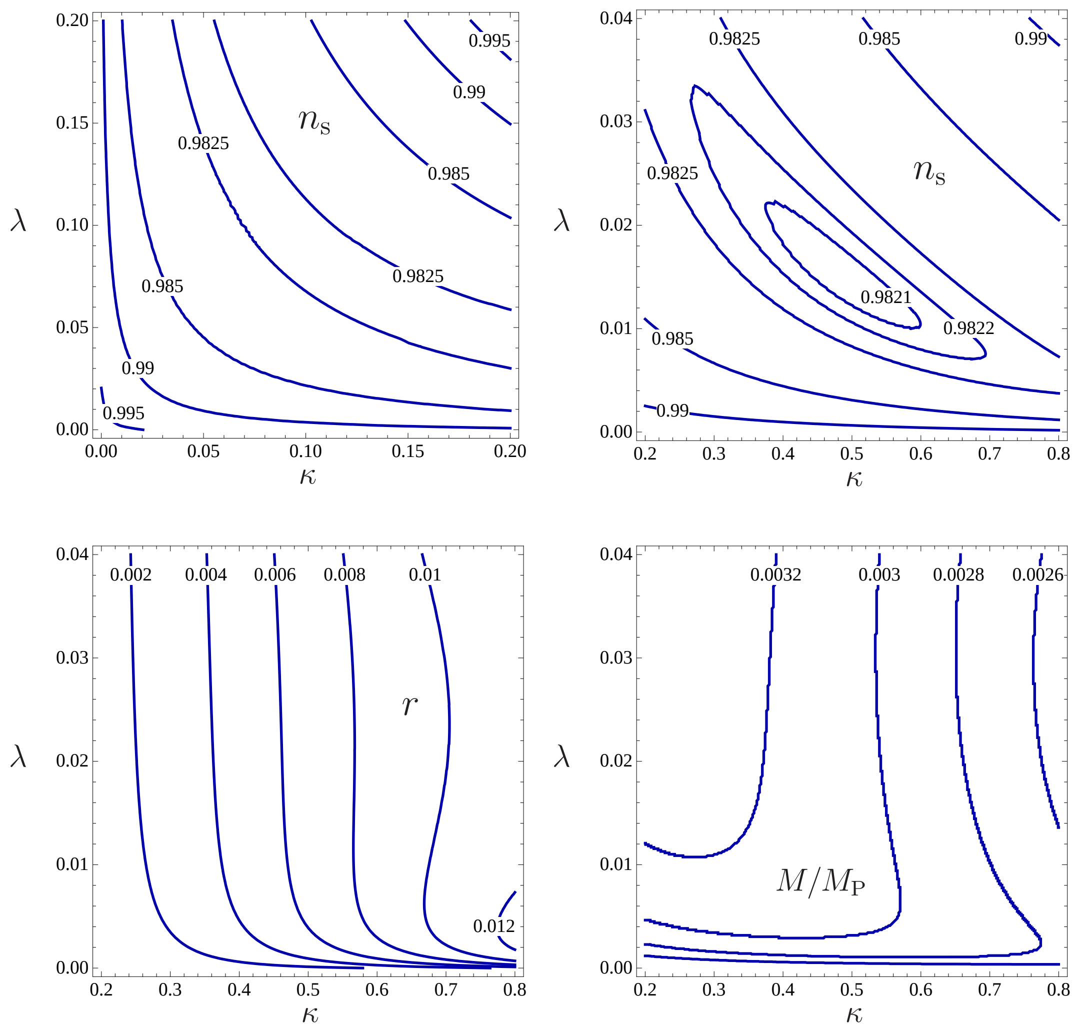

Figure 6.8: Contours of the predicted values of $n_{\rm s}$, r and M depending on κ and λ.

Chapter 7

Chaotic Inflation Models

Chaotic inflation models with quadratic potentials shall be considered within both the global SUSY as well as the SUGRA context in this chapter. The original non-SUSY version of chaotic inflation [23] can be extended to the framework of global SUSY in a trivial way. Nevertheless, when trying to find SUGRA embeddings of such a scenario, trouble is inevitable.

First of all, as explained in Sec. 5.1, the η-problem always poses a threat to inflation models in SUGRA. But with large inflaton field values well above the Planck scale $\phi > M_{\mathrm{P}}$, as necessary for chaotic inflation, the η-problem is much more severe. This can be seen by looking at Eq. (5.2). For a canonical Kähler potential, any term of higher order in the SUGRA expansion $\sim \exp(|\phi|^2/M_{\mathrm{P}}^2)$ contributes an even more severe η-problem than all the terms of lower order. Therefore, perturbatively the expansion does not make sense anymore. Solutions to the η-problem by a tuning of the parameters in a general expansion of the Kähler potential, as we have discussed in the context of hybrid inflation in Sec. 6.1.1 and tribrid inflation in Sec. 6.2.1, are thus not possible for large field models.

Hence, the application of fundamental symmetries in the Kähler potential is the only way of saving chaotic inflation in SUGRA. As we have already discussed extensively in Ch. 6, two promising candidate symmetries are shift symmetry and Heisenberg symmetry. Chaotic inflation in SUGRA using a shift symmetry in the Kähler potential has been introduced in Refs. [106, 134]. We have shown that viable chaotic inflation can also be realized imposing a Heisenberg symmetry on the Kähler potential while the additional modulus field can be stabilized during slow-roll inflation [3].

This chapter is organized as follows. In Sec. 7.1, we briefly explain how a quadratic inflaton potential can be accounted for in global SUSY and point out why it is problematic to realize SUGRA chaotic inflation with the same setup. Sec. 7.2 is dedicated to a viable realization of quadratic inflaton potentials in SUGRA. For this purpose we introduce a different superpotential which has the suitable properties to be combined with Kähler symmetries, similar as for tribrid inflation, cf. (6.23). As explicit examples, we review the shift symmetry solution of Kawasaki et al. [106, 134] in Sec. 7.2.1 and present our Heisenberg symmetry realization in Sec. 7.2.2.

7.1 Supersymmetric Chaotic Inflation

The simplest choice of a superpotential for realizing chaotic inflation with a quadratic potential is just a mass term for a single inflaton chiral supermultiplet Φ given by

$$W = m\,\Phi^2\,. \tag{7.1}$$

Here, m shall denote a dimensionful mass parameter. In the absence of gauge interactions, the scalar potential derived from Eq. (7.1) stems from F-term contributions only. Thus, using Eq. (3.38) and introducing the canonically normalized real scalar inflaton $\phi = \sqrt{2}\,|\Phi|$ we obtain

$$V_F = m^2|\Phi|^2 = \frac{1}{2}\,m^2\phi^2\,. \tag{7.2}$$

What we have found is exactly the simple quadratic potential (2.43) which for $N_{\mathrm{e}} = 60$ generically predicts $n_{\mathrm{s}} \approx 0.97$ in good agreement with observations and a rather large $r \approx 0.13$ while $\phi_{60} > 1$ and $m \approx 10^{-6}$.

So far this is all good, however things start to look less promising when we consider the above superpotential in SUGRA [106]. A minimal Kähler potential $K = |\Phi|^2$ is unacceptable due to severeness of the η-problem when $|\Phi| > 1$. Therefore, all we are left with are symmetry solutions. The simplest case to consider which also provides us with canonical kinetic terms is to apply a shift symmetry as introduced in Eq. (5.5) in the Kähler potential given by

$$K = \frac{1}{2}\,(\Phi + \Phi^*)^2\;. \tag{7.3}$$

Eq. (7.3) protects the imaginary part $\phi = \sqrt{2}\,\mathrm{Im}(\Phi)$ as potential inflaton from the η-problem, while it provides the real part $\mathrm{Re}(\Phi) = (\Phi + \Phi^*)/2$ with large SUGRA mass corrections stabilizing it at zero during inflation. This can be seen by looking at the F-term potential calculated with the use of Eq. (3.67) to

$$V_F = \mathrm{e}^{2\,\mathrm{Re}(\Phi)^2}\left[\left|\left(2\,m\,\Phi + 2\,m\,\mathrm{Re}(\Phi)\,\Phi^2\right)\right|^2 - 3\,m^2|\Phi|^4\right]\,. \tag{7.4}$$

For $\phi > 1$ the potential (7.4) obviously contains the mass terms required to stabilize $\mathrm{Re}(\Phi) = 0$. Effectively, we thus end up with a tree level scalar potential

$$V_F \simeq 2\,m^2\phi^2 - \frac{3}{4}\,m^2\phi^4\,, \tag{7.5}$$

in which for $|\phi| > 2/\sqrt{3}$, the term $\sim \phi^4$ dominates and $V_{\mathrm{tree}} \to -\infty$ as $\phi \to \infty$ which does not allow for viable slow-roll inflation. We therefore conclude that with the superpotential (7.1) it is not possible to solve the η-problem of chaotic inflation in SUGRA using a shift symmetric Kähler potential.

7.2 Supergravity Chaotic Inflation

In this section, we introduce a different superpotential, which turns out to be more promising for realizing chaotic inflation in SUGRA with a quadratic potential. Let us first allude the general properties of the superpotential in global SUSY before coming to explicit SUGRA realizations in Sec. 7.2.1 and Sec. 7.2.2. The superfield content $\{X, \Phi\}$ of the setup under consideration is extended by one further chiral superfield X in addition to the inflaton superfield Φ, whose interactions in terms of the superpotential read

$$W = m\, X\, \Phi\,, \tag{7.6}$$

where m is a real mass parameter. This superpotential has first been considered in [106].

Starting with Eq. (7.6), we can derive the F-term scalar potential using (3.38) which in the absence of gauge interactions is given by

$$V_F = m^2|\Phi|^2 + m^2|X|^2\,. \tag{7.7}$$

As we can see from Eq. (7.7), at the global SUSY level and having again $m \ll \mathcal{O}(1)$, both X and Φ are potential inflaton directions with degenerate masses. However, as we show explicitly in Sec. 7.2.1 and Sec. 7.2.2, the mass degeneracy can be broken by SUGRA effects. Without any symmetry in the Kähler potential for X, the latter receives large SUGRA mass corrections of the order of the Hubble scale $m_X \approx \mathcal{H}$ which keep it stabilized in its potential minimum at $X = 0$. The inflaton DOF in Φ on the other hand can be subject to some symmetry in the Kähler potential. Hence it retains its small mass m, making it the only viable inflaton candidate $\phi = \sqrt{2}\,|\Phi|$ with an effective potential

$$V_F \simeq \frac{1}{2}\, m^2 \phi^2\,, \tag{7.8}$$

just as in Eq. (7.2).

Still there are crucial differences which are responsible for making the superpotential in Eq. (7.6) compatible with symmetric Kähler potentials, while the one presented in Eq. (7.1) fails to be realized in SUGRA. Very similar as for the hybrid and tribrid inflation models in Ch. 6 the relevant properties are related to the superpotential and its derivatives during inflation [129]. In the inflationary patch $X = 0$ and $\Phi \neq 0$, Eq. (7.6) implies

$$W = 0\,, \qquad W_\Phi = 0\,, \qquad W_X \neq 0\,. \tag{7.9}$$

On the contrary, the simpler superpotential Eq. (7.1) can neither fulfill the first nor the second property of Eq. (7.9) during inflation when $\Phi \neq 0$. Nevertheless, it is exactly the above conditions (7.9) which avoid problematic terms as the negative one in the scalar potential (7.4) when gauging global SUSY. In the following, we demonstrate in two explicit realizations using different symmetries in the Kähler potential that Eq. (7.6) can indeed give rise to sufficient chaotic inflation.

7.2.1 Chaotic Inflation with Shift Symmetry

In the following, we briefly review the idea of combining the superpotential in Eq. (7.6) with a shift symmetry invariant Kähler potential based on Refs. [106, 134]. They have shown that imposing a shift symmetry as given in Eq. (5.5) on the Kähler potential in the inflaton sector Φ, however with a minimal Kähler potential term for the additional superfield X, namely

$$K = \frac{1}{2}\left(\Phi + \Phi^*\right)^2 + |X|^2\,, \tag{7.10}$$

viable chaotic inflation can be realized in the direction of the imaginary part of the complex $\Phi = (\phi_{\mathrm{R}} + \mathrm{i}\,\phi)/\sqrt{2}$. Eqs. (7.6) and (7.10) are justified by a $U(1)_{\mathrm{R}}$ symmetry under which $X \to \mathrm{e}^{-2\,\mathrm{i}\,\alpha}X$ and $\Phi \to \Phi$ and a discrete $\mathbb{Z}_2$ symmetry under which they transform as $X \to -X$ and $\Phi \to -\Phi$. The above Kähler potential gives rise to canonical kinetic terms, since the Kähler metric derived from it is described by the unit matrix $K_{i\bar{j}} = \delta_{i\bar{j}}$.

Note that the superpotential (7.6) violates the shift symmetry (5.5). However, the symmetry is enhanced in the limit $m \to 0$ and the small breaking parameter $m \ll \mathcal{O}(1)$ should find its explanation in a more fundamental theory. Hence, the model is natural in 't Hooft's sense [125].

The SUGRA F-term scalar potential (3.67) applied to Eqs. (7.6) and (7.10) is given by

$$\begin{aligned} V_F &= \mathrm{e}^K\left[\left|(W_\Phi + W K_\Phi)\right|^2 + \left|(W_X + W K_X)\right|^2 - 3\,|W|^2\right] \\ &= m^2\,\mathrm{e}^{\phi_{\mathrm{R}}^2 + |X|^2}\left[\left(1 - |\Phi|^2 + 2\,\phi_{\mathrm{R}}^2 + 2\,\phi_{\mathrm{R}}^2|\Phi|^2\right)|X|^2 + \left(1 + |X|^4\right)|\Phi|^2\right]\,. \end{aligned} \tag{7.11}$$

In Eq. (7.11), the exponential factor e^K enforces field values $\phi_{\mathrm{R}}, |X| < \mathcal{O}(1)$ while the inflaton ϕ value is not restricted due to the shift symmetry protecting it. Assuming this, we can effectively write the potential as

$$V_F \simeq \frac{1}{2}\,m^2\phi^2\left(1 + \phi_{\mathrm{R}}^2\right) + m^2|X|^2\,. \tag{7.12}$$

The system is assumed to be in a chaotic initial situation (2.42) which implies initially large $\phi(0) \approx m^{-1} \gg 1$. In Eq. (7.12) this generates a very large effective mass for ϕ_{R} which thus settles to zero very rapidly. Now the ϕ-field dominates the potential due to Eq. (7.12) and drives inflation with a Hubble scale $\mathcal{H} \approx m\,\phi/\sqrt{6}$. The X-field however also satisfies slow-roll conditions, $\epsilon_X \ll 1$, $|\eta_X| \ll 1$, and therefore evolves according to the slow-roll EOM (2.16) such that one can estimate

$$\frac{X(t)}{X(0)} \approx \frac{\phi(t)}{\phi(0)}\,, \tag{7.13}$$

where the initial field values are denoted by $X(0)$ and $\phi(0)$. Consequently, X decreases faster than ϕ and we typically have the situation $X \ll \phi$ which effectively satisfies (7.9) According to [135], the amplitude of the curvature perturbation is therefore only determined by ϕ with a scale of inflation $m \approx 10^{-5}$ as derived from the WMAP normalization in Eq. (2.41).

We would like to note that invoking a higher dimensional operator $\sim |X|^4$ in the Kähler potential can provide a SUGRA mass for X which prevents it from slow-rolling in the first place. This helps to fulfill (7.9) during inflation.

7.2.2 Chaotic Inflation with Heisenberg Symmetry

The idea of using a Heisenberg symmetry [109] in the Kähler potential to solve the η-problem of chaotic inflation in SUGRA with the superpotential (7.6) is presented below. It is based on the treatment in Ref. [3].

First of all, let us introduce the chiral field content which consists of three superfields $\{X, \Phi, T\}$. The driving field supposed to stay at zero during inflation which generates the inflaton potential by its F-term is the scalar component of X. The supermultiplet Φ contains the slowly rolling inflaton and the presence of the modulus supermultiplet T is required by the invariance of the Kähler potential under the Heisenberg symmetry (5.7). Having only one inflaton in Φ, the Heisenberg invariant combination (5.8) becomes

$$\rho = T + T^* - |\Phi|^2 \,. \tag{7.14}$$

Such an additional Kähler modulus field T is immediately subject to the moduli stabilization problem as described in Sec. 5.2 and we have to make sure that T, or equivalently ρ, is sufficiently stable during slow-roll inflation.

The Kähler potential we propose respects the Heisenberg symmetry, since it only depends on the invariant field combination ρ defined in Eq. (7.14) and is given by

$$K = \left(1 + \kappa_X\, |X|^2 + \kappa_\rho\, \rho\right) |X|^2 + f(\rho) \,. \tag{7.15}$$

At this point we should note that just as in Sec. 6.2.3 there is no need for distinguishing between the effective and fundamental dimensionless parameters $\kappa_X = \tilde{\kappa}_X$, $\kappa_\rho = \tilde{\kappa}_\rho$. Without loss of generality, we assume that the higher dimensional effective operators in Eq. (7.15) are generated at the gravity scale and hence $\Lambda = 1$. This is also the reason for not explicitly writing down the suppression by appropriate powers of Λ in the above Kähler potential. As we will see, the parameter κ_X helps to give a large SUGRA induced mass to X which keeps it in its potential minimum at zero during inflation. Similarly, κ_ρ generates a potential minimum for ρ with a mass of the order of the Hubble scale, thereby solving the moduli stabilization problem.

In this framework, having $X = 0$, we realize inflation with conditions (7.9) satisfied. An attractive feature of satisfying these conditions during inflation is that it typically cancels several couplings between the inflaton sector and any other possibly existing scalar field sector in the theory.[1] We note that the superpotential (7.6) breaks the Heisenberg symmetry respected by the Kähler potential (7.15) and gives rise to a quadratic tree level potential for the absolute value $\phi = \sqrt{2}\,|\Phi|$. Again, this model is "natural" in the sense that setting the small breaking parameter to zero allows us to realize enhanced symmetry [125].

[1]This has been studied, e.g., in the context of hybrid inflation with shift symmetry, see Refs. [2, 132].

Imposing a Heisenberg symmetry on the Kähler potential allows us to implement super-Planckian values for the inflaton field in SUGRA theories, as can be seen by looking at the tree level scalar potential (3.67). Due to the Heisenberg symmetry the exponential factor e^K is independent of the inflaton field and therefore we can realize field values larger than M_P as required for chaotic inflation.

The Kähler metric can be calculated as the second derivatives of the Kähler potential (7.15) w.r.t. the superfields and their conjugates and, along the direction $X = 0$ in the (X, Φ, T)-basis, it is of block-diagonal form

$$\left(K_{i\bar{j}}\right) = \begin{pmatrix} 1 + \kappa_\rho\,\rho & 0 & 0 \\ 0 & f''(\rho)\,|\Phi|^2 - f'(\rho) & -f''(\rho)\,\Phi^* \\ 0 & -f''(\rho)\,\Phi & f''(\rho) \end{pmatrix} . \tag{7.16}$$

Notice the great similarity to the Kähler metric (6.55) in the case of tribrid inflation which is due to the Heisenberg symmetry. Assuming $X = 0$, which accounts for (7.9) the F-term scalar potential reads

$$V_F \simeq e^K\, K^{X\bar{X}}\, |W_X|^2 . \tag{7.17}$$

Now we make a particular choice of $f(\rho)$ of the no-scale form [112] which is given by

$$f(\rho) = -3 \ln \rho . \tag{7.18}$$

We would like to remark that for our approach to work this specific form of the Kähler potential is not required, however, it is well motivated from string theory and serves well to illustrate the modulus stabilization mechanism via the coupling between ρ and X in the Kähler potential. After inflation, when $X \simeq \Phi \approx 0$, another sector of the model will be responsible for SUSY breaking which may lead, for instance, to an effective no-scale model with radiatively induced gravitino mass. With $f(\rho)$ of the above no-scale form, the potential (7.17) is given by

$$V_F = \frac{m^2 |\Phi|^2}{\rho^3 (1 + \kappa_\rho\,\rho)} . \tag{7.19}$$

For any fixed value of ρ the potential is just a mass term for the ϕ field that is suitable for chaotic inflation. The scalar potential has a minimum at[2]

$$\langle\rho\rangle = -\frac{3}{4\,\kappa_\rho} . \tag{7.20}$$

Assuming that ρ is positive, κ_ρ must be negative to generate a minimum and, as an example, we choose its value to be $\kappa_\rho = -1$. Other than that, the form of the potential has a pole located at $\rho = -1/\kappa_\rho$ and for values $\rho > -1/\kappa_\rho$ the potential is negative and has a runaway behavior. Viable inflation thus occurs in the range of initial field values $0 < \rho < -1/\kappa_\rho$ only, which we assume for all our considerations below. To have a picture of the modulus potential in mind, we note that for fixed inflaton values ϕ it is very similar to Fig. 6.5.

[2]For a general function $f(\rho)$ the VEV is determined from $f'(\langle\rho\rangle)\,(1 + \kappa_\rho\,\langle\rho\rangle) = \kappa_\rho$.

At the start of inflation $m_\rho^2 \sim |W_X|^2$ and the Hubble scale squared is also of the same order. On the other hand $m_\phi \simeq m \ll \mathcal{H}$ during inflation. Therefore the ρ field settles to its minimum very quickly whereas the inflaton field, being light, slowly rolls along its potential. When ρ has settled to its minimum and ϕ is slowly rolling, the vacuum energy dominates and drives inflation. In fact, the coupling κ_ρ between ρ and X in the Kähler potential induces a mass for the ρ field proportional to the vacuum energy during inflation and it allows the modulus to be stabilized very quickly before inflation.[3] For $\kappa_\rho = 0$ in the expression for the scalar potential (7.19), the ρ field would acquire a runaway potential. Although $\langle\rho\rangle$ is independent of ϕ, the ρ field is not absolutely fixed at the minimum of the potential due to the presence of effects from non-canonical kinetic terms. We will verify these qualitative statements below by deriving and simulating the full evolution equations of the fields. Note furthermore that when X and ρ settle to their respective minima, i.e., $X = 0$ and $\rho = \langle\rho\rangle$, the potential has the same form as the corresponding global SUSY potential (7.8). For example in our case, once the modulus is stabilized at the minimum during inflation the tree level potential reduces to a quadratic potential, as can be seen from Eq. (7.19). Next, we discuss the scalar field dynamics and the scalar mass spectrum.

Above, we have discussed the form of the potential when the X field has settled to its minimum and argued qualitatively that the ρ field will also get stabilized quickly such that successful chaotic inflation can be realized. Let us now perform a full numerical simulation to investigate the dynamics of the fields and calculate their masses afterwards. We begin by writing down the kinetic terms for the fields. With $X = X^* = 0$ in the Kähler metric the kinetic terms read

$$\mathcal{L}_{\text{kin}} = (1+\kappa_\rho\,\rho)\,|\partial_\mu X|^2 + \frac{3}{\rho}\,|\partial_\mu\Phi|^2 + \frac{3}{4\,\rho^2}\,(\partial_\mu\rho)^2 + \frac{3}{4\,\rho^2}\,(I_\mu)^2\,, \tag{7.21}$$

where we have defined the four-vector

$$I_\mu = \mathrm{i}\,[\partial_\mu(T-T^*) + \Phi\,\partial_\mu\Phi^* - \Phi^*\partial_\mu\Phi]\ . \tag{7.22}$$

The phases of the scalar fields Φ, X as well as $\mathrm{Im}(T)$ very quickly approach a constant value in an expanding universe and subsequently decouple from the absolute values and $\mathrm{Re}(T)$ in the EOMs, as discussed in App. B.1. Therefore, we only take into account the absolute values $\phi = \sqrt{2}\,|\Phi|$, $x = \sqrt{2}\,|X|$ and the real field ρ in what follows. In this case I_μ vanishes identically.

The evolution equations for the background field values ϕ, x and ρ in an expanding universe derived from Eq. (7.21) are given by

$$\begin{aligned}
\ddot{\phi} + 3\,\mathcal{H}\,\dot{\phi} - \frac{\dot{\rho}\,\dot{\phi}}{\rho} + \frac{\rho}{3}\,V_\phi &= 0\,,\\
\ddot{x} + 3\,\mathcal{H}\,\dot{x} + \frac{1}{(1+\kappa_\rho\,\rho)}\,(\kappa_\rho\,\dot{\rho}\,\dot{x} + V_x) &= 0\,,\\
\ddot{\rho} + 3\,\mathcal{H}\,\dot{\rho} - \frac{\dot{\rho}^2}{\rho} + \dot{\phi}^2 - \frac{\kappa_\rho}{3}\,\rho^2\,\dot{x} + \frac{2}{3}\,\rho^2\,V_\rho &= 0\,,
\end{aligned} \tag{7.23}$$

[3]The role of this coupling as a mechanism to stabilize ρ during inflation is very similar to what we have studied in Sec. 6.2.3 in the context of tribrid inflation with Heisenberg symmetry.

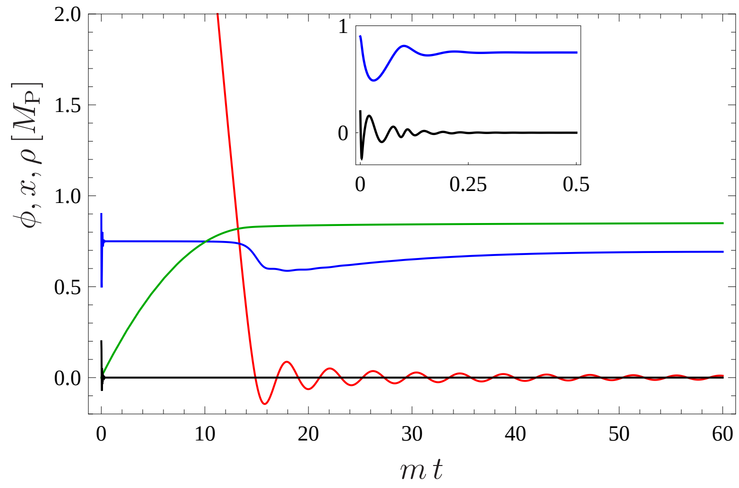

Figure 7.1: Evolution of the scalar fields. The green curve represents the number of e-folds in $N_\mathrm{e}/100$, the red curve represents the inflaton ϕ, the blue curve depicts the evolution of ρ and the black curve the evolution of x.

where V_ϕ, V_x, V_ρ are the derivatives of the potential w.r.t. the fields, and $\mathcal{H}$ denotes the Hubble expansion rate. For large enough initial values of ϕ the field follows a slow-roll trajectory with x and ρ being practically fixed at the minimum of their potential. In Fig. 7.1 we show the dynamics of the fields for generic initial conditions. As one can see, the x field settles to its minimum followed by the ρ field, whereas the inflaton field remains slowly rolling for more than 60 e-folds of inflation. As the inflaton field rolls down towards its minimum, the vacuum energy density decreases and thus also the mass of the ρ field. When the slow-roll conditions are violated, the inflaton field acquires a large velocity and this provides a kick to the evolution of ρ due to the $\dot{\phi}^2$ term in its EOM. At the end of inflation the ϕ field starts oscillating, the velocity term gets damped and finally the ρ field settles at a slightly different field value. At this epoch we expect some other modulus stabilization mechanism to take over. In App. C, we discuss the implications of one-loop corrections to the effective potential. As it turns out, they have a negligible effect for the predictions of the model since their contribution provides an additional mass term which just leads to a mass renormalization in order to fit the WMAP normalization on $\mathcal{P}_\mathcal{R}(k)$. The loop-effect on the field dynamics is depicted in Fig. C.1.

In what follows, we denote as physical fermions and scalars those with canonical kinetic terms. According to Eq. (3.63) the scalar kinetic Lagrangian in terms of the original fields $Z^i = \{X, \Phi, T\}$ reads

$$\mathcal{L}_\mathrm{kin} = K_{i\bar{j}}\, \partial_\mu Z^i\, \partial^\mu Z^{*\bar{j}}\,, \tag{7.24}$$

and similarly for the fermions. We can canonically normalize the fields by expanding the Lagrangian around the minimum. In particular during inflation we have $X = 0$ and $\rho = \langle\rho\rangle = -3/4\,\kappa_\rho$, while ϕ is varying slowly with time. The field ρ can be redefined as the canonically normalized field $\tilde{\rho}$ by

$$\tilde{\rho} = \sqrt{\frac{3}{2}}\,\ln\rho\,. \tag{7.25}$$

The remaining non-canonical factors in Eq. (7.21) only depend on ρ (or $\tilde{\rho}$) and we can define the physical states during inflation by expanding those factors around $\langle\rho\rangle$ giving

$$\mathcal{L}_{\rm kin} = (1+\kappa_\rho\langle\rho\rangle)\,|\partial_\mu X|^2 + \frac{3}{\langle\rho\rangle}\,|\partial_\mu\Phi|^2 + \frac{1}{2}\,(\partial_\mu\tilde{\rho})^2 + \ldots\,. \tag{7.26}$$

Then, with normalization factors $(1+\kappa_\rho\langle\rho\rangle\,,\,3/\langle\rho\rangle\,,\,1)$, for $(\tilde{x},\,\tilde{\phi},\,\tilde{\rho})$ the physical scalar masses squared are given by

$$\begin{aligned}
m_{\tilde{x}}^2 &= \frac{64}{27}\,\kappa_\rho^2\,m^2\left(1+32\,\kappa_X\,\kappa_\rho\,\tilde{\phi}^2\right)\,,\\
m_{\tilde{\phi}}^2 &= -\frac{64}{27}\,\kappa_\rho^3\,m^2\,,\\
m_{\tilde{\rho}}^2 &= -\frac{256}{27}\,\kappa_\rho^3\,m^2\tilde{\phi}^2\,.
\end{aligned} \tag{7.27}$$

Fields with a tilde are canonically normalized. To calculate their masses we have assumed that ρ has settled to its minimum.[4] First we note that the canonically normalized inflaton field has a mass smaller than the Hubble scale $\mathcal{H}$ during inflation when its field value is super-Planckian. For κ_X negative, the mass of the $\tilde{x}$ field is larger than the mass of $\tilde{\rho}$ and both masses are larger than the Hubble scale $\mathcal{H}\simeq\sqrt{V_F/3}$ (with positive masses squared provided that κ_ρ is negative). Therefore, as we have shown in the numerical simulation the x field settles quickly to its VEV at $\langle x\rangle = 0$ followed by the ρ field which settles to $\langle\rho\rangle$.

Similar to the conventional chaotic inflation model with purely quadratic potential the predictions for the spectral index $n_{\rm s}$ and the tensor-to-scalar ratio r are given by

$$n_{\rm s}\simeq 1-\frac{2}{N_{\rm e}}\,,\qquad r\simeq\frac{8}{N_{\rm e}}\,, \tag{7.28}$$

where $N_{\rm e}$ is the number of e-folds before the end of inflation where the observable scales have crossed the horizon. For $N_{\rm e}=60$ the predicted value of the spectral index $n_{\rm s}\approx 0.97$ is well consistent with the available cosmological data (2.41). In addition, the tensor-to-scalar ratio is predicted to be $r\approx 0.13$. This might be probed by the Planck satellite [9] which is taking data at the time of writing.

[4] Here κ_ρ has been left general instead of setting it to -1 as done before.

Part IV

Inflation in Grand Unification

Chapter 8

Gauge Non-Singlet Tribrid Inflation

This part of the dissertation is concerned with realizing inflation in SUSY GUTs. In this context, hybrid or tribrid inflation models turn out to be particularly appealing, since there the waterfall field that ends inflation can simultaneously play the role of the Higgs field breaking the GUT. Thus a direct link between the GUT breaking phase transition and the end of inflation can be established, bringing together particle physics and cosmology. In contrast to other treatments based on SUSY hybrid inflation [45, 47, 46, 136], where the inflaton typically remains a gauge singlet, we propose a model based on tribrid inflation in which the inflaton can carry a charge under the gauge group of the GUT. Due to the initial displacement of the inflaton VEV, many generic problems of inflation in GUTs such as the violation of slow-roll inflation via gauge interactions and the formation of stable topological defects after inflation can be avoided in this framework. The work presented in this part is based on [4].

This chapter is dedicated to motivating the concept of a GNS inflaton and making the reader familiar with it. After motivating our work in Sec. 8.1, we introduce the basic idea of SUSY tribrid inflation with a GNS inflaton, focusing on the example of an Abelian gauge group $G = U(1)$ in Sec. 8.2. In Ch. 9 we discuss a realistic model of this kind based on the SUSY PS gauge group. The general model is proposed in Sec 9.1 and subsequently in Sec. 9.2 we derive the conditions under which inflation can proceed along D-flat directions. We specialize to the case of the right-handed sneutrino inflationary trajectory in Sec. 9.3. Within this particular scenario, the issues associated with radiative corrections for a GNS inflaton at one- and two-loop level are confronted in Sec. 9.4. Finally, in Ch. 10 we are concerned with embedding the preceding PS model into a SUSY $SO(10)$ GUT on the one hand and into SUGRA on the other hand. The generalization to $SO(10)$ is demonstrated in Sec. 10.1, whereas in Sec. 10.2 we apply a generalization of the Heisenberg symmetry proposed in part III to incorporate GNS fields.

8.1 Motivation: Inflation meets Particle Physics

A long standing question in inflation models is: Who is the inflaton? We are still far from answering this question. Indeed it is still unclear whether the inflaton, the (presumed) scalar field responsible for inflation, should originate from the observable (matter) sector or the hidden (e.g. moduli) sector of the theory. However, the connection between inflation and particle physics is rather difficult to achieve in the observable sector due to the lack of understanding of physics beyond the SM and in the hidden sector due to the lack of understanding of the string vacuum. Over the past dozen years there has been a revolution in particle physics due to the experimental discovery of neutrino mass and mixing [137], and this improves the prospects for finding the inflaton in the observable sector. Indeed, if the SM is extended to include the seesaw mechanism [39, 40, 41, 42, 43] and SUSY [138], the right-handed sneutrinos, the superpartners of the right-handed neutrinos, become excellent inflaton candidates. Motivated by such considerations, the possibility of chaotic (large field) inflation with a sneutrino inflaton [139] was revisited [140]. Subsequently it has been suggested that one (or more) of the singlet sneutrinos could be the inflaton of hybrid inflation [44].

Despite the unknown identity of the inflaton, conventional wisdom dictates that it must be a gauge singlet since otherwise gauge interactions could spoil the required flatness of the inflaton potential. For example in SUSY models, scalar components of gauge non-singlet superfields have quartic terms in their potential, due to the D-terms, leading to violations of the slow-roll conditions which are inconsistent with recent observations by WMAP. In addition, gauge non-singlet inflatons would be subject to one-loop Coleman Weinberg corrections from loops with gauge fields which could easily lead to large radiative corrections that induce an unacceptably large slope of the inflaton potential. Furthermore a charged inflaton is in general also subject to two-loop corrections to its mass which can easily be larger than the Hubble scale [48]. Such a contribution is in principle large enough to spoil inflation for any gauge non-singlet scalar field, leading to a sort of gauge η-problem.

In the following chapters we shall argue that, contrary to conventional wisdom, the inflaton may in fact be a GNS. For definiteness we shall confine ourselves to examples of SUSY tribrid inflation[1] as introduced in Sec. 6.2 and show that the scalar components of GNS superfields, together with fields in conjugate representations, may form a D-flat direction suitable for inflation. Along this D-flat trajectory the usual F-term contributes the large vacuum energy.

We emphasize that, in sneutrino inflation models, the right-handed sneutrino has previously been taken to be a gauge singlet, as for example in SUSY GUTs based on $SU(5)$ rather than $SO(10)$. However, one of the attractive features of SUSY $SO(10)$ is that it predicts right-handed neutrinos which carry a charge under a gauged $B-L$ symmetry. The right-handed sneutrinos of SUSY $SO(10)$, being charged under a gauged $B-L$ symmetry, have not previously been considered as suitable inflaton candidates, but here they may be. Indeed, assuming the sneutrino inflationary trajectory, we calculate the one-loop

[1] We note that GNS inflation may be applied to other types of inflation other than SUSY tribrid inflation.

Coleman–Weinberg corrections and the two-loop corrections, which usually give rise to the gauge η-problem and show that both corrections are compatible with slow-roll inflation. In addition we show that the monopole problem [49] of $SO(10)$ GUTs can be resolved. We shall also show in Sec. 10.2 that the usual η-problem arising from SUGRA [31, 32, 97] may be resolved using a Heisenberg symmetry [109] with stabilized modulus along the lines of the mechanism proposed in Sec. 6.2.3.

8.2 Toy Model and Basic Ideas

The crucial differences between SUSY inflation models of the hybrid and the tribrid type in the absence of gauge interactions have been pointed out in Ch. 6. Here however, we are working with gauge representations. Therefore, let us briefly compare the associated superpotentials again under this premiss. SUSY hybrid inflation is typically[2] based on the superpotential [38]

$$W_0 = \kappa\, S \left(H\bar{H} - M^2 \right) \tag{8.1}$$

where the superfield S is a singlet under some gauge group G, while the superfields H and $\bar{H}$ reside in conjugate reps of G. The F-term of S provides the vacuum energy to drive inflation, the scalar component of the singlet S is identified as the slowly rolling inflaton, and the scalar components of H and $\bar{H}$ are waterfall Higgs fields which take zero values during inflation but acquire a non-vanishing VEV when the inflaton reaches some critical value, ending inflation and breaking the gauge group G at their global minimum $\langle H \rangle = \langle \bar{H} \rangle^* = M$. Typically G is identified as a GUT group and H, $\bar{H}$ are the Higgs which break that group [38].

In contrast, we study the following simple tribrid inflation extension of the superpotential in Eq. (8.1) given by

$$W = W_0 + \frac{\zeta}{\Lambda}\,(\phi\,\bar{\phi})(H\bar{H})\,, \tag{8.2}$$

where we have included an additional pair of GNS superfields ϕ and $\bar{\phi}$ in conjugate reps of G which couple to the waterfall Higgs superfields via a non-renormalizable coupling controlled by a dimensionless coupling constant ζ and a scale Λ.[3] At first glance, we might expect that the presence of the effective operator in Eq. (8.2), which we have added to the superpotential W_0 (8.1), does not perturb the usual SUSY hybrid inflation scenario described above. Nevertheless, its presence allows for the new possibility that inflation is realized via slowly rolling scalar fields contained in the superfields ϕ and $\bar{\phi}$ with the singlet field S staying fixed at zero during (and after) inflation. In a SUGRA framework, non-canonical terms for S in the Kähler potential can readily provide a large mass for S such that it quickly settles at $S = 0$, see Sec. 6.2. On the other hand, large SUGRA mass

[2] Although slight modifications of this superpotential such as shifted hybrid inflation [47] or smooth hybrid inflation [45, 46] have been proposed to resolve the monopole problem.

[3] For illustrative purposes in this chapter we only consider the single operator contraction shown, even though other distinct operators with different contractions are expected. A fully realistic model of this type will be presented in the next chapter.

contributions can be avoided for ϕ and $\bar{\phi}$ using a Heisenberg symmetry, see Sec. 6.2.3. The generalization of this mechanism to the GNS inflaton case will be briefly discussed in Sec. 10.2.

While the singlet field S is held at a zero value by SUGRA corrections, the scalar components of ϕ, $\bar{\phi}$, having no such SUGRA corrections, are free to take non-zero values during the inflationary epoch. The non-zero ϕ, $\bar{\phi}$ field values provide positive mass squared contributions to all components of the waterfall fields H and $\bar{H}$ during inflation, thus stabilizing them at zero by the F-term of the second term in Eq. (8.2). As in standard SUSY hybrid inflation, the F-term of S, arising from W_0 in Eq. (8.1), yields the large vacuum energy density $V_0 = \kappa^2 M^4$ which drives inflation and breaks SUSY. Since ϕ, $\bar{\phi}$ are the only fields which are allowed to take non-zero values during inflation, they may be identified as inflaton fields provided that their potential is sufficiently flat. Due to the fact that both ϕ and $\bar{\phi}$ carry gauge charges under G, their VEVs break G already during inflation. Thus, although ϕ and $\bar{\phi}$ are GNS fields under the original gauge group G, they are clearly gauge singlets under the surviving subgroup of $G' \subset G$ respected by inflation. This trivial observation will help to protect the masses of ϕ and $\bar{\phi}$ against large radiative corrections, as we shall see later. Another key feature is that the quartic term in the ϕ, $\bar{\phi}$ potential arising from D-term gauge interactions is avoided in a D-flat valley in which the fields ϕ and $\bar{\phi}^*$ in conjugate reps take equal VEVs.

Let us assume that the potential of ϕ, $\bar{\phi}$ is sufficiently flat to enable them to be inflaton fields, and that the dominant contribution to the slope of the inflaton potential arises from quantum corrections due to SUSY breaking which make ϕ and $\bar{\phi}$ slowly roll towards zero. Then the waterfall mechanism which ends inflation works in a familiar way, as follows. Once a critical value of ϕ and $\bar{\phi}$ is reached, the negative mass squared contributions to the scalar components of H and $\bar{H}$ from W_0 in Eqs. (8.1) and (8.2) dominate, destabilizing them to fall towards their true vacuum. In this phase transition, the breaking of G is basically taken over by the Higgs VEVs $\langle \bar{H} \rangle^* = \langle H \rangle = M$ and at the same time inflation ends due to a violation of the slow-roll conditions. The vacuum energy is approximately cancelled by the Higgs VEVs and SUSY is approximately restored at the global minimum.

Let us now explicitly calculate the tree-level global SUSY potential for the model in Eq. (8.2), assuming an Abelian gauge group $G = U(1)$. From Eq. (3.38) we know that any SUSY gauge theory gives rise to a scalar potential

$$V = V_F + V_D = F_i^* F^i + \frac{1}{2}\, D^a D^a \,. \tag{8.3}$$

For $G = U(1)$ and equal charges for ϕ and H we find $D = -g\left(|\phi|^2 - |\bar{\phi}|^2 + |H|^2 - |\bar{H}|^2\right)$, where the index a has disappeared because a $U(1)$ has only one generator with g being the gauge coupling constant. Thus we obtain a D-term contribution[4]

$$V_D = \frac{g^2}{2}\left(|\phi|^2 - |\bar{\phi}|^2 + |H|^2 - |\bar{H}|^2\right)^2 \,, \tag{8.4}$$

[4]Setting a possible Fayet–Iliopoulos term to zero.

which in the inflationary trajectory $\langle H\rangle = \langle \bar{H}\rangle^* = 0$ has a D-flat direction $|\phi| = |\bar{\phi}|$. Under the assumption that the D-term potential Eq. (8.4) has already stabilized the fields in the D-flat valley, the remaining potential is generated from the F-term part

$$\begin{aligned} V_F = &\left|\kappa\left(H\bar{H} - M^2\right)\right|^2 + \left|\frac{\zeta}{\Lambda}\,\bar{\phi}\,(H\bar{H})\right|^2 + \left|\frac{\zeta}{\Lambda}\,\phi\,(H\bar{H})\right|^2 \\ &+ \left|\kappa\, S\,\bar{H} + \frac{\zeta}{\Lambda}\,(\phi\,\bar{\phi})\,\bar{H}\right|^2 + \left|\kappa\, S\, H + \frac{\zeta}{\Lambda}\,(\phi\,\bar{\phi})\,H\right|^2 , \end{aligned} \tag{8.5}$$

which can be calculated with the EOMs (3.25). Plugging the D-flatness condition $|\phi| = |\bar{\phi}|$ into Eq. (8.5) and setting $S = 0$, the F-term potential reduces to

$$V_F = \kappa^2 \left|\left(M^2 - H\bar{H}\right)\right|^2 + 2\,\frac{|\zeta|^2}{\Lambda^2}|\phi|^2|H|^2|\bar{H}|^2 + \frac{|\zeta|^2}{\Lambda^2}|\phi|^4|H|^2 + \frac{|\zeta|^2}{\Lambda^2}|\phi|^4|\bar{H}|^2\,. \tag{8.6}$$

The upper panel of Fig. 8.1 depicts the F-term scalar potential within the D-flat valley for all model parameters set to unity. Obviously, in the inflationary valley $S = H = \bar{H} = 0$ it has a flat inflaton direction $|\phi|$ and a tachyonic waterfall direction below some critical value $|\phi_c|$.

One potential problem that arises if the waterfall is associated with the breaking of a non-Abelian unified gauge group G is the possibility of copiously producing topological defects [49] like magnetic monopoles in the waterfall transition at the end of inflation. For such topological defects to form it is necessary that at the critical value when the waterfall occurs several different vacuum directions have degenerate masses and none is favored over the other. If the same vacuum is chosen everywhere in space, no topological defects can form. In this respect, it is crucial to note that the VEV of the inflaton field already breaks the gauge symmetry G. Due to this breaking, effective operators containing terms like[5] $H^n\bar{H}^m\phi^p\,\bar{\phi}^q$ can lead to a deformation of the potential which can force the waterfall to take place in a particular field direction everywhere in space, avoiding the production of potentially problematic topological defects. This is illustrated in the lower plot of Fig. 8.1 for the Abelian example (even though no monopoles can be created in this case; domain walls, however, can.). We will discuss this in more detail in section 9.3.

Yet another potential problem may arise when the inflaton is a GNS. It is due to two-loop corrections to the inflaton mass that is generically larger than the Hubble scale during inflation and would thus spoil slow-roll inflation [48]. As we argue in Sec. 9.4.2, due to the breaking of the gauge symmetry during inflation these corrections to the inflaton potential are not problematic in our model. This comes from the fact that the corrections get suppressed by powers of the large gauge boson masses induced by the inflaton VEV.

Since the two-loop corrections turn out to be negligible, it is enough to consider the effective potential up to one-loop level when calculating predictions for the observable quantities. The slope at one-loop arises due to inflaton-dependent, SUSY breaking waterfall masses. Diagonalizing the mass matrices in the $(H, \bar{H})$-basis we calculate the eigenvalues

[5] With general powers n, m, p, q such that $n + p = m + q$ ensures gauge invariance.

from Eqs. (8.2) and (8.6). There is one Dirac fermion and there are two complex scalars with squared mass given by

$$m_{\mathrm{F}}^2 = |\zeta|^2|\phi|^4/\Lambda^2\,, \qquad m_{\mathrm{S}}^2 = |\zeta|^2|\phi|^4/\Lambda^2 \pm |\kappa|^2 M^2\,. \tag{8.7}$$

In particular for a single field model as in the case $G = U(1)$, the relevant inflationary predictions are given by N_{e}, $\mathcal{P}_{\mathcal{R}}$, n_{s}, r and $\mathrm{d}n_{\mathrm{s}}/\mathrm{d}\ln k$ as defined in Eqs. (2.19), (2.36), (2.38), (2.39) and (2.40). Calculated from the CW one-loop effective potential (6.4), they are indistinguishable from what we have estimated in Sec. 6.2.

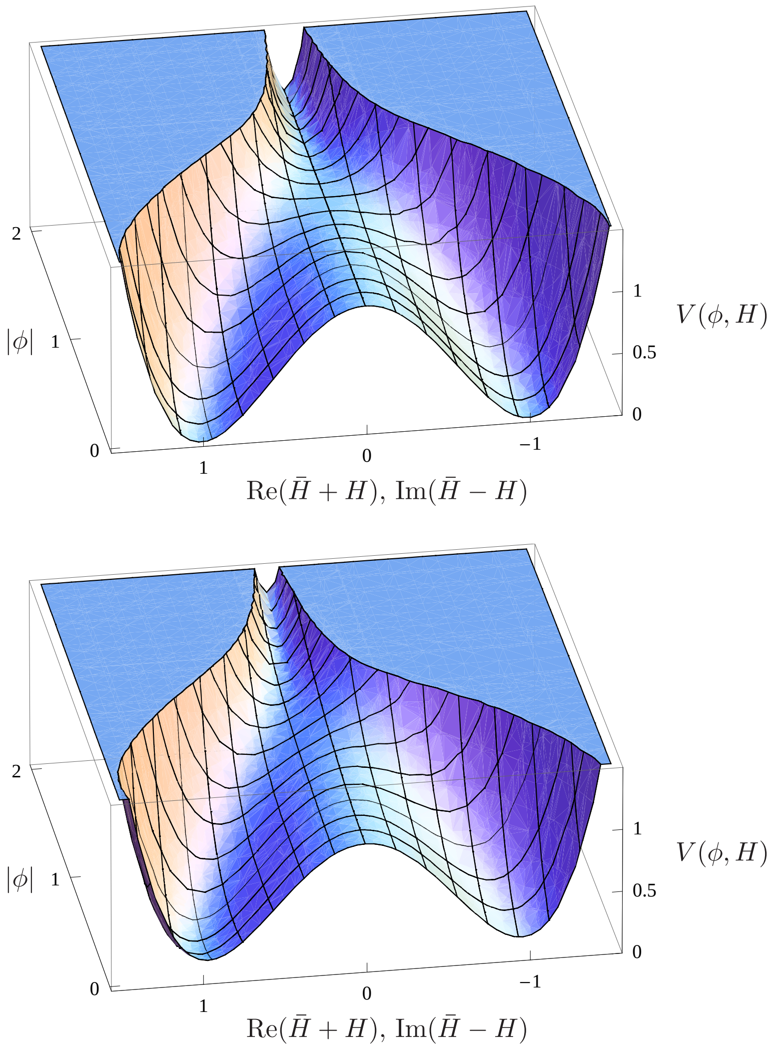

Figure 8.1: Plot of the F-term tribrid inflation potential in the D-flat valleys $\phi = \bar{\phi}^*$, $H = \bar{H}^*$, with (lower plot) and without (upper plot) deformations by higher dimensional effective operators. The lower plot displays the deformed potential where an effective superpotential term $H\,\bar{\phi}$ has been switched on. This term gives rise to a slope at $H = \bar{H} = 0$ that forces the field into the global minimum at positive M.

Chapter 9

Matter Inflation in Pati–Salam

In this chapter we discuss a fully realistic example of SUSY tribrid inflation with a GNS inflaton where $G = G_{\text{PS}}$ is identified with the SUSY PS gauge group. Following the general ideas presented in the previous chapter, in the model under construction inflation will proceed along a trajectory in field space where the D-term contribution to the scalar potential vanishes and the F-term contribution dominates the vacuum energy density.

In addition to that we want to associate the inflaton field to the *matter sector* of the theory so that the model is closely related to low energy particle physics. Typically if there are only matter fields in the (CP conjugated) right-handed PS reps R_i^c this would lead to large D-term contributions incompatible with inflation. Therefore, in addition to the matter fields we also introduce another field $\bar{R}^c$ in the conjugate rep of the gauge group. For simplicity, we discuss the case where $i = 1, \ldots, 4$ and where there is only one $\bar{R}^c$. As we will see, the introduction of $\bar{R}^c$ is necessary in order to keep all the waterfall directions stabilized during inflation. The presence of $\bar{R}^c$ also facilitates inflation to proceed along a D-flat valley. After inflation, one linear combination of the fields R_i^c will pair with $\bar{R}^c$ and become heavy, while three other combinations remain light and contain the three generations of MSSM fields.

Furthermore, the superfields containing the right-handed neutrinos of the seesaw mechanism will obtain their large masses after inflation. In addition to the introduction of the model in this chapter, we also work out an example in full detail where the inflaton moves along a flat direction such that both $\bar{R}^c$ and one of the R_i^c have a VEV in the sneutrino direction. Based on this explicit example of *sneutrino inflation* we also study one- and two-loop corrections to the flat tree-level inflaton potential.

9.1 The Model

As an explicit realization of the idea of having a GNS inflaton, we consider the PS gauge group $G_{\text{PS}} = SU(4)_C \times SU(2)_L \times SU(2)_R$ [83, 84] as introduced in Sec. 4.2. We focus on the right sector of the theory only, i.e. fields that are charged under $SU(2)_R$. From the point of view of the Higgs sector breaking PS to the SM this is sufficient, since VEVs of one

	G_{PS}	R	$\mathbb{Z}_{10}$
S	$(\mathbf{1},\mathbf{1},\mathbf{1})$	1	0
X	$(\mathbf{1},\mathbf{1},\mathbf{1})$	0	7
H^c	$(\bar{\mathbf{4}},\mathbf{1},\bar{\mathbf{2}})$	0	1
$\bar{H}^c$	$(\mathbf{4},\mathbf{1},\mathbf{2})$	0	2
R_i^c	$(\bar{\mathbf{4}},\mathbf{1},\bar{\mathbf{2}})$	1/2	3
$\bar{R}^c$	$(\mathbf{4},\mathbf{1},\mathbf{2})$	1/2	4
H	$(\mathbf{4},\mathbf{2},\mathbf{1})$	0	1
$\bar{H}$	$(\bar{\mathbf{4}},\bar{\mathbf{2}},\mathbf{1})$	0	2
L_i	$(\mathbf{4},\mathbf{2},\mathbf{1})$	1/2	3
$\bar{L}$	$(\bar{\mathbf{4}},\bar{\mathbf{2}},\mathbf{1})$	1/2	4

Table 9.1: Superfield content of the model and associated symmetries.

$(\mathbf{4},\mathbf{1},\mathbf{2})$ and one $(\bar{\mathbf{4}},\mathbf{1},\bar{\mathbf{2}})$ are enough for this purpose. Let us first introduce the left-chiral $SU(2)_R$ doublet matter superfields and their conjugate rep given by

$$\begin{aligned} R_i^c &= (\bar{\mathbf{4}},\mathbf{1},\bar{\mathbf{2}}) = \begin{pmatrix} u_i^c & u_i^c & u_i^c & \nu_i^c \\ d_i^c & d_i^c & d_i^c & e_i^c \end{pmatrix}, \\ \bar{R}^c &= (\mathbf{4},\mathbf{1},\mathbf{2}) = \begin{pmatrix} \bar{u}^c & \bar{u}^c & \bar{u}^c & \bar{\nu}^c \\ \bar{d}^c & \bar{d}^c & \bar{d}^c & \bar{e}^c \end{pmatrix}, \end{aligned} \tag{9.1}$$

where we have omitted color indices for convenience and i denotes a generation index. Here, the R_i^c multiplets contain the $SU(2)_L$ singlet fields under the SM gauge group defined in Eq. (4.13). The waterfall Higgs superfields breaking PS to the SM after inflation reside in the multiplets introduced in Eq. (4.23). In addition, we introduce two further gauge singlet fields dubbed S and X. The symmetry assignments for all the fields are given in the upper half of Tab. 9.1. We have introduced two additional symmetries: an R-symmetry and a discrete $\mathbb{Z}_{10}$ symmetry. The lower half of Tab. 9.1 can be ignored until we introduce the $SU(2)_L$ doublets in a more general framework in section 10.1.1. We would also like to remark at this point that the symmetries and charge assignments of Tab. 9.1 are not unique and should mainly serve the purpose to illustrate that it is possible to obtain the desired form of the superpotential by symmetry.

Indeed, with the symmetry assignments of Tab. 9.1 the allowed terms in the superpotential up to dimension five operators are the following,

$$\begin{aligned} W = &\kappa\, S\left(\frac{\langle X\rangle}{\Lambda}\, H^c\bar{H}^c - M^2\right) + \frac{\zeta_i}{\Lambda}\,(R_i^c\,\bar{R}^c)(H^c\bar{H}^c) \\ &+ \frac{\gamma}{\Lambda}\,(\bar{R}^cH^c)(\bar{R}^cH^c) + \frac{\xi_i}{\Lambda}\,(R_i^c\bar{H}^c)(\bar{R}^cH^c) + \frac{\lambda_{ij}}{\Lambda}\,(R_i^c\bar{H}^c)(R_j^c\,\bar{H}^c)\,, \end{aligned} \tag{9.2}$$

where two multiplets enclosed in brackets are contracted with their respective $SU(4)_C$ and $SU(2)_R$ indices. To keep matters simple we only consider effective operators generated by the exchange of singlet messenger fields (for a detailed discussion, see App. D.2).

The tasks of the superfields in this model are the following. S is the gauge singlet contributing the large vacuum energy density during inflation by its F-term, i.e $W_S \neq 0$. It stays at zero both during and after inflation. A large mass for S that keeps the field at zero can be generated by SUGRA effects due to higher order terms in the Kähler potential, see Sec. 6.2. The right-doublets H^c, $\bar{H}^c$ contain the waterfall fields as scalar components which are zero during inflation and become tachyonic subsequently, ending inflation and breaking $G_{\rm PS}$ to $G_{\rm SM}$ by their VEVs. The $SU(2)_R$-charged matter multiplets R_i^c together with $\bar{R}^c$ provide the slow-roll inflaton directions as scalar components.

After the end of inflation we want all components of three generations R_i^c, except for their right-handed neutrino, to be light, whereas all components of $\bar{R}^c$ need to be heavy. This is achieved by the introduction of several generations of R_i^c fields. With the number of generations of R_i^c larger than the one of $\bar{R}^c$ by three (i.e., $i = 1, \ldots, 4$), all the $\bar{R}^c$ fields pair up with some R_i^c and form Dirac-type mass terms at the GUT scale and decouple from the theory. Only three R_i^c generations remain light.

Finally, let us discuss the superpotential given in Eq. (9.2) in more detail. The terms proportional to the ζ_i provide universal masses for all components of the H^c and $\bar{H}^c$ fields during inflation when R_i^c and $\bar{R}^c$ get VEVs. Looking at the superpotential we can easily convince ourselves that without the presence of the $\bar{R}^c$ fields, not all of the waterfall squared masses are positive during inflation and their immediate destabilization would not allow for slow-roll dynamics. The introduction of the superfield X is motivated as follows: We have imposed the discrete $\mathbb{Z}_{10}$ symmetry to forbid a direct mass term for the R_i^c and $\bar{R}^c$ fields, therefore charging $R_i^c\,\bar{R}^c$ under the symmetry. On the other hand, we have allowed the operator $R_i^c\,\bar{R}^c\,H^c\bar{H}^c$ in Eq. (9.2), thus $H^c\bar{H}^c$ cannot be invariant under this discrete symmetry. Hence a superpotential term of the form $S\,H^c\bar{H}^c$ is forbidden. However, in the presence of the gauge singlet field X which acquires a VEV around the Planck scale and breaks the discrete $\mathbb{Z}_{10}$ symmetry spontaneously, a similar term $S\,\frac{X}{\Lambda}\,H^c\bar{H}^c$ is allowed and it effectively generates the desired term after X gets its VEV. To allow the term $S\,\frac{X}{\Lambda}\,H^c\bar{H}^c$, the X field carries a charge equal to the charge of the product $R_i^c\bar{R}^c$ under the discrete symmetry, as can be seen in Tab. 9.1.

9.2 D-Flat Inflaton Directions

The inflationary epoch is determined by the scalar potential given by both F-term and D-term contributions of all chiral superfields. For the sake of simplicity, within this chapter we investigate only the global SUSY limit and extend our framework to SUGRA in Ch. 10.

At the basic level tribrid inflation requires a large vacuum energy density responsible for a quasi-exponential expansion of the scale factor and a nearly flat direction whose quantum fluctuations generate the metric perturbations. In our model inflation proceeds along a trajectory in the field space of R_i^c and $\bar{R}^c$ along which the D-term contributions vanish. In such a D-flat valley, the F-term contribution of the S field provides the necessary vacuum energy density. Due to large F-term contributions to the masses of the waterfall fields, they remain at zero during inflation, i.e., $H^c = \bar{H}^c = 0$. In turn, both the R_i^c and $\bar{R}^c$ fields do not have any tree-level F-term mass contributions. Therefore, in our PS framework the tree-level F-term inflaton potential becomes $V_F \approx \kappa^2 M^4$, whereas the D-term potential reduces to

$$V_D = \frac{g^2}{2} \sum_{a=1}^{18} \left(\bar{R}^{c\dagger} \mathcal{T}^a \bar{R}^c - R_i^{c\dagger} \mathcal{T}^{a*} R_i^c \right)^2 . \qquad (9.3)$$

We denote the eighteen relevant generators by $\mathcal{T}^a$, with $a = 1, \ldots, 18$, which have been explicitly listed in Eq. (D.1). Furthermore, we assume $g \equiv g_C = g_R$ around the GUT scale. Thus, the D-flatness conditions from Eq. (9.3) give the more specific conditions in the PS case

$$R_i^{c\dagger} \mathcal{T}^{a*} R_i^c = \bar{R}^{c\dagger} \mathcal{T}^a \bar{R}^c \, , \qquad (9.4)$$

where the sum over all generations i has to be taken into account in each of the eighteen equations. During inflation, our D-flat trajectory is thus constrained by the conditions in Eq. (9.4) which have to be imposed on the F-term scalar potential.

Using Eq. (9.4) it can be shown that several flat directions exist in this model. All these directions can in principle be valid trajectories for inflation to occur. During inflation R_i^c and $\bar{R}^c$ acquire VEVs along one of these directions and break the PS symmetry. The gauge fields coupled to this particular direction in field space become massive. This direction is classically flat and lifted only by radiative corrections such that it is suitable for inflation.

On the other hand, other flat directions in field space along which the gauge symmetry is not broken and the gauge fields are still massless, acquire large two-loop mass corrections as will be clarified in Sec. 9.4.2. Such large mass contributions essentially lift these other flat directions strongly and drive their VEVs to zero. After inflation, the breaking of G_{PS} is realized by the VEVs of H^c and $\bar{H}^c$. In the next section we will explicitly consider inflation along the right-handed sneutrino directions ν^c and $\bar{\nu}^c$, which is one possible D-flat direction in field space. We show explicitly that in this case the waterfall is triggered in such a way that generically the VEVs of H^c and $\bar{H}^c$ are aligned in the right-handed sneutrino direction as well. Thus an example model of sneutrino inflation is realized with the inflaton being in a non-singlet rep of G_{PS}. It is important to emphasize that although the inflaton belongs to a non-singlet rep, it effectively behaves like a singlet since the gauge group G_{PS} is broken to G_{SM} during inflation. As already mentioned, this proves to be important w.r.t. quantum corrections to the inflaton potential.

9.3 Explicit Example: Sneutrino Inflation

As we have mentioned in the last section, the model has several tree-level flat directions in the $R_i^c, \bar{R}^c$ field space and in principle inflation can proceed along any of them. In this section we would like to discuss the inflationary scenario in which the inflaton fields acquire VEVs along the sneutrino direction. In this context we also study the waterfall mechanism in more detail. It turns out to be an interesting feature of this particular flat direction that, at the end of inflation, and for generic choices of parameters, the waterfall fields H^c and $\bar{H}^c$ generically acquire VEVs along the corresponding right-handed sneutrino directions

ν_H^c and $\bar{\nu}_H^c$ as well. This preferred waterfall direction helps to avoid the production of topologically stable monopoles after inflation.

As an explicit example inflaton trajectory we consider a simple case where only one of the $R^c \equiv R_1^c \neq 0$ is slowly rolling while all the others remain at zero $R_{i\neq 1}^c = 0$. In addition, we want to realize inflation along the sneutrino direction, i.e.

$$R^c = \begin{pmatrix} 0 & 0 & 0 & \nu^c \\ 0 & 0 & 0 & 0 \end{pmatrix} , \qquad \bar{R}^c = \begin{pmatrix} 0 & 0 & 0 & \bar{\nu}^c \\ 0 & 0 & 0 & 0 \end{pmatrix} . \tag{9.5}$$

This reduces our inflationary superpotential in Eq. (9.2) to the effective form

$$\begin{aligned} W_{\text{inf}} =& \kappa\, S \left(H^c \bar{H}^c - M^2 \right) \\ &+ \lambda\, (\nu^c\, \bar{\nu}_H^c)^2 + \gamma\, (\bar{\nu}^c\, \nu_H^c)^2 + \xi\, (\nu^c\, \bar{\nu}^c)\, \nu_H^c \bar{\nu}_H^c + \zeta\, (\nu^c\, \bar{\nu}^c)\, H^c \bar{H}^c \,, \end{aligned} \tag{9.6}$$

where we have absorbed $\langle X \rangle$ and Λ into the definition of the parameters. Due to the VEVs in Eq. (9.5), G_{PS} is already broken to G_{SM} during inflation. If we can also ensure that the waterfall is forced into the ν_H^c and $\bar{\nu}_H^c$ directions in field space, no monopoles will be produced after inflation.

Since R^c and $\bar{R}^c$ point in the right-handed sneutrino direction, the D-term potential projects out only the part proportional to the generators $\mathcal{T}^{15}$ and $\mathcal{T}^{18}$ of G_{PS}. Hence, the global SUSY D-term potential reads

$$V_D = \frac{5}{16}\, g^2 \left(|\nu^c|^2 - |\bar{\nu}^c|^2 \right)^2 . \tag{9.7}$$

This potential obviously has a flat direction $|\nu^c| = |\bar{\nu}^c|$. From now on, we assume that inflation occurs in this D-flat valley. Therefore the scalar potential during inflation has to be calculated in the inflationary trajectory $S = H^c = \bar{H}^c = 0$ with the D-flatness condition $|\nu^c| = |\bar{\nu}^c|$ imposed.

For the D-flat direction $\langle \nu^c \rangle = \langle \bar{\nu}^c \rangle$, assuming real VEVs, the field combination[1] $\text{Re}(\delta\bar{\nu}^c - \delta\nu^c)$ having mass $5\, g^2 \langle \nu^c \rangle^2 / 2$ is orthogonal to the flat direction $\text{Re}(\delta\bar{\nu}^c + \delta\nu^c)$ which remains massless. On the other hand, for the other D-flat direction $\langle \nu^c \rangle = -\langle \bar{\nu}^c \rangle$, the field combination $\text{Re}(\delta\bar{\nu}^c + \delta\nu^c)$ acquires a mass of $5\, g^2 \langle \nu^c \rangle^2 / 2$ and is orthogonal to the flat direction $\text{Re}(\delta\bar{\nu}^c - \delta\nu^c)$. The complete mass spectrum of the inflaton sector is listed in Tab. 9.2.

Now we discuss how the waterfall mechanism works in our particular example. We decompose all complex scalar fields into canonically normalized real and imaginary components as $\nu_H^c = \left(\text{Re}(\tilde{\nu}_H^c) + \text{i}\, \text{Im}(\tilde{\nu}_H^c) \right) / \sqrt{2}$ and $\bar{\nu}_H^c = \left(\text{Re}(\tilde{\bar{\nu}}_H^c) + \text{i}\, \text{Im}(\tilde{\bar{\nu}}_H^c) \right) / \sqrt{2}$ and analogous for all the other waterfall fields. Here and in the following, a tilde denotes canonically normalized fields and we define the sneutrino inflaton fields $\nu^c = |\tilde{\nu}^c| / \sqrt{2}$ and $\bar{\nu}^c = |\tilde{\bar{\nu}}^c| / \sqrt{2}$.

[1] Where the quantum field is expanded about the background VEV as $\phi \to \langle \phi \rangle + \delta\phi$.

The full F-term potential calculated with the use of Eq. (3.38) is given by

$$\begin{aligned} V_F = &\left|\kappa\left(H^c\bar{H}^c - M^2\right)\right|^2 + \left|2\,\lambda\,(\nu^c)^2\bar{\nu}^c_H + \xi\,(\nu^c\,\bar{\nu}^c)\,\nu^c_H + \zeta\,(\nu^c\,\bar{\nu}^c)\,\nu^c_H\right|^2 \\ &+ \left|\kappa\,S\,\bar{H}^c + \zeta\,(\nu^c\,\bar{\nu}^c)\,\bar{H}^c\right|^2 + \left|2\,\gamma\,(\bar{\nu}^c)^2\nu^c_H + \xi\,(\nu^c\,\bar{\nu}^c)\,\bar{\nu}^c_H + \zeta\,(\nu^c\,\bar{\nu}^c)\,\bar{\nu}^c_H\right|^2 \\ &+ \left|\kappa\,S\,H^c + \zeta\,(\nu^c\,\bar{\nu}^c)\,H^c\right|^2 + \left|2\,\gamma\,\bar{\nu}^c\,(\nu^c_H)^2 + \xi\,\nu^c\,(\nu^c_H\bar{\nu}^c_H) + \zeta\,\nu^c\,(H^c\bar{H}^c)\right|^2 \\ &+ \left|2\,\lambda\,\nu^c\,(\bar{\nu}^c_H)^2 + \xi\,\bar{\nu}^c\,(\nu^c_H\bar{\nu}^c_H) + \zeta\,\bar{\nu}^c\,(H^c\bar{H}^c)\right|^2 , \end{aligned} \tag{9.8}$$

where terms containing single H^c and $\bar{H}^c$ superfields have to be summed over all components of the PS multiplet. In terms like $(H^c\bar{H}^c)$ all gauge indices are contracted.

Due to large F-term contributions to their masses from the VEVs of the inflaton fields, cf. Eq. (9.8), the waterfall fields get fixed at zero during inflation. As the inflaton fields slowly roll to smaller values, the masses of the waterfall fields decrease and finally one or more directions in field space become tachyonic. The $H^c, \bar{H}^c$ fields now quickly "fall" to their true minima and inflation ends by the waterfall. We now discuss into which direction in field space the waterfall gets triggered, i.e., which direction becomes tachyonic first.

Both scalar as well as pseudoscalar squared mass matrices are block-diagonal with universal sub-blocks for the $(u^c_H, \bar{u}^c_H)$, $(d^c_H, \bar{d}^c_H)$ and $(e^c_H, \bar{e}^c_H)$ parts respectively coming from the couplings κ and ζ. The scalar and pseudoscalar squared mass matrices are given by

$$\begin{aligned} \mathcal{M}^2_{\mathrm{Re}(u^c_H,\bar{u}^c_H)} &= \begin{pmatrix} \frac{1}{4}\,|\zeta|^2\,|\tilde{\nu}^c|^4 & -|\kappa|^2M^2 \\ -|\kappa|^2M^2 & \frac{1}{4}\,|\zeta|^2\,|\tilde{\nu}^c|^4 \end{pmatrix} , \\ \mathcal{M}^2_{\mathrm{Im}(u^c_H,\bar{u}^c_H)} &= \begin{pmatrix} \frac{1}{4}\,|\zeta|^2\,|\tilde{\nu}^c|^4 & |\kappa|^2M^2 \\ |\kappa|^2M^2 & \frac{1}{4}\,|\zeta|^2\,|\tilde{\nu}^c|^4 \end{pmatrix} , \end{aligned} \tag{9.9}$$

with two eigenvalues each. For example, the normalized field directions $\mathrm{Re}(u^c_H + \bar{u}^c_H)$ and $\mathrm{Im}(\bar{u}^c_H - u^c_H)$ have unstable squared masses

$$m^2_{u,1} = \frac{1}{4}\,|\zeta|^2\,|\tilde{\nu}^c|^4 - |\kappa|^2M^2\,, \tag{9.10}$$

whereas the stable directions $\mathrm{Re}(\bar{u}^c_H - u^c_H)$ and $\mathrm{Im}(\bar{u}^c_H + u^c_H)$ have squared masses

$$m^2_{u,2} = \frac{1}{4}\,|\zeta|^2\,|\tilde{\nu}^c|^4 + |\kappa|^2M^2\,. \tag{9.11}$$

Exactly the same mass spectra hold for the equivalent combinations of $(d^c_H, \bar{d}^c_H)$ and $(e^c_H, \bar{e}^c_H)$.

Due to the additional contributions from the non-universal couplings λ, γ and ξ, the SM-singlet directions $(\nu^c_H, \bar{\nu}^c_H)$ obtain different mass matrices for the real scalar components

$$\mathcal{M}^2_{\mathrm{R}} = \begin{pmatrix} \frac{1}{4}\left(|\zeta+\xi|^2 + 4|\gamma|^2\right)|\tilde{\nu}^c|^4 & \frac{1}{2}\mathrm{Re}\left((\gamma+\lambda)^*(\zeta+\xi)\right)|\tilde{\nu}^c|^4 - |\kappa|^2M^2 \\ \frac{1}{2}\mathrm{Re}\left((\gamma+\lambda)^*(\zeta+\xi)\right)|\tilde{\nu}^c|^4 - |\kappa|^2M^2 & \frac{1}{4}\left(|\zeta+\xi|^2 + 4|\lambda|^2\right)|\tilde{\nu}^c|^4 \end{pmatrix} , \tag{9.12}$$

and for the pseudoscalar components

$$\mathcal{M}_{\mathrm{P}}^2 = \begin{pmatrix} \frac{1}{4}\left(|\zeta+\xi|^2+4|\gamma|^2\right)|\tilde{\nu}^c|^4 & \frac{1}{2}\mathrm{Re}\left((\gamma+\lambda)^*(\zeta+\xi)\right)|\tilde{\nu}^c|^4+|\kappa|^2M^2 \\ \frac{1}{2}\mathrm{Re}\left((\gamma+\lambda)^*(\zeta+\xi)\right)|\tilde{\nu}^c|^4+|\kappa|^2M^2 & \frac{1}{4}\left(|\zeta+\xi|^2+4|\lambda|^2\right)|\tilde{\nu}^c|^4 \end{pmatrix}. \tag{9.13}$$

Setting $\gamma = \lambda$, we obtain the following mass eigenvalues for the real scalar parts

$$\begin{aligned} m^2_{\mathrm{Re}(\nu),1} &= \frac{|\zeta+\xi+2\gamma|^2}{4}|\tilde{\nu}^c|^4 - |\kappa|^2M^2\,, \\ m^2_{\mathrm{Re}(\nu),2} &= \frac{|\zeta+\xi-2\gamma|^2}{4}|\tilde{\nu}^c|^4 + |\kappa|^2M^2\,. \end{aligned} \tag{9.14}$$

For the pseudoscalar parts, we obtain the mass eigenvalues

$$\begin{aligned} m^2_{\mathrm{Im}(\nu),1} &= \frac{|\zeta+\xi-2\gamma|^2}{4}|\tilde{\nu}^c|^4 - |\kappa|^2M^2\,, \\ m^2_{\mathrm{Im}(\nu),2} &= \frac{|\zeta+\xi+2\gamma|^2}{4}|\tilde{\nu}^c|^4 + |\kappa|^2M^2\,. \end{aligned} \tag{9.15}$$

In Eqs. (9.14) and (9.15), the first one can give rise to an instability in both cases and corresponds to the directions $\mathrm{Re}(\bar{\nu}^c_H - \nu^c_H)$, $\mathrm{Im}(\bar{\nu}^c_H - \nu^c_H)$, respectively. The second, stable eigenvalues correspond to $\mathrm{Re}(\bar{\nu}^c_H + \nu^c_H)$ and $\mathrm{Im}(\bar{\nu}^c_H + \nu^c_H)$. All these masses are listed in Tab. 9.3 with the complete waterfall mass spectrum.

The critical values at which the system gets destabilized can be calculated by setting the dynamical masses to zero. For the $\mathrm{Re}(u^c_H + \bar{u}^c_H)$-, $\mathrm{Im}(\bar{u}^c_H - u^c_H)$-, …directions we find

$$|\tilde{\nu}^c_{\mathrm{crit}}| = \sqrt{\frac{2\,|\kappa|\,M}{|\zeta|}}\,, \tag{9.16}$$

and for the $\mathrm{Re}(\bar{\nu}^c_H - \nu^c_H)$- and $\mathrm{Im}(\bar{\nu}^c_H - \nu^c_H)$-directions we find the real, positive solutions

$$|\tilde{\nu}^c_{\mathrm{crit}}| = \sqrt{\frac{2\,|\kappa|\,M}{|\zeta+\xi+2\gamma|}}\,, \qquad |\tilde{\nu}^c_{\mathrm{crit}}| = \sqrt{\frac{2\,|\kappa|\,M}{|\zeta+\xi-2\gamma|}}\,. \tag{9.17}$$

For generic non-zero values of γ and, e.g., small ξ, either the $\mathrm{Re}(\bar{\nu}^c_H - \nu^c_H)$- or the $\mathrm{Im}(\bar{\nu}^c_H - \nu^c_H)$-direction will become tachyonic for larger values of the inflaton VEV than the $\mathrm{Re}(u^c_H + \bar{u}^c_H)$-, …directions. Consequently, it destabilizes first and the waterfall occurs in the corresponding direction in field space.

We note that with the effective operators in Eq. (9.2) included in this discussion, there is still the possibility of domain wall formation associated with the $\mathbb{Z}_2$ symmetry $\nu^c_H \to -\nu^c_H$ and $\bar{\nu}^c_H \to -\bar{\nu}^c_H$. However, additional higher dimensional effective operators that contain odd powers of H^c and $\bar{H}^c$ (in particular terms linear in H^c and $\bar{H}^c$) can efficiently lift this degeneracy and force the waterfall to take place in one unique direction. An example for such a deformed inflaton potential is shown in Fig. 8.1. For different possibilities to evade the cosmological domain wall problem, the reader is referred to [141].

In summary, since the gauge symmetry is already broken by the inflaton VEVs during inflation, higher dimensional operators allow to force the waterfall to occur in one single direction in field space such that a particular vacuum is chosen everywhere in space and the production of topological defects such as monopoles can be avoided.

9.4 Radiative Corrections

Based on the scenario of sneutrino tribrid inflation in PS as introduced in Sec. 9.3, we now describe radiative corrections to the flat tree-level inflaton potential. Sec. 9.4.1 is dedicated to discussing the one-loop CW corrections [58, 59, 60]. We summarize the full mass spectrum during inflation as calculated in detail in App. D.1 and Sec. 9.3. As it turns out, in the absence of soft SUSY breaking mass terms, only the fields of the waterfall sector show a splitting between the inflaton-dependent masses of the scalar and fermionic components and hence contribute to the lifting of the flat direction at one-loop level. In Sec. 9.4.2, we give estimates for potentially dangerous two-loop corrections pointed out in [48] and show that they are small and can be neglected in our model.

9.4.1 One-Loop Corrections

Typically, tree-level flat directions get lifted by the CW one-loop radiative corrections to the effective potential as given in Eq. (6.4). In terms of our real sneutrino inflaton in the D-flat valley $\nu^c = \bar{\nu}^{c*}$ we obtain the one-loop correction

$$V_{\text{loop}}(\nu^c) = \frac{1}{64\,\pi^2}\,\text{Str}\left[\mathcal{M}^4(\nu^c)\left(\ln\left(\frac{\mathcal{M}^2(\nu^c)}{Q^2}\right) - \frac{3}{2}\right)\right]\,, \tag{9.18}$$

where Q is the renormalization scale. Since the supertrace is taken over all fermionic and bosonic DOFs we have to take into account the full mass spectrum.

We have already calculated the one-loop contributions to the inflaton potential due to the inflaton field-dependent masses of the scalar and fermionic components of the waterfall sector superfields in different scenarios in Ch. 6. The calculation here can be performed analogously. However, in addition to the chiral superfield sector we have to consider the vector superfield sector of the theory for the one-loop contributions, i.e., the contributions from inflaton field-dependent masses of gauge bosons and gauginos. Plugged into Eq. (9.18) we end up with the effective potential in our sneutrino inflation scenario.

Let us start with the gauge sector masses of the model. Since we are effectively working in global SUSY and do not include any soft SUSY breaking terms, this sector is not directly affected by the breaking. In the generalized SUGRA framework which we have in mind, see Ch. 10, gaugino mass terms as in Eq. (3.74) can provide a source for SUSY breaking in the gauge sector. The presence (or absence) of such gaugino masses depends on the details of the SUGRA model. If, for instance, the gauge kinetic function is diagonal and constant $f_{ab} = \delta_{ab}$, or more precisely, independent of fields that obtain a non-zero F-term, such as S in our model, then the contributions vanish. Tab. 9.2 summarizes the mass

Quantum Fields	Squared Masses m^2
8 gauge bosons	$g^2 \langle\nu^c\rangle^2$
1 gauge boson	$5\,g^2 \langle\nu^c\rangle^2/2$
8 Dirac fermions	$g^2 \langle\nu^c\rangle^2$
1 Dirac fermion	$5\,g^2 \langle\nu^c\rangle^2/2$
8 real scalars	$g^2 \langle\nu^c\rangle^2$
1 real scalar	$5\,g^2 \langle\nu^c\rangle^2/2$

Table 9.2: Gauge sector mass spectrum.

Quantum Fields	Squared Masses m^2
7 Dirac fermions	$\lvert\zeta\rvert^2 \langle\nu^c\rangle^4$
1 Majorana fermion	$\lvert 2\,\gamma-\zeta-\xi\rvert^2 \langle\nu^c\rangle^4$
1 Majorana fermion	$\lvert 2\,\gamma+\zeta+\xi\rvert^2 \langle\nu^c\rangle^4$
7 complex scalars	$\lvert\zeta\rvert^2 \langle\nu^c\rangle^4 - \lvert\kappa\rvert^2 M^2$
7 complex scalars	$\lvert\zeta\rvert^2 \langle\nu^c\rangle^4 + \lvert\kappa\rvert^2 M^2$
1 real scalar	$\lvert\zeta+\xi-2\gamma\rvert^2 \langle\nu^c\rangle^4 + \lvert\kappa\rvert^2 M^2$
1 real scalar	$\lvert\zeta+\xi-2\gamma\rvert^2 \langle\nu^c\rangle^4 - \lvert\kappa\rvert^2 M^2$
1 real scalar	$\lvert\zeta+\xi+2\gamma\rvert^2 \langle\nu^c\rangle^4 + \lvert\kappa\rvert^2 M^2$
1 real scalar	$\lvert\zeta+\xi+2\gamma\rvert^2 \langle\nu^c\rangle^4 - \lvert\kappa\rvert^2 M^2$

Table 9.3: Waterfall sector mass spectrum.

eigenvalues of the gauge bosons, the gaugino-chiral fermion mixings and the D-term real scalars. Lacking a mass splitting, the supertrace over these contributions vanishes and they do not contribute to Eq. (9.18).

Hence SUSY breaking, $\langle\nu^c\rangle$-dependent contributions arise from the waterfall sector masses only. The corresponding squared masses are listed in Tab. 9.3 and carry the mass splittings $\mu = \kappa\, M$. Thus they contribute to the one-loop inflaton potential via Eq. (9.18), lifting the tree-level flat direction.

For an example set of parameters, we have checked that the one-loop effective potential has the typical shape of the CW potential in tribrid inflation as displayed by the dotted red curve in Fig. 6.3. Since in the case considered here, the inflationary trajectory is a straight line in field space, we are effectively dealing with a single-field model and the inflationary predictions can be directly calculated using Eqs. (2.36)-(2.40). The negative curvature of the potential gives rise to a spectral index below one (typically $n_\mathrm{s} \approx 0.98$), while the tensor-to-scalar ratio $r \lesssim 10^{-2}$ as usual in SUSY hybrid and tribrid models. The WMAP normalization $P_\mathcal{R}^{1/2} \approx 5 \cdot 10^{-5}$ fixes the scale of inflation M and as before, we have assumed $N_\mathrm{e} = 60$. Furthermore, as mentioned above, the inflationary trajectory is not curved in field space and therefore we do not expect large non-gaussianities.[2]

We note that in the SUGRA context the prediction for n_s can be further lowered and thus brought even closer to the best fit value of the latest WMAP results [5]. This can be explained if a Kähler potential coupling between the S field and the waterfall fields is taken into account[3] as we have investigated in some detail in Sec. 6.2.2.

9.4.2 Two-Loop Corrections

It has been pointed out by Dvali in [48] that during inflation GNS scalar fields generically obtain two-loop mass corrections which are of the order of the Hubble scale $\mathcal{H}$. For the inflaton field such large masses are incompatible with slow-roll inflation. In the following, we discuss why the two-loop corrections do not endanger inflation in our type of models. First of all, we state the problem in general terms and subsequently demonstrate how such two-loop corrections get suppressed in our case of sneutrino inflation.

For a GNS inflaton the problem arises under a simple condition. There has to be one singlet superfield S, which contributes the large vacuum energy density by its F-term $W_S \neq 0$ and it couples to some GNS superfields, in our case H^c, $\bar{H}^c$. The relevant superpotential terms read, for example,

$$W \supset \kappa\, S \left(H^c \bar{H}^c - M^2 \right) . \tag{9.19}$$

If these premises are given, any GNS direction ϕ will receive two-loop contributions to its effective mass of the order

$$\delta m^2 \sim \frac{g^4}{(4\pi)^4} \frac{|W_S|^2}{m_F^2} , \tag{9.20}$$

where g is the gauge coupling constant and m_F refers to the SUSY conserving mass of the H^c, $\bar{H}^c$ superfields. In Fig. 9.1 we have displayed the Feynman diagrams contributing to these mass corrections.

Typically, a contribution as in Eq. (9.20) is large enough to provide an inflaton mass that exceeds the Hubble scale, i.e., $\delta m > \mathcal{H}$ and thus slow-roll conditions are violated. Hence, the problem can in some sense be considered a *two-loop gauge η-problem* since it implies $|\eta| \approx 1$ due to radiative corrections from gauge interactions. In our model introduced in Sec. 9.1, we are thus interested in $\phi = \{R^c, \bar{R}^c\}$.

We emphasize that the mass correction (9.20) cannot be applied to our scenario, since the inflaton VEV already breaks the gauge group G_PS during inflation. Indeed, Eq. (9.20) is calculated under the assumption that the gauge bosons A_μ mediating the loops are massless, which is not the case in our model. As we argue in the following, the broken gauge symmetry during inflation implies large gauge boson masses that suppress the two-loop corrections corresponding to the diagrams depicted in Fig. 9.1.

[2]We note that for more complicated trajectories, non-gaussianities may arise.

[3]We note that a lower spectral index in SUSY hybrid inflation models can also be achieved by different means [142].

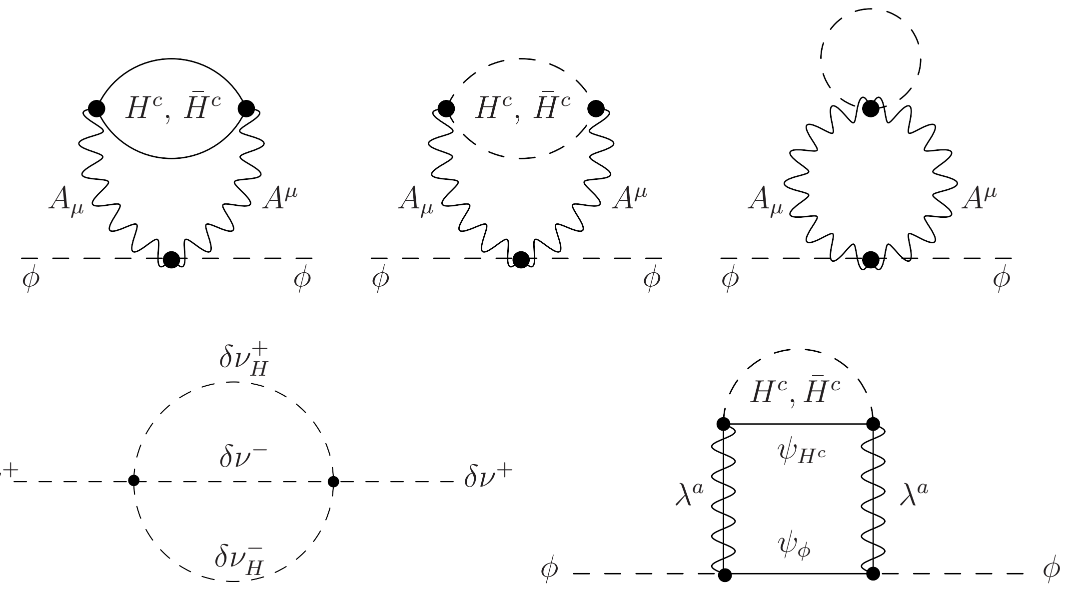

Figure 9.1: Two-loop Feynman diagrams contributing to the gauge η-problem pointed out in [48]. In the fourth diagram, we have defined mass eigenstates $\delta\nu^+ = \mathrm{Re}(\delta\bar{\nu}^c + \delta\nu^c)$, $\delta\nu^- = \mathrm{Re}(\delta\bar{\nu}^c - \delta\nu^c)$, $\delta\nu_H^+ = \mathrm{Re}(\delta\bar{\nu}_H^c + \delta\nu_H^c)$ and $\delta\nu_H^- = \mathrm{Re}(\delta\bar{\nu}_H^c - \delta\nu_H^c)$ which appear in the D-term part of the scalar potential.

More explicitly, for $\phi = \{\nu^c, \bar{\nu}^c\}$, the gauge bosons in Fig. 9.1 are contained in the coset $G_{\mathrm{PS}}/G_{\mathrm{SM}}$, thus corresponding to the massive ones, which is why their contributions get suppressed. Another way to say this is that the effective gauge symmetry during inflation is G_{SM}, under which the inflaton direction ϕ is a singlet. All other directions $\phi = \{u^c, d^c, e^c, \bar{u}^c, \bar{d}^c, \bar{e}^c\}$ couple to gauge bosons that are still massless, which allows the use of Eq. (9.20). As a consequence, they just obtain additional mass contributions helping to keep them at zero during the inflationary epoch.

Let us now estimate the typical size of the two-loop corrections in our explicit example in the large gauge boson mass limit[4] $M_g \gg p$. For the SUSY-splitted waterfall masses, we have plugged in

$$m_+^2 = m_F^2 + \mu^2 \,, \qquad m_-^2 = m_F^2 - \mu^2 \,, \tag{9.21}$$

where $m_F^2 \simeq \zeta^2 \langle \nu^c \rangle^4 / M_{\mathrm{P}}^2$ is the mass of the waterfall superpartner chiral fermion and $\mu = \kappa\, M$ is the SUSY breaking scale. Due to the non-renormalization theorem, all contributions not proportional to powers of μ must cancel such that in the SUSY limit $\mu \to 0$ the sum of all loop contributions vanishes. Thus we expand the final loop integrals in terms of μ. A list of all relevant loop integrals can be found in the appendix of [143].

In analogy to the calculations in [144] we find that in the large gauge boson mass limit

[4]Here, p denotes the momentum of the gauge boson.

the Feynman diagrams in Fig. 9.1 lead to two-loop inflaton mass contributions of the orders

$$\begin{aligned} \delta m_1^2 &\sim \frac{g^4}{(4\pi)^4}\,\frac{m_F^2\,\mu^4}{M_g^4}\,,\\ \delta m_2^2 &\sim \frac{g^4}{(4\pi)^4}\,\frac{\mu^4}{M_g^2}\,,\\ \delta m_3^2 &\sim \frac{g^4}{(4\pi)^4}\,\frac{m_F\,\mu^4}{M_g^3}\,. \end{aligned} \tag{9.22}$$

Using the values $\kappa = 0.05$, $\zeta = 0.2$, $g = 0.5$, $M = 3.4 \cdot 10^{-3}$ and $\langle \nu^c \rangle = 0.36$ at about 50 e-folds before the end of inflation taken from an example used in Sec. 6.2.3 where we analyze a similar tribrid inflation superpotential, we can further estimate

$$\begin{aligned} \frac{\delta m_1^2}{\mathcal{H}^2} &\sim \frac{3\,\zeta^2\kappa^2}{(4\pi)^4} \simeq \mathcal{O}(10^{-8})\,,\\ \frac{\delta m_2^2}{\mathcal{H}^2} &\sim \frac{3\,g^2\kappa^2}{(4\pi)^4}\left(\frac{M_{\mathrm{P}}}{\langle\nu^c\rangle}\right)^2 \simeq \mathcal{O}(10^{-6})\,,\\ \frac{\delta m_3^2}{\mathcal{H}^2} &\sim \frac{3\,g\,\zeta\,\kappa^2}{(4\pi)^4}\left(\frac{M_{\mathrm{P}}}{\langle\nu^c\rangle}\right) \simeq \mathcal{O}(10^{-7})\,. \end{aligned} \tag{9.23}$$

The Hubble scale during inflation is given by $\mathcal{H}^2 \simeq \kappa^2 M^4/3M_{\mathrm{P}}^2$. We can thus conclude that the two-loop contributions can be neglected in our PS sneutrino tribrid inflation scenario. Therefore, the one-loop corrections summarized in Sec. 9.4.1 are enough to calculate the predictions of the model.

In summary, for the explicit example of sneutrino inflation in PS we have presented a full viable GNS inflation model where monopole production after inflation can be avoided, while two-loop corrections to the inflaton mass are suppressed by the mass of the heavy gauge bosons. Finally, let us point out that although we have calculated such corrections explicitly in the sneutrino inflation trajectory only, we expect that the same mechanism protects any general inflaton direction from the two-loop gauge η-problem.

Chapter 10

Grand Unification and Supergravity

After introducing the model of matter inflation in SUSY PS in the last chapter with a detailed study of the sneutrino inflaton trajectory, we turn to generalizations of the framework in this chapter. The chapter is divided into two main parts.

Firstly, since the PS model is not based on a simple gauge group, it can only be an intermediate step towards grand unification. Therefore, in Sec. 10.1, we present a possible embedding of the model into SUSY $SO(10)$. We restrict ourselves to sketching how the field content can be contained within reps of $SO(10)$ and present symmetry assignments which explain the desired superpotential. Details of the breaking of $SO(10)$ to the PS group, the required field content and related problems that may arise would be interesting to study in future works, but they are beyond the scope of this work.

The second part of this chapter is concerned with the generalization of the model, which has so far been discussed in the global SUSY context, to SUGRA. This is particularly important since in the global SUSY context the driving field S, which provides the vacuum energy density to drive inflation, is still a flat direction and could just as well play the role of a SUSY hybrid inflaton. Therefore, in order to realize our scenario of GNS tribrid inflation, a SUGRA mass that keeps the driving field fixed at zero is inevitable. In Sec. 10.2 we use the Heisenberg symmetry explained in part III to obtain a consistent picture of *SUGRA GNS tribrid inflation*.

10.1 Generalization to $SO(10)$

We now turn to the embedding of the PS model of matter inflation presented in the last chapter into a SUSY $SO(10)$ GUT. Starting with the field content of the PS model, first of all we extend it to become explicitly left-right symmetric in Sec. 10.1.1. Subsequently in Sec. 10.1.2, we describe how the extended field content can reside in reps of $SO(10)$. In order to construct the desired superpotential, we point out how avoiding potentially dangerous operators for inflation could be related to having flavor symmetry breaking after inflation.

10.1.1 Left-Right Extension of the Pati–Salam Model

In order to make our example model of the previous chapter explicitly left-right-symmetric, we need to add left-handed[1] supermultiplets to the theory. In addition to the right-handed matter fields and their conjugates, defined in Eq. (9.1), we therefore introduce left-chiral $SU(2)_L$-doublet matter fields contained in the G_{PS} multiplets

$$\begin{aligned} L_i = (\mathbf{4}, \mathbf{2}, \mathbf{1}) &= \begin{pmatrix} u_i & u_i & u_i & \nu_i \\ d_i & d_i & d_i & e_i \end{pmatrix} , \\ \bar{L} = (\bar{\mathbf{4}}, \bar{\mathbf{2}}, \mathbf{1}) &= \begin{pmatrix} \bar{u} & \bar{u} & \bar{u} & \bar{\nu} \\ \bar{d} & \bar{d} & \bar{d} & \bar{e} \end{pmatrix} , \end{aligned} \tag{10.1}$$

where we have omitted the color indices for convenience and i denotes a generation index as before. The waterfall Higgs superfields which break G_{PS} to G_{SM} by the VEVs of their scalar components are defined in Eq. (4.23). Making the field content left-right symmetric, we have to include their left-handed counterparts as well, which read

$$\begin{aligned} H = (\mathbf{4}, \mathbf{2}, \mathbf{1}) &= \begin{pmatrix} u_H & u_H & u_H & \nu_H \\ d_H & d_H & d_H & e_H \end{pmatrix} , \\ \bar{H} = (\bar{\mathbf{4}}, \bar{\mathbf{2}}, \mathbf{1}) &= \begin{pmatrix} \bar{u}_H & \bar{u}_H & \bar{u}_H & \bar{\nu}_H \\ \bar{d}_H & \bar{d}_H & \bar{d}_H & \bar{e}_H \end{pmatrix} . \end{aligned} \tag{10.2}$$

The symmetry assignments are listed in Tab. 9.1. Note that at this stage the model contains two copies of the inflaton sector discussed in Ch. 9, one charged under $SU(2)_R$ and one charged under $SU(2)_L$. The interactions in the superpotential would contain terms which couple each sector separately as well as additional couplings between the two sectors. In the absence of a discrete left-right symmetry we would expect the couplings in the left and right sector to be not exactly equal. With two potential sectors for inflation, it may take place in both of them with the respective sneutrinos playing the role of the inflaton. Thus we might have an "inflaton race" between the two sectors. Once the waterfall is triggered in one of them[2], inflation ends since the vacuum energy density given by the F_S-term vanishes. At the same time the masses of the matter fields get fixed by the VEVs of the waterfall fields and the couplings between the left and the right sector. When the waterfall phase transition has taken place, we *(re)name* the corresponding sector as the $SU(2)_R$ sector under the SM gauge group. Before the breaking of G_{PS} to G_{SM} the names right-handed and left-handed were arbitrary and a renaming is always possible at this stage. Thus, without loss of generality, we can assume that PS is broken to the SM by the VEV of a right-handed PS Higgs field.

[1]By left- and right-handed we refer to non-trivial transformation under $SU(2)_L$ and $SU(2)_R$.

[2]With different couplings in each sector, which breaks left-right symmetry, we do not expect this to happen simultaneously.

10.1.2 Embedding into the $SO(10)$ Framework

As described in Sec. 4.3, one attractive feature of $SO(10)$ GUTs is that all matter fields of a family, including right-handed neutrinos, are contained in one **16** rep of $SO(10)$. If we furthermore consider SUSY $SO(10)$, these fields are accompanied by their scalar superpartners. It is then tempting to try to realize matter inflation by one or more of the scalar fields belonging to such a **16** supermultiplet. In terms of the PS framework considered in the preceding sections, each family of the left- and right-handed matter superfields is unified into a **16** rep and their conjugate counterparts into a $\overline{\mathbf{16}}$ rep according to the decompositions

$$\begin{aligned} \mathbf{16} &= (\mathbf{4},\mathbf{2},\mathbf{1}) \oplus (\bar{\mathbf{4}},\mathbf{1},\bar{\mathbf{2}})\,, \\ \overline{\mathbf{16}} &= (\bar{\mathbf{4}},\bar{\mathbf{2}},\mathbf{1}) \oplus (\mathbf{4},\mathbf{1},\mathbf{2})\,. \end{aligned} \tag{10.3}$$

In addition, the SM Higgs can be embedded into a **10** rep which under PS decomposes as

$$\mathbf{10} = (\mathbf{1},\mathbf{2},\mathbf{2}) \oplus (\mathbf{6},\mathbf{1},\mathbf{1})\,. \tag{10.4}$$

However, one immediately encounters a potential problem for realizing inflation connected to the Yukawa couplings of the matter reps to the **10** Higgs rep. If the theory contains renormalizable Yukawa interactions, i.e. terms of the form

$$y\,\mathbf{16}.\mathbf{10}.\mathbf{16}\,, \tag{10.5}$$

then the F-term of the **10** yields a contribution to the scalar potential

$$\sim \left|y\,\mathbf{16}^2\right|^2 . \tag{10.6}$$

Such a term represents quartic couplings of the inflaton field(s) which is, unless y is extremely small, strongly disfavored by the WMAP data.

On the other hand, in many flavor models based on GUTs combined with family symmetries, the Yukawa couplings, especially the ones for the first two families, do not arise from renormalizable couplings but rather from higher dimensional operators. The suppression of the higher dimensional operators allows to explain the hierarchical structure of the charged fermion masses. The Yukawa couplings are then generated after some family symmetry breaking Higgs field θ, called flavon, gets its VEV. Such Yukawa couplings can be schematically written as

$$y\,\frac{\langle\theta\rangle}{\Lambda}\,\mathbf{16}.\mathbf{10}.\mathbf{16}\,, \tag{10.7}$$

where Λ stands for the generation scale of the effective operator and $\langle\theta\rangle$ is the family symmetry breaking scale. Eq. (10.7) represents, in a simplified notation, the typically more complicated flavor sector of the theory, whose detailed discussion is beyond the scope of this thesis. As long as the flavon field θ obtains its VEV after inflation and has zero VEV during inflation, the potentially problematic coupling in Eq. (10.5) is effectively absent during inflation. We assume this situation in the following.

	$SO(10)$	R	$\mathbb{Z}_{10}$	$\mathbb{Z}_2$
S	$\mathbf{1}$	1	0	$+$
X	$\mathbf{1}$	0	7	$+$
H	$\mathbf{16}$	0	1	$+$
$\bar{H}$	$\overline{\mathbf{16}}$	0	2	$+$
F_i	$\mathbf{16}$	1/2	3	$+$
$\bar{F}$	$\overline{\mathbf{16}}$	1/2	4	$+$
h	$\mathbf{10}$	0	4	$-$
θ	$\mathbf{1}$	0	0	$-$

Table 10.1: Example of $SO(10)$ superfield content and associated symmetries.

The next issue we would like to address is how $SO(10)$ gets broken down to the SM, and how this breaking is connected to the monopole problem. Since monopoles would be disastrous if they survived until today, it is clear that either their production has to be avoided altogether[3] or they have to be diluted by a subsequent stage of inflation. The breaking of $SO(10)$ can take place via various hierarchies of intermediate subgroups [145]. The possibility corresponding to the strategy followed in this work is via the intermediate PS group as displayed in Eq. (4.26). In this pattern, monopoles can in principle be produced in the first and in the second stage of the breaking. In Sec. 9.3 we have already discussed how the monopole production at the second stage of the breaking from PS to the SM can be avoided in our model of sneutrino inflation. If we assume that $SO(10)$ is broken to G_{PS} before the last observable stage of inflation, the monopoles produced at this first stage of the breaking get diluted.

We also note that the breaking via PS is not the only possible breaking pattern compatible with GNS sneutrino inflation. For example, one could break $SO(10)$ to the minimal left-right symmetric group $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ and then to the SM, avoiding monopole production completely at the second stage. Since, apart from this, the discussion would be analogous to the one in the PS framework, we do not dwell on this in any more detail.

Keeping these points in mind, let us now turn to the formulation of the model in the $SO(10)$ framework. As described above, we unify the left- and right-handed multiplets into $\mathbf{16}$ and $\overline{\mathbf{16}}$ reps, cf. Eq. (10.3). The matter fields containing the SM fermions and their superpartners are denoted by $F_i = \mathbf{16}_i$ according to the definition in Eq. (4.27), while the conjugate rep is introduced as $\bar{F} = \overline{\mathbf{16}}$. The waterfall Higgs fields are unified into the $SO(10)$ representations $H = \mathbf{16}$ and $\bar{H} = \overline{\mathbf{16}}$. Basically, the symmetry assignments are chosen as in the previous chapter, Tab. 9.1. An example superfield content with associated symmetry assignments is displayed in Tab. 10.1.

[3]This is mandatory for phase transitions after inflation.

Up to dimension seven operators, the allowed superpotential terms read

$$\begin{aligned} W =& \kappa\, S \left(\frac{\langle X \rangle}{\Lambda}\, H\bar{H} - M^2 \right) + \frac{\lambda_{ij}}{\Lambda}\, F_i F_j \bar{H}\bar{H} + \frac{\zeta_i}{\Lambda}\, F_i \bar{F} H \bar{H} + \frac{\gamma}{\Lambda}\, \bar{F}\bar{F} H H \\ &+ y_{ij}\, \frac{\langle \theta \rangle}{\Lambda}\, F_i\, h\, F_j + \tilde{y}\, \frac{\langle \theta \rangle}{\Lambda^3}\, h^2 \bar{F}\, h\, \bar{F} + \dots\,, \end{aligned} \tag{10.8}$$

where $h = \mathbf{10}$ contains the SM Higgs superfields. Like in the PS version of the model, we assume that X has already acquired its large VEV $\langle X \rangle \simeq \Lambda$ before inflation has started. Furthermore we assume $\langle \theta \rangle = 0$ during inflation as explained above.

The part of the superpotential of our model relevant for inflation has the form

$$W_{\text{inf}} = \kappa\, S \left(H\bar{H} - M^2 \right) + \frac{\lambda_{ij}}{\Lambda}\, F_i F_j \bar{H}\bar{H} + \frac{\zeta_i}{\Lambda}\, F_i \bar{F} H \bar{H} + \frac{\gamma}{\Lambda}\, \bar{F}\bar{F} H H + \dots\,. \tag{10.9}$$

Under our aforementioned assumption, $SO(10)$ is broken to G_{PS} before inflation. Therefore, inflation as well as the waterfall after inflation are realized as in Sec. 9.3. Let us emphasize at this point that the minimalist field content and the choice of symmetries mainly serves the purpose of giving a proof of existence that GNS inflation can in principle be realized in SUSY $SO(10)$. In a fully realistic model which, e.g., might also contain a full flavor sector, different symmetries may have to be chosen and the field content may have to be extended.

10.2 Generalization to Supergravity

So far, we have investigated the proposed model within the context of global SUSY only. The purpose of this section is to outline how GNS inflation can be generalized in a SUGRA framework. When dealing with inflation model building in SUGRA, a typical problem that arises and with which one has to cope is the η-problem which we have described in Ch. 5. We have outlined possible solutions to this problem for tribrid inflation in Sec. 6.2. One such solution is the use of a fundamental Heisenberg symmetry [109] in the Kähler potential as proposed in Sec. 6.2.3. In the following, we combine the latter with our GNS inflation model, since it is also of the tribrid-type satisfying (6.23).

Therefore, in addition to the superpotentials treated in this part we introduce a Kähler potential that is invariant under a Heisenberg symmetry. In this approach an additional modulus field T is introduced, which transforms under the non-compact Heisenberg group transformations (5.7) together with the candidate inflaton directions. Note that now one has to take the sum over all generation indices, gauge indices and reps. Using the $SO(10)$ field content of the previous section, we obtain an invariant combination given by

$$\rho = T + T^* - F_i^\dagger F_i - \bar{F}^\dagger \bar{F}\,, \tag{10.10}$$

where the dagger indicates complex conjugation and summation over all gauge indices.

Following the framework presented in Sec. 6.2.3, a suitable Heisenberg symmetry invariant Kähler potential is given by

$$K = f(\rho) + \big(\, 1 + \kappa_S \,|S|^2 + \kappa_\rho \, \rho \,\big) \, |S|^2 + H^\dagger H + \bar{H}^\dagger \bar{H} + h^\dagger h \, . \tag{10.11}$$

Note that the function $f(\rho)$ can be a general function which is only constrained by the requirement that the resulting potential has a stable minimum $\rho_{\text{min}} = \langle \rho \rangle$ in which ρ can settle during inflation and that $f'(\rho_{\text{min}}) < 0$ to obtain positive kinetic terms for the inflaton fields. An important feature of Eq. (10.11) is the term $\kappa_S |S|^4$. For negative κ_S, it gives a large mass to the S field which stabilizes it at zero during inflation.[4] We can choose ρ and the components of F_i and $\bar{F}$ to be the independent DOFs and eliminate the one of the modulus T. Then, the F-term potential in the inflationary minimum, where S, H, $\bar{H}$ and h vanish, is of the form in Eq. (6.59) and thus flat at tree level in the direction of the F_i and $\bar{F}$ components. Thereby, the additional coupling κ_ρ in the Kähler potential is essential to generate the stabilizing minimum for the modulus field ρ which is possible for negative κ_ρ.

In a SUGRA framework, under the assumption of a constant diagonal gauge kinetic function $f_{ab} = \delta_{ab}$, the D-term potential (3.76) is also ρ-dependent and of the form[5]

$$V_D \simeq \frac{g^2}{2} \, f'(\rho)^2 \sum_a \Big(F_i^\dagger \, \mathcal{T}^a F_i - \bar{F}^\dagger \, \mathcal{T}^{a*} \bar{F} \Big)^2 \, . \tag{10.12}$$

The basic difference to the global SUSY D-term contribution (9.3) is the global factor of $f'(\rho)^2$. Due to the fact that the modulus quickly acquires its minimum at the very beginning of inflation, $f'(\rho_{\text{min}})^2$ soon approaches a constant value and the D-flatness conditions basically do not change w.r.t. the global SUSY ones, cf. Eq. (9.4).

We emphasize that the Heisenberg symmetry is not meant to be an exact symmetry of the theory, but rather an approximate one. It is even necessary to break the Heisenberg symmetry at some level since otherwise the inflaton potential would be exactly flat and inflation could not end. In our model, the Heisenberg symmetry is broken by the effective operators with parameters λ_{ij}, ζ_i, γ in the superpotential (10.9) as well as by the gauge interactions. At tree level, the latter effects vanish in the D-flat valley and the gauge loop effects have been discussed in detail in Sec. 9.4. Thus, the breaking of the Heisenberg symmetry in our scenario is capable of generating the desired slope of the inflaton potential but does not endanger the solution to the η-problem. Furthermore, we emphasize that the Heisenberg symmetry approach is especially suitable for solving the η-problem for GNS inflation in SUGRA, in contrast to other approaches applicable to gauge singlet inflation. For example, in Sec. 6.2.2 we have proposed a shift symmetry in the Kähler potential to solve the η-problem in a tribrid inflation model with a gauge singlet inflaton field. Clearly, a shift symmetry $F_i \to F_i + \mathrm{i}\,\mu$ cannot be applied to GNS inflation since it does not respect the gauge symmetry.

[4]Which has only been assumed throughout the preceding chapters.

[5]This form is only valid if there are small contributions to $W \neq 0$ which are in general present.

In summary, the use of a Heisenberg symmetry in the Kähler potential is particularly suitable for realizing GNS inflation in SUGRA, because it allows to solve the η-problem in a way that is compatible with a charged inflaton.

Part V

Summary and Conclusions

Chapter 11

Summary and Conclusions

In this dissertation, we have followed the guiding principle of exploring possible connections between early universe cosmology and particle physics beyond the SM. To accomplish this, we have focused on two aspects of inflation models. One aspect concerns their realization in SUGRA, the other aspect concerns their embedding into the matter sector of a SUSY GUT. On top of that, we have combined both frameworks within the scenario of tribrid inflation to give rise to a working model of inflation in the matter sector of SUGRA $SO(10)$.

In the first part of this thesis, we have studied hybrid, tribrid and chaotic inflation models with a special emphasis on different solutions to the η-problem in SUGRA. One such possible solution is to use a general expansion of the Kähler potential, wherein appropriate tuning of the expansion parameters ensures a flat inflaton potential. This method always works for inflation models with field values below the Planck scale (such as hybrid or tribrid inflation), however, fails when super-Planckian field values are involved (as in chaotic inflation). A more elegant way to forbid operators which are subject to the η-problem is to impose fundamental symmetries on the Kähler potential. Symmetry solutions are appealing since they can even protect a direction with large field values where an expansion in terms of effective operators breaks down. Implementations with either a shift symmetry or a Heisenberg symmetry have been discussed as specific examples.

Model Class	Kähler Expansion	Shift Symmetry	Heisenberg Symmetry
Hybrid	✓	×	×
Tribrid	✓	✓	✓
Chaotic	×	✓	✓

Table 11.1: Comparison of the hybrid, tribrid and chaotic models of inflation concerning their compatibility with different solutions to the η-problem in the Kähler potential. A blue checkmark (red cross) indicates (in)compatibility of the combination. See Refs. [44, 105, 106, 122, 123].

As for hybrid-type inflation models, a new characterization in terms of the respective superpotential has been proposed. During inflation, the superpotential of the well-known SUSY hybrid inflation models, as well as its derivative with respect to the inflaton superfield are non-vanishing, cf. Eq. (6.2). Unlike this, models of tribrid inflation, inspired by sneutrino hybrid inflation, have vanishing superpotential and vanishing derivative with respect to the inflaton superfield, cf. Eq. (6.23). This latter trait turns out to be crucial regarding compatibility with the aforementioned symmetry solutions to the η-problem. We have summarized these compatibilities in Tab. 11.1.

The failure of hybrid inflation to combine with symmetries in the Kähler potential is due to the fact that a non-vanishing superpotential can give rise to instabilities in the scalar potential. We have explicitly reviewed this issue for a shift symmetric Kähler potential. Also for a combination with a Heisenberg symmetric Kähler potential, a stabilization of the modulus is not possible without involving new fields.

Unlike this, tribrid inflation has a built-in prevention of such instabilities owing to its vanishing superpotential. Furthermore, this feature helps to avoid many potentially problematic couplings between the inflaton sector and other sectors of the theory. Specifically, adding a moduli stabilization superpotential to the tribrid inflation superpotential works out fine, in contrast to hybrid inflation.

As a first viable example, we have explicitly studied the tribrid inflation superpotential with a shift symmetry in the Kähler potential. In this setup, the shift symmetry ensures an exactly flat tree level potential, while a symmetry breaking effective operator in the superpotential induces a slope at one-loop level. A non-minimal term in the Kähler potential induces a large SUGRA mass for the driving field, which contributes the vacuum energy density by its F-term, and stabilizes it at zero. We have further investigated the effect of a new term in the Kähler potential, which couples the waterfall field to the driving field. This coupling changes the quantum loop corrections, and the shape of the scalar potential can become hilltop-type. According to that, a reduction of the spectral index, compared to the prediction $n_\mathrm{s} \gtrsim 0.98$ in the limit of a minimal Kähler potential, is achieved. Without any tuning of parameters, the seven-year WMAP best fit value $n_\mathrm{s} \approx 0.96$ can be realized, while the tensor-to-scalar ratio is generically very small, $r \lesssim 10^{-2}$. To match the amplitude of the observed curvature power spectrum, the scale of inflation has to be close to the GUT scale.

Following our approach, it has been shown in Ref. [132] that the setup can be successfully combined with a KL-type modulus sector. The authors conclude that the model has the nice phenomenology of combining low scale SUSY breaking with GUT scale inflation. In Ref. [146], we have applied the model to sneutrino hybrid inflation with additional lepton Yukawa couplings and studied combined constraints from successful inflation and nonthermal leptogenesis on the seesaw parameters.

As a second viable example, we have examined a realization of the same superpotential combined with a Heisenberg symmetric Kähler potential. Such a combination is motivated both by string theory considerations, since it can include the commonly encountered case of no-scale SUGRA, as well as from particle physics, since the right-handed sneutrino is again a natural inflaton candidate. A notable feature of the Heisenberg symmetry is that, in a basis of redefined degrees of freedom, it allows for diagonal kinetic terms facilitating the canonical normalization procedure. As in the first example, the inflaton potential is flat at tree level and lifted radiatively by a small term in the superpotential, which does not respect the Heisenberg symmetry. The driving field is stabilized at zero as in the previous case. A new mechanism stabilizes the modulus field associated with the Heisenberg symmetry without any tuning of parameters. This is realized by a term in the Kähler potential coupling modulus and driving field. Both, the driving field and the modulus field acquire SUGRA masses, larger than the Hubble scale, keeping them fixed during inflation. To confirm this, we have simulated the full dynamics with generic initial conditions. The predictions are in good agreement with the seven-year WMAP data.

Rather than considering the job completely done, we do not want to refrain from drawing the reader's attention to one open issue in this setup. Due to the fact that the modulus field is stabilized by the help of the vacuum energy density during inflation, the same stabilization mechanism cannot account for a modulus mass after inflation, once the vacuum energy has disappeared. Therefore, one has to engage a different modulus stabilization mechanism after inflation. It is an interesting issue for future study to investigate if combining the setup with such an additional sector can be successfully realized.

Furthermore, we have devoted our attention to chaotic inflation models in SUGRA. After having described why the naive superpotential quadratic in the inflaton superfield fails to solve the η-problem by symmetries, we have reviewed the model first considered by Kawasaki et al. [106] which works with a shift symmetry in the Kähler potential.

We have demonstrated that, as a viable alternative, the same superpotential linear in both a driving and the inflaton superfield also succeeds to solve the η-problem via the Heisenberg symmetry. In this scenario, the driving field and the modulus field become stabilized by the same mechanisms as in the tribrid case. At tree level, the small Heisenberg symmetry breaking parameter in the superpotential gives rise to a quadratic inflaton potential. Additionally, we have calculated the one-loop radiative corrections which conserve the quadratic potential and can therefore be absorbed in a redefinition of the inflaton mass. Sufficient slow-roll inflation with super-Planckian field values is possible with the driving field and modulus settling to their respective minima at the very beginning of inflation. In the explicit realization with no-scale modulus Kähler potential, we have demonstrated this by simulating the evolution of all participating fields. The predictions for the minimal setup discussed perfectly correspond to those of standard chaotic inflation with a quadratic potential, namely a spectral index $n_{\mathrm{s}} \approx 0.97$ and large tensor-to-scalar ratio $r \approx 0.13$.

The second part of this thesis is devoted to tribrid inflation in the matter sector of a SUSY GUT based on the Pati–Salam gauge group. For this purpose, inflation has to occur in non-singlet representations under the unifying gauge group. Together with fields in the conjugate representation, the scalar components of the matter superfields may form a D-flat direction suitable for inflation.

We have first demonstrated that inflation with a charged inflaton is possible within the tribrid setup in a simple toy model based on the Abelian gauge group $U(1)$. This toy model has served as means to introduce the basic ideas.

Moreover, we have investigated matter inflation in a more realistic model where the inflaton transforms non-trivially under the right-handed part of the Pati–Salam group. Along the explicit D-flat right-handed sneutrino direction, we have examined the F-term scalar potential including one- and two-loop radiative corrections. The fact that the inflaton vacuum expectation value breaks the Pati-Salam group already during inflation helps to enforce the preferred SM direction for the waterfall vacuum expectation value ending inflation. This avoids the production of stable magnetic monopoles, which would be a cosmological disaster. In addition, since, due to its vacuum expectation value, the inflaton is effectively a gauge-singlet during inflation, potentially problematic two-loop mass corrections get strongly suppressed by the mass of the heavy gauge bosons. After inflation, the model can give rise to the three light generations of right-handed quarks and charged leptons while the neutrinos obtain GUT scale masses. This is achieved by having an excess of three matter representations with respect to the conjugate ones. Except for those three, all other generations pair up with some conjugate representation and form Dirac-type mass terms at the GUT scale, thus decoupling from the theory.

Finally, we have sketched how the previous Pati–Salam framework can be generalized to grand unification and supergravity. For this purpose, first we have extended the field content by representations transforming under the left-handed part of Pati–Salam and subsequently embedded it into SUSY $SO(10)$. Furthermore, we have shown that a combined SUGRA $SO(10)$ framework is feasible when relying on the Heisenberg symmetry, since the latter preserves gauge invariance.

Our considerations pave the way towards a fully realistic model of matter inflation in SUGRA $SO(10)$. Such a model will have to incorporate the complete Higgs sector breaking $SO(10)$ with successful solutions to related problems and could even be extended by family symmetries to account for the flavor structure. These appear to be interesting topics for future investigations, and might have implications for nonthermal leptogenesis and the resulting low energy phenomenology.

In conclusion, we point out that the new class of tribrid inflation models is particularly promising concerning connections between early universe cosmology and particle physics beyond the SM. Especially, its association to the neutrino mass generation after inflation via the seesaw mechanism establishes a link between fundamental parameters of the inflaton potential and parameters of the low energy particle physics. With new generations of high precision experiments in observational cosmology and neutrino physics, we might be able to either verify or rule out such possibilities in the near future.

Part VI

Appendix

Appendix A

Notations and Conventions

In this appendix we summarize the notations and conventions used throughout the thesis. We work in natural units where $\hbar = c = 1$. In addition, we set the reduced Planck scale $M_{\mathrm{P}} = 1$. Except for in Sec. 2.3, we use the FLRW metric with line element (2.10) and signature $(+,-,-,-)$. The sum over repeated indices is implied.

A.1 Pauli and Dirac Matrices

For the three Pauli matrices, as the generators of $SU(2)$, we use the convention

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma^2 = \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}, \qquad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{A.1}$$

As the simplest compact Lie algebra, they satisfy the commutation relations

$$[\sigma^i, \sigma^j] = 2\,\mathrm{i}\,\epsilon^{ijk}\sigma^k\,, \tag{A.2}$$

where ϵ^{ijk} is the totally antisymmetric tensor with $i,j,k = 1,2,3$ and $\epsilon^{123} = +1$.

In terms of the Pauli matrices, the 4×4 Dirac matrices in Weyl representation can be defined as

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}, \qquad \gamma^5 = \mathrm{i}\,\gamma^0\,\gamma^1\,\gamma^2\,\gamma^3 = \begin{pmatrix} -\mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix}, \tag{A.3}$$

where $\bar{\sigma}^0 = \sigma^0 = \mathbb{1}$ and $\bar{\sigma}^i = -\sigma^i$ with $\mu = 0,\ldots,3$. Each entry has to be understood as 2×2 matrices. They obey the Clifford algebra

$$\{\gamma^\mu, \gamma^\nu\} = 2\,g^{\mu\nu} \times \mathbb{1}_{n\times n}\,, \tag{A.4}$$

in $n = 4$ dimensions. Furthermore, the γ^μ matrices anticommute with the γ^5 matrix, therefore

$$\{\gamma^\mu, \gamma^5\} = 0\,. \tag{A.5}$$

We can immediately write down generators of the Lorentz group in Weyl representation given by

$$S^{\mu\nu} = \frac{\mathrm{i}}{4}\,[\gamma^\mu, \gamma^\nu]\,. \tag{A.6}$$

A.2 Weyl Spinors

The anticommuting two-component left-chiral Weyl spinors χ_α and their right-chiral conjugate $\chi^{\dagger\dot{\alpha}}$ transform irreducibly under Lorentz transformations as

$$\chi_\alpha \to S(\Lambda)_\alpha^{\ \beta}\,\chi_\beta\,, \qquad \chi^{\dagger\dot{\alpha}} \to \bar{S}(\Lambda)^{\dot{\alpha}}_{\ \dot{\beta}}\,\chi^{\dagger\dot{\beta}}\,, \tag{A.7}$$

which from Eq. (A.6) can be generated respectively by

$$\sigma^{\mu\nu} = \frac{1}{4}\left(\sigma^\mu\bar{\sigma}^\nu - \sigma^\nu\bar{\sigma}^\mu\right)\,, \qquad \bar{\sigma}^{\mu\nu} = \frac{1}{4}\left(\bar{\sigma}^\mu\sigma^\nu - \bar{\sigma}^\nu\sigma^\mu\right)\,, \tag{A.8}$$

such that

$$S(\Lambda) = \exp\left(\frac{1}{2}\,\omega_{\mu\nu}\,\sigma^{\mu\nu}\right)\,, \qquad \bar{S}(\Lambda) = \exp\left(\frac{1}{2}\,\omega_{\mu\nu}\,\bar{\sigma}^{\mu\nu}\right)\,, \tag{A.9}$$

where the $\omega_{\mu\nu}$ parametrize boosts and rotations. The spinor indices $\alpha, \beta = 1, 2$ and $\dot{\alpha}, \dot{\beta} = 1, 2$ can be raised and lowered using the totally antisymmetric tensor $\epsilon_{\alpha\beta} = \epsilon_{\dot{\alpha}\dot{\beta}} = -\epsilon^{\alpha\beta} = -\epsilon^{\dot{\alpha}\dot{\beta}}$ where $\epsilon_{12} = 1$. Via Hermitian conjugation, we can relate left- and right-chiral spinors.

As a convention, contracted spinor indices can be suppressed, in particular

$$\begin{aligned} \xi\,\chi &= \xi^\alpha\chi_\alpha = \chi^\alpha\xi_\alpha = \chi\,\xi\,,\\ \xi^\dagger\chi^\dagger &= \xi^\dagger_{\dot{\alpha}}\chi^{\dagger\dot{\alpha}} = \chi^\dagger_{\dot{\alpha}}\xi^{\dagger\dot{\alpha}} = \chi^\dagger\xi^\dagger = (\xi\,\chi)^\dagger = (\chi\,\xi)^\dagger\,. \end{aligned} \tag{A.10}$$

In the following, we list a few identities used in Ch. 3 given by

$$\begin{aligned} \xi^\dagger\bar{\sigma}^\mu\chi &= -\chi\,\sigma^\mu\xi^\dagger = \left(\chi^\dagger\bar{\sigma}^\mu\xi\right)^* = -\left(\xi\,\sigma^\mu\chi^\dagger\right)^*\,,\\ \xi\,\sigma^\mu\bar{\sigma}^\nu\chi &= \chi\,\sigma^\nu\bar{\sigma}^\mu\xi = \left(\chi^\dagger\bar{\sigma}^\nu\sigma^\mu\xi^\dagger\right)^* = \left(\xi^\dagger\bar{\sigma}^\mu\sigma^\nu\chi^\dagger\right)^*\,, \end{aligned} \tag{A.11}$$

as well as the so-called Fierz rearrangement identity

$$\chi_\alpha(\xi\,\psi) = -\xi_\alpha(\psi\,\chi) - \psi_\alpha(\chi\,\xi)\,. \tag{A.12}$$

Furthermore, some of the useful reduction identities read

$$\begin{aligned} \sigma^\mu_{\alpha\dot{\alpha}}\,\bar{\sigma}^{\dot{\beta}\beta}_\mu &= -2\,\delta^\beta_\alpha\,\delta^{\dot{\beta}}_{\dot{\alpha}}\,,\\ \left(\sigma^\mu\bar{\sigma}^\nu + \sigma^\nu\bar{\sigma}^\mu\right)_\alpha^{\ \beta} &= -2\,g^{\mu\nu}\delta^\beta_\alpha\,,\\ \left(\bar{\sigma}^\mu\sigma^\nu + \bar{\sigma}^\nu\sigma^\mu\right)^{\dot{\beta}}_{\ \dot{\alpha}} &= -2\,g^{\mu\nu}\delta^{\dot{\beta}}_{\dot{\alpha}}\,. \end{aligned} \tag{A.13}$$

Since typically, the spinors and their transformations are defined locally in flat Minkowski spacetime, when adopting the notation for a flat FLRW universe, we work in the comoving frame.

A.3 Dirac and Majorana Spinors

A four component Dirac spinor Ψ is a reducible rep of the Lorentz group and is composed of two Weyl spinor objects, one left-chiral spinor χ_α and one right-chiral spinor $\xi^{\dagger\dot{\alpha}}$ and reads

$$\Psi = \begin{pmatrix} \chi_\alpha \\ \xi^{\dagger\dot{\alpha}} \end{pmatrix} , \qquad \bar{\Psi} = \Psi^\dagger \gamma^0 = (\xi^\alpha, \chi^\dagger_{\dot{\alpha}}) , \tag{A.14}$$

where $\bar{\Psi}$ is the Dirac conjugate of Ψ. Using the chiral projection operators

$$P_{L/R} = \frac{1}{2} \left(\mathbb{1} \mp \gamma^5 \right) , \tag{A.15}$$

we can see that the names left- and right-chiral fit, because applied to the general Dirac spinor, we obtain

$$\Psi_L = P_L \Psi = \begin{pmatrix} \chi_\alpha \\ 0 \end{pmatrix} , \qquad \Psi_R = P_R \Psi = \begin{pmatrix} 0 \\ \xi^{\dagger\dot{\alpha}} \end{pmatrix} . \tag{A.16}$$

A Dirac field describes a charged spin-1/2 particle such as the electron or the quarks. Therefore, it is useful to define the charge conjugated Dirac spinor

$$\Psi^c = C \bar{\Psi}^T = \begin{pmatrix} \xi_\alpha \\ \chi^{\dagger\dot{\alpha}} \end{pmatrix} \tag{A.17}$$

where the charge conjugation matrix $C = \mathrm{i}\, \gamma^0 \gamma^2$ satisfies $C^{-1} \gamma^\mu C = -(\gamma^\mu)^T$. A mass term of a Dirac spinor with mass m is given by

$$\mathcal{L}_\mathrm{D} = -m\, \bar{\Psi}\, \Psi = -m \left(\chi\, \xi + \chi^\dagger \xi^\dagger \right) , \tag{A.18}$$

which breaks chiral symmetry.

If a four component spinor Ψ fulfills the Majorana condition

$$\Psi^c = \Psi , \tag{A.19}$$

it has to consist of two identical Weyl spinors

$$\Psi = \begin{pmatrix} \chi_\alpha \\ \chi^{\dagger\dot{\alpha}} \end{pmatrix} , \qquad \bar{\Psi} = (\chi^\alpha, \chi^\dagger_{\dot{\alpha}}) . \tag{A.20}$$

Such a spinor is called a Majorana spinor and the corresponding particle is its own antiparticle. A Majorana mass term with a mass M is written

$$\mathcal{L}_\mathrm{M} = -\frac{1}{2}\, M\, \bar{\Psi}\, \Psi = -\frac{1}{2}\, M \left(\chi\, \chi + \chi^\dagger \chi^\dagger \right) . \tag{A.21}$$

Appendix B

Tribrid Inflation Supplement

B.1 Detailed Background Evolution

In Sec. 6.2.3, we have used the assumption that the evolution of the imaginary parts of the scalar components of all chiral superfields can be neglected. Here, we show explicitly that this is justified for the phase of Φ and the imaginary part of the modulus T from the full EOMs given that $s = h = 0$. Using Eq. (3.63) with the Kähler metric of Eq. (6.55), we obtain the relevant kinetic Lagrangian terms

$$\begin{aligned}\mathcal{L}_{\rm kin} = &\left[f''(\rho)\,|\Phi|^2 - f'(\rho)\right]\,\partial_\mu\Phi\,\partial^\mu\Phi^* + f''(\rho)\,\partial_\mu T\,\partial^\mu T^* \\ &- f''(\rho)\,\Phi^*\,\partial_\mu\Phi\,\partial^\mu T^* - f''(\rho)\,\Phi\,\partial_\mu\Phi^*\,\partial^\mu T\,.\end{aligned} \tag{B.1}$$

In the following we use the no-scale form (6.60) and decompose T into its real and imaginary part. Furthermore, we write Φ in terms of its modulus ϕ and phase θ and introduce ρ in terms of the real scalar DOFs:

$$T = \frac{1}{\sqrt{2}}\left(t_{\rm R} + {\rm i}\,t_{\rm I}\right)\,, \quad \Phi = \frac{1}{\sqrt{2}}\,\phi\exp\left({\rm i}\,\theta\right), \quad \rho = \sqrt{2}\,t_{\rm R} - \frac{1}{2}\,\phi^2\,. \tag{B.2}$$

Note that using the definition of ρ, we can fully eliminate $t_{\rm R}$. The full system is thus described by $(t_{\rm I}, \theta, \rho, \phi)$ with the kinetic terms given by

$$\mathcal{L}_{\rm kin} = \frac{3}{2\,\rho^2}\left[\frac{\dot\rho^2}{2} + \dot t_{\rm I}^2 + \rho\,\dot\phi^2 + \frac{1}{2}\,\phi^4\,\dot\theta^2 + \rho\,\phi^2\,\dot\theta^2 - \sqrt{2}\,\phi^2\,\dot\theta\,\dot t_{\rm I}\right]\,. \tag{B.3}$$

Since neither the tree-level nor the one-loop potential depend on $t_{\rm I}$ and θ, these are flat directions and we have to make sure that they get *effectively frozen* very quickly due to the *cosmic viscosity* and their EOMs decouple from the ρ- and ϕ-evolution. As the effective potential we apply Eq. (6.72) and obtain the coupled set of EOMs

$$\begin{aligned}
\ddot{t}_{\rm I} + 3\,\mathcal{H}\,\dot{t}_{\rm I} - 2\,\frac{\dot\rho}{\rho}\,\dot{t}_{\rm I} - \frac{3}{\sqrt{2}}\,\mathcal{H}\,\phi^2\,\dot\theta - \sqrt{2}\,\phi\,\dot n\,\dot\theta - \frac{1}{\sqrt{2}}\,\phi^2\,\ddot\theta + \sqrt{2}\,\frac{\dot\rho}{\rho}\,\phi^2\,\dot\theta &= 0\,, \\
\left(1 + 2\,\frac{\rho}{\phi^2}\right)\left[\ddot\theta + 3\,\mathcal{H}\,\dot\theta - 2\,\frac{\dot\rho}{\rho}\,\dot\theta\right] + & \\
\left[\left(4\,\frac{\dot\phi}{\phi} + 2\,\frac{\dot\rho}{\phi^2} + 4\,\frac{\rho}{\phi^3}\,\dot\phi\right)\dot\theta + \sqrt{2}\left(2\,\frac{\dot\rho}{\rho\,\phi^2} - 2\,\frac{\dot\phi}{\phi^3} - \frac{3\,\mathcal{H}}{\phi^2}\right)\dot{t}_{\rm I} - \sqrt{2}\,\frac{\ddot{t}_{\rm I}}{\phi^2}\right] &= 0\,, \qquad \text{(B.4)}\\
\ddot\rho + 3\,\mathcal{H}\,\dot\rho - \frac{\dot\rho^2}{\rho} + \dot\phi^2 + \frac{2\,\rho^2}{3}\,\frac{\partial V_{\rm eff}}{\partial\rho} + 2\,\frac{\dot{t}_{\rm I}^2}{\rho} + \phi^2\,\dot\theta^2 + \frac{\phi^4\,\dot\theta^2}{2\,\rho} - \frac{2\sqrt{2}}{\rho}\,\phi^2\,\dot\theta\,\dot{t}_{\rm I} &= 0\,, \\
\ddot\phi + 3\,\mathcal{H}\,\dot\phi - \frac{\dot\rho}{\rho}\,\dot\phi + \frac{\rho}{3}\,\frac{\partial V_{\rm eff}}{\partial\phi} - \phi\,\dot\theta^2 - \frac{\phi^3}{\rho}\,\dot\theta^2 + \sqrt{2}\,\frac{\phi}{\rho}\,\dot\theta\,\dot{t}_{\rm I} &= 0\,.
\end{aligned}$$

Note from the last two equations that in the limit $\dot{t}_{\rm I} \to 0$ and $\dot\theta \to 0$, the evolution of ϕ and ρ decouple from $t_{\rm I}$ and θ and we recover Eqs. (6.58).

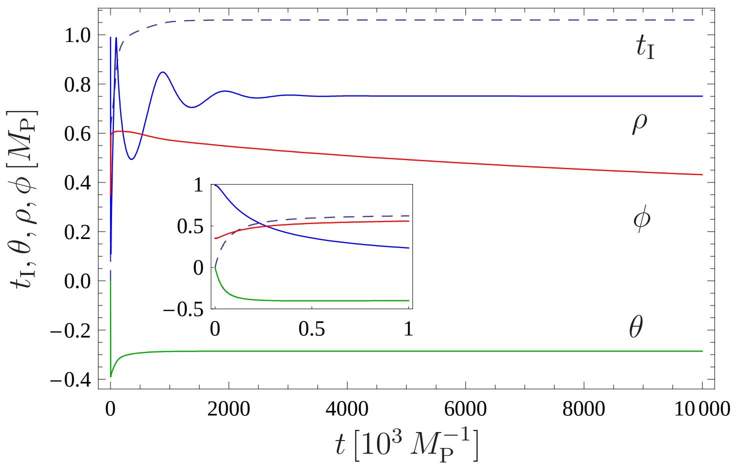

Figure B.1: Full evolution including the imaginary part $t_{\rm I}$ and the phase θ. The purpose of the inlay is to show that for small t, the evolution of the fields is perfectly smooth.

In the following, we simulate the full evolution described by Eq. (B.4) for some generic initial conditions. With the same renormalization scale Q and model parameters as in in Sec. 6.2.3, the field evolution is plotted versus cosmic time in Fig. B.1. As initial conditions for the fields, we have chosen the values $(t_{\rm I}, \theta, \rho, \phi)\,|_{t=0} = (0, 0, 0.99, 0.25)$ and the velocities $(\dot{t}_{\rm I}, \dot\theta, \dot\rho, \dot\phi)|_{t=0} = (0.01, -0.01, 0, 0)$. As can be seen from the plot, both $t_{\rm I}$ and θ obtain initial velocities in opposite directions. During this time period, the evolution of ρ and ϕ is influenced by them. However, due to the strong damping by $\mathcal{H}$ from the expansion of the Universe, after a very short period of time, both the imaginary part and the phase get frozen and stay constant subsequently. Thereafter, the ρ and ϕ trajectories are not affected by $t_{\rm I}$ and θ anymore.

From this we conclude that putting the phase of Φ and $\mathrm{Im}(T)$ to zero initially and using the decoupled Eqs. (6.58) for the absolute value and $\mathrm{Re}(T)$ is justified. Similar conclusions have been drawn in [147].

B.2 Toy Model with Shift Symmetry

The toy model presented here serves as a proof of existence of the tribrid inflation setup with a shift symmetric Kähler potential proposed in Sec. 6.2.2. We would like to outline how the simple superpotential and Kähler potential of Eqs. (6.25) and (6.43) can be realized in an explicit model and, in particular, how additional unwanted terms[1] can be avoided.

Therefore, we assume three additional gauge singlet chiral superfields X, Y and Z which are heavy and, during the period of inflation, have already acquired VEVs in their bosonic components. Next we impose the discrete symmetries $\mathbb{Z}_2$, $\mathbb{Z}_4$, $\mathbb{Z}_6$ as well as an R-symmetry under which the superpotential carries unit charge. The field content of the toy model together with the assigned charges are displayed in Tab. B.1.

	H	S	Φ	X	Y	Z
$\mathbb{Z}_2$	$-$	$+$	$-$	$+$	$+$	$+$
$\mathbb{Z}_4$	0	2	0	1	1	0
$\mathbb{Z}_6$	2	0	1	4	0	5
R	1/2	0	0	0	1/2	0

Table B.1: Toy model superfield content and imposed symmetries.

With these symmetries and associated charge assignments, terms in the superpotential up to dimension six operators and Kähler potential terms allowed are given by

$$\begin{aligned} W &= S\left(H^2\langle X\rangle^2 - \langle Y\rangle^2\right) + \Phi^2 H^2 + \langle Y\rangle^2 S^3\,, \\ K &= \left(\Phi\,\langle Z\rangle + \Phi^*\langle Z\rangle^*\right)^2 + \ldots\,. \end{aligned} \tag{B.5}$$

Since we have assumed the fields X, Y and Z as heavy and already at their minima, we can neglect them for the dynamics of inflation and just insert their VEVs. Realizing that for a given complex and spatially constant VEV $\langle Z\rangle$ we can always perform global phase redefinitions of the fields to make $\langle Z\rangle$ real, we can recover the simple superpotential and Kähler potential of Eqs. (6.25) and (6.43) with one additional operator $\langle Y\rangle^2 S^3$ in the superpotential. Since $S = 0$ during inflation, however, this term has no effect on the inflationary dynamics. Accordingly, we reproduce our setup for the VEVs given by $\langle X\rangle = \langle Z\rangle \simeq M_{\mathrm{P}}$ and $\langle Y\rangle \simeq M$.

[1]For instance a direct mass term Φ^2 in the superpotential.

Appendix C

Chaotic Inflation Supplement

In Sec. 7.2.2, we have proposed a model of chaotic inflation in SUGRA in which the η-problem of the classical potential can be resolved by imposing a Heisenberg symmetry on the Kähler potential. Here we discuss why the quantum corrections to this potential have negligible effects. For this purpose, we calculate the CW one-loop effective potential using the formulae given in [110, 148]. Introducing a cutoff scale $\Lambda = M_{\mathrm{P}}$ in the theory, the one-loop correction to the effective potential is given by

$$V_{\text{loop}} = \frac{\Lambda^2}{32\,\pi^2}\,\mathrm{Str}\,\mathcal{M}^2 + \frac{1}{64\,\pi^2}\,\mathrm{Str}\,\mathcal{M}^4\,\ln\left(\frac{\mathcal{M}^2}{\Lambda^2}\right)\,. \tag{C.1}$$

Since to fit observations, the mass parameter m should be of the order $m = \mathcal{O}(10^{-5})$, we can safely ignore the logarithmic part of the loop-correction.

For the dominant quadratic part, the supertrace can be written in the form

$$\mathrm{Str}\,\mathcal{M}^2 = 2\,(N-1)\,V_F + 2\,|W|^2\,\mathrm{e}^K\left(N - 1 - G^i\,R_{i\bar{j}}\,G^{\bar{j}}\right)\,, \tag{C.2}$$

where $N = 3$ is the total number of chiral superfields and the Kähler function is defined in Eq. (3.59). The Ricci tensor of the Kähler manifold contributing in Eq. (C.2) is defined by

$$R_{i\bar{j}} = \partial_i\,\partial_{\bar{j}}\,\ln\,\det\left(G_{m\bar{n}}\right)\,. \tag{C.3}$$

Taking $\kappa_X < 0$, the curvature along the X-direction is large and positive during inflation when $\phi \neq 0$ so that the field will quickly go to zero. For the sake of simplicity, we set $x = 0$ which is justified by the simulation with the full x-dependent potential as depicted in Fig. 7.1. Plugging the superpotential and Kähler potential of Eqs. (7.6) and (7.15) into Eq. (C.2), we end up with the ρ- and ϕ-dependent loop-potential

$$V_{\text{loop}} = \frac{m^2\phi^2}{32\,\pi^2\,(3\,\rho^3)}\left[\frac{2\,(3 + 4\,\kappa_\rho\,\rho)}{(1+\kappa_\rho\,\rho)^2} - \frac{(\kappa_\rho\,\rho + 12\,\kappa_X)}{(1+\kappa_\rho\,\rho)^3}\right]\,. \tag{C.4}$$

Therefore, the presence of loop-corrections just has the effect of shifting the minimum of the potential in ρ-direction, cf. Fig. C.1 where the field dynamics have been plotted

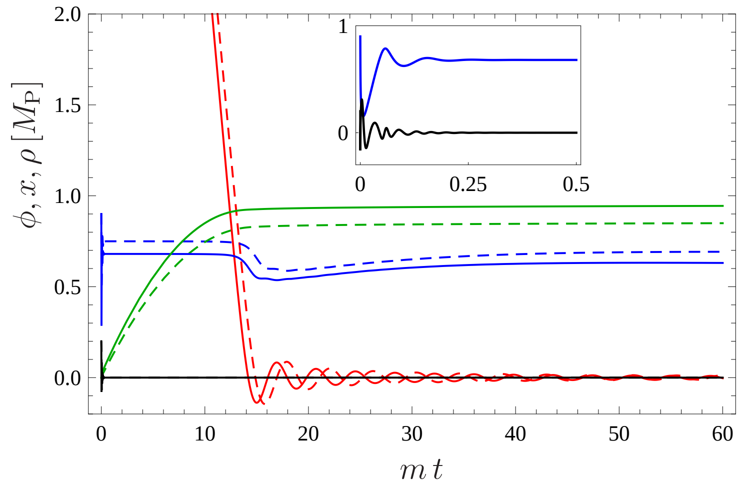

Figure C.1: Evolution of the scalar fields. The green curve represents the number of e-folds in $N_{\mathrm{e}}/100$, the red curve represents the inflaton direction ϕ, the blue curve depicts the evolution of ρ and the black curve the evolution of x. Solid lines represent the evolution of the same fields if in addition to the tree-level potential, quantum corrections are taken into account.

with and without the loop-corrections. For example with $\kappa_\rho = -1$ and $\kappa_X = -1$ the loop-corrected minimum during inflation shifts to $\langle\rho\rangle = 0.68$ from its tree-level value of $\langle\rho\rangle = 0.75$.

It is important to note that similar to the tree-level potential, the loop-corrected potential also has a ϕ-independent minimum $\langle\rho\rangle$. Once the ρ-field gets stabilized in its new minimum, we obtain a different factor in front of the inflaton mass squared. In order to fit the amplitude of the curvature perturbation $\mathcal{P}_{\mathcal{R}}$, one simply absorbs this factor in m^2 and adjusts the new effective mass squared. The amplitude of the primordial spectrum is given by $\mathcal{P}_{\mathcal{R}}^{1/2} \simeq (N_{\mathrm{e}}/\sqrt{6}\,\pi)\,m_{\mathrm{eff}}$, and the WMAP normalization by $\mathcal{P}_{\mathcal{R}}^{1/2} \approx 5 \cdot 10^{-5}$ then gives $m_{\mathrm{eff}} \approx 6 \cdot 10^{-6}$. Thus we conclude that the loop-corrections do not change the predictions calculated from the tree-level potential in Sec. 7.2.2 but instead just lead to a mass-renormalization of the inflaton field.

Appendix D

Matter Inflation Supplement

D.1 Mass Spectrum during Inflation

In this appendix we calculate the masses of the relevant fields during inflation for the model of Sec. 9.3. In particular, we calculate the gauge boson masses, the fermion masses corresponding to the chiral superfields H^c and $\bar{H}^c$ and the fermion masses arising from the mixing between the chiral and gauge multiplets. The results have been summarized in the main text in Tab. 9.2 and Tab. 9.3 and they have been used in calculating the one loop radiative corrections in Sec. 9.4.1. The scalar masses for the waterfall sector have been calculated in the main text, Sec. 9.3.

D.1.1 Gauge Boson Masses

We now calculate the gauge boson masses corresponding to the gauge factors $SU(2)_R$ and $SU(4)_C$ of the Pati–Salam gauge group. As we will see, some of the gauge fields become massive when the inflaton fields acquire VEVs during inflation.

In our calculation, we set the coupling constants $g_R = g_C \equiv g$ close to the GUT scale and we use the following generators

$$\begin{aligned}
\mathcal{T}^a &= T^a \otimes \mathbb{1}_{2\times 2}\,, \\
\mathcal{T}^{16} &= \mathbb{1}_{4\times 4} \otimes \frac{1}{2}\,\sigma^1\,, \\
\mathcal{T}^{17} &= \mathbb{1}_{4\times 4} \otimes \frac{1}{2}\,\sigma^2\,, \\
\mathcal{T}^{18} &= \mathbb{1}_{4\times 4} \otimes \frac{1}{2}\,\sigma^3\,.
\end{aligned} \tag{D.1}$$

Here, σ^b with $b = 1, 2, 3$ are the Pauli matrices given by (A.1) and T^a with $a = 1, \ldots, 15$ are the 15 generators of $SU(4)_C$ displayed in Tab. D.1.

$$T^1 = \tfrac{1}{2}\begin{pmatrix}0&1&0&0\\1&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix} \quad T^2 = \tfrac{1}{2}\begin{pmatrix}0&-\mathrm{i}&0&0\\\mathrm{i}&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix} \quad T^3 = \tfrac{1}{2}\begin{pmatrix}1&0&0&0\\0&-1&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}$$

$$T^4 = \tfrac{1}{2}\begin{pmatrix}0&0&1&0\\0&0&0&0\\1&0&0&0\\0&0&0&0\end{pmatrix} \quad T^5 = \tfrac{1}{2}\begin{pmatrix}0&0&-\mathrm{i}&0\\0&0&0&0\\\mathrm{i}&0&0&0\\0&0&0&0\end{pmatrix} \quad T^6 = \tfrac{1}{2}\begin{pmatrix}0&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&0\end{pmatrix}$$

$$T^7 = \tfrac{1}{2}\begin{pmatrix}0&0&0&0\\0&0&-\mathrm{i}&0\\0&\mathrm{i}&0&0\\0&0&0&0\end{pmatrix} \quad T^8 = \tfrac{1}{2\sqrt{3}}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-2&0\\0&0&0&0\end{pmatrix} \quad T^9 = \tfrac{1}{2}\begin{pmatrix}0&0&0&1\\0&0&0&0\\0&0&0&0\\1&0&0&0\end{pmatrix}$$

$$T^{10} = \tfrac{1}{2}\begin{pmatrix}0&0&0&-\mathrm{i}\\0&0&0&0\\0&0&0&0\\\mathrm{i}&0&0&0\end{pmatrix} \quad T^{11} = \tfrac{1}{2}\begin{pmatrix}0&0&0&0\\0&0&0&1\\0&0&0&0\\0&1&0&0\end{pmatrix} \quad T^{12} = \tfrac{1}{2}\begin{pmatrix}0&0&0&0\\0&0&0&-\mathrm{i}\\0&0&0&0\\0&\mathrm{i}&0&0\end{pmatrix}$$

$$T^{13} = \tfrac{1}{2}\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix} \quad T^{14} = \tfrac{1}{2}\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&-\mathrm{i}\\0&0&\mathrm{i}&0\end{pmatrix} \quad T^{15} = \tfrac{1}{2\sqrt{6}}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&-3\end{pmatrix}$$

Table D.1: Fifteen $SU(4)_C$ generators.

The masses for the gauge bosons are given by the following term in the Lagrangian

$$\mathcal{L}_{GB} = \left| \sum_{a=1}^{18} g\, A_\mu^a\, \mathcal{T}^a \langle R^c \rangle \right|^2 + \text{ terms for } \langle \bar{R}^c \rangle \,, \tag{D.2}$$

where $\langle R^c \rangle$ and $\langle \bar{R}^c \rangle$ are the VEVs of the sneutrinos acting as inflatons, cf. Eq. (9.5).

We can easily see that the gauge fields corresponding to the generators $\mathcal{T}^1, \ldots, \mathcal{T}^8$ remain massless. On the other hand, for the gauge fields corresponding to the generators $\mathcal{T}^9$ and $\mathcal{T}^{10}$ we find

$$\mathcal{L}_{GB} \supset \frac{1}{2}\, g^2 \left\langle \nu^c \right\rangle^2 \left[(A_\mu^9)^2 + (A_\mu^{10})^2 \right] \,. \tag{D.3}$$

This yields

$$m_9^2 = m_{10}^2 = g^2 \left\langle \nu^c \right\rangle^2 . \tag{D.4}$$

Similarly, the gauge bosons corresponding to the generators $\mathcal{T}^{11}, \ldots, \mathcal{T}^{14}$ as well as $\mathcal{T}^{16}$ and $\mathcal{T}^{17}$ acquire the same mass. The generators $\mathcal{T}^{18}$ and $\mathcal{T}^{15}$ are diagonal and the corresponding gauge bosons mix. We find

$$\mathcal{L}_{GB} \supset g^2 \frac{\langle \nu^c \rangle^2}{4} \left(A_\mu^{18} - \sqrt{\frac{3}{2}}\, A_\mu^{15} \right)^2 + \text{ terms for } \langle \bar{R}^c \rangle \,. \tag{D.5}$$

Defining the new normalized field

$$Z_\mu^{\parallel} \equiv \sqrt{\frac{2}{5}} \left(A_\mu^{18} - \sqrt{\frac{3}{2}} A_\mu^{15} \right) , \tag{D.6}$$

this becomes

$$\mathcal{L}_{GB} \supset \frac{5}{4}\, g^2 \, \langle \nu^c \rangle^2 \, (Z_\mu^{\parallel})^2 \,. \tag{D.7}$$

The combination orthogonal to $Z_\mu^{\parallel}$, i.e.,

$$Z_\mu^{\perp} \equiv \sqrt{\frac{2}{5}} \left(A_\mu^{15} + \sqrt{\frac{3}{2}} A_\mu^{18} \right) , \tag{D.8}$$

remains massless and it is the gauge boson of $U(1)_Y$. All gauge boson masses have been summarized in Tab. 9.2.

D.1.2 Fermion Masses

In global SUSY as introduced in Sec. 3.1, there are two contributions to the fermion masses, one coming directly from the superpotential and another one from the mixing between the chiral and the gauge multiplets.

The contribution from the superpotential is given by

$$\mathcal{L}_1 = -\frac{1}{2} \frac{\delta^2 W}{\delta\phi_i \, \delta\phi_j} \left(\chi_i \, \chi_j + \chi_i^\dagger \, \chi_j^\dagger \right) \,. \tag{D.9}$$

Here, ϕ_i and χ_i are the scalar boson and chiral fermion contained in the chiral superfield $\Phi_i \supset \phi_i, \chi_i$ and W is the superpotential regarded as a function of the scalar fields only.

Using the form of the superpotential in Eq. (9.2) and keeping in mind that the VEVs of the scalar components of H^c and $\bar{H}^c$ remain at zero during inflation, we conclude that Eq. (D.9) does not contribute to the fermion masses corresponding to the chiral multiplets R^c and $\bar{R}^c$. But it does contribute to the fermion masses corresponding to H^c and $\bar{H}^c$, namely due to

$$\begin{aligned} \mathcal{L}_1 = &- \zeta \, \langle \nu^c \rangle^2 \left[\chi_{u_{1H}^c} \chi_{\bar{u}_{1H}^c} + \, \ldots \, + \chi_{d_{3H}^c} \chi_{\bar{d}_{3H}^c} + \chi_{e_H^c} \chi_{\bar{e}_H^c} + \text{h.c.} \right] \\ &- \frac{1}{2} \, \langle \nu^c \rangle^2 \left[2 \, \gamma \, \chi_{\nu_H^c} \chi_{\nu_H^c} + 2 \, (\zeta + \xi) \, \chi_{\nu_H^c} \chi_{\bar{\nu}_H^c} + 2 \, \lambda \, \chi_{\bar{\nu}_H^c} \chi_{\bar{\nu}_H^c} + \text{h.c.} \right] . \end{aligned} \tag{D.10}$$

Combining two chiral spinors to a Dirac spinor

$$\Psi_{u^c_{1H}} = \begin{pmatrix} \chi_{u^c_{1H}} \\ \chi^\dagger_{\bar{u}^c_{1H}} \end{pmatrix} , \ldots \tag{D.11}$$

the first part becomes

$$\mathcal{L}_1 \supset -\zeta \, \langle \nu^c \rangle^2 \left[\bar{\Psi}_{u^c_{1H}} \Psi_{u^c_{1H}} + \ \ldots + \bar{\Psi}_{d^c_{3H}} \Psi_{d^c_{3H}} + \bar{\Psi}_{e^c_H} \Psi_{e^c_H} \right] . \tag{D.12}$$

Diagonalizing the mass matrix of the second part, we find

$$\mathcal{L}_1 \supset -\frac{1}{2} \, \langle \nu^c \rangle^2 \left[(2\,\gamma - \zeta - \xi) \, \chi_a \, \chi_a \, + \, (2\,\gamma + \zeta + \xi) \, \chi_b \, \chi_b + \text{ h.c.} \right] , \tag{D.13}$$

where

$$\begin{pmatrix} \chi_a \\ \chi_b \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \chi_{\bar{\nu}^c_H} - \chi_{\nu^c_H} \\ \chi_{\bar{\nu}^c_H} + \chi_{\nu^c_H} \end{pmatrix} , \tag{D.14}$$

and we have set $\gamma = \lambda$ for simplicity.

Finally, defining the two Majorana spinors

$$\Psi_a = \begin{pmatrix} \chi_a \\ \chi_a^\dagger \end{pmatrix} , \qquad \Psi_b = \begin{pmatrix} \chi_b \\ \chi_b^\dagger \end{pmatrix} , \tag{D.15}$$

this becomes

$$\mathcal{L}_1 \supset -\frac{1}{2} \, \langle \nu^c \rangle^2 \left[(2\,\gamma - \zeta - \xi) \ \bar{\Psi}_a \Psi_a \, + \, (2\,\gamma + \zeta + \xi) \ \bar{\Psi}_b \Psi_b \right] . \tag{D.16}$$

The resulting masses have been summarized in Tab. 9.3.

Next, we turn to the second contribution due to the mixings between the chiral fermions χ_i of the chiral superfields and the gauginos λ^a. It is given by

$$\mathcal{L}_2 = -\sqrt{2}\, g \sum_a \left(\phi^*_{R^c} \, \mathcal{T}^a \, \chi_{R^c} \right) \, \lambda^a - \sqrt{2}\, g \sum_a \lambda^{\dagger a} \left(\chi^\dagger_{R^c} \, \mathcal{T}^a \, \phi_{R^c} \right) \, + \text{ terms for } \bar{R}^c , \tag{D.17}$$

where ϕ_{R^c} and χ_{R^c} are the scalar and fermionic fields contained in the supermultiplet R^c.

Plugging in the VEVs of the R^c and $\bar{R}^c$ fields we end up with

$$\begin{aligned} \mathcal{L}_2 = -\frac{g}{\sqrt{2}} \, \langle \nu^c \rangle \Big[& \chi_{u^c_1} \left(-\lambda^9 + \mathrm{i}\,\lambda^{10} \right) + \chi_{\bar{u}^c_1} \left(\lambda^9 + \mathrm{i}\,\lambda^{10} \right) + \ \ldots \ + \\ & \chi_{u^c_3} \left(-\lambda^{13} + \mathrm{i}\,\lambda^{14} \right) + \chi_{\bar{u}^c_3} \left(\lambda^{13} + \mathrm{i}\,\lambda^{14} \right) + \\ & \chi_{e^c} \left(-\lambda^{16} - \mathrm{i}\,\lambda^{17} \right) + \chi_{\bar{e}^c} \left(\lambda^{16} - \mathrm{i}\,\lambda^{17} \right) + \\ & \chi_{\nu^c} \left(\sqrt{\frac{3}{2}} \, \lambda^{15} - \lambda^{18} \right) + \chi_{\bar{\nu}^c} \left(-\sqrt{\frac{3}{2}} \, \lambda^{15} + \lambda^{18} \right) + \text{ h.c.} \Big] . \end{aligned} \tag{D.18}$$

Defining the following normalized left-chiral fields

$$
\begin{aligned}
\xi_1 &= \frac{1}{\sqrt{2}}\left(-\lambda^9 + \mathrm{i}\,\lambda^{10}\right) & \xi_2 &= \frac{1}{\sqrt{2}}\left(\lambda^9 + \mathrm{i}\,\lambda^{10}\right) \\
&& \dots && \\
\xi_{e^c} &= -\frac{1}{\sqrt{2}}\left(\lambda^{16} + \mathrm{i}\,\lambda^{17}\right) & \xi_{\bar{e}^c} &= \frac{1}{\sqrt{2}}\left(\lambda^{16} - \mathrm{i}\,\lambda^{17}\right) \\
\chi_\nu^{\parallel} &= \frac{1}{\sqrt{2}}(\chi_{\nu^c} - \chi_{\bar{\nu}^c}) & \chi_\nu^{\perp} &= \frac{1}{\sqrt{2}}(\chi_{\nu^c} + \chi_{\bar{\nu}^c}) \\
\xi_{\nu^c}^{\parallel} &= \sqrt{\frac{2}{5}}\left(\sqrt{\frac{3}{2}}\lambda^{15} - \lambda^{18}\right) & \xi_{\nu^c}^{\perp} &= \sqrt{\frac{2}{5}}\left(\sqrt{\frac{3}{2}}\lambda^{18} + \lambda^{15}\right)
\end{aligned}
\tag{D.19}
$$

we can combine these with the chiral fermion fields from the R^c and $\bar{R}^c$ superfields to form the following Dirac spinors

$$
\Psi_1 = \begin{pmatrix} \chi_{u_1^c} \\ \xi_1^\dagger \end{pmatrix}, \quad \Psi_2 = \begin{pmatrix} \chi_{\bar{u}_1^c} \\ \xi_2^\dagger \end{pmatrix}, \quad \dots \quad, \quad \Psi_{\nu^c}^{\parallel} = \begin{pmatrix} \chi_{\nu^c}^{\parallel} \\ \xi_{\nu^c}^{\dagger\parallel} \end{pmatrix}, \quad \Psi_{\nu^c}^{\perp} = \begin{pmatrix} \chi_{\nu^c}^{\perp} \\ \xi_{\nu^c}^{\dagger\perp} \end{pmatrix}. \tag{D.20}
$$

With these, we can now write

$$
\mathcal{L}_2 = -\,g\,\langle\nu^c\rangle\left[\bar{\Psi}_1\Psi_1 + \dots + \bar{\Psi}_6\Psi_6 + \bar{\Psi}_{e^c}\Psi_{e^c} + \bar{\Psi}_{\bar{e}^c}\,\Psi_{\bar{e}^c}\right] - \sqrt{\frac{5}{2}}\,g\,\langle\nu^c\rangle\,\bar{\Psi}_{\nu^c}^{\parallel}\Psi_{\nu^c}^{\parallel}\,. \tag{D.21}
$$

The mass spectrum has been listed in Tab. 9.2.

D.2 Effective Dimension Five Operators

In our simple Pati–Salam model of Sec. 9.1 we want to consider all effective dimension five operators which are generated by the exchange of singlet messenger fields and are allowed by the imposed R and $\mathbb{Z}_{10}$ symmetries.

To begin with, let us focus on the $SU(4)_C$ gauge structure. Under $SU(4)_C$ we have $\bar{R}^c, \bar{H}^c \sim \mathbf{4}$, whereas $R^c, H^c \sim \bar{\mathbf{4}}$. We know that

$$
\begin{aligned}
\mathbf{4}\otimes\bar{\mathbf{4}} &= \mathbf{1}\oplus\mathbf{15} \\
\mathbf{4}\otimes\mathbf{4} &= \mathbf{10}\oplus\bar{\mathbf{6}} \\
\bar{\mathbf{4}}\otimes\bar{\mathbf{4}} &= \bar{\mathbf{10}}\oplus\mathbf{6}
\end{aligned}
\tag{D.22}
$$

To form a singlet messenger we therefore have to couple one field transforming as a $\mathbf{4}$ to one transforming as a $\bar{\mathbf{4}}$. Coupling two such fields will also yield a singlet under $SU(2)_R$, since in our model they transform as $\mathbf{2}$ respectively $\bar{\mathbf{2}}$ under this symmetry. The allowed fundamental vertices are shown in Fig. D.1.

When combining two of these fundamental vertices to form an effective $d = 5$ operator, we have to introduce a mass insertion into the diagram, cf. Fig. D.2. The corresponding

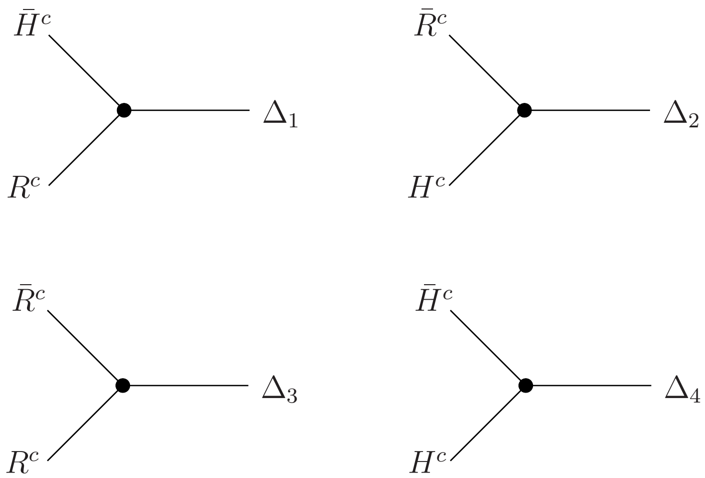

Figure D.1: Interaction vertices yielding singlet messenger fields.

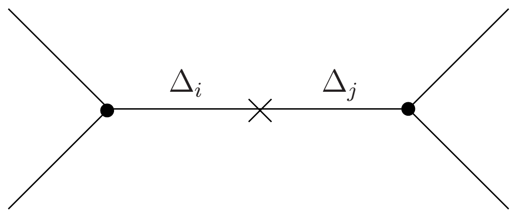

Figure D.2: Feynman diagram generating the effective $d = 5$ operators.

term in the superpotential reads

$$W \supset \Lambda\, \Delta_i \Delta_j \; . \tag{D.23}$$

From this we see that the R and $\mathbb{Z}_{10}$ quantum numbers of the messenger fields involved have to add up to 1 respectively a multiple of 10. These quantum numbers can be found in Tab. D.2.

Thus, we can couple Δ_1 and Δ_2 to themselves, Δ_1 to Δ_2 and finally Δ_3 to Δ_4. After integrating out the heavy messengers, the following effective operators are generated, where

Messenger	R	$\mathbb{Z}_{10}$
Δ_1	$1/2$	5
Δ_2	$1/2$	5
Δ_3	0	3
Δ_4	1	7

Table D.2: Quantum numbers of the singlet messenger fields.

round brackets denote contraction of the $SU(4)_C$ and $SU(2)_R$ indices

$$\begin{aligned}\mathcal{O}_1^{d=5} &= \frac{\lambda}{\Lambda}\left(R^c\bar{H}^c\right)\left(R^c\bar{H}^c\right)\,,\\ \mathcal{O}_2^{d=5} &= \frac{\gamma}{\Lambda}\left(\bar{R}^cH^c\right)\left(\bar{R}^cH^c\right)\,,\\ \mathcal{O}_3^{d=5} &= \frac{\zeta}{\Lambda}\left(R^c\bar{R}^c\right)\left(H^c\bar{H}^c\right)\,,\\ \mathcal{O}_4^{d=5} &= \frac{\xi}{\Lambda}\left(R^c\bar{H}^c\right)\left(\bar{R}^cH^c\right)\,.\end{aligned} \tag{D.24}$$

The corresponding effective vertices are depicted in Fig. D.3.

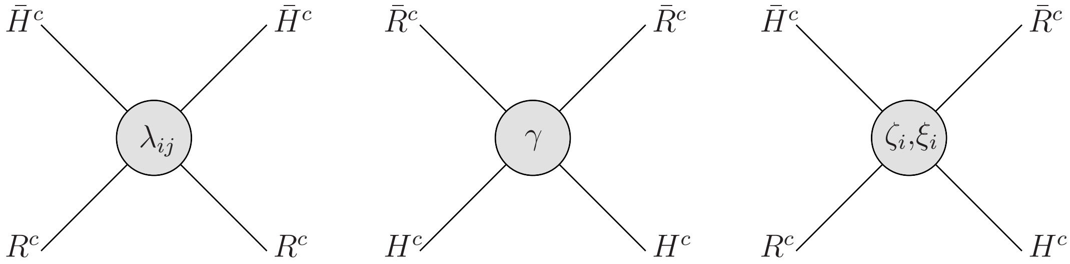

Figure D.3: Generated effective $d = 5$ operators.

The complete effective superpotential resulting from the symmetry assignments with singlet messenger exchange now reads

$$\begin{aligned}W = \kappa\, S &\left(\frac{\langle X\rangle}{\Lambda}\,H^c\bar{H}^c - M^2\right)\\ &+ \frac{\lambda}{\Lambda}\,(R^c\bar{H}^c)(R^c\bar{H}^c) + \frac{\gamma}{\Lambda}\,(\bar{R}^cH^c)(\bar{R}^cH^c)\\ &+ \frac{\zeta}{\Lambda}\,(R^c\bar{R}^c)(H^c\bar{H}^c) + \frac{\xi}{\Lambda}\,(R^c\bar{H}^c)(\bar{R}^cH^c)\,.\end{aligned} \tag{D.25}$$

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