Four Essays on Imperfect Competition: Strategic Information Acquisition, Product Choice under Government Regulation, and Forward Trading

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To Chrissi and Nico

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Chapter 1

Introduction

Industrial organization (IO) is as old as economics. Many of the best-remembered parts of the *Wealth of Nations* concern the central subject matter of the field: the effects of monopoly and collusion, the determinants of size and structure of firms, and so forth.[1] Historically, the field has focused on how markets depart from idealized conditions of perfect competition, e.g., because of scale economies, transaction costs, strategic behavior, or other factors.

Modern research in industrial organization has been largely a response to two challenges that the field faced into the 1970s. The first was a lack of compelling theoretical models for studying imperfectly competitive markets. The second challenge was a lack of good data and of convincing empirical strategies for evaluating hypotheses about competition or industry structure. The first problem was largely reversed by the game theory revolution of the 1980s which permitted much sharper modeling and analysis of problems such as product differentiation, network effects, barriers to entry, pricing strategies, and the effect of asymmetric information in product markets. The reversal of the second problem was so pronounced that nowadays most of the IO articles that are published in leading journals are empirical studies.

A third perhaps not less serious challenge to industrial organization was put forward by Peltzman (1991) in his review of the Handbook of Industrial Organization.[2] The narrow interpretation of his argument is that, back then, IO was plagued by a wide gap between theoretical and empirical work. Yet, a more general point can be deduced from his statement: IO researchers were pretty reluctant to look at other fields, e.g. finance, in order to learn from

[1] See Smith (1776), e.g., "Of the Natural and Market Price of Commodities" (chapter I.7) and "Of the wages of labor" (chapter I.8).

[2] Schmalensee and Willig, eds (1989)

their experience. Though, even in the early 1990s it was not hard to assess how empirical – e.g. Mehra and Prescott (1985) – and theoretical work can stimulate each other.

My view is that, with the notable exception of "New Behavioral IO", the general point of the third challenge has not been tackled in a satisfying way due to the following reasons:

(i) Theoretical models in IO are still predominantly deterministic. However, in reality, firms usually face a considerable amount of uncertainty with respect to demand and/or cost. This questions, to some degree, how instructive these models are about real-world phenomena. On top of this, deterministic theories are probably not well suited for guiding empirical work.

(ii) Still, it takes a considerably long time until innovative and relevant approaches from other fields find their way to IO.

(iii) Especially in the discrete choice theory of product differentiation most of the theoretical work is only to a minor degree applicable for the analysis of non-standard industries.

This dissertation means to contribute to tackling the challenge put forward by Peltzman (1991). It comprises four self-contained chapters that add to different research areas in the field of imperfect competition, and each chapter addresses at least one of the perceived "shortcomings" mentioned above. The second and the third chapter analyze imperfect competition in a stochastic framework. The second and the fifth chapter employ innovative and relevant approaches developed in other fields, i.e. the theory of rational inattention and forward trading, respectively. Finally, the fourth chapter extends, among other things, an important theoretical contribution to the discrete choice theory of product differentiation, i.e. the paper by Caplin and Nalebuff (1991a), in order to analyze the Chilean pension fund market.

In the second chapter – a joint study with Markus Reisinger – we focus on the question whether oligopolistic firms choose to synchronize or to stagger planning decisions. Since the seminal papers on dynamic duopoly by Maskin and Tirole (1987; 1988a;b) this question attracted considerable attention by researchers. The main body of this literature analyzed the question by considering how commitment power of firms shapes this decision, see e.g., the papers by Maskin and Tirole, De Fraja (1993) or Lau (2001). In these papers there is a physical friction that hinders firms from adjusting their plans each period.

However, the approach relying on commitment power is not satisfactory in three important aspects: First, firms usually face a considerable amount of uncertainty with respect to demand and/or costs, which heavily influences their adjustment decision. This uncertainty is neglected in the above literature. Second, firms can often change their prices almost costlessly, and so the commitment assumption is somewhat artificial in many markets. Third, the length between two consecutive adjustments is prespecified in the above models to either one or two periods. However, firms are usually more flexible in choosing this length.

We approach this question by drawing on a recent development in macroeconomics, i.e., the theory of rational inattention put forward by Reis (2006b): in a stochastic environment a firm can choose the dates at which it updates its information set to recalculate its optimal price. However, it has to incur a cost to acquire, absorb and process this information, i.e., it is costly for a firm to replan its optimal action. Overall, empirical evidence seems to suggest that the costly information processing framework fits many industries better than the commitment model. For example, Zbaracki et al. (2004), using data from a large U.S. manufacturing firm, find that the managerial planning costs of price adjustments are often much larger than the physical costs of price adjustments. Thus, it is often much harder to determine the optimal price in an uncertain environment than to change the price.

Reis (2006b) analyzes the optimal length of a monopolist's inattentiveness period. The paper that is closest to ours is Hellwig and Veldkamp (2009). Using the costly information acquisition and processing framework of Reis (2006b), they derive the conditions under which non-strategic firms want to synchronize or stagger their planning decisions. Our contribution to this strand of the literature is that we approach the question of synchronization vs. non-synchronization of firms' planning decisions in a setting with strategic firms.

In particular, we consider an infinite horizon continuous time model of competition between two firms who produce differentiated goods and – at each instant – face stochastic demands and costs. Since planning is costly, each firm chooses to plan only at some points in time and stays (rationally) inattentive in the meantime. During the inattentiveness period uncertainty builds up in the system. Thus, when choosing the sequence of planning dates each firm balances the costs of planning and the gains from having a re-optimized plan. At a planning date a firm observes the history of the game and the current shock realizations. Given this information it chooses a price path up to its next planning date. This implies that at each instant firms play a one-shot price competition game with potentially imperfect and different information. As a consequence, there is no commitment possibility. Instead, what matters for synchronization or non-synchronization of planning decisions is how the decision of one firm to plan affects the other firm's gain of planning.

We derive the following results. First, non-synchronized planning equilibria can only exist if products are strategic substitutes while synchronized planning equilibria can only exist if products are strategic complements. This result is in sharp contrast to the predictions obtained by models that assume commitment power of firms. In these models strategic complementarity leads to non-synchronization while strategic substitutability tends to lead to synchronization. Interestingly, we show that for both classes of equilibria, alternating and synchronous planning, there exist multiple inattentiveness lengths that can be supported as an equilibrium. For synchronous planning this result is already known from Hellwig and Veldkamp (2009), who analyze a model with a continuum of firms. However, the result is new for the class of alternating equilibria. In addition, we characterize under which conditions an alternating planning equilibrium arises endogenously. In this respect we go beyond most papers in the previous literature which impose that firms have already reached an alternating move structure and focus on the stationary equilibrium.

In the third chapter – a joint study with Markus Reisinger – we analyze the question whether oligopolistic firms choose to compete in quantities or in prices under uncertainty. Singh and Vives (1984) and Cheng (1985) show that quantity-setting is a dominant action for both firms in a deterministic two-stage game in which duopolists first choose their strategy variable and compete afterwards. However, as mentioned above, a deterministic model is not fully appropriate because firms often face uncertainty at the time the strategy variable has to be chosen.

We address this question using the same game structure as Singh and Vives (1984) and Cheng (1985), namely firms first select their strategy variable independently of each other and then compete. In contrast to these authors, we consider stochastic demand. More specifically, we consider a linear demand system where a shock affects the slope of the demand curves. We find that in equilibrium the relative magnitude of demand uncertainty and the degree of substitutability determines firms' variable choice. Firms commit to prices if demand uncertainty is high compared to the degree of substitutability and to quantities if the reverse holds true. The reason for this result is that demand uncertainty and the degree of substitutability have countervailing effects on variable choice: Prices adapt better to uncertainty while quantities induce softer competition. If no effect dominates, firms choose different strategy variables in equilibrium.

The only paper that analyzes the choice of prices versus quantities under uncertainty in an oligopolistic setting is Klemperer and Meyer (1986). They consider a one-stage duopoly game in which a firm chooses the strategy variable and its magnitude at the same time. Klemperer and Meyer (1986) already identify the uncertainty-based benefit of price-setting if market size is uncertain. Yet, as their game has a simultaneous structure, each firm acts as a monopolist given its expected residual demand curve. Thus, firms choose to set a price because it adapts optimally to demand uncertainty. The competitive advantage of quantity-setting, however, is not present. Even if uncertainty is small, firms select prices although they induce harsher competition. By contrast, in our setting the relative *magnitude* of the strategic benefit of quantities and the uncertainty-based benefit of prices is crucial for variable choice. Thus, it is the amount of uncertainty and not only its mere presence that matters.

We extend our analysis by introducing a shock to the intercept that might be correlated with the shock to the slope. This case is arguably more relevant in reality because uncertainty usually affects both market size and reservation price distribution. We find that now it is the covariance that in addition to the variance of the slope shock drives firms' choices of their strategy variables. Firms commit to a price rather than to a quantity if the covariance is sufficiently large compared to the degree of substitutability, and vice versa. The "hybrid" outcome in which firms choose different strategy variables in equilibrium still arises but only for sufficiently high degrees of substitutability.

In the fourth chapter – a joint study with Elena Krasnokutskaya and Petra Todd – we investigate how the Chilean government's regulation of the pension fund industry affects its operation. Chile has been at the forefront of pension reforms, having switched to a private retirement accounts system twenty-eight years ago. Numerous other Latin American countries followed suit, building on the Chilean model. Proposed plans for pension reform in the US and in Europe have many features in common with Chile's current system. They outline a system under which all workers are mandated to contribute a pre-specified part of their income to their pension account, which is managed by money manager(s) (either a government owned company or a competitive industry of money managers). The government serves as a last resort guarantor, supplementing pension income if accumulations are insufficient upon retirement (below pre-specified minimal level), either because of low income or unfavorable investment returns. All these features are present in the Chilean pension fund system, called the *Administradoras de Fondos de Pensiones* (AFPs). Workers are mandated to contribute 10% of their earnings to a retirement account, and those who contribute for at least 20 years receive a minimum pension benefit guarantee from the government.

Chile has a competitive pension fund industry overseeing pension investment that is subject to government regulations designed to limit fees, to facilitate switching among funds by promoting transparency of fees and pension fund returns and to limit the riskiness of the investment products offered. A particularly important regulation is a return requirement which makes money managers responsible with their capital for delivering a rate of return that is no less than two percentage points below the industry average return. This regulation essentially shifts some of the risk of investment from consumers to the pension fund firm. Another important regulation restricts firms to charge fees only on new contributions and not on existing balances.

We study the operation of the Chilean privatized pension fund industry under the existing regulation and under alternative regulatory schemes. To this end, we develop an equilibrium demand and supply model of consumers' decisions about whether to contribute to their pension accounts and where to invest their funds, and pension fund firms' decisions with regard to pricing and portfolio characteristics. Because our set-up differs significantly from the types of frameworks previously considered in the literature, we prove existence of a pricing and location equilibrium for our model, extending results of Caplin and Nalebuff (1991b) and Anderson et al. (1997).

We estimate the model using data from multiple sources: (i) longitudinal survey data on a random sample of working age Chileans gathered in 2002 and 2004, (ii) administrative data on contributions and fund choices from 1981-2004 that were obtained from the pension fund regulatory agency and have been linked to the household survey data, (iii) market data on the performance of the various funds, and (iv) a data series on the fees charged by funds as well as (v) accounting cost data. The survey data come from the 2002 *Historia Laboral y*

Seguridad Social (HLLS) survey and the 2004 *Enquesta Proteccion Sociale (EPS)* follow up survey. The data contain demographic and labor market information on 17,246 individuals of age 18 or older, including information on household demographics, work history, pension plan participation, and savings, as well as information on health, assets, disability status and utilization of medical services.

The estimated model is used to investigate how alternative regulatory schemes affect firm pricing behavior, consumer choices, consumer balances and government obligations. We find that in the absence of any pension fund return regulation, the industry offers very risky products, which induces volatile consumer balances and government obligations. Another finding is that the government's regulation that requires firms to deliver returns close to the industry average has the intended consequence of limiting product diversity but it is not very effective in reducing the riskiness of the industry's products. Moreover, it discourages individuals from participating in the pension system. An alternative form of regulation that is found to be more effective places explicit restrictions on pension fund firms' portfolio riskiness.

In the fifth chapter I address the question whether forward trading has anti- or pro-competitive effects in an infinitely-repeated oligopoly. Static oligopoly models yield contrasting results with respect to the question whether forward trading makes markets more or less competitive. Ultimately, the diverging findings are rooted in the mode of competition in the (product) spot market: If firms compete in strategic substitutes (complements), forward trading makes markets more (less) competitive.

Liski and Montero (2006) add to the debate by putting this question in the context of an infinitely repeated oligopoly with perfect information in which forward and spot stages alternate. They show that forward trading considerably expands the range of discount factors for which collusion can be sustained in equilibrium. Their result does not depend on the mode of competition in the spot market.

I show that the result of Liski and Montero (2006) is not entirely correct for the case in which firms compete in quantities in the spot market. More specifically, I find that the presence of forward markets reduces the firms' ability to collude relative to the pure-spot infinitely repeated Cournot duopoly if the cartel's forward position is sufficiently long.

Moreover, the range of discount factors for which collusion can be sustained in equilibrium is maximal for moderate long position. In this respect, the corrected result is consistent with empirical evidence pointed out by Mahenc and Salanié (2004) that in the history of alleged manipulation of commodity markets unreasonably high prices often resulted from moderate long positions held by a cartel of producers.

Chapter 2

Dynamic Duopoly with Inattentive Firms*

2.1 Introduction

It is a heavily discussed question in the economics literature whether oligopolistic firms choose to synchronize the adjustment of their prices (or quantities) or if they adjust them at different points in time. Since the seminal papers on dynamic duopoly by Maskin and Tirole (1987; 1988a;b) this question attracted considerable attention by researchers. The main body of this literature analyzed the question by considering how commitment power of firms shapes this decision, see e.g., the papers by Maskin and Tirole, De Fraja (1993) or Lau (2001). In these papers there is a physical friction that hinders firms from adjusting their plans each period. Thus, firms are either exogenously equipped with commitment power or can decide to be committed for some time period. The mechanism that then drives the results concerning synchronization versus non-synchronization is rooted in the mode of strategic interaction, that is, if firms' strategy variables are strategic complements or substitutes.

However, the approach relying on commitment power is not satisfactory in three important aspects: First, firms usually face a considerable amount of uncertainty with respect to demand and/or costs, which heavily influences their adjustment decision. This uncertainty is neglected in the above literature. Second, firms can often change their prices almost costlessly, and so the commitment assumption is somewhat artificial in many markets. Third, the length between two consecutive adjustments is prespecified in the above models to either one or two periods. However, firms are usually more flexible in choosing this length.

*This chapter is joint work with Markus Reisinger.

In this chapter, we approach this question by drawing on recent developments in the theory of rational inattention by Reis (2006b) and Hellwig and Veldkamp (2009). In particular, we consider a model in which firms' demand and costs, at each point in time, are hit by a shock. As in Reis (2006b) and Hellwig and Veldkamp (2009) a firm can choose to become informed about this shock and recalculate its optimal price. However, it has to incur a cost to acquire, absorb and process this information, i.e., it is costly for a firm to replan its optimal action.[1] If a firm chooses to plan, it sets a price path until its next planning date. The prices during this path are fully flexible, that is, they can differ at different points in time. This implies that firms have no commitment power.[2] Finally, a firm is free to choose the optimal length for which it stays inattentive, i.e., the time period in which it does not plan. Overall, this framework seems to fit many industries in a better way than the commitment model since it is often much harder to determine the optimal price in an uncertain environment than to change the price. For example, Zbaracki et al. (2004), using data from a large U.S. manufacturing firm, find that the managerial planning costs of price adjustments are often much larger than the physical costs of price adjustments.

More specifically, we consider an infinite horizon continuous time model of competition between two firms who produce differentiated goods and – at each instant – face stochastic demands and costs. At the beginning of the game both firms are informed about the state of demand and costs and independently from each other choose an infinite sequence of dates at which they choose to plan, where planning means that they acquire information about demand and cost and re-optimize their prices. Since planning is costly, each firm chooses to plan only at some points in time and stays (rationally) inattentive in the meantime. During the inattentiveness period uncertainty builds up in the system. Thus, when choosing the sequence of planning dates each firm balances the costs of planning and the gains from having a re-optimized plan. At a planning date a firm observes the history of the game and the current shock realizations. Given this information it chooses a price path up to its next planning date. This implies that at each instant firms play a one-shot price competition game with potentially imperfect and different information. As a consequence, there is no commitment possibility. Instead, what matters for synchronization or non-synchronization

[1] This is a realistic assumption since, as e.g., Radner (1992) points out, absorbing and processing the relevant information for decision making is an important goal of many managerial occupations.

[2] This is line with recent empirical work. For example, the work by Bils and Klenow (2004) seems to contradict the finding that prices are adjusted only infrequently.

of planning decisions is how the decision of one firm to plan affects the other firm's benefit from planning.

In this set-up we derive the following results. First, non-synchronized planning equilibria can only exist if prices are strategic substitutes while synchronized planning equilibria can only exist if prices are strategic complements. This result is in sharp contrasts to the predictions obtained by models that suppose commitment power of firms. As shown by Maskin and Tirole (1987; 1988b) and Lau (2001), in these models strategic complementarity leads to non-synchronization while strategic substitutability tends to lead to synchronization. The reason is that in the case in which products are strategic complements and decisions are synchronized each firm has an incentive to undercut the price of its rival which leads to low prices and profits. By contrast, in a sequential game the overall level of prices is higher. Thus, firms choose non-synchronization in equilibrium. The reverse argument holds if products are strategic substitutes.

The intuition behind our result is rooted in two different effects, the strategic effect and the externality effect. The strategic effect determines how a firm's incentive to plan at some instant changes if the rival plans at this instant. If firm i stays inattentive, its price is inaccurate compared to the full information price due to the variance of demand and costs. Now suppose that the rival firm j plans. If the demand realization is high, firm j sets a higher price than in case of non-planning. If products are strategic complements, firm i's optimal price is then larger than in the case in which firm j did not plan. By the same argument, if the demand realization is low, firm i's optimal price is lower than without planning of firm j. As a consequence, firm i's price when being inattentive becomes more inaccurate, which increases the incentive for firm i to plan. By the reverse argument, if products are strategic substitutes, planning of firm j lowers the incentive for firm i to plan. As a consequence, there is a tendency to synchronize planning decisions under strategic complementarity and to non-synchronize these decisions under strategic substitutability.

In addition, there is an effect that occurs because by acquiring information firm j exerts an externality on its rival. This is due to the fact that an informed firm's price reacts to the shocks and is thus a random variable. Now, by planning firm i induces firm j to change its price and therewith firm i can influence this externality. We refer to this effect as the "externality effect". The absolute value of this externality is, in expectation, the higher the larger is the difference between firm i's last planning date and the next planning date of firm j. Thus, firm i can reduce the extent of the expected externality by moving closer to firm j's planning date. This is the case because less time has elapsed since firm i's last planning date, which reduces the uncertainty about firm i's demand and cost. Now suppose that the externality is positive and products are strategic complements. In this case firm i prefers to set its planning dates relatively far away from the ones of firm j to increase the expected externality, thereby providing a tendency towards alternating planning. By a similar argument, if the externality is negative and products are strategic substitutes, there is a tendency towards synchronized planning decisions. In sum, we find that in our framework both effects in combination exclude any non-synchronous planning equilibria in case of strategic complementarity and any synchronous planning equilibria in case of strategic substitutability.

Interestingly, we show that for both classes of equilibria, alternating and synchronous planning, there exist multiple inattentiveness lengths that can be supported as an equilibrium. For synchronous planning this result is natural – and known from Hellwig and Veldkamp (2009) – since if both firms plan at the same instant, the objective function of each firm involves a discontinuity at this instant thereby giving rise to multiple equilibria. However, the result is new for the class of alternating equilibria. In this type of equilibrium firm j remains inattentive at a planning date of firm i. Thus, one may expect that firm i's incentive to exceed or shorten its inattentiveness period is the same. However, we show that this is not the case. The reason is that firm j expects firm i to set different planning dates in the future in the case in which firm i exceeded its inattentiveness lengths than in the case in which it shortened its inattentiveness length. Due to this, firm j will in the future react differently to the different changes of the inattentiveness length of firm i, which changes the objective function of firm i in a discontinuous way. As a consequence, multiple equilibria emerge even in the class of alternating equilibria.

We also characterize how the inattentiveness length changes with the degree of strategic complementarity. Here we find that the period lengths become shorter, the larger the degree of strategic complementarity. This is the case because if firm j chooses to plan it adjusts its price in the direction of the shock realizations, i.e., if demand or costs are high, firm j raises its price and vice versa. Now, by the argument explained above, if the degree of strategic complementarity rises, firm i's price when being inattentive becomes more inaccurate, and therefore, it has an incentive to plan earlier. As a consequence, the equilibrium inattentiveness lengths become shorter.

In our model we also characterize under which conditions an alternating planning equilibrium arises endogenously without imposing this structure of planning dates. To do so we allow one firm to set its first inattentiveness interval at a different length than the others which enables firm to reach a non-synchronized equilibrium. Thus, in this respect we go beyond most papers in the previous literature which impose that firms have already reached an alternating move structure and focus on the stationary equilibrium.

As mentioned, the previous literature on dynamic duopoly that analyzes if firms synchronize their decisions is mainly concerned how commitment power affects this decision. The seminal papers in this literature are the ones by Maskin and Tirole (1987; 1988a;b). The first two are on quantity competition (strategic substitutability) while the last one is on price competition (strategic complementarity). In the main parts of these papers Maskin and Tirole suppose that firms are committed to a particular price or quantity for two periods, and that the firms adjust their prices or quantities in a alternating manner. Maskin and Tirole then endogenize the timing by analyzing if firms indeed choose to adjust their variables in an alternating manner but keep the assumption that firms are committed for two periods. The authors find that this is indeed the case for price competition but not for standard quantity competition. The reason is that if products are strategic complements, the market is less competitive if one of the firms acts as the Stackelberg leader and the other one as the follower compared to the simultaneous move case. However, the reverse holds true for strategic substitutes.[3] Lau (2001) extends the price competition model by first allowing for differentiated goods and, second, by giving each firm the choice to set its commitment length for either one or two periods. He shows that even in this case firms prefer to move alternatingly and to be committed for two periods. By allowing for a commitment length of one period, Lau (2001) shows how firms can endogenously reach this alternating structure, i.e., one firm chooses a one-period commitment at the beginning and then sets a two-period commitment forever while the second firm chooses a two-period commitment from the beginning. Although

[3]De Fraja (1993) and Lau (1996) reach a similar conclusion in a model of wage setting in which the wages are fixed for two periods. Both papers find that the sum of the wage setters' profits is larger under staggered than under synchronized wage setting.

the mechanism in these papers are very different to the ones we consider, the goal of the analyses – i.e., finding conditions for synchronization and non-synchronization – is similar. As mentioned, our framework yields opposing results compared to the literature relying on commitment.[4]

There are other papers that also address the question of synchronization versus non-synchronization but consider a continuum of players. These papers identify different aspects that can be of relevance for this decision. For example, Bhaskar (2002) develops a model with different industries each comprised of a continuum of firms that act non-strategically. There is aggregate strategic complementarity across industries but the degree of strategic complementarity within an industry is larger. Firms are committed to their price levels for two periods. Bhaskar (2002) shows that there is a (strict) Nash equilibrium that involves staggered price setting in which some firms adjust their prices in odd and others in even periods. This is the case since each firm prefers to set its price together with its industry rivals although the aggregate number of firms that adjusts its price in this periods may be relatively small. In a different framework Ball and Romer (1990) consider a model with aggregate and firm-specific shocks in which the firm-specific shocks arrive at different times for different firms, and firms can adjust their prices every two periods. They show that due to the difference in shock arrival, staggered price setting is a Nash equilibrium in which each firm adjusts its price at the time its demand is hit by a shock.[5]

The way we model rational inattentiveness of firms was developed by Reis (2006b).[6] He analyzes the optimal length of a monopolist's inattentiveness period and derives an approximate solution in a general setting.[7] He also tests the model's prediction by using US inflation data and finds that his recursive state-dependent approach fits the data better than previous state-contingent models do. The paper that is closest to ours is the one by Hellwig and Veldkamp (2009). They answer the question under which condition firms want to acquire the same information as their rivals. To do so they analyze a model with a continuum of firms in which costly information acquisition and processing is modeled as in Reis (2006b). The objective of each firm is to set its price close to a target price that consists of a weighted

[4]For a general treatment of a dynamic duopoly with adjustment costs that provide an exogenous commitment, see Jun and Vives (2004).

[5]For a paper with a similar structure, see Ball and Cecchetti (1988).

[6]For a model that focusses on rational inattentiveness of consumers, see Reis (2006a).

[7]For an extension of this analysis, see Jinnai (2007).

average of the shock realization and the average price of competitors. The higher is the degree of strategic complementarity between firms, the larger is the weight on the average price in this target price. In this framework Hellwig and Veldkamp (2009) show that there is a unique staggered planning equilibrium for any degree of strategic complementarity while there are multiple synchronized planning equilibria that only exist if prices are strategic complements. In contrast to Hellwig and Veldkamp (2009), we consider a duopoly model instead of a competitive monopolists model with a continuum of firms. In addition, the objective function of firms in our model is substantially different since we consider a standard duopoly model without a target price. Therefore, the strategic incentives in our model are different, e.g., the externality effect described above cannot be present in their model due to the assumption of a continuum of firms. We also obtain different results concerning the alternating planning equilibrium, i.e., we find that there are multiple alternating planning equilibria while in their model it is unique, and show that the comparative static result concerning the degree of strategic complementarity are opposite to theirs. We will explain the differences and similarities to work of Hellwig and Veldkamp (2009) in detail repeatedly throughout this chapter.

This chapter proceeds as follows. The next section provides an easily accessible two-stage information-choice-then-pricing game that sheds light on how the nature of product market competition and uncertainty about the shocks determine whether the duopolists' decisions to acquire information are strategic substitutes or complements. Section 2.3 presents the dynamic model in which firms first choose their planning dates and then compete on the product market, and derives the conditions for existence of an alternating and a synchronous planning equilibrium, respectively. Section 2.4 concludes.

2.2 Two-Stage Information-Choice-then-Pricing Game

There are two firms denoted by 1 and 2 that produce differentiated goods and compete in prices. Each firm i faces the demand function

$$q_i = \alpha + \theta - p_i + \gamma p_j, \quad i \neq j, \ i, j = 1, 2,$$

where the intercept α and the degree of strategic interaction γ are known constants, with $\alpha > 0$ and $-1 \leq \gamma \leq 1$. If $\gamma > 0$, firms produce substitutable goods and strategy variables are strategic complements, if $\gamma < 0$, firms produce complementary goods and strategy variables are strategic substitutes, and if $\gamma = 0$, firms' demands are unrelated. Demand is stochastic, which is captured by the random variable θ that represents a common shock to each firm's demand. Without loss of generality we assume that $E[\theta] = 0$ and $Var[\theta] = \sigma_\theta^2 > 0$. Each firm faces marginal production costs that consist of a deterministic part, c, and a stochastic part, ξ, which are also common to both firms.[8] We assume that $E[\xi] = 0$ and $Var[\xi] = \sigma_\xi^2 > 0$. Thus, the production costs of firm i are given by of $q_i(c+\xi)$. The two shocks can be correlated with correlation coefficient ρ. This implies that $E[\theta\xi] = \rho\sigma_\theta\sigma_\xi$. Therefore, the profit function of firm i for the case of full information, that is, when firm i knows the realization of both shocks, is given by

$$\Pi_i = (p_i - c - \xi)\left(\alpha + \theta - p_i + \gamma p_j\right). \tag{2.1}$$

Before competing in the product market each firm chooses whether to acquire information about the realizations of the shocks. The decision to acquire information comes at a cost of $K > 0$ for each firm.[9]

The timing of the game is as follows: In the first stage, both firms decide independently of each other whether to acquire information or not, and these decisions become publicly observable. In the second stage, firms choose their optimal prices conditional on the information choices in the first stage.

This timing and information structure is suitable to elucidate the effects that are at work in the dynamic game in an accessible way. In order to focus on the interplay of firms' information acquisition decisions, we assume in this two-stage framework that a firm knows the information status of its rival. A justification for the assumption in the two-stage game is that planning is likely to be a process that may need consultation of outside agencies and where, in large firms, several managerial layers are involved. Thus, firms may not be able

[8] Arguably, the situation of common demand and cost shocks is a relatively realistic scenario since both firms are in the same market and are likely to procure the input from the same supply industry.

[9] For a model that considers costless information acquisition but in which firms can share their acquired information, see e.g., Raith (1996).

to keep this decision secret.[10] It is important to note that this assumption is not needed in the dynamic framework in which a firm (more realistically) observes the history of the game when acquiring information but not the simultaneous planning choice of its rival.

We solve the game by backward induction and look for perfect Bayesian equilibria. In the following we denote by D_i the choice of firm i to acquire information in the first stage: $D_i = 1$ if firm i chooses to become informed and $D_i = 0$ otherwise. In the second stage three different scenarios can occur conditional on D_i and D_j, $i \neq j$, $i, j = 1, 2$: both firms are uninformed, $(D_i = 0, D_j = 0)$, both are informed $(D_i = 1, D_j = 1)$, and an asymmetric situation where firm i is informed while firm j is not $(D_i = 1, D_j = 0)$. We consider these scenarios in turn. In the following we denote by $p_i(D_i, D_j)$ and $\Pi_i(D_i, D_j)$, the equilibrium price and the equilibrium profit of firm i for given information choices.

Case 1: $D_i = 0, D_j = 0$

First, consider the case in which no firm is informed. Since $E[\theta] = E[\xi] = 0$ and $E[\theta\xi] = \rho\sigma_\theta\sigma_\xi$, from (2.1) we have that the expected profit of firm i can be written as

$$E[\Pi_i] = (p_i - c)(\alpha - p_i + \gamma p_j) - \rho\sigma_\theta\sigma_\xi.$$

Deriving the Nash equilibrium yields equilibrium prices of

$$p_i(0,0) = p_j(0,0) = p(0,0) = \frac{\alpha + c}{2 - \gamma}.$$

Inserting these prices into the profit function, we obtain the expected profit in case that both firms abstain from acquiring information in the first stage:

$$\Pi_i(0,0) = \Pi_j(0,0) = \Pi(0,0) = \frac{(\alpha - c(1-\gamma))^2}{(2-\gamma)^2} - \rho\sigma_\theta\sigma_\xi.$$

Case 2: $D_i = 1, D_j = 1$

Second, we look at the case in which both firms choose to acquire information. In this case, the realizations of θ and ξ are observed by both firms. Therefore, in the second stage each

[10] This two-stage game structure is also standard in several oligopoly models. A prominent example is Singh and Vives (1984) in which firms first publicly announce their strategy variable (price or quantity) and then compete in the chosen strategy variables.

firm maximizes

$$\Pi_i = (p_i - c - \xi)(\alpha + \theta - p_i + \gamma p_j).$$

Deriving the Nash equilibrium yields equilibrium prices of

$$p_i(1,1) = p_j(1,1) = p(1,1) = \frac{\alpha + c}{2-\gamma} + \frac{\theta + \xi}{2-\gamma}. \tag{2.2}$$

Substituting these prices into the profit function, simplifying and taking expectations from the perspective of stage 1 we obtain the expected profit in the case in which both firms choose to acquire information:

$$\Pi_i(1,1) = \Pi_j(1,1) = \Pi(1,1) = \frac{(\alpha - c(1-\gamma))^2}{(2-\gamma)^2} + \frac{\sigma_\theta^2 + \sigma_\xi^2(1-\gamma)^2 - \rho\sigma_\theta\sigma_\xi(2-\gamma)}{(2-\gamma)^2}.$$

Case 3: $D_1 = 1, D_2 = 0$

Finally, we consider the asymmetric case in which one firm is informed while the other one is not. Let us denote by firm 1 the firm that is informed and by firm 2 the one that abstained from acquiring information. In the second stage the profit function of firm 1 is then given by

$$\Pi_1 = (p_1 - c - \xi)(\alpha + \theta - p_1 + \gamma p_2)$$

while the expected profit of firm 2 is

$$E[\Pi_2] = E[(p_2 - c - \xi)(\alpha + \theta - p_2 + \gamma p_1)].$$

The reaction functions of firm 1 and firm 2 can then be written as

$$p_1(p_2) = \frac{\alpha + c + \theta + \xi + \gamma p_2}{2}$$

and

$$p_2(E[p_1]) = \frac{\alpha + c + E[\theta] + E[\xi] + \gamma E[p_1]}{2},$$

respectively. Since firm 1 is informed, it conditions its price on the shock realizations, while firm 2 cannot do so due to its lack of information. As a consequence, firm 2 must form expectations about the shock realizations and about the price of firm 1. This is due to the fact that this price is a random variable for firm 2 because it depends on the shock realizations which firm 2 cannot observe. Solving for the Nash equilibrium and using that $E[\theta] = E[\xi] = 0$ we obtain

$$p_1(1,0) = \frac{\alpha + c}{2 - \gamma} + \frac{\theta + \xi}{2} \tag{2.3}$$

and

$$p_2(0,1) = \frac{\alpha + c}{2 - \gamma}.$$

Thus, from the perspective of firm 2 the expected equilibrium price of firm 1 is $E[p_1] = (\alpha + c)/(2 - \gamma)$.

We can now determine the profits of both firms. Inserting the equilibrium prices into the profit function of firm 1 and taking expectations we obtain

$$\Pi_1(1,0) = \frac{(\alpha - c(1-\gamma))^2}{(2-\gamma)^2} + \frac{\sigma_\theta^2 + \sigma_\xi^2 - 2\rho\sigma_\theta\sigma_\xi}{4}.$$

Since firm 2 is the uninformed firm its profit can be written as

$$E[\Pi_2] = (p_2 - c)(\alpha - p_2 + \gamma E[p_1]) - E[\xi(\theta + \gamma p_1)]. \tag{2.4}$$

It is evident from the last term on the right-hand side, $E[\xi(\theta + \gamma p_1)]$, that the covariance between ξ and p_1 affects firm 2's expected profits. This is due to the fact that p_1 depends on the realizations of θ and ξ and, therefore, $E[\xi p_1]$ depends on the variance of ξ and the covariance between θ and ξ. Inserting the equilibrium prices into (2.4) yields the expected profit of firm 2:

$$\Pi_2(0,1) = \frac{(\alpha - c(1-\gamma))^2}{(2-\gamma)^2} + \frac{\gamma(\sigma_\theta\sigma_\xi\rho + \sigma_\xi^2) + 2\rho\sigma_\theta\sigma_\xi}{2}. \tag{2.5}$$

We are now in a position to proceed to the information acquisition stage.

Information acquisition stage

We set out by determining the benefit from acquiring information for a firm conditional on the information acquisition decision of its rival. Let $\Delta(1) := \Pi_i(1,1) - \Pi_i(0,1)$ and

$\Delta(0) := \Pi_i(1,0) - \Pi_i(0,0)$. Suppose that firm j is informed. Then firm i's benefit from acquiring information is

$$\Delta(1) = \frac{2(\sigma_\theta^2 + \rho\sigma_\theta\sigma_\xi + \sigma_\xi^2) - \gamma^2(2-\gamma)(\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2)}{2(2-\gamma)^2}. \tag{2.6}$$

If instead firm j is not informed, firm i's benefit is given by

$$\Delta(0) = \frac{\sigma_\theta^2 + \sigma_\xi^2 + 2\rho\sigma_\theta\sigma_\xi}{4} > 0. \tag{2.7}$$

Taking the difference between (2.6) and (2.7), we see that the benefit of information acquisition is larger in the case in which the other firm is also informed if and only if

$$\frac{\gamma\left(4(\sigma_\theta^2 + 2\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2) - \gamma(\sigma_\theta^2 + 6\rho\sigma_\theta\sigma_\xi + 5\sigma_\xi^2) + 2\gamma^2(\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2)\right)}{4(2-\gamma)^2} > 0. \tag{2.8}$$

Rearranging (2.8) yields the following result:

Proposition 1 *Information acquisition decisions are strategic complements, i.e.,* $\Delta(1) - \Delta(0) > 0$, *if and only if*

$$\gamma > 0 \quad \text{and} \quad \rho > \max\left[-\frac{\sigma_\theta^2(4-\gamma) + \sigma_\xi^2(4 - 5\gamma + 2\gamma^2)}{2\sigma_\theta\sigma_\xi(4 - 3\gamma + \gamma^2)}, -1\right] \tag{2.9}$$

or

$$\gamma < 0 \quad \text{and} \quad \rho < \max\left[-\frac{\sigma_\theta^2(4-\gamma) + \sigma_\xi^2(4 - 5\gamma + 2\gamma^2)}{2\sigma_\theta\sigma_\xi(4 - 3\gamma + \gamma^2)}, -1\right]. \tag{2.10}$$

Information acquisition decisions are strategic substitutes, i.e., $\Delta(1) - \Delta(0) < 0$, *if and only if neither* (2.9) *nor* (2.10) *hold.*

Proposition 1 implies that, if (2.9) or (2.10) hold, there is strategic complementarity in information acquisition decisions. This implies that there exists a range of acquisition costs K, i.e.,

$$\frac{\sigma_\theta^2 + \sigma_\xi^2 - 2\rho\sigma_\theta\sigma_\xi}{4} < K < \frac{2\sigma_\theta^2 + \sigma_\xi^2(2 - 2\gamma^2 + \gamma^3) - \rho\sigma_\theta\sigma_\xi(4 - 2\gamma^2 + \gamma^3)}{2(2-\gamma)^2}, \tag{2.11}$$

such that either both firms are informed or no firm is informed.[11] If none of the two conditions hold, then information acquisition decisions are strategic substitutes. In this case there is a range of K, i.e.,

$$\frac{2\sigma_\theta^2 + \sigma_\xi^2(2 - 2\gamma^2 + \gamma^3) - \rho\sigma_\theta\sigma_\xi(4 - 2\gamma^2 + \gamma^3)}{2(2-\gamma)^2} < K < \frac{\sigma_\theta^2 + \sigma_\xi^2 - 2\rho\sigma_\theta\sigma_\xi}{4}, \tag{2.12}$$

in which an asymmetric equilibrium emerges, that is, one firm acquires information while the other one does not.

Whether an asymmetric equilibrium emerges or not depends on how a firm's decision to acquire information changes the other firm's incentive to become informed. There are two effects that change this incentive.

We first turn to the effect to which we will refer as the "strategic effect" in the remainder. Suppose that firm j has acquired information. Firm j adjusts its price in direction of the shock realizations, that is, it sets a high price if the shock to the demand or to the cost is high and a low one if the reverse holds true. This can be seen from the equilibrium prices of an informed firm, which are given by

$$p(1,1) = \frac{\alpha + c}{2 - \gamma} + \frac{\theta + \xi}{2 - \gamma} \quad \text{and} \quad p(1,0) = \frac{\alpha + c}{2 - \gamma} + \frac{\theta + \xi}{2}, \tag{2.13}$$

where $p(1,1)$ is the price for the case in which the other firm is informed as well while $p(1,0)$ is the price for the case in which the other firm is not informed. Now suppose for the sake of exposition that the shock to the cost is relatively small. Thus, firm j's price follows the demand shock. Since the shock is a common shock, firm i when being informed would also set a high price if θ is large and a low one if θ is small. Now, if products are strategic complements, i.e., $\gamma > 0$, firm i's optimal price is higher if θ is large compared to the case without strategic interaction because firm j sets a high price if θ is large. Similarly, if θ is low, firm i's optimal price with strategic interaction is lower than without. Taken together, this implies that the variance of firm i's full information price has gone up. As a consequence, firm i's price when being uninformed becomes more inaccurate, which renders information acquisition more profitable for firm i. Thus, if products are strategic complements, firm

[11] Clearly, if K is larger than the term on the right-hand side of (2.11), it is optimal for both firms not to acquire information, while if K is smaller than the term on the left-hand side, acquiring information is optimal for both firms.

i's incentive to acquire information increases if firm j chooses to become informed due to the strategic effect. By a similar argument, if product are strategic substitutes, the reverse holds true, i.e., firm i's benefit from becoming informed is lower if firm j chooses to acquire information.

In addition to this strategic effect, there is a second effect that we will refer to as the "externality effect". This effect occurs because a firm exerts by acquiring information an externality on its rival. In the remainder we refer to this externality as the "information-induced externality". In order to illustrate it, suppose that firm j does acquire information. In this case firm j exerts – by planning – an externality on firm i. This is due to the fact that now firm j's price is a random variable. Thus, in the profit function of firm i firm j induces a covariance between ξ and p_j, $E[\xi p_j]$, and therewith an information-induced externality that is given by $-\gamma E[\xi p_j]$.

We first characterize how the sign of $E[\xi p_j]$ is determined by the sign of the correlation coefficient and the relation between σ_ξ and σ_θ. The next corollary follows immediately from inspection of an informed firm's optimal price.

Corollary 1 *Suppose that firm j acquires information. Then, $E[\xi p_j] > 0$, if either $\rho > 0$ or $\rho < 0$ and $\sigma_\theta < \sigma_\xi$. If $\rho < 0$ and $\sigma_\theta > \sigma_\xi$, then $E[\xi p_j] < 0$. If the shocks are uncorrelated, i.e., $\rho = 0$, then $E[\xi p_j] = 0$.*

By acquiring information firm i can alter the information-induced externality that firm j exerts. If firm i acquires information it affects firm j's price. As can be seen from comparing the two prices in (2.13), for $\gamma > 0$ the denominator of the second term in both expressions is smaller for $p(1,1)$ than for $p(1,0)$. This implies that firm j's equilibrium price reacts more strongly to the realization of the shocks if firm i is informed than if firm i is not informed. This is the case because, as explained above, if products are strategic complements each firm amplifies the reaction of the other firm. The opposite holds true for strategic substitutes.

We can now determine how this effect changes firm i's expected profit. Suppose for example that $\gamma > 0$ and that $E[\xi p_j] < 0$. Since the expected externality is given by $-\gamma E[\xi p_j]$, it is positive in this case. Now if firm i acquires information, we know from above that firm j's optimal price reacts more strongly on the realizations of the shocks. Since, from

Corollary 1, $E[\xi p_j] < 0$ can only occur if $\sigma_\theta > \sigma_\xi$ and $\rho < 0$, we have that $E[\xi p_j]$ becomes more negative if firm i acquires information. Thus, firm i increases the positive information-induced externality if it acquires information. Therefore, in this case the strategic and the externality effect go in the same direction.

However, as can be seen from Proposition 1 if ρ is sufficiently negative, information acquisition decision are strategic substitutes although $\gamma > 0$. Thus, in this case both effects have to work in opposite directions and the externality effect has to dominate the strategic effect. To see this note first that the situation

$$-1 \leq \rho < -\frac{\sigma_\theta^2(4-\gamma) + \sigma_\xi^2(4 - 5\gamma + 2\gamma^2)}{2\sigma_\theta\sigma_\xi(4 - 3\gamma + \gamma^2)}$$

can only occur if $\sigma_\xi > \sigma_\theta$. From Corollary 1 we know that the covariance between ξ and p_j is positive if $\rho < 0$ and $\sigma_\xi > \sigma_\theta$. This implies that the information-induced externality is negative. Now, if products are strategic complements, firm i can ameliorate this externality by choosing to stay uninformed. In addition, if the correlation between the two shocks is highly negative, an informed firm's price in each state is relatively close to the optimal price of an uninformed firm. But this implies that the component on which the strategic effect operates is relatively small. Therefore, the externality effect dominates the strategic effect in this case, and information acquisition decision are strategic substitutes. A similar effect occurs if $\gamma < 0$, $\sigma_\xi > \sigma_\theta$ and ρ is highly negative. Then, information acquisition decision are strategic complements although prices are strategic substitutes.

The strategic effect of information acquisition is at the heart of the analysis of Hellwig and Veldkamp (2009). This is due to the fact that their framework considers a continuum of players which implies that the information-induced externality that a planning firm exerts on all other firms is negligible. Therefore, the externality effect cannot arise in their environment and it is solely the strategic effect that determines the mode of strategic interaction in the information choice stage.

We will now go on and analyze a fully dynamic model with an infinite number of planning dates and show that the main insights obtained in this simple two-stage game carry over and how the interplay between strategic and externality effect plays out in a dynamic framework.

2.3 Dynamic Information Choices

2.3.1 The Model

We consider two firms, each producing a non-storable good in continuous time. Each instant firms face the linear demand system

$$\begin{aligned} q_i(t) &= \alpha + \theta(t) - p_i(t) + \gamma p_j(t), && (2.14) \\ q_j(t) &= \alpha + \theta(t) - p_j(t) + \gamma p_i(t), && (2.15) \end{aligned}$$

where again α and γ are known constants, with $\alpha > 0$ and $-1 \leq \gamma \leq 1$. The random variable $\theta(t)$ follows a Brownian motion with zero drift. More specifically, we assume that

$$\theta(t) = \sigma_\theta Z(t), \tag{2.16}$$

where $\sigma_\theta > 0$ and $Z(t)$ is a standard Wiener process. Without loss of generality, we set $\theta(0) = 0$. The instantaneous profit function of firm i is given by

$$\Pi_i(t) = \Big(\alpha + \theta(t) - p_i(t) + \gamma p_j(t)\Big)\Big(p_i(t) - (c + \xi(t))\Big), \tag{2.17}$$

where $c > 0$ denotes the firm's constant marginal cost of production and $\xi(t)$ denotes a cost shock. The cost shock's evolution is described by

$$\xi(t) = \sigma_\xi Y(t), \tag{2.18}$$

where $\sigma_\xi > 0$ and $Y(t)$ is a standard Wiener process. Again we set $\xi(0) = 0$ without loss of generality. The processes $Z(t)$ and $Y(t)$ are (potentially) correlated and we denote the instantaneous correlation coefficient by ρ, where $-1 \leq \rho \leq 1$. The statistical properties of the processes $\theta(t)$ and $\xi(t)$ are common knowledge.[12]

We follow Reis (2006a;b) and Hellwig and Veldkamp (2009) in the way in which we incorporate the feature of costly information processing into our setting: A firm incurs a fixed cost

[12] For reasons that will become clear when we turn to a firm's objective function we do not impose non-negativity constraints on (expected) prices or (expected) quantities.

$K > 0$ each time it chooses to process the available information, i.e. to plan, in order to (re-)compute its optimal actions. As a consequence, time will be endogenously partitioned into planning and non-planning dates. Let $D_i(n) : \mathbb{N}_0 \to \mathbb{R}^+$ denote the process that determines the dates at which firm i chooses to process information, with $D_i(0) = 0$, $i = 1, 2$.

We distinguish between firm 1 and firm 2. At the outset of the game both firms simultaneously and irrevocably choose the length of their inattentiveness periods. This choice cannot be observed by the rival. Firm 1 chooses its inattentiveness interval $d_1 \in \mathbb{R}^+$ which induces an infinite sequence of planning dates, given by $\{d_1, 2d_1, 3d_1, ...\}$. Firm 2 also chooses its inattentiveness interval $d_2 \in \mathbb{R}^+$ and, simultaneously, has to decide between two planning modes:

$$(A) \qquad \left\{\frac{d_2}{2}, \frac{3d_2}{2}, \frac{5d_2}{2}, ...\right\},$$

$$(S) \qquad \{d_2, 2d_2, 3d_2, ...\}.$$

A planning mode translates d_2 into a sequence of planning dates.

Thus, we restrict each firm to a once-and-for-all decision about the length of its respective inattentiveness interval. As will become clear below this restriction does not alter the equilibrium outcome of the game. This is due to the fact that we formulate the model so that it is stationary. Hence, it would indeed not be profitable for a firm to vary its optimal inattentiveness period, even if it had the possibility to do so at the outset.

The reason for giving firm 2 the opportunity to choose between two planning modes is the following. We are particularly interested whether equilibria exist in which firms plan in an alternating and sequential order. In our setting firms can only attain this planning pattern if firm 2 chooses planning mode (A) since we assume that both firms acquire information simultaneously at $t = 0$. In this mode, the first inattentiveness period of firm 2 is exactly half as long as its future ones. A consequence of the assumptions concerning the planning modes is that we require the firms to reach the alternating planning scenario in a single step. Therefore, if (stationary) alternating and sequential planning equilibria exist in this setting, they will very likely exist as well if we allow for more sophisticated convergence patterns.

At a planning date a firm sets a sequence of prices for each instant until its consecutive planning date. These prices are set so that they are measurable with respect to the available information. At a non-planning date a firm does not process information and sets the previously determined price.

Now, we characterize the information that is available for firm i at a planning date $D_i(n)$. We denote by $H_{D_i(n)}$ the set of all possible histories up to, but not including instant $D_i(n)$. An element $h_{D_i(n)} \in H_{D_i(n)}$ includes all past prices $p(s) = (p_1(s), p_2(s))$, $0 \leq s < D_i(n)$, all past planning dates $D_i(m) < D_i(n)$ and $D_j(k) < D_i(n)$, and all past shock realizations $\theta(s)$ and $\xi(s)$, $0 \leq s < D_i(n)$. The information available to a firm i which plans at instant $D_i(n)$ is

$$\Xi_{D_i(n)} = \left(h_{D_i(n)}, \theta(D_i(n)), \xi(D_i(n))\right).$$

Put differently, at a planning date the planning firm observes the current shock realizations in addition to the complete history. Since a firm is inattentive in between its planning dates, it cannot update its available information. Hence, $\Xi(t) = \Xi(D_i(n))$ at any instant t, such that $D_i(n) \leq t < D_i(n+1)$. As a consequence, the firms' histories are of unequal length if the latest planning dates of the firms differ.

We restrict firms to (pure) stationary Markov strategies with the firms' beliefs about the realizations of $\theta(t)$ and $\xi(t)$ as the state variables since these variables are the only payoff relevant ones. Because the latest planning dates of both firms determine their beliefs about the realization of the shocks at date t, the economy is at each instant t characterized by these planning dates. If firm i last planned at date $D_i(n)$, it enters date $t > D_i(n)$ with the information set $I_t = I_{D_i(n)} = \left(\theta_{D_i(n)}, \xi_{D_i(n)}\right)$. If firm i plans at the current date t, its new information set contains the shock realizations of the current date: $I_t = I_{D_i(n+1)} = \left\{\theta_{D_i(n+1)}, \xi_{D_i(n+1)}\right\}$. Thus, given that the latest planning dates of firm i and j are $D_i(n)$ and $D_j(m)$, respectively, the Markov state of firm i is

$$\omega_i = \begin{cases} \left(I_{D_i(n)}, I_{D_j(m)}\right) & \text{if} \quad D_i(n) > D_j(m), \\ I_{D_i(n)} & \text{if} \quad D_i(n) \leq D_j(m). \end{cases} \tag{2.19}$$

If firm i is better informed than firm j, i.e., if $D_i(n) > D_j(m)$, firm j's information set is included in the Markov state of firm i since firm i can observe the complete history. Instead, if firm i and firm j are equally informed or if firm i is worse informed than firm j, i.e., $D_i(n) \leq D_j(m)$, only the information set of firm i constitutes its Markov state. Let Ω_i be the set of all possible Markov states. Then, a stationary Markov strategy p_i for firm i is defined as $p_i : \Omega_i \rightarrow \mathbf{R}$, that is, it maps the state space into actions.

We adopt the solution concept of Markov Perfect Bayesian Equilibrium (MPBE). The reason for using this solution concept is that it is inherent in the assumption of costly information processing that a firm cannot observe its rival's actions while it is inattentive. A feature of the MPBE is that a firm's beliefs about the future behavior of its rival are arbitrary once it observed an out-of-equilibrium action of its competitor. By restricting each firm's strategies to irrevocably choosing one inattentiveness interval at the outset of the game we get a grip on the arbitrariness of out-of-equilibrium beliefs to a certain degree. This is due to the fact that once firm i observes that firm j has chosen an inattentiveness length of d'_j, although the equilibrium called for a length of d_j, firm i knows that firm j will set d'_j forever.

However, this construction cannot restrict the firm's beliefs at the planning dates at which it detects a deviation but does not observe its rival's actual out-of-equilibrium strategy. This happens, if e.g., firm i plans at some instant and observes that firm j has not planned yet although firm j should have done so in equilibrium. At these planning dates we assume that firm i believes that firm j chose an inattentiveness interval that induces the same consecutive planning date as the inattentiveness interval that was prescribed in equilibrium. Although, this is only one possible belief, we argue that it is among the most reasonable. This is due to the fact that this out-of-equilibrium belief formalizes mistakes of the following kind: in equilibrium the planning unit is supposed to meet every Wednesday. However, the announcement is mistaken and states that the meeting takes place every other Wednesday.

The objective of firm i is to minimize the loss function

$$\mathcal{L}_i = \mathbb{E}_0 \left\{ \sum_{n=0}^{\infty} \left(\int_{D_i(n)}^{D_i(n+1)} e^{-rt} \left(\Pi_i^{FI}(t) - \Pi_i(t) \right) dt + e^{-rD_i(n+1)} K \right) \right\}, \tag{2.20}$$

via prescribing an infinite sequence of planning dates at the outset of the game, taking as given the sequence of its rival. It follows from our formulation of the strategy space that firms differ in the way in which they determine their respective sequences: firm 1 induces its infinite sequence of planning dates by choosing one inattentiveness interval, denoted by d_1. Firm 2 establishes its sequence by simultaneously setting a planning mode in addition to an inattentiveness interval, denoted by d_2. In (2.20), $\Pi_i^{FI}(t)$ denotes the (hypothetical) full information profit that firm i would earn if it planned at instant t, for given actions of firm j. The firms' common discount rate is denoted by $r > 0$.

We set up the firms' problem in terms of a loss function which we construct by scaling a firm's instantaneous expected profit by the firm's (hypothetical) instantaneous expected full information profit. This objective function delivers the same results in terms of instantaneous optimal prices and the equilibrium pattern of planning dates as a formulation which considers only the firm's profit function. This is due to the fact that $\Pi_i^{FI}(t)$ is a constant at a given instant. One way to think about the instantaneous loss function is that it measures, for given actions of its rival, a firm's expected instantaneous opportunity cost of choosing its prescribed actions, i.e., information status and associated price, relative to processing information.

The merit of working with the loss and not with the profit function is that the former specification guarantees that the model is stationary. This is the case because a firm's expected loss is zero at any planning date, irrespective of the absolute time that elapsed between a planning date and $t = 0$. Thus, a firm's expected loss depends at each instant only on the time that passed since its latest planning date and its rival's actions. Therefore a firm's incentive to plan is invariant with respect to absolute time. To the contrary, a firm's expected instantaneous profit at a given instant depends on the absolute time distance between this instant and the starting point of the model. This is due to the fact that at the outset firms have to evaluate the sequence of expected profits that their respective sequence of planning dates delivers. Thus, firms have to form expectations from the perspective of instant $t = 0$. Since the second moments of the stochastic processes increase linearly in time, expected instantaneous profits and therewith the firms' incentive to plan depend on absolute time.

The fact that the model is stationary makes the analysis tractable. In addition, it implies that we can, without loss of generality, restrict the strategy space so that each firm irrevocably chooses exactly one inattentiveness interval. As mentioned before, given the stationarity of planning incentives, a firm would not find it optimal to set inattentiveness periods of unequal length, even if we explicitly allowed for this.

To sum up, at the outset, both firms observe the initial state $\theta_0 = 0$ and $\xi_0 = 0$. Each firm can then calculate the per-instant price equilibrium for any combination of planning dates. Given this, firm 1 chooses it inattentiveness interval d_1 to minimize the loss function $\mathcal{L}_1$, taking as given the sequence of firm 2, while firm 2 simultaneously chooses its inattentiveness interval d_2 and its planning mode in order to minimize the loss function $\mathcal{L}_2$, taking as given the sequence of firm 1.

2.3.2 Equilibrium Prices and Loss Functions

We solve the model by backward induction. First, we determine the optimal prices and corresponding expected losses for any combination of planning dates chosen by the firms at the outset. Second, we derive the equilibrium planning pattern and the respective inattentiveness intervals.

We set out by characterizing the firms' equilibrium prices. As will become evident below, each firm's best response is linear in the shock realizations. In conjunction with our assumption that the stochastic processes have zero drift, it will turn out that each firm's optimal price is constant over time until the subsequent planning date of any firm.

We start with the case in which firms' planning dates are asynchronous. Denote by v the latest and by v' the subsequent planning date of firm i, i.e., $v := D_i(n)$ and $v' := D_i(n+1)$, and by w the latest and by w' the next planning date of firm j, i.e., $w := D_j(m)$ and $w' := D_j(m+1)$.

Lemma 2 *Suppose that firm i planned for the last time at v and that firm j planned for the last time at date w, where $w < v$. Firm j believes (correctly) that firm i plans at v. In the Markov Perfect Bayesian Equilibrium, prices at $v \leq t < \min\{v', w'\}$ are*

$$p_i^\star(t) = \frac{\alpha + c + \theta(v) + \xi(v)}{2-\gamma} - \frac{\gamma}{2(2-\gamma)}\Big(\theta(v) - \theta(w) + \xi(v) - \xi(w)\Big), \tag{2.21}$$

$$p_j^\star(t) = \frac{\alpha + c + \theta(w) + \xi(w)}{2-\gamma}. \tag{2.22}$$

The firms' full information prices are

$$\begin{aligned} p_i^{FI}(t) &= \frac{\alpha+c+\theta(t)+\xi(t)}{2-\gamma} - \frac{\gamma}{2(2-\gamma)}\Big(\theta(t)-\theta(w)+\xi(t)-\xi(w)\Big), \quad (2.23) \\ p_j^{FI}(t) &= \frac{\alpha+c+\theta(t)+\xi(t)}{2-\gamma} - \frac{\gamma}{2(2-\gamma)}\Big(\theta(t)-\theta(v)+\xi(t)-\xi(v)\Big) + \\ &+\frac{\gamma^2}{4(2-\gamma)}\Big(\theta(v)-\theta(w)+\xi(v)-\xi(w)\Big). \quad (2.24) \end{aligned}$$

Proof We start with the optimization problem of firm i. Firm i's expected instantaneous profit for $v \leq t < \min\{v', w'\}$ is

$$E\Big[\Big(\alpha+\theta(t)-p_i(t)+\gamma p_j(t)\Big)\Big(p_i(t)-(c+\xi(t))\Big)\Big|I_v, I_w\Big],$$

which can be written as

$$(\alpha+\theta(v)-p_i(t)+\gamma p_j(t))(p_i(t)-c)-\xi(v)(\alpha-p_i(t)+\gamma p_j(t))-E[\theta(t)\xi(t)|I_v,I_w]. \quad (2.25)$$

From (2.25) it becomes evident that $p_j(t)$ is, from the perspective of firm i at v, a non-random variable. This is due to the fact that firm j's optimal price is based on information that is known to firm i at v. Differentiating (2.25) with respect to $p_i(t)$ yields firm i's reaction function

$$p_i(t) = \frac{\alpha+c+\gamma p_j(t)+\theta(v)+\xi(v)}{2}. \quad (2.26)$$

Now we turn to firm j. The expected instantaneous profit of firm j at date t is

$$E\Big[\Big(\alpha+\theta(t)-p_j(t)+\gamma p_i(t)\Big)\Big(p_j(t)-(c+\xi(t))\Big)\Big|I_w\Big]. \quad (2.27)$$

Firm j's information set contains all shock realizations up to and including its latest planning date w. Now, taking into account that firm j observed $\theta(w)$ and $\xi(w)$ when it planned for the last time, it follows from (2.16) and (2.18) that $E[\theta(t)|I_w]=\theta(w)$ and $E[\xi(t)|I_w]=\xi(w)$. Thus (2.27) can be represented as

$$\Big(\alpha+\theta(w)-p_j(t)+\gamma E[p_i(t)|I_w]\Big)\Big(p_j(t)-c\Big)-\xi(w)\Big(\alpha-p_j(t)\Big)-E\Big[\xi(t)\Big(\theta(t)+\gamma p_i(t)\Big)\Big|I_w\Big]. \quad (2.28)$$

From the perspective of firm j with information set I_w, the price that firm i sets at time t is a random variable. This is due to the fact that firm i updated its information set for the last time at $v > w$. Thus, firm i acts on more recent information which firm j has to infer.

Differentiating (2.28) with respect to $p_j(t)$ and rearranging yields

$$p_j(t) = \frac{\alpha + c + \gamma E[p_i(t)|I_w] + \theta(w) + \xi(w)}{2}. \tag{2.29}$$

Now, we derive the belief that firm j holds about firm i's price at date t. Though firm j does not observe $\theta(t)$ and $\xi(t)$, it knows that firm i's reaction function is given by (2.26). Forming expectations about $\theta(t)$ and $\xi(t)$ reveals that firm j expects the following reaction function of firm i at date t:

$$E[p_i(t)|I_w] = \frac{\alpha + c + \gamma p_j(t) + \theta(w) + \xi(w)}{2}. \tag{2.30}$$

Solving (2.29) and (2.30) for $E[p_i(t)|I_w]$ and $p_j(t)$ yields

$$E[p_i(t)|I_w] = p_j^\star(t) = \frac{\alpha + c + \theta(w) + \xi(w)}{2 - \gamma}, \tag{2.31}$$

which is (2.22). Firm i knows firm j's rationale which implies that firm i knows that firm j sets its price according to (2.31). Then, inserting (2.31) in (2.26) yields (2.21).

Finally, we derive the full information prices. We start with $p_i^{FI}(t)$, the price that firm i would optimally set, if it planned at t. The full information profit of firm i at date t is given by

$$(\alpha + \theta(t) - p_i(t) + \gamma p_j(t))(p_i(t) - (c + \xi(t))) \tag{2.32}$$

Differentiating (2.32) with respect to $p_i(t)$ and rearranging yields

$$p_i(t) = \frac{\alpha + c + \gamma p_j(t) + \theta(t) + \xi(t)}{2}. \tag{2.33}$$

Now, inserting $p_j^\star(t)$ as given in (2.22) yields (2.23).

By the same logic the full information price of firm j is given by

$$p_j(t) = \frac{\alpha + c + \gamma p_i(t) + \theta(t) + \xi(t)}{2}. \tag{2.34}$$

Inserting $p_i^\star(t)$ as given in (2.21) yields (2.24). ■

Next, we characterize prices for the case of synchronous planning.

Lemma 3 *Suppose that firm j and firm i planned for the last time at date v, and each firm believes that the other also planned at v. Then, in the Markov Perfect Bayesian Equilibrium, prices at $v \leq t < \min\{v', w'\}$ are*

$$p_i^\star(t) = p_j^\star(t) = \frac{\alpha + c + \theta(v) + \xi(v)}{2-\gamma}, \tag{2.35}$$

while full information prices are

$$p_i^{FI}(t) = p_j^{FI}(t) = \frac{\alpha + c + \theta(t) + \xi(t)}{2-\gamma} - \frac{\gamma}{2(2-\gamma)}\Big(\theta(t) - \theta(v) + \xi(t) - \xi(v)\Big). \tag{2.36}$$

Proof Firm i's expected instantaneous profit at t is given by (2.25). Thus, firm i's reaction function is (2.26). Solving (2.26) for $p_i(t)$ using symmetry yields (2.35). Similarly the full information price is derived analogously to (2.23). ■

Before we move on with the analysis we point out an important observation that follows immediately from Lemma 2 and Lemma 3.

Corollary 2 *Suppose that the latest planning dates of firms i and j are v and w, respectively, with $v \geq w$. Then, firm j's optimal price path $\{p_j^\star(t) : v \leq t < w'\}$ remains constant until firm j's consecutive planning date.*

Put differently, a firm sets identical prices in two situations that are informationally not equivalent: equal and worse information. The reason is the following: If firm j has more outdated information than firm i, it forms expectations about the shock realizations that firm i will observe at its next planning date. Since the stochastic processes have zero drift, firm j's best estimates of future shock realizations are the ones that it observed at its latest planning date. Thus, from the perspective of firm j, the firms' information sets are identical in expectation. In addition, the optimal price of each firm is linear in the shock realizations.

Therefore, firm j expects firm i to set the same price as in the case in which firm j has the same information as firm i. This implies that firm j sets the same price in both situations.

After having characterized the optimal and full information prices, we are in a position to derive the instantaneous expected losses under synchronous and asynchronous planning. We obtain the following result:

Lemma 4 *Suppose that firm i planned for the last time at v and that firm j planned for the last time at w, where $w \leq v$. Firm j (correctly) believes that firm i plans at v. The instantaneous expected loss of firm i at instant t with $v \leq t < \min\{v', w'\}$ is given by*

$$L_i := E\Big[\Pi_i^{FI}(t) - \Pi_i(t)|I_v, I_w\Big] = \frac{1}{4}\Big(\sigma_\theta^2 + 2\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2\Big)(t-v), \tag{2.37}$$

and the instantaneous expected loss of firm j for $v \leq t < \min\{v', w'\}$ is given by

$$L_j := E\Big[\Pi_j^{FI}(t) - \Pi_j(t)|I_w\Big] = \frac{1}{4}\Big(\sigma_\theta^2 + 2\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2\Big)(t-w) + \frac{\gamma(4+\gamma)}{16}\Big(\sigma_\theta^2 + 2\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2\Big)(v-w). \tag{2.38}$$

Proof First, we derive firm i's profit under full information. Using (2.23) and (2.21) in (2.32) yields

$$\Pi_i^{FI}(t) = \frac{1}{4}\Big(\alpha - c + \theta(t) - \xi(t) + \frac{\gamma(a + c + \theta(w) + \xi(w))}{2-\gamma}\Big)^2.$$

Thus, the expected full information profit of firm i at instant t with $v \leq t < \min\{v', w'\}$ is given by

$$E\Big[\Pi_i^{FI}(t)|I_v, I_w\Big] = \frac{1}{4}\Big(\sigma_\theta^2 - 2\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2\Big)(t-v) + \chi, \tag{2.39}$$

where

$$\chi := \left(\frac{\alpha + \theta(v) - c - \xi(v) + \gamma c - \frac{\gamma}{2}\Big(\theta(v) - \xi(v) - (\theta(w) + \xi(w))\Big)}{2-\gamma}\right)^2.$$

Next, using (2.21) and (2.22) in (2.25) yields firm i's expected profit if it planned for the last time at $v \leq t$:

$$E\Big[\Pi_i(t)|I_v, I_w\Big] = -\rho\sigma_\theta\sigma_\xi + \frac{(\theta(v) - \xi(v))^2}{4} + \chi. \tag{2.40}$$

Subtracting (2.40) from (2.39) yields (2.37).

Now we turn to firm j. Using (2.21) and (2.24) in the full information profit that is given by

$$(\alpha + \theta(t) - p_j(t) + \gamma p_i(t))(p_j(t) - (c + \xi(t)))$$

and taking expectations from the perspective of date w yields

$$\begin{aligned} E\Big[\Pi_j^{FI}(w)|I_w\Big] &= \frac{1}{4}\Big(\sigma_\theta^2 - 2\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2\Big)(t-w) + \frac{\gamma}{4}\left(\sigma_\theta^2 - \sigma_\xi^2 + \frac{\gamma}{4}\Big(\sigma_\theta^2 + 2\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2\Big)\right)(v-w) \\ &+ \left(\frac{\alpha + \theta(w) - c - \xi(w) + \gamma(c + \xi(w))}{2-\gamma}\right)^2. \end{aligned} \tag{2.41}$$

If instead firm j planned for the last time at w, then its expected profit at t is given by

$$\begin{aligned} E\Big[\Pi_j(t)|I_w\Big] &= \Big(\alpha + \theta(w) - p_j(t) + \gamma E[p_i(t)|I_w]\Big)(p_j(t) - c) \\ &- \xi(w)\Big(\alpha - p_j(t)\Big) - E\Big[\xi(t)\Big(\theta(t) + \gamma p_i(t)\Big)|I_w\Big] \end{aligned} \tag{2.42}$$

We already derived that

$$E_j[p_i(t)|I_w] = p_j(t) = \frac{\alpha + c + \theta(w) + \xi(w)}{2-\gamma}.$$

Using

$$p_i(t) = \frac{\alpha + c + \theta(v) + \xi(v)}{2-\gamma} - \frac{\gamma}{2(2-\gamma)}\Big(\theta(v) - \theta(w) + \xi(v) - \xi(w)\Big),$$

in (2.42) and taking expectations delivers that

$$\begin{aligned} E\Big[\Pi_j(t)|I_w\Big] &= -\rho\sigma_\theta\sigma_\xi(t-w) - \frac{\gamma}{2}\Big(\sigma_\xi^2 + \rho\sigma_\theta\sigma_\xi\Big)(v-w) \\ &+ \left(\frac{\alpha + \theta(w) - c - \xi(w) + \gamma(c + \xi(w))}{2-\gamma}\right)^2. \end{aligned} \tag{2.43}$$

Subtracting (2.43) from (2.42) yields (2.38). ■

After having characterized the expected instantaneous loss functions, we can determine how a firm affects the loss of its rival via planning. A comparison of (2.37) and (2.38) then reveals the following result:

Corollary 3 *Suppose that the latest planning dates of firms i and j are v and w, respectively, with $w \leq v$, and that firms i and j plan for the next time at v' and w', respectively, with $v < w' < v'$. At w' firm i's instantaneous expected loss increases discretely if the firms' strategy variables are strategic complements, i.e., if $\gamma > 0$. If firms compete in strategic substitutes, i.e., $\gamma < 0$, the instantaneous expected loss of firm i decreases by a discrete amount at w'.*

The intuition for this result is that in the considered situation only the strategic effect is at work. This is the case because the strategic effect, unlike the externality effect, is triggered solely by the rival's planning decision: when firm j plans, it changes its price in accordance with the realizations of the shocks. Whether this move decreases or increases the inaccurateness of the price that the uninformed firm i sets at this instant depends on the mode of strategic interaction: the inaccurateness increases if $\gamma > 0$, whereas it decreases if $\gamma < 0$. Suppose for example that $\gamma > 0$. In this case firm j raises its price if the realizations of θ and ξ are large. But since the shocks are common to both firms, this implies that, due to the strategic complementarity, firm i's full information price is now even larger as compared to the case in which firm j did not plan. By a similar argument, if the realizations of θ and ξ are low, firm i's full information price is lower than in the case in which firm j did not plan. Thus, the price that firm i sets when being uninformed becomes more inaccurate. The reason why the externality effect does not appear is that by construction of the loss function the action of the rival is held constant. Thus, firm i does not change the information-induced externality.

2.3.3 Structure of Planning Dates

After having determined the expected instantaneous losses for every pattern of planning dates, we now derive the equilibrium structure of planning dates and the respective inattentiveness periods. Here our aim is to characterize the conditions under which an alternating or synchronized equilibrium in planning dates exist. First, we turn to the analysis of an alternating planning equilibrium.

Alternating Planning

In our framework the issue of existence of an alternating planning equilibrium is particularly interesting. This is due to the fact that each firm's first planning date is at $t = 0$. Thus, in the beginning firms are by assumption in a synchronous planning pattern. As a consequence, we characterize under which conditions an alternating planning equilibrium arises endogenously without imposing this structure of planning dates.[13] Given that an alternating planning equilibrium exists, we determine whether it is unique.

We restrict our attention to the case of symmetric alternating planning equilibria, that is, to equilibria in which the inattentiveness period denoted by d is the same for both firms and each firm plans exactly in the middle of the inattentiveness period of the other firm. Put differently, the time that elapses between a planning date of firm i and a planning date of firm j is $d/2$.[14] Given our assumption on the strategy space such an equilibrium can only arise if firm 1's sequence of planning dates is given by $\{d, 2d, 3d, ...\}$ and firm 2 chooses planning mode (A), that is, its sequence of planning dates is

$$\left\{\frac{d}{2}, \frac{3d}{2}, \frac{5d}{2}, ...\right\}.$$

Before we proceed with the analysis we introduce some notation. Let

$$\begin{aligned}
\Lambda_1 &:= \frac{\sigma_\theta^2 + 2\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2}{4}, \\
\Lambda_2 &:= \frac{\sigma_\theta^2 + 2\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2}{(2-\gamma)^2}, \\
\Gamma_1 &:= \frac{2(\sigma_\theta^2 + 2\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2) - \gamma^2(2-\gamma)(\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2)}{2(2-\gamma)^2}, \\
\Gamma_2 &:= \Lambda_2\left(1 - \frac{\gamma}{2}\right) - \frac{\gamma^2\left(2(1-\gamma)(\sigma_\xi^2 + \rho\sigma_\theta\sigma_\xi) + \sigma_\xi^2 - \sigma_\theta^2\right)}{8(2-\gamma)^2}.
\end{aligned}$$

[13] The only other paper that pursues a similar goal is Lau (2001). However, as mentioned above, he uses a deterministic framework in which each firm can choose to be committed to a price for one or two periods. Thus, an asynchronous equilibrium implies that each firm chooses a commitment length of two periods and adjusts its price exactly in periods when the rival does not.

[14] Most papers in the literature focussed on this equilibrium adjustment pattern. For example, Hellwig and Veldkamp (2009) consider a similar planning structure in their model. Maskin and Tirole (1988a;b) and Lau (2001) also confine their attention to the case of symmetric commitment lengths.

Now, suppose that $d_a^\star$ is the length of an inattentiveness interval in an alternating planning equilibrium. We first analyze for which $d_a^\star$ neither firm 1 nor firm 2 have an incentive to deviate by marginally shortening or extending this interval to $d_a^\star - \epsilon$ or $d_a^\star + \epsilon$.

Lemma 5 *Suppose that an alternating planning equilibrium exists and let $d_a^\star$ denote the common equilibrium inattentiveness period. Firm 1 has no incentive to deviate marginally from $d_a^\star$ if and only if $\underline{d}_a^1 \leq d_a^\star \leq \bar{d}_a^1$, where $\underline{d}_a^1$ is the unique solution to*

$$\frac{\underline{d}'_a \Lambda_2 (8-4\gamma+\gamma^2)}{8e^{r\underline{d}'_a}} - \frac{re^{r\underline{d}'_a}K}{(e^{r\underline{d}'_a}-1)^2} - \frac{\Lambda_1 e^{r\underline{d}'_a/2}+\Gamma_1}{r(e^{r\underline{d}'_a}-1)(e^{r\underline{d}'_a/2}+1)} + \frac{\Gamma_2 \underline{d}'_a (2e^{r\underline{d}'_a}-1)}{e^{r\underline{d}'_a}(e^{r\underline{d}'_a}-1)^2} = 0, \quad (2.44)$$

and $\bar{d}_a^1$ is the unique solution to

$$\frac{\bar{d}'_a \Lambda_1 (8+4\gamma+\gamma^2)}{8e^{r\bar{d}'_a}} - \frac{re^{r\bar{d}'_a}K}{(e^{r\bar{d}'_a}-1)^2} - \frac{\Lambda_1 e^{r\bar{d}'_a/2}+\Gamma_1}{r(e^{r\bar{d}'_a}-1)(e^{r\bar{d}'_a/2}+1)} + \frac{\Gamma_2 \bar{d}'_a (2e^{r\bar{d}'_a}-1)}{e^{r\bar{d}'_a}(e^{r\bar{d}'_a}-1)^2} = 0. \quad (2.45)$$

Firm 2 has no incentive to deviate marginally from $d_a^\star$ if and only if $\underline{d}_a^2 \leq d_a^\star \leq \bar{d}_a^2$, where $\underline{d}_a^2$ is the unique solution to

$$\frac{\Lambda_1 \underline{d}'_a}{2e^{r\underline{d}'_a}} - \frac{\Lambda_1 e^{r\underline{d}'_a/2}(e^{r\underline{d}'_a/2}-1)}{r(e^{r\underline{d}'_a}-1)} - \frac{rKe^{r\underline{d}'_a}}{e^{r\underline{d}'_a}-1} - \frac{\Gamma_1(e^{r\underline{d}'_a/2}-1)}{r\left(e^{r\underline{d}'_a}-1\right)} - \quad (2.46)$$

$$-\Gamma_2 \underline{d}'_a \left(\frac{1}{e^{r\underline{d}'_a}(e^{r\underline{d}'_a}-1)} + \frac{\ln(1-e^{-r\underline{d}'_a/2}) - \ln(1+e^{-r\underline{d}'_a/2})}{e^{-r\underline{d}'_a/2}} \right) = 0,$$

and $\bar{d}_a^2$ is the unique solution to

$$\frac{\Lambda_1 \bar{d}'_a}{2e^{r\bar{d}'_a}} - \frac{\Lambda_1 e^{r\bar{d}'_a/2}(e^{r\bar{d}'_a/2}-1)}{r(e^{r\bar{d}'_a}-1)} - \frac{\Gamma_1(2e^{r\bar{d}'_a/2}-2-r\bar{d}'_a)}{2re^{r\bar{d}'_a/2}(e^{r\bar{d}'_a}-1)} - \frac{rKe^{r\bar{d}'_a}}{e^{r\bar{d}'_a}-1} = 0. \quad (2.47)$$

Proof See the Appendix.

In Lemma 5 we derive for each firm lower and upper bounds on the potential equilibrium inattentiveness period $d_a^\star$. More specifically, $d_a^\star$ must be larger than $\underline{d}_a^i$ since otherwise firm i would have an incentive to deviate to $d_a^\star + \epsilon$. Similarly, the upper bound $\bar{d}_a^i$ prevents deviations to $d_a^\star - \epsilon$. Since the two firms have to choose different planning sequences in order to attain the alternating pattern, the bounds for firm 1 and firm 2 differ.

An interesting observation is that for each firm the conditions that determine the upper and the lower bound on potential equilibrium inattentiveness lengths differ. Due to fact that firm j is inattentive at a planning date of firm i one may expect that firm i's incentive to marginally shorten or extend its inattentiveness length are identical. However, the proposition shows that this is not the case. The reasons for this result are inherent in the dynamic nature of the model and in the fact that firms are strategic players in our set-up. Consider the case in which firm 1 deviates from an inattentiveness length d to $d+\epsilon$. This deviation is detected by firm 2 at its next planning date. Due to our assumption on the firms' strategy spaces, firm 2 knows that firm 1's inattentiveness length will be $d+\epsilon$ for the rest of the game. This implies that the time that elapses between a planning date of firm 2 and the next planning date of firm 1 becomes longer and longer over time. Thus, after firm 2 detected firm 1's deviation, it knows that it will be the better informed firm over a longer time horizon, i.e., one which is longer than $d/2$. As a consequence, firm 2 is the worse informed firm for a shorter time. The reverse is true in the case in which firm 1 deviated to $d-\epsilon$. As can be seen from (2.21) and (2.22), a firm sets different prices when it is better informed than when it is worse informed than the competitor. This implies that firm 2 reacts, after it detects a deviation at its next planning date, differently to the two deviations. Hence, marginally shortening or prolonging the inattentiveness period are differently profitable for firm 1. In other words, the streams of expected losses that both deviations induce differ discretely. Therefore, the inattentiveness length that prevents one or the other deviation must differ as well.

A necessary condition for the existence of alternating planning equilibria is that for each firm $\underline{d}_a^i \leq \bar{d}_a^i$. The following lemma states the conditions under which this is the case.

Lemma 6 *For firm 1, $\underline{d}_a^1 < \bar{d}_a^1$ for all ρ and $\gamma \neq 0$, while $\underline{d}_a^1 = \bar{d}_a^1$ if $\gamma = 0$. For firm 2,*

$$\bar{d}_a^2 \gtreqless \underline{d}_a^2 \quad \text{if} \quad \rho \gtreqless \max\left[\hat{\rho}, -1\right],$$

where $\hat{\rho}$ is the unique solution to

$$-\frac{\Lambda_1}{2(1-e^{-rd_a})} - \frac{\Gamma_2\left(\ln(1+e^{-rd_a/2}) - \ln(1-e^{-rd_a/2})\right)}{e^{-rd_a/2}} = 0, \tag{2.48}$$

and d_a is the solution to (2.46) *and* (2.47) *at $\rho = \hat{\rho}$.*[15]

[15] Note that the left-hand side of (2.48) depends on ρ since Λ_1 and Γ_2 depend on ρ.

Proof See the Appendix.

The result stated in Lemma 6 shows that unless $\gamma = 0$ there exists a range of potential equilibrium inattentiveness periods for firm 1 that are robust against a marginal deviation. This is not true for firm 2: If ρ is sufficiently negative, this range might be empty. The reason for this result is the following. Generally, a negative correlation implies longer equilibrium inattentiveness intervals because the shocks partially offset each other. Now, this effect has a different impact on the bounds of firm 2 since its first planning interval is only half the length of its future ones under planning mode (A). Thus, firm 2 has a stronger incentive to extend its planning period than to reduce it. This implies that the lower bound shifts upward more strongly than the upper bound as the correlation between the shocks decreases. Now, if the correlation is sufficiently negative, it may happen that the range of potential equilibrium inattentiveness intervals is empty.

Moreover, it is important to note that for $\gamma = 0$, i.e., the case in which the firms' demands are unrelated, there is a range of potential optimal inattentiveness periods for firm 2, whereas for firm 1 there is a unique optimal solution. For firm 1 this result is a natural implication of the fact that $\gamma = 0$. Since the firm is a monopolist, there is no interaction with firm 2 on the product market. Therefore, its deviation incentives are the same for marginally extending or shortening its inattentiveness period. In this situation, i.e., for $\gamma = 0$, the firms are in principle alike. Thus, we should expect the same result for firm 2. However, in order to reach the alternating planning pattern we oblige firm 2 to choose planning mode (A). This implies that for firm 2, even though there is no strategic interaction with firm 1, there are multiple inattentiveness periods that are robust against a marginal deviation. This finding highlights the importance of considering non-marginal deviations in our set-up. In this particular case, i.e. $\gamma = 0$, we get the result that firm 2 will choose the same inattentiveness period as firm 1 and the planning mode (S).

Finally, we combine the existence conditions for both firms. This is done in the next lemma:

Lemma 7 *Marginal deviations do not preclude the existence of an alternating planning equilibrium if and only if* $\rho \geq max\,[\check{\rho}, -1]$ *where* $\check{\rho} \geq \hat{\rho}$ *solves*

$$\Gamma_2 d_a \left(\frac{2}{e^{rd_a}(e^{rd_a}-1)^2} + \frac{\ln(1-e^{-rd_a/2}) - \ln(1+e^{-rd_a/2})}{e^{-rd_a/2}} \right) - \frac{\Lambda_1 d_a (8+4\gamma+\gamma^2 - e^{rd_a}(2+\gamma)^2)}{8e^{rd_a}(e^{rd_a}-1)} = 0, \qquad (2.49)$$

and d_a *is the solution to* (2.46) *and* (2.47) *at* $\rho = \check{\rho}$.

Proof See the Appendix.

So, when only considering marginal deviations, we obtain that an alternating planning equilibrium exists if ρ is not too negative. The reason is the same as the one explained after Lemma 6. However, the critical ρ is now given by $\check{\rho}$, which is weakly larger than $\hat{\rho}$ as determined in Lemma 6. This is due to the fact that the upper bound of firm $1'$s equilibrium range, $\bar{d}_a^1$, is strictly below firm $2'$s upper bound, $\bar{d}_a^2$. As shown in Lemma 6, if ρ is sufficiently negative, $\underline{d}_a^2$ may become larger than $\bar{d}_a^2$. But the fact that $\bar{d}_a^1 < \bar{d}_a^2$ implies that $\underline{d}_a^2$ is larger than $\bar{d}_a^1$ as well. Thus, the requirement that an alternating planning equilibrium exists if and only if the ranges $[\underline{d}_a^1, \bar{d}_a^1]$ and $[\underline{d}_a^2, \bar{d}_a^2]$ overlap implies a tighter lower bound on ρ compared to the condition that ensures that firm $2'$s range is non-empty.

So far we only considered marginal deviations. Now, we turn to the analysis of non-marginal deviations. Here we consider deviations for firm 1 to period lengths of $d[1 \pm (l/m)]$, for natural numbers $l \leq m$. The reason why we confine the analysis to fractional deviations is the following. With fractions l/m, we can approximate any real number between zero and one arbitrarily closely. In addition, the stream of expected losses that is induced by the fractional deviations allows us to concisely characterize the bounds on the range of equilibrium inattentiveness periods. Similarly, for firm 2, we consider deviations to period lengths of $d[1 \pm (l/m)]$ both in the (S) and the (A) mode.

The following lemma states that for r close to 0 we can without loss of generality concentrate on one particular type of non-marginal deviation: firm 2 deviates to planning mode (S) but keeps $d_a^\star$. In the remainder, we will refer to this type of deviation as the "RTS deviation".[16]

[16] The acronym RTS abbreviates "return to synchronous" planning.

Lemma 8 *Any non-marginal deviation by firm* 1 *and firm* 2 *in which firm* 2 *remains in the* (A) *planning mode does not exclude potential alternating planning equilibria for* r *close to 0. Denote by* $\mathbb{S}$ *the set of potential alternating planning equilibria that survive the RTS deviation and by* $\mathbb{S}'$ *the set of potential alternating planning equilibria that survive deviations of the following type: Firm* 2 *deviates to the* (S) *planning mode and chooses* $(1+l/m)d$*. Then,* $\mathbb{S}=\mathbb{S}'$ *for* r *close to 0.*
In particular, the RTS deviation does not exclude $d_a^\star$ *if*

$$rK \leq \frac{d_a^\star}{8e^{3rd_a^\star}(2-\gamma)^2}\Big(e^{5rd_a^\star/2}\left(4\gamma^2(\sigma_\theta^2+\rho\sigma_\theta\sigma_\xi)+2\gamma^3(\sigma_\xi^2+\rho\sigma_\theta\sigma_\xi)-4\Lambda_1(\gamma^4+4)\right) \tag{2.50}$$

$$+8\Lambda_1 e^{2rd_a^\star}(2-\gamma)^2+4(1+e^{rd_a^\star/2}-e^{rd_a^\star})\gamma^2\left(\Lambda_1\gamma^2-(\sigma_\xi^2+\rho\sigma_\theta\sigma_\xi)\gamma+\sigma_\xi^2-\sigma_\theta^2\right)\Big).$$

Proof See the Appendix.

The result stated in Lemma 8 allows us, in case r is sufficiently close to 0, to concentrate on the deviation to the sequence $\{d_a^\star, 2d_a^\star, 3d_a^\star, ...\}$ by firm 2 when considering non-marginal deviations. This allows us to characterize the conditions for equilibrium existence in a concise way. It is intuitive that the deviation of firm 2 to the planning mode (S) with the optimal inattentiveness length of firm 1 eliminates many potential equilibria. This is due to the combination of the following two facts: First, firm 2 is per se more prone to deviate from the alternating planning equilibrium as its first inattentiveness period is only half the length of its future ones. Second, the RTS deviation constitutes the only possibility for firm 2 to unilaterally implement a synchronous planning pattern at a potential alternating equilibrium planning frequency.

The result that we can concentrate on the RTS deviation can be shown analytically for $r \to 0$ and, by continuity, this argument extends to r in a neighborhood of zero. However, computations suggest that even for $r > 0$ the RTS deviation excludes the largest set of potential equilibria.

Before we proceed we define ρ^+ as the ρ for which (2.50) holds with equality. Now, we combine the marginal and non-marginal “no-deviation” conditions in order to state the main result of this subsection:

Proposition 2

(i) For r close to zero, an alternating planning equilibrium does not exist if $\gamma \geq 0$.

(ii) Suppose $\gamma < 0$. For r close to zero, an alternating planning equilibrium exists if $\rho \geq \max[\check{\rho}, \rho^+]$, and the maximum range of equilibrium inattentiveness periods is given by $d_a^\star \in [\underline{d}_a^1, \bar{d}_a^1]$.

(iii) For $r \to 0$, the alternating planning equilibrium is unique, that is $\underline{d}_a^1 = \bar{d}_a^1 = d_a^\star$, and exists if and only if $\gamma < 0$ and

$$\rho \geq \max\left[-\frac{\sigma_\theta^2(4-\gamma)+\sigma_\xi^2(4-5\gamma+2\gamma^2)}{2\sigma_\theta\sigma_\xi(4-3\gamma+\gamma^2)}, -\frac{4(4-\gamma)(\sigma_\theta^2+\sigma_\xi^2)-\gamma^2(11\sigma_\xi^2-\sigma_\theta^2)+6\gamma^3\sigma_\theta^2}{2\sigma_\theta\sigma_\xi(16-4\gamma-5\gamma^2+3\gamma^3)}, -1\right].$$

Proof See the Appendix.

The proposition consists of three parts. First, it states that an alternating planning equilibrium cannot exist if products are strategic complements. This is the case because the non-marginal deviation characterized in Lemma 8 eliminates all potential equilibria with $\gamma \geq 0$. The reason for this closely resembles the strategic effect described above, i.e., that an uninformed firm's optimal price becomes more inaccurate when the rival plans. Thus, for $\gamma > 0$ there is a tendency to synchronize planning decisions. This is particularly pronounced if the degree of strategic complementarity is large. In addition, if γ is relatively small in absolute value, firm 2 prefers a planning sequence with equal period lengths, that is, it prefers the (S) to the (A) mode since this would be its optimal choice if it was a monopolist. In conjunction, these two effects imply that for $\gamma > 0$ there exists no alternating planning equilibrium.

Parts (ii) and (iii) of the proposition state that if products are strategic substitutes, an alternating planning equilibrium exists if the correlation between the shocks is not too negative. The intuition behind this result is a combination of the strategic and the externality effect, similar to the one explained in Section 2.2. First, due to the strategic effect, if $\gamma < 0$ an uninformed firm's price becomes more accurate when the rival plans. Thus, there is an inherent tendency towards alternating planning. Second, by planning, firm j induces an externality on firm i. Suppose for example that $\sigma_\theta > \sigma_\xi$. In this case the informed firm j adjusts its price (in expectation) in the direction of the shock to the intercept. Thus, the covariance between p_j and the realization of the cost shock is negative, which implies, together with $\gamma < 0$, that firm j exerts a negative (expected) externality on firm i. Since the variance of the shocks increases linearly in the time that elapses after a planning date of firm i, this externality is the larger the longer the time distance between firm i's latest planning date and the next planning date of firm j. In order to reduce this externality, firm i is inclined to move closer to the planning date of firm j. Thereby, it destroys the potential equilibrium.

Interestingly, as can be seen from statement (iii), in the limit as r tends to zero, one of the conditions that ρ has to meet so that an alternating planning equilibrium exists is precisely the one for which information acquisition decisions are strategic substitutes in the example presented in Section 2.2. This shows that the interplay between the strategic and the externality effect is present in the dynamic game as well and that its implications resemble the ones given in the example.

Part (iii) of the proposition also states that the alternating planning equilibrium is unique for $r \to 0$. In this case the two marginal "no-deviation" conditions of player 1 are identical. The reason for this is the following: If the future is undiscounted, the difference between marginally extending and shortening the inattentiveness period vanishes since the time period for which this difference matters – the present – becomes negligible relative to the future. Of course, this is not the case if $r > 0$, i.e., in this case there are multiple alternating planning equilibria as stated in Part (ii) and the maximum range is the one that is bounded by the marginal "no-deviation" conditions of player 1.

The proposition also implies that $\gamma < 0$ is only a necessary but not a sufficient condition for the existence of an alternating planning equilibrium. For example, one can check numerically that if γ is close to zero, such an equilibrium does not exist since (2.50) can never be satisfied. However, our numerical computations confirm that the range of γ for which alternating planning equilibria exist is quite sizeable, even if r is strictly positive, and often starts at relatively small absolute values of γ, such as -0.1.

Having characterized the equilibria of the game and under which conditions they exist we are now in a position to determine how the equilibrium inattentiveness lengths change with the degree of strategic substitutability. This is of interest since it helps to discuss in more detail how strategic interaction shapes the equilibrium outcome given that there is a finite number of players. In the exposition we focus on the equilibria in which the bounds of the equilibrium range are determined by the marginal deviations of player 1. We obtain the following results:

Proposition 3 *Suppose that $\gamma < 0$ and ρ are so that the range of alternating planning equilibria is non-empty and bounded by the marginal "no-deviation" conditions of firm* 1.
(i) For $r \to 0$, $d_a^\star$ is strictly increasing in $|\gamma|$.

(ii) For r positive but small the equilibrium range of inattentiveness periods, $[\underline{d}_a^1, \bar{d}_a^1]$, becomes strictly larger as $|\gamma|$ increases.

Proof See the Appendix.

The first result states that, in the limit as r tends to zero, $d_a^\star$ increases as $|\gamma|$ increases. Put differently, if the degree of strategic substitutability rises, i.e., γ becomes more negative, firms choose longer inattentiveness lengths in equilibrium. Although we can only show this analytically for $r \to 0$, the result that the equilibrium range of inattentiveness periods shifts upward as $|\gamma|$ increases is for $r > 0$ computationally robust for most parameter constellations.

The comparative static effect with respect to the degree of strategic substitutability differs from the one in Hellwig and Veldkamp (2009). They find that an increase in the degree of strategic substitutability decreases equilibrium inattentiveness lengths. This difference stems from the fact that the degree of strategic substitutability has different implications on the impact of uncertainty in the two models. In their model a firm's objective is to minimize the expected distance between its price and a target price. The latter is a weighted average of the shock realization and the average of the other firms' prices where the weights are determined by the substitutability parameter. Now, as the degree of substitutability increases, the target price puts more weight on the shock realization. Ultimately, this implies that in the alternating planning equilibrium inattentiveness periods become shorter. To the contrary, in our model the equilibrium inattentiveness periods become longer as the degree of strategic substitutability increases. The intuition behind our result is the following. At its planning date firm j will adjust its price to the realization of the shocks. Since $\gamma < 0$ the strategic effect is so that firm j's planning decision renders an uninformed firm's price less inaccurate which reduces firm i's incentive to plan. This effect increases in the degree of strategic substitutability. As a consequence, the firms' inattentiveness length in an alternating planning equilibrium increases in the degree of strategic substitutability.

As mentioned above, one can demonstrate computationally that even if r is strictly positive, an increase in $|\gamma|$ increases both $\underline{d}_a^1$ and $\bar{d}_a^1$. The only exception can occur in the case in which both ρ and γ are sufficiently negative and $\sigma_\xi << \sigma_\theta$. In this case $\underline{d}_a^\star$ decreases in $|\gamma|$. This result is driven by the externality effect. By planning firm j exerts a negative (expected) externality on firm i, and firm i can ameliorate this externality by moving closer to the planning date(s) of firm j. Thus, firm i has a stronger incentive to set a marginally shorter inattentiveness length, and this incentive increases in $|\gamma|$. As a consequence, the lower bound falls as $|\gamma|$ increases.

Finally, the second result of Proposition 3 states that the range of equilibria widens as the degree of strategic substitutability increases. Thus, although both $\underline{d}_a^1$ and $\bar{d}_a^1$ usually increase as $|\gamma|$ increases, $\bar{d}_a^1$ increases by a larger extent than $\underline{d}_a^1$. The intuition behind this result can be easily gained from the discussion following Lemma 5. As pointed out there, the difference between the two bounds stems from the fact that firm j reacts in the future in a different way if firm i shortens or extends its inattentiveness interval. Clearly, the extent of this difference depends on the degree of strategic interaction, i.e., on $|\gamma|$. If $|\gamma|$ becomes larger, firm j reacts more strongly to each deviation. Ultimately, this increases the difference between the two bounds.

After having characterized the conditions for existence of an alternating planning equilibrium, we now turn to the analysis of synchronous planning equilibria.

Synchronized Planning

In order to characterize the set of inattentiveness intervals for which synchronous planning is potentially an equilibrium we set out by considering unilateral marginal deviations. More specifically, we analyze whether a firm has an incentive to deviate from a synchronous planning pattern with an inattentiveness interval of length d by either marginally shortening or extending the current inattentiveness period. The set of synchronous inattentiveness periods for which marginal deviations are not profitable is characterized in the following lemma.

Lemma 9 *Suppose that a synchronized planning equilibrium exists and let $d_s^\star$ denote a common synchronous equilibrium inattentiveness period. Then $d_s^\star$ is bounded above by $\bar{d}_s$, where $\bar{d}_s$ solves*

$$\bar{d}_s\Lambda_1 - rK - \left(\frac{e^{r\bar{d}_s} - 1}{re^{r\bar{d}_s}}\right)\Gamma_1 = 0, \tag{2.51}$$

and $d_s^\star$ is bounded below by $\underline{d}_s$, where $\underline{d}_s$ solves

$$e^{-r\underline{d}_s}\left(\underline{d}_s\Lambda_2 - \frac{\Lambda_1}{r}\right) - \frac{rK}{e^{r\underline{d}_s} - 1} + \frac{\underline{d}_s e^{-r\underline{d}_s}}{e^{r\underline{d}_s} - 1}\Gamma_1 = 0. \tag{2.52}$$

Proof See the Appendix.

It is evident from Lemma 9 that a synchronous planning equilibrium may exist only if $\bar{d} \geq \underline{d}$. We formulate the condition that has to be met so that a synchronous planning equilibrium potentially exists in terms of a threshold correlation stated in the following lemma.

Lemma 10 *If strategy variables are strategic substitutes and the intercept shock is more volatile than the cost shock, i.e., $\gamma < 0$ and $\sigma_\theta > \sigma_\xi$, then $\bar{d}_s \geq \underline{d}_s$ if $-1 \leq \rho \leq \hat{\rho}$, where $\hat{\rho}$ is bounded above by*

$$\rho' = -\frac{\sigma_\theta^2(4-\gamma) + \sigma_\xi^2(4 - 5\gamma + 2\gamma^2)}{2\sigma_\theta\sigma_\xi(4 - 3\gamma + \gamma^2)}. \tag{2.53}$$

If strategy variables are strategic complements and the intercept shock is less volatile than the cost shock, i.e., $\gamma > 0$ and $\sigma_\theta < \sigma_\xi$, then $\bar{d}_s \geq \underline{d}_s$ if $\rho \geq \max\{-1, \hat{\rho}\}$.
If $\gamma > 0$ and $\sigma_\theta > \sigma_\xi$ then $\bar{d}_s \geq \underline{d}_s$ for all $\rho \in [-1, 1]$.
Finally, if $\gamma < 0$ and $\sigma_\theta < \sigma_\xi$ then $\bar{d}_s \leq \underline{d}_s$ for all $\rho \in [-1, 1]$.

Proof See the Appendix.

Put differently, Lemma 10 states that if strategy variables are strategic complements and the intercept shock is less volatile than the cost shock, the synchronous equilibrium does not exist for a sufficiently negative correlation, that is for $-1 \leq \rho \leq \hat{\rho}$. In addition, if strategy variables are strategic substitutes and the intercept shock is more volatile than the cost shock, simultaneous planning can be an equilibrium if $-1 \leq \rho \leq \hat{\rho}$.

The reason for our result can be most easily seen by alluding to the two-stage game analyzed in Section 2.2: the upper bound of the threshold correlation coincides with the condition that identifies the parameter regions in which the externality effect overcompensates the strategic effect. Thus, if firms compete in strategic complements, the expected negative externality that firms exert on each other when planning jointly may be so large that one firm is, for every inattentiveness interval, better off by acquiring information either shortly before or after the other firm. To the contrary, if strategy variables are strategic substitutes, it may be profitable for a firm to synchronize its planning decision because this reduces the expected negative externality that it has to endure.

Lemma 9 and Lemma 10 identify and characterize the candidates for a stationary synchronized planning equilibrium. As mentioned before, the prerequisite in deriving the bounds on the optimal inattentiveness period was that for a given behavior of one firm the other firm may only deviate marginally from the candidate inattentiveness interval. Thus, it remains to analyze whether the candidate equilibria are robust to non-infinitesimal deviations.

Combining the marginal and non-marginal "no-deviation" conditions allows us to state the main result of this subsection:

Proposition 4 *A synchronized planning equilibrium exists for r close to zero if and only if $\gamma > 0$ and $\bar{d}_s > \underline{d}_s$. In such an equilibrium, any (common) inattentiveness period is given by $d_s^\star \in [d_s^l, d_s^u]$, where*

$$d_s^l = \frac{2\sqrt{6}(2-\gamma)\sqrt{Km^\star}}{\sqrt{\Theta(m^\star+1)}}, \tag{2.54}$$

$$d_s^u = \frac{8\sqrt{6}\sqrt{Km^\star}}{\sqrt{\Theta(m^\star+1)}}, \tag{2.55}$$

with

$$\begin{aligned}\Theta &= (\rho\sigma_\theta\sigma_\xi+\sigma_\xi^2)(2+m^\star)2\gamma^3 - ((7+5m^\star)\sigma_\xi^2 + 6\rho\sigma_\theta\sigma_\xi(m^\star+1)+\sigma_\theta^2(m^\star-1))\gamma^2 \\ &\quad +4(\sigma_\theta^2+2\rho\sigma_\theta\sigma_\xi+\sigma_\xi^2)(m^\star-1)(\gamma(m^\star-1)+3), \\ m^\star &= \left[\sqrt{\frac{4(\rho\sigma_\theta\sigma_\xi+\sigma_\xi^2)\gamma^3+(\sigma_\theta^2-6\rho\sigma_\theta\sigma_\xi-7\sigma_\xi^2)+4(\sigma_\theta^2+2\rho\sigma_\theta\sigma_\xi+\sigma_\xi^2)(3-\gamma)}{\gamma(2(\rho\sigma_\theta\sigma_\xi+\sigma_\xi^2)\gamma^2-(\sigma_\theta^2+6\rho\sigma_\theta\sigma_\xi+5\sigma_\xi^2)\gamma+4(\sigma_\theta^2+2\rho\sigma_\theta\sigma_\xi+\sigma_\xi^2))}}\right]\end{aligned} \tag{2.56}$$

that is, $m^\star$ is the closest integer to the right-hand side of (2.56).

Proof See the Appendix.

Proposition 4 states that a synchronous planning equilibrium cannot exist if products are strategic substitutes. Here, the non-marginal deviations eliminate all potential synchronous planning equilibria for $\gamma < 0$. Reversely, when products are strategic complements, the proposition implies that for $r \to 0$ the non-marginal deviations do not exclude the existence of a synchronous planning equilibrium whenever it is robust against the marginal deviations. Instead, the non-marginal deviations only tighten the upper and lower bounds of the range of equilibrium inattentiveness periods, that is, the range of synchronous equilibria shrinks from $[\underline{d}_s, \bar{d}_s]$ to $[d_s^l, d_s^u]$. By continuity, this argument extends to discount rates that lie in a neighborhood of zero.

Though we can show existence of the synchronous planning equilibrium analytically only for r in a neighborhood of zero, numerical simulations suggest that the result stated in Proposition 4 is robust for r strictly larger than zero. More specifically, it turns out that even for $r > 0$, d_s^l and d_s^u remain to be the tightest bounds on the range of common inattentiveness periods that constitute synchronous planning equilibria.

To sum up, our analysis shows that after combining marginal and non-marginal deviations synchronous planning equilibria exist only if $\gamma > 0$ while alternating planning equilibria exist only if $\gamma < 0$. Thus, the prediction of this model for the existence of synchronous and alternating planning equilibria is clear-cut. In addition, both types of equilibria only exist if the correlation between the demand and the cost shock is not too negative.

2.4 Conclusion

In this chapter we considered an infinite-horizon dynamic duopoly model in which firms can choose to costly acquire and process information about the realization of a common demand and a common cost shock. We identify the effects that are decisive for the firms' choice to synchronize or stagger their planning decision. These are the strategic effect and the externality effect. The strategic effect determines if a firm's price, when being inattentive, becomes more or less inaccurate when the rival plans at the same instant. The externality effect determines how a firm, via planning, can change the expected externality that its rival induces. This externality effect is inherent in the nature of a model with a discrete number of players and cannot occur with a continuum of players. We show that due to the combination of these two effects, alternating planning equilibria exist only if products are strategic substitutes while synchronous planning equilibria exist only if products are strategic complements. In addition, we find that in both classes multiple equilibria with different inattentiveness lengths exist. Finally, we show that the equilibrium inattentiveness length tends to be shorter, the larger is the degree of strategic complementarity. This is the case because if firm j plans and adjusts its price in accordance with the shocks, firm i's price when staying inattentive becomes more incorrect, and so it has an incentive to plan earlier.

We restricted our attention to the case of common demand and cost shocks. As mentioned above, this is reasonable if firms operate in the same market area and procure inputs from the same suppliers. However, in some markets firms may face idiosyncratic shocks, i.e., because they buy inputs from different suppliers or, due to location differences, their demands are affected in different ways. An interesting direction for future research would be to allow for such idiosyncratic shocks. In this case, when planning, a firm observes its own shock realizations but cannot fully infer the shock realizations of its rival from this information. It is of interest to analyze how this affects the extent of the strategic and the externality effect and, more specifically, if the equilibrium inattentiveness lengths become shorter or longer, and how this is affected by the degree of strategic complementarity.

A second extension of our analysis could be to consider the case in which firms can coordinate their planning decisions. One could imagine for example that firms are still rivals at the product market at each instant but have sourced out their planning decisions to a third party that acts as a consultant and is payed according to profits. In this case, the third party may act as a collusion device in planning dates. It is then possible to determine if and how the structure of planning dates that maximizes joint profits differs from the equilibrium one. This can give new insights into the interplay of the strategic and the externality effect, in particular, if the effects are favorable or detrimental to firms.

As mentioned in the introduction, the analysis presented in this chapter is the first to study the implications of rational inattention in a model with a finite number of players. To do so we used the formulation of inattentiveness developed in Reis (2006b). In this formulation firms, when deciding to be attentive, become aware of all relevant information. Although extreme, this feature has the advantage of making the model highly tractable. However, a different theory of rational inattention was proposed by Sims (2003). In his model agents cannot attend to all information because of limited capacity. This model was used e.g., by Moscarini (2004) to study optimal sampling of a decision maker and by Mackowiak and Wiederholt (2009) who analyze a model with an infinite number of firms which face an aggregate and an idiosyncratic shock but cannot fully absorb both shocks due to limited capacity. A similar idea as in Mackowiak and Wiederholt (2009) can also be incorporated in our structure, i.e., firms can only absorb the cost or the demand shock at each instant but not both. It is then possible to analyze to which shock firms pay more attention and how this is affected by the competitiveness of the market. However, such a model does not allow us to draw conclusions about the optimal inattentiveness period. Nevertheless, it is of interest to compare the two approaches and to determine if the results concerning the mode of strategic interaction and the equilibrium structure of planning dates obtained in this chapter are also valid under this alternative formulation of rational inattention.

2.5 Appendix

Proof of Lemma 5

Let us consider the following candidate equilibrium in which firms plan in an alternating and sequential order: The inattentiveness length of each firm is given by $d \in \mathbb{R}^+$ and firm i plans exactly in the middle of firm j's inattentiveness period, that is the time that elapses between a planning date of firm i and a planning date of firm j is $d/2$. This sequence of planning dates can only be an equilibrium if an infinitesimal deviation is not profitable for firm i. There are two forms of infinitesimal deviations. The first is that firm i chooses a longer inattentiveness period, that is it deviates to $d' = d + \triangle$, with $\triangle > 0$. The second infinitesimal deviation is that firm i chooses a shorter inattentiveness period $d'' = d - \triangle$.

Firm 1

We start with firm 1. In order to derive the "no-deviation" conditions we have to compare firm $1'$s expected loss from following the proposed equilibrium sequence with the expected losses from the sequences $\mathcal{D}' = \{d', 2d', 3d', ...\}$ and $\mathcal{D}'' = \{d'', 2d'', 3d'', ...\}$.

Expected loss for $\mathcal{D}$

When determining the expected loss from following the equilibrium strategy we have to distinguish between the case where firm 1 is the better informed firm since it was the last one that planned, and the case where it is the worse informed firm since firm 2 was the last to plan. In the first case we know from Lemma 4 that firm $1'$s expected instantaneous loss at any instant t, with $nd \le t < (n + 1/2)d$, for all $n \in \mathbb{N}_0$, is given by

$$E[\mathcal{L}_1^e|I_0] = \Lambda_1 \tau,$$

where τ denotes the time that elapsed since the last planning date, that is $\tau = t - nd$, for all $n \in \mathbb{N}_0$. In the second case, we know from Lemma 4 that firm $1'$s expected instantaneous loss at any instant t, with $(n + 1/2)d \le t < (n + 1)d$, for all $n \in \mathbb{N}_0$, is given by

$$E[\mathcal{L}_2^e|I_0] = \Lambda_1 \tau + \frac{(2+\gamma)^2}{4} \Lambda_1 \frac{d}{2},$$

where τ denotes the time that elapsed since the last planning date of firm 2, that is $\tau = t-(n+1/2)d$, for all $n \in \mathbb{N}_0$.

Calculating the expected loss implied by the candidate equilibrium inattentiveness interval we obtain

$$E[\mathcal{L}(\mathcal{D})|I_0] = \frac{8e^{rd} + rd\gamma(4+\gamma)(e^{rd/2}-1) - 8(1+rd)}{8r^2(e^{rd}-1)}\Lambda_1 + \frac{K}{e^{rd}-1}. \tag{2.57}$$

Expected loss for $\mathcal{D}'$

In order to consistently transform expected profits into expected losses we have to scale the expected stream of profits under the infinitesimal deviations by the equilibrium expected full information profits. For that reason we first write out these full information profits. In case firm 1 is the better informed firm, that is, it has planned at nd, $n \in \mathbb{N}$ while firm 2 has planned at $(n-1/2)d$, $n \in \mathbb{N}$, and we look at an instant t with $nd \leq t < (n+1/2)d$ the per instant full information profit can be written as

$$E[\Pi^{FI}(t)\,|\,nd \leq t < (n+1/2)d] = \frac{(\alpha-(1-\gamma)c)}{(2-\gamma)^2} + \Lambda_1(nd+\tau) \tag{2.58}$$

$$+\gamma\Big(\frac{4(\sigma_\theta^2-\sigma_\xi^2)+2\gamma\rho\sigma_\theta\sigma_\xi+\gamma(3\sigma_\xi^2-\sigma_\theta^2)}{4(2-\gamma)^2}\Big)\frac{nd}{2},$$

where $\tau = t-nd$. Similarly, in case firm 1 has planned at nd, $n \in \mathbb{N}$ while firm 2 has planned at $(n+1/2)d$, $n \in \mathbb{N}$, and we look at an instant t with $(n+1/2)d/2 \leq t < (n+1)d$ the per instant full information profit can be written as

$$E[\Pi^{FI}(t)\,|\,(n+1/2)d/2 \leq t < (n+1)d] = \frac{(\alpha-(1-\gamma)c)}{(2-\gamma)^2} + \Lambda_1((n+1/2)d+\tau) \tag{2.59}$$

$$+\gamma\Big(\frac{\sigma_\theta^2-\sigma_\xi^2+\Lambda_1}{4}\Big)\frac{(2n+1)d}{2} + \gamma^2\Big(\frac{2(\sigma_\theta^2-\sigma_\xi^2)+2\gamma(\sigma_\xi^2+\rho\sigma_\theta\sigma_\xi)+\gamma^2\Lambda_1}{4(2-\gamma)^2}\Big)nd,$$

where $\tau = t-(n+1/2)d$.

Before we turn to the different intervals we introduce some notation. Let $\tilde{m} \in \mathbb{N}$ denote a natural number for which $\tilde{m} \leq d/(2\triangle)$ and $(\tilde{m}+1) > d/(2\triangle)$.

In the following we derive the expected loss from the sequence $\mathcal{D}'$. First, we turn to the expected instantaneous loss that firm 1 incurs in the interval $[d, d']$ if it deviates to the sequence $\mathcal{D}'$.[17]

From d to $d' = d + \triangle$

If firm 1 deviates to $\mathcal{D}'$ then its expected profit function at instant $d+\tau$, $\tau \in [0, \triangle)$ is given by

$$\begin{aligned} E[\Pi(d+\tau)|I_0^1] &= (\alpha - p_1(d+\tau) + \gamma E[p_2(d/2+\tau)|I_0^1])(p_i(d+\tau) - c) \\ &- E[\xi(d+\tau)(\theta(d+\tau) + \gamma p_2(d/2+\tau))|I_0^i]. \end{aligned} \tag{2.60}$$

In this interval firm 1 is the firm that has more outdated information. However, since firm 2 has last planned at $d/2$, it has not yet detected firm $1'$s deviation. Thus, firm 2 still believes that firm 1 plans after d periods and, therefore, 2 thinks that the information set of firm 1 is I_d. Hence, as shown in Lemma 3 firm $2's$ optimal price at $d+\tau$ is given by

$$p_2^\star(d+\tau) = \frac{\alpha + \theta(d/2) + c + \xi(d/2)}{2-\gamma}. \tag{2.61}$$

Therefore, since firm 1 has worse information than firm 2 at instant $d+\tau$, $\tau \in [0, \triangle)$, firm 1's optimal price is

$$p_1^\star(d+\tau) = E[p_2^\star(d+\tau)|I_0^1, I_0^2] = \frac{\alpha + c}{2-\gamma}. \tag{2.62}$$

Using (2.62) and (2.61) in (2.60) yields that firm $1'$s expected instantaneous profit under $\mathcal{D}'$ in the interval $t \in [d, d']$ is given by

$$E[\Pi^\star(d+\tau)|I_0^1, I_0^2] = -\frac{\gamma(2\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2)}{2-\gamma} d - \rho\sigma_\theta\sigma_\xi\tau + \frac{(\alpha - (1-\gamma)c)}{(2-\gamma)^2}. \tag{2.63}$$

Subtracting (2.63) from (2.58) with $n = 1$ yields that firm $1'$s expected instantaneous loss in the period $[d, d+\triangle)$ is

$$E[\mathcal{L}_1'|I_0] = \Lambda_1(d+\tau) + \frac{\gamma(4-\gamma)}{4}\Lambda_2\frac{d}{2}, \tag{2.64}$$

[17] Note that the expected loss from the candidate equilibrium sequence $\mathcal{D}$ and from the deviation sequence $\mathcal{D}'$ in the interval $t \in [0, d)$ is the same since $\mathcal{D}'$ means that planning is postponed by $\triangle$.

with $\tau \in [0, \triangle)$.

From d' to $3d/2$

In this time interval firm 2 still believes that firm 1 sticks to the proposed equilibrium strategy. Thus, its optimal price is at each instant given by (2.61). Firm 1 acquires new information at instant $d' = d + \triangle$. Therefore, its expected profit function in this interval is given by

$$\begin{aligned} E[\Pi(d'+\tau)|I^1_{d'}] &= (\alpha - p_1(d'+\tau) + \gamma p_2(d/2+\tau))(p_1(d'+\tau) - c) \\ &- E[\xi(d'+\tau)(\theta(d'+\tau) + \gamma p_2(d/2+\tau))|I^1_{d'}], \end{aligned} \tag{2.65}$$

with $\tau \in [0, d/2 - \triangle)$. Proceeding in the same way as before, we obtain that

$$p_1^\star(d'+\tau) = \frac{\alpha + \theta(d') + c + \xi(d')}{2} + \frac{\gamma\Big(\alpha + \theta(d) + c + \xi(d)\Big)}{2(2-\gamma)} \tag{2.66}$$

Using (2.66) and (2.61) in (2.65), taking expectations and subtracting the resulting expression from (2.58) with $n = 1$ yields for the interval $[d', 3d/2)$ the following instantaneous loss function

$$E[\mathcal{L}_2'|I_0] = \Lambda_1 \tau, \tag{2.67}$$

with $\tau \in [0, d/2 - \triangle)$.

From $3/2d$ to $2d$

At $3d/2$ firm 2 detects that firm 1 deviated to the sequence $\mathcal{D}'$. This implies that firm 2 knows that firm $1'$s consecutive planning dates will be at nd', for $n \in \mathbb{N} \setminus \{1\}$. As firm 2 is better informed in the interval $[3/2d, 2d)$ than firm 1, the latter's expected profit function equals

$$\begin{aligned} E[\Pi(3d/2+\tau)|I^1_{d'}] &= (\alpha - p_1(3d/2+\tau) + \gamma E[p_2(3d/2+\tau)|I^1_{d'}](p_1(3d/2+\tau) - c) \\ &- E[\xi(3d/2+\tau)(\theta(3d/2+\tau) + \gamma p_j(3d/2+\tau))|I^1_{d'}], \end{aligned} \tag{2.68}$$

with $\tau \in [0, d/2)$. As a consequence the firms' optimal prices in this period are given by

$$p_2^\star(3d/2+\tau) = \frac{\alpha+\theta(3d/2)+c+\xi(3d/2)}{2} + \frac{\gamma\Big(\alpha+\theta(d')+c+\xi(d')\Big)}{2(2-\gamma)}, \tag{2.69}$$

$$p_1^\star(3d/2+\tau) = E[p_2^\star(3d/2+\tau)|I_{d'}^1] = \frac{\alpha+\theta(d')+c+\xi(d')}{2-\gamma}. \tag{2.70}$$

Using (2.70) and (2.69) in (2.68), taking expectations and subtracting the resulting expression from the corresponding full information profit, which is given by (2.59) with $n=1$, yields the instantaneous expected loss in this interval, which is given by

$$E[\mathcal{L}_3'|I_0] = \Lambda_1\frac{(2+\gamma)^2}{4}\frac{d}{2} - \Gamma_1\triangle + \Lambda_1\tau, \tag{2.71}$$

with $\tau \in [0, d/2)$.

Due to the fact that firm 2 knows firm $1'$s consecutive planning dates, the instantaneous expected loss in each interval $[(n+1/2)d, (n+1)d]$ is given by

$$E[\mathcal{L}_3'|I_0] = \Lambda_1\frac{(2+\gamma)^2}{4}\frac{d}{2} - \Gamma_1 n\triangle + \Lambda_1\tau,$$

with $\tau \in [0, d/2)$ and $n \in \{2, ..., \tilde{m}\}$.

From $2d$ to $2d' = 2(d+\triangle)$

Since firm 2 detects at $3d/2$ that firm 1 deviated and firm 2 does not acquire new information between $2d$ to $2d'$, the expected profit that firm 1 earns between $2d$ to $2d'$ is again given by (2.68) and the optimal prices are as in (2.69) and (2.70). However, when determining the expected loss in this interval, we need to use the full information profit in (2.58) with $n=2$. Doing so we obtain

$$E[\mathcal{L}_4'|I_0] = \Gamma_2 d - \Gamma_1\triangle + \Lambda_1\tau, \tag{2.72}$$

with $\tau \in [0, 2\triangle)$.

As above, since firm 2 knows firm $1'$s consecutive planning dates, the instantaneous expected loss in each interval $[nd, nd']$, for $n \in \{2, ..., \tilde{m}\}$, is given by

$$E[\mathcal{L}_4'|I_0] = \Gamma_2 d - \Gamma_1 n\triangle + \Lambda_1\tau,$$

with $\tau \in [0, n\triangle)$ and $n \in \{2, ..., \tilde{m}\}$.

From $nd' = n(d + \triangle)$ **to** $(n + 1/2)d$**, for** $n \in \{2, ..., \tilde{m}\}$

In these intervals firm $2'$s information set is more outdated. This was also the case in the interval between d' and $3d/2$. But from Lemma 2 and Lemma 3 we know that the firm with the worse information sets the same price independent of the exact planning date of the better informed firm. Therefore, the expected instantaneous loss of firm 1 is in every interval $[nd', (n + 1/2)d)$, for $n \in \{2, ..., \tilde{m}\}$, given by

$$E[\mathcal{L}_5'|I_0] = \Lambda_1\tau, \tag{2.73}$$

with $\tau \in [0, d - n\triangle)$ and $n \in \{2, ..., \tilde{m}\}$.

We can now compare the expected loss from the sub-sequence $\mathcal{D}'_{\tilde{m}} = \{d', ..., \tilde{m}d'\}$ with the one of the equilibrium sequence. The expected loss from the sub-sequence $\mathcal{D}'_{\tilde{m}}$ is given by

$$\begin{aligned} E[\mathcal{L}'(\mathcal{D}'_{\tilde{m}})|I_0] \;&=\; e^{-rd}\int_{\tau=0}^{\triangle} e^{-r\tau}\Big(\Lambda_1(d+\tau) + \frac{\gamma(4-\gamma)}{4}\Lambda_2\frac{d}{2}\Big)\,d\tau + K\sum_{n=1}^{\tilde{m}} e^{-rnd'} \\ &+\; \sum_{n=1}^{\tilde{m}}\Bigg(e^{-rnd'}\int_{\tau=0}^{d/2-n\triangle} e^{-r\tau}\Lambda_1\tau\,d\tau\Bigg) \\ &+\; \sum_{n=1}^{\tilde{m}}\Bigg(e^{-r(n+1/2)d}\int_{\tau=0}^{d/2} e^{-r\tau}\Big(\Lambda_1\frac{(2+\gamma)^2}{4}\frac{d}{2} - \Gamma_1 n\triangle + \Lambda_1\tau\Big)d\tau\Bigg) \\ &+\; \sum_{n=2}^{\tilde{m}}\Bigg(e^{-rnd}\int_{\tau=0}^{n\triangle} e^{-r\tau}\Big(\Gamma_2 d - \Gamma_1 n\triangle + \Lambda_1\tau\Big)d\tau\Bigg). \end{aligned} \tag{2.74}$$

Subtracting the expected loss implied by the equilibrium sub-sequence $\mathcal{D}_{\tilde{m}} = \{d, ..., \tilde{m}d\}$, denoted by $E[\mathcal{L}(\mathcal{D}_{\tilde{m}})|I_0]$, from (2.74) yields

$$\begin{aligned} &e^{-rd}\int_{\tau=0}^{\triangle} e^{-r\tau}\Big(\Lambda_1 d + \frac{\gamma(4-\gamma)}{4}\Lambda_2\frac{d}{2}\Big)\,d\tau + K\Big(\sum_{n=1}^{\tilde{m}} e^{-r(nd')} - \sum_{n=1}^{\tilde{m}} e^{-r(nd)}\Big) \\ &-\sum_{n=1}^{\tilde{m}}\Bigg(e^{-rnd'}\int_{\tau=0}^{d/2-n\triangle} e^{-r\tau}\Lambda_1 n\triangle\,d\tau\Bigg) - \sum_{n=1}^{\tilde{m}}\Bigg(e^{-r(n+1/2)d}\int_{\tau=0}^{d/2} e^{-r\tau}\Big(\Gamma_1 n\triangle\Big)d\tau\Bigg) \\ &+\sum_{n=2}^{\tilde{m}}\Bigg(e^{-rnd}\int_{\tau=0}^{n\triangle} e^{-r\tau}\Big(\Gamma_2 d - \Gamma_1 n\triangle\Big)d\tau\Bigg) + e^{-r\tilde{m}d}\Upsilon, \end{aligned} \tag{2.75}$$

where Υ denotes the difference in expected losses beyond date $\tilde{m}d$. This difference in expected losses is at each instant $t > \tilde{m}d$ bounded below by the expected instantaneous loss implied by the sequence $\mathcal{D}$. We get this by assuming the best possible case for the deviation, that is, the expected instantaneous loss that is implied by the sequence $\mathcal{D}'$ is zero from date $\tilde{m}d$ onwards. In addition, we know that the instantaneous loss from the sequence $\mathcal{D}$ is finite since firm i optimally chooses to plan after some time length. Thus, at each instant $t > \tilde{m}d$ the difference in expected losses is bounded below by

$$-e^{-r\tau}\left(1+\frac{(2+\gamma)^2}{4}\right)\Lambda_1\frac{d}{2}$$

for $\tau \in (0, d]$ and by

$$-e^{-rd}K$$

at a planning date d. Both expressions are finite because d is finite.

In order to determine the per instant difference in expected losses in the time period before date $\tilde{m}d$, we divide the first five terms in (2.75) by $n\triangle$, where n is chosen appropriately for the different intervals. Then, we take the limit $\triangle \to 0$. The last term in (2.75) $e^{-r\tilde{m}d}\Upsilon$ vanishes as $\triangle \to 0$ implies that $\tilde{m} \to \infty$ since the future is discounted at rate $r > 0$. Therefore, we can concentrate on the first five terms when determining the critical inattentiveness length such that deviating to the sequence $\mathcal{D}'$ is profitable.

We obtain that the loss from deviating to $\mathcal{D}'$ is lower than the lifetime expected loss from the proposed equilibrium sequence $\mathcal{D}$ if $d \leq \underline{d}_a^1$, where $\underline{d}_a^1$ solves

$$\frac{\underline{d}_a'\Lambda_2(8-4\gamma+\gamma^2)}{8e^{r\underline{d}_a'}} - \frac{re^{r\underline{d}_a'}K}{(e^{r\underline{d}_a'}-1)^2} - \frac{\Lambda_1 e^{r\underline{d}_a'/2}+\Gamma_1}{r(e^{r\underline{d}_a'}-1)(e^{r\underline{d}_a'/2}+1)} + \frac{\Gamma_2(2e^{r\underline{d}_a'}-1)}{e^{r\underline{d}_a'}(e^{r\underline{d}_a'}-1)^2} = 0. \quad (2.76)$$

We now turn to existence and uniqueness of $\underline{d}_a^1$. Consider first the case of $\underline{d}_a' \to 0$. In this case the second and the fourth term of the left-hand side of (2.76) are the dominating terms. Since $rK > 0$, the second term goes to $-\infty$ as $\underline{d}_a' \to 0$ while the fourth term goes to ∞ or $-\infty$ dependent on the sign of Γ_2. So we have to determine which of the two terms tends to the extreme value at a faster rate. To do so we differentiate the two terms with respect to $\underline{d}_a'$. Here we obtain

$$\frac{r^2Ke^{r\underline{d}_a'}(e^{r\underline{d}_a'}+1)}{(e^{r\underline{d}_a'}-1)^3} \quad (2.77)$$

for the second term and

$$-\frac{\Gamma_2\left(3e^{r\underline{d}'_a}-2e^{2r\underline{d}'_a}-1+r\underline{d}'_a(1+4e^{2r\underline{d}'_a}-3e^{r\underline{d}'_a})\right)}{8e^{r\underline{d}'_a}(e^{r\underline{d}'_a}-1)^3} \tag{2.78}$$

for the fourth term. Since the numerator of (2.78) goes to zero as $\underline{d}'_a \to 0$ while this is not the case for (2.77), we have that the second term tends to $-\infty$ at a faster rate than the fourth term to ∞ or $-\infty$. As a consequence, the left-hand side of (2.76) goes to $-\infty$ as $\underline{d}'_a \to 0$. Conversely, if $\underline{d}'_a \to \infty$, the left-hand side of (2.76) goes to 0 from above. This is the case because the last three terms go to zero at a faster rate than the first term, and the first term is strictly positive since $\Lambda_2 > 0$. Thus, there exists a solution to (2.76) at which $\underline{d}'_a > 0$. It remains to show that this solution is unique. To do so we differentiate (2.76) with respect to $\underline{d}'_a$ to get

$$-\frac{\underline{d}'_a\Lambda_2(8-4\gamma+\gamma^2)(r\underline{d}'_a-1)}{8e^{r\underline{d}'_a}}+\frac{r^2Ke^{r\underline{d}'_a}(e^{r\underline{d}'_a}+1)}{(e^{r\underline{d}'_a}-1)^3}+\frac{\Lambda_1e^{r\underline{d}'_a/2}\left(e^{r\underline{d}'_a}+e^{3r\underline{d}'_a/2}\right)}{2(e^{r\underline{d}'_a}-1)^2(e^{r\underline{d}'_a/2}+1)^2} \tag{2.79}$$

$$+\frac{\Gamma_1e^{r\underline{d}'_a/2}\left(3e^{r\underline{d}'_a/2}-1\right)}{2(e^{r\underline{d}'_a}-1)^2(e^{r\underline{d}'_a/2}+1)}-\frac{\Gamma_2\left(3e^{r\underline{d}'_a}-2e^{2r\underline{d}'_a}-1+r\underline{d}'_a(1+4e^{2r\underline{d}'_a}-3e^{r\underline{d}'_a})\right)}{8e^{r\underline{d}'_a}(e^{r\underline{d}'_a}-1)^3}.$$

Now, for $\underline{d}'_a \to 0$, (2.79) goes to ∞ since the second term dominates the remaining terms, while for $\underline{d}'_a \to \infty$, (2.79) goes to 0 from below. This is the case because the first term goes to zero at a slower rate than the other terms, and this term is negative. We now look at the five terms of (2.79) in turn. The first term changes its sign from positive to negative as $\underline{d}'_a$ increases. This, occurs at $\underline{d}'_a = 1/r$. The second and the third are strictly positive and strictly decrease as $\underline{d}'_a$ rises. The fourth and the fifth term are either positive or negative for any $\underline{d}'_a$, depending on the signs of Γ_1 and Γ_2. As the second and the third terms, they become strictly smaller in absolute value as $\underline{d}'_a$ rises, and they do so at a faster rate than the second term. Thus, we have that the first term is the only term in (2.79) that changes its sign as $\underline{d}'_a$ increases. In addition, this term becomes the dominant one as $\underline{d}'_a$ gets larger and larger. This is the case because the numerator of the first term includes $(\underline{d}'_a)^2$ and the denominator is $e^{r\underline{d}'_a}$. This is not the case for any other term. As a consequence, there exists a unique value of $\underline{d}'_a$ at which (2.79) changes its sign from positive to negative. But this, in combination with the fact that the left-hand side of (2.79) is negative at $\underline{d}'_a = 0$ and positive at $\underline{d}'_a \to \infty$, implies that there must exist a unique solution to (2.79).

As a consequence, we have that if $d \geq \underline{d}_a^1$ then the loss from deviating to $\mathcal{D}'$ is larger than the lifetime expected loss from the proposed equilibrium sequence $\mathcal{D}$.

Expected loss for $\mathcal{D}''$

From $d'' = d - \triangle$ to d

In this interval, firm 2 believes that firm 1 will acquire new information at d. However, firm 1 updates its information set at d'' and is therefore the better informed firm. As a consequence the firm $1'$s expected profit function in this interval is given by

$$\begin{aligned} E[\Pi(d''+\tau)|I_{d''}] &= (\alpha - p_1(d''+\tau) + \gamma p_2(d''+\tau))(p_1(d''+\tau) - c) \\ &- E[\xi(d''+\tau)(\theta(d''+\tau) + \gamma p_2(d''+\tau))]. \end{aligned} \tag{2.80}$$

Proceeding in a similar way as before, we derive the firms' optimal prices

$$p_1^\star(d''+\tau) = \frac{\alpha + c + \theta(d'') + \xi(d'')}{2-\gamma} + \frac{\gamma((2-\gamma)(\theta(d/2) + \xi(d/2) - 2(\theta(d'') + \xi(d'')))}{4(2-\gamma)}, \tag{2.81}$$

$$p_2^\star(d''+\tau) = \frac{2(\alpha + c + \theta(d/2) + \xi(d/2)) - \gamma(\tau(d/2) + \xi(d/2))}{2(2-\gamma)}. \tag{2.82}$$

Using (2.81) and (2.82) in (2.80), taking expectations and subtracting the resulting expression from the corresponding expected full information profit, which is obtained by replacing n by 0 in (2.59), yields that the expected instantaneous loss in this interval is given by

$$E[\mathcal{L}_1''|I_0] = \Lambda_1 \tau. \tag{2.83}$$

From d to $3/2d$

In this interval firm 2 expects that firm 1 has planned at d—although firm 1 in fact plans at d''—and so it expects to be the worse informed firm. Therefore, the profit of firm 1 is the same as in (2.74) but the price of firm 2 has changed. Here we get that

$$p_1^\star(d''+\tau) = \frac{\alpha + c + \theta(d'') + \xi(d'')}{2-\gamma} + \frac{\tau(d/2) + \xi(d/2) - \theta(d'') - \xi(d''}{2(2-\gamma)}, \tag{2.84}$$

$$p_2^\star(d''+\tau) = \frac{\alpha + c + \theta(d/2) + \xi(d/2)}{2-\gamma}. \tag{2.85}$$

In the same way as above we can calculate the expected instantaneous loss to get

$$E[\mathcal{L}_2''|I_0] = \Lambda_1(\tau + \triangle). \tag{2.86}$$

From $(n + 1/2)d$ **to** $(n + 1)(d - \triangle)$**, with** $n \in \mathbb{N}$

In this intervals firm 2 realized that firm 1 deviated. The profit function of firm 1 is then given by

$$\begin{aligned} E[\Pi(d'' + \tau)|I_{d''}] &= (\alpha - p_1(d'' + \tau) + \gamma E[p_2(d'' + \tau)|I_{d''}])(p_1(d'' + \tau) - c) \\ &- E[\xi(d'' + \tau)(\theta(d'' + \tau) + \gamma p_2(d'' + \tau))|I_{d''}]. \end{aligned} \tag{2.87}$$

Optimal prices in this case are

$$p_1^\star = \frac{\alpha + c + \theta(n(d - \triangle)) + \xi(n(d - \triangle))}{2 - \gamma} \tag{2.88}$$

and

$$p_2^\star(d'' + \tau) = \frac{\alpha + c}{2 - \gamma} + \frac{\theta(n(d - \triangle)) + \xi(n(d - \triangle)) + \theta((n + 1/2)d) + \xi((n + 1/2)d)}{2(2 - \gamma)}, \tag{2.89}$$

which gives an expected loss of

$$E[\mathcal{L}_3''|I_0] = \Gamma_1 n\triangle + \Lambda_1\tau + \frac{(2 + \gamma)^2}{4}\Lambda_1\frac{d}{2}. \tag{2.90}$$

From $(n + 1)(d - \triangle)$ **to** $(n + 1)d$**, with** $n \in \mathbb{N}$

In contrast to the interval from d'' to d, in the intervals from $(n+1)(d-\triangle)$ to $(n+1)d$ firm 2 realized that firm i deviated. The profit function of firm 1 can then be written as

$$\begin{aligned} &E[\Pi((n + 1)(d - \triangle) + \tau)|I_{(n+1)(d-\triangle)}] = \\ = \;& (\alpha - p_1((n + 1)(d - \triangle) + \tau) + \gamma p_2((n + 1)(d - \triangle) + \tau))(p_1((n + 1)(d - \triangle) + \tau) - c) \\ - \;& E[\xi((n + 1)(d - \triangle) + \tau)(\theta((n + 1)(d - \triangle) + \tau) + \gamma p_2((n + 1)(d - \triangle) + \tau))]. \end{aligned} \tag{2.91}$$

Calculating optimal prices in the same way as above yields

$$p_1^\star = \frac{\alpha+c}{2-\gamma} + \frac{\tau((n+1/2)d) + \xi((n+1/2)d) + \theta((n+1)(d-\triangle)) + \xi((n+1)(d-\triangle))}{2(2-\gamma)} \tag{2.92}$$

and

$$p_2^\star(d''+\tau) = \frac{\alpha + c + \theta((n+1/2)d) + \xi((n+1/2)d)}{2-\gamma}. \tag{2.93}$$

We then obtain an expected instantaneous loss of

$$E[\mathcal{L}_4''|I_0] = \Lambda_1\tau + \gamma^2\Big(\gamma^2\frac{\Lambda_2}{16} + \frac{\sigma_\xi^2 - \sigma_\theta^2 - \gamma(\sigma_\xi^2 + \rho\sigma_\theta\sigma_\xi)}{2(2-\gamma)^2}\Big)\frac{d}{2}. \tag{2.94}$$

From $(n+1)d$ to $(n+3/2)d$, with $n \in \mathbb{N}$

The difference in these intervals to the interval from d to $3/2d$ is just that firm 2 knows that firm 1 has deviated. However, since firm 2 always expects to be the worse informed firm in these intervals, we know from Lemma 2 and 3 that the price of firm 2 is the same independent of the fact that it is aware of the deviation or not. As a consequence, the expected instantaneous loss is given by

$$E[\mathcal{L}_5''|I_0] = \Lambda_1(\tau + (n+1)\triangle). \tag{2.95}$$

In the same way as in the case of $\mathcal{D}'$ we can now calculate the expected loss from the sequence $\mathcal{D}''$. Subtracting the expected loss of the equilibrium sequence from the expected loss from the sequence $\mathcal{D}''$, dividing the difference by $n\triangle$ and taking the limit $\triangle \to 0$ yields that the expected loss from deviating to $\mathcal{D}''$ exceeds the lifetime expected loss from the proposed equilibrium sequence $\mathcal{D}$ if $d \leq \bar{d}_a^1$, where $\bar{d}_a^1$ solves

$$\frac{\bar{d}_a'\Lambda_1(8+4\gamma+\gamma^2)}{8e^{r\bar{d}_a'}} - \frac{re^{r\bar{d}_a'}K}{(e^{r\bar{d}_a'}-1)^2} - \frac{\Lambda_1 e^{r\bar{d}_a'/2} + \Gamma_1}{r(e^{r\bar{d}_a'}-1)(e^{r\bar{d}_a'/2}+1)} + \frac{\Gamma_2(2e^{r\bar{d}_a'}-1)}{e^{r\bar{d}_a'}(e^{r\bar{d}_a'}-1)^2} = 0. \tag{2.96}$$

We can show existence and uniqueness in exactly the same way as they were shown for $\underline{d}_a$.

Firm 2

We now turn to firm 2. In an asynchronous equilibrium, firm 2 chooses the sequence $\mathcal{D} = \{d/2, 3d/2, 5d/2, ...\}$. Thus, when considering marginal deviations we need to check that the firm has no incentive to deviate to the sequences $\mathcal{D}' = \{(d+\triangle)/2, 3(d+\triangle)/2, 5(d+\triangle)/2, ...\}$ and $\mathcal{D}'' = \{(d-\triangle)/2, 3(d-\triangle)/2, 5(d-\triangle)/2, ...\}$. We can now proceed in the same way as above, namely first calculating the instantaneous expected loss if firm 2 chooses the sequence $\mathcal{D}$ and compare it with the one of the sequences $\mathcal{D}'$ and $\mathcal{D}''$ and then let $\triangle \to 0$.

Again we start with the equilibrium candidate sequence $\mathcal{D}$. By calculations in the same way as for firm 1 we obtain that the expected instantaneous loss between time 0 and $d/2$ is given by

$$E[\mathcal{L}_1^e|I_0] = \Lambda_1\tau, \tag{2.97}$$

where τ denotes the time that elapsed since date 0. Similarly, the expected instantaneous loss at t with $(n+1/2)d \leq t < (n+1)d$, $n \in \mathbb{N}_0$ is also given by (2.97) with $\tau = t-(n+1/2)d$. Finally, the expected instantaneous loss at t with $(n+1)d \leq t < (n+3/2)d$, $n \in \mathbb{N}_0$ for all $n \in \mathbb{N}_0$, is given by

$$E[\mathcal{L}_2^e|I_0] = \Lambda_1\tau + \frac{(2+\gamma)^2}{4}\Lambda_1\frac{d}{2},$$

where τ denotes the time that elapsed since the last planning date of firm 1, that is $\tau = t-(n+1)d$.

Now let us look at the deviation sequence $\mathcal{D}'$. As above in this case the expected instantaneous loss between time 0 and $(d+\triangle)/2$ is given by $E[\mathcal{L}_1'|I_0] = \Lambda_1\tau$ with $\tau = t$ and the expected instantaneous loss at t, with $(d+\triangle)/2 \leq t < d$ is given by $E[\mathcal{L}_2'|I_0] = \Lambda_1\tau$ with $\tau = t - 1/2(d+\triangle)$. Now at $t = d$ firm 1 observes that firm 2 has deviated to $(d+\triangle)/2$. Thus, is it will update its belief that the new sequence of firm 2 is $\mathcal{D}'$ and sets its prices accordingly. As was calculated above the instantaneous expected loss of firm 2 from d to $3d/2$ is given by

$$E[\mathcal{L}_3'|I_{d'}^1, I_{3d/2}^2] = \Lambda_1\frac{(2+\gamma)^2}{4}\frac{d}{2} - \Gamma_1\frac{\triangle}{2} + \Lambda_1\tau, \tag{2.98}$$

with $\tau \in [0,d)$, and, more generally, the instantaneous expected from nd to $(n+1/2)d$, $n \in \mathbb{N}$ can be written as

$$E[\mathcal{L}_3'|I_0] = \Lambda_1\frac{(2+\gamma)^2}{4}\frac{d}{2} - \Gamma_1\left(n+\frac{1}{2}\right)\triangle + \Lambda_1\tau. \tag{2.99}$$

Finally, from $(n+1/2)d$ to $(n+1/2)(d+\triangle)$, $n \in \mathbb{N}$, the instantaneous expected loss of firm 2 is given by

$$E[\mathcal{L}_4'|I_0] = \Gamma_2 d - \Gamma_1 \left(n + \frac{1}{2}\right) \triangle + \Lambda_1 \tau, \tag{2.100}$$

with $\tau \in [0, (n+1/2)\triangle)$.

We can now compare the difference between the expected loss from the sequence $\mathcal{D}$ with the one of $\mathcal{D}'$. As above, dividing the respective losses by $(n+1/2)\triangle$, where $n \in \mathbb{N}_0$ is appropriately chosen for the respective intervals, and then taking the limit $\triangle \to 0$, we obtain that the loss from deviating to $\mathcal{D}'$ exceeds the lifetime expected loss from the proposed equilibrium sequence $\mathcal{D}$ if $d \geq \underline{d}_a^2$, where $\underline{d}_a^2$ solves

$$\frac{\Lambda_1 \left(\underline{d}_a' r(e^{r\underline{d}_a'} - 1) - 2e^{r\underline{d}_a'/2}\left(e^{r\underline{d}_a'/2} - 1\right)\right)}{2r(e^{r\underline{d}_a'} - 1)} - \frac{rKe^{r\underline{d}_a'}}{e^{r\underline{d}_a'} - 1} - \frac{\Gamma_1(e^{r\underline{d}_a'} - 1)}{r\left(e^{r\underline{d}_a'/2} - 1\right)} - \tag{2.101}$$

$$-\Gamma_2 \underline{d}_a' \left(\frac{1}{e^{r\underline{d}_a'}(e^{r\underline{d}_a'} - 1)} + \frac{\ln(1 - e^{-r\underline{d}_a'/2}) - \ln(1 + e^{-r\underline{d}_a'/2})}{e^{-r\underline{d}_a'/2}}\right) = 0.$$

Existence and uniqueness of $\underline{d}_a^2$ follow from similar arguments as in the case of firm 1.

Turning to the comparison of $\mathcal{D}$ with $\mathcal{D}''$ we can proceed exactly in the same way as in above, i.e., first calculating the expected instantaneous losses for the two sequences, then dividing the respective losses by $(n+1/2)\triangle$, where $n \in \mathbb{N}_0$ and then taking the limit $\triangle \to 0$. Here we obtain that the expected loss from deviating to $\mathcal{D}''$ exceeds the expected loss from the proposed equilibrium sequence $\mathcal{D}$ if $d \leq \bar{d}_a^2$, where $\bar{d}_a^2$ solves

$$\frac{\Lambda_1 \bar{d}_a'}{2e^{r\bar{d}_a'}} - \frac{\Lambda_1 e^{r\bar{d}_a'/2}(e^{r\bar{d}_a'/2} - 1)}{r(e^{r\bar{d}_a'} - 1)} + \frac{\Gamma_1(2 + r\bar{d}_a' - 2e^{r\bar{d}_a'/2})}{2re^{r\bar{d}_a'/2}(e^{r\bar{d}_a'/2} - 1)} - \frac{rKe^{r\bar{d}_a'}}{e^{r\bar{d}_a'} - 1} = 0. \tag{2.102}$$

Existence and uniqueness of $\bar{d}_a^2$ follow from similar arguments as in the case of firm 1. ■

Proof of Lemma 6

Suppose first that $\underline{d}_a^1 = \bar{d}_a^1 = d$. Subtracting the left-hand side of (2.45) from the left-hand side of (2.44) we obtain

$$\frac{\gamma^2 d e^{dr}(8 - \gamma^2)(\sigma_\theta^2 + 2\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2)}{32}.$$

Thus, the two sides are equal if and only if $\gamma = 0$ or $\rho = -(\sigma_\theta^2 + \sigma_\xi^2)/(2\sigma_\theta\sigma_\xi)$, where $-(\sigma_\theta^2 + \sigma_\xi^2)/(2\sigma_\theta\sigma_\xi) \leq -1$. But the latter equality can never be fulfilled since $\rho \geq -1$, and for $\rho = -1$ and $\sigma_\theta^2 = \sigma_\xi^2$, i.e., the case in which $-(\sigma_\theta^2 + \sigma_\xi^2)/(2\sigma_\theta\sigma_\xi) = -1$. we have that firms never plan since their profit at each instant is certain due to the perfect negative correlation of the shocks. Since planning only incurs costs, they have no incentive to plan. Hence, $\underline{d}_a^1 = \bar{d}_a^1$ if and only if $\gamma = 0$.

Now let us look at the case in which $\gamma \neq 0$. Here we know that the left-hand side of (2.45) is lower than the left-hand side of (2.44) if $\underline{d}_a^1 = \bar{d}_a^1$. From the proof of Proposition 1 we also know that the derivatives of the left-hand sides of (2.44) and (2.45) at their respective solutions are strictly positive. But from this it follows that $\bar{d}_a^1 > \underline{d}_a^1$ for all $\gamma \neq 0$.

We can conduct a similar analysis for firm 2. Suppose that $\underline{d}_a^2 = \bar{d}_a^2 = d$. By subtracting the left-hand side of (2.46) from the left-hand side of (2.47) and simplifying we obtain that this can only occur if

$$-\frac{\Lambda_1}{2(1-e^{-rd})} - \frac{\Gamma_2\left(\ln(1+e^{-rd/2}) - \ln(1-e^{-rd/2})\right)}{e^{-rd/2}} = 0, \tag{2.103}$$

The first term on the left-hand side of (2.103) is negative while the second one is negative if Γ_2 is positive. Solving $\Gamma_2 = 0$ for ρ wo obtain that $\Gamma_2 \geq 0$ if

$$\rho \geq -\frac{4(\sigma_\theta^2 + \sigma_\xi^2)(2-\gamma) + \gamma^2(\sigma_\theta^2 - 3\sigma_\xi^2 + 2\gamma\sigma_\xi^2)}{2\sigma_\theta\sigma_\xi(8 - 4\gamma - \gamma^2 + \gamma^3)}. \tag{2.104}$$

It is easy to check that the right-hand side of (2.104) is larger than -1 if and only if $\sigma_\xi > \sigma_\theta$. Thus, for $\sigma_\theta > \sigma_\xi$, (2.104) is for sure fulfilled which implies that the second term on the left-hand side of (2.103) is negative. Thus, in this case $\underline{d}_a^2$ can not be equal to $\bar{d}_a^2$. Since the left-hand side of (2.47) is increasing in $\bar{d}_a^2$ and the left-hand side of (2.46) is increasing in $\underline{d}_a^2$, we obtain that in this case $\bar{d}_a^2 > \underline{d}_a^2$ because the difference between (2.47) and (2.46) at $\bar{d}_a^2 = \underline{d}_a^2$ is negative. If, on the other hand, $\sigma_\xi > \sigma_\theta$, (2.104) may not fulfilled. Then, the left-hand side of (2.103) consists of two countervailing terms. In this case $\bar{d}_a^2$ is smaller than $\underline{d}_a^2$ if $\rho < \hat{\rho}$, where $\hat{\rho}$ is the solution to (2.103). Since ρ is bounded below by -1, the result stated in the lemma follows.

It remains to show that $\hat{\rho}$ is the unique solution to (2.103). To do so we first use equations (2.47) and (2.46) the build the implicit functions $d\bar{d}_a^2/d\rho$ and $d\underline{d}_a^2/d\rho$. Then subtracting $d\underline{d}_a^2/d\rho$ from $d\bar{d}_a^2/d\rho$ and evaluating it at $\underline{d}_a^2 = \bar{d}_a^2 = d$, we obtain

$$\frac{\sigma_\theta \sigma_\xi d(1-e^{-rd})}{\Lambda_1(2e^{-rd} + rd\,(3-e^{-rv}))} \tag{2.105}$$

$$+\frac{d(1-e^{-rd})(\sigma_\theta\sigma_\xi(8-4\gamma-\gamma^2+\gamma^3))\left(\ln(1+e^{-rd/2})-\ln(1-e^{-rd/2})\right)}{4\Gamma_2(2-\gamma)^2\left((\ln(1+e^{-rd/2})-\ln(1-e^{-rd/2}))\,(1+rd)(e^{-rd}-1)-rde^{-rd/2}\right)}.$$

The first term of (2.105) is positive while the second term is positive only if $\Gamma_2 < 0$. But we know from above that $\underline{d}_a^2 = \bar{d}_a^2$ can only occur if (2.104) is not fulfilled which implies that $\Gamma_2 < 0$. Therefore, at $\underline{d}_a^2 = \bar{d}_a^2$ we have that both terms of (2.105) are positive which implies that $d\bar{d}_a^2/d\rho > d\underline{d}_a^2/d\rho$. It follows that $\hat{\rho}$, i.e., the value of ρ at which $\underline{d}_a^1 = \bar{d}_a^1$, is unique. ∎

Proof of Lemma 7

We start with a comparison of $\bar{d}_a^1$ with $\bar{d}_a^2$. To simplify this comparison we first multiply (2.47) by $(e^{r\bar{d}_a^2}-1)^{-1}$. Clearly, this does not change the optimal solution of this equation. However, it is helpful because it eliminates K when comparing the left-hand sides of (2.47) and (2.45). Suppose now that $\bar{d}_a^1 = \bar{d}_a^2 = d$. Then, subtracting the left-hand side of (2.47) from the left-hand side of (2.45), we obtain

$$\frac{\Lambda_1 d(8-4\gamma-\gamma^2+e^{rd}(2+\gamma)^2)}{8e^{rd}(e^{rd}-1)} + \frac{\Gamma_2 d(2e^{rd}-1)}{e^{rd}(e^{rd}-1)^2} + \frac{\Gamma_1 d}{(e^{rd}-1)^2}, \tag{2.106}$$

where the terms involving K are eliminated since (2.47) was multiplied by $(e^{r\bar{d}_a^2}-1)^{-1}$. One can easily check that (2.106) is strictly positive. Since we know that the left-hand sides of (2.47) and (2.45) are strictly increasing at their respective solutions $\bar{d}_a^1$ and $\bar{d}_a^2$, it follows that $\bar{d}_a^1 < \bar{d}_a^2$.

From Lemma 6 we know that $\bar{d}_a^2 < \underline{d}_a^2$ if ρ is sufficiently negative. Since $\bar{d}_a^1 < \bar{d}_a^2$, there must exist a critical ρ such that for all ρ below this critical ρ we have $\bar{d}_a^1 < \underline{d}_a^2$. To determine this critical ρ we multiply the left-hand side of (2.46) by $(e^{r\underline{d}_a^2}-1)^{-1}$ and then set it equal to the left-hand side (2.47). We obtain that the two sides are equal if and only if (2.49) holds where d_a is the solution to (2.46) and (2.47) at the critical ρ. In the same way as in the proof of Lemma 6 we can show that the critical ρ is unique. Finally, denoting the maximum of the critical ρ and -1 by $\check{\rho}$, it follows that $\check{\rho} \leq \hat{\rho}$, since $\bar{d}_a^1 < \bar{d}_a^2$. ■

Proof of Lemma 8

Firm 1

We start with firm 1. We confine our attention to the case of a non-marginal deviation of the form in which firm 1 lowers (or extends) the inattentiveness period by $1/m$. We will show later that $m \to \infty$ yields the tightest bounds in the case in which firms plan in an alternating manner. Thus, even if firm 1 deviates to an inattentiveness period of the form l/m, we obtain the same result. It is therefore without loss of generality to consider deviations of the form $d(1 - 1/m)$ instead of $d(1 - l/m)$, $l > 1$.

Suppose that firm 1 deviates to a sequence $d - d/m$, where m is an odd number, whereas firm 2 sticks to the equilibrium sequence $\{d/2, 3/2d, 5/2d, ...\}$. Before we proceed, we introduce some notation. We define

$$\Gamma_3 := \frac{\gamma^2(\gamma^2(\sigma_\theta^2 + 2\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2) - 8\gamma(\sigma_\theta^2 + \rho\sigma_\theta\sigma_\xi + \sigma_\xi^2) + 8(\sigma_\xi^2 - \sigma_\theta^2))}{32(2-\gamma)^2},$$

and $k \in \mathbb{N}_0$.

The deviation of type $d(1 - 1/m)$ where m is an odd number induces a stream of expected losses that is composed of seven different instantaneous expected losses:

$$\mu_1 := \Lambda_1 \left(\tau + \frac{d(2+\gamma)^2}{8} \right), \tag{2.107}$$

$$\mu_2 := \Lambda_1 \tau, \tag{2.108}$$

$$\mu_3 := \Lambda_1 \tau + \Gamma_3 d, \tag{2.109}$$

$$\mu_4 := \Lambda_1 \left(\tau + \frac{(m-1)d}{2m} \right), \tag{2.110}$$

$$\mu_5 \quad := \quad \Lambda_1\left(\tau+\frac{d(2+\gamma)^2}{8}\right)+\Gamma_1\frac{dk}{2m}, \tag{2.111}$$

$$\begin{aligned}\mu_6 \quad := \quad & \Lambda_1\tau+\Gamma_1\frac{dk}{2m}-d\left(\frac{\Lambda_2}{32}(16-\gamma^4)\right) \\ & -d\left(\frac{\gamma^2\left(3\sigma_\xi^2+2\rho\sigma_\theta\sigma_\xi-\sigma_\theta^2+8\gamma(\sigma_\xi^2+\rho\sigma_\theta\sigma_\xi)\right)}{4(2-\gamma)^2}\right),\end{aligned} \tag{2.112}$$

$$\mu_7 \quad := \quad \Lambda_1\tau+\Gamma_1\frac{dk}{2m}-\gamma d\left(\frac{\Lambda_2}{2}-\gamma\frac{\sigma_\theta^2+6\rho\sigma_\theta\sigma_\xi-4\sigma_\xi^2-2\gamma(\sigma_\xi^2+\rho\sigma_\theta\sigma_\xi)}{8(2-\gamma)^2}\right). \tag{2.113}$$

Here μ_1 is the expected instantaneous loss in periods in which firm 2 was the last to plan, e.g., at $d/2$, up to the next planning date of firm 1 under deviation, that is, at $d(1-1/m)$. The component μ_2 captures the instantaneous expected loss in periods in which firm 1 is better informed under the deviation than on the equilibrium path, e.g., at $d(1-1/m)$, up to the next planning date under non-deviation, e.g., at d. However, firm 2 has not yet detected the deviation of firm 1. Similarly, μ_3 captures the same instantaneous expected loss but in periods in which firm 2 has detected the deviation. μ_4 displays the instantaneous expected loss in which firm 1 is better informed than firm 2 on the equilibrium and also under deviation but its latest planning date after deviation was before the one on equilibrium. This occurs between the periods $((m-1)/2-1)d$ and $((m-1)/2)(1-1/m)d$.

The instantaneous expected loss μ_5 occurs in periods in which firm 1 is worse informed after a deviation but was also worse informed on the equilibrium path. However, the firm's latest planning date after a deviation was before the one on the equilibrium path. This occurs for example in periods between $3/2d$ and $2(1-1/m)d$, in which firm 1 on the equilibrium path last planned at d but after deviation at $(1-1/m)d$. The instantaneous expected loss μ_6 is similar to μ_5, i.e., firm 1 is worse informed both on the equilibrium and after a deviation, but in this case firm 1 latest planning date in the deviation case was after the one on equilibrium. This occurs in periods from $((m-1)/k-1/2+k)d$ to $((m-1)/2+k)d$. Finally, μ_7 captures the loss in periods in which firm 1 is worse informed after deviation but better informed on the equilibrium path. This occurs in periods from $((m-1)/2+k)d$ up to $((m-1)/2+k)(1-1/m)d$.

We can now turn to the expected stream of losses. Given the expected instantaneous losses (2.107) to (2.113) a deviant's expected stream of losses is given by

$$\begin{aligned}
E[\mathcal{D}_1|\theta_0,\xi_0] \;=\;& e^{-rd/2}\int_{\tau=0}^{\left(\frac{1}{2}-\frac{1}{m}\right)d} e^{-r\tau}\mu_1 d\tau + e^{-r\left(1-\frac{1}{m}\right)d}\int_{\tau=0}^{d/m} e^{-r\tau}\mu_2 d\tau \\
&+\sum_{k=\frac{m-1}{2}}^{m-2}\left(e^{-r(k+1)\left(1-\frac{1}{m}\right)d}\int_{\tau=0}^{\left(k+\frac{1}{2}-(k+1)\left(1-\frac{1}{m}\right)\right)d} e^{-r\tau}\mu_2 d\tau\right) \\
&+\sum_{k=1}^{\frac{m-1}{2}-1}\left(e^{-r(k+1)\left(1-\frac{1}{m}\right)d}\int_{\tau=0}^{\left(\frac{k+1}{m}\right)d} e^{-r\tau}\mu_3 d\tau\right) \\
&+\sum_{k=1}^{\frac{m-1}{2}-1}\left(e^{-r\left(k+\frac{1}{2}\right)d}\int_{\tau=0}^{(k+1)\left(1-\frac{1}{m}-k-\frac{1}{2}\right)d} e^{-r\tau}\mu_5 d\tau\right) + \sum_{k=\frac{m+1}{2}}^{m-2}\left(e^{-r\left(k-\frac{1}{2}\right)d}\int_{\tau=0}^{d/2} e^{-r\tau}\mu_6 d\tau\right) \\
&+\sum_{n=1}^{\infty}\left(e^{-r\left(n(m-1)+\frac{1}{2}\right)d}\left(\int_{\tau=0}^{\left(\frac{1}{2}-\frac{1}{m}\right)d} e^{-r\tau}\mu_1 d\tau\right)\right) + \sum_{n=1}^{\infty}\left(e^{-rn(k-1)d}\left(\int_{\tau=0}^{d/2} e^{-r\tau}\mu_2 d\tau\right)\right) \\
&+\sum_{n=0}^{\infty}\sum_{k=\frac{m-1}{2}+1}^{m-1}\left(e^{-r\left(n(m-1)+\left(1-\frac{1}{m}\right)k\right)d}\left(\int_{\tau=0}^{\left(\frac{1}{2}+k-1-k\left(1-\frac{1}{m}\right)\right)d} e^{-r\tau}\mu_2 d\tau\right)\right) \\
&+\sum_{n=0}^{\infty}\sum_{k=0}^{\frac{m-3}{2}}\left(e^{-r\left(n(m-1)+\left(1-\frac{1}{m}\right)(k+1)\right)d}\left(\int_{\tau=0}^{\left(\frac{k+1}{m}\right)d} e^{-r\tau}\mu_3 d\tau\right)\right) \\
&+\sum_{n=0}^{\infty}\left(e^{-r(m-1)(1/2+n)d}\left(\int_{\tau=0}^{\left(1-\frac{1}{m}-\frac{m-1}{2m}\right)d} e^{-r\tau}\mu_4 d\tau\right)\right) \\
&+\sum_{n=0}^{\infty}\sum_{k=0}^{\frac{m-1}{2}-1}\left(e^{-r\left(n(m-1)+\frac{k}{2}\right)d}\left(\int_{\tau=0}^{\left(\frac{1}{2}-\frac{k+1}{m}\right)d} e^{-r\tau}\mu_5 d\tau\right)\right) \\
&+\sum_{n=0}^{\infty}\sum_{k=\frac{m+1}{2}}^{m-2}\left(e^{-r\left(n(m-1)+k-\frac{1}{2}\right)d}\left(\int_{\tau=0}^{d/2} e^{-r\tau}\mu_6 d\tau\right)\right) \\
&+\sum_{n=0}^{\infty}\sum_{k=0}^{\frac{m-2}{2}-2}\left(e^{-r\left((m-1)(n+\frac{1}{2})+1+k\right)d}\left(\int_{\tau=0}^{\left(\left(1-\frac{1}{m}\right)\left(m-\frac{1}{2}+k+2\right)-\left(m-\frac{1}{2}+k+1\right)\right)d} e^{-r\tau}\mu_7 d\tau\right)\right) \\
&+K\sum_{n=1}^{\infty}\left(e^{-rn\left(1-\frac{1}{m}\right)d}\right). \qquad (2.114)
\end{aligned}$$

Now, we subtract from this fractional-deviation induced expected stream of losses the equilibrium expected stream of losses which is appropriately adapted to the considered time horizon. This yields

$$E[\mathcal{D}_1|\theta_0,\xi_0] - E[\mathcal{S}_1|\theta_0,\xi_0], \qquad (2.115)$$

where

$$E[\mathcal{S}_1|\theta_0,\xi_0] = \Lambda_1 \frac{e^{rd/2}(8+rd(2+\gamma)^2)-8-rd(8+4\gamma+\gamma^2)}{16r^2(e^{rd}-1)} + \Lambda_1 \frac{2e^{rd/2}-2-rd}{4e^{rd/2}r^2(e^{rd}-1)} + \frac{K}{e^{rd}-1}.$$

Thus, it is profitable for a firm to deviate from a proposed alternating equilibrium inattentiveness period d in a non-marginal way if (2.115) is negative for at least one $m \in \mathbb{N}$.

Now building the difference quotient by dividing the respective terms of (2.115) by x/m, where x is chosen appropriately for the different intervals, and then letting $r \to 0$, we obtain that (2.115) is negative if

$$K < d^2 \left(\Lambda_2 \frac{4-\gamma}{8} + \frac{\gamma^2 \left(6\gamma(\sigma_\xi^2 + \rho\sigma_\theta\sigma_\xi^2) + \sigma_\theta^2 - 10\rho\sigma_\theta\sigma_\xi - 11\sigma_\xi\right)}{32} \right) \times \tag{2.116}$$

$$\left(\frac{8\Lambda_1(40+12\gamma)m^3 + \Lambda_1(4-\gamma)m^2\Gamma_1(36-24\gamma)m}{8\Lambda_1(40+12\gamma)m^3 + \Lambda_2(8-4\gamma+\gamma^2)m^2 + \Gamma_1(132-42\gamma)m + \Lambda_2 + \gamma^2(\sigma_\xi^2+\rho\sigma_\theta\sigma_\xi)} \right).$$

Differentiating the second term of the right-hand side of (2.116) with respect to m, we obtain

$$\begin{aligned} &24\Lambda_1(40+12\gamma)\Lambda_2(4-3\gamma+\gamma^2)m^4 + 16\Lambda_1(40+12\gamma)\Gamma_1(32+6\gamma-11\gamma^2+2\gamma^3)m^3 \\ &+ \left(24\Lambda_1(40+12\gamma)(\Lambda_2+\gamma^2(\sigma_\xi^2+\rho\sigma_\theta\sigma_\xi)) + 6\Gamma_1(152-122\gamma+46\gamma^2-7\gamma^3)\right) m^2 \\ &+ (2\Lambda_1(4-\gamma)m + \Gamma_1(36-24\gamma)) \left(\Lambda_2 + \gamma^2(\sigma_\xi^2+\rho\sigma_\theta\sigma_\xi)\right). \end{aligned}$$

It is easy to see that this term is strictly positive. Thus, (2.116) is strictly increasing in m. Therefore, letting $m \to \infty$ is the most profitable deviation for firm 1. We can then write (2.116) as

$$K < d^2 \left(\Lambda_2 \frac{4-\gamma}{8} + \frac{\gamma^2 \left(6\gamma(\sigma_\xi^2 + \rho\sigma_\theta\sigma_\xi^2) + \sigma_\theta^2 - 10\rho\sigma_\theta\sigma_\xi - 11\sigma_\xi\right)}{32} \right). \tag{2.117}$$

Now letting $r \to 0$ in (2.45), i.e., in the condition that gives the upper bound for any potential equilibrium from the marginal deviations of firm 1, $\bar{d}_a^1$, we obtain that $\bar{d}_a^1$ is (implicitly) given by

$$K = (\bar{d}_a^1)^2 \left(\Lambda_2 \frac{4-\gamma}{8} + \frac{\gamma^2 \left(6\gamma(\sigma_\xi^2 + \rho\sigma_\theta\sigma_\xi^2) + \sigma_\theta^2 - 10\rho\sigma_\theta\sigma_\xi - 11\sigma_\xi\right)}{32} \right). \tag{2.118}$$

But since only $d \leq \bar{d}_a^1$ are potential equilibria, combining (2.117) and (2.118), we obtain that (2.117) can never be satisfied for any $d \leq \bar{d}_a^1$. This implies that the non-marginal deviation of firm 1 to a period length of $d - d/m$ with m being an odd number does not eliminate potential equilibria.

We can proceed in the same way for non-marginal deviations in which firm 1 chooses a period length of $d - d/m$ with m being an even number, and also for non-marginal deviations in which firm 1 sets a period length of $d + d/m$ with m being an odd or even number. In all cases we obtain that in the limit as $r \to 0$, the deviation to $m \to \infty$ excludes the largest set of inattentiveness periods, and that this set is equivalent to the one that is excluded by the marginal deviations. Thus, non-marginal deviations of firm 1 do not exclude potential equilibria in the limit as $r \to 0$. By continuity, if r is positive but sufficiently close to 0, the result applies as well.

Firm 2

Now, we turn to firm 2. Here we derive the expected stream of losses for the case in which firm $2'$s deviates to an inattentiveness interval of length $d + d/m$, where m is an even number, but sticks to the (A) mode. Before we proceed, we introduce some notation. We define $k \in \{0, .., (m-2)/2\}$.

The considered deviation of firm 2 induces a stream of expected losses that is composed of nine different expected instantaneous losses:

$$\vartheta_1 := \Lambda_1 \left(\tau + \frac{d}{2}\right), \tag{2.119}$$

$$\vartheta_2 := \Lambda_1 \tau, \tag{2.120}$$

$$\vartheta_3 := \Lambda_1 \tau + \Gamma_3 d, \tag{2.121}$$

$$\vartheta_4 := \Lambda_1 \left(\tau + \left(\frac{1}{2} - \frac{1+2k}{2m}\right)\right), \tag{2.122}$$

$$\vartheta_5 := \Lambda_1 \left(\tau + \frac{(2+\gamma)^2}{4}\frac{d}{2}\right) - \Gamma_1 \left(\frac{1+k}{2m}\right)\frac{d}{2}, \tag{2.123}$$

$$\begin{aligned}
\vartheta_6 &:= \Lambda_1\tau - \Gamma_1\left(\frac{1+2k}{m}\right)\frac{d}{2} + \Gamma_2 d, && (2.124)\\
\vartheta_7 &:= \Lambda_1\tau + \Gamma_1\left(\frac{1-2k}{m}\right)\frac{d}{2} + \Gamma_2 d, && (2.125)\\
\vartheta_8 &:= \Lambda_1\left(\tau + \frac{(2+\gamma)^2}{4}\frac{d}{2}\right) + \Gamma_1\left(\frac{d}{2m}\right), && (2.126)\\
\vartheta_9 &:= \Lambda_1\tau + \Gamma_1\left(\frac{d}{2m}\right) + \Gamma_2 d, && (2.127)
\end{aligned}$$

The first four expected instantaneous losses—(2.119) to (2.122)—capture firm 2′s losses when it is the better informed firm. The expected instantaneous loss in the period that elapses between the first planning date on the equilibrium path and the first planning date under the deviation is given by (2.119). In this period, firm 2′s deviation is undetected by firm 1. The component (2.120) captures firm 2′s expected instantaneous losses in the time period in which it is better informed, irrespective of whether the deviation was detected or not.

If firm 2 deviates to a frequency of $(1 + 1/m)\,d$ then it happens that firm 1 plans twice between two consecutive planning dates of firm 2. This takes place for the first time between the $m/2$th and the $(m+2)/2$th planning date of firm 2 and occurs thereafter between every $m(1/2 + (n+1))$th and $((m+2)/2 + (n+1)m)$th, $n \in \mathbb{N}_0$, planning date of firm 2. At this point, firm 1 has already detected firm 2′s deviation. Moreover, the $((m+2)/2)$th planning date of firm 2 happens at an instant at which it would have been the worse informed firm if it followed the equilibrium planning horizon. The corresponding expected instantaneous loss is captured by (2.121). After $(1/2 - (1+2k)/(2m))$ periods firm 2 is still the better informed firm but now in a time period in which it would have also been the better informed firm on the equilibrium path. The corresponding loss is captured by (2.122).

Note that after each mth repetition of firm 2′s planning horizon the relative time distance to firm 1′s planning dates is the same. Thus, in the following we refer to this as a cycle. The components (2.123) and (2.124) capture firm 2′s losses from being worse informed in the first half of the cycle, i.e. the $(1/2 + k + nm)$th to the $((m-1)/2 + nm)$th repetition of the planning horizon $(1 + 1/m)d$. The difference between (2.123) and (2.124) is that the former captures the losses in a time period in which firm 2 was the worse informed and the latter the losses in which it was the better informed firm on the equilibrium path. In this part of the cycle, firm 2 is relative to the equilibrium path worse informed for a shorter time.

After firm 1 repeated its planning horizon d for the $((m-1)+k+n(m+1))$th time, firm 2 is the worse informed firm in a time period in which it was already worse informed on the equilibrium path until it repeated its planning horizon for the $((m+1)/2+k+nm)$th time. The expected instantaneous losses in this periods are caught by (2.125). The cycle is completed after firm 2 repeated its inattentiveness interval for the $(m+1/2+n(m+1))$th time. In this last repetition the components (2.126) and (2.127) pick up firm $2'$ expected instantaneous losses from being the worse informed firm. Again, the former component captures the losses in a time period in which firm 2 was the worse informed and the latter the losses in which it was the better informed firm on the equilibrium path.

Given the expected instantaneous expected losses (2.119) to (2.127) firm $2'$s expected stream of losses from a deviation to $(1+1/m)d$ can be written as

$$
\begin{aligned}
E[\mathcal{D}_2|\theta_0,\xi_0] \;=\;& e^{-r\frac{d}{2}}\int_0^{\frac{d}{2m}} e^{-r\tau}\vartheta_1 d\tau + \sum_{n=0}^{\infty}\sum_{k=0}^{\frac{m-2}{2}} e^{-rd\left(n(m+1)+\frac{(1+2k)(1+m)}{2m}\right)}\left(\int_{\tau=0}^{\left(\frac{m-(1+2k)}{2m}\right)d} e^{-r\tau}\vartheta_2 d\tau\right)\\
&+\sum_{n=0}^{\infty}\sum_{k=0}^{\frac{m-2}{2}} e^{-rd\left(n(m+1)+\frac{(1+m+2k)(1+m)}{2m}\right)}\left(\int_{\tau=0}^{\left(\frac{m-(1+2k)}{2m}\right)d} e^{-r\tau}\vartheta_3 d\tau\right)\\
&+\sum_{n=0}^{\infty}\sum_{k=0}^{\frac{m-2}{2}} e^{-rd\left(n(m+1)+1+\frac{1+m+2k}{2}\right)}\left(\int_{\tau=0}^{\frac{d}{2}} e^{-r\tau}\vartheta_4 d\tau\right)\\
&+\sum_{n=0}^{\infty}\sum_{k=0}^{\frac{m-2}{2}} e^{-rd(n(m+1)+k+1)}\left(\int_{\tau=0}^{\frac{d}{2}} e^{-r\tau}\vartheta_5 d\tau\right)\\
&+\sum_{n=0}^{\infty}\sum_{k=0}^{\frac{m-4}{2}} e^{-rd\left(n(m+1)+k+\frac{3}{2}\right)}\left(\int_{\tau=0}^{\frac{d}{2}\frac{3+k(2-m)}{m}} e^{-r\tau}\vartheta_6 d\tau\right)\\
&+\sum_{n=0}^{\infty} e^{-rd\left(n(m+1)+\frac{m-1}{2}\right)}\left(\int_{\tau=0}^{\frac{d}{2}} e^{-r\tau}\vartheta_6 d\tau\right)\\
&+\sum_{n=0}^{\infty}\sum_{k=0}^{\frac{m-2}{2}} e^{-rd\left(n(m+1)+\frac{m+2}{2}+k\right)}\left(\int_{\tau=0}^{\frac{d}{2}} e^{-r\tau}\vartheta_7 d\tau\right)\\
&+\sum_{n=1}^{\infty} e^{-rd(n(m+1))}\left(\int_{\tau=0}^{\frac{d}{2}} e^{-r\tau}\vartheta_8 d\tau + e^{-r\frac{d}{2}}\int_{\tau=0}^{\frac{d}{2m}} e^{-r\tau}\vartheta_9 d\tau\right)\\
&+K\sum_{n=1}^{\infty}\left(e^{-rd\left(\frac{1}{2}+n\left(1+\frac{1}{m}\right)\right)}\right).
\end{aligned}
\tag{2.128}
$$

In the same way as for player 1 we can now subtract from this fractional-deviation induced expected stream of losses the equilibrium expected stream of losses which is appropriately adapted to the considered time horizon. This yields

$$E[\mathcal{D}_2|\theta_0,\xi_0] - E[\mathcal{S}_2|\theta_0,\xi_0], \tag{2.129}$$

where

$$E[\mathcal{S}_2|\theta_0,\xi_0] = \Lambda_1\frac{2e^{rd/2}-2-rd}{e^{rd}r^2(e^{rd}-1)} + \Lambda_1\frac{e^{rd/2}(8+rd(2+\gamma)^2)-8-rd(8+4\gamma+\gamma^2)}{4e^{rd/2}r^2(e^{rd}-1)} + \frac{Ke^{rd/2}}{e^{rd}-1}.$$

Thus, it is profitable for firm 2 to deviate from a proposed synchronous inattentiveness period d in a non-marginal way if (2.129) is negative for at least one $m \in \mathbb{N}$.

Now building the difference quotient by dividing the respective terms of (2.129) by x/m, where again x is chosen apprppriately for the different intervals, and then letting $r \to 0$, we obtain that (2.129) is negative if

$$K > d^2\left(\Lambda_2(8-4\gamma) - \frac{\gamma^2\left(3\sigma_\xi^2+2\rho\sigma_\theta\sigma_\xi-\sigma_\theta^2-2\gamma^2(\sigma_\xi^2+\rho\sigma_\theta\sigma_\xi)\right)}{(2-\gamma)^2}\right)\times \tag{2.130}$$

$$\left(\frac{\Lambda_2\left(\frac{(2+\gamma)^2}{16}\right)m^3+\left(\Lambda_1\frac{1}{2}+\Gamma_2\frac{(2-\gamma)^2}{32}\right)m^2+\left(\Lambda_2\frac{(2-\gamma)^2}{4}+\Gamma_1\frac{(2+\gamma)^2}{16}\right)m+\Lambda_2\left(\frac{12+2\gamma}{(2-\gamma)^2}\right)+\Gamma_3\left(\frac{8+4\gamma+\gamma^2}{32}\right)}{\Lambda_2\left(\frac{(2+\gamma)^2}{16}\right)m^3+\Lambda_2\left(\frac{4+4\gamma+\gamma^2}{16}\right)m^2+\Gamma_2\left(\frac{(2+\gamma)^2(2-\gamma)}{32}\right)m}\right).$$

As above, via differentiating the second term on the right-hand side of (2.116), one can check that it is strictly decreasing in m. Therefore, letting $m \to \infty$ is the most profitable deviation for firm 2. We can then write (2.130) as

$$K > d^2\left(\Lambda_2(8-4\gamma) - \frac{\gamma^2\left(3\sigma_\xi^2+2\rho\sigma_\theta\sigma_\xi-\sigma_\theta^2-2\gamma^2(\sigma_\xi^2+\rho\sigma_\theta\sigma_\xi)\right)}{(2-\gamma)^2}\right). \tag{2.131}$$

Now letting $r \to 0$ in (2.46) stated in Lemma 5, i.e., in the condition that gives the lower bound from the marginal deviation of player, $\underline{d}_a^2$, we obtain that $\underline{d}_a^2$ is (implicitly) given by

$$K = (\underline{d}_a^2)^2\left(\Lambda_2(8-4\gamma) - \frac{\gamma^2\left(3\sigma_\xi^2+2\rho\sigma_\theta\sigma_\xi-\sigma_\theta^2-2\gamma^2(\sigma_\xi^2+\rho\sigma_\theta\sigma_\xi)\right)}{(2-\gamma)^2}\right). \tag{2.132}$$

But since only $d \geq \underline{d}_a^2$ can be potential equilibria, combining (2.131) and (2.132), we obtain that (2.131) can never be satisfied for any $d \geq \underline{d}_a^2$. This implies that the non-marginal deviation of firm 2 to a period length of $d + d/m$ with m being an even number does not eliminate any potential equilibria. By the same argument, even if we consider deviations of firm 2 to period lengths of $d(1+l/m)$, $l \leq m$, this does not eliminate any potential equilibria since $m \to \infty$ is the most profitable deviation in this case either.

We can proceed in the same way for non-marginal deviations in which firm 2 chooses a period length of $d + d/m$ with m being an odd number, and sticks to the (A) mode and also for non-marginal deviations in which firm 2 sets a period length of $d - d/m$ with m being an odd or even number, and sticks to the (A) mode. In all cases we obtain that in the limit as $r \to 0$, the deviation to $m \to \infty$ excludes the largest set of inattentiveness period length, and that this set is equivalent to the one that is excluded by the marginal deviations. Thus, non-marginal deviations in which firm 2 chooses the (A) mode do not exclude any potential equilibria in the limit as $r \to 0$. By continuity, if r is positive but sufficiently close to 0, the result applies as well.

We now turn to the case in which firm 2 deviates to the (S) mode, that is, to sequence of planning dates

$$\left\{ d\left(1 \pm \frac{1}{m}\right), 2d\left(1 \pm \frac{1}{m}\right), ... \right\}.$$

Proceeding in the same way as above, i.e., building the differential quotient between the expected stream of losses and the equilibrium loss and letting r go to zero, we obtain that independent of the inattentiveness length being $d(1-1/m)$ or $d(1+1/m)$ the most profitable deviation is the one in which $m \to \infty$, i.e., the sequence $\{d, 2d, ...\}$.

Now, we determine the condition under which it is not profitable for firm 2 to deviate to the RTS deviation. We do so by determining the differential quotient between the RTS and the equilibrium sequence for r in a neighborhood of zero. This yields that deviating to the RTS sequence is not profitable if and only if

$$rK \leq \frac{d}{8e^{3rd}(2-\gamma)^2}\Big(e^{5rd/2}\left(4\gamma^2(\sigma_\theta^2 + \rho\sigma_\theta\sigma_\xi) + 2\gamma^3(\sigma_\xi^2 + \rho\sigma_\theta\sigma_\xi) - 4\Lambda_1(\gamma^4 + 4)\right) \tag{2.133}$$

$$+8\Lambda_1 e^{2rd}(2-\gamma)^2 + 4(1 + e^{rd/2} - e^{rd})\gamma^2\left(\Lambda_1\gamma^2 - (\sigma_\xi^2 + \rho\sigma_\theta\sigma_\xi)\gamma + \sigma_\xi^2 - \sigma_\theta^2\right)\Big).$$

Thus, only those $d_a^\star$ that satisfy (2.133) are not eliminated by the RTS deviation. Since no potential equilibrium inattentiveness period is eliminated by the non-marginal deviations of firm 1 and the other non-marginal deviations of firm 2, the result of the lemma follows. ■

Proof of Proposition 2

On (i): Taking the limit $r \to 0$ of (2.133) this inequality can be written as

$$0 \leq -\frac{\gamma d_a^\star \left(4(\sigma_\theta^2 + 2\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2) - \gamma(\sigma_\theta^2 + 6\rho\sigma_\theta\sigma_\xi + 5\sigma_\xi^2) + 2\gamma^2(\sigma_\xi^2 + \rho\sigma_\theta\sigma_\xi)\right)}{8(2-\gamma)^2}. \tag{2.134}$$

Therefore, in the limit as $r \to 0$ an alternating planning equilibrium can only exist for $\gamma > 0$ if and only if

$$\rho < \max\left[-\frac{\sigma_\theta^2(4-\gamma) + \sigma_\xi^2(4 - 5\gamma + 2\gamma^2)}{2\sigma_\theta\sigma_\xi(4 - 3\gamma + \gamma^2)}, -1\right]. \tag{2.135}$$

From the proof of Lemma 8 we also know that, as $r \to 0$, marginal deviations are not profitable if

$$K \leq (d_a^\star)^2 \left(\Lambda_2 \frac{4-\gamma}{8} + \frac{\gamma^2 \left(6\gamma(\sigma_\xi^2 + \rho\sigma_\theta\sigma_\xi^2) + \sigma_\theta^2 - 10\rho\sigma_\theta\sigma_\xi - 11\sigma_\xi\right)}{32}\right).$$

Suppose for the moment that $K \to 0$. Then solving the last inequality for ρ we obtain that marginal deviations are not profitable if

$$\rho \geq -\frac{4(4-\gamma)(\sigma_\theta^2 + \sigma_\xi^2) - \gamma^2(11\sigma_\xi^2 - \sigma_\theta^2) + 6\gamma^3\sigma_\theta^2}{2\sigma_\theta\sigma_\xi(16 - 4\gamma - 5\gamma^2 + 3\gamma^3)}. \tag{2.136}$$

We can now check if conditions (2.135) and (2.136) can be jointly satisfied. To do so we calculate

$$-\frac{4(4-\gamma)(\sigma_\theta^2 + \sigma_\xi^2) - \gamma^2(11\sigma_\xi^2 - \sigma_\theta^2) + 6\gamma^3\sigma_\theta^2}{2\sigma_\theta\sigma_\xi(16 - 4\gamma - 5\gamma^2 + 3\gamma^3)} - \left(-\frac{\sigma_\theta^2(4-\gamma) + \sigma_\xi^2(4 - 5\gamma + 2\gamma^2)}{2\sigma_\theta\sigma_\xi(4 - 3\gamma + \gamma^2)}\right),$$

which yields

$$-\frac{\gamma(2-\gamma)^2(\sigma_\xi - \sigma_\theta)(\sigma_\xi + \sigma_\theta)}{\sigma_\theta\sigma_\xi(16 - 4\gamma - 5\gamma^2 + 3\gamma^3)(4 - 3\gamma + \gamma^2)}.$$

For both conditions to be jointly satisfied at $\gamma > 0$ we must have that the latter term is strictly negative which can only hold true if $\sigma_\xi > \sigma_\theta$. But we have that at $\sigma_\xi > \sigma_\theta$

$$-\frac{\sigma_\theta^2(4-\gamma)+\sigma_\xi^2(4-5\gamma+2\gamma^2)}{2\sigma_\theta\sigma_\xi(4-3\gamma+\gamma^2)} < -1,$$

thereby ruling out an alternating planning equilibrium. For $K > 0$ condition (2.136) becomes even tighter. Thus, an alternating planning equilibrium does not exists for this case either. We have shown the result for $r \to 0$. However, by continuity, we must have that even if r is positive but close to 0, an alternating planning equilibrium can not exist for $\gamma > 0$.

On (ii): From part (i) of this proposition it follows that for r close to zero an alternating planning equilibrium can only exist for $\gamma < 0$. Lemmas 7 and 8 imply that even for $\gamma < 0$ the equilibrium only exists if $\rho \geq \max[\check{\rho}, \rho^+]$. Finally, it follows from Lemmas 5 and 6 that the maximum range of this equilibrium is given by the marginal deviations of player 1, that is $d_a^\star \in [\underline{d}_a^1, \bar{d}_a^1]$.

On (iii): The equilibrium inattentiveness lengths $\underline{d}_a^\star$ and $\bar{d}_a^\star$ are characterized by (2.44) and (2.45). Solving each of the two equations for K and then taking the limit $r \to 0$, we obtain that $\underline{d}_a^\star = \bar{d}_a^\star = d_a^\star$, where $d_a^\star$ is characterized by

$$K = (d_a^\star)^2 \left(\Lambda_2 \frac{4-\gamma}{8} + \frac{\gamma^2 \left(6\gamma(\sigma_\xi^2 + \rho\sigma_\theta\sigma_\xi^2) + \sigma_\theta^2 - 10\rho\sigma_\theta\sigma_\xi - 11\sigma_\xi\right)}{32} \right). \tag{2.137}$$

The right-hand side of (2.137) can only be positive if

$$\rho \geq -\frac{4(4-\gamma)(\sigma_\theta^2+\sigma_\xi^2) - \gamma^2(11\sigma_\xi^2 - \sigma_\theta^2) + 6\gamma^3\sigma_\theta^2}{2\sigma_\theta\sigma_\xi(16-4\gamma-5\gamma^2+3\gamma^3)}. \tag{2.138}$$

From the proof of part (i) of this proposition we know that this is also the tightest bound on ρ such that marginal deviations are not profitable. In addition, an alternating equilibrium can only exist if (2.134) is satisfied. The proof of part (i) of this proposition implies that (2.134) and (2.138) can jointly only be satisfied if $\gamma < 0$ and

$$\rho \geq -\frac{\sigma_\theta^2(4-\gamma)+\sigma_\xi^2(4-5\gamma+2\gamma^2)}{2\sigma_\theta\sigma_\xi(4-3\gamma+\gamma^2)}.$$

As a consequence, the alternating planning equilibrium exists if and only if $\gamma < 0$ and

$$\rho \geq \max\left[-\frac{\sigma_\theta^2(4-\gamma)+\sigma_\xi^2(4-5\gamma+2\gamma^2)}{2\sigma_\theta\sigma_\xi(4-3\gamma+\gamma^2)}, -\frac{4(4-\gamma)(\sigma_\theta^2+\sigma_\xi^2)-\gamma^2(11\sigma_\xi^2-\sigma_\theta^2)+6\gamma^3\sigma_\theta^2}{2\sigma_\theta\sigma_\xi(16-4\gamma-5\gamma^2+3\gamma^3)}, -1\right].$$

■

Proof of Proposition 3

On (i): We know that for $r \to 0$, $\underline{d}_a^\star = \bar{d}_a^\star = d_a^\star$, where $d_a^\star$ is defined by (2.137). The left-hand side of (2.137) is constant in $d_a^\star$ and γ while the right-hand side of (2.137) is strictly increasing in $d_a^\star$. Now differentiating the right-hand side with respect to γ we obtain

$$\frac{3d_a^\star\left(4(\sigma_\theta^2+2\rho\sigma_\theta\sigma_\xi\sigma_\xi^2)-\gamma(8-6\gamma+\gamma^2)(\sigma_\xi+\rho\sigma_\theta\sigma_\xi)\right)}{16(2-\gamma)^3} > 0,$$

where the inequality sign stems from the fact that at the equilibrium $\gamma < 0$. It therefore follows that after an increase in γ, (2.137) can only be fulfilled if $d_a^\star$ falls, yielding that $dd_a^\star/d\gamma < 0$.

On (ii): Now consider the case where $r > 0$ but small such that the difference between $\underline{d}_a^\star$ and $\bar{d}_a^\star$ is small. We know that $\underline{d}_a^\star$ and $\bar{d}_a^\star$ are determined by (2.44) and (2.45). As is easy to see the last three terms on the left-hand side of both equations have the same structure and only differ because $\underline{d}_a^\star$ and $\bar{d}_a^\star$ differ while the first term is different even if $\underline{d}_a^\star$ and $\bar{d}_a^\star$ were the same. Since we consider the case in which $\underline{d}_a^\star$ and $\bar{d}_a^\star$ are very close to each other, we can concentrate on the difference in the respective first terms in determining if $\underline{d}_a^\star$ and $\bar{d}_a^\star$ change differently with γ.

Totally differentiating the first term of (2.44) we obtain

$$\frac{d\underline{d}_a^\star}{d\gamma} = -\frac{8\underline{d}_a^\star}{(2-\gamma)(8-4\gamma+\gamma^2)(1-r\underline{d}_a^\star)} < 0,$$

while totally differentiating the first term of (2.45) we obtain

$$\frac{d\bar{d}_a^\star}{d\gamma} = -\frac{\bar{d}_a^\star(8+4\gamma+\gamma^2)}{2(2+\gamma)(1-r\bar{d}_a^\star)} < 0.$$

Now subtracting $d\underline{d}_a^\star/d\gamma$ from $d\bar{d}_a^\star/d\gamma$ and taking into account that $\underline{d}_a^\star$ is close to $\bar{d}_a^\star$ we obtain

$$\frac{d\bar{d}_a^\star}{d\gamma} - \frac{d\underline{d}_a^\star}{d\gamma} \approx -\frac{\bar{d}_a^\star(96 - 80\gamma + 2\gamma^4 - \gamma^5)}{2(2+\gamma)(2-\gamma)(8-4\gamma+\gamma^2)(1-r\bar{d}_a^\star)}.$$

Since r is small, the denominator is positive and so the whole expression is negative. But, since both $d\underline{d}_a^\star/d\gamma$ and $d\bar{d}_a^\star/d\gamma$ are strictly decreasing in γ, this implies that $\bar{d}_a^\star$ decreases by more than $\underline{d}_a^\star$ if γ rises. Thus, the equilibrium range of inattentiveness periods shrinks as γ rises. ■

Proof of Lemma 9

Suppose that firm i expects that firm j chooses an inattentiveness period of length $d \in \mathbb{R}^+$ at $D_i(0) = D_j(0) = 0$. The sequence of planning dates induced by d can only be an equilibrium if neither of the following infinitesimal deviations are profitable for firm i: choose a longer inattentiveness period, that is deviate to $d' = d + \triangle$, with $\triangle > 0$, or a shorter period, i.e. $d'' = d - \triangle$. In order to simplify the exposition we assume that the shock realizations at 0 are given by $\theta(0) = 0$ and $\xi(0) = 0$. This assumption is without loss of generality because the shock realizations do not influence a firms' deviation incentive.

In order to derive the "no-deviation" conditions we have to compare firm i's expected loss from following the proposed equilibrium sequence with the expected losses from the sequences $\mathcal{D}' = \{d', 2d', 3d', ...\}$ and $\mathcal{D}'' = \{d'', 2d'', 3d'', ...\}$.

Lifetime expected loss for $\mathcal{D}$

We know from Lemma 5 that firm i's expected instantaneous loss from following the proposed equilibrium sequence at any instant t, with $nd \leq t < (n+1)d$, for all $n \in \mathbb{N}_0$, is given by

$$E[\mathcal{L}^e|I_0] = \Lambda_1\tau,$$

where τ denotes the time that elapsed since the last planning date, that is $\tau = t - nd$, for all $n \in \mathbb{N}_0$.

Thus, the lifetime expected loss implied by any candidate equilibrium inattentiveness interval is

$$E[\mathcal{L}(\mathcal{D})|I_0] = \frac{e^{rd} - 1 - rd}{e^{rd}r^2(e^{rd} - 1)}\Lambda_1 + \frac{K}{e^{rd} - 1}. \tag{2.139}$$

In order to consistently transform expected profits into expected losses we scale the expected stream of profits under the infinitesimal deviations by the equilibrium expected full information profits. In the synchronous planning equilibrium the expected full information profit a firm earns

$$\begin{aligned} E[\Pi^{FI}(t)|I_0] &= \Big(\frac{\sigma_\theta^2 - 2\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2}{4}\Big)\tau + \Big(\frac{\sigma_\theta^2 - 2\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2 - \gamma\sigma_\xi^2(2-\gamma) + 2\gamma\rho\sigma_\theta\sigma_\xi}{(2-\gamma)^2}\Big)nd^\star \\ &\quad + \frac{(\alpha - (1-\gamma)c)}{(2-\gamma)^2}. \end{aligned} \tag{2.140}$$

Lifetime expected loss for $\mathcal{D}'$

We set out by introducing some notation. Let $\tilde{m} \in \mathbb{N}$ denote a natural number for which $\tilde{m}\triangle \leq d$ and $(\tilde{m}+1)\triangle > d$.

In the following we derive the lifetime expected loss from the sequences $\mathcal{D}'$. First, we turn to the expected instantaneous loss that firm i incurs in the interval $[d, d']$ if it deviates to the sequence $\mathcal{D}'$.

From d to $d' = d + \triangle$

If firm i deviates to $\mathcal{D}'$ then its expected profit function at instant $d+\tau$, $\tau \in [0, \triangle)$ is given by

$$\begin{aligned} E[\Pi(d+\tau)|I_0^i, I_0^j] &= (\alpha - p_i(d+\tau) + \gamma E[p_j(d+\tau)|I_0^i, I_0^j])(p_i(d+\tau) - c) \\ &- E[\xi(d+\tau)(\theta(d+\tau) + \gamma p_j(d+\tau))|I_0^i, I_0^j]. \end{aligned} \tag{2.141}$$

In this interval firm i is the firm with the more outdated information est. However, firm j has not detected firm i's deviation. Thus, firm j still believes that both firms share the same information set I_d. Hence, it follows that firm $j's$ optimal price at $d+\tau$ is given by

$$p_j^\star(d+\tau) = \frac{\alpha + \theta(d) + c + \xi(d)}{2-\gamma}. \tag{2.142}$$

Therefore,

$$p_i^\star(d+\tau) = E[p_j^\star(d+\tau)|I_0^i, I_0^j] = \frac{\alpha+c}{2-\gamma}. \tag{2.143}$$

Using (2.143) and (2.142) in (2.141) yields that firm i's expected instantaneous profit under $\mathcal{D}'$ in the interval $t \in [d, d']$ is given by

$$E[\Pi^\star(d+\tau)|I_0^i, I_0^j] = -\frac{2\rho\sigma_\theta\sigma_\xi + \gamma\sigma_\xi^2}{2-\gamma}d - \rho\sigma_\theta\sigma_\xi\tau + \frac{(\alpha-(1-\gamma)c)}{(2-\gamma)^2}. \tag{2.144}$$

Subtracting (2.144) from (2.140) yields that firm i's expected instantaneous loss in the period $[d, d+\triangle)$ is

$$E[\mathcal{L}_1'|I_0] = \Lambda_2 d + \Lambda_1\tau, \tag{2.145}$$

with $\tau \in [0, \triangle)$.

From d' to $2d$

In this time interval firm j still believes that firm i sticks to the proposed equilibrium strategy. Thus, its optimal price is at each instant given by (2.142). Firm i acquires new information at instant $d' = d + \triangle$. Therefore, its expected profit function is in this interval given by

$$\begin{aligned} E[\Pi(d'+\tau)|I_{d'}^i, I_d^j] &= (\alpha - p_i(d'+\tau) + \gamma p_j(d'+\tau))(p_i(d'+\tau) - c) \\ &- E[\xi(d'+\tau)(\theta(d'+\tau) + \gamma p_j(d'+\tau))|I_{d'}^i, I_d^j], \end{aligned} \tag{2.146}$$

with $\tau \in [0, d-\triangle)$. Proceeding in the same way as before, we obtain that

$$p_i^\star(d'+\tau) = \frac{\alpha + \theta(d') + c + \xi(d')}{2} + \frac{\gamma\Big(\alpha + \theta(d) + c + \xi(d)\Big)}{2(2-\gamma)}. \tag{2.147}$$

Using (2.147) and (2.142) in (2.146), taking expectations and subtracting the resulting expression from (2.140) yields for the interval $[d', 2d)$ the following instantaneous loss function

$$E[\mathcal{L}_2'|I_0] = \Lambda_1\tau, \tag{2.148}$$

with $\tau \in [0, d-\triangle)$.

From $2d$ to $2d'$

At $2d$ firm j detects, that firm i deviated to the sequence $\mathcal{D}'$. This implies that firm j knows that firm i's consecutive planning dates will be at nd', for $n \in \mathbb{N} \setminus \{1\}$. As firm j is in the interval $[2d, 2d')$ better informed than firm i, the latter's expected profit function equals

$$\begin{aligned} E[\Pi(2d+\tau)|I^i_{d'}, I^j_{d'}] &= (\alpha - p_i(2d+\tau) + \gamma E[p_j(2d+\tau)|I^i_{d'}, I^j_{d'}](p_i(2d+\tau) - c) \\ &- E[\xi(2d+\tau)(\theta(2d+\tau) + \gamma p_j(2d+\tau))|I^i_{d'}, I^j_{d'}], \end{aligned} \tag{2.149}$$

with $\tau \in [0, 2\triangle)$. As a consequence the firms' optimal prices in this period are given by

$$p^\star_j(2d+\tau) = \frac{\alpha + \theta(2d) + c + \xi(2d)}{2} + \frac{\gamma\Big(\alpha + \theta(d') + c + \xi(d')\Big)}{2(2-\gamma)}, \tag{2.150}$$

$$p^\star_i(2d+\tau) = E[p^\star_j(2d+\tau)|I^i_{d'}, I^j_{d'}] = \frac{\alpha + \theta(d') + c + \xi(d')}{2-\gamma}. \tag{2.151}$$

Using (2.151) and (2.150) in (2.149), taking expectations and subtracting the resulting expression from the corresponding full information profit, which is obtained by replacing d by $2d$ in (2.140), yields the instantaneous expected loss in this interval, which is given by

$$E[\mathcal{L}'_3|I^i_{d'}, I^j_{d'}] = \Gamma_1(d - \triangle) + \Lambda_1\tau, \tag{2.152}$$

with $\tau \in [0, 2\triangle)$. Due to the fact that firm j knows firm i's consecutive planning dates, the instantaneous expected loss is in each interval in which firm i is the firm with worse information, that is in $[nd, nd']$, for $n \in \{2, ..., \tilde{m}\}$, given by

$$E[\mathcal{L}'_3|I_0] = \Gamma_1(d - n\triangle) + \Lambda_1\tau,$$

with $\tau \in [0, d - n\triangle)$ and $n \in \{2, ..., \tilde{m}\}$.

From nd' to $(n+1)d$, for $n \in \{2, ..., \tilde{m}\}$

These intervals share the feature that firm j's information set is more outdated. It is easy to see that the firm with the worse information in an alternating planning pattern sets the same optimal price as a firm in the synchronous planning pattern. Thus, it follows immediately, that the expected instantaneous loss is in every interval $[nd', (n+1)d)$, for $n \in \{2, ..., \tilde{m}\}$, given by

$$E[\mathcal{L}_2'|I_0] = \Lambda_1\tau, \tag{2.153}$$

with $\tau \in [0, d - n\triangle)$ and $n \in \{2, ..., \tilde{m}\}$.

The above analysis implies, that the expected loss from the sub-sequence $\mathcal{D}'_{\tilde{m}} = \{d', ..., \tilde{m}d'\}$ is given by

$$\begin{aligned} E[\mathcal{L}'(\mathcal{D}'_{\tilde{m}})|I_0] &= e^{-rd}\int_{\tau=0}^{\triangle} e^{-r\tau}(\Lambda_2 d + \Lambda_1\tau)\,d\tau \\ &+ \sum_{n=1}^{\tilde{m}}\left(e^{-rnd'}\int_{\tau=0}^{d-n\triangle} e^{-r\tau}\Lambda_1\tau d\tau\right) + K\sum_{n=1}^{\tilde{m}} e^{-rnd'} \\ &+ \sum_{n=2}^{\tilde{m}}\left(e^{-rnd}\int_{\tau=0}^{n\triangle} e^{-r\tau}\Big(\Gamma_1(d-n\triangle)+\Lambda_1\tau\Big)d\tau\right). \end{aligned} \tag{2.154}$$

Subtracting the expected loss implied by the equilibrium sub-sequence $\mathcal{D}_{\tilde{m}} = \{d, ..., \tilde{m}d\}$, denoted by $E[\mathcal{L}(\mathcal{D}_{\tilde{m}})|I_0]$, from (2.154) yields

$$\begin{aligned} & e^{-rd}\int_{\tau=0}^{\triangle} e^{-r\tau}(\Lambda_2 d)\,d\tau - \sum_{n=1}^{\tilde{m}}\left(e^{-rnd'}\int_{\tau=0}^{d-n\triangle} e^{-r\tau}\Lambda_1 n\triangle d\tau\right) + K\Big(\sum_{n=1}^{\tilde{m}} e^{-r(nd')} - \sum_{n=1}^{\tilde{m}} e^{-r(nd)}\Big) \\ + & \sum_{n=2}^{\tilde{m}}\left(e^{-rnd}\int_{\tau=0}^{n\triangle} e^{-r\tau}\Gamma_1(d-n\triangle)d\tau\right) + e^{-r\tilde{m}d}\Upsilon. \end{aligned} \tag{2.155}$$

where Υ denotes the difference in expected losses beyond date $\tilde{m}d$. This difference in expected losses is at each instant $t > \tilde{m}d$ bounded below by the expected instantaneous loss implied by the sequence $\mathcal{D}$. Again we get this by assuming the best possible case for the deviation, that is, the expected instantaneous loss that is implied by the sequence $\mathcal{D}'$ is zero from date $\tilde{m}d$ onwards. In addition, we know that the instantaneous loss from the sequence $\mathcal{D}$ is finite since firm i optimally chooses to plan after some time length. Thus, at each instant $t > \tilde{m}d$ the difference in expected losses is bounded below by

$$-e^{-r\tau}\Lambda_1 d,$$

for $\tau \in (0, d]$ and

$$-e^{-rd}K,$$

at a planning date d. Both expressions are finite because d is finite.

In order to determine the per instant difference in expected losses in the time period before date $\tilde{m}d$, we divide the first four terms in (2.155) by $n\triangle$, where n is chosen appropriately for the different intervals. Then, we take the limit $\triangle \to 0$. The last term in (2.155) $e^{-r\tilde{m}d}\Upsilon$ vanishes as $\triangle \to 0$ implies that $\tilde{m} \to \infty$ and the future is discounted at rate $r > 0$. Therefore, we can concentrate on the first four terms when determining the critical inattentiveness length such that deviating to the sequence $\mathcal{D}'$ is profitable.

We get that the lifetime expected loss from deviating to $\mathcal{D}'$ is lower than the lifetime expected loss from the proposed equilibrium sequence $\mathcal{D}$ if $d \leq \underline{d}_s$, where $\underline{d}_s$ solves

$$e^{-r\underline{d}_s}\underline{d}_s\Lambda_2 - e^{-r\underline{d}_s}\frac{\Lambda_1}{r} - \frac{rK}{e^{r\underline{d}_s}-1} + \frac{\underline{d}_s e^{-r\underline{d}_s}}{e^{r\underline{d}_s}-1}\Gamma_1 = 0. \tag{2.156}$$

If $\underline{d}_s \to 0$, the left-hand side of (2.156) goes to $-\infty$ because the term involving $-rK < 0$ is dominating term. Conversely, if $\underline{d}_s \to \infty$, the left-hand side of (2.156) goes to 0 from above. This is the case because the last three terms go to zero at a faster rate than the first term and the first term is strictly positive since $\Lambda_2 > 0$. Thus, there exists a solution to (2.156) at which $\underline{d}_s > 0$. It remains to show that this solution is unique. To do so we differentiate (2.156) with respect to $\underline{d}_s$ to get

$$e^{-r\underline{d}_s}\Lambda_1 + \frac{r^2 e^{r\underline{d}_s}K}{(e^{r\underline{d}_s}-1)^2} - \Gamma_1\left(\frac{e^{-r\underline{d}_s} + r\underline{d}_s(2 - e^{-r\underline{d}_s} - 1)}{(e^{r\underline{d}_s}-1)^2}\right) + \Lambda_2 e^{-r\underline{d}_s}(1 - r\underline{d}_s). \tag{2.157}$$

It is easy to check that for $\underline{d}_s \to 0$, (2.157) goes to ∞ since $K > 0$, while for $\underline{d}_s \to \infty$, (2.157) goes to 0 from below. This is the case because the term $-\Lambda_2 e^{-r\underline{d}_s} r\underline{d}_s < 0$ goes to zero at a slower rate than the other terms. We now look at the four terms of (2.157) in turn. The first two terms are strictly positive and strictly decrease as $\underline{d}_s$ rises. The third term is either positive or negative for any $\underline{d}_s$, dependent on Γ_1 being positive or negative. As the frist two terms, it becomes strictly smaller in absolute value as $\underline{d}_s$ increases but it does so at a higher rate than the first two terms. Finally, the fourth term is the only term in (2.157) that changes its sign from positive to negative as $\underline{d}_s$ increases which occurs at $\underline{d}_s = 1/r$. In addition, this fourth term becomes the dominant term as $\underline{d}_s$ gets larger and larger. As a consequence, there exist a unique value of $\underline{d}_s$ at which (2.157) changes its sign from positive to negative. But this, in combination with the fact that the left-hand side of (2.156) is negative at $\underline{d}_s = 0$ and positive at $\underline{d}_s \to \infty$, implies that there must exist a unique solution to (2.156). As a consequence, we have that if $d \geq \underline{d}_s$ then the lifetime expected loss from deviating to $\mathcal{D}'$ exceeds the lifetime expected loss from the proposed equilibrium sequence $\mathcal{D}$.

Using a similar derivation we obtain that the lifetime expected loss from deviating to $\mathcal{D}''$ exceeds the lifetime expected loss from the proposed equilibrium sequence $\mathcal{D}$ if $d \leq \bar{d}_s$, where $\bar{d}_s$ solves

$$\bar{d}_s\Lambda_1 - rK - \left(\frac{e^{r\bar{d}_s} - 1}{re^{r\bar{d}_s}}\right)\Gamma_1 = 0. \tag{2.158}$$

Existence and uniqueness of $\bar{d}_s$ are implied by arguments similar to the ones developed for $\underline{d}_s$. ■

Proof of Lemma 10

It follows from (2.51) and (2.52) that $\bar{d}_s = \underline{d}_s = \hat{d}_s$ if $\rho = \hat{\rho}$, where

$$\hat{\rho} = \frac{\delta_1 e^{r\hat{d}_s} + \delta_2}{2\sigma_\theta\sigma_\xi\Big((4(1 + r\hat{d}_s) + \gamma^2 - \gamma(3 + r\hat{d}_s))e^{r\hat{d}_s} + \delta_3\Big)}, \tag{2.159}$$

with

$$\begin{aligned}
\delta_1 &:= (\gamma(r\hat{d}_s + 5)\sigma_\xi^2 + (r\hat{d}_s + 1)\sigma_\theta^2) - 2\gamma^2\sigma_\xi^2 - 4(\sigma_\theta^2 + \sigma_\xi^2)(r\hat{d}_s + 1),\\
\delta_2 &:= 4(\sigma_\theta^2 + \sigma_\xi^2) + \gamma((4r\hat{d}_s - 5)\sigma_\xi^2 - \sigma_\theta^2) - 2\gamma^2\sigma_\xi^2(r\hat{d}_s - 1),\\
\delta_3 &:= -4 - \gamma(2r\hat{d}_s - 3) - \gamma^2(1 - r\hat{d}_s).
\end{aligned}$$

Differentiating $\hat{\rho}$ with respect to r yields

$$\frac{\hat{d}_s\gamma(\sigma_\theta^2 - \sigma_\xi^2)(2-\gamma)(4-\gamma)(1 + e^{2r\hat{d}_s} + (\hat{d}_s r)^2 e^{r\hat{d}_s} - 2e^{r\hat{d}_s})}{2\sigma_\theta\sigma_\xi\Big((4(1 + r\hat{d}_s) + \gamma^2 - \gamma(3 + r\hat{d}_s))e^{r\hat{d}_s} + \delta_3\Big)^2}. \tag{2.160}$$

Thus,

$$\mathrm{sign}\Big\{\frac{\partial\hat{\rho}}{\partial r}\Big\} = \mathrm{sign}\Big\{\gamma(\sigma_\theta^2 - \sigma_\xi^2)\Big\}.$$

Therefore, $\hat{\rho}$ is decreasing in r if either $\sigma_\theta > \sigma_\xi$ and $\gamma < 0$ or $\sigma_\theta < \sigma_\xi$ and $\gamma > 0$. If these conditions are not met, then $\hat{\rho}$ is increasing in r.

If $\hat{\rho}$ is decreasing in r, then we obtain the upper bound of $\hat{\rho}$, which we denote as $\bar{\rho}$, by taking the limit $r \to 0$. We get that

$$\bar{\rho} = \lim_{r \to 0} \hat{\rho} = -\frac{\sigma_\theta^2(4-\gamma) + \sigma_\xi^2(4 - 5\gamma + 2\gamma^2)}{2\sigma_\theta\sigma_\xi(4 - 3\gamma + \gamma^2)}.$$

Thus, $\bar{\rho} = \rho'$.

If $\hat{\rho}$ is increasing in r, we obtain the upper bound of $\hat{\rho}$, which we denote as $\tilde{\rho}$, by taking the limit $r \to \infty$. We get

$$\tilde{\rho} = \lim_{r \to \infty} \hat{\rho} = -\frac{\sigma_\theta^2 + \sigma_\xi^2}{2\sigma_\theta\sigma_\xi}.$$

It is easy to show that $\tilde{\rho} < -1$. ■

Proof of Proposition 4

We constructed the range of candidate equilibrium inattentiveness intervals by assuming that a firm may only deviate marginally from the proposed equilibrium planning horizon. Thus, we have to complement the analysis by considering non-infinitesimal deviations.

Suppose for the remainder of the proof that $\rho > \hat{\rho}$ so that $\underline{d} \leq \bar{d}$. As in the alternating planning scenario we restrict the exposition to fractional deviations: firm j sticks to the proposed inattentiveness period $d \in [\underline{d}, \bar{d}]$ and firm i chooses to remain inattentive for a period of $d\,(1 \pm l/m)$, $m \in \mathbb{N}$ and $l \in \{l \in \mathbb{N} : l \leq m\}$.

The proof proceeds as follows. First, we show that the fractional deviations of the type $d\,(1 \pm 1/m)$ induce the tightest bounds on the range of potential equilibria. Second, we derive a firm's expected lifetime loss from deviating non-infinitesimally from any candidate equilibrium inattentiveness interval. Third, we characterize the tightest bounds that this type of non-marginal deviation imposes on the range of inattentiveness periods that are robust against marginal deviations.

Comparison of the $d(1 \pm l/m)$, $l > 1$, and $d(1 \pm 1/m)$ deviation

Generally, a deviation to a sequence $d(1 + l/m)$, $l > 1$ induces higher expected losses net of planning cost than a deviation to the sequence $d(1 + 1/m)$. This is due to the fact that with the former type of deviation firm i is the worse informed firm for a longer time. The advantage of the former over the latter lies in the planning cost reduction. Thus, firm i prefers the $d(1 + 1/m)$ to the $d(1 + l/m)$, $l > 1$, deviation if

$$E\left[\mathcal{S}\left(\left(1+\frac{l}{m}\right)d\right)\right] - E\left[\mathcal{S}\left(\left(1+\frac{1}{m}\right)d\right)\right] - K\frac{e^{\frac{rd(m+l)}{m}} - e^{\frac{rd(m+1)}{m}}}{\left(e^{\frac{rd(m+1)}{m}}-1\right)\left(e^{\frac{rd(m+l)}{m}}-1\right)} \geq 0, \tag{2.161}$$

where $E\left[\mathcal{S}((1 + l/m)\, d)\right]$ denotes the expected losses net of planning cost induced by the sequence $(1 + l/m)\, d$ and $E\left[\mathcal{S}((1 + 1/m)\, d)\right]$ denotes the expected losses net of planning cost induced by the sequence $(1 + 1/m)\, d$.

For $d \to 0$ and $d \to \infty$ the net expected losses induced by both sequences are identical. In the first case both firms plan at each instant whereas in the case both firms never plan. Thus, in both cases the difference between the net expected losses is zero. Thus, for $d \to 0$ the left-hand side of (2.161) converges to $-\infty$. For $d \to \infty$ the left-hand side of (2.161) goes to zero. We know from the arguments developed in Lemma 9 concerning existence and uniqueness that the left-hand side of (2.161) is first increasing and then decreasing in d. Thus, there exists a unique d, denoted by $\hat{d}$, so that the inequality in (2.161) holds for all $d > \hat{d}$. Thus, for given m the fractional deviation of type $d(1+1/m)$ induces the largest lower bound on the set of potential equilibrium inattentiveness periods as compared to fractional deviations of the type $d(1 + l/m)$, $l > 1$.

Similar arguments imply that for given m the fractional deviation of type $d(1 - 1/m)$ induces the lowest upper bound on the set of potential equilibrium inattentiveness periods as compared to fractional deviations of the type $d(1 - l/m)$, $l > 1$.

Now, we derive firm i's expected stream of losses if it deviates to an inattentiveness interval of length $d(1 + 1/m)$, $m \in \mathbb{N}$.

$d(1+1/m)$: Expected instantaneous losses

Suppose that firm i deviated to $d + d/m$, $m \in \mathbb{N}$, whereas firm j sticks to the equilibrium planning horizon d. This deviation induces a stream of expected losses that is composed of three different instantaneous expected losses:

$$\nu_1 \;:=\; \Lambda_1 \tau + \Lambda_2 d, \tag{2.162}$$

$$\nu_2 \;:=\; \Lambda_1 \tau, \tag{2.163}$$

$$\nu_3 \;:=\; \Lambda_1 \tau + \Gamma_1 \Big(\frac{m-k}{m}\Big) d,\; k \in \mathbb{N},\; k < m. \tag{2.164}$$

The expected instantaneous loss in the period that elapses between the first planning date on the equilibrium path and the first planning date under the deviation are given by (2.162). In this period, firm i's deviation is undetected by firm j. The component (2.163) captures firm i's expected instantaneous losses in the time period in which it is the better informed firm, irrespective of whether the deviation was detected or not. (2.164) captures firm i's expected instantaneous losses in the time period in which it is the worse informed firm after its deviation has been detected.

The counter $((m-k)/m)d$, $k \in \{0, ..., m-1\}$ in (2.164) measures the time that elapses between a planning date of firm i and the consecutive planning date of firm j. If, e.g., $m = 5$ and $k = 4$, then the time that elapses between the kth planning date of firm i and the $(k+1)$th planning date of firm j is given by $(k+1)d - k(1+1/m)d = 5d - 4(1+1/5)d = 1/5d$. It also becomes evident from this example that the firms plan simultaneously every $(m+1)d$ periods: firm i repeated its planning horizon exactly m times and firm j planned for the $(m+1)$th time at this date. This is due to the fact that we consider deviations in which the deviant expands its inattentiveness period by a fraction.

$d(1+1/m)$: Expected stream of losses

Given the expected instantaneous expected losses (2.162) to (2.164) a deviant's expected stream of losses is given by

$$
\begin{aligned}
E[\underline{\mathcal{S}}|\theta_0,\xi_0] \;=\;& e^{-rd}\int_0^{\frac{d}{m}} e^{-r\tau}\nu_1 d\tau + \sum_{k=1}^{m-1}\left(e^{-r\left(1+\frac{1}{m}\right)kd}\left(\int_{\tau=0}^{\left(\frac{m-k}{m}\right)d} e^{-r\tau}\nu_2 d\tau\right)\right) \\
&+\sum_{n=1}^{\infty}\sum_{k=0}^{m-1}\left(e^{-r\left(n(m+1)+\left(1+\frac{1}{m}\right)k\right)d}\left(\int_{\tau=0}^{\left(\frac{m-k}{m}\right)d} e^{-r\tau}\nu_2 d\tau\right)\right) \\
&+\sum_{k=1}^{m-1}\left(e^{-r(1+k)d}\left(\int_{\tau=0}^{\left(\frac{k+1}{m}\right)d} e^{-r\tau}\nu_3 d\tau\right)\right) \\
&+\sum_{n=1}^{\infty}\sum_{k=0}^{m-1}\left(e^{-r(n(m+1)+1+k)d}\left(\int_{\tau=0}^{\left(\frac{k+1}{m}\right)d} e^{-r\tau}\nu_3 d\tau\right)\right) \\
&+K\sum_{n=1}^{\infty}\left(e^{-rn\left(1+\frac{1}{m}\right)d}\right). \qquad (2.165)
\end{aligned}
$$

Now, we derive the deviant's expected stream of losses if it chooses an inattentiveness interval of length $d - d/m$, $m \in \mathbb{N}$ instead of d.

$d(1-1/m)$: Expected instantaneous losses

Suppose that firm i deviated to $d - d/m$, $m \in \mathbb{N}$, whereas firm j sticks to the equilibrium planning horizon d. This deviation induces a stream of expected losses that is composed of two different instantaneous expected losses:

$$
\begin{aligned}
\phi_1 \;&:=\; \Lambda_1\tau, \qquad (2.166)\\
\phi_2 \;&:=\; \Lambda_1\tau + \Gamma_1\left(\frac{k}{m}\right)d,\; k\in\mathbb{N},\; k<m. \qquad (2.167)
\end{aligned}
$$

In this scenario, the deviation is detected by firm j at its first planning date. The component (2.166) captures firm i's expected instantaneous losses in the time period in which it is the better informed firm, irrespective of whether the deviation was detected or not. Correspondingly, (2.167) captures firm i's expected instantaneous losses in the time period in which it is the worse informed firm.

The counter $(k/m)d$, $k \in \mathbb{N}$, $k < m$ in (2.167) measures the time that elapses between a planning date of firm i and the consecutive planning date of firm j. In order to illustrate this, suppose again that $m = 5$ and $k = 3$. Then the time that elapses between the kth planning date of firm i and the kth planning date of firm j is given by $(k)d - k(1-1/m)d =$

$3d - 3(1-1/5)d = 3/5d$. It also becomes evident from this example that the firms plan simultaneously every $(m-1)d$ periods: firm i repeated its planning horizon exactly $m-1$ times and firm j planned for the mth time at this date. Again, this is due to the fact that we consider deviations in which the deviant expands its inattentiveness period by a fraction.

$d(1-1/m)$: Expected stream of losses

Given the expected instantaneous expected losses (2.166) and (2.167) a deviant's expected stream of losses is given by

$$\begin{aligned} E[\bar{\mathcal{S}}|\theta_0,\xi_0] &= \sum_{n=0}^{\infty}\sum_{k=1}^{m-1}\left(e^{-r\left(k-\frac{k}{m}+n(m-1)\right)d}\left(\int_0^{\left(\frac{k}{m}\right)d} e^{-r\tau}\phi_1 d\tau\right)\right)+\sum_{n=0}^{\infty}\left(e^{-r(n(m-1)d)}\int_{\tau=0}^{\left(1-\frac{1}{m}\right)d}\phi_1 d\tau\right) \\ &\quad +\sum_{n=0}^{\infty}\sum_{k=1}^{m-2}\left(e^{-r(n(m-1)+1+k)d}\left(\int_{\tau=0}^{\left(1-\frac{k+1}{m}\right)d} e^{-r\tau}\phi_2 d\tau\right)\right) \\ &\quad +K\sum_{n=1}^{\infty}\left(e^{-rn\left(1-\frac{1}{m}\right)d}\right). \end{aligned} \tag{2.168}$$

Now, we turn to the third part of the proof and characterize the equilibrium range of synchronous inattentiveness intervals.

To do so, we set out by deriving a firm's incentive to deviate for each type of fractional deviation. Subtracting from each fractional-deviation induced expected stream of losses – (2.165) to (2.168) – the equilibrium expected stream of losses which is appropriately adapted to the considered time horizon yields

$$E[\underline{\mathcal{S}}|\theta_0,\xi_0] \quad - \quad E[\mathcal{S}_1|\theta_0,\xi_0], \tag{2.169}$$

$$E[\bar{\mathcal{S}}|\theta_0,\xi_0] \quad - \quad E[\mathcal{S}_2|\theta_0,\xi_0], \tag{2.170}$$

where

$$\begin{aligned} E[\mathcal{S}_1|\theta_0,\xi_0] &= \frac{e^{rd}-1-rd}{e^{rd}r^2(e^{rd}-1)}\Lambda_1+\frac{K}{e^{rd}-1}, \\ E[\mathcal{S}_2|\theta_0,\xi_0] &= e^{-r\left(1-\frac{1}{m}\right)d}\int_{\tau=0}^{\frac{d}{m}} e^{-r\tau}\left(\Lambda_1\left(\left(1-\frac{1}{m}\right)d+\tau\right)\right)d\tau+E[\mathcal{S}_1|\theta_0,\xi_0]. \end{aligned}$$

Thus, it is profitable for a firm to deviate from a proposed synchronous inattentiveness period d in a non-marginal way if either (2.169) or (2.170) are negative for at least one $m\in\mathbb{N}$.

Using (2.169) and (2.170) we can show that synchronous planning cannot be an equilibrium for $\gamma < 0$ and $-1 \leq \rho \leq \hat{\rho}$. Thus, according to Lemma 10 a synchronous planning equilibrium can only exist for $\gamma > 0$ and $\rho \geq \max\{-1, \hat{\rho}\}$.

In order to derive the tightest bounds that the non-marginal "no-deviation" conditions impose on the range of potential equilibria we proceed as follows: we treat, for the sake of simplicity, m as a real non-negative number and minimize (2.169) and (2.170) with respect to m. Denote the real numbers that solve the minimization problem of (2.169) and (2.170) by $\underline{m}$ and $\bar{m}$. It can be shown that $\underline{m} = \bar{m}$. Accordingly, denote by $m^\star = [\bar{m}]$ the integer which is closest to $\bar{m}$. This yields for each direction the fractional deviation, i.e., $\left(1 - \frac{1}{m^\star}\right) d$ and $\left(1 + \frac{1}{m^\star}\right) d$, that induces the smallest expected stream of losses. As a consequence, the equilibrium inattentiveness intervals for which (2.169) and (2.170) are zero for $m = m^\star$ constitute the tightest bounds that characterize the equilibrium range.

For the sake of brevity, we limit the presentation to the derivation of $\underline{m}$. Solving (2.169) with respect to K and taking $r \to 0$ in the solution delivers

$$\begin{aligned} K \;=\; & \frac{d^2}{24(2-\gamma)^2 m}\Big(\left(2(\sigma_\xi^2 + \rho\sigma_\theta\sigma_\xi)\gamma^2 - (\sigma_\theta^2 + 6\rho\sigma_\theta\sigma_\xi + 5\sigma_\xi^2)\gamma + 4(\sigma_\theta^2 + 2\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2)\right)\gamma m^2 \\ & + (2(\sigma_\theta^2 + 2\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2) - (2-\gamma)(\sigma_\xi^2 + \rho\sigma_\theta\sigma_\xi)\gamma^2)6m \\ & + 4(3-\gamma)(\sigma_\theta^2 + 2\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2) + (\sigma_\theta^2 - 6\rho\sigma_\theta\sigma_\xi - 7\sigma_\xi^2)\gamma^2\Big). \end{aligned} \tag{2.171}$$

Differentiating (2.171) with respect to m yields

$$\underline{m} = \sqrt{\frac{4(\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2)\gamma^3 + (\sigma_\theta^2 - 6\rho\sigma_\theta\sigma_\xi - 7\sigma_\xi^2) + 4(\sigma_\theta^2 + 2\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2)(3-\gamma)}{\gamma(2(\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2)\gamma^2 - (\sigma_\theta^2 + 6\rho\sigma_\theta\sigma_\xi + 5\sigma_\xi^2)\gamma + 4(\sigma_\theta^2 + 2\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2))}}. \tag{2.172}$$

Solving the second derivative of (2.169) with respect to m for K, taking $r \to 0$ in the solution and evaluating the resulting expression at $\underline{m}$ yields that this is indeed a minimum. Thus, it follows from (2.171) and (2.169) that a firm has no incentive to deviate to an inattentiveness period of length $d + \frac{d}{m} \forall m \in \mathbb{N}$, where

$$d_s^l = \frac{2\sqrt{6}(2-\gamma)\sqrt{Km^\star}}{\sqrt{\Theta(m^\star + 1)}},$$

and

$$\begin{aligned}
\Theta &= (\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2)(2+m^\star)2\gamma^3 - ((7+5m^\star)\sigma_\xi^2 + 6\rho\sigma_\theta\sigma_\xi(m^\star+1) + \sigma_\theta^2(m^\star-1))\gamma^2 \\
&\quad +4(\sigma_\theta^2 + 2\rho\sigma_\theta\sigma_\xi + \sigma_\xi^2)(m^\star-1)(\gamma(m^\star-1)+3), \\
m^\star &= \lfloor m \rfloor .
\end{aligned}$$

Similar arguments imply that a firm has no incentive to deviate to an inattentiveness period of length $d - \frac{d}{m}\,\forall m \in \mathbb{N}$ if $d \leq d_s^u$, where

$$d_s^u = \frac{8\sqrt{6}\sqrt{Km^\star}}{\sqrt{\Theta(m^\star+1)}}.$$

Obviously, $d_s^u > d_s^l$ for $\gamma > 0$.

A similar argument yields that the bounds that are implied by player 2 choosing planning mode (A) and an fractional deviation of the type $(1 + l/m)\,d$ are not tighter than d_s^l and d_s^u. Hence, every common $d \in [d_s^l, d_s^u]$ is a synchronous equilibrium inattentiveness period. ■

Chapter 3

The Choice of Prices vs. Quantities under Uncertainty†

3.1 Introduction

The two classic papers in the theory of strategic interaction among firms are those by Cournot (1838) and Bertrand (1883). The former proposes quantities as the strategy variable while the latter suggests prices. Since then it has been well known that quantities induce a lower degree of competition if goods are substitutes. So if firms were free to choose their strategy variable, they would prefer quantities rather than prices. This result was first confirmed by Singh and Vives (1984) and Cheng (1985) in a deterministic two-stage game in which duopolists first choose their strategy variable and compete afterwards. They show that quantity-setting is a dominant action for both firms.[1,2]

However, a deterministic model is not fully appropriate because firms often face uncertainty at the time the strategy variable has to be chosen. For example, firms may be uncertain about the size of the market or about the distribution of consumers' valuations. Our analysis

†This chapter is joint work with Markus Reisinger and provides an extended version of Reisinger and Ressner (2009).

[1]This result still tends to hold if there are more than two firms. For example, Vives (1985) and Häckner (2000) analyze models with N firms and predetermined strategy variables. They show that profits under quantity competition are higher than under price competition if firms are not too asymmetric. Recently, Tanaka (2001) and Tasnádi (2006) consider two-stage models in which firms select the strategy variable. Both find that quantity-setting is the subgame perfect equilibrium if firms are symmetric and if there are no capacity limits.

[2]Singh and Vives (1984) also show that if goods were complements, price-setting would be a dominant action for each firm. In the remainder, we concentrate on the case of substitutable goods and briefly discuss the case of complementary goods at the end of Section 3.3.

incorporates this aspect by introducing uncertainty via shocks that affect the slope and the intercept of the demand curve. We show that the dominance of quantity-setting no longer holds since a higher amount of uncertainty lowers firms' profits under quantity competition and thus favors prices. So if uncertainty is high compared to the degree of substitutability, price-setting is a dominant action for each firm. Moreover, we find that for an intermediate amount of uncertainty the equilibrium involves one price-setting and one quantity-setting firm.

Our analysis employs the same game structure as Singh and Vives (1984) and Cheng (1985), namely firms first select their strategy variable independently of each other and then compete. In contrast to these authors, we consider stochastic demand. We set out by developing the main insights in the simplest possible framework. Specifically, we consider a linear demand system where a shock affects the slope of the demand curves. To focus purely on the strategic effects of variable choice we assume constant marginal cost. We demonstrate that there is a relative advantage of price-setting due to uncertainty. The reason is that with a slope shock the market size is uncertain while the distribution of reservation prices is not. As a consequence, the ex post optimal quantity depends more strongly on the realization of the shock than the ex post optimal price. Thus, a quantity-setting firm incurs a larger loss in expected profit relative to the ex post optimal level. This uncertainty-based benefit of price-setting is the larger, the larger is the variance of the shock. On the other hand, the relative advantage of quantity-setting is that is induces a smaller degree of competition. This becomes more pronounced the larger the degree of substitutability. Therefore, firms commit to a price if uncertainty is high relative to the degree of substitutability and commit to a quantity if the reverse holds true. Moreover, for every degree of substitutability there exists an intermediate range of uncertainty in which no effect dominates, and the equilibrium outcome involves one firm committing to a price and the other one to a quantity.

We extend our analysis by introducing a shock to the intercept that might be correlated with the shock to the slope. This case is arguably more relevant in reality because uncertainty usually affects both market size and reservation price distribution. Since intercept and slope are now stochastic, both price- and quantity-setting are associated with a profit loss relative to the ex post optimal level. Still, the same line of reasoning as before applies. However, now it is the covariance that in addition to the variance of the slope shock drives firms' choices of their strategy variables. If the covariance is not too negative, market size varies more than the distribution of reservation prices. In this case, the expected loss is smaller under price-setting. Thus, firms commit to a price rather than to a quantity if the covariance is sufficiently large compared to the degree of substitutability, and vice versa. The "hybrid" outcome in which firms choose different strategy variables in equilibrium still arises but only for sufficiently high degrees of substitutability.

To demonstrate the empirical relevance of our results consider the industry for wine, brandy, and brandy spirits (SIC 2084) and the industry for distilled and blended liquor (SIC 2085).[3] In a recent paper, de Jong et al. (2008) find, by using a modified version of the methodology developed in Sundaram et al. (1996),[4] that these comparable industries differ with respect to their mode of competition: Firms engage in price competition in the wine and brandy industry but in quantity competition in the liquor industry.

Arguably, firms in both industries have a meaningful choice of their strategy variable. Vintners, for example, can control the supplied quantity by changing their acreage. Moreover, both industries produce distilled goods, which implies that the technology is such that firms can to some degree satisfy market demand by expanding their production. According to de Jong et al. (2008), however, the wine and brandy industry faces a higher degree of demand uncertainty than the liquor industry. This seems reasonable as the quality of wine is more sensitive to weather conditions than that of liquor. Thus, consumers' willingness to pay for wine should be more volatile than that for of-the-shelf liquor. Concerning the competitiveness, we would expect that liquors are at least as substitutable as wines. Hence, it is likely that the amount of demand uncertainty matters for the type of strategic interaction

[3] A description of the functioning of the industries that we mention in this example can be found at http://www.referenceforbusiness.com/industries/.

[4] Sundaram et al. (1996) develop a proxy – the competitive strategy measure (CSM) – for whether firms compete in strategic complements or substitutes, i.e. in prices or quantities. The CSM is defined as the correlation between (i) the ratio of the change in a firm's profit and the change in its sales, and (ii) the change in the firm's rivals' combined sales. If an industry's CSM is positive (negative), the firms in this industry are classified as competing in strategic complements (substitutes). A potential problem of the methodology that Sundaram et al. (1996) use to estimate the CSM is that it might yield biased estimates due to industry-wide shocks (see e.g. Lyandres (2006)). In order to mitigate this potential flaw, de Jong et al. (2008) estimate the CSM using quarterly data during a relatively short time period, vary the time periods in order to check the robustness of their estimates, and use only those industries for their analysis which have a CSM that is statistically significant at the 10% confidence level.

in the industry: Demand for wine is more uncertain than that for liquor, and so the wine producing industry competes in prices and the liquor distilling industry in quantities.[5]

A different pair of industries is household audio and video equipment (SIC 3651) and prerecorded audio tapes and disks (SIC 3652). Here, de Jong et al. (2008) find that firms compete in prices in the former but in quantities in the latter. However, according to de Jong et al. (2008), demand uncertainty is very similar. In both industries firms have some leeway in choosing their strategy variable. In the tape and disk industry, a recording company contracts with several hundred artists from different music categories. So it clearly has the option to vary the production levels of some genres, but it can also set different prices for the recorded product. In the audio and video equipment industry, technology does not restrict manufacturing firms to produce on demand but they could also produce a quantity and let the price clear the market.

Yet, the degree of substitutability seems to be very different in both industries. In the tape and disk industry, products are relatively close substitutes and often only different promotion strategies determine the number of sales.[6] In the audio and video equipment industry, consumers often have brand preferences, e.g. for domestic or foreign producers or because of different product experience. This implies that the degree of substitutability in this industry is relatively low. As a consequence, the finding of de Jong et al. (2008) that firms compete in prices in the electronic equipment industry but in quantities in the tapes and disks industry is consistent with the prediction of our model that the industry which exhibits a higher degree of substitutability should be more likely to compete in quantities if demand uncertainty is similar.

One potential caveat of the examples is that there might be more characteristics that differ between the compared industries besides demand uncertainty (in the first example) and degree of substitutability (in the second example). For instance, de Jong et al. (2008) observe that Cournot industries consist of less firms. This could be part of the explanation as well. It is, however, neither theoretically nor empirically clear that industries with more firms

[5]In general, de Jong et al. (2008) find that in the industries that compete in prices demand uncertainty is on average higher than in the industries that compete in quantities.

[6]The above cited website describes the tape and disc industry as "intensely competitive" compared to other industries.

are more prone to compete in strategic complements. We argue that our theory helps to explain the contrasting empirical findings in the two pairs of closely related industries over and above other (un)observed industry characteristics.

The only paper that analyzes the choice of prices versus quantities under uncertainty in an oligopolistic setting is Klemperer and Meyer (1986).[7] They consider a one-stage duopoly game in which a firm chooses the strategy variable and its magnitude at the same time. Klemperer and Meyer (1986) already identify the uncertainty-based benefit of price-setting if market size is uncertain. The intuition is the same as in our work. Yet, as their game has a simultaneous structure, each firm acts as a monopolist given its expected residual demand curve. Thus, firms choose to set a price because it adapts optimally to demand uncertainty.[8] The competitive advantage of quantity-setting, however, is not present. Even if uncertainty is small, firms select prices although they induce harsher competition. By contrast, in our setting the relative *magnitude* of the strategic benefit of quantities and the uncertainty-based benefit of prices is crucial for variable choice. Thus, it is the amount of uncertainty and not only its mere presence that matters.[9]

This chapter proceeds as follows. Section 3.2 sets out the model. In Section 3.3 we solve for the subgame perfect equilibrium in case of a shock affecting the slope. In Section 3.4 we augment the model by incorporating a shock to the intercept. Section 3.5 concludes.

[7]Even in single-decision making problems there are only few papers that deal with this choice . See, e.g Weitzman (1974) who analyzes the incentives of a social planner, or Reis (2006b) who considers the choice of a monopolist under general demand conditions.

[8]Here, we refer to the analysis of Klemperer and Meyer (1986) with constant marginal cost. In addition, they consider more general cost functions and show that quantity-setting becomes optimal if the cost function is sufficiently convex. Yet, as explained above, in our analysis the optimality of quantity-setting is driven by a different effect.

[9]There are other papers that consider a deterministic set-up but deviate from the structure in Singh and Vives (1984). For example, Correa-López (2007) incorporates the input market and Lambertini (1997) analyzes an infinitely repeated game after variable choice.

3.2 The Model

Consider a duopoly with differentiated products. Assume that firms face the linear inverse demand system

$$p_i = \alpha - \frac{\beta}{\theta} q_i - \frac{\gamma}{\theta} q_j, \tag{3.1}$$

$$p_j = \alpha - \frac{\beta}{\theta} q_j - \frac{\gamma}{\theta} q_i, \tag{3.2}$$

with $\alpha > 0$ and $\beta > \gamma \geq 0$.[10] When $\gamma \to \beta$, products become perfect substitutes, whereas with $\gamma = 0$, they are independent. θ is a random variable with $E[\theta] = 1$ and $\mathrm{Var}(\theta) = \sigma_\theta^2 > 0$. We denote $E[1/\theta]$ by z. By Jensen's inequality, $z > 1$ and it increases in σ_θ^2. To avoid unnecessary complications, we require the support of θ to be sufficiently small that no equilibria emerge in which a price-setting firm sells a negative quantity or a quantity-setting firm receives a negative price. We further assume that firms have zero marginal costs.[11]

Competition between firms takes the form of a two-stage game. In stage 1 firms simultaneously and irrevocably choose their strategy variables, i.e. they offer a price or a quantity contract. With a price contract a firm commits to supply consumers' demand at a predetermined price in the second stage while a quantity contract commits a firm to supply a predetermined quantity in the second stage. Each firm observes the other firm's contract choice and competes in stage 2 contingent on the chosen strategy variables. Thereafter, the shock realizes, markets clear, and profits accrue.

3.3 Solution to the Model

We solve for the subgame perfect equilibrium by backward induction.

[10] The demand system and the way in which the shock affects the slopes of the inverse demand curves are the same as in Klemperer and Meyer (1986).

[11] As Singh and Vives (1984) show, the analysis would not change if firms faced positive constant marginal costs c because this would only lower the effective intercept from α to $\alpha - c$.

3.3.1 The Second Stage

First, suppose that both firms offer a price contract. Solving the equations in (3.1) and (3.2) for q_i and q_j gives firm i's demand curve

$$q_i = \frac{\theta(\alpha(\beta-\gamma) - \beta p_i + \gamma p_j)}{\beta^2 - \gamma^2}. \tag{3.3}$$

As $E[\theta] = 1$, firm i thus solves

$$\max_{p_i} \quad p_i\left(\frac{\alpha(\beta-\gamma) - \beta p_i + \gamma p_j}{\beta^2 - \gamma^2}\right).$$

Computing the solution to the maximization problems of firm i and j, solving for the equilibrium prices and inserting these prices into the profit function yields an expected profit for each firm of

$$\Pi^{pp} = \frac{\alpha^2(\beta-\gamma)\beta}{(\beta+\gamma)(2\beta-\gamma)^2}. \tag{3.4}$$

Next, suppose that both firms offer the quantity contract. Since $E[1/\theta] = z$, firm i solves

$$\max_{q_i} \quad q_i(\alpha - z(\beta q_i + \gamma q_j)). \tag{3.5}$$

Proceeding as before we obtain that each firm's expected profit is

$$\Pi^{qq} = \frac{\alpha^2\beta}{z(2\beta+\gamma)^2}. \tag{3.6}$$

Last, if firm i commits to a quantity while firm j commits to a price, firm i's inverse demand curve and firm j's demand curve are given by

$$p_i = \alpha\Big(1 - \frac{\gamma}{\beta}\Big) - \frac{1}{\theta}\Big(\frac{\beta^2-\gamma^2}{\beta}\Big)q_i + \frac{\gamma}{\beta}p_j. \tag{3.7}$$

and

$$q_j = \alpha\frac{\theta}{\beta} - \frac{\theta}{\beta}p_j - \frac{\gamma}{\beta}q_i \tag{3.8}$$

Calculating expected profits yields

$$\Pi^{qp} = \frac{\alpha^2 z(\beta^2-\gamma^2)(2\beta-\gamma)^2}{(4z(\beta^2-\gamma^2)+\gamma^2)^2\beta} \quad \text{and} \quad \Pi^{pq} = \frac{\alpha^2(\beta-\gamma)^2(2z(\beta+\gamma)-\gamma)^2}{(4z(\beta^2-\gamma^2)+\gamma^2)^2\beta},$$

where Π^{qp} is the quantity-setting firm's profit and Π^{pq} is the price-setting firm's profit.

Before we continue with the analysis, we introduce some notation.[12] Conditional on firm j offering a quantity contract the difference in profits of firm i between choosing a price and a quantity contract is defined as $\Delta\Pi^q(\gamma, z) := \Pi^{pq} - \Pi^{qq}$. If firm j offers a price contract, this difference is defined as $\Delta\Pi^p(\gamma, z) := \Pi^{pp} - \Pi^{qp}$.

3.3.2 The First Stage

As spelled out before, if the game is deterministic ($\sigma_\theta^2 = 0$), it is the dominant action for firms to commit to a quantity in the first stage since they induce softer competition.[13] The following lemma shows that this is no longer true under uncertainty.

Lemma 1 *For every $\gamma \in (0, \beta)$, there exists a $z^q(\gamma)$ such that*

$$\Delta\Pi^q(\gamma, z) \gtreqless 0 \quad \textit{if} \quad z \gtreqless z^q(\gamma),$$

and there exists a $z^p(\gamma)$ such that

$$\Delta\Pi^p(\gamma, z) \gtreqless 0 \quad \textit{if} \quad z \gtreqless z^p(\gamma).$$

$z^q(\gamma)$ and $z^p(\gamma)$ are strictly increasing with $\lim_{\gamma\to 0} z^q(\gamma) = \lim_{\gamma\to 0} z^p(\gamma) = 1$ and $z^p(\gamma) > z^q(\gamma)$ for all $\gamma \in (0, \beta)$.

Proof See the Appendix.

[12] In the following analysis, we hold α and β fixed and consider only changes in γ and z to point out the tension between the degree of substitutability and the amount of uncertainty.

[13] This is easy to check since for all $\gamma \in (0, \beta)$, $\Delta\Pi^q(\gamma, 1) < 0$ and $\Delta\Pi^p(\gamma, 1) < 0$.

The result that $z^q(\gamma)$ and $z^p(\gamma)$ are strictly increasing highlights the trade-off between uncertainty and the degree of substitutability. Let us explain this in detail.

First we turn to the strategic advantage of quantity-setting, namely that competing by choosing a quantity lowers the competitive pressure a firm faces in the second stage, irrespective of the strategy variable that its rival competes in. The economic reasoning behind this strategic advantage is the following: If firm j competes by choosing a price, a reduction in the price of firm i leaves the price of firm j unchanged. If, to the contrary, firm j competes by setting a quantity, a price cut of firm i induces firm j to reduce its price in order to keep its quantity fixed. Since the firms' products are substitutes, it follows that a price reduction by firm i will increase its demand by less if firm j competes by quantity-setting than if it competes by price-setting. A similar argument applies if firm i competes by fixing a quantity, as a quantity increase of firm i corresponds to a reduction of its price. Formally, the above reasoning translates into the fact that firm i's expected inverse residual demand curve is more steeply negatively sloped when firm j is fixing a quantity than when it is fixing a price.[14] As can be seen from (3.5), the slope is $-z\beta$ in case firm j fixes a quantity, while, from (3.7), it is $-z\Big((\beta^2-\gamma^2)/\beta\Big)$ if firm j fixes a price. As a consequence, if firm j competes by setting a quantity, firm i's incentive to expand its market share is lower than if firm j competes by setting a price. Thus, for a given degree of substitutability, competition is the softest if both firms compete in quantities, it is fiercer if one firm competes in quantities and the other firm competes in prices, and it is the fiercest if both firms compete in prices.

Now we explain that the strategic advantage of quantity-setting increases in the degree of substitutability. As explained above, if firm j competes by choosing a quantity, a price cut of firm i causes a decrease in firm j's price to keep its quantity fixed. The amount by which firm j has to reduce its price is the larger, the larger the degree of substitutability between the products. The reason is that if goods are closer substitutes, firm j's demand decreases by more after a price cut of firm i. Thus, a larger reduction in p_j is required to keep firm j's quantity fixed. In the limit, as goods become perfect substitutes, this reduction in p_j is such that firm i's demand will not increase if it cuts its price. In mathematical terms, this can again be seen from the slopes of firm i's expected inverse residual demand curves, which is $-z\beta$, if firm j competes in quantities, and $-z\Big((\beta^2-\gamma^2)/\beta\Big)$, if it competes in prices.

[14] As laid out before, the strategic advantage is independent of whether firm i competes by setting a price or a quantity itself. Thus, it suffices to compare its residual demand curves.

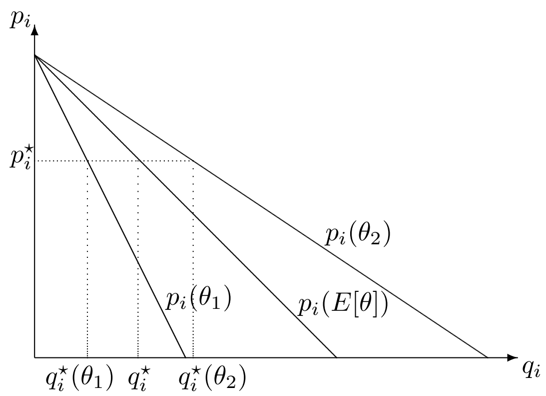

Figure 3.1: Residual demand curves under one-dimensional uncertainty

The absolute difference between the former and the latter, which constitutes the strategic advantage of quantity-setting, increases in the degree of substitutability.

Now we turn to the uncertainty-based benefit of price-setting, namely that the relative attractiveness of fixing a price is the higher, the less variable is the ex post optimal price compared to the ex post optimal quantity. Suppose that firm j commits to choose a price. In this case, the inverse residual demand of firm i is, for a given realization of θ, specified by (3.7). It is easy to see that the effective intercept, $\alpha(1-\gamma/\beta)+p_j(\gamma/\beta)$, is independent of θ. As a consequence, firm i's ex post optimal price is not affected by the shock. The ex post optimal quantity, however, does depend on the shock realization. This is due to the fact that the slope of firm i's inverse residual demand, $-(\beta^2-\gamma^2)/(\theta\beta)$, varies with θ. We illustrate this by means of an example depicted in Figure 3.1, where θ can take on two values, either θ_1 or θ_2, with $\theta_2 > \theta_1$. This implies that firm i incurs a profit loss relative to the ex post optimal level by fixing a quantity while this is not the case when it fixes a price. Figure 1 does not apply exactly if firm j commits to a choose a quantity, because in this case the effective intercept of firm i's inverse demand depends on θ as well. However, the ex post optimal price is still less variable than the ex post optimal quantity, and so firm i's profit loss from quantity-setting is larger than from fixing a price. Thus, the uncertainty-based benefit of price-setting is independent of the rival firm's choice of the nature and value of its strategy variable, and, ceteris paribus, it increases in the amount of uncertainty.

From the preceding arguments it follows that for z smaller than both $z^q(\gamma)$ and $z^p(\gamma)$, the strategic benefit of quantity-setting dominates the uncertainty-based benefit of price-setting for each firm, whichever strategy variable the rival firm chooses. Similarly for z larger than both $z^q(\gamma)$ and $z^p(\gamma)$, the uncertainty-based benefit of choosing a price dominates for each firm the strategic benefit of choosing a quantity, irrespective of the strategy variable chosen by the rival.

We now provide the intuition for the result that $z^q(\gamma) < z^p(\gamma)$. From the above arguments we know that the competitive disadvantage of price-setting changes with the strategy variable chosen by the rival firm. Suppose that firm j offers the quantity contract. In order to make firm i indifferent between price- and quantity-setting, the amount of uncertainty, i.e. the value of z, has to be sufficiently large so that the uncertainty-based advantage of price-setting exactly offsets its competitive disadvantage. Now, if firm i offers the price contract, the competitive disadvantage of price-setting for firm j increases by a discrete amount as competition is fiercer if both firms are price-setters. Thus, the amount of uncertainty that is needed to make a firm indifferent between price- and quantity-setting has to be strictly larger if the rival firm is setting a price than if it is choosing a quantity. This implies that $z^q(\gamma) < z^p(\gamma)$.

Lemma 1 immediately yields the following result.[15]

Proposition 1 *The subgame perfect equilibrium outcome of the two-stage game under uncertainty is the following: If $z < z^q(\gamma)$, both firms offer the quantity contract in the first stage. If $z = z^q(\gamma)$, at least one firm offers the quantity contract in the first stage. If $z^p(\gamma) > z > z^q(\gamma)$, one firms offers the price contract and the other firm the quantity contract in the first stage. If $z = z^p(\gamma)$, at most one firm offers the quantity contract in the first stage. If $z > z^p(\gamma)$, both firms offer the price contract in the first stage.*

The equilibrium outcome is displayed in Figure 3.2. Graphically, the analysis of Singh and Vives (1984) corresponds to the ordinate ($z = 1$) where quantity-setting is the dominant action for both firms. To get a sense for the parameter range in which the hybrid equilibrium emerges, consider the following numerical example. Let $\alpha = 1$ and $\beta = 1$. Then, for $\gamma = 0.6$ the hybrid equilibrium emerges if $1.09 \leq z \leq 1.20$ and for $\gamma = 0.85$ it emerges if $1.45 \leq z \leq 3.43$.

[15] We restrict attention to pure-strategy equilibria.

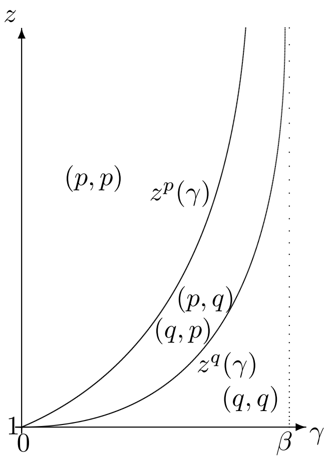

Figure 3.2: Equilibrium outcome under one-dimensional uncertainty

In our analysis the trade-off between the greater flexibility of prices and the lower degree of competition induced by quantities is the driving force behind firms' variable choice. This logic does not apply in the one-stage game of Klemperer and Meyer (1986). There, a firm only maximizes against its expected residual demand curve which it knows in equilibrium. Therefore, a firm only cares about the optimality of its strategy variable with respect to uncertainty. So firms set a price even for a small amount of uncertainty although this results in lower profits. By contrast, in our setting the relative *size* of the uncertainty-based benefit of prices and the strategic benefit of quantities determines firms' variable choice.

We have restricted our attention to the case of substitutes, i.e. $\gamma \geq 0$. Here, we note briefly that if products were complements, i.e. $\gamma < 0$, price-setting would be the dominant action for both firms. The reason is that both the strategic and the uncertainty effect favor prices. First, if $\gamma < 0$, price-setting is the dominant action in a deterministic environment. Second, the relative advantage of the price contract with respect to uncertainty is still present since it is independent of the sign of γ.

3.4 Two-Dimensional Uncertainty

So far we considered a shock affecting the slope of the inverse demand curve. This implies that a firm knows the range of consumers' reservation prices but it does not know the market size. In reality, however, the range of reservation prices might also be uncertain. Therefore, it is important to establish that the basic trade-off between the uncertainty-based benefit of price-setting and the competitive advantage of quantity-setting which we identified in the last section extends to this arguably more relevant situation.

In order to incorporate this aspect, we now additionally consider a shock to the intercept. The inverse demand system is then given by

$$p_i = \alpha + \epsilon - \frac{\beta}{\theta} q_i - \frac{\gamma}{\theta} q_j,$$

$$p_j = \alpha + \epsilon - \frac{\beta}{\theta} q_j - \frac{\gamma}{\theta} q_i,$$

where ϵ is a random variable.[16] Without loss of generality, we set $E[\epsilon] = 0$ and $\mathrm{Var}(\epsilon) = \sigma_\epsilon^2 > 0$. We denote the covariance between the shocks by $\sigma_{\theta\epsilon}$.

Proceeding in the same way as before now yields expected equilibrium profits of

$$\Pi^{pp} = \frac{(\alpha + \sigma_{\theta\epsilon})^2(\beta - \gamma)\beta}{(\beta + \gamma)(2\beta - \gamma)^2} \tag{3.9}$$

in the case in which both firms offer the price contract in the first stage. If both firms commit to a quantity, expected equilibrium profits are the same as in the case of one-dimensional uncertainty, namely

$$\Pi^{qq} = \frac{\alpha^2\beta}{z(2\beta + \gamma)^2}.$$

If firms select different strategy variables, the price-setting firm receives an expected equilibrium profit of

$$\Pi^{pq} = \frac{(\beta - \gamma)^2(2z(\beta + \gamma)(\alpha + \sigma_{\theta\epsilon}) - \alpha\gamma)^2}{(4z(\beta^2 - \gamma^2) + \gamma^2)^2\beta}$$

while the quantity-setting firm receives an expected equilibrium profit of

$$\Pi^{qp} = \frac{(\beta^2 - \gamma^2)z(\alpha(2\beta - \gamma) + \gamma\sigma_{\theta\epsilon})^2}{(4z(\beta^2 - \gamma^2) + \gamma^2)^2\beta}.$$

[16] We do not consider the case of a shock to the intercept alone. The reason is that, as was shown by Klemperer and Meyer (1986), price-setting and quantity-setting adapt equally well to uncertainty in this case. Thus in the two-stage game, only the strategic effects operates rendering quantity-setting the dominant action for both firms.

As before, we require the support of the shocks to be sufficiently small such that realized prices and quantities are positive. With two-dimensional uncertainty this can only be fulfilled if $\sigma_{\theta\epsilon} > \hat{\sigma}_{\theta\epsilon}(\gamma) := -\alpha\Big(1 - \gamma/(2z(\beta+\gamma))\Big)$.[17]

In contrast to the setting with one-dimensional uncertainty, we keep σ_θ and therewith z fixed and consider only changes in γ and $\sigma_{\theta\epsilon}$. To be more precise, we restrict changes in $\sigma_{\theta\epsilon}$ to stem from changes in σ_ϵ and in the correlation coefficient. The reason is that σ_θ enters z indirectly and $\sigma_{\theta\epsilon}$ directly, and so it is more convenient to work with $\sigma_{\theta\epsilon}$. Still, our results hold for any z, and we discuss after Proposition 2 how the results change qualitatively if z varies.

Before proceeding to the first stage, we define $\Delta\Pi^q(\gamma, \sigma_{\theta\epsilon}) := \Pi^{pq} - \Pi^{qq}$ and $\Delta\Pi^p(\gamma, \sigma_{\theta\epsilon}) := \Pi^{pp} - \Pi^{qp}$, where $\Delta\Pi^q(\gamma, \sigma_{\theta\epsilon})$ denotes the difference in expected profits of firm i if firm j chooses the quantity contract in the first stage, while $\Delta\Pi^p(\gamma, \sigma_{\theta\epsilon})$ denotes the difference conditional on firm j offering the price contract.

Lemma 2 *For any $\gamma \in [0, \beta)$ there exists a $\sigma^q_{\theta\epsilon}(\gamma) > \hat{\sigma}_{\theta\epsilon}(\gamma)$ such that*

$$\Delta\Pi^q(\gamma, \sigma_{\theta\epsilon}) \gtreqqless 0 \quad \textit{if} \quad \sigma_{\theta\epsilon} \gtreqqless \sigma^q_{\theta\epsilon}(\gamma).$$

Moreover, there exists a unique $\gamma^+ \in (0, \beta)$ such that for any $\gamma \in [0, \gamma^+)$ there exists a $\sigma^p_{\theta\epsilon}(\gamma) > \hat{\sigma}_{\theta\epsilon}(\gamma)$ such that

$$\Delta\Pi^p(\gamma, \sigma_{\theta\epsilon}) \gtreqqless 0 \quad \textit{if} \quad \sigma_{\theta\epsilon} \gtreqqless \sigma^p_{\theta\epsilon}(\gamma).$$

For any $\gamma \in (\gamma^+, \beta)$, $\Delta\Pi^p(\gamma, \sigma_{\theta\epsilon}) < 0$ for all $\sigma_{\theta\epsilon} > \hat{\sigma}_{\theta\epsilon}(\gamma)$.

Finally, $\sigma^q_{\theta\epsilon}(\gamma) = \sigma^p_{\theta\epsilon}(\gamma)$ for $\gamma \in \{0, \beta(1 - 1/\sqrt{z})\}$, while $\sigma^q_{\theta\epsilon}(\gamma) > \sigma^p_{\theta\epsilon}(\gamma)$ for $\gamma \in (0, \beta(1 - 1/\sqrt{z}))$ and $\sigma^q_{\theta\epsilon}(\gamma) < \sigma^p_{\theta\epsilon}(\gamma)$ for $\gamma \in (\beta(1 - 1/\sqrt{z}), \gamma^+)$.

Proof See the Appendix.

[17]The restriction on $\sigma_{\theta\epsilon}$ stems from the equilibrium price of the price-setting firm in the hybrid game which is given by $p_i = ((\beta - \gamma)(2z(\beta+\gamma)(\alpha + \sigma_{\theta\epsilon}) - \alpha\gamma)) \,/\, \big(4z(\beta^2 - \gamma^2) + \gamma^2\big)$.

This lemma immediately yields the following result.[18]

Proposition 2 *If $0 < \gamma < \beta\left(1 - 1/\sqrt{z}\right)$, the subgame perfect equilibrium outcome of the two-stage game with two-dimensional uncertainty is the following: If $\sigma_{\theta\epsilon} < \sigma^p_{\theta\epsilon}(\gamma)$, both firms offer the quantity contract in the first stage. If $\sigma^q_{\theta\epsilon}(\gamma) \geq \sigma_{\theta\epsilon} \geq \sigma^p_{\theta\epsilon}(\gamma)$, either both firms offer the price contract or both firms offer the quantity contract in first stage. If $\sigma_{\theta\epsilon} > \sigma^q_{\theta\epsilon}(\gamma)$, both firms offer the price contract in the first stage*

If $\gamma > \beta\left(1 - 1/\sqrt{z}\right)$, the unique subgame perfect equilibrium outcome of the two-stage game with two-dimensional uncertainty is the following: If $\sigma_{\theta\epsilon} < \sigma^q_{\theta\epsilon}(\gamma)$, both firms offer the quantity contract in the first stage. If $\sigma_{\theta\epsilon} = \sigma^q_{\theta\epsilon}(\gamma)$, at least one firm offers the quantity contract in the first stage. If $\sigma^p_{\theta\epsilon}(\gamma) > \sigma_{\theta\epsilon} > \sigma^q_{\theta\epsilon}(\gamma)$, one firm offers the price contract and the other firm offers the quantity contract in the first stage. If $\sigma_{\theta\epsilon} = \sigma^p_{\theta\epsilon}(\gamma)$, at most one firm offers the quantity contract in the first stage. If $\sigma_{\theta\epsilon} > \sigma^p_{\theta\epsilon}(\gamma)$, both firms offer the price contract in the first stage.

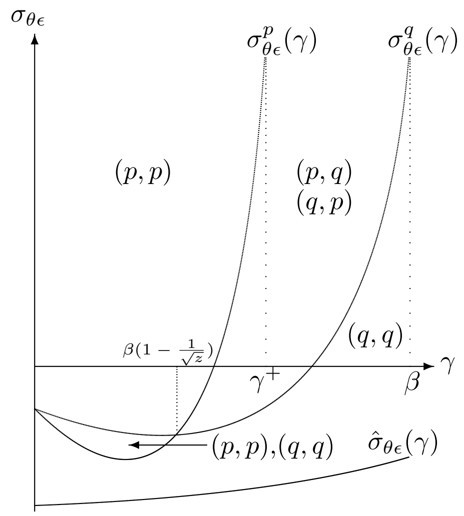

Figure 3.3: Equilibrium outcome under two-dimensional uncertainty

The equilibrium outcome of the game with two-dimensional uncertainty is depicted in Figure 3.3. As one can see, the basic trade-off between the uncertainty-based benefit of price-setting and the strategic benefit of quantity-setting can be identified in this case as well if the correlation between the shocks is positive, which is arguably the more relevant situation.[19]

[18] Again, we only consider pure-strategy equilibria.

[19] If consumers have a high willingness to pay (positive shock to the intercept), it is likely that the market size becomes larger as well (positive shock to the slope) or at least does not shrink.

The qualitative nature of the outcome is not affected by z but the exact position of the curves $\sigma^p_{\theta\epsilon}(\gamma)$ and $\sigma^q_{\theta\epsilon}(\gamma)$ is. If, for example, z increases, both curves shift to the south-east. Thus, the region in which price-setting is optimal becomes larger. So an increase in σ_θ has similar effects as in the case of one-dimensional uncertainty.

The theoretical importance of this result is that our analysis is the first to jointly analyze a shock to slope and intercept. In the seminal work of Klemperer and Meyer (1986) both shocks are treated separately.[20] This allows us to determine under which conditions a higher variance of each of the shocks favors price-setting: Ceteris paribus, a higher σ_ϵ increases the relative attractiveness of choosing the price contract if the correlation coefficient is positive and vice versa. Instead, as pointed out above, an increase of σ_θ unambiguously favors price-setting.

The intuition for the equilibrium can be gained along similar lines as in the previous section. Since there is a shock to the intercept, the distribution of valuations is uncertain and so ex post optimal prices depend on the realization of ϵ. Thus, equilibrium prices are no longer ex post optimal, irrespective of the strategy variable chosen by the competitor. This reduces the attractiveness of the price contract. Yet, the ex post optimal quantity depends on the realization of both shocks, i.e. on θ and the product of θ and ϵ. Now suppose that the correlation is positive. This means that a positive shock on the intercept goes along with a flatter expected slope of the inverse residual demand curve and vice versa. This implies that the variation of ex post optimal quantities relative to the variation of ex post optimal prices is the larger, the larger the covariance. As a consequence, firms prefer to offer the price contract if the covariance is not too negative and the degree of substitutability is relatively small. If the covariance is sufficiently negative, the variance of ex post optimal quantities is relatively low. Thus, firms prefer to choose the quantity contract in equilibrium for every degree of substitutability.

There are two differences to the case with one-dimensional uncertainty. First, if γ is large, i.e. $\gamma > \gamma^+$, there does not exist an equilibrium in which both firms offer the price contract. The reason is that with an intercept shock, price-setting is less attractive. As a consequence,

[20] In addition, Klemperer and Meyer (1986) provide a general approximate analysis in which they determine the advantages of price- versus quantity-setting for a small amount of uncertainty. As mentioned above, in a two-stage game the size of the shock is important for variable choice and so their analysis cannot be extended to this case.

if products are close substitutes, competition will be too aggressive to induce both firms to commit to a price. Second, if γ is small and $\sigma_{\theta\epsilon}$ is sufficiently, but not too negative, there is a region in which both firms either commit to a price or to a quantity in equilibrium. The reason for this result is the following: In this parameter range the degree of substitutability and the covariance are such that it depends on the rival firm's strategy variable whether the uncertainty-based benefit of prices dominates the competitive advantage of quantities or vice versa. Let us explain this in more detail. Suppose that firm j competes by setting a price. As explained above, compared to the situation in which firm j sets a quantity, the inverse residual demand curve of firm i is flatter. This implies that the variance of firm i's ex post optimal quantities is relatively large. As a consequence, for firm i the uncertainty-based benefit of price-setting is so pronounced that it dominates the competitive advantage of quantities. Put differently, by setting a price, firm j renders price-setting optimal for firm i and vice versa. Now, suppose that firm j competes in quantities. In this case firm i's inverse residual demand curve is relatively steep which implies that its ex post optimal quantities vary to a smaller extent. Thus, by choosing a quantity firm j decreases the uncertainty-based benefit of price-setting for firm i so that it is dominated by the competitive advantage of quantity-setting. Therefore, by committing to a quantity firm j renders quantity-setting optimal for firm i and the other way round.

3.5 Conclusion

We show that the superiority of quantity competition for firms might no longer hold if there is a substantial amount of uncertainty concerning demand conditions. In the setting with a shock affecting the slope, we find that if uncertainty is high relative to the degree of substitutability, firms prefer price-setting to quantity-setting. Moreover, for an intermediate range of uncertainty the equilibrium outcome is that firms choose different strategy variables. We also demonstrate that if there is a shock to the intercept in addition to the slope shock, the covariance between the shocks is crucial for variable choice. Nevertheless, the basic intuition from the analysis with one-dimensional uncertainty carries over to this case as well, and the qualitative results are similar.

Throughout the analysis we incorporated the shock(s) into the inverse demand system and assumed symmetric uncertainty. Yet, all our insights remain valid if we change both assumptions. First, feeding the shock(s) into the direct demand system yields an analysis that is akin to the one in Section 3.4. As there, the relation between the degree of substitutability and the covariance of the shocks determines the equilibrium outcome and all qualitative results remain unchanged. Second, if uncertainty is asymmetric, the firm that faces a larger demand variance is more inclined to offer the price contract. Thus, there exists a region with a unique hybrid equilibrium in which the firm with the larger (smaller) uncertainty offers a price (quantity) contract. But the main insights and intuitions of our analysis carry over to this case as well.

3.6 Appendix

Proof of Lemma 1

Existence and Uniqueness of $z^q(\gamma)$ and $z^p(\gamma)$

First, we show existence and uniqueness of $z^q(\gamma)$ and $z^p(\gamma)$ for any $\gamma \in (0, \beta)$.

The profit functions are rational functions defined on the domain $\gamma \in [0, \beta)$ and $z > 1$. Thus, $\Delta\Pi^q(\gamma, z)$ and $\Delta\Pi^p(\gamma, z)$ are continuous in γ and z and at least once continuously differentiable.

For arbitrary $\gamma \in (0, \beta)$,

$$\lim_{z\to 1} \Delta\Pi^q(\gamma, z) = \frac{\alpha^2\gamma^3(6\gamma^2\beta - 8\beta^3 + \gamma^3)}{(4\beta^2 - 3\gamma^2)^2\beta(2\beta + \gamma)^2} < 0,$$

and

$$\lim_{z\to\infty} \Delta\Pi^q(\gamma, z) = \frac{\alpha^2}{4\beta} > 0.$$

Since

$$\frac{\partial\Delta\Pi^q(\gamma, z)}{\partial z} = \frac{4\alpha^2(\beta-\gamma)^2(2z(\beta+\gamma)-\gamma)(\gamma+\beta)\gamma(2\beta-\gamma)}{(4z(\beta^2-\gamma^2)+\gamma^2)^3\beta} + \frac{\alpha^2\beta}{(2\beta+\gamma)^2 z^2} > 0,$$

we have shown that $z^q(\gamma)$ exists and that it is unique. An immediate consequence of this and the facts that $\lim_{z\to 1} \Delta\Pi^q(\gamma, z) < 0$ and $\lim_{z\to\infty} \Delta\Pi^q(\gamma, z) > 0$ is that $\Delta\Pi^q(\gamma, z) \gtreqless 0$ if $z \gtreqless z^q(\gamma)$.

Now we turn to the existence and uniqueness of $z^p(\gamma)$. For an arbitrary $\gamma \in (0, \beta)$

$$\lim_{z\to 1} \Delta\Pi^p(\gamma, z) = -\frac{(\beta-\gamma)\alpha^2\gamma^3(8\beta^3 - 6\gamma^2\beta + \gamma^3)}{(\beta+\gamma)(2\beta-\gamma)^2(4\beta^2-3\gamma^2)^2\beta} < 0,$$

and

$$\lim_{z\to\infty} \Delta\Pi^p(\gamma, z) = \frac{\beta(\beta-\gamma)\alpha^2}{(\beta+\gamma)(2\beta-\gamma)^2} > 0.$$

Since

$$\frac{\partial \Delta\Pi^p(\gamma, z)}{\partial z} = \frac{\alpha^2(2\beta-\gamma)^2(\beta^2-\gamma^2)(4z(\beta^2-\gamma^2)-\gamma^2)}{(4z(\beta^2-\gamma^2)+\gamma^2)^3\beta}, \tag{3.10}$$

which is negative for $z < \tilde{z}(\gamma) := \frac{\gamma^2}{4(\beta^2-\gamma^2)}$ and positive for $z > \tilde{z}(\gamma)$, we have shown that for every γ there exists a unique $z^p(\gamma) > \tilde{z}(\gamma)$. Combining the uniqueness of $z^p(\gamma)$ with the facts that $\lim_{z\to 1} \Delta\Pi^p(\gamma, z) < 0$ and $\lim_{z\to\infty} \Delta\Pi^p(\gamma, z) > 0$ yields $\Delta\Pi^p(\gamma, z) \gtreqless 0$ if $z \gtreqless z^p(\gamma)$.

If $\gamma \to 0$, then

$$\lim_{\gamma\to 0} \Delta\Pi^q(\gamma, z) = \frac{\alpha^2(z-1)}{4\beta z}$$

and

$$\lim_{\gamma\to 0} \Delta\Pi^p(\gamma, z) = \frac{\alpha^2(z-1)}{4\beta z}.$$

Thus, for $\gamma \to 0$, $z^q(\gamma) \to 1$ and $z^p(\gamma) \to 1$.

Characterization of $z^q(\gamma)$ and $z^p(\gamma)$

In the following we show that $\frac{\partial z^q(\gamma)}{\partial\gamma} > 0$ and $\frac{\partial z^p(\gamma)}{\partial\gamma} > 0$. This is done via the Implicit Function Theorem.[21]

We already know that $\frac{\partial\Delta\Pi^q(\gamma,z)}{\partial z}$ is globally strictly positive. So it is also strictly positive when evaluated at $z^q(\gamma)$. Thus, $\left.\frac{\partial\Delta\Pi^q(\gamma,z)}{\partial z}\right|_{z=z^q(\gamma)} > 0$.

In the following, we show that the derivative of $\Delta\Pi^q(\gamma, z)$ with respect to γ is negative if it is evaluated at $z^q(\gamma)$.

Differentiating $\Delta\Pi^q(\gamma, z)$ with respect to γ yields

$$-\frac{2\alpha^2(\beta-\gamma)(2z(\beta+\gamma)-\gamma)(4z(\beta^2-\gamma\beta+\gamma^2)-\gamma^2)}{(4z(\beta^2-\gamma^2)+\gamma^2)^3} + \frac{\alpha^2\beta}{(2\beta+\gamma)^3 z}.$$

Evaluating $\frac{\partial\Delta\Pi^q(\gamma,z)}{\partial\gamma}$ at $z^q(\gamma)$ yields

$$\frac{2\alpha^2(\beta-\gamma)(2z^q(\gamma)(\beta+\gamma)-\gamma)}{(4z^q(\gamma)(\beta^2-\gamma^2)+\gamma^2)^3\beta(2\beta+\gamma)}\,\phi(\gamma, z^q(\gamma)) \tag{3.11}$$

[21] In principle, we could solve for $z^q(\gamma)$ and $z^p(\gamma)$ explicitly. However, the resulting expressions are not accessible and determining the signs of their derivatives directly is impossible.

with

$$\phi(\gamma,z)=8(\beta^2-\gamma^2)^2z^2+2(\beta^2\gamma^2-4\beta^4-3\gamma^4)z+\gamma^2(\gamma^2+2\beta^2).$$

Since the first factor of (3.11) is strictly bigger than zero, the sign of the derivative is determined by the sign of $\phi(\gamma,z)$ at $z^q(\gamma)$. Since $\phi(\gamma,z)$ is a quadratic function in z with a positive leading term, it is convex and has two real roots. The one that involves values of $z>1$ is denoted by $\hat{z}(\gamma)$, where

$$\hat{z}(\gamma)=\frac{4\beta^4-\beta^2\gamma^2+3\gamma^4+\sqrt{\chi(\gamma)}}{8(\beta^2-\gamma^2)^2},$$

with

$$\chi(\gamma)=49\beta^4\gamma^4-24\beta^6\gamma^2-6\beta^2\gamma^6+16\beta^8+\gamma^8.$$

It can be shown that $\chi(0)$ and $\chi(\beta)$ are strictly positive. Furthermore, for $\gamma\in(0,\beta)$, the minimum value of $\chi(\gamma)$ is $13\beta^8>0$. Thus, $\hat{z}(\gamma)$ is well defined.

In the following, we compare $\hat{z}(\gamma)$ with $z^q(\gamma)$ and use the fact that $\Delta\Pi^q(\gamma,z^q(\gamma))=0$. Evaluating $\Delta\Pi^q(\gamma,z)$ at an arbitrary $\gamma\in(0,\beta)$ and the corresponding $\hat{z}(\gamma)$ yields

$$\Delta\Pi^q(\gamma,\hat{z}(\gamma))=\frac{(4\gamma^3\beta-\gamma^4+3\gamma^2\beta^2-4\gamma\beta^3+4\beta^4+\sqrt{\chi(\gamma)})^2\alpha^2}{4\beta(4\beta^4+\gamma^2\beta^2+\gamma^4+\sqrt{\chi(\gamma)})^2}$$
$$-\frac{8\alpha^2\beta(\beta^2-\gamma^2)^2}{(2\beta+\gamma)^2(4\beta^4-\gamma^2\beta^2+3\gamma^4+\sqrt{\chi(\gamma)})}.$$

To see that $\Delta\Pi^q(\gamma,\hat{z}(\gamma))>0$ for all $\gamma\in(0,\beta)$, we rewrite the right hand side of the previous equation as

$$\gamma^2\varphi_1(\gamma)\Big(\sqrt{\chi(\gamma)}(36\beta^6\gamma^2+40\gamma^3\beta^5+47\beta^4\gamma^4+96\beta^8+64\gamma\beta^7+40\gamma^5\beta^3+2\beta^2\gamma^6-\gamma^8)$$
$$+960\gamma^4\beta^8+123\gamma^8\beta^4+24\gamma^9\beta^3+\gamma^{12}+\varphi_2(\gamma)-\varphi_3(\gamma)\Big)$$

with

$$\begin{aligned}
\varphi_1(\gamma) &= \frac{4\alpha^2}{\beta(\gamma^4+\beta^2\gamma^2+4\beta^4+\sqrt{\chi(\gamma)})^2(2\beta+\gamma)^2(4\beta^4-\beta^2\gamma^2+3\gamma^4+\sqrt{\chi(\gamma)})} > 0,\\
\varphi_2(\gamma) &= 600\beta^7\gamma^5+80\beta^5\gamma^7+256\beta^{11}\gamma+384\beta^{12} > 0,\\
\varphi_3(\gamma) &= 127\beta^6\gamma^6+21\beta^2\gamma^{10}+240\beta^{10}\gamma^2+96\beta^9\gamma^3 \geq 0.
\end{aligned}$$

Obviously $\gamma = 0$ is one of the roots of $\Delta\Pi^q(\gamma, \hat{z}(\gamma))$. Now we need to show that it has none for $\gamma \in (0, \beta)$. Since

$$\varphi_2(\gamma) > 127\beta^7\gamma^5+21\beta^5\gamma^7+240\beta^{11}\gamma+96\beta^{12} > \varphi_3(\gamma),$$

for $\gamma \in (0, \beta)$, we have shown that $\Delta\Pi^q(\gamma, \hat{z}(\gamma))$ has no real root for $\gamma \in (0, \beta)$. Thus, $\Delta\Pi^q(\gamma, \hat{z}(\gamma)) > 0$ for all $\gamma \in (0, \beta)$. As $\Delta\Pi^q(\gamma, z)$ is increasing in z, $z^q(\gamma) < \hat{z}(\gamma)$ for every $\gamma \in (0, \beta)$. Thus, $\phi(\gamma, z^q(\gamma)) < 0$ and thereby the derivative of $\Delta\Pi^q(\gamma, z^q(\gamma))$ with respect to γ is negative.

Since $\Delta\Pi^q_\gamma(\gamma, z^q(\gamma)) < 0$ and $\Delta\Pi^q_z(\gamma, z^q(\gamma)) > 0$ for $\gamma \in (0, \beta)$, the Implicit Function Theorem implies that

$$\frac{dz^q(\gamma)}{d\gamma} = -\frac{\Delta\Pi^q_\gamma(\gamma, z^q(\gamma))}{\Delta\Pi^q_z(\gamma, z^q(\gamma))} > 0.$$

Now we turn to the function $\Delta\Pi^p(\gamma, z) = 0$. If (3.10) is evaluated at $z^p(\gamma)$, it is strictly positive, since $z^p(\gamma) > \tilde{z}(\gamma)$. Thus, $\frac{\partial\Delta\Pi^p(\gamma,z)}{\partial z}\Big|_{z=z^p(\gamma)} > 0$.

In the following, we show that $\frac{\partial\Delta\Pi^p(\gamma,z)}{\partial\gamma}\Big|_{z=z^p(\gamma)} < 0$. Differentiating $\Delta\Pi^p(\gamma, z)$ with respect to γ yields

$$2\alpha^2\left(\frac{(2\beta-\gamma)z(4z(\beta^3-2\beta^2\gamma-\beta\gamma^2+2\gamma^3)-\gamma(2\gamma^2+\beta\gamma+4\beta^2))}{(4z(\beta^2-\gamma^2)+\gamma^2)^3} - \frac{\beta(\beta^2-\gamma\beta+\gamma^2)}{(\beta+\gamma)^2(2\beta-\gamma)^3}\right).$$

Evaluating $\frac{\partial\Delta\Pi^p(\gamma,z)}{\partial\gamma}$ at $z^p(\gamma)$ yields

$$\frac{2\alpha^2(2\beta-\gamma)(\beta+\gamma)\gamma}{(4z''(\gamma)(\beta^2-\gamma^2)+\gamma^2)^3\beta}\,\psi(\gamma, z^p(\gamma)), \tag{3.12}$$

with

$$\psi(\gamma, z) = (4z - 1)(6\beta - \gamma)\gamma - (5z - 1)4\beta^2.$$

Since the first factor in (3.12) is bigger than zero for all $\gamma \in (0, \beta)$, the sign of this derivative is negative if $\psi(\gamma, z^p(\gamma))$ is negative.

In order to check the sign of $\psi(\gamma, z^p(\gamma))$, we solve $\Delta\Pi^p(\gamma, z^p(\gamma)) = 0$ to obtain

$$z^p(\gamma) = \frac{\kappa(\gamma) + 8\beta^2\gamma^2(\beta - \gamma) + (2\beta - \gamma)^2\sqrt{\kappa(\gamma)}\sqrt{\beta + \gamma}}{32\beta^2(\beta - \gamma)^2(\beta + \gamma)}$$

with

$$\kappa(\gamma) = 16\beta^5 - 8\beta^2\gamma(2\beta^2 + 3\beta\gamma - 4\gamma^2) - \gamma^4(7\beta - \gamma).$$

Obviously, $\kappa(0)$ and $\kappa(\beta)$ are strictly positive. Moreover, for $\gamma \in (0, \beta)$ this expression attains its minimum at $\gamma \approx 0.88\beta$. Evaluating $\kappa(\gamma)$ at this value yields $1.47\beta^5$. Thus, $\kappa(\gamma) > 0$.

Inserting $z^p(\gamma)$ into $\psi(\gamma, z)$ yields

$$-\frac{(2\beta - \gamma)}{8(\beta^2 - \gamma^2)\beta^2}\Big(24\beta^5 + 31\beta^2\gamma^3 + \gamma^5 - 2\gamma\beta(6\beta^3 + 13\beta^2\gamma + 5\gamma^3) + (5\beta - \gamma)(2\beta - \gamma)\sqrt{\kappa(\gamma)}\sqrt{\beta + \gamma}\Big).$$

Since

$$24\beta^5 + 31\beta^2\gamma^3 + \gamma^5 - 2\gamma\beta(6\beta^3 + 13\beta^2\gamma + 5\gamma^3) > \kappa(\gamma) > 0,$$

$\psi(\gamma, z^p(\gamma))$ and $\left.\frac{\partial\Delta\Pi^p(\gamma,z)}{\partial\gamma}\right|_{z=z^p(\gamma)}$ are negative.

The Implicit Function Theorem implies that

$$\frac{dz^p(\gamma)}{d\gamma} = -\frac{\Delta\Pi^p_\gamma(\gamma, z^p(\gamma))}{\Delta\Pi^p_z(\gamma, z^p(\gamma))} > 0.$$

Relation of $z^q(\gamma)$ and $z^p(\gamma)$

Consider an arbitrary $\gamma \in (0, \beta)$ and the associated $z^p(\gamma)$. Evaluating $\Delta\Pi^q(\gamma, z)$ at that $z^p(\gamma)$ yields

$$\frac{4\alpha^2\gamma^2(2\beta - \gamma)^2}{\beta(2\beta + \gamma)^2}\lambda_1(\gamma)\Big(\sqrt{\kappa(\gamma)}\sqrt{\beta + \gamma}(2\beta - \gamma)\lambda_2(\gamma) + \lambda_3(\gamma)\Big), \tag{3.13}$$

with

$$\lambda_1(\gamma) = \Big((16\beta^3(\beta^2 - \beta\gamma - \gamma^2) + \gamma^3(24\beta^2 - 7\gamma\beta + \gamma^2) + (2\beta - \gamma)^2\sqrt{\kappa(\gamma)}\sqrt{\beta + \gamma})$$
$$(8\beta^3(2\beta^2 - 2\beta\gamma - \gamma^2) + \gamma^3(16\beta^2 - 7\gamma\beta + \gamma^2) + (2\beta - \gamma)^2\sqrt{\kappa(\gamma)}\sqrt{\beta + \gamma})^2\Big)^{-1},$$

$$\lambda_2(\gamma) = (288\beta^7 - 304\beta^6\gamma - 176\beta^5\gamma^2 + 304\beta^4\gamma^3 - 70\beta^3\gamma^4 - 13\beta^2\gamma^5 + 8\beta\gamma^6 - \gamma^7),$$

and

$$\begin{aligned}\lambda_3(\gamma) = {} & \gamma^{11} - 13\beta\gamma^{10} + 67\beta^2\gamma^9 - 95\beta^3\gamma^8 - 592\beta^4\gamma^7 + 2832\beta^5\gamma^6 - 3536\beta^6\gamma^5 \\ & - 1920\beta^7\gamma^4 + 7168\beta^8\gamma^3 - 3072\beta^9\gamma^2 - 2560\beta^{10}\gamma + 1792\beta^{11}.\end{aligned}$$

Now we need to determine the sign of $\lambda_1(\gamma)$, $\lambda_2(\gamma)$, and $\lambda_3(\gamma)$. Since

$$\begin{aligned}16\beta^3(\beta^2 - \beta\gamma - \gamma^2) + \gamma^3(24\beta^2 - 7\gamma\beta + \gamma^2) &\geq \kappa(\gamma) > 0 \text{ and} \\ (8\beta^3(2\beta^2 - 2\beta\gamma - \gamma^2) + \gamma^3(16\beta^2 - 7\gamma\beta + \gamma^2) &\geq \kappa(\gamma) > 0,\end{aligned}$$

$\lambda_1(\gamma)$ is strictly bigger than zero.

It can be shown that $\lambda_2(\gamma)$ and $\lambda_3(\gamma)$ have no root in $(0, \beta)$. Since $\lambda_2\left(\frac{\beta}{2}\right) \approx 125\beta^7 > 0$ and $\lambda_3\left(\frac{\beta}{2}\right) \approx 449\beta^{11} > 0$, both expressions are strictly bigger than zero. This implies that (3.13) is bigger than zero. Since $\Delta\Pi^q(\gamma, z)$ is increasing in its second argument, $z^q(\gamma)$ has to be smaller than $z^p(\gamma)$ in order for this condition to hold. ■

Proof of Lemma 2

Existence and Uniqueness of $\sigma^q_{\theta\epsilon}(\gamma)$ and $\sigma^p_{\theta\epsilon}(\gamma)$

First, consider the case in which firm j sets a quantity. Firm i is indifferent between setting a price or a quantity if

$$\frac{(\beta - \gamma)^2(2z(\beta + \gamma)(\alpha + \sigma_{\theta\epsilon}) - \alpha\gamma)^2}{(4z(\beta^2 - \gamma^2) + \gamma^2)^2\beta} = \frac{\alpha^2\beta}{z(2\beta + \gamma)^2}. \tag{3.14}$$

The solutions of (3.14) are

$$\sigma_{\theta\epsilon}^{(1)}(\gamma) = \frac{\alpha(\sqrt{z}\beta(4z(\beta^2-\gamma^2)+\gamma^2) - z(2\beta+\gamma)(\beta-\gamma)(2z(\beta+\gamma)-\gamma))}{2z^2(\beta^2-\gamma^2)(2\beta+\gamma)}$$

and

$$\sigma_{\theta\epsilon}^{(2)}(\gamma) = -\frac{\alpha(\sqrt{z}\beta(4z(\beta^2-\gamma^2)+\gamma^2) - z(2\beta+\gamma)(\beta-\gamma)(2z(\beta+\gamma)-\gamma))}{2z^2(\beta^2-\gamma^2)(2\beta+\gamma)}.$$

We have to check how $\sigma_{\theta\epsilon}^{(1)}(\gamma)$ and $\sigma_{\theta\epsilon}^{(2)}(\gamma)$ relate to $\hat{\sigma}_{\theta\epsilon}(\gamma)$. In order to do that we simply subtract $\hat{\sigma}_{\theta\epsilon}(\gamma)$ from both threshold covariances. It turns out that $\sigma_{\theta\epsilon}^{(1)}(\gamma) - \hat{\sigma}_{\theta\epsilon}(\gamma) = \Delta$, and $\sigma_{\theta\epsilon}^{(2)}(\gamma) - \hat{\sigma}_{\theta\epsilon}(\gamma) = -\Delta$, where Δ is given by

$$\frac{\alpha(\sqrt{z}\beta(4z(\beta^2-\gamma^2)+\gamma^2)}{2z^2(\beta^2-\gamma^2)(2\beta+\gamma)} > 0.$$

Thus, $\sigma_{\theta\epsilon}^{(2)}(\gamma)$ is never in the admissible range of $\sigma_{\theta\epsilon}$ while $\sigma_{\theta\epsilon}^{(1)}(\gamma)$ is. We denote $\sigma_{\theta\epsilon}^{(1)}(\gamma)$ by $\sigma_{\theta\epsilon}^{q}(\gamma)$.

Now we have to determine under which conditions firm i prefers to set a price or a quantity conditional on firm j choosing a quantity. Comparing Π^{pq} with Π^{qq} it is easy to see that firm i sets a price if $\sigma_{\theta\epsilon}(\gamma) > \sigma_{\theta\epsilon}^{q}(\gamma)$ since Π^{pq} is increasing in $\sigma_{\theta\epsilon}(\gamma)$ while Π^{qq} is independent of $\sigma_{\theta\epsilon}(\gamma)$. Thus, $\Delta\Pi^{q}(\gamma, \sigma_{\theta\epsilon}) \gtreqless 0$ if $\sigma_{\theta\epsilon} \gtreqless \sigma_{\theta\epsilon}^{q}(\gamma)$.

Now, suppose firm j sets a price. Then, firm i is indifferent between choosing a price or a quantity if

$$\frac{(\alpha+\sigma_{\theta\epsilon})^2(\beta-\gamma)\beta}{(\beta+\gamma)(2\beta-\gamma)^2} = \frac{(\beta^2-\gamma^2)z(\alpha(2\beta-\gamma)+\gamma\sigma_{\theta\epsilon})^2}{(4z(\beta^2-\gamma^2)+\gamma^2)^2\beta}. \tag{3.15}$$

The solutions of (3.15) are

$$\tilde{\sigma}_{\theta\epsilon}^{(1)}(\gamma) = \frac{\alpha\sqrt{z}(\beta^2-\gamma^2)(2\beta-\gamma)}{h(\gamma)}\Big(2\sqrt{z}\gamma(\beta+\gamma) + \beta(4z(\beta^2-\gamma^2)+\gamma^2)\Big) - \alpha$$

and

$$\tilde{\sigma}_{\theta\epsilon}^{(2)}(\gamma) = \frac{\alpha\sqrt{z}(\beta^2-\gamma^2)(2\beta-\gamma)}{h(\gamma)}\Big(2\sqrt{z}\gamma(\beta+\gamma) - \beta(4z(\beta^2-\gamma^2)+\gamma^2)\Big) - \alpha,$$

with

$$h(\gamma) := 16\beta^2(\beta^2-\gamma^2)^2z^2 + \gamma^2(\beta+\gamma)(4\beta^3 - 8\beta^2\gamma + 3\beta\gamma^2 - \gamma^3)z + \beta^2\gamma^4.$$

It is easy to show that $h(0) = 16z^2\beta^6 > 0$, $h(\beta) = \beta^6(1-4z) < 0$, and $\frac{\partial h(\gamma)}{\partial\gamma} < 0$. This implies that there exists a unique $\gamma \in (0,\beta)$, labeled γ^+, that solves $h(\gamma) = 0$. Thus, $\tilde{\sigma}^{(1)}_{\theta\epsilon}(\gamma)$ and $\tilde{\sigma}^{(2)}_{\theta\epsilon}(\gamma)$ are defined on $\gamma \in [0,\beta) \setminus \{\gamma^+\}$.

Now we analyze how $\tilde{\sigma}^{(1)}_{\theta\epsilon}(\gamma)$ relates to $\hat{\sigma}_{\theta\epsilon}(\gamma)$. Subtracting the latter from the former yields

$$\frac{\alpha(4z(\beta^2-\gamma^2)+\gamma^2)}{2z(\beta+\gamma)h(\gamma)}\Big((4z^{\frac{3}{2}}(\beta+\gamma)\beta(2\beta-\gamma)(\beta^2-\gamma^2)+\gamma^2(2z\beta(2\beta^2-\gamma^2)+\beta^2\gamma(z-1)+z\gamma^3)\Big). \tag{3.16}$$

The numerator of (3.16) is nonnegative for all $\gamma \in [0,\beta)$ while the sign of the denominator depends on the sign of $h(\gamma)$. The argument above implies that $h(\gamma) > 0$ for $\gamma \in [0,\gamma^+)$ and $h(\gamma) < 0$ for $\gamma \in (\gamma^+,\beta)$. As a consequence, we have $\tilde{\sigma}^{(1)}_{\theta\epsilon}(\gamma) > \hat{\sigma}_{\theta\epsilon}(\gamma)$ if and only if $\gamma < \gamma^+$.

Now we investigate the relation between $\tilde{\sigma}^{(2)}_{\theta\epsilon}(\gamma)$ and $\hat{\sigma}_{\theta\epsilon}(\gamma)$. Subtracting the minimum of the latter $(-\alpha)$ from the former yields

$$\frac{\alpha\sqrt{z}(\beta^2-\gamma^2)(2\beta-\gamma)}{h(\gamma)}\Big(2\sqrt{z}\gamma(\beta+\gamma)-\beta(4z(\beta^2-\gamma^2)+\gamma^2)\Big).$$

This expression is equal to zero for $\gamma = \beta$ and smaller than zero for all $\gamma \in [0,\beta)$.[22] Thus, $\tilde{\sigma}^{(2)}_{\theta\epsilon}(\gamma)$ is never in the admissible range of $\sigma_{\theta\epsilon}$. So for every $\gamma \in [0,\gamma^+)$, we denote $\tilde{\sigma}^{(1)}_{\theta\epsilon}(\gamma)$ by $\sigma^p_{\theta\epsilon}(\gamma)$.

Now we show that firm i prefers to set a price contingent on firm j choosing a price if $\sigma_{\theta\epsilon} > \sigma^p_{\theta\epsilon}(\gamma)$. Firm i sets a price if Π^{pp} is bigger than Π^{qp}. Differentiating $\Delta\Pi^p(\gamma,\sigma_{\theta\epsilon})$ with respect to $\sigma_{\theta\epsilon}$, and evaluating this difference at $\Delta\Pi^p(\gamma,\sigma_{\theta\epsilon}) = 0$ yields

$$\frac{\partial(\Pi^{pp}-\Pi^{qp})}{\partial\sigma_{\theta\epsilon}}\Big|_{\Delta\Pi^p(\gamma,\sigma_{\theta\epsilon})=0} = \frac{4\sqrt{z}\alpha(\beta-\gamma)^2}{(2\beta-\gamma)(4z(\beta^2-\gamma^2)+\gamma^2)} > 0.$$

This implies that at $\sigma^p_{\theta\epsilon}(\gamma)$ firm i's marginal benefit from an increase in the covariance is bigger if it selects a price than if it chooses a quantity. Thus, $\Delta\Pi^p(\gamma,\sigma_{\theta\epsilon}) \gtreqless 0$ if $\sigma_{\theta\epsilon} \gtreqless \sigma^p_{\theta\epsilon}(\gamma)$.

Finally, for $\gamma \in (\gamma^+,\beta]$ there exists no $\sigma_{\theta\epsilon} > \hat{\sigma}_{\theta\epsilon}$ such that (3.15) holds. As a consequence, the right hand side of (3.15) is bigger than the left hand side and so $\Delta\Pi^p(\gamma,\sigma_{\theta\epsilon}) < 0$ for any $\gamma \in (\gamma^+,\beta]$.

[22] Here, both the numerator and the denominator are zero at $\gamma = \gamma^+$, and one can show by using the Rule of L'Hospital that the expression is negative at $\gamma = \gamma^+$.

Relation of $\sigma^q_{\theta\epsilon}(\gamma)$ and $\sigma^p_{\theta\epsilon}(\gamma)$

In the following we determine how $\sigma^q_{\theta\epsilon}(\gamma)$ and $\sigma^p_{\theta\epsilon}(\gamma)$ relate to each other. It is easy to check that at $\gamma = 0$, $\sigma^q_{\theta\epsilon}(0) = \sigma^p_{\theta\epsilon}(0) = -\alpha\Big(1 - \frac{1}{\sqrt{z}}\Big)$. Subtracting $\sigma^p_{\theta\epsilon}(\gamma)$ from $\sigma^q_{\theta\epsilon}(\gamma)$, setting the difference equal to zero, and solving for γ reveals that there exists a unique $\gamma \in (0, \beta)$, namely $\gamma = \beta\Big(1 - \frac{1}{\sqrt{z}}\Big)$, for which both threshold covariances are equal. It remains to show that $\sigma^q_{\theta\epsilon}(\gamma)$ and $\sigma^p_{\theta\epsilon}(\gamma)$ cross at $\gamma = \beta\Big(1 - \frac{1}{\sqrt{z}}\Big)$. Differentiating $\sigma^q_{\theta\epsilon}(\gamma)$ with respect to γ and evaluating the derivative at the intersection point yields

$$\frac{\alpha\Big(1 - 3\sqrt{z} - 3z + 15z^{\frac{3}{2}} - 10z^2\Big)}{2\beta\sqrt{z}\Big(2\sqrt{z} - 1\Big)^2\Big(3\sqrt{z} - 1\Big)^2}, \tag{3.17}$$

while differentiating $\sigma^p_{\theta\epsilon}(\gamma)$ with respect to γ and evaluating the derivative at the intersection point yields

$$\frac{\alpha\Big(1 - 7\sqrt{z} + 13z + 25z^{\frac{3}{2}} - 128z^2 + 106z^{\frac{5}{2}} + 251z^3 - 523z^{\frac{7}{2}} + 101z^4 + 491z^{\frac{9}{2}} - 430z^5 + 100z^{\frac{11}{2}}\Big)}{2\beta z^{\frac{3}{2}}\Big(15z^2 - 11z^{\frac{3}{2}} - 4z + 5\sqrt{z} - 1\Big)^2}. \tag{3.18}$$

Since $z > 1$, it is easy to check that (3.18) is strictly bigger than (3.17). This shows that at the intersection point $\sigma^p_{\theta\epsilon}(\gamma)$ crosses $\sigma^q_{\theta\epsilon}(\gamma)$ from below and so $\sigma^p_{\theta\epsilon}(\gamma) < \sigma^q_{\theta\epsilon}(\gamma)$ for $0 < \gamma < \beta\Big(1 - \frac{1}{\sqrt{z}}\Big)$ and $\sigma^p_{\theta\epsilon}(\gamma) > \sigma^q_{\theta\epsilon}(\gamma)$ for $\beta\Big(1 - \frac{1}{\sqrt{z}}\Big) < \gamma < \gamma^+$. ■

Chapter 4

Product Choice under Government Regulation: The Case of Chile's Privatized Pension System[‡]

4.1 Introduction

The United States and many European countries are currently considering how best to reform their pay-as-you-go social security systems. Demographic trends indicate rising numbers of pensioners per worker and pending insolvency of many social security systems. The kinds of reforms being considered include increasing the required social security contribution per worker, raising the standard retirement age, or overhauling the system by transiting to a private retirement accounts system. Chile has been at the forefront of pension reforms, having switched to a private retirement accounts system twenty-eight years ago.[1] Numerous other Latin American countries followed suit, building on the Chilean model. These include (with years of adoption in parentheses): Peru (1993), Colombia (1994), Argentina (1994), Uruguay (1996), Bolivia (1997), Mexico (1997), El Salvador (1998), Costa Rica (2001), the Dominican Republic (2003), Nicaragua (2004), and Ecuador (2004).[2]

‡This chapter is joint work with Elena Krasnokutskaya and Petra Todd. Both are at the University of Pennsylvania, USA.

1University of Chicago economists played a role in the early adoption of the privatized account system.

2Cogan and Mitchell (2003) discuss prospects for funded individual defined contributions account pensions in the United States.

Previous research on Chile mainly examined the impact of pension reforms on the macroeconomy, capital markets and aggregate savings.[3] It found substantial benefits of moving to a private retirement accounts system in promoting the development of well-functioning capital markets and stimulating economic growth. However, critics of Chile's pension system point to low coverage rates and commissions and fees that are thought to be excessive.[45]

Proposed plans for pension reform in the US and in Europe have many features in common with Chile's current system. They outline a system under which all workers are mandated to contribute a pre-specified part of their income to their pension account, which is managed by money manager(s) (either a government owned company or a competitive industry of money managers). The government serves as a last resort guarantor, supplementing pension income if accumulations are insufficient upon retirement (below pre-specified minimal level), either because of low income or unfavorable investment returns. All these features are present in the Chilean pension fund system, called the *Administradoras de Fondos de Pensiones* (AFPs). Workers are mandated to contribute 10% of their earnings to a retirement account, and those who contribute for at least 20 years receive a minimum pension benefit guarantee from the government.[6]

Two important concerns have been raised about an individual retirement accounts pension system. The first is that government obligations can be large, particularly in years with unfavorable market returns on investments. Second, the government guarantee of minimal support may induce moral hazard problems by providing incentives for consumers with low income to choose risky investment options. If the system is run by a competitive industry, then money managers may offer products to meet this riskier demand, which, in turn, can raise government obligations. This is an undesirable feature of a competitive pension fund industry, although competition can also provide incentives for cost efficiency, quality improvement, and efficient pricing.

[3] Many have written on the Chilean pensions system (e.g. Cheyre (1988); Iglesias and Acuña (1991); Baeza et al. (1995), and SAFP, 1998). Some of the literature is summarized in Arenas de Mesa et al. (2006).

[4] A recent critique citing the problem of low coverage rates is Holzmann et al. (2005). Low coverage rates are mainly due to the presence of a significant informal sector of the economy, where workers do not contribute to the pension system, and to low labor force participation among some groups in the population, such as women.

[5] See Arenas de Mesa et al. (2006).

[6] For additional discussion of Chile's personal accounts system and its origins, see Mitchell, Todd and Bravo (2008).

Chile has a competitive pension fund industry overseeing pension investment that is subject to government regulations designed to limit fees, to facilitate switching among funds by promoting transparency of fees and pension fund returns and to limit the riskiness of the investment products offered. A particularly important regulation is a return requirement which makes money managers responsible with their capital for delivering a rate of return that is no less than two percentage points below the industry average return.[7] This regulation essentially shifts some of the risk of investment from consumers to the pension fund firm. Another important regulation restricts firms to charge fees only on new contributions and not on existing balances.

In this chapter we investigate how the government's regulation of the pension fund industry affects its operation. To this end, we develop and estimate an equilibrium demand and supply model of the pension investment market and use the model to study the effects of existing regulatory rules and of hypothetical alternative forms of regulation. The question of whether and to what extent governmental regulations imposed on a privatized account system can protect investors from risk and high fees without too greatly compromising investment returns is pertinent not only for Chile but for any other country considering a move to a privatized account system.

Our empirical analysis combines data from multiple sources: (i) longitudinal survey data on a random sample of working age Chileans gathered in 2002 and 2004, (ii) administrative data on contributions and fund choices from 1981-2004 that were obtained from the pension fund regulatory agency and have been linked to the household survey data, (iii) market data on the performance of the various funds, and (iv) a data series on the fees charged by funds as well as (v) accounting cost data. The survey data come from the 2002 *Historia Laboral y Seguridad Social (HLLS)* survey and the 2004 *Enquesta Proteccion Sociale (EPS)* follow up survey. The data contain demographic and labor market information on 17,246 individuals of age 18 or older, including information on household demographics, work history, pension

[7]There is also a regulation that requires AFPs that earn returns in excess of two percentage points above the market average to keep the excess in a reserve fund to be used in the event of reaching the lower return boundary. In practice, the upper limit was only reached once by two firms (Fomenta and Valora). In that case, the excess return was paid out to the investors when the firms merged. In this chapter, we ignore the upper bound on AFP returns, which essentially assumes that consumers get any excess returns. The upper bound was eliminated in 2008.

plan participation, and savings, as well as information on health, assets, disability status and utilization of medical services.

We develop a demand model of consumer choices among AFP funds that assumes that the consumer chooses an AFP to manage his/her pension savings at the beginning of each period (annually). Under the Chilean pension system, consumers are required to invest all their pension accumulations with one money manager, but they can freely move their savings from one money manager to another. The funds are not allowed to charge a fee to set up an account or to withdraw money. Our model assumes that the consumer's choice of investment fund at a given point in time depends on the product's characteristics (mean return and risk) and on the fees charged by that fund, and that consumers choose where to invest their funds on an annual basis. Chilean pension funds charge fixed and variable fees that depend on contribution levels. Hence, a fund with a high fixed fee but a low variable fee might be the one that comes at the lowest cost for a consumer with a high contribution level, whereas a fund with zero fixed fee but a high variable fee might be the one that comes at the lowest cost for someone with a low contribution level. In addition to making a choice about where to invest pension accumulations, consumers also decide each period whether to actively contribute to their pension plan by working in the formal sector where pension contributions are mandated. Thus, consumers choose jointly where to invest pension accumulations and whether to contribute.

The demand-side model is specified as a random coefficients random utility model, which leads to a multinomial discrete choice framework. Our model allows for both observable and unobservable sources of heterogeneity across consumers as well as unobservable attributes of pension fund firms that may, in addition to fund performance, affect the consumers' perceptions of fund quality. Repeated pension fund choices over time along with realized investment returns determine the consumers' balance accumulation. Aggregation over consumers generates the market demand for an AFP's product.

The supply side of the market is modeled as an oligopolistic environment in which AFPs sequentially choose their location (the mean return and the variance of their portfolio) and choose their two-part (fixed and variable) fee structure, taking into account the distribution of consumers' preferences and consumer types. At the time of our data collection, each AFP firm offered a distinct investment product. After 2002, however, there was a regulatory reform that introduced a new regulation requirement that firms have to offer five different investment products that vary in risk characteristics. In this chapter we focus on the regulatory environment that was in place prior to 2002, because allowing each firm to offer multiple products and for consumers to make portfolio choices within firms substantially increases both the modeling and the computational complexity. We view our study of the effects of the regulation in the simpler pre-2002 environment of each firm offering one product as a first step towards a more general understanding of how the regulation of a privatized pension industry affects its operation. Our study also provides some insight, though, into why the regulatory reform was needed to increase the diversity of products offered. As previously noted, another regulation on firms that was instituted in 1988 and remains in place today is that firms are allowed to charge fees only on new contributions and not on balances. Our analysis of the pension fund industry shows why fees charged to consumers are lower with this type of regulatory restriction.

There are a few features of the pension fund industry that make it inherently different from a standard product market. One is that once an individual accumulates some pension funds, he/she is essentially forced to consume one of the industry's products by choosing an investment fund.[8] On the firm side, firms service consumers who may not be actively contributing and therefore not paying any fees. Another nonstandard feature of the industry relative to a standard product market is that the relative costs of different products is consumer-specific, because the costs depend on the contributed amounts. Because our set-up differs significantly from the types of frameworks previously considered in the literature, we prove existence of a pricing and location equilibrium for our model, extending results of Caplin and Nalebuff (1991a) to this different environment.

The parameters of our demand model are estimated by simulated method of moments technique McFadden (1989). The estimates are based on micro-moments that capture the contribution of different consumer characteristics to the consumer's propensity to make a specific choices. We examine the goodness of fit of the model both to the moments used in estimation and with respect to aggregate statistics on market balance shares that were not used in the estimation and find that the model has a good fit. The demand side estimation

[8]Consumers cannot easily opt out of participating, because there are stringent rules in place that prohibit early withdrawal of pension funds, except in cases where individuals can demonstrate very high accumulations to assure that they are not at any risk of relying on minimum pension or welfare pension support.

recovers coefficients of risk aversion which are in line with estimates reported in other studies. We find that the risk aversion is inverse U-shaped with the youngest and oldest being relatively more willing to take on risk.

The supply side of the model is estimated using standard panel data techniques. The pension fund cost function is specified as a flexible translog function, which allows for scale economies with respect to the number of consumers and total balance managed by the fund. The estimates point to the existence of scale economies and also to an optimal scale both with respect to the number of consumers and total balance managed. Interestingly, we do not find evidence of increasing returns to scale throughout, supporting the role for more than one firm in supplying the pension fund market.

After estimating the parameters of the demand and supply model, i.e. the distribution of consumers' tastes and companies' cost functions, we use the model to perform counterfactual experiments that investigate firm and consumer behavior under four different regulatory environments. As a benchmark, we first study behavior in the absence of any regulation on firms' pension fund returns. Second, we consider the set of products that would be offered by a social planner, under the constraint that the level of government obligations is fixed at the level implied by the data. We consider this to be a constrained optimal set of products. Third, we analyze the existing regulation that requires AFP firms to deliver returns that are within two percentage points of the industry average. Fourth, we evaluate the effect of an alternative regulation that explicitly regulates the choice of investment instruments, by placing an upper limit on the riskiness of the firm's portfolio (the CAPM beta). Our analysis yields the following key findings:

(i) In the absence of regulation on pension fund returns, the industry offers a very risky set of products which, in turn, implies a large variance in consumer balances and in the level of government obligations.

(ii) The actual governmental regulation does not achieve its goal of reducing the riskiness of industry products. Rather, it results in relatively risky products, low product diversity and low pension plan participation rates.

(iii) A simple regulation in the form of an explicit restriction on the portfolio riskiness is more effective than the actual regulation in that it more closely resembles the constrained optimum.

The chapter is organized as follows. Section 4.2 provides some background information on the Chilean private accounts system and reviews related literature. Section 4.3 describes the consumer's choice problem and outlines the oligopolistic model of the firms' price and location decisions. Section 4.4 describes the estimation strategy. Section 4.5 presents the empirical results. Section 4.6 uses the estimated model to evaluate the effects of several alternative regulatory regimes and Section 4.7 concludes.

4.2 Industry Description and Related Literature

4.2.1 Industry Description

As previously described, investors in the Chilean pension system are permitted to hold their money in only one AFP at a time. The rules governing switching between money managers changed several times over the years, but beginning in 1984 investors could switch funds without incurring any monetary costs. Pension funds charge fees for their services. Initially, the fee was a three part non-linear tariff consisting of a fixed fee, a variable fee proportional to the participant's contribution, and a fee proportional to the participant's balance. Some companies also charged fees for withdrawal of funds, but in 1984 the government passed a regulation to disallow fees on the balance or on withdrawal. Currently, most AFPs charge a two-part tariff consisting of a fixed fee and a fee that is proportional to the participant's contribution.

From the inception of the private accounts system, the government exerted some control over the investment choices. Initially, pension investments could only be held in government bonds, but over time the options expanded to include riskier assets, such as stocks and foreign investments. As an additional measure to reduce the riskiness of the system, the government required that AFPs deliver a real return within a 2 percentage point band around the industry average, making the AFP firms responsible for covering low realizations of returns with their own capital. During the period after 1987, a number of AFPs had financial difficulties because of these restrictions and had to exit the market.[9]

Up until 2000, each AFP firm essentially offered a single investment product. Starting in 2000, however, they were allowed to offer five instruments which differ according to the riskiness of the investment.[10] As previously noted, our analysis is based on data from the time period prior to offering multiple investment instruments to simplify the modeling of the firm's choice of their product characteristics.

4.2.2 Related literature

Our study is, to the best of our knowledge, the first to jointly analyze consumer choice among money managing companies and the money managing companies' pricing and product choice decisions.

There exists a substantial literature based on US data that studies to what extent the performance of mutual fund managers, stock analysts etc. can be predicted from publicly available data on their characteristics and past performance. However, we are aware of only one study by Hortacsu and Syverson (2004) that focused on consumer choice among money managing companies. It explores consumer choices of S&P 500 index funds, which exhibit return homogeneity and sizable dispersion in fund fees. They find that consumer choices are largely driven by search costs, i.e. the cost of acquiring information about a fund which would be indicative of the fund's future performance. The authors conclude that this property combined with consumer heterogeneity in search costs and a large proportion of consumers with high search costs leads to a large dispersion in fees.

The literature on consumer choice of services/products in the presence of switching costs emphasizes entry barriers that arise as a result of switching costs, low incentives to invest in quality and adverse selection which arises if the switching cost is private information of the consumer. Some of the empirical and theoretical papers in this area include Beggs and

[9] In each case, the exit was organized as a merger with one of the existing AFPs. The clients of an exiting AFP were transferred to its merging partner, though they could easily switch funds afterwards.

[10] Each of these instruments has a targeted age group. An investor's contributions are allocated by default into an age-appropriate fund unless he/she chooses otherwise.

Klemperer (1992), Calem et al. (2005), Gravelle and Masiero (2000), Kiser (2002), Klemperer (1987), Knittel (1997), Rhoades (2000), Stango (2002).

We do not explicitly formalize consumer switching and search cost. The reason for this is twofold: First, before the time period under consideration the Chilean regulation systematically removed all financial and salient psychological barriers to switching AFPs. E.g., the last salient psychological barrier to switching was removed in 1988 when the government eliminated the requirement to request in person the change from one AFP to another. Second, contrary to the US mutual fund industry, the Chilean industry consisted in the time period under consideration only of twelve funds. In addition, AFPs have to meet rigorous information disclosure requirements.[11,12]

4.3 Model

In this section, we develop a formal model of how consumers make an AFP choice and of how AFP firms make pricing and portfolio (location) decisions, taking into account the specific regulations governing the Chilean private account system. We set out by describing the basic features of the environment, then formalize the demand side of the model and subsequently develop the supply side.

4.3.1 Framework

We assume that market participants are symmetrically informed. There are J AFPs, $J \in \mathbb{N}$. A consumer considers fund j as being characterized in period t by its return, $\tilde{R}_{jt}$, its fees, and other non-portfolio characteristics, e.g., features related to the convenience of obtaining service from this fund. Each AFP j offers a single product denoted by ϕ_j. All products lie in a $p+s$-dimensional space of portfolio and non-portfolio characteristics, $\Phi^{p+s} \subset \mathbb{R}^{p+s}$, where

[11] They report daily to the supervisory agency their investment transactions and submit monthly reports on their financial position and overall performance. They are also required to provide regular statements (three times a year) to their affiliates disclosing the last four monthly contributions paid by employers, the financial performance of the pension fund and the accumulated balance and rate of return on their individual account.

[12] However, we cannot exclude that consumers are e.g. "mentally attached" to an AFP and are therefore less willing to switch.We try to capture this in the estimation by allowing for idiosyncratic consumer tastes.

p denotes the "portfolio dimension" and s denotes the non-portfolio or "service dimension". We concentrate on the AFPs' portfolio choice and subsume the non-portfolio characteristics of AFP j's product by fund fixed effects ξ_{jt}.

Each AFP j constructs a portfolio out of the assets in the investment opportunity set. The random (gross) return of AFP j's portfolio is denoted by $\tilde{R}_{jt}$, which is realized at the end of period t. We assume that $\tilde{R}_{jt}$ is distributed according to a continuous probability density function denoted by $g(\cdot)$ for all $j \in \{1, ..., J\}$. Moreover, $g(\cdot) > 0$ on its domain and the associated c.d.f is denoted by $G(\cdot)$. We assume that consumers have mean-variance preferences over end-of-period retirement wealth. Thus, the portfolio characteristics of AFP j's product are completely described by $E\left[\tilde{R}_{jt}\right]$ and $E\left[\tilde{R}^2_{jt}\right]$.

We assume that consumers know the portfolio compositions of all AFPs and, therefore, the joint and marginal distributions of AFPs' returns. As noted in the introduction AFPs are subject to a minimum return (or profitability) requirement for the pension fund under their management. The return requirement is set in relation to the average performance of all pension funds over any twelve-month period. More specifically, if the real investment return is lower than the average by 2 percentage points, the AFP is required to make up the difference from its investment reserves.

We assume that consumers rationally incorporate this minimum return requirement into their decision process. This implies that a consumer anticipates that if he/she deposits her retirement savings at AFP j, then his/her actual return given the minimum return requirement in period t is

$$\tilde{R}^r_{jt} = \max\left\{\underline{\tilde{\mathrm{R}}}_t, \tilde{R}_{jt}\right\},$$

with $\underline{\tilde{\mathrm{R}}}_t := \frac{\sum_{n=1}^{J} \tilde{R}_{nt}}{J} - 0.02$.

AFP firms can set their fees freely, but the types of fees that they are allowed to charge are regulated. Authorized fees include a fixed fee per contribution and a pro rata fee on wages on which contributions are based. Because of the pro rata fee, different consumers may face different fees for participating in a particular AFP. More specifically, denote the fee that AFP j charges a customer i in period t by

$$\bar{p}_{jt} + \hat{p}_{jt} y_{i1t}, \tag{4.1}$$

where $\bar{p}_{jt}$ denotes the fixed fee, $\hat{p}_{jt}$ denotes AFP j's pro-rata fee, and y_{i1t} denotes the mandatory contribution of consumer i who is employed in the formal sector, denoted 1, in period t. The Chilean regulation stipulates that individuals which are employed in the formal sector have to contribute 10% of their annual labor income, i.e. $y_{i1t} = 0.1Y_{i1t}$, where Y_{i1t} denotes the annual labor income individual i earns in period t.

Another regulation is that the fee that a fund may charge a consumer cannot exceed the consumer's contribution amount that period and no fees may be charged during periods when the consumer is not contributing. Thus, the fee that an AFP may charge customer i can be represented as

$$p_{ijt} = \min\{y_{i1t}, \bar{p}_{jt} + \hat{p}_{jt} y_{i1t}\}. \tag{4.2}$$

Both the mandatory contribution and the fee are deducted from an individual's current income. Thus, if individual i is employed in the formal sector and if he/she is affiliated to AFP j, then his/her disposable income, denoted z_{ijt}, is given by

$$z_{ijt} = Y_{i1t} - y_{i1t} - p_{ijt}. \tag{4.3}$$

4.3.2 Demand Side

If an individual already participates in the pension program, then at the beginning of each period he/she chooses where to allocate his/her current pension balance and whether he/she should actively contribute to his/her account. If an individual never participated in the pension program, which implies that he/she has never worked in the formal sector, then he/she decides whether to remain outside of the formal sector (either not working or working in the informal sector) and, if choosing to work in the formal sector, where to invest his/her contributions. We assume that pension plan decisions are made on an annual basis.[13]

Below, we formalize the multinomial choice problem that an individual faces each period.

Choice Set An individual is, in a given period, either not affiliated, $\omega = 0$, or affiliated to the system. If an individual is affiliated he/she has to choose an AFP $j \in \{1, ..., J\}$ to

[13] The administrative pension fund data recorded daily accounting transactions to the individual pension accounts. We aggregated the data to create an annual data series.

manage his/her retirement savings. In addition, he/she has to decide whether to contribute actively to her account, denoted $(j,1)$, or not, denoted $(j,0)$. Thus, each individual's choice set is Ω, $\Omega := \{0\} \cup \mathbb{D}$, where

$$\mathbb{D} := \bigcup_{j=1}^{J} \Big\{ (j,1), (j,0) \Big\}.$$

We denote a general element of the set Ω by ω and a general element of the set $\mathbb{D}$ by d. In the remainder, we refer to the choice $(j,1)$ as "active affiliation" to AFP j, to the choice $(j,0)$ as " passive affiliation" to AFP j, and to the choice 0 as "non-participation".

Next, we characterize the expected utility that an individual i derives from each possible choice in period t.

Consumer (Expected) Utility We assume that an affiliated individual i at time t receives utility from two different components: current consumption and retirement wealth. We assume that utility is logarithmic in current consumption and that individuals have mean-variance preferences over end-of-period retirement wealth. This assumption implies that consumers are myopic with respect to their AFP choice and is common in the literature on delegated portfolio choice.[14] The parameter τ_{it}, which varies across consumers, reflects how the individual trades off current consumption and future retirement consumption at a given point in time.

Models of horizontal product differentiation generally assume that a consumer's utility function is composed of a part that measures his/her utility from consuming a numeraire commodity, i.e. current consumption, and a transport cost function that measures his/her disutility from not consuming his/her most preferred product, i.e. the portfolio that he/she would choose if he/she could invest on his/her own. In this respect our formulation is more general. Instead of exogenously imposing a transport cost function we employ a utility function that is commonly used in (delegated) portfolio choice theory to endogenously derive an individual's disutility from not consuming his/her most preferred portfolio.

At the beginning of period t, i.e. the time of deciding on affiliation and/or where to invest the retirement savings, we assume that individuals are uncertain about end-of-period investment

[14] See, e.g. Hugonnier and Kaniel (2010) and Kapur and Timmermann (2005).

returns but that they know their potential annual earnings (Y_{i0t}, Y_{i1t}) from participating in the informal and formal sectors.[15]

Expected Utility from Active Affiliation (j,1): The expected utility that individual i derives from the choice $(j,1)$ at the beginning of period t is given by

$$\begin{aligned} E\left[U_{i(j,1)t}\right] &= \tau_{it}\ln(Y_{i1t} - y_{i1t} - p_{ijt}) + w_{i(j,1)t}E\left[\tilde{R}^r_{jt}\right] \\ &- \gamma_{it}w^2_{i(j,1)t}E\left[\left(\tilde{R}^r_{jt}\right)^2\right] + \xi_{jt} + \epsilon_{i(j,1)t}, \end{aligned} \tag{4.4}$$

where $w_{i(j,1)t} = b_{it} + y_{i1t}$ denotes the beginning-of-period retirement wealth in period t and b_{it} represents the retirement balance of individual i, i.e. the sum of past mandatory contributions including interest. The parameter $\gamma_{it} > 0$ determines an individual i's coefficient of risk aversion and the parameter $\tau_{it} > 0$ determines the elasticity of substitution between current and retirement consumption. In the remaining discussion, we refer to τ as the elasticity of substitution and to γ as the risk aversion parameter. The unobservable component of consumer i's preferences is $\epsilon_{i(j,1)t}$ and the unobserved product-specific fixed effects is ξ_{jt}. As mentioned before, in our set-up market participants, i.e. AFP firms and consumers, are symmetrically informed. Thus, we assume that both $\epsilon_{i\omega t}$ and ξ_{jt} are observable to consumers and AFP firms but are unobservable to the econometrician.

Expected Utility from Passive Affiliation (j,0): Each affiliated individual has the option to not make a current contribution by not working in the formal sector where contributions are mandatory. We assume that if an individual ceases to contribute actively, he/she obtains some annual income in the informal sector. As previously noted, Chilean regulations governing pension fund fees stipulate that AFPs are not permitted to levy commissions on inactive accounts. Thus, the expected utility a customer i derives from choosing $(j,0)$ is

$$E\left[U_{i(j,0)t}\right] = \tau_{it}\ln(Y_{i0t}) + w_{i(j,0)t}E\left[\tilde{R}^r_{jt}\right] - \gamma_{it}w^2_{i(j,0)t}E\left[\left(\tilde{R}^r_{jt}\right)^2\right] + \xi_{jt} + \epsilon_{i(j,0)t}, \tag{4.5}$$

where Y_{i0t} denotes the annual income consumer i earns in the informal sector and $w_{i(j,0)t} = b_{it}$. Because the individual does not contribute in the current period its beginning-of-period

[15] In the data, only one of the earnings in one of the sectors (the sector chosen) will be observed. We describe in Appendix 4.8.C the procedure and the selection correction adjustment that we use to impute earnings in the sector that is not observed.

retirement wealth in period t corresponds to the sum of past mandatory contributions including interest.

Finally, we state an unaffiliated individual's utility at time t. An individual is unaffiliated if he/she has never worked previously in the formal sector.

Expected Utility from Non-participation (0): If an individual has never been affiliated to the pension system before t and does not choose to become affiliated in period t either, his/her (expected) utility is given by

$$E\left[U_{i0t}\right] = \tau_{it} \ln(Y_{i0t}) + \epsilon_{i0t}.$$

Corresponding to the set of possible choices, the expected utility of consumer i at time t can be represented as:[16]

$$\begin{aligned} E[U_{i(j,1)}] &= \tau_i \ln(Y_{i1} - y_{i1} - p_{ij}) + \alpha_{1,i(j,1)} E[\tilde{R}^r_j] && (4.6)\\ &+ \alpha_{2,i(j,1)} E[(\tilde{R}^r_j)^2] + \xi_j + \epsilon_{i(j,1)}, \\ E[U_{i(j,0)}] &= \tau_i \ln(Y_{i0}) + \alpha_{1,i(j,0)} E[\tilde{R}_j] + \alpha_{2,i(j,0)} E[(\tilde{R}^r_j)^2] + \xi_j + \epsilon_{i(j,0)}, && (4.7)\\ E[U_{i0}] &= \tau_i \ln(Y_{i0}) + \epsilon_{i0}, && (4.8) \end{aligned}$$

where

$$\begin{aligned} \alpha_{1,id} &= w_{id}, \\ \alpha_{2,id} &= -\gamma_i w^2_{id}, \\ \alpha_{3,i\omega} &= \epsilon_{i\omega}, \\ \alpha_{1,i0} = \alpha_{2,i0} &= 0. \end{aligned}$$

From inspection of (4.6) to (4.8) it is evident that the regularity condition that Caplin and Nalebuff (1991a) require for individual preferences is only partly fulfilled in our set-up: Expected utility is linear in the preference parameters $\alpha_{l,i\omega}$, $l \in \{1, 2, 3\}$, but – and this is where we depart – the utility of current consumption is weighted with the individual specific characteristic τ, reflecting the willingness to trade-off current consumption for future

[16] In the remainder of this subsection we suppress the time index t for the ease of exposition.

retirement wealth.[17] Due to this reason the price equilibrium existence proof provided in Caplin and Nalebuff (1991a) does not extend to our setting.

Note that the identity of an AFP does not affect an individual's beginning-of-period retirement wealth w_i, i.e. $w_{ij} = w_i \,\forall\, j$. However, we carry along the "j" subscript to unambiguously identify an individual's choice.

From the previous analysis it follows that a consumer i in period t is characterized by the $(7+(2J+1))$-tuple $(\alpha, \tau, Y_0, Y_1) \in \mathbb{B}$ where $\mathbb{B}$ is a convex subset of $\mathbb{R}^{7+(2J+1)}$. The α parameters are associated with the weights that an individual assigns to an AFP firm's characteristics, i.e. on expected return and variance of an AFP's portfolio, and an individual's idiosyncratic preference for an AFP. The parameter τ is associated with the trade-off between current consumption and future retirement wealth and (Y_0, Y_1) capture an individual's potential incomes in the informal and formal sectors.

We complete the description of the demand side by an assumption concerning the joint density of consumer preference parameters that is important for establishing equilibrium existence in the location and pricing stages.

Assumption A1 *The logarithm of the joint density of consumers' utility parameters, elasticity of substitution and income,* $\ln[f(\alpha, \tau, Y_0, Y_1)]$, *is a concave function in* (α, τ, Y_0, Y_1) *over its support,* $\mathbb{B}$, *which is a convex subset of* $\mathbb{R}^{7+(2J+1)}$.

This assumption is not too restrictive. In fact, the best known multivariate probability distributions (and densities) are log-concave: the multivariate Beta, Dirichlet, Exponential, Gamma, Laplace, Lognormal, Normal, Uniform, Weibull, and Wishart distribution.[18]

4.3.3 Supply Side

The demand model described above assumes that consumers have mean-variance preferences over retirement wealth. In this context, the mutual fund separation theorem implies that money managers – the AFPs – can satisfy consumers by choosing a convex combination

[17] In addition, consumers are heterogeneous with respect to income and individual preferences are nonlinear in current consumption. In Caplin and Nalebuff (1991a), consumer income heterogeneity is treated in a setting in which consumer preferences are linear in income.

[18] See Prékopa (1995) for an in-depth treatment of multivariate log-concave probability distributions and densities.

of the risk-free asset and the market portfolio. Therefore, the moments of AFP j's return distribution are uniquely determined by x_{jt}, the share of assets that AFP j invests in the market portfolio in period t. We refer to this share as AFP j's product choice or "location".

In order to proceed to the profit function of AFP j we have to characterize the demand for its product. It follows from the discussion of the demand side that consumers can, in principle, choose between two modes of investing their retirement wealth at AFP j: active or passive affiliation. In the discussion below, we refer to individual (aggregate) demand for active affiliation as individual (aggregate) active demand and to individual (aggregate) demand for passive affiliation as individual (aggregate) passive demand. In addition, we refer to the sum of individual (aggregate) passive and active demand as total individual (aggregate) demand.

We begin the characterization of an AFP's demand components by establishing that aggregate active, aggregate passive, and therewith total aggregate demand are continuous in the AFPs' prices.

Proposition 1 *Under Assumption A1 and preferences given by* (4.6) - (4.8)*, aggregate active, aggregate passive, and total aggregate demand are continuous in the price vector* $(\bar{p}_j, \hat{p}_j)_{j=1}^J$ *whenever the* J *products are distinct.*

Proof See Appendix 4.8.B.

We establish in Appendix 4.8.A that individual (active) demand features the reservation price property. There are two reasons for proceeding in this way: First, this property implies that aggregate (active) demand for a given firm's product is non-increasing in the respective firm's price vector. Second, the reservation price functions put structure on the set of consumers which demand an AFP firm's portfolio at a given price.

In the remaining exposition we denote by k the AFP that an individual chooses from the set $\{1, ..., J\}$ and by j a general element from this set.

Formally, type $(\alpha_\omega, \tau, Y_0, Y_1)$ maximizes his/her contemporaneous utility by choosing $(k, 1)$ if and only if

$$p_{ik} \leq R_k^a\Big(\alpha_\omega, \tau, \bar{x}, Y_0, z_{j\neq k} | \, \omega \in \Omega \setminus \{(k,1)\}\Big),$$

where $\bar{x} = \{x_1, ..., x_J\}$ denotes the set of products in the market which is, for now, assumed to be constant, and he/she maximizes his/her contemporaneous utility by investing his/her retirement wealth at AFP k if and only if

$$\begin{aligned} \psi_{ik} &:= (1 - \mathbb{I}_{(\alpha_{(k,1)}, \alpha_{(k,0)}, \tau, Y_0, Y_1)}) p_{ik} \\ &\leq R_k\Big(\alpha_\omega, \tau, \bar{x}, Y_0, z_{j \neq k} | \, \omega \in \Omega \setminus \{(k,1),(k,0)\}\Big), \end{aligned}$$

where

$$\mathbb{I}_{(\alpha_{(k,1)}, \alpha_{(k,0)}, \tau, Y_0, Y_1)} = \begin{cases} 1 & \text{if } \max\{E[U_{(k,1)}], E[U_{(k,0)}]\} = E[U_{(k,1)}] \\ 0 & \text{if } \max\{E[U_{(k,1)}], E[U_{(k,0)}]\} = E[U_{(k,0)}] \end{cases}.$$

We define $R_k^a(\cdot)$ and $R_k(\cdot)$ in Appendix 4.8.A. With continuous demand functions, the reservation price functions provide a concise representation of aggregate demand for a given firm's product. Once the firms' locations and the other firms' prices are fixed, demands for AFP k may be viewed from the perspective of a monopolist facing consumers with the respective reservation price functions $R_k^a(\cdot)$ and $R_k(\cdot)$. More specifically, in a sub-market that is characterized by consumers with identical $(\bar{\tau}, \bar{Y}_0, \bar{Y}_1)$ aggregate active demand and aggregate demand for AFP k's product at prices $\bar{p}_k$ and $\hat{p}_k$, are given by

$$D_k^a(\bar{p}_k, \hat{p}_k | \, \bar{\tau}, \bar{Y}_0, \bar{Y}_1) = \int_{(\alpha, \bar{\tau}, \bar{Y}_0, \bar{Y}_1): R_k^a \geq p_{ik}} f(\alpha_{(k,1)} | \bar{\tau}, \bar{Y}_0, \bar{Y}_1) d\alpha_{(k,1)}, \tag{4.9}$$

and

$$D_k(\bar{p}_k, \hat{p}_k | \, \bar{\tau}, \bar{Y}_0, \bar{Y}_1) = \int_{(\alpha, \bar{\tau}, \bar{Y}_0, \bar{Y}_1): R_k \geq \psi_{ik}} f(\alpha_{(k,1)}, \alpha_{(k,0)} | \bar{\tau}, \bar{Y}_0, \bar{Y}_1) d\alpha_{(k,1)} d\alpha_{(k,0)}. \tag{4.10}$$

Denote by $\mathbb{B}'$, which is a convex subset of $\mathbb{B}$, the set of all sub-markets where each sub-market is characterized by consumers with identical $(\bar{\tau}, \bar{Y}_0, \bar{Y}_1)$. Aggregating over all sub-markets yields that aggregate active demand and total aggregate demand of AFP k are given by

$$D_k^a(\bar{p}_k, \hat{p}_k) = \int_{(\tau, Y_0, Y_1) \in \mathbb{B}'} f(\tau, Y_0, Y_1) D_k^a(\bar{p}_k, \hat{p}_k | \, \bar{\tau}, \bar{Y}_0, \bar{Y}_1) \, d\tau \, dY_0 \, dY_1, \tag{4.11}$$

and

$$D_k(\bar{p}_k, \hat{p}_k) = \int_{(\tau, Y_0, Y_1) \in \mathbb{B}'} f(\tau, Y_0, Y_1) D_k(\bar{p}_k, \hat{p}_k | \, \bar{\tau}, \bar{Y}_0, \bar{Y}_1) \, d\tau \, dY_0 \, dY_1. \tag{4.12}$$

The following proposition establishes that aggregate active demand (4.11) and total aggregate demand (4.12) are log-concave functions of AFP k's price vector in the relevant range.

Proposition 2 *Consider preferences represented by* (4.6) *to* (4.8) *and* $f(\alpha, \tau, Y_0, Y_1)$ *a log-concave probability density function on* $\mathbb{R}^{7+(2J+1)}$ *with convex support* $\mathbb{B}$*. Active aggregate demand* (4.11) *and total aggregate demand* (4.12) *are log-concave over the price interval in which demand is strictly positive.*

Proof See Appendix 4.8.B.

After having characterized the demand for an AFP's product we complete the description of an AFP's objective function by formulating the production process that an AFP uses to provide its service. We think of the output of an AFP as being two-dimensional, consisting of the number of customers that receive its service as well as the total retirement wealth that the AFP manages on their behalf. We assume that all AFPs have access to the same technology, which may have economies of scale related to both inputs as well as the relative level of inputs. Additionally, we assume that AFPs may have firm-specific cost factors that reflect, for example, managerial talent or any other firm-specific productivity factors.

We denote by $D^{\mathrm{w}}_{jt}(\bar{p}_k, \hat{p}_k) := D^{a,y_1}_j(\bar{p}_k, \hat{p}_k) + D^b_j(\bar{p}_k, \hat{p}_k)$, the total retirement wealth that AFP j manages on behalf of its customers, where $D^{a,y_1}_k(\cdot)$ captures the aggregate current contribution to AFP k at prices $\bar{p}_k$ and $\hat{p}_k$ and $D^b_k(\cdot)$ represents the aggregate current retirement balance that AFP k manages if it charges $\bar{p}_k$ and $\hat{p}_k$ for its service. Denote by $\mathbb{B}'' \subset \mathbb{B}$ the set of sub-markets that are characterized by identical $(\bar{b}, \bar{\tau}, \bar{Y_0}, \bar{Y_1})$, then

$$\begin{aligned} D^{a,y_1}_k(\bar{p}_k, \hat{p}_k) &= \int_{(\tau, Y_0, Y_1) \in \mathbb{B}'} y_{1i} f(\tau, Y_0, Y_1) D^a_k(\bar{p}_k, \hat{p}_k)\, d\tau\, dY_0\, dY_1, \\ D^b_k(\bar{p}_k, \hat{p}_k) &= \int_{(b, \tau, Y_0, Y_1) \in \mathbb{B}''} b_i f(b, \tau, Y_0, Y_1) D_k(\bar{p}_k, \hat{p}_k)\, db\, d\tau\, dY_0\, dY_1. \end{aligned}$$

Concerning the characteristics of $D^{a,y_1}_k(\cdot)$ and $D^b_k(\cdot)$ it follows immediately from A1 and the arguments developed in the course of the proof of Proposition 2 (presented in Appendix 4.8.B) that both are log-concave in $(\bar{p}_k, \hat{p}_k)$.

We specify an AFP j's cost of producing its product as a translog function $C(D_{jt}(\bar{p}_j,\hat{p}_j), D^{\mathrm{w}}_{jt}(\bar{p}_j,\hat{p}_j))$ such that

$$\begin{aligned}\ln(C(D_{jt}(\bar{p}_j,\hat{p}_j), D^{\mathrm{w}}_{jt}(\bar{p}_j,\hat{p}_j))) &= \varphi_1 \ln(D_{jt}(\bar{p}_j,\hat{p}_j)) + \varphi_2 \ln(D^{\mathrm{w}}_{jt}(\bar{p}_j,\hat{p}_j)) \\ &+\varphi_3(\ln(D_{jt}(\bar{p}_j,\hat{p}_j)))^2 + \varphi_4(\ln(D^{\mathrm{w}}_{jt}(\bar{p}_j,\hat{p}_j)))^2 \\ &+\varphi_5 \ln(D_{jt}(\bar{p}_j,\hat{p}_j)) \ln(D^{\mathrm{w}}_{jt}(\bar{p}_j,\hat{p}_j)) + \nu_j + \eta_{jt},\end{aligned} \tag{4.13}$$

where ν_j is a firm fixed effect, and η_{jt} is an idiosyncratic shock to the cost of AFP j in period t.

As previously described, Chile's pension fund regulations require firms to provide their customers a minimum return which is no less than the average industry return minus 2 percentage points. If an AFP does not achieve this return than it is required to make up the difference between the realized and the minimum return from its investment reserve. Thus, an AFP faces an additional regulatory cost which is, in expectation, equal to

$$C^{reg}_{jt}(\bar{p}_j,\hat{p}_j) = E[(\tilde{R}_{jt} - \underline{\tilde{\mathrm{R}}}_t)|\tilde{R}_{jt} < \underline{\tilde{\mathrm{R}}}_t](D^{a,y_1}(\bar{p}_j,\hat{p}_j)_j + D^b_j(\bar{p}_j,\hat{p}_j)), \tag{4.14}$$

in period t.

Thus, in period t the expected profit of an AFP j that chooses location x_j and charges fees $p_j = (\bar{p}_j,\hat{p}_j)$ when its competitors locate at $\mathbf{x}_{-j}$ and charge $\mathbf{p}_{-j}$ is

$$\begin{aligned}E[\Pi_j(p_j,\mathbf{p}_{-j},x_j,\mathbf{x}_{-j})] &= \bar{p}_j\, D_j(p_j,\mathbf{p}_{-j},x_j,\mathbf{x}_{-j}) + \hat{p}_j\, D^{a,y_1}_j(p_j,\mathbf{p}_{-j},x_j,\mathbf{x}_{-j}) \\ &- C(D_j(p_j,\mathbf{p}_{-j},x_j,\mathbf{x}_{-j}), D^{\mathrm{w}}_j(p_j,\mathbf{p}_{-j},x_j,\mathbf{x}_{-j})) \\ &- C^{reg}_j(p_j,\mathbf{p}_{-j},x_j,\mathbf{x}_{-j}),\end{aligned} \tag{4.15}$$

where we assumed that p_j and y_{i1} are so that $p_{ij} = \bar{p}_j + \hat{p}_j y_{i1} \leq y_{i1}$, for all types i for the ease of exposition. The last term in (4.15) is zero in the absence of the minimum return requirement.

Proposition 3 *Under Assumption A1 and preferences given by* (4.6) - (4.8) *each AFP j's expected profit function is quasi-concave in* $(\bar{p}_j,\hat{p}_j)$.

Proof See Appendix 4.8.B.

4.3.4 Equilibrium Conditions

At a given point in time, competition between AFPs takes the form of a two-stage game. In the first stage, we assume that AFPs simultaneously and irrevocably choose their portfolio characteristics. Thus, portfolio characteristics are fixed at the second stage when AFPs engage in price competition. That is, given the set of portfolios, $\bar{x} = \{x_1, .., x_J\}$, funds simultaneously choose prices given locations. Thereafter, the rate of return on the market portfolio is realized, interest is paid on consumers' retirement wealth, and AFP firms' profits accrue. We solve for the subgame perfect equilibrium by backward induction.

Pricing Stage Having established that each AFPs profit function is quasi-concave we are in a position to derive an important result, stated in Proposition 4, concerning the existence of a Nash equilibrium.

Proposition 4 *Under Assumption A1 and preferences given by* (4.6) *to* (4.8)*, there exists a pure strategy Bertrand-Nash equilibrium for any J firms and arbitrary portfolios $\bar{x}$.*

Proof Given the continuity of firms' profit functions with respect to own prices, rival prices, and the quasi-concavity of a firm's profit with respect to fixed and variable fee, an equilibrium necessarily exists (by Kakutani's fixed point theorem) as the set of feasible price combinations is compact and convex. ■

Proposition 4 guarantees the existence of an equilibrium in the pricing stage, but the equilibrium is not necessarily unique. We take the potential non-uniqueness of second stage equilibria into account in our choice of the estimation strategy and when we use the estimated model to perform the counterfactual experiments.

Specifically, we assume in the counterfactual simulations that if multiple equilibria exist firms select the payoff-dominant equilibrium. The solution of the above problem yields equilibrium prices as a function of AFPs' locations, i.e. $\bar{p}^{\star}_{jt}(x_{jt}, \mathbf{x}_{-jt})$ and $\hat{p}^{\star}_{jt}(x_{jt}, \mathbf{x}_{-jt})$.

Location Stage Given the other AFP's locations and taking into account the impact on prices, firm $j \in \{1, .., J\}$ maximizes its expected profit by choosing the location $x_{jt} = x^{\star}_{jt}$, where $x^{\star}_{jt}$ solves

$$x^{\star}_{jt} = \arg\max E[\Pi_{jt}(\bar{p}^{\star}_{jt}(x_{jt}, \mathbf{x}_{-jt}), \hat{p}^{\star}_{jt}(x_{jt}, \mathbf{x}_{-jt}); \bar{\mathbf{p}}^{\star}_{-jt}(x_{jt}, \mathbf{x}_{-jt}), \hat{\mathbf{p}}^{\star}_{-jt}(x_{jt}, \mathbf{x}_{-jt})]. \quad (4.16)$$

Due to the fact that the pricing-stage equilibrium is not necessarily unique we can provide a proper existence and uniqueness proof of the two-stage location-then-pricing game only for versions of the model in which the second-stage equilibrium is unique. This is the case, e.g., in a version of the model in which we assume that consumers have to be actively affiliated to an AFP firm.[19] However, we confirm by numerical methods that there exists an equilibrium in the "full" two-stage location-then-pricing game outlined before.

4.4 Estimation Approach

This section describes how we estimate the funds' locations, the parameters of consumer preferences and of the industry cost structure from the data. The funds' location choices are not directly observed and therefore need to be inferred from the data. Our estimation procedure consists of two steps. In the first step we estimate the funds' locations from data on investment returns and then, in the second step, we use the estimated locations to recover consumer preferences and the firms' cost structure. We take the two-step nature of our estimator into account in estimating the variance of the estimated coefficients.

4.4.1 Recovering Funds' Locations

The pension funds' location choice (the riskiness of a fund's portfolio) is an important component of both the demand and supply side models. As noted, we use the funds' estimated CAPM beta to represent its location choice. More specifically, we work with a model of

[19] Even in the restricted version we add to the literature by our existence and uniqueness proof. This is due to the fact that we consider general multi-dimensional log-concave consumer densities. So far, the only paper that analyzed the conditions under which an unique equilibrium of the two-stage location-then-price oligopoly game exists for general log-concave consumer densities is Anderson et al. (1997). However, they only consider one-dimensional consumer characteristics.

time varying beta and GARCH errors to recover the funds' betas, which approximates the consumers' forecast of the funds' expected returns and return volatilities. Our approach to estimating the CAPM betas is based on Bollerslev at al.'s (1989) CAPM model with time varying covariances. Denote by $E_{j,t}$ the excess return of fund j at time t and by $E_{m,t}$ the industry average excess return at time t. We assume that the vector ($E_{j,t}$, $E_{m,t}$) changes over time according to

$$\begin{aligned} E_{j,t} &= b_j + \delta h_{jm,t} + \epsilon_{jt} \\ E_{m,t} &= b_m + \epsilon_{mt} \end{aligned} \tag{4.17}$$

where $\epsilon_t = (\epsilon_{jt}, \epsilon_{mt})$ is distributed according to $N(0, H_t)$ and the elements of the variance-covariance matrix are:

$$\begin{aligned} h_{jj,t} &= \gamma_{jj} + \alpha_{jj}\epsilon^2_{j,t-1} + \beta_{jj}h_{jj,t-1} \\ h_{mm,t} &= \gamma_{mm} + \alpha_{mm}\epsilon^2_{m,t-1} + \beta_{mm}h_{mm,t-1} \\ h_{jm,t} &= \gamma_{jm} + \alpha_{jm}\epsilon_{j,t-1}\epsilon_{m,t-1} + \beta_{jm}h_{jm,t-1} \end{aligned} \tag{4.18}$$

Model parameters are estimated via maximum likelihood. The beta values and forecasts are obtained using rolling 18 months window.

4.4.2 Estimating Consumer Preference Parameters

The demand side of the model formalizes a consumer's choice among multiple pension investment fund alternatives. As noted in the previous section, his/her preference for alternative options is described by a random utility model, where utility depends on balances, income, fees, and the location of the firm. Consumers differ in risk aversion and in the relative weight they place on current versus future (retirement) consumption in a way that depends on observable demographics as well as on unobservables. Under the set-up described above, this yields a random coefficients model. Our demand model also incorporates alternative-specific fixed effects to accommodate, for example, unobserved differences in the perception of fund quality.

The demand model is estimated using using McFadden's (1989) simulated method of moments (SMM) approach. The parameter vector θ is recovered as

$$\theta = \arg\min_{\theta}(d - P(\theta))'W'W(d - P(\theta)), \tag{4.19}$$

where d denotes the $Jn \times 1$ vector of consumer choices with $d_{ij} = 1$ if individual i chose alternative j and $P(\theta)$ represents the predicted choice given a vector of coefficients θ. Therefore, $d - P(\theta)$ is a vector of residuals stacked by alternatives for a given individual and by individuals. The weighting matrix W is an $K \times Jn$ array of instruments of rank $K \geq k$ where k is the length of the parameter vector.

The choice probability is estimated using a frequency simulator. McFadden (1989) shows that with a suitable choice of a simulator and a matrix of instruments which is proportional to $\partial \ln(P(\theta^*)/\partial\theta$, the method is asymptotically efficient. We implement the method using an iterative process. First, we find an 'initial consistent' estimator of θ^* using a matrix of non-optimal instruments (X_{kij}, X_{kij}^2). Then we use the first-stage estimator to construct optimal instruments which are used to obtain the final estimate of θ^*.[20]

We compute the variance-covariance matrix of the estimated coefficients according to

$$\Sigma = (R'R)^{-1}R'GR(R'R)^{-1} \tag{4.20}$$

where

$$R = \lim_{n\to\infty} n^{-1}\, WP_\theta(\theta^*) \tag{4.21}$$

and

$$G = \lim_{n\to\infty} n^{-1}\,(1 + r^{-1})\sum_{t=1}^{t=n}\sum_{j=1}^{j=J}(P(j|\theta^*, X)W_{jt}W_{jt}' - W_{\cdot t}W_{\cdot t}' \ . \tag{4.22}$$

Here, r is the number of draws used in the frequency simulator.

[20]Mcfadden's SMM method requires to use two independent sets of random draws in estimation for consistency: the first set to construct the instruments and the second set to simulate the choice probabilities.

4.4.3 The Cost Function

The cost function parameters are estimated using annual data on various components of firms' operational costs. This estimation is informative on potential scale effects both with respect to the number of customers served and the total balance managed by a particular fund, while allowing for the interdependence of these two factors in determining costs. We estimate the cost function using a flexible 'translog' functional form, where costs depend on the number of customers (D_{jt}) and the total retirement wealth of AFP j's customers (D^{w}_{jt}). Specifically, we assume that

$$\begin{aligned}\ln(C(D_{jt}, D^{\mathrm{w}}_{jt})) &= \varphi_1 \ln(D_{jt}) + \varphi_2 \ln(D^{\mathrm{w}}_{jt}) + \varphi_3(\ln(D_{jt}))^2 \\ &\quad +\varphi_4(\ln(D^{\mathrm{w}}_{jt}))^2 + \varphi_5 \ln(D_{jt}) \ln(D^{\mathrm{w}}_{jt}) + \nu_j + \eta_{jt},\end{aligned} \tag{4.23}$$

where D_y are year effects. The parameters φ are estimated using standard panel data methods.

4.5 Empirical Results

4.5.1 Data Description

As previously described, this paper combines data from multiple sources. They include longitudinal survey data on a random sample of working-age Chileans that was collected in 2002 and 2004, linked administrative data from the pension fund regulatory agency on contributions, balances, and fees charged by the various AFPs, market data on the performance of various funds, a data series on fees charged by the funds, and accounting cost data.[21]

Our longitudinal data contain demographic, labor market and pension related information on 17,246 individuals age 15 or older. We restrict our analysis, however, to individuals who are 65 years old or younger who have not yet entered the pension withdrawal phase.[22] This

[21] The 2002 survey was called *Historia Laboral y Seguridad Social (HLLS)* and the 2004 survey was called *Enquesta Proteccion Sociale*

[22] Age 65 is a typical retirement age for men in Chile

Table 4.1: Descriptive Statistics

	25% centile	median	75% centile
age	27	34	42
years contributing	1.41	3.83	7.58
annual income (all ages)	$1230	$1342	$3196
annual income (age=30)	$1127	$1250	$2916
annual income (age=40)	$1342	$1512	$3705
annual income (age=55)	$760	$834	$3235
balance (all ages)	$783	$1090	$3381
balance (age=30)	$590	$930	$2231
balance (age=40)	$1500	$2500	$6210
balance (age=55)	$1653	$2670	$7161

is because our model pertains to pension fund investment decisions and does not include, for example, decisions that people typically face at retirement, such as whether to use the funds to purchase an annuity or the rate at which to draw down the funds.

4.5.2 Descriptive statistics

Table 4.1 presents some descriptive statistics derived from the administrative pension data, which includes men and women. The pension plan participant sample is fairly young, with a median age of 34 and an interquartile range of 27-42. The median years of contribution is 3.83 years with an interquartile range of 1.41-7.58. This indicates that consumers do not contribute in every year that they are affiliated with the pension system. The median pension fund balance is close to the median of one year's annual income. There is rising dispersion with age, particularly over the age 35-45 range. At subsequent ages, the dispersion remains roughly constant. There is also increasing dispersion in income up through age 40, as exhibited by the interquartile range, after which it declines. The dispersion in income is not as large as the dispersion in balances, which might be expected given that balances represent a stock measure and income a flow measure.

We next turn to the descriptive characteristics of the AFP firms. Table 4.2 shows the fixed and variable fees charged by the AFP firms in year 1999 and reveals substantial variation in the fees charged across firms. A number of funds do not charge any fixed fee. The AFP firm Habitat has the lowest variable fee at 2.84% of monthly contributions and no fixed fee. The firm Concordia has the highest fixed fee at 3.48% and also a relatively high fixed fee at 230 pesos per month. The variation in fees indicates that products are differentiated.

Table 4.2: Fees charged by AFP firms in 1999

AFP	Percent Fee	Fixed Fee
Concordia	3.48	230
Cuprum	2.99	0
Habitat	2.84	0
Planvital	3.45	280
Provida	2.85	195
Santa Maria	3.15	100
Summa	3.15	230
Magister	3.4	220
Union	3.7	290
Proteccion	2.94	0
Futuro	3.25	0
Formenta	3.25	0

In Table 4.3, we compare the market shares in 1999 of the different funds in terms of the share of clients and the share of the total market balance under each firm's management. The table also shows the estimated CAPM-beta, with lower betas indicating lower risk. The fund with the largest market share both in terms of customers and balances is Provida, which manages pensions for about one-third of all pension plan participants. Provida is also one of the least risky funds.

The AFP firm Santa Maria has the second largest market share in terms of clients but ranks lower in terms of balance shares. Its portfolio allocation is in the median risk range. The firm with the lowest fees, Habitat, is relatively low ranking in terms of numbers of share of customers but is in the top three in terms of share of total balances. There is also a number of funds in the market with very low shares of customers and of balances. For example, Fomenta has the riskiest portfolio, measured in terms of beta, and also attracts few clients. In summary, there is substantial heterogeneity across firms in fee structures, in shares of clients and in shares of balances.

Table 4.3: Market Structure in 1999

	N_Share	B_Share	Beta
Concordia	0.191	0.223	0.356
Cuprum	0.018	0.005	0.540
Habitat	0.058	0.133	0.400
Planvital	0.027	0.011	0.525
Provida	0.345	0.450	0.376
Santa Maria	0.207	0.074	0.330
Summa	0.076	0.053	0.500
Magister	0.014	0.008	0.530
Union	0.051	0.024	0.520
Proteccion	0.013	0.013	0.550
Futuro	0.000	0.001	0.380
Fomenta	0.001	0.005	0.640

4.5.3 Model Estimates

Tables 4.4, 4.5, 4.6 and 4.7 present the estimated model parameters and evidence on goodness of fit.

We allow the intertemporal substitution parameter, τ, to depend on age, the number of months with active participation, and on unobservable sources of heterogeneity.

Table 4.4: Demand Estimation: Elasticity of Substitution (τ)

		Parameter	Std. Error
age ≥ 35	contr years ≤ 6	1.86	0.46
$35 \leq$ age≤ 50	contr years ≤ 6	1.64	0.35
age ≥ 50	contr years ≤ 6	2.86	0.51
age ≥ 35	$6 \leq$ contr years ≤ 14	1.66	0.43
$35 \leq$ age≤ 50	$6 \leq$ contr years ≤ 14	1.54	0.28
age ≥ 50	$6 \leq$ contr years ≤ 14	1.65	0.51
age ≥ 35			
$35 \leq$ age≤ 50	$14 \leq$ contr years ≤ 20	1.04	0.32
age ≥ 50	$14 \leq$ contr years ≤ 20	1.66	0.52
σ_τ		1.02	0.32

We find that the parameter which measures intertemporal substitution is non-linear in age with middle-aged people less willing to substitute than very young and older people. Similarly, people with higher active participation (below 20 years) are more willing to contribute than people at a lower level of active participation.

Similarly, we allow the risk aversion parameter, γ, to depend on age and an unobservable component intended to capture unobservable sources of heterogeneity in people's attitudes towards risk.

Table 4.5: Demand Estimation: Risk Aversion (γ)

	Parameter	Std. Error
mean		
age ≥ 35	3.36	1.23
$35 \leq$ age≤ 50	7.34	1.34
age ≥ 50	5.36	1.51
σ_γ	2.05	0.95

Theoretical models of dynamic savings decisions[23] would suggest that age is a determinant of risk aversion, although its net effect on risk aversion is ambiguous. Older individuals are typically less willing to take on investment risk, because of a shorter time horizon until retirement, but may also be more willing to take on risk, because they have higher balances.

Table 4.6: Implied Risk Aversion

Age	Absolute Risk Aversion	Relative Risk Aversion
30	0.030	2.24
40	0.048	2.98
50	0.031	2.11

Table 4.6 presents estimates of the Arrow-Pratt measure of absolute and relative risk aversion at different ages. As seen in the table, people are estimated to be more risk averse at age 40 than at age 30 or 50.

[23]See e.g., Brunnermeier and Nagel (2008) and Li (2007)

In Table 4.7, we examine the importance of unobservables to the fit of the model. Specifically, we evaluate the fit of the moments under the original model and under two restricted versions of the model, one that sets the alternative-specific fixed effects to zero (i.e. shuts down permanent unobservable firm heterogeneity) and one that, in addition, suppresses the utility shock. We find that the fit of the moments is not greatly compromised by shutting down unobservable sources of heterogeneity, although the fit is certainly improved by including these components.

Table 4.7: Role of Unobservables

	Proportion explained
Observable part of utility function	75%
Observable part plus fixed effects	80%
Observable part plus fixed effects plus Weibull errors	97%

Table 4.8 compares the model's aggregate predicted shares of annual contributions to pension funds to the empirical shares. In the estimation we only used moments pertaining to shares of customers and shares of balances. The moments related to shares of contributions were not used in estimating the model parameters, i.e., γ and τ, so this comparison could be viewed as a form of model validation. Generally, the model is able to identify the top five AFP firms in terms of shares of contributions and is fairly accurate in terms of predicting the actual contribution share for four of the five funds. The AFP firm Provida had the largest contribution share in the data, which is also predicted by the model. For the third (Concordia), though, the model overpredicts the contribution share.

Table 4.9 provides the estimated parameters of an AFP's cost function that are derived from panel data on firms' costs and cost components. We assume that costs depend on the number of clients served, and the total balance under management. We specify the cost function flexibly as a function of linear and interaction effects in these variables. According to the estimates, once the pension fund reaches a certain size there are decreasing returns to scale. This implies that the market is efficiently served by more than one pension fund firm.

Table 4.8: Aggregate Fit to Contribution Shares

	Actual	Predicted
Concordia	18.5%	30.4%
Cuprum	1.3%	0.8%
Habitat	12.1%	8.4%
Planvital	2.0%	1.3%
Provida	29.0%	30.7%
Santa Maria	18.6%	22.3%
Summa	9.3%	8.5%
Magister	1.7%	1.3%
Union	4.4%	2.8%
Proteccion	2.5%	1.0%
Futuro	0.1%	0.01%

Table 4.9: Parameter Estimates of the Translog Cost Function

	Parameter	Std. Error
Constant	-4.5	0.95
log(affiliates)	0.98	0.28
log(assets)	0.45	0.14
$(\log(\text{affiliates}))^2$	0.015	0.021
$(\log(\text{assets}))^2$	0.038	0.012
log(affiliates)*log(assets)	-0.086	0.027

4.6 Policy Experiments

Next, we use our estimated demand- and supply-side model to evaluate the effects of several regulatory regimes on AFP product choices, consumer participation, balance accumulation, and the level of government obligations. For reasons of computational difficulty, our current results are for a market with three firms only (the largest firms in the market).

We study four regulatory regimes: (a) unrestricted competition or no regulation, (b) constrained optimization, (c) current regulation, (d) an alternative regulation that places an upper limit on the riskiness of the firm's portfolio (the CAPM beta). Constrained optimization describes a set of products which would be chosen by a social planner while holding the government guarantees fixed at the current level.

Table 4.10 shows the averages and standard deviations of the CAPM betas for the portfolios offered under the four regulatory regimes. We also separately investigate the industry equilibrium during the periods of high and low market volatility. In a market with low volatility, we find that under no regulation the industry chooses to offer a very risky but well diversified set of portfolios. The selection of portfolios in the constrained optimum is less risky but still reasonably well diversified. However, the current regulation that requires rates of return within 2% of the industry average induces a portfolio choices that are very similar to each other. This effect arises because the regulation penalty is related to the factors that affect a firm's profitability, i.e. the amount of contributions received by the firm. In a market with high volatility and no regulation, there is a move into even riskier products than under low volatility, catering to the demand from low income consumers seeking a high-risk gamble. The products offered under the constrained optimization regime reflect the planner's intention to control overall risk in the system by reducing the exposure to aggregate risk when it is high. Interestingly, the current regulation leads to similar outcomes as the no-regulation regime with the industry choosing higher-risk products relative to constrained optimum. The products also exhibit low diversification. We also consider an alternative regulation that places explicit restrictions on the riskiness of portfolios (the CAPM betas), which leads to product characteristics that are similar to those chosen under the constrained optimum.

Table 4.10: Product Characteristics Under Different Regulatory Regimes

	β, average	β, std. dev.
Low market volatility,	$\sigma_m = 0.10$	
No regulation	1.05	0.35
Current Regulation	0.53	0.05
Portfolio Restrictions	0.65	0.18
Constrained Optimum	0.71	0.27
High market volatility,	$\sigma_m = 0.25$	
No regulation	1.25	0.47
Current Regulation	0.85	0.09
Portfolio Restrictions	0.58	0.11
Constrained Optimum	0.56	0.15

Table 4.11 compares the proportion of consumers participating in the pension plan under the four different cases considered previously. As noted in the introduction, a major concern of the Chilean government is the low contribution rate, with a substantial fraction of workers opting to work outside the formal sector of the economy and therefore to not participate in the pension program. The estimates in Table 4.11 show that the current regulation results in the lowest participation rate both in the high and low volatility markets. It is significantly lower than under a constrained optimum or under the alternative regulation that restricts portfolio risk directly. The lower contribution rate arises because of the low product diversity induced by the current regulation.

Table 4.11: Pension Plan Participation Rates under Alternative Regulatory Schemes

	Participation Rate	Balance (Mean)	Balance (Std. Dev.)
Low market volatility,	$\sigma_m = 0.10$		
No Regulation	65%	1.25	1.35
Current Regulation	58%	0.8	0.75
Portfolio Restrictions	75%	0.93	0.93
Constrained Optimum	75%	1	1
High market volatility,	$\sigma_m = 0.25$		
No Regulation	55%	1.35	1.60
Current Regulation	56%	1.21	1.18
Portfolio Restrictions	72%	1.01	1.01
Constrained Optimum	70%	1	1

Table 4.12 analyzes the size of government obligations under the four regulatory regimes. We find that the no regulation regime results in extremely high volatility of government obligations. The current regulation induces high expected government liabilities as well as substantial variation in government liabilities. The first effect arises because of the low participation rate whereas the second effect arises because of the high riskiness of the portfolios offered under the current regulation.

Table 4.12: Government Obligations

	Retirees on support (mean)	Retirees on support (std. dev.)	Total amount (mean)	Total amount (mean)
Low market volatility,	$\sigma_m = 0.10$			
No Regulation	28%	10%	0.476	0.17
Current Regulation	25%	3%	0.425	0.051
Portfolio Restrictions	23%	5.6%	0.37	0.089
Constrained Optimum	20%	5%	0.34	0.085
High market volatility,	$\sigma_m = 0.25$			
No Regulation	34%	25%	0.731	0.425
Current Regulation	32%	15%	0.544	0.255
Portfolio Restrictions	27%	9.8%	0.453	0.144
Constrained Optimum	25%	9%	0.425	0.136

The amount of government obligations is given in billions of US dollars.

4.7 Conclusion

Chile has one of the oldest individual-account pension systems and therefore provides a unique opportunity to study firm and consumer behavior under a well-established private accounts system. The design of the Chilean pension system includes insurance features, in the form of a minimum return guarantee and a minimum pension guarantees, that are intended to protect investors against low levels of pension accumulations.

In this chapter, we developed a demand and supply model of the Chilean pension fund market and used the model to understand the effects of different kinds of regulations on the operation of the pension fund market. In the demand model, a consumer chooses an AFP to manage his/her investments, taking into account pension fund fees and historical pension fund performance. Consumers are heterogeneous in terms of their wealth, demographics, risk aversion and in the relative weight they place on current consumption verses future retirement wealth. The supply side is modeled as an oligopolistic environment in which AFP firms sequentially choose product location (mean and variance of the return), and the fixed and variable fees they charge for service, taking into account consumers' preferences.

After estimating the parameters of the model, we use the model to assess the impact of government regulations on pension funds' choices of locations. We also study the implications of this regulations for the consumers' accumulated balances, and for the government's obligations. We find that the Chilean regulatory rule that mandates firms to guarantee returns within 2% of the industry average creates incentives for the AFP firms to invest in riskier portfolios than they would choose under an alternative regulation that instead restricts the riskiness of their portfolio. Surely, this is an unanticipated effect of the regulation. Because the firms' portfolio choices are riskier, fewer people participate in the pension program, which is a particularly worrisome finding considering that the government places a high priority on increasing coverage rates. Also, the choice of the portfolios under the current regulation is riskier than the selection of portfolios that a social planner would choose. Not surprisingly, it leads to a higher than desirable (by the social planner) volatility in accumulated balances. We find that from the point of view of social welfare, an alternative regulation that directly restricted the investment instruments of pension funds rather than requiring them to achieve a performance near the mean would be more effective.

4.8 Appendix

4.8.A Reservation Price Property of Individual Demand

In the course of the derivation, we focus on a given type of individual, as characterized by (α, τ, Y_0, T_1). For ease of exposition, we suppress the subscripts i and t. We consider the set of products in the market, $\bar{x} = \{x_1, .., x_J\}$, to be constant. In addition, we denote by k the AFP that an individual chooses from the set $\{1, ..., J\}$ and by j a general element from this set.

First, we turn to individual active demand. That individual active demand features the reservation price property results directly from unit demand and the strict monotonicity of preferences in contemporaneous consumption, that is in $z_j = 0.9Y_1 - p_j$, where $p_j = \min\{y_1, \bar{p}_j + \hat{p}_j y_1\}$. Put differently, preferences are non-increasing in p_j. Thus, each individual has a reservation price, denoted R_k^a, for contributing actively to AFP k. Formally, type $(\alpha_\omega, \tau, Y_0, Y_1)$ maximizes his/her contemporaneous utility by choosing $(k, 1)$ if and only if

$$p_k \leq R_k^a\Big(\alpha_\omega, \tau, \bar{x}, Y_0, z_{j\neq k} | \, \omega \in \Omega \setminus \{(k,1)\}\Big). \tag{4.24}$$

To further characterize the reservation price, we denote the best alternative utility level of type $(\alpha_{(j,1)}, \tau, Y_0, Y_1)$ as $A_k^a(\alpha_\omega, \tau, \bar{x}, Y_0, z_{j\neq k} | \, \omega \in \Omega \setminus \{(k,1)\}\Big)$, defined as

$$\max \left\{ \max_{j \neq k} E[U(\alpha_{(j,1)}, \tau, x_j, z_j)], \max_j E[U(\alpha_{(j,0)}, \tau, x_j, Y_0)], E[U(\alpha_0, \tau, Y_0)] \right\}$$

Thus, individual i's reservation price for active affiliation is

$$R_k^a\Big(\alpha_\omega, \tau, \bar{x}, Y_0, z_j | \omega \in \Omega \setminus \{(k,1)\}\Big) = \begin{cases} y_1 & \text{if } E[U(\alpha_{(k,1)}, \tau, x_k, 0.8Y_1)] > A_k^a; \\ p_k & \text{if } p_k \text{ solves} \\ & E[U(\alpha_{(k,1)}, \tau, x_k, z_k)] = A_k^a; \\ -\infty & \text{if } E[U(\alpha_{(k,1)}, \tau, x_k, z)] \; < \; A_k^a, \\ & \forall \, z \; \in \; [0.8Y_1, 0.9Y_1]. \end{cases}$$

The first case reads as follows: Even if AFP k charges consumer i the maximum fee, consumer i will prefer active affiliation to AFP k at this price to all other alternatives. Thus, the reservation price of consumer i is the maximum fee y_1.

The amount $0.8Y_1$ in the reservation price formulation comes about as follows: Only individuals in the formal sector contribute. These individuals earn an annual income of Y_1. The mandatory contribution amounts to 10% of their annual income. In addition, AFPs charge consumers a fee for their services, and the maximum fee that a fund may charge a consumer is an amount equal to the mandatory contribution. Thus, if an AFP charges an actively contributing individual the maximum allowable fee, the individual's disposable income would amount to $0.8Y_1$.

The reservation price property implies that aggregate active demand is non-increasing in $(\bar{p}_j, \hat{p}_j)$. In a similar way, we can establish the reservation price property of individual demand for AFP k. In order to formalize this, we introduce the indicator function $\mathbb{I}_{\alpha_{(k,1)},\alpha_{(k,0)},\tau,Y_0,Y_1)}$, where

$$\mathbb{I}_{(\alpha_{(k,1)},\alpha_{(k,0)},\tau,Y_0,Y_1)} = \begin{cases} 1 & \text{if } \max\{E[U_{(k,1)}], E[U_{(k,0)}]\} = E[U_{(k,1)}] \\ 0 & \text{if } \max\{E[U_{(k,1)}], E[U_{(k,0)}]\} = E[U_{(k,0)}] \end{cases}$$

Now, type $(\alpha_\omega, \tau, Y_0, Y_1)$ maximizes his/her contemporaneous utility by choosing AFP k's product if and only if

$$\begin{aligned} \psi_k &:= (1 - \mathbb{I}_{(\alpha_{(k,1)},\alpha_{(k,0)},\tau,Y_0,Y_1)})p_k \\ &\leq R_k\Big(\alpha_\omega, \tau, \bar{x}, Y_0, z_{j\neq k} | \, \omega \in \Omega \setminus \{(k,1),(k,0)\}\Big). \end{aligned}$$

Correspondingly, the best alternative utility level of type $(\alpha_{(k,1)}, \alpha_{(k,0)}, \tau, Y_0, Y_1)$, which is denoted by $A_k\Big(\alpha_\omega, \tau, \bar{x}, Y_0, z_{j\neq k} | \, \omega \in \Omega \setminus \{(k,1),(k,0)\}\Big)$, is defined as

$$\max\left\{ \max_{j\neq k} E[U(\alpha_{(j,1)}, \tau, x_j, z_j)], \max_{j\neq k} E[U(\alpha_{(j,0)}, x_j, \tau, Y_0)], E[U(\alpha_0, \tau, Y_0)] \right\}.$$

Hence, individual i's reservation price for being affiliated to AFP k is

$$R_k\Big(\alpha_\omega, \tau, \bar{x}, Y_0, z_{j\neq k} | \, \omega \in \Omega \setminus \{(k,1),(k,0)\}\Big) = \begin{cases} y_1 & \text{if } \max\{E[U_{(k,1)}], E[U_{(k,0)}]\} \\ & = E[U(\alpha_{(k,1)}, \tau, x_k, 0.8Y_1)] > A_k; \\ \psi_k & \text{if } \psi_k \text{ solves} \\ & \max\{E[U_{(k,1)}], E[U_{(k,0)}]\} = A_k; \\ -\infty & \text{if } \max\{E[U_{(k,1)}], E[U_{(k,0)}]\} < A_k \\ & \forall\, z \in [0.8Y_1, 0.9Y_1]. \end{cases}$$

This implies that aggregate demand – i.e., the sum of aggregate active and passive demand – is non-increasing in $\bar{p}_j$ and $\hat{p}_j$. The reasoning behind this is the following. Suppose that AFP j raises either $\bar{p}_j$ or $\hat{p}_j$. Now, it might happen that those consumers who no longer maximize utility with $(j,1)$, i.e. actively contributing to AFP j, due to this price increase switch to $(j,0)$, i.e. passively contributing. This switch does not alter aggregate demand.

4.8.B Price Equilibrium Existence Proof

In this appendix, we prove existence of the pricing equilibrium. Our proof proceeds along lines similar to Caplin and Nalebuff (1991b) but is different because their regularity conditions are not fully satisfied in our case. One of the main points of departure from their framework is that we weight the utility of current consumption with the consumer characteristic τ, reflecting the willingness to trade-off current consumption for future retirement wealth. In addition, consumers are heterogeneous with respect to income and preferences are non-linear in current consumption. In Caplin and Nalebuff (1991b), consumer income heterogeneity is analyzed in a setting in which consumer preferences are linear in income.

Our set-up is different for two other main reasons that relate to the fact that the regulations that are in place in the Chilean system make the pension fund market inherently different from the standard product market considered in Caplin and Nalebuff (1991a). First, consumers have the option to be passively affiliated to an AFP and in that case do not pay fees. Second, AFPs charge two-part tariffs.

As will become clear below, the existence of the price equilibrium depends solely on consumer preferences and the joint distribution of consumer preference parameters. Thus, the price equilibrium existence proof is valid for all regulatory settings, i.e. with or without the minimum return requirement, and under the counterfactual regulatory experiments that we perform, which affect the portfolio characteristics of AFP j.

The proof proceeds as follows: First, we proof that the aggregate demand for a firm's product is continuous (Proposition 1) and log-concave in an AFP's price vector (Proposition 2). Second, given the log-concavity of each demand component, we provide the proof of Proposition 3 that shows that an AFP's expected profit function is quasi-concave.

Proof of Proposition 1

It follows from (4.6) that individuals who are indifferent between contributing actively to AFP k and AFP j, $j \neq k$ are defined by

$$\alpha\eta_{k,j} = \phi_{k,j}, \tag{4.25}$$

where

$$\begin{aligned}
\alpha &= \begin{pmatrix} \alpha_{1,i(k,1)} \\ \alpha_{2,i(k,1)} \end{pmatrix}, \\
\eta_{k,j} &= \begin{pmatrix} E[\tilde{R}_k] - E[\tilde{R}_j] \\ E[\tilde{R}_k^2] - E[\tilde{R}_j^2] \end{pmatrix},
\end{aligned}$$

$$\phi_{k,j} = \epsilon_{i(k,1)} - \epsilon_{i(j,1)} + \xi_k - \xi_j + \tau_i\Big(\ln(z_{ik}) - \ln(z_{ij})\Big).$$

By assumption $\eta_{k,j} \neq 0$, so that (4.25) defines a hyperplane in $\mathbb{R}^n$.

In addition, the hyperplane that describes the consumers indifferent between contributing actively to AFP k and the outside options is defined by

$$E[U(\alpha_{i,(k,1)}, \tau_i, x_k, z_{ik})] = \max\left\{\max_j E[U(\alpha_{i(j,0)}, \tau_i, x_j, Y_{i0})], E[U(\alpha_{i0}, \tau_i, Y_{i0})]\right\}. \tag{4.26}$$

It follows from Assumption A1 that the distribution of consumer types is hyperdiffuse. Thus, the sets of indifferent consumers – (4.25) and (4.26) – have zero measure. The continuity of demand for active contribution to AFP k then follows from the continuity of the $\ln(\cdot)$ function.

Similar arguments imply the continuity of aggregate passive demand and therewith, the continuity of aggregate demand. ■

Proof of Proposition 2

The proof proceeds as follows: First, we show that demand functions are log-concave in a market segment in which consumers have identical elasticities of substitution and income, that is for fixed $(\bar{\tau}, \bar{Y}_0, \bar{Y}_1)$. Second, we establish that aggregate demands – which are generated by aggregating the demand functions of all market segments – are log-concave under Assumption A1.

Active Aggregate Demand In the first part of the proof, the strategy is to show the following: Suppose that an individual of type $(\alpha, \bar{\tau}, \bar{Y}_0, \bar{Y}_1)$ prefers choice $(k, 1)$ when the price of AFP k's product is $p_k = (\bar{p}_k, \hat{p}_k)$ and that the same choice is preferred for type $(\alpha', \bar{\tau}, \bar{Y}_0, \bar{Y}_1)$ when the price of k's product is $p'_k = (\bar{p}'_k, \hat{p}'_k)$, with $p'_k > p_k$. Then it follows that $(k, 1)$ is among the most preferred products for type $(\alpha^\lambda, \bar{\tau}, \bar{Y}_0, \bar{Y}_1)$, with $\alpha^\lambda = \lambda\alpha + (1-\lambda)\alpha'$, at price $p^\lambda_k = [\lambda\bar{p}_k + (1-\lambda)\bar{p}'_k, \lambda\hat{p}_k + (1-\lambda)\hat{p}'_k]$.

Thus, in principle we have to check this condition for active demand in three different cases, i.e.

(1) customers prefer $(k, 1)$ to $(h, 1)$, $h \neq k$; prices of other products are fixed at $\mathbf{p}_h$,

(2) customers prefer $(k, 1)$ to $(j, 0)$, for all $j \in \{1, ..., J\}$,

(3) customers prefer $(k, 1)$ to (0).

For the sake of exposition we focus on case (1). Arguments similar to the one developed in case (1) imply that the required condition holds in the cases (2) and (3) as well.

Case (1) Consider the comparison between actively contributing to AFP k and another AFP h, $h \neq k$. Substitution in the expected utility function (4.6) yields the following inequalities:

$$\begin{aligned}\sum_{l=1}^{3} \alpha_l t_l(x_k) + \bar{\tau}\ln(z_{ik}) + t_4(x_k) &\geq \sum_{l=1}^{3} \alpha_l t_l(x_h) + \bar{\tau}\ln(z_{ih}) + t_4(x_h), \\ \sum_{l=1}^{3} \alpha'_l t_l(x_k) + \bar{\tau}\ln(z'_{ik}) + t_4(x_k) &\geq \sum_{l=1}^{3} \alpha'_l t_l(x_h) + \bar{\tau}\ln(z_{ih}) + t_4(x_h),\end{aligned}$$

where $t_1(x_j) = E[\tilde{R}^r_j]$, $t_2(x_j) = E[(\tilde{R}^r_j)^2]$, $t_3(x_j) = 1$, and $t_4(x_j) = \xi_j$.

Combining these inequalities yields

$$\begin{aligned}\sum_{l=1}^{3} \alpha^\lambda_l t_l(x_k) + \lambda\bar{\tau}\ln(z_{ik}) + (1-\lambda)\bar{\tau}\ln(z'_{ik}) + t_4(x_k) &\geq \\ \sum_{l=1}^{3} \alpha^\lambda_l t_l(x_h) + \bar{\tau}\ln(z_{ih}) + t_4(x_h). &\end{aligned} \tag{4.27}$$

It follows from concavity that

$$\bar{\tau}\ln(z^\lambda_{ik}) \geq \lambda\bar{\tau}\ln(z_{ik}) + (1-\lambda)\bar{\tau}\ln(z'_{ik})$$

Thus the portfolio of AFP k at price p^λ_k is among the most preferred products for consumers of type $(\alpha^\lambda, \bar{\tau}, \bar{Y}_0, \bar{Y}_1)$.

When products k and h, $h \neq k$ are distinct, the mass of consumers for which (4.27) is an equality is zero. Hence, the Prékopa-Borell Theorem applies.[24]

If observed and unobserved characteristics of the products k and h are identical, product k must still be the most preferred product at price p^λ_k for type $(\alpha^\lambda, \bar{\tau}, \bar{Y}_0, \bar{Y}_1)$. This is due to the fact that $p^\lambda_k < p'_k$ by definition and that $p_h \geq p'_k$ as otherwise type $(\alpha', \bar{\tau}, \bar{Y}_0, \bar{Y}_1)$ would have strictly preferred AFP h's portfolio. Hence the set of individuals investing at AFP k contains the Minkowski average of consumers choosing $(k, 1)$ at prices p_k and p'_k. Thus, the Prékopa-Borell Theorem applies directly.

Similar arguments imply that cases (2) and (3) are covered as well.

[24] See Caplin and Nalebuff (1991a) for an in-depth treatment of the Prékopa-Borell Theorem.

Aggregate demand Aggregate demand includes customers who prefer to be passively affiliated to an AFP. It follows directly from the preceding analysis that AFP k's product is the most preferred choice for every type that can be represented as a linear combination of types whose most preferred choice is passive affiliation to AFP k and types whose most preferred alternative is either non-affiliation or affiliation to another AFP.

In addition, we have to account for types whose most preferred alternative is active affiliation to AFP k. In order to see that the required condition also holds in this case, suppose that type $(\alpha, \bar{\tau}, \bar{Y}_0, \bar{Y}_1)$ prefers AFP k's product if it is priced at p_k and that type $(\alpha', \bar{\tau}, \bar{Y}_0, \bar{Y}_1)$ also prefers k's product if it is priced at p'_k, with $p'_k > p_k$. Moreover, the former type prefers $(k, 1)$ to $(k, 0)$ and the latter type prefers $(k, 0)$ to $(k, 1)$ at the corresponding prices.

By construction, type $(\alpha', \bar{\tau}, \bar{Y}_0, \bar{Y}_1)$ also prefers $(k, 1)$ at price p'_k to the other alternatives. Thus, affiliation to AFP k is among the most preferred products for type $(\alpha^\lambda, \bar{\tau}, \bar{Y}_0, \bar{Y}_1)$ at price $p^\lambda_k = [\lambda\bar{p}_k + (1-\lambda)\bar{p}'_k, \lambda\hat{p}_k + (1-\lambda)\hat{p}'_k]$.

Hence, the set of individuals who are affiliated to AFP k contains the Minkowski average of the consumers who prefer affiliation to AFP k at prices p_k and p'_k. Thus, the Prékopa-Borell Theorem applies.

The arguments developed above imply that aggregate active demand and aggregate demand in a sub-market characterized by identical $(\bar{\tau}, \bar{Y}_0, \bar{Y}_1)$, which are given by

$$D^a_k(\bar{p}_k, \hat{p}_k|\, \bar{\tau}, \bar{Y}_0, \bar{Y}_1) = \int_{\alpha, \bar{\tau}, \bar{Y}_0, \bar{Y}_1 : R^a_k \geq p_{ik}} f(\alpha_{(k,1)}|\bar{\tau}, \bar{Y}_0, \bar{Y}_1) d\alpha_{(k,1)}, \tag{4.28}$$

and

$$D_k(\bar{p}_k, \hat{p}_k|\, \bar{\tau}, \bar{Y}_0, \bar{Y}_1) = \int_{\alpha, \bar{\tau}, \bar{Y}_0, \bar{Y}_1 : R_k \geq \psi_{ik}} f(\alpha_{(k,1)}, \alpha_{(k,0)}|\bar{\tau}, \bar{Y}_0, \bar{Y}_1) d\alpha_{(k,1)} d\alpha_{(k,0)}, \tag{4.29}$$

are log-concave. This completes the first part of the proof.

In the second part we show that aggregate active and total aggregate demand are log-concave under Assumption A1. Denote by $\mathbb{B}'$ a convex subset of $\mathbb{B}$, then aggregating both $D^a_k(\bar{p}_k, \hat{p}_k|\bar{\tau}, \bar{Y}_0, \bar{Y}_1)$ and $D_k(\bar{p}_k, \hat{p}_k|\bar{\tau}, \bar{Y}_0, \bar{Y}_1)$ over all sub-markets characterized by identical incomes and elasticities yields

$$D_k^a(\bar{p}_k, \hat{p}_k) = \int_{(\tau, Y_0, Y_1)\,\in\,\mathbb{B}'} f(\tau, Y_0, Y_1) D_k^a(\bar{p}_k, \hat{p}_k | \,\bar{\tau}, \bar{Y}_0, \bar{Y}_1)\, d\tau\, dY_0\, dY_1, \tag{4.30}$$

and

$$D_k(\bar{p}_k, \hat{p}_k) = \int_{(\tau, Y_0, Y_1)\,\in\,\mathbb{B}'} f(\tau, Y_0, Y_1) D_k(\bar{p}_k, \hat{p}_k | \,\bar{\tau}, \bar{Y}_0, \bar{Y}_1)\, d\tau\, dY_0\, dY_1. \tag{4.31}$$

Due to A1 it follows from Theorem 6 in Prékopa (1973) that $f(\tau, Y_0, Y_1)$, i.e. the marginal density of consumer incomes and elasticity of substitution, is also log-concave. Thus, each term of the products $f(\tau, Y_0, Y_1)D_k^a(\bar{p}_k, \hat{p}_k)$ and $f(\tau, Y_0, Y_1)D_k(\bar{p}_k, \hat{p}_k)$ is log-concave. From this it follows that (4.30) and (4.31) are log-concave as well. ■

Proof of Proposition 3

We set out with the case in which $\bar{p}_j$, $\hat{p}_j$, and y_{i1} are so that $p_{ij} = \bar{p}_j + \hat{p}_j y_{i1} \leq y_{i1}$, for all types i. Then the expected profit function of AFP j is given by

$$\begin{aligned} E[\Pi_j(\bar{p}_j, \hat{p}_j)] \;&=\; \bar{p}_j\, D_j(\bar{p}_j, \hat{p}_j) + \hat{p}_j\, D_j^{a,y_1}(\bar{p}_j, \hat{p}_j) \\ &\quad -C(D_j(\bar{p}_j, \hat{p}_j), D_j^{\mathrm{w}}(\bar{p}_j, \hat{p}_j)) - C_j^{reg}(\bar{p}_j, \hat{p}_j), \end{aligned}$$

The profit function is monotonically increasing in $p_j = (\bar{p}_j, \hat{p}_j)$ in the region in which $E[\Pi_j(\bar{p}_j, \hat{p}_j)] < 0$; an increase in $\bar{p}_j$ and $\hat{p}_j$ reduces sales and per-unit losses. Hence we can restrict attention to p_j so that $E[\Pi_j(p_j)] \geq 0$. Because we consider exclusively the profit function of AFP j, we will suppress the index j in the remainder of the proof.

Denote by $p' := (\bar{p}', \hat{p}')$ and by $p^\lambda := (\lambda\bar{p} + (1-\lambda)\bar{p}', \lambda\bar{p} + (1-\lambda)\hat{p}')$, with $0 < \lambda < 1$, then a failure of quasi-concavity requires that there exist $\bar{p} < \bar{p}'$ and $\hat{p} < \hat{p}'$ so that

$$\bar{p}\, D(p) + \hat{p}\, D^{a,y_1}(p) - C(D(p), D^{\mathrm{w}}(p)) - C^{reg}(p) \;>$$
$$\bar{p}^\lambda\, D(p^\lambda) + \hat{p}^\lambda\, D^{a,y_1}(p^\lambda) - C(D(p^\lambda), D^{\mathrm{w}}(p^\lambda)) - C^{reg}(p^\lambda), \tag{4.32}$$
$$\bar{p}'\, D(p') + \hat{p}'\, D^{a,y_1}(p') - C(D(p'), D^{\mathrm{w}}(p')) - C^{reg}(p') \;>$$
$$\bar{p}^\lambda\, D(p^\lambda) + \hat{p}^\lambda\, D^{a,y_1}(p^\lambda) - C(D(p^\lambda), D^{\mathrm{w}}(p^\lambda)) - C^{reg}(p^\lambda). \tag{4.33}$$

Now, suppose that $\bar{p}$, $\bar{p}'$, $\hat{p}$ and $\hat{p}'$ are such that

$$\begin{aligned}\hat{p}\, D^{a,y_1}(p) - C(D(p), D^{\mathrm{w}}(p)) - C_j^{reg}(p) &< \\ \hat{p}^{\lambda}\, D^{a,y_1}(p^{\lambda}) - C(D(p^{\lambda}), D^{\mathrm{w}}(p^{\lambda})) - C_j^{reg}(p^{\lambda}), \\ \hat{p}'\, D^{a,y_1}(p') - C(D(p'), D^{\mathrm{w}}(p')) - C_j^{reg}(p') &< \\ \hat{p}^{\lambda}\, D^{a,y_1}(p^{\lambda}) - C(D(p^{\lambda}), D^{\mathrm{w}}(p^{\lambda})) - C_j^{reg}(p^{\lambda}).\end{aligned}$$

Then, it follows from (4.32) and (4.33) that a failure of quasi-concavity requires that

$$\bar{p}\, D(p) \;>\; \bar{p}^{\lambda} D(p^{\lambda}), \tag{4.34}$$

$$\bar{p}'\, D(p)') \;>\; \bar{p}^{\lambda} D(p^{\lambda}). \tag{4.35}$$

Taking logs, weighting the logged inequalities (4.34) and (4.35) with λ and $1-\lambda$, respectively, and adding the left- and right-hand sides yields

$$\lambda \ln \Big(\bar{p}\, D(p)\Big) + (1-\lambda) \ln \Big(\bar{p}' D(p')\Big) > \ln \Big(\bar{p}^{\lambda} D(p^{\lambda})\Big), \tag{4.36}$$

which is contradicted by the log-concavity of $D(p)$.

Next, suppose that $\bar{p}$, $\bar{p}'$, $\hat{p}$ and $\hat{p}'$ are such that

$$\begin{aligned}\hat{p}\, D^{a,y_1}(p) - C(D(p), D^{\mathrm{w}}(p)) - C_j^{reg}(p) &> \\ \hat{p}^{\lambda}\, D^{a,y_1}(p^{\lambda}) - C(D(p^{\lambda}), D^{\mathrm{w}}(p^{\lambda})) - C_j^{reg}(p^{\lambda}), \\ \hat{p}'\, D^{a,y_1}(p') - C(D(p'), D^{\mathrm{w}}(p')) - C_j^{reg}(p') &< \\ \hat{p}^{\lambda}\, D^{a,y_1}(p^{\lambda}) - C(D(p^{\lambda}), D^{\mathrm{w}}(p^{\lambda})) - C_j^{reg}(p^{\lambda}).\end{aligned}$$

Then, it follows from (4.32), (4.33), and the preceding analysis, i.e. (4.34) to (4.36), that a failure of quasi-concavity requires that

$$\bar{p}\, D(p) \;<\; \bar{p}^{\lambda} D(p^{\lambda}), \tag{4.37}$$

$$\bar{p}' D^{a}(p') \;>\; \bar{p}^{\lambda} D^{a}(p^{\lambda}). \tag{4.38}$$

In conjunction, (4.37) and (4.38) imply that there exists a $\lambda \in (0,1)$, denoted by $\hat{\lambda}$ so that

$$\hat{\lambda}\ln\Big(\bar{p}\,D(p)\Big) + (1-\hat{\lambda})\ln\Big(\bar{p}'D(p')\Big) = \ln\Big(\bar{p}^{\hat{\lambda}}D(p^{\hat{\lambda}})\Big),$$

which is contradicted by the log-concavity of $D(p)$.

Finally, suppose that $\bar{p}$, $\bar{p}_f$, $\hat{p}$ and $\hat{p}'$ are such that

$$\begin{aligned} \hat{p}\,D^{a,y_1}(p) - C(D(p), D^{\mathrm{w}}(p)) - C_j^{reg}(p) &> \\ \hat{p}^{\lambda}\,D^{a,y_1}(p^{\lambda}) - C(D(p^{\lambda}), D^{\mathrm{w}}(p^{\lambda})) - C_j^{reg}(p^{\lambda}), & \qquad (4.39) \\ \hat{p}'\,D^{a,y_1}(p') - C(D(p'), D^{\mathrm{w}}(p')) - C_j^{reg}(p') &> \\ \hat{p}^{\lambda}\,D^{a,y_1}(p^{\lambda}) - C(D(p^{\lambda}), D^{\mathrm{w}}(p^{\lambda})) - C_j^{reg}(p^{\lambda}). & \qquad (4.40) \end{aligned}$$

By repeatedly applying the steps developed above, we can show that (4.39) and (4.40) are in conjunction contradicted by the log-concavity of $D^{a,y_1}(p)$, $D(p)$, $D^b(p)$, and $C(D(p), D^{\mathrm{w}}(p))$. We omit this for the sake of brevity.

Conducting the same analysis for each demand and cost component of an AFP's profit function yields the desired result. Obviously, the result also holds in the absence of the minimum return requirement, i.e. $C_j^{reg}(p) = 0$ and $\tilde{R}_{jt}^{r} = \tilde{R}_{jt}$.

Similar arguments imply that an AFP's expected profit function is also quasi-concave if $p_{ij} = y_{i1} \leq \bar{p}_j + \hat{p}_j y_{i1}$, for some types i. ■

4.8.C Procedure for imputing potential earnings in the sector that is not observed

In the demand model, individuals choose whether or not to contribute to their pension account by working in the formal sector where contributions are mandatory. Their decision is based on potential earnings in the two sectors, on fund fees, and on their expectations about AFP firms' returns. Our model assumed that individuals know their potential earnings in each sector, with the earnings in the two sectors denoted by (Y_0, Y_1). In the data, however, we only observe earnings in the sector in which they are actually working. Thus, to implement our demand model we impute the earnings that are not unobserved. For this purpose, we estimate a Roy (1951) type of model, with log wage equations for the two sectors and a selection equation representing the decision to work that period in the formal sector that is used to control for sector selectivity. Our sample correction procedure is the standard Heckman (1974) two-step procedure, which assumes that the error terms in the sector choice equation and in the outcome equations are jointly normally distributed.

In the first stage, we estimate a probit model for the decision to work in the formal sector. This decision would be expected to depend on factors that determine own earnings capacities as well as on spousal characteristics. We assume that that decision to work in the formal sector depends on the following variables: experience, experience squared, years of education, whether the individual has incomplete high school, marital status (married or not), spouse's years of education, spouse's age, an indicator for whether spouse's education is missing, an indicator for whether spouse's age is missing, an indicator for whether own education is missing, and an interaction between own education and own experience.[25] In the second stage, we estimate a model for log annual earnings, using 2004 wages data, and including the mill's ratio term to correct for sample selectivity. Log wages are assumed to be linear in years of education, experience, experience squared, an indicator for married, an indicator for whether high school is incomplete, an indicator for whether years of education is missing, an interaction term between experience and education, and the mills ratio term (that is a function of the probability of working in the formal sector). The estimated coefficients are then used to predict earnings for the sector that is not observed in the data, including the selectivity correction terms.[26]

[25]The experience is potential experience, which equals age-years-5.

[26] The income equations in both sectors only use data on workers and we do not attempt to model the decision about whether to work. Most men work, but roughly two thirds of women do not work. In not incorporating nonworkers into our analysis, we are implicitly assuming that the women who choose to work in the formal sector would also be working in the informal sector and vice versa and that the decision about whether to work at all is unaffected by changes in pension fund fees (which seems likely).

Chapter 5

Forward Trading and Collusion in Oligopoly: Comment[§]

Liski and Montero (2006) add to the debate whether forward trading has anti- or pro-competitive effects by putting this question in the context of an infinitely repeated oligopoly with perfect information. In order to introduce forward trading, they give the following structure to the constituent game: In the first stage firms trade forward contracts with competitive, risk-neutral speculators. In the second stage firms compete in the spot market.

The authors prove that access to forward markets facilitates collusion relative to the infinitely repeated pure-spot duopoly if firms compete in prices in the spot market. They state in Proposition 3 that this result also holds if firms set quantities. This claim rests on their finding that firms will – irrespective of the cartel's forward market position – never find it profitable to deviate in the forward market.

Yet, the result that it is never profitable for firms to deviate in the forward market is not correct if the cartel takes a long position. Liski and Montero (2006) explicitly allow for this. In this chapter the corrected Proposition for the case of quantity competition in the spot market is derived. Moreover, it proves that the presence of forward markets reduces the firms' ability to collude relative to the pure-spot infinitely repeated Cournot duopoly if the cartel's forward position is sufficiently long.

The game structure is given by the following assumptions: Time is infinite and discrete and the per time-unit discount factor is denoted by δ, with $0 < \delta < 1$. Each time-unit consists of two periods. Each time-unit's first period corresponds to a forward market and

[§]This work presented in this chapter is the predecessor of Ressner et al. (2010).

its second period corresponds to a spot market. Thus, forward markets open in the even periods ($t = 0, 2, ...$), spot markets open in the odd periods ($t = 1, 3, ...$), and each spot market is preceded by at least one forward market.

There are two symmetric firms, denoted by 1 and 2, that produce a homogenous good at constant marginal cost c. In a forward market firms simultaneously buy or sell forward contracts – possibly for all succeeding spot markets – that call for the physical delivery of the good in the future spot market. Let $f_i^{t,t+k}$, with $t = 0, 2, ...$ and $k = 1, 3, ...$, denote the forward position that firm i acquires in period t which calls for delivery in period $t + k$. If $f_i^{t,t+k} > 0$, firm i sells forwards (that is, it takes a short position), and if $f_i^{t,t+k} < 0$, firm i buys forward contracts (that is, it takes a long position). Given their forward market obligations firms serve the spot market in period $t + k$ by simultaneously choosing the quantities q_1^{t+k} and q_2^{t+k} for production that cover their spot market sales and forward positions for this period. The spot price in period $t + k$, which is denoted by p_s^{t+k}, is given by the inverse demand function $p_s^{t+k} = a - (q_1^{t+k} + q_2^{t+k})$.

In order to explore the effect that forward trading has on the firms' ability to sustain collusion, Liski and Montero (2006) consider the following (symmetric) strategies in which firms are partially or fully contracted only one period ahead: In period 0, firm i trades $f_i^{0,1} = xq^m/2$ and $f_i^{0,k+2} = 0$, where $x \in [-1, 1]$ and $q^m = (a - c)/2$. Depending on whether t corresponds to a spot or forward opening, firm i operates as follows: if t is an odd period (that is a spot opening), firm i sets $q_i^t = (1 - x)q^m/2$, if before t both firms have chosen $(1 - x)q^m/2$ in the odd periods and have forward contracted $xq^m/2$ one period ahead in the even periods; otherwise firms play according to the Allaz and Vila (1993) equilibrium thereafter. If t is an even period (that is a forward opening), firm i trades $f_i^{t,t+1} = xq^m/2$ and $f_i^{t,t+k+2} = 0$, if before t both firms have chosen $(1 - x)q^m/2$ in the odd periods and have forward contracted $xq^m/2$ one period ahead in the even periods; otherwise firms play according to the Allaz and Vila (1993) equilibrium thereafter.

The main implication of Proposition 3 (Liski and Montero, 2006, p. 222) is that access to forward markets facilitates collusion if quantity-setting firms follow the aforementioned strategies. More specifically, the authors claim that irrespective of the cartel's forward market position, that is for every $x \in [-1, 1]$, firms will only deviate in the spot market and that the critical discount factor that solves the "no deviation in the spot market condition"

(Liski and Montero, 2006, equation (9), p. 222) never exceeds 9/17, which is the critical discount factor of the infinitely repeated pure-spot Cournot duopoly.

Yet, this Proposition rests on the authors' claim that firms will never find it profitable to deviate in the forward market. This claim is based on the observation that the profit a firm realizes in the time-unit of deviation,

$$\frac{(a-c-x\frac{q^m}{2})^2}{8},$$

never exceeds the collusive profit that it could have earned in this time-unit,

$$\frac{(a-c)^2}{8}.$$

Obviously, this statement is not correct if firms take long positions while colluding, that is if $x \in [-1, 0)$. Therefore, Proposition 3 should be revised as follows.

Revised Proposition 3 *For $x \in [-1, 1]$, the above strategies constitute a subgame perfect equilibrium if $\delta \geq \hat{\delta}(x) := \max\{\delta_s(x), \delta_f(x)\}$, where $\delta_s(x)$ is implicitly defined by the "no deviation in the spot market condition"*

$$\frac{1-x+x\delta_s(x)}{8(1-\delta_s(x))} = \frac{(3-x)^2}{64} + \sum_{N=1}^{\infty} \frac{(1+N)(\delta_s(x))^N}{(3+2N)^2}, \tag{5.1}$$

and $\delta_f(x)$ is implicitly defined by the "no deviation in the forward market condition"

$$\frac{1}{2(1-\delta_f(x))} = \frac{(4-x)^2}{32} - \frac{\delta_f(x)+\ln(1-\delta_f(x))}{(\delta_f(x))^2} - \frac{1}{2}. \tag{5.2}$$

$\delta_s(x)$ is strictly increasing for all $x \in [-1, 1]$, with $\delta_s(-1) = 0$, $\delta_s(0) = 0.238$, and $\delta_s(1) = 0.512 < 9/17$. $\delta_f(x)$ is strictly decreasing for all $x \in [-1, 0]$, with $\delta_f(-1) = 0.545 > 9/17$ and $\delta_f(0) = 0$. If $x \in (0, 1]$, there exists no $\delta \in [0, 1)$ that solves (5.2). Thus, there is a unique x, denoted by $\tilde{x}$, where $\tilde{x} \approx -0.161$, so that $\hat{\delta}(x) = \delta_f(x)$ if $x \leq \tilde{x}$ and $\hat{\delta}(x) = \delta_s(x)$ if $x \geq \tilde{x}$.

Finally,

$$\hat{\delta}(x) \gtreqless \frac{9}{17} \text{ if } x \lesseqgtr \bar{x}, \text{where } \bar{x} \approx -0.932.$$

Proof The derivation of (5.1) can be found in Liski and Montero (2006, p. 222 f.). In addition, the authors show (Liski and Montero, 2006, p. 227 f.) that the forward market deviation profit is given by

$$\frac{(a-c)^2}{4}\left(\frac{(4-x)^2}{32}\right),$$

and that the deviant's per time-unit punishment profit is

$$\frac{(a-c)^2}{4(2+N)},$$

where $N \geq 1$. Since

$$\sum_{N=1}^{\infty} \frac{\delta^N (a-c)^2}{4(2+N)} = -\frac{(a-c)^2}{4}\left(\frac{\delta + \ln(1-\delta)}{\delta^2} + \frac{1}{2}\right),$$

it follows that the "no deviation in the forward market condition" is given by (5.2). Clearly, for a firm it is neither profitable to deviate in the forward nor in the spot market if for given $x \in [-1, 1]$, δ is at least as large as the maximum of $\delta_f(x)$ and $\delta_s(x)$, which is denoted by $\hat{\delta}(x)$.

Exact values of $\delta_s(x)$ can only be obtained numerically. Liski and Montero (2006, p. 223 f.) already show that $\delta_s(x)$ is strictly increasing for all $x \in [-1, 1]$, with $\delta_s(-1) = 0$, $\delta_s(0) = 0.238$, and $\delta_s(1) = 0.512 < 9/17$.

Now, it is shown that $\delta_f(x)$ is strictly decreasing for $x \in [-1, 0)$. Using the implicit function theorem yields that,

$$\frac{d\delta_f(x)}{dx} = \frac{(4-x)(1-\delta_f(x))^2 \delta_f(x)^3}{8(4\ln(1-\delta_f(x))(1-\delta_f(x))^2 + \delta_f(x)((2-\delta_f(x))^2 - 2\delta_f(x)))} < 0,$$

since the denominator is negative for $\delta \in (0, 1)$. Moreover, $\delta_f(-1) = 0.545$ and $\delta_f(0) = 0$. From the discussion preceding the revised Proposition it is evident that there exists no $\delta \in [0, 1)$ that solves the "no deviation in the forward market condition" if $x \in (0, 1]$.

The preceding arguments imply that there exists a unique x, denoted by $\tilde{x}$, at which $\delta_f(x)$ and $\delta_s(x)$ intersect. It can be shown that $\tilde{x} \approx -0.1618$ and $\delta_f(x) > (<)\ \delta_s(x)$ if $x < (>)\ \tilde{x}$. As mentioned before, $\hat{\delta}(-1) = 0.544 > 9/17$. Hence, if the cartel takes extremely long positions, collusion will be harder to sustain as compared to the infinitely repeated pure-spot Cournot

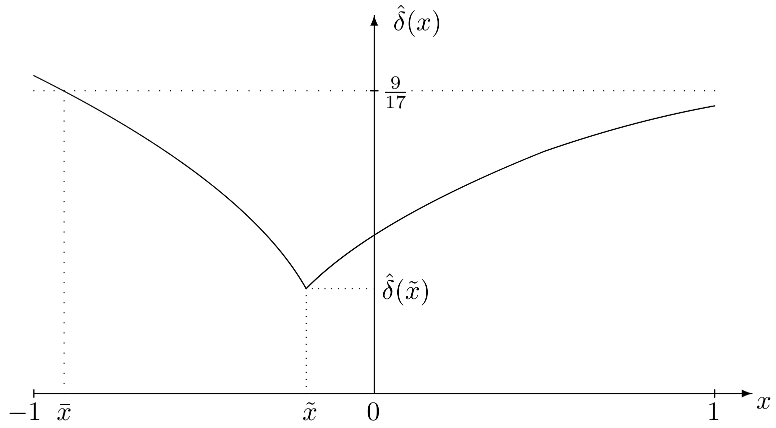

Figure 5.1: Critical discount factor

duopoly. By continuity, this also holds in a neighborhood of $x = -1$. Numerically, it can be shown that $\hat{\delta}(x) \geq 9/17$ if $x \leq \bar{x}$, where $\bar{x} \approx -0.932$. ■

The critical discount factor is depicted in Figure 5.1. As can be seen from (5.2) a forward market deviation becomes more profitable as the cartel's one period ahead long position increases. The intuition for this result is the following. If firm i deviates in the forward market t, it does so by selling forwards for delivery for all subsequent spot markets. The reason for this is that the firms' strategy variables are strategic substitutes. Thus, by building up a short position for all future spot markets, firm i credibly commits to higher future production. Therewith it forces firm j to accommodate by contracting its spot market sales.

In addition, there is an effect which is peculiar for the time-unit of deviation. More specifically, the deviating firm – firm i – rationally anticipates that firm j will behave, ceteris paribus, less competitively in the spot market that opens right after the deviation period. This is due to the fact that firm j inherits the cartel's long position for this particular spot market. Thus, it has a strong incentive to stabilize the spot price in period $t + 1$, as p_s^{t+1} is the price at which those who sold firm j's production short in t, have to close their positions. Firm j's incentive to stabilize is the larger, the higher its leverage, that is the longer the cartel's forward market position. This "leverage-effect" amplifies the deviating firm's incentive to act more aggressively in the spot market $t + 1$. As the cartel is merely contracted one period ahead, the leverage effect is not present for future spot markets.

Formally, the above arguments translate into the result that the cartel's forward position impacts only the deviating firm's profit in the time-unit of deviation. Moreover, for $x = 0$ the deviation profit corresponds to the profit of a Stackelberg-leader and increases in the cartel's long position, that is it increases as x decreases. The "leverage-effect" is so pronounced that for a sufficiently long position of the cartel, i.e. $x \leq \tilde{x} \approx -0.161$ it is more tempting for a firm to deviate in a forward than in a spot market. Moreover, for $x \leq \bar{x} \approx -0.932$, the profit from a forward market deviation is so that in the Liski and Montero (2006) framework tacit collusion is harder to sustain relative to the infinitely repeated pure-spot Cournot duopoly.

In the following it is discussed how the revised result adds to the debate whether forward trading has anti- or pro-competitive effects.

On the one hand, the main result of Liski and Montero (2006) remains qualitatively unaffected for a wide range of possible forward positions that a cartel can choose, that is for $x > -0.932$. Moreover, the lowest discount factor for which collusion can be sustained as the subgame-perfect equilibrium in the infinite-horizon setting is $\hat{\delta}(\tilde{x}) \approx 0.184$, which is far below the critical discount factor of the infinitely repeated pure-spot Cournot duopoly. Thus, if firms that have access to forward markets are pessimistic about the cartel's stability, their first choice would be to take a moderate long position. In this respect, the revised result is consistent with empirical evidence pointed out by Mahenc and Salanié (2004) that in the history of alleged manipulation of commodity markets unreasonably high prices often resulted from moderate long positions held by a cartel of producers.

On the other hand, the main qualification to the argument put forward by Liski and Montero (2006) is that access to forward markets can have a detrimental effect on cartel stability. This is the case if quantity-setting firms form a cartel that has to take – due to regulatory restrictions – a sufficiently long position in the forward market.

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Ich versichere hiermit eidesstattlich, dass ich die vorliegende Arbeit selbständig und ohne fremde Hilfe verfasst habe. Die aus fremden Quellen direkt oder indirekt übernommenen Gedanken sowie mir gegebene Anregungen sind als solche kenntlich gemacht.

Die Arbeit wurde bisher keiner anderen Prüfungsbehörde vorgelegt und auch noch nicht veröffentlicht.

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