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Applications of the D-Instanton Calculus in Type IIB Orientifold Compactifications

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Zusammenfassung

In dieser Dissertation werden Stringkompaktifizierungen im Formalismus der Typ IIB Stringtheorie mit großem Volumen (engl. "LARGE volume models") untersucht. Diese Klasse von Kompaktifizierungen besitzt eine in vielerlei Hinsicht phänomenologisch interessante effektive Niederenergiefeldtheorie. Thema dieser Arbeit ist die Weiterentwicklung dieser Modelle motiviert durch neuere Erkenntnisse im D-Branen-Instantonkalkül der Stringtheorie.

Nach einer kurzen, allgemeinen Einführung in die Stringtheorie und insbesondere in Typ IIB Orbifolds und deren Konsistenzbedingungen in Kapitel 1 und 2 werden in Kapitel 3 die Modelle mit großem Volumen ausführlich vorgestellt und die bisherigen Erkenntnisse zu deren Phänomenologie – wie Skalenhierarchien, Eichkopplungen, Supersymmetriebrechung und kosmologische Fragestellungen – besprochen.

Ein wesentlicher Bestandteil in der Konstruktion der Modelle mit großem Volumen ist das Stabilisieren von Modulifeldern mit Hilfe von nicht-perturbativen Beiträgen zum Superpotential in der effektiven Niederenergiefeldtheorie, die von D-Branen-Instantonen oder Gauginokondensaten hervorgerufen werden. Mit neueren Erkenntnissen im D-Branen-Instantonkalkül wird in Kapitel 4 gezeigt, dass die Modulistabilisierung mit dem bisher angewendeten Mechanismus nicht mit der Existenz von chiralen Fermionen, wie sie im Standardmodell der Elementarteilchenphysik vorkommen, verträglich ist. Es wird ein modifizierter Mechanismus vorgeschlagen, bei dem die Modulifelder durch Hinzunahme von D-Termen stabilisiert werden.

In Kapitel 5 wird durch sog. „Polyinstantonkorrekturen" zur eichkinetischen Funktion ein neues Szenario mit großem Volumen konstruiert, bei dem die Stringskala ohne Feinabstimmung nicht in einem wie in diesen Modellen üblichen intermediären Bereich von etwa 10^{11} GeV liegt, sondern bei 10^{16} GeV. Somit wird diese Konstruktion auch für große vereinheitlichte Theorien (GUT-Theorien) mit $SU(5)$- oder $SO(10)$-Eichgruppen interessant. Dies wird an expliziten Modellen vorgeführt.

Zuletzt wird in Kapitel 6 Supersymmetriebrechung in Szenarien mit großem Volumen behandelt. Durch den neuen Mechanismus zur Modulistabilisierung wird nahegelegt, dass die Supersymmetriebrechung durch ein von den MSSM-Branen völlig isoliertes D-Branen-Instanton hervorgerufen wird. Die Beiträge von unterschiedlichen Mediationsmechanismen zu den Softtermen des MSSM werden detailliert berechnet und verglichen.

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1. Introduction

One might call the twentieth century the century of physics. Never before in the history of science, the understanding of the fundamental laws of nature grew faster than in the last decades, and never before such an effort was made to drive our knowledge even further.

The reductionistic approach to describe natural phenomena was tremendously successful: in a multitude of collider experiments, only a handful of basic building blocks of matter could be identified, organized in three families of elementary particles, and there are only four fundamental forces governing their interactions: gravity, electromagnetism, the strong and the weak force. Their dynamics are described by two theories which constitute the two pillars of today's theoretical physics: Einstein's general theory of relativity, published in 1915, which is a classical theory and the standard model of particle physics, formulated in its final form in the seventies by Glashow, Weinberg and Salam, described within the framework of quantum field theory. Together they provide a very detailed and accurate description of nature, including the processes taking place a split second after the Big Bang, the structure formation in the universe up to the complicated collision events in the detectors of high-energy experiments.

The concept of unification of existing theories merging in a more fundamental one was proven to be a fruitful concept to gain new insight into physical processes: Isaac Newton realized in his famous work "Philosophiæ Naturalis Principia Mathematica", published in 1687, that the motion of celestial bodies and of objects on Earth can be described by the same physical laws. As a consequence, a unified theory of motion and gravitation could be derived. In 1873, James Clerk Maxwell merged the at that time completely disconnected theories of electric and magnetic phenomena into a new theory of electromagnetism. With the help of its equations, Maxwell conjectured the existence of electromagnetic waves.

In the context of modern particle physics, the term "unification" is used in a more special sense: quantum field theories (the theoretical framework, the standard model of particle physics is formulated in) rely on so-called gauge symmetry groups. At lower energies, the gauge group of the theory which matches experiments is the product group $SU(3)_{\mathrm{c}} \times U(1)_{\mathrm{e.\,m.}}$. Unification means here that this group structure can be understood as being the result of a symmetry breaking of a larger group. The weak interaction can be unified with the electromagnetic force into the electroweak interaction. The resulting theory has the gauge group $SU(3)_{\mathrm{c}} \times SU(2)_{\mathrm{L}} \times U(1)_Y$. Symmetry breaking is believed to be triggered by a till today hypothetical field — the Higgs field. The detection of the corresponding particle is perhaps the most urgent task of experimental high-energy physics and hoped to be achieved within the next couple of years with help of the Large Hadron Collider (LHC) at CERN in Geneva.

It is obvious to ask the question if also the two remaining fundamental interactions, i. e. the strong force and gravity, could be unified in this way with the other two. Concerning the strong interaction, this can be achieved quite easily. The resulting class of theories go by the name of "grand unified theories" (GUTs). The most popular gauge groups are $SU(5)$ or $SO(10)$ [1, 2]. As symmetry breaking presumably would occur at energies as high as 10^{16} GeV, direct experimental evidence for grand unified theories will not be available in the near future.

For the case of gravity, the situation is more difficult. Being a classical theory, incorporating it into the framework of the standard model would require to reformulate it as a quantum field theory. It turns out that scattering amplitudes involving the graviton contain infinities. As usual in quantum field theory, these can be absorbed by redefining some parameters of the theory. However, in each higher order of the perturbation series, more and more infinities appear requiring more and more parameters — the theory is not renormalizable and thus not a meaningful quantum field theory.

There is a remarkably wide hierarchy in the relative strength of the four fundamental interactions. Gravity is by far the weakest of all. Compared to the next strongest one — the weak interaction — it has a relative strength of 10^{-25}. On the other hand, there is also discrepancy in their long-distance behavior: the strong and the weak interaction have only a range comparable to the size of a nucleus. Electromagnetism is in principle infinitely long ranged, but charged matter organizes itself such that on large scales, the charges cancel and there is no net electric charge. Thus, the only relevant force on large, i. e. cosmological scales, is gravity.

Given these facts, it appears that there is a well defined segregation in the scope of application between the classical general theory of relativity and the three remaining interactions, described by a quantum theory. The former describes the large scale dynamics of space as a whole where the others play no rôle because they are only short ranged, whereas the latter apply to microscopic processes only, where gravity can be neglected anyway due to its weakness.

In view of the fact that these two theory frameworks complement one another in such a successful way and on the other hand seem to be mathematically incompatible, one might wonder if the dream of a single unified theory of all elementary physical processes, sometimes exaggeratedly also called a "theory of everything", is only a wishful thinking and nature is just such that two coexisting theories are necessary to describe it.

However, there are a couple of hints which point to the existence of such a underlying, more fundamental theory of quantum gravity and the other interactions. First of all, in principle there is no upper bound in the energy to which particles can be accelerated to in colliders (or also by natural processes taking place somewhere in the universe). At an energy of roughly 10^{19} GeV, the Planck scale, the Compton wavelength of a particle becomes comparable to its Schwarzschild radius and classically should form a black hole. Clearly, a theory which describes gravity in a quantum mechanical manner is necessary in this situation. Similar high energies prevailed in the very early universe in the first few Planck times after the Big Bang.

Moreover, there exists the common belief among the vast majority of high energy physicists that the standard model, besides the lack of a gravitational sector, is an incomplete theory also already well below the Planck scale. This is on the one hand due to the fact that a couple of parameters must be "tuned" throughout a huge number of digits in order to explain experimentally measured data. Such a fine-tuning is usually regarded as being "unnatural" and rather considered as an artifact of the lack of an underlying theory. On the other hand, there exist also a couple of physical phenomena which cannot be described by the standard model in a satisfactory way at all.

1.1. Problems of the Standard Model

The most pressing problem of the standard model is related to the mass of the Higgs particle. Experiments and theoretical considerations bound its value between roughly 115 GeV and 180 GeV. It turns out that this mass parameter receives loop corrections of the order of the ultraviolet cutoff scale. Assuming that the standard model is valid up to very high energies, this scale could be the (reduced) Planck scale $m_{\mathrm{P}} \approx 10^{18}$ GeV. In order to get a physical value for the mass within the experimental bounds, the bare mass parameter must be fine-tuned up to with no less than 30 orders of magnitude. As the need for this high amount of fine-tuning stems ultimately from the large hierarchy between the weak scale and the Planck scale, this puzzle goes by the name "hierarchy problem". Clearly, an extension of the standard model which resolves this issue in a more natural way would be preferable.

A similar situation can be found in QCD. Unlike in the electroweak sector, CP-symmetry is unbroken to very high precision. Also here, this turns out to be an example of fine-tuning, if no further mechanisms beyond the standard model are at work. A CP-violating term in the QCD Lagrangian is characterized by an angular parameter θ. As such, its natural value is expected to be in the order of one. This CP-odd term would cause an electric dipole moment of the neutron, which is however experimentally found to be compatible with zero. In turn, the upper bound which can be put on the angular parameter θ with help of this data is as small as $\theta < 10^{-9}$. For this so-called "strong CP-problem" a solution was proposed in the seventies by Peccei and Quinn: by the introduction of a new scalar particle, the "axion", the parameter θ is set to zero dynamically. Interestingly, particles showing up these properties appear naturally in string theories. Though intensive searches, there is no experimental evidence for axions yet.

The most distinct example for fine-tuning in physics shows up when coupling the standard model Lagrangian to gravity in a canonical way. It turns out that vacuum fluctuations due to virtual particles create an effective cosmological constant term in the order of m_{P}^4. The actual value was measured in 1998 by observing the redshifts of type Ia supernovæ [3]. Indeed, the universe undergoes a phase of accelerated expansion at the moment. Quantitatively this can be described by an effective cosmological constant of approximately $10^{-120}\, m_{\mathrm{P}}^4$, a value which is 120 orders of magnitude smaller than expected by the theory!

It should be noted that here, in contrast to the first two examples of fine-tuning, the cosmological constant problem is not a fine-tuning problem of the standard model by itself as the latter does not contain any gravity related terms. Therefore, the large numerical discrepancy between the theoretical and the experimental value of the cosmological constant could be interpreted as a hint for a missing theory of quantum gravity.

Still, the origin of the accelerated expansion of the universe is completely unknown. Classically in can be modeled by a perfect fluid with negative pressure. As such it constitutes roughly three-quarter of the total energy density of the universe. Due to its mysterious nature, this phenomenon has been given the name "dark energy".

About 22 % of the remaining energy density of the universe consists of an unknown form of matter, called "dark matter". This is well supported by observations of the galactic rotation curves which suggest that about half of the mass of galaxies is concentrated in the dark halo. Such a mass distribution cannot be explained with ordinary baryonic matter which tends to clump together and ultimately to form stars. Also theories of structure formation in the early universe as well as further independent observational data point to a similar amount of dark matter in space. This form of matter must be composed of very weakly interacting but massive particles (WIMPs). The standard model does not contain a particle with these properties.

In summary it can be said that the bigger part of the energy density of the universe cannot be described by the standard model of particle physics. In addition, it suffers from severe fine-tuning problems and last but not least, with its nearly 20 free parameters — some with hierarchically different numerical values — possesses a high degree of arbitrariness.

Evidently, a more fundamental theory of particle physics and of gravity is likely to exist.

1.2. Supersymmetry

During the last decades there have been various proposals for extensions of the standard model which solve more or less of the problems mentioned in the last section. The most prominent extension and the one which is considered to be the most likeliest to be proven soon at the LHC is "supersymmetry".[1]

The idea is to extend the gauge symmetry algebra of the standard model by a set of anti-

1 In fact, supersymmetry is an export hit of string theory.

commuting generators with the consequence that the particle content of the theory is doubled: every elementary particle comes with a "superpartner" which is equally charged but differing in half of a unit in spin.

Though it was not the original motivation, the most convincing strength of supersymmetry is the fact that it solves the hierarchy problem of the standard model in an elegant way: for every divergent loop diagram entering the Higgs mass, in supersymmetric models there is a corresponding diagram involving the respective superpartner. Due to the reversed spin-statistics, this diagram contributes with the opposite sign compared to the original one and thus cancels the unwanted quantum correction to the Higgs mass of the standard model.

Supersymmetry implies that the superpartners and the known particles of the standard model have the same mass. If supersymmetry were an exact symmetry of nature, the superpartners would have been detected a long time ago already. Thus, if it ought to exist, supersymmetry must be broken spontaneously. This has however two consequences which are quite welcome: firstly, the lightest supersymmetric particle (LSP) may be stable and can serve as the so far unknown constituent of dark matter. Secondly, the running of gauge couplings is altered above the superpartner's masses in such a way that they meet closer at 10^{16} GeV than in non-supersymmetric theories. One can say that gauge unification and supersymmetry go well together.

What appeals about supersymmetry at first glance is embittered though when going into detail: very general sum-rules can be derived from the supersymmetry algebra which constrain the mass of the superpartners to a range where they are excluded experimentally already. The loophole is to introduce a so-called "hidden sector" of particles which do not share any gauge interactions with the standard model ones. Supersymmetry is broken there and "communicated" by a mediation mechanism to the standard model sector. It is not hard to imagine that this reintroduces a high degree of arbitrariness in all supersymmetric models.

1.3. String Theory

Among the candidates for a quantum theory of gravity, the most promising and the most highly developed is in all likelihood string theory [4–11]. In contrast to all usual quantum theories, its basic entity is not point-like but extended in one dimension, like a string.

Originally it was applied in the sixties for the description of mesons. Being a bound state of a quark and an anti-quark, the strong force acts like a little spring between the two constituents, giving rise to a one-dimensionally extended object. With string theory it was possible to reproduce some of the meson's behavior, in particular the characteristic linear relation between energy and spin known as Regge trajectories.

However, with the advent of quantum chromodynamics (QCD) in the mid-seventies, string theory was disfavored as theory of the strong interaction. At the same time it was realized that the properties of a closed string match with those of a graviton. This suggests that string theory should be rather interpreted as a theory of quantum gravity.

The first breakthrough occurred in 1984 when Green and Schwarz showed that type I string theory is anomaly free and therefore suitable as a quantum theory. Only one year later, heterotic string theory was discovered. Exhibiting gauge symmetry with gauge group $E_8 \times E_8$ or $SO(32)$ and containing chiral fermions, it became evident that string theory is not only a candidate for a theory of quantum gravity but for a unified theory of all interactions.

A lot of remarkable achievements have been made since then which support string theory. The most celebrated is perhaps the derivation of the Bekenstein–Hawking entropy formula of black holes $S_{\rm BH} = A/4$. In classical general relativity, the "no-hair theorem" states that a black hole in fact does not have any microstates. Hence the concept of entropy does not apply. In string theory — in contrast to usual quantum field theories — as being a quantum theory of gravity, black holes can be described in terms of dynamical microscopical objects and hence it should be possible to regain the Bekenstein–Hawking entropy. Indeed this was achieved for the first time in 1996 by Strominger and Vafa for a special class of extremal five-dimensional black holes [12].

Another interesting development of string theory started in 1998 when Maldacena proposed a duality of string theory and certain lower dimensional quantum field theories [13]. By "duality" usually an equivalence of two theories is understood where a common expansion parameter is inverted. For instance, string theory in an negatively curved space–time ($AdS_5 \times S^5$) with string coupling g is dual to a superconformal Yang-Mills theory in four dimensions with coupling $1/g$. This is of special appeal since the equivalence allows to study the properties of a strongly coupled field theory, which is usually not under computational control, with the help of a weakly coupled string theory which is controllable. Applications of this remarkable conjecture are strongly coupled phenomena in QCD such as the quark–gluon plasma but also even in condensed matter physics such as superconductivity. The AdS/CFT correspondence has become an important branch of research by itself.

1.4. The Vacuum Problem of String Theory

Quantization of string theory is only consistent in 26 or 10 space–time dimensions in the purely bosonic or the supersymmetric formulation respectively. For obvious reasons, this is phenomenologically not acceptable. The loophole is not to assume a flat and infinitely extended space–time but to "compactify" the extra dimensions to small sizes. Surprisingly, under certain circumstances, the effective four-dimensional theory is still in agreement with experiments if the typical radius of the extra dimensions is as large as the tenth part of a millimeter [14]. Still, the presence of extra dimensions has far-reaching consequences.

A very interesting fact is that the Planck scale in the four-dimensional effective theory is given by the actual, ten-dimensional Planck scale multiplied by the volume of the compactification space. Given that the latter is rather large, it is quite possible that the fundamental scale of quantum gravity is much lower than expected from purely four-dimensional theories. An interesting scenario using this fact was proposed in [15, 16]: the question why the weak scale is so many orders of magnitude smaller than the Planck scale and the resulting hierarchy problem can be nullified by assuming large extra dimensions such that the TeV scale is also the fundamental Planck scale. This has also various other appealing phenomenological implications so that the concept of large extra dimensions has become popular also among non-string-theorists. If this scenario were true, we would have the exciting prospect to probe string theory in a direct way at the LHC already. A couple of experimental signals of this scenario have been worked out in [17, 18].

Size and shape of the compactification space determine various parameters of the four-dimensional effective theory such as the particle spectrum and couplings. Though subject to a multitude of consistency conditions, the geometry of the compactification space is probably not unique. On the contrary: its size and shape are described by dynamical fields in a hilly potential, the so-called "landscape". It is believed that there exists a incredibly large, but *finite* number of minima in this potential for the geometrical fields, giving rise to an equal number of four-dimensional effective field theories. A commonly quoted estimate is in the order of 10^{500} [19].[2]

Given the enormous number of vacua, it is likely that there exists not only one four-dimensional effective theory which resembles the standard model of particle physics but a large set thereof, possibly all with different behavior at higher energies.[3] Critics decry this as complete loss of predictivity of string theory, disqualifying it as scientific theory. However, at the fundamental string scale at the latest, specific and model-independent effects will show up which cannot be explained by a usual quantum field theory. On the other hand, the high number of minima in the landscape may be an explanation for the arbitrariness of the parameters of the standard model: in this picture, they do not have a deeper meaning but are just "environmental" values. It might be possible to gain new insights in this regard by considering statistical methods [22]. Fine-tuning of parameters can be explained by invoking the weak anthropic principle. A prerequisite for this to make sense is to have a large enough ensemble (here vacua) where the parameter in question is scanned. This line of argument was firstly proposed in 1987 by Weinberg for the cosmological constant [23]. The concept of the landscape bears the correct set-up to implement this idea [24].[4]

2 In an earlier work [20] even the number of 10^{1500} was proposed.

3 The abundance of standard model like vacua within the landscape was estimated in [21] and found to be less than one in a billion in a toy set-up.

4 The anthropic principle is disfavored by many physicists — though unjustified. Applied in this precise sense, the AP makes sense very well. The same argumentation can be applied when asking for the reason why the distance between Earth and Sun is fine-tuned in the way it is.

1.5. Approaches to Model Building

At this stage, there exists only a perturbative formulation of string theory. It is therefore impossible to tell whether upon the inclusion of all non-perturbative effects, the vacuum problem will persist or not. Though this is certainly an interesting and important question, the most obvious hurdle string theory has to clear is to prove that the standard model of particle physics (or its supersymmetric extensions) is contained in the landscape at all.

The phenomenological properties of a quantum field theory can be tuned in a relatively easy way by choosing a couple of parameters. Hence it is not difficult to find the Lagrangian among the infinitely many which fits the experimental data. In four-dimensional compactifications of string theory however, the parameters of the low-energy effective theory are encoded in a complicated way in the geometry and topology of the compactification manifold. A tuning in the parameters of the latter may have a large impact on the phenomenology in four dimension. As a consequence, experimental data cannot be fitted in such a way this is possible in field theory and this is precisely the reason why there is no candidate for a fully realistic string theory model till this day.

In principle there are two ways how a realistic vacuum could be obtained in string theory: in the top–down approach, one firstly chooses a compactification manifold and then determines the phenomenological properties from first principles. As these cannot be "predicted", in face of the huge number of vacua within the landscape it is very unlikely to find the standard model in the foreseeable future in this way.

On the other hand, many physical properties do not depend on the global compactification geometry but can be locally "engineered". The four-dimensional effective gauge theories are realized on D-branes, which are higher dimensional planes on which open strings can end on. This is usually much easier than constructing a globally consistent model. Once having an interesting local set-up at hand, the hope is to embed it finally in an appropriate compactification space, promoting it to a globally consistent model. This kind of bottom–up like ansatz has attracted attention recently, in particular in the context of "F-theory" [25–28], a twelve-dimensional geometrical framework describing a particular (type IIB to be precise) superstring theory in its non-perturbative regime.

1.6. Moduli Stabilization and the LARGE volume scenario

An example for a recent success of string phenomenology is the solution of the problem of "moduli stabilization": the afore mentioned fields encoding size and shape of the compactification space possess a flat potential at lowest order in perturbation theory and are therefore called "moduli fields". As being massless scalars in the four-dimensional effective theory, they give rise to a fifth, long-ranged force which disagrees with observations. Massive scalar particles however give rise to short-ranged interactions and are compatible with experiments as long as they mass is higher than roughly 10^{-3} eV [14]. From cosmological considerations one can derive a mass bound from even 100 TeV as otherwise, these moduli fields may spoil nucleosynthesis.

Other moduli fields determine the gauge coupling of the four-dimensional gauge theory. Without a minimum in the potential, the resulting large fluctuations of these fields contradict constant gauge couplings. Clearly, stabilized moduli fields are a prerequisite for realistic string compactifications and hence this question has become a central issue of string phenomenology.

During the last years, several mechanisms have been proposed which stabilize various classes of moduli: higher p-form fluxes, loop and non-perturbative effects generate a potential and thus freeze the moduli fields. An important milestone was the model of Kachru, Kallosh, Linde and Trivedi (KKLT) which was the first one with all moduli stabilized and positive cosmological constant in the minimum of the potential [29].

A refinement for a scenario with stabilized scenario was proposed recently by Balasubramanian, Berglund, Conlon and Quevedo [30]. As opposed to the KKLT construction where only non-perturbative effects cause moduli freezing, here also perturbative contributions to the moduli potential were taken into account, making this scenario more generic. Under certain circumstances, the resulting moduli potential is such that the overall volume of the compactification space is stabilized at exponentially large values. For this reason, this class of models goes by the name "LARGE volume scenario".

Further investigations revealed that the LARGE volume scenario has a couple of very interesting phenomenological properties. In particular supersymmetry breaking can be achieved in a controlled way, giving rise to realistic patterns of moduli and superpartner masses [31–33].

1.7. Overview over this Thesis

Quite recently, there were new results concerning non-perturbative effects in the moduli potential: Euclidean D-branes populating the compactification space, known as D-brane instantons (or D-instantons), may contribute to the potential of the low-energy effective theory. The number of zero modes of open strings stretching between these instantonic branes and branes supporting the gauge theory determine the form of these non-perturbative terms [34, 35]. An important point is that these instanton effects are necessarily present and cannot be switched on or off by will.

Subject of this thesis is to investigate the influence of these instanton effects in the context of the LARGE volume scenario, especially with respect to the new results of [34, 35] we mentioned before.

Therefore we start in chapter 2 with a review of orientifold compactifications of type IIB string theory, the language in which the LARGE volume scenario is formulated. We put emphasis on the question of how to extract the four-dimensional effective theory from the compactification data and which consistency conditions have to be met in a global model. We also present the newer results of the D-instanton calculus to the extent we need it for its application to the LARGE volume scenario.

In chapter 3, we review the LARGE volume scenario up to the level of knowledge before the publication of the new results in the D-instanton calculus. We discuss the perturbative corrections in α' and the string coupling constant to the low-energy effective action and show how these lead to the LARGE volume minimum using the well-known example of the compactification space $\mathbb{P}^4_{[1,1,1,6,9]}$[18]. Also some of the particle physical and astrophysical features such as mass scales, SUSY breaking terms, inflation and the cosmological moduli problem (CMP) are elaborated.

The remaining four chapters cover the author's field of research. In chapter 4 we deal with the following puzzle: according to the D-brane instanton calculus, the non-perturbative term which is assumed in the original literature of the LVS has a different form when chiral matter shall be present in the four-dimensional effective theory. This spoils moduli stabilization. This conflict must be resolved since both aspects are important for a meaningful model. We show that this can be achieved with the help of D-terms. This chapter covers the paper [36].

D-brane instantons generate corrections for the gauge kinetic function of the four-dimensional effective theory. In chapter 5, which is based on [37], we show how this effect can be used in order to obtain TeV scale soft terms, coexisting with a string scale at the usual GUT scale of 10^{16} GeV in a natural way. This kind of GUT models is not possible in the original LARGE volume scenario unless significant fine-tuning is accepted. Several mechanisms of SUSY breaking mediation are studied within this set-up.

Inspired by the nice results of chapter 5, we investigate SUSY breaking in GUT-like scenarios more carefully in chapter 6, which is based on [38]. We focus in particular on set-ups in which the SUSY breaking sector is in some sense "sequestered" from the matter sector. These are related to models realized on D3-branes at singularities and also to F-theory models. The soft terms are calculated in detail and we find an interesting suppression of the contributions induced by gravity mediation and also anomaly mediation therein. This suggests that gauge mediation might be the dominant mechanism for SUSY breaking in this set-up. We discuss the phenomenological implications of these findings.

Finally we conclude and discuss the results of this thesis in chapter 7.

2. Type IIB String Theory and Model Building

In this chapter we introduce all techniques and ingredients we need for building semi-realistic four-dimensional string compactifications. After a short overview over the ten-dimensional type IIB superstring theory and its low-energy effective action we describe how to compactify it to four dimensions. Three-form fluxes provide a mechanism to stabilize phenomenologically unwanted massless fields, so-called moduli. However, we prove that in pure Calabi–Yau compactifications it is not possible to have non-trivial flux. An alternative are compactifications on orientifolds of Calabi–Yau manifolds, which are introduced afterwards. In this kind of models we may also include D-branes, which can give rise to chiral matter fields in four dimensions. Related to D-branes are D-brane instantons which furnish a non-perturbative contribution to the low-energy effective action and provide with it another mechanism to stabilize moduli fields. We close with the introduction of the so-called KKLT scenario, a four-dimensional model with all moduli stabilized and positive cosmological constant.

2.1. Ten Dimensional Low-Energy Effective Action

The spectrum of the type II superstring contains a finite number of massless modes as well as an infinite tower of massive excitations. Many phenomenological questions can be addressed by using an effective supergravity theory of the massless modes only. Intuitively this is clear: provided strings are small enough, their "stringy" nature is not visible from larger distances and thus they appear as point particles at the (low) energies, accessible with todays particle colliders. Instead of establishing an effective field theory for all modes and then integrating out the massive ones, a much simpler approach is to write down a field theory for only the massless modes from the beginning. This is justified as the massive string excitations are of the order of the string scale $m_{\mathrm{s}} = \ell_{\mathrm{s}}^{-1} = \frac{1}{2\pi\sqrt{\alpha'}}$, which is usually assumed to be of the order of the Planck scale $m_{\mathrm{P}} \approx 10^{19}\,\mathrm{GeV}$. Also in the case of compactifications to lower dimensions, the Kaluza–Klein modes have masses of the order of the inverse compactification radii which are typically also of the order of the string scale.[1] Thus if one is interested in the phenomenology of string theory at energies accessible with particle colliders, it is (and will be in the near future) completely sufficient to consider only the massless string modes.

We start with the ten-dimensional action for the superstring in order to identify the massless fields. As type II string theory exhibits $\mathcal{N} = 2$ space–time supersymmetry in ten dimensions,

1 This picture dramatically changes if one considers compactifications with large extra dimensions. Here the string scale can be as low as a TeV. We will not consider such models in this thesis.

it is natural to construct a supergravity theory as low-energy effective field theory. Supergravity theories are very constrained and the high amount of super- and gauge symmetries in fact fixes the action as soon as we know the field content of the theory.

A superstring sweeps out a two-dimensional surface in the ten-dimensional target space-time. This so-called world-sheet can be described by a two-dimensional superconformal field theory with the embedding coordinates X^M, $M = 0, \dots, 9$ as dynamical fields. For world-sheet supersymmetry, their fermionic superpartners $\Psi^M = (\psi^M, \widetilde{\psi}^M)^T$ have to be included. There is an obvious proposal for the action of a world-sheet Σ propagating freely in flat ten-dimensional space–time. It reads:

$$S = \frac{1}{2\pi\alpha'} \int_\Sigma \mathrm{d}^2 z \left(\partial X^M \bar{\partial} X_M + \frac{\alpha'}{2} \left(\psi^M \bar{\partial} \psi_M + \widetilde{\psi}^M \partial \widetilde{\psi}_M \right) \right). \tag{2.1}$$

Type II string theory contains closed strings only. The equations of motion reveal that the world-sheet field can be split up into independent left- and right-moving (chiral and anti-chiral) sectors: $X^M = X_L^M(z) + X_R^M(\bar{z})$, $\psi^M = \psi^M(z)$, $\widetilde{\psi}^M = \widetilde{\psi}^M(\bar{z})$.

The boundary conditions allow for two different choices of periodicity of the closed string world-sheet fermions:

$$\begin{aligned} \psi^M(w + 2\pi) &= +\psi^M(w) \qquad \text{Ramond sector (R)}, \\ \psi^M(w + 2\pi) &= -\psi^M(w) \qquad \text{Neveu–Schwarz sector (NS)}. \end{aligned} \tag{2.2}$$

So far, we treated the string world-sheet as a purely classical object. In order to formulate the world-sheet field theory as a quantum theory, we expand the fields it contains into their oscillation modes:

$$\begin{aligned} X_L^M(z) &= \frac{x^M}{2} - i\frac{\alpha'}{2} p_L^M \ln(z) + i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \frac{\alpha_n^M}{n} z^{-n}, \\ X_R^M(\bar{z}) &= \frac{x^M}{2} - i\frac{\alpha'}{2} p_R^M \ln(\bar{z}) + i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \frac{\tilde{\alpha}_n^M}{n} \bar{z}^{-n}, \\ \psi_{L,\mathrm{R}}^M(z) &= \sum_{n\in\mathbb{Z}} d_n^M z^{-n-1/2}, \qquad \psi_{L,\mathrm{NS}}^M(z) = \sum_{r\in\mathbb{Z}+1/2} b_r^M z^{-r-1/2}, \\ \widetilde{\psi}_{R,\mathrm{R}}^M(\bar{z}) &= \sum_{n\in\mathbb{Z}} \tilde{d}_n^M \bar{z}^{-n-1/2}, \qquad \widetilde{\psi}_{R,\mathrm{NS}}^M(\bar{z}) = \sum_{r\in\mathbb{Z}+1/2} \tilde{b}_r^M \bar{z}^{-r-1/2}. \end{aligned} \tag{2.3}$$

In analogy to the harmonic oscillator, the oscillation modes are subject to the usual canonical (anti-)commutation relations and the Hilbert space of excitations is constructed by acting with the raising operators on the vacuum in the respective sector. This is done best in light cone coordinates and upon imposing the light cone gauge condition, in which negative norm states, so-called "ghosts" do not appear. In light cone coordinates, the dynamical degrees of freedom are given by the transversal directions only, counted by the space–time indices $M = 2, \ldots, 9$. The ground state in the Ramond sector is special in that it turns out to be degenerate: it is described by a real 16-component spinor, transforming in a spin representation of $SO(8)$.

At this stage, the theory suffers from a severe inconsistency: the ground state in the NS sector is a tachyon indicating an instability of the vacuum. In the R sector the ground state is massless, showing that the theory cannot be supersymmetric. Both shortcomings[2] can be cured by truncating the spectrum with respect to a projection operator called G-parity [39, 40]:

$$\begin{aligned} G &= (-1)^{F+1} = (-1)^{\sum_{r=1/2}^{\infty} b_{-r}^M b_r^M + 1} \quad &&\text{(NS)}, \\ G &= \Gamma_{11} (-1)^{\sum_{n=1}^{\infty} d_{-n}^M d_n^M} &&\text{(R)}, \end{aligned} \tag{2.4}$$

where F is the world-sheet fermion number. In the NS sector, only the states with positive G-parity are kept. As the ground state has fermion number zero it has negative G-parity and is projected out. In the R sector one has the freedom of choice whether the G-odd oder -even states are projected out. One is left with either a left- or right-handed Majorana–Weyl spinor as the R ground state, exhibiting eight real degrees of freedom. This procedure of truncating the spectrum goes under the name "GSO projection". It may appear rather ad hoc at this stage, however the necessity of truncating the spectrum would appear again when demanding modular invariance of the one- and two-loop partition functions.

For the closed string, the different combinations of Ramond and Neveu–Schwarz fermions in the left- and right-moving sectors respectively give rise to four different closed string sectors. The states in the NS–NS and R–R sectors are space–time bosons whereas the states in the NS–R and R–NS sectors are space–time fermions. The G-parity of the Ramond sector can be chosen equal or same for the left- and right-movers. This leads to the two different consistent superstring theories: type IIA and type IIB.

After the GSO projection, the massless states in type IIB string theory, on which we focus from now on, are given by:

$$\begin{aligned} &\tilde{b}_{-1/2}^M |0\rangle_{\text{NS}} \otimes b_{-1/2}^N |0\rangle_{\text{NS}}, \\ &|\alpha\rangle_{\text{R}} \otimes |\alpha\rangle_{\text{R}}, \\ &\tilde{b}_{-1/2}^M |0\rangle_{\text{NS}} \otimes |\alpha\rangle_{\text{R}}, \\ &|\alpha\rangle_{\text{R}} \otimes b_{-1/2}^M |0\rangle_{\text{NS}}. \end{aligned} \tag{2.5}$$

We denoted the Ramond ground state, which is an eight-component spinor, by $|\alpha\rangle_{\text{R}}$. Ap-

2 Absence of supersymmetry itself is of course not a shortcoming. However the spectrum contains also a massless gravitino and therefore the interacting theory cannot be consistent without supersymmetry.

parently in each sector there are $8 \times 8 = 64$ degrees of freedom. They decompose as follows: in the NS–NS sector, there is an antisymmetric rank-two tensor B_{MN} (Kalb–Ramond field) with 28 states, a symmetric traceless rank-two tensor g_{MN} (graviton) with 35 states and a scalar ϕ (dilaton). In the R–R sector, one obtains a zero-, a two- and a four-form field with self-dual field strength (C_0, C_2 and C_4 with 1, 28 and 35 degrees of freedom). The NS–R and R–NS sectors contain a spin $3/2$ (gravitino with 56 states) and a spin $1/2$ fermion (dilatino with 8 states) each.

As the number of bosonic and fermionic degrees of freedom agree, it suggests itself that type IIB string theory is space–time supersymmetric. We omit the proof here and simply state the fact that it indeed possesses $\mathcal{N} = 2$ supersymmetry in agreement with the fact that there are two gravitinos in the massless spectrum.

The low-energy effective field theory action for the massless modes can in principle be constructed by calculating various string scattering amplitudes and finding a field theory which reproduces the same amplitudes as the string theoretic calculation in the limit $\alpha' \to 0$. However, there are only few candidate field theories which come into question as supersymmetric theories in ten dimensions are highly constrained. Requiring gauge invariance and closure of the supersymmetry algebra, up to the second derivative level, there are basically only two consistent $\mathcal{N} = 2$ supersymmetric theories including gravity. They are called type IIA and type IIB supergravity. Indeed, they turn out to be the correct low-energy effective field theories for type IIA and type IIB string theory.

As we already said, we are interested in a regime where the string length can be neglected and energy is low compared to the string scale and thus supergravity is a good approximation for any string theory model we will build. The type IIB supergravity action is well-known. Its bosonic part reads (in Einstein frame):

$$\begin{aligned} S_{\text{IIB}} =& \frac{1}{2\kappa_{10}^2} \int \mathrm{d}^{10}x \, \sqrt{-g} \left(R - \frac{\partial_M \tau \, \partial^M \bar{\tau}}{2(\operatorname{Im} \tau)^2} - \frac{1}{2} \frac{|G_3|^2}{\operatorname{Im} \tau} - \frac{1}{2} |F_5|^2 \right) \\ &+ \frac{1}{8i\kappa_{10}^2} \int \frac{1}{\operatorname{Im} \tau} C_4 \wedge G_3 \wedge \overline{G}_3, \end{aligned} \tag{2.6}$$

where we have redefined the fields in the following way: $\tau = C_0 + ie^{-\phi}$, $G_3 = F_3 - \mathrm{d}B_2$, $F_3 = \mathrm{d}C_2 - C_0 \wedge \mathrm{d}B_2$ and $F_5 = \mathrm{d}C_4 - \frac{1}{2} C_2 \wedge \mathrm{d}B_2 + \frac{1}{2} B_2 \wedge \mathrm{d}C_2$. The action alone does not give the full equations of motion. It has to be completed with a self-duality constraint: $F_5 = \star_{10} F_5$. Notably, the action exhibits an $SL(2, \mathbb{R})$ symmetry, i. e. it is invariant under the transformations:

$$\tau \to \frac{a\tau + b}{c\tau + d}, \qquad G_3 \to \frac{G_3}{c\tau + d}, \tag{2.7}$$

where $a, b, c, d \in \mathbb{R}$ and $ad - bc = 1$.

There exists an equivalent formulation of the equations of motion stemming from (2.6), in which one introduces the additional R–R potentials C_6 and C_8. The additional physical degrees of freedom are reduced to the original amount by imposing two additional constraints on the field strengths: $F_1 = \star_{10} F_9$ and $F_3 = -\star_{10} F_7$. This so-called "democratic formulation" [41, 42] of the Ramond sector has certain advantages as soon as we discuss D-branes in section 2.6.

2.2. Compactifications on Calabi–Yau Manifolds

The conformal symmetry of the classical string action is in general broken at the quantum level. Only in the special case of ten space–time dimensions, this conformal anomaly is absent. So the consistency of string theory seems to imply a definite dimensionality of space–time, the strings are propagating in, namely ten.[3] However from our experience — and also from collider experiments, which are more sensitive to the detection of extra dimensions than the eye — it is clear that there cannot be more than four non-compact space–time dimensions. Nevertheless this does not rule out string theory. The loophole is to assume that six of the nine spatial dimensions are not infinitely extended but curled up to small sizes such that the dynamics of particles in that space appears effectively four-dimensional. Such an asymmetric ten-dimensional space–time $\mathcal{M}_{10}$ can be understood as a product of a four-dimensional non-compact space $\mathcal{M}_4$ and a compact "internal" space $\mathcal{X}$, for instance:

$$\mathcal{M}_{10} = \mathcal{M}_4 \times \mathcal{X}. \tag{2.8}$$

In order to respect homogeneity and isotropy of the known universe, the metric on this ten-dimensional space–time should be such, that it respects the symmetry group of $\mathcal{M}_4$. The internal space $\mathcal{X}$ may be highly anisotropic and curved. The most general metric compatible with these requirements on the geometry of space–time reads:

$$\mathrm{d}s^2 = e^{2A(y)} g_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu + e^{-2A(y)} g_{mn} \mathrm{d}y^m \mathrm{d}y^n, \tag{2.9}$$

where x^μ and y^m are coordinates for four-dimensional space–time and the internal space respectively. Notably, a warp factor $e^{2A(y)}$ can be admitted in the four-dimensional part. However, in many cases the warping is small such that the exponential prefactors can be neglected.[4]

In order not to destroy the supersymmetry of the ten-dimensional string or low-energy effective supergravity theory, the internal space has to fulfill certain geometrical requirements.

3 Strictly speaking there is also the possibility to cancel the conformal anomaly with a non-linear dilaton background, allowing then also a space–time with less than ten dimensions. However, non-linear dilaton backgrounds are not well understood and we will consider critical string theory only.

4 There is an interesting application of this fact in the so-called Randall–Sundrum scenario [43] where the warp factor was used to explain the large hierarchy between the Planck scale and the weak scale.

$$
\begin{array}{ccccccc}
 & & & h^{(0,0)} & & & \\
 & & h^{(1,0)} & & h^{(0,1)} & & \\
 & h^{(2,0)} & & h^{(1,1)} & & h^{(0,2)} & \\
h^{(3,0)} & & h^{(2,1)} & & h^{(1,2)} & & h^{(0,3)} \\
 & h^{(3,1)} & & h^{(2,2)} & & h^{(1,3)} & \\
 & & h^{(3,2)} & & h^{(2,3)} & & \\
 & & & h^{(3,3)} & & &
\end{array}
\xrightarrow[\text{CY three-fold}]{}
\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 0 & & 0 & & \\
 & 0 & & h^{(1,1)} & & 0 & \\
1 & & h^{(2,1)} & & h^{(2,1)} & & 1 \\
 & 0 & & h^{(1,1)} & & 0 & \\
 & & 0 & & 0 & & \\
 & & & 1 & & &
\end{array}
$$

Table 2.1.: Hodge diamond for a Calabi–Yau three-fold

In particular, the compactification manifold has to admit a covariantly constant spinor. This constrains the manifold to exhibit strict $SU(3)$ holonomy. Such manifolds are called "Calabi–Yau" manifolds and can alternatively be characterized as being complex Kähler manifolds which are Ricci flat or having vanishing first Chern class of the tangent bundle $c_1(\mathrm{T}\mathcal{X}) = 0$. Compactifications on Calabi–Yau manifolds exhibit $\mathcal{N} = 2$ supersymmetry in four dimensions.

The 4d massless spectrum is affected by the compactification space. Indeed, it is determined by the zero modes of various wave operators on the internal space. We demonstrate this with the example of a ten-dimensional massless scalar $\Phi(x, y)$, where we already distinguished between coordinates on $\mathcal{M}_4$ and $\mathcal{X}$. If we neglect warping for a moment, the ten-dimensional Laplacian Δ_{10} can be decomposed in a four-dimensional and an internal six-dimensional part, i. e. $\Delta_{10} = \Delta_4 + \Delta_6$. Thus the ten-dimensional Laplace equation for the scalar $\Delta_{10}\Phi = 0$ can be rewritten like:

$$\Delta_4\Phi + \Delta_6\Phi = 0. \tag{2.10}$$

Clearly a non-zero eigenvalue of Φ with respect to Δ_6 will serve as a mass term for the four-dimensional field and correspondingly, the number zero modes of Δ_6, i. e. the number of harmonic functions on $\mathcal{X}$, determines the number of massless four-dimensional fields stemming from the ten-dimensional scalar Φ.

The number of zero modes of differential operators on compact manifolds is given by the topology of the manifold $\mathcal{X}$ in question. The number of independent harmonic (p, q) forms is counted by the dimension of various cohomology groups, the so-called Hodge numbers $h^{(p,q)} = \dim H^{(p,q)}$. Not all of them are independent, but are related due to the so-called Poincaré duality $H^{(p,q)} \sim H^{(d-p),(d-q)}$, where d is the complex dimension of $\mathcal{X}$. For the special case of a Calabi–Yau three-fold, which, due to the vanishing first Chern class, necessarily has $h^{(1,0)} = h^{(0,1)} = 0$, the Hodge numbers, taking into account dualities, are listed in table 2.1.

Thus, there are only two independent Hodge numbers: $h^{(1,1)}$ and $h^{(2,1)}$. In each cohomology class, one can choose a unique harmonic representative. This collection forms a basis of the different cohomology groups. With ω_A we denote the basis of harmonic $(1,1)$-forms (which induce by Poincaré duality also a basis for $H^{(2,2)}$, denoted by $\widetilde{\omega}^B$, such that $\int_{\mathcal{X}} \omega_A \wedge \widetilde{\omega}^B = \delta_A^B$). Similarly, one can choose a basis of harmonic three-forms (α_K, β^L) for H^3.

Let us now trace the fate of the massless fields of ten-dimensional supergravity when compactifying to four-dimensions on a Calabi–Yau three-fold. Therefore one has to decompose all fields in a four-dimensional and an internal part, where the internal part is expanded in terms of the basis of the appropriate harmonic forms as needed for a massless field in four dimensions.

We start with the higher p-form fields:

$$\begin{aligned} B_2 &= B_2(x) + b^A(x)\omega_A, \qquad C_2 = C_2(x) + c^A(x)\omega_A, \qquad && A = 1,\dots,h^{(1,1)}, \\ C_4 &= D_2^A(x) \wedge \omega_A + V^K(x) \wedge \alpha_K - U_K(x) \wedge \beta^K \\ &\quad + \rho_A(x)\widetilde{\omega}^A, && K = 0,\dots,h^{(1,2)}. \end{aligned} \tag{2.11}$$

Apparently we find in the four-dimensional spectrum scalars $b^A(x)$, $c^A(x)$, $\rho_A(x)$, one-forms $V^K(x)$, $U_K(x)$ and two-forms $B_2(x)$, $C_2(x)$, $D_2^A(x)$.

The metric g_{MN} decomposes into the usual four-dimensional metric $g_{\mu\nu}$ and the internal metric g_{mn}. The off-diagonal entries $g_{\mu N}$ must vanish as they are one-forms from the internal point of view and hence do not exist on Calabi–Yau manifolds. The variation of the metric of the internal space has to respect the Ricci flatness of $\mathcal{X}$. Therefore the variations $\delta g_{m\bar{n}}$ have to fulfill a differential equation, the so-called Lichnerowicz equation. Its solutions can be associated to the $h^{(1,1)}$ harmonic $(1,1)$-forms and $h^{(2,1)}$ harmonic $(2,1)$-forms on $\mathcal{X}$. The former describe deformations of the Kähler form $J = i g_{m\bar{n}} \mathrm{d}x^m \wedge \mathrm{d}x^{\bar{n}}$ and are therefore called Kähler moduli. The latter are associated to deformations of the complex structure and hence are called complex structure moduli.

Finally, the ten-dimensional scalars ϕ and C_0 also appear as scalars in four dimensions of course.

As the choice of a Calabi–Yau manifold as compactification space guarantees the preservation of $\mathcal{N} = 2$ supersymmetry, all the bosonic fields we found arrange together with their superpartners in $\mathcal{N} = 2$ supermultiplets as summarized in table 2.2.

The effective four-dimensional action can be obtained by inserting the expansions (2.11) into the ten-dimensional action (2.6) and integrating over the internal space $\mathcal{X}$. For brevity, we omit the details here and state that after dualizing the two-forms B_2 and C_2 to scalars and applying further simplifications, the action can be expressed in the usual $\mathcal{N} = 2$ supergravity

type of multiplet	mult.	fields
gravity	1	$(g_{\mu\nu}, V^0)$
vector	$h^{(2,1)}$	(V^K, z^K)
hyper	$h^{(1,1)}$	(v^A, b^A, c^A, ρ_A)
double-tensor	1	(B_2, C_2, ϕ, C_0)

Table 2.2.: Type IIB $\mathcal{N} = 2$ supermultiplets in a Calabi–Yau compactification to four dimensions.

form (see [44–48] for details):

$$\begin{aligned} S_{\text{IIB},4d} = \int_{\mathcal{M}_4} & -\frac{1}{2}R + \frac{1}{4}\operatorname{Re} M_{KL}F^K \wedge F^L + \frac{1}{4}\operatorname{Im} M_{KL}F^K \wedge \star_4 F^L \\ & - G_{K\bar{L}}\mathrm{d}z^K \wedge \mathrm{d}\bar{z}^L - h_{pq}\mathrm{d}\tilde{q}^p \wedge \mathrm{d}\tilde{q}^q . \end{aligned} \tag{2.12}$$

The scalars span a moduli space which is a product of a quaternionic manifold $\mathcal{M}^Q$ and a special Kähler manifold $\mathcal{M}^{\text{SK}}$. The former is spanned by the fields q^p and its metric is h_{pq} (an explicit expression for h_{pq} can be found in [49, 50]). The latter, $\mathcal{M}^{\text{SK}}$, is spanned by the scalars z^K and its metric $G_{K\bar{L}}$ can be shown to be given by the Kähler potential

$$\mathcal{K}_{\text{cs}} = -\ln\left(i\int_{\mathcal{X}} \Omega \wedge \overline{\Omega}\right), \qquad G_{KL} = \partial_{z^K}\partial_{z^L}\mathcal{K}_{\text{cs}}. \tag{2.13}$$

The matrix M_{KL} is given by various products of the basis (α_K, β^L) of the cohomology group $H^{(3)}$. Ω is the unique holomorphic three-form of $\mathcal{X}$.

2.3. A No-Go Theorem for Fluxes

In the last section, we learned that in Calabi–Yau compactifications there necessarily appear geometric moduli in the four-dimensional spectrum, i. e. massless scalars which stem from geometric deformations of the internal space. They can be divided into two classes: $h^{(1,1)}$ Kähler moduli, of which there are $h^{(1,1)}$, and complex structure moduli, counted by $h^{(2,1)}$. Phenomenologically they lead to a long-ranged force in four dimensions, which is not observed. Thus finding a mechanism for giving a mass to the geometric moduli is of utmost importance in order to construct a realistic string compactification.

One path to achieve this is to switch on appropriate NS–NS and R–R fluxes in the internal space. Heuristically it is obvious that this could give a mass to the moduli: any non-trivial field configuration carries energy–momentum. This implies that the flux will lead to a certain back-reaction on the geometry. Likewise it will then cost energy to deform the geometry which is equivalent to saying that the geometric moduli, which describe the fluctuations of the internal geometry, become massive.

Unfortunately, for Calabi–Yau compactifications, there exists a strong no-go theorem which excludes a non-trivial form field flux [51–53]. We derive now this no-go theorem.

Four-dimensional Poincaré invariance allows for a non-vanishing three-form flux G_3 along internal directions only and a five-form flux of the form

$$F_5 = (1+\star)(\mathrm{d}\alpha \wedge \mathrm{d}x^0 \wedge \mathrm{d}x^1 \wedge \mathrm{d}x^2 \wedge \mathrm{d}x^3), \tag{2.14}$$

where α is an arbitrary function on the compactification space. Now, we consider the trace reversed ten-dimensional Einstein equation:

$$R_{MN} = \kappa_{10}^2 \big(T_{MN} - \tfrac{1}{8} g_{MN} T\big). \tag{2.15}$$

With the energy–momentum tensor stemming from the type IIB supergravity action (2.6), the four-dimensional components read:

$$R_{\mu\nu} = -g_{\mu\nu}\left(\frac{G_{mnp}\overline{G}^{mnp}}{48\,\mathrm{Im}\,\tau} + \frac{e^{-8A}}{4}\partial_m\alpha\,\partial^m\alpha\right). \tag{2.16}$$

The ansatz for the metric (2.9) yields the components of the Ricci tensor:

$$R_{\mu\nu} = -\eta_{\mu\nu} e^{4A}\widetilde{\nabla}^2 A = -\frac{1}{4}\eta_{\mu\nu}\big(\widetilde{\nabla}^2 e^{4A} - e^{-4A}\partial_m e^{4A}\,\partial^m e^{4A}\big). \tag{2.17}$$

Inserting this in (2.16) and taking the trace finally gives:

$$\widetilde{\nabla}^2 e^{4A} = e^{2A}\frac{G_{mnp}\overline{G}^{mnp}}{12\,\mathrm{Im}\,\tau} + e^{-6A}\big(\partial_m\alpha\,\partial^m\alpha + \partial_m e^{4A}\,\partial^m e^{4A}\big). \tag{2.18}$$

When integrating eq. (2.18) over the internal space, which we assume to be a compact manifold without boundary, the left hand side of the equation vanishes. The right hand side is obviously positive semi-definite. Consequently the fluxes must vanish and the warp factor must be constant.

As most no-go theorems, it can be circumvented if the assumptions are changed. If we allow a further term in the action which describes localized sources, i. e. $S = S_{\mathrm{IIB}} + S_{\mathrm{loc}}$, then eq. (2.18) is extended by a new term:

$$\widetilde{\nabla}^2 e^{4A} = e^{2A}\frac{G_{mnp}\overline{G}^{mnp}}{12\,\mathrm{Im}\,\tau} + e^{-6A}\big(\partial_m\alpha\,\partial^m\alpha + \partial_m e^{4A}\,\partial^m e^{4A}\big) + \frac{\kappa_{10}^2}{2}e^{2A}\big(T^m_m - T^\mu_\mu\big)^{\mathrm{loc}}, \tag{2.19}$$

where the energy–momentum tensor of the localized source is obtained as usual by varying the corresponding action with respect to the metric:

$$T_{MN}^{\text{loc}} = -\frac{2}{\sqrt{-g}} \frac{\delta S_{\text{loc}}}{\delta g^{MN}}. \tag{2.20}$$

For all localized objects, so-called p-branes, wrapping a $(p-3)$-cycle Σ of the internal space, the contribution to the right hand side of (2.19) can be shown to be:

$$\left(T_m^m - T_\mu^\mu\right)^{\text{loc}} = (7-p)T_p\delta(\Sigma), \tag{2.21}$$

where T_p is the tension ob the brane. The new term in the action is justified because in string theory, there do exist such localized objects: D-branes, which have positive tension, and orientifold planes with negative tension. In order to cancel the positive definite flux terms in (2.19) we hence need orientifold planes in the compactification.

2.4. Compactifications on Orientifolds of Calabi–Yau Manifolds

By "orientifolding" [54–58] we mean modding out a particular combination of symmetries of the original theory, in our case type IIB string theory compactified on a Calabi–Yau three-fold. The symmetry group to be modded out can involve geometrical symmetries of the target space of the theory, which for the sake of four-dimensional Poincaré invariance can only be symmetries of the internal space $\mathcal{X}$. We denote such a symmetry by σ. The set of fixed points of σ is called orientifold plane or On-plane, where n denotes the number of its space-like dimensions.

Furthermore, symmetries acting on the world-sheet can be admitted. In particular for orientifolds, the world-sheet parity Ω_p operation, which renders the world-sheets unorientable must be included. Usually it is dressed by the operator $(-1)^{F_L}$, where F_L denotes the space–time fermion number in the left-moving sector. All operations in question have to be assembled such that their combined action becomes a symmetry of the theory.

As the two gravitinos of type II string theory stem from the left- and the right-moving sector respectively, it is clear that by identifying both sectors by Ω_p, we loose half of supersymmetry and thus compactifications on orientifolds of Calabi–Yau spaces exhibit $\mathcal{N} = 1$ supersymmetry at most.[5]

The various fields in type IIB string theory behave like:

$$\begin{aligned} \Omega_p: &\qquad \text{even: } \phi, g, C_2, &\qquad \text{odd: } C_0, B_2, C_4, \\ (-1)^{F_L}: &\qquad \text{even: } \phi, g, B_2, &\qquad \text{odd: } C_0, C_2, C_4. \end{aligned} \tag{2.22}$$

Concerning the symmetry map σ of the target space, it is required that it is an isometric and

5 In general, the inclusion of D-branes can further reduce the amount of supersymmetry.

holomorphic involution of the internal space $\mathcal{X}$ [56–58]. By definition, an isometry leaves the metric of the manifold invariant. Since a Calabi–Yau manifold is Kähler, it follows in particular that the Kähler form J is invariant, i. e. $\sigma^* J = J$. Moreover, since σ is holomorphic, it respects the Hodge decomposition of $H^*(\mathcal{X})$. As a consequence, $\sigma^* H^{(3,0)} \sim H^{(3,0)}$. This is only true for the cohomology group as a whole, not for each class itself. But since σ is an involution and hence $(\sigma^*)^2 = \text{id}$, there are only two possibilities for how it can act on the holomorphic three-form $\Omega \in H^{(3,0)}$:

$$\sigma^* \Omega = \pm \Omega. \tag{2.23}$$

Correspondingly, there are finally two possible combinations of symmetry operations G, leading to different sets of O-planes in the resulting theory:

$$G = \begin{cases} (-1)^{F_L} \Omega_p \sigma & \text{with } \sigma^* \Omega = -\Omega \qquad \text{O3- and/or O7-planes,} \\ \Omega_p \sigma & \text{with } \sigma^* \Omega = +\Omega \qquad \text{O5- or O9-planes.} \end{cases} \tag{2.24}$$

The combined actions of Ω_p and/or $(-1)^{F_L}$ do not leave all fields invariant for themselves. In view of eq. (2.22), we get only an invariant spectrum if in addition the fields transform under σ like:

$$\begin{array}{llll} \text{all cases:} & \sigma^* \phi = \phi, & \sigma^* g = g, & \sigma^* B_2 = -B_2, \\ \text{O3/O7:} & \sigma^* C_0 = C_0, & \sigma^* C_2 = -C_2, & \sigma^* C_4 = C_4, \\ \text{O5/O9:} & \sigma^* C_0 = -C_0, & \sigma^* C_2 = C_2, & \sigma^* C_4 = -C_4. \end{array} \tag{2.25}$$

In view of the involutive character of σ, i. e. $(\sigma^*)^2 = \text{id}$, all cohomology groups of $\mathcal{X}$ can be split up into a direct sum of two eigenspaces of σ with eigenvalues ± 1, also called even and odd eigenspaces:

$$H^{(p,q)} = H_+^{(p,q)} \oplus H_-^{(p,q)}. \tag{2.26}$$

The dimensions of the even and odd eigenspaces are affected by the properties of σ: the Hodge $\star$-operator commutes with σ^* as it preserves the orientation. Thus, Poincaré duality also holds separately, i. e. $h_\pm^{(1,1)} = h_\pm^{(2,2)}$. The holomorphy of σ leads to $h_\pm^{(2,1)} = h_\pm^{(1,2)}$. For the holomorphic three-form, there are two possibilities as indicated in eq. (2.23) and thus, depending on whether we have O3/O7- or O5/O9-planes, we have:

$$\begin{array}{lll} \text{O3/O7:} & h_+^{(3,0)} = h_+^{(0,3)} = 0, & h_-^{(3,0)} = h_-^{(0,3)} = 1, \\ \text{O5/O9:} & h_+^{(3,0)} = h_+^{(0,3)} = 1, & h_-^{(3,0)} = h_-^{(0,3)} = 0. \end{array} \tag{2.27}$$

The volume form of $\mathcal{X}$ is proportional to $\Omega \wedge \overline{\Omega}$ and is thus invariant under σ^* in both cases.

	O3/O7		O5/O9	
type of multiplet	mult.	fields	mult.	fields
gravity	1	$g_{\mu\nu}$	1	$g_{\mu\nu}$
vector	$h_+^{(2,1)}$	V^λ	$h_-^{(2,1)}$	V^k
chiral	$h_-^{(2,1)}$	z^k	$h_+^{(2,1)}$	z^λ
	$h_-^{(1,1)}$	(b^A, c^A)	$h_+^{(1,1)}$	(v^A, c^A)
	1	(ϕ, l)	—	—
chiral/linear	$h_+^{(1,1)}$	(v^A, ρ_A)	$h_-^{(1,1)}$	(b^A, ρ_A)
	—	—	1	(ϕ, C_2)

Table 2.3.: Type IIB $\mathcal{N} = 1$ supermultiplets in orientifold compactifications to four dimensions

Accordingly, $h_+^{(3,3)} = h_+^{(0,0)} = 1$ and $h_-^{(3,3)} = h_-^{(0,0)} = 0$.

All fields can now be expanded in bases of $H_\pm^{(p,q)}$ according to their transformation properties under σ^*, listed in 2.25. It is clear that this leads to a reduced spectrum compared to the Calabi–Yau case (2.11). In detail, we have $B_2 = b^a(x)\omega_a$, $a = 1, \ldots, h_-^{(1,1)}$, where $\{\omega_a\}$ is a basis of $H_-^{(1,1)}$. Apparently, the four-dimensional two-form is projected out. For the O3/O7 system, the R–R forms are expanded as:

$$\begin{aligned} C_2 &= c^a(x)\omega_a, & a &= 1, \ldots, h_-^{(1,1)}, \\ C_4 &= D_2^\alpha(x) \wedge \omega_\alpha + V^\kappa(x) \wedge \alpha_\kappa + U_\kappa(x) \wedge \beta^\kappa & \alpha &= 1, \ldots, h_+^{(1,1)}, \\ &\quad + \rho_\alpha(x)\widetilde{\omega}^\alpha, & \kappa &= 1, \ldots, h_+^{(1,2)}. \end{aligned} \tag{2.28}$$

The two-form $C_2(x)$ in four dimensions vanishes because C_2 is odd under σ^*. This is in contrast to the axion C_0 which remains in the spectrum. In the O5/O9 system, the situation is vice versa. The expansion of the fields reads:

$$\begin{aligned} C_2 &= C_2(x) + c^\alpha(x)\omega_\alpha, & \alpha &= 1, \ldots, h_+^{(1,1)}, \\ C_4 &= D_2^a(x) \wedge \omega_a + V^k(x) \wedge \alpha_k - U_k(x) \wedge \beta^k & a &= 1, \ldots, h_-^{(1,1)}, \\ &\quad + \rho_a(x)\widetilde{\omega}^a, & k &= 1, \ldots, h_-^{(1,2)}. \end{aligned} \tag{2.29}$$

In this case, the axion C_0 is projected out, whereas the two-form $C_2(x)$ is kept. The complete spectrum is summarized in table 2.3.

The low-energy effective action is again obtained by inserting the field expansions (2.28) or (2.29) in the ten-dimensional action (2.6) and integrating over the internal space. As half of the supersymmetry is broken by performing the orientifold projection, the component fields of the former $\mathcal{N} = 2$ supermultiplets have to be reassembled into $\mathcal{N} = 1$ multiplets and the four-dimensional low-energy effective action ought to be recast in the usual $\mathcal{N} = 1$ form, which is given by a Kähler potential $\mathcal{K}$, a holomorphic superpotential W and holomorphic gauge-kinetic functions f:

$$\begin{aligned} S = -\int_{\mathcal{M}_4} \frac{1}{2}R + \mathcal{K}_{a\bar{b}}\, DM^a \wedge \star_4 D\overline{M}^{\bar{b}} + \frac{1}{2}\,\mathrm{Re}\, f_{\kappa\lambda}\, F^\kappa \wedge \star_4 F^\lambda \\ + \frac{1}{2}\,\mathrm{Im}\, f_{\kappa\lambda}\, F^\kappa \wedge F^\lambda + V, \end{aligned} \tag{2.30}$$

with the scalar potential

$$V = V_F + V_D = e^{\mathcal{K}}\big(\mathcal{K}^{a\bar{b}} D_a W\, D_{\bar{b}}\overline{W} - 3|W|^2\big) + \frac{1}{2}(\mathrm{Re}\, f)^{-1\,\kappa\lambda} D_\kappa D_\lambda. \tag{2.31}$$

The fields M^a are the complex scalars in the chiral multiplets, $\mathcal{K}_{a\bar{b}} = \partial_a \bar{\partial}_{\bar{b}} \mathcal{K}$ is the Kähler metric associated with the Kähler potential $\mathcal{K}$ and the covariant derivative in the scalar potential is Kähler-covariant, i. e. $D_a = \partial_a + \partial_a \mathcal{K}$.

We do not perform the reduction to $\mathcal{N} = 1$ here explicitly but state only the results. For a detailed derivation see [59]. At this stage, i. e. before the inclusion of non-trivial background fluxes and non-perturbative effects, no scalar potential appears in the low-energy effective action, hence $W = 0$, $D_\alpha = 0$. The gauge kinetic function is given in terms of the gauge kinetic matrix of the $\mathcal{N} = 2$ action (2.12):

$$f_{\kappa\lambda} = -\tfrac{i}{2} \overline{M}_{\kappa\lambda}|_{z^\kappa = \bar{z}^\kappa = 0}. \tag{2.32}$$

The Kähler potential for the complex structure moduli turns out to still have the form (2.13). The potential for the dilaton and the Kähler moduli depends on which orientifold projection we choose. In the case of the O3/O7-system, the complete Kähler potential reads:

$$\mathcal{K} = \mathcal{K}_{\mathrm{cs}} + \mathcal{K}^Q = -\ln\Big(i \int_{\mathcal{X}} \Omega \wedge \overline{\Omega}\Big) - \ln\big(-i(\tau - \bar{\tau})\big) - 2\ln\big(\mathrm{vol}_E(\mathcal{X})\big), \tag{2.33}$$

where $\mathrm{vol}_E(\mathcal{X})$ denotes the volume of the internal space in Einstein frame. It should be noted, that it contains the dilaton in its definition, which in consequence renders the Kähler metric non-block-diagonal in the Kähler and dilaton sector. The Kähler potential for the O5/O9-system is given by:

$$\mathcal{K} = \mathcal{K}_{\mathrm{cs}} + \mathcal{K}^Q = -\ln\Big(i \int_{\mathcal{X}} \Omega \wedge \overline{\Omega}\Big) - \ln\Big(\tfrac{1}{48}\kappa_{\alpha\beta\gamma}(t+\bar{t})^\alpha (t+\bar{t})^\beta (t+\bar{t})^\gamma\Big). \tag{2.34}$$

Here it is necessary to define new Kähler coordinates $t^\alpha = e^{-\phi} v^\alpha - i c^\alpha$. The $\kappa_{\alpha\beta\gamma}$ are the triple intersection numbers of the internal space, i. e. $\kappa_{\alpha\beta\gamma} = \int_{\mathcal{X}} \omega_\alpha \wedge \omega_\beta \wedge \omega_\gamma$.

2.5. Compactifications with Non-Trivial Flux

When we compactify on an orientifold of a Calabi–Yau manifold, we are free to allow non-zero G_3-flux in the internal space (five-form flux would have to be supported on non-trivial five-cycles in the internal space, which do not exist on Calabi–Yau manifolds). In this section we analyze further conditions on the G_3-flux and motivate the superpotential arising in the presence of flux.

The three-forms F_3 and H_3 appearing in G_3 have to be expanded in terms of a basis of $H^3_\pm(\mathcal{X})$ such that they show the correct parity under the orientifold projection as listed in eq. (2.25). From the definition of F_5 in (2.6) we see that the Bianchi identity for this field is modified in the presence of three-form flux:

$$\mathrm{d}F_5 = H_3 \wedge F_3. \tag{2.35}$$

Upon dimensionally reducing the ten-dimensional action (2.6) one finds the following expression stemming from the kinetic term of G_3:

$$S_G = \frac{1}{4\kappa_{10}^2 \operatorname{Im}\tau} \int_{\mathcal{X}} G_3 \wedge \star_6 \overline{G}_3. \tag{2.36}$$

This term can be rewritten by splitting up G_3 in imaginary self-dual and anti self-dual parts, i. e. $G_3 = G_3^+ + G_3^-$ with $G_3^\pm = \frac{1}{2}(G_3 \pm i\star_6 G_3)$ and $\star_6 G_3^\pm = \mp i G_3^\pm$. With this definition, eq. (2.36) reads:

$$S_G = \frac{1}{2\kappa_{10}^2 \operatorname{Im}\tau} \int_{\mathcal{X}} G_3^+ \wedge \star_6 \overline{G}_3^+ - \frac{i}{4\kappa_{10}^2 \operatorname{Im}\tau} \int_{\mathcal{X}} G_3 \wedge \overline{G}_3. \tag{2.37}$$

The second term is topological, i. e. it is proportional to an integer $\sim \mu_3 N_{\text{flux}}$, where μ_3 is the D3-brane tension. It contributes to the D3-brane tadpole (2.51a) and cancels in a consistent set-up against the contributions from D3-branes and O3-planes. The first term can be interpreted as part of the $\mathcal{N} = 1$ scalar F-term potential V_F in four dimensions.

From now on, we have to distinguish between the O3/O7- and the O5/O9-system in that we have to expand the three-form G_3^+ in the sub-space of $H^3(\mathcal{X})$ exhibiting the correct parity under σ.

In the O3/O7-system, according to (2.25), H_3 and F_3 are odd forms. Thus, we have to expand G_3^+ in a basis of $H^3_-(\mathcal{X})$. Due to the decomposition in imaginary (anti) self-dual parts, only the subspace $H^{(3,0)}_-(\mathcal{X}) \oplus H^{(1,2)}_-(\mathcal{X})$ comes into question. As we will see later, in order to preserve supersymmetry, there are further restrictions on the flux components.

In the chosen basis of $H_-^{(3,0)}(\mathcal{X}) \oplus H_-^{(2,1)}(\mathcal{X})$, denoted by $\{\Omega, \chi_k\}$, the expansion reads:

$$G_3^+ = -\frac{1}{\int \Omega \wedge \overline{\Omega}} \Big(\Omega \int_{\mathcal{X}} \overline{\Omega} \wedge G_3 + G^{lk} \bar{\chi}_k \int_{\mathcal{X}} \chi_l \wedge G_3 \Big), \tag{2.38}$$

where G^{kl} is the inverse of the metric in eq. (2.13). Inserting this expansion in (2.37) one obtains the following expression for the scalar potential V_F:

$$V_F = \frac{i}{2\,\mathrm{Im}\,\tau \kappa_{10}^2 \int_{\mathcal{X}} \Omega \wedge \overline{\Omega}} \Big(\int_{\mathcal{X}} \Omega \wedge \overline{G}_3 \int_{\mathcal{X}} \overline{\Omega} \wedge G_3 + G^{kl} \int_{\mathcal{X}} \chi_k \wedge G_3 \int_{\mathcal{X}} \bar{\chi}_l \wedge \overline{G}_3 \Big). \tag{2.39}$$

As we formulate the low-energy effective action in the formalism of $\mathcal{N} = 1$ supergravity, we have to find a holomorphic superpotential W which reproduces the scalar potential as in eq. (2.31). It is not difficult to check that the Gukov–Vafa–Witten superpotential [60] indeed reproduces (2.39):

$$W = \frac{1}{\kappa_{10}^2} \int_{\mathcal{X}} G_3 \wedge \Omega. \tag{2.40}$$

The case of the O5/O9-system is different in that the three-form fields F_3 and H_3 have different parity under the orientifold projection. Therefore, F_3 is an element of $H_+^{(3)}(\mathcal{X})$, whereas $H_3 \in H_-^{(3)}(\mathcal{X})$. Consequently, G_3 is expanded in a different set of harmonic three-forms and the resulting scalar potential reads:

$$\begin{aligned} V = {} & \frac{i}{2\,\mathrm{Im}\,\tau \kappa_{10}^2 \int_{\mathcal{X}} \Omega \wedge \overline{\Omega}} \Big(\int_{\mathcal{X}} \Omega \wedge F_3 \int_{\mathcal{X}} \overline{\Omega} \wedge F_3 + G^{\kappa\lambda} \int_{\mathcal{X}} \chi_\kappa \wedge F_3 \int_{\mathcal{X}} \bar{\chi}_\lambda \wedge F_3 \Big) \\ & - \frac{\mathrm{Im}\,\tau}{4\kappa_{10}^2} \big[m_H^k (\mathrm{Im}\, M)_{kl} m_H^l + \big(e_k^H - (m_H \,\mathrm{Re}\, M)_k \big) (\mathrm{Im}\, M)^{-1\,kl} \big(e_l^H - (m_H \,\mathrm{Re}\, M)_l \big) \big], \end{aligned} \tag{2.41}$$

where the m_H denote the expansion coefficients of the NS–NS three-from H_3. It turns out that only R–R fluxes generate a superpotential, whereas the NS–NS fluxes generate a D-term [59]. Thus the GVW superpotential in the case of O5/O9-planes is obtained by setting $H_3 = 0$ in (2.40):

$$W = \frac{1}{\kappa_{10}^2} \int_{\mathcal{X}} F_3 \wedge \Omega. \tag{2.42}$$

For unbroken $\mathcal{N} = 1$ supersymmetry, all F-terms have to vanish. This gives further constraints on which components of G_3 are allowed. In particular the F-term conditions for the Kähler, dilaton and complex structure moduli imply:

$$\begin{aligned} D_{T^A}W &= (\partial_{T^A})\mathcal{K}\int_{\mathcal{X}} G_3\wedge\Omega_3 = 0 &&\Rightarrow G_3^{(0,3)} = 0,\\ D_\tau W &= \frac{1}{\tau-\bar\tau}\int_{\mathcal{X}} \overline{G}_3\wedge\Omega_3 = 0 &&\Rightarrow G_3^{(3,0)} = 0,\\ D_{U^k}W &= \int_{\mathcal{X}} G_3\wedge\chi_k = 0 &&\Rightarrow G_3^{(1,2)} = 0. \end{aligned} \tag{2.43}$$

These are $2h_-^{(2,1)}+4$ real equations, compared to $2h^{(2,1)}+2$ real parameters (τ, U^k) to solve for. In general, G_3-flux will thus not preserve supersymmetry.

The same result can be obtained by analyzing the supersymmetry conditions with fluxes directly in ten dimensions or by studying the equations of motion. In both cases it turns out that the three-form flux G_3 must be a primitive $(2,1)$-form satisfying the imaginary self-duality condition.[6]

We close this section with an important consideration concerning the moduli fields. The scalar potential of the low-energy effective action in type IIB flux compactifications with O3- and O7-planes is given by (2.31) with Kähler potential (2.33) and superpotential (2.40). The volume of the internal space can be written in terms of four-cycle volumes which are the correct Kähler variables for the $\mathcal{N}=1$ low-energy effective action. For the case of one Kähler modulus, the relation is:

$$\mathrm{vol}_E(\mathcal{X}) = \mathrm{Re}\,(T)^{3/2}. \tag{2.44}$$

Thus, the corresponding term in the Kähler potential can be rewritten:

$$\mathcal{K} = -2\ln\big(\mathrm{vol}_E(\mathcal{X})\big) = -3\ln(T+\overline{T}). \tag{2.45}$$

It is not difficult to derive that if we perform the sum over all fields in (2.31), the term with the Kähler modulus T, i. e. $\mathcal{K}^{T\overline{T}}D_TW\,D_{\overline{T}}\overline{W}$ exactly cancels against $-3|W|^2$. We are left with a positive semi-definite F-term potential of the form

$$V_F = e^{\mathcal{K}}\mathcal{K}^{i\bar{\jmath}}D_iW\,D_{\bar{\jmath}}\overline{W}, \tag{2.46}$$

where the indices $i, \bar{\jmath}$ run over all superfields *but* the Kähler modulus T. The situation remains unchanged in the more general case of having more than one Kähler modulus. Such models are well-known in supergravity and go under the name "no-scale models" [61, 62]. The associated Lagrangian is said to posses a "no-scale structure" (for the Kähler moduli). The minimum of (2.46) is at $V_F=0$, i. e. it is a supersymmetric minimum since the F-terms

6 On Calabi–Yau manifolds, $(2,1)$-forms are always primitive.

vanish: $D_i W = 0$. At this particular point, the prefactor $e^{\mathcal{K}}$ is undetermined and as the only dependence on T is in the Kähler potential, we conclude that with the help of three-form fluxes, it is possible to lift the complex structure moduli and the dilaton, as it should be clear from (2.39), but not the Kähler moduli.

2.6. D-Branes

Strings are not the only dynamical objects in string theory. When studying their equations of motion it turns out that it is possible to allow also open strings in the theory, provided suitable boundary conditions on their endpoints are imposed.

There are two types of boundary conditions which are compatible with the equations of motion:

$$\begin{aligned} \partial_\sigma X^a(\tau,\sigma)|_{\sigma=0,\pi} &= 0, \qquad \text{Neumann boundary condition,} \\ \delta X^n(\tau,\sigma)|_{\sigma=0,\pi} &= 0, \qquad \text{Dirichlet boundary condition.} \end{aligned} \tag{2.47}$$

When imposing Neumann boundary conditions along the directions X^a, $a = 0, \ldots, p$ and Dirichlet boundary conditions on the remaining X^n, $n = p+1, \ldots, 9$, the motion of the string endpoints is confined to a p-dimensional plane, called Dirichlet brane or D-brane.[7]

In [64] it was shown that D-branes carry Ramond–Ramond charge. They couple in a canonical way to the R–R potentials. This tells us that in type IIB there can only exist stable D(−1)-, D1-, D3-, D5-, D7- and D9-branes as for the even numbered ones, the corresponding R–R potentials are absent.

Redoing the quantization procedure for an open superstring attached to a single Dp-brane shows that all excitations with bosonic oscillators are massive. The NS and R sector give rise to a massless $U(1)$ vector and a Majorana–Weyl spinor in ten dimensions,

$$A_M \psi^M_{-1/2}|0\rangle_{\text{NS}} \oplus \chi_\alpha|\alpha\rangle_{\text{R}}, \tag{2.48}$$

transforming in the $\mathbf{8}_V \oplus \mathbf{8}_s$ of ten-dimensional space–time. The collection of the fields (A_M, χ_α) form together an $\mathcal{N} = 1$ vector multiplet under ten-dimensional supersymmetry.

The vector A_M can be decomposed in components tangential and normal to the brane. The $9 - p$ transverse components are scalars from the D-brane world-volume point of view and describe the transversal fluctuations of the brane in space.

This suggests that a D-brane is a dynamical rather than a static and rigid object in space–time. As such, it has couplings to the bulk closed string fields, described by the Dirac–Born–Infeld action for the NS–NS fields and the Chern–Simons action for the R–R fields. In its most

7 Another strong motivation for the existence of D-branes in string theory comes from T-duality [63].

general form, the action for single Dp-brane reads:

$$S = S_{\text{DBI}} + S_{\text{CS}} = -\,\mu_p \int_{\mathcal{W}} \mathrm{d}^{p+1}\xi\, e^{-\phi(X)} \sqrt{-\det\big(g_{MN}(X) + B_{MN} + 2\pi\alpha' F_{MN}(X)\big)} \\ -\,\mu_p \int_{\mathcal{W}} \mathrm{ch}(2\pi\alpha' F) \wedge \sqrt{\frac{\hat{A}(\mathcal{R}_T)}{\hat{A}(\mathcal{R}_N)}} \wedge \bigoplus_q C_q, \tag{2.49}$$

where the ξ are coordinates of the $p+1$-dimensional world-volume $\mathcal{W}$ of the brane, F is the field strength of the $U(1)$ vector field A_M and $\hat{A}(\mathcal{R}_{T,N})$ is the A-roof-genus of the tangent and normal bundle respectively.

As we are interested in compactifications to four dimensions which preserve four-dimensional Poincaré invariance, D-branes have to be placed in space–time such that they fill out all four non-compact directions and wrap a cycle of the compactification manifold[8], which in order to preserve supersymmetry is required to be a holomorphic submanifold of the Calabi–Yau [65]. Thus we are left with D3-, D5-, D7- and D9-branes at our disposal. If we compactify on an orientifold, we furthermore have to ensure that for each brane there is a corresponding "mirror brane" such that the whole configuration is invariant under the geometric involution σ. Moreover, the parity of the R–R potentials under the orientifold projection (2.25) further restricts the type of D-brane that can appear, depending on the type of orientifold projection (2.23). In the O3/O7-system, only D3- and D7-branes are invariant objects, whereas in the O5/O9-system only D5- and D9-branes are possible.

From the four-dimensional point of view, the 10d vector A_M decomposes into a 4d vector A_μ and six scalars A_i. The 10d Majorana–Weyl spinor χ_α decomposes as $(\mathbf{2},\mathbf{4})\oplus(\bar{\mathbf{2}},\bar{\mathbf{4}})$ under the $SO(3,1)\otimes SO(6)$ space–time symmetry group, thus in summary there are four 4d Weyl fermions. All fields together form a 4d $\mathcal{N}=4$ vector multiplet.

If we include more than one D-brane in the compactification, several cases have to be distinguished: multiple D-branes of the same dimension can be placed on top of each other forming a what is called stack of branes, they can be parallel to each other or they can intersect.

In the first case, the D-branes are at the same position in space. The end points of open strings are endowed with so-called Chan–Paton labels λ^A, indicating on which of the D-branes within the stack they end. Instead of an abelian $U(1)$ gauge theory, there now arises a non-abelian $U(N)$ gauge theory on the world-volume of the stack of the D-branes, where N is the number of branes lying on top of each other.[9] Likewise, from the four-dimensional point of view, the vector A_μ and the spinor χ_α now transform in the adjoint representation

8 The situation in which D-branes wrap internal cycles only will be considered in section 2.7

9 If the stack of D-branes lies on top of the orientifold plane, the gauge group is instead $SO(2N)$ or $SP(2N)$, depending on the details of the orientifold projection.

Representation	Multiplicity
$(\overline{N}_a, N_b)$	I_{ab}
(N_a, N_b)	$I_{a'b}$
A_a	$\frac{1}{2}(I_{a'a} + 2I_{O7\,a})$
S_a	$\frac{1}{2}(I_{a'a} - 2I_{O7\,a})$

Table 2.4.: Chiral spectrum for intersecting D7-branes

of $U(N)$.

If the N D-branes in the stack are shifted away from each other along the normal direction, we end up with a configuration of N parallel branes. Intuitively it is clear that the open strings stretching between two different branes are massive and thus are no longer present in the low-energy effective theory. Only the modes stemming from open string starting and ending on the same brane remain massless and we end up with a $U(1)^N$ gauge theory. The process of separating a stack of D-branes can be interpreted from the field theory point of view as giving the transversal scalars a vev, breaking the $U(N)$ gauge symmetry down to $U(1)^N$.

The most interesting case is the configuration of two intersecting (stacks of) D-branes. From now on, we restrict ourselves to the case of intersecting (magnetized) D7-branes, as this is the configuration we will consider in the later chapters. They wrap four-cycles in the internal space and hence two D7-branes generically intersect over a two-dimensional subspace within the Calabi–Yau. In addition to the massless modes we encountered in the case of a single stack, there now appear massless modes stemming from open strings stretching from one stack to the other along the intersection locus.

To be more concrete, we consider two stacks a and b with N_a and N_b branes on top of each other respectively, wrapping the cycles D_a, D_b and carrying the gauge fields V_a and V_b. In an orientifold compactification, their image branes under the geometric involution σ have to be included as well. They wrap the cycles $D_{a',b'}$. Last but not least, the orientifold cycle is simply denoted by O7. Then we compute the various chiral indices as:

$$I_{ab} = \int_{D_a \cap D_b} \big(c_1(V_a) - c_1(V_b)\big) = \int_{\mathcal{X}} \big(c_1(V_a) - c_1(V_b)\big) \wedge [D_a] \wedge [D_b], \qquad (2.50)$$

where $[D_{a,b}]$ denote the Poincaré dual two-forms corresponding to the four-cycles $D_{a,b}$. The resulting chiral spectrum appearing at the intersection in the internal space is listed in table 2.4.

Note that for the first time, we encounter chiral fermions in the spectrum — one of the most outstanding features of the standard model. As we are interested in constructing models which resemble the standard model as close as possible, intersecting D7-branes will be a necessary ingredient in the compactifications we consider.

We discussed already that D-branes carry Ramond–Ramond charge. This charge has to be canceled with corresponding negative contributions, furnished by the charge of the orientifold planes. This is analogous to classical electrostatics: a single positive charge placed on a compact space without boundary would violate Gauß's law. Similarly, in string theory, the correct charge cancellation condition can be obtained if we consider the equations of motion for the Ramond–Ramond potentials, i. e. set to zero the variation of the total action with respect to C_q.

In the case of the O3/O7-system, the condition for C_4 reads (after taking the orientifold quotient):

$$N_{\mathrm{D3}} + N_{\mathrm{flux}} - \sum_a N_a \int_{D_a} \mathrm{ch}_2(V_a) = \frac{N_{\mathrm{O3}}}{4} + \sum_a \frac{N_a}{24} \int_{D_a} c_2(T_{D_a}) + \frac{1}{12} \int_{D_{\mathrm{O7}}} c_2(T_{\mathrm{O7}}). \tag{2.51a}$$

Here $N_{\mathrm{D3,O3}}$ denotes the number of D3-branes and point-like (in the internal space) orientifold planes respectively. As anticipated from (2.37) there is a contribution from the fluxes, denoted by N_{flux}. From the Chern–Simons action (2.49) it is clear that a Dp-brane couples also to Ramond–Ramond potentials of lower rank. Thus we have a contribution also from D7-branes, wrapping four-cycles D_a and carrying gauge bundles V_a.

For the same reason, there is a similar condition for C_6, even if no D5-branes are present:

$$\sum_a \mathrm{ch}_1(V_a) \wedge D_a \wedge \omega_a = 0, \tag{2.51b}$$

where $\{\omega_a\}$ is a basis of $H_-^{(1,1)}(\mathcal{X})$ and hence, the above condition will automatically be satisfiyed provided $h_-^{(1,1)} = 0$. In [66, 67] is was pointed out that this condition plays an important rôle in the cancellation of various chiral anomalies.

Finally there is a simpler condition for C_8:

$$\sum_a N_a D_a = 4 D_{\mathrm{O7}}, \tag{2.51c}$$

where D_{O7} is the four-cycle of the orientifold plane in the internal space. These three conditions (2.51a)–(2.51c) go by the name "tadpole cancellation conditions", as non-zero tadpole diagrams for the moduli fields in the low-energy effective field theory arise if they are not fulfilled. This indicates an instability.

We conclude this section with some remarks on various anomalies that can occur when branes are present in a compactification. From (2.50) it is clear that in order to have chiral fermions in the spectrum, at least one of the D7-branes has to carry a non-trivial $U(N)$ gauge bundle. Generically it contains an anomalous $U(1)$ subgroup giving rise to non-abelian, mixed abelian–non-abelian, cubic abelian and mixed abelian–gravitational anomalies.

The cubic non-abelian anomaly vanishes if the D7- and D5-tadpole conditions (2.51b) and (2.51c) are fulfilled [67]. The anomalous diagrams for the mixed abelian–non-abelian, cubic abelian and mixed abelian–gravitational anomalies are canceled via a generalized Green-Schwarz mechanism: when reducing the Chern–Simons action, there arise couplings between the anomalous gauge field F and four-dimensional axions, stemming from the dimensional reduction of the Ramond–Ramond potential C_4, schematically:

$$S_{\text{CS}} \sim \int_{\mathcal{M}_4 \times D_a} C_4 \wedge F \wedge F. \tag{2.52}$$

From this, such couplings can be generated which precisely cancel the chiral anomalies. In addition, a Stückelberg mass term four the anomalous gauge symmetry is generated.

Finally, it was shown in [68] that another anomaly can appear if there is three-form flux present: with non-vanishing H_3, the Bianchi identity for the diagonal $U(1)$ gauge field strength F on D-branes is modified:

$$\mathrm{d}F = -H_3. \tag{2.53}$$

By integration over a three-cycle Σ_3 we get the condition

$$\int_{\Sigma_3} H_3 = 0. \tag{2.54}$$

Its fulfillment guarantees that the so-called Freed–Witten anomaly is absent.

2.7. D-Brane Instantons

Instead of filling out four-dimensional space–time and wrapping a cycle of the compactification manifold, it is also possible that a D-brane wraps an internal cycle only such that it is point-like from the four-dimensional point of view. One then speaks of a (Euclidean) D-brane instanton or E-instanton as it affects the calculation of correlation functions in a similar fashion as the well-known instantons in field theory. In fact, special configurations of D-branes and D-brane instantons precisely reproduce in the low-energy limit a four-dimensional gauge theory including the effects of gauge instantons [69].

As in the field theory case, the effects of D-brane instantons come with the typical exponential factor of $\sim \exp(-S_{\text{inst}})$ and are thus highly suppressed. Only if certain couplings are absent in perturbation theory, for instance due to non-renormalization theorems, instanton effects may furnish the leading order contribution. The exponential suppression can potentially be used to explain some of the small and large hierarchies present in the standard model and its supersymmetric extensions. Examples include Majorana masses for the right-handed neutrinos [34, 70, 71], the MSSM μ-term [72, 73] and top quark Yukawa couplings in $SU(5)$ GUT theories [74, 75].

In this work, we will be mainly interested in the generation of mass terms for the so far massless Kähler moduli. Intuitively it is clear that this should be possible as the instanton action depends on the volume of the internal cycle it wraps.

As there does not exist a complete second quantized version of string theory yet, the instanton calculus cannot be derived from first principles. Instead one relies on analogies to the field theory case. Doing so, we expect a contribution to the four-dimensional effective action of the form:

$$S_{\text{n.p.}} = \int \mathrm{d}\mathcal{M}\, e^{-S_E^{(0)} - S_E^{\text{int.}}(\mathcal{M})}, \tag{2.55}$$

where $\mathcal{M}$ denotes the collection of all instanton zero modes, $S_E^{(0)}$ is the classical instanton effective action and $S_E^{\text{int.}}$ its interaction part. The integral over the fermionic zero modes vanishes if more than one zero mode is pulled down from the exponent as they are described by Graßmann variables. Hence only instantons with a very specific fermionic zero mode structure can contribute to the effective action. We encounter a similar distinction of cases as with D-branes since various E-instantons can intersect with themselves, D-branes and orientifold planes, giving rise to different sets of zero modes. In detail, there exist:

Universal zero modes. They arise from strings starting and ending on the same instanton. There are four bosonic ones x^μ parameterizing the (point-like) position of the instanton in four-dimensional space–time. They can be understood as being the Goldstone bosons corresponding to the breakdown of the four-dimensional translational invariance. Universal fermionic zero modes arise from the breakdown of supersymmetries. Here one has to distinguish several cases: firstly, the instanton can wrap a cycle which is not invariant under the orientifold projection and thus an image instanton has to be included. In such a configuration, there are two chiral Goldstinos θ^α as well as anti-chiral ones $\bar{\tau}^{\dot{\alpha}}$. Consequently, a contribution to an F-term can only arise if the extra $\bar{\tau}^{\dot{\alpha}}$ are saturated. If the instanton wraps a cycle already populated by a D-brane, the extra anti-chiral modes are soaked up in a way reproducing the celebrated ADHM constraints for gauge instantons. Such a configuration is in fact equivalent to an ordinary gauge instanton in the field theory living on the D-brane in question. For an $SU(N_c)$ $\mathcal{N}=1$ supersymmetric QCD with $N_f = N_c - 1$ flavors engineered on a stack of D-branes it was shown in [76–79] that indeed the Affleck–Dine–Seiberg superpotential is generated by an E-instanton. Finally, the D-brane instanton may be on top of the orientifold plane. A stack of ordinary D-branes on such a cycle can have gauge group $SO(N)$ or $SP(N)$. For a D-brane instanton on this cycle the gauge group is swapped to $SP(N)$ or $SO(N)$. In the first case, there are $N(N-1)/2$ zero modes θ^α and $N(N+1)/2$ $\bar{\tau}^{\dot{\alpha}}$s. In the latter case one gets $N(N+1)/2$ θ^α and $N(N-1)/2$ $\bar{\tau}^{\dot{\alpha}}$ zero modes. Hence the case of most interest are $O(1)$ instantons on top of an orientifold plane because we end up with the right number of two universal zero modes θ^α. For the case of E3-instantons wrapping a four-cycle Γ_4 in the internal space, which we will mainly consider in the later chapters, the contribution to the superpotential of the low-energy effective action was already pioneered in [80] and turns out to be of the form:

$$W_{\text{inst}} \sim e^{-2\pi T}, \tag{2.56}$$

where T is the Kähler modulus corresponding to the four-cycle Γ_4.

Deformation zero modes. They stem from the transversal modes of open strings starting and ending on a particular D-brane instanton, describing infinitesimal deformations of the instantonic brane. Each complex valued deformation leads to one complex bosonic zero mode. One chiral and one anti-chiral Weyl spinor comprise the fermionic zero modes. Whether a cycle can be deformed or not is a question of topology. The number of deformations is counted by the Betti numbers b^1 and b^2 of the cycle in question. They count the Wilson-line moduli and the transversal deformations respectively. Since these zero modes come in addition to the universal ones, instantons on cycles with deformations contribute only if the extra zero modes are soaked up, or lifted by flux [81–83, 83–85]. The extra modes are absent if the cycle simply has no deformations, i. e. is rigid.

Charged zero modes appear at the intersection of a D-brane instanton with an ordinary, space-filling D-brane. They are called "charged" as one end of the open strings at the intersection locus is attached to the (stack of) D-brane(s) and hence is charged under the four-dimensional gauge group of it. We specialize now to the case of an $O(1)$ instanton. The number of charged zero modes is then counted by the intersection number I_{E,D_a}, defined in (2.50). They carry the total $U(1)_a$ charge $Q_a(E) = N_a(I_{E,D_a} - I_{E,D'_a})$. In order to contribute to the superpotential, the additional charged zero modes λ have to be soaked up. This works with the help of couplings of the form $\lambda_{E\,a_i}\Phi_{a_i b_i}\lambda_{b_i\,E}$ appearing in $S_E^{\text{int.}}$. Hence the saturation of the λ zero modes pulls down charged matter fields $\Phi_{a_i b_i}$ in the fermionic integral in such a way that the $U(1)_a$ charges are preserved. Consequently there appear terms in the superpotential of the form

$$W = \prod_{i=1}^{M} \Phi_{a_i b_i} e^{-S_E}. \tag{2.57}$$

Here, the charge of the product of the matter fields is canceled by the sum of the charges of the zero modes:

$$\sum_{i=1}^{M} Q_a(\Phi_{a_i b_i}) = -N_a(I_{E,D_a} - I_{E,D'_a}), \tag{2.58}$$

such that the whole expression is gauge invariant. The consequences for model building of a charged superpotential (2.57) will be discussed in chapter 4 in detail.

Multi-instanton zero modes. In the same way a D-brane instanton can intersect a D-brane, it can also intersect another E-instanton. Then zero modes arise at the intersection locus and are therefore called multi-instanton zero modes. For the case we are interested in, namely an E3-instanton with gauge group $O(1)$ (hence lying on the orientifold plane), the number of zero modes is given by the same expression as for D-brane case, given in table 2.4.

2.8. De Sitter Vacua in String Theory

For a completely realistic and full-fledged string vacuum, one has not only to engineer the particle content and interactions of the standard model. In addition, all moduli have to be stabilized with a mass of at least 10^{-3} eV in order not to be in conflict with fifth-force experiments [14] and to have definitive values for the couplings. Moreover, as there is evidence for a small, but non-zero positive cosmological constant, the four-dimensional non-compact part of the ten-dimensional space–time should have the geometry of de Sitter space.

There are no-go theorems showing that de Sitter solutions do not exist in string theory if one only takes the lowest order terms in the effective supergravity action into account [51, 52]. Fortunately, the inclusion of g_s- and α'-corrections improves the situation.

Kachru, Kallosh, Linde and Trivedi (KKLT) first proposed a construction in which both, stabilization of all moduli and a positive cosmological constant can be obtained [29]. In a first step, the complex structure moduli and the dilaton are stabilized with background fluxes as described in section 2.5. Then, one takes into account non-perturbative effects resulting either from D-brane instantons or from gaugino condensation in the effective superpotential in order to break the no-scale structure for the Kähler moduli. As a result, a scalar potential is generated for them, stabilizing their vacuum expectation value. Finally, one introduces anti D3-branes in the internal space, which explicitly break supersymmetry and uplift the vacuum energy to a positive value.

Non-perturbative effects

At the perturbative level, no corrections to the superpotential can appear. The reason is the following: the string coupling is given by the expectation value of the dilaton: $g_s = \langle e^{\phi} \rangle$. As the superpotential is by definition a holomorphic function of the superfields, higher powers of g_s can appear in the superpotential only in conjunction with the universal axion, the field with it is combined into a complex scalar $S = e^{-\phi} + iC_0$. A polynomial dependence on C_0 would however break its perturbative shift symmetry.

At the non-perturbative level, there are two sources for corrections to the superpotential. Euclidean D-brane instantons, wrapping a four-cycle in the internal space generate for the case of E3-instanton a new term

$$W = A(U)e^{-2\pi T}, \tag{2.59}$$

where $A(U)$ is a complex structure moduli dependent one-loop determinant and T is the Kähler modulus corresponding to the four-cycle, the D-brane is wrapping. In section 2.7 we have seen that the generation of such a term highly depends on the zero mode structure of open string modes stretching between the instanton and other D-branes.

A similar non-perturbative term can arise from gaugino condensates on D7-branes: Consider a stack of N_c coincident D7-branes, wrapping a rigid four-cycle in the internal space. On their word-volume they support a $U(N_c)$ gauge theory with four-dimensional gauge coupling

$$\frac{8\pi^2}{g_{\rm YM}^2} = 2\pi\tau, \tag{2.60}$$

where τ denotes the volume of the internal four-cycle Γ, i. e. $\tau = \frac{\mathrm{vol}(\Gamma)}{\ell_s^4}$. At low energies, this is effectively a pure $\mathcal{N} = 1$ supersymmetric gauge theory which undergoes gaugino condensation. The correction to the effective superpotential reads

$$W = \Lambda^3 = Ae^{-\frac{2\pi T}{N_c}}, \tag{2.61}$$

where Λ is the dynamical scale of the gauge theory. Thus both effects yield a similar correction to the superpotential. They lead to a dependence of the superpotential on the Kähler moduli, breaking the no-scale structure of tree-level flux compactifications, discussed in section 2.5.

Supersymmetric anti de Sitter vacua

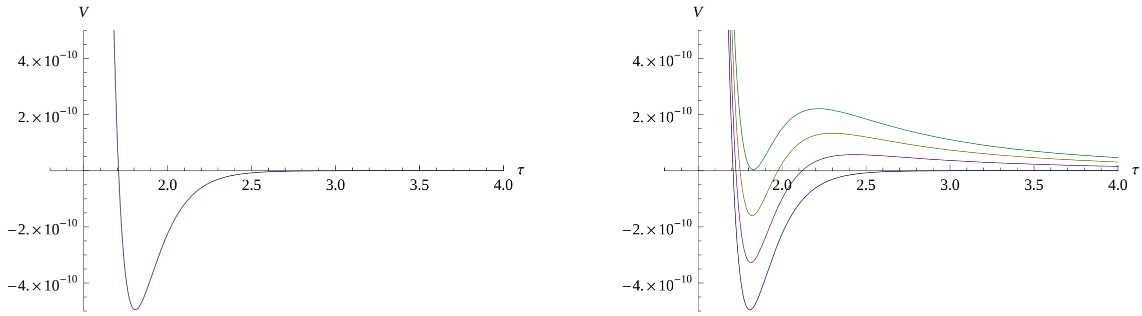
Now we assume the tree-level no-scale Kähler potential for the Kähler moduli:

$$K = -3\log(T + \overline{T}) \tag{2.62}$$

and a non-perturbative correction to the superpotential of the form

$$W = W_0 + Ae^{-aT}, \tag{2.63}$$

where a can be fractional and thus can account for both, instantons and gaugino condensates. For the moment, we ignore the axion belonging to the complex scalar T. From (2.31) it is obvious that there exists a supersymmetric minimum in the scalar potential if $D_T W =$



(a) AdS minimum before uplift (b) dS minimum after uplift

Figure 2.1.: Minima in the KKLT construction

$\partial_T W + \partial_T K\, W = 0$. One easily finds the solution

$$W_0 = -Ae^{-a\tau_0}\left(1 + \frac{2}{3}a\tau_0\right) \tag{2.64}$$

and the vacuum energy turns out to be negative:

$$V_0 = -\frac{a^2 A^2 e^{-2a\tau_0}}{6\tau_0}. \tag{2.65}$$

Thus we have found a supersymmetry preserving AdS vacuum. A plot of the scalar potential can be seen in figure 2.1(a). In general, fluxes generate a value of $W_0 \sim \mathcal{O}(1)$ in the superpotential. From (2.64) it is clear that this implies $a\tau_0 \sim \mathcal{O}(1)$ which comes into conflict with the supergravity approximation where $\tau \gg 1$ is required. If the compactification manifold supports a large enough number of three-cycles, the discretuum of fluxes becomes dense enough to allow also for much smaller values of W_0 [19]. Moreover, in chapter 5 we will see how so-called racetrack potentials can generate a small value for W_0 without fine-tuning.

Uplifting to de Sitter vacua

In [86] it was shown that anti D3-branes contribute positively to the vacuum energy:

$$\delta V = 2\frac{a_0^4 T_3}{g_s^4}\frac{1}{\tau^3}. \tag{2.66}$$

The inclusion of these objects may uplift the anti de Sitter minimum of the last section to a minimum with positive vacuum energy. As anti D3-branes preserve a different set of supersymmetries as the background, supersymmetry will be broken in this construction by an amount proportional to the brane tension.

The branes do not introduce new translational moduli as the ISD flux generates a potential for the world-volume scalars. In fact they are driven to the tip of the Klebanov–Strassler throat. This is reflected in the value of a_0 in (2.66) which will be exponentially small. Its value can be tuned discretely by different choices of flux quanta.

In figure 2.1(b) the uplifting of the AdS minimum is illustrated for different values of a_0. Apparently, the position of the minimum for the Kähler modulus is nearly not affected. This guarantees that we stay in a large volume regime also after the uplift procedure, if this was the case before.

The plot of the scalar potential shows also that the de Sitter minimum is only metastable. The lifetime of the minimum before decompactifying to ten-dimensional Minkowski space should be at least as long as the lifetime of the universe in order to be a sensible model for the phenomenological description of the current state of accelerated expansion of space. Computations in the thin-wall approximation as well as the no-wall approximation show that the lifetime is as long as

$$t_{\text{decay}} \approx e^{10^{122}}, \tag{2.67}$$

measured in multiples of the Planck time. Thus, for all practical purposes, the de Sitter minimum can be considered as stable.

3. The LARGE Volume Scenario

In section 2.8 we got to know the KKLT construction as a proposal how to stabilize all moduli in a string compactification. In particular the Kähler moduli received a mass upon including non-perturbative effects like those of D-brane instantons or gaugino condensates, whereas the complex structure moduli, the dilaton and also the transversal D-brane moduli were frozen by a tree-level effect, namely three-form fluxes.

A shortcoming of the KKLT construction is the fact that an unnaturally small value of the flux superpotential $W_0 \ll 1$ is required in order to be in a regime where the supergravity approximation is valid. However, the three-form flux induces a value for W_0 of rather $\mathcal{O}(1)$. In view of the plethora of string vacua there certainly exist compactifications where the flux superpotential takes a value in the range suitable for KKLT accidentally, but clearly this is not the generic situation.

Instead of relying on non-perturbative effects only for the stabilization of the Kähler moduli, it suggests itself to revisit if perturbative corrections to the low-energy effective supergravity action could also do the job. We argued already that due to its holomorphy, corrections to the superpotential are highly constrained and in fact, due to strong non-renormalization theorems, there are no perturbative corrections to it. However, we will see in section 3.1 that the Kähler potential — a non-holomorphic quantity — is subject to perturbative corrections in both expansion parameters, g_s and α', which will also break the no-scale structure of the Kähler moduli [87]. It will turn out that in particular the α'-correction competes with the non-perturbative corrections to the superpotential in such a way that under certain circumstances, the volume of the compactification manifold is driven to exponentially large values [30]. Compactifications of this type go under the name "LARGE volume scenario" (LVS).

Blowing up the compactification manifold usually has the effect that all two- and four-cycles of the manifold also grow (as on a torus for example). This is however unfavorable as the size of such a cycle determines the coupling constant of the four-dimensional gauge theory living on the world-volume of a D-brane filling out the non-compact space and wrapping the cycle in question in the internal space. Hence, too large internal cycles gives rise to too weakly coupled gauge theories in four dimensions. We will see in section 3.3 that there exist also Calabi–Yau manifolds on which the four-cycles correspond to "holes" in the geometry. They can be kept small when growing the rest of the manifold and therefore, the aforementioned problem does not arise. For obvious reasons, this kind of manifold was given the name "swiss cheese" manifolds and they are an important ingredient of the LARGE volume scenario.

Besides the fact that the LVS does not rely on a small value of the flux-induced superpotential (and is in that respect more generic than the KKLT construction), it has a lot of very appealing particle physical and also astrophysical properties. These will be explored in detail in the later sections of this chapter.

3.1. Perturbative Corrections in α' to the Kähler potential

We start with perturbative α'-corrections to the Kähler potential. These have been explored in [87]. The corrections arise upon taking into account higher derivative terms in the ten-dimensional effective action (2.6). There is no complete calculation of all higher order interactions (see [88] for an overview), however we are interested only in those giving corrections to the Kähler moduli space and thus finally to the Kähler potential of the $\mathcal{N}=1$ effective action. The relevant additional two terms to the ten-dimensional $\mathcal{N}=2$ supergravity action were shown to be [87]:

$$\Delta S = -\frac{1}{2\kappa_{10}} \int \mathrm{d}^{10}x\, e^{-2\phi} \alpha'^3 \left(\frac{\zeta(3)}{3 \cdot 2^{11}} J_0 + (\nabla^2\phi) Q \right), \tag{3.1}$$

where ζ is the Riemann zeta function, J_0 is a particular contraction of the Riemann–Christoffel tensor $R^M{}_{NOP}$ and a generalization of the Euler density, denoted by E_8:

$$J_0 = t^{M_1 N_1 \ldots M_4 N_4} t_{M_1' N_1' \ldots M_4' N_4'} R^{M_1' N_1'}{}_{M_1 N_1} \ldots R^{M_4' N_4'}{}_{M_4 N_4} + \frac{1}{4} E_8. \tag{3.2}$$

The second summand in (3.1) does not modify the ten-dimensional equations of motion to order $\mathcal{O}(\alpha'^3)$ but it is necessary in the derivation of the four-dimensional effective action. It contains a generalization of the six-dimensional Euler integrand: $\int_{\mathcal{X}} \mathrm{d}^6x\, \sqrt{g} Q = \chi(\mathcal{X})$.

The most important point one should bear in mind here is that the higher derivative terms determine the α'^3-corrections to the ten-dimensional effective action. They encode geometrical information of the compactification manifold in form of the Euler characteristic.

When compactifying the ten-dimensional effective action including the corrections (3.1) we get a four-dimensional $\mathcal{N}=2$ supersymmetric action which has, compared to the original case (2.12), a modified metric of the moduli space of the Kähler deformations. Finally we are interested again in a compactification on an orientifold of a Calabi–Yau manifold and thus, the perturbative corrections to the hypermultiplet action have to be translated into appropriate type IIB variables and truncated to an $\mathcal{N}=1$ sub-sector.

This procedure is rather involved. A notable consequence is that a constant dilaton in the internal space is not a solution to the equations of motion anymore. As a consequence, upon compactifying to four dimensions, where we have to integrate over the internal space, the four-dimensional dilaton is corrected. In string frame it now reads:

$$e^{-2\phi} = e^{-2\phi_0}\big(\mathcal{V} + \tfrac{1}{2}\xi\big), \tag{3.3}$$

with $\mathcal{V}$ denoting the volume of the internal space and $\xi = -\frac{1}{2}\chi(\mathcal{X})\zeta(3)$. As we will work mainly in Einstein frame later, we give here the same relation, transformed to Einstein frame:

$$e^{-2\phi} = e^{-\phi_0/2}\big(\widehat{\mathcal{V}} + \tfrac{1}{2}\hat{\xi}\big), \tag{3.4}$$

where we defined $\widehat{\mathcal{V}} = \mathcal{V}e^{-3\phi_0/2}$ and $\hat{\xi} = \xi e^{-3\phi_0/2}$.

Using all redefined variables as well as the modified dilaton solution, one finally can derive the $\mathcal{N} = 1$ Lagrangian, allowing to read off the corrected Kähler potential. It takes the form:

$$\mathcal{K} = -2\ln\left(\widehat{\mathcal{V}} + \frac{\xi}{2}\left(\frac{-i(\tau-\bar{\tau})}{2}\right)^{3/2}\right) - \ln\big(-i(\tau-\bar{\tau})\big) - \ln\Big(-i\int_{\mathcal{X}} \Omega\wedge\overline{\Omega}\Big). \tag{3.5}$$

Though it would be consequential to derive the perturbative corrections in the string coupling g_s to the Kähler potential at this stage and then continue to analyze the minima of the resulting scalar potential, we postpone the string loop corrections to section 3.4 because we can understand their effects in a better way, when we know already the stabilization mechanism of the LVS.

3.2. The Scalar Potential

We consider now a flux compactification on an orientifold of a Calabi–Yau manifold and include non-perturbative corrections to the superpotential as well as the α'-corrections to the Kähler potential. The effective $\mathcal{N} = 1$ theory for the closed string sector is thus given by the Kähler potential (3.5) and the superpotential

$$W = W_{\text{GVW}} + W_{\text{n.p.}} = \int_X G_3 \wedge \Omega_3 + \sum_n A_n e^{-a_n T_n}. \tag{3.6}$$

The non-perturbative contribution to the superpotential can either stem from a D3-brane instanton or from gaugino condensation. Both can be accounted for in the expression (3.6) by choosing the factor a_n to be 2π or $2\pi/N_c$ respectively.

The (string frame) volume of the compactification space can be written generically in terms of two-cycle volumes t^i and the triple intersection form κ_{ijk}:

$$\mathcal{V} = \int_{\mathcal{X}} J^3 = \frac{1}{6}\kappa_{ijk}t^i t^j t^k. \tag{3.7}$$

There exists a relation to the dual four-cycle volumes:

$$\tau_i = \partial_{t_i}\mathcal{V} = \frac{1}{2}\kappa_{ijk}t^k t^k. \tag{3.8}$$

In the class of compactification spaces we consider, eq. (3.8) can be inverted and plugged into (3.7) so that we can express the volume in (3.5) directly in terms of four-cycle volumes. This is advantageous since the four-cycle volumes are the real parts of the Kähler variables $T_i = \tau_i + ib_i$ entering the effective supergravity action, making it easy to calculate the Kähler metric.

It is clear that α'-corrections in (3.5) will modify the inverse Kähler metric for the Kähler moduli $\mathcal{K}^{T_a\overline{T}_b}$ such that in addition to the non-perturbative corrections to the superpotential they furnish another source for the breaking of the no-scale structure. The new expression for the Kähler metric was first derived in [89]. It can be expressed in terms of the Hessian matrix of the volume function depending on the four-cycle volumes τ_i [37]. In Einstein frame it reads:

$$\mathcal{K}^{\tau_a\tau_b} = -2\left(\widehat{\mathcal{V}} + \frac{\hat{\xi}}{2}\right)\left(\frac{\partial^2\widehat{\mathcal{V}}}{\partial\tau_a\partial\tau_b}\right)^{-1} + \tau_a\tau_b\frac{4\widehat{\mathcal{V}} - \hat{\xi}}{\widehat{\mathcal{V}} - \hat{\xi}}. \tag{3.9}$$

We are now in a position to start to derive the scalar F-term potential (2.31). We assume that the complex structure moduli and the dilaton are stabilized via the conditions $D_U W = D_S W = 0$ and therefore can be integrated out.[1] Using (3.9), the following expression for F-term potential arises:

$$\begin{aligned} V_F = e^{\mathcal{K}}\Big[&\mathcal{K}^{T_i\overline{T}_j}\big(a_iA_ia_j\bar{A}_je^{-(a_iT_i+a_j\overline{T}_j)} - (a_iA_ie^{-a_iT_i}\overline{W}\partial_{\overline{T}_j}\mathcal{K} + a_j\bar{A}_je^{-a_j\overline{T}_j}W\partial_{T_i}\mathcal{K})\big) \\ &+ 3\xi\frac{(\xi^2+7\xi\mathcal{V}+\mathcal{V}^2)}{(\mathcal{V}-\xi)(2\mathcal{V}+\xi)^2}|W|^2\Big] \\ =:\ & V_{\text{np1}} + V_{\text{np2}} + V_{\alpha'}, \end{aligned} \tag{3.10}$$

where for the sake of legibility, we dropped the "hats" over the symbols $\mathcal{V}$ and ξ. All quantities are understood to be in Einstein frame. Now we argue that there exists a minimum at exponentially large volume with negative cosmological constant. Therefore we do not minimize the full scalar potential (3.10) but expand each of the three terms in the large $\mathcal{V}$ limit and minimize the simplified potential.[2] One might be tempted to argue that the non-perturbative terms V_{np1} and V_{np2} are exponentially suppressed and can be neglected compared to the perturbative α' term. We assume now that the compactification space

1 In section 3.5 we will give a justification for this approach.

2 This procedure can also be cross-checked by numerically minimizing the full potential. Doing so one finds the same minimum at exponentially large volume.

is such that the overall volume can grow arbitrarily whereas at least one of the four-cycle volumes can remain relatively small. Then the non-perturbative terms can compete with the perturbative α' term. To simplify matters, we assume that there is just one such small four-cycle and denote its volume by τ_s. All other four-cycle volumes are assumed to grow with the volume, i. e. $\tau_i \to \infty$.

We start with V_{np1} in the large volume limit:

$$V_{\text{np1}} = e^{\mathcal{K}} \mathcal{K}^{T_i \overline{T}_j} a_i A_i a_j \bar{A}_j e^{-(a_i T_i + a_j \overline{T}_j)}. \tag{3.11}$$

It is clear that all terms involving the large four-cycles are exponentially suppressed compared to the term with the small four-cycle. Hence it is sufficient to keep only the term with $i = j = s$:

$$V_{\text{np1}} = e^{\mathcal{K}} \mathcal{K}^{T_s \overline{T}_s} a_s^2 |A_s|^2 e^{-2a_s \tau_s}. \tag{3.12}$$

In the large volume limit the Kähler metric (3.9) simplifies to:

$$\mathcal{K}^{T_s \overline{T}_s} = -2\mathcal{V} \left(\frac{\partial^2 \mathcal{V}}{\partial \tau_s \partial \tau_s} \right)^{-1}. \tag{3.13}$$

The volume is a homogeneous function of degree $3/2$ of the four-cycle volumes. Thus we can expect the entry of the Hessian matrix to be of degree $1/2$. Moreover it must be negative definite such that the whole expression (3.13) is positive. This is because (3.12), before inserting the explicit form of the superpotential, reads $K^{T_s \overline{T}_s} \partial_{T_s} W \partial_{\overline{T}_s} \overline{W}$; it is the length of the vector $\partial_{T_s} W$ in terms of the Kähler metric. Provided we are inside the Kähler cone, this must be positive definite. We write (3.13) symbolically as

$$\mathcal{K}^{T_s \overline{T}_s} = \mathcal{V} \sqrt{\tau_s}, \tag{3.14}$$

having in mind that at this stage we need only the correct scaling behavior in the large volume limit. Finally we can expand the Kähler potential (3.5) itself and find the approximation:

$$e^{\mathcal{K}} \xrightarrow[\mathcal{V} \to \infty]{} \frac{e^{\mathcal{K}_{\text{c.s.}}}}{\mathcal{V}^2}, \tag{3.15}$$

where $\mathcal{K}_{\text{c.s.}} = -\ln\big(-i(\tau - \bar{\tau})\big) - \ln\Big(-i \int_{\mathcal{X}} \Omega \wedge \overline{\Omega}\Big)$. Putting all approximations together we finally find:

$$V_{\text{np1}} \approx \frac{a_s^2 |A_s|^2 \sqrt{\tau_s} e^{-2a_s \tau_s} e^{\mathcal{K}_{\text{c.s}}}}{\mathcal{V}} + \mathcal{O}\left(\frac{e^{-2a_s \tau_s}}{\mathcal{V}^2} \right). \tag{3.16}$$

Now we come to V_{np2}. Also here we have to keep only the term involving the small four-cycle τ_s:

$$\begin{aligned} V_{\mathrm{np2}} &= -e^{\mathcal{K}}\mathcal{K}^{T_s\overline{T}_s}\big(a_sA_se^{-a_sT_s}\overline{W}\partial_{\overline{T}_s}\mathcal{K} + a_s\bar{A}_se^{-a_s\overline{T}_s}W\partial_{T_s}\mathcal{K}\big) \\ &= -e^{\mathcal{K}}\mathcal{K}^{T_s\overline{T}_s}\big(a_sA_se^{-a_s\tau_s}\overline{W}\partial_{\overline{T}_s}\mathcal{K}e^{ia_s\tau_s} + c.c.\big) \\ &= 2\,\mathrm{Re}\big(-e^{\mathcal{K}}\mathcal{K}^{T_s\overline{T}_s}a_sA_se^{-a_s\tau_s}\overline{W}\partial_{\overline{T}_s}\mathcal{K}e^{ia_s\tau_s}\big). \end{aligned} \tag{3.17}$$

This expression is the only term where the axion a_s appears. Thus when we minimize the F-term potential with respect to a_s, its value will be set such that V_{np2} is rendered real and negative. From (3.5) it is easy to see that symbolically:

$$\partial_{\overline{T}_s}\mathcal{K} = -\frac{2}{\mathcal{V}+\frac{\xi}{2}}\frac{\partial\mathcal{V}}{\partial\overline{T}_s} \xrightarrow[\mathcal{V}\to\infty]{} -\frac{2\sqrt{\tau_s}}{\mathcal{V}}, \tag{3.18}$$

where we used again the expected scaling behavior of the volume depending on the four-cycle volumes. We insert now this and also the expression for $\mathcal{K}^{T_s\overline{T}_s}$ (3.14) as well as the approximation (3.15) in (3.17) and obtain the following result:

$$V_{\mathrm{np2}} \approx -\frac{a_s\tau_se^{-a_s\tau_s}}{\mathcal{V}^2}|A_sW_0|e^{\mathcal{K}_{\mathrm{c.s}}} + \mathcal{O}\left(\frac{e^{-a_s\tau_s}}{\mathcal{V}^3}\right), \tag{3.19}$$

where we used that the flux part dominates the superpotential (3.6) giving a vev of order one: $W \approx W_0 \sim \mathcal{O}(1)$ such that the non-perturbative contributions can be neglected here. We also dropped numerical prefactors.

In the large volume limit, the α' term in (3.10) scales like:

$$V_{\alpha'} \approx \frac{\xi}{\mathcal{V}^3}|W_0|^2e^{\mathcal{K}_{\mathrm{c.s.}}} + \mathcal{O}\big(\mathcal{V}^{-4}\big). \tag{3.20}$$

The complete F-term potential in the large volume limit is thus given by:

$$V_F = \frac{a_s^2|A_s|^2\sqrt{\tau_s}e^{-2a_s\tau_s}e^{\mathcal{K}_{\mathrm{c.s}}}}{\mathcal{V}} - \frac{a_s|A_sW_0|\tau_se^{-a_s\tau_s}}{\mathcal{V}^2}e^{\mathcal{K}_{\mathrm{c.s}}} + \frac{\xi}{\mathcal{V}^3}|W_0|^2e^{\mathcal{K}_{\mathrm{c.s.}}} + \ldots \tag{3.21}$$

Now we choose a particular direction in the Kähler moduli space in which the minimum at large volume of (3.21) becomes obvious. This direction is given by:

$$\mathcal{V} \to \text{LARGE} \qquad \text{and} \qquad a_s\tau_s = \ln(\mathcal{V}). \tag{3.22}$$

If we drop in addition the common prefactor $e^{\mathcal{K}_{\mathrm{c.s.}}}$, in this limit the F-term potential (3.21) reads:

$$V_F = \frac{a_s^{3/2}|A_s|^2\sqrt{\ln(\mathcal{V})}}{\mathcal{V}^3} - \frac{|A_sW_0|\ln(\mathcal{V})}{\mathcal{V}^3} + \frac{\xi}{\mathcal{V}^3}|W_0|^2. \tag{3.23}$$

All terms scale with $\mathcal{V}^{-3}$. However the second one has the strongest dependence on $\mathcal{V}$ in the numerator. Thus it will overcome the first and the third term in the limit $\mathcal{V} \to \infty$. Consequently in this limit, the potential approaches zero from below. On the other hand for small values of $\mathcal{V}$, either the first or the third term will dominate the potential, depending on the value of τ_s. In order to guarantee the existence of a minimum we have to require now that the third term is positive definite. This is the case if $\xi > 0$ and thus $\chi(\mathcal{X}) < 0$ which is equivalent to the condition $h^{(2,1)} > h^{(1,1)}$. Then it is ensured that at small values of $\mathcal{V}$ the value of the potential is positive.[3] The conclusion is that there must be an anti de Sitter minimum at large values of $\mathcal{V}$. Coming from smaller values of the volume we reach the minimum as soon as the second term in (3.23) becomes dominant. This happens for "large" values of $\ln(\mathcal{V})$ which means that we can expect the minimum to be located at exponentially large volume.

So far, we found a minimum in one particular direction of the moduli space given by $a_s\tau_s = \ln(\mathcal{V})$. It remains to be shown that this is also a minimum in all other directions and not a saddle point. This can be seen when looking at the full scalar potential (3.21) again: for a fixed value of $\mathcal{V}$, as we decrease the value of other four-cycles, the first term will start to dominate as it has the lowest power of the volume in the denominator and the exponential suppression decreases.[4] Since this is a positive definite term, the value of the potential grows at the boundary of the Kähler cone and we are driven back to the AdS minimum at large volume. On the other hand if we increase various four-cycle volumes, the exponential suppression of the first two terms increases and the positive third term starts to dominate, resulting in an increase of the value of the scalar potential.

Let us now study a few properties of this newly found minimum at large volume. First of all, the value of the scalar potential in the minimum is of the order $V_F \sim \mathcal{O}(\mathcal{V}^{-3})$. The general expression for the F-term potential (2.31) tells us that, since the second term $-3e^{\mathcal{K}}|W|^2$ is $\sim \mathcal{O}(\mathcal{V}^{-2})$, consequently $D_{T_s}W \neq 0$ and thus supersymmetry is broken spontaneously. This is in contrast to the KKLT scenario where the minimum before uplifting to de Sitter is supersymmetric. Another interesting property of the LVS distinguishing it from the KKLT scenario is the stability of the gravitino mass: whereas in KKLT the particular relation between the value of the flux superpotential and the volume (i. e. the Kähler modulus T) is given by (2.64), leading to a direct dependence of the gravitino mass $m_{3/2} = e^{\mathcal{K}/2}|W|$ on the choice of fluxes, in the LVS the scaling behavior is precisely such that the dependence on

3 For the case $\xi < 0$, the existence of a minimum at large volume is not excluded. Only the behavior at small values of $\mathcal{V}$ is not obvious and we cannot directly assume that the scalar potential is positive there.

4 In the expression (3.21) there is only one term of this form because we assumed only one modulus to be small from the beginning. Remember that in principle there is such a term for each modulus which cannot be neglected anymore if the corresponding four-cycle volume becomes small.

W_0 is canceled. This can easily be seen if we define $\widetilde{\mathcal{V}}:=\frac{|A_s|\mathcal{V}}{W_0}$ and rewrite (3.21):

$$V_F \sim \left(\frac{A_s^3}{W_0}\right)\left[\frac{a_s^2\sqrt{\tau_s}e^{-2a_s\tau_s}}{\widetilde{\mathcal{V}}} - \frac{a_s\tau_s e^{-a_s\tau_s}}{\widetilde{\mathcal{V}}^2} + \frac{\xi}{\widetilde{\mathcal{V}}^3}\right]. \tag{3.24}$$

The minimum of (3.24) as a function of $\widetilde{\mathcal{V}}$ is independent of W_0 and we can write:

$$\mathcal{V} \sim \frac{W_0}{A_s} f(a_s, \mathcal{X}) + \text{sub-leading terms}, \tag{3.25}$$

where f is function depending on the geometry of the compactification space $\mathcal{X}$. The gravitino mass is then:

$$m_{3/2} = e^{\mathcal{K}/2}|W| \approx \frac{A_s}{f(a_s, \mathcal{X})}, \tag{3.26}$$

independent from W_0. As the gravitino mass sets the SUSY breaking scale it is an important phenomenological parameter. Concerning this point the LVS can be said to be more predictive than the KKLT scenario.

The anti de Sitter minimum can be uplifted to de Sitter one by the same mechanisms proposed for the KKLT scenario: anti D3-branes or also magnetic fluxes on D7-branes [90]. A possible uplift potential is given by:

$$V_{\text{uplift}} = \frac{\epsilon}{\mathcal{V}^2}. \tag{3.27}$$

Notably as it scales with $\mathcal{V}^{-2}$, quite an amount of fine-tuning is required in order not to destroy the large volume minimum.

3.3. The $\mathbb{P}^4_{[1,1,1,6,9]}[18]$ example

In this section be briefly check the existence of the large volume minimum on an explicit example. We consider as compactification space the degree 18 hypersurface in the weighted projective space $\mathbb{P}^4_{[1,1,1,6,9]}$. Some of its geometric properties were studied in [91]. It has two Kähler moduli and 272 complex structure moduli. This meets the requirement of a negative Euler characteristic $\chi = 2(h^{(1,1)} - h^{(2,1)}) = -540$. The volume given in terms of four-cycle volumes reads:

$$\mathcal{V} = \frac{1}{9\sqrt{2}}\left(\tau_b^{3/2} - \tau_s^{3/2}\right). \tag{3.28}$$

One important property of this space is that the overall volume is controlled by one single modulus: the value of $\mathcal{V}$ can be made arbitrarily large by increasing τ_b while leaving τ_s at small values. On the other hand, an increase of the four-cycle volume τ_s decreases the overall volume. Its value is bounded from above since the volume cannot be negative. One can imagine the geometry as a cheese with holes. The parameter τ_b controls the size of the piece of cheese whereas τ_s describes the size of a hole. Inspired by this example, we call any manifold with a relation between the overall volume and the four-cycle volumes of the following form:

$$\mathcal{V} = (\eta_b\tau_b)^{3/2} - \sum_i (\eta_i\tau_i)^{3/2} \tag{3.29}$$

a *swiss cheese* manifold.

Having at hand the explicit expression for the volume (3.28) it is easy to calculate the Kähler metric by inserting it into (3.5) and taking the derivative of the resulting expression twice. As the four-cycle volumes appear with the power $3/2$, in the large volume limit, the (inverse) Kähler metric has indeed the form we assumed already in (3.14) and (3.18). Together with the superpotential

$$W = W_0 + A_s e^{-a_s T_s} \tag{3.30}$$

we find indeed a scalar potential of the anticipated form:

$$V_F = \frac{\lambda\sqrt{\tau_s}e^{-2a_s\tau_s}}{\mathcal{V}} - \frac{\mu}{\mathcal{V}^2}\tau_s e^{-a_s\tau_s} + \frac{\nu}{\mathcal{V}^3} \tag{3.31}$$

with prefactors:

$$\lambda = 3\sqrt{2}a_s^2|A_s|^2, \qquad \mu = \frac{1}{2}a_s|A_sW_0|, \qquad \nu = \xi|W_0|^2, \qquad \xi = 1.31. \tag{3.32}$$

Unfortunately it is not possible to find the minimum of (3.31) analytically. One has to apply further approximations. We start by setting to zero the derivative of V_F with respect to $\mathcal{V}$:

$$\frac{\partial V_F}{\partial \mathcal{V}} = -\frac{\lambda\sqrt{\tau_s}e^{-2a_s\tau_s}}{\mathcal{V}^2} + \frac{\mu}{\mathcal{V}^3}\tau_s e^{-a_s\tau_s} - \frac{\nu}{\mathcal{V}^4} \stackrel{!}{=} 0. \tag{3.33}$$

This easily can be solved for $\mathcal{V}$:

$$\mathcal{V} = \frac{\mu}{\lambda}\sqrt{\tau_s}e^{-a_s\tau_s}\left(1 \pm \sqrt{1 - \frac{3\nu\lambda}{\mu^2\tau_s^{3/2}}}\right). \tag{3.34}$$

From taking the derivative with respect to τ_s we get the equation:

$$\frac{\partial V_F}{\partial \tau_s} = \frac{\lambda\mathcal{V}e^{-a_s\tau_s}}{\tau_s^{1/2}}\left(\frac{1}{2} - 2a_s\tau_s\right) - \mu(1 - a_s\tau_s) \stackrel{!}{=} 0. \tag{3.35}$$

Here we insert (3.33) for the volume and obtain an implicit equation for the small modulus:

$$\left(1 \pm \sqrt{1 - \frac{3\nu\lambda}{\mu^2 \tau_s^{3/2}}}\right)\left(\frac{1}{2} - 2a_s\tau_s\right) = (1 - a_s\tau_s). \tag{3.36}$$

This equation cannot be solved for τ_s analytically. Now we require that $a_s\tau_s \gg 1$ in order to be able to ignore higher instanton corrections. We can use this condition to simplify (3.36):

$$1 \pm \sqrt{1 - \frac{3\nu\lambda}{\mu^2 \tau_s^{3/2}}} = \frac{1}{2}. \tag{3.37}$$

Now, this can be easily solved for τ_s. Inserting the resulting expression into (3.34) gives then also the solution for the volume:

$$\tau_s = \left(\frac{4\nu\lambda}{\mu^2}\right)^{2/3}, \qquad \mathcal{V} = \frac{\mu}{2\lambda}\left(\frac{4\nu\lambda}{\mu^2}\right)^{2/3} e^{a_s\left(\frac{4\nu\lambda}{\mu^2}\right)^{2/3}}. \tag{3.38}$$

If we restore the prefactors according to (3.32) we can read off the scaling behavior of the large volume minimum depending on the parameters of the model:

$$\tau_s \sim (4\xi)^{2/3}, \qquad \mathcal{V} \sim \frac{\xi^{1/3}|W_0|}{a_s A_s} e^{a_s\tau_s}. \tag{3.39}$$

We see that the volume scales exponentially with the geometry depending parameter ξ. Moreover it scales linearly in W_0, making the gravitino mass independent of the choice of flux, as already anticipated in section 3.2.

It is interesting to insert typical values for all parameters. We assume that the non-perturbative superpotential is generated by D-brane instantons, hence $a_s = 2\pi$. For $A_s = 1$, $W_0 = 10$ and $g_s = 1/10$ one obtains $\mathcal{V} \approx 3 \cdot 10^{10}$, a gravitino mass of $m_{3/2} \approx 10^9$ GeV and a string scale of $m_s = m_P g_s/\sqrt{\mathcal{V}} \approx 10^{12}$ GeV. This is an explicit realization of the intermediate string scale scenario without fine-tuning, which has been claimed to have some phenomenological virtues [92, 93].

Let us summarize the most important properties of the LARGE volume scenario, especially in what aspects it is different from the KKLT construction: we have shown that there exists a minimum of the scalar potential in the limit where the overall volume is exponentially large, whereas one modulus scales like $a\tau_s \approx \ln\mathcal{V}$. In this limit, the non-perturbative corrections to the superpotential compete with α'-corrections to the Kähler potential. The latter were not taken into account in the KKLT construction. The LARGE volume minimum exists on a quite wide class of compactification manifolds. A necessary condition is that the overall volume can be blown up independently from at least one four-cycle volume. A prototype class of manifolds with this property was introduced and given the name "swiss cheese manifolds". A sufficient condition for the existence of the minimum is then a negative Euler characteristic ($h^{(2,1)} > h^{(1,1)} > 1$).

As we have an exponentially large volume, this scenario naturally gives rise to a scenario where the fundamental string scale can be hierarchically smaller than the Planck scale since $m_s \sim m_P/\sqrt{\mathcal{V}}$. In the minimum, supersymmetry is broken and the gravitino mass is unaffected by the value of W_0, i. e. not dependent of the choice of fluxes. This is in contrast to KKLT, where the minimum of the potential supersymmetric. SUSY is broken explicitly by introducing anti D3-branes, which furnish also as uplift mechanism to a de Sitter minimum. There is no need for fine-tuning any parameters, especially not the flux superpotential W_0 which has to be as small as 10^{-4} in the KKLT construction.

3.4. String Loop Corrections to the Kähler Potential

As the Kähler potential is not protected by non-renormalization theorems like the superpotential and also not constrained by holomorphy, it is natural to ask, whether further perturbative corrections might compete with the α'-corrections. Here we focus on those perturbative corrections in g_s which arise in the presence of sources (D-branes and orientifold planes). Indeed, the string loop corrections to the Kähler potential scale as $\mathcal{O}(g_s^2\alpha'^2)$ in string frame. By dimensional analysis we expect therefore that in the $\mathcal{V}^{-1}$ expansion in Einstein frame they scale as $\Delta\mathcal{K} \sim (g_s\mathcal{V}^{-2/3})$ as opposed to the α'-corrections which a priory imply a correction of $\Delta\mathcal{K} \sim \mathcal{O}(g_s^{-3/2}\mathcal{V}^{-1})$. Hence, naïvely one would await that the g_s-corrections even dominate the α'-corrections at large volume. However it will turn out when calculating the F-term potential that certain cancellations occur such that we are left with a term $\Delta V_{g_s} \sim \mathcal{O}(g_s\mathcal{V}^{-3})$, to be compared with the α'-term $\Delta V_{\alpha'} \sim \mathcal{O}(g_s^{-1/2}\mathcal{V}^{-3})$. Still, if g_s is at not too small values, the loop corrections could compete with the α'-corrections and significantly modify the form of the potential. Unavoidably we have to study in detail the influence of these corrections.

Unfortunately the corrections we want to study are not known for the case of a smooth Calabi–Yau orientifold. They are known however for toroidal orbifolds, as in this case, explicit string amplitude calculations can be applied [94]. Assuming a similar scaling behavior of the corrections, we can carry over the results to the more general case.

String loop corrections stem from Kaluza–Klein (K) modes between D7-branes, O7-planes, D3-branes or O3-planes. Moreover, winding strings (W) can be exchanged between intersecting stacks of D7-branes (or between D7-branes and O7-planes) if there are non-contractible one-cycles within the intersection locus. The Kähler potential capturing these effects at lowest order on a toroidal orientifold was shown to be [94, 95]:

$$\begin{aligned}\mathcal{K} &= -2\ln\left(\mathcal{V} + \xi\left(\frac{-i(\tau-\bar{\tau})}{2}\right)^{3/2}\right) + \mathcal{K}_{\text{c.s.}} + \sum_{i=1}^{3} g_{\text{s}}\frac{\mathcal{E}_i^{(\text{K})}(U,\overline{U})}{4\tau_i} + \sum_{i\neq j\neq k}^{3}\frac{\mathcal{E}_k^{(\text{W})}(U,\overline{U})}{4\tau_i\tau_j}\\ &\approx -2\ln(\mathcal{V}) - \frac{\xi}{\mathcal{V}} + \sum_{i=1}^{3} g_{\text{s}}\frac{\mathcal{E}_i^{(\text{K})}(U,\overline{U})}{4\tau_i} + \sum_{i\neq j\neq k}^{3}\frac{\mathcal{E}_k^{(\text{W})}(U,\overline{U})}{4\tau_i\tau_j} + \mathcal{K}_{\text{c.s.}},\end{aligned} \tag{3.40}$$

where the prefactors $\mathcal{E}^{(\text{K},\text{W})}$ in principle do depend on the complex structure moduli U. However, the precise functional dependence is not relevant here.

Given the expression (3.40) it suggests itself that also in the general case of a smooth Calabi–Yau orientifold, there will appear terms which are suppressed by a single Kähler modulus:

$$\Delta\mathcal{K}_{g_{\text{s}}} \sim g_{\text{s}}\frac{\mathcal{E}}{\tau_i}. \tag{3.41}$$

Clearly such terms would dominate the term induced by the α'-correction $\xi/\mathcal{V}$ for $i \neq b$. However we have to bear in mind that the torus is very special concerning the dependence of the overall volume on the two- and four-cycle volumes. Indeed the volume can be written as $\mathcal{V} \sim \tau_i t_i$, where no summation over i is understood. This is not compatible with the swiss cheese form (3.29). Consequently the authors of [95] proposed that the new terms in the Kähler potential for the case of swiss cheese manifolds do not scale as in (3.40) but rather like:

$$\Delta\mathcal{K}_{g_{\text{s}}} \sim \sum_a g_{\text{s}}\frac{g_{\text{K}}^a(t,g_{\text{s}})\mathcal{E}_a^{(\text{K})}}{\mathcal{V}} + \sum_q \frac{g_W^q(t,g_{\text{s}})\mathcal{E}_q^{(W)}}{\mathcal{V}}, \tag{3.42}$$

accounting for the special form of intersection numbers.[5] The dependence on the complex structure moduli cannot be obtained by analogy arguments from the toroidal case. It is encoded in the unknown functions $\mathcal{E}(U,\overline{U})$. From (3.42) it is clear that it depends on the so far not specified functions $g_{\text{K}}^a(t,g_{\text{s}})$ and $g_{\text{W}}^q(t,g_{\text{s}})$, whether or not the g_{s}-corrections will dominate the α'-corrections. They depend linearly or reciprocally on a linear combination of the two-cycle volumes $t = \sum_i c_i t_i$ respectively. This linear combination can contain the dual of the large four-cycle $\tau_b \sim \mathcal{V}^{2/3}$ and this fixes the scaling with respect to $\mathcal{V}$ of the terms in question and ultimately if they dominate or not.

Now we consider again the example of section 3.3, i. e. the $\mathbb{P}^4_{[1,1,1,6,9]}[18]$ hypersurface. In [97] it was show that the four-cycles Γ_b and Γ_s do not intersect. Consequently there will

5 The form of (3.42) was confirmed in [96] by field theoretical reasoning: the requirement that the loop corrected kinetic terms for a Kähler modulus should be suppressed by a factor of g^2 (with g being the coupling constant for the gauge theory on the branes wrapping the four-cycle in question) fixes the scaling of $\mathcal{V}$ and τ in $\Delta\mathcal{K}_{g_{\text{s}}}$.

no corrections from exchange of winding strings. Also neglecting flux corrections to the KK mass spectrum, the Kähler and superpotential we have to consider are of the form:

$$\begin{aligned} \mathcal{K} &= -2\ln(\mathcal{V}) - \frac{\xi}{\mathcal{V}} + g_{\mathrm{s}} \frac{\sqrt{\tau_b}\mathcal{E}_b^{(\mathrm{K})}}{\mathcal{V}} + g_{\mathrm{s}} \frac{\sqrt{\tau_s}\mathcal{E}_s^{(\mathrm{K})}}{\mathcal{V}} + \mathcal{K}_{\mathrm{c.s.}}, \\ W &= W_0 + A_s e^{-a_s T_s}. \end{aligned} \tag{3.43}$$

Given these two functions, we can derive the scalar F-term potential in analogy to (3.10). At leading order in the $\mathcal{V}^{-1}$ expansion we get:

$$\begin{aligned} V_{\mathrm{np}1} &= e^{\mathcal{K}_{\mathrm{c.s.}}} \frac{24 a_s^2 |A_s|^2 \tau_s^{3/2} e^{-2a_s\tau_s}}{\mathcal{V}\Delta}, \\ V_{\mathrm{np}2} &= -e^{\mathcal{K}_{\mathrm{c.s.}}} g_{\mathrm{s}} \frac{2 a_s |A_s W_0| \tau_s e^{-a_s\tau_s}}{\mathcal{V}^2} \left(1 + \frac{6\mathcal{E}_s^{(\mathrm{K})}}{\Delta}\right), \\ V_{\alpha'} &= \frac{3 e^{\mathcal{K}_{\mathrm{c.s.}}} |W_0|^2}{8\mathcal{V}^3} \left(\xi + g_{\mathrm{s}}^2 \frac{4\left(\mathcal{E}_s^{(\mathrm{K})}\right)^2 \sqrt{\tau_s}}{\Delta}\right), \end{aligned} \tag{3.44}$$

where the second term is again rendered negative upon minimizing the axion b_s and we have used the definition

$$\Delta = \sqrt{2} g_{\mathrm{s}}^{-1} \tau_s - 3\mathcal{E}_s^{(\mathrm{K})}. \tag{3.45}$$

The range of validity of this expansion is limited to those regions in the moduli space where the denominator Δ does not become too small.

It is interesting to note that the loop corrections affect $V_{\mathrm{np}1}$ and $V_{\mathrm{np}2}$ at leading order in the $\mathcal{V}^{-1}$ expansion. This is in contrast to the α'-correction, which appears only at order $\mathcal{V}^{-3}$. At first sight this is surprising as both corrections are equally suppressed with $\mathcal{V}^{-1}$ in the Kähler potential (3.43). The reason for this is that the loop correction also explicitly depends on τ_s and not on the overall volume only. The terms containing the loop corrections from the big four-cycle are sub-leading. They show up at order $\mathcal{O}(\mathcal{V}^{-10/3})$:

$$V_{10/3} = 2 g_{\mathrm{s}}^3 \frac{6^{1/3} |W_0|^2 e^{\mathcal{K}_{\mathrm{c.s.}}}}{\mathcal{V}^{10/3}} \left(\left(\mathcal{E}_b^{(\mathrm{K})}\right)^2 + \frac{3}{4} \partial_\alpha \mathcal{E}_b^{(\mathrm{K})} \partial_{\bar\alpha} \mathcal{E}_b^{(\mathrm{K})} \mathcal{K}_{\mathrm{c.s.}}^{\alpha\bar\alpha} \right). \tag{3.46}$$

The volume dependence of all terms of the F-term potential can be summarized by the following rule: firstly the common prefactor $e^{\mathcal{K}}$ gives an overall suppression of $\mathcal{V}^{-2} = \tau_b^{-3}$. For a quantum correction proportional to $\tau_b^{-\lambda}$, a correction appears in $V_{\alpha'}$ at order $1/\tau_b^{\lambda+3}$ for all values of λ, except for $\lambda = 1$. In the latter case, there is a cancellation at leading order and they appear only at order $1/\tau_b^{2+3}$.

The scalar potential (3.44) now has to be minimized by solving $\partial_{\mathcal{V},\tau_s} V_F = 0$. It is not instructive to do an algebraic analysis as we did in the last section for the α'-corrections only.

We consider the results of a numerical minimization procedure done in [95]. There it was found that for a range of the parameters $g_s^{-1} \in [8, 11]$ and $\mathcal{E}_s^{(K)} \in [20, 40]$, the minimum of the F-term potential is described with very good accuracy by

$$\begin{aligned} \log_{10} \mathcal{V} &= 1.720\, g_s^{-1} - 0.1208\, \mathcal{E}_s^{(K)} - 3.437, \\ \tau_s &= 5.000\, g_s^{-1} - 0.3581\, \mathcal{E}_s^{(K)} - 8.638. \end{aligned} \tag{3.47}$$

The reader should remember from (3.39) that the terms proportional to g_s^{-1} were already there without the loop corrections. The new terms are those proportional to $\mathcal{E}_s^{(K)}$ only. As an interesting fact we note that the value of $\mathcal{V}$ and τ_s in the minimum now depend also on the complex structure moduli through $\mathcal{E}_s^{(K)}$. This is contrast to the original case where they depend on the parameter $\hat{\xi}$, and thus scale with the Euler characteristic and g_s, only.

We conclude that the influence on the position of the minimum is after all roughly 20 % but still, as the qualitative behavior of the three terms of the F-term potential does not change, it is fair to say that the LARGE is robust against string loop corrections. This result has been derived on the basis of a concrete example for the sake of clearness. However in [96] it was shown that this indeed holds whenever the g_s-corrections to the Kähler potential are homogeneous functions of the two-cycle volumes t of degree -2. In the example we considered, we obtained precisely this form of the corrections by making an educated guess. The correctness of this ansatz was confirmed also in [96] by comparing the so obtained scalar potential with the Coleman–Weinberg one-loop effective potential.

3.5. Scales and Moduli Masses

Now we come to the phenomenological properties of the LARGE volume scenario. We start with summarizing the mass scales and list the moduli spectrum. We consider again the concrete example on the swiss cheese manifold $\mathbb{P}^4_{[1,1,1,6,9]}$[18].

In the low-energy effective action we included the lightest fields only. As we consider now a concrete model, it is possible to check the extent of validity of this approach by studying the mass scales of the heavy fields, not included in the effective action. We express the masses in units of the string mass $m_s = \frac{1}{\ell_s} = \frac{1}{2\pi\sqrt{\alpha'}}$. It is related to the four-dimensional Planck mass m_P by:

$$m_s = \frac{g_s}{\sqrt{4\pi\mathcal{V}}} m_P. \tag{3.48}$$

The nth stringy excitation then simply has the mass $m_S^2 = \frac{n}{\alpha'}$ such that the scale for the lightest excitation is:

$$m_S \sim 2\pi m_s. \tag{3.49}$$

For toroidal backgrounds the Kaluza–Klein and winding modes have a mass squared of:

$$m^2 = \frac{n^2}{R^2} + \frac{w^2 R^2}{\alpha'^2}, \tag{3.50}$$

with $R = R_s \ell_s$ the dimensionful Kaluza–Klein radius. In terms of the string scale $m_s = \ell_s^{-1}$ this is:

$$m_{\mathrm{KK}} \sim \frac{m_s}{R_s} \qquad \text{and} \qquad m_{\mathrm{W}} \sim (2\pi)^2 R_s m_s. \tag{3.51}$$

As we are in the supergravity regime where $R_s \gg 1$, the relevant modes are the KK modes. Expressed in terms of four-cycle volumes, their masses for the different cycles scale like $m_{\mathrm{KK}}^4 \sim 1/\tau_i$. The lightest KK excitation is hence correlated with the "big cycle" $\tau_b \sim \mathcal{V}^{2/3}$ such that:

$$m_{\mathrm{KK}} \sim \frac{2\pi}{\mathcal{V}_s^{1/6}} m_s, \tag{3.52}$$

where the volume has been dressed with a small 's' in the index, indicating that its string frame value is understood.

The next lightest fields are the complex structure moduli. In order to calculate their masses, we consider again the $\mathcal{N} = 1$ supergravity action with the scalar potential (2.31). In leading order we can still rely on the no-scale structure such that the potential for the complex structure moduli with all prefactors restored reads:

$$V = \frac{g_s^4 m_{\mathrm{P}}^4}{8\pi \mathcal{V}_s^2} \int \mathrm{d}^4 x \sqrt{-g_E}\, e^{\mathcal{K}_{\mathrm{c.s.}}} \left(\mathcal{K}^{a\bar{b}} D_a W \, D_{\bar{b}} \overline{W} \right). \tag{3.53}$$

For an analytic expression one would need the inverse of the Kähler metric

$$\mathcal{K}_{a\bar{b}} = \partial_a \partial_{\bar{b}} \ln\left(-i \int_{\mathcal{X}} \Omega \wedge \overline{\Omega} \right) \tag{3.54}$$

which in turn requires the knowledge of all the periods of the Calabi–Yau. However since we are only interested in orders of magnitude, we simply observe that the Kähler metric (3.54) is independent of the dilaton and the Kähler moduli. Hence the inverse of it will not introduce new factors of g_s and $\mathcal{V}$ in (3.53) and we can read off the mass squared as being of the order:

$$m_{\mathrm{c.s.}}^2 \sim \frac{g_s^4 N^2 m_{\mathrm{P}}^2}{\mathcal{V}_s^2}. \tag{3.55}$$

Here, N counts the number of flux quanta and is of the order $\sim \mathcal{O}(\sqrt{\frac{\chi}{24}})$. Expressed in terms of the string scale, this can be rewritten to:

$$m_{\text{c.s.}} \sim \frac{g_s N}{\sqrt{\mathcal{V}_s}} m_s. \tag{3.56}$$

By comparing this with (3.52) we see than in the LARGE limit, there is a small hierarchy between the masses of the complex structure moduli and the first Kaluza–Klein excitations. The latter set the upper bound on the energy scale, where the low-energy effective action can be trusted.

For the Kähler moduli one has to be more explicit as we can await a large hierarchy between the modulus controlling the overall volume τ_b and the small modulus τ_b. It is possible to canonically normalize the fields analytically and then taking the derivative of the scalar potential twice. We do not carry out the calculation but simply give the results, derived in [98]:

$$\begin{aligned} m_{\tau_b} &\sim \frac{g_s^2 W_0}{\sqrt{a_s \tau_s} \mathcal{V}_s^{3/2}} m_P \sim \frac{1}{\sqrt{\ln \mathcal{V}}\, \mathcal{V}^{3/2}} m_P, \qquad m_{b_b} \sim \exp(-\tau_b) m_P, \\ m_{\tau_s} &\sim m_{b_s} \sim \frac{g_s a_s \tau_s^{1/4} W_0}{\mathcal{V}_s} m_P \sim \frac{\ln \mathcal{V}}{\mathcal{V}} m_P. \end{aligned} \tag{3.57}$$

The alert reader will have noticed that the mass of the small modulus is comparable to the mass of the complex structure moduli (3.55) so the question arises if it was consistent to integrate out the latter and to minimize only the potential for the Kähler moduli exclusively as we did in section 3.2. But there is simple argument showing that the minimum we found in this way must be a minimum also of the full potential including the complex structure moduli. This can easily be understood by noticing that the latter enter the potential with term $\frac{DW\,DW}{\mathcal{V}^2}$ which vanishes in the minimum but is of the order $\mathcal{O}(\mathcal{V}^{-2})$ in principle. The vacuum energy in the minimum is of the order $-\mathcal{O}(\mathcal{V}^{-3})$. Now, if the complex structure moduli move away from their minimum where $DW = 0$, they contribute to the vacuum energy positively with $\mathcal{V}^{-2}$, which cannot be compensated with the Kähler moduli term (3.23) only, since they are of order $\mathcal{O}(\mathcal{V}^{-3})$ and so this will definitely increase the vacuum energy. Hence, the minimum we found by integrating out the complex structure moduli must be minimum also of the full potential.

3.6. Soft SUSY Breaking Terms for Chiral Matter

Having worked out the masses of the moduli fields, we now turn to the phenomenologically very interesting soft terms for the chiral matter sector, supported on the D7 branes wrapping the small cycle with volume τ_s. In particular the mass terms of the superpartners of the standard model fermions and gauge bosons may be of direct relevance for upcoming experiments at the LHC.

A naïve estimate for the mass scale of the superpartners is given by the gravitino mass $m_{3/2} = e^{\mathcal{K}/2}W$, here being roughly

$$m_{3/2} \sim \frac{W_0}{\mathcal{V}} m_{\mathrm{P}}. \tag{3.58}$$

For a volume of $\mathcal{V} \sim 10^{15}$, a typical value for the LARGE volume scenario, the mass of the superpartners are at the phenomenologically interesting TeV scale.

In the LVS, supersymmetry is broken in the Kähler moduli sector. There are several mediation mechanisms which come in question to induce soft terms in the MSSM sector: mediation by Planck scale suppressed operators (gravity mediation) and by certain one-loop diagrams associated to the super-Weyl anomaly (anomaly mediation) are two mechanism which are always present and it has to be checked, which one dominates. For gauge mediation, a suitable messenger sector with appropriate couplings to the MSSM fields has to be included. In this sense, this mechanism is not as generic as the two others and we do not consider it yet, at this stage.

The first step for the computation of the soft terms is to expand the Kähler and superpotential as well as the gauge kinetic function of the matter fields C^α and the Higgs doublets H_1, H_2 with respect to the supersymmetry breaking fields, here the moduli denoted by Φ:

$$\begin{aligned} W &= \widehat{W}(\Phi) + \mu(\Phi) H_1 H_2 + \frac{1}{6} Y_{\alpha\beta\gamma}(\Phi) C^\alpha C^\beta C^\gamma + \dots, \\ \mathcal{K} &= \widehat{\mathcal{K}}(\Phi, \overline{\Phi}) + \widetilde{\mathcal{K}}_{\alpha\beta}(\Phi, \overline{\Phi}) C^\alpha C^{\bar\beta} + \left[Z(\Phi, \overline{\Phi}) H_1 H_2 + h.c. \right] + \dots, \\ f_a &= f_a(\Phi). \end{aligned} \tag{3.59}$$

Apparently we need to know the Kähler metric of the chiral matter fields. This metric is unfortunately only known for simple toroidal models where explicit string scattering computations can be carried out. In [99] a technique was developed which allows to figure out the leading order modular dependence of the matter Kähler metrics in the LVS by simple scaling arguments. This knowledge suffices to estimate the order of magnitude of the soft terms.

The modular dependence of the matter metric can be determined in the following way: starting point are the physical Yukawa couplings $\widehat{Y}_{\alpha\beta\gamma}$. They are related to the unnormalized ones $Y_{\alpha\beta\gamma}$ by:[6]

$$\widehat{Y}_{\alpha\beta\gamma} = e^{\mathcal{K}/2} \frac{Y_{\alpha\beta\gamma}}{(\widetilde{\mathcal{K}}_\alpha \widetilde{\mathcal{K}}_\beta \widetilde{\mathcal{K}}_\gamma)^{1/2}}. \tag{3.60}$$

From (3.60) their modular dependence can be extracted by the help of the following two

6 Here we assumed the matter metric to be diagonal: $\widetilde{\mathcal{K}}_{\alpha\beta} = \widetilde{\mathcal{K}}_\alpha \delta_{\alpha\beta}$. The result holds however also in the non-diagonal case.

arguments: Firstly, under certain circumstances, the modular scaling behavior for the physical Yukawa couplings is known. Concerning the "big" cycle in the in the LARGE volume scenario, we can argue that the physical Yukawa couplings should be independent of the overall volume $\mathcal{V} \sim \tau_b^{3/2}$. This is because the gauge theory of the MSSM is localized on D7-branes, wrapping the small four-cycle τ_s. The swiss cheese structure of the compactification manifold guarantees that the overall volume can be increased without affecting the volume of τ_s. Consequently, the physics on the D7-branes should be untouched, when increasing the volume. Another way to state this is to say that we consistently decouple gravity by taking the limit $m_{\mathrm{P}}/m_{\mathrm{s}} \to \infty$.

Secondly, the unnormalized Yukawa couplings $Y_{\alpha\beta\gamma}$ appear in the superpotential W. In type IIB flux compactifications, the four-cycle volumes τ_i cannot appear in the tree-level superpotential. This because the Peccei–Quinn shift symmetry of the axions $b_i = \mathrm{Im}(T_i)$, $T_i \to T_i + i\epsilon_i$ is unbroken perturbatively. A dependence on the four-cycle volume $\tau_i = \mathrm{Re}(T_i)$ would therefore induce by holomorphy a dependence on the $\mathrm{Im}(T_i) = b_i$ which would break this symmetry. Consequently the physical Yukawa couplings cannot depend on the τ_i.

Now we expand the matter Kähler metric in a power series in τ_i:

$$\widetilde{\mathcal{K}}_\alpha = \tau_i^\lambda \widetilde{K}_0(U) + \tau_i^{\lambda-1}\widetilde{\mathcal{K}}_1(U) + \dots \tag{3.61}$$

It should be noted that τ_i contains a factor of $e^{-\phi} = g_{\mathrm{s}}^{-1}$ and therefore this series can be also regarded as a loop expansion. We consider at first the leading order dependence on the volume $\mathcal{V} = \tau_b^{3/2}$ by making the ansatz

$$\widetilde{\mathcal{K}}_\alpha = \tau_b^{-p_\alpha} k_\alpha(\tau_i, U, \phi), \tag{3.62}$$

where by τ_i we denote all other four-cycle volumes except τ_b. Inserting this into (3.60), we get:

$$\widehat{Y}_{\alpha\beta\gamma} = \frac{x Y_{\alpha\beta\gamma}}{\big(k_\alpha k_\beta k_\gamma(\tau_i, U, \phi)\big)^{1/2}} \tau_b^{\frac{-3+(p_\alpha+p_\beta+p_\gamma)}{2}}, \tag{3.63}$$

where x is defined by $x\mathcal{V} = \tau_b^{3/2}(1 + \dots)$. As argued before, the left hand side must be independent of τ_b. This implies that

$$p_\alpha + p_\beta + p_\gamma = 3. \tag{3.64}$$

As the chiral matter stems from open strings stretching between different stacks of D7-branes, which in the simplest model wrap the same four-cycle τ_s, it is natural to expect that the exponents p_α are universal and finally:

$$p_\alpha = 1 \qquad \forall\alpha. \tag{3.65}$$

For the modular dependence on the small Kähler modulus one makes a similar ansatz:

$$\widetilde{\mathcal{K}}_\alpha = \frac{\tau_s^\lambda}{\mathcal{V}^{2/3}} k_\alpha(U,\phi). \tag{3.66}$$

The difference here is, that the physical Yukawa couplings *do* scale with the size of τ_s. In principle, the Yukawa couplings are given by the overlap integral of normalized wave functions, localized on the intersection locus of the D7-branes supporting the fields in question. For the case here, where all D7-branes wrap the same four-cycle one can easily show that under a rescaling $\tau_s \to \beta\tau_s$, the normalized wave functions scale as $\psi \to \frac{\psi}{\sqrt{\beta}}$ and consequently also

$$\widehat{Y}_{\alpha\beta\gamma} \to \widehat{Y}'_{\alpha\beta\gamma} \sim \int_{\Gamma_4} (\beta \mathrm{d}^4 y) \left(\frac{\psi_\alpha}{\sqrt{\beta}}\right)\left(\frac{\psi_\beta}{\sqrt{\beta}}\right)\left(\frac{\psi_\gamma}{\sqrt{\beta}}\right) = \frac{\widehat{Y}_{\alpha\beta\gamma}}{\sqrt{\beta}} \tag{3.67}$$

which can be rephrased as $\widehat{Y}_{\alpha\beta\gamma} \to \widehat{Y}'_{\alpha\beta\gamma} = \frac{\widehat{Y}_{\alpha\beta\gamma}}{\sqrt{\tau_s}}$. Now we can insert again (3.66) into (3.60) and as we know now the scaling of the physical Yukawa couplings we can conclude:

$$\widetilde{\mathcal{K}}_\alpha \sim \frac{\tau_s^{1/3}}{\mathcal{V}^{2/3}} k_\alpha(U,\phi), \tag{3.68}$$

with an unknown function of the complex structure moduli and dilaton $k_\alpha(U,\phi)$, which will however turn out not to be relevant for the soft terms.

For diagonal matter Kähler metric, the most general potential for the canonically normalized matter fields $\widehat{C}^\alpha$, gauginos $\widehat{\lambda}^a$ and Higgs doublets $\widehat{H}_1$, $\widehat{H}_2$ with supersymmetry breaking terms not inducing quadratic divergences for the scalar masses, i. e. the soft supersymmetry breaking potential of the MSSM is given by:

$$V_{\text{soft}} = \frac{1}{2}\left(M_a \widehat{\lambda}^a \widehat{\lambda}^a + h.c.\right) - m_\alpha^2 \widehat{C}^\alpha \overline{\widehat{C}}^{\bar{\alpha}} - \left(\frac{1}{6} A_{\alpha\beta\gamma} \widehat{Y}_{\alpha\beta\gamma} \widehat{C}^\alpha \widehat{C}^\beta \widehat{C}^\gamma + B\hat{\mu}\widehat{H}_1\widehat{H}_2 + h.c.\right). \tag{3.69}$$

In the case of gravity mediation, which we consider now at first, the soft terms m_α, $A_{\alpha\beta\gamma}$, B and $\hat{\mu}$ are induced by the non-zero moduli F-terms $F^m = e^{\widehat{\mathcal{K}}/2}\widehat{\mathcal{K}}^{m\bar{n}} D_{\bar{n}}\overline{\widehat{W}}$, $m = \tau_b, \tau_s, \ldots$ They arise by expanding the $\mathcal{N}=1$ supergravity scalar potential (2.31) in terms of the non-zero F-terms and their Kähler metric $\widehat{\mathcal{K}}_m$, the matter metric $\widetilde{\mathcal{K}}_\alpha$ and superpotential. The result was computed in [100, 101] and reads:

$$
\begin{aligned}
m_0^2 &= \left(m_{3/2}^2 + V_0\right) - F^{\bar m} F^n \partial_{\bar m}\partial_n \log \widetilde{\mathcal{K}}_\alpha,\\
A_{\alpha\beta\gamma} &= F^m \Big[\widehat{\mathcal{K}}_m + \partial_m \log Y_{\alpha\beta\gamma} - \partial_m \log(\widetilde{\mathcal{K}}_\alpha \widetilde{\mathcal{K}}_\beta \widetilde{\mathcal{K}}_\gamma)\Big],\\
B &= \hat\mu^{-1}(\widetilde{\mathcal{K}}_{H_1}\widetilde{\mathcal{K}}_{H_2})^{-1/2}\Big\{ e^{\widehat{\mathcal{K}}/2}\mu\Big(F^m\big[\widehat{\mathcal{K}}_m + \partial_m \log\mu - \partial_m \log(\widetilde{\mathcal{K}}_{H_1}\widetilde{\mathcal{K}}_{H_2})\big] - m_{3/2}\Big)\\
&\quad + \left(2m_{3/2}^2 + V_0\right)Z - m_{3/2}\overline{F}^{\bar m}\partial_{\bar m} Z + m_{3/2}F^m\Big[\partial_m Z - Z\partial_m \log(\widetilde{\mathcal{K}}_{H_1}\widetilde{\mathcal{K}}_{H_2})\Big]\\
&\quad - \overline{F}^{\bar m} F^n\Big[\partial_{\bar m}\partial_n Z - (\partial_{\bar m} Z)\partial_n \log(\widetilde{\mathcal{K}}_{H_1}\widetilde{\mathcal{K}}_{H_2})\Big]\Big\},\\
\hat\mu &= (\widetilde{\mathcal{K}}_{H_1}\widetilde{\mathcal{K}}_{H_2})^{-1/2}\left(e^{\widehat{\mathcal{K}}/2}\mu + m_{3/2}Z - \overline{F}^{\bar m}\partial_{\bar m} Z\right)
\end{aligned}
\tag{3.70}
$$

and

$$
m_{1/2} = \frac{1}{2}(\mathrm{Re}\, f_a)^{-1} F^m \partial_m f_a. \tag{3.71}
$$

Although the formulæ are rather complicated, for the simple scaling behavior of the matter metric (3.68) and the assumption that the superpotential Yukawa couplings $Y_{\alpha\beta\gamma}$ do not depend on the Kähler moduli, the result of inserting all expressions in (3.70) turns out to yield the surprisingly simple expressions:

$$
\begin{aligned}
m_{1/2} &= \frac{F^{\tau_s}}{2\tau_s},\\
m_0^2 &= \frac{1}{\sqrt{3}} m_{1/2},\\
A_{\alpha\beta\gamma} &= -m_{1/2},\\
B &= -\frac{4}{3} m_{1/2}.
\end{aligned}
\tag{3.72}
$$

As an interesting fact it should be noted that though most of the soft terms in (3.70) scale with $m_{3/2}$, the final expressions (3.72) all scale with $\frac{F^{\tau_s}}{2\tau_s} \sim m_{3/2}/\log(m_{\mathrm{P}}/m_{3/2})$. This small hierarchy between the gravitino mass and the soft terms is the result of a cancellation taking place in (3.70) whose origin will be studied in detail in chapter 6.

The suppression of the gravity mediated soft terms with respect to the gravitino mass raises the question, if anomaly mediation [102] could be the dominant mechanism for the generation of soft terms. In [103] a formula for the gaugino masses in anomaly mediation was given:

$$m_{1/2} = -\frac{g^2}{16\pi^2}\left[(3T_G - T_R)m_{3/2} - (T_G - T_R)\mathcal{K}_i F^i - \frac{T_R}{d_R}(\ln\det\mathcal{K}|''_R)_i F^i\right]. \tag{3.73}$$

Here, $\mathcal{K}|''_R$ denotes the Kähler metric restricted to the matter fields in the representation R. T_R and d_r are the corresponding Dynkin index and dimension; T_G denotes the Dynkin index of the respective adjoint representation. For the gauge groups $SU(N)$ they are normalized to $T_G = N$ and $T_R = 1/2$. With the matter metric as before, the determinant in the last term for d_R matter fields is given by:

$$\det\mathcal{K}|''_R = \frac{\tau_s^{\frac{1}{3}d_R}}{\mathcal{V}^{\frac{2}{3}d_R}}X(\phi), \tag{3.74}$$

with an unknown function of the complex structure moduli $X(\phi)$ which however drops out when taking the derivative of the logarithm of this expression:

$$\frac{1}{d_R}(\ln\det\mathcal{K}|''_R)_i F^i = \frac{F^s}{6\tau_s} - \frac{F^b}{2\tau_b}. \tag{3.75}$$

Furthermore there is the important relation:

$$\mathcal{K}_i F^i = -\frac{3F^b}{2\tau_b}\big(1 + \mathcal{O}(\mathcal{V}^{-1})\big) = 3m_{3/2}\big(1 + \mathcal{O}(\mathcal{V}^{-1})\big), \tag{3.76}$$

where i runs over the Kähler moduli. This is a consequence of the no-scale structure and will be discussed in much more detail in chapter 6 again. When putting everything together, also here cancellations occur and we have again a simple result:

$$m_{1/2}^{\text{anom.}} = \frac{g^2}{16\pi^2}\frac{1}{3}\left(\frac{F^s}{2\tau_s}\right) = \frac{g^2}{16\pi^2}\frac{1}{3}m_{1/2}^{\text{grav.}}. \tag{3.77}$$

We see that the anomaly mediated contribution to the gaugino mass is suppressed with respect to the gravity mediated one by the usual loop factor $\frac{g^2}{16\pi^2}$ and is thus sub-dominant. The cancellation can be traced back to the relation (3.76) and hence considered to be a consequence of the no-scale structure at leading order. One can await therefore a similar cancellation for the other soft terms induced by anomaly mediation, too. Their calculation is quite involved and (see [104, 105]) we do not carry out an analysis of them here.

3.7. Moduli Inflation

We have seen in the last section, that the no-scale structure of the Kähler moduli sector — though broken — can still lead to certain cancellation effects. For instance, the soft terms are suppressed with respect to their natural value of $m_{3/2}$ by a factor of $\log(m_{\mathrm{P}}/m_{3/2})$. Heuristically one might say that this is due to the fact that the mechanisms breaking the no-scale structure are not at tree-level, but at higher lever in the α'-expansion of the effective field theory or even at the non-perturbative level in g_s. Moreover, the exponentially large volume can create hierarchies which are not expected in the generic case.

Thus, it might be fruitful to seek for other situations where small, fine-tuned values of a quantity are necessary for whatever reason, but the theory predicts a much larger value for the generic case.[7] In the context of cosmology of supergravity theories, the so-called η-problem is of this kind: the slow-roll parameter $\eta = \frac{V''}{V}$ must be small in order for slow-roll inflation to work. However generically, in F-term inflation its value is expected to be of order $\mathcal{O}(1)$, unless a finely tuned cancellation is at work. Indeed the "pseudo no-scale structure" of the LVS can help here.

The prospect of implementing inflation in the context of the LVS was pioneered in [106]. A typical slow-roll potential for a field τ is given by:

$$V_{\text{infl.}} = V_0\big(1 - Ae^{-\tau} + \ldots\big). \tag{3.78}$$

The scalar potential of the LARGE volume scenario contains such exponentially flat terms for the small moduli and in fact in can be recasted precisely in the form (3.78) as we will show now. Therefore, we consider again the scalar potential (3.21):

$$V_F = \frac{a_s^2|A_s|^2\sqrt{\tau_s}e^{-2a_s\tau_s}e^{\mathcal{K}_{\text{c.s}}}}{\mathcal{V}} - \frac{a_s|A_sW_0|\tau_s e^{-a_s\tau_s}}{\mathcal{V}^2}e^{\mathcal{K}_{\text{c.s}}} + \frac{\xi}{\mathcal{V}^3}|W_0|^2e^{\mathcal{K}_{\text{c.s.}}}. \tag{3.79}$$

For obvious reasons we assume the small modulus τ_s to be the inflaton field. At the begin of inflation it can be assumed to have a value far away from its minimum value: $a_s\tau_s \gg \ln\mathcal{V}$. In this limit, the first term scaling with $e^{-2a_s\tau_s}$ can be neglected. The second term, scaling with $e^{-a_s\tau_s}$, as being negative, decreases the potential energy when decreasing the value of τ_s. Provided that all other moduli, in particular the overall volume, are fixed, the exponential flatness in τ_s can drive inflation.

However, displacing τ_s from its minimum nullifies its contribution to the scalar potential, producing a runaway direction for the overall volume. Thus, we need at least one more small Kähler modulus, which we assume to have fallen already into its minimum, where it becomes heavy and decouples from inflationary dynamics. This induces a contribution to

7 The most obvious quantity falling in this category is the cosmological constant. As it is well-known, it is also the most resistant to simple explanations for its tiny value. We will comment on this subject in section 5.3.

the scalar potential of the order $\mathcal{O}(\mathcal{V}^{-3})$, stabilizing the overall volume. We have now:

$$V_F = V_0 - \frac{|W_0|}{\mathcal{V}^2} 4a_s|A_s|\tau_s e^{-a_s\tau_s} \qquad \text{with} \qquad V_0 \sim \frac{\beta W_0^2}{\mathcal{V}^3}, \tag{3.80}$$

which has a similar form like (3.78). However that is only half the truth. The Kähler metric for the small modulus $\mathcal{K}_{ss} = \frac{3\lambda}{8\sqrt{\tau_s}\mathcal{V}}$ is non-trivial, so τ_s has to be canonically normalized. After doing so, the scalar potential looses its simple form and we ought to analyze the slow-roll parameter:

$$\epsilon = \frac{m_{\mathrm{P}}^2}{2}\left(\frac{V'}{V}\right)^2, \qquad \eta = m_{\mathrm{P}}^2\frac{V''}{V}, \qquad \xi = m_{\mathrm{P}}^4\frac{V'V'''}{V^2}, \tag{3.81}$$

where the derivatives have to be taken with respect to the canonically normalized field τ_s^c. One finds:

$$\begin{aligned}
\epsilon &= \frac{32\mathcal{V}^3}{3\beta^2 W_0^2} a_s^2 A_s^2 \sqrt{\tau_s}(1 - a_s\tau_s)^2 e^{-2a_s\tau_s}, \\
\eta &= -\frac{4a_s A_s \mathcal{V}^2}{3\lambda\sqrt{\tau_s}\beta W_0}\left[\left(1 - 9a_s\tau_s + 4(a_s\tau_s)^2\right)e^{-a_s\tau_s}\right], \\
\xi &= \frac{-32(a_s A_s)^2\mathcal{V}^4}{9\beta^2\lambda^2 W_0^2\tau_s}(1 - a_s\tau_s)\left(1 + 11a_s\tau_s - 30(a_s\tau_s)^2 + 8(a_s\tau_s)^3\right)e^{-2a_s\tau_s},
\end{aligned} \tag{3.82}$$

and we see that the slow-roll conditions $\xi \ll \epsilon$ and in particular $\eta \ll 1$ are satisfied, as indeed for large initial values of τ_s we surely have $e^{-a_s\tau_s} \ll \frac{1}{\mathcal{V}^2}$.

The requirement that that the number of e-foldings $N_e = \int_{\phi_{\mathrm{end}}}^{\phi} \frac{V}{V'}\mathrm{d}\phi$ should be in the range 50 to 60 and the COBE normalization for the density fluctuations $\delta_H = 1.92 \times 10^{-5}$ should be matched, eventually fixes several parameters of the model. It turns out that the influence of the threshold correction A_s and the tree-level superpotential value W_0 are rather unimportant at this and it is only the overall volume which is constrained in the narrow range

$$10^5 \leq \mathcal{V} \leq 10^6. \tag{3.83}$$

This is clearly in tension with the value of $\mathcal{V} \sim 10^{15} - 10^{16}$ which is needed for TeV scale supersymmetry breaking.

3.8. Cosmological Moduli Problem

Inflation is not the only cosmological stage, where moduli can play a rôle. They exhibit only very weak, gravitational-strength interactions with ordinary matter. Therefore they may decay in the later history of the universe, bringing up the question, whether they can spoil nucleosynthesis or overclose the universe [107].

In contrast to the section before we assume now again a compactification volume of $\mathcal{V} \sim 10^{15}$, giving rise to TeV scale supersymmetry breaking — the case where the post-inflationary cosmological development is well-known.

We study now the decay of the Kähler moduli into a pair of photons. The relevant Lagrangian in the vicinity of the moduli minimum $\tau_i = \langle \tau_i \rangle + \delta\tau_i$ is simply given by:

$$\mathcal{L} = \mathcal{K}_{ij}\partial_\mu(\delta\tau_i)\partial^\mu(\delta\tau_j) - V_0 - (M^2)_{ij}(\delta\tau_i)(\delta\tau_j) - \mathcal{O}(\delta\tau^3) - \kappa\frac{\tau_\alpha}{m_{\mathrm{P}}}F_{\mu\nu}F^{\mu\nu}, \tag{3.84}$$

where the gauge kinetic function $f_{U(1)} = \kappa\tau_\alpha$ for the $U(1)$ part of the standard model, realized on D7-branes wrapping the small cycle α was used. The couplings can be read off after canonically normalizing the moduli fields in (3.84) and rewriting the Lagrangian in terms of the mass eigenstates of $(\mathcal{K}^{-1})_{ij}(M^2)_{jk}$, in the following denoted by Φ and χ. Written in terms of the new fields, the Lagrangian is:

$$\begin{aligned}\mathcal{L} =& \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - V_0 - \frac{1}{2}m_\Phi^2\Phi^2 - \frac{1}{2}m_\chi^2\chi^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ &- \frac{\big(\Phi(v_\Phi)_s + \chi(v_\chi)_s\big)}{4\sqrt{2}\langle\tau_s\rangle m_{\mathrm{P}}}F_{\mu\nu}F^{\mu\nu}\end{aligned} \tag{3.85}$$

and likewise, in terms of the eigenvectors v_Φ and v_χ, the couplings λ of the moduli to photons can be read off. They are:

$$\lambda_{\Phi\gamma\gamma} = \frac{(v_\Phi)_s}{\sqrt{2}\langle\tau_s\rangle}, \qquad \lambda_{\chi\gamma\gamma} = \frac{(v_\chi)_s}{\sqrt{2}\langle\tau_s\rangle}. \tag{3.86}$$

The explicit diagonalization of the respective matrices can be found in the appendix of [107]. The mass eigenvalues turn out to be:

$$\begin{aligned} m_\Phi^2 &\sim \mathrm{Tr}(\mathcal{K}^{-1}M^2) \sim \big(2m_{3/2}\ln(m_{\mathrm{P}}/m_{3/2})\big)^2 \sim \left(\frac{\ln\mathcal{V}}{\mathcal{V}}\right)^2 \sim (1000\,\mathrm{TeV})^2, \\ m_\chi^2 &\sim \frac{\det(\mathcal{K}^{-1}M^2)}{\mathrm{Tr}(\mathcal{K}^{-1}M^2)} \sim \frac{1}{(\ln\mathcal{V})\,\mathcal{V}^3} \sim (1\,\mathrm{MeV})^2. \end{aligned} \tag{3.87}$$

By comparing comparing (3.87) with (3.57) we see that the heavy eigenstate Φ is mostly given by τ_s and the light one by τ_b. However, by re-expressing the original fields $\delta\tau_b$ and $\delta\tau_s$ in terms of Φ and χ it is revealed that there is a small mixing between the fields. This has the interesting consequence that the light field, though mostly aligned with τ_b has a coupling to the photons localized on τ_s, which would not be the case for τ_b alone. This has an apparent effect on the couplings which come out to be:

$$\lambda_{\chi\gamma\gamma} = \frac{\sqrt{6}}{2m_{\mathrm{P}} \ln(m_{\mathrm{P}}/m_{3/2})}, \qquad \lambda_{\Phi\gamma\gamma} = \frac{2}{\sqrt{3}} \frac{\langle\tau_b\rangle^{3/4}}{\langle\tau_s\rangle^{3/4} m_{\mathrm{P}}} \sim \frac{\sqrt{\mathcal{V}}}{m_{\mathrm{P}}} \sim \frac{1}{m_{\mathrm{s}}}. \tag{3.88}$$

The coupling of the light field is suppressed with respect to the naïve expectation of $\lambda \sim 1/m_{\mathrm{P}}$ with a factor of $\ln(m_{\mathrm{P}}/m_{3/2})$ suppressing the decay rate with $\ln(m_{\mathrm{P}}/m_{3/2})^2 \sim 1000$. Even more striking is the fact that the coupling of the heavy field Φ is suppressed with the string scale only instead of the Planck scale. As $m_{\mathrm{s}} \ll m_{\mathrm{P}}$, it will decay very rapidly, possibly imposing phenomenological problems in the early universe as we will see in a moment.

Another decay channel of the moduli is the one into an e^+e^- pair. The corresponding couplings can be obtained in a similar way as for the photons upon using the matter Kähler metric (3.68). One finds for the couplings:

$$\lambda_{\chi ee} \sim \left(1 + \frac{1}{a\langle\tau_s\rangle}\right) \frac{m_e}{\sqrt{6} m_{\mathrm{P}}}, \qquad \lambda_{\Phi ee} \sim \frac{\sqrt{\mathcal{V}} m_e}{m_{\mathrm{P}}}. \tag{3.89}$$

Finally, moduli may also decay into gravitini. Generically in models with heavy moduli, one encounters a large branching ratio for this channel of the order $\mathcal{O}(1)$ [108, 109]. This is problematic as an overproduction of gravitini in the early universe may spoil nucleosynthesis or exceed the observed abundance of dark matter. However, also here the LVS turns out to be very special: as the gravitino is a bulk field, the decay of the heavy modulus $\Phi \to 2\psi_{3/2}$, as being mostly τ_s, is suppressed by a factor of $(m_{\mathrm{s}}/m_{\mathrm{P}})^2 = \mathcal{V}^{-1}$. Moreover, the dominant F-term in this model is the one associated with the light modulus τ_b and not with the small and heavy modulus τ_s giving another factor of $\mathcal{V}$. This results in a double suppression of the $\Phi \to 2\psi_{3/2}$ decay mode and so, the branching ratio can estimated by $\mathrm{Br}(\Phi \to 2\psi_{3/2}) \sim \mathcal{V}^{-2} \sim 10^{-30}$.

For cosmology, the lifetimes of the moduli are of interest. For a particular decay mode, they are basically given by the inverse square of the corresponding couplings: $\tau = \frac{64\pi m_{\mathrm{P}}^2}{\lambda^2 m^3}$. Inserting the numerical values, we find:

$$\begin{aligned} &\tau_{\chi\to\gamma\gamma} \sim 6 \times 10^{25}\,\mathrm{s}\,, \qquad \tau_{\chi\to e^+e^-} \sim 1.7 \times 10^{24}\,\mathrm{s}, \\ &\tau_{\Phi\to e^+e^-} \sim 10^{-17}\,\mathrm{s}, \end{aligned} \tag{3.90}$$

where we listed only the dominant decay channel for the heavy field Φ.

The small mass in the MeV range of the field χ is a result of the large overall volume and thus a generic feature of the LVS. It is well-known that light moduli fields can be problematic for early-universe cosmology: low masses imply long life-times and after inflation they may thus dominate the energy density of the universe or spoil nucleosynthesis. After inflation, the light modulus fields starts oscillating around its minimum. As radiation decreases with a^{-4}, but the energy in these coherent oscillations only with a^{-3}, it will become the dominant energy density of the universe. At the temperature T_D, the field decays, converting this energy density back into radiation of the temperature:

$$T_{\text{RH}} \sim \left(\rho(T_D)\right)^{1/4} \sim \left(\frac{m_{\text{P}}}{\tau}\right)^{1/2} \sim \left(\frac{m_\phi^3}{m_{\text{P}}}\right)^{1/2}. \tag{3.91}$$

A too low reheating temperature of $T_{\text{RH}} \lesssim 10\,\text{MeV}$ causes a too large increase in the entropy of the universe given by:

$$\Delta = \left(\frac{T_{\text{RH}}}{T_D}\right)^3. \tag{3.92}$$

This washes out any previously generated baryon asymmetry. The bound on the reheating temperature can be converted into a mass bound: moduli fields with a mass lower than $m_\phi \lesssim 100\,\text{TeV}$ will spoil nucleosynthesis. This is the so-called cosmological moduli problem.

From (3.56) we see that the complex structure and dilaton moduli have a mass of approximately $\sim 10\,\text{TeV}$ and are apparently subject to the CMP a priory. However their lower suppressed contribution to the scalar potential of $\mathcal{V}^{-2}$ (in contrast to $\mathcal{V}^{-3}$ for the Kähler moduli) keeps the fields trapped at the minimum, evading the dangerous oscillations.

The heavy modulus field has a mass of $\sim 10^3\,\text{TeV}$. Its coupling to ordinary matter is suppressed with the string scale only rather than the Planck scale and thus its lifetime is very short. We have seen that its branching ratio for $\Phi \to 2\psi_{3/2}$ is doubly suppressed with the volume and thus 30 orders of magnitude smaller than the usual expectation of $\mathcal{O}(1)$. In all it does not suppose any cosmological problems.

However, the light field χ with its mass in the MeV range *is* clearly subject to the CMP, raising an inherent problem of the LARGE volume scenario. A possible loophole is the so-called thermal inflation [110]. By this one denotes a class of models which after the usual slow-roll inflation enter again in a short period of low-energy inflation. This can be usefully applied to the LVS as the overproduced light modulus field can be sufficiently diluted during this phase.

The mechanism driving thermal inflation works as follows: in supersymmetric theories, there are many flat directions in field space which are lifted only after supersymmetry breaking. For such a field σ usually a vacuum expectation value much larger than its mass is assumed. In thermal equilibrium with matter, its scalar potential is corrected by finite temperature effects:

$$V = V_0 + (T^2 - m_\sigma^2)\sigma^2 + \ldots \tag{3.93}$$

At temperatures $T > T_c = m_\sigma$, the field is trapped at the origin, which is however only a false vacuum. When the potential energy density V_0 begins to dominate as the temperature falls below $T \sim V_0^{1/4} > T_c$, a short period of inflation develops which ends at $T = T_c$ when the field σ becomes tachyonic at the origin and runs into its zero temperature minimum. The number of e-foldings during this period can be estimated to be:

$$N \sim \log\big(V_0^{1/4}/T_C\big) \sim \log(M_*/m_\sigma)^{1/2}, \tag{3.94}$$

where M_* denotes the zero temperature minimum of the potential. Reasonable values proposed for the so-called "flaton" field σ are $m_\sigma \sim 1\,\text{TeV}$ and $M_* \sim 10^{11}\,\text{GeV}$, giving rise to about ten e-foldings. This is a preferred value as it is just enough to dilute the overproduced moduli and thus solving the CMP without interfering the density perturbations from the original slow-roll inflation.

It happens that the two scales M_* and m_σ are automatically present in the examples for the LVS we considered: these are the string scale m_s and the supersymmetry breaking scale $\sim m_{3/2}$. Natural candidates for flaton fields in such string compactifications could be for example open string moduli stemming from the D7-branes supporting the standard model.

The lightness of the moduli field χ may be challenging for the LARGE volume scenario. However, once having overcome the cosmological moduli problem — for instance with a phase of thermal inflation — it may also have interesting applications in the later history of the universe: as being very long lived, it may serve as a dark matter component. We saw in (3.90) that the decay channel $\chi \to e^+e^-$ is dominant. Being so, these decays could be responsible for the $511\,\text{keV}$ line observed from the galactic center. Observations of the gamma ray spectrum puts a bound on the positron injection energy of $\lesssim 3\,\text{MeV}$. This indeed fits nicely to the estimates for the mass of χ of $\sim 1\,\text{MeV}$.

4. Moduli Stabilization and Chirality

In the last chapter, we introduced the LARGE volume scenario and pointed out some of its attractive features. After having stabilized all moduli, it is possible to address cosmological questions and to calculate the low-energy phenomenological parameters (at least their order of magnitude) as gauge couplings and particle spectra.

It must be admitted that so far, we have to not been very rigorous concerning the numerous string theoretical consistency conditions discussed in chapter 2. In particular we simply assumed that the standard model or its supersymmetric extension is realized in a consistent way on a stack of D7-branes wrapping one single small internal four-cycle. Even this alone is an extreme challenging and to this day still not completely solved problem in the type IIB context (for progress into this direction in heterotic model building see for instance [111, 112] and related work).

At the latest now, after having demonstrated that within the LVS it is possible to tackle a lot of interesting phenomenological questions, it is time to set value on accuracy and to show that this scenario can really be constructed with all consistency conditions within string theory.

Along this way, we will encounter a serious problem: an important implicit assumption made so far was that it is a valid procedure to split the construction of such string models into two steps. The first one is to fix all moduli by a combination of fluxes and non-perturbative effects. After that has been achieved, the second step is to introduce the *module* of the MSSM on some intersecting respectively magnetized D7-branes. However it will turn out that there is a fundamental problem when combining a chiral MSSM like module with the moduli stabilization module: Firstly it leads to the generation of a D-term potential, which appears at lower order in the $1/\mathcal{V}$ expansion of the scalar potential. Therefore, there is the danger of destabilizing the large volume minimum. Secondly, as anticipated in section 2.7, on the intersection of the D7-branes with the E3-brane instantons, extra charged *chiral* fermionic zero modes can appear, possibly spoiling the generation of an uncharged superpotential.

As we will show, due to the chirality of the D7-brane sector, in our MSSM case the D7-branes and the E3-branes should better wrap in some sense “orthogonal” four-cycles. This means that not all the sizes of the D7-branes are fixable by instanton induced F-terms. However, it is precisely the sizes of these cycles on which some of the low-energy MSSM parameters depend. Therefore, by this effect we seem to loose part of the very predictive power of the models.

However, the combination of both aspects mentioned in the previous paragraph provides a natural solution for the problem of fixing all Kähler moduli for an MSSM like model. Non-perturbative effects fix some of the Kähler moduli except the ones controlling the size of the MSSM branes. These latter are fixed by the vanishing of the D-term potential. Since it is a D-term, there could also be charged matter contributions. Again, by requiring not to break the MSSM gauge symmetry already at the high scale they should better have vanishing vacuum expectation values.

As we are forced so separate the moduli stabilizing instanton sector from the MSSM sector on different four-cycles, in order to stick on our intent to be more rigorous, we need to consider another compactification manifold as the standard example considered so far, the $\mathbb{P}_{[1,1,1,6,9]}[18]$, supports only one small four-cycle. This will be done in section 4.2, where we investigate some of the geometrical and topological aspects of the swiss cheese type Calabi–Yau manifold defined via the resolution of the singular hypersurface in a weighted projective space $\mathbb{P}_{[1,3,3,3,5]}[15]_{(3,75)}$. In appendix B, we will also provide the geometric data for a second new swiss cheese Calabi–Yau, namely the resolution of $\mathbb{P}_{[1,1,3,10,15]}[30]_{(5,251)}$.

4.1. Instantons and Chirality

At first, in this section, we are going to investigate the aforementioned interplay between a chiral theory realized by intersecting and magnetized D7-branes and the E3-brane instantons. More specifically, we assume again for the moment that some version of the MSSM can be described by a configuration of D7-branes wrapping four-cycles in the Calabi–Yau manifold.

To prepare our discussion let us stress the following important points: In order for an E3-brane instanton to actually generate a superpotential term like in (3.6), to be precise a term:

$$W_{\text{n.p.}} = A(S,U)e^{-\sum_i a_i T_i}, \tag{4.1}$$

according to section 2.7, the zero mode structure must be of a special nature.[1] For the case of the $\mathbb{P}_{[1,1,1,6,9]}[18]$ example, in the original literature it was argued that such a contribution must be present as the small divisor τ_s has an F-theory uplift to a six-dimensional divisor in the Calabi–Yau fourfold with $\chi(D,\mathcal{O}) = 1$. By the zero mode criterion derived in [80], an instanton along such a cycle may contribute to the superpotential. This is however only a necessary condition. A detailed analysis of the zero mode structure along the lines of section 2.7 must be performed in order to decide whether such a contribution is really generated or not. We do so in the next section.

1 This and likewise the problems that will follow hold equally for the KKLT construction which relies on a similar non-perturbative term in the superpotential.

4.1.1. E3-brane Instantons

We are now going to investigate E3-brane instanton effects in more detail. In particular, we turn our attention to additional zero modes which can arise from D-branes supporting chiral matter on the same cycle. These may spoil a generation of the form (4.1). We will also comment on the case that we freeze the Kähler moduli by gaugino condensates on a stack of N_c D7-branes wrapping a four-cycle D_G.

Following what we have said in 2.7, in order to contribute in whatever way to the superpotential, the following points have to be respected:

- A single, isolated instanton must wrap a four-cycle invariant under the orientifold projection and must carry an $O(1)$ gauge symmetry [78, 79, 113]. In the case of $h_{-}^{(1,1)}(\mathcal{X}) = 0$, this implies that the instanton carries a trivial gauge bundle.

- Next, we have to worry about deformation zero modes of the E3-instanton. These are clearly absent, if the E3-brane wrapping the four-dimensional divisor D does not have any further moduli. That is, there are no Wilson lines counted by $H^1(D, \mathcal{O})$ or transverse deformations counted by $H^2(D, \mathcal{O})$. If this sufficient condition is not satisfied, then fluxes or curvature on the moduli space might soak up some of the zero modes, but a more careful analysis is necessary [85, 114]. Similarly, for gaugino condensation many adjoint matter fields counted by $H^i(D_G, \mathcal{O})$ with $i = 1, 2$ spoil asymptotic freedom of the gauge theory on the D7$_G$-branes.

- If, as in our case, there are additional space–time filling D7-branes present, there can appear extra charged fermionic zero modes from the intersection of the E3-instanton and the D7-branes [34]. The chiral index of these fermionic zero modes is

$$Z_a = N_a \int_{D_a \cap D_{\text{E3}}} c_1(\mathcal{L}_a) = N_a \int_{\mathcal{X}} c_1(\mathcal{L}_a) \wedge [D_a] \wedge [D_{\text{E3}}]. \tag{4.2}$$

  In order to soak up these additional fermionic zero modes, one has to pull down charged matter fields in the instanton computation. The pure exponential term as in (4.1) is then multiplied by products of charged matter superfields Φ_i as [34]

$$W_{\text{string}} \sim \Big[\prod_i \Phi_i\Big] e^{-S_{\text{inst}}}. \tag{4.3}$$

  Remember that such instantons are not gauge instantons and therefore often called "stringy" or "exotic" instantons.

- For the special case when the E3-instanton lies right on top of the D7-branes[2], it is

2 Note that in [73, 115] it was shown that for an instanton on top of a single D-brane also a superpotential of the form (4.3) can be generated.

possible to have non-trivial gauge bundles on the instanton. It can then be regarded as a gauge instanton from the perspective of the D7-brane gauge theory and additional bosonic and non-chiral fermionic zero modes arise parameterizing the ADHM instanton super moduli space [69, 76, 78, 79]. The effect of such instantons is of the same nature as gaugino condensates for the gauge theory on stacks of $D7_G$-branes, so that we can discuss them together. In order to soak up the ADHM zero modes one needs extra non-chiral (with respect to the $U(N_G)$ gauge group) matter zero modes from the intersection of the E3-instanton with the other D7-branes [76, 116]. If we end up with an $SU(N_c)$ gauge group with effectively N_f flavors, then for $N_f < N_c$ the contribution to the superpotential is

$$W_{\text{gauge}} \sim \frac{1}{\det_{ff'}\left[\widetilde{\Phi}^c_f \Phi_{cf'}\right]} e^{-S_{\text{inst}}}. \tag{4.4}$$

In writing this, it is assumed that we are on the Higgs branch, where the determinant is non-vanishing and so the flavor gauge group is completely broken. Such a configuration is not part of the MSSM and therefore the instanton respectively the D_G branes should better not have any intersection with the D-branes supporting the MSSM.

4.1.2. Moduli Stabilization for Chiral Models

We will now argue that given the structure and constraints from the previous discussion, for chiral orientifolds not all Kähler moduli can be frozen by instantons. In particular, some of the moduli controlling the size of the chiral D7-brane sector are left unfixed by the E3-brane instantons.

Let us first summarize the possible matter fields which can be present in the configurations we are considering.

- We assume that the chiral MSSM like matter fields, denoted as Φ_{SM}, are part of the chiral matter spectrum arising on a set of intersecting D7-branes carrying initial gauge group $G = \prod_{a=1}^{K} U(N_a)$. Typical examples discussed in the literature are $G = U(5) \times U(1)$, $G = U(4) \times U(2) \times U(2)$ or $G = U(3) \times U(2) \times U(1) \times U(1)$.
- There can also be additional (chiral) fields, which also arise from the same set of intersecting D7-branes leading to so-called exotic matter fields. There can exist exotic matter fields transforming in non-trivial representations of the non-abelian part of the MSSM gauge group. These are denoted as Φ_{exo}.
- However, since in D-brane models we genuinely have these extra $U(1)$ gauge factors, there might be fields which are not charged under the MSSM gauge group $SU(3) \times$

$SU(2) \times U(1)_Y$ but carry non-trivial charges with respect to $U(1)$s orthogonal to $U(1)_Y$. These we denote as Φ_{abel}.

- In addition there can in principle be further hidden sector matter fields Φ_{H}, whose D-terms and F-terms however do not mix with the standard model ones. Therefore, we will not focus on those in the following. However, this sector might be important for the eventual uplift of the AdS minimum to de Sitter with small cosmological constant.

Consider now an E3-instanton wrapping a four-cycle which gives rise to extra standard model charged zero modes. These can either be chiral fermionic zero modes coming from stringy instantons or non-chiral zero modes from gauge instantons. To soak up all these zero modes, the superpotential coupling must contain products of the standard model matter fields Φ_{SM} and, since they appear on the same D7-branes, also products of the additional fields Φ_{exo} and Φ_{abel}

$$W \sim \prod_i \Phi_{\text{SM}}^{(i)} \prod_j \Phi_{\text{exo}}^{(j)} \prod_k \Phi_{\text{abel}}^{(k)}\, e^{-T_{\text{E3}}}. \tag{4.5}$$

Note that for gauge instantons or gaugino condensates there will be determinants of the matter fields in the denominator. Furthermore, in the equation above $T_{\text{E3}} = \sum_i m^i T_i$ denotes the Kähler modulus corresponding to the instanton on the cycle $D_{\text{E3}} = \sum_i m^i D_i$.

The important point is now that, for phenomenological reasons, at this high scale we do not want to break the MSSM gauge symmetry by giving vevs to these fields. If we allow for vevs of charged matter fields, the D-term potential (4.13), which we will consider in the next section, generates a mass of the generic order $m_{\text{matter}} = m_{\text{P}}/\sqrt{\mathcal{V}} = m_{\text{s}}$ for them, i. e. the matter fields become very heavy. The MSSM gauge symmetry breaking and mass generation should occur as usual at the low scale in the process of supersymmetry breaking. Therefore, we are only interested in vacua with $\langle \Phi_{\text{SM}} \rangle = \langle \Phi_{\text{exo}} \rangle = 0$, so that effectively the contribution of such an instanton to the superpotential vanishes and the F-term potential V_F does not depend explicitly on T_{E3}. What could be possible in principle is to allow vevs for GUT Higgs fields.

Of course this argumentation is not really satisfying as in a fully realistic moduli stabilization scenario, we also would like to have these charged matter fields stabilized *dynamically*. But our point of view is, that it is very likely that in a given concrete model the four contributions:[3]

- the soft supersymmetry breaking mass terms $V_{\text{soft}} = m^2 \Phi_{\text{SM}}^2$,

- the perturbative and instanton induced superpotential contributions of the form $W = \prod \Phi_{\text{SM}}$,

- the D-terms and

3 See for instance [117, 118] for a recent discussion of matter fields moduli stabilization.

- the generic absence of gauge instantons or gaugino condensates for MSSM fields, i. e. terms like $W_{\text{gauge}} \sim \frac{1}{\det(\Phi_{\text{SM}})} e^{-S_{\text{inst}}}$

suffice to freeze to MSSM matter fields at $\langle \Phi_{\text{SM}} \rangle = \langle \Phi_{\text{exo}} \rangle = 0$. If such a mechanism is indeed at work, then, since they appear in the same open string sector, also the fields Φ_{abel} are likely to be frozen at vanishing vevs. However, just from phenomenology these vevs could be non-vanishing, a fact to be kept in mind when we will mainly discuss the case $\langle \Phi_{\text{abel}} \rangle = 0$.

Therefore, if we want to fix the size of the four-cycle the E3-instanton is wrapping, it should not have any zero modes charged under the standard model gauge symmetry. Recall that we derived the analogous condition also for moduli freezing via gaugino condensates on a stack of D7-branes wrapping a four-cycle D_G. There too, D_G should not have any charged matter fields from intersections with branes supporting the MSSM.

Recalling then equation (4.2), we have to satisfy the necessary condition

$$N_a \int_{\mathcal{X}} c_1(\mathcal{L}_a) \wedge [D_a] \wedge [D_{\text{E3}}] = 0, \tag{4.6}$$

for standard model branes wrapping the divisor D_a with line bundle $\mathcal{L}$. Furthermore, not only the chiral instanton zero modes have to be absent but also those which are vector-like. For determining them one has to compute the cohomology classes

$$H^i\Big(D_a \cap D_{\text{E3}}, \mathcal{L}_a \otimes \mathcal{K}_{D_a}^{\frac{1}{2}} \otimes \mathcal{K}_{D_{\text{E3}}}^{\frac{1}{2}}\Big) \qquad \text{for } i = 0, 1, \tag{4.7}$$

where $\mathcal{K}_D$ denotes the canonical line bundle of the divisor $D \subset \mathcal{X}$. If these cohomology classes are non-trivial, extra pairs of instanton zero modes are present and the resulting term in the superpotential will be of the form (4.5). However we will mainly be concerned with chiral zero modes and generically do not explicitly determine the vector-like ones. But one has to keep in mind that they might be present and one has to worry about soaking them up.

Coming back to equation (4.6), we can expand the Poincaré dual of the instanton cycle $[D_{\text{E3}}]$ in a basis $\{\omega_i\}$ of two-forms in $H^{1,1}(\mathcal{X})$

$$[D_{\text{E3}}] = \sum_i m^i \omega_i. \tag{4.8}$$

Then, we define the following matrix

$$M_{a,i} = \int_{\mathcal{X}} c_1(\mathcal{L}_a) \wedge [D_a] \wedge \omega_i, \tag{4.9}$$

with $i = 1, \dots, h^{(1,1)}(\mathcal{X})$ and $a = 1, \dots, K$ where K is the number of MSSM supporting

D7-branes carrying $U(N)$ gauge symmetry. To not over-constrain the system, we can assume that $K \leq h^{(1,1)}$ and so the maximal number of linear independent E3-brane instantons $N_{\rm E3}$ one is allowed to introduce is given by the kernel of the matrix $M_{a,i}$.

Since the kernel of the matrix (4.9) is not equal to $h^{(1,1)}(\mathcal{X})$ because of the chirality of the MSSM, it is clear that not all Kähler moduli can be stabilized by E3-brane instantons. Here we want to stress again that this equally holds for the KKLT model, as soon as a chiral matter sector is included.

4.1.3. The Chiral D7-brane Sector

The formula for the chiral spectrum between two D7-branes (2.50) implies that in order to obtain chirality, it is necessary that at least one of the D7-branes carries a non-trivial $U(N)$ gauge bundle. For our purposes, it is not crucial to have a complete MSSM sector, but we will just take one of the main features of the standard model, namely its chirality, and assume the minimal chiral configuration. We consider K stacks of N_a D7-branes wrapping the cycle D_a with vector bundle V_a. However, in order to avoid stability issues of higher rank vector bundles and vector bundle moduli, from now on we just choose line bundles $\mathcal{L}_a$ on the D7-branes.

For such chiral intersecting D-brane models, it is known that generically they contain anomalous $U(1)$ gauge symmetries. For D7-branes, these anomalies are canceled by the four-dimensional axions

$$\rho_a = \int_{D_a} C_4, \tag{4.10}$$

arising from the dimensional reduction of the R–R four-form along the four-cycle D_a. Indeed, the Chern–Simons action for a D7-brane on a four-cycle D_a contains terms of the form

$$S_{\rm CS} \sim \int_{\mathbb{R}^{1,3}\times D_a} C_4 \wedge F \wedge F, \tag{4.11}$$

which give rise to the following Green–Schwarz couplings. First, there is the mass term for the gauge field obtained by choosing two legs of C_4 along D_a and F to be the curvature of the internal line bundle $\mathcal{L}$. Second, the $\rho - A^2$ vertex arises from choosing all four legs of C_4 along D_a. Such a gauging of the axionic shift symmetry leads to a Fayet–Iliopoulos term for a $U(1)$, which in our case turns out to be

$$\xi_a = \frac{1}{\widehat{\mathcal{V}}} \int_{\mathcal{X}} c_1(\mathcal{L}_a) \wedge [D_a] \wedge \hat{J}. \tag{4.12}$$

Therefore, a chiral D7-brane sector necessarily gives rise to a D-term potential V_D of the following form

$$V_D = \sum_{a=1}^{K} \frac{1}{\mathrm{Re}(f_a)} \Big(\sum_i Q_i^{(a)} |\phi_i|^2 - \xi_a \Big)^2 , \tag{4.13}$$

where $m_{\mathrm{P}} = 1$ and $Q_i^{(a)}$ are the $U(1)_a$ charges of the canonically normalized matter fields ϕ_i. Furthermore, $\mathrm{Re}(f_a)$ denotes the real part of the gauge kinetic function for the corresponding D-brane. It is effectively the DBI action of a supersymmetric E3-brane instanton along the cycle D_a and reads

$$\mathrm{Re}(f_a) = e^{-\phi} \frac{1}{2} \int_{D_a} J \wedge J - e^{-\phi} \int_{D_a} \mathrm{ch}_2(B + \mathcal{L}_a) = \hat{\tau}_a - \mathrm{Re}(S) c_a . \tag{4.14}$$

Here, c_a denotes the integrated second Chern character of $B + \mathcal{L}_a$ on the respective D7-brane and $\hat{\tau}_a$ is the (Einstein frame) volume of D_a.

Note that this D-term is generically only of order $\mathcal{V}^{-2}$ in the volume expansion (3.21) so that an additional (natural) D-term supersymmetry breaking destabilizes the large volume minimum found at order $\mathcal{V}^{-3}$. Therefore, for preserving the large volume minimum we will require that the D-term vanishes, i. e. $V_D = 0$. The other option is to allow for significant fine-tuning and use this D-term in a hidden sector for up-lifting the AdS minimum to a small and positive vacuum energy [90, 119–121].

4.1.4. Moduli Stabilization with D-Terms

After having shown that the D-terms stemming from the chiral matter sector do not destabilize the LARGE volume minimum, we now explore their impact on moduli stabilization. Therefore let us expand the Kähler form J in the basis $\{\omega_i\}$ as $J = \sum_i t^i \omega_i$. Recalling then equation (4.12), we find that the Fayet–Iliopoulos parameter can be expressed as

$$\xi_a = \frac{1}{\widehat{\mathcal{V}}} \int_{\mathcal{X}} c_1(\mathcal{L}_a) \wedge [D_a] \wedge \hat{J} = \frac{1}{\widehat{\mathcal{V}}} \sum_i M_{a,i} \hat{t}^i , \tag{4.15}$$

so that the Kähler moduli will also appear in the D-terms. The vanishing of the D-terms then provides additional restrictions on the t^i. Indeed, the number of moduli fixed through these equations is given by the rank of the matrix $M_{a,i}$, defined in (4.9), which satisfies $\mathrm{rk}(M) \geq K_{\mathrm{anom}}$ where K_{anom} denotes the number of anomalous $U(1)$ gauge factors supported on the MSSM branes. To be more precise, the Kähler moduli counted by the defect of $M_{a,i}$ are fixed by the D-term. These are orthogonal to the ones possibly fixed by E3-instantons and are in the kernel of $M_{a,i}$. Since the MSSM matter spectrum is chiral, it is clear from the definition of $M_{a,i}$ that there must be at least one anomalous $U(1)$ gauge factor.

To summarize: at most $h^{(1,1)} - K_{\mathrm{anom}}$ Kähler moduli can be fixed by E3-instantons whereas for the remaining moduli, which control the size of the D7-branes supporting the MSSM sector, there appears a D-term potential. For not destabilizing the large volume minimum due to the $1/\mathcal{V}^2$ factor in front, this D-term has to vanish. Therefore, despite our initial concern, with sufficient rigid instantons being present in a model, we have enough constraints to fix all Kähler moduli.

For the case that we cannot fix all remaining Kähler moduli via instantons and D-terms, there still exists the possibility that they are frozen similar to the volume $\mathcal{V}$ by perturbative corrections to the F-term scalar potential.

Clearly, the general arguments presented above need to be investigated more carefully in every concrete model. As in this section, we focus more on the question of moduli stabilization rather than the detailed realization of a complete standard model sector with D7-branes, from now on, though not dynamically proven but at least phenomenologically motivated, we generally assume

$$\langle \Phi_{\text{SM}} \rangle = \langle \Phi_{\text{exo}} \rangle = \langle \Phi_{\text{abel}} \rangle = 0, \tag{4.16}$$

so that the vanishing of the D-terms in the MSSM sector effectively implies the vanishing of the Fayet–Iliopoulos parameters (4.15). We will mention at certain points the changes once vevs of Φ_{abel} are non-vanishing, but as we stressed already so far we do not have a complete theory to dynamically freeze these fields.

4.1.5. F-term Scalar Potential

From the discussion in the previous sections it is clear now that within the framework of the LVS, just one small four-cycle is not enough to account for both, moduli stabilization and a chiral matter sector. At least one additional small four-cycle without chiral intersections with the cycle supporting the matter branes is needed to stabilize the overall volume.

In the original work about the large volume scenario [30], only the case with *one* E3-brane instanton along *one* small four-cycle was studied in detail. Later it was argued that similar results carry over to configurations where more than one four-cycle stays small supporting instantons [122]. For our purpose it is useful and illustrative to start again from a general set-up and perform the steps along the lines of [30], including now also the possibility that the cycles that the instantons and D-branes wrap can be (effective) linear combinations of the base cycles.

Similarly to section 3.2, we assume that the complex structure moduli U and the axio-dilaton S have been fixed by fluxes via $D_U W = D_S W = 0$ and the value of the Gukov–Vafa–Witten superpotential (2.40) in the minimum will again be denoted by W_0. For the stabilization of the Kähler moduli we use the usual α'-corrected Kähler potential (3.5) and introduce E3-instantons. However, we we allow for instantons wrapping general four-cycles $D_\alpha = M_\alpha^i D_i$ where M_α^i are the wrapping numbers of the instanton α and $\{D_i\}$ is a basis of four-cycles on $\mathcal{X}$. The superpotential then takes the form

$$W = W_0 + \sum_\alpha A_\alpha e^{-2\pi M_\alpha^i T_i}, \tag{4.17}$$

where the sum is over all contributing instantons in the large radius limit. Computing the Kähler metric similarly to [89], we can write the scalar F-term potential as

$$\begin{aligned} V_F = e^{\mathcal{K}} \Bigg(& -\frac{(2\pi)^2}{2}(2\widehat{\mathcal{V}} + \hat{\xi}) \sum_{\alpha,\beta} \mathrm{vol}(D_\alpha \cap D_\beta) A_\alpha \bar{A}_\beta e^{-2\pi M_\alpha^i T_i} e^{-2\pi M_\beta^j \overline{T}_j} \\ & + \frac{(2\pi)^2}{4} \frac{4\widehat{\mathcal{V}} - \hat{\xi}}{\widehat{\mathcal{V}} - \hat{\xi}} \sum_{\alpha,\beta} \hat{\tau}_\alpha \hat{\tau}_\beta A_\alpha \bar{A}_\beta e^{-2\pi M_\alpha^i T_i} e^{-2\pi M_\beta^j \overline{T}_j} \\ & + \frac{2\pi}{2} \frac{4\widehat{\mathcal{V}}^2 + \widehat{\mathcal{V}}\hat{\xi} + 4\hat{\xi}^2}{(2\widehat{\mathcal{V}} + \hat{\xi})(\widehat{\mathcal{V}} - \hat{\xi})} \sum_\alpha \hat{\tau}_\alpha \Big(A_\alpha e^{-2\pi M_\alpha^i T_i} \overline{W} + \bar{A}_\alpha e^{-2\pi M_\alpha^i \overline{T}_i} W \Big) \\ & + 3\hat{\xi} \frac{\widehat{\mathcal{V}}^2 + 7\widehat{\mathcal{V}}\hat{\xi} + \hat{\xi}^2}{(2\widehat{\mathcal{V}} + \hat{\xi})^2 (\widehat{\mathcal{V}} - \hat{\xi})} |W|^2 \Bigg). \end{aligned} \tag{4.18}$$

Here we have used $\widehat{\mathcal{V}}$ and $\hat{\tau}_\alpha$ to respectively denote in Einstein frame the volume of the Calabi–Yau manifold and the volume of the four-cycle wrapped by the instanton α. Furthermore, to simplify the formulas we used

$$\mathrm{vol}(D_\alpha \cap D_\beta) = M_\alpha^i M_\beta^j \kappa_{ijk} \hat{t}^k \tag{4.19}$$

for the volume of the intersection of two four-cycles D_α and D_β (in Einstein frame) and we have defined $\hat{\xi} = \xi / g_s^{3/2}$.

Let us now perform the large volume expansion of V_F. Note that in this limit the second term in (4.18) is sub-leading. Keeping also only the leading term W_0 in the superpotential, we find up to an overall constant

$$\begin{aligned} V_F \simeq & -\frac{(2\pi)^2}{\widehat{\mathcal{V}}} \sum_{\alpha,\beta} \mathrm{vol}(D_\alpha \cap D_\beta) A_\alpha \bar{A}_\beta e^{-2\pi M_\alpha^i T_i} e^{-2\pi M_\beta^j \overline{T}_j} \\ & + \frac{2\pi}{\widehat{\mathcal{V}}^2} \sum_\alpha \hat{\tau}_\alpha \Big(A_\alpha e^{-2\pi M_\alpha^i T_i} \overline{W}_0 + \bar{A}_\alpha e^{-2\pi M_\alpha^i \overline{T}_i} W_0 \Big) + \frac{3}{4} \frac{\hat{\xi}}{\widehat{\mathcal{V}}^3} |W_0|^2. \end{aligned} \tag{4.20}$$

In the one instanton case, the second term in equation (4.20) was the only place where the axion corresponding to the instanton appeared. Recalling $T_i = \hat{\tau}_i + i\rho_i$, such a term could be written as $Xe^{i\rho} + \overline{X}e^{-i\rho}$ and upon minimizing the potential with respect to ρ, it was rendered real and negative [30, 98]. The negativity of this term was crucial for the existence of the minimum of the F-term potential at exponentially large volume.

In the general case of more than one instanton, the first term in (4.20) also depends on the axions, provided the volume of the intersection locus of the respective instanton cycles is non-vanishing. In this case, a more careful analysis of V_F is needed. We do not consider this case here but require instead that

$$\mathrm{vol}(D_\alpha \cap D_\beta) = 0, \tag{4.21}$$

for all pairs of instantons with $\alpha \neq \beta$. This guarantees that the respective axions are stabilized in the way described above by the second term in (4.20).

For the following, we will restrict ourselves to the case of an instanton wrapping a general four-cycle D_{E3} in the Calabi–Yau manifold. Employing then the stabilization of the axion associated to the instanton illustrated above, the F-term potential for one E3-brane instanton simplifies to

$$\begin{aligned} V_F \simeq &- \frac{(2\pi)^2}{\widehat{\mathcal{V}}}\,\mathrm{vol}(D_{\mathrm{E3}} \cap D_{\mathrm{E3}})|A_{\mathrm{E3}}|^2 e^{-4\pi\hat{\tau}_{\mathrm{E3}}} \\ &- \frac{4\pi}{\widehat{\mathcal{V}}^2}\hat{\tau}_{\mathrm{E3}} e^{-2\pi\hat{\tau}_{\mathrm{E3}}}|A_{\mathrm{E3}}W_0| + \frac{3}{4}\frac{\hat{\xi}}{\widehat{\mathcal{V}}^3}|W_0|^2. \end{aligned} \tag{4.22}$$

This expression is nearly similar to the well-known expression of V_F (3.21) in the original LARGE volume scenario. The only difference is the first term. If we find that

$$\mathrm{vol}(D_{\mathrm{E3}} \cap D_{\mathrm{E3}}) \simeq -\sqrt{\hat{\tau}_{\mathrm{E3}}}, \tag{4.23}$$

then as shown in section 3.2, we are guaranteed to find a minimum of V_F at exponentially large values of $\mathcal{V}$ and with $\tau_{\mathrm{E3}} \simeq \log(\mathcal{V})$. However, in general the minima of V_F will depend on the concrete model and on the way the moduli are stabilized.

Let us summarize the results of this section. Performing the LARGE volume expansion of the scalar F-term potential for a general instanton configuration leads to an expression where the axions corresponding to the instantons cannot be stabilized easily. We did not attempt to address this question but restricted us to the case of one instanton along a general four-cycle.

The main question is now whether it is indeed possible to freeze the Kähler moduli controlling the size of the MSSM D7-branes via the D-terms of the $U(1)$ gauge factors supported on these D7-branes and whether these sizes are of the same order of magnitude as the instantonic four-cycles. Let us collect the formal constraints we have to successfully implement in a concrete model for this scenario to work:

- Find a Calabi–Yau of swiss cheese type with one large four-cycle controlling the size of the manifold and small cycles typically arising from resolutions of singularities.[4]

- Define an orientifold projection of this space leading to O3- and O7-planes and freeze the complex structure and dilaton moduli by G_3-form flux. This latter will contribute to the D3-brane tadpole.

- Introduce a set of intersecting (magnetized) D7-branes supporting the chiral MSSM spectrum and a hidden D7-brane sector such that the D7- and D3-brane tadpole cancellation conditions are satisfied. Moreover, the D7-branes must be free of Freed–Witten anomalies [68].

- Classify all E3-instantons on this space which from the zero mode structure can contribute to the uncharged superpotential. For this, a sufficient condition is that the instanton is rigid and has no other chiral or vector-like zero modes from E3–D7 intersections. Furthermore, one also needs to ensure that the instantons are free of Freed–Witten anomalies [123].

- Compute the effective F- and D-term potential and analyze whether the combination of both freezes all Kähler moduli inside the Kähler cone with the size of the D7-branes coming out of the same order as the sizes of the instantons $\tau \simeq \log(\mathcal{V})$.

Moreover, since in the non-supersymmetric large volume minimum the D-terms vanish, we still only have F-term supersymmetry breaking and the soft-terms can be computed in the usual way [101].

In the next section, we will explicitly carry out some of the steps mentioned above for a concrete Calabi–Yau orientifold model. Our simple (toy) model is neither realistic nor can all conditions mentioned above be met explicitly, but it nevertheless shows how this program can partly be realized even on a simple Calabi–Yau manifold.

4.2. An Example on the $\mathbb{P}_{[1,3,3,3,5]}[15]$ Calabi–Yau

From the previous section it is now clear that in the LVS, we need at least three Kähler moduli to have both, stabilization of all moduli by D-brane instantons and a chiral D7-brane sector. The exponentially large cycle, controlling the overall size of the manifold, is usually frozen by the competing effects of the leading order α'-corrections to the Kähler potential and the E3-instanton contribution. On the small cycles of a swiss cheese type Calabi–Yau, the instantons and the D7-branes will be distributed.

4 It would be interesting to investigate whether also for instance Calabi–Yaus with a fibration structure can lead to large volume moduli freezing. For these the volume can usually be brought to the schematic form $\mathcal{V} = \tau_1\sqrt{\tau_2} - \sum_I \tau_I^{3/2}$.

A well studied class of Calabi–Yau three-folds is given by hypersurfaces in weighted projective spaces. A candidate which actually is of swiss cheese type and has more than one small blow-up cycle is the resolution of the $\mathbb{P}_{[1,3,3,3,5]}[15]$ manifold. It will turn out that this Calabi–Yau is still not rich enough to allow for complex structure moduli stabilization by fluxes and a complete MSSM sector, but serves as a simply toy model to give a proof of principle how the combination of F- and D-term moduli stabilization can work in more realistic models. We describe now the algebraic geometry of this Calabi–Yau in more detail in the next subsections.

4.2.1. The Topology of $\mathbb{P}_{[1,3,3,3,5]}[15]$

Toric Resolution

The $\mathbb{P}_{[1,3,3,3,5]}[15]$ manifold has a $\mathbb{Z}_3$ singularity along the complex line $x_1 = x_5 = 0$, which is met by the hypersurface constraint. The resolution of this A_2 orbifold singularity introduces two intersecting $\mathbb{P}^1$s over the line.

This resolution is easily described invoking the methods of toric geometry. Besides the five divisors $v_1^* = (1,0,0,0)$, $v_2^* = (0,1,0,0)$, $v_3^* = (0,0,1,0)$, $v_4^* = (0,0,0,1)$, $v_5^* = (-3,-3,-3,-5)$ one introduces the two blowing-up divisors $v_6^* = (-2,-2,-2,3)$ and $v_7^* = (-1,-1,-1,-1)$. The unique maximal triangulation is then given by

$$\begin{aligned}\text{Triangle} =\Big\{&[1,2,3,4],\ [1,2,3,5],\ [1,2,4,7],\ [1,2,6,7],\ [1,2,5,6],\ [1,3,4,7],\\ &[1,3,6,7],\ [1,3,5,6],\ [2,3,4,7],\ [2,3,6,7],\ [2,3,5,6]\Big\}.\end{aligned} \tag{4.24}$$

The data of the associated linear sigma model is the following. We have seven complex coordinates x_i with three $U(1)$ symmetries. The corresponding charges are shown in (4.25).

x_1	x_2	x_3	x_4	x_5	x_6	x_7	p
3	3	3	5	1	0	0	15
2	2	2	3	0	1	0	10
1	1	1	1	0	0	1	5

(4.25)

The divisors D_i are defined by the constraints $x_i = 0$ and the resulting Stanley–Reisner ideal, found with the Maple package *Schubert*, reads:

$$\text{SR} = \{x_4x_5,\ x_4x_6,\ x_5x_7,\ x_1x_2x_3x_6,\ x_1x_2x_3x_7\}. \tag{4.26}$$

The triple intersection numbers in the basis $\eta_1 = D_5$, $\eta_2 = D_6$, $\eta_3 = D_7$ are calculated as

$$I_3 = 9\eta_1^3 - 40\eta_2^3 - 40\eta_3^3 - 15\eta_1^2\eta_2 + 25\eta_1\eta_2^2 - 5\eta_2^2\eta_3 + 15\eta_2\eta_3^2. \tag{4.27}$$

From section 3.2 we recall that the volume τ_i of the divisor D_i and the overall volume of the manifold (in string frame) are expressed in terms of the Kähler form in the following way

$$\tau_i = \frac{1}{2}\int_{\mathcal{X}} [D_i] \wedge J \wedge J, \qquad \mathcal{V} = \frac{1}{6}\int_{\mathcal{X}} J \wedge J \wedge J. \tag{4.28}$$

Expanding then the Kähler form in the basis $\{\eta_1, \eta_2, \eta_3\}$ from above as $J = \sum_{i=1}^{3} t_i[\eta_i]$ we find for the volumes of the divisors D_5, D_6 and D_7

$$\begin{aligned} \tau_5 &= \frac{1}{2}(3t_1 - 5t_2)^2, \\ \tau_6 &= \frac{5}{6}\Big[(3t_3 - t_2)^2 - (5t_2 - 3t_1)^2\Big], \\ \tau_7 &= -\frac{5}{2}(t_2 - 4t_3)(t_2 - 2t_3). \end{aligned} \tag{4.29}$$

The Kähler Cone

Next, we are going to determine the Kähler cone, which is defined by the condition that the volumes of all effective curves $\mathcal{C}$ are positive. The first step is to compute the cone of all effective curves, which is called the Mori cone and then deduce from this the Kähler cone by the condition $\int_{\mathcal{C}} J > 0$. The resulting constraints describing the Kähler cone are

$$t_2 - 2t_3 > 0, \qquad t_1 - 2t_2 + t_3 > 0, \qquad -3t_1 + 5t_2 > 0. \tag{4.30}$$

These conditions ensure also that the overall volume $\mathcal{V}$ is positive, that all volumes of effective divisors are positive and, by construction, that all volumes of holomorphic curves are positive.

Swiss Cheese Structure

For a large volume compactification we want to make one four-cycle large while keeping the others small. Let us therefore take a closer look at the volume. Using the Kähler cone restrictions above, we find that $\mathcal{V}$ can be written as

$$\mathcal{V} = \sqrt{\frac{2}{45}}\Big((5\tau_5 + 3\tau_6 + \tau_7)^{3/2} - \tfrac{1}{3}(5\tau_5 + 3\tau_6)^{3/2} - \tfrac{\sqrt{5}}{3}\tau_5^{3/2}\Big). \tag{4.31}$$

From this expression we see that this model admits a swiss cheese structure. Indeed, we can make τ_7 large so that the total volume $\mathcal{V}$ becomes large while keeping the four-cycles volumes τ_5 and τ_6 small. On the latter ones the D-branes supporting the MSSM will be wrapped.

In such a set-up, we are thus not allowed to wrap D-branes supporting the MSSM on (some combination involving) D_7 because then the gauge coupling $1/g^2_{\rm YM} \sim \tau_7$ would be too small. Similarly, we ignore instantons along this divisor, because its contribution to the superpotential is exponentially suppressed. We are then left with the two divisors D_5 and D_6. Note however that not all combinations of D_5 and D_6 are allowed. We have to wrap D-branes and instantons along effective cycles, i. e. positive linear combinations of the divisors.

Rigid Cycles

Furthermore, we require the instanton to be rigid in the sense that no extra fermionic zero modes from the deformations of the cycle or from Wilson lines along one-cycles do appear. The transverse deformations of a holomorphic four-cycle D are counted by the global sections of the normal bundle N of D. By the adjunction formula and Serre duality on D we get $H^0(D, N_D) = H^2(D, \mathcal{O}_D)$. The Wilson lines are counted by the non-contractable one-cycles on D, which are counted by $H^1(D, \mathcal{O}_D)$. Therefore, for an instanton to not have additional deformation zero modes we will require

$$H^0(D, \mathcal{O}_D) = 1, \qquad H^i(D, \mathcal{O}_D) = 0, \qquad \text{for } i = 1, 2. \tag{4.32}$$

A necessary criterion for this is that the Euler characteristic of the trivial line bundle over D is equal to one, i. e.

$$\chi(D, \mathcal{O}_D) = \sum_{i=0}^{2} (-1)^i H^i(D, \mathcal{O}_D) = 1. \tag{4.33}$$

Employing the Koszul sequence

$$0 \to \mathcal{O}_{\mathcal{X}}[-D] \to \mathcal{O}_{\mathcal{X}} \to \mathcal{O}_D \to 0, \tag{4.34}$$

and the resulting long exact sequence in cohomology, one obtains the relation $\chi(D, \mathcal{O}_D) = \chi(\mathcal{X}, \mathcal{O}[-D])$.

In our concrete example, for a four-cycle $D = m\eta_1 + n\eta_2 + l\eta_3$ the Euler characteristic is calculated as

$$\begin{aligned} \chi(D, \mathcal{O}_D) = \tfrac{15}{2} n l^2 + \tfrac{25}{2} m n^2 - \tfrac{5}{2} n^2 l - \tfrac{15}{2} m^2 n - \tfrac{20}{3} l^3 \\ - \tfrac{20}{3} n^3 + \tfrac{5}{3} n + \tfrac{5}{3} l + \tfrac{3}{2} m^3 - \tfrac{1}{2} m. \end{aligned} \tag{4.35}$$

Looking via a computer search for combinations with $\chi(D, \mathcal{O}_D) = 1$ and $l = 0$ we have found the solutions

$$(m, n, l) = \big\{(1,0,0),\ (1,1,0),\ (2,1,0),\ (2,2,0)\big\}. \tag{4.36}$$

In order to compute the precise cohomology classes $H^i(D, \mathcal{O}_D)$, we use the cohomology

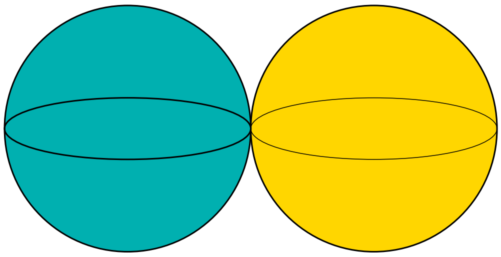


Figure 4.1.: Singular rigid divisors

classes of general line bundles on the toric ambient space shown in appendix A and then run them through the Koszul sequences for the restrictions on the Calabi–Yau hypersurface and the divisors D. The result is that the first four divisors in (4.36) really have $H^i(D, \mathcal{O}_D) = (1, 0, 0)$, i. e. these are irreducible effective divisors without any Wilson lines or transverse deformations.

One comment is in order here. Note that the three rigid divisors $(1, 1, 0)$, $(2, 1, 0)$, $(2, 2, 0)$ are singular. Let us explain this for the first one $D_5 + D_6$. The only constraint one can write down of this degree is $Q = x_5 x_6 = 0$. This defines two complex divisors $x_5 = 0$ and $x_6 = 0$ intersecting along the curve $x_5 = x_6 = 0$, where the manifold becomes singular. Since the four-cycle has no deformations, the singularity cannot be smoothed out. A lower dimensional analogy is shown in figure 4.1. In the following we allow E3-instantons and D7-branes to also wrap these rigid cycles, though we know that strictly speaking, our method for the calculation of the Euler characteristic is not valid here. However, in order to demonstrate our ideas we tacitly ignore this.

Diagonal Basis

In the following it will be more convenient to work in a basis where the volume $\mathcal{V}$ as well as the triple intersection numbers become particularly simple. Guided by (4.31), we introduce the new basis of divisors as

$$D_a = 5D_5 + 3D_6 + D_7, \qquad D_b = 5D_5 + 3D_6, \qquad D_c = D_5, \tag{4.37}$$

for which the triple intersection numbers diagonalize

$$I_3 = 5D_a^3 + 45D_b^3 + 9D_c^3. \tag{4.38}$$

The total volume in terms of the divisor volumes τ_a, τ_b and τ_c reads

$$\mathcal{V} = \sqrt{\frac{2}{45}}\Big(\tau_a^{3/2} - \tfrac{1}{3}\tau_b^{3/2} - \tfrac{\sqrt{5}}{3}\tau_c^{3/2}\Big). \tag{4.39}$$

Expanding also the Kähler form in this diagonal basis as $J = t_a D_a - t_b D_b - t_c D_c$, we find that the Kähler cone conditions have the very simple form

$$\frac{1}{3}t_a > t_b > t_c > 0. \tag{4.40}$$

As one can see from the above, the large divisor is now simply D_a. For the gauge couplings not to be unrealistically small, we do not wrap the D7-branes supporting the MSSM along the large cycle. Moreover, significant E3-instanton contributions only arise from instantons wrapped on the small four-cycles. Therefore, we can make the general ansatz for the D-brane and instanton cycles

$$D_{\text{D7}} = n_b D_b + n_c D_c, \qquad D_{\text{E3}} = m_b D_b + m_c D_c, \tag{4.41}$$

where now the wrapping numbers n and m need not be integer. They are related to the wrapping numbers n_i in the $\{\eta_i\}$ basis by

$$n_b = \frac{1}{3}n_2, \qquad n_c = n_1 - \frac{5}{3}n_2, \tag{4.42}$$

and similarly for (m_b, m_c).

4.2.2. Moduli Stabilization

Now that we have collected all the topological data, we can develop our model further. It will turn out that the tadpole cancellation conditions for the present set-up impose strong restrictions so that we cannot consider a full MSSM set-up but only a chiral toy model. We will have two stacks of D7-branes wrapping rigid four-cycles D_A and D_B where only on the first one a non-trivial line bundle $\mathcal{L}_A$ is turned on. We consider the standard model as being part of the $U(N_A)$ gauge group on the first stack of branes (even though in the eventual model it will not have large enough gauge group). Then we get MSSM matter from the intersections AA' and AB where the prime denotes the orientifold image. Connecting to our discussion in section 4.1.2, we in general allow the gauge group $U(N_A)$ to be larger than just the MSSM gauge group. Then from the two intersections AA' and AB we get matter Φ_{SM} which is part of the standard model. Furthermore, we get other matter Φ_{abel} transforming in singlet representations of the MSSM gauge group, but carrying certain charges under abelian $U(1)$s orthogonal to $U(1)_Y$. In addition, in order for satisfying the D7-brane tadpoles we need extra hidden sector branes.

Before we give the complete model, let us first elaborate on the D- and F-term constraints.

D-Term Constraints

In section 4.1.3 we have explained that the D-terms in large volume scenarios should vanish in order not to spoil the $1/\mathcal{V}$ expansion of the scalar F-term potential and the resulting minimum. The D-term contains the Fayet–Iliopoulos parameter ξ and the possible matter fields $\Phi_{\rm SM}$, $\Phi_{\rm exo}$ and $\Phi_{\rm abel}$. However, as argued previously, for the simple reason that the SM gauge symmetry is unbroken at low energies, at least the vevs of the first two matter fields have to vanish and for $\Phi_{\rm abel}$ it is likely to vanish. For the MSSM sector, we are thus left with the requirement that $\xi_A = \xi_B = 0$. Recalling the precise form of the FI-parameter (4.12), the condition $\xi_A = 0$ reads

$$0 = \int c_1(\mathcal{L}_A) \wedge [D_{{\rm D7}_A}] \wedge J. \tag{4.43}$$

For the second D7-brane the condition $\xi_B = 0$ is trivially satisfied because of $c_1(\mathcal{L}_B) = 0$. Next, we consider the (chiral) zero mode constraint from the D7–E3 intersections. The only non-trivial equation comes from D7$_A$ and reads

$$0 = \int c_1(\mathcal{L}_A) \wedge [D_{{\rm D7}_A}] \wedge [D_{\rm E3}]. \tag{4.44}$$

Using then our ansatz (4.41) in the diagonal basis, we find that the only suitable solution to the two equations above is

$$J = t_a[D_a] - t[D_{\rm E3}]. \tag{4.45}$$

Let us note that this solution implies $t_b = \frac{1}{3}m_b t$ and $t_c = \frac{1}{3}m_c t$. Comparing with the Kähler cone constraint $t_b > t_c > 0$ and going back to the basis in $\{\eta_1, \eta_2, \eta_3\}$, we see that only wrapping numbers with $2m_2 > m_1 > \frac{5}{3}m_2$ are possible. This cannot be solved by any of the rigid cycles $(m_1, m_2, m_3) \in \{(1,0,0),\ (1,1,0), (2,1,0),\ (2,2,0)\}$. However, the choice $(m_1, m_2, m_3) = (2,1,0)$, i. e. $D_{\rm E3} = \frac{1}{3}(D_b + D_c)$, is at least on the boundary of the Kähler cone at $t_b = t_c$. Of course, we cannot choose instantons at will but have to take all of them into account. But we can arrange our set-up in such a way that only an instanton along the cycle $(m_1, m_2, m_3) = (2,1,0)$ contributes to the stabilization of the Kähler moduli. We will come back to this point after we specified the D-brane configuration in our model. Note furthermore, by allowing a non-vanishing vev for $\Phi_{\rm abel}$, it might be possible to fix t_b and t_c on a ray inside the Kähler cone via the instanton above.

Let us now choose the stacks of D7-branes to wrap the rigid four-cycles

$$D_{\mathrm{D7}_A} = D_5 + D_6 = \frac{1}{3}(D_b - 2D_c), \qquad D_{\mathrm{D7}_B} = D_5 = D_c, \tag{4.46}$$

with the line bundles

$$\mathcal{L}_A = \frac{1}{3}(2D_b + 5D_c), \qquad \mathcal{L}_B = \mathcal{O}. \tag{4.47}$$

With this choice, as shown above, there are no chiral zero modes on the D7–E3 intersections. However, similar to [124], we expect both vector-like bosonic and fermionic zero modes, because, as shown in figure 4.1, the rigid E3-instanton actually contains both $D_{\mathrm{D7}_A} = D_5 + D_6$ and $D_{\mathrm{D7}_B} = D_5$ as a sub-locus. One way to get rid of these zero modes, would be to turn on discrete Wilson lines or discrete displacement on the D7-brane resp. E3-instanton. It is beyond the scope of this thesis to analyze mathematically this possibility for these divisors. From now on, we proceed by assuming that such non-chiral zero modes can be made massive so that indeed the E3-instanton on $D_{\mathrm{E3}} = 2D_5 + D_6$ contributes to the uncharged superpotential.

Before concluding this part, let us note that the vanishing of the D-term gives rise to a minimum of the scalar D-term potential. Moreover, we have argued that the F-term potential does not depend on at least one linear combination of Kähler moduli $\bar{\tau}$ which however appears in the D-term. For moduli stabilization this means that $\partial V/\partial\bar{\tau} = \partial V_D/\partial\bar{\tau} = 0$ is solved by the vanishing D-term and thus in our set-up fixes

$$t_b = t_c =: t. \tag{4.48}$$

In the diagonal basis this solution implies that D_6 shrinks to zero size but D_5 stays finite. Note first, our standard model branes do both involve D_5 and so their volume is always non-zero. Second, for a non-vanishing vev of Φ_{abel} we expect the volume of D_6 to be finite.

F-Term Constraints

Let us now go on and study the F-term potential. Since we only have a single instanton contributing to the potential, we can refer to equation (4.22). Using then the concrete data of our model, we find $\mathrm{vol}(D_{\mathrm{E3}} \cap D_{\mathrm{E3}}) = -5t_b - t_c$ and therefore

$$V_F \simeq \frac{(2\pi)^2}{\widehat{\mathcal{V}}}(5\hat{t}_b + \hat{t}_c)|A_{\mathrm{E3}}|^2 e^{-4\pi\hat{\tau}_{\mathrm{E3}}} - \frac{4\pi}{\widehat{\mathcal{V}}^2}\hat{\tau}_{\mathrm{E3}} e^{-2\pi\hat{\tau}_{\mathrm{E3}}}|A_{\mathrm{E3}}W_0| + \frac{3}{4}\frac{\hat{\xi}}{\widehat{\mathcal{V}}^3}|W_0|^2. \tag{4.49}$$

The first term cannot be expressed as a square root of $\hat{\tau}_{\mathrm{E3}} = \frac{1}{6}(45\hat{t}_b^2 + 9\hat{t}_c^2)$ and so the analysis of section 3.2 for the minimum of V_F at large volumes is not applicable. However, employing equation (4.48), we find the following relation between the volume of the instanton cycle and the volume of its self-intersection

$$\mathrm{vol}(D_{\mathrm{E3}} \cap D_{\mathrm{E3}}) = -6\hat{t} = -2\sqrt{\hat{\tau}_{\mathrm{E3}}}. \tag{4.50}$$

Note that this volume formally is negative, which simply reflects the fact that the four-cycle D_{E3} is exceptional with a self-intersection not corresponding to an effective two-cycle. Using this relation, the above expression becomes

$$V_F \simeq \frac{8\pi^2}{\widehat{\mathcal{V}}}\sqrt{\hat{\tau}_{\mathrm{E3}}}|A_{\mathrm{E3}}|^2 e^{-4\pi\hat{\tau}_{\mathrm{E3}}} - \frac{4\pi}{\widehat{\mathcal{V}}^2}\hat{\tau}_{\mathrm{E3}} e^{-2\pi\hat{\tau}_{\mathrm{E3}}}|A_{\mathrm{E3}}W_0| + \frac{3}{4}\frac{\hat{\xi}}{\widehat{\mathcal{V}}^3}|W_0|^2. \tag{4.51}$$

Recalling our discussion in section 4.1.5, the $1/\widehat{\mathcal{V}}$ expansion of the F-term potential is of the form which allows for a minimum of V_F at large values of $\widehat{\mathcal{V}}$.

We can then treat these variables as fixed and use their relation to the Kähler moduli. We obtain

$$t_b = t_c = t = \frac{1}{3}\sqrt{\tau_{\mathrm{E3}}}, \qquad t_a = \left(\frac{6}{5}\mathcal{V}_0 + \frac{2}{5}\tau_{\mathrm{E3}}^{3/2}\right)^{1/3}, \tag{4.52}$$

where we denoted the value of $\mathcal{V}$ in the minimum by $\mathcal{V}_0$. Therefore, in this model all Kähler moduli have been stabilized. To be more precise, we have seen that all coefficients t_a in the expansion of J are fixed and so are the real parts of the Kähler moduli T_i. Furthermore, through the F-term potential the axion corresponding to the instanton cycle is stabilized and via the D-term and Green–Schwarz mechanism the axion associated with the matter sector gets massive.

For the Kähler moduli, we now get three different mass scales. Since the D-term vanishes in the minimum, the mass of the large volume modulus and the small cycle fixed by the instanton do not change. Just keeping track of the $1/\mathcal{V}_0$ factor they scale like $m_{\tau_b} \simeq m_{\mathrm{P}}/\mathcal{V}_0^{3/2}$ and $m_{\tau_s} \simeq m_{\mathrm{P}}/\mathcal{V}_0$ [98]. The orthogonal Kähler modulus fixed by the D-term then has mass $m_{\tau_D} \simeq m_{\mathrm{P}}/\sqrt{\mathcal{V}_0}$, which being of string scale size is much heavier than the other two.

Numerical Analysis

In order to explicitly check that the LARGE volume minimum of the full scalar potential persists in our model, we have numerically evaluated equation (4.49). Installing the appropriate factors of 2π and g_s, and choosing $|A_{E3}| = 1$, $|W_0| = 5$, we minimized the function[5]

$$\begin{aligned} V_{F+D}(\mathcal{V}, \tau_b, \tau_c) = \; & \frac{18.6}{\mathcal{V}}\left(\sqrt{5\tau_b} + \sqrt{\tau_c}\right) g_s e^{-\frac{4\pi}{3}\frac{1}{g_s}(\tau_b+\tau_c)} \\ & - \frac{20.9}{\mathcal{V}^2}(\tau_b + \tau_c) g_s^2 e^{-\frac{2\pi}{3}\frac{1}{g_s}(\tau_b+\tau_c)} + \frac{6.5}{\mathcal{V}^3} g_s^3 \\ & + \frac{13.3}{\mathcal{V}^2}\frac{1}{\tau_b - 2\tau_c} g_s^3 \left(\sqrt{5\tau_c} - \sqrt{\tau_b}\right)^2 . \end{aligned} \tag{4.53}$$

Note that we have not yet fixed the value of g_s which is determined by the vev of the dilaton. We have assumed that it is stabilized by fluxes and since we did not perform an explicit analysis of this mechanism, we choose $g_s = 1/10$ for convenience. However, as noted in [31], the stabilized volume $\mathcal{V}$ will depend exponentially on g_s through $\mathcal{V} \sim e^{c/g_s}$ where c is some constant. Thus, a more careful analysis of the flux sector is inevitable.

Coming back to the potential above, we observe that the dominant part of (4.53) is given by the D-term potential fixing the combination $\tau_b = 5\tau_c$. On top of that direction, we found a minimum of the potential in the variables $\mathcal{V}$ and τ_b. In figures 4.2 and 4.3, we have plotted two sections through the parameter space showing the potential in the vicinity of the minimum. The numerical values (in string units) in the minimum are $\mathcal{V} \approx 2.2 \cdot 10^{16}$ and the four-cycle volumes are stabilized at $\tau_b \approx 1.63$, $\tau_c \approx 0.33$.[6] For the volume of the standard model cycles we find $\tau_{\mathrm{SM}} \simeq 0.33$ and the value of the scalar potential in the minimum is of the order $V_{\min} \simeq -10^{-54}\, m_{\mathrm{P}}^4$.

The stabilized four-cycle volumes are in a region where we have to worry whether we can trust the supergravity approximation. Let us investigate more closely what the numerical reason is. Recall from section 3.3 the approximate formulas for the volume $\mathcal{V}$ and the four-cycle in the minimum

$$\mathcal{V} \simeq \frac{\mu g_s |W_0|}{2\lambda a_s |A_s|}\left(\frac{4\nu\lambda\xi}{\mu^2}\right)^{1/3} e^{\frac{a_s}{g_s}\left(\frac{4\nu\lambda\xi}{\mu^2}\right)^{2/3}}, \qquad \tau \simeq \left(\frac{4\nu\lambda\xi}{\mu^2}\right)^{2/3}, \tag{4.54}$$

where we have used the notation from equation (3.32). Note that λ contains the information about the intersection of the instanton cycles and thus depends on the topology of the manifold and on the cycles suitable for instantons. Furthermore, ξ is proportional to the Euler characteristic χ and so the above formulas depend strongly on the topology of the compactification manifold.

For our present model, using the data after D-term fixing but leaving the Euler character-

5 A very similar potential appeared in [125], but without the D-term part.

6 If we minimize the potential (4.18) instead of its large volume expansion (4.53), we find the minimum at $\tau_c \simeq 0.53$, $\tau_b \simeq 2.64$ and $\mathcal{V} \simeq 1.1 \cdot 10^{13}$ for $g_s = 1/10$. The difference in the value of $\mathcal{V}$ can be compensated by arranging $g_s = 1/12$ so that the minimum is at $\tau_c \simeq 0.53$, $\tau_b \simeq 2.64$ and $\mathcal{V} \simeq 7 \cdot 10^{15}$.

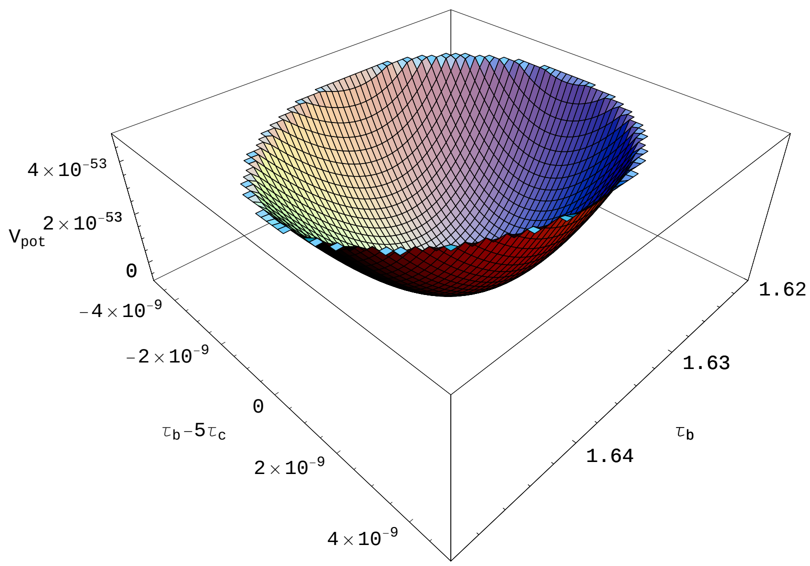


Figure 4.2.: The potential $V(\mathcal{V}, \tau_b, \tau_c)$ *for* $\mathcal{V} = 2.15 \cdot 10^{16}$

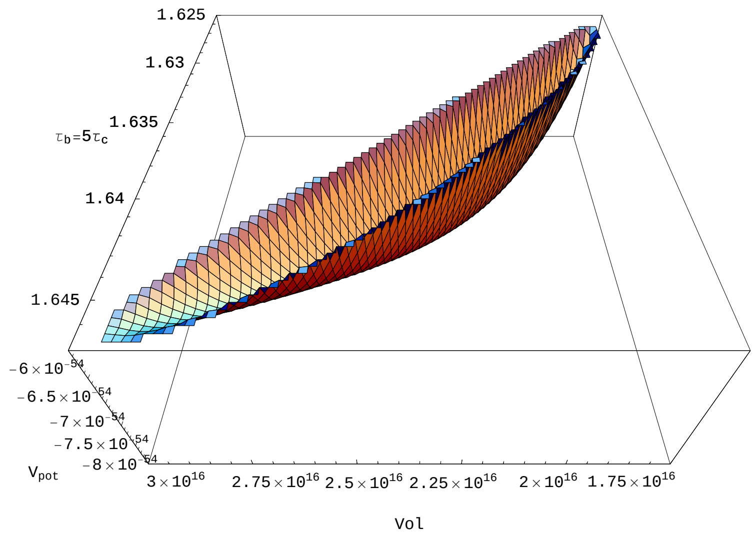


Figure 4.3.: The potential $V(\mathcal{V}, \tau_b, \tau_c)$ *for* $\tau_c = 0.33$

istic χ and the string coupling g_s unspecified, we obtain

$$\mathcal{V} \simeq 6.1 \cdot 10^{-2} g_s (-\chi)^{1/3} e^{0.145 \frac{1}{g_s} (-\chi)^{2/3}}, \qquad \tau_{\mathrm{SM}} \simeq 1.2 \cdot 10^{-2} (-\chi)^{2/3}. \tag{4.55}$$

Therefore the prefactor of order 10^{-2} in (4.55) and the smallness of the Euler characteristic $\chi = -144$ of our Calabi–Yau manifold are the reasons for the string frame four-cycle volume τ_{SM} to come out so small.

Just as a rough estimate, let us analyze for which values of g_s and χ the formulas (4.55) give more realistic values of the Kähler moduli. Choosing for instance $\tau_{\mathrm{SM}} = 1.2$ leads to $\chi \simeq -1000$. For the string coupling $g_s = 3/8$ we then get $\mathcal{V} = 5 \cdot 10^{15}$. This points towards choosing Calabi–Yau manifolds with Euler characteristics just at the limit of presently known examples for χ.

To conclude we have demonstrated at a specific swiss cheese Calabi–Yau manifold with $h^{(1,1)}(\mathcal{X}) = 3$ that a combination of E3-instantons and D7-brane D-terms can fix all three Kähler moduli in the large volume regime with all small cycles wrapped by D7-branes of order $\log(\mathcal{V})$. In our case, the D-terms (only) fixed the moduli on the boundary of the Kähler cone, where the four-cycle D_6 collapses. Furthermore, we have argued that out of the rigid divisors (4.36) only $D_{\mathrm{E3}} = 2\eta_1 + \eta_2$ contributes to the uncharged superpotential. However, actually the complete vector-like zero mode spectrum has to be computed for such overlapping singular divisors and presumably also discrete Wilson lines and displacements have to be included. We do not perform this complete mathematical investigation for this specific model here, as our intention was rather to exemplify for a concrete Calabi–Yau that the F- and D-term freezing scenario has a good chance to be realizable in concrete LARGE volume type IIB model with a chiral D7-brane sector.

5. GUT Models in the LARGE Volume Scenario

In the last chapter we investigated the robustness of the LARGE volume (and also the KKLT) scenario upon taking serious the D-brane instanton calculus. We learned that *charged zero modes*, which are present if the instanton cycle is also populated by D7-branes supporting chiral matter, spoil the generation of the usually assumed non-perturbative term in the superpotential (4.1). As a consequence, the usual mechanism stabilizing the Kähler moduli with a combination of instanton-generated non-perturbative contribution to the superpotential and α'-corrections to the Kähler potential does not work for all Kähler moduli at the same time.

Recent work on D-brane instanton calculus revealed that also the gauge kinetic function receives non-perturbative corrections induced by euclidean D3-branes. We will see that with the help of those, it is relatively easy to obtain a racetrack-like superpotential, generating effective parameters W_0^{eff} and A^{eff}, i. e.

$$W = W_0^{\text{eff}} + A^{\text{eff}} e^{-aT}, \tag{5.1}$$

which can be tuned to exponentially small values without fine-tuning. The strength of this fact is the following: remember that in the LVS, the two crucial scales m_{s} and $m_{3/2}$ depend on the model parameters like this:

$$m_{\text{s}} \sim \frac{m_{\text{P}}}{\sqrt{\mathcal{V}}}, \qquad m_{3/2} \sim \frac{|W_0|}{\mathcal{V}} m_{\text{P}}. \tag{5.2}$$

Up to now it was always assumed that the natural value for W_0, generated by G_3 flux is of the order $\mathcal{O}(1) - \mathcal{O}(10)$. Hence the two scales are controlled by only one parameter, the overall volume $\mathcal{V}$. If we require for phenomenological reasons TeV scale supersymmetry breaking, the string scale is already fixed in the intermediate range of $m_{\text{s}} \sim 10^{11}$ GeV. Though this scenario may have its virtues, it would be nice if we could also realize a conventional supersymmetric GUT scenario with $m_{\text{s}} \sim m_X \sim 10^{16}$ GeV. From (5.2) it is clear that therefore an overall volume of $\mathcal{V} \sim 10^5$ is required. If we require in addition TeV scale supersymmetry breaking, a value of W_0 as small as $\sim 10^{-10}$ is necessary. For a purely flux-generated W_0, although possible, this is probably not natural.

In section 5.1 we will see how poly-instanton corrections to the superpotential can induce exponentially small values of W_0 without fine-tuning. We will then explore this kind of models with the help of three examples in section 5.2. In the first one, the dominating mediation mechanism is gravity mediation. While the gravitino mass is at about $m_{3/2} \simeq 1\,\text{TeV}$, the soft terms come out by far to small. This is the result of a hierarchical suppressions which occurs when the D7-branes wrap a cycle where gaugino condensates generate a racetrack superpotential. The second one makes use of precisely this fact. Here, the gravitino and soft scalar masses are chosen in an intermediate regime while the gaugino masses are found at the weak scale, dominantly generated by anomaly mediation. This is a dynamical realization of split supersymmetry [126]. The third example is a modification of the first one, in which our original goal of TeV scale soft terms is realized again with gravity mediation only. Finally we comment on the cosmological constant problem in section 5.3.

5.1. Instanton Corrections and Gaugino Condensates

As usual in the LARGE volume scenario, we assume non-trivial R–R and NS–NS fluxes, combined into the three-form $G_3 = F_3 + SH_3$. They give rise to a tree-level superpotential of the form:

$$W_{\text{flux}} = \int_{\mathcal{X}} G_3 \wedge \Omega_3, \tag{5.3}$$

which in general stabilizes all complex structure moduli U_i together with the axio-dilaton S by the supersymmetry conditions $D_{U_i} W_{\text{flux}} = D_S W_{\text{flux}} = 0$.

Now, in contrast to the usual LVS set-up (and also to the KKLT scenario), we consider the special case that the flux-induced value of the superpotential vanishes in the minimum, i. e.

$$W_{\text{flux}}|_{\text{min.}} = 0. \tag{5.4}$$

Although this condition (5.4) together with the supersymmetry conditions mentioned above is an over-constrained system of equations, the set of solutions are not highly suppressed. Indeed it was suggested in [127] that the number of minima fulfilling (5.4) compared to all flux minima is roughly given by

$$\frac{\#(W_{\text{flux}}|_{\text{min.}} = 0)}{\#(\text{tot})} \sim \frac{1}{L^{n/2}}, \tag{5.5}$$

with L the upper limit on the flux quanta and n an integer.

Taking this additional assumption as satisfied, we use again the fact that the stabilization of the complex structure moduli and the axio-dilaton takes places at a higher order in the $\mathcal{V}^{-1}$ expansion of the scalar potential than the Kähler moduli such that it is a valid procedure to stabilize the two different classes of moduli independently of each other.

For reasons that will become clear in a moment, we require the compactification space to have three Kähler moduli and the geometry to be of the swiss cheese form such that we can write the overall volume as:

$$\mathcal{V} = (\eta_b \tau_b)^{3/2} - (\eta_1 \tau_1)^{3/2} - (\eta_2 \tau_2)^{3/2}. \tag{5.6}$$

Here, as usual, τ_b controls the overall volume $\mathcal{V}$ and $\tau_{(1,2)}$ are small holes in this geometry. The constants η_b, η_1, η_2 are determined by a specific choice of a compactification manifold.

As we want to engineer a racetrack superpotential [91, 128, 129], we need to place gaugino condensates in the model. The respective gauge theories shall be realized on two stacks of D7-branes supporting a $U(N)$ and a $U(M)$ gauge theory respectively. They wrap the four-cycle Γ_1 corresponding to the Kähler modulus T_1.[1] As it is well-known, this leads to a racetrack superpotential containing exponentials of the two gauge kinetic functions:

$$W_{\text{np}} = A e^{-a f_a} + B e^{-b f_b}. \tag{5.7}$$

The constants $a = \frac{2\pi}{N}$ and $b = \frac{2\pi}{M}$ are determined by the ranks N and M of the two gauge groups on the two D7-branes.

In further developing the D-brane instanton calculus, it has been argued in [131, 132] that also the gauge kinetic function receives non-perturbative corrections from euclidean D-brane instantons. In analogy to the case of D9- and D5-branes, analyzed in [132] for the gauge theory on the D7-branes we make the following ansatz for the instanton generated non-perturbative correction:

$$\Delta_{\text{np}} f_a = g(U) e^{2\pi T_{\text{E3}}}, \tag{5.8}$$

where T_{E3} denotes the Kähler modulus corresponding to the cycle wrapped by the instanton.

For such an instanton to contribute, it must have a zero mode structure specified by $h^{(2,0)}(\Gamma_{E3}) = 1$ and $h^{(1,0)}(\Gamma_{E3}) = 0$. Let us assume that such corrections can indeed arise from an instanton wrapping the four-cycle Γ_2 with Kähler modulus T_2. Note that, because of its zero mode structure, this instanton will not contribute as a single instanton and so the superpotential, after integrating out the complex structure moduli, reads at leading order (see also [133])

$$W_{\text{np}} = A e^{-a\left(T_1 + C_1 e^{-2\pi T_2}\right)} - B e^{-b\left(T_1 + C_2 e^{-2\pi T_2}\right)}, \tag{5.9}$$

with all Kähler moduli in Einstein frame. The exponential of the non-perturbative correction can be developed into a power series. Only including the leading terms, we obtain:

$$W_{\text{np}} \approx \left[A e^{-aT_1} - B e^{-bT_1}\right] - \left[A C_1 a e^{-aT_1} - B C_2 b e^{-bT_1}\right] e^{-2\pi T_2}. \tag{5.10}$$

From this it is clear that after stabilization of the modulus T_1, the superpotential is of the

1 On the type I side, such a set-up can be realized for instance by discrete Wilson lines as it has been shown in [130].

form (5.1) where the first term in brackets can play the rôle of an effective W_0^{eff} and the second term that of A^{eff}. Because of their exponential dependence on T_1 it should be possible to drive them to very small values without fine-tuning.

Now, we are going to estimate the LARGE volume minimum of this scenario analytically. We would like to emphasize that the specific models of the following sections have been analyzed also numerically for the Kähler potential (3.5) and superpotential (5.9) *without* any approximations. We start from the scalar F-term potential $V_F = e^{\mathcal{K}}\big(|DW_{\text{np}}|^2 - 3|W_{\text{np}}|^2\big)$ with U and S stabilized. Since we are interested in a minimum at large $\mathcal{V}$, we expand this expression in powers of $1/\mathcal{V}$ and keep only the leading order term in T_1:

$$V_F \sim \frac{\sqrt{\tau_1}|\partial_{T_1} W_{\text{np}}|^2}{\mathcal{V}} + \mathcal{O}\big(\mathcal{V}^{-2}, e^{-4\pi\tau_2}\big). \tag{5.11}$$

The minimum of (5.11) is determined by $\partial_{T_1} W_{\text{np}} = \partial_{\overline{T}_1} \overline{W}_{\text{np}} = 0$ stabilizing T_1 at

$$\tau_1^* \simeq \frac{1}{a-b}\ln\left(\frac{Aa}{Bb}\right), \qquad \rho_1^* = 0, \tag{5.12}$$

where without loss of generality we assumed $A > B$ are real and $a > b$.

We proceed and study the resulting effective potential for $\mathcal{V}$ and T_2 with T_1 stabilized at values (5.12). As for the LARGE volume scenario, we take the limit $\mathcal{V} \gg 1$ and keep only the leading term in $\mathcal{V}$ at each order of $\exp(-2\pi\tau_2)$. The resulting potential then reads (in Einstein frame)

$$\begin{aligned} V_F \sim & \frac{8}{3}\left(\frac{\sqrt{\tau_1^*}}{\eta_1}\big|\gamma W_0^{\text{eff}}\big|^2 + (2\pi)^2\frac{\sqrt{\tau_2}}{\eta_1}\big|A^{\text{eff}}\big|^2\right)\frac{e^{-4\pi\tau_2}}{\mathcal{V}} \\ & - 4\big|W_0^{\text{eff}}\big|\big|\tau_1^*\gamma W_0^{\text{eff}} + 2\pi\tau_2 A^{\text{eff}}\big|\frac{e^{-2\pi\tau_2}}{\mathcal{V}^2} \\ & + \frac{3\hat{\xi}}{4}\big|W_0^{\text{eff}}\big|^2\frac{1}{\mathcal{V}^3}, \end{aligned}$$

with $\gamma = (aC_1 - bC_2)\big(\frac{1}{a} - \frac{1}{b}\big)^{-1}$ and the axion ρ_2 stabilized such that the second term in (5.13) is minimized. The analytical analysis of this potential reveals that $\mathcal{V}$ is fixed at

$$\mathcal{V}^* = P\big(A^{\text{eff}}, W_0^{\text{eff}}, \tau_1^*, \tau_2^*, \ldots\big)e^{2\pi\tau_2^*}, \tag{5.13}$$

where P is a rather complicated algebraic function of various quantities and τ_2^* is determined by an implicit equation. However, using the scaling $2\pi\tau_2 \sim \ln\mathcal{V}$ obtained from (5.13), we observe that those terms in (5.13) involving τ_1^* scale as $\mathcal{V}^{-3}$ and can therefore be absorbed into $\hat{\xi}$. The resulting potential is of a form as in the LVS and so we expect to find a non-supersymmetric AdS minimum for large values of $\mathcal{V}$.

In order to obtain a positive cosmological constant, eventually the AdS minimum has to be uplifted. This is achieved for instance by anti D3-branes at the bottom of a Klebanov–Strassler warped throat [134] as usual. The corresponding uplift potential has the form

$$V_{\text{up}} \sim \frac{a^4}{\mathcal{V}^2} m_{\text{P}}^4 \qquad \text{and} \qquad a = e^{-\frac{2\pi K}{3 g_s M}} \tag{5.14}$$

is the warp factor at the bottom of the throat with M, K the A- respectively B-cycle flux-quanta.

5.2. Supersymmetric GUT Scenarios

In the previous section, we described a set-up in which we have exponential control over the effective parameters W_0^{eff} and A^{eff} in the superpotential.

As we will show in the following, by tuning the initial parameters mostly at the order of 10 %, it is possible to dynamically fix the moduli such that $\mathcal{V} \sim 10^5$ and $\left|W_0^{\text{eff}}\right| \sim 10^{-10}$. As already anticipated in the introduction to this section, these values give rise to a string scale at the GUT scale $m_{\text{s}} \simeq m_X \simeq 1.2 \cdot 10^{16}$ GeV and $m_{3/2}$ in the TeV range. Note that for the usual LVS, a high degree of tuning is needed to have $m_{\text{s}} \simeq m_X$ while keeping the SUSY breaking scale in the TeV regime.

So war we focused only on the D7-branes supporting the gauge theories undergoing gaugino condensation. They wrap the small cycle Γ_1. Let us now investigate where in such a scenario the MSSM respectively the Grand Unified Theory might be localized.

- If the D7-branes supporting the MSSM wrap the big four-cycle Γ_b with Kähler modulus T_b, the gauge coupling $\alpha^{-1} \simeq \tau_b \sim \mathcal{V}^{2/3}$ is by far too small. This is similar to the original LVS set-up.

- A second possibility is to place the branes on Γ_2 with Kähler modulus T_2 giving roughly with the value of the overall volume we have chosen $\alpha^{-1} \simeq \tau_2 \simeq \frac{1}{2\pi} \ln \mathcal{V} \simeq 1.8$. This differs from the GUT gauge coupling $\alpha_X^{-1} = 25$ by an order of magnitude.

- Last but not least, we can wrap the D7-branes along Γ_1 with gauge coupling $\alpha^{-1} \simeq \tau_1 \sim -\ln \left|W_0^{\text{eff}}\right| \sim 21$ which is in the right ballpark.

Unfortunately, the most appealing third possibility clashes with the "chirality problem" described in chapter 4: as the cycle Γ_1 is already populated by D-branes supporting gaugino condensates, further D7-branes with a chiral matter spectrum would spoil the generation of the superpotential we consider here. In principle, a solution is be to consider a manifold with a further small Kähler modulus T_4 and wrapping the branes along $\Gamma_1 + \Gamma_4$ with chiral intersection on Γ_4. The additional modulus then has to be stabilized by D-terms or, as

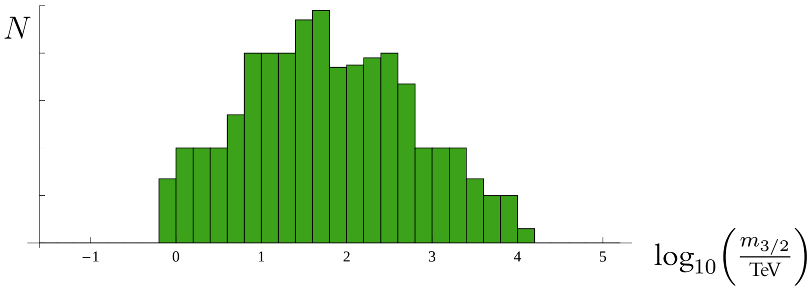


Figure 5.1.: Distribution of $m_{3/2}$ with $\tau_1 = 25 \pm 0.25$ and $\mathcal{V}/\sqrt{g_s} \simeq 1.26 \cdot 10^5$ for a scan over natural values of a, b, A, B. The statistical mean of this distribution is $\left\langle \log_{10}\left(m_{3/2}/\text{TeV}\right)\right\rangle = 1.79$ and the standard deviation is $\sigma = 0.98$.

suggested in [135], by string loop effects [95]. As we do not want to further complicate our set-up, we ignore the chirality issue from now on, keeping in mind that it can be solved with the aid of another small four-cycle.

Having identified a suitable choice for the MSSM branes in our set-up, let us now take a different point of view and fix $\mathcal{V}/\sqrt{g_s} \simeq 1.26 \cdot 10^5$ together with $\tau_1 \simeq \alpha_X^{-1} \simeq 25$. By scanning the parameters $a = \frac{2\pi}{N}$, $b = \frac{2\pi}{M}$, A, B in a natural range, $M \in [2, 12]$, $0 < N < M$ and $\ln A \in [\ln 0.1, \ln 10]$, $\ln B \in [\ln \frac{A}{10}, \ln A)$ in equidistant steps, we find the distribution of resulting values for $m_{3/2}$ shown in figure 5.1. This illustrates that in our set-up with input $m_s = m_X$ and $\alpha^{-1} = 25$, a gravitino mass in the TeV region is obtained rather naturally.

In the following subsections, we investigate the scalar F-term potential resulting from the Kähler potential (3.5) and the superpotential (5.9) numerically, that is we are searching for minima at large volume with $\ln \mathcal{V} \sim 2\pi\tau_2$. We analyze three different models where for each one, we calculate the masses scales of the complex structure and Kähler moduli, given by (see also section 3.5):

$$
\begin{aligned}
m_U &\sim \frac{m_P}{\mathcal{V}}, & m_{T_1} &\sim \left|W_0^{\text{eff}}\right| m_P, \\
m_{T_2} &\sim \frac{\left|W_0^{\text{eff}}\right|}{\mathcal{V}} m_P, & m_{T_b} &\sim \frac{\left|W_0^{\text{eff}}\right|}{\mathcal{V}^{3/2}} m_P.
\end{aligned}
\tag{5.15}
$$

We also compare the gravity and anomaly mediated gaugino masses given by (3.71) and (3.73) respectively in order to identify the dominant mediation mechanism.[2] For the Kähler metric of the chiral matter we assume the form (3.66). The parameter λ therein can take values between 0 and 1 in principle. We will use the value $\lambda = 1/3$ of the minimal swiss cheese set-up [31]. The scalar masses for gravity mediation of supersymmetry breaking are given in (3.70) where we set the potential in the minimum V_0 is set to zero. Generically, the two-

2 We can safely ignore the effect of the uplifting [136] which gives only sub-dominant contributions to the soft terms.

loop generated scalar masses for an anomaly mediated scenario are always smaller than the supergravity mediated masses. Therefore, we do not calculate them here.

5.2.1. Model 1: A Starter

In the following subsections, we investigate the scalar F-term potential resulting from the Kähler potential (3.5) and the superpotential (5.9) numerically, that is we are searching for minima at large volume with $\ln \mathcal{V} \sim 2\pi\tau_2$.

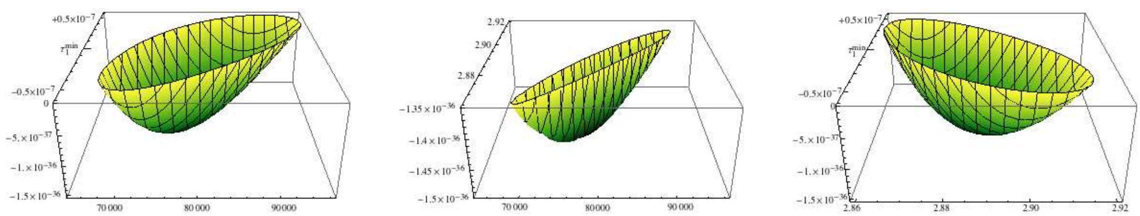
Taking the observations from the beginning of this section into account, we first assume that the MSSM is localized on D7-branes wrapping the cycle Γ_1 associated to the racetrack modulus T_1 with size $\tau_1 \simeq 25$. As easily to be checked by inserting the following values in (5.12), a particular set of parameters realizing this set-up without fine-tuning is the following

$$\begin{aligned} &A = 1.6, \quad B = 0.2, \quad C_1 = 1, \qquad C_2 = 3, \quad a = \frac{2\pi}{8}, \qquad b = \frac{2\pi}{9}, \\ &g_s = \frac{2}{5}, \quad \eta_b = 1, \qquad \eta_1 = \frac{1}{53}, \quad \eta_2 = \frac{1}{6}, \quad \chi = -136. \end{aligned} \tag{5.16}$$

For realizing $\tau_1 \sim 25$ we have a great freedom of choice of course. We determined the parameters by requiring that they are in their natural order of magnitude only and chose them randomly apart from that. Then we computed the scalar potential using the Kähler potential (3.5) and the superpotential (5.9) without any approximations and employed the computer program *Mathematica* to determine the minimum with high numerical precision. The resulting values are

$$\mathcal{V}^* = 78559, \qquad T_1^* = 25.18, \qquad T_2^* = 2.88, \qquad V_F^* = -1.5 \cdot 10^{-36}\, m_{\mathrm{P}}^4, \tag{5.17}$$

which is indeed the AdS LARGE volume minimum argued for in the last section. Three plots showing different sections of the potential in the vicinity of the minimum can be found in figures 5.2.



(a) $V_F(\mathcal{V}, \tau_1)$ (b) $V_F(\mathcal{V}, \tau_2)$ (c) $V_F(\tau_1, \tau_2)$

Figure 5.2.: F-term potential of the GUT model 1 in the vicinity of the minimum.

The various masses are computed using the formulas, mentioned above. Let us however emphasize that for the gaugino and scalar masses it is crucial to work with very high numerical precision for T_1^*, T_2^* and $\mathcal{V}^*$, as there appear certain cancellations. The origin of these will concern us later.

Fundamental masses	Moduli masses	Soft masses
$m_s = 1.2 \cdot 10^{16}\,\mathrm{GeV}$	$m_U = 3.1 \cdot 10^{13}\,\mathrm{GeV}$	$m_{1/2}^{\text{gravity}} = 1.1 \cdot 10^{-5}\,\mathrm{GeV}$
$m_{3/2} = 1.6 \cdot 10^{4}\,\mathrm{GeV}$	$m_{T_1} = 1.2 \cdot 10^{9}\,\mathrm{GeV}$	$m_{1/2}^{\text{anomaly}} = 1.2 \cdot 10^{-3}\,\mathrm{GeV}$
	$m_{T_2} = 1.6 \cdot 10^{4}\,\mathrm{GeV}$	$m_0^{\text{gravity}} = 64\,\mathrm{GeV}$
	$m_{T_b} = 56\,\mathrm{GeV}$	

Let us comment on these scales:

- By construction, the string scale m_s coincides with the GUT scale and the gravitino mass $m_{3/2}$ is in the TeV regime.

- The closed sector moduli masses are ordered from heavy to light and take acceptable values except for T_b which is too small. For $m_s = m_X$, the lower bound for moduli masses not to be subject to the cosmological moduli problem is around 1 TeV as described in section 3.8. Hence this model cannot be convincing when taking serious cosmology.

- In addition, the gaugino as well as the scalar masses are far too small. The main reason is that we realized the MSSM on the cycle Γ_1 which is related to the racetrack modulus T_1. For this modulus we observe numerical cancellations giving $F^1 \simeq 2 \cdot 10^{-22}\, m_P \sim 10^{-8}\, m_{3/2}$, i. e. supersymmetry breaking on the racetrack cycle is eight orders of magnitude smaller than naïvely expected. The explanation is that the exact racetrack minimum, in leading order given by the globally supersymmetric minimum (5.12), is almost supersymmetric. For the gravity mediated gaugino masses we therefore obtain a strong suppression

$$m_{1/2}^{\text{gravity}} = \frac{1}{2\tau_1} F^1 \sim \frac{1}{2 \cdot 25}\, 2 \cdot 10^{-22}\, m_P \sim 10^{-5}\,\mathrm{GeV}. \tag{5.18}$$

- For the anomaly mediated gaugino masses $m_{1/2}^{\text{anomaly}}$, we find the expected cancellation of $m_{3/2}$ at leading order (see [31] for a detailed derivation), however the sub-leading order in $\mathcal{V}$ dominates over F^1

$$
\begin{aligned}
m_{1/2}^{\text{anomaly}} &\sim \frac{\alpha_a}{4\pi}\left(3T_G m_{3/2}\big(1-1+\mathcal{O}(\mathcal{V}^{-1})\big)+2\lambda T_R\frac{F^1}{2\tau_1}\right)\\
&\sim \frac{1}{300}\left(3\cdot 3\cdot 10^4\,\text{GeV}\cdot 10^{-5}+2\lambda\cdot 6\cdot\frac{2\cdot 10^{-22}\,m_{\text{P}}}{2\cdot 25}\right)\\
&\sim 10^{-3}\,\text{GeV}.
\end{aligned}
\tag{5.19}
$$

This value is of course still too small but note it is larger than the gravity mediated term. The gaugino masses are thus dominantly generated via anomaly mediation.

- A similar mechanism is at work for the scalar masses where $m_{3/2}^2$ is canceled at leading order and sub-leading corrections in $\mathcal{V}$ give the main contribution (see again [31] for a detailed derivation of this expression)

$$
\begin{aligned}
\left(m_0^{\text{gravity}}\right)^2 &\sim m_{3/2}^2\big(1-1+\mathcal{O}(\mathcal{V}^{-1})\big)+\lambda\left(\frac{F^1}{2\tau_1}\right)^2\\
&\sim \left(10^4\,\text{GeV}\right)^2\cdot 10^{-5}+\lambda\left(\frac{2\cdot 10^{-22}\,m_{\text{P}}}{2\cdot 25}\right)^2\\
&\sim \left(10^{3/2}\,\text{GeV}\right)^2.
\end{aligned}
\tag{5.20}
$$

In conclusion, although we were able to easily arrange for $m_{\text{s}} \simeq m_X$, $\alpha^{-1} \simeq \alpha_X^{-1} \simeq 25$ and a gravitino mass in the TeV range, the soft terms are much too small.

In order to obtain realistic soft masses, two options seem viable: either we take the present set-up and scale the mass parameters by a factor of 10^6, or we wrap the MSSM branes not only along Γ_1 but on $\Gamma_1+\Gamma_2$ giving also a contribution from F^2 to the soft terms. In the following two subsections, we discuss these two possibilities in more detail.

5.2.2. Model 2: A Mixed Anomaly–Gravity Mediated Model

Recall that the minimum for the racetrack modulus T_1 is approximately supersymmetric. To construct a model with gaugino masses in the TeV range, we use the same set of parameters (5.16) of the previous set-up but scale A and B to the admittedly more unrealistic values of

$$
A = 1.6\times 8\cdot 10^5, \qquad B = 0.2\times 8\cdot 10^5. \tag{5.21}
$$

In view of (5.12) it is obvious that a common scaling of A and B does not change the value of τ_1 in the minimum, which determines the gauge coupling of the GUT theory on the matter branes. On the other hand, the value of the superpotential (5.9) in minimum is increased by a factor of 10^5. This should affect the soft-masses by the same amount as F^1 is proportional to W in the minimum. Numerically minimizing the scalar potential with the new set of parameters gives:

$$\mathcal{V}^* = 78559, \qquad T_1^* = 25.18, \qquad T_2^* = 2.88, \qquad V_F^* = -9.5 \cdot 10^{-25}\, m_{\rm P}^4. \tag{5.22}$$

Three plots showing the potential in the vicinity of the minimum can be found in figures 5.3 on page 106. The mass scales for the new set-up are the following:

Fundamental masses	Moduli masses	Soft masses
$m_{\rm s} = 1.2 \cdot 10^{16}$ GeV	$m_U = 3.1 \cdot 10^{13}$ GeV	$m_{1/2}^{\rm gravity} = 8.6$ GeV
$m_{3/2} = 1.3 \cdot 10^{10}$ GeV	$m_{T_1} = 9.9 \cdot 10^{14}$ GeV	$m_{1/2}^{\rm anomaly} = 962$ GeV
	$m_{T_2} = 1.3 \cdot 10^{10}$ GeV	$m_0^{\rm gravity} = 5.1 \cdot 10^7$ GeV
	$m_{T_b} = 4.5 \cdot 10^7$ GeV	

We again comment on these scales:

- Since the value of the volume modulus is not changed compared to the previous set-up, we similarly obtain $m_{\rm s} \simeq m_X$. However, as intended, the gravitino mass is in an intermediate regime due to the change in $W_0^{\rm eff}$.

- With the gravitino mass in the intermediate regime, the cosmological moduli problem has been evaded, as T_b is much heavier then the TeV scale now. We also see that $m_{T_1} > m_U$, but since we have a well-defined expansion in $1/\mathcal{V}$ we can safely stabilize U and then study the stabilization of T_1.

- Concerning the gravity mediated gaugino mass, the scaling (5.21) results in a value of $W_0^{\rm eff}$ which is $8 \cdot 10^5$ times larger than in the previous set-up leading to $F^1 \sim 10^{-16}\, m_{\rm P}$. The gravity mediated gaugino mass $m_{1/2}^{\rm gravity}$ is still too small but the anomaly mediated masses are now in the TeV regime.

- As expected from the first example, the gravity mediated scalar masses are at $m_0^{\rm gravity} \sim 10^7$ GeV and therefore much heavier than the gaugino masses. However, they come in complete $SU(5)$ multiplets and therefore do not spoil gauge coupling unification.

- With this hierarchy between the gaugino masses and the scalar masses, we have a dynamical realization of the split supersymmetry scenario [126]. The Higgs sector masses, i. e. the canonically normalized $\hat{\mu}$-term and the soft term μB, are expected to be of the same order of magnitude as the scalar masses, simply for the reason that here both $F^b \sim m_{3/2}$ and F^1 contribute. Therefore, in order to keep these at the weak scale, a fine-tuning of the supersymmetric μ-term is necessary.

To summarize, tuning the initial parameters such that $\alpha^{-1} \simeq \alpha_X^{-1} \simeq 25$ and $m_s = m_X$, and localizing the MSSM solely on the racetrack cycle Γ_1, we find a high suppression of the gravity mediated gaugino masses due to the quasi-supersymmetry of the racetrack minimum. Fixing the gaugino masses at the TeV scale leads to an intermediate supersymmetry breaking scenario with gravitino masses and scalar masses in the intermediate regime. Due to the large value of $m_{3/2}$, this evades the CMP and gives a stringy realization of the split supersymmetry scenario proposed in [126].[3]

5.2.3. Model 3: An LVS like Supergravity Mediated Model

We now consider the second possibility from the end of section 5.2.1 which indeed realizes our initial goal, namely to naturally find large volume minima with the string scale at the GUT scale, gauge coupling unification and soft masses in the TeV regime. This provides a concrete moduli stabilization scenario for which the analysis of [33] is applicable. There, the computation of soft masses and running to the weak scale has been studied quite systematically for moduli dominated supersymmetry breaking in F-theory respectively type IIB orientifold compactifications realizing the MSSM.

We modify our original set-up such that the soft terms are dominantly generated via T_2 similarly to the original LVS. To do so, we place the MSSM D7-branes on the combination of cycles $\Gamma_1 + \Gamma_2$. A concrete set of parameters realizing this set-up without fine-tuning is

$$\begin{aligned} &A = 1.5, \quad B = 0.25, \quad C_1 = 1, \quad C_2 = 3, \quad a = \frac{2\pi}{8}, \quad b = \frac{2\pi}{9}, \\ &g_s = \frac{2}{5}, \quad \eta_b = 1, \quad \eta_1 = \frac{1}{40}, \quad \eta_2 = \frac{1}{6}, \quad \chi = -153. \end{aligned} \tag{5.23}$$

The minimum of the scalar F-term potential is again determined with the help of *Mathematica* giving

$$\mathcal{V}^* = 92158, \qquad T_1^* = 21.58, \qquad T_2^* = 2.91, \qquad V_F^* = -1.4 \cdot 10^{-34}\, m_{\rm P}^4, \tag{5.24}$$

so that the gauge coupling of the D7-branes $\alpha^{-1} = \tau_1 + \tau_2 = 24.8$ is again the unified gauge coupling at the GUT scale. Three plots showing the potential in the vicinity of the minimum can be found in figures 5.4 on page 106. The mass scales in this set-up are calculated as follows:

3 For a local realization of split supersymmetry see [137], and a realization by mixed anomaly-D-term mediation has been reported in [119].

Fundamental masses	Moduli masses	Soft masses
$m_s = 1.1 \cdot 10^{16}\,\text{GeV}$	$m_U = 2.6 \cdot 10^{13}\,\text{GeV}$	$m_{1/2}^{\text{gravity}} = 819\,\text{GeV}$
$m_{3/2} = 168\,\text{TeV}$	$m_{T_1} = 1.5 \cdot 10^{10}\,\text{GeV}$	$m_{1/2}^{\text{anomaly}} = 10\,\text{GeV}$
	$m_{T_2} = 168\,\text{TeV}$	$m_0^{\text{gravity}} = 817\,\text{GeV}$
	$m_{T_b} = 553\,\text{GeV}$	

Let us again comment on the various scales arising in this LARGE volume minimum:

- We arranged the parameters for the present set-up such that $m_s \simeq m_X$ together with the gravitino mass in the TeV range.

- The closed sector moduli masses take (almost) acceptable values with m_{T_b} close to the regime where the cosmological moduli problem may be avoided. (Note that we were not careful with factors of 2π.)

- The soft masses are dominantly generated via the supersymmetry breaking of T_2 specified by $F^2 \sim 10^{-14}\, m_P$. Since T_2 is stabilized as in the original LARGE volume scenario, similar mechanisms generating the soft masses are at work [31]. In particular, using the fact that $F^1 \sim 10^{-20}\, m_P \ll F^2$, the common term determining the gaugino as well as the scalar masses can be expressed as

$$\frac{F^1 + F^2}{2(\tau_1 + \tau_2)} \sim \frac{F^2}{2(\tau_1 + \tau_2)} \sim \frac{\tau_2}{\tau_1 + \tau_2} \frac{m_{3/2}}{\ln(m_P/m_{3/2})}. \tag{5.25}$$

Here we used that $F^2 \simeq 2\tau_2 m_{3/2}/\ln(m_P/m_{3/2})$ which was obtained in [138]. For the gravity mediated gaugino mass we thus find

$$m_{1/2}^{\text{gravity}} = \frac{F^1 + F^2}{2(\tau_1 + \tau_2)} \sim \frac{3}{25} \frac{1.7 \cdot 10^5\,\text{GeV}}{\ln(10^{18} \cdot 10^{-5})} \sim 700\,\text{GeV}. \tag{5.26}$$

- For the anomaly mediated gaugino mass we use equation (5.19) to obtain

$$\begin{aligned} m_{1/2}^{\text{anomaly}} &\sim \frac{\alpha_a}{4\pi}\left(3T_G m_{3/2}\big(1 - 1 + \mathcal{O}(\mathcal{V}^{-1})\big) + 2\lambda T_R \frac{F^1 + F^2}{2(\tau_1 + \tau_2)}\right) \\ &\sim \frac{1}{300}\big(3 \cdot 3 \cdot 10^5\,\text{GeV} \cdot 10^{-6} + 2\lambda \cdot 6 \cdot 700\,\text{GeV}\big) \\ &\sim 10\,\text{GeV}, \end{aligned} \tag{5.27}$$

with $\lambda = 1/3$ [31]. Note that again the contribution of $m_{3/2}$ is canceled at leading order but the sub-leading correction $m_{3/2}/\mathcal{V}$ is suppressed compared to F^2.

Furthermore, in this supersymmetry breaking scheme, the anomaly mediated gaugino masses are significantly smaller than the gravity mediated ones. This is contrast to the so-called mirage mediation scheme arising for instance in the original KKLT scenario [139–141], where both are of the same order of magnitude.

- Finally, we consider the gravity mediated scalar masses. Referring to formula (5.20), we obtain

$$\begin{aligned} \left(m_0^{\text{gravity}}\right)^2 &\sim m_{3/2}^2\big(1-1+\mathcal{O}(\mathcal{V}^{-1})\big)+\lambda\left(\frac{F^1+F^2}{2(\tau_1+\tau_2)}\right)^2 \\ &\sim \left(10^5\,\text{GeV}\right)^2\cdot 10^{-6}+\lambda\left(700\,\text{GeV}\right)^2 \\ &\sim \left(10^2\,\text{GeV}\right)^2. \end{aligned} \tag{5.28}$$

  Note that for the scalar masses, the contribution from $m_{3/2}$ is canceled at leading order but now the sub-leading corrections scale as $m_{3/2}/\sqrt{\mathcal{V}}$. Therefore, by accident, the two terms in the equation above are of the same order which is in contrast to the relation $m_0^{\text{gravity}}=m_{1/2}^{\text{gravity}}/\sqrt{3}$ obtained in the original LVS [31].

- Assuming that the supersymmetric μ-term vanishes, the canonical normalized Higgs parameters are also in the TeV regime. This would solve the μ-problem via the Giudice–Masiero mechanism [142].

In summary, by wrapping the MSSM supporting D7-branes along $\Gamma_1+\Gamma_2$, we obtain a LARGE volume scenario with supergravity mediated soft masses in the TeV region and $m_{\text{s}}=m_X$. This is different compared to the original LVS where the string scale is usually at an intermediate scale. It would be interesting to calculate the soft-terms at the weak scale along the lines of [32, 33] to see whether distinctive patterns for the supersymmetric phenomenology to be tested at the LHC can be obtained.

5.3. Comment on the Cosmological Constant

For the GUT set-up discussed in the previous section, the tree-level cosmological constant is $V_F^*=-1.4\cdot 10^{-34}\,m_{\text{P}}^4$ and therefore a high degree of fine-tuning in the uplift potential (5.14) is needed to obtain the observed value of $\Lambda\simeq +10^{-120}\,m_{\text{P}}^4$. Note that after such a fine-tuning has been achieved, a LARGE volume scenario with $m_{\text{s}}\simeq m_X$ contains a natural candidate serving as a quintessence field [143, 144]: indeed, taking into account also non-perturbative corrections corresponding to the large four-cycle Γ_b, the scalar potential depending on the (canonically normalized) axion σ_b takes the form

$$\frac{V_Q}{m_{\text{P}}^4}\simeq\left(\frac{m_{\text{P}}\,m_{3/2}}{m_X^2}\right)e^{-\frac{2\pi}{L}\tau_b}\left(1-\cos\left(\frac{2\pi}{L}\sigma_b\right)\right). \tag{5.29}$$

For the minimum $\mathcal{V}^* \simeq \tau_b^{2/3} \simeq 92158$ of our model from section 5.2.3 and L of the order $L = 40, \ldots, 50$, the prefactor in (5.29) is of the right order of magnitude.

Although the GUT models from the previous section may contain a quintessence field, let us now take a different point of view. Since in our scenario we have exponential control over W_0^{eff} and A^{eff}, one might ask whether it is possible to dynamically find a minimum $V_F^* \simeq -\left|W_0^{\text{eff}}\right|^2 \mathcal{V}^{-3} \simeq -10^{-120}\, m_{\text{P}}^4$ realized without fine-tuning. Ignoring the weak scale for a moment, for the string scale we have $m_{\text{s}} \sim \mathcal{V}^{-1/2}\, m_{\text{P}} > \text{TeV}$ which implies that $\mathcal{V} < 10^{30}$. To keep the tuning of a, b, A, B moderate, we identify $\left|W_0^{\text{eff}}\right| \sim 10^{-15}$ and thus $\mathcal{V} \sim 10^{30}$ as a natural choice to realize $V_F^* \simeq -10^{-120}\, m_{\text{P}}^4$. A set of parameters dynamically leading to such values is for instance

$$\begin{aligned} &A = 1, \quad B = 0.1, \quad C_1 = 1, \quad C_2 = 3, \quad a = \frac{2\pi}{13}, \quad b = \frac{2\pi}{14}, \\ &g_{\text{s}} = \frac{1}{5}, \quad \eta_b = 1, \quad \eta_1 = \frac{1}{30}, \quad \eta_2 = \frac{1}{6}, \quad \chi = -452, \end{aligned} \tag{5.30}$$

and the plots showing the potential in the vicinity of the minimum can be found in figures 5.5 on page 106. The numerical values specifying the minimum are

$$\mathcal{V}^* = 6.4 \cdot 10^{28}, \qquad T_1^* = 68.84, \qquad T_2^* = 11.53, \qquad V_F^* = -7.8 \cdot 10^{-121}\, m_{\text{P}}^4, \tag{5.31}$$

and because the AdS minimum is at $V_F^* \sim -10^{-120}\, m_{\text{P}}^4$, the warp factor $a = 10^{-15}$ in the uplift potential (5.14) does not involve any fine-tuning of the flux parameters K and M.

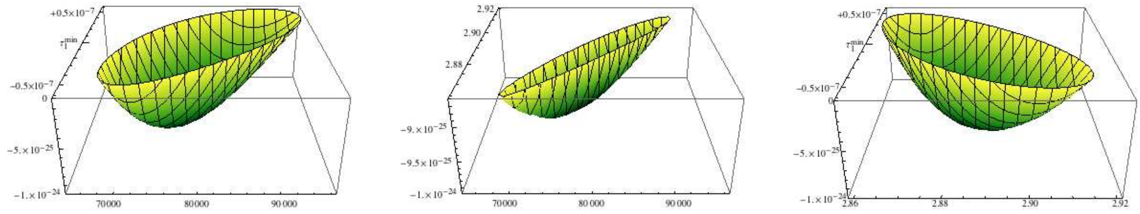
We conclude that there exist vacua of the scalar potential (5.13) whose tree-level cosmological constant has the right order of magnitude. However, this clearly does not solve the cosmological constant problem, as we have not yet identified the standard model and the origin of the weak scale. Once we try to introduce the MSSM into this set-up, we are confronted with the usual problems. Let us briefly explain three possibilities:

- Localizing the MSSM on D7-branes wrapping the cycles $\Gamma_{(1,2)}$ leads to soft masses below the gravitino mass scale $m_{3/2} \simeq 10^{-18}$ eV which itself is ridiculously small.

- Since $m_{\text{s}} \sim$ TeV, we could break supersymmetry at the string scale and place a non-supersymmetric anti D3-brane configuration realizing the MSSM at the bottom of a throat, i. e. on the TeV brane in the RS scenario. The uncanceled NS–NS tadpole of the non-supersymmetric brane configuration would be the red-shifted uplift term. However, all mass scales in the throat are red-shifted as well [145] so that the stringy excitations such as squarks have masses $m_0 \simeq a m_{\text{s}} \simeq \Lambda^{1/4} = 10^{-3}$ eV.

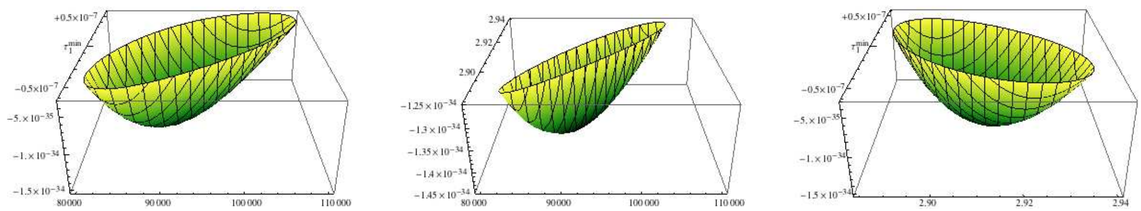
- A third option is to place an explicitly supersymmetry breaking D-brane configuration in the bulk, i. e. on the Planck brane in the RS scenario. Then the superpartners have string scale masses in the TeV region, but we get an additional positive contribution $V \sim \mathcal{O}(m_s^4)$ to the scalar potential.

In conclusion, even though we have exponential control over the effective parameters W_0^{eff} and A^{eff}, the cosmological constant problem is not even touched. It can be phrased as the problem of hiding the TeV scale supersymmetry breaking of the standard model such that it does not induce a large contribution to the tree-level value Λ_0.



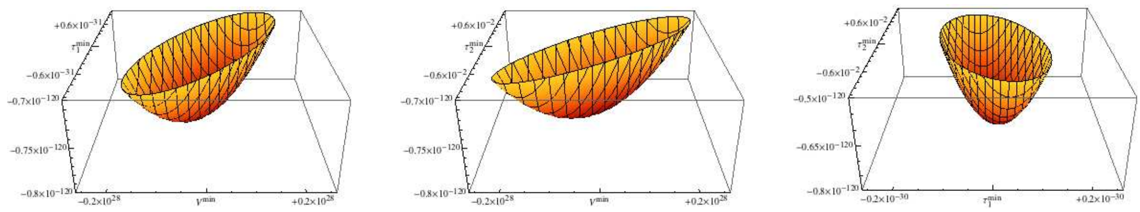
(a) $V_F(\mathcal{V}, \tau_1)$ (b) $V_F(\mathcal{V}, \tau_2)$ (c) $V_F(\tau_1, \tau_2)$

Figure 5.3.: F-term potential of the GUT model 2 in the vicinity of the minimum.



(a) $V_F(\mathcal{V}, \tau_1)$ (b) $V_F(\mathcal{V}, \tau_2)$ (c) $V_F(\tau_1, \tau_2)$

Figure 5.4.: F-term potential of the GUT model 3 in the vicinity of the minimum.



(a) $V_F(\mathcal{V}, \tau_1)$ (b) $V_F(\mathcal{V}, \tau_2)$ (c) $V_F(\tau_1, \tau_2)$

Figure 5.5.: F-term potential of the Λ model in the vicinity of the minimum.

6. SUSY Breaking in Local String/F-Theory Models

In the previous chapter we analyzed a modified LARGE volume scenario in which the string scale is at the GUT scale $m_s \sim m_X \sim 10^{16}$ GeV. In order to get a suitable value for the gauge coupling in these GUT scenarios, the matter D7-branes had to wrap the same four-cycle in the internal space as the D7-branes supporting the gaugino condensates, needed for moduli stabilization.

In the numerical calculation of the soft terms in the visible sector, we found a rather unexpected cancellation leading to a hierarchical suppression of the soft masses compared to the gravitino mass. We credited this with the fact that the minimum of the scalar potential for the Kähler modulus Γ_1 associated to this particular four-cycle was almost supersymmetric, and correspondingly the F-term F^1 was smaller than expected. Still, under certain circumstances, gravity mediation was the dominant supersymmetry breaking mediation mechanism.

In this chapter we investigate supersymmetry breaking in these kinds of set-ups more carefully. As a refinement compared to the models in chapter 5, we no longer assume that the branes from the matter sector and the moduli stabilizing sector populate the same four-cycle in the internal space, accounting for the "chirality problem" described in chapter 4. Remember, we found that the four-cycles supporting the chiral matter sector cannot be stabilized directly by the F-term potential induced by non-perturbative effects as usually done in the LARGE volume or KKLT scenario, but rather by D-terms. Consequently, the corresponding F-term is zero and gravity mediated soft terms are induced only by F-terms associated to other Kähler moduli or the dilaton. Hence we await a similar, or even stronger kind of suppression of the soft terms as in the models in chapter 5.

Beside the motivation from the chirality problem, such "sequestered" set-ups can also be found in other contexts: a way of realizing interesting gauge theories is given by D3-branes at the singular point as discussed in [146, 147]. Various low-energy models were studied on the first two del Pezzo surfaces dP_0 and dP_1, allowing for both GUT-like and extended MSSM scenarios. From the effective field theory point of view, these are quite similar to the models we studied in chapter 4. Remember that there, the D-term potential was such that Kähler modulus associated to the chiral matter branes was stabilized at zero size (without taking into account higher α'-corrections).

Another related scenario was proposed in the context of F-theory [25]. Model building on elliptically fibered Calabi–Yau fourfolds has come into vogue recently [26–28, 148–152]. The appeal of these constructions is due to the fact that F-theory is genuinely non-perturbative and as such admits in a relatively easy way to engineer spinor representations of an $SO(10)$

or the top quark Yukawa coupling $\mathbf{10}\,\mathbf{10}\,\mathbf{5}_H$ in $SU(5)$. Both is not possible in perturbative orientifold constructions. Another advantage is that these F-theory models allow for an essentially local treatment [27] if there exists a limit in which gravity decouples from the gauge theory on the GUT brane. This is the case if the space transverse to the brane can become arbitrary large which equivalently means that the four-cycle the branes are wrapping can shrink to zero size. A class which is often considered for these divisors is given again by del Pezzo surfaces. If gravity can be decoupled from the GUT sector, usually a suppression for the gravity induced soft terms of the order m_s/m_P relative to their general values is expected, suggesting that gauge mediation is the dominant mechanism for the mediation of SUSY breaking. This kind of models has a number of interesting phenomenological properties [153, 154].

In view of the similarity to the F-theory models, we await a similar suppression of the gravity mediated soft terms in our LARGE volume set-up. Indeed, after introducing the geometric framework of local GUTs in section 6.1, we compute the gravity mediated soft terms in section 6.2 and find a cancellation at $\mathcal{O}(m_{3/2})$. Sub-leading contributions to the soft terms appear at order $\mathcal{O}\big(m_{3/2}/\sqrt{\mathcal{V}}\big) = \mathcal{O}\Big(m_{3/2}^{3/2}/m_P^{1/2}\Big)$. In certain circumstances these contributions can also cancel and we give a set of well posed assumptions when this can occur. In section 6.3 we discuss the implications from these soft terms for both gauge mediation and the cosmological moduli problem.

6.1. Effective Field Theories and Moduli Stabilization

Let us describe in more detail the set-up we are going to investigate in this chapter. We require a Calabi–Yau three-fold with at least three four-cycles: one large cycle and two small del Pezzo four-cycles, i. e. the three-fold is of the (strong) swiss cheese type. One of the del Pezzos supports the $SU(5)$/MSSM gauge theory while the other supports a D3-brane instanton. We exclude chiral intersections between the two small four-cycles along the lines of chapter 4. Then the D3-brane instanton induces a non-perturbative contribution to the superpotential of the form (4.1), such that there exists the non-supersymmetric AdS-type LARGE volume minimum. Since the GUT brane is localized on a del Pezzo surface "orthogonal" to the instantonic del Pezzo and the size of the GUT brane is fixed by D-terms at small values, the previous computations of the gravity induced soft terms should be modified. The same calculation is also necessary for the case that the GUT cycle is collapsed at the quiver locus.

Obviously there are two basic regimes where the effective field theory (EFT) for light modes is reliable:

- The geometric regime, where all of the four-cycles, including the standard model or GUT cycle are larger than the string scale.

- The size of the standard model cycle is much smaller than the string scale. It is a blow-up mode expanded around its vanishing value corresponding to the del Pezzo singularity. Fortunately string theory is under control at the singularity and the EFT can be safely defined in an expansion on the blow-up mode.

Since the D-term conditions tend to prefer a small value of the standard model cycle, it is important to understand the physics in both regimes of validity of EFT. It is clear that these are two different effective field theories for standard model physics. But, as we will see, since the standard model cycle does not participate in the breaking of supersymmetry, the structure of soft breaking terms will be the same in both cases.

Gauge Couplings on the GUT Brane

In orientifold models we can realize a GUT theory on a stack of five D7-branes giving rise to the Chan–Paton gauge group $U(5)$. This allows for a non-vanishing gauge flux $\mathcal{F}_a$ in the diagonal $U(1) \subset U(5)$. We assumed the four-cycles to be del Pezzo surfaces. These are rigid and do not even contain any discrete Wilson lines. The gauge symmetry is broken to $SU(3) \times SU(2) \times U(1)_Y$ by a non-trivial $U(1)_Y$ gauge flux $\mathcal{F}_Y$ supported on a two-cycle $C_a \in H_2(D_a, \mathbb{Z})$ which is trivial in $H_2(\mathcal{X}, \mathbb{Z})$ [27, 28]. As explained in [155], this way of breaking the $SU(5)$ gauge group leads to a specific pattern of MSSM gauge couplings at the unification scale

$$f_i = T_a - \frac{1}{2}\kappa_i S, \qquad i \in \{1, 2, 3\}, \tag{6.1}$$

with

$$\begin{aligned} \kappa_3 &= \int_{D_a} \mathcal{F}_a^2, \qquad \kappa_2 = \int_{D_a} \mathcal{F}_a^2 + \mathcal{F}_Y^2 + 2\mathcal{F}_a\mathcal{F}_Y, \\ \kappa_1 &= \int_{D_a} \mathcal{F}_a^2 + \tfrac{3}{5}\big(\mathcal{F}_Y^2 + 2\mathcal{F}_a\mathcal{F}_Y\big). \end{aligned} \tag{6.2}$$

For concreteness we are using these orientifold relations in the following.

In the limit that the cycle is collapsed to the singularity, the gauge kinetic function takes a similar form:

$$f_i = \delta_i S + s_{ik} T_k, \tag{6.3}$$

where now T_k has to be understood as the blow-up modes that resolve the singularity. For $\mathbb{Z}_n$ singularities δ_i is universal; however for more complicated singularities δ_i can be non-universal. For applications to unification, we are interested in singularities where the different gauge groups have universal couplings at the singularity.

For both classes of local models the GUT unification scale and string scale differ significantly by a factor of the bulk radius. More precisely, the GUT unification scale m_X is given by $m_X = Rm_s$, where $R \sim \mathcal{V}^{1/6}$ is the bulk radius of the Calabi–Yau in string units. This can be seen through the Kaplunovsky–Louis relation between physical and holomorphic gauge couplings [156]

$$g_a^{-2}(\Phi,\bar{\Phi},\mu) = \mathrm{Re}\big(f_a(\Phi)\big) + \frac{\big(\sum_r n_r T_a(r) - 3T_a(G)\big)}{8\pi^2}\ln\left(\frac{m_{\mathrm{P}}}{\mu}\right) + \frac{T(G)}{8\pi^2}\ln g_a^{-2}(\Phi,\bar{\Phi},\mu)$$
$$+ \frac{\big(\sum_r n_r T_a(r) - T(G)\big)}{16\pi^2}\widehat{K}(\Phi,\bar{\Phi}) - \sum_r \frac{T_a(r)}{8\pi^2}\ln\det Z^r(\Phi,\bar{\Phi},\mu). \tag{6.4}$$

Using the IIB Kähler potential $\widehat{K} = -2\ln\mathcal{V}$ and the behavior for local models $\widehat{Z} = \mathcal{V}^{-2/3}$ we obtain

$$g_a^{-2}(\mu) - \frac{T(G)}{8\pi^2}\ln g_a^{-2}(\mu) = \mathrm{Re}\big(f_a(\Phi)\big) + \beta_a \ln\left(\frac{(Rm_{\mathrm{s}})^2}{\mu^2}\right), \tag{6.5}$$

giving effective unification at Rm_{s}. As described in [157, 158], at the string level this dependence arises from the presence of tadpoles that are sourced in the local model but are only canceled globally. This comes from the fact that the $U(1)_Y$ flux that breaks the GUT group is on a two-cycle that is non-trivial in $H_2(D_a,\mathbb{Z})$ and trivial in $H_2(\mathcal{X},\mathbb{Z})$. Locally the $U(1)_Y$ flux sources an RR tadpole, which is in fact absent globally due to the triviality of the cycle. The finiteness of threshold corrections is tied to the absence of RR tadpoles, but the triviality of C_a requires knowledge of the global geometry, leading to the presence of the scale Rm_{s}.

Moduli Stabilization

We take the zero mode arguments of chapter 4 serious and assume that the GUT branes wrap a four-cycle Γ_a which has no chiral intersection with the instanton cycle in order to avoid the generation of charged term in the non-perturbative superpotential. As mentioned, in type IIB orientifolds we allow for an additional gauge flux $\mathcal{F}_a$ in the diagonal $U(1)_a \subset U(5)$ perturbative Chan–Paton gauge group. Vanishing of the Fayet–Iliopoulos $U(1)_a$ D-term constraint (at order $\mathcal{V}^{-2}$)

$$\int_{D_a} J \wedge \mathcal{F}_a = 0 \tag{6.6}$$

implies that that the volume of the cycle Γ is driven to zero, i. e. $\tau_a \to 0$, so that we are at the quiver locus where α'-corrections cannot be ignored. In the EFT, the condition (6.6) is essentially that the field dependent FI-term vanishes $\mathcal{K}_{T_a} = 0$. Using the Kähler potential (3.5) in both the geometric and quiver regimes, this condition show explicitly a dynamical preference for a collapsed cycle $\tau_a \to 0$ as for a swiss cheese manifold with volume (3.29) we have

$$\mathcal{K}_{T_a} = \frac{3}{2}\frac{\eta_a^{3/2}\tau_a^{1/2}}{\mathcal{V}+\frac{\hat{\xi}}{2}}. \tag{6.7}$$

The F-term of the field T_a is of the form

$$F_a = e^{\mathcal{K}/2}\big(W_{T_a} + W\mathcal{K}_{T_a}\big). \tag{6.8}$$

Since the superpotential W does not depend on the modulus T_a and the D-term condition implies $\mathcal{K}_{T_a} = 0$, we can see that this field does *not* break supersymmetry, i. e. $F_a = 0$. Notice that this conclusion will not be modified by including perturbative and non-perturbative corrections to the Kähler potential since these corrections will equally modify the D- and F-terms.

Now it is instructive to take a look at the Kähler metric derived from (3.5):

$$\begin{aligned}
\mathcal{K}^{a\bar{b}} &= \pm 2\Big(\mathcal{V}+\tfrac{\hat{\xi}}{2}\Big)\frac{3}{16\eta_a^{3/2}}\tau_a^{1/2}\delta_{ab} + \frac{4\mathcal{V}-\hat{\xi}}{\mathcal{V}-\hat{\xi}}\tau_a\tau_b,\\
\mathcal{K}^{a\bar{S}} &= -\frac{3}{2}\big(S+\bar{S}\big)\frac{\hat{\xi}}{\mathcal{V}-\hat{\xi}}\tau_a,\\
\mathcal{K}^{S\bar{S}} &= \frac{\big(S+\bar{S}\big)^2}{4}\frac{4\mathcal{V}-\hat{\xi}}{\mathcal{V}-\hat{\xi}},
\end{aligned} \tag{6.9}$$

where we included this time also the dilaton part. Since $\tau_a = 0$ it is clear from (6.9) that also $F^a = e^{\mathcal{K}/2}\mathcal{K}^{a\bar{j}}F_{\bar{j}} = 0$. This is a very important conclusion, as it indicates that the standard model is somehow sequestered from the sources of supersymmetry breaking.

A loophole to this argument is that it implicitly assumes that the standard model fields, charged under the corresponding $U(1)$, will not get a vev. Otherwise they would contribute to the D-terms and cancel the contribution from the FI-term. Even though this is desirable phenomenologically to avoid a large scale breaking of the standard model symmetries, such as color, it should be the outcome of a calculation. We illustrate in the appendix in a toy model that this is actually the case as long as the soft scalar masses are not tachyonic.

A direct consequence is that the soft terms on the GUT brane can only be generated at "sub-leading" order by F^b, F^s and F^S, i. e. by moduli which are sort of sequestered from the GUT brane.

Including Matter Fields

So far we have concentrated only on the EFT for the moduli fields and their stabilization. In order to study soft-supersymmetry breaking we need to properly introduce the matter field dependence in the EFTs in both the geometric and singular cycle regimes. The important term to be included is the matter fields' Kähler potential $\widetilde{K} = Z_{\alpha\beta}\varphi_\alpha\varphi^*_\beta + \cdots$ with $Z_{\alpha\beta}$ a function of the moduli fields.

At this state, only the dependence on τ_b and τ_s is relevant, as all the other fields do not break supersymmetry (to leading order). Z should only depend on τ_b, S and the Kähler modulus of the GUT brane τ_a, so $Z = Z(\tau_b, \tau_a, S)$. We found the leading order expression for Z in section 3.6 to be $Z \sim 1/\mathcal{V}^{2/3}$. This applies to both chiral matter at magnetized D7-branes and to the better understood fractional D3-branes at singularities. Since the α'-corrections to the Kähler potential are crucial to determine the large volume vacuum, consistency requires that these corrections should also be included in the matter field Kähler potential. Unfortunately these corrections are not known at present but as in the tree-level case, we are mostly interested on their overall volume dependence.

Let us parametrize the α'-corrections by a so far unknown function f:

$$Z_\alpha = \frac{k_\alpha}{\tau_b}\left(1 + f\left(\frac{\mathrm{Re}(S)}{\tau_b}\right)\right). \tag{6.10}$$

The dependence of f on the variables can only be in the indicated way in order to have the right power in g_s. Now consider the next-to-leading order correction in α' to the tree-level result, which, we claim, must be of the form:

$$Z_\alpha = \frac{k_\alpha}{\tau_b}\left(1 - \delta\left(\frac{\mathrm{Re}(S)}{\tau_b}\right)^{\frac{n}{2}} + \cdots\right), \tag{6.11}$$

with $n = 1, 2, \ldots$ denoting the $(\alpha')^n$ order of this term. The question now is at which order in $(\alpha')^n$ the first correction appears. Since we are only interested in the correction which does not include τ_a, we can use a scaling argument like in section 3.6. Assuming that the physical Yukawa couplings do not depend on the overall volume of the space and taking into account the Kähler potential (3.5), the leading order correction to the Kähler metrics were shown to scale as $\frac{k_\alpha}{\tau_b}$. Then it is expected that also at next-to-leading order the scalings must match, which means that also the Kähler metrics are corrected at order $(\alpha')^3$. This argument shows that $n = 3$ is the smallest expected correction in (6.11) and then

$$Z_\alpha = \frac{k_\alpha}{\tau_b}\left(1 - \delta\left(\frac{\mathrm{Re}(S)}{\tau_b}\right)^{3/2}\right). \tag{6.12}$$

Summary of EFTs

We can finally summarize the expressions for the EFTs we are using for the two relevant regimes:

1. In the geometric regime the EFT is determined by:

$$\mathcal{K} = -2\ln\Big(\mathcal{V} + \tfrac{\hat{\xi}}{2}\Big) - \ln\big(S + \bar{S}\big) + \mathcal{K}_{\text{CS}} + Z\varphi\varphi^* + \cdots, \tag{6.13}$$

$$W = W_0 + Ae^{-aT_s} + W_{\text{matter}}, \tag{6.14}$$

$$f_i = T_a - \frac{1}{2}\kappa_i S, \tag{6.15}$$

where $\mathcal{V} = (\eta_b\tau_b)^{3/2} - (\eta_s\tau_s)^{3/2} - (\eta_a\tau_a)^{3/2}$ and $Z = k\Big(1 - \delta\big(\text{Re}(S)\big)^{3/2}/\mathcal{V}\Big)/\mathcal{V}^{2/3}$.

2. In the singular cycle (blow-up) regime there is a slight change in the standard model cycle dependence of $\mathcal{K}$:

$$\mathcal{K} = -2\ln\Big(\mathcal{V} + \tfrac{\hat{\xi}}{2}\Big) + \alpha\frac{\tau_a^2}{\mathcal{V}} - \ln\big(S + \bar{S}\big) + \mathcal{K}_{\text{CS}} + Z\varphi\varphi^* + \cdots, \tag{6.16}$$

$$W = W_0 + Ae^{-aT_s} + W_{\text{matter}}, \tag{6.17}$$

$$f = \delta_i S + s_{ik}T_k, \tag{6.18}$$

with now $\mathcal{V} = (\eta_b\tau_b)^{3/2} - (\eta_s\tau_s)^{3/2}$ and $Z = \big(\beta - \delta/\mathcal{V} + \gamma\tau_a^m\big)/\mathcal{V}^{2/3}$ with $m > 0$.

Since in both cases the standard model/GUT cycle does not break supersymmetry, the structure of soft breaking terms will be essentially the same.

6.2. Gravity Mediated Soft Terms

As we have seen, the LARGE volume minimum of the scalar potential breaks supersymmetry, so that this breaking induces soft supersymmetry breaking terms on the GUT brane. There are two sources which are relevant here. First, there are of course the gravity mediated soft terms. However, since the GUT brane is sequestered from the non-supersymmetric bulk one might expect that anomaly mediation is the leading order contribution. In this section we compute the gravity mediated soft terms, i. e. the gaugino- and sfermion-masses as well as the μ-, A- and B-terms. Moreover, we compute the anomaly mediated gaugino masses. Let us emphasize again that the scenario differs from the usual intermediate scale LARGE volume scenario in that the string scale is much higher (we assume $m_s \sim 10^{15}$ GeV for consistency with unification at $m_X \sim 10^{16}$ GeV), and that the GUT or MSSM branes are wrapping a four-cycle completely sequestered from the four-cycles supporting D3-brane instantons.

From the relation between the string scale and the Planck scale (3.48) we see that for $m_s = 10^{15}$ GeV, which implies in our set up the usual unification scale at $m_X \sim 1.2 \cdot 10^{16}$ GeV, an internal volume of roughly $\mathcal{V} = \mathcal{O}(10^6 - 10^7)$ is needed (in Einstein frame). This is a value large enough to trust the $\mathcal{V}^{-1}$ expansion. Moreover, we immediately realize that the LARGE volume expansion parameter is directly related to the local GUT expansion parameter, i. e. $\mathcal{V}^{-1/2} \simeq m_s/m_P$.

6.2.1. Structure of Soft Terms

We are now in a position to compute each of the gravity mediated soft supersymmetry breaking terms in this class of scenarios.

Gaugino Masses

Remember, for gravity mediated supersymmetry breaking, the gaugino masses are calculated as

$$m_{1/2} = \frac{1}{2\,\mathrm{Re}(f_i)} F^I \partial_I f_i, \tag{6.19}$$

for $i = 3, 2, 1$, where for the gauge kinetic functions we use (6.1) with $\tau_a \simeq 0$ due to the D-term constraint.

The distinctive feature of our set-up is the fact that the GUT brane is sequestered from the bulk, which implies $F^a = 0$. The only contribution to the gaugino masses can come from the dilaton F-term $F^S = e^{\mathcal{K}/2}\mathcal{K}^{S\bar{J}}\bar{F}_{\bar{J}}$.

Here, a subtlety arises concerning $F_S = D_S W = \partial_S W + W(\partial_S \mathcal{K})$: in the LARGE volume scenario, the Kähler potential depends on the dilaton not only in the usual way via $-\ln(S+\bar{S})$, but there is also a contribution in the α'-correction in the Kähler moduli part (see (3.5)). Thus, $\partial_S\mathcal{K}$ receives $\mathcal{V}^{-1}$ corrections:

$$D_S W \approx \partial_S W_0 - \frac{g_s}{2} W_0 - \frac{3}{4}\frac{\xi}{g_s^{1/2}}\frac{W_0}{\mathcal{V}} + \mathcal{O}(\mathcal{V}^{-2}). \tag{6.20}$$

Also as a consequence of the α'-corrections, the minimum of the scalar potential for the dilaton is shifted away from the supersymmetric locus $D_S W = 0$ at order $\mathcal{V}^{-1}$. In order to determine the new minimum, one would have to minimize the full potential, before integrating out the dilaton. However, since we do not have an explicit model with a full flux sector, in order to capture this effect, we assume that the two leading order terms in (6.20) cancel and keep only the next-to-leading order terms in the $\mathcal{V}^{-1}$ expansion. The expression we get in this way has certainly the correct order in $\mathcal{V}$ and we include an order one constant γ' in the expression for the F-term, comprising the uncertainty about the true location of the new minimum:

$$D_S W \approx -\frac{3}{4}\gamma' \frac{\xi}{g_s^{1/2}}\frac{W_0}{\mathcal{V}}. \tag{6.21}$$

In the sum over $D_I W$ in the dilaton F-Term $F^S = e^{\mathcal{K}/2}\mathcal{K}^{SJ} D_J W$, there are finally two contributions at order $\mathcal{V}^{-2}$: one from $\mathcal{K}^{Sb}F_b$ and another one from $\mathcal{K}^{SS}F_S$. They have both the same dependence on $W_0/\mathcal{V}^2$ and ξ/g_s^2. We combine the former constant γ' with other order one constants into a new variable γ such that we can write the dilaton F-term as follows:

$$F^S \approx \frac{3}{2\sqrt{2}}\gamma\frac{\xi}{g_s^2}\frac{W_0}{\mathcal{V}^2}. \tag{6.22}$$

This can be inserted into the expression for the gravity mediated gaugino masses (6.19). The result reads:

$$m_{1/2} = \frac{3}{4\sqrt{2}}\gamma\frac{\xi}{g_s}\frac{|W_0|m_P}{\mathcal{V}^2} = \frac{3}{4}\gamma\frac{\xi}{g_s^{3/2}}\frac{m_{3/2}}{\mathcal{V}}. \tag{6.23}$$

We see that this is independent of the MSSM gauge group factor, as the factor κ_i in (6.1) cancels. Here we have assumed that the D-term fixes the size of the GUT four-cycle at small volume in string units, so that the leading contribution to $\mathrm{Re}(f_i) \simeq 25$ comes from the gauge flux induced correction $\simeq \kappa_i \,\mathrm{Re}(S)$.

Squark/Slepton Masses

For convenience we repeat the formula for the scalar masses in gravity mediation of supersymmetry breaking:

$$m_0^2 = m_{3/2}^2 + V_0 - F^I \bar{F}^{\bar{J}} \partial_I \partial_{\bar{J}} \ln Z_\alpha. \tag{6.24}$$

From now on we assume that the potential in the minimum V_0 is already uplifted so that $V_0 \simeq 0$.

Let us first discuss the tree-level term in Z_α as deduced in (3.68). In this case and with the assumption $V_0 \simeq 0$, (6.24) reduces to

$$m_0^2 = m_{3/2}^2 - \frac{\left(F^b\right)^2}{4\tau_b^2}. \tag{6.25}$$

For the F-term F^b we have to perform again a careful analysis, as it will turn out that at leading order there is a precise cancellation between the gravitino mass squared and the second term in (6.25).

For this purpose, we start with the derivation of several approximation formulas, which we will need later. Consider the scalar potential (3.21), where we introduce again the usual prefactors $\lambda = \frac{g_s}{2}\frac{8}{3\eta_s^{3/2}}$, $\mu = 2g_s$ and $\nu = \frac{3}{8}$. Upon minimizing it with respect to the two independent variables τ_s and $\mathcal{V}$, we get two expressions: first, from the condition $\frac{\partial V_F}{\partial \tau_s} = 0$, it follows:

$$e^{-a\tau_s} = \frac{\mu}{\lambda}\frac{|W_0|}{aA\mathcal{V}}\frac{1}{\sqrt{\tau_s}}\frac{(1-a\tau_s)}{\left(-2a+\frac{1}{2\tau_s}\right)}. \tag{6.26}$$

After developing the denominator in powers of $1/(a\tau_s)$ and inserting the expressions for μ

and λ we get

$$e^{-a\tau_s} \approx \frac{3}{4}\frac{\eta_s^{3/2}}{aA}\sqrt{\tau_s}\frac{W_0}{\mathcal{V}}\left(1-\frac{3}{4a\tau_s}\right). \tag{6.27}$$

The second approximation formula arises upon solving $\frac{\partial V_F}{\partial \mathcal{V}} = 0$ for $\tau_s^{3/2}$ and thereby using (6.27). The result is

$$\tau_s^{3/2} \approx \frac{\hat{\xi}}{2\eta_s^{3/2}}\left(1+\frac{1}{2a\tau_s}\right). \tag{6.28}$$

Another useful approximation which we need in the following is given by:

$$\mathcal{K}^{ab}(\partial_b\mathcal{K}) = -\frac{4\mathcal{V}^2+\mathcal{V}\hat{\xi}+4\hat{\xi}^2}{2\left(\mathcal{V}-\hat{\xi}\right)\left(\mathcal{V}+\frac{\hat{\xi}}{2}\right)}\tau_a \approx -2\tau_a - \frac{3}{2}\hat{\xi}\frac{\tau_a}{\mathcal{V}}, \tag{6.29}$$

where the sum runs only over Kähler moduli. The first equality can be derived using the expressions for the Kähler metric and the derivatives of the Kähler potential with respect to the moduli in terms of two-cycle volumes t^a instead of four-cycle volumes τ_a (see [30, 89] for details).

We are now in a position to calculate F^b:

$$F^b = e^{\mathcal{K}/2}\mathcal{K}^{bJ}D_JW = e^{\mathcal{K}/2}\left(\mathcal{K}^{b\tau_j}(\partial_{\tau_j}\mathcal{K})W + \mathcal{K}^{bs}(\partial_sW) + \mathcal{K}^{bS}D_SW\right). \tag{6.30}$$

The term involving D_SW turns out to be sub-leading in the $\mathcal{V}^{-1}$ expansion with respect to the other terms and thus can be neglected (see below). The derivative of the superpotential with respect to T_s undergoes a sign-flip due to the minimization with respect to the corresponding axion as it was already described in section 3.2. We insert now the approximations (6.27), (6.28) and (6.29) in (6.30) and we get:

$$F^b = -2\tau_b\frac{\sqrt{g_s}}{\sqrt{2}}\frac{W_0}{\mathcal{V}} - \frac{3}{8\sqrt{2}}\frac{\tau_b}{a\tau_s}\left(1+\frac{3}{2a\tau_s}\right)\frac{W_0}{\mathcal{V}^2} + \mathcal{O}(\mathcal{V}^{-3}). \tag{6.31}$$

For the later comparison with the other soft-terms, it is instructive to express this result in terms of the gravitino mass $m_{3/2} = e^{\mathcal{K}/2}W \sim \frac{\sqrt{g_s}}{\sqrt{2}}\frac{W_0}{\mathcal{V}}$ and the gaugino mass (6.23):

$$F^b = -2\tau_b m_{3/2} - \frac{\tau_b}{2a\tau_s}\left(1+\frac{3}{2a\tau_s}\right)m_{1/2} + \mathcal{O}(\mathcal{V}^{-3}). \tag{6.32}$$

From (6.27) we can be estimate that $a\tau_s \approx \ln\mathcal{V} \approx 10$. Thus, for the sake of shorter formulæ, we may also neglect the second term in the parenthesis such that:

$$F^b \approx -2\tau_b m_{3/2} - \frac{\tau_b}{2a\tau_s}m_{1/2}. \tag{6.33}$$

We square now (6.31) and clearly see the cancellation at leading order in the scalar mass formula (6.25):

$$\left(F^b\right)^2 \approx 4\tau_b^2\left[m_{3/2}^2 + \frac{3}{8a\tau_s}\frac{\xi}{g_s^{3/2}}\left(1+\frac{3}{2a\tau_s}\right)\frac{m_{3/2}^2}{\mathcal{V}}\right]. \tag{6.34}$$

We insert this into (6.25) and finally get for the soft sfermion masses squared

$$\begin{aligned} m_0^2 &= -\frac{3}{16a\tau_s}\frac{\xi}{g_s^{1/2}}\left(1+\frac{3}{2a\tau_s}\right)\frac{|W_0|^2 m_{\mathrm{P}}^2}{\mathcal{V}^3} \\ &= -\frac{3}{8a\tau_s}\frac{\xi}{g_s^{3/2}}\frac{m_{3/2}^2}{\mathcal{V}}, \end{aligned} \tag{6.35}$$

which at this stage come out tachyonic.

Next we need to discuss the higher α'-corrections in (6.11). The term with the highest power in $1/\mathcal{V}$ is the one with $\left(F^b\right)^2\partial_b\partial_b\log\cdots$. It is straightforward, that for $\tau_a/\tau_b \ll 1$ this simplifies to

$$F^m F^n \partial_m\partial_n \log\left(1-\delta\left(\frac{\mathrm{Re}\,S}{\tau_b}\right)^{\frac{n}{2}}+\cdots\right) \simeq F^b\,F^b\frac{\delta n(n+2)(\mathrm{Re}\,S)^{\frac{n}{2}}}{4\tau_b^{\frac{n}{2}+2}} \sim \frac{\delta}{g_s^{\frac{n-2}{2}}}\frac{|W_0|^2 m_{\mathrm{P}}^2}{\mathcal{V}^{\left(2+\frac{n}{3}\right)}}. \tag{6.36}$$

Therefore, if there were corrections of order $n=1,2$, they would dominate over the corrections in (6.35). It is precisely the third order corrections in α' which contribute to the sfermion masses at the same order in $1/\mathcal{V}$. Including also the other moduli fields in (6.36), the overall value of the squared scalar masses will then be proportional to $\delta-\xi/3$:

$$m_0^2 = m_{3/2}^2\left(-\frac{1}{4a\tau_s}\frac{\xi}{g_s^{3/2}\mathcal{V}} + \frac{15(\delta-\xi/3)}{4g_s^{3/2}\mathcal{V}}\right). \tag{6.37}$$

Depending on the relative size of these two contributions one can get tachyonic or non-tachyonic sfermion masses. Moreover, it also shows that for $\delta = \xi/3$ there are further cancellations taking place at this order. This is precisely the value one expects from the above mentioned scaling argument of the physical Yukawa couplings.

Later we will give an argument under which quite general assumptions such cancellations should occur. One of the assumptions will be that really the uplifting sector is correctly taken into account, which leads to a further dependence of the Kähler metric on a supersymmetry breaking field. Note that indeed the soft sfermion masses (6.35) are of the same order as the AdS vacuum energy $V_0 \sim W_0^2/\mathcal{V}^3$, indicating that in these computations the uplift sector cannot be neglected.

$\hat{\mu}/\hat{\mu}B$-terms

The formula for the $\hat{\mu}$-term is

$$\hat{\mu} = \left(e^{\mathcal{K}/2}\mu + m_{3/2}Z - \bar{F}^{\bar{I}}\partial_{\bar{I}}Z\right)\left(Z_{H_1}Z_{H_2}\right)^{-1/2}, \tag{6.38}$$

where μ denotes the supersymmetric μ-parameter, which we keep for completeness, although it can be argued to vanish under very general assumptions [99]. We assume again the Kähler metric (6.11) for the Higgs fields as well as for Z. Here again, a cancellation of the second and the third term occurs. Note, if the μ parameter is not equal to zero, it dominates over the sub-leading terms stemming from F^b. Dropping the factors of order one, we are left with:

$$\hat{\mu} \approx \frac{\sqrt{g_s}}{\sqrt{2}}\frac{\tau_b}{\mathcal{V}}\mu - \frac{m_{1/2}}{4a\tau_s}. \tag{6.39}$$

The expression for $B\hat{\mu}$ is more complicated:

$$\begin{aligned} B\hat{\mu} =&(Z_{H_1}Z_{H_2})^{-1/2}\Big(e^{\mathcal{K}/2}\mu\big(F^I\partial_I\mathcal{K} + F^I\partial_I\log\mu - F^I\partial_I\log(Z_{H_1}Z_{H_2}) - m_{3/2}\big) \\ &+ (2m_{3/2}^2 + V_0)Z - m_{3/2}\bar{F}^{\bar{I}}\partial_{\bar{I}}Z + m_{3/2}F^I\big(\partial_I Z - Z\partial_I\log(Z_{H_1}Z_{H_2})\big) \\ &- F^{\bar{I}}F^J\big(\partial_{\bar{I}}\partial_J Z - (\partial_{\bar{I}}Z)\partial_J\log(Z_{H_1}Z_{H_2})\big)\Big). \end{aligned} \tag{6.40}$$

However, due to the simple Kähler metric and assuming that μ is just an input parameter without any moduli dependence, after a long but straightforward calculation, the result is rather simple:

$$B\hat{\mu} = -\left(\frac{\sqrt{g_s}}{\sqrt{2}}\frac{\tau_b}{\mathcal{V}}\mu + \frac{m_{3/2}}{2a\tau_s}\right)m_{1/2}, \tag{6.41}$$

where we have dropped again the order one constants k_{H_i} and z.

A-terms

The A-terms are given by:

$$A_{\alpha\beta\gamma} = F^I(\partial_I\mathcal{K}) + F^I\partial_I\log Y_{\alpha\beta\gamma} - F^I\partial_I\log Z_\alpha Z_\beta Z_\gamma. \tag{6.42}$$

The Peccei–Quinn shift symmetry forbids a dependence of the holomorphic superpotential on the axio-dilaton or Kähler moduli, thus the Yukawa couplings $Y_{\alpha\beta\gamma}$ can only depend on the complex structure moduli and they drop out.

There is a cancellation of F^b in the remaining two sums and we are left with

$$A_{\alpha\beta\gamma} = F^s(\partial_s\mathcal{K}) + F^S(\partial_S\mathcal{K}). \tag{6.43}$$

The first term, $F^s(\partial_s\mathcal{K})$ is suppressed with respect to the second one $F^S(\partial_S\mathcal{K})$ by a factor of $1/a\tau_s$. As we are interested only in orders of magnitude, we keep only the latter term and get as result:

$$A_{\alpha\beta\gamma} \approx F^S(\partial_S\mathcal{K}) = -\frac{3}{4\sqrt{2}}\frac{\xi}{g_s}\frac{|W_0|}{\mathcal{V}^2}m_{\mathrm{P}} = -m_{1/2}. \tag{6.44}$$

Anomaly Mediated Gaugino Masses

Let us also now estimate the anomaly mediated gaugino mass. It is clear that, for such a sequestered observable sector, one would have guessed that not gravity mediation but anomaly mediation induces the leading order soft terms. General formulæ for all the different soft terms are not available, so that in this section we just compute the anomaly mediated gaugino masses. Remember from eq. (3.73) that the formula for $m_{1/2}^{\text{anom.}}$ contains terms proportional to $m_{3/2}$ such that one would await a gaugino mass of the order $W_0/\mathcal{V}$. However also here, there is a cancellation at leading order stemming from the F-term $F^b \approx -2\tau_b m_{3/2} - \frac{\tau_b}{2a\tau_s}m_{1/2}$. The gravitino mass drops out and the final expression for the anomaly mediated gaugino mass for a $SU(N)$ gauge group is:

$$m_{1/2}^{\text{anom}} = -\frac{g^2}{16\pi^2}\left[\left(N-\tfrac{1}{2}\right) - \frac{1}{4a\tau_s}\left(3N-\tfrac{1}{2}\right)\right]m_{1/2}. \tag{6.45}$$

The surprising conclusion is: though the gravity mediated contribution to the gaugino mass is suppressed with respect to the gravitino mass by a factor of $(m_X/m_{\mathrm{P}})^2$, anomaly mediation is not the dominating source for the gaugino mass. It is suppressed by the usual one-loop factor with respect to the gravity mediated contribution. We expect a similar suppressed behavior for the other soft terms, so that anomaly mediation is sub-leading to gravity mediation.

6.2.2. Summary of Gravity Mediated Soft Masses

In the above computation of soft terms we have seen that the leading terms cancel and that we need to include higher order corrections in $\mathcal{V}^{-1}$. Since this scale is directly correlated with $\zeta = m_s/m_{\mathrm{P}}$, we can express these gravity mediated soft terms in terms of the scales $m_{3/2}$ and $\zeta = m_s/m_{\mathrm{P}}$. The results are listed in table 6.1, where we have set the supersymmetric μ parameter to zero and estimated

$$\sqrt{\frac{\pi}{3\xi}} \simeq \sqrt{\frac{400}{\chi(\mathcal{X})}} \simeq 1, \qquad \text{and} \qquad g_s \simeq 1. \tag{6.46}$$

All soft terms in table 6.1 are suppressed by $(m_s/m_{\mathrm{P}})^2$ relative to the naïve expectation

soft term	scale
$m_{1/2}$	$\frac{1}{4} m_{3/2} \zeta^2$
m_0^2	$\frac{1}{16 \log \zeta} m_{3/2}^2 \zeta^2$
$\hat{\mu}$-term	$\frac{1}{8 \log \zeta} m_{1/2}$
$B\hat{\mu}$-term	$\frac{1}{4 \log \zeta} m_{3/2}\, m_{1/2}$
A-term	$-m_{1/2}$

Table 6.1.: Classical gravity mediated soft terms for a naïve computation of soft terms. Here the expansion parameter is $\zeta = m_s/m_P$. We have assumed the supersymmetric μ-term to vanish [99].

$m_{3/2}^n$ with $n = 1, 2$ depending on the mass-dimension. This explicitly demonstrates that gravity effects from the bulk are suppressed on the shrinkable GUT cycles, which is *the* main assumption of the local F-theory GUTs.

However, as seen in the text in certain cases there can be more cancellations leading to even higher suppressions. Indeed so far we have neglected the uplift sector, but have seen that the sfermion masses are actually of the same order of magnitude as the uplift so that it should better not be neglected. We now discuss under which well posed assumptions further cancellations are present.

6.2.3. Uplift and Cancellations

In the last section we have computed the gravity induced soft terms on the GUT brane. As we have explained, the computation relies on assumptions about the expansions of the matter metrics at higher orders in α'. While such corrections must surely be present, it is difficult to know the precise form of these corrections. We have explicitly seen for the sfermion masses that these corrections contribute at the same order in $1/\mathcal{V}$ as the next-to-leading order contributions from F^b. Indeed, as seen in eq. (6.37) there can potentially be further cancellations at this order. We have also computed the soft terms under the assumption of $V_0 = 0$, but have not taken into account the contribution of the supersymmetry breaking from the uplifting sector to the soft terms. To consider these possibilities, let us argue in this section, how one can arrive at quite general statements by making some well posed assumptions and exploiting the consequences of using the supergravity formalism.

Recall that the physical Yukawas are given by

$$\widehat{Y}_{\alpha\beta\gamma} = e^{\mathcal{K}/2} \frac{Y_{\alpha\beta\gamma}}{\sqrt{Z_\alpha Z_\beta Z_\gamma}}. \qquad (6.47)$$

The shift symmetries of the Kähler moduli imply that they do not appear perturbatively in the superpotential Yukawa couplings $Y_{\alpha\beta\gamma}$. Let us make the assumption that the physical Yukawas, being local renormalizable couplings, do not depend on the fields breaking supersymmetry. This includes the volume and also the hidden sector fields that are responsible for uplifting and giving vanishing cosmological constant. We also assume pure F-term uplifting.

Such supersymmetry breaking fields appear in the overall Kähler potential, and the constraints of holomorphy then imply that in order for the physical Yukawas to be independent of such fields,

$$Z_\alpha = e^{\mathcal{K}/3}. \tag{6.48}$$

Note that this includes the tree-level behavior of local matter fields, $Z_\alpha \sim \frac{1}{\mathcal{V}^{2/3}} \sim \frac{1}{(T_b + \bar{T}_b)}$. In this case it follows that

$$\begin{aligned} m_0^2 &= V_0 + m_{3/2}^2 - F^m \bar{F}^{\bar{n}} \partial_m \partial_{\bar{n}} \ln Z_\alpha \\ &= V_0 + m_{3/2}^2 - F^m \bar{F}^{\bar{n}} \frac{\mathcal{K}_{m\bar{n}}}{3} = \frac{2}{3} V_0 = 0, \end{aligned} \tag{6.49}$$

for the case of vanishing cosmological constant.

The A-terms also vanish under this assumption. In this case the A-terms can be most intuitively written as

$$A_{\alpha\beta\gamma} Y_{\alpha\beta\gamma} = F^I \partial_I \widehat{Y}_{\alpha\beta\gamma}, \tag{6.50}$$

with $\widehat{Y}_{\alpha\beta\gamma}$ the physical Yukawa couplings. So it immediately follows that if the physical Yukawa couplings do not depend on the fields breaking supersymmetry, the A-terms all vanish.

The anomaly mediated contribution for gaugino masses gives

$$\begin{aligned} m_{1/2}^{\text{anom}} &= \frac{b_a}{16\pi^2} m_{3/2} - \frac{\left(\sum_r n_r T_a(r) - T(G)\right)}{16\pi^2} F^m \partial_m \mathcal{K}(\Phi, \bar{\Phi}) \\ &\quad + \sum_r \frac{n_r T_a(r)}{8\pi^2} F^m \partial_m \ln Z^r(\Phi, \bar{\Phi}) \\ &= \frac{b_a}{16\pi^2} m_{3/2} - \frac{\left(\sum_r n_r T_a(r) - 3T(G)\right)}{16\pi^2} \frac{F^m \partial_m \mathcal{K}(\Phi, \bar{\Phi})}{3} \\ &= \frac{b_a}{16\pi^2} \left(m_{3/2} - \tfrac{1}{3} F^m \partial_m \mathcal{K}\right), \end{aligned} \tag{6.51}$$

where we have used $Z = e^{\mathcal{K}/3}$. The size of the anomaly mediated contributions to gaugino masses then depends on the size of $m_{3/2} - \frac{1}{3} F^m \partial_m \mathcal{K}$. The no-scale cancellation for τ_b implies the $\mathcal{O}(\mathcal{V}^{-1})$ terms cancel with non-vanishing terms at $\mathcal{O}(\mathcal{V}^{-2})$. However (6.51) also includes the hidden uplifting sector, which must have $\mathcal{K}_{\phi\bar{\phi}} F^\phi F^{\bar{\phi}} \sim \frac{1}{\mathcal{V}^3}$ (in order to uplift the vacuum energy to Minkowski). At this level we therefore cannot rule out that

$F^{\phi}\partial_{\phi}\mathcal{K} \sim \mathcal{V}^{-3/2}$, giving gaugino masses of order $\frac{g^2}{16\pi^2}\frac{1}{\mathcal{V}^{3/2}}$.

For the μ-term, we obtain

$$\hat{\mu} = e^{\mathcal{K}/6}\mu + \left(m_{3/2} - \tfrac{1}{3}F^I\partial_I\mathcal{K}\right). \tag{6.52}$$

For the B-term, we have (assuming no moduli dependence in μ)

$$\begin{aligned}(B\mu) = (Z_{H_1}Z_{H_2})^{-1/2}\Big(& e^{\mathcal{K}/2}\mu\big(F^I\partial_I\mathcal{K} - F^I\partial_I\ln(Z_{H_1}Z_{H_2}) - m_{3/2}\big)\\ &+ (2m_{3/2}^2 + V_0)Z - m_{3/2}F^{\bar{I}}\partial_{\bar{I}}Z + m_{3/2}F^I\big(\partial_I Z - Z\partial_I\ln(Z_{H_1}Z_{H_2})\big)\\ &- F^I F^{\bar{J}}\big(\partial_{\bar{I}}\partial_J Z - (\partial_{\bar{I}}Z)\partial_J\ln(Z_{H_1}Z_{H_2})\big)\Big).\end{aligned} \tag{6.53}$$

If we take $Z = Z_{H_1} = Z_{H_2} = e^{\mathcal{K}/3}$ then we eventually obtain

$$B\mu = e^{\mathcal{K}/6}\mu\Big(\tfrac{1}{3}F^I\partial_I\mathcal{K} - m_{3/2}\Big) + \Big|\tfrac{1}{3}F^I\partial_I\mathcal{K} - m_{3/2}\Big|^2. \tag{6.54}$$

This implies that the μ- and B-terms involve the same expression as appeared in the anomaly mediated expression (6.51) and that the μ- and $B\mu$-term are of the same order as required for successful electroweak symmetry breaking.

As we do not currently know the form of α'-corrections to the matter metrics, we do not know whether the form $Z = e^{\mathcal{K}/3}$ is correct. However it is a natural choice in the sense that it simply says that the physical Yukawa couplings, being local, do not depend on the value of bulk fields. In the context of the $\zeta(3)\chi(\mathcal{X})\alpha'^3$-correction that entered the moduli stabilization, this is equivalent to the statement that physical Yukawa couplings do not alter if you perform a conifold transition in the bulk (which alters the Calabi–Yau Euler number).

The advantage of phrasing the computation in this way is that we can say that moduli generate soft scalar masses to the extent to which the physical Yukawa couplings depend on the moduli. While not straightforward, it is in principle easier to compute the dependence of physical Yukawa couplings on the moduli. String CFT computations give the directly physical couplings and therefore one could analyze for certain local models (for example for a stack of D3-branes at an orbifold singularity in a compact space) whether the physical couplings do depend on the volume through a direct vertex operator string computation.

We can also use (6.49) to compute the minimal value of the soft scalar masses. The complete cancellation in (6.49) arose from the assumption that the physical Yukawa couplings has no dependence on all fields with non-zero F-terms. However we know this statement is not true. The dilaton has an irreducible F-term of $\mathcal{O}(\mathcal{V}^{-2})$ and enters the physical Yukawas. This provides a minimal value for the scale of the physical Yukawa couplings.

6.3. Consequences for Supersymmetry Breaking

In this previous section we have seen that both, gravity and anomaly mediated contributions to soft terms occur at levels far lower than naïve expectation. This gives novel phenomenological consequences for various aspects of supersymmetry breaking, which we now discuss.

6.3.1. Gauge Mediated Scenarios

In local F-theory models an interesting proposal was made assuming a model of gauge mediation to dominate supersymmetry breaking in the observable sector. This is very interesting since it incorporates the positive properties of gauge mediation, such as positive squared scalar masses and flavor universality and yet address its problems, such as the $\mu/B\mu$ problem. This proposal though requires the following implicit assumptions:

1. The mechanism responsible for moduli stabilization, which was not considered, fixes moduli at a high mass and decouples from supersymmetry breaking.

2. Introduce a new matter sector that breaks supersymmetry dynamically and a set of messengers that communicate this breaking to the standard model fields.

3. An anomalous $U(1)$ was proposed to communicate both sectors and address the $\mu/B\mu$ problem of gauge mediation. The anomalous $U(1)$ is naturally as heavy as the string scale but has low-energy implications after being integrated out.

These conditions look at first sight too strong and unnatural. Achieving moduli stabilization without supersymmetry breaking and small cosmological constant is a very strong assumption not realized in any of the moduli stabilization scenarios so far. It is known that a supersymmetric vacuum in supergravity, such as in KKLT before the uplifting, is naturally anti de Sitter since in that case the vacuum energy is $V_0 = -3m_{3/2}^2 m_{\rm P}^2$, which is very large unless the superpotential is tuned in such a way that it almost vanishes at the supersymmetric minimum. Also a positive cosmological constant has to be induced after supersymmetry breaking. If the local supersymmetry breaking is responsible for this lifting then its effect should not have been neglected for moduli stabilization in the first place. Finally, it is not consistent to consider the low-energy effects of a very heavy anomalous $U(1)$ without also including the effects of the moduli fields which are generically much lighter than the string or compactification scale. In particular the Fayet–Iliopoulos term of the anomalous $U(1)$ is a function of the moduli.

Nevertheless, our explicit results here show that a scenario similar to this may not be impossible to realize. The main point is that although moduli are stabilized at a non-supersymmetric point, the breaking of supersymmetry is suppressed by inverse powers of the volume or equivalently by powers of $m_{\rm s}/m_{\rm P}$. This makes the first point above approximately correct. The second point still has to be assumed as in all models of gauge mediation and requires an explicit realization. Here the relevant observation is to compare the strength of gauge mediation F_X/x to the strength of gravity mediation which is usually taken to be $m_{3/2}$. However as we have seen the proper comparison is between F_X/x with the size of the gravity mediation soft breaking terms which are much smaller than the gravitino mass. Regarding the third point an explicit analysis should be performed in which both the anomalous $U(1)$ and the moduli are taken into account in the process of moduli stabilization and supersymmetry breaking.

Very similar to the recently discussed local F-theory models, we may expand our model assuming that there exists a source for gauge mediation, which is parametrized by the vacuum expectation values of a scalar field $\langle X\rangle = x + \theta^2 F_X$. This supersymmetry breaking happens in a sector hidden from the GUT brane and is being mediated by messenger fields, which are charged under the GUT gauge group. In order not to spoil gauge coupling unification, this is generically assumed to be a vector-like pair in the $\mathbf{5}+\bar{\mathbf{5}}$ representation of $SU(5)$. For our purposes, we won't present a viable dynamical stringy realization of this supersymmetry breaking, but just assume that there exists an extra sector, which stabilizes the new moduli such that just the field F_X develops a non-zero vev without spoiling the LARGE volume minimum for the bulk moduli. This is clearly a strong assumption, as a dynamical realization of gauge mediation is known to be challenging [159–161]. We will comment more on this towards the end of this section.

The gauge mediated gaugino and sfermion masses are of order

$$m_{1/2}^{\text{gauge}} \sim m_0^{\text{gauge}} = \frac{\alpha_X}{4\pi}\frac{F_X}{x}, \tag{6.55}$$

where the $\alpha_X/4\pi$ prefactor is due to the fact that these masses are induced via a one-loop effect for the gauginos and via a two-loop diagram for the sfermions. Note that these formulæ use a canonical normalized superfield X.

Now, we would like these gauge mediated soft masses to dominate the gravity mediated ones. In particular, we want the gauge mediated sfermion masses to dominate over the gravity mediated ones. To get a first impression of the numerology we get, we also impose the strong constraint that the supersymmetry breaking F_X already uplifts the negative vacuum energy of the LARGE volume minimum. By inserting the approximation relations found in section 6.2.1 into the scalar potential (3.21) we find the precise value of the vacuum energy to be:

$$V_0 = -\frac{3}{16a\tau_s}\frac{\xi}{g_s^{3/2}}\frac{W_0^2}{\mathcal{V}^3} \approx -\frac{m_{3/2}^2}{16\log\left(\frac{m_P}{m_s}\right)}\frac{m_s^2}{m_P^2}. \tag{6.56}$$

We therefore require

$$\frac{F_X^2}{m_P^2} \simeq \frac{m_{3/2}^2}{16\log\left(\frac{m_P}{m_s}\right)}\frac{m_s^2}{m_P^2}, \tag{6.57}$$

leading to the relation

$$F_X \simeq \frac{1}{4\sqrt{\log\left(\frac{m_{\rm P}}{m_{\rm s}}\right)}} m_{3/2}\, m_{\rm s}. \tag{6.58}$$

Remember from table 6.1 that $m_0^{\rm grav} \simeq \frac{m_{3/2}}{4\sqrt{\log\left(\frac{m_{\rm P}}{m_{\rm s}}\right)}} \frac{m_{\rm s}}{m_{\rm P}}$. Requiring now that $m_0^{\rm gauge} > \left|m_0^{\rm grav}\right|$ leads to the moderate bound

$$x < \frac{\alpha_X}{4\pi} m_{\rm P} \simeq 10^{16}\,{\rm GeV}. \tag{6.59}$$

If there is a further suppression, i. e. $m_0 \simeq m_{3/2}/\mathcal{V}$, then this bound becomes even more relaxed. For solving the hierarchy problem, one also needs $F_X/x \simeq 10^5\,{\rm GeV}$. Once one has specified the favorite values for x and F_X, one can use (6.58) to determine the value of the gravitino mass, which we would like to stress will be gravity-dominated. Let us discuss two examples.

- In the local F-theory models, it was argued that the best values are

$$x \simeq 10^{12}\,{\rm GeV}, \qquad F_X \simeq 10^{17}\,{\rm GeV}^2, \tag{6.60}$$

  which lead to $m_{3/2} \simeq 1\,{\rm TeV}$, which needs a certain amount of tuning of W_0. However, the light modulus τ_b has a mass of the order

$$m_{\tau_b} \simeq m_{3/2} \frac{m_{\rm s}}{m_{\rm P}}, \tag{6.61}$$

  which in this case gives $m_{\tau_b} \simeq 1\,{\rm GeV}$. For such a light modulus, we expect to face the cosmological moduli problem.

- Let us now require that the light modulus avoids the CMP by having a mass $m_{\tau_b} \simeq 100\,{\rm TeV}$. Then according to (6.61), the gravity mediated gravitino mass has to be of the order $m_{3/2} \simeq 10^5\,{\rm TeV}$. Using (6.58), this leads to $F_X \simeq 10^{22}\,{\rm GeV}^2$. For gauge mediated soft masses of the order $500\,{\rm GeV}$, we therefore get $x \simeq 5 \cdot 10^{16}\,{\rm GeV}$, which is slightly beyond the stronger limit (6.59). For further suppression of the sfermion masses there is no problem.

- In the first case one could ameliorate this problem by allowing for a certain tuning of the Higgs mass, so that the supersymmetry breaking scale for the visible sector can be larger than $500\,{\rm GeV}$. Let us still have $F_X \simeq 10^{22}\,{\rm GeV}^2$ to avoid the CMP and require $x \simeq 5 \cdot 10^{14}\,{\rm GeV}$ to satisfy the constraint (6.59) for gauge mediation dominance. Then the gauge mediated soft masses are of the order $50\,{\rm TeV}$.

Finally, let us discuss in which way this simple model of gauge mediation needs to be improved in order to show that it can really be embedded into string theory. As we already mentioned, we did not dynamically explain where the SUSY breaking field X gets its vev from. Recently, various kinds of models have been suggested, which are not completely convincing from a string theory point of view. One promising model is the so-called Fayet–Polonyi model. It combines an anomalous Peccei–Quinn symmetry with a linear superpotential in X generated by another D3-instanton wrapping a del Pezzo surface of size T_{FP}. This gives rise both to a D-term potential with a T_{FP} dependent Fayet–Iliopoulos term and an F-term potential form the linear superpotential. Note, that the latter also depends on T_{FP}. Now, also taking the Kähler potentials into account one has to show that dynamically really supersymmetry can be broken in such a way that the desired values for x and F_X arise.[1] Moreover, one expects that also $F_{T_{\mathrm{FP}}} \neq 0$, which gives another source of supersymmetry breaking. Finally, one has to ensure that the moduli stabilization in the bulk, i. e. of the τ_b and τ_s moduli and the moduli stabilization of the local X and T_{FP} moduli decouple.

6.3.2. Implications for the Cosmological Moduli Problem

Let us comment on the cosmological moduli problem, discussed in section 3.8. Remember that moduli fields are expected to be displaced from their minimum during the inflationary epoch, subsequently oscillating about their minimum and red-shifting as matter. The lifetime of such moduli is $\tau \sim \frac{m_{\mathrm{P}}^2}{m_\phi^3} \gg 1\,\mathrm{s}$ for $m_\phi \lesssim 1$ TeV. They come to dominate the energy density of the universe, but if they decay too late then they fail to reheat the universe to temperatures sufficient for nucleosynthesis.

The results in this chapter suggest a novel approach to this problem. One of the properties of local LARGE volume GUTs with D-term stabilization is that the soft terms appear at a scale hierarchically smaller than the gravitino mass. Depending on the extent of cancellations, we have seen that soft terms appear at an order not larger than $m_{\mathrm{soft}} \sim m_{3/2}^{3/2}/m_{\mathrm{P}}^{1/2}$, in the case when the dilaton F-term is responsible for uplifting. In all other cases gaugino masses will be further suppressed, with at least an extra loop factor as in anomaly mediation, and possibly even as far as $m_{\mathrm{soft}} \sim m_{3/2}^2/m_{\mathrm{P}}$. For the two extreme cases the gravitino mass appropriate to TeV soft terms is

$$m_{\mathrm{soft}} \sim \frac{m_{3/2}^{3/2}}{m_{\mathrm{P}}^{1/2}} \longrightarrow m_{3/2} \sim 10^8\,\mathrm{GeV}, \qquad m_{\mathrm{soft}} \sim \frac{m_{3/2}^2}{m_{\mathrm{P}}} \longrightarrow m_{3/2} \sim 10^{11}\,\mathrm{GeV}. \tag{6.62}$$

Instead of solving the moduli problem by making the moduli heavy and keeping soft terms

1 It was shown in [160], that this model with a simple choice of the Kähler potential actually still posses supersymmetric minima.

comparable to the gravitino mass, this suggests making the gravitino heavy and having soft terms much lighter than the gravitino mass.

In the LARGE volume models the volume modulus T_b is relatively light and has a mass $m_{T_b} \sim m_{3/2}^{3/2}/m_{\mathrm{P}}^{1/2}$, while all other moduli have masses comparable to $m_{3/2}$. In the first case listed above, with a gravitino mass of around 10^8 GeV, the volume modulus has $m \sim 1$ TeV and still poses cosmological problems. However in the other cases m_{T_b} is sufficiently large to decay before nucleosynthesis. In the case of maximal suppression, with $m_{3/2} \sim 10^{11}$ GeV, then we have $m_{T_b} \sim 10^7$ GeV with no cosmological problems. In all cases the other moduli (for example dilaton and complex structure moduli) have masses comparable to the gravitino mass and decay very rapidly.

It would also be interesting to study whether these suppressed soft terms would affect the thermal behavior of the LARGE volume models studied in [162].

6.3.3. Implications for Model Building

Several scenarios regarding gravity and anomaly mediation are possible and which of these is actually realized may be model dependent. The main possibilities are:

- If the F-term of the dilaton field is responsible for the uplifting to de Sitter space, then $F^S \sim \mathcal{V}^{-3/2}$ and all the soft masses are of order $\frac{m_{\mathrm{P}}}{\mathcal{V}^{3/2}} \sim \frac{m_{3/2}}{\sqrt{\mathcal{V}}}$. This is of the same order as the mass of the lightest modulus, the volume modulus, and this field remains dangerous for the cosmological moduli problem.

- If any other field is responsible for the de Sitter uplifting, the dilaton induces gravity mediated gaugino masses of order $\frac{m_{\mathrm{P}}}{\mathcal{V}^2}$ or from anomaly mediation, barring any further cancellation, of order $\alpha\frac{m_{\mathrm{P}}}{\mathcal{V}^{3/2}}$ where α is a loop factor. In both of these cases, identifying the gaugino masses with the TeV scale, the cosmological moduli problem is absent since the volume modulus would be at least as heavy as 10 TeV.

- For each of the two cases of the previous item, gravity mediated scalar masses, if not tachyonic, are of order $\frac{m_{\mathrm{P}}}{\mathcal{V}^{3/2}}$ and therefore hierarchically heavier than the gaugino masses, indicating a minor version of split supersymmetry [126, 163, 164]. However if we have perfect sequestering in the sense that physical Yukawa couplings do not depend on the Kähler moduli fields that break supersymmetry, such terms will cancel. However scalar masses will always receive a contribution from the dilaton F-term at order $\frac{m_{\mathrm{P}}}{\mathcal{V}^2}$.

- Since leading order gravity and anomaly mediation contributions to the soft terms are suppressed, then other effects have to be considered. In particular string loop corrections could be relevant, e. g. as in [94,95], (giving potential contributions to scalar masses of order $\frac{m_{\mathrm{P}}}{\mathcal{V}^{4/3}}$ [165]) but also a novel scenario may be conceived in which the main source of supersymmetry breaking for the observable sector is gauge mediation, however the gravitino mass remains very large and unlike previous models of gauge mediation, the LSP is no longer the gravitino but can be a more standard neutralino.

Even though there are several scenarios, we can still extract some general conclusions from this analysis. First, as emphasized in [166], the effects of the de Sitter uplifting play an important rôle on the soft breaking terms. This is unlike previous scenarios based on the LARGE volume in which they were negligible. Second, in all scenarios the gravitino mass is much heavier than the TeV scale $m_{3/2} \geq 10^8$ GeV which relaxes the cosmological problems associated to low-energy supersymmetry. Generically (except in the case that the dilaton is responsible for uplifting) the lightest modulus is heavier than the soft terms and therefore cosmologically harmless also.

There is an interesting conclusion we can draw concerning the overall volume of the compactification manifold: even though there are several cancellations that reduce the value of the volume to have the TeV scale, there is a minimum value that can be extracted from this analysis. Namely, the universal source of gaugino masses due to the dilaton dependence of the gauge kinetic function, implies that the gaugino masses cannot be smaller than $\frac{m_{\rm P}}{\mathcal{V}^2}$. The same limit appears for scalar masses for the case of perfect sequestering ($Z = e^{\mathcal{K}/3}$). This provides a bound for the size of the overall volume. In string units it is given by

$$\mathcal{V} \sim 10^6 - 10^7. \tag{6.63}$$

This corresponds to a string scale of order $m_{\rm s} \sim 10^{15}$ GeV. Combining this with the recent result [157] that in local models the GUT unification scale is given by $m_{\rm GUT} \sim m_{\rm s}\mathcal{V}^{1/6}$ this gives a unification scale of the same order as the one expected for supersymmetric GUT models from LEP precision results of

$$m_{\rm GUT} \sim 10^{16}\,\text{GeV}. \tag{6.64}$$

If this scenario is actually realized it would provide an example in which a string model addresses simultaneously the two positive properties of the MSSM, namely the full hierarchy problem, without tuning, and obtaining the preferred scale of gauge unification.

Interestingly, this value of the volume is also of the order of magnitude preferred by models of inflation in order for the inflaton to give rise to density perturbations of the right amplitude, normalized by COBE. In particular a volume $\mathcal{V} \sim 10^5 - 10^7$ was needed to achieve Kähler moduli inflation as we have seen in section 3.7. It also ameliorates the gravitino mass problem pointed out in [167, 168].

7. Summary and Discussion

Since its proposal in 2005, the LARGE volume scenario has shaped up as phenomenologically very interesting set-up for both, particle and astrophysics. The topic of this thesis was to explore the implications of recent developments in the D-brane instanton calculus, in particular the influence of charged zero modes and poly-instanton effects, to the LVS. On our path, we gained new insights into supersymmetry breaking in local GUT models or corresponding F-theory constructions.

Models in the context of the LARGE volume scenario are usually described in an effective supergravity formalism. In chapter 2 we reviewed in detail its origin, type IIB string theory. Starting with the world-sheet action of a single superstring, we explained how to extract the low-energy effective action and identified the relevant degrees of freedom. Moreover, we introduced all concepts appearing in the LVS including compactification to four space–time dimensions, orientifolding, fluxes, D-branes, instantons and moduli stabilization.

We further set the stage in chapter 3, where we showed that α'-corrections to the Kähler potential in a flux vacuum shift the minimum of the scalar potential for the Kähler moduli to exponentially large volume, the LARGE volume minimum. This scenario requires special geometrical properties of the compactification manifold. The class of appropriate spaces has been given the name "swiss cheese manifolds". A concrete example was studied on the hypersurface $\mathbb{P}^4_{[1,1,1,6,9]}$[18], which has been the only one so far in the literature. After showing that the new minimum is robust against quantum corrections, we presented some of the most prominent phenomenological virtues in particle physical and astrophysical applications.

In chapter 4, for the first time we brought together our knowledge about the D-brane instanton calculus and the LARGE volume scenario. We entered into the question of combining Kähler moduli stabilization by instantons resp. gaugino condensation with a chiral D7-brane sector carrying the unbroken chiral gauge theory which we would like to have in four dimensions. Clearly, in order to make progress in deriving viable and predictive string compactifications, this question is of utmost importance.

We argued quite generally, employing both string consistency conditions as well as phenomenological input, that for chiral D7-brane sectors only a combination of F- and D-terms can fix all Kähler moduli. Then we investigated whether the unquestionable nice features of the LARGE volume scenario can be preserved once these D-terms are taken into account. We showed that for more than one E3-instanton also the F-term scalar potential contains new terms containing the axionic fields, which potentially destabilize the LARGE volume minimum. Requiring these terms to be absent means that the instanton cycles should not intersect. Moreover, we also allowed for singular four-cycles, which homologically are linear combinations of the elementary ones. These also induce a different moduli dependence in the F-term scalar potential.

All these general arguments about F-and D-terms were exemplified by constructing a concrete type IIB model on a new swiss cheese type Calabi–Yau manifold with three Kähler moduli. Ignoring the details of the three-form flux sector, we constructed an example which showed all the features we do expect for a realistic model including a chiral intersecting D7-brane sector. Due to chirality there was an induced D-term, fixing (for vanishing vevs of matter fields) one combination of the Kähler moduli at the boundary of the Kähler cone. We had one rigid small cycle unoccupied by the D7-branes, so that a stringy $O(1)$ E3-instanton wrapped on this cycle contributed to the superpotential. Then the F- and D-terms together fixed the overall volume $\mathcal{V}$ at large values and the two diagonal small ones at size $\tau_i \simeq \log(\mathcal{V})$ in a such a way that another four-cycle collapsed.

This simple toy model can be considered as a proof of principle that the LVS can be robust enough that also chiral D7-brane sectors can be introduced. Of course, phenomenologically it was not satisfying yet: the gauge group and matter content is not realistic and the D3-brane tadpole constraint leaves probably not enough freedom to fix all complex structure moduli by three-form fluxes. Moreover, our analysis of the non-chiral zero modes was not complete. Presumably, these shortcoming only reflect the simplicity of the used Calabi–Yau space. Using Calabi–Yau manifolds with larger Euler characteristics will remedy these problems. To this end, it would be very important to know which of the toric Calabi–Yau manifolds in the list of [169] have a swiss cheese like structure, respectively can lead to large volume moduli stabilization. It might be technically very challenging but would be a major step forward to really build completely predictive concrete string compactifications with fluxes and intersecting D7-branes on such more involved Calabi–Yau manifolds.

In chapter 5, another application of a D-brane instanton effect in the context of the LARGE volume scenario was studied: we considered a compactification manifold of swiss cheese type where gaugino condensation on two stacks of D7-branes leads to a racetrack superpotential. Taking into account D-brane instanton corrections to the gauge kinetic function, manifesting themselves as poly-instanton corrections to the racetrack superpotential, we constructed a scenario featuring exponential control over the parameters W_0^{eff} and A^{eff} in an effective superpotential of the form $W^{\text{eff}} = W_0^{\text{eff}} - A^{\text{eff}} \exp(-aT)$. In contrast to the usual LVS (and also KKLT) set-up, these can be arranged for exponential small values *without* fine-tuning.

Within this set-up, we were able to find minima of the resulting scalar potential realizing supersymmetric GUT scenarios with the string scale at the GUT scale $m_s = m_X \simeq 1.2 \cdot 10^{16}$ GeV — in contrast to the usual LVS, where the string scale is at $m_s \simeq 10^{11}$ GeV. Also the gauge coupling took the usual GUT value of $\alpha^{-1} \simeq \alpha_X^{-1} \simeq 25$ and the gravitino mass was found to be in the TeV region without fine-tuning. However, despite the phenomenological interesting value of $m_{3/2}$, the soft terms in our first set-up were strongly suppressed. The reason was that the racetrack modulus is stabilized in a nearly supersymmetric minimum giving F-terms which are much smaller than expected. We proposed two resolutions of this issue and constructed the corresponding set-ups:

- First, we scaled our parameters such that effectively $W_0^{\rm eff}$ is scaled by a factor of 10^6 leading to a larger gravitino mass but also to larger soft masses. The gaugino masses are dominantly generated by anomaly mediation while the scalar masses are generated by gravity mediation leading to a stringy realization of split supersymmetry if the Higgs and Higgsino masses are tuned to small values.

- The second possibility we considered was to generate the soft terms not by the F-term of the racetrack modulus but also by the F-term of the small LVS Kähler modulus. The gaugino as well as the scalar masses are then generated by gravity mediation similarly to the original LARGE volume scenario. The lightest modulus was on the edge of posing problems with cosmology (CMP) and the μ-problem could be solved by the Giudice–Masiero mechanism.

We showed also that the exponential tuning of the effective superpotential parameters allows to construct a set-up with tree-level cosmological constant at the order of $\Lambda_0 \sim -10^{-120}\, m_{\rm P}^4$ which can be uplifted to a positive value without fine-tuning. However, although we obtained an encouraging value for Λ_0, introducing the standard model in this set-up will spoil this feature.

Clearly, a more detailed analysis of the phenomenological implications of these set-ups along the lines of [32, 33] would be very interesting. A drawback was however, that we did not account for the problem concerning the intrinsic tension between moduli stabilization via instantons and a chiral matter sector raised in chapter 4.

Inspired by the cancellations appearing in the soft terms in these set-ups, we studied in chapter 6 in great detail the structure of gravity mediated soft terms that arise when combining LARGE volume moduli stabilization with *local* GUT-like theories. The motivation for local models came from our considerations on the chirality problem in chapter 4: the stabilization of the cycle supporting the GUT branes via D-terms is such that the corresponding four-cycle volume is driven to zero size. A similar geometry can be be found in local F-theory models.

We found that the modulus determining the size of the standard model cycle does not break supersymmetry and therefore the scale of gravity mediated soft terms is highly suppressed compared to the gravitino mass. Both "standard" gravity mediated terms of $\mathcal{O}(m_{3/2})$ and also known anomaly mediated terms of $\mathcal{O}(g^2 m_{3/2}/16\pi^2)$ vanished. The first non-zero terms appear to arise at $\mathcal{O}(m_{3/2}/\sqrt{\mathcal{V}}) \simeq m_{3/2}^{3/2}/m_{\rm P}^{1/2}$. However it is possible that additional cancellations occur and suppress the soft terms even further than this down to $\mathcal{O}(m_{3/2}^2/m_{\rm P})$.

The appearance of these further cancellations is related to the (in)dependence of the physical Yukawa couplings on the fields breaking supersymmetry.

The cancellation of contributions to the soft masses of order $m_{3/2}$ introduces several subtleties. In particular, as the soft terms occur at a scale parametrically smaller than the gravitino mass, effects which are normally negligible become important. We have tried to include all known effects and have given general arguments as to when cancellations will take place. Nonetheless, it is important to look for any further possible contributions to soft terms which could possibly be dangerous. In this respect one would ideally like a direct stringy computation of soft terms that would bypass the need to go through the supergravity formalism.

Depending on the extent of cancellations in the soft terms, the volume modulus may or may not be subject to the cosmological moduli problem. We identified several possible scenarios with different sources for uplifting to de Sitter and considered also gauge mediation as supersymmetry breaking mediation mechanism. In all scenarios, the gravitino mass is as large as $m_{3/2} \geq 10^8$ GeV, relaxing the cosmological moduli problem for the light volume modulus in most cases. We were able to identify a minimal value for the overall volume of $\mathcal{V} \sim 10^6 - 10^7$ which is not only the range favored for moduli inflation but also implies that the unification scale is at the usual value of $m_{\rm GUT} \sim 10^{16}$ GeV.

We consider our results bring closer local string/F-theory models to honest-to-God string compactifications since we incorporate the main properties of such models regarding supersymmetry breaking and moduli stabilization. Many questions remain open. Concrete examples where the cancellations illustrated here are realized, including an uplifting term, loop corrections, etc. are desirable. The presence of such sub-leading contributions to soft terms can be recast in the presence of corrections to the physical Yukawa couplings. Specifically, the scale of the soft terms can be related to the extent to which the (local) physical Yukawa couplings depend on the (bulk) supersymmetry breaking fields. In the limit of perfect sequestering the Kähler moduli contribution to soft masses vanish. It may be possible to study this issue more precisely using the techniques of orbifold CFT. Furthermore, for F-theory constructions, even though in general they are treated in a way similar to orientifold constructions, the 4d effective field theory for F-theory models is less under control. In particular the α'-corrections which are crucial in the large volume scenario, need to be computed for F-theory compactifications.

It is fair to say that with the appearance of the LARGE volume scenario, the discipline of string phenomenology received new impetus. Today's state of the art techniques suffice already to extract from the few well motivated assumptions one has to make a lot of distinctive features of the models we considered, both, particle physical and cosmological ones. The prospects of discovering some of those in the near future are better then ever. Still, a common expressed criticism concerning string theory is that it does not produce any unique, testable predictions. However one should bear in mind that string theory is still in an early stage of development. The merits of this long-term research programme are only now starting to come to fruition.

A. Cohomology classes of line bundles

In this appendix we combinatorically compute the cohomology classes of general line bundles $\mathcal{L} = \mathcal{O}(m, n, l)$ over the resolution $\mathcal{M}$ of the ambient space $\mathbb{P}_{[1,3,3,3,5]}$. The corresponding classes on the hypersurface $\mathcal{X}$ can then be computed via the Koszul sequence.

$$0 \to \mathcal{L} \otimes \mathcal{O}(-15, -10, -5)_{\mathcal{M}} \to \mathcal{L}_{\mathcal{M}} \to \mathcal{L}_{\mathcal{X}} \to 0. \tag{A.1}$$

Let us recall the resolution

$$\begin{array}{ccccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\ \hline 3 & 3 & 3 & 5 & 1 & 0 & 0 \\ 2 & 2 & 2 & 3 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 1 \end{array} \tag{A.2}$$

Then the classes $H^i(\mathcal{M}, \mathcal{L})$ can be computed by counting monomials of degree (m, n, l) [170, 171] as listed in table A.1. This can be easily put on a computer. We have checked for many examples that the results are consistent with the Euler characteristic $\chi(\mathcal{X}, \mathcal{L})$ in eq. (4.35).

Cohomology	Monomials of degree (m, n, l)
$H^0(\mathcal{M}, \mathcal{L})$	$P(x_1, x_2, x_3, x_4, x_5, x_6, x_7)$
$H^1(\mathcal{M}, \mathcal{L})$	$\frac{P(x_1,x_2,x_3,x_6,x_7)}{x_4x_5Q(x_4,x_5)}$ $\frac{P(x_1,x_2,x_3,x_5,x_7)}{x_4x_6Q(x_4,x_6)}$ $\frac{P(x_1,x_2,x_3,x_4,x_6)}{x_5x_7Q(x_5,x_7)}$ $\frac{P(x_1,x_2,x_3,x_7)}{x_4x_5x_6Q(x_4,x_5,x_6)}$ $\frac{P(x_1,x_2,x_3,x_6)}{x_4x_5x_7Q(x_4,x_5,x_7)}$
$H^2(\mathcal{M}, \mathcal{L})$	0
$H^3(\mathcal{M}, \mathcal{L})$	$\frac{P(x_4,x_5)}{x_1x_2x_3x_6x_7Q(x_1,x_2,x_3,x_6,x_7)}$ $\frac{P(x_4,x_6)}{x_1x_2x_3x_5x_7Q(x_1,x_2,x_3,x_5,x_7)}$ $\frac{P(x_5,x_7)}{x_1x_2x_3x_4x_6Q(x_1,x_2,x_3,x_4,x_6)}$ $\frac{P(x_4,x_5,x_6)}{x_1x_2x_3x_7Q(x_1,x_2,x_3,x_7)}$ $\frac{P(x_4,x_5,x_7)}{x_1x_2x_3x_6Q(x_1,x_2,x_3,x_6)}$
$H^4(\mathcal{M}, \mathcal{L})$	$\frac{1}{x_1x_2x_3x_4x_5x_6x_7Q(x_1,x_2,x_3,x_4,x_5,x_6,x_7)}$

Table A.1.: Cohomology groups and corresponding monomials for $\mathbb{P}_{[1,3,3,3,5]}[15]$

B. The $\mathbb{P}_{[1,1,3,10,15]}[30]$ Calabi–Yau

Here we will briefly summarize some properties of the Calabi–Yau $\mathbb{P}_{[1,1,3,10,15]}[30]$ as another example of a swiss cheese like manifold. It has five Kähler moduli out of which four are toric. In the following we collect the toric data for the resolution of the toric singularities.

- The manifold is specified by the resolution

$$\begin{array}{cccccccc|c} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & p \\ \hline 15 & 10 & 3 & 1 & 1 & 0 & 0 & 0 & 30 \\ 5 & 3 & 1 & 0 & 0 & 1 & 0 & 0 & 10 \\ 3 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 6 \\ 6 & 4 & 1 & 0 & 0 & 0 & 0 & 1 & 12 \end{array} \tag{B.1}$$

- The Stanley–Reisner ideal reads

$$\begin{aligned} \text{SR} = \{&x_2x_7,\ x_2x_8,\ x_3x_8,\ x_1x_3x_6,\ x_1x_6x_7, \\ &x_1x_6x_8,\ x_2x_4x_5,\ x_3x_4x_5,\ x_4x_5x_7\}. \end{aligned} \tag{B.2}$$

- The triple triple intersection numbers in the basis $\eta_1 = D_5$, $\eta_2 = D_6$, $\eta_3 = D_7$, $\eta_4 = D_8$ are encoded in

$$\begin{aligned} I_3 = &- \eta_1^3 + 18\eta_2^3 + 8\eta_3^3 + 9\eta_4^3 + 2\eta_1^2\eta_2 + \eta_1^2\eta_4 - 6\eta_1\eta_2^2 \\ &- 2\eta_1\eta_3^2 + \eta_3^2\eta_4 - 3\eta_1\eta_4^2 - 3\eta_3\eta_4^2 + \eta_1\eta_3\eta_4. \end{aligned} \tag{B.3}$$

- If one expands the Kähler form in the basis $\{\eta_1, \eta_2, \eta_3, \eta_4\}$ as

$$J = t_1[\eta_1] + t_2[\eta_2] + t_3[\eta_3] + t_4[\eta_4], \tag{B.4}$$

  then the volumes of the basis divisors are

$$\begin{aligned} \tau_1 &= \frac{1}{2}\big(-t_1^2 + 4t_1t_2 - 6t_2^2 - 2t_3^2 + 2(t_1+t_3)t_4 - 3t_4^2\big), \\ \tau_2 &= (t_1 - 3t_2)^2, \\ \tau_3 &= \frac{1}{2}(-2t_1 + 4t_3 + 3t_4)(2t_3 - t_4), \\ \tau_4 &= \frac{1}{2}(t_1 + t_3 - 3t_4)^2. \end{aligned} \tag{B.5}$$

- The Kähler cone is found by imposing $\int_{\mathcal{C}} J > 0$ which gives the following conditions on the $\{t_i\}$

$$3t_2+t_3-3t_4 > 0, \qquad t_1-3t_2 > 0, \qquad t_4-t_3 > 0, \qquad -t_1+2t_2+t_4 > 0. \quad \text{(B.6)}$$

- Using these restrictions, the overall volume is expressed in terms of the four-cycle volumes as

$$\mathcal{V} = \frac{\sqrt{2}}{45}\Big((15\tau_1 + 5\tau_2 + 3\tau_3 + 6\tau_4)^{3/2} - (3\tau_3 + \tau_4)^{3/2} - \frac{5}{\sqrt{2}}\tau_2^{3/2} - 5\tau_4^{3/2}\Big). \quad \text{(B.7)}$$

From this we see that by making τ_1 large while keeping the others small, we obtain a swiss cheese like structure.

- The Euler characteristic χ for the cycle $D = m\eta_1 + n\eta_2 + p\eta_3 + q\eta_4$ is

$$\begin{aligned}\chi(\mathcal{X}, \mathcal{O}_D) = &- 3mn^2 + \tfrac{3}{2}q^3 + 3n^3 - \tfrac{1}{6}m^3 + \tfrac{1}{2}p^2q + mpq - \tfrac{3}{2}pq^2mp^2 \\ &- \tfrac{3}{2}mq^2 + \tfrac{4}{3}p^3 + \tfrac{1}{2}m^2q + m^2n - n - \tfrac{1}{3}p - \tfrac{1}{2}q + \tfrac{13}{6}m,\end{aligned} \quad \text{(B.8)}$$

where $\mathcal{X}$ stands for $\mathbb{P}_{[1,1,3,10,15]}[30]$. The interesting combinations for the present set-up are those with $\chi = 1$ and $m = 0$. Up to wrapping numbers 100, these are

$$(m, n, p, q) = (0, 0, 0, 1),\ (0, 0, 1, 0),\ (0, 0, 1, 1). \quad \text{(B.9)}$$

- It is more convenient to work in a diagonal basis which we define guided by the form of the volume (B.7)

$$\begin{aligned} D_a &= 15D_1 + 5D_2 + 3D_3 + 6D_4, & D_b &= 3D_3 + D_4, \\ D_c &= D_2, & D_d &= D_4. \end{aligned} \quad \text{(B.10)}$$

In this basis the total volume reads

$$\mathcal{V} = \frac{\sqrt{2}}{45}\Big(\tau_a^{3/2} - \tau_b^{3/2} - \frac{5}{\sqrt{2}}\tau_c^{3/2} - 5\tau_d^{3/2}\Big), \quad \text{(B.11)}$$

and the triple intersection numbers again diagonalize

$$I_3 = 225D_a^3 + 225D_b^3 + 18D_c^3 + 9D_d^3. \quad \text{(B.12)}$$

- Expanding also the Kähler form in this diagonal basis as $J = t_a[D_a] - t_b[D_b] - t_c[D_c] -$

$t_d[D_d]$, we find that the Kähler cone is defined by

$$5t_b > t_d > t_c > 0, \qquad t_a > t_b + 2t_c + t_d. \tag{B.13}$$

- We finally present a list of monomials to be counted in order to determine the cohomology classes $H^i(\mathcal{M}, \mathcal{L})$ on the ambient toric variety. We use the shorthand notation $(1, 2, 4, 5, 7, 8|3, 6)$ for all monomials of the form $\frac{P(x_1,x_2,x_4,x_5,x_7,x_8)}{x_3x_6Q(x_3,x_6)}$ and similarly for the others.

Cohomology	Monomials of degree (m, n, p, q)
$H^0(\mathcal{M}, \mathcal{L})$	$(1, 2, 3, 4, 5, 6, 7, 8\|)$
$H^1(\mathcal{M}, \mathcal{L})$	$(1, 2, 4, 5, 7, 8\|3, 6)$ $(1, 2, 4, 5, 6, 8\|3, 7)$ $(1, 2, 3, 4, 5, 7\|6, 8)$ $(1, 2, 4, 5, 8\|3, 6, 7)$ $(1, 2, 4, 5, 7\|3, 6, 8)$
$H^2(\mathcal{M}, \mathcal{L})$	$(3, 4, 5, 6, 8\|1, 2, 7)$ $(3, 4, 5, 6, 7\|1, 2, 8)$ $(2, 3, 4, 5, 8\|1, 6, 7)$ $(1, 3, 6, 7, 8\|2, 4, 5)$ $(1, 2, 6, 7, 8\|3, 4, 5)$ $(1, 2, 3, 6, 7\|4, 5, 8)$ $(1, 2, 7, 8\|3, 4, 5, 6)$ $(1, 2, 6, 7\|3, 4, 5, 8)$ $(2, 3, 4, 5\|1, 6, 7, 8)$ $(2, 4, 5, 8\|1, 3, 6, 7)$ $(1, 2, 3, 7\|4, 5, 6, 8)$ $(1, 2, 6, 8\|3, 4, 5, 7)$ $(3, 4, 5, 6\|1, 2, 7, 8)$ $(3, 4, 5, 8\|1, 2, 6, 7)$ $(1, 6, 7, 8\|2, 3, 4, 5)$ $(1, 3, 6, 7\|2, 4, 5, 8)$ $(4, 5, 6, 8\|1, 2, 3, 7)$ $(3, 4, 5, 7\|1, 2, 6, 8)$ $(1, 2, 7\|3, 4, 5, 6, 8)$ $(1, 2, 8\|3, 4, 5, 6, 7)$ $(1, 6, 7\|2, 3, 4, 5, 8)$ $(2, 4, 5\|1, 3, 6, 7, 8)$ $(3, 4, 5\|1, 2, 6, 7, 8)$ $(4, 5, 8\|1, 2, 3, 6, 7)$
$H^3(\mathcal{M}, \mathcal{L})$	$(3, 6\|1, 2, 4, 5, 7, 8)$ $(3, 7\|1, 2, 4, 5, 6, 8)$ $(6, 8\|1, 2, 3, 4, 5, 7)$ $(3, 6, 7\|1, 2, 4, 5, 8)$ $(3, 6, 8\|1, 2, 4, 5, 7)$
$H^4(\mathcal{M}, \mathcal{L})$	$(\|1, 2, 3, 4, 5, 6, 7, 8)$

Table B.1.: Cohomology groups and corresponding monomials for $\mathbb{P}_{[1,1,3,10,15]}[30]$

C. Vanishing D-terms Including Matter

We will consider in this appendix a concrete example with a generic D-term including not only the field dependent FI-term but also a charged matter field. In general vanishing D-terms do not imply vanishing FI-term but a cancellation between the two terms entering the D-term potential. We argue here (following [172]) that once soft supersymmetry breaking terms are included, as long as the square of the scalar masses is positive the minimum of the scalar potential is for vanishing both matter field vev and FI-term.

Since in local models the standard model cycle is a del Pezzo surface that can and usually prefers to shrink to small size, it is dangerous to work in the regime where the cycle size is larger than the string scale. Even though at sizes of the order of the string scale the spectrum and couplings of the model are not understood, the regime close to a del Pezzo singularity is under a much better control, the spectrum is determined by the extended quiver diagrams and the low-energy effective theory can be reliably used in an expansion in the small blow-up mode.

This effective field theory has been recently discussed in [147]. We start with the same background geometry as before including one large τ_1 and two small cycles τ_2, τ_3. On the rigid cycle τ_2 we have the standard non-perturbative effect. Being at the singular locus for τ_3, the effective field theory can be approximated by the following supergravity set-up:

$$\begin{aligned} \mathcal{K} &= -2\log\left(\mathcal{V}+\tfrac{\hat{\xi}}{2}\right)+\frac{\alpha\tau_3^2}{\mathcal{V}}+Z|\varphi|^2, \\ W &= W_0+Ae^{-aT_2}, \\ f &= dT_3+S, \end{aligned} \tag{C.1}$$

where φ denotes a matter field that is charged under an anomalous $U(1)$ on the standard model cycle, as is the cycle volume itself. As discussed in [147], the effective theory for τ_3 differs from the standard treatment for relatively large values of τ_3 since we are working close to the singularity. The anomalous $U(1)$ generates a D-term potential with a Fayet–Iliopoulos term:

$$V_D = \frac{1}{2(d\tau_3+s)}\left(Q_\varphi Z|\varphi|^2+\frac{Q_{\tau_3}\tau_3}{\mathcal{V}}\right)^2. \tag{C.2}$$

The matter metric Z is taken to have the general form

$$Z = \frac{1}{\mathcal{V}^{2/3}}\left(\beta+\gamma\tau_3^\lambda-\frac{\delta}{\mathcal{V}}\right), \tag{C.3}$$

where the constants β, δ can in principle depend on the dilaton and complex structure moduli.

The D-term potential determines the size of τ_3 and implies

$$\tau_3 \sim |\varphi|^2 \mathcal{V}^{1/3}. \tag{C.4}$$

For a vanishing vev of φ this implies as previously $\tau_3 = 0$. Expanding around $\varphi = 0$, the scalar potential is given by the standard LARGE volume potential and at next-to-leading order by a contribution quadratic in φ:

$$\begin{aligned} V = &\frac{1}{\left(\mathcal{V}+\frac{\xi}{2}\right)^2}\Bigg(\frac{8}{3}|aA|^2\sqrt{\tau_2}\tau_1^{3/2}e^{-2a\tau_2} - 4W_0aA\tau_2e^{-a\tau_2} + \frac{W_0^2 3\xi}{4\tau_1^{3/2}} + Y \\ &- \frac{\beta|\varphi|^2}{3\tau_1}\left(\frac{8}{3}|aA|^2\sqrt{\tau_2}\tau_1^{3/2}e^{-2a\tau_2} - 4W_0aA\tau_2e^{-a\tau_2} + \frac{9W_0^2(5\frac{\delta}{\beta}+2\xi)}{4\tau_1^{3/2}}\right)\Bigg) \\ &+ \frac{\beta|\varphi|^2}{\tau_1\left(\mathcal{V}+\frac{\xi}{2}\right)^2}\left(\frac{8}{3}|aA|^2\sqrt{\tau_2}\tau_1^{3/2}e^{-2a\tau_2} - 4W_0aA\tau_2e^{-a\tau_2} + \frac{3W_0^2\xi}{4\tau_1^{3/2}} + Y\right), \end{aligned} \tag{C.5}$$

where the last term arises from the expansion of $e^{\mathcal{K}}$ and Y denotes the F-term uplifting term, which allows for a stabilization at zero vacuum energy.

With zero vacuum energy, the mass of φ is given by

$$\begin{aligned} m_\varphi^2 &= \mathcal{K}_{\varphi\bar{\varphi}}^{-1}\frac{-\beta}{3\tau_1^4}\left(\frac{8}{3}|aA|^2\sqrt{\tau_2}\tau_1^{3/2}e^{-2a\tau_2} - 4W_0aA\tau_2e^{-a\tau_2} + \frac{9W_0^2\left(2\xi-5\frac{\delta}{\beta}\right)}{4\tau_1^{3/2}}\right) \\ &= -\frac{1}{3\tau_1^3}\left(V_{\text{min}} + \frac{45W_0^2\left(\frac{\xi}{3}-\frac{\delta}{\beta}\right)}{4\tau_1^{3/2}}\right) \approx \frac{15W_0^2\left(\frac{\delta}{\beta}-\frac{\xi}{3}\right)}{4\tau_1^{9/2}}. \end{aligned} \tag{C.6}$$

Different ratios of δ/β allow for tachyonic, zero or positive masses at this order. In particular:

$$\frac{\delta}{\beta}\begin{cases} < \frac{\xi}{3} & \text{tachyonic,} \\ = \frac{\xi}{3} & \text{zero,} \\ > \frac{\xi}{3} & \text{positive.} \end{cases} \tag{C.7}$$

With respect to the matter metric the condition $\frac{\delta}{\beta} = \frac{\xi}{3}$ can be understood as follows: The case of vanishing masses corresponds to the following matter metric:

$$Z = \frac{\beta}{\mathcal{V}^{2/3}}\left(1 - \frac{\xi}{3\mathcal{V}}\right) \approx \frac{\beta}{\left(\mathcal{V}+\frac{\xi}{2}\right)^{2/3}} = \beta e^{\mathcal{K}/3}, \tag{C.8}$$

which is the condition found in section 6.2.3 for extreme sequestering and cancellation of scalar masses at the $1/\mathcal{V}^{3/2}$ level. Without the uplifting term, the effect of the term arising from the expansion of $e^{\mathcal{K}}$ is generally sub-leading to the other contribution since it is suppressed with $1/a\tau_2$.

For positive scalar masses we can clearly see that combining the term $m_\varphi^2\varphi^2$ with the D-term potential, both the vev of φ and the FI-term vanish at the minimum as desired. For the tachyonic case this would indicate as usual that at the minimum the scalar field and the FI-term would be non-vanishing. If φ is a field charged under the standard model gauge group this is undesirable since it would break the standard model symmetries at high energies. If the condition $\frac{\delta}{\beta} = \frac{\xi}{3}$ is satisfied the positivity of the squared scalar masses at a higher order would have to be determined.

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