ECONOMIC POLICY AND THE

HETEROGENEOUS FIRM

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The accelerating development of the world economy during the last decades has challenged government economic policy. Heterogeneous firms participating in global production are stimulated to select sophisticated integration strategies to exploit profit opportunities. Among others, the best possible answers to political decisions motivate firms to modify their integration status dynamically. Considerations and tasks of policy makers are multifaceted, including the anticipation of the behavior of heterogeneous firms, the choice of policy instrument, the design of optimal trade policy, and interaction with other jurisdictions.

The incorporation of heterogeneity in firm productivity into models of monopolistic competition with international trade and multinational firms is a recent innovation in the trade literature. The first models of horizontal or vertical integration strategies of multinational firms were developed assuming homogeneous productivities between all plants in a market.¹ This was followed by theoretical work focused on the study of optimal integration strategies of complex firms in the presence of firm heterogeneity in terms of total factor productivity.² One main insight was that the integration strategy of a firm is dependent on its productivity level. Given productivity differences across firms, alternative integration modes coexisted, driven by the notion of firm heterogeneity.

Starting in 1995, the body of empirical literature has included evidence that firms serving the domestic market are less productive than firms trading internationally.³ Empirical studies have indicated that exporters and multinational enterprises (MNEs) are larger and more productive and differ in input characteristics compared with domestic firms.⁴ Additionally, the activity of MNEs has been among the most dynamic economic activities, followed by international trade in goods and services.⁵

Intuitively, the degree to which firm heterogeneity is present in the jurisdiction of a government not only determines the distribution of firms in terms of integration strategies but also influences policy outcomes. In this context, governments anticipate the behavior of heterogeneous firms that are motivated to modify their

As in Markusen (1984) and Helpman (1984).

² Compare with Helpman, Melitz and Yeaple (2004).

³ For a survey, see Bernard, Jensen and Lawrence (1995).

⁴ As in Barba Navaretti et al. (2003); and Criscuolo and Martin (2003); and Clerides, Lach and Tybout

^{(1998).} ⁵ In 2006 global, FDI inflows grew for the third consecutive year and reached the level of \$1.306 trillion being slightly below the record level of \$1.411 trillion in 2000, as in UNCTAD (2008) and World Bank Institute (2007).

integration status dynamically, depending on political decisions. The increasing mobility of firms and their potential option to modify their integration status influence government policy outcomes. The efficiency of trade policy is influenced by the possibilities of firms relocating their production, shifting profits, bypassing trade barriers by engaging in a multinational strategy, and exiting the market. Thus, governments are challenged by multifarious considerations to be incorporated in the design of optimal trade policy.

They are confronted with the task of identifying optimal trade policy instruments and developing welfare-maximizing economic environments using these instruments.

This thesis is an analysis of the implications of ad-valorem tariffs, on the one hand, and profit taxation, on the other hand. The optimality and efficiency of policy instruments mainly depend on the preferences of households, the presence of firm heterogeneity, and the ability of firms to modify their integration status.

Furthermore, the cooperation or noncooperation of governments with each other is central to explaining policy outcomes. Whereas a social planner maximizes welfare considering all countries (i.e., welfare maximization from a world welfare perspective), single governments behave noncooperatively, without policy coordination. Noncooperative behavior implies that countries find best responses to the policies of their neighbour countries to increase their own welfare at the expense of the other countries. The potential gain of single governments may be the potential loss of households in the other countries. Therefore, international policy coordination has entitlement in avoiding, for example, a prisoner's dilemma outcome in policy competition.⁶ Empirical work has resulted in support for this view in context with tariff setting. In their study, the authors have found evidence that the United States has set higher tariffs on goods when no constraints from the World Trade Organization (WTO) exist, which explains the coordinating role of that organization.⁷

This thesis contains three sections, all based on a similar, numerically solvable, model.

In section 1 and 2, the model is focused on optimal trade policy of benevolent governments that maximize welfare under the optimal choice of ad-valorem tariff

⁶ Compare with Nash (1951).

⁷ As in Broda et al. (2008). In contrast, see Rose (2004), who does not find significant reduction in tariffs due to WTO membership.

rates. Section 3 focuses on withholding profit tax rates levied on MNE profits. Both benevolent governments provide lump-sum transfers to households in their own jurisdictions when levying tariffs on imports or taxes on MNE profits.

We have set up a model that consists of two countries, A and B, in which only one factor, labor (L), is used for production and firm or plant setup. Households in A and B share the same love-for-variety preferences and benefit from consumption of homogeneous goods and differentiated goods. The homogeneous good is supplied under perfect competition. We have focused on parameter configurations that ensure diversification of production so that the homogeneous good is produced in both countries in equilibrium and may be traded at zero costs across national borders. Heterogeneous firms in the differentiated sector produce under monopolistic competition.

Profit maximization results in different integration strategies, depending on the individual productivity of the firms (i.e., domestic production to serve domestic consumers only, producing in one country and serving consumers in both countries from there (exporting), or engaging in multiplant production and serving consumers locally through domestic and foreign subsidiaries [MNEs]). Furthermore, the mass of firms that enter the market in equilibrium depends on the level of ad-valorem tariff or tax rate. Hence, the mass of firms in equilibrium is determined endogenously. This endogeneity results in corresponding endogenous market sizes.

In section 1, Ad-Valorem Tariff and the Heterogeneous Firm, we have focused on welfare maximization of social planners that endogenously determine the ad-valorem tariff rates of the two countries τ .⁸ In this first section, neither governments has the possibility of optimally reacting to the policy of the neighboring country.

Depending on the tariff rate, not only MNEs and exporters decide on entering the market depending on the tariff. Trade policy also influences the mass of domestic firms. Given a certain tariff rate, the composition of prevailing integration strategies is due to the constitution of competition.

In this context, the emphasis in the empirical work has been that cuts in tariffs by the United States and Canada induce a stronger export orientation in some of the

⁸ For seminal contributions, see Torrens (1933), Mill (1948), and Bickerdike (1907).

Canadian affiliates of U.S. parent firms.⁹ Further empirical work has revealed an emphasis on countries setting tariffs according to their market power.¹⁰ For example. an increasing tariff rate induces fewer exporters to enter the market and, depending on the size of fixed costs f_i, may also cause them to refrain from becoming MNEs. Consequently, fewer firms supply demand in this country and expected profits increase. Therefore, the output of each single firm is influenced; and more domestic firms can enter the market, competing expected profits to zero.¹¹ Results of numerical analysis show that social planners maximize the welfare of households, determining a free trade scenario. This result is optimal from a world welfare perspective (i.e., welfare of both countries is maximized).

Section 2, Best-Response Tariffs with Endogenous Market Size and Economic Integration, extends the analysis of section 1, incorporating transport costs. We have determined optimal tariff rates set by benevolent planners (i.e., when countries behave cooperatively) and contrasted them to optimal best-response tariff rates (i.e., when countries behave noncooperatively). As in the previous section social planners determine free trade scenarios to be optimal. In a noncooperative setting, a government has a unilateral incentive to deviate from a free trade scenario. This behavior can be anticipated by the other government; therefore, both deviate from zero tariff scenarios. This results in inefficiently high tariff rates, which are stable Nash equilibria. These Nash equilibria are characterized by lower welfare for both countries than in the social planner's scenario without tariffs. The welfare-superior free trade scenario can only be obtained under reliable policy coordination. Hence, section 2 provides rationale for the existence of ad-valorem tariffs in a model of heterogeneous firms.

In contrast to previous chapters, section three, Best-Response Tax Rates on Profits of Multinational Firms: A Numerical Approach, is a study of the implications of an alternative policy instrument. We have analyzed taxes on profits of MNEs. Empirical work indicates that FDIs react sensitively to variations in tax rates. In his work, Hines determines the elasticity of FDIs subject to taxes to be -0.6.¹² Further work has

⁹ As in Feinberg and Keane (2001).

¹⁰ For a survey, see Broda et al. (2008). ¹¹ See Davies, Egger and Egger (2009).

¹² Compare with Hines (1999).

indicated support for Hines but with weaker reactions according to taxes.¹³ Social welfare-maximizing governments levy withholding taxes on MNE profits earned by subsidiaries producing in their jurisdictions. The generated tax revenue is spent for lump-sum transfers to the households in their countries.

As in section 2, we have distinguished between the perspectives of social planners and single governments. Additionally, we have derived optimal tax rates with identical country sizes and contrasted our results to the outcome of the model assuming marginally differing country sizes. Furthermore, in this section, we have pointed to competition brought to single governments that maximizes welfare, levying withholding taxes on MNE profits. However, this model does not join standard tax competition models. Because governments levy withholding taxes on MNE profits earned in their jurisdictions, they do not compete for the same tax base. Still, the selected tax rate of a foreign government influences welfare of the representative household in the home country, inducing this government to react with another tax rate (i.e., best-response withholding tax rate). This is because the integration strategies chosen by firms are influenced by the tax rate of the other country. Empirical work has shown support for an increase in withholding tax rates inducing a decline of MNE investments in this jurisdiction.¹⁴ This coherence is consistent with the findings of Hines (1999) or Devereux and Griffith (2003).¹⁵

Results of numerical analysis show that a social planner's cooperative approach maximizes world welfare, resulting in efficient tax rates. Welfare maximization of a single government (i.e., governments behave noncooperatively), results in inefficient, high tax rates in equilibrium. Because the social planner's tax rates are unstable in a noncooperative setting, both governments have a unilateral incentive to deviate from this efficient equilibrium. We have show that coordination of governmental decisions helps to avoid a prisoner's dilemma and results in efficient tax rates in equilibrium.¹⁶

The following three sections are studies of the outlined topics of economic policy and the intentions of government welfare maximization and contain the author's

¹³ As in Devereux and Griffith (2003). In contrast, older literature indicated negligible effects from tax policies on FDI. See Brainard (1997); and Wheeler and Mody (1992) for a survey. For further work in this context see Grubert and Mutti (1991); Maskus (1998); Blonigen and Davies (2000); and in Egger, Egger and Greenaway (2008).

¹⁴ Evidence for this can be found in Devereux (2006) and in Hines and Rice (1994).

¹⁵ Early empirical work finds negligible effects of tax policies on FDI. See Brainard (1997) and Wheeler and Mody (1992) for a survey.

¹⁶ As in Nash (1951).

contribution to this lively debate on trade policy.¹⁷ These sections may also be read independently of the others.

¹⁷ The last chapter three is based on joint work with Julia Lichtenberg.

Chapter 1

AD-VALOREM TARIFF AND THE HETEROGENEOUS FIRM

1.1 Introduction

The idea that countries can profit from protection has a long tradition. In this context, the concept of optimal tariff setting is built on the argument that a tariff results in production and consumption distortions. However, it also results in terms-of-trade benefits, depending on the market power of importers.¹⁸

Observed empirically, applied tariff levels show variations among different groups of countries. Generally, tariffs decrease with an increasing degree of industrialization of countries (e.g., developed countries effectively apply 2.1% on imports from the world, whereas developing countries apply 4.9%).¹⁹ Due to this fact, the purpose of this paper is to answer the question of whether tariffs undermine the idea of global free trade or if there is evidence of positive welfare implications.

To meet the requirements of increasing economic integration, we have put emphasis on recent innovations in the trade literature of incorporating heterogeneity of firmproductivity into models of monopolistic competition with international trade and multinational firms.²⁰ In this context, the studies in theoretical work are focused on optimal integration strategies of complex integrated firms in the presence of firm heterogeneity in terms of total factor productivity.²¹ Firm heterogeneity appears in various layers, such as productivity, size, and integration status.²² One key finding is that differences in productivity levels across firms often result in a variety of optimal integration strategies, which result in domestic production, exporting operations, and multinational activities being elements of economic trading activities.²³ Empirical work indicates that the activity of multinational enterprises is among the most dynamic economic activities, followed by international trade in goods and services.²⁴ The

¹⁸ For a survey of seminal contributions, see Torrens (1833), Mill (1844), and Johnson (1954). Latest literature, as in Broda, Limão and Weinstein (2008), are empirical studies of the coherence of market power and tariff setting.

¹⁹ As in UNCTAD (2007a).

²⁰ Initially, models of vertical or horizontal integration strategies of multinational firms were developed under the assumption of homogeneous productivities between all plants in a market. For a survey, see Markusen (1984) and Helpman (1984).

²¹ Compare with Helpman, Melitz and Yeaple (2004).

²² As in Clerides, Lach and Tybout (1998).

²³ As in Bernard et al. (2007).

²⁴ In 2006, global FDI inflows grew for the third consecutive year and reached the level of \$1.306 trillion, slightly below the record level of \$1.411 trillion in 2000. As in UNCTAD (2008) and World Bank Institute (2007).

average annual growth rate of foreign affiliate sales, for instance, was 8.4% during the period 1996-2000 and was even 16.2% in 2006.²⁵

To incorporate the outlined topics of optimal tariff setting, firm heterogeneity, and the increasing importance of MNEs, we set up a numerically solvable model of heterogeneous firms that select their optimal integration strategies from a menu of three options: domestic operation, exporting operation, or horizontal MNE activities.²⁶ Empirical analysis of integration strategies of multinational enterprises (MNEs) indicates support for this approach through indirect evidence that is mainly in favor of horizontal MNE models in contrast to vertical MNE models.²⁷ We assume that firms in the manufacturing sector supply a variety of differentiated goods under monopolistic competition.

In our model, benevolent policy makers cooperatively maximize welfare of two symmetric countries by endogenously selecting an optimal tariff rate (i.e., a social planner maximizes welfare). The generated tariff revenue is spent on a lump-sum transfer to the households in the jurisdictions of the respective governments. Furthermore, the integration strategies that heterogeneous firms select as optimal are affected by the tariff rate. Not only increasing fixed costs, such as market entry costs, but also increasing tariff rates induce exporters to leave the market or, depending on their productivity levels, to become horizontal MNEs. Emphasized in the empirical work is that cuts in tariffs by the United States and Canada induce stronger export orientations in some Canadian affiliates of U.S. parent firms.²⁸ Further empirical evidence reveals confirmation that the decision of firms to export is dependent on the market-entry cost and plant heterogeneity.²⁹

Another objective of this paper is to highlight the role of heterogeneity in monopolistic competition trade models. The question is whether firm heterogeneity is an inevitable feature or whether the assumption of homogeneous firms is a sufficient determinant

²⁵ In the same time, the gross product of foreign affiliates increased 7.3% p.a. in the years 1996-2000 and rose by 16.2% in 2006. Exports of foreign affiliates showed an increase of 3.3% p.a. in 1996-2000 and rose by 12.2% in 2006. As in UNCTAD (2008).

²⁶ In contrast to Davies and Eckel (2007), assuming mobile firms; and in contrast to Jørgensen and Schröder (2007a) and Jørgensen and Schröder (2007b), not focusing on utility maximization and monopolistic competition.

²⁷ As supported by Markusen and Maskus (2001) and Brainard (1993a).

 $^{^{28}}$ As in Feinberg and Keane (2001).

²⁹ As in Bernard and Jensen (2004).

for welfare-maximizing tariff rates. For this reason, we ease the assumption that productivity follows a distribution function, and, instead examine the model with homogeneous firms.

The remainder of this paper is structured as follows. Section 1.2 describes the setup of the model and explicitly introduces the preferences of the consumers and the resulting demand in section 1.2.1. Section 1.2.2 presents the production process of heterogeneous firms through an explanation of relevant production parameters for the reader. Section 1.3 describes the behavior of firms and their respective integration strategies. Section 1.3.1 shows the strategy allocation of firms with $\tau = 0$; section 1.3.2 determines the integration strategies of firms with $\tau > 0$. Section 1.3.3 shows the effects of the tariff on the mass of firms and on the available different varieties in each country. Section 1.3.4 is an analysis of the extensive border effects that a tariff has on the export sector. Section 1.4 addresses the aspects of welfare maximization. After showing the effects through the introduction of an ad-valorem tariff, we maximize the utility of the consumers with respect to τ using a numerical approach, in section 1.5. The results are presented in chapter 1.6. In section 1.7, we flesh out the role of heterogeneity in monopolistic competition trade models. Finally, in section 1.8, we conclude the results and discuss the findings.

1.2 The setup of the model

1.2.1 Demand

In this model, we use a quasi-linear approach to reflect consumers' preferences.

Because all consumers share the same preferences, a representative consumer is used to clarify utility. The utility function is represented by:

$$U_{j} = X_{0} + \frac{1}{\mu} \left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} X_{j}(i)^{\alpha} di \right)^{\mu}, j \in \{A, B\}$$
(1)

The representative household in A and B benefits from consumption of the homogeneous good x_o , which is taken as the numéraire for convenience. Each of the two countries hosts a second industry that produces differentiated goods under monopolistic competition. $x_j(i)$ is the consumption of output of the i-th firm, which is $i \in \{0, ..., \theta_{max}\}$. The condition $0 < \alpha < 1$ being constant results in a constant elasticity of substitution (C.E.S.) of $\sigma = 1/(1-\alpha) > 1$ between any pair of differentiated goods. This

expression reflects standard properties of love-for-variety preferences in which a richer supply of differentiated goods results in increased utility. μ is a constant with $0 < \mu < \alpha < 1$ and reflects the preference for the differentiated industry over the homogenous industry in the utility function of the representative household. At a certain level of differentiated products supplied in one country, an additional unit shows diminishing marginal utility. The consumption of differentiated products is

represented by the expression $X = \left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} x_{j}(i)^{\alpha} di \right)$, the subutility of the differentiated

sector. Obviously, the utility function is linear in x_0 , but nonlinear in the differentiated varieties. This implies that the demand for differentiated products depends on prices of differentiated goods but not on earnings. Consumers of different countries show the same love-for-variety preferences and, therefore, apply the same elasticity of substitution σ .

To derive the demand for variety $x_j(i)$ of a single household in country j, we consider the utility function in (1) and satisfy the standard side condition $m_j \ge p_0 \cdot x_0 + \int_0^{\theta_{max}} p_j(i) \cdot x_j(i)$. Labor income m is spent on the homogeneous goods, where we set $p_0 = 1$, and on differentiated goods. This results in the demand of a single household for differentiated goods of³⁰

$$\mathbf{x}_{j}(\mathbf{i}) = \frac{1}{\left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} \mathbf{x}_{j}(\mathbf{i})^{\alpha} d\mathbf{i}\right)^{\frac{1-\mu}{1-\alpha}}} \mathbf{p}_{j}(\mathbf{i})^{\frac{1}{1-\alpha}}$$
(2)

or

$$p_{j}(\mathbf{i}) = \frac{1}{\left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} \mathbf{x}_{j}(\mathbf{i})^{\alpha} d\mathbf{i}\right)^{1-\mu}} \mathbf{x}_{j}(\mathbf{i})^{1-\alpha}}.$$
(3)

The demand of a single household in country j for differentiated goods of the i-th firm depends on the price that firm i sets, on the substitutability of any pair of differentiated goods for another through α , on μ , and on the subutility of consumption

³⁰ See Appendix 1.9.1.

$$X = \left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} x_{j}(i)^{\alpha} di\right).$$
 The impact of an increasing α is that products in the

differentiated sector become closer substitutes for one another, which reduces the market power of a single firm. As μ increases, the benefit of differentiated goods decreases. The marginal utility of a further unit of differentiated goods becomes smaller. An increasing X reduces the distribution of the single firms as competition between the firms intensifies. As can be seen from equations (2) and (3) the size of X is determined endogenously. For this reason, X can also be interpreted as the market size for differentiated goods and demands for specification. X depends on the strategic alignment of heterogeneous firms.

Therefore, we distinguish between different scenarios. In the first case, market size X consists of the market of domestic firms, foreign firms exporting their goods from abroad (henceforth referred to as exporters), and, firms choosing horizontal MNE activity. Then, market size X in equilibrium is defined as:

$$X_{d,ex,i} = \underbrace{\left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} x_{j}(i)^{\alpha} di\right)}_{\text{domestic (d)}} + \underbrace{\left(\int_{\theta_{d/ex}}^{\theta_{ex/i}} \frac{1}{\alpha} x_{j}(i)^{\alpha} di\right)}_{\text{exp orter (ex)}} + \underbrace{\left(\int_{\theta_{ex/i}}^{\theta_{max}} \frac{1}{\alpha} x_{j}(i)^{\alpha} di\right)}_{\text{MNE (i)}}$$
(4)

Alternatively, the export strategy does not exist (i.e., is not profitable); and firms choose either supplying domestically or acting as MNEs. This specified market size in this scenario shows:

$$X_{d,i} = \underbrace{\left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} x_{j}(i)^{\alpha} di\right)}_{\text{domestic}} + \underbrace{\left(\int_{\theta_{d/i}}^{\theta_{max}} \frac{1}{\alpha} x_{j}(i)^{\alpha} di\right)}_{\text{MNE}}$$
(5)

Finally, MNE activity may be nonprofitable so that market size consists of demand from domestic and exporting producers only. The specified market size in this case shows:

$$X_{d,ex} = \underbrace{\left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} x_{j}(i)^{\alpha} di\right)}_{domestic} + \underbrace{\left(\int_{\theta_{d/ex}}^{\theta_{max}} \frac{1}{\alpha} x_{j}(i)^{\alpha} di\right)}_{exporter}$$
(6)

Figure 1 is a visualization of market size under alternative integration strategies of heterogeneous firms.



Figure 1. The market size X, depending on different integration strategies.

1.2.2 Production

Each of the two symmetric small countries, $j=\{A,B\}$, hosts two industries.³¹ The subscript j is the identifier for the country in which the economic activity takes place. We focus on equilibria with diversification of production so that each of the two countries, $j=\{A,B\}$, hosts the two industries. One industry provides a homogeneous good x_0 that is traded under competitive conditions and is the numéraire in this model. Firms in the other industry produce differentiated goods under monopolistic competition.

Let us assume that Countries A and B are endowed with a fixed amount of internationally immobile labor, L. Because the homogeneous good is freely tradable, used as the numéraire, and uses one unit of L for one unit of output, there is international wage equalization at unitary wages (i.e., $w_j=1$) as long as diversification of production prevails.

Firms are located symmetrically in each country and produce under monopolistic competition. Hence, each firm in the differentiated sector produces a single variety of a certain good. The differentiated goods available in a country j are provided by different sources. Consumers in j buy goods produced by national producers in j, imports from the other country, and goods from subsidiaries in j that have their origin

³¹ The countries being small imply that they cannot influence prices.

in the other country (MNEs). Hence, the mass of firms in the world equals the amount of differentiated goods potentially available.

Firms in the differentiated sector differ with respect to their productivity, but ex-ante all these firms are identical. If they expect positive earnings from the production process, they pay sunk entry costs f_d upfront, which are measured in units of labor. As long as firms expect positive profits, they enter the market. It is assumed that the individual productivity levels of the firms in each country are independent draws from a cumulative productivity distribution function $F(\theta)$. The fee f_d allows the firms to independently draw their productivity from the distribution $F(\theta)$ with support over (0, θ_{max}). With this procedure, firms located in the home country are guaranteed to produce domestically, even with very low productivity, to reduce the loss of f_d . The time line in figure 2 shows the logical sequence from the moment prior to entry, when all firms are identical, to the moment when firms in the industry decide their integration strategies and outputs.

					⊾ t
0	A-priori, all firms are identical.	Firm expects positive earnings and pays market entry costs f _d .	Productivity θ is revealed.	Domestic production and its output are decided.	If θ sufficient, output decision for additional export or MNE activity is made.

Figure 2. Steps towards the choice of integration of the firm.

According to their productivity $\theta(i)$, firms choose their integration strategies. In their domestic countries all firms start as domestic producers. If their productivity is low, a firms will not enter the foreign market, neither through exports nor through foreign plant setup. If productivity is high enough, firms have the additional choice to serve foreign markets via exports or foreign affiliate production (the latter being referred to as horizontal MNE activity). The choice between exporting and setting up foreign plants is driven by the proximity-concentration trade-off, characterized by the fact that MNE activity relative to exports saves trading costs as reflected by the tariff τ for cross-border trade of differentiated varieties.³² On the other hand, foreign plant setup has fixed costs f_i in terms of units of labor that are higher than fixed costs for

³² See, for example, Horstmann and Markusen (1992), Brainard (1993b), or Markusen and Venables (2000) for a survey.

exporters f_{ex} because production facilities must be duplicated.³³ For this reason, f_d<f_{ex}<f_i is assumed.

Beyond these fixed costs, firms pay variable costs, depending on their productivity levels $\theta(i)$ [i.e., $x(i)/\theta(i)$] and their integration strategies (i.e., exporters' activities are subject to the tariff τ). According to x(i)/ θ (i), when comparing two firms with the same amount of output, the firm with higher productivity θ (i) must bear lower variable costs. 34

Furthermore, governments may choose positive ad-valorem tariff rates τ subject to imports (i.e., the tariff is a percentage of the value of one unit of the imported good). With $\tau_A > 0$, these firms (i.e., exporters from B importing to A) consider τ_A an additional factor influencing profits and vice versa. If tariff revenue in j is positive, it is passed on to households in j as a lump-sum transfer.

Given the preferences in (1), the demand of households in (2), and the price consumption curve in (3), it is straightforward to compute maximum attainable profits of a firm in j serving its domestic market:³⁵

$$\pi_{d,j} = p_{j}(i)_{d} x_{j}(i)_{d} - \frac{x_{j}(i)_{d}}{\theta(i)} - f_{d}$$

$$= \frac{1}{\left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} x_{j}(i)^{\alpha} di\right)^{(1-\mu)} x_{j}(i)_{d}^{(1-\alpha)}} x_{j}(i)_{d} - \frac{x_{j}(i)_{d}}{\theta(i)} - f_{d}$$

The derivative with respect to $x_i(i)$ is an expression for the profit-maximizing output of a domestic firm i in its domestic market j, $j \in \{A, B\}$:³⁶

$$\mathbf{x}_{j}(\mathbf{i})_{d}^{*} = \frac{(\alpha \theta)^{\frac{1}{(1-\alpha)}}}{\left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} \mathbf{x}_{j}(\mathbf{i})^{\alpha}\right)^{\frac{(1-\mu)}{(1-\alpha)}}}_{\text{marketsize X}},$$
(7)

 ³³ As in Helpman, Melitz and Yeaple (2004).
 ³⁴ As in Grossman, Helpman and Szeidl (2006); Melitz (2203); Schröder (2007); and in Helpman, Melitz and Rubinstein (2007). ³⁵ See Appendix 1.9.1.

³⁶ See Appendix 1.9.2.

associated with the optimal price³⁷

$$p_{j}(i)_{d}^{*} = \frac{1}{\alpha \theta(i)}.$$
(8)

The optimal output of a firm in the domestic market depends on market size X.³⁸ According to (7), the optimal output level a single firm is negatively correlated with X due to competitive conditions. Furthermore, the productivity level of a firm is positively correlated with its output.

Because there is monopolistic competition, the market power of a single producer depends on the elasticity of substitution σ between two varieties of differentiated goods. Therefore, firms maximize their profits by charging the mill price (i.e., $p(\theta(i)) = \frac{1}{\alpha} \frac{w}{\theta(i)}$, where w=1 as assumed and $1/\alpha$ reflects the mark-up on the price).³⁹

This is the standard markup pricing in which a greater elasticity of substitution is associated with a smaller markup. Producers in a market in which differentiated goods are close substitutes associated with a higher α only apply small markups on their prices because their market power is infinitesimally small. Accordingly, maximum attainable profits of a domestic firm i in j are given by:⁴⁰

$$\pi_{j}(i)_{d}^{*} = \frac{\left(\alpha\theta(i)\right)^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}} \left(\frac{1}{\alpha} - 1\right) - f_{d}$$
(9)

Analogously, we can now derive profits of firms with an export strategy. Profits of exporters from Country A are defined by: ⁴¹

$$\pi_{A}(i)_{ex} = \underbrace{p_{B}(i)_{ex}(1+\tau)x_{B}(i)_{ex} - \frac{x_{B}(i)_{ex}}{\theta(i)} - f_{ex}}_{\text{engage in exports if }>0} + \pi_{A}(i)_{d}$$
(10)

Profits of exporters from Country B are defined by:

$$\pi_{\rm B}(i)_{\rm ex} = \underbrace{p_{\rm A}(i)_{\rm ex}(1+\tau)x_{\rm A}(i)_{\rm ex} - \frac{x_{\rm A}(i)_{\rm ex}}{\theta(i)} - f_{\rm ex}}_{\text{engage in exports if > 0}} + \pi_{\rm B}(i)_{\rm d}$$
(11)

³⁷ See Appendix 1.9.2.

³⁸ The market size X has to be specified according to $X_{d,ex}$, $X_{d,ex,i}$, or $X_{d,i}$.

³⁹ This follows from the derivative of the profit function with respect to the price as in Appendix B.

⁴¹ See Appendix 1.9.3.

The exporting firm has two sources of earnings: domestic sales and export activity. For a firm i from j, the expression in (10) or (11) results in optimal output in the other country (output for exporting):⁴²

$$x_{j}(i)_{ex}^{*} = \frac{(\alpha\theta)^{\frac{1}{(1-\alpha)}}}{(1+\tau)^{\frac{1}{(1-\alpha)}} X^{\frac{(1-\mu)}{(1-\alpha)}}},$$
(12)

associated with the optimal price for exports $q(i)_{ex}^{*} = p(i)_{ex}^{*} (1+\tau)$:⁴³

$$q_{j}(i)_{ex}^{*} = \frac{(1+\tau)}{\alpha \theta(i)}$$
(13)

In addition to the previous analysis, one can see that the optimal output and price for exports depend on the tariff τ in contrast to the optimal output and price when supplying domestic demand. Because raising a tariff results in increased prices for imports $q(i)_{ex}^{*}$, the supply of an exporting firm $x_i(i)_{ex}^{*}$ decreases. The representative household is not willing to pay any higher price for imported goods to satisfy the lovefor-variety preference. Hence, the demand for imported goods decreases more than proportional to increases in the tariff τ .

Accordingly, maximum attainable profits of an exporting firm i in j are given by:⁴⁴

$$\pi_{j}(i)_{ex}^{*} = \frac{(\alpha\theta(i))^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}(1+\tau)^{\frac{1}{(1-\alpha)}}\theta(i)} \left(\frac{1}{\alpha} - 1\right) - f_{ex} + \pi_{j}(i)_{d}^{*}$$
(14)

Now, it is straightforward to compute maximum attainable profits of a firm engaged in MNE activities. As the firm produces goods for both markets locally, the tariff rate is not relevant. Instead, a firm i from Country A opens an affiliate in B and becomes a horizontal MNE. Profits of an MNE i headquartered in Country A are defined by:⁴⁵

$$\pi_{A}(i)_{i} = \underbrace{p_{B}(i)_{i} x_{B}(i)_{i} - \frac{x_{B}(i)_{i}}{\theta(i)} - f_{i}}_{\text{engage in MNE if } > 0} + \pi_{A}(i)_{d}$$
(15)

Analogously, this can be derived for a firm i from B building up a subsidiary in A.

An MNE expects at least zero profits from running both domestic and foreign subsidiaries.

⁴² See Appendix 1.9.3.

⁴³ See Appendix 1.9.3. ⁴⁴ See Appendix 1.9.3.

⁴⁵ See Appendix 1.9.4.

Profit-maximizing plant output of an MNE i headquartered in j is shown by:⁴⁶

$$\mathbf{x}_{j}(\mathbf{i})_{i}^{*} = \frac{\left(\alpha \theta(\mathbf{i})\right)^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}},$$
(16)

associated with the optimal price:47

$$p_{j}(i)_{i}^{*} = \frac{1}{\alpha} \frac{1}{\theta(i)}$$
(17)

Accordingly, maximum attainable profits of an MNE i headquartered in j are given by:⁴⁸

$$\pi_{j}(\mathbf{i})_{i}^{*} = \frac{(\alpha\theta(\mathbf{i}))^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}\theta(\mathbf{i})} \left(\frac{1}{\alpha} - 1\right) - f_{i} + \pi_{j}(\mathbf{i})_{d}^{*}$$
(18)

Firms choose their integration strategies based on the knowledge of their productivity levels. This results in cut-off levels being determinants of minimum levels of productivity for a firm i to generate zero profits additionally when ex-ante selecting strategies with more than domestic production. In general, more productive firms are more successful in all three strategies. The least productive firms only serve the domestic market through domestic production. Because of their low productivity, their variable costs are too high so that higher fixed costs to operate in an additional market cannot be covered.

The first cut-off occurs when the productivity of a firm is such that additional profits of exporting exactly results in zero profits. This can be derived from (10) and applies for:49

$$\theta_{d/ex} = \frac{(1+\tau)^{\frac{1}{\alpha}} \cdot f_{ex} \frac{(1-\alpha)}{\alpha} \cdot X^{\frac{(1-\mu)}{\alpha}}}{\alpha(1-\alpha)^{\frac{(1-\alpha)}{\alpha}}}$$
(19)

The market size X results endogenously, according to X_{d.ex.i}. A firm with productivity $\boldsymbol{\theta}_{d/ex}$ generates zero profits from exporting. Hence, this firm is indifferent between only selling domestically or additionally engaging in exports. A firm with productivity just above this level already earns positive profits from exporting and will definitely

 ⁴⁶ See Appendix 1.9.4.
 ⁴⁷ See Appendix 1.9.4.
 ⁴⁸ See Appendix 1.9.4.

⁴⁹ See Appendix 1.9.5.

engage in exporting. The critical productivity level $\theta_{d/ex}$ is positively correlated with τ , fex, and market size X. Hence, the indifferent firm must be more productive to break even. In other words, higher productivity yields lower variable costs of production. Furthermore, conditional on the existence of the export strategy, productivity levels exist that ensure the profits of exporters exceed the profits of MNEs. This results in the next threshold where profits of an exporting firm equal profits of an MNE (i.e., $\pi(i)_{ex} = \pi(i)_i$).⁵⁰ This applies to the following expression:

$$\theta_{ex/i} = \frac{(f_i - f_{ex}) \cdot X^{\frac{(1-\mu)}{\alpha}}}{\alpha (1-\alpha)^{\frac{(1-\alpha)}{\alpha}} \left(1 - \frac{1}{(1+\tau)^{\frac{1}{(1-\alpha)}}}\right)^{\frac{(1-\alpha)}{\alpha}}}$$
(20)

Only firms with $\theta(i) > \theta_{ex/i}$ gain positive profits from serving foreign markets through building subsidiaries instead of exporting their goods. $\theta_{ex/i}$ depends on the difference in fixed costs $(f_i - f_{ex}) > 0$, which can be interpreted as overhead and setup costs of an MNE subsidiary. The higher the overhead costs $(f_i - f_{ex})$ for a foreign subsidiary, the more productive the indifferent firm must be to engage in MNE activity (i.e., the cut-off level $\theta_{ex/i}$ takes over a higher value). As the tariff τ increases, the firm becomes more likely to engage in the MNE strategy, which also results in a lower value of $\theta_{ex/i}$. Furthermore, if $\tau = 0$, the threshold $\theta_{ex/i}$ is infinite. Hence, $\theta_{ex/i}$ is only defined for $\tau > 0$. Intuitively, firms do not engage in MNE activities if the tariff is $\tau = 0$. The MNE strategy does not exist under this constellation.

Alternatively, certain configurations of parameters may result in a situation in which domestic firms directly integrate as MNEs instead of choosing the export strategy. The following cut-off level is the relevant productivity threshold when for example the tariff τ reaches a level at which firms do not choose the export strategy anymore (i.e., $\theta_{d/i} < \theta_{d/ex}$). The associated cut-off level results from $\pi_i(i)_i \ge 0$ and is given by:⁵¹

$$\theta_{d/i} = \frac{f_i^{\frac{(1-\alpha)}{\alpha}} X^{\frac{(1-\mu)}{\alpha}}}{(1-\alpha)^{\frac{(1-\alpha)}{\alpha}} \alpha}$$
(21)

⁵⁰ See Appendix 1.9.6. ⁵¹ See Appendix 1.9.7.

The associated market size X in this scenario endogenously results in $X_{d,i}$. As can be seen, this setup ensures that all three strategies can coexist and are determined by the individual productivity of the firm.

1.3 The behavior of the firms

Consumers in the home country can buy all the goods being provided by domestically located firms. Furthermore, consumers can buy goods from foreign firms. The decisions by firms to export to foreign markets or to build foreign subsidiaries depend on their specific productivity levels associated with corresponding parameter configurations (e.g., the tariff τ , variable costs, and fixed costs). The influence of a tariff on firm integration strategies can be demonstrated by analyzing the bahavior of the firms if $\tau = 0$ and by comparing it to the situation at $\tau > 0$ or $\tau_1 < \tau_2$.

1.3.1 Integration strategies at $\tau=0$

Because exporting activities are not linked to additional transport costs and $\tau = 0$, firms will not benefit from building subsidiaries. This will result in higher fixed costs, (i.e., $f_i > f_{ex}$) but will not have any further upside for heterogeneous firms. At $\tau = 0$, the cut-off level $\theta_{ex/i}$ that separates exporting firms from MNEs is infinite because

$$\left(1 - \left(\frac{1}{(1+\tau)}\right)^{\frac{1}{(1-\alpha)}}\right)^{\frac{(1-\alpha)}{\alpha}} = 0.$$

MNE activities do not exist. Hence, no firms engage in FDI. Instead, firms export to foreign markets. Figure 3 is representative of the allocation of foreign firms at $\tau = 0$.



Figure 3. The allocation of foreign firms, τ =0.

The productivity level $\theta_{d/ex}$ that at least guarantees zero profits for exporters in this scenario (i.e., $\tau = 0$) is represented by:

$$\theta_{d/ex}\big|_{\tau=0} = \frac{f_{ex}^{\frac{(1-\alpha)}{\alpha}} \cdot X^{\frac{(1-\mu)}{\alpha}}}{\alpha(1-\alpha)^{\frac{(1-\alpha)}{\alpha}}}.$$
(19a)

The threshold $\theta_{d/ex}$ requires a productivity level sufficient to cover the fixed costs f_{ex} . Furthermore, $\theta_{d/ex}$ is positively correlated with the market size $X_{d,ex}$. Therefore, an increasing market size results in a decreasing demand for the products of the i-th firm. The indifferent firm must be more productive. A situation in which governments do not apply ad-valorem tariffs on imports results in a scenario in which foreign goods are solely imported.

1.3.2 Integration strategies at $\tau > 0$

A tariff is an additional decision parameter when firms choose their integration strategies. $\tau > 0$ decreases consumers' demand for imports which results in an increasing value of the cut-off level $\theta_{d,ex}$. In contrast to the scenario at $\tau = 0$, the indifferent firm must be more productive to break even. The tariff forces low-productivity firms to exit the export strategy and harms consumers by supplying the market with fewer varieties. The tariff also affects exporters with higher productivity levels. Highly productive exporters are now in favor of engaging in MNE activities. Although fixed costs associated with MNE integration are higher than in the export strategy ($f_{i}>f_{ex}$), the MNE activity bypasses the tariff. Hence, consumers do not face distorted prices that affect their decisions. Firms in the export strategy face reduced demand due to the tariff, whereas the same firms using an MNE strategy are not confronted with such a consequence.

The determined cut-off levels (19), (20), and (21) give information about the allocation of firms utilizing the different integration strategies. From the perspective of the home country, all firms located in the home country manufacture differentiated goods as domestic producers. In addition, it is valuable to know the distribution of foreign firms across the other strategies. This is shown in figure 4.





Firms desiring to export goods must be at least as productive as the cut-off level defined in (19). The next threshold (20) gives the productivity level of the highest-productivity exporting firms. Above this level, firms engage in MNE activities to satisfy demand.

Alternative parameter configurations may result in situations in which the export strategy disappears and foreign firms directly integrate as MNEs. This is the case as soon as the condition $\theta_{d/i} < \theta_{d/ex}$ holds, which is given by:⁵²

$$\tau \ge \left(\frac{f_i}{f_{ex}}\right)^{(1-\alpha)} - 1$$

The expression depends on the ratio of fixed costs and the tariff rate τ . Figure 5 is a representation of that situation.



Figure 5. The allocation of foreign firms, $\theta_{d/l} \le \theta_{d/ex}$.

The coexistence of both international strategies disappears and foreign firms directly choose the MNE strategy.

1.3.3 The effect of the tariff on the mass of firms

Now, it is straightforward to calculate the mass of firms following the different strategies. The mass of exporting firms supplying the market in j through imports can be derived by calculating $\theta_{ex/i} - \theta_{d/ex}$, which is given by:

$$n_{j,exp} = \frac{X^{\frac{(1-\mu)}{\alpha}} \left(-f_{ex} \frac{(1-\alpha)}{\alpha} (1+\tau)^{\frac{1}{\alpha}} + (f_{i} - f_{ex})^{\frac{(1-\alpha)}{\alpha}} \right)}{\alpha (1-\alpha)^{\frac{(1-\alpha)}{\alpha}} \left(1 - (1+\tau)^{\frac{1}{(1-\alpha)}} \right)^{\frac{(1-\alpha)}{\alpha}}}$$
(22)

The mass of exporters in the home country is positively dependent on the fixed costs of MNEs f_i , negatively on f_{ex} , and positively on the consumption index as denoted by

⁵² See Appendix 1.9.7.

X. Although an increase in X stimulates an increase in exporters to sell their goods competition among them intensifies and results in lower demand per firm. This competition effect can be clarified using the expression in (12). The derivative subject to X results in the following negative expression:

$$\frac{\partial x_{j}(i)_{ex}}{\partial X} = -\frac{(1-\mu)X^{(-1)-\frac{(1-\mu)}{(1-\alpha)}}(\alpha\theta)^{\frac{1}{(1-\alpha)}}(1+\tau)^{\frac{-1}{(1-\alpha)}}}{1-\alpha}$$
(23)

The single exporter importing to j faces a reduced demand due to increased available varieties from which to choose. Thus, consumers buy fewer goods from a single firm.

Analogously, the mass of firms engaging in FDI is defined by $n_{j,i} = n_j - \theta_{ex/i}$ and results in the following expression:

$$n_{j,i} = n_j - \frac{\left(1 - \alpha\right)^{\frac{(\alpha - 1)}{\alpha}} \left(f_i - f_{ex}\right)^{\frac{1 - \alpha}{\alpha}} X^{\frac{(1 - \mu)}{\alpha}} \left(1 - \left(1 + \tau\right)^{\frac{1}{(1 - \alpha)}}\right)^{\frac{(\alpha - 1)}{\alpha}}}{\alpha}$$
(24)

The mass of MNEs in j is positively correlated with the tariff τ , the fixed costs associated with the export strategy f_{ex} , and the mass of firms in country j, n_j . An opposite effect can be seen if fixed costs for MNE activity f_i or the general market size X increases.

1.3.4 The effects of the tariff on the export sector: Extensive border effects

Because a tariff has different implications for the behavior of heterogeneous firms, the export sector is analyzed explicitly. First, we examine the effects on less productive exporters. Their relevant threshold $\theta_{d/ex}$, without the application of a tariff, is given by (19a). Hence, the mass of firms leaving the strategy due to the tariff τ can be derived from the following expression:

$$\theta_{d/ex}|_{\tau>0} - \theta_{d/ex}|_{\tau=0} = \frac{f_{ex} \frac{(1-\alpha)}{\alpha} X^{\frac{(1-\mu)}{\alpha}} \left((1+\tau)^{\frac{1}{\alpha}} - 1 \right)}{\alpha (1-\alpha)^{\frac{(1-\alpha)}{\alpha}}}$$
(25)

This way of calculating the mass of firms is applicable because we assume the firms to be distributed following a uniform distribution.



Figure 6 shows the effect of a tariff on the lower productivity threshold.

Figure 6. Extensive border effect on low-productivity exporters.

The mass of firms affected by introduction of a tariff is denoted as C in the figure and forces firms in this interval to remain domestic producers in their home countries. An increase in the tariff results in increases in the cut-off level and to the mass of firms in C. This is because consumer demand decreases due to higher prices. Therefore, firms in the interval C must be more productive to break even. As explained previously, an increase in the market size X also enlarges the interval in C as competition intensifies.

To analyze the implications of an ad-valorem tariff on the behavior of highly productive exporters, we analyze the effect assuming $\tau_1 < \tau_2$. Figure 7 visualizes how the tariff affects the integration strategy of highly productive exporters:



Figure 7. Extensive border effect on high-productivity exporters, $\tau_1 < \tau_2$.

The tariff $\tau_2 > \tau_1$ lowers sales of exporters and equalizes the difference in the cost structure between the export and the MNE strategies. As a result, the mass of D firms changes its strategy and engages in FDI because this strategy requires a lower minimum productivity as seen in (20). The mass of firms affected by a higher tariff can be derived by $D = \theta_{ex/i} |_{\tau_2} - \theta_{ex/i} |_{\tau_1}$, which is given by:

$$D = \frac{\left(f_{i} - f_{ex}\right)^{\frac{(1-\alpha)}{\alpha}} X^{\frac{(1-\mu)}{\alpha}}}{\alpha (1-\alpha)^{\frac{(1-\alpha)}{\alpha}}} \left(1 - \frac{1}{\left(1 - \left(\frac{1}{1+\tau}\right)^{\frac{1}{(1-\alpha)}}\right)^{\frac{(1-\alpha)}{\alpha}}}\right)$$
(26)

An increase in τ makes MNE activities more likely. Therefore, the mass in D increases. Hence, the higher the tariff τ , the more firms change their strategies and engage in MNE activities.

The opposite effect may be found if the fixed costs f_i are relatively more expensive than f_{ex} . The mass of firms in interval D being affected by the tariff decreases. Fewer firms change their strategies to engage in FDI. Therefore, the decision of firms is not solely affected by the fixed costs of one strategy but by the fixed costs f_i relative to fixed costs f_{ex} .

An increase in the market size X also has the effect of decreasing the mass in D. Table 1 is a summary of the effects on the strategies of firms with respect to the previously analyzed cases.

	au = 0	$\tau > 0$
Varieties available	Maximum in imports	Less imports due to tariff
Export strategy	Yes	Yes
Multinational strategy	No	Yes

Table 1. Comparison of $\tau = 0$ versus $\tau > 0$.

At $\tau = 0$, imports to the home country are maximized. The mass of foreign goods available in the home country decreases when $\tau > 0$. The introduction of the tariff causes low-productivity exporters to exit the market. As a result, consumers forfeit the varieties of those suppliers. The export strategy remains the only international strategy when $\tau = 0$. In this scenario, the MNE strategy requires higher fixed costs but has no further upside to firms. The situation changes when $\tau > 0$. The tariff induces high-productivity firms to start MNE activities. The tariff reduces the demand of exporting firms, which reduces firm profits. Sufficiently productive firms engage in MNE strategies to bypass the tariff. They generate higher fixed costs to build subsidiaries but profit from higher sales volumes.

1.4 The welfare-maximization process

The objective of the government is to evaluate the effects of an ad-valorem tariff on the welfare of households. Such a policy evokes reactions in the integration strategies of firms influencing the utility of the household. Therefore, the implications of these reactions must be considered in the welfare-maximization process. Anticipating the behavior of firms, governments maximize consumer welfare by endogenously determining ad-valorem tariffs τ . Because in this paper we assume two completely symmetric, small countries, it is sufficient to show the decisionmaking process of only one country. The objective of a government is to maximize the expression $\frac{\partial U}{\partial \tau}$ to find the level of the tariff rate τ maximizing welfare. In the following, the relevant effects are described for the general case that three strategies exist (i.e., domestic producers, exporters and horizontal MNEs). This implies that $\theta_{ex/i}$ is finite as ensured by $\tau \neq 0$, and that $\theta_{d/ex} < \theta_{ex/i}$ holds. Of course, the market size X may endogenously have different outcomes and must be specified according to (4), (5), and (6), depending on parameter configuration. Furthermore, to guarantee a continuous solution, firms are ranked according to their individual productivity levels, starting with low-productivity firms.

The utility of the households is positively dependent on consumption of the numéraire. This monetary effect is given by the difference in labor income m_j and expenses for differentiated goods. Without transfer, utility from x_0 is shown by:

$$\mathbf{m}_{j} - \left(\int_{0}^{\theta_{max}} \mathbf{p}_{j}(\mathbf{i})_{d}^{*} \mathbf{x}_{j}(\mathbf{j})_{d}^{*} d\mathbf{i} + \int_{\theta_{d/ex}}^{\theta_{ex/i}} \mathbf{q}_{j}(\mathbf{i})_{ex}^{*} \mathbf{x}_{j}(\mathbf{j})_{ex}^{*} d\mathbf{i} + \int_{\theta_{ex/i}}^{\theta_{max}} \mathbf{p}_{j}(\mathbf{i})_{i}^{*} \mathbf{x}_{j}(\mathbf{j})_{i}^{*} d\mathbf{i}\right)$$
(27)

Additionally, and in contrast to the scenario at $\tau = 0$, the government in j generates revenues that are transferred to consumers in j. This monetary effect enables consumers in j to buy more of the numéraire good x_0 , having a positive effect on the utility. Considering the utility function in (1) and the profit-maximizing output of exporters in (12), this effect is given by:

$$\int_{\theta_{d/ex}}^{\theta_{ex/i}} \frac{(\alpha\theta(i))^{\frac{1}{(1-\alpha)}}}{(1+\tau)^{\frac{1}{(1-\alpha)}} X^{\frac{(1-\mu)}{(1-\alpha)}}} \cdot \tau \, di$$
(28)
_

The government in j generates tariff revenues for its consumers in j on all imports to the market in j.

The next utility generating effect stems from consumption of domestic goods:

$$\frac{1}{\mu} \left[\int_{0}^{\theta_{max}} \frac{1}{\alpha} \left(\frac{(\alpha \theta(i))^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}} \right)^{\alpha} di \right]^{\mu}$$
(29)

Furthermore, the utility of consumers in j consists of consumption of imported goods. This effect is depicted by:

$$\frac{1}{\mu} \left[\int_{\theta_{d/ex}}^{\theta_{ex/i}} \frac{1}{\alpha} \left(\frac{\left(\alpha\theta(i)\right)^{\frac{1}{(1-\alpha)}}}{\left(1+\tau\right)^{\frac{1}{(1-\alpha)}} X^{\frac{(1-\mu)}{(1-\alpha)}}} \right)^{\alpha} di \right]^{\mu}$$
(30)

By maximizing utility, the government considers the ambiguous effect of the tariff τ . On the one hand, an increase in the tariff τ stimulates highly productive exporters to engage in MNE activities. This makes the affected varieties cheaper because the goods of MNEs are not subject to the trade barrier. On the other hand, lowproductivity exporters are forced to leave the market because their productivity θ (i) is not sufficient. The products of these suppliers are no longer available to consumers in j, which has a negative influence on the utility of the households in j.

The next utility-generating element is represented by consumption of goods of MNEs. In formal accounts, this effect can be described by the following expression:

$$\frac{1}{\mu} \left[\int_{\theta_{ex/i}}^{\theta_{max}} \frac{1}{\alpha} \left(\frac{(\alpha \theta(i))^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}} \right)^{\alpha} di \right]^{\mu}$$
(31)

The demand of households for goods of MNEs is not directly affected by the tariff τ . However, an increasing tariff rate τ influences the lower limit of integration because highly productive exporters may change their integration status following MNE activities. Therefore, the area described by the integral increases (i.e., $\theta_{ex/i}$ decreases), having positive implications on utility.

Governments in j consider that all the described elements influence utility of the representative household interdependently. Therefore, the expression governments in j maximize subject to the tariff τ is shown by: ⁵³

$$U_{j} = m_{j} - \left(\int_{0}^{\theta_{max}} p_{j}(i)_{d}^{*} x_{j}(i)_{d}^{*} di + \int_{\theta_{d/ex}}^{\theta_{ex/i}} q_{j}(i)_{ex}^{*} x_{j}(i)_{ex}^{*} di + \int_{\theta_{ex/i}}^{\theta_{max}} p_{j}(i)_{i}^{*} x_{j}(i)_{i}^{*} di\right) + \int_{\theta_{d/ex}}^{\theta_{ex/i}} x_{j}(i)_{ex}^{*} \cdot \tau di$$

$$+ \frac{1}{\mu} \left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} (x_{j}(i)_{d}^{*})^{\alpha} di + \int_{\theta_{d/ex}}^{\theta_{ex/i}} \frac{1}{\alpha} (x_{j}(i)_{ex}^{*})^{\alpha} di + \int_{\theta_{ex/i}}^{\theta_{max}} \frac{1}{\alpha} (x_{j}(i)_{i}^{*})^{\alpha} di\right)^{\mu}$$
(32)

or

$$\begin{split} U_{j} &= m_{j} - \left(\int_{0}^{\theta_{max}} \frac{1}{\alpha \theta(i)} \frac{\left(\alpha \theta(i)\right)^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}} di + \int_{\theta_{d/ex}}^{\theta_{ex/i}} \frac{(1+\tau)t}{\alpha \theta(i)} \frac{\left(\alpha \theta(i)\right)^{\frac{1}{(1-\alpha)}}}{(1+\tau)^{\frac{1}{(1-\alpha)}} X^{\frac{(1-\mu)}{(1-\alpha)}}} di + \int_{\theta_{ex/i}}^{\theta_{max}} \frac{1}{\alpha \theta(i)} \frac{\left(\alpha \theta(i)\right)^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}} di \right) \\ &+ \int_{\theta_{d/ex}}^{\theta_{ex/i}} \frac{\left(\alpha \theta(i)\right)^{\frac{1}{(1-\alpha)}}}{(1+\tau)^{\frac{1}{(1-\alpha)}} X^{\frac{(1-\mu)}{(1-\alpha)}}} \cdot \tau di \\ &+ \frac{1}{\mu} \left[\int_{0}^{\theta_{max}} \frac{1}{\alpha} \left(\frac{\left(\alpha \theta(i)\right)^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}} \right)^{\alpha} di + \int_{\theta_{d/ex}}^{\theta_{ex/i}} \frac{1}{\alpha} \left(\frac{\left(\alpha \theta(i)\right)^{\frac{1}{(1-\alpha)}}}{(1+\tau)^{\frac{1}{(1-\alpha)}} X^{\frac{(1-\mu)}{(1-\alpha)}}} \right)^{\alpha} di + \int_{\theta_{ex/i}}^{\theta_{max}} \frac{1}{\alpha} \left(\frac{\left(\alpha \theta(i)\right)^{\frac{1}{(1-\alpha)}}}{(1+\tau)^{\frac{1}{(1-\alpha)}} X^{\frac{(1-\mu)}{(1-\alpha)}}} \right)^{\alpha} di + \int_{\theta_{ex/i}}^{\theta_{max}} \frac{1}{\alpha} \left(\frac{\left(\alpha \theta(i)\right)^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}} \right)^{\alpha} di \right]^{\mu} \end{split}$$

In this expression, we also consider that $\theta(i) = \frac{i}{n}\overline{\theta}$ and that the market size results endogenously depending on the integration strategies of heterogeneous firms. The endogeneity of market size X results in a situation in which every parameter configuration results in a different level of X. As well endogenous, the mass of firms n in equilibrium varies with associated dependent variables. Expected profits for the coexistence of the domestic, the export, and the MNE strategies are depicted by:⁵⁴

(33)

⁵³ Utility of the representative household under alternative parameter configurations resulting in $X_{d/ex'}$ $X_{d/i}$ and X_d is shown in Appendix 1.9.8. ⁵⁴ Expected profits result endogenously according $X_{d/ex/i}$, $X_{d/ex}$, $X_{d/i}$ or X_d .

$$\begin{split} \mathsf{E}\pi_{j} &= \int_{0}^{\theta_{max}} \frac{\left(\alpha\theta(i)\right)^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}\theta(i)} \left(\frac{1}{\alpha} - 1\right) - f_{d} + \int_{\theta_{d/ex}}^{\theta_{ex/i}} \frac{\left(\alpha\theta(i)\right)^{\frac{1}{(1-\alpha)}}}{\left(1+\tau\right)^{\frac{1}{(1-\alpha)}}X^{\frac{(1-\mu)}{(1-\alpha)}}\theta(i)} \left(\frac{1}{\alpha} - 1\right) - f_{ex} \\ &+ \int_{\theta_{ex/i}}^{\theta_{max}} \frac{\left(\alpha\theta(i)\right)^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}\theta(i)} \left(\frac{1}{\alpha} - 1\right) - f_{i} \end{split}$$
(34)

Furthermore, both variables, the market size X and the mass of firms n in equilibrium, are interdependent. The market size X and the mass of firms n induce further interdependences to other equilibrium determining expressions (i.e., the cut-off levels and the demand of the households). Even without the complexity being induced by the linkages of the different variables, the maximization of the previously mentioned expression, $\frac{\partial U_i}{\partial \tau}$, results in a problem with a dimensionality higher than fourth degree.⁵⁵ These aspects preclude an analytical solution of $\frac{\partial U_i}{\partial \tau}$ and suggest using numerical analysis to determine the welfare-maximizing tariff rates τ and their interactions with other variables.

1.5 The setup of the numerical framework

To derive a solution to the problem discussed previously and to find a welfaremaximizing expression for the tariff τ , we use Mathematica 6.0 to set up the numerical framework that represents the theory of the model as derived in previous sections.⁵⁶

1.5.1 Definitions

The coding of the numerical framework begins with defining variables and making assumptions. Analogously to the assumptions of the model, the fixed costs, θ_{max} , as well as α and μ , are defined as constant numerical values, considering $f_d < f_{ex} < f_i$ and $0 < \mu < \alpha < 1$.

 ⁵⁵ A derivation of a unique solution with dimensionality higher than fourth degree cannot be provided. This is proved by the theory of E. Galois. For a survey, see Taton (1983).
 ⁵⁶ See Appendix D for the full input sheet. For a survey of numerical mathematics and methods, see

⁵⁰ See Appendix D for the full input sheet. For a survey of numerical mathematics and methods, see Knorrenschild (2008) and Spelucci (1993).

After generating the market-entry costs of f_d , firms draw their individual productivity θ . We apply a uniform distribution and specify the density as $f[\theta_{-}]:=1/\theta$ max. The associated distribution function is computed as $F[\theta_{-}]:=\theta/\theta$ max. To guarantee continuous results, firms are ranked according to their individual productivity, starting with low-productivity firms. This is reflected by the expression $\theta[i_{-},n_{-}]$. The productivity $\theta[i_{-},n_{-}]$ depends on the rank i of the i-th firm, given a mass of firms in the economy n.

For further analysis, a function is computed to provide the rank of indifferent firms between two strategies. This expression is given by $inr[\theta_n, n_n]$ and reports the rank of each firm, given the productivity θ and the mass of firms n founded in the country.

The demand of the representative household, as in (2), results in optimal output for the firms, as derived previously. The computation of the profit-maximizing output is represented by $xdi[\Theta_, x_]$ for domestic firms and MNEs and by $x[\Theta_, x_, \tau_]$ for firms in the export sector.

The choice of the integration strategy of each firm is driven by cut-off productivity thresholds. The first threshold separates domestic producers from exporters and is computed as $\Theta de[X_, \tau_]$, considering X and τ in the country in which differentiated goods are sold. The associated firm number is reported by $ide[n_, X_, \tau_]:=inr[\Theta de[X, \tau], n]$. The expression calculates the rank of the indifferent firm in means of productivity, depending on the mass of firms n, the market size X, and the tariff τ . For example, by entering ide[50, 1000, 0.03], the system calculates that with 50 firms in the country j, a market size of 1000, and a tariff of 3% in j, the firm that just has the cut-off productivity $\theta_{d/ex}$ is the 22.8084nd firm. Hence, it is the 23rd firm out of 50 that exports for sure.

Analogously, we compute the threshold productivity $\theta_{ex/i}$ as $\Theta ei[X_, \tau_]$, associated with the expression for the firm that produces with the cut-off productivity $iei[n_, X_, \tau_] := inr[\Theta ei[X, \tau], n]$, considering X and τ in the country in which the economic activity takes place. The same notion is used to compute the threshold $\theta_{d/i}$ as $\Theta di[X_, \tau_]$, associated with its rank $idi[X_, \tau_]$, considering X and τ in the country in which the economic activity takes place.

1.5.2 Consistency of market size X

The market size X depends on the mass of firms n, the tariff τ , and household demand x(i). It is given by $X = \left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} x_{j}(i)^{\alpha}\right)$. The inclusion of the endogenously

defined market size X from a demand perspective as in section 1.2.1, into the numerical model does not result in the consistency needed to derive results. The proof of inconsistency starts with computing the market size of country j from a supply perspective for firms active in the different strategies in j, $j \in \{A,B\}$ (i.e., Yd [supply of a domestic firm from A and vice versa from B], Yex [supply of an exporting firm from A and vice versa from B], Yex [supply of an exporting firm from A and vice versa for MNEs with origin in B and vice versa with origin in A]). The market size for domestic producers in A referring to the representative household yields

 $Yd[n_,X_,\tau_]:=NIntegrate[(1/\alpha)(xdi[\theta[i,n],X])^{\alpha}, \{i,0,n\}].$

Yd depends on the mass of firms n in A, the market size X in A, and the tariff τ being applied in A. It is characterized by the integral over the output of all domestic firms i.

Analogously, we compute the market size of firms that export from B to A. From the perspective of firms producing in B and exporting to A, the export market size referring to the representative household in A is given by

 $\begin{aligned} &\operatorname{Yex}[n_,X_{-},\tau_{-}] := &\operatorname{If}[ide[n,X,\tau] < &\operatorname{Min}[iei[n,X,\tau],n], &\operatorname{NIntegrate}[(1/\alpha)] \\ &) &\operatorname{X}[\ \theta[i,n],X,\tau])^{\alpha}, \\ &\{i,ide[n,X,\tau], &\operatorname{Min}[iei[n,X,\tau],n]\}], 0]. \end{aligned}$

Yex depends on the mass of firms n in country B, the market size X in A, and the tariff τ in A. The definition of Yex in the numerical analysis also considers the scenario that possibly no exporters exist based on the implemented conditions.

The expression starts with a condition for Yex to exist as depicted by

 $If[ide[n,X,\tau] < Min[iei[n,X,\tau],n].$

Only if the number of the cut-off firm ide is smaller than the smaller value of either the number of the cut-off firm iei or the mass of firms n, does an export market exist. If ide<iei <n, the export strategy and the MNE strategy coexist. If ide<n<iei, the MNE strategy does not exist. The reverse is that ide>iei or ide>n, providing intuition that the export sector does not exist, which results in ouput 0 for the export strategy.

The condition being satisfied gives the system the indication to integrate the output x of all firms i in the export sector. The integration has its lower limit given by the cut-off

firm ide and its upper limit given by the smaller value of either iei or n. The included conditions are essential for the result to exist because this procedure guarantees that the lower limit of integration is always smaller than the upper limit. Otherwise, the program can under certain circumstances, erroneously generate misleading results.

The market size for MNEs in terms of the representative household is defined by $Yin[n_,X_,\tau_]:=NIntegrate[(1/\alpha)(xdi[\theta[i,n],X])^{\alpha}, \{i,Min[n,iei[n,X,\tau]],n\}].$

The market for MNEs in Country A depends on the mass of firms being located in Country B, the market size X in A, and the tariff rate τ . Also, here, the coding includes conditions to guarantee that the system integrates correctly regarding prevailing integration strategies.

The entire market size from a supply perspective, referring to the representative household in A, is determined as the sum of all three market segments and is represented by

 $Y[n_,X_,\tau_]:=Yd[n,X,\tau]+Yex[n,X,\tau]+Yin[n,X,\tau].$

The inconsistency of the market size from a supply (Y) and a demand (X) perspective can be clarified by figure 8.



Figure 8. The inconsistency of the market size.

The inconsistency of the market size in figure 8 is considerably apparent. The curve of the market size X has a different progression than the curve Y. The intersection of

both graphs gives the true market size, given the mass of firms n and the tariff τ (i.e., 343,524).

To achieve the essential consistency, the computation is performed in two steps. We implement FindRoot[Y[1000,X,.2]==X, {X,6}], an expression to find the exact market size for given values for the mass of firms n and the tariff τ . The result in this case for X is 1338.65. To program the correct and consistent market size Xm, we must define the expression

 $Xm[n_, \tau_]$:=ReplaceAll[X,FindRoot[Y[n,X, τ]-X==0,{X,10}]], to replace all values for X with the results of the FindRoot command.⁵⁷ For further analysis, the expression Xm indicates the consistent market size and, therefore, is used in the following section.

1.5.3 The mass of firms in equilibrium

The decision of firms to enter the market is based on the expectation of future profits. Heterogeneous firms enter production as long as their future earnings expectations are positive. Hence, the mass of firms in the market is determined by expected profits being equal to zero. After computing the profit function of the single firm i with its particular strategy, expected profits are determined by the profits of all firms in the specific market. The profit functions of the single firm i in the different strategies are computed. The profit function of a firm with rank i following a domestic integration strategy is computed as $Gd[i,n,\tau]$. Similarly, profits of a firm in the export sector are computed as $Gex[i,n,\tau]$, and the code $Gin[i,n,\tau]$ defines the profit function of a firm with an MNE strategy.

Expected profits (EG) in an economy result from the integration of profits over all firms in the different strategies. They are given by

 $EG[n_, \tau_]$:= $EGd[n, \tau]$ + $EGex[n, \tau]$ + $EGin[n, \tau]$. The computation of expected profits includes conditions to ensure that profits are only integrated if the associated strategy exists. Finally, coding the mass of firms in equilibrium results from expected profits being competed to zero by firms entering the market.

The computed loop calculates the mass of firms n in country j given a tariff rate $\,\tau\,.\,$

⁵⁷ The test of Xm[1000,0.2] approves the approach and yields 1338.65.

This expression is given by firms[τ_{-}]. The process to find the null is coded by the instruction to test several values in defined steps. After determining the first negative value of expected profits, the program returns to approach the null exactly while the width of the steps is permanently reduced. For example, a tariff of $\tau = 0.1\%$ results in 19728.7 firms in equilibrium with associated expected profits of 1.61949×10^{-6.58} This is support for the previously described method.

1.5.4 The maximization of welfare

Considering the utility function in (1), households in the two symmetric small countries benefit from consumption of the homogeneous good x_0 and the differentiated goods. To implement the utility maximization process, we first consider the stand-alone contribution of x_0 . Without the existence of a differentiated goods sector, the representative household only generates utility by consuming x_0 . Hence, the benefit of one unit of a differentiated good is constituted by its net contribution (i.e., additional utility versus additional costs). The computation of the equilibrium utilizes this notion to implement the utility-maximizing process. The equilibrium of the model depends on a given value of the tariff τ and is computed so that the system delivers data describing the equilibrium.⁵⁹

The expression describing the equilibrium is labelled as Equilibrium[τ_{-}] and indicates the expression is dependent on a given value of the tariff τ .⁶⁰

At first, the program delivers a value for the mass of firms in equilibrium Ngg=N[firms[τ]], which delivers a numerical value of the expression firms[τ]. The next value of interest is the market size in equilibrium Xmgg=Xm[Ngg, τ], which depends on the derived value for the mass of firms in equilibrium Ngg and the tariff τ . The utility in equilibrium is defined as Ugg and sums up the contribution of consumption of domestically produced goods, imports, and goods supplied by MNEs.⁶¹

The computation of the utility in equilibrium Ugg utilizes conditions to guarantee the limits of integration to be valid, delivering stable results. Analogously, we derive the expenses of a single household in equilibrium Egg. They are computed based on

⁵⁸ The codes are N[firms[.1]]and EG[19728.666015625`,.1].

⁵⁹ See Appendix 1.9.9 for the computation of the equilibrium.

⁶⁰ The notation for values in equilibrium consists of "gg" as additional two letters (e.g. Ugg is the utility in equilibrium).

⁶¹ See Appendix 1.9.9 for the computation of the utility in equilibrium.

demand in a single household for domestic products, imports, and goods produced by MNEs linked to their associated prices.⁶² Finally, welfare in equilibrium W_{gg} is calculated as the net contribution of the differentiated goods sector (i.e., W_{gg} = $U_{gg}-E_{gg}$).

1.6 The results of the numerical model

Using the knowledge of the behavior of firms concerning their integration strategies, the requirement of this paper is to find coherence of the tariff τ and the welfare of households from a social planner's perspective. Therefore, we examine equilibria of the model resulting from a variation of the tariff rate τ .

To analyze the numerical output, we focus on the mass of firms in equilibrium n, which indicates the mass of differentiated goods available in the country to satisfy the consumers' love-for-variety preferences. The market size in equilibrium Xmgg is utilized to derive utility Ugg and expenses Egg in equilibrium. Finally, the welfare in equilibrium Wgg is derived by calculating the saldo of Ugg and Egg (i.e., Wgg=Ugg-Egg). Table 2 is a summary of the results of the model in equilibrium utilizing different levels of the tariff τ .

Tariff	Ngg	Xmgg	Ugg	Egg	Wgg	ide	iei	idi	strategies
0	23600	5164.00	281.608	126.723	154.884	18213	1591040	19706	exporter
0.01	23543	5114.82	279.993	125.522	154.471	18317	34302	19556	exporter
0.02	23503	5068.22	278.460	124.400	154.060	18438	27269	19429	exporter
0.03	23480	5024.19	277.006	123.354	153.651	18575	23879	19321	exporter
0.035	23314	5004.66	276.359	123.193	153.166	18524	22565	19144	exporter/ multinational
0.037	23238	4998.58	276.158	123.207	152.951	18500	22100	19070	exporter/ multinational
0.04	23149	4991.05	275.908	123.244	152.664	18485	21484	18981	exporter/ multinational
0.05	23002	4976.15	275.413	123.478	151.935	18574	19941	18831	exporter/ multinational
0.06	23013	4971.78	275.268	123.834	151.434	18810	18909	18831	exporter/ multinational
0.07	21989	4963.86	275.005	123.752	151.253	18184	17279	17978	multinational
0.08	21078	4944.16	274.350	123.457	150.892	17611	15927	17196	multinational
0.09	20338	4971.78	275.268	123.834	151.434	17158	14847	16642	multinational
0.1	19729	4894.03	272.677	122.705	149.972	16800	13967	16008	multinational

Table 2. Summarized results of the model in equilibrium.

The summary of the generated results shows the configuration of equilibriadescribing parameters. Furthermore, the allocation of firms on the different integration strategies is clarified by columns ide, iei, and idi. The last column gives intuition about existing integration strategies in equilibrium. In the following, we explicitly examine important equilibria:

⁶² See Appendix 1.9.9 for the computation of the expenses in equilibrium Egg.

1.6.1 Tariff $\tau = 0\%$

If the level of the tariff τ a social planner chooses is τ =0%, the welfare (Wgg=154.884) of the representative household, and therefore, of the whole country is maximized in this scenario. The equilibrium is characterized by a maximum of differentiated available varieties (Ngg=23600) being linked to a market size Xmgg of 5164.08. The derived values create utility Ugg of 281.608 and expenses Egg of 126.723.

In addition to the values describing the equilibrium, the allocation of firms producing with the different strategies is the focus of this analysis. Here, the firm with cut-off productivity $\theta_{d/ex}$ (i.e., ide=18212.8) or the 18212.9th firm out of 23600 firms, starts to engage in an export strategy. In the case of τ =0, the least productive MNE is iei= 1591040. Hence, in this economy, no heterogeneous firm represents the MNE strategy because founding an MNE is wasteful in terms of overhead fixed costs (f_i-f_{ex}). In this economy, it is always advantageous to supply foreign markets via exports. Furthermore, there are 5387.2 exporting and, consequently importing firms. This results in an overall share of differentiated varieties of 22.83% being imported by foreign firms.

1.6.2 Tariff $\tau = 3.5\%$

With an increase in the tariff to τ =3.5%, the relevant parameters decrease (i.e., the mass of firms in equilibrium Ngg, market size Xmgg, utility Ugg, expenses Egg, and welfare in equilibrium). In this equilibrium, welfare Wgg=153.166. In contrast to the equilibrium with τ =0, this is equivalent to a reduction of welfare of 1.11%. τ =3.5% has an influence on firm integration strategies. The tariff τ reduces household demand for imported goods, affecting profits of exporters and making the MNE strategy attractive. This equilibrium hosts both international strategies (i.e., exporters and MNEs). The share of foreign differentiated varieties is 20.54%.

1.6.3 Tariff $\tau = 7\%$

A further increase in the tariff τ results in a further decrease of the parameters affecting the welfare in equilibrium Wgg. With τ =7%, welfare Wgg=151.253 is equivalent to a reduction in welfare of 2.34% compared with the optimal policy.

The integration strategies are affected so that the export strategy disappears. Because idi<ide, heterogeneous foreign firms directly integrate as MNEs because τ =7% reduces the profit of exporting firms, making it beneficial to accept higher overhead fixed costs and to engage in MNE strategies instead. The indifferent MNE, producing with the cut-off productivity, has the number idi=17978. Hence, in this economy, the share of foreign firms and, therefore, of foreign goods is at 18.24%.

1.7 The role of heterogeneity

To analyze the role of heterogeneity in this model, we examine the previous model, assuming that the productivity θ (i) does not follow a distribution function F(θ) and that all firms have the same productivity $\overline{\theta}$. The choice of integration strategy by the firm still focuses on productivity thresholds, thereby maximizing profit.

However, the choice of $\overline{\theta}$ predetermines the outcome of the model, thereby defining prevailing integration strategies. The firms being homogeneous result in an equilibrium in which all firms charge the same price and choose the same strategy. In general, the model can generate four scenarios, depending on the choice of $\overline{\theta}$, which will be discussed in the following section.

1.7.1 Autarky

We study the scenario in which productivity $\overline{\theta}$ is chosen so that $\overline{\theta} \leq \theta_{d/ex} \leq \theta_{d/i}$.

Satisfying the condition for $\overline{\theta}$, the market size is given by $X = \int_{0}^{n} \frac{1}{\alpha} \left(\frac{(\alpha \overline{\theta})^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}} \right)^{\alpha}$, which

results in

$$X_{d} = \left(n\frac{1}{\alpha}\left(\alpha\overline{\theta}\right)^{\frac{\alpha}{(1-\alpha)}}\right)^{\frac{(1-\alpha)}{(1-\alpha\mu)}}.$$
(35)

The market size X_d is a constant, increasing with the mass of firms $n=n^A=n^B$. With $\overline{\theta}$ below the minimum productivity requirement for an international strategy, all firms in j remain domestic producers for the domestic market in j, which results in the economy operating in autarky. The utility of the household is given by

$$U_{d} = \frac{1}{\mu} \left[\left(n \frac{1}{\alpha} \left(\alpha \overline{\theta} \right)^{\frac{\alpha}{(1-\alpha)}} \right)^{\frac{(1-\alpha)}{(1-\alpha\mu)}} \right]^{\mu} , \qquad (36)$$

and expenses are given by

$$\mathsf{E}_{\mathsf{d}} = \mathsf{n} \left(\frac{\left(\alpha \overline{\theta} \right)_{(1-\alpha)}^{1}}{X^{\frac{(1-\mu)}{(1-\alpha)}}} \right) \left(\frac{1}{\alpha \overline{\theta}} \right).$$
(37)

Welfare of the representative household can be derived as the saldo of utility and expenses, also giving a constant value.

1.7.2 Domestic firms and exporters

To generate a scenario in which firms are active in the domestic and the export market, productivity $\overline{\theta}$ is chosen so that $\theta_{d/ex} \le \theta_{d/i} \le \overline{\theta} \le \theta_{ex/i}$. Satisfying this condition, market size X_{ex} consists of the market of domestic producers, depending on the mass of firms in the home country, A. Furthermore, the mass of firms in Country B n^B exports to the home country. X_{ex} can be derived

using
$$X_{ex} = \int_{0}^{n^{A}} \frac{1}{\alpha} \left(\frac{\left(\alpha \overline{\theta}\right)^{1}}{X^{(1-\alpha)}} \right)^{\alpha} + \int_{0}^{n^{B}} \frac{1}{\alpha} \left(\frac{\left(\alpha \overline{\theta}\right)^{1}}{X^{(1-\alpha)}} - \frac{1}{(1-\alpha)} \right)^{\alpha}$$

Because the two countries are symmetric, we apply $n=n^A=n^B$; the market size X_{ex} is represented by

$$X_{ex} = \left[\frac{n}{\alpha} \left(\alpha \overline{\theta}\right)^{\frac{\alpha}{(1-\alpha)}} \left(1 + \frac{1}{(1+\tau)^{\frac{\alpha}{(1-\alpha)}}}\right)\right]^{\frac{(1-\alpha)}{(1-\alpha\mu)}}.$$
(38)

The corresponding utility of the representative household is given by

$$U_{ex} = \frac{1}{\mu} \left[\left[\frac{n}{\alpha} \left(\alpha \overline{\theta} \right)^{\frac{\alpha}{(1-\alpha)}} \left(1 + \frac{1}{(1+\tau)^{\frac{\alpha}{(1-\alpha)}}} \right) \right]^{\frac{(1-\alpha)}{(1-\alpha\mu)}} \right]^{\mu}.$$
(39)

Expenses for consumption of domestic goods are $E = n \left(\frac{\left(\alpha \overline{\theta} \right)^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}} \right) \left(\frac{1}{\alpha \overline{\theta}} \right)$, and

expenses for exports are $E = n \left(\frac{\left(\alpha \overline{\theta} \right)^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}} \left(1+\tau \right)^{\frac{\alpha}{(1-\alpha)}}} \right) \left(\frac{1}{\alpha \overline{\theta}} \right)$. The sum of both positions

gives total expenses. The welfare W_{ex} results as the saldo of U_{ex} - E_{ex} .

The stability of the constellation that all firms export to the foreign market is mainly dependent on the condition that $\overline{\theta} \ge \theta_{d/ex}$. If the parameter configuration changes (e.g., the tariff τ is increased), the export market crunches and the economy returns to the autarky scenario. This happens if $\overline{\theta} < \theta_{d/ex}$, which results in the following coherence, depending on the tariff τ :

$$(1+\tau) > \left(\frac{\overline{\theta}\alpha(1-\alpha)^{(1-\alpha)}}{f_{ex}^{\frac{(1-\alpha)}{\alpha}}X_{ex}}\right)^{\alpha}$$
(40)

1.7.3 Domestic firms and MNEs

The homogeneous model may result in a situation in which $\overline{\theta}$ is chosen so that only domestic and MNE firms exist. The productivity of all firms in this scenario is $\overline{\theta} \ge \theta_{ex/i}$, which results in the following condition for equilibrium to exist in the described way:

$$(1+\tau) \ge 1 - \frac{\alpha^{\alpha} (1-\alpha)^{(1-\alpha)} \overline{\Theta}^{\alpha}}{(f_i - f_{ex})^{(1-\alpha)} X_i^{1-\mu}}.$$
(41)

If $(1+\tau)$ satisfies the condition, the economy only consists of domestic firms and

MNEs, which results in a market size $X_i = \int_0^{n_A} \frac{1}{\alpha} x(i)^{\alpha} + \int_0^{n_B} \frac{1}{\alpha} x(i)^{\alpha}$ or

$$X_{i} = \left(2n\frac{1}{\alpha}\left(\alpha\overline{\theta}\right)^{\frac{\alpha}{(1-\alpha)}}\right)^{\frac{(1-\alpha)}{(1-\alpha\mu)}}.$$
(42)

The associated utility is given by

$$U_{i} = \frac{1}{\mu} \left[\left(2n \frac{1}{\alpha} \left(\alpha \overline{\theta} \right)^{\frac{\alpha}{(1-\alpha)}} \right)^{\frac{(1-\alpha)}{(1-\alpha\mu)}} \right]^{\mu} , \qquad (43)$$

in which expenses are $E_i=2E_d$. The stability of this constellation is dependent on the above condition in (41). If $(1+\tau)$ does not satisfy the condition in (41), the market for MNEs crunches and only domestic firms and exporters produce their goods.

Alternatively, $\overline{\theta}$ may be fixed so that $\overline{\theta} \ge \theta_{d/i}$. This scenario generates a market size constituted by domestic firms and MNEs. The utility and, therefore, expenses and welfare do not differ from the previous results; but equilibrium returns to the autarky scenario if $\overline{\theta} < \theta_{d/i}$.

1.7.4 The market size with homogeneous firms and the role of heterogeneity

After discussing all potential outcomes of the model assuming homogeneous firms, the market size under all alternative scenarios in formal accounts is given by

$$X = \begin{cases} X_{d} = \left(n\frac{1}{\alpha}\left(\alpha\overline{\theta}\right)^{\frac{\alpha}{(1-\alpha)}}\right)^{\frac{(1-\alpha)}{(1-\alpha\mu)}} & \text{if } \overline{\theta} \le \theta_{d/ex} \le \theta_{d/i} \\ X_{ex} = \left[\frac{n}{\alpha}\left(\alpha\overline{\theta}\right)^{\frac{\alpha}{(1-\alpha)}}\left(1 + \frac{1}{(1+\tau)^{\frac{\alpha}{(1-\alpha)}}}\right)\right]^{\frac{(1-\alpha)}{(1-\alpha\mu)}} & \text{if } \theta_{d/i} \le \theta_{d/ex} \le \overline{\theta} \le \theta_{ex/i} \\ X_{i} = \left(2n\frac{1}{\alpha}\left(\alpha\overline{\theta}\right)^{\frac{\alpha}{(1-\alpha)}}\right)^{\frac{(1-\alpha)}{(1-\alpha\mu)}} & \text{if } \overline{\theta} \ge \theta_{ex/i} \text{ or } \overline{\theta} \ge \theta_{d/i} \end{cases}$$
(44)

The relaxation of the assumption that the productivity levels of the firms follow a distribution function $F(\theta)$ results in a model in which possible equilibria have punctiform characteristics predetermining the framework of later equilibria. The market size depicts the different outcomes, depending on the strategic alignment of the firms (e.g., the market size ends in an autarky scenario with X_d if the productivity $\overline{\theta}$ is below the minimum requirement of productivity for an international strategy). Similarly, the market size X results in X_{ex} or X_i, depending on the choice of $\overline{\theta}$.

In contrast to the model with heterogeneous firms, the coexistence of both the export and the MNE strategy disappears. The gradual characteristics of the model get lost. A social planner is able to make an either-or decision on the preferred strategic alignment of firms in the country with limited further influence.

However, the model is hardly adequate to build reality because the existence of a multitude of possible integration strategies of heterogeneous firms is an omnipresent phenomenon. Finally, the assumption of heterogeneity in monopolistic trade models is unavoidable.

1.8 Conclusion

The objective of this paper is to model a two-country trade model with firm-level heterogeneity. Exporting is linked to an ad-valorem tariff τ , which reduces household demand for those varieties. To avoid the tariff, firms may select a horizontal MNE strategy.

The requirement of a social planner is to maximize the welfare of the representative household by endogenously determining an optimal positive level of the tariff τ .

The integration strategies heterogeneous firms choose as optimal are dependent on exogenously given parameters. This choice is represented by cut-off productivity levels determined by exogenously given values for fixed costs and the tariff rate. Their relative size and ratio influence the choice of integration status of heterogeneous firms.⁶³

Additionally, market entry and market size are treated endogenously. Therefore, we are able to analyze the implications of national and international policy decisions on integration modi of heterogeneous firms. Not only MNEs and exporters decide on entering the market depending on the tariff, but also the mass of domestic firms is influenced by trade policy. Given a certain tariff rate, the composition of prevailing integration strategies is due to the constitution of competition. For example, an increasing tariff rate induces fewer exporters to enter the market and, depending on the size of fixed costs f_i, may also cause them to refrain from becoming MNEs. Consequently, fewer firms will supply demand in this country and expected profits will increase. Therefore, the output of each single firm is influenced; and more domestic firms can enter the market competing expected profits to zero.⁶⁴

This paper is a study of the implications of our model, assuming the firms to be homogeneous. We show that the assumption of heterogeneity is central to analyze equilibria consisting of different integration strategies. In the homogeneous model, all

firms select the same integration strategy as optimal, depending on the choice of $\overline{\theta}$. Additionally, firm level-heterogeneity in our model follows a uniform distribution $F(\theta)$, influencing our results. The specification of an alternative distribution function, therefore, may induce differing results. The assumption of a distribution $G(\theta)$, according to which the mass of firms increases with productivity so that many MNEs and few domestic firms exist, slows the stimulating effect on domestic firms of entering the market if positive tariff rates are selected.

 ⁶³ See Davies and Eckel (2007).
 ⁶⁴ See Davies, Egger and Egger (2009).

These aspects of several interdependent endogenous variables preclude an analytical solution of $\frac{\partial U_j}{\partial \tau}$ and suggest using numerical analysis to determine the welfare-maximizing tariff rates τ and their interactions with other variables. Utilizing numerical analysis, we show that welfare is maximized when the tariff τ =0. This equilibrium is characterized by foreign goods being imported solely. MNEs do not exist is this scenario. The exclusive existence of the export strategy in this equilibrium can be explained by the wastefulness caused by the overhead fixed costs of the MNE strategy. Furthermore, the optimality of the tariff τ =0 is reasonable because exporting is not linked to further trading costs (e.g., transport costs).

The result is mainly motivated by a variety effect induced by the trade barrier (i.e., increasing tariff rates decrease the mass of firms in equilibrium and, therefore, available varieties). This loss in utility cannot be compensated by monetary transfer of tariff revenues. Consumers prefer a rich supply of differentiated varieties to a monetary transfer they spend on homogeneous goods. The intervention into market outcomes, thereby relieving consumers of available varieties, does not have the appreciated effect.

Finally, our model shows the implications of welfare-maximizing governments from a social planner's perspective. Resulting tariff rates in both countries are identical. Additionally, the governments do not have the possibility of reacting optimally to the trade policy of the government in neighboring countries.

A perspective for further research is to study best-response tariffs in the same setting. Furthermore, our model does not include the implication of negative tariff rates (i.e. subsidies), which also may be an objective of further studies.

1.9 Appendix

1.9.1 Demand

We use the utility function in (1), $U_j = x_0 + \frac{1}{\mu} \left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} x_j(i)^{\alpha} di \right)^{\mu}$ and the standard side

condition to derive the demand of a representative household for goods of the ith firm:

$$L = \mathbf{x}_{0} + \frac{1}{\mu} \left[\int_{0}^{\theta_{max}} \frac{1}{\alpha} (\mathbf{x}_{j}(\mathbf{i}) + \beta \varsigma_{j}(\mathbf{i}))^{\alpha} d\mathbf{i} \right]^{\mu} + \lambda \left[\mathbf{m}_{j} - \mathbf{x}_{0} - \int_{0}^{\theta_{max}} \mathbf{p}_{j}(\mathbf{i}) (\mathbf{x}_{j}(\mathbf{i}) + \beta \varsigma_{j}(\mathbf{i})) d\mathbf{i} \right]$$

$$\begin{split} \frac{\partial L}{\partial x_{0}} &= 1 - \lambda = 0 \quad \Rightarrow \lambda = 1 \\ \frac{\partial L}{\partial \lambda} &= m_{j} - x_{0} - \int_{0}^{\theta_{max}} p_{j}(i)(x_{j}(i) + \beta\varsigma_{j}(i))di = 0 \quad \Rightarrow x_{0} = m_{j} - \int_{0}^{\theta_{max}} p_{j}(i)(x_{j}(i) + \beta\varsigma_{j}(i))di \\ \frac{\partial L}{\partial \beta} &= \left[\int_{0}^{\theta_{max}} \frac{1}{\alpha}(x_{j}(i) + \beta\varsigma_{j}(i))^{\alpha}di\right]^{(\mu - 1)} \left[\int_{0}^{\theta_{max}} (x_{j}(i) + \beta\varsigma_{j}(i))^{(\alpha - 1)}\varsigma_{j}(i)di\right] - \int_{0}^{\theta_{max}} p_{j}(i)\varsigma_{j}(i)di = 0 \\ \text{If } \beta = 0: \\ \frac{\partial L}{\partial \beta} &= \left[\int_{0}^{\theta_{max}} \frac{1}{\alpha}(x_{j}(i) + \beta\varsigma_{j}(i))^{\alpha}di\right]^{(\mu - 1)} \left[\int_{0}^{\theta_{max}} e_{max}\right]^{(\mu - 1)} \left[\int_$$

$$\frac{\partial L}{\partial \beta} = \left[\int_{0}^{\sigma_{max}} \frac{1}{\alpha} x_{j}(i)^{\alpha} di \right] \int_{0}^{\sigma_{max}} x_{j}(i)^{(\alpha-1)} \zeta(i) di - \int_{0}^{\sigma_{max}} p_{j}(i) \zeta_{j}(i) di = 0$$
$$\Rightarrow \int_{0}^{\theta_{max}} \left[\frac{x_{j}(i)^{(\alpha-1)}}{\left[\int_{0}^{\theta_{max}} \frac{1}{\alpha} x_{j}(i)^{\alpha} di \right]^{(1-\mu)}} - p_{j}(i) \right] \zeta_{j}(i) di = 0$$

$$\Rightarrow \mathbf{x}_{j}(\mathbf{i})^{(\alpha-1)} = \mathbf{p}_{j}(\mathbf{i})\mathbf{X}^{(1-\mu)}$$
$$\Rightarrow \frac{1}{\mathbf{x}_{j}(\mathbf{i})^{(1-\alpha)}} = \mathbf{p}_{j}(\mathbf{i})\mathbf{X}^{(1-\mu)}$$
$$\Rightarrow \mathbf{x}_{j}(\mathbf{i})^{(1-\alpha)} = \frac{1}{\mathbf{X}^{(1-\mu)}\mathbf{p}_{j}(\mathbf{i})}$$
$$\Rightarrow \mathbf{x}_{j}(\mathbf{i}) = \frac{1}{\mathbf{X}^{(1-\mu)}\mathbf{p}_{j}(\mathbf{i})^{\frac{1}{(1-\alpha)}}}$$
$$\Rightarrow \mathbf{p}_{j}(\mathbf{i}) = \frac{1}{\mathbf{X}^{(1-\mu)}\mathbf{x}_{j}(\mathbf{i})^{(1-\alpha)}}$$

This paper applies $X = \left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} x_{j}(i)^{\alpha} di \right)$, which denotes the sub-utility and market size specifiable as $X_{d,ex,i}$ in (4), $X_{d,i}$ in (5) and $X_{d,ex}$ in (6).

1.9.2 The domestic firm

The derivation of the profit-maximizing output is shown in the following:

$$\pi_{j}(\mathbf{i})_{d} = \mathbf{p}_{j}(\mathbf{i})_{d} \mathbf{x}_{j}(\mathbf{i})_{d} - \frac{\mathbf{x}_{j}(\mathbf{i})_{d}}{\theta(\mathbf{i})} - \mathbf{f}_{d}$$

$$= \frac{1}{\left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} x_{j}(i)^{\alpha} di\right)^{(1-\mu)} x_{j}(i)_{d}^{(1-\alpha)}} x_{j}(i)_{d} - \frac{x_{j}(i)_{d}}{\theta(i)} - f_{d}}{\frac{\partial \pi_{j}(i)_{d}}{\partial x_{j}(i)_{d}}} = \alpha x_{j}(i)_{d}^{(\alpha-1)} \frac{1}{\left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} x_{j}(i)^{\alpha} di\right)^{(1-\mu)}} - \frac{1}{\theta(i)} = 0$$
$$x_{j}(i)_{d}^{(\alpha-1)} = \frac{1}{\theta(i)} \frac{X^{(1-\mu)}}{\alpha}$$
$$x_{j}(i)_{d}^{(\alpha-1)} = \frac{X^{(1-\mu)}}{\alpha\theta(i)}$$
$$x_{j}(i)_{d}^{*} = \frac{(\alpha\theta(i))^{(1-\alpha)}}{X^{(1-\alpha)}}$$

The derivation of the optimal price:

$$\begin{aligned} \pi_{j}(i)_{d} &= p_{j}(i)_{d} x_{j}(i)_{d} - \frac{x_{j}(i)_{d}}{\theta(i)} - f_{d} \\ \pi_{j}(i)_{d} &= p_{j}(i)_{d} \frac{1}{\left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} x_{j}(i)^{\alpha} di\right)^{\left(\frac{1-\mu}{1-\alpha}\right)} p_{j}(i)_{d}^{\frac{1}{(1-\alpha)}} - \left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} x_{j}(i)^{\alpha} di\right)^{\left(\frac{1-\mu}{1-\alpha}\right)} \frac{1}{\theta(i)} - f_{d} \\ \frac{\partial \pi_{j}(i)_{d}}{\partial p_{j}(i)_{d}} &= \frac{-\alpha}{(1-\alpha)} p_{j}(i)_{d}^{-\frac{1}{(1-\alpha)}} \frac{1}{\left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} x_{j}(i)^{\alpha} di\right)^{\left(\frac{1-\mu}{1-\alpha}\right)} + \frac{1}{(1-\alpha)} p_{j}(i)_{d}^{(-1)} \frac{1}{\left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} x_{j}(i)^{\alpha} di\right)^{\left(\frac{1-\mu}{1-\alpha}\right)} p_{j}(i)_{d}^{-\frac{1}{(1-\alpha)}} p_{j}(i)_{d}^{-\frac{1}{(1-\alpha)}} p_{j}(i)_{d}^{-\frac{1}{(1-\alpha)}} p_{j}(i)_{d}^{-\frac{1}{(1-\alpha)}} \frac{1}{\theta(i)} = 0 \\ \alpha \frac{1}{\left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} x_{j}(i)^{\alpha} di\right)^{\left(\frac{1-\mu}{1-\alpha}\right)}} = p_{j}(i)_{d}^{(-1)} \frac{1}{\left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} x_{j}(i)^{\alpha} di\right)^{\frac{(1-\mu)}{(1-\alpha)}} \frac{1}{\theta(i)}} \frac{1}{\left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} x_{j}(i)^{\alpha} di\right)^{\frac{(1-\mu)}{(1-\alpha)}}} \frac{1}{\theta(i)} \\ p_{j}(i)_{d}^{*} = \frac{1}{\alpha} \frac{1}{\theta(i)} \end{aligned}$$

The domestic firm i applies the price $p_j(i)$, which is the standard mill price. The factor $\frac{1}{\alpha}$ expresses the markup on the price.

The derivation of maximum attainable:

$$\pi_{j}(i)_{d}^{*} = \frac{(\alpha\theta(i))^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}} \frac{1}{\alpha} \frac{1}{\theta(i)} - \frac{(\alpha\theta(i))^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}\theta(i)} - f_{d} = \frac{(\alpha\theta(i))\frac{1}{(1-\alpha)}}{X^{\frac{(1-\mu)}{(1-\alpha)}}\theta(i)} \left(\frac{1}{\alpha} - 1\right) - f_{d}$$

1.9.3 The exporter

$$\begin{split} x_{j}(i)_{ex} &= \frac{1}{X^{\frac{(1-\mu)}{(1-\alpha)}} (p_{j}(i)(1+\tau))^{\frac{1}{(1-\alpha)}}} \\ \pi_{j}(i)_{ex} &= p_{j}(i)_{ex} x_{j}(i)_{ex} - \frac{x_{j}(i)_{ex}}{\theta(i)} - f_{ex} + \pi_{j}(i)_{d} \\ \pi_{j}(i)_{ex} &= p_{j}(i)_{ex} \frac{1}{X^{\frac{(1-\mu)}{(1-\alpha)}} (p_{j}(i)(1+\tau))^{\frac{1}{(1-\alpha)}}} - \frac{1}{X^{\frac{(1-\mu)}{(1-\alpha)}} (p_{j}(i)(1+\tau))^{\frac{1}{(1-\alpha)}}} \frac{1}{\theta(i)} - f_{ex} \\ \pi_{j}(i)_{ex} &= \frac{1}{X^{\frac{(1-\mu)}{(1-\alpha)}} (1+\tau)^{\frac{1}{(1-\alpha)}} p_{j}(i)_{ex} \frac{-\alpha}{(1-\alpha)}} - \frac{1}{X^{\frac{(1-\mu)}{(1-\alpha)}} (1+\tau)^{\frac{1}{(1-\alpha)}}} p_{j}(i)_{ex} \frac{-\alpha}{(1-\alpha)}} \frac{1}{\theta(i)} - f_{ex} \\ \frac{\partial \pi_{j}(i)_{ex}}{\partial p_{j}(i)_{ex}} &= -\frac{\alpha}{1-\alpha} p_{j}(i)_{ex} \frac{-1}{(1-\alpha)} \frac{1}{X^{\frac{(1-\mu)}{(1-\alpha)}} (1+\tau)^{\frac{1}{(1-\alpha)}}} + \frac{1}{1-\alpha} p_{j}(i)_{ex} \frac{-(2+\alpha)}{(1-\alpha)} \frac{1}{X^{\frac{(1-\mu)}{(1-\alpha)}} (1+\tau)^{\frac{1}{(1-\alpha)}}} \frac{1}{\theta(i)} \\ \frac{\alpha}{1-\alpha} p_{j}(i)_{ex} \frac{-1}{(1-\alpha)} \frac{1}{X^{\frac{(1-\mu)}{(1-\alpha)}} (1+\tau)^{\frac{1}{(1-\alpha)}}} = \frac{1}{1-\alpha} p_{j}(i)_{ex} \frac{-(2+\alpha)}{(1-\alpha)} \frac{1}{X^{\frac{(1-\mu)}{(1-\alpha)}} (1+\tau)^{\frac{1}{(1-\alpha)}}} \frac{1}{\theta(i)} \\ p_{j}(i)_{ex}^{+} &= \frac{1}{\alpha} \frac{1}{\theta(i)} \end{split}$$

$$q_{j}(i)_{ex}^{*} = (1 + \tau)p_{j}(i)_{ex}^{*}$$

The derivation of optimal output is derived, applying $p_j(i)_{ex}^*$ into $x_j(i)_{ex}$ and is shown by:

$$\mathbf{x}_{j}(\mathbf{i})_{ex}^{*} = \frac{(\alpha \theta(\mathbf{i}))^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}(1+\tau)^{\frac{1}{(1-\alpha)}}}$$

The derivation of maximum attainable profits:

$$\pi_{j}(i)_{ex}^{*} = \frac{(\alpha\theta(i))^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}(1+\tau)^{\frac{1}{(1-\alpha)}}} \frac{1}{\alpha} \frac{t}{\theta(i)} - \frac{(\alpha\theta(i))^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}(1+\tau)^{\frac{1}{(1-\alpha)}}} - f_{ex} + \pi_{j}(i)_{d}^{*}$$

$$=\frac{\left(\alpha\theta(i)\right)^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}(1+\tau)^{\frac{1}{(1-\alpha)}}\theta(i)}\left(\frac{1}{\alpha}-1\right)-f_{ex}+\pi_{j}(i)_{d}^{*}$$

1.9.4 The MNE

The profit function of a firm that engages in an MNE strategy is similar to the profit function of a domestic firm. Only fixed costs in this strategy are higher $f_i > f_{ex}$. The derivation of the profit-maximizing price:

$$\begin{aligned} \pi_{j}(\mathbf{i})_{i} &= p_{j}(\mathbf{i})_{i} \mathbf{x}_{j}(\mathbf{i})_{i} - \frac{\mathbf{x}_{j}(\mathbf{i})_{i}}{\theta(\mathbf{i})} - f_{i} \\ \pi_{j}(\mathbf{i})_{i} &= p_{j}(\mathbf{i})_{i} \frac{1}{\left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} \mathbf{x}_{j}(\mathbf{i})^{\alpha} d\mathbf{i}\right)^{\frac{(1-\mu)}{(1-\alpha)}} p_{j}(\mathbf{i})_{i}^{\frac{1}{(1-\alpha)}} - \frac{1}{\left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} \mathbf{x}_{j}(\mathbf{i})^{\alpha} d\mathbf{i}\right)^{\frac{(1-\mu)}{(1-\alpha)}} - \int_{0}^{\theta_{max}} \frac{1}{\alpha} \mathbf{x}_{j}(\mathbf{i})^{\alpha} d\mathbf{i} \int_{0}^{\frac{(1-\mu)}{(1-\alpha)}} \frac{1}{\theta(\mathbf{i})} - f_{i} \\ \frac{\partial \pi_{j}(\mathbf{i})_{i}}{\partial p_{j}(\mathbf{i})} &= -\alpha p_{j}(\mathbf{i})_{i}^{\frac{-1}{(1-\alpha)}} \frac{1}{\left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} \mathbf{x}_{j}(\mathbf{i})^{\alpha} d\mathbf{i}\right)^{\frac{(1-\mu)}{(1-\alpha)}} + \frac{1}{1-\alpha} p_{j}(\mathbf{i})_{i} \frac{1}{\left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} \mathbf{x}_{j}(\mathbf{i})^{\alpha} d\mathbf{i}\right)^{\frac{(1-\mu)}{(1-\alpha)}} + \frac{1}{\theta(\mathbf{i})} = 0 \\ \alpha p_{j}(\mathbf{i})_{i}^{\frac{-1}{(1-\alpha)}} \frac{1}{\left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} \mathbf{x}_{j}(\mathbf{i})^{\alpha} d\mathbf{i}\right)^{\frac{(1-\mu)}{(1-\alpha)}}} &= \frac{1}{1-\alpha} p_{j}(\mathbf{i})_{i} \frac{1}{\left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} \mathbf{x}_{j}(\mathbf{i})^{\alpha} d\mathbf{i}\right)^{\frac{(1-\mu)}{(1-\alpha)}} p_{j}(\mathbf{i})_{i}^{\frac{-(2+\alpha)}{(1-\alpha)}} \frac{1}{\theta(\mathbf{i})} \\ p_{j}(\mathbf{i})_{i}^{i} &= \frac{1}{\alpha} \frac{1}{\theta(\mathbf{i})} \end{aligned}$$

Analogously, we find profit-maximizing output by applying $p_j(i)_i^*$ into $x_j(i)$ which is shown by:

$$\mathbf{x}_{i}(\mathbf{i})_{i}^{*} = \frac{(\alpha \theta(\mathbf{i}))^{\frac{1}{(1-\alpha)}}}{\mathbf{x}^{\frac{(1-\mu)}{(1-\alpha)}}}$$

Maximum attainable profits are given by:

$$\pi_{j}(i)_{i}^{*} = \frac{(\alpha\theta(i))^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}} \frac{1}{\alpha} \frac{1}{\theta(i)} - \frac{(\alpha\theta(i))^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}\theta(i)} - f_{i} + \pi_{j}(i)_{d}^{*} = \frac{(\alpha\theta(i))^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}\theta(i)} \left(\frac{1}{\alpha} - 1\right) - f_{i} + \pi_{j}(i)_{d}^{*}$$

<u>1.9.5 Cut-off level $\theta_{d/ex}$ </u>

The productivity level that at least guarantees zero profit from exporting is derived in the following:

$$\begin{split} \pi_{j}(i)_{ex}^{*} &= x_{j}(i)_{ex}^{*} p_{j}(i)_{ex}^{*} - \frac{x_{j}(i)_{ex}^{*}}{\theta(i)} - f_{ex} \geq 0 \\ \pi_{j}(i)_{ex}^{*} &= \frac{\left(\alpha \theta(i)\right)^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}(1+\tau)^{\frac{1}{(1-\alpha)}}} \frac{1}{\alpha} \frac{1}{\theta(i)} - \frac{\left(\alpha \theta(i)\right)^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}(1+\tau)^{\frac{1}{(1-\alpha)}} \theta(i)} - f_{ex} \geq 0 \\ \frac{\left(\alpha \theta(i)\right)^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}(1+\tau)^{\frac{1}{(1-\alpha)}} \theta(i)} \left(\frac{1}{\alpha} - 1\right) \geq f_{ex} \\ \frac{\left(\alpha \theta(i)\right)^{\frac{1}{(1-\alpha)}}}{(1+\tau)^{\frac{1}{(1-\alpha)}} X^{\frac{(1-\mu)}{(1-\alpha)}} \theta(i)} \left(\frac{1-\alpha}{\alpha}\right) \geq f_{ex} \\ \frac{\left(\alpha \theta(i)\right)^{\frac{\alpha}{(1-\alpha)}}(1-\alpha)}{(1+\tau)^{\frac{1}{(1-\alpha)}} X^{\frac{(1-\mu)}{(1-\alpha)}}} \geq f_{ex} \\ \frac{\alpha^{\frac{\alpha}{(1-\alpha)}} \theta(i)^{\frac{\alpha}{(1-\alpha)}}(1-\alpha)}{(1+\tau)^{\frac{1}{(1-\alpha)}} X^{\frac{(1-\mu)}{(1-\alpha)}}} \geq f_{ex} \\ \frac{\theta(i)^{\frac{\alpha}{(1-\alpha)}} \chi^{\frac{(1-\mu)}{(1-\alpha)}} X^{\frac{(1-\mu)}{(1-\alpha)}}}{\alpha^{\frac{\alpha}{(1-\alpha)}}(1-\alpha)} \geq f_{ex} \\ \theta_{d/ex} &= \frac{f_{ex}^{\frac{(1-\alpha)}{\alpha}} X^{\frac{(1-\mu)}{\alpha}}(1+\tau)^{\frac{1}{\alpha}}}{\alpha(1-\alpha)^{\frac{(1-\alpha)}{\alpha}}} \end{split}$$

Firms that are at least as productive as $\theta_{d/ex}$ engage in export strategies.

<u>1.9.6 Cut-off level $\theta_{ex/i}$ </u>

The next threshold is characterized by a productivity level at which exporting and MNEs have the same profits. Firms with productivity levels above this level engage in an MNE activity.

$$\begin{aligned} \pi_{j}(i)_{ex} &\leq \pi_{j}(i)_{i} \\ \frac{(\alpha\theta(i))^{\frac{\alpha}{(1-\alpha)}}(1-\alpha)}{(1+\tau)^{\frac{1}{(1-\alpha)}} X^{\frac{(1-\mu)}{(1-\alpha)}}} - f_{ex} &\leq \frac{(\alpha\theta(i))^{\frac{\alpha}{(1-\alpha)}}(1-\alpha)}{X^{\frac{(1-\mu)}{(1-\alpha)}}} - f_{i} \\ \frac{(\alpha\theta(i))^{\frac{\alpha}{(1-\alpha)}}(1-\alpha)}{X^{\frac{(1-\mu)}{(1-\alpha)}}} \left(1 - \left(\frac{1}{(1+\tau)^{\frac{1}{(1-\alpha)}}}\right) \right) &\geq f_{i} - f_{ex} \\ \theta_{ex/i}^{\frac{\alpha}{(1-\alpha)}} &\geq \frac{(f_{i} - f_{ex})X^{\frac{(1-\mu)}{(1-\alpha)}}}{\alpha^{\frac{\alpha}{(1-\alpha)}}(1-\alpha) \left(1 - \left(\frac{1}{(1+\tau)^{\frac{1}{(1-\alpha)}}}\right) \right)} \\ \theta_{ex/i} &\geq \frac{(f_{i} - f_{ex})^{\frac{(1-\alpha)}{\alpha}} X^{\frac{(1-\mu)}{\alpha}}}{\alpha(1-\alpha)^{\frac{(1-\alpha)}{\alpha}} \left(1 - \left(\frac{1}{(1+\tau)^{\frac{1}{(1-\alpha)}}}\right) \right)^{\frac{(1-\alpha)}{\alpha}}} \end{aligned}$$

Firms producing with productivity $\theta_{ex/i}$ are indifferent whether to choose MNE or exporting strategies. A firm with productivity $\theta(i)$ just above $\theta_{ex/i}$ engages in an MNE strategy and generates positive profits from this activity.

$\underline{1.9.7 \ Cut\text{-off level}} \theta_{d/i}$

The next cut-off level characterizes a situation where the export strategy does not exist. Firms in this scenario directly integrate their firm following MNE activities. The resulting cut-off level can be derived as follows:

$$\begin{aligned} &\pi_{i} \geq 0 \\ &\frac{(\alpha \theta(i))^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}} \left(\frac{1-\alpha}{\alpha}\right) - f_{i} \geq 0 \\ &\frac{(\alpha \theta(i))^{\frac{\alpha}{(1-\alpha)}}(1-\alpha)}{X^{\frac{(1-\mu)}{(1-\alpha)}}} \geq f_{i} \end{aligned}$$

$$\theta(i)^{\frac{\alpha}{(1-\alpha)}} \geq \frac{f_{i} \cdot X^{\frac{(1-\mu)}{(1-\alpha)}}}{(1-\alpha)\alpha^{\frac{\alpha}{(1-\alpha)}}}$$
$$\theta_{d/i} \geq \frac{f_{i}^{\frac{(1-\alpha)}{\alpha}} \cdot X^{\frac{(1-\mu)}{(1-\alpha)}}}{(1-\alpha)^{\frac{(1-\alpha)}{\alpha}}\alpha}$$

Firms satisfying the following condition integrate their firm as an MNE, export strategies do not exist:

$$\begin{split} & \frac{f_{i}^{\frac{(1-\alpha)}{\alpha}}X^{\frac{(1-\mu)}{\alpha}}}{\alpha(1-\alpha)^{\frac{(1-\alpha)}{\alpha}}} \leq \frac{f_{ex}^{\frac{(1-\alpha)}{\alpha}}X^{\frac{(1-\mu)}{\alpha}}(1+\tau)^{\frac{1}{\alpha}}}{\alpha(1-\alpha)^{\frac{(1-\alpha)}{\alpha}}} \\ & f_{i}^{\frac{(1-\alpha)}{\alpha}} \leq f_{ex}^{\frac{(1-\alpha)}{\alpha}}(1+\tau)^{\frac{1}{\alpha}} \\ & (1+\tau) \geq \left(\frac{f_{i}}{f_{ex}}\right)^{(1-\alpha)} \end{split}$$

1.9.8 Utility of the representative household at X_{d,i}, X_{d/ex} and X_d

For the constellation that ensures the market size to be $X_{d,i}$, the following condition $\theta_{d/i} < \theta_{d/ex}$ has to hold and utility is given by:

$$U_{j} = m_{j} - \left(\int_{0}^{\theta_{max}} p_{j}(i)_{d}^{*} x_{j}(i)_{d}^{*} di + \int_{\theta_{d/i}}^{\theta_{max}} p_{j}(i)_{i}^{*} x_{j}(i)_{i}^{*} di\right)$$
$$+ \frac{1}{\mu} \left[\int_{0}^{\theta_{max}} \frac{1}{\alpha} \left(x_{j}(i)_{d}^{*}\right)^{\alpha} di + \int_{\theta_{d/i}}^{\theta_{max}} \frac{1}{\alpha} \left(x_{j}(i)_{i}^{*}\right)^{\alpha} di\right]^{\mu} + 0$$

Since the export strategy does not exist, a transfer cannot be provided.

For that the market size is $X_{d,ex}$, the condition $\theta_{d/ex} < \theta_{max}$ has to hold to ensure that the export strategy exists. Furthermore the MNE strategy does not exist if $\tau = 0$ since

$$\theta_{ex/i}$$
 is infinite $\left(1 - \left(\frac{1}{(1+\tau)}\right)^{\frac{1}{(1-\alpha)}}\right)^{\frac{(1-\alpha)}{\alpha}} \bigg|_{\tau=0} = 0$ which results in utility with transfer:

$$\begin{split} U_{j} &= m_{j} - \left(\int_{0}^{\theta_{max}} p_{j}(i)_{d}^{*} x_{j}(i)_{d}^{*} di + \int_{\theta_{d/ex}}^{\theta_{max}} p_{j}(i)_{ex}^{*} x_{j}(i)_{ex}^{*} di \right) + \int_{\theta_{d/ex}}^{\theta_{max}} x_{j}(i)_{ex}^{*} \cdot \tau di + \\ \frac{1}{\mu} \left[\int_{0}^{\theta_{max}} \frac{1}{\alpha} \left(x_{j}(i)_{d}^{*} \right)^{\alpha} di + \int_{\theta_{d/ex}}^{\theta_{max}} \frac{1}{\alpha} \left(x_{j}(i)_{ex}^{*} \right) \right]^{\mu} \end{split}$$

Only the domestic strategy exists if $\theta_{d/ex} > \theta_{max}$ and $\theta_{d/i} > \theta_{max}$ hold. This results in following utility for the household:

$$\boldsymbol{U}_{j} = \boldsymbol{m}_{j} - \left(\int_{0}^{\boldsymbol{\theta}_{max}} \boldsymbol{p}_{j}(\boldsymbol{i})_{d}^{*} \boldsymbol{x}_{j}(\boldsymbol{i})_{d}^{*} d\boldsymbol{i}\right) + \frac{1}{\mu} \left(\int_{0}^{\boldsymbol{\theta}_{max}} \frac{1}{\alpha} (\boldsymbol{x}_{j}(\boldsymbol{i})_{d}^{*})^{\alpha} d\boldsymbol{i}\right)^{\mu}$$

1.9.9 Mathematica input and plots

fex:=.0015 fin:=.0019 α **:=0.75** μ **:=0.6** ⊖max:=30 f[⊖_]:=1/⊖max F[0_]:=0/0max $inr[\theta_n, n_]:=n F[\theta]$ ⊖[i_,n_]:=i ⊖max/n $x[\Theta_{, X_{, \tau_{}}]:=(\alpha \Theta)^{(1/(1-\alpha))/((1+\tau)^{(1/(1-\alpha))} X^{((1-\mu)/(1-\alpha)))}$ xdi[$\ominus_$, X_]:=($\alpha \ominus$)^(1/(1- α))/(X^((1- μ)/(1- α))) $Plot[x[15,X, 0.1], \{X, 1, 50\}, AxesLabel \rightarrow \{X, x\}]$ х 700⊦ 600 500 400 300 200 100 0 $\frac{1}{50}$ X 10 20 30 40 $\Theta de \left[X_{\tau_{1}} \right] := fex^{((1-\alpha) / \alpha)} X^{((1-\mu) / \alpha)} (1+\tau)^{(1/\alpha)/(\alpha (1-\alpha)^{((1-\alpha)/\alpha)})}$



 $Plot[\{ \Theta de[30, \ \tau], \ \Theta ei[30, \ \tau] \}, \{\tau, 0, 2\}, \ PlotRange \rightarrow \{\{0, \ .5\}, \ \{0, \ 10\}\}, \ AxesLabel \rightarrow \{\tau, \Theta\}]$







FindRoot[Y[1000,X,.2]==X,{X,6}]

100

 $Xm[n_, \tau_]:=ReplaceAll[X,FindRoot[Y[n,X, \tau]-X==0, \{X, 10\}]]$

300

Test[n_, $\tau_$]:={Xm[n, τ],Y[n,Xm[n, τ], τ]}

200

 $Gdom[i_,n_,\tau_]:=(xdi[\Theta[i,n],Xm[n,\tau]])^{\alpha}/(Xm[n,\tau])^{(1-\mu)}-xdi[\Theta[i,n],Xm[n,\tau]]/\Theta[i,n]-0.001$

400

Χ

500

 $Gd[i_,n_,\tau_]:=xdi[\Theta[i,n],Xm[n,\tau]] (1/(\alpha \ \Theta[i,n]))-xdi[\Theta[i,n],Xm[n,\tau]]/\Theta[i,n]-0.001$ $Gin[i_,n_,\tau_]:=xdi[\Theta[i,n],Xm[n,\tau]] (1/(\alpha \ \Theta[i,n]))-xdi[\Theta[i,n],Xm[n,\tau]]/\Theta[i,n]-fin$

```
Gex[i_,n_,τ_]:=(1+τ)^(-1/(1-α)) xdi[\Theta[i,n],Xm[n,τ]] (1/(α \Theta[i,n]))- (1+τ)^(-1/(1-α))
xdi[∂[i,n],Xm[n,τ]]/∂[i,n]-fex
ide[7,Xm[7,0.2],0.2]
EGd[n_,\tau]:=NIntegrate[Gd[i,n,\tau],{i,0,n}]
EGex[n_,\tau_]:=If[ide[n,Xm[n,\tau],\tau]<Min[iei[n,Xm[n,\tau],\tau],n],NIntegrate[Gex[i,n,\tau],{i,ide[n
,Xm[n,τ],τ],Min[iei[n,Xm[n,τ],τ],n]}],0]
EGin[n_,\tau_]:=NIntegrate[Gin[i,n,\tau],{i,Min[n,iei[n,Xm[n,\tau],\tau]],n}]
EG[n_{\tau}] = EGd[n, \tau] + EGex[n, \tau] + EGin[n, \tau]
firms[τ_]:=(N1=1;K=0;While[EG[N1,τ]>0,N1=2 N1;K=K+1];K=K-2;N1=N1-
2^K;While[K>-9,K=K-1;If[EG[N1, z]>0,N1=N1+2^K,N1=N1-2^K]];Return[N1])
N[firms [.1]]
128585/32
EG[23170.091796875`,.1]
-5.98913×10<sup>-7</sup>
N[firms [\tau]]
Equilibrium[\tau_]:=(Ngg=N[firms[\tau]];Xmgg=Xm[Ngg,\tau];Ugg=(1/\mu) (NIntegrate[(1/\alpha)
(xdi[\Theta[i,Ngg],Xmgg])^{\alpha},\{i,0,Ngg\}]+If[ide[Ngg,Xmgg,\tau]<Min[iei[Ngg,Xmgg,\tau],Ngg]+NInt
egrate[(1/\alpha)(x[\Theta[i,Ngg],Xmgg,\tau])^{\alpha},{i,ide[Ngg,Xmgg,\tau],Min[iei[Ngg,Xmgg,\tau],Ngg]}],0]
+NIntegrate[(1/\alpha) (xdi[\partial[i,Ngg],Xmgg])^{\alpha},{i,Min[Ngg,iei[Ngg,Xmgg,\tau]],Ngg}])^{\mu};
 Egg=NIntegrate[ xdi[\partial[i,Ngg],Xmgg] 1/(\alpha \partial[i,Ngg]),{i,0,Ngg}]
   +lf[ide[Ngg,Xmgg,τ]<Min[iei[Ngg,Xmgg,τ],Ngg]+NIntegrate[x[θ[i,Ngg],Xmgg,τ]
1/(\alpha \in [i,Ngg]), \{i,ide[Ngg,Xmgg,\tau],Min[iei[Ngg,Xmgg,\tau],Ngg]\},0]
   +NIntegrate[ xdi[\partial[i,Ngg],Xmgg] 1/(\alpha
\Theta[i,Ngg]),{i,Min[Ngg,iei[Ngg,Xmgg,\tau]],Ngg}];Wgg=Ugg-
Egg;{Ngg,Xmgg,Ugg,Egg,Wgg})
```

Chapter 2

BEST-RESPONSE TARIFFS WITH ENDOGENOUS MARKET SIZE AND ECONOMIC INTEGRATION

2.1 Introduction

Empirical observations of applied tariff levels show that developed countries effectively apply 2.1% on world-wide imports; developing countries even 4.9%.⁶⁵ An approach to justify the application of tariffs uses the concept of optimal tariff setting that builds on the argument that a tariff generates production and consumption distortions. However, it also generates terms-of-trade benefits, depending on the market power of importers.⁶⁶

This paper identifies further rationale to explain the application of ad-valorem tariffs. What is the optimal tariff a social planner determines that maximizes the welfare of countries (i.e., countries behave cooperatively)? What is the optimal best-response tariff if countries behave uncooperatively?

To answer these questions, we have considered the existence of increasing economic integration, emphasizing recent innovations in the trade literature in which heterogeneity in firm productivity has been incorporated into models of monopolistic competition with international trade and MNEs.⁶⁷ In this context, the theoretical work has focused on optimal integration strategies of complex, integrated firms in the presence of firm heterogeneity in terms of total factor productivity.⁶⁸ Firm heterogeneity appears in various layers, such as productivity, size, and integration status.⁶⁹ One key finding is that differences in productivity levels across firms often result in a variety of optimal integration strategies, which result in domestic production, exporting operations and MNE activities being elements of economic trading activities.⁷⁰ Empirical work has shown support for MNE activity being among the most dynamic economic activities, followed by international trade in goods and

⁶⁵ As in UNCTAD (2007a).

⁶⁶ For a survey of seminal contributions, see Torrens (1833), Mill (1844), and Johnson (1954). Latest literature, as in Broda, Limão and Weinstein (2008), are empirical studies of the coherence of market power and tariff setting.

⁶⁷ Initially, models of vertical or horizontal integration strategies of multinational firms were developed under the assumption of homogeneous productivities between all plants in a market. For a survey, see Markusen (1984) and Helpman (1984).

⁶⁸ Compare with Helpman, Melitz and Yeaple (2004).

⁶⁹ As in Clerides, Lach and Tybout (1998).

⁷⁰ As in Bernard et al. (2007).

services.⁷¹ The average annual growth rate of foreign affiliate sales, for instance, was 8.4% during the period of 1996-2000 and even 16.2% in 2006.⁷²

To incorporate the outlined topics of optimal tariff setting with cooperative and noncooperative behavior of countries, firm heterogeneity, and the increasing importance of MNEs, we have set up a numerically solvable model of heterogeneous firms that select their optimal integration strategies from a menu of three options: domestic operations, exporting operations, or horizontal MNE activities.⁷³

Empirical analysis of integration strategies of MNEs has resulted in indirect evidence more in favor of horizontal MNE models more so than vertical MNE models.⁷⁴ It has been assumed that firms in the manufacturing sector supply a variety of differentiated goods under monopolistic competition.

In our model, a social planner cooperatively maximizes welfare of two symmetric countries by endogenously selecting an optimal tariff rate. To contrast this approach of welfare maximization of a benevolent planner, we have determined the best-response tariffs countries that behave uncooperatively may select. Governments in each country spend the generated tariff revenue on a lump-sum transfer to the households in their jurisdictions. Furthermore, the integration strategies that heterogeneous firms select as optimal are affected by the tariff rate. Not only increasing fixed costs, such as market entry costs, but also increasing tariff rates induce exporters to leave the market or to become horizontal MNEs, depending on their productivity levels. Empirical work has emphasized that cuts in tariffs by the United States and Canada have induced a stronger export orientation in some Canadian affiliates of U.S. parent firms.⁷⁵ Further empirical evidence has indicated confirmation that firm decisions to export are dependent on market entry costs and plant heterogeneity.⁷⁶

⁷¹ In 2006, global FDI inflows grew for the third consecutive year and reached the level of \$1.306 trillion, being slightly below the record level of \$1.411 trillion in 2000, as in UNCTAD (2008) and World Bank Institute (2007).

⁷² In the same time, the gross product of foreign affiliates increased 7.3% p.a. during 1996-2000 and rose by 16.2% in 2006. Exports of foreign affiliates showed an increase of 3.3% p.a. in 1996-2000 and rose by 12.2% in 2006. As in UNCTAD (2008).

⁷³ In contrast to Davies and Eckel (2007), assuming mobile firms, and in contrast to Jørgensen and Schröder (2007a) and Jørgensen and Schröder (2007b), not focusing on utility maximization and monopolistic competition.

⁷⁴ As supported by Markusen and Maskus (2001) and Brainard (1993a).

 $^{^{75}}$ As in Feinberg and Keane (2001).

⁷⁶ As in Bernard and Jensen (2004).

The remainder of this paper is structured as follows: Section 2.2 describes the setup of the model and explicitly introduces the preferences of the consumers and the resulting demand in section 2.2.1. Section 2.2.2 presents the production process of heterogeneous firms and the derived optimal integration strategies of firms in the differentiated sector. These depend on the relative size of fixed costs for plant setup, market sizes, firm productivity, transport costs, and ad-valorem tariffs. Section 2.3 describes the behavior of firms and prevailing integration strategies under alternative combinations of transport costs and ad-valorem tariff rates.

After presenting welfare maximization and the objective of governments, we set up a numerical framework in section 2.5. In contrast to related theoretical work, we endogenously derive the mass of firms entering markets as well as the market size itself.⁷⁷ The results in terms of both cooperative and noncooperative behavior in the maximization process are presented in section 2.6. In section 2.7 we flesh out the main differences of this approach relative to recent theoretical work. Finally, section 2.8 contains the conclusion and implications of our findings referring to optimal tariff setting and economic outcome.

2.2 The setup of the model

2.2.1 Demand

In this model, we use a quasi-linear approach to reflect consumers' preferences. Because all consumers share the same preferences, a representative consumer is used to clarify utility. The utility function is represented by:

$$U_{j} = X_{0} + \frac{1}{\mu} \left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} X_{j}(i)^{\alpha} di \right)^{\mu}, j \in \{A, B\}$$
(45)

The representative household in A and B benefits from consumption of the homogeneous good x_o , which is taken as the numéraire for convenience. Each of the two countries hosts a second industry that produces differentiated goods under monopolistic competition. x(i) is the consumption of output of the i-th firm, which is $i \in \{0,...,\theta_{max}\}$.

The condition $0 < \alpha < 1$ being constant results in a constant elasticity of substitution (C.E.S.) of $\sigma = 1/(1-\alpha) > 1$ between any pair of differentiated goods. This expression reflects standard properties of love-for-variety preferences in which a richer supply of

⁷⁷ In contrast see Grossman, Helpman and Szeidl (2006) or Davies, Egger and Egger (2009).

differentiated goods results in increased utility. μ is a constant with $0 < \mu < \alpha < 1$ and reflects the preference for the differentiated industry over the homogenous industry in the utility function of the representative household. At a certain level of differentiated products supplied in one country, an additional unit shows diminishing marginal utility.

The consumption of differentiated products is represented by the expression $X = \left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} x_{j}(i)^{\alpha} di\right)$, the subutility of the differentiated sector. Obviously,

the utility function is linear in x_0 but nonlinear in the differentiated varieties. This implies that the demand for differentiated products depends on prices of differentiated goods but not on earnings. Consumers of different countries show the same love-for-variety preferences and, therefore, apply the same elasticity of substitution σ .

To derive demand for variety $x_j(i)$ of a single household in country j, we consider the utility function in (45) and satisfy the standard side condition $m_j \ge p_0 \cdot x_0 + \int_0^{\theta_{max}} p_j(i) \cdot x_j(i)$. Labor income m is spent on the homogeneous goods, where we set $p_0 = 1$, and on differentiated goods. This results in the demand of a

where we set $p_0 = 1$, and on differentiated goods. This results in the demand of a single household for differentiated goods of⁷⁸

$$\mathbf{x}_{j}(\mathbf{i}) = \frac{1}{\left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} \mathbf{x}(\mathbf{i})^{\alpha} d\mathbf{i}\right)^{\frac{(1-\mu)}{(1-\alpha)}}} \mathbf{p}_{j}(\mathbf{i})^{\frac{1}{(1-\alpha)}}$$
(46)

or

$$p_{j}(\mathbf{i}) = \frac{1}{\left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} \mathbf{x}(\mathbf{i})^{\alpha} d\mathbf{i}\right)^{(1-\mu)}} \mathbf{x}_{j}(\mathbf{i})^{(1-\alpha)}}$$
(47)

The demand of a single household in country j for differentiated goods of the i-th firm depends on the price that firm i sets, on the substitutability of any pair of differentiated goods for another through α , on μ , and on the subutility of consumption

$$X = \left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} x_{j}(i)^{\alpha} di\right).$$
 The impact of an increasing α is that products in the

⁷⁸ See Appendix 2.9.1.

differentiated sector become closer substitutes for one another, which reduces the market power of a single firm. As μ increases, the benefit of differentiated goods decreases. The marginal utility of a further unit of differentiated goods becomes smaller. An increasing X reduces the distribution of single firms as competition between the firms intensifies.

As can be seen from equations (46) and (47), the size of X is determined endogenously. For this reason, X can also be interpreted as the market size for differentiated goods and demands for specification. X depends on the strategic alignment of heterogeneous firms. Therefore, we distinguish between three different scenarios.

In the first case, market size X consists of the market of domestic firms, foreign firms exporting their goods from abroad (henceforth referred to as exporters), and firms choosing horizontal MNE activity. Then, market size X for the representative household is given by:

$$X_{d,ex,i} = \underbrace{\left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} x_{j}(i)^{\alpha} di\right)}_{domestic (d)} + \underbrace{\left(\int_{\theta_{d/ex}}^{\theta_{ex/i}} \frac{1}{\alpha} x_{j}(i)^{\alpha} di\right)}_{exp orter (ex)} + \underbrace{\left(\int_{\theta_{ex/i}}^{\theta_{max}} \frac{1}{\alpha} x_{j}(i)^{\alpha} di\right)}_{MNE(i)}$$
(48)

Alternatively, the export strategy does not exist (i.e., is not profitable); and firms choose either domestic supply or MNE. This scenario results in a market size of:

$$X_{d,i} = \underbrace{\left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} x_{j}(i)^{\alpha} di\right)}_{\text{domestic}} + \underbrace{\left(\int_{\theta_{d/i}}^{\theta_{max}} \frac{1}{\alpha} x_{j}(i)^{\alpha} di\right)}_{\text{MNE}}.$$
(49)

Finally, MNE activity may be nonprofitable so that market size consists of demand from domestic and exporting producers only. The specified market size in this case shows:

$$X_{d,ex} = \underbrace{\left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} x_{j}(i)^{\alpha} di\right)}_{domestic} + \underbrace{\left(\int_{\theta_{d/ex}}^{\theta_{max}} \frac{1}{\alpha} x_{j}(i)^{\alpha} di\right)}_{exporter}$$
(50)

Figure 9 is a visualization of market size under alternative integration strategies of heterogeneous firms.



Figure 9. The market size X, depending on different integration strategies.

2.2.2 Production

Each of the two symmetric small countries, $j=\{A,B\}$, hosts two industries.⁷⁹ The subscript j is the identifier for the country in which the economic activity takes place. We focus on equilibria with diversification of production so that each of the two countries, $j=\{A,B\}$, hosts the two industries. One industry provides a homogeneous good x_0 that is traded under competitive conditions and is the numéraire in this model. Firms in the other industry produce differentiated goods under monopolistic competition. Thereby we implicitly assume that a firm in the differentiated sector does not produce in the homogeneous goods sector.

We assume that Countries A and B are endowed with a fixed amount of internationally immobile labor, L. Because the homogeneous good is freely tradable, used as the numéraire, and uses one unit of L for one unit of output, there is international wage equalization at unitary wages (i.e., $w_j = 1$) as long as diversification of production prevails. This will be the case as long as the numéraire good is produced in every country and can be traded at no cost.

Firms in the differentiated sector can be founded in each country and every firm in the differentiated sector produces a single variety under monopolistic competition. The differentiated goods available in a country j are provided by different sources. Consumers in j buy goods produced by national producers in j, imports from the other country, and goods from subsidiaries in j that have their origin in the other country

⁷⁹ The countries being small imply that they cannot influence prices.

(MNEs). Hence, the mass of firms in the world equals the amount of differentiated goods potentially available. Firms in the differentiated sector differ with respect to their productivity, but ex-ante all these firms are identical. If they expect positive earnings from the production process, they pay sunk entry costs f_d upfront, which are measured in units of labor. As long as firms expect positive profits, they enter the market. It is assumed that the individual productivity levels of the firms in each country are independent draws from a cumulative productivity distribution function $F(\theta)$. The fee f_d allows the firms to independently draw their productivity from the distribution $F(\theta)$ with support over $(0, \theta_{max})$. With this procedure, firms located in the home country are guaranteed to produce domestically, even with very low productivity, to reduce the loss of f_d . The time line in figure 10 shows the logical sequence from the moment prior to entry, when all firms are identical, to the moment when firms in the industry decide their integration strategies and outputs.

0	A-priori, all firms are identical.	Firm expects positive earnings and pays market entry costs f _{d.}	Productivity θ is revealed.	Domestic production and its output are decided.	If θ sufficient, output decision for additional export or MNE activity is made.

Figure 10. Steps towards the choice of integration of the firm.

According to their productivity $\theta(i)$, firms choose their integration strategies. In their domestic countries all firms start as domestic producers. If their productivity is low, firms will not enter the foreign market, neither through exports nor through foreign plant setup. If productivity is high enough, firms have the additional choice to serve foreign markets via exports or foreign affiliate production (the latter being referred to as horizontal MNE activity). The choice between exporting and setting up foreign plants is driven by the proximity-concentration trade-off, characterized by the fact that MNE activity relative to exports saves trading costs as reflected by iceberg transport costs t for cross-border trade of differentiated varieties.⁸⁰ The idea of iceberg transport costs is that to deliver one unit of differentiated goods, the producer must ship t \geq 1 units to the distant point of sale. On the other hand, foreign plant setup has fixed costs f_i in terms of units of labor that are higher than fixed costs for exporters f_{ex}

⁸⁰ See, for example, Horstmann and Markusen (1992), Brainard (1993b), or Markusen and Venables (2000) for a survey.

because production facilities must be duplicated.⁸¹ For this reason, $f_d < f_{ex} < f_i$ is assumed.

Beyond these fixed costs, firms pay variable costs, depending on their productivity levels θ (i) [i.e., x(i)/] θ (i)] and their integration strategies (i.e., exporters pay transport costs t>1). According to $x_i(i)/\theta(i)$, when comparing two firms with the same amount of output in one country, the firm with higher productivity θ (i) must be r lower variable costs.⁸²

Furthermore, governments may choose positive ad-valorem tariff rates τ subject to imports (i.e., the tariff is a percentage of the value of one unit of the imported good). With $\tau_A > 0$, these firms (i.e., exporters from B importing to A) consider τ_A and additional factor influencing profits and vice versa. If tariff revenue in j is positive, it is passed on to households in j as a lump-sum transfer.

Given the preferences in (45), the demand of households in (46), and the price consumption curve in (47) it is straightforward to compute maximum attainable profits of a firm i in j serving its domestic market:⁸³

$$\pi_{j}(i)_{d} = p_{j}(i)_{d} x_{j}(i)_{d} - \frac{x_{j}(i)_{d}}{\theta(i)} - f_{d}$$

$$= \frac{1}{\left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} x(i)^{\alpha} di\right)^{(1-\mu)} x_{j}(i)_{d}^{(1-\alpha)}} x_{j}(i)_{d} - \frac{x_{j}(i)_{d}}{\theta(i)} - f_{d}$$
(51)

The derivative with respect to $x_i(i)_d$ is an expression for the profit-maximizing output of a domestic firm i in its domestic market j, $j \in \{A, B\}$:⁸⁴

$$\mathbf{x}_{j}(\mathbf{i})_{d}^{*} = \frac{(\alpha\theta)^{\frac{1}{(1-\alpha)}}}{\left(\int\limits_{0}^{\theta_{max}} \frac{1}{\alpha} \mathbf{x}(\mathbf{i})^{\alpha}\right)^{\frac{(1-\mu)}{(1-\alpha)}}}_{\max \text{ marketsize } \mathbf{x}}}$$
(52)

associated with the optimal price:⁸⁵

 ⁸¹ As in Helpman, Melitz and Yeaple (2004).
 ⁸² As in Grossman, Helpman and Szeidl (2006); Melitz (2003); Schröder (2007) and Helpman; Melitz and Rubinstein (2007).

 ⁸³ See Appendix 2.9.2.
 ⁸⁴ See Appendix 2.9.2.

⁸⁵ See Appendix 2.9.2.
$$p_{j}(i)_{d}^{*} = \frac{1}{\alpha \theta(i)}$$
(53)

The optimal output of a firm i in its domestic market j depends on market size X.⁸⁶ According to (52), the optimal output of a single firm level is negatively correlated with X due to competitive conditions. Furthermore, the productivity level of a firm is positively correlated with its output.

Because there is monopolistic competition, the market power of a single producer depends on the elasticity of substitution σ between two varieties of differentiated goods. Therefore, firms maximize their profits by charging the mill price $(i.e., p(\theta(i)) = \frac{1}{\alpha} \frac{w}{\theta(i)}$, where w=1 as assumed and $1/\alpha$ reflects the mark-up on the price).⁸⁷ This is the standard markup pricing in which greater elasticity of substitution is associated with a smaller markup. Producers in a market in which differentiated goods are close substitutes associated with a higher α only apply small markups on their prices because their market power is infinitesimally small. Accordingly, maximal attainable profits of a domestic firm i in j are given by:⁸⁸

$$\pi_{j}(\mathbf{i})_{d}^{*} = \frac{(\alpha\theta(\mathbf{i}))^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}\theta(\mathbf{i})} \left(\frac{1}{\alpha} - 1\right) - f_{d}$$
(54)

Analogously, we can now derive profits of firms with an export strategy. Profits of exporters from Country A are defined by:⁸⁹

$$\pi_{A}(i)_{ex} = \underbrace{p_{B}(i)_{ex}(1+\tau)x_{B}(i)_{ex} - \frac{x_{B}(i)_{ex} \cdot t}{\theta(i)} - f_{ex}}_{\text{engage in exports if }>0} + \pi_{A}(i)_{d}$$
(55)

Profits of exporters from Country B are defined by:

$$\pi_{B}(i)_{ex} = \underbrace{p_{A}(i)_{ex}(1+\tau)x_{A}(i)_{ex} - \frac{x_{A}(i)_{ex} \cdot t}{\theta(i)} - f_{ex}}_{\text{engage in exports if > 0}} + \pi_{B}(i)_{d}$$
(55a)

The exporting firm has two sources of earnings: domestic sales and export activity. For a firm i from j, the expression in (55) and (55a) results in optimal output in the other country (output for exporting):⁹⁰

 $^{^{86}}$ The market size X has to be specified according to $X_{d,ex}$, $X_{d,ex,i}$ or $X_{d,i}$. 87 This follows from the derivative of the profit function with respect to the price as in Appendix B.

⁸⁸ See Appendix 2.9.2.

⁸⁹ See Appendix 2.9.3.

⁹⁰ See Appendix 2.9.3.

$$x_{j}(i)_{ex}^{*} = \frac{(\alpha\theta)^{\frac{1}{(1-\alpha)}}}{(1+\tau)^{\frac{1}{(1-\alpha)}} \left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} x(i)^{\alpha}\right)^{\frac{(1-\mu)}{(1-\alpha)}} t^{\frac{1}{(1-\alpha)}}} = \frac{(\alpha\theta)^{\frac{1}{(1-\alpha)}}}{(1+\tau)^{\frac{1}{(1-\alpha)}} X^{\frac{(1-\mu)}{(1-\alpha)}} t^{\frac{1}{(1-\alpha)}}},$$
(56)

associated with the optimal price for exports $q_i(i)_{ex}^* = p_i(i)_{ex}^* (1+\tau)$

1

$$q_{j}(i)_{ex}^{*} = \frac{(1+\tau) \cdot t}{\alpha \theta(i)}$$
(57)

In addition to the previous analysis, one can see that the optimal output and price for exports depend on the tariff τ and on transport costs t in contrast to the optimal output and price when supplying domestic demand. Because raising a tariff results in increased prices for imports $q_j(i)_{ex}^*$, the supply of an exporting firm $x_j(i)_{ex}^*$ decreases. The representative household is not willing to pay any higher price for imported goods to satisfy the love-for-variety preference. Hence, demand for imported goods decreases more than proportional to increases in the tariff τ . Accordingly, maximum attainable profits of an exporting firm i in j are given by:⁹²

$$\pi_{j}(\mathbf{i})_{ex}^{*} = \frac{(\alpha\theta(\mathbf{i}))^{\overline{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}(1+\tau)^{\frac{1}{(1-\alpha)}}\theta(\mathbf{i})t^{\frac{\alpha}{(1-\alpha)}}} \left(\frac{1}{\alpha} - 1\right) - f_{ex} + \pi_{j}(\mathbf{i})_{d}^{*}$$
(58)

Now, it is straightforward to compute maximum attainable profits of a firm engaged in MNE activities. As the firm produces goods for both markets locally, transport costs do not occur. Instead, a firm i from Country A opens an affiliate in B and becomes a horizontal MNE. Profits of an MNE i headquartered in Country A are defined by:⁹³

$$\pi_{A}(i) = \underbrace{p_{B}(i)_{i} x_{B}(i)_{i} - \frac{x_{B}(i)_{i}}{\theta(i)} - f_{i}}_{\text{engage in MNE if >0}} + \pi_{A}(i)_{d}$$
(59)

Analogously, this can be derived for a firm i from B building up a subsidiary in A.

An MNE expects at least zero profits from running both domestic and foreign subsidiaries.

Profit-maximizing plant output of an MNE i headquartered in j is shown by:⁹⁴

⁹¹ See Appendix 2.9.3.

 ⁹² See Appendix 2.9.3.
 ⁹³ See Appendix 2.9.4.

⁹⁴ See Appendix 2.9.4.

$$\mathbf{x}_{j}(\mathbf{i})_{i}^{*} = \frac{\left(\alpha \theta(\mathbf{i})\right)^{\frac{1}{(1-\alpha)}}}{\mathbf{X}^{\frac{(1-\mu)}{(1-\alpha)}}}$$
(60)

associated with the optimal price:95

$$p_{j}(i)_{i}^{*} = \frac{1}{\alpha} \frac{1}{\theta(i)}$$
(61)

Accordingly, maximum attainable profits of an MNE i in j are given by:⁹⁶

$$\pi_{j}(\mathbf{i})_{i}^{*} = \frac{(\alpha\theta(\mathbf{i}))^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}\theta(\mathbf{i})} \left(\frac{1}{\alpha} - 1\right) - f_{i} + \pi_{j}(\mathbf{i})_{d}^{*}$$
(62)

Firms choose their integration strategies based on the knowledge of their productivity levels. This results in cut-off levels being determinants of minimum levels of productivity for a firm i to generate zero profits when ex-ante selecting strategies with more than domestic production. In general, more productive firms are more successful in all three strategies. The least productive firms only serve the domestic market through domestic production. Because of their low productivity, their variable costs are too high so that higher fixed costs to operate in an additional market cannot be covered.

The first cut-off characterizes occurs when the productivity of a firm is such that additional profits of exporting exactly result in zero profits. This can be derived from (55) and applies for:⁹⁷

$$\theta_{d/ex} = \frac{(1+\tau)^{\frac{1}{\alpha}} \cdot f_{ex}^{\frac{(1-\alpha)}{\alpha}} \cdot \left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} x(i)^{\alpha} di\right)^{\frac{(1-\mu)}{\alpha}}}{\alpha(1-\alpha t)^{\frac{(1-\alpha)}{\alpha}}}$$
(63)

The market size X results endogenously, according to X_{d.ex,i}. A firm with productivity $\theta_{d/ex}$ generates zero profits from exporting. Hence, this firm is indifferent between only selling domestically or additionally engaging in exports. A firm with productivity just above this level already earns positive profits from exporting and will definitely engage in exporting. The critical productivity level $\theta_{d/ex}$ is positively correlated with τ ,

 ⁹⁵ See Appendix 2.9.4.
 ⁹⁶ See Appendix 2.9.4.
 ⁹⁷ See Appendix 2.9.5.

t, f_{ex}, and market size X. Hence, the indifferent firm must be more productive to break even. In other words, higher productivity yields lower variable costs of production. Furthermore, conditional on the existence of the export strategy, productivity levels exist that ensure the profits of exporters exceed the profits of MNEs.

This results in the next threshold where profits of an exporting firm equal profits of an MNE (i.e., $\pi(i)_{ex} = \pi(i)_i$).⁹⁸ This applies to the following expression:

$$\theta_{ex/i} = \frac{(f_i - f_{ex})^{\frac{(1-\alpha)}{\alpha}} \cdot X^{\frac{(1-\mu)}{\alpha}}}{\alpha(1-\alpha)^{\frac{(1-\alpha)}{\alpha}} \left(1 - \frac{1}{t^{\frac{\alpha}{(1-\alpha)}}(1+\tau)^{\frac{1}{(1-\alpha)}}}\right)^{\frac{(1-\alpha)}{\alpha}}}$$
(64)

Only firms with $\theta(i) > \theta_{ex/i}$ gain positive profits from serving foreign markets through building subsidiaries instead of exporting their goods. $\theta_{ex/i}$ depends on the difference in fixed costs ($f_i - f_{ex}$) > 0, which can be interpreted as overhead and set up costs of an MNE subsidiary. The higher the overhead costs ($f_i - f_{ex}$) for a foreign subsidiary, the more productive the indifferent firm must be to engage in MNE activity (i.e., the cut-off level $\theta_{ex/i}$ takes a higher value). The higher the transport costs t are, the more likely firms are to engage in the MNE integration strategy. Higher t, therefore, results in a lower value of $\theta_{ex/i}$. As the tariff τ increases, the firm becomes more likely to engage in the MNE strategy, which also results in a lower value of $\theta_{ex/i}$. Furthermore, if $\tau = 0$, the threshold $\theta_{ex/i}$ is infinite. Hence, $\theta_{ex/i}$ is only defined for $\tau > 0$. Intuitively, firms do not engage in MNE activities if the tariff is $\tau = 0$. The MNE strategy does not exist under this constellation.

Additionally, if $t^{\frac{\alpha}{(1-\alpha)}}(1+\tau)^{\frac{1}{(1-\alpha)}} \le 1$ the expression is infinite which can be reduced to $t^{\alpha} \le \frac{1}{(1+\tau)}$. If the parameter configuration of the transport costs t, the tariff τ , and α satisfies the condition, the MNE activity does not exist.

Alternatively, certain configurations of parameters may result in a situation in which domestic firms directly integrate as MNEs instead of choosing the export strategy. The following cut-off level is the relevant productivity threshold when for example, the tariff τ reaches a level at which firms do not choose the export strategy anymore

⁹⁸ See Appendix 2.9.6.

(i.e.,
$$\theta_{d/i} < \theta_{d/ex}$$
) which results in $(1 + \tau) \ge \left(\frac{f_i}{f_{ex}}\right)^{(1-\alpha)} t^{\alpha}$.⁹⁹ The associated cut-off level

results from $\pi_i(i)_i \ge 0$ and is given by:¹⁰⁰

$$\theta_{d/i} = \frac{f_i^{\frac{(1-\alpha)}{\alpha}} X^{\frac{(1-\mu)}{\alpha}}}{(1-\alpha)^{\frac{(1-\alpha)}{\alpha}} \alpha}$$
(65)

The associated market size X in this scenario endogenously results in X_{d,i}. As can be seen, this setup ensures that all three strategies can coexist and are determined by the individual productivity of the firm.

2.3 The behavior of the firms

Consumers in the home country can buy all the goods provided domestically. Furthermore, they can buy goods from foreign firms. Firm decisions to export to foreign markets or to build foreign subsidiaries are dependent on their specific productivity levels associated with corresponding parameter configurations (e.g., the tariff τ , transports costs t, variable costs, and fixed costs). Consequently, we analyze the behavior of firms under different parameter configurations regarding the tariff rate and transport costs.

2.3.1 Integration strategies at t=1 and τ =0

The influence of a tariff on the integration strategies of firms can be demonstrated by analyzing firm behavior at t=1 and $\tau = 0$. In these situations, firms have no advantage to engaging in MNE activities. This strategy requires higher fixed costs (i.e., $f_i > f_{ex}$) without having any further upside for the firms. At $\tau = 0$ and t=1, the cut-off level $\theta_{ev/i}$

in (64) is infinite because
$$\left(1 - \frac{1}{t^{\frac{\alpha}{(1-\alpha)}}(1+\tau)^{\frac{1}{(1-\alpha)}}}\right)^{\frac{(1-\alpha)}{\alpha}} = 0.$$

Hence, no firm with sufficient productivity will engage in MNE activities but will deliver foreign markets via exports.

Figure 11 shows the allocation of foreign firms for t=1 and $\tau = 0$.

⁹⁹ See Appendix 2.9.7. ¹⁰⁰ See Appendix 2.9.7.



Figure 11. The allocation of foreign firms, $\tau = 0$, t = 1.

The productivity level that at least guarantees zero profits for exporters is given by (63) in which, in this special case, t=1 and $\tau = 0$. Hence, the threshold requires a productivity that is able to cover the fixed costs f_{ex} . Furthermore, the productivity of the firm takes the market size X into account. Referring to (46), an increasing market size results in a decreasing demand for goods of the i-th firm. Hence, the indifferent firm must be more productive. In this scenario, consumers benefit from a maximum of available differentiated products. Thus, this situation results in a scenario in which foreign goods are solely imported.

2.3.2 Integration strategies at t=1 and τ >0

A tariff is an additional decision parameter when firms choose their integration strategies. $\tau > 0$ decreases consumers' demand for imports, which results in an increasing value of the cut-off level $\theta_{d,ex}$. In contrast to the scenario at $\tau = 0$, the indifferent firm must be more productive to break even. The tariff forces low-productivity firms to exit the export strategy and harms consumers by supplying the market with fewer varieties.

The tariff also affects exporters with higher productivity levels. Highly productive exporters are now in favor of engaging in MNE activities. Although fixed costs associated with MNEs are higher than in the export strategy ($f_{i>f_{ex}}$), the MNE activity bypasses the tariff. Hence, consumers do not face distorted prices that affect their decisions. In contrast to MNEs, firms in the export strategy face reduced demand due to the tariff.

The determined cut-off levels (63) and (64) give information about the allocation of firms utilizing the different integration strategies. From the perspective of the home country, all firms located in the home country manufacture differentiated goods as domestic producers. Figure 12 shows the allocation of foreign firms among the international strategies.



Figure 12. The allocation of foreign firms, $\tau > 0$ and t = 0.

Firms desiring to export goods must be at least as productive as the cut-off level determined in (63) with t=1. The next threshold as determined in (64) gives the productivity level of the highest-productivity exporting firm, again with t=1. Above this level, firms engage in MNE activities to satisfy demand.

2.3.3 Integration strategies with t>1 and τ =0

This scenario describes a situation in which exporting, in contrast to MNE activity, is linked to transport costs t>1.Transport costs are modelled as iceberg costs (i.e., t>1), which increase variable costs of the firm. The considerations of firms according their integration strategies include consequences arising from the different cost structures. According to their integration strategies, firms face the proximity-concentration trade-off.¹⁰¹ Foreign plant setup requires higher fixed costs because production facilities are duplicated but saves trading costs as modelled by t.

The barrier that separates exporting firms from MNEs is reflected by the productivity threshold in (64) and, in this case, is given by:

$$\theta_{ex/i} = \frac{(f_i - f_{ex})^{\frac{(1-\alpha)}{\alpha}} \cdot X^{\frac{(1-\mu)}{\alpha}}}{\alpha(1-\alpha)^{\frac{(1-\alpha)}{\alpha}} \left(1 - \frac{1}{t^{\frac{\alpha}{(1-\alpha)}}}\right)^{\frac{(1-\alpha)}{\alpha}}}$$
(64a)

An increase in t makes the export strategy more expensive and, hence, stimulates firms to engage in MNE activity, as $\frac{\partial \theta_{ex/i}}{\partial t} < 0$.

Furthermore, low productivity-exporters are forced to leave the market because consumers' demand decreases due to higher prices caused by an increase in transport costs t. The associated cut-off level, as in (63) in the situation at t>1 and τ =0, shows

¹⁰¹ See, for example, Horstmann and Markusen (1992), Brainard (1993b), or Markusen and Venables (2000) for a survey.

$$\theta_{d/ex} = \frac{f_{ex}^{\frac{(1-\alpha)}{\alpha}} \cdot \left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} x(i)^{\alpha}\right)^{\frac{(1-\mu)}{\alpha}}}{\alpha(1-\alpha t)^{\frac{(1-\alpha)}{\alpha}}} \text{ and it is } \frac{\partial \theta_{d/ex}}{\partial t} > 0 \,.$$

Additionally if $\frac{1}{\alpha} = t$ holds, the expression is infinite and the export strategy disappears. Then, certain parameter configurations may result in a scenario in which $\theta_{d/i} \leq \theta_{d/ex}$ holds. This results in a situation in which domestic firms directly integrate their firms as MNEs.

2.3.4 Integration strategies at t>1 and τ >0

At t>1 and τ >0, firms in the export strategy face two effects reducing the demand of the single firm and, therefore, their profits $\pi_j(i)_{ex}$. Both, the tariff τ and the transport costs t increase consumers' prices for imported differentiated goods. This decreases household demand for those goods, which forces producers to reduce output. In this scenario, both international strategies potentially can coexist.

Comparative statics of the relevant cut-off level in (64) with respect to the tariff τ and the transport costs show the direction of the effects an increase of the tariff τ or the transport costs t have. The derivative of $\theta_{ex/i}$ with respect to t is given by:

$$\frac{\partial \theta_{ex/i}}{\partial t} = -\frac{(f_i - f_{ex})^{\frac{(1-\alpha)}{\alpha}} X^{\frac{(1-\mu)}{\alpha}} (1-\alpha)(t\alpha - 1)^{\frac{(1-\alpha)}{\alpha}-1} (1+\tau)^{\frac{1}{\alpha}}}{\left((1-\alpha)^{\frac{(1-\alpha)}{\alpha}} \alpha + (t\alpha - 1)^{\frac{(1-\alpha)}{\alpha}}\right)^2} < 0$$
(66)

For the expression to be finite, it must hold that $t\alpha > 1$. Furthermore, the expression is negative, which intuitively is support for the notion that MNE activity becomes more attractive with an increase in t.

The effect an increase of the tariff τ has on the productivity threshold can be demonstrated by finding the derivative of (64) subject to τ , which is given by:

$$\frac{\partial \theta_{\text{ex/i}}}{\partial \tau} = \frac{\left(f_{\text{i}} - f_{\text{ex}}\right)^{\frac{(1-\alpha)}{\alpha}} X^{\frac{(1-\mu)}{\alpha}} (1+\tau)^{\frac{(1-\alpha)}{\alpha}}}{\alpha \left(\left(1-\alpha\right)^{\frac{(1-\alpha)}{\alpha}} \alpha + \left(t\alpha - 1\right)^{\frac{(1-\alpha)}{\alpha}}\right)} < 0$$
(67)

Again, it must hold that $t\alpha > 1$ for the derivative to be defined. As the tariff τ increases, MNE activity becomes more likely, which decreases the cut-off level as seen in (64).

2.4 Welfare of the representative household and decisions of governments

The objective of governments is to evaluate the effects of ad-valorem tariffs on household welfare. Such policies evoke reactions in the integration strategies of firms, influencing the utility of the household. Therefore, the implications of these reactions must be considered in the welfare-maximization process. Anticipating the behavior of firms, governments maximize welfare of their consumers by endogenously determining ad-valorem tariffs τ . Because this model assumes two small, symmetric countries, this paper analyzes the utility-maximizing process of only one country. The objective of a government is to maximize the expression $\frac{\partial U}{\partial r}$ to find the level of the tariff rate τ maximizing welfare. In the following, the relevant effects are described for the general case that the domestic, the export, and the MNE strategies exist. This implies the relevant limits of integration to be $\theta_{d/ex}$ as the lower and $\theta_{\text{ex/i}}$ as the upper limits of integration. Of course, the market size X may endogenously have different outcomes and must be specified according (48), (49), or (50). Furthermore, to guarantee a continuous solution, firms are ranked according to their individual productivity, starting with low-productivity firms. Hence, we integrate over i, thereby considering $F(\theta)=i/n$ and $\theta(i)=F^{-1}\left(\frac{i}{n}\right)$, respectively $\theta(i)=\frac{i}{n}\overline{\theta}$ or *(*.)

$$i = \frac{n\theta(i)}{\overline{\theta}}$$
.

The utility of households is positively dependent on consumption of the numéraire and on all the other goods. This monetary effect is given by the difference of labor income m and expenses for differentiated goods. Without transfer, utility from x_0 is shown by:

$$m_{j} - \int_{0}^{\theta_{max}} p_{j}(i)_{d}^{*} x_{j}(i)_{d}^{*} di + \int_{\theta_{d/ex}}^{\theta_{ex/i}} q_{j}(i)_{ex}^{*} x_{j}(i)_{ex}^{*} di + \int_{\theta_{ex/i}}^{\theta_{max}} p_{j}(i)_{i}^{*} x_{j}(i)_{i}^{*} di$$
(68)

In contrast to the scenario at $\tau = 0$, the government in j generates revenues that are transferred to consumers in j. This monetary effect enables consumers in j to buy more of the numéraire good x_0 , having a positive effect on utility. Considering the utility function in (45) and the profit-maximizing output of exporters in (56), this effect is given by:

$$\int_{\theta_{d/ex}}^{\theta_{ex/i}} \frac{\left(\alpha\theta(i)\right)^{\frac{1}{(1-\alpha)}}}{\left(1+\tau\right)^{\frac{1}{(1-\alpha)}} X^{\frac{(1-\mu)}{(1-\alpha)}} t^{\frac{1}{(1-\alpha)}}} \cdot \tau \, di$$
(69)

The governments in A and B generate tariff revenues for their consumers on all imports to their home countries. The effect applies to expression (68), considering the cut-off levels (63) and (64). All imported goods in the area defined by the cut-off levels are subject to the trade barrier.

The next utility generating effect stems from consumption of domestic goods, which is shown by:

$$\frac{1}{\mu} \left[\int_{0}^{\theta_{max}} \frac{1}{\alpha} \left(\frac{(\alpha \theta(i))^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}} \right)^{\alpha} di \right]^{\mu} .$$
(70)

Furthermore, the utility of the consumers in A and B is constituted by consumption of imported goods. This is depicted by:

$$\frac{1}{\mu} \left[\int_{\theta_{d/ex}}^{\theta_{ex/i}} \frac{1}{\alpha} \left(\frac{\left(\alpha\theta(i)\right)^{\frac{1}{(1-\alpha)}}}{\left(1+\tau\right)^{\frac{1}{(1-\alpha)}} X^{\frac{(1-\mu)}{(1-\alpha)}} t^{\frac{1}{(1-\alpha)}}} \right)^{\alpha} di \right]^{\mu}$$
(71)

By maximizing the utility, the governments in A and B consider the ambiguous effect of the tariff τ . On the one hand, an increase in the tariff τ stimulates highly productive exporters to engage in MNE activity. This makes the affected varieties cheaper because the goods of MNEs are not subject to the trade barrier. On the other hand, low-productivity exporters are forced to leave the market because their productivity θ (i) is not sufficient to break even. The products of these suppliers are no longer available for consumers, which has a negative influence on the utility of households.

The next utility generating element is represented by the consumption of goods of MNEs. In formal accounts, this effect can be described by the following expression:

$$\frac{1}{\mu} \left[\int_{\theta_{ex/i}}^{\theta_{max}} \frac{1}{\alpha} \left(\frac{\left(\alpha \theta(i) \right)^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}} \right)^{\alpha} di \right]^{\mu}$$
(72)

The household demand for goods of MNEs is not directly affected by the tariff τ . However, the tariff rate τ influences consumers' utility because the lower limit of integration decreases with an increase in τ . Hence, the area described by the integral increases, which increases utility.

All the effects that have influence on the utility of the representative household can be summarized by the following expression:¹⁰²

$$U_{j} = m_{j} - \left(\int_{0}^{\theta_{max}} p_{j}(i)_{d}^{*} x_{j}(i)_{d}^{*} di + \int_{\theta_{d/ex}}^{\theta_{ex/i}} q_{j}(i)_{ex}^{*} x_{j}(i)_{ex}^{*} di + \int_{\theta_{ex/i}}^{\theta_{max}} p_{j}(i)_{i}^{*} x_{j}(i)_{i}^{*} di\right) + \int_{\theta_{d/ex}}^{\theta_{ex/i}} x_{j}(i)_{ex}^{*} \cdot \tau di$$

$$+ \frac{1}{\mu} \left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} (x_{j}(i)_{d}^{*})^{\alpha} di + \int_{\theta_{d/ex}}^{\theta_{ex/i}} \frac{1}{\alpha} (x_{j}(i)_{ex}^{*})^{\alpha} di + \int_{\theta_{ex/i}}^{\theta_{max}} \frac{1}{\alpha} (x_{j}(i)_{i}^{*})^{\alpha} di\right)^{\mu}$$
(73)

or

$$\begin{aligned} & (74) \\ U_{j} = m_{j} - \left(\int_{0}^{\theta_{max}} \frac{1}{\alpha\theta(i)} \frac{(\alpha\theta(i))^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{1-\alpha}}} di + \int_{\theta_{d/ex}}^{\theta_{ex/i}} \frac{(1+\tau)t}{\alpha\theta(i)} \frac{(\alpha\theta(i))^{\frac{1}{(1-\alpha)}}}{(1+\tau)^{\frac{1}{(1-\alpha)}} X^{\frac{(1-\mu)}{1-\alpha}}} di + \int_{\theta_{ex/i}}^{\theta_{max}} \frac{1}{\alpha\theta(i)} \frac{(\alpha\theta(i))^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{1-\alpha}}} di \right) \\ & + \int_{\theta_{d/ex}}^{\theta_{ex/i}} \frac{(\alpha\theta(i))^{\frac{1}{(1-\alpha)}}}{(1+\tau)^{\frac{1}{(1-\alpha)}} X^{\frac{(1-\mu)}{1-\alpha}}} \cdot \tau di \\ & + \frac{1}{\mu} \left[\int_{0}^{\theta_{max}} \frac{1}{\alpha} \left(\frac{(\alpha\theta(i))^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{1-\alpha}}} \right)^{\alpha} di + \int_{\theta_{d/ex}}^{\theta_{ex/i}} \frac{1}{\alpha} \left(\frac{(\alpha\theta(i))^{\frac{1}{(1-\alpha)}}}{(1+\tau)^{\frac{1}{(1-\alpha)}} X^{\frac{(1-\mu)}{1-\alpha}}} \right)^{\alpha} di + \int_{\theta_{ex/i}}^{\theta_{max}} \frac{1}{\alpha} \left(\frac{(\alpha\theta(i))^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{1-\alpha}}} \right)^{\alpha} di \right]^{\mu} \end{aligned}$$

To maximize the expression in (74), we must find the derivative subject to the tariff τ . In this expression we also consider that $\theta(i) = \frac{i}{n}\overline{\theta}$ and that the market size has different outcomes, depending on the integration strategies heterogeneous firms

- - >

 $^{^{102}}$ For this constellation to exist, it must hold that $\theta_{d/ex} < \theta_{ex/i}$ and $\tau \neq 0$, which ensures that $\theta_{ex/i}$ is finite. Furthermore, the utility of the household under alternative parameter configurations resulting in $X_{d/ex}, X_{d/i}$, and X_d is shown in Appendix 2.9.8.

choose. This endogeneity of the market size X results in a situation in which every parameter configuration results in a different level of X. The mass of firms n in equilibrium as well is endogenous and varies with the associated dependent variables.

Because profits are competed to zero, expected profits are depicted by:¹⁰³

$$E\pi_{j} = \int_{0}^{\theta_{max}} \frac{(\alpha\theta(i))^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}\theta(i)} \left(\frac{1}{\alpha} - 1\right) - f_{d} + \int_{\theta_{d/ex}}^{\theta_{ex/i}} \frac{(\alpha\theta(i))^{\frac{1}{(1-\alpha)}}}{(1+\tau)^{\frac{1}{(1-\alpha)}}X^{\frac{(1-\mu)}{(1-\alpha)}}t^{\frac{1}{(1-\alpha)}}\theta(i)} \left(\frac{1}{\alpha} - 1\right) - f_{ex}$$

$$+ \int_{\theta_{ex/i}}^{\theta_{max}} \frac{(\alpha\theta(i))^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}\theta(i)} \left(\frac{1}{\alpha} - 1\right) - f_{i}$$
(75)

Both variables, the market size X and the mass of firms n in equilibrium, are interdependent and induce further interdependences to other equilibrium-determining expressions (i.e. the cut-off levels and the household demand).

Even without the complexity induced by the linkages of the different variables, the maximization of the expression above, $\frac{\partial U_j}{\partial \tau}$, results in a problem with dimensionality

higher than fourth degree.¹⁰⁴ These aspects preclude an analytical solution of $\frac{\partial U_j}{\partial \tau}$ and suggest using numerical analysis to determine the welfare-maximizing tariff rate

 $\boldsymbol{\tau}_i$ and its interactions with other variables.

2.5 The setup of the numerical framework

To derive a solution to this problem and to find a welfare maximizing expression for the tariff rate τ_j we use Mathematica 6.0. This program is utilized to set up the numerical framework that represents the theory of the model as derived in previous sections.¹⁰⁵

2.5.1 Definitions

The coding of the numerical framework begins with defining variables and making assumptions. Analogously to the assumptions of the model, the fixed costs, θ_{max} , as

¹⁰³ Expected profits result endogenously, according to $X_{d/ex/l}$, $X_{d/ex}$, $X_{d/l}$, or X_d . Furthermore, all firms in the market together generate zero profits. Low-productivity firms produce to minimize the loss of f_d . ¹⁰⁴ This is proved by the theory of E. Galois. For a survey, see Taton (1983) and Stroth (1998).

¹⁰⁵ See Appendix 2.9.9 for the full input sheet.

well as α and μ , are set to constant numerical values, considering $f_d < f_{ex} < f_i$ and $0 < \mu < \alpha < 1$.

After paying market-entry costs of f_d , firms draw their individual productivity levels θ . We apply a uniform distribution of the firms over θ , specified as $F[\Theta_{-}]$. The distribution function is defined piecewise to ensure that $F[\Theta_{-}]$ takes the value 0 if the distribution is not reached and takes the value 1 in the boundaries of θ_{min} and θ_{max} . To guarantee continuous results, firms are ranked according to their individual productivity, starting with low-productivity firms. This is reflected by the expression $\theta[i_{-},n_{-}]$. The productivity of a single firm $\theta[i_{-},n_{-}]$ depends on the rank i of the i-th firm, given a mass of firms in the economy n.

For further analysis, a function is computed to provide the rank of indifferent firms between two strategies. This expression is given by $inr[\theta_n,n_n]$ and reports the rank of the firms, given the productivity θ and the mass of firms n founded in the country.

The demand of the representative household as in (46), results in optimal output for the firms, as derived previously. Therefore, the computation of the profit-maximizing output is represented by $x[\theta_, x_, \tau_, t_]$. The optimal output of a firm i depends on its productivity θ , the market size X, the tariff τ , and the transport costs t. As the tariff τ is only relevant for the export strategy, we must consider $\tau=0$ for the domestic and the MNE strategies.

The choice of integration strategy is driven by cut-off productivity thresholds. The first threshold separates domestic producers from exporters from j and is computed as $\Theta de[X_, \tau_, t_]$, considering X and τ in the country in which the differentiated goods are sold. The associated firm number is reported by $ide[n_, X_, \tau_, t_]$. The expression calculates the rank of the indifferent firm in terms of productivity, depending on the endogenous mass of firms n in j, the endogenous market size in the other country, the tariff rate τ , and the transport costs t. For example, by entering ide[25000, 5000, 0.03, 1.04], the system calculates the rank of the indifferent firm ide with cut-off productivity $\theta_{d/ex}$ to be the 20515.2nd firm, given a mass of 25000 firms in this country, a market size of 5000, a tariff of 3%, and transport costs of 1.04. Hence, it is the 20515.3rd firm out of 25000 that exports for sure.

Analogously, we compute the threshold productivity $\theta_{ex/i}$ as $\Theta ei[X_, \tau_, t_]$ with the associated rank of the firm producing with cut-off productivity $\theta_{ex/i}$ as $iei[n_, X_, \tau_, t_]$. This expression considers the mass of firms in the country j and X, τ , and t where the economic activity takes place. The same notion is used to code the cut-off level $\theta_{d/i}$ as $\Theta di[X_, \tau_, t_]$ linked to $idi[n_, X_, \tau_, t_]$.

2.5.2 Consistency of market size X

The market size X depends on the mass of firms n, the tariff τ , the household demand x(i) and is endogenously given by $X = \left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} x_{j}(i)^{\alpha}\right)$, with its adequate specification dependent on the strategies represented in the economy. The inclusion

of the endogenously defined market size X from a demand perspective, as in section 2.2.1, in the numerical model does not result in consistency, which is needed to derive results. The proof of inconsistency starts with computing the market size of country j from a supply perspective for firms active in the different strategies in j, $j \in \{A,B\}$ (i.e., Yd [supply of domestic firms from A and vice versa from B], Yex [supply of exporting firms from B and vice versa from A] and Yi [supply of MNEs with origin in B and vice versa with origin in A]). The market size for domestic producers in A, referring to the representative household, shows $Yd[nA_, X_, \tau_, t_]$. Yd depends on the mass of firms in the domestic market nA, the market size x in A, the tariff τ , and the transport costs t. It is characterized by the integral over the output of all domestic firms i.

Analogously, we compute the market size of firms that export from Country B to Country A. From the perspective of firms producing in B and exporting to A, the export market, referring to the representative household in A, is given by $Yex[nB_, X_, \tau_, t_]$. The size of the export market of firms from Country B in Country A depends on the mass of firms located in Country B, as given by nB, on the market size x in Country A, the tariff τ , and the transport costs t. The definition of Yex in the numerical analysis also considers the scenario that possibly no exporters exist.

The market size for MNEs in terms of the representative household is defined by $Yin[nB_, X_, \tau_, t_]$. The market for MNEs in Country A depends on the mass of

firms being located in Country B, nB, the market size x in A, the tariff τ in A, and the transport costs t. Also, the coding includes conditions to guarantee that the system integrates correctly regarding prevailing integration strategies. The entire market size from a supply perspective, referring to the representative household in A, is determined as the sum of all three market segments and is represented by:

 $Y[nA_,nB_,X_,\tau_,t_]$:=Yd[nA,X, τ ,t]+Yex[nB,X, τ ,t]+Yin[nB,X, τ ,t]

Inconsistency in market size will result in differing outcomes regarding market size from both supply (Y) and demand (X) perspectives. If the configuration is consistent, we may expect a result, for example, of Y=5000 if X=5000. However, in using the code defined previously and inserting X=5000, Y=5000 does not necessarily occur. For example, Y[20000,20000,5000,1,0.03,1.03] results in a market size Y= 4820.77. The inconsistency in market size is clarified in figure 13.



Figure 13. The inconsistency of the market size.

The inconsistency of the market size in figure 13 is apparent. The curve of the market size X has a different progression than the curve Y. The intersection of both curves gives the true market size for the given values.

To achieve the essential consistency of market size, the computation uses a quasi-Newton method, which is computed as $Xm[nA_,nB_,\tau_,t_]$.¹⁰⁶ The method is named quasi-Newton because we use an approximation for the slope, using the

¹⁰⁶ As in Spelucci (1993) and Knorrenschild (2008).

gradient of the secant of the function x_m for which we search. Figure 14 is a visualization of this method:¹⁰⁷



Figure 14. Visualization of the Quasi-Newton method.

The program is coded to find the null, starting with calculating the secant at a value of 1000 (in figure 14, this corresponds to x_0) and assuming a width of 20 (in figure 14, this corresponds to the second value x_1). The slope of the secant results in a null, which is the next starting value (in figure 14, this corresponds to x_2). The slope of the secant associated with this new starting value gives a new null (in figure 14, this corresponds to x_3). This iteration is repeated until the exact null is found. Meanwhile, the width in which boundaries the slope of the secants is calculated is reduced stepwise.

2.5.3 The mass of firms n in equilibrium

Firm decisions to enter the market are based on the expectation of future profits. Heterogeneous firms enter production as long as their future earnings expectations are positive. Hence, the mass of firms in the market is determined by expected profits being equal to 0. After computing the profit function of single firms i with their particular strategies, expected profits are determined by the profits of all firms in the specific market.

¹⁰⁷ As in Knorrenschild (2008).

For firms selecting a domestic strategy in Country A, we apply $Gd[i_,nA_,nB_,\tau_,t_]$. We must consider that the tariff τ must be set to 0 because the trade barrier is not relevant in the domestic market. The same applies for transport costs t, which must be set to 1. To compute the profit of a firm from A with rank i, we must consider the mass of firms in A and B because of competitive conditions.

The profit of a firm in the export strategy is computed as $Gex[i_,nA_,nB_,\tau B_,t_]$. Profits of exporting firms i from Country A to Country B depend on the mass of firms in Countries A and B, nA and nB, the tariff being applied in Country B (i.e., τB), and transport costs t.

The profit function of MNEs originally located in Country A with subsidiaries in B is coded as $Gin[i_,nA_,nB_,\tau B_,\tau B_,t_]$. Profits of a MNEs, i, from Country A (being MNEs) in Country B are also dependent on the mass of firms in A and B, nA and nB, the tariff rate τB , and the transport costs t.

Expected profits (EG) in an economy result from the integration of profits over all firms in the different strategies. For Country A, they are given by:

 $EG[nA_,nB_,\tau A_,\tau B_,t_] := EGd[nA,nB,\tau A,t] + EGex[nA,nB,\tau B,t] + EGin[nA,nB,\tau B,t].$

The computation of expected profits includes conditions to ensure that profits are only integrated if the associated strategy exists.

Finally, coding the mass of firms in equilibrium results from firms entering the market competing the expected profits to zero. For firms in A, this is given by:

Firms [nB_, τ A_, τ B_, t_]. The process to find the null is coded with the instruction to test several values in defined steps. After determining the first negative value of expected profits, the program returns to approach the null exactly, while the width of the steps is permanently reduced. For example, the instruction to calculate the mass of firms in equilibrium in Country A given nB=20000, τ A=3%, τ B=3%, and transport costs t=1.03 is depicted by Firms[20000,0.03,0.03,1.03]. The example results in ≈25577 firms in Country A given 20000 firms in Country B, with associated expected profits of -4.06419×10⁻⁶. This is support for the previously described method.

2.5.4 The maximization of welfare

Considering the utility function in (45), households in the two symmetric countries benefit from consumption of homogeneous goods x₀ and differentiated goods. To implement the utility maximization process, we must first consider the stand-alone contribution of x_0 . Without the existence of a differentiated sector, the representative household only generates utility by consuming x₀. Hence, the benefit of one unit of differentiated goods is constituted by its net contribution (i.e., additional utility versus additional costs). The computation of the equilibrium utilizes this notion to implement the utility-maximizing process. The equilibrium of this model is labelled Equilibrium [τA_{τ} , τB_{τ} , t_] and is dependent on the given value of the tariff τ in Countries A and B and the transport costs t. The equilibrium is computed so that the system delivers data that describe the equilibrium. Given the exogenous variables, the system endogenously determines the equilibrium mass of firms in A and B. The tariff rates being unequal (i.e., $\tau_A \neq \tau_B$) results in the mass of firms differing in both countries $n^{A} \neq n^{B}$. The programming of the mass of firms in equilibrium is visualized in figure 15.



Figure 15. The computation of the mass of firms, $\tau^{A} \neq \tau^{B}$.

In the first step, the computation to find the correct value for the mass of firms begins with nB=1 and searches for the corresponding mass of firms in Country A conditioned on nB=1 [i.e., nA(nB)]. In the figure, this is denoted as a. Given nA(1) firms in Country A, the iteration proceeds by calculating the associated mass of firms

in Country B, denoted as a*. Analogously, we assume nA=1 and search for the associated value for the mass of firms in B, denoted as b [i.e., nB(nA)]. Given nB(1) firms in B, the system calculates the corresponding mass of firms in A, denoted as b*. The intersection of the two resulting graphs gives the new starting value, denoted as C. This loop is repeated until the difference between the new starting value minus the old starting value ≤ 10 (i.e., nB3-nB0 ≤ 10) in the program or C in the figure. The computation proceeds by using calculations of expected profits for Country A and B. The system calculates expected profits and all further key figures for the different possible integration strategies and sums them up afterwards. Hence, EGewA denotes expected profits in A; EGewB denotes expected profits in B.

The next relevant variable is the consistent market size for Country A, which is calculated using $Xm[nA_, nB_, \tau_, t_]$ as defined in section 2.5.2, and analogously for B. The results show the contribution of the different strategies to total market size and separately for Countries A and B. For example, the share of output of all domestic firms in its market in A is computed as $YDA=Yd[NA, XMA, \tau A, t]$, where capital letters denote equilibria values. The expression is dependent on the mass of firms in Country A, NA; on the overall market size in A, XMA; and on the transport costs t.¹⁰⁸

The sum of expenses for differentiated goods in the representative household in A is represented by MA=MYDA+MYEXA+MYINA. MYDA=MYd[NA,XMA,TA,t] denotes expenses of the representative household in A for goods from domestic producers,

MYEXA=MYex[NB,XMA, τ A,t] are expenses for imports from Country B to country A consumed by households in Country A, and MYINA=MYin[NB,XMA, τ A,t] is the calculation for expenses for MNE goods from B of the representative household in Country A. The analogous notion is used to compute expenses for households in Country B, (MB).

Equilibria and, therefore, welfare are constituted by the utility of consumption of differentiated goods. Again, to determine utility, we distinguish between Countries A and B and between the different strategies. For Country A, we compute the utility of consumption of differentiated goods from domestic firms, from imports from Country B, and from MNEs in A originally located in B. Then, the overall utility of the

¹⁰⁸ Analogously, we compute YEXA, YINA for the market size of the export and the MNE strategy in Country A. The analogue computation for Country B is given as YDB, YEXB, and YINB.

representative household from differentiated good consumption in A is given, dependent on the sum of the three subfunctions:

UA=1/ μ (YCDA+YCINA+YCEXA) $^{\mu}$, referring to (45) and respectively for Country B. Finally, welfare in A is given as WA=UA-MA for Country A and WB=UB-MB for Country B.

2.6 The results of numerical analysis

Using the knowledge of the behavior of firms concerning their integration strategies, governments maximize welfare (measured per capita) of the representative households in their jurisdictions by optimally choosing a tariff τ . Therefore, we examine equilibria of the model resulting from a variation of the tariff τ . All things being equal, this results in equilibria for each of the two countries, A and B.

To analyze the numerical output, we focus on the mass of firms in equilibrium labelled as N in the following tables, thereby indicating the mass of differentiated goods in the country. Furthermore, we focus on the consistent market size Xm and its contribution by the output of firms selecting different integration strategies.

Table 3 is a summary of equilibria of the model using different levels of the tariff τ assuming that the governments in A and B behave cooperatively.

Tariff	Ν	Xm	Xd	Xex	Xi	U	E	W=U-E	international strategies
0	23662	5130	3172	1958	0	280.5	126.22	154.27	exporter
0.01	23601	5081	3200	1881	0	278.89	125.04	153.85	exporter
0.02	23567	5036	3230	1806	0	277.39	123.95	153.44	exporter
0.03	23275	4998	3219	1132	647	276.14	123.45	152.70	exporter, multinational
0.04	23056	4980	3203	485	1292	275.54	123.53	152.01	exporter, multinational
0.05	23025	4974	3203	90.7	1680	275.33	123.79	151.54	exporter, multinational
0.06	23040	4973	3206	0	1768	275.32	123.89	151.43	multinational
0.07	23040	4973	3206	0	1768	275.32	123.89	151.43	multinational
0.08	23040	4973	3206	0	1768	275.32	123.89	151.43	multinational
0.09	23040	4973	3206	0	1768	275.32	123.89	151.43	multinational
0.1	23040	4973	3206	0	1768	275.32	123.89	151.43	multinational
0.11	23040	4973	3206	0	1768	275.32	123.89	151.43	multinational

Table 3. Symmetric results, $\tau^{A} = \tau^{B}$, nA=nB, t=1.01.

Here, the welfare maximum is reached if both Countries A and B choose $\tau = \tau^{A} = \tau^{B} = 0$. Welfare of the representative household in both countries is W=WA=WB=154.27. Furthermore, the equilibrium at $\tau = 0$ is characterized by a mass of firms of N=NA=NB=23662 and a market size of Xm=XmA=XmB=5130. In line with theoretical findings, foreign differentiated goods are imported and the MNE strategy

does not exist. Increasing tariff rates τ results in a decrease of the mass of firms N in equilibrium at the expense of exporters, accompanied by stimulation of MNEs and a decrease in welfare.

At $\tau = 0.03$, highly productive exporters change their integration status and found MNEs. In the range $\tau = 0.03$ to $\tau = 0.05$ both international strategies coexist. Where exporters are increasingly squeezed out of the market in this range, founding MNEs becomes more attractive. At the tariff of $\tau = 0.06$, protectionism has reached a level where the export strategy does not exist anymore and foreign differentiated goods are provided by MNEs. Welfare of the representative household is at W=151.43 in both Countries A and B. Because exporters have already disappeared at $\tau = 0.06$, an increase in the tariff rate τ is ineffective. Hence, welfare remains at W= 151.43, independent of increasing tariff levels τ .

Because policy makers of Country A attempt to maximize the welfare of households in their country, they do not consider welfare in Country B and vice versa (i.e., they behave noncooperatively). Given any certain tariff level τ^{B} , there is incentive to determine the welfare-maximizing best-response tariff τ^{A} and vice versa.

We implement this approach in the numerical model to find equilibrium because the mass of firms in A (i.e., NA) settles at a level that guarantees expected profits in both countries being competed to zero. The same iteration is repeated for the second country. Using the knowledge of the existing masses of firms in both countries given the tariff rates τ_A and τ_B , we determine equilibrium-describing variables and study welfare. Given the condition of consistency regarding NA and NB, with the outcome of numerical analysis in terms of welfare in A and B, we derive Nash equilibria concerning the tariff rates τ_A and τ_B .¹⁰⁹ This can be done for any combination of full-percentage tariff rates.

¹⁰⁹ Furthermore, tests of stability of this model confirm this notion of equilibrium. A convergence of equilibria still appears if firms in A and B alternately enter and exit the market. For a survey, see Nash (1951).

2.6.1 Best-response tariffs

Given a scenario where Country B chooses $\tau_B = 0$, the best-response tariff of Country A is $\tau_A = 5\%$.¹¹⁰ In this situation, welfare in Country A is WA=158.68 at the expense of welfare in Country B (i.e., WB=148.15).¹¹¹ The equilibrium is characterized by NA= 39099 varieties available in Country A versus NB=6998 varieties of differentiated goods in Country B. The domestic market in Country A is at XmA=5375, associated with a market share of 92% of domestic producers. The tariff $\tau_A = 5\%$ protects the market from imports while stimulating the foundation of MNEs and market entry of domestic producers. The market share of MNEs in A is 7.23% compared with a market share of 0.0005% of exporters. Furthermore, households benefit because differentiated goods of MNEs are cheaper than imports subject to the tariff.

Given the preceding scenario, we find the best-response tariff to $\tau_A = 5\%$ at $\tau_B = 3\%$.¹¹² Hence, the constellation of $\tau_B = 0$ and $\tau_A = 5\%$ is not a stable equilibrium. If Country B deviates from choosing $\tau_B = 0$ and instead determines $\tau_B = 3\%$, the welfare of the representative household increases to WB=151.81. Welfare of the neighboring Country, A, is at WA=152.56. The equilibrium is characterized by a market size in Country A XmA=4946 in contrast to the XmB=5030 associated with a mass of firms in Country A NA=21136, compared with NB=25194. Hence, consumers in Country B benefit from a richer supply of differentiated goods compared with households in Country A.

To determine the unique, stable equilibrium of this model, we find the best-response tariff to $\tau_B = 3\%$ at $\tau_A = 4\%$.¹¹³ In this equilibrium, the representative household of Country A has a welfare of WA=152.775, compared with welfare in Country B WB=152.04. The higher level of protectionism in Country A stimulates not only the foundation of MNEs but also gives incentives to domestic firms in Country A to enter the market. The market share of exporters in Country A is 9.17% in contrast to

¹¹⁰ Because the countries are symmetric, the response can also be interpreted as the response of Country B to a given tariff rate in Country A.

¹¹¹ See Appendix 2.9.10 for a table summarizing all responses to τ_B =0%.

 $^{^{112}}$ See Appendix 2.9.11 for a table summarizing all responses to $\,\tau_{\rm B}$ =5%.

 $^{^{113}}$ See Appendix 2.9.12 for a table summarizing all responses to $\,\tau_{\rm B}$ =3%.

23.99% in Country B. Because imports are more expensive than the same goods provided by MNEs, consumers in Country A benefit from comparably low imports, which explains WA>WB.

Finally, given $\tau_A = 4\%$, Country B has incentive to respond optimally with $\tau_B = 3\%$ when Country A analogously has a welfare of WA=152.75%. The stability of the equilibrium $\tau_A = 3\%$ and $\tau_B = 4\%$, respectively $\tau_A = 4\%$ and $\tau_B = 3\%$, is guaranteed because tariff rates of $\tau = 3\%$ and $\tau_A = 4\%$, respectively, are best responses for one another.¹¹⁴ Figure 16 is a summary of the results of best-response tariffs, showing the unique, stable equilibrium at $\tau_A = 3\%$ and $\tau_B = 4\%$ and $\tau_B = 4\%$ and vice versa.



Figure 16. Mutual best-response tariffs.

Table 4 is a summary of the notion of best response tariffs.

Given τ	Best-response τ	W(given tariff)	W (best-response tariff)
0%	5%	148.15	158.68
1%	4%	150.13	156.22
2%	4%	151.47	154.20
3%	4%	152.04	152.75
4%	3%	152.75	152.04
5%	3%	152.56	151.81
6%	3%	152.46	151.80
7%	3%	152.46	151.80
8%	3%	152.46	151.80
9%	3%	152.46	151.80
10%	3%	152.46	151.80

 $^{^{114}}$ See Appendix 2.9.13 for a table summarizing all responses given $\,\tau_{B}^{}$ =4%.

Table 4. Best-response tariffs.

Thus, it can be summarized that although the constellation at $\tau_A = \tau_B = 0$ is welfare superior for both countries, the instability of this equilibrium is characterized by mutual incentives to deviate until the stable equilibrium at $\tau_A = 3\%$ and $\tau_B = 4\%$ is reached. From both individual and world welfare perspectives, a noncooperative tariff setting results in an outcome characterized by inefficiently high tariff rates.

Governments are completely informed when setting tariff rates. For this reason, they both know that the other has an incentive to deviate from a zero tariff setting. Considering this, welfare in its own jurisdiction is maximized considering the tariff rate the other country will select. The zero tariff rate scenario can only be obtained under reliable cooperation (i.e., with a social planner) because each single government has incentive to deviate.

2.7 Outline

These derived results of inefficient tariff rates selected by governments in a noncooperative tariff setting are supported by findings in other trade literature and empirical studies.¹¹⁵ Our analysis uses an alternative approach dependent on exogenously given parameters, such as transport costs and fixed costs, and the resulting endogeneity of integration strategies, endogenous market entry, and heterogeneity of firms.

2.7.1 The role of exogenously given parameters

Our model derives cut-off levels between the different integration strategies (domestic producers, exporters and horizontal MNEs), dependent on exogenously given parameters and their constellation to each other. At the first cut-off level $\theta_{d/ex}$ in (63), firm productivity is such that additional profits of exporting exactly result in zero profits. The productivity in (63) increases with increasing fixed costs, market size, tariff rate, and transport costs.

At the next threshold $\theta_{ex/i}$ in (64), the productivity level is such that profits of an exporting firm equal the profits of an MNE.

¹¹⁵ As in Broda et al. (2008).

The critical productivity level in (64) increases with increasing overhead costs ($f_i - f_{ex}$), an increasing market size X, decreasing transport costs t, and tariff rate τ . An increasing $\theta_{ex/i}$ is associated with a smaller mass of firms selecting MNE strategies. The existence of prevailing integration strategies depends on the constellation of

these exogenously given parameters. For example, the MNE strategy does not exist if $\tau = 0$ because the expression in (64) is infinite. Additionally, the threshold is infinite if exogenously given parameters satisfy the condition $t^{\alpha} \leq \frac{1}{(1+\tau)}$. Alternatively, if

exogenously given parameters satisfy the condition $(1 + \tau) \ge \left(\frac{f_i}{f_{ex}}\right)^{(1-\alpha)} t^{\alpha}$, it implies that

 $\theta_{d/i} < \theta_{d/ex}$ and heterogeneous firms directly integrate as horizontal MNEs. Therefore, based on the cut-off levels, the mass of firms selecting integration strategies as well as which strategies are optimal to select at all, depends on exogenously given parameters. These dependencies in a setting with heterogeneous firms distinguish this model from the latest literature.¹¹⁶

2.7.2 The role of exogenous market entry and market size

In this model the mass of firms in equilibrium results endogenously because expected profits are competed to zero until the last firm entering the market generates zero profits. With the inclusion of this endogeneity, we can analyze the implications of national and international policy decisions on integration modi of heterogeneous firms. Furthermore, both the decisions of MNEs and exporting firms to enter the market and the mass of domestic firms are dependent on the tariff. Given a specific tariff rate, the exact composition of prevailing integration strategies in this country is due to the constitution of competition. For example, if a tariff rate increases, fewer exporters enter the market; depending on the size of transport costs, they also may refrain from becoming exporters. Then, fewer firms will supply demand in this country, and expected profits will increase. Therefore, the output of each single firm is influenced. Also, more domestic firms and MNEs may enter the market, competing expected profits to zero. Hence, equilibria with different tariff rates are determined by other compositions of integration strategies and other masses of firms producing individual optimal output. This endogenous market size and

¹¹⁶ As in Davies and Eckel (2007).

especially further entry of domestic firms in a setting with heterogeneous firms distinguishes this model from the latest literature.¹¹⁷

2.7.3 The role of heterogeneity

In our model, stable equilibria are obtained with a 3%-4% tariff setting scenario. This result is driven by the incentive of each government to deviate unilaterally from free trade to induce positive impacts on welfare in its jurisdiction. This influence on welfare is characterized by the following implications: The mass of firms in its country increases. Increased welfare, therefore, results in more available varieties of differentiated goods for consumers there. More goods from domestic firms and MNEs are supplied; but in contrast, fewer products from exporters are available for them induced by the tariff. For the unilaterally deviating country, the overall impact in this scenario is that the tariff induces positive welfare implications due to love-for-variety preferences and positive tariff revenue, even though fewer varieties of exporters are supplied.

The extent of more domestic firms and MNEs entering the market in this analysis also depends on the distribution of firms over productivity levels. In this analysis, a uniform distribution $F(\theta)$ is assumed. The specification of an alternative distribution function, therefore, may induce differing results. The assumption of a distribution $G(\theta)$ in which the mass of firms increases with productivity so that many MNEs and few domestic firms exist slows the stimulating effect on domestic firms to enter the market if positive tariff rates are selected.

Hence, an increase in the tariff rate of a single government has the following implications: As an increase in the tariff rate induces some exporters not to enter the market and expected profits are competed to zero, alternatively, integrated firms can enter the market and single firm adjust their outputs. Depending on the distribution function, the composition of the mass of firms selecting different integration strategies then differs. For this reason, if $G(\theta)$ instead of $F(\theta)$ is applied, fewer domestic firms can enter the market and single optimal output adjusts according to endogenous market entry conditions. Obviously, the extent of the resulting implications depends on exact parameter configurations. However, the impact of $F(\theta)$ with more firms with lower single output always is positive for consumers due to love-for-variety preferences. If, instead of $F(\theta)$, $G(\theta)$ is applied, this impact on welfare concerning

¹¹⁷ As in Davies, Egger and Egger (2009).

more available varieties is dampened. Instead, outputs of single firm output will be increasingly influenced.

Another positive impact on utility of the representative household is achieved by providing a lump-sum transfer. Because an additional lump-sum transfer is used only to finance the consumption of x_0 and the homogeneous goods are appreciated less than the differentiated goods, according to (45), the impact of transfer on welfare is not extensive.

The third impact of a tariff on welfare concerns some highly productive exporters that do not enter the market. Instead, these firms integrate as MNEs, providing cheaper goods. This has positive implications for the representative household because consumers benefit from relatively cheap varieties supplied by MNEs.

In an analysis with the herein described distribution function $G(\theta)$, love-for-variety preferences will be less satisfied than in the analysis with $F(\theta)$. Previously, in the analysis with $F(\theta)$, the positive impact on welfare by unilaterally deviating from the cooperative free trade scenario is mainly driven by higher satisfaction of these preferences. With this alternative distribution of firms $G(\theta)$, fewer domestic firms will enter the market; and far more cheap varieties supplied by MNEs will be available for consumers in the jurisdiction of this government.

Hence, in contrast to $F(\theta)$, this distribution function $G(\theta)$ more likely results in a negative impact due to the tariff (i.e., the negative impact of fewer varieties provided by exporters can be more influential than the positive implication given by tariff revenue and stimulated market entry satisfying love-for-variety preferences). Obviously, the result depends on exact parameter configurations; but focusing on configurations ensuring this described impact of $G(\theta)$ free trade will result in a stable equilibrium. In the previous analysis, free trade is optimal from a world welfare perspective. Unfortunately, it is unstable in a noncooperative tariff setting. Hence, the results in this analysis are mainly constituted by the exact specification of the distribution of firms over productivity and, therefore, are due to heterogeneity.

2.8 Conclusion

The requirement of this paper is to provide rationale for the existence of ad-valorem tariffs in a model of heterogeneous firms. We determine optimal tariff rates set by

benevolent planners (i.e., when countries behave cooperatively and contrast this to optimal best-response tariff rates if countries behave noncooperatively).

To derive welfare maximizing ad-valorem tariff rates set by benevolent planners, we develop a model with heterogeneous firms. When determining optimal tariff rates, governments take their impact on optimal integration strategies for firms, as well as on market entry and the mass of firms, into account. The integration strategies heterogeneous firms choose as optimal depend on their individual productivity levels. Given their individual productivity levels, firms maximize profits considering relative sizes of fixed costs, size of transport costs, market sizes, per-unit variable costs, and the tariff level. Therefore, each firm individually either selects domestic production, an exporting strategy, or MNE activities as optimal, where the composition of prevailing strategies is determined endogenously, depending on the ad-valorem tariff rates chosen by the governments. These described behavioral modifications of integration strategies of heterogeneous firms responding to economic policy interventions are included in the government considerations. Due to the incorporation of several endogenous variables, especially market entry and market sizes, this utilitarian maximization of welfare is solved numerically in this analysis. Results of numerical analysis with cooperative behavior of countries show that a social planner determines a free trade scenario to be optimal from a world welfare perspective (i.e. welfare of both countries is maximized). In a noncooperative setting, the government jurisdiction has unilateral incentive to deviate from a free trade scenario. This behavior can be anticipated by the other government, resulting in both governments deviating from a zero tariff scenario, which results in inefficiently high tariff rates, which are stable Nash equilibria. These Nash equilibria are characterized by lower welfare for both countries than in a social planner's scenario without tariffs.

In a noncooperative setting, the social planner's free trade scenario is not stable because each government has a unilateral incentive to deviate, even though it generates the highest welfare from a world welfare perspective.

In conclusion, our model provides rationale for the existence of ad-valorem tariffs in a model of heterogeneous firms. The instability of a free trade scenario with countries behaving noncooperatively is given by the incentive of a single government to deviate. The welfare-superior free trade scenario can only be obtained under reliable policy coordination.

2.9 Appendix

2.9.1 Demand

We use the utility function in (45), $U_j = x_0 + \frac{1}{\mu} \left(\int_0^{\theta_{max}} \frac{1}{\alpha} x_j(i)^{\alpha} di \right)^{\mu}$ and the standard side

condition to derive the demand of a representative household for the goods of the ith firm:

$$\begin{split} \mathsf{L} &= \mathsf{x}_0 + \frac{1}{\mu} \Biggl[\overset{\mathsf{h}_{max}}{\int} \frac{1}{\alpha} (\mathsf{x}_1(i) + \beta \varsigma_1(i))^{\alpha} di \Biggr]^{\mu} + \lambda \Biggl[\mathsf{m}_i - \mathsf{x}_0 - \overset{\mathsf{h}_{max}}{\int} \overset{\mathsf{h}_{max}}{\mathcal{D}} \mathsf{p}_1(i) (\mathsf{x}_1(i) + \beta \varsigma_1(i)) di \Biggr] \\ \frac{\partial \mathsf{L}}{\partial \mathsf{x}_0} &= 1 - \lambda = 0 \quad \Rightarrow \lambda = 1 \\ \frac{\partial \mathsf{L}}{\partial \lambda} &= \mathsf{m}_i - \mathsf{x}_0 - \overset{\mathsf{h}_{max}}{\int} \overset{\mathsf{h}_{max}}{\mathcal{D}} \mathsf{p}_1(i) (\mathsf{x}_1(i) + \beta \varsigma_1(i)) di = 0 \quad \Rightarrow \mathsf{x}_0 = \mathsf{m}_i - \overset{\mathsf{h}_{max}}{\int} \overset{\mathsf{h}_{max}}{\mathcal{D}} \mathsf{p}_1(i) (\mathsf{x}_1(i) + \beta \varsigma_1(i)) di \Biggr] \\ \frac{\partial \mathsf{L}}{\partial \beta} &= \Biggl[\overset{\mathsf{h}_{max}}{\int} \frac{1}{\alpha} (\mathsf{x}_1(i) + \beta \varsigma_1(i))^{\alpha} di \Biggr] \overset{(\mu^{-1})}{\int} \Biggl[\overset{\mathsf{h}_{max}}{\int} (\mathsf{x}_1(i) + \beta \varsigma_1(i))^{(\alpha^{-1})} \varsigma_1(i) di \Biggr] - \overset{\mathsf{h}_{max}}{\int} \overset{\mathsf{h}_{max}}{\mathcal{D}} \mathsf{p}_1(i) (\mathsf{x}_1(i) + \beta \varsigma_1(i))^{\alpha} di \Biggr] \\ = \mathsf{O} : \\ \frac{\partial \mathsf{L}}{\partial \beta} &= \Biggl[\overset{\mathsf{h}_{max}}{\int} \frac{1}{\alpha} \mathsf{x}_1(i)^{\alpha} di \Biggr] \overset{(\mu^{-1})}{\int} \overset{\mathsf{h}_{max}}{\mathcal{D}} \mathsf{x}_1(i)^{(\alpha^{-1})} \varsigma_1(i) di \Biggr] \overset{\mathsf{h}_{max}}{\mathcal{D}} \mathsf{p}_1(i) (\mathsf{h}_1(i) \mathsf{h}_1) = \mathsf{O} \\ \Rightarrow \overset{\mathsf{h}_{max}}{\int} \Biggl[\dfrac{\mathsf{x}_1(i)^{(\alpha^{-1})}}{\int} \overset{\mathsf{h}_{max}}{\mathcal{D}} \mathsf{p}_1(i) \mathsf{p}_1(i) \mathsf{p}_1(i) \mathsf{p}_1) \Biggr] \mathsf{p}_1(i) \mathsf{p}_1(i) \mathsf{p}_1 \mathsf{p}_1(i) \mathsf{p}_1(i) \mathsf{p}_1(i) \mathsf{p}_1 \mathsf{p}_1(i) \mathsf{p}_1(i$$

$$\Rightarrow \mathbf{x}_{j}(\mathbf{i})^{(1-\alpha)} = \frac{1}{\mathbf{X}^{(1-\mu)}\mathbf{p}_{j}(\mathbf{i})}$$

$$\Rightarrow x_{j}(i) = \frac{1}{X^{\frac{(1-\mu)}{(1-\alpha)}} p_{j}(i)^{\frac{1}{(1-\alpha)}}}$$
$$\Rightarrow p_{j}(i) = \frac{1}{X^{(1-\mu)} x_{j}(i)^{(1-\alpha)}}$$

This paper applies $X = \left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} x_{j}(i)^{\alpha} di \right)$, which denotes the subutility or market size as specifiable as $X_{d,ex,i}$ in (48), $X_{d,i}$ in (49) and $X_{d,ex}$ in (50).

2.9.2 The domestic firm

The derivation of the profit-maximizing output:

$$\begin{aligned} \pi_{j}(i)_{d} &= p_{j}(i)_{d} x_{j}(i)_{d} - \frac{x_{j}(i)_{d}}{\theta(i)} - f_{d} \\ &= \frac{1}{\left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} x_{j}(i)^{\alpha} di\right)^{1-\mu} x_{j}(i)_{d}^{(1-\alpha)}} x_{j}(i) - \frac{x_{j}(i)_{d}}{\theta(i)} - f_{d} \\ \frac{\partial \pi_{j}(i)_{d}}{\partial x_{j}(i)} &= \alpha x_{j}(i)_{d}^{(\alpha-1)} \frac{1}{\left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} x_{j}(i)^{\alpha} di\right)^{(1-\mu)}} - \frac{1}{\theta(i)} = 0 \\ x_{j}(i)_{d}^{(\alpha-1)} &= \frac{1}{\theta(i)} \frac{X^{(1-\mu)}}{\alpha} \\ x_{j}(i)_{d}^{(\alpha-1)} &= \frac{X^{(1-\mu)}}{\alpha\theta(i)} \\ x_{j}(i)_{d}^{(\alpha-1)} &= \frac{X^{(1-\mu)}}{\alpha\theta(i)} \end{aligned}$$

The derivation of the optimal price:

$$\pi_{j}(i)_{d} = p_{j}(i)_{d} x_{j}(i)_{d} - \frac{x_{j}(i)_{d}}{\theta(i)} - f_{d}$$

$$\pi_{j}(i)_{d} = p_{j}(i)_{d} \frac{1}{\left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} x_{j}(i)^{\alpha} di\right)^{\frac{(1-\mu)}{(1-\alpha)}} p_{j}(i)_{d}^{\frac{1}{(1-\alpha)}}} - \frac{1}{\left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} x_{j}(i)^{\alpha} di\right)^{\frac{(1-\mu)}{(1-\alpha)}} p_{j}(i)_{d}^{\frac{1}{(1-\alpha)}}} \frac{1}{\theta(i)} - f_{d}$$

$$\frac{\partial \pi_{j}(\mathbf{i})_{d}}{\partial p_{j}(\mathbf{i})} = \frac{-\alpha}{(1-\alpha)} p_{j}(\mathbf{i})_{d} \frac{1}{(1-\alpha)} \mathbf{s}_{j} \frac{1}{\left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} \mathbf{x}_{j}(\mathbf{i})^{\alpha} d\mathbf{i}\right)^{\left(\frac{1-\mu}{\alpha}\right)}}{\left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} \mathbf{x}_{j}(\mathbf{i})^{\alpha} d\mathbf{i}\right)^{\left(\frac{1-\mu}{\alpha}\right)}} + \frac{1}{(1-\alpha)} p_{j}(\mathbf{i})_{d}^{(-1)} \frac{1}{\left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} \mathbf{x}_{j}(\mathbf{i})^{\alpha} d\mathbf{i}\right)^{\left(\frac{1-\mu}{\alpha}\right)}} p_{j}(\mathbf{i})_{d} \frac{1}{\left(\frac{1-\mu}{\alpha}\right)} p_{j}(\mathbf{i})_{d} \frac{1}{(1-\alpha)}} p_{j}(\mathbf{i})_{d} \frac{1}{(1-\alpha)} p_{j}(\mathbf{i}$$

The domestic firm i applies the price $p_j(i)$, which is the standard mill price. The factor $\frac{1}{\alpha}$ expresses the mark-up. The closer differentiated goods substitute, the higher α , the smaller is the market power of the single firm.

Maximum attainable profits are:

$$\pi_{j}(i)_{d}^{*} = \frac{(\alpha\theta(i))^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}} \frac{1}{\alpha} \frac{1}{\theta(i)} - \frac{(\alpha\theta(i))^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}\theta(i)} - f_{d} = \frac{(\alpha\theta(i))^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}\theta(i)} \left(\frac{1}{\alpha} - 1\right) - f_{d}$$

2.8.3 The exporter

The derivation of the optimal output of an exporting firm from country A:

$$\begin{split} x_{j}(i)_{ex} &= \frac{1}{X^{\frac{(1-\mu)}{(1-\alpha)}} (p_{j}(i)(1+\tau))^{\frac{1}{(1-\alpha)}}} \\ \pi_{j}(i)_{ex} &= p_{j}(i)_{ex} x_{j}(i)_{ex} - \frac{x_{j}(i)_{ex}t}{\theta(i)} - f_{ex} + \pi_{j}(i)_{d} \\ \pi_{j}(i)_{ex} &= p_{j}(i)_{ex} \frac{1}{X^{\frac{(1-\mu)}{(1-\alpha)}} (p_{j}(i)(1+\tau))^{\frac{1}{(1-\alpha)}}} - \frac{1}{X^{\frac{(1-\mu)}{(1-\alpha)}} (p_{j}(i)(1+\tau))^{\frac{1}{(1-\alpha)}}} \frac{t}{\theta(i)} - f_{ex} \\ \pi_{j}(i)_{ex} &= \frac{1}{X^{\frac{(1-\mu)}{(1-\alpha)}} (1+\tau)^{\frac{1}{(1-\alpha)}}} p_{j}(i)_{ex}^{\frac{-\alpha}{(1-\alpha)}} - \frac{1}{X^{\frac{(1-\mu)}{(1-\alpha)}} (1+\tau)^{\frac{1}{(1-\alpha)}}} p_{j}(i)_{ex}^{\frac{-1}{(1-\alpha)}} \frac{t}{\theta(i)} - f_{ex} \\ \frac{\partial \pi_{j}(i)_{ex}}{\partial p(i)_{j}} &= -\frac{\alpha}{1-\alpha} p_{j}(i)_{ex}^{\frac{-1}{(1-\alpha)}} \frac{1}{X^{\frac{(1-\mu)}{(1-\alpha)}} (1+\tau)^{\frac{1}{(1-\alpha)}}} + \frac{1}{1-\alpha} p_{j}(i)_{ex}^{\frac{-(2+\alpha)}{(1-\alpha)}} \frac{1}{X^{\frac{(1-\mu)}{(1-\alpha)}} (1+\tau)^{\frac{1}{(1-\alpha)}}} \frac{t}{\theta(i)} \end{split}$$

$$\frac{\alpha}{1-\alpha} p_{j}(i)_{ex}^{\frac{-1}{(1-\alpha)}} \frac{1}{X^{\frac{(1-\mu)}{(1-\alpha)}}(1+\tau)^{\frac{1}{(1-\alpha)}}} = \frac{1}{1-\alpha} p_{j}(i)_{ex}^{\frac{-(2+\alpha)}{(1-\alpha)}} \frac{1}{X^{\frac{(1-\mu)}{(1-\alpha)}}(1+\tau)^{\frac{1}{(1-\alpha)}}} \frac{t}{\theta(i)}$$
$$p_{j}(i)_{ex}^{*} = \frac{1}{\alpha} \frac{t}{\theta(i)}$$

or

$$q_{j}(i)_{ex}^{*} = (1 + \tau)p_{j}(i)_{ex}^{*}$$

The optimal output is derived by applying $p_{j}(i)_{_{ex}}^{*}$ into $x_{j}(i)_{_{ex}}$ and is shown by:

$$\mathbf{x}_{j}(\mathbf{i})_{ex}^{*} = \frac{(\alpha \theta(\mathbf{i}))^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}(1+\tau)^{\frac{1}{(1-\alpha)}}t^{\frac{1}{(1-\alpha)}}}$$

The maximum attainable profits therefore are:

$$\pi_{j}(i)_{ex}^{*} = \frac{(\alpha\theta(i))^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}(1+\tau)^{\frac{1}{(1-\alpha)}}t^{\frac{1}{(1-\alpha)}}} \frac{1}{\alpha} \frac{t}{\theta(i)} - \frac{(\alpha\theta(i))^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}(1+\tau)^{\frac{1}{(1-\alpha)}}t^{\frac{1}{(1-\alpha)}}\theta(i)} - f_{ex} + \pi_{j}(i)_{d}^{*}$$
$$= \frac{(\alpha\theta(i))^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}(1+\tau)^{\frac{1}{(1-\alpha)}}t^{\frac{1}{(1-\alpha)}}\theta(i)} \left(\frac{1}{\alpha} - 1\right) - f_{ex} + \pi_{j}(i)_{d}^{*}$$

2.9.4 The MNE

The profit function of a firm that engages in an MNE strategy is similar to the profit function of a domestic firm. Only fixed costs in this strategy are higher $f_i > f_{ex}$. The derivation of the profit-maximizing price is shown in the following:

$$\begin{aligned} \pi_{j}(\mathbf{i})_{i} &= p_{j}(\mathbf{i})_{i} \mathbf{x}_{j}(\mathbf{i})_{i} - \frac{\mathbf{x}_{j}(\mathbf{i})_{i}}{\Theta(\mathbf{i})} - f_{i} \\ \pi_{j}(\mathbf{i})_{i} &= p_{j}(\mathbf{i})_{i} \frac{1}{\left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} \mathbf{x}_{j}(\mathbf{i})^{\alpha} d\mathbf{i}\right)^{\frac{(1-\mu)}{(1-\alpha)}} p_{j}(\mathbf{i})_{i}^{\frac{1}{(1-\alpha)}} - \frac{1}{\left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} \mathbf{x}_{j}(\mathbf{i})^{\alpha} d\mathbf{i}\right)^{\frac{(1-\mu)}{(1-\alpha)}} p_{j}(\mathbf{i})_{i}^{\frac{1}{(1-\alpha)}} \frac{1}{\Theta(\mathbf{i})} - f_{i} \\ \frac{\partial \pi_{j}(\mathbf{i})_{i}}{\partial p_{j}(\mathbf{i})_{i}} &= -\alpha p_{j}(\mathbf{i})_{i}^{\frac{-1}{(1-\alpha)}} \frac{1}{\left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} \mathbf{x}_{j}(\mathbf{i})^{\alpha} d\mathbf{i}\right)^{\frac{(1-\mu)}{(1-\alpha)}} + \frac{1}{1-\alpha} p_{j}(\mathbf{i})_{i} \frac{1}{\left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} \mathbf{x}_{j}(\mathbf{i})^{\alpha} d\mathbf{i}\right)^{\frac{(1-\mu)}{(1-\alpha)}}} p_{j}(\mathbf{i})_{i}^{\frac{(2+\alpha)}{(1-\alpha)}} \frac{1}{\Theta(\mathbf{i})} = 0 \end{aligned}$$

$$\alpha p_{j}(i)_{i}^{\frac{-1}{(1-\alpha)}} \frac{1}{\left(\int\limits_{0}^{\theta_{max}} \frac{1}{\alpha} x_{j}(i)^{\alpha} di\right)^{\frac{(1-\mu)}{(1-\alpha)}}} = \frac{1}{1-\alpha} p_{j}(i)_{i} \frac{1}{\left(\int\limits_{0}^{\theta_{max}} \frac{1}{\alpha} x_{j}(i)^{\alpha} di\right)^{\frac{(1-\mu)}{(1-\alpha)}}} p_{j}(i)_{i}^{\frac{-(2+\alpha)}{(1-\alpha)}} \frac{1}{\theta(i)}$$

$$p_{j}(i)_{i}^{*} = \frac{1}{\alpha} \frac{1}{\theta(i)}$$

Analogously, we find profit-maximizing output by applying $p_i(i)_i^*$ into $x_j(i)$:

$$\mathbf{x}_{j}(\mathbf{i})_{i}^{*} = \frac{(\alpha \theta(\mathbf{i}))^{\frac{1}{(1-\alpha)}}}{\mathbf{X}^{\frac{(1-\mu)}{(1-\alpha)}}}$$

Maximum attainable profits are given by:

$$\pi_{j}(i)_{i}^{*} = \frac{(\alpha\theta(i))^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}} \frac{1}{\alpha} \frac{1}{\theta(i)} - \frac{(\alpha\theta(i))^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}\theta(i)} - f_{i} + \pi_{j}(i)_{d}^{*} = \frac{(\alpha\theta(i))^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}\theta(i)} \left(\frac{1}{\alpha} - 1\right) - f_{i} + \pi_{j}(i)_{d}^{*}$$

2.9.5 Derivation of cut-off level $\theta_{d/ex}$

We derive the productivity level that at least additionally guarantees zero profits from exporting:

$$\begin{split} \pi_{j}(i)_{ex}^{*} &= x_{j}(i)_{ex}^{*} p_{j}(i)_{ex}^{*} - \frac{x_{j}(i)_{ex}^{*}}{\theta(i)} - f_{ex} \geq 0 \\ \pi_{j}(i)_{ex} &= \frac{(\alpha\theta(i))^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}(1+\tau)^{\frac{1}{(1-\alpha)}}t^{\frac{1}{(1-\alpha)}}} \frac{1}{\alpha} \frac{t}{\theta(i)} - \frac{(\alpha\theta(i))^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}(1+\tau)^{\frac{1}{(1-\alpha)}}\theta(i)}} - f_{ex} \geq 0 \\ \frac{(\alpha\theta(i))^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}t^{\frac{1}{(1-\alpha)}}\theta(i)}} \left(\frac{1}{\alpha} - 1\right) \geq f_{ex} \\ \frac{(\alpha\theta(i))^{\frac{1}{(1-\alpha)}}}{(1+\tau)^{\frac{1}{(1-\alpha)}}X^{\frac{(1-\mu)}{(1-\alpha)}t^{\frac{\alpha}{(1-\alpha)}}\theta(i)}} \left(\frac{1-\alpha}{\alpha}\right) \geq f_{ex} \\ \frac{(\alpha\theta(i))^{\frac{1}{(1-\alpha)}}}{(1+\tau)^{\frac{1}{(1-\alpha)}}X^{\frac{(1-\mu)}{(1-\alpha)}t^{\frac{\alpha}{(1-\alpha)}}} \in f_{ex} \\ \frac{(\alpha\theta(i))^{\frac{\alpha}{(1-\alpha)}}(1-\alpha)}{(1+\tau)^{\frac{1}{(1-\alpha)}}X^{\frac{(1-\mu)}{(1-\alpha)}t^{\frac{\alpha}{(1-\alpha)}}} \geq f_{ex} \\ \frac{\alpha^{\frac{\alpha}{(1-\alpha)}}\theta(i)^{\frac{\alpha}{(1-\alpha)}}(1-\alpha)}{(1+\tau)^{\frac{1}{(1-\alpha)}}X^{\frac{(1-\mu)}{(1-\alpha)}t^{\frac{\alpha}{(1-\alpha)}}} \geq f_{ex} \end{split}$$

$$\begin{split} \theta(i)^{\frac{\alpha}{(1-\alpha)}} &\geq \frac{f_{ex}\left(1+\tau\right)^{\frac{1}{(1-\alpha)}} X^{\frac{(1-\mu)}{(1-\alpha)}t^{\frac{\alpha}{(1-\alpha)}}}}{\alpha^{\frac{\alpha}{(1-\alpha)}} (1-\alpha)} \\ \theta_{d/ex} &= \frac{f_{ex}^{\frac{(1-\alpha)}{\alpha}} X^{\frac{(1-\mu)}{\alpha}} (1+\tau)^{\frac{1}{\alpha}}t}{\alpha (1-\alpha)^{\frac{(1-\alpha)}{\alpha}}} \end{split}$$

Firms that are at least as productive as $\,\theta_{_{d/ex}}$ engage in the export strategy.

$\underline{\text{2.9.6 Derivation of cut-off level }} \theta_{\text{ex/i}}$

The next threshold is a productivity level at which exporting and multinational firms have same profits. Firms with productivity levels above this level engage in a MNE activity.

$$\begin{split} & \pi_{j}(i)_{ex} \leq \pi_{j}(i)_{i} \\ & \frac{\left(\alpha\theta(i)\right)^{\frac{\alpha}{(1-\alpha)}}\left(1-\alpha\right)}{\left(1+\tau\right)^{\frac{1}{(1-\alpha)}} X^{\frac{(1-\mu)}{(1-\alpha)}} t^{\frac{\alpha}{(1-\alpha)}}} - f_{ex} \leq \frac{\left(\alpha\theta(i)\right)^{\frac{\alpha}{(1-\alpha)}}\left(1-\alpha\right)}{X^{\frac{(1-\mu)}{(1-\alpha)}}} - f_{i} \\ & \frac{\left(\alpha\theta(i)\right)^{\frac{\alpha}{(1-\alpha)}}\left(1-\alpha\right)}{X^{\frac{(1-\mu)}{(1-\alpha)}}} \left(1 - \left(\frac{1}{\left(1+\tau\right)^{\frac{1}{(1-\alpha)}} t^{\frac{\alpha}{(1-\alpha)}}}\right)\right) \geq f_{i} - f_{ex} \\ & \theta_{ex/i}^{\frac{\alpha}{(1-\alpha)}} \geq \frac{\left(f_{i} - f_{ex}\right)X^{\frac{(1-\mu)}{(1-\alpha)}}}{\alpha^{\frac{\alpha}{(1-\alpha)}}\left(1-\alpha\right) \left(1 - \left(\frac{1}{\left(1+\tau\right)^{\frac{1}{(1-\alpha)}} t^{\frac{\alpha}{(1-\alpha)}}}\right)\right)} \\ & \theta_{ex/i} \geq \frac{\left(f_{i} - f_{ex}\right)^{\frac{(1-\alpha)}{\alpha}} X^{\frac{(1-\mu)}{\alpha}}}{\alpha\left(1-\alpha\right)^{\frac{(1-\alpha)}{\alpha}} \left(1 - \left(\frac{1}{\left(1+\tau\right)^{\frac{1}{(1-\alpha)}} t^{\frac{\alpha}{(1-\alpha)}}}\right)\right)^{\frac{(1-\alpha)}{\alpha}}} \end{split}$$

Firms producing with productivity $\theta_{ex/i}$ are indifferent whether to choose the MNE or exporting strategy or not. A firm with productivity $\theta(i)$ just above $\theta_{ex/i}$ engages in a MNE strategy and generates positive profits from this activity.

2.9.7 Derivation and analysis of cut-off level $\theta_{\text{d/i}}$

The next cut-off level characterizes a situation in which the export strategy does not exist. Firms in this scenario directly integrate their firm following a MNE activity. The resulting cut-off level can be derived as follows:

$$\begin{split} &\pi_{j}(i)_{i} \geq 0 \\ &\frac{\left(\alpha\theta(i)\right)^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}} \left(\frac{1-\alpha}{\alpha}\right) - f_{i} \geq 0 \\ &\frac{\left(\alpha\theta(i)\right)^{\frac{\alpha}{(1-\alpha)}}\theta(i)}{X^{\frac{(1-\mu)}{(1-\alpha)}}} \geq f_{i} \\ &\frac{\left(\alpha\theta(i)\right)^{\frac{\alpha}{(1-\alpha)}}\left(1-\alpha\right)}{X^{\frac{(1-\mu)}{(1-\alpha)}}} \geq \frac{f_{i} \cdot X^{\frac{(1-\mu)}{(1-\alpha)}}}{(1-\alpha)\alpha^{\frac{\alpha}{(1-\alpha)}}} \\ &\theta_{d/i} \geq \frac{f_{i}^{\frac{(1-\alpha)}{\alpha}} \cdot X^{\frac{(1-\mu)}{(1-\alpha)}}}{(1-\alpha)^{\frac{(1-\alpha)}{\alpha}}\alpha} \end{split}$$

Firms that satisfy the following condition integrate their firm as an MNE, the export strategy does not exist:

$$\begin{split} & \theta_{d/i} < \theta_{d/ex} \\ & \frac{f_i^{\frac{(1-\alpha)}{\alpha}} X^{\frac{(1-\mu)}{\alpha}}}{\alpha (1-\alpha)^{\frac{(1-\alpha)}{\alpha}}} \leq \frac{f_{ex}^{\frac{(1-\alpha)}{\alpha}} X^{\frac{(1-\mu)}{\alpha}} (1+\tau)^{\frac{1}{\alpha}} t}{\alpha (1-\alpha)^{\frac{(1-\alpha)}{\alpha}}} \\ & f_i^{\frac{(1-\alpha)}{\alpha}} \leq f_{ex}^{\frac{(1-\alpha)}{\alpha}} (1+\tau)^{\frac{1}{\alpha}} t \\ & (1+\tau) \geq \left(\frac{f_i}{f_{ex}}\right)^{(1-\alpha)} t^{\alpha} \end{split}$$

2.9.8 Utility of the representative household at $X_{d,i}$, $X_{d/ex}$ and X_{d}

For the constellation that ensures the market size to be $X_{d,i}$, the following condition $\theta_{d/i} < \theta_{d/ex}$ must hold and utility is given by:

$$U_{j} = m_{j} - \left(\int_{0}^{\theta_{max}} p_{j}(i)_{d}^{*} x_{j}(i)_{d}^{*} di + \int_{\theta_{d/i}}^{\theta_{max}} p_{j}(i)_{i}^{*} x_{j}(i)_{i}^{*} di\right)$$
$$+ \frac{1}{\mu} \left[\int_{0}^{\theta_{max}} \frac{1}{\alpha} \left(x_{j}(i)_{d}^{*}\right)^{\alpha} di + \int_{\theta_{d/i}}^{\theta_{max}} \frac{1}{\alpha} \left(x_{j}(i)_{i}^{*}\right)^{\alpha} di\right]^{\mu} + 0$$

Because the export strategy does not exist, a transfer cannot be provided. For that the market size is $X_{d,ex}$, the condition $\theta_{d/ex} < \theta_{max}$ must hold. Furthermore, the MNE strategy does not exist if $\tau = 0$ because $\theta_{ex/i}$ is infinite $\left(1 - \left(\frac{1}{(1+\tau)}\right)^{\frac{1}{(1-\alpha)}}\right)^{\frac{1}{\alpha}} = 0$ which results in utility with transfer: $U_j = m_j - \left(\int_{0}^{\theta_{max}} p_j(i)_d^* x_j(i)_d^* di + \int_{\theta_{d/ex}}^{\theta_{max}} p_j(i)_{ex}^* x_j(i)_{ex}^* di\right) + \int_{\theta_{d/ex}}^{\theta_{max}} x_j(i)_{ex}^* \cdot \tau di + \frac{1}{\mu} \left[\int_{0}^{\theta_{max}} \frac{1}{\alpha} (x_j(i)_d^*)^{\alpha} di + \int_{\theta_{d/ex}}^{\theta_{max}} \frac{1}{\alpha} (x_j(i)_{ex}^*)^{\alpha}\right]^{\mu}$

Only the domestic strategy exists if $\theta_{d/ex} > \theta_{max}$ and $\theta_{d/i} > \theta_{max}$ hold. This results in following utility for the household:

$$U_{j} = m_{j} - \left(\int_{0}^{\theta_{max}} p_{j}(i)_{d}^{*} x_{j}(i)_{d}^{*} di\right) + \frac{1}{\mu} \left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} (x_{j}(i)_{d}^{*})^{\alpha} di\right)^{\mu}$$

2.9.9 Mathematica input and plots
- $\Theta de[X_, \tau_, t_] := (a = t fex^{((1-\alpha) / \alpha)} X^{((1-\mu) / \alpha)} (1+\tau)^{(1/\alpha)/(\alpha(1-\alpha))} ((1-\alpha))^{(1-\alpha)/(\alpha(1-\alpha))} (1-\alpha))^{(1-\alpha)/(\alpha(1-\alpha))} (1-\alpha)^{(1-\alpha)/(\alpha(1-\alpha))} (1-\alpha)^{(1-\alpha)/(\alpha)} (1-$
- α)/ α));If[a< \ominus min,a= \ominus min,If[a> \ominus max,a= \ominus max]];a)
- $ide[n_,X_, _, _, t_]:=inr[\ominus de[X, _, t], n]$
- $\Theta ei \quad [X_{\tau_{t}},t_{l}]:=(If[t^{\alpha \leq 1/(1+\tau)}, \Theta max, Min[\Theta max, (fin-fex)^{((1-\alpha)/\alpha)} X^{((1-\mu)/\alpha)/(\alpha(1-\mu)/\alpha)}) X^{((1-\mu)/\alpha)/(\alpha(1-\mu)/\alpha)} X^{(1-\mu)/\alpha}) X^{(1-\mu)/\alpha} X^{(1-\mu)/\alpha} X^{(1-\mu)/\alpha}) X^{(1-\mu)/\alpha} X^{(1-\mu)/\alpha} X^{(1-\mu)/\alpha} X^{(1-\mu)/\alpha} X^{(1-\mu)/\alpha} X^{(1-\mu)/\alpha} X^{(1-\mu)/\alpha}) X^{(1-\mu)/\alpha} X^{(1-$
- $\alpha))^{((1-\alpha) / \alpha)} (1 (1/(t^{\alpha} (1+\tau)))^{(1/(1-\alpha))} ^{((1-\alpha) / \alpha))]])$
- $iei[n_,X_, __,t_]:=inr[\ominus ei[X, _,t],n]$
- $\Theta di[X_, \tau_, t_] := Min[\Theta max, fin^{((1-\alpha)/\alpha)} X^{((1-\mu)/\alpha)/(\alpha(1-\alpha))^{((1-\alpha)/\alpha)})]$
- $idi[n_,X_, __,t_]:=inr[\ominus di[X, _,t],n]$
- $Yd[nA_,X_,\tau_,t_]:=NIntegrate[(1/\alpha) (x[\ominus[i,nA],X,0,1])^{\alpha},\{i,0,nA\}]$
- $Yex[nB_,X_,\tau_,t_]:=If[ide[nB,X,\tau,t]<Min[iei[nB,X,\tau,t],nB],NIntegrate[(1/\alpha) (x[$
- $\Theta[i,nB], X, \tau, t])^{\Lambda} \alpha, \{i, ide[nB, X, \tau, t], Min[iei[nB, X, \tau, t], nB]\}], 0]$
- $\label{eq:constraint} Yin[nB_,X_,___,t_]:=NIntegrate[(1/\alpha)(x[\varTheta[i,nB],X,0,1])^{\alpha},\{i,Min[nB,Max[iei[nB,X,_,t],idi[nB,X,_,t],idi[nB,X,$
- $nB,X,\tau,t]]],nB\}]$
- $Y[nA_,nB_,X_,__,t_]:=Yd[nA,X,_,t]+Yex[nB,X,_,t]+Yin[nB,X,_,t]$
- $Xm[nA_,nB_,\tau_,t_]:=(k=0;X0=1000;Z0=Y[nA,nB,X0,\tau,t]-X0;d=20*2^{-1})$
- k);X1=X0+d;Z1=Y[nA,nB,X1, t,t]-X1;X2=N[(d Z0+X0 Z0-X0 Z1)/(Z0-
- Z1)];While[Abs[X2-X0]>.00001&&k<15,X0=X2;k++;Z0=Y[nA,nB,X0, t,t]-X0;d=20*2^(-
- k);X1=X0+d;Z1=Y[nA,nB,X1, τ ,t]-X1;X2=N[(d Z0+X0 Z0-X0 Z1)/(Z0-Z1)]];X2)
- $Gd[i_,nA_,nB_, _,t_]:=x[\ominus[i,nA], Xm[nA,nB, _,t], 0, 1] (1/(\alpha \ominus[i,nA]))-$
- $x[\ominus[i,nA],Xm[nA,nB,\tau,t],0,1]/\ominus[i,nA]\text{-}fd$
- $x[\ominus[i,nA],Xm[nB,nA, \tau B,t],0,1]/\ominus[i,nA]$ -fin
- $Gex[i_,nA_,nB_, \tau B_,t_] := x[\ominus[i,nA], Xm[nB,nA, \tau B,t], \tau B,t] (t/(\alpha \ominus[i,nA])) (t/(\alpha \ominus[i,nA]))$
- $x[\ominus[i,nA],Xm[nB,nA,\tau B,t],\tau B,t] \ t/\ominus[i,nA]\text{-fex}$
- $EGd[nA_,nB_, \tau A_,t_] := NIntegrate[Gd[i,nA,nB, \tau A,t], \{i,0,nA\}]$
- $EGex[nA_,nB_, \tau B_, t_]:=If[ide[nA,Xm[nB,nA, \tau B,t], \tau B,t] < Min[iei[nA,Xm[nB,nA, \tau B,t], \tau B,t] < Min[ie$
- B,t],nB],NIntegrate[Gex[i,nA,nB, τ B,t],{i,ide[nA,Xm[nB,nA, τ B,t], τ B,t],Min[iei[nA,Xm[nB,nA, τ B,t], τ B,t],nA]}],0]
- $$\begin{split} & \mathsf{EGin[nA_,nB_, \tau B_,t_]:=NIntegrate[Gin[i,nA,nB, \tau B,t], {i,Min[nA,Max[iei[nA,Xm[nB,nA, \tau B,t], \tau B,t],idi[nA,Xm[nB,nA, \tau B,t], \tau B,t]]],nB}] \end{split}$$
- $EG[nA_,nB_, \tau A_, \tau B_, t_] := EGd[nA, nB, \tau A, t] + EGex[nA, nB, \tau B, t] + EGin[nA, nB, \tau B, t]$
- $Firms[nB_{,} \tau A_{,} \tau B_{,} t_{]}:=(N1=2^{10}; K=10; While[EG[N1, nB, \tau A, \tau B, t]>0, N1=2)$
- ;N1;K=K+1];K=K-2;N1=N1-2^K;While[K>-6,K=K-
- 1;If[EG[N1,nB, *t*A, *t*B,t]>0,N1=N1+2^K,N1=N1-2^K]];Return[N1])

```
MYd[nA_,X_,\tau_,t_]:=NIntegrate[1/(\alpha \in [i,nA]) x[\in [i,nA],X,0,1],\{i,0,nA\}]
MYex[nB_,X_,\tau_,t_]:=If[ide[nB,X,\tau,t]<Min[iei[nB,X,\tau,t],nB],NIntegrate[ t/(\alpha \ \theta[i,nB]) (x[
\Theta[i,nB],X,\tau,t]),{i,ide[nB,X,\tau,t],Min[iei[nB,X,\tau,t],nB]}],0]
MYin[nB_,X_,\tau_,t_]:=1/s NIntegrate[1/(\alpha \in [i,nB]) (x[
Θ[i,nB],X,0,1]),{i,Min[nB,Max[iei[nB,X,τ,t],idi[nB,X,τ,t]]],nB}]
Country maximizes own welfare:
Equilibrium [\tau A, \tau B, t]:=(nA0=1; nB0=1; J=0; nA1=N[Firmenanzahl[nB0, \tau A, \tau B, t]]; nB1
=N[Firms[nA0,\tauB,\tauA,t]];nA2=N[Firms[nB1,\tauA,\tauB,t]];nB2=N[Firms[nA1,\tauB,\tauA,t]];nA3
=N[nA2-nA1 nA2-nA1 nB1+nA1 nA2 nB1+nA1 nB2-nA2 nB2/(1-nA1-nB1+nA2
nB1+nA1 nB2-nA2 nB2)];nB3=N[(nB1-1)/(nA2-nA1) (nA3nA1)+1];
Print[{J,N[nA3],N[nB3]}];While[(Abs[nA3-nA0]>10||Abs[nB3-nB0]>50)&&J<10,
nA0=nA3;nB0=nB3;nA1=Firms[nB0, \tau A, \tau B, t];nB1=Firms[nA0, \tau B, \tau A, t];nA2=Firms[nB0, \tau A, \tau B, t];nA1=Firms[nB0, \tau A, \tau B, t];nA1=Firms[nB0, \tau A, \tau B, t];nA1=Firms[nB0, \tau A, \tau B, t];nB1=Firms[nB0, \tau A, \tau B, t];nA1=Firms[nB0, \tau A, \tau B, t];nB1=Firms[nB0, \tau A, \tau B, t];nA1=Firms[nB0, \tau A, \tau B, t];nB1=Firms[nB0, t
1, \tau A, \tau B, t];nB2=Firms[nA1, \tau B, \tau A, t];nA3=N[nA2-nA1 nA2-nA1 nB1+nA1 nA2
nB1+nA1 nB2-nA2 nB2/(1-nA1-nB1+nA2 nB1+nA1 nB2-nA2 nB2)];nB3=N[(nB1-
1)/(nA2-nA1) (nA3-nA1)+1]; J++;Print[{J,nA3,nB3}]]; Print[{"Equilibrium",J,nA3,nB3}];
NA=nA3;NB=nB3;EGewA=EG[NA,NB, TA, TB,t];EGewB=EG[NB,NA, TB, TA,t];XMA=X
m[NA,NB, \approx A,t];XMB=Xm[NB,NA, \approx B,t];YINA=Yin[NB,XMA, \approx A,t];YINB=Yin[NA,XMB, \approx
B,t];YEXA=Yex[NB,XMA, \tau A,t];YEXB=Yex[NA,XMB, \tau B,t];YDA=Yd[NA,XMA, \tau A,t];YD
B=Yd[NB,XMB, zB,t];MYDA=MYd[NA,XMA, zA,t];MYDB=MYd[NB,XMB, zB,t];MYEXA
=MYex[NB,XMA, tA,t];MYEXB=MYex[NA,XMB,tB,t];MYINA=MYin[NB,XMA,tA,t];MY
INB=MYin[NA,XMB, zB,t];YCDA=Yd[NA,XMAzA,t];YCDB=Yd[NB,XMB,zB,t];YCINA=
Yin[NB,XMA, zA,t];YCINB=Yin[NA,XMB,zB,t];YCEXA=Yex[NB,XMA,zA,t];YCEXB=Ye
x[NA, XMB, \tau B, t]; UA=1/\mu(YCDA+YCINA+YCEXA)^{\mu}; UB=1/\mu(YCDB+YCINB+YCEXB)
^{\mu};MA=MYDA+MYEXA+MYINA;MB=MYDB+MYEXB+MYINB;WA=UA-MA;WB=UB-
MB;result={DateString[], TA, TB, t, NA, NB, EGewA, EGewB, XMA, XMB, YDA, YDB, YEXA,
YEXB,YINA,YINB,MYDA,MYDB,MYEXA,MYEXB,MYINA,MYINB,YCDA,YCDB,YCEX
A,YCEXB,YCINA,YCINB,UA,UB,MA,MB,WA,WB};Print[result];PutAppend[result,targ]
etfile])
```

Best-response tariff:

Equilibrium[$\tau A_, \tau B_, t_]$:=(nA0=1;nB0=1;J=0;nA1=N[Firms[nB0, $\tau A, \tau B, t]$];nB1=N[Firms[nA0, $\tau B, \tau A, t$]];nA2=N[Firms[nB1, $\tau A, \tau B, t$]];nB2=N[Firms[nA1, $\tau B, \tau A, t$]];nA3=N[nA2-nA1 nA2-nA1 nB1+nA1 nA2 nB1+nA1 nB2-nA2 nB2/(1-nA1-nB1+nA2 nB1+nA1 nB2-nA2 nB2)];nB3=N[(nB1-1)/(nA2-nA1) (nA3-nA1)+1]; Print[{J,N[nA3],N[nB3]}];

MB;result={DateString[] *z*A, *z*B,t,NA,NB,EGewA,EGewB,XMA,XMB,YDA,YDB,YEXA, YEXB,YINA,YINB,MYDA,MYDB,MYEXA,MYEXB,MYINA,MYINB,YCDA,YCDB,YCEX A,YCEXB,YCINA,YCINB,UA,UB,MA,MB,WA,WB};Print[result];PutAppend[result, targetfile])

<u>2.9.10 Response tariffs of country A given $\tau_{B} = 0$ </u>

T = ====	NIA	ND	V A	VD	V-LA	VJD	V A	V····D	V: A	V:D				50	14/ 4	
Tariff	NA	NB	XMA	XMR	XQA	хав	хеха	хехв	XIA	XIR	UA	UB	EA	EB	WA	WB
0	23662	23662	5130	5130	3172	3172	1958	1958	0	0	280.50	280.50	126.22	126.22	154.27	154.27
0.01	29385	17782	5205	5015	3871	2449	1333	2566	0	0	282.94	276.71	127.00	124.52	155.94	152.19
0.02	33863	13061	5275	4924	4391	1839	884	3084	0	0	285.21	273.68	127.92	123.16	157.29	150.52
0.03	37008	9455	5329	4848	4739	1357	488	3491	102	0	286.98	271.13	128.80	122.01	158.18	149.12
0.04	38531	7653	5363	4809	4897	1109	173	3700	293	0	288.06	269.88	129.47	121.42	158.60	148.40
0.05	39099	6998	5375	4795	4956	1017	30	3778	389	0	288.45	269.36	129.77	121.21	158.68	148.15
0.06	39140	6960	5375	4795	4961	1011	0	3783	415	0	288.20	269.34	129.81	121.20	158.66	148.14
0.07	39140	6960	5375	4795	4961	1011	0	3783	415	0	288.20	269.34	129.81	121.20	158.66	148.14
0.08	39140	6960	5375	4795	4961	1011	0	3783	415	0	288.20	269.34	129.81	121.20	158.66	148.14

<u>2.9.11 Response tariffs of country A given</u> $\tau_{B} = 5\%$

Toriff	NA	ND	٧m٨	VmD	V d A	VAD	VovA	VovD	VIA	ViD	114	IID	E۸	ED	\A/ A	W/D
Tarin	INA	IND			лия	лир	VEXA	Verp	AIA	VID	UA	UB	EA	ED	WA	WD
0	6998	39099	4795	5375	1017	4956	3778	30	0	389	269.36	288.45	121.21	129.77	148.15	158.68
0.01	13036	33437	4849	5233	1870	4377	2980	54	0	802	271.18	283.85	121.29	127.67	149.89	156.18
0.02	18129	28477	4901	5114	2567	3832	2334	73	0	1208	272.92	279.96	121.67	125.89	151.25	154.06
0.03	21136	25194	4946	5030	2960	3458	1214	84	772	1488	274.41	277.19	122.60	124.64	151.81	152.56
0.04	22476	23642	4967	4989	3132	3277	497	89	1339	1624	275.12	275.85	123.33	124.03	151.79	151.82
0.05	23025	23025	4974	4974	3203	3203	91	91	1680	1680	275.33	275.33	123.79	123.79	151.54	151.54
0.06	23109	22994	4976	4974	3214	3198	0	91	1762	1685	275.39	275.36	123.93	123.80	151.47	151.55
0.07	23109	22994	4976	4974	3214	3198	0	91	1762	1685	275.39	275.36	123.93	123.80	151.47	151.55
0.08	23109	22994	4976	4974	3214	3198	0	91	1762	1685	275.39	275.36	123.93	123.80	151.47	151.55

2.9.12 Response tariffs of country A given $\tau_B = 3\%$

Tariff	NA	NB	XmA	XmB	XdA	XdB	XexA	XexB	XiA	XiB	UA	UB	EA	EB	WA	WB
0	9455	37008	4848	5329	1357	4739	3491	488	0	102	271.13	286.98	122.01	128.80	149.12	158.18
0.01	15337	31438	4903	5194	2171	4152	2732	773	0	269	272.96	282.59	122.16	126.61	150.81	155.98
0.02	20254	26599	4954	5081	2831	3607	2124	1000	0	474	274.69	278.88	122.57	124.78	152.12	154.11
0.03	23275	23275	4998	4998	3219	3219	1132	1132	647	647	276.15	276.15	123.45	123.45	152.70	152.70
0.04	24633	21734	5022	4960	3388	3034	461	1190	1173	736	276.93	274.87	124.18	122.83	152.75	152.04
0.05	25194	21136	5030	4946	3458	2960	84	1214	1488	772	277.19	274.41	124.64	122.60	152.56	151.81
0.06	25256	21087	5030	4945	3466	2954	0	1216	1564	775	277.21	274.38	124.74	122.59	152.46	151.80
0.07	25256	21087	5030	4945	3466	2954	0	1216	1564	775	277.21	274.38	124.74	122.59	152.46	151.80
0.08	25256	21087	5030	4945	3466	2954	0	1216	1564	775	277.21	274.38	124.74	122.59	152.46	151.80

2.9.13 Response tariffs of country A given $\tau_B = 4\%$

Tariff	NA	NB	XmA	XmB	XdA	XdB	XexA	XexB	XiA	XiB	UA	UB	EA	EB	WA	WB
0	7653	38531	4809	5363	1109	4897	3700	173	0	293	269.88	288.06	121.42	129.47	148.4	158.60
0.01	13658	32875	4863	5223	1952	4314	2911	301	0	608	271.64	283.52	121.51	127.30	150.13	156.22
0.02	18713	27931	4915	5105	2641	3766	2274	403	0	936	273.38	279.67	121.90	125.47	151.47	154.20
0.03	21734	24633	4960	5022	3034	3388	1190	461	736	1173	274.87	276.93	122.83	124.18	152.04	152.75
0.04	23056	23056	4980	4980	3203	3203	485	485	1292	1292	275.54	275.54	123.53	123.53	152.01	152.01
0.05	23642	22476	4989	4967	3277	3132	89	497	1624	1339	275.85	275.12	124.03	123.33	151.82	151.79
0.06	23706	22426	4990	4967	3286	3125	0	498	1704	1343	275.86	275.10	124.14	123.32	151.72	151.78
0.07	23706	22426	4990	4967	3286	3125	0	498	1704	1343	275.86	275.10	124.14	123.32	151.72	151.78
0.08	23706	22426	4990	4967	3286	3125	0	498	1704	1343	275.86	275.10	124.14	123.32	151.72	151.78

Chapter 3

BEST-RESPONSE TAX RATES ON PROFITS OF FOREIGN MULTINATIONAL FIRMS: A NUMERICAL APPROACH

3.1 Introduction

In a recent innovation to trade literature, heterogeneity of firm-productivity has been incorporated into models of monopolistic competition with international trade and multinational firms. Initially, models of vertical or horizontal integration strategies of multinational firms were developed under the assumption of homogeneous productivities between all plants in a market.¹¹⁸ Later, theoretical work was focused on the study of optimal integration strategies of such complex firms in the presence of firm heterogeneity in terms of total factor productivity.¹¹⁹ One key finding was that the optimal integration strategy for a firm depends on its productivity. In addition, given productivity differences across firms – coexistence of alternative modes of integration is based on the notion of firm heterogeneity.

Empirically, the activity of multinational enterprises is among the most dynamic economic activities (followed by international trade in goods and services).¹²⁰ For instance, the average annual growth rate of foreign affiliate sales was 8.4% during the period of 1996-2000 and was 16.2% in 2006.¹²¹ The focus of empirical analyses of integration strategies of multinational enterprises (MNEs) has been on whether purely vertical or horizontal strategies are prevalent in data on foreign direct investments (FDI). As a result of such work, indirect evidence has favored horizontal MNE models more so than vertical MNE models.¹²²

Work on the role of profit taxes on FDIs has suggested that FDI react sensitively to changes in tax rates.¹²³ The latter indicates that the debate on optimal taxation should be of key interest to policy makers. Researchers have also pointed out the importance of corporate taxation in influencing firm location and production decisions.¹²⁴ Empirical evidence in support of this has suggested the relevance of taxation to location and volume of FDIs (i.e. production decisions of MNEs). The various impacts include the impact of a corporate tax rate in the parent country on inbound FDI, the impact of the corporate tax rate in the host country on outbound

¹¹⁸ As in Markusen (1984) and Helpman (1984).

¹¹⁹ Compare with Helpman, Melitz and Yeaple (2004).

¹²⁰ In the year 2006 global FDI inflows grew for the third consecutive year and reached the level of \$1.306 trillion being slightly below the record level of \$1.411 trillion in 2000. As in UNCTAD (2008) and World Bank Institute (2007).

¹²¹ In the same time, the gross product of foreign affiliates increased 7.3% p.a. in the years of 1996-2000 and rose by 16.2% in 2006. Exports of foreign affiliates showed an increase of 3.3% p.a. in 1996-2000 and rose by 12.2% in 2006. As in UNCTAD (2008).

¹²² As supported by Markusen and Maskus (2001) and Brainard (1993a).

¹²³ As in Grubert and Mutti (1991) and Blonigen and Davies (2004).

¹²⁴ See Hines (1999) or Gresik (2001) for a survey.

FDIs, and the effects of parent and host country taxation in terms of different methods of double taxation relief.¹²⁵

Research regarding statutory tax rates and their impact on FDIs is abundant, containing diverse distinctions between different methods of double taxation relief and the impact of statutory corporate tax rates on MNE activities.¹²⁶ In analyzing the impact of withholding tax rates on MNE activities, we have seen that they are independent of the method of double taxation relief. For example, if foreign-earned profits are subject to withholding taxes levied, increasing withholding tax rates reduces MNE activities in the host country.¹²⁷

Although diverse implications of withholding tax rates are cogitable, these and the impacts on tax rates on MNE activities of heterogeneous firms have hardly been studied in theoretical work.

To focus on the topics of firm heterogeneity, the increasing importance of MNEs, and the impact of corporate taxation on MNE activities, we have set up a model of heterogeneous firms that select their strategies from a menu of three options: domestic operations, exporting operations, or horizontal MNE activities.¹²⁸ We have assumed that manufacturing firms supply varieties of differentiated goods under monopolistic competition.

In our model, social welfare-maximizing governments levy withholding taxes on MNE profits earned by subsidiaries producing in the jurisdiction of the particular government. Furthermore, the generated tax revenue is spent for a lump-sum transfer to the households there. These corporate tax rates affect the integration strategies of heterogeneous firms. Of course the economic structure and the nature of competition are essential for this to be a welfare-maximizing policy.¹²⁹

We also have distinguished between the perspectives of a social planner and a single government on maximization because a single government only maximizes its own national welfare. An increase in withholding tax rates, nevertheless, induces a

¹²⁵ To see the impact on inbound FDI, see Head, Ries and Swenson (1999) for a survey; to see the impact on outbound FDI see Mutti and Grubert (2004) for a survey; and for the impact of parent and host country taxation, see Swenson (1994).

¹²⁶ We can distinguish between the credit, exemption, and deduction methods. See Egger et al. (2006b) for a survey.

¹²⁷ As in Egger et al. (2006b).

¹²⁸ In contrast to Davies and Eckel (2007) assuming mobile firms.

¹²⁹ As in Dixit and Grossman (1986), Venables (1985), or Helpman and Flam (1986).

decline of MNE investments in this jurisdiction.¹³⁰ This coherence is consistent with the findings of Hines (1999) or Devereux and Griffith (2003).¹³¹ Governments are completely informed and consider the implications of taxation on the integration strategies of heterogeneous firms and the resulting impacts on the utility of the representative household in their own jurisdictions. Social planners consider welfare implications in both countries.

The remainder of this chapter is structured as follows: Section 3.2 outlines the model and derives optimal integration strategies of firms in the differentiated sector. These are dependent on the relative size of fixed costs for plant setup, market, country sizes, firm productivities, transport costs, and corporate taxation. After presenting welfare maximization and governments objectives, we set up a numerical framework in section 3.2.5. In contrast to related theoretical work, we endogenously derive the mass of firms entering markets as well as the market size itself.¹³² We study the results of this numerical analysis with special emphasis on the role of country size. In section 3.2.6, we flesh out the main differences of this approach relative to recent theoretical work. Finally, in section 3.3, we conclude and point to the implications of our findings in terms of optimal taxation and economic outcome.

3.2 The model

The following partial analysis is a description of optimal integration strategies of heterogeneous firms, with particular emphasis on the role of profit taxation. We focus on the optimal tax policy of governments providing a lump-sum transfer to households in their jurisdictions, depending on the integration strategies chosen by heterogeneous firms.

First, consider a simple model with two countries, A and B, in which only one factor, labor (L), is used for production and firm or plant setup. L is assumed to be mobile between sectors but immobile across national borders. Goods may be consumed from local or foreign producers. The latter results in goods trade, which invokes iceberg-type trade costs. With regard to integration strategies, firms choose between

¹³⁰ Evidence for this can be found in Devereux (2006) and in Hines and Rice (1994).

¹³¹ Early empirical work finds negligible effects of tax policies on FDI. See Brainard (1997) and Wheeler and Mody (1992) for a survey.

¹³² In contrast, see Grossman, Helpman and Szeidl (2006) or Davies, Egger and Egger (2009).

three options: locating in one country and serving only domestic consumers, concentrating production in one country and serving consumers worldwide from there (exporting), or engaging in multiplant production and serving consumers locally through domestic and foreign subsidiaries (MNEs).

There are two industries. One of them produces a homogeneous good x_0 ; the other industry produces differentiated goods. The homogeneous good is supplied under perfect competition. For the sake of elegance, let us assume that one unit of labor is needed to fabricate one unit of the homogeneous good. We focus on parameter configurations, which ensure diversification of production, so that the homogeneous good is produced in both countries in equilibrium and may be traded at zero costs across national borders.

Varieties of the differentiated good are supplied under monopolistic competition. Each firm in the differentiated sector acts as a monopolist in supplying its variety. However varieties are substitutable at an elasticity of $\sigma > 1$, which also reflects the elasticity of demand. Consequently, firms in that sector charge a fixed markup over marginal costs.

To enter the differentiated industry, the amount of f_d units of labor, which are sunk costs, must be invested. These can be considered firm setup costs. With this investment a firm in this heterogeneous sector discovers its own potential productivity level (θ). The productivity level drawn by a firm is a random variable.

3.2.1 Demand

We assume that the preferences of households are quasi-linear and that households are identical with respect to their preferences. In formal accounts, the utility function of the representative household is represented by:

$$U_{j} = X_{0} + \frac{1}{\mu} \left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} X_{j}(i)^{\alpha} di \right)^{\mu}, j \in \{A, B\}$$
(76)

The representative households in A and B benefit from consumption of the homogeneous good x_o , which is taken as the numéraire for convenience. Furthermore, each of the two countries hosts a second industry that produces differentiated goods under monopolistic competition. $x_i(i)$ is the consumption of

output of the i-th firm, which is $i \in \{0, ..., \theta_{max}\}$. The condition $0 < \alpha < 1$ being constant results in a constant elasticity of substitution (C.E.S.) of $\sigma = 1/(1-\alpha) > 1$ between any pair of differentiated goods. This expression reflects standard properties of love for variety preferences, where a broader supply of differentiated goods results in increased utility. μ is a constant with $0 < \mu < \alpha < 1$ and reflects the preference for the differentiated industry over the homogenous industry in the utility function of the representative household. At a certain level of differentiated products supplied in one country, an additional unit shows diminishing marginal utility. The consumption of

differentiated products is represented by the expression $X = \left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} x_{j}(i)^{\alpha} di \right)$, the sub-

utility of the differentiated sector.

Obviously, the utility function is linear in x_0 but nonlinear in the differentiated varieties. This implies that the demand for differentiated products depends on prices of differentiated goods but not on earnings.

To derive demand of a single household demand for the variety $x_j(i)$ in country j, we consider the utility function in (76) and satisfy the standard side condition $m_j \ge p_0 \cdot x_0 + \int_0^{\theta_{max}} p_j(i) \cdot x_j(i)$. Labor income m is spent on the homogeneous good, where we set $p_0 = 1$, and on differentiated goods. This results in the demand of a single household for differentiated goods of¹³³

$$\mathbf{x}_{j}(\mathbf{i}) = \frac{1}{\left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} \mathbf{x}_{j}(\mathbf{i})^{\alpha} d\mathbf{i}\right)^{\frac{1-\mu}{1-\alpha}}} p_{j}(\mathbf{i})^{\frac{1}{1-\alpha}}$$
(77)

or,

$$p_{j}(\mathbf{i}) = \frac{1}{\left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} \mathbf{x}_{j}(\mathbf{i})^{\alpha} d\mathbf{i}\right)^{1-\mu} \mathbf{x}_{j}(\mathbf{i})^{1-\alpha}}$$
(78)

respectively.

¹³³ See 3.4.1 in the Appendix.

The demand of a single household in country j for differentiated goods of the i-th firm depends on the price firm i sets, on how any pair of differentiated goods can be substituted for another through α , on μ , and on the subutility of consumption

$$X = \left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} x_{j}(i)^{\alpha} di\right).$$
 The impact of an increasing α is that products of the

differentiated sector become closer substitutes for one another, which results in reduced market power for a single firm.

As can be seen from equations (77) and (78), the size of X is determined endogenously. For this reason, X can also be interpreted as the market size for differentiated goods and demands for specification. X depends on the strategic alignment of heterogeneous firms.

We also distinguish between different scenarios. In the first case, market size X consists of the market of domestic firms, foreign firms exporting their goods from abroad (henceforth referred to as exporters), and of firms choosing horizontal MNE activity. Market size X in equilibrium is defined as:

$$X_{d,ex,i} = \underbrace{\left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} x_{j}(i)^{\alpha} di\right)}_{\text{domestic (d)}} + \underbrace{\left(\int_{\theta_{d/ex}}^{\theta_{ex/i}} \frac{1}{\alpha} x_{j}(i)^{\alpha} di\right)}_{\text{exporter (ex)}} + \underbrace{\left(\int_{\theta_{ex/i}}^{\theta_{max}} \frac{1}{\alpha} x_{j}(i)^{\alpha} di\right)}_{\text{MNE (i)}}$$
(79)

Alternatively, in another scenario, the export strategy does not exist (i.e., is not profitable). Firms choose either supplying domestically or acting as MNEs. This scenario results in a market size of:

$$X_{d,i} = \underbrace{\left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} x_{j}(i)^{\alpha} di\right)}_{\text{domestic}} + \underbrace{\left(\int_{\theta_{d/i}}^{\theta_{max}} \frac{1}{\alpha} x_{j}(i)^{\alpha} di\right)}_{\text{MNE}}$$
(80)

Finally, MNE activity may be nonprofitable so that market size consists of demand from domestic and exporting producers only. The specified market size in this case shows:

$$X_{d,ex} = \underbrace{\left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} x_{j}(i)^{\alpha} di\right)}_{domestic} + \underbrace{\left(\int_{\theta_{d/ex}}^{\theta_{max}} \frac{1}{\alpha} x_{j}(i)^{\alpha} di\right)}_{exp \text{ orter}}$$
(81)

Figure 17 shows market size under the alternative integration strategies of heterogeneous firms.



Figure 17. The market size X depending on different integration strategies.

3.2.2 Production

As mentioned before, we focus on equilibria with diversification of production so that each of the two countries, $j=\{A,B\}$, hosts the two industries. The associated country size of A and B is reflected by s_A and s_B .

We assume that countries A and B are endowed with a fixed amount of internationally immobile labor, L. Because the homogeneous good is freely tradable, is used as the numéraire, and uses one unit of L for one unit of output, there is international wage equalization at unitary wages (i.e., $w_j = 1$) as long as diversification of production prevails.

The differentiated goods available in a country j are provided by different sources. Consumers in j buy goods produced by national producers in j, imports from the other country, and goods from subsidiaries in j where the origin of these firms is in the other country (MNEs). Hence, the mass of firms in the world equals the amount of differentiated goods potentially available.

Firms in the differentiated sector differ with respect to their productivity, but ex-ante all firms are identical. If they expect positive earnings from the production process, they pay sunk entry costs f_d upfront, which are measured in units of labor. As long as firms expect positive profits, they enter the market. It is assumed that the individual productivity levels of the firms in each country are independent draws from a cumulative productivity distribution function $F(\theta)$. Because of the fee f_d , the firms may independently draw their productivity from the distribution $F(\theta)$ with support over (0,

 θ_{max}). With this procedure, firms located in the home country, even with very low productivity, will produce domestically to reduce the loss of f_d . The time line in figure 18 shows the logical sequence from the moment prior to entry, where all firms are identical, to the moment where firms in the industry decide on their integration strategies and output:

					— t
0	A-priori, all firms are identical	Firm expects positive earnings and pays market entry costs f _d	Productivity θ is revealed	Domestic production and its output decision	If θ sufficient: Output decision for additional export or MNE activity

Figure 18. Steps towards the choice of integration of the firm.

Firms choose their integration strategy according to their productivity $\theta(i)$. In their domestic country, all firms start as domestic producers. If productivity is low, a firm will not enter the foreign market, neither through exports nor through foreign plant setup. If productivity is high enough, a firm has the choice to serve foreign markets additionally via exports or foreign affiliate production (the latter being referred to as horizontal MNE activity). The choice between exporting and foreign plant setup is driven by the proximity-concentration trade-off, characterized by the savings in trading costs for MNE activity relative to exports, reflected by iceberg transport costs t for cross-border trade of differentiated varieties.¹³⁴ The idea of iceberg transport costs is that to deliver one unit of differentiated goods, the producer must ship t ≥ 1 units to the distant point of sale. On the other hand, in foreign plant setup, the fixed costs f_i in terms of units of labor must be higher than the fixed costs for exporters f_{ex} because production facilities must be duplicated.¹³⁵ For this reason $f_d < f_{ex} < f_i$ is assumed.

In addition to these fixed costs, firms pay variable costs, depending on their own productivity levels θ (i), on the integration strategies (i.e., exporters pay transport costs t>1), and on country size [i.e. $s_j x_j(i)t/\theta$ (i)]. Hence, country size s_j reflects the total demand for variety i in j and t=1 for domestic producers and MNEs. Given two firms with the same amount of output in one country, the firm with higher productivity θ (i) must bear lower variable costs, according to $s_j x_j(i)/\theta$ (i).

¹³⁴ See e.g. Horstmann and Markusen (1992), Brainard (1993b), or Markusen and Venables (2000) for a survey.

¹³⁵ As in Helpman, Melitz and Yeaple (2004).

Furthermore, governments may choose positive profit tax rates subject to foreign MNEs to maximize welfare in their own jurisdictions. If tax revenue in j is positive, it is passed on to households in j as a lump-sum transfer. In this analysis, a government in j can levy taxes on profits earned by MNEs in j. Because these MNEs are headquartered in the other country, only the profits earned from production in the plant in j can be taxed by the government in j (i.e., the location of tax payment is identical with an MNE subsidiary location. Therefore, in this setting, double taxation is not the problem in the analysis.¹³⁶

For this reason, γ_A denotes a withholding tax rate of the government in A on profits of an MNE plant in A, where the origin of this firm is in B. With $\gamma_A > 0$, these firms consider γ_{A} an additional factor influencing profits. γ_{B} denotes a withholding tax rate of the government in B on profits of an MNE plant in B, where the origin of this firm is in A.

Because γ_j describes a withholding tax rate on profits earned by subsidiaries, taxation is not considered for domestic and export profits (i.e., γ_{A} and γ_{B} are relevant parameters considering profits of MNEs only).

Given the household demand in (77) and the price consumption curve in (78), it is straightforward to compute maximum attainable profits for a firm in j serving its domestic market:

$$\pi_{j}(i)_{d} = s_{j}p_{j}(i)_{d}x_{j}(i)_{d} - \frac{s_{j}x_{j}(i)_{d}}{\theta(i)} - f_{d}$$
$$= s_{j}\frac{1}{X^{(1-\mu)}x_{j}(i)_{d}}^{(1-\mu)}x_{j}(i)_{d} - \frac{s_{j}x_{j}(i)_{d}}{\theta(i)} - f_{d}$$

The derivative with respect to $x_i(i)$ yields:

$$\frac{\partial \pi_{j}(i)_{d}}{\partial x_{j}(i)_{d}} = s_{j} \alpha x_{j}(i)_{d}^{(\alpha-1)} \frac{1}{\chi^{(1-\mu)}} - \frac{s_{j}}{\theta(i)} = 0$$

This finally results in an expression for the profit-maximizing output of a firm i in its domestic market j, $j \in \{A, B\}$,¹³⁷

 ¹³⁶ As in Egger et al. (2006a).
 ¹³⁷ See 3.4.2 in the Appendix.

$$x_{j}(i)_{d}^{*} = \frac{\left(\alpha\theta(i)\right)^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}}$$
(82)

associated with the optimal price¹³⁸

$$p_{j}(i)_{d}^{*} = \frac{1}{\alpha \theta(i)} .$$
(83)

The optimal output of a firm in the domestic market depends on market size X.¹³⁹ According to (82), the optimal output level of a single firm is negatively correlated with X due to competitive conditions. Furthermore, the productivity level of a firm is positively correlated with its output.

In setting the price set, firms follow standard markup pricing in which higher productivity is associated with smaller price. The markup is represented by the factor $\frac{1}{2}$.

Accordingly, maximum attainable profits of a domestic firm in j are given by:¹⁴⁰

$$\pi_{j}(\mathbf{i})_{d}^{*} = \frac{\mathbf{s}_{j}(\alpha\theta(\mathbf{i}))^{\frac{1}{(1-\alpha)}}}{X^{\frac{1-\mu}{1-\alpha}}\theta(\mathbf{i})} \left(\frac{1}{\alpha} - 1\right) - \mathbf{f}_{d}$$
(84)

Analogously, we now can derive profits of firms with export strategies. Profits of exporters from Country A are defined by:

$$\pi_{A}(i)_{ex} = \underbrace{s_{B}p_{B}(i)_{ex}x_{B}(i)_{ex} - \frac{s_{B}x_{B}(i)_{ex}t}{\theta(i)} - f_{ex}}_{\text{engage in exports if }>0} + \underbrace{s_{A}p_{A}(i)_{d}x_{A}(i)_{d} - \frac{s_{A}x_{A}(i)_{d}}{\theta(i)} - f_{d}}_{\text{domestic profits}}$$
(85)

Profits of exporters from Country B are defined by:

$$\pi_{B}(i)_{ex} = \underbrace{s_{A}p_{A}(i)_{ex}x_{A}(i)_{ex} - \frac{s_{A}x_{A}(i)_{ex}t}{\theta(i)} - f_{ex}}_{engage in exports if > 0} + \underbrace{s_{B}p_{B}(i)_{d}x_{B}(i)_{d} - \frac{s_{B}x_{B}(i)_{d}}{\theta(i)} - f_{d}}_{domestic profits}$$
(86)

An exporting firm has two sources of earnings. The company generates profits from domestic sales and from export activity. The variable costs for exports depend on t.

¹³⁸ See 3.4.2 in the Appendix. ¹³⁹ The market size X has to be specified according to $X_{d,ex}$, $X_{d,ex,i}$ or $X_{d,i}$. ¹⁴⁰ See 3.4.2 in the Appendix.

For a firm i from j, the expression (85) and (86) above result in optimal output in the other country (output for exporting),¹⁴¹

$$\mathbf{x}_{j}(\mathbf{i})_{ex}^{*} = \frac{\left(\alpha\theta(\mathbf{i})\right)^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}\mathbf{t}^{\frac{1}{(1-\alpha)}}}$$
(87)

associated with the optimal price for exports,¹⁴²

$$p_{j}(i)_{ex}^{*} = \frac{t}{\alpha \theta(i)}.$$
(88)

In addition to the previous analysis, we can see that the optimal output and price for exports depend on transport costs t in contrast to the optimal output and price when supplying domestic demand. Accordingly, maximum attainable profits of a firm i from A, exporting to B, are given by:¹⁴³

$$\pi_{A}(\mathbf{i})_{ex}^{*} = \frac{\mathbf{s}_{B}(\alpha\theta(\mathbf{i}))^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}\mathbf{t}^{\frac{\alpha}{(1-\alpha)}}\theta(\mathbf{i})} \left(\frac{1}{\alpha} - 1\right) - \mathbf{f}_{ex} + \pi_{A}(\mathbf{i})_{d}^{*}$$
(89)

Analogously, this can be derived for a firm i from B exporting to A.

Now, it is straightforward to compute maximum attainable profits for firms engaged in multinational activities. As they produce goods for both markets locally, transport costs do not occur. Instead a firm i from Country A opens an affiliate in B and becomes a horizontal MNE.

To maximize social welfare, a government in j may choose to levy withholding taxes on profits of foreign MNEs earned by subsidiaries in its jurisdiction. Profits of an MNE headquartered in Country A are defined by:

$$\pi_{A}(\mathbf{i})_{i} = \underbrace{\left(s_{B}p_{B}(\mathbf{i})_{i}x_{B}(\mathbf{i})_{i} - \frac{s_{B}x_{B}(\mathbf{i})_{i}}{\theta(\mathbf{i})}\right)\left(1 - \gamma_{B}\right) - f_{i}}_{\text{engage in MNE if } > 0} + \pi_{A}(\mathbf{i})_{d}$$
(90)

Profits of an MNE headquartered in Country B are defined by:

$$\pi_{\rm B}(\mathbf{i})_{i} = \underbrace{\left(\mathbf{s}_{\rm A} \mathbf{p}_{\rm A}(\mathbf{i})_{i} \mathbf{x}_{\rm A}(\mathbf{i})_{i} - \frac{\mathbf{s}_{\rm A} \mathbf{x}_{\rm A}(\mathbf{i})_{i}}{\mathbf{\theta}(\mathbf{i})} \right) (1 - \gamma_{\rm A}) - \mathbf{f}_{i}}_{\text{engage in MNE if } > 0} + \pi_{\rm B}(\mathbf{i})_{\rm d}$$
(91)

 $^{^{141}}$ See 3.4.3 in the Appendix. 142 See 3.4.3 in the Appendix.

¹⁴³ See 3.4.3 in the Appendix.

An MNE expects at least zero profits from running both domestic and foreign subsidiaries. Profit maximizing plant output¹⁴⁴

$$\mathbf{x}_{j}(\mathbf{i})_{i}^{*} = \frac{\left(\alpha \boldsymbol{\theta}(\mathbf{i})\right)_{(1-\alpha)}^{1}}{X^{\frac{(1-\mu)}{(1-\alpha)}}}$$
(92)

is associated with optimal price¹⁴⁵

$$p_{j}(i)_{i}^{*} = \frac{1}{\alpha} \frac{1}{\theta(i)}.$$
(93)

Accordingly, the maximum attainable profits of a multinational firm i headquartered in Country A are given by¹⁴⁶

$$\pi_{A}(i)_{i}^{*} = \frac{S_{B}(\alpha\theta(i))^{\frac{1}{(1-\alpha)}}(1-\gamma_{B})}{X^{\frac{(1-\mu)}{(1-\alpha)}}\theta(i)} \left(\frac{1}{\alpha}-1\right) - f_{i} + \pi_{A}(i)_{d}^{*}.$$
(94)

Analogously, this can be derived for MNEs from B with subsidiaries in A.

Firms choose their integration strategies based on the knowledge of their productivity levels. This results in the cut-off levels being the determinants of minimum levels of productivity for a firm i to generate zero profits additionally when ex-ante selecting a strategy with more than domestic production. In general, more productive firms are more successful in all three strategies.

The least productive firms only serve the domestic market through domestic production. Because of their low productivity, their variable costs are too high. Therefore, the higher fixed costs of operating in an additional market cannot be covered.

At this point, the cut-off levels must be analyzed. At the first cut-off, firm productivity is such that additional profits of exporting result in exactly zero profits.

For a firm i from A exporting to B, this is derived from:

$$\pi_{A}(i)_{ex} = \underbrace{s_{B}p_{B}(i)_{ex}^{*} x_{B}(i)_{ex}^{*} - \frac{s_{B}x_{B}(i)_{ex}^{*} t}{\theta(i)} - f_{ex}}_{=D} + \underbrace{\pi_{A}(i)_{d}}_{\text{domestic profits}}$$

Analogously, this holds true for a firm i from B exporting to A.

With $D \ge 0$, this applies for:¹⁴⁷

¹⁴⁴ See 3.4.4 in the Appendix. ¹⁴⁵ See 3.4.4 in the Appendix.

¹⁴⁶ See 3.4.4 in the Appendix.

$$\theta_{d/ex} = \frac{f_{ex}^{\frac{(1-\alpha)}{\alpha}} t X^{\frac{(1-\mu)}{\alpha}}}{\alpha (s_{j}(1-\alpha))^{\frac{(1-\alpha)}{\alpha}}}$$
(95)

The market size X results endogenously, according to X_{d.ex.i}. Furthermore, the cut-off productivity depends on the country size s_i. The larger the foreign country s_i, the smaller the productivity of a firm has to be for the export strategy to become reasonable. A firm with productivity $\theta_{d/ex}$ generates zero profits from exporting. Hence, this firm is indifferent in terms of only selling domestically or engaging in exports in addition to domestic sales. A firm with productivity just above this level is already earning positive profits from exporting and will definitely engage in exporting.

The critical productivity level in (95) is positively correlated with t, f_{ex}, and market size X. Hence, the indifferent firm must be more productive to break even. In other words, a higher productivity yields lower variable costs of production. Furthermore, conditional on the existence of the export strategy, productivity levels exist that ensure that profits of exporters exceed profits of MNEs.

At the next productivity level threshold, profits of an exporting firm equal profits of an MNE (i.e. $\pi(i)_{ex} = \pi(i)_i$).¹⁴⁸

This applies to
$$\theta_{ex/i} = \frac{(f_i - f_{ex})^{\frac{(1-\alpha)}{\alpha}} \cdot X^{\frac{(1-\mu)}{\alpha}}}{s_j^{\frac{(1-\alpha)}{\alpha}} \alpha (1-\alpha)^{\frac{(1-\alpha)}{\alpha}} \left[(1-\gamma_j) - t^{\frac{-\alpha}{(1-\alpha)}} \right]^{\frac{(1-\alpha)}{\alpha}}}.$$
 (96)

Only firms with $\theta(i) > \theta_{ex/i}$ gain positive profits from serving foreign markets through building subsidiaries instead of exporting their goods. $\theta_{ex/i}$ depends on the difference in fixed costs $(f_i - f_{ex}) > 0$, which can be interpreted as overhead and set-up costs of an MNE subsidiary. The higher the overhead costs $(f_i - f_{ex})$ for a foreign subsidiary are, the more productive the indifferent firm must be to engage in MNE activity (i.e., the cut-off level $\theta_{ex/i}$ takes over a higher value). The higher the transport costs t are, the more likely firms are to engage in the MNE integration strategy. Higher t, therefore, results in a lower value of $\theta_{ex/i}$. The larger the foreign country s_i, the

¹⁴⁷ See 3.4.5 in the Appendix.
¹⁴⁸ See 3.4.6 in the Appendix.

smaller the productivity of the firm must be for the export strategy to become reasonable. Furthermore, only if $(1-\gamma_i) > t^{\frac{-\alpha}{(1-\alpha)}}$ holds does a real and unique solution exist. If the parameter configuration of the transport costs t, the tax rate γ , and α do not satisfy this condition, MNE activities do not exist.

Alternatively, certain configurations of parameters may result in a situation in which domestic firms integrate directly as MNEs instead of choosing the export strategy. The following cut-off level is the relevant productivity threshold if $\theta_{d/i} < \theta_{d/ex}$. The associated cut-off level results from

$$\pi_{A}(i)_{i} = \underbrace{\left(s_{B}p_{B}(i)_{i}^{*} x_{B}(i)_{i}^{*} - \frac{s_{B}x_{B}(i)_{i}^{*}}{\theta(i)}\right)(1 - \gamma_{B}) - f_{i}}_{=E} + \pi_{A}(i)_{d} \text{ or}$$

$$\pi_{B}(i)_{i} = \underbrace{\left(s_{A}p_{A}(i)_{i}^{*} x_{A}(i)_{i}^{*} - \frac{s_{A}x_{A}(i)_{i}^{*}}{\theta(i)}\right)(1 - \gamma_{A}) - f_{i}}_{=E} + \pi_{B}(i)_{d}, \text{ with E>0, and shows}^{149}$$

$$\theta_{d/i} = \frac{f_{i}\frac{(1 - \alpha)}{\alpha} \chi^{\frac{(1 - \mu)}{\alpha}}}{\alpha(s_{j}(1 - \gamma_{j})(1 - \alpha))^{\frac{(1 - \alpha)}{\alpha}}}.$$
(97)

The associated market size X in this scenario endogenously results in X_{d,i}. The larger the foreign country s_i, the smaller the productivity of a firm must be for the MNE strategy to become reasonable. For $\theta_{d/i} \! < \! \theta_{d/ex}$ so that this cut-off level exists, the following condition must hold:¹⁵⁰

$$(1-\gamma_{j}) \ge \frac{f_{i}}{f_{ex}t^{\frac{\alpha}{(1-\alpha)}}}$$
(98)

3.2.3 Welfare maximization and the objective of the government

In the following, governments can set profit taxes in a first stage and cannot rescind their offers by assumption. Then, firms decide upon their optimal integration strategies, whereas the governments take this into account when setting tax rates. A government chooses a withholding tax rate, $0 < \gamma_j < 1$, $j \in \{A,B\}$, to capture profits of foreign MNEs earned in plants in j. Hence, MNE profits from production in j are taxed

 $^{^{149}}$ See 3.4.7 in the Appendix. 150 See 3.4.7 in the Appendix.

by the government in j. This tax revenue is passed on to the households within that jurisdiction. When selecting an optimal tax rate γ_j , to be optimal, the government in j maximizes the utility of the representative household in its country.

Furthermore, transport costs t reduce exporting firm profits and are given exogenously. Taxation reduces MNE profits, where tax rates are set endogenously by both governments. The set of optimal integration strategies is influenced by these transport costs and profit taxes, both of which have an impact on the mass of firms choosing the different optimal integrations strategies.

3.2.3.1 The objective of the governments

In this section, cases are analyzed, in which the governments of both countries, A and B, can levy withholding taxes γ_A and γ_B , which are taken into account in the MNE profit functions. In this setting, taxes are paid on MNE profits either in A or in B. The location of tax payments depends on the production location of a firm.

Furthermore, by assumption, households do not know the underlying tax basis for provision of the lump-sum transfer so that the composition of consumption of differentiated goods is not distorted.

The price for the homogeneous product is $p_0=1$; and prices for differentiated products are shown by $p_j(i)$, where $p_j(i)$ is the price for variety i in country j.

As already shown, the representative household utility in country j, $j \in \{A; B\}$, is given by (76), which can also be shown by:

$$U_{j} = m_{j} - \int_{0}^{\theta_{max}} p_{j}(i) x_{j}(i) di + \frac{1}{\mu} \left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} x_{j}(i)^{\alpha} di \right)^{\mu}$$
(99)

Representative household utility increases in m_i and declines in $p_i(i)$.

For a government to pass on a lump-sum transfer to the households in its jurisdiction j and, therefore, to select $\gamma_j > 0$, the representative household utility with lump-sum transfer may not be smaller than the utility in (99), considering the implications of profit taxation in the other jurisdiction. Hence, government tax revenue depends on the strategies chosen by the firms. For this reason, the firm profit functions dependent on tax rates must be examined to consider the associated utility function,

including lump-sum transfer. To show these welfare implications, optimal integration strategies with taxation are examined in the following subsection.

3.2.3.2 Strategic alignment

According to alternative parameter configurations, strategic alignments of firms and their impact on welfare with $\gamma_i > 0$ are examined in the following:

1. For all integration strategies to coexist these conditions must hold: $\theta_{d/ex} < \theta_{ex/i}$, which results in a relation between the fixed costs; transport costs t; and the tax rate γ_i :

$$\left(1-\gamma_{j}\right) < \frac{1}{t^{\frac{\alpha}{(1-\alpha)}}} \left(\frac{\left(f_{i}-f_{ex}\right)}{f_{ex}}+1\right)$$
(100)

Furthermore, for the MNE strategy to exist, $(1 - \gamma_i) > t^{\frac{-\alpha}{(1-\alpha)}}$ must hold.¹⁵¹ The resulting utility function of the representative household with lump-sum transfer is given by:

$$\begin{split} U_{j} &= m_{j} - \left(\int_{0}^{\theta_{max}} p_{j}(i)_{d}^{*} x_{j}(i)_{d}^{*} di + \int_{\theta_{d/ex}}^{\theta_{ex/i}} p_{j}(i)_{ex}^{*} x_{j}(i)_{ex}^{*} di + \int_{\theta_{ex/i}}^{\theta_{max}} p_{j}(i)_{i}^{*} x_{j}(i)_{i}^{*} di \right) \\ &+ \frac{1}{\mu} \left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} (x_{j}(i)_{d}^{*})^{\alpha} di + \int_{\theta_{d/ex}}^{\theta_{ex/i}} \frac{1}{\alpha} (x_{j}(i)_{ex}^{*})^{\alpha} di + \int_{\theta_{ex/i}}^{\theta_{max}} \frac{1}{\alpha} (x_{j}(i)_{i}^{*})^{\alpha} di \right)^{\mu} \\ &+ \frac{\gamma_{j}}{s_{j}} \int_{\theta_{ex/i}}^{\theta_{max}} \left(s_{j} p_{j}(i)_{i}^{*} x_{j}(i)_{i}^{*} - \frac{s_{j} x_{j}(i)_{i}^{*}}{\theta(i)} \right) di \end{split}$$

$$(101)$$

2. Alternative parameter configurations can result in a situation in which only domestic and MNE firms enter production. For this constellation to exist, the following conditions must hold: $\theta_{d/i} < \theta_{d/ex}$ which results in a relation between the fixed costs; transport costs t; and the tax rate γ_i :¹⁵²

$$(1 - \gamma_j) \ge \frac{f_i}{f_{ex}t^{\frac{\alpha}{(1 - \alpha)}}}$$

Furthermore, to ensure that the MNE strategy exists, it follows from $\theta_{\text{d/i}}$ that $(1-\gamma_i)(1-\alpha)s_i \neq 0$ must hold. Therefore, governments must select $\gamma_i < 1$. The resulting utility function of the representative household with lump-sum transfer is given by:

 $^{^{151}}$ See 3.4.7 in the Appendix. 152 See 3.4.7 in the Appendix.

$$U_{j} = m_{j} - \left(\int_{0}^{\theta_{max}} p_{j}(i)_{d}^{*} x_{j}(i)_{d}^{*} di + \int_{\theta_{d/i}}^{\theta_{max}} p_{j}(i)_{i}^{*} x_{j}(i)_{i}^{*} di\right)$$

$$+ \frac{1}{\mu} \left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} (x_{j}(i)_{d}^{*})^{\alpha} di + \int_{\theta_{d/i}}^{\theta_{max}} \frac{1}{\alpha} (x_{j}(i)_{i}^{*})^{\alpha} di\right)^{\mu}$$

$$+ \frac{\gamma_{j}}{s_{j}} \int_{\theta_{d/i}}^{\theta_{max}} \left(s_{j}p_{j}(i)_{i}^{*} x_{j}(i)_{i}^{*} - \frac{s_{j}x_{j}(i)_{i}^{*}}{\theta(i)}\right) di$$

$$(102)$$

3. Alternative parameter configurations may result in a situation in which no MNEs enter production. Then, the MNE strategy only exists if $(1-\gamma_j) > t^{\frac{-\alpha}{(1-\alpha)}}$ holds. Otherwise, only domestic and exporting strategies are chosen. Additionally, to guarantee that the export strategy exists, $\theta_{d/ex} < \theta_{max}$ must hold. This can also be written as:

$$\frac{f_{ex}\frac{(1-\alpha)}{\alpha}X^{\frac{(1-\mu)}{\alpha}}t}{\alpha(s_{j}(1-\alpha))^{\frac{(1-\alpha)}{\alpha}}} < \theta_{max}$$
(103)

The resulting utility function of the representative household is given by:

$$U_{j} = m_{j} - \left(\int_{0}^{\theta_{max}} p_{j}(i)_{d}^{*} x_{j}(i)_{d}^{*} di + \int_{\theta_{d/ex}}^{\theta_{max}} p_{j}(i)_{ex}^{*} x_{j}(i)_{ex}^{*} di\right)$$

$$+ \frac{1}{\mu} \left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} (x_{j}(i)_{d}^{*})^{\alpha} di + \int_{\theta_{d/ex}}^{\theta_{max}} \frac{1}{\alpha} (x_{j}(i)_{ex}^{*})^{\alpha} di\right)^{\mu}$$

$$+ \frac{\gamma_{j}}{s_{j}} \cdot 0$$
(104)

Without firms selecting MNE activities, a tax base does not exist. For this reason, a lump-sum transfer to the representative household cannot be provided independent of the size of γ_i .

4. Alternative parameter configurations may result in a situation in which only domestic firms enter production. For this constellation to exist, the following conditions have to hold: $\theta_{d/ex} > \theta_{max}$ and $\theta_{d/i} > \theta_{max}$

The resulting utility function of the representative household is given by:

$$U_{j} = m_{j} - \left(\int_{0}^{\theta_{max}} p_{j}(i)_{d}^{*} x_{j}(i)_{d}^{*} di\right)$$

+
$$\frac{1}{\mu} \left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} (x_{j}(i)_{d}^{*})^{\alpha} di\right)^{\mu}$$

+
$$\frac{\gamma_{i}}{s_{j}} \cdot 0$$
 (105)

Without firms selecting MNE activities, a tax base does not exist. For this reason, a lump-sum transfer to the representative household cannot be provided independent of the size of γ_i .

3.2.3.3 The decision of the government

The decision of the government depends on the exogenous variables, such as transport costs, country size, variable costs, fixed costs, and firm productivity resulting in a set of optimal integration strategies.

First, the prices of a single firm do not change because of the taxes levied. These are only influenced by transport costs.¹⁵³ Transport costs are passed on to the households, whereas taxes are paid by the firms; and prices in A and B for goods from the same firm only differ if transport costs exist. If a strategy of production in A and B is reasonable, not all profits of a firm are taxed. Instead, only profits generated by MNEs in the foreign market are subject to taxation of the foreign government. Hence, the government in B taxes the profits gained in B of MNEs that have their origins in A and vice versa.

Governments consider the following aspects when setting their optimal tax rate, $0 < \gamma_i < 1$:

1. Firm profits, including taxation, only change if levied taxes influence firms to choose strategies other than MNE activities as optimal. If this is true, government tax revenues also change because the mass of firms choosing MNE activities is influenced by the size of γ_i .

2. Prices in the differentiated sector depend on the chosen strategies of firms, and the mass of firms selecting the MNE strategy depends on γ_j . If $\theta_{d/ex} < \theta_{ex/i}$, this impact on the utility of households in j increases if firms select the export strategy instead of MNE activity as optimal because of taxation in j.

¹⁵³ For optimal prices $p_i(i)^*$ see 3.4.2, 3.4.3, and 3.4.4 in the Appendix.

3. The degree of taxation influences the mass of firms entering the market in j because of profit taxation in j. The utility of households in j is affected by this variety effect.

4. In general, a lump-sum transfer is additional income for households in that country and is spent on x_0 .

5. If firms in the differentiated sector select strategies other than at γ_j =0, the working income of households is not lowered because they can work in the homogeneous sector.

6. When selecting $\gamma_j > 0$, the tax rate selected by one government depends on the tax rate of the other.

7. The market size X results endogenously and depends on the selected γ_i .

Only if the positive impacts of taxation outweigh the negative ones are governments acting as benevolent planners interested in selecting $0 < \gamma_j < 1$. For this reason, each

government solves $\frac{\partial U}{\partial \gamma_j}$, as in (101) and (102). In these expressions, the market size

has different outcomes depending on the integration strategies heterogeneous firms choose. This endogeneity of the market size X results in a situation in which every parameter configuration results in a corresponding level of X. This implies that the mass of firms in equilibrium varies endogenously.

Due to market entry conditions, expected profits according to (101), (102), (104), and (105) are competed to zero. As an example, consider the situation in (101). Expected profits for all firms headquartered in A are defined by:

$$\begin{aligned} \mathsf{E}\pi_{\mathsf{A}} &= \int_{0}^{\theta_{\max}} \frac{\mathsf{s}_{\mathsf{A}} \left(\alpha \theta(i) \right)^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}} \theta(i)} \left(\frac{1}{\alpha} - 1 \right) - f_{\mathsf{d}} + \int_{\theta_{\mathsf{d}/\mathsf{ex}}}^{\theta_{\mathsf{ex}/i}} \frac{\mathsf{s}_{\mathsf{B}} \left(\alpha \theta(i) \right)^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}} \theta(i)} \left(\frac{1}{\alpha} - 1 \right) - f_{\mathsf{ex}} \\ &+ \int_{\theta_{\mathsf{ex}/i}}^{\theta_{\max}} \frac{\mathsf{s}_{\mathsf{B}} \left(\alpha \theta(i) \right)^{\frac{1}{(1-\alpha)}} \left(1 - \gamma_{\mathsf{j}} \right)}{X^{\frac{(1-\mu)}{(1-\alpha)}} \theta(\mathsf{i})} \left(\frac{1}{\alpha} - 1 \right) - f_{\mathsf{i}} = 0 \end{aligned}$$
(106)

Vice versa, this also accounts for all firms headquartered in B. Furthermore, market size X and the mass of firms in equilibrium are interdependent. The market size X and the mass of firms induce further interdependences to other equilibrium determining expressions (i.e., the cut-off levels and the demand of the households).

Even without the complexity induced by the linkages of the different variables, the maximization of welfare in (101), (102), (104), and (105), $\frac{\partial U}{\partial v_i}$, results in a problem with a dimensionality higher than fourth degree.¹⁵⁴ These aspects preclude an analytical solution of $\frac{\partial U}{\partial \gamma}$ and suggest using numerical analysis to determine the welfare-maximizing tax rates $\boldsymbol{\gamma}_j$ and their interactions with other variables.

3.2.4 The numerical framework setup

To derive a solution to this problem and to find a welfare-maximizing expression for the tax rate γ_i , we use Mathematica 7.0. This program is utilized to set up the numerical framework that represents the theory of the model as derived in previous sections.155

3.2.4.1 Definitions

The coding of the numerical framework starts with defining variables and making assumptions. Analogously to the assumptions of the model, the fixed costs, θ_{max} , α , and μ are set to constant numerical values, considering $f_d < f_{ex} < f_i$ and $0 < \mu < \alpha < 1$.

After paying the market-entry costs of f_d, a firm draws its individual productivity level θ . We apply a uniform distribution of the firms over θ , specified as $\mathbb{F}[\theta_{-}]$. The distribution function is defined piecewise to ensure that $F[\Theta_]$ takes the value 0 if the distribution is not reached and takes the value 1 in the boundaries of θ_{min} and θ_{max} . To guarantee continuous results, firms are ranked according to individual productivity, starting with low-productivity firms, reflected by the expression θ [i ,n]. The productivity of the single firm θ [i_,n_] depends on the rank i of the i-th firm, given a mass of firms in the economy n.

For further analysis, a function to provide the rank that is between two strategies is computed using the expression $inr[\theta_n, n_n]$, which reports the rank of the firm given the productivity θ and the mass of firms n founded in the country.

¹⁵⁴ A derivation of a unique solution with dimensionality higher than fourth degree cannot be provided. This is proved by the theory of Galois. For a survey see Taton (1983). ¹⁵⁵ See 3.4.8 in the Appendix for the full input sheet, with given values for specific variables.

The demand of the representative household as in (77), results in optimal output for the firms as derived previously. Therefore, the computation of the profit-maximizing output is represented by $x[\Theta_, x_, t_, \gamma_]$. The optimal output of a firm i depends on its productivity θ , the market size X, the transport costs t, and the withholding tax rate γ_j . As the tax rate of country j, γ_j , is only relevant for the MNE strategy, we must consider γ_j =0 for both the domestic and the exporting strategies.

The choice of the integration strategy of firms is driven by cut-off productivity thresholds. They are coded as follows: The first threshold separates domestic producers from exporters from j and is computed as $\theta de[X_, s_, t_]$, considering X and s in the country in which the differentiated goods are sold. The associated firm number is reported by $ide[n_, X_, s_, t_]$. With this expression, the rank of the indifferent firm is calculated in terms of productivity, depending on the endogenous mass of firms n in j, the endogenous market size in the other country, its country size, and the transport costs t. Becasuse the domestic and exporting strategy both are independent of the tax rate; $ide[n_, X_, s_, t_]$ does not depend on it. For example, by entering ide[25000, 5000, 1, 1.05], the system calculates the rank of the indifferent firm ide with cut-off productivity $\theta_{d/ex}$ to be the 19912nd firm, given a mass of 25000 firms in this country, a market size of 5000, a country size of 1, and transport costs of 1,05. Hence, it is the 19913rd firm out of 25000 exports for sure.

Analogously, we compute the threshold productivity $\theta_{ex/i}$ as $\theta ei[X_, s_, t_, \gamma_]$ with the associated rank $iei[n_, X_, s_, t_, \gamma_]$, considering X, s, and γ in the country in which the economic activity takes place. The same notion is used to code the cut-off level $\theta_{d/i}$ as $\theta di[X_, s_, t_, \gamma_]$, linked to the rank $idi[n_, X_, s_, t_, \gamma_]$, considering X, s, and γ in the country in which the economic activity takes place.

3.2.4.2 Consistency of market size X

The inclusion of the endogenously defined market size X from a demand perspective as in section 3.2.2, into the numerical model does not result in the consistency needed to derive results. The proof of inconsistency starts with computing market size of country j from a supply perspective for firms active in the different strategies in j, j \in {A,B} (i.e., Yd [supply of domestic firms from A vice versa from B], Yex [supply of exporting firms from B vice versa from A], and Yin [supply of MNEs with origin in B and vice versa with origin in A]). The market size for domestic producers in A, referring to the representative household, yields Yd[nA_,X_,t_]. Yd depends on the mass of firms in the domestic market nA, the market size X in A, and transport costs t. It is characterized by the integral over the output of all domestic firms i.

Analogously, we compute the market size of firms that export from Country B to Country A. From the perspective of firms producing in B and exporting to A, the export market size, referring to the representative household in A, is given by $Yex[nB_, X_, s_, t_, \gamma_]$. The size of the export market of firms from B in A depends on the mass of firms located in B (nB), the market size X in A, the country size s in A, transport costs t, and the tax rate γ in A. The definition of Yex in the numerical analysis also considers the scenario that possibly no exporters exist.

The market size for multinational firms in terms of the representative household is defined by $Yin[nB_, X_, s_, t_, \gamma_]$. The market for MNEs in country A depends on the mass of firms located in B, nB, the market size X in A, the country size s in A, transport costs t, and the tax rate γ in A. This coding also includes conditions to guarantee that the system integrates correctly regarding prevailing integration strategies.

The entire market size from a supply perspective, referring to the representative household in A, is determined as the sum of all three market segments and is represented by:

Y[nA_,nB_,X_,s_,t_,γ_]:=Yd[nA,X,t]+Yex[nB,X,s,t,γ]+Yin[nB,X,s,t ,γ]

Inconsistency in the market size will result in differing outcomes regarding the market size from supply (Y) and demand (X) perspectives. If the configuration is consistent, we should expect a result of Y=5000, for example, if X is 5000. However, using the code defined previously, inserting X=5000, Y=5000 does not necessarily occur. For example, Y[20000, 20000, 5000, 1, 1.05, 0.08] results in a market size of Y= 4192.26. The inconsistency of the market size can be clarified with the figure 19 in which the inconsistency is obvious.¹⁵⁶ The curve of the market size X has a different

¹⁵⁶ The plot can be implemented using the code:

progression than the curve Y. The intersection of both curves gives the true market size for the given values, Xm=4691.47.



Figure 19. The inconsistency of the market size.

To achieve the essential consistency of market size, the computation uses a quasi-Newton method, computed as $Xm[nA_, nB_, s_, t_, \gamma_-]$.¹⁵⁷ The method is so named because we use an approximation for the slope using the gradient of the secant of the function Xm, for which we search. Figure 20 shows the visualization of the method.¹⁵⁸ The program is coded to find the null, starting the calculation of the secant at a value of 1000 (in figure 20, this corresponds to x_0), assuming a width of 20 (in figure 20, this corresponds to the second value x_1). The slope of the secant results in a null, which is the next starting value (in figure 20, this corresponds to x_2). The slope of the secant associated with this new starting value results in a new null (in figure 20, this corresponds to x_3). This iteration is repeated until the exact null is found. Meanwhile, the width in which the boundaries of the slope of the secants are calculated is reduced stepwise.



Figure 20. Visualization of the quasi-Newton method.

3.2.4.3 The mass of firms in equilibrium

Firm decisions to enter the market are based on the expectation of future profits. Heterogeneous firms enter production as long as their future earnings expectations are positive. Hence, the mass of firms in the market is determined by expected profits being equal to zero. After computing the profit function of the single firm i with its particular strategy, expected profits are determined by the profits of all firms in the specific market. The profit functions of the single firms in the different strategies are computed.

For firm selecting domestic Α. а the strategy in we apply $Gd[i_n, nA_nB_s, t_n\gamma_n]$. We must consider that the tax rate γ has to be set to 0 and transport costs to 1 because they both are not relevant for domestic producers. Furthermore, the rank of the i-th firm, the mass of firms in A and B, and the country size of A must be considered to compute Gd of the i-th firm. Although this firm selects the domestic strategy in Country A, the mass of firms in B must be considered because of competitive conditions.

The profit of a firm in the export strategy is computed as $Gex[i_,nA_,nB_,sB_,t_,\gamma B_]$. Profits of an exporting firm, i, from A to B depend on the mass of firms in A and B (nA and nB), the size of B, transport costs t that apply for the export strategy, and the tax rate being applied in B (γ B for MNEs).

The profit function of MNEs originally located in A with subsidiaries in B is coded as $Gin[i_,nA_,nB_,sB_,t_,\gamma B_]$. Profits of an MNE, i, from A, being an MNE in B, also depend on the mass of firms in A and B (nA and nB), the size of B (sB), transport costs t that apply for the export strategy, and the tax rate on profits of MNEs earned in B being applied in country B(γ B).

Expected profits (EG) in an economy result from the integration of profits over all firms in the different strategies. For A, they are given by:

 $EG[nA_,nB_,sA_,sB_,t_,\gamma A_,\gamma B_] := EGd[nA,nB,sA,t,\gamma A] + EGex[nA,nB,sB,t,\gamma B] + EGin[nA,nB,sB,t,\gamma B]$

The computation of expected profits includes conditions to ensure that profits are only integrated if the associated strategy exists.

Finally, coding the mass of firms in equilibrium results from the expected profits being competed to zero by firms entering the market.

For firms in A, this is given by:

Firms [nB_, sA_, sB_, t_, γ A_, γ B_]. The process to find the null is coded by the instruction to test several values in defined steps. After determining the first negative value of expected profits, the program exactly approaches the null while the width of the steps is permanently reduced.

For example, the instruction to calculate the mass of firms in equilibrium in A given nB=18000, sA=1, sB=1, t=1.05, γ A=0.08, and γ B=0.08 is depicted by

Firms [18000,1,1,1.05,0.08,0.08]. The example results in 12852.1 firms in A, given 18000 firms in Country B, with associated expected profits of -0.00004, thus supporting the previously described method.

3.2.4.4 Equilibria

Considering the utility function in (76), households in the two countries benefit from consumption of homogeneous good x_0 and differentiated goods. To implement the utility maximization process, we must first consider the stand-alone contribution of x_0 . Without the existence of a differentiated sector, the representative household only generates utility by consuming x_0 . Hence, the benefit of one unit of differentiated goods is constituted by its net contribution (i.e., additional utility versus additional costs). The computation of the equilibrium utilizes this notion to implement the utility-

maximizing process. In this computation, the equilibrium of the model is labeled as $Equilibrium[sA_, sB_, t_, \gamma A_, \gamma B_]$ and depends on the given values of country sizes, transport costs, and tax rates in A and B. It is computed so that the system delivers data describing the equilibrium.

Given the exogenous variables, the system endogenously determines the equilibrium mass of firms in A and B. The tax rates being unequal, $\gamma_A \neq \gamma_B$, results in the mass of firms differing in both countries, nA•nB. The programming of the mass of firms in equilibrium can be visualized in figure 21.





In computing to find the correct value for the mass of firms, nB=1 is the starting point. The program searches for the corresponding mass of firms in A conditioned on nB=1 (i.e. nA(nB). In the figure, this is denoted as a. Given nA(1) firms in A, the iteration proceeds by calculating the associated mass of firms in B, denoted as a^{*}. Analogously, we assume nA=1 and search for the associated value for the mass of firms in B, denoted as b (i.e. nB(nA)). Given nB(1) firms in B, the system calculates the corresponding mass of firms in A, denoted as b^{*}. In the intersection of the two resulting graphs, the new starting value is given, C. This loop is repeated until the

difference between the new starting value minus the old starting value is \leq 20 (i.e., nB3-nB0 \leq 20) in the program or C in the figure.¹⁵⁹

The computation proceeds using calculations of expected profits for A and B. The system calculates expected profits and all other key figures for the different possible integration strategies and sums them up afterwards. Hence, EGewA denotes expected profits in A; EGewB denotes expected profits in B.

The next relevant variable is the consistent market size for A, which is calculated using $Xm[nA_,nB_,sA_,t_,\gamma A]$, as defined in section 3.2.4.2, and analogously for B. Afterwards, the contributions of the different strategies to total market size are shown separately for A and B. For example, the share of output of all domestic firms in its market in A is computed as YDA=Yd[NA,XMA,t]; the capital letters denote equilibrium values. The expression depends on the mass of firms in A (NA), the overall market size in A (XMA), and transport costs t.¹⁶⁰

The representative household sum of expenses for differentiated goods in A is represented by MA=MYDA+MYEXA+MYINA. MYDA=MYd[NA,XMA,sA,t] denotes expenses of the representative household in A for goods from domestic producers,

 $\texttt{MYEXA=MYex[NB, XMA, sA, t, \gammaA]} \text{ are expenses for imports from B to A, and}$

$$\begin{split} \texttt{MYINA=MYin[NB,XMA,sA,t,\gammaA]} & \text{is the calculation expenses of the} \\ \texttt{representative household in A for goods from MNEs from B. The analogous notion} \\ (\texttt{MB}) & \text{is used to compute expenses for the representative household in B.} \end{split}$$

Equilibria and, therefore, welfare are constituted by the utility of consumption of differentiated goods. Again, to determine utility, we distinguish between A and B and between the different strategies. For A, we compute the utility of consumption of differentiated goods from domestic firms, from imports from B and from MNEs in A originally located in B. Then, the overall utility of the representative household from differentiated good consumption in A is given, dependent on the sum of the three subfunctions, resulting in $UA=1/\mu(YDA+YINA+YEXA)^{\mu}$, referring to (76) and vice versa for B.

In addition, the representative household benefits from a lump-sum transfer financed by profit taxation of foreign MNE production in subsidiaries. A single MNE with origin

 $^{^{159}}$ Compare with the computation in the input sheet as in derivation 3.4.8 in the Appendix for nB3-nB0 \leq 20.

¹⁶⁰ Analogously, we compute YEXA for the share of output of all exporting firms located in B supplying A's market. The same notion is used to compute YINA, which denotes the share of output of all MNEs headquartered in B supplying the market in A by a subsidiary in A. The analogue computation for Country B is given as YDB, YEXB, and YINB.

in A pays taxes according to its profits: TinB[i_,nA_,nB_,sB_,t_, \B_]:=
(Gin[i,nA,nB,sB,t, \B]+fin)*\B/(1-\B).

Because this is tax revenue paid on profits of a single MNE with origin in A gained by its subsidiary in B and is subject to taxation in B, total tax revenue in the jurisdiction of B is defined by:

ETinB[nA_,nB_,sB_, τ B_,t_, γ B_]:=NIntegrate[TinB[i,nA,nB,sB, τ B,t , γ B],{i,Min[nA,Max[iei[nA,Xm[nB,nA,sB, τ B,t, γ B],sB, τ B,t, γ B],idi [nA,Xm[nB,nA,sB, τ B,t, γ B],sB, τ B,t, γ B]]],nA}]and analogously for A. That is, ETinB is given by the integral over all firms selecting the MNE strategy and paying profit taxes in B and vice versa in A.

A household in B obviously receives a lump-sum transfer TB=1/sB*ETinB and a household in A receives TA=1/sA*ETinA from its government. Finally, welfare is given as WB=UB-MB+TB for B and WA=UA-MA+TA for A.

3.2.4.5 Results of numerical analysis

Using the knowledge of the behavior of firms concerning their integration strategies, governments maximize welfare (measured per capita) in their jurisdictions by optimally choosing withholding tax rates γ_j . Therefore, we examine equilibrium of the model resulting from a variation of the tax rates γ_j (c.p., this results in equilibrium for each of the two countries, A and B).

To analyze the numerical output, we focus on the mass of firms in equilibrium, labeled NA and NB, for the mass of firms in each country, A or B, thereby indicating the mass of differentiated goods in each country. Furthermore, we focus on the consistent market sizes in each country, XmA and XmB, and their contributions based on the output of firms selecting different integration strategies.

3.2.4.5.1 Results with identical country sizes

In a scenario in which the two countries, A and B, behave cooperatively, referring to a social planner's perspective, the resulting numerical analysis indicates that welfare is maximized for both jurisdictions with the choice of $\gamma_A = \gamma_B = 0.^{161}$ In this case, the welfare of the representative household in each country results in

¹⁶¹ See derivation 3.4.8 in Appendix for the full input sheet with given values for specific variables.

WA=WB=139.023.¹⁶² In this scenario, an equilibrium mass of firms of NA=NB=15239 and an equilibrium market size of XmA=XmB=4313.14 come to the fore. NA=NB is constituted by ≈11139 domestic firms, no exporters from B, and ≈44100 MNEs. Furthermore, the market size in equilibrium is constituted by a domestic market share of YDA=YDB=2515.59 (≈58.32%) and а market of **MNEs** share of YINA=YINB=1797.54 (≈41.68%). On the one hand, these market shares consider the mass of firms; on the other hand, they also consider the output of the firms selecting each strategy. Separating these two impacts, we find 73.1% of the firms select the domestic strategy and 26.9% select the MNE strategy. Because the governments do not levy taxes on profits of MNEs, the export strategy does not exist in this equilibrium because costs associated with an MNE activity are lower than costs of exporting due to transport costs. Obviously, a lump-sum transfer to the households cannot be provided in this equilibrium because $\gamma_{A} = \gamma_{B} = 0$.

Because policy makers of A attempt to maximize the welfare of households in their country, they do not consider the welfare in B and vice versa. Hence, they behave uncooperatively. Given any certain tax rate γ_B , there is incentive to determine the welfare-maximizing best-response tax rate γ_A and vice versa. We implement this approach in the numerical model, finding equilibrium because the mass of firms in A (NA) settles to a level that guarantees expected profits in both countries are competed to zero. The same iteration is repeated for the second country, B.

Using the knowledge of the existing masses of firms in both countries given the tax rates γ_A and γ_B , we determine equilibrium describing variables, and study welfare. Given this condition of consistency, regarding NA and NB, with the outcome of numerical analysis in terms of welfare in A and B, we can derive Nash equilibria concerning tax rates γ_A and γ_B .¹⁶³ This can be done for any combination of full-percentage tax rates.

¹⁶² See 3.4.9 in the Appendix for a table summarizing welfare implications resulting from any combination of γ_{A} and γ_{B} .

¹⁶³ Compare with Nash (1951). Furthermore tests of stability of this model confirm this notion of equilibrium. A convergence of equilibria still appears if firms in A and B alternately enter and exit the market.

Given a scenario in which both countries, A and B, have $\gamma_j=0$, the best response tax rate of one country, given zero taxation in the other country, is $\gamma_j=8\%$.¹⁶⁴ For this country, this results in welfare in equilibrium of Wj=142.555. This obvious increase of welfare from Wj=139.023 to Wj=142.555 is achieved at the expense of the other country because its welfare declines to Wj=135.183 without the application of taxation in this jurisdiction. Instead, a social planner will refrain from this solution because total welfare declines in this scenario compared to the zero taxation setting described previously.

Assuming that A is the deviator, choosing $\gamma_A=8\%$, this scenario is characterized by NA=25974 varieties available in A versus NB=4316 varieties of differentiated goods in B. NA is constituted by ≈19543 domestic firms, ≈5287 exporters from B and ≈1144 MNEs. NB is constituted by ≈3077 domestic firms, no exporters from A, and ≈1239 MNEs.

This tax rate constellation shows the following implications: The market in A is given by XmA=4494, associated with an increased market share of 90.8% domestic producers. The market share of MNEs in A declines to 2.5%; and, in this constellation, differentiated goods also are imported to A. The market share of these exporting firms from B in A is 6.7%. In contrast with zero taxation, this strategy is not existent. Again, on the one hand, these market shares consider the mass of firms; on the other hand, they also consider the output of the firms selecting each strategy. Separating these two impacts of output and masses of firms selecting single strategies, we find 75.24% of firms selecting the domestic strategy, 20.36% from B exporting, and 4.4% selecting the MNE strategy. In comparison to a zero taxation scenario, the mass of MNEs declines and the mass of exporters from B and domestic firms increases.

Households in A benefit from more firms entering the market. This increase is associated with a larger market size and is due to the love for variety preferences. Furthermore a positive per-capita lump-sum transfer to households in A is achieved. Also, a negative impact of γ_A =8% is generated because fewer cheap differentiated goods from MNEs are available. Imports are more expensive than goods supplied by MNEs.

 $^{^{164}}$ See 3.4.10 in the Appendix for a table summarizing all responses to $\,\gamma_B$ =0%.

In total, the positive impact outweighs the negative for households in A. For this reason, welfare in A increases because of $\gamma_A=8\%$ and $\gamma_B=0\%$, compared to a scenario without taxation.

In contrast, the market in Country B is given by XmB=4116, associated with a market share of 18.3% of domestic producers. The market share of MNEs in A increases to 81.7% and still no goods are imported to B. Households in B suffer from a decreased mass of firms entering the market there because the mass of domestic firms in B declines dramatically. On the one hand, more cheap goods from MNEs are available for households in B; but, on the other hand, fewer national firms enter the market in B. Hence households suffer from fewer available varieties of goods due to market entry conditions (i.e., expected profits are competed to zero). In addition to fewer domestically produced varieties being available, the increased market share of MNEs reflects not only more varieties from MNEs but also single MNE output, which does not increase households' utility due to love for variety preferences. Because the increased market share of MNEs does not increase welfare to the same extent, the negative impact of market shares cannot be compensated by an increased market share of MNEs in B due to love for variety preferences. Because $\gamma_{\rm B}$ =0% is selected, a per-capita lump-sum transfer in B does not arise. In sum, welfare in B declines compared to the situation without profit taxation in both countries.

Given the described situation for B, we find the best-response tax rate to $\gamma_A=8\%$ to be $\gamma_B=7\%$.¹⁶⁵ Hence the constellation of $\gamma_A=8\%$ and $\gamma_B=0\%$ is not stable. Country B, therefore, does not select $\gamma_B=0\%$; instead $\gamma_B=7\%$ is the best response given $\gamma_A=8\%$. The welfare of the representative household increases to WB=138.082 compared to WB=135.183 in the $\gamma_A=8\%$ and $\gamma_B=0\%$ scenario.

The welfare of the neighbor country, A, then decreases to WA=138.319 instead of WA=142.555 in the γ_A =8% and γ_B =0% scenario. The equilibrium is characterized by a market size in A of XmA=4260 in contrast to XmB=4238 associated with a mass of firms NA=16156 compared to NB=14233. NA is constituted by ≈11814 domestic firms, ≈248 exporters from B, and ≈4094 MNEs; and NB is constituted by ≈10379 domestic firms, ≈180 exporters from A, and ≈3674 MNEs.

 $^{^{165}}$ See 3.4.11 in the Appendix for a table summarizing all responses to $~\gamma_{\text{A}}$ =8%.
Compared to the $\gamma_A=8\%$ and $\gamma_B=0\%$ scenario, this indicates an increased market size XmB as well as a considerable increase in the mass of firms NB.

Increased welfare in B, therefore, is due to more available varieties of differentiated goods for consumers in B. Fewer products from MNEs are available for them because of taxation, but more goods from domestics and exporters are supplied. In this scenario, taxation induces positive welfare implications due to love for variety preferences and positive tax revenue for households in B compared to the previous scenario.

The constellation of $\gamma_A=8\%$ and $\gamma_B=7\%$ is a stable equilibrium with a noncooperative tax setting. This can be seen because the best response for A, given $\gamma_B=7\%$, again is $\gamma_A=8\%$. Hence, this combination of tax rates is a best response for one another.¹⁶⁶ Because the countries are identical, obviously $\gamma_A=7\%$ with $\gamma_B=8\%$ is also a stable equilibrium. Even though both countries generate lower welfare than without taxation, these are stable equilibria in contrast to a zero taxation scenario because every country has incentive to deviate.

Figure 22 summarizes the results of best-response taxes and shows the two stable equilibria:



Figure 22. Best-response tax rates with $s_A = s_B$.

 $^{^{166}}$ See 3.4.12 in the Appendix for a table summarizing all responses to $\,\gamma_B$ =7%.

From both individual and world welfare perspectives, a noncooperative tax setting results in inefficiently high tax rates. Governments are completely informed when setting tax rates. For this reason, they both know that the other has incentive to deviate from zero taxation. Considering this, welfare in its own jurisdiction is maximized considering the tax rate the other country is to select. Hence, zero taxation is not a stable choice for either country, even though it will deliver the highest welfare for each of them.

Therefore, the zero taxation scenario should only be obtained under reliable cooperation (i.e., with a social planner) because each single government has incentive to deviate.

3.2.4.5.2 Results with different country sizes

In a scenario assuming A is marginally larger than B and in which the two countries behave cooperatively, the results of numerical analysis indicate that welfare is maximized in both jurisdictions with the choice of $\gamma_A = \gamma_B = 0$. In this case, the welfare of the representative household in each country results in WA=139.829 and WB=139.201.¹⁶⁷ Compared to the scenario with identical country sizes, this result shows that already the marginally larger country size of A delivers a positive welfare implication that also occurs for the smaller country, B. In this scenario, an equilibrium mass of firms of NA=16235 and NB=14671 and an equilibrium market size of XmA=4355 and XmB=4322 come to the fore. NA is constituted by ≈11889 domestic firms, no exporters from B, and ≈4346 MNEs; NB is constituted by ≈10735 domestic firms, no exporters from A, and ≈3936 MNEs.

The market size in equilibrium is constituted by a domestic market share of YDA=2649 (\approx 60.08%) and YDB=2416 (\approx 55.9%) and a market share of MNEs of YINA=1706 (\approx 39.2%) and YINB=1907 (\approx 44.1%). These market shares include the mass of firms selecting a strategy as well as the output of these single firms. Because the governments do not levy taxes on profits of MNEs, the export strategy does not exist in this equilibrium because the costs associated with MNE activity are lower than the costs of exporting due to transport costs and fixed costs relations. Obviously, a lump-sum transfer to the households cannot be provided in this equilibrium because $\gamma_A=\gamma_B=0$.

¹⁶⁷ See 3.4.13 in the Appendix for a table summarizing welfare implications resulting from any combination of γ_{A} and γ_{B} .

In comparison to the analysis with identical country sizes, the mass of firms increases in A and decreases in B. The market sizes of both countries increase, but the impact of country size results in a stronger magnitude for the market size in A than in B. Furthermore, the constitution of market shares changes. The domestic market share increases in A and declines in B in contrast to the market shares of MNEs that decline in A and increase in B in comparison to the analysis with equally sized countries.

The impact of country size results in the following implications for country A: Because country size is larger, the demand is also larger, resulting in more firms entering the market in A. Hence, NA and XMA are greater than in the previous analysis with identical country sizes; and more domestic firms are founded. Households in A benefit because more varieties are available. Furthermore, the foundation of domestic firms is stimulated because competition given by MNEs from B is not as intense. This is because fewer firms decide to enter the market in B because it is the smaller country. For these reasons, welfare in A is not higher because of the availability of cheaper differentiated goods from MNEs but because of a higher satisfaction of love for variety preferences.

The impact of country size results in the following implications for country B: The assumption of the larger country size of A results in more firms being stimulated to enter the market in A. This is because firms in A face greater national demand. For this reason, more firms supplying demand in B with MNE activities also enter the market in A. This intensifies competition in B. For this reason, fewer domestic firms enter the market in B. Hence, NB is smaller than with identical country sizes. Although fewer varieties of differentiated goods are available in B, XMB is greater than before; and the impact on welfare is positive compared to the previous analysis. Hence, all things being equal, if country size in A exogenously is given marginally as being bigger than in B, welfare in B is higher, even though it is the smaller country in this analysis.

Because policy makers of countries attempt to maximize the welfare of their households, they do not consider welfare in the other jurisdictions. Given any certain tax rate γ_B , there is incentive to determine the welfare-maximizing best-response tax rate γ_A and vice versa.

With the outcome of the numerical analysis in terms of welfare in A and B, we can derive Nash equilibria concerning the tax rates γ_A and γ_B .¹⁶⁸ This can be done for any combination of full-percentage tax rates.

Given this scenario in which both countries have $\gamma_j=0$, the best-response tax rate of one country, given zero taxation in the other country, is $\gamma_j=8\%$.¹⁶⁹ The deviator achieves a welfare increase at the expense of the other country (i.e., WA= 143.078 with WB=135.185 if A deviates and WB=142.566 with WA=135.687 if B deviates). Instead, a social planner will refrain from this solution because total welfare declines in this scenario compared to the zero taxation setting described previously.

Selecting γ_j =8%, the deviator achieves a welfare increase in its jurisdiction because households in this country benefit from a bigger mass of firms entering the market. This is associated with a larger market size and is due to love for variety preferences. Furthermore a positive per-capita lump-sum transfer to households in this country is achieved.

Also a negative impact of γ_j =8% is generated in this jurisdiction because fewer cheap differentiated goods from MNEs are available. Although imports also occur, these are more expensive than goods supplied by MNEs. In total, the positive impact outweighs the negative. For this reason, welfare in j increases, because of γ_j =8%, if profits in the other country are not taxed.

In this scenario, households in the other country suffer from fewer firms entering the market there because the mass of their domestic firms declines dramatically. On the one hand, more cheap goods from MNEs are available for households in this country. On the other hand, as expected profits are competed to zero, fewer national firms enter this market; and households suffer from fewer available varieties. In addition to fewer domestically produced varieties being available, the increased market share of the MNEs reflects not only more varieties from MNEs but also single MNE output, which does not increase household utility due to love for variety preferences. Because the increased market share of MNEs does not increase welfare to the same extent, the negative impact of market shares cannot be compensated by an

¹⁶⁸ Furthermore tests of stability of this model confirm this notion of equilibrium. A convergence of equilibria still appears if firms in A and B alternately enter and exit the market. Because this model consists of one period only, entry and exit happens immediately off the reel.

¹⁶⁹ See 3.4.14 and 3.4.15 in the Appendix for tables summarizing all responses to $\gamma_B = 0\%$ or $\gamma_A = 0\%$.

increased market share of MNEs in B due to love for variety preferences. As taxation in this country is not applied, a per-capita lump-sum transfer does not arise here. In sum, welfare in this country declines compared to the situation without profit taxation in both countries.

Given the described situation for the heretofore nondeviating country, we find the best-response tax rate to γ_j =8% at 7% instead of zero taxation.¹⁷⁰ Hence the constellation of γ_j =8% as response to zero taxation is not stable. The country responding on γ_j =8% achieves a welfare increase at the expense of the other country (i.e., WA= 138.555 with WB=138.326 if A responds with γ_j =7%, and WB=138.129 with WA=138.847 if B responds with γ_j =7%).

Increased welfare in the country selecting a tax rate of 7% instead of zero taxation, therefore, is due to more available varieties of differentiated goods for consumers. Because of taxation, fewer products from MNEs are available for them, but more goods from domestic and exporting firms are supplied. In this scenario, taxation induces positive welfare implications due to love for variety preferences and positive tax revenue compared to the previous scenario without taxation in this jurisdiction.

The 8%-7% tax rate constellation is a stable equilibrium. This can be seen because the best response for A given γ_B =7% again is γ_A =8%, and the best response for B given γ_A =7% again is γ_B =8%. Hence, this combination of tax rates is the best response for one another.¹⁷¹ Although both countries generate lower welfare than without taxation, these are stable equilibria in contrast to a zero taxation scenario because every country has incentive to deviate.

Figure 23 is a summary of the results of best-response taxes and shows the two stable equilibria.

¹⁷⁰ See derivations 3.4.16 and 3.4.17 in the Appendix for tables summarizing all responses to $\gamma_B = 8\%$ and $\gamma_A = 8\%$.

¹⁷¹ See derivations 3.4.18 and 3.4.19 the Appendix for tables summarizing all responses to $\gamma_B = 7\%$ and $\gamma_A = 7\%$.



Figure 23. Best-response tax rates with $s_A \neq s_B$.

These stable equilibrium tax rates are identical with those for symmetric countries; but the conditions of consistency, foremost the implications on welfare, differ in magnitude to the previous analysis. Obviously, the exact values of optimal tax rates, as well as the results of these values in both settings with symmetric and asymmetric countries, only occur due to the selected parameter configurations.

From both individual and world welfare perspectives, a noncooperative tax setting results in inefficiently high tax rates. Governments are completely informed when setting tax rates. For this reason, they both know that the other has an incentive to deviate from zero taxation. Considering this, welfare in its own jurisdiction is maximized considering the tax rate the other country is to select. Hence zero taxation is not a stable choice for either country, even though it would deliver highest welfare for each of them.

Therefore the zero taxation scenario could only be obtained under reliable cooperation (i.e., with a social planner) because each single government has incentive to deviate.

3.2.5 Outline

Support for the here derived result of inefficient tax rates selected by governments in a noncooperative tax setting is found in other trade literature and empirical studies.¹⁷²

¹⁷² For a survey see Davies and Eckel (2007), Zodrow (2003), Wilson (1999), Sinn (1990), and Razin and Sadka (1991).

In this analysis, we utilize an alternative approach dependent on exogenously given parameters such as transport costs and fixed costs and the resulting endogeneity of integration strategies, endogenous market entry, and heterogeneity of firms.

3.2.5.1 The role of the constellation of exogenously given parameters

In our model, cut-off levels are derived between domestic, export and MNE producers. These depend on exogenously given parameters. At the first cut-off, firm productivity is such that additional profits of exporting exactly result in zero profits:

$$\theta_{d/ex} = \frac{f_{ex}^{\frac{(1-\alpha)}{\alpha}} t X^{\frac{(1-\mu)}{\alpha}}}{\alpha (s_i(1-\alpha))^{\frac{(1-\alpha)}{\alpha}}}.$$
(95)

The critical productivity level in (95) increases with increasing t, f_{ex} , and market size X. Hence, the indifferent firm must be more productive to break even. Additionally, the cut-off productivity depends on the country size s_j . The larger the foreign country s_j , the smaller firm productivity must be for the export strategy to become reasonable. The next threshold is the productivity level at which profits of an exporting firm equal the profits of an MNE:

$$\theta_{ex/i} = \frac{(f_i - f_{ex})^{\frac{(1-\alpha)}{\alpha}} \cdot \chi^{\frac{(1-\mu)}{\alpha}}}{s_j^{\frac{(1-\alpha)}{\alpha}} \alpha (1-\alpha)^{\frac{(1-\alpha)}{\alpha}} \left((1-\gamma_j) - t^{\frac{-\alpha}{(1-\alpha)}}\right)^{\frac{(1-\alpha)}{\alpha}}}$$
(96)

The critical productivity level in (96) increases with increasing overhead costs ($f_i - f_{ex}$), an increasing market size X, decreasing transport costs t, increasing γ_j , and a decreasing s_j . An increasing $\theta_{ex/i}$ is associated with a smaller mass of firms selecting MNE strategies. Furthermore, only if $(1 - \gamma_j) > t^{\frac{-\alpha}{(1 - \alpha)}}$ does a real and unique solution exist. If the parameter configuration of the transport costs t, the tax rate γ_j , and α does not satisfy this condition, MNE activities do not exist.

Alternatively, certain configurations of parameters may result in a situation in which domestic firms integrate directly as MNEs instead of choosing the export strategy. The following cut-off level is the relevant productivity threshold if $\theta_{d/i} < \theta_{d/ex}$:

$$\theta_{d/i} = \frac{f_i^{\frac{(1-\alpha)}{\alpha}} X^{\frac{(1-\mu)}{\alpha}}}{\alpha (s_j(1-\gamma_j)(1-\alpha))^{\frac{(1-\alpha)}{\alpha}}}$$
(97)

The critical productivity level in (97) increases with increasing f_i and an increasing market size X. Additionally the cut-off productivity depends on the country size s_j and γ_j . The smaller the foreign country s_j and the higher the tax rate γ_j , the higher firm productivity must be for the MNE strategy to become reasonable.

For $\theta_{d/i} < \theta_{d/ex}$ so that this cut-off level exists, the following condition must hold:

$$(1-\gamma_{j}) \ge \frac{f_{i}}{f_{ex}t^{\frac{\alpha}{(1-\alpha)}}}$$
(98)

As can be seen from the cut-off levels, the mass of firms selecting an integration strategy, as well as which strategies are optimal to select at all, depends on exogenously given parameters. These dependencies in a setting with heterogeneous firms distinguish this model from the latest literature.¹⁷³

3.2.5.2 The role of endogenous market entry and market size

In this model, the mass of firms in equilibrium results endogenously because expected profits are competed to zero until the last firm entering the market generates zero profits. With the inclusion of this endogeneity we can analyze the implications of national and international policy decisions on integration modi of heterogeneous firms. Taxation influences not only entry of MNEs and exporting firms into the market but also the mass of domestic firms.

Given a specific tax rate, the exact composition of prevailing integration strategies in this country is due to the constitution of competition. For example, if a tax rate increases, fewer MNEs enter the market; and, depending on the size of transport costs, they also may refrain from becoming exporters. Then, fewer firms will supply demand in this country and expected profits will increase. Therefore, the output of each single firm is influenced; and more domestic firms can enter the market, competing expected profits to zero. Hence, equilibria with different tax rates are determined by other compositions of integration strategies and other masses of firms

¹⁷³ As in Davies and Eckel (2007).

producing individual optimal outputs. This endogenous market size and especially further entry of domestic firms in a setting with heterogeneous firms distinguish this model from the latest literature.¹⁷⁴

3.2.5.3 The role of heterogeneity

In our model, stable equilibria are obtained with a 8%-7% tax setting scenario. This is driven by the incentive of each government to deviate unilaterally from zero taxation to induce positive impacts on welfare in its jurisdiction. This influence on welfare is characterized by following implications: The mass of firms in its country increases. Increased welfare, therefore, results in more available varieties of differentiated goods for consumers there. Although more goods from domestic and export firms are supplied, fewer products from MNEs are available for them, induced by taxation. For the unilaterally deviating country, the overall impact in this scenario is that taxation induces positive welfare implications due to love for variety preferences and positive tax revenue, even though fewer cheap goods supplied by MNEs are available. The extent of more domestic firms entering the market in this analysis also depends on the distribution of firms over productivity levels. In this analysis, a uniform distribution $F(\theta)$ is assumed. The specification of an alternative distribution function, therefore, might induce different results.

The assumption of a distribution $G(\theta)$ in which the mass of firms increases with productivity so that many MNEs and few domestic firms exist, will slow the stimulating effect on domestic firms to enter the market if positive tax rates are selected. Hence, an increase in the profit tax rate of a single government has the following implications: Because an increase in the profit tax rate induces some MNEs not to enter the market and expected profits are competed to zero, alternatively integrated firms can enter the market and the outputs of single firms are adjusted. Depending on the distribution function, the composition of the mass of firms selecting different integration strategies then differs. For this reason, if G(θ) instead of F(θ) is applied, fewer domestic firms can enter the market and single optimal output adjusts according to endogenous market entry conditions. Obviously, the extent of the resulting implications depends on the exact parameter configuration. However, the impact of F(θ) with more firms with lower single output is always positive for consumers due to love for variety preferences. If, instead of F(θ), G(θ) is applied, this

¹⁷⁴ As in Davies, Egger and Egger (2009).

impact on welfare concerning more available varieties is dampened. Instead, the outputs of single firm will be increasingly influenced.

Another positive impact on utility of the representative household is achieved by providing a lump-sum transfer. As an additional lump-sum transfer only is used to finance the consumption of x_0 and the homogeneous good is appreciated less than differentiated goods. According to (76), the impact of transfer on welfare is not extensive.

The third impact of taxation on welfare concerns some MNEs providing cheaper goods not entering the market. This has negative implications for the representative household because these relatively cheap varieties are not supplied.

In an analysis with the here described distribution function $G(\theta)$, love for variety preferences is satisfied less than in the analysis with $F(\theta)$. Previously, in the analysis with $F(\theta)$, the positive impact on welfare by unilaterally deviating from the cooperative zero tax setting scenario is mainly driven by higher satisfaction of these preferences. With this alternative distribution of firms $G(\theta)$, fewer domestic firms will enter the market and far fewer cheap varieties supplied by MNEs will be available for consumers in this jurisdiction.

Hence, in contrast to $F(\theta)$, this distribution function $G(\theta)$ will more likely result in a negative impact of taxation (i.e. the negative impact of fewer cheaper varieties provided by MNEs can be more influential than the positive implication given by tax revenue and stimulated market entry satisfying love for variety preferences). Obviously, the results depend on the exact parameter configurations; but configurations that ensure this described impact of $G(\theta)$ zero taxation will result in a stable equilibrium. In the previous analysis, zero taxation is optimal from a world welfare perspective but, unfortunately, is unstable in a noncooperative tax setting. Hence, the results in this analysis are mainly constituted by the exact specification of the distribution of firms over productivity and, therefore, due to heterogeneity.

3.3 Conclusion

We develop a model with heterogeneous firms to derive welfare-maximizing profit tax rates set by benevolent planners. Heretofore, governments levy withholding tax rates on profits earned by MNE subsidiaries located in their countries. When selecting these optimal tax rates, governments take their impact on the optimal integration strategies of firms, as well as on market entry and market sizes, into account.

The integration strategies chosen depend on the individual productivity levels of firms. Given their productivity levels, firms maximize profits, considering relative sizes of fixed costs, size of transport costs, country and market sizes, per-unit variable costs, and degree of government profit taxation. Therefore, each firm individually either selects domestic production, an export strategy, or MNE activities as optimal; the composition of the prevailing strategies is determined endogenously, dependent on the withholding profit tax rates chosen by the governments. These described behavioral modifications of integration strategies of heterogeneous firms responding to economic policy interventions are included in the considerations of the government. Due to the incorporation of several endogenous variables, especially market entry and market size, this utilitarian maximization of welfare is solved numerically in this analysis, considering identical and differing country sizes.

Numerical analysis with identical country sizes results in a zero taxation scenario that can only obtained under cooperation of governments (i.e., from a social planner's perspective). An incentive for governments to deviate unilaterally from a zero taxation scenario is given in a noncooperative setting. Because this can be anticipated by the other government, both governments deviate from the zero taxation scenario, resulting in inefficiently high tax rates, which are stable Nash equilibria. The constellation of these profit tax rates is characterized by lower welfare for both jurisdictions than without taxation. Because of the unilateral incentive to deviate from a scenario without taxation, the zero taxation scenario is not stable in a noncooperative setting, although it generates the highest welfare from a world welfare perspective.

Numerical analysis, assuming one country to be marginally larger than the other, results in the same optimal tax rates in equilibrium (i.e., a social planner selects a zero taxation scenario to be optimal considering welfare in both countries; and in a noncooperative tax setting, stable Nash equilibria with inefficiently high tax rates are obtained). In comparison to the analysis with identical country sizes, we emphasize the implication of these differing country sizes on welfare of the representative households in both countries. Our main finding in this context is that not only the welfare of the assumed marginally larger country is higher but also the welfare in the

exogenously given smaller country is higher because of a larger world demand than in the numerical analysis with identical country sizes.

In conclusion, using our model, we derive inefficiently high tax rates in a noncooperative setting and zero taxation from a social planner's perspective when governments act as benevolent planners and set withholding tax rates on profits earned by subsidiaries of MNEs in their countries. Based on our results and the existence of only a little research regarding withholding tax rates with MNE activity, we are motivated to do further research.

3.4 Appendix

3.4.1 Demand

We use the utility function in (76), $U_j = x_0 + \frac{1}{\mu} \left(\int_0^{\theta_{max}} \frac{1}{\alpha} x_j(i)^{\alpha} di \right)^{\mu}$ and the standard side condition to derive the demand of a representative household for the goods of the ith

condition to derive the demand of a representative household for the goods of the i^m firm:

$$\begin{split} \mathsf{L} &= \mathsf{x}_{0} + \frac{1}{\mu} \Bigg[\int_{0}^{\theta_{max}} \frac{1}{\alpha} (\mathsf{x}_{j}(i) + \beta\varsigma_{j}(i))^{\alpha} di \Bigg]^{\mu} + \lambda \Bigg[\mathsf{m}_{j} - \mathsf{x}_{0} - \int_{0}^{\theta_{max}} \mathsf{p}_{j}(i) (\mathsf{x}_{j}(i) + \beta\varsigma_{j}(i)) di \Bigg] \\ \frac{\partial \mathsf{L}}{\partial \mathfrak{x}_{0}} &= \mathsf{1} - \lambda = \mathsf{0} \quad \Rightarrow \lambda = \mathsf{1} \\ \frac{\partial \mathsf{L}}{\partial \lambda} &= \mathsf{m}_{j} - \mathsf{x}_{0} - \int_{0}^{\theta_{max}} \mathsf{p}_{j}(i) (\mathsf{x}_{j}(i) + \beta\varsigma_{j}(i)) di = \mathsf{0} \quad \Rightarrow \mathsf{x}_{0} = \mathsf{m}_{j} - \int_{0}^{\theta_{max}} \mathsf{p}_{j}(i) (\mathsf{x}_{j}(i) + \beta\varsigma_{j}(i)) di \Bigg] \\ \frac{\partial \mathsf{L}}{\partial \beta} &= \Bigg[\int_{0}^{\theta_{max}} \frac{1}{\alpha} (\mathsf{x}_{j}(i) + \beta\varsigma_{j}(i))^{\alpha} di \Bigg]^{(\mu-1)} \Bigg[\int_{0}^{\theta_{max}} (\mathsf{x}_{j}(i) + \beta\varsigma_{j}(i))^{(\alpha-1)} \varsigma_{j}(i) di \Bigg] - \int_{0}^{\theta_{max}} \mathsf{p}_{j}(i) \varsigma_{j}(i) di = \mathsf{0} \\ \mathsf{If} \ \beta = \mathsf{0} : \\ \frac{\partial \mathsf{L}}{\partial \beta} &= \Bigg[\int_{0}^{\theta_{max}} \frac{1}{\alpha} \mathsf{x}_{j}(i)^{\alpha} di \Bigg]^{(\mu-1)} \int_{0}^{\theta_{max}} \mathsf{x}_{j}(i)^{(\alpha-1)} \varsigma_{j}(i) di - \int_{0}^{\theta_{max}} \mathsf{p}_{j}(i) \varsigma_{j}(i) di = \mathsf{0} \\ \Rightarrow \int_{0}^{\theta_{max}} \Bigg[\frac{\mathsf{x}_{j}(i)^{(\alpha-1)}}{\int_{0}^{\theta_{max}} \mathsf{x}_{j}(i)^{\alpha} di \Bigg]^{(1-\mu)}} - \mathsf{p}_{j}(i) \Bigg] \mathsf{s}_{j}(i) di = \mathsf{0} \end{split}$$

$$\Rightarrow \mathbf{x}_{j}(\mathbf{i})^{(\alpha-1)} = \mathbf{p}_{j}(\mathbf{i})\mathbf{X}^{(1-\mu)}$$
$$\Rightarrow \frac{1}{\mathbf{x}_{j}(\mathbf{i})^{(1-\alpha)}} = \mathbf{p}_{j}(\mathbf{i})\mathbf{X}^{(1-\mu)}$$
$$\Rightarrow \mathbf{x}_{j}(\mathbf{i})^{(1-\alpha)} = \frac{1}{\mathbf{X}^{(1-\mu)}\mathbf{p}_{j}(\mathbf{i})}$$
$$\Rightarrow \mathbf{x}_{j}(\mathbf{i}) = \frac{1}{\mathbf{X}^{\frac{(1-\mu)}{(1-\alpha)}}\mathbf{p}_{j}(\mathbf{i})^{\frac{1}{(1-\alpha)}}}$$
$$\Rightarrow \mathbf{p}_{j}(\mathbf{i}) = \frac{1}{\mathbf{X}^{(1-\mu)}\mathbf{x}_{j}(\mathbf{i})^{(1-\alpha)}}$$

This paper applies $X = \left(\int_{0}^{\theta_{max}} \frac{1}{\alpha} x_{j}(i)^{\alpha} di \right)$ which denotes the subutility, respectively the market size as specifiable as $X_{d,ex,i}$ in (79), $X_{d,i}$ in (80) and $X_{d,ex}$ in (81).

3.4.2 The derivation of the profit-maximizing output:

$$\begin{aligned} \pi_{j}(i)_{d} &= s_{j}p_{j}(i)_{d}x_{j}(i)_{d} - \frac{s_{j}x_{j}(i)_{d}}{\theta(i)} - f_{d} \\ &= s_{j}\frac{1}{X^{(1-\mu)}x_{j}(i)_{d}^{(1-\alpha)}}x_{j}(i)_{d} - \frac{s_{j}x_{j}(i)_{d}}{\theta(i)} - f_{d} \\ \frac{\partial\pi_{j}(i)_{d}}{\partial x_{j}(i)_{d}} &= s_{j}\alpha x_{j}(i)_{d}^{(\alpha-1)}\frac{1}{X^{(1-\mu)}} - \frac{s_{j}}{\theta(i)} = 0 \\ x_{j}(i)_{d}^{(\alpha-1)} &= \frac{s_{j}}{\theta(i)}\frac{X^{(1-\mu)}}{\alpha s_{j}} \\ x_{j}(i)_{d}^{(\alpha-1)} &= \frac{X^{(1-\mu)}}{\alpha \theta(i)} \\ x_{j}(i)_{d}^{*} &= \frac{(\alpha \theta(i))\frac{1}{(1-\alpha)}}{X^{(1-\alpha)}} \end{aligned}$$

The derivation of the optimal price:

$$\pi_{j}(i)_{d} = s_{j}p_{j}(i)_{d}x_{j}(i)_{d} - \frac{s_{j}x_{j}(i)_{d}}{\theta(i)} - f_{d}$$

$$\pi_{j}(i)_{d} = s_{j}p_{j}(i)_{d}\frac{1}{X^{\frac{(1-\mu)}{(1-\alpha)}}p_{j}(i)^{\frac{1}{(1-\alpha)}}} - \frac{1}{X^{\frac{(1-\mu)}{(1-\alpha)}}p_{j}(i)^{\frac{1}{(1-\alpha)}}}\frac{s_{j}}{\theta(i)} - f_{d}$$

$$\begin{aligned} \frac{\partial \pi_{j}(i)_{d}}{\partial p_{j}(i)_{d}} &= \frac{-\alpha}{(1-\alpha)} p_{j}(i)_{d}^{\frac{-1}{(1-\alpha)}} s_{j} \frac{1}{X^{\frac{(1-\mu)}{(1-\alpha)}}} + \frac{1}{(1-\alpha)} p_{j}(i)_{d}^{(-1)} \frac{1}{X^{\frac{(1-\mu)}{(1-\alpha)}}} p_{j}(i)_{d}^{\frac{-1}{(1-\alpha)}} \frac{s_{j}}{\theta(i)} = 0 \\ \alpha s_{j} \frac{1}{X^{\frac{(1-\mu)}{(1-\alpha)}}} &= p_{j}(i)_{d}^{(-1)} \frac{1}{X^{\frac{(1-\mu)}{(1-\alpha)}}} \frac{s_{j}}{\theta(i)} \\ p_{j}(i)_{d}^{*} &= \frac{1}{\alpha} \frac{1}{\theta(i)} \end{aligned}$$

The domestic firm i applies the price $p_j(i)$ in country j.

The maximum attainable profits for a domestic firm i from country j are given by:

$$\pi_{j}(i)_{d}^{*} = \frac{s_{j}(\alpha\theta(i))^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}} \frac{1}{\alpha} \frac{1}{\theta(i)} - \frac{s_{j}(\alpha\theta(i))^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}\theta(i)} - f_{d} = \frac{s_{j}(\alpha\theta(i))^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}\theta(i)} \left(\frac{1}{\alpha} - 1\right) - f_{d}$$

3.4.3 The derivation of the optimal output of exports of a firm i from country A:

$$\begin{split} \pi_{A}(i)_{ex} &= s_{B}x_{B}(i)_{ex}p_{B}(i)_{ex} - \frac{s_{B}tx_{B}(i)_{ex}}{\theta(i)} - f_{ex} + \pi_{A}(i)_{d} \\ &= s_{B}x_{B}(i)_{ex} \frac{1}{X^{1-\mu}x_{B}(i)_{ex}^{(1-\mu)}} - \frac{s_{B}tx_{B}(i)_{ex}}{\theta(i)} - f_{ex} + \pi_{A}(i)_{d} \\ &= s_{B}x_{B}(i)_{ex}^{\alpha} \frac{1}{X^{1-\mu}} - \frac{s_{B}tx_{B}(i)_{ex}}{\theta(i)} - f_{ex} + \pi_{A}(i)_{d} \\ \frac{\partial \pi_{A}(i)_{d}}{\partial x_{B}(i)_{ex}} &= \frac{\alpha s_{B}}{X^{(1-\mu)}} x_{B}(i)_{ex}^{(\alpha-1)} - \frac{s_{B}t}{\theta(i)} = 0 \\ x_{B}(i)_{ex}^{(\alpha-1)} &= \frac{s_{B}t}{X^{(1-\mu)}} \frac{X^{(1-\mu)}}{\alpha s_{B}} \\ x_{B}(i)_{ex} &= \left(\frac{s_{B}t}{\theta(i)} \frac{X^{(1-\mu)}}{\alpha s_{B}}\right)^{\frac{1}{(\alpha-1)}} \\ x_{B}(i)_{ex}^{*} &= \left(\frac{\theta(i)}{t} \frac{\alpha}{X^{(1-\mu)}}\right)^{\frac{1}{(1-\alpha)}} \\ x_{j}(i)_{ex}^{*} &= \frac{(\alpha \theta(i))^{\frac{1}{(1-\alpha)}}}{X^{(1-\mu)}t^{\frac{1}{(1-\alpha)}}} \end{split}$$

The derivation of the optimal price for exports to B of a firm i from A:

$$\pi_{A}(i)_{ex} = s_{B}p_{B}(i)_{ex}x_{B}(i)_{ex} - \frac{s_{B}x_{B}(i)_{ex}t}{\theta(i)} - f_{ex} + \pi_{A}(i)_{d}$$

$$\begin{aligned} \pi_{A}(i)_{ex} &= s_{B}p_{B}(i)_{ex} \frac{1}{X^{\frac{(1-\mu)}{(1-\alpha)}}p_{B}(i)_{ex}^{\frac{1}{(1-\alpha)}}} - \frac{1}{X^{\frac{(1-\mu)}{(1-\alpha)}}p_{B}(i)_{ex}^{\frac{1}{(1-\alpha)}}} \frac{s_{B}t}{\theta(i)} - f_{ex} + \pi_{A}(i)_{d} \\ \pi_{A}(i)_{ex} &= \frac{s_{B}}{X^{\frac{(1-\mu)}{(1-\alpha)}}}p_{B}(i)_{ex}^{\frac{-\alpha}{(1-\alpha)}} - \frac{1}{X^{\frac{(1-\mu)}{(1-\alpha)}}}p_{B}(i)_{ex}^{\frac{-1}{(1-\alpha)}} \frac{s_{B}t}{\theta(i)} - f_{ex} + \pi_{A}(i)_{d} \\ \frac{\partial \pi_{A}(i)_{ex}}{\partial p_{B}(i)_{ex}} &= -\left(\frac{\alpha}{1-\alpha}\right)p_{B}(i)_{ex}^{\frac{-1}{(1-\alpha)}} \frac{s_{B}}{X^{\frac{(1-\mu)}{(1-\alpha)}}} + \left(\frac{1}{1-\alpha}\right)p_{B}(i)_{ex}^{(-1)}p_{B}(i)_{ex}^{\frac{-1}{(1-\alpha)}} \frac{1}{X^{\frac{(1-\mu)}{(1-\alpha)}}} \frac{s_{B}t}{\theta(i)} = 0 \\ \alpha &= \frac{1}{\theta(i)}tp_{B}(i)_{ex}^{(-1)} \\ p_{B}(i)_{ex}^{*} &= \frac{t}{\alpha\theta(i)} \\ p_{j}(i)_{ex}^{*} &= \frac{t}{\alpha\theta(i)} \end{aligned}$$

Maximum attainable profits of an exporting firm i from country A:

$$\pi_{A}(i)_{ex}^{*} = \frac{s_{B}(\alpha\theta(i))^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}t^{\frac{\alpha}{(1-\alpha)}}\theta(i)} \left(\frac{1}{\alpha} - 1\right) - f_{ex} + \pi_{A}(i)_{d}^{*}$$

3.4.4 The MNE

In contrast to a domestic firm, MNE profits bear higher fixed costs $f_i > f_d$ and an MNE pays taxes γ_j on the difference between sales and variable costs.

The derivation of the optimal output of an MNE from A supplying the market in B by a subsidiary in B is given by:

$$\begin{aligned} \pi_{A}(i)_{i} &= \left(s_{B} p_{B}(i)_{i} x_{B}(i)_{i} - \frac{s_{B} x_{B}(i)_{i}}{\theta(i)} \right) (1 - \gamma_{B}) - f_{i} + \pi_{A}(i)_{d} \\ &= \left(s_{B} \frac{1}{X^{(1-\mu)} x_{B}(i)_{i}^{(1-\alpha)}} x_{B}(i)_{i} - \frac{s_{B} x_{B}(i)_{i}}{\theta(i)} \right) (1 - \gamma_{B}) - f_{i} + \pi_{A}(i)_{d} \\ \frac{\partial \pi_{A}(i)_{i}}{\partial x_{B}(i)_{i}} &= s_{B} \alpha x_{B}(i)_{i}^{(\alpha-1)} \frac{1}{X^{(1-\mu)}} (1 - \gamma_{B}) - \frac{s_{B}}{\theta(i)} (1 - \gamma_{B}) = 0 \\ x_{B}(i)_{i}^{(\alpha-1)} &= \frac{s_{B}(1 - \gamma_{B})}{\theta(i)} \frac{X^{(1-\mu)}}{\alpha s_{B}(1 - \gamma_{B})} \end{aligned}$$

$$x_{B}(i)_{i}^{(\alpha-1)} = \frac{X^{(1-\mu)}}{\alpha\theta(i)}$$
$$x_{B}(i)_{i}^{*} = \frac{(\alpha\theta(i))^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}}$$

The derivation of the optimal price supplying the market in B:

$$\pi_{A}(i)_{i} = \left(s_{B}p_{B}(i)_{i}x_{B}(i)_{i} - \frac{s_{B}x(i)_{i}}{\theta(i)}\right)(1 - \gamma_{B}) - f_{i} + \pi_{A}(i)_{d}$$

$$\pi_{A}(i)_{i} = \left(s_{B}\frac{1}{\frac{(1-\mu)}{X^{(1-\alpha)}p_{B}(i)_{i}^{\frac{1}{(1-\alpha)}}}}\left(p_{B}(i)_{i} - \frac{1}{\theta(i)}\right)\right)(1 - \gamma_{B}) - f_{i} + \pi_{A}(i)_{d}$$

$$\frac{\partial \pi_{A}(i)_{i}}{\partial p_{B}(i)_{i}} = \left(\frac{-\alpha}{(1-\alpha)}p_{B}(i)_{i}^{\frac{-1}{(1-\alpha)}}s_{B}\frac{1}{\frac{1}{X^{(1-\mu)}}}\right)(1 - \gamma_{B}) + \left(\frac{1}{(1-\alpha)}p_{B}(i)_{i}^{(-1)}\frac{1}{\frac{1}{X^{\frac{(1-\mu)}{(1-\alpha)}}}p_{B}(i)_{i}^{\frac{-1}{(1-\alpha)}}s_{B}\frac{1}{\theta(i)}}\right)(1 - \gamma_{B}) = 0$$

$$\alpha s_{B} \frac{1}{X^{(1-\mu)}} = p_{B}(i)_{i}^{(-1)} \frac{1}{X^{(1-\mu)}} \frac{s_{B}}{\theta(i)}$$
$$p_{B}(i)_{i}^{*} = \frac{1}{\alpha} \frac{1}{\theta(i)}$$

Maximum attainable profits of an MNE i from Country A therefore are given by:

$$\pi_{A}(i)_{i}^{*} = \left(\frac{S_{B}(\alpha\theta(i))^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}}\frac{1}{\alpha}\frac{1}{\theta(i)} - \frac{S_{B}(\alpha\theta(i))^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}\theta(i)}\right)(1-\gamma_{B}) - f_{i} + \pi_{A}(i)_{d}^{*}$$
$$= \frac{S_{B}(\alpha\theta(i))^{\frac{1}{(1-\alpha)}}(1-\gamma_{B})}{X^{\frac{(1-\mu)}{(1-\alpha)}}\theta(i)}\left(\frac{1}{\alpha}-1\right) - f_{i} + \pi_{A}(i)_{d}^{*}$$

3.4.5 Derivation of cut-off level $\theta_{d/ex}$

We derive the productivity level that at least guarantees zero profits from exporting. The exporting firm generates zero profits from D; and hence, is indifferent whether to engage in export activities or not. Firms with productivity levels just above this threshold benefit from exporting:

$$\begin{split} \mathsf{D} &= \mathsf{s}_{j} \mathsf{x}_{j}(\mathsf{i})_{\mathsf{ex}}^{*} \mathsf{p}_{j}(\mathsf{i})_{\mathsf{ex}}^{*} - \frac{\mathsf{s}_{j} \mathsf{x}_{j}(\mathsf{i})_{\mathsf{ex}}^{*} \mathsf{t}}{\theta(\mathsf{i})} - \mathsf{f}_{\mathsf{ex}} \geq 0 \\ &= \frac{\mathsf{s}_{j} (\alpha \theta(\mathsf{i}))^{\frac{1}{(1-\alpha)}}}{\mathsf{X}^{\frac{(1-\mu)}{(1-\alpha)}} \mathsf{t}^{\frac{\alpha}{(1-\alpha)}} \theta(\mathsf{i})} \left(\frac{1}{\alpha} - 1 \right) - \mathsf{f}_{\mathsf{ex}} \geq 0 \\ \frac{\mathsf{s}_{j} (\alpha \theta(\mathsf{i}))^{\frac{1}{1-\alpha}}}{\mathsf{X}^{\frac{(1-\mu)}{(1-\alpha)}} \mathsf{t}^{\frac{\alpha}{(1-\alpha)}} \theta(\mathsf{i})} \left(\frac{1-\alpha}{\alpha} \right) \geq \mathsf{f}_{\mathsf{ex}} \\ \theta(\mathsf{i})^{\frac{\alpha}{(1-\alpha)}} \frac{\mathsf{s}_{j} \alpha^{\frac{\alpha}{(1-\alpha)}} (1-\alpha)}{\mathsf{x}^{\frac{(1-\mu)}{(1-\alpha)}} \mathsf{t}^{\frac{\alpha}{(1-\alpha)}}} \geq \mathsf{f}_{\mathsf{ex}} \\ \theta(\mathsf{i})^{\frac{\alpha}{(1-\alpha)}} \geq \frac{\mathsf{f}_{\mathsf{ex}} \mathsf{X}^{\frac{(1-\mu)}{(1-\alpha)}} \mathsf{t}^{\frac{\alpha}{(1-\alpha)}}}{\mathsf{x}^{\frac{(1-\mu)}{(1-\alpha)}} \mathsf{t}^{\frac{\alpha}{(1-\alpha)}}} \\ \theta_{\mathsf{d}/\mathsf{ex}} \geq \frac{\mathsf{f}_{\mathsf{ex}} \mathsf{X}^{\frac{(1-\alpha)}{\alpha}} \mathsf{X}^{\frac{(1-\mu)}{\alpha}}}{\alpha(\mathsf{s}_{j}(1-\alpha))^{\frac{(1-\alpha)}{\alpha}}} \end{split}$$

Firms that are at least as productive as $\theta_{d/ex} < \theta_{d/i}$ engage in export strategies.

<u>3.4.6 Derivation of cut-off level</u> $\theta_{ex/i}$

The next threshold is a productivity level at which exporting and multinational firms have the same profits. Firms with productivity levels above this level engage in a MNE activity.

$$\begin{split} &\frac{x_{ex} \leq \pi_{i}}{\frac{s_{j}(\alpha\theta(i))^{\frac{1}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}t^{\frac{\alpha}{(1-\alpha)}}} \left(\frac{1-\alpha}{\alpha}\right) - f_{ex} \leq \frac{s_{j}(\alpha\theta(i))^{\frac{1}{(1-\alpha)}}(1-\gamma_{j})}{X^{\frac{(1-\mu)}{(1-\alpha)}}\theta(i)} \left(\frac{1-\alpha}{\alpha}\right) - f_{i} \\ &\frac{s_{j}(\alpha\theta(i))^{\frac{\alpha}{(1-\alpha)}}(1-\alpha)}}{X^{\frac{(1-\mu)}{(1-\alpha)}}} \left((1-\gamma_{j}) - t^{-\frac{\alpha}{(1-\alpha)}}\right) \geq f_{i} - f_{ex} \end{split}$$

$$\begin{split} \theta(i)^{\frac{\alpha}{(1-\alpha)}} &\geq \frac{(f_i - f_{ex})X^{\frac{(1-\mu)}{(1-\alpha)}}}{s_j \alpha^{\frac{\alpha}{(1-\alpha)}} (1-\alpha) \left((1-\gamma_j) - t^{-\frac{\alpha}{(1-\alpha)}}\right)} \\ \theta_{ex/i} &\geq \frac{(f_i - f_{ex})^{\frac{(1-\alpha)}{\alpha}} \cdot X^{\frac{(1-\mu)}{\alpha}}}{\alpha(s_j(1-\alpha))^{\frac{(1-\alpha)}{\alpha}} \left((1-\gamma_j) - t^{-\frac{\alpha}{(1-\alpha)}}\right)^{\frac{(1-\alpha)}{\alpha}}} \end{split}$$

Firms producing with productivity $\theta_{ex/i}$ are indifferent whether to choose the MNE or exporting strategy or not. A firm with productivity $\theta(i)$ just above $\theta_{ex/i}$ engages in a MNE strategy and generates positive profits from this activity.

3.4.7 Derivation and analysis of cut-off level $\theta_{d/i}$

The next cut-off level characterizes a situation where the export strategy does not exist. Firms in this scenario directly integrate their firm following a MNE activity. The following condition must be satisfied:

$$\begin{split} & E \geq 0 \\ & \frac{s_j(\alpha \theta(i))^{\frac{1}{(1-\alpha)}} (1-\gamma_j)}{X^{\frac{(1-\mu)}{(1-\alpha)}} \theta(i)} \left(\frac{1-\alpha}{\alpha}\right) - f_i \geq 0 \\ & \frac{s_j(\alpha \theta(i))^{\frac{\alpha}{(1-\alpha)}} \theta(i)}{X^{\frac{(1-\mu)}{(1-\alpha)}} (1-\alpha)(1-\gamma_j)} \geq f_i \\ & \frac{\theta^{\frac{\alpha}{(1-\alpha)}}}{X^{\frac{(1-\mu)}{(1-\alpha)}}} \geq \frac{f_i \cdot X^{\frac{(1-\mu)}{(1-\alpha)}}}{(1-\alpha)\alpha^{\frac{\alpha}{(1-\alpha)}} s_j(1-\gamma_j)} \\ & \theta_{d/i} \geq \frac{f_i^{\frac{(1-\alpha)}{\alpha}} \cdot X^{\frac{(1-\mu)}{\alpha}}}{((1-\gamma_j)(1-\alpha)s_j)^{\frac{(1-\alpha)}{\alpha}} \alpha} \end{split}$$

Firms that satisfy the following condition integrate their firm as an MNE, the export strategy does not exist:

$$\begin{split} & \frac{\theta_{d/i} < \theta_{d/ex}}{f_i^{\frac{(1-\alpha)}{\alpha}} \cdot X^{\frac{(1-\mu)}{\alpha}}} \\ & \frac{f_i^{\frac{(1-\alpha)}{\alpha}} \cdot X^{\frac{(1-\mu)}{\alpha}}}{((1-\gamma_j)(1-\alpha)s_j)^{\frac{(1-\alpha)}{\alpha}}\alpha} \leq \frac{f_{exi}^{\frac{(1-\alpha)}{\alpha}} X^{\frac{(1-\mu)}{\alpha}}t}{\alpha(s_j(1-\alpha))^{\frac{(1-\alpha)}{\alpha}}} \\ & \frac{f_i^{\frac{(1-\alpha)}{\alpha}}}{(1+\gamma_j)^{\frac{(1-\alpha)}{\alpha}}} \leq f_{ex}^{\frac{(1-\alpha)}{\alpha}}t \\ & (1-\gamma_j) \geq \frac{f_i}{f_{ex}t^{\frac{\alpha}{(1-\alpha)}}} \end{split}$$

3.4.8 Mathematica- Input

fd:=0.0014

fex:=.0015

fin:=.0017

α :=**0.75**

μ **:=0.6**

⊖min:=0

⊖max:=30

```
F[⊖_]:=(⊖-⊖min)/(⊖max-⊖min)
```

inr[*⊖*_,n_]:=n F[*⊖*]

```
\Theta[i\_,n\_]:=Piecewise[\{\{-1,i<0||i>n\},\{i\ (\ominus max-\ominus min)/n\ +\ominus min,i\ge 0\ \&\&i\le n\}\}]
```

 $x[\Theta_{, X_{,t_]}:= (\alpha \Theta)^{(1/(1-\alpha))/((t)^{(1-\alpha)})} X^{((1-\mu)/(1-\alpha))}$

```
\Theta de[X_{s_{1}}] := t fex^{((1-\alpha) / \alpha)} X^{((1-\mu) / \alpha) / (\alpha (s (1-\alpha))^{((1-\alpha)/\alpha)})
```

```
ide[n_X_s_t]:=inr[\ominus de[X,s,t],n]
```

```
\Theta ei [X_,s_,t_,\gamma_]:=If[\gamma \ge 1-t^{(-\alpha/(1-\alpha)),\infty}, (fin-fex)^{((1-\alpha)/\alpha)} X^{((1-\mu)/\alpha)/(\alpha (s (1-\alpha))^{((1-\alpha)/\alpha)})} X^{((1-\alpha)/\alpha)} X^{((1-\alpha)/\alpha)/(\alpha (s (1-\alpha))^{(1-\alpha)/\alpha})} X^{((1-\alpha)/\alpha)} X^{((1-\alpha)/\alpha)/(\alpha (s (1-\alpha))^{(1-\alpha)/\alpha})} X^{(1-\alpha)/\alpha})
```

 α) $/\alpha$) (1- γ -t^(- $\alpha/(1-\alpha)$))^(((1- $\alpha)/\alpha$))]

```
iei[n_X_s_t_\gamma]:=inr[\ominus ei[X,s,t,\gamma],n]
```

```
\Theta di[X_,s_,t_,\gamma_] := fin^{((1-\alpha)/\alpha)} X^{((1-\mu)/\alpha)/(\alpha (s (1-\gamma) (1-\alpha))^{((1-\alpha)/\alpha)})}
```

 $idi[n_X_s_t_\gamma]:=inr[\ominus di[X,s,t,\gamma],n]$

```
Yd[nA_,X_,t_]:=NIntegrate[(1/\alpha) (x[ \ominus[i,nA],X,0,1])^{\alpha},\{i,0,nA\}]
```

```
Yex[nB_,X_,s_,t_,\gamma_]:=If[ide[nB,X,s,t]<Min[iei[nB,X,s,t,\gamma],nB],NIntegrate[(1/\alpha) (x[
```

```
\ominus [i,nB],X,t])^{\Lambda} \alpha, \{i,ide[nB,X,s,t],Min[iei[nB,X,s,t,\gamma],nB]\}],0]
```

 $Yin[nB_,X_,s_,t_,\gamma_] := NIntegrate[(1/\alpha) (x[$

 $\label{eq:inb} \varTheta[i,nB],X,0,1]) \land \alpha, \{i,Min[nB,Max[iei[nB,X,s,t,\gamma],idi[nB,X,s,t,\gamma]]],nB\}]$

 $\label{eq:constraint} Y[nA_,nB_,X_,s_,t_,\gamma_] := Yd[nA,X,t] + Yex[nB,X,s,t,\gamma] + Yin[nB,X,s,t,\gamma]$

Xm[nA_,nB_,s_,t_,\gram_]:=(k=0;X0=1000;Z0=Y[nA,nB,X0,s,t,\gram]-X0;d=20*2^(-

k);X1=X0+d;Z1=Y[nA,nB,X1,s,t,γ]-X1;X2=N[(d Z0+X0 Z0-X0 Z1)/(Z0-

X0;d=20*2^(-k);X1=X0+d;Z1=Y[nA,nB,X1,s, t, γ]-X1;X2=N[(d Z0+X0 Z0-X0 Z1)/(Z0-Z1)]];X2)

Gd[i_,nA_,nB_,s_,t_,γ_]:=s x[θ[i,nA],Xm[nA,nB,s,t,γ],0,1] (1/(α θ[i,nA]))-s

 $x[\ominus[i,nA],Xm[nA,nB,s,t,\gamma],0,1]/\ominus[i,nA]\text{-}fd$

 $Gin[i_,nA_,nB_,sB_,t_,\gammaB_]:=(1-\gamma B) (sB x[\ominus[i,nA],Xm[nB,nA,sB,t,\gamma B],0,1] (1/(\alpha B)) (sB x[\ominus[i,nA],Xm[nB,nA,sB,t,\gamma B],0,1] (sB x[a,i],Xm[nB,iA,iA)) (sB x[a,i],Xm[nB,iA)) (sB x[a,i],Xm$

 \ominus [i,nA]))-sB x[\ominus [i,nA],Xm[nB,nA,sB,t, γ B],0,1]/ \ominus [i,nA])-fin

$$\begin{split} & \mathsf{Gex}[i_,\mathsf{nA}_,\mathsf{nB}_,\mathsf{sB}_,\mathsf{t}_,\gamma\mathsf{B}_] \coloneqq \mathsf{sB} \ \mathsf{x}[\ominus[\mathsf{i},\mathsf{nA}],\mathsf{Xm}[\mathsf{nB},\mathsf{nA},\mathsf{sB},\mathsf{t},\gamma\mathsf{B}],\tau\mathsf{B},\mathsf{t}] \ (\mathsf{t}/(\alpha \ominus[\mathsf{i},\mathsf{nA}])) \text{-} \ \mathsf{sB} \ \mathsf{x}[\ominus[\mathsf{i},\mathsf{nA}],\mathsf{Xm}[\mathsf{nB},\mathsf{nA},\mathsf{sB},\mathsf{t},\gamma\mathsf{B}],\mathsf{t}] \ \mathsf{t}/\ominus[\mathsf{i},\mathsf{nA}] \text{-} \mathsf{fex} \end{split}$$

 $EGd[nA_,nB_,sA_,t_,\gammaA_] := NIntegrate[Gd[i,nA,nB,sA,t,\gammaA],\{i,0,nA\}]$

$$\begin{split} & \mathsf{EGex}[\mathsf{nA}_\mathsf{,nB}_\mathsf{,sB}_\mathsf{,t}_\mathsf{,}\gamma\mathsf{B}_]:=\mathsf{lf}[\mathsf{ide}[\mathsf{nA},\mathsf{Xm}[\mathsf{nB},\mathsf{nA},\mathsf{sB},\mathsf{t},\gamma\mathsf{B}],\mathsf{sB},\mathsf{t}]<\mathsf{Min}[\mathsf{iei}[\mathsf{nA},\mathsf{Xm}[\mathsf{nB},\mathsf{nA},\mathsf{sB},\mathsf{t},\gamma\mathsf{B}],\mathsf{sB},\mathsf{t},\mathsf{sB},\mathsf{t},\mathsf{sB},\mathsf{sB},\mathsf{t},\mathsf{sB}],\mathsf{sB},\mathsf{t},\mathsf{sB},\mathsf{sB},\mathsf{t},\mathsf{sB}],\mathsf{sB},\mathsf$$

$$\begin{split} & \mathsf{EGin}[\mathsf{nA}_{\mathsf{n}}\mathsf{B}_{\mathsf{n}}\mathsf{sB}_{\mathsf{n}}\mathsf{t}_{\mathsf{n}}\mathcal{V}\mathsf{B}_{\mathsf{n}}] \coloneqq \mathsf{NIntegrate}[\mathsf{Gin}[\mathsf{i},\mathsf{nA},\mathsf{nB},\mathsf{sB},\mathsf{t},\mathcal{V}\mathsf{B}],\mathsf{i},\mathsf{Min}[\mathsf{nA},\mathsf{Max}[\mathsf{iei}[\mathsf{nA},\mathsf{Xm}[\mathsf{nB},\mathsf{nA},\mathsf{sB},\mathsf{t},\mathcal{V}\mathsf{B}],\mathsf{sB},\mathsf{t},\mathcal{V}\mathsf{B}],\mathsf{sB},\mathsf{t},\mathcal{V}\mathsf{B}],\mathsf{sB},\mathsf{t},\mathcal{V}\mathsf{B}],\mathsf{sB},\mathsf{t},\mathcal{V}\mathsf{B}],\mathsf{sB},\mathsf{t},\mathcal{V}\mathsf{B}]],\mathsf{nA}\}] \end{split}$$

Firms[nB_,sA_,sB_,t_,γA_,γB_]:=(N1=2^10;K=10;While[EG[N1,nB,sA,sB,t,γA,γB]>0,

N1=2 N1;K=K+1];K=K-2;N1=N1-2^K;While[K>-4,K=K-1;

 $If[EG[N1,nB,sA,sB,t,\gamma A,\gamma B] > 0, N1 = N1 + 2^{K}, N1 = N1 - 2^{K}]; Return[N1])$

 $MYd[nA_,X_s_,t_] := NIntegrate[1/(\alpha \ \ominus[i,nA]) x[\ \ominus[i,nA],X,0,1],\{i,0,nA\}]$

 $MYex[nB_,X_,s_,t_,\gamma A_]:=If[ide[nB,X,s,t] < Min[iei[nB,X,s,t,\gamma A],nB], NIntegrate[t/(\alpha A_{A})) = NIntegrate[t/(\alpha A_{A}))$

 Θ [i,nB]) (x[Θ [i,nB],X,t]),{i,ide[nB,X,s,t],Min[iei[nB,X,s,t, γ A],nB]}],0]

 $MYin[nB_,X_,s_,t_,\gamma A_]:= NIntegrate[1/(\alpha \ \ominus[i,nB]) (x[$

 $\ominus [i,nB], X,0,1]), \{i,Min[nB,Max[iei[nB,X,s,t,\gamma A],idi[nB,X,s,t,\gamma A]]],nB\}]$

 $TinB[i_,nA_,nB_,sB_,t_,\gamma B_]:=(Gin[i,nA,nB,sB,t,\gamma B]+fin)^*\gamma B/(1-\gamma B)$

ETinB[nA_,nB_,sB_,t_,yB_]:=NIntegrate[TinB[i,nA,nB,sB,t,yB],{i,Min[nA,Max[iei[nA,X

 $m[nB, nA, sB, t, \gamma B], sB, t, \gamma B], idi[nA, Xm[nB, nA, sB, t, \gamma B], sB, t, \gamma B]]], nA\}]$

Targetfile="DatenPIII.dat"

Equilibrium[sA_,sB_,t_,yA_,yB_]:=(nA0=2000;nB0=2000;J=0;nA1=N[Firms[nB0,sA,s B,t, γ A_, γ B_]];nB1=N[Firmsl[nA0,sB,sA,t, γ B, γ A]];nA2=N[Firms[nB1,sA,sB,t, γ A, γ B]];n B2=N[Firms[nA1,sB,sA,t,yB,yA]];nA3=N[nA2-nA1 nA2-nA1 nB1+nA1 nA2 nB1+nA1 nB2-nA2 nB2/(1-nA1-nB1+nA2 nB1+nA1 nB2-nA2 nB2)];nB3=N[(nB1-1)/(nA2-nA1) (nA3-nA1)+1];Print[{J,N[nA3],N[nB3]}];While[(Abs[nA3-nA0] >20]| Abs [nB3nB0]>20)&&J<50, nA0=nA3; nB0=nB3; nA1=Firms[nB0,sA,sB,t, γA, γB]; $nB1=Firms[nA0,sB,sA,t,\gamma B,\gamma A];nA2=Firms[nB1,sA,sB,t,\gamma A_,\gamma B_];nB2=Firms[nA1,s];nA2=Firms[nA1,s];nA2=Firms[nA1,s];nA2=Firms[nA1,s];nA1=Firm$ B,sA,,t, yB, yA];nA3=N[nA2-nA1 nA2-nA1 nB1+nA1 nA2 nB1+nA1 nB2-nA2 nB2/(1nA1-nB1+nA2 nB1+nA1 nB2-nA2 nB2)];nB3=N[(nB1-1)/(nA2-nA1) (nA3-nA1)+1]; J++;Print[{J,nA3,nB3}]];Print[{"Equilibrium",J,nA3,nB3}];NA=nA3;NB=nB3;EGewA=E G[NA,NB,sA,sB,t, \vee A, \vee B];EGewB=EG[NB,NA,sB,sA,t, \vee B, \vee A];XMA=Xm[NA,NB,sA,t, \vee A]; A];XMB=Xm[NB,NA,sB,t, yB];YINA=Yin[NB,XMA,sA,t, yA];YINB=Yin[NA,XMB,sB,t, yB] ;YEXA=Yex[NB,XMA,sA,t,γA];YEXB=Yex[NA,XMB,sB,t,γB];YDA=Yd[NA,XMA,t];YDB =Yd[NB,XMB,t];MYDA=MYd[NA,XMA,sA,t];MYDB=MYd[NB,XMB,sB,t];MYEXA=MYe x[NB,XMA,sA,t,γA];MYEXB=MYex[NA,XMB,sB,t,γB];MYINA=MYin[NB,XMA,sA,t,γA] ;MYINB=MYin[NA,XMB,sB,t, γ B];UA=1/ μ (YDA+YINA+YEXA) $^{\mu}$;UB=1/ μ (YDB+YINB+YEXB)^ μ ;MA=MYDA+MYEXA+MYINA;MB=MYDB+MYEXB+MYINB;W A=UA-MA+(1/sA) ETinB[NB,NA,sA,t, A];WB=UB-MB+(1/sB) ETinB[NA,NB,sB,t, \Brack{B}];Ergebnis={DateString[],sA,sB,t, \Brack{A}, \Brack{B},NA,NB,EGewA,EGewB,

XMA,XMB,YDA,YDB,YEXA,YEXB,YINA,YINB,MYDA,MYDB,MYEXA,MYEXB,MYINA ,MYINB,(1/sA) ETinB[NB,NA,sA,t,γA],(1/sB) ETinB[NA,NB,sB,t,γB], UA,UB,MA,MB,WA,WB};Print[Result];PutAppend[Result,Targetfile])

3.4.9 Welfare implications with identical country sizes

γ Α/ γΒ	0%	1%	2%	3%	4%	5%
0%	139.023; 139.023	138.508; 139.793	137.843; 140.411	137.204; 140.970	136.631; 141.449	136.128; 141.862
1%	139.793; 138.508	138.934; 138.934	138.442; 139.654	137.825; 140.219	137.256; 140.701	136.752; 141.118
2%	140.411; 137.843	139.654; 138.442	138.842; 138.842	138.388; 139.508	137.836; 139.994	137.337; 140.404
3%	140.970; 137.204	140.219; 137.825	139.508; 138.388	138.742; 138.742	138.337; 139.326	137.854; 139.740
4%	141.449; 136.631	140.701; 137.256	139.994; 137.836	139.326; 138.337	138.633; 138.633	138.281; 139.132
5%	141.862; 136.128	141.118; 136.752	140.404; 137.337	139.740; 137.854	139.132; 138.281	138.519; 138.519
6%	142.194; 135.707	141.443; 136.329	140.729; 136.912	140.057; 137.432	139.463; 137.875	138.916; 138.223
7%	142.432; 135.386	141.678; 135.999	140.955; 136.580	140.281; 137.097	139.675; 137.541	139.130; 137.903
8%	142.555; 135.183	141.788; 135.791	141.049; 136.365	140.369; 136.876	139.747; 137.318	139.155; 137.607
9%	142.545; 135.134	141.744; 135.737	140.986; 136.302	140.274; 136.808	139.616; 137.247	139.046; 137.607
1 0 %	142.544; 135.135	141.754; 135.734	140.987; 136.303	140.275; 136.810	139.617; 137.254	139.041; 137.611

γ Α/ γΒ	6%	7%	8%	9%	10%
0%	135.707; 142.194	135.386; 142.432	135.183; 142.555	135.134; 142.545	135.135; 142.544
1%	136.329; 141.443	135.999; 141.678	135.791; 141.788	135.737; 141.744	135.734; 141.754
2%	136.912, 140.729	136.580; 140.955	136.365; 141.049	136.302; 140.986	136.303; 140.987
3%	137.432; 140.057	137.097; 140.281	136.876; 140.369	136.808; 140.274	136.810; 140.275
4%	137.875; 139.463	137.541; 139.675	137.318; 139.747	137.247; 139.616	137.254; 139.617
5%	138.223; 138.916	137.903; 139.130	137.681; 139.197	137.607; 139.046	137.611; 139.041
6%	138.356; 138.356	138.162; 138.662	137.953; 138.727	137.875; 138.553	137.881; 138.533
7%	138.662; 138.162	138.204; 138.204	138.082; 138.319	138.023; 138.153	138.033; 138.118
8%	138.727; 137.953	138.319; 138.083	138.002; 138.002	137.996; 137.853	138.010; 137.804
9%	138.553; 137.875	138.153; 138.023	137.853; 137.996	137.689; 137.689	137.712; 137.634
1 0%	138.533; 137.881	138.118; 138.033	137.804; 138.010	137.634; 137.712	137.635; 137.635

<u>3.4.10 Best-response tax rates given $\gamma_B = 0\%$ </u>

Tax rate	NA	NB	XmA	XmB	YdA	YdB	YexA	YexB	YINA	YINB	TinA/sa	TinB/sb	WA	WB
0%	15239	15239	4313	4313	2516	2516	0	0	1798	1798	0	0	139.023	139.023
1%	17038	13538	4348	4287	2786	2251	0	0	1562	2035	0.102	0	139.793	138.508
2%	18784	11742	4376	4252	3047	1972	0	0	1329	2281	0.174	0	140.411	137.843
3%	20454	10014	4403	4219	3293	1697	49	0	1061	2522	0.208	0	140.971	137.204
4%	21954	8455	4428	4190	3511	1445	99	0	818	2745	0.213	0	141.449	136.631
5%	23292	7065	4451	4164	3703	1216	144	0	604	2948	0.197	0	141.862	136.128
6%	24427	5887	4470	4143	3863	1020	188	0	419	3123	0.163	0	142.194	135.707
7%	25328	4960	4485	4127	3990	863	236	0	259	3263	0.118	0	142.432	135.386
8%	25974	4316	4494	4116	4081	753	301	0	112	3363	0.058	0	142.555	135.183
9%	26251	4070	4497	4114	4122	711	375	0	0	3403	0	0	142.545	135.134
10%	26250	4071	4497	4114	4122	711	375	0	0	3403	0	0	142.544	135.135

<u>3.4.11 Best-response tax rates given $\gamma_A = 8\%$ </u>

Tax rate	NA	NB	XmA	XmB	YdA	YdB	YexA	YexB	YINA	YINB	TinA/sa	TinB/sb	WA	WB
0%	25974	4316	4494	4116	4081	753	301	0	112	3363	0.058	0	142.555	135.183
1%	24374	4316	4452	4137	3873	1026	409	0	170	3111	0.088	0.208	141.787	135.789
2%	24374	4316	4412	4157	3662	1290	513	0	237	2867	0.124	0.384	141.049	136.365
3%	21277	8991	4374	4176	3453	1542	612	99	308	2534	0.161	0.507	140.369	136.876
4%	19832	10445	4340	4195	3249	1782	706	220	385	2193	0.202	0.585	139.747	137.317
5%	18501	11740	4307	4209	3059	1996	788	358	460	1855	0.243	0.617	139.155	137.607
6%	17267	13073	4283	4227	2874	2211	873	522	536	1494	0.283	0.596	138.727	137.95
7%	16156	14233	4260	4238	2707	2400	946	728	608	1110	0.322	0.516	138.319	138.082
8%	15225	15225	4242	4242	2564	2564	1008	1008	671	671	0.356	0.356	138.002	138.002
9%	16236	14538	4250	4241	2728	2449	1431	1074	91	717	0.054	0.381	137.852	137.995
10%	16356	14469	4250	4241	2748	2437	1502	1082	0	722	0	0.383	137.804	138.01

<u>3.4.12 Best-response tax rates given $\gamma_B = 7\%$ </u>

Tax rate	NA	NB	XmA	XmB	YdA	YdB	YexA	YexB	YINA	YINB	TinA/sa	TinB/sb	WA	WB
0%	4960	25328	4127	4485	863	3990	0	236	3263	259	0	0.118	135.386	142.432
1%	6624	23641	4148	4443	1146	3766	0	312	3002	364	0.201	0.166	135.999	141.678
2%	8254	21984	4169	4402	1419	3541	0	385	2749	476	0.367	0.218	136.581	140.955
3%	9838	20380	4189	4364	1682	3317	95	456	2412	592	0.482	0.271	137.097	140.281
4%	11332	18877	4208	4330	1927	3102	210	521	2072	707	0.551	0.325	137.541	139.674
5%	12732	17474	4226	4299	2154	2896	339	582	1733	822	0.576	0.379	137.903	139.129
6%	14005	16212	4240	4272	2360	2707	492	636	1389	929	0.553	0.431	138.161	138.661
7%	15084	15084	4246	4246	2537	2537	681	681	1028	1028	0.477	0.477	138.204	138.204
8%	16156	14233	4260	4238	2707	2400	946	728	608	1110	0.322	0.516	138.319	138.082
9%	17143	13490	4266	4233	2867	2277	1333	772	66	1184	0.039	0.550	138.153	138.023
10%	17235	13434	4266	4234	2882	2268	1384	776	0	1190	0	0.550	138.117	138.032

	3.4.13 Welfare im	plications	with as	vmmetric countr	v sizes
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γ Α/ γΒ	0%	1%	2%	3%	4%	5%
0%	139.829; 139.201	139.002; 139.787	138.353; 140.421	137.714; 140.981	137.141; 141.461	136.637; 141.876
1%	140.322; 138.524	139.521; 139.065	138.928; 139.653	138.327; 140.232	137.758; 140.725	137.255; 141.132
2%	140.938; 137.847	140.179; 138.464	139.425; 138.975	138.863; 139.495	138.332; 140.013	137.835; 140.421
3%	141.486; 137.206	140.742; 137.836	140.016; 138.414	139.298; 138.858	138.803; 139.313	138.344; 139.758
4%	141.965; 136.631	141.215; 137.265	140.503; 137.854	139.832; 138.367	139.188; 138.754	138.741; 139.118
5%	142.371; 136.131	141.623; 136.761	140.914; 137.352	140.246; 137.876	139.634; 138.316	139.071; 138.632
6%	142.706; 135.712	141.952; 136.338	141.233; 136.928	140.566; 137.452	139.967; 137.901	139.418; 138.261
7%	142.948; 135.388	142.194; 136.008	141.463; 136.595	140.794; 137.115	140.184; 137.565	139.636; 137.933
8%	143.078; 135.185	142.306; 135.798	141.573; 136.376	140.883; 136.895	140.348; 137.289	139.711; 137.707
9%	143.075; 135.136	142.282; 135.741	141.521; 136.312	140.796; 136.826	140.236; 137.215	139.561; 137.631
10%	143.075; 135.137	142.281; 135.742	141.521; 136.313	140.797; 136.828	140.240; 137.218	139.557; 137.635

γ Α/ γΒ	6%	7%	8%	9%	10%
0%	136.278; 142.132	135.893; 142.452	135.687; 142.565	135.642; 142.537	135.642; 142.537
1%	136.832; 141.456	136.503; 141.689	136.293; 141.791	136.236; 141.747	136.238; 141.747
2%	137.410; 140.744	137.078; 140.966	136.860; 141.062	136.798; 140.993	136.799; 140.990
3%	137.924; 140.083	137.591; 140.294	137.370; 140.373	137.304; 140.267	137.306; 140.269
4%	138.359; 139.468	138.031;139.689	137.808; 139.757	137.738; 139.620	137.741; 139.622
5%	138.687; 138.918	138.387; 139.145	138.167; 139,.204	138.097; 139.042	138.101; 139.034
6%	138.919; 138.481	138.623; 138.663	138.428; 138.721	138.362; 138.547	138.369; 138.526
7%	139.166; 138.199	138.766; 138.319	138.555; 138.326	138.508; 138.149	138.513; 138.095
8%	139.226; 137.978	138.847; 138.129	138.541; 138.083	138.452; 137.823	138.473; 137.773
9%	139.077; 137.901	138.675; 138.052	138.375; 138.029	138.243; 137.754	138.234; 137.673
10%	139.045; 137.907	138.628; 138.058	138.328; 138.042	138.185; 137.763	138.183; 137.695

3.4.14 Best-response tax rates given $\gamma_B = 0\%$ (sB=1, sA=1.01)

Tax rate	NA	NB	XmA	XmB	YdA	YdB	YexA	YexB	YINA	YINB	TinA/sa	TinB/sb	WA	WB
0%	16235	14671	4355	4322	2649	2416	0	0	1706	1907	0	0	139.829	139.201
1%	17781	13061	4375	4287	2885	2164	0	0	1490	2123	0.098	0	140.322	138.524
2%	19550	11191	4404	4252	3147	1879	0	0	1257	2373	0.164	0	140.938	137.847
3%	21212	9464	4431	4220	3390	1604	46	0	995	2616	0.195	0	141.486	137.206
4%	22717	7897	4456	4190	3606	1350	91	0	758	2841	0.197	0	141.965	136.631
5%	24039	6518	4478	4165	3793	1122	132	0	553	3042	0.179	0	142.371	136.131
6%	25170	5340	4496	4143	3951	925	169	0	377	3218	0.147	0	142.706	135.712
7%	26060	4419	4512	4127	4075	769	208	0	229	3358	0.104	0	142.948	135.388
8%	26688	3787	4522	4117	4162	661	263	0	97	3455	0.050	0	143.078	135.185
9%	26945	3555	4525	4114	4199	621	325	0	0	3493	0	0	143.075	135.136
10%	26944	3557	4525	4114	4199	622	325	0	0	3493	0	0	143.075	135.136

<u>3.4.15 Best-response tax rates given $\gamma_A = 0\%$ (sB=1, sA=1.01)</u>

Tax rate	NA	NB	XmA	XmB	YdA	YdB	YexA	YexB	YINA	YINB	TinA/sa	TinB/sb	WA	WB
0%	16235	14671	4355	4322	2649	2416	0	0	1706	1907	0	0	139.829	139.201
1%	14230	16540	4312	4347	2350	2705	0	0	1962	1642	0	0.108	139.002	139.787
2%	12435	18300	4278	4376	2073	2969	0	0	2206	1407	0	0.184	138.354	140.423
3%	10700	19980	4246	4403	1800	3217	0	52	2446	1134	0	0.222	137.714	140.981
4%	9129	21494	4216	4428	1548	3438	0	107	2668	883	0	0.233	137.141	141.461
5%	7723	22850	4190	4450	1320	3633	0	157	2871	660	0	0.215	136.637	141.876
6%	6529	24004	4168	4469	1122	3796	0	207	3046	464	0	0.181	136.216	142.207
7%	5567	24945	4152	4485	962	3929	0	265	3190	291	0	0.132	135.889	142.449
8%	4902	25619	4142	4494	850	4025	0	342	3292	127	0	0.066	135.687	142.566
9%	4655	25903	4140	4496	807	4068	0	429	3333	0	0	0	135.641	142.537
10%	4656	25901	4140	4496	807	4067	0	429	3332	0	0	0	135.642	142.537

<u>3.4.16 Best-response tax rates given $\gamma_B = 8\%$ (sB=1, sA=1.01)</u>

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l ax rate	NA	NB	XmA	XmB	YdA	YdB	YexA	YexB	YINA	YINB	TinA/sa	TinB/sb	WA	WB
0%	4902	25619	4142	4494	850	4025	0	342	3292	127	0	0.066	135.687	142.566
1%	6520	23992	4162	4452	1123	3813	0	451	3039	188	0.203	0.098	136.293	141.791
2%	8078	22422	4183	4412	1383	3602	0	555	2799	256	0.374	0.134	136.864	141.062
3%	9613	20881	4203	4374	1637	3389	97	655	2469	330	0.493	0.173	137.374	140.374
4%	11068	19430	4221	4340	1875	3184	214	748	2133	408	0.567	0.215	137.808	139.753
5%	12430	18086	4238	4309	2095	2989	347	835	1796	486	0.596	0.256	138.168	139.209
6%	13693	16850	4253	4282	2299	2806	506	914	1448	562	0.576	0.297	138.428	138.721
7%	14821	15768	4263	4260	2481	2642	705	985	1077	633	0.499	0.336	138.555	138.326
8%	15825	14889	4271	4246	2643	2505	979	1048	649	696	0.343	0.368	138.544	138.083
9%	16820	14172	4277	4242	2805	2387	1385	1113	87	742	0.052	0.394	138.375	138.029
10%	16938	14102	4277	4242	2824	2375	1453	1121	0	746	0	0.396	138.328	138.043

<u>3.4.17 Best-response tax rates given $\gamma_A = 8\%$ (sB=1, sA=1.01)</u>

Tax rate	NA	NB	XmA	XmB	YdA	YdB	YexA	YexB	YINA	YINB	TinA/sa	TinB/sb	WA	WB
0%	26688	3787	4522	4117	4162	661	263	0	97	3455	0	0	143.078	135.185
1%	25075	5396	4480	4137	3955	936	371	0	154	3200	0.081	0.215	142.306	135.798
2%	23500	6962	4440	4157	3747	1201	474	0	219	2956	0.114	0.396	141.573	136.376
3%	21952	8508	4402	4179	3536	1460	575	102	290	2615	0.152	0.524	140.881	136.895
4%	20693	9787	4372	4193	3361	1671	658	230	354	2292	0.186	0.611	140.348	137.289
5%	19150	11348	4337	4213	3141	1927	757	351	440	1915	0.231	0.637	139.711	137.707
6%	17893	12646	4310	4227	2956	2138	838	541	514	1548	0.272	0.617	139.226	137.978
7%	16791	13825	4288	4239	2791	2330	912	757	585	1153	0.309	0.536	138.847	138.129
8%	15825	14889	4271	4246	2643	2505	979	1048	649	696	0.343	0.368	138.544	138.083
9%	15088	15866	4265	4248	2525	2667	1041	1485	698	96	0.371	0.057	138.452	137.823
10%	15022	15990	4266	4249	2513	2688	1050	1561	702	0	0.372	0	138.473	137.773

<u>3.4.18 Best-response tax rates given $\gamma_B = 7\%$ (sB=1, sA=1.01)</u>

Tax rate	NA	NB	XmA	XmB	YdA	YdB	YexA	YexB	YINA	YINB	TinA/sa	TinB/sb	WA	WB
0%	5567	24945	4152	4485	962	3929	0	265	3190	291	0	0.132	135.889	142.445
1%	7255	23233	4174	4442	1246	3702	0	342	2928	399	0.196	0.182	136.503	141.689
2%	8891	21567	4195	4402	1518	3474	0	415	2677	512	0.357	0.235	137.078	140.966
3%	10478	19957	4215	4364	1778	3248	93	485	2345	630	0.468	0.290	137.591	140.294
4%	11972	18451	4235	4330	2020	3032	204	550	2010	748	0.534	0.344	138.031	139.689
5%	13366	17049	4251	4299	2244	2825	328	610	1678	862	0.557	0.399	138.387	139.145
6%	14620	15783	4265	4271	2446	2636	475	664	1344	971	0.534	0.453	138.623	138.66
7%	15720	14734	4276	4251	2622	2475	661	711	993	1066	0.463	0.495	138.766	138.316
8%	16791	13825	4288	4239	2791	2330	912	757	585	1153	0.309	0.536	138.847	138.129
9%	17750	13088	4293	4234	2946	2209	1284	799	63	1225	0.038	0.570	138.675	138.052
10%	17823	13042	4293	4234	2959	2201	1334	802	0	1230	0	0.572	138.628	138.058

Tax rate	NA	NB	XmA	XmB	YdA	YdB	YexA	YexB	YINA	YINB	TinA/sa	TinB/sb	WA	WB
0%	26060	4419	4512	4127	4075	769	208	0	229	3358	0.104	0	142.948	135.388
1%	24360	6099	4470	4148	3852	1055	285	0	333	3093	0.151	0.207	142.193	136.008
2%	22688	7745	4429	4169	3627	1332	359	0	443	2837	0.202	0.379	141.463	136.595
3%	21086	9331	4392	4189	3406	1595	429	99	557	2496	0.255	0.499	140.794	137.115
4%	19565	10845	4357	4209	3190	1844	495	218	679	2147	0.309	0.572	140.184	137.565
5%	17049	13365	4298	4251	2825	2244	610	328	862	1678	0.398	0.557	139.145	138.387
6%	16857	13574	4299	4241	2793	2286	612	511	894	1443	0.413	0.575	139.166	138.199
7%	15720	14734	4276	4251	2622	2475	661	711	993	1066	0.463	0.495	138.766	138.316
8%	14821	15768	4263	4260	2481	2642	705	985	1077	633	0.499	0.336	138.555	138.326
9%	14074	16776	4259	4266	2359	2806	750	1390	1151	69	0.534	0.041	138.508	138.149
10%	14022	16855	4259	4265	2350	2820	753	1445	1156	0	0.536	0	138.513	138.095

<u>3.4.19 Best-response tax rates given $\gamma_A = 7\%$ (sB=1, sA=1.01)</u>

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