Three Essays on Liquidity Crisis, Monetary Policy, and Banking Regulation

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Jin Cao

To my parents

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Preface

This monograph emerges from the project started three years ago, when the world was still abundant with liqiudity and investors were eyeing on new models from Porsche instead of the financial hurricane gathering on the remote horizon. Then hard time came suddenly, and the entire world has been stuck in quagmire.

This monograph presents the lessons I've learned so far in such a rare opportunity, but surely the research project will go on, as long as there are still unknowns to be discovered:

> *"Grau, teurer Freund, ist alle Theorie,*
> *Und grün des Lebens goldner Baum."*[1]
>
> — Johann Wolfgang von Goethe

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[1] "All theory, dear friend, is gray, but the golden tree of life springs ever green."

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Finally, my greatest gratitude goes to my parents — your love and care have been invaluable during the whole of my life.

J. C.

Acronyms

CDO	Collateralized Debt Obligation
DSGE	Dynamic Stochastic General Equilibrium
FT	Financial Times
LoLR	Lender of Last Resort
SEC	(U.S.) Securities and Exchange Commission
WSJ	The Wall Street Journal

Part I

Introduction

The issues

Liquidity, the ease of converting assets to cash, is perhaps one of the most mysterious terms in both finance and macroeconomics. In economic booms the world is abundant with liquidity, but when crisis hits liquidity drains out immediately as if it didn't exist at all. The reason why financial institutions hold liquid — while low yielding — assets, has been extensively explored since (at least as early as) Keynes (1936) — the liquid assets enable agents to better weather shocks in their liabilities. Therefore, when agents face liquidity shortages, they have to sell their illiquid — yet high yielding — assets, at a cost ("bid-ask spread"). The burgeoning financial innovations in the past decade, people argue, help eliminate such cost and push the entire world closer to the perfect Arrow-Debreu economy.

Unfortunately, the subprime crisis that erupted in 2007 turns out to be a nightmare in nirvana, once again showing how imperfect the financial world can be. In existing banking literature many works focus on the consequences of liquidity crises and liquidity shocks are thus often assumed to be exogenous, which lack the explanation why systemic liquidity shortages come into being. Instead, this monograph, which has been started developing before the crisis, presents a compact model showing how liquidity shortages emerge as endogenous systemic risks, driven by the free-riding incentives of rational agents even if there are only illiquidity shocks.

However, there are already a few works on other mechanisms (for example, the global game approach such as Goldstein & Pauzner, 2005) leading to systemic liquidity risks, so this monograph doesn't stop at providing a "me-too" explanation. It has been long aware of that financial market has an increasingly huge power on macro economy, however, when one takes a look at economic research, finance and macroeconomics are actually two much insulated fields — generally there's little concern on macro policy in financial research, and in the dominating DSGE monetary economics there's hardly any role for financial sector. This monograph is going to take the challenge of bridging these two fields, and carefully examine the proper macro policy in liquidity crises and its impact on financial market.

Inevitably, severe financial crises are echoed by a resonance of draconian re-regulation. The world wide crisis triggered in 2007 will be no exception. After the meltdown of financial markets in September 2008, politicians and voters from Washington to Warsaw, from Berlin to Beijing, joined in a unanimous call for drastic regulation of greedy financial institutions that stole jobs and held the entire global economy to ransom. Old fashioned proposals such as narrow banking and banning of short selling, which for a long time have been intentionally desecrated, deliberately forgotten, or cautiously disguised in the regulators' reports, regained reputation and momentum.

Regulatory rules should, however, be based on sound economic analysis. First, regulators need to fully understand the driving forces behind misallocations in the market economy before designing adequate rules. Second, the benefit and cost of various regulatory schemes need to be quantified so that the optimal one can be picked up. Third, regulators have to go beyond current crisis measures in order not to run the risk of fighting the last war but rather to be able to design robust policies, addressing market's incentives to circumvent latest regulation.

A key lesson from the current crisis has been that a sound regulatory and supervisory framework requires a system-wide approach: the macroeconomic impact of risk across exposure across financial institutions needs to be taken into account. Regulation based purely on the soundness of individual institutions misses a crucial dimension of financial stability — the fact that risky activities undertaken by individual institutions may get amplified on the aggregate level. Among academics, this "macro-prudential" perspective has been the focus of intensive research for quite some time, stressing the need to cope with the pro-cyclicality of capital regulation (see Danielsson *et al.*, 2001 and Borio, 2003). Several recent studies surveyed in the following section provide a deeper understanding of the nature of externalities creating a tendency for financial intermediaries to lean towards excessive correlation, resulting in exposure to systemic risk. Most of these studies concentrate on solvency issues and capital adequacy regulation. As emphasized by Acharya (2009), externalities creating incentives to raise systemic risk justify charging a higher capital requirement against exposure to general risk factors: capital adequacy requirements should be increasing not just in individual risks, but also in the correlation of risks across banks.

Surprisingly, however, there are hardly any studies of the systemic impact of liquidity regulation. Given the recent massive unprecedented scale of central bank intervention in the market for liquidity, a careful analysis of incentives for private and public liquidity provision seems to be warranted. Presumably, one of the reasons for neglecting this issue is the notion that central bank intervention is the perfect instrument to cope with problems of systemic liquidity crises. Following several studies (in particular, Holmström & Tirole, 1998 and Allen & Gale, 1998), the public provision of emergency liquidity is frequently considered to be an efficient response to aggregate liquidity shocks. Central bank's Lender of Last Resort policy is seen as optimal insurance mechanism against these shocks. In this view, private provision of the public good of emergency liquidity would be costly and wasteful.

But as we will show, this notion is no longer correct if the exposure of financial institutions to systemic shocks is affected by decisions of these institutions themselves. In Holmström & Tirole (1998), aggregate liquidity shocks are assumed to be exogenous. We show, however, that incentives affect endogenously the exposure of financial institutions to systemic liquidity shocks. Based on Cao & Illing (2008, 2009a), we demonstrate that externalities result in excessive investment in illiquid assets (maturity mismatch), creating systemic liquidity risk. These externalities may be reinforced by central bank intervention. *Ex ante* liquidity regulation (the requirements to reduce maturity mismatch) can raise investor's payoff.

Another key lesson from the current crisis is how the ambiguity between illiquidity and insolvency problems complicates the crisis policy as well as banking regulation. Usually illiquidity and insolvency have been studied as separate phenomena and there are a couple of traditional solutions for either of them. However, it is argued in this monograph that they have been becoming joint plagues in financial market as modern financial innovations are rapidly blurring the boundary in between. Such new feature brings new challenges to both market practitioners and banking regulators. If there's no ambiguity between illiquidity and insolvency, conventional wisdoms work well: with pure liquidity risks banks can get enough liquidity from the central bank with their long-term assets as collateral; with pure insolvency risks equity holding can be a self-sufficient cushion for the banks to get rid of the losses. However, if there's uncertainty about the banks' true trouble, things become complicated — banks cannot get enough liquidity because the collateral, in the presence of insolvency risk, is no longer considered to be good, therefore pure liquidity regulation may fail; on the other hand equity requirement may be inefficient as well because the dual problems make equity holding even costlier. This monograph is thus going to step into the troubled water, hoping to shed some light on understanding the market failure and designing proper regulatory rules via extending the basic framework.

Most related literature: A very brief survey

Although there will be sections of literature review in each of the following chapters, let us take a very brief survey here on the most related existing literature.

The need for banking regulation is based on the inherent fragility of financial intermediation. Whereas traditional models focus on coordination failures of a representative bank triggered by runs (Diamond & Dybvig, 1983), recent research analyzes endogenous incentives for systemic risk arising from correlation of asset returns held by different banks. As shown by Acharya (2009), risk-shifting incentives for banks may result in over-investment in correlated risk activities, thereby increasing economy-wide aggregate risk.

In Acharya (2009), these incentives arise from limited liability of banks and the presence of a negative externality of one bank's failure on the health of other banks. If this effect dominates the strategic benefit of surviving banks from the failure of other banks (expansion and increase in scale), banks find it optimal to increase the probability of surviving and failing together. Thus, capital adequacy requirements should be increasing not just in individual risks, but also in the correlation of risks across banks.

The correlation of portfolio selection is also explored by Acharya & Yorulmazer (2005). Here, incentives to correlate arise from informational spillovers. Starting from a two-bank economy, when the returns of bank's investments have a systemic factor, the failure of one bank conveys negative information about this factor which makes market participants skeptical about the health of the banking industry, inflating the borrowing cost of the surviving bank and increasing its probability to fail. Since such informational spillover is costly for banks, they herd *ex ante* (i.e. they choose perfectly correlated portfolio) to boost the likelihood of joint survival, given that bankers' limited liability mitigates concerns about their joint failure. Again, systemic risk arises out of excessive correlations.

Wagner (2009) considers a financial market with a continuum of banks, all offering fixed deposit contracts, their portfolios being invested in two types of assets. A bank is run when it cannot meet the contract. Liquidation costs increases with the number of the banks run. However, since each bank is atomistic in this economy, the marginal liquidation cost when one more bank fails is zero. Therefore, when deciding about its investment portfolio, each single bank never internalizes its impact on the social cost of bank runs, imposing a negative externality on the banking industry. As a result, the banks equilibrium portfolios correlate in an inefficient way. Therefore, small banking failures may ripple to a large amount of banks with similar investment strategies. Optimal banking regulation should take correlation of the banks assets into account, encouraging heterogeneous investment.

In Korinek (2008), endogenous systemic risk arises from the feedback between incomplete financial markets and the real economy. Adverse shocks tighten individuals' credit constraints, triggering the contraction of economic activities. This depresses the prices of productive assets, hence the net worth of their owners, and worsens their credit constraints. The financial accelerator amplifies negative shocks to the economy, giving rise to externalities: atomistic agents take the level of asset prices in the economy as given. In their demand for productive assets, they do not internalize the externalities that arise when aggregate shocks lead to aggregate fluctuations. So decentralized agents undervalue social benefits of having stronger buffers when financial constraints are binding, taking on too much systemic risk in their investment strategies. Again, capital requirements need to address the externality so as to implement the constrained efficient allocation.

All studies surveyed look at endogenous incentives to create systemic solvency risk, arising from excessive correlation of assets invested. In contrast, this monograph analyzes endogenous incentives to create systemic liquidity risk. Our model attempts to capture the unease many market participants felt for a long time about abundant liquidity being available, before liquidity suddenly dried out world-wide in August 2007. We characterize incentives of financial intermediaries to rely on liquidity provided by other intermediaries and the central bank. Traditional models of liquidity shortages claim that provision of liquidity by the central bank is the optimal response to systemic shocks. We argue, however, that this view neglects the endogenous nature of liquidity provision. As we will show, incentives to rely on liquidity provided by the market may result in excessively illiquid investment. Enforcing strict liquidity requirements *ex ante* can tackle the externalites involved.

The classic paper about private and public provision of liquidity is Holmström & Tirole (1998). In their model, liquidity shortages arise when financial institutions and industrial companies scramble for, and cannot find the cash required to meet their most urgent needs or undertake their most valuable projects. They show that credit lines from financial intermediaries are sufficient for implementing the socially optimal (second-best) allocation, as long as there is no aggregate uncertainty. In the case of aggregate uncertainty, however, the private sector cannot cope with its own liquidity needs. In that case, according to Holmström & Tirole (1998), the government needs to inject liquidity. The government can provide (outside) liquidity (additional resources) by committing future lump sum tax revenue to back up the reimbursements. In their model, public provision of liquidity is a pure public good in the presence of aggregate shocks, causing no moral hazard effects. The reason is that aggregate liquidity shocks are modeled as exogenous events. The aggregate amount of liquidity available is not determined endogenously by the investment choice of financial intermediaries. Furthermore, according to Holmström & Tirole (1998) and also Fahri & Triole (2009), the Lender of Last Resort can redirect resources *ex post* at not cost via lump sum taxation. Allowing for lump sum taxation *ex post*, however, amounts to liquidity constraints becoming effectively irrelevant *ex ante*.

Allen & Gale (1998) analyze a quite different mechanism for public provision of liquidity, closer to current central bank practice. They allow for nominal deposit contracts. The injection of public liquidity works via adjusting the price level in an economy with nominal contracts: the public liquidity the central bank injects, the lower the real value of nominal deposits. Diamond & Rajan (2006) adopt this mechanism to characterize post crisis intervention in an elegant framework of financial intermediation with bank deposits and bank runs triggered by real illiquidity. Similar to Holmström & Tirole (1998), however, shocks to real liquidity are again assumed to be exogenous.

Any *ex post* intervention, however, usually has profound impact on the industry players' *ex ante* incentives. Financial intermediaries relying on being bailed out by the central bank in case of illiquidity may be encouraged to cut down on investing in liquid assets. If so, taking liquidity shocks as exogenously given and concentrating on crisis intervention misses a decisive part of the problem: *ex post* effective intervention may exacerbate the problems *ex* *ante* that lead to the turmoil. So policy implications from models based on exogenous liquidity shocks may be seriously misleading.

Concerning introducing joint problems of both illiquidity and insolvency risks, the mostly closely related work is probably the model considered in Bolton, Santos and Scheinkman (2009a, a.k.a. BSS as in the following). The feature that the market participants can hardly distinguish between illiquidity and insolvency is captured in their model, but they mainly focus on the supply side of liquidity, i.e. liquidity from financial institutions' own cash reserve (*inside liquidity*) or from the proceeds from asset sales to the other investors with longer time preference (*outside liquidity*) and the timing perspective of liquidity trading. This monograph takes BSS's view that (outside) liquidity shortage arises from the banks' coordinative failure, but the timing of liquidity trading is not going to be my focus. Rather, this monograph provide a different explanation of systemic liquidity risk, i.e. liquidity underprovision may come from the banks' incentive of free-riding on each others' liquidity supply, which is not covered in BSS (in which they restrict attention to pure strategy equilibria); and clear-cut results from a more compact and flexible model in this monograph make it easier to be applied on banking regulation. What's more, since financial contracts in BSS are real, they (BSS, 2009b) conclude that efficiency can be restored by central banks' credible supporting (real) asset prices. However, in contrast, this monograph shows that the introduction of (more realistic) nominal contracts may alter the policy implications drastically — nominal liquidity injection from central banks may crowd out market liquidity supply without improving efficiency, therefore policy makers should take a more careful view on designing regulatory rules and bailout policies.

Key contributions

This monograph contributes to the existing research for the following three aspects:

First, we endogenize systemic liquidity risk in an intuitive and tractable way. We provide a baseline model for regulatory analysis of pure liquidity shocks. We show that even with rational financial market participants, no asymmetric information and pure illiquidity risk the free-riding incentive on liquidity provision may be large enough to generate bankers' excessive appetite for risks, at a cost of the stability of the financial market. Our framework captures two major sources of inefficiency: (a) competitive forces encourage bankers with limited liability to take on more risk, resulting in inferior mixed strategy equilibrium and (b) bank runs forcing inefficient liquidation impose social costs. The mix of both externalities creates a role for liquidity regulation;

Second, following Diamond & Rajan (2006), we extend the baseline model by allowing for nominal deposit contracts. This captures the popular notion that central banks can ease nominal liquidity constraints using the stroke of a pen. Doing so, central banks don't produce real wealth. Instead, their intervention works via redistribution of real wealth. Flooding the market with nominal liquidity in times of crisis may help to prevent *ex post* inefficient bank runs; at the same time, however, it encourages financial intermediaries to invest excessively on high yielding, but illiquid projects, lowering liquid resources available for investors. We show that with unconditional liquidity support by central banks, all banks will free ride on liquidity in equilibrium, reducing the expected payoff for investors substantially. In contrast, *ex ante* liquidity regulation combined with *ex post* Lender of Last Resort policy can implement the constrained second-best outcome from the investor's point of view. Further on, we explicitly compare the effectiveness of various regulatory schemes. To the best of my knowledge, this is the first work providing such an analysis;

Third, we extend the baseline model of pure illiquidity risks by including insolvency risks. Generally, allowing both plagues in one single model brings out many difficulties in endogenizing the systemic risks — as such setting explodes the state space and usually ends up with intractable mixed strategy equilibria. However, in this monograph the problem is avoided by a designed trick of trimming off less interesting states, allowing the author to capture the kernel of the problem without loss of generality and arrive at clear-cut analytical results. This enables the author to go further with quantitative policy analysis and propose hybrid regulatory schemes of lower cost. To the best of my knowledge, this is the first work providing such an analysis based on the mixture of both illiquidity and insolvency risks.

The structure of the monograph

Part II, Chapter 1 (adapted from Cao & Illing, 2008) presents the baseline framework of pure illiquidity risks and the feedback mechanism between monetary policy and financial market. Part III extends the baseline model: Chapter 2 (adapted from Cao & Illing, 2009a, b) compares the effectiveness of various regulatory schemes in the baseline framework; and Chapter 3 re-examines the schemes in an extended framework with co-existence of illiquidity and insolvency risks, then proposes policies with lower regulatory cost. Part IV concludes.

Part II

Liquidity Shortages as Endogenous Systemic Risks

1

Liquidity Shortages and Monetary Policy

Moral hazard fundamentalists misunderstand the insurance analogy.

—Lawrence Summers, *Financial Times*, Sept. 24th, 2007

Just as imprudent banks have been saved from their mistakes by indulgent central bankers, so CDO-makers could be rewarded for the mess that they helped to create. ... The creators of CDOs and conduits may end the year with new Porsches. Vroom-vroom.

—Croesus's cousins, *The Economist*, Sept. 22nd, 2007

1.1 INTRODUCTION

1.1.1 The issues

For quite some time, at least a few market participants had the feeling that financial markets have been susceptible to excessive risk taking, encouraged by extremely low risk spreads. There was the notion of abundant liquidity, stimulated by a "savings glut"; by an "investment drought" or by central banks running too-loose monetary policies. In that context, some brave economists warned against the rising risk of a liquidity squeeze which might force central banks to ease policy again (compare, for example, *A fluid concept*, *The Economist*, February 2007). Frequently it was argued that it was exactly the anticipation of such a central bank reaction which encouraged further excessive risk taking: the belief in "abundant" provision of aggregate liquidity might have resulted in overinvestment in activities creating systemic risk.

Since August 2007, liquidity indeed has dried out worldwide. There has been an unprecedented freeze on the money markets, triggering desperate calls within the financial sector to lower interest rates[1]. Initially, central banks have been split over how to respond to the credit squeeze. Some central banks immediately pumped billions of extra money into the financial system; some even lowered interest rates. Others warned of the hazards of providing central bank insurance to those institutions that have engaged in reckless lending. Mervyn King, Bank of England governor, argued (FT, Sept. 12 2007): *"The provision of large liquidity facilities penalises those financial institutions that sat out the dance, encourages herd behavior and increases the intensity of future crises."*

[1]The eruption in credit market turmoil has taken some by surprise — see Alan Greenspans remark *"I ask you if anybody in early June could contemplate what we are now confronted with?"* WSJ September 7, 2007. Others have been puzzled that it took so long to trigger fire sales — see the references in Illing (2007).

The current problems in financial markets provoked a heated debate on causes and potential solutions. At Jackson Hole, James Hamilton (2007) called for regulatory and supervisory reforms, pointing out that significant negative externalities have been created. This chapter tries to shed some light on a crucial type of externality involved: the incentive of financial intermediaries to free-ride on liquidity. This chapter, the main part having been written before the outbreak of the crisis, concentrates on a particular, but — from our point of view — key issue: it focuses on the interaction between risk taking in the financial sector and central bank policy. For that purpose, we analyze an economy with pure illiquidity risk. Intuition suggests that injection of public liquidity should always be welfare improving in that highly unrealistic case. A surprising result of this chapter is that even for pure illiquidity risk, intuition turns out not to be correct in general.

We prove that insuring against aggregate risks will result in a higher share of less liquid projects funded. So liquidity provision as public insurance does indeed encourage higher risk taking. But one has to be careful about the impact on welfare: this effect will not necessarily result in "excessive" risk. For some parameter values, liquidity provision turns out to be welfare improving (as suggested in the traditional literature on lender of last resort, see Goodhart & Illing, 2002). In the presence of aggregate risk, banks may prefer to take no precaution against the risk of being run in bad states, when these states are highly unlikely. If so, public provision of liquidity to prevent inefficient bank runs improves upon the allocation, even though it encourages more risk taking (less liquid investment) by private banks. So liquidity provision by central banks provides an insurance against aggregate risk in an incomplete market economy, encouraging investment in projects which give a higher return, but at the same time exhibit higher risk of illiquidity.

But, unfortunately, this result does not hold in general. As we will show, the incentive of financial intermediaries to free-ride on liquidity in good states may result in excessively low liquidity in bad states. In the prevailing mixed strategy equilibrium, depositors are worse off than if banks would coordinate on more liquid investment. When the mixed strategy equilibrium prevails, public liquidity injection would increase the incentive to free-ride, making the free-riding problem even worse. If that case prevails, the central bank should commit to abstain from intervening in order to discourage free-riding. The results derived show that liquidity injection is a delicate issue possibly creating severe moral hazard problems.

The present chapter builds on the set up of Diamond & Rajan (2006) and extends it to capture the feedback from liquidity provision to risk taking incentives of financial intermediaries. As in Diamond & Rajan, deposit contracts solve a hold up problem for impatient lenders investing in illiquid projects: these contracts give banks as financial intermediaries a credible commitment mechanism not to extract rents from their specific skills. But at the same time deposit contracts make non-strategic default very costly. Consequently, negative aggregate shocks may trigger bank runs with serious costs for the whole economy, thus destroying the commitment mechanism. Diamond & Rajan (2006) show that monetary policy can alleviate this problem in an economy with nominal deposits: via open market operations, the central bank can mimic state contingent real debt contracts by adjusting the nominal price level to the size of the aggregate shock.

This chapter extends the set up of Diamond & Rajan in several ways. In their model, the type of risky projects is exogenously given. Banks can either invest in risky, possibly illiquid projects or invest instead in a safe liquid asset with inferior return. In the equilibrium they characterize, banks invest all resources either in illiquid or liquid assets. They do not analyze the feedback mechanism from monetary policy towards the risk taking of financial intermediaries when central bank policy works as an insurance mechanism against aggregate risk.

In contrast, the present chapter determines endogenously the aggregate level of illiquidity out of private investments. As in Diamond & Rajan, illiquidity is captured by the notion that some fraction of projects turns out to be realized late. In contrast to their approach, however, we allow banks to choose the proportion of funds invested in less liquid projects continuously. These projects have a higher expected return, but at the same time also a higher probability of late realization. Because of that feature, some banks will have an incentive to free-ride on liquidity. Banks investing a larger share in illiquid projects with higher, yet delayed returns will always be more profitable as long as they stay solvent. Yet there is an economic role for liquidity to satisfy the need for early withdrawals by investors in our model. The problem is that ″naughty″ free-riding banks can always attract funds away from those prudent banks which had invested in more liquid, but less profitable assets (to use the poetic phrase by Mervyn King: *those financial institutions that sat out the dance*).

In times of a liquidity crisis, the ″naughty″ banks will run into trouble. They would have to leave the market, to make sure that *ex ante* expected returns for depositors are the same for all banks. If, however, the central bank provides liquidity to the market in bad states, this helps ″naughty″ banks to survive, allowing them to indeed pay out high returns later. The at first sight surprising, but at second thought quite intuitive reason is that ″naughty″ banks are always in a better position to attract funds even in a crisis — as long as policy helps them to stay solvent (Note that in this chapter we abstract from insolvency except if triggered by illiquidity). The problem is that relying on such interventions *ex ante* will give all banks strong incentives to behave ″naughty″, so liquidity is bound to dry out in the sense that there will be insufficient supply of real goods in the intermediate period. Of course, a commitment not to intervene in these cases is not really credible, as sadly has been demonstrated in the UK in September 2007, when Northern Rock (a mortgage bank in the UK which promised high deposit rates as a way to finance attractive investment in real estate) smashed the credibility of the Bank of England just the day after Mervyn King reconfirmed his brave statements in a letter to the chancellor.

1.1.2 Related literature

Liquidity provision has been mainly analyzed in the context of models with real assets — see Diamond & Dybvig (1983), Bhattacharya & Gale (1987), Diamond & Rajan (2001, 2005), Fecht & Tyrell (2005) and for a survey the reader of Goodhart & Illing (2002). Only a few recent papers explicitly include nominal assets and so are able to address monetary policy, such as Allen & Gale (1998), Diamond & Rajan (2006), Skeie (2006) and Sauer (2007). Skeie (2006) shows that nominal demand deposits, repayable in money, can prevent self-fulfilling bank runs of the Diamond / Dybvig type, when interbank lending is efficient.

Here, we are concerned with bank runs triggered by real shocks as in Diamond & Rajan (2006). Demand deposits provide a credible commitment mechanism. A related, but quite different mechanism has been analyzed by Holmström & Tirole (1998). They model credit lines as a way to mitigate moral hazard problems on the side of firms. In their model, Holmström & Tirole also characterize a role for public provision of liquidity, but again they do not consider feedback mechanisms creating endogenous aggregate risk.

Apart from Diamond & Rajan (2006), the paper most closely related is Sauer (2007). Building on the cash-in-the-market pricing model of Allen & Gale (2004), Sauer analyzes liquidity provision by financial markets and characterizes a trade-off between avoiding real losses by injecting liquidity and the resulting risks to price stability in an economy with agents subject to a cash-in-advance constraint. The present chapter uses the more traditional framework with banks as financial intermediaries. This framework can capture the impact of financial regulation of leveraged institutions in a straightforward way.

1.1.3 Sketch of the chapter

Section 3.2 presents the basic settings of the model. Let us here sketch the structure informally. There are three types of agents, and all agents are assumed to be risk neutral.

1. *Entrepreneurs*. They have no funds, just ideas for productive projects. Each project needs one unit of funding in the initial period 0 and will either give a return early (at date 1) or late (at date 2). There are two types of entrepreneurs: entrepreneurs of type 1 with projects maturing for sure early at date 1, yielding a return $R_1 > 1$ and entrepreneurs of type 2 with projects yielding a higher return $R_2 > R_1 > 1$. The latter projects, however, may be delayed: with probability $1 - p$, they turn out to be illiquid and can only be realized at date 2. For projects being completed successfully, the specific skills of the entrepreneur are needed. Human capital being not alienable entrepreneurs can only commit to pay a fraction $\gamma R_i > 1$ to lenders. They earn a rent $(1-\gamma)R_i$ for their specific skills. Entrepreneurs are indifferent between consuming early or late;

2. *Investors*. They have funds, but no productive projects on their own. They can either store their funds (with a meagre return 1) or invest in the projects of entrepreneurs. Investors are impatient and want to consume early (in period 1). Resources being scarce, there are less funds available than projects of either type. In the absence of commitment problems, investors would put all their funds in early projects R_1 and capture the full return; Entrepreneurs would receive nothing. But financial intermediaries are needed to overcome commitment problems. In addition to the entrepreneur's commitment problem, specific collection skills are needed to transfer the return to the lender. As shown in Diamond & Rajan (2001, 2005), by issuing deposit contracts designed with a collective action problem (the risk of a bank run), bank managers can credibly commit to use their collection skills to pass on to depositors the full amount received from entrepreneurs. So limited commitment motivates a role for banks as intermediaries;

3. *Banks*. Due to their fragile structure, bank managers are committed to pay out deposits as long as banks are not bankrupt. Holding capital (eq-

uity), which will be allowed in the next chapter, can reduce the fragility of banks, but it allows bank managers to capture a rent (assumed to be half of the surplus net of paying out depositors) and so lowers the amount of pledgeable funds. Like entrepreneurs, bank managers are indifferent between consuming early or late.

Banks offer deposit contracts. There is assumed to be perfect competition among bank managers, so investors deposit their funds at those banks offering the highest expected return at the given market interest rate. Most of the time (see Footnote 3), we assume that investors are able to monitor all bank's investment. So if, in a mixed strategy equilibrium, banks differ with respect to their investment strategy, the expected return from deposits must be the same across all banks.

Except for introducing two types of entrepreneurs, the structure of the model is essentially the same as the set up of Diamond & Rajan (2006). By assuming that depositors (investors) value consumption only at $t = 1$, all relevant elements are captured in the most tractable way: at date 1, there is intertemporal liquidity trade with inelastic liquidity demand. Banks competing for funds at date 0 are forced to offer conditions which maximize expected consumption of investors at the given expected interest rates. Whereas Diamond & Rajan (2006) just present numerical examples for illustrating relevant cases, we fully characterize the type of equilibria as a function of parameter values. Furthermore, we derive endogenously the extent of financial fragility as a function of the parameter values.

As a reference point, Section 1.3 analyzes the case of pure idiosyncratic risk. It is shown that banks will choose their share of investment in safe projects such that all banks will be always solvent, given that there is liquid trading on the inter bank market. Section 1.4 introduces aggregate shocks. The outcome strongly depends on the probability of a bad aggregate shock occurring. If this probability is low, banks care only for the good state (Proposition 1.4.1 a)) and accept the risk of failure with costly liquidation in the bad state. In contrast, banks play safe if the probability of a bad shock is very high (Proposition 1.4.1 b)). For an intermediate range, however (Proposition 1.4.2), financial intermediaries have an incentive to free-ride on excess liquidity available in the good state. This leads to low liquidity in bad states. In the prevailing mixed strategy equilibrium, depositors are worse off than if banks would coordinate on more liquid investment.

Section 1.5 analyzes central bank intervention. With nominal bank contracts, monetary policy can help to prevent costly runs by injecting additional money before $t = 1$. The real value of deposits will be reduced such that banks on the aggregate level are solvent despite the negative aggregate shock. It turns out that if the probability of a bad aggregate shock is low enough, central bank intervention is welfare improving, even though banks relying on liquidity injection will invest more in illiquid late projects. If, however, the probability of a bad aggregate shock is high, central bank intervention will make the free-riding problem even worse. In any case, the central bank needs to be able to commit to restrict liquidity provision only to prudent banks. Otherwise, free-riding crowds out all prudent banks in equilibrium. Such a commitment, however, is not dynamically consistent. So liquidity injection is a delicate issue, possibly creating severe moral hazard problems. Section 1.6 concludes.

1.2 THE MODEL — BASIC SETTINGS

1.2.1 Agents, technologies and preferences

There is a continuum of risk-neutral investors with unit endowment at $t = 0$ who want to consume at $t = 1$. They have access only to a storage technology with return 1, i.e. their wealth may be simply stored without perishing for future periods. As an alternative, they can lend their funds to finance profitable long term investments of entrepreneurs. Due to commitment problems, lending has to be done via financial intermediation.

There are two types of entrepreneurs who have ideas for projects: when funded, type $i = 1, 2$ entrepreneurs can produce:

- Type 1: safe projects, yielding $R_1 > 1$ for sure early at date 1;
- Type 2: risky projects, yielding $R_2 > R_1 > 1$ either early at date 1 with probability p (and $pR_2 < R_1$), or late at date 2 with probability $1 - p$.

Borrowing and lending is done via competitive and risk-neutral banks of a limited number N, who have no endowment at $t = 0$. Banks use the investors' funds (obtained via deposits or equity) to finance and monitor entrepreneurs' projects. They have a special collection technology such that they can capture a constant share $0 < \gamma < 1$ of the projects' return. The fragile banking structure allows them to commit to pass those funds which have been invested as deposits back to investors (see below). For funds obtained via equity (to be explored in the next chapter), banks are able to capture a rent (assumed to be $\frac{1}{2}$ of the captured return net of deposit claims).

Entrepreneurs and banks are indifferent between consumption at $t = 1$ or $t = 2$. Because only banks have the special skills in collecting deposits from investors and returns from entrepreneurs, entrepreneurs cannot contract with investors directly; instead, they can only get projects funded via bank loans.

Resources are scarce in the sense that there are more projects than aggregate endowment of investors. This excludes the possibility that entrepreneurs might bargain with banks on the level of γ.

1.2.2 Timing

There are 4 periods:

1. $t = 0$. The banks offer deposit contract to investors, promising fixed payment d_0 in the future for each unit of deposit. The investors deposit their endowments if $d_0 > 1$. The banks then decide the share α of total funds to be invested in safe projects. Funded entrepreneurs receive loans and start their projects. d_0 and α are observable to all the agents, but p may be unknown at that date.

   The *fixed payment deposit contract* has the following features:

- Investors can claim a fixed payment d_0 for each unit of deposit at any date after $t = 0$;
- Banks have to meet investors' demand with all resources available. If liquidity at hand is not sufficient, delayed projects have to be liquidated at a cost: premature liquidation yields only c ($0 < c < 1 < \gamma R_1$) for each unit.

These contracts are adopted in the banking industry as a commitment mechanism. Since collecting returns from entrepreneurs requires specific skills, the bank managers would have an incentive to renegotiate with lenders at $t = 1$ in order to exploit rents. So a standard contract would break down. As shown in Diamond & Rajan (2001), the debt contract can solve the problem of renegotiation: whenever the investors anticipate a bank might not pay the promised amount, they will run and the bank's rent is completely destroyed by the costly liquidation. Therefore the banks will commit to the contract.

2. $t = \frac{1}{2}$. At that intermediate date, p is revealed and so the investors can calculate the payment from the banks. If a banks resources are not sufficient to meet the deposit contract, i.e. the investors' expected average payment at $t = 1$ is $d_1 < d_0$ for each unit of deposit, all investors will run the bank already at $t = \frac{1}{2}$ in the attempt to be the first in the line, and so still being paid d_0. When a bank is run at $t = \frac{1}{2}$, it is forced to liquidate all projects immediately (even those which would be realized early) trying to satisfy the urgent demand of depositors — so in the case of a run, the bank will not be able to recover more than c from each project.

   To concentrate on runs triggered by real shocks, we exclude self fulfilling panics: as soon as $d_1 > d_0$ investors are assumed never to run and to believe that the others don't run either.

3. $t = 1$. If the investors didn't run in the previous period, they withdraw and consume. The banks collect a share γ from the early projects.

But as long as entrepreneurs are willing to deposit their rents at $t = 1$ at banks, banks can pay out more resources to investors. Since early entrepreneurs retain the share $1-\gamma$ of the returns and they are indifferent between consumption at $t = 1$ or $t = 2$, the banks can borrow from them against the return of late projects at the market interest rate $r \geq 1$. r clears market by matching aggregate liquidity demand with aggregate liquidity supply. We assume that there is a perfectly liquid inter bank market at $t = 1$, so even if early entrepreneurs trade with other banks, the initial bank will be able to borrow the liquidity needed to refinance delayed projects as long as it is not bankrupt.

4. $t = 2$. Banks collect return from late projects and repay the liquidity providers at $t = 1$. Both early and late entrepreneurs consume.

Table 1.1 summarizes the basic elements of the model, and and Fig. 1.1 summarizes the timing of the game.

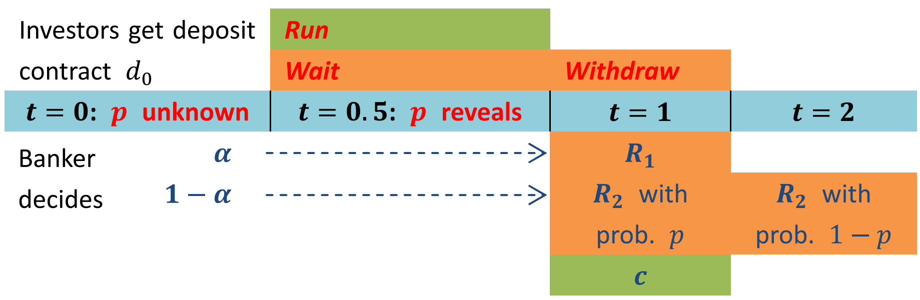

Fig. 1.1 The timing of the game

In the following sections we analyze the outcomes of the game in various scenarios.

1.3 PURE IDIOSYNCRATIC SHOCKS

As a baseline, consider the case in which p is deterministic and known to all the agents at $t = 0$. Equilibrium is characterized by the share α of funds banks

Table 1.1 The basic elements of the model: Agents, technologies, and preferences

Investors	• Unit $t = 0$ endowment — stored or invested in projects; • Investors want to consume at $t = 1$.
Entrepreneurs	• With type 1 project — Return $R_1 > 1$, safely realized at $t = 1$; • With type 2 project — Highest return $R_2 > R_1$, risky. It may return at $t = 1$, but may also be delayed to $t = 2$.
Banks	• Engage in Bertrand competition; • Expertise to collect $0 < \gamma < 1$ from projects return; • Offer deposit contracts — Commitment device not to abuse the expertise, and — Making banking industry fragile; • Risk of bank runs: poor liquidation return $0 < c < 1$.

choose to invest in safe projects and the interest rate r for deposits invested at $t = 1$. The outcome is captured in the following lemma:

Lemma 1.3.1 *When p is deterministic, there exists a symmetric non-idle pure strategy equilibrium in which*

1. *All the banks set*

$$\alpha_i(p,r) = \alpha^*(p,r) = \frac{\gamma\frac{1-p}{r} - (1-\gamma)p}{\gamma\frac{1-p}{r} + (1-\gamma)\left(\frac{R_1}{R_2} - p\right)}, \forall i \in \{1, \ldots, N\};$$

2. *Interest rate r is determined by*

$$r\sum_{i=1}^{N}(1-\gamma)\left[\alpha_i R_1 + (1-\alpha_i)pR_2\right] = \sum_{i=1}^{N}\gamma(1-\alpha_i)(1-p)R_2$$

and

$$r \leq \frac{R_2}{R_1}.$$

What's more, there exists no equilibrium in mixed strategies. ∎

Proof See Appendix A.1.1. ∎

By Lemma 1.3.1 multiple equilibria exist for all $1 \leq r \leq \frac{R_2}{R_1}$. To make the analysis interesting, we introduce the following equilibrium selection criterion:

Definition An *optimal symmetric pure strategy equilibrium* profile $\alpha^*(p,r^*)$ is given by

$$\begin{aligned}
&(1) \qquad r^* = \arg\max_r \kappa_i = \alpha_i(p,r)R_1 + (1-\alpha_i(p,r))pR_2;\\
&(2) \qquad \forall \alpha_i'(p,r^*) \neq \alpha^*(p,r^*) \text{ with } \alpha_{-i}(p,r^*) = \alpha^*(p,r^*),\\
&\qquad\quad\; \kappa_i(\alpha^*(p,r^*)) \geq \kappa_i(\alpha_i'(p,r^*),\alpha_{-i}(p,r^*)) \text{ with } -i \in \{1,\dots,N\}\backslash\{i\}.
\end{aligned}$$ ∎

The optimal symmetric equilibrium is the dominant equilibrium from the investor's point of view. Resources of investors being scarce, the market solution would maximize the payoff of investors in the absence of commitment problems (that is, in the case of $\gamma = 1$). Since the optimal symmetric equilibrium comes closest to achieving that market outcome, from now on we will focus our analysis on this specific equilibrium.

Lemma 1.3.2 *When p is deterministic, there exists a unique optimal symmetric equilibrium of pure strategy in which*

1. *All the banks set*

$$\alpha^*(p, r^*) = \frac{\gamma - p}{\gamma - p + (1-\gamma)\frac{R_1}{R_2}}, \forall i \in \{1, \ldots, N\};$$

2. *Interest rate* $r^* = 1$. ∎

Proof See Appendix A.1.2. ∎

From now on, denote $\alpha^*(p, r^*)$ by $\alpha(p)$ for simplicity. Then if the risks are purely idiosyncratic, the equilibrium outcome is given by:

Corollary 1.3.3 *When there are idiosyncratic risks such that for one bank i the probability p_i follows i.i.d. with pdf $f(p_i)$ with a non-empty support $\Omega \subseteq [0, \gamma]$, then there exists a unique optimal symmetric equilibrium of pure strategy in which*

1. *All the banks set*

$$\alpha(\mathbb{E}[p_i]) = \frac{\gamma - \mathbb{E}[p_i]}{\gamma - \mathbb{E}[p_i] + (1-\gamma)\frac{R_1}{R_2}}, \forall i \in \{1, \ldots, N\};$$

2. *Interest rate* $r^* = 1$. ∎

This is pretty intuitive: as long as there are just idiosyncratic shocks, banks are always solvent via trade on the liquid inter bank market.

In the absence of aggregate risk, the optimal equilibrium can thus be characterized in a straightforward way. When there is only idiosyncratic risk, a share p of risky projects will always be realized early in the aggregate economy. The representative bank chooses the share α^* of funds invested in safe projects such that in period 1, it is able to pay out depositors and equity to all investors. Otherwise, the bank would be bankrupt and forced to liquidate late projects at high costs (liquidation gives an inferior return of $c < 1$).

Depositors having a claim of $\gamma\mathbb{E}[R(\alpha, r)] = \gamma\left[\alpha(\overline{p}, r)R_1 + (1 - \alpha(\overline{p}, r))R_2\right]$ per unit deposited, the total amount to be paid out via deposits at $r^* = 1$ is $\gamma\left[\alpha R_1 + (1-\alpha)R_2\right]$. The representative bank will choose α^* such that at $t = 1$, there are just enough resources available to pay out all depositors, taking into account that early entrepreneurs are reinvesting their rents at banks as deposits at $t = 1$. The condition $\alpha R_1 + (1-\alpha)\overline{p}R_2 = \gamma\left[\alpha R_1 + (1-\alpha)R_2\right]$ gives as solution for α^* as a function of $\overline{p}$ (see Fig. 1.2):

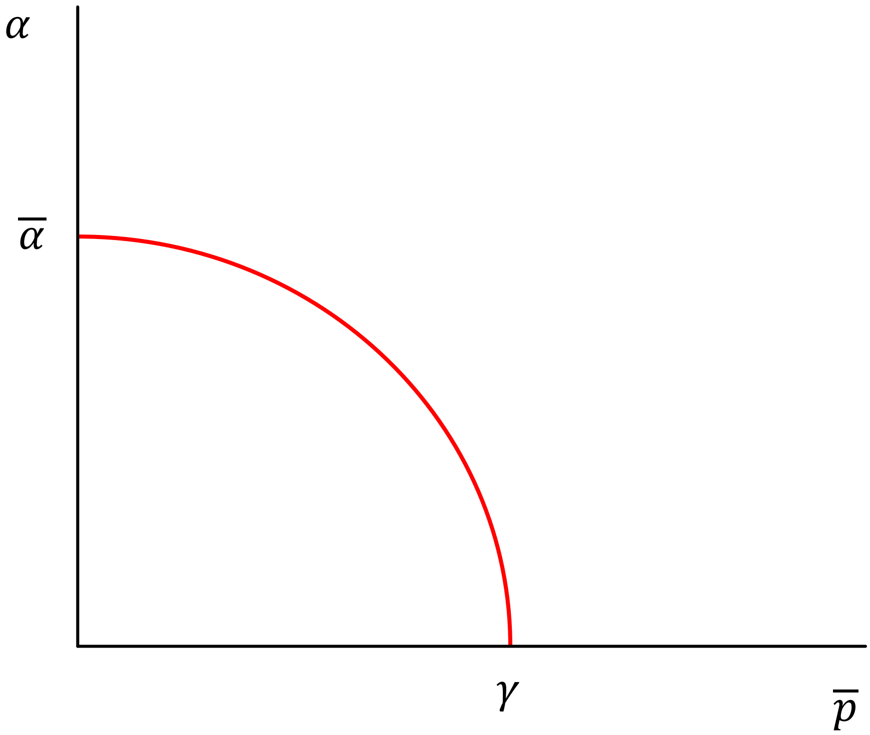

Fig. 1.2 α^* (the optimal share of funds invested in safe projects) as a function of $\overline{p}$ (the aggregate share of type 2 projects realized early)

$$\alpha^*(\overline{p}) = \frac{\gamma - \overline{p}}{\gamma - \overline{p} + (1-\gamma)\frac{R_1}{R_2}}$$

with

$$\frac{\partial \alpha^*}{\partial \overline{p}} = \frac{-(1-\gamma)\frac{R_1}{R_2}}{\left[\gamma - \overline{p} + (1-\gamma)\frac{R_1}{R_2}\right]^2} < 0; \frac{\partial \alpha^*}{\partial\left(\frac{R_1}{R_2}\right)} > 0.$$

And also

$$\alpha^* \in [0, \overline{\alpha}] \text{ with } \alpha^*(\overline{p} = 0) = \overline{\alpha} = \frac{\gamma}{\gamma + (1-\gamma)\frac{R_1}{R_2}};$$
$$\alpha^*(\overline{p} = \gamma) = 0; \alpha^*(\gamma = 1) = 1.$$

The higher $\overline{p}$ (the larger the share of early projects with a high payoff R_2), the lower the share of funds invested in projects of type R_1. If $\overline{p} > \gamma$, the representative bank would be solvent at $t = 1$ even when all funds were invested in risky type 2 projects. But even if $\overline{p} = 0$, there will be some investment in projects with a high payoff R_2 as long as $\gamma < 1$. The reason is that all early entrepreneurs, deferring consumption until $t = 2$, provide liquidity at $t = 1$. They are willing to deposit their rents at solvent banks. Thus, banks are able to pay out investors all funds available at $t = 1$ as long as they stay solvent. This way, with low interest rates, investors can also gain from the higher payoff of late projects, so $\alpha^*(\overline{p} = 0) > \gamma$. With $\frac{R_1}{R_2}$ increasing, the share α invested in safe projects will rise, allowing investors to profit from higher returns of less liquid projects already at $t = 1$. Again, this is due to the fact that early entrepreneurs provide liquidity. Note, however, that there would be no funding of risky projects at all in the absence of a commitment problem (that is, if $\gamma = 1$).

1.4 THE CASE OF AGGREGATE RISK

The interesting case is the case of aggregate risk. Assume that p is now unknown to all the agents at $t = 0$ and realizes at $t = \frac{1}{2}$ as an aggregate risk. We assume that

1. p can take just two possible values p_H or p_L with $0 < p_L < p_H < \gamma$;
2. p_H realizes with probability π and p_L with probability $1 - \pi$.

In the presence of aggregate risk, a bank has several options available: the bank may just take care for provisions in the good state, choosing $\alpha^* = \alpha(p_H)$ and may take no precaution against the risk of a bank run in the bad state p_L.

If so, the bank is run when p_L realizes and is forced to liquidate all projects. Obviously, this does not make sense if the probability of the bad state is high enough. Instead, the bank may increase the share of safe assets to $\alpha^* = \alpha(p_L)$ trying to prevent insolvency. If all banks would follow that strategy, there would be excess supply of liquidity in the good state p_H. This may give banks an incentive to free-ride on the provision of liquidity by other banks, and a pure strategy equilibrium may not exist[2]. So a careful analysis of all cases is required. We will now show that there are 3 types of equilibria, depending on the probability π — the probability that a high share of early projects is realized:

1. If π is high enough (for $\pi \in [\overline{\pi}_2, 1]$, all banks will choose $\alpha^* = \alpha(p_H)$. With that strategy, banks will be run at p_L, so depositors get only the return c if the share of early projects with high yields turns out to be unpleasantly low. All agents in the economy being risk neutral, it is more profitable for banks to take that risk into account in order to gain from the high returns in aggregate state p_H, as long as that event is not very likely;

2. If π is low enough (for $\pi \in [0, \overline{\pi}_1]$, all banks will choose $\alpha^* = \alpha(p_L)$. In that case, banks will never be bankrupt, so they will be able to payout all depositors at $t = 1$ even if the share of delayed projects is high. But if the share of delayed projects is low (in the state p_H), there will be excess liquidity floating around at $t = 1$;

3. For some parameter constellations (for the intermediate range $\pi \in (\overline{\pi}_1, \overline{\pi}_2)$), banks will be tempted to free-ride[3] on the excess liquidity in state p_H. These banks invest all their funds in the risky projects

[2] Banks may also hold some equity in order to cushion shocks. We will discuss this in the next chapter but ignore equity here.

[3] Bhattacharya & Gale (1987) have already shown that there is free-riding on liquidity provision when investors cannot monitor the amount of projects invested by the intermediaries. Footnote 4 confirms their argument in our context. But we derive a stronger result. We show that for an

($\alpha = 0$), trying to profit from the high returns available in case a large share of profitable projects happens to be realized early. The high expected returns in this case compensate depositors *ex ante* for the risk of getting just c in the other aggregate state of the world.

These equilibria are summarized in the following propositions.

Proposition 1.4.1 *Given p_H and p_L, and suppose that α's are observable*[4] *to all investors, a) There is a unique optimal symmetric equilibrium of pure strategy such that all the banks set $\alpha^* = \alpha(p_H)$ and $d_0^* = \gamma\left[\alpha(p_H)R_1 + (1-\alpha(p_H))R_2\right] = \alpha(p_H)R_1 + (1-\alpha(p_H))pR_2$ as soon as the probability of p_H satisfies $\pi > \overline{\pi}_2 = \frac{\gamma\mathbb{E}[R_L]-c}{\gamma\mathbb{E}[R_H]-c}$, in which $\mathbb{E}\left[R_s\right] = \alpha(p_s)R_1 + (1-\alpha(p_s))pR_2$, $s \in \{H, L\}$; b) When $0 \leq \pi < \frac{\gamma\mathbb{E}[R_L]-c}{\gamma R_2 - c} = \overline{\pi}_1$, there exists a unique optimal symmetric equilibrium of pure strategy such that all the banks set $\alpha^* = \alpha(p_L)$ and $d_0^* = \gamma\left[\alpha(p_L)R_1 + (1-\alpha(p_L))R_2\right] = \alpha(p_L)R_1 + (1-\alpha(p_L))pR_2$.*

Proof See Appendix A.1.3. ▮

The intuition behind Proposition 1.4.1 is the following: when it is very unlikely that the low state realizes, i.e. π is very high, then the cost of a bank run is too small relative to the high return in the high state. So the best strategy for the banks is to exploit the maximum return from the high state

intermediate range of parameter values, even with perfect monitoring of banks, some banks have an incentive to free-ride on liquidity in good states, giving rise to a mixed strategy equilibrium, resulting in excessively low liquidity in bad states. In the prevailing mixed strategy equilibrium, depositors are worse off than if banks would coordinate on more liquid investment.

[4] This condition is crucial for $\pi \in [0, \overline{\pi}_2]$. If α's were not observable to investors in this range, $\alpha(p_L)$ would fail to be a symmetric equilibrium of pure strategy. The reason is straightforward: suppose that all the banks coordinate and set $\alpha^* = \alpha(p_L)$, then there is always incentive for one single bank i to deviate and set $\alpha_i = \alpha(p_H)$ because it earns positive profit at p_H, i.e. $\gamma\left[\alpha(p_H)R_1 + (1-\alpha(p_H))R_2\right] - \gamma\left[\alpha(p_L)R_1 + (1-\alpha(p_L))R_2\right] > 0$, and at p_L it is run with zero profit because of limited liability. In the end its expected profit is positive, which is larger than its peers who get zero profit because of perfect competition. Anticipating this, the banks would never coordinate to set $\alpha^* = \alpha(p_L)$.

and neglect the cost in the low state. On the contrary, when it is very likely that the low state realizes, then the cost of bank run is too high relative to the high return in the high state. Therefore the best strategy for the banks is to stick to the safest strategy and avoid the high cost in the low state. The interesting outcome takes place for intermediate π such that the cost of bank run is also intermediate and return from liquidity free-riding is sufficiently high in the high state:

Proposition 1.4.2 *When $\frac{\gamma\mathbb{E}[R_L]-c}{\gamma R_2-c} < \pi < \frac{\gamma\mathbb{E}[R_L]-c}{\gamma\mathbb{E}[R_H]-c}$, there exists no symmetric equilibrium of pure strategies. What's more, given $p_H R_2 < R_1$ and c not too high ($c < 1$) there exists a unique equilibrium of mixed strategies such that for a representative bank*

1. *With probability θ the bank chooses to be risky — it sets $\alpha_r^* = 0$, and with probability $1-\theta$ to be safe — it sets $\alpha_s^* > 0$;*
2. *Interest rates at states p_H and p_L are $r_H > r_L > 1$;*
3. *At $t = 0$ a risky bank offers a deposit contract with $d_0^r = \gamma\left[p_H R_2 + \frac{(1-p_H)R_2}{r_H}\right]$ and a safe bank with $d_0^s = \gamma\left[\alpha_s^* R_1 + (1-\alpha_s^*)\,p_H R_2 + \frac{(1-\alpha_s^*)(1-p_H)R_2}{r_H}\right]$;*
4. *Equal return condition: $\kappa_r = \pi d_0^r + (1-\pi)c = d_0^s = \kappa_s$;*
5. *Market clearing conditions:*
   (a) *At p_H: $\theta D_r + (1-\theta)D_s = \theta S_r + (1-\theta)S_s$, in which*
   $$\begin{cases} D_r = d_0^r - \gamma p_H R_2, \\ D_s = d_0^s - \gamma\left[\alpha_s^* R_1 + (1-\alpha_s^*)\,p_H R_2\right], \\ S_r = (1-\gamma)p_H R_2, \\ S_s = (1-\gamma)\left[\alpha_s^* R_1 + (1-\alpha_s^*)\,p_H R_2\right]; \end{cases}$$
   (b) *At p_L: $r_L(1-\gamma)\left[\alpha_s^* R_1 + (1-\alpha_s^*)\,p_L R_2\right] = \gamma(1-\alpha_s^*)(1-p_L)R_2$, i.e. $\alpha_s^* = \alpha^*(p_L, r_L)$.*

Proof See Appendix A.1.4. ∎

Though complicated, the intuition behind is still not difficult to see (To help the reader see the insight a numerical example is provided in Appendix A.2). Suppose that we increase π from 0 where all the banks set $\alpha_i = \alpha(p_L)$. When π just gets higher than $\overline{\pi}_1$ free-riding on liquidity provision becomes profitable because

1. The cost of bank run is no longer too high;

2. At p_H the early entrepreneurs have excess liquidity supply. Therefore, an arbitrary bank i can free-ride and set its $\alpha_i' = 0$. By doing so it can trade liquidity at $t = 1$ from early entrepreneurs with high return from its late projects and promise $d_0' = \gamma R_2 > \gamma \mathbb{E}[R_L]$ to the investors. The higher return in state p_H compensates the fact that it is surely run at p_L due to liquidity shortage.

But if every bank would behave as a free-rider, there would not be sufficient liquidity supply. So free-riders and prudent banks must co-exist, i.e. the equilibrium is of mixed strategies.

The free-riding behavior results in two consequences:

1. As more banks become free-riders, the interest rate r_H is bid higher;

2. The prudent banks set lower $\alpha_s^* < \alpha(p_L)$ in order to cut down the opportunity cost of investing in safe projects.

And in the end, r_H and α_s^* are adjusted such that depositors are indifferent between the two types of banks.

On the aggregate level the probability of being a free-rider is determined by market clearing conditions for both states.

The resulting inefficiency is captured by the following corollary:

Corollary 1.4.3 *For the equilibrium of mixed strategies defined by* Proposition *1.4.2, the banks are worse off than the case if they coordinate and choose* $\alpha_i = \alpha(p_L)$ *and* $d_0^* = \gamma\left[\alpha(p_L)R_1 + (1 - \alpha(p_L))R_2\right] = \alpha(p_L)R_1 + (1 - \alpha(p_L))pR_2$.

Proof The banks return is equal to $d_0^s = \kappa\left(\alpha^*\left(p_L, r_L\right), d_0^s\right) < \kappa\left(\alpha^*, d_0^*\right)$, in which $\alpha^* = \alpha(p_L)$ and $d_0^* = \gamma\left[\alpha(p_L)R_1 + (1-\alpha(p_L))R_2\right] = \alpha(p_L)R_1 + (1-\alpha(p_L))pR_2$, by Lemma 1.3.2, given the fact that $r_L > 1$. ∎

Fig. 1.3 illustrates the expected payoff for investors as characterized in Proposition 1.4.1 and 1.4.2. When π (the probability that a large share of type 2 projects will be realized early) is very high, banks prefer to exploit the higher profitability of these projects rather than to self insure against the risk of a bank run. As long as the probability of a bank run is small enough (less than $1 - \overline{\pi}_2$), the risky strategy gives investors a higher expected return (Proposition 1.4.1 a).

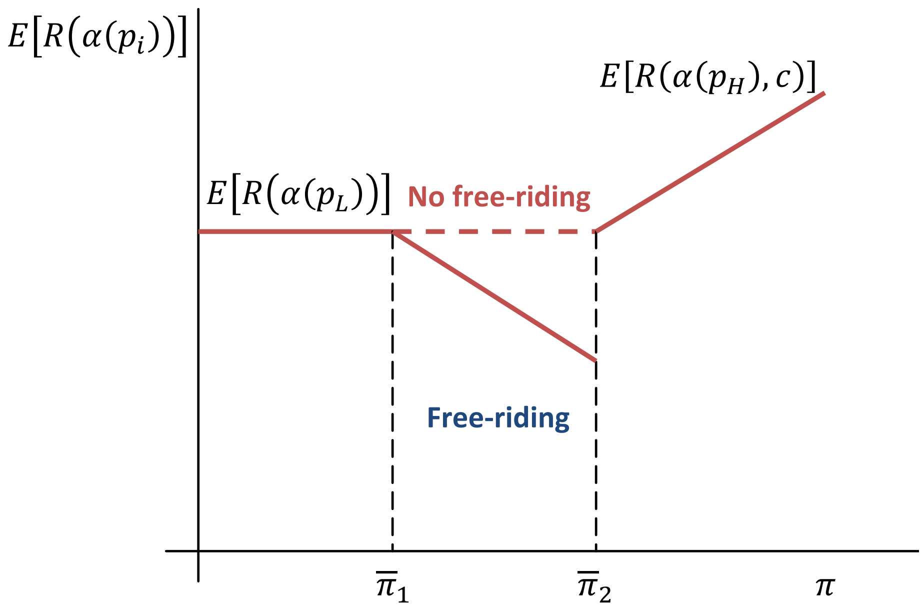

Fig. 1.3 Expected payoff for investors as a function of π (the probability that the share of early type 2 projects is high)

In contrast, when it is very likely that the bad state (with a low share of early type 2 projects) occurs, banks prefer to play safe by investing sufficiently in type 1 projects so as to make sure to be never run (Proposition 1.4.1 b). Whenever $\pi < \overline{\pi}_2$, such a strategy would give investors the highest expected payoff, even though the safe strategy results in excess liquidity if the good state turns out to be realized. But as shown in PROPOSITION 1.4.2, for π high enough there is an incentive of naughty banks to free-ride on the excess supply of liquidity available in the good state. In the range $\overline{\pi}_1 < \pi < \overline{\pi}_2$, a mixed strategy equilibrium prevails with inferior return to investors, free-riding down the expected payoff for investors below the level feasible in the absence of free-riding in this range.

1.5 CENTRAL BANK INTERVENTION

Let us now consider the role of monetary policy. Suppose that central bank is now the fourth player in the game. We make some slight changes to the original game in the following way

1. At $t = 0$ the banks provide *nominal* deposit contracts to investors, promising a fixed nominal payment d_0 in the future. The central bank announces a minimum level $\underline{\alpha}$ of investment on safe projects required to be eligible for liquidity support in times of a crisis;
2. At $t = \frac{1}{2}$ the banks decide whether to borrow liquidity from central bank. If yes, the central bank commits to provide liquidity for banks provided they fulfil the requirement $\underline{\alpha}$;
3. At $t = 1$ the central bank supports those banks having fulfilled the requirement defined in (2) by injecting money at the low borrowing rate $r^{CB} = 1$ if asked for.

For simplicity we assume that one unit of money is of equal value to one unit real good in payment. And the price level is determined by *cash-in-the-market principle* (Allen & Gale, 2005), i.e. the ratio of amount of liquidity (the sum of money and real goods) in the market to amount of real goods.

Central bank intervention may help to prevent inefficient liquidation in the case of aggregate shocks for high values of π (the probability of a high share of early projects p_H being high enough). This intervention reduces the critical threshold $\overline{\pi}_2$ to the left (to $\overline{\pi}_2'$) and so expands the range of parameter values for which it is optimal to choose the risky strategy $\alpha^* = \alpha(p_H)$. To avoid incentives for free-riding, however, central bank intervention has to be made contingent on banks having investing a minimum level $\underline{\alpha}$ in safe projects at stage 0.

By injecting liquidity in case of a crisis, the central bank can prevent inefficient liquidation of early projects via bank runs, raising expected returns of those banks choosing a risky strategy $\alpha^* = \alpha(p_H)$ when p_L is realized. So let us consider the case of aggregate shocks when π is high and the central bank sets $\underline{\alpha} = \alpha(p_H)$. In this case banks will set $\alpha^* = \alpha(p_H)$ and borrow liquidity from central bank only at p_L. Given this the investors will no longer run at p_L because they can only get c real goods plus $d_0 - c$ money for each unit of deposit. Instead if they wait till $t = 1$, they will get $\kappa\left[R_H|p_L\right] = \alpha(p_H)R_1 + (1-\alpha(p_H))p_L R_2$ real goods plus $d_0 - \kappa\left[R_H|p_L\right] = (1-\alpha(p_H))(p_H - p_L)R_2$ money, and they are better off by waiting.

Now the lower bound for $\alpha^* = \alpha(p_H)$ being the dominant strategy is shifted towards:

$$\overline{\pi}_2' = \frac{\gamma\mathbb{E}[R_L] - \kappa\left[R_H|p_L\right]}{\gamma\mathbb{E}[R_H] - \kappa\left[R_H|p_L\right]} < \frac{\gamma\mathbb{E}[R_L] - c}{\gamma\mathbb{E}[R_H] - c} = \overline{\pi}_2.$$

So free-riding is partially deterred and the investors are better off with higher return,

$$\pi\gamma\mathbb{E}[R_H] + (1-\pi)\kappa\left[R_H|p_L\right] > \pi\gamma\mathbb{E}[R_H] + (1-\pi)c.$$

For high enough π ($\pi > \overline{\pi}_2'$), injection of money before $t = 1$ can help to improve the allocation. Since it prevents costly runs, obviously, banks relying on central intervention will invest more in illiquid late projects. So the range of parameter values for which it is optimal to choose the risky strategy $\alpha^* = \alpha(p_H)$ is expanded. Nevertheless, liquidity provision improves the allocation in an incomplete market economy, provided central bank intervention is made contingent on banks having investing a minimum level $\underline{\alpha}$ in safe projects at stage 0. The central bank's goal is consistent with the banks' strategies since $\underline{\alpha} = \alpha(p_H)$.

Suppose now that π is low enough ($0 \le \pi \le \overline{\pi}_1$, with $\overline{\pi}_1$ being the same as that in Proposition 1.4.1). The equilibrium is very similar to Proposition 1.4.1 b. The central bank can simply announce $\underline{\alpha} = \alpha(p_L)$. The banks would coordinate to meet this requirement, since the cost of free-riding is too high (anyone who sets a lower α would not be bailed out by the central bank and is run at p_L).

When π is intermediate ($\overline{\pi}_1 < \pi < \overline{\pi}'_2$) the equilibrium is again of mixed strategies, as in Proposition 1.4.2. The difference is that the prudent banks now have an outside option to obtain cheap liquidity from the central bank when the market rate is bid up. Given that the central bank announces $\underline{\alpha} = \alpha(p_L)$ and a prudent bank i sets $\alpha_i = \underline{\alpha}$, when at p_H the market rate is bid up by naughty banks, the prudent bank is able to obtain liquidity from the central bank instead of buying expensive liquidity from early entrepreneurs. In contrast, the naughty banks have to obtain liquidity at the higher market rate $r^M >> 1$ from early entrepreneurs. Naughty banks will be run at p_L. In the end, the expected nominal returns from both types of banks have to be equal in equilibrium.

The targeted injection is designed so as not to save the naughty bank: due to the competitive banking service, the prudent banks will be forced to transfer all injected liquidity to their investors. So in the bad state, the naughty banks cannot obtain liquidity via the inter-bank market. Consequently, the prudent banks can meet their nominal deposit contract with cheap liquidity provided by the central banks, whereas the naughty banks would be punished struggling in vain to get liquidity from the market at a higher rate. Such a policy might work, provided the central bank has perfect knowledge about the type of banks. But if there is the slightest doubt whether a bank is really prudent or not, such a scheme runs the risk to fail.

Surely the central bank's intervention improves allocation when π is high, which seems to make the intervention justified. However, the welfare improvement for intermediate π is limited, or at least ambiguous, in comparison to the laissez-faire equilibrium as stated in Proposition 1.4.2. Remember that what makes free-riding attractive there is the abundant liquidity supply at p_H. Here the prudent banks simply ask the central bank for liquidity, and all their early entrepreneurs have to go to the market seeking for buyers. This makes the market more abundant in liquidity at p_H, which makes free-riding more attractive. It lures more banks to be naughty. In the end, in comparison to the laissez-faire mixed strategy equilibrium, the share of naughty banks may increase — implying that there is less investment in safe project, hence less aggregate real return in $t = 1$ and more paper money — making the investors' welfare inferior.

On the other hand, the prudent banks set $\alpha^P = \underline{\alpha} = \alpha(p_L)$ as required by the central bank. This is higher than the α_S^* in the laissez-faire equilibrium. So the real early return from an individual prudent bank is higher. However, as just argued, the share of prudent banks is reduced. Thus, the aggregate level of real return to investors is ambiguous, likely to be lower.

Fig. 1.4 illustrates the effects of targeted central bank liquidity provision on investors expected payoff. In the expanded range $\pi > \overline{\pi}_2'$ for which the risky strategy is dominant, liquidity injection prevents bank runs with inefficient liquidation of early projects and thus raises expected returns to investors. In contrast, in the intermediate range with mixed strategy equilibria $\overline{\pi}_1 < \pi < \overline{\pi}_2'$, the payoff of laissez-faire equilibrium is likely to be lower as compared to the outcome in the absence of liquidity provision by the central bank.

In reality, however, things are likely to be even much worse because of a serious time inconsistency problem. It makes free-riding even more attractive in the case of central bank intervention: since banks face a pure illiquidity problem (all projects are known to be realized at some stage), illiquid banks can always credibly promise to pay back later. Therefore, *ex post* it is always welfare improving for the central bank to support the naughty banks, avoiding costly bank runs. Obviously, anticipating this behavior *ex ante* increases incentives for free-riding: naughty banks, having invested all their

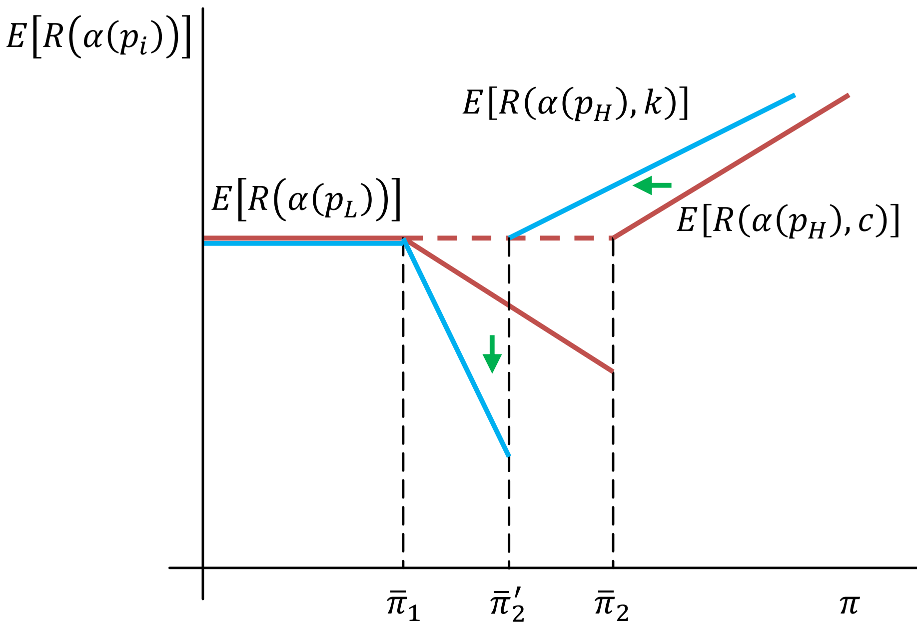

Fig. 1.4 Expected payoff for investors under targeted central bank liquidity provision

funds in the risky projects ($\alpha = 0$), can always afford to pay early investors a higher rate of return as long as central bank intervention helps to prevent bankruptcy. The problem is that naughty (free-riding) banks have a higher average return than prudent banks, provided that they will be bailed out by central bank intervention. Because the naughty banks are absolutely better off than prudent banks when central bank money is provided, the incentive to free-ride will be aggravated.

In formal terms, the time inconsistency problem turns liquidity provision (as defined at the beginning of this section) into liquidity flooding: the central bank just floods the market with liquidity via open market operation to keep the market rate at $r^M = 1$. This seems to be a fair description of the strategy central banks usually follow in times of crises. It may, however, have disastrous effects. The central bank is flooding the market for the following reasons:

1. A central bank has limited instruments for implementation. Rather than provide liquidity to specific targeted types of banks, open market operation is the central bank's most effective (and simplest) device. It acts in its good faith that the city of Sodom should be spared from the destruction if a few righteous are found within (Genesis, 18:26);

2. When crisis hits, the naughty banks are those crying first. If the central bank gives in to their pressures too early, most of the liquidity injected is likely to be directed towards the naughty banks instead of the prudent ones.

In the end, the market (which is only needed for the naughty banks at this time) is flooded by liquidity as a result.

In the end, liquidity flooding will crowd out all the prudent banks in equilibrium, as we prove now in Proposition 1.5.1 for a special case.

Proposition 1.5.1 *Assume that* $\pi p_H R_2 + (1-\pi)c \geq 1$ *and that for* $\overline{\pi}_1 < \pi < \overline{\pi}_2$, $d_0^j = \gamma R_2 > \pi p_H R_2 + (1-\pi)c$. *If the central bank is willing to provide liquidity to the entire market in times of crisis, all banks have an incentive to play naughty, choosing* $\alpha_j = 0$.

Proof Suppose that a representative bank chooses to be prudent with $\alpha_i = \underline{\alpha}$, and promises a nominal deposit contract $d_0^i = \gamma\left[\underline{\alpha}R_1 + (1-\underline{\alpha})R_2\right]$ in order to maximize its investors return. Then when the bad state with high liquidity needs is realized, the central bank has to inject enough liquidity into the market to keep interest rate at $r = 1$ in order to ensure bank i's survival. However, given $r = 1$, a naughty bank j can always profit from setting $\alpha_j = 0$, promising the nominal return $d_0^j = \gamma R_2 > d_0^i$ to its investors. Thus, surely the banks prefer to play naughty. For other parameter values, there may not exist any equilibrium at all with liquidity injection, suggesting that liquidity provision makes the world more vulnerable, driving banks to corner solutions (see Appendix A.1.5). ∎

Proposition 1.5.1 shows that providing market liquidity can be quite dangerous. Abraham argued in Genesis, God should save the entire city because a few good men are living in it. The problem with this advice is that such a rescue simply makes the naughty men (banks) better off without suffering the punishment (the bank runs) they deserve. So in order not to encourage even more free-riding, the central bank should commit to abstain from bailing out naughty banks. It should stick firmly to its commitment as credible ”lender to quality” instead of playing ”lender of last resort”. Obviously, such a commitment is not really credible during a crisis: once the bad state has been realized, liquidity injecting can prevent investors from running the banks, so *ex post* it will always be welfare improving. The efficient solution (targeting only prudent banks) is dynamically inconsistent.

These results show that liquidity injection is a delicate issue possibly creating severe moral hazard problems. It casts serious doubt on the desirability of central bank intervention. This argument seems to be very robust. It would be straightforward to introduce vulture funds in the model trying to buyout some of the bankrupt naughty banks in the bad state, financed by liquidity provision of early entrepreneurs. These vulture funds could at least partly mitigate the social costs involved with bank runs. We plan to do this in a future extension. Obviously, public liquidity provision will prevent the market price of failed banks from falling sufficiently to be profitable for vulture funds.

The current setup models pure illiquidity risk. With asymmetric information about insolvency risk, intuition suggests that the moral hazard problem is likely to become even worse. It is left for Chapter 3 to find out whether this notion is indeed true.

As is often the case with economic models, some policy conclusions are not clear cut: assume the central bank would be really able to strictly commit to targeted liquidity provision. If that is the case, liquidity injection could definitely be welfare improving for some range of parameter values (for very high π); for lower values, however, it turns out to have ambiguous effects. Which case is more relevant? The sets with different ranges of local equilibria are the result of the discrete probability space.

A natural extension would be to extend the set up to a continuous probability distribution for p. Our intuition is that the generic outcome for the continuous case is captured by the mixed strategy equilibrium for the following reason: the set-up is characterized by serious non-convexities which are likely to result in mixed strategy equilibria even for continuous state space. We plan to analyze this in future research. In any (or rather in the realistic) case, if commitment is not feasible, liquidity provision is haunted by moral hazard issues with disastrous impact.

Furthermore, liquidity injection may also impede the role of money as a medium to facilitate ordinary transactions. This question is left for future work (see Sauer, 2007 for a first analysis of the trade-off between financial stability and price stability).

1.6 CONCLUSION

This chapter analyzes the interaction between risk taking in the financial sector and central bank policy in an economy with pure illiquidity risk. We extend the model of Diamond & Rajan (2006) to capture the feedback from liquidity provision to risk taking incentives of banks. We show that liquidity provision encourages higher risk taking: insuring against aggregate risks results in a higher share of less liquid projects funded.

It turns out that the impact on welfare is ambiguous: assume first the central bank is able to strictly commit to targeted liquidity provision. For some parameter values, liquidity provision turns out to be welfare improving, allowing banks to take more socially valuable risks. But we show that liquidity provision has ambiguous effects for other parameter values. More seriously: central banks need to be able to commit to abstain from providing liquidity via open market operations in order discourage free-riding. Such a commitment, however, is not credible. In the absence of commitment, provision of public liquidity may have disastrous effects. It increases the incentive of financial intermediaries to free-ride on liquidity in good states, resulting in excessively low liquidity in bad states. There is a serious dynamic consistency problem.

The surprising result is that — contrary to prevailing intuition — the moral hazard problem is inherent even in an economy with pure illiquidity risk. Of course in reality, unlike in models, there is no clear cut distinction between illiquidity and insolvency risk. It should be fairly straightforward to make the model more realistic and introduce asymmetric information about solvency of the financial intermediaries. A promising route might be to follow Brunnermeier & Pederson (2009). With private information about solvency risk the moral hazard problem is likely to become more serious. CHAPTER 3 will explore this issue based on an extended version of current framework.

In the model presented, the optimal policy response depends to some extent on specific parameter values (the probability of the bad state occurring). This is an artifact of the discrete probability space. Our conjecture is that the generic outcome in continuous state space is the mixed strategy equilibrium with commitment to no intervention as optimal solution. Again, we leave this to future research.

How should the dynamic consistency problem be solved? Do we really suggest not to intervene during an acute crisis? Following the "Austrian hangover theory" some argue that creating a recession might be necessary to purge the excesses of previous booms, leaving the economy in a healthier state. The "winds of creative destruction" would cause healthy pain. We don't think this is a sensible solution to the problem. Bad investments in the past should not require the unemployment of good workers in the present. Rather, we think the incentive problem needs to be addressed in other ways — by stronger regulation or alternative instruments. Just as in standard dynamic consistency problems, the right approach is to tackle the externalities directly. Currently, central banks are caught in a trap reminding of a Greek tragedy. It was a humiliating experience to see the credibility of the Bank of England being smashed by a Northern rock engaged in reckless lending.

The key challenge, of course, is the question what instruments should be used in order to address the underlying externalities. The current set-up provides some foundation to analyze this question: it is flexible enough to incorporate equity and the role of capital requirements. We will do this in the next chapters.

Some may argue that a banking model cannot address realities of a modern economy with highly securitized markets. In our view, this is a misunderstanding: following Diamond & Rajan (2006), we analyzed the impact of liquidity injection in a sound model based on an explicit optimal contract for the underlying commitment problem which turns out to be a fragile banking system. As impressively demonstrated by Northern Rock, a run on markets with the risk of fire sales can be at least as devastating as a run on traditional banks, whenever there are leveraged institutions borrowing short and lending long. We have, however, serious doubts that the securitization arrangements in the US subprime markets have been based on an optimal principal agent contract addressing the inherent incentive problems in an adequate way. We are still waiting for an optimal contract model of securitization and are happy to analyze the impact of liquidity provision again in such a model whenever it will be available.

Appendix

A.1 PROOFS

A.1.1 Proof of Lemma 1.3.1

The proof is done by the following steps:

Claim 1 *Any non-idle equilibrium must be symmetric.*

Since the banks are competitive, therefore in equilibrium no bank is able to make strictly positive profit. Without restriction there exists a kind of equilibria in which some banks stay idle with zero profit by taking inferior strategies and getting no deposit at all. To make the results interesting, we exclude such equilibria throughout the chapter.

As a direct conclusion, a representative bank i being active must achieve the same expected return $d_{0i} = \gamma\left[\alpha_i R_1 + (1-\alpha_i)pR_2\right] + \frac{\gamma(1-\alpha_i)(1-p)R_2}{r}$ (otherwise any bank j with d_{0j} smaller than the others will lose all its business). Given equilibrium outcome r being equal for all banks (since r is determined by aggregate liquidity demand and supply), all of them should take the same (α_i, d_{0i}) (so far we don't require α_i be pure strategy).

Claim 2 *Any non-idle symmetric equilibrium of pure strategy takes the form stated in* Lemma *1.3.1.*

Since the equilibrium is symmetric, we can simply denote the equilibrium strategy profile by $\left(\alpha^*, d_0^*\right)$ for all the banks. Market clearing condition requires that the market interest rate is determined by the aggregate liquidity supply and demand, i.e.

$$rN(1-\gamma)\left[\alpha^* R_1 + (1-\alpha^*)pR_2\right] = N\gamma(1-\alpha^*)(1-p)R_2,$$

and the equilibrium interest rate r^* is thus given by

$$r^* = \frac{\gamma(1-\alpha^*)(1-p)R_2}{(1-\gamma)\left[\alpha^* R_1 + (1-\alpha^*)pR_2\right]}.$$

Express α^* as a function of r^*,

$$\alpha^*(p, r^*) = \frac{\gamma\frac{1-p}{r^*} - (1-\gamma)p}{\gamma\frac{1-p}{r^*} + (1-\gamma)\left(\frac{R_1}{R_2} - p\right)}.$$

A bank manager's expected return under r^* is

$$\kappa^* = \gamma\left[\alpha^* R_1 + (1-\alpha^*)pR_2\right] + \frac{\gamma(1-\alpha^*)(1-p)R_2}{r^*} = \alpha^* R_1 + (1-\alpha^*)pR_2,$$

which is the upper bound of d_0^*. Since Bertrand competition allows zero profit for the bank managers,

$$d_0^* = \kappa^* = \alpha^* R_1 + (1-\alpha^*)pR_2.$$

Now it's clear that r^* and d_0^* are uniquely determined for any given α^*, and α^* may take its value in $[0, \overline{\alpha}]$ (such that $r(0, \overline{\alpha}) \leq \frac{R_2}{R_1}$). So we guess that $\forall \alpha^* \in [0, \overline{\alpha}]$, with $d_0^* = \alpha^* R_1 + (1-\alpha^*) p R_2$ and $r^* = \frac{\gamma(1-\alpha^*)(1-p)R_2}{(1-\gamma)\left[\alpha^* R_1 + (1-\alpha^*)pR_2\right]}$, such $\left(\alpha^*, d_0^*\right)$ is an equilibrium strategic profile.

To see that such strategy profile is indeed in equilibrium, suppose that bank i deviates by setting $\left(\alpha_i'(p, r'), d_0'\right) \neq \left(\alpha^*(p, r^*), d_0^*\right)$. Then, should both the deviator and the rest get any deposit from the investors,

1. If $\alpha_i'(p, r') < \alpha^*(p, r^*)$, by market clearing condition

$$\begin{aligned} & r' \left\{(1-\gamma)\left[\alpha_i R_1 + (1-\alpha_i)pR_2\right] + (N-1)(1-\gamma)\left[\alpha^* R_1 + (1-\alpha^*)pR_2\right]\right\} \\ = \; & \left[\gamma(1-\alpha_i)(1-p)R_2 + (N-1)\gamma(1-\alpha^*)(1-p)R_2\right], \end{aligned}$$

   it's clear that $r' > r^*$, given $\gamma > p$, i.e. the deviator bids up the market interest rate at $t = \frac{1}{2}$ by investing less on safe assets. For the non-deviators, the expected return now becomes

$$\begin{aligned} \kappa_{-i} &= \gamma\left[\alpha^* R_1 + (1-\alpha^*)pR_2\right] + \frac{\gamma(1-\alpha^*)(1-p)R_2}{r'} \\ &< \gamma\left[\alpha^* R_1 + (1-\alpha^*)pR_2\right] + \frac{\gamma(1-\alpha^*)(1-p)R_2}{r^*} \\ &= d_0^*, \end{aligned}$$

   i.e. they'll not be able to meet d_0^* and will be run by investors at $t = \frac{1}{2}$. In the end, the deviator can only get liquidity from its own entrepreneurs at $t = 1$, and at $t = 0$ set d_0' at most as

$$d_0' = \alpha' R_1 + (1-\alpha')pR_2 < \alpha^* R_1 + (1-\alpha^*)pR_2 = d_0^*$$

   — but this implies that the deviator is never able to get any business at $t = 0$ (because all the investors would choose to deposit at the non-deviators, ensuring the return d_0^*) and the deviation is not optimal;

2. If $\alpha_i'(p,r') > \alpha^*(p,r^*)$, given $r \leq \frac{R_2}{R_1}$, it's clear that $r' < r^*$ with the non-deviators' return becoming

$$\begin{aligned}\kappa_{-i} &= \gamma\left[\alpha^* R_1 + (1-\alpha^*)pR_2\right] + \frac{\gamma(1-\alpha^*)(1-p)R_2}{r'} \\ &> \gamma\left[\alpha^* R_1 + (1-\alpha^*)pR_2\right] + \frac{\gamma(1-\alpha^*)(1-p)R_2}{r^*} \\ &= d_0^*,\end{aligned}$$

and they'll all survive at $t = \frac{1}{2}$. However, the deviator's own expected return

$$\begin{aligned}\kappa_i &= \gamma\left[\alpha_i' R_1 + (1-\alpha_i')pR_2\right] + \frac{\gamma(1-\alpha_i')(1-p)R_2}{r'} \\ &< \gamma\left[\alpha^* R_1 + (1-\alpha^*)pR_2\right] + \frac{\gamma(1-\alpha^*)(1-p)R_2}{r'} \\ &= \kappa_{-i}\end{aligned}$$

implying that such deviation is not profitable. To see that "$<$" in the second line holds, note that κ is linear in α, so if any $\alpha_i'(p,r') > \alpha^*(p,r^*)$ achieves higher κ_i, it must be $\alpha_i' = 1$. That is, the highest expected return for such deviator must be $\kappa_i\left(\alpha_i' = 1\right) = \gamma R_1$.

On the other hand, the worst expected return for non-deviators is achieved under $r' = \frac{R_2}{R_1}$, and such worse case corresponds to

$$\begin{aligned}\underline{\kappa}_{-i} &= \gamma\left[\alpha^* R_1 + (1-\alpha^*)pR_2\right] + \frac{\gamma(1-\alpha^*)(1-p)R_2}{r'} \\ &= \gamma\left[\alpha^* R_1 + (1-\alpha^*)pR_2\right] + \gamma(1-\alpha^*)(1-p)R_1.\end{aligned}$$

If we can show $\underline{\kappa}_{-i} \geq \kappa_i\left(\alpha_i' = 1\right)$, then the deviating strategy is dominated in any case. This is to show

$$\begin{aligned}\gamma\left[\alpha^* R_1 + (1-\alpha^*)pR_2\right] + \gamma(1-\alpha^*)(1-p)R_1 &\geq \gamma R_1, \\ \gamma pR_2 - \gamma\alpha^* pR_2 - \gamma pR_1 + \gamma\alpha^* pR_1 &\geq 0, \\ (R_2 - R_1)(1-\alpha^*) &\geq 0,\end{aligned}$$

and the last step holds for sure, meaning that such deviation is indeed inferior.

Therefore no unilateral deviation is profitable.

Claim 3 *There exists no equilibrium of mixed strategies.*

First, notice that even under mixed strategies the expected return d_0 should be the same across all the banks because of Bertrand competition, given that p is deterministic. Therefore we can simply concentrate on mixed strategies with respect to α. Suppose that there exists an equilibrium of mixed strategies in which a representative bank i takes a mixed strategy σ_i with $\#\mathrm{supp}\sigma_i \geq 2$. Take two arbitrary elements $\alpha_i^1, \alpha_i^2 \in \mathrm{supp}\sigma_i$ and $\alpha_i^1 \neq \alpha_i^2$, given σ_{-i} and equilibrium outcome r the following equation must hold

$$\kappa_i\left(\alpha_i^1, \sigma_{-i}\right) = \kappa_i\left(\alpha_i^2, \sigma_{-i}\right)$$

meaning that $\alpha_i^1 = \alpha_i^2$. A contradiction. ∎

A.1.2 Proof of Lemma 1.3.2

Since $\frac{\partial \kappa_i}{\partial r} = \frac{\partial \kappa_i}{\partial \alpha_i(p,r)} \frac{\partial \alpha_i(p,r)}{\partial r} < 0$ and $r \geq 1$, so $r^* = 1$ maximizes κ_i. Also in symmetric case r^* is directly determined by α^*, and d_0^* depends on r^* and α^*, the problem then boils down to the bank managers' decision on α^* that maximizes their expected return via getting liquidity at the lowest price, i.e. $r^* = 1$.

Suppose now bank i sets $\alpha_i' \neq \alpha^*(p, r^*)$, then the liquidity it can borrow from early entrepreneurs is given by

$$\frac{\gamma\left(1 - \alpha_i'\right)(1-p)R_2}{r}$$

because of the resource constraint. Then

1. For $\alpha_i' > \alpha^*(p, r^*)$, $r = r^* = 1$,

$$\kappa_i\left(\alpha_i', \alpha_{-i}(p, r^*)\right) = \gamma\left[\alpha_i' R_1 + (1-\alpha_i')R_2\right]$$

$$\begin{aligned} &< \gamma\left[\alpha^*(p,r^*)R_1 + (1-\alpha^*(p,r^*))R_2\right] \\ &= \kappa_i(\alpha^*(p,r^*)); \end{aligned}$$

2. For $\alpha_i' < \alpha^*(p,r^*)$, r is bid up so that all the other banks are run,,

$$\begin{aligned} \kappa_i\left(\alpha_i', \alpha_{-i}(p,r^*)\right) &= \alpha_i' R_1 + (1-\alpha_i')pR_2 \\ &< \alpha_{-i}(p,r^*)R_1 + (1-\alpha_{-i}(p,r^*))pR_2 \\ &= \kappa_i(\alpha^*(p,r^*)). \end{aligned}$$

So there doesn't exist any $\alpha_i'(p,r^*) \neq \alpha^*(p,r^*)$ such that $\kappa_i(\alpha^*(p,r^*)) < \kappa_i\left(\alpha_i'(p,r^*), \alpha_{-i}(p,r^*)\right)$.
∎

A.1.3 Proof of Proposition 1.4.1

By Lemma 1.3.2 $\alpha(p_H)$ and $\alpha(p_L)$ maximize the banks' expected return at p_H and p_L respectively. The banks' expected return at p_H is higher than that at p_L because

$$\begin{aligned} \kappa(\alpha(p_H), p_H) &= \gamma\left[\alpha(p_H)R_1 + (1-\alpha(p_H))R_2\right] = \gamma\mathbb{E}[R_H] \\ &> \kappa(\alpha(p_L), p_L) = \gamma\mathbb{E}[R_L]. \end{aligned}$$

However banks with $\alpha(p_H)$ are run at p_L and only get return of c, because

$$\begin{aligned} \kappa(\alpha(p_H), p_L) &= \alpha(p_H)R_1 + (1-\alpha(p_H))p_L R_2 \\ &< \alpha(p_H)R_1 + (1-\alpha(p_H))p_H R_2 = \kappa(\alpha(p_H), p_H). \end{aligned}$$

So the banks prefer $\alpha(p_H)$ to $\alpha(p_L)$ only if $\gamma\mathbb{E}[R_H]\pi + (1-\pi)c > \gamma\mathbb{E}[R_L]$, solve to get

$$\pi > \frac{\gamma\mathbb{E}[R_L] - c}{\gamma\mathbb{E}[R_H] - c} = \overline{\pi}_2.$$

When $\pi = 0$ the problem degenerates to deterministic case, so $\alpha^* = \alpha(p_L)$ is still the unique optimal symmetric pure strategy equilibrium.

When $0 < \pi < \overline{\pi}_2$ any strategic profile α^* in which all banks set $\alpha^* \neq \alpha(p_L)$ cannot be an optimal symmetric pure strategy equilibrium:

1. For $\alpha^* \in (\alpha(p_H), \alpha(p_L))$, the maximum return one bank can obtain at p_L is $\alpha^* R_1 + (1-\alpha^*)p_L R_2 < \alpha(p_L)R_1 + (1-\alpha(p_L))p_L R_2 = \kappa\left(\alpha(p_L)\right)$, and the maximum return one bank can obtain at p_H is $\gamma\left[\alpha^* R_1 + (1-\alpha^*)R_2\right] > \gamma\left[\alpha(p_L)R_1 + (1-\alpha(p_L))R_2\right] = \kappa\left(\alpha(p_L)\right)$. Given this fact, the banks are run at p_L and only get an actual return of $\gamma\left[\alpha^* R_1 + (1-\alpha^*)R_2\right]\pi + (1-\pi)c$, but one can deviate by setting $\alpha_i = \alpha(p_H)$ making a higher expected return;

2. For $\alpha^* \in [0, \alpha(p_H))$ in which the banks are run at p_L (because $\alpha^* R_1 + (1-\alpha^*)p_H R_2 > \alpha^* R_1 + (1-\alpha^*)p_L R_2$), α^* is dominated by the optimal symmetric pure strategy equilibrium $\alpha^* = \alpha(p_H)$ for deterministic p_H;

3. For $\alpha^* = \alpha(p_H)$, by Proposition 1.4.1 α^* is dominated by $\alpha^* = \alpha(p_L)$;

4. For $\alpha^* \in (\alpha(p_L), 1]$ in which the banks survive at both states, α^* is dominated by $\alpha^* = \alpha(p_L)$ because $\gamma\left[\alpha^* R_1 + (1-\alpha^*)R_2\right] < \gamma\left[\alpha(p_L)R_1 + (1-\alpha(p_L))R_2\right]$.

Now suppose that $\pi = \delta > 0$ and the banks still stick to $\alpha^* = \alpha(p_L)$. Then when p_H realizes with probability π, all early entrepreneurs have excess liquidity supply

$$\begin{aligned}
& \underbrace{(1-\gamma)\left[\alpha(p_L)R_1 + (1-\alpha(p_L))p_H R_2\right]}_{(A)} - \underbrace{\gamma(1-\alpha(p_L))(1-p_H)R_2}_{(B)} \\
= \; & (1-\gamma)\left[\alpha(p_L)R_1 + (1-\alpha(p_L))p_L R_2\right] - \gamma(1-\alpha(p_L))(1-p_L)R_2 \\
= \; & 0
\end{aligned}$$

in which term (A) is the entrepreneurs' rent from early projects and term (B) is the deposit from early entrepreneurs in $t = 1$. Knowing this, one bank i can exploit this opportunity by setting $\alpha_i < \alpha(p_L)$ because all its liquidity shortage can be fulfilled by early entrepreneurs' deposit given $r^* = 1$. In this case $\alpha_i = 0$ maximizes its return at p_H, i.e. $\kappa_i\left(0, \alpha_{-i}(p_L)\right) = \gamma R_2 > \gamma \mathbb{E}[R_L] = \kappa_i\left(\alpha_i(p_L)\right)$.

However any deviation $\alpha_i < \alpha(p_L)$ makes bank i run at p_L. Since α_i is observable by its depositors, its expected return for its investors is now

$$\begin{aligned} \gamma R_2 \pi + (1-\pi)c &> \gamma \mathbb{E}[R_L], \\ \pi &> \frac{\gamma \mathbb{E}[R_L] - c}{\gamma R_2 - c}. \end{aligned}$$

Otherwise all the banks would stick to $\alpha^* = \alpha(p_L)$. ∎

A.1.4 Proof of Proposition 1.4.2

The proposition is proved by construction.

Claim 1 *When* $\frac{\gamma \mathbb{E}[R_L]-c}{\gamma R_2 - c} < \pi < \frac{\gamma \mathbb{E}[R_L]-c}{\gamma \mathbb{E}[R_H]-c}$, *there exists no optimal symmetric pure strategy equilibrium.*

Proposition 1.4.1 already shows that for $\frac{\gamma \mathbb{E}[R_L]-c}{\gamma R_2 - c} < \pi < \frac{\gamma \mathbb{E}[R_L]-c}{\gamma \mathbb{E}[R_H]-c}$ there exists no optimal symmetric pure strategy equilibrium because profitable unilateral deviation is always possible.

Claim 2 *If an equilibrium of mixed strategies exist, the equilibrium can only have a two-point support* $\{\alpha_r^*, \alpha_s^*\}$ *such that one bank survives at both states by choosing* α_s^* *and survives at only one state by choosing* α_r^*.

Suppose that α_1 and α_2 ($\alpha_1 \neq \alpha_2$) are two arbitrary elements in the support of the mixed strategies equilibrium, r_H and r_L are the corresponding equilibrium interest rates at p_H and p_L respectively. One bank shall be indifferent between choosing α_1 and α_2.

Suppose that one bank survives at both states by choosing either α_1 and α_2. So its expected return should be the same for both strategies,

$$\begin{aligned} & \gamma \left[\alpha_1 R_1 + (1-\alpha_1) p_H R_2 + \frac{(1-\alpha_1)(1-p_H) R_2}{r_H} \right] \\ = \; & \gamma \left[\alpha_2 R_1 + (1-\alpha_2) p_H R_2 + \frac{(1-\alpha_2)(1-p_H) R_2}{r_H} \right], \end{aligned}$$

i.e. $\alpha_1 = \alpha_2$, a contradiction. Therefore there is at most one strategy by which one bank survives at both states.

Suppose that by choosing either α_1 and α_2 one bank survives at one state but is run in the other, so its expected return should be the same for both strategies:

$$\begin{aligned} & \gamma\left[\alpha_1 R_1 + (1-\alpha_1)\, p_H R_2 + \frac{(1-\alpha_1)(1-p_H)\, R_2}{r_H}\right]\pi + (1-\pi)c \\ = \; & \gamma\left[\alpha_2 R_1 + (1-\alpha_2)\, p_H R_2 + \frac{(1-\alpha_2)(1-p_H)\, R_2}{r_H}\right]\pi + (1-\pi)c, \end{aligned}$$

i.e. $\alpha_1 = \alpha_2$, a contradiction.

Suppose that by choosing α_1 one bank survives at p_H and is run at p_L, and by choosing α_2 one bank survives at p_L and is run at p_H. This implies that

$$\begin{aligned} & \gamma\left[\alpha_1 R_1 + (1-\alpha_1)\, p_H R_2 + \frac{(1-\alpha_1)(1-p_H)\, R_2}{r_H}\right] \\ > \; & \gamma\left[\alpha_1 R_1 + (1-\alpha_1)\, p_L R_2 + \frac{(1-\alpha_1)(1-p_L)\, R_2}{r_L}\right], \end{aligned}$$

i.e. $p_H R_2 + \frac{(1-p_H)R_2}{r_H} > p_L R_2 + \frac{(1-p_L)R_2}{r_L}$, as well as

$$\begin{aligned} & \gamma\left[\alpha_2 R_1 + (1-\alpha_2)\, p_H R_2 + \frac{(1-\alpha_2)(1-p_H)\, R_2}{r_H}\right] \\ < \; & \gamma\left[\alpha_2 R_1 + (1-\alpha_2)\, p_L R_2 + \frac{(1-\alpha_2)(1-p_L)\, R_2}{r_L}\right], \end{aligned}$$

i.e. $p_H R_2 + \frac{(1-p_H)R_2}{r_H} < p_L R_2 + \frac{(1-p_L)R_2}{r_L}$, a contradiction.

Therefore there is at most one strategy by which one bank survives at one state and is run at the other. The equilibrium profile of mixed strategies is supported by $\{\alpha_r^*, \alpha_s^*\}$ such that one bank survives at both states by choosing α_s^* and survives at only one state by choosing α_r^*.

Claim 3 *In such equilibrium, interest rates at states p_H and p_L are $r_H > r_L > 1$.*

By choosing α_s^* one bank should have equal return at both states: $d_0^s = d_0^s(p_H) = d_0^s(p_L)$, i.e.

$$\gamma\left[\alpha_s^* R_1 + (1-\alpha_s^*)\, p_H R_2 + \frac{(1-\alpha_s^*)(1-p_H)\, R_2}{r_H}\right]$$

$$= \gamma\left[\alpha_s^* R_1 + (1-\alpha_s^*)\, p_L R_2 + \frac{(1-\alpha_s^*)\,(1-p_L)\, R_2}{r_L}\right].$$

With some simple algebra this is equivalent to

$$\frac{1}{r_H} = \frac{1-p_L}{1-p_H}\frac{1}{r_L} - \frac{p_H - p_L}{1-p_H}.$$

Plot $\frac{1}{r_H}$ as a function of $\frac{1}{r_L}$, as in Fig. A.1

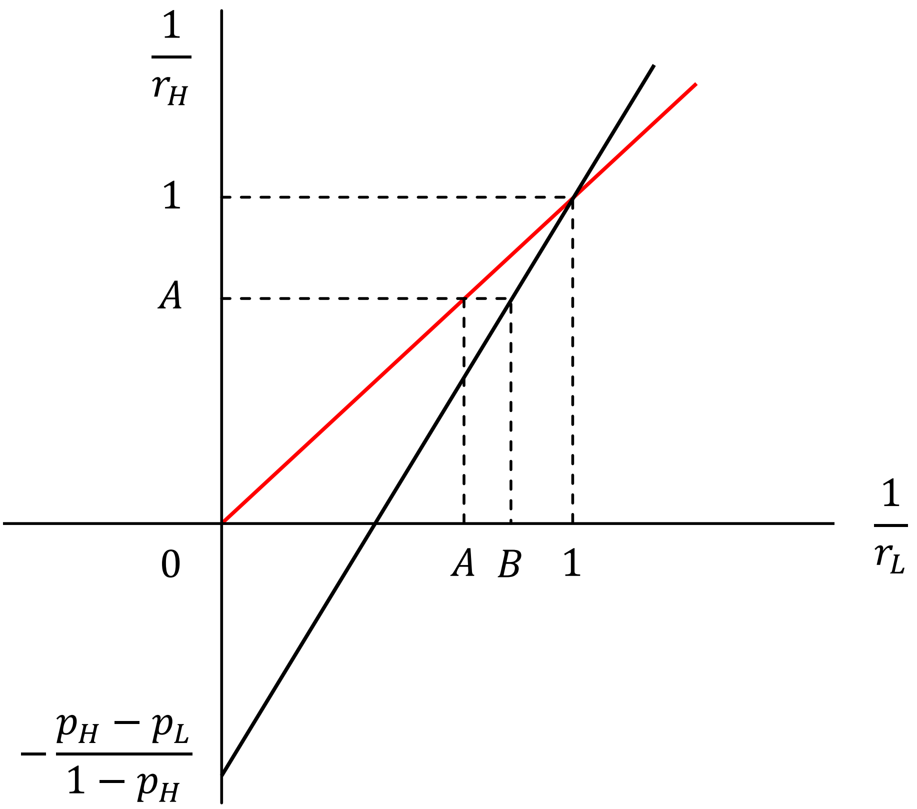

Fig. A.1 $\frac{1}{r_H}$ as a function of $\frac{1}{r_L}$

The slope $\frac{1-p_L}{1-p_H} > 1$ and intercept $-\frac{p_H-p_L}{1-p_H} < 0$, and the line goes through $(1,1)$. But $r_H = r_L = 1$ cannot be equilibrium outcome here, because $\alpha(p_L)$ is dominant strategy in this case and subject to deviation. So whenever

$r_H > 1$ (suppose $\frac{1}{r_H} = A$ in the graph), there must be $r_H > r_L > 1$ (because $\frac{1}{r_H} < \frac{1}{r_L} = B < 1$).

Claim 4 *In such equilibrium, risky banks set $\alpha_r^* = 0$ and safe banks $\alpha_s^* > 0$. Risky banks promise $d_0^r = \gamma \left[p_H R_2 + \frac{(1-p_H)R_2}{r_H} \right]$ and are run at p_L; safe banks survive at both states by promising $d_0^s = \gamma \left[\alpha_s^* R_1 + (1-\alpha_s^*)\, p_k R_2 + \frac{(1-\alpha_s^*)(1-p_k)R_2}{r_k} \right]$ in which $k \in \{H, L\}$. Moreover, $\pi d_0^r + (1-\pi)c = d_0^s$.*

Since $\frac{(1-\alpha_s^*)(1-p_H)R_2}{r_H} < \frac{(1-\alpha_s^*)(1-p_L)R_2}{r_L}$, i.e. the safe banks get less liquidity from their early entrepreneurs at p_H, and also these early entrepreneurs have higher liquidity supply at p_H (because $(1-\gamma)\left[\alpha_s^* R_1 + (1-\alpha_s^*)\, p_H R_2\right] > (1-\gamma)\left[\alpha_s^* R_1 + (1-\alpha_s^*)\, p_L R_2\right]$), therefore there must be excess liquidity supply from these early entrepreneurs at p_H. This excess liquidity supply must be absorbed at r_H by the risky banks. As a result, the risky banks survive at p_H by free-riding excess liquidity supply and are run at p_L.

At r_H by setting α_r^* the risky banks get a return of

$$d_0^r = \gamma \left[\alpha_r^* R_1 + (1-\alpha_r^*)\, p_H R_2 + \frac{(1-\alpha_r^*)\,(1-p_H)\, R_2}{r_H} \right].$$

Since the banks are risk-neutral, the risky banks maximize the expression above by setting either $\alpha_r^* = 0$ or $\alpha_r^* = 1$ depending on all the other parameters. $\alpha_r^* = 1$ is excluded because if so the banks become autarkic and survive at both states. Therefore for $p_H R_2$ not too small and r_H not too big the risky banks maximize their return at r_H with $\alpha_r^* = 0$. This determines d_0^r in the claim.

Moreover the expected return should be equal for both types of banks, $\pi d_0^r + (1-\pi)c = d_0^s$, to deter the deviation between types.

Claim 5 *In such equilibrium, the strategy for the safe banks is given by $\alpha_s^* = \alpha^*(p_L, r_L)$, i.e. $r_L(1-\gamma)\left[\alpha_s^* R_1 + (1-\alpha_s^*)\, p_L R_2\right] = \gamma\,(1-\alpha_s^*)\,(1-p_L)\, R_2$.*

Since the risky banks are run and safe banks survive at p_L, given r_L the safe banks maximize their return by setting $\alpha_s^* = \alpha^*(p_L, r_L)$ by exhausting all liquidities provided by early entrepreneurs. By the proof of Lemma 1.3.1 any unilateral deviation can only make lower return.

Claim 6 *There exists proper solution of α_s^* for such equilibrium profile of mixed strategies.*

By $\pi d_0^r + (1-\pi)c = d_0^s$,

$$\begin{aligned} & \gamma\left[p_H R_2 + \frac{(1-p_H)R_2}{r_H}\right]\pi + (1-\pi)c \\ = \quad & \gamma\left[\alpha_s^* R_1 + (1-\alpha_s^*)\, p_H R_2 + \frac{(1-\alpha_s^*)\,(1-p_H)\,R_2}{r_H}\right]. \end{aligned} \tag{A.1}$$

By $d_0^s = d_0^s(p_H) = d_0^s(p_L)$,

$$\begin{aligned} & \gamma\left[\alpha_s^* R_1 + (1-\alpha_s^*)\, p_H R_2 + \frac{(1-\alpha_s^*)\,(1-p_H)\,R_2}{r_H}\right] \\ = \quad & \alpha_s^* R_1 + (1-\alpha_s^*)\, p_L R_2. \end{aligned} \tag{A.2}$$

From (A.1) and (A.2), solve to get

$$\frac{\gamma\,(1-p_H)\,R_2}{r_H} = \frac{\alpha_s^* R_1 + (1-\alpha_s^*)\, p_L R_2 - (1-\pi)c - \pi\gamma p_H R_2}{\pi}. \tag{A.3}$$

Apply (A.3) into (A.2), by some simple algebra we get a quadratic equation of α_s^*

$$\begin{aligned} & (R_1 - p_L R_2)\,\alpha_s^{*2} - \left[\pi\,(\gamma R_1 - c) - (p_L R_2 - c) + (1-\pi)\,(R_1 - p_L R_2)\right]\alpha_s^* \\ & -(p_L R_2 - c)\,(1-\pi) = 0. \end{aligned} \tag{A.4}$$

Define the left hand side of equation (A.4) as a function of α_s^*, $f\,(\alpha_s^*) = \omega\alpha_s^{*2} + \phi\alpha_s^{*2} + \psi$ in which

$$\begin{cases} \omega = R_1 - p_L R_2 > 0, \\ \phi = -\left[\pi\,(\gamma R_1 - c) - (p_L R_2 - c) + (1-\pi)\,(R_1 - p_L R_2)\right], \\ \psi = -\,(p_L R_2 - c)\,(1-\pi) < 0. \end{cases}$$

Since $\phi^2 - 4\omega\psi > 0$, the quadratic equation has two real roots, denoted by $\alpha_{s,2}^* < \alpha_{s,1}^*$. And by $\frac{\psi}{\omega} < 0$ and $f(0) = \psi < 0$, we know $\alpha_{s,2}^*\alpha_{s,1}^* < 0$, i.e. $\alpha_{s,2}^* < 0 < \alpha_{s,1}^*$.

Moreover we find that

$$\begin{aligned} f(1) &= \omega + \phi + \psi \\ &= R_1 - p_L R_2 - \left[\pi\left(\gamma R_1 - c\right) - \left(p_L R_2 - c\right) + (1-\pi)\left(R_1 - p_L R_2\right)\right] \\ &\quad - \left(p_L R_2 - c\right)(1-\pi) \\ &= \pi(1-\gamma)R_1 \\ &> 0, \end{aligned}$$

we know that $\alpha^*_{s,2} < 0 < \alpha^*_{s,1} < 1$.

And again we can find that

$$\begin{aligned} f(1-\pi) &= \left(R_1 - p_L R_2\right)(1-\pi)^2 - \left[\pi\left(\gamma R_1 - c\right) - \left(p_L R_2 - c\right) + \right. \\ &\quad \left.(1-\pi)\left(R_1 - p_L R_2\right)\right](1-\pi) - \left(p_L R_2 - c\right)(1-\pi) \\ &= -\pi\left(\gamma R_1 - c\right)(1-\pi) \\ &< 0, \end{aligned}$$

we know that $\alpha^*_{s,2} < 0 < 1-\pi < \alpha^*_{s,1} < 1$. This implies that in current settings, there always exists a plausible solution: $\alpha^*_{s,1} \in (1-\pi, 1)$.

All the arguments above can be captured by Fig. A.2.

By equation (A.1) we already know that when $\pi = \overline{\pi}_1$, $\alpha^*_s = \alpha(p_L)$ and $r_H = 1$. When $\pi = \overline{\pi}_1 + \delta$, $\alpha^*_s \in (1-\pi, 1)$, then r_H has to be larger than 1 to make the equation still hold. From Claim 3, this implies that $r_H > r_L > 1$.

Claim 7 *Given features described in previous claims, there exists no profitable unilateral deviation.*

Suppose that one bank i deviates by choosing $\alpha_i \neq \alpha^*_s$ and $\alpha_i \neq 0$. Then by doing so there are three possible consequences:

1. The bank survives at both states. But by Claim 5 its return at p_L must be lower than d^s_0. If it survives at both states, it cannot promise $d^i_0 > d^s_0$. Given this, no investor would deposit at all;

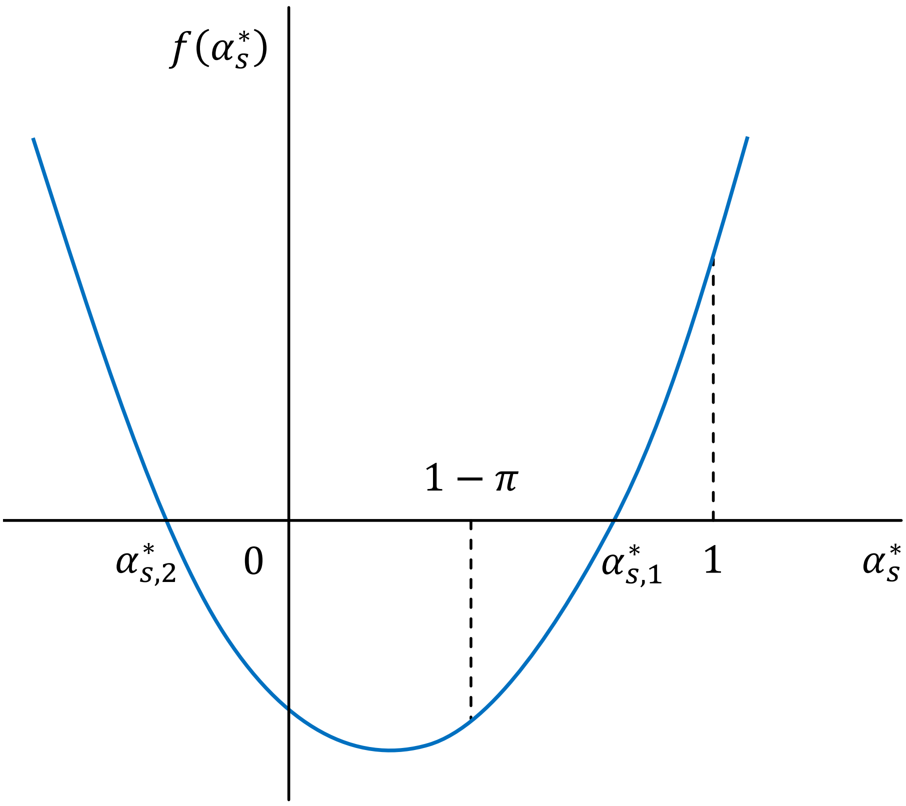

Fig. A.2 The existence of the proper solution for α_s^*

2. It survives at p_H but is run at p_L. Since $\alpha_i > 0$ by Claim 4 its return at p_H must be lower than d_0^r;

3. It survives at p_L but is run at p_H. By (1) its return is $d_0^i < d_0^s$ at p_L and c at p_H. Its expected return is $d_0^i\pi + (1-\pi)c < d_0^s$.

Therefore strategic profile σ_i cannot be a profitable unilateral deviation such that σ_i contains $\alpha_i \neq \alpha_s^*$ and $\alpha_i \neq 0$ with probability $p \in (0, 1]$.

And section 5 of the proposition is simply market clearing condition balancing aggregate liquidity supply and demand. ∎

A.1.5 Addendum to Proposition 1.5.1

For those parameter values such that $\pi p_H R_2 + (1-\pi) p_L R_2 < 1$ there exists no equilibrium with liquidity injection. The reason is the following:

1. Any symmetric strategic profile cannot be equilibrium, because
   (a) If there is no trade under such strategic profile, i.e. α is so small that the real return is less than 1, one bank can deviate by setting $\alpha = 1$ and trading with investors;
   (b) If there is trade under such strategic profile, i.e. $\alpha > 0$ for all the banks, then one bank can deviate by setting $\alpha = 0$ and getting higher nominal return than the other banks.
2. Any asymmetric strategic profile, or profile of mixed strategies, cannot be equilibrium, because
   (a) If there is no trade under such strategic profile, then the argument of 1 (a) applies here;
   (b) If there is trade under such strategic profile, then one bank can deviate by choosing a pure strategy, $\alpha = 0$, and get better off — there is no reason to mix with the other dominated strategies.

A.2 A NUMERICAL EXAMPLE FOR THE EQUILIBRIUM OF MIXED STRATEGIES

Suppose that $p_H = 0.4$, $p_L = 0.3$, $\gamma = 0.6$, $R_1 = 2$, $R_2 = 4$, $c = 0.8$. Then

$$\begin{aligned}
\alpha(p_H) &= \frac{\gamma - p_H}{\gamma - p_H + (1-\gamma)\frac{R_1}{R_2}} = \frac{1}{2}, \\
\alpha(p_L) &= \frac{\gamma - p_L}{\gamma - p_L + (1-\gamma)\frac{R_1}{R_2}} = 0.6, \\
\mathbb{E}[R_H] &= \alpha(p_H) R_1 + (1-\alpha(p_H)) R_2 = 3, \\
\mathbb{E}[R_L] &= \alpha(p_L) R_1 + (1-\alpha(p_L)) R_2 = 2.8, \\
\overline{\pi}_2 &= \frac{\gamma \mathbb{E}[R_L] - c}{\gamma \mathbb{E}[R_H] - c} = 0.88,
\end{aligned}$$

$$\overline{\pi}_1 = \frac{\gamma \mathbb{E}[R_L] - c}{\gamma R_2 - c} = 0.55.$$

Take $\pi = 0.7 \in (\overline{\pi}_1, \overline{\pi}_2)$ and by $\pi d_0^r + (1-\pi)c = d_0^s$

$$\begin{aligned}
&\gamma\left[p_H R_2 + \frac{(1-p_H)R_2}{r_H}\right]\pi + (1-\pi)c \\
= \ &\gamma\left[\alpha_s^* R_1 + (1-\alpha_s^*)\,p_H R_2 + \frac{(1-\alpha_s^*)\,(1-p_H)\,R_2}{r_H}\right], \\
&0.6\left[0.4 \times 4 + \frac{0.6 \times 4}{r_H}\right] \times 0.7 + 0.3 \times 0.8 \\
= \ &0.6\left[\alpha_s^* \times 2 + (1-\alpha_s^*) \times 0.4 \times 4 + \frac{(1-\alpha_s^*) \times 0.6 \times 4}{r_H}\right].
\end{aligned} \tag{A.5}$$

By $d_0^s = d_0^s(p_H) = d_0^s(p_L)$,

$$\begin{aligned}
&\gamma\left[\alpha_s^* R_1 + (1-\alpha_s^*)\,p_H R_2 + \frac{(1-\alpha_s^*)\,(1-p_H)\,R_2}{r_H}\right] \\
= \ &\alpha_s^* R_1 + (1-\alpha_s^*)\,p_L R_2, \\
&0.6\left[\alpha_s^* \times 2 + (1-\alpha_s^*) \times 0.4 \times 4 + \frac{(1-\alpha_s^*) \times 0.6 \times 4}{r_H}\right] \\
= \ &\alpha_s^* \times 2 + (1-\alpha_s^*) \times 0.3 \times 4.
\end{aligned} \tag{A.6}$$

Solve equations (A.5) and (A.6) to get $\alpha_s^* = 0.47 < \alpha(p_H) < \alpha(p_L)$, $r_H = 1.519$.

And $d_0^s = \alpha_s^* R_1 + (1-\alpha_s^*)\,p_L R_2 = 1.576$, $d_0^r = \frac{d_0^s - (1-\pi)c}{\pi} = 1.908$.

Market clearing at p_L:

$$\begin{aligned}
r_L(1-\gamma)\left[\alpha_s^* R_1 + (1-\alpha_s^*)\,p_L R_2\right] &= \gamma\,(1-\alpha_s^*)\,(1-p_L)\,R_2, \\
r_L \times 0.4\,[0.47 \times 2 + 0.53 \times 0.3 \times 4] &= 0.6 \times 0.53 \times 0.7 \times 4,
\end{aligned}$$

solve to get $r_L = 1.414$.

Market clearing at p_H:

$$\begin{cases}
D_r = d_0^r - \gamma p_H R_2 = 0.948, \\
D_s = d_0^s - \gamma\left[\alpha_s^* R_1 + (1-\alpha_s^*)\,p_H R_2\right] = 0.503, \\
S_r = (1-\gamma) p_H R_2 = 0.64, \\
S_s = (1-\gamma)\left[\alpha_s^* R_1 + (1-\alpha_s^*)\,p_H R_2\right] = 0.715,
\end{cases}$$

as well as $\theta D_r + (1-\theta) D_s = \theta S_r + (1-\theta) S_s$, solve to get $\theta = 0.402$.

Part III

Endogenous Systemic Liquidity Risk and Banking Regulation

2

Endogenous Systemic Liquidity Risk

The events earlier this month leading up to the acquisition of Bear Stearns by JP Morgan Chase highlight the importance of liquidity management in meeting obligations during stressful market conditions. ... The fate of Bear Stearns was the result of a lack of confidence, not a lack of capital. ... At all times until its agreement to be acquired by JP Morgan Chase during the weekend, the firm had a capital cushion well above what is required to meet supervisory standards calculated using the Basel II standard.

—Chairman Cox, SEC, March 20, 2008

Bear Stearns never ran short of capital. It just could not meet its obligations. At least that is the view from Washington, where regulators never stepped in to force the investment bank to reduce its high leverage even after it became clear Bear was struggling last summer. Instead, the regulators issued repeated reassurances that all was well. Does it sound a little like a doctor emerging from a funeral to proclaim that he did an excellent job of treating the late patient?

—Floyd Norris, *New York Times*, April 4, 2008

2.1 INTRODUCTION

For a long time, presumably starting in 2004, financial markets seemed to have been awash with excessive liquidity. But suddenly, in August 2007, liquidity dried out nearly completely as a response to doubts about the quality of subprime mortgage-backed securities. Despite massive central bank interventions, the liquidity freeze did not melt away, but rather spread slowly to other markets such as those for auction rate bonds. On March 16th, 2008, the investment bank Bear Sterns which — according to the SEC chairman — was adequately capitalized even a week before had to be rescued via a Fed-led takeover by JP Morgan Chase.

Following the turmoil on financial markets, there has been a strong debate about the adequate policy response. Some have warned that central bank actions may encourage dangerous moral hazard behavior of market participants in the future. Others instead criticized central banks of responding far too cautiously. The most prominent voice has been Willem Buiter who — jointly with Ann Sibert — right from the beginning of the crisis in August 2007 strongly pushed the idea that in times of crises, central banks should act as market maker of last resort. As adoption of the Bagehot principles to modern times with globally integrated financial systems, central banks should actively purchase and sell illiquid private sector securities and so play a key role in assessing and pricing credit risk. In his FT blog "Maverecon", Willem Buiter stated the intellectual arguments behind such a policy very clearly on December 13th, 2007:

> *"Liquidity is a public good. It can be managed privately (by hoarding inherently liquid assets), but it would be socially inefficient for private banks and other financial institutions to hold liquid assets on their balance sheets in amounts sufficient to tide them over when markets become disorderly. They are meant to intermediate short maturity liabilities into long maturity assets and (normally) liquid liabilities into illiquid assets. Since central banks can create unquestioned liquidity at the drop of a hat, in any amount and at zero cost, they should be*

> *the liquidity providers of last resort, both as lender of last resort and as market maker of last resort. There is no moral hazards as long as central banks provide the liquidity against properly priced collateral, which is in addition subject to the usual 'liquidity haircuts' on this fair valuation. The private provision of the public good of emergency liquidity is wasteful. It's as simple as that."*

Buiter's statement represents the prevailing main stream view that there is no moral hazard risk as long as the Bagehot principles are followed as best practice in liquidity management.

According to the Bagehot principles, a Lender of Last Resort policy should target liquidity provision to the market, but not to specific banks. Central banks should "lend freely at a high rate against good collateral." This way, public liquidity support is supposed to be targeted towards solvent yet illiquid institutions, since insolvent financial institutions should be unable to provide adequate collateral to secure lending. This chapter wants to challenge the view that a policy following Bagehot principle does not create moral hazard. The key argument is this view neglects the endogeneity of aggregate liquidity risk. Starting with Allen & Gale (1998) and Holmström & Tirole (1998), there have been quite a few models recently analyzing private and public provision of liquidity. But as far as we know, in all these models except Chapter 1 or our companion paper Cao & Illing (2008), aggregate systemic risk is assumed to be an exogenous probability event.

In Holmström & Tirole (1998), for instance, liquidity shortages arise when financial institutions and industrial companies scramble for, and cannot find the cash required to meet their most urgent needs or undertake their most valuable projects. They show that credit lines from financial intermediaries are sufficient for implementing the socially optimal (second-best) allocation, as long as there is no aggregate uncertainty. In the case of aggregate uncertainty, however, the private sector cannot satisfy its own liquidity needs, so the existence of liquidity shortages vindicates the injection of liquidity by the government. In their model, the government can provide (outside) liquidity by committing future tax income to back up the reimbursements.

In the model of Holmström & Tirole (1998), the Lender of Last Resort indeed provides a free lunch: public provision of liquidity in the presence of aggregate shocks is a pure public good, with no moral hazard involved. The reason is that aggregate liquidity shocks are modelled as exogenous events; there is no endogenous mechanism determining the aggregate amount of liquidity available. The same holds in Allen & Gale (1998), even though they analyze a quite different mechanism for public provision of liquidity: the adjustment of the price level in an economy with nominal contracts. We adopt Allen & Gale's mechanism. But we show that there is no longer a free lunch when private provision of liquidity affects the likelihood of an aggregate (systemic) event.

The basic idea of our model is fairly straightforward: financial intermediaries can choose to invest in more or less (real) liquid assets. We model illiquidity in the following way: some fraction of projects turns out to be realized late. The aggregate share of late projects is endogenous; it depends on the incentives of financial intermediaries to invest in risky, illiquid projects. This endogeneity allows us to capture the feedback from liquidity provision to risk taking incentives of financial intermediaries. We show that the anticipation of unconditional central bank liquidity provision will encourage excessive risk taking (moral hazard). It turns out that in the absence of liquidity requirements, there will be overinvestment in risky activities, creating excessive exposure to systemic risk.

In contrast to what the Bagehot principle suggests, unconditional provision of liquidity to the market (lending of central banks against good collateral) is exactly the wrong policy: it distorts incentives of banks to provide the efficient amount of private liquidity. In our model, we concentrate on pure illiquidity risk: there will never be insolvency unless triggered by illiquidity (by a bank run). Illiquid projects promise a higher, yet possibly retarded return. Relying on sufficient liquidity provided by the market (or by the central bank), financial intermediaries are inclined to invest more heavily in high yielding, but illiquid long term projects. Central banks liquidity provision, helping to prevent bank runs with inefficient early liquidation, encourages bank to invest more in illiquid assets. At first sight, this seems to work fine, even if systemic risk increases: after all, public insurance against aggregate risks should allow agents to undertake more profitable activities with higher social return. As long as public insurance is a free lunch, there is nothing wrong with providing such a public good.

The problem, however, is that due to limited liability some banks will be encouraged to free-ride on liquidity provision. This competition will force other banks to reduce their efforts for liquidity provision, too. Chuck Prince, at that time chief executive of Citigroup, stated the dilemma posed in fairly poetic terms on July 10th 2007 in a (in-) famous interview with Financial Times[1]:

> *"When the music stops, in terms of liquidity, things will be complicated. But as long as the music is playing, you've got to get up and dance. We're still dancing."*

The naughty dancing banks simply enjoy liquidity provided in good states of the world and just disappear (go bankrupt) in bad states. The incentive of financial intermediaries to free-ride on liquidity in good states results in excessively low liquidity in bad states. Even worse: as long as they are not run, naughty "dancing" banks can always offer more attractive collateral in bad states — so they are able to outbid prudent banks in a liquidity crisis. For that reason, the Bagehot principle, rather than providing correct incentives, is

[1]The key problem is best captured by the following remark about Citigroup in the New York Times report "Treasury Dept. Plan Would Give Fed Wide New Power" on March 29, 2008: *"Mr. Frank said he realized the need for tighter regulation of Wall Street firms after a meeting with Charles O. Prince III, then chairman of Citigroup. When Mr. Frank asked why Citigroup had kept billions of dollars in 'structured investment vehicles' off the firm's balance sheet, he recalled, Mr. Prince responded that Citigroup, as a bank holding company, would have been at a disadvantage because investment firms can operate with higher debt and lower capital reserves."*

the wrong medicine in modern times with a shadow banking system relying on liquidity being provided by other institutions.

This chapter extends a model developed in the last chapter, i.e. Cao & Illing (2008). There we did not allow for banks holding equity, so we could not analyze the impact of equity requirements. As we will show, imposing equity requirements can be inferior even relative to the outcome of a mixed strategy equilibrium with free-riding (dancing) banks. In contrast, imposing binding liquidity requirements *ex ante* combined with Lender of Last Resort policy *ex post* is able to implement the optimal second best outcome. In our model, it yields a strictly superior outcome compared to imposing equity requirements. We also prove that ñarrow banking" (banks being required to hold sufficient equity so as to be able to pay out demand deposits in all states of the world) is inferior relative to *ex ante* liquidity regulation.

Allen & Gale (2007, p 213f) notice that the nature of market failure leading to systemic liquidity risk is not yet well understood. They argue that "a careful analysis of the costs and benefits of crises is necessary to understand when intervention is necessary." In this chapter, we try to fill this gap, providing a cost / benefit analysis of different forms of banking regulation to better understand what type of intervention is desired. We explicitly compare the impact of both liquidity and capital requirements. To the best of our knowledge, this is the first work providing such an analysis.

Our argument also seems to be valid for the modelling approach used in Goodfriend & McCallum (2007). They introduce a banking sector in the standard new Keynesian framework to reconsider the role of money and banking in monetary policy analysis. Goodfriend & McCallum show that "banking accelerator" transmission effects work via an "external finance premium." In their model, the central bank should react more aggressively to problems in the banking sector. This result may need to be qualified if these problems within the banking sector are generated endogenously rather than being the result of exogenous shocks.

2.2 THE STRUCTURE OF THE MODEL

In the economy, there are three types of agents: investors, banks (run by bank managers, or bankers) and entrepreneurs. All agents are risk neutral. The economy extends over 3 periods. We assume that there is a continuum of investors each initially (at $t = 0$) endowed with one unit of resources. The resource can be either stored (with a gross return equal to 1) or invested in the form of bank equity or bank deposits. Using these funds, banks as financial intermediaries can fund projects of entrepreneurs. There are two types i of entrepreneurs ($i = 1, 2$), characterized by their projects return R_i. Projects of type 1 are realized early at period $t = 1$ with a safe return $R_1 > 1$. Projects of type 2 give a higher return $R_2 > R_1 > 1$. With probability p, these projects will also be realized at $t = 1$, but they may be delayed (with probability $1-p$) until $t = 2$. In the aggregate, the share p of type 2 projects will be realized early. The aggregate share p, however is not known at $t = 0$. It will be revealed between 0 and 1 at some intermediate period $t = \frac{1}{2}$. Investors are impatient: they want to consume early (at $t = 1$). In contrast, both entrepreneurs and bank managers are indifferent between consuming early ($t = 1$) or late ($t = 2$).

Resources of investors are scarce in the sense that there are more projects of each type available than the aggregate endowment of investors. Thus, in the absence of commitment problems, total surplus would go to the investors. In the absence of commitment problems, investors would simply put all their funds in early projects and capture the full return. We take this frictionless market outcome as reference point and analyze those equilibria coming closest to implement that market outcome. Since there is a market demand for liquidity only if investors' funds are the limiting factor, we concentrate on deviations from the frictionless market outcome and consider investors payoff as the relevant criterion.

Due to hold up problems as modelled in Hart & Moore (1994), entrepreneurs can only commit to pay a fraction $\gamma R_i > 1$ of their return. Banks as financial intermediaries can pool investment; they have superior collection skills (a higher γ). Following Diamond & Rajan (2001), banks offer deposit contracts with a fixed payment d_0 payable at any time after $t = 0$ as a credible commitment device not to abuse their collection skills. The threat of a bank run disciplines bank managers to fully pay out all available resources pledged in the form of bank deposits. There are a finite number of active banks engaged in Bertrand competition. Banks compete by choosing the share α of deposits invested in type 1 projects, taking their competitors choice as given. Investors have rational expectations about each banks default probability; they are able to monitor all banks investment. So if, in a mixed strategy equilibrium, banks differ with respect to their investment strategy, the expected return from deposits must be the same across all banks. Due to Bertrand competition, all banks will earn zero profit in equilibrium. In the absence of aggregate risk, financial intermediation via bank deposits can implement a second best allocation, given the hold up problem posed by entrepreneurs.

Note that because of the hold up problem, entrepreneurs retain a rent — their share $(1 - \gamma)R_i$. Since early entrepreneurs are indifferent between consuming at $t = 1$ or $t = 2$, they are willing to provide liquidity (using their rent to buy equity and to deposit at banks at $t = 1$ at the market rate r). Banks use the liquidity provided to pay out depositors. This way, impatient investors can profit indirectly from investment in high yielding long term projects. So banking allows transformation between liquid claims and illiquid projects.

At date 0, banks competing for funds offer deposit contracts with payment d_0 and equity claims which maximize expected consumption of investors at the given expected interest rates. Investors put their funds into those assets promising the highest expected return among all assets offered. So in equilibrium, expected return from deposits and equity must be equal across all active banks. At date $t = 1$, banks and early entrepreneurs trade at a perfect market for liquidity, clearing at interest rate r. As long as banks are liquid, the payoff structure is described as in Fig. 2.1.

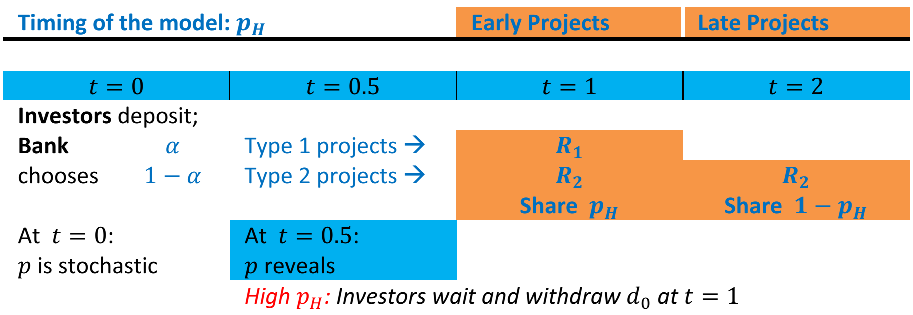

Fig. 2.1 Timing and payoff structure, when banks are liquid

Deposit contracts, however, introduce a fragile structure into the economy: whenever depositors have doubts about their bank's liquidity (the ability to pay depositors the promised amount d_0 at $t = 1$), they run the bank early (they run already at the intermediate date $t = \frac{1}{2}$), forcing the bank to liquidate all its projects (even those funding safe early entrepreneurs) at high costs: early liquidation of projects gives only the inferior return $c < 1$. We do not consider pure sunspot bank runs of the Diamond & Dybvig type. Instead we concentrate on runs happening if liquid funds (given the interest rate r) are not sufficient to payout depositors.

If the share p of type 2 projects realized early is known at $t = 0$, there is no aggregate uncertainty. Banks will invest such that — on aggregate — they are able to fulfil depositor's claims in period 1, so there will be no run. But we are interested in the case of aggregate shocks. We model them in the simplest way: the aggregate share of type 2 projects realized early can take on just two values: either p_H or p_L with $p_H > p_L$. The "good" state with a high share of early type 2 projects (the state with plenty of liquidity) will be realized with probability π. Note that the aggregate liquidity available depends on the total share of funds invested in liquid type 1 projects. Let α be this share. If α is so low that banks cannot honor deposits when p_L occurs, depositors will run at $t = \frac{1}{2}$. The payoff is captured in Fig. 2.2.

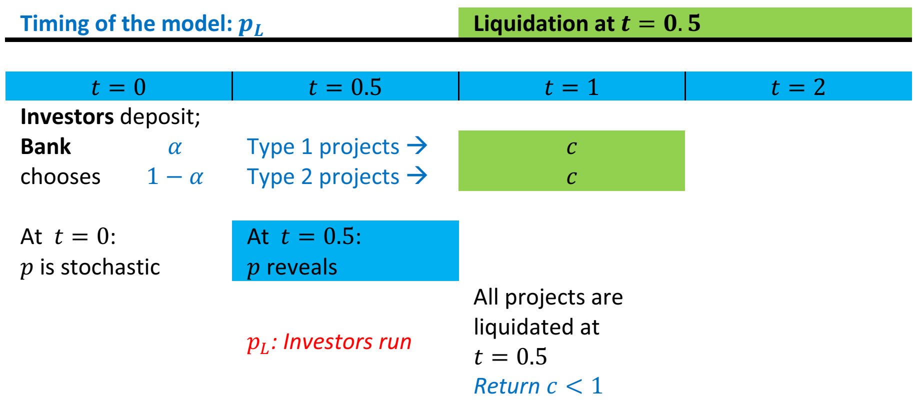

Fig. 2.2 Timing and payoff structure, when banks are illiquid

Given this structure, a bank seems to have just two options available: it may either invest so much in safe type 1 projects that it will be able to pay out its depositors all the time (that is, even if the bad state occurs). Let us call this share $\alpha(p_L)$. Alternatively, it may invest just enough, $\alpha(p_H)$, so as to pay out depositors in the good state. If so, the bank will be run in the bad state. Obviously, the optimal share depends on what other banks will do (since that determines aggregate liquidity available at $t = 1$ and so the interest rate for liquid funds between period 1 and 2), but also on the probability π for the good state. To gain some intuition, let us first assume that all banks behave the same — just as a representative bank. If so, it will not pay to take precautions against the bad state if the likelihood for that outcome is considered to be very low. Thus, if π is very high, the representative bank will obviously invest only a small share $\alpha(p_H)$ — just enough to pay out depositors in the good state. Alternatively, if π is very low (close to 0), it always pays to be prepared for the worst case, so the representative bank will invest a high share $\alpha(p_L) > \alpha(p_H)$ in safe projects. Since $\alpha(p_s)$ is the share invested in safe projects with return R_1, the total payoff out of investment strategy $\alpha(p_s)$ is: $\mathbb{E}[R_s] = \alpha(p_s)R_1 + [1 - \alpha(p_s)]R_2$ with $\mathbb{E}[R_H] > \mathbb{E}[R_L]$.

With a high share $\alpha(p_L)$ of safe projects, the banks will be able to pay out depositors in all states. There will never be a bank run. So independent of π, the expected payoff for depositors is $\gamma\mathbb{E}[R_L]$ (assuming that the gross interest rate between $t = 1$ and $t = 2$ is $r = 1$, which is the case maximizing the investors payoffs).

With $\alpha(p_H)$ there will be a bank run in the bad state, giving just the bankruptcy payoff c with probability $1-\pi$. So strategy $\alpha(p_H)$ gives $\pi\gamma\mathbb{E}[R_H] + (1-\pi)c$, increasing in π. Depositors prefer $\alpha(p_H)$, if $\pi\gamma\mathbb{E}[R_H]+(1-\pi)c > \gamma\mathbb{E}[R_L]$ or

$$\pi > \overline{\pi}_2 = \frac{\gamma\mathbb{E}[R_L] - c}{\gamma\mathbb{E}[R_H] - c}.$$

Obviously, for π below $\overline{\pi}_2$ depositors are better off with the safe strategy, so they prefer banks to choose $\alpha(p_L)$ rather than to exploit high profitability of type 2 entrepreneurs. The intuition is straightforward: when π is not high enough, the high return R_2 will come too late most of the time, triggering frequent bank runs in period 1. So depositors rather prefer banks to play the safe strategy in the range. In contrast, for $\pi > \overline{\pi}_2$ it would be inefficient for private banks to hold enough liquid assets on their balance sheets to prevent disasters when markets become disorderly. As long as all banks play according to the strategies outlined above, depositors' payoff is characterized by the dotted red line in Fig. 2.3.

Up to now, we simply assumed that all banks follow the same strategy, maximising depositor's payoff. But when all banks choose the strategy $\alpha(p_L)$, there will be excess liquidity at $t = 1$ if the good state occurs (with a large share of type 2 projects realized early). A bank anticipating this event has a strong incentive to invest all their funds in type 2 projects, reaping the benefit of excess liquidity in the good state. As long as the music is playing, such a deviating bank gets up and dances. Having invested only in high yielding projects, the naughty dancing bank can always credibly extract entrepreneur's excess liquidity at $t = 1$, promising to pay back at $t = 2$ out of

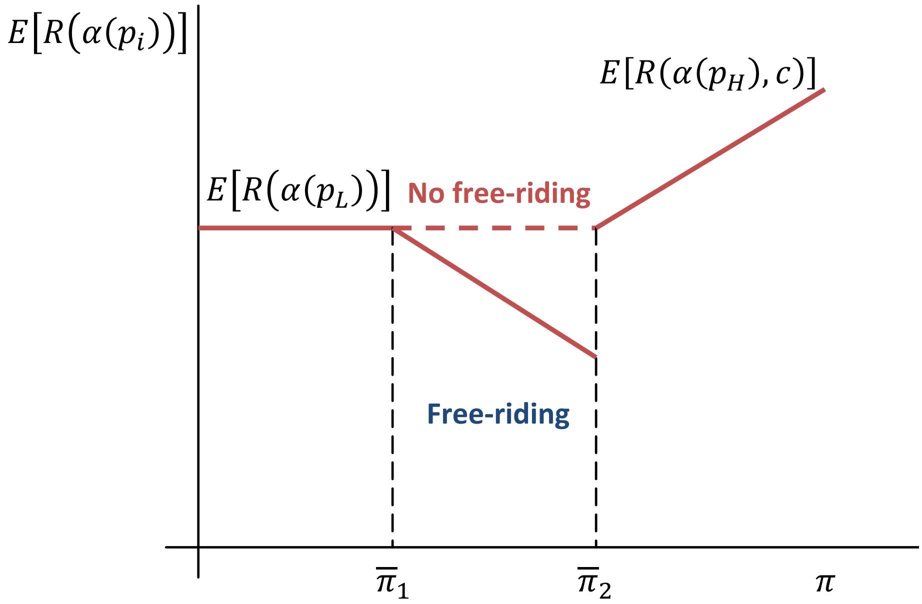

Fig. 2.3 Depositors' expected return

highly profitable projects. After all, at that stage, this bank, free-riding on liquidity, can offer a capital cushion with expected returns well above what prudent banks are able to promise. Of course, if the bad state happens, there is no excess liquidity. The naughty "dancing" banks would just bid up the interest rates, urgently trying to get funds. Rational depositors, anticipating that these banks won't succeed, will already trigger a bank run on these banks at $t = \frac{1}{2}$.

When the music stops, in terms of liquidity, things get complicated. As long as the naughty dancing banks are not supported in the bad state, they are driven out of the market, providing just the return c. Nevertheless, a bank free-riding on liquidity in the good state can on average offer the attractive return $\pi\gamma R_2 + (1 - \pi)c$ as expected payoff for depositors. Thus, a free-riding bank will always be able to outbid a prudent bank whenever the probability π for the good state is not too low. The condition is

$$\pi > \overline{\pi}_1 = \frac{\gamma \mathbb{E}[R_L] - c}{\gamma R_2 - c}.$$

Since $R_2 > \mathbb{E}[R_H]$, it pays to dance within the range $\overline{\pi}_1 \leq \pi < \overline{\pi}_2$.

Obviously, there cannot be equilibrium in pure strategies within that range. As long as the music is playing, all banks would like to get up and dance. But then, there would be no prudent bank left providing the liquidity needed to be able to dance. In the resulting mixed strategy equilibrium, a proportion of banks behave prudent, investing some amount $\alpha_s < \alpha(p_L)$ in liquid assets, whereas the rest free-rides on liquidity in the good state, choosing $\alpha = 0$. Prudent banks reduce $\alpha_s < \alpha(p_L)$ in order to cut down the opportunity cost of investing in safe projects. Interest rates and α_s adjust such that depositors are indifferent between the two types of banks. At $t = 0$, both prudent and naughty dancing banks offer the same expected return to depositors. The proportion of free-riding banks is determined by aggregate market clearing conditions in both states. Naughty dancing banks are run for sure in the bad state, but the high return $R_2 > \mathbb{E}[R_s]$ compensates depositors for that risk.

As shown in Proposition 2.2.1, free-riding drives down the return for investors (see Fig. 2.3). They are definitely worse off than if all banks would coordinate on the prudent strategy $\alpha(p_L)$. As illustrated in Fig. 2.3, the effective return on deposits for investors deteriorates in the range $\overline{\pi}_1 \leq \pi < \overline{\pi}_2$ as a result of free-riding behavior.

Proposition 2.2.1 *In the mixed strategy equilibrium, investors are worse off than if all banks would coordinate on the prudent strategy $\alpha(p_L)$.* ∎

Proof See Appendix A.1.1. ∎

2.3 LENDER OF LAST RESORT POLICY

A Lender of Last Resort cannot create real liquidity at period one. But a central bank can add nominal liquidity at the stroke of a pen. Following Allen & Gale (1998, 2004) and Diamond & Rajan (2006), assume from now on that deposit contracts are arranged in nominal terms. The liquidity injection is done such that the banks are able to honor their nominal contracts, reducing the real value of deposits just to the amount of real resources available at that date. This intervention raises the real payoff of depositors compared to inefficient liquidation, increasing expected payoff of the risky strategy $\alpha(p_H)$.

Consider that the central bank injects liquidity in order to prevent bank runs if the bad state (with low payoffs at $t = 1$) occurs. Such a policy, preventing inefficient costly liquidation, seems to raise investor's expected payoff and so definitely improve upon the allocation for high values $\pi > \overline{\pi}_2$. Essentially, nominal deposits allow the central bank to implement state contingent payoffs. This argument seems to confirm the view that Lender of Last Resort indeed is a free lunch, providing a public good at no cost. It turns out, however, that the anticipation of these actions has an adverse impact on the amount of aggregate liquidity provided by the private sector, affecting endogenously the exposure to systemic risk.

The incentive for free-riding prevalent in modern times of competitive financial markets complicates the picture dramatically. In the model presented, a Lender of Last Resort, providing liquidity support to the market requesting good collateral as the only condition, will drive out all prudent banks. Just as in Gresham's law, all banks are encouraged to dance and choose the risky strategy $\alpha(p_H)$, knowing that they can get liquidity support against good collateral. The public provision of emergency liquidity results in serious moral hazard. It's as simple as that.

Proposition 2.3.1 *Assume that $\pi p_H R_2 + (1-\pi) p_L R_2 \geq 1$ and that for $\pi \in (\overline{\pi}_1, \overline{\pi}_2)$, $d_0^j = \gamma R_2 > \pi p_H R_2 + (1-\pi)c$. If the central bank is willing to provide liquidity to the entire market in times of crisis, all banks have an incentive to dance, choosing $\alpha_j = 0$.* ▮

Proof See Appendix A.1.2. ▮

The reason for this surprising result is the following: by purpose, we concentrate on the case of pure illiquidity risk. In our model, the liquidity shock just retards the realization of high yielding projects: in the end (at $t = 2$), all projects will certainly be realized. So there is no doubt about solvency of the projects, unless insolvency is triggered by illiquidity. Central bank support against allegedly good collateral, creating artificial liquidity at the drop of a hat, destroys all private incentives to care about *ex ante* liquidity provision. The key problem with the Bagehot principle here is that naughty dancing banks do invest in projects with higher return, as long as they have not to be terminated. In reality, there is no clear-cut distinction between insolvency and illiquidity. We leave it to the next chapter to allow for the risk of insolvency. But we'll show that our basic argument will not be much affected.

So what policy options should be taken? One might argue that a central bank should provide liquidity support only to prudent banks (so conditional on banks having invested sufficiently in liquid assets). As shown in the last chapter, such a policy may improve the allocation at least to some extent. But we argued that such a commitment is simply not credible: as emphasised by Rochet (2004, 2008), there is a serious problem of dynamic consistency.

Rather than relying on an implausible commitment mechanism, the obvious solution would be a mix between two instruments: *ex ante* liquidity regulation combined with *ex post* Lender of Last Resort policy. It seems to be rather surprising that perceived wisdom argues that central banks can pursue both price stability and financial stability using just one tool, interest rate policy. Instead, the second best outcome from the investors' point of view needs to be implemented by the following policy: in a first step, a banking regulator has to impose *ex ante* liquidity requirements. Requesting minimum investment in liquid type 1 assets of at least $\alpha(p_L)$ for $\pi < \overline{\pi}_2'$ and $\alpha(p_H)$ for $\pi > \overline{\pi}_2'$ would give investors the highest expected payoff as characterized in Fig. 2.4. For $\pi < \overline{\pi}_2'$, playing safe gives investors the highest payoff. In contrast, for $\pi > \overline{\pi}_2'$ investors are better off if banks invest in liquid assets as low as $\alpha(p_H)$ as long as Lender of Last Resort policy helps to prevent runs. Since such a rule would not allow banks to operate when liquidity holdings are less than required, it could get rid of incentives for free-riding. Given that the *ex ante* imposed liquidity requirements have been fulfilled, *ex post* the central bank can safely play its role as lender in the range $\pi > \overline{\pi}_2'$ whenever the bad state turns out to be realized. Note that this policy raises expected payoff for investors, even though it increases the range of parameter values with systemic risk.

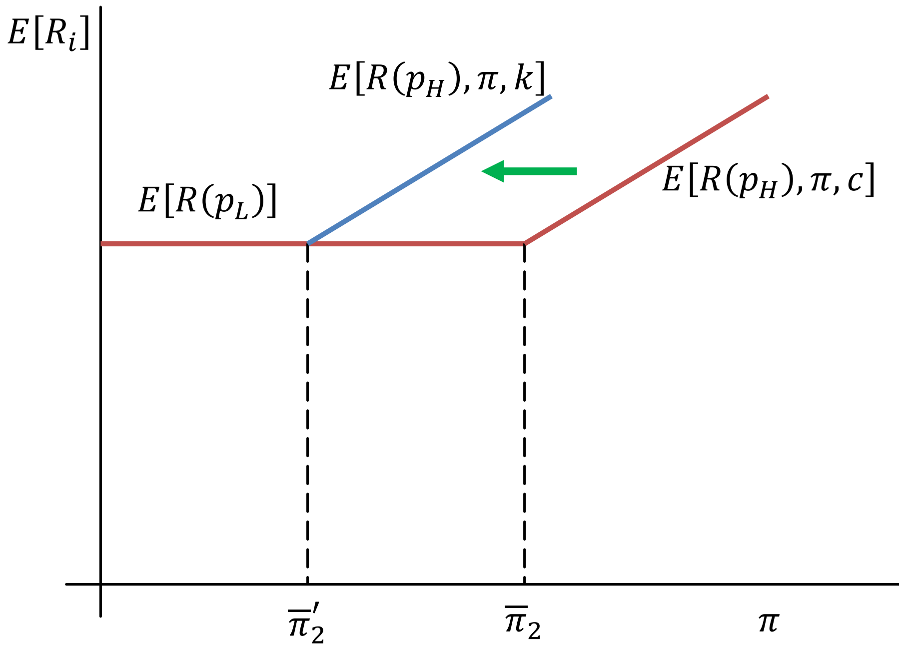

Fig. 2.4 Depositors' expected return with *ex ante* liquidity regulation and *ex post* LoLR policy ($\mathbb{E}\left[R(p_H),\pi,\kappa\right]$) versus the expected return in the *laissez-faire* economy ($\mathbb{E}\left[R(p_H),\pi,c\right]$) when π is high

The key task for regulators and the central bank is to cope with free-riding incentives. An alternative mechanism compared to *ex ante* liquidity regulation, the central bank might commit to try to mop up the excessive liquidity available in the good state. If that can be done, potential free-riders would have no chance to survive. We doubt, however, that the central will be able to implement such a policy.

As further alternative, one might impose narrow banking in the sense that banks are required to hold sufficient liquid funds so as to pay out in all contingencies. Finally, one might expect that imposing equity, or capital requirements are sufficient to provide a cushion against liquidity shocks. As shown in the next section, both these options turn out to be strictly worse than imposing minimum liquidity standards *ex ante* combined with Lender of Last Resort policy. They are even likely to be inferior relative to the outcome of a mixed strategy equilibrium with free-riding (dancing) banks.

2.4 THE ROLE OF EQUITY AND NARROW BANKING

Let us now introduce equity requirements in the model, i.e. banks are required to hold some equity in their assets. Keep the same settings as before with the presence of aggregate uncertainty, except that instead of pure fixed deposit contract, the banks issue a mixture of deposit contract and equity for the investors (Diamond & Rajan, 2000, 2005, 2006). To make it clear, equity is a claim that can be renegotiated such that the bank managers and the capital holders (here the investors) split the residual surplus after the deposit contract has been paid. The mixture of deposit contract and equity seems to be a quite artificial setting at the first sight. But actually it turns out to be a convenient modelling device. In particular, in the symmetric equilibria of the banks, such a mixture will exactly be the portfolio held by a representative agent out of the homogenous investors. In other words, whenever investors are homogenous, it's not necessary to separate equity holders from the depositors.

Equity can reduce the fragility, but it allows the bank manager to capture a rent. Being a renegotiatable claim, equity is always subject to the hold-up problem, i.e. equity holders can only get a share of ζ ($\zeta \in [0,1]$) from the surplus. To make it simpler, in the following we simply assume that $\zeta = \frac{1}{2}$.

With $\zeta = \frac{1}{2}$ the bank managers get a rent of $\frac{\gamma\mathbb{E}[R]-d_0}{2}$, sharing the surplus over deposits equally with the equity holders. Suppose that all the banks have to meet the level of equity k which comes from the central bank's regulatory rules, then if a bank i is not run k is defined as

$$k = \frac{\frac{\gamma\mathbb{E}[R_{s,i}]-d_{0,i}}{2}}{\frac{\gamma\mathbb{E}[R_{s,i}]-d_{0,i}}{2} + d_{0,i}}$$

in which $R_{s,i}$ is bank i's return achieved under state s.

One additional, but crucial assumptions concerning timing are that (1) the dividend of the equity is paid *after* the payment of $d_{0,i}$ and (2) capital requirement has to be met till the last minute before the dividend payment — this deters the bank managers' incentive to transfer their dividend income to the investors *ex post*, which increases $d_{0,i}$ *ex ante*.

Solve for $d_{0,i}$ to get

$$d_{0,i} = \frac{1-k}{1+k}\gamma\mathbb{E}[R_{s,i}].$$

Then one would ask: under what conditions would it make sense to introduce equity requirements? It is easy to see that introducing equity will definitely reduce investor's payoff in the absence of aggregate risk. Somewhat counterintuitive, capital requirements even reduces the share α invested in the safe project in that case. The reason is that with equity, bank managers get a rent of $\frac{\gamma\mathbb{E}[R]-d_0}{2}$, sharing the surplus over deposits equally with the equity holders. So investors providing funds in form of both deposits and equity to the banks will get out at $t = 1$ just $\frac{1}{1+k}\gamma\mathbb{E}[R] < \gamma\mathbb{E}[R]$. Since return at $t = 2$ is higher than at $t = 1$, bank managers prefer to consume late, so the amount of resources needed at $t = 1$ is lower in the presence of equity. Consequently, the share α will be reduced. Of course, banks holding no equity provide more attractive conditions for investors, so equity could not survive. This at first sight counterintuitive result simply demonstrates that there is no role (or rather only a payoff reducing role) for costly equity in the absence of aggregate risk.

But when there is aggregate risk, equity helps to absorb the aggregate shock. In the simple 2-state set up, equity holdings need to be just sufficient to cushion the bad state. So with equity, the bank will chose $\alpha^* = \alpha(p_H)$. The level of equity k needs to be so high that, given $\alpha^* = \alpha(p_H)$, the bank just stays solvent in the bad state — it is just able to payout the fixed claims of depositors, whereas all equity will be wiped out.

With equity k, the total amount that can be pledged to both depositors and equity in the good state is $\frac{1}{1+k}\gamma\mathbb{E}[R_H]$ with claims of depositors being $d_0 = \frac{1-k}{1+k}\gamma\mathbb{E}[R_H]$ and equity $EQ = \frac{k}{1+k}\gamma\mathbb{E}[R_H]$. In the bad state, a marginally solvent bank can pay out to depositors $d_0 = \alpha(p_H)R_1 + (1-\alpha(p_H))p_L R_2$. So k is determined by the condition:

$$\frac{1-k}{1+k}\gamma\mathbb{E}[R_H] = \alpha(p_H)R_1 + (1-\alpha(p_H))p_L R_2,$$

and solve to get

$$k = \frac{\gamma\mathbb{E}[R_H] - d_0}{\gamma\mathbb{E}[R_H] + d_0}. \tag{2.1}$$

It's observed that k is decreasing in p_L: the higher p_L, the lower the equity k needed to stay solvent in the bad state. $k = 0$ for $p_L = p_H$, and for p_L close to p_H equity holding is superior to the strategy $\alpha^* = \alpha(p_H)$. That is if

$$d_0 \geq \gamma\mathbb{E}[R_H]\pi + (1-\pi)c.$$

Such (d_0, k) is the equilibrium for the banks. The reason is easy to see: first, no banks are willing to set higher k_i — because equity holding is costly and she is not able to compete the other banks for $(d_{0,i}, k_i)$; second, no banks are able to set higher $d_{0,i}$ given (d_0, k) set by all the other banks — because k has to be met when $d_{0,i}$ is paid, the only thing the deviator can do is to bid up interest rate and this leads to bank runs across the whole banking industry — the deviation is not profitable.

From the regulator's point of view, the unique optimal equity requirement k it imposes is exactly the k determined by condition (2.1), which is so high that the bank just stays solvent in the bad state — it is just able to payout the fixed claims of depositors, whereas all equity will be wiped out. The reason is simple: since equity holding is costly, the only reason for the central bank to make it sensible is to eliminate the costly bank run. Therefore neither too low k (which is purely a cost and doesn't prevent any bank run) nor too high k (which prevent bank runs, but incurs a too high cost of holding equity) is optimal. Thus from now on we can concentrate on such level of k without loss of generality.

Now the interesting question is: can capital requirement improve the allocation in this economy, in comparison to the *laissez-faire* outcome we studied before?

Definition Define a representative depositor's expected return function without equity requirements as $\Pi(\pi, \cdot)$, such that

$$\Pi(\pi, \cdot) = \begin{cases} \gamma \mathbb{E}[R_L], & \text{if} \quad \pi \in [0, \overline{\pi}_1]\,; \\ \alpha_s^* R_1 + (1 - \alpha_s^*)\, p_L R_2, & \text{if} \quad \pi \in (\overline{\pi}_1, \overline{\pi}_2)\,; \\ \gamma \mathbb{E}[R_H]\pi + (1 - \pi)c, & \text{if} \quad \pi \in [\overline{\pi}_2, 1] \end{cases}$$

and her expected return function under equity requirements as $\Pi_e(\pi, \cdot)$, as well as the set S in which the investor's payoff is improved under equity requirement, such that

$$S := \{\hat{\pi} | \Pi_e(\hat{\pi}, \cdot) \geq \Pi(\hat{\pi}, \cdot)\}\,. \quad \blacksquare$$

The blue lines of Fig. 2.5 describe the *laissez-faire* outcome $\Pi(\pi, \cdot)$, and the red line shows the depositors expected return $\Pi_e(\pi, \cdot) = d_0 + \frac{\Pi}{2}\pi$ under capital requirement, which consists of two terms:

- The deposit payment d_0;

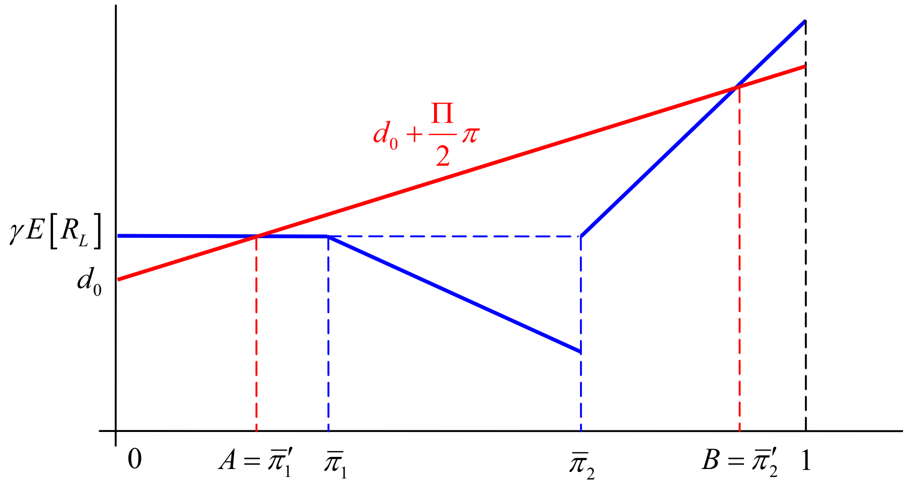

Fig. 2.5 Expected return with / without equity — Case 1

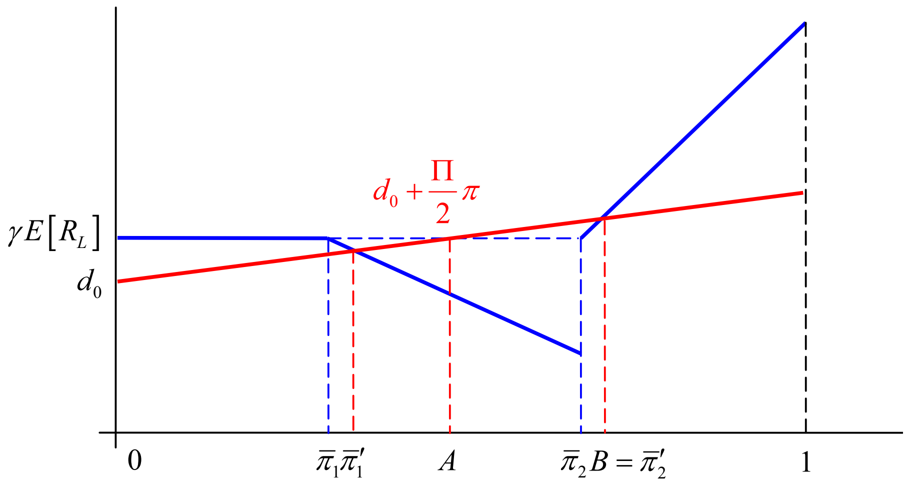

Fig. 2.6 Expected return with / without equity — Case 2

- The dividend of equity holdings $\frac{\Pi}{2}$, which is only achieved in the good state, and its value is determined by

$$\frac{\Pi}{2} = \frac{\gamma\mathbb{E}[R_H] - d_0}{2} = \frac{\gamma\mathbb{E}[R_H] - \frac{1-k}{1+k}\gamma\mathbb{E}[R_H]}{2} = \frac{k}{1+k}\gamma\mathbb{E}[R_H].$$

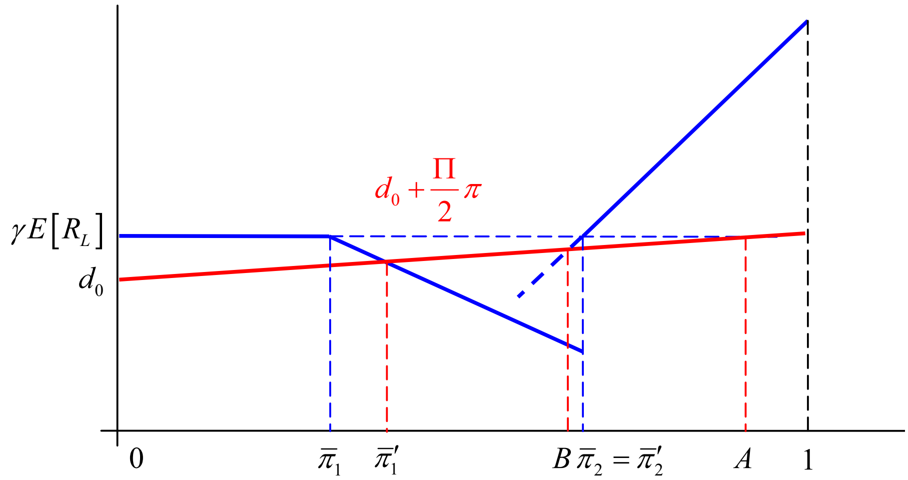

Fig. 2.7 Expected return with / without equity — Case 3

Denote the intersection of $\Pi_e(\pi, \cdot) = d_0 + \frac{\Pi}{2}\pi$ and $\gamma\mathbb{E}[R_L]$ by A, which is equal to (see Appendix A.1.4 for detail)

$$A = \frac{2(R_1 - p_L R_2)}{(1-\gamma)R_1 + (\gamma - p_L)R_2},$$

as well as the intersection of $\Pi_e(\pi, \cdot) = d_0 + \frac{\Pi}{2}\pi$ and $\gamma\mathbb{E}[R_H]\pi + (1-\pi)c$ by B, which is equal to (see Appendix A.1.4 for detail)

$$B = \frac{2\left[(1-\gamma)(cR_1 - p_L R_1 R_2) + (\gamma - p_H)(cR_2 - R_1 R_2)\right]}{2(1-\gamma)cR_1 + 2(\gamma - p_H)cR_2 + \left[\gamma(p_H - 1) - (\gamma - p_H) - (1-\gamma)p_L\right] R_1 R_2}.$$

Now it's straightforward to compare investor's payoff under equity requirements with the *laissez-faire* free-riding equilibrium for some extreme values:

Lemma 2.4.1 *The depositors' expected return under equity requirement is lower than the* laissez-faire *outcome when* $\pi = 0$ *or* $\pi = 1$. ▮

Proof See Appendix A.1.3. ▮

The intuition of Lemma 2.4.1 is straightforward: there is no uncertainty when $\pi = 0$ or $\pi = 1$, so it's inferior to hold costly equities as we already explained before.

Then Proposition 2.4.2 characterizes the improvement in investor's payoff achievable by introducing equity requirements.

Proposition 2.4.2 *Given equity requirement k imposed by the regulator,*

- *When* $A \in (0, \overline{\pi}_1]$, *i.e.*

  $$(2\gamma R_2 - \gamma \mathbb{E}[R_H] - d_0)(\gamma \mathbb{E}[R_L] - d_0) + (2\gamma \mathbb{E}[R_L] - \gamma \mathbb{E}[R_H] - d_0)(d_0 - c) \leq 0,$$

  then $S = [A, B] \supseteq [\overline{\pi}_1, \overline{\pi}_2]$;

- *When* $A \in (\overline{\pi}_1, \overline{\pi}_2]$, *i.e.*

  $$(2\gamma R_2 - \gamma \mathbb{E}[R_H] - d_0)(\gamma \mathbb{E}[R_L] - d_0) + (2\gamma \mathbb{E}[R_L] - \gamma \mathbb{E}[R_H] - d_0)(d_0 - c) > 0,$$

  and

  $$\gamma(\mathbb{E}[R_H] - \mathbb{E}[R_L])(d_0 - c) \geq (\gamma \mathbb{E}[R_H] - c)(\gamma \mathbb{E}[R_L] - d_0),$$

  then $S = [\tilde{\pi}, B]$ *in which* $\tilde{\pi} \in (\overline{\pi}_1, \overline{\pi}_2]$ *and* $S \bigcap [\overline{\pi}_1, \overline{\pi}_2] = [\tilde{\pi}, \overline{\pi}_2]$;

- *When* $A \in (\overline{\pi}_2, 1]$, *i.e.*

  $$2(\gamma \mathbb{E}[R_L] - d_0)(\gamma \mathbb{E}[R_H] - c) \geq (\gamma \mathbb{E}[R_H] - d_0)(\gamma \mathbb{E}[R_L] - c),$$

  then $S \subseteq [\tilde{\pi}, B]$ *in which* $\tilde{\pi} \in (\overline{\pi}_1, \overline{\pi}_2]$ *and* $S \bigcap [\overline{\pi}_1, \overline{\pi}_2] = [\tilde{\pi}, \overline{\pi}_2]$. ∎

Proof See Appendix A.1.4. ∎

The three possible cases are characterized in Fig. 2.5, 2.6 and 2.7, respectively. Numerical examples simulating these cases are presented in the Appendix A.2.

Equity requirements give investors a higher payoff than the *laissez-faire* market outcome whenever their payoff with a safe bank holding sufficient equity exceeds the payoff of the mixed strategy equilibrium with free-riding banks for all parameter values. This case is captured as case 1, shown in Fig. 2.5. Since free-riding partly destroys the value of deposits held by prudent banks (forcing them to hold a riskier portfolio), it seems obvious that imposing equity requirements will always dominate the *laissez-faire* outcome with mixed strategies. Unfortunately, this need not be the case. It is quite likely that equity requirements result in inferior payoffs for some range of parameter values (as shown in case 2 — see Fig. 2.6). It might even be that imposing equity requirements makes investors worse than *laissez-faire* for all parameter values. This is shown in Fig. 2.7, representing case 3.

The intuition behind this at first surprising result is that holding equity can be quite costly; if so, it may be superior to accept the fact that systemic risk is a price to be paid for higher returns on average.

The mix of *ex ante* liquidity requirements with *ex post* Lender of Last Resort policy is always dominating equity requirements. See Fig. 2.8. The reason is as following: consider that the banks are required to hold $\underline{\alpha} = \alpha(p_H)$ when π is high. Then when p_H reveals, the investor's real return is $\gamma\mathbb{E}[R_H]$; and when p_L reveals, the investor's real return is $\alpha(p_H)R_1 + (1-\alpha(p_H))p_L R_2$. Therefore the investor's overall expected return turns out to be

$$\Pi_m = \gamma\mathbb{E}[R_H]\pi + (1-\pi)\left[\alpha(p_H)R_1 + (1-\alpha(p_H))p_L R_2\right],$$

which is linear in π, as the green line of Fig. 2.8 shows. Note that when $\pi = 1$, $\Pi_m = \gamma\mathbb{E}[R_H] > d_0 + \frac{\Pi}{2}\pi$; and when $\pi = 0$, $\Pi_m = \alpha(p_H)R_1 + (1-\alpha(p_H))p_L R_2 = d_0$. Therefore, Π_m line is above $d_0 + \frac{\Pi}{2}\pi$, $\forall\pi \in (0,1]$, i.e. the mix of liquidity requirements with Lender of Last Resort policy is always dominating equity requirements when aggregate uncertainty exists.

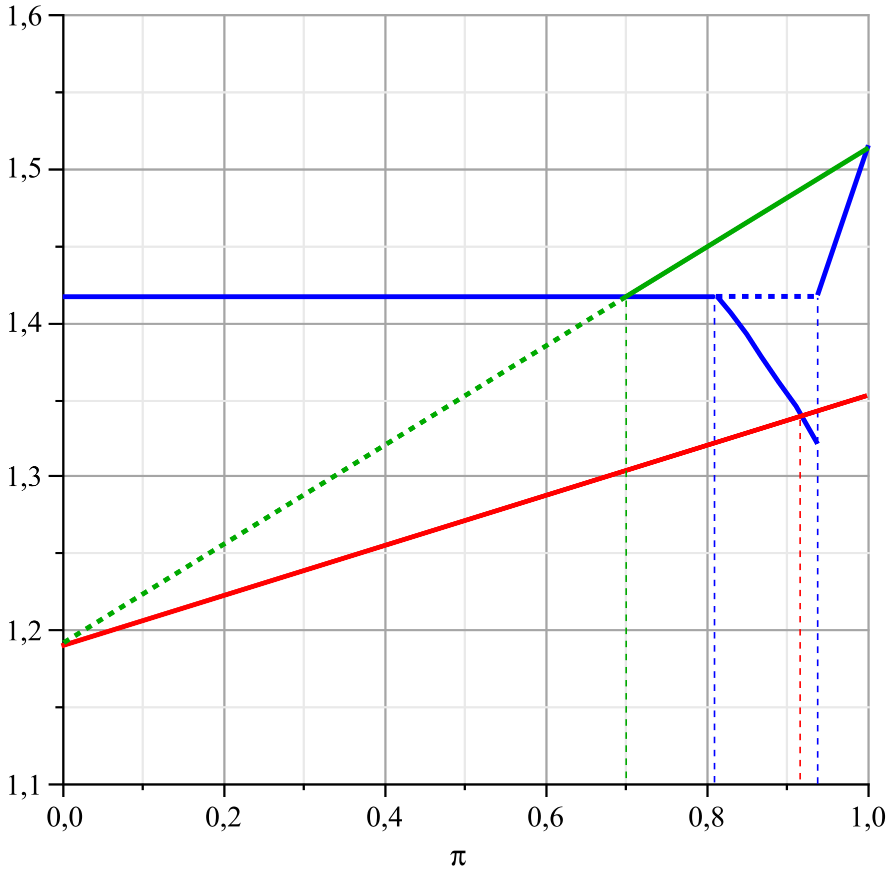

Fig. 2.8 Expected return with credible liquidity injections (for the case of Fig. A.4)

In times of crises, frequently there are calls to go back to narrow banking in order to avoid the risk of runs. Under narrow banking, institutions with deposits would be required to hold as assets only the most liquid instruments so as to be always able to meet any deposit withdrawal by selling its assets. Obviously, narrow banking can be extremely costly. In our model, banks would be required to hold sufficient liquid funds to pay out in all contingencies: $\alpha > \alpha(p_L)$. As Fig. 2.9 illustrates, under narrow banking investor's payoff can be much lower for high π compared to *ex ante* liquidity regulation combined with *ex post* Lender of Last Resort policy. Just as with equity requirements, narrow banking (imposing the requirement that banks hold sufficient equity so as to be able to pay out demand deposits in all states of the world) can be quite inferior: if the bad state is a rare probability event, it simply makes no sense to dispense with all the efficiency gains out of investing in high yielding illiquid assets despite its impact on systemic risk.

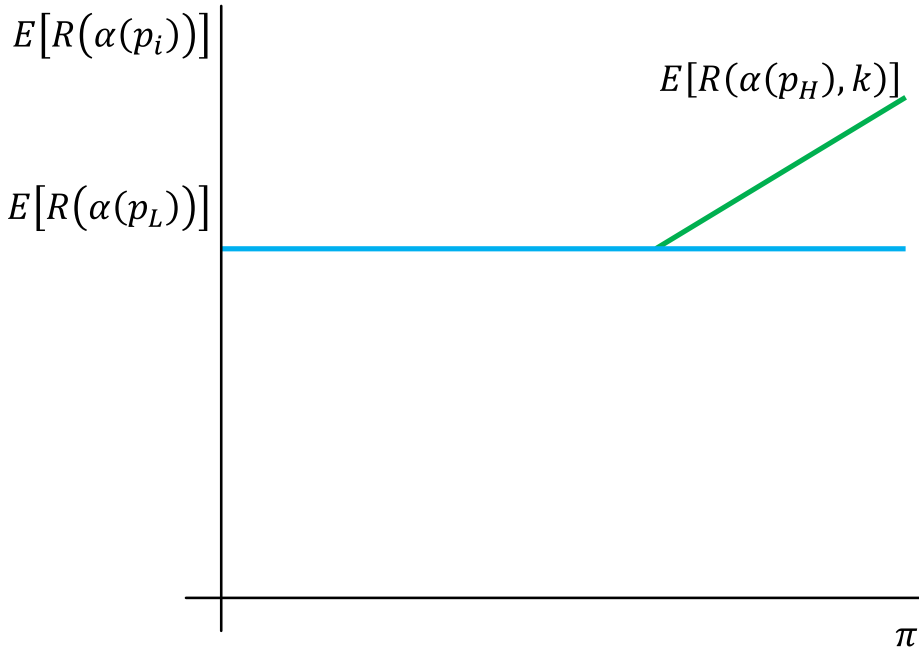

Fig. 2.9 Expected return with narrow banking compared to *ex ante* liquidity regulation

2.5 CONCLUSION

Traditionally, aggregate liquidity shocks have been modelled as exogenous events. In this chapter, we derive the aggregate share of liquid projects endogenously. It depends on the incentives of financial intermediaries to invest in risky, illiquid projects. This endogeneity allows us to capture the feedback between financial market regulation and incentives of private banks, determining the aggregate amount of liquidity available.

We model (real) illiquidity in the following way: liquid projects are realized early. Illiquid projects promise a higher return, but a stochastic fraction of these type of projects will be realized late. We concentrate on pure illiquidity risk: there will never be insolvency unless triggered by illiquidity (by a bank run). Financial intermediaries choose the share invested in high yielding but less liquid assets. As a consequence of limited liability, banks are encouraged to free-ride on liquidity provision. Relying on sufficient liquidity provided by the market, they are inclined to invest excessively in illiquid long term projects.

Liquidity provision by central banks can help to prevent bank runs with inefficient early liquidation. In the last chapter, we showed that the anticipation of unconditional liquidity provision results in overinvestment in risky activities (moral hazard), creating excessive exposure to systemic risk.

Extending our previous work, this chapter analyzes the adequate policy response to endogenous systemic liquidity risk, providing a cost / benefit analysis of different forms of banking regulation to better to understand what type of intervention is desired. We explicitly compare the impact both of liquidity and equity requirements.

We show that it is crucial for efficient Lender of Last Resort policy to impose *ex ante* minimum liquidity standards for banks. In addition, we analyze the impact of equity requirements in the following sense: banks are required to hold sufficient equity so as to pay out fixed claims of depositors in all contingencies. We prove that such a policy is strictly inferior to imposing minimum liquidity standards *ex ante* combined with Lender of Last Resort policy. We show that it is even likely to be inferior relative to the outcome of a mixed strategy equilibrium with free-riding banks. For similar reasons, imposing narrow banking (require banks to hold sufficient liquid funds to pay out in all contingencies) turns out to be strictly inferior relative to the combination of liquidity requirements with Lender of Last Resort policy.

By purpose, our model focuses on the case of pure liquidity risk. Since the return of all projects is non-stochastic as long as they finally can be realized, there is no insolvency unless triggered by illiquidity. Given that insolvency is not an issue, it may not be surprising that there is no role for equity requirements. After all, in our set up equity is always costly, since it allows bank managers to extract rents. We expect that equity requirements can improve the allocation when we allow solvency to be of concern (by making return of illiquid projects at period 2 stochastic). We leave it for the next chapter to analyze that issue.

Following Diamond & Rajan (2006), we model financial intermediation via traditional banks offering fragile deposit contracts. Systemic risk is triggered by bank runs. In modern economies, a significant part of intermediation is provided by the shadow banking sector. These institutions (like hedge funds and investment banks) are not financed via deposits, but they are highly leveraged. Incentives to dance (to free-ride on liquidity provision) seem to be even stronger for the shadow banking industry. So imposing liquidity requirements only for the banking sector will not be sufficient to cope with free-riding. In future work, we plan to analyze incentives for leveraged institutions within our framework.

Appendix

A.1 PROOFS

A.1.1 Proof of Proposition 2.2.1

The mixed strategy equilibrium is characterized as Proposition 1.4.2 of the last chapter. By chooseing α_s^* a prudent bank should have equal return at both states, $d_0^s = d_0^s(p_H) = d_0^s(p_L)$, i.e.

$$\begin{aligned} & \gamma\left[\alpha_s^* R_1 + (1-\alpha_s^*)p_H R_2 + \frac{(1-\alpha_s^*)(1-p_H)R_2}{r_H}\right] \\ = \; & \gamma\left[\alpha_s^* R_1 + (1-\alpha_s^*)p_L R_2 + \frac{(1-\alpha_s^*)(1-p_L)R_2}{r_L}\right]. \end{aligned}$$

With some simple algebra this is equivalent to

$$\frac{1}{r_H} = \frac{1-p_L}{1-p_H}\frac{1}{r_L} - \frac{p_H - p_L}{1-p_H}.$$

Plot $\frac{1}{r_H}$ as a function of $\frac{1}{r_L}$ as Fig. A.1 shows.

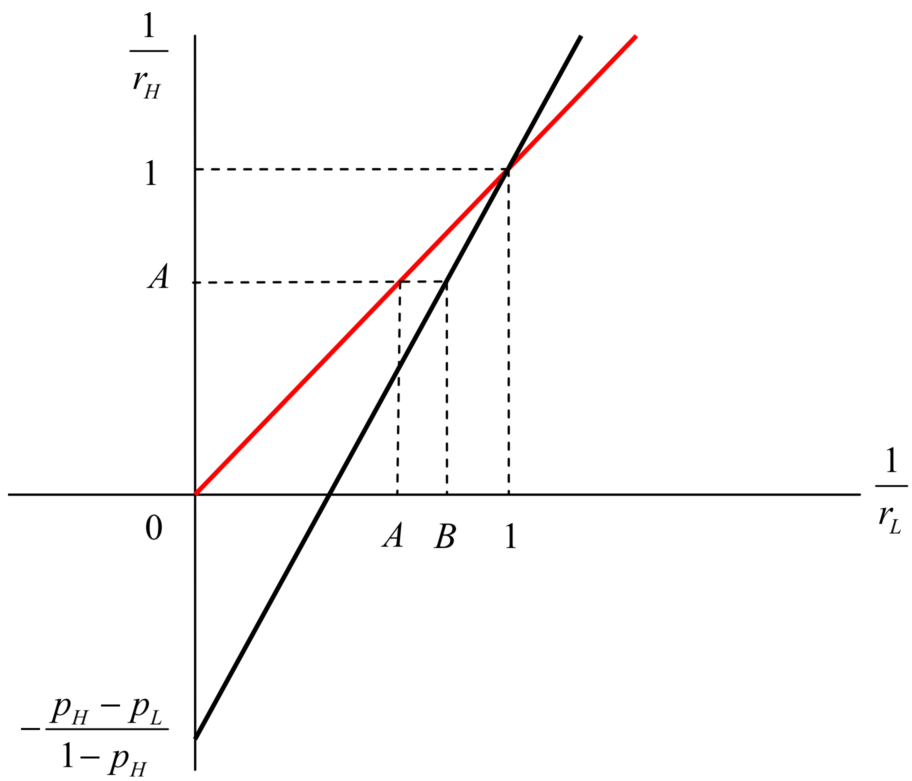

Fig. A.1 Higher interest rates in the mixed strategy equilibrium

The slope $\frac{1-p_L}{1-p_H} > 1$ and intercept $-\frac{p_H-p_L}{1-p_H} < 0$, and the line goes through $(1, 1)$. But $r_H = r_L = 1$ cannot be equilibrium outcome here, because $\alpha(p_L)$ is dominant strategy in this case and subject to deviation. So whenever $r_H > 1$ (suppose $\frac{1}{r_H} = A$ in the graph), there must be $r_H > r_L > 1$ (because $\frac{1}{r_H} < \frac{1}{r_L} = B < 1$).

At p_L, given that $r_L > 1$ the prudent bank's return is equal to $d_0^s = \kappa(\alpha_s^*(p_L, r_L)) < \kappa(\alpha(p_L))$, since the latter maximizes the bank's expected return with $r^* = 1$ by Lemma 1.3.2 of the last chapter. Therefore in the mixed strategy equilibrium, investors are worse off than if all banks would coordinate on the prudent strategy $\alpha(p_L)$. ∎

A.1.2 Proof of Proposition 2.3.1

Suppose that a representative bank chooses to be prudent with $\alpha_i = \underline{\alpha}$, and promises a nominal deposit contract $d_0^i = \gamma\left[\underline{\alpha}R_1 + (1-\underline{\alpha})R_2\right]$ in order to maximize its investors return. Then when the bad state with high liquidity needs is realized, the central bank has to inject enough liquidity into the market to keep interest rate at $r = 1$ in order to ensure bank i's survival. However, given $r = 1$, a naughty bank j can always profit from setting $\alpha_j = 0$, promising the nominal return $d_0^j = \gamma R_2 > d_0^i$ to its investors. Thus, surely the banks prefer to play naughty.

For those parameter values such that $\pi p_H R_2 + (1-\pi)p_L R_2 < 1$ there exists no equilibrium with liquidity injection. The reason is the following:

1. Any symmetric strategic profile cannot be equilibrium, because
   (a) If there is no trade under such strategic profile, i.e. α is so small that the real return is less than 1, one bank can deviate by setting $\alpha = 1$ and trading with investors;
   (b) If there is trade under such strategic profile, i.e. $\alpha > 0$ for all the banks, then one bank can deviate by setting $\alpha = 0$ and getting higher nominal return than the other banks.
2. Any asymmetric strategic profile, or profile of mixed strategies, cannot be equilibrium, because
   (a) If there is no trade under such strategic profile, then the argument of 1 (a) applies here;
   (b) If there is trade under such strategic profile, then one bank can deviate by choosing a pure strategy, $\alpha = 0$, and get better off — there is no reason to mix with the other dominated strategies. ∎

A.1.3 Proof of Lemma 2.4.1

When $\pi = 0$,

$$\begin{aligned} d_0 + \frac{\Pi}{2} \cdot 0 &= \alpha(p_H) R_1 + (1 - \alpha(p_H)) p_L R_2 \\ &< \alpha(p_L) R_1 + (1 - \alpha(p_L)) p_L R_2 \\ &= \gamma \mathbb{E}[R_L]; \end{aligned}$$

When $\pi = 1$,

$$\begin{aligned} d_0 + \frac{\Pi}{2} &= \frac{\alpha(p_H) R_1 + (1 - \alpha(p_H)) p_L R_2 + \alpha(p_H) R_1 + (1 - \alpha(p_H)) p_H R_2}{2} \\ &< \alpha(p_H) R_1 + (1 - \alpha(p_H)) p_H R_2 \\ &= \gamma \mathbb{E}[R_H]. \quad \blacksquare \end{aligned}$$

A.1.4 Proof of Proposition 2.4.2

Generically, there are three cases concerning the relative positions of $\Pi(\pi, \cdot)$ and $\Pi_e(\pi, \cdot)$:

1. As Fig. 2.5 shows, the intersection A lies between 0 and $\overline{\pi}_1$;

2. As Fig. 2.6 shows, the intersection A lies between $\overline{\pi}_1$ and $\overline{\pi}_2$;

3. As Fig. 2.7 shows, the intersection A lies between $\overline{\pi}_2$ and 1.

The intersection A takes the value of π, such that

$$\gamma \mathbb{E}[R_L] = d_0 + \frac{\Pi}{2} \pi.$$

Solve to get

$$A = \frac{2(\gamma \mathbb{E}[R_L] - d_0)}{\gamma \mathbb{E}[R_H] - d_0} = \frac{2(R_1 - p_L R_2)}{(1 - \gamma) R_1 + (\gamma - p_L) R_2}.$$

The intersection B takes the value of π, such that

$$\gamma\mathbb{E}[R_H]\pi + (1-\pi)c = d_0 + \frac{\Pi}{2}\pi.$$

Solve to get

$$\begin{aligned} B &= \frac{d_0 - c}{\frac{\gamma\mathbb{E}[R_H]+d_0}{2} - c} \\ &= \frac{2\left[(1-\gamma)(cR_1 - p_L R_1 R_2) + (\gamma - p_H)(cR_2 - R_1 R_2)\right]}{2(1-\gamma)cR_1 + 2(\gamma - p_H)cR_2 + \left[\gamma(p_H - 1) - (\gamma - p_H) - (1-\gamma)p_L\right]R_1 R_2}. \end{aligned}$$

Then the set S can be determined in each case:

1. As Fig. 2.5 shows, when $A \in (0, \overline{\pi}_1]$,

$$\frac{2\left(\gamma\mathbb{E}[R_L] - d_0\right)}{\gamma\mathbb{E}[R_H] - d_0} \le \overline{\pi}_1 = \frac{\gamma\mathbb{E}[R_L] - c}{\gamma R_2 - c},$$

rearrange to get

$$\begin{aligned} &\left(2\gamma R_2 - \gamma\mathbb{E}[R_H] - d_0\right)\left(\gamma\mathbb{E}[R_L] - d_0\right) + \left(2\gamma\mathbb{E}[R_L] - \gamma\mathbb{E}[R_H] - d_0\right)\left(d_0 - c\right) \\ \le\ & 0. \end{aligned}$$

Since $\Pi_e(\pi, \cdot)$ is strictly increasing in π, then

$$\begin{aligned} &\Pi_e(\pi,\cdot)|_{\pi=B} > \Pi_e(\pi,\cdot)|_{\pi=A} \ge \gamma\mathbb{E}[R_L]|_{\pi=\overline{\pi}_1} = \left(\gamma\mathbb{E}[R_H]\pi + (1-\pi)c\right)|_{\pi=\overline{\pi}_2} \\ \ge\ & \Pi(\pi,\cdot)|_{\pi\in[\overline{\pi}_1,\overline{\pi}_2]}, \end{aligned}$$

which implies $S = [A, B] \supseteq [\overline{\pi}_1, \overline{\pi}_2]$;

2. As Fig. 2.6 shows, when $A \in (\overline{\pi}_1, \overline{\pi}_2]$,

$$\overline{\pi}_1 = \frac{\gamma\mathbb{E}[R_L] - c}{\gamma R_2 - c} < \frac{2\left(\gamma\mathbb{E}[R_L] - d_0\right)}{\gamma\mathbb{E}[R_H] - d_0},$$

rearrange to get

$$\begin{aligned}&(2\gamma R_2 - \gamma\mathbb{E}[R_H] - d_0)(\gamma\mathbb{E}[R_L] - d_0) + (2\gamma\mathbb{E}[R_L] - \gamma\mathbb{E}[R_H] - d_0)(d_0 - c)\\ > \quad & 0.\end{aligned}$$

What's more, in this case $B \in [\overline{\pi}_2, 1]$, and this is equivalent to

$$\frac{\gamma\mathbb{E}[R_L] - c}{\gamma\mathbb{E}[R_H] - c} = \overline{\pi}_2 < \frac{d_0 - c}{\frac{\gamma\mathbb{E}[R_H]+d_0}{2} - c},$$

rearrange to get

$$\gamma\left(\mathbb{E}[R_H] - \mathbb{E}[R_L]\right)(d_0 - c) \geq (\gamma\mathbb{E}[R_H] - c)(\gamma\mathbb{E}[R_L] - d_0).$$

Similarly,

$$\begin{aligned}&\Pi_e(\pi,\cdot)|_{\pi\leq A} \leq \gamma\mathbb{E}[R_L]|_{\pi=\overline{\pi}_1} = (\gamma\mathbb{E}[R_H]\pi + (1-\pi)c)\,|_{\pi=\overline{\pi}_2} \leq \Pi(\pi,\cdot)|_{\pi\in[\overline{\pi}_2,B]}\\ \leq \quad & \Pi_e(\pi,\cdot)|_{\pi\geq B},\end{aligned}$$

which implies $S = [\tilde{\pi}, B]$ in which $\tilde{\pi} \in (\overline{\pi}_1, \overline{\pi}_2]$ and $S \bigcap [\overline{\pi}_1, \overline{\pi}_2] = [\tilde{\pi}, \overline{\pi}_2]$;

3. As Fig. 2.7 shows, when $A \in (\overline{\pi}_2, 1]$,

$$\overline{\pi}_2 = \frac{\gamma\mathbb{E}[R_L] - c}{\gamma\mathbb{E}[R_H] - c} < \frac{2\left(\gamma\mathbb{E}[R_L] - d_0\right)}{\gamma\mathbb{E}[R_H] - d_0},$$

rearrange to get

$$2\left(\gamma\mathbb{E}[R_L] - d_0\right)(\gamma\mathbb{E}[R_H] - c) \geq (\gamma\mathbb{E}[R_H] - d_0)(\gamma\mathbb{E}[R_L] - c).$$

Similarly,

$$\Pi_e(\pi,\cdot)|_{\pi\le B} < \Pi_e(\pi,\cdot)|_{\pi\ge A} \le \gamma\mathbb{E}[R_L]|_{\pi=\overline{\pi}_1} = (\gamma\mathbb{E}[R_H]\pi + (1-\pi)c)\,|_{\pi=\overline{\pi}_2},$$

which implies $S \subseteq [\tilde{\pi}, B]$ in which $\tilde{\pi} \in (\overline{\pi}_1, \overline{\pi}_2]$ and $S \bigcap [\overline{\pi}_1, \overline{\pi}_2] = [\tilde{\pi}, \overline{\pi}_2]$.
∎

A.2 RESULTS OF NUMERICAL SIMULATIONS

The following figures present numerical simulations representing the three different cases.

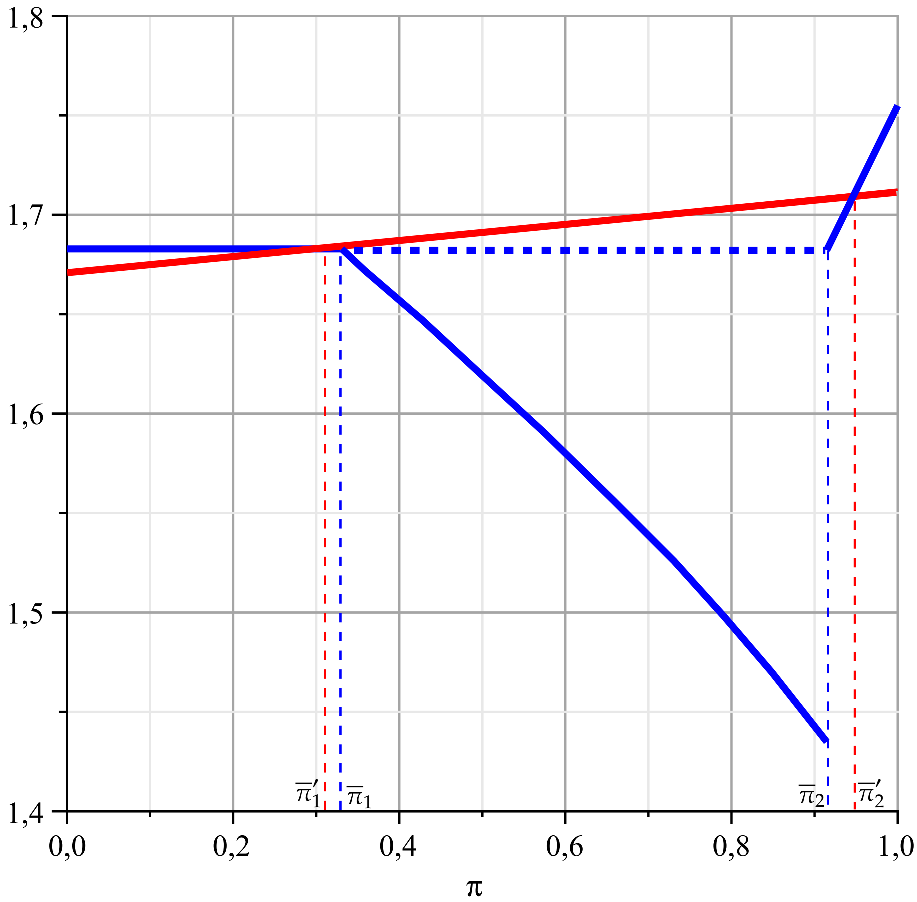

Fig. A.2 Expected return with / without equity, with $p_H = 0.3$, $p_L = 0.25$, $\gamma = 0.6$, $R_1 = 1.8$, $R_2 = 5.5$, $c = 0.9$

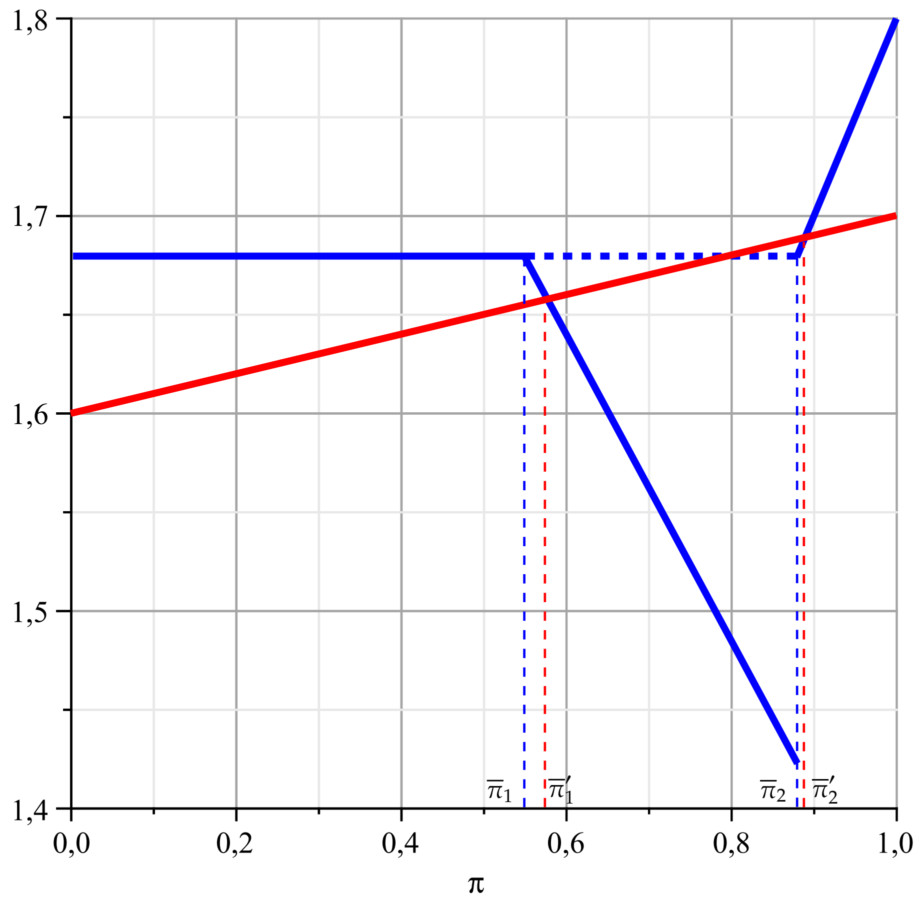

Fig. A.3 Expected return with / without equity, with $p_H = 0.4$, $p_L = 0.3$, $\gamma = 0.6$, $R_1 = 2$, $R_2 = 4$, $c = 0.8$

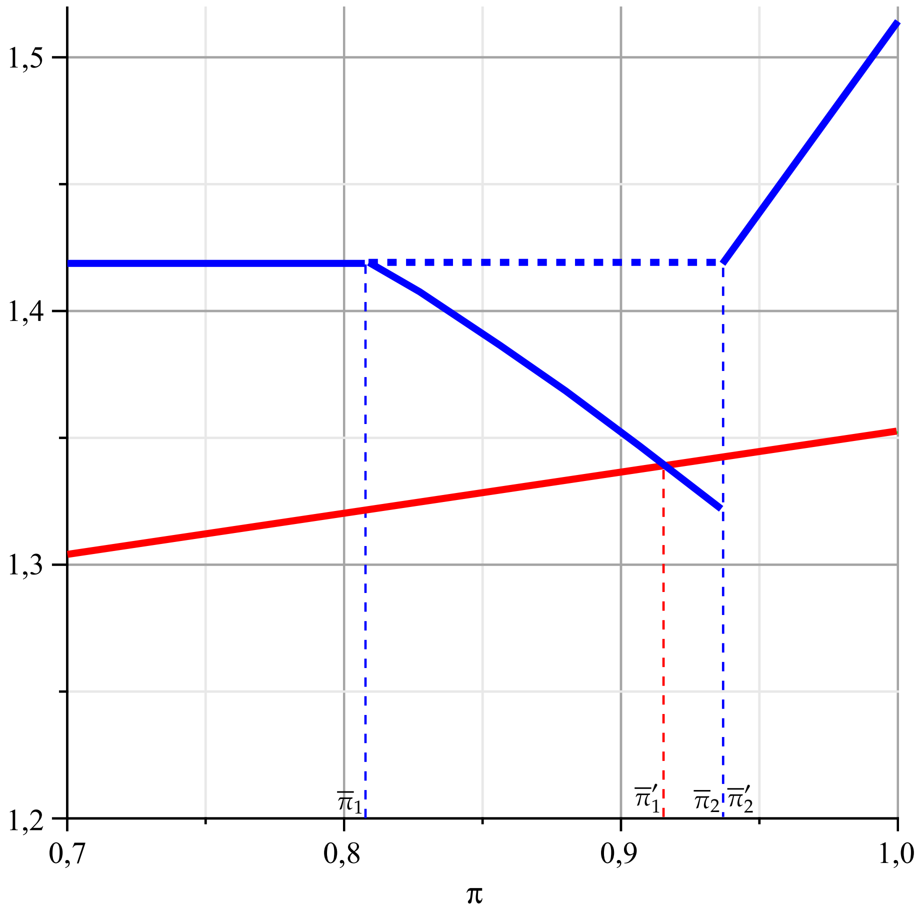

Fig. A.4 Expected return with / without equity, with $p_H = 0.5$, $p_L = 0.25$, $\gamma = 0.7$, $R_1 = 1.8$, $R_2 = 2.5$, $c = 0$

3

Illiquidity, Insolvency, and Banking Regulation

There is no single big remedy for the banks' flaws. But better rules — and more capital — could help.

—"Three trillion dollars later ...". *The Economist*, May 14th, 2009

3.1 INTRODUCTION

In banking literature, illiquidity and insolvency problems have been intensively studied for decades. Illiquidity means that one financial institution is not able to meet its short term liability via monetizing the future gains from its long term projects — in other words, there's only a mismatch between the time when the long term projects return and the time when its liability is due, i.e. it's "cash flow trapped" but "balance sheet solvent". In contrast, insolvency of one financial institution generally means in its balance sheet liabilities exceed assets, i.e. it is not able to meet due liabilities even by perfectly monetizing the future gains from its long term projects. Existing banking models usually focuses on either problems; and if a financial firm's ailment is diagnosed to be one of them, the solution is then (at least intuitively) clear. For example, illiquid banks may be bailed out by central bank's liquidity injection (against their illiquid assets "good" collateral, see CHAPTER 1, 2 or Cao & Illing, 2009a, b), and insolvent banks have to be closed down for avoiding contagion (see Freixas, Parigi and Rochet, 2004).

Since mid-2007, the world has seen one of the worst financial crises in history, which has stolen millions of jobs and held the entire global economy to ransom. As is observed in the past two years, one prominent feature about this crisis is the ambiguity in the financial institutions' health, especially the daunting question whether the problem for the large banks is illiquidity or insolvency. Financial innovation in the past two decades doesn't only help improve market efficiency, but also creates high complexity (hence, asymmetric information) which blurs the boundary between illiquidity and insolvency. The over complicated financial products, as Gorton (2009) states, finally "could not be penetrated by most investors or counterparties in the financial system to determine the location and size of the risks." For example, subprime mortgages, a financial innovation and from which the current crisis broke out, were designed to finance riskier long-term borrowers via short-term funding. So when the trend of continuing US house price appreciation started to stagger and giant investment banks came into trouble, the trouble seemed to be mere illiquidity problem — as long as house price goes higher in the future, the long-term yields of subprime mortgages related assets will be juicy as well. However, since the location and size of the risks in these complicated financial products could not be fully perceived even by the designer banks themselves, there was a probability that these financial institutions were insolvent. With this ambiguity banks could hardly get sufficient liquidity from market and the crisis erupted.

Such new feature brings new challenges to both market practitioners and banking regulators. If there's no ambiguity between illiquidity and insolvency, conventional wisdoms work well: if the problem is just illiquidity, then liquidity regulation works perfectly — banks can get enough liquidity from the central bank with their long-term assets as collateral, since the high yields from these assets will return in the future for sure. If the problem is just insolvency, then equity holding can be a self-sufficient solution for the banks to get rid of the losses. However, if there's uncertainty about the banks' trouble, things become complicated — banks cannot get enough liquidity because the collateral, in the presence of insolvency risk, is no longer considered to be good, therefore liquidity regulation may fail; on the other hand equity requirement may be inefficient as well because the dual problems make equity holding even costlier. This chapter is thus going to step into the troubled water, hoping to shed some light on understanding the market failure and designing proper regulatory rules with a compact and flexible model.

3.1.1 Summary of the chapter

In this chapter, banks are intermediaries financing entrepreneurs' short-term (safe) and long-term (risky) projects via short-term deposit contracts, as the standard view such as Diamond & Rajan (2006). Illiquidity is modelled in the following way as Chapter 2, or Cao & Illing (2009a): some fraction of risky projects turns out to be realized late. The aggregate share of late projects is endogenous; it depends on the incentives of financial intermediaries to invest in risky, illiquid projects. This endogeneity captures the feedback from liquidity provision to risk taking incentives of financial intermediaries.

Departing from models with pure illiquidity or insolvency problems, in the intermediate period the market participants only observe the aggregate amount of early returns from the risky projects, but they don't know whether these risky assets are just illiquid (i.e. the majority of high yield risky projects will return late) or the banks are insolvent (i.e. the substantial amount of the risky projects will fail in the next period). The introduction of such ambiguity has both significant impacts on equilibrium outcome and new implications for banking regulation.

Given the same structure of the banking game as in Chapter 1, or Cao & Illing (2008, 2009a), the equilibria in this extended model are similar: two intervals for pure strategy equilibria — the banks coordinate to be risky when the sun always shines and be prudent when it always rains, and mixed strategy equilibrium for mid-range case. However the gap between the expected return from the risky projects in good state and that in bad state gets higher with the ambiguity between the dual plagues — asset price is more inflated in good state because of the probability that the risky assets are just illiquid, while asset price is more depressed in bad state because of the probability that the banks are going to be insolvent. The bigger gap makes the interval for mixed strategy equilibrium wider in current setting, making free-riding more attractive (more excessive liquidity supply when time is good).

New insights have been discovered for banking regulation. Solution for pure illiquidity risk as proposed in Cao & Illing (2009b), *ex ante* liquidity requirement with *ex post* conditional bailout, is not sufficient now. The reason is simple: because the central bank doesn't have superior knowledge to market participants, i.e. it isn't able to distinguish between illiquidity and insolvency risks as well, the value of the banks' collaterals in the bad state cannot be as high as that at that in the good state for differences in their insolvency risks. Therefore, the banks cannot get sufficient liquidity from the central bank in the bad state even they do observe the *ex ante* liquidity requirement. Bank run is thus not avoided any more.

Such finding suggests that the additional insolvency problem implies an extra cost for stabilizing financial system, i.e. the regulator needs extra resources to hedge against the insolvency risk. Therefore, a counter-cyclical deposit insurance mechanism will work. The proposal is as following: the banks have to be taxed away part of their revenue in the good state, and the taxation revenue can be used to cover the gap in central bank's liquidity provision in the bad state.

On the other hand, equity requirement as a typical solution for pure insolvency risk seems to be suboptimal as well. The co-existence of two banking plagues means higher capital ratio, hence higher cost, should be imposed for banking industry.

Since it's hard to catch two rabbits at the same time, it would be optimal to combine the advantages of several instruments. A hybrid regulatory scheme is therefore proposed in this chapter, allowing liquidity regulation to discourage the inferior mixed strategy equilibrium (which leads to liquidity shortage) and equity requirement to absorb the loss from insolvency.

3.1.2 Review of literature

This chapter is a natural extension of the baseline model from the previous chapters, or Cao & Illing (2008, 2009a, 2009b), in a more realistic context. It has been shown that when there is only pure illiquidity risk, there's an incentive for a financial institution to free-ride on liquidity provision from the others, resulting in excessively low liquidity in bad states. Since illiquidity is the only risk, conditional (with *ex ante* liquidity requirements for banks' entry to the financial market) liquidity injection from the central bank fully eliminates the risk of bank runs when bad states are less likely, and the outcome of such conditional bailout policy dominates that of capital requirement scheme since the banks have to incur a substantially high cost of holding equity in order to fully stabilize the system. However, one may ask what happens if there's an additional risk of insolvency. Indeed, when insolvency is mixed with illiquidity and market participants cannot distinguish between them, banks would have difficulties in raising sufficient liquidity using their assets as collateral. This may have profound impacts on both equilibrium outcomes and policy implications, and exploring these issues is the main task of this chapter.

This chapter differs from the main contributions in the existing literature in two respects:

1. This chapter addresses the systemic liquidity risk as an endogenous phenomenon from the joint illiquidity-insolvency problem;

2. Central bank intervention and banking regulation are examined under nominal contracts.

Although illiquidity and insolvency problems have been intensively studied respectively in the banking literature, the endogenous systemic liquidity risk arising from the co-existence of both problems has been rarely investigated. Most past works that analyze these two problems in one model mainly focus on *how* banking crises evolve, rather than *why* the banking industry arrives at the brink of collapse. Therefore, liquidity shortage is usually introduced as an exogenous shock, instead of a strategic outcome. For example, Freixas, Parigi and Rochet (2000, 2004) model systemic liquidity risk out of coordinative failure from the interbank market, and a banking crisis may be triggered by an exogenous insolvency shock; therefore, closing insolvent banks helps cut off the contagion chain and save the system. Taking liquidity risk as (partially) exogenously given surely works well for understanding the development of banking crisis, however, one has to be cautious when applying these models on banking regulation. As is stated in Acharya (2009), "... Such partial equilibrium approach has a serious shortcoming from the standpoint of understanding sources of, and addressing, inefficient systemic risk... " In other words, if we admit that it is equally important to establish proper regulatory rules *ex ante* as to bailout the failing banks *ex post*, it should be equally crucial to ask what causes the failure as to tell how severe the crisis can be, i.e. systemic liquidity risk should be an endogenous phenomenon.

It seems that an increasing number of recent works start analyzing endogenous incentives for systemic risk. Acharya (2009) and Acharya & Yorulmazer (2008) define such incentive as the correlation of portfolio selection, i.e. when the return of a bank's investment has a "systemic factor", the failure of one bank conveys negative information about this factor which makes the market participants worry about the health of the entire banking industry, increasing the bank's probability to fail. The concern of such "informational spillover" induces the banks to herd *ex ante*, leading to an inefficiently high correlation in the banks' portfolio choices. These insights are similar in spirit (but quite different in modelling) as in this chapter (for example, the inefficiently high correlation corresponds to the mixed strategy equilibrium and public information about the early returns means perfect informational spillover); however, since illiquidity problem is not explicitly modelled in their works, liquidity regulation doesn't play any role (in contrast to this chapter).

Recent endogenous approaches to modelling systemic liquidity risk include Wagner (2009, in which inefficiency comes from the externalities of bank runs), Korinek (2008, in which inefficiency comes from the fact that financial institutions don't internalize the impact of asset prices on the production sector), etc. However, to the best of my knowledge, works addressing joint illiquidity-insolvency problem and its impact on macro policy still seem to be rare, if not absent. In this sense, this chapter contributes to understanding the new features in current credit crunch and the lessons for banking regulation.

The mostly closely related work is probably the model considered in Bolton, Santos and Scheinkman (2009a, a.k.a. BSS as in the following). The feature that the market participants can hardly distinguish between illiquidity and insolvency is captured in their model, while they mainly focus on the supply side of liquidity, i.e. liquidity from financial institutions' own cash reserve (*inside liquidity*) or from the proceeds from asset sales to the other investors with longer time preference (*outside liquidity*), and the timing perspective of liquidity trading. This chapter takes BSS's view that (outside) liquidity shortage arises from the banks' coordinative failure, but the timing of liquidity trading is not going to be my focus. Rather, I provide a different explanation of systemic liquidity risk, i.e. liquidity under-provision may come from the banks' incentive of free-riding on each others' liquidity supply, which is not covered in BSS (in which they restrict attentions to pure strategy equilibria); and clear-cut results from a more compact and flexible model in this chapter make it easier to be applied on banking regulation. What's more, since financial contracts in BSS are real, they (BSS, 2009b) conclude that efficiency can be restored by central banks' credible supporting (real) asset prices. However, in contrast, this chapter shows that the introduction of (more realistic) nominal contracts may alter the policy implications drastically — nominal liquidity injection from central banks may crowd out market liquidity supply without improving efficiency, therefore policy makers should take a more careful view on designing regulatory rules and bailout policies.

In banking literature, such inside-outside liquidity approach has been much explored in Holmström and Tirole (1998, 2008), etc. (although their focuses and methodologies are quite different from this chapter). In these works. it has been argued that since private liquidity supply is inefficient, public provision of emergency (real) liquidity as a pure public good improves allocation in the presence of aggregate shocks. However, central banks usually lack the capability of redirecting the economy's real resources to financial sector via lump sum taxation; instead, more likely they can only achieve redistribution through nominal instruments. This view is in line with Allen & Gale (1998), in which public liquidity intervention works through nominal contracts and the price level is adjusted via *cash-in-the-market principle*. Diamond & Rajan (2006) explores this mechanism further, however, unlike this chapter it focuses on monetary policy in banking crisis — liquidity shocks are thus taken as exogenously given.

3.1.3 Structure of the chapter

Section 3.2 presents the baseline model with real deposit contracts. Section 3.2.1 shows the equilibrium when liquidity and solvency shocks are both deterministic, then Section 3.2.2 extends the results with uncertainty in the types of shocks and Section 3.2.3 gives the equilibria of such *laissez-faire* economy. The failure of liquidity regulation is analyzed in Section 3.3.1, and an alternative scheme with additional taxation is proposed. It is shown in Section 3.4 that equity requirement becomes too costly in the presence of dual problems, therefore an improved regulatory scheme combining liquidity regulation and minimum level of capital ratio is discussed. Section 3.5 concludes.

3.2 THE MODEL

In this section the deposit contracts are restricted to be real, i.e. central bank as fiat money issuer is absent in the game. The model is almost the same as that from Chapter 1, or Cao & Illing (2008); the differences are (1) the payoff structure of the risky assets; (2) the information. The basic elements of the game are summarized in Table 3.1 and Fig. 3.1

Agents with different time preferences Three types of risk neutral agents: a continuum of investors (each endowed with unit of resources), N banks (run by bank managers or bankers, engaging in Bertrand competition) and a continuum of entrepreneurs. Impatient investors want to consume one period after investing their endowments, while entrepreneurs and bank managers are indifferent between consuming early or late;

Technologies Investors only have access to inferior storage technology so that they will take the deposit contract if the expected gross return rate from the deposit is higher than 1. Two types of entrepreneurs with different projects: safe (liquid) projects returning $R_1 > 1$ for sure at $t = 1$, risky (illiquid) projects as explained later. Bank managers have

Table 3.1 The basic elements of the extended model: Agents, technologies, and preferences

Investors	• Unit $t = 0$ endowment — stored or invested in projects; • Investors want to consume at $t = 1$.
Entrepreneurs	• With type 1 project — Return $R_1 > 1$, safely realized at $t = 1$; • With type 2 project — Highest return $R_2 > R_1$, risky. It may return at $t = 1$, · but also may be delayed to $t = 2$, or · fail with zero return.
Banks	• Engage in Bertrand competition; • Expertise to collect $0 < \gamma < 1$ from projects return; • Offer deposit contracts — Commitment device not to abuse the expertise, and — Making banking industry fragile; • Risk of bank runs: poor liquidation return $0 < c < 1$.

the expertise in collecting a share γ of the projects' return — a motivation for intermediation;

Timing At $t = 0$ banks compete for investors by providing a take-it-or-leave-it deposit contract $\left(\alpha_i, d_0^i\right)$ in which α_i is the share of bank i's investments on safe projects and d_0^i the promised $t = 1$ return for investors. The illiquid projects' riskiness is unknown at $t = 0$ but partially revealed at $t = \frac{1}{2}$, at which time the investors decide whether to run the banks or wait till $t = 1$. If run, both safe and risky projects have to be liquidated with a poor return $0 < c < 1$;

Limited liability All the financial contracts only have to be met with the debtors' entire plausible resources. For the deposit contracts between investors and banks, when bank run happens only the early withdrawers can get promised d_0^i with the bank's run value; for the liquidity contracts between banks and entrepreneurs at $t = 1$, although in equilibrium the contracted interest rate is bid up by the competing banks to the level that the entrepreneurs seize all the return from the risky projects in the good state of the world at $t = 2$ (the details will be explained later), the entrepreneurs cannot claim more than the actual yields in the bad state.

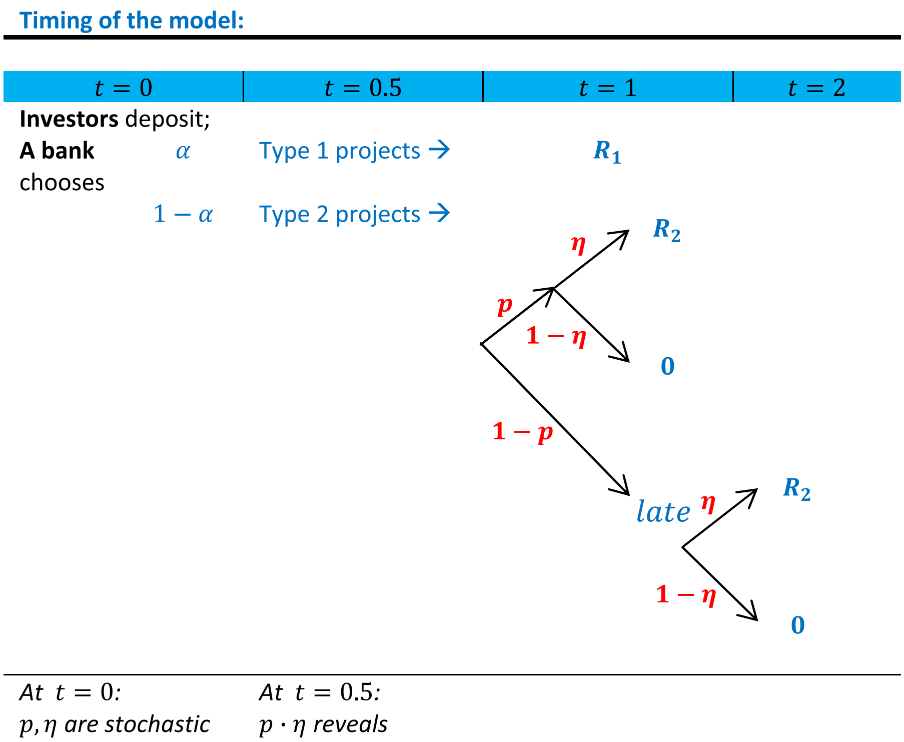

Fig. 3.1 The timing of the game

Here the risky project has the following special features, as shown in Fig. 3.1

1. With probability p the project returns early. For projects with early returns
   (a) With probability η the return is as high as R_2;
   (b) With probability $1 - \eta$ the return is as low as 0;
2. With probability $1 - p$ the project returns late. For projects with late returns
   (a) With probability η the return is as high as R_2;
   (b) With probability $1 - \eta$ the return is as low as 0.

p can take three values, $p_L < \overline{p} < p_H$. η can take three values as well, $\eta_L < \overline{\eta} < \eta_H$. Assume that $\eta R_2 > R_1$ such that the expected return for each unit of risky asset invested at $t = 0$ is higher than the that for safe asset.

At $t = \frac{1}{2}$, $p \cdot \eta$, or the early return from the risky projects, becomes public information. However, no player, even the bank managers themselves, knows the exact values of p and η. Further, assume that there can be only one shock at $t = 1$, i.e. it's only possible that either p or η takes its "extreme" value, but not both; and assume there are only two possible values [1] for $p \cdot \eta$

[1] There are both technical and practical reasons for such assumption. Recall that what simplifies Chapter 1, or Cao & Illing (2008) most is assuming two states of the world, making computing mixed strategy equilibrium straightforward; otherwise with more states the solution gets more complicated (Chapter 1, or Cao & Illing (2008) has a brief discussion in the end) while contributes no more insights. Now back to current settings, two states for p and two states for η make four states of the world, which makes it tricky to apply the previous exercises. However, current market turmoil suggests that one of the most problematic features of modern financial crisis is that one can hardly distinguish between illiquidity and insolvency (even the financial institutions fail to do so in the presence of excessive securitization), therefore among the entire four states, $p_H \cdot \eta_H$ and $p_L \cdot \eta_L$ are actually trivial and non-interesting. So without much loss of generality we may concentrate on the two states in which players aren't able to tell insolvency from illiquidity; this makes the research, in the author's opinion, both technically tractable and practically appealing.

and $(p \cdot \eta)_L = \overline{p} \cdot \eta_L = \overline{\eta} \cdot p_L < \overline{p} \cdot \eta_H = \overline{\eta} \cdot p_H = (p \cdot \eta)_H$. The higher early return, $(p \cdot \eta)_H$, occurs with probability π and the lower early return, $(p \cdot \eta)_L$, occurs with probability $1 - \pi$. Therefore,

1. If one observes a high $p \cdot \eta$, it may comes from either p_H (with probability σ) or η_H (with probability $1 - \sigma$);
2. If one observes a low $p \cdot \eta$, it may comes from either p_L (with probability σ) or η_L (with probability $1 - \sigma$).

Such p - η setting captures the dual concerns in banking industry. p defines how likely the cash flow is materialized earlier, i.e. the liquidity of the risky projects, and η defines how successful the projects are — or, how likely the banks stay solvent.

In the following, let's first analyze the baseline case in which there's no uncertainty concerning the values of p and η. Then the model is extended to the case in which the true reason for a liquidity shock is not discernable.

3.2.1 The baseline result (when p and η are deterministic)

Suppose that both p and η are deterministic. In this case, the expected return for each unit of risky asset invested at $t = 0$ is

$$\mathbb{E}[R_2] = p\eta R_2 + (1 - p)\eta R_2 = \eta R_2.$$

Then for each unit deposit the bank manager collects, her liability to her depositors is

$$\alpha\gamma R_1 + (1 - \alpha)\gamma\mathbb{E}[R_2] = \alpha\gamma R_1 + (1 - \alpha)\gamma\eta R_2;$$

and at $t = 1$ the aggregate liquidity available is

$$\alpha R_1 + (1 - \alpha)p\eta R_2.$$

The optimal symmetric equilibrium is therefore the α that equates these two terms, i.e.

$$\alpha\gamma R_1 + (1-\alpha)\,\gamma\eta R_2 = \alpha R_1 + (1-\alpha)\,p\eta R_2,$$

solve to get

$$\alpha = \frac{\gamma - p}{(\gamma - p) + (1-\gamma)\frac{R_1}{\eta R_2}} = \frac{1}{1 + (1-\gamma)\frac{R_1}{\eta R_2(\gamma - p)}}. \tag{3.1}$$

When $\eta = 1$, i.e. no insolvency risk, equation (3.1) degenerates to the baseline case in Chapter 1, or Cao & Illing (2008). It can be seen that $\frac{\partial\alpha}{\partial\eta} > 0$, i.e. when insolvency is less severe, illiquidity problem dominates so that more funds should be invested on the safe assets; similarly, since $\frac{\partial\alpha}{\partial p} < 0$, more funds should be invested on the safe assets when the long term projects get riskier.

3.2.2 Introducing aggregate risk (when p and η are stochastic)

Now suppose that at $t = \frac{1}{2}$, the value $p \cdot \eta$ is stochastic, i.e. either $(p \cdot \eta)_H$ or $(p \cdot \eta)_L$ is observed. Then when $(p \cdot \eta)_H$ reveals

- If the true state is p_H with $\overline{\eta}$, then the expected return from the late risky projects at $t = 2$ is $(1 - p_H)\,\overline{\eta}R_2$;
- If the true state is η_H with $\overline{p}$, then the expected return from the late risky projects at $t = 2$ is $(1 - \overline{p})\,\eta_H R_2$.

So the expected return at $t = 2$ is given by

$$\begin{aligned} R_2^H &= \left[(1-p_H)\,\overline{\eta}\sigma + (1-\overline{p})\,\eta_H(1-\sigma)\right] R_2 \\ &= \left[\overline{\eta}\sigma + (1-\overline{p}-\sigma)\,\eta_H\right] R_2 \\ &= [(1-\overline{p})\,\overline{\eta} + (1-\overline{p}-\sigma)\,\underbrace{(\eta_H - \overline{\eta})}_{>0}]R_2, \end{aligned} \tag{3.2}$$

and the aggregate expected return from the risky projects is

$$\mathbb{E}\left[R_2|(p\cdot\eta)_H\right] = (p\cdot\eta)_H R_2 + [(1-\overline{p})\,\overline{\eta} + (1-\overline{p}-\sigma)\,(\eta_H - \overline{\eta})]R_2. \tag{3.3}$$

Similarly when $(p\cdot\eta)_L$ is observed at $t=\frac{1}{2}$, then

- If the true state is p_L with $\overline{\eta}$, then the expected return from the late risky projects at $t=2$ is $(1-p_L)\,\overline{\eta}R_2$;
- If the true state is η_L with $\overline{p}$, then the expected return from the late risky projects at $t=2$ is $(1-\overline{p})\,\eta_L R_2$.

So the expected return from the late risky projects at $t=2$ is given by

$$\begin{aligned} R_2^L &= \left[(1-p_L)\,\overline{\eta}\sigma + (1-\overline{p})\,\eta_L(1-\sigma)\right]R_2 \\ &= \left[\overline{\eta}\sigma + (1-\overline{p}-\sigma)\,\eta_L\right]R_2 \\ &= [(1-\overline{p})\,\overline{\eta} + (1-\overline{p}-\sigma)\underbrace{(\eta_L-\overline{\eta})}_{<0}]R_2, \end{aligned} \tag{3.4}$$

and the aggregate expected return from the risky projects is

$$\mathbb{E}\left[R_2|(p\cdot\eta)_L\right] = (p\cdot\eta)_L R_2 + [(1-\overline{p})\,\overline{\eta} + (1-\overline{p}-\sigma)\,(\eta_L - \overline{\eta})]R_2. \tag{3.5}$$

To make our analysis interesting, assume that

$$\begin{aligned} \mathbb{E}\left[R_2|(p\cdot\eta)_H\right] &> \mathbb{E}\left[R_2|(p\cdot\eta)_L\right], \\ (p\cdot\eta)_H - (p\cdot\eta)_L &> (1-\overline{p}-\sigma)\,(\eta_L-\eta_H). \end{aligned}$$

If there's only illiquidity risk as in CHAPTER 1, or Cao & Illing (2008, 2009a), the expected return from the late risky projects is just R_2 (the only thing that matters is the timing of cash flow). Now with co-existence of insolvency risk, such return is determined by the probability and scale of insolvency, as (3.2) and (3.4) suggest:

1. When time is good, the confidence in the risky assets (less likely to be insolvent) raises future expected return (hence asset price at $t = 1$);

2. When time is bad, the lack of confidence in the risky assets (more likely to be insolvent) depresses future expected return (hence asset price at $t = 1$).

In the following sections, we'll see that this new feature in late risky projects' expected return makes current model depart from Chapter 1, 2 or Cao & Illing (2008, 2009a) in many ways.

3.2.3 Equilibria for the *laissez-faire* economy

Suppose that only $(p \cdot \eta)_H$ is the only intermediate state of the world and all the bank managers set their α, call it α_H, according to that. Then the equilibrium should be the α_H under which the banks get the cheapest liquidity without bank runs, i.e.

$$\begin{aligned}
\alpha_H \gamma R_1 + (1-\alpha_H)\gamma \mathbb{E}\left[R_2|(p\cdot\eta)_H\right] &= \gamma \underbrace{\left\{\alpha_H R_1 + (1-\alpha_H)\,\mathbb{E}\left[R_2|(p\cdot\eta)_H\right]\right\}}_{\mathbb{E}[R_H]} \\
&= \alpha_H R_1 + (1-\alpha_H)(p\cdot\eta)_H R_2 \\
\alpha_H &= \frac{1}{1+(1-\gamma)\frac{R_1}{\gamma\mathbb{E}\left[R_2|(p\cdot\eta)_H\right]-(p\cdot\eta)_H R_2}}.
\end{aligned}$$

Similar as in Chapter 1, assume that $\gamma\mathbb{E}\left[R_2|(p\cdot\eta)_H\right] > (p\cdot\eta)_H R_2$ to ensure that banks need to hold both liquid and illiquid assets.

If $(p\cdot\eta)_L$ is the only intermediate state of the world and all the bank managers set their α, call it α_L, according to that, then

$$\alpha_L = \frac{1}{1+(1-\gamma)\frac{R_1}{\gamma\mathbb{E}\left[R_2|(p\cdot\eta)_L\right]-(p\cdot\eta)_L R_2}}.$$

Similar as before, assume that $\gamma\mathbb{E}\left[R_2|(p\cdot\eta)_L\right] > (p\cdot\eta)_L R_2$.

To simplify the notations in the following, denote

$$\mathbb{E}\left[R_H\right] = \alpha_H R_1 + (1-\alpha_H)\,\mathbb{E}\left[R_2|(p\cdot\eta)_H\right],$$

as well as

$$\mathbb{E}\left[R_L\right] = \alpha_L R_1 + (1-\alpha_L)\,\mathbb{E}\left[R_2|(p\cdot\eta)_L\right].$$

The equilibria for the *laissez-faire* economy are then summarized in the following proposition:

Proposition 3.2.1 *The equilibria for the* laissez-faire *economy depend on the value of π, such that*

1. *There is a unique optimal symmetric equilibrium of pure strategy such that all the banks set $\alpha^* = \alpha_H$ as long as the probability of $(p\cdot\eta)_H$ satisfies $\pi > \overline{\pi}_2 = \frac{\gamma\mathbb{E}[R_L]-c}{\gamma\mathbb{E}[R_H]-c}$. In addition,*
   - *(a) At $t = 0$ the banks offer the investors a deposit contract with $d_0 = \gamma\mathbb{E}\left[R_H\right]$;*
   - *(b) The banks survive at $(p\cdot\eta)_H$, but are run at $(p\cdot\eta)_L$;*
   - *(c) The investors' expected return is $\mathbb{E}\left[R\left(\alpha_H, c\right)\right] = \pi d_0 + (1-\pi)c$;*
2. *There exists a unique optimal symmetric equilibrium of pure strategy such that all the banks set $\alpha^* = \alpha_L$ as long as the probability of $(p\cdot\eta)_H$ satisfies $0 \le \pi < \overline{\pi}_1 = \frac{\gamma\mathbb{E}[R_L]-c}{\gamma\mathbb{E}\left[R_2|(p\cdot\eta)_L\right]-c}$. In addition,*
   - *(a) At $t = 0$ the banks offer the investors a deposit contract with $d_0 = \gamma\mathbb{E}\left[R_L\right]$;*
   - *(b) The banks survive at both $(p\cdot\eta)_H$ and $(p\cdot\eta)_L$;*
   - *(c) The investors' expected return is $\mathbb{E}\left[R\left(\alpha_L\right)\right] = d_0$;*
   - *(d) At $(p\cdot\eta)_H$ the bank managers get a rent of $\gamma(1-\alpha_L)\left(\mathbb{E}\left[R_2|(p\cdot\eta)_H\right] - \mathbb{E}\left[R_2|(p\cdot\eta)_L\right]\right)$ for each unit of deposit;*
3. *When $\pi \in [\overline{\pi}_1, \overline{\pi}_2]$ there exists no symmetric equilibrium of pure strategies. What's more, there exists a unique equilibrium of mixed strategies in which for a representative bank manager*

(a) *With probability θ the bank chooses to be naughty — those who set $\alpha_r^* = 0$, offer high return for investors at $(p \cdot \eta)_H$ and are run at $(p \cdot \eta)_L$; and with probability $1 - \theta$ to be prudent — those who set $\alpha_s^* > 0$ and survive both $(p \cdot \eta)_H$ and $(p \cdot \eta)_L$;*

(b) *At $t = 0$ a naughty bank offers a deposit contract with higher return $d_0^r = \gamma\left[(p \cdot \eta)_H R_2 + \frac{R_2^H}{r_H}\right]$, but the banks is run when $(p \cdot \eta)_L$ is observed; a prudent bank offers a deposit contract with lowerer return $d_0^s = \gamma\left[\alpha_s^* R_1 + (1 - \alpha_s^*)(p \cdot \eta)_H R_2 + \frac{(1-\alpha_s^*)R_2^H}{r_H}\right]$, but the banks survive in both states;*

(c) *The expected returns for both types are equal, i.e. $\pi d_0^r + (1 - \pi)c = d_0^s$, and the probability θ is determined by market clearing condition, which equates liquidity supply and demand in both states;*

(d) *The expected returns for prudent banks are equal at both states. Especially, at $(p \cdot \eta)_L$,*

$$d_0^s = \min\left\{\alpha_s^* R_1 + (1 - \alpha_s^*)(p \cdot \eta)_L R_2, \gamma\left[\alpha_s^* R_1 + (1 - \alpha_s^*)(p \cdot \eta)_L R_2 + (1 - \alpha_s^*) R_2^L\right]\right\}.$$

Moreover, $r_L = 1$ with $\alpha_s^* \geq \alpha_L$ when

$$d_0^s = \gamma\left[\alpha_s^* R_1 + (1 - \alpha_s^*)(p \cdot \eta)_L R_2 + (1 - \alpha_s^*) R_2^L\right];$$

and $r_L \geq 1$ with $\alpha_s^* \leq \alpha_L$ when

$$d_0^s = \alpha_s^* R_1 + (1 - \alpha_s^*)(p \cdot \eta)_L R_2.$$

$\alpha_s^* = \alpha_L$ only when

$$\gamma\left[\alpha_s^* R_1 + (1 - \alpha_s^*)(p \cdot \eta)_L R_2 + (1 - \alpha_s^*) R_2^L\right] = \alpha_s^* R_1 + (1 - \alpha_s^*)(p \cdot \eta)_L R_2.$$

Proof See Appendix A.1.1.

So far the results seem to be similar as those in Chapter 1, or Cao & Illing (2008). Although the ambiguity between illiquidity and insolvency problems makes four states of the world at $t = 2$, namely $(p_H, \overline{\eta})$, $(\eta_H, \overline{p})$, $(p_L, \overline{\eta})$, and $(\eta_L, \overline{p})$, only two signals are actually observed in $t = 1$. As long as the equilibria are still driven by just two $t = 1$ signals, the outcomes should be of similar pattern.

The difference here lies in the mixed strategy equilibrium, i.e. what Proposition 3.2.1 (3d) shows. Recall that in the presence of pure illiquidity risks, the expected return of the risky assets remains the same (i.e. R_2) in both states because the only problem there is the timing of getting the fractions of the yields. But if there are additional insolvency risks as in current settings, the expected return of the risky assets differs in both states, i.e. $\mathbb{E}\left[R_2|(p \cdot \eta)_L\right] < \mathbb{E}\left[R_2|(p \cdot \eta)_H\right]$ as shown in equations (3.3) and (3.5). Therefore at $(p \cdot \eta)_H$ there's a trade-off for prudent banks now:

1. $(p \cdot \eta)_H$ implies a lower probability of insolvency at $t = 2$, therefore the value of risky assets gets higher. With higher net worth of illiquid assets, the banks are able to pledge more liquidity in liquidity market (hence, offer higher d_0^s at $t = 0$). Such "income effect" encourages prudent banks to set higher α_s^*;

2. $(p \cdot \eta)_H$ implies higher early return from the risky projects, making it easier to fulfill d_0^s. Such "substitution effect" discourages prudent banks to set higher α_s^*.

The equilibrium value α_s^* then depends on the cost of the banks' liquidity financing at $t = 1$, i.e. the interest rate r_H. Since r_H is bid up by the free-riders, or the naughty banks, its value reflects the incentive for free-riding, which hinges on the probability of being in a good state, π

1. When π is just a bit higher than $\overline{\pi}_1$, the profitability of free-riding is not much higher than being prudent. Therefore, there won't be many free-riders and r_H won't be that high. The prudent banks can thus pledge more liquidity with their risky assets, i.e. they can get higher early

return while they need less high yield risky assets to fulfill d_0^s. In this case "substitution effect" dominates and prudent banks will choose to set a higher α_s^*;

2. When π is much higher than $\overline{\pi}_1$, the profitability of free-riding is much higher than being prudent. Therefore, there will be many free-riders and r_H will be high. The prudent banks thus cannot pledge more liquidity with their risky assets, i.e. they have to fulfill d_0^s by competing for liquidity. In this case "income effect" dominates and prudent banks will choose to set a lower α_s^*.

The investors' expected return in equilibrium as a function of π is summarized in Fig. 3.2.

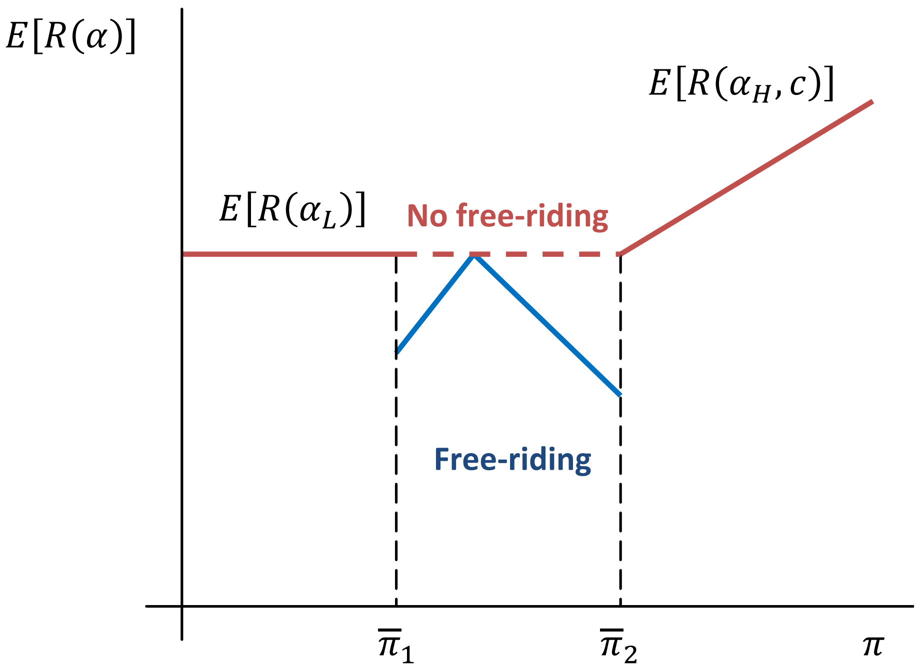

Fig. 3.2 Investors' expected return in *laissez-faire* economy

All in all, in current settings with co-existence of insolvency problem the inefficiencies are again (1) inferior mixed strategy equilibrium — the investors' expected return is lower whenever $\alpha_s^* \neq \alpha_L$, $\forall \pi \in [\overline{\pi}_1, \overline{\pi}_2]$ and (2) the costly bank runs, which are to be fixed by proper regulatory rules. However, when it comes to banking regulation, typical (one-handed) schemes may be no longer optimal or even become infeasible when insolvency gets involved, as the next section shows.

3.3 LIQUIDITY REGULATION, NOMINAL CONTRACT AND LENDER OF LAST RESORT POLICY

Similar as CHAPTER 1, 2, or Cao & Illing (2008, 2009a), now we introduce central bank as a fourth player. Banks are required to invest a minimum level $\underline{\alpha}$ on safe projects, and only those who observe the rule of game will be offered the lifeboat when there's liquidity shortage. Liquidity injection is implemented via creating fiat money, and the timing of the game is summarized as Fig. 3.3. The key elements in the settings are as following:

Nominal contracts Since central banks don't produce real goods, rather, they increase liquidity supply by printing fiat money at zero cost, therefore in this section all financial contracts have to be nominal, i.e. one unit of money is of equal value to one unit real good in payment and central bank's liquidity injection inflates the nominal price by *cash-in-the-market principle* à la Allen & Gale (2004) — the nominal price is equal to the ratio of amount of liquidity (the sum of money and real goods) in the market to amount of real goods;

Liquidity regulation At $t = 0$ a minimum level $\underline{\alpha}$ of investment on safe projects is announced by the central bank;

Conditional entry and bailout In Cao & Illing (2009b) two scenarios are considered concerning the role of liquidity requirement $\underline{\alpha}$: either (1) it's both a requirement for entry to the banking industry and a prerequisite for getting liquidity injection; or (2) it's a voluntary option for the banks, but only those who observe it get the lifeline from the central bank. In this chapter, we concentrate on the first scenario. However, later it can be seen that the same conclusions mostly hold when the second one is considered as well.

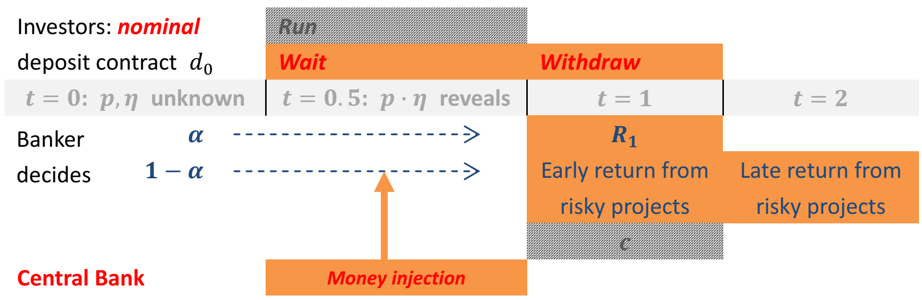

Fig. 3.3 The timing of the game with central bank

3.3.1 Liquidity regulation with conditional bailout

Remember that in the presence of pure illiquidity risk liquidity injection eliminates the costly bank run, reducing inefficiency, as Chapter 2, or Cao & Illing (2009a) suggests. Suppose the same policy is applied that at $t = 0$ all banks are required to invest $\underline{\alpha} = \alpha_H$ when $\pi > \overline{\pi}_2$, and will be bailed out by the liquidity injected against their assets as collateral when necessary. Then when $(p \cdot \eta)_H$ is indeed observed, the banks can meet the depositors' demand without the need for liquidity injection, i.e.

$$d_0 = \alpha_H \gamma R_1 + (1 - \alpha_H) \gamma \mathbb{E} \left[R_2 | (p \cdot \eta)_H \right].$$

However, when $(p \cdot \eta)_L$ is observed, the nominal contract on d_0 cannot be met purely by the banks' expected real return so that they need to apply for central bank's liquidity injection with their assets as collateral. However, since there's a positive probability that the banks may be insolvent, the central bank can only inject liquidity up to the fair value of the the risky projects, i.e.

the expected return of the risky assets, or, in this case the maximum nominal payoff the depositors can get

$$\begin{aligned} d_0|_{(p\cdot\eta)_L} &= \alpha_H \gamma R_1 + (1-\alpha_H)\,\gamma\mathbb{E}\left[R_2|(p\cdot\eta)_L\right] \\ &< d_0 \end{aligned} \tag{3.6}$$

— the banks will still be run even they obtain the promised lifeboat from the central bank, and the outcome is no different from that in the *laissez-faire* economy. The scheme fails to eliminate the inefficient bank runs for $\pi > \overline{\pi}_2$, and the outcome is the same as that in the *laissez-faire* economy.

For $0 \leq \pi \leq \overline{\pi}_2$, the liquidity requirement should be $\underline{\alpha} = \alpha_L$. Since $\underline{\alpha}$ is also the entry requirement for the entire banking industry, it is no longer possible to free-ride for intermediate value of π; the inferior mixed strategy equilibrium is thus eliminated, which improves efficiency. On the other hand, banks survive on both contingencies by setting $\underline{\alpha} = \alpha_L$, so there will be no need for liquidity injection.

As a conclusion, in contrast to Cao & Illing (2009b), with the additional insolvency risk this scheme can only eliminate the inefficiency from the mixed strategy equilibrium, but fails to avoid the high cost from bank runs. It's effectiveness is rather limited.

If the liquidity requirement $\underline{\alpha}$ is voluntary instead of obligatory, even such limited effectiveness will disappear. The outcomes under $\pi > \overline{\pi}_2$ and $0 \leq \pi < \overline{\pi}_1$ maintains, however, for intermediate $\pi \in [\overline{\pi}_1, \overline{\pi}_2]$ the prudent banks are guaranteed with cheap injected liquidity, leaving their entrepreneurs to sell liquidity in the market, which generates even more excessive liquidity supply when time is good. This only makes free-riding more attractive — in the end, the scheme aggravates the inefficiency in the mixed strategy equilibrium instead of fixing it!

3.3.2 Conditional liquidity injection with procyclical taxation

The failure of this scheme comes from the following fact: the insolvency risk brings a wedge between the expected return of the late risky projects at $(p \cdot \eta)_H$ and that at $(p \cdot \eta)_L$; therefore, even if the banks are guaranteed with liquidity injection when time is bad, they are not able to obtain as much liquidity as they need — in other words, the potential insolvency risk adds an extra cost to stabilizing the financial system. This suggests that the regulator needs to find a second instrument for covering such cost, for example, an additional procyclical taxation may help solve this problem by imposing a tax at $t = 0$ on the banks' revenue when $(p \cdot \eta)_H$ is observed, and bailing out the troubled banks with liquidity injection plus such tax revenue when $(p \cdot \eta)_L$ is observed.

The proposed augmented scheme works as following. At $t = 0$ a minimum liquidity requirement, the minimum share $\underline{\alpha}_T$ of the funds invested on the safe projects, is imposed on all banks; and at $t = 1$ the banks are taxed away a certain amount $T_H \geq 0$ out of their revenue when $(p \cdot \eta)_H$ is observed. The banks are bailed out with liquidity injection (with their assets as collateral) plus the tax revenue when $(p \cdot \eta)_L$ is observed — surely in this case the banks pay no tax, $T_L = 0$.

$\underline{\alpha}_T$ and T_H are determined by π, i.e. regulatory policies are only introduced where there are inefficiencies

1. For $\pi \geq \overline{\pi}_2$, a positive tax $T_H > 0$ is levied at $(p \cdot \eta)_H$ and the revenue is used as bailout funds at $(p \cdot \eta)_H$. Bank managers have to set $(\alpha_{H,T}, d_{0,T})$ at $t = 0$ by internalizing T_H as an additional cost at $t = 1$. In this case, costly bank run is the source of inefficiency which is to be entirely eliminated by the conditional liquidity injection and the tax;

2. For $0 \leq \pi \leq \overline{\pi}_1$, banks are required to set $\underline{\alpha}_T = \alpha_L$ as an entry condition. Since the inefficient mixed strategy equilibrium is deterred by imposing such obligation, and the banks always survive in this case, no safety funds are necessary. Therefore, $T_H = 0$.

Now we have to examine whether this scheme works; and if yes, how much T_H should be imposed. Let's concentrate on the case where $T_H > 0$, i.e. $\pi \geq \overline{\pi}_2$. $(\alpha_{H,T}, d_{0,T})$ is set by

$$\begin{aligned}\alpha_{H,T}\gamma R_1 + (1-\alpha_{H,T})\,\gamma\mathbb{E}\left[R_2|(p\cdot\eta)_H\right] - T_H &= \alpha_{H,T}R_1 + (1-\alpha_{H,T})\,(p\cdot\eta)_H R_2 \\ &= d_{0,T}. \qquad (3.7)\end{aligned}$$

The liquidity requirement $\underline{\alpha}_T$ should be so high that the banks are just able to utilize the resources optimally (as equation (3.7) shows), i.e. $\underline{\alpha}_T = \alpha_{H,T}$, and the conditional bailout policy must make sure that the banks are not to be run in the worst case, i.e.

$$\begin{aligned}&\alpha_{H,T}\gamma R_1 + (1-\alpha_{H,T})\,\gamma\mathbb{E}\left[R_2|(p\cdot\eta)_H\right] - T_H \\ = \;& \alpha_{H,T}\gamma R_1 + (1-\alpha_{H,T})\,\gamma\mathbb{E}\left[R_2|(p\cdot\eta)_L\right] + T_H\frac{\pi}{1-\pi}. \qquad (3.8)\end{aligned}$$

$\alpha_{H,T}$, $d_{0,T}$, and T_H are determined by equations (3.7) and (3.8), solve to get

$$\begin{cases} \alpha_{H,T} = \dfrac{(p\cdot\eta)_H R_2 - \gamma\pi\mathbb{E}\left[R_2|(p\cdot\eta)_H\right] + \gamma\pi\mathbb{E}\left[R_2|(p\cdot\eta)_L\right] - \gamma\mathbb{E}\left[R_2|(p\cdot\eta)_L\right]}{\gamma R_1 - R_1 + (p\cdot\eta)_H R_2 - \gamma\pi\mathbb{E}\left[R_2|(p\cdot\eta)_H\right] - \gamma\mathbb{E}\left[R_2|(p\cdot\eta)_L\right] + \gamma\pi\mathbb{E}\left[R_2|(p\cdot\eta)_L\right]}, \\ d_{0,T} = -\dfrac{\gamma R_1\left\{\pi\left(\mathbb{E}\left[R_2|(p\cdot\eta)_H\right] - \mathbb{E}\left[R_2|(p\cdot\eta)_L\right]\right) + \mathbb{E}\left[R_2|(p\cdot\eta)_L\right] - (p\cdot\eta)_H R_2\right\}}{\gamma R_1 - R_1 + (p\cdot\eta)_H R_2 - \gamma\pi\mathbb{E}\left[R_2|(p\cdot\eta)_H\right] - \gamma\mathbb{E}\left[R_2|(p\cdot\eta)_L\right] + \gamma\pi\mathbb{E}\left[R_2|(p\cdot\eta)_L\right]}, \\ T_H = \dfrac{\gamma R_1(\pi-1)(1-\gamma)\left(\mathbb{E}\left[R_2|(p\cdot\eta)_H\right] - \mathbb{E}\left[R_2|(p\cdot\eta)_L\right]\right)}{\gamma R_1 - R_1 + (p\cdot\eta)_H R_2 - \gamma\pi\mathbb{E}\left[R_2|(p\cdot\eta)_H\right] - \gamma\mathbb{E}\left[R_2|(p\cdot\eta)_L\right] + \gamma\pi\mathbb{E}\left[R_2|(p\cdot\eta)_L\right]}. \end{cases}$$

To get rid of complications, further assume that $\gamma\mathbb{E}\left[R_2|(p\cdot\eta)_L\right] > (p\cdot\eta)_H R_2$, i.e. even in the worst case, it is still appealing for the banks to hold both liquid and illiquid assets.

The effectiveness of the scheme is summarized in the following proposition:

Proposition 3.3.1 *When $\pi \geq \overline{\pi}_2$, with $\underline{\alpha}_T$ as both the requirement for entry to the banking industry and a prerequisite for getting liquidity injection from the central bank, as well as an additional tax T_H charged at $(p\cdot\eta)_H$ as safety funds for rescuing banks at $(p\cdot\eta)_H$, the required $\underline{\alpha}_T$ should be so high that $\underline{\alpha}_T > \alpha_H$ and the corresponding* *investors' expected return is (weakly) higher than that in the* laissez-faire *economy under the same π, as long as c is sufficiently small.*

Proof See Appendix A.1.2.

The intuition behind the proposition is fairly straightforward. The gain from such modified scheme is to avoid the costly bank runs, however, the scheme also adds additional direct and indirect costs for banking business. The direct one comes from T_H, the "safety funds" to make up the losses in bad time as equation (3.8) shows, i.e. to distribute the tax revenue in the downturn, $T_H \frac{\pi}{1-\pi}$; the indirect one comes from $\alpha_{H,T}$ — at $t = 0$ the banks have to invest more on the safe projects to pay the tax at $t = 1$, leaving less resources for risky, but high yield projects. When π is sufficiently high and the bad state seldom happens, the regulator doesn't need to charge too high T_H and the regulatory cost is comparatively lower than the economy's gain from the regulation, and this is more likely to hold when the gain from avoiding bank runs (i.e. when c is sufficiently small) is sufficiently large.

Fig. A.1 (Appendix A.2) visualizes the results by numerical simulation. When the cost of bank runs is fairly high (too low c), this scheme significantly improves efficiency when π is high, where T_H doesn't need to be high and the opportunity cost from investing on higher $\alpha_{H,T}$ is much lower than the gain from completely avoiding bank runs.

However, in reality such safety funds via procyclical taxation are certainly subject to implementation difficulties. The funds have to be accumulated to a sufficient amount before they are in need, i.e. when a crisis hits. Otherwise, when a crisis comes before the funds are fully established, the government must face a public deficit which can only be covered by the future taxation revenue. Usually raising public deficits implies political debates and compromises, substantially restricting the effectiveness of such scheme. In this sense, a "self-sufficient" solution such as equity holding may be superior, which is to be studied in the next section.

3.4 INSOLVENCY RISK AND EQUITY REQUIREMENT

As seen above, with dual plagues the scheme of liquidity requirement with conditional bailout only works if an additional cost is introduced. Such cost can be either "external", for example, establishing safety funds via taxation as the past section suggested, or "internal", for example, covering the cost with equity holdings. In current settings, introducing equity requirement may not be so costly as in Chapter 2, or Cao & Illing (2009a) since the cheaper stabilizing instrument there ceases to work here. Therefore, comparing with the bigger cost caused by bank runs, taking a costly equity requirement may be the lesser of two evils.

3.4.1 Pure equity requirement and narrow banking

Now suppose equity requirement is adopted as a sole instrument for the regulator to stabilize financial system in a self-sufficient way, i.e. all the losses will be absorbed by equity holders. Here equity is introduced à la Diamond & Rajan (2005) such that the banks issue a mixture of deposit contract and equity for the investors. Assume that the equity holders (investors) and the bank managers share the profit equally (that is, to set ζ in Chapter 2, or Cao & Illing (2009a) to be 0.5), i.e. in the good time the level of equity k is

$$k = \frac{\frac{\gamma\mathbb{E}[R_H]-d_{0,E}}{2}}{\frac{\gamma\mathbb{E}[R_H]-d_{0,E}}{2} + d_{0,E}}, d_{0,E} = \frac{1-k}{1+k}\gamma\mathbb{E}\left[R_H\right].$$

The minimum equity requirement k should make the banks just able to survive from bank runs in the worst contingency, i.e. all the equity is wiped out when $(p \cdot \eta)_L$ is observed,

$$\frac{1-k}{1+k}\gamma\mathbb{E}\left[R_H\right] = \underbrace{\alpha_H R_1 + (1-\alpha_H)\,(p \cdot \eta)_L R_2}_{\mathbb{E}\left[R_H|(p\cdot\eta)_L\right]} = d_{0,E}, \tag{3.9}$$

or,

$$k = \frac{\gamma \mathbb{E}[R_H] - d_{0,E}}{\gamma \mathbb{E}[R_H] + d_{0,E}}.$$

Since $\frac{\partial k}{\partial (p \cdot \eta)_L} < 0$ by equation (3.9), banks need higher equity ratio to survive in the worst contingency when both (or either) of the two plagues get(s) more severe, implying a higher regulatory cost.

Fig. A.2 (Appendix A.2) visualizes the results by numerical simulation. Again, as Chapter 2 or Cao & Illing (2009a) shows, holding equity is costly when π is high (i.e. less funds are available for the risky assets with relatively safe, high yields, although costly bank runs are completely eliminated). Holding equity may be superior to mixed strategy equilibrium of *laissez-faire* economy depending on parameter values, but is inferior to conditional liquidity injection with procyclical taxation — because taxation revenue is entirely returned to investors as bailout funds while in current scheme part of profits goes to bank managers as dividends. However, concerning the implementation difficulties of imposing an extra tax, this may be a necessary cost for both investors and regulators.

3.4.2 Combining equity requirement with liquidity regulation

Liquidity requirement with conditional liquidity injection works best with pure illiquidity risk, but the scheme fails when there's additional insolvency risk; on the other hand, pure equity requirement is able to stabilize the system under both settings at a relatively high cost. Now the question is: is it possible to design a regulatory scheme that combines the advantages of these two at a minimum cost?

The answer is yes. Look at the right hand side of equation (3.9). If the banks are required to maintain the financial stability in a self-sufficient way, in all contingencies the depositors can only get the same expected return as in the worst case, i.e. the total $t = 1$ liquidity when time is bad. However, since there's a positive probability that the risky assets are simply illiquid, the expected future return from the risky assets can be higher, i.e. the "fair" value of the risky assets (as the right hand side of equation (3.6) shows) is higher. Therefore, liquidity injection from the central bank enables the banks to pledge for bailout funds up to the fair value of their late risky assets. However, as Section 3.3.1, 3.3.2 argued, without imposing extra costs such as taxation these bailout funds won't be enough for the banks to avoid the costly bank runs, as long as there's still a positive probability that the banks will be insolvent. The regulator can impose equity requirement to cover this part of cost. By doing so, since the banks need equity to cover only part of the regulatory cost, it'll be much less costly for the banks to carry equity.

The regulatory scheme is as following. First, all the banks are required to invest $\underline{\alpha}_E = \alpha_H$ of their funds on safe assets at $t = 0$ for high π, and $\underline{\alpha}_E = \alpha_L$ for low π (the cutoff value of π is different from $\overline{\pi}_2$, and we'll compute it later); second, all the banks are required to meet a minimum equity ratio k' for high π[2]. Then the banks are bailed out by liquidity injection in the form of fiat money provision when time is bad. In this case, the regulator only needs to set k' to fill in the gap after liquidity injection when $(p \cdot \eta)_L$ is observed, i.e.

$$\frac{1-k'}{1+k'}\gamma\mathbb{E}\left[R_H\right] = \alpha_H\gamma R_1 + (1-\alpha_H)\,\gamma\mathbb{E}\left[R_2|(p\cdot\eta)_L\right] \tag{3.10}$$

in which $k' < k$ since the right hand side of (3.10) is higher than that of (3.9). Then when $(p \cdot \eta)_H$ is observed, the investors' real expected return is $\frac{1-k'}{1+k'}\gamma\mathbb{E}\left[R_H\right]$; however, when $(p \cdot \eta)_L$ is observed, the investors' real expected return is $\mathbb{E}\left[R_H|(p\cdot\eta)_L\right]$ (the right hand side of (3.9)) and the liquidity is injected for the banks to meet the nominal deposit contract. Therefore, the investors' real expected return is

[2]For sufficiently low π the banks coordinate on the safe strategy, therefore there will be no bank runs and no need for liquidity injection, hence no need for equity to cover the gap in bailout funds.

$$\begin{aligned}
& \frac{1-k'}{1+k'}\gamma\mathbb{E}\left[R_H\right]\pi + (1-\pi)\mathbb{E}\left[R_H|(p\cdot\eta)_L\right] \\
= \quad & \left\{\alpha_H\gamma R_1 + (1-\alpha_H)\,\gamma\mathbb{E}\left[R_2|(p\cdot\eta)_L\right]\right\}\pi + (1-\pi)\mathbb{E}\left[R_H|(p\cdot\eta)_L\right]. \quad (3.11)
\end{aligned}$$

For sufficiently low π the banks coordinate on the safe strategy, i.e. $\alpha^* = \underline{\alpha}_E = \alpha_L$, and the investors' expected return is $\gamma\mathbb{E}\left[R_L\right]$. It pays off for the banks to choose α_L instead of α_H only if they get higher expected real return than (3.11), i.e. when

$$\begin{aligned}
\gamma\mathbb{E}\left[R_L\right] \quad & > \quad \frac{1-k'}{1+k'}\gamma\mathbb{E}\left[R_H\right]\pi + (1-\pi)\mathbb{E}\left[R_H|(p\cdot\eta)_L\right], \\
\gamma\mathbb{E}\left[R_L\right] \quad & > \quad \left\{\alpha_H\gamma R_1 + (1-\alpha_H)\,\gamma\mathbb{E}\left[R_2|(p\cdot\eta)_L\right]\right\}\pi + (1-\pi)\mathbb{E}\left[R_H|(p\cdot\eta)_L\right]. \quad (3.12)
\end{aligned}$$

The solution gives the cutoff value $\overline{\pi}_2'$, which can be solved from (3.12) when it holds with equality

$$\overline{\pi}_2' = \frac{\gamma\mathbb{E}\left[R_L\right] - \mathbb{E}\left[R_H|(p\cdot\eta)_L\right]}{\alpha_H\gamma R_1 + (1-\alpha_H)\,\gamma\mathbb{E}\left[R_2|(p\cdot\eta)_L\right] - \mathbb{E}\left[R_H|(p\cdot\eta)_L\right]}.$$

Fig. A.3 (Appendix A.2) visualizes the results by numerical simulation. Such hybrid scheme indeed effectively reduces regulatory cost in comparison to pure equity requirement, since the banks do not have to hold that much equity to stabilize the system, i.e. regulator needs two instruments to deal with dual plagues.

Fig. A.4 (Appendix A.2) compares the investors' returns under all schemes. Again, the outcome under conditional liquidity injection with procyclical taxation is superior to all the others, since all the profits that are levied as the safety tax will be entirely returned to the investors. However, when the political cost is too high to impose an extra tax and raise public deficit, combining the advantages of liquidity regulation and equity requirement is the best self-sufficient scheme.

3.5 CONCLUSION

In existing banking literature, illiquidity and insolvency shocks are usually insulated in the sense that market participants have perfect knowledge about the type of the shock. This chapter attempts to model the new feature of modern finance that financial innovation makes it harder to tell whether a financial institution is illiquid or insolvent. Such ambiguity doesn't only alter the equilibrium outcomes under *laissez-faire* economy, but also significantly complicates the regulator's roadmaps.

In order to capture the core of the problem in a relatively tractable framework, it is assumed that the only uncertainty in the economy is that market participants cannot distinguish between illiquidity and insolvency shocks. That is, when some intermediate signal, say, a negative shock in intermediate outcome, has been observed, nobody can tell whether it's because more risky projects return late (a liquidity shock) or more risky projects fail (a solvency shock). So in this stage, when pricing the illiquid assets market players have to take into account the risk that the financial institution is going to be insolvent in the future, therefore, such price should be lower than that in an economy under pure illiquidity risks where the only problem is the timing of return.

Though more complicated than the prototype model, the equilibrium outcomes under *laissez-faire* economy still look similar. When either of the two signals has been observed in $t = 1$, there's a price of liquidity associated with it, i.e. the value of risky assets can be uniquely determined. Therefore, the banks coordinate to be safe when the probability of bad weather is too high, and to be risky otherwise. In the intermediate range, there's a free-riding incentive to exploit the excessive liquidity supply in the good state of the world, and the outcome here is a prevailing mixed strategy equilibrium with both prudent and naughty banks.

However, the mixed strategy equilibrium is made a bit different by the additional insolvency risk. A good signal doesn't only mean a higher intermediate output, but also a lower risk of future insolvency which inflates the value of illiquid assets and makes the banks able to pledge more liquidity in $t = 1$, and vice versa. Therefore, the prudent banks have the trade-off between these two effects, and the balance depends on the cost of funding, which is driven by the free-riders. However, the strategic profiles of the banks in equilibrium deviate from the coordinate solution which maximizes their expected payoffs, the mixed strategy equilibrium is inferior, anyway.

Again the inefficiencies under current settings are the inferior mixed strategy equilibrium and the costly bank runs, which are to be fixed by properly designed regulatory rules. However, with the mixture of both illiquidity and insolvency risks, traditional regulatory rules need to be carefully reviewed. First, it has been shown that under current settings, liquidity requirement with conditional lender of last resort policy, which was the optimal scheme when there's only illiquidity risk, ceases to work. The reason is fairly straightforward: when bad state comes, since there's a risk that the banks in trouble may be insolvent in the future, the price of the illiquidity assets is thus depressed. When the banks turn to the central bank for help, they cannot get sufficient liquidity as needed because the collaterals, i.e. illiquid assets, don't worth that much as in the good state. Therefore, the banks will be run anyway, even if they do observe the rules of liquidity!

The fact that the illiquidity assets worth less in the bad state implies that in the presence of insolvency risk an extra informational cost is needed for both bailing out banks *ex post* and making regulatory rules *ex ante*. One proposal, suggested by the author, is to set up a safety funds via procyclical taxation, as a complement for conditional liquidity rules. The tax revenue, which is levied in the good state, is used in the bad state to fill in the gap which is left by pure liquidity injection. Under such scheme efficiency is improved: the costly bank runs are thus entirely eliminated and the mixed strategy equilibrium is deterred by the industry's entry requirement. However, if crisis hits before the funds are fully established, a public deficit has to be initiated. Considering the political cost of increasing public deficit, it may be tricky to implement such scheme in reality.

An alternative approach to covering the informational cost is the self-sufficient way, i.e. the banking industry stabilize itself by issuing equities. The investors and bank managers share the profit in the good state, but the equity is eliminated in the bad state. As a regulatory requirement, the minimum equity level to stabilize the economy is the amount which is just sufficient to make the banks survive in the bad state. Because of the additional informational rent more equity is required under current settings; and since holding equity is costly, the outcome is inferior to the market solution when the probability of the bad state is very low.

Now it is known that equity holding is able to cushion the financial shocks at a cost, and liquidity requirement together with conditional liquidity injection is able to partly cover the liquidity shortage in economic downturn; therefore, regulators may combine the advantages from both instruments to achieve higher efficiency. Indeed, it is shown that given that banks observe the liquidity requirement as well as the minimum equity holding, they can pledge the liquidity from the central bank up to the value of their collaterals, and the rest of the cost to stay solvent is shouldered by the shareholders; and the corresponding outcome dominates that under pure equity requirement.

However, investors achieve the highest expected return under the scheme of conditional liquidity injection with procyclical taxation because here the profit taxed away in the good state will be fully refunded in the bad state, instead of being pocketed by the bank managers under the schemes with equity holdings. But self-sufficient schemes can be implemented at a much lower political cost, which seem to be more attractive for regulators in reality.

Appendix

A.1 PROOFS

A.1.1 Proof of Proposition 3.2.1

Proof Given that under current settings there are still two $t = 1$ states of the world, the equilibria of the game can be easily constructed following the same method as in the proofs for Proposition 1.4.1 and 1.4.2, Chapter 1. The only necessary step here is to clarify the mixed strategy equilibrium.

When $(p \cdot \eta)_H$ reveals, the prudent banks get a high early return from their risky assets, i.e $(p \cdot \eta)_H R_2$. On the other hand, the value of the late assets, R_2^H, gets higher as well because of lower probability of insolvency and this allows them to get more liquidity in the market at $t = 1$ with market rate r_H. So the trade-off for the prudent banks here is whether to invest more on liquid assets (increase α_s^*) or to invest more on illiquid assets (decrease α_s^*), and the reference point is α_L.

The market rate r_H is pinned down by $t = 1$ liquidity demand and supply, and these are jointly determined by the number of both prudent and naughty banks (note that naughty banks only survive at $(p \cdot \eta)_H$), i.e.

1. When r_H is low, i.e. the free-riding incentive is not high, or π is not high, prudent banks are able to get market liquidity at a lower cost. Therefore, there's no need to invest in more illiquid assets and it's preferable for the prudent banks to reap the early harvest, i.e. $\alpha_s^* > \alpha_L$ in this case. And $r_L = 1$ because of the overinvestment in liquid assets;

2. When r_H is high, i.e. the free-riding incentive is high, or π is high, prudent banks are no longer able to get market liquidity at a low cost. Therefore, they have to invest in more illiquid assets to compete with naughty banks on $t = 1$ market liquidity, i.e. $\alpha_s^* < \alpha_L$ in this case. And $r_L > 1$ because of the underinvestment in liquid assets. ∎

A.1.2 Proof of Proposition 3.3.1

Proof To show that $\underline{\alpha}_T > \alpha_H$, we only have to show

$$
\begin{aligned}
& \frac{(p\cdot\eta)_H R_2 - \gamma\pi\mathbb{E}\left[R_2|(p\cdot\eta)_H\right] + \gamma\pi\mathbb{E}\left[R_2|(p\cdot\eta)_L\right] - \gamma\mathbb{E}\left[R_2|(p\cdot\eta)_L\right]}{\gamma R_1 - R_1 + (p\cdot\eta)_H R_2 - \gamma\pi\mathbb{E}\left[R_2|(p\cdot\eta)_H\right] - \gamma\mathbb{E}\left[R_2|(p\cdot\eta)_L\right] + \gamma\pi\mathbb{E}\left[R_2|(p\cdot\eta)_L\right]} \\
> \;& \frac{1}{1 + (1-\gamma)\frac{R_1}{\gamma\mathbb{E}\left[R_2|(p\cdot\eta)_H\right] - (p\cdot\eta)_H R_2}},
\end{aligned}
$$

simplify to get

$$
\begin{aligned}
& \left\{ \frac{-\gamma\pi\mathbb{E}\left[R_2|(p\cdot\eta)_L\right] + \gamma\pi\mathbb{E}\left[R_2|(p\cdot\eta)_H\right] + \pi\mathbb{E}\left[R_2|(p\cdot\eta)_L\right] - \pi\mathbb{E}\left[R_2|(p\cdot\eta)_H\right]}{\gamma R_1 - R_1 + (p\cdot\eta)_H R_2 - \gamma\pi\mathbb{E}\left[R_2|(p\cdot\eta)_H\right] - \gamma\mathbb{E}\left[R_2|(p\cdot\eta)_L\right] + \gamma\pi\mathbb{E}\left[R_2|(p\cdot\eta)_L\right]} \right. \\
& \left. + \frac{-\gamma\mathbb{E}\left[R_2|(p\cdot\eta)_H\right] - \mathbb{E}\left[R_2|(p\cdot\eta)_L\right] + \gamma\mathbb{E}\left[R_2|(p\cdot\eta)_L\right] + \mathbb{E}\left[R_2|(p\cdot\eta)_H\right]}{\gamma R_1 - R_1 + (p\cdot\eta)_H R_2 - \gamma\pi\mathbb{E}\left[R_2|(p\cdot\eta)_H\right] - \gamma\mathbb{E}\left[R_2|(p\cdot\eta)_L\right] + \gamma\pi\mathbb{E}\left[R_2|(p\cdot\eta)_L\right]} \right\} \\
& \cdot \frac{\gamma R_1}{-\gamma\mathbb{E}\left[R_2|(p\cdot\eta)_H\right] + (p\cdot\eta)_H R_2 + \gamma R_1 - R_1} \\
> \;& 0. \qquad (A.1)
\end{aligned}
$$

It can be seen that

$$
\begin{aligned}
& -\gamma\mathbb{E}\left[R_2|(p\cdot\eta)_H\right] + (p\cdot\eta)_H R_2 + \gamma R_1 - R_1 \\
= \;& \gamma\left(R_1 - \mathbb{E}\left[R_2|(p\cdot\eta)_H\right]\right) + \left((p\cdot\eta)_H R_2 - R_1\right) \\
< \;& 0, \qquad (A.2)
\end{aligned}
$$

as well as

$$
\begin{aligned}
& \gamma R_1 - R_1 + (p\cdot\eta)_H R_2 - \gamma\pi\mathbb{E}\left[R_2|(p\cdot\eta)_H\right] - \gamma\mathbb{E}\left[R_2|(p\cdot\eta)_L\right] + \gamma\pi\mathbb{E}\left[R_2|(p\cdot\eta)_L\right] \\
= \;& R_1(\gamma-1) + \left((p\cdot\eta)_H R_2 - \gamma\mathbb{E}\left[R_2|(p\cdot\eta)_L\right]\right) + \gamma\pi\left(\mathbb{E}\left[R_2|(p\cdot\eta)_L\right] - \mathbb{E}\left[R_2|(p\cdot\eta)_H\right]\right) \\
< \;& 0 \qquad (A.3)
\end{aligned}
$$

since each term is negative. What's more,

$$
\begin{aligned}
& -\gamma\pi\mathbb{E}\left[R_2|(p\cdot\eta)_L\right] + \gamma\pi\mathbb{E}\left[R_2|(p\cdot\eta)_H\right] + \pi\mathbb{E}\left[R_2|(p\cdot\eta)_L\right] - \pi\mathbb{E}\left[R_2|(p\cdot\eta)_H\right] \\
& -\gamma\mathbb{E}\left[R_2|(p\cdot\eta)_H\right] - \mathbb{E}\left[R_2|(p\cdot\eta)_L\right] + \gamma\mathbb{E}\left[R_2|(p\cdot\eta)_L\right] + \mathbb{E}\left[R_2|(p\cdot\eta)_H\right] \\
= \;& (1-\pi)(1-\gamma)\left(\mathbb{E}\left[R_2|(p\cdot\eta)_H\right] - \mathbb{E}\left[R_2|(p\cdot\eta)_L\right]\right) \\
> \;& 0
\end{aligned}
$$

since each term is positive. Given that the sign of each part of inequality (A.1)'s left hand side has been determined, it's easily seen that inequality (A.1) indeed holds.

To show that $d_{0,T} \geq \mathbb{E}\left[R\left(\alpha_H, c\right)\right] = \pi d_0 + (1-\pi)c$, we only have to show

$$d_{0,T} - \pi d_0 + (1-\pi)c \geq 0. \tag{A.4}$$

Define the left hand side of inequality (A.4) as a function of c, i.e.

$$g(c) = d_{0,T} - \pi d_0 + (1-\pi)c.$$

Insert the expressions for $d_{0,T}$ and d_0, and evaluate $g(c)$ at $c = 0$ and $c = R_1$ respectively, one can get

$$\begin{aligned} g(0) \quad = \quad & -\frac{1}{-\gamma\mathbb{E}\left[R_2|(p\cdot\eta)_H\right] + (p\cdot\eta)_H R_2 + \gamma R_1 - R_1} \\ & \cdot\frac{1}{\gamma R_1 - R_1 + (p\cdot\eta)_H R_2 - \gamma\pi\mathbb{E}\left[R_2|(p\cdot\eta)_H\right] - \gamma\mathbb{E}\left[R_2|(p\cdot\eta)_L\right] + \gamma\pi\mathbb{E}\left[R_2|(p\cdot\eta)_L\right]} \\ & \cdot\Big(-R_1^2\gamma\mathbb{E}\left[R_2|(p\cdot\eta)_L\right] + \pi^2\gamma^2 R_1\mathbb{E}^2\left[R_2|(p\cdot\eta)_H\right] + R_1^2\gamma(p\cdot\eta)_H R_2 - R_1\gamma(p\cdot\eta)_H^2 R_2^2 \\ & + R_1^2\gamma\mathbb{E}\left[R_2|(p\cdot\eta)_L\right]\pi - \pi^2\gamma^2 R_1(p\cdot\eta)_H R_2\mathbb{E}\left[R_2|(p\cdot\eta)_H\right] + 2\pi\gamma^2 R_1\mathbb{E}\left[R_2|(p\cdot\eta)_H\right] \\ & \cdot\mathbb{E}\left[R_2|(p\cdot\eta)_L\right] + \pi\gamma^2 R_1^2(p\cdot\eta)_H R_2 - \pi\gamma R_1^2(p\cdot\eta)_H R_2 - R_1\gamma\mathbb{E}\left[R_2|(p\cdot\eta)_L\right]\pi(p\cdot\eta)_H R_2 \\ & + R_1\gamma\mathbb{E}\left[R_2|(p\cdot\eta)_L\right]\pi(p\cdot\eta)_H R_2 + \pi\gamma R_1(p\cdot\eta)_H^2 R_2^2 - \pi\gamma^2 R_1(p\cdot\eta)_H R_2\mathbb{E}\left[R_2|(p\cdot\eta)_L\right] \\ & + \pi^2\gamma^2 R_1(p\cdot\eta)_H R_2\mathbb{E}\left[R_2|(p\cdot\eta)_L\right] - \pi^2\gamma^2 R_1\mathbb{E}\left[R_2|(p\cdot\eta)_H\right]\mathbb{E}\left[R_2|(p\cdot\eta)_L\right] \\ & - R_1\gamma^2\mathbb{E}^2\left[R_2|(p\cdot\eta)_H\right]\pi + R_1\gamma^2(p\cdot\eta)_H R_2\mathbb{E}\left[R_2|(p\cdot\eta)_H\right] - R_1^2\gamma^2\mathbb{E}\left[R_2|(p\cdot\eta)_L\right]\pi \\ & - R_1^2\gamma^2(p\cdot\eta)_H R_2 - R_1\gamma^2\mathbb{E}\left[R_2|(p\cdot\eta)_L\right]\mathbb{E}\left[R_2|(p\cdot\eta)_H\right] + R_1^2\gamma^2\mathbb{E}\left[R_2|(p\cdot\eta)_L\right]\Big). \end{aligned}$$

Inequality (A.2) shows that the first term, $-\frac{1}{-\gamma\mathbb{E}\left[R_2|(p\cdot\eta)_H\right]+(p\cdot\eta)_H R_2+\gamma R_1-R_1}$, is positive, and Inequality (A.3) shows that the second term,

$$\frac{1}{\gamma R_1 - R_1 + (p\cdot\eta)_H R_2 - \gamma\pi\mathbb{E}\left[R_2|(p\cdot\eta)_H\right] - \gamma\mathbb{E}\left[R_2|(p\cdot\eta)_L\right] + \gamma\pi\mathbb{E}\left[R_2|(p\cdot\eta)_L\right]},$$

is negative. Further, the fact that $\pi \geq \overline{\pi}_2 = \frac{\gamma\mathbb{E}[R_L]-c}{\gamma\mathbb{E}[R_H]-c}$ implies that the third term is non-positive as well. Therefore, $g(0) \geq 0$.

Similarly, when $c = R_1$

$$
\begin{aligned}
g(R_1) = & -\frac{1}{-\gamma\mathbb{E}\left[R_2|(p\cdot\eta)_H\right] + (p\cdot\eta)_H R_2 + \gamma R_1 - R_1} \\
& \cdot \frac{1}{\gamma R_1 - R_1 + (p\cdot\eta)_H R_2 - \gamma\pi\mathbb{E}\left[R_2|(p\cdot\eta)_H\right] - \gamma\mathbb{E}\left[R_2|(p\cdot\eta)_L\right] + \gamma\pi\mathbb{E}\left[R_2|(p\cdot\eta)_L\right]} \\
& \cdot\Big(-2R_1^2\gamma\mathbb{E}\left[R_2|(p\cdot\eta)_L\right] + 2\pi^2\gamma^2 R_1\mathbb{E}^2\left[R_2|(p\cdot\eta)_H\right] + 2R_1^2(p\cdot\eta)_H R_2 - 2R_1^3\pi\gamma \\
& + R_1\pi(p\cdot\eta)_H^2 R_2^2 - R_1^2\gamma(p\cdot\eta)_H R_2 - R_1\gamma(p\cdot\eta)_H^2 R_2^2 + 3R_1^2\gamma\pi\mathbb{E}\left[R_2|(p\cdot\eta)_L\right] \\
& + R_1^3\pi\gamma^2 + R_1^2\gamma^2\mathbb{E}\left[R_2|(p\cdot\eta)_H\right] - R_1^2\mathbb{E}\left[R_2|(p\cdot\eta)_H\right]\gamma - \pi^2\gamma^2 R_1 R_2(p\cdot\eta)_H \\
& \cdot\mathbb{E}\left[R_2|(p\cdot\eta)_H\right] + 4\pi\gamma^2 R_1\mathbb{E}\left[R_2|(p\cdot\eta)_H\right]\mathbb{E}\left[R_2|(p\cdot\eta)_L\right] + R_1\gamma^2\mathbb{E}\left[R_2|(p\cdot\eta)_L\right]\pi^2 \\
& - R_1^3\gamma^2 - R_1^2\gamma^2\pi^2\mathbb{E}\left[R_2|(p\cdot\eta)_H\right] + R_1 R_2\gamma(p\cdot\eta)_H\mathbb{E}\left[R_2|(p\cdot\eta)_H\right] + \pi\gamma^2 R_1^2 R_2(p\cdot\eta)_H \\
& + \pi\gamma R_1^2 R_2(p\cdot\eta)_H - 3R_1 R_2\gamma\pi(p\cdot\eta)_H\mathbb{E}\left[R_2|(p\cdot\eta)_L\right] + 2R_1 R_2\gamma(p\cdot\eta)_H\mathbb{E}\left[R_2|(p\cdot\eta)_L\right] \\
& + \pi\gamma R_1 R_2^2(p\cdot\eta)_H^2 - \pi\gamma^2 R_1 R_2(p\cdot\eta)_H\mathbb{E}\left[R_2|(p\cdot\eta)_L\right] + \pi^2\gamma^2 R_1 R_2(p\cdot\eta)_H\mathbb{E}\left[R_2|(p\cdot\eta)_L\right] \\
& - 2\pi^2\gamma^2 R_1\mathbb{E}\left[R_2|(p\cdot\eta)_H\right]\mathbb{E}\left[R_2|(p\cdot\eta)_L\right] + 2R_1^3\gamma - R_1 R_2^2(p\cdot\eta)_H^2 - R_1^3 + R_1 R_2\gamma \\
& \cdot\pi^2(p\cdot\eta)_H\mathbb{E}\left[R_2|(p\cdot\eta)_L\right] - R_1 R_2\gamma\pi^2(p\cdot\eta)_H\mathbb{E}\left[R_2|(p\cdot\eta)_H\right] - 2R_1\gamma^2\pi\mathbb{E}^2\left[R_2|(p\cdot\eta)_H\right] \\
& + R_1 R_2\gamma^2(p\cdot\eta)_H\mathbb{E}\left[R_2|(p\cdot\eta)_H\right] + R_1^3\pi - 2R_1^2 R_2\pi(p\cdot\eta)_H - R_1^2\gamma\pi^2\mathbb{E}\left[R_2|(p\cdot\eta)_L\right] \\
& + R_1^2\pi^2\gamma\mathbb{E}\left[R_2|(p\cdot\eta)_H\right] - 3R_1^2\gamma^2\pi\mathbb{E}\left[R_2|(p\cdot\eta)_L\right] - R_1^2 R_2\gamma^2(p\cdot\eta)_H - 2R_1\gamma^2 \\
& \cdot\mathbb{E}\left[R_2|(p\cdot\eta)_H\right]\mathbb{E}\left[R_2|(p\cdot\eta)_L\right] + 2R_1^2\gamma^2\mathbb{E}\left[R_2|(p\cdot\eta)_L\right]\Big).
\end{aligned}
$$

The first two terms are the same as those in $g(0)$, and the fact that $\pi \geq \overline{\pi}_2 = \frac{\gamma\mathbb{E}[R_L]-c}{\gamma\mathbb{E}[R_H]-c}$ implies that the third term is non-negative. Therefore, $g(R_1) \leq 0$.

Since $g(c)$ is continuous and monotone in c, then there exists a $c_0 \in [0, R_1]$ such that $d_{0,T} \geq \mathbb{E}\left[R(\alpha_H, c)\right]$, $\forall c \in [0, c_0]$. ∎

A.2 RESULTS OF NUMERICAL SIMULATIONS

The following figures present numerical simulations for various regulatory schemes.

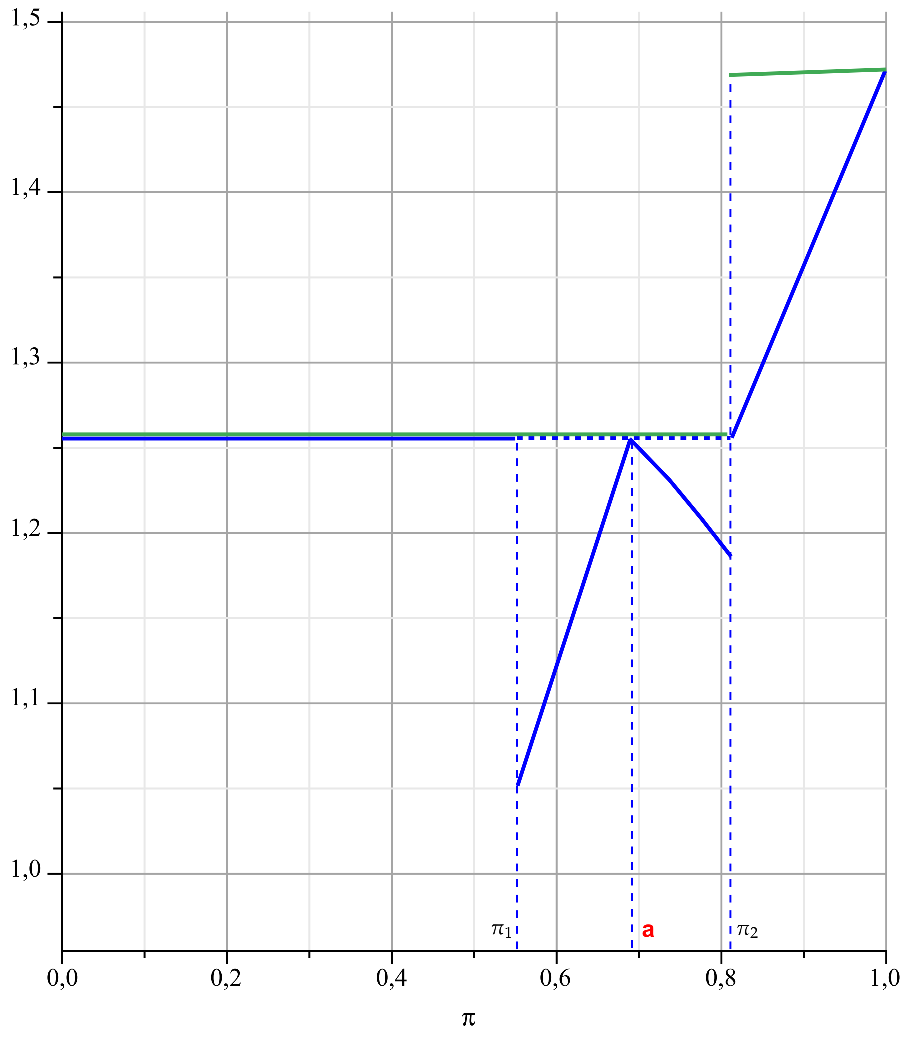

Fig. A.1 Investors' expected return in equilibrium: *laissez-faire* economy (solid blue line) versus economy with conditional liquidity injection & procyclical taxation (solid green line). Parameter values: $(p \cdot \eta)_H = 0.36$, $(p \cdot \eta)_L = 0.24$, $\gamma = 0.6$, $R_1 = 1.5$, $R_2 = 4$, $c = 0.3$, $\overline{\eta} = 0.8$, $\eta_H = 0.9$, $\eta_L = 0.6$, $\overline{p} = 0.4$, $p_H = 0.45$, $p_L = 0.3$, $\sigma = 0.5$.

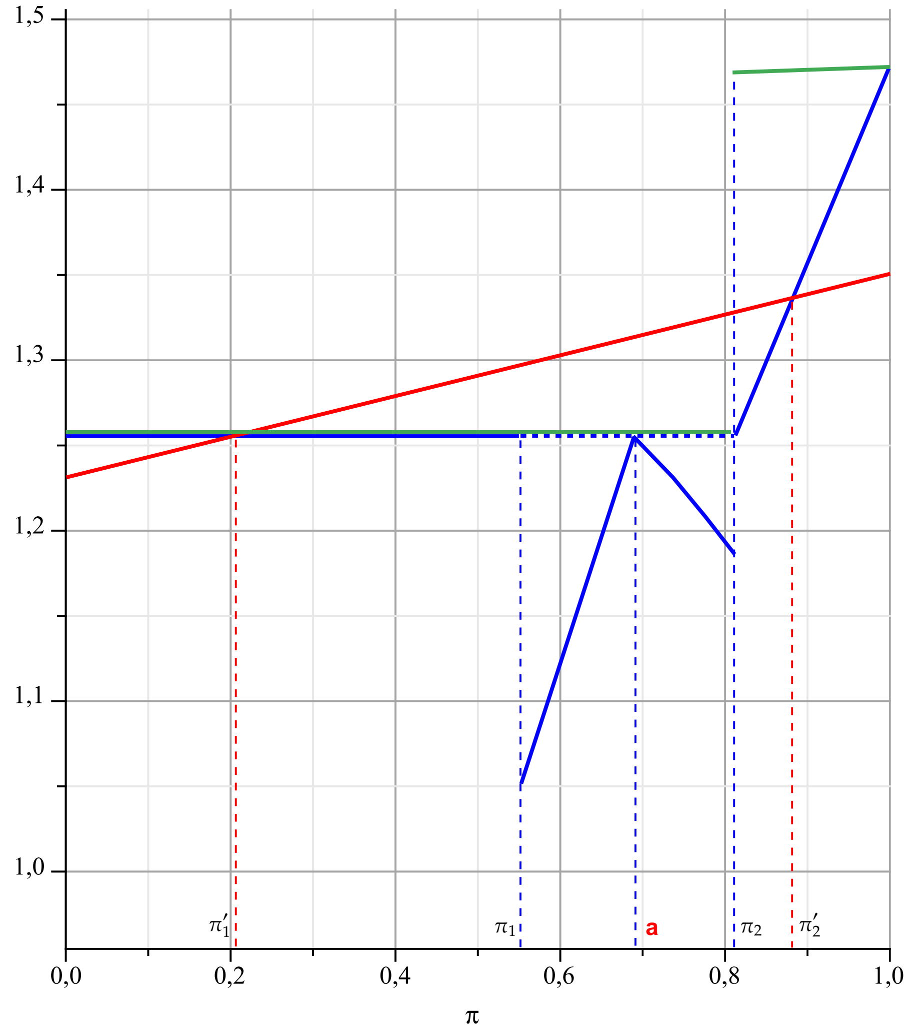

Fig. A.2 Investors' expected return in equilibrium: *laissez-faire* economy (solid blue line) versus economy with (1) equity requirement (solid red line) (2) conditional liquidity injection & procyclical taxation (solid green line). Parameter values: $(p\cdot\eta)_H = 0.36$, $(p\cdot\eta)_L = 0.24$, $\gamma = 0.6$, $R_1 = 1.5$, $R_2 = 4$, $c = 0.3$, $\overline{\eta} = 0.8$, $\eta_H = 0.9$, $\eta_L = 0.6$, $\overline{p} = 0.4$, $p_H = 0.45$, $p_L = 0.3$, $\sigma = 0.5$, $\zeta = 0.5$. The outcome under equity requirement is superior to that of *laissez-faire* economy for $\pi \in \left[\overline{\pi}_1', \overline{\pi}_2'\right]$.

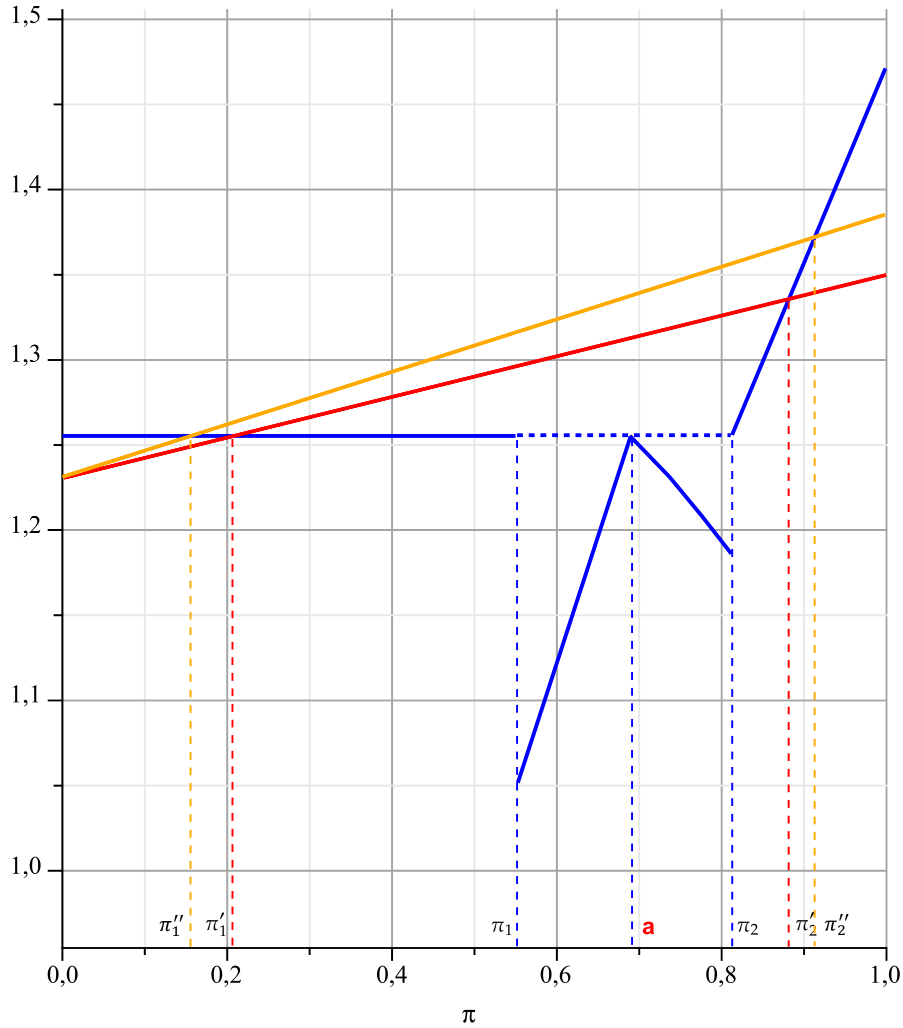

Fig. A.3 Investors' expected return in equilibrium: *laissez-faire* economy (solid blue line) versus economy with (1) pure equity requirement (solid red line) (2) equity requirement & liquidity regulation (solid orange line). Parameter values: $(p \cdot \eta)_H = 0.36$, $(p \cdot \eta)_L = 0.24$, $\gamma = 0.6$, $R_1 = 1.5$, $R_2 = 4$, $c = 0.3$, $\overline{\eta} = 0.8$, $\eta_H = 0.9$, $\eta_L = 0.6$, $\overline{p} = 0.4$, $p_H = 0.45$, $p_L = 0.3$, $\sigma = 0.5$, $\zeta = 0.5$. The outcome under equity requirement & liquidity regulation is superior to that of *laissez-faire* economy for $\pi \in \left[\overline{\pi}_1'', \overline{\pi}_2''\right]$.

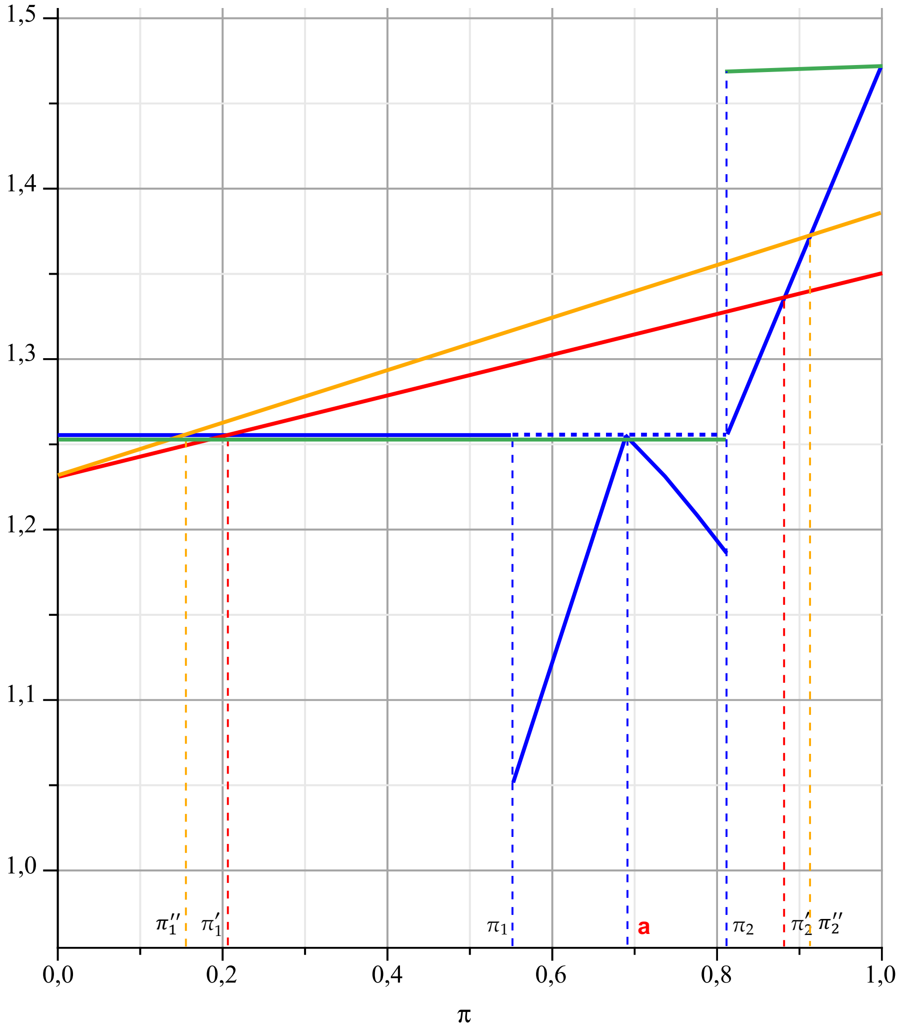

Fig. A.4 Investors' expected return in equilibrium: *laissez-faire* economy (solid blue line) versus economy with (1) conditional liquidity injection & procyclical taxation (solid green line) (2) pure equity requirement (solid red line) (3) equity requirement & liquidity regulation (solid orange line). Parameter values: $(p \cdot \eta)_H = 0.36$, $(p \cdot \eta)_L = 0.24$, $\gamma = 0.6$, $R_1 = 1.5$, $R_2 = 4$, $c = 0.3$, $\overline{\eta} = 0.8$, $\eta_H = 0.9$, $\eta_L = 0.6$, $\overline{p} = 0.4$, $p_H = 0.45$, $p_L = 0.3$, $\sigma = 0.5$, $\zeta = 0.5$.

Part IV

Epilogue

Our long voyage of discovery is over and our bark has drooped her weary sails in port at last. Once more we take the road to Nemi. It is evening, and as we climb the long slope of the Appian Way up to the Alban Hills, we look back and see the sky aflame with sunset, its golden glory resting like the aureole of a dying saint over Rome and touching with a crest of fire the dome of St. Peter's. The sight once seen can never be forgotten, but we turn from it and pursue our way darkling along the mountain side, till we come to Nemi and look down on the lake in its deep hollow, now fast disappearing in the evening shadows. . . . But Nemi's woods are still green, and as the sunset fades above them in the west, there comes to us, borne on the swell of the wind, the sound of the church bells of Aricia ringing the Angelus. *Ave Maria!* Sweet and solemn they chime out from the distant town and die lingeringly away across the wide Campagnan marshes. *Le roi est mort, vive le roi! Ave Maria!*

—Sir James Frazer (1922), *Farewell to Nemi*

The monograph deviates from the standard Arrow-Debreu economy by allowing some minimal frictions in the financial market:

1. Collecting returns from the projects requires specific skills, which motivates the banks' intermediation between investors and entrepreneurs. However, the bank managers would have an incentive to renegotiate with investors to exploit rents, making standard contracts break down. Therefore fixed payment deposit contract serves as a credible commitment device, and the costly bank runs punish those bank managers who abuse their collection skills;

2. If the projects have to be terminated before they mature, only a small part of their value can be recovered;

3. Financial institutions only have limited liabilities in their debts.

Now since financial institutions finance the long-term and high yield projects via short term borrowing, they have to hold sufficient liquid assets to meet investors' short term demand (to avoid the costly bank runs). Assume that illiquidity is the only risk in the economy, i.e. some projects are likely to return late. The first chapter shows that even under pure illiquidity risks banks have strong incentive to free-ride on the others' liquidity provision in the mixed strategy equilibrium, — to maximize their revenue in the good state without fully shouldering the cost in the bad state, — leading to underinvestment in liquid assets across the entire economy. This result doen't only explain why there's still liquidity shortage even when market participants have a perfect information about the likelihood of the bad weather, but also sheds some light on monetary policy and banking regulation. With modelling the feedback between Lender of Last Resort policy and incentives of private banks, the second chapter shows that minimum liquidity standards for banks *ex ante* are a crucial requirement for sensible Lender of Last Resort policy. In the presence of pure illiquidity risks, it's not surprising that imposing equity requirement is a strictly inferior solution.

Modern financial innovations are rapidly blurring the boundary between illiquidity and insolvency risks, which creates another dimension of complexity. The potential risk of insolvency inflates asset price in the good state, but depresses it in the bad state, making the troubled financial institutions even harder to get funding. Then the third chapter shows that with the co-existence of both risks, *ex ante* liquidity regulation with *ex post* Lender of Last Resort policy fails to work. In order to cover the informational cost for banking regulation, regulators have to find additional resources via either public solutions, e.g. establishing safety funds by taxing the banks' revenue in the good state, or private solutions, e.g. making the financial system stabilize itself by imposing minimum equity requirement. Regulatory cost can be reduced by combining the advantages of different schemes.

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