Forecasting Macroeconomic Aggregates Pooling of Forecasts and Pooling of Information

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Johannes Mayr

Referent: Prof. Dr. Gebhard Flaig

Korreferent: Prof. Dr. Kai Carstensen

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Nomenclature

ADL	Autoregressive Distributed Lag
AIC	Akaike Information Criterion
AR	Autoregressive
BC	Business Climate
BE	Business Expectations
BIC	Bayesian Information Criterion
BMA	Bayesian Model Averaging
BS	Business Situation
DGP	Data Generating Process
Disc	Discounted
DSGE	Dynamic Stochastic General Equilibrium
ESI	Economic Sentiment Indicator
Ex	Excluded
GDP	Gross Domestic Product
HLN	Harvey Leybourne Newbold
HP	Hodrick Prescott
IPI	Industrial Production Index
ISIC	International Standard Industrial Classification of all Economic Activities
LARS	Least Angle Regression
LASSO	Least Absolute Shrinkage and Selection Operator
MSE	Mean Squared Error

NACE	Nomenclature Statistique des Activités Économiques dans la Communauté Européenne
NBER	National Bureau of Economic Research
OECD	Organisation for Economic Co–operation and Development
OLS	Ordinary Least Squares
OPI	Optimal Pooling of Information
PC	Principal Components
PF	Pooling of Forecasts
PI	Pooling of Information
PLS	Partial Least Squares
tol	Tolerance
VAR	Vector–Autoregressive
WES	World Economic Survey
WZ	Wirtschaftszweig

Preface

Research on economic forecasting historically has focused on models with only a handful of variables. In contrast, economists in central and commercial banks, governments and related agencies whose daily business is to track the swings of the economy and to provide decision–makers with forecasts in real time have long examined a large number of time series. Nowadays, improvements in computing and electronic data availability have made it feasible to put the forecasts on a broader information basis. In fact, given the growing number of indicators developed, each forecast exercise can be described as being conducted in a data–rich environment. In general, two approaches have been developed to deal with the plurality of information: Pooling of information (PI) and pooling or combining of forecasts (PF). Pooling of information densifies the available information into a manageable number of variables which are used as predictors in a forecast equation. This corresponds to a single model approach of the forecast exercise. In a situation where the number of candidate predictor variables is rather limited, a simple least squares regression poses a straightforward approach to pool the available information. In contrast, pooling of forecasts is a multi model approach where the available dataset is split into subsets, each of them building the basis for a forecast of the target series. These single predictions are densified in a second step to form one final prediction. A priori, it is an open question which of the two frameworks fits a certain empirical forecast exercise. This dissertation compares both forecast frameworks in a general setting and contributes to each of them in various aspects described below.

Combining has a long history in science that predates its use in economic forecasting. The idea that averaging improves the quality of an outcome had been formally developed by Laplace (1818): "In combining the results of these two methods, we can obtain a result for which the probability of errors is more rapidly decreasing." Galton (1879) used photographic equipment to combine many portraits and concluded that "all of the composites are better looking than their components because the averaged portrait of many persons is free from the irregularities that variously blemish the look of each of them." In other words, the more average, the better looking.

However, it was not until the pioneering work of Bates and Granger (1969) that the idea of combining or pooling of forecasts was established. Since then, a growing number of studies have been reviewed and applied in a variety of research fields including economics, management, systematics, biomedicine, meteorology and climatology. The logic of pooling of forecasts is that instead of searching the single best method, one asks which methods would help to improve accuracy, assuming that each method has something to contribute. The different methods are combined using some sort of rule that can be replicated. In the literature on economic forecasting, the single predictions are typically obtained by estimating a number of alternative models over the same sample period. The individual models differ in the predictor variables they use to forecast the target series. However, the employed forecast equations can differ not only in what information they use, i.e. what variables are employed as predictors but also in how the information is used. Pesaran and Timmermann (2007) demonstrate that pooled forecasts obtained using the same model but estimated across different observation windows improves accuracy in the presence of structural breaks. In an empirical application for a wide range of US macroeconomic aggregates, Clark and McCracken (2009b) find similar results in pooling forecast from recursive and rolling estimation windows. Restricting their analysis to random walk models with drift, Pesaran and Pick (2008) find that forecasts of inflation in 21 OECD countries gain accuracy if the drift parameters are averaged over models across different estimation windows. Apart from estimation samples, the idea of improving performance by combining can be extended to various dimensions. Regarding dynamic time series approaches, averaging forecasts from models estimated with different lag polynomials or different data transformations are promising areas of future research.

In practice, pooled forecasts increasingly gain attention by both, by forecasters themselves as well as by the public. A growing number of private and public organizations such as Consensus Economics (Consensus Forecast), the Federal Reserve Bank of Philadelphia (Survey of Professional Forecasters and the Livingston Survey) and Aspen Publishers (Blue Chip Economic Indicators) publishes surveys of macroeconomic forecasts. As a comprehensive figure of each publication, the agencies calculate the simple average of all respondents' forecasts. Several of the institutes further report the empirical distribution of all predictions as a measure of the uncertainty among the forecasters. These consensus forecasts have gained wide acceptance and are frequently used as a benchmark in forecast evaluation exercises. A larger academic literature has studied the benefits of pooling the forecasts from professional agencies. Batchelor and Dua (1995) showed that Blue Chip Economic Indicators forecasts for main US economic aggregates outperformed more than 70% of the panelists in the 1980s, confirming earlier findings of Zarnowitz (1984) and McNees (1987). Although the vast majority of articles confirms the predominance of pooling for model based forecasts, the results remain heterogenous regarding the size of gains. As there are numerous sources for the large variation of the gains, it is difficult to estimate the improvement in forecast accuracy in a given forecast situation ex ante based on empirical findings. To fill this gap, Chapter 1 of this dissertation analytically analyzes the benefits of pooling of forecasts compared to pooling of information and derives conditions where the former outperforms the latter. To provide a guideline to practitioners, Chapter 3 estimates the minimum gains of pooling of forecasts that are achievable in any forecast situation. We employ a Monte Carlo study based on a standard DSGE model to mirror the characteristics of major economies and estimate the gains from pooling of VAR forecasts that are obtained under strict lab conditions. Keeping control of the data–generating process, our Monte Carlo experiment further allows us to decompose the forecast errors and analyze where the observed gains stem from.

Theoretically, pooling of information and pooling of forecast strategies can be used at any forecast horizon. In fact, meteorologists employ what they call "ensemble forecasting" in particular for larger forecast horizons. By slightly varying the initial conditions of their models, they produce a range of forecasts and calculate the "ensemble mean" or average which is expected to have more skill because it averages over the many possible initial states and essentially smoothes the chaotic nature of climate. In contrast, for macroeconomic target series, pooling of forecasts and pooling of information gains relevance especially for short–run predictions. This is due to the fact that most forecasts of macroeconomic time series are based on business cycle indicators that exhibit a rather short lead over the economic activity or even track the target series contemporaneously. Again, Monte Carlo techniques enable us to answer the question whether pooling of forecasts makes sense in the medium and long run as well. Chapter 3 of this dissertation tracks the gains of pooling as the forecast horizon grows.

Predicting the current quarter of macroeconomic aggregates has recently gained increasing attention. In the course of the financial crisis and the successive crash of the world economy, the focus has somehow shifted from longer run perspectives to the current state of the economy. One reason is that policy decision makers as well as central banks – in their decision to implement fiscal stimulus packages or monetary policy actions – strongly rely on assessments of the current unobservable state of the economy. This is of great importance as the effectiveness of certain fiscal and monetary measures greatly depends on the right timing. However, as there are numerous indicators that potentially track the current unobservable state of the economy, solely focussing on one predictor variable or one single forecast model is frequently considered as being venturesome. The main reason is that during exceptional rapid movements of the economy, the ex post observed and estimated relation of the indicator or the set of indicators to the reference series is intensified exposed to structural breaks or other forms of non–linearities. Pooling of forecasts can be regarded as an easy to handle but still informative alternative way of dealing with the potentially very rich set of short–term indicators. Instead of specifying one single forecast model, an approach developed and implemented during this dissertation builds on the idea of pooling the forecasts from small subset models based on all possible combinations of the predictors. Each of the indicator combinations is used to estimate a simple linear forecast equation and predictions for the current period are derived. The combined forecast is calculated as a simple average of the single predictions. Two major advantages emerge from the approach. First, the final prediction is based on a broader information set and the approach guarantees insurance against breakdowns of single relationships between indicators and the reference series. Second, the empirical distribution of the single predictions provides a measure of uncertainty associated with the indicator selection process. Chapter 1 of this dissertation presents the approach in forecasting the current monthly value of German industrial production based on various branch level indices.

Despite the undeniable success of pooling of forecasts in empirical exercises, various researchers object its use in general. From a statistical point of view, combining forecasts and models plays havoc with traditional statistical procedures, such as the calculation of statistical significance levels. A strong headwind also comes from frequentists who believe there is a one right model to forecast. The issue of how to choose "best" among a number of candidate models is however an open question. Notably, statisticians and econometricians have different preferences over this. While econometricians tend to carry out a series of tests to compare competing model specifications, statisticians more likely choose a model by selecting a general class of models and then selecting a member of this family so as to minimize a statistical information criterion. Tests are only used to check the residuals from the chosen "best" model. In contrast, bayesian statisticians will avoid tests and attempt to assess the strength of evidence as between competing models by calculating posterior odds ratios.

One widely accepted approach in econometrics is the step–wise general–to–specific (Gets) procedure. Hendry (1980) provides an overview. The Gets approach employs t–tests to remove individual variables with statistically insignificant coefficient estimates. While computationally attractive, the approach suffers from two major drawbacks. Insignificant estimates can arise not only because the true parameters are small but also because the predictive variables are highly multicollinear what increases the variance in the estimates of the parameters. Furthermore, re–estimating the model after removal of some parameters and examining statistical significant levels, the method becomes path dependent. Additionally, this "all or nothing" approach of either using the OLS estimate or omitting the regressor may be too restrictive. Finally, given a potentially very large number of candidate predictor variables, the Gets approach seems rather infeasible in the data–rich environment of macroeconomic forecasting. A number of approaches that smooth the zero one decision to a softer threshold have been proposed in literature, including bayesian estimation techniques.

Although pooling of information incorporates the Gets approach, it can be defined more broadly as a framework that densifies the available information into a manageable number of predictors that are used to forecast the variable of interest. In recent years, a growing number of pooling of information techniques have been developed. The majority of these frameworks deals with the trade–of between an adequate reduction of the dimension of the information set and capturing all important variations of the different variables, the so–called bias variance trade–off.

One increasingly popular way to pool the information from a large dimensional data set of predictors is to extract some sort of common latent factors that are linear combinations of the total set of variables. Incorporating these factors in the forecast equation bases the prediction on a broader information set without running into estimation problems. A large number of empirical as well as theoretical studies has demonstrated the potential of these approaches especially for very large datasets, see Eickmeier and Ziegler (2008) for a meta–analysis. The major drawback of the classical factor models in terms of forecasting is that they are exclusively in–sample focused and do not take the correlation towards the target series into account. In Chapter 1 of this dissertation, we develop a new method of pooling of information that explicitly weights the relevant predictors with respect to an out–of–sample based loss function. The approach builds on the idea of predictive modeling. This optimal pooling of information algorithm (OPI) is evaluated in an empirical forecast setting predicting German industrial production. To test the applicability of the approach in an international setting, Chapter 2 compares the OPI approach with several competing pooling of information techniques in predicting real GDP in the euro area and opposes the results to pooling of forecasts.

More precisely, Chapter 1 analytically analyzes the relative benefits of pooling of forecasts compared to pooling of information. We demonstrate that pooling of information theoretically dominates pooling of forecasts for perfectly measured explanatory variables. However, in the more realistic scenario where the explanatory variables that form the data–generating process are unobservable and the forecasts rely on noisy indicators of the variables, pooling of information potentially looses its predominance. To compare pooling of forecasts and pooling of information in an empirical experiment, we forecast German industrial production based on Ifo survey data. We propose a new method to aggregate the set of candidate predictor variables that weights the individual series with respect to an out–of–sample based loss function. We find that the algorithm performs considerably well and outperforms economically weighted indices by attributing a non–zero weight to only a smaller number of candidate predictors. However, in general, the analysis confirms the analytical findings that pooling of forecasts likely dominates pooling of information in small samples and under imperfect measurement. We find that pooling of forecasts each based only on a smaller subset of the predictors improves accuracy by up to 40% compared to the benchmark model that combines the entire information by means of economic weights.

Chapter 2 applies the OPI approach in an empirical experiment of forecasting euro area quarterly real GDP.[1] Choosing three different indicators that are available on the national and on the area–wide level, we compare the performance of OPI against the published area–wide benchmark indices as well as against different pooling of forecast strategies. In an out–of–sample experiment we find that OPI outperforms alternative forecasting methods in terms of mean squared forecast error (MSE). Again, only a reduced number of national indicators are attributed a non–zero weight and enter the newly generated area–wide indices.

Chapter 3 quantifies the gains from pooling of VAR forecasts under strict lab conditions where accidental effects such as breaks in the data–generating process that bias the results in favor of combination approaches in empirical studies are explicitly excluded.[2] Employing a Monte Carlo study based on a standard DSGE model, we

[1]The chapter relies on Hülsewig et al. (2008) which is available as *CESifo Working Paper 2371*.
[2]The chapter relies on Henzel and Mayr (2009) which is available as *ifo Working Paper 65*.

mimic a macroeconomic forecast situation and obtain the business cycle behavior of the most relevant variables such as GDP, inflation and interest rates for flexible as well as more persistent economies. Dynamically forecasting real economic activity from parsimonious VAR models, each built only on a subset of the relevant information we find that pooling of forecasts leads to a substantial improvement in accuracy of about 20 percent, which is comparable to the effect of an increase of the estimation sample from 25 to 1000 observations. Most notably, this gain is already obtained with an average of about four different forecasts and is higher for more persistent economies.

Chapter 1

The Use of Plural Information in Forecasting: Pooling of Forecasts and Pooling of Information

In this paper, we analyze the relative merits of pooling of forecasts compared to pooling of information. In an analytical part, we demonstrate that for perfectly measured explanatory variables, pooling of information theoretically dominates pooling of forecasts but looses its virtues in the more realistic scenario where only noisy measures of the explanatory variables are at hand. In accordance with the analytical findings, our empirical experiment of forecasting German industrial production based on Ifo survey data confirms the advantages of pooling of forecasts. We find that pooling predictions each based only on a small subset of branch level indices improves forecast accuracy by up to 40% compared to the benchmark model that pools the entire information by means of economically weighted indices. Relating to the framework of predictive modeling, we further develop the OPI approach as an alternative pooling of information strategy that optimizes the weights of the sub–indices specifically with respect to a certain forecast exercise. We find that OPI performs considerably well and poses a serious alternative to the dominant pooling of forecasts strategies. Most notably, only a small subset of branch level indices is assigned a non–zero weight and thus relevant in forecasting industrial production.

1.1 Introduction

Forecasting a variable of interest, one is usually faced with a broader set of candidate predictor variables. The question that arises is how to use this set. There are generally two directions one can proceed: Pooling of forecasts and pooling of information. Pooling of information integrates the available predictor variables into one model whereas pooling of forecasts combines the predictions of various models each based only on a subset of the predictor variables at hand.

Hendry and Clements (2004) as well as Huang and Lee (2006) analytically analyze the merits of pooling of forecasts compared to single subset models as well as compared to larger models incorporating all relevant predictor variables. These studies assume that the predictor variables available to the forecaster correspond to the explanatory variables that build the data–generating process. In other words, the explanatory variables are perfectly measured, i.e. they are directly observable. Pooling of information then corresponds to estimating a model including all explanatory variables and mirrors the data–generating process.

In practice however, the true explanatory variables are often not directly observable but are only measured by certain indicator variables that additionally incorporate noise components. The lower the signal–to–noise ratio of these indices, the higher the resulting forecast error with respect to the target series. The question that arises is whether pooling of information maintains its theoretical predominance if all models are build on noisy measures of unobservable explanatory variables.

In this paper, we follow Hendry and Clements (2004) and Huang and Lee (2006) and show analytically that pooling of information dominates pooling of forecasts for observable explanatory variables in the absence of sample uncertainty. Extending the analysis to the more relevant case where the explanatory variables are only imperfectly measured, we introduce additional noise into the system. We find that the relative merits of pooling of information decrease with increasing noise as well as for increasing correlation of the shocks that hit the observable indicators of the explanatory variables.

As an empirical application, we predict the German industrial production index on basis of survey data of the Ifo business climate for manufacturing industry. We compare the economically weighted Ifo series as one straightforward pooling of information strategy with alternative PI and PF approaches based on disaggregate branch level data. We develop the OPI algorithm that pools the available time series by means of out–of–sample optimized weights. We find that the economically weighted indices have considerable predictive content and pose an adequate aggregation scheme to forecast industrial production. However, reweighting the branch level data by means of PI and PF strategies additionally improves forecast accuracy by up to 40%. The OPI algorithm performs best for all PI approaches under consideration attributing a non–zero weight only to a subset of the disaggregate branch level series. In general, PF strategies that weight the single forecasts by means of their discounted past performance perform best and reduce the MSE considerably.

This paper is organized as follows. Section 1.2 analyzes analytically the relative merits of pooling of forecasts compared to pooling of information. Section 1.2.1 derives expressions of the MSE for PF and PI under perfectly measured explanatory variables and compares theoretically optimal and equal PF weights. Section 1.2.2 extends the analysis to imperfectly measured explanatory variables by introducing noise into the system. Section 1.3 compares the relative merits of PI and PF strategies in an empirical application predicting German industrial production via survey data of the Ifo business climate for manufacturing industry. Section 1.4 concludes.

1.2 Analytical Analysis

In this section, we analyze analytically the relative merits of pooling of forecasts compared to pooling of information. We follow Hendry and Clements (2004) and Huang and Lee (2006) and define the DGP as

$$\begin{aligned} y_t &= \beta_1 x_{1,t} + \beta_2 x_{2,t} + e_t \\ &= \beta X_t + e_t \end{aligned} \tag{1.1}$$

where $e_t \sim N(0, \Omega_{ee})$. y_t denotes the target series and $x_{1,t}$ and $x_{2,t}$ are strictly exogenous explanatory variables.[1] We assume that all variables of the DGP have been reduced to weak stationarity by appropriate transformations. The explanatory variables are defined as

$$X_t = \begin{pmatrix} x_{1,t} \\ x_{2,t} \end{pmatrix} = \begin{pmatrix} \phi_{1,t} \\ \phi_{2,t} \end{pmatrix} + \begin{pmatrix} \xi_{1,t} \\ \xi_{2,t} \end{pmatrix} \tag{1.2}$$

where $\phi_{1,t}$ and $\phi_{2,t}$ are fixed functions of past variables and

$$\begin{pmatrix} \xi_{1,t} \\ \xi_{2,t} \end{pmatrix} \sim IN_k \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix} \right]. \tag{1.3}$$

Abstracting from any dynamic and deterministic factors in the DGP, we set $\phi_{1,t} = \phi_{2,t} = 0$. We assume that this is known to the forecaster, so intercepts and further lags are omitted in all forecast equations. To simplify the algebra later on, we assume that $\beta_2 = 1 - \beta_1$, i.e. the coefficients in the DGP sum up to unity.[2] We aim to

[1] The assumption of strict exogeneity rules out lagged dependent variables only to simplify the algebra. The results can be extended to a more complicated model without the strict exogeneity assumption.

[2] This mirrors an economic context where the growth rate y_t of a certain target series can be calculated by aggregating the growth rates of its components. The weights assigned to the components' growth rates reflect their relative shares in the target series.

forecast the unknown current value of scalar variable y_t on basis of the information available up to time t.[3]

1.2.1 Perfect Measurement

In the first part of the analytical analysis, we assume that the current values of the explanatory variables are observable to the forecaster, i.e. we forecast the unknown value of scalar variable y_t on basis of an information set given as $I_t = (X_s)_{s=0}^t$. Build on this assumption, we derive expressions of the MSE as the relevant loss function and compare the performance of PF against the dominant single subset model as well as against PI. This analysis gives insights into the relative merits of PF and PI under strict lab conditions.

1.2.1.1 Forecast Framework I

Pooling of information corresponds to a forecast equation given as

$$\begin{aligned} \hat{y}_{PI,t} &= \hat{a}_1 x_{1,t} + \hat{a}_2 x_{2,t} \\ &= \hat{A} X_t \end{aligned} \tag{1.4}$$

where all relevant explanatory variables form part of the model and the parameters are estimated by ordinary least squares.[4] In contrast, the pooling of forecasts approach can be written as

$$\hat{y}_{PF,t} = w_1 \hat{y}_{1,t} + w_2 \hat{y}_{2,t} \tag{1.5}$$

where the single predictions result from two non–nested misspecified forecast equations given as

$$\begin{aligned} \hat{y}_{1,t} &= \hat{b}_1 x_{1,t} \\ \hat{y}_{2,t} &= \hat{b}_2 x_{2,t} \end{aligned} \tag{1.6}$$

[3]Forecasting the value of a reference series for the current period is frequently referred to as nowcast. Preponing the index of the target series in the forecast equations to $t+h$ adjusts the setting to a forecast of a h–step ahead value of the target series.

[4]In the empirical part of the study, we extend the comparison to PI schemes that abstract from coefficients estimated by least squares but pool the information via alternative techniques.

and w_i $(i = 1, 2)$ denote the combination weights of the two predictions respectively. For the analytical analysis, we assume that the combination weights are exogenously given such that no additional estimation uncertainty enters. Given the structure of the DGP and to simplify the algebra, we set $w_1 = w$ and $w_2 = 1 - w$, assuming that the combination weights sum up to unity.

1.2.1.2 Forecast Error Comparison I

To compare the forecast performance of PF against the single models as well as compared to PI, we derive analytical expressions of the MSEs for each of the frameworks in Appendix 1.B. The MSEs of the different forecast approaches are given as

$$\begin{aligned}
MSE_1 &= T^{-1}\Omega_{ee} + \beta_2^2\Omega_{\eta_{21}} + \Omega_{ee} \\
MSE_2 &= T^{-1}\Omega_{ee} + \beta_1^2\Omega_{\eta_{12}} + \Omega_{ee} \\
MSE_{PF} &= \Omega_{ee} + T^{-1}\Omega_{ee}(w^2 + (1-w)^2) + 2w(1-w)E[\delta_{\hat{b}_1} x_{1,t} x_{2,t} \delta_{\hat{b}_2}] + w^2\beta_2^2\Omega_{\eta_{21}} \\
&\quad + (1-w)^2\beta_1^2\Omega_{\eta_{12}} + 2w(1-w)\beta_1\beta_2 E[\eta_{21,t}\eta_{12,t}] \\
MSE_{PI} &= 2T^{-1}\Omega_{ee} + \Omega_{ee}
\end{aligned}$$

where T represents the estimation sample size, $\eta_{12,t}$ and $\eta_{21,t}$ are the idiosyncratic components of $x_{1,t}$ and $x_{2,t}$ respectively, $\Omega_{\eta_{12}}$ and $\Omega_{\eta_{21}}$ are the corresponding variances and $\delta_{\hat{b}_i} = \hat{b}_i - E[\hat{b}_i]$ for $i = 1, 2$ are the sample variabilities of the estimators.

To illustrate the dominance of PF against the best single subset model and thus to justify its use in general, we assume that the forecasts from Model 2 are on average more accurate than those of Model 1, i.e. $MSE_2 < MSE_1$. The pooled prediction predominates the more accurate forecast model, that is $MSE_{PF} < MSE_2$ if the following condition holds:

$$T^{-1}\Omega_{ee}(w^2+(1-w)^2)+2w(1-w)E[\delta_{\hat{b}_1}x_{1,t}x_{2,t}\delta_{\hat{b}_2}]+w^2\beta_2^2\Omega_{\eta_{21}}$$
$$+(1-w)^2\beta_1^2\Omega_{\eta_{12}}+2w(1-w)\beta_1\beta_2E[\eta_{21,t}\eta_{12,t}]$$
$$<$$
$$T^{-1}\Omega_{ee}+\beta_1^2\Omega_{\eta_{12}}$$

This condition can be simplified by defining $\beta_1^2\Omega_{\eta_{12}} = k\beta_2^2\Omega_{\eta_{21}}$ with $k < 1$ as a measure of the predominance of Model 2 against Model 1. In the absence of estimation uncertainty, i.e. for $T \to \infty$, it follows that $MSE_{PF} < MSE_2$ holds if

$$(w^2+(1-w)^2k-k)\beta_2^2\Omega_{\eta_{21}} - 2w(1-w)\beta_1\beta_2\Omega_{12}(I_n - \prod_{12}\prod_{21}) \quad < \quad 0 \qquad (1.7)$$

where $\prod_{12} = \Omega_{22}^{-1}\Omega_{12}$ and $\prod_{21} = \Omega_{11}^{-1}\Omega_{12}$. In the special case of uncorrelated explanatory variables, i.e. $\Omega_{12} = 0$, the condition simplifies to

$$w \quad < \quad \frac{2k}{1+k} \qquad (1.8)$$

and

$$k \quad > \quad \frac{w}{2-w}. \qquad (1.9)$$

Hence, an improvement over the dominant single model is achieved for a certain range of the combination weight w. The weight attributed to the worse model must be less than a threshold value that is proportional to k. In other words, the larger the difference in forecast performance between the two models and thus the smaller k, the higher the weight attributed to the superior model must be for PF to dominate.

Now we turn to the comparison of PF and PI. To simplify the MSE equations given above, we assume that the variances of the subseries equal unity, i.e $\Omega_{11} = \Omega_{22} = 1$ in the following. Pooling the forecasts from the two parsimonious and misspecified models given in Equation (1.18) dominates the forecasts from the PI model given in Equation (1.4), i.e. $MSE_{PF} < MSE_{PI}$ if the following condition holds:[5]

[5]Note: For $\Omega_{11} = \Omega_{22} = 1$, we can reformulate $E[\delta_{\hat{b}_1}x_{1,t}x_{2,t}\delta_{\hat{b}_2}] = E[x_{1,t}\Omega_{\delta_{\hat{b}_1}}x_{2,t}] = E[x_{1,t}\Omega_{\delta_{\hat{b}_2}}x_{2,t}] = T^{-1}\Omega_{ee}\Omega_{12}$.

$$w^2(T^{-1}\Omega_{ee} + \beta_2^2(1-\Omega_{12}^2)) + (1-w)^2(T^{-1}\Omega_{ee} + \beta_1^2(1-\Omega_{12}^2))$$
$$+2w(1-w)(T^{-1}\Omega_{ee}\Omega_{12} - \beta_1\beta_2\Omega_{12}(1-\Omega_{12}^2))$$
$$<$$
$$2T^{-1}\Omega_{ee}$$

This inequality condition can be rewritten as:

$$T^{-1}\Omega_{ee} \quad > \quad \frac{(1-\Omega_{12}^2)(w^2\beta_2^2 + (1-w)^2\beta_1^2 - 2w(1-w)\beta_1\beta_2\Omega_{12})}{1 + 2w(1-w)(1-\Omega_{12})} \tag{1.10}$$

PF dominates PI more likely for small values of T and large values of Ω_{ee} as well as for $|\Omega_{12}| \to 1$, i.e. for a high positive or negative correlation of the explanatory variables. This is due to the parameter estimation error that is larger in the PI equation then in the single equations that build the basis for the PF approach. With an increasing sample size, i.e. for $T \to \infty$, the left hand side of Equation (1.10) converges to zero and $MSE_{PI} \leq MSE_{PF}$ necessarily holds. The large sample predominance of PI is due to the two–stage estimation strategy of PF that filters the available information set through individual forecast models first. This strategy introduces the usual efficiency loss by ignoring any correlations between the underlying information sources. More precisely, when estimating the single models as given in Equation (1.18), the estimated coefficients $\hat{b}_i$ are based on the covariances Ω_{iy} but do not take the covariances Ω_{jy} and Ω_{ij} into account, where $i, j = 1, 2$ and $i \neq j$. In the absence of estimation uncertainty, this implies that PI necessarily predominates PF.

In case of PF, the MSE is a complex function of the weights w assigned to the forecasts as well as of the dynamics of the subseries, the variance of the noise in the DGP and the estimation uncertainty that depends on the size of the estimation sample T. To better understand the relative merits of PF, we focus on MSE minimizing weights and on equal weights as the most frequently applied weighting schemes in literature.[6] Abstracting from estimation uncertainty, i.e. for $T \to \infty$,

[6] In the empirical part of the chapter, we present a more detailed discussion of relevant PF

the MSE minimizing optimal weights w^* are given as

$$w^* = \frac{\beta_1\left(\beta_1\left(\Omega_{11}-\Omega_{12}\right)+\Omega_{12}\right)}{\Omega_{22}\left(\beta_1-1\right)^2+\beta_1\left(\beta_1\Omega_{11}-2\left(\beta_1-1\right)\Omega_{12}\right)} \tag{1.11}$$

and the corresponding MSE^*_{PF} amounts to

$$MSE^*_{PF} = \frac{\left(\beta_1-1\right)^2\beta_1^2\left(\Omega_{12}^2-\Omega_{11}\Omega_{22}\right)^2}{\Omega_{11}\Omega_{22}\left(\Omega_{22}\left(\beta_1-1\right)^2+\beta_1\left(\beta_1\Omega_{11}-2\left(\beta_1-1\right)\Omega_{12}\right)\right)}+\Omega_{ee} \tag{1.12}$$

Assuming equal variances of the subseries, i.e. $\Omega_{11}=\Omega_{22}=1$, Figure 1.1 illustrates w^* and the corresponding MSE^*_{PF} as a function of the correlations of the subseries Ω_{12} and of the coefficients β_1 and β_2 in the DGP.[7] As long as $\Omega_{12}<1$, w^* is a s–shaped function of the coefficient β_1, i.e. the weight attributed to a forecast model increases disproportionately with the weight of the respective explanatory variable in the DGP. As mentioned above, PF generally benefits from a high positive or negative correlation of the explanatory variables, i.e. $MSE_{PF}\rightarrow\Omega_{ee}$ for $|\Omega_{12}|\rightarrow 1$.[8] Independent of the correlation of the explanatory variables, MSE^*_{PF} reaches a maximum for $\beta_1=0.5$.

Figure 1.2 illustrates the MSE for equal weights again as a function of the correlations of the subseries Ω_{12} and of the economic weights β_1. Assigning equal weights to the single forecasts, the MSE is minimized and equals MSE^* for $\beta_1=0.5$. Regarding the correlation of the explanatory variables, the MSEs reach a maximum for $\Omega_{12}=-\frac{1}{3}$ for both weighting schemes considered.

weighting schemes.

[7]As w^* tends to infinite values for perfectly negative correlated variables and $\beta_1=0.5$, we limit the graphical illustration of w^* to values of Ω_{12} between 1 and -0.75.

[8]Note that Ω_{ee} does not influence the dynamics of the MSE function as it enters additively. To simplify matters, we set $\Omega_{ee}=0$ for the graphical illustrations.

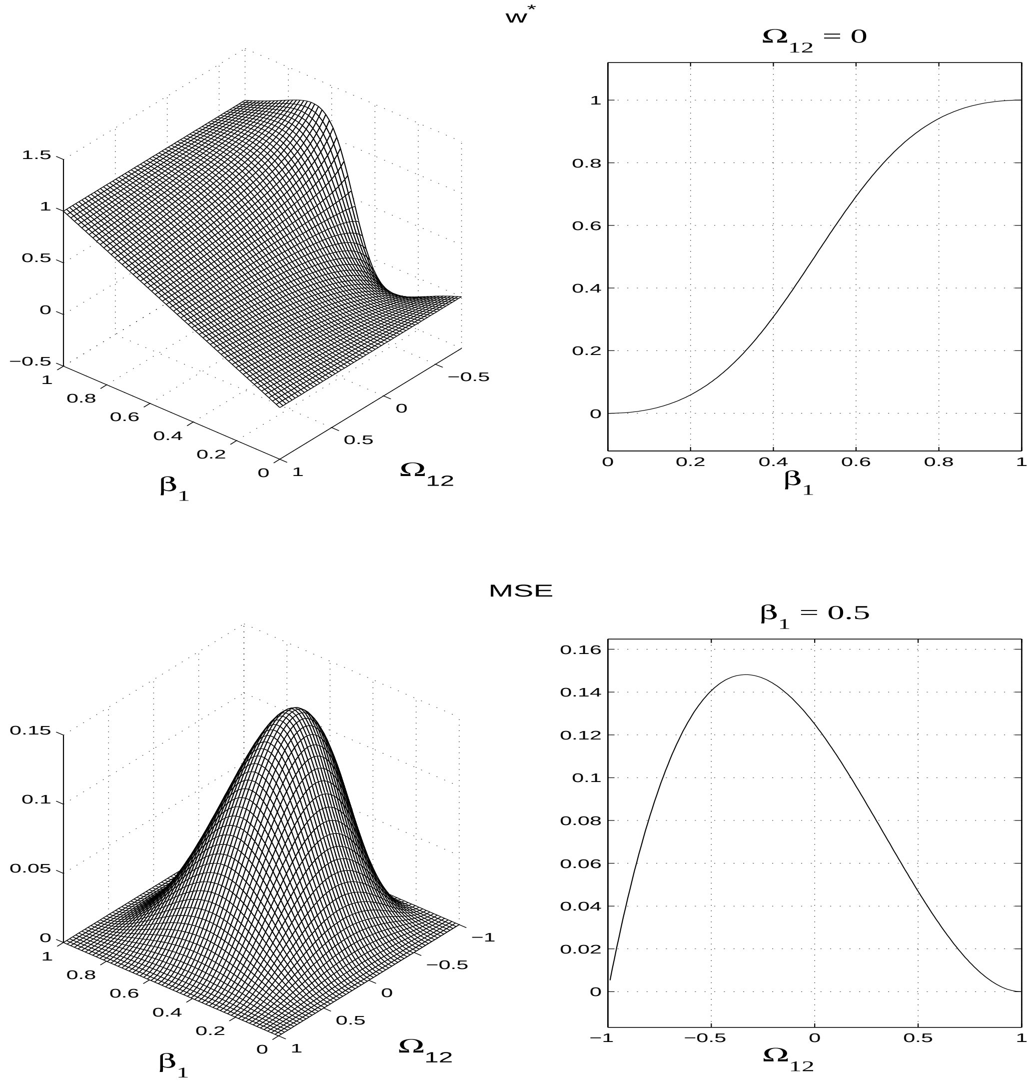


Figure 1.1: Pooling of Forecasts - Optimal Weights

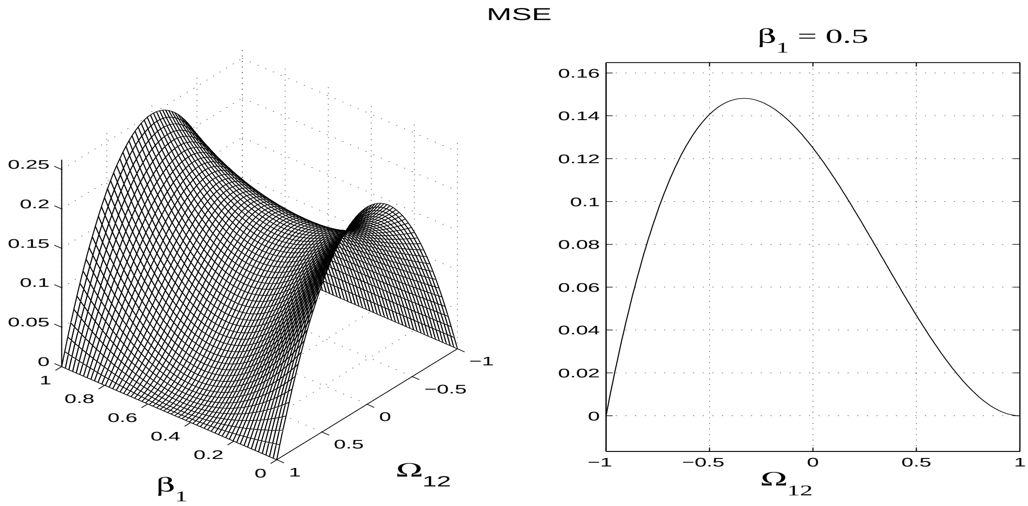


Figure 1.2: Pooling of Forecasts - Equal Weights

1.2.2 Imperfect Measurement - The Role of Indicators

In the second part of the analytical analysis we turn to a more realistic scenario and assume that the current values of the explanatory variables are not observable to the forecaster but that the predictions are based only on noisy measures of x_1 and x_2. The observable proxy variables can be regarded as indicators, each measuring a certain part of the DGP.

1.2.2.1 Indicators of Subseries

The indicator variables are defined as

$$\tilde{X}_t = \delta + CX_t + \epsilon_t \tag{1.13}$$

where δ denotes a (2×1) vector of fixed intercept terms, C is a (2×2) diagonal matrix of fixed coefficients and

$$\begin{pmatrix} \epsilon_{1,t} \\ \\ \epsilon_{2,t} \end{pmatrix} \sim IN_k \left[\begin{pmatrix} 0 \\ \\ 0 \end{pmatrix}, \begin{pmatrix} \Omega_{\epsilon_1\epsilon_1} & \Omega_{\epsilon_1\epsilon_2} \\ \\ \Omega_{\epsilon_2\epsilon_1} & \Omega_{\epsilon_2\epsilon_2} \end{pmatrix} \right] \tag{1.14}$$

is an innovation process that comprises all noisy information affecting the indicator series.[9] The quality of indicator i (for $i = 1, 2$) is measured by means of a signal–to–noise ratio, given as

$$\kappa_i = \frac{\Omega_{ii}}{\Omega_{\epsilon_i\epsilon_i}}. \tag{1.15}$$

Thus, a high value of κ_i corresponds to an indicator that poses a proper measure of an economic aggregate whereas a low value of κ_i corresponds to a noisy indicator that contains less relevant information.

1.2.2.2 Forecast Framework II

Forecasts of the unknown value of scalar variable y_t are based on the information set available up to time t that now comprises the indicators instead of the unobservable explanatory variables, i.e. $I_t = (\tilde{X}_s)_{s=0}^t$. The forecaster is faced with the question of how to optimally use the information set available to forecast y_t. In contrast to the situation of observable explanatory variables as described in section 1.2.1, pooling the information by means of the coefficients of Equation (1.43) is not necessarily optimal even in the absence of estimation uncertainty.

In fact, various PI and PF approaches have been proposed in literature to deal with this issue and we take a closer look at the most relevant ones in the empirical application presented in section 1.3. Figure 1.3 illustrates the links between the observable indicators $\tilde{X}_t$ and the aggregate target series y_t in a general setting.

Three major frameworks to forecast y_t can be distinguished. 1.) The observable indicators are aggregated by means of the coefficients β of the DGP. In a macroe-

[9] To simplify the algebra, we assume that the observable indicators are mean–adjusted, i.e. we set $\delta_1 = \delta_2 = 0$ and that C is an identity matrix.

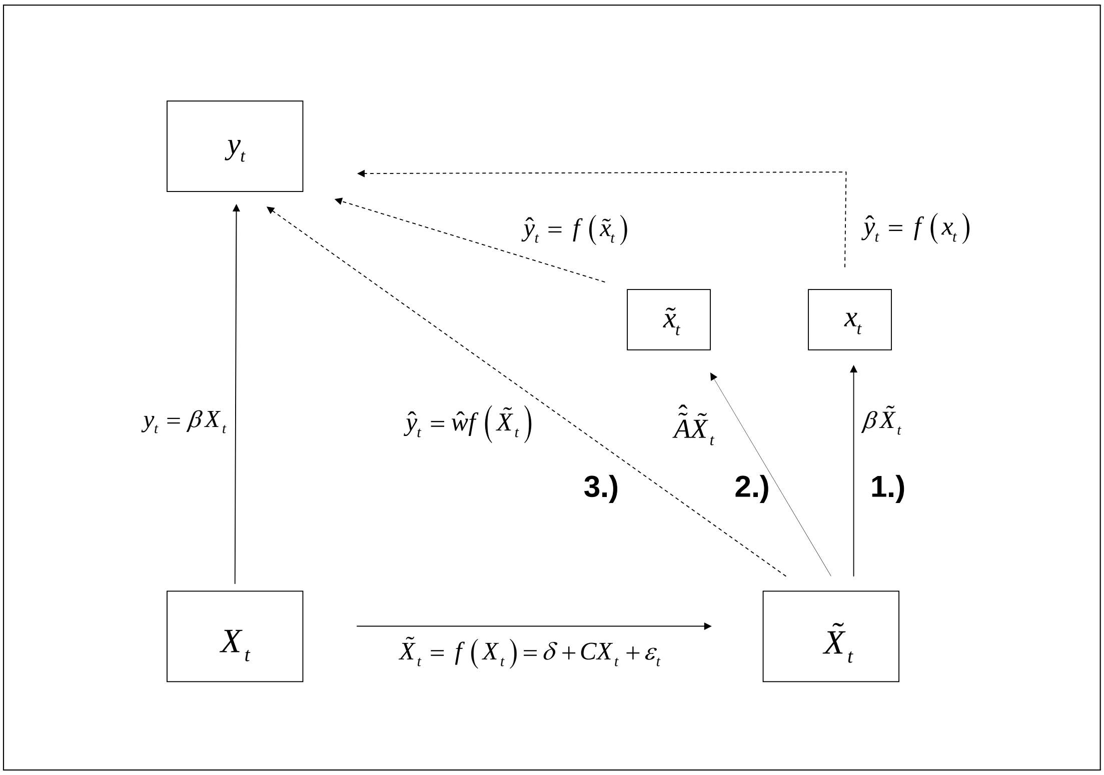


Figure 1.3: Structure of the Forecast Problem

conomic context, these coefficients correspond to observable economic weights, e.g. relative shares in gross–value added. 2.) The observable indicators are aggregated by alternative PI weighting schemes. A larger number of econometric approaches has been developed to extract or summarize the information contained in various indicators. The majority of these frameworks computes an aggregate predictor by means of a linear combination of the indicators with estimated weights $\hat{\tilde{A}}$. The weights potentially deviate from the coefficients in the DGP even in the absence of estimation uncertainty as certain indicators potentially fail to track the respective explanatory variables and exhibit a lower signal–to–noise ratio. 3.) The observable indicators are used as predictor variables in various subset model. The single predictions are pooled based on estimated or ad–hoc attributed combination weights $\tilde{w}$.

Analogously to Section 1.2.1, the PI forecast equation based on OLS estimated weights is given as

$$\begin{aligned}\hat{\hat{y}}_{PI,t} &= \hat{\hat{a}}_1\tilde{x}_{1,t} + \hat{\hat{a}}_2\tilde{x}_{2,t} \\ &= \hat{\hat{A}}\tilde{X}_t\end{aligned} \tag{1.16}$$

where all observable indicators form part of the model. The PF approach based on indicator variables is given as

$$\hat{\hat{y}}_{PF,t} = w_1\hat{\hat{y}}_{1,t} + w_2\hat{\hat{y}}_{2,t} \tag{1.17}$$

where the single predictions result from two forecast equations given as

$$\begin{aligned}\hat{\hat{y}}_{1,t} &= \hat{\hat{b}}_1\tilde{x}_{1,t} \\ \hat{\hat{y}}_{1,t} &= \hat{\hat{b}}_2\tilde{x}_{2,t}\end{aligned} \tag{1.18}$$

1.2.2.3 Forecast Error Comparison II

To analyze the effects of noisily measured indicators on the relative merits of PF and PI, we restrict the analysis to a stylized case where the subseries $x_{1,t}$ and $x_{2,t}$ are uncorrelated and have unity variance, i.e. $\Omega_{12} = 0$ and $\Omega_{11} = \Omega_{22} = 1$. The variances of the noise components of the indicators are determined by the corresponding signal–to–noise ratios κ_i. To simplify the algebra, we further abstract from estimation uncertainty and assume equal coefficients in the DGP, i.e. $\beta_1 = \beta_2 = 0.5$. The MSE expressions for the single models as well as for PF and PI are obtained by replacing the unobservable explanatory variables $x_{1,t}$ and $x_{2,t}$ with the corresponding indicator variables $\tilde{x}_{1,t}$ and $\tilde{x}_{2,t}$ in the MSE derivations shown in Appendix 1.B.[10]

To illustrate the effect of increasing values of $\Omega_{\epsilon_1\epsilon_2}$, we first assume a signal–to–noise ratio of unity for both indicator series, i.e $\kappa_1 = \kappa_2 = 1$. The left graphic of Figure 1.4 illustrates the effects of increasingly correlated shocks on the forecast performance of PI and PF measured by the MSE. Assuming that the indicators are

[10] We abstract from presenting the resulting expressions of the MSEs and of the combination weights as they are complex expressions of the dynamics of the subseries as well as of the noise components of the indicators. All expressions have been computed using the software package Mathematica 5.2 and are available from the author upon request.

disturbed by uncorrelated shocks, i.e. $\Omega_{\epsilon_1\epsilon_2} = 0$, the right graphic illustrates the effects of increasing shares of noise measured by the indicators. To simplify matters, we assume symmetric measurement errors, i.e. $\kappa = \kappa_1 = \kappa_2$.

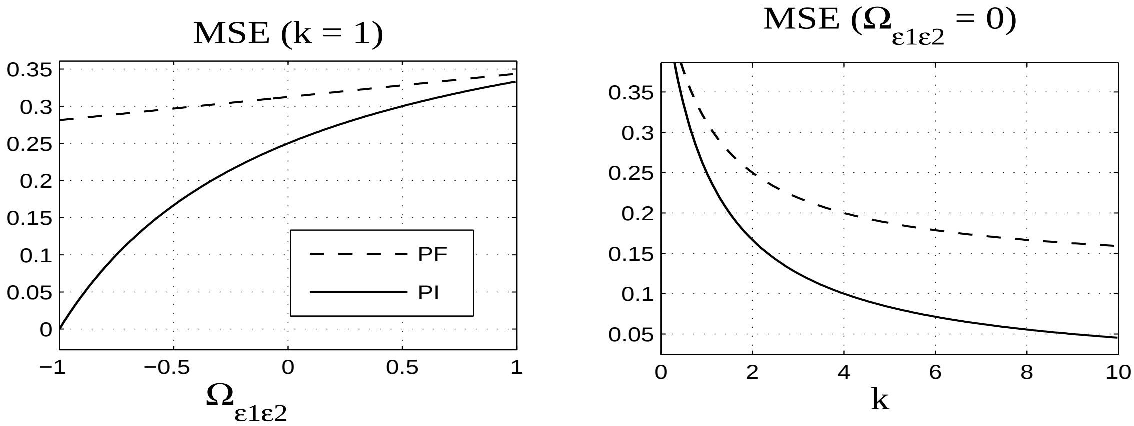


Figure 1.4: Effects of noise on PF and PI

Notably, PF relatively benefits from positively correlated noise components whereas the dominance of PI increases for negative values of $\Omega_{\epsilon_1\epsilon_2}$. Regarding the size of the signal–to–noise ratios, the MSEs converge for $\kappa \to 0$, i.e. for indicators that contain only little relevant information. Obviously, although not explicitly shown in the analysis, pooling the information by means of the coefficients β of the DGP can be regarded optimal only for symmetric measurement errors, i.e. for $\kappa_1 = \kappa_2$. As soon as $\kappa_1 \neq \kappa_2$, a deviation from these coefficients by down weighting the indicator that exhibits the lower signal–to–noise ratio potentially improves forecast accuracy.

The analytical analysis shows that PI looses its merits compared to PF for highly correlated shocks to the indicators as well as for noisily measured subseries. This potentially adds to the success of pooled predictions from parsimonious models frequently found in empirical applications. In practice, indicators measuring different parts of the economy are frequently disturbed by global shocks or shocks that are highly correlated interfering the estimation of larger models.

1.3 Empirical Study - Forecasting Industrial Production with Ifo Business Survey Data for Germany

In this section, we study the relative performance of different PF and PI techniques in predicting the German industrial production index (IPI). As explanatory variables, we employ data of the Ifo business survey of manufacturing industry for Germany, both at the branch level as well as at the sectoral level. We explore whether the construction of sectoral indicators based on economic weights – as used by the Ifo institute – poses an adequate strategy or whether an alternative weighting scheme improves forecast accuracy. Furthermore, we evaluate the merits of pooled forecasts based on the disaggregated branch level series. Throughout this chapter, X_t denotes the (N $\times$ 1) dimensional vector of stationary predictor variables with observations from $t = 1, \ldots, T$ and y_t is the stationary target series.

1.3.1 Ifo Business Survey

The widely noticed Ifo Business Climate – published by the Ifo Institute in the last week of each month – is based on micro–level data that is aggregated in several steps.[11] At the different levels, the aggregation is carried out according to the German version (WZ 2003) of the European statistical classification scheme (NACE) by means of shares in gross–value added. The resulting indices are thus comparable to official economic data published by the German authorities.[12]

At each level of the aggregation hierarchy, the *Business Climate (BC)* is calculated

[11] The deadline for the return of the questionnaires ends about two days before the release day of the indicator that is set by the European Commission as major principal. As the distribution of the questionnaires for a given month already starts in the last days of the previous months, the level of information of the respondents varies between the last week of the previous month and the first three weeks of the current month.

[12] Currently, the aggregation scheme is modified to fit the German version (WZ 2008) of the NACE revision 2. The implementation will be finished by May 2010.

as the geometric mean of the balances of the current *Business Situation (BS)* and the *Business Expectations (BE)*:[13]

$$BC = \sqrt{(BS + 200)(BE + 200)} - 200 \tag{1.19}$$

The balance value of the current *Business Situation* is the difference in percentage shares of the responses "good" and "poor". The balance value of the *Business Expectations* is the difference in percentage shares of the responses "more favorable" and "less favorable".[14] To derive the indices, the transformed balances are normalized to the average of a base year, currently the year 2000.

$$Index_t = \frac{Balance_t + 200}{Balance_{2000} + 200} 100 \tag{1.20}$$

Figure 1.5 illustrates the aggregation scheme of the Ifo Business Climate.

Focussing on manufacturing industries, in a first step, the responses of participants are aggregated to results for 300 product groups. Weights are assigned to the individual responding unit in accordance with the logarithmic function of the number of employees involved in the production of the product or products to which the response refers.[15] The resulting indices of four–digit level product groups are then aggregated to the three–digit and two–digit level based on relative shares in gross–value added. The two–digit level series represent the 22 major branches in manufacturing such as *chemicals and chemical products* or *machinery and equipment*. The indices of these major branches are aggregated to the sectoral level, again based on relative shares in gross–value added. The sectoral time series for manufacturing, construction, wholesaling and retailing are seasonally adjusted by means of the so–

[13] *BS* refers to the firms' assessment of the current business situation and *BE* refers to their business expectations for the next six months.

[14] The enterprises can give one of three categorical answers ("1" positive, "2" neutral, "3" negative) per standard question.

[15] See Ruppert (2007) for details.

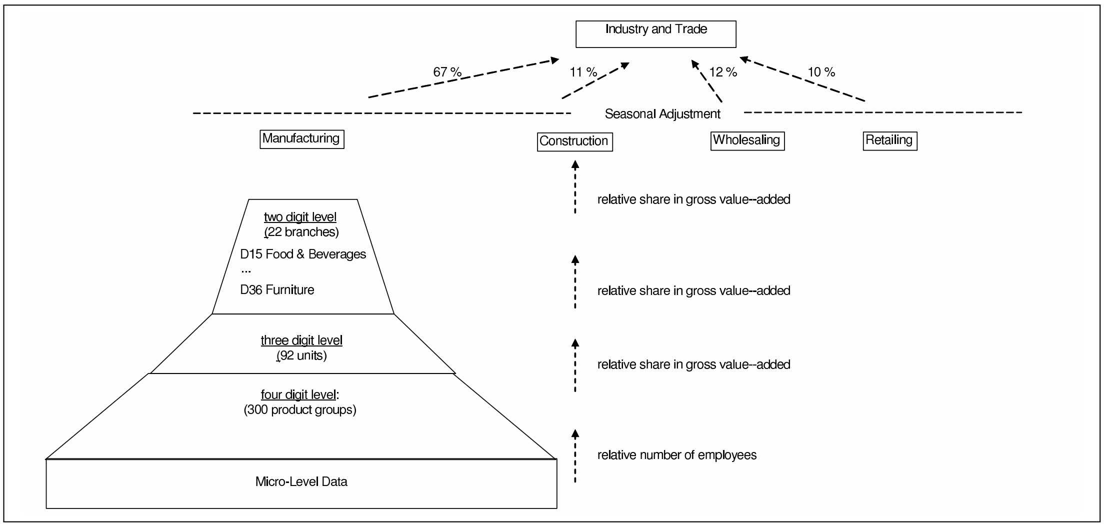


Figure 1.5: Aggregation of the Ifo Business Climate

called ASA II procedure.[16] The seasonally adjusted series are aggregated to the *Ifo Business Climate of industry and trade* again by means of relative shares in gross–value added. Table 1.1 reports the relative economic weights of the 22 branches on the two–digit level within the sectoral indices of manufacturing industry.

[16]See Goldrian (1993) and Goldrian and Lehne (1998) for details.

D Manufacturing Industries	
Branches (two digits)	Relative Weights
DA15 Food products and beverage	8.5
DA16 Tobacco Products	0.4
DB17 Textiles	1.6
DB18 Wearing apparel	1.0
DC Leather and Leather products	0.3
DD Wood and wood products	2.0
DE21 Pulp, paper and paper products	2.3
DE22 Publishing, printing and reproduction of recorded media	3.0
DF Coke, refines petroleum	0.6
DG Chemicals and chemical products	11.0
DH Rubber and plastic products	5.1
DI Non-metallic mineral products	4.8
DJ27 Basic metals	4.7
DJ28 Fabricated metals products	9.1
DK Machinery and equipment	15.1
DL30 Office machinery and computers	1.1
DL31 Electrical machinery	7.8
DL32 Radio, television and communication equipment	2.1
DL33 Medical, precision and optical instruments	3.2
DM34 Motor vehicles	12.3
DM35 Other transport	0.8
DN36 Furniture	3.1
Sum	100.0

Notes: The weights of the two-digit branches in Ifo business surveys represent their relative share in gross–value added of manufacturing industry.

Table 1.1: Relative weights of two–digit branches in manufacturing industry

The relative weights assigned to the different branches within the manufacturing industries vary significantly. The largest weights are assigned to *machinery and equipment*, *motor vehicles* and *chemicals and chemical products* whereas e.g. *tobacco products* or *leather and leather products* are of less economic importance.

Aggregating the branch level data by means of economic weights as given in Table 1.1 poses a simple and straight forward form of PI.[17] The use of economic weights guarantees that important industries are highly represented within the aggregate sectoral indices.

[17]Economic weights are frequently used in the construction of economic indicators for the euro area, see e.g. European Commission (2007) for the Economic Sentiment Indicator (ESI) and Stangl (2007) for the World Economic Survey (WES).

However, in a forecast framework, the adequacy of the weighting scheme builds on the assumptions that

- the target series for the forecast exercise is proportional to the weighting scheme
- the subindices have comparable forecast performance regarding the disaggregate reference series
- the subindices have similar time series characteristics

In case of violation of one of these assumptions, a deviation from economic weights potentially improves forecast performance. If the target to be predicted is not a measure of output but the demand for commodities, employment or credit, the weights of the branch level indices should not refer to relative shares in gross–value added but rather to some measure of relative energy, labor or debt financing of the industry.[18] If the branch level indices significantly differ in their ability to track the corresponding components of production, again a deviation from economic weights potentially increases forecast performance. These differences can either be caused by different qualities of the indices or different signal–to–noise ratios of the production series themselves.

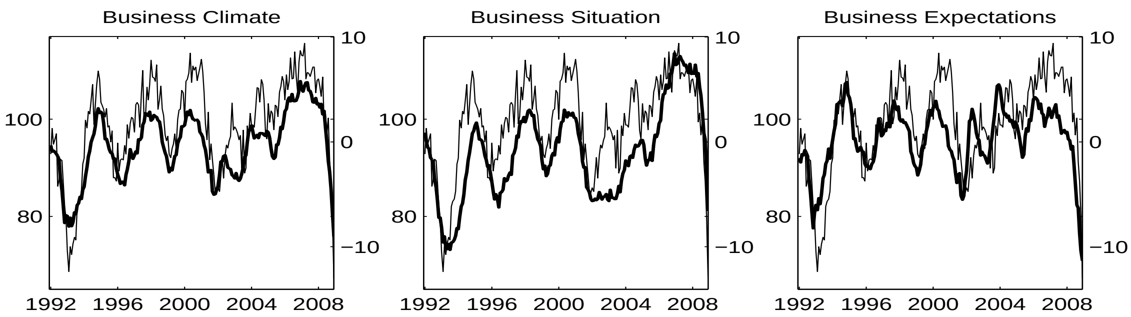
1.3.2 Industrial Production

To evaluate the forecast performance of different PI and PF schemes, we choose monthly data of IPI as reference series to assess the state of the economy. The data is obtained from the Federal Statistical Office (Destatis). As described in section 1.3.1, the weights used by Destatis to aggregate the branch level production to the sectoral IPI series are used for the construction of the sectoral Ifo series as given in Table 1.1. Although the share of the industrial sector in total gross–value added amounts to not more than one forth, IPI mirrors the state of the economy comparably well and shows a high correlation to quarterly values of real GDP. Hinze (2003) shows

[18]See the Ifo Employment Barometer for employment as target and the Ifo Credit Constraint indicator for credit supply as target.

that the relative performance of business–cycle indicators is mostly independent of the chosen measure for aggregate output. In fact, due to the higher variance of production, the industrial sector is frequently referred to as the *cyclemaker* of the German economy.[19] Its monthly publication is a major advantage of choosing IPI as reference since the Ifo business survey series – as most business–cycle indicators – are published on a monthly frequency as well. The monthly set–up prevents any information loss due to quarterly aggregation.

Since the Ifo business survey series fluctuate around stationary values, we detrend the target series to satisfy stationarity conditions. As we aim at evaluating the quantitative forecast performance of several PI and PF strategies, IPI is transformed to growth rates.[20] As shown in Figure 1.6, the Ifo business survey series for manufacturing industry exhibit a high correlation to the annual growth rates of IPI.



Notes: The thick lines represent the Ifo Business surveys (LS) and the thin line gives the annual growth rate of IPI (RS).

Figure 1.6: Ifo indices for manufacturing industry and annual growth rate of IPI

Hence, we follow Hüfner and Schröder (2002), Dreger and Schumacher (2005), Fritsche and Kuzin (2005) and the analysis of the Sachverständigenrat (2005) who choose annual growth of IPI as target series.

In contrast to symmetric statistical filter approaches as the HP filter or other band–pass filters, growth rates can be interpreted as asymmetric filters, implicating that

[19]See e.g. Sachverständigenrat (2005).

[20]To evaluate the performance of indicators for the detection of turning points in the business cycle, the reference series are frequently detrended by means of symmetric filter techniques, see e.g. Abberger and Nierhaus (2008) for an application with Ifo survey data.

the resulting series do not change when new observations are added. This comes along with the drawback that asymmetric filters lead to phase shifts of the reference time series towards the beginning, i.e. peaks and troughs are dated earlier. Thus, leading indicators potentially loose their lead character. Additionally, as the cyclical component of a reference series can be regarded as the combination of various ideal cycles of different cycle length, calculating growth rates increases the relative size of shorter cyclical components.[21]

Forecasting month–to–month growth rates of IPI via month–to–month changes of the Ifo business survey series removes the phase shift problem and the predictor variables maintain their potential qualities as leading indicators. This comes along with the drawback that the resulting series are noisier and the correlation between the indices and the reference series is considerably lower. As it is a priori not clear which of the transformations is optimal, we present the corresponding results for month–to–month changes in Appendix 1.C.2.

In the present study, we exclusively use latest–available (final) data of IPI instead of real–time data.[22] We thus circumvent the problem of which vintage to choose as target series. Figure 1.7 shows the overlapping time series of the monthly vintages from March 2000 to March 2009 for annual and monthly growth rates of IPI and displays the correlation coefficients of the first vintage with the subsequent vintages. Obviously, the revisions regarding IPI in Germany are only minor such that the results of our forecast experiment should hold whatever vintage is used as estimation

[21] Let the complex cyclical function be represented by a weighted sum of i sine waves of varying cycle length

$$c(y) = \sum_i a_i sin(k_i y)$$

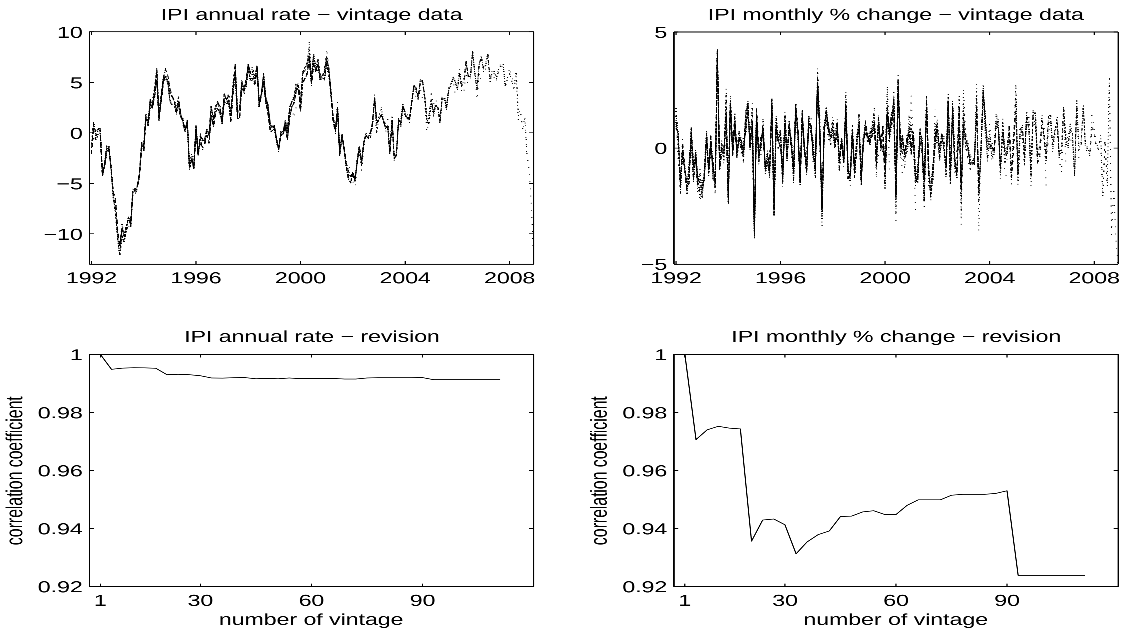
with k_i determining the frequency and a_i the weight in the total cycle of subcycle i. Thus, the greater k_i, the shorter is the length of subcycle i. The derivative of the cycle corresponds to the short term growth rate:

$$c'(y) = \sum_i a_i k_i cos(k_i y)$$

As the derivatives of sine waves are cosine waves, the transformation corresponds to a shift to the left. After the differentiation, the $k_i s$ modify the original weights a_i of the subcycles, over–weighting the shorter cycles.

[22] Schumacher and Breitung (2008) show that data revisions do not affect the forecast performance regarding macroeconomic aggregates for Germany.

and realization base. In fact, the bivariate correlation decreases with an increasing distance between the vintages but remains above 0.99 for the annual growth rate and 0.92 for the more volatile month–on–month rate.

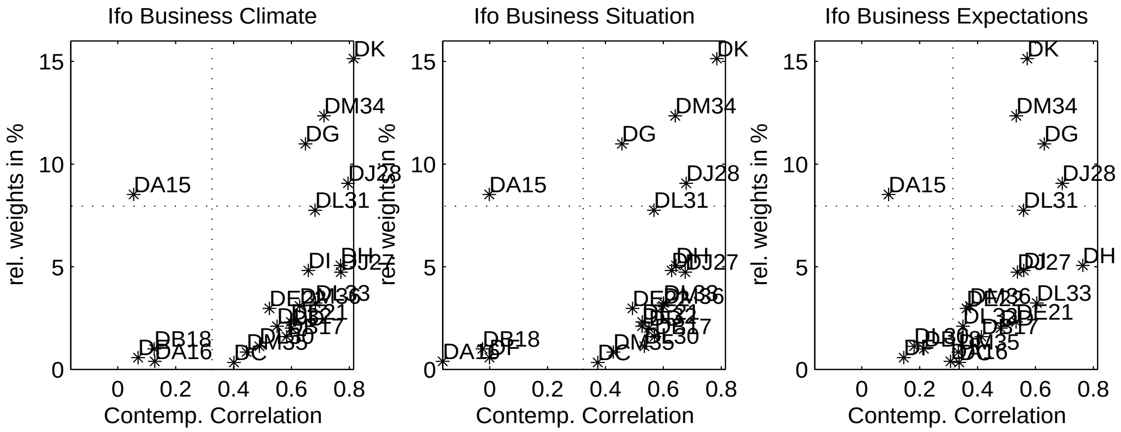


Notes: The figures show monthly overlapping vintages of y–y and m–m growth of IPI from March 2000 to March 2009 and the corresponding bivariate correlation coefficients of each vintage to the first vintage.

Figure 1.7: Real–time data of industrial production

1.3.3 Analysis of Branch Level Data

To obtain an assessment of the quality of the Ifo survey data on branch level, Figure 1.15 in Appendix 1.C.1 shows the annual growth rates of the production series with the corresponding Ifo indices of the 22 branches of manufacturing industry. Obviously, some branches, such as *rubber* or *basic metals*, are very well tracked by the Ifo survey series whereas others, such as *food products and beverage* or *tobacco products*, show a larger idiosyncratic part not captured by the related indices. To extract critical branches that likely affect forecast accuracy of the sectoral Ifo series of manufacturing industry, Figure 1.8 plots – for each of the branches – the contemporaneous correlation of the indices to the corresponding production series against its relative economic weight.

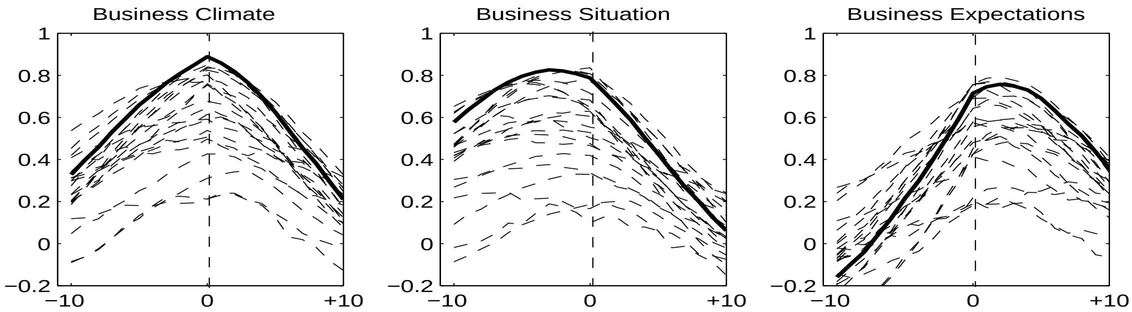


Notes: The figure plots the contemporaneous correlation of branch level indices to annual growth of the corresponding production series against the relative weight of the branches.

Figure 1.8: Performance of Ifo branch level indices

The branches in the upper right quadrant are heavily weighted within the sectoral index for manufacturing industry and measure the underlying production properly. In contrast, branches to the lower left are not very well tracked by the survey series and do not contribute much to the sectoral index. In general, branches located in the left quadrants fail to indicate movements of the respective production series. Particularly critical are branches in the upper left quadrant, such as *food products and beverages*, that are assigned an unproportional high weight. Overall, the spread of the branches in Figure 1.8 suggests that the sectoral Ifo business survey series for manufacturing industry are not optimal in terms of forecasting IPI growth and that some form of re–weighting likely improves forecast accuracy.

Focussing on sectoral IPI as target, the time dependencies between the reference series and the Ifo survey series of manufacturing industry on branch level as candidate predictor variables are shown in Figure 1.9 by means of the cross–correlations. With a peak in the cross–correlations at a lead or lag of zero, *BC* can be regarded as a coincident indicator for almost all branches. *BS* is coincident for some branches but shows a lag of up to two months for the majority of branches and for the sectoral index. In contrast, *BE* leads year–on–year IPI growth for 2 months on average. These correlations remain mainly unchanged when using the HP detrended reference series as shown in Figure 1.16 in Appendix 1.C.2.[23] The structure of the cross–correlation functions suggests that the performance of *BS* and *BE* in forecasting IPI as well as the optimal weights for the branches within an aggregate index depend on the forecast horizon. Notably, the sectoral indices do not show the highest correlations to the target series for various lead and lag constellations.



Notes: The solid lines represents the cross–correlations of year–on–year growth of IPI with the survey series for manufacturing industry and the dotted lines give the correlations to the 22 branch series.

Figure 1.9: Cross–Correlation of Ifo surveys and annual growth of IPI

1.3.4 PI Strategies

As shown in the analytical part in Section 1.2.2, one straightforward form of PI is to estimate the forecast equation including all $N = 22$ branch indices at once. The point estimates reflect the weights of the subindices within the aggregate indicator. However, due to their multitude and their high correlation, including all variables simultaneously in an unrestricted forecast equation is not feasible and leads to the well–known problem of overfitting which detrimentally affects the forecast performance.[24] Thus, the PI strategies described below deal with a large dimension dataset by either shrinking the variance of parameters in estimated equations or by sum-

[23]Notably, the phase shift associated with the calculation of growth rates contributes only minor to the observed lag of *BS* which might be explained by the time structure of the responses as described in section 1.3.1.

[24]The average bivariate correlation amounts to about 0.55 for *BC* but varies considerably between -0.04 and 0.93 among the branches.

marizing the information into a reduced number of common factors or both. One major advantage of PI compared to PF in the context of the present study is that it results in a new representable index that tracks the current or future monthly values of IPI.

1.3.4.1 Economic Weights

The use of economic weights to combine the information of disaggregate indicators is widely spread among the publishing agencies. As described in Section 1.2.2, the approach ignores any correlation between the indices as well as towards the reference series and assumes that the weights are exogenously given. It further builds on a set of assumptions described in Section 1.3.1. As described in Section 1.3.1, the aggregate Ifo indices for manufacturing industry that pose the benchmark in our study are aggregated by means of economic weights.

1.3.4.2 Ad–Hoc Restrictions

Figure 1.8 suggests that the exclusion of branches located in the upper and lower left quadrants potentially improves forecast accuracy of the sectoral indices. Traditionally, the Ifo institute additionally reports the survey data for manufacturing industry excluding *food products and beverages* and *tobacco products* in its monthly publication. Figure 1.15 in Appendix 1.C.1 shows that the growth rates of these production series are less persistent and do not exhibit well–defined business cycles and that the corresponding branch indices are only very noisy measures. As one of the candidate PI strategies, we thus analyze whether ad–hoc attributing zero weights to these branches improves forecast accuracy of the aggregate index.[25]

1.3.4.3 Factor Models

Factor models pose an alternative framework to aggregate the branch indices without running into overfitting problems. The motivation to use factor models in the present

[25] Abberger (2006) finds that excluding these branches slightly reduces forecast errors of month–on–month growth rates of IPI.

context is that the variation of the branch indices can be explained by a small number of common factors or shocks. Factor models exploit the co–movement of the branch level series and reduce the dimension of the dataset to a smaller set of underlying unobservable factors. The resulting factors can be interpreted as the driving forces of the survey responses for all branches an thus be entered into the forecast equation to derive a prediction of the target series. In factor models, the vector of observed variables X is represented as the sum of two mutually orthogonal unobservable components, the common component χ and the idiosyncratic component Ξ. The common component is driven by a small number of factors common to all variables in the model. In contrast, the idiosyncratic component is driven by variable specific shocks. This idea has a long tradition in macroeconomics. For example, the notion of a common business cycle underlies the classic work of Burns and Mitchell (1947) and the indices of leading and coincident indicators originally developed at the National Bureau of Economic Research (NBER). In the literature, forecasting using factor models has provided a formal way to systematically handle larger information sets of potentially relevant predictor variables. The dynamic factor model representation for variable x_{it} as an element of X_t is given as

$$x_{i,t} = \lambda_i(L) f_t + \xi_{i,t} \tag{1.21}$$

where f_t is the $(q \times 1)$ vector of latent factors to be estimated from the data. $\lambda_i(L)$ is a lag polynomial with non–negative power of the lag operator. Thus, in a general setting it is allowed that lags of the factors affect the current movement of the variables. ξ_t is the idiosyncratic component of variable i. For estimation purposes, it is convenient to reformulate the model. If the lag polynomial has finite order p, the model can be rewritten as

$$X_t = \Lambda F_t + \Xi_t. \tag{1.22}$$

In case of a static factor model, F_t is the $(r \times 1)$ vector of common factors, Λ is the $(N \times r)$ matrix of factor loadings and Ξ_t is the $(N \times 1)$ vector of idiosyncratic components which can be weakly mutually and serially correlated as shown by Bai and Ng (2002). The factors are used to replace the original predictor variables in the forecast equation.

The main advantage of this static representation is that the factors F_t and the factor loadings Λ can be estimated using principal components (PC). This corresponds to solving the following optimization problem

$$V(r) = min_{\Lambda,F_t} \frac{1}{NT} \sum_{i=1}^{N} \sum_{t=1}^{T} (x_{i,t} - \lambda_i' F_t)^2 \tag{1.23}$$

subject to $\sum_{t=1}^{T} \frac{F_t F_t'}{T} = I_r$ and $\Lambda'\Lambda$ being diagonal. One unique solution is given by an eigenvalue–eigenvector decomposition of $\sum_{t=1}^{T} \frac{X_t' X_t}{T}$. Estimates for Λ and F_t are given by

$$\hat{\Lambda} = \hat{V} \tag{1.24}$$

and

$$\hat{F}_t = \hat{V}' X_t \tag{1.25}$$

where $\hat{V}$ is the $(N \times r)$ matrix of eigenvectors corresponding to the r largest eigenvalues of $\sum_{t=1}^{T} \frac{X_t' X_t}{T}$. The number of factors r has to be set exogenously. To identify the factors up to a rotation, the data are usually normalized to have zero mean and unit variance prior to the estimation, see e.g. Stock and Watson (2002a) and Bai (2003). The above described estimation of the latent factors via PC is essentially a static exercise although leads and lags of the original variables can be added to X_t, see e.g. Grenouilleau (2004).

Forni et al. (2005) propose a weighted version of the principal component estimator where the time series are weighted according to their signal–to–noise ratio, estimated in the frequency domain. They estimate the covariance matrices of common and idiosyncratic components with dynamic principal components analysis. This involves estimating the spectral density matrix of X_t, $\Sigma_X(\Theta)$ which has rank q. The largest q eigenvalues and corresponding eigenvectors of $\Sigma_X(\Theta)$ are computed for each frequency Θ and the spectral density matrix of the common components $\hat{\Sigma}_\chi(\Theta)$ is estimated. The corresponding spectral density matrix of the idiosyncratic components is calculated as $\hat{\Sigma}_\Xi(\Theta) = \hat{\Sigma}_X(\Theta) - \hat{\Sigma}_\chi(\Theta)$. The time–domain autocovariances $\hat{\Gamma}_\chi(p)$ and $\hat{\Gamma}_\Xi(p)$ of the common and the idiosyncratic component for lag p are derived by the inverse Fourier transform. Since dynamic PC corresponds to a two–sided filter of the time series, this approach alone is not suited for forecasting. In a second step, the r linear combinations of X_t that maximize the contemporaneous covariance explained by the common factors $\hat{G}_j'\hat{\Gamma}_\chi(0)\hat{G}_j$ with $j = 1, \ldots, r$ are calculated. This optimization problem is subject to the normalization $\hat{G}_j'\hat{\Gamma}_\Xi(0)\hat{G}_i = 1$ for $i = j$ and 0 else. It can be reformulated as the generalized eigenvalue problem $\hat{\Gamma}_\chi(0) = \hat{\mu}_j\hat{\Gamma}_\chi(0)\hat{G}_j$ where $\hat{\mu}_j$ denotes the j–th generalized eigenvalue and $\hat{G}_j$ the corresponding $N \times 1$ eigenvector. The factor estimates are obtained as $\hat{F}_t^{dyn} = \hat{G}'X_t$.[26]

Another alternative to estimate the factors relevant for smaller data samples is to use parametric state–space frameworks. This technique builds on early work of Stock and Watson (1989) who use state space models to extract factors via the Kalman filter and maximum likelihood estimation. Kapetanios and Marcellino (2003) apply a subspace algorithm which allows the factors to be estimated without specifying and identifying the full state space model. This essentially uses OLS to obtain estimates of the matrix coefficient in a multivariate regression of leads of X_t on lags of X_t. Then a reduced rank approximation to this estimated coefficient matrix provides estimates for the factors.

Comparing the different dynamic factor estimation frameworks, it is frequently found that the easy to implement static PC approach proposed by Stock and Watson (2002b) performs comparably well in terms of forecasting macroeconomic variables based on a larger set of potential predictors.[27] In the present study, we thus focus on static PC estimation.

[26]The difference between static and dynamic PC analysis can be explained by means of a model given as

$$x_{i,t} = \lambda_1 F_t + \lambda_2 F_{t-1} + \xi_{i,t}.$$

Static PC views the model as having two static factors, F_t and F_{t-1} though there is only one common source of variation. Dynamic PC deals with such a shifted relation between the factor and $x_{i,t}$ via evaluation of the periodogram at different frequencies. However, if such a shifted relation between F_t and $x_{i,t}$ is not present in the data, unnecessary estimation of the spectral density matrices potentially induce efficiency losses.

[27]Schumacher (2005) forecasts quarterly German GDP and finds no statistical significant differences in terms of accuracy between the three methods described above. Boivin and Ng (2005) compare static and dynamic PC via Monte Carlo simulations as well as for forecasting eight US macroeconomic times series such as industrial production. They find that static PC dominates in the majority of settings and favor the method on practical grounds.

1.3.4.4 Partial Least Squares

Partial least squares regression (PLS) has been developed and is used primarily by chemometricians for predicting a response variable from an often large number of highly correlated explanatory variables. The origins of the technique can be traced to Wold (1966). The basic idea is that factors or components which are linear combinations of the original predictor variables are used as regressors. However, in contrast to PC as described above, the relationship between the explanatory variables and the dependent variable is explicitly considered in constructing the factors. The PLS factors are those linear combinations of the predictor variables that give maximum covariance towards the dependent variable while being orthogonal to each other.[28]Generalizing the PC approach, in a first step PLS simultaneously decomposes both X and Y as a product of a common set of orthogonal latent factors Z and a set of specific loadings.[29] As for PC, the data are normalized to have zero mean and unit variance prior to the estimation. The predictor variables are decomposed as

$$X = PZ \tag{1.26}$$

where Z is the (r $\times$ 1) vector of latent factors, P is the (N $\times$ r) matrix of factor loadings and $Z'Z$ is an identity matrix. Likewise, Y is estimated as

$$\hat{Y} = CBZ \tag{1.27}$$

where C is the (1 $\times$ r) weight vector of the dependent variable and B is a (r $\times$ r) diagonal matrix with the regression weights as the diagonal elements. $r \leq rank(X)$ is the limit on the number of latent factors in the regression and is exogenously set. As any set of orthogonal vectors spanning the column space of X could be used as latent factors, additional conditions are required to identify Z.

[28]In general, PLS allows for multiple independent variables. As we restrict our analysis on IPI as target series, we set $Y = y$ in the following.

[29]A detailed description of the PLS algorithm is given in Appendix 1.C.3.

This amounts to finding two sets of weights, w and c, to create a linear combination of the columns of X and Y, i.e.

$$\begin{aligned} z &= Xw \\ u &= Yc \end{aligned}$$

such that their covariance $z'u$ is maximal with $w'w = 1$ and $z'z = 1$. When the first latent vector z is found, it is subtracted from X and Y and the procedure is reiterated until X is a null matrix.[30]

1.3.4.5 Lasso Regression

The lasso regression (least absolute shrinkage and selection operator) is a shrinkage and selection method for linear regression proposed by Tibshirani (1996). By construction, the underlying algorithm combines variable selection and parameter estimation. The estimator depends in a nonlinear manner on the variable to be predicted what potentially has advantages in empirical applications. De Mol et al. (2008) show that lasso regressions as well as ridge regressions can be described as Bayesian regression methods shrinking the parameters via Gaussian and double–exponential priors respectively.

In case of ridge regressions, the Gaussian prior yields non–zero coefficients for all variables in the panel. Ridge regressions amount to solving a penalized least–squares problem with a penalty proportional to the sum of the squared regression coefficients.[31]

$$w^{ridge} = \arg\min_{w}[\sum_{i=1}^{t}(\hat{y}_i - y_i)^2 + \nu||w||^2]. \tag{1.28}$$

where $||w||^2 = \sum_{j=1}^{N} w_j^2$.

The procedure gives non–zero weights to all candidate predictor variables under the constraint that $||w||^2 < s$ where s is determined by the regularization param-

[30] Groen and Kapetanios (2008) provide an overview of candidate algorithms and give an example for using PLS in macroeconomic forecasting in data–rich environments.

[31] In what follows, we will denote by $||.||$ the Euclidean norm of the respective vector of weights.

eter ν. As for principal components, the regressors are linear combinations of all variables in the panel. However, unlike PC where unit weight is imposed to the dominant eigenvalue of the covariance matrix and zero to the others, the Gaussian prior gives decreasing weight to the ordered eigenvalues. In case of the standard ridge regression, the coefficients can be estimated via a closed form.

As a modification of the ridge regression, the lasso regression penalizes proportional to the sum of the absolute values of the coefficients

$$w^{lasso} = \arg\min_{w}[\sum_{i=1}^{t}(\hat{y}_i - y_i)^2 + \nu||w||^1] \tag{1.29}$$

where $||w||^1 = \sum_{j=1}^{N}|w_j|$. In contrast to the classical ridge regressions, the algorithm sets certain coefficients to zero. Hence, compared to principal components or partial least squares, the lasso approach is a regression on a few variables rather than on a few aggregates of the variables. Under high collinearity, a few variables, if appropriately selected, should capture the essence of the covariation of the data and approximately span the space of the pervasive common factors. However, the selection should also be unstable and rather sensitive to minor perturbations of the data preventing a clearer economic interpretation of the results. As there are no analytical solutions to the minimization problem stated in Equation (1.29), we employ the LARS (Least Angle Regression) algorithm developed by Efron et al. (2004). The algorithm starts with all coefficients set to zero and finds the predictor that is highest correlated with y. The corresponding coefficient is stepwise increased in the direction of the sign of its correlation with y and the vector of residuals is calculated at each step. The algorithm includes an additional predictor once it is as high correlated with the remaining residuals as the previous. The coefficients of both predictors are increased in their joint least squares direction until another predictor enters. The algorithm stops at an exogenously set limit of regressors to include. We denote the number of coefficients that are assigned a non–zero weight with r.[32]

[32] All the procedures are applied to standardized data. Mean and variance are re–attributed to the forecasts accordingly.

1.3.4.6 Optimal Pooling of Information

Relating to the framework of predictive modeling, we propose a method we name optimal pooling of information. OPI poses an alternative PI approach where the weighting scheme is specifically optimized with respect to a certain forecast exercise.

In the present context, based on a non–linear numerical optimization routine, the branch level indices are aggregated by weights that minimize the MSE of the forecast equation over a rolling window of past observations. To describe the algorithm, we assume a simple static forecast equation of the form

$$y_{t+h} = \delta + cx_t + \epsilon_{t+h}. \tag{1.30}$$

where y_t is the target series and x_t is an aggregated index. We aim to forecast y_{t+h} based on information of the predictor variables available at time t. The determination of the corresponding OPI weights includes the following steps. We begin with an initial guess for the weights w and compute the aggregate index as $x_t = wX_t$. Employing a recursive procedure, we estimate the forecast equation based on a reduced estimation window of size τ with $\tau = t - M - h, \dots, t - 1 - h$ and forecast the respective h–step ahead value of the target series $y_{\tau+h}$ at each iteration step. This leaves us with M ex–post forecasts and respective forecast errors that build the basis for the optimization of the OPI weights. The MSE as the loss function to minimize is calculated on basis of these M forecasts that can be evaluated at time t.[33] The optimized weights are restricted to sum up to unity and to be non–negative, i.e. $w'1_K = 1$ and $w_i \geqq 0 \ \forall i$.

[33] As M is a constant number, the weights are calculated by means of a rolling window approach.

More precisely, at forecast origin t and for each forecast horizon h, the iterative steps j of the algorithm are given as:

1. Step $w = w_j$ (Update the weighting scheme)

2. Step $x_t = wX_t$ (Calculate the aggregate index)

3. Step For $\tau = t - M - h, \ldots, t - 1 - h$, estimate Equation 1.30 and derive the h–step ahead forecast of $y_{\tau+h+1}$ based on $x_{\tau+1}$ (Evaluate aggregated index for past M observations of y_t)

4. Step Calculate the objective function as $MSE^*_{t,j} = \frac{1}{M}\sum_{\tau=t-M-h}^{t-1-h}(\hat{y}_\tau - y_\tau)^2$

If $|MSE^*_{t,j-1} - MSE^*_{t,j}| > tol$, go back to 1.*Step*. Else, the algorithm has converged and the current vector of optimal weights has been found, i.e. $w^* = w$. The corresponding OPI index is calculated as $x^*_t = w^* X_t$ and used as predictor in the forecast equation to derive the h–steps ahead prediction $\hat{y}_{t+h,t}$ of the target series.

Restricting the weights to be non–negative and to sum up to unity promotes sparse portfolios of branch level indices attributing a weight larger zero only to a reduced number of variables. While being analytically simple, the solution of this problem can still be challenging in practice, depending on the nature of X_t. For highly correlated candidate predictors, a non–regularized numerical procedure will lead to unstable and unreliable estimates of the weights w.

To obtain meaningful and stable results for such an ill–conditioned problem, we adopt a regularization procedure similar to the one proposed by Brodie et al. (2007). We augment the objective function of the optimization procedure with a penalty term ℓ_t, yielding a minimization problem given as

$$\begin{aligned} w^* &= \arg\min_w \left[\frac{1}{M} \sum_{\tau=t-M-h}^{t-1-h} (\hat{y}_{\tau+h,\tau} - y_{\tau+h})^2 + \ell_t \right] \qquad (1.31) \\ s.t. \ & w' 1_K = 1 \\ & w_i \geqq 0 \quad \forall i \end{aligned}$$

where

$$\ell_t = \psi \frac{1}{K} \sum_{i=1}^{K} (w_{i,t} - w_{i,t-1})^2. \qquad (1.32)$$

Thus, we introduce a term penalizing any deviation of the current weights from the weights assigned to the indices one period before. ψ is a parameter that allows us to adjust the relative importance of the penalization in our optimization. Attributing large values to ψ corresponds to stabilizing the weights as any deviation of past weights is penalized more heavily. For $\psi \to \infty$, the algorithm yields constant weights, determined by the starting values.[34] In contrast, for $\psi \to 0$, the weights adjust very quickly to the most recent forecast errors and are thus more flexible.

The main advantages of the OPI algorithm compared to the above described in–sample approaches is that it can easily be adjusted to various forms of penalty terms and the weights can simultaneously be restricted to be non–negative and to sum up to unity. Most important, OPI is strictly out–of–sample focused, i.e. the forecast errors rather than in–sample residuals are minimized. This feature potentially contributes to a sparser weighting scheme that is more robust. On the other hand, splitting the data sample into an estimation and an optimization period potentially leads to less consistent estimates of the respective coefficients.

[34] As the starting values gain relevance only for large values of ψ, we initialize the weights of the branch level indices with the relative shares in gross–value added as reported in Table 1.1. Thus, the MSE of the OPI algorithm converges to the MSE of the sectoral index for $\psi \to \infty$.

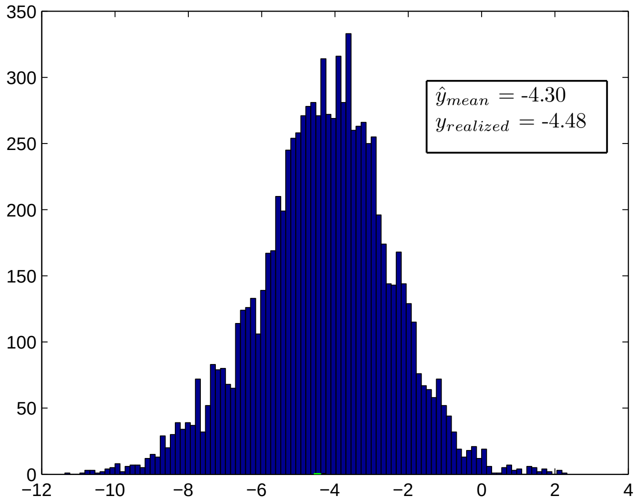
1.3.5 PF Strategies

Given the set of 22 candidate explanatory variables, pooling of forecasts corresponds to the estimation of a larger set of parsimonious models, each based on a subset of the total information. As it is a priori impossible to discard a certain variable combination, we follow Mayr and Ulbricht (2007) and derive all permutations of the candidate predictors. To prevent overfitting, we exclude combinations with more than 4 indicators. This procedure leaves us with $n = 9108$ different combinations of the branch level indices. Each of these variable combinations is used to forecast the target series for $h = 0, \ldots, 2$. The combined forecast is derived as a weighted average of the single predictions, i.e.

$$\hat{y}_{t+h} = \sum_{i=1}^{K} w_i \hat{y}_{t+h,i} = w\hat{Y}_{t+h}. \tag{1.33}$$

The empirical distribution of the single forecasts gives an insight into the variation of the assessment within the 22 branches and provides a measure of model uncertainty related to the selection of relevant branch indices. Figure 1.10 exemplarily shows the empirical distributions of the single forecasts for January 2002. The label on the abscissae represents the realized value of IPI growth as published by Destatis.

The difference between the mean forecast and the realized value corresponds to the forecast error of the equally weighted average of all single predictions. To generate pooled forecasts, we focus on a limited selection of the different weighting schemes proposed in literature. Diebold and Lopez (1996), Newbold and Harvey (2007) and Timmermann (2006) provide surveys on candidate forecast combination techniques. These methods differ in the way they use historical information to compute the combination weights. A theme that is common across estimators of the combination weights w is that estimation errors in forecast combinations are generally important especially when the number of forecasts is large relative to the length of the data sample.



Notes: The figure illustrates the empirical distribution of forecasts of year–on–year growth of IPI for January 2002 based on Ifo branch level data of manufacturing industry.

Figure 1.10: Empirical distribution of IPI forecasts

1.3.5.1 Theoretically Optimal Weights

Assuming a MSE based loss function that exclusively depends on the forecast error of the pooled forecast, optimal weights in the sense of Bates and Granger (1969) are chosen to solve the problem:

$$w^* = \arg \min_w [w' \Sigma_e w] \tag{1.34}$$

which gives

$$w^* = \frac{\Sigma_e^{-1} 1_n}{1_n' \Sigma_e^{-1} 1_n}. \tag{1.35}$$

where Σ_e denotes the $(n \times n)$ covariance matrix of the forecast errors e_i (for $i = 1, \ldots, n$) of the single models. In practice, the elements of Σ_e are unknown and have to be estimated which introduces an additional source of uncertainty. Imprecise estimates of Σ_e potentially deteriorate forecast performance. Assuming a linear–in–weights model, the combination weights can be estimated via OLS, regressing realizations of the target series on the n–vector of forecasts, using ex–post data.

Three different versions of the basic least squares projection have been considered by Granger and Ramanathan (1984).

(1) $y_{t+h} = w_{0,h} + w_h' \hat{y}_{t+h,t} + \epsilon_{t+h}$

(2) $y_{t+h} = w_h' \hat{y}_{t+h,t} + \epsilon_{t+h}$

(3) $y_{t+h} = w_h' \hat{y}_{t+h,t} + \epsilon_{t+h}, \quad s.t. \quad w_h' \iota = 1$

The first and second equation can be estimated by standard least–squares whereas estimation of the third equation is done via constrained least–squares. The second and third equation omit an intercept term, assuming unbiasedness of the single forecasts.

1.3.5.2 Relative Performance Weights

As the number of single forecasts is large in the present study, we abstract from estimating all elements of the covariance matrix of the forecast errors. We follow Stock and Watson (1998) who propose to disregard the off–diagonal elements of Σ_e and employ relative performance weights, assuming uncorrelated errors. This yields a weighting scheme based on the inverse MSE of the single models relative to the sum of those of all models as a measure of relative past forecast performance.

$$w = \frac{MSE_{i,t|t-h}^{-K}}{\sum_{i=1}^{n} MSE_{i,t|t-h}^{-K}} \tag{1.36}$$

The parameter K sets the inverse MSE to various powers. A value of $K = 0$ corresponds to an equal weighted average whereas $K \rightarrow \infty$ assigns a weight of one to the best performing model whereas all other models are assigned a weight of zero. It can thus be considered as a special case of MSE weighted averages, where at each forecast origin, the model with the lowest historical MSE is identified and exclusively used to forecast into the future. Clark and McCracken (2009a) refer to this scheme as predictive least squares.

Frequently, the historical forecast errors are discounted, i.e. from the forecast origin t, the squared forecast errors in the preceding periods are discounted by a certain factor (see e.g. Rapach and Stauss (2008)). Instead of the common MSE, an adjusted measure of past forecast performance is used in Equation 1.36, given as

$$MSE_{i,t}^{adj.} = \frac{1}{M} \sum_{s=T_0}^{t-h} d^{t-h-s} (\hat{y}_{s+h|s} - y_{s+h})^2 \tag{1.37}$$

where d is the discount factor with $0 < d \leq 1$. For values of $d < 1$, more distant squared errors are assigned a lower weight within the calculation of the weighting scheme. For $d = 1$, no discounting is implemented and all squared errors are weighted equally regardless of the distance to the forecast origin. For both approaches, the measure of past forecast performance can be calculated in a recursive manner as well as be means of a rolling window. As we employ a recursive strategy to estimate the forecast equations, our learning period for the combination weights corresponds to a rolling window of fixed length.

1.3.5.3 Simple Combinations

The simplest weighting scheme is the mean forecast which corresponds to an equally weighted average of the single predictions. The mean can either be calculated on the unadjusted set of forecasts, or on an adjusted set truncating a certain percentage of extreme large and small predicted values. This technique is frequently referred to as trimming (see e.g. Stock and Watson (1999) and Clark and McCracken (2009a)). The trimmed mean is computed with symmetric trimming a certain percentage of forecasts. In that way, the median forecast which corresponds to the 0.5 quantile of the forecast distribution equals a trimmed forecast with a trimming parameter of 50 percent. In case of highly volatile predictions, trimming likely improves forecast accuracy as less importance is attached to predictions within the tails of the distribution.

1.3.5.4 Nonparametric Combination Schemes

Rank based weighting schemes let the combination weights be inversely proportional to the models' ranks:

$$w_{i,t} = \frac{R_{t,t-h,i}^{-K}}{\sum_{i=1}^{n} R_{t,t-h,i}^{-K}} \tag{1.38}$$

The parameter K again sets the inverse performance measure to various powers. As the MSE weighted averages, the combination scheme ignores correlations across forecast errors. Since ranks tend to be less sensitive to outliers, these approaches are likely more robust than weighting schemes directly based on measures of past forecast performance.

1.3.5.5 Bayesian Model Averaging

Bayesian model averaging (BMA) can be thought of as a bayesian approach to pooling of forecasts. In BMA, the weights assigned to the single forecasts are computed as formal posterior probabilities that the respective models are correct. In addition, the individual forecasts in BMA are the posterior means of the variable to forecast, conditional of the selected model. The posterior probabilities $P(M_i|D)$ of model i can be derived as

$$P(M_i|D) = \frac{P(D|M_i)P(M_i)}{\sum_{j=1}^{n} P(D|M_j)P(M_j)} \tag{1.39}$$

where

$$P(D|M_i) = \int P(D|\Theta, M_i)P(\Theta|M_i)d\Theta \tag{1.40}$$

is the marginal likelihood, $P(\Theta|M_i)$ is the prior density of the parameter vector in model i and $P(D|\Theta, M_i)$ is the likelihood. The model priors $P(M_i)$ and the parameter priors $P(\Theta|M_i)$ need to be specified ad–hoc.[35]

[35] For a detailed description of BMA in forecasting macroeconomic aggregates, see e.g. Stock and Watson (2006).

Assuming diffuse priors of the model parameters, the marginal likelihood can be approximated by means of the BIC criterion (see e.g. Clark and McCracken (2009a)). Assuming that all single models are equally likely, i.e. $P(M_i) = \frac{1}{n}$, the BMA weights correspond to the relative BIC values of all models in the framework.[36]

1.3.6 Settings of the Forecast Experiment

Timely information on the current state of economic activity is of great interest for policy makers as well as for market analysts and central banks. As described in section 1.3.2, the monthly published IPI mirrors the dynamics of the economy comparably well as it shows a high correlation to real GDP. However, it is only released with a lag of about 38 days after the end of the reference month. During this interval of time, decision makers are faced with considerable uncertainty about the economic conditions.

To provide a consistent and prompt picture of the state of the economy, nowcasting the current monthly value has become an important task. Additionally, forecasting the missing monthly values of the current quarter is essential when IPI itself is used as predictor in bridge equations to nowcast real GDP.[37] Thus, the present study not only evaluates the Ifo business survey data for the IPI nowcast, i.e. $h = 0$, but extends the forecast horizon to $h = 1$ and $h = 2$.

[36] As an alternative, Wright (2003) uses routine integration to approximate the marginal likelihood.

[37] Bridging stands for linking monthly data of economic indicators such as IPI, typically released early in the quarter, with quarterly data such as GDP or its components. The bridge equation is estimated from quarterly aggregates of monthly observations. Predictions of the lower frequency target variable are derived in two steps. First, the monthly indicators are forecasted over the remainder of the quarter to obtain forecasts of their quarterly aggregates. Second, the resulting values are used as regressors in the bridge equation to generate a forecast of the target series. Due to the different timing of data releases, the number of missing monthly values differs across series and models. Nowcasts of real GDP growth based on bridge equations are usually derived under incomplete information, i.e. when the indicators are only partially known for the respective quarter and one to three monthly values are missing. In contrast to the present study where we use IPI as the reference series to assess the state of the economy, industrial production is frequently used itself as a business cycle indicator of a higher frequency, predicting a broader measure of a lower frequency such as real GDP, see e.g. Banerjee et al. (2005) and Angelini et al. (2008) who derive nowcasts of quarterly real GDP growth in the Euro area based on monthly indicators including IPI.

The set of explanatory variables used to forecast IPI contains the Ifo business survey indices, namely *Business Climate*, *Business Situation* and *Business Expectations* for manufacturing industry as well as for the 22 branches shown in Table 1.1. All time series are seasonally adjusted by means of ASA II methodology and cover a time span from January 1992 to December 2008. Although the published series for aggregate manufacturing industry are seasonally adjusted after aggregation, weighting the branch level data by means of the relative shares in gross–value added fits the sectoral counterpart as published very well.[38]

The forecasts based on the candidate predictors are computed using h–step ahead projections. Our forecast equations are specified as autoregressive distributed lag models (ADL) of the form

$$y_{t+h} = c + A(L)y_t + B(L)x_t + \epsilon_{t+h} \tag{1.41}$$

where

$$\begin{aligned} A(L) &= \alpha_1 L + \alpha_2 L^2 + \ldots + \alpha_p L^p \\ B(L) &= \beta_0 + \beta_1 L + \beta_2 L^2 + \ldots + \beta_q L^q \end{aligned}$$

are lag polynomials. The forecasts are derived as direct step predictions. The general specification of ADL models as given in Equation (1.41) nests a larger number of special cases as the static regression model, the univariate autoregression model or the general error–correction model.

To assess the forecast performance of different PI and PF frameworks, the data sample is split into an estimation period and an evaluation period. The first is used to estimate the forecasting relationships, the second to evaluate the forecast performance of competing approaches. Forecasting with a constant lead time implies that the information set on which the forecast is based is updated as the forecast moves through the evaluation sub–sample. It is an open question whether and, if so, how the estimated relationships should also be updated. The three possibilities are re-

[38] The correlation coefficients amount to 0.9997 for *BC* and to 0.9995 and 0.9996 for *BS* and *BE* respectively.

ferred to as fixed, recursive and rolling estimation scheme.[39] Due to the shortness of our dataset, we focus on the recursive estimation technique, updating the regression coefficients based on the total available information set at any point on the timeline. Consequently, for PF and PI strategies that build on weights estimated by means of out–of–sample measures, a fixed length rolling optimization window is split from the sample used to estimate the model coefficients.

For $h = 0$, starting with an estimation sample from January 1992 to December 1999, we nowcast the following month of IPI based on the current value of the respective predictors. Employing the recursive estimation strategy, the estimation sample is augmented by one month afterwards and the procedure is repeated. The most recent predictions are derived for December 2008 yielding a forecast window of 108 observations.[40] Following Hüfner and Schröder (2002), in the first part of the forecast experiment, the lag orders p and q of Equation (1.41) are optimized in a two–step procedure. First, the lag length of a purely autoregressive model of the reference series is selected via the Akaike information criterion (AIC). Second, given the specified AR–order, the size of the B(L) polynomial is optimized again via AIC.[41] The purely autoregressive model is thus necessarily nested in the unrestricted ADL equation based on the indicator series.

[39]In general, reducing the estimation sample to reduce heterogeneity increases the variance of the parameter estimates what maps into the forecast errors and causes the MSE to increase. However, in the presence of structural breaks, using older data that follows a data–generating process uncorrelated to the present potentially leads to biased parameter estimates and forecasts. This trade–off has to be dealt with when deciding between a recursive and a rolling estimation sample.

[40]Note: For $h > 0$, the initial estimation sample is shortened by h–periods such that the size of the forecast window is held constant.

[41]Note: The contemporaneous value of the predictor x_t is always incorporated in the model.

1.3.7 Results for Manufacturing Indices

To evaluate whether the Ifo survey series for manufacturing industry are useful in forecasting IPI at all and whether certain branches exceptionally track IPI, Table 1.2 (annual growth rates) and Table 1.6 in Appendix 1.C.2 (month–on–month growth rates) compare the forecast accuracy of unrestricted ADL models as given in Equation (1.41) based on sectoral indices and based on branch level indices as listed in Table 1.1 to purely autoregressive models where $\beta_i = 0$ for $i = 0, \ldots, p$.

The columns present the results for BC and its components as predictors separately as well as for including both, BS and BE simultaneously in the forecast equation. We report relative MSEs together with t–statistics of the test proposed by Clark and West (2007) comparing predictive accuracy of the respective model to the restricted univariate benchmark model.[42]

[42] Assuming that model 1 is the parsimonious model and Model 2 is the larger model that nests model 1, we test for equal mean–squared prediction errors by regressing $\hat{f}_{t+h}$, given as

$$\hat{f}_{t+h} = (y_{t+h} - \hat{y}_{1,t+h})^2 - [(y_{t+h} - \hat{y}_{2,t+h})^2 - (\hat{y}_{1,t+h} - \hat{y}_{2,t+h})^2]. \tag{1.42}$$

on a constant and using the resulting t–statistic for a zero coefficient. We reject if this statistic is greater than +1.28 (for a one-sided 0.10 test) or +1.65 (for a one-sided 0.05 test). For the one–step–ahead forecast errors, the usual least squares error can be used. For autocorrelated forecasting errors, we use the Newey–West heteroscedasticity–autocorrelation consistent standard errors. Note: As the lag orders of the forecast equations are optimized in a two–step procedure as described in section 1.3.6, the tested models necessarily nest the univariate benchmark model and the test is applicable.

	h=0				h=1				h=2			
	BC	BS	BE	BS/BE	BC	BS	BE	BS/BE	BC	BS	BE	BS/BE
AR	3.64	3.64	3.64	3.64	5.19	5.19	5.19	5.19	6.92	6.92	6.92	6.92
Manufacturing	0.81	0.72	0.79	0.74	0.66	0.62	0.61	0.60	0.63	0.62	0.58	0.57
	(2.85)	(2.36)	(2.40)	(2.64)	(3.09)	(3.11)	(2.76)	(3.07)	(3.19)	(3.17)	(2.84)	(3.16)
DA15	1.03	1.06	0.99	1.02	1.04	1.04	0.99	1.01	1.03	1.03	1.07	1.09
	(-0.87)	(-0.63)	(0.92)	(-0.38)	(-0.24)	(-0.05)	(1.12)	(0.54)	(0.80)	(-0.24)	(0.24)	(0.04)
DA16	1.02	1.04	1.04	1.09	1.10	1.06	1.04	1.07	1.11	1.05	1.02	1.04
	(-0.55)	(-0.60)	(-0.53)	(-1.01)	(-0.45)	(-0.25)	(-0.30)	(-0.77)	(-0.60)	(-1.11)	(0.14)	(-0.28)
DB17	1.04	1.05	0.97	1.00	0.93	0.93	0.96	0.93	0.91	0.94	0.94	0.92
	(1.24)	(0.34)	(1.62)	(1.44)	(2.43)	(3.02)	(1.52)	(1.88)	(2.21)	(2.61)	(1.54)	(1.84)
DB18	0.98	1.02	0.89	0.85	0.94	0.97	0.84	0.79	0.93	0.99	0.83	0.78
	(1.16)	(-1.56)	(2.07)	(1.85)	(2.20)	(0.81)	(2.40)	(2.12)	(2.91)	(0.59)	(3.03)	(2.39)
DC	0.97	1.01	0.89	0.90	0.95	0.99	0.87	0.88	0.91	0.99	0.84	0.84
	(1.41)	(-1.63)	(1.61)	(1.41)	(1.61)	(1.20)	(1.58)	(1.41)	(1.55)	(1.18)	(1.73)	(1.61)
DD	1.05	1.11	0.99	1.00	1.02	1.08	0.91	0.95	0.93	1.00	0.89	0.93
	(0.20)	(-0.19)	(1.53)	(1.74)	(1.77)	(0.22)	(2.53)	(2.15)	(2.69)	(1.57)	(2.30)	(2.32)
DE21	0.91	0.88	0.84	0.82	0.87	0.93	0.76	0.78	0.82	0.92	0.72	0.73
	(3.42)	(2.79)	(2.82)	(2.92)	(3.02)	(2.56)	(2.84)	(2.97)	(2.92)	(2.22)	(2.81)	(2.97)
DE22	0.97	1.02	0.85	0.88	1.02	1.12	0.79	0.82	0.94	1.03	0.84	0.89
	(1.62)	(-1.35)	(1.94)	(1.83)	(1.30)	(0.45)	(2.18)	(2.02)	(1.85)	(1.46)	(2.28)	(2.03)
DF	1.03	1.05	0.92	0.98	1.00	1.05	0.86	0.97	1.02	1.05	0.85	1.00
	(0.47)	(-1.10)	(1.29)	(0.92)	(1.33)	(0.12)	(1.45)	(1.31)	(1.50)	(0.09)	(1.74)	(1.72)
DG	0.96	0.95	0.85	0.90	0.85	0.94	0.73	0.81	0.90	0.93	0.72	0.81
	(2.89)	(2.26)	(2.12)	(2.39)	(2.89)	(2.86)	(2.42)	(2.63)	(2.82)	(2.93)	(2.40)	(2.74)
DH	0.79	0.78	0.77	0.76	0.73	0.86	0.66	0.66	0.72	0.82	0.64	0.63
	(2.59)	(2.32)	(2.54)	(2.70)	(2.84)	(3.21)	(2.73)	(3.03)	(2.80)	(2.99)	(2.94)	(3.06)
DI	1.02	1.07	0.92	0.91	1.00	1.09	0.84	0.83	1.00	1.06	0.82	0.90
	(0.54)	(-0.19)	(2.63)	(2.53)	(2.16)	(0.96)	(3.22)	(2.80)	(2.05)	(1.66)	(2.89)	(2.48)
DJ27	0.84	0.89	0.87	0.87	0.77	0.85	0.80	0.79	0.83	0.95	0.81	0.82
	(2.77)	(1.88)	(2.60)	(2.78)	(2.95)	(2.45)	(2.75)	(2.97)	(3.01)	(2.16)	(2.55)	(2.79)
DJ28	0.86	0.80	0.79	0.75	0.83	0.80	0.72	0.68	0.86	0.84	0.72	0.72
	(1.95)	(1.80)	(2.48)	(2.06)	(2.23)	(2.00)	(2.81)	(2.47)	(2.85)	(2.62)	(2.96)	(2.70)
DK	0.80	0.79	0.71	0.75	0.68	0.68	0.55	0.56	0.64	0.68	0.53	0.55
	(3.46)	(2.31)	(2.86)	(2.90)	(3.66)	(3.74)	(3.11)	(3.39)	(3.97)	(3.98)	(3.36)	(3.84)
DL30	0.99	1.01	1.03	0.99	1.03	1.00	1.07	1.11	1.01	0.99	1.03	1.05
	(1.30)	(-0.30)	(-0.61)	(0.85)	(0.59)	(0.15)	(0.42)	(-0.15)	(1.37)	(0.81)	(1.00)	(0.96)
DL31	0.90	0.85	0.86	0.88	0.73	0.80	0.75	0.88	0.71	0.77	0.70	0.81
	(2.82)	(2.00)	(2.88)	(2.80)	(2.70)	(1.97)	(3.08)	(2.78)	(2.83)	(2.08)	(3.29)	(3.04)
DL32	0.78	0.93	0.81	0.79	0.79	0.91	0.77	0.75	0.79	0.93	0.75	0.74
	(2.46)	(1.50)	(2.26)	(2.25)	(2.56)	(1.52)	(2.18)	(2.32)	(2.57)	(1.82)	(1.95)	(2.20)
DL33	0.84	0.90	0.83	0.83	0.79	0.96	0.80	0.77	0.71	0.90	0.71	0.73
	(1.68)	(1.59)	(2.12)	(1.89)	(2.44)	(1.68)	(2.55)	(2.30)	(3.06)	(2.14)	(3.03)	(2.59)
DM34	0.85	0.85	0.91	0.88	0.83	0.82	0.88	0.85	0.77	0.79	0.90	0.85
	(1.82)	(1.70)	(2.11)	(1.91)	(1.69)	(1.59)	(1.96)	(1.92)	(1.89)	(1.78)	(1.98)	(1.60)
DM35	1.00	1.07	0.94	1.01	0.98	1.07	0.94	0.98	0.96	1.10	0.88	0.99
	(0.92)	(-1.62)	(2.88)	(0.96)	(1.35)	(-1.01)	(2.61)	(2.11)	(2.08)	(-0.99)	(4.12)	(2.35)
DN36	0.92	1.01	0.84	0.89	0.74	0.93	0.72	0.80	0.77	0.95	0.69	0.77
	(2.18)	(1.20)	(2.48)	(2.17)	(3.22)	(2.74)	(2.76)	(2.92)	(2.68)	(2.24)	(2.73)	(2.81)

Notes: The first line reports the absolute MSE values for the univariate benchmark process. For the Ifo indices, we report the relative MSE values relative to the benchmark. The figures in parentheses represent the t–statistics of a one–sided test for predictive accuracy for nested models as proposed by Clark and West (2007). A t–statistic greater than +1.28 (10 percent significance level) or +1.65 (5 percent significance level) indicates that the unrestricted model which additionally contains Ifo survey series yields a significant smaller MSE than the autoregressive benchmark model. The standard errors are heteroscedastic and autocorrelation robust (Newey–West).

Table 1.2: Relative MSE of Ifo indices against AR benchmark for annual growth of IPI

For the annual growth rates, in general, incorporating Ifo business survey data significantly improves forecast accuracy compared to the purely autoregressive models. On average, the improvement slightly increases with a growing forecast horizon. Focusing on aggregate manufacturing industry, *BS* performs best for $h = 0$ whereas *BE* is dominant as the forecast horizon grows. The calculation of *Business Climate* as the geometric mean of *BS* and *BE* cannot be considered optimal in terms of predicting IPI for any of the considered forecast horizons. In fact, incorporating *BS* and *BE* simultaneously in the forecast equation improves accuracy compared to *BC* and yields a MSE comparable to the better of *BS* and *BE*.[43]

Regarding the disaggregate predictor variables, certain indicators at the branch level such as *machinery and equipment* and *rubber and plastic products* significantly improve forecast accuracy compared to autoregressive models and even slightly outperform the sectoral indices of manufacturing industry.[44] In particular, *BE* of *machinery and equipment* yields the lowest MSE regarding IPI for a forecast horizon of $h = 0, \ldots, 2$. For the month–on–month growth rates, incorporating sectoral Ifo survey data improves forecast accuracy significantly for $h = 0$ whereas the gains are minor for $h = 1$ and $h = 2$. For the nowcast, *BS* again clearly dominates the aggregated *Business Climate*. As for the annual growth rates of IPI, *machinery and equipment* and *rubber and plastic products* are among those branches that significantly outperform the univariate benchmark and yield a smaller MSE than sectoral aggregated indices of manufacturing industry.

To examine the stability of our results, we divide the evaluation period in half and

[43] Entorf (1991) finds a similar result. He specifies BC as

$$BC = BE^{\alpha}(BS/BE)^{\beta} = BE^{\alpha-\beta}BS^{\beta}$$

and evaluates the forecast performance regarding industrial production of a parameter setting $\alpha = 1$ and $\beta = 0.5$ (which corresponds to the aggregation scheme used by the Ifo institute) against a setting where the parameters are freely estimated via the log–linearized equation

$$IND_{t+h} = c + (\alpha - \beta)log(BE_t) + \beta log(BS_t).$$

IND_{t+h} thereby corresponds to the trend adjusted logarithmic value of industrial production. He finds that, for $h = 3$, a deviation of equal weights significantly improves forecast accuracy.

[44] Abberger (2006) predicts month–on–month grows rates of IPI based on first differences of the indices and finds similar results.

compute the MSEs over the two subsamples from January 2000 to June 2004 and from July 2004 to December 2008. A stable and potent forecast model exhibits a MSE less than the benchmark in both subsamples, whereas an unstable model is characterized by a relative MSE less than one in one period but greater than one in the other period. Figure 1.11 exemplarily shows scatterplots of the relative MSEs of the forecasts in the first versus the second sub–sample for $h = 0$.[45]

The points represent pairs of relative MSEs of the branch level data as well as the sectoral indices. Stable forecast relations correspond to points scattered around the 45°–line. Points in the lower left quadrant indicate an improvement over the univariate benchmark in both periods whereas points in the upper right quadrant correspond to branches that yield a worsening of forecast accuracy in both forecast periods. Evidently, the sectoral index of manufacturing industry – which is marked with a circle – as well as a larger number of branch level indices show considerable stability, outperforming the benchmark model in both sub–samples. As there are barely any points above the horizontal line, the Ifo survey series on average exhibit a slightly stronger forecast performance in the second subsample compared to the first. Obviously, incorporating survey data helps predicting IPI particularly in times of volatile movements as the sharp decline of IPI starting in September 2008. These results hold for all forecast horizons of annual growth of IPI. For the month–on–month growth of IPI, the Ifo data contains valuable information in both subsamples only for the nowcast, i.e. for $h = 0$ whereas for $h = 1$ and $h = 2$, the relative MSEs are clustered around the origin, indicating predictors that have negligible marginal predictive content for month–on–month growth of IPI above and beyond that in lags of the reference series.

Overall, the analysis suggests that economically weighted indices for manufacturing industry – as published by the Ifo institute – pose an adequate measure of industrial output and are capable to forecast future monthly values of annual IPI growth. However, disaggregate survey data on branch level contains additional information that can be used to improve forecast accuracy. Although *BS* is favorable when

[45]The scatterplots for annual growth for $h = 1$ and $h = 2$ as well as for month–on–month growth for $h = 0, \ldots, 2$ are shown in Figure 1.17 to 1.21 in Appendix 1.C.1

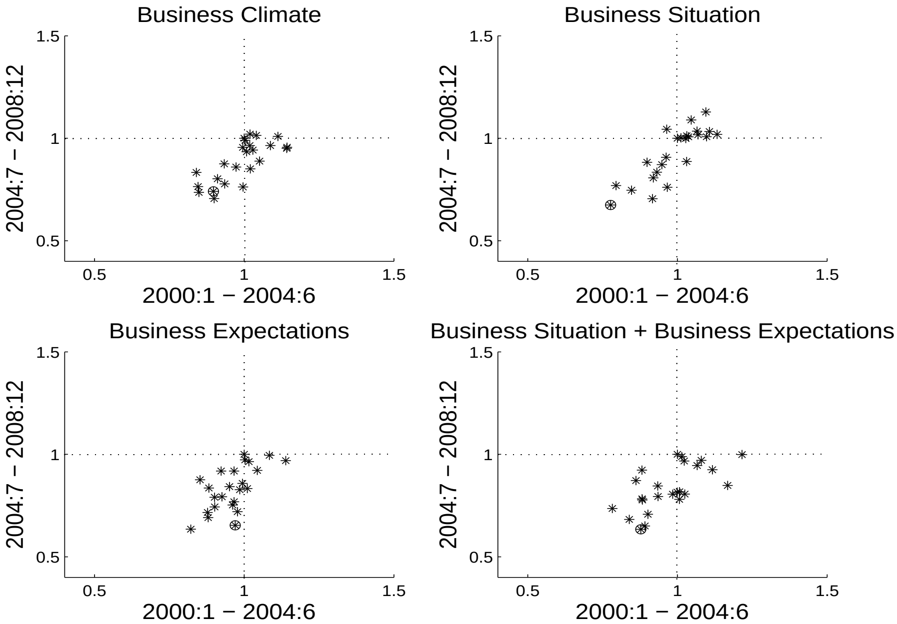


Figure 1.11: Scatter plot of relative MSEs against univariate benchmark model for annual growth of IPI ($h = 0$).

forecasting the current monthly value of IPI and *BE* slightly dominates as the forecast horizon grows, calculating *BC* as the geometric mean of *BS* and *BE* yields a comparable proper forecast performance for each of the horizons considered.

1.3.8 Results for PI and PF

For the comparison of the different PI and PF frameworks, we focus on *Business Climate* as predictor as it performs considerably well for each of the forecast horizons $h = 0, \ldots, 2$. We follow De Mol (2008) and report the results of the competing approaches for a fixed value of $p = 0$, i.e. we abstract from lags of the regressors. Hence, we do not evaluate the incremental forecasting power of the competing approaches but compare the performance in terms of direct out–of–sample Granger causality. The exclusion of lagged predictor variables guarantees comparability of the approaches in terms of how a given information set is used to predict the target series. Furthermore, a constant lag choice is important for the OPI approach as varying numbers of lags potentially interfere with the convergence of the algorithm. In general, the OPI algorithm can be used for any given number of lags.[46] Additionally, the extraction of PLS factors is complicated when lagged endogenous variables are included as one needs to control for the effect of these lagged terms on the covariances between the predictor and the reference time series. For the PF and PI frameworks that require information on ex–post (historical) forecast performance, a training (optimization) period is split from the model estimation sample at each forecast origin. The training sample is used as a rolling window with a size of $N = 48$ periods. Table 1.3 shows the performance of the different PI and PF frameworks for the full evaluation period, spanning 108 forecasts for annual growth of IPI.[47] We report the relative MSEs together with t–statistics of a one–sided test for forecast accuracy for non–nested models proposed by Harvey et al. (1997) – denoted HLN hereafter – comparing predictive accuracy of the respective model to a benchmark model based on the published indices of manufacturing industry.[48]

Notably, the majority of PI and PF frameworks outperforms the economically weighted sectoral indices in terms of forecast accuracy. As indicated by the low correlations of their indices and the corresponding production series, ad–hoc attributing a weight of zero to volatile branches as *food products and beverages* and *tobacco products* reduces the MSE of the aggregate index. However, these gains are not significant for any of the forecast horizons.

[46]Chapter 2 of this dissertation illustrates of how OPI can handle potentially varying numbers of optimal lags in an ADL forecast equation.

[47]Table 1.7 in Appendix 1.C.2 reports the results for month–on–month growth of IPI as reference series.

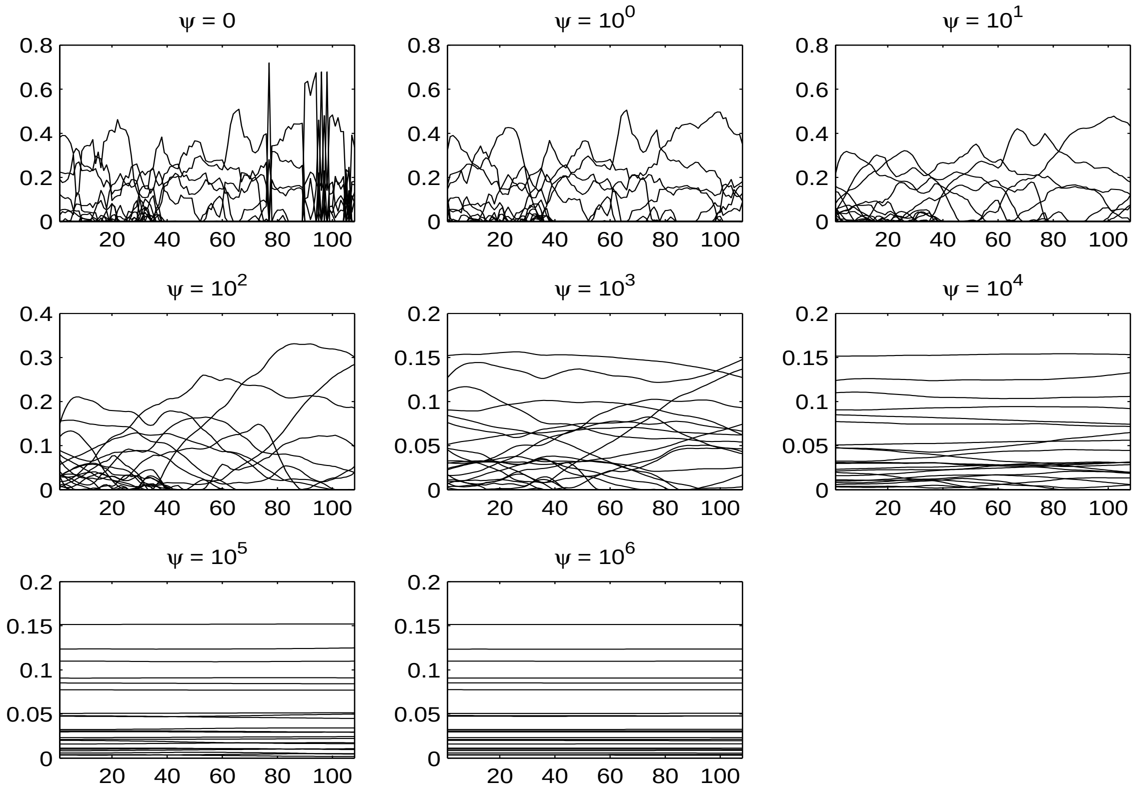
[48]Note that the tested models do not nest the benchmark model which uses the economically sectoral index as predictor variable.

Pooling of Information (PI)	h=0	h=1	h=2	**Pooling of forecasts (PF)**	h=0	h=1	h=2
	BC	BC	BC		BC	BC	BC
Ex DA15 DA16	0.98	0.98	0.98	Mean	0.79	0.86	0.89
	(1.16)	(1.12)	(0.94)		(3.34)	(1.76)	(1.52)
PC (r=1)	1.04	1.09	1.10	Median	0.82	0.86	0.89
	(-0.60)	(-1.03)	(-1.04)		(2.53)	(1.79)	(1.67)
PC (r=2)	1.04	1.07	1.07	Mean trimmed 5 %	0.80	0.86	0.89
	(-0.58)	(-0.71)	(-0.68)		(3.34)	(1.79)	(1.54)
PC (r=3)	0.80	0.78	0.82	Mean trimmed 10 %	0.80	0.86	0.89
	(3.56)	(2.88)	(2.43)		(3.29)	(1.78)	(1.53)
PC (r=4)	0.84	0.84	0.86	BMA K=1	0.79	0.86	0.89
	(2.56)	(2.05)	(1.82)		(3.40)	(1.82)	(1.54)
PLS (r=1)	1.06	1.10	1.09	BMA K=2	0.79	0.86	0.89
	(-0.93)	(-1.13)	(-1.03)		(3.45)	(1.86)	(1.56)
PLS (r=2)	0.87	0.90	0.95	BMA K=4	0.80	0.86	0.89
	(1.60)	(0.89)	(0.43)		(3.52)	(1.93)	(1.56)
PLS (r=3)	0.90	0.85	0.89	BMA K=10	0.80	0.86	0.90
	(1.46)	(1.65)	(1.22)		(3.51)	(1.98)	(1.46)
PLS (r=4)	0.98	0.92	1.02	MSE K=1	0.79	0.85	0.89
	(0.23)	(0.87)	(-0.17)		(3.63)	(2.03)	(1.63)
OPI ($\psi = 0$)	0.78	0.82	0.87	MSE K=2	0.79	0.85	0.89
	(2.52)	(1.81)	(1.70)		(3.72)	(2.13)	(1.58)
OPI ($\psi = 10^0$)	0.75	0.84	0.88	MSE K=4	0.79	0.86	0.91
	(3.01)	(1.59)	(1.54)		(3.58)	(2.11)	(1.32)
OPI ($\psi = 10^1$)	0.78	0.89	0.86	MSE K=10	0.80	0.88	0.95
	(2.78)	(1.10)	(1.03)		(2.73)	(1.72)	(0.66)
OPI ($\psi = 10^2$)	0.90	0.97	0.89	Predictive Least Squares	0.90	0.95	1.04
	(1.60)	(0.27)	(1.30)		(1.01)	(0.51)	(-0.42)
OPI ($\psi = 10^3$)	0.97	0.94	0.92	Disc MSE (d=0.95, K=1)	0.77	0.81	0.85
	(0.94)	(1.52)	(1.86)		(3.93)	(2.64)	(2.24)
OPI ($\psi = 10^4$)	1.00	0.98	0.98	Disc MSE (d=0.95, K=2)	0.75	0.78	0.82
	(0.29)	(1.26)	(1.52)		(4.23)	(3.07)	(2.51)
OPI ($\psi = 10^5$)	1.02	1.01	1.00	Disc MSE (d=0.95, K=4)	0.73	0.75	0.78
	(-2.13)	(-1.03)	(-0.10)		(4.44)	(3.24)	(2.51)
OPI ($\psi = 10^6$)	1.03	1.01	1.00	Disc MSE (d=0.95, K=10)	0.70	0.71	0.74
	(-2.45)	(-1.17)	(-0.20)		(4.22)	(2.82)	(2.17)
LASSO (r=1)	3.26	3.62	3.17	Disc MSE (d=0.90, K=1)	0.75	0.78	0.81
	(-4.80)	(-3.64)	(-3.13)		(4.24)	(3.08)	(2.57)
LASSO (r=2)	2.77	2.74	2.43	Disc MSE (d=0.90, K=2)	0.72	0.73	0.76
	(-4.15)	(-3.08)	(-2.89)		(4.59)	(3.41)	(2.72)
LASSO (r=4)	1.15	1.20	1.08	Disc MSE (d=0.90, K=4)	0.69	0.68	0.70
	(-1.21)	(-1.20)	(-0.30)		(4.74)	(3.27)	(2.51)
LASSO (r=6)	0.95	1.02	0.92	Disc MSE (d=0.90, K=10)	0.65	0.64	0.65
	(0.60)	(-0.12)	(0.35)		(4.39)	(2.75)	(2.17)
LASSO (r=8)	0.96	1.02	0.91	Rank (K=1)	0.64	0.67	0.68
	(0.54)	(-0.13)	(0.36)		(4.82)	(2.96)	(2.32)
LASSO (r=10)	0.97	1.02	0.91	Rank (K=2)	0.63	0.72	0.68
	(0.42)	(-0.16)	(0.32)		(3.72)	(1.94)	(1.71)
				Rank (K=4)	0.62	0.75	0.69
					(3.55)	(1.69)	(1.56)
				Rank (K=10)	0.62	0.75	0.69
					(3.51)	(1.64)	(1.53)

Notes: We report the relative MSE values for the PI and PF approaches compared to the economically weighted sectoral index as benchmark. The figures in parentheses represent the t–statistics of a one–sided test for predictive accuracy for non–nested models as proposed by Harvey et al. (1997). A t–statistic greater than +1.28 (10 percent significance level) or +1.65 (5 percent significance level) indicates that the tested approach yields a significant smaller MSE than the benchmark.

Table 1.3: Relative MSE of PI and PF approaches against economically weighted benchmark for annual growth of IPI

Regarding PI, the largest improvements are achieved by the OPI algorithm for smaller values of ψ between 0 and 10, depending on the forecast horizon. These gains fade away for large values of ψ as the optimized weights converge to their economic counterparts. For $\psi = 10^0$, the OPI algorithm improves forecast accuracy by about 25% for $h = 0$ and still more than 10% for $h = 1$ and $h = 2$. Figure 1.12 illustrates the weights attributed to the branch level indices within the OPI approach at the different forecast origins $t = 1, \ldots, 108$ for the respective values of ψ for the nowcast.



Notes: The lines represent the weights attributed to the branch level indices at the different forecast origins $t = 1, \ldots, 108$ for $h = 0$.

Figure 1.12: OPI weights for annual growth of IPI ($h = 0$)

In general, the weights gain persistence with an increasing value of ψ. In the absence of a penalty term, i.e. for $\psi = 0$, the weights are very volatile whereas they show a rather persistent behavior for $\psi > 10^3$. As the relative MSEs reported in Table

1.3 show, the trade–off between persistence and flexibility is optimal for $\psi = 10^0$ in terms of forecast accuracy. Table 1.4 compares the relative economic weights of the branches to the average weights attributed to the branch level indices by the OPI approach for $\psi = 10^0$. The columns give the average weights for the forecast horizons $h = 0, \ldots, 2$ and the corresponding MSE of an aggregate index build on these weights relative to the sectoral benchmark index.[49]

For each horizon, only a smaller number of branch level indices is attributed a significant weight. As the relative MSEs show, focussing on a core group of indices consisting of about 8 branches improves the forecast performance by about 30% on average. Forecast accuracy can be further improved by introducing a five percent threshold, i.e. setting all weights less than 5% exogenously to zero. The resulting index for $h = 0$ as shown in the last column of Table 1.4 reduces the MSE by around 40% and yields comparable improvements even for $h = 1$ and $h = 2$. This means that a weighted index based only on a small subset of branch indices including *textiles* and *basic metals* has a considerable higher forecast performance regarding aggregate industrial production. Most notably, *machinery and equipment*, frequently referred to as the key branch of German industry is not included in this index.

Pooling the branch level information via the extraction of principal components improves accuracy for $r = 3$ and $r = 4$ by about 20% for each forecast horizon. The PLS approach works best for $r = 2$ and $r = 3$, reducing MSE compared to the economically weighted sectoral index by about 10%. Hence, for both approaches, a rather small number of 3 common factors captures most of the dynamics of the manufacturing sector. The predominance of PC compared to PLS indicates that the correlations between the predictor variables and the reference series are rather volatile such that excluding these relations in the extraction of common factors improves stability and forecast accuracy.

The lasso regressions work best for $r = 6$, i.e. incorporating 6 branch level indices simultaneously in a forecast equation yields the best forecast performance. However, the resulting gains are significant only for the month–on–month growth rate of IPI.

[49] Note that the MSE ratio shows a hypothetical improvement as the average weights are calculated using ex ante information that would not have been available to the forecaster in real–time.

Branches (two digits)	Relative Weights	Mean Coefficients			
		h=0	h=1	h=2	5 % threshold
DA15 Food products and beverage	8.5	0.0	2.4	0.4	0.0
DA16 Tobacco Products	0.4	1.6	0.9	5.5	0.0
DB17 Textiles	1.6	20.0	13.7	11.1	21.5
DB18 Wearing apparel	1.0	0.2	0.1	0.0	0.0
DC Leather and Leather products	0.3	0.3	3.9	2.0	0.0
DD Wood and wood products	2.0	0.1	1.2	0.1	0.0
DE21 Pulp, paper and paper products	2.3	0.7	6.9	6.9	0.0
DE22 Publishing, printing and reproduction of recorded media	3.0	0.5	3.8	1.4	0.0
DF Coke, refines petroleum	0.6	0.1	1.0	1.9	0.0
DG Chemicals and chemical products	11.0	0.1	1.5	6.1	0.0
DH Rubber and plastic products	5.1	8.5	1.6	2.5	9.1
DI Non-metallic mineral products	4.8	0.0	0.9	1.8	0.0
DJ27 Basic metals	4.7	17.6	18.1	9.4	18.8
DJ28 Fabricated metals products	9.1	0.3	0.8	0.1	0.0
DK Machinery and equipment	15.1	0.1	0.7	0.0	0.0
DL30 Office machinery and computers	1.1	1.4	0.3	0.5	0.0
DL31 Electrical machinery	7.8	6.4	8.3	5.2	6.9
DL32 Radio, television and communication equipment	2.1	16.2	11.3	8.3	17.4
DL33 Medical, precision and optical instruments	3.2	9.2	10.3	4.8	9.8
DM34 Motor vehicles	12.3	10.1	5.5	15.4	10.8
DM35 Other transport	0.8	5.3	5.1	8.7	5.7
DN36 Furniture	3.1	1.3	1.9	7.9	0.0
relative MSE		0.65	0.80	0.77	

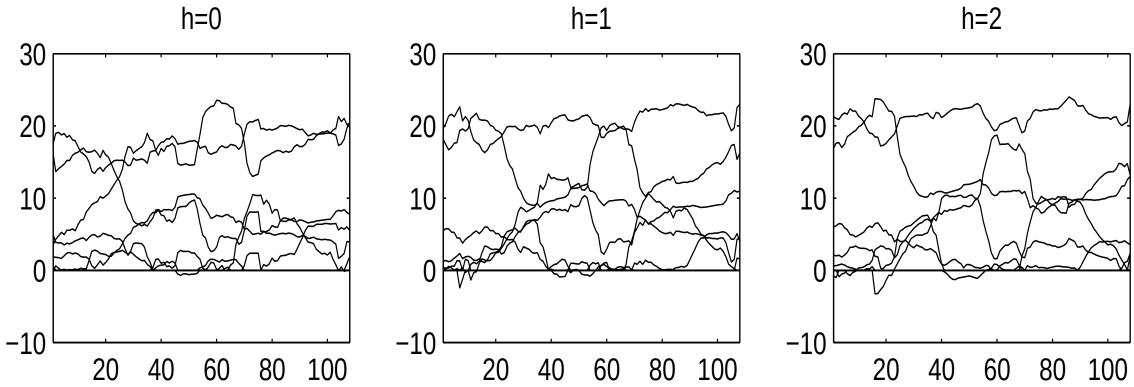
Notes: The reported figures are the mean weights attributed to the 22 branches in manufacturing by the OPI algorithm over 108 forecasts.

Table 1.4: Average OPI weights for annual growth of IPI

Figure 1.13 illustrates the corresponding coefficients of the branch level indices in the lasso equation for $h = 0, \ldots, 2$ and Table 1.5 reports the average coefficients for the 108 forecast steps. Notably, compared to the OPI approach, the coefficients exhibit a rather persistent behavior over the sample.[50] In fact, only a smaller subset of branch level indices are attributed a coefficient larger than zero in any of the estimation steps, including *paper products*, *chemicals* and *rubber*. Although the number of branches is comparable to the OPI selection, different branches are chosen on average.

Regarding the PF strategies, the simple mean and the median forecast reduce MSE by 20% on average for $h = 0$ and by 10% for $h = 1, 2$ compared to the sectoral index. Trimming the 5% and 10% highest and lowest forecasts as well as weighting predictions by means of their in–sample and out–of–sample performance does not significantly change the results. In contrast, discounting the past prediction errors increases forecast performance by another 10% on average. These gains grow with a decreasing discount factor d. This indicates again that the relative performance of

[50]Note that at each forecast origin, the lasso equation is estimated over the full ex post estimation sample. As the optimal branch selection potentially varies over time, a reduced estimation sample by means of a rolling window could improve forecast performance.



Notes: The lines represent the coefficients attributed to the branch level indices at the different forecast origins $t = 1, \ldots, 108$.

Figure 1.13: Lasso coefficients for annual growth of IPI (h=0)

Branches (two digits)	Relative Weights	Mean Coefficients		
		h=0	h=1	h=2
DA15 Food products and beverage	8.5	0.0	-0.1	-0.1
DA16 Tobacco Products	0.4	0.0	0.0	0.0
DB17 Textiles	1.6	4.5	4.7	3.7
DB18 Wearing apparel	1.0	0.0	0.1	0.4
DC Leather and Leather products	0.3	0.0	0.0	0.0
DD Wood and wood products	2.0	0.0	0.0	0.0
DE21 Pulp, paper and paper products	2.3	9.3	11.7	12.8
DE22 Publishing, printing and reproduction of recorded media	3.0	0.0	0.0	0.0
DF Coke, refines petroleum	0.6	1.4	1.7	1.8
DG Chemicals and chemical products	11.0	17.6	20.6	21.2
DH Rubber and plastic products	5.1	15.6	10.6	8.5
DI Non-metallic mineral products	4.8	0.0	0.0	0.0
DJ27 Basic metals	4.7	1.4	0.8	0.6
DJ28 Fabricated metals products	9.1	0.0	0.0	0.0
DK Machinery and equipment	15.1	2.1	2.3	2.3
DL30 Office machinery and computers	1.1	0.0	0.0	0.0
DL31 Electrical machinery	7.8	0.0	0.0	0.0
DL32 Radio, television and communication equipment	2.1	0.0	0.0	0.2
DL33 Medical, precision and optical instruments	3.2	0.0	0.0	0.0
DM34 Motor vehicles	12.3	0.3	0.1	0.0
DM35 Other transport	0.8	0.0	0.0	0.0
DN36 Furniture	3.1	0.7	1.2	1.0

Notes: The reported figures are the mean coefficients for the 22 branches in manufacturing over 108 estimations of the lasso regression.

Table 1.5: Average lasso coefficients for annual growth of IPI

branch level indices regarding IPI growth changes over time. The highest improvements in forecast accuracy are obtained for weighting the models by means of their inverse ranks over the past periods. As this approach disregards the absolute size of the differences in forecast accuracy, it is less sensitive to outliers and the weighting scheme is more robust than weighting schemes directly based on measures of past forecast performance. The proper forecast record of the approach in the present study further indicates that single models that perform well on average are hit by extra ordinary forecast failure in certain periods.

To examine the stability of these high dimensional forecasts, we divide the evaluation period again in half and compute the MSEs over the two periods from January 2000 to June 2004 and from July 2004 to December 2008.[51]

For the different horizons, most scatter points are located in the lower left quadrant around the 45°–line. This indicate an dominant and stable forecast accuracy of the corresponding PI and PF approaches compared to the economically weighted sectoral index as benchmark in both periods.

Regarding the month–on–month growth rates of IPI, again OPI as well as the rank weighted PF approach are the best performing frameworks. For OPI, a value of $\psi = 10^2$ yields the highest forecast accuracy in the short run whereas the approach fails to outperform the benchmark for $h = 2$. In contrast, the lasso regression significantly outperforms the benchmark model for $k > 4$ and $h = 2$.[52] Ad–hoc attributing a weight of zero to *food products and beverages* and *tobacco products* significantly improves forecast performance for $h = 0$. Extracting common factors via PC and PLS does not increase forecast accuracy for any of the settings under consideration. The results for the competing PF strategies are less encouraging as well as none of the weighting schemes significantly outperforms the benchmark model. Focussing on OPI, as for the annual growth rates, only a smaller number of branch level indices is attributed a significant weight.[53] Again, a smaller core

[51] The OPI forecast for $\psi = 10^0$ which dominates the PI schemes for the total forecast sample is marked with a circle.

[52] Table 1.9 and Figure 1.24 in Appendix 1.C.2 report the respective coefficients of the lasso approach for month–on–month growth of IPI.

[53] Table 1.8 in Appendix 1.C.2 reports the average weights attributed to the branch level indices

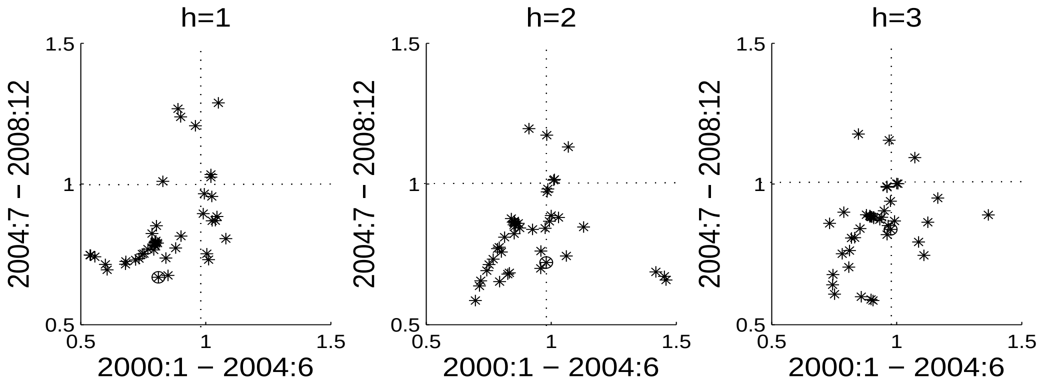


Figure 1.14: Scatter plot of relative MSEs of PI and PF against economic weighted benchmark for annual growth of IPI (h=0)

group of indices significantly improves the forecast performance of the resulting index. Introducing a five percent threshold yields an aggregate index that reduces the MSE by around 10% for $h = 0$ whereas these gains fade away for $h = 1$ and $h = 2$.

The empirical forecast evaluation leads to a number of general observations. First, when predicting IPI growth, the use of the economically weighted sectoral indices cannot be regarded optimal in terms of MSE loss. Instead, predictively pooling the information via the OPI algorithm significantly improves forecast performance. As the corresponding weights fluctuate considerably over time, introducing a regularization term that penalizes deviations from previous weights increases forecast performance. Pooling forecast from models that build on a small subset of branch level indices poses a robust alternative. Although the equal weighted average yields a significant improvement in forecast accuracy, the method can be boosted by discounting past prediction errors or weighting the predictions by means of the past inverse ranks of the respective models. Overall, in case of predicting annual growth of IPI, pooling forecasts from parsimonious models based on a smaller number of branch level indices poses the dominant forecast strategy and can be favored on practical grounds.

for $\psi = 10^2$ and gives the corresponding MSE relative to the sectoral benchmark index. Figure 1.23 tracks the weights over the different forecast origins.

1.4 Conclusion

In this paper, we analyze the relative merits of pooling of forecasts compared to pooling of information. In an analytical part, we demonstrate that PI theoretically dominates PF in the absence of estimation uncertainty and for perfectly measured explanatory variables. In the more realistic scenario where only noisy measures of the explanatory variables are at hand, PI looses its virtues as the noise components increase and for highly correlated shocks.

In an empirical exercise, we evaluate the performance of various candidate PI and PF strategies predicting German industrial production based on Ifo business survey data. As a first result of the experiment, we find that economically weighted indices – as published by the Ifo institute – exhibit significant predictive content regarding monthly industrial production. Although the *Business Climate* index, calculated as geometric mean of the *Business Situation* and the *Business Expectations* performs reasonably well, this form of aggregation cannot be regarded optimal in terms of forecasting. In fact, *BS* is favorable when forecasting the current monthly value of IPI and *BE* dominates as the forecast horizon grows.

In accordance with the analytical findings, our empirical study confirms the advantages of pooling a larger number of single forecasts, each based only on a subset of the available information. Comparing various PF frameworks, we find that weighting single predictions based on branch level indices by means of their past forecast performance improves forecast accuracy by up to 40% compared to economically weighted indices. The best results are obtained for weights measured in terms of relative ranks and discounted MSEs of the single forecasts.

Although PF poses the dominant strategy, we find that alternative PI approaches also improve forecast accuracy compared to the economically weighted benchmark index. Relating to the framework of predictive modeling, we develop the OPI approach that optimizes the weighting scheme specifically with respect to a certain forecast exercise. We find that OPI performs considerably well and poses a serious alternative to the dominant PF strategies.

Appendix

1.A Analytical Derivations - DGP

The data–generating process is given as

$$\begin{aligned} y_t &= \beta_1 x_{1,t} + \beta_2 x_{2,t} + e_t \\ &= \beta X_t + e_t \end{aligned} \tag{1.43}$$

with

$$X_t = \begin{pmatrix} x_{1,t} \\ \\ x_{2,t} \end{pmatrix} = \begin{pmatrix} \phi_{1,t} \\ \\ \phi_{2,t} \end{pmatrix} + \begin{pmatrix} \xi_{1,t} \\ \\ \xi_{2,t} \end{pmatrix} \tag{1.44}$$

where $\phi_{1,t}$ and $\phi_{2,t}$ are fixed functions of past variables and

$$\begin{pmatrix} \xi_{1,t} \\ \\ \xi_{2,t} \end{pmatrix} \sim IN_k \left[\begin{pmatrix} 0 \\ \\ 0 \end{pmatrix}, \begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \\ \Omega_{21} & \Omega_{22} \end{pmatrix} \right]. \tag{1.45}$$

The explanatory variables can be described as

$$x_{1,t} = [\Omega_{22}^{-1}\Omega_{12}]x_{2,t} + \eta_{12,t}$$

$$x_{2,t} = [\Omega_{11}^{-1}\Omega_{12}]x_{1,t} + \eta_{21,t}$$

where $\eta_{1,2,t}$ and $\eta_{2,1,t}$ are the idiosyncratic components with

$$\begin{aligned} E[x_{2,t}\eta_{12,t}] &= 0 \\ E[x_{1,t}\eta_{21,t}] &= 0 \end{aligned}$$

The variances and covariances of the explanatory variables and their idiosyncratic parts can be written as:

$$
\begin{aligned}
\Omega_{11} &= [\Omega_{22}^{-1}\Omega_{12}]\Omega_{22}[\Omega_{22}^{-1}\Omega_{12}] + \Omega_{\eta_{12}} \\
&= \Omega_{12}\Omega_{22}^{-1}\Omega_{12} + \Omega_{\eta_{12}} \\
\Omega_{22} &= [\Omega_{11}^{-1}\Omega_{12}]\Omega_{11}[\Omega_{11}^{-1}\Omega_{12}] + \Omega_{\eta_{21}} \\
&= \Omega_{12}\Omega_{11}^{-1}\Omega_{12} + \Omega_{\eta_{21}} \\
\Omega_{\eta_{12}} &= \Omega_{11} - \Omega_{12}\Omega_{22}^{-1}\Omega_{12} \\
\Omega_{\eta_{21}} &= \Omega_{22} - \Omega_{12}\Omega_{11}^{-1}\Omega_{12}
\end{aligned}
$$

$$
\begin{aligned}
E[\eta_{12,t}\eta_{21,t}] &= E[(x_{2,t} - \Omega_{11}^{-1}\Omega_{12}x_{1,t})(x_{1,t} - \Omega_{22}^{-1}\Omega_{12}x_{2,t})] \\
&= E[(x_{2,t}x_{1,t} - \Omega_{22}^{-1}\Omega_{12}x_{2,t}^2 - \Omega_{11}^{-1}\Omega_{12}x_{1,t}^2 + \Omega_{11}^{-1}\Omega_{22}^{-1}\Omega_{12}^2 x_{1,t}x_{2,t})] \\
&= E[x_{2,t}x_{1,t}] - \Omega_{22}^{-1}\Omega_{12}E[x_{2,t}^2] - \Omega_{11}^{-1}\Omega_{12}E[x_{1,t}^2] + \Omega_{11}^{-1}\Omega_{22}^{-1}\Omega_{12}^2 E[x_{1,t}x_{2,t}] \\
&= \Omega_{12} - \Omega_{22}^{-1}\Omega_{12}\Omega_{22} - \Omega_{11}^{-1}\Omega_{12}\Omega_{11} + \Omega_{11}^{-1}\Omega_{22}^{-1}\Omega_{12}^3 \\
&= -\Omega_{12} + \Omega_{11}^{-1}\Omega_{22}^{-1}\Omega_{12}^3 \\
&= -\Omega_{12}(I_n - \Omega_{11}^{-1}\Omega_{12}\Omega_{22}^{-1}\Omega_{12}) \\
&= -\Omega_{12}(I_n - \prod_{12}\prod_{21})
\end{aligned}
$$

where

$$
\begin{aligned}
\prod_{12} &= \Omega_{22}^{-1}\Omega_{12} \\
\prod_{21} &= \Omega_{11}^{-1}\Omega_{12}
\end{aligned}
$$

1.B Analytical Derivations - Perfect Measurement

1.B.1 Single Forecasts

The single forecast equations are given as:

$$y_{1,t} = b_1 x_{1,t} + u_t$$

$$y_{2,t} = b_2 x_{2,t} + v_t$$

Employing ordinary least squares, the estimated coefficients can be written as

$$\hat{b}_1 = E[\hat{b}_1] + \delta_{\hat{b}_1}$$

$$\hat{b}_2 = E[\hat{b}_2] + \delta_{\hat{b}_2}$$

where the expected values of the estimators are given as

$$\begin{aligned} E[\hat{b}_1] &= E[\Omega_{11}^{-1}\Omega_{1y}] \\ &= \beta_1 + \Omega_{11}^{-1}\Omega_{12}\beta_2 \\ E[\hat{b}_2] &= E[\Omega_{22}^{-1}\Omega_{2y}] \\ &= \beta_2 + \Omega_{22}^{-1}\Omega_{12}\beta_1. \end{aligned}$$

The latter terms of these expression are the corresponding omitted variable biases for each of the estimators.

The variances of the estimators are given as

$$\Omega_{\hat{b}_1} = \Omega_{\delta_{\hat{b}_1}} = T^{-1}\Omega_{ee}\Omega_{11}^{-1}$$

$$\Omega_{\hat{b}_2} = \Omega_{\delta_{\hat{b}_2}} = T^{-1}\Omega_{ee}\Omega_{22}^{-1}$$

and depend on the sample size, the variance of the errors as well as the variance of the explanatory variable in the forecast equation.

The forecast errors of the single models can be calculated as

$$
\begin{aligned}
\hat{u}_t &= y_t - \hat{y}_{1,t} \\
&= (\beta_1 - \hat{b}_1)x_{1,t} + \beta_2 x_{2,t} + e_t \\
&= (\beta_1 - E[\hat{b}_1] - \delta_{\hat{b}_1})x_{1,t} + \beta_2 x_{2,t} + e_t \\
&= (\beta_1 - (\beta_1 + \Omega_{11}^{-1}\Omega_{12}\beta_2) - \delta_{\hat{b}_1})x_{1,t} + \beta_2(\Omega_{11}^{-1}\Omega_{12}x_{1,t} + \eta_{21,t}) + e_t \\
&= \beta_2\eta_{21,t} - \delta_{\hat{b}_1}x_{1,t} + e_t \\
\hat{v}_t &= y_t - \hat{y}_{2,t} \\
&= \beta_1 x_{1,t} + (\beta_2 - \hat{b}_2)x_{2,t} + e_t \\
&= \beta_1 x_{1,t} + (\beta_2 - E[\hat{b}_2] - \delta_{\hat{b}_2})x_{2,t} + e_t \\
&= \beta_1(\Omega_{22}^{-1}\Omega_{12}x_{2,t} + \eta_{12,t}) + (\beta_2 - (\beta_2 + \Omega_{22}^{-1}\Omega_{12}\beta_1) - \delta_{\hat{b}_2})x_{2,t} + e_t \\
&= \beta_1\eta_{12,t} - \delta_{\hat{b}_2}x_{2,t} + e_t.
\end{aligned}
$$

with expected values given as

$$
\begin{aligned}
E[\hat{u}_t|x_{1,t}, x_{2,t}] &= \beta_2\eta_{21,t} \\
E[\hat{v}_t|x_{1,t}, x_{2,t}] &= \beta_1\eta_{12,t}.
\end{aligned}
$$

The expected values of the single models' forecast errors correspond to the idiosyncratic component of the missing variable weighted by its share in the DGP. Assuming unbiased forecast errors, the MSEs of the single forecast equations are calculated as

$$\begin{aligned}
MSE_1 &= E[\hat{u}_t^2 | x_{1,t}, x_{2,t}] \\
&= x_{1,t}\Omega_{\hat{b}_1} x_{1,t} + \beta_2^2 \Omega_{\eta_{21}} + \Omega_{ee} \\
&= T^{-1}\Omega_{ee} + \beta_2^2 \Omega_{\eta_{21}} + \Omega_{ee} \\
MSE_2 &= E[\hat{v}_t^2 | x_{1,t}, x_{2,t}] \\
&= x_{2,t}\Omega_{\hat{b}_2} x_{2,t} + \beta_1^2 \Omega_{\eta_{12}} + \Omega_{ee} \\
&= T^{-1}\Omega_{ee} + \beta_1^2 \Omega_{\eta_{12}} + \Omega_{ee}.
\end{aligned}$$

and depend on the sample size, the variance of the noise in the DGP as well as on the variance of the idiosyncratic component of the missing variable weighted by its squared share in the DGP.

1.B.2 PF Forecast

Assuming exogenously given weights w, the pooled forecast can be derived as

$$\begin{aligned}
\hat{y}_{PF,t} &= w\hat{y}_{1,t} + (1-w)\hat{y}_{2,t} \\
&= w(\hat{y}_{1,t} - \hat{y}_{2,t}) + \hat{y}_{2,t}.
\end{aligned}$$

This translates into a PF forecast error of

$$\begin{aligned}
\hat{e}_{PF,t} &= y_t - \hat{y}_{cf,t} \\
&= (y_t - \hat{y}_{2,t}) - w(\hat{y}_{1,t} - \hat{y}_{2,t}) \\
&= \hat{v}_t - w(y_t - \hat{u}_t - y_t + \hat{v}_t) \\
&= \hat{v}_t + w(\hat{u}_t - \hat{v}_t) \\
&= \hat{v}_t + w\hat{u}_t - w\hat{v}_t \\
&= w\hat{u}_t + (1-w)\hat{v}_t.
\end{aligned}$$

Inserting the forecast errors of the single models, the corresponding MSE of pooling of forecasts can be calculated as

$$
\begin{aligned}
MSE_{PF} &= E[\hat{e}^2_{PF,t}|x_{1,t}, x_{2,t}] \\
&= E[((w\hat{u}_t) + (1-w)\hat{v}_t)^2] \\
&= E[(w(\beta_2\eta_{21,t} - \delta_{\hat{b}_1}x_{1,t} + e_t) + (1-w)(\beta_1\eta_{12,t} - \delta_{\hat{b}_2}x_{2,t} + e_t))^2] \\
&= \Omega_{ee} + E[(w(\beta_2\eta_{21,t} - \delta_{\hat{b}_1}x_{1,t}) + (1-w)(\beta_1\eta_{12,t} - \delta_{\hat{b}_2}x_{2,t}))^2] \\
&= \Omega_{ee} + E[(w^2(\beta_2\eta_{21,t} - \delta_{\hat{b}_1}x_{1,t})^2 + (1-w)^2(\beta_1\eta_{12,t} - \delta_{\hat{b}_2}x_{2,t})^2 + \\
&\quad 2w(1-w)(\beta_2\eta_{21,t} - \delta_{\hat{b}_1}x_{1,t})(\beta_1\eta_{12,t} - \delta_{\hat{b}_2}x_{2,t}))] \\
&= \Omega_{ee} + w^2E[(\beta_2\eta_{21,t} - \delta_{\hat{b}_1}x_{1,t})^2] + (1-w)^2E[(\beta_1\eta_{12,t} - \delta_{\hat{b}_2}x_{2,t})^2] + \\
&\quad 2w(1-w)E[(\beta_2\eta_{21,t} - \delta_{\hat{b}_1}x_{1,t})(\beta_1\eta_{12,t} - \delta_{\hat{b}_2}x_{2,t})] \\
&= \Omega_{ee} + w^2(T^{-1}\Omega_{ee} + \beta_2^2\Omega_{\eta_{21}}) + (1-w)^2(T^{-1}\Omega_{ee} + \beta_1^2\Omega_{\eta_{12}}) + \\
&\quad 2w(1-w)E[\beta_2\eta_{21,t}\eta_{12,t}\beta_1 + \delta_{\hat{b}_1}x_{1,t}x_{2,t}\delta_{\hat{b}_2}] \\
&= \Omega_{ee} + T^{-1}\Omega_{ee}(w^2 + (1-w)^2) + 2w(1-w)E[\delta_{\hat{b}_1}x_{1,t}x_{2,t}\delta_{\hat{b}_2}] + w^2\beta_2^2\Omega_{\eta_{21}} \\
&\quad + (1-w)^2\beta_1^2\Omega_{\eta_{12}} + 2w(1-w)\beta_2E[\eta_{21,t}\eta_{12,t}]\beta_1
\end{aligned}
$$

1.B.3 PI Forecast

Pooling of information corresponds to a forecast model that incorporates all relevant explanatory variables. Hence, the PI forecast equation is given as

$$
\begin{aligned}
y_{PI,t} &= a_1x_{1,t} + a_2x_{2,t} + e_{PI,t} \\
&= AX_t + e_{PI,t}.
\end{aligned}
$$

The estimated coefficients are defined as

$$\begin{aligned}\hat{a}_1 &= E[\hat{a}_1] + \delta_{\hat{a}_1} \\ \hat{a}_2 &= E[\hat{a}_2] + \delta_{\hat{a}_2}\end{aligned}$$

and their expected values equal the true parameters of the DGP, i.e.

$$\begin{aligned}E[\hat{a}_1] &= \beta_1 \\ E[\hat{a}_2] &= \beta_2 \\ E[\hat{A}] &= \beta.\end{aligned}$$

This translates into a forecast error of

$$\begin{aligned}\hat{e}_{PI,t} &= (\beta_1 - \hat{a}_1)x_{1,t} + (\beta_2 - \hat{a}_2)x_{2,t} + e_t \\ &= (\beta - \hat{A})X_t + e_t \\ &= (\beta - E[\hat{A}] - \delta_{\hat{A}})X_t + e_t \\ &= -\delta_{\hat{A}}X_t + e_t\end{aligned}$$

with an expected value given as

$$E[\hat{e}_{PI,t}|x_{1,t}, x_{2,t}] = 0$$

The corresponding MSE can be written as

$$
\begin{aligned}
MSE_{PI} &= E[\hat{e}^2_{PI,t}|x_{1,t}, x_{2,t}] \\
&= E[(-\delta_{\hat{A}} X_t)(-\delta_{\hat{A}} X_t)] + \Omega_{ee} \\
&= E[X_t \Omega_{\delta_{\hat{A}}} X_t'] + \Omega_{ee} \\
&= E[X_t \Omega_{\hat{A}} X_t'] + \Omega_{ee} \\
&= E[X_t (T^{-1} \Omega_{ee} \Omega_{XX}^{-1}) X_t'] + \Omega_{ee} \\
&= E[X_t \Omega_{XX}^{-1} X_t'] T^{-1} \Omega_{ee} + \Omega_{ee} \\
&= tr(\Omega_{XX}^{-1} E[X_t' X_t]) T^{-1} \Omega_{ee} + \Omega_{ee} \\
&= tr(\Omega_{XX}^{-1} \Omega_{XX}) T^{-1} \Omega_{ee} + \Omega_{ee} \\
&= tr(I_k) T^{-1} \Omega_{ee} + \Omega_{ee} \\
&= k T^{-1} \Omega_{ee} + \Omega_{ee} \\
&= 2 T^{-1} \Omega_{ee} + \Omega_{ee} \\
&= \Omega_{ee}(1 + \tfrac{2}{T})
\end{aligned}
$$

1.C Empirical Study

1.C.1 Time Series Characteristics

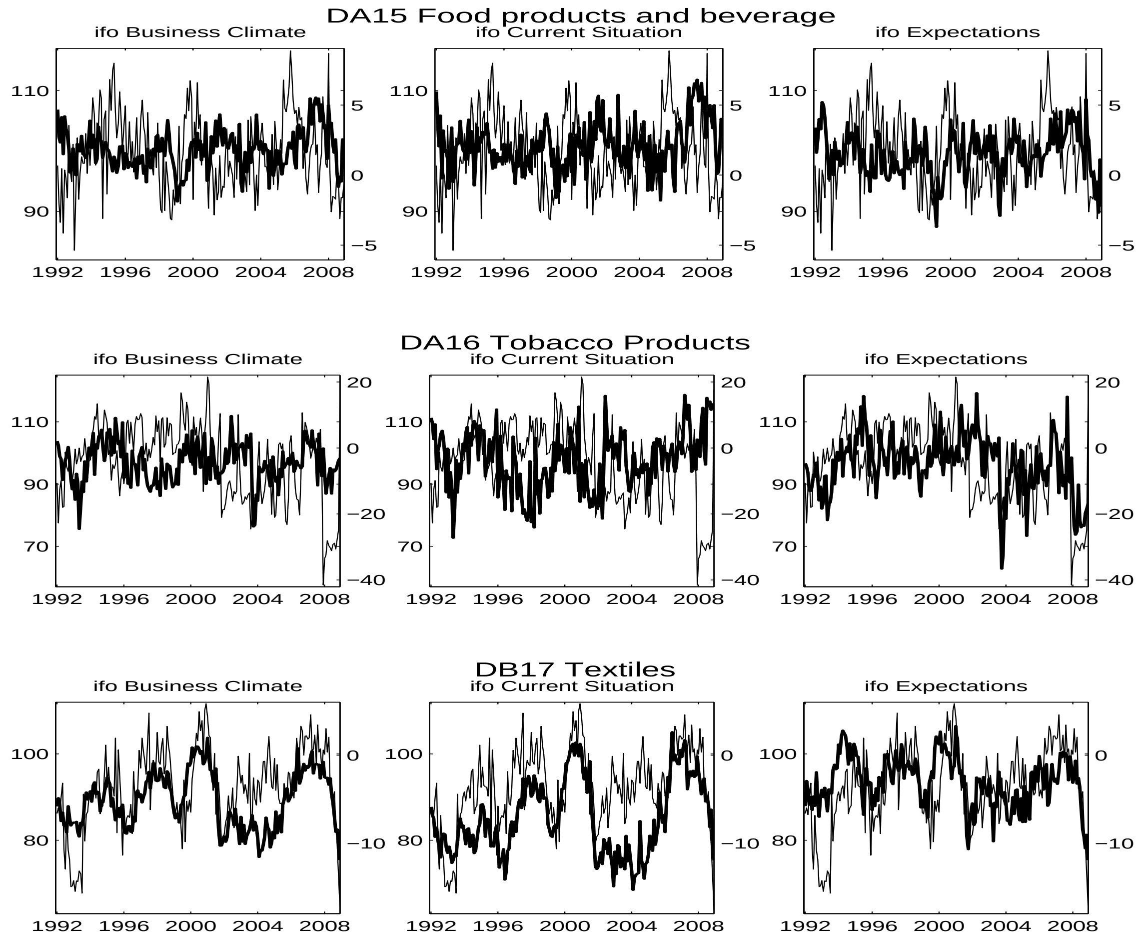
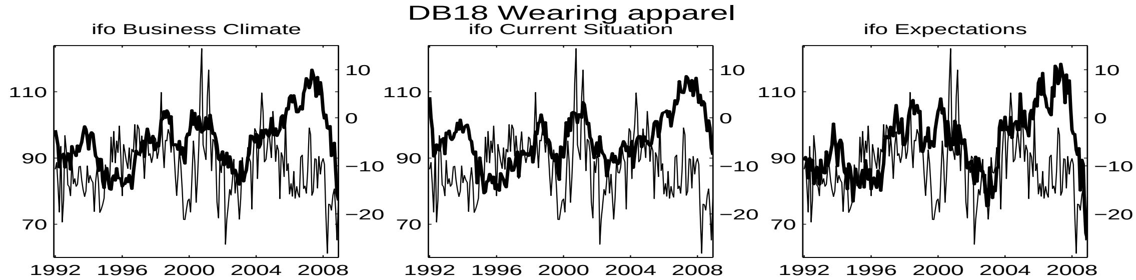
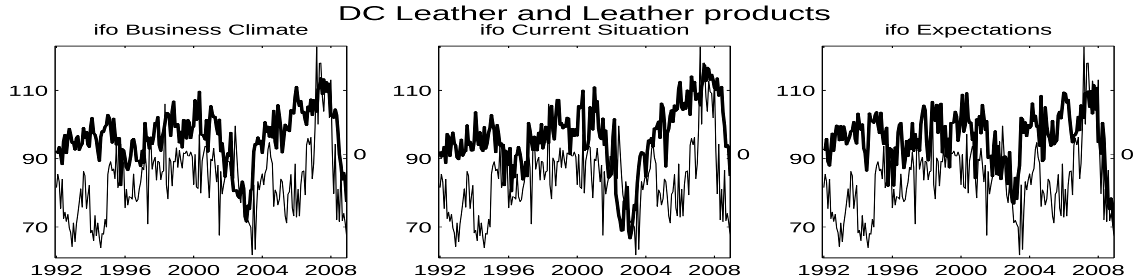
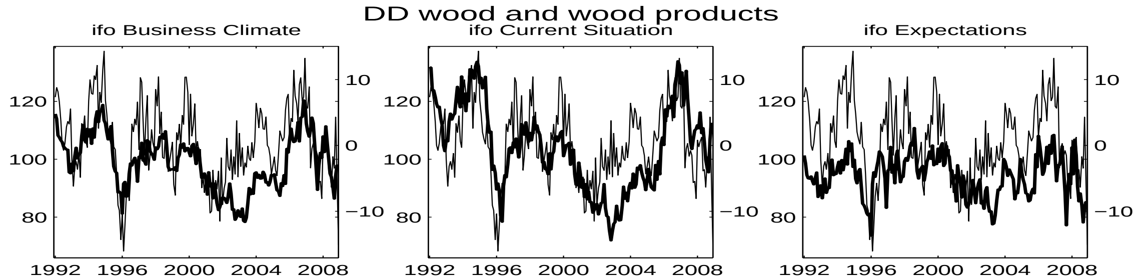
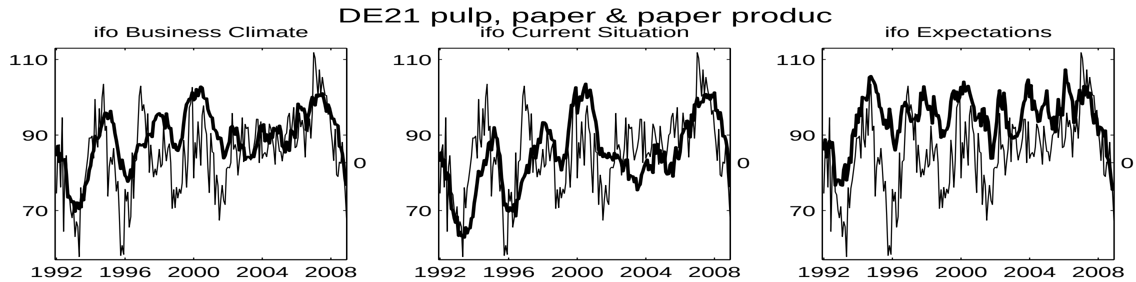


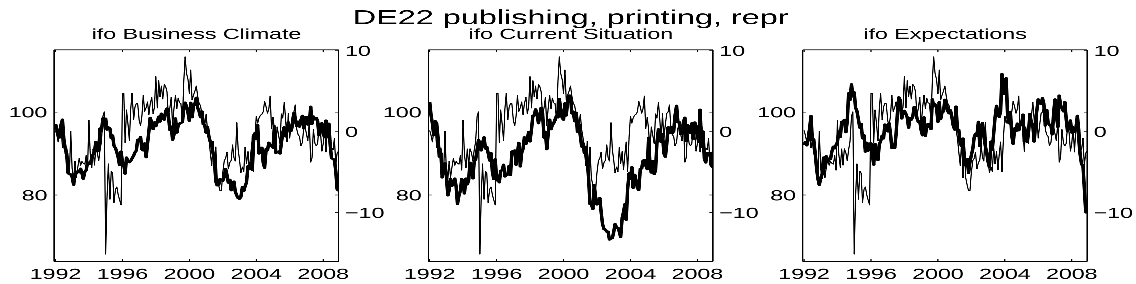
Figure 1.15: Ifo branch level survey and annual growth rate of production

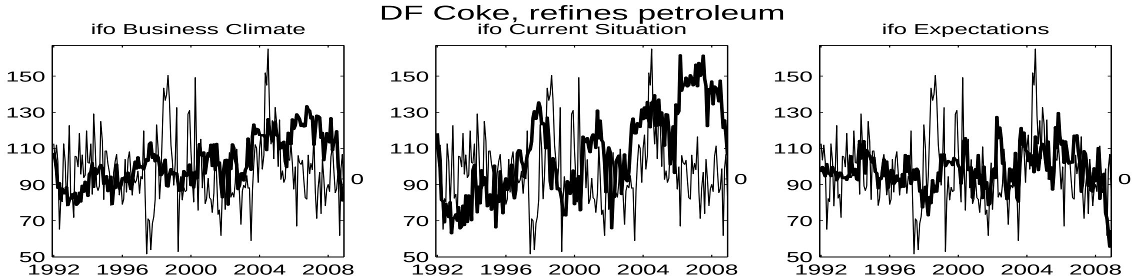

DB18 Wearing apparel
ifo Business Climate
ifo Current Situation
ifo Expectations
1992 1996 2000 2004 2008

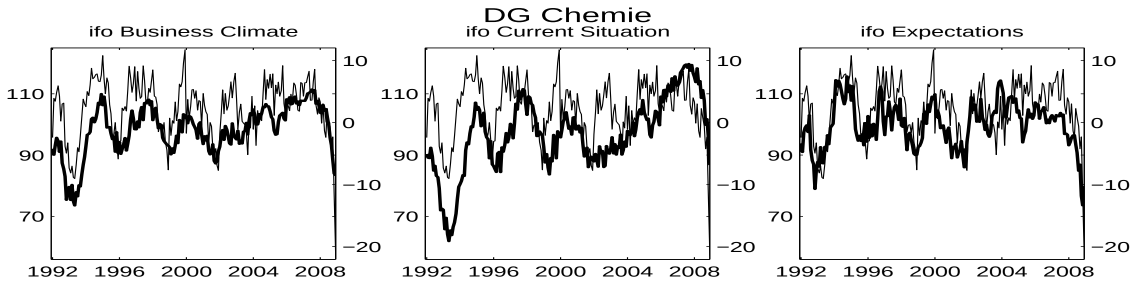

DC Leather and Leather products
ifo Business Climate
ifo Current Situation
ifo Expectations
1992 1996 2000 2004 2008

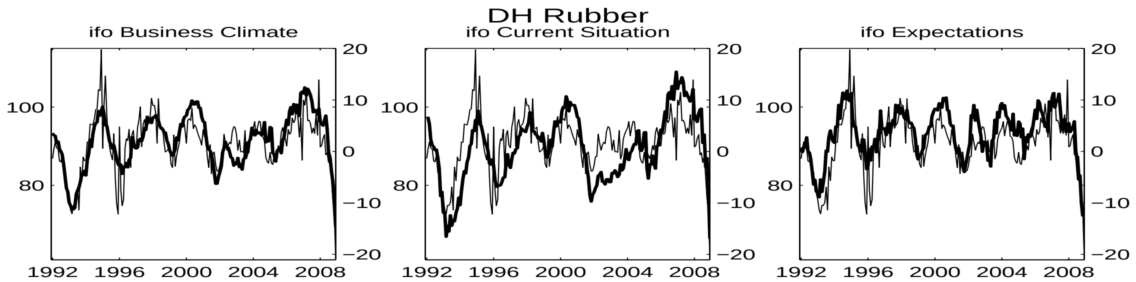

DD wood and wood products
ifo Business Climate
ifo Current Situation
ifo Expectations
1992 1996 2000 2004 2008

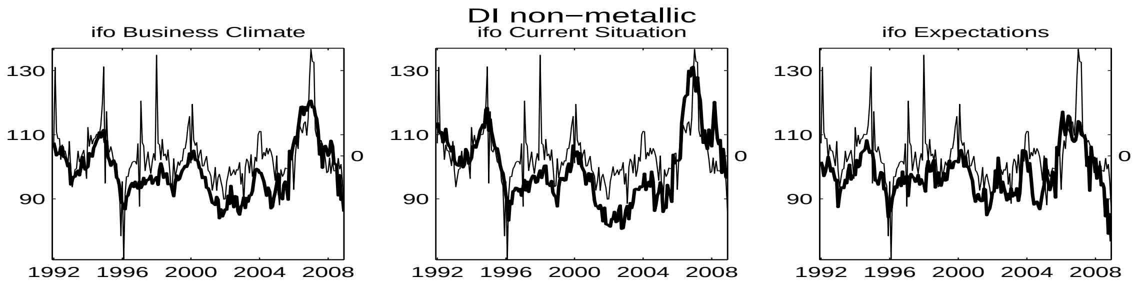

DE21 pulp, paper & paper produc
ifo Business Climate
ifo Current Situation
ifo Expectations
1992 1996 2000 2004 2008

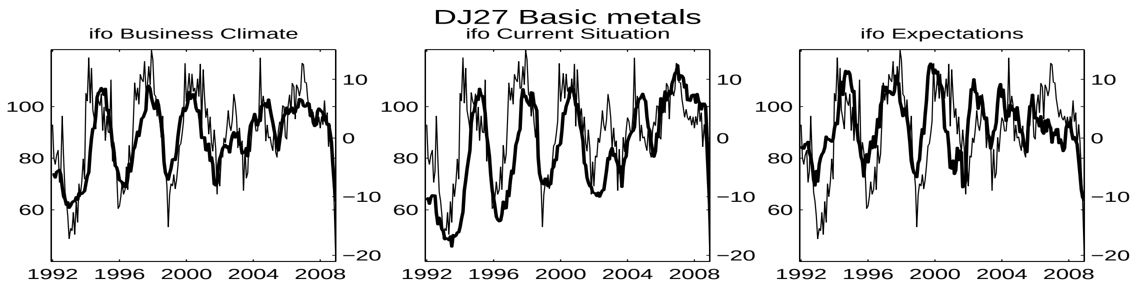

DE22 publishing, printing, repr
ifo Business Climate
ifo Current Situation
ifo Expectations
1992 1996 2000 2004 2008

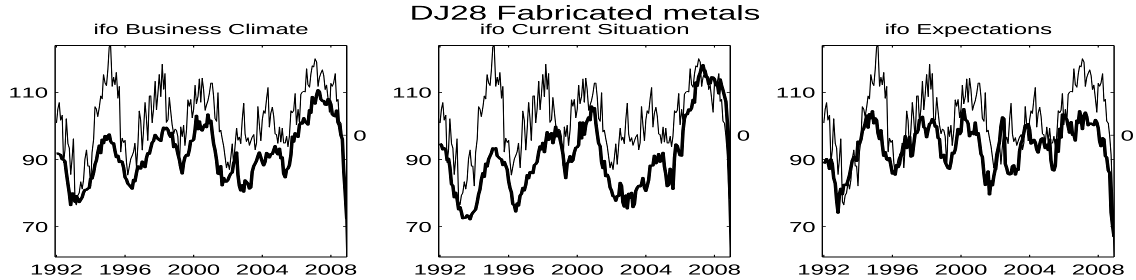

DF Coke, refines petroleum
ifo Business Climate
ifo Current Situation
ifo Expectations

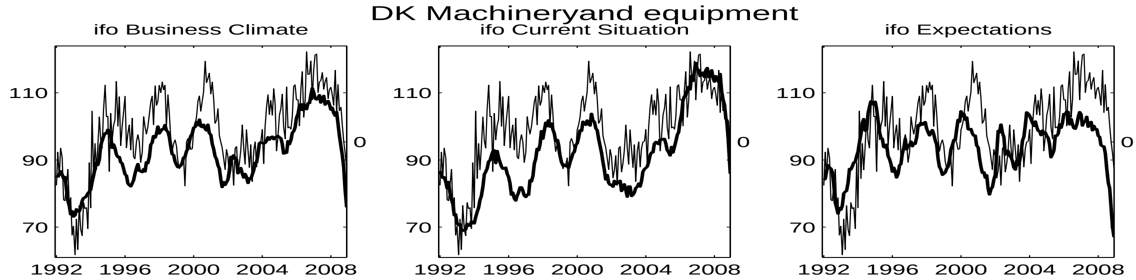

DG Chemie
ifo Business Climate
ifo Current Situation
ifo Expectations

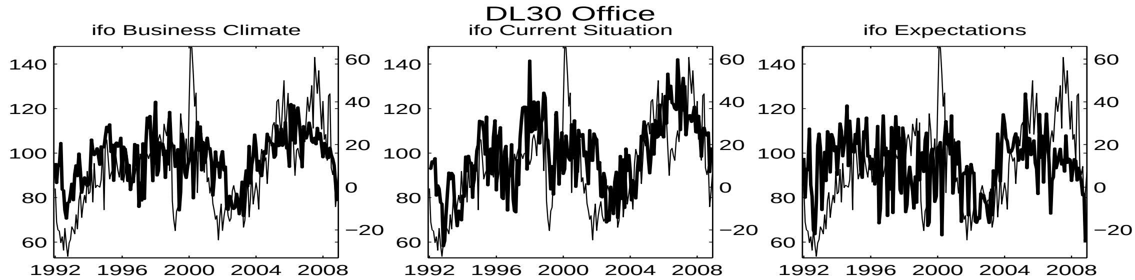

DH Rubber
ifo Business Climate
ifo Current Situation
ifo Expectations

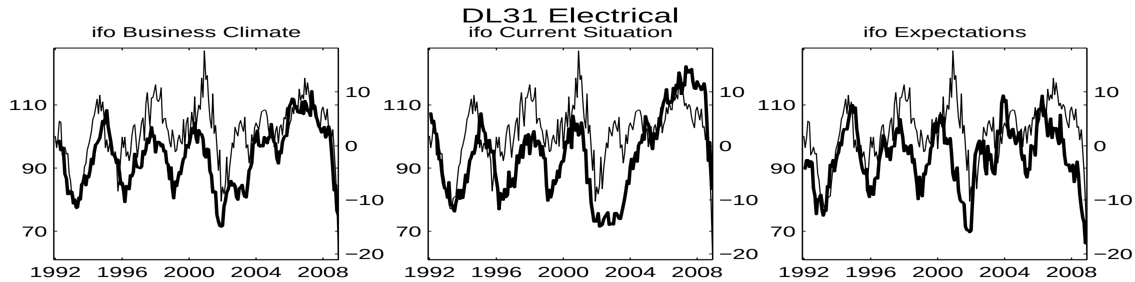

DI non−metallic
ifo Business Climate
ifo Current Situation
ifo Expectations

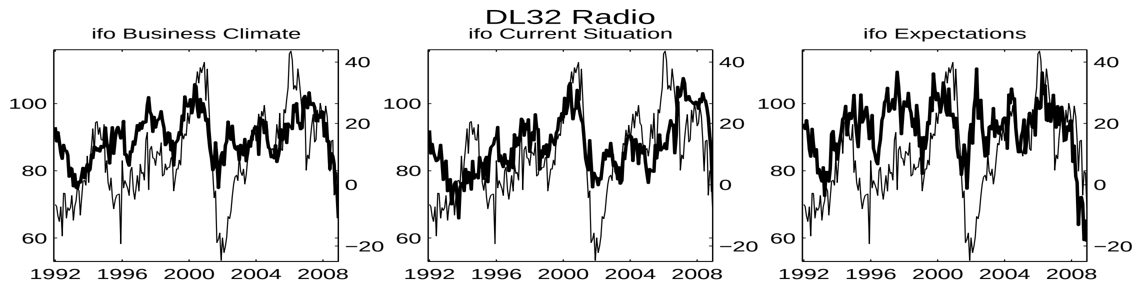

DJ27 Basic metals
ifo Business Climate
ifo Current Situation
ifo Expectations

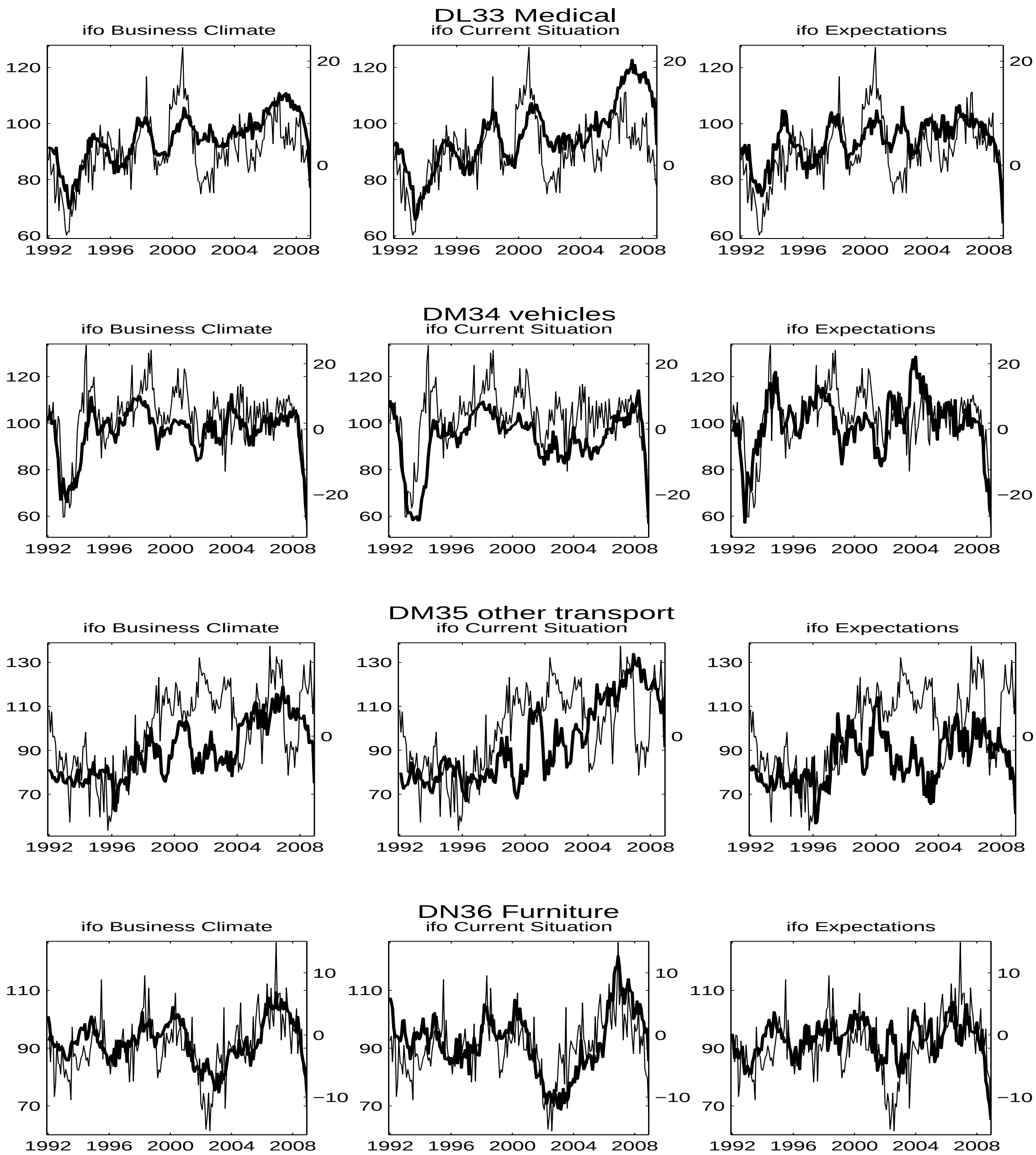

DJ28 Fabricated metals
ifo Business Climate
ifo Current Situation
ifo Expectations
110
90
70
0
1992 1996 2000 2004 2008


DK Machineryand equipment
ifo Business Climate
ifo Current Situation
ifo Expectations
110
90
70
0
1992 1996 2000 2004 2008


DL30 Office
ifo Business Climate
ifo Current Situation
ifo Expectations
140
120
100
80
60
60
40
20
0
−20
1992 1996 2000 2004 2008

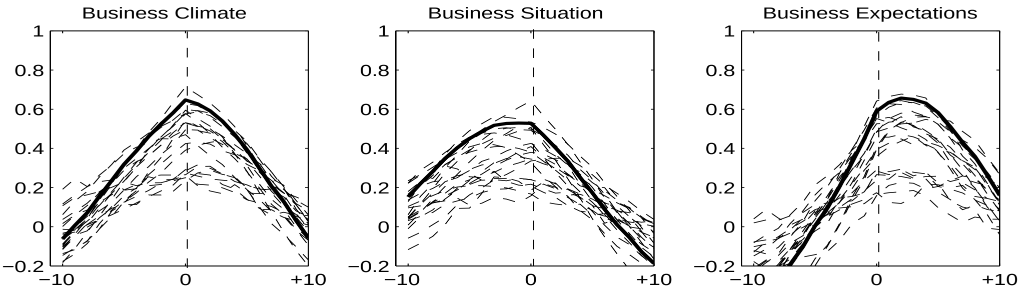

DL31 Electrical
ifo Business Climate
ifo Current Situation
ifo Expectations
110
90
70
10
0
−10
−20
1992 1996 2000 2004 2008


DL32 Radio
ifo Business Climate
ifo Current Situation
ifo Expectations
100
80
60
40
20
0
−20
1992 1996 2000 2004 2008



Notes: The figures show Ifo *Business Climate*, *Business Situation* and *Business Expectations* for the 22 branches of manufacturing industry (thick line, LS) against the annual growth rate of industrial production in the corresponding industry (thin line, RS).

1.C.2 Robustness Checks



Notes: The solid lines represents the cross–correlations of HP detrended IPI with the survey series for manufacturing industry and the dotted lines give the correlations to the 22 branch series.

Figure 1.16: Cross–Correlation of Ifo surveys and HP detrended IPI

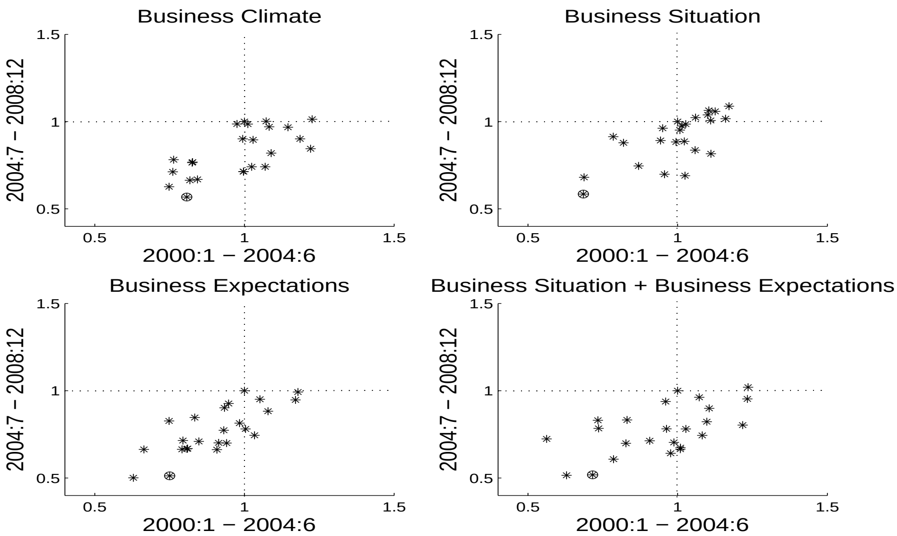


Figure 1.17: Scatter plot of relative MSEs against univariate benchmark model for annual growth of IPI ($h = 1$).

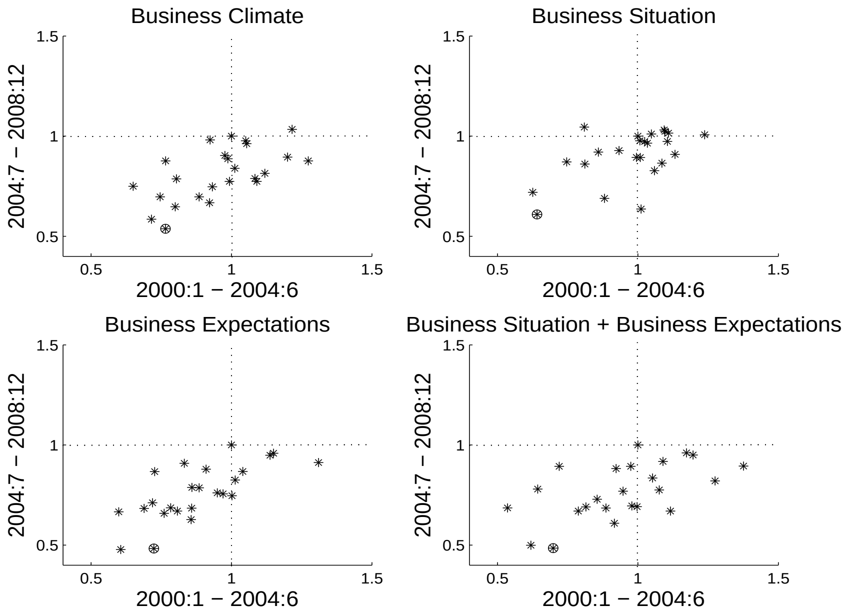


Figure 1.18: Scatter plot of relative MSEs against univariate benchmark model for annual growth of IPI ($h = 2$).

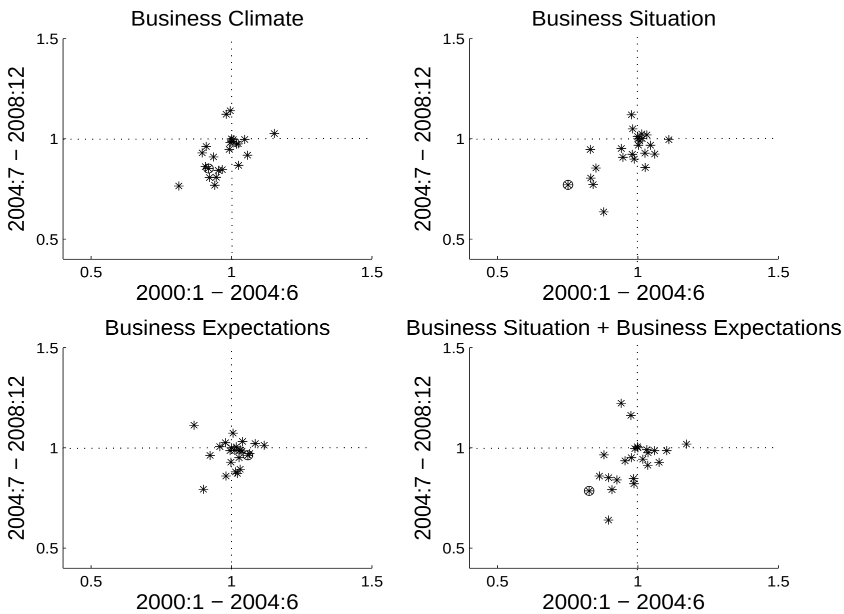


Figure 1.19: Scatter plot of relative MSEs against univariate benchmark model for month–on–month growth of IPI ($h = 0$).

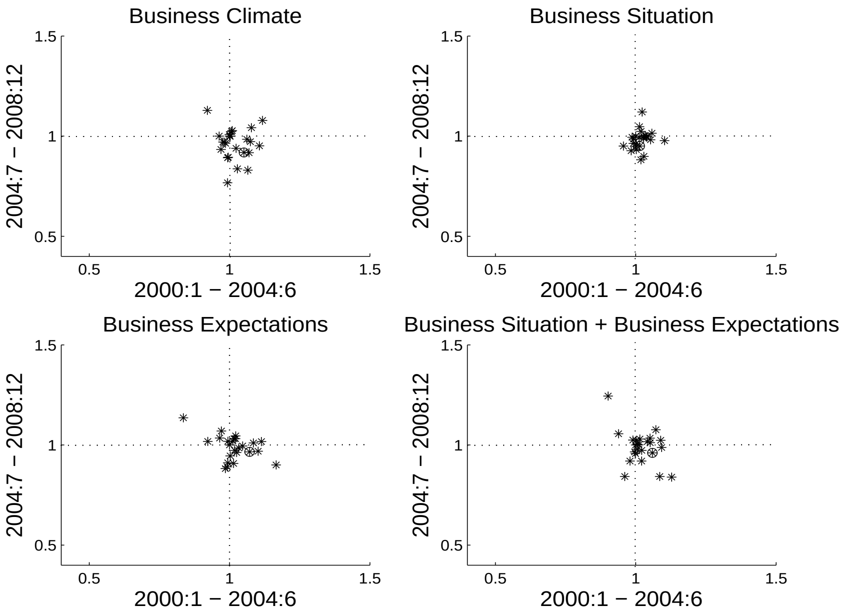


Figure 1.20: Scatter plot of relative MSEs against univariate benchmark model for month–on–month growth of IPI ($h = 1$).

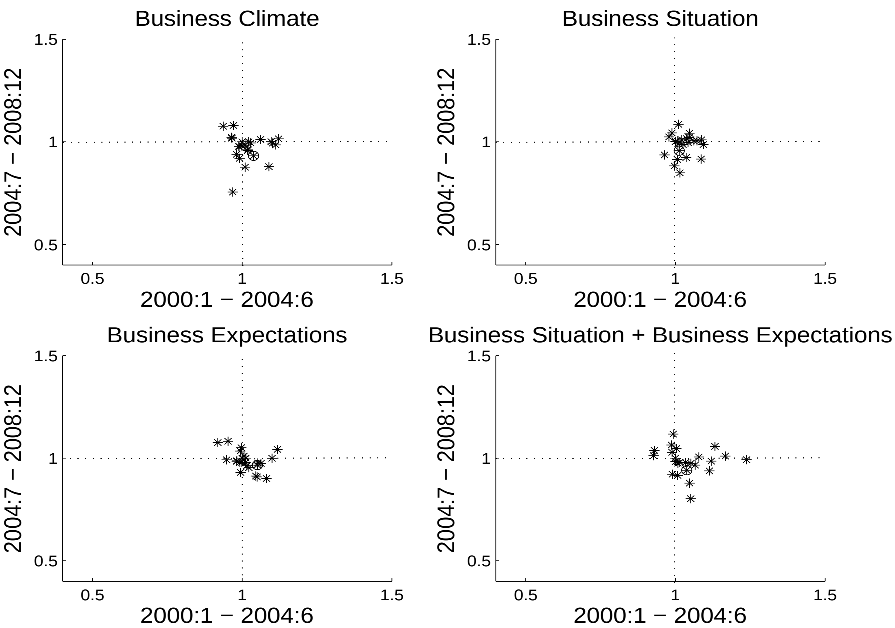


Figure 1.21: Scatter plot of relative MSEs against univariate benchmark model for month–on–month growth of IPI ($h = 2$).

	h=0				h=1				h=2			
	BC	BS	BE	BS/BE	BC	BS	BE	BS/BE	BC	BS	BE	BS/BE
AR	2.09	2.09	2.09	2.09	2.00	2.00	2.00	2.00	2.05	2.05	2.05	2.05
Manufacturing	0.88	0.76	1.00	0.80	0.98	0.98	1.01	1.00	0.98	0.98	1.00	0.98
	(1.67)	(2.01)	(0.76)	(1.87)	(0.97)	(1.16)	(0.41)	(0.80)	(1.03)	(1.16)	(0.71)	(1.01)
DA15	1.00	0.98	1.05	1.00	1.02	0.99	1.06	1.05	1.06	1.01	1.07	1.07
	(0.40)	(1.49)	(-0.96)	(0.56)	(0.50)	(1.16)	(-0.34)	(-0.40)	(-0.72)	(-0.77)	(-0.48)	(-0.64)
DA16	1.07	1.04	1.05	1.08	1.10	1.03	1.04	1.07	1.03	1.01	1.03	1.03
	(0.18)	(0.31)	(-0.96)	(-0.12)	(-0.48)	(0.52)	(-0.19)	(0.06)	(0.13)	(0.48)	(-0.03)	(0.26)
DB17	0.99	1.01	1.01	1.01	0.97	0.99	0.99	1.01	0.99	1.04	0.99	0.99
	(1.33)	(0.81)	(0.60)	(0.70)	(1.76)	(1.38)	(0.97)	(0.53)	(1.49)	(-1.17)	(1.26)	(0.99)
DB18	0.96	1.02	0.96	0.97	1.02	0.97	1.02	1.02	0.98	0.95	0.99	0.99
	(1.18)	(-1.38)	(1.18)	(0.90)	(-1.05)	(1.19)	(-1.54)	(-1.04)	(1.20)	(1.56)	(0.83)	(0.63)
DC	0.99	1.00	1.01	0.99	1.00	1.01	1.03	1.01	0.99	1.03	0.98	1.01
	(0.64)	(0.73)	(0.03)	(0.86)	(0.10)	(-1.04)	(-1.12)	(-0.47)	(0.61)	(-1.49)	(1.30)	(0.28)
DD	1.08	1.07	1.05	1.09	1.02	1.03	1.03	1.00	0.98	1.02	1.01	1.03
	(-0.37)	(0.17)	(-0.16)	(0.85)	(-0.43)	(0.06)	(0.29)	(1.43)	(1.18)	(0.13)	(-1.36)	(-0.02)
DE21	0.92	0.94	0.95	0.96	0.98	1.02	0.98	1.01	1.01	1.00	1.02	1.00
	(2.35)	(2.11)	(1.83)	(1.85)	(1.36)	(0.29)	(1.41)	(0.91)	(0.49)	(0.59)	(0.41)	(0.57)
DE22	1.02	1.02	1.00	1.00	0.87	0.94	1.02	0.96	0.84	0.92	0.98	0.91
	(-0.04)	(-2.10)	(0.48)	(0.35)	(2.53)	(2.10)	(0.60)	(1.41)	(2.33)	(1.99)	(1.10)	(2.01)
DF	0.93	0.98	1.01	0.96	0.94	0.96	1.00	0.98	1.00	1.01	1.01	0.99
	(1.51)	(1.81)	(0.06)	(1.29)	(1.43)	(2.25)	(0.69)	(1.41)	(0.44)	(-1.17)	(0.37)	(0.80)
DG	0.97	0.97	1.04	1.03	1.02	1.01	1.04	1.03	1.04	1.04	1.01	1.04
	(0.94)	(1.22)	(-0.83)	(0.07)	(0.01)	(0.12)	(-1.79)	(-0.42)	(-0.74)	(-1.67)	(0.02)	(-0.82)
DH	0.85	0.82	0.93	0.87	0.98	1.03	0.95	0.96	0.97	0.99	0.97	0.95
	(1.67)	(1.82)	(1.40)	(1.47)	(0.77)	(0.51)	(1.27)	(1.04)	(0.87)	(0.92)	(0.84)	(1.08)
DI	1.07	1.02	1.02	1.11	1.04	1.08	1.00	1.10	1.02	1.05	1.03	1.07
	(0.66)	(0.98)	(1.61)	(0.31)	(1.24)	(-0.78)	(1.81)	(0.88)	(1.64)	(-0.21)	(0.88)	(0.76)
DJ27	0.89	0.93	0.95	0.88	0.92	0.98	0.93	0.95	1.04	1.00	1.04	1.09
	(1.47)	(1.03)	(1.67)	(1.27)	(1.39)	(1.00)	(1.93)	(1.75)	(-0.19)	(0.56)	(1.17)	(0.59)
DJ28	0.83	0.73	0.91	0.74	0.93	0.96	0.93	0.89	1.04	1.03	0.99	1.01
	(1.57)	(1.79)	(1.59)	(1.80)	(1.39)	(1.14)	(1.61)	(1.59)	(-0.71)	(-1.76)	(1.03)	(0.45)
DK	0.86	0.85	0.93	0.87	0.94	0.95	0.95	0.95	0.93	0.95	0.97	0.95
	(1.77)	(2.08)	(1.36)	(1.64)	(1.49)	(1.77)	(1.18)	(1.49)	(1.42)	(1.65)	(1.06)	(1.45)
DL30	0.99	1.01	1.00	1.01	1.00	1.01	1.01	1.03	1.01	1.03	1.01	1.04
	(1.24)	(-3.02)	(0.26)	(-0.41)	(-0.04)	(-0.74)	(0.25)	(-0.24)	(-1.86)	(-1.05)	(-0.91)	(-2.12)
DL31	0.94	0.90	0.99	0.93	0.98	0.98	1.01	0.98	0.98	0.97	0.98	1.01
	(1.56)	(2.53)	(1.09)	(2.12)	(0.93)	(1.27)	(1.19)	(1.08)	(0.86)	(0.76)	(1.39)	(0.72)
DL32	0.89	0.92	0.99	0.90	1.02	1.00	1.02	1.03	0.99	1.02	0.99	0.99
	(1.64)	(1.10)	(1.18)	(1.41)	(-0.17)	(-0.19)	(0.20)	(-1.85)	(1.10)	(-0.76)	(0.98)	(1.02)
DL33	0.78	0.92	0.83	0.84	0.95	1.02	0.97	1.01	0.95	1.03	0.96	1.01
	(1.87)	(1.85)	(1.65)	(1.71)	(1.63)	(0.36)	(1.20)	(0.76)	(1.69)	(0.22)	(1.75)	(1.03)
DM34	0.88	0.80	1.00	0.86	0.97	0.95	1.02	0.99	0.96	0.93	0.98	0.95
	(1.62)	(1.64)	(0.92)	(1.57)	(1.29)	(1.60)	(-0.05)	(1.32)	(1.33)	(1.53)	(0.94)	(1.48)
DM35	0.99	0.95	1.01	0.94	1.06	1.02	1.03	1.04	1.03	1.02	1.01	1.09
	(0.94)	(1.67)	(0.76)	(1.68)	(-0.41)	(-1.73)	(0.49)	(-1.84)	(0.26)	(-0.79)	(1.05)	(-0.36)
DN36	0.92	1.00	0.98	0.98	0.97	1.01	0.99	0.99	1.00	1.00	0.97	0.98
	(1.89)	(1.30)	(1.00)	(1.29)	(1.45)	(0.71)	(0.97)	(1.23)	(0.96)	(0.93)	(1.22)	(1.49)

Notes: The first line reports the absolute MSE values for the univariate benchmark process. For the Ifo indices, we report the relative MSE values relative to the benchmark. The figures in parentheses represent the t–statistics of a one–sided test for predictive accuracy for nested models as proposed by Clark and West (2007). A t–statistic greater than +1.28 (10 percent significance level) or +1.65 (5 percent significance level) indicates that the unrestricted model which additionally contains Ifo survey series yields a significant smaller MSE than the autoregressive benchmark model. The standard errors are heteroscedastic and autocorrelation robust (Newey–West).

Table 1.6: Relative MSE of Ifo indices against AR benchmark for month–on–month growth of IPI

Pooling of Information (PI)	h=0	h=1	h=2	Pooling of forecasts (PF)	h=0	h=1	h=2
	BC	BC	BC		BC	BC	BC
Ex DA15 DA16	0.98	1.00	0.99	Mean	1.00	1.03	1.01
	(1.79)	(-0.05)	(0.65)		(0.06)	(-1.02)	(-0.21)
PC (r=1)	1.09	1.11	1.03	Median	1.00	1.03	1.01
	(-1.14)	(-1.40)	(-0.50)		(-0.01)	(-1.00)	(-0.22)
PC (r=2)	1.10	1.02	1.05	Mean trimmed 5 %	1.00	1.03	1.01
	(-1.39)	(-0.45)	(-0.89)		(0.04)	(-1.00)	(-0.20)
PC (r=3)	1.10	1.03	1.08	Mean trimmed 10 %	1.00	1.03	1.01
	(-1.49)	(-0.65)	(-1.47)		(0.02)	(-0.99)	(-0.20)
PC (r=4)	0.97	0.99	1.06	BMA K=1	1.00	1.03	1.01
	(0.68)	(0.26)	(-1.33)		(0.07)	(-1.01)	(-0.21)
PLS (r=1)	0.98	0.99	0.99	BMA K=2	1.00	1.03	1.01
	(0.91)	(0.68)	(0.59)		(0.07)	(-1.01)	(-0.21)
PLS (r=2)	1.02	1.13	1.04	BMA K=4	1.00	1.03	1.01
	(-0.32)	(-1.35)	(-0.55)		(0.09)	(-1.01)	(-0.21)
PLS (r=3)	0.98	1.10	1.05	BMA K=10	1.00	1.03	1.01
	(0.21)	(-1.61)	(-0.61)		(0.12)	(-1.00)	(-0.21)
PLS (r=4)	0.98	1.09	1.05	MSE K=1	1.00	1.03	1.01
	(0.17)	(-1.54)	(-0.64)		(0.10)	(-1.03)	(-0.21)
OPI ($\psi = 0$)	0.99	0.97	1.02	MSE K=2	1.00	1.03	1.01
	(0.30)	(0.70)	(-0.32)		(0.13)	(-1.04)	(-0.20)
OPI ($\psi = 10^0$)	0.99	0.94	1.04	MSE K=4	0.99	1.03	1.01
	(0.14)	(1.41)	(-0.59)		(0.20)	(-1.06)	(-0.18)
OPI ($\psi = 10^1$)	1.00	0.97	1.06	MSE K=10	0.99	1.03	1.00
	(0.00)	(1.12)	(-0.70)		(0.40)	(-1.11)	(-0.13)
OPI ($\psi = 10^2$)	0.96	0.97	1.08	Predictive Least Squares	1.00	1.03	1.01
	(1.11)	(1.68)	(-1.73)		(0.11)	(-1.00)	(-0.20)
OPI ($\psi = 10^3$)	0.99	0.98	1.01	Disc MSE (d=0.95, K=1)	1.00	1.02	1.01
	(1.27)	(1.57)	(-0.55)		(0.15)	(-0.99)	(-0.18)
OPI ($\psi = 10^4$)	1.00	0.99	1.00	Disc MSE (d=0.95, K=2)	0.99	1.02	1.01
	(0.12)	(1.33)	(0.25)		(0.23)	(-0.93)	(-0.15)
OPI ($\psi = 10^5$)	1.00	0.99	1.00	Disc MSE (d=0.95, K=4)	0.99	1.01	1.00
	(-0.16)	(1.25)	(0.35)		(0.38)	(-0.47)	(-0.10)
OPI ($\psi = 10^6$)	1.00	0.99	1.00	Disc MSE (d=0.95, K=10)	1.00	1.02	1.01
	(-0.19)	(1.24)	(0.36)		(0.12)	(-1.00)	(-0.18)
LASSO (r=1)	1.07	1.07	0.98	Disc MSE (d=0.90, K=1)	1.00	1.02	1.01
	(-1.07)	(-1.48)	(0.73)		(0.18)	(-0.97)	(-0.15)
LASSO (r=2)	1.00	1.06	0.98	Disc MSE (d=0.90, K=2)	0.99	1.02	1.00
	(-0.05)	(-1.35)	(0.83)		(0.29)	(-0.89)	(-0.10)
LASSO (r=4)	0.99	1.05	0.96	Disc MSE (d=0.90, K=4)	0.99	1.00	1.00
	(0.26)	(-1.28)	(1.46)		(0.44)	(-0.23)	(-0.04)
LASSO (r=6)	0.98	1.01	0.94	Disc MSE (d=0.90, K=10)	0.97	1.00	1.01
	(0.78)	(-0.52)	(2.06)		(0.75)	(0.11)	(-0.14)
LASSO (r=8)	0.97	1.01	0.94	Rank (K=1)	0.96	1.00	1.02
	(0.82)	(-0.49)	(2.11)		(0.81)	(0.08)	(-0.36)
LASSO (r=10)	0.98	1.02	0.93	Rank (K=2)	0.96	0.99	1.03
	(0.63)	(-0.69)	(2.04)		(0.79)	(0.15)	(-0.45)
				Rank (K=4)	0.96	0.99	1.04
					(0.78)	(0.19)	(-0.48)
				Rank (K=10)	0.95	1.04	1.05
					(0.84)	(-0.57)	(-0.91)

Notes: We report the relative MSE values for the PI and PF approaches compared to the economically weighted sectoral index as benchmark. The figures in parentheses represent the t–statistics of a one–sided test for predictive accuracy for non–nested models as proposed by Harvey et al. (1997). A t–statistic greater than +1.28 (10 percent significance level) or +1.65 (5 percent significance level) indicates that the tested approach yields a significant smaller MSE than the benchmark.

Table 1.7: Relative MSE of PI and PF approaches against economically weighted benchmark for month–on–month growth of IPI

Branches (two digits)	Relative Weights	Optimized Weights			
		h=0	h=1	h=2	5 % threshold
DA15 Food products and beverage	8.5	2.8	14.8	2.3	0.0
DA16 Tobacco Products	0.4	0.0	0.1	6.2	0.0
DB17 Textiles	1.6	0.4	4.7	4.0	0.0
DB18 Wearing apparel	1.0	3.6	0.1	2.2	0.0
DC Leather and Leather products	0.3	3.8	0.8	1.7	0.0
DD Wood and wood products	2.0	0.6	1.8	2.8	0.0
DE21 Pulp, paper and paper products	2.3	0.9	7.0	0.6	0.0
DE22 Publishing, printing and reproduction of recorded media	3.0	1.2	1.9	2.8	0.0
DF Coke, refines petroleum	0.6	1.0	1.8	9.2	0.0
DG Chemicals and chemical products	11.0	4.8	8.6	4.1	0.0
DH Rubber and plastic products	5.1	5.4	5.0	1.9	7.9
DI Non-metallic mineral products	4.8	2.1	3.6	3.5	0.0
DJ27 Basic metals	4.7	2.9	7.5	2.0	0.0
DJ28 Fabricated metals products	9.1	9.7	11.0	7.2	14.1
DK Machinery and equipment	15.1	12.9	14.1	16.6	18.8
DL30 Office machinery and computers	1.1	2.1	0.0	1.1	0.0
DL31 Electrical machinery	7.8	8.7	4.9	1.4	12.6
DL32 Radio, television and communication equipment	2.1	8.6	0.2	0.9	12.6
DL33 Medical, precision and optical instruments	3.2	8.0	4.0	0.5	11.7
DM34 Motor vehicles	12.3	15.4	4.9	16.0	22.4
DM35 Other transport	0.8	4.2	0.3	1.1	0.0
DN36 Furniture	3.1	0.8	3.0	12.0	0.0
relative MSE		0.92	1.02	0.99	

Notes: The reported figures are the mean weights attributed to the 22 branches in manufacturing by the OPI algorithm over 108 forecasts.

Table 1.8: Average OPI weights for month–on–month growth of IPI

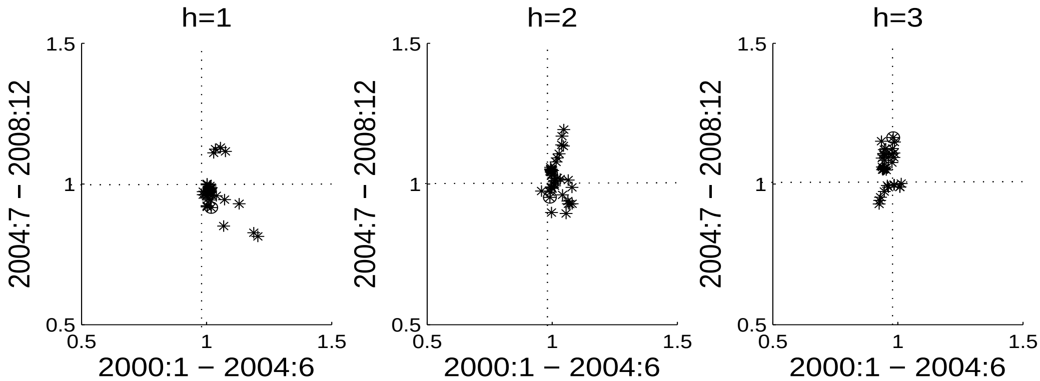
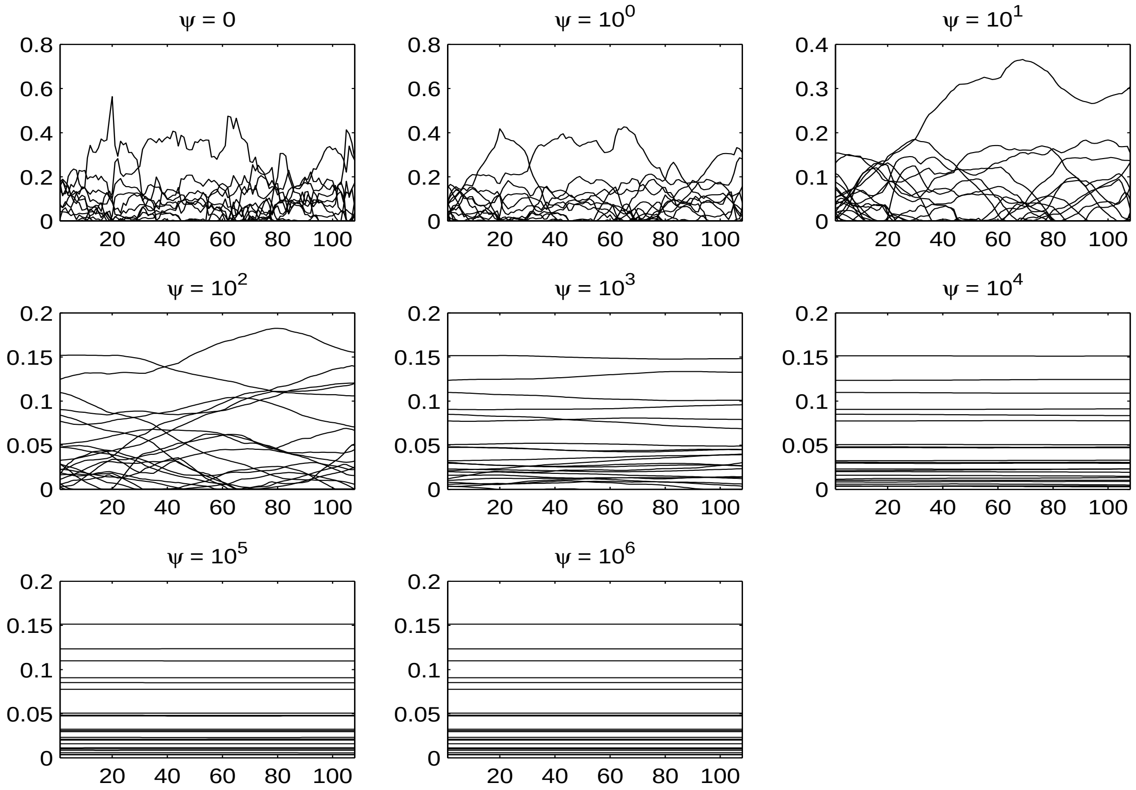


Figure 1.22: Scatter plot of relative MSEs of PI and PF against economic weighted benchmark for month–on–month growth of IPI (h=0)

Branches (two digits)	Relative Weights	Mean Coefficients		
		h=0	h=1	h=2
DA15 Food products and beverage	8.5	2.4	0.0	0.0
DA16 Tobacco Products	0.4	0.0	0.8	0.0
DB17 Textiles	1.6	0.4	1.1	0.0
DB18 Wearing apparel	1.0	0.0	0.0	0.0
DC Leather and Leather products	0.3	-0.2	0.0	0.0
DD Wood and wood products	2.0	0.0	0.0	0.0
DE21 Pulp, paper and paper products	2.3	7.2	4.0	0.0
DE22 Publishing, printing and reproduction of recorded media	3.0	0.1	0.0	1.0
DF Coke, refines petroleum	0.6	0.2	0.5	0.0
DG Chemicals and chemical products	11.0	-0.5	0.0	0.0
DH Rubber and plastic products	5.1	0.0	0.4	3.4
DI Non-metallic mineral products	4.8	0.0	0.0	0.9
DJ27 Basic metals	4.7	1.0	0.9	0.8
DJ28 Fabricated metals products	9.1	4.6	0.7	1.1
DK Machinery and equipment	15.1	0.0	5.8	10.7
DL30 Office machinery and computers	1.1	-0.7	0.0	0.0
DL31 Electrical machinery	7.8	0.0	0.0	0.0
DL32 Radio, television and communication equipment	2.1	0.0	0.0	0.0
DL33 Medical, precision and optical instruments	3.2	0.0	0.0	3.3
DM34 Motor vehicles	12.3	-0.4	0.0	0.4
DM35 Other transport	0.8	0.0	0.0	0.0
DN36 Furniture	3.1	0.4	2.3	0.0

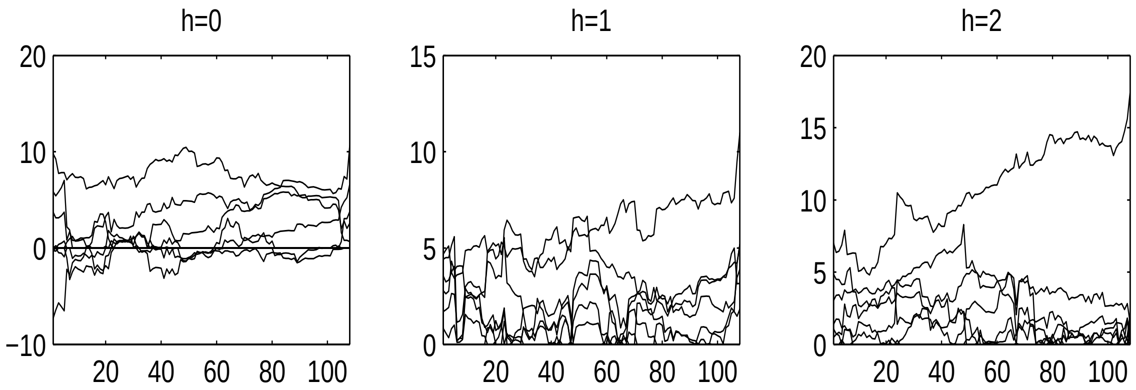
Notes: The reported figures are the mean coefficients for the 22 branches in manufacturing over 108 estimations of the lasso regression.

Table 1.9: Average lasso coefficients for month–on–month growth of IPI



Notes: The lines represent the weights attributed to the branch level indices at the different forecast origins $t = 1, \ldots, 108$ for $h = 0$.

Figure 1.23: OPI weights for month–on–month growth of IPI ($h = 0$)



Notes: The lines represent the coefficients attributed to the branch level indices at the different forecast origins $t = 1, \ldots, 108$.

Figure 1.24: Lasso coefficients for month–on–month growth of IPI (h=0)

1.C.3 PLS Algorithm

Create the matrices $\tilde{X} = X$ and $\tilde{Y} = Y$. $\tilde{X}$ and $\tilde{Y}$ are column centered and normalized. Starting the algorithm, the Y score vector u and the X weighting matrix w are initialized as $u = Y$ and $w_j = \tilde{X}'u$ and we set $i = j = 1$. The algorithm is given as:

1. Step $w = w_j$ (initialize the X weight vector)

2. Step $z = \tilde{X}w$ (compute the X score vector)

3. Step $c = \tilde{Y}'z$ (compute the Y loadings vector)

4. Step $u = \tilde{Y}c$ (compute the Y score vector)

5. Step $w_j = \tilde{X}'u$ (update the X weight matrix)

If $|w - w_j| > tol$, set $j = j+1$ and go back to 1.*Step*. Else, continue to 6.*Step* as the inner algorithm has converged and two new score vectors z and u have been found.

6. Step $p = (z'z)^{-1}z'\tilde{X}$ (compute the X loading vector from relationship $X = pz$)

7. Step $b = (z'z)^{-1}z'\tilde{Y}$ (compute the Y coefficients that explain Y as linear combination of the already found factors z $Y = bz$)

8. Step $\tilde{X}_i = \tilde{X} - zp'$ (adjust X for the already found factors)

9. Step $\tilde{Y}_i = \tilde{Y} - zbc'$ (adjust Y for the already found factors)

If $|\tilde{Y}_i| > tol$, set $i = i+1$ and set $\tilde{Y} = \tilde{Y}_i$ and $\tilde{X} = \tilde{X}_i$ and go back to *Step* 1. Else, the outer algorithm has converged and all score vectors have been found.

The dependent variables Y can be predicted using the multivariate regression formula as $\hat{Y} = CBZ = XB_{PLS}$ with $B_{PLS} = P'_+BC$ where P'_+ is the Moore–Penrose pseudo–inverse of P'.

Chapter 2

Forecasting Euro Area Real GDP: Optimal Pooling of Information

This paper proposes a new method of forecasting euro area quarterly real GDP that uses area–wide indicators, which are derived by optimally pooling the information contained in national indicator series. Following the ideas of predictive modeling, we construct the area–wide indicators by utilizing weights that minimize the variance of the out–of–sample forecast errors of the area–wide target variable. In an out–of–sample forecast experiment we find that our optimal pooling of information (OPI) approach outperforms alternative forecasting methods in terms of forecast accuracy.

2.1 Introduction

Since Eurostat publishes the first official release of euro area quarterly real GDP several weeks after the end of each quarter, an early assessment of the actual state of the economy is appreciable. Timely information is contained in business cycle indicators – e.g. industrial production, confidence surveys or composite indicators – that are more promptly available. Forecasts of euro area quarterly real GDP are frequently derived by means of bridge models that explicitly incorporate such business cycle indicators.

In the euro area, business cycle indicators are typically collected at a national level by national statistical agencies or national survey institutes. In such a data–rich environment professional forecasters who aim at predicting euro area quarterly real GDP, can choose between two forecast strategies: pooling of forecasts and pooling of information. In the case of predicting euro area wide aggregates, pooling of forecasts uses national indicator series as predictors in the bridge model equations. One strategy is to generate a number of forecasts of euro area real GDP growth rates by employing various parsimonious models and to combine them to a single forecast of the area–wide target variable. The optimal weighting scheme thereby takes the correlations of the forecast errors of each model into account. Alternatively, real GDP growth rates of each euro area member country can be forecasted separately and then be aggregated to a single euro area real GDP growth rate by using the relative economic weight of each member country as proposed by Marcellino et al. (2003).

Pooling of information generates a projection of euro area real GDP growth rates by using area–wide indicators as predictors that combine the information of the national indicators. Thus, the number of regressions is reduced to one. The simplest strategy is to employ area–wide indicators which are provided by Eurostat or other institutions – e.g. the European Commission or the OECD – and which are economically weighted averages of national indicators. Alternatively, professional forecasters might combine the set of national information by extracting common dynamic factors or principal components (see e.g. Forni et al. (2000) and Stock and

Watson (2002b)).

This paper proposes the OPI algorithm as a new method of forecasting euro area quarterly real GDP that uses area–wide indicators, which are derived by optimally pooling the information contained in national indicator series. Following the ideas of predictive modeling, we construct the area–wide indicators by utilizing weights that minimize the MSE of the aggregate target variable. By allowing a pre–aggregation of individual information to national indicator series, the optimal pooling of information problem is reduced to a manageable number of variables, which avoids the construction of a “super model” (Timmermann, 2006) whose computation is often deemed to be prohibitively costly or even impossible.

To evaluate the forecast performance of the OPI approach for the euro area, we focus on three business cycle indicators, which are all available at both the area–wide and the national level: the Industrial Production Index (IPI), the Economic Sentiment Indicator (ESI) of the European Commission and the CESifo World Economic Survey (WES) indicator for the euro area. The forecast models are specified as Autoregressive Distributed Lag (ADL) models, which are estimated by employing a model averaging strategy in order to reduce the problems associated with selecting a certain lag length. In a first step, we evaluate the potential gain of the OPI approach in a forecast exercise using ex ante information. Our main result is that optimally pooled area–wide indicators potentially reduce the out–of–sample mean squared forecast errors for euro area quarterly real GDP growth by 25% on average compared to economically weighted indicators. An analysis of the optimal weighting schemes shows that only a limited number of national indicators is attributed a weight larger than zero. As we find that the optimal weights derived from shorter optimization windows significantly vary over time, we introduce a stabilization factor to make our approach feasible in practice.

In a second step, we evaluate the applicability of the OPI approach in real–time by employing a pseudo out–of–sample forecast experiment, in which optimally pooled area–wide indicators are computed using only ex–post information that would have been available in real–time. The optimized weights are derived from a recursively growing optimization window, which is then excluded from the forecast evaluation process. The performance of OPI is compared to a number of alternative forecast methods, which include pooling of forecast and competing pooling of information strategies. We find that the OPI approach generally outperforms the alternative forecast methods in terms of forecast accuracy as measured by the out–of–sample forecast MSE.

The remainder of the paper is structured as follows. Section 2.2 reviews the traditional forecast strategies. In Section 2.3 we introduce our OPI approach. In Section 2.4 we present the forecast experiment. We describe the forecast models applied, introduce the data set and discuss the empirical results, which refer to (i) the use of ex–ante information and (ii) to the use of ex–post information. Section 2.5 summarizes and concludes.

2.2 Review of Traditional Forecast Strategies

For an overview of the traditional forecast strategies we introduce the following notations. Suppose we forecast the aggregate target variable Y_t – i.e. euro area quarterly real GDP growth – using a broad set of disaggregate information variables, denoted by $X_{i,t}$, where t is time and i refers to the disaggregate unit, i.e. the member states of the currency area. The number of disaggregate units is given by K. The data sample that is available for the forecast experiment ranges from $t = 1, \ldots, \Theta_2$. The forecast model is estimated recursively over the estimation window $[1, T]$, with T gradually increasing from Θ_0 to $\Theta_2 - 1$, where $1 < \Theta_0 < \Theta_2 - 1$.

The current quarter forecasts of the area–wide target variable, denoted by $\widehat{Y}_{T+1|T+1}$, are computed for $T+1$ using the national information already available at $T+1$.[1] As T increases from Θ_0 to $\Theta_2 - 1$, the maximum number of out–of–sample forecasts is given by $\Theta_2 - \Theta_0$. In the first part of our experiment, the performance of the different forecast strategies is evaluated by computing the MSE for each model over the total out–of–sample window $[\Theta_0 + 1, \Theta_2]$ on the basis of the out–of–sample forecast errors $\widehat{\varepsilon}_{T+1|T+1} = Y_{T+1} - \widehat{Y}_{T+1|T+1}$. In the second part of our experiment, we mimic a

[1]Since in our set–up the current quarter of the target is predicted, the literature often uses the notion “nowcast” instead of forecast (Domenico et al. (2006)).

forecast situation in real time and hence optimize the different forecast models only on basis of ex post information that would have been available to the forecaster. We thus separate a so–called optimization window of size $[T+1, \Theta_1]$ such that the evaluation period is reduced to $[\Theta_1+1, \Theta_2]$.

Figure 2.1 summarizes the time structure of the estimation, optimization and forecast evaluation windows.

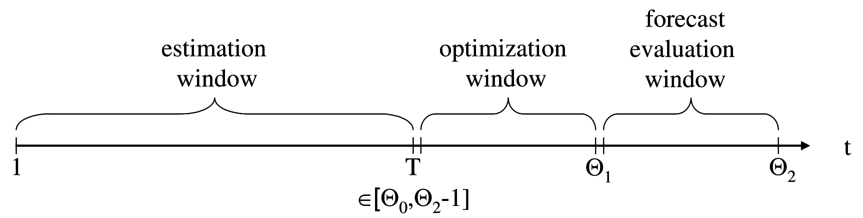


Figure 2.1: Time structure of the estimation and forecasting procedures I

Notice that in the following we use a static structure of the forecasting models to keep the review as simple as possible. Later in the empirical part of the paper, we allow for more dynamics.

2.2.1 Pooling of Forecasts

Pooling of forecasts summarizes the combination of two or more individual forecasts to generate one single, pooled forecast. The idea of improving the accuracy of predictions regarding a certain target variable by combining the forecasts of different models was first proposed by Bates and Granger (1969) and mainly follows the ideas of portfolio optimization and diversification gains. A large number of theoretical and empirical studies – see e.g. Timmermann (2006) and Stock and Watson (2004) – have shown the superiority of combined model based predictions compared to individual models.[2]

In the context of forecasting euro area quarterly real GDP, three strategies have been

[2] For a more detailed description of different pooling of forecast strategies, see Section 1.3.5 in Chapter 1.

proposed for combining single forecasts, which are derived from national indicator series using a multiple equation set–up. The crucial issue in all strategies is the determination of an adequate weighting scheme.

2.2.1.1 Optimal Combination of Area–Wide GDP Forecasts

In the first strategy the following forecasting model is estimated for each of the K national indicators over the period $t = 1, ..., T$:

$$Y_t = \delta + c_i X_{i,t} + \varepsilon_{i,t}, \tag{2.1}$$

where δ is a constant term, c_i denote parameter matrices and $\varepsilon_{i,t}$ are error terms. The K forecasts resulting from the models are then linearly combined to a single forecast for the area–wide target variable according to:

$$\widehat{Y}_{T+1|T+1} = \sum_{i=1}^{K} \omega_i \widehat{Y}^i_{T+1|T+1}, \tag{2.2}$$

where the superscript i attached to $\widehat{Y}_{T+1|T+1}$ denotes the forecast of the area–wide target variable obtained from the model using the national indicator $X_{i,t}$.

The optimal weights ω_i of the single forecasts, and hence the weights attributed to each model, depend on the model's out–of–sample performance. Under the assumption that the forecasts are unconditionally unbiased, the $\Theta_2 - \Theta_0$ out–of–sample forecast errors of model i, $\widehat{\varepsilon}^i_{T+1|T+1} = Y_{T+1} - \widehat{Y}^i_{T+1|T+1}$ with $T = \Theta_0, \ldots, \Theta_2 - 1$, are distributed around zero with variance σ_i^2 and covariance $\rho_{ij}\sigma_i\sigma_j$ for $j = 1, ..., K$, where ρ_{ij} denotes the correlation coefficient of the forecast errors from the forecast models i and j. Defining $\boldsymbol{\omega}$ as the $K \times 1$ vector containing the weights of each model and $\boldsymbol{\Sigma}_{\widehat{\varepsilon}}$ as the $K \times K$ variance–covariance matrix of the out–of–sample forecast errors, the optimal weights are obtained from minimizing the variance of the combined out–of–sample forecast error:

$$\boldsymbol{\omega}^{opt} = \arg\min_{\omega} \left[\boldsymbol{\omega}' \boldsymbol{\Sigma}_{\widehat{\varepsilon}} \boldsymbol{\omega} \right], \tag{2.3}$$

which gives:

$$\boldsymbol{\omega}^{opt} = \frac{\boldsymbol{\Sigma}^{-1}_{\widehat{\varepsilon}} \mathbf{1}_N}{\mathbf{1}'_N \boldsymbol{\Sigma}^{-1}_{\widehat{\varepsilon}} \mathbf{1}_N}. \tag{2.4}$$

A major benefit of the combination of forecasts approach is the possibility of including a large number of candidate regressors in forecasting a certain target series without running into the problem of overparametrization or overfitting. However, as the data generating process is typically unknown, the need to specify a large number of parsimonious regression models may lead to high specification errors (Lütkepohl, 1987). The major challenge of the approach is the estimation of the variance–covariance matrix $\mathbf{\Sigma}_{\widehat{\varepsilon}}$. Assuming linear relationships, the optimal weights can be estimated by ordinary least squares, regressing realizations of the target variable Y_t on the K–vector of forecasts $\widehat{Y}^i_{T+1|T+1}$ and the constant term (Granger and Ramanathan, 1984). However, for the computation of the optimal weights problems arise if the number of models K becomes too large.

2.2.1.2 Equally Weighted Combination of Area–Wide GDP Forecasts

A simplification of the optimal combination approach is the use of equal weights, which particularly solves the computation problem. Concerning the forecast performance of equally weighted combinations, Timmermann (2006) derives conditions, under which the simple average of a number of forecasts outperforms single model based forecasts as well as more elaborated weighting schemes. Among others, Stock and Watson (2004) provide evidence for the superiority of the equal weighting scheme in a broad empirical application, thereby confirming the so–called *forecast combination puzzle.*

2.2.1.3 Aggregation of National GDP Forecasts

Following Marcellino et al. (2003) the third strategy is to aggregate national real GDP forecasts to a single euro area real GDP forecast. The forecast model is estimated for each member country of the monetary union $i = 1, \ldots, K$ over the period $t = 1, \ldots, T$:

$$Y_{i,t} = \delta_i + c_i X_{i,t} + \varepsilon_{i,t}, \tag{2.5}$$

and forecasts of euro area real GDP growth are generated by computing weighted averages of the national predictions:

$$\widehat{Y}_{T+1|T+1} = \sum_{i=1}^{K} \omega_i \widehat{Y}_{i,T+1|T+1} = \sum_{i=1}^{K} \omega_i \widehat{\delta}_i + \sum_{i=1}^{K} \omega_i \widehat{c}_i X_{i,T+1}. \tag{2.6}$$

The economic weights ω_i reflect the relative importance of country i in the monetary union (e.g. GDP shares).

In contrast to the optimal combination approach, the weighting of national information $X_{i,t}$ is not derived from the minimization of the variance of the out–of–sample forecast error, but is influenced by both, the in–sample fit of the disaggregate model for country i and the economic weight of country i (see equation (2.6)). As before, the approach hardly suffers from the problem of overfitting. However, due to the need to specify of a large number of parsimonious models, it faces the drawback of larger specification errors when the data generating processes are unknown.

2.2.2 Pooling of Information

Pooling of information generates a projection of euro area quarterly real GDP by using area–wide indicators as predictors that combine all national information. In contrast to the multi–equation approaches of forecast pooling, pooling of information reduces the number of regressions to one and – as a consequence – the problem of running into specification errors is reduced. The crucial issue of the pooling of information approach is again the weighting scheme applied to derive area–wide indicators from the national indicator series.[3]

2.2.2.1 Economic Weights

A straightforward strategy is to use area–wide indicators officially provided by statistical agencies as regressors of the forecasting model:

$$Y_t = \delta + cX_t + \varepsilon_t, \tag{2.7}$$

[3] For a more detailed description of different pooling of information strategies, see Section 1.3.4 in Chapter 1.

where $t = 1, \ldots, T$. The area–wide indicator X_t is computed as a weighted average of national indicators:

$$X_t = \sum_{i=1}^{K} \omega_i X_{i,t}, \tag{2.8}$$

where the ω_i's typically reflect country i's relative economic weight in the currency area.

Employing economic weights to construct a single aggregate indicator series implies that these weights are exogenously given. Thus, any correlation between the national indicator series is ignored. Furthermore, the approach does not take into account any correlations between the resulting indicator series and the area–wide target variable.

2.2.2.2 OLS Weights

The use of OLS weights circumvents this drawback. Estimating the forecast model:

$$Y_t = \delta + \sum_{i=1}^{K} c_i X_{i,t} + \varepsilon_t \tag{2.9}$$

over the period $t = 1, \ldots, T$, the weighting of national information is given by the point estimates for c_i, which are derived from the minimization of the in–sample residuals. Thus, the in–sample fit of this approach with respect to the aggregate target variable must be superior to a multiple equation approach (see Section 2.2.1.3). The problem of this approach is, however, that with an increasing number of disaggregate information variables K, the regression model more likely suffers from overfitting. As overparametrization leads to higher estimation uncertainty in finite samples, the out–of–sample performance of the OLS weighting approach is likely to worsen.

2.2.2.3 Factor Models

The use of factor models attempts to mitigate the problem of parameter proliferation. While the forecasting model has the same structure as in equation (2.7), it is preceded by a factor model that pools disaggregate information over the estimation window $[1, T]$ to a common factor X_t, which is used to forecast the target variable Y_t.

The intuition behind factor models in the context of macroeconomic forecasting is that the co–movement in economic time series, in our case the co–movement in the national indicator series, is arising largely from a small set of common factors or even from a single common factor. A number of estimation techniques have been applied in the literature. The simplest method of constructing latent factors proposed by Stock and Watson (2002b) is the static principal components analysis (PC). In our case, the single common factor thereby corresponds to the first principal component, which accounts for as much of the variability in the disaggregate indicators as possible. The weights ω_i are the squared elements of the eigenvector, which is associated with the first principal component. If the resulting common factor explains a large part of the variance of $X_{i,t}$, then $X_{i,t}$ is attributed a high weight.

In the context of business cycle analysis a useful extension of the static version of the factor model is the generalized dynamic factor model of Forni et al. (2000), which takes into account phase differences between disaggregate indicator time series by appropriately weighting leading and lagging variables. The advantage of factor models is that information of a possibly large set of indicators is pooled by taking into account the in–sample covariances between the candidate regressors. The main drawback of common factor models is that the construction of the common factor ignores any correlation between the disaggregate indicator variables X_t and the area–wide target series Y_t. Thus, the weighting of national information only reflects in–sample correlation patterns between the national indicators and is independent of the forecasting model.

2.3 Optimal Pooling of Information

The OPI approach forecasts euro area quarterly real GDP by area–wide indicators that are constructed from national indicator series using optimal weights, which minimize the MSE of the area–wide target variable.[4] The procedure involves a non–

[4]For a more detailed description of the algorithm, see Section 1.3.4.6 in Chapter 1.

linear numerical optimization routine, which accounts for correlations between both, the national indicator series and the area–wide target series.

The determination of the optimal weights includes the following steps. We begin with an initial guess for the weights $\boldsymbol{\omega} = (\omega_1, ..., \omega_K)^{'}$. We then compute the area–wide indicator X_t according to equation (2.8) and estimate equation (2.7) over the period $t = 1, ..., T$. Finally, we compute the out–of–sample forecasts:

$$\widehat{Y}_{T+1|T+1}(\boldsymbol{\omega}) = \widehat{\delta} + \widehat{c}\sum_{i=1}^{K} \omega_i X_{i,T+1|T+1} = \widehat{\delta} + \widehat{c}\boldsymbol{X}_{T+1|T+1}^{'}\boldsymbol{\omega}, \tag{2.10}$$

where $\boldsymbol{X}_{T+1|T+1}$ is a $K \times 1$ vector containing the national indicators at time $T + 1$. The optimal weights then result from a minimization problem given as:

$$\begin{aligned} \boldsymbol{\omega}^{opt} \quad &= \quad \arg\min_{\omega} \left[\frac{1}{N} \sum_{\tau=T+1}^{\Theta_1} (\hat{Y}_{\tau,\tau} - Y_\tau)^2 + \ell_t \right] \\ s.t. \quad & \sum_{i=1}^{K} \omega_i = 1 \\ & \omega_i \geqq 0 \quad \forall i \end{aligned} \tag{2.11}$$

where

$$\ell_t = \psi \frac{1}{K} \sum_{i=1}^{K} (\omega_{i,\Theta_1} - \omega_{i,\Theta_1 - 1})^2. \tag{2.12}$$

and Θ_1 marks the end of the window used to optimize the weights.[5]

Following the idea of sparse and stable portfolio optimization, we regularize our objective function by restricting the weights to be non–negative and to sum up to unity. Additionally, we introduce a penalty term ℓ_t for any deviation of the current weights from previous weights. ψ is a parameter that allows us to adjust the relative importance of the penalization in our optimization. Attributing large values to ψ corresponds to stabilizing the weights as any deviation of past weights is penalized

[5]Note: We use ex ante information to optimize the weights in the first part of the empirical experiment such that $\Theta_1 = \Theta_2$.

more heavily. For $\psi \to \infty$, the algorithm yields constant weights, determined by the starting values.

Introducing the penalty term leads to a stabilization of the optimization problem and promotes sparse portfolios by attributing a weight of zero to a number of national indicators.

The main advantage of the optimal pooling of information approach is that it takes into account correlations between both, predictors as well as predictors and the target variable. In contrast to other pooling of information strategies these correlations refer to the out–of–sample performance of the forecast model. Thus, as a way of predictive modeling optimal pooling of information poses a way to handle the bias–variance trade–off that typically appears when specifying a forecast model. A major drawback of the approach is that the computation of optimal weights may become difficult with an increasing number of disaggregate information K. One way to circumvent the construction of such a “super model” (Timmermann, 2006) is to pre–aggregate individual information to national indicator series, which reduces the optimal pooling of information problem to a manageable number of variables.[6]

2.4 Empirical Results

2.4.1 Forecast Model Specification

Following Banerjee et al. (2005), we generate forecasts of euro area quarterly real GDP by using bridge models that are specified as:

$$A(L)Y_t = \delta + B(L)X_t + \varepsilon_t, \tag{2.13}$$

where Y_t denotes real GDP expressed in quarterly growth rates, δ is a constant term, X_t describes the quarterly values of a business cycle indicator, $A(L)$ and $B(L)$ are lag polynomials and ε_t denotes the error terms.[7] Quarterly projections of

[6]In case of consumer or business surveys, the number of disaggregate information K can be very large as the approach could in principle be tracked down to the level of single survey respondents.

[7]Notice that in cases where national information enters the bridge model equation (2.13) and/or if national real GDP growth rates are used as dependent variables, the following model specification applies: $A(L)Y_{i,t} = \delta + B(L)X_{i,t} + \varepsilon_{i,t}$.

real GDP growth are derived by exploiting the timely information contained in the contemporaneous business cycle indicator in addition to the information provided by past realizations.

An important issue in specifying bridge models for forecasting purposes is the choice of the number of lags of the endogenous and exogenous variables included. Since traditional lag selection approaches – such as in–sample and out–of–sample criteria – suffer from shortcomings, e.g. problems of overparametrization or the use of ex post information that would not have been available in real time, we do not restrict the model specifications to a certain lag length but implement a model averaging strategy that allows us to consider different lag orders. Accordingly, we follow the notion that it is a priori impossible to discard a certain lag order from the forecasting exercise. We derive forecasts from a business cycle indicator within each forecast model by considering a certain maximum number of lags of the exogenous and endogenous variables. The different model specifications are built by permutating the candidate regressors and imposing the restriction that the contemporaneous value of the business–cycle indicator forms part of each model.[8] One–step–ahead forecasts from every model specification are computed. Since simple pooling schemes perform comparably well (see e.g. Timmermann, 2005, and Stock and Watson, 2004), the forecasts are then combined using equal weights.

2.4.2 Data Set

Our data set includes real GDP and several business cycle indicators. The data is collected for both, the euro area and the member states, over the period from 1990Q1 to 2008Q4. Real GDP is taken from the OECD's Main Economic Indicators Original Release Data and Revisions Database that comprises vintage data, which is published each month since January 2000. Since the availability of real GDP for the euro area member states is limited in the sample period under consideration, we focus on a subset of member states – as summarized in Table 2.1 – that cover

[8]In the following we specify the forecasting models with a maximum lag length of two, which means that we obtain 16 different model specifications for each business–cycle indicator.

almost 95% of area–wide economic activity.[9]

Table 2.1: Selected euro area member states

Rank	Country	Share of GDP in %
1.	Germany	26.9
2.	France	21.1
3.	Italy	17.0
4.	Spain	11.9
5.	Netherlands	6.3
6.	Belgium	3.8
7.	Austria	3.1
8.	Finland	2.0
9.	Portugal	1.8

Notes: Euro area member states considered. Share of country GDP in area–wide economic activity evaluated in the year 2008.

In order to get a balanced panel of real GDP data, missing values for the period from 1990Q1 to 1994Q4 were completed with real GDP data from the first vintage available of the OECD database. Vintages of real GDP that exhibit a seasonal pattern are seasonally adjusted by means of Census X–12. All series are converted into quarterly growth rates to satisfy stationarity conditions.

For a business cycle indicator to be selected the following criteria had to be met: (1) It is published both at the area–wide and at the national level. (2) It is a leading or a coincident indicator of economic activity and therefore suited to forecast real GDP growth. (3) The indicator is published quarterly or at a higher frequency. (4) It covers a sufficient time span, starting at least in 1990. (5) It is either not revised or vintage data is available covering the total time span. Keeping these guidelines in mind, we end up with three business cycle indicators, namely the Industrial Production Index (IPI), the Economic Sentiment Indicator (ESI) of the European Commission and the CESifo World Economic Survey (WES).

The IPI provides a measure of the volume of value added generated by production

[9]Real time data for Ireland, Luxembourg and Greece starts considerably later in the OECD database such that we excluded these countries – as well as the new member states – from our data set.

units classified under the industrial sectors, i.e. C (mining), D (manufacturing) and E (electricity, gas and water) of the International Standard Industrial Classification of all Economic Activities (ISIC Rev.3). It is released on a monthly basis so that the quarterly value is derived from the monthly average. In the euro area data are collected by the national statistical offices and aggregated by Eurostat to an area–wide index. The country weights used for the aggregation are value added at factor costs; they are revised every five years (Eurostat, 2006). As the indicator is subject to data revisions, vintage data is provided by the OECD's Main Economic Indicators Original Release Data and Revisions Database from 1990 onwards.

The ESI combines the weighted information contained in confidence indicators of different sectors – namely industry, services, construction, retail trade and consumers – that are in turn constructed from survey data. Since the indicator is published on a monthly basis, the quarterly value is computed as an average of the monthly releases within the survey quarter. The ESI is built in two steps. In a first step, the area–wide confidence indicators of each sector are derived by aggregating the individual country sector confidence indicators. The weights are the shares of each of the member states in an area–wide reference series – here GDP growth – and are smoothed by calculating a two year moving average. In a second step, the area–wide confidence indicators are combined by using survey weights, which are based on two criteria: (i) the importance of the corresponding sector in the overall economy, and (ii) the ability of tracking the movements of the reference series (European Commission, 2007).

Finally, the WES summarizes the judgement of economic experts about the economic situation of the country they inhabit by revealing their appraisals and expectations. It is exclusively based on qualitative information and is timely released within the survey quarter on a quarterly basis. The WES is collected for each member state of the euro area, whereby the aggregate area–wide index is calculated as a weighted average of the individual country indices. The weighting scheme adopted refers to the share of a single country in total world trade (Stangl, 2007).[10]

[10] The calculation of the national trade volumes is based on the foreign trade statistic published by the United Nations. The weighting scheme is readjusted once a year.

2.4.3 Forecast Experiment Using Ex–Ante Information

We generate forecasts of euro area quarterly real GDP by estimating bridge models for each business cycle indicator recursively. We focus on the entire forecast evaluation window that ranges from Θ_0 = 1999Q4 to Θ_2 = 2008Q4. The projections are derived as nowcasts for every quarter following the end of the estimation window T, which is gradually extended from 1999Q3 to 2008Q3.[11]

Since we seek to evaluate the full forecast potential of the optimal pooling of information approach, the computation of the optimal weights draws on the 37 out–of–sample forecast errors of the entire forecast evaluation window. As the forecast evaluation window and the optimization window coincide, we explicitly use so–called ex–ante information to optimize the weights, which means that we use information that would not have been available in real time.[12] Notwithstanding this analysis allows us to gain an insight into the composition of the weighting schemes that result from the optimization algorithm.

Table 2.2: Forecast performance of optimally pooled indicators relative to economically weighted indicators

		MSE ratio	HLN p–value
Industrial Production	IPI	0.75	0.03
Economic sentiment	ESI	0.69	0.04
CESifo Economic Climate	WES	0.82	0.10

Notes: The MSE ratios are calculated as the MSE resulting from optimally pooled area–wide indicators relative to the MSE resulting from economically weighted area–wide indicators. The HLN p–value was calculated from a Student's t–distribution with $\Theta_2 - \Theta_0 - 1 = 36$ degrees of freedom. Note that ψ is set too zero.

Analyzing the full forecast potential of optimal pooling of information, the results in Table 2.2 indicate that forecast accuracy in terms of the out–of–sample MSE

[11]Note that we use the first release of real GDP available after the end of the respective quarter as the relevant realization for computing the forecast errors. As our data set ranges from 1990Q1 to 2008Q4 the last projection is generated for 2008Q4.

[12]For a comparison of the forecast performance of different forecast strategies in a real–time experiment, the optimization window should be separated from the evaluation window in order to avoid any informational advantages. We perform such an experiment in Section 2.4.4.

calculated over the entire forecast evaluation window is on average improved by around 25% compared to the economically weighted indicators. The test of forecast accuracy by Harvey et al. (1997) confirms the significance of the improvement at the 10% level.[13]

For an insight into the composition of the optimally pooled area–wide indicators, Table 2.3 depicts the weights that the optimization algorithm attributes to the single national indicator series. For the IPI almost all national indicators are considered – the only exception are the Belgian and the Portuguese indicators – while for the ESI and the WES a smaller number of national indicator series are selected. In the case of the IPI high weights are attributed to Germany, France and Italy, which also constitute the largest economies in the currency area. In the cases of the ESI and the WES, a large weight is assigned to Germany, but also to a subset of indicators of economically smaller countries, such as the Netherlands and Portugal. Surprisingly, for the ESI the Dutch indicator series obtains a weight that lies far above the Dutch share in euro area economic activity, which is currently around 6%. For the WES the same holds for the Portuguese indicator series. Even more surprisingly, despite the eminent economic role of France and Italy within the euro area, in the cases of both, the ESI and the WES, the indicators of these countries obtain weights which are close to or even equal to zero.

In order to analyze why certain national indicators enter the optimally pooled area–wide indicators, we calculated the out–of–sample MSE resulting from area–wide models using only a single national indicator as predictor relative to the MSE resulting from an area–wide model using the economically weighted area–wide indicators. The results are shown in Table 2.4 in which the best–performing national indicators are marked in bold. A comparison of the relative MSEs with the results reported in Table 2.3 shows that the optimization algorithm attributes a high weight to those national indicators that exhibit a high degree of forecast accuracy regarding area–wide real GDP. For the ESI (WES), the Dutch (Spanish) indicator that is heavily

[13]The null hypothesis of the HLN test is that the difference between the squared out–of–sample forecast error resulting from optimally pooled area–wide indicators and the squared out–of–sample forecast error resulting from economically weighted area–wide indicators is not less than zero.

Table 2.3: Optimal weighting schemes

National Indicator Series	IPI	ESI	WES
Austria	0.05	0.00	0.00
Belgium	0.00	0.00	0.00
Finland	0.08	0.06	0.00
France	0.17	0.00	0.03
Germany	0.24	0.26	0.23
Italy	0.34	0.00	0.00
Netherlands	0.03	0.25	0.05
Portugal	0.00	0.17	0.32
Spain	0.09	0.25	0.37

Notes: The weights are derived by minimizing the out–of–sample MSE resulting from 37 one–step ahead forecasts.

weighted by the optimization algorithm performs as well as the area–wide counterpart. In contrast, there is no national IPI series that shows a comparable forecast record for the euro area. This hints to the fact that a larger number of national IPI series are needed to capture the predictive information regarding area–wide real GDP.

Table 2.4: Area–wide ADL–models using single national indicators as predictors

National Indicator Series	IPI	ESI	WES
	MSE ratio		
Austria	3.45	1.65	1.20
Belgium	2.67	1.54	1.12
Finland	3.83	1.42	1.49
France	**1.76**	1.46	1.19
Germany	2.13	1.13	1.25
Italy	2.01	1.65	1.35
Netherlands	4.83	**0.99**	1.31
Portugal	4.13	1.32	1.28
Spain	1.84	1.44	**1.04**

Notes: The MSE ratios are calculated as the MSE resulting from national indicators relative to the MSE resulting from economically weighted area–wide indicators. MSE ratios in bold label the best performing nation indicators.

Apart from looking one–dimensionally at the mean forecast error, the theory of portfolio optimization highlights the role of correlations for the determination of

Table 2.5: Correlations of forecast errors of the area–wide models using single national indicators as predictors

IPI	Aus	Bel	Fin	Fra	Ger	Ita	Net	Por	Spa
Aus	1.00	0.85	0.93	**0.63**	0.37	0.88	0.85	0.93	0.88
Bel	0.85	1.00	0.86	0.61	0.48	0.88	0.82	0.85	0.81
Fin	0.93	0.86	1.00	**0.64**	0.33	0.87	0.84	0.93	0.86
Fra	0.63	0.61	0.64	1.00	0.54	0.67	0.65	0.71	0.77
Ger	0.37	0.48	0.33	**0.54**	1.00	0.49	0.42	0.44	0.51
Ita	0.88	0.88	0.87	**0.67**	0.49	1.00	0.83	0.93	0.91
Net	0.85	0.82	0.84	**0.65**	0.42	0.83	1.00	0.89	0.80
Por	0.93	0.85	0.93	0.71	0.44	0.93	0.89	1.00	0.91
Spa	0.88	0.81	0.86	**0.77**	0.51	0.91	0.80	0.91	1.00

ESI	Aus	Bel	Fin	Fra	Ger	Ita	Net	Por	Spa
Aus	1.00	0.91	0.93	0.94	0.88	0.94	0.85	0.88	0.84
Bel	0.91	1.00	0.87	0.91	0.89	0.89	0.88	0.87	0.73
Fin	0.93	0.87	1.00	0.95	0.85	0.94	0.83	0.88	0.87
Fra	0.94	0.91	0.95	1.00	0.83	0.96	0.81	0.90	0.90
Ger	0.88	0.89	0.85	0.83	1.00	0.83	**0.83**	0.84	0.66
Ita	0.94	0.89	0.94	0.96	0.83	1.00	0.83	0.89	0.92
Net	0.85	0.88	0.83	0.81	0.83	0.83	1.00	0.87	0.67
Por	0.88	0.87	0.88	0.90	0.84	0.89	**0.87**	1.00	0.80
Spa	0.84	0.73	0.87	0.90	0.66	0.92	**0.67**	0.80	1.00

WES	Aus	Bel	Fin	Fra	Ger	Ita	Net	Por	Spa
Aus	1.00	0.95	0.97	0.96	0.87	0.97	0.96	0.96	0.91
Bel	0.95	1.00	0.94	0.96	0.85	0.94	0.89	0.91	0.95
Fin	0.97	0.94	1.00	0.98	0.84	0.99	0.97	0.97	0.93
Fra	0.96	0.96	0.98	1.00	0.85	0.98	0.94	0.95	0.95
Ger	0.87	0.85	0.84	0.85	1.00	0.84	0.85	0.82	**0.79**
Ita	0.97	0.94	0.99	0.98	0.84	1.00	0.97	0.97	0.95
Net	0.96	0.89	0.97	0.94	0.85	0.97	1.00	0.96	**0.89**
Por	0.96	0.91	0.97	0.95	0.82	0.97	0.96	1.00	**0.90**
Spa	0.91	0.95	0.93	0.95	0.79	0.95	0.89	0.90	1.00

Notes: Figures in bold label the national indicator series that additionally enter the newly constructed area–wide indicators besides the dominant ones.

the optimal weighting scheme. An analysis of the correlations of the forecast errors resulting from area–wide models that only use a single national indicator as predictor, might in particular be helpful in explaining why some of the rather poorly–performing national indicator series enter the optimally pooled indicators in addition to the best performing ones. Table 2.5 reveals that the optimization algorithm attributes a weight larger than zero to those national indicator series whose forecast errors are less correlated with the best–performing national indicators. Consider the ESI as an example where the Dutch series poses the dominant single indicator in terms of forecasting area–wide real GDP. The Spanish index is assigned a weight that is far greater than the relative economic share of its economy in the euro area although it performs rather poorly when it comes to forecasting euro area real GDP growth. Evidently, its high MSE ratio which is greater than that of the Finish and the Portuguese indicators is overcompensated by the low correlation between its forecast errors and those resulting from the dominant Dutch indicator. The same holds for the German WES indicator that is attributed a weight comparable to its economic importance although its forecast performance regarding area–wide GDP is rather limited. Again, its forecast errors are relatively low correlated with those of the dominant Spanish index.

2.4.4 Forecast Experiment Using Ex–Post Information

The critical point of the optimal pooling of information approach is the use of the out–of–sample MSE as the target function of the optimization algorithm since this requires to rely on (pseudo) ex–ante information. By exploiting information stemming from the forecast evaluation window, the approach is advantaged compared to competing forecasting methods.

Hence, this section evaluates the performance of OPI in a real–time forecast experiment where only ex–post information enters the optimization frameworks. Given the results based on ex–ante information as shown in Table 2.2, OPI necessarily outperforms the economically weighted indices if the optimized weights reported in Table 2.3 remain stable over time. In this context stability means that the weights attributed to each national indicator series are robust against variations of the length and the initial date of the optimization window.

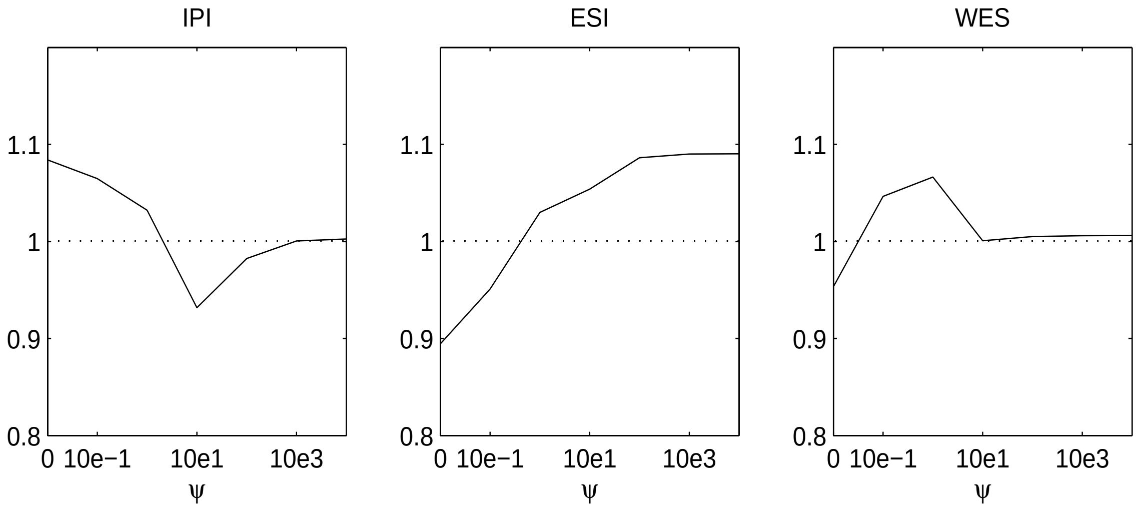
In the following we derive optimal weights by focusing on shorter optimization windows that are strictly separated from the evaluation window. Figure 2.1 presents an overview of the timing of events. Table 2.6 shows the optimal weights that are computed from rolling fixed length optimization windows with 10 and 15 forecast errors. Given that the number of potential out–of–sample forecasts in the experiment is equal to $\Theta_2 - \Theta_0 = 37$, we end up with 27 and 22 fixed–length optimization windows, which can be used to derive the weights. A comparison of Table 2.6 with Table 2.3 shows that the means of the weights deviate from the weights computed from the complete optimization window for various indices. Although the variation, measured in terms of standard deviations, decreases with an increasing optimization window on average, the weights still vary considerably over time such that the practicability of the approach has to be evaluated in a real–time experiment.

Table 2.6: Optimal weighting schemes derived from rolling optimization windows with 10 and 15 forecast errors

	window size 10			window size 15		
	IPI	ESI	WES	IPI	ESI	WES
Aus	0.04	0.01	0.09	0.02	0.00	0.03
	(0.04)	(0.02)	(0.12)	(0.02)	(0.00)	(0.05)
Bel	0.09	0.04	0.00	0.05	0.02	0.00
	(0.13)	(0.08)	(0.01)	(0.09)	(0.05)	(0.00)
Fin	0.06	0.06	0.00	0.07	0.07	0.01
	(0.05)	(0.09)	(0.03)	(0.05)	(0.11)	(0.07)
Fra	0.14	0.03	0.11	0.15	0.01	0.16
	(0.15)	(0.08)	(0.17)	(0.13)	(0.02)	(0.19)
Ger	0.20	0.44	0.40	0.19	0.50	0.41
	(0.09)	(0.29)	(0.30)	(0.10)	(0.13)	(0.26)
Ita	0.19	0.00	0.00	0.24	0.00	0.00
	(0.13)	(0.01)	(0.00)	(0.10)	(0.00)	(0.00)
Net	0.10	0.17	0.10	0.08	0.21	0.14
	(0.10)	(0.20)	(0.12)	(0.08)	(0.16)	(0.08)
Por	0.07	0.07	0.16	0.06	0.04	0.14
	(0.09)	(0.15)	(0.21)	(0.08)	(0.07)	(0.18)
Spa	0.12	0.18	0.13	0.15	0.15	0.11
	(0.15)	(0.17)	(0.18)	(0.11)	(0.20)	(0.17)

Notes: The Table shows the mean of 27 and 22 optimal weights derived from rolling optimization windows with 10 and 15 forecast errors. The figures in parentheses denote the standard deviations around the mean.

One straight forward way to increase stability of the weights over time is to employ a recursive optimization window instead of a rolling window. Another approach is to add an additional term to the minimization function that penalizes a deviation of the optimized weights from those computed one period before. For small values of this penalty term ψ, the weights are allowed to vary whereas larger values force the algorithm to keep the weights more constant. This trade–off is analyzed in Figure 2.2 which gives the relative MSE of OPI to the economically weighted indices for various values of the penalization coefficient ranging from $\psi = 0$ to $\psi = 10e5$.



Notes: The lines represent the relative MSE of OPI to the economically weighted indices for the different values of ψ.

Figure 2.2: Relative MSE of OPI for varying values of ψ

The results show that the stability of the weights is sufficiently high for the ESI and the WES to outperform the benchmark indices in a real–time experiment and the introduction of a penalty term does not increase forecast performance. In contrast, for the IPI, a value of $\psi = 10e1$ is optimal in terms of forecasting area–wide GDP since the weights are too volatile in absence of the penalty term. This confirms the analysis of the optimal weighting scheme for a rolling window as presented in Table 2.6. For $\psi \to \infty$, the weights are kept constant at the level of the nations relative economic share and the relative MSE converge to unity for the IPI and the WES. This does not hold for the ESI where the area–wide index is aggregated differently as described in Section 2.4.2.

Although economic weights pose the most popular aggregation scheme, a number of alternative benchmark models are at disposal. In order to take our optimal pooling approach to a tougher test we compare its forecast accuracy in the following with the competing economic and econometric weighting schemes and prediction approaches presented in Section 2.2 in a real–time experiment. In addition, we also derive forecasts from an univariate forecast model. The competing forecast models are thereby estimated using the same area–wide and national business cycle indicators at disposal. The optimized weights are derived from a recursively growing optimization window, which is excluded from the forecast evaluation process. For the first iteration, the optimized indicator is calculated by minimizing the sum of the first 10 out–of–sample squared forecast errors and the forecast of euro area real GDP growth for second quarter 2002 is generated. At each iteration, the optimization window is expanded one quarter and the weights are updated using this recursive approach. The same setting is used to derive the weights for the optimal pooling of area–wide forecasts as described in Section 2.2.1.1 in detail. Again, the weighting scheme is solely based on ex–post information. Furthermore, at each iteration a static factor model as well as a dynamic factor model are employed to extract an area–wide indicator which is used to forecast current quarter's real GDP growth for the euro area. The area–wide indicator thereby corresponds to the first common factor extracted.[14] For the aggregation of national GDP forecasts, we employ economic weights based on the countries' shares of national GDP in euro area real activity.

Table 2.7 reports the forecast MSE of the optimal pooling of information approach relative to those of the alternative forecast approaches for $\psi = 10e1$ in case of IPI and for $\psi = 0$ in case of the ESI and the WES. The results can be summarized as follows: 1) The optimal pooling of information approach results in general in a

[14]The number of common factors extracted as well as the lag–window size used in the dynamic factor model are optimized regarding the ex–post forecast performance of the resulting area–wide indicator.

Table 2.7: Forecast performance of optimal pooling of information relative to traditional forecast strategies

		MSE ratio	HLN p–value
IPI	Univariate approach	0.19	0.01
	Pooling of information		
	Economic weights	0.93	0.24
	OLS weights	0.47	0.01
	Principal component analysis	0.74	0.19
	Dynamic factor model	0.62	0.09
	Pooling of forecasts		
	Optimal weighting of area–wide forecasts	0.54	0.11
	Equal weighting of area–wide forecasts	0.39	0.02
	Aggregation of national forecasts	1.06	0.78
ESI	Univariate approach	0.48	0.02
	Pooling of information		
	Economic weights	0.89	0.26
	OLS weights	1.08	0.62
	Principal component analysis	0.92	0.25
	Dynamic factor model	0.57	0.02
	Pooling of forecasts		
	Optimal weighting of area–wide forecasts	0.59	0.00
	Equal weighting of area–wide forecasts	0.73	0.00
	Aggregation of national forecasts	0.84	0.27
WES	Univariate approach	0.69	0.03
	Pooling of information		
	Economic weights	0.95	0.35
	OLS weights	0.79	0.04
	Principal component analysis	0.94	0.35
	Dynamic factor model	0.87	0.25
	Pooling of forecasts		
	Optimal weighting of area–wide forecasts	0.82	0.25
	Equal weighting of area–wide forecasts	0.85	0.10
	Aggregation of national forecasts	0.94	0.36

Notes: The MSE ratios are calculated as the MSE resulting from the optimal pooling of information approach relative to the MSE resulting from respective forecast strategy. The HLN p–value was calculated from a Student's t–distribution with $\Theta_2 - \Theta_0 - 1 = 26$ degrees of freedom.

lower forecast error, i.e. the MSE ratios are below unity. Only in two cases – for the IPI and the ESI – the MSE ratios are above unity. 2) The optimal pooling of information approach dominates the univariate forecast model significantly in all cases. 3.) The optimal pooling of information approach yields a smaller MSE than the economic weighting schemes for all three indicators under consideration, which confirms the results obtained in Section 2.4.3 where we allowed for the use of ex–ante information. However, the improvements are not significant. 4) The HLN test shows at the 10% significance level that the optimal pooling of information approach significantly dominates 4 of the competing forecast approaches in the case of the IPI and the ESI, and 3 of the competing forecast approaches in the case of the WES. 5) In those cases where the MSE ratio is above unity, the optimal pooling of information approach is not systematically beaten by the competing forecast method.

2.5 Conclusion

This paper proposes a new method of forecasting euro area quarterly real GDP that uses area–wide indicators, which are derived by optimally pooling the information contained in national indicator series. Following the ideas of predictive modeling, the area–wide indicators are computed by applying weights that minimize the variance of the out–of–sample forecast error of the aggregate target variable. We evaluate the forecast performance of our optimal pooling of information approach by focusing on three business cycle indicators, namely the Industrial Production Index (IPI), the economic sentiment indicator (ESI) of the European Commission and the CESifo World Economic Survey (WES) indicator for the euro area, which are all available at the area–wide and country–specific level.

Our results show that short–term forecasts of euro area quarterly real GDP are improved by using area–wide indicators based on optimal weights rather than economic weights. The optimally pooled area–wide indicators reduce the out–of–sample MSE by 25% on average. Since the optimal weights can vary considerably over time depending on the indicator used, the introduction of a penalty term that helps to stabilize the weights potentially promotes the practicability of the approach in real–time.

In an out–of–sample forecast experiment we compare the forecast performance of the optimal pooling of information approach with that of a number of competing forecasting strategies. The optimally pooled area–wide indicators are constructed using only information that would have been available in real–time. We find that our OPI approach outperforms competing methods in terms of forecast accuracy and that optimized indices for the euro area consist of only a rather small number of national indices .

Chapter 3

The Virtues of VAR Forecast Pooling - A DSGE Model Based Monte Carlo Study

In the presence of model uncertainty, pooling different forecasts tends to outperform individual forecasts. However, any empirical forecast evaluation approach suffers from numerous unknown detrimental effects that deteriorate the results. Hence, the magnitude of benefits attainable is still unclear. Consequently, we use Monte Carlo techniques which enable us to identify the virtues from pooling VAR forecasts which can be traced back to a well–defined form of mis–specification. To mimic a forecast situation, we derive the data from a common DSGE model. Given strict lab conditions, the results are allowed to vary with respect to the structure of the model economy, number of pooled forecasts, forecast weighting scheme, forecast horizon, estimation sample size and the noisiness of data available. As the setup assumes a form of mis–specification inevitably present in VAR forecast approaches, the experiment yields a quantification of the virtues obtainable in any forecast situation. We find that pooling of VAR forecasts leads to a substantial improvement in accuracy of about 20 percent, which is comparable to the effect of the elimination of estimation uncertainty. Most notably, this gain is already obtained with an average of about four different forecasts and is higher for more persistent economies.

3.1 Introduction

Since the seminal article of Bates and Granger (1969), a large number of studies have shown that pooling different forecasts of the same event tends to outperform individual forecasts in terms of forecast accuracy.[1] Empirical evidence suggests that in many cases it is possible to improve performance considerably by simple averaging the forecasts (Clemen (1989)). For instance, Makridakis et al. (1982) find that combinations of as little as six time–series forecasts outperform most individual forecasts. In addition to assessing the predominance of pooled predictions, a number of papers quantify their empirical gains compared to single forecasts. Although the vast majority of articles confirms the predominance of pooled forecasts, the results remain heterogenous regarding the size of gains. Armstrong (2001) reviews 30 empirical studies on forecast combinations and reports an average reduction in forecast errors of 12.5 percent, ranging between 3 and 24 percent each. As there are numerous sources for the large variation of the resulting gains, it is difficult to estimate the improvement in accuracy in a given forecast situation ex–ante based on empirical findings. A broad range of detrimental effects such as structural breaks or outliers and revisions of the data potentially impact empirical findings and account for the differences of gains reported.

To provide a common guideline to the forecaster, we employ a Monte Carlo study and simulate the data sets from standard DSGE models. This has the advantage that we deal with a well–specified dynamic structure that lends itself to an economically meaningful interpretation. We calibrate three different data generating processes (DGPs) to mirror the characteristics of major economies as commonly done by central banks. As a result, we obtain the business cycle behavior of the most relevant variables such as GDP, inflation and interest rates.

Naturally, if the true DGP is known, there is no scope to downsize forecast errors by means of different forecasting approaches. However, in practice the forecaster is not equipped with the true model. This is why the present paper is based on the

[1]See e.g. Marcellino (2004), Kuzin et al. (2009), Stock and Watson (2004), Clark and McCracken (2009a) and Clark and McCracken (2009b).

more realistic assumption that the DGP is unknown to the forecaster and thus all forecast models at hand necessarily suffer from some form of mis–specification. To be more precise we dynamically forecast from parsimonious VAR models, each built only on a subset of the relevant information. Thus, as we keep control of the DGP, our results rely on well–defined forms of mis–specification of the forecast models and we explicitly exclude any accidental effects that might bias the results in favor of combination approaches. Hence, Monte Carlo techniques enable us to estimate the gains of pooling under strict lab conditions.

The findings have practical relevance as the forms of mis–specification we assume are likely to occur in any real forecast situation where forecasts are conducted with the help of VAR models. As presented in Section 3.2, there are numerous sources of mis–specification which tend to make pooling beneficial. Naturally, the exact form of mis–specification is not known ex–ante. Hence, we exclude any effects like structural breaks that might occur accidentally and bias results in favor of pooling approaches. In that sense, the reported results can be interpreted as some sort of minimum gain that is obtained from pooling of forecasts. To provide guidance if pooling is beneficial in a specific forecast situation, we vary the setting with respect to different dimensions. First, altering the DGP leads to conclusions about the influence economic structures exert on the virtues of combined forecasts. Second, we analyze the effects of a growing number of forecasts included in the combination approach. Third, we track the performance of pooling as the forecast horizon grows. Forth, we explicitly assess estimation uncertainty by varying the estimation sample size. Moreover, we contrast a simple average with theoretically optimal pooling techniques as in Timmermann (2006) and answer the question in which situations the former outperforms the latter. We also evaluate how pooling performs if economic variables of the DGP are not directly observable and the forecasts rely on noisily measured indicators. Moreover, we also compare the performance of pooled forecasts from parsimonious VARs to benchmark VARs including all relevant variables. Finally, we check if gains from equally weighted forecasts prevail if the benchmark model is chosen by a statistical information criterion. Additionally, we analyze the composition of the resulting mean squared forecast errors (MSEs) in order to clarify where the gains from pooling originate.

Our analysis shows that pooling leads to a substantial reduction of the MSE that amounts to about 20 percent for more rigid economies. In fact, the gain from pooling is approximately comparable to an extinction of estimation uncertainty associated with a forecast model. Most notably, this gain is already obtained with a simple average of about four different forecasts. In contrast, the estimation of theoretically optimal weights is advisable only for very large data sets hardly available in practice. Thus, given our results, we recommend the use of equally weighted VAR predictions as an easy to implement forecast approach that most likely improves accuracy compared to single VAR predictions.

The structure of the paper is given as follows. Section 3.2 gives the framework for the theoretical and empirical gains of pooling of forecasts. Section 3.3 describes the model economy which constitutes the DGP. Section 3.4 introduces the VAR forecast framework and presents the combination schemes used. Section 3.4.3 describes the settings of our Monte Carlo experiment. Section 3.5 presents the results and Section 3.6 concludes.

3.2 The Gains from Pooling

In general, the use of pooled forecast is motivated by portfolio diversification or hedging arguments, guaranteeing insurance against very large forecast errors. Obviously, if the true DGP is known to the forecaster, there are no gains to be made by pooling forecasts from different models. Instead, as argued by Timmermann (2006), pooling the information and thus constructing one "super model" yields the best forecast performance. However, in practice none of the models at hand coincides with the unobservable DGP. Moreover, Diebold and Lopez (1995) point out, that in many forecast situations, particularly in real time, pooling of information sets is either impossible or prohibitively costly. Thus, some form of mis–specification or mis–estimation will be present and contribute to the resulting forecast error. To be more precise, it may be the case that the forecast model omits relevant variables or information in general. Moreover, there is a risk of employing a biased model especially when there are unobserved structural breaks in the DGP. In fact, pooled forecasts can even dominate the best individual device. When using VARs for forecasting, we necessarily introduce inefficiencies by omitting cross–equation restrictions from the DGP. This negatively effects the forecast performance for finite estimation samples.

Hendry and Clements (2004) describe a set of potential explanations for the gains achieved by combining individual forecasts regarding the forecast MSE. If single predictions are differently biased – i.e. upwards biased and downwards biased – pooling them might improve forecast accuracy. However, reasonably constructed forecast models prevent systematically biased predictions. A source of improvement more relevant in practice results from unexpected breaks in the DGP. As each forecast model is affected differently by breaks, i.e. each model is differentially mis–specified, pooling the resulting predictions guarantees insurance and might again improve forecast accuracy. In the presence of estimation uncertainty, another potential source of gain follows from a reduction of parameter proliferation due to overfitting. Forecasts from the true but estimated DGP do not encompass forecasts from competing mis–specified models in general, especially when the sample size is short in relation to the number of parameters to be estimated in the true model. As a result, pooling forecasts from parsimonious models that omit a subset of explanatory variables might even outperform forecasts from the true model.

Given there are numerous situations where pooling is advantageous, we briefly discuss the general framework building the theoretical motivation for the analysis. As in Batchelor and Dua (1995), we want to forecast h–period ahead future values of some target variable y whose realization is given by y_{t+h}. The forecast based on some model i is denoted by $\hat{y}_{i,t+h}$, and the resulting forecast error is given by $e_{i,t+h} = \hat{y}_{i,t+h} - y_{t+h}$. The presumed loss function is MSE loss. Assuming unbiased predictions, i.e. $E(e_{i,t+h}) = 0$, $MSE_{i,h}$ equals the variance of the forecast errors $(\sigma^2_{i,h})$. Given a total number M of different single forecasts, the expected error variance of a single randomly selected forecast k can be calculated as the average of the error variances of all single forecasts, i.e.

$$E(\sigma_{k,h}^2) = \overline{\sigma}_h^2 = \frac{1}{M}\sum_{i=1}^{M}\sigma_{i,h}^2. \tag{3.1}$$

The expected error variance of an equally weighted average of a set of m randomly selected single forecasts can be calculated as

$$E(\sigma_h^2(m)) = \overline{\tau}_h^2(m) = \frac{1}{m}\overline{\sigma}_h^2 + \frac{m-1}{m}\overline{\vartheta_h}, \tag{3.2}$$

where $\overline{\vartheta_h}$ is the average bivariate covariance between all pairs of single forecast errors for horizon h.[2] The percentage reduction in expected error variance by pooling m forecasts is thus given by

$$\frac{\overline{\tau}_h^2(m) - \overline{\sigma}_h^2}{\overline{\sigma}_h^2} = \frac{1-m}{m}(1 - \frac{\overline{\vartheta_h}}{\overline{\sigma}_h^2}). \tag{3.3}$$

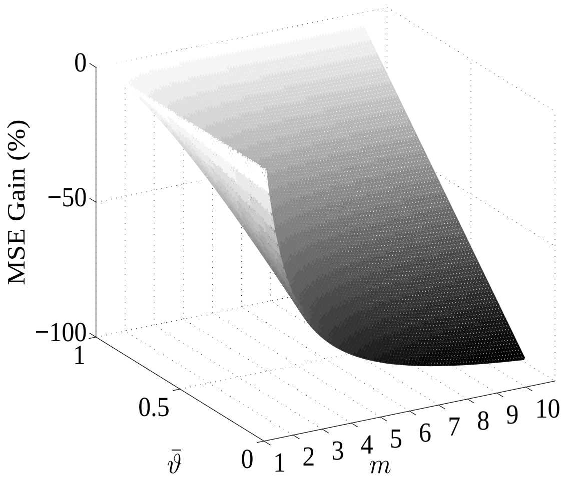
Since $\overline{\sigma}_h^2 \geq \overline{\vartheta_h}$, this term is negative and takes a value of zero only if forecast errors are perfectly correlated. Figure 3.1 shows the percentage reduction in expected error variance for $\overline{\sigma}_h^2 = 1$ as a function of the number m of forecasts combined and of the average bivariate covariance $\overline{\vartheta_h}$ of forecast errors.

The improvement in forecast accuracy increases with the number of forecasts combined and with a decreasing average bivariate covariance of the forecast errors and converges to $\frac{\overline{\vartheta_h}}{\overline{\sigma}_h^2} - 1$ for large values of m. Figure 3.1 supports the conclusion of Armstrong (2001), that the combination of five forecasting methods is sufficient to achieve most of the possible reduction in forecast error variance and that the inclusion of further forecasts generates only minor additional gains.

In the present paper, we largely follow the framework of Batchelor and Dua (1995). Similarly, here, the analysis relies on randomly drawn benchmark models which has the advantage that results are not dependent on the performance of some model selection process.[3] In fact, following Equation (3.3), documented changes in forecast accuracy represent the average effect resulting from a condensation of information contained in many economic time series.

[2] For the derivation of the above expression, see Appendix 3.A.1.

[3] See Section 3.5.5 for a further discussion on that issue.



Notes: The figure presents the gain in MSE of pooling m randomly chosen forecasts as % of one single randomly chosen model for forecast horizon h.

Figure 3.1: Theoretical MSE Gain from pooling m randomly chosen forecasts

3.3 The Model Economy

For our controlled experiment, we choose a common New–Keynesian type model to simulate economic relationships. Generally speaking, New-Keynesian DSGE models have been developed to replicate distinct features of economic data on a business cycle frequency. Consequently, they are most commonly estimated on quarterly data of aggregate measures such as real GDP, inflation and money–market rates. In recent years, they have become a popular tool – e.g. for central banks – not only for policy analysis, but also for forecasting.[4] In contrast to VAR models, the behavior of all variables is traceable to a set of fundamental assumptions about the underlying structure of the model economy. In other words, the forecasts lend themselves to an economic interpretation as the dynamics of variables are the result of economic decisions taken at the micro–level. Thus, given that economic theory is in any case meaningful, employing a New-Keynesian DSGE model to simulate the data guarantees that the forecasting experiment is based on data which shares the

[4]See among others Smets and Wouters (2003), Harrison et al. (2005) or Murchison and Rennison (2006).

distinct features of aggregate economic data on a business cycle frequency.

The model we choose is very similar to the one presented in Canova and Paustian (2007). Featuring staggered wages and prices, it is very much in the spirit of Erceg et al. (2000). We allow for habit formation as in Fuhrer (2000) and for indexation as in Rabanal and Rubio-Ramirez (2005). The model also captures interest rate smoothing of the central bank as in Clarida et al. (2000). The approach has the advantage that many simpler models are nested in our baseline scenario and we can vary the degree of persistence of the system by changing well specified parameter values that have a structural interpretation. The linearized model equations are (in log deviations from steady state):

$$\lambda_t = E_t\lambda_{t+1} + E_t[r_t - \pi_{t+1}] \tag{3.4}$$
$$\lambda_t = \xi_t^b - \frac{\sigma_c}{(1-h_y)}(y_t - h_y y_{t-1}) \tag{3.5}$$
$$y_t = \xi_t^z + (1-\alpha)n_t \tag{3.6}$$
$$mc_t = w_t + n_t - y_t + err_2 \tag{3.7}$$
$$mrs_t = -\lambda_t + \gamma n_t \tag{3.8}$$
$$w_t = w_{t-1} + \pi_t^w - \pi_t + err_1 \tag{3.9}$$
$$\pi_t^w - \mu_w\pi_{t-1} = \kappa_w(mrs_t - w_t) + \beta(E_t\pi_{t+1}^w - \mu_w\pi_t) \tag{3.10}$$
$$\pi_t - \mu_p\pi_{t-1} = \kappa_p(mc_t + \xi_t^\mu) + \beta(E_t\pi_{t+1} - \mu_p\pi_t) \tag{3.11}$$
$$r_t = \rho_r r_{t-1} + (1-\rho_r)(\gamma_\pi\pi_t + \gamma_y y_t) + \xi_t^r \tag{3.12}$$
$$\xi_t^b = \rho_b\xi_{t-1}^b + \nu_t^b \quad \nu_t^b \sim N(0,\sigma_b^2) \tag{3.13}$$
$$\xi_t^z = \rho_z\xi_{t-1}^z + \nu_t^z \quad \nu_t^z \sim N(0,\sigma_z^2) \tag{3.14}$$
$$\xi_t^\mu \sim N(0,\sigma_\mu^2) \tag{3.15}$$
$$\xi_t^r \sim N(0,\sigma_r^2) \tag{3.16}$$

Equations (3.4) and (3.5) describe the demand side of the economy where λ_t is the marginal utility of consumption which depends on the expected real interest rate as the difference of the nominal rate r_t and inflation π_{t+1} and h_y measures the degree of habit formation in total demand y_t. Moreover, demand is subject to a taste shock ξ_t^b. Equation (3.6) is the linearized production function where $1-\alpha$ denotes labor share in production and n_t is labor input (hours worked). The process is subject to a productivity shock ξ_t^z. Equations (3.7) and (3.8) define marginal cost and the marginal rate of substitution, respectively. Here, w_t is the real wage and γ measures the substitution effect of a change in hourly wages on labor supply. The real wage is defined in Equation (3.9), with real wage inflation being the difference between nominal wage inflation π_t^w and price inflation π_t. The wage Phillips curve is presented in Equation (3.10), where indexation is measured by μ_w and κ_w is the slope. The parameter that determines the dynamics of the wage equation κ_w can also be calculated from deep parameters; i.e. by the probability of keeping wages fixed $1-\zeta_w$, the discount factor β, the elasticity of the labor bundler ψ and the elasticity of labor supply with respect to wages γ:

$$\kappa_w = \frac{(1-\zeta_w)(1-\beta\zeta_w)}{\zeta_w(1+\psi\gamma)}. \tag{3.17}$$

Analogously, Equation (3.11) defines the Phillips curve for prices with indexation parameter μ_p and slope κ_p. The slope of the Phillips curve can be shown to depend upon the probability of keeping prices fixed $1-\zeta_p$, the discount factor β, the elasticity of the goods bundler ϵ and the labor share in production $1-\alpha$ in the following way:

$$\kappa_p = \frac{(1-\zeta_p)(1-\beta\zeta_p)}{\zeta_p}\frac{1-\alpha}{(1-\alpha+\alpha\epsilon)}. \tag{3.18}$$

In addition, marginal cost is also driven by an exogenous mark–up shock ξ_t^μ. The nominal interest rate is set by the central bank according to a Taylor–type rule (3.12) with interest rate smoothing of degree ρ_r. γ_π and γ_y capture the response of the central bank to inflation and output, respectively. The remaining four equations define the emergence of exogenous shocks, where some persistence is allowed for the taste shock and the productivity shock. Moreover, to avoid perfect multicollinearity, real wages and marginal cost are assumed to be measured with error err_1 and err_2, respectively. The parameter values assigned during the experiment are reported in Table 3.1.

In principle, we adopt the values from Canova and Paustian (2007) but allow for

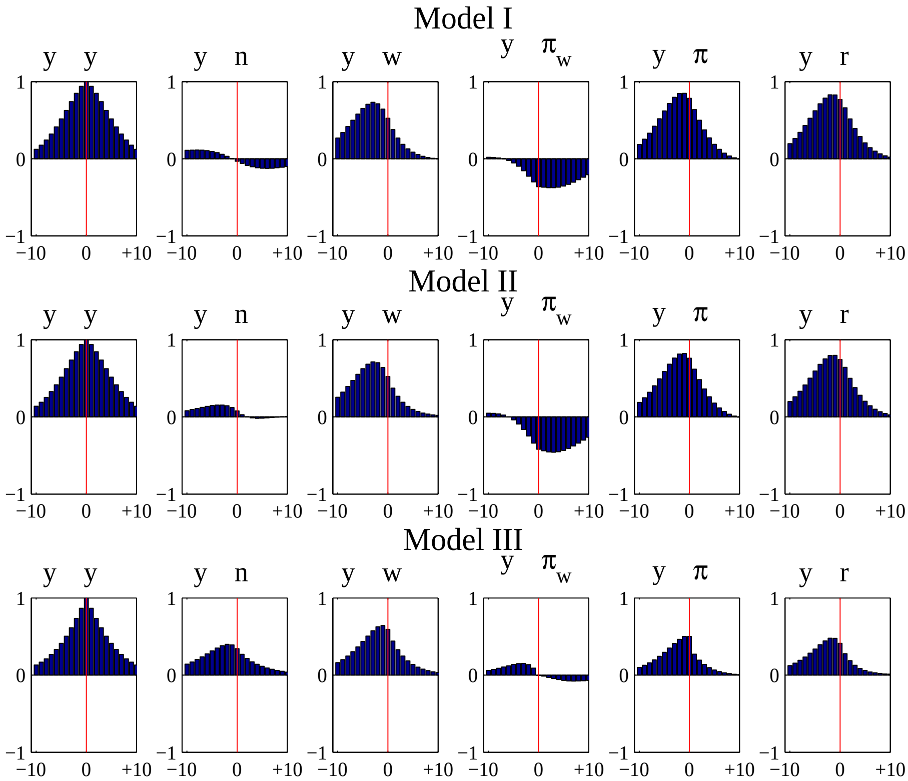
	Model parameter	*Model I*	*Model II*	*Model III*
β	discount factor	0.99	0.99	0.99
ϵ	elasticity (goods)	6.00	6.00	6.00
ψ	elasticity (labor)	6.00	6.00	6.00
σ_c	risk aversion	8.33	8.33	8.33
γ	inverse Frish elasticity of labor supply	1.74	1.74	1.74
h_y	degree of habit formation	0.90	0.00	0.00
ζ_p	1-probability of keeping prices fixed	0.75	0.75	0.75
ζ_w	1-probability of keeping wages fixed	0.62	0.62	0.62
μ_p	rule–of–thumb price setters	0.70	0.70	0.00
μ_w	rule–of–thumb wage setters	0.80	0.80	0.00
α	1-labor share in production function	0.36	0.36	0.36
ρ_r	interest rate smoothing	0.74	0.74	0.74
γ_y	reaction to output in Taylor rule	0.26	0.26	0.26
γ_π	reaction to inflation in Taylor rule	1.08	1.08	1.08
ρ_b	persistence of taste shock	0.82	0.82	0.82
ρ_z	persistence of productivity shock	0.74	0.74	0.74
σ_b	std of taste shock	0.1188	0.1188	0.1188
σ_z	std of productivity shock	0.0388	0.0388	0.0388
σ_μ	std of markup shock	0.3167	0.3167	0.3167
σ_r	std of monetary policy shock	0.0033	0.0033	0.0033
σ_{err1}	std of measurement error 1	0.0001	0.0001	0.0001
σ_{err2}	std of measurement error 2	0.0001	0.0001	0.0001

Table 3.1: Calibration of the model economy

three different types of economies. In *Model I*, we allow for a considerable degree of backward–lookingness and habit formation. This results in a more sluggish response of GDP to all kinds of shocks when compared to *Model II* and *III*. The setup of *Model II* abstracts from habit formation, i.e. $h_y = 0$, and, thus, output is less persistent when compared to *Model I*. A productivity shock results in a more pronounced, but somewhat earlier response. *Model III* mimics a more flexible economy where neither prices nor wages are subject to indexation, i.e. $h_y = 0$, $\mu_p = 0$ and $\mu_w = 0$. This leads to quite similar responses of GDP as in Model II. However, differences occur with respect to price and wage reactions to shocks. As intended, prices tend to adjust quicker – especially in response to markup–shocks.

When it comes to forecasting, the differences between *Models I* to *III* stem from the fact that interdependencies between variables differ. As lags and leads are important, we also depict the cross–correlations of the relevant variables with the target variable y in Figure 3.2 for ± 10 lags. In *Model I* and *II*, for instance, output

y has only little correlation with employment n, whereas in *Model III*, there is a rather strong correlation with lags of employment. Also wage inflation π_w shows a strong negative correlation in *Model I* and *II*, whereas in *Model III* there is almost no relationship. Naturally, correlations among variables are lower for the less persistent economy given by *Model III*. On the whole, the procedure introduces economically meaningful cross–equation restrictions and a well–defined cross–correlation structure between variables.



Note: The plots show the correlation of the first variable with the lags of the second variable, e.g. corr(y_t, n_{t+k}). Lags are given on the abscissa and correlations are depicted on the ordinate, where the numbers represent the mean of a Monte–Carlo simulation based on 10000 replications of the data set.

Figure 3.2: Mean of cross–correlation functions of the DGP

3.4 Setting up the Simulation Study

3.4.1 VAR Forecast Framework

To analyze the gains from pooling of forecasts, we choose to predict the target series y employing VAR models. The use of VAR models for macroeconomic forecasting has first been introduced by Sims (1980) to address the common structural identification problem inherent in simultaneous equation models. It follows the idea of exploiting the dynamic correlation patterns among observed time series without imposing restrictions. As all variables are determined endogenously, no a–priori knowledge is used except to decide which variables should enter the system. Thus, the VAR approach is often referred to as being atheoretical. An important feature of VAR forecasts is their unbiasedness. Dufour (1985) shows that, as long as the DGP considered has an autoregressive representation and satisfies the symmetry condition, a VAR estimated by least squares will yield unbiased forecasts even if the lag length of the estimated VAR is lower than the actual one and even if explanatory variables are missing. Due to their comparable good forecast record, the comparatively low computational effort involved and their ability to generate iterative multi–step predictions VAR models are frequently used in macroeconomic forecasting.[5]

In the presence of model uncertainty, the researcher has to choose about the lag length and the variables to include into the VAR. We restrict the analysis to small scale VAR models with $K = 2$ endogenous variables and a lag length of $p = 1$ simply because this is the simplest forecasting model available and keeps the dimension of the analysis tractable.[6] Furthermore, given the structure and the dynamics of the

[5]Examples of small scale VAR–systems used to forecast output, prices and interest rates are numerous, including Litterman (1986), Del-Negro and Schorfheide (2004), Favero and Marcellino (2005) and Clark and McCracken (2009a). See Lütkepohl (2006) for a detailed discussion of VARs in macroeconomic forecasting.

[6]To check if the lag length is appropriate, we calculated the BIC for each bivariate model and each replication. It delivered a lag length of one in more than 90% of the cases. The reason why we do not implement this approach, is that we need to compare VARs that suffer from an identical degree of estimation uncertainty. Consequently, we choose identical lag lengths for all VARs.

DGPs described in Section 3.3, each of these VARs will provide reasonable forecasts and – by holding p and K constant for each VAR – we compare forecast approaches that are subject to the same degree of estimation uncertainty.[7] Most important, as any larger VAR – i.e. choosing $K > 2$ – would still be only an approximation to the true DGP – in our Monte Carlo experiment as well as in practice – it will again necessarily be mis–specified in our sense. In general, the size of the VAR should only be of minor importance for our results, as the model specification is identical also for the pooling approach.

However, a bivariate VAR will completely omit information present in cross correlations among y, the additional variable and observable variables that are not included in the VAR. Moreover, any cross equation restrictions from the DSGE model are omitted. Thus, information contained in the cross–correlation of the additional observable variable with y is necessarily inefficiently used by the forecaster. In fact, we focus on a form of mis–specification that essentially arises when VAR models are employed to forecast structural processes. In detail, each VAR model only incorporates the target series plus one of the remaining 5 variables. Here, we assume that past values of y, r, n, w, π and π^w are directly observable by the forecaster. This reflects a practical situation, as these quantities are usually provided by national accounts or – in case of r – are observable from financial markets. Given an estimation sample of size T, we estimate VAR models with $T - (K^2p + K)$ degrees of freedom. Using this simplest form of multivariate forecasting models at hand leaves us with $M = 5$ different forecast models. Finally, we calculate $h = 1, \dots, 100$ step predictions of variable y for each VAR model to see how far we can forecast until the differences between forecasting approaches eventually die out.

3.4.2 Pooling Techniques

One crucial issue when pooling different forecasts is the weighting scheme that defines how single predictions are combined into one pooled forecast of the target variable. Various methods for the estimation of appropriate weighting schemes have

[7]For a further discussion on this issue see Section 3.5.4.

been proposed and their relative performance depends on the underlying assumptions. As Winkler (1989) points out: "The better we understand which sets of underlying assumptions are associated with which combining rules, the more effective we will be at matching combining rules to forecasting situations." We focus on two schemes which define the upper and the lower bound of complexity.[8] On the one hand we introduce theoretically optimal weighting of forecasts, which relies on the covariance structure of forecast errors from different models. On the other hand, we employ a simple average which is particularly easy to implement and no additional information is needed for the computation of weights. Note that all remaining weighting schemes discussed in the literature rely on some form of trade–off between the advantages of these two schemes.

Optimal weighting scheme Assuming a MSE based loss function that exclusively depends on the forecast error of the pooled forecast, $e^c = y_{t+h} - g(y_{i,t+h};\omega)$, optimal weights are chosen to solve the problem:

$$\boldsymbol{\omega}^* = \arg\ \min_{\omega}[\boldsymbol{\omega}'\boldsymbol{\Sigma}_e\boldsymbol{\omega}], \tag{3.19}$$

which gives

$$\boldsymbol{\omega}^* = \frac{\boldsymbol{\Sigma}_e^{-1}\mathbf{1}_N}{\mathbf{1}_N'\boldsymbol{\Sigma}_e^{-1}\mathbf{1}_N}. \tag{3.20}$$

where Σ_e denotes the covariance matrix of the forecast errors e_i of the single models. In practice, the elements of Σ_e are unknown and have to be estimated which introduces an additional source of uncertainty. Imprecise estimates of Σ_e potentially deteriorate forecast performance.

Equal weighting scheme Another straightforward pooling approach is the use of equal weights, which particularly solves the estimation problem. In empirical applications, simple averages of forecasts tend to outperform more elaborated weighting schemes, a phenomenon often referred to as *forecast combination puzzle* (see e.g.

[8]For a more detailed description of different pooling of forecast strategies, see Section 1.3.5 in Chapter 1.

Stock and Watson (2004)). As Timmermann (2006) shows, equal weights are theoretically optimal if the individual forecast errors have the same variance and identical pair–wise correlations, i.e. $\boldsymbol{\omega}^* = \frac{1}{N}\boldsymbol{\iota}$, where $\boldsymbol{\iota}$ is an $N \times 1$ column vector of ones. This case gains relevance by the fact that forecasts of a certain variable based on differently specified VAR models often show similar characteristics. As forecasts converge to the unconditional mean of the process when the forecast horizon grows, the respective moments, variances and pair–wise correlations of the forecast errors also converge.

3.4.3 Monte Carlo Simulations

We employ each of the DSGE Models described in Section 3.3 to simulate the path of the six observable variables (y, r, n, w, π, π^w) for 1100 periods. The Monte Carlo experiment relies on 10000 replications. Each time, we draw $m = 1, \ldots, M$ random bivariate VAR models from all possible bivariate VARs. The respective model is estimated from a sample of length T. We then derive $h = 1, \ldots, 100$ step ahead forecasts of the target variable y from each single VAR ($m = 1$) as well as from $m = 2, \ldots, M$ pooled VAR forecasts. As the forecast exercise builds on the assumption that VAR operators are stable and forecasts are stationary, we exclude iterations that yield estimated VAR models with unstable roots.[9] Single VAR forecasts are pooled employing equal weights as well as optimized weights calculated from the estimated covariance matrix of forecast errors as already described in Section 3.4.2. In the latter case, for each forecast horizon h, the covariance matrix is estimated from a subset of N forecast errors of the total of 10000 draws. This keeps the computational effort tractable and makes it possible to simulate weights for different degrees of estimation uncertainty by varying N. To rule out randomness when estimating the weights, we also draw the subset N 1000 times, which ensures that weights converge to their average values.

In a second step, we assume that macroeconomic aggregates are imperfectly mea-

[9]Note: If an unstable root is detected in any of the estimated single VAR models, the entire draw is excluded from the analysis.

sured. This means that observed values are noisy indicators – e.g. some sort of business or consumer confidence – of the underlying economic activity and useless information enters the procedure. In fact, we inflate the variables by measurement error. To be more precise, we define a signal–to–noise ratio s that relates the variance of the respective economic variable to the variance of the added measurement error. This introduces another dimension to the simulations. Theoretically, s might run from infinity (no measurement error) to zero (infinite variance of the measurement error). For the analysis we add values ranging from $s = 10^4$ (the signal variance is 10000 times the error variance) to $s = 10^{-6}$ (the signal is completely overlaid with noise).

To quantify the virtues of a reduction of the mis–specification problem, as introduced in Section 3.2, we compare the performance of a randomly chosen single VAR to the performance of a pooled prediction from m randomly chosen VARs with $2 \leq m \leq M$. The random draw of one or more forecasts from a broader set reflects the situation of a practical forecaster who is faced with choosing between a possibly large number of different reasonable models and thus forecasts. [10]

3.5 Discussion of the Results

3.5.1 Forecast Errors from Single VARs

To receive a first impression of the results, we analyze forecast errors from single models. For a deeper understanding of the characteristics of forecast errors, it seems worthwhile to analyze their structure by breaking down the MSE into three

[10] In empirical applications, variable selection is frequently based on some form of information criterion. We do not pursue this approach here, because different criteria might indicate different specifications and, as a consequence, results are distorted by the performance of the respective selection criterion. Moreover, in such a framework, one cannot ignore the information supplied by the selection criterion when implementing the pooling approach. In effect, one could use the ranking of forecasts delivered by the selection criterion to overweight outperforming models in the pooling approach. In Section 3.5.5 we look a little closer at this issue and use R^2 to select the benchmark model.

components: the bias part, the variance part, and the covariance part. Following Theil (1966), the h–step forecast MSE can be written as:

$$MSE_{t+h} = (\bar{\hat{y}}_{t+h} - \bar{y}_{t+h})^2 + (\sigma_{\hat{y}_{t+h}} - \sigma_{y_{t+h}})^2 + 2(1-\rho)\sigma_{\hat{y}_{t+h}}\sigma_{y_{t+h}}, \quad (3.21)$$

where $\hat{y}_{t+h}$ and y_{t+h} are the forecast and actual values of some variable y for period $t+h$, $\bar{y}_{t+h}$, $\bar{\hat{y}}_{t+h}$, $\sigma_{y_{t+h}}$ and $\sigma_{\hat{y}_{t+h}}$ are the respective mean and standard deviation. The correlation between them is ρ.[11] The first term on the right hand side shows the deviation of the forecast mean from the mean of the actual series and is identified as the bias part. In the present setup, we can neglect the bias part because it is virtually zero. The second term, which is labeled the variance part, reports the deviation of forecast variation from the variation of the actual series. The last term on the right hand side is named the covariance part and reflects the general co–movement of forecast and realized values.

As Table 3.2 in Appendix 3.B.1 shows, the MSE decreases with an increasing estimation sample size T. This is, of course, due to a decline of estimation uncertainty associated with the forecast. With T growing from 25 to 1000, the reduction in the MSE is quite considerable and converges to about 17 percent (*Model I* and *II*) and 19 percent (*Model III*) for $h = 1$. As the forecast horizon grows, benefits from precisely estimated model coefficients accumulate to around 40 percent (*Model I* and *II*) and 35 percent (*Model III*) for $h = 100$. Comparing the predictability of the three models, the levels of MSE decrease from *Model I* to *Model III* independent of the estimation sample size and the forecast horizon.[12] Figure 3.3 exemplarily illustrates the development of the MSE components mentioned above for different DGPs and $T = 1000$.

The MSE of a single randomly chosen VAR rises with the forecast horizon for all DGPs. Of course, this is in line with theoretical considerations, as the MSE should converge to the variance of the process if forecasts are unbiased. The covariance

[11] For the derivation of the above expression see Appendix 3.A.2.

[12] This hints at the fact that a forecasting exercise is more difficult in less flexible economies because the variation of the target series and, hence, the unavoidable part of the MSE tends to be larger.

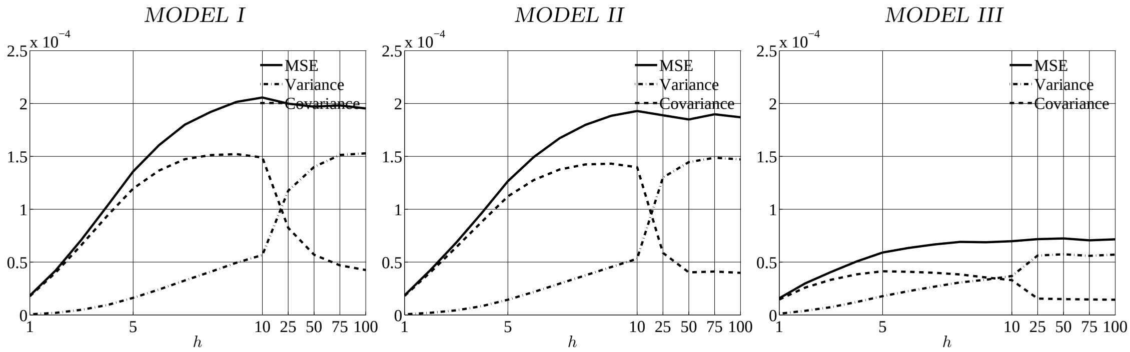


Figure 3.3: MSE and its components for a single VAR forecast ($T = 1000$)

part, though, does not increase monotonically with the forecast horizon. For larger estimation samples it shows a peak during the first 10 forecast periods and declines to some lower values afterwards. The larger the estimation sample T, the earlier is the turning point and the steeper is the decline of the covariance part as the horizon h grows. This movement can be explained by the ambivalent effect of converging forecasts. As equation (3.21) shows, on the one hand, the covariance part increases with a growing forecast horizon as the correlation ρ between the predicted and the actual values decreases. On the other hand, the standard deviation of the predicted values $\sigma_{\hat{y}_{t+h}}$ decreases for larger values of h reducing the covariance part. As can be inferred from the second and third column of Table 3.2 in Appendix 3.B, for shorter horizons up to about $h = 10$ (*Model I* and *II*) and $h = 3$ (*Model III*), the rise of the MSE is largely attributed to a rise of the covariance part. In contrast, the variance part increases monotonically until a certain level is reached. In the absence of estimation uncertainty, its contribution to the MSE eventually dominates that of the covariance part for horizons greater than $h = 19$ (*Model I*), $h = 17$ (*Model II*) and $h = 10$ (*Model III*). Thus, if we wish to forecast on a business cycle frequency, we should make an effort to reduce the covariance part of the forecast errors in the first place. In fact, this is done by pooling of forecasts. In contrast, for longer horizons the variance part plays the crucial role as far as forecast accuracy is concerned.[13]

[13]For details refer to results presented in Section 3.5.2.1.

3.5.2 Pooling and Perfectly Measured Economic Variables

In the following, we present results for pooling of forecasts in case only relevant information enters the candidate models. In effect, this coincides with a situation where only variables are considered which are indeed useful for forecasting as they emerge from the DGP. In particular, variables are perfectly measured and there is no noisy information. Tables 3.3 to 3.5 in Appendix 3.B.2 show the results of Monte Carlo simulations for *Model I* to *Model III* in turn. Columns titled 'AVERAGE' correspond to equal weighting of forecasts whereas 'OPTIMAL WEIGHTS' refers to the optimal weighting scheme.

3.5.2.1 Equally Weighted Forecasts

We first focus on equally weighted forecasts and look at the performance relative to a single VAR. The percentage changes in the MSE are given with respect to four dimensions: sample size T, forecast horizon h, number of pooled forecasts m and the type of the simulated economy (*Model I* to *Model III*). A negative value, thus, represents an improvement in forecast accuracy.[14] To clarify how the virtues of pooling develop, we also present the respective contribution of the variance and the covariance part to the total MSE change.

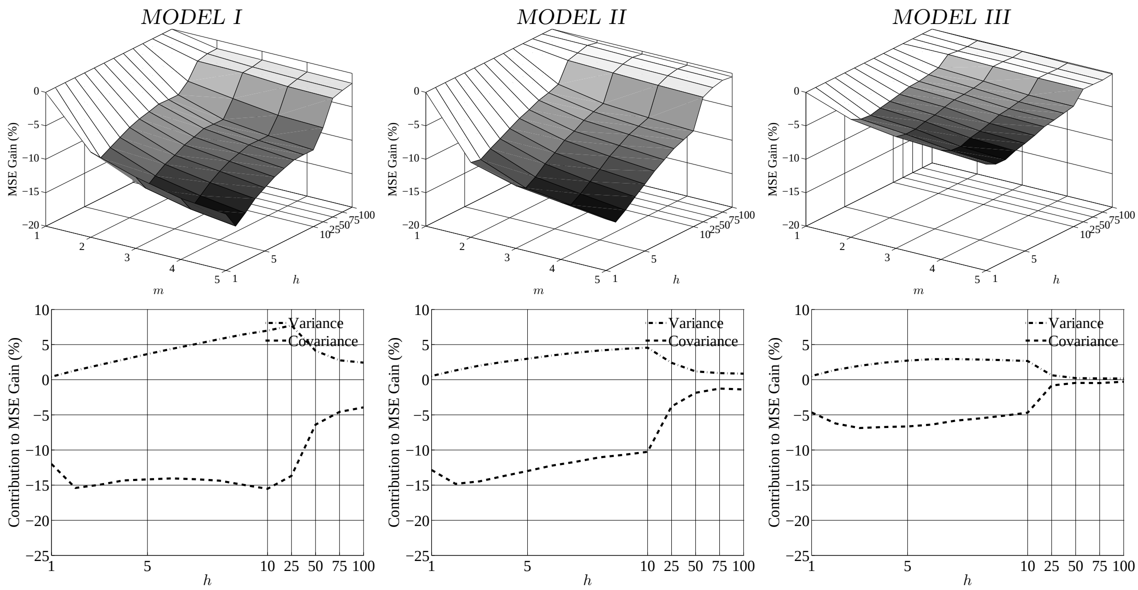
It turns out that even the small number of five pooled VAR forecasts yields a significant reduction of the MSE. The results are very much in line with theoretical gains described in Section 3.2, as a combination of only $m = 4$ single predictions already guarantees the major proportion of improvement. Most notably, for shorter horizons, when combining $m = 5$ forecasts the decline of the MSE is comparable to the decline achieved by an extension of the sample size for a single VAR from $T = 25$ to $T = 1000$.[15] Thus, the gain from pooling is approximately comparable to the abolition of estimation uncertainty. This has practical relevance when forecasting quarterly macroeconomic aggregates, as one usually has to deal with a maximum number of observations no more than 200.

[14]The presentation, here follows equation (3.3) in Section 3.2.
[15]Compare the results presented in Section 3.5.1.

Figure 3.4 exemplarily illustrates the reduction of the MSE when combining $m = 5$ VARs for $T = 100$ as the forecast horizon h grows. It also gives the corresponding contributions of the variance and covariance part to the MSE change respectively.

In terms of the variance part, pooling is clearly not favorable as it reduces the variation of the forecast compared to the single VAR prediction. Although this is intended in most applications – because pooling is often seen as an insurance against extreme forecast errors – it leads to a positive contribution to the MSE change. By contrast, pooling considerably reduces the covariance part. This over–compensates for the increase of the variance part and leads to a decline of the total MSE, as the covariance part dominates for VAR forecasts in absolute terms. With respect to different DGPs, reduction in the MSE is considerably higher for *Model I* and *Model II* than for *Model III*. Note that gains are remarkably persistent for *Model I* and prevail until $h = 100$. In more flexible economies like *Model III*, benefits die out for $h > 25$. This indicates that pooling of forecasts is particularly beneficial in less flexible economies.

As presented in Tables 3.3 to 3.5 in Appendix 3.B.2, we find that the improvement is higher for smaller samples. This holds for all DGPs. If we consider $T = 25$, we find that the MSE reduction due to pooling of five VARs ranges from 17 percent (*Model I*) to 9 percent (*Model III*) for $h = 1$. It becomes clear that the gain is quite substantial and it increases with the number of pooled forecasts. With growing estimation samples, gains decrease and reach 12 percent and 4 percent for $T = 100$, respectively. And even for $T = 1000$, i.e. when there is virtually no estimation uncertainty, the gain still amounts to 9 percent (*Model I*) and 3.5 percent (*Model III*). It is worth noting that, irrespective of the DGP and the estimation sample size, the gain reaches a maximum for $h = 2$ and gradually decreases with the forecast horizon. This result supports the findings of equation (3.3), as the average bivariate covariance of the VAR forecast errors is minimal for $h = 2$ in the present study. Whereas gains remain significant for small samples, for larger estimation samples they vanish as h increases. Most notably, as there remains a small sample bias in the estimation of the unconditional mean for each model, the gain does not disappear for $T < 1000$. Thus, averaging long–term forecasts can



Note: The upper panel depicts MSE gain compared to a single VAR for $m = 1, \ldots, 5$ and $h = 1, \ldots, 100$. For better readability, the lower panel concentrates on contributions of variance and covariance to total MSE change for $m = 5$ VARs.

Figure 3.4: MSE gains from pooling of $m = 5$ VARs for $T = 100$.

be regarded as an insurance against a mis–estimation of the unconditional mean of the DGP. On the contrary, when the estimation sample is large, all forecasts of bivariate VAR(1) processes asymptotically converge to an unconditional mean of zero and thus, pooling these predictions does not reduce the MSE compared to single forecasts. Regarding the number of pooled predictions, the variance part of the MSE increases and the covariance part decreases monotonically with m. The reduction of the total MSE due to pooling can thus be attributed to a higher covariance of the forecast and the target variable.

3.5.2.2 Optimally Weighted Forecasts

We now turn to optimized weights which adds an additional dimension to the analysis. Here, we present gains for different numbers N of forecast errors used to estimate the covariance matrix Σ_e. N is referred to as the size of the optimization window and grows from $N = 10$ to $N = 1000$ observations. In a practical forecast exercise, one has to rely on ex–post forecast errors to estimate Σ_e. Thus, the total length of the data set available has to be split into an estimation sample used to estimate the models' coefficients and an optimization window used to estimate Σ_e. As the total number of observations is usually limited when forecasting macroeconomic aggregates, sizes of the optimization window of $N = 10$ to $N = 50$ gain practical relevance. In contrast, a size of $N = 1000$ imitates a situation without estimation uncertainty regarding Σ_e and, thus, estimated optimal weights will approximate their true values.

From Tables 3.3 to 3.5 in Appendix 3.B.2, it becomes clear that optimal pooling benefits from an increasing N. This holds for all sizes T and for all forecast horizons h. For $N = 10$, optimal pooling yields a MSE way above its single VAR counterpart for all models and all estimation samples. The relative inferiority increases with the forecast horizon. For *Model I* and $T = 1000$, it amounts to about 56 percent for $h = 1$ and reaches about 100 percent for $h = 100$. This most likely explains empirical studies finding that pooling forecasts based on optimized weights frequently yields a poor forecast performance.[16]

At shorter horizons, the optimal pooling approach dominates the single VARs for $N = 25$ ($N = 50$) and beats the equally weighted average for $N = 50$ ($N = 100$) for *Model I and II* (*Model III*). However, these gains fade away for longer horizons. When there is no estimation uncertainty regarding the optimal weights, i.e. for $N = 1000$, optimally pooling 5 single VAR forecasts improves forecast accuracy for $h = 1$ compared to a single forecast by about 20 percent on average for *Model I* and *II* and by about 10 percent on average for *Model III*. These gains slightly increase for $h = 2$ and then decline with the forecast horizon. However, even for $h = 10$, an improvement of about 10 percent on average for *Model I* and *II* and of about 3 percent on average for *Model III* remains.

In the absence of estimation uncertainty with respect to Σ_e, benefits of using optimized weights mainly result from a reduction of the covariance part of the MSE. Given that the estimated parameters of Σ_e approximate their true values, the covariance part is considerably reduced for short– and mid–term forecasts. Hence, for $h = 1$ to $h = 10$ it decreases by about 20 percent (10 percent) for *Model I* and *II* (*Model III*).

The variance part is slightly smaller for optimized weights when compared to a simple average for small values of h, but it increases faster as h grows. Interestingly, we observe, that a larger optimization window N comes along with a larger variance part. However, it converges to the value of the benchmark approach only in the absence of uncertainty about Σ_e. For values of $N \leq 200$ the variance part stays below these levels even for $h = 100$. Hence, employing optimized weights in practice, i.e. for $N \leq 50$, likely reduces the variance part of the forecast errors provided that the underlying estimation samples are sufficiently long. Thus, the gain, if any, from estimating optimal weights in practice most likely originates from a reduction of the variance part.

Having two different pooling approaches at hand, we now further evaluate which

[16] Starting their recursive optimization window with a size of $N = 17$, Clark and McCracken (2009a) find that pooling based on optimized weights yields a root mean squared forecast error (RMSE) twice as high as an equally weighted average.

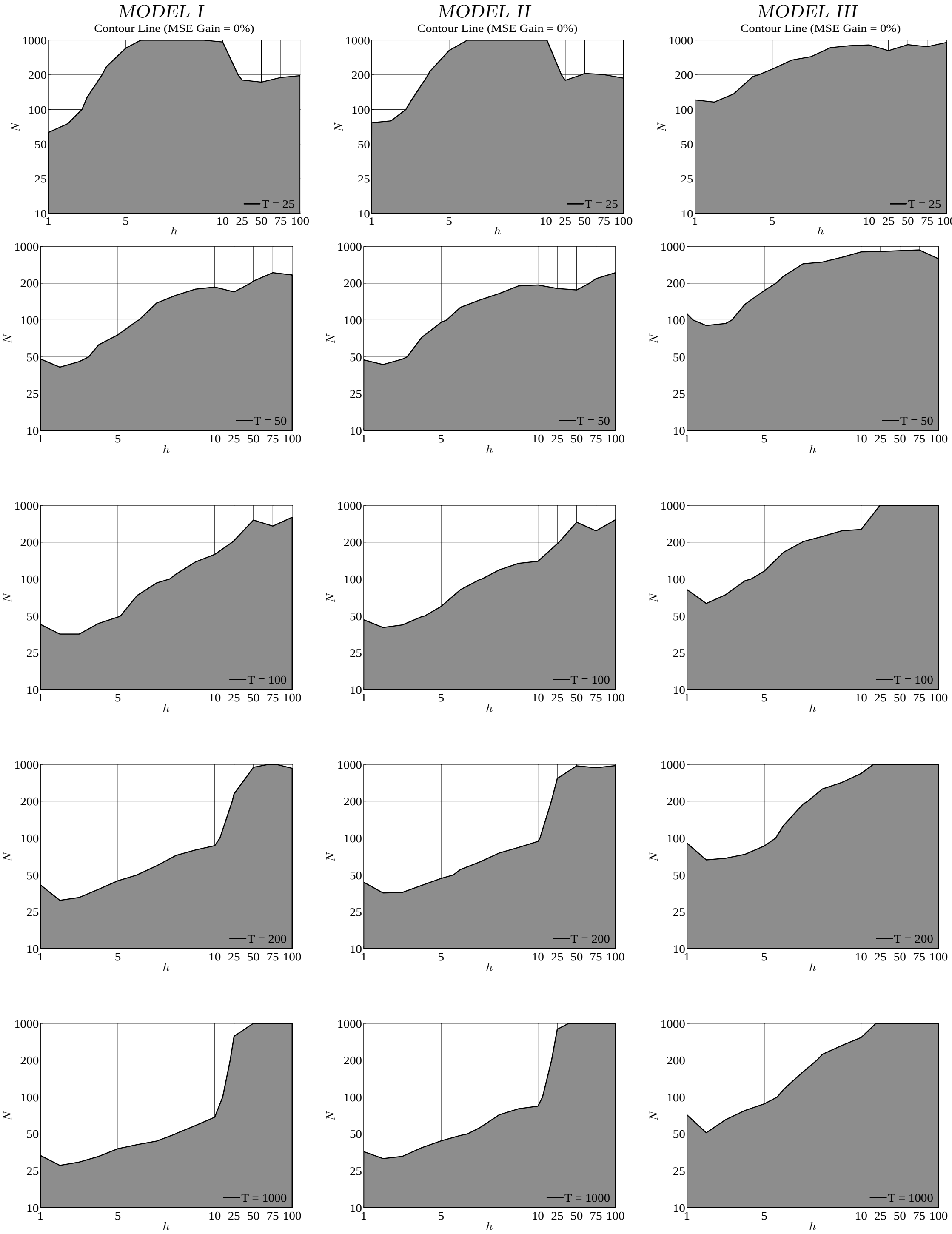
of the two dominates under certain conditions. This can be done by comparing relative gains of optimal to equally weighted predictions. In order to save space, we restrict this analysis to $m = 5$ pooled forecasts. A graphical representation is given in Figure 3.5 by way of a contour plot.

The displayed line marks those combinations of h and N which lead to an equal performance of both approaches in terms of MSE gain. The simple average dominates for parameter combinations that fall into the gray area.[17] For all values of T, the frontier shows a positive relationship between h and N, i.e. the larger the forecast horizon, the larger the optimization window N needs to be to dominate the equal weighted average. As a rule of thumb, the use of optimized weights based on the covariance matrix of the forecast errors seems advisable only for an adequate length of the optimization window of $N \geq 50$ and only for predictions up to $h = 10$. This holds for all DGPs analyzed here. For larger values of h, the equally weighted average clearly dominates the more enhanced weighting scheme for values of N that have practical relevance. Interestingly, the optimized framework works best for less flexible economies that are harder to forecast in general as represented by *Model I* and *Model II*.

3.5.3 Pooling and Noisily Measured Economic Variables

So far, all observable variables at hand solely carried relevant information. This stacks the cards in favor of pooling as, by construction, all observable variables of the model economy are necessarily useful to forecast y and should thus be considered in a forecast model. In a practical situation, however, the forecaster is faced with observables that do not directly emerge from the DGP. Instead, she most likely uses indicator variables – e.g. business sentiment – to measure the state of a certain

[17] As we alter parameters in discrete steps during the simulation, those combinations that lead to an equal performance are approximated by interpolation in order to obtain a suitable graphical representation of the results. As a consequence, it might happen that whenever the surface of the considered data is flat, then approximated contour lines will show a rather volatile behavior. If a line is not displayed in the respective contour plot this indicates that, for those parameter combinations, the threshold lies beyond the simulated values.



Notes: The figure compares the MSEs of optimized weights versus equal weights for $m = 5$. The gray area describes those parameter combinations of h and N where the simple average dominates the optimized weighting scheme.

Figure 3.5: Contour plots for optimized versus equal weights (m=5)

unobservable economic variable. Hence, this section deals with the question whether pooling of forecasts still has benefits if the additional variables used to forecast the target series are poorly measured and, thus, do not correlate perfectly with their underlying economic counterparts. More precisely, we evaluate how much valuable information has to be present for pooling to carry its virtues. Of course, if all additional variables are pure noise without any correlation to the DGP, then pooling of forecasts will loose its benefits. This relates to the fact that no valuable information enters the system apart from y. Still, it seems worthwhile to estimate the amount of noise that builds the threshold for the benefits of pooling to prevail. As described in Section 3.4.3, the signal–to–noise ratio s thereby measures the variance of an economic variable in relation to the variance of a measurement error. A high value of s corresponds to a precise indicator whereas an imprecise indicator is associated with a low value of s.

3.5.3.1 Equally Weighted Forecasts

As in Section 3.5.2.1, we analyze the performance of an equally weighted forecast relative to a forecast from a single VAR. With the signal–to–noise ratio varying from $s = \infty$ to $s = 10^{-6}$, this adds an additional dimension to our results. Tables 3.6 to 3.8 in Appendix 3.B.3 compare gains of equally pooled forecasts to one randomly chosen VAR for various values of the signal–to–noise ratio in detail. Figure 3.6 illustrates the main findings and shows the relative gains in the MSE of equally pooling $m = 5$ VARs as a function of the signal–to–noise ratio s and the forecast horizon h for a sample size of $T = 100$. A negative value, again, corresponds to an improvement in forecast accuracy.

It becomes visible that the benefit of pooling decreases with a decreasing value of s. However, the MSE gains basically remain unaffected for values of $s \geq 100$. Interestingly, even for a value of $s = 1$, for $h = 1$ the gain is still half the size when compared to a situation without noise. For larger values of s, the benefits decline rapidly. Thus, $s = 1$ poses the threshold value for the benefits of equally pooling short–run forecasts. For larger horizons, the corresponding threshold values

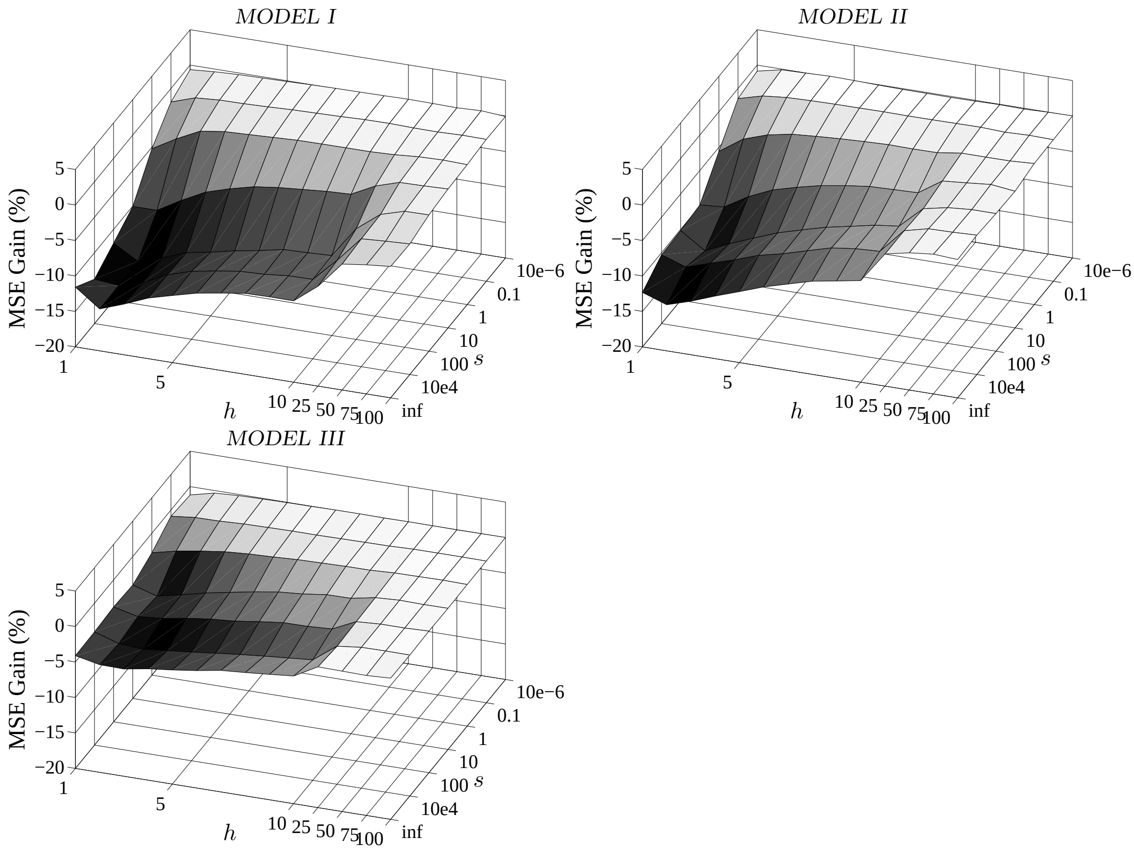


Figure 3.6: Gains from pooling of $m = 5$ VARs with noisy information ($T = 100$)

are higher. This holds for all estimation samples T. For values of s found in practice and very small sample sizes, the gains persist even for $h = 100$. Even for $s = 10^{-6}$, i.e. in a situation where the indicators carry very little information, pooled forecasts outperform the benchmark by about 3 percent on average. Like before, gains are larger for *Model I* and *II* in general. In a nutshell, results indicate that, even in case additional variables carry to some extent useless information, virtues from pooling still prevail.

3.5.3.2 Optimally Weighted Forecasts

We now turn to pooling of forecasts with weights based on the covariance matrix of the errors. As optimal weights are estimated, introducing noise potentially alters the performance of the approach shown in Section 3.5.2.2. Tables 3.9 to 3.11 in Appendix 3.B.3 compare gains of $m = 5$ optimally pooled forecasts to one randomly chosen VAR in the presence of noise in detail. Compared to a single random VAR, it becomes obvious, that a rather large N is needed. For very little noise, results qualitatively coincide with those of Section 3.5.2.2. If, however, there is less information provided by the data, the performance of a single VAR improves relative to optimal weighting. When the signal–to–noise ratio approaches zero, then, eventually, even for $N = 1000$, optimal pooling loses its virtues for almost all sample sizes T we consider. Only for small values of T and short horizons, we obtain an improvement.[18] In other words, when forecasting with noisy indicators, N has to be larger in order to outperform the simple average. We interpret this as the explanation of the so–called *forecast combination puzzle* as described in Section 3.4.2. Illustrating the performance of optimal relative to equal weights for $m = 5$, Figure 3.7 shows the contour lines for a zero gain.

Thus, all points to the lower left of the displayed lines represent combinations of s and h where optimal pooling outperforms the equally weighted average.[19] In general,

[18]Note: In practice, a combination of small T and large N is not of importance as the forecaster would trade–off the estimation window and the optimization window when sample size is limited. However, we present results for these cases for completeness.

[19]For very low signal–to–noise ratios, the performance of both approaches is approximately

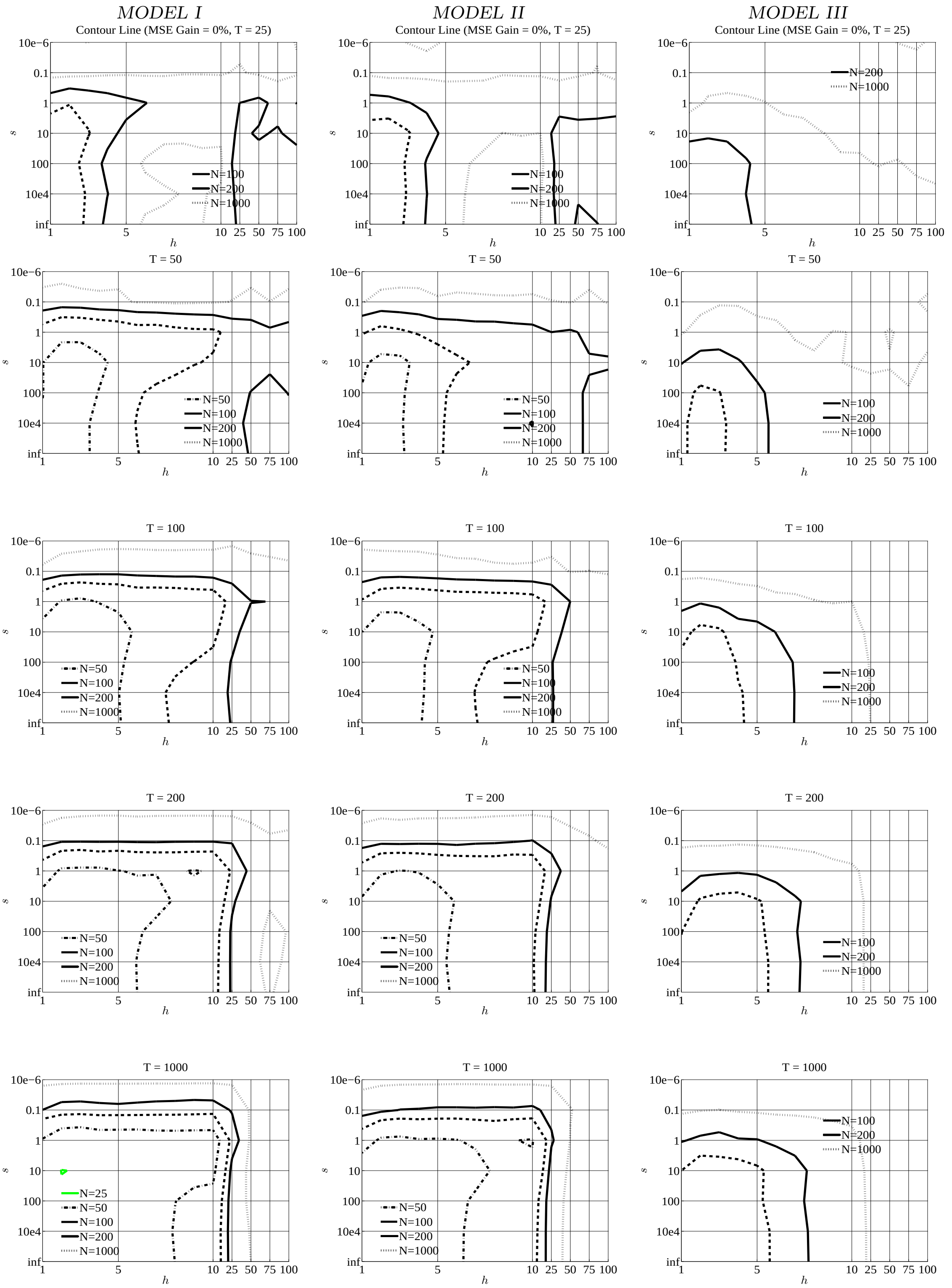
the contour lines show a negative relationship between s and h, i.e. the more noise overlays the signals, the shorter the maximum forecast horizon for optimal pooling to outperform equal weights. As a rule of thumb, a signal–to–noise ratio of $s = 1$ and a forecast horizon of $h = 10$ pose threshold values. For $s > 1$ or $h > 10$, the size N of the optimization window necessary for optimal pooling do predominate exceeds values relevant in practice. This leads to the conclusion that, in practice, equal pooling is more advisable the larger the forecast horizon and the lower the signal–to–noise ratio are.

3.5.4 Incorporating all Variables as Benchmark

In the present section, we simulate a situation without model uncertainty, i.e. where all economic aggregates are known to the forecaster. In such a situation, it would be possible to combine all relevant variables from the DGP into one large model. Naturally, as the large model uses information in an efficient way, pooling forecasts from parsimonious and thus mis–specified VARs is asymptotically less advantageous. However, when facing a real forecasting situation, such a large VAR simply is infeasible because of the multitude of variables we would have to include. In fact, as in the large model, more parameters have to be estimated from a given number of observations, estimates will be less precise than those of a smaller (subset) model. As a consequence, incorporating all variables can also lead to an inferior performance in finite samples. Also note, that we depart from the proceeding in previous sections when analyzing models with different degrees of estimation uncertainty. In effect, the present section is not a strict quantification of the gains that emerge from pooling of forecasts. It gives rather a hint under which circumstances pooling would lose its virtues. Note, that in our understanding, the large model is not exactly the correct benchmark the pooling approach from bivariate VARs should be contrasted with. It is easy to imagine the use of a number of larger VARs also within the pooling exercise and contrast these with the large model.[20]

identical. This leads to a rather flat surface of the plotted data and translates into a volatile behavior of (interpolated) contour lines.

[20] Compare also Section 3.4.1.



Notes: The figure compares the MSEs of optimized weights versus equal weights for $m = 5$ under noisy information. All points to the lower left of the displayed lines represent parameter combinations of s and h where optimized weights outperform equal weights for the respective value of N.

Figure 3.7: Contour plots for optimized weights with noisy information (m=5)

In detail, we specify the benchmark model as a VAR(1) that consists of all observable variables (y, r, n, w, π, π^w) and compute predictions h steps ahead.[21] Table 3.12 to 3.14 in Appendix 3.B.4 compare the MSE of a single randomly chosen ($m = 1$) and pooled ($m = 5$) bivariate VAR prediction to the MSE of a benchmark model that contains all observable variables for different sample sizes T and signal–to–noise ratios s. Figure 3.8 illustrates the performance of $m = 5$ equally weighted forecasts compared to the large model as a function of h and s for *Model I* to *Model III* and for $T = 25$ (first row) to $T = 1000$ (last row). The contour line shows those combinations of h and s that correspond to an identical performance of both approaches. All points on the white area represent combinations where pooled forecasts are beneficial.

For $T = 25$, the pooling approach outperforms the large model for all combinations of h and s and all models, as all contour lines are off the scale.[22] For T growing from 50 to 1000, the space of advantageous combinations of h and s shrinks successively. Still, for sample sizes relevant in practice ($T < 100$), pooling $m = 5$ forecasts derived from bivariate VARs is beneficial. Only if the number of observations exceeds 100, the large model tends to outperform the pooling approach. Results are similar for *Model II*. In contrast, for *Model III* pooled forecasts dominate the benchmark model even for $T = 1000$. Again, the differences between the approaches die out as the signal–to–noise ratio s decreases. To put it in a nutshell, for sample sizes T relevant in practice, pooling of forecasts from parsimonious and thus mis–specified models poses a competitive approach even if all relevant variables are known to the forecaster. This claim is supported by the fact that in a practical situation, there will be uncertainty about the model and it will be likely that the forecaster omits variables that should be included in the large model.

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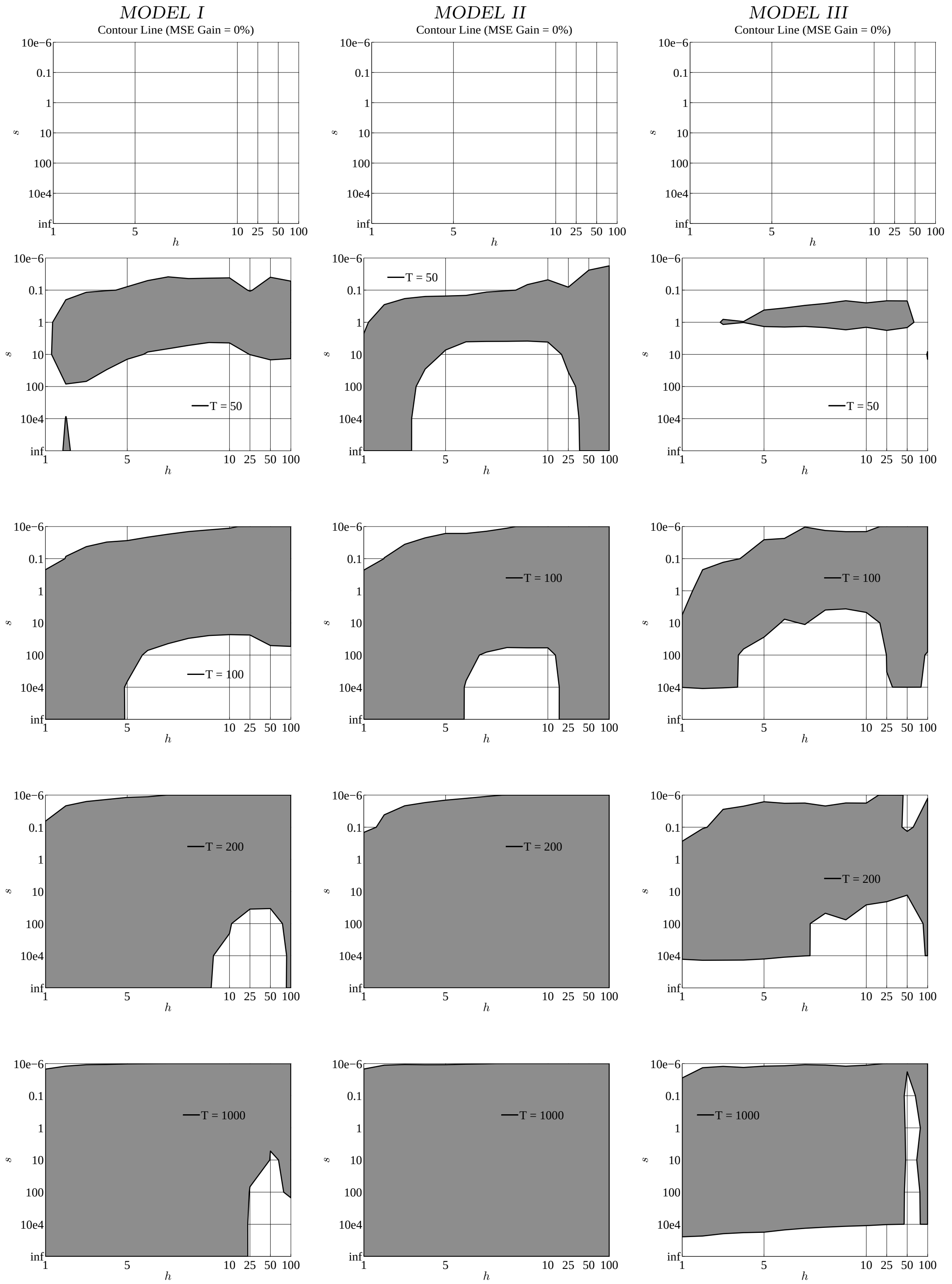
[21] Given the setting of our experiment, we abstract from including higher lag orders in the benchmark process.

[22] For very small samples, even the randomly chosen single VAR model outperforms the benchmark. For more detailed results see Tables 3.12 to 3.14 in Appendix 3.B.4 were we contrast the MSE of one randomly chosen VAR ($m = 1$) and an average of $m = 5$ randomly chosen models to the MSE of the large model for different sample sizes T and signal–to–noise ratios s.



Notes: The figure compares the MSEs of equal weights for $m = 5$ versus a large VAR incorporating all relevant variables. The white area describes those parameter combinations of s and h where the simple average dominates the large VAR.

Figure 3.8: Contour plots for equal weights versus large VAR (m=5)

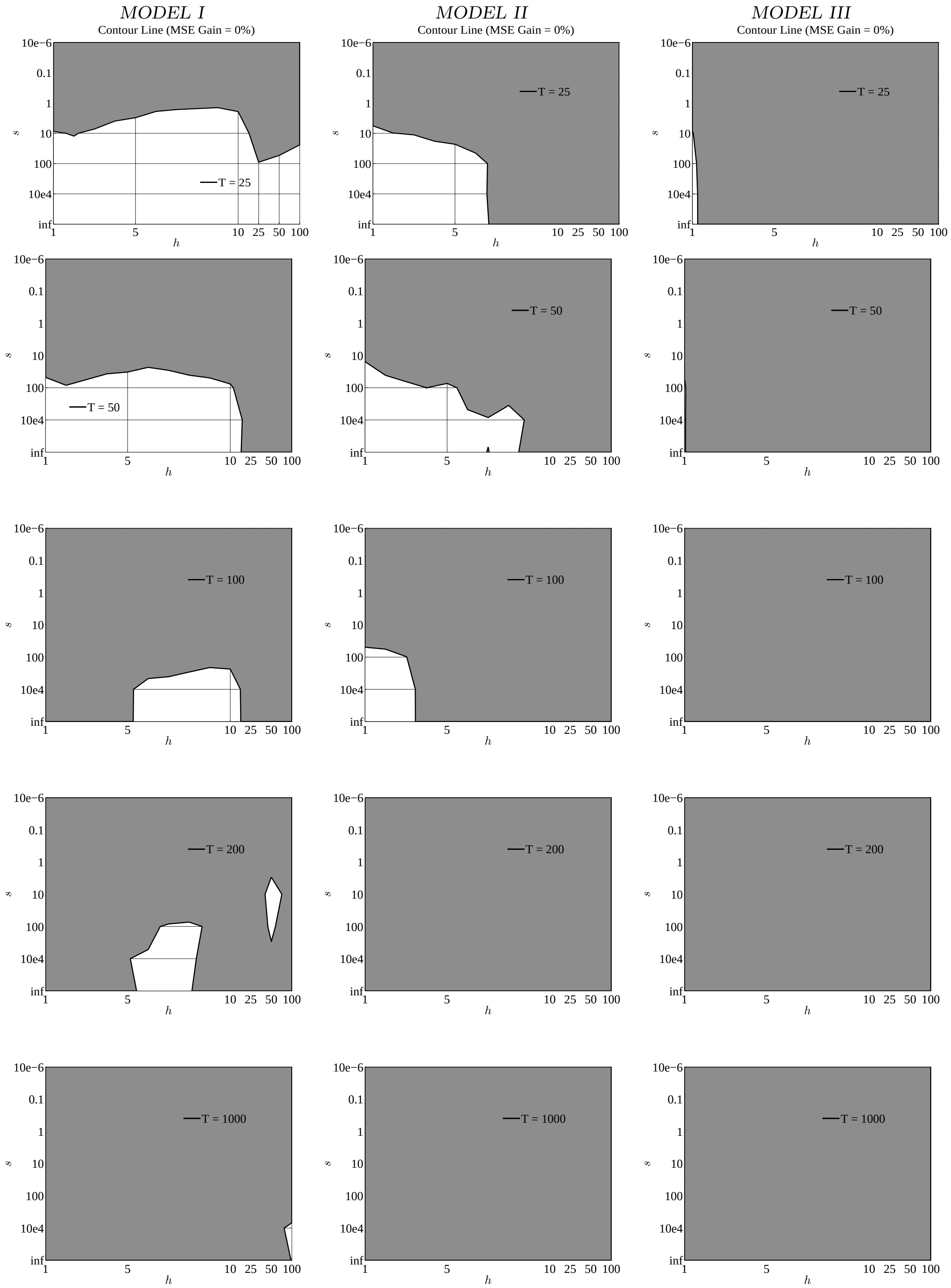
3.5.5 Non–Random Benchmark Models

In practice, the assumption of perfect knowledge of the variables to include in a forecast model is quite unrealistic. As none of the models at hand coincides with the DGP, the decision is one of selecting between mis–specified models which most likely omit relevant information. A researcher would rather choose the set of variables to include in a model with the help of some statistical model selection criterion.[23] The R^2 of the forecast equation of the VAR provides us with a measure of the in–sample fit of the target series. When comparing pooled forecast from randomly chosen models to benchmark models chosen by some form of statistical model selection criterion, the results have to be interpreted differently from previous sections. Gains from pooling cannot be traced back to a broader information base as intended in the present study but are also affected by the performance of the selection criterion.[24] Again, the present section rather gives a hint under which circumstances pooling would lose its virtues.

Tables 3.15 to 3.17 in Appendix 3.B.5 compare the MSE of a single randomly chosen VAR ($m = 1$) and $m = 5$ pooled VAR predictions to the MSE of a benchmark model selected by R^2 for different sample sizes T and signal–to–noise ratios s. A graphical representation is given in Figure 3.9.

[23]Most commonly, for lag length selection, Akaike's Information Criterion (AIC) and the Bayesian Information Criterion (BIC) are applicable. These criteria refer to all equations of the system simultaneously. More precisely, both criteria build on the full system log likelihood that depends on the determinant of the residual covariance. However, in the present setup, additional variables in the VAR are characterized by different variances. Hence, calculated BIC or AIC values are not comparable across different bivariate VARs and a selection based on these values potentially leads to models that poorly perform in forecasting the target series.

[24]In fact, selecting models according to their in–sample performance can be understood as a specific form of weighting scheme. The selected model is given a weight of one, whereas all competing models are given a weight of zero. Alternatively, one could think about implementing a weighting scheme according to R^2. Attaching a non–zero or, say, declining weight to competing models would leave us with some sort of trimming of pooled forecasts in the sense of Aiolfi and Timmermann (2006) or Granger and Jeon (2004).



Notes: The figure compares the MSEs of equal weights for $m = 5$ versus a VAR chosen by means of the R^2 criterion. The white area describes those parameter combinations of s and h where the simple average dominates the R^2–selected VAR.

Figure 3.9: Contour plots for equal weights versus R^2 selected VAR (m=5)

White areas mark situations in which pooling is still beneficial. It becomes clear that the benefits from equally weighted forecasts still prevail for smaller samples up to $T = 50$ for *Model I* and *Model II* and $T = 100$ for *Model III*. Moreover, they tend to increase with a growing forecast horizon.[25] This means, that pooling forecast from a larger number of reasonable VARs tends to dominate the selection of one "best" model via model selection criteria.

3.6 Conclusion

This paper employs a Monte Carlo study based on a standard DSGE model to quantify the gains from pooling of forecasts. Given strict lab conditions, we identify the virtues of pooling and explicitly exclude any accidental effects – present in empirical applications – that may bias the results in favor of combination approaches. Thus, reported gains should be realizable in almost any forecast situation encountered in practice. Built on simulated data sets, we specify parsimonious VAR models to derive h–step ahead predictions of output. Single forecasts are combined using equal weighting as well as theoretically optimal weighting as two boundary cases. In addition, forecast errors are decomposed into variance and covariance part. Given our setup, we identify constellations where pooling yields a higher gain in forecast accuracy and show how different combination schemes work under certain economic structures. Additionally, we show how the number of pooled forecasts effects the performance and analyze the size of the gains as the forecast horizon and estimation sample size vary.

Allowing for noisy information, we analyze how the gains of the different pooling schemes are effected by noise and how much valuable information has to be present for pooling to carry its virtues. We thus estimate the signal–to–noise ratio that builds the threshold value for the benefits of the different pooling schemes. Moreover, we simulate the hypothetical scenario that a forecaster knows with certainty

[25] Choosing according to R^2 minimizes the 1–step ahead prediction error. Given an iterative procedure to generate h–step ahead forecasts, not taking into account the fit of additional equations translates into higher forecast errors as the horizon increases.

which variables carry relevant information. Hence, we compare the performance of pooled forecasts from parsimonious models to benchmark VARs including all relevant variables. Finally, we check if gains from equally weighted forecasts prevail if the benchmark model is chosen by a model selection criterion.

Our results show that pooling forecasts is clearly beneficial. Despite the fact that the gain in forecast accuracy increases with the number of forecasts pooled, we find that a combination of about four predictions is sufficient to achieve most of the possible gain. Most notably, the decline in the MSE is comparable to the decline achieved by a considerable enlargement of the length of the estimation sample. This gains practical relevance, as one usually has to deal with a rather limited number of observations yielding a relative high estimation uncertainty when forecasting macroeconomic aggregates. Regarding the structure of the underlying DGP, the largest benefits are achieved for rigid economies that are harder to forecast in general. Interestingly, the benefits reach a maximum for the 2–steps ahead predictions and decrease with a growing forecast horizon. However, in finite estimation samples, they remain significant even for very long horizons reflecting the estimation uncertainty regarding the unconditional mean of the process. We find that – under lab conditions – pooling leads to a substantial reduction of the MSE of up to 20 percent. Decomposing the MSE, results show that gains due to pooling mainly reflect a better forecast performance with respect to the covariance part whereas the variance part of the MSE increases. We show that the estimation of optimized weights built on the covariance matrix of the forecast errors yields a substantial improvement over a single model only in the absence of corresponding sample uncertainty. For reasonable sizes of the underlying optimization window, estimation uncertainty is too large for benefits of optimal pooling to outperform equal weights. This leads to the conclusion that optimal pooling can be discarded for most practical applications.

Allowing for noisy information, we find that a signal–to–noise ratio of $s = 1$ poses a threshold value for pooling to carry on its virtues. Hence, in case additional variables carry to some extent useless information the virtues from pooling still prevail. We also find that, for reasonable sample sizes T, pooling the forecasts from randomly selected, parsimonious VAR models tends to outperform forecasts from models that incorporate all relevant variables. For small samples and little noise, pooling by equal weights dominates even the “best” bivariate model as selected by R^2. To sum up, the use of equally weighted VAR predictions provides an easy to implement forecast approach that most likely improves accuracy substantially.

Appendix

3.A Theoretical Deviations

3.A.1 Quantification of the Gain from Pooling of Forecasts

The expected error variance $E(\sigma_m^2)$ of an equally weighted average of a set of m randomly selected single forecasts can be derived as follows:

$$\begin{aligned}
E(\sigma_h^2(m)) &= E[var(e_h(m))] \\
&= var(E[e_h(m)]) \\
&= var(\tfrac{1}{m}\textstyle\sum_{i=1}^{m} e_h(i)) \\
&= \tfrac{1}{m^2} var(\textstyle\sum_{i=1}^{m} e_h(i)) \\
&= \tfrac{1}{m^2}(\textstyle\sum_{i=1}^{m} var(e_h(i)) + 2\sum_{i=1}^{m-1}\sum_{j=i+1}^{m} cov(e_h(i), e_h(j))) \\
&= \tfrac{1}{m}\overline{\sigma}_h^2 + \tfrac{2}{m^2}\textstyle\sum_{i=1}^{m-1}\sum_{j=i+1}^{m} cov(e_h(i), e_h(j)) \\
&= \tfrac{1}{m}\overline{\sigma}_h^2 + \tfrac{2}{m^2}\tfrac{m(m-1)}{2}\tfrac{\sum_{i=1}^{m-1}\sum_{j=i+1}^{m} cov(e_h(i), e_h(j))}{\frac{m(m-1)}{2}} \\
&= \tfrac{1}{m}\overline{\sigma}_h^2 + \tfrac{m^2-m}{m^2}\overline{\vartheta}_h \\
&= \tfrac{1}{m}\overline{\sigma}_h^2 + \tfrac{m-1}{m}\overline{\vartheta}_h
\end{aligned}$$

3.A.2 Decomposition of the MSE

The mean squared error (MSE) can be derived as follows:

$$\begin{aligned}
MSE_{t+h} &= E[e^2_{t+h}] \\
&= (E[e_{t+h}])^2 + var(e_{t+h}) \\
&= (E[\hat{y}_{t+h} - y_{t+h}])^2 + var(\hat{y}_{t+h} - y_{t+h}) \\
&= (E[\hat{y}_{t+h} - y_{t+h}])^2 + \sigma^2_{\hat{y}_{t+h}} + \sigma^2_{y_{t+h}} - 2\rho\sigma_{\hat{y}_{t+h}}\sigma_{y_{t+h}} \\
&= (E[\hat{y}_{t+h}] - E[y_{t+h}])^2 + \sigma^2_{\hat{y}_{t+h}} + \sigma^2_{y_{t+h}} - 2\rho\sigma_{\hat{y}_{t+h}}\sigma_{y_{t+h}} \\
&= (E[\hat{y}_{t+h}] - E[y_{t+h}])^2 + (\sigma_{\hat{y}_{t+h}} - \sigma_{y_{t+h}})^2 - 2\rho\sigma_{\hat{y}_{t+h}}\sigma_{y_{t+h}} + 2\sigma_{\hat{y}_{t+h}}\sigma_{y_{t+h}} \\
&= (E[\hat{y}_{t+h}] - E[y_{t+h}])^2 + (\sigma_{\hat{y}_{t+h}} - \sigma_{y_{t+h}})^2 + 2(1-\rho)\sigma_{\hat{y}_{t+h}}\sigma_{y_{t+h}}
\end{aligned}$$

3.B Results – Tables

3.B.1 Single VARs

		MODEL I														
		MSE (10e-4)					VARIANCE (10e-4)					COVARIANCE (10e-4)				
	T	25	50	100	200	1000	25	50	100	200	1000	25	50	100	200	1000
h	1	0.221	0.200	0.186	0.190	0.183	0.001	0.004	0.003	0.003	0.006	0.220	0.196	0.184	0.187	0.177
	2	0.542	0.485	0.439	0.436	0.421	0.006	0.015	0.013	0.012	0.020	0.535	0.470	0.426	0.424	0.401
	3	0.942	0.843	0.737	0.730	0.712	0.013	0.043	0.028	0.031	0.049	0.928	0.800	0.709	0.698	0.663
	4	1.406	1.217	1.081	1.040	1.032	0.024	0.071	0.056	0.061	0.094	1.382	1.145	1.025	0.980	0.938
	5	1.805	1.518	1.368	1.342	1.357	0.031	0.095	0.091	0.099	0.161	1.774	1.422	1.276	1.243	1.195
	6	2.191	1.844	1.637	1.629	1.606	0.040	0.132	0.135	0.150	0.240	2.151	1.711	1.500	1.479	1.366
	7	2.504	2.074	1.869	1.861	1.800	0.045	0.155	0.184	0.215	0.326	2.460	1.917	1.685	1.646	1.473
	8	2.772	2.248	2.045	2.060	1.921	0.051	0.177	0.236	0.289	0.409	2.720	2.071	1.807	1.771	1.512
	9	2.977	2.382	2.150	2.182	2.015	0.059	0.195	0.280	0.350	0.494	2.916	2.186	1.869	1.830	1.521
	10	3.154	2.449	2.227	2.256	2.057	0.069	0.207	0.315	0.411	0.567	3.084	2.241	1.911	1.843	1.490
	25	3.218	2.706	2.351	2.166	1.999	0.061	0.324	0.588	0.801	1.175	3.157	2.382	1.762	1.365	0.824
	50	3.364	2.542	2.218	2.125	1.971	0.057	0.357	0.738	1.042	1.400	3.307	2.184	1.480	1.083	0.570
	100	3.361	2.459	2.316	2.052	1.953	0.038	0.360	0.834	1.094	1.528	3.322	2.100	1.481	0.958	0.425
		MODEL II														
		MSE (10e-4)					VARIANCE (10e-4)					COVARIANCE (10e-4)				
	T	25	50	100	200	1000	25	50	100	200	1000	25	50	100	200	1000
h	1	0.225	0.204	0.195	0.185	0.185	0.002	0.003	0.003	0.004	0.005	0.223	0.202	0.191	0.181	0.180
	2	0.527	0.491	0.452	0.431	0.425	0.006	0.012	0.014	0.015	0.021	0.521	0.479	0.438	0.415	0.404
	3	0.896	0.837	0.753	0.719	0.684	0.016	0.031	0.032	0.037	0.044	0.880	0.806	0.720	0.682	0.640
	4	1.280	1.175	1.057	1.018	0.968	0.025	0.057	0.056	0.071	0.084	1.255	1.118	1.000	0.947	0.884
	5	1.628	1.462	1.323	1.296	1.265	0.034	0.082	0.085	0.113	0.144	1.594	1.379	1.237	1.183	1.121
	6	1.933	1.734	1.568	1.535	1.494	0.041	0.113	0.127	0.170	0.217	1.892	1.619	1.439	1.365	1.276
	7	2.188	1.942	1.792	1.720	1.672	0.043	0.138	0.181	0.236	0.296	2.145	1.803	1.610	1.484	1.376
	8	2.453	2.120	1.966	1.899	1.798	0.055	0.172	0.239	0.311	0.375	2.396	1.947	1.727	1.588	1.424
	9	2.610	2.225	2.061	1.985	1.884	0.060	0.193	0.285	0.370	0.453	2.547	2.031	1.777	1.615	1.431
	10	2.711	2.299	2.111	2.057	1.929	0.063	0.212	0.325	0.441	0.531	2.646	2.086	1.786	1.616	1.397
	25	2.929	2.449	2.192	1.976	1.888	0.062	0.347	0.682	0.919	1.300	2.867	2.101	1.511	1.057	0.588
	50	3.052	2.398	2.118	1.966	1.849	0.075	0.373	0.747	1.042	1.446	2.976	2.023	1.370	0.924	0.403
	100	3.153	2.409	2.163	1.931	1.870	0.076	0.389	0.785	1.038	1.472	3.076	2.018	1.378	0.893	0.398
		MODEL III														
		MSE (10e-4)					VARIANCE (10e-4)					COVARIANCE (10e-4)				
	T	25	50	100	200	1000	25	50	100	200	1000	25	50	100	200	1000
h	1	0.196	0.177	0.167	0.166	0.158	0.005	0.009	0.010	0.011	0.012	0.191	0.168	0.157	0.155	0.147
	2	0.365	0.342	0.312	0.305	0.298	0.014	0.029	0.035	0.035	0.040	0.351	0.313	0.277	0.269	0.259
	3	0.529	0.479	0.437	0.431	0.406	0.022	0.051	0.060	0.071	0.073	0.507	0.428	0.377	0.359	0.332
	4	0.647	0.596	0.535	0.537	0.507	0.028	0.070	0.087	0.117	0.123	0.619	0.526	0.447	0.419	0.384
	5	0.741	0.678	0.600	0.612	0.591	0.035	0.091	0.114	0.160	0.178	0.706	0.586	0.486	0.452	0.413
	6	0.818	0.737	0.663	0.651	0.633	0.040	0.107	0.149	0.193	0.225	0.778	0.629	0.514	0.458	0.408
	7	0.860	0.779	0.732	0.670	0.667	0.039	0.118	0.187	0.221	0.268	0.820	0.660	0.545	0.448	0.399
	8	0.915	0.809	0.762	0.710	0.691	0.044	0.126	0.209	0.256	0.308	0.870	0.682	0.553	0.453	0.383
	9	0.935	0.815	0.755	0.712	0.688	0.047	0.133	0.217	0.272	0.333	0.888	0.682	0.538	0.439	0.355
	10	0.971	0.829	0.751	0.724	0.698	0.051	0.141	0.227	0.297	0.368	0.920	0.687	0.524	0.427	0.329
	25	1.079	0.854	0.784	0.744	0.717	0.050	0.169	0.310	0.411	0.562	1.028	0.685	0.474	0.333	0.155
	50	1.040	0.874	0.770	0.759	0.724	0.045	0.183	0.304	0.426	0.574	0.995	0.692	0.466	0.333	0.150
	100	1.102	0.856	0.796	0.726	0.715	0.047	0.176	0.321	0.406	0.571	1.055	0.679	0.475	0.320	0.144

Table 3.2: MSE decomposition of a single VAR

3.B.2 Perfectly Measured Economic Variables

MODEL I

T=25

	MSE (%) AVERAGE				MSE (%) OPTIMAL WEIGHTS N=10	N=25	N=50	N=100	N=200	N=1000	VARIANCE (%) AVERAGE				VARIANCE (%) OPTIMAL WEIGHTS N=10	N=25	N=50	N=100	N=200	N=1000	COVARIANCE (%) AVERAGE				COVARIANCE (%) OPTIMAL WEIGHTS N=10	N=25	N=50	N=100	N=200	N=1000
m	2	3	4	5	5	5	5	5	5	5	2	3	4	5	5	5	5	5	5	5	2	3	4	5	5	5	5	5	5	5
h 1	-10.21	-14.09	-15.24	-16.60	70.47	-3.98	-15.38	-19.03	-20.78	-22.06	0.13	0.25	0.29	0.30	4.51	-0.08	0.12	0.21	0.26	0.31	-10.34	-14.33	-15.53	-16.92	65.95	-3.92	-15.51	-19.25	-21.05	-22.39
2	-11.46	-16.22	-17.52	-18.84	77.27	-4.72	-16.55	-20.43	-22.27	-23.53	0.59	0.94	1.11	1.19	11.21	0.01	0.53	0.83	1.00	1.11	-12.06	-17.16	-18.63	-20.04	66.05	-4.75	-17.10	-21.27	-23.28	-24.66
3	-10.44	-14.96	-16.41	-17.67	106.69	-0.47	-12.25	-17.08	-18.76	-20.14	0.98	1.49	1.77	1.90	23.22	0.26	0.90	1.46	1.70	1.94	-11.42	-16.44	-18.19	-19.57	83.47	-0.73	-13.15	-18.55	-20.46	-22.07
4	-9.53	-13.27	-14.58	-15.66	133.85	6.82	-8.26	-13.19	-15.33	-16.71	1.31	1.94	2.31	2.46	39.39	0.19	1.07	1.82	2.24	2.53	-10.84	-15.21	-16.88	-18.12	94.45	6.62	-9.33	-15.01	-17.56	-19.24
5	-9.02	-12.27	-13.55	-14.57	158.49	9.27	-6.04	-11.44	-13.48	-14.90	1.50	2.18	2.60	2.76	52.05	0.52	1.11	2.07	2.54	2.90	-10.51	-14.44	-16.15	-17.34	106.43	8.75	-7.15	-13.51	-16.02	-17.80
6	-8.56	-11.45	-12.63	-13.62	184.78	12.56	-3.99	-9.76	-12.00	-13.52	1.63	2.34	2.79	2.96	69.16	0.39	1.08	2.04	2.62	3.08	-10.19	-13.80	-15.43	-16.58	115.58	12.16	-5.07	-11.80	-14.62	-16.61
7	-8.13	-10.58	-11.77	-12.63	189.21	16.10	-2.17	-8.18	-10.80	-12.45	1.66	2.37	2.81	2.97	74.12	0.94	0.84	1.93	2.61	3.13	-9.79	-12.95	-14.59	-15.60	115.04	15.15	-3.01	-10.11	-13.42	-15.59
8	-7.47	-9.68	-10.70	-11.53	237.48	21.00	-0.12	-7.04	-9.67	-11.41	1.69	2.40	2.83	2.99	107.14	1.38	0.65	1.89	2.60	3.17	-9.16	-12.08	-13.55	-14.53	130.22	19.61	-0.78	-8.93	-12.27	-14.58
9	-6.97	-9.02	-9.96	-10.69	287.92	23.81	1.60	-5.77	-8.83	-10.68	1.73	2.45	2.87	3.03	144.40	1.56	0.50	1.71	2.61	3.27	-8.70	-11.47	-12.84	-13.72	143.34	22.23	1.10	-7.48	-11.44	-13.95
10	-6.63	-8.38	-9.18	-9.84	323.81	28.04	3.36	-4.89	-7.88	-9.95	1.78	2.49	2.91	3.07	167.36	2.46	0.34	1.68	2.56	3.34	-8.40	-10.88	-12.10	-12.91	156.19	25.55	3.01	-6.57	-10.45	-13.29
25	-6.80	-8.04	-9.00	-9.34	375.38	30.74	2.50	-6.46	-9.85	-12.12	1.59	2.14	2.37	2.53	215.43	3.48	0.39	1.57	2.54	3.40	-8.39	-10.19	-11.36	-11.87	159.84	27.24	2.10	-8.04	-12.40	-15.53
50	-6.67	-8.34	-9.15	-9.64	429.19	41.05	3.33	-6.55	-10.47	-13.02	1.64	2.29	2.57	2.76	246.18	7.32	0.77	1.81	2.86	3.85	-8.31	-10.64	-11.72	-12.40	182.87	33.71	2.57	-8.36	-13.33	-16.87
100	-6.75	-9.62	-10.12	-10.97	487.27	45.58	4.79	-6.59	-11.07	-13.96	1.59	2.28	2.56	2.75	297.51	9.53	1.30	1.79	2.86	3.89	-8.34	-11.90	-12.70	-13.73	189.60	36.03	3.48	-8.39	-13.94	-17.86

T=50

	MSE (%) AVERAGE				MSE (%) OPTIMAL WEIGHTS N=10	N=25	N=50	N=100	N=200	N=1000	VARIANCE (%) AVERAGE				VARIANCE (%) OPTIMAL WEIGHTS N=10	N=25	N=50	N=100	N=200	N=1000	COVARIANCE (%) AVERAGE				COVARIANCE (%) OPTIMAL WEIGHTS N=10	N=25	N=50	N=100	N=200	N=1000
m	2	3	4	5	5	5	5	5	5	5	2	3	4	5	5	5	5	5	5	5	2	3	4	5	5	5	5	5	5	5
h 1	-7.96	-10.47	-12.18	-12.79	70.14	-2.42	-13.40	-17.27	-18.97	-20.22	0.28	0.60	0.51	0.62	1.76	-0.51	-0.08	0.12	0.19	0.26	-8.25	-11.07	-12.69	-13.41	68.37	-1.92	-13.32	-17.39	-19.15	-20.48
2	-9.10	-12.54	-14.53	-15.15	75.38	-7.94	-17.92	-21.67	-23.38	-24.59	0.87	1.49	1.44	1.63	7.00	-0.46	0.35	0.77	0.93	1.08	-9.97	-14.03	-15.96	-16.79	68.37	-7.48	-18.27	-22.44	-24.31	-25.67
3	-8.39	-11.41	-13.20	-14.09	70.75	-4.69	-15.46	-19.52	-21.26	-22.55	1.55	2.49	2.50	2.78	6.36	-0.54	0.94	1.72	2.04	2.37	-9.94	-13.90	-15.71	-16.87	64.37	-4.16	-16.41	-21.25	-23.31	-24.92
4	-7.40	-10.26	-11.80	-12.72	84.06	-0.02	-11.25	-15.73	-17.57	-18.95	2.00	3.11	3.21	3.52	8.86	-0.48	1.49	2.64	3.17	3.58	-9.40	-13.36	-15.01	-16.23	75.18	0.45	-12.75	-18.38	-20.74	-22.53
5	-7.34	-9.83	-11.36	-12.20	97.01	2.32	-9.61	-13.93	-15.81	-17.16	2.36	3.60	3.78	4.12	17.19	-0.39	2.12	3.34	4.06	4.60	-9.69	-13.42	-15.14	-16.31	79.78	2.69	-11.75	-17.29	-19.88	-21.77
6	-7.17	-9.30	-10.83	-11.49	110.41	5.72	-6.79	-11.58	-13.40	-14.79	2.71	4.07	4.34	4.70	21.30	-0.91	2.13	4.04	4.83	5.56	-9.87	-13.36	-15.17	-16.19	89.09	6.61	-8.93	-15.63	-18.24	-20.36
7	-6.73	-9.01	-10.46	-10.99	127.99	7.88	-5.53	-10.03	-12.10	-13.57	2.94	4.37	4.71	5.10	31.05	-1.01	2.32	4.25	5.42	6.23	-9.65	-13.37	-15.17	-16.08	96.91	8.88	-7.85	-14.28	-17.52	-19.80
8	-6.74	-9.10	-10.21	-10.69	142.45	9.98	-4.28	-9.38	-11.32	-12.93	3.14	4.64	5.04	5.46	43.07	-1.23	2.37	4.90	5.93	7.04	-9.87	-13.73	-15.25	-16.14	99.35	11.20	-6.64	-14.29	-17.25	-19.97
9	-6.69	-9.09	-10.10	-10.53	151.03	10.65	-3.06	-8.68	-10.89	-12.36	3.30	4.83	5.29	5.73	43.25	-1.72	2.32	5.09	6.53	7.65	-9.99	-13.92	-15.39	-16.26	107.74	12.37	-5.38	-13.77	-17.42	-20.00
10	-6.52	-9.03	-9.81	-10.21	136.61	14.20	-3.08	-8.22	-10.45	-11.94	3.45	5.00	5.51	5.96	34.28	-1.76	2.65	5.43	6.90	8.09	-9.97	-14.03	-15.31	-16.18	102.31	15.95	-5.74	-13.65	-17.34	-20.03
25	-5.34	-6.21	-6.97	-7.05	283.31	28.32	3.35	-4.89	-7.71	-9.46	3.34	4.42	4.89	5.20	114.50	-4.90	-1.25	3.09	5.33	7.03	-8.67	-10.63	-11.87	-12.24	168.70	33.22	4.61	-7.97	-13.03	-16.48
50	-3.62	-4.20	-4.48	-4.66	304.24	34.47	7.39	-1.17	-4.54	-6.60	2.65	3.56	3.77	3.98	119.48	-6.83	-3.00	1.42	3.95	5.91	-6.26	-7.75	-8.25	-8.64	184.65	41.31	10.41	-2.57	-8.47	-12.49
100	-2.98	-4.14	-4.38	-4.51	270.26	32.14	6.93	-0.77	-4.02	-6.19	2.53	3.39	3.62	3.81	94.82	-6.95	-2.84	1.21	3.73	5.73	-5.50	-7.52	-7.99	-8.32	175.31	39.07	9.75	-1.99	-7.75	-11.93

T=100

	MSE (%) AVERAGE				MSE (%) OPTIMAL WEIGHTS N=10	N=25	N=50	N=100	N=200	N=1000	VARIANCE (%) AVERAGE				VARIANCE (%) OPTIMAL WEIGHTS N=10	N=25	N=50	N=100	N=200	N=1000	COVARIANCE (%) AVERAGE				COVARIANCE (%) OPTIMAL WEIGHTS N=10	N=25	N=50	N=100	N=200	N=1000
m	2	3	4	5	5	5	5	5	5	5	2	3	4	5	5	5	5	5	5	5	2	3	4	5	5	5	5	5	5	5
h 1	-7.27	-9.36	-10.89	-11.57	66.19	-3.66	-13.92	-17.89	-19.75	-21.10	0.27	0.36	0.44	0.43	1.36	-0.62	-0.41	-0.31	-0.25	-0.21	-7.54	-9.72	-11.32	-11.99	64.82	-3.03	-13.50	-17.57	-19.49	-20.88
2	-8.86	-11.72	-13.22	-14.10	58.12	-8.93	-19.12	-22.64	-24.40	-25.65	0.84	1.10	1.31	1.33	2.68	-0.90	-0.32	-0.06	0.08	0.20	-9.67	-12.79	-14.50	-15.40	55.47	-7.99	-18.75	-22.53	-24.44	-25.80
3	-8.05	-10.62	-11.94	-12.87	66.42	-7.94	-17.64	-21.36	-23.16	-24.46	1.33	1.74	2.04	2.10	6.51	-0.66	0.14	0.62	0.86	1.07	-9.36	-12.33	-13.96	-14.95	59.93	-7.26	-17.75	-21.95	-23.99	-25.50
4	-7.25	-9.63	-10.70	-11.47	64.54	-3.13	-13.60	-17.71	-19.55	-20.93	1.83	2.38	2.77	2.88	4.14	-0.69	0.92	1.82	2.24	2.57	-9.07	-11.99	-13.45	-14.33	60.41	-2.42	-14.50	-19.52	-21.77	-23.48
5	-6.50	-9.00	-9.73	-10.55	95.63	0.08	-10.90	-15.03	-16.81	-18.33	2.33	3.01	3.50	3.65	21.99	-0.43	1.88	3.14	3.80	4.35	-8.80	-11.98	-13.21	-14.18	73.64	0.53	-12.76	-18.15	-20.59	-22.66
6	-5.96	-8.19	-8.97	-9.68	85.60	4.70	-7.39	-11.49	-13.57	-15.08	2.80	3.61	4.19	4.39	9.64	-0.67	2.83	4.39	5.44	6.25	-8.73	-11.77	-13.14	-14.04	75.97	5.38	-10.20	-15.87	-19.00	-21.31
7	-5.59	-7.71	-8.47	-9.11	88.26	7.22	-4.37	-9.66	-11.57	-13.16	3.23	4.15	4.82	5.07	8.82	-0.76	3.05	5.52	6.59	7.60	-8.80	-11.84	-13.27	-14.16	79.43	7.98	-7.41	-15.18	-18.16	-20.75
8	-5.18	-7.20	-7.98	-8.60	107.05	9.72	-3.38	-8.32	-10.38	-11.89	3.69	4.72	5.47	5.78	15.45	-1.63	3.31	6.35	7.82	9.05	-8.85	-11.90	-13.44	-14.37	91.58	11.35	-6.69	-14.67	-18.19	-20.93
9	-5.24	-7.26	-8.02	-8.53	110.63	10.60	-2.66	-7.56	-9.64	-11.29	4.11	5.25	6.08	6.45	14.98	-1.89	3.59	6.87	8.58	10.09	-9.33	-12.49	-14.09	-14.96	95.62	12.48	-6.25	-14.43	-18.22	-21.39
10	-5.26	-7.29	-8.05	-8.54	119.42	11.52	-2.13	-7.13	-9.23	-10.75	4.45	5.67	6.57	6.99	17.95	-2.49	3.69	7.36	9.46	10.94	-9.71	-12.96	-14.62	-15.52	101.43	14.01	-5.83	-14.49	-18.70	-21.69
25	-3.95	-4.89	-5.72	-5.95	213.76	22.29	3.23	-3.15	-5.87	-7.53	5.22	6.33	7.19	7.73	53.56	-10.69	-2.24	4.02	7.83	10.69	-9.18	-11.22	-12.91	-13.68	160.16	32.97	5.46	-7.17	-13.70	-18.21
50	-1.68	-1.75	-2.17	-2.26	211.60	28.68	7.88	1.54	-1.22	-2.96	2.86	3.36	3.96	4.13	33.29	-18.25	-8.01	-1.42	2.47	5.52	-4.54	-5.11	-6.13	-6.39	178.29	46.93	15.89	2.96	-3.69	-8.47
100	-1.26	-1.30	-1.43	-1.49	168.95	25.97	8.60	2.30	-0.23	-2.09	1.64	2.03	2.37	2.44	13.46	-20.50	-10.51	-3.73	0.05	3.41	-2.91	-3.33	-3.80	-3.93	155.48	46.46	19.11	6.03	-0.28	-5.51

Table 3.3: Gains from pooling VAR forecasts – no noise (*Model I*)

		T=200																												
		MSE (%)										VARIANCE (%)										COVARIANCE (%)									
		AVERAGE				OPTIMAL WEIGHTS						AVERAGE				OPTIMAL WEIGHTS						AVERAGE				OPTIMAL WEIGHTS					
						N=10	N=25	N=50	N=100	N=200	N=1000					N=10	N=25	N=50	N=100	N=200	N=1000					N=10	N=25	N=50	N=100	N=200	N=1000
	m	2	3	4	5	5	5	5	5	5	5	2	3	4	5	5	5	5	5	5	5	2	3	4	5	5	5	5	5	5	5
h	1	-6.72	-8.82	-9.88	-10.60	61.39	-3.02	-13.43	-17.29	-19.12	-20.47	0.37	0.40	0.54	0.51	0.54	-0.59	-0.40	-0.27	-0.23	-0.20	-7.09	-9.23	-10.43	-11.11	60.83	-2.44	-13.04	-17.03	-18.90	-20.28
	2	-7.78	-10.64	-12.02	-12.94	55.81	-9.99	-19.53	-23.12	-24.86	-26.08	0.87	1.06	1.32	1.32	2.57	-0.71	-0.21	0.06	0.17	0.27	-8.66	-11.71	-13.35	-14.27	53.22	-9.29	-19.34	-23.19	-25.03	-26.36
	3	-7.20	-9.44	-10.82	-11.89	58.82	-8.10	-17.87	-21.52	-23.19	-24.46	1.37	1.75	2.12	2.16	3.00	-0.52	0.41	0.91	1.14	1.32	-8.58	-11.19	-12.95	-14.06	55.81	-7.59	-18.29	-22.43	-24.34	-25.79
	4	-6.31	-8.61	-9.78	-10.71	67.18	-4.69	-14.54	-17.99	-19.88	-21.21	1.79	2.34	2.79	2.87	3.85	-0.16	1.57	2.31	2.76	3.07	-8.10	-10.95	-12.58	-13.57	63.32	-4.54	-16.11	-20.30	-22.64	-24.27
	5	-5.44	-7.50	-8.43	-9.20	69.80	-0.18	-10.90	-14.69	-16.53	-17.94	2.18	2.89	3.40	3.52	4.72	0.15	2.61	3.69	4.39	4.93	-7.62	-10.39	-11.83	-12.73	65.06	-0.33	-13.51	-18.39	-20.92	-22.86
	6	-4.99	-7.04	-7.80	-8.49	82.42	2.59	-8.47	-12.54	-14.34	-15.77	2.57	3.42	4.00	4.16	6.79	0.19	3.79	5.48	6.51	7.19	-7.57	-10.48	-11.81	-12.67	75.61	2.39	-12.26	-18.02	-20.85	-22.97
	7	-4.74	-6.82	-7.52	-8.09	83.15	4.07	-7.03	-11.30	-13.28	-14.70	3.04	4.07	4.72	4.94	6.63	-0.14	4.45	6.81	8.20	9.20	-7.79	-10.90	-12.25	-13.04	76.49	4.20	-11.48	-18.11	-21.48	-23.90
	8	-4.74	-6.63	-7.29	-7.78	82.12	6.96	-5.40	-9.89	-11.79	-13.24	3.50	4.71	5.44	5.71	2.94	-0.26	4.91	8.16	9.99	11.19	-8.26	-11.36	-12.74	-13.50	79.16	7.22	-10.31	-18.05	-21.78	-24.43
	9	-4.43	-6.28	-7.03	-7.44	87.52	7.67	-4.33	-8.93	-10.86	-12.43	3.95	5.34	6.14	6.47	3.17	-1.27	5.21	9.22	11.45	13.08	-8.41	-11.66	-13.20	-13.95	84.33	8.94	-9.54	-18.15	-22.31	-25.51
	10	-4.34	-6.14	-6.85	-7.28	93.92	7.93	-3.73	-8.19	-10.29	-11.81	4.43	5.99	6.87	7.27	4.69	-1.48	5.69	10.28	12.81	15.07	-8.79	-12.16	-13.76	-14.57	89.21	9.41	-9.42	-18.47	-23.10	-26.87
	25	-3.08	-3.72	-4.25	-4.47	179.81	22.06	3.98	-1.87	-4.14	-5.78	6.09	7.94	9.30	9.95	20.33	-17.03	-4.09	4.28	8.97	13.28	-9.16	-11.65	-13.55	-14.42	159.45	39.08	8.07	-6.16	-13.11	-19.07
	50	-1.00	-1.02	-1.48	-1.44	174.24	28.71	10.06	2.66	0.26	-1.59	2.87	3.54	4.20	4.43	-0.51	-28.46	-15.50	-5.21	-0.29	4.61	-3.87	-4.56	-5.68	-5.88	174.74	57.16	25.56	7.87	0.55	-6.20
	100	-0.44	-0.44	-0.54	-0.55	143.12	22.92	8.43	3.28	0.98	-0.74	0.93	1.02	1.21	1.25	-17.19	-29.53	-16.67	-8.79	-4.03	0.54	-1.37	-1.46	-1.74	-1.81	160.27	52.44	25.10	12.07	5.01	-1.27

		T=1000																												
		MSE (%)										VARIANCE (%)										COVARIANCE (%)									
		AVERAGE				OPTIMAL WEIGHTS						AVERAGE				OPTIMAL WEIGHTS						AVERAGE				OPTIMAL WEIGHTS					
						N=10	N=25	N=50	N=100	N=200	N=1000					N=10	N=25	N=50	N=100	N=200	N=1000					N=10	N=25	N=50	N=100	N=200	N=1000
	m	2	3	4	5	5	5	5	5	5	5	2	3	4	5	5	5	5	5	5	5	2	3	4	5	5	5	5	5	5	5
h	1	-5.69	-8.14	-8.71	-9.31	55.78	-5.42	-14.81	-18.72	-20.29	-21.53	0.15	0.36	0.48	0.50	-1.02	-1.67	-1.43	-1.25	-1.21	-1.18	-5.84	-8.51	-9.19	-9.80	56.79	-3.75	-13.38	-17.47	-19.08	-20.35
	2	-6.85	-9.56	-10.28	-10.93	52.87	-9.57	-18.94	-22.39	-23.96	-25.16	0.56	0.94	1.19	1.25	2.79	-2.12	-1.47	-1.25	-1.10	-1.04	-7.41	-10.50	-11.46	-12.18	50.06	-7.45	-17.48	-21.15	-22.86	-24.13
	3	-6.32	-8.63	-9.45	-10.16	54.45	-8.00	-17.17	-21.08	-22.86	-24.14	1.00	1.52	1.88	2.00	0.15	-2.43	-1.32	-0.84	-0.56	-0.34	-7.33	-10.16	-11.34	-12.17	54.26	-5.60	-15.87	-20.27	-22.33	-23.83
	4	-5.71	-7.61	-8.07	-8.86	51.41	-5.02	-14.86	-18.73	-20.51	-21.85	1.38	1.99	2.45	2.62	-1.67	-2.58	-0.70	0.15	0.57	0.94	-7.10	-9.61	-10.52	-11.48	53.06	-2.46	-14.17	-18.90	-21.10	-22.80
	5	-4.71	-6.18	-6.63	-7.24	70.70	-0.88	-11.52	-15.53	-17.26	-18.71	1.74	2.45	2.99	3.20	2.91	-2.99	-0.26	1.24	2.06	2.59	-6.45	-8.63	-9.61	-10.44	67.77	2.11	-11.26	-16.78	-19.34	-21.31
	6	-4.14	-5.23	-5.65	-6.18	66.40	1.48	-9.32	-13.46	-15.38	-16.74	2.12	2.93	3.55	3.81	-1.25	-2.97	1.00	3.07	4.03	5.02	-6.27	-8.16	-9.20	-9.99	67.63	4.44	-10.33	-16.54	-19.42	-21.77
	7	-3.99	-4.94	-5.35	-5.88	69.16	3.01	-8.03	-11.77	-13.72	-15.13	2.52	3.43	4.13	4.44	-3.94	-3.13	2.36	4.84	6.45	7.57	-6.51	-8.38	-9.49	-10.32	73.08	6.13	-10.41	-16.62	-20.18	-22.71
	8	-3.32	-4.37	-4.80	-5.26	72.47	6.04	-5.21	-9.35	-11.25	-12.80	2.94	3.98	4.76	5.13	-6.68	-3.14	3.80	7.05	8.76	10.67	-6.26	-8.35	-9.56	-10.39	79.13	9.17	-9.02	-16.41	-20.02	-23.48
	9	-3.30	-4.18	-4.63	-5.07	79.35	6.88	-4.16	-8.17	-10.20	-11.70	3.37	4.56	5.42	5.85	-6.18	-3.76	4.52	8.71	11.36	13.35	-6.67	-8.74	-10.05	-10.91	85.51	10.62	-8.69	-16.89	-21.57	-25.06
	10	-3.17	-3.99	-4.48	-4.86	85.46	8.36	-2.86	-7.24	-9.20	-10.64	3.83	5.18	6.14	6.63	-7.57	-4.60	5.01	10.56	13.72	16.39	-7.00	-9.17	-10.63	-11.48	93.01	12.95	-7.88	-17.81	-22.92	-27.04
	25	-1.61	-1.64	-2.19	-2.34	110.54	19.80	6.35	1.18	-1.21	-2.95	5.96	8.66	10.29	11.20	-34.01	-28.52	-13.19	-2.08	5.82	14.45	-7.58	-10.30	-12.48	-13.54	144.54	48.31	19.54	3.26	-7.03	-17.40
	50	-0.49	-0.70	-0.91	-0.92	115.26	22.22	8.52	3.12	0.89	-0.91	3.02	4.43	5.14	5.55	-44.44	-40.33	-24.44	-12.53	-5.01	3.99	-3.51	-5.13	-6.05	-6.47	159.69	62.55	32.96	15.64	5.90	-4.91
	100	0.00	-0.04	-0.06	-0.06	99.50	21.80	9.43	3.96	1.87	0.26	0.22	0.42	0.48	0.51	-57.73	-45.36	-30.34	-18.00	-10.77	-2.34	-0.22	-0.46	-0.54	-0.57	157.21	67.15	39.77	21.96	12.64	2.60

Table 3.3 continued: Gains from pooling VAR forecasts – no noise (*Model I*)

MODEL II

T=25

		MSE (%)										VARIANCE (%)										COVARIANCE (%)									
		AVERAGE				OPTIMAL WEIGHTS						AVERAGE				OPTIMAL WEIGHTS						AVERAGE				OPTIMAL WEIGHTS					
						N=10	N=25	N=50	N=100	N=200	N=1000					N=10	N=25	N=50	N=100	N=200	N=1000					N=10	N=25	N=50	N=100	N=200	N=1000
	m	2	3	4	5	5	5	5	5	5	5	2	3	4	5	5	5	5	5	5	5	2	3	4	5	5	5	5	5	5	5
h	1	-10.83	-13.11	-14.64	-15.68	79.44	-2.45	-13.07	-17.29	-19.10	-20.41	0.35	0.54	0.60	0.64	5.57	-0.01	0.29	0.47	0.55	0.62	-11.18	-13.66	-15.25	-16.33	73.85	-2.46	-13.38	-17.77	-19.66	-21.05
	2	-10.55	-13.10	-14.86	-15.91	89.09	-1.03	-12.91	-17.39	-19.30	-20.61	0.74	1.10	1.25	1.36	12.14	0.07	0.61	0.96	1.13	1.26	-11.29	-14.22	-16.12	-17.28	76.92	-1.12	-13.54	-18.36	-20.44	-21.89
	3	-9.05	-11.54	-13.28	-14.15	119.63	4.73	-9.25	-13.70	-15.74	-17.23	1.09	1.59	1.82	1.98	28.84	0.00	0.78	1.33	1.59	1.84	-10.13	-13.14	-15.10	-16.14	90.77	4.73	-10.04	-15.03	-17.33	-19.07
	4	-7.86	-10.49	-11.98	-12.65	139.74	10.15	-5.48	-10.17	-12.50	-14.02	1.23	1.79	2.06	2.24	37.94	-0.04	0.75	1.40	1.82	2.15	-9.09	-12.29	-14.04	-14.89	101.77	10.19	-6.23	-11.57	-14.33	-16.17
	5	-7.06	-9.35	-10.72	-11.27	167.07	14.80	-2.34	-7.88	-10.20	-11.73	1.28	1.88	2.15	2.34	54.57	0.07	0.57	1.45	1.94	2.29	-8.34	-11.23	-12.88	-13.61	112.45	14.73	-2.91	-9.33	-12.13	-14.01
	6	-5.63	-7.83	-9.03	-9.52	194.64	19.37	0.93	-5.28	-7.81	-9.49	1.27	1.86	2.14	2.32	71.21	0.24	0.34	1.29	1.83	2.25	-6.91	-9.68	-11.16	-11.83	123.37	19.12	0.58	-6.56	-9.64	-11.73
	7	-5.31	-6.96	-8.25	-8.71	250.20	21.53	2.65	-4.16	-6.87	-8.47	1.17	1.71	1.96	2.13	109.74	0.58	0.14	1.15	1.70	2.11	-6.48	-8.66	-10.20	-10.82	140.39	20.96	2.51	-5.30	-8.56	-10.57
	8	-5.01	-6.25	-7.56	-7.91	262.55	25.11	3.48	-3.22	-5.87	-7.63	1.14	1.66	1.91	2.06	118.44	0.62	-0.07	0.94	1.60	2.08	-6.14	-7.89	-9.45	-9.96	144.04	24.49	3.56	-4.15	-7.46	-9.70
	9	-4.26	-5.48	-6.68	-6.99	335.30	27.94	5.48	-2.23	-4.83	-6.75	1.06	1.54	1.78	1.91	173.73	0.89	-0.32	0.82	1.43	1.99	-5.31	-6.99	-8.43	-8.88	161.49	27.06	5.81	-3.02	-6.25	-8.72
	10	-3.80	-5.19	-6.21	-6.47	307.81	29.89	5.93	-1.35	-4.38	-6.30	0.98	1.41	1.64	1.76	151.47	1.45	-0.27	0.60	1.34	1.89	-4.77	-6.57	-7.82	-8.21	156.29	28.46	6.22	-1.92	-5.70	-8.17
	25	-1.99	-2.32	-3.28	-3.19	385.97	39.05	8.12	-0.33	-3.73	-6.41	0.37	0.60	0.81	0.83	205.12	3.95	-0.56	0.19	0.90	1.66	-2.36	-2.92	-4.09	-4.02	180.77	35.08	8.68	-0.52	-4.63	-8.07
	50	-1.26	-2.34	-3.02	-3.07	419.64	47.10	10.73	0.71	-2.96	-5.62	0.40	0.66	0.92	0.95	228.19	5.81	-0.65	0.27	1.14	2.01	-1.66	-3.00	-3.93	-4.01	191.36	41.29	11.39	0.44	-4.10	-7.63
	100	-1.42	-2.70	-3.64	-3.65	496.34	52.06	10.84	-0.07	-4.02	-7.15	0.40	0.68	0.97	1.00	291.45	8.17	-0.40	0.23	1.13	2.14	-1.82	-3.38	-4.61	-4.64	204.79	43.89	11.25	-0.30	-5.15	-9.28

T=50

		MSE (%)										VARIANCE (%)										COVARIANCE (%)									
		AVERAGE				OPTIMAL WEIGHTS						AVERAGE				OPTIMAL WEIGHTS						AVERAGE				OPTIMAL WEIGHTS					
						N=10	N=25	N=50	N=100	N=200	N=1000					N=10	N=25	N=50	N=100	N=200	N=1000					N=10	N=25	N=50	N=100	N=200	N=1000
	m	2	3	4	5	5	5	5	5	5	5	2	3	4	5	5	5	5	5	5	5	2	3	4	5	5	5	5	5	5	5
h	1	-7.62	-10.06	-11.61	-12.17	75.04	-3.14	-12.95	-16.76	-18.66	-20.00	0.31	0.37	0.40	0.47	3.67	-0.41	-0.14	-0.01	0.07	0.13	-7.93	-10.44	-12.01	-12.64	71.35	-2.74	-12.82	-16.76	-18.74	-20.13
	2	-8.54	-10.84	-12.66	-13.36	73.11	-5.26	-15.47	-19.21	-21.17	-22.40	0.79	1.04	1.13	1.29	5.57	-0.47	0.22	0.57	0.74	0.89	-9.33	-11.88	-13.79	-14.65	67.51	-4.80	-15.70	-19.79	-21.92	-23.30
	3	-8.13	-10.43	-11.65	-12.42	83.22	-1.20	-13.12	-17.24	-19.01	-20.30	1.22	1.64	1.79	2.02	11.81	-0.55	0.72	1.44	1.74	1.97	-9.35	-12.07	-13.44	-14.44	71.38	-0.66	-13.86	-18.69	-20.76	-22.28
	4	-6.88	-8.99	-10.19	-10.91	108.03	3.44	-8.58	-12.96	-14.93	-16.31	1.59	2.15	2.34	2.62	20.15	-0.61	1.13	2.16	2.69	3.10	-8.47	-11.13	-12.53	-13.52	87.85	4.04	-9.72	-15.14	-17.63	-19.42
	5	-5.86	-7.86	-9.02	-9.70	114.83	8.29	-5.38	-9.98	-12.11	-13.63	1.86	2.52	2.74	3.05	23.34	-0.73	1.35	2.74	3.39	3.96	-7.72	-10.38	-11.75	-12.75	91.45	9.00	-6.74	-12.73	-15.51	-17.59
	6	-5.08	-7.05	-7.87	-8.52	119.87	10.51	-2.89	-7.79	-9.90	-11.44	2.07	2.82	3.06	3.39	24.15	-1.42	1.33	2.98	3.81	4.59	-7.15	-9.87	-10.92	-11.90	95.68	11.91	-4.23	-10.78	-13.71	-16.04
	7	-4.67	-6.60	-7.30	-7.88	139.42	12.41	-1.99	-6.74	-8.83	-10.40	2.20	3.00	3.25	3.59	33.37	-1.81	1.29	3.12	4.12	4.97	-6.87	-9.60	-10.53	-11.46	106.01	14.21	-3.28	-9.87	-12.96	-15.37
	8	-4.24	-6.01	-6.46	-7.02	143.71	15.10	-0.06	-5.39	-7.65	-9.26	2.33	3.18	3.44	3.79	34.94	-2.11	0.72	3.05	4.32	5.30	-6.58	-9.19	-9.89	-10.81	108.73	17.21	-0.77	-8.44	-11.97	-14.56
	9	-3.93	-5.58	-6.02	-6.51	176.69	17.69	1.09	-4.49	-6.67	-8.36	2.39	3.25	3.51	3.86	53.24	-2.74	0.53	3.19	4.54	5.61	-6.32	-8.83	-9.52	-10.37	123.41	20.43	0.56	-7.68	-11.21	-13.96
	10	-3.74	-5.24	-5.66	-6.10	196.94	19.78	2.26	-3.95	-6.21	-7.90	2.39	3.27	3.52	3.87	65.76	-2.95	0.20	3.10	4.51	5.66	-6.13	-8.51	-9.18	-9.97	131.15	22.73	2.06	-7.04	-10.72	-13.55
	25	-1.66	-2.71	-2.71	-3.10	370.35	34.04	8.01	-0.82	-3.48	-5.65	1.41	2.05	2.16	2.36	171.88	-6.89	-3.65	0.75	2.75	4.74	-3.07	-4.76	-4.87	-5.46	198.39	40.92	11.65	-1.57	-6.22	-10.39
	50	-1.56	-1.98	-2.26	-2.35	329.82	36.51	8.59	0.13	-2.90	-4.94	1.09	1.59	1.72	1.85	131.73	-7.47	-4.37	-0.11	2.19	4.09	-2.65	-3.57	-3.96	-4.20	198.05	43.99	12.97	0.25	-5.09	-9.02
	100	-1.27	-1.93	-2.00	-2.14	304.40	37.06	10.37	1.65	-1.55	-3.62	1.03	1.50	1.62	1.75	115.96	-8.21	-4.92	-0.45	2.06	3.93	-2.28	-3.43	-3.61	-3.89	188.39	45.27	15.30	2.11	-3.60	-7.55

T=100

		MSE (%)										VARIANCE (%)										COVARIANCE (%)									
		AVERAGE				OPTIMAL WEIGHTS						AVERAGE				OPTIMAL WEIGHTS						AVERAGE				OPTIMAL WEIGHTS					
						N=10	N=25	N=50	N=100	N=200	N=1000					N=10	N=25	N=50	N=100	N=200	N=1000					N=10	N=25	N=50	N=100	N=200	N=1000
	m	2	3	4	5	5	5	5	5	5	5	2	3	4	5	5	5	5	5	5	5	2	3	4	5	5	5	5	5	5	5
h	1	-8.82	-10.65	-11.50	-12.32	67.12	-3.10	-13.39	-17.39	-19.17	-20.50	0.32	0.40	0.49	0.53	0.94	-0.49	-0.15	0.00	0.08	0.14	-9.15	-11.04	-11.98	-12.84	66.16	-2.61	-13.24	-17.38	-19.24	-20.63
	2	-9.11	-11.70	-12.59	-13.51	60.50	-6.25	-16.84	-20.34	-22.24	-23.45	0.80	1.04	1.23	1.32	2.14	-0.53	0.25	0.55	0.75	0.87	-9.92	-12.73	-13.81	-14.82	58.35	-5.72	-17.08	-20.88	-22.99	-24.31
	3	-8.22	-10.79	-11.65	-12.45	67.15	-4.48	-15.02	-18.98	-20.66	-22.08	1.23	1.60	1.88	2.02	5.76	-0.40	0.87	1.50	1.75	2.06	-9.46	-12.38	-13.52	-14.46	61.37	-4.07	-15.89	-20.48	-22.41	-24.13
	4	-7.46	-9.55	-10.54	-11.17	80.07	-2.00	-11.35	-15.80	-17.60	-19.03	1.55	2.03	2.37	2.54	8.07	-0.12	1.54	2.60	3.05	3.48	-9.02	-11.58	-12.90	-13.71	71.98	-1.88	-12.89	-18.40	-20.65	-22.50
	5	-6.66	-8.63	-9.45	-10.01	76.08	3.72	-9.00	-12.91	-14.83	-16.32	1.83	2.40	2.78	2.99	6.62	-0.19	2.60	3.76	4.47	5.04	-8.50	-11.02	-12.22	-12.99	69.42	3.90	-11.60	-16.67	-19.30	-21.35
	6	-5.88	-7.50	-8.23	-8.78	97.35	6.11	-5.65	-10.04	-12.01	-13.51	2.12	2.77	3.20	3.45	12.34	-0.61	2.87	4.60	5.67	6.42	-8.01	-10.27	-11.43	-12.23	84.97	6.71	-8.53	-14.64	-17.68	-19.93
	7	-5.31	-6.85	-7.34	-7.84	98.76	8.98	-3.06	-7.89	-9.83	-11.44	2.37	3.10	3.57	3.85	13.28	-1.09	2.79	5.28	6.41	7.53	-7.68	-9.95	-10.90	-11.68	85.45	10.06	-5.85	-13.18	-16.24	-18.97
	8	-4.62	-5.96	-6.43	-6.88	116.12	11.39	-1.42	-6.37	-8.42	-9.93	2.57	3.36	3.86	4.17	17.33	-2.03	2.79	5.77	7.38	8.59	-7.19	-9.31	-10.28	-11.05	98.75	13.42	-4.22	-12.14	-15.79	-18.52
	9	-4.23	-5.40	-5.91	-6.29	117.97	12.51	-0.53	-5.35	-7.55	-9.12	2.70	3.54	4.06	4.40	17.13	-2.64	2.52	6.19	8.03	9.51	-6.93	-8.94	-9.97	-10.69	100.79	15.14	-3.06	-11.54	-15.58	-18.63
	10	-3.67	-4.90	-5.36	-5.70	123.19	15.35	0.68	-4.76	-6.70	-8.30	2.80	3.67	4.20	4.56	15.14	-4.06	2.28	6.36	8.31	10.07	-6.47	-8.57	-9.56	-10.26	108.02	19.40	-1.60	-11.12	-15.01	-18.38
	25	-0.85	-1.06	-1.25	-1.39	282.45	28.49	8.35	1.04	-1.49	-3.30	1.45	1.90	2.12	2.41	84.86	-16.74	-8.21	-1.24	2.23	5.28	-2.29	-2.96	-3.38	-3.80	197.56	45.23	16.56	2.29	-3.72	-8.58
	50	-0.46	-0.48	-0.54	-0.67	243.05	33.02	11.24	3.22	0.40	-1.57	0.72	0.94	1.04	1.18	51.42	-20.23	-11.82	-4.33	-0.42	2.93	-1.18	-1.42	-1.58	-1.85	191.61	53.24	23.06	7.55	0.82	-4.49
	100	-0.39	-0.41	-0.42	-0.52	176.08	28.93	9.66	3.16	0.57	-1.21	0.52	0.69	0.78	0.87	12.76	-21.24	-11.09	-4.51	-0.71	2.42	-0.91	-1.11	-1.20	-1.38	163.30	50.17	20.75	7.67	1.28	-3.64

Table 3.4: Gains from pooling VAR forecasts – no noise (*Model II*)

T=200

	MSE (%)									
	AVERAGE				OPTIMAL WEIGHTS					
					N=10	N=25	N=50	N=100	N=200	N=1000
m	2	3	4	5	5	5	5	5	5	5
h 1	-6.50	-8.17	-9.58	-10.13	63.36	-1.93	-12.15	-16.14	-17.87	-19.20
2	-8.28	-10.54	-12.26	-12.83	52.69	-8.20	-17.26	-21.16	-22.70	-24.00
3	-7.83	-9.86	-11.27	-11.97	64.08	-7.19	-16.28	-19.98	-21.69	-22.90
4	-7.18	-8.84	-10.02	-10.65	65.66	-3.89	-13.33	-17.50	-18.94	-20.33
5	-6.29	-7.66	-8.60	-9.21	72.63	0.64	-10.24	-14.15	-15.98	-17.36
6	-5.82	-6.88	-7.70	-8.14	77.06	2.82	-7.52	-11.69	-13.46	-14.89
7	-5.18	-6.25	-6.85	-7.24	82.42	5.35	-5.83	-9.82	-11.73	-13.21
8	-4.62	-5.66	-6.14	-6.44	92.03	8.82	-4.06	-8.07	-10.24	-11.64
9	-4.44	-5.53	-5.88	-6.19	88.53	9.09	-3.02	-7.27	-9.46	-10.87
10	-4.17	-5.13	-5.38	-5.68	95.98	10.09	-1.45	-6.09	-8.07	-9.65
25	-0.49	-0.69	-0.92	-0.93	198.03	26.94	9.17	2.75	0.19	-1.63
50	-0.07	-0.10	-0.13	-0.12	171.58	29.74	11.74	4.53	1.69	-0.20
100	0.05	0.00	0.03	-0.01	148.68	26.22	9.97	4.24	1.74	-0.07

	VARIANCE (%)									
	AVERAGE				OPTIMAL WEIGHTS					
					N=10	N=25	N=50	N=100	N=200	N=1000
m	2	3	4	5	5	5	5	5	5	5
h 1	0.34	0.58	0.54	0.61	0.55	-0.89	-0.61	-0.50	-0.43	-0.39
2	0.82	1.27	1.28	1.41	1.00	-0.99	-0.47	-0.14	0.01	0.11
3	1.27	1.87	1.94	2.12	3.16	-0.95	0.06	0.65	0.85	1.08
4	1.66	2.36	2.51	2.72	3.07	-0.98	0.78	1.74	2.08	2.45
5	1.97	2.72	2.96	3.19	4.76	-0.92	1.83	2.95	3.61	4.22
6	2.29	3.09	3.41	3.67	4.23	-0.82	2.54	4.57	5.44	6.21
7	2.59	3.43	3.84	4.12	2.33	-1.14	3.28	5.85	7.02	8.21
8	2.81	3.68	4.17	4.47	5.43	-2.62	3.69	6.56	8.50	9.80
9	3.01	3.90	4.47	4.78	0.51	-3.06	3.42	7.25	9.36	11.05
10	3.20	4.11	4.76	5.08	0.25	-4.15	3.13	7.93	10.12	12.41
25	1.79	2.29	2.73	2.90	15.97	-25.68	-13.20	-4.31	0.75	5.41
50	0.35	0.48	0.57	0.62	-7.34	-31.74	-19.55	-9.86	-4.27	0.63
100	0.14	0.22	0.25	0.28	-16.62	-31.18	-18.41	-9.74	-4.63	0.07

	COVARIANCE (%)									
	AVERAGE				OPTIMAL WEIGHTS					
					N=10	N=25	N=50	N=100	N=200	N=1000
m	2	3	4	5	5	5	5	5	5	5
h 1	-6.83	-8.74	-10.11	-10.73	62.82	-1.02	-11.52	-15.62	-17.41	-18.80
2	-9.09	-11.79	-13.51	-14.22	51.72	-7.17	-16.75	-20.99	-22.66	-24.07
3	-9.10	-11.72	-13.21	-14.08	60.93	-6.23	-16.33	-20.62	-22.53	-23.97
4	-8.84	-11.20	-12.53	-13.38	62.58	-2.92	-14.12	-19.25	-21.02	-22.78
5	-8.26	-10.38	-11.55	-12.41	67.86	1.56	-12.08	-17.11	-19.59	-21.58
6	-8.10	-9.98	-11.11	-11.81	72.79	3.63	-10.08	-16.27	-18.91	-21.11
7	-7.77	-9.69	-10.70	-11.37	80.06	6.46	-9.13	-15.70	-18.77	-21.45
8	-7.45	-9.35	-10.33	-10.93	86.56	11.41	-7.77	-14.67	-18.77	-21.47
9	-7.45	-9.43	-10.36	-10.98	87.99	12.14	-6.45	-14.53	-18.84	-21.94
10	-7.37	-9.24	-10.14	-10.76	95.71	14.23	-4.59	-14.03	-18.20	-22.07
25	-2.28	-2.98	-3.64	-3.84	182.04	52.62	22.36	7.05	-0.57	-7.04
50	-0.43	-0.58	-0.70	-0.74	178.90	61.48	31.29	14.38	5.96	-0.83
100	-0.09	-0.22	-0.22	-0.30	165.28	57.39	28.37	13.98	6.37	-0.15

T=1000

	MSE (%)									
	AVERAGE				OPTIMAL WEIGHTS					
					N=10	N=25	N=50	N=100	N=200	N=1000
m	2	3	4	5	5	5	5	5	5	5
h 1	-6.36	-9.17	-9.81	-10.29	53.04	-5.47	-14.73	-18.37	-20.17	-21.45
2	-7.71	-10.59	-11.28	-11.84	51.61	-8.70	-18.23	-21.92	-23.63	-24.82
3	-7.00	-9.62	-10.38	-10.85	52.04	-7.47	-16.14	-19.96	-21.62	-22.91
4	-5.70	-7.97	-8.69	-9.14	64.01	-2.90	-12.85	-16.80	-18.57	-19.90
5	-5.22	-6.82	-7.50	-7.96	69.02	0.44	-9.91	-13.88	-15.83	-17.20
6	-4.28	-5.61	-6.35	-6.71	71.82	2.74	-7.11	-11.66	-13.46	-14.92
7	-3.75	-4.71	-5.46	-5.79	81.35	5.13	-5.06	-9.33	-11.18	-12.57
8	-3.62	-4.23	-4.92	-5.21	80.96	7.58	-2.99	-7.27	-9.17	-10.69
9	-3.42	-3.89	-4.59	-4.85	80.48	8.04	-1.75	-6.30	-8.24	-9.74
10	-3.12	-3.44	-4.07	-4.32	89.13	10.09	-0.89	-5.43	-7.55	-9.02
25	-0.54	-0.79	-0.78	-0.78	119.21	22.08	8.09	2.94	0.62	-1.04
50	0.03	0.05	0.04	0.04	147.71	25.66	10.10	4.56	1.95	0.23
100	0.01	0.01	-0.01	-0.01	94.83	21.14	9.30	4.16	2.00	0.28

	VARIANCE (%)									
	AVERAGE				OPTIMAL WEIGHTS					
					N=10	N=25	N=50	N=100	N=200	N=1000
m	2	3	4	5	5	5	5	5	5	5
h 1	0.35	0.48	0.53	0.59	-0.61	-1.20	-0.91	-0.79	-0.71	-0.66
2	0.89	1.18	1.32	1.44	-0.03	-1.70	-1.03	-0.70	-0.53	-0.42
3	1.36	1.77	1.98	2.15	0.03	-1.57	-0.41	0.09	0.41	0.55
4	1.80	2.32	2.60	2.81	0.65	-1.57	0.21	1.28	1.77	2.09
5	2.18	2.78	3.12	3.37	1.95	-1.74	0.94	2.52	3.35	3.87
6	2.57	3.27	3.66	3.95	-1.92	-1.80	1.98	4.14	5.24	6.15
7	2.91	3.71	4.15	4.46	-2.24	-1.54	3.56	6.34	7.85	9.14
8	3.22	4.11	4.59	4.94	-5.62	-1.99	4.34	8.06	10.21	11.79
9	3.51	4.49	5.02	5.39	-7.98	-2.72	5.03	9.75	12.48	14.51
10	3.79	4.88	5.45	5.86	-4.32	-4.45	4.93	10.65	14.15	16.59
25	2.69	3.69	4.10	4.38	-40.64	-38.27	-22.18	-10.63	-3.09	5.19
50	0.14	0.21	0.22	0.24	-42.84	-48.36	-31.53	-19.99	-11.62	-3.12
100	0.03	0.06	0.05	0.05	-59.66	-45.15	-30.82	-18.85	-11.47	-2.63

	COVARIANCE (%)									
	AVERAGE				OPTIMAL WEIGHTS					
					N=10	N=25	N=50	N=100	N=200	N=1000
m	2	3	4	5	5	5	5	5	5	5
h 1	-6.73	-9.67	-10.35	-10.89	53.64	-4.28	-13.83	-17.58	-19.46	-20.80
2	-8.61	-11.77	-12.60	-13.28	51.63	-7.00	-17.20	-21.22	-23.11	-24.41
3	-8.35	-11.38	-12.36	-13.00	52.01	-5.90	-15.72	-20.05	-22.03	-23.45
4	-7.48	-10.28	-11.28	-11.94	63.37	-1.32	-13.05	-18.06	-20.33	-21.98
5	-7.40	-9.61	-10.62	-11.33	67.07	2.17	-10.85	-16.40	-19.18	-21.07
6	-6.84	-8.88	-10.01	-10.66	73.74	4.55	-9.09	-15.80	-18.69	-21.06
7	-6.65	-8.42	-9.60	-10.25	83.59	6.67	-8.62	-15.67	-19.04	-21.70
8	-6.84	-8.34	-9.51	-10.15	86.58	9.57	-7.34	-15.32	-19.38	-22.48
9	-6.93	-8.38	-9.60	-10.24	88.45	10.77	-6.78	-16.05	-20.71	-24.24
10	-6.90	-8.31	-9.51	-10.17	93.45	14.55	-5.81	-16.07	-21.68	-25.59
25	-3.23	-4.48	-4.88	-5.16	159.80	60.34	30.25	13.56	3.71	-6.24
50	-0.11	-0.16	-0.18	-0.19	190.49	74.02	41.63	24.55	13.58	3.36
100	-0.03	-0.05	-0.06	-0.06	154.47	66.29	40.11	23.01	13.47	2.91

Table 3.4 continued: Gains from pooling VAR forecasts – no noise (*Model II*)

MODEL III

T=25

	MSE (%)										VARIANCE (%)										COVARIANCE (%)									
	AVERAGE				OPTIMAL WEIGHTS						AVERAGE				OPTIMAL WEIGHTS						AVERAGE				OPTIMAL WEIGHTS					
					N=10	N=25	N=50	N=100	N=200	N=1000					N=10	N=25	N=50	N=100	N=200	N=1000					N=10	N=25	N=50	N=100	N=200	N=1000
m	2	3	4	5	5	5	5	5	5	5	2	3	4	5	5	5	5	5	5	5	2	3	4	5	5	5	5	5	5	5
h 1	-5.95	-7.39	-8.09	-8.51	124.52	10.79	-2.82	-7.96	-9.96	-11.48	0.57	0.74	0.78	0.82	15.53	-1.07	-0.29	0.17	0.38	0.54	-6.52	-8.14	-8.86	-9.32	108.99	11.87	-2.51	-8.12	-10.32	-12.01
2	-6.10	-8.10	-8.90	-9.45	133.01	9.97	-4.19	-9.00	-11.10	-12.62	0.93	1.30	1.39	1.49	25.92	-1.51	-0.41	0.30	0.63	0.93	-7.03	-9.40	-10.30	-10.93	107.07	11.48	-3.78	-9.30	-11.73	-13.55
3	-5.29	-7.04	-7.78	-8.26	144.69	13.50	-1.78	-7.34	-9.39	-11.00	1.13	1.58	1.70	1.83	35.94	-1.72	-0.66	0.34	0.81	1.18	-6.41	-8.62	-9.48	-10.10	108.71	15.20	-1.12	-7.69	-10.21	-12.19
4	-4.57	-6.05	-6.82	-7.22	176.08	18.66	1.06	-4.84	-7.34	-9.00	1.23	1.69	1.84	1.98	53.88	-1.43	-0.77	0.37	1.07	1.58	-5.79	-7.74	-8.66	-9.21	122.14	20.07	1.82	-5.23	-8.42	-10.58
5	-3.85	-5.28	-5.95	-6.31	270.83	21.40	2.97	-3.47	-6.02	-7.79	1.29	1.74	1.91	2.06	119.32	-1.59	-0.96	0.37	1.12	1.71	-5.14	-7.03	-7.86	-8.37	151.45	22.98	3.92	-3.84	-7.14	-9.50
6	-3.05	-4.18	-4.92	-5.23	281.44	27.71	5.32	-1.72	-4.53	-6.19	1.28	1.70	1.88	2.02	119.21	-1.22	-1.20	0.33	1.17	1.79	-4.33	-5.88	-6.80	-7.25	162.15	28.93	6.52	-2.04	-5.69	-7.97
7	-2.71	-3.49	-4.16	-4.46	366.73	31.48	6.24	-0.77	-3.52	-5.31	1.20	1.57	1.75	1.87	183.88	-0.72	-1.23	0.24	1.09	1.74	-3.91	-5.06	-5.91	-6.33	182.78	32.21	7.48	-1.00	-4.60	-7.04
8	-2.28	-2.92	-3.53	-3.88	408.70	36.86	9.52	0.99	-2.28	-4.32	1.17	1.51	1.69	1.80	210.81	-0.43	-1.64	-0.02	0.98	1.77	-3.45	-4.43	-5.22	-5.68	197.80	37.29	11.17	1.02	-3.25	-6.09
9	-2.36	-2.97	-3.51	-3.81	432.14	41.33	11.04	0.96	-2.11	-4.13	1.12	1.44	1.62	1.73	222.42	0.55	-1.76	-0.03	0.98	1.78	-3.48	-4.40	-5.12	-5.53	209.63	40.78	12.81	1.00	-3.08	-5.90
10	-2.04	-2.65	-3.11	-3.40	467.08	45.01	11.36	1.90	-1.61	-3.69	1.07	1.36	1.54	1.64	246.47	1.34	-1.97	-0.25	0.86	1.72	-3.11	-4.02	-4.66	-5.04	220.53	43.66	13.33	2.16	-2.47	-5.40
25	-1.56	-1.85	-2.39	-2.41	648.52	63.22	16.60	4.04	-0.21	-3.37	0.64	0.83	0.94	0.98	398.97	7.34	-2.18	-1.11	0.00	1.14	-2.20	-2.68	-3.33	-3.38	249.35	55.87	18.79	5.16	-0.20	-4.50
50	-2.18	-2.76	-2.98	-3.02	775.99	72.32	20.71	5.05	0.15	-3.49	0.73	0.90	1.04	1.06	497.63	10.69	-1.67	-1.10	0.03	1.29	-2.91	-3.66	-4.02	-4.08	278.16	61.60	22.37	6.13	0.11	-4.79
100	-2.57	-2.96	-3.27	-3.28	770.90	82.52	22.90	5.83	0.11	-3.51	0.84	1.00	1.15	1.16	489.79	14.32	-1.55	-1.11	0.12	1.40	-3.41	-3.96	-4.42	-4.44	280.91	68.18	24.45	6.94	-0.01	-4.90

T=50

	MSE (%)										VARIANCE (%)										COVARIANCE (%)									
	AVERAGE				OPTIMAL WEIGHTS						AVERAGE				OPTIMAL WEIGHTS						AVERAGE				OPTIMAL WEIGHTS					
					N=10	N=25	N=50	N=100	N=200	N=1000					N=10	N=25	N=50	N=100	N=200	N=1000					N=10	N=25	N=50	N=100	N=200	N=1000
m	2	3	4	5	5	5	5	5	5	5	2	3	4	5	5	5	5	5	5	5	2	3	4	5	5	5	5	5	5	5
h 1	-3.87	-4.62	-5.23	-5.76	108.10	10.88	-0.60	-5.42	-7.43	-8.99	0.31	0.41	0.49	0.55	4.86	-1.98	-0.99	-0.44	-0.22	0.00	-4.18	-5.02	-5.72	-6.30	103.24	12.87	0.40	-4.96	-7.20	-8.98
2	-4.22	-5.43	-6.06	-6.58	109.24	10.01	-2.45	-7.30	-9.17	-10.74	0.75	1.04	1.20	1.34	8.74	-3.47	-1.76	-0.76	-0.35	0.04	-4.97	-6.48	-7.26	-7.92	100.47	13.47	-0.69	-6.54	-8.83	-10.79
3	-3.10	-4.58	-5.08	-5.53	127.78	11.55	-1.29	-5.96	-7.99	-9.60	1.08	1.51	1.72	1.91	18.43	-4.78	-2.25	-1.00	-0.52	0.15	-4.18	-6.10	-6.81	-7.44	109.30	16.30	0.94	-4.98	-7.48	-9.78
4	-2.98	-4.14	-4.84	-5.21	128.97	14.49	0.92	-4.29	-6.42	-8.02	1.24	1.74	1.98	2.20	19.42	-5.24	-2.74	-0.89	0.01	0.68	-4.22	-5.89	-6.83	-7.41	109.50	19.71	3.64	-3.43	-6.45	-8.72
5	-3.08	-4.09	-4.75	-5.08	143.88	17.53	2.01	-3.06	-5.57	-7.14	1.38	1.93	2.19	2.44	23.62	-6.39	-3.08	-0.96	0.39	1.25	-4.45	-6.04	-6.96	-7.53	120.18	23.87	5.04	-2.15	-6.00	-8.43
6	-2.67	-3.54	-4.26	-4.48	154.05	19.48	3.88	-1.91	-4.15	-5.83	1.42	1.99	2.26	2.51	26.99	-6.93	-3.66	-0.74	0.60	1.78	-4.09	-5.54	-6.54	-7.01	126.99	26.36	7.50	-1.21	-4.78	-7.65
7	-2.33	-3.02	-3.66	-3.92	182.56	22.90	5.49	-0.36	-2.99	-4.75	1.40	1.96	2.24	2.49	46.34	-7.63	-3.73	-0.89	0.94	2.29	-3.74	-5.00	-5.91	-6.42	136.14	30.48	9.18	0.49	-3.96	-7.08
8	-2.04	-2.73	-3.23	-3.49	216.45	25.86	6.42	0.04	-2.48	-4.25	1.36	1.90	2.18	2.41	58.07	-8.00	-3.83	-0.60	1.09	2.51	-3.40	-4.65	-5.42	-5.92	158.31	33.83	10.22	0.62	-3.59	-6.78
9	-1.49	-2.21	-2.62	-2.84	238.90	27.86	7.79	0.99	-1.60	-3.36	1.33	1.85	2.13	2.35	74.06	-8.71	-4.43	-0.87	1.02	2.49	-2.82	-4.08	-4.76	-5.20	164.74	36.54	12.20	1.84	-2.64	-5.88
10	-1.30	-2.05	-2.36	-2.56	300.67	31.45	8.53	1.76	-0.94	-2.84	1.28	1.78	2.05	2.26	118.34	-8.98	-4.72	-1.19	0.94	2.46	-2.58	-3.84	-4.41	-4.83	182.25	40.40	13.24	2.94	-1.90	-5.32
25	-0.42	-0.75	-0.97	-0.91	447.25	53.96	15.65	4.82	1.10	-1.24	0.60	0.88	1.07	1.12	200.77	-11.08	-8.59	-3.52	-0.66	1.54	-1.02	-1.63	-2.04	-2.04	246.40	65.03	24.23	8.35	1.77	-2.77
50	-0.62	-0.79	-0.87	-0.91	434.98	47.84	15.74	5.53	1.33	-1.21	0.44	0.66	0.80	0.84	197.05	-12.41	-9.63	-4.66	-1.50	0.96	-1.06	-1.45	-1.68	-1.75	237.83	60.23	25.36	10.19	2.82	-2.17
100	-0.18	-0.26	-0.50	-0.52	347.38	43.76	13.80	4.35	1.05	-1.33	0.41	0.62	0.74	0.78	137.14	-12.89	-8.99	-4.02	-1.45	0.74	-0.59	-0.88	-1.24	-1.30	210.15	56.63	22.77	8.36	2.49	-2.08

T=100

	MSE (%)										VARIANCE (%)										COVARIANCE (%)									
	AVERAGE				OPTIMAL WEIGHTS						AVERAGE				OPTIMAL WEIGHTS						AVERAGE				OPTIMAL WEIGHTS					
					N=10	N=25	N=50	N=100	N=200	N=1000					N=10	N=25	N=50	N=100	N=200	N=1000					N=10	N=25	N=50	N=100	N=200	N=1000
m	2	3	4	5	5	5	5	5	5	5	2	3	4	5	5	5	5	5	5	5	2	3	4	5	5	5	5	5	5	5
h 1	-2.38	-3.32	-3.70	-4.12	100.55	11.60	-0.88	-5.41	-7.50	-9.06	0.23	0.39	0.56	0.54	2.03	-2.55	-1.50	-0.98	-0.74	-0.56	-2.61	-3.71	-4.26	-4.65	98.49	14.16	0.64	-4.42	-6.74	-8.48
2	-3.07	-3.66	-4.43	-4.83	97.94	8.37	-3.35	-7.73	-9.73	-11.17	0.70	1.06	1.37	1.40	3.23	-4.79	-2.94	-2.01	-1.46	-1.16	-3.77	-4.72	-5.81	-6.22	94.68	13.17	-0.40	-5.71	-8.26	-10.00
3	-3.23	-3.63	-4.57	-4.88	104.23	8.99	-2.27	-6.81	-8.90	-10.36	1.04	1.52	1.92	1.99	7.50	-5.72	-3.17	-1.90	-1.29	-0.73	-4.26	-5.16	-6.49	-6.87	96.71	14.72	0.92	-4.89	-7.59	-9.60
4	-2.88	-3.24	-4.03	-4.31	108.27	12.69	0.16	-4.52	-6.66	-8.16	1.30	1.87	2.32	2.42	4.44	-6.68	-3.25	-1.52	-0.25	0.37	-4.18	-5.11	-6.34	-6.73	103.80	19.38	3.42	-2.99	-6.39	-8.52
5	-2.60	-2.97	-3.72	-3.93	125.53	15.36	1.35	-3.44	-5.66	-7.34	1.48	2.10	2.59	2.72	12.07	-8.53	-3.88	-1.62	-0.45	0.69	-4.08	-5.08	-6.31	-6.64	113.44	23.89	5.25	-1.81	-5.19	-8.02
6	-2.26	-2.53	-3.29	-3.46	126.21	18.22	2.82	-1.92	-4.03	-5.70	1.60	2.26	2.76	2.92	7.43	-10.12	-4.47	-1.52	0.12	1.37	-3.86	-4.79	-6.05	-6.38	118.74	28.35	7.30	-0.39	-4.13	-7.06
7	-1.86	-2.16	-2.75	-2.88	132.31	19.52	4.50	-0.73	-2.85	-4.55	1.62	2.27	2.77	2.94	6.56	-12.14	-5.38	-1.62	0.34	2.08	-3.48	-4.44	-5.51	-5.82	125.72	31.66	9.88	0.90	-3.19	-6.62
8	-1.78	-2.08	-2.56	-2.63	144.52	20.57	5.83	-0.04	-2.37	-3.98	1.60	2.22	2.70	2.88	12.83	-13.41	-6.49	-1.45	1.32	3.10	-3.38	-4.30	-5.26	-5.50	131.64	33.98	12.32	1.41	-3.69	-7.07
9	-1.52	-1.87	-2.26	-2.33	162.28	22.75	6.16	0.48	-1.84	-3.41	1.56	2.16	2.61	2.79	16.74	-14.84	-6.50	-1.28	1.42	3.47	-3.08	-4.03	-4.87	-5.12	145.49	37.58	12.66	1.75	-3.27	-6.89
10	-1.21	-1.56	-1.99	-2.01	153.86	24.11	7.26	0.88	-1.43	-3.11	1.50	2.06	2.49	2.67	8.40	-15.58	-7.26	-1.63	1.21	3.56	-2.71	-3.62	-4.48	-4.68	145.42	39.67	14.51	2.51	-2.66	-6.68
25	0.05	-0.09	-0.18	-0.18	255.30	37.12	12.43	4.65	1.81	-0.14	0.39	0.46	0.60	0.64	49.51	-23.76	-14.61	-6.94	-2.79	0.68	-0.35	-0.56	-0.78	-0.82	205.74	60.87	27.03	11.59	4.60	-0.82
50	-0.16	-0.10	-0.18	-0.22	186.50	32.99	11.22	4.48	1.84	-0.14	0.16	0.14	0.22	0.23	13.77	-24.53	-14.26	-7.33	-3.44	0.04	-0.32	-0.24	-0.40	-0.45	172.69	57.52	25.47	11.81	5.29	-0.18
100	-0.08	-0.03	-0.12	-0.10	174.70	26.87	10.12	4.30	1.72	-0.10	0.12	0.09	0.17	0.18	11.23	-23.55	-13.67	-7.15	-3.30	0.01	-0.20	-0.13	-0.29	-0.28	163.44	50.42	23.78	11.45	5.02	-0.11

Table 3.5: Gains from pooling VAR forecasts – no noise (*Model III*)

	T=200																														
		MSE (%)										VARIANCE (%)										COVARIANCE (%)									
		AVERAGE				OPTIMAL WEIGHTS						AVERAGE				OPTIMAL WEIGHTS						AVERAGE				OPTIMAL WEIGHTS					
						N=10	N=25	N=50	N=100	N=200	N=1000					N=10	N=25	N=50	N=100	N=200	N=1000					N=10	N=25	N=50	N=100	N=200	N=1000
	m	2	3	4	5	5	5	5	5	5	5	2	3	4	5	5	5	5	5	5	5	2	3	4	5	5	5	5	5	5	5
h	1	-2.38	-3.14	-3.27	-3.66	92.99	12.64	0.51	-4.32	-6.35	-7.98	0.35	0.43	0.52	0.54	0.75	-2.71	-1.64	-1.11	-0.87	-0.70	-2.73	-3.57	-3.79	-4.20	92.24	15.35	2.15	-3.21	-5.47	-7.28
	2	-2.93	-3.86	-4.12	-4.47	91.21	8.34	-2.53	-7.29	-9.24	-10.67	0.92	1.20	1.39	1.46	0.86	-4.92	-3.29	-2.31	-1.88	-1.54	-3.85	-5.06	-5.51	-5.93	90.34	13.26	0.77	-4.98	-7.35	-9.13
	3	-3.06	-3.81	-4.10	-4.38	82.61	9.23	-2.33	-6.85	-8.95	-10.44	1.45	1.93	2.21	2.33	-4.19	-7.27	-4.59	-3.13	-2.35	-1.80	-4.50	-5.74	-6.30	-6.70	86.80	16.50	2.26	-3.71	-6.59	-8.63
	4	-3.04	-3.59	-3.80	-4.02	95.72	10.39	-1.31	-6.15	-8.21	-9.70	1.90	2.54	2.89	3.05	-2.70	-9.61	-6.23	-4.33	-3.23	-2.46	-4.94	-6.13	-6.68	-7.07	98.41	19.99	4.91	-1.82	-4.98	-7.24
	5	-2.57	-3.10	-3.39	-3.58	90.65	12.19	-0.08	-4.53	-6.52	-8.06	2.23	2.99	3.38	3.58	-7.93	-11.45	-6.99	-4.69	-3.60	-2.37	-4.81	-6.09	-6.77	-7.16	98.57	23.63	6.91	0.16	-2.92	-5.69
	6	-2.24	-2.84	-3.12	-3.28	101.17	14.49	2.40	-2.60	-4.55	-6.16	2.48	3.32	3.73	3.96	-8.06	-13.07	-7.39	-4.06	-2.42	-1.28	-4.73	-6.16	-6.85	-7.24	109.21	27.55	9.79	1.46	-2.13	-4.89
	7	-2.06	-2.42	-2.64	-2.86	107.94	18.30	4.00	-0.94	-3.02	-4.63	2.65	3.55	3.97	4.22	-7.35	-15.09	-7.72	-3.57	-1.55	0.02	-4.72	-5.97	-6.61	-7.08	115.27	33.38	11.71	2.62	-1.48	-4.66
	8	-1.78	-2.14	-2.25	-2.45	115.47	20.86	5.53	0.39	-1.93	-3.47	2.67	3.56	3.97	4.23	-8.17	-17.15	-7.71	-2.79	0.13	2.20	-4.45	-5.70	-6.23	-6.68	123.62	38.00	13.22	3.18	-2.07	-5.68
	9	-1.35	-1.73	-1.87	-2.01	133.68	20.08	6.07	0.89	-1.16	-2.83	2.65	3.53	3.93	4.19	-2.38	-17.53	-8.55	-3.07	-0.20	2.52	-4.00	-5.26	-5.80	-6.20	136.05	37.60	14.62	3.96	-0.96	-5.35
	10	-1.01	-1.29	-1.43	-1.52	135.10	21.74	7.18	2.00	-0.23	-1.94	2.57	3.41	3.78	4.04	-6.85	-20.16	-9.87	-3.53	-0.06	2.74	-3.58	-4.70	-5.21	-5.57	141.94	41.90	17.04	5.53	-0.18	-4.69
	25	-0.19	-0.15	-0.19	-0.19	186.32	31.72	11.39	4.68	1.87	0.07	0.46	0.57	0.62	0.67	-7.04	-33.82	-20.31	-10.90	-5.09	-0.23	-0.65	-0.71	-0.82	-0.86	193.35	65.53	31.69	15.57	6.96	0.29
	50	-0.01	0.01	-0.02	-0.02	152.63	24.19	10.44	4.26	1.83	0.15	0.05	0.09	0.10	0.11	-17.32	-32.15	-20.54	-11.22	-5.98	-1.38	-0.07	-0.07	-0.11	-0.13	169.93	56.33	30.98	15.48	7.81	1.54
	100	0.04	-0.02	-0.02	-0.02	132.83	23.88	9.28	4.09	1.74	0.15	0.04	0.07	0.08	0.09	-23.78	-31.65	-18.77	-10.31	-5.29	-0.92	0.00	-0.10	-0.10	-0.11	156.59	55.52	28.04	14.40	7.02	1.07

	T=1000																														
		MSE (%)										VARIANCE (%)										COVARIANCE (%)									
		AVERAGE				OPTIMAL WEIGHTS						AVERAGE				OPTIMAL WEIGHTS						AVERAGE				OPTIMAL WEIGHTS					
						N=10	N=25	N=50	N=100	N=200	N=1000					N=10	N=25	N=50	N=100	N=200	N=1000					N=10	N=25	N=50	N=100	N=200	N=1000
	m	2	3	4	5	5	5	5	5	5	5	2	3	4	5	5	5	5	5	5	5	2	3	4	5	5	5	5	5	5	5
h	1	-2.15	-2.96	-3.34	-3.53	86.96	9.74	-1.38	-5.54	-7.72	-9.28	0.31	0.44	0.44	0.48	-1.30	-3.05	-2.06	-1.53	-1.32	-1.16	-2.46	-3.40	-3.78	-4.01	88.25	12.78	0.68	-4.01	-6.40	-8.11
	2	-2.22	-3.70	-3.82	-4.18	77.05	6.51	-4.05	-7.99	-10.05	-11.53	0.85	1.15	1.23	1.32	-4.74	-5.91	-4.12	-3.28	-2.87	-2.54	-3.06	-4.85	-5.05	-5.50	81.78	12.42	0.06	-4.71	-7.18	-8.99
	3	-2.00	-3.61	-3.93	-4.19	89.85	8.95	-2.50	-6.87	-8.87	-10.41	1.37	1.85	2.00	2.15	-1.92	-7.98	-5.51	-4.04	-3.45	-2.85	-3.37	-5.46	-5.94	-6.34	91.75	16.91	3.00	-2.84	-5.43	-7.57
	4	-1.64	-3.06	-3.40	-3.63	88.75	10.62	-0.61	-5.36	-7.21	-8.83	1.86	2.49	2.72	2.92	-7.53	-10.32	-6.62	-4.45	-3.80	-2.90	-3.50	-5.55	-6.12	-6.55	96.26	20.93	6.01	-0.92	-3.41	-5.93
	5	-1.68	-2.83	-3.16	-3.34	92.75	11.41	0.50	-4.19	-6.28	-7.77	2.27	3.02	3.32	3.57	-8.96	-12.20	-7.98	-5.21	-3.66	-2.65	-3.95	-5.85	-6.48	-6.90	101.69	23.60	8.47	1.01	-2.63	-5.13
	6	-1.82	-2.68	-3.05	-3.22	88.06	13.89	1.79	-2.79	-4.81	-6.41	2.65	3.51	3.86	4.16	-18.46	-14.35	-8.10	-4.38	-2.74	-1.44	-4.47	-6.19	-6.91	-7.38	106.50	28.24	9.89	1.59	-2.07	-4.97
	7	-1.55	-2.26	-2.66	-2.83	93.33	15.47	3.06	-1.39	-3.49	-5.07	2.94	3.87	4.27	4.60	-16.75	-16.07	-7.61	-3.17	-1.50	0.50	-4.49	-6.13	-6.94	-7.43	110.05	31.54	10.67	1.78	-1.99	-5.57
	8	-1.42	-1.96	-2.25	-2.41	95.98	15.91	4.75	-0.16	-2.15	-3.81	3.16	4.15	4.59	4.94	-23.08	-17.04	-7.96	-2.04	0.37	3.66	-4.59	-6.11	-6.84	-7.35	119.03	32.95	12.71	1.88	-2.51	-7.46
	9	-1.30	-1.76	-2.02	-2.17	94.60	17.97	5.37	0.68	-1.53	-3.13	3.35	4.39	4.86	5.23	-28.03	-20.40	-8.78	-2.31	2.01	5.62	-4.65	-6.15	-6.88	-7.40	122.60	38.36	14.14	2.99	-3.54	-8.75
	10	-0.98	-1.60	-1.80	-1.95	102.36	18.54	6.56	1.22	-0.89	-2.61	3.45	4.51	5.00	5.38	-30.48	-23.20	-11.24	-2.44	1.95	6.38	-4.44	-6.11	-6.81	-7.33	132.81	41.74	17.80	3.66	-2.83	-8.98
	25	-0.09	-0.09	-0.10	-0.12	133.96	25.19	9.74	4.32	1.78	0.09	0.65	0.83	0.93	0.98	-46.36	-47.50	-30.82	-19.03	-10.66	-1.99	-0.75	-0.92	-1.03	-1.11	180.28	72.68	40.56	23.35	12.45	2.08
	50	0.00	0.00	0.00	0.00	109.65	23.12	9.28	4.36	2.04	0.29	0.01	0.01	0.01	0.02	-51.92	-47.10	-31.01	-20.22	-12.25	-3.24	-0.01	-0.01	-0.02	-0.02	161.54	70.21	40.29	24.58	14.30	3.53
	100	0.00	0.00	0.00	0.00	107.67	21.53	8.59	4.08	1.98	0.30	0.00	0.00	0.01	0.01	-56.14	-46.46	-29.93	-19.16	-11.77	-2.89	0.00	0.00	-0.01	-0.01	163.77	67.99	38.52	23.24	13.75	3.19

Table 3.5 continued: Gains from pooling VAR forecasts – no noise (*Model III*)

3.B.3 Noisily Measured Economic Variables

		MODEL I																							
		T = 25																							
		Signal to noise = 10e4				Signal to noise = 100				Signal to noise = 10				Signal to noise = 1				Signal to noise = 0.1				Signal to noise = 10e-6			
	nVAR	2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5
h	1	-11.39	-14.27	-16.31	-16.94	-8.34	-11.71	-13.65	-14.15	-10.18	-11.87	-14.59	-15.38	-3.59	-5.10	-6.66	-6.97	-2.64	-2.67	-4.07	-4.36	-2.18	-2.90	-3.20	-3.79
	2	-12.24	-15.66	-18.19	-18.83	-8.31	-12.86	-14.55	-16.20	-8.09	-10.11	-13.03	-13.78	-2.02	-3.00	-3.66	-4.07	-1.42	-1.00	-1.82	-1.95	-1.13	-1.44	-1.21	-1.53
	3	-10.72	-14.30	-16.51	-17.17	-8.63	-12.39	-13.80	-15.26	-6.61	-8.16	-11.26	-11.88	-1.08	-1.89	-2.23	-2.50	-0.87	-0.68	-1.27	-1.20	-0.89	-1.02	-0.81	-0.75
	4	-10.36	-13.28	-14.95	-15.59	-8.36	-11.72	-12.38	-13.92	-5.71	-6.63	-9.59	-10.09	-0.79	-1.17	-1.13	-1.57	-0.65	-0.21	-0.75	-0.46	-0.77	-0.93	-0.63	-0.58
	5	-9.53	-12.44	-14.06	-14.55	-6.83	-10.00	-10.79	-12.11	-4.80	-5.59	-8.08	-8.26	-0.66	-0.98	-0.76	-1.30	-0.24	0.07	-0.40	0.00	-0.83	-0.86	-0.68	-0.53
	6	-9.06	-11.58	-13.10	-13.55	-6.45	-9.08	-9.86	-11.13	-4.24	-4.55	-7.10	-7.09	-0.61	-0.82	-0.57	-1.05	-0.21	0.03	-0.41	-0.03	-0.98	-0.93	-0.78	-0.60
	7	-7.94	-10.56	-12.04	-12.35	-5.47	-7.64	-8.60	-9.70	-3.97	-4.31	-6.71	-6.48	-0.64	-0.83	-0.59	-1.10	-0.15	0.14	-0.24	0.21	-1.09	-1.09	-0.84	-0.64
	8	-7.35	-9.88	-11.24	-11.69	-4.92	-7.09	-7.77	-9.23	-3.79	-3.62	-6.14	-5.82	-0.57	-0.65	-0.47	-1.07	-0.21	0.09	-0.34	0.21	-1.00	-1.06	-0.79	-0.85
	9	-6.95	-9.58	-10.90	-11.29	-4.34	-6.91	-7.45	-8.56	-3.29	-3.27	-5.29	-5.17	-0.37	-0.40	-0.38	-0.97	-0.35	0.05	-0.33	0.12	-0.92	-1.16	-0.89	-1.15
	10	-6.81	-9.38	-10.55	-10.92	-3.89	-6.21	-6.81	-7.78	-3.11	-3.04	-4.74	-4.92	-0.44	-0.37	-0.37	-0.99	-0.16	-0.07	-0.40	0.00	-0.96	-1.27	-1.12	-1.38
	25	-5.18	-8.06	-8.93	-8.97	-3.33	-3.72	-4.65	-4.84	-2.23	-3.73	-3.26	-3.26	-0.41	-0.10	-1.46	-1.25	0.02	0.29	-0.37	0.57	-1.77	-1.00	-1.08	-1.23
	50	-5.92	-7.89	-9.70	-9.85	-4.66	-5.61	-5.77	-6.74	-2.67	-3.97	-4.68	-5.04	-1.43	-0.48	-1.79	-1.91	0.53	1.15	0.12	0.83	-1.95	-1.28	-1.42	-1.74
	100	-7.71	-9.03	-10.95	-11.48	-3.77	-5.14	-6.27	-6.91	-3.51	-4.51	-4.63	-5.08	-1.38	-1.34	-2.71	-2.46	-0.68	-0.19	-0.86	-0.01	-3.15	-2.83	-2.19	-2.29
		T = 50																							
		Signal to noise = 10e4				Signal to noise = 100				Signal to noise = 10				Signal to noise = 1				Signal to noise = 0.1				Signal to noise = 10e-6			
	nVAR	2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5
h	1	-8.68	-11.23	-12.68	-13.50	-6.48	-9.28	-10.79	-11.67	-6.33	-9.17	-9.91	-10.93	-2.58	-4.30	-5.43	-5.95	-1.27	-1.98	-2.07	-2.46	-0.76	-1.32	-1.77	-1.57
	2	-9.27	-12.03	-13.88	-14.87	-7.94	-10.93	-13.29	-13.56	-6.57	-8.75	-10.02	-11.02	-1.64	-2.32	-3.33	-3.53	-0.55	-0.89	-0.74	-0.99	-0.46	-0.60	-0.78	-0.67
	3	-10.34	-12.43	-14.03	-14.96	-7.64	-10.21	-12.18	-12.92	-5.59	-7.37	-8.27	-9.23	-1.01	-1.21	-1.64	-1.87	-0.24	-0.46	-0.23	-0.58	-0.36	-0.44	-0.38	-0.40
	4	-9.49	-11.43	-12.75	-13.62	-7.25	-9.88	-11.60	-12.11	-4.61	-6.26	-6.98	-7.53	-0.55	-0.78	-1.31	-1.27	-0.14	-0.24	-0.12	-0.43	-0.34	-0.35	-0.23	-0.30
	5	-9.23	-10.86	-12.14	-13.02	-6.32	-8.97	-10.58	-11.32	-3.94	-5.25	-5.94	-6.30	-0.47	-0.67	-1.18	-1.11	-0.27	-0.33	-0.22	-0.47	-0.27	-0.22	-0.23	-0.16
	6	-8.76	-10.33	-11.21	-12.19	-5.44	-8.10	-9.27	-9.88	-3.28	-4.24	-5.14	-5.30	-0.29	-0.62	-0.98	-0.90	-0.19	-0.19	-0.12	-0.47	-0.26	-0.16	-0.26	-0.14
	7	-8.13	-9.61	-10.61	-11.57	-5.74	-8.02	-8.76	-9.67	-2.85	-3.73	-4.58	-4.65	-0.19	-0.58	-0.80	-0.65	-0.05	-0.19	-0.04	-0.32	-0.44	-0.15	-0.37	-0.21
	8	-7.29	-9.14	-9.97	-11.01	-6.05	-8.06	-8.69	-9.72	-2.78	-3.55	-4.39	-4.23	-0.31	-0.62	-0.79	-0.62	-0.16	-0.36	-0.07	-0.48	-0.28	-0.14	-0.29	-0.12
	9	-6.58	-8.45	-9.31	-10.29	-5.96	-7.77	-8.69	-9.43	-2.76	-3.63	-4.35	-4.12	-0.42	-0.73	-0.77	-0.71	-0.27	-0.34	-0.08	-0.32	-0.25	-0.28	-0.33	-0.03
	10	-6.29	-8.18	-9.06	-9.88	-5.99	-7.89	-8.51	-9.26	-2.50	-3.56	-3.94	-3.65	-0.29	-0.58	-0.73	-0.65	-0.20	-0.24	-0.07	-0.16	-0.42	-0.34	-0.37	-0.06
	25	-4.43	-5.38	-6.53	-6.69	-3.06	-3.64	-3.92	-4.84	-1.00	-2.45	-2.50	-2.03	-0.46	-0.83	-1.00	-0.54	0.13	0.33	-0.21	-0.02	-0.09	-0.41	-0.33	-0.14
	50	-2.73	-3.18	-4.01	-4.31	-2.34	-2.92	-3.68	-3.59	-0.31	-1.13	-1.73	-1.44	-0.48	-0.56	-0.34	-0.15	0.02	-0.21	0.06	0.24	0.25	0.08	-0.15	-0.12
	100	-3.54	-3.97	-4.38	-4.71	-1.52	-2.14	-3.16	-3.28	0.15	-0.85	-1.46	-1.08	0.12	-0.32	-0.11	0.38	0.21	0.04	-0.10	0.03	-0.23	-0.24	-0.01	-0.14
		T = 100																							
		Signal to noise = 10e4				Signal to noise = 100				Signal to noise = 10				Signal to noise = 1				Signal to noise = 0.1				Signal to noise = 10e-6			
	nVAR	2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5
h	1	-9.27	-11.82	-12.68	-13.74	-7.38	-9.67	-10.84	-11.95	-6.04	-8.44	-9.51	-10.07	-2.74	-4.00	-4.85	-5.23	-1.37	-1.43	-1.86	-1.92	-0.35	-0.47	-0.47	-0.63
	2	-9.11	-11.62	-13.00	-14.07	-8.64	-11.59	-12.59	-13.95	-5.87	-8.64	-9.48	-10.06	-1.36	-2.42	-2.93	-3.28	-0.57	-0.62	-0.83	-0.78	-0.27	-0.27	-0.25	-0.39
	3	-8.49	-10.94	-12.14	-13.12	-8.16	-11.03	-11.79	-12.93	-4.71	-6.92	-7.79	-8.11	-0.46	-1.23	-1.47	-1.63	-0.36	-0.51	-0.61	-0.53	-0.20	-0.19	-0.17	-0.23
	4	-7.29	-9.36	-10.67	-11.47	-7.92	-9.81	-10.69	-11.65	-3.31	-5.21	-6.15	-6.40	-0.36	-0.86	-0.96	-1.15	-0.33	-0.48	-0.52	-0.50	-0.15	-0.14	-0.08	-0.17
	5	-6.24	-8.50	-9.61	-10.43	-7.67	-9.59	-10.10	-11.01	-2.61	-4.35	-5.18	-5.37	-0.48	-0.95	-0.76	-1.01	-0.20	-0.43	-0.42	-0.39	-0.18	-0.08	-0.04	-0.10
	6	-5.75	-8.02	-9.12	-9.82	-7.28	-9.11	-9.79	-10.28	-2.05	-3.51	-4.33	-4.53	-0.53	-0.96	-0.65	-0.86	-0.13	-0.35	-0.29	-0.25	-0.11	0.02	0.03	-0.02
	7	-5.45	-7.61	-8.86	-9.32	-6.87	-8.61	-9.31	-9.69	-1.80	-3.10	-3.80	-4.09	-0.53	-0.83	-0.50	-0.71	-0.05	-0.33	-0.21	-0.18	-0.09	0.09	0.07	0.03
	8	-5.40	-7.50	-8.73	-9.20	-6.24	-8.12	-8.63	-8.98	-1.53	-2.89	-3.46	-3.81	-0.40	-0.78	-0.41	-0.69	0.00	-0.28	-0.15	-0.10	-0.10	0.15	0.10	0.07
	9	-4.94	-7.31	-8.52	-8.85	-6.11	-7.96	-8.33	-8.88	-1.40	-2.69	-3.20	-3.53	-0.34	-0.67	-0.37	-0.66	0.02	-0.24	-0.04	0.01	-0.10	0.17	0.11	0.11
	10	-4.94	-7.40	-8.47	-8.80	-5.98	-7.70	-8.23	-8.74	-1.35	-2.47	-3.16	-3.38	-0.30	-0.64	-0.34	-0.67	0.00	-0.24	-0.02	0.03	-0.07	0.23	0.17	0.16
	25	-3.37	-4.64	-5.45	-5.57	-3.45	-3.84	-4.07	-4.40	-0.93	-1.33	-1.63	-1.68	-0.21	-0.48	-0.39	-0.54	-0.06	-0.17	-0.13	-0.05	0.04	0.17	0.14	0.12
	50	-0.85	-1.72	-2.09	-1.98	-1.47	-1.49	-1.91	-1.83	0.13	-0.13	-0.08	-0.61	-0.45	-0.37	-0.33	-0.28	-0.11	-0.07	-0.01	-0.07	0.00	0.03	-0.08	0.06
	100	-1.27	-1.47	-1.45	-1.60	-0.01	-0.64	-0.30	-0.80	-0.31	-0.27	-0.49	-0.43	-0.12	-0.30	-0.24	-0.28	-0.12	0.00	-0.02	0.00	-0.07	0.04	0.08	-0.06

Table 3.6: Gains from equal pooling VAR forecasts – noise (*Model I*)

		T = 200																							
		Signal to noise = 10e4				Signal to noise = 100				Signal to noise = 10				Signal to noise = 1				Signal to noise = 0.1				Signal to noise = 10e-6			
	nVAR	2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5
h	1	-7.38	-9.34	-10.10	-10.72	-7.01	-8.23	-9.89	-10.75	-6.04	-7.85	-9.06	-9.62	-3.13	-3.52	-4.65	-4.68	-0.62	-0.69	-1.01	-1.17	-0.28	-0.33	-0.33	-0.49
	2	-8.04	-10.21	-11.45	-12.21	-7.41	-10.19	-11.57	-12.65	-6.49	-8.53	-9.35	-10.15	-2.03	-1.99	-2.70	-2.84	-0.10	-0.03	-0.27	-0.39	-0.22	-0.10	-0.15	-0.27
	3	-7.85	-9.43	-10.55	-11.41	-6.90	-9.66	-10.90	-11.99	-4.88	-6.61	-7.28	-7.88	-1.23	-1.25	-1.63	-1.80	0.14	0.14	0.00	-0.09	-0.12	-0.04	-0.04	-0.12
	4	-6.34	-8.37	-9.19	-9.96	-6.61	-9.08	-10.21	-11.18	-3.82	-4.97	-5.72	-6.20	-0.74	-0.77	-1.01	-1.19	0.21	0.14	0.05	-0.01	-0.11	-0.05	-0.05	-0.11
	5	-5.53	-7.54	-8.33	-8.87	-5.50	-7.70	-8.67	-9.40	-2.93	-3.77	-4.38	-4.87	-0.53	-0.64	-0.76	-0.89	0.16	0.07	-0.02	-0.06	-0.11	-0.04	-0.05	-0.09
	6	-4.69	-6.41	-7.13	-7.68	-5.12	-6.79	-7.69	-8.33	-2.33	-2.99	-3.53	-3.94	-0.33	-0.52	-0.59	-0.68	0.12	0.03	-0.05	-0.04	-0.08	-0.03	-0.04	-0.07
	7	-4.19	-6.01	-6.47	-7.03	-4.41	-5.63	-6.49	-7.11	-2.01	-2.52	-2.99	-3.34	-0.30	-0.48	-0.47	-0.55	0.08	0.00	-0.07	-0.07	-0.06	-0.02	-0.03	-0.05
	8	-3.65	-5.23	-5.71	-6.27	-4.33	-5.18	-5.99	-6.56	-1.90	-2.34	-2.76	-3.05	-0.13	-0.36	-0.25	-0.37	0.05	-0.04	-0.08	-0.08	-0.06	-0.02	-0.02	-0.04
	9	-3.48	-5.04	-5.56	-5.98	-3.94	-4.73	-5.33	-5.94	-1.68	-2.14	-2.52	-2.81	-0.11	-0.33	-0.23	-0.34	0.04	-0.05	-0.07	-0.07	-0.04	-0.01	-0.01	-0.03
	10	-3.41	-4.99	-5.38	-5.81	-3.87	-4.40	-5.05	-5.63	-1.56	-1.94	-2.30	-2.59	-0.11	-0.27	-0.22	-0.32	0.04	-0.04	-0.05	-0.05	-0.03	0.00	0.01	-0.01
	25	-3.02	-4.16	-4.70	-4.81	-1.82	-2.15	-2.39	-2.65	-0.86	-0.99	-0.92	-1.05	0.02	-0.12	-0.10	-0.15	-0.03	-0.06	-0.05	-0.05	0.00	-0.01	-0.02	-0.01
	50	-0.37	-1.07	-1.10	-1.22	-0.32	-0.33	-0.66	-0.18	0.12	-0.04	-0.11	-0.16	-0.01	-0.07	-0.04	-0.05	0.04	-0.02	0.02	0.01	0.03	0.03	0.02	0.03
	100	-0.23	-0.33	-0.12	-0.25	0.06	-0.28	-0.06	-0.10	-0.13	-0.03	-0.04	-0.06	-0.02	-0.03	-0.01	-0.01	0.01	0.00	-0.01	-0.01	0.01	0.00	0.00	0.00
		T = 1000																							
		Signal to noise = 10e4				Signal to noise = 100				Signal to noise = 10				Signal to noise = 1				Signal to noise = 0.1				Signal to noise = 10e-6			
	nVAR	2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5
h	1	-6.62	-8.97	-9.98	-10.50	-7.87	-8.75	-9.97	-10.67	-4.47	-6.02	-6.88	-7.38	-1.90	-3.09	-3.47	-3.99	-0.55	-0.82	-0.80	-0.74	-0.04	-0.11	-0.08	-0.05
	2	-8.15	-10.38	-11.84	-12.50	-7.68	-9.00	-10.62	-11.29	-4.53	-6.34	-7.35	-7.78	-1.18	-1.88	-2.24	-2.48	-0.29	-0.45	-0.40	-0.41	-0.05	-0.07	-0.07	-0.03
	3	-6.75	-9.06	-10.47	-11.06	-6.94	-8.29	-9.36	-9.99	-3.40	-5.28	-6.10	-6.35	-0.71	-1.29	-1.43	-1.49	-0.15	-0.22	-0.23	-0.22	-0.02	-0.04	-0.05	-0.01
	4	-5.51	-7.27	-8.50	-9.14	-5.87	-7.28	-8.14	-8.48	-2.52	-3.92	-4.60	-4.90	-0.19	-0.61	-0.72	-0.81	-0.09	-0.16	-0.14	-0.15	-0.01	-0.02	-0.02	0.00
	5	-4.20	-6.07	-7.08	-7.58	-4.72	-6.09	-6.95	-7.18	-1.95	-3.23	-3.66	-3.86	0.04	-0.41	-0.37	-0.49	-0.06	-0.11	-0.11	-0.13	-0.01	-0.01	-0.01	0.01
	6	-3.60	-5.24	-6.14	-6.58	-4.36	-5.31	-6.11	-6.35	-1.74	-2.84	-3.11	-3.33	0.11	-0.28	-0.29	-0.36	-0.08	-0.10	-0.11	-0.12	0.01	0.00	-0.01	0.01
	7	-3.01	-4.70	-5.40	-5.73	-3.70	-4.74	-5.33	-5.63	-1.43	-2.39	-2.59	-2.82	0.08	-0.26	-0.23	-0.31	-0.07	-0.08	-0.09	-0.11	0.00	0.00	0.00	0.01
	8	-2.54	-4.30	-4.86	-5.12	-3.30	-4.29	-5.00	-5.26	-1.29	-2.12	-2.27	-2.50	0.01	-0.20	-0.18	-0.28	-0.08	-0.04	-0.08	-0.10	0.00	0.00	0.00	0.01
	9	-2.43	-4.08	-4.52	-4.84	-2.63	-3.69	-4.45	-4.70	-1.20	-1.96	-2.08	-2.31	0.04	-0.11	-0.07	-0.15	-0.06	-0.01	-0.06	-0.08	-0.01	-0.01	-0.01	0.00
	10	-2.13	-3.72	-4.14	-4.48	-2.32	-3.53	-4.17	-4.44	-1.14	-1.73	-1.84	-2.04	0.01	-0.10	-0.07	-0.12	-0.06	0.00	-0.06	-0.07	-0.01	-0.01	-0.01	-0.01
	25	-1.77	-2.36	-2.83	-3.09	-1.27	-1.87	-2.09	-2.23	-0.34	-0.50	-0.67	-0.73	-0.02	0.07	0.05	0.04	0.01	0.00	0.02	0.02	-0.01	-0.01	-0.01	-0.01
	50	-0.54	-0.45	-0.71	-0.79	-0.18	-0.34	-0.39	-0.43	-0.01	-0.02	-0.04	-0.05	-0.02	-0.03	0.00	-0.01	-0.01	0.00	-0.01	-0.01	0.00	0.00	0.00	0.00
	100	-0.14	-0.14	-0.16	-0.20	-0.02	0.02	-0.01	0.02	-0.01	-0.01	-0.01	-0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 3.6 continued: Gains from equal pooling VAR forecasts – noise (*Model I*)

MODEL II

T = 25

		Signal to noise = 10e4				Signal to noise = 100				Signal to noise = 10				Signal to noise = 1				Signal to noise = 0.1				Signal to noise = 10e-6			
	nVAR	2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5
h	1	-10.96	-13.17	-14.36	-15.56	-7.23	-10.94	-13.07	-13.44	-9.18	-11.87	-12.68	-13.80	-6.44	-7.93	-8.11	-9.88	-2.27	-2.59	-3.24	-3.51	-2.22	-3.03	-3.84	-3.86
	2	-11.23	-14.32	-15.76	-16.69	-8.95	-12.05	-15.07	-15.22	-8.44	-10.88	-11.76	-12.69	-3.84	-4.78	-4.85	-6.05	-0.73	-0.92	-0.88	-1.42	-0.68	-0.87	-1.10	-1.29
	3	-9.20	-12.71	-13.83	-14.56	-8.53	-11.22	-13.64	-14.03	-7.19	-9.29	-10.07	-10.80	-2.41	-3.40	-3.14	-4.05	-0.45	-0.46	-0.41	-0.82	-0.56	-0.47	-0.73	-0.85
	4	-7.39	-10.88	-11.96	-12.59	-7.52	-10.22	-12.07	-12.61	-6.29	-7.52	-8.55	-8.93	-1.49	-2.51	-1.97	-2.73	-0.19	-0.18	-0.20	-0.60	-0.56	-0.28	-0.51	-0.55
	5	-5.73	-9.08	-10.18	-10.75	-6.86	-9.37	-10.74	-11.21	-5.20	-6.55	-7.12	-7.73	-1.11	-1.87	-1.40	-2.09	0.19	0.24	-0.05	-0.47	-0.52	0.06	-0.28	-0.36
	6	-4.81	-7.93	-9.00	-9.49	-6.02	-8.45	-9.69	-10.19	-4.50	-5.67	-6.11	-6.61	-0.81	-1.44	-0.86	-1.70	0.09	0.31	-0.01	-0.31	-0.57	0.17	-0.18	-0.20
	7	-4.16	-6.80	-8.00	-8.33	-5.28	-7.73	-8.69	-9.24	-3.64	-5.29	-5.39	-5.88	-0.45	-1.12	-0.82	-1.41	-0.04	0.33	-0.17	-0.32	-0.41	0.41	0.16	-0.04
	8	-3.86	-5.98	-7.12	-7.39	-4.43	-6.67	-7.40	-8.08	-2.78	-4.79	-4.56	-5.08	-0.41	-1.12	-0.88	-1.23	-0.24	0.28	-0.30	-0.31	-0.46	0.32	0.12	0.10
	9	-3.27	-5.10	-6.31	-6.40	-3.99	-5.94	-6.64	-7.33	-2.29	-4.25	-3.91	-4.29	-0.70	-1.19	-0.93	-1.35	-0.55	0.09	-0.56	-0.45	-0.35	0.20	0.10	0.27
	10	-3.15	-4.80	-5.76	-5.86	-4.01	-5.57	-6.15	-7.01	-2.45	-4.23	-3.75	-4.09	-0.80	-1.21	-0.88	-1.29	-0.69	0.17	-0.60	-0.45	-0.16	0.23	0.26	0.40
	25	-1.73	-2.87	-3.49	-3.66	-2.16	-2.21	-2.88	-3.40	-2.20	-3.17	-2.90	-3.05	-1.01	-0.81	-0.58	-0.91	-0.91	-0.53	-0.72	-0.85	-1.11	-0.76	-0.33	-0.70
	50	-2.18	-3.00	-3.64	-3.71	-1.70	-2.58	-3.21	-3.11	-2.82	-3.72	-3.66	-3.53	-1.30	-1.16	-2.22	-1.70	-1.05	-0.83	-0.65	-0.74	-0.96	-0.77	0.00	-0.66
	100	-1.60	-3.38	-3.58	-3.89	-2.40	-3.19	-2.90	-3.26	-4.07	-4.78	-3.54	-4.68	-1.88	-1.06	-1.38	-1.33	-0.81	-0.93	-0.53	-0.86	-0.99	-0.23	-0.16	-0.94

T = 50

		Signal to noise = 10e4				Signal to noise = 100				Signal to noise = 10				Signal to noise = 1				Signal to noise = 0.1				Signal to noise = 10e-6			
	nVAR	2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5
h	1	-6.83	-9.19	-10.80	-11.31	-7.71	-10.13	-11.51	-12.17	-6.68	-9.16	-10.70	-11.02	-3.52	-4.65	-5.25	-5.69	-1.82	-2.09	-2.69	-2.52	-1.34	-1.65	-1.52	-1.70
	2	-8.15	-10.54	-12.05	-12.73	-8.62	-10.61	-12.29	-12.98	-6.22	-8.89	-10.21	-10.60	-2.03	-2.94	-3.10	-3.20	-0.56	-0.60	-1.11	-0.75	-0.49	-0.84	-0.66	-0.65
	3	-7.72	-9.87	-11.17	-11.85	-7.65	-9.24	-11.12	-11.59	-5.24	-7.82	-8.47	-8.87	-1.27	-1.80	-2.05	-1.97	0.00	-0.16	-0.39	-0.18	-0.27	-0.43	-0.30	-0.24
	4	-6.78	-8.65	-9.98	-10.61	-6.69	-7.91	-9.85	-10.24	-4.20	-6.35	-6.94	-7.20	-0.97	-1.21	-1.69	-1.58	0.07	-0.15	-0.25	-0.16	-0.31	-0.48	-0.26	-0.28
	5	-6.27	-8.16	-9.34	-9.90	-5.91	-6.93	-8.89	-9.24	-3.78	-5.34	-5.76	-6.20	-0.45	-0.91	-1.19	-1.28	-0.09	-0.12	-0.42	-0.36	-0.46	-0.42	-0.29	-0.38
	6	-5.89	-7.52	-8.59	-9.09	-5.33	-6.01	-7.89	-8.12	-3.38	-4.82	-5.20	-5.60	-0.23	-0.81	-1.07	-1.09	-0.16	-0.10	-0.46	-0.44	-0.42	-0.41	-0.20	-0.29
	7	-5.63	-7.14	-8.05	-8.54	-5.18	-5.83	-7.37	-7.52	-3.20	-4.46	-4.67	-5.09	-0.04	-0.55	-0.71	-0.87	-0.03	0.01	-0.41	-0.24	-0.35	-0.28	-0.15	-0.10
	8	-5.06	-6.36	-7.30	-7.76	-4.94	-5.36	-6.61	-6.89	-3.20	-4.24	-4.47	-4.92	0.06	-0.44	-0.62	-0.77	-0.22	0.03	-0.57	-0.32	-0.28	-0.21	-0.07	-0.08
	9	-4.75	-5.82	-6.79	-7.23	-4.35	-4.82	-5.93	-6.20	-3.27	-4.22	-4.38	-4.82	0.08	-0.51	-0.56	-0.77	-0.26	0.08	-0.54	-0.23	-0.26	-0.13	-0.06	-0.18
	10	-4.54	-5.63	-6.52	-6.96	-4.19	-4.68	-5.47	-5.86	-3.21	-4.07	-4.22	-4.76	0.02	-0.44	-0.46	-0.83	-0.22	0.12	-0.49	-0.16	-0.28	-0.11	-0.11	-0.28
	25	-2.06	-2.60	-3.04	-3.15	-0.83	-0.94	-1.55	-1.69	-2.25	-2.47	-2.57	-2.93	-0.20	-0.36	0.05	-0.42	-0.04	-0.37	-0.28	-0.11	-0.45	-0.14	-0.28	-0.76
	50	-1.21	-1.54	-1.63	-1.80	-0.96	-1.22	-1.97	-1.90	-1.09	-1.68	-1.67	-2.01	-0.66	-0.54	-0.33	-0.46	-0.54	-0.47	-0.80	-0.51	-0.16	-0.10	-0.06	0.05
	100	-0.83	-1.20	-1.55	-1.69	-1.10	-1.07	-1.44	-1.39	-1.33	-1.85	-1.82	-2.26	-0.19	-0.49	-0.04	-0.35	-0.18	-0.20	-0.39	-0.03	-0.39	-0.27	-0.40	-0.25

T = 100

		Signal to noise = 10e4				Signal to noise = 100				Signal to noise = 10				Signal to noise = 1				Signal to noise = 0.1				Signal to noise = 10e-6			
	nVAR	2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5
h	1	-5.82	-8.66	-9.55	-10.30	-6.05	-8.97	-9.64	-10.17	-6.65	-8.58	-9.27	-9.86	-4.32	-4.81	-5.38	-5.59	-0.75	-1.32	-1.54	-1.45	-1.03	-0.86	-1.04	-0.85
	2	-6.41	-9.58	-10.75	-11.48	-8.03	-11.49	-12.05	-12.47	-6.32	-8.51	-9.17	-9.59	-2.36	-2.70	-3.00	-3.30	-0.27	-0.59	-0.59	-0.51	-0.17	-0.17	-0.32	-0.11
	3	-6.15	-8.56	-9.81	-10.41	-7.01	-9.96	-10.43	-10.93	-4.78	-6.62	-7.23	-7.59	-1.63	-1.75	-2.03	-2.17	-0.06	-0.21	-0.20	-0.17	0.02	-0.10	-0.14	-0.02
	4	-6.03	-7.66	-8.83	-9.37	-6.40	-8.70	-9.16	-9.59	-3.71	-5.24	-5.75	-6.08	-1.18	-1.25	-1.55	-1.52	-0.01	-0.12	-0.08	-0.07	0.12	-0.01	0.02	0.09
	5	-5.42	-6.62	-7.68	-8.18	-5.31	-7.40	-7.93	-8.29	-3.18	-4.49	-4.78	-5.15	-0.98	-1.03	-1.40	-1.27	0.07	-0.02	-0.06	-0.01	0.12	0.03	0.01	0.04
	6	-5.13	-5.90	-6.88	-7.41	-4.87	-6.59	-6.98	-7.33	-2.80	-3.84	-4.09	-4.37	-0.77	-0.90	-1.16	-1.04	0.15	0.00	-0.03	-0.01	0.07	0.02	-0.04	-0.05
	7	-4.43	-4.96	-5.96	-6.43	-4.19	-5.57	-6.13	-6.40	-2.35	-3.40	-3.60	-3.82	-0.67	-0.85	-1.01	-0.89	0.14	0.04	-0.04	0.02	0.08	0.02	-0.04	-0.11
	8	-3.53	-4.14	-5.00	-5.47	-3.89	-5.02	-5.57	-5.73	-2.21	-3.12	-3.23	-3.39	-0.51	-0.75	-0.90	-0.78	0.05	0.03	-0.04	-0.01	0.07	-0.01	-0.06	-0.12
	9	-3.25	-3.82	-4.59	-5.04	-3.18	-4.35	-4.91	-5.00	-2.27	-3.07	-3.11	-3.27	-0.61	-0.84	-0.91	-0.83	0.05	0.03	-0.03	0.00	0.06	-0.06	-0.08	-0.15
	10	-3.20	-3.79	-4.41	-4.85	-2.90	-3.96	-4.56	-4.60	-2.22	-2.93	-3.06	-3.17	-0.50	-0.81	-0.83	-0.80	0.08	0.04	0.00	0.03	0.04	-0.07	-0.06	-0.11
	25	-0.97	-1.02	-1.16	-1.39	-1.17	-1.49	-1.58	-1.57	-0.69	-0.89	-0.97	-0.96	0.17	-0.12	-0.20	-0.32	0.03	0.05	0.06	0.05	0.07	-0.04	-0.02	-0.06
	50	-0.20	-0.48	-0.49	-0.57	-0.56	-0.57	-0.73	-0.87	-0.37	-0.60	-0.60	-0.67	-0.20	-0.29	-0.34	-0.33	-0.08	-0.08	-0.07	-0.09	0.06	-0.06	-0.09	-0.11
	100	-0.13	-0.26	-0.30	-0.36	-0.52	-0.48	-0.62	-0.61	-0.57	-0.63	-0.74	-0.74	-0.06	-0.17	-0.15	-0.12	0.00	-0.05	-0.03	-0.06	0.01	0.01	0.00	0.02

Table 3.7: Gains from equal pooling VAR forecasts – noise (*Model II*)

		T = 200																							
		Signal to noise = 10e4				Signal to noise = 100				Signal to noise = 10				Signal to noise = 1				Signal to noise = 0.1				Signal to noise = 10e-6			
	nVAR	2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5
h	1	-7.84	-9.72	-11.14	-11.45	-6.60	-8.91	-9.92	-10.53	-5.33	-7.17	-8.54	-8.94	-3.48	-4.02	-4.68	-5.00	-0.69	-1.14	-1.02	-1.26	-0.43	-0.60	-0.60	-0.49
	2	-9.45	-11.81	-13.01	-13.54	-7.16	-9.82	-11.10	-11.78	-5.72	-7.20	-8.48	-9.18	-2.19	-2.42	-2.94	-2.89	-0.20	-0.45	-0.44	-0.46	-0.19	-0.30	-0.30	-0.23
	3	-8.29	-10.80	-11.86	-12.42	-6.86	-8.62	-9.85	-10.49	-4.77	-6.34	-7.10	-7.76	-1.24	-1.35	-1.67	-1.59	-0.09	-0.34	-0.25	-0.31	-0.19	-0.20	-0.20	-0.18
	4	-7.26	-9.45	-10.41	-10.92	-5.87	-7.46	-8.51	-9.12	-3.53	-4.84	-5.36	-5.76	-0.62	-0.76	-0.95	-0.79	-0.01	-0.19	-0.14	-0.17	-0.15	-0.14	-0.13	-0.10
	5	-6.28	-8.14	-9.17	-9.55	-4.86	-6.61	-7.32	-7.89	-2.64	-4.11	-4.44	-4.76	-0.36	-0.56	-0.64	-0.49	-0.04	-0.18	-0.12	-0.14	-0.12	-0.11	-0.12	-0.09
	6	-5.06	-6.79	-7.71	-7.95	-3.97	-5.69	-6.29	-6.77	-2.09	-3.24	-3.60	-3.91	-0.31	-0.48	-0.57	-0.43	-0.01	-0.11	-0.09	-0.13	-0.10	-0.09	-0.09	-0.05
	7	-4.28	-5.92	-6.76	-6.99	-3.27	-4.93	-5.51	-5.88	-1.60	-2.76	-3.07	-3.33	-0.30	-0.36	-0.55	-0.37	0.00	-0.09	-0.07	-0.13	-0.08	-0.06	-0.07	-0.03
	8	-3.49	-4.98	-5.80	-5.98	-2.86	-4.28	-4.83	-5.12	-1.18	-2.23	-2.52	-2.75	-0.21	-0.25	-0.42	-0.24	0.00	-0.10	-0.07	-0.12	-0.05	-0.03	-0.04	-0.01
	9	-3.41	-4.65	-5.42	-5.57	-2.69	-3.74	-4.37	-4.55	-0.84	-1.75	-2.09	-2.31	-0.08	-0.11	-0.27	-0.12	0.01	-0.09	-0.05	-0.12	-0.05	-0.03	-0.03	-0.01
	10	-3.03	-4.14	-4.77	-4.93	-2.28	-3.12	-3.77	-3.93	-0.85	-1.65	-1.92	-2.07	-0.01	-0.02	-0.18	-0.07	0.02	-0.05	-0.03	-0.09	-0.05	-0.02	-0.03	-0.01
	25	-1.11	-1.43	-1.45	-1.49	-0.17	-0.30	-0.43	-0.48	-0.14	-0.19	-0.39	-0.48	0.09	-0.02	-0.07	-0.01	0.04	-0.03	-0.04	-0.01	-0.04	-0.04	-0.05	-0.03
	50	-0.12	-0.06	-0.15	-0.16	-0.13	-0.07	-0.19	-0.21	-0.12	-0.17	-0.24	-0.26	-0.14	-0.12	-0.11	-0.09	-0.01	0.00	-0.01	0.00	-0.01	-0.01	0.00	-0.01
	100	-0.06	-0.01	-0.04	-0.04	-0.06	-0.05	-0.03	-0.05	-0.07	-0.07	-0.11	-0.11	-0.04	-0.04	-0.02	-0.04	0.00	-0.01	-0.02	0.00	-0.01	-0.01	-0.01	-0.01

		T = 1000																							
		Signal to noise = 10e4				Signal to noise = 100				Signal to noise = 10				Signal to noise = 1				Signal to noise = 0.1				Signal to noise = 10e-6			
	nVAR	2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5
h	1	-6.02	-7.43	-8.46	-9.09	-6.43	-8.35	-9.09	-9.59	-4.72	-7.13	-7.56	-8.15	-2.46	-2.60	-3.43	-3.39	-0.77	-0.78	-0.68	-0.70	-0.04	-0.06	-0.05	-0.06
	2	-7.07	-8.92	-9.86	-10.46	-7.40	-9.85	-10.61	-11.42	-4.40	-6.72	-7.03	-7.64	-1.30	-1.38	-1.92	-1.73	-0.33	-0.43	-0.34	-0.32	-0.10	-0.08	-0.07	-0.08
	3	-6.59	-8.63	-9.75	-10.06	-6.64	-9.02	-9.85	-10.51	-3.29	-5.25	-5.68	-6.15	-0.80	-0.83	-1.29	-1.13	-0.21	-0.26	-0.16	-0.16	-0.08	-0.05	-0.04	-0.04
	4	-5.33	-7.25	-8.34	-8.46	-5.68	-7.75	-8.37	-8.88	-2.68	-4.34	-4.60	-4.91	-0.40	-0.60	-0.81	-0.75	-0.13	-0.16	-0.09	-0.09	-0.06	-0.05	-0.03	-0.05
	5	-4.13	-5.88	-6.78	-6.82	-4.55	-6.24	-6.71	-7.18	-1.90	-3.25	-3.53	-3.79	-0.28	-0.50	-0.61	-0.59	-0.09	-0.11	-0.04	-0.06	-0.05	-0.04	-0.03	-0.04
	6	-3.34	-4.93	-5.68	-5.75	-3.94	-5.46	-5.81	-6.14	-1.44	-2.63	-2.82	-3.06	-0.22	-0.46	-0.60	-0.55	-0.06	-0.08	-0.04	-0.04	-0.04	-0.03	-0.02	-0.03
	7	-2.95	-4.43	-4.99	-5.04	-3.64	-4.95	-5.26	-5.50	-1.15	-2.24	-2.41	-2.66	-0.20	-0.39	-0.50	-0.45	-0.03	-0.06	-0.04	-0.03	-0.04	-0.03	-0.03	-0.03
	8	-2.67	-4.01	-4.48	-4.49	-3.60	-4.52	-4.86	-5.10	-0.93	-2.00	-2.02	-2.28	-0.14	-0.34	-0.45	-0.36	-0.01	-0.05	-0.05	-0.05	-0.05	-0.03	-0.03	-0.04
	9	-2.64	-3.60	-4.07	-4.10	-3.12	-3.82	-4.18	-4.43	-1.03	-1.91	-1.89	-2.13	-0.12	-0.31	-0.38	-0.31	-0.04	-0.07	-0.07	-0.08	-0.05	-0.03	-0.03	-0.03
	10	-2.54	-3.31	-3.75	-3.79	-3.01	-3.43	-3.79	-4.04	-1.14	-1.81	-1.74	-2.01	-0.03	-0.26	-0.30	-0.23	-0.03	-0.06	-0.07	-0.08	-0.04	-0.03	-0.03	-0.03
	25	-0.49	-0.74	-0.63	-0.72	-0.39	-0.36	-0.63	-0.66	-0.50	-0.52	-0.46	-0.50	-0.08	-0.10	-0.13	-0.12	-0.02	-0.04	-0.03	-0.03	0.00	-0.01	0.00	-0.01
	50	0.03	0.02	0.00	0.00	-0.02	-0.03	-0.02	-0.02	0.02	-0.02	-0.01	-0.01	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	100	0.00	0.00	0.00	0.00	0.00	-0.01	-0.01	-0.01	0.00	-0.01	-0.01	-0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 3.7 continued: Gains from equal pooling VAR forecasts – noise (*Model II*)

		MODEL III																							
		T = 25																							
		Signal to noise = 10e4				Signal to noise = 100				Signal to noise = 10				Signal to noise = 1				Signal to noise = 0.1				Signal to noise = 10e-6			
	nVAR	2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5
h	1	-5.18	-6.70	-7.60	-7.97	-5.30	-6.15	-7.25	-7.91	-3.52	-5.93	-6.24	-6.50	-2.61	-3.62	-4.58	-4.20	-3.16	-4.02	-3.90	-4.31	-3.30	-3.68	-4.37	-4.33
	2	-6.29	-7.68	-8.37	-9.02	-6.05	-6.99	-8.22	-8.58	-4.46	-6.38	-6.45	-6.94	-2.44	-3.07	-3.30	-3.47	-1.72	-1.94	-1.80	-2.11	-1.64	-2.06	-1.87	-2.06
	3	-5.46	-6.80	-7.32	-7.90	-5.62	-6.67	-7.71	-8.08	-3.87	-5.60	-6.01	-6.44	-1.93	-2.38	-2.64	-2.62	-0.99	-1.13	-1.09	-1.36	-1.31	-1.61	-1.43	-1.58
	4	-4.25	-5.64	-6.13	-6.65	-5.06	-6.27	-7.12	-7.18	-3.54	-4.86	-5.47	-5.64	-1.89	-2.11	-2.45	-2.30	-0.78	-0.88	-0.65	-0.94	-1.05	-1.30	-1.20	-1.33
	5	-3.25	-4.38	-5.14	-5.54	-4.41	-5.62	-6.41	-6.55	-2.98	-4.21	-4.59	-4.75	-1.32	-1.92	-1.72	-1.71	-0.48	-0.61	-0.58	-0.88	-0.85	-1.27	-0.96	-1.18
	6	-3.14	-4.20	-4.88	-5.15	-3.87	-4.68	-5.22	-5.70	-2.44	-3.64	-4.03	-4.27	-1.02	-1.62	-1.49	-1.57	-0.30	-0.46	-0.53	-0.69	-1.06	-1.19	-1.01	-1.11
	7	-2.99	-3.91	-4.55	-4.81	-3.27	-4.13	-4.75	-5.17	-2.15	-3.17	-3.67	-3.69	-0.50	-1.08	-0.94	-1.17	-0.48	-0.49	-0.76	-0.92	-1.04	-1.09	-0.95	-0.99
	8	-2.23	-3.25	-3.83	-4.06	-3.14	-3.53	-4.19	-4.65	-2.03	-2.84	-3.22	-3.51	-0.27	-0.79	-0.72	-1.05	-0.51	-0.53	-0.69	-1.10	-1.19	-1.18	-1.11	-1.10
	9	-1.97	-3.01	-3.53	-3.71	-3.12	-3.10	-3.90	-4.21	-1.87	-2.55	-2.86	-3.21	-0.41	-0.85	-0.72	-1.03	-0.37	-0.30	-0.42	-1.01	-1.03	-0.99	-0.93	-1.09
	10	-2.05	-2.87	-3.31	-3.44	-2.70	-2.56	-3.48	-3.55	-2.00	-2.37	-2.59	-3.15	-0.33	-0.90	-0.86	-1.14	-0.54	-0.54	-0.56	-1.25	-1.00	-0.87	-0.94	-1.35
	25	-1.34	-1.92	-2.08	-2.20	-1.90	-1.72	-2.09	-2.70	-1.15	-1.76	-1.62	-2.33	-0.44	-0.19	-0.93	-1.07	-0.69	-0.49	-0.78	-1.07	-0.52	-0.90	-0.57	-1.01
	50	-1.85	-2.51	-2.85	-2.97	-3.37	-2.31	-2.80	-3.02	-1.09	-1.55	-1.45	-2.34	-0.30	0.10	-0.85	-0.64	-0.54	-0.32	-0.83	-1.33	-0.59	-0.37	-0.80	-0.84
	100	-1.74	-1.92	-2.60	-2.69	-3.97	-3.25	-3.58	-4.12	-0.32	-1.17	-1.04	-2.55	-0.41	-0.18	-1.10	-1.85	0.45	0.25	-0.13	-0.38	-1.27	-1.99	-1.52	-1.64
		T = 50																							
		Signal to noise = 10e4				Signal to noise = 100				Signal to noise = 10				Signal to noise = 1				Signal to noise = 0.1				Signal to noise = 10e-6			
	nVAR	2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5
h	1	-3.62	-5.02	-5.36	-5.71	-4.78	-5.81	-6.48	-6.59	-3.41	-4.12	-4.77	-5.15	-2.34	-3.31	-3.80	-3.83	-0.58	-1.05	-1.63	-1.96	-0.84	-1.38	-1.40	-1.65
	2	-4.52	-5.72	-6.36	-6.75	-4.49	-6.04	-6.71	-6.63	-3.20	-4.29	-4.77	-5.19	-1.70	-2.31	-2.74	-2.42	-0.15	-0.32	-0.53	-0.78	-0.34	-0.55	-0.66	-0.65
	3	-4.47	-5.13	-5.79	-6.11	-3.92	-5.04	-6.07	-6.14	-2.58	-3.57	-4.26	-4.60	-1.18	-1.91	-2.03	-1.76	-0.04	-0.26	-0.35	-0.37	-0.29	-0.41	-0.48	-0.49
	4	-3.29	-3.70	-4.39	-4.71	-2.94	-3.99	-5.02	-5.06	-1.94	-2.97	-3.39	-3.74	-1.04	-1.41	-1.48	-1.25	0.07	-0.17	-0.24	-0.26	-0.25	-0.31	-0.29	-0.32
	5	-3.04	-3.07	-3.80	-4.08	-2.63	-3.24	-4.18	-4.22	-1.12	-2.33	-2.49	-2.74	-0.76	-1.03	-1.03	-0.85	0.14	-0.07	-0.13	-0.22	-0.23	-0.28	-0.22	-0.26
	6	-2.61	-2.93	-3.59	-3.79	-2.50	-2.91	-3.65	-3.71	-1.17	-2.28	-2.31	-2.53	-0.46	-0.52	-0.62	-0.44	-0.03	0.02	-0.18	-0.26	-0.10	-0.20	-0.19	-0.23
	7	-2.06	-2.37	-2.96	-3.21	-1.80	-2.32	-2.98	-3.07	-1.04	-2.13	-2.01	-2.24	-0.25	-0.28	-0.41	-0.34	-0.02	0.02	-0.16	-0.20	-0.15	-0.21	-0.17	-0.23
	8	-1.87	-2.15	-2.78	-2.96	-1.45	-1.80	-2.46	-2.60	-1.26	-2.13	-2.00	-2.15	-0.17	-0.36	-0.34	-0.31	0.03	0.10	-0.05	-0.08	-0.03	-0.10	-0.02	-0.15
	9	-1.96	-2.24	-2.64	-2.88	-1.44	-1.72	-2.34	-2.49	-0.97	-1.71	-1.72	-1.80	-0.34	-0.43	-0.32	-0.27	0.08	0.00	0.07	-0.03	-0.02	-0.13	0.03	-0.09
	10	-1.88	-2.28	-2.53	-2.73	-1.16	-1.58	-2.18	-2.37	-0.92	-1.41	-1.64	-1.65	-0.34	-0.39	-0.27	-0.24	0.12	0.09	0.20	0.08	-0.04	-0.15	0.07	-0.04
	25	-0.49	-0.47	-0.69	-0.69	-0.41	-0.62	-0.89	-0.91	-0.53	-0.66	-0.65	-0.82	-0.16	-0.38	-0.34	-0.22	0.11	0.06	0.08	0.02	-0.08	-0.06	0.05	-0.07
	50	-0.81	-0.67	-0.74	-0.83	-0.28	-0.54	-0.59	-0.74	-0.38	-0.50	-0.53	-0.64	-0.37	-0.23	-0.16	-0.32	0.17	0.18	0.13	0.21	0.05	0.07	0.15	0.05
	100	-0.36	-0.40	-0.61	-0.58	-0.67	-0.69	-0.70	-0.85	-0.37	-0.56	-0.55	-0.54	-0.24	0.18	-0.64	-0.60	0.24	0.05	0.40	0.61	-0.45	-0.26	0.06	-0.14
		T = 100																							
		Signal to noise = 10e4				Signal to noise = 100				Signal to noise = 10				Signal to noise = 1				Signal to noise = 0.1				Signal to noise = 10e-6			
	nVAR	2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5
h	1	-2.58	-3.61	-3.75	-4.05	-1.86	-3.23	-3.57	-3.82	-2.35	-3.30	-3.75	-4.01	-2.37	-2.42	-2.64	-2.76	-0.40	-0.48	-0.89	-0.89	-1.00	-0.70	-1.17	-1.17
	2	-3.19	-4.38	-4.66	-5.13	-2.37	-3.85	-4.27	-4.62	-3.06	-4.21	-4.78	-5.03	-1.77	-1.66	-1.74	-1.97	-0.32	-0.30	-0.45	-0.51	-0.27	-0.18	-0.26	-0.35
	3	-3.44	-4.73	-4.96	-5.39	-2.00	-3.46	-3.94	-4.33	-2.43	-3.58	-3.93	-4.30	-1.11	-1.18	-1.15	-1.41	-0.34	-0.30	-0.40	-0.47	-0.14	-0.16	-0.18	-0.17
	4	-3.24	-4.23	-4.50	-5.03	-1.64	-2.97	-3.47	-3.77	-1.92	-2.88	-3.06	-3.49	-0.81	-0.79	-0.68	-0.95	-0.24	-0.14	-0.26	-0.32	-0.09	-0.12	-0.13	-0.09
	5	-3.14	-3.98	-4.22	-4.69	-1.62	-2.66	-2.99	-3.33	-1.56	-2.46	-2.45	-2.84	-0.53	-0.68	-0.49	-0.71	-0.20	-0.15	-0.25	-0.28	-0.03	-0.03	-0.04	-0.04
	6	-2.95	-3.73	-3.93	-4.29	-1.75	-2.43	-2.80	-3.06	-1.17	-1.82	-1.86	-2.21	-0.44	-0.55	-0.43	-0.59	-0.23	-0.12	-0.19	-0.22	0.01	-0.02	0.00	-0.03
	7	-2.82	-3.32	-3.70	-3.93	-1.62	-2.12	-2.31	-2.58	-0.81	-1.44	-1.45	-1.68	-0.32	-0.43	-0.38	-0.41	-0.23	-0.13	-0.20	-0.22	0.01	0.01	0.01	0.01
	8	-2.64	-3.04	-3.44	-3.60	-1.34	-1.78	-1.91	-2.17	-0.66	-1.25	-1.20	-1.41	-0.27	-0.39	-0.31	-0.34	-0.21	-0.12	-0.16	-0.19	0.01	0.03	0.02	0.03
	9	-2.48	-2.78	-3.19	-3.31	-1.41	-1.79	-1.88	-2.14	-0.44	-0.88	-0.83	-0.96	-0.22	-0.37	-0.23	-0.26	-0.15	-0.12	-0.14	-0.16	-0.02	-0.01	0.00	-0.01
	10	-2.07	-2.59	-2.92	-3.02	-1.29	-1.65	-1.71	-1.96	-0.50	-0.88	-0.82	-0.92	-0.26	-0.40	-0.24	-0.24	-0.13	-0.08	-0.12	-0.13	-0.02	-0.01	0.02	0.01
	25	-0.38	-0.47	-0.52	-0.55	-0.23	-0.39	-0.25	-0.38	-0.31	-0.28	-0.28	-0.27	-0.09	-0.09	-0.08	-0.07	-0.05	-0.03	-0.04	-0.01	0.00	0.00	0.01	0.01
	50	-0.06	-0.09	-0.21	-0.16	-0.07	-0.16	-0.10	-0.16	-0.10	-0.09	-0.10	-0.08	-0.06	-0.06	-0.06	-0.04	0.00	0.00	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01
	100	-0.06	-0.15	-0.14	-0.14	0.00	-0.02	-0.02	-0.05	-0.20	-0.10	-0.11	-0.12	-0.07	-0.07	-0.09	-0.05	-0.02	-0.02	-0.02	-0.03	0.01	0.01	0.01	0.01

Table 3.8: Gains from equal pooling VAR forecasts – noise (*Model III*)

		T = 200																							
		Signal to noise = 10e4				Signal to noise = 100				Signal to noise = 10				Signal to noise = 1				Signal to noise = 0.1				Signal to noise = 10e-6			
	nVAR	2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5
h	1	-3.48	-3.77	-4.10	-4.39	-2.90	-3.34	-3.94	-4.15	-1.75	-2.50	-2.89	-3.12	-1.98	-2.50	-2.13	-2.33	-0.72	-0.88	-0.91	-0.85	-0.28	-0.31	-0.38	-0.40
	2	-3.29	-4.19	-4.66	-4.96	-2.95	-3.80	-4.37	-4.72	-2.18	-3.05	-3.55	-3.71	-1.30	-1.63	-1.35	-1.60	-0.32	-0.38	-0.36	-0.44	-0.16	-0.20	-0.24	-0.25
	3	-3.32	-4.00	-4.69	-4.96	-3.01	-4.15	-4.55	-4.89	-1.69	-2.71	-3.11	-3.24	-0.62	-0.79	-0.78	-0.88	-0.25	-0.25	-0.31	-0.29	-0.10	-0.09	-0.10	-0.15
	4	-2.64	-3.85	-4.39	-4.54	-2.44	-3.58	-3.94	-4.28	-1.07	-1.96	-2.36	-2.46	-0.38	-0.43	-0.54	-0.58	-0.18	-0.15	-0.21	-0.19	-0.05	-0.07	-0.05	-0.09
	5	-2.18	-3.34	-3.86	-3.85	-1.94	-2.83	-3.08	-3.37	-0.92	-1.68	-1.93	-2.11	-0.42	-0.40	-0.39	-0.44	-0.17	-0.07	-0.13	-0.13	-0.04	-0.05	-0.03	-0.07
	6	-1.93	-2.80	-3.39	-3.38	-2.01	-2.63	-2.95	-3.19	-0.81	-1.32	-1.52	-1.72	-0.31	-0.36	-0.28	-0.31	-0.11	-0.06	-0.08	-0.09	-0.04	-0.06	-0.04	-0.08
	7	-1.76	-2.61	-3.23	-3.19	-1.90	-2.46	-2.70	-2.91	-1.20	-1.43	-1.70	-1.91	-0.28	-0.30	-0.22	-0.23	-0.05	0.01	-0.04	-0.03	-0.08	-0.11	-0.06	-0.11
	8	-1.23	-1.88	-2.51	-2.48	-1.50	-2.00	-2.27	-2.42	-1.07	-1.28	-1.43	-1.68	-0.10	-0.10	-0.08	-0.10	-0.01	0.03	0.00	-0.01	-0.09	-0.09	-0.05	-0.10
	9	-0.90	-1.47	-2.06	-2.03	-1.08	-1.49	-1.69	-1.87	-1.04	-1.11	-1.26	-1.40	-0.12	-0.13	-0.06	-0.10	0.00	0.02	0.01	0.02	-0.08	-0.08	-0.05	-0.09
	10	-0.77	-1.22	-1.72	-1.73	-1.02	-1.33	-1.51	-1.66	-0.86	-0.80	-0.91	-1.05	-0.07	-0.11	-0.05	-0.08	0.00	0.00	-0.01	0.01	-0.05	-0.06	-0.04	-0.07
	25	-0.04	-0.11	-0.12	-0.15	-0.05	-0.02	-0.03	-0.07	-0.11	-0.09	-0.07	-0.11	-0.05	-0.04	-0.05	-0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	50	0.01	0.00	-0.02	-0.02	-0.09	-0.03	-0.07	-0.08	-0.02	-0.04	-0.03	-0.04	0.02	0.03	0.02	0.02	0.00	0.00	0.00	0.00	-0.01	-0.01	-0.01	-0.01
	100	-0.05	-0.03	0.00	-0.02	0.04	0.05	0.03	0.03	-0.02	-0.03	-0.04	-0.04	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
		T = 1000																							
		Signal to noise = 10e4				Signal to noise = 100				Signal to noise = 10				Signal to noise = 1				Signal to noise = 0.1				Signal to noise = 10e-6			
	nVAR	2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5
h	1	-2.09	-2.71	-3.15	-3.37	-2.31	-2.86	-3.01	-3.12	-1.11	-2.32	-2.50	-2.75	-1.25	-1.50	-1.92	-1.93	-0.04	-0.12	-0.25	-0.25	-0.01	0.02	0.04	0.03
	2	-3.02	-3.83	-4.29	-4.53	-2.72	-3.32	-3.65	-3.93	-1.59	-2.55	-2.84	-3.09	-0.57	-0.81	-0.88	-1.07	-0.18	-0.19	-0.15	-0.13	0.06	0.05	0.06	0.04
	3	-2.64	-3.58	-4.08	-4.25	-2.61	-3.23	-3.73	-3.92	-1.73	-2.53	-2.84	-3.02	-0.49	-0.65	-0.74	-0.75	-0.08	-0.03	-0.06	-0.07	0.02	0.00	0.01	0.00
	4	-2.25	-2.93	-3.39	-3.65	-2.26	-2.56	-3.24	-3.39	-1.84	-2.36	-2.81	-2.83	-0.47	-0.51	-0.50	-0.62	-0.06	-0.04	-0.04	-0.07	0.00	0.02	0.02	0.02
	5	-2.25	-2.81	-3.21	-3.39	-1.71	-1.99	-2.59	-2.77	-1.05	-1.61	-1.86	-2.00	-0.33	-0.31	-0.23	-0.34	-0.07	-0.03	-0.03	-0.06	0.01	0.02	0.03	0.03
	6	-1.59	-2.30	-2.72	-2.89	-1.67	-1.81	-2.43	-2.56	-1.21	-1.67	-1.76	-1.91	-0.24	-0.23	-0.20	-0.27	-0.03	0.00	0.00	-0.02	0.01	0.02	0.02	0.02
	7	-1.22	-1.94	-2.20	-2.34	-1.54	-1.66	-2.06	-2.24	-1.27	-1.57	-1.64	-1.77	-0.20	-0.13	-0.16	-0.15	-0.04	-0.02	0.00	-0.02	0.00	0.00	0.01	0.00
	8	-0.88	-1.69	-1.80	-1.95	-0.96	-1.19	-1.51	-1.66	-1.19	-1.47	-1.45	-1.53	-0.14	-0.06	-0.10	-0.07	-0.05	-0.04	-0.01	-0.04	0.00	-0.01	0.00	0.00
	9	-0.67	-1.49	-1.59	-1.70	-0.56	-0.87	-1.10	-1.25	-1.11	-1.18	-1.25	-1.30	-0.14	-0.10	-0.13	-0.09	-0.03	-0.02	0.01	-0.03	0.01	-0.01	0.00	-0.01
	10	-0.57	-1.17	-1.28	-1.38	-0.49	-0.69	-0.97	-1.08	-0.76	-0.78	-0.87	-0.92	-0.07	-0.03	-0.07	-0.04	-0.04	-0.03	0.00	-0.04	0.01	0.00	0.01	0.00
	25	-0.07	-0.15	-0.15	-0.17	-0.19	-0.27	-0.23	-0.25	-0.04	-0.08	-0.07	-0.07	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	50	0.01	-0.01	0.00	0.00	-0.01	0.00	0.00	0.00	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	100	0.00	0.00	0.00	0.00	-0.01	-0.01	-0.01	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 3.8 continued: Gains from equal pooling VAR forecasts – noise (*Model III*)

MODEL I

T = 25

		Signal to noise = 10e4						Signal to noise = 10						Signal to noise = 1						Signal to noise = 0.1						Signal to noise = 10e-6					
		N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000
	m	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
h	1	75.92	-3.62	-15.36	-19.56	-21.24	-22.60	84.03	-2.14	-13.16	-17.09	-18.91	-20.17	131.73	11.54	-0.91	-5.97	-8.07	-9.62	132.30	22.18	5.58	0.11	-2.03	-3.83	160.75	19.66	5.61	0.12	-2.06	-3.61
	2	76.27	-5.71	-16.57	-20.78	-22.56	-23.83	94.93	-0.47	-11.59	-16.05	-18.05	-19.39	165.11	19.40	1.75	-3.91	-6.24	-7.94	189.52	28.69	9.12	2.96	0.47	-1.37	188.83	25.77	8.88	2.60	0.14	-1.52
	3	94.65	0.13	-12.38	-16.75	-18.66	-20.00	129.81	7.04	-6.95	-12.03	-14.01	-15.37	203.26	22.48	4.81	-1.47	-4.08	-5.78	210.02	29.61	9.81	3.57	1.11	-0.76	187.68	28.03	9.43	3.48	0.93	-0.75
	4	118.97	5.77	-8.35	-13.67	-15.62	-17.01	173.91	14.19	-2.95	-8.35	-10.45	-11.96	234.98	28.05	6.31	0.02	-2.55	-4.34	251.66	33.68	11.03	4.27	1.61	-0.19	262.05	29.41	10.43	4.11	1.50	-0.29
	5	144.63	9.20	-6.33	-11.42	-13.77	-15.19	212.47	19.03	0.52	-5.41	-7.95	-9.43	282.51	30.22	8.91	1.19	-1.72	-3.61	280.41	35.11	12.07	4.79	2.04	0.20	262.25	32.22	12.13	4.69	1.72	-0.10
	6	173.50	14.52	-4.34	-10.03	-12.22	-13.75	218.13	21.65	3.40	-3.59	-5.89	-7.50	341.19	33.06	9.42	1.68	-1.10	-3.03	302.82	36.62	12.64	5.11	2.22	0.20	276.19	37.94	11.98	4.80	1.72	-0.15
	7	221.74	15.69	-2.22	-8.30	-10.88	-12.45	259.17	24.29	4.72	-2.11	-4.83	-6.64	348.93	35.58	9.84	2.63	-0.52	-2.64	303.72	37.12	13.66	5.54	2.37	0.41	314.74	37.94	13.22	5.06	1.97	-0.10
	8	284.47	19.66	-0.07	-7.14	-9.84	-11.66	281.97	28.71	5.68	-1.56	-4.38	-6.06	449.34	36.39	11.43	2.99	-0.16	-2.31	338.17	40.83	13.79	5.60	2.41	0.28	326.78	40.78	14.13	5.36	2.07	-0.10
	9	264.12	24.77	1.43	-6.46	-9.28	-11.25	296.76	31.62	6.64	-0.90	-3.72	-5.48	427.21	41.00	12.17	2.87	-0.34	-2.39	420.07	44.56	14.24	5.66	2.45	0.20	376.40	41.39	14.13	5.54	2.12	-0.17
	10	306.18	27.92	2.30	-5.75	-9.03	-11.03	354.40	32.65	6.53	-0.32	-3.26	-5.09	399.95	40.16	11.98	3.06	-0.26	-2.46	385.23	47.77	15.07	6.14	2.34	0.13	404.54	45.16	14.90	5.06	1.80	-0.51
	25	356.46	35.52	3.21	-6.13	-9.81	-12.13	430.87	41.00	9.76	-0.55	-3.83	-6.21	572.41	55.03	15.25	2.86	-1.26	-3.97	741.54	73.26	23.06	8.45	3.75	0.40	829.37	70.14	22.94	7.84	2.51	-0.80
	50	444.01	41.33	3.65	-6.55	-10.69	-13.15	466.91	42.72	9.61	-0.93	-4.72	-7.41	876.05	63.44	16.11	1.96	-2.90	-6.00	1130.37	95.38	28.99	12.26	5.59	1.19	1055.85	102.88	28.09	10.35	3.65	-0.83
	100	429.31	42.54	3.92	-7.63	-11.84	-14.74	556.26	55.60	11.84	0.24	-4.04	-7.29	1097.54	85.36	19.15	2.66	-2.50	-6.11	1225.96	115.96	35.47	13.15	5.76	0.34	1181.23	119.06	34.96	11.67	2.97	-2.41

T = 50

		Signal to noise = 10e4						Signal to noise = 10						Signal to noise = 1						Signal to noise = 0.1						Signal to noise = 10e-6					
		N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000
	m	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
h	1	72.64	-3.53	-13.96	-18.08	-19.75	-20.99	74.30	-0.36	-10.83	-15.16	-17.02	-18.31	111.68	9.82	-2.67	-7.22	-9.23	-10.66	141.27	20.98	6.66	1.13	-1.10	-2.78	144.00	23.25	8.27	2.70	0.57	-1.23
	2	64.93	-7.59	-17.71	-21.38	-23.04	-24.28	79.09	-3.92	-13.74	-17.91	-19.58	-20.97	111.25	10.40	-2.06	-7.03	-9.24	-10.70	161.96	25.36	8.54	2.68	0.28	-1.34	151.38	25.34	9.12	3.83	1.35	-0.44
	3	70.32	-5.51	-16.27	-20.19	-21.98	-23.28	96.09	0.45	-11.66	-15.72	-17.64	-18.97	133.44	15.64	-0.57	-5.35	-7.67	-9.17	161.32	25.32	9.14	3.30	0.82	-0.93	156.92	26.69	9.70	4.37	1.81	0.04
	4	88.05	-1.31	-12.18	-16.42	-18.34	-19.70	103.58	5.10	-8.44	-12.56	-14.63	-16.00	131.96	17.99	1.05	-3.96	-6.20	-7.85	179.73	26.01	9.51	3.51	1.21	-0.66	176.88	25.30	10.03	4.35	1.90	0.11
	5	109.85	2.85	-10.28	-14.46	-16.50	-17.86	119.87	8.53	-5.09	-10.05	-11.94	-13.56	153.48	18.33	2.27	-3.45	-5.44	-7.08	194.32	26.87	10.29	3.88	1.16	-0.64	174.96	26.41	10.41	4.57	1.84	0.08
	6	116.46	5.02	-7.94	-12.03	-13.98	-15.43	135.91	12.35	-1.92	-7.07	-9.25	-10.73	187.10	20.11	3.17	-2.30	-4.63	-6.23	203.47	28.14	10.20	4.11	1.43	-0.40	173.22	27.79	10.79	4.28	1.89	0.11
	7	122.99	7.39	-5.71	-10.54	-12.59	-14.00	165.23	14.15	-0.88	-5.96	-8.11	-9.70	173.04	21.31	3.84	-2.17	-4.42	-6.11	205.64	28.99	11.14	4.42	1.73	-0.21	176.00	28.58	10.88	4.46	2.01	0.06
	8	126.88	9.72	-4.55	-9.25	-11.55	-13.04	181.64	16.41	0.45	-4.97	-7.24	-8.91	194.68	22.66	4.36	-1.59	-4.23	-5.89	214.97	31.86	11.91	4.64	1.81	-0.20	197.80	30.16	11.37	4.76	2.11	0.15
	9	128.96	11.29	-2.95	-8.10	-10.35	-11.86	161.22	18.48	1.28	-4.27	-6.41	-8.11	201.66	23.64	4.98	-1.34	-3.99	-5.70	244.11	33.62	11.86	5.04	2.01	-0.11	200.60	29.36	11.78	4.81	2.15	0.13
	10	152.02	12.37	-2.26	-7.64	-9.92	-11.51	180.93	20.51	2.35	-3.38	-5.98	-7.57	210.36	23.59	5.23	-1.20	-3.59	-5.52	238.06	34.68	12.38	5.12	2.09	-0.02	217.10	32.16	12.65	5.09	2.27	0.20
	25	281.25	29.21	3.38	-4.23	-7.05	-8.83	299.55	32.65	7.28	-0.31	-3.42	-5.48	329.33	38.02	9.23	0.29	-2.85	-5.06	428.64	52.72	18.10	7.12	2.87	-0.05	381.37	49.10	17.33	7.24	3.06	0.37
	50	301.28	34.82	7.84	-0.44	-4.05	-6.21	320.30	37.14	8.96	0.72	-2.53	-4.52	479.65	51.88	12.51	1.84	-1.80	-4.08	622.28	62.50	19.69	7.14	2.68	-0.32	545.95	65.68	20.89	7.99	3.66	0.53
	100	258.48	33.25	7.47	-0.90	-4.28	-6.33	306.67	36.90	9.73	1.13	-1.97	-4.24	569.37	49.29	14.14	3.17	-0.79	-3.44	698.21	70.84	20.40	7.26	2.34	-0.82	603.51	70.49	24.02	9.69	4.54	0.99

T = 100

		Signal to noise = 10e4						Signal to noise = 10						Signal to noise = 1						Signal to noise = 0.1						Signal to noise = 10e-6					
		N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000
	m	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
h	1	59.72	-6.29	-15.83	-20.06	-21.68	-22.99	73.28	-1.68	-11.88	-15.69	-17.64	-18.97	100.75	9.94	-2.71	-7.26	-9.33	-10.82	131.72	21.95	7.28	1.84	-0.26	-2.04	146.69	25.00	9.57	3.85	1.55	-0.19
	2	58.25	-10.03	-18.76	-22.68	-24.42	-25.63	63.24	-4.34	-15.22	-19.45	-21.21	-22.56	99.08	8.17	-3.60	-8.31	-10.28	-11.80	127.23	22.49	8.11	2.83	0.42	-1.38	144.84	25.12	10.01	4.11	1.79	0.05
	3	63.53	-7.36	-17.59	-21.60	-23.36	-24.65	68.65	-3.93	-14.54	-18.43	-20.26	-21.64	102.82	9.98	-2.69	-7.28	-9.33	-10.93	146.93	22.30	8.27	2.78	0.40	-1.28	165.93	24.73	10.61	4.31	1.92	0.16
	4	79.01	-2.76	-13.52	-17.63	-19.44	-20.83	81.66	0.08	-10.06	-14.43	-16.55	-18.03	105.84	12.26	-0.92	-5.68	-7.89	-9.55	140.44	23.01	8.36	2.72	0.25	-1.43	141.49	25.11	10.20	4.50	1.95	0.19
	5	78.36	-0.16	-10.53	-14.63	-16.63	-18.09	97.27	4.14	-7.43	-12.12	-14.06	-15.62	114.66	14.22	0.16	-5.10	-7.37	-9.05	138.22	23.54	7.88	2.76	0.33	-1.42	159.01	26.50	10.26	4.54	2.02	0.28
	6	85.63	4.27	-7.05	-11.44	-13.48	-15.08	96.06	8.59	-3.70	-8.63	-10.81	-12.39	117.57	14.30	1.09	-3.87	-6.38	-8.04	157.58	24.08	8.69	3.29	0.64	-1.13	164.74	27.69	10.80	4.62	2.16	0.31
	7	87.38	8.61	-4.97	-9.75	-11.66	-13.25	108.03	10.72	-1.99	-6.90	-9.18	-10.86	119.60	15.56	1.87	-3.69	-6.07	-7.80	156.85	25.30	9.60	3.29	0.82	-0.98	165.51	28.84	10.64	4.81	2.31	0.37
	8	100.02	8.10	-4.20	-8.76	-10.70	-12.37	105.30	12.75	-0.52	-5.72	-8.16	-9.73	130.76	16.21	2.12	-3.65	-5.98	-7.71	186.69	25.81	9.70	3.56	1.02	-0.86	158.33	29.14	11.03	4.98	2.35	0.40
	9	105.42	9.89	-3.32	-7.85	-9.89	-11.48	123.87	14.28	0.78	-4.93	-7.19	-8.91	129.07	16.72	2.03	-3.37	-6.02	-7.63	153.56	26.58	10.18	3.99	1.15	-0.74	178.43	28.57	11.67	5.09	2.29	0.42
	10	109.54	11.17	-2.64	-7.10	-9.35	-10.86	126.79	14.84	0.90	-4.04	-6.53	-8.16	156.77	18.39	2.48	-3.12	-5.53	-7.35	173.65	27.48	10.74	3.95	1.35	-0.67	170.74	28.93	11.50	5.15	2.57	0.45
	25	194.72	20.79	3.51	-2.94	-5.40	-7.12	241.84	27.72	6.37	0.23	-2.37	-4.30	265.94	31.00	8.08	0.68	-2.34	-4.42	307.45	38.91	13.10	4.64	1.18	-1.12	287.03	40.95	14.39	6.06	2.61	0.34
	50	203.10	28.96	8.93	1.84	-0.95	-2.79	242.05	34.28	11.26	3.57	0.43	-1.62	359.30	43.60	13.31	3.32	-0.33	-2.47	410.28	53.28	16.77	5.95	1.97	-0.68	426.16	48.73	18.43	7.64	3.21	0.49
	100	166.22	26.92	9.03	2.36	-0.32	-2.20	195.48	28.76	10.91	3.23	0.40	-1.52	293.80	41.19	12.57	3.53	0.35	-1.82	392.01	45.69	17.26	5.95	2.14	-0.23	373.98	49.11	17.15	6.98	2.92	0.35

Table 3.9: Gains from optimal pooling VAR forecasts – noise (*Model I*)

T = 200

		Signal to noise = 10e4						Signal to noise = 10						Signal to noise = 1						Signal to noise = 0.1						Signal to noise = 10e-6					
		N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000
	m	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
h	1	57.95	-3.80	-13.57	-17.45	-19.30	-20.67	68.13	-1.93	-11.71	-15.64	-17.34	-18.74	89.79	10.23	-1.99	-6.56	-8.80	-10.34	125.05	20.75	7.23	2.22	-0.16	-1.81	131.10	22.96	10.01	4.07	1.76	0.05
	2	57.50	-9.25	-18.98	-22.51	-24.28	-25.53	59.38	-6.41	-16.66	-20.64	-22.37	-23.65	94.75	9.02	-3.54	-8.17	-10.14	-11.71	135.97	20.71	7.19	2.38	-0.08	-1.68	144.83	25.66	9.46	4.06	1.88	0.16
	3	64.63	-7.32	-17.41	-21.26	-22.81	-24.08	59.35	-5.61	-14.97	-18.95	-20.75	-22.10	99.54	9.08	-2.75	-7.44	-9.51	-11.10	144.28	21.50	7.62	2.44	0.20	-1.48	153.97	25.58	9.85	4.37	1.97	0.22
	4	66.82	-3.64	-13.21	-17.43	-19.19	-20.57	74.66	-0.56	-11.86	-15.79	-17.47	-18.82	102.82	10.18	-2.15	-6.24	-8.59	-10.10	130.85	21.29	7.83	2.86	0.31	-1.38	134.06	24.60	9.55	4.59	2.03	0.18
	5	83.00	0.25	-10.65	-14.42	-16.30	-17.68	74.50	3.71	-8.49	-12.73	-14.52	-15.91	102.94	12.06	-0.98	-6.11	-8.16	-9.65	129.42	21.72	7.72	2.59	0.23	-1.49	141.67	25.29	9.69	4.41	1.89	0.22
	6	79.62	3.49	-7.57	-11.76	-13.67	-15.10	89.47	5.61	-5.29	-9.87	-11.89	-13.31	104.23	12.84	-0.43	-5.09	-7.14	-8.67	130.31	22.55	8.39	2.67	0.30	-1.35	149.30	25.26	9.92	4.34	2.06	0.27
	7	92.81	5.22	-6.34	-10.39	-12.35	-13.78	91.71	7.81	-4.27	-8.47	-10.49	-11.96	103.46	12.32	-0.42	-5.06	-7.24	-8.81	136.00	23.14	8.26	2.81	0.30	-1.49	158.52	26.49	9.68	4.42	2.06	0.27
	8	95.67	7.13	-3.93	-8.24	-10.39	-11.86	107.85	9.68	-2.79	-6.95	-9.16	-10.66	104.07	14.15	-0.02	-4.75	-6.93	-8.59	140.95	23.43	7.83	2.68	0.19	-1.53	151.71	24.72	10.61	4.63	2.12	0.28
	9	95.24	9.23	-3.15	-7.66	-9.63	-11.07	99.44	10.79	-1.72	-6.39	-8.43	-9.93	98.40	12.25	-0.54	-4.95	-7.06	-8.75	154.97	22.48	8.29	2.61	0.17	-1.64	155.41	25.71	10.72	4.72	2.18	0.31
	10	101.46	9.37	-1.95	-6.86	-8.90	-10.47	108.43	12.23	-0.92	-5.79	-7.80	-9.34	102.98	13.17	0.01	-4.87	-7.08	-8.66	156.18	22.18	8.16	2.51	0.20	-1.64	170.24	26.52	10.30	5.06	2.23	0.34
	25	175.05	20.86	3.70	-2.19	-4.46	-6.11	172.19	26.17	6.87	1.11	-1.36	-3.10	202.99	25.74	6.90	0.34	-2.50	-4.25	216.30	31.26	9.88	2.98	0.19	-1.74	222.93	33.08	12.35	5.52	2.49	0.40
	50	175.32	27.07	9.70	3.00	0.38	-1.37	218.89	31.58	11.28	4.00	1.30	-0.73	285.25	36.16	12.98	4.18	0.63	-1.61	324.29	40.88	13.50	5.09	1.53	-0.70	357.26	42.32	14.55	6.19	2.79	0.51
	100	135.06	23.85	9.35	3.81	1.31	-0.35	148.68	25.80	9.78	3.73	1.33	-0.38	213.87	33.17	11.60	4.42	1.33	-0.61	268.81	39.13	13.07	5.06	1.89	-0.22	287.66	37.08	14.66	5.93	2.50	0.40

T = 1000

		Signal to noise = 10e4						Signal to noise = 10						Signal to noise = 1						Signal to noise = 0.1						Signal to noise = 10e-6					
		N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000
	m	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
h	1	56.56	-6.34	-15.55	-19.71	-21.40	-22.69	63.89	-3.23	-13.22	-16.61	-18.50	-19.80	77.59	6.57	-4.38	-8.77	-10.67	-12.16	106.58	19.11	6.31	1.32	-0.76	-2.35	154.31	25.66	9.55	4.42	2.11	0.35
	2	43.40	-12.09	-19.96	-23.83	-25.29	-26.51	54.32	-7.90	-16.95	-20.63	-22.32	-23.57	72.99	2.90	-6.57	-10.87	-12.71	-14.22	113.86	18.59	6.04	1.15	-1.14	-2.69	142.47	23.94	10.00	4.39	2.03	0.35
	3	49.83	-9.41	-18.35	-21.82	-23.63	-24.93	55.34	-6.05	-15.86	-19.54	-21.24	-22.55	72.92	5.04	-5.81	-10.08	-11.91	-13.41	103.36	18.95	5.55	1.15	-1.08	-2.72	165.57	25.30	10.12	4.38	2.17	0.37
	4	50.10	-5.84	-15.51	-18.98	-20.81	-22.12	64.71	-2.85	-12.74	-16.78	-18.60	-20.03	79.39	6.08	-4.22	-8.67	-10.76	-12.29	109.02	18.09	6.43	1.49	-0.77	-2.41	156.91	25.98	10.25	4.51	2.05	0.35
	5	65.84	-1.18	-11.79	-15.85	-17.67	-19.06	67.09	0.90	-9.99	-14.02	-15.97	-17.36	76.99	6.62	-3.81	-8.08	-9.97	-11.48	111.11	18.54	6.06	1.42	-0.64	-2.36	151.27	24.66	9.92	4.63	2.04	0.36
	6	64.40	1.66	-9.81	-13.87	-15.64	-17.14	64.30	1.53	-8.31	-12.61	-14.48	-15.91	82.05	8.03	-3.84	-8.04	-10.22	-11.68	113.12	18.93	6.16	1.42	-0.77	-2.48	152.38	25.02	10.14	4.47	2.03	0.37
	7	77.66	2.64	-7.45	-11.54	-13.56	-15.04	77.10	3.47	-6.68	-11.20	-12.96	-14.40	93.97	6.95	-3.53	-7.93	-9.99	-11.50	105.23	18.65	6.63	1.32	-0.97	-2.58	137.73	26.09	10.04	4.61	2.18	0.37
	8	79.40	6.07	-4.82	-9.37	-11.28	-12.74	77.09	6.15	-4.13	-9.27	-11.15	-12.49	76.01	8.47	-3.11	-7.78	-9.75	-11.22	110.99	19.61	6.04	1.28	-0.99	-2.54	162.20	25.64	9.94	4.35	2.08	0.37
	9	81.52	7.16	-3.80	-8.13	-10.06	-11.51	78.28	6.74	-3.65	-8.15	-10.11	-11.64	87.25	7.48	-3.29	-7.67	-9.72	-11.26	105.09	19.18	6.31	1.18	-1.08	-2.64	144.04	24.99	9.67	4.53	2.02	0.35
	10	88.85	8.49	-2.22	-6.77	-8.78	-10.31	84.09	8.15	-3.10	-7.52	-9.46	-10.94	88.70	8.82	-3.19	-7.83	-9.81	-11.28	107.45	17.90	6.00	1.06	-1.04	-2.66	142.97	25.40	10.18	4.48	2.08	0.35
	25	127.02	18.80	5.88	0.39	-1.88	-3.56	108.43	20.36	7.10	2.21	0.01	-1.72	116.10	19.54	5.94	1.12	-1.12	-2.73	120.20	22.09	7.68	2.37	0.17	-1.60	157.67	26.14	10.42	4.68	2.11	0.33
	50	121.10	22.82	9.06	3.30	1.00	-0.74	135.46	24.45	9.61	4.44	2.00	0.30	150.61	24.29	9.73	4.37	1.99	0.26	150.63	25.89	9.83	4.50	1.98	0.20	169.60	28.42	10.93	4.76	2.13	0.37
	100	114.48	21.92	9.11	3.99	1.78	0.16	106.31	21.90	9.14	4.07	1.95	0.23	117.48	22.51	9.65	4.22	2.05	0.23	133.91	24.12	9.59	4.01	1.93	0.24	160.28	25.36	9.79	4.55	2.11	0.34

Table 3.9 continued: Gains from optimal pooling VAR forecasts – noise (*Model I*)

Model II

T = 25

		Signal to noise = 10e4						Signal to noise = 10						Signal to noise = 1						Signal to noise = 0.1						Signal to noise = 10e-6					
		N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000
	m	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
h	1	85.54	-1.20	-13.05	-17.22	-19.07	-20.33	80.33	1.52	-10.92	-14.97	-16.82	-18.19	119.15	9.03	-3.69	-8.29	-10.54	-12.01	151.64	21.78	6.47	0.77	-1.59	-3.24	154.02	20.80	5.66	0.17	-2.18	-3.84
	2	94.24	-2.26	-14.19	-18.26	-20.20	-21.50	104.45	2.45	-9.78	-14.30	-16.19	-17.64	164.49	17.71	1.40	-4.23	-6.61	-8.28	180.55	27.67	9.92	3.17	0.75	-0.91	162.49	26.90	9.91	3.34	0.71	-1.14
	3	124.93	4.02	-9.91	-14.41	-16.41	-17.84	150.50	8.04	-6.01	-11.05	-13.25	-14.74	189.32	22.94	4.42	-1.69	-4.13	-6.03	206.34	30.93	10.51	4.39	1.45	-0.37	187.85	28.76	9.90	3.74	0.99	-0.87
	4	142.93	8.36	-5.22	-10.39	-12.61	-14.08	166.02	14.43	-2.38	-7.40	-9.70	-11.34	243.54	25.24	6.28	0.14	-2.33	-4.27	216.97	32.44	10.85	4.30	1.71	-0.17	206.01	29.07	10.13	3.97	1.14	-0.72
	5	177.72	14.32	-1.90	-7.40	-9.72	-11.27	198.47	18.79	0.83	-5.01	-7.26	-8.95	269.50	29.36	8.99	1.64	-1.40	-3.19	252.84	33.31	12.17	4.67	1.85	-0.01	213.76	34.35	11.11	4.30	1.71	-0.30
	6	203.28	18.43	0.37	-5.32	-7.85	-9.49	236.91	21.75	3.40	-3.08	-5.54	-7.29	294.46	32.96	8.73	2.22	-0.68	-2.62	281.40	34.52	12.63	4.66	2.05	0.05	230.08	33.75	12.66	4.75	2.03	0.02
	7	243.32	22.49	2.65	-3.89	-6.41	-8.12	281.99	27.94	5.50	-1.79	-4.35	-6.17	331.77	33.53	10.06	2.52	-0.47	-2.30	321.42	38.52	12.72	5.18	1.92	0.00	305.93	35.92	13.30	5.21	2.30	0.25
	8	275.40	24.11	4.51	-2.51	-5.19	-7.09	272.18	29.20	6.97	0.11	-3.11	-5.06	312.66	37.91	11.03	2.83	0.01	-2.07	394.24	39.01	12.69	5.16	1.86	-0.24	353.63	40.16	13.92	5.55	2.36	0.25
	9	294.45	28.17	5.73	-1.44	-4.21	-6.23	315.22	33.64	8.48	1.10	-2.26	-4.34	408.33	37.37	10.86	3.11	-0.07	-2.11	345.86	41.35	13.73	5.18	1.85	-0.35	420.32	44.11	15.13	5.92	2.61	0.38
	10	305.82	30.25	6.51	-1.04	-3.66	-5.71	348.22	37.01	8.60	0.99	-1.93	-4.08	426.37	43.39	11.64	3.07	-0.30	-2.33	383.73	44.23	13.86	5.32	1.97	-0.31	447.68	44.85	15.48	6.11	3.01	0.63
	25	340.14	39.77	8.47	-1.20	-4.59	-7.07	427.40	40.20	8.04	-1.17	-4.66	-6.95	650.40	59.99	17.14	4.82	0.43	-2.45	684.20	69.87	21.30	8.36	3.08	-0.31	669.63	79.20	21.92	7.82	2.57	-0.57
	50	453.80	43.39	8.94	0.20	-3.77	-6.45	552.40	48.93	10.11	-0.85	-4.79	-7.27	785.04	70.47	18.97	4.91	-0.11	-3.32	1007.33	91.33	28.96	9.48	3.47	-0.52	987.48	103.15	28.46	10.47	3.90	-0.30
	100	497.25	52.61	11.44	-0.73	-4.67	-7.68	627.64	53.78	9.77	-2.12	-6.29	-8.96	1014.76	87.36	23.86	5.63	-0.02	-3.94	1275.55	115.18	35.07	12.84	5.13	-0.01	1259.69	121.67	37.04	13.42	5.28	-0.19

T = 50

		Signal to noise = 10e4						Signal to noise = 10						Signal to noise = 1						Signal to noise = 0.1						Signal to noise = 10e-6					
		N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000
	m	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
h	1	71.36	-1.48	-11.36	-16.11	-17.82	-19.23	82.60	0.68	-10.24	-14.38	-16.34	-17.69	102.29	12.77	-0.81	-5.41	-7.58	-9.15	143.10	21.63	7.03	1.49	-0.85	-2.47	135.45	25.42	8.30	2.96	0.53	-1.22
	2	76.59	-3.62	-15.06	-18.78	-20.55	-21.85	83.54	-1.98	-12.00	-16.41	-18.22	-19.55	123.62	15.86	0.76	-4.13	-6.49	-8.17	157.45	24.46	9.11	3.06	0.70	-1.02	157.78	25.74	9.48	3.89	1.46	-0.25
	3	85.65	-1.22	-12.29	-16.81	-18.55	-19.81	88.86	2.38	-10.21	-14.22	-16.04	-17.39	148.23	17.61	2.94	-2.39	-4.72	-6.39	164.99	26.45	9.97	3.75	1.29	-0.45	171.83	24.44	10.52	4.27	1.93	0.07
	4	98.95	3.11	-8.35	-12.54	-14.57	-15.99	110.54	7.89	-5.95	-10.38	-12.30	-13.78	162.90	20.09	4.50	-1.29	-3.72	-5.31	164.59	26.19	9.60	3.86	1.36	-0.40	164.31	27.00	10.30	4.42	1.82	0.01
	5	123.30	6.92	-5.55	-10.32	-12.30	-13.75	136.51	10.03	-2.36	-7.66	-9.79	-11.34	173.19	22.13	5.62	-0.24	-2.59	-4.35	185.13	27.30	10.08	4.13	1.34	-0.44	182.38	26.98	10.60	4.41	1.78	-0.05
	6	119.88	10.45	-3.87	-8.26	-10.42	-11.88	136.77	13.48	-1.08	-6.04	-8.20	-9.78	170.66	22.28	6.16	0.33	-2.31	-4.03	201.64	27.79	9.43	4.03	1.34	-0.59	188.23	27.33	11.16	4.42	2.02	0.05
	7	129.69	12.71	-2.09	-7.21	-9.33	-10.91	162.10	15.30	0.27	-4.90	-7.20	-8.81	217.09	24.96	7.03	0.77	-1.84	-3.69	187.94	30.31	11.03	4.24	1.55	-0.38	197.13	29.39	11.46	4.92	2.19	0.27
	8	151.98	14.73	-1.10	-5.97	-8.27	-9.87	170.02	15.58	1.04	-4.22	-6.69	-8.34	207.77	26.24	7.18	1.01	-1.72	-3.55	213.03	30.78	11.34	4.42	1.48	-0.44	223.66	33.31	12.02	5.37	2.40	0.35
	9	190.22	15.99	0.49	-5.10	-7.30	-8.95	183.06	19.09	2.31	-3.76	-6.10	-7.79	206.43	27.81	7.85	1.09	-1.57	-3.46	274.65	32.45	11.64	4.66	1.69	-0.39	222.38	32.63	12.93	5.16	2.61	0.39
	10	167.36	17.24	1.32	-4.54	-6.95	-8.64	196.44	21.02	2.66	-3.29	-5.64	-7.36	242.52	30.73	8.61	1.54	-1.45	-3.43	245.12	33.75	11.63	4.80	1.72	-0.40	245.56	32.87	13.19	5.44	2.51	0.40
	25	333.21	33.05	7.52	-0.74	-3.72	-5.71	329.40	36.48	7.55	-0.39	-3.33	-5.34	422.58	49.29	13.27	3.39	-0.42	-2.74	513.06	57.00	17.47	6.95	2.54	-0.16	433.27	50.70	18.16	6.58	2.89	0.10
	50	337.10	37.86	9.22	0.79	-2.30	-4.38	361.48	40.33	10.29	0.97	-2.52	-4.55	485.77	54.96	15.28	3.37	-0.75	-3.20	616.44	67.79	20.19	7.59	2.92	-0.47	657.89	69.59	22.01	8.60	3.34	0.27
	100	300.87	39.58	10.08	1.92	-1.15	-3.16	340.21	35.23	10.57	0.73	-2.40	-4.40	527.22	57.01	16.82	4.88	0.23	-2.17	651.57	75.22	22.51	8.51	3.31	0.05	762.66	74.22	24.32	9.14	3.61	0.17

T = 100

		Signal to noise = 10e4						Signal to noise = 10						Signal to noise = 1						Signal to noise = 0.1						Signal to noise = 10e-6					
		N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000
	m	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
h	1	69.06	-1.86	-11.48	-15.39	-17.34	-18.67	76.34	1.06	-9.81	-13.74	-15.61	-17.02	97.91	11.74	-1.32	-5.85	-8.00	-9.56	132.79	21.89	7.59	2.20	-0.04	-1.75	140.79	25.11	8.60	3.40	1.08	-0.72
	2	67.14	-4.35	-14.79	-18.68	-20.35	-21.70	86.51	-1.49	-12.59	-16.56	-18.23	-19.56	111.36	10.48	-1.56	-5.81	-7.81	-9.39	138.71	23.63	8.90	3.06	0.67	-1.08	146.83	24.08	9.69	4.32	1.91	0.16
	3	70.41	-2.94	-12.90	-17.09	-18.93	-20.28	77.94	0.71	-10.63	-14.86	-16.65	-18.02	119.22	13.29	-0.37	-5.09	-7.26	-8.79	146.92	23.89	9.00	3.33	1.02	-0.68	145.08	24.53	9.84	4.35	1.92	0.24
	4	79.65	0.64	-9.87	-14.16	-15.94	-17.37	90.33	4.01	-7.31	-12.02	-13.92	-15.42	112.90	13.95	1.13	-4.23	-6.34	-7.95	149.74	23.80	9.23	3.72	1.20	-0.55	171.00	26.41	10.56	4.36	1.92	0.36
	5	89.00	5.15	-6.88	-11.05	-13.13	-14.62	92.29	6.46	-4.74	-9.63	-11.66	-13.17	115.63	15.16	1.05	-3.50	-5.78	-7.44	146.08	25.08	9.51	3.82	1.39	-0.35	166.25	26.65	10.04	4.54	2.00	0.31
	6	97.36	7.21	-3.82	-8.55	-10.64	-12.22	109.58	10.68	-2.46	-7.44	-9.50	-11.08	119.70	16.57	2.27	-3.09	-5.40	-6.95	147.45	26.32	9.45	4.38	1.61	-0.23	157.91	26.81	10.27	4.22	1.99	0.23
	7	99.38	11.31	-1.87	-6.36	-8.51	-10.08	102.90	12.54	-0.80	-5.53	-7.88	-9.48	132.54	17.84	2.28	-2.79	-4.98	-6.72	148.62	25.88	10.18	4.24	1.64	-0.24	171.13	26.75	11.13	4.49	2.19	0.26
	8	109.85	13.56	0.09	-4.68	-6.98	-8.58	112.11	13.80	0.46	-4.79	-6.86	-8.45	134.27	16.69	2.91	-2.54	-4.78	-6.46	158.47	26.00	10.64	4.35	1.78	-0.17	151.45	27.68	10.26	4.69	2.30	0.28
	9	116.52	14.42	1.07	-4.32	-6.34	-7.90	119.71	14.38	0.78	-4.25	-6.48	-8.07	146.69	18.91	3.03	-2.57	-4.83	-6.48	164.06	26.75	10.83	4.59	1.88	-0.14	169.32	27.53	10.84	5.00	2.15	0.28
	10	121.45	16.23	1.23	-3.71	-5.89	-7.48	126.39	15.91	1.43	-3.83	-5.98	-7.66	153.65	19.49	3.34	-2.20	-4.48	-6.23	166.12	28.46	11.29	4.62	1.92	-0.14	157.38	28.30	11.31	5.07	2.42	0.32
	25	239.27	30.81	8.50	1.18	-1.49	-3.32	250.23	30.26	7.78	0.88	-1.80	-3.67	288.33	31.01	7.51	0.35	-2.52	-4.45	279.42	39.72	14.22	5.17	1.82	-0.44	330.75	40.71	14.36	6.12	2.72	0.48
	50	207.51	31.40	10.87	3.30	0.40	-1.47	250.10	35.47	10.72	3.19	0.02	-1.98	427.57	42.07	13.90	3.30	-0.32	-2.62	462.10	52.81	17.37	6.77	2.72	-0.03	395.30	52.41	17.93	7.75	3.34	0.55
	100	185.95	28.71	9.60	3.25	0.66	-1.07	191.36	28.83	9.39	2.70	0.03	-1.79	287.89	36.61	11.34	3.34	0.00	-1.94	384.73	49.12	17.03	6.38	2.81	0.15	344.67	49.83	16.53	6.79	2.95	0.40

Table 3.10: Gains from optimal pooling VAR forecasts – noise (*Model II*)

		T = 200																													
		Signal to noise = 10e4						Signal to noise = 10						Signal to noise = 1						Signal to noise = 0.1						Signal to noise = 10e-6					
		N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000
	m	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
h	1	60.06	-2.67	-13.44	-17.40	-19.10	-20.40	70.13	1.55	-9.45	-13.53	-15.38	-16.82	92.50	9.63	-2.03	-6.30	-8.43	-9.96	127.29	20.15	7.05	2.24	-0.11	-1.83	143.99	24.64	9.61	3.97	1.66	-0.06
	2	61.24	-8.62	-17.92	-21.69	-23.39	-24.61	64.70	-2.60	-12.65	-16.29	-18.14	-19.49	87.06	9.11	-2.37	-6.87	-8.95	-10.42	134.29	21.90	7.89	2.47	0.28	-1.43	157.55	23.85	9.94	3.96	1.87	0.13
	3	55.48	-7.45	-16.63	-20.32	-22.08	-23.29	72.42	-0.90	-11.27	-15.33	-17.14	-18.48	96.90	12.03	-1.71	-6.23	-8.25	-9.75	128.06	22.08	8.34	2.84	0.55	-1.20	156.51	23.98	10.16	4.29	1.96	0.23
	4	61.31	-3.54	-13.65	-17.47	-19.25	-20.58	75.17	1.86	-8.36	-12.67	-14.53	-15.91	95.22	12.23	-0.63	-5.04	-7.10	-8.69	132.98	22.50	8.26	2.94	0.56	-1.14	145.23	24.98	9.99	4.31	1.99	0.26
	5	72.89	-0.83	-10.36	-14.52	-16.40	-17.69	83.26	4.18	-6.35	-10.42	-12.26	-13.79	100.51	13.33	0.83	-3.94	-5.97	-7.60	123.95	23.03	8.21	2.94	0.52	-1.20	140.99	26.72	10.09	4.25	1.95	0.28
	6	73.46	3.73	-6.95	-11.42	-13.30	-14.74	87.94	7.02	-3.66	-8.21	-10.21	-11.66	101.50	14.29	1.46	-3.42	-5.37	-7.01	133.48	22.70	8.90	2.90	0.70	-1.12	148.74	24.56	10.08	4.39	2.11	0.29
	7	82.08	5.76	-5.08	-9.58	-11.54	-12.99	89.14	8.43	-2.41	-7.04	-9.07	-10.55	105.17	14.24	1.57	-3.11	-5.45	-7.03	130.13	22.83	8.06	2.78	0.45	-1.30	158.05	25.17	10.26	4.63	2.03	0.33
	8	89.85	8.53	-3.22	-7.74	-9.75	-11.23	88.37	9.87	-1.26	-6.08	-7.93	-9.47	110.80	15.05	2.22	-2.99	-5.17	-6.82	146.95	23.05	8.07	2.82	0.34	-1.49	153.61	25.67	10.63	4.58	2.18	0.33
	9	87.49	9.54	-2.31	-6.67	-8.81	-10.30	89.33	11.29	-0.71	-5.26	-7.28	-8.86	110.43	15.17	1.20	-3.29	-5.47	-7.17	140.12	22.47	8.09	2.56	0.13	-1.62	155.31	26.52	10.69	4.50	2.12	0.32
	10	94.39	11.67	-0.37	-5.20	-7.43	-8.94	100.71	13.05	1.04	-4.09	-6.34	-7.87	105.56	14.12	2.33	-2.70	-5.13	-6.80	142.19	22.63	8.24	2.24	-0.11	-1.83	162.53	27.72	10.31	4.80	2.18	0.31
	25	194.23	26.17	8.04	2.09	-0.45	-2.19	182.23	26.20	8.61	2.38	-0.27	-1.99	194.27	27.62	7.83	1.33	-1.12	-2.91	214.41	30.85	10.41	3.53	0.81	-1.16	205.73	33.93	12.52	5.93	2.49	0.30
	50	181.63	30.90	11.36	4.45	1.69	-0.25	194.46	32.54	11.85	4.55	1.50	-0.59	296.69	40.58	14.14	4.45	1.04	-1.14	276.17	40.29	13.30	5.18	1.86	-0.30	335.96	44.66	14.67	6.29	2.70	0.35
	100	139.92	25.17	9.80	4.08	1.67	-0.11	154.64	26.72	10.37	4.10	1.57	-0.20	206.28	34.05	12.63	4.79	1.53	-0.50	228.00	35.04	13.36	5.22	2.35	0.16	249.01	38.12	13.96	5.77	2.49	0.38

		T = 1000																													
		Signal to noise = 10e4						Signal to noise = 10						Signal to noise = 1						Signal to noise = 0.1						Signal to noise = 10e-6					
		N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000
	m	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
h	1	61.29	-4.07	-13.64	-17.28	-19.11	-20.42	62.15	-1.31	-11.25	-14.90	-16.68	-18.04	84.45	9.60	-1.08	-5.44	-7.62	-9.03	105.65	20.41	7.82	2.74	0.35	-1.27	138.92	23.99	9.25	4.26	1.90	0.24
	2	57.60	-8.38	-17.10	-20.80	-22.39	-23.65	60.38	-4.58	-13.92	-17.67	-19.33	-20.68	84.11	8.80	-2.43	-6.80	-8.73	-10.21	106.61	19.81	7.26	2.27	0.12	-1.53	140.72	24.22	9.80	4.28	1.97	0.23
	3	53.79	-6.86	-15.74	-19.35	-20.93	-22.23	61.64	-2.41	-12.23	-16.36	-17.94	-19.31	85.09	8.56	-2.13	-6.68	-8.60	-10.06	114.22	19.94	6.85	2.06	-0.19	-1.85	144.53	26.75	9.89	4.29	1.99	0.30
	4	62.39	-3.01	-12.64	-16.24	-17.99	-19.31	65.52	-0.08	-10.06	-14.02	-15.82	-17.18	93.81	9.98	-1.07	-5.78	-7.80	-9.27	109.30	19.61	6.94	2.15	-0.18	-1.85	153.85	24.95	9.71	4.11	1.98	0.28
	5	72.33	0.87	-9.14	-12.88	-14.77	-16.18	77.18	3.65	-7.63	-11.63	-13.47	-14.87	80.52	10.13	-1.01	-5.38	-7.46	-8.95	100.89	20.18	6.96	1.86	-0.25	-1.88	148.75	24.96	9.45	4.34	1.95	0.30
	6	74.50	3.08	-6.45	-10.68	-12.58	-14.04	74.44	4.71	-6.18	-10.27	-12.04	-13.49	83.06	10.15	-0.68	-5.15	-7.22	-8.73	111.20	19.83	6.76	1.78	-0.24	-1.93	140.87	24.88	9.84	4.16	2.00	0.29
	7	82.30	6.19	-3.87	-8.48	-10.46	-11.89	80.07	6.97	-3.91	-8.22	-10.19	-11.71	84.03	11.75	0.08	-4.57	-6.68	-8.28	115.89	18.95	7.22	1.93	-0.20	-1.95	150.40	25.93	9.98	4.31	2.01	0.31
	8	84.63	10.05	-2.19	-6.58	-8.51	-10.04	81.09	8.77	-1.86	-6.71	-8.55	-10.08	86.17	12.10	0.14	-4.31	-6.39	-8.02	106.16	20.28	7.08	2.20	-0.26	-1.91	139.98	25.68	9.29	4.26	2.00	0.32
	9	92.14	9.78	-1.08	-5.52	-7.55	-9.03	78.62	9.95	-1.60	-5.86	-8.00	-9.48	88.04	11.64	-0.21	-4.62	-6.50	-8.09	109.67	19.01	6.73	1.81	-0.27	-2.00	155.35	24.83	10.43	4.58	2.17	0.34
	10	88.96	10.08	-0.78	-5.11	-6.98	-8.55	93.09	10.43	-1.19	-5.81	-7.82	-9.28	89.44	10.50	-0.48	-5.11	-7.14	-8.67	117.40	18.97	6.71	1.78	-0.41	-2.06	145.52	26.06	10.39	4.60	2.10	0.34
	25	133.40	20.71	8.05	3.05	0.65	-0.98	116.42	21.91	7.59	2.57	0.37	-1.22	108.76	20.04	7.13	1.63	-0.37	-2.07	127.42	21.87	7.77	2.69	0.42	-1.24	146.83	24.10	10.35	4.60	2.02	0.30
	50	130.32	23.44	10.73	4.52	2.00	0.18	146.79	23.91	9.68	4.38	1.85	0.17	131.19	25.66	9.46	4.32	1.73	0.04	132.77	26.62	10.18	3.97	1.61	-0.04	174.99	28.08	10.57	4.71	2.13	0.36
	100	101.59	22.99	8.95	4.22	1.92	0.30	105.50	21.70	9.27	4.13	1.99	0.29	117.44	23.11	9.22	4.32	2.01	0.32	127.56	23.97	9.94	4.40	2.03	0.32	149.56	27.70	10.50	4.24	2.00	0.33

Table 3.10 continued: Gains from optimal pooling VAR forecasts – noise (*Model II*)

Model III

T = 25

		Signal to noise = 10e4						Signal to noise = 10						Signal to noise = 1						Signal to noise = 0.1						Signal to noise = 10e-6					
		N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000
	m	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
h	1	119.18	10.04	-2.53	-7.36	-9.37	-10.92	123.82	13.95	1.26	-3.79	-6.04	-7.56	130.66	19.79	6.15	0.24	-2.01	-3.72	135.92	22.28	5.68	0.08	-2.28	-3.85	142.17	19.83	5.38	-0.33	-2.46	-4.14
	2	125.54	10.08	-3.59	-8.37	-10.57	-12.09	136.30	15.64	0.63	-4.51	-6.72	-8.36	170.59	24.77	5.97	0.45	-2.05	-3.64	181.83	25.17	8.75	2.75	0.27	-1.52	178.29	26.66	8.67	2.26	-0.02	-1.78
	3	181.08	14.35	-1.04	-6.41	-8.91	-10.60	160.53	17.23	1.61	-3.97	-6.22	-7.88	227.67	26.19	8.14	1.50	-1.12	-2.90	213.96	30.33	10.97	3.73	1.21	-0.78	211.50	29.46	9.87	3.35	0.51	-1.26
	4	186.55	19.62	1.83	-4.33	-6.65	-8.36	227.87	20.38	3.40	-2.70	-4.97	-6.67	249.96	31.04	8.75	1.91	-0.56	-2.46	277.04	36.30	11.60	4.21	1.43	-0.40	252.24	34.14	11.24	3.86	1.04	-0.90
	5	218.30	21.77	3.96	-2.86	-5.27	-7.00	264.58	27.53	5.60	-1.14	-3.79	-5.57	326.97	34.05	11.27	3.06	0.23	-1.73	308.47	38.22	12.09	4.65	1.64	-0.42	322.14	36.59	12.12	4.33	1.19	-0.73
	6	320.16	27.57	5.48	-1.76	-4.41	-6.12	326.07	29.05	6.76	-0.36	-3.01	-4.81	373.77	38.45	12.75	4.06	0.86	-1.22	394.95	46.29	14.19	5.15	1.83	-0.34	413.68	40.59	13.05	4.65	1.40	-0.72
	7	313.50	30.59	6.50	-1.21	-3.80	-5.69	356.68	33.02	7.71	0.46	-2.41	-4.29	493.89	45.10	13.29	4.75	1.64	-0.58	437.09	48.13	14.28	5.08	1.67	-0.51	434.86	44.85	13.43	4.51	1.11	-0.85
	8	408.51	35.25	8.99	0.50	-2.59	-4.50	363.62	37.33	10.13	1.76	-1.57	-3.55	481.02	53.23	14.91	5.47	1.92	-0.23	459.88	53.64	15.72	5.56	1.89	-0.40	594.96	48.54	13.83	4.81	1.19	-0.91
	9	431.47	41.42	10.93	1.24	-2.03	-4.06	504.81	42.67	11.53	2.65	-0.85	-2.99	575.98	51.08	16.31	5.67	2.20	-0.15	579.83	55.80	17.23	6.24	2.25	-0.08	700.13	51.86	15.98	5.08	1.55	-0.73
	10	510.83	47.25	11.56	1.80	-1.47	-3.76	528.35	48.27	12.94	2.89	-0.56	-2.67	624.67	58.59	18.70	6.03	2.43	0.05	623.25	56.78	17.65	6.32	2.19	-0.23	693.82	58.93	16.46	5.31	1.41	-0.94
	25	600.44	64.39	17.40	4.56	-0.15	-3.18	658.70	64.35	19.22	5.35	0.95	-2.33	965.44	94.07	27.23	10.81	3.91	0.27	991.84	105.49	29.05	9.95	3.50	-0.26	1101.97	101.90	29.43	10.01	3.21	-0.52
	50	651.99	68.96	20.18	4.88	-0.10	-3.47	762.95	83.58	23.15	8.54	3.22	-0.93	1185.26	119.53	35.04	13.91	5.96	1.20	1335.63	135.90	35.71	13.35	4.87	-0.20	1330.60	138.00	36.55	12.78	4.07	-0.88
	100	785.94	90.43	25.07	6.77	0.67	-2.98	902.14	103.01	28.78	8.90	3.00	-1.33	1424.64	137.04	40.55	15.04	6.65	1.23	1427.45	144.52	42.52	15.88	6.99	1.01	1587.15	156.70	45.60	14.18	4.37	-1.29

T = 50

		Signal to noise = 10e4						Signal to noise = 10						Signal to noise = 1						Signal to noise = 0.1						Signal to noise = 10e-6					
		N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000
	m	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
h	1	106.77	12.95	-0.64	-5.35	-7.45	-8.99	117.34	15.18	2.41	-2.90	-5.07	-6.59	118.46	18.65	5.16	0.19	-2.10	-3.73	142.88	21.59	8.04	2.83	0.32	-1.37	137.92	23.55	8.03	2.84	0.55	-1.19
	2	109.32	10.22	-2.38	-7.51	-9.42	-10.89	110.31	13.69	1.42	-3.93	-5.97	-7.53	145.80	20.77	6.46	1.10	-1.15	-2.87	161.73	25.43	9.03	3.77	1.21	-0.44	157.30	25.52	10.03	3.96	1.38	-0.39
	3	121.69	12.86	-1.49	-6.64	-8.65	-10.19	126.27	15.49	1.96	-3.10	-5.26	-6.89	145.03	23.03	7.40	1.48	-0.86	-2.58	155.42	25.85	9.69	4.17	1.44	-0.26	169.63	26.76	9.68	4.03	1.64	-0.12
	4	137.91	15.41	1.39	-3.79	-5.95	-7.55	139.95	19.06	3.82	-1.55	-3.87	-5.47	179.96	24.69	8.02	2.24	-0.12	-1.89	210.29	27.58	10.33	4.14	1.68	-0.16	192.88	27.84	10.77	4.44	1.88	0.04
	5	146.95	18.30	3.34	-2.21	-4.51	-6.21	173.17	22.26	6.13	0.07	-2.19	-4.02	211.86	27.26	9.55	3.37	0.63	-1.19	218.84	30.23	10.88	4.64	1.93	0.08	210.70	29.39	11.59	4.79	1.90	0.09
	6	191.13	23.26	4.69	-1.17	-3.50	-5.14	195.51	22.91	6.93	0.86	-1.54	-3.29	260.25	29.71	10.42	3.75	1.16	-0.66	251.96	33.86	12.18	4.72	1.97	0.09	246.89	32.70	10.83	4.86	2.04	0.12
	7	194.51	23.95	6.77	0.09	-2.32	-4.08	215.31	27.73	8.87	1.91	-0.64	-2.53	272.34	34.77	12.76	4.84	1.77	-0.23	253.67	34.72	13.44	5.18	2.12	0.13	284.66	32.69	12.71	5.10	2.24	0.18
	8	232.17	27.42	6.50	0.35	-2.00	-3.78	258.06	29.98	8.73	2.13	-0.45	-2.28	297.40	38.07	12.86	4.90	2.00	-0.11	296.31	40.63	13.29	5.78	2.54	0.38	317.84	37.85	13.47	5.82	2.52	0.31
	9	247.10	28.42	7.80	0.91	-1.70	-3.48	235.51	32.55	10.05	2.80	0.14	-1.87	312.54	40.97	13.69	4.91	1.82	-0.28	314.06	42.25	14.43	5.97	2.56	0.42	295.47	40.35	14.02	5.98	2.65	0.38
	10	270.54	30.87	9.40	1.55	-1.15	-3.03	276.72	37.30	10.85	3.52	0.55	-1.58	386.55	42.18	14.18	5.66	1.93	-0.24	427.56	46.91	16.14	6.48	2.83	0.48	376.85	42.88	14.27	6.24	3.05	0.46
	25	436.03	55.92	14.83	5.09	1.35	-1.01	391.73	47.08	14.70	4.97	1.53	-0.76	661.02	71.79	21.40	7.42	2.67	-0.09	701.86	67.51	21.97	8.03	3.33	0.30	631.95	66.31	21.71	8.01	3.40	0.55
	50	371.64	47.80	15.99	5.65	1.22	-1.09	369.11	45.43	15.67	5.84	1.75	-0.61	570.03	70.08	22.01	7.78	2.61	-0.36	562.09	71.79	24.35	9.10	3.73	0.59	602.88	67.22	22.02	8.81	3.77	0.70
	100	430.64	44.98	13.68	4.53	0.97	-1.47	297.56	45.37	13.74	5.27	1.45	-0.78	513.67	64.69	19.19	7.38	2.80	-0.16	527.93	67.10	20.24	8.00	3.34	0.40	453.36	64.09	22.06	8.58	3.72	0.46

T = 100

		Signal to noise = 10e4						Signal to noise = 10						Signal to noise = 1						Signal to noise = 0.1						Signal to noise = 10e-6					
		N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000
	m	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
h	1	104.31	10.98	-0.58	-5.46	-7.56	-9.01	112.93	12.93	1.34	-3.24	-5.55	-6.99	109.89	18.82	4.96	0.18	-2.06	-3.75	143.74	22.12	8.59	3.39	1.11	-0.53	162.34	23.51	8.53	3.34	0.98	-0.80
	2	91.40	8.09	-3.57	-7.93	-10.04	-11.48	104.75	10.37	-0.86	-5.71	-7.60	-9.17	123.70	19.42	5.64	0.37	-1.79	-3.40	152.31	26.09	9.55	3.99	1.60	-0.09	139.79	24.92	9.49	4.19	1.73	-0.02
	3	91.71	7.77	-3.02	-7.33	-9.35	-10.86	109.21	13.01	0.65	-4.67	-6.45	-8.04	123.66	21.50	6.95	1.41	-0.85	-2.48	159.67	24.55	9.58	4.24	1.71	-0.01	159.52	25.53	10.34	4.18	1.74	0.07
	4	108.96	12.18	-0.13	-5.15	-7.34	-8.86	105.23	15.57	2.50	-2.30	-4.30	-5.94	136.40	20.98	7.40	2.45	0.15	-1.54	157.04	25.60	9.71	4.41	1.82	0.10	166.31	25.30	9.46	4.36	1.83	0.10
	5	115.68	13.83	0.85	-4.30	-6.52	-8.08	117.91	17.33	3.81	-1.34	-3.52	-5.17	158.26	24.00	8.72	2.85	0.64	-1.11	181.99	26.69	9.86	4.06	1.80	0.11	168.22	26.52	9.68	4.20	1.83	0.14
	6	120.55	16.65	2.26	-2.77	-4.89	-6.52	136.03	18.52	5.48	0.19	-2.18	-3.79	146.23	25.76	8.68	3.43	0.94	-0.73	193.71	26.63	9.81	4.14	1.76	0.11	181.50	27.70	10.70	4.25	1.90	0.22
	7	143.91	18.23	3.48	-1.71	-3.91	-5.60	135.72	21.47	6.75	1.26	-1.14	-2.85	167.94	26.31	9.58	3.69	1.29	-0.53	201.10	28.23	11.25	4.63	1.81	0.12	189.56	27.94	10.54	4.54	2.19	0.30
	8	148.04	17.90	4.31	-1.19	-3.34	-4.97	138.75	24.38	7.55	1.96	-0.47	-2.14	173.88	27.70	9.78	4.18	1.46	-0.36	197.65	30.21	10.71	4.34	1.92	0.15	205.34	29.48	11.50	4.86	2.26	0.35
	9	140.36	20.43	5.10	-0.29	-2.70	-4.40	153.92	23.69	7.91	2.72	0.22	-1.45	192.48	27.70	10.42	4.49	1.62	-0.23	217.75	30.62	11.43	4.63	2.02	0.20	208.13	30.58	11.72	4.85	2.29	0.36
	10	156.51	21.29	5.66	-0.09	-2.43	-4.11	180.21	26.34	8.84	2.58	0.44	-1.34	223.19	30.52	11.22	4.32	1.58	-0.24	239.29	32.77	11.72	4.75	2.13	0.19	242.01	31.93	11.71	5.17	2.28	0.40
	25	252.46	35.21	11.23	4.29	1.51	-0.49	280.80	36.22	12.87	4.95	1.97	-0.02	298.94	42.02	14.59	5.63	2.24	0.09	345.84	45.72	14.90	6.01	2.53	0.32	324.11	44.71	14.49	5.96	2.53	0.39
	50	180.96	31.49	11.16	4.59	1.73	-0.10	184.58	28.75	11.14	4.67	2.04	0.11	209.36	34.86	12.92	5.46	2.23	0.24	234.01	37.04	14.11	5.81	2.43	0.35	238.80	36.09	13.06	5.84	2.55	0.39
	100	174.47	26.96	10.52	4.51	1.72	-0.12	167.75	28.18	10.43	4.37	1.89	0.04	196.50	31.01	11.24	4.83	2.05	0.20	251.06	34.06	12.44	5.42	2.42	0.32	198.38	33.50	12.14	5.09	2.23	0.32

Table 3.11: Gains from optimal pooling VAR forecasts – noise (*Model III*)

		T = 200																													
		Signal to noise = 10e4						Signal to noise = 10						Signal to noise = 1						Signal to noise = 0.1						Signal to noise = 10e-6					
		N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000
	m	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
h	1	93.96	11.75	-0.45	-4.83	-6.99	-8.63	105.07	16.18	3.32	-1.43	-3.53	-5.17	117.62	19.45	5.65	0.79	-1.47	-3.08	133.30	22.44	8.77	3.30	1.08	-0.62	141.35	23.94	9.32	3.68	1.51	-0.17
	2	90.55	8.71	-3.51	-7.61	-9.63	-11.11	96.00	11.32	0.65	-4.00	-5.86	-7.48	107.63	19.09	6.07	1.05	-1.17	-2.77	141.33	22.94	9.22	3.76	1.43	-0.19	136.54	24.40	9.62	4.18	1.74	0.05
	3	86.25	8.23	-2.92	-7.36	-9.53	-10.97	95.16	11.94	0.66	-4.01	-6.04	-7.58	128.98	18.83	6.48	1.50	-0.56	-2.27	134.67	23.68	9.18	3.86	1.59	-0.01	142.96	25.89	10.21	4.55	1.93	0.23
	4	89.18	8.43	-2.51	-6.39	-8.74	-10.18	108.81	14.26	1.55	-3.45	-5.57	-7.13	115.90	20.73	7.07	1.88	-0.37	-1.99	139.01	24.86	9.16	3.91	1.66	0.01	157.27	26.32	10.41	4.46	2.10	0.28
	5	100.07	10.83	0.13	-4.73	-6.70	-8.32	110.08	14.88	2.44	-2.32	-4.48	-6.08	124.06	19.70	7.64	2.24	-0.09	-1.76	162.64	23.85	9.68	4.06	1.86	0.12	142.07	26.36	9.83	4.70	2.10	0.31
	6	111.07	14.68	2.03	-2.77	-4.69	-6.26	101.64	16.36	3.88	-0.96	-2.90	-4.54	119.84	22.41	7.47	2.62	0.41	-1.23	139.99	25.70	9.70	4.34	1.83	0.15	162.80	25.34	9.93	4.54	2.10	0.31
	7	106.92	16.86	3.66	-1.28	-3.43	-4.95	107.89	18.31	4.94	0.05	-2.15	-3.74	128.38	22.69	8.12	2.97	0.88	-0.85	153.16	26.50	10.25	4.28	1.98	0.22	175.18	27.75	10.33	4.63	2.10	0.30
	8	129.30	19.34	5.67	0.37	-1.88	-3.47	126.41	19.55	6.03	1.16	-1.16	-2.77	135.44	24.00	8.41	3.49	1.25	-0.57	159.74	26.60	10.23	4.46	2.02	0.28	170.87	27.42	10.30	4.63	2.16	0.28
	9	121.68	18.80	5.94	1.01	-1.13	-2.83	126.41	20.65	6.70	1.60	-0.62	-2.30	141.67	25.11	9.11	3.68	1.36	-0.28	168.09	28.69	9.92	4.42	1.98	0.29	169.14	30.40	10.42	4.63	2.09	0.25
	10	134.41	21.19	7.17	1.59	-0.56	-2.15	134.04	22.35	7.24	2.27	0.06	-1.60	147.81	24.16	9.29	3.68	1.55	-0.18	168.05	28.38	10.49	4.49	2.13	0.34	202.45	28.66	10.53	4.78	2.17	0.31
	25	203.78	30.02	11.32	4.58	1.93	0.11	200.98	31.01	11.05	4.68	2.08	0.22	223.26	32.07	11.80	4.70	2.06	0.14	277.20	36.67	13.53	5.44	2.51	0.37	246.13	34.47	12.34	5.31	2.17	0.35
	50	131.47	25.23	10.13	4.11	1.87	0.14	131.67	24.54	10.00	4.24	1.96	0.17	159.36	24.51	10.14	4.58	2.05	0.24	173.64	28.26	10.86	4.65	2.16	0.33	197.08	26.92	10.59	4.63	2.16	0.32
	100	125.05	23.70	9.22	4.15	1.93	0.16	125.40	23.38	9.38	4.26	1.92	0.22	136.47	22.74	9.90	4.22	1.96	0.27	161.53	25.48	10.14	4.15	2.00	0.32	171.25	26.11	10.04	4.41	2.08	0.26

		T = 1000																													
		Signal to noise = 10e4						Signal to noise = 10						Signal to noise = 1						Signal to noise = 0.1						Signal to noise = 10e-6					
		N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000	N=10	N=25	N=50	N=100	N=200	N=1000
	m	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
h	1	82.77	10.60	-1.37	-5.52	-7.55	-9.14	84.96	13.60	1.84	-2.75	-4.89	-6.42	98.88	16.98	4.89	0.25	-1.80	-3.45	123.46	22.03	8.91	3.80	1.67	-0.02	144.85	24.50	9.90	4.42	1.98	0.38
	2	75.91	7.28	-4.00	-8.55	-10.36	-11.84	81.46	9.85	-1.01	-5.01	-6.96	-8.50	104.67	17.36	5.16	0.93	-1.39	-2.95	116.74	21.31	8.84	3.87	1.68	-0.08	143.81	24.94	10.17	4.55	2.16	0.42
	3	81.82	8.15	-2.63	-7.01	-8.92	-10.43	86.85	11.01	0.08	-4.39	-6.63	-8.12	102.53	17.42	6.05	0.92	-1.35	-2.90	120.87	22.04	8.83	3.71	1.60	-0.08	155.57	24.26	9.92	4.33	2.13	0.36
	4	85.61	10.87	-0.72	-5.39	-7.31	-8.85	87.87	12.20	0.52	-4.07	-6.05	-7.57	99.88	18.28	6.15	1.50	-0.74	-2.40	114.83	22.68	9.46	3.84	1.82	0.04	142.15	25.21	9.95	4.42	2.06	0.36
	5	83.09	11.55	0.16	-4.22	-6.31	-7.80	96.17	13.76	2.28	-2.39	-4.49	-6.04	103.16	17.67	6.43	1.68	-0.40	-2.07	120.13	24.02	9.19	4.15	1.78	0.12	146.38	25.09	9.51	4.41	2.07	0.37
	6	89.78	13.75	2.10	-2.46	-4.55	-6.08	100.46	15.59	3.83	-1.18	-3.19	-4.81	101.00	19.64	7.49	2.31	0.06	-1.57	114.40	22.49	9.42	4.28	1.93	0.22	144.32	25.16	9.77	4.24	2.08	0.34
	7	101.96	14.92	3.97	-0.77	-2.97	-4.58	94.90	17.36	4.66	-0.11	-2.42	-3.95	104.00	20.28	7.75	2.83	0.53	-1.10	118.13	22.97	9.20	4.13	1.86	0.19	144.52	24.07	9.87	4.50	2.00	0.34
	8	99.48	17.91	5.22	0.30	-1.65	-3.33	103.68	16.76	5.92	1.16	-1.15	-2.85	109.37	20.88	8.19	3.07	0.84	-0.77	123.27	22.68	9.51	4.12	1.85	0.19	154.42	24.70	9.68	4.19	2.06	0.34
	9	108.30	18.02	5.82	1.04	-1.15	-2.68	103.05	19.33	6.39	1.75	-0.48	-2.17	118.60	19.83	8.64	3.31	1.11	-0.57	132.34	22.43	10.14	4.23	1.92	0.24	157.01	24.21	10.18	4.43	2.14	0.34
	10	108.47	20.05	7.00	1.96	-0.34	-2.03	103.63	18.57	7.10	2.31	0.11	-1.58	104.23	22.40	8.76	3.62	1.40	-0.28	118.62	22.63	9.56	4.15	1.97	0.24	149.16	23.24	9.97	4.68	2.16	0.37
	25	136.96	23.27	9.67	4.10	1.78	0.05	131.87	24.21	9.81	4.47	1.96	0.27	146.77	25.90	10.07	4.43	2.10	0.32	170.38	25.99	10.45	4.40	2.09	0.32	197.92	29.02	11.03	4.63	2.19	0.32
	50	113.88	22.76	9.24	4.32	1.96	0.28	104.25	22.32	9.23	4.19	1.97	0.25	116.71	21.35	9.61	4.43	2.00	0.33	131.08	25.19	9.92	4.43	2.13	0.35	155.71	26.25	10.25	4.52	2.06	0.32
	100	110.03	21.16	8.91	4.14	1.94	0.31	105.69	21.48	9.30	4.17	2.06	0.35	111.38	21.50	9.32	4.17	1.94	0.36	121.12	22.73	9.22	4.23	1.96	0.28	159.87	24.48	9.96	4.69	2.08	0.36

Table 3.11 continued: Gains from optimal pooling VAR forecasts – noise (*Model III*)

3.B.4 Incorporating all Variables as Benchmark

		MODEL I											
		T = 25											
	s	∞		10e4		10		1		0.1		10e-6	
	m	1	5	1	5	1	5	1	5	1	5	1	5
h	1	-4.18	-20.31	-6.56	-20.37	-3.96	-16.32	-6.70	-14.03	-14.39	-18.70	-19.30	-21.78
	2	-2.49	-20.42	-3.33	-20.44	2.69	-9.79	0.68	-4.34	-7.35	-8.66	-10.06	-10.94
	3	-5.87	-22.05	-5.30	-21.86	-0.02	-10.16	0.53	-3.29	-4.89	-5.73	-6.87	-7.46
	4	-7.40	-21.30	-6.23	-21.19	-2.46	-11.02	0.45	-2.58	-3.37	-3.75	-4.80	-5.45
	5	-10.67	-22.64	-8.68	-22.35	-4.26	-11.56	-0.03	-2.42	-3.14	-3.23	-3.61	-4.27
	6	-11.88	-22.71	-9.97	-22.46	-5.01	-11.56	-0.72	-2.48	-3.05	-2.97	-2.91	-3.54
	7	-13.08	-22.65	-11.09	-22.38	-5.02	-11.03	-0.81	-2.24	-3.26	-3.04	-2.47	-3.10
	8	-12.21	-21.04	-10.41	-21.05	-5.64	-11.08	-1.20	-2.65	-3.17	-3.09	-2.26	-2.99
	9	-10.22	-18.52	-8.39	-18.78	-5.25	-10.54	-0.91	-2.20	-2.77	-2.62	-1.81	-2.80
	10	-9.35	-16.98	-7.58	-17.37	-5.85	-10.67	-0.76	-1.91	-2.57	-2.67	-1.66	-2.64
	25	-9.51	-16.39	-9.16	-17.48	-3.08	-6.31	-1.70	-2.88	-2.68	-3.86	-3.77	-4.12
	50	-10.19	-16.96	-10.69	-19.89	-4.30	-8.81	-3.31	-3.64	-5.01	-5.23	-4.78	-5.62
	100	-11.08	-18.62	-13.18	-23.65	-3.58	-9.06	-3.05	-2.90	-6.26	-7.57	-5.68	-6.58
		T = 50											
	s	∞		10e4		10		1		0.1		10e-6	
	m	1	5	1	5	1	5	1	5	1	5	1	5
h	1	12.84	-1.17	13.11	-1.63	11.10	-2.45	3.64	-3.25	-7.06	-9.11	-9.10	-10.65
	2	18.17	0.36	18.08	-0.04	20.25	5.72	10.74	6.06	-1.70	-2.69	-4.19	-5.01
	3	14.66	-0.90	15.86	-1.27	19.49	7.66	11.32	8.39	0.04	-0.64	-2.49	-2.85
	4	6.85	-6.47	7.95	-6.89	14.73	5.13	9.91	7.92	0.33	-0.19	-1.70	-1.85
	5	0.21	-11.64	1.14	-12.02	10.14	1.85	8.29	6.80	0.50	0.09	-1.56	-1.68
	6	-5.39	-15.74	-4.47	-16.11	6.71	-0.40	6.87	5.58	0.96	0.55	-1.34	-1.49
	7	-8.47	-18.05	-7.40	-18.29	5.10	-1.30	6.16	5.11	1.15	0.71	-1.23	-1.39
	8	-10.38	-19.59	-9.41	-19.71	3.66	-1.94	5.06	4.19	1.18	0.62	-1.34	-1.68
	9	-11.48	-20.03	-10.61	-20.21	2.32	-2.35	4.39	3.64	0.97	0.45	-1.53	-1.98
	10	-12.53	-20.56	-11.20	-20.65	2.12	-2.33	4.10	3.58	0.87	0.30	-1.49	-1.99
	25	-5.27	-10.29	-5.06	-10.90	1.78	-0.60	2.67	2.42	-0.51	-0.47	-1.01	-1.36
	50	-2.76	-6.68	-2.70	-7.16	2.42	0.76	3.70	3.26	0.19	0.10	-1.19	-1.21
	100	-3.75	-6.66	-4.05	-7.23	2.23	1.24	3.69	3.00	0.48	0.16	-1.13	-1.45
		T = 100											
	s	∞		10e4		10		1		0.1		10e-6	
	m	1	5	1	5	1	5	1	5	1	5	1	5
h	1	18.72	4.21	17.94	4.62	21.19	10.13	13.16	6.73	-1.63	-3.32	-3.81	-4.65
	2	26.44	8.48	26.85	8.95	29.10	16.52	17.80	13.65	0.76	0.19	-1.60	-2.08
	3	24.53	7.88	24.35	7.97	26.43	16.70	16.70	13.54	1.03	0.76	-1.06	-1.28
	4	19.37	5.01	18.57	4.75	21.45	13.55	14.17	12.16	1.11	0.93	-0.66	-0.82
	5	11.77	-0.41	10.75	-0.58	16.88	10.60	12.37	10.72	0.86	0.79	-0.57	-0.66
	6	4.89	-5.97	3.56	-5.96	12.23	6.98	10.39	9.03	0.77	0.75	-0.42	-0.50
	7	1.50	-8.13	0.05	-8.24	10.29	5.67	9.12	7.93	0.67	0.70	-0.32	-0.39
	8	-0.95	-9.64	-1.91	-9.82	8.56	4.32	8.18	7.10	0.59	0.68	-0.19	-0.29
	9	-2.33	-10.51	-3.05	-10.60	7.50	3.82	7.66	6.76	0.61	0.72	-0.17	-0.26
	10	-2.78	-11.05	-3.62	-11.03	6.90	3.53	7.27	6.49	0.64	0.79	-0.13	-0.25
	25	-1.64	-6.10	-1.42	-5.90	4.03	2.44	5.50	5.08	0.78	0.84	-0.01	-0.14
	50	-0.22	-2.26	-0.36	-2.12	2.53	1.56	2.72	2.17	0.24	0.33	0.13	0.05
	100	0.39	-0.94	0.41	-1.01	1.75	1.31	2.55	2.21	0.68	0.79	0.10	0.07
		T = 200											
	s	∞		10e4		10		1		0.1		10e-6	
	m	1	5	1	5	1	5	1	5	1	5	1	5
h	1	25.23	11.93	25.52	11.77	23.22	11.87	14.54	8.15	1.32	0.62	-1.87	-2.33
	2	36.99	18.99	36.28	18.84	31.00	18.73	18.95	14.63	2.48	2.14	-0.87	-0.93
	3	34.33	17.59	33.73	17.63	28.92	19.35	17.60	14.68	2.39	2.19	-0.48	-0.47
	4	27.69	14.14	27.18	14.18	23.51	16.53	14.89	12.98	1.97	1.92	-0.26	-0.23
	5	20.13	8.65	19.84	8.91	18.82	13.59	12.98	11.61	1.78	1.74	-0.14	-0.10
	6	14.49	5.01	14.60	5.45	15.04	10.74	11.29	10.21	1.55	1.58	-0.10	-0.09
	7	11.79	3.30	11.86	3.93	13.17	9.28	10.43	9.72	1.36	1.41	-0.08	-0.05
	8	8.63	0.82	8.94	1.47	11.42	7.80	9.57	9.03	1.35	1.39	0.04	0.06
	9	7.17	-0.35	7.03	0.13	10.88	7.31	9.29	8.85	1.40	1.45	0.09	0.11
	10	6.14	-0.92	6.05	-0.58	9.97	6.64	8.82	8.47	1.45	1.49	0.13	0.16
	25	0.54	-3.83	0.81	-3.64	3.17	2.09	4.63	4.39	0.87	0.89	-0.16	-0.12
	50	0.02	-1.70	-0.24	-1.82	1.09	0.66	1.67	1.58	0.48	0.47	-0.08	-0.06
	100	0.48	0.12	0.35	-0.02	0.42	0.40	0.76	0.69	0.20	0.21	0.01	0.00
		T = 1000											
	s	∞		10e4		10		1		0.1		10e-6	
	m	1	5	1	5	1	5	1	5	1	5	1	5
h	1	26.73	14.76	29.00	15.23	28.44	16.87	16.21	11.32	2.94	2.54	-0.42	-0.51
	2	37.13	21.06	38.37	21.26	36.03	23.64	18.90	16.33	3.19	3.08	-0.22	-0.24
	3	32.19	17.41	32.37	17.42	34.42	24.87	17.48	15.96	2.77	2.72	-0.08	-0.09
	4	30.45	17.60	30.15	17.66	30.40	23.32	14.78	13.89	2.35	2.35	-0.07	-0.06
	5	27.66	17.28	27.83	17.18	25.91	20.43	12.90	12.28	2.05	2.02	-0.03	-0.02
	6	26.05	16.91	26.18	16.81	22.87	18.46	11.59	11.17	1.90	1.87	-0.02	-0.01
	7	22.98	15.01	23.42	15.10	20.34	16.56	10.56	10.24	1.79	1.78	0.00	0.01
	8	19.12	11.93	19.61	12.06	17.52	14.04	9.66	9.49	1.64	1.65	0.03	0.03
	9	16.34	9.95	16.67	10.02	15.90	12.75	9.34	9.19	1.61	1.62	0.03	0.04
	10	14.94	9.05	15.50	9.04	14.41	11.61	9.20	9.02	1.58	1.59	0.05	0.06
	25	1.29	-0.91	1.92	-0.86	1.99	1.41	2.86	2.75	0.80	0.76	0.05	0.04
	50	-3.73	-4.61	-3.21	-4.17	0.01	-0.04	0.12	0.11	0.06	0.06	0.02	0.01
	100	-0.89	-0.99	-0.83	-0.88	0.05	0.07	0.06	0.05	0.01	0.01	0.00	0.00

Table 3.12: Gains from equal VAR pooling relative to large VAR (*Model I*)

		MODEL II											
		T = 25											
	s	∞		10e4		10		1		0.1		10e-6	
	m	1	5	1	5	1	5	1	5	1	5	1	5
h	1	-5.14	-18.02	-5.08	-18.28	-0.79	-15.47	-7.49	-14.89	-15.78	-19.67	-18.63	-21.29
	2	-1.93	-16.32	-1.22	-16.44	4.11	-9.82	-0.25	-5.14	-9.87	-11.29	-11.25	-12.19
	3	-5.62	-18.29	-5.22	-18.24	0.17	-10.70	0.17	-3.19	-6.59	-7.50	-6.97	-7.69
	4	-9.84	-20.37	-8.98	-20.21	-4.33	-13.08	-0.70	-2.83	-5.03	-5.50	-5.14	-5.60
	5	-12.60	-21.61	-11.72	-21.53	-6.57	-13.76	-0.74	-2.53	-4.05	-4.19	-3.96	-4.39
	6	-14.40	-22.13	-13.60	-21.98	-8.26	-14.33	-1.62	-3.06	-3.46	-3.49	-3.03	-3.44
	7	-14.75	-21.66	-14.38	-21.53	-8.78	-14.09	-1.92	-2.97	-3.21	-3.15	-2.61	-2.89
	8	-13.64	-19.81	-13.44	-19.66	-8.75	-13.24	-2.27	-3.25	-2.86	-2.82	-1.97	-2.24
	9	-11.93	-17.69	-12.15	-17.55	-7.95	-11.95	-2.10	-3.29	-2.78	-2.81	-1.74	-2.11
	10	-10.96	-16.47	-11.33	-16.25	-7.47	-11.19	-2.11	-2.99	-2.89	-3.05	-1.62	-1.94
	25	-6.19	-10.40	-7.76	-10.08	-3.46	-5.92	-1.08	-1.46	-3.02	-3.06	-2.76	-3.08
	50	-6.29	-11.21	-7.68	-10.69	-6.03	-7.55	-3.06	-4.11	-4.37	-5.30	-4.45	-4.50
	100	-5.27	-10.29	-6.35	-9.59	-3.94	-6.50	-2.41	-3.77	-5.14	-6.75	-3.84	-4.33
		T = 50											
	s	∞		10e4		10		1		0.1		10e-6	
	m	1	5	1	5	1	5	1	5	1	5	1	5
h	1	15.80	2.44	15.64	2.42	15.40	2.30	5.38	-1.50	-7.49	-9.25	-6.43	-9.07
	2	20.08	4.59	20.91	4.56	19.96	6.73	8.76	4.43	-2.98	-3.69	-3.40	-4.51
	3	15.30	1.77	15.96	1.73	15.47	5.39	8.11	5.33	-1.18	-1.97	-2.00	-2.72
	4	7.75	-3.57	8.53	-3.58	10.07	1.84	6.68	4.55	-0.68	-1.16	-1.50	-1.85
	5	2.25	-7.68	2.74	-7.68	6.44	-0.65	5.41	3.66	-0.44	-0.78	-1.17	-1.32
	6	-1.12	-9.81	-1.01	-9.80	3.82	-2.05	4.80	3.05	-0.35	-0.64	-0.93	-0.98
	7	-2.80	-10.52	-2.76	-10.51	3.02	-2.19	4.89	3.28	-0.21	-0.40	-0.62	-0.71
	8	-3.08	-10.26	-3.02	-10.26	2.08	-2.38	4.84	3.44	-0.18	-0.28	-0.59	-0.64
	9	-3.18	-9.68	-3.02	-9.67	1.50	-2.28	4.35	3.24	-0.15	-0.21	-0.59	-0.69
	10	-2.30	-8.54	-2.35	-8.52	1.39	-1.84	4.32	3.30	-0.13	-0.14	-0.44	-0.68
	25	0.29	-1.78	1.23	-1.71	2.63	1.19	4.41	3.04	-0.22	-0.40	-0.88	-0.92
	50	2.22	0.95	3.31	1.14	3.50	2.49	4.03	3.18	0.38	0.39	-1.11	-0.85
	100	1.41	0.10	2.43	0.18	2.73	1.36	3.36	2.92	0.38	-0.05	-0.62	-0.39
		T = 100											
	s	∞		10e4		10		1		0.1		10e-6	
	m	1	5	1	5	1	5	1	5	1	5	1	5
h	1	21.94	9.49	21.64	9.50	20.75	8.83	10.54	5.42	-2.00	-3.39	-4.80	-5.55
	2	26.50	11.44	26.52	11.46	27.40	14.12	15.01	11.31	0.57	0.00	-1.84	-2.17
	3	24.26	10.14	23.23	10.16	25.11	14.74	14.12	11.86	1.22	0.87	-0.96	-1.11
	4	18.32	5.79	17.29	5.80	19.17	11.54	11.35	9.93	1.25	0.97	-0.49	-0.59
	5	12.98	2.59	12.74	2.60	14.40	8.76	9.91	8.75	1.25	0.98	-0.24	-0.33
	6	8.47	-0.25	8.30	-0.25	11.00	6.44	8.53	7.78	1.14	0.93	-0.23	-0.29
	7	5.90	-1.67	5.73	-1.68	8.82	5.13	7.73	7.19	1.12	0.91	-0.13	-0.16
	8	3.72	-2.49	3.86	-2.52	7.21	4.05	6.98	6.62	1.24	1.02	-0.02	-0.08
	9	3.24	-2.35	3.51	-2.38	6.54	3.79	6.96	6.61	1.31	1.08	0.13	0.04
	10	2.64	-2.28	3.25	-2.31	6.23	3.70	6.92	6.69	1.44	1.26	0.25	0.16
	25	3.49	1.72	3.19	1.71	4.40	3.23	5.10	4.63	0.90	0.70	0.29	0.20
	50	1.03	0.51	1.15	0.49	1.97	1.24	2.60	2.35	0.74	0.40	0.12	0.11
	100	1.21	0.91	1.18	0.90	1.82	1.44	1.99	1.94	0.67	0.46	0.28	0.27
		T = 200											
	s	∞		10e4		10		1		0.1		10e-6	
	m	1	5	1	5	1	5	1	5	1	5	1	5
h	1	26.98	13.14	26.86	13.09	22.62	11.53	11.16	6.27	-0.12	-1.01	-2.94	-3.00
	2	34.40	16.80	33.72	16.79	28.84	16.72	15.37	12.02	1.16	0.84	-1.26	-1.21
	3	31.64	15.08	30.42	15.08	28.13	18.19	14.56	12.52	1.74	1.54	-0.80	-0.74
	4	25.66	11.95	24.65	11.95	24.03	16.27	12.68	11.42	1.65	1.55	-0.53	-0.49
	5	19.51	8.62	18.95	8.63	19.64	13.76	11.23	10.32	1.70	1.59	-0.35	-0.33
	6	14.89	5.91	14.82	5.92	15.38	10.75	9.92	9.23	1.62	1.58	-0.23	-0.21
	7	11.99	4.37	12.00	4.38	13.03	9.48	9.24	8.75	1.58	1.55	-0.11	-0.11
	8	10.09	3.59	10.24	3.60	11.16	8.29	8.66	8.30	1.53	1.53	-0.02	-0.03
	9	9.21	3.37	9.34	3.39	10.50	7.98	8.53	8.25	1.53	1.54	0.04	0.03
	10	7.41	2.45	7.67	2.47	9.00	6.90	7.89	7.65	1.56	1.53	0.07	0.04
	25	1.99	0.68	1.73	0.68	2.35	1.86	3.82	3.62	1.02	1.03	0.21	0.20
	50	0.36	0.37	0.50	0.37	0.84	0.67	1.34	1.37	0.53	0.51	0.10	0.10
	100	0.37	0.19	0.22	0.19	0.44	0.36	0.61	0.57	0.20	0.18	0.04	0.04
		T = 1000											
	s	∞		10e4		10		1		0.1		10e-6	
	m	1	5	1	5	1	5	1	5	1	5	1	5
h	1	27.58	17.59	29.75	17.59	25.32	15.58	15.84	10.76	3.24	2.39	-0.45	-0.53
	2	34.65	22.25	38.03	22.27	32.59	22.36	19.49	16.47	3.42	2.90	-0.17	-0.17
	3	31.59	19.88	34.52	19.88	30.95	22.10	17.51	15.78	2.75	2.48	-0.10	-0.09
	4	26.07	16.44	28.68	16.45	26.08	19.14	15.03	13.90	2.29	2.13	-0.09	-0.08
	5	21.32	13.28	23.06	13.30	21.78	16.21	12.68	11.86	1.98	1.88	-0.07	-0.06
	6	18.23	11.32	19.26	11.34	19.15	14.27	11.17	10.46	1.84	1.75	-0.04	-0.03
	7	15.07	9.11	15.77	9.12	16.19	12.15	10.16	9.65	1.74	1.66	-0.02	-0.01
	8	12.63	7.17	12.83	7.18	13.71	10.46	9.47	9.03	1.62	1.55	0.01	0.01
	9	10.84	6.23	11.32	6.23	12.32	9.66	9.01	8.64	1.47	1.46	0.03	0.03
	10	9.80	5.80	10.25	5.80	11.46	9.14	8.84	8.46	1.46	1.46	0.04	0.03
	25	1.29	0.50	1.37	0.50	1.70	1.29	2.45	2.47	0.66	0.64	0.02	0.02
	50	0.07	0.07	0.10	0.07	0.16	0.11	0.27	0.21	0.09	0.10	0.00	0.00
	100	0.04	0.03	0.06	0.03	0.01	0.03	0.02	0.03	0.01	0.01	0.00	0.00

Table 3.13: Gains from equal VAR pooling relative to large VAR (*Model II*)

		MODEL III											
		T = 25											
	s	∞		10e4		10		1		0.1		10e-6	
	m	1	5	1	5	1	5	1	5	1	5	1	5
h	1	-40.91	-45.71	-15.68	-22.92	-15.65	-21.76	-13.05	-18.51	-18.37	-21.55	-18.09	-21.32
	2	-39.62	-44.63	-11.89	-19.10	-9.63	-16.35	-7.54	-11.32	-10.65	-12.32	-10.32	-11.82
	3	-38.59	-43.18	-10.26	-17.18	-8.56	-14.11	-5.98	-9.17	-7.21	-8.41	-6.50	-7.57
	4	-35.96	-40.20	-11.59	-17.50	-8.27	-13.04	-4.71	-7.55	-5.10	-5.84	-4.87	-5.78
	5	-33.61	-37.39	-11.54	-16.95	-7.67	-11.95	-3.23	-5.57	-4.85	-5.08	-3.79	-4.52
	6	-28.45	-32.36	-11.63	-16.33	-8.00	-11.33	-3.16	-5.11	-4.27	-4.20	-3.69	-4.38
	7	-28.51	-32.08	-10.09	-14.49	-6.52	-9.76	-2.92	-4.79	-4.17	-4.12	-3.33	-3.90
	8	-29.36	-32.73	-9.30	-13.14	-6.13	-8.79	-2.75	-4.14	-3.78	-3.80	-2.96	-3.55
	9	-30.47	-33.27	-7.61	-11.23	-5.06	-7.67	-2.37	-3.70	-3.39	-3.61	-2.80	-3.37
	10	-30.37	-32.87	-7.29	-10.38	-4.72	-6.95	-2.04	-3.24	-3.49	-3.77	-3.18	-3.71
	25	-44.82	-46.39	-4.06	-6.10	-4.56	-5.78	-2.65	-3.33	-3.04	-3.28	-3.62	-4.07
	50	-60.03	-61.48	-7.02	-8.73	-6.00	-7.30	-2.78	-3.32	-5.06	-4.84	-5.34	-5.86
	100	-65.70	-67.00	-6.36	-8.68	-6.30	-7.10	-3.24	-3.26	-5.01	-5.87	-6.29	-7.02
		T = 50											
	s	∞		10e4		10		1		0.1		10e-6	
	m	1	5	1	5	1	5	1	5	1	5	1	5
h	1	-36.20	-39.91	-1.45	-7.28	-0.83	-6.67	-3.36	-7.39	-7.67	-9.40	-10.09	-11.61
	2	-33.65	-38.10	0.42	-6.16	1.17	-3.99	1.50	-1.49	-3.30	-4.18	-5.29	-5.63
	3	-32.53	-36.48	-0.01	-5.79	1.35	-3.17	2.16	0.11	-1.83	-2.27	-3.52	-3.78
	4	-31.61	-34.78	-1.94	-7.29	-0.58	-4.10	1.44	-0.15	-1.26	-1.49	-2.40	-2.56
	5	-30.25	-33.12	-2.89	-7.98	-1.06	-4.02	1.75	0.43	-0.89	-0.99	-1.77	-1.99
	6	-29.93	-32.75	-4.28	-8.63	-1.69	-4.07	1.72	0.48	-0.86	-0.88	-1.38	-1.51
	7	-28.96	-31.40	-4.55	-8.66	-1.75	-3.97	1.56	0.52	-0.53	-0.58	-1.05	-1.18
	8	-27.31	-29.57	-3.72	-7.50	-0.87	-2.77	1.52	0.47	-0.37	-0.44	-0.94	-1.00
	9	-29.11	-31.26	-3.23	-6.68	-0.72	-2.35	1.50	0.47	-0.35	-0.42	-0.89	-1.03
	10	-27.76	-29.75	-3.17	-6.11	-0.94	-2.24	1.15	0.14	-0.24	-0.40	-0.90	-1.06
	25	-31.98	-32.44	-0.25	-1.38	-0.17	-0.83	0.69	0.12	-0.09	-0.23	-0.86	-0.97
	50	-42.29	-42.57	-0.59	-1.21	-0.39	-0.88	0.57	0.05	0.01	-0.28	-0.55	-0.66
	100	-53.76	-54.05	0.44	-0.60	0.18	-0.18	0.30	-0.07	-0.52	-0.62	-0.57	-0.88
		T = 100											
	s	∞		10e4		10		1		0.1		10e-6	
	m	1	5	1	5	1	5	1	5	1	5	1	5
h	1	-33.23	-36.44	4.80	0.26	5.26	0.52	1.17	-1.87	-2.52	-3.86	-4.16	-5.30
	2	-34.72	-38.33	7.60	1.64	7.18	2.24	3.74	1.43	-0.29	-0.88	-1.53	-2.01
	3	-29.95	-33.39	6.22	0.70	5.78	1.84	3.34	1.85	0.03	-0.17	-0.84	-1.08
	4	-27.30	-30.33	4.30	-0.29	4.33	1.15	2.53	1.53	0.14	0.07	-0.50	-0.66
	5	-25.26	-27.96	3.35	-0.94	3.69	0.75	2.18	1.50	0.43	0.37	-0.13	-0.32
	6	-21.70	-24.07	2.07	-1.64	2.34	0.01	1.49	1.05	0.34	0.23	-0.01	-0.16
	7	-20.94	-22.63	1.20	-1.54	2.11	0.20	1.39	1.13	0.48	0.32	0.09	-0.02
	8	-17.72	-19.37	0.33	-2.06	0.95	-0.44	1.16	0.83	0.46	0.31	0.01	-0.08
	9	-18.51	-20.11	-0.18	-2.19	0.62	-0.52	1.05	0.75	0.45	0.32	0.00	-0.09
	10	-18.70	-20.10	-0.30	-2.12	0.57	-0.37	1.04	0.78	0.41	0.30	0.00	-0.09
	25	-48.36	-48.63	0.32	-0.03	0.45	0.18	0.30	0.23	0.06	0.10	0.06	0.03
	50	-72.55	-72.62	0.16	0.06	0.13	0.14	0.27	0.23	0.05	0.02	0.03	0.04
	100	-74.15	-74.17	-0.06	-0.03	0.12	0.12	0.20	0.17	0.08	0.09	0.01	0.01
		T = 200											
	s	∞		10e4		10		1		0.1		10e-6	
	m	1	5	1	5	1	5	1	5	1	5	1	5
h	1	-22.14	-24.76	7.14	3.08	7.18	2.97	4.34	2.44	-1.07	-1.84	-1.93	-2.37
	2	-27.64	-30.39	10.77	5.00	9.82	5.79	5.31	4.46	0.19	-0.21	-0.49	-0.81
	3	-25.71	-28.44	9.58	4.62	9.14	5.45	5.28	4.66	0.90	0.67	-0.37	-0.51
	4	-26.03	-28.52	8.75	4.43	8.08	5.16	4.43	4.09	0.68	0.60	-0.23	-0.35
	5	-25.05	-27.42	7.07	3.12	6.14	3.93	3.37	3.16	0.51	0.46	-0.05	-0.13
	6	-24.68	-27.03	4.87	1.30	4.24	2.41	2.63	2.43	0.31	0.26	-0.03	-0.08
	7	-24.36	-26.51	3.28	0.14	2.83	1.42	1.97	1.85	0.30	0.24	-0.05	-0.08
	8	-23.69	-25.30	2.31	-0.43	2.05	0.92	1.62	1.47	0.25	0.20	-0.11	-0.12
	9	-19.23	-20.77	2.56	-0.18	1.97	1.08	1.32	1.22	0.27	0.24	-0.09	-0.09
	10	-21.06	-22.56	1.36	-0.80	1.26	0.52	1.11	1.05	0.21	0.17	-0.07	-0.07
	25	-18.33	-18.43	0.34	-0.02	0.05	0.02	0.12	0.10	0.00	0.01	0.04	0.05
	50	-18.93	-18.98	-0.06	-0.05	0.03	0.02	0.02	0.03	0.00	-0.01	-0.01	-0.01
	100	-25.64	-25.66	0.03	0.01	0.06	0.03	0.07	0.05	0.01	0.01	0.00	0.00
		T = 1000											
	s	∞		10e4		10		1		0.1		10e-6	
	m	1	5	1	5	1	5	1	5	1	5	1	5
h	1	-6.39	-9.97	10.11	6.38	9.68	6.10	6.06	3.97	1.05	0.51	-0.25	-0.48
	2	-10.06	-14.60	13.72	8.44	12.48	8.24	7.57	5.95	1.53	1.29	-0.13	-0.23
	3	-12.12	-16.45	11.99	6.86	11.25	7.29	6.47	5.49	1.04	0.95	-0.05	-0.11
	4	-11.89	-15.78	9.46	5.57	9.63	6.18	5.17	4.57	0.83	0.81	-0.10	-0.13
	5	-10.64	-13.86	7.71	4.55	7.74	5.04	4.22	3.78	0.79	0.77	-0.07	-0.08
	6	-12.22	-15.11	5.72	3.23	5.79	3.79	3.45	3.17	0.73	0.69	-0.05	-0.06
	7	-12.37	-15.22	4.59	2.18	3.98	2.66	2.59	2.52	0.64	0.63	-0.02	-0.03
	8	-10.41	-12.71	3.35	1.28	2.96	1.76	1.93	1.92	0.49	0.47	-0.03	-0.03
	9	-13.40	-15.37	2.81	1.04	2.39	1.35	1.61	1.57	0.40	0.39	-0.03	-0.03
	10	-13.10	-14.87	2.29	0.67	2.01	1.11	1.41	1.35	0.32	0.32	-0.01	-0.02
	25	-7.46	-7.53	0.17	0.08	0.10	0.07	0.08	0.09	0.02	0.02	0.00	0.00
	50	-0.50	-0.51	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	0.00	0.00	0.00	0.00
	100	-6.62	-6.62	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00

Table 3.14: Gains from equal VAR pooling relative to large VAR (*Model III*)

3.B.5 Non–Random Benchmark Models

		MODEL I											
		T = 25											
	S	∞		10e4		10		1		0.1		10e-6	
	m	1	5	1	5	1	5	1	5	1	5	1	5
h	1	11.22	-7.25	12.22	-6.79	7.72	-8.84	-3.20	-9.95	-6.28	-10.37	-6.34	-9.90
	2	12.97	-8.32	13.77	-7.65	9.25	-5.81	-0.05	-4.11	-1.53	-3.45	-2.56	-4.06
	3	8.26	-10.87	8.38	-10.23	6.89	-5.81	0.75	-1.76	-0.72	-1.91	-1.74	-2.48
	4	6.04	-10.56	6.51	-10.09	4.99	-5.60	0.75	-0.83	-0.35	-0.81	-1.24	-1.81
	5	4.85	-10.43	4.98	-10.30	3.70	-4.87	0.96	-0.35	-0.08	-0.08	-0.83	-1.36
	6	3.24	-10.82	3.23	-10.76	2.32	-4.94	1.17	0.11	0.09	0.07	-0.45	-1.05
	7	2.76	-10.21	2.62	-10.05	2.05	-4.56	1.54	0.42	0.37	0.59	-0.12	-0.76
	8	2.56	-9.27	2.82	-9.20	1.63	-4.28	1.62	0.53	0.58	0.79	0.04	-0.81
	9	2.66	-8.31	3.34	-8.33	1.45	-3.79	1.77	0.78	0.74	0.85	0.10	-1.05
	10	3.22	-6.94	4.55	-6.87	1.86	-3.15	1.74	0.73	0.68	0.67	0.42	-0.96
	25	7.48	-2.56	6.97	-2.62	6.39	2.92	3.78	2.48	0.86	1.44	2.63	1.38
	50	9.67	-0.90	9.64	-1.17	7.72	2.29	4.55	2.55	1.48	2.32	2.55	0.77
	100	7.88	-3.95	8.15	-4.26	8.39	2.88	7.35	4.71	2.43	2.42	4.09	1.71
		T = 50											
	S	∞		10e4		10		1		0.1		10e-6	
	m	1	5	1	5	1	5	1	5	1	5	1	5
h	1	13.12	-1.35	14.47	-0.99	7.82	-3.96	-0.27	-6.21	-2.20	-4.60	-3.61	-5.13
	2	16.66	-1.02	16.65	-0.69	12.87	0.44	2.75	-0.87	-0.22	-1.20	-1.12	-1.78
	3	13.82	-2.22	15.16	-2.07	11.94	1.61	2.82	0.90	0.17	-0.41	-0.62	-1.02
	4	10.16	-3.85	11.47	-3.71	10.28	1.98	2.74	1.43	0.27	-0.16	-0.29	-0.59
	5	8.64	-4.61	9.85	-4.45	8.14	1.33	2.91	1.77	0.51	0.03	-0.21	-0.37
	6	6.89	-5.39	7.96	-5.21	6.04	0.42	3.05	2.12	0.52	0.05	-0.10	-0.24
	7	6.76	-4.97	7.62	-4.83	5.60	0.69	3.30	2.63	0.64	0.31	0.04	-0.17
	8	7.56	-3.93	7.93	-3.96	5.50	1.04	3.51	2.86	0.73	0.25	0.03	-0.09
	9	7.93	-3.43	7.53	-3.53	5.55	1.21	3.75	3.02	0.82	0.50	0.15	0.11
	10	8.61	-2.49	8.15	-2.54	5.39	1.55	3.80	3.12	0.88	0.73	0.27	0.21
	25	9.95	2.20	9.38	2.07	4.80	2.68	4.61	4.04	1.29	1.27	0.31	0.17
	50	6.71	1.74	6.36	1.78	3.79	2.30	3.37	3.22	1.55	1.80	0.18	0.05
	100	6.71	1.90	7.03	1.99	3.08	1.97	2.85	3.24	1.40	1.42	0.25	0.11
		T = 100											
	S	∞		10e4		10		1		0.1		10e-6	
	m	1	5	1	5	1	5	1	5	1	5	1	5
h	1	15.70	2.32	18.52	2.23	12.65	1.30	4.33	-1.13	-0.39	-2.30	-1.73	-2.35
	2	19.46	2.62	19.44	2.63	15.92	4.26	6.56	3.06	0.65	-0.13	-0.56	-0.95
	3	17.58	2.45	18.11	2.61	15.39	6.03	5.74	4.02	0.91	0.37	-0.30	-0.53
	4	15.05	1.85	15.19	1.97	12.72	5.51	5.18	3.97	0.79	0.28	-0.18	-0.35
	5	12.18	0.35	12.27	0.56	10.79	4.83	4.73	3.67	0.77	0.37	-0.09	-0.20
	6	9.69	-0.93	9.97	-0.83	8.74	3.82	4.33	3.43	0.72	0.47	-0.05	-0.07
	7	8.64	-1.26	8.94	-1.21	7.99	3.57	4.08	3.34	0.77	0.59	-0.03	0.01
	8	7.80	-1.47	8.51	-1.47	7.27	3.18	3.92	3.21	0.72	0.62	-0.06	0.01
	9	7.64	-1.53	8.18	-1.40	6.83	3.06	3.79	3.10	0.70	0.71	-0.06	0.05
	10	7.82	-1.38	8.15	-1.36	6.69	3.09	3.71	3.02	0.78	0.81	-0.09	0.07
	25	7.72	1.31	7.30	1.33	4.02	2.28	3.09	2.54	0.71	0.66	0.15	0.28
	50	3.21	0.87	2.87	0.83	1.66	1.04	1.53	1.25	0.54	0.47	0.16	0.22
	100	2.14	0.62	2.20	0.57	1.51	1.08	1.14	0.85	0.33	0.33	0.21	0.15
		T = 200											
	S	∞		10e4		10		1		0.1		10e-6	
	m	1	5	1	5	1	5	1	5	1	5	1	5
h	1	15.21	3.00	15.45	3.07	12.60	1.77	3.49	-1.35	0.45	-0.72	-1.07	-1.55
	2	18.65	3.30	17.86	3.47	16.88	5.02	5.66	2.67	0.76	0.38	-0.34	-0.61
	3	16.61	2.75	15.96	2.73	15.36	6.27	5.72	3.82	0.62	0.53	-0.20	-0.31
	4	13.18	1.06	12.15	0.98	13.10	6.10	5.00	3.75	0.50	0.49	-0.10	-0.21
	5	10.28	0.13	9.89	0.14	10.97	5.57	4.53	3.60	0.63	0.57	-0.03	-0.12
	6	9.09	-0.17	8.19	-0.12	9.12	4.83	4.05	3.33	0.66	0.63	-0.01	-0.08
	7	8.51	-0.27	7.38	-0.17	8.34	4.72	3.91	3.34	0.73	0.66	0.02	-0.03
	8	8.32	-0.10	6.61	-0.07	8.00	4.71	3.61	3.23	0.73	0.64	0.06	0.02
	9	8.78	0.68	7.19	0.78	7.78	4.75	3.49	3.14	0.73	0.66	0.09	0.06
	10	9.10	1.16	7.49	1.25	7.40	4.63	3.48	3.15	0.73	0.68	0.06	0.06
	25	6.27	1.52	6.64	1.51	2.86	1.77	2.18	2.02	0.65	0.61	0.09	0.08
	50	1.83	0.36	1.53	0.29	0.76	0.60	0.88	0.83	0.30	0.31	0.04	0.06
	100	0.71	0.15	0.53	0.28	0.31	0.25	0.30	0.29	0.12	0.12	0.01	0.02
		T = 1000											
	S	∞		10e4		10		1		0.1		10e-6	
	m	1	5	1	5	1	5	1	5	1	5	1	5
h	1	16.31	5.49	18.01	5.61	12.48	4.18	5.42	1.21	0.62	-0.12	-0.27	-0.32
	2	17.63	4.78	19.77	4.80	17.73	8.57	7.41	4.74	0.98	0.57	-0.06	-0.10
	3	15.04	3.35	16.20	3.36	17.34	9.89	7.05	5.46	0.80	0.57	0.00	-0.01
	4	12.06	2.14	12.41	2.14	14.80	9.18	6.02	5.16	0.71	0.55	0.00	0.01
	5	9.91	1.96	10.35	1.99	12.89	8.53	5.17	4.65	0.64	0.51	0.00	0.01
	6	8.99	2.25	9.49	2.28	11.87	8.14	4.61	4.23	0.62	0.50	0.00	0.01
	7	8.82	2.42	8.68	2.45	10.56	7.43	4.28	3.95	0.60	0.49	0.01	0.03
	8	8.07	2.38	7.92	2.39	9.41	6.68	3.90	3.61	0.56	0.45	0.03	0.03
	9	8.16	2.68	7.89	2.67	8.79	6.28	3.63	3.48	0.54	0.46	0.04	0.04
	10	8.20	2.95	7.76	2.93	8.22	6.02	3.49	3.37	0.56	0.48	0.05	0.04
	25	3.31	0.89	4.08	0.86	1.83	1.09	1.30	1.34	0.22	0.24	0.02	0.01
	50	1.22	0.29	1.04	0.25	0.04	-0.01	0.07	0.06	0.02	0.01	0.00	0.00
	100	0.05	-0.01	0.11	-0.09	0.04	0.03	0.03	0.03	0.00	0.01	0.00	0.00

Table 3.15: Gains from equal VAR pooling relative to R^2 selected VAR (*Model I*)

		Model II											
		T = 25											
	S	∞		10e4		10		1		0.1		10e-6	
	m	1	5	1	5	1	5	1	5	1	5	1	5
h	1	9.89	-7.34	9.56	-7.48	5.04	-9.45	-2.10	-11.77	-6.65	-9.92	-6.60	-10.20
	2	9.15	-8.22	10.02	-8.35	5.57	-7.83	0.42	-5.66	-2.45	-3.83	-2.82	-4.08
	3	5.98	-9.02	6.44	-9.06	5.06	-6.29	0.49	-3.58	-1.55	-2.35	-1.47	-2.31
	4	3.61	-9.49	3.38	-9.63	3.02	-6.18	-0.10	-2.83	-0.79	-1.39	-0.72	-1.26
	5	2.13	-9.38	1.42	-9.48	1.98	-5.91	-0.18	-2.26	-0.58	-1.06	-0.73	-1.09
	6	0.57	-9.01	0.40	-9.12	1.38	-5.33	-0.07	-1.78	-0.27	-0.59	-0.55	-0.75
	7	0.85	-7.93	0.33	-8.03	1.45	-4.51	0.20	-1.21	0.03	-0.29	-0.54	-0.58
	8	1.39	-6.63	0.79	-6.65	1.41	-3.73	0.49	-0.75	0.36	0.05	-0.47	-0.37
	9	2.02	-5.11	1.39	-5.09	1.65	-2.71	0.90	-0.47	0.70	0.25	-0.44	-0.17
	10	2.90	-3.76	2.27	-3.72	2.08	-2.10	1.18	-0.12	0.64	0.18	-0.64	-0.23
	25	4.85	1.51	5.52	1.65	5.45	2.24	2.46	1.53	1.94	1.07	0.34	-0.36
	50	4.92	1.70	5.78	1.85	7.04	3.26	3.77	2.01	2.14	1.38	1.02	0.35
	100	6.75	2.86	7.31	3.14	8.22	3.16	3.27	1.89	2.15	1.27	0.23	-0.72
		T = 50											
	S	∞		10e4		10		1		0.1		10e-6	
	m	1	5	1	5	1	5	1	5	1	5	1	5
h	1	12.80	-0.92	11.59	-1.03	9.32	-2.73	0.81	-4.93	-1.80	-4.28	-3.45	-5.09
	2	15.15	-0.23	14.30	-0.25	11.32	-0.48	3.39	0.08	-0.02	-0.76	-1.42	-2.07
	3	13.59	-0.52	12.97	-0.42	11.39	1.51	3.46	1.43	0.16	-0.02	-0.92	-1.15
	4	9.94	-2.06	9.63	-2.00	8.97	1.13	3.05	1.42	0.35	0.19	-0.44	-0.72
	5	7.32	-3.09	7.57	-3.08	7.67	1.00	2.65	1.34	0.59	0.23	0.02	-0.36
	6	5.80	-3.22	6.40	-3.27	7.22	1.22	2.26	1.15	0.77	0.33	0.19	-0.10
	7	5.25	-3.04	5.90	-3.15	7.07	1.62	2.14	1.25	0.86	0.62	0.21	0.11
	8	5.02	-2.36	5.66	-2.54	7.07	1.80	2.09	1.31	1.19	0.87	0.28	0.20
	9	4.99	-1.84	5.58	-2.05	6.97	1.82	2.22	1.43	1.33	1.09	0.51	0.33
	10	5.07	-1.34	5.81	-1.55	6.90	1.81	2.19	1.35	1.33	1.17	0.74	0.46
	25	5.51	2.24	5.61	2.28	5.49	2.40	3.01	2.58	1.35	1.24	0.95	0.19
	50	4.74	2.28	4.20	2.32	4.71	2.60	3.04	2.57	1.60	1.08	0.74	0.78
	100	3.88	1.65	3.47	1.72	4.49	2.13	2.39	2.03	1.31	1.28	0.61	0.36
		T = 100											
	S	∞		10e4		10		1		0.1		10e-6	
	m	1	5	1	5	1	5	1	5	1	5	1	5
h	1	13.12	-0.81	10.46	-0.92	10.24	-0.62	5.41	-0.48	-0.64	-2.08	-2.17	-3.00
	2	15.47	-0.12	12.83	-0.13	13.40	2.52	5.64	2.15	0.68	0.16	-0.90	-1.01
	3	14.86	0.56	12.31	0.62	12.96	4.39	5.71	3.41	0.72	0.54	-0.55	-0.58
	4	13.10	0.46	11.02	0.62	11.06	4.31	4.82	3.23	0.69	0.62	-0.52	-0.42
	5	10.97	-0.14	8.90	-0.01	9.37	3.73	4.30	2.98	0.58	0.57	-0.32	-0.29
	6	9.00	-0.57	7.44	-0.52	7.75	3.04	4.00	2.92	0.55	0.54	-0.14	-0.20
	7	8.22	-0.26	6.64	-0.21	7.03	2.95	3.73	2.81	0.53	0.54	-0.06	-0.18
	8	7.72	0.31	6.14	0.34	6.81	3.19	3.56	2.75	0.52	0.51	-0.03	-0.15
	9	7.68	0.91	6.30	0.94	6.92	3.42	3.53	2.67	0.57	0.58	0.06	-0.08
	10	7.53	1.40	6.57	1.40	7.04	3.65	3.47	2.64	0.56	0.59	0.11	0.00
	25	3.17	1.74	3.18	1.74	3.29	2.30	2.28	1.96	0.53	0.58	0.22	0.17
	50	1.58	0.90	1.48	0.90	1.92	1.24	1.60	1.26	0.65	0.55	0.13	0.02
	100	1.25	0.73	1.11	0.75	1.70	0.95	1.04	0.92	0.47	0.41	0.07	0.10
		T = 200											
	S	∞		10e4		10		1		0.1		10e-6	
	m	1	5	1	5	1	5	1	5	1	5	1	5
h	1	14.11	2.56	15.87	2.60	12.11	2.09	5.78	0.49	-0.33	-1.58	-0.64	-1.13
	2	19.63	4.29	20.61	4.28	16.97	6.23	7.01	3.92	0.64	0.18	-0.13	-0.35
	3	17.89	3.78	18.40	3.69	16.04	7.03	6.28	4.59	0.68	0.37	0.01	-0.17
	4	14.75	2.53	15.07	2.50	13.57	7.03	5.05	4.22	0.74	0.56	0.06	-0.04
	5	12.71	2.33	13.16	2.36	11.76	6.43	4.43	3.93	0.75	0.60	0.08	-0.01
	6	10.72	1.71	10.54	1.76	9.94	5.64	3.90	3.45	0.73	0.60	0.02	-0.04
	7	9.50	1.57	9.23	1.59	9.09	5.46	3.69	3.31	0.77	0.64	-0.01	-0.04
	8	8.76	1.75	8.19	1.72	8.12	5.15	3.35	3.10	0.78	0.66	0.00	-0.01
	9	8.77	2.04	8.00	1.98	7.58	5.10	3.16	3.04	0.80	0.68	0.02	0.01
	10	7.89	1.77	6.97	1.70	6.81	4.59	2.91	2.83	0.75	0.66	0.00	-0.02
	25	1.67	0.72	2.27	0.74	1.85	1.36	1.52	1.51	0.48	0.48	0.10	0.07
	50	0.47	0.34	0.51	0.34	0.76	0.50	0.66	0.57	0.21	0.21	0.04	0.04
	100	0.19	0.18	0.21	0.18	0.30	0.19	0.26	0.21	0.10	0.10	0.03	0.02
		T = 1000											
	S	∞		10e4		10		1		0.1		10e-6	
	m	1	5	1	5	1	5	1	5	1	5	1	5
h	1	16.86	4.84	15.34	4.86	13.11	3.89	6.30	2.70	1.58	0.87	-0.20	-0.25
	2	22.48	7.98	20.62	8.00	17.75	8.76	7.41	5.55	1.36	1.03	-0.03	-0.11
	3	21.41	8.24	20.42	8.30	17.12	9.92	6.73	5.52	1.15	0.99	-0.03	-0.07
	4	18.41	7.59	17.62	7.66	14.94	9.29	5.66	4.86	1.10	1.01	-0.02	-0.07
	5	16.02	6.79	14.67	6.85	12.54	8.27	4.95	4.34	0.93	0.87	-0.01	-0.05
	6	13.51	5.89	12.44	5.97	10.50	7.12	4.40	3.83	0.85	0.81	0.01	-0.03
	7	11.28	4.84	10.52	4.95	9.00	6.10	3.94	3.47	0.76	0.73	0.01	-0.02
	8	9.80	4.07	9.08	4.18	7.71	5.25	3.63	3.26	0.73	0.68	0.02	-0.02
	9	8.96	3.67	8.21	3.77	7.14	4.86	3.39	3.07	0.72	0.64	0.03	0.00
	10	8.30	3.62	7.79	3.70	6.90	4.75	3.26	3.02	0.67	0.58	0.03	0.00
	25	1.22	0.43	1.16	0.44	1.43	0.92	1.16	1.03	0.24	0.21	0.01	0.01
	50	0.02	0.06	0.06	0.06	0.10	0.09	0.12	0.12	0.02	0.02	0.00	0.00
	100	0.03	0.02	0.02	0.02	0.03	0.02	0.01	0.02	0.00	0.00	0.00	0.00

Table 3.16: Gains from equal VAR pooling relative to R^2 selected VAR (*Model II*)

		Model III											
		T = 25											
	S	∞		10e4		10		1		0.1		10e-6	
	m	1	5	1	5	1	5	1	5	1	5	1	5
h	1	-1.60	-9.98	-2.23	-10.02	-4.79	-10.98	-7.25	-11.14	-6.65	-10.68	-6.91	-10.94
	2	0.93	-8.60	0.32	-8.73	0.11	-6.84	-2.12	-5.52	-2.40	-4.46	-2.49	-4.50
	3	1.17	-7.19	0.61	-7.34	0.52	-5.96	-1.04	-3.63	-1.17	-2.51	-1.40	-2.96
	4	1.02	-6.28	0.36	-6.31	0.48	-5.19	-0.19	-2.48	-0.73	-1.66	-1.09	-2.40
	5	1.41	-4.99	0.64	-4.94	0.48	-4.30	0.02	-1.69	-0.56	-1.44	-0.82	-1.99
	6	0.22	-5.03	0.27	-4.90	0.39	-3.89	0.42	-1.15	-0.21	-0.91	-0.76	-1.86
	7	-0.36	-4.80	0.13	-4.69	0.18	-3.52	0.25	-0.92	0.06	-0.86	-0.51	-1.50
	8	-0.36	-4.23	-0.05	-4.11	0.62	-2.91	-0.02	-1.08	0.32	-0.78	-0.66	-1.76
	9	0.64	-3.20	0.68	-3.05	0.94	-2.30	0.20	-0.83	0.25	-0.76	-0.58	-1.66
	10	0.54	-2.88	0.73	-2.73	1.09	-2.09	0.28	-0.86	0.14	-1.11	-0.49	-1.84
	25	0.49	-1.93	0.45	-1.76	1.04	-1.31	0.09	-0.99	-0.06	-1.13	-0.53	-1.53
	50	2.04	-1.04	2.25	-0.78	1.26	-1.10	-0.41	-1.05	0.04	-1.29	-1.04	-1.87
	100	2.10	-1.25	1.62	-1.11	0.59	-1.98	-0.59	-2.42	-0.75	-1.13	-0.35	-1.99
		T = 50											
	S	∞		10e4		10		1		0.1		10e-6	
	m	1	5	1	5	1	5	1	5	1	5	1	5
h	1	0.43	-5.35	0.33	-5.40	-0.58	-5.70	-0.49	-4.30	-3.36	-5.25	-2.77	-4.37
	2	1.10	-5.55	1.34	-5.50	0.18	-5.02	0.15	-2.28	-1.37	-2.14	-1.04	-1.69
	3	1.27	-4.33	1.95	-4.28	1.01	-3.63	0.81	-0.96	-0.63	-0.99	-0.30	-0.79
	4	1.23	-4.04	0.65	-4.10	1.03	-2.74	0.87	-0.39	-0.26	-0.52	-0.23	-0.55
	5	0.60	-4.51	-0.49	-4.55	0.16	-2.59	0.82	-0.03	-0.15	-0.38	-0.33	-0.59
	6	0.04	-4.44	-0.72	-4.48	0.32	-2.22	0.37	-0.07	0.05	-0.21	-0.28	-0.51
	7	-0.33	-4.24	-1.07	-4.24	0.51	-1.74	-0.02	-0.36	0.04	-0.16	-0.22	-0.45
	8	-0.15	-3.63	-0.66	-3.60	0.86	-1.31	0.03	-0.29	-0.14	-0.23	-0.22	-0.37
	9	-0.44	-3.26	-0.39	-3.26	0.80	-1.01	0.12	-0.15	-0.21	-0.24	-0.29	-0.38
	10	-0.61	-3.16	-0.46	-3.17	0.49	-1.16	0.27	0.03	-0.35	-0.27	-0.34	-0.39
	25	0.36	-0.56	0.22	-0.48	0.58	-0.25	-0.05	-0.27	0.05	0.07	-0.12	-0.19
	50	-0.03	-0.94	-0.13	-0.96	0.02	-0.62	0.06	-0.26	-0.03	0.19	0.01	0.06
	100	0.67	0.14	0.84	0.25	0.86	0.32	-0.07	-0.67	-0.11	0.49	0.10	-0.04
		T = 100											
	S	∞		10e4		10		1		0.1		10e-6	
	m	1	5	1	5	1	5	1	5	1	5	1	5
h	1	3.01	-1.23	3.00	-1.17	1.63	-2.45	-0.47	-3.21	-1.56	-2.43	-1.65	-2.81
	2	3.14	-1.84	3.47	-1.84	2.77	-2.40	0.56	-1.43	-0.20	-0.71	-1.03	-1.38
	3	2.35	-2.65	2.86	-2.68	2.01	-2.37	0.77	-0.66	0.31	-0.16	-0.60	-0.77
	4	0.89	-3.46	1.63	-3.48	1.38	-2.15	0.86	-0.10	0.16	-0.17	-0.33	-0.42
	5	1.10	-2.87	1.87	-2.90	1.40	-1.48	0.77	0.06	0.24	-0.03	-0.21	-0.25
	6	0.60	-2.88	1.48	-2.88	0.99	-1.24	0.57	-0.02	0.34	0.12	-0.11	-0.14
	7	0.71	-2.20	1.86	-2.14	1.09	-0.61	0.72	0.30	0.34	0.12	-0.02	-0.01
	8	0.94	-1.71	2.02	-1.65	0.80	-0.63	0.60	0.26	0.21	0.01	0.03	0.06
	9	1.10	-1.25	2.18	-1.20	0.62	-0.35	0.44	0.18	0.13	-0.03	0.05	0.04
	10	1.16	-0.87	2.29	-0.80	0.80	-0.13	0.41	0.17	0.13	0.00	0.08	0.10
	25	0.24	0.06	0.60	0.05	0.34	0.07	0.14	0.07	0.01	0.00	0.05	0.06
	50	0.34	0.13	0.27	0.12	0.12	0.03	0.07	0.03	0.02	0.02	-0.01	-0.02
	100	0.21	0.11	0.26	0.12	0.19	0.07	0.13	0.08	0.05	0.02	0.00	0.01
		T = 200											
	S	∞		10e4		10		1		0.1		10e-6	
	m	1	5	1	5	1	5	1	5	1	5	1	5
h	1	3.83	0.03	4.60	0.01	2.99	-0.23	1.06	-1.29	-0.31	-1.16	-1.02	-1.41
	2	5.27	0.57	5.83	0.58	4.44	0.56	1.25	-0.37	0.25	-0.19	-0.47	-0.71
	3	4.95	0.36	5.59	0.35	3.87	0.50	0.98	0.08	0.39	0.09	-0.29	-0.44
	4	4.13	-0.05	4.70	-0.06	2.54	0.02	0.83	0.24	0.41	0.21	-0.11	-0.20
	5	3.50	-0.20	3.79	-0.20	2.25	0.09	0.78	0.34	0.34	0.21	-0.06	-0.13
	6	3.03	-0.35	3.16	-0.33	1.82	0.06	0.66	0.35	0.22	0.13	-0.06	-0.14
	7	2.43	-0.50	2.79	-0.49	1.86	-0.09	0.68	0.45	0.10	0.07	0.02	-0.09
	8	2.30	-0.21	2.33	-0.21	1.73	0.01	0.44	0.34	0.04	0.03	0.05	-0.05
	9	1.79	-0.25	1.81	-0.26	1.42	0.00	0.39	0.29	0.04	0.06	0.04	-0.05
	10	1.07	-0.47	1.28	-0.48	0.85	-0.21	0.33	0.25	0.00	0.01	0.04	-0.03
	25	0.21	0.02	0.16	0.02	0.14	0.03	0.05	0.02	0.00	0.00	-0.01	0.00
	50	0.07	0.05	0.08	0.06	0.07	0.03	0.02	0.04	0.01	0.01	-0.01	-0.02
	100	0.06	0.04	0.06	0.04	0.03	-0.01	-0.01	-0.01	-0.01	-0.01	0.00	0.00
		T = 1000											
	S	∞		10e4		10		1		0.1		10e-6	
	m	1	5	1	5	1	5	1	5	1	5	1	5
h	1	6.36	2.61	6.20	2.61	4.96	2.08	0.44	-1.49	-0.01	-0.26	-0.08	-0.05
	2	6.94	2.48	7.35	2.49	5.09	1.84	0.94	-0.14	0.26	0.13	-0.12	-0.08
	3	6.07	1.63	6.15	1.64	4.78	1.62	1.02	0.27	0.07	0.00	-0.04	-0.05
	4	4.84	1.04	4.87	1.04	4.40	1.45	1.05	0.43	0.08	0.01	-0.04	-0.03
	5	4.21	0.74	4.27	0.74	3.15	1.09	0.78	0.45	0.16	0.10	-0.03	0.00
	6	3.79	0.45	3.44	0.45	2.66	0.70	0.63	0.36	0.07	0.06	-0.02	0.00
	7	3.06	0.15	2.54	0.15	2.24	0.43	0.48	0.33	0.05	0.03	0.02	0.01
	8	2.52	0.04	2.03	0.04	1.96	0.39	0.38	0.32	0.05	0.01	0.00	0.00
	9	2.19	-0.03	1.69	-0.03	1.55	0.22	0.37	0.28	0.04	0.01	0.00	-0.01
	10	2.06	0.08	1.47	0.08	1.22	0.29	0.25	0.21	0.05	0.01	-0.01	-0.01
	25	0.14	0.02	0.19	0.02	0.10	0.03	0.02	0.02	0.00	0.00	0.00	0.00
	50	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	100	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 3.17: Gains from equal VAR pooling relative to R^2 selected VAR (*Model III*)

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Eidesstattliche Versicherung

Ich versichere hiermit eidesstattlich, dass ich die vorliegende Arbeit selbstständig und ohne fremde Hilfe verfasst habe. Die aus fremden Quellen direkt oder indirekt übernommenen Gedanken sowie mir gegebene Anregungen sind als solche kenntlich gemacht. Die Arbeit wurde bisher keiner anderen Prüfungsbehörde vorgelegt und auch noch nicht veröffentlicht.

München, 24. September 2009

Johannes Mayr

Curriculum Vitae

04/2006 – 09/2009	Ph.D. Student at the University of Munich and Junior Researcher at the Ifo institute for Economic Research
10/1999 – 11/2005	Diploma in Economics and Business Administration at University of Augsburg
08/1998	Abitur Staatliches Gymnasium Kaufbeuren
22.7.1978	Born in Mering, Germany

München, 24. September 2009