On the effective theory of type II string compactifications on nilmanifolds and coset spaces

Claudio Caviezel

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Claudio Caviezel
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Contents

Chapter 1

Introduction

The seminal findings of theoretical physics of the 20th century are the standard model of particle physics and the general theory of relativity. The standard model of particle physics describes the world at small length scales and predicts with an impressive accuracy how particles such as quarks, electrons and neutrinos interact. On the other hand, the theory of general relativity provides us with a detailed explanation for astrophysical observations at very large scales.

The discovery of the standard model was guided by quantum electrodynamics. Attempts were made to construct a gauge theory of the weak interaction, and in the mid-1960s the hypothesized charged intermediate vector bosons ($W^{\pm}$) were complemented with a neutral partner, the Z-boson. The incorporation of the Higgs mechanism into the electroweak theory solved the problem of having both a gauge theory and massive mediators of the weak interaction. Completed with the theory of the strong interaction, the standard model of particle physics was born, a theory of three of the four known fundamental interactions and the elementary particles that take part in these interactions. Experimentally, the standard model (extended by massive neutrinos) has been tested to a very high precision and the only missing ingredient to be discovered is the scalar Higgs particle [1]. However, physicists have little doubt that this discovery will happen in the LHC experiment.

On the other hand, despite the success of the standard model in all its confrontations with experimental results, it leaves us with a whole bunch of fundamental theoretical questions. The most important drawback is that it is not a complete theory of fundamental interactions since it does not include gravity. However, the constant progress of physics towards unification of all interactions is a strong indication that a theory in which all the forces are treated on the same footing may exist. Another weak point of the standard model is that it requires a large number of unrelated and arbitrary numerical parameters put in by hand, mostly related to the ad-hoc introduction of the Higgs and Yukawa sectors in the theory. And there is the famous *hierarchy problem* of the standard model: via the Higgs mechanism all masses of the standard model particles are proportional to the Higgs mass m_{H}, which is expected, from measurements of the mass of the W-bosons, to be of order of the electroweak scale, $m_{\mathrm{H}} \sim 100$ GeV. The problem is that m_{H}^2 receives quantum corrections quadratic in the cutoff scale Λ from the virtual effects of every particle that couples, directly or indirectly, to the Higgs field. If we assume the standard model to be valid up to a scale of order of the Planck scale, this quantum correction to m_{H}^2 is some 30 orders of magnitude larger than the experimentally favored value of the Higgs mass. The possibility to fine tune the bare value of the Higgs mass in order to cancel the quantum corrections except for some value of the order of the electroweak scale is very unnatural and unsatisfactory. On the other hand, one could simply assume Λ to be of the order of the electroweak scale and replace the standard model by another theory above the energy scale Λ. However, not to run into the same fine tuning problems in this new theory, this theory should explain how a tiny Higgs mass can be protected from quantum corrections quadratic in the cutoff Λ' of the new theory.

A proposal for such a new theory beyond the standard model is to postulate a new symmetry that relates bosons with fermions - called supersymmetry. In fact, supersymmetry is the only possible extension of the known space-time symmetries, circumventing the Coleman and Mandula theorem [2] by allowing anticommuting symmetry generators [3]. The minimal supersymmetric standard model (MSSM) formulated in 1981 [4] is the simplest supersymmetric extension of the standard model containing the superpartners of all standard model particles. It offers a natural solution to some of the problems of the standard model. In particular, since there is a relative minus sign between fermion loop and boson loop corrections to the squared Higgs mass m_{H}^2, the radiative corrections quadratic in the cutoff Λ' neatly cancel. This allows the Higgs mass to be of the order of the electroweak scale also in a theory with a higher mass scale, without the need of some unnatural fine-tuning. Furthermore, the renormalization group flow predicts a unification of the electroweak and strong interactions. As supersymmetry is not directly observed in nature, it must be broken, which, in general, leads to a mass split between bosonic and fermionic partners. If this mass split is roughly of order of the electroweak scale, i.e. $m_{\mathrm{split}} \sim 100$ GeV, the supersymmetric partners of the standard model particles could be heavy enough not to be observed in experiments so far. One hopes that the LHC experiment detects some of the superpartners in the TeV region in the foreseeable future, verifying the so far only theoretical concept of supersymmetry. However, even when such superparticles are detected, it would still remain to unify the supersymmetric standard model with general relativity.

General relativity, on the other hand, is a classical theory which does not take into account the quantum mechanical nature of matter as described in the standard model. Since the Einstein equation relates geometry with matter, we can not treat matter quantum mechanically without a quantum mechanical theory of gravity. The construction of a renormalizable quantum field theory that treats gravity quantum mechanically has not yet been carried out, even though there are several suggestions (see for instance [5]). This constraints the validity of general relativity to physics, where quantum mechanical effects are of negligible importance. However, there are circumstances where a quantum theory of gravity is needed, for instance for the physics of the very early universe.

Interestingly, string theory provides a natural way of including gravity in a quantum theory of matter: in string theory one replaces the ordinary point particles with a quantum theory of small one-dimensional extended objects - the strings that can be both closed or open. These strings have various vibrational modes corresponding to different particles, whose interaction is described by the splitting or joining of the strings. As a matter of fact, every consistent such string theory necessarily contains among the possible vibrating modes a massless spin-two mode which is a natural candidate for the graviton whose long-distance interactions are described by general relativity. Upon quantization this provides us with a consistent quantum theory of gravity. Ultra-violet divergences of graviton scattering amplitudes are evaded, since the extended character of the string smears out the location of the interaction.

The Planck mass, $M_P = 1.2 \times 10^{19}\,\mathrm{GeV}$, is a natural first guess for a rough estimate of the fundamental string mass scale m_s [1]. Thus, the extended structure of the strings only becomes apparent at the Planck scale, far beyond our abilities to measure in the laboratory (for comparison, the LHC experiment should reach a collision energy of 14 TeV). At energies far below the Planck scale strings can be accurately approximated by point-like particles. This low-energy theory is well described by an effective field theory that describes the massless modes of string theory (the first massive vibrational modes have masses of order the string scale m_s, which we assume to be of the order of the Planck scale, such that they can be integrated out in an effective theory). However, the effective theory inherits supersymmetry as well as the massless spin-two graviton mode from string theory. This limit is called supergravity and is thus a supersymmetric extension of general relativity, where the nonrenormalizability of the supergravity is cured by the extended nature of the string.

Let us very briefly sketch how to determine the spectrum of the strings and how to determine the action for its low-energy supergravity limit. For details we refer the reader to the literature (see for instance [7, 8, 9, 10, 11]).

A one-dimensional string sweeps out a two-dimensional surface when it propagates through D-dimensional space-time. We call this surface the world-sheet Σ. In analogy to the description of a point particle by its world-line $X^M(\tau)$, we describe a string by the embedding of the string world-sheet into space-time, i.e., by a map $X^M(\tau, \sigma) : \Sigma \to \mathcal{M}_D$, where τ and σ parameterize the points on the world-sheet. For a closed string the variable σ is periodic, and its world-sheet describes a tube in space-time, whereas for an open string σ covers a finite interval, and the world-sheet is a surface with boundaries. To describe the dynamics of the string we need an action, and the simplest action that comes to mind is the so-called Nambu-Goto action, which is a straightforward generalization of the relativistic action for a point particle moving in

[1] Let us mention that in "large extra dimension" scenarios, the string scale can be much lower, namely at the order of TeV. This is because the four-dimensional Planck mass M_P and the string mass m_s are related by the compactification volume [6]. We will not consider these scenarios further in this thesis and assume the string scale to be of order of the Planck scale.

D-dimensional Minkowski space-time,

$$S_{\mathrm{NG}} = \frac{1}{2\pi\alpha'} \int_\Sigma \mathrm{d}\sigma \mathrm{d}\tau \sqrt{\det\left(-\eta_{MN}\partial_\alpha X^M \partial_\beta X^N\right)}\,, \tag{1.1}$$

where $\alpha, \beta = \tau, \sigma$. Here, $T = 1/2\pi\alpha'$ is the string tension related with the string mass scale by $m_s = (\alpha')^{-1/2}$. However, for quantization this action is not very useful as it contains a square root. One thus makes use of a classically equivalent action by introducing an auxiliary world-sheet metric $h_{\alpha\beta}$ such that

$$S_\sigma = \frac{1}{4\pi\alpha'} \int_\Sigma \mathrm{d}\sigma \mathrm{d}\tau \sqrt{-h} h^{\alpha\beta} \partial_\alpha X \cdot \partial_\beta X\,. \tag{1.2}$$

This so-called Polyakov action has important symmetries. In addition to the global Poincaré invariance there are two local symmetries of the action. It is invariant under reparameterization of the world-sheet coordinates τ, σ and under Weyl transformations, i.e. $h_{\alpha\beta} \to e^{\phi(\tau,\sigma)} h_{\alpha\beta}$ for an arbitrary function $\phi(\tau, \sigma)$. Using these local symmetries to make a convenient gauge choice and taking care of the appropriate boundary conditions, we end up with the equations of motion for the world-sheet coordinates $X^M(\tau, \sigma)$. These equations have the structure of a wave equation with a general solution that contains vibrational modes. For a closed string, the solution is a sum of left- and right-movers. In the quantization process, the world-sheet coordinates and correspondingly the vibrational modes are promoted to operators satisfying essentially the algebra of raising and lowering operators of the quantum mechanical harmonic oscillator. The spectrum is constructed by applying raising operators on the ground state. Skipping the details, it turns out that the obtained spectrum contains a tachyon but no states that transform as spinors under the D-dimensional Lorentz group, which could be interpreted as space-time fermions. However, we can cure this by introducing supersymmetry. In the so-called Ramond-Neveu-Schwarz (RNS) approach, we introduce in a supersymmetric way anti-commuting world-sheet fermions ψ^M into the Polyakov action (1.2). For the fermionic fields, however, the variation of the action allows two possibilities to satisfy the boundary conditions: it is possible to impose periodic (Ramond) boundary conditions or anti-periodic (Neveu-Schwarz) boundary conditions. For closed strings, corresponding to the different pairings of the left- and right-movers, we thus distinguish four different sectors. All the states in these sectors carry quantum numbers of the D-dimensional Lorentz group, and it turns out that we can interpret the states in the NS-NS and R-R sector as space-time bosons, while states in the NS-R and R-NS sector are space-time fermions.

Note that the dimension D of the space-time is *not* arbitrary. Due to the indefinite signature of space-time, the spectrum contains negative norm states, violating causality and unitarity. On the other hand, one can show that in the particular case of $D = 10$ these negative norm states can be decoupled from the physical spectrum. Nevertheless, there is still a tachyon in the spectrum and the spectrum is not space-time supersymmetric (the number of fermionic degrees of freedom in not equal to the number of bosonic degrees of freedom). We can turn the RNS string theory into a consistent theory by truncating the spectrum in a very specific way that eliminates the tachyon and leads to a supersymmetric theory in ten-dimensional space-time, known as the GSO-projection [2]. The remaining spectrum consists of a set of massless particles and an infinite tower of massive excitations with masses quantized in units of the string scale m_s. As we assume the string scale to be of order of the Planck mass, these states are extremely heavy.

It turns out, demanding modular invariance of the one loop partition function and anomaly cancellation of the gauge symmetries coming from non-Abelian gauge potentials in the spectrum of the string theory, that one can only construct five consistent string theories in $D = 10$ Minkowski space-time. These five theories are type I string theory, consisting of unoriented open and closed strings with a gauge group SO(32), type IIA and IIB string theory, made of closed strings, and two heteroric string theories that have closed strings only, one with gauge group SO(32), and one with gauge group $E_8 \times E_8$. However, these five theories are related by a web of dualities and are nowadays viewed as different corners of one fundamental theory - referred to as M-theory. Even though a full description of the theory is yet unknown, the uniqueness of M-theory makes it a very promising theory.

Let us focus on the massless spectrum of the two type II theories, since the type II theories will be of particular interest for this thesis. Both theories contain closed strings only [3], and their massless bosonic spectrum includes from the NS-NS sector a graviton g_{MN}, a scalar called the dilaton Φ and an antisymmetric tensor field B_{MN}. In addition, each of these theories has its individual bosonic excitations living in the R-R sector. In the type IIA theory the R-R one- and three-form, in the type IIB theory the R-R zero-, two- and four-form. In addition we have massless fermions from the NS-R and R-NS sector. Each of these sectors contains a spin-3/2 gravitino and a spin-1/2 dilatino. In type IIB the two gravitini have the same chirality, whereas in the type IIA they have opposite chirality. It follows that type II string theories have $\mathcal{N} = 2$ supersymmetry.

How does one construct an action for the low-energy limit of string theory, describing the massless states in the string spectrum? To find a space-time action for these theories, one can use the constraints implied by the Weyl symmetry of the string action. Note that so far we only considered strings moving in ten-dimensional Minkowski space-time. For a more realistic situation, we generalize the Polyakov action by the fields obtained in the various spectra of the five theories. For instance, for the theories based on closed strings only (heterotic and type II string theories) this reads for the NS-NS sector

$$S_\sigma = \frac{1}{4\pi\alpha'} \int_\Sigma \mathrm{d}\sigma \mathrm{d}\tau \sqrt{-h} \left[\left(h^{\alpha\beta} g_{MN}(X) + i\epsilon^{\alpha\beta} B_{MN}(X) \right) \partial_\alpha X^M \partial_\beta X^N + \alpha' R \Phi(X) \right] , \tag{1.3}$$

[2] The GSO-projection may appear to be an ad-hoc condition. Actually, it is also possible to derive it by demanding one-loop and two-loop modular invariance.

[3] We will see in a moment, how one introduces also open strings in these theories.

where R is the two-dimensional curvature. The NS-NS fields can be interpreted as coupling functions in the two-dimensional world-sheet field theory. However, not all field configurations preserve Weyl symmetry at the quantum level. The Weyl anomaly is absent if the β-function for each of the couplings vanishes, i.e., $\beta^g_{MN} = \beta^B_{MN} = \beta^\Phi = 0$. This leads to a set of equations that take the form of equations of motion for the space-time fields g_{MN}, B_{MN} and Φ. The supergravity action - the low-energy limit of string theory - is the action that reproduces these equations at lowest order [4]. Let us mention that the same procedure for the fields in the R-R sector is not straightforward in the RNS-formulation. However, there is an equivalent formulation, the Green-Schwarz formulation, where the p-form fields of the R-R sector can be included as well.

Compactification and moduli stabilization

As we pointed out, a consistent string theory lives in a ten-dimensional space-time. The observable world, however, is four-dimensional. To make contact with our four-dimensional world we need a mechanism to hide the extra six dimensions from observation - such a mechanism is called dimensional reduction. One way to achieve a dimensional reduction is by choosing these extra dimensions very small and compact such that they are too small to be detected in present-day experiments.

In fact, the idea of geometric compactification is very old and goes back to the 1920s. Kaluza [12] and Klein [13, 14] suggested a unification of the gravitational and electromagnetic interaction by postulating an extra, fifth, dimension of space-time. Choosing this extra dimension to be topologically S^1 yields a very simple explanation for the compactness of the gauge group and hence the quantization of the electric charge. However, the theory contains one more degree of freedom, the radius R of the extra-dimensional circle. Since the classical Einstein equations are scale invariant, there is no preferred value for this radius R and Kaluza and Klein simply postulated a value for it consistent with experimental bounds.

Even though the motivation has changed, the idea of Kaluza and Klein can be generalized to the reduction of ten-dimensional string theory from ten to four dimensions. In the Kaluza-Klein reduction one starts with an ansatz for the background space-time. The specific ansatz we will use here is that space-time has a product structure of the following form

$$\mathcal{M}^{(9,1)} = \mathcal{M}^{(3,1)} \times \mathcal{M}_6 \,, \tag{1.4}$$

where $\mathcal{M}^{(3,1)}$ is our four-dimensional non-compact space-time and $\mathcal{M}_6$ is a six-dimensional compact manifold. If $\mathcal{M}_6$ is chosen small enough, these six additional dimensions are not visible in experiments with present-day accelerators. This type of dimensional reduction is alternatively called compactification.

[4] An alternative way to derive equations of motion for the massless fields is to consider n-point functions in the two-dimensional world-sheet theory using the supersymmetric version of the action (1.3). The classical scattering amplitudes of the effective space-time action, from which we derive the equations of motion, should then reproduce these n-point functions.

At this point, however, much of the uniqueness of ten-dimensional string theory (and ten-dimensional supergravity as its low-energy limit) gets lost, since the compactification mechanism yields a very large number of possible four-dimensional solutions with inequivalent four-dimensional physics. The reason is twofold: first, making any concrete Kaluza-Klein reduction requires making a choice for a compactification manifold with a given topology around which to expand in the Kaluza-Klein reduction, and no principle suggest that there is a particular preferred manifold. Second, as we will explain in the following, even having chosen a particular compactification manifold, one has many free parameters which enter into observable predictions and no particular values of these parameters appear to be preferred.

The appearance of the free parameters is explained as follows. Just like the classical Einstein-Maxwell equations, the classical supergravity equations are scale invariant. Thus, if one finds any solution to the supergravity equations, by rescaling the size R of the compactification manifold, one obtains a one-parameter family of solutions, differing only in the value of R. Hence, the choice of R is unconstraint by the equations of motion and thus appears as a massless neutral scalar field in four dimensions. Depending on the choice of the internal manifold, the situation is even worse, and there are much more massless scalars in the theory, corresponding to parameters such as the shape of the internal manifold. They label the continuous degeneracy of the internal manifold $\mathcal{M}_6$ and are generally not driven to a particular value. One calls these massless scalars moduli fields.

The emergence of massless scalars is a serious problem for string theory that aspires to be a fundamental theory predicting the values for the fundamental constants [5]. Suppose we want to compute physical predictions by performing a Kaluza-Klein reduction on a given compactification manifold $\mathcal{M}_6$. The resulting fundamental constants will depend on the details of the chosen manifold and the values of the moduli fields. Since the choice of the compactification manifold is not unique and the values for the moduli fields are completely arbitrary, how do particular values we observe for the fundamental constants of physics actually emerge from string theory?

Apart from the specific choice of a compactification manifold, the predictivity of string theory could be improved if one provides a mechanism which induces a potential for the moduli fields - called moduli stabilization [6]. Finding such a potential offers the

[5] Let us stress that a moduli field *does not* correspond to a massless Goldstone mode. The origin of the Goldstone mode in symmetry breaking implies that the physics of any constant configuration of this field must be the same (since all are related by symmetry). On the other hand, moduli fields can exist without a symmetry and the physics usually depends on their values.

[6] In principle, quantum corrections can already generate masses for the moduli fields. However, in supersymmetric theories there are non-renormalization theorems excluding corrections to the superpotential to all orders in perturbation theory. In theories that do not admit non-perturbative corrections, moduli fields are thus natural. After supersymmetry breaking, all scalar fields, including the moduli, receive mass. However, as an upper bound on these masses, depending on the particular model of supersymmetry breaking, one finds a moduli mass of the order of 1 TeV. This turns out to be problematic for phenomenological reasons: light moduli fields would be problematic in the present universe, as they mediate fifth forces of gravitational strength. In addition, they cause a Polonyi problem: the

possibility to fix their values in a (possibly metastable) vacuum and make them sufficiently massive such that they can be discarded from the observed spectrum. Indeed, there are such mechanism to generate a potential for the moduli fields and the most popular ones are the inclusion of background fluxes, instanton corrections and gaugino condensates. In this thesis we will focus on the mechanism of including background fluxes in the extra dimensions, which is referred to as flux compactification. The energy of such a field depends on the moduli and thus generates a contribution to the effective potential for the moduli fields.

Let us consider an example. As we have seen, type II string theories contain among other fields the NS-NS two-form potential B_2. We define its field strength by $H_3 = \mathrm{d}B_2$. Suppose now that we choose a compactification manifold with a non-trivial three-cycle Σ. We can consider a flux configuration with a non-zero flux of the field strength,

$$\frac{1}{l_s^2}\int_\Sigma H_3 = n \neq 0\,, \tag{1.5}$$

where n is an integer for proper quantization of the flux and $l_s = 2\pi\sqrt{\alpha'}$ the string length scale. Note that by insisting on maximal four-dimensional symmetry, we can only turn on non-trivial fluxes in the internal dimensions. The key point is that because the flux is threading the internal cycle Σ, changing the internal geometry will cost energy - in other words, we generate a potential for the geometric moduli. If this potential has favorable characteristics, we can determine the possible (metastable) vacuum states of the theory as the local minima of the potential.

However, on a qualitative level, the mechanism works for fairly generic nonzero choices of quantized flux and one finds a huge number of possible discrete ground states. At present, there is no known mechanism that would single out one or a subset of these vacua as the preferred candidates to describe our universe. Any sufficiently long-lived vacuum which fits all the data of observations would be an equally good candidate to describe our universe. It seems that the request on string theory as a fundamental theory of nature to allow only for a single solution explaining all physical phenomena was too ambitious. However, given our limited understanding of both general principles of quantum gravity and of its microscopic definition, all has not been said and done.

At present, however, all we can do is comparing possible solutions with observational data. Let us mention in the following some of the observational requirements we impose on an acceptable solution:

- Phenomenologically, an $\mathcal{N}=1$ matter sector with spontaneously broken supersymmetry at low energies may be preferred. This offers a natural extension of the standard model and helps solving the hierarchy problem, offers an explanation of coupling unification and contains a possible dark matter candidate, the lightest supersymmetric particle. In addition, as a technical argument, supersymmetry simplifies the computation of the four-dimensional low-energy effective action.

oscillations of such a field about the minima of their potential, in a cosmological setting, will overclose the universe [15]. To safely avoid these problems, we should look for physics of moduli stabilization at energy scales ~ 100 TeV and above.

- As we discussed, we require a large positive mass for all the moduli fields to fix their vacuum expectation values. In fact, the moduli fields should receive masses of the order 100 TeV and above to avoid phenomenological problems (see discussion in footnote 6).
- At low energies, string theory should reproduce the standard model of particle physics, in particular the standard model gauge group SU(3)×SU(2)×U(1) should emerge from a viable string theory.
- To fit the present cosmological observations of a spatially flat universe with its energy density dominated by 74% dark energy behaving very similar to a positive cosmological constant, we look for a string theory vacuum with small positive cosmological constant.
- The observed cosmic microwave background (CMB) radiation including its small density fluctuations could be elegantly explained by an inflation scenario in the early universe. A viable model should therefore offer the possibility to realize such an inflation scenario.

Most of the early attempts to construct viable four-dimensional $\mathcal{N} = 1$ vacua were done by compactifications of the $E_8 \times E_8$ heterotic string theory on Calabi-Yau manifolds (following the work of [16]), with the intention to break one of the E_8 gauge groups to the standard model or GUT gauge group. In contrast to heterotic SO(32) and $E_8 \times E_8$ and type I theories, both type II string theories do not apriori contain non-Abelian gauge groups.

However, with the discovery of D-branes as non-perturbative BPS objects in the middle of the 90's [17], also the type II theories were found to naturally include non-trivial gauge theories. More precise, D-branes are extended objects defined as hyperplanes in the ten-dimensional space-time on which open strings can end. Additionally, they constitute the sources for the higher dimensional R-R p-form fluxes (the NS-NS two-form B_2 couples to the string world-sheet). A U(1) gauge field emerges then from an open string ending with both ends on the same D-brane. By putting a stack of n D-branes on top of each other, the gauge group gets enhanced to U(n), modeling at lowest order a Yang-Mills gauge theory in the low-energy effective action. Compactifications involving space-time filling D-branes, non-vanishing vacuum expectation values for background fluxes and non-perturbative effects such as instanton corrections are an attractive setup for model building in particle physics as well as cosmology (see for instance [18] and references therein).

Motivation and organization of this thesis

An essential step to further study phenomenological properties of string vacua is to determine the four-dimensional low-energy effective theory for particular compactification models. An important application comprises the viability of these models for phenomenological applications. This is in particular an interesting question in type IIA string theory, since D-brane model building on space-time filling D6-branes made much progress in the last years [18, 19]. A viable compactification in type IIA, say one that has solutions with all moduli stabilized and with small positive cosmological constant as well as an epoch of inflation, would be of extraordinary interest.

In this thesis we will focus on flux compactifications in type IIA and type IIB string theory with the intention to derive the four-dimensional low-energy effective theory on a large class of new compactification manifolds (for reviews on flux compactifications and a more complete list of references, see, e.g., [20, 21, 18]). One aspect that complicates the derivation of these effective actions is that p-form fluxes generally back-react on the geometry of the compactification manifold, deforming them away from well understood classes such as Ricci-flat Calabi-Yau manifolds. The back-reaction can be rather mild, as, e.g., in type IIB orientifolds with D3/D7-branes [22], where the internal space is still conformal to a Calabi-Yau manifold. In these comparatively simple models, however, the fluxes turn out to stabilize only the dilaton and the complex structure moduli, while the Kähler moduli stabilization requires the use of quantum effects, e.g., along the lines of [23].

We will instead be interested in a different class of flux compactifications for which the back-reaction of the fluxes on the geometry is less trivial. Concretely, we will derive the four-dimensional low-energy effective action for a large class of models, where $\mathcal{M}_6$ is a six-dimensional compact space that is either a *nilmanifold* or a *coset space*. Some of these models allow for an $\mathcal{N} = 1$ supersymmetric solutions to AdS_4. Being compactifications to AdS_4 space-time, these models do not appear realistic as such, but they can serve as starting points for the construction of more realistic setups or have other applications. In particular, we will investigate for these models whether the potential already has meta-stable de Sitter vacua away from the AdS_4 vacuum or whether there are regions suitable for inflation.

Let us mention that one could also think to use these models to study other phenomenological applications, e.g. after the inclusion of an additional uplifting potential so as to construct meta-stable de Sitter vacua in the spirit of the IIB models discussed in [23]. In addition, replacing fluxes by branes, the AdS_4 vacua can be potentially obtained as near-horizon geometries of intersecting branes [24]. AdS_4 flux vacua of the type we will consider may admit a full non-perturbative definition via a dual three-dimensional CFT [25]. The above-mentioned brane solutions also correspond to domain walls that interpolate between different flux vacua. The existence of these domain walls may correspond to interesting transitions in the landscape of flux vacua.

To render the analysis tractable, we will only consider structures and fluxes which are constant in the basis of *left-invariant* one-forms (see chapter 2 and chapter 4 for an introduction to G-structure manifolds and left-invariant forms, respectively). A general problem is that an explicit computation of the low-energy theory of a given compactification requires a suitable choice of expansion basis for the 'light' fluctuations. Unfortunately, it is still unclear how to construct such a basis *in general*. In generic flux compactifications, the set of harmonic forms would be unsuitable as expansion forms, as, e.g., the forms J and Ω that define the SU(3)-structure (and which enter the supergravity expressions for the Kähler- and superpotential) are no longer closed (see e.g. [26, 27, 28] for a few proposals). A detailed discussion of the general constraints on such a basis appeared in [29]. In the special case of nilmanifolds and coset manifolds, however, the set of left-invariant forms (with the appropriate behaviour under the orientifold action) readily presents itself as the natural choice and obeys the requirements of [29] [7].

For the compactifications we study in this thesis, we will introduce D-brane and orientifold sources. The reason is that, in some of the models we study, the Bianchi identities cannot be satisfied without orientifold sources. A further reason is that we are interested in four-dimensional, $\mathcal{N}=1$ supersymmetric effective theories, for which the orientifold sources are necessary [8]. In addition, as we discuss further in chapter 2, the sources potentially allow for a hierarchy of scales between the size of the internal manifold and the AdS_4 curvature, thereby providing a possibility to decouple the tower of Kaluza-Klein modes from the light modes.

A somewhat delicate feature of our models is that the orientifolds have to be smeared. The reason for this is that the supersymmetry conditions of [31] (for non-vanishing Romans mass) force the warp factor to be constant. Considering the back-reaction of a localized orientifold, on the other hand, one would expect a non-constant warp factor, at least close to the orientifold source [9]. A helpful interpretation of the smearing of a localized source, whose Poincaré dual is given roughly-speaking by a delta-function, is that it corresponds to Fourier-expanding the delta-function and discarding all but the zero mode. We will adopt the pragmatic point of view that the smeared orientifolds are an unavoidable feature of our models that is consistent with a Kaluza-Klein reduction in the approximation where only the lowest modes are kept. The question of how to associate orientifold involutions to a smeared source turns out to be somewhat subtle. We will make the natural assumption that the different orientifolds correspond to the decomposable (simple) terms in the orientifold current (see further the discussion in appendix D).

This thesis is organized in three main parts. The aim of the first part is to provide the formalism and the techniques needed to analyse type II string theory compactifications to four dimensions. We start in *chapter 2* with the $\mathcal{N}=1$ supersymmetry compactification ansatz. Demanding that not all supersymmetries are broken in the four-dimensional effective theory places strong topological constraints on the internal manifold. For instance, the structure group of the tangent bundle of the internal mani-

[7] Since the left-invariant forms are constant over the moduli space, this basis satisfies requirements *7-*9 of [29] rather trivially. Note that left-invariant forms are not in general harmonic: they are eigenmodes of the Laplacian to eigenvalues of the order of the geometric flux.

[8] For a discussion of the $\mathcal{N}=2$ theory arising from type IIA theory on nearly-Kähler manifolds *without* orientifolds see [30].

[9] A possible way around this contradiction is that taking into account α'-corrections might allow for a non-constant warp factor (see also [32] for an alternative discussion), or one has to consider more general vacua with SU(3)×SU(3)-structure instead [33].

fold is reduced to SU(3) or a subgroup thereof. We further discuss the conditions on supersymmetric massive type IIA AdS_4 solutions with strict SU(3)-structure. As already mentioned, these conditions force the warp factor and dilaton to be constant. However, we will provide a generalization of supersymmetric type IIA AdS_4 compactifications by allowing for a non-constant warp factor and dilaton, provided that the Romans mass is set to zero. In *chapter 3* we discuss how to obtain the four-dimensional low-energy effective action for a given compactification manifold. We start by discussing the direct approach, the Kaluza-Klein reduction. The modern approach, however, is the effective supergravity approach where one calculates the superpotential and the Kähler potential by means of geometrical data of the compactification manifold and the background fluxes. We review the techniques for this approach in the generalized geometry language and specialize the expressions then to strict SU(3)-structure in type IIA and static SU(2)-structure in type IIB theory. We end this chapter by a discussion on how to choose the most general ansatz for the background fluxes to label the disconnected bubbles of moduli space. In *chapter 4* we then turn to the description of the two classes of six-dimensional manifolds we study in this thesis. These are six-dimensional nilmanifolds and coset spaces. *Chapter 5* discusses the phenomenological aspects of this thesis, in particular the question whether our models are valid candidates to allow for a de Sitter solution or to realize inflation scenarios (at tree level, without additional perturbative or non-perturbative quantum effects). In string theory, the moduli fields of the compactification are natural inflaton candidates. We will thus first review the important aspects of cosmology and the Hot Big Bang model and give a brief overview of the necessary conditions for a particular inflation scenario, the so-called slow-roll inflation. However, for type IIA compactifications at tree level, there exist quite strong no-go theorems against de Sitter vacua and slow-roll inflation. We will review and slightly modify these theorems such that we can apply them to our models.

In the second part of this thesis we apply the techniques studied so far to the class of nilmanifolds. A systematic scan yields exactly two nilmanifolds that satisfy the necessary and sufficient conditions for (massive) type IIA $\mathcal{N} = 1$ compactifications to AdS_4 discussed in chapter 2. We present these solutions in *chapter 6*. As a matter of fact, these solutions are related (for some values of the parameters) by T-duality along two directions. We also find a type IIB solution with static SU(2)-structure on a different nilmanifold, which forms the intermediate step after one T-duality. Interestingly, as shown in section 6.4, for the same range of the parameter space for which the T-dualities above are valid, the solutions admit an interpretation as near-horizon geometries of intersecting brane configurations, as in [24]. From this point of view, the nilmanifold vacua in this range are nothing but near-horizon geometries of intersections of Kaluza-Klein-monopoles with other branes in flat space. One of the main goals of this part of the thesis is to provide a check on the effective supergravity approach, in particular on the explicit expressions of the superpotential and Kähler potential given in the literature. To do so, we perform in *chapter 7* an explicit Kaluza-Klein reduction around the two type IIA solutions of chapter 6 and compute the mass spectrum of the moduli fields. On the other hand, in *chapter 8* we analyse the same two models by means of the effective supergravity techniques and compute again the mass spectrum of the moduli fields around the supersymmetric solution. We find perfect agreement providing an important consistency check between both approaches. Having performed this consistency check for these models, we will restrict ourselves to the effective supergravity approach in the following.

In the third part of this thesis we focus on coset manifolds, where we first examine in *chapter 9* the geometry of the coset models that are suitable for supersymmetric compactifications to four dimensions. In the following *chapter 10* we present the coset models that satisfy the necessary and sufficient conditions for an $\mathcal{N}=1$ compactification to AdS_4. We closely follow in these two chapters [34]. We also comment on a possible supersymmetric AdS_4 solution with non-constant warp factor and dilaton. The main results of this part of the thesis are the following three chapters: in *chapter 11* we compute the four-dimensional type IIA low-energy effective theory for a large class of coset models. In each case, we compute the superpotential and the Kähler potential for the most general choice of background fluxes in order to cover the whole moduli space. For the models with a supersymmetric AdS_4 vacuum we compute the mass spectrum around this vacuum and find that for all the coset models, except for one, all moduli are stabilized at tree level. For some models we comment on how to identify the number of possible $\mathcal{N}=1$ AdS_4 vacua in a particular bubble of moduli space. In *chapter 12* we compute the effective theory for type IIB SU(2)-structure compactifications on the coset models allowing for a strict SU(2)-structure. Finally, in *chapter 13*, we discuss phenomenological applications for the coset models. In particular, we apply the no-go theorems of chapter 5 to the coset models and we study whether the models we consider in this thesis are interesting candidates for inflation or have stable de Sitter minima.

Finally, we give some technical details in different appendices. In particular to mention is appendix C, where we give a short introduction to the framework of generalized geometry.

Part I

Formalism

Chapter 2

G-structure manifolds and supersymmetric vacua

As we discussed in the introduction, the ten-dimensional type IIA/IIB supergravities, which are low-energy theories of type IIA/IIB string theory, have $\mathcal{N} = 2$ supersymmetry in ten dimensions. One way to connect string theory to four-dimensional real-world physics is to compactify it from ten dimensions to four dimensions using a compactification ansatz as in eq. (1.4), where we choose the internal manifold $\mathcal{M}_6$ small and compact, such that the six additional dimensions are not detectable in present-day experiments.

The structure of the four-dimensional theory so-obtained strongly depends on the chosen internal manifold $\mathcal{M}_6$. For instance, the properties of $\mathcal{M}_6$ determine the amount of preserved four-dimensional supersymmetry. In this thesis we will focus on four-dimensional $\mathcal{N} = 1$ effective theories. Let us mention some reasons for this requirement. As we discussed in the introduction, supersymmetry suggests natural extensions of the standard model such as the minimal supersymmetric standard model. Some of the phenomenologically attractive features of these models are the following: they offer a possible solution of the hierarchy problem, they can explain the gauge coupling unification and they may provide a candidate for dark matter, the lightest supersymmetric particle. Another reason is that supersymmetry provides a comparatively easy way to obtain solutions of the full equations of motion, since the supersymmetry conditions are much easier to solve as the equations of motion. It can be shown that solutions to the supersymmetry conditions, completed with the Bianchi identities for the form fields, automatically provide solutions to the full equations of motion. Of course, after one has constructed a supersymmetric solution to the supergravity equations of motion, one has to provide additional mechanisms to break supersymmetry spontaneously at low energies.

As we will discuss in this chapter, demanding that not all supersymmetries are broken in the four-dimensional effective theory imposes very stringent requirements on the internal manifold $\mathcal{M}_6$. The existence of four-dimensional supersymmetry parameters (this is required to obtain a four-dimensional supersymmetric theory) reduces the structure group of the internal manifold $\mathcal{M}_6$, which is a topological constraint, whereas the existence of a supersymmetric vacuum of the theory further imposes differential constraints on the geometry of the internal manifold. See, e.g., [20, 21, 18, 35] for reviews and more references.

2.1 Supersymmetric effective theories and G-structures

We assume a product structure for the 10-dimensional space-time as follows

$$\mathcal{M}^{(9,1)} = \mathcal{M}^{(3,1)} \times \mathcal{M}_6 \,, \tag{2.1}$$

where $\mathcal{M}_6$ is the six-dimensional compact internal manifold. Motivated by phenomenology, we consider the four-dimensional space-time $\mathcal{M}^{(3,1)}$ to admit maximal space-time symmetry, i.e., flat Minkowski, anti-de Sitter (AdS_4) or de Sitter (dS_4). These have Poincaré, SO(1,4) and SO(2,3) invariance, respectively. With this symmetry requirement, the most general ansatz for a ten-dimensional metric is given by

$$\mathrm{d}s^2 = g_{\mu\nu}(x)\mathrm{d}x^\mu \mathrm{d}x^\nu + g_{mn}(y)\mathrm{d}y^m \mathrm{d}y^n \,, \tag{2.2}$$

where the external metric $g_{\mu\nu}$ is a Minkowski, dS_4 or an AdS_4 metric. More generally, one can allow for a non-trivial warp factor $e^{2A(y)}$ that only depends on the internal coordinates y^m, $m = 1, \ldots, 6$, into the ansatz (2.2). This amounts to replace $g_{\mu\nu}(x) \;\to\; e^{2A(y)} g_{\mu\nu}(x)$, which is the most general ansatz consistent with four-dimensional maximal symmetry [36, 37].

The product structure of the space-time background (2.1) implies a decomposition of the Lorentz group Spin(9,1) $\supset$ Spin(3,1)$\times$ Spin(6) and an associated decomposition of the spinor representation $\mathbf{16} \in$ Spin(9,1) according to $\mathbf{16} \to (\mathbf{2}, \mathbf{4}) \oplus (\bar{\mathbf{2}}, \bar{\mathbf{4}})$. In order to obtain an $\mathcal{N} = 1$ four-dimensional effective theory, which has one four-dimensional supersymmetry parameter ζ, we make the following ansatz [38] [1]

$$\begin{aligned} \epsilon_1 &= \zeta_+ \otimes \eta_+^{(1)} \; + \, \zeta_- \otimes \eta_-^{(1)} \;, \\ \epsilon_2 &= \zeta_+ \otimes \eta_\mp^{(2)} \; + \, \zeta_- \otimes \eta_\pm^{(2)} \;, \end{aligned} \tag{2.3}$$

for IIA/IIB, where $\zeta_\pm$ are four-dimensional and $\eta_\pm^{(1,2)}$ six-dimensional Weyl spinors. The Majorana conditions for $\epsilon_{1,2}$ imply the four- and six-dimensional reality conditions $(\zeta_+)^* = \zeta_-$ and $(\eta_+^{(1,2)})^* = \eta_-^{(1,2)}$.

Let us first concentrate on the special case where the two internal spinors $\eta^{(1)}$ and $\eta^{(2)}$ are parallel everywhere: $\eta^{(1)} \propto \eta^{(2)} \propto \eta$. For the decomposition of the ten-dimensional

[1] In the concrete models we study in this thesis we introduce orientifold sources. The orientifold projection forces the four-dimensional supersymmetry generators ζ to be the same in both lines of the ansatz (2.3), ruling out an $\mathcal{N} = 2$ ansatz based on independent ζs in the two lines. See appendix C for more details.

supercurrents as in eq. (2.3) to be possible, we require the existence of a spinor η that is globally defined on the internal manifold $\mathcal{M}_6$. The existence of such a spinor imposes a non-trivial topological condition on the internal manifold. A globally defined spinor must be the same in different patches and thus invariant under the transition functions defining the structure group. The spinor representation is in the **4** of Spin(6) $\simeq$ SU(4), which can be further decomposed in representations of SU(3) as $\mathbf{4} \to \mathbf{3} + \mathbf{1}$. There is therefore an SU(3) singlet in the decomposition, and we conclude that the topological condition for a globally defined spinor is the requirement that the internal manifold has reduced SU(3)-structure.

Further reducing the structure group of the internal manifold $\mathcal{M}_6$ to a group smaller than SU(3) results in a larger number of globally defined internal spinors. For instance, if the structure group is reduced to SU(2) there are two independent globally defined spinors on $\mathcal{M}_6$ such that a general decomposition as in (2.3) is possible. Combining the terminology of [38] and [39], the following classification can be made:

- strict SU(3)-structure: $\eta^{(1)}$ and $\eta^{(2)}$ are parallel everywhere;
- static SU(2)-structure: $\eta^{(1)}$ and $\eta^{(2)}$ are orthogonal everywhere;
- intermediate SU(2)-structure: $\eta^{(1)}$ and $\eta^{(2)}$ are at a fixed angle, but neither a zero angle nor a right angle;
- dynamic SU(3)×SU(3)-structure: the angle between $\eta^{(1)}$ and $\eta^{(2)}$ varies, possibly becoming a zero angle or a right angle at a special locus.

In this thesis we will study compactifications with strict SU(3)-structure and static SU(2)-structure. In the following sections we give more details on these two cases. However, there exists a unifying mathematical description of all manifolds having the structures classified above. This description is obtained by a generalization of ordinary complex geometry, called generalized geometry. It turns out that the formalism of generalized geometry is very convenient to calculate quantities such as the induced metric and the Kähler potential. We therefore give a brief introduction to generalized geometry in appendix C. In the following two sections we describe the special cases of strict SU(3)-structure and static SU(2)-structure that we want to consider in this thesis. More details can be found in the above mentioned appendix.

2.1.1 Strict SU(3)-structure

If the structure group of the internal manifold is SU(3) and can not be further reduced into a subgroup of SU(3) we call this a manifold with strict SU(3)-structure. For such a manifold we have one globally defined spinor such that the supersymmetry generators of (2.3) are proportional

$$\eta_+^{(2)} = (b/a)\eta_+^{(1)} , \tag{2.4}$$

with $|\eta_+^{(1)}|^2 = |a|^2$ and $|\eta_+^{(2)}|^2 = |b|^2$. In the following, we will assume $|a| = |b|$ such that $b/a = e^{i\theta}$ is just a phase. This condition is actually imposed by supersymmetry in compactifications to AdS_4 [2]. Let us define a normalized spinor η_+ such that $\eta_+^{(1)} = a\eta_+$ and $\eta_+^{(2)} = b\eta_+$ and moreover we choose the phase of η such that $a = b^*$.

Given an internal manifold with reduced structure group SU(3), we can decompose other SO(6) representations under SU(3). For a vector we have $\mathbf{6} \to \mathbf{3} + \bar{\mathbf{3}}$, for a two-form this reads $\mathbf{15} \to \mathbf{8} + \mathbf{3} + \bar{\mathbf{3}} + \mathbf{1}$ and for a three-form $\mathbf{20} \to \mathbf{6} + \bar{\mathbf{6}} + \mathbf{3} + \bar{\mathbf{3}} + \mathbf{1} + \mathbf{1}$. Thus, there further exists a two-form and a complex three-form both non-vanishing and globally defined, but no invariant vectors (or equivalently five-forms). These forms provide us with an equivalent description of a strict SU(3)-structure. Indeed, with the SU(3)-invariant spinor η we can construct the real non-degenerate two-form J and the complex decomposable three-form Ω as follows

$$J_{mn} = i\eta_+^\dagger \gamma_{mn}\eta_+ \,, \qquad \Omega_{mnp} = \eta_-^\dagger \gamma_{mnp}\eta_+ \,. \tag{2.5}$$

These forms satisfy the SU(3)-structure conditions

$$\Omega \wedge J = 0 \,, \tag{2.6a}$$

$$\Omega \wedge \Omega^* = \frac{4i}{3} J^3 \neq 0 \,, \tag{2.6b}$$

since there is no invariant five-form and there is only one invariant six-form (this can also be shown using Fierz identities). Up to a choice of orientation, a volume normalization is defined as

$$\frac{1}{6} J^3 = -\frac{i}{8}\Omega \wedge \Omega^* = \mathrm{vol}_6 \,. \tag{2.7}$$

Equivalently, the equations (2.6) and (2.7) completely specify an SU(3)-structure on a six-dimensional manifold, provided that the associated metric to J and Ω is positive definite [3].

The existence of a globally defined everywhere non-vanishing spinor is a topological condition that reduces the structure group to SU(3). As we will explain in section 2.2.1 in more detail, the conditions for a supersymmetric vacuum imposes further differential constraints on the spinor. In the simplest case, where no background fluxes are turned on, a supersymmetric solution requires the internal spinor to be covariantly constant with respect to the Levi-Civita connection, $\nabla_m \eta = 0$. From eq. (2.5) we thus obtain

$$\mathrm{d}J = \mathrm{d}\Omega = 0 \,. \tag{2.8}$$

Such a manifold has SU(3)-holonomy [4] and is called a Calabi-Yau manifold.

[2] As a matter of fact, the condition $|a| = |b|$ is also implied by the orientifold projection that we will impose in our concrete models (see further appendix C).

[3] In appendix C it is explained in term of generalized geometry how to obtain the metric associated to J and Ω.

[4] The holonomy group is the group generated by transformations induced by parallel transport around loops. The covariant constant spinor remains the same by parallel transport around a loop. Following the same arguments as above, the holonomy group is reduced to SU(3).

These conditions change drastically in the presence of fluxes, where the supersymmetry conditions imply that the spinors are not covariantly constant with respect to the Levi-Civita connection. The failure of the manifold to be of special holonomy or equivalently the deviation from closure of J and Ω is parameterized by the intrinsic torsion. To be more precise, on a manifold with SU(3)-structure there is always a metric compatible connection ∇' (i.e., a connection with $\nabla'_m g_{np} = 0$) with or without torsion that has SU(3)-holonomy, $\nabla'_m \eta = 0$. In case this connection is torsionless, the manifold is Calabi-Yau. The part of the torsion which is independent of the choice of ∇' is known as the intrinsic torsion and can be used to classify the types of SU(3)-structures. The intrinsic torsion tensor can be decomposed in terms of SU(3) representations as follows

$$\begin{aligned} T_{mn}{}^p &\in (\mathbf{3} \oplus \bar{\mathbf{3}}) \otimes (\mathbf{1} \oplus \mathbf{3} \oplus \bar{\mathbf{3}}) \\ &= (\mathbf{1} \oplus \mathbf{1}) \oplus (\mathbf{8} \oplus \mathbf{8}) \oplus (\mathbf{6} \oplus \bar{\mathbf{6}}) \oplus 2(\mathbf{3} \oplus \bar{\mathbf{3}}) \\ &\quad\;\; \mathcal{W}_1 \qquad\quad \mathcal{W}_2 \qquad\quad \mathcal{W}_3 \qquad\quad \mathcal{W}_4\ \mathcal{W}_5\,, \end{aligned} \tag{2.9}$$

where the $\mathcal{W}_i$ are the torsion classes [40, 41]. Here, $\mathcal{W}_1$ is a scalar, $\mathcal{W}_2$ is a primitive (1,1)-form, $\mathcal{W}_3$ is a real primitive $(1,2) + (2,1)$-form, $\mathcal{W}_4$ is a real one-form and $\mathcal{W}_5$ a complex (1,0)-form.

It follows that $\mathrm{d}J$ and $\mathrm{d}\Omega$ can be decomposed using these torsion classes in the following way

$$\begin{aligned} \mathrm{d}J &= \frac{3}{2}\mathrm{Im}\,(\mathcal{W}_1\Omega^*) + \mathcal{W}_4 \wedge J + \mathcal{W}_3\,, \\ \mathrm{d}\Omega &= \mathcal{W}_1 J \wedge J + \mathcal{W}_2 \wedge J + \mathcal{W}_5^* \wedge \Omega\,, \end{aligned} \tag{2.10}$$

A classification of special manifolds in terms of vanishing torsion classes is given in table 2.1 [20]. For example, a manifold is *complex* if the first two torsion classes vanish, $\mathcal{W}_1 = \mathcal{W}_2 = 0$. Indeed, if this is valid, $\mathrm{d}\Omega$ is a (3,1)-form, the only possibility on a complex manifolds, since Ω is a (3,0)-form. For a *symplectic* manifold, the fundamental two-form J has to be closed and one has therefore $\mathcal{W}_1 = \mathcal{W}_3 = \mathcal{W}_4 = 0$. A *Kähler* manifold is complex and symplectic, which implies that $\mathcal{W}_5$ is the only non-vanishing torsion class, and J is called the Kähler form. In this case, the manifold has U(3)-holonomy that is reduced to SU(3)-holonomy by further choosing $\mathcal{W}_5 = 0$ so that all the torsion classes vanish and we are left with a *Calabi-Yau* manifold.

Of special interest for this thesis are manifolds for which only the classes $\mathcal{W}_1$ and $\mathcal{W}_2$ are non-vanishing and in this class the special case where also $\mathcal{W}_2$ vanishes, the so-called *nearly-Kähler* manifolds. Further, as we will see in section 2.3, a supersymmetric solution with non constant warp factor/dilaton implies a manifold with non-vanishing fifth torsion class $\mathcal{W}_5$.

For later convenience, let us also mention that the pure spinors associated to a strict SU(3)-structure are given as follows (see also appendix C)

$$\Psi_- = -\Omega\,, \qquad \text{and} \qquad \Psi_+ = e^{-i\theta} e^{iJ}\,, \tag{2.11}$$

where J and Ω are defined in eq. (2.5).

Torsion classes	Name of manifold
$W_1 = W_2 = 0$	Complex
$W_1 = W_3 = W_4 = 0$	Symplectic
$\mathrm{Im}\, W_1 = \mathrm{Im}\, W_2 = W_4 = W_5 = 0$	Half-flat
$W_1 = W_2 = W_4 = W_5 = 0$	Special Hermitean
$W_2 = W_3 = W_4 = W_5 = 0$	Nearly-Kähler
$W_1 = W_3 = W_4 = W_5 = 0$	Almost-Kähler
$W_1 = W_2 = W_3 = W_4 = 0$	Kähler
$W_1 = W_2 = W_3 = W_4 = W_5 = 0$	Calabi-Yau
$W_1 = W_2 = W_3 = 3W_4 - 2W_5 = 0$	"Conformal" Calabi-Yau

Table 2.1: Classification of manifolds from vanishing torsion classes.

2.1.2 Static SU(2)-structure

Further reducing the structure group to SU(2), we have two independent non-vanishing, globally defined spinors, $\eta^{(1,2)}$. In the following we assume that $\eta^{(1)}$ and $\eta^{(2)}$ are orthogonal everywhere such that we have a static SU(2)-structure as defined in section 2.1. The static SU(2)-structure is a special case of the more general SU(3)×SU(3)-structure. In appendix C we will give an introduction into the language of generalized geometry and SU(3)×SU(3)-structures. In this section we will discuss the main formulas needed to deal with static SU(2)-structure compactifications.

Having two nowhere vanishing and orthogonal spinors $\eta^{(1)}$ and $\eta^{(2)}$, we can, just as for the SU(3)-structure, define the SU(2)-structure in terms of SU(2)-invariant forms. Following [42, 43, 39], we choose to parameterize the two orthogonal spinors as follows

$$\eta_+^{(1)} = a\eta_+ \,, \tag{2.12a}$$

$$\eta_+^{(2)} = bV^i\gamma_i\eta_- \,, \tag{2.12b}$$

where $|\eta_+^{(1)}|^2 = |a|^2$ and $|\eta_+^{(2)}|^2 = |b|^2$, which imposes $|V|^2 = 1/2$. Again, we choose in the following $|a| = |b|$, which is implied by the orientifold projection [43] and we choose the relative phases of the spinors such that $a = b^*$ and $b/a = e^{i\theta}$, where only the phase θ is physical. We further define a normalized spinor $\tilde{\eta}_+ = \eta_+^{(2)}/b$, i.e.

$$\tilde{\eta}_+ = V^i\gamma_i\eta_- \,, \tag{2.13}$$

and one constructs the one-form associated to the vector V in terms of the spinors as

$$V_k = \frac{1}{2}\eta_-^\dagger\gamma_k\tilde{\eta}_+ \,. \tag{2.14}$$

In addition we can construct from bilinears of the spinor fields a real two-form ω_2 and a holomorphic two-form Ω_2 as follows

$$\omega_{2\,ij} = i\eta_+^\dagger \gamma_{ij} \eta_+ - i\tilde{\eta}_+^\dagger \gamma_{ij} \tilde{\eta}_+ \,, \tag{2.15a}$$

$$\Omega_{2\,ij} = \tilde{\eta}_+^\dagger \gamma_{ij} \eta_+ \,. \tag{2.15b}$$

These forms are obviously SU(2)-invariant. Using Fierz identities these forms satisfy the following structure conditions [42, 39]

$$\omega_2 \wedge \omega_2 = \frac{1}{2} \Omega_2 \wedge \Omega_2^* \neq 0 \tag{2.16a}$$

$$\omega_2 \wedge \Omega_2 = 0 \,, \qquad \Omega_2 \wedge \Omega_2 = 0 \,, \tag{2.16b}$$

$$\iota_V \Omega_2 = 0 \,, \qquad \iota_V \omega_2 = 0 \,. \tag{2.16c}$$

Equivalently, forms ω_2, Ω_2 and V satisfying eq. (2.16) completely specify a static SU(2)-structure, provided that the associated metric is positive definite. We explain how to obtain this associated metric in appendix C.

Note that the SU(2)-structure is naturally embedded in the SU(3)-structure defined by η_+ in eq. (2.5). We get from the eqs. (2.14) and (2.15)

$$J = \omega_2 - 2iV \wedge V^* \,, \qquad \Omega = 2V \wedge \Omega_2 \,, \tag{2.17}$$

and one has the reverse relations

$$\omega_2 = J + 2iV \wedge V^* \,, \qquad \Omega_2 = \iota_{V^*} \Omega \,. \tag{2.18}$$

We then find for the pure spinors associated to a static SU(2)-structure as explained in appendix C

$$\Psi_+ = -e^{-i\theta} e^{2V \wedge V^*} \Omega_2 \,, \tag{2.19a}$$

$$\Psi_- = -2\, V \wedge e^{i\omega_2} \,. \tag{2.19b}$$

In the following, it will be convenient to absorb the phase $e^{-i\theta}$ into Ω_2.

2.2 Supersymmetric solutions

Demanding maximal symmetry of a vacuum of the theory, only the bosonic fields can have non-vanishing expectation values. Thus, the supersymmetry variations of the bosonic fields that always contain a fermionic field are automatically vanishing. Hence, we have just to consider the variation of the fermionic fields. For a supersymmetric vacuum we require that the vacuum expectation value of the supersymmetric variation of all fermionic fields χ vanish, $\langle \delta\chi \rangle = 0$.

As we have seen in the introduction, the fermionic fields in type II theories are two gravitini ψ_M^1 and ψ_M^2 and two dilatini λ^1 and λ^2. We can combine these Majorana-Weyl spinors in a column vector $\psi_M = \left(\psi^1_M, \psi^2_M\right)^{\mathrm{T}}$, and similarly for λ. In the supergravity approximation, the bosonic parts of the supersymmetry variation of the gravitini and dilatini read [20] (in string frame) [5]

$$\delta\psi_M = D_M\epsilon = \left(\nabla_M + \frac{1}{4}\underline{H_M}\mathcal{P} + \frac{e^\Phi}{16}\sum_n \underline{F_n}\Gamma_M\mathcal{P}_n\right)\epsilon\,, \tag{2.20a}$$

$$\delta\lambda = \left(\underline{\partial}\Phi + \frac{1}{2}\underline{H}\mathcal{P} + \frac{e^\Phi}{8}\sum_n(-1)^n(5-n)\underline{F_n}\mathcal{P}_n\right)\epsilon\,, \tag{2.20b}$$

respectively, where underline denotes the contraction with gamma-matrices as defined in eq. (A.18), and

$$\mathrm{IIA}:\ \mathcal{P} = \Gamma_{(10)}\,, \qquad \mathrm{IIB}:\ \mathcal{P} = -\sigma_3, \tag{2.21a}$$

$$\mathrm{IIA}:\ \mathcal{P}_n = (\Gamma_{(10)})^{\frac{n}{2}}\sigma_1\,,\quad \mathrm{IIB}:\ \mathcal{P}_n = \sigma_1\ \left(\frac{n+1}{2}\,\mathrm{even}\right),\ i\sigma_2\ \left(\frac{n+1}{2}\,\mathrm{odd}\right). \tag{2.21b}$$

It is sometimes convenient to use the modified dilatino variation

$$\Gamma^M\delta\psi_M - \delta\lambda = \left(\underline{\nabla} - \underline{\partial}\Phi + \frac{1}{4}\underline{H}\mathcal{P}\right)\epsilon\,. \tag{2.22}$$

A type II geometry will preserve supersymmetry if and only if there is at least one ϵ for which all the supersymmetry variations (2.20) vanish. The number of such ϵ's determine the number of supercharges and thus the amount of supersymmetry in four dimensions. As we will see, these conditions place strong constraints on the geometry.

To preserve maximal four-dimensional symmetry, we are allowed to turn on only those fluxes which have either no leg or four legs along four-dimensional space-time. We require

$$F = \hat{F} + \mathrm{vol}_4 \wedge e^{4A}\tilde{F}\,, \tag{2.23}$$

with vol_4 the (unwarped) four-dimensional volume form such that $\hat{F}$ and $\tilde{F}$ are purely internal forms. This allows us to write the supersymmetry variations in terms of internal fluxes only. The Hodge duality (B.1) (here in string frame) then implies the following relation

$$\tilde{F} = \mp\alpha(\star_6\hat{F})\,, \tag{2.24}$$

for IIA/IIB, and the operator α reversing the order of the indices is defined in appendix A. In the following we will drop the hat symbol and hope that the notation is clear.

[5]Here we use the democratic formulation of [44]. See appendix B for our conventions.

2.2.1 Supersymmetric backgrounds without fluxes

For the simplest case when no fluxes are present, the gravitino variation (2.20a) requires the existence of a covariantly constant spinor on the ten-dimensional manifold, $\nabla_M \epsilon = 0$. The four-dimensional space-time component of this condition, $\nabla_\mu \epsilon = 0$, implies, using integrability conditions, that the warp factor has to be zero and the four-dimensional manifold can only be Minkowski space [20]. When no fluxes are present, we can decompose the ten-dimensional supersymmetry generators as follows (we assume here strict SU(3)-structure)

$$\begin{aligned} \epsilon_1 &= \zeta^1_+ \otimes \eta_+ + \zeta^1_- \otimes \eta_- \,, \\ \epsilon_2 &= \zeta^2_+ \otimes \eta_\mp + \zeta^2_- \otimes \eta_\pm \,, \end{aligned} \tag{2.25}$$

for IIA/IIB, where $\zeta^A_\pm$, $A = 1, 2$ are two four-dimensional Weyl spinors. This compactification preserve eight supercharges which implies $\mathcal{N} = 2$ in four dimensions.

We solve the internal component of the gravitino variation, $\nabla_m \epsilon = 0$, with the decomposition ansatz (2.25) provided that

$$\nabla_m \eta_\pm = 0\,, \tag{2.26}$$

which means that the internal manifold has to admit the existence of a non-vanishing, globally defined six-dimensional spinor that is covariantly constant (with respect to the Levi-Civita connection). As mentioned earlier, this condition implies that the internal manifold not only has SU(3)-structure but also SU(3)-holonomy and is a Calabi-Yau manifold.

2.2.2 Supersymmetric backgrounds in the presence of fluxes

If we turn on background fluxes, the supersymmetry conditions (2.20) relate the two supersymmetry parameters ϵ_1 and ϵ_2 in (2.25). It turns out that the four-dimensional supersymmetry parameters ζ^1 and ζ^2 cannot be chosen independently anymore, breaking four-dimensional supersymmetry to $\mathcal{N} = 1$. Demanding maximal four-dimensional symmetry, ζ^1 and ζ^2 should be proportional, and we arrive at the most general ansatz for $\mathcal{N} = 1$ in four dimensions given in eq. (2.3).

In the following, we will specify the necessary and sufficient conditions for $\mathcal{N} = 1$ compactifications of (massive) type IIA supergravity to AdS_4 with the strict SU(3)-structure ansatz.

Type IIA strict SU(3)-structure supersymmetry conditions

The necessary and sufficient conditions for $\mathcal{N} = 1$ compactifications of (massive) type IIA supergravity to AdS_4 with the strict SU(3)-structure ansatz (2.4) were first given in [31]. These vacua require constant warp factor, A, and constant dilaton, Φ. The solutions of [31] are given by [6]:

$$H = \frac{2m}{5} e^{\Phi} \mathrm{Re}\,\Omega\,, \tag{2.27a}$$

$$F_2 = \frac{f}{9} J + F_2'\,, \tag{2.27b}$$

$$F_4 = f \mathrm{vol}_4 + \frac{3m}{10} J \wedge J\,, \tag{2.27c}$$

$$W e^{i\theta} = -\frac{1}{5} e^{\Phi+A} m + \frac{i}{3} e^{\Phi+A} f\,, \tag{2.27d}$$

where H is the NSNS three-form, and F_n denote the RR-forms. In the following, we will set the warp factor equal A to zero. Furthermore, f and m are constants parameterizing the solution: f is the Freund-Rubin parameter, while m is the mass of Romans' supergravity [45] – which can be identified with F_0 in the 'democratic' formulation [44].

The constant W is defined by the following relation for the AdS_4 Killing spinors, $\zeta_\pm$,

$$\nabla_\mu \zeta_- = \frac{1}{2} W \gamma_\mu \zeta_+\,, \tag{2.28}$$

so that the radius of AdS_4 is given by $|W|^{-1}$. The two-form F_2' is the primitive part of F_2 (i.e. it is in the **8** of SU(3)).

The intrinsic torsion of the internal manifold is constrained by supersymmetry and the Bianchi identities. The only non-zero torsion classes are $\mathcal{W}_1$ and $\mathcal{W}_2$, and they are purely imaginary what we indicate with a minus superscript, i.e., $\mathcal{W}_{1,2} = \mathcal{W}_{1,2}^- = i\mathrm{Im}\mathcal{W}_{1,2}^-$. The forms J and Ω thus satisfy (see the definition of the torsion classes in eq. (2.10))

$$\mathrm{d}J = -\frac{3}{2} i\, \mathcal{W}_1^- \mathrm{Re}\,\Omega\,, \tag{2.29a}$$

$$\mathrm{d}\Omega = \mathcal{W}_1^- J \wedge J + \mathcal{W}_2^- \wedge J\,, \tag{2.29b}$$

where the torsion classes are given by:

$$\mathcal{W}_1^- = -\frac{4i}{9} e^{\Phi} f\,, \qquad \mathcal{W}_2^- = -i e^{\Phi} F_2'\,. \tag{2.30}$$

From eq. (2.27) and eq. (2.30) we immediately conclude that F_2' is constrained by the Bianchi identity for F_2 (see eq. (B.9a)):

$$\mathrm{d}F_2' = (\frac{2}{27} f^2 - \frac{2}{5} m^2) e^{\Phi} \mathrm{Re}\,\Omega - j^6\,, \tag{2.31}$$

[6]As opposed to [31] we do not use superspace conventions. Furthermore we use here the string frame and put $m = -2m_{\mathrm{there}}$, $H = -H_{\mathrm{there}}$, $J = -J_{\mathrm{there}}$, $F_2 = -2m_{\mathrm{there}}B'$ and $F_4 = -G$.

where we have added a source, j^6, for D6-branes/O6-planes on the right-hand side. In addition, for vanishing sources, equation (2.31) yields $\mathrm{d}\mathcal{W}_2^- \propto \mathrm{Re}\,\Omega$. It is convenient to define the following proportionality constants c_1 and c_2

$$\mathrm{d}J = -\frac{3}{2}i\,\mathcal{W}_1^-\,\mathrm{Re}\,\Omega = c_1\mathrm{Re}\,\Omega\,, \tag{2.32a}$$

$$\mathrm{d}\mathcal{W}_2^- = ic_2\,\mathrm{Re}\,\Omega\,, \tag{2.32b}$$

where we show in appendix C

$$c_2 = -\frac{1}{8}|\mathcal{W}_2^-|^2\,. \tag{2.33}$$

The absolute value of a two-form Θ is defined as $|\Theta|^2 := \Theta^*_{mn}\Theta^{mn}$.

For the sourceless case it was proven by analyzing integrability conditions that a background that is supersymmetric and whose fluxes satisfy the Bianchi identities and equations of motion is a solution to the full equations of motion (whenever there are no mixed external-internal components of the Einstein tensor, which we will assume) [46, 47, 31]. Turning on sources, the Bianchi identities get modified by these sources. Assuming these sources to be supersymmetric (they must be generalized calibrated as in [48]) it can similarly be shown that, under mild assumptions, supersymmetry guarantees the appropriately source-modified Einstein and dilaton equation of motion to be automatically satisfied if these source-modified Bianchi identities and form equations of motion are satisfied [43].

For vanishing source, we find from the eqs. (2.31), (2.32) and (2.33) that the following bound on $(\mathcal{W}_1^-, \mathcal{W}_2^-)$ has to be satisfied for a geometry to be a supersymmetric background

$$\frac{16}{5}e^{2\Phi}m^2 = 3|\mathcal{W}_1^-|^2 - |\mathcal{W}_2^-|^2 \geq 0\,. \tag{2.34}$$

This is a very restrictive condition for a manifold to be suitable for a supersymmetric solution. Let us note that condition (2.34) turns out to be too stringent to be satisfied for any nilmanifold whose only non-zero torsion classes are $\mathcal{W}_{1,2}^-$ [49].

To relax the this restrictive constraint (2.34) we may allow for a brane/orientifold source, $j^6 \neq 0$. The simplest source we can consider is one proportional to $\mathrm{Re}\,\Omega$ [50]:

$$j^6 = -\frac{2}{5}e^{-\Phi}\mu\mathrm{Re}\,\Omega\,, \tag{2.35}$$

where μ is a *discrete*, real parameter of dimension $(\text{mass})^2$, so that $-\mu$ is proportional to the orientifold/D6-brane charge (μ is positive for net orientifold charge and negative for net D6-brane charge) [7]. For the choice (2.35) the source wraps supersymmetric cycles,

[7] To be more precise, the charge of a D6-brane is $\mu_6 = (2\pi)^{-6}\alpha'^{-7/2}$, whereas the charge of a O6-plane is $-2\mu_6$. An orientifold plane is not a genuine supergravity object, but defined by the string compactification, where the orientifold plane is the fixed point locus of the involution σ^*. Thus, for net D-brane charge, $\mu < 0$ is an arbitrary, discrete parameter (proportional to the number of D6-branes), whereas for net orientifold charge, $\mu > 0$ is fixed by the charge of the orientifold. However, for the explicit calculations in this thesis, we take the pragmatic point of view that we can enrich the supergravity action by an object with arbitrary negative charge [38].

which is easily verified by looking at the calibration conditions for D6-branes/O6-planes:

$$j^6 \wedge \mathrm{Re}\,\Omega = 0\,, \qquad j^6 \wedge J = 0\,, \tag{2.36}$$

which are satisfied for (2.35). The constraint coming from (2.31) now reads

$$e^{2\Phi} m^2 = \mu + \frac{5}{16}\left(3|\mathcal{W}_1^-|^2 - |\mathcal{W}_2^-|^2\right) \geq 0\,. \tag{2.37}$$

Since we assume that μ is arbitrary, the above equation can always be satisfied, and therefore no longer imposes any constraint on the torsion classes of the manifold. With orientifolds sources there are possible solutions on nilmanifolds as we will demonstrate in chapter 6.

There is a more general choice as (2.35) for the source that satisfies the calibration condition (2.36):

$$j^6 = -\frac{2}{5} e^{-\Phi} \mu \mathrm{Re}\,\Omega + w_3\,, \tag{2.38}$$

where w_3 has to be a primitive (1,2)+(2,1)-form. For this choice we relax the constraint that $\mathrm{d}\mathcal{W}_2^- \propto \mathrm{Re}\,\Omega$ such that

$$\mathcal{W}_2^- = ic_2 \mathrm{Re}\,\Omega + ie^{\Phi} w_3\,. \tag{2.39}$$

The condition (2.34) is still the same, since it involves only the (3,0) and (0,3)-part of eq. (2.31).

As we have mentioned, for some of our models we will study, the inclusion of smeared orientifold sources is required to relax the bound from (2.34) to (2.37) and to allow for a supersymmetric AdS_4 solution. In appendix D we explain how to associate orientifold involutions to a smeared source. Under each orientifold involution the dilaton, metric and fluxes must transform as follows [43]:

$$\begin{aligned} &\text{Even}: \quad && \sigma^* e^{\Phi} = e^{\Phi}\,, \quad && \sigma^* F_0 = F_0\,, \quad && \sigma^* F_4 = F_4\,, \\ &\text{Odd}: && \sigma^* H = -H\,, && \sigma^* F_2 = -F_2\,, && \end{aligned} \tag{2.40a}$$

whereas the SU(3)-structure transforms as

$$\begin{aligned} &\text{Even}: \quad && \sigma^* \mathrm{Im}\,\Omega = \mathrm{Im}\,\Omega\,, && \\ &\text{Odd}: && \sigma^* \mathrm{Re}\,\Omega = -\mathrm{Re}\,\Omega\,, \quad && \sigma^* J = -J\,. \end{aligned} \tag{2.40b}$$

Let us mention that there is no $\mathcal{N}=1$ AdS_4 solution for a compactification in type IIA with static SU(2)-structure, as was already noted in [42]. We provide a very simple proof for this statement in terms of generalized geometry in appendix C. The same type of argument is also applicable for the type IIB side, where it is easy to see that there is no $\mathcal{N}=1$ AdS_4 solution for type IIB and strict SU(3)-structure possible. We summarize these results in table 2.2.

$\mathcal{N}=1$ AdS$_4$ solution	type IIA	type IIB
strict SU(3)	possible	not possible
static SU(2)	not possible	possible

Table 2.2: Possible $\mathcal{N}=1$ AdS$_4$ solution for type IIA/IIB with strict SU(3)-structure or static SU(2)-structure.

Hierarchy of scales

For a solution satisfying the type IIA conditions given in this section to be a valid supergravity approximation, we have to verify that the string loops can be safely ignored and that we can ignore α'-corrections. We thus have to show that we can consistently take the string coupling constant to be small ($g_s = e^{\Phi} \ll 1$), and that the volume of the internal manifold is large in string units ($L_{\text{int}}/l \gg 1$, where L_{int} is the characteristic length of the internal manifold).

As we will show in the following, we can always choose the background fluxes in a way that the supergravity approximation is valid. In the full quantum theory, all the fluxes have to be quantized according to

$$\frac{1}{l^{p-1}} \int_{\mathcal{C}_p} F_p = n_p \,, \tag{2.41}$$

where $l := 2\pi\sqrt{\alpha'}$, $\mathcal{C}_p$ is a cycle in the internal manifold, and $n_p \in \mathbb{Z}$. For the supersymmetric solutions we will study in part II and III of this thesis, the NSNS three-form H turns out to be exact (in fact, since $H \propto \text{Re}\Omega \propto \text{d}J$ this follows from the supersymmetry conditions in section 2.2.2, see first equation in (2.27) and eq. (2.29a)), hence its integral over any internal three-cycle vanishes, and it therefore suffices to impose (2.41) for the RR-fluxes.

Concretely, in chapter 8 and 11, where we will study the mass spectrum around the supersymmetric solutions for our models, we choose conventions such that

$$J \sim L_{\text{int}}^2 \,, \quad \Omega \sim L_{\text{int}}^3 \,. \tag{2.42}$$

We immediately conclude from (2.27), (2.29) and (2.31) the following scalings [8]

$$F_p \sim \frac{1}{g_s L_{\text{int}}} L_{\text{int}}^p \,, \quad |\mathcal{W}_i|^2 \sim L_{\text{int}}^{-2} \,. \tag{2.43}$$

We thus define $f_p/(g_s L_{\text{int}})$ as the norm of the flux density F_p, where f_p is some number depending on the geometry. The quantization condition (2.41) then implies

$$f_p g_s^{-1} L_{\text{int}}^{p-1} = l^{p-1} n_p \,, \tag{2.44}$$

[8] In our conventions, the structure constants are dimensionless such that the derivative does not influence the scaling.

from which one easily derives the following equations

$$\begin{aligned} g_s = (f_0^3 f_4)^{\frac{1}{4}} (n_0^3 n_4)^{-\frac{1}{4}} \,, \quad \frac{L_{\text{int}}}{l} &= \left(\frac{f_0}{f_4}\right)^{\frac{1}{4}} \left(\frac{n_4}{n_0}\right)^{\frac{1}{4}} \,, \\ \frac{n_2}{\sqrt{n_0 n_4}} = \frac{f_2}{\sqrt{f_0 f_4}} \,, \quad \frac{n_0 n_6}{n_2 n_4} &= \frac{f_0 f_6}{f_2 f_4} \,. \end{aligned} \tag{2.45}$$

Given a solution $\{n_p\}$ to the quantization conditions (2.44), there are several different possible scalings $n_p \to N^{\lambda_p} n_p$, for $N, \lambda_p \in \mathbb{N}$, which leave the f_p's invariant and, at the same time, ensure that g_s is parametrically small while L_{int}/l is parametrically large (with large parameter N). For instance, assume the rescaling

$$n_0 \to N^4 n_0 \,, \qquad n_2 \to N^6 n_2 \,, \qquad n_4 \to N^8 n_4 \,, \qquad n_6 \to N^{10} n_6 \,, \tag{2.46}$$

and it is easy to verify that the f_p's are invariant whereas $g_s \propto N^{-5}$ and $L_{\text{int}}/l \propto N$. Despite the fact that we are allowing for large flux quanta, it can be shown that higher-order flux corrections can also be neglected. Indeed it is not difficult to see that the parameter $|g_s F_p|^2$, which controls the size of these corrections, scales with a negative power of the large parameter N [51].

Decoupling of Kaluza-Klein modes

A further consistency requirement is that the Kaluza-Klein tower can be decoupled, i.e., we have to establish that the lightest excitations above the Breitenlohner-Freedman bound with mass squares m^2_{LM} are much lighter than the Kaluza-Klein excitations with mass square m^2_{KK}. This is the problem of separation of scales. One can take the point of view that this problem should not be discussed until the model is uplifted to a phenomenologically viable model with a small, positive cosmological constant - a procedure that also changes the mass spectrum of the lightest modes such that it is necessary to re-address the separation of scales.

However, in the following we will study the conditions for the separation of scales even before the uplifting. It will actually turn out that the separation of scales is difficult to establish and will not be possible for most of the models we study in this thesis such that one may hope that after an uplift procedure the scales are properly separated. Nevertheless, let us study the conditions for decoupling the Kaluza-Klein modes.

As we will discuss in section 3.1, the mass squares of the lightest excitations above the Breitenlohner-Freedman bound are of order $|W|^2$ whereas the massive states of the Kaluza-Klein tower have mass squares of the order L^{-2}_{int}. The necessary condition to have a hierarchy between the lightest excitations and the Kaluza-Klein tower can thus be rewritten as follows

$$|W|^2 L^2_{\text{int}} \ll 1 \,. \tag{2.47}$$

Using (2.27d) we find that to decouple the Kaluza-Klein scale we must impose

$$|W|^2 L_{\rm int}^2 = \frac{1}{25}(g_s)^2 m^2 L_{\rm int}^2 + \frac{1}{9}(g_s)^2 f^2 L_{\rm int}^2 \ll 1\,, \tag{2.48}$$

which means that each of the two terms on the right-hand side of the equal sign must be separately much smaller than one.

Let us first consider the second square in the condition for a decoupling of the Kaluza-Klein scale (2.48). This requires $(g_s)^2 f^2 L_{\rm int}^2 \ll 1$. From eq. (2.30) this conditions reads

$$|\mathcal{W}_1^-| L_{\rm int} \ll 1\,, \tag{2.49}$$

which requires manifolds for which $\mathcal{W}_1^-$ vanishes (and only $\mathcal{W}_2^-$ is possibly non-zero). Such manifolds are called 'nearly Calabi-Yau' (NCY), see e.g. [52]. Hence, for the decoupling of the Kaluza-Klein scales, the internal manifold must admit an SU(3)-structure which is sufficiently close to the NCY limit.

The first square of condition (2.48) yields the condition $(g_s)^2 m^2 L_{\rm int}^2 \ll 1$. Using eq. (2.37) this condition is equivalent to

$$\mu L_{\rm int}^2 + \frac{5}{16}\left(3|\mathcal{W}_1^-|^2 - |\mathcal{W}_2^-|^2\right) L_{\rm int}^2 \ll 1\,. \tag{2.50}$$

Note that without source terms it is not possible to satisfy this condition (unless $3|\mathcal{W}_1^-|^2 - |\mathcal{W}_2^-|^2 \ll L_{\rm int}^{-2}$). However, with source terms we just need to show that we can choose μ so that it is close to its bound to satisfy (2.50). The discrete parameter μ, which is, for $\mu < 0$, proportional to the net number of D6-branes $n_{\rm D6}$, scales as (up to numerical factors of order one)

$$\mu \sim -n_{\rm D6} g_s l L_{\rm int}^{-3}\,, \tag{2.51}$$

as can be seen from the quantization condition for F_2, and the Bianchi identity for F_2 (B.9a). With eq. (2.43) we can rewrite this equation schematically as follows:

$$-n_{\rm D6} g_s \left(\frac{l}{L_{\rm int}}\right) + a \ll 1\ , \tag{2.52}$$

where a is a number of order one. Since $g_s\left(\frac{l}{L_{\rm int}}\right) \ll 1$, we can then satisfy this bound, provided that a is positive, by choosing some large integer $n_{\rm D6}$.

Once a solution for $n_{\rm D6}$ is obtained in this way, we are free to rescale $n_{\rm D6} \to N^q n_{\rm D6}$ leaving (2.52) invariant, provided we take: $q = (\lambda_0 + \lambda_4)/2 \in \mathbb{N}$. For the example (2.46) we add $n_{\rm D6} \to N^6 n_{\rm D6}$ which leaves eq. (2.52) and all the f_p's in eq. (2.45) invariant and ensures that g_s is parametrically small while $L_{\rm int}/l$ is parametrically large (with large parameter N).

On the other hand, when a turns out to be strictly negative, we can not accomplish (2.52) with $-n_{\rm D6} \to n_{\rm O6}$, which corresponds to net orientifold charge, since the number of orientifolds is not freely adjustable (see also footnote 7). It depends thus on the model we study, whether the Kaluza-Klein modes can be decoupled or not.

2.3 Supersymmetric type IIA solution with non-constant warp factor and dilaton

The (massive) type IIA supersymmetry conditions for $\mathcal{N} = 1$ compactifications to AdS_4 given in section 2.2.2 assumed constant warp factor A and constant dilaton Φ. The condition of constant warp factor/dilaton follows from the supersymmetry equations and the Bianchi identity for F_0. However, we can allow for non-constant warp factor/dilaton provided that we set the Romans mass m to zero [9]. To analyse this in more detail, we rederive the type IIA supersymmetry conditions given in section 2.2.2. It will be convenient for the analysis to use the language of generalized geometry, which we review in appendix C, where the conditions for a supersymmetric solution take a very concise form.

Let us start by first combining eq. (C.43b) and (C.43c) to give

$$\mathrm{d}_H \left[e^{3A-\Phi}\Psi_2\right] = 2We^{2A-\Phi}\mathrm{Re}\,\Psi_1\,, \tag{2.53}$$

where we used the fact that W is a constant. The second equation which we have to solve includes the RR-fluxes (collectively summarized in the polyform $\tilde{F}$) and is given in eq. (C.43a). It reads

$$\mathrm{d}_H \left(e^{4A-\Phi}\mathrm{Im}\,\Psi_1\right) = 3e^{3A-\Phi}\mathrm{Im}\,(W^*\Psi_2) + e^{4A}\tilde{F}\,, \tag{2.54}$$

where $\tilde{F}$ is defined in eq. (2.24).

For a strict SU(3)-structure the pure spinors are given in eq. (2.11), where for type IIA $\Psi_1 = \Psi_-$ and $\Psi_2 = \Psi_+$. We first solve eq. (2.53) which imposes constraints on the geometry. It consists of a one-, three- and five-form part. The one-form part reads

$$\mathrm{d}(e^{3A-\Phi}e^{-i\theta}) = \mathrm{d}(e^{3A-\Phi}W^*e^{-i\theta}) = 0\,, \tag{2.55}$$

where we used in the second equation that W is constant. In the following it will be convenient to include the phase of W into the angle θ' as follows

$$W^*e^{-i\theta} = |W|e^{-i\theta'}\,. \tag{2.56}$$

The conditions resulting from eq. (2.55) are thus

$$\mathrm{d}\theta' = 0\,, \tag{2.57a}$$

$$3\mathrm{d}A - \mathrm{d}\Phi = 0\,. \tag{2.57b}$$

The three-form part of eq. (2.53) is rewritten with eq. (2.57) as follows

$$i\mathrm{d}J + H = -2e^{-A}|W|e^{-i\theta'}\mathrm{Re}\,\Omega\,, \tag{2.58}$$

[9] Recently, this was also emphasized in [53] and in [54, 33] such $\mathcal{N} = 2$ type IIA solutions are constructed from M-theory backgrounds on seven dimensional Sasaki-Einstein manifolds reduced to type IIA.

such that we get

$$\mathrm{d}J = -2e^{-A}|W| \sin\theta' \mathrm{Re}\,\Omega = c_1 \mathrm{Re}\,\Omega\,, \tag{2.59a}$$

$$H = -2e^{-A}|W| \cos\theta' \mathrm{Re}\,\Omega\,, \tag{2.59b}$$

where we introduced the proportionality constant c_1 as in eq. (2.32). Note that eq. (2.59a) implies the vanishing of the torsion classes $\mathcal{W}_3$ and $\mathcal{W}_4$ and constrains the first torsion class to be purely imaginary,

$$\mathcal{W}_1^- = -\frac{4i}{3} e^{-A}|W| \sin\theta'\,. \tag{2.60}$$

The five-form part of eq. (2.53) is very easy, it reads using the result (2.57),

$$\mathrm{d}\left(-\frac{1}{2} J \wedge J\right) + iH \wedge J = 0\,, \tag{2.61}$$

and is automatically satisfied as can easily be seen from eq. (2.59) and the compatibility conditions for a strict SU(3)-structure (2.6).

Let us now analyse the second condition (2.54) involving the RR-fields. It consists of a zero-, two-, four- and six-form part. We first analyse the zero-, two- and six-form part of this equation and analyse the two-form part later. The conditions read, respectively,

$$\begin{aligned} \tilde{F}_0 &= 3e^{-\Phi} e^{-A}|W| \sin\theta'\,, \\ \tilde{F}_2 &= -3e^{-\Phi} e^{-A}|W| \cos\theta' J \\ \tilde{F}_6 &= 3e^{-\Phi} e^{-A}|W| \cos\theta' \mathrm{vol}_6 - e^{-\Phi} H \wedge \mathrm{Im}\,\Omega\,, \end{aligned} \tag{2.62}$$

where we defined the volume vol_6 in eq. (2.7). Using eq. (2.24) [10], these equations translate into

$$\begin{aligned} F_0 &= m\,, \\ F_4 &= \frac{3m}{10} J \wedge J\,, \\ F_6 &= -f \mathrm{vol}_6\,, \end{aligned} \tag{2.63}$$

where we defined

$$\begin{aligned} m &= -5e^{-\Phi} e^{-A}|W| \cos\theta'\,, \\ f &= 3e^{-\Phi} e^{-A}|W| \sin\theta'\,. \end{aligned} \tag{2.64}$$

So far we obtained exactly the conditions given in section 2.2.2, which were first derived in [31]. The crucial point is that the Bianchi identity for F_0 (see eq. (B.9a)) reads

$$\mathrm{d}F_0 = \mathrm{d}(-5e^{-\Phi} e^{-A}|W| \cos\theta') = 0\,, \tag{2.65}$$

[10] Note that we drop the hat on F in the following.

which is equivalent, using eq. (2.57a), to

$$\mathrm{d}\Phi + \mathrm{d}A = 0\,. \tag{2.66}$$

Together with eq. (2.57b), this implies that the warp factor A and the dilaton Φ have to be constant. If, on the other hand, the Romans mass m is vanishing, we do not have this condition from the Bianchi of F_0 and the warp factor A and the dilaton Φ are not constant anymore but still satisfy eq. (2.57b). Hence we choose in the following $\theta' = \pm\frac{\pi}{2}$ such that [11]

$$m = 0 \quad \text{and} \quad f = \pm 3 e^{-\Phi} e^{-A} |W|\,. \tag{2.67}$$

This has now consequences for the geometry, as can be seen as follows. From eq. (2.59a) we obtain

$$0 = \mathrm{d}^2 J = \mathrm{d}(-2e^{-A}|W|\mathrm{Re}\,\Omega)\,, \tag{2.68}$$

such that

$$\mathrm{dRe}\,\Omega = \mathrm{d}A \wedge \mathrm{Re}\,\Omega\,. \tag{2.69}$$

Comparing this with the definitions of the torsion classes in eq. (2.10), we have a non-vanishing fifth torsion class $\mathcal{W}_5$, for which the real part is given by

$$\mathrm{Re}\mathcal{W}_5 = \mathrm{d}A \neq 0\,, \tag{2.70}$$

whereas $\mathrm{Re}\mathcal{W}_1 = \mathrm{Re}\mathcal{W}_2 = 0$. This implies for $\mathrm{Im}\,\Omega$ the following

$$\mathrm{dIm}\,\Omega = -i\mathcal{W}_1^- J \wedge J - i\mathcal{W}_2^- \wedge J + \mathrm{d}A \wedge \mathrm{Im}\,\Omega\,. \tag{2.71}$$

We are now ready to analyse the missing four-form part of eq. (2.54). This equation reads then

$$\tilde{F}_4 = -\frac{3}{2} e^{-\Phi} e^{-A} |W| J \wedge J + e^{-A-\Phi} \mathrm{d}(e^A \mathrm{Im}\Omega)\,, \tag{2.72}$$

which translates, using eq. (2.24) and eq. (2.71), in

$$F_2 = \frac{f}{9} J + F_2' + 2e^{-\Phi} \star_6 (\mathrm{d}A \wedge \mathrm{Im}\,\Omega)\,, \tag{2.73}$$

where we defined

$$F_2' = i e^{-\Phi} \mathcal{W}_2^-\,. \tag{2.74}$$

Let us in the following briefly summarize the results of this analysis. Putting the Romans mass m to zero, there are solutions to the strict SU(3)-structure supersymmetry

[11] In the following, we will choose the plus sign in eq. (2.67).

conditions with non-constant warp factor and dilaton and the following non-vanishing torsion classes

$$\begin{aligned}
\mathcal{W}_1^- &= -\frac{4i}{9} e^{-\Phi} f \,, \\
\mathcal{W}_2^- &= -i e^{\Phi} F_2' \,, \\
\mathrm{Re}\mathcal{W}_5 &= \mathrm{d}A \,.
\end{aligned} \tag{2.75}$$

The background fluxes are given by

$$\begin{aligned}
H &= 0 \,, \\
F_0 &= 0 \,, \\
F_2 &= \frac{f}{9} J + F_2' - 2e^{-\Phi} \star_6 (\mathrm{d}A \wedge \mathrm{Im}\,\Omega) \,, \\
F_4 &= 0 \,, \\
F_6 &= -f \mathrm{vol}_6 \,.
\end{aligned} \tag{2.76}$$

Let us stress again that this time, f is not a constant but $f = 3e^{-\Phi} e^{-A} |W|$. The warp factor A and the dilaton Φ are related by eq. (2.57b).

Chapter 3

Low-energy four-dimensional physics

As we mentioned in the introduction, the idea of geometric compactification is very old. Kaluza [12] and Klein [13, 14] suggested a unification of the gravitational and electromagnetic interaction by postulating an extra compact dimension of space-time. As we will explain in this chapter, the compactification of extra dimensions into a small internal manifold results in an infinite tower of scalar, vector and tensor modes with masses quantized in units of the inverse radius of the internal manifold. In the early days of Kaluza and Klein, however, it was far from clear how to interpret these massive particles, since with the electric charge set equal to its experimentally observed value these masses turned out to be very heavy. The acceptance of extra dimensions was therefore very low.

The discovery of string theory provided another way of introducing higher dimensions into physics. String theory requires a ten-dimensional space-time to be a consistent theory. To make contact with our observed four-dimensional world we need a mechanism to hide six of these ten dimensions from present-day experiments and the idea of Kaluza and Klein gained immediately new interest. Hence, we choose these six extra dimensions to be small and compact such that they are not detectable in present-day experiments. For a given background, the ten-dimensional theory can then be reduced to four dimensions by a Kaluza-Klein reduction, resulting in an infinite tower of Kaluza-Klein modes. Choosing the internal manifold small enough not to be observed, the higher modes of the Kaluza-Klein tower become very heavy and can be integrated out. We end up with an effective four-dimensional theory for the lightest Kaluza-Klein modes. As we mentioned in the introduction, the scalars in this light spectrum correspond to the moduli fields. If there are no background fluxes or metric fluxes present, these moduli will be massless and unstabilized.

For supersymmetric theories, on the other hand, one can compute the four-dimensional effective theory in a more elaborate approach that relies on supersymmetry. We will refer to this approach as the effective supergravity approach. Concretely, for an

$\mathcal{N} = 1$ supergravity, one determines the Kähler potential $\mathcal{K}$ and the superpotential $\mathcal{W}$ in terms of geometrical data of the internal manifold and the background fluxes [1]. By means of these expressions we can straightforwardly construct the effective action, as we will review in the following. We will use the expressions for the Kähler potential and the superpotential derived in [55, 27, 56, 57, 58, 59]. However, we will not blindly adopt these expressions without checking them carefully. We will perform this check by calculating the mass spectrum for two of our explicit models we study in this thesis, both by a direct Kaluza-Klein reduction as well as in the effective supergravity approach. We obtain exactly the same results in both cases such that we will restrict ourselves to the effective supergravity approach for the other manifolds we study.

In this chapter we first give a short survey of the Kaluza-Klein recipe, for a detailed review see e.g. [60]. Next we turn to the description of the effective supergravity approach and comment on the possible choices of background fluxes.

3.1 Kaluza-Klein reduction

In the Kaluza-Klein reduction for a $D = (4 + d)$-dimensional space $\mathcal{M}_D$ to our observed four-dimensional non-compact space-time $\mathcal{M}^{(3,1)}$, one assumes a (warped) product structure for the manifold $\mathcal{M}_D$

$$\mathcal{M}_D = \mathcal{M}^{(3,1)} \times \mathcal{M}_d \,, \tag{3.1}$$

where $\mathcal{M}_d$ represents the d-dimensional compact internal manifold. Let x and y be space-time and internal-manifold coordinates, respectively. The most general ansatz for the background metric is given in eq. (2.2) and reads

$$\mathrm{d}s^2 = e^{2\hat{A}(y)} \hat{g}_{\mu\nu}(x) \mathrm{d}x^\mu \mathrm{d}x^\nu + \hat{g}_{mn}(y) \mathrm{d}y^m \mathrm{d}y^n \,, \tag{3.2}$$

where hatted fields denote a vacuum, i.e. a particular solution of the equations of motion of ten-dimensional supergravity. The requirement of maximal symmetry for the four-dimensional space-time $\mathcal{M}^{(3,1)}$ restricts us to spaces of constant curvature, i.e. to a de Sitter (dS) space for positive curvature, Minkowski for flat space-time and anti de Sitter (AdS) for negative curvature. Maximal space-time symmetry allows the 'warped-product' ansatz including the a warp factor $A(y)$ in eq. (3.2).

Moreover, we denote by $\hat{\Psi}(x, y)$ a 'vacuum' for the different matter fields such as the dilaton, the NSNS two-form B_2 potential or the different RR p-form potentials. The Kaluza-Klein reduction consists in expanding all ten-dimensional fields $g_{MN}(x, y)$ and $\Psi(x, y)$ in 'small' fluctuations around the vacuum:

$$g_{MN}(x, y) = \hat{g}_{MN}(x, y) + \delta g_{MN}(x, y) \,, \tag{3.3a}$$

$$\Psi(x, y) = \hat{\Psi}(x, y) + \delta\Psi(x, y) \,. \tag{3.3b}$$

[1] In our concrete models, there are no vector multiplets such that we will not consider D-terms.

To determine the spectrum of the four-dimensional theory we substitute this expansion in the equations of motion keeping only terms up to linear order in $\delta g_{MN}(x,y)$ and $\delta\Psi(x,y)$ (corresponding to at most quadratic terms in the Lagrangian). Each fluctuation, collectively denoted by $\delta\Phi(x,y)$, is decomposed as a sum of terms of the form

$$\delta\Phi(x,y) = \sum_n \phi_n(x)\omega_n(y)\,, \tag{3.4}$$

where $\phi_n(x)$ are four-dimensional space-time fields and the $\omega_n(y)$'s form a basis of eigenforms of the Laplacian operator [2] $\Delta_d = \mathrm{d}\mathrm{d}^\dagger + \mathrm{d}^\dagger\mathrm{d}$ in the internal d-dimensional space $\mathcal{M}_d$,

$$\Delta_d\omega_n(y) = m_n^2\omega_n(y)\,. \tag{3.5}$$

From the four-dimensional point of view this results in an effective four-dimensional theory with an infinite tower of massive states with masses m_n quantized as $1/L_{\text{int}}$ where L_{int} is the 'radius' of internal manifold such that its volume is of order L_{int}^d. For small internal manifolds these masses will become very heavy and can be integrated out.

In the following we will truncate all the higher Kaluza-Klein modes in the harmonic expansion (3.4) and keep only those $\omega_n(y)$'s in (3.4) that are left-invariant on $\mathcal{M}_d$. The resulting modes are not in general harmonic, but can be combined into eigenvectors of the Laplacian whose eigenvalues are of order of the geometric fluxes.

Plugging the ansatz (3.3)-(3.4) into the ten-dimensional equations of motion and keeping at most linear-order terms in the fluctuations, one can read off the masses of the space-time fields, i.e. the 'spectrum'. In the present case, this is accomplished by comparing with the equations of motion for non-interacting fields propagating in AdS_4. Let M and Λ be the mass of the field and the cosmological constant of the AdS space, respectively, such that

$$\text{Scalar}: \qquad \Delta\phi + \left(M^2 + \frac{2}{3}\Lambda\right)\phi = 0\,, \tag{3.6a}$$

$$\text{Vector}: \qquad \Delta\phi_\mu + \nabla_\mu\nabla^\nu\phi_\nu + M^2\phi_\mu = 0\,, \tag{3.6b}$$

$$\text{Metric}: \qquad \Delta_L h_{\mu\nu} + 2\nabla_{(\mu}\nabla^\rho h_{\nu)\rho} - \nabla_{(\mu}\nabla_{\nu)}h^\rho{}_\rho + (M^2 - 2\Lambda)h_{\mu\nu} = 0\,, \tag{3.6c}$$

where Δ_L is the Lichnerowicz operator defined by:

$$\Delta_L h_{\mu\nu} = -\nabla^2 h_{\mu\nu} - 2R_{\mu\rho\nu\sigma}h^{\rho\sigma} + 2R_{(\mu}{}^\rho h_{\nu)\rho}\,. \tag{3.7}$$

The definition of mass as in eq. (3.6) is such that the massless state, $M = 0$, corresponds to a gauge field with only two degrees of freedom for the metric and vectors and scalars propagating on the light cone [61, 60].

[2] See appendix A for our conventions.

With the above definitions, the Breitenlohner-Freedman bound [61] is simply

$$M^2 \geq 0\,, \tag{3.8}$$

for the metric and the vectors. For the scalars, however, a negative mass-squared is allowed:

$$M^2 \geq \frac{\Lambda}{12} = -\frac{|W|^2}{4}\,, \tag{3.9}$$

where W was defined in eq. (2.28). Actually, we will present the results for the mass spectrum of the scalars in terms of

$$\tilde{M}^2 = M^2 + \frac{2}{3}\Lambda\,, \tag{3.10}$$

for which the Breitenlohner-Freedman bound reads

$$\tilde{M}^2 \geq -\frac{9|W|^2}{4}\,. \tag{3.11}$$

We will take $\tilde{M} = 0$ as the definition of an unstabilized modulus since from (3.6a) we see that then, if it were not for the boundary conditions of AdS_4, a constant shift of ϕ would be a solution to the equations of motion. Therefore a constant shift of ϕ leads to a new vacuum solution.

To determine the spectrum of the four-dimensional theory, we plug the expansion ansatz (3.3) in the equations of motion, where the NSNS- and RR-field strength appear. We would thus like to express the fluctuations of the RR-field strengths δF in terms of the fluctuations of the potentials δC in such a way that the Bianchi identity $d_H F = -j$ is automatically satisfied. How this can be done is explained in the next section.

3.1.1 Bianchi identities

The recipe for the Kaluza-Klein reduction tells us to expand all the fields in 'small' fluctuations around the vacuum. The Bianchi identities for the gauge flux have to be satisfied for the background as well as for the background plus fluctuation, i.e.,

$$(\mathrm{d} + \hat{H})\hat{F} = -j\,, \tag{3.12a}$$

$$(\mathrm{d} + \hat{H} + \delta H)(\hat{F} + \delta F) = -j\,, \tag{3.12b}$$

where we assumed that the source does not fluctuate, since it is associated to smeared orientifolds.

The integrability equations read

$$(\mathrm{d} + \hat{H})j = 0\,, \tag{3.13a}$$

$$(\mathrm{d} + \hat{H} + \delta H)j = 0\,, \tag{3.13b}$$

from which follows

$$\delta H \wedge j = 0 \,. \tag{3.14}$$

From eq. (3.14) and the integrability conditions (3.13a) we show that

$$(\mathrm{d} + \hat{H})(e^{\delta B} \wedge j) = 0 \,, \tag{3.15}$$

so that, subtracting (3.13a), we can define (locally)

$$-(e^{\delta B} - 1) \wedge j = (\mathrm{d} + \hat{H})\delta\omega \,. \tag{3.16}$$

Now, for orientifold sources the left hand side of this equation always vanishes. This follows because the pull-back of δB to the orientifold, $\delta B|_\Sigma$, must be zero, which implies using (D.2):

$$\delta B \wedge j = 0 \,, \tag{3.17}$$

and the same for all powers of δB. This implies that we can also choose $\delta\omega = 0$.

The difference between (3.12a) and (3.12b) gives the Bianchi identity for the fluctuations

$$\left(\mathrm{d} + \hat{H} + \delta H\right) \delta F + \delta H \wedge \hat{F} = 0 \,. \tag{3.18}$$

This equation can be rewritten as

$$\left(\mathrm{d} + \hat{H}\right) \left(e^{\delta B} \delta F\right) + \delta H \wedge e^{\delta B} \hat{F} = 0 \,. \tag{3.19}$$

We now introduce the potentials δC to solve this equation. The solution reads

$$e^{\delta B} \delta F = (\mathrm{d} + \hat{H})\delta C - (e^{\delta B} - 1)\hat{F} + \delta\omega \,, \tag{3.20}$$

where we can set $\delta\omega = 0$.

Expanding this expression, we obtain for type IIA the fluctuations

$$\begin{aligned}
&\delta F_0 = 0 \,, \\
&\delta F_2 = \mathrm{d}\delta C_1 - m\delta B \,, \\
&\delta F_4 = \mathrm{d}\delta C_3 + \hat{H} \wedge \delta C_1 - \delta B \wedge (\hat{F}_2 + \delta F_2) - \frac{1}{2} m (\delta B)^2 \,, \\
&\delta F_6 = \mathrm{d}\delta C_5 + \hat{H} \wedge \delta C_3 - \delta B \wedge (\hat{F}_4 + \delta F_4) - \frac{1}{2}(\delta B)^2 \wedge (\hat{F}_2 + \delta F_2) - \frac{1}{3!} m (\delta B)^3 \,,
\end{aligned} \tag{3.21}$$

and for type IIB

$$\begin{aligned}
&\delta F_1 = \mathrm{d}\delta C_0 \,, \\
&\delta F_3 = \mathrm{d}\delta C_2 + \hat{H} \wedge \delta C_0 - \delta B \wedge (\hat{F}_1 + \delta F_1) \,, \\
&\delta F_5 = \mathrm{d}\delta C_4 + \hat{H} \wedge \delta C_2 - \delta B \wedge (\hat{F}_3 + \delta F_3) - \frac{1}{2}(\delta B)^2 \wedge (\hat{F}_1 + \delta F_1) \,.
\end{aligned} \tag{3.22}$$

For the Kaluza-Klein reduction we will only need the terms linear in the fluctuations while for an analysis of finite fluctuations using the Kähler potential and superpotential we need higher orders too.

For the NSNS-flux we can just write

$$H = \hat{H} + \delta H = \hat{H} + \mathrm{d}\delta B \,. \tag{3.23}$$

3.2 Effective supergravity

As already mentioned in the introduction of this chapter, the $\mathcal{N} = 1$ effective four-dimensional action [3] can be obtained from the superpotential $\mathcal{W}$ and the Kähler potential $\mathcal{K}$. The part of the effective four-dimensional action containing the graviton and the scalars reads

$$S = \int \mathrm{d}^4x \sqrt{-g_4} \left(\frac{M_P^2}{2} R - M_P^2 \mathcal{K}_{i\bar{j}} \partial_\mu \phi^i \partial^\mu \bar{\phi}^{\bar{j}} - V(\phi, \bar{\phi}) \right) , \tag{3.24}$$

where M_P is the four-dimensional Planck mass. The scalar potential is given in terms of the superpotential via

$$V(\phi, \bar{\phi}) = M_P^{-2} e^{\mathcal{K}} \left(\mathcal{K}^{i\bar{j}} D_i \mathcal{W}_{\mathrm{E}} D_{\bar{j}} \mathcal{W}_{\mathrm{E}}^* - 3|\mathcal{W}_{\mathrm{E}}|^2 \right) , \tag{3.25}$$

where $D_i \mathcal{W}_{\mathrm{E}} \equiv \partial_i \mathcal{W}_{\mathrm{E}} + (\partial_i \mathcal{K}) \mathcal{W}_{\mathrm{E}}$.

The superpotential and Kähler potential of the effective $\mathcal{N} = 1$ supergravity have been derived in various ways. This is done most generally in terms of pure spinors in the framework of generalized geometry [4]. Here we will present the results of these derivations and then specialize the expressions to strict SU(3)- and static SU(2)-structure. As we mentioned, we will verify these expressions by performing a consistency check between this effective supergravity approach and the direct Kaluza-Klein approach that does not rely on supersymmetry. This is done by calculating the mass spectrum for some of our models, both by direct Kaluza-Klein reduction (chapter 7) as well as in the effective supergravity approach (chapter 8), obtaining exactly the same results in both cases (see also [62] for related work).

In [27, 56] (based on earlier work of [55]) the superpotential has been computed at the level of the fermionic effective action. One uses the fact that the superpotential $\mathcal{W}$ appears linearly in a four-dimensional $\mathcal{N} = 1$ supergravity theory as the mass term of the gravitino ψ_μ,

$$S \propto \int \mathrm{d}^4x \sqrt{-g} e^{\mathcal{K}/2} \left(\mathcal{W} \bar{\psi}_\mu \bar{\sigma}^{\mu\nu} \bar{\psi}_\nu + \text{c.c.} \right) . \tag{3.26}$$

Thus, focusing on the mass terms of ψ_μ in the explicit reduction of the fermionic part of the ten-dimensional effective action provides us with an explicit expression for

[3] In our concrete models, there are no vector fields in the spectrum such that we will not consider gauge kinetic couplings and D-terms.

[4] See appendix C for details on pure spinors and generalized geometry.

the superpotential $\mathcal{W}$. We skip here the details of the calculation and just quote the result in eq. (3.27). However, let us mention that the expression for the superpotential obtained in this way agrees with the expression obtained in [57], where the authors derive the superpotential using a Gukov-Vafa-Witten type argument involving domain walls, which was also used in [63] in the specific case of Calabi-Yau compactifications. The argument relies on the tension of a BPS domain wall. From the four-dimensional point of view, the domain wall separates two supersymmetric flux configurations. By energy conservation one gets a relation between the tension of the domain wall and the difference of the superpotential on both sides of the domain wall, $T_{\rm DW} \propto |\Delta\mathcal{W}|$. On the other hand, the tension of the domain wall is obtained as the integral of the calibration over the internal generalized cycle which the domain wall wraps. By further demanding holomorphicity, the authors of [57] proposed an explicit expression for the superpotential.

In both approaches one arrives at the following expressions for the superpotential in Einstein frame

$$\mathcal{W}_{\rm E} = \frac{-i}{4\kappa_{10}^2}\int_M \langle \Psi_2, F + i\,\mathrm{d}_H(e^{-\Phi}\mathrm{Im}\,\Psi_1)\rangle \ , \tag{3.27}$$

where $\langle\cdot,\cdot\rangle$ indicates the Mukai pairing (C.10) and Ψ_1 and Ψ_2 are the pure spinors describing the geometry. Using the expansion in background and fluctuations of the eqs. (3.21)-(3.23) we can rewrite the superpotential as follows

$$\mathcal{W}_{\rm E} = \frac{-i}{4\kappa_{10}^2}\int_M \langle \Psi_2 e^{\delta B}, \hat{F} + i\,\mathrm{d}_{\hat{H}}(e^{\delta B}e^{-\Phi}\mathrm{Im}\,\Psi_1 - i\delta C)\rangle \ , \tag{3.28}$$

where we used the property (C.11) of the Mukai pairing. This shows how the fields organize in complex multiplets $\Psi_2 e^{\delta B}$ and $e^{-\Phi}\mathrm{Im}\,\Psi_1 - i\delta C$, which will be clearer in concrete examples.

The Kähler potential reads [5]

$$\mathcal{K} = -\ln i\int_M \langle \Psi_2, \bar{\Psi}_2\rangle - 2\ln i\int_M \langle t, \bar{t}\rangle + 3\ln(8\kappa_{10}^2 M_P^2)\,, \tag{3.29}$$

where we defined $t = e^{-\Phi}\Psi_1$. As we discuss in appendix C in more detail, the real part of a pure spinor is actually a function of its imaginary part. For instance, $\mathrm{Re}t$ is obtained from $\mathrm{Im}t$ via the Hitchin procedure. To take this relation properly into account we use the fact that the Kähler potential for the t-sector may be written as (see eq. (C.15))

$$\mathcal{K}_t = -2\ln 4\int_M H(\mathrm{Im}t)\,, \tag{3.30}$$

where $H(\mathrm{Im}t)$ is the Hitchin functional. More details on how to compute the Hitchin functional are given in appendix C.

[5] The constant last term makes $e^{\mathcal{K}}$ dimensionless.

Note that we have the freedom of a Kähler transformation

$$\mathcal{W}'_{\mathrm{E}} = f^{-3}\mathcal{W}_{\mathrm{E}}\,, \qquad \mathcal{K}' = \mathcal{K} + 3\ln f + 3\ln f^*\,, \tag{3.31}$$

where $f = f(\phi)$ is an arbitrary holomorphic function.

We will later compare the results of an explicit Kaluza-Klein reduction on some of our models with the results obtained from the effective supergravity approach explained in this chapter. To do that, we also have to take into account that the results from the Kaluza-Klein reduction were in the *ten-dimensional* Einstein frame (B.2), whereas using the techniques of this section we get the result in the *four-dimensional* Einstein frame (3.24). To compute the relation between the masses computed in these two frames we note the following relation

$$g_{\mathrm{E}4}\frac{M_P^2}{2} = \frac{e^{2A}}{2\kappa_{10}^2}g_{\mathrm{E}10}\mathrm{Vol}_{\mathrm{E}}\,, \tag{3.32}$$

which we get from (B.2) by integration over the internal manifold and comparing this with (3.24). Thus we have

$$m_{\mathrm{E}4}^2 = \frac{g_{\mathrm{E}10}}{g_{\mathrm{E}4}}m_{\mathrm{E}10}^2 = \kappa_{10}^2 M_P^2 e^{-2A}\mathrm{Vol}_{\mathrm{E}}^{-1}\, m_{\mathrm{E}10}^2\,. \tag{3.33}$$

In the following we specialize the expressions obtained in the generalized geometry language to the specific cases of a strict SU(3)-structure and a static SU(2)-structure.

Type IIA, strict SU(3)-structure

Specializing to the type IIA SU(3) case with pure spinors (2.11), the superpotential takes the form

$$\mathcal{W}_{\mathrm{E}} = \frac{-ie^{-i\theta}}{4\kappa_{10}^2}\int_M \langle e^{i(J-i\delta B)}, \hat{F} - i\mathrm{d}_{\hat{H}}\left(e^{\delta B}e^{-\Phi}\mathrm{Im}\Omega + i\delta C_3\right)\rangle\,, \tag{3.34}$$

and the Kähler potential is given by

$$\mathcal{K} = -\ln\int_M \frac{4}{3}J^3 - 2\ln\int_M 2\,e^{-\Phi}\mathrm{Im}\,\Omega\wedge e^{-\Phi}\mathrm{Re}\,\Omega + 3\ln(8\kappa_{10}^2 M_P^2)\,, \tag{3.35}$$

where $e^{-\Phi}\mathrm{Re}\,\Omega$ should be seen as a function of $e^{-\Phi}\mathrm{Im}\Omega$. On the fluctuations we must impose the orientifold projections (2.40). It turns out that for all the concrete models we will study

$$\delta B\wedge\mathrm{Im}\,\Omega = 0\,, \tag{3.36}$$

since there are no odd five-forms [6]. By expanding in a suitable basis of even and odd expansion forms (which have to be identified separately for each case), we find that the

[6] In fact, for some of the models we will choose the orientifold projections appropriately to project out the one- and five-forms. This is to automatically satisfy the compatibility condition (2.6a) of the strict SU(3)-structure also for the fluctuations.

fluctuations organize naturally in complex scalars

$$J_c = J - i\delta B = (k^i - ib^i)Y_i^{(2-)} = t^i Y_i^{(2-)}\,, \tag{3.37a}$$

$$e^{-\Phi}\mathrm{Im}\,\Omega + i\delta C_3 = (u^i + ic^i)Y_i^{(3+)} = z^i Y_i^{(3+)}\,. \tag{3.37b}$$

Type IIB, static SU(2)-structure

Specializing to the case of type IIB static SU(2)-structure with pure spinors (2.19), the superpotential becomes

$$\mathcal{W}_{\mathrm{E}} = \frac{i}{4\kappa_{10}^2}\int_M \langle 2V \wedge e^{i(\omega_2 - i\delta B)}, \hat{F} - i\mathrm{d}_{\hat{H}}\left(e^{\delta B}e^{-\Phi}\mathrm{Im}\,(e^{2V\wedge\bar{V}}\wedge\Omega_2) + i\delta C\right)\rangle\,, \tag{3.38}$$

and the Kähler potential

$$\mathcal{K} = -\ln\left(-2i\int_M 2V\wedge 2\bar{V}\wedge\omega_2^2\right) - 2\ln\int_M 2\,\langle \mathrm{Re}t, \mathrm{Im}t\rangle + 3\ln(8\kappa_{10}^2 M_P^2)\,, \tag{3.39}$$

where again $\mathrm{Re}t$ should be considered as a function of $\mathrm{Im}t = -\mathrm{Im}\left(e^{-\Phi}e^{2V\wedge\bar{V}}\Omega_2\right)$.

As discussed in appendix C, we can obtain the action of the orientifold involution on the SU(2)-structure quantities from the action of the orientifold on the pure spinors. We find from (C.40) the following

$$\begin{aligned} O5: &\quad \sigma^* V = -V\,, \quad \sigma^*\omega_2 = -\omega_2\,, \quad \sigma^*\Omega_2 = -\Omega_2^*\,, \quad \sigma^*\delta B = -\delta B\,, \\ O7: &\quad \sigma^* V = V\,, \quad \sigma^*\omega_2 = -\omega_2\,, \quad \sigma^*\Omega_2 = \Omega_2^*\,, \quad \sigma^*\delta B = -\delta B\,, \end{aligned} \tag{3.40}$$

and for the RR-sector [43]

$$\begin{aligned} O5: &\quad \sigma^*\delta C_2 = \delta C_2\,, \quad \sigma^*\delta C_4 = -\delta C_4\,, \\ O7: &\quad \sigma^*\delta C_2 = -\delta C_2\,, \quad \sigma^*\delta C_4 = \delta C_4\,. \end{aligned} \tag{3.41}$$

Again we find that the fluctuations organize naturally in complex scalars

$$\omega_c = \omega_2 - i\delta B = (k^i - ib^i)Y_i^{(2--)} = t^i Y_i^{(2--)}\,, \tag{3.42a}$$

$$e^{-\Phi}\mathrm{Im}\,\Omega_2 + i\delta C_2 = (u^i + ic^i)Y_i^{(2+-)} = z^i Y_i^{(2+-)}\,, \tag{3.42b}$$

$$-ie^{-\Phi}2V\wedge\bar{V}\wedge\mathrm{Re}\,\Omega_2 + i\delta C_4 = (v^i + ih^i)Y_i^{(4-+)} = w^i Y_i^{(4-+)}\,, \tag{3.42c}$$

$$2V = C(iY_1^{(1-+)} - \tau Y_2^{(1-+)})\,, \tag{3.42d}$$

where we define $\tau = x + iy$, and each time the first/second sign of the Y_i indicates the behavior under the O5/O7-involution. Note that C is a complex overall factor that is not a degree of freedom. As we will see in the concrete examples, we can eliminate C by performing a Kähler transformation (3.31).

3.3 Choice of background fluxes and bubbles of moduli space

To evaluate the expressions for the superpotential for type IIA strict SU(3)-structure or type IIB static SU(2)-structure, (3.34) or (3.38) respectively, we have to make a choice for the background fluxes $\hat{H}$ and $\hat{F}$. However, since we fluctuate the gauge fields, two choices of background fluxes may be equivalent if they are related by a fluctuation of the moduli fields. To classify distinct choices we have to find configurations that are not related by pure fluctuations of the moduli fields. We label these different configurations as disconnected bubbles of the moduli space, i.e. these bubbles are such that it is not possible to reach another bubble by finite fluctuations of the moduli fields. In the following we will classify these different bubbles for type IIA and type IIB, respectively.

Type IIA

Classifying the different bubbles in terms of fluxes amounts to finding configurations that solve the Bianchi identities

$$\mathrm{d}\hat{H} = 0\,, \tag{3.43a}$$

$$\mathrm{d}\hat{F}_0 = 0\,, \tag{3.43b}$$

$$\mathrm{d}\hat{F}_2 + m\hat{H} = -j_3\,, \tag{3.43c}$$

$$\mathrm{d}\hat{F}_4 + \hat{H}\wedge\hat{F}_2 = 0\,, \tag{3.43d}$$

while two configurations are considered equivalent if they are related by a fluctuation of the moduli fields, which after imposing the orientifold projection (and assuming it removes one-forms) is given by (see section 3.1.1)

$$\delta H = \mathrm{d}\delta B\,, \tag{3.44a}$$

$$\delta F_0 = 0\,, \tag{3.44b}$$

$$\delta F_2 = -m\delta B\,, \tag{3.44c}$$

$$\delta F_4 = \mathrm{d}\delta C_3 - \delta B\wedge(\hat{F}_2 + \delta F_2) - \frac{1}{2}m(\delta B)^2\,, \tag{3.44d}$$

$$\delta F_6 = \hat{H}\wedge\delta C_3 - \delta B\wedge(\hat{F}_4 + \delta F_4) - \frac{1}{2}(\delta B)^2\wedge(\hat{F}_2 + \delta F_2) - \frac{1}{3!}m(\delta B)^3\,. \tag{3.44e}$$

In other words, we want to find representatives of the cohomology of the Bianchi identities (3.43) modulo pure fluctuations of the potentials (3.44).

Let us first consider the case $\hat{F}_0 \neq 0$. From eqs. (3.43a), (3.43b), (3.44a) and (3.44b) follows immediately that $\hat{H} \in H^3(M,\mathbb{R})$ and $\hat{F}_0$ constant. This determines δB only up to a closed form, we call it δB^c. It can be used to analyse (3.43c) and (3.44c): the closed part of F_2 is pure fluctuation, so that we choose $\hat{F}_2$ as the most general non-closed odd two-form, which then determines the source j_3. At this point, we completely specified δB. Moving on to $\hat{F}_4$, we find that in eq. (3.43d) $\hat{H} \wedge \hat{F}_2 = 0$, since we assumed there were no even five-forms under all the orientifold involutions. Moreover, with the fluctuations δC_3 we can remove the exact part of $\hat{F}_4$ so that $\hat{F}_4 \in H^4(M, \mathbb{R})$. This, however, leaves the closed part of δC_3 undetermined, which we call δC_3^c. If we have chosen $\hat{H}$ non-trivial, we can then use the closed part of δC_3^c in (3.44e) to put $\hat{F}_6 = 0$, provided that $\delta C_3^c \wedge \hat{H} \propto \mathrm{vol}_6$. This is not possible for $\hat{H}$ trivial and we have to choose a non-zero $\hat{F}_6$ proportional to the volume form.

For the case $\hat{F}_0 = 0$ we choose for $\hat{F}_2$ the most general two-form since there are no fluctuations left in eq. (3.44c). $\hat{H}$ is still in $H^3(M, \mathbb{R})$ and $\hat{F}_0$ constant. We thus still have the closed part of δB at disposal for $\hat{F}_4$ in (3.44d) such that we can choose $\hat{F}_4 \in H^4(M, \mathbb{R})$ and put to zero the part proportional to $\delta B^c \wedge \hat{F}_2$. Similar for $\hat{F}_6$: we can put it to zero if $\delta C_3^c \wedge \hat{H} \propto \mathrm{vol}_6$ for non-trivial $\hat{H}$ or if δB^c is not fixed completely up to now and $\delta B^c \wedge \hat{F}_4 \propto \mathrm{vol}_6$.

Type IIB

The analysis in type IIB is quite similar to the analysis in type IIA. In the following we will be interested in models with O5/O7 orientifolds such that we assume here these orientifold projections. From the fluctuations in (3.22) this then implies (since a scalar is always even under O5/O7 but δC_0 should be odd/even, there is no δC_0)

$$\delta F_1 = 0\,, \tag{3.45}$$

and we choose the most general one-form for $\hat{F}_1$, which then determines the source $j_{\mathrm{O}7}$. We will assume that $\mathrm{d}\hat{H} = 0$ such that [7]

$$\delta H = \mathrm{d}\delta B\,. \tag{3.46}$$

This allows to choose $\hat{H} \in H^{3--}(M, \mathbb{R})$ and fixes δB up to closed forms. Let us first assume that there is no closed part in δB (which is actually the case in our concrete models). This then implies for F_3 from (3.22)

$$\delta F_3 = \mathrm{d}\delta C_2\,, \tag{3.47}$$

such that we choose $\hat{F}_3$ up to exact forms, which determines the source $j_{\mathrm{O}5}$. This fixes δC_2 up to closed forms. For F_5, which has to be closed (the volume-form is even/even under O5/O7), we are left with

$$\delta F_5 = \mathrm{d}\delta C_4 + \hat{H} \wedge \delta C_2^c\,, \tag{3.48}$$

[7] For the models we study in chapter 12 there is room for $\mathrm{d}\hat{H} = j_{\mathrm{NS5}} \neq 0$, but we will set to zero this contribution since we do not know whether the proposed expression for the superpotential (3.38) takes the NS5-source properly into account.

hence we choose $\hat{F}_5 \in H^{5-+}(M, \mathbb{R})$ and we can put to zero the part of $\hat{F}_5$ that is proportional to $\hat{H} \wedge \delta C_2^c$.

If there is a part of δB that is closed, we have to take into account that

$$\delta F_3 = \mathrm{d}\delta C_2 - \delta B^c \wedge \hat{F}_1 \,, \tag{3.49a}$$

$$\delta F_5 = \mathrm{d}\delta C_4 + \hat{H} \wedge \delta C_2^c - \delta B^c \wedge (\hat{F}_3 + \delta F_3) - \frac{1}{2}(\delta B^c)^2 \wedge \hat{F}_1 \,, \tag{3.49b}$$

such that we can put to zero the part in $\hat{F}_3$ that is proportional to $\delta B^c \wedge \hat{F}_1$, and, if this does not fix δB^c completely, we can also put to zero the parts in $\hat{F}_5$ that are proportional to the last two terms in eq. (3.49b) where δB^c is the part not fixed by δF_3.

Chapter 4

Nilmanifolds and coset spaces

There are few explicit examples of six-dimensional manifolds suitable for compactifications to four dimensions. In [64] a systematic search for $\mathcal{N} = 1$ Minkowski vacua of type II string theories on compact six-dimensional nil- and solvmanifolds was performed yielding very few examples. These solutions require the presence of orientifold planes, typically smeared, due to a no-go theorem [36, 37] that rules out vacua in which the four-dimensional space is Minkowski and the internal compact manifold has non-zero background fluxes and no sources. This no-go theorem can be circumvented for $\mathcal{N} = 1$ compactifications to four-dimensional AdS space-time.

The oldest constructions of $\mathcal{N} = 1$ AdS_4 compactifications arise by considering the Hopf reductions of eleven-dimensional supergravity considered by Nilsson and Pope [65] that lead to supersymmetric type IIA compactifications with a non-vanishing second torsion class $\mathcal{W}_2$ [66, 67, 68] without the need of sources. As these solutions come from the reduction of eleven-dimensional supergravity, they have vanishing Romans mass. On the other hand, another simple type IIA construction with no need of orientifolds considers manifolds that are nearly-Kähler, such that the deviation from the Calabi-Yau metric is expressed by a non-vanishing first torsion class $\mathcal{W}_1$ [69]. These manifolds are also Einstein, where the scalar curvature is proportional to $|\mathcal{W}_1|^2$. In [51] the compactifications were constructed that interpolate between the vanishing Romans mass solutions and the nearly-Kähler solutions on two special coset manifolds, which can be described using twistor space techniques.

However, there are also type IIA constructions involving sources. First examples of compactifications to $\mathcal{N} = 1$ in type IIA with all moduli stabilized and in which possible corrections are parametrically under control were constructed in [70, 62, 71, 72] using orientifold planes and Calabi-Yau manifolds. From a purely ten-dimensional perspective these vacua are interpreted as a low-energy approximation in which the orientifolds are effectively smeared [50].

A systematic search for more type IIA examples of $\mathcal{N} = 1$ AdS_4 compactifications on six-dimensional coset spaces with a strict SU(3)-structure ansatz was performed in [34]. The authors identify four coset spaces that satisfy the necessary and sufficient conditions for $\mathcal{N} = 1$ compactifications to AdS_4 in the absence of sources, whereas in the presence of smeared brane/orientifold sources there is one possibility more. We will come back to these solutions in part III of this thesis, where we derive the corresponding effective actions for these compactifications.

Allowing for smeared sources, the conditions for type IIA $\mathcal{N} = 1$ AdS_4 vacua can actually be solved for some nilmanifolds [49] (as we pointed out in section 2.2.2, this is not possible without a source term for the nilmanifolds). There are two nilmanifolds, the torus and the Iwasawa manifold, that solve the type IIA equations. In addition, there is a further type IIB static SU(2)-structure solution on a different nilmanifold.

In this chapter we give a brief review of the spaces we consider in this thesis. We first begin with some well-known facts about group manifolds to set up the notation. We then introduce the coset space construction with special emphasis on the material that we will need in the following. For the interested reader, there are many good reviews [73, 74, 75]. In the end of this chapter, we will describe the six-dimensional nilmanifolds, which are special cases of group manifolds.

4.1 Group manifolds

In order to fix our notation and ideas, let us start with a group manifold, i.e., with a Lie group G of dimension $d = \dim(G)$ viewed as a manifold. We denote the generators of the Lie group G as T_a with $a = 1, \dots, d$ and they obey the algebra

$$[T_a, T_b] = f^c{}_{ab} T_c \,, \tag{4.1}$$

with $f^c{}_{ab}$ the structure constants of the group G. Let us mention that the structure constants are often referred to as geometric fluxes in the context of flux compactifications.

Let $U \in G$ be an arbitrary element of the group manifold G that we parameterize in terms of coordinates y^m, where $m = 1, \dots, d$. We define d left-invariant one-forms e^a on G by

$$U(y)^{-1} \mathrm{d}U(y) = e^a(y) T_a \,. \tag{4.2}$$

The left-invariance of e^a is easily seen: under $U \to AU$, where $A \in G$ is constant, the one-forms e^a defined in eq. (4.2) do not change. Taking the exterior derivative of eq. (4.2) we see

$$\mathrm{d}e^c T_c = -U^{-1}\mathrm{d}U \wedge U^{-1}\mathrm{d}U = -e^a T_a \wedge e^b T_b = -\frac{1}{2} e^a \wedge e^b [T_a, T_b] \,, \tag{4.3}$$

such that with the structure constants (4.1) we obtain the Maurer-Cartan equation

$$\mathrm{d}e^c = -\frac{1}{2} f^c{}_{ab} e^a \wedge e^b \,. \tag{4.4}$$

The Jacobi identity for the structure constants of a Lie algebra, $f^c{}_{a[b}f^a{}_{de]} = 0$, assures that $\mathrm{d}^2 e^c = 0$. The Maurer-Cartan equation (4.4) turns out to be very useful for our calculations since differential equations are reduced to algebraic ones. As the duals of the left-invariant one-forms we get left-invariant vector fields $L_a = L_a{}^m \partial/\partial y^m$ defined via

$$\langle e^a, L_b \rangle = \delta^a{}_b \,, \tag{4.5}$$

which satisfy

$$[L_a, L_b] = f^c{}_{ab} L_c \,. \tag{4.6}$$

We can classify Lie algebras according to Levi's theorem: an arbitrary Lie algebra $\mathfrak{g}$ is a semidirect sum of a semi-simple algebra and of a solvable algebra. The definition of a solvable algebra $\mathfrak{g}$ is as follows. Consider the series defined recursively by

$$\mathfrak{g}^{(0)} = \mathfrak{g} \,, \quad \mathfrak{g}^{(1)} = [\mathfrak{g}, \mathfrak{g}] \,, \quad \mathfrak{g}^{(i)} = [\mathfrak{g}^{(i-1)}, \mathfrak{g}^{(i-1)}] \,. \tag{4.7}$$

If this series becomes zero after a finite number of steps, the Lie algebra is said to be solvable. There is an equivalent criterion for a Lie algebra to be solvable or semi-simple: a Lie-algebra is semi-simple if and only if the Killing metric is nondegenerate, whereas it is solvable if and only if the Killing metric is identically zero. A special class of solvable Lie algebras are nilpotent algebras that are defined as follows. The recursively defined series

$$\mathfrak{g}^{0} = \mathfrak{g} \,, \quad \mathfrak{g}^{1} = [\mathfrak{g}, \mathfrak{g}] \,, \quad \mathfrak{g}^{i} = [\mathfrak{g}, \mathfrak{g}^{i-1}] \,. \tag{4.8}$$

converges after a finite number of steps to zero. The number of steps is called the nilpotency degree of the manifold. Comparing the definition (4.8) to the definition of a solvable algebra (4.7) we see that the series (4.8) decreases more slowly as the series for a solvable algebra (4.7) and may not reach zero even if the solvable algebra did, i.e. every nilpotent algebra is a solvable algebra but the converse is not true.

In this thesis, we will consider two different classes of six-dimensional compactification manifolds: nilmanifolds and coset manifolds based on semi-simple and U(1) groups. They are somehow opposite due to Levi's decomposition. The reason to consider nilmanifolds is that the mathematics is better known, in particular the criteria for compactness, and there is a complete classification of all six-dimensional nilmanifolds [76]. However, for the coset spaces we consider semi-simple groups. Here, the classification of [34] tells us which groups are needed to end up with a six-dimensional coset space and these groups are well known in the literature.

4.2 Geometry of coset spaces

We define coset spaces as the quotient G/H, where G is a Lie-group [1] and H is a compact Lie subgroup of G. The elements of G/H are equivalence classes of the form gH for left cosets, which we will consider in the following. The action of G on the coset is transitive, i.e., any point of G/H can be transformed to any other point by a G-transformation.

To describe coset spaces of the form G/H we may proceed as we did for group manifolds. To do so, we divide the generators of the group G, $\mathcal{G}_i \in \mathfrak{g}$, in two sets: a set of generators of the group H and a set of generators of the complement of H inside G, denoted by K. We label the corresponding elements of the Lie algebras $\mathfrak{h}$ and $\mathfrak{k}$ (such that $\mathfrak{g} = \mathfrak{k} \oplus \mathfrak{h}$) as follows: $\{\mathcal{H}_a\}$, where $a = 1, \ldots, \dim(H)$, and $\{\mathcal{K}_i\}$, where $i = 1, \ldots, \dim(G) - \dim(H)$, respectively.

The structure constants are then defined as

$$
\begin{aligned}
[\mathcal{H}_a, \mathcal{H}_b] &= f^c{}_{ab}\mathcal{H}_c \, , \\
[\mathcal{H}_a, \mathcal{K}_i] &= f^j{}_{ai}\mathcal{K}_j + f^b{}_{ai}\mathcal{H}_b \, , \\
[\mathcal{K}_i, \mathcal{K}_j] &= f^k{}_{ij}\mathcal{K}_k + f^a{}_{ij}\mathcal{H}_a \, .
\end{aligned}
\tag{4.9}
$$

For H compact, or connected and semi-simple, one can choose the subspace $\mathfrak{k}$ of $\mathfrak{g}$ such that

$$
h\mathfrak{k}h^{-1} \subset \mathfrak{k} \, , \quad \forall h \in H \, , \tag{4.10}
$$

i.e., $[\mathcal{H}, \mathcal{K}] \subset \mathcal{K}$ and therefore the structure constants $f^b{}_{ai}$ vanish. Such a coset is called reductive. Since we will need $H \subseteq SU(3)$, i.e. compactness, this will always be the case in our examples.

We label the coordinates on G/H as y^m, $m = 1, \ldots, \dim(G) - \dim(H)$. Let $L(y)$ be a representative element of each H-equivalence class. This leads to a corresponding decomposition of the one-forms

$$
\Theta \equiv L(y)^{-1}\mathrm{d}L(y) = e^i(y)\mathcal{K}_i + \omega^a(y)\mathcal{H}_a \, . \tag{4.11}
$$

The $e^i(y)$ defines in this way a coframe on the coset G/H. It is easily shown that

$$
\begin{aligned}
\mathrm{d}\Theta &= \mathrm{d}L(y)^{-1}\mathrm{d}L(y) = -\Theta \wedge \Theta \\
&= -\frac{1}{2}\Big(e^i \wedge e^j[\mathcal{K}_i, \mathcal{K}_j] + 2\omega^a \wedge e^i[\mathcal{H}_a, \mathcal{K}_i] + \omega^a \wedge \omega^b[\mathcal{H}_a, \mathcal{H}_b]\Big) \, ,
\end{aligned}
\tag{4.12}
$$

such that using the definition of the structure constants (4.9) we derive the exterior derivative acting on the one-forms

$$
\mathrm{d}e^i = -\frac{1}{2}f^i{}_{jk}e^j \wedge e^k - f^i{}_{aj}\omega^a \wedge e^j \, , \tag{4.13a}
$$

[1] In this section, G is an arbitrary Lie-group. We thus can generalize some results of this section to the nilmanifolds in the next section. For our concrete examples of coset spaces we will later restrict G to be a product of semi-simple and U(1)-groups.

$$d\omega^a = -\frac{1}{2} f^a{}_{bc} \omega^b \wedge \omega^c - \frac{1}{2} f^a{}_{ij} e^i \wedge e^j \,. \tag{4.13b}$$

For our applications we will just need the exterior derivative on the e^i one-forms.

Consider the left action $g \in G$ on a coset representative $L(y)$. This will give another element $z \in G$, which in general will belong to a different equivalence class, whose representative we denote by $L(z)$. Since $z = L(z)h$ for some $h \in H$ this can be expressed as

$$gL(y) = L(z)h(g, y) \,, \quad \text{where} \quad g \in G \,,\, h \in H \,. \tag{4.14}$$

This equation determines both z and h as a function of y and g. To determine how the coframe e^i transforms under the left action of G on G/H we derive from eq. (4.11) and (4.14)

$$\begin{aligned} L(z)^{-1} dL(z) &= e^i(z)\mathcal{K}_i + \omega^a(z)\mathcal{H}_a \\ &= hL(y)^{-1} dL(y)h^{-1} + h dh^{-1} \\ &= e^i(y) h\mathcal{K}_i h^{-1} + \omega^a(y) h\mathcal{H}_a h^{-1} + h dh^{-1} \,. \end{aligned} \tag{4.15}$$

Since we assume that we have a reductive coset space (H is compact in our examples) we know the relation (4.10) and can define

$$h\mathcal{K}_i h^{-1} = D_i{}^j(h^{-1})\mathcal{K}_j \,, \tag{4.16}$$

such that eq. (4.14) leads to the transformation rule for the coframe

$$e^j(z) = e^i(y) D_i{}^j(h^{-1}) \,. \tag{4.17}$$

With the transformation rule for the coframe we can write down the condition for G-invariance for any tensor. As an example, for any metric on G/H that can locally be written in terms of the coset frame as

$$g = g_{ij} e^i \otimes e^j \,, \tag{4.18}$$

the condition for G-invariance amounts to $g_{ij} =$ constant and

$$g_{ij} = g_{kl} D_i{}^k(h) D_j{}^l(h) \,, \qquad \forall h \in H \,. \tag{4.19}$$

For an infinitesimal version of eq. (4.19) we note from eq. (4.16) and the definition of the structure constants (4.9) that

$$D_i{}^j(h)\mathcal{K}_j = (\delta_i^j - t^a f^j{}_{ai})\mathcal{K}_j \,, \tag{4.20}$$

where we defined $h = e^{t^a \mathcal{H}_a}$. The infinitesimal version for a G-invariant metric on the coset space G/H then reads

$$f^j{}_{a(l} g_{k)j} = 0 \,. \tag{4.21}$$

For an arbitrary p-form

$$\alpha = \frac{1}{p!}\alpha_{i_1\ldots i_p} e^{i_1} \wedge \ldots \wedge e^{i_p} \,, \tag{4.22}$$

we show similarly that the condition for G-invariance is that the components $\alpha_{i_1\ldots i_p}$ are constant and

$$f^j{}_{a[i_1}\alpha_{i_2\ldots i_p]j} = 0 \,. \tag{4.23}$$

Let us give an intuitive explanation for the condition (4.23). From eq. (4.13a) we see that, taking the exterior derivative on an arbitrary p-form, we obtain contributions including ω^a forms. Condition (4.23) ensures that the part coming from the second term in (4.13a) drops out, and we get again a G-invariant form.

The Maurer-Cartan equations are very useful to calculate various quantities relevant to characterize the geometry of the manifold, such as the connection and the curvature. The Levi-Civita connection one-forms $\omega^i{}_j$ of a metric are uniquely determined by two equations

$$\mathrm{d}g_{ij} - \omega^k{}_i g_{kj} - \omega^k{}_j g_{ik} = 0 \,, \qquad \text{(metric compatibility)} \,, \tag{4.24a}$$

$$\mathrm{d}e^i + \omega^i{}_j \wedge e^j = 0 \,, \qquad \text{(vanishing torsion)} \,. \tag{4.24b}$$

For a G-invariant metric, the metric compatibility in (4.24a) is the condition

$$\omega_{ij} \equiv g_{ik}\omega^k{}_j = -\omega_{ji} \,. \tag{4.25}$$

Choosing e^i to be the coset frame given in (4.11) and using the structure constants defined in (4.13a), the solution of (4.24) is given by [77]

$$\omega^i{}_j = f^i{}_{aj}\omega^a + D^i_{kj} e^k \,, \tag{4.26}$$

where

$$D^i_{kj} = g^{im}\left(\frac{1}{2} f^l{}_{mj} g_{lk} + f^l{}_{k[j} g_{m]l}\right) \,. \tag{4.27}$$

We now have all the data we need to calculate the curvature $R = \mathrm{d}\omega + \omega \wedge \omega$, which is done in [75]. We only display here the Ricci scalar, which we find by contracting indices:

$$R = -g^{ij} f^k{}_{ai} f^a{}_{kj} - \frac{1}{2} g^{ij} f^k{}_{li} f^l_{kj} - \frac{1}{4} g_{ij} g^{kl} g^{mn} f^i{}_{km} f^j{}_{ln} \,. \tag{4.28}$$

In chapter 2 we have seen that the requirement of four-dimensional supersymmetry imposes a condition on the six-dimensional internal manifold, namely that the structure group is reduced to SU(3) or a subgroup thereof. This requirement imposes a constraint on the possible choices of coset spaces of the form G/H that are suitable for supersymmetric compactifications. As is shown in [34], a necessary requirement that the coset space G/H has reduced structure group SU(3) translates into the requirement that the group H should be contained in SU(3), and all possible six-dimensional manifolds $M = G/H$ of this type [2] are listed in table 4.1, taken from [34].

In our concrete models of compactifications on coset spaces (see part III of the thesis), we will allow for orientifold sources. If we introduce orientifolds, the structure constant tensor

$$f = \frac{1}{2} f^i{}_{jk} V_i \otimes e^j \wedge e^k + f^i{}_{aj} V_i \otimes \omega^a \wedge e^j + \frac{1}{2} f^a{}_{ij} U_a \otimes e^i \wedge e^j + \frac{1}{2} f^a{}_{bc} U_a \otimes \omega^b \wedge \omega^c \,, \tag{4.29}$$

where the V_i, U_a are dual to the e^i, ω^a defined as in eq. (4.5), has to be even under the orientifold involution (for some suitable extension of the involution to the ω^a) in order to ensure that the exterior derivative is even.

G	H
G_2	SU(3)
SU(3)×SU(2)^2	SU(3)
Sp(2)	S(U(2)×U(1))
SU(3)×U(1)^2	S(U(2)×U(1))
SU(2)^3×U(1)	S(U(2)×U(1))
SU(3)	U(1)×U(1)
SU(2)^2×U(1)^2	U(1)×U(1)
SU(3)×U(1)	SU(2)
SU(2)^3	SU(2)
SU(2)^2×U(1)	U(1)
SU(2)^2	1

Table 4.1: All six-dimensional manifolds of the type $M = G/H$, where H is a subgroup of SU(3) and G and H are both products of semi-simple and U(1)-groups. To be precise, this list should be completed with the cosets obtained by replacing any number of SU(2) factors in G by U(1)^3.

4.3 Geometry of nilmanifolds

The second class of manifolds we will consider in this thesis are nilmanifolds. A nilmanifold is a quotient of a nilpotent Lie group G by a discrete subgroup Γ, $M = G/\Gamma$. In [79] it is shown that all six-dimensional nilmanifolds admit generalized complex structures, making them interesting for our purposes.

[2] These coset spaces were already considered in the construction of heterotic string compactifications in [78].

As a special case of a group manifold, G has a set of globally defined one-forms e^i satisfying the Maurer-Cartan equations (4.4). For an illustration, let us discuss the simple and often cited example for a nilpotent algebra: the Heisenberg algebra with the only non-vanishing structure constant ${f^3}_{12} = -n$. From eq. (4.4) we get

$$\mathrm{d}e^1 = 0\,; \quad \mathrm{d}e^2 = 0\,; \quad \mathrm{d}e^3 = n e^1 \wedge e^2\,. \tag{4.30}$$

We will use in the following the notation $(0, 0, n\,12)$ to specify the structure constants. We can choose a gauge for the coordinates which satisfies the algebra (4.30) as follows

$$e^1 = \mathrm{d}y^1\,; \quad e^2 = \mathrm{d}y^2\,; \quad e^3 = \mathrm{d}y^3 + n y^1 e^2\,. \tag{4.31}$$

To compactify G, we can make the identifications $(y^1, y^2, y^3) \simeq (y^1, y^2 + 1, y^3) \simeq (y^1, y^2, y^3 + 1)$, but we need to twist the identification for y^1, $(y^1, y^2, y^3) \simeq (y^1 + 1, y^2, y^3 - n y^2)$, to render e^3 single-valued. Hence, the space $M = G/\Gamma$ is topologically distinct from a three-torus T^3, namely an S^1 fibration over T^2 whose first Chern class is $c_1 = n$. Such a manifold M is called a nilmanifold and a general nilmanifold is always an iteration of torus fibrations. Nilmanifolds are often called twisted tori in the physical literature, and the structure constants are referred to as metric or geometric fluxes.

Let us note that there are infinitely many algebras of the form (4.30), since n is a free parameter. However, these algebras are all isomorphic via a rescaling of e^3. When we talk about nilmanifolds in the following, we mean actually isomorphism classes of nilmanifolds. However, since we work with a basis of left-invariant forms, the choice of the representant of the isomorphism class does not matter for the analysis.

The nilpotent Lie groups up to dimension 7 have been classified and the list of six-dimensional nilpotent Lie groups is finite [76]. The complete list of the 34 isomorphism classes of simply-connected six-dimensional nilpotent Lie groups is given in table 4.7 of [64], where the authors scanned all these nilmanifolds to find $\mathcal{N} = 1$ Minkowski solutions. We will use this list to scan for AdS_4 solutions in part II of this thesis.

The question arises whether all of these six-dimensional Lie groups can be compactified by modding out a discrete compact subgroup Γ as in the example of the Heisenberg algebra above. A necessary condition on the structure constants is ${f^j}_{ij} = 0$. The reason is simple: if ${f^j}_{ij} \neq 0$, the volume form $\mathrm{vol}_6 = e^1 \wedge \ldots \wedge e^6$ would be exact, since for the left-invariant five-form $\alpha = \epsilon_{i_1 \ldots i_6} \alpha^{i_1} e^{i_2} \wedge \ldots \wedge e^{i_6}$, with α^{i_1} constant, we have $\mathrm{d}\alpha = ({f^j}_{ij} \alpha^i)\mathrm{vol}_6$. Hence, there is no top-form non-trivial in cohomology which is of course required for a compact manifold G/Γ. One can also show that this condition is sufficient, provided that the structure constants are rational in some basis [80]. Since these conditions are satisfied for the structure constants of all the 34 classified six-dimensional nilpotent Lie groups, they all admit a discrete subgroup Γ such that $M = G/\Gamma$ is compact.

The Ricci scalar for a nilmanifold is a special case of the metric of the coset space in that the first term in eq. (4.28) obviously vanishes, as well as the second term which is the Killing metric (for a nilpotent algebra, the Killing metric vanishes). We are left with

$$R = -\frac{1}{4} g_{ij} g^{kl} g^{mn} f^i{}_{km} f^j{}_{ln} \,, \tag{4.32}$$

which is always negative. Nilmanifolds are thus non Ricci-flat and therefore suitable for compactifications in the presence of fluxes.

Chapter 5

Cosmology and inflation

One of the legitimate criticism on string theory, which aspires to be a fundamental theory of quantum gravity, are the very restricted possibilities to confront the theory with observations. Assuming the string scale to be of the order of the Planck scale, it is very unlikely that we can ever construct a high energy accelerator providing enough power in order to test the Planck scale predictions of the theory. At low energies, of course, string theory has to reveal the standard model of particle physics. Nevertheless, there is a possibility to observe physics at very high energies, even if this physics happened billions of years before our time: the earliest moments of our universe involved such extreme energies, and the fingerprints of its birth are revealed today by precision measurements of the cosmic microwave background (CMB) and the large-scale structure of the universe. The ability of string theory to reproduce the observed cosmology thus provides us with a highly non-trivial test of string theory.

That this is possible is due to the astrophysical measurements over the recent years, which provide us with fascinating data about the large scale structure of our universe. In particular, the universe is found to be spatially flat, $|\Omega - 1| \ll 1$, and the latest CMB data from WMAP5 agree with an almost scale-invariant spectrum with scalar spectral index $n_s = 0.96 \pm 0.013$ [81]. As we will discuss in the following, an epoch of cosmic inflation in the early universe is the dominant paradigm to explain these data [82, 83]. For string theory to be a valid theory of quantum gravity, it should be able to realize inflation.

Another important cosmological observation of the past decade is that the present universe is in a state of accelerated expansion [84], apparently driven by a non-vanishing vacuum energy with an equation of state very close to that of a small and positive cosmological constant Λ. In an effective field theory setup, an asymptotic de Sitter phase induced by a constant vacuum energy would correspond to a positive local minimum of the potential.

The moduli fields of string theory compactifications provide us with natural inflaton candidates. These models can roughly be divided into closed string inflation models, in which the string is a closed string modulus [85], and open string (or brane-) inflation models, where a scalar describing some relative brane distance or orientation plays the role of the inflaton [86]. Mixtures of open and closed string moduli have also been considered as inflaton candidates, e.g., in some variations of D3/D7-brane inflation [87]. Let us mention that there are also other possibilities including [88].

There has been remarkable progress constructing various plausible models of inflation in string theory, mostly within type IIB string theory (following the work of [22, 23, 89]). Turning on only p-form fluxes in the type IIB theory, one can not stabilize all the moduli fields (the Kähler moduli are not fixed by the fluxes) [22]. In [23], a solution to this difficulty was proposed by turning on non-perturbative effects such as gluino condensation and instantons, yielding a supersymmetric AdS_4 vacua. The inclusion of a small number of anti-D3 branes breaks supersymmetry and allows one to uplift the AdS_4 minimum and make it a metastable de Sitter ground state. Starting from this model, the authors of [89] tried to construct, using brane moduli, an inflation model.

However, these and related models in type IIB share a common property: they are not entirely explicit constructions as they involve, besides the classical effects in the potential (which are easily computed by supergravity techniques as we discussed in section 3.2), also quantum effects, whose existence is well established, but for which precise calculations are often difficult. On the other hand, in type IIA compactifications, all geometrical moduli can be already stabilized at the classical level by fluxes in a well-controlled regime (corresponding to large volume and small string coupling, such that quantum corrections are small) with power law parametric control. This explicitness of type IIA compactifications makes these models very interesting for phenomenology. Let us further mention that type IIA orientifolds with intersecting D6-branes offer good prospects for deriving the standard model form string theory [18, 19, 90]. If cosmological aspects can likewise be modeled in type IIA, one may study questions such as reheating much more explicitly.

In this thesis, we will derive the explicit four-dimensional low-energy effective potential for a large class of type IIA compactifications. To render these models interesting for phenomenological applications we would like to examine whether these models support inflation. However, there are a number of simple but very strong no-go theorems against inflation in type IIA string theory at tree level [91, 92]. These theorems already exclude most of the explicitly known compactification models for type IIA, in particular models where only the standard NSNS H_3-flux and RR-fluxes F_p, $(p = 0, 2, 4, 6)$ as well as contributions from O6/D6 sources are turned on. As we will review in this chapter, the minimal requirements for an inflation model in classical type IIA compactifications are non-vanishing Romans mass and non-vanishing geometric fluxes.

Let us mention that in type IIB, where F_1 flux can be turned on, the above mentioned no-go theorems do not apply. In fact, we will also examine some type IIB compactifications with static SU(2)-structure. However, as we will see in chapter 12, most of our models are related by T-duality to type IIA models we study in chapter 11, and we can then apply the type IIA no-go theorems to these models.

In the next section, we give a brief introduction to inflation. Of course this can not be done in its full completeness. For the interested reader, we refer to the extensive literature, see e.g., [82, 83, 93]. We will then review the relevant no-go theorems against an epoch of slow-roll inflation that turn out to be very useful for analysing our particular compactification manifolds that we will describe in part III of this thesis.

5.1 Inflation

The current understanding of cosmology is described by the Hot Big Bang model, which starts as a hot soup of elementary particles, whose temperature was once at least 10 billion degrees. The history of the universe then describes the cooling of this initial state as the universe expands. However, this model can not explain the current observations if there were not very special initial conditions. An epoch of inflation - a period with exponential expansion of the universe even before the Hot Big Bang model starts - may provide exactly these initial conditions. In this chapter, we will first start with a description of the geometry of space-time on which the Hot Big Bang and an inflation model relies. In the following, we will show that a period of exponential expansion can be driven by a scalar field φ. As we will explain, for such a regime to work, sufficient conditions on the scalar field potential (but not necessary ones, there are other possibilities to drive inflation) are the so-called slow-roll conditions on the potential of the inflaton.

5.1.1 Cosmology and Hot Big Bang model

Based on large scale observations of the distribution of matter and radiation within the universe we see around us, we can assume the universe to be homogeneous and isotropic at large distance scales. For instance, the observed temperature fluctuations of the CMB are of order $\delta T/T \sim 10^{-5}$. This motivates to consider the most general four-dimensional geometry which is consistent with isotropy and homogeneity of its spatial slices. Such a geometry is described by a Friedmann-Robertson-Walker (FRW) geometry with a metric given by

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\left(\frac{\mathrm{d}r^2}{1-\kappa r^2} + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\phi^2\right) , \tag{5.1}$$

where $\kappa = 1, 0, -1$ describes a spherical, flat or hyperbolic universe, respectively. The factor $a(t)$ is called the scale factor and we define the Hubble parameter

$$H(t) = \frac{\dot{a}(t)}{a(t)} , \tag{5.2}$$

where the dot denotes derivation with respect to time. The time evolution of the scale factor $a(t)$ is obtained from the Einstein equation

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu} , \tag{5.3}$$

specialized to the FRW-metric (5.1). We obtain two equations, the Friedmann equation

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{\kappa}{a^2} = \frac{\rho}{3M_P^2}\,, \quad \text{where} \quad M_P^{-2} = 8\pi G\,, \tag{5.4}$$

and the Raychaudhuri equation

$$\frac{\ddot{a}}{a} = -\frac{1}{6M_P^2}\left(\rho + 3p\right)\,. \tag{5.5}$$

Here we assumed the most general form for the energy-momentum stress tensor $T_{\mu\nu}$ of the universe's matter content consistent with homogeneity and isotropy

$$T_{\mu\nu} = \begin{pmatrix} \rho & 0 \\ 0 & pg_{ij} \end{pmatrix}\,, \tag{5.6}$$

where i, j run over the spatial coordinates. It turns out to be useful to derive a first order equation from the eqs. (5.4) and (5.5) that expresses energy conservation,

$$3\left(\rho + p\right)\frac{\dot{a}}{a} + \dot{\rho} = 0 \quad \Leftrightarrow \quad \frac{\mathrm{d}}{\mathrm{d}t}\left(\rho a^3\right) = -p\frac{\mathrm{d}}{\mathrm{d}t}a^3\,. \tag{5.7}$$

How the scale factor $a(t)$ evolves with time depends on what kind of matter the universe contains. Present observations give evidence that there are the following kinds of cosmic fluids, each coming with a different equation of states:

- *Radiation*: The contribution from relativistic particles in the universe, namely photons and cosmic relic neutrinos (whose masses are small enough to be considered as relativistic particles), satisfying the equation of state of a weakly interacting gas

$$p_{\mathrm{rad}} = \frac{1}{3}\rho_{\mathrm{rad}}\,. \tag{5.8}$$

- *Baryons and dark matter*: Ordinary matter (electrons, nuclei, atoms) that is non-relativistic such that the rest mass dominates over the average kinetic energy (which corresponds to its pressure), implying $p \approx 0$. Observations infer further the presence of a large amount of non-observed (at least not by its electromagnetic radiation), non-relativistic matter which gravitates just like ordinary baryons do. This so-called dark matter has the same equation of state as the baryons. Together these two contributions form the non-relativistic matter content of the universe with equation of state

$$p_{\mathrm{m}} \approx 0\,. \tag{5.9}$$

- *Dark energy*: Observations further motivate the existence of yet another type of invisible "matter". Main evidence is that the overall expansion rate of the universe seems to be increasing at present time. From eq. (5.5) it is clear that in order to obtain an accelerating universe, i.e. $\ddot{a} > 0$, matter with sufficiently negative pressure, $p < -\rho/3$, is required. Since this is not true for radiation nor for non-relativistic matter, we need something whose pressure is negative and at present time dominates that of the other forms of matter - the so-called dark energy - which behaves very similarly to a positive cosmological constant and whose equation of states is predicted to be

$$p_{\mathrm{DE}} \approx -\rho_{\mathrm{DE}} \,. \tag{5.10}$$

Note that each of these equations of state implies the time-independent ratio $w_i = p_i/\rho_i$, and we easily integrate eq. (5.7), obtaining

$$\rho_i = \rho_{i0} \left(\frac{a_0}{a}\right)^{3(1+w_i)} \,. \tag{5.11}$$

Given an initial density ρ_0 and the initial fraction of the different contributions $f_i = \rho_{i0}/\rho_0$, we obtain

$$\rho(a) = \rho_0 \left(f_{\mathrm{DE}} + f_{\mathrm{m}} \left(\frac{a_0}{a}\right)^3 + f_{\mathrm{rad}} \left(\frac{a_0}{a}\right)^4 \right) , \tag{5.12}$$

which implies that the energy content of the universe was first dominated by a radiation epoch, followed by an epoch of matter domination and then by dark energy, explaining the presently observed accelerated expansion of the universe.

Even though the Hot Big Bang model is very successful, it leaves many important questions unanswered. To mention the most important, the Hot Big Bang model can not explain the currently observed flatness of our universe (observations indicate that the quantity κ/a^2 is at present consistent with zero) and the homogeneity of our universe (the temperature fluctuations of the CMB only arise at the level of one part in 10^5, no matter from which direction we receive this radiation), unless we start the universe off with a very special kind of initial conditions.

The first problem is called the *flatness problem*. To see why this is a problem, let us first divide the Friedmann equation (5.4) by H^2 to obtain

$$1 + \frac{\kappa}{(aH)^2} = \frac{8\pi G\rho}{3H^2} \equiv \Omega(a) \,. \tag{5.13}$$

Observations indicate that, at present, $\Omega = \Omega_0$ is equal to unity up to 4%. But since the product aH decreases with time for both matter and radiation domination, the curvature term in eq. (5.13) becomes more and more important as time passes. Exact calculations show that at the epoch of Big Bang Nucleosynthesis, Ω has to be unity with an accuracy of roughly one part in 10^{18} in order to reproduce the value $\Omega_0 \approx 1$ seen today. We thus have to explain this very special initial condition.

The second problem is called the *horizon problem*. To explain the homogeneous distribution of temperature of the CMB (up to one part in 10^5), the whole universe had to be causality connected at the time of recombination - the epoch in which the universe first became transparent for photons, which is the origin of the CMB. However, assuming a matter dominated universe, $a(t) = a_0(t/t_0)^{1/2}$, one shows that the proper distance that a light signal can travel by the time of recombination - the physical horizon size - is $L_{\rm rec} = H_{\rm rec}^{-1} = H_0^{-1}(a_{\rm rec}/a_0)^{3/2}$. When we look at the CMB, we are observing the universe at a scale factor $a_0/a_{\rm rec} \approx 1100$, which is at a proper distance of approximately $D_0 = 2H_0^{-1}(1 - \sqrt{a_{\rm rec}/a_0}) \approx 2H_0^{-1}$. At the time of last scattering, this was in a distance of $D_{\rm rec} = (a_{\rm rec}/a_0)D_0$. Hence, if we observe two parts of the CMB separated by more than an angle $\theta \approx L_{\rm rec}/D_{\rm rec} \approx 1^\circ$, they will have non-overlapping horizons and were causally disconnected at recombination.

Inflationary cosmology provides a solution to the flatness and horizon problem. The idea is to assume that there was a period in the very early universe during which the scale factor was accelerating, i.e. $\ddot{a} > 0$, which requires an equation of states $p < -\rho/3$. The simplest models of inflation assume

$$p \approx -\rho\,, \tag{5.14}$$

and we see from eq. (5.7), for the case $p = -\rho$, that $\rho = \rho_\star$ has to be constant. By integration of the Friedmann eq. (5.4) (neglecting the curvature term) we obtain an exponential expansion

$$a(t) = a_0 e^{\sqrt{\frac{\rho_\star}{3M_P^2}}(t-t_0)}\,, \tag{5.15}$$

and a constant Hubble length

$$H^{-1} = \frac{a(t)}{\dot{a}(t)} = \sqrt{\frac{3M_P^2}{\rho_\star}} = H_\star^{-1}\,. \tag{5.16}$$

With this assumption, aH grows exponentially such that it does not take long for any initial curvature $\kappa/(aH)^2$ to be diluted to extremely small values - providing a solution to the flatness problem. For a phase of exponential expansion of the scale factor, the horizon size, $L_{\rm hor}(t) = a(t)r_{\rm hor}$, grows more quickly than the Hubble length $H_\star^{-1}$. Modes which were at the beginning of inflation shorter than the Hubble length may be stretched to be larger than the Hubble length, and homogeneity over a very small patch is enough to solve the horizon problem. Quantum fluctuations make it impossible for inflation to smooth out the universe with perfect precision, explaining the observed approximately scale free spectrum of primordial density perturbations. Once these primordial density fluctuations have been amplified, they seed the formation of galaxies through gravitational collapse. Measurements of galaxy distributions provide us thus with precise experimental data to test the different inflation models.

5.1.2 Slow-roll inflation

We have seen that, in order to have the right initial conditions to start with the Hot Big Bang model, we need a phase of accelerated expansion, i.e. we look for an equation of states that satisfies eq. (5.14). This is possible for a scalar field whose equation of motion satisfies some special conditions. The dynamics of a scalar field φ with canonically normalized kinetic term and potential $V(\varphi)$ moving in an FRW-universe is described by the action

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V(\varphi)\right)\,, \tag{5.17}$$

where the metric is given by eq. (5.1). From the action (5.17), we calculate the equation of motion for φ as

$$\ddot{\varphi} + 3H\dot{\varphi} = -V'(\varphi)\,, \tag{5.18}$$

where $V'(\varphi) = \mathrm{d}V/\mathrm{d}\varphi$. The variation of (5.17) with respect to $\delta g^{\mu\nu}$ and the definition (5.6) gives us the expression for the energy density and the pressure as follows

$$\rho = \frac{1}{2}\dot{\varphi}^2 + V(\varphi)\,, \tag{5.19a}$$

$$p = \frac{1}{2}\dot{\varphi}^2 - V(\varphi)\,. \tag{5.19b}$$

We thus can obtain the regime of interest (5.14) when the kinetic energy of φ is negligible compared with its potential energy (the field φ has to roll slowly)

$$\frac{1}{2}\dot{\varphi}^2 \ll V(\varphi)\,, \tag{5.20}$$

such that $p \approx -V \approx -\rho$ and, from (5.16),

$$H^2 \approx \frac{V}{3M_P^2}\,, \tag{5.21}$$

would be approximately constant. The slow-roll condition (5.20) remains a good approximation for an appreciable time provided $\dot{\varphi}$ changes slowly, such that we demand $\ddot{\varphi} \ll H\dot{\varphi}$. This allows us to neglect the $\ddot{\varphi}$-term in eq. (5.18) such that

$$\dot{\varphi} \approx -\frac{V'}{3H}\,. \tag{5.22}$$

Using the slow-roll condition (5.20), we conclude that V must satisfy $V'^2/9H^2V \ll 1$ and with eq. (5.21)

$$\epsilon \equiv \frac{1}{2}\left(\frac{M_P V'}{V}\right)^2 \ll 1\,. \tag{5.23}$$

To justify the slow-roll approximation in eq. (5.20) throughout the inflation period, we require $\ddot{\varphi}$ to remain small. Differentiating eq. (5.22) with respect to time, we get (using that H is approximately constant) $\ddot{\varphi} \approx V''\dot{\varphi}/3H$, which has to be much smaller compared with $3H\dot{\varphi}$. This gives (in absolute values) $\left|V''/(3H)^2\right| \ll 1$, or

$$|\eta| \ll 1\,, \quad \text{where} \quad \eta = \frac{M_P^2 V''}{V}\,. \tag{5.24}$$

The conditions (5.23) and (5.24) are called the first and second flatness condition, respectively. They are sufficient conditions to have a region of exponential expansion. However, the models we will study in this thesis are much more complicate than the one field model we just studied. The generic kind of scalar dynamics for the real scalar fields φ^a which emerge in the low-energy limit of string theory is (in Einstein frame)

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{M_P^2}{2}R - \frac{1}{2}g_{ab}(\varphi)\partial_\mu\varphi^a\partial^\mu\varphi^b - V(\varphi)\right), \tag{5.25}$$

where $g_{ab}(\varphi)$ is the Kähler metric in real coordinates (see section 3.2). We thus generalize the expressions for ρ and p of eq. (5.19) to

$$\rho = \frac{1}{2}g_{ab}(\varphi)\dot{\varphi}^a\dot{\varphi}^b + V(\varphi)\,, \tag{5.26a}$$

$$p = \frac{1}{2}g_{ab}(\varphi)\dot{\varphi}^a\dot{\varphi}^b - V(\varphi)\,. \tag{5.26b}$$

As before, a sufficient condition for inflation is $\frac{1}{2}g_{ab}\dot{\varphi}^a\dot{\varphi}^b \ll V$ and by similar arguments as for the single field case we obtain the conditions for slow-roll inflation

$$\epsilon \ll 1 \quad \text{where} \quad \epsilon = \frac{M_P^2 g^{ab}\partial_{\varphi^a}V\partial_{\varphi^b}V}{2V^2} = \frac{\mathcal{K}^{i\bar{j}}\partial_{\phi^i}V\partial_{\bar{\phi}^{\bar{j}}}V}{V^2}\,, \tag{5.27}$$

where we passed in the last equation from real coordinates to complex coordinates, and

$$|\eta| \ll 1 \quad \text{where} \quad \eta = \text{min eigenvalue}\left(\frac{M_P^2\nabla^a\partial_{\varphi^b}V}{V}\right), \tag{5.28}$$

where we take the covariant derivative with respect to the Kähler metric g_{ab}.

These are the conditions we have to satisfy for a period of inflation. However, there are strong no-go theorems that results in an upper bound for ϵ, thus excluding an epoch of slow-roll inflation. In the following sections, we review some of the no-go theorems that we will use in chapter 13 to study our models.

5.2 No-go theorem in the volume-dilaton plane

The first no-go theorem we want to study was constructed in [91] extending earlier work of [94]. This no-go theorem excludes slow-roll inflation and de Sitter vacua for the simplest compactifications of massive type IIA supergravity with p-form fluxes and D6/O6-sources. The no-go theorem relies on the scaling behavior with respect to the overall volume modulus, $\rho = (\mathrm{Vol})^{1/3}$, and the four-dimensional dilaton modulus, $\tau = e^{-\Phi}\sqrt{\mathrm{Vol}}$, of the different contributions of the fluxes and sources to the four-dimensional effective potential, where the volume is defined as (see eq. (2.7))

$$\mathrm{Vol} = \frac{1}{6}\kappa_{abc}k^a k^b k^c\,, \tag{5.29}$$

where κ_{abc} denotes the triple intersection number, given in terms of the odd two-forms $Y_a^{(2-)}$ as

$$\kappa_{abc} = \int_M Y_a^{(2-)} \wedge Y_b^{(2-)} \wedge Y_c^{(2-)} \,. \tag{5.30}$$

As we have explained in section 5.1, inflation requires $\epsilon \ll 1$, where ϵ is defined as

$$\epsilon = \frac{\mathcal{K}^{i\bar{j}}\partial_{\phi^i}V\partial_{\bar{\phi}^{\bar{j}}}V}{V^2} = \frac{\mathcal{K}^{i\bar{j}}\left(\partial_{\mathrm{Re}\,\phi^i}V\partial_{\mathrm{Re}\,\phi^j}V + \partial_{\mathrm{Im}\,\phi^i}V\partial_{\mathrm{Im}\,\phi^j}V\right)}{4V^2} \,. \tag{5.31}$$

The expression for the Kähler metric appears in the kinetic terms for the moduli fields

$$\begin{aligned} T &= -\mathcal{K}_{i\bar{j}}\partial_\mu\phi^i\partial^\mu\bar{\phi}^{\bar{j}} \\ &= -\frac{1}{4}\frac{\partial^2\mathcal{K}}{\partial\mathrm{Re}\,\phi^i\partial\mathrm{Re}\,\phi^j}\left(\partial_\mu\mathrm{Re}\,\phi^i\partial^\mu\mathrm{Re}\,\phi^j + \partial_\mu\mathrm{Im}\,\phi^i\partial^\mu\mathrm{Im}\,\phi^j\right) \,, \end{aligned} \tag{5.32}$$

and thus

$$\mathcal{K}_{i\bar{j}} = \frac{1}{4}\frac{\partial^2\mathcal{K}}{\partial\mathrm{Re}\,\phi^i\partial\mathrm{Re}\,\phi^j} \,. \tag{5.33}$$

As we will explicitly see in chapter 11, the Kähler potential for our models is given by

$$\mathcal{K}_k = -\ln\left(\frac{4}{3}\kappa_{abc}k^a k^b k^c\right) = -\ln(8\rho^3) \,, \tag{5.34a}$$

$$\mathcal{K}_c = -4\ln\tau \,, \tag{5.34b}$$

for the Kähler sector and complex structure sector, respectively.

Since ρ and τ are the real components of the corresponding complex moduli, we derive from eq. (5.33) the following kinetic terms for ρ and τ

$$T = -\left(\frac{3}{4\rho^2}(\partial_\mu\rho)^2 + \frac{1}{\tau^2}(\partial_\mu\tau)^2 + \cdots\right) \,, \tag{5.35}$$

where the dots stand for other manifest positive kinetic terms for the remaining moduli fields.

Let us mention that this short derivation is not as trivial as we just showed. As an example, let us consider the Kähler sector, where we define the complex Kähler modulus as $t^i = k^i - ib^i$. The kinetic energy is given by

$$T = -\mathcal{K}_{i\bar{j}}\left(\partial_\mu k^i\partial^\mu k^j + \partial_\mu b^i\partial^\mu b^j\right) \,. \tag{5.36}$$

From eq. (5.34a), we derive

$$\mathcal{K}_{i\bar{j}} = \frac{1}{16\mathrm{Vol}^2}(\kappa_{iab}k^a k^b)(\kappa_{jab}k^a k^b) - \frac{1}{4\mathrm{Vol}}(\kappa_{ija}k^a) \,. \tag{5.37}$$

We now make the change of coordinates from k^a to the overall volume ρ and a set of angular variables γ^a via

$$k^a = \rho\gamma^a\,, \qquad \text{where} \quad \kappa_{abc}\gamma^a\gamma^b\gamma^c = 6\,. \tag{5.38}$$

The constraint on the angular variables γ^a ensures that $\mathrm{Vol} = \frac{1}{6}\kappa_{abc}k^a k^b k^c = \rho^3$. From this constraint it follows that $\partial_\mu(\kappa_{abc}\gamma^a\gamma^b\gamma^c) = 0$, and hence $\kappa_{abc}(\partial_\mu\gamma^a)\gamma^b\gamma^c = 0$. With this we easily calculate the explicit expression for the kinetic terms from eq. (5.36) as

$$T = -\left(\frac{3(\partial_\mu\rho)^2}{4\rho^4} - \frac{1}{4}\kappa_{abc}\gamma^c\partial_\mu\gamma^a\partial^\mu\gamma^b + \frac{\kappa_{acd}\gamma^c\gamma^d\kappa_{bef}\gamma^e\gamma^f - 4\kappa_{abc}\gamma^c}{16\rho^2}\partial_\mu b^a\partial^\mu b^b\right)\,. \tag{5.39}$$

We conclude from this more careful derivation that there are no cross-terms involving $\partial_\mu\rho\partial^\mu\gamma^a$ and that we exactly get the proposed kinetic term for ρ in eq. (5.35). Additionally, each of the three terms in eq. (5.39) has to be positive, since in the physical region the total kinetic energy must be positive. For the complex structure/dilaton sector, the derivation is similar to that in the Kähler sector.

With the explicit kinetic terms for the moduli ρ and τ in eq. (5.35), we derive for ϵ from eq. (5.31) the following inequality (note that all contributions from the other moduli to ϵ are positive so that we obtain an inequality)

$$\epsilon \geq \frac{1}{V^2}\left(\frac{1}{3}\left(\rho\frac{\partial V}{\partial\rho}\right)^2 + \frac{1}{4}\left(\tau\frac{\partial V}{\partial\tau}\right)^2\right)\,. \tag{5.40}$$

We now subtract the positive quantity $\frac{4}{13}\left(\rho\frac{\partial V}{\partial\rho} - \frac{1}{4}\tau\frac{\partial V}{\partial\tau}\right)^2$ from eq. (5.40) and we arrive at

$$\epsilon \geq \frac{1}{39V^2}\left(\rho\frac{\partial V}{\partial\rho} + 3\tau\frac{\partial V}{\partial\tau}\right)^2\,. \tag{5.41}$$

In the following we will specify the necessary requirements such that the following holds

$$DV \equiv \left(-\rho\frac{\partial}{\partial\rho} - 3\tau\frac{\partial}{\partial\tau}\right)V \geq 9V\,. \tag{5.42}$$

If we now assume that we are in a region where $V > 0$, which is necessary for inflation, we can plug the square of eq. (5.42) in eq. (5.41) such that

$$\epsilon \geq \frac{27}{13}, \qquad \text{whenever} \quad V > 0\,, \tag{5.43}$$

which implies that slow-roll inflation and de Sitter vacua are excluded.

Provided that we can show the inequality (5.42), we have a no-go theorem against slow-roll inflation and de Sitter vacua. The proof is remarkably simple and uses only the scaling properties of the scalar potential with respect to the fields ρ and τ. Concretely, the classical four-dimensional scalar potential may receive contributions from the NSNS H_3-flux, the RR-fluxes F_p, where $p = 0, 2, 4, 6$, the geometric fluxes as well as from the sources (O6-orientifolds/D6-branes), such that, respectively,

$$V = V_3 + \sum_p V_p + V_f + V_{\mathrm{O6/D6}}\,, \tag{5.44}$$

where $V_3, V_p \geq 0$ since these contributions come from quadratic terms in the ten-dimensional action (B.2) and V_f and $V_{\mathrm{O6/D6}}$ can have either sign. From the ten-dimensional action (B.2) one easily deduces how the different contributions in the potential scales with respect to ρ and τ. As an example, let us examine the scaling properties of the NSNS H_3-flux. The energy arising from H_3 comes from the term in (B.2) that is proportional to H_3^2. From (B.3) it follows that H_3^2 is contracted with three factors of $g^{\mu\nu}$, the inverse internal metric, which scales with as $g^{-1} \propto \rho^{-1}$. This implies $V_3 \propto \rho^{-3}$. Concerning the scaling behavior of this term with respect to the four-dimensional dilaton τ we have to be careful to transform correctly from ten-dimensional Einstein frame to four-dimensional Einstein frame. Taking further into account the relation between the metric in string frame and in Einstein frame $g_{\mathrm{E}} = e^{-\Phi/2} g_{\mathrm{s}}$ we arrive at $V_3 \propto \tau^{-2}$. Similarly we derive the scalings for the other terms. Note that the contribution V_f in the four-dimensional potential V comes from the Einstein-Hilbert term in the ten-dimensional action,

$$V_f = -\frac{1}{2} M_P^4 \kappa_{10}^2 e^{2\Phi} \mathrm{Vol}^{-1} R = -\frac{1}{2} M_P^4 \kappa_{10}^2 \tau^{-2} R\,, \tag{5.45}$$

where R is the scalar curvature of the internal manifold (the explicit expression in given in eq. (4.28)). We conclude that $V_f \propto \rho^{-1}\tau^{-2}$, since $R \propto g^{-1} \propto \rho^{-1}$.

In summary, we obtain the general scaling behavior with respect to ρ and τ of the different contributions in the scalar potential as follows,

$$V_3 \propto \rho^{-3}\tau^{-2}, \qquad V_p \propto \rho^{3-p}\tau^{-4}, \qquad V_{O6/D6} \propto \tau^{-3}, \qquad V_f \propto \rho^{-1}\tau^{-2}\,. \tag{5.46}$$

Plugging these scalings in eq. (5.42) implies for the scalar potential

$$-\rho\frac{\partial V}{\partial \rho} - 3\tau\frac{\partial V}{\partial \tau} = 9V + \sum_{p=2,4,6} pV_p - 2V_f. \tag{5.47}$$

Hence, the necessary requirement to satisfy inequality (5.42) is that the contribution from the metric fluxes is zero or negative (recall that $V_p \geq 0$). This then translates in the above-mentioned bound $\epsilon \geq \frac{27}{13}$ ruling out slow-roll inflation and de Sitter vacua. We see from eq. (5.47) that one can avoid this no-go theorem if $V_f > 0$ for some region in the moduli space.

As we have seen in eq. (5.45), the contribution from the geometric fluxes is proportional to the negative scalar curvature of the internal manifold, $V_f \propto -R$. Avoiding the no-go theorem is thus equivalent to demanding that the internal space has negative curvature for some region in the moduli space.

Let us mention that we also require $V_{\mathrm{O6/D6}} < 0$. The reason is that we want to avoid a runaway of the potential in τ-direction. As all terms in eq. (5.46) scale with negative power of τ, for $V_{\mathrm{O6/D6}} \geq 0$ all terms would have positive coefficients (since we also required $V_f > 0$), leading to a runaway direction.

Let us further mention that for any vacuum we have $\partial V/\partial\rho = \partial V/\partial\tau = 0$ such that the right hand side of eq. (5.47) vanishes. For vanishing geometric fluxes $V_f = 0$ and assuming $V_p > 0$ for at least one $p = 2, 4, 6$ this implies $V = -(\sum pV_p)/9$, thus ruling out Minkowski vacua as well.

Let us summarize this result:

$$\epsilon \geq \frac{27}{13} \quad \text{whenever } V > 0 \text{ and } V_f \leq 0\,. \tag{5.48}$$

With the scaling properties of the different terms in eq. (5.46), we can find other combinations of derivatives with respect to ρ and τ that sets a bound for ϵ, e.g.

$$-\rho\frac{\partial V}{\partial\rho} - \tau\frac{\partial V}{\partial\tau} = 3V + 2V_3 - 2V_0 + 2V_4 + 4V_6 \geq 3V - 2V_0\,. \tag{5.49}$$

This combination is interesting for the case of vanishing mass parameter, $m = 0$, since for this case we have $V_0 \propto m = 0$. If we subtract the positive quantity $\frac{1}{84}\left(4\rho\frac{\partial V}{\partial\rho} - 3\tau\frac{\partial V}{\partial\tau}\right)^2$ from the right hand side in eq. (5.40), we obtain

$$\epsilon \geq \frac{1}{7V^2}\left(\rho\frac{\partial V}{\partial\rho} + \tau\frac{\partial V}{\partial\tau}\right)^2 \geq \frac{9}{7}\,, \tag{5.50}$$

where the second inequality comes from eq. (5.49) assuming vanishing Romans mass. This is a no-go against inflation for the case of vanishing Romans mass [1].

We learned in this section that the minimal ingredients for slow-roll inflation or de Sitter vacua are

$$\begin{aligned}&V_f > 0\,, m \neq 0\,, \quad \text{(Necessary conditions for slow-roll inflation or de Sitter vacua)}\,,\\ &V_{\mathrm{O6/D6}} < 0\,, \quad \text{(Condition to avoid a runaway direction)}\,.\end{aligned} \tag{5.51}$$

Strictly speaking, the only real restriction is that we have an internal manifold with negative curvature since we can always turn on F_0 flux and do an orientifold projection.

The nilmanifolds, which always have negative scalar curvature (see section 4.3) and, apart from the torus example, non-vanishing geometric fluxes, avoid these no-go theorems. As we will further show in part III of this thesis, some of the coset models we will study also have regions in moduli space with negative scalar curvature avoiding these no-go theorems. This makes these models interesting candidates for inflation and de Sitter vacua (without additional perturbative or non-perturbative quantum effects as in type IIB). However, one can formulate a stronger no-go theorem to further study the coset models. These redefined no-go theorems were proposed in [92], and we will review them in the next section. As we will see, we have slightly to adjust the proposed no-go theorems for the coset models [96].

[1] A different derivation for this no-go was recently given in [95].

5.3 Refined no-go theorems in the $(\tau,\,\sigma)$-plane

The models we want to study in this thesis all have special intersection numbers: the volume (5.29) depends only linearly on at least one of the Kähler moduli k^a. In the following, we denote this linear factor in the volume as k^0. Models with this property have intersection numbers that split into $\{0, a\}$, where a runs over the remaining Kähler moduli, such that the only non-vanishing intersection numbers are

$$\kappa_{0ab} \equiv \lambda_{ab}\,. \tag{5.52}$$

The refined no-go theorem of [92], which is quite similar to the no-go theorem of section 5.2, makes use of these special intersection numbers.

For the no-go theorem in the previous section 5.2, we split the Kähler moduli into an overall volume variable ρ and a set of angular variables γ^a. In the case where the volume factorizes, it turns out that it is useful to keep k^0 and then split the remaining Kähler moduli by

$$k^a = \sigma\chi^a\,, \tag{5.53}$$

where the angular variables are constrained by

$$\lambda_{ab}\chi^a\chi^b = 2\,, \tag{5.54}$$

to ensure that the volume of the internal space is $\mathrm{Vol} = k^0\sigma^2$. From eq. (5.37) we obtain the Kähler metric adapted to the special intersection numbers (5.52) as

$$\mathcal{K}_{i\bar{j}} = \begin{pmatrix} \frac{1}{4(k^0)^2} & 0 \\ 0 & \frac{1}{4\sigma^2}\left(\lambda_{ac}\chi^c\lambda_{bd}\chi^d - \lambda_{ab}\right) \end{pmatrix}. \tag{5.55}$$

With this Kähler metric we calculate the kinetic terms (again using $\partial_\mu(\lambda_{ab}\chi^a\chi^b) = 0$ from which follows that there are no mixed terms of the form $\partial_\mu\sigma\partial^\mu\chi^a$),

$$T = -\left(\frac{1}{4(k^0)^2}(\partial_\mu k^0)^2 + \frac{1}{2\sigma^2}(\partial_\mu\sigma)^2 + \frac{1}{4}\left(\lambda_{ac}\chi^c\lambda_{bd}\chi^d - \lambda_{ab}\right)\partial_\mu\chi^a\partial^\mu\chi^b + \cdots\right), \tag{5.56}$$

where the dots stand for additional manifestly positive kinetic terms for the other moduli fields. We now plug the kinetic term for σ in the definition for ϵ (5.31) and obtain the inequality

$$\epsilon \geq \frac{1}{V^2}\left(\frac{1}{2}\left(\sigma\frac{\partial V}{\partial\sigma}\right)^2 + \frac{1}{4}\left(\tau\frac{\partial V}{\partial\tau}\right)^2\right), \tag{5.57}$$

where the τ dependence is as in eq. (5.40). We again can subtract a positive quantity from the right hand side of eq. (5.57),

$$\frac{1}{2}\left(\sigma\frac{\partial V}{\partial\sigma}\right)^2 + \frac{1}{4}\left(\tau\frac{\partial V}{\partial\tau}\right)^2 - \frac{1}{36}\left(4\rho\frac{\partial V}{\partial\rho} - \tau\frac{\partial V}{\partial\tau}\right)^2 = \frac{1}{18}\left(\sigma\frac{\partial V}{\partial\sigma} + 2\tau\frac{\partial V}{\partial\tau}\right)^2, \tag{5.58}$$

and obtain for ϵ the inequality

$$\epsilon \geq \frac{1}{18V^2}\left(\sigma\frac{\partial V}{\partial\sigma}+2\tau\frac{\partial V}{\partial\tau}\right)^2 . \tag{5.59}$$

If we can show that

$$DV \equiv \left(-\sigma\frac{\partial}{\partial\sigma}-2\tau\frac{\partial}{\partial\tau}\right)V \geq 6V, \tag{5.60}$$

we would get for $V>0$ the following bound on ϵ

$$\epsilon \geq 2\,, \qquad \text{whenever} \quad V>0\,, \tag{5.61}$$

and slow-roll and de Sitter vacua are excluded.

Similar to the proof of the no-go theorem in section 5.2, we study the scaling properties with respect to σ and τ of the different contributions to the scalar potential. These scalings are computed in [55, 92] and can be summarized as follows

$$V_3 \propto \sigma^{-2}\tau^{-2}, \qquad V_0 \propto \sigma^2\tau^{-4}, \qquad V_6 \propto \sigma^{-2}\tau^{-4}, \qquad V_{O6/D6} \propto \tau^{-3}\,, \tag{5.62}$$

whereas V_2 and V_4 contain two terms, respectively,

$$V_2 = C_1\sigma^2\tau^{-4}+C_2\tau^{-4}\,, \qquad V_4 = C_3\sigma^{-2}\tau^{-4}+C_4\tau^{-4}\,, \tag{5.63}$$

where the coefficients C_i, $i=1,\dots,4$ depend on the fluxes and the other moduli and one can show that the two terms in V_2 and the two terms in V_4 are all separately positive [2].

From these scalings we compute

$$\begin{aligned} DV_3 &= 6V_3\,, \\ DV_{\mathrm{O6/D6}} &= 6V_{\mathrm{O6/D6}}\,, \\ DV_0 &= 6V_0\,, \\ DV_2 &= 6V_2 + \text{positive terms}\,, \\ DV_4 &= 8V_4 + \text{positive terms}\,, \\ DV_6 &= 10V_6\,. \end{aligned} \tag{5.64}$$

This implies that whenever we can show that $DV_f \geq 6V_f$, the no-go theorem (5.60), and hence $\epsilon \geq 2$, is applicable, which rules out slow-roll inflation and de Sitter vacua.

In [92], a condition was given such that $DV_f = 6V_f$ is satisfied automatically excluding slow-roll inflation and de Sitter vacua. Let us define the matrices r_{iI} as follows [97]

$$\mathrm{d}Y_i^{(2-)} = r_{iI}Y^{(3-)I}\,. \tag{5.65}$$

[2]We refer the reader to [92] for the explicit form of these coefficients.

where $Y_i^{(2-)}$ are the odd two-forms defined in eq. (3.37) and $Y^{(3-)I}$ is a basis of odd three-forms, such that $\int Y^{(3-)I} \wedge Y_J^{(3+)} = \delta_J^I$. The authors of [92] showed that the extra condition $r_{iI} = 0$ would ensure that $DV_f = 6V_f$, implying the no-go theorem. In the coset examples we will discuss in part III of the thesis, however, one has $r_{aI} \neq 0$. Therefore, we will explicitly check for each case separately whether $DV_f \geq 6V_f$ is satisfied or not.

To this end, it is convenient to define the variable U as follows

$$V_f = \frac{1}{2\tau^2 \mathrm{Vol}} U \,, \tag{5.66}$$

so that

$$DV_f = 6V_f + \frac{1}{2\tau^2 \mathrm{Vol}} DU = 6V_f + \frac{1}{2\tau^2 \mathrm{Vol}} (-\sigma \partial_\sigma) U \,, \tag{5.67}$$

and the no-go theorem applies if we can show that

$$-\sigma \frac{\partial U}{\partial \sigma} = -k^a \frac{\partial U}{\partial k^a} \geq 0 \,. \tag{5.68}$$

Furthermore, if the inequality (5.68) is strictly valid, Minkowski vacua are ruled out as well. This can be seen as follows. Using the eqs. (5.64) and (5.67), we obtain

$$DV = 6V + 2V_4 + 4V_6 + \frac{1}{2\tau^2 \mathrm{Vol}} (-\sigma \partial_\sigma) U + \text{positive terms} \,, \tag{5.69}$$

so that for a vacuum, $DV = 0$, we find with eq. (5.68)

$$V = -\frac{1}{6} \left(2V_4 + 4V_6 + \frac{1}{2\tau^2 \mathrm{Vol}} (-\sigma \partial_\sigma) U + \text{positive terms} \right) \leq 0 \,. \tag{5.70}$$

So, if the inequality (5.68) strictly holds, (5.70) strictly holds as well and Minkowski vacua are ruled out.

With the help of the no-go theorems discussed in this and the last section we will examine in chapter 13 whether the coset models constructed in part III of this thesis are valuable candidates for slow-roll inflation and de Sitter minima. All of the coset spaces have non-vanishing geometric fluxes and non-vanishing Romans mass. We thus have to check if the scalar curvature of the coset models is negative, and, if this is the case, if $-\sigma \partial_\sigma U \geq 0$. The nilmanifolds on the other hand always have negative scalar curvature and we can turn on non-vanishing Romans mass, thus circumventing the no-go theorem of section 5.2. However, in [92] no-go theorems like the one described in this section were applied to the class of twisted tori. The authors showed that for all these twisted tori the epsilon parameter is bounded from below by numbers of order unity ruling out slow-roll inflation and de Sitter minima for these models. Since the nilmanifolds can be identified with twisted tori, they are not valuable candidates for inflation and de Sitter vacua, and we will thus only study for the coset models of part III whether they allow slow-roll inflation or de Sitter solutions.

5.3.1 A comment on extra ingredients

Some ingredients that are not taken into account in the original no-go theorem of [91], nor in the no-go theorems of section 5.3 [92] are KK-monopoles, NS5-branes, D4-branes and D8-branes. Some of these ingredients were used in constructing simple de Sitter-vacua in [98, 99]. KK-monopoles would drastically change the topology and geometry of the internal manifold so that their introduction makes it difficult to obtain a clear ten-dimensional picture, hence we will not discuss this possibility further in this thesis. NS5-branes, D4-branes and D8-branes would contribute through their respective currents $j_{\rm NS5}$, $j_{\rm D4}$ and $j_{\rm D8}$ as follows to the Bianchi identities

$$\begin{aligned} \mathrm{d}H &= -j_{\rm NS5}\,, \\ \mathrm{d}F_4 + H \wedge F_2 &= -j_{\rm D4}\,, \\ \mathrm{d}F_0 &= -j_{\rm D8}\,. \end{aligned} \tag{5.71}$$

Since H and F_2 should be odd, and F_0 and F_4 even under all the orientifold involutions, we find that $j_{\rm NS5}$ is an odd four-form, $j_{\rm D4}$ an even five-form and $j_{\rm D8}$ an even one-form. In the approximation of left-invariant SU(3)-structure which we use in this thesis, one should also impose these brane-currents to be left-invariant (making the branes itself smeared branes). For the concrete type IIA models of chapter 11 there are no such currents $j_{\rm NS5}$, $j_{\rm D4}$ or $j_{\rm D8}$ with the appropriate properties under all orientifold involutions, implying that NS5-branes, D4- and D8-branes cannot be used in these models.

Let us briefly mention that an F-term uplifting along the lines of O'KKLT [100, 101] by combining the coset models with the quantum corrected O'Raifeartaigh model will not be a promising possibility either. The O'Raifeartaigh model is given by $\mathcal{W}_{\rm O} = -\mu^2 S$ and $\mathcal{K}_{\rm O} = S\bar{S} - \frac{(S\bar{S})^2}{\Lambda^2}$. The model has a de Sitter minimum for $S = 0$ where $V_{\rm O} \approx \mu^4$. We combine the two models as follows (the subscript IIA refers to the previously discussed flux and brane contributions)

$$\mathcal{W} = \mathcal{W}_{\rm IIA} + \mathcal{W}_{\rm O}\,, \qquad \mathcal{K} = \mathcal{K}_{\rm IIA} + \mathcal{K}_{\rm O}\,. \tag{5.72}$$

In lowest order in S the total potential is then given by

$$V \approx V_{\rm IIA} + e^{\mathcal{K}_{\rm IIA}} V_{\rm O} + \dots\,. \tag{5.73}$$

$$V_{\rm up} = \frac{A_{\rm up}}{\tau^4\,{\rm Vol}}\,. \tag{5.74}$$

Since we assume a positive uplift potential, $V_{\rm up} > 0$, the fact that $V_{\rm up}$ scales like F_6 tells us that adding this uplift potential does not help in circumventing the no-go theorems of section 5.2 or section 5.3.

Part II

Application to Nilmanifolds

Summary

For many phenomenological applications the exact knowledge of the full four-dimensional low-energy effective potential is required. For instance, one can search for phenomenologically interesting stable de Sitter solutions (the stability is checked by calculating the mass spectrum around the solution) or check whether the potential satisfies the necessary conditions for an inflationary epoch (this amounts to finding regions in moduli space with small values for ϵ and η as described in chapter 5). If the model admits a supersymmetric AdS_4 solution, one can explicitly calculate the mass spectrum of the moduli fields around the supersymmetric solution. To construct phenomenological attractive models, one can try to uplift such an AdS_4 solution by adding uplifting-terms to the potential, e.g. along the lines of the KKLT scenario [23].

One of the main motivations of this thesis is to provide the techniques to derive the four-dimensional low-energy effective field theory for a given compactification manifold. In the first part of this thesis we discussed the formal premises for such a project and it is now time for applying the developed techniques to concrete compactification manifolds. We first want to consider a large class of possible six-dimensional compact manifolds, the nilmanifolds which we described in section 4.2. As we discussed there, there are 34 isomorphism classes of six-dimensional nilmanifolds. In the following, we refer to these 34 isomorphism classes simply as “the nilmanifolds”. The complete list of these nilmanifolds can be found in table 4 of [64] and we will adopt their numbering.

We first want to construct type IIA AdS_4 $\mathcal{N}=1$ solutions on these nilmanifolds. In section 2.2.2 we discussed the necessary and sufficient conditions for such supersymmetric vacua. We first have to scan for nilmanifolds whose only non-vanishing torsion classes are $\mathcal{W}^-_{1,2}$. If we do not turn on source terms, we have to satisfy condition (2.34), which follows from the Bianchi identities. This condition turns out to be too restrictive for all the 34 nilmanifolds. We thus have to allow for D-brane/orientifold smeared sources. The Bianchi condition is then relaxed to condition (2.37), which indeed can be satisfied. Additionally, we have to check the positivity of the metric induced by J and Ω.

As a matter of fact, there are (only) two nilmanifolds among the 34 nilmanifolds that satisfy all the necessary and sufficient conditions [49], the six-torus and the nilmanifold 4.7 of table 4 of [64]. The nilmanifold 4.7 is also known under the name of the Iwasawa manifold. Essentially, the Iwasawa solution is the twisted torus $T^6/(\mathbb{Z}_2\times\mathbb{Z}_2)$ example examined in [70, 62, 72] [3].

We will describe these solutions in chapter 6. As we will see, the torus and the Iwasawa solution are related by T-duality along two directions (at least for some values of the parameters). Interestingly, as the intermediate step after one T-duality, there is a type IIB solution with static SU(2)-structure on the nilmanifold 5.1 of table 4 in [64].

Remarkably, for the same range of the parameter space for which the T-dualities above are valid, the solutions admit an interpretation as near-horizon geometries of intersecting brane configurations, [24]. From this point of view, the nilmanifold vacua in this range are nothing but near-horizon geometries of intersections of KK-monopoles with other branes in flat space. This nice feature of the 'brane picture' is summarized in table 6.1. Each solution in this table is related to the one in the column next to it by a T-duality. For the three nilmanifolds that provide a solution to $\mathcal{N} = 1$ AdS$_4$ we

IIA	IIB	IIA
T^6	nilmanifold 5.1	Iwasawa
D4/D8/NS5	D3/D5/D7/NS5/KK	D2/D6/KK

Table 6.1: Brane picture

next study in detail the four-dimensional low-energy effective field theory. The usual approach to construct the four-dimensional effective action is by using four-dimensional effective supergravity techniques which rely on supersymmetry. As we reviewed in section 3.2, this boils down to calculate the Kähler potential and the superpotential.

However, the direct approach to derive the four-dimensional effective action is by performing a Kaluza-Klein reduction. We reviewed the Kaluza-Klein recipe in section 3.1. The main result of this part of the thesis is a comparison of the results obtained by the direct Kaluza-Klein reduction with the results obtained via the effective supergravity approach. This provides us with an important consistency check between the two approaches.

To do so, we will first explicitly perform, in chapter 7, a Kaluza-Klein reduction on the torus and the Iwasawa manifold around the supersymmetric solution of chapter 6 and derive the mass spectrum for all the moduli fields.

In the following chapter 8, we derive the effective low-energy potential by means of the supergravity techniques and again derive the mass spectrum for the moduli fields around the supersymmetric solution. Comparing these results with the masses obtained from the direct Kaluza-Klein analysis, we find perfect agreement - showing that we can rely on the effective supergravity techniques also in the presence of metric fluxes.

Note that the results of this part of the thesis are published in [49].

[3] In the Iwasawa model there are four orientifolds. These can be equivalently described as a single orientifold supplemented with its images under a certain geometric $\mathbb{Z}_2 \times \mathbb{Z}_2$ group acting on the internal manifold.

Chapter 6

AdS_4 solutions on nilmanifolds

By taking the internal six-dimensional manifold to be a nilmanifold, we can construct explicit examples of type IIA $\mathcal{N} = 1$ compactifications to AdS_4 described in section 2.2.2. A systematic scan yields exactly two possibilities in type IIA satisfying the necessary and sufficient conditions: the torus T^6 and the Iwasawa manifold 4.7 of table 4 of [64], which (for some values of the parameters) are related by T-duality along two directions. We also find a type IIB solution with static SU(2)-structure which forms the intermediate step after one T-duality [1]. In this chapter we describe these solutions.

6.1 Type IIA solution on the T^6

Our first type IIA solution is obtained by taking the internal manifold to be a six-dimensional torus. Let us define a left-invariant basis $\{e^i\}$ such that:

$$\mathrm{d}e^i = 0, \qquad i = 1, \ldots, 6\,. \tag{6.1}$$

On the torus we can just choose $e^i = \mathrm{d}y^i$, where y^i are the internal coordinates. The SU(3)-structure is given by

$$\begin{aligned} J &= e^{12} + e^{34} + e^{56}\,, \\ \Omega &= (ie^1 + e^2) \wedge (ie^3 + e^4) \wedge (ie^5 + e^6)\;, \end{aligned}$$

which can indeed be seen to satisfy eqs. (2.6), (2.7) and (2.1) for $f = 0$, putting $\mathrm{vol}_6 = e^{1\ldots 6}$. It readily follows that all torsion classes vanish in this case. Note, however, that there are non-vanishing H and F_4 fields given by eq. (2.27)

$$\begin{aligned} H &= \frac{2}{5} e^{\Phi} m \left(e^{246} - e^{136} - e^{145} - e^{235}\right)\;, \\ F_4 &= \frac{3}{5} m \left(e^{1234} + e^{1256} + e^{3456}\right)\;. \end{aligned} \tag{6.2}$$

[1] In the case of type IIB, we did not make a complete scan so there might be more solutions of this type.

From the Bianchi identity in eq. (B.9a) we compute for the source term

$$j^6 = -\frac{2}{5}e^{\Phi}m^2(e^{246} - e^{136} - e^{145} - e^{235})\,, \tag{6.3}$$

such that, from eq. (2.37), we find that there is an orientifold source of the type (2.35) with $\mu = e^{2\Phi}m^2$. This source term corresponds to smeared orientifolds along $(1,3,5)$, $(2,4,5)$, $(2,3,6)$ and $(1,4,6)$ (see also the discussion in appendix D). The corresponding orientifold involutions are [2]

$$\begin{aligned}
\text{O6}: &\quad e^2 \to -e^2\,, \quad e^4 \to -e^4\,, \quad e^6 \to -e^6\,,\\
\text{O6}: &\quad e^1 \to -e^1\,, \quad e^3 \to -e^3\,, \quad e^6 \to -e^6\,,\\
\text{O6}: &\quad e^1 \to -e^1\,, \quad e^4 \to -e^4\,, \quad e^5 \to -e^5\,,\\
\text{O6}: &\quad e^2 \to -e^2\,, \quad e^3 \to -e^3\,, \quad e^5 \to -e^5\,.
\end{aligned} \tag{6.4}$$

For the torus, since we have vanishing torsion classes, we can decouple the tower of Kaluza-Klein masses (see discussion in section 2.2.2) when we take $m^2(e^{2\Phi}L^2_{\text{int}}) \ll 1$.

6.2 Type IIA solution on the Iwasawa manifold

The second type IIA solution is obtained by taking the internal manifold to be the Iwasawa manifold. The left-invariant basis is defined by:

$$\begin{aligned}
\mathrm{d}e^a &= 0, \quad a = 1,\ldots,4\,,\\
\mathrm{d}e^5 &= e^{13} - e^{24}\,,\\
\mathrm{d}e^6 &= e^{14} + e^{23}\,,
\end{aligned} \tag{6.5}$$

and is usually denoted by $(0,0,0,0,13-24,14+23)$. Up to basis transformations there is a unique SU(3)-structure satisfying the supersymmetry conditions of section 2.2.2:

$$\begin{aligned}
J &= e^{12} + e^{34} + \beta^2 e^{65}\,,\\
\Omega &= \beta\,(ie^5 - e^6)\wedge(ie^1 + e^2)\wedge(ie^3 + e^4)\,.
\end{aligned} \tag{6.6}$$

In the left-invariant basis, the metric is given by $g = \mathrm{diag}(1,1,1,1,\beta^2,\beta^2)$, and the torsion classes can be read off from $\mathrm{d}J$, $\mathrm{d}\Omega$, taking eq. (2.1) into account:

$$\begin{aligned}
\mathcal{W}_1^- &= -\frac{2i}{3}\beta\,,\\
\mathcal{W}_2^- &= -\frac{4i}{3}\beta\,\left(e^{12} + e^{34} + 2\,\beta^2\,e^{56}\right)\,,
\end{aligned} \tag{6.7}$$

[2] Each orientifold can be represented as $\Omega_p\sigma$, where Ω_p acts as a reflection on the world-sheet and σ is a purely geometrical operation acting on the target space. The composition of two six-orientifold actions $\Omega_p\sigma_1$ and $\Omega_p\sigma_2$ is purely geometrical, given by $\sigma_1\circ\sigma_2$, since $\Omega_p^2 = 1$. Similarly, the action of any number of orientifolds can be thought of equivalently as being generated by a single orientifold together with a purely geometrical action of a discrete group. In the case at hand, the four orientifold six-planes can be equivalently thought of as a single orientifold together with an orbifolding of the internal manifold by $\mathbb{Z}_2\times\mathbb{Z}_2$.

while all other torsion classes vanish. The fluxes can be read off from eq. (2.27) by plugging in $f = \frac{3}{2}e^{-\Phi}\beta$, while we can find m from eq. (2.37). We can verify that $d\mathcal{W}_2^-$ is proportional to $\mathrm{Re}\Omega$:

$$d\mathcal{W}_2^- = -\frac{8i}{3}\beta^2 \mathrm{Re}\Omega \ . \tag{6.8}$$

From the second line of eq. (6.7) we can read off: $|\mathcal{W}_2^-|^2 = 64\beta^2/3$. Comparing with eq. (2.37), taking $|\mathcal{W}_1^-|^2 = 4\beta^2/9$ into account – as follows from the first line of (6.7) – we therefore find a non-zero net orientifold six-plane charge:

$$\mu \geq \frac{25}{4}\beta^2 \ . \tag{6.9}$$

The solution (6.6) has one continuous parameter, β, corresponding essentially to the first torsion class $\mathcal{W}_1^-$. An additional second parameter can be introduced by noting that the defining SU(3)-structure equations (2.6) are invariant under the rescaling

$$J \to \gamma^2 J; \qquad \Omega \to \gamma^3 \Omega \ . \tag{6.10}$$

The additional scalar γ is related to the volume modulus via $\mathrm{vol}_6 = -\gamma^6\beta^2 e^{1\ldots6}$, as can be seen from eq. (2.7).

For the case $m = 0$, for which the bound (6.9) is saturated, the above example can also be obtained by performing two T-dualities on the torus solution of section 6.1, as can be checked explicitly. We find then that $\beta = \frac{2}{5}m_T e^{\Phi}$ where m_T is the mass parameter of the dual torus solution. The limit of decoupling the Kaluza-Klein tower corresponds to taking $\beta L_{\mathrm{int}} \ll 1$.

6.3 Type IIB solution on the nilmanifold 5.1

This solution is related, via a single T-duality, to both T^6 and the Iwasawa manifold. Indeed, let us perform a T-duality on the six-torus example of section 6.1 using the T-duality rules of e.g. [102] (see also [103] for a discussion of the action of T-duality on the pure spinors of a SU(3)×SU(3)-structure) [3]. After rescaling and relabeling the left-invariant forms we find the nilmanifold 5.1 described by (0,0,0,0,0,12+34). For the SU(2)-structure quantities described in section 2.1.2 we obtain

$$\begin{aligned} e^{i\theta}V &= \frac{1}{2}\left(\beta e^6 + ie^5\right) , \\ \omega_2 &= e^{13} - e^{24} , \\ \Omega_2 &= -ie^{i\theta}(ie^1 + e^3) \wedge (ie^4 + e^2) . \end{aligned} \tag{6.11}$$

[3]Note that it does not matter along which direction one performs the T-duality since all six perpendicular directions are equivalent. For the second T-duality (from which we obtain the Iwasawa solution of the previous section), only one direction leading to a geometric background is possible.

The metric is given by $g = \mathrm{diag}(1,1,1,1,1,\beta^2)$, and for the fluxes we have

$$\begin{aligned} H &= -\beta\left(e^{235}+e^{145}\right) , \\ e^{\Phi}F_1 &= \frac{5}{2}\beta^2 e^6 , \\ e^{\Phi}F_3 &= \frac{3}{2}\beta\left(e^{135}-e^{245}\right) , \\ e^{\Phi}F_5 &= \frac{3}{2}\beta^2 e^{12346} . \end{aligned} \tag{6.12}$$

Again we find that β is related to the mass parameter of the torus example via $\beta = \frac{2}{5}m_T e^{\Phi}$.

6.4 The brane picture

Following [24], it is possible to interpret the solutions presented in sections 6.1-6.3, from the perspective of intersecting branes. Namely, we would like to recover these solutions as near-horizon limits of domain walls in four non-compact dimensions, corresponding to systems of (orthogonally) intersecting branes (we will henceforth use the term 'brane' to refer to either a Dp-brane, an NS5-brane, or a KK-monopole).

More specifically, we will impose the following requirements on our brane configurations:

1. All configurations should consist of branes in ten-dimensional flat space, of which four directions are non-compact and six directions form a six-torus.
2. All branes should have exactly the same two spatial directions along the non-compact space.
3. All branes should intersect orthogonally, and we do not consider world-volume gauge fields.
4. The resulting configuration should preserve $\mathcal{N}=1$ supersymmetry in D=3, and should admit a regular near-horizon geometry with an AdS$_4$ factor.
5. Each configuration should include the maximum number of branes compatible with requirements 1-4.

Before we come to the description of explicit configurations satisfying the above requirements, let us note that, as we will see in the following, only brane configurations that lead to strict SU(3)-structure (as well as their T-dual configurations leading to static SU(2)-structures) arise in this way; this is the same class of backgrounds considered in chapter 2. The easiest way to arrive at this conclusion is to first determine which types of SU(3)×SU(3)-structure [4] are compatible with each brane separately. Indeed,

[4] See appendix C for a brief introduction to the language of generalized geometry and SU(3)×SU(3)-structure compactifications.

using their corresponding κ-symmetry projectors, it is straightforward to analyse what relations between the internal supersymmetry generators $\eta^{(1)}$ and $\eta^{(2)}$ of eq. (2.3) are possible, which leads to the following table of branes and their corresponding compatible types of structure: [5]

Brane	Structure type
D2	strict SU(3)
D3	static SU(2)
D4	SU(3)×SU(3)
D5	SU(3)×SU(3)
D6	SU(3)×SU(3)
D7	static SU(2)
D8	strict SU(3)
NS5	SU(3)×SU(3)
KK	SU(3)×SU(3)

See section 2.1 for the terminology. It turns out, that the configuration always needs to have D-branes to get a regular near-horizon AdS_4 limit. From the above table it follows, that if one of these D-branes is a D2, D3, D7 or D8 we already find strict SU(3)- or static SU(2)-structure. If not, let us consider the SU(3)-structure associated to $\eta^{(1)}$ as in eq. (2.5). Let us also define the complex coordinates z^i associated with this SU(3)-structure as well as their real and imaginary parts: $z^i = x^i + iy^i$. Because all the branes defining this SU(3)-structure intersect orthogonally (requirement 3), for each brane the x^i and y^i directions will be either along or perpendicular to the brane, i.e., there are no angles other than right angles. Now the relation between $\eta^{(1)}$ and $\eta^{(2)}$, which we can get from the κ-symmetry conditions of one of the D-branes, will contain gamma-matrices for directions that are also parallel or orthogonal to the x^i and y^i directions. Exhausting then all possibilities for the resulting structure shows that it can only be strict SU(3)- or static SU(2)-structure. It follows that if one is interested in constructing a configuration with general SU(3)×SU(3)-structure, one should restrict to D4, D6, D5, NS5 and KK-branes *and* put these branes at non-orthogonal angles.

Let us make a few comments concerning the requirements 1-5 above. The first one anticipates the fact that, as it will turn out, the internal nilmanifolds in the solutions of section 6.2-6.3 can be thought of as intersections of KK-monopoles in flat space. It therefore suffices to consider branes in flat space. The second requirement is of course just the requirement that the configuration should correspond to a domain wall in four space-time dimensions. The requirement of orthogonality was imposed for simplicity. It would be interesting to consider branes/monopoles intersecting at angles, but it would be quite difficult to construct the corresponding geometry because one could no longer use the harmonic superposition rules for branes [104]. The first part of the

[5] We also refer to table 1 of [43] which represents the allowed types of structure too, but now for space-filling orientifolds. Orientifolds have the same supersymmetry properties as D-branes with vanishing world-volume gauge field, however the difference of space-filling versus domain wall basically shifts the table.

fourth requirement is equivalent to demanding that the domain wall, viewed from the point of view of four-dimensional space-time, should be supersymmetric. Indeed, the minimal supersymmetry a domain wall in four dimensions can preserve, is one-half of $\mathcal{N} = 1$ in $D = 4$. This is equal to two real supercharges, i.e. $\mathcal{N} = 1$ in $D = 3$. Note that this implies that exactly one-sixteenth of the original supersymmetry of type II supergravity in $D = 10$ should be preserved. As each brane breaks supersymmetry by (at most) one-half, there will be (at least) four branes in the configuration. The final requirement is imposed because a configuration that does not include the maximum number of branes compatible with requirements 1-4, turns out not to have a regular AdS$_4$ near-horizon limit.

The rules for supersymmetric, orthogonally-intersecting branes were formulated some time ago [104, 105]. For the type of configurations we are considering, they can be summarized as follows:

intersecting branes	# of relative transverse directions
Dp/Dq	0 mod 4
NS5/NS5	0 mod 4
Dp/NS5	$7 - p$ or $11 - p$
Dp/KK	$5 - p$ or $9 - p$
KK/KK	0 mod 4
NS5/KK	4 or 8

The requirements 1-5 listed above severely restrict the set of admissible intersecting-brane configurations. It is in fact straightforward to show that all possible such configurations are related to each other by T-dualities. The brane configurations comprising the ‘nodes’ of this T-duality web, listed in table 6.1, are analyzed in the following [6].

D4/D8/NS5

This is the type IIA solution given in [24] and corresponds to the following system of intersecting D4/NS5/D8-branes:

[6] Without the second part of the fourth requirement there are three more configurations connected to each other by T-duality: D5/NS5, D6/D4/NS5/KK and D5/KK. Because they do not admit a regular near-horizon limit with AdS$_4$ factor, they are not of interest to us here, and we do not consider them.

	x^0	x^1	x^2	x^3	y^1	y^2	y^3	y^4	y^5	y^6
D4	$\otimes$	$\otimes$	$\otimes$		$\otimes$	$\otimes$				
D4$'$	$\otimes$	$\otimes$	$\otimes$				$\otimes$	$\otimes$		
D4$''$	$\otimes$	$\otimes$	$\otimes$						$\otimes$	$\otimes$
NS5	$\otimes$	$\otimes$	$\otimes$		$\otimes$		$\otimes$		$\otimes$	
NS5$'$	$\otimes$	$\otimes$	$\otimes$		$\otimes$			$\otimes$		$\otimes$
NS5$''$	$\otimes$	$\otimes$	$\otimes$			$\otimes$		$\otimes$	$\otimes$	
NS5$'''$	$\otimes$	$\otimes$	$\otimes$			$\otimes$	$\otimes$			$\otimes$
D8	$\otimes$	$\otimes$	$\otimes$		$\otimes$	$\otimes$	$\otimes$	$\otimes$	$\otimes$	$\otimes$

The full solution of [24] patches two asymptotic regions: a near-horizon $\text{AdS}_4\times\text{T}^6$ region and a flat region at infinity. Here we will concentrate on the near-horizon limit of the solution where the brane system above is replaced by fluxes. After rescaling of the coordinates, it can be written as:

$$\begin{aligned}
&\mathrm{d}s^2_{10} = \mathrm{d}s^2_{\text{AdS}_4} + \sum_{i=1}^{6}(\mathrm{d}y^i)^2; \quad \Phi = \text{const.}\,; \\
&H_{y^2y^4y^6} = H_{y^2y^5y^3} = H_{y^1y^6y^3} = H_{y^1y^5y^4} = a\,, \\
&F_{y^3y^4y^5y^6} = F_{y^1y^2y^5y^6} = F_{y^1y^2y^3y^4} = \frac{3}{2}e^{-\Phi}a\,, \qquad F_0 = \frac{5}{2}e^{-\Phi}a\ ,
\end{aligned} \tag{6.13}$$

where a and e^{Φ} are given in terms of the brane quanta in [24], and the SU(3)-structure is given by:

$$\begin{aligned}
&J = \mathrm{d}y^1\wedge\mathrm{d}y^2 + \mathrm{d}y^3\wedge\mathrm{d}y^4 + \mathrm{d}y^5\wedge\mathrm{d}y^6\,, \\
&\Omega = \left(i\mathrm{d}y^1 + \mathrm{d}y^2\right)\wedge\left(i\mathrm{d}y^3 + \mathrm{d}y^4\right)\wedge\left(i\mathrm{d}y^5 + \mathrm{d}y^6\right)\ .
\end{aligned} \tag{6.14}$$

We can readily see that, in the language of section 2.2.2, the present solution corresponds to setting $F_2' = 0$, $f = 0$ and $m = a$ with a source term:

$$j^{O6} = -\frac{2a^2}{5}e^{-\Phi}\text{Re}\,\Omega\ . \tag{6.15}$$

So while the original brane configuration has disappeared in the near-horizon limit, we have to introduce a set of smeared orientifold sources in order to satisfy the tadpole conditions:

	x^0	x^1	x^2	x^3	y^1	y^2	y^3	y^4	y^5	y^6
O6	$\otimes$	$\otimes$	$\otimes$	$\otimes$	$\otimes$		$\otimes$		$\otimes$	
O6$'$	$\otimes$	$\otimes$	$\otimes$	$\otimes$	$\otimes$			$\otimes$		$\otimes$
O6$''$	$\otimes$	$\otimes$	$\otimes$	$\otimes$		$\otimes$		$\otimes$	$\otimes$	
O6$'''$	$\otimes$	$\otimes$	$\otimes$	$\otimes$		$\otimes$	$\otimes$			$\otimes$

Indeed, as follows from eq. (2.30), in this limit, all torsion classes of the internal manifold vanish, as they should for T^6. Moreover, this is exactly the solution of section 6.1.

D3/D5/D7/NS5/KK

By applying a T-duality on the solution of the previous subsection, we obtain the following configuration (we do not display the non-compact directions anymore, but let us keep in mind that they form domain walls):

	y^1	y^2	y^3	y^4	y^5	y^6
D7		$\otimes$	$\otimes$	$\otimes$	$\otimes$	$\otimes$
D3		$\otimes$				
D5′	$\otimes$		$\otimes$	$\otimes$		
D5″	$\otimes$				$\otimes$	$\otimes$
NS5	$\otimes$		$\otimes$		$\otimes$	
NS5′	$\otimes$			$\otimes$		$\otimes$
KK″	$\bullet$	$\otimes$		$\otimes$	$\otimes$	
KK‴	$\bullet$	$\otimes$	$\otimes$			$\otimes$

Without loss of generality, we have taken the T-duality to be along y^1. Let us only describe the salient features of this model.

First of all, an analysis of the κ-symmetry conditions of the D-branes reveals that for this configuration the internal spinors satisfy

$$\eta_+^{(2)} = -e^{-i\theta}\gamma_{\underline{1}}\eta_-^{(1)} \ , \tag{6.16}$$

where $e^{-i\theta}$ is a phase describing the supersymmetry preserved by the domain wall in four dimensions, or, after taking the near-horizon limit, the phase of the superpotential W of AdS. So we see that we have static SU(2)-structure, which is also the only possibility for type IIB as mentioned in section 2.2.2 and explained in more detail in appendix C.

Secondly, when one goes to the near-horizon limit, the effect of the KK-monopoles is to twist the S^1 of direction 1 over the T^4 corresponding to the directions $(3,4,5,6)$, which is indicated with a bullet in the tables. This means that we find for the metric, after rescaling,

$$\mathrm{d}s^2_{10} = \mathrm{d}s^2_{\mathrm{AdS}_4} + \sum_{i=1}^{6}(e^i)^2 \, , \tag{6.17}$$

with

$$\begin{aligned} e^1 &:= \mathrm{d}y^1 + a(y^6\mathrm{d}y^3 + y^5\mathrm{d}y^4) \, , \\ e^i &:= \mathrm{d}y^i \ ; \quad i = 2, \dots, 6 \, , \end{aligned} \tag{6.18}$$

where a is the same parameter as in the T-dual. This means we have

$$\begin{aligned} \mathrm{d}e^1 &= a(e^{63} + e^{54})\,, \\ \mathrm{d}e^i &= 0\,, \end{aligned} \tag{6.19}$$

which, in fact, is equivalent to nilmanifold 5.1. So we see that just like the other branes the KK-monopoles disappear in the near-horizon limit and are replaced by flux, in this case the geometric flux a.

It turns out that in addition to the fluxes we have O5/O7 orientifold planes along the following directions:

	x^0	x^1	x^2	x^3	y^1	y^2	y^3	y^4	y^5	y^6
O5	⊗	⊗	⊗	⊗			⊗		⊗	
O5′	⊗	⊗	⊗	⊗				⊗		⊗
O7	⊗	⊗	⊗	⊗	⊗	⊗		⊗	⊗	
O7′	⊗	⊗	⊗	⊗	⊗	⊗	⊗			⊗

After appropriate rescaling and relabeling, this solution corresponds to the solution on the nilmanifold 5.1 of section 6.3.

D2/D6/KK

Starting from the type IIB configuration above, there is exactly one possibility left for a T-duality, i.e. along y^2. This is because T-dualizing along a direction perpendicular to a KK-monopole would result in a non-geometric background.

	y^1	y^2	y^3	y^4	y^5	y^6
D6			⊗	⊗	⊗	⊗
D2						
D6′	⊗	⊗	⊗	⊗		
D6″	⊗	⊗			⊗	⊗
KK	⊗	•	⊗		⊗	
KK′	⊗	•		⊗		⊗
KK″	•	⊗		⊗	⊗	
KK‴	•	⊗	⊗			⊗

An analysis of the κ-symmetry conditions of the branes reveals that this model has again strict SU(3)-structure. The four KK-monopoles result in a near-horizon geometry for which the T^2 along the directions $(1,2)$ is twisted over the base T^4 along $(3,4,5,6)$. The metric reads

$$\mathrm{d}s^2_{10} = \mathrm{d}s^2_{\mathrm{AdS}_4} + \sum_{i=1}^{6} (e^i)^2\ , \tag{6.20}$$

where we have defined

$$\begin{aligned} e^1 &:= \mathrm{d}y^1 + a(y^6 \mathrm{d}y^3 + y^5 \mathrm{d}y^4)\,, \\ e^2 &:= \mathrm{d}y^2 + a(y^5 \mathrm{d}y^3 - y^6 \mathrm{d}y^4)\,, \\ e^i &:= \mathrm{d}y^i \ ; \quad i = 3, \dots, 6\ , \end{aligned} \tag{6.21}$$

such that

$$\begin{aligned} \mathrm{d}e^1 &= a(e^{63} + e^{54})\,, \\ \mathrm{d}e^2 &= a(e^{53} + e^{46})\,, \\ \mathrm{d}e^i &:= \mathrm{d}y^i \ ; \quad i = 3, \dots, 6\ . \end{aligned} \tag{6.22}$$

After rescaling and relabeling we find the solution of section 6.2 for $m = 0$. For $m \neq 0$ the latter solution does not have a dual brane picture.

Finally note that in order to satisfy the tadpole conditions we have again O6-planes along the following directions:

	x^0	x^1	x^2	x^3	y^1	y^2	y^3	y^4	y^5	y^6
O6	$\otimes$	$\otimes$	$\otimes$	$\otimes$		$\otimes$	$\otimes$		$\otimes$	
O6$'$	$\otimes$	$\otimes$	$\otimes$	$\otimes$		$\otimes$		$\otimes$		$\otimes$
O6$''$	$\otimes$	$\otimes$	$\otimes$	$\otimes$	$\otimes$			$\otimes$	$\otimes$	
O6$'''$	$\otimes$	$\otimes$	$\otimes$	$\otimes$	$\otimes$		$\otimes$			$\otimes$

This completes the overview of brane configurations of table 6.1.

Chapter 7

Kaluza-Klein reduction

In this chapter, we will explicitly perform a Kaluza-Klein reduction described in section 3.1 on the torus solution of section 6.1 and the Iwasawa solution with $m = 0$ of section 6.2 and calculate the mass spectrum of the moduli fields around the supersymmetric vacuum. In the next chapter we will again derive the mass spectrum using effective supergravity techniques and compare the results. If we find agreement, this provides an important consistency check between the to approaches.

7.1 Expansion of the fields

For the Kaluza-Klein reduction on T^6 and the Iwasawa manifold, we will expand the fluctuations of the various fields in the following basis:

$$\delta B(x,y) = b^{i,\vec{n}}(x)\mathcal{Y}^{(2)}_{i,\vec{n}}(y) + b_1^{i,\vec{n}}(x)\mathcal{Y}^{(1)}_{i,\vec{n}}(y) + b_2^{\vec{n}}(x)\mathcal{Y}^{(0)}_{\vec{n}}(y)\,, \tag{7.1a}$$

$$\delta\phi(x,y) = \delta\phi^{\vec{n}}(x)\mathcal{Y}^{(0)}_{\vec{n}}(y)\,, \tag{7.1b}$$

$$\delta C^{(1)}(x,y) = c^{(1)i,\vec{n}}(x)\mathcal{Y}^{(1)}_{i,\vec{n}}(y) + c_1^{(1)\vec{n}}(x)\mathcal{Y}^{(0)}_{\vec{n}}(y)\,, \tag{7.1c}$$

$$\begin{aligned}\delta C^{(3)}(x,y) =& c^{(3)i,\vec{n}}(x)\mathcal{Y}^{(3)}_{i,\vec{n}}(y) + c_1^{(3)i,\vec{n}}(x)\mathcal{Y}^{(2)}_{i,\vec{n}}(y) + c_2^{(3)i,\vec{n}}(x)\mathcal{Y}^{(1)}_{i,\vec{n}}(y)\\ &+ c_3^{(3)\vec{n}}(x)\mathcal{Y}^{(0)}_{\vec{n}}(y)\,,\end{aligned} \tag{7.1d}$$

$$\delta g(x,y) = h^{i,\vec{n}}(x)\mathcal{X}^{(2)}_{i,\vec{n}}(y) + h_1^{i,\vec{n}}(x)\mathcal{Y}^{(1)}_{i,\vec{n}}(y) + h_2^{\vec{n}}(x)\mathcal{Y}^{(0)}_{\vec{n}}(y)\,. \tag{7.1e}$$

The functions $\mathcal{Y}^{(l)}_{i,\vec{n}}(y)$ are the l-eigenforms of the Laplacian operator and are given by

$$\mathcal{Y}^{(l)}_{i,\vec{n}}(y) = Y^{(l)}_i e^{i\vec{p}\cdot\vec{y}}\,, \qquad \vec{p} = \frac{\vec{n}}{L_{\rm int}}\,, \quad \vec{n}\in\mathbb{Z}^6\,, \tag{7.2}$$

where the $Y^{(l)}_i$ form a basis of harmonic l-forms on T^6. $\mathcal{X}^{(2)}$ are symmetric two-tensors

$$\mathcal{X}^{(2)}_{i,\vec{n}}(y) = X^{(2)}_i e^{i\vec{p}\cdot\vec{y}}\,, \qquad \vec{p} = \frac{\vec{n}}{L_{\rm int}}\,, \quad \vec{n}\in\mathbb{Z}^6\,, \tag{7.3}$$

Since we will restrict our analysis to the zero modes ($\vec{p}=0$), we only keep $\mathcal{Y}^{(l)}_{i,\vec{n}=0}(y) = Y^{(l)}_i$ and $\mathcal{X}^{(2)}_{i,\vec{n}=0}(y) = X^{(2)}_i$ in the expansions above and derivatives only act on the external fields. For the Iwasawa manifold, we will use for the expansion forms $Y^{(l)}_i$ left-invariant forms, which will not necessarily be all harmonic. When exterior derivatives act on these forms terms will be generated of the order of the geometric fluxes.

7.2 Kaluza-Klein expansion on AdS$_4\times$T^6

On the torus we can just choose $e^i = \mathrm{d}y^i$, where y^i are the internal coordinates, due to eq. (6.1). The harmonic l-forms in which we will expand the fields according to eq. (7.1) are thus of the form $e^{m_1\cdots m_l} = \mathrm{d}y^{m_1}\wedge\cdots\mathrm{d}y^{m_l}$, $l=1,\dots,6$. Since there is an orientifold projection present in our compactification, suitable expansion forms must be even or odd under all the orientifold involutions. The set of even/odd forms of different degree under all the orientifold involutions given in eq. (6.4) is

type	basis	name
odd 2-form	$e^{12},\ e^{34},\ e^{56}$	$Y^{(2-)}_i$
even 3-form	$e^{135},\ e^{146},\ e^{236},\ e^{245}$	$Y^{(3+)}_i$
odd 3-form	$e^{136},\ e^{145},\ e^{235},\ e^{246}$	$Y^{(3-)}_i$
even 4-form	$e^{1234},\ e^{1256},\ e^{3456}$	$Y^{(4+)}_i$
even symmetric 2-tensor	$e^1\otimes e^1, e^2\otimes e^2,\dots,e^6\otimes e^6$	$X^{(2)}_i$

Table 7.1: List of invariant forms for the torus solution

In particular, we find that there are no one- and five-forms nor even two-form. All external fields are even under the orientifold involutions (the orientifolds span the whole four-dimensional space-time). We find from eqs. (2.40a) and (2.40b) that Φ, g, F_0, C_3 are even, and B, C_1 are odd. The allowed terms of the expansion (7.1) are therefore

$$\delta B(x,y) = b^i(x) Y^{(2-)}_i\,, \tag{7.4a}$$

$$\delta\Phi(x,y) = \Phi(x)\,, \tag{7.4b}$$

$$\delta C^{(3)}(x,y) = c^{(3)i}(x) Y^{(3+)}_i + c^{(3)}_3(x)\,, \tag{7.4c}$$

$$\delta g(x,y) = h^i(x) X^{(2)}_i + h_2(x)\,. \tag{7.4d}$$

From eq. (3.21) we find the linear fluctuations of the field strengths (remember for the torus that $\hat{F}_2 = 0$)

$$\delta F_2 = -m\delta B\,, \tag{7.5a}$$

$$\delta F_4 = \mathrm{d}\delta C_3\,, \tag{7.5b}$$

and

$$\delta H = \mathrm{d}\delta B\,. \tag{7.6}$$

We want to derive the mass matrix for the four-dimensional fields. To achieve this, we compute the equations of motion for the four-dimensional fluctuations, which have to be of the form (3.6a) for the scalar fields $b^i(x), \Phi(x), c^{(3)i}(x)$ and $h^i(x)$, whereas for the external metric fluctuation $h_2(x)$ it will be of the form (3.6c). We thus first compute the variation of all the equations of motion (B.7a),(B.7b),(B.9b) and (B.10) to first order. In these equations we plug in the background values and the expansion of the fields (7.4), which gives us the equations of motion for the fluctuations.

The calculation is straightforward but rather lengthy. The variation of the Hodge star $\star$ complicates the calculation, since the metric appears in the Hodge star [1]. We can derive an expression for its variation

$$(\delta\star)F_l = \left(\frac{1}{2}g^{MN}\delta g_{MN}\right)\star F_l - \star[\delta g\cdot F_l]\,, \tag{7.7}$$

where

$$[\delta g\cdot F_l]_{M_1\dots M_l} = l\cdot\delta g_{[M_1|A}g^{AB}F_{B|M_2\dots M_l]}\,. \tag{7.8}$$

Let us look at one of the equations in more detail to explain the important steps in the calculation. The variation of the equation of motion for H, eq. (B.10), takes the form

$$0 = \mathrm{d}\star\mathrm{d}\delta B - (\mathrm{d}\delta\Phi)\wedge\star\hat{H} + \mathrm{d}\left[(\delta\star)\hat{H}\right] + \hat{F}_4\wedge\delta F_4 - (\star\hat{F}_4)\wedge\delta F_2 - m\star\delta F_2\,, \tag{7.9}$$

where we used the freedom to set $e^{-\hat{\Phi}} = 1$ in the torus solution, eq. (B.1) to remove the redundant RR-fields coming from the democratic formulation and that $\hat{F}_2 = 0$, $\mathrm{d}(\star\hat{H}) = 0$ and $\psi_n\wedge\alpha(j)|_8 = 0$ in the torus solution. Terms like the second one in eq. (7.9) vanish since $\star\hat{H} = \mathrm{vol}_4\wedge\star_6\hat{H}$ and $\mathrm{d}\delta\Phi(x)\wedge\mathrm{vol}_4 = 0$. Remember that we are only considering the zero internal modes and hence that the torus derivatives only act on the external fields. For the third term in eq. (7.9) we use eq. (7.7). Plugging the fluctuations (7.5) in eq. (7.9) and applying a Hodge star operation, we arrive at the following equation for the scalars b^i, which has [external/internal] index structure [0,2]:

$$0 = \Delta b^i Y_i^{(2-)} - \star(\hat{F}_4\wedge\mathrm{d}c_3^{(3)}) - m\star(\star\hat{F}_4\wedge b^i Y_i^{(2-)}) + m^2 b^i Y_i^{(2-)}\,. \tag{7.10}$$

Here we used, following the conventions summarized in appendix A [2],

$$\star_4\mathrm{d}\star_4\mathrm{d} = -\Delta\,. \tag{7.11}$$

[1] See appendix A for our conventions for the Hodge star. Further note that in this chapter, $\star$ denotes the *ten-dimensional* Hodge star, whereas the *four-* and *six-dimensional* Hodge star are indicated as $\star_4$ and $\star_6$, respectively.

[2] Note that $\mathrm{d}^\dagger b^i(x) = \star_4\mathrm{d}\star_4 b^i(x) = 0$ for $b^i(x)$ an external scalar field.

Similarly, we derive from the variation of the equation of motion of F_4 a [0,3]-equation and a [1,6]-equation, respectively,

$$0 = \Delta c^{(3)i} Y_i^{(3+)} - \star(\hat{H} \wedge \mathrm{d}c_3^{(3)}) \,, \tag{7.12a}$$

$$0 = \mathrm{d} \star \mathrm{d}c_3^{(3)} + \mathrm{d}b^i \wedge Y_i^{(2-)} \wedge \hat{F}_4 + \hat{H} \wedge \mathrm{d}c^{(3)i} \wedge Y_i^{(3+)} \,, \tag{7.12b}$$

and from the variation of the equation of motion of F_2 a $[4,5]$- and $[3,6]$-equation

$$0 = \hat{H} \wedge \star \left[h^i X_i^{(2)} \cdot \hat{F}_4 \right] \,, \tag{7.13a}$$

$$0 = \hat{H} \wedge \star(\mathrm{d}c^{(3)i} \wedge Y_i^{(3+)}) \,. \tag{7.13b}$$

Note that the equations for the RR-fields and H do not mix with the dilaton and the metric. The equations (7.13) are automatically satisfied using the orientifold projection. Indeed, the right-hand sides should have contained an even internal five-form respectively six-form under all orientifold involutions, which do not exists, so they must vanish.

To solve the eqs. (7.12), we integrate eq. (7.12b) and put the integration constant to zero (this would correspond to changing the background value of f). Taking the Hodge star of the integrated equation we get an expression for $\mathrm{d}c_3^{(3)}$ that we can put in eq. (7.12a) and also in eq. (7.10). This procedure corresponds to dualizing $c_3^{(3)}$, as explained in [106, 55]. Indeed, one may wonder why the three-form part $c_3^{(3)}$ of $\delta C^{(3)}$ appears in the equations of motion for the scalars but is easily integrated out. The reason is that we defined $\mathrm{d}c_3^{(3)}$ to describe the variation of the external part of δF_4. By means of the duality (B.1),

$$F_6 = e^{\frac{1}{2}\Phi} \star F_4 \,, \tag{7.14}$$

we can equivalently describe the external part of F_4 by the internal part of F_6. The variation of eq. (7.14) reads

$$\delta F_{6,\mathrm{int}} = \frac{1}{2} e^{\frac{1}{2}\Phi} f \, (\delta g^{\mu}{}_{\mu} - \delta g^{m}{}_{m} - \delta\Phi) \wedge \mathrm{vol}_6 + e^{\frac{1}{2}\Phi} \star \mathrm{d}c_3^{(3)} \,. \tag{7.15}$$

If we now plug in the general variation of the equation of motion of F_4,

$$\begin{aligned} e^{\frac{1}{2}\Phi} \star_4 \mathrm{d}c_3^{(3)} \wedge \mathrm{vol}_6 = &+ \frac{1}{2} e^{\frac{1}{2}\Phi} f \, (\delta g^{\mu}{}_{\mu} - \delta g^{m}{}_{m} - \delta\Phi) \wedge \mathrm{vol}_6 \\ &+ c^{(3)i} \hat{H} \wedge Y_i^{(3+)} - b^i \wedge Y_i^{(2-)} \wedge \hat{F}_4 + \delta f \,, \end{aligned} \tag{7.16}$$

we find

$$\delta F_{6,\mathrm{int}} = c^{(3)i} \hat{H} \wedge Y_i^{(3+)} - b^i \wedge Y_i^{(2-)} \wedge \hat{F}_4 \,, \tag{7.17}$$

which exactly corresponds to the part of δF_6 in eq. (3.21) that is first order in the fluctuations.

We are now ready to put in the expansion forms given in table 7.1 and solve the equations for the fluctuations of the RR-fields and H. To display the results it is convenient to make an appropriate choice of the expansion forms as follows

$$Y_0^{(3+)} = \mathrm{Im}\,\Omega\,, \tag{7.18a}$$

$$Y_i^{(3+)}\,, \quad i = 1,2,3: \quad 3 \text{ real } (2,1)+(1,2) \text{ forms}\,, \tag{7.18b}$$

and the odd two-forms

$$Y_0^{(2-)} = J\,, \tag{7.19a}$$

$$Y_i^{(2-)}\,, \quad i = 1,2: \quad 2 \text{ primitive real 2-forms}\,, \tag{7.19b}$$

where a primitive two-form is defined in (C.31). As explained in section 3.1, we display the result for the eigenvalues of the mass-matrix $\tilde{M}^2 = M^2 + 2/3\Lambda$:

mass eigenmode	mass (in units $m^2/25$)
$b^i, \quad i=1,2$	10
$c^i, \quad i=1,2,3$	0
$b^0 - 4c^{(3)0}$	10
$3b^0 + c^{(3)0}$	88

We now come to the dilaton and the Einstein equation. Let us first look at the dilaton equation (B.7a). The tricky part is the variation of the source term,

$$\star\langle\Psi_n, j\rangle\,, \tag{7.20}$$

where, according to eq. (B.6), $\Psi_7 = -\mathrm{vol}_4 \wedge e^{-\Phi}\mathrm{Im}\,\Omega$. The variation of $\mathrm{Im}\,\Omega$ can be done by looking at the variation of the vielbeins,

$$\Omega_{nmp} = e^{\underline{a}}{}_n e^{\underline{b}}{}_m e^{\underline{c}}{}_p \Omega_{\underline{abc}}\,, \tag{7.21}$$

where the underlined indices are flat indices. We can use the following relation

$$\delta e^{\underline{a}}{}_n = \frac{1}{2}\delta g_{nm} g^{mp} e^{\underline{a}}{}_p\,, \tag{7.22}$$

and we obtain

$$\delta \mathrm{Im}\,\Omega = \frac{1}{2}[\delta g \cdot \mathrm{Im}\,\Omega]\,. \tag{7.23}$$

The rest of the calculation is straightforward and we arrive at

$$0 = (\Delta + \frac{67m^2}{25})\delta\Phi + \frac{7m^2}{25}\sum_{i=1}^{6} h^i\,. \tag{7.24}$$

To compute the variation of the internal Einstein equation around the vacuum, we use the same methods as described above resulting in

$$\delta R_{mn} = \frac{1}{2}\Delta_L \delta g_{mn} + \nabla_{(m}\nabla^P \delta g_{n)P} - \frac{1}{2}\nabla_m \nabla_n \delta g^Q{}_Q \,, \tag{7.25}$$

where Δ_L is the Lichnerowicz operator defined in eq. (3.7), and all covariant derivatives and contractions are with respect to the background metric. For the flat torus vacuum this is easy evaluated to give

$$\delta R_{mn} = \frac{1}{2}\Delta \delta g_{mn} \,, \tag{7.26}$$

and the variation of the internal Einstein equation reads

$$0 = \Delta h^i + \frac{8m^2}{25}h^i + \frac{7m^2}{50}g_{ii}\delta\Phi + \frac{m^2}{50}g_{ii}\sum_{j=1}^{6} h^j + \frac{2m^2}{5}g_{ii}h^{i-(-1)^i} \,. \tag{7.27}$$

The result of diagonalizing the mass matrix is

mass eigenmode	mass (in units $m^2/25$)
$-h_{z^1\bar{z}^1} + h_{z^2\bar{z}^2} = -h^1 - h^2 + h^3 + h^4$	18
$-h_{z^1\bar{z}^1} + h_{z^3\bar{z}^3} = -h^1 - h^2 + h^5 + h^6$	18
$-3\,\delta\Phi + 7\sum h_i$	18
$7\,\delta\Phi + \sum h_i$	70
$\mathrm{Re}\, h_{z^1z^1} = -h^1 + h^2$	-2
$\mathrm{Re}\, h_{z^2z^2} = -h^3 + h^4$	-2
$\mathrm{Re}\, h_{z^3z^3} = -h^5 + h^6$	-2

The external contribution of the variation of the metric, $h_{\mu\nu}(x)$ in eq. (7.4d), is expected to describe a massless graviton . To verify this, we calculate the variation of the external Einstein equation. This results in

$$\frac{1}{2}\Delta_L h_{\mu\nu} + \nabla_{(\mu}\nabla^\rho h_{\nu)\rho} - \frac{1}{2}\nabla_{(\mu}\nabla_{\nu)}h^P{}_P + \frac{3m^2}{25}h_{\mu\nu} - \frac{3m^2}{20}g_{\mu\nu}\sum h_i - \frac{21m^2}{100}g_{\mu\nu}\delta\Phi = 0 \,. \tag{7.28}$$

At this point we have to take into account that so far we worked in the *ten*-dimensional Einstein frame. From eq. (3.32) we find that the conversion to the *four*-dimensional Einstein frame is as follows

$$g_{\mathrm{E}\mu\nu} = c\sqrt{g_6}\, g_{\mu\nu} \,, \tag{7.29}$$

where the constant factor $c = M_P^{-2}\kappa_{10}^{-2}V_s$ does not matter here, so that

$$c^{-1}h_{\mathrm{E}\mu\nu} = \sqrt{g_6}\, h_{\mu\nu} + \frac{1}{2}\sqrt{g_6}\, g_{\mu\nu}\sum_i h_i \,. \tag{7.30}$$

Plugging this into eq. (7.28) and using eq. (7.27) we find for $h_{\mathrm{E}\mu\nu}$ exactly equation (4.28) with $M^2 = 0$ so that $h_{\mathrm{E}\mu\nu}$ indeed describes a *massless* graviton.

7.3 Kaluza-Klein expansion on the Iwasawa manifold

The background for the Iwasawa manifold with $m = 0$, around which we expand the fields, is given in section 6.2. In contrast to the torus, some elements of a basis of left-invariant forms are no longer closed. We thus expand the fields not only in harmonic forms. This complicates the Kaluza-Klein computation since the derivatives not only act on the external fields but also on the internal zero-modes $Y^{(l)}$ of the left-invariant basis. When exterior derivatives act on these non-harmonic forms, terms will be generated of the order of the geometric fluxes.

The basis for these left-invariant forms that are even or odd under the orientifold involution turns out to be the same as for the torus, but now in the left-invariant basis appropriate to the Iwasawa manifold. This basis is given in table 7.1. Again Φ, g, F_0, C_3 are even, while B, C_1 are odd, resulting in the same expansion (7.4) as for the torus. From eq. (3.21) we get for the linear fluctuations of the field strengths

$$\delta F_2 = 0\,, \tag{7.31a}$$

$$\delta F_4 = \mathrm{d}\delta C_3 - \delta B \wedge \hat{F}_2\,. \tag{7.31b}$$

Expanding the equation of motion for H around the Iwasawa solution, we obtain

$$\begin{aligned} 0 =&\Delta b^i\, Y_i^{(2-)} + b^i \left(\star_6 \mathrm{d} \star_6 \mathrm{d}Y_i^{(2-)}\right) - c^{(3)i} \star_6 (\star_6 \mathrm{d}Y_i^{3+} \wedge \hat{F}_2) \\ &+ b^i \star_6 \left[\star_6 \left(Y_i^{(2-)} \wedge \hat{F}_2\right) \wedge \hat{F}_2\right] + f c^{(3)i} \star_6 \mathrm{d}Y_i^{3+} - b^i f \star_6 \left(Y_i^{(2-)} \wedge \hat{F}_2\right)\,, \end{aligned} \tag{7.32}$$

while the equation of motion for F_4 splits in $[1,6]$- and $[4,3]$-index structure

$$0 = \mathrm{d} \star_4 \mathrm{d}c_3^{(3)} + \frac{1}{2} f \mathrm{d}\left(\delta g^\mu{}_\mu - \delta g^m{}_m - \delta\Phi\right)\,, \tag{7.33a}$$

$$0 = \Delta c^{(3)i}\, Y_i^{(3+)} + c^{(3)i} \left(\star_6 \mathrm{d} \star_6 \mathrm{d}Y_i^{(3+)}\right) + f b^i \star_6 \mathrm{d}Y_i^{(2-)} - b^i \star_6 \mathrm{d} \star_6 \left(Y_i^{(2-)} \wedge \hat{F}_2\right)\,. \tag{7.33b}$$

The equation for $c_3^{(3)}$ mixes this time with the dilaton and the metric. Just as in the torus case, we integrate eq. (7.33a), put the integration constant to zero and plug the result for $\mathrm{d}c_3^{(3)}$ into the equations for the dilaton and the metric which we derive below.

We proceed by choosing the expansion forms. We take the same three-forms as in eq. (7.18), while for the two-forms we choose

$$Y_0^{(2-)} = \beta^2 e^{56}\,, \tag{7.34a}$$

$$Y_1^{(2-)} = e^{12} + e^{34}\,, \tag{7.34b}$$

$$Y_2^{(2-)} = e^{12} - e^{34}\,. \tag{7.34c}$$

As already mentioned, this time $Y_0^{(3+)}$ and $Y_0^{(2-)}$ are not closed. Defining m_T such that $\beta = \frac{2}{5} e^\Phi m_T$ (this is of course the Romans mass of the T-dual torus solution), we get the following masses:

mass eigenmode	mass (in units $m_T^2/25$)
$c^i, \quad i=1,2,3$	0
b^0+b^1	10
b^2	10
$8c^{(3)0}+5b^0+3b^1$	10
$c^{(3)0}-b^0+2b^1$	88

Due to T-duality the mass eigenvalues are the same as for the torus solution.

The equation for the variation of the dilaton around the background reads

$$\nabla^2\delta\Phi - \frac{99m_T^2}{100}\delta\Phi - \frac{3m_T^2}{100}\sum_{i=1}^{4}h^i + \frac{9m_T^2}{20}\sum_{i=5}^{6}h^i - \frac{9m_T^2}{100}\delta g^\mu{}_\mu - \frac{f}{2}\mathrm{d}c_3^{(3)}\cdot \mathrm{vol}_4\,. \tag{7.35}$$

We now plug in the result of integrating eq. (7.33a) and arrive at the following equation for the variation of the dilaton

$$0 = (\Delta + \frac{27m_T^2}{25})\delta\phi - \frac{9m_T^2}{25}\sum_{i=5}^{6}h^i + \frac{3m_T^2}{25}\sum_{i=1}^{4}h^i\,. \tag{7.36}$$

For the Einstein equation, we use again the variation of the Ricci tensor given in eq. (7.25). This time, however, we have non-trivial spin connections so that the calculation is not as simple as in the flat torus case. By explicitly deriving the spin connections one can show that the last two terms in eq. (7.25) vanish, whereas the Lichnerowitz operator (3.7) gets non-trivial contributions. The final result for the variation of the Einstein equation around the vacuum reads

$$0 = \Delta h^i + \frac{49m_T^2}{50}h^i + \frac{53m_T^2}{50}h^{i-(-1)^i} - \frac{11m_T^2}{50}\sum_{j=1}^{4}h^j - \frac{33m_T^2}{50}\delta\phi \quad \text{for} \quad i=5,6\,, \tag{7.37a}$$

and, for $i=1,\dots,4$,

$$0 = \Delta h^i + \frac{8m_T^2}{25}h^i + \frac{2m_T^2}{5}h^{i-(-1)^i} - \frac{3m_T^2}{10}\sum_{j=5}^{6}h^j + \frac{m_T^2}{10}\sum_{j=1}^{4}h^j + \frac{3m_T^2}{10}\delta\phi\,. \tag{7.37b}$$

Diagonalizing the mass matrix we find the following eigenmodes with corresponding masses:

mass eigenmode	mass (in units $m_T^2/25$)
$-h_{z^1\bar z^1}+h_{z^2\bar z^2} = -h^1-h^2+h^3+h^4$	18
$11h_{z^1\bar z^1}+5h_{z^3\bar z^3} = 11(h^1+h^2)+5(h^5+h^6)$	18
$5\delta\Phi-3(h^1+h^2)$	18
$3\delta\Phi-3(h^5+h^6)+(h^1+h^2+h^3+h^4)$	70
$\mathrm{Re}\,h_{z^1z^1} = -h^1+h^2$	-2
$\mathrm{Re}\,h_{z^2z^2} = -h^3+h^4$	-2
$\mathrm{Re}\,h_{z^3z^3} = -h^5+h^6$	-2

Once again, we find the same masses as in the torus example.

Let us summarize the results of the Kaluza-Klein reduction on the six-torus and the Iwasawa manifold. In both cases we obtain the following mass eigenvalues (in units $m_T^2/25$) for the scalar fields

Complex structure	$-2,\ -2,\ -2$
Kähler & dilaton	70, 18, 18, 18
Three axions of δC_3	0, 0, 0
δB & one more axion	88, 10, 10, 10

That we obtain exactly the same mass spectrum for both manifolds is the expected result, since the two solutions are related by T-duality. An interesting observation is that all three axions correspond to massless moduli, a feature that is also discussed in [72]. It is argued there that, when one introduces D6-branes, these axions can provide Stückelberg masses to some of the U(1) gauge fields on the D-brane. In any case, we will see later that most of the coset examples we will study in the third part of this thesis do have all moduli stabilized. We also notice that some masses are tachyonic, which is allowed because they are still above the Breitenlohner-Freedman bound (3.11).

Chapter 8

Effective supergravity

In chapter 7 we derived the masses of the scalar fields by means of an explicit Kaluza-Klein reduction for the torus and the Iwasawa solution. The widely-used approach to derive the four-dimensional effective action is by using $\mathcal{N} = 1$ effective supergravity techniques based on the superpotential and Kähler potential. We reviewed the supergravity techniques in section 3.2. In this section we will use this approach and again derive the masses of the scalar fields around the supersymmetric solution. Comparing these results with the results obtained with the direct Kaluza-Klein reduction we perform an important cross-check for the expressions for the superpotential and the Kähler potential to handle geometric fluxes.

8.1 Type IIA on T^6

Given the orientifold projection (6.4) we choose the following basis of odd two-forms and even three-forms

$$\begin{aligned} Y_i^{(2-)}: &\qquad e^{12}, e^{34}, e^{56}\,, \\ Y_i^{(3+)}: &\qquad -e^{135}, e^{146}, e^{236}, e^{245}\,, \end{aligned} \tag{8.1}$$

as expansion forms in eq. (3.37) such that

$$\begin{aligned} J_c = t^1 e^{12} + t^2 e^{34} + t^3 e^{56}\,, \\ e^{\Phi}\mathrm{Im}\,\Omega + i\delta C_3 = e^{-\hat{\Phi}}(-z^1 e^{135} + z^2 e^{146} + z^3 e^{236} + z^4 e^{245})\,, \end{aligned} \tag{8.2}$$

where we took out the background $e^{-\hat{\Phi}}$ from the definition of z^i for further convenience.

Using the expression (3.34) and the background fluxes in eq. (6.2) to derive the superpotential, we immediately find

$$\mathcal{W}_{\mathrm{E,Torus}} = \frac{e^{-i\theta}}{4\kappa_{10}^2} V_s m \left[-t^1 t^2 t^3 + \frac{3}{5}(t^1 + t^2 + t^3) - \frac{2}{5}(z^1 + z^2 + z^3 + z^4) \right]\,, \tag{8.3}$$

where V_s is a standard volume $V_s = \int e^{1...6}$, which does not depend on the moduli. Moreover, with eq. (3.35) and the Hitchin procedure explained in appendix C, the Kähler potential reads:

$$\mathcal{K} = \mathcal{K}_k + \mathcal{K}_c + 3\ln(8\kappa_{10}^2 M_P^2 V_s^{-1} e^{4\hat{\Phi}/3}) \,, \tag{8.4a}$$

where

$$\mathcal{K}_k = -\ln\left(\prod_{i=1}^{3}(t^i + \bar{t}^i)\right) \,, \tag{8.4b}$$

is the Kähler potential in the Kähler-moduli sector and

$$\mathcal{K}_c = -\ln\left(4\prod_{i=1}^{4}\left(z^i + \bar{z}^i\right)\right) \,, \tag{8.4c}$$

is the Kähler potential in the complex structure moduli sector.

We are now ready to calculate the mass spectrum of the scalar fields around the supersymmetric solution. Using the expressions for the superpotential (8.3) and the Kähler potential (8.4), it is straightforward to calculate the four-dimensional Einstein-frame action (3.25). From this action we compute the equation of motion for the scalar fields

$$\Delta\phi^k + M_P^{-2}(\hat{\mathcal{K}}^{-1}\hat{M})^k{}_i\phi^i = 0 \,, \tag{8.5}$$

where $\hat{M}_{ij} = \frac{1}{2}\frac{\partial^2 V}{\partial\phi^i\partial\phi^j}|_{\text{background}}$ is the mass matrix and $\hat{\mathcal{K}}_{ij}$ is the Kähler metric in real coordinates in the background. Therefore, to compare the results for the masses in the analysis with the superpotential and the Kähler potential with the results from the Kaluza-Klein reduction we need to diagonalize the matrix $M_P^{-2}\hat{\mathcal{K}}^{-1}\hat{M}$. Remember that the results from the Kaluza-Klein reduction were in the ten-dimensional Einstein frame, whereas using the effective supergravity approach of this section we get the results in four-dimensional Einstein frame such that we have to use eq. (3.33) to compare the results of both approaches. Upon noting that in the Kaluza-Klein analysis we set the background values for the warp factor and the dilaton equal to zero and $\text{Vol} = V_s$, we find exactly the same result for the mass spectrum as in section 7.2.

8.2 Type IIA on the Iwasawa manifold

For convenience we choose this time the following expansion basis:

$$\begin{aligned} Y^{(2-)}: &\qquad e^{12}, e^{34}, -\beta^2 e^{56}, \\ Y^{(3+)}: &\qquad -\beta e^{135}, -\beta e^{146}, -\beta e^{236}, \beta e^{245} \,. \end{aligned} \tag{8.6}$$

This implies that $\mathrm{d}Y_i^{(3+)} = -\beta e^{1234}$ for all $i = 1, \dots, 4$.

We find the superpotential

$$\mathcal{W}_{\mathrm{E,Iwasawa}} = \frac{-ie^{-i\theta}}{4\kappa_{10}^2} m_T V_s \left[\frac{3}{5} - \frac{2}{5} t^3 (z^1 + z^2 + z^3 + z^4) + \frac{3}{5}(t^1 t^3 + t^2 t^3) - t^1 t^2 \right] , \tag{8.7}$$

where $V_s = \int -\beta^2 e^{1\dots 6}$ is again a standard volume and $m_T = \frac{5}{2} e^{-\hat{\Phi}} \beta$ the Romans mass of the T-dual torus solution. We note here the following relation

$$\mathcal{W}_{\mathrm{E,Iwasawa}} = -it^3 \mathcal{W}_{\mathrm{E,Torus}}(t^3 \to \frac{1}{t^3}) , \tag{8.8}$$

which follows from T-duality [1]. The Kähler potential for the Iwasawa manifold is the same as in eq. (8.4).

In the end, we find exactly the same masses as on the torus, as expected from T-duality, and thus also the same masses as in the Kaluza-Klein approach for the Iwasawa manifold. Let us stress again that this provides an important consistency check on the ability of the superpotential/Kähler potential approach to handle geometric fluxes.

If we now turn on $m \neq 0$ in the Iwasawa solution, we get extra terms in the superpotential that look exactly like the torus superpotential, so we find:

$$\mathcal{W}_{\mathrm{E,Iwasawa},m\neq 0} = \mathcal{W}_{\mathrm{E,Iwasawa}}(m_T) + \mathcal{W}_{\mathrm{E,Torus}}(m) . \tag{8.9}$$

The mass spectrum is the same upon replacing $m_T^2 \to m^2 + m_T^2$.

8.3 Type IIB on nilmanifold 5.1

For our analysis we will need expansion forms with the following behaviour under O5 and O7-planes

type under O5/O7	basis	name
odd/even 1-form	$e^5, -\beta e^6$	$Y_i^{(1-+)}$
even/odd 2-form	e^{14}, e^{23}	$Y_i^{(2+-)}$
odd/odd 2-form	$e^{13}, -e^{24}$	$Y_i^{(2--)}$
odd/even 4-form	$\beta e^{1256}, \beta e^{3456}$	$Y_i^{(4-+)}$

and choose the standard volume $V_s = \int \beta e^{123456}$.

The superpotential is given by [2]:

$$\mathcal{W}_{\mathrm{E,nil}} = -\frac{m_T V_s C}{4\kappa_{10}^2} \left(\frac{3}{5} - \frac{2}{5}\tau (z^1 + z^2 + w^1 + w^2) + \frac{3}{5}\tau (t^1 + t^2) - t^1 t^2 \right) , \tag{8.10}$$

[1] Note that in order to keep the form of the Kähler potential, we transform the superpotential as $\mathcal{W} \to t\mathcal{W}$.

[2] Here, it turns out to be convenient to take out the background $e^{-\hat{\Phi}}$ from the definition of z^i and w^i, i.e., we expand as follows $e^{-\Phi}\mathrm{Im}\Omega_2 + i\delta C_2 = z^i e^{-\hat{\Phi}} Y_i^{(2+-)}$ and $-ie^{-\Phi} 2V \wedge \bar{V} \wedge \mathrm{Re}\Omega_2 + i\delta C_4 = w^i e^{-\hat{\Phi}} Y_i^{(4-+)}$.

where $V_s = \int \beta e^{123456}$ is the standard volume. The Kähler potential reads:

$$\begin{aligned}\mathcal{K} = &- \ln \left((\tau + \bar{\tau}) \prod_{i=1}^{2} (t^i + \bar{t}^i) \right) - \ln \left(4 \prod_{i=1}^{2} (z^i + \bar{z}^i) \prod_{i=1}^{2} (w^i + \bar{w}^i) \right) \\ &+ 3 \ln (8\kappa_{10} M_P^2 V_s^{-1} e^{4\hat{\Phi}/3}) - \ln |C|^2 \,.\end{aligned} \tag{8.11}$$

We can eliminate the complex scalar C by performing a Kähler transformation (3.31). Again, by T-duality, we expect the same mass spectrum as for the torus and the Iwasawa manifold, which indeed turns out to be the case. This implies that the proposed expressions for the complex scalars and the superpotential and Kähler potential for the static SU(2)-structure proposed in section 3.2 yield sensible results.

Part III

Application to Coset Spaces

Summary

The second class of compactification manifolds we want to consider in this thesis are the six-dimensional coset spaces which we described in chapter 4. Examples of compactifications on coset spaces in other contexts have already appeared in [65, 78], whereas particular $\mathcal{N} = 1$ AdS_4 solutions on type IIA string theory appeared in [107, 69, 108]. A systematic search for type IIA solutions on coset spaces was performed recently in [34].

The aim in this part of the thesis is to compute the four-dimensional low-energy effective theory for compactifications on coset spaces. We discussed the necessary procedure for this in chapter 3. After having established the consistency between the direct Kaluza-Klein reduction and the effective supergravity techniques for the example of nilmanifolds, we will rely in this part of the thesis on the supergravity techniques to derive the effective theory.

In the first two chapters of this part, we will, following [34], discuss the geometry on the different coset spaces of table 4.1 and identify the coset spaces that allow to define a strict SU(3)-structure. Furthermore, we will review the $\mathcal{N} = 1$ AdS_4 solutions on the coset spaces and comment on a possible solution with non-constant warp factor and dilaton.

In the following chapter we then come to the derivation of the four-dimensional effective action for the coset spaces in question. In particular, we will derive the superpotential and the Kähler potential for the most general choice of background fluxes. As an application of the effective action, we compute the mass spectrum of the moduli fields around the supersymmetric AdS_4 solution (if the coset allows for a solution) and comment for two models on how to identify the number of supersymmetric solutions in a given bubble of the moduli space. The subsequent chapter studies type IIB compactifications with static SU(2)-structure on the coset models. Most of these compactifications turn out to be related by a T-duality to type IIA strict SU(3)-structure compactifications that we already studied. However, one model is new since it is related by T-duality to a type IIA strict SU(3)-structure compactification with *non-geometric* fluxes.

Finally, in chapter 13, we study the phenomenological aspects of the compactifications on the coset models. As we discussed in chapter 5 there are in classical type IIA strong no-go theorems against slow-roll inflation and de Sitter vacua. We will thus systematically analyse whether the coset compactifications are able to avoid the no-go theorems. In fact, there are two coset compactifications that are not directly ruled out by any known no-go theorem (one of them is the type IIB compactification with non-geometric T-dual). For these models a numerical analysis is necessary.

As a general remark, we note that none of our models contain light bulk gauge fields in the spectrum.

In the following two tables we summarize some of the important results. Table 8.1 summarizes type IIA strict SU(3)-structure compactifications, whereas table 8.2 summarizes type IIB static SU(2)-structure compactifications.

Coset space	Moduli fields	AdS_4 solution	Unstabilized moduli	Avoids no-go
$\frac{\mathrm{G}_2}{\mathrm{SU}(3)}$	4	yes	0	no
$\frac{\mathrm{Sp}(2)}{\mathrm{S}(\mathrm{U}(2)\times\mathrm{U}(1))}$	6	yes	0	no
$\frac{\mathrm{SU}(3)}{\mathrm{U}(1)\times\mathrm{U}(1)}$	8	yes	0	no
$\frac{\mathrm{SU}(3)\times\mathrm{U}(1)}{\mathrm{SU}(2)}$	8	yes	0	no
$\mathrm{SU}(2)\times\mathrm{SU}(2)$	14	yes	1	yes
$\frac{\mathrm{SU}(2)^2}{\mathrm{U}(1)}\times\mathrm{U}(1)$	10	no	-	no
$\mathrm{SU}(2)\times\mathrm{U}(1)^3$	14	no	-	no

Table 8.1: Results for type IIA strict SU(3)-structure compactifications on coset models. Indicated are the number of moduli fields, whether the compactification allows an $\mathcal{N}=1$ AdS_4 solution and if so, the number of unstabilized moduli in this solution. In addition indicated is whether the coset model avoids the no-go theorems against inflation and de Sitter vacua.

Coset space	Moduli fields	Type IIA T-dual	Avoids no-go
$\frac{\mathrm{SU}(3)\times\mathrm{U}(1)}{\mathrm{SU}(2)}$	8	yes	no
$\frac{\mathrm{SU}(2)^2}{\mathrm{U}(1)}\times\mathrm{U}(1)$	10	yes	no
$\mathrm{SU}(2)\times\mathrm{SU}(2)$	14	no	yes
$\mathrm{SU}(2)\times\mathrm{U}(1)^3$	14	yes	no

Table 8.2: Results for type IIB static SU(2)-structure compactifications on coset models. Indicated are the number of moduli fields and whether the compactification is related by T-duality to a type IIA strict SU(3)-structure compactification of table 8.1. In addition indicated is whether the coset model avoids the no-go theorems against inflation and de Sitter vacua.

Note that the results of this part of the thesis are published in parts in [49, 96]. In particular, some of the results of chapter 11 can be found in [49], whereas the results of chapter 13 are published in [96].

Chapter 9

Geometry of coset spaces that admit a strict SU(3)-structure

In this chapter, we describe the six-dimensional coset spaces based on semi-simple and U(1)-groups that are suitable for supersymmetric compactifications to four space-time dimensions. We discussed in chapter 2 the necessary condition for a six-dimensional compact manifold to allow for a supersymmetric four-dimensional effective theory, namely that the structure group of the manifold is reduced to SU(3) [1]. As the authors of [34] showed, this condition translates into the necessary requirement that the group H of a coset space $M = G/H$ should be contained in SU(3). The list of all six-dimensional coset spaces based on semi-simple and U(1)-groups of this type was given in that paper and is summarized in table 4.1.

To decide whether a coset space satisfying the necessary condition $H \subseteq \mathrm{SU}(3)$ actually admits a left-invariant strict SU(3)-structure we will proceed as follows: as explained in section 4.2, we specify the structure constants by examining the corresponding Lie-algebras of G and H. Next we compute the set of G-invariant forms using condition (4.23). With these forms, we can write down the most general ansatz for J and Ω and check whether it is possible to satisfy the conditions for a strict SU(3)-structure (2.6), to obtain a well defined Hitchin functional and whether the induced metric can be chosen to be positive definite. The coset spaces satisfying these conditions are summarized in table 9.1.

In this and the next chapter, we will closely follow [34], where the authors presented $\mathcal{N} = 1$ supersymmetric AdS_4 solutions on the coset spaces with left-invariant strict SU(3)-structure. Supersymmetric AdS_4 solutions are possible on the first five cosets presented in table 9.1. However, the last two coset spaces in table 9.1 were not explicitly pointed out as possible candidates admitting a left-invariant SU(3)-structure, since they do not allow for a supersymmetric AdS_4 solution.

In the following we will assume that the algebra $\mathfrak{g}$ of G is generated by the set of

[1] We will discuss compactifications on coset models with static SU(2)-structure in chapter 12.

Coset space	admit SU(3)-structure	AdS_4 solution
$\frac{\mathrm{G}_2}{\mathrm{SU}(3)}$	yes	yes
$\frac{\mathrm{Sp}(2)}{\mathrm{S}(\mathrm{U}(2)\times\mathrm{U}(1))}$	yes	yes
$\frac{\mathrm{SU}(3)}{\mathrm{U}(1)\times\mathrm{U}(1)}$	yes	yes
$\frac{\mathrm{SU}(3)\times\mathrm{U}(1)}{\mathrm{SU}(2)}$	yes	yes
$\mathrm{SU}(2)\times\mathrm{SU}(2)$	yes	yes
$\frac{\mathrm{SU}(2)^2}{\mathrm{U}(1)}\times\mathrm{U}(1)$	yes	no
$\mathrm{SU}(2)\times\mathrm{U}(1)^3$	yes	no

Table 9.1: List of coset models with SU(3)-structure. Further indicated are the coset models that allow for an $\mathcal{N}=1$ AdS_4 solution.

generators E_A, $A=1,\dots\dim(G)$, where

$$[E_A, E_B] = f^C{}_{AB} E_C\,. \tag{9.1}$$

We choose the generators such that the E_A with $A=1,\dots,6$ correspond to the $\mathcal{K}_i$ and the E_A with $A=7\dots,6+\dim(H)$, correspond to the $\mathcal{H}_a$ (see also the discussion in section 4.2).

The coset $\frac{\mathbf{G}_2}{\mathbf{SU(3)}}$

The structure constants for the group G_2 are given by [109, 78]:

$$\begin{aligned}
&f^1{}_{63} = f^1{}_{45} = f^2{}_{53} = f^2{}_{64} = \frac{1}{\sqrt{3}}\,,\\
&f^7{}_{36} = f^7{}_{45} = f^8{}_{53} = f^8{}_{46} = f^9{}_{56} = f^9{}_{34} = f^{10}{}_{16} = f^{10}{}_{52}\\
&= f^{11}{}_{51} = f^{11}{}_{62} = f^{12}{}_{41} = f^{12}{}_{32} = f^{13}{}_{31} = f^{13}{}_{24} = \frac{1}{2}\,,\\
&f^{14}{}_{43} = f^{14}{}_{56} = \frac{1}{2\sqrt{3}}\,, \qquad f^{14}{}_{21} = \frac{1}{\sqrt{3}}\,,\\
&f^{i+6}{}_{j+6,k+6} = \tilde{f}_{ijk}\,,
\end{aligned} \tag{9.2}$$

where $E^7,\dots,E^{14}$ generate the su(3) subalgebra and $\tilde{f}_{ijk}$ are the corresponding Gell-Mann structure constants. The G-invariant forms satisfying condition (4.23) are [2]

$$\begin{aligned}
\text{two-forms}: &\quad \{e^{12} - e^{34} + e^{56}\}\,,\\
\text{three-forms}: &\quad \{\rho = e^{245} + e^{135} + e^{146} - e^{236}, \hat{\rho} = -e^{235} - e^{246} + e^{145} - e^{136}\}\,,
\end{aligned} \tag{9.3}$$

[2] We only display G-invariant one-, two-, and three-forms since the G-invariant forms of higher degree can be obtained by duality.

and there are no G-invariant one-forms. With the structure constants and eq. (4.4), it is straightforward to derive the Betti numbers of this manifold:

$$b_1 = b_2 = b_3 = 0\,. \tag{9.4}$$

In the following we will impose an orientifold projection. Note that there are only two invariant three-forms, so that one has to be even and one odd under the orientifold projection. The requirement that the structure constant tensor (4.29) be even under the orientifold projection only allows one choice: that $\hat{\rho}$ is even and ρ is odd. Since there is only one odd three-form there is no room for a source not proportional to $\mathrm{Re}\,\Omega$.

The $\frac{\mathbf{Sp(2)}}{\mathbf{S(U(2)\times U(1))}}$ coset

As shown in [34], the maximal embedding of su(2)⊕su(1) in sp(2) leads to a coset space that does not allow any G-invariant one- or three-form. We have to exclude this possibility for an SU(3)-structure solution.

The non-maximal embedding is given by embedding su(2)⊕su(1) into an so(4) subgroup of sp(2). The structure constants are totally antisymmetric, and the non-zero ones are given by

$$\begin{gathered} f^5{}_{41} = f^5{}_{32} = f^6{}_{13} = f^6{}_{42} = \frac{1}{2}\,, \quad f^7{}_{56} = f^{10}{}_{89} = -1\,, \\ f^7{}_{21} = f^7{}_{43} = f^8{}_{14} = f^8{}_{32} = f^9{}_{13} = f^9{}_{24} = f^{10}{}_{34} = f^{10}{}_{21} = \frac{1}{2}\,. \end{gathered} \tag{9.5}$$

This space is topologically equivalent to $\mathbb{CP}^3$ and can also be viewed as the twistor space $\mathrm{Tw}(\mathrm{S}^4)$ [51].

The G-invariant forms are spanned by

$$\begin{aligned} &\text{two-forms}: \quad \{e^{12} + e^{34}, e^{56}\}\,, \\ &\text{three-forms}: \quad \{\rho = e^{245} - e^{135} - e^{146} - e^{236}, \hat{\rho} = e^{235} + e^{246} + e^{145} - e^{136}\}\,, \end{aligned} \tag{9.6}$$

and there are no G-invariant one-forms. Again, the source (if present) must be proportional to $\mathrm{Re}\,\Omega$.

The Betti numbers of this coset space are

$$b_1 = 0\,, \quad b_2 = 1\,, \quad b_3 = 0\,. \tag{9.7}$$

The $\frac{\mathbf{SU(3)}}{\mathbf{U(1)\times U(1)}}$ coset

Using the permutation (12456738) of the Gell-Mann structure constants $\tilde{f}_{ijk}$, the structure constants of SU(3) are given by

$$\begin{aligned} & f^1{}_{54} = f^1{}_{36} = f^2{}_{46} = f^2{}_{35} = f^3{}_{47} = f^5{}_{76} = \frac{1}{2}\,, \\ & f^1{}_{27} = 1\,, \quad f^3{}_{48} = f^5{}_{68} = \frac{\sqrt{3}}{2}\,, \text{ all cyclic.} \end{aligned} \tag{9.8}$$

The U(1)×U(1) is then generated by E^7 and E^8. This space is also known as the flag manifold $\mathbb{F}(1,2:3)$ or the twistor space $\mathrm{Tw}(\mathbb{CP}^2)$ [51].

This time, the G-invariant two- and three-forms are given by

$$\begin{aligned} \text{two-forms}: &\quad \{e^{12}, e^{34}, e^{56}\}\,, \\ \text{three-forms}: &\quad \{\rho = e^{245} + e^{135} + e^{146} - e^{236}, \hat{\rho} = e^{235} + e^{136} + e^{246} - e^{145}\}\,, \end{aligned} \tag{9.9}$$

respectively. The condition (4.23) excludes the existence of G-invariant one-forms. With the given three-forms, there is no possibility for a source (if present) not proportional to $\mathrm{Re}\,\Omega$.

The Betti numbers of $\frac{\mathrm{SU(3)}}{\mathrm{U(1)\times U(1)}}$ are easily calculated and read

$$b_1 = 0\,, \quad b_2 = 2\,, \quad b_3 = 0\,. \tag{9.10}$$

The $\frac{\mathbf{SU(3)\times U(1)}}{\mathbf{SU(2)}}$ coset

The most general case corresponds to taking

$$\begin{aligned} & E_i = G_{i+3}\,, \quad i = 1, \ldots, 5; \quad E_6 = M; \\ & E_7 = G_1; \quad E_8 = G_2; \quad E_9 = G_3\,, \end{aligned} \tag{9.11}$$

where the G_i's are the Gell-Mann matrices generating su(3). M generates a u(1) and the su(2) subalgebra is generated by E_7, E_8 and E_9. It follows that the SU(2) subgroup is embedded entirely inside the SU(3), so that the total space is given by $\frac{\mathrm{SU(3)}}{\mathrm{SU(2)}} \times \mathrm{U(1)} \simeq S^5 \times S^1$.

We find the following cyclic structure constants

$$\begin{aligned} & f^7{}_{89} = 1, \quad f^7{}_{14} = f^7{}_{32} = f^8{}_{13} = f^8{}_{24} = f^9{}_{12} = f^9{}_{43} = 1/2\,, \\ & f^5{}_{12} = f^5{}_{34} = \frac{\sqrt{3}}{2}, \quad \text{all cyclic}\,. \end{aligned} \tag{9.12}$$

This time, the coset space allows for G-invariant one-forms given by

$$\text{one-forms}: \quad \{e^5, e^6\}\,, \tag{9.13}$$

and the following two- and three-forms, respectively,

$$\begin{aligned} \text{two-forms}: \quad & \{e^{12}+e^{34}, e^{13}-e^{24}, e^{14}+e^{23}, e^{56}\}\,, \\ \text{three-forms}: \quad & \{e^{145}+e^{235}, e^{135}-e^{245}, e^{126}+e^{346}, \\ & e^{146}+e^{236}, e^{136}-e^{246}, e^{125}+e^{345}\}\,, \end{aligned} \tag{9.14}$$

The Betti numbers of this coset are

$$b_1 = 1\,, \quad b_2 = 0\,, \quad b_3 = 0\,. \tag{9.15}$$

The SU(2) × SU(2) coset

Even though SU(2) × SU(2) is not a coset space, it will be convenient to henceforth refer to it as a coset space; it is a trivial coset space. The structure constants in this case are

$$f^1{}_{23} = f^4{}_{56} = 1\,, \qquad \text{cyclic}\,. \tag{9.16}$$

On SU(2) × SU(2) all the left-invariant forms of different degree are trivially G-invariant. The Betti numbers of this coset space are

$$b_1 = 0\,, \quad b_2 = 0\,, \quad b_3 = 2\,. \tag{9.17}$$

The $\frac{\mathbf{SU(2)}^2}{\mathbf{U(1)}} \times$ U(1) coset

It was shown in [34] that if the U(1) factor does not sit completely in the SU(2)^2, the resulting coset is equivalent (with its SU(3)-structure) to SU(2) × SU(2), so we exclude this possibility here, as the above notation already suggests. The resulting coset space in then equivalent to T^{1,1}× U(1) [110]. In this case one can choose the following generators

$$\begin{aligned} & E_i = L_i\,, \quad i=1,2,3\,; \quad E_{i+3} = L'_i\,, \quad i=1,2; \quad E_6 = M; \\ & E_7 = L'_3 - aL_3\,, \end{aligned} \tag{9.18}$$

where we denote the generators of the two su(2) algebras as $\{L_i\}$ and $\{L'_i\}$ and M generates a u(1) and $a \in \mathbb{R}$. The structure constants then read

$$\begin{aligned} f^1{}_{23} = f^7{}_{45} = 1, \quad \text{cyclic,} \\ f^3{}_{45} = f^2{}_{17} = f^1{}_{72} = a. \end{aligned} \tag{9.19}$$

As a matter of fact, it turns out that only for $a=1$ there exists a well defined SU(3)-structure. For another choice of a, the Hitchin functional turns out to be imaginary.

The G-invariant forms on $\frac{\mathrm{SU}(2)^2}{\mathrm{U}(1)} \times \mathrm{U}(1)$ are then given by

$$\begin{aligned} \text{one-forms}: &\quad \{e^3, e^6\}\,, \\ \text{two-forms}: &\quad \{e^{12}, e^{36}, e^{45}, e^{25} - e^{14}, e^{15} + e^{24}\}\,, \\ \text{three-forms}: &\quad \{e^{123}, e^{126}, e^{345}, e^{456}, e^{235} - e^{134}, \\ &\quad e^{135} + e^{234}, e^{256} - e^{146}, e^{156} + e^{246}\}\,, \end{aligned} \tag{9.20}$$

The Betti numbers of this coset read

$$b_1 = 1\,, \quad b_2 = 1\,, \quad b_3 = 2\,. \tag{9.21}$$

The $\mathrm{SU}(2) \times \mathrm{U}(1)^3$ coset

This space is again a trivial coset space. The structure constants in this case are

$$f^1{}_{23} = 1\,, \qquad \text{cyclic}\,. \tag{9.22}$$

All the forms are G-invariant. The Betti numbers of this coset are

$$b_1 = 3\,, \quad b_2 = 3\,, \quad b_3 = 2\,. \tag{9.23}$$

The other coset spaces

Let us shortly mention why there is no well-defined strict SU(3)-structure possible on the other coset models of table 4.1. For the explicit structure constants of the models see [34].

For the coset model $\frac{\mathrm{SU}(3)\times\mathrm{U}(1)^2}{\mathrm{SU}(2)\times\mathrm{U}(1)}$, it turns out that with the set of G-invariant three-forms it is not possible to define $\mathrm{Im}\,\Omega$ such that the Hitchin functional is not vanishing, excluding this model for our analysis.

Similar for the coset models $\frac{\mathrm{SU}(2)^2\times\mathrm{U}(1)^2}{\mathrm{U}(1)^2}$ and $\frac{\mathrm{SU}(2)\times\mathrm{U}(1)^4}{\mathrm{U}(1)}$. For these models, the Hitchin functional turns out to be imaginary.

The coset spaces $\frac{\mathrm{SU}(2)^3}{\mathrm{SU}(2)}$ (where the SU(2) is embedded in the last two SU(2) factors) and the coset space $\frac{\mathrm{SU}(2)^3\times\mathrm{U}(1)}{\mathrm{SU}(2)\times\mathrm{U}(1)}$ only allow for G-invariant two-forms which can not satisfy the normalisation condition (2.6b).

The last two possibilities in table 4.1 are $\frac{\mathrm{SU}(2)^3}{\mathrm{SU}(2)}$ (where the SU(2) is diagonally embedded in $\mathrm{SU}(2)^3$) and $\frac{\mathrm{SU}(3)\times\mathrm{SU}(2)^2}{\mathrm{SU}(3)}$. These two possibilities are shown in [34] to be equivalent to the $\mathrm{SU}(2)\times\mathrm{SU}(2)$ model such that we will also exclude these models from our analysis.

Chapter 10

Type IIA AdS$_4$ $\mathcal{N} = 1$ solutions

We described in chapter 9 the six-dimensional coset spaces that allow to define a left-invariant strict SU(3)-structure. For some of these coset spaces one can actually solve the conditions for an AdS$_4$ $\mathcal{N} = 1$ solution described in section 2.2.2. These solutions were systematically analyzed in [34] and also incorporate some solutions that were already known [65, 69, 108, 51]. In this section we review the coset solutions listed in [34].

In the subsequent chapter 11, we will derive the four-dimensional effective theory of compactifications on the coset models analyzed in chapter 9. As an interesting application of this effective theory we will compute for each of the supersymmetric AdS$_4$ solutions of this chapter the mass spectrum of the scalars around the supersymmetric solution. As we will see, in all models except SU(2) $\times$ SU(2), all moduli are stabilized.

As explained in section 2.2.2, the only non-vanishing torsion classes for a supersymmetric AdS$_4$ solution are $\mathcal{W}_1^-$ and $\mathcal{W}_2^-$. With the given structure constants and eq. (4.13a), one derives dJ and dΩ, where one assume the most general ansatz for J and Ω compatible with the set of G-invariant forms. If eq. (2.29) can be satisfied, we read off the torsion classes $\mathcal{W}_1^-$ and $\mathcal{W}_2^-$. The Bianchi identity (2.31) determines if there is a source to be present for a solution. It then remains to check whether the metric is positive definite.

10.1 The $\frac{\mathbf{G_2}}{\mathbf{SU(3)}}$ solution

With the given set of G-invariant forms (9.3), the most general ansatz for J and Ω is

$$
\begin{aligned}
J &= a(e^{12} - e^{34} + e^{56})\,, \\
\Omega &= d\left[(e^{245} + e^{146} + e^{135} - e^{236}) + i(e^{145} - e^{246} - e^{235} - e^{136})\right]\,,
\end{aligned}
\tag{10.1}
$$

with a, the overall scale, the only free parameter. The conditions for a SU(3)-structure (2.6), metric positivity and the supersymmetry conditions (2.29) and (2.31) are solved for

$$
\begin{aligned}
a &> 0\,, \qquad \text{metric positivity}\,,\\
d^2 &= a^3, \qquad \text{normalization of } \Omega\,,\\
c_1 &:= -\frac{3i}{2}\mathcal{W}_1^- = -\frac{2}{3}e^{\Phi}f = -\frac{\sqrt{3}a}{d}\,,\\
\mathcal{W}_2^- &= 0\,,\\
e^{2\Phi}m^2 - \mu &= \frac{5}{12}c_1^2\,.
\end{aligned}
\tag{10.2}
$$

Since the second torsion class is vanishing, the only possibility for this coset is the nearly-Kähler geometry.

With the help of eq. (2.27) we now easily obtain the background fluxes in terms of the geometric data (10.2). It will be convenient to isolate the scale a and introduce the reduced flux parameters

$$
\tilde{m} = a^{1/2}e^{\Phi}m\,, \qquad \tilde{f} = a^{1/2}e^{\Phi}f\,, \qquad \tilde{\mu} = a\mu\,. \qquad \tilde{c}_1 = a^{1/2}c_1\,. \tag{10.3}
$$

In terms of these redefinitions the background fluxes and the source take the form

$$
\begin{aligned}
H &= \frac{2\tilde{m}}{5}a(e^{245}+e^{135}+e^{146}-e^{236})\,,\\
e^{\Phi}F_2 &= \frac{a^{1/2}}{2\sqrt{3}}\left(e^{12}-e^{34}+e^{56}\right)\,,\\
e^{\Phi}F_4 &= a^{-1/2}\tilde{f}\mathrm{vol}_4 - \frac{3}{5}\tilde{m}a^{3/2}\left(e^{1234}-e^{1256}+e^{3456}\right)\,,\\
e^{\Phi}j^6 &= -\frac{2}{5}a^{1/2}\tilde{\mu}(e^{245}+e^{135}+e^{146}-e^{236})\,.
\end{aligned}
\tag{10.4}
$$

As mentioned before, $\mu > 0$ corresponds to net orientifold charge. Solutions with $\mu \leq 0$ — i.e. with net D-brane charge — are possible, but in that case we still assume that smeared orientifolds are present, which then should be compensated by introducing enough smeared D-branes. It can be easily read off from j^6 that the orientifolds are along the directions $(1,3,6),(2,4,6),(2,3,5)$ and $(1,4,5)$, leading to four orientifold involutions (see also the discussion in appendix D)

$$
\begin{aligned}
&\text{O6}: \qquad e^2\to -e^2\,, \quad e^4 \to -e^4\,, \quad e^5\to -e^5\,,\\
&\text{O6}: \qquad e^1\to -e^1\,, \quad e^3 \to -e^3\,, \quad e^5\to -e^5\,,\\
&\text{O6}: \qquad e^1\to -e^1\,, \quad e^4 \to -e^4\,, \quad e^6\to -e^6\,,\\
&\text{O6}: \qquad e^2\to -e^2\,, \quad e^3 \to -e^3\,, \quad e^6\to -e^6\,.
\end{aligned}
\tag{10.5}
$$

One easily checks that all fields and the SU(3)-structure transform as in (2.40) under *each* of the orientifold involutions. Also, the structure constant tensor (4.29) is even.

10.2 The $\frac{\mathbf{Sp(2)}}{\mathbf{S(U(2)\times U(1))}}$ solution

The most general ansatz for J and Ω with the given G-invariant forms (9.6) is

$$\begin{aligned} J &= a(e^{12}+e^{34}) - ce^{56}\,, \\ \Omega &= d\left[(e^{245}-e^{236}-e^{146}-e^{135}) + i(e^{246}+e^{235}+e^{145}-e^{136})\right]\,, \end{aligned} \tag{10.6}$$

with a and c two free parameters. We compute the following conditions for the geometry

$$\begin{aligned} a>0\,, \quad c>0, &\qquad \text{metric positivity}\,, \\ d^2 = a^2 c, &\qquad \text{normalization of } \Omega\,, \\ c_1 := -\frac{3i}{2}\mathcal{W}_1^- &= -\frac{2}{3}e^{\Phi} f = \frac{2a+c}{2d}\,, \\ \mathcal{W}_2^- &= -\frac{2i}{3d}\left[a(a-c)(e^{12}+e^{34}) + 2c(a-c)e^{56}\right]\,, \\ c_2 := -\frac{1}{8}|\mathcal{W}_2^-|^2 &= -\frac{2}{3a^2c}(a-c)^2\,, \\ \frac{2}{5}(e^{2\Phi}m^2-\mu) = c_2 + \frac{1}{6}c_1^2 &= \frac{1}{8a^2c}\left(-4a^2-5c^2+12ac\right)\,. \end{aligned} \tag{10.7}$$

The nearly-Kähler limit corresponds to setting $a=c$. The two parameters correspond to the overall scale a and a parameter $\sigma \equiv c/a$ that measures the deviation from the nearly-Kähler limit [1].

For the background fluxes and sources, we find from eq. (2.27) in terms of the reduced flux parameters (10.3):

$$\begin{aligned} H &= \frac{2\tilde{m}}{5}a\sigma^{1/2}(e^{245}-e^{135}-e^{146}-e^{236})\,, \\ e^{\Phi}F_2 &= \frac{a^{1/2}}{4}\sigma^{-1/2}\left[(2-3\sigma)(e^{12}+e^{34}) + (6\sigma-5\sigma^2)e^{56}\right]\,, \\ e^{\Phi}F_4 &= a^{-1/2}\tilde{f}\mathrm{vol}_4 + \frac{3}{5}a^{3/2}\tilde{m}\left(e^{1234}-\sigma e^{1256}-\sigma e^{3456}\right)\,, \\ e^{\Phi}j^6 &= -\frac{2}{5}a^{1/2}\tilde{\mu}\sigma^{1/2}(e^{245}-e^{135}-e^{146}-e^{236})\,. \end{aligned} \tag{10.8}$$

Let us stress that the parameters a and σ are not moduli fields since they also appear in the expressions for the background fluxes and are thus quantized.

From the source, we read off the same orientifold involutions as in eq. (10.5) and check that all fields and the structure constants transform as expected.

[1] Let us mention that this solution was also presented in [51] using an alternative description in terms of twistor bundles. The relation of the solution given here with the results of [51] is given in [34].

10.3 The $\frac{\mathbf{SU(3)}}{\mathbf{U(1)\times U(1)}}$ solution

The set of G-invariant forms allows the following general ansatz for J and Ω

$$\begin{aligned} J &= -ae^{12} + be^{34} - ce^{56}\,, \\ \Omega &= d\left[(e^{245} + e^{135} + e^{146} - e^{236}) + i(e^{235} + e^{136} + e^{246} - e^{145})\right]\,, \end{aligned} \tag{10.9}$$

with a, b and c three free parameters and

$$\begin{aligned} &a > 0,\ b > 0,\ c > 0\,, \qquad \text{metric positivity}\,, \\ &d^2 = abc, \qquad \text{normalization of } \Omega\,, \\ &c_1 := -\frac{3i}{2}\mathcal{W}_1^- = -\frac{2}{3}e^{\Phi}f = -\frac{a+b+c}{2d}\,, \\ &\mathcal{W}_2^- = -\frac{2i}{3d}\left[a(2a-b-c)e^{12} + b(a-2b+c)e^{34} + c(-a-b+2c)e^{56}\right]\,, \\ &c_2 := -\frac{1}{8}|\mathcal{W}_2^-|^2 = -\frac{2}{3abc}\left(a^2+b^2+c^2-(ab+ac+bc)\right)\,, \\ &\frac{2}{5}(e^{2\Phi}m^2 - \mu) = c_2 + \frac{1}{6}c_1^2 = \frac{1}{8abc}\left[-5(a^2+b^2+c^2) + 6(ab+ac+bc)\right]\,. \end{aligned} \tag{10.10}$$

Putting $a = b$ we end up with a model that is very similar to the one of section 10.2, while further putting $a = b = c$ corresponds to the nearly-Kähler limit. Next to the overall scale a we have this time two shape parameters $\rho \equiv b/a$ and $\sigma \equiv c/a$ [2].

Introducing again the reduced flux parameters (10.3), we find for the fluxes and the source

$$\begin{aligned} H &= \frac{2\tilde{m}}{5}a\sqrt{\rho\sigma}(e^{245} + e^{135} + e^{146} - e^{236})\,, \\ e^{\Phi}F_2 &= \frac{a^{1/2}}{4\sqrt{\rho\sigma}}\left[(5-3\rho-3\sigma)e^{12} + (3\rho-5\rho^2+3\rho\sigma)e^{34} + (-3\sigma-3\rho\sigma+5\sigma^2)e^{56}\right]\,, \\ e^{\Phi}F_4 &= a^{-1/2}\tilde{f}\mathrm{vol}_4 - \frac{3}{5}a^{3/2}\tilde{m}\left(\rho e^{1234} - \sigma e^{1256} + \rho\sigma e^{3456}\right)\,, \\ e^{\Phi}j^6 &= -\frac{2}{5}a^{1/2}\tilde{\mu}\sqrt{\rho\sigma}(e^{135} + e^{146} + e^{245} - e^{236})\,, \end{aligned} \tag{10.11}$$

while the orientifold involutions are still as in eq. (10.5), such that all fields and structure constants transform as is expected.

[2] Also this space has an alternative description in terms of twistor bundles, see [51]. However, that description does not allow to describe the complete parameter space.

10.4 The $\frac{\mathbf{SU(3)\times U(1)}}{\mathbf{SU(2)}}$ solution

Let us first note that, as we have seen in eq. (9.13), this coset space allows G-invariant one- and five-forms. The strict SU(3)-structure condition (2.6a) is therefore not automatically satisfied. However, one can nevertheless find a solution satisfying the strict SU(3)-structure conditions (2.6) as follows [34]

$$\begin{aligned} J &= -a(e^{13}-e^{24})+b(e^{14}+e^{23})+ce^{56}\,, \\ \Omega &= -\frac{\sqrt{3}}{2c_1}\Big\{\,\big[2a(e^{145}+e^{235})+2b(e^{135}-e^{245})+c(e^{126}+e^{346})\big] \\ &\quad -\frac{i}{\sqrt{a^2+b^2}}\big[ac(e^{146}+e^{236})+bc(e^{136}-e^{246})-2(a^2+b^2)(e^{125}+e^{345})\big]\Big\}\,, \end{aligned} \tag{10.12}$$

with a, b and c three free parameters and

$$\begin{aligned} &c>0\,, \quad a^2+b^2\neq 0\,, \qquad \text{metric positivity}\,, \\ &\frac{1}{(c_1)^2} = \frac{2}{3}\sqrt{a^2+b^2}, \qquad \text{normalization of }\Omega\,, \\ &c_1 := -\frac{3i}{2}\mathcal{W}_1^- = -\frac{2}{3}e^{\Phi}f\,, \\ &\mathcal{W}_2^- = \frac{i}{2\,c_1\sqrt{a^2+b^2}}\big[-a(e^{13}-e^{24})+b(e^{14}+e^{23})-2ce^{56}\big]\,, \\ &d\mathcal{W}_2^- = -\frac{i\sqrt{3}}{2\,c_1\sqrt{a^2+b^2}}\big[a(e^{145}+e^{235})+b(e^{135}-e^{245})-c(e^{126}+e^{346})\big]\,, \\ &3|\mathcal{W}_1^-|^2-|\mathcal{W}_2^-|^2=0\,. \end{aligned} \tag{10.13}$$

By a suitable change of basis we can always arrange for $a>0$ and $b>0$, which we will assume from now on. Note that $\mathrm{d}\mathcal{W}_2^-$ is not proportional to $\mathrm{Re}\,\Omega$, hence the source is not of the form (2.35). Interestingly, if we take the part of the source along $\mathrm{Re}\,\Omega$ to be zero, i.e. $j^6\wedge\mathrm{Im}\,\Omega=0$, we find from the last equation in (10.13) that $m=0$. This would amount to a combination of smeared D6-branes and O6-planes such that the total tension is zero. Allowing for negative total tension (more orientifolds), we could have $m>0$.

For an arbitrary m we find the background

$$\begin{aligned} H &= -\frac{\sqrt{3}\tilde{m}}{5\tilde{c}_1}a\big[2(e^{145}+e^{235})+2\rho(e^{135}-e^{245})+\sigma(e^{126}+e^{346})\big]\,, \\ e^{\Phi}F_2 &= \frac{1}{2}a^{1/2}\tilde{c}_1\big[(e^{13}-e^{24})-\rho(e^{14}+e^{23})+\sigma e^{56}\big]\,, \\ e^{\Phi}F_4 &= a^{-1/2}\tilde{f}\mathrm{vol}_4+\frac{3}{5}a^{3/2}\tilde{m}\big[(1+\rho^2)e^{1234}-\sigma(e^{1356}-e^{2456})+\rho\sigma(e^{1456}+e^{2356})\big]\,, \end{aligned} \tag{10.14}$$

where we defined the shape parameters $\rho = b/a$ and $\sigma = c/a$ and used again eq. (10.3).

From eq. (2.31) we compute for the source

$$\begin{aligned} e^{\Phi} j^6 = &-\frac{\sqrt{3}}{10\tilde{c}_1} a^{1/2}\left(5\tilde{c}_1^2 - 4\tilde{m}^2\right)\left[e^{145} + e^{235} + \rho(e^{135} - e^{245})\right] \\ &+ \frac{\sqrt{3}}{20\tilde{c}_1} a^{1/2}\sigma\left(5\tilde{c}_1^2 + 4\tilde{m}^2\right)\left(e^{126} + e^{346}\right) . \end{aligned} \tag{10.15}$$

One can check that for the background the source satisfies the calibration conditions (2.36). However, this time it is not immediately obvious how to choose the orientifold projection. Choosing them naively along the six terms in the source (10.15) leads to the fields and structure constants having the wrong transformation properties. In appendix D we outline how to find the orientifold involutions associated to a smeared source in general. As explained in that appendix in detail, the procedure boils down to find an appropriate coordinate transformation compatible with the structure constants (i.e., the structure constants read the same in the new basis) for which the source contains at most four *decomposable* three-forms which we then identify with the orientifold involutions. For the case at hand, we make the following coordinate transformation which is compatible with the structure constants [3]

$$\begin{aligned} &e^{1'} = e^1\,, \quad e^{2'} = e^2\,, \quad e^{5'} = e^5\,, \quad e^{6'} = e^6\,, \\ &e^{3'} = \frac{1}{\sqrt{1+\rho^2}}(\rho e^3 + e^4)\,, \quad e^{4'} = \frac{1}{\sqrt{1+\rho^2}}(-e^3 + \rho e^4)\,, \end{aligned} \tag{10.16}$$

and we see that j^6 is a sum of four decomposable terms

$$\begin{aligned} e^{\Phi} j^6 = &-\frac{\sqrt{3}}{10\tilde{c}_1} a^{1/2}\left(5\tilde{c}_1^2 - 4\tilde{m}^2\right)\sqrt{1+\rho^2}\left(e^{1'3'5'} - e^{2'4'5'}\right) \\ &+ \frac{\sqrt{3}}{20\tilde{c}_1} a^{1/2}\sigma\left(5\tilde{c}_1^2 + 4\tilde{m}^2\right)\left(e^{1'2'6'} + e^{3'4'6'}\right) , \end{aligned} \tag{10.17}$$

to which we can associate four orientifold involutions. Note that this model does not allow for a type IIA solution without orientifold sources.

10.5 The SU(2) × SU(2) solution

Since all the left-invariant forms e^i are G-invariant on this space (SU(2) × SU(2) is a trivial coset such that eq. (4.23) is satisfied for every form), the most general ansatz

[3] Note that in order to obtain a coordinate transformation compatible with the structure constants (9.12), we also need the following transformations: $e^{7'} = \frac{1}{\sqrt{1+\rho^2}}(\rho e^7 - e^8)$, $e^{8'} = \frac{1}{\sqrt{1+\rho^2}}(e^7 + \rho e^8)$, $e^{9'} = e^9$.

for J would consist of a sum of 15 two-forms. However, it was shown in [111] that due to the symmetry of the structure constants (9.16) there always exists a change of basis preserving the form of the structure constants that brings J in diagonal form

$$J = ae^{14} + be^{25} + ce^{36} \,. \tag{10.18}$$

Using this observation, the most general solution to the conditions in section 2.2.2 was given in [34] and reads

$$\begin{aligned}
J &= ae^{14} + be^{25} + ce^{36} \,, \\
\Omega &= -\frac{1}{c_1}\Bigg\{ a(e^{234} - e^{156}) + b(e^{246} - e^{135}) + c(e^{126} - e^{345}) \\
&\quad - \frac{i}{h}\Big[-2\,abc(e^{123} + e^{456}) + a(b^2 + c^2 - a^2)(e^{234} + e^{156}) + b(a^2 + c^2 - b^2)(e^{153} + e^{426}) \\
&\quad + c(a^2 + b^2 - c^2)(e^{345} + e^{126})\Big]\Bigg\} \,,
\end{aligned} \tag{10.19}$$

with a, b and c three free parameters and

$$\begin{aligned}
&abc > 0 \,, \qquad \text{metric positivity} \,, \\
&h = \sqrt{2\,a^2b^2 + 2\,b^2c^2 + 2\,a^2c^2 - a^4 - b^4 - c^4} \,, \\
&\quad \text{and thus} \quad 2\,a^2b^2 + 2\,b^2c^2 + 2\,a^2c^2 - a^4 - b^4 - c^4 > 0 \,, \\
&c_1^2 = \frac{4}{9}e^{2\Phi}f^2 = \frac{h}{2abc} \,, \\
&\mathcal{W}_2^- = -\frac{2i}{3hc_1}\Bigg[\frac{(b^2 - c^2)^2 + a^2(-2a^2 + b^2 + c^2)}{bc} e^{14} \\
&\qquad + \frac{(c^2 - a^2)^2 + b^2(-2b^2 + c^2 + a^2)}{ac} e^{25} + \frac{(a^2 - b^2)^2 + c^2(-2c^2 + a^2 + b^2)}{ab} e^{36} \Bigg] \,.
\end{aligned} \tag{10.20}$$

By a suitable change of basis we can always arrange for $a > 0, b > 0$ and $c > 0$, which we will assume in the following. In terms of the reduced flux parameters (10.3), to which we add

$$\tilde{h} = a^{-2}h \,, \tag{10.21}$$

we find for the fluxes from eq. (2.27)

$$
\begin{aligned}
H &= \frac{2\tilde{m}}{5\tilde{c}_1} a \left[(e^{156} - e^{234}) + \rho(e^{135} - e^{246}) + \sigma(e^{345} - e^{126}) \right] , \\
e^{\Phi} F_2 &= \frac{\tilde{c}_1 a^{1/2}}{2\tilde{h}^2} \Big\{ \left[3(\rho^4 + \sigma^4) - 5 + 2(\rho^2 + \sigma^2) - 6\rho^2\sigma^2 \right] e^{14} \\
&\quad + \rho \left[3(1 + \sigma^4) - 5\rho^4 + 2\rho^2(1 + \sigma^2) - 6\sigma^2 \right] e^{25} \\
&\quad + \sigma \left[3(1 + \rho^4) - 5\sigma^4 + 2\sigma^2(1 + \rho^2) - 6\rho^2 \right] e^{36} \Big\} , \\
e^{\Phi} F_4 &= a^{-1/2} \tilde{f} \mathrm{vol}_4 - a^{3/2} \frac{3\tilde{m}}{5} (\rho e^{1245} + \sigma e^{1346} + \rho\sigma e^{2356}) .
\end{aligned}
\tag{10.22}
$$

From eq. (2.31) we derive j^6,

$$
\begin{aligned}
e^{\Phi} j^6 &= -i\mathrm{d}\mathcal{W}_2^- + \left(\frac{2}{27} f^2 - \frac{2}{5} m^2 \right) e^{2\Phi} \mathrm{Re}\,\Omega \\
&= j_1(e^{234} - e^{156}) + j_2(e^{246} - e^{135}) + j_3(e^{126} - e^{345}) ,
\end{aligned}
\tag{10.23}
$$

where j_1, j_2 and j_3 are some complicated factors depending on a, b and c whose exact form does not matter for the moment. It contains the same terms as $\mathrm{Re}\,\Omega$ but with different coefficients. In fact, one can check that j^6 is *not* proportional to $\mathrm{Re}\,\Omega$ unless $a = b = c$, which reduces the solution to a nearly-Kähler geometry.

Also for this model, the source (10.23) contains six three-form terms. Following the procedure described in appendix D, we find the orientifold involutions associated to this smeared source. In order to present the resulting involutions, it is convenient to define complex one-forms as follows

$$
\begin{aligned}
e^{z^1} &= \pm \frac{e^{\frac{i3\pi}{4}}}{2c_1\sqrt{bc(2bc - h)}} \left\{ [2bc - h + i(a^2 - b^2 - c^2)]e^1 + [a^2 - b^2 - c^2 + i(2bc - h)]e^4 \right\} , \\
e^{z^2} &= \pm \frac{e^{\frac{i3\pi}{4}}}{2c_1\sqrt{ac(2ac - h)}} \left\{ [2ac - h + i(b^2 - a^2 - c^2)]e^2 + [b^2 - a^2 - c^2 + i(2ac - h)]e^5 \right\} , \\
e^{z^3} &= \pm \frac{e^{\frac{i\pi}{4}}}{2c_1\sqrt{ab(2ab - h)}} \left\{ [2ab - h + i(c^2 - a^2 - b^2)]e^3 + [c^2 - a^2 - b^2 + i(2ab - h)]e^6 \right\} ,
\end{aligned}
\tag{10.24}
$$

where the signs must be chosen such that $\Omega = e^{z^1 z^2 z^2}$. Defining further the associated x and y one-forms $e^{z^i} = e^{x^i} - ie^{y^i}$, the orientifold involutions are given as in eq. (D.10).

10.6 Some comments on solutions with non-constant warp factor and dilaton

The solutions on the coset spaces we analyzed so far in this chapter all assumed constant warp factor and dilaton. However, in section 2.3 we analyzed the conditions for a supersymmetric $\mathcal{N} = 1$ solution with non-constant warp factor/dilaton. We have seen that such a solution is indeed possible, provided that the Romans mass m is chosen to be zero.

In the literature there are already a few sourceless $\mathcal{N} = 2$ solutions with non-constant warp factor/dilaton, based on M-theory reductions of seven-dimensional Sasaki-Einstein manifolds to type IIA (see for instance [54, 33]). As an application of the analysis of section 2.3, we want to study in this section whether one of the coset solutions of this chapter can be deformed into a new sourceless solution with non-constant warp factor/dilaton. To this end, we will try to find an infinitesimal fluctuation around the supersymmetric solution turning on an infinitesimal non-constant warp factor/dilaton.

For this to work, however, we have to leave the convenient notion of left-invariant forms, since the left-invariant ansatz drops the explicit coordinate dependence that is necessary to describe a non-trivial warp factor/dilaton. This makes the analysis rather complicated. However, we can make use of the observation that one can describe one of our coset spaces, namely $\frac{\mathrm{Sp}(2)}{\mathrm{S}(\mathrm{U}(2)\times\mathrm{U}(1))} = \mathbb{CP}^3$, as a foliation with transversal coordinate ξ, with the leaves taking the form of a five-dimensional coset space [112]. In this way, we have an explicit coordinate ξ at our disposal for the ansatz for a non-constant warp factor/dilaton, but can still apply the convenient techniques of coset spaces for the other five coordinates.

10.6.1 Adapted coordinates for the background

As the background around we want to deform, we choose the sourceless solution on $\frac{\mathrm{Sp}(2)}{\mathrm{S}(\mathrm{U}(2)\times\mathrm{U}(1))} = \mathbb{CP}^3$ for $\sigma = c/a = 2$, implying vanishing Romans mass m, as can easily be seen from the last equation in eq. (10.7). This is the $\mathcal{N} = 6$ solution with the standard Fubini-Study metric coming from the $\mathcal{N} = 8$ M-theory background $\mathrm{AdS}_4\times\mathrm{S}^7$ reduced to type IIA, as it was constructed a long time ago in [65].

We use the observation that one can consider $\mathbb{CP}^3$ locally as a foliation where the leaves take the form of the five-dimensional coset manifold [112]

$$\mathcal{N}^{1,-1} = \frac{\mathrm{SO}(4)}{\mathrm{U}(1)}\,. \tag{10.25}$$

Following [53], an intuitive way to see this foliation is the following. The splitting $\mathbb{C}^4 = \mathbb{R}^8 = \mathbb{R}^4 \times \mathbb{R}^4$ allows one to realize S^7 as a fibration of $S^3 \times S^3 = \mathrm{SO}(4)$ on a segment. We parameterize the two S^3 with the coordinates $(\theta_i, \phi_i, \psi_i)$, $i = 1, 2$, and the segment as an angle $0 \leq \xi \leq \pi/2$. The metric for the S^7 reads

$$\mathrm{d}s^2_{S^7} = d\xi^2 + \sin^2\xi\,\mathrm{d}s^2_{S^3_1} + \cos^2\xi\,\mathrm{d}s^2_{S^3_2}\,, \tag{10.26}$$

where the radii of the two S^3s are $\sin\xi$ and $\cos\xi$. Corresponding to this description, appropriate coordinates for $\mathbb{C}^4\setminus\{0\}$ are thus

$$\begin{aligned}
Z^1 &= t\sin\xi\cos\frac{\theta_1}{2}\exp\frac{i}{2}\left(\psi_1+\phi_1\right) ,\\
Z^2 &= t\sin\xi\sin\frac{\theta_1}{2}\exp\frac{i}{2}\left(\psi_1-\phi_1\right) ,\\
Z^3 &= t\cos\xi\cos\frac{\theta_2}{2}\exp\frac{i}{2}\left(-\psi_2+\phi_2\right) ,\\
Z^4 &= t\cos\xi\sin\frac{\theta_2}{2}\exp\frac{i}{2}\left(-\psi_2-\phi_2\right) ,
\end{aligned} \tag{10.27}$$

where $t>0$ is the overall radius and $0\le\theta_{1,2}<\pi$, $0\le\phi_{1,2}<2\pi$, $0\le\psi_{1,2}<2\pi$ are the spherical coordinates for the two S^3s. We can now rearrange $\psi_1=\psi+\lambda$ and $\psi_2=\psi-\lambda$, and reduce on the angle ψ. In this way, for each value $\xi=\xi_0$, the SO(4) gets reduced to the coset (10.25). The factor $te^{i/2\lambda}$ in each of the Z^i of (10.27) corresponds to α in the identification

$$(Z^1,Z^2,Z^3,Z^4)\cong\alpha(Z^1,Z^2,Z^3,Z^4)\,,\qquad \text{where}\quad \alpha\in\mathbb{C}\setminus\{0\}\,. \tag{10.28}$$

for the homogeneous coordinates on $\mathbb{CP}^3$. Thus, we have realized $\mathbb{CP}^3$ as a foliation where the leaves take the form of the coset $\mathcal{N}^{1,-1}$, and the homogeneous coordinates are

$$\begin{aligned}
&Z^1=\sin\xi\cos\frac{\theta_1}{2}\exp\frac{i}{2}\left(\psi+\phi_1\right) ,\quad Z^2=\sin\xi\sin\frac{\theta_1}{2}\exp\frac{i}{2}\left(\psi-\phi_1\right) ,\\
&Z^3=\cos\xi\cos\frac{\theta_2}{2}\exp\frac{i}{2}\left(-\psi+\phi_2\right) ,\quad Z^4=\cos\xi\sin\frac{\theta_2}{2}\exp\frac{i}{2}\left(-\psi-\phi_2\right) .
\end{aligned} \tag{10.29}$$

In [113], the properties of the five-dimensional coset $\mathcal{N}^{1,-1}$ are worked out in detail and we here just cite the results we need for our analysis. The structure constants are given by

$$\begin{aligned}
&f^5{}_{12}=f^6{}_{12}=1\,,\quad f^5{}_{34}=-f^6{}_{34}=-1\,,\quad f^1{}_{25}=f^2{}_{51}=\frac{1}{2}\,,\quad f^3{}_{45}=f^4{}_{53}=-\frac{1}{2}\,,\\
&f^1{}_{26}=f^2{}_{61}=\frac{1}{2}\,,\quad f^3{}_{46}=f^4{}_{63}=\frac{1}{2}\,,
\end{aligned} \tag{10.30}$$

and one can choose the following coordinate representation for the one-forms (e^i,ω^a)

$$\begin{aligned}
e^1 &= -\sin\psi\,\mathrm{d}\theta_1+\sin\theta_1\cos\psi\,\mathrm{d}\phi_1\,,\\
e^2 &= \cos\psi\,\mathrm{d}\theta_1+\sin\theta_1\sin\psi\,\mathrm{d}\phi_1\,,\\
e^3 &= -\sin\psi\,\mathrm{d}\theta_2-\sin\theta_2\cos\psi\,\mathrm{d}\phi_2\,,\\
e^4 &= -\cos\psi\,\mathrm{d}\theta_2+\sin\theta_2\sin\psi\,\mathrm{d}\phi_2\,,\\
e^5 &= -\left[2\,\mathrm{d}\psi+\cos\theta_1\,\mathrm{d}\phi_1-\cos\theta_2\,\mathrm{d}\phi_2\right] ,\\
\omega^1 &= -\left[\cos\theta_1\,\mathrm{d}\phi_1+\cos\theta_2\,\mathrm{d}\phi_2\right] ,
\end{aligned} \tag{10.31}$$

where $(\theta_{1,2}, \phi_{1,2})$ are the remaining spherical coordinates on the S^3s and ψ describes the U(1). The set of the relevant left-invariant forms, obtained by the condition (4.23), are spanned by

$$\begin{aligned} \text{one-forms}: &\quad e^5\,, \\ \text{two-forms}: &\quad e^{12}, e^{34}, e^{14} - e^{23}, e^{13} + e^{24}\,, \\ \text{three-forms}: &\quad e^{125}, e^{345}, e^{145} - e^{235}, e^{135} + e^{245}\,. \end{aligned} \tag{10.32}$$

In these coordinates, the Fubini-Study metric, which reads in homogeneous coordinates

$$\mathrm{d}s^2 = \frac{\mathrm{d}Z^a \mathrm{d}\bar{Z}^{\bar{b}}}{\sum_a |Z^a|^2}\left(\delta_{a\bar{b}} - \frac{\bar{Z}^{\bar{a}} Z^b}{\sum_a |Z^a|^2}\right)\,, \tag{10.33}$$

becomes

$$\begin{aligned} a^{-1}\mathrm{d}s^2 = \mathrm{d}\xi^2 &+ \frac{\sin^2\xi}{4}\left[(\mathrm{d}\theta_1)^2 + \sin^2\theta_1(\mathrm{d}\phi_1)^2\right] + \frac{\cos^2\xi}{4}\left[(\mathrm{d}\theta_2)^2 + \sin^2\theta_2(\mathrm{d}\phi_2)^2\right] \\ &+ \sin^2\xi\cos^2\xi\left(\mathrm{d}\psi + \frac{1}{2}\cos\theta_1\mathrm{d}\phi_1 - \frac{1}{2}\cos\theta_2\mathrm{d}\phi_2\right)^2\,, \\ &= \mathrm{d}\xi^2 + \frac{\sin^2\xi}{4}(e^1\otimes e^1 + e^2\otimes e^2) + \frac{\cos^2\xi}{4}(e^3\otimes e^3 + e^4\otimes e^4) \\ &+ \frac{\sin^2\xi\cos^2\xi}{4}e^5\otimes e^5\,, \end{aligned} \tag{10.34}$$

where $a > 0$ is an overall scale, and the transversal coordinate ξ is chosen such that $g_{\xi\xi} = a$ is constant. At this point the metric in these new coordinates seems to be irregular for the points $\xi \to 0$ and $\xi \to \pi/2$, where one of the two S^3s shrink to zero. For instance, in the limit $\xi \to 0$, the problematic terms read

$$a^{-1}\mathrm{d}s^2 = \mathrm{d}\xi^2 + \frac{\xi^2}{4}\left[(\mathrm{d}\theta_1)^2 + \sin^2\theta_1(\mathrm{d}\phi_1)^2 + (2\mathrm{d}\psi + \cos\theta_1\mathrm{d}\phi_1 - \cos\theta_2\mathrm{d}\phi_2)^2\right] + \dots\,. \tag{10.35}$$

However, one can show that for (ξ, θ_2, ϕ_2) constant the second term is the standard metric for an S^3 with radius ξ and volume $4\pi^2\xi^2$ such that the terms in the metric (10.35) approach flat $\mathbb{R}^4$ as ξ tends to zero, described in spherical coordinates [114]. The same argument shows the regularity at $\xi \to \pi/2$. The regularity of the metric of the deformed solution will be of particular interest in the following.

Since we know the metric (10.34) explicitly, one easily derives the corresponding SU(3)-structure quantities J and Ω in these new coordinates. This is done by considering the most general ansatz for J and $\mathrm{Re}\,\Omega$ expanded in the corresponding left-invariant forms (10.32) and the extra left-invariant one-form $\mathrm{d}\xi$. Solving the necessary conditions for a supersymmetric vacuum of section 2.2.2, the result is [113]

$$\begin{aligned}
a^{-1}J =& -\frac{\sin^2\xi\cos 2\xi}{4}e^{12} + \frac{\cos^2\xi\cos 2\xi}{4}e^{34} + \frac{\sin 2\xi}{4}\mathrm{d}\xi\wedge e^5 \\
&+\frac{\sin^2 2\xi}{8}\left[\cos\theta\left(e^{13}+e^{24}\right)+\sin\theta\left(e^{14}-e^{23}\right)\right]\,, \\
a^{-3/2}\Omega =& \left(\mathrm{d}\xi - \frac{i}{4}\sin 2\xi\, e^5\right)\wedge\frac{1}{2}\left[\cos^2\xi\,(ie^3+e^4)-\sin^2\xi\, e^{-i\theta}(e^1-ie^2)\right] \\
&\wedge\frac{1}{4}\sin 2\xi\left[ie^{i\theta}\left(e^1+ie^2\right)+e^3+ie^4\right]\,,
\end{aligned} \tag{10.36}$$

where θ is constant and a free parameter of the solution. In the following we will choose the gauge $\theta=0$. We find from eq. (2.32) that

$$c_1 = \frac{4}{\sqrt{a}} \quad \text{and} \quad c_2 = -\frac{1}{6}(c_1)^2\,. \tag{10.37}$$

The second torsion class reads in these coordinates

$$\begin{aligned}
\mathcal{W}_2^- =& \, i\frac{\sqrt{a}}{6}\big(\sin^2\xi(\cos 2\xi-3)e^{12}-\cos^2\xi(\cos 2\xi+3)e^{34} \\
&+2\sin 2\xi\mathrm{d}\xi\wedge e^5 - 2\cos^2\xi\sin^2\xi(e^{13}+e^{24})\big)\,.
\end{aligned} \tag{10.38}$$

10.6.2 First order perturbation

We now come to a small deformation of the background of the previous section. The aim is to turn on a non-constant warp factor and dilaton. For this we first need to specify the deformations $J\to J+\delta J$ and $\Omega\to\Omega+\delta\Omega$ that still satisfy the strict SU(3)-structure conditions (2.6) [4]. Given these constraints we make the following ansatz [27]

$$\begin{aligned}
\delta\Omega &= M_{(2,1)} - 4v^{(1,0)}\wedge J + \lambda\Omega\,, \\
\delta J &= K_{(1,1)} + \iota_{v^{(1,0)}}\Omega + \iota_{v^{(0,1)}}\bar{\Omega} + \frac{2}{3}\mathrm{Re}\lambda J\,,
\end{aligned} \tag{10.39}$$

where $K_{(1,1)}$, $M_{(2,1)}$ and $v^{(1,0)}$ are arbitrary left-invariant forms such that $K_{(1,1)}$ is a primitive real $(1,1)$-form, $M_{(2,1)}$ is a primitive $(2,1)$-form (i.e. $M_{(2,1)}\wedge J=0$) and $v^{(1,0)}$ is a $(1,0)$-vector. λ is a complex function. These fluctuations guarantee that $\Omega+\delta\Omega$ is decomposable and it is easy to verify that the compatibility conditions (2.6) are automatically satisfied.

[4] In this section, we denote the fluctuation of e.g. J with δJ, whereas the background is denoted by J.

We parameterize the most general left-invariant forms described above as follows:

$$
\begin{aligned}
M_{(2,1)} = & \,(u_1(\xi) + iu_2(\xi))\,\frac{a^{3/2}}{8}\sin 2\xi\,\big[(1-\cos 2\xi)\mathrm{d}\xi\wedge e^{12} - (1+\cos 2\xi)\mathrm{d}\xi\wedge e^{34} \\
& + \cos 2\xi \mathrm{d}\xi\wedge(e^{13}+e^{24}) - i\mathrm{d}\xi\wedge(e^{14}-e^{23}) + \frac{i}{8}\big((2\sin 2\xi - \sin 4\xi)e^{125} \\
& -(2\sin 2\xi + \sin 4\xi)e^{345} + \sin 4\xi(e^{13}+e^{24})\wedge e^5 - 2i\sin 2\xi(e^{14}-e^{23})\wedge e^5\big)\big] \\
& + (u_3(\xi)+iu_4(\xi))\frac{ia^{3/2}}{2}\,\big[\sin^2\xi \mathrm{d}r\wedge e^{12} + \cos^2\xi\wedge e^{34} \\
& -\frac{i}{4}\sin 2\xi(\sin^2\xi e^{125} + \cos^2\xi e^{345})\Big]\,,
\end{aligned}
\tag{10.40}
$$

$$
\begin{aligned}
K_{(1,1)} = & \,a\,(u_5(\xi)+u_6(\xi))\sin 2\xi\left[\mathrm{d}\xi\wedge e^5 - \frac{1}{4}\sin 2\xi(e^{13}+e^{24})\right] \\
& + a\left[\sin^2\xi(\cos^2\xi u_6(\xi) - \sin^2\xi u_5(\xi))e^{12} + \cos^2\xi(\sin^2\xi u_6(\xi) - \cos^2\xi u_5(\xi))e^{34}\right]\,,
\end{aligned}
\tag{10.41}
$$

$$
v^{(1,0)} = \sqrt{a}\,(u_7(\xi)+iu_8(\xi))\,(\mathrm{d}\xi - \frac{i}{4}\sin 2\xi e^5)\,, \tag{10.42}
$$

$$
\lambda = u_9(\xi) + iu_{10}(\xi)\,, \tag{10.43}
$$

where $u_i(\xi)$, $i = 1, \dots, 10$, are arbitrary real functions of ξ. For the fluctuation of the second torsion class we take the most general ansatz for a left-invariant two-form with arbitrary functions $w_i(\xi)$, $i = 1, \dots, 5$,

$$
\delta\mathcal{W}_2^- = i\left[w_1(\xi)e^{12} + w_2(\xi)e^{34} + w_3(\xi)\mathrm{d}\xi\wedge e^5 + w_4(\xi)(e^{14}-e^{23}) + w_5(\xi)(e^{13}+e^{24})\right]\,. \tag{10.44}
$$

The crucial point is that we now allow for a non-constant warp factor, i.e. $\delta A = \delta A(\xi)$. Since the warp factor always appears in the combination $e^{-A}|W|$, it is convenient to introduce a new variable $\widetilde{W}$ as follows

$$
\widetilde{W} \equiv e^{-A}|W|\,, \tag{10.45}
$$

and, according to eq. (2.59a),

$$
c_1 = -2\widetilde{W}\,. \tag{10.46}
$$

For the fluctuation away from the background c_1 becomes ξ dependent such that we make the ansatz

$$
\delta c_1 = c(\xi)\,, \tag{10.47}
$$

where $c(\xi)$ is an arbitrary real function of ξ. This choice implies $e^{-A}|W| = \widetilde{W} = -\frac{1}{2}c_1$ such that $\delta\widetilde{W} = -\delta A\widetilde{W} + e^{-A}\delta|W|$ and

$$\mathrm{d}\delta A = -\frac{\mathrm{d}\delta\widetilde{W}}{\widetilde{W}} = -\frac{\mathrm{d}\delta c_1}{c_1}\,, \tag{10.48}$$

since we assume that the background values A and $\widetilde{W}$ are constants (recall that we also assume $|W|$ and $\delta|W|$ to be constants).

The conditions we have to solve for the functions $u_i(\xi)$, $i = 1,\dots,10$, $w_i(\xi)$, $i = 1,\dots,5$ and $c(\xi)$ are the variations of the eqs. (2.59a), (2.69) and (2.71). These variations read, using eq. (10.48), respectively [5]

$$\mathrm{d}\delta J = \delta c_1 \mathrm{Re}\,\Omega + c_1 \delta\mathrm{Re}\,\Omega\,, \tag{10.49a}$$

$$\mathrm{d}\delta\mathrm{Re}\,\Omega = -\frac{\mathrm{d}\delta c_1}{c_1}\wedge\mathrm{Re}\,\Omega\,, \tag{10.49b}$$

$$\mathrm{d}\delta\mathrm{Im}\,\Omega = \frac{2}{3}\delta c_1 J\wedge J + \frac{4}{3}c_1\delta J\wedge J - i\delta\mathcal{W}_2^-\wedge J - i\mathcal{W}_2^-\wedge\delta J - \frac{\mathrm{d}\delta c_1}{c_1}\wedge\mathrm{Im}\,\Omega\,. \tag{10.49c}$$

Further we have to solve the Bianchi identity for the variation of F_2 in eq. (2.73). To derive the variation of F_2 we make use of the relation between the warp factor and the dilaton from eq. (2.57b). This implies $\mathrm{d}\delta\Phi = -3\frac{\mathrm{d}\delta c_1}{c_1}$ such that

$$\delta\Phi = -3\frac{\delta c_1}{c_1} + K\,, \tag{10.50}$$

where K is an integration constant. We arrive at

$$\begin{aligned}\delta F_2 = e^{-\Phi}\bigg(&-\frac{2}{3}\delta c_1 J - \frac{1}{6}c_1\delta J + 3i\frac{\delta c_1}{c_1}\mathcal{W}_2^- + i\delta\mathcal{W}_2^- - 2\star_6\left(\frac{\mathrm{d}\delta c_1}{c_1}\wedge\mathrm{Im}\,\Omega\right)\\ &-K(-\frac{1}{6}c_1 J + i\mathcal{W}_2^-)\bigg)\,.\end{aligned} \tag{10.51}$$

Note that the last term in eq. (10.51) with the integration constant K is nothing else than the background F_2 and hence does not contribute the the Bianchi identity for δF_2, since $\mathrm{d}F_2 = 0$ for the sourceless background. Plugging all the ansätze for δJ, $\delta\Omega$, $\delta\mathcal{W}_2^-$ and δc_1 in eq. (10.51), we look for a solution of the sourceless Bianchi identity

$$\mathrm{d}\delta F_2 = 0\,. \tag{10.52}$$

10.6.3 Solving the conditions

We first want to solve condition (10.49a). As it turns out, this condition is relatively easy to solve and already specifies most of the unknown functions $u_i(\xi)$ and $c(\xi)$ in

[5] Note that we also assume $\delta m = 0$ such that we do not turn on m or H-flux with the fluctuation. This is the case for $\delta\theta' = 0$ (as can easily be seen from eq. (2.64)) which we will assume in the following.

terms of only four remaining functions, $u_5(\xi)$, $u_6(\xi)$, $u_8(\xi)$ and $u_9(\xi)$ and derivatives thereof. The explicit solution reads

$$
\begin{aligned}
u_1(\xi) &= \frac{1}{24}\left[2\cot 2\xi\left(-24\cot 2\xi(u_5(\xi)+2u_6(\xi))+3u_5'(\xi)+9u_6'(\xi)+u_9'(\xi)\right)\right.\\
&\quad\left.+3u_5''(\xi)+3u_6''(\xi)-u_9''(\xi)\right]\,,\\
u_2(\xi) &= \cot 2\xi u_8(\xi)-\frac{1}{2}u_8'(\xi)\,,\\
u_3(\xi) &= 0\,,\\
u_4(\xi) &= (\tan\xi-\cot\xi)u_5(\xi)+\frac{1}{12}\left(-48\cot 2\xi u_6(\xi)-6u_6'(\xi)+u_9'(\xi)\right)\,,\\
u_7(\xi) &= \cot 2\xi(u_5(\xi)+2u_6(\xi))+\frac{1}{4}(u_5'(\xi)+u_6'(\xi))-\frac{1}{12}u_9'(\xi)\,,\\
u_{10}(\xi) &= -3\cot 2\xi u_8(\xi)-\frac{1}{2}u_8'(\xi)\,,\\
c(\xi) &= \frac{1}{6\sqrt{a}}\left[\frac{24}{\sin^2 2\xi}(u_6(\xi)+\cos 4\xi(2u_5(\xi)+3u_6(\xi))-8u_9(\xi)\right.\\
&\quad\left.+6\cot 2\xi\left(5u_5'(\xi)+7u_6'(\xi)-u_9'(\xi)\right)+3u_5''(\xi)+3u_6''(\xi)-u_9''(\xi)\right]\,,
\end{aligned}
\tag{10.53}
$$

where prime denotes the derivative with respect to ξ. Note that the function $u_8(\xi)$ corresponds to choosing another gauge for θ in eq. (10.36) and none of the equations (10.49) puts a constraint on $u_8(\xi)$.

With this solution we automatically solve the second condition (10.49b). We can use the condition (10.49c) to fix the unknown functions $w_i(\xi)$, $i=1,\ldots,5$ of the variation of the second torsion class (10.44). The solution is not difficult but rather lengthy such that we will not display it here. Let us mention that all the unknown functions $w_i(\xi)$, $i=1,\ldots,5$ are functions of $u_5(\xi)$, $u_6(\xi)$, $u_8(\xi)$ and $u_9(\xi)$ and derivatives thereof. Note that the solution for $\delta\mathcal{W}_2^-$ also satisfies the following conditions

$$
\begin{aligned}
0 &= \delta\mathcal{W}_2^-\wedge\Omega+\mathcal{W}_2^-\wedge\delta\Omega\,,\\
0 &= \delta\mathcal{W}_2^-\wedge J\wedge J+2\mathcal{W}_2^-\wedge\delta J\wedge J\,,
\end{aligned}
\tag{10.54}
$$

which follow from the condition that $\mathcal{W}_2^-$ is a primitive $(1,1)$-form (see eq. (C.31)).

Next we turn to the solution of the Bianchi identity for δF_2 (10.52). First we note that this condition gives us two independent equations for the three unknown functions $u_5(\xi), u_6(\xi), u_9(\xi)$ and derivatives thereof. We try to solve these equations with the following ansatz

$$\begin{aligned}
u_5(\xi) &= \frac{1}{12}\left(6\cot 2\xi\, l(\xi) + l'(\xi)\right) + g(\xi)\,,\\
u_6(\xi) &= \frac{1}{12}\left(-6\cot 2\xi\, l(\xi)\right) + l'(\xi) + h(\xi)\,,\\
u_9(\xi) &= \frac{1}{2}\left(6\cot 2\xi\, l(\xi) + l'(\xi)\right)\,,
\end{aligned} \tag{10.55}$$

where $l(\xi)$, $g(\xi)$ and $h(\xi)$ are arbitrary functions. This ansatz is motivated by the observation that we have the freedom to reparameterize $\xi \to l(\xi)$, which also corresponds to a fluctuation, and it turns out that this fluctuation is given by eq. (10.55) with vanishing $g(\xi)$ and $h(\xi)$. This ansatz simplifies the two independent equations coming from eq. (10.52) considerably and we are left with a pair of coupled differential equations for $g(\xi)$ and $h(\xi)$,

$$\begin{aligned}
0 &= 32(13+3\cos 8\xi)g(\xi) + 32(17+12\cos 4\xi + 3\cos 8\xi)h(\xi)\\
&\quad - 2\sin 2\xi\,\big[4(-11\cos 2\xi + 7\cos 6\xi)g'(\xi) - 4(\cos 2\xi - 5\cos 6\xi)h'(\xi)\\
&\quad +2\sin 2\xi\,\left\{4(3+2\cos 4\xi)g''(\xi) + 4(4+3\cos 4\xi)h''(\xi) + \sin 4\xi(g'''(\xi)+h'''(\xi))\right\}\big]\,,
\end{aligned} \tag{10.56a}$$

$$\begin{aligned}
0 &= 16(\cos 2\xi + 2\cos 6\xi)g'(\xi) - 2\sin 2\xi\,\big[-64g(\xi) + 64h(\xi) + 10g''(\xi)\\
&\quad +2\cos 4\xi(96(g(\xi)+h(\xi)) + g''(\xi)) + 8\sin^2 2\xi h''(\xi) + \sin 4\xi(56h'(\xi) + g'''(\xi))\big]\,.
\end{aligned} \tag{10.56b}$$

These differential equations are actually not so easy to solve but, with some patience, we obtain the solution for $g(\xi)$

$$g(\xi) = \frac{3C_1}{16} - \frac{C_2}{2} + \frac{C_1}{12}\cos 2\xi + \frac{-5C_1+24C_2-6C_3}{48\sin^2\xi} - \frac{C_3}{8\cos^2\xi} + C_4\,, \tag{10.57}$$

and $h(\xi)$

$$\begin{aligned}
h(\xi) &= \frac{1}{24\sin^4\xi}\big[-9C_3 - 12C_4 + (48C_4 - 8C_2)\cos^2\xi\\
&\quad +3(C_1+4C_2-8C_4)\cos^4\xi - 3C_1\cos^6\xi + \frac{12C_6}{\cos^2\xi} + \frac{24C_5}{\cos^4\xi}\big]\,,
\end{aligned} \tag{10.58}$$

where C_i, $i=1,\dots,6$ denote integration constants. Plugging these expressions into eq. (10.55) and eq. (10.53), we checked explicitly that this is a solution for all the conditions (10.49) and the Bianchi identity (10.52). For instance, the solution for $\delta c_1(\xi) = c(\xi)$ reads

$$\begin{aligned}
\delta c_1(\xi) &= \frac{1}{6\sqrt{a}}\left(9C_1 - 24C_2 + 48C_4 + C_1\cos 2\xi + \frac{-5C_1+24C_2-6C_3}{\sin^2\xi} - \frac{6C_3}{\cos^2\xi}\right)\\
&= -\frac{4}{\sqrt{a}}\delta A(\xi) - 2e^{-A}\delta|W|\,,
\end{aligned} \tag{10.59}$$

where the second equation comes from the definition of $\delta c_1 = -2\widetilde{W}$ and eq. (10.37). We indeed have a non-constant warp factor.

However, we still have to check whether our solution leads to regular expressions at the special points $\xi = 0$ and $\xi = \pi/2$, where respectively the first and the second S^3 collapse. Let us first consider the solution for $c(\xi)$ in eq. (10.59), which contains the non-constant warp factor. From this expression we immediately see that it is not regular at $\xi = 0$ nor at $\xi = \pi/2$. However, we can fix this by choosing appropriate boundary conditions as follows

$$C_2 = \frac{5C_1}{24}\,, \quad C_3 = 0\,. \tag{10.60}$$

Unfortunately, the regularity of the metric is problematic. For instance, let us consider the components g_{33} and g_{55} of the solution for the metric (with the constraint (10.60)). The first terms in the expansion around $\xi = 0$ read

$$g_{33} = a\frac{7C_1 - 72C_4 - 144C_5}{144\xi^4} + O(\xi^0)\,, \qquad g_{55} = -a\frac{C_1 - 12C_4 + 6C_6}{24\xi^4} + O(\xi^0)\,. \tag{10.61}$$

Choosing

$$C_1 = 36(4C_5 + C_6) \qquad C_4 = \frac{1}{2}(-24C_5 - 7C_6)\,, \tag{10.62}$$

we can make these and the other terms of the metric regular at $\xi = 0$. However, for the regularity at $\xi = \pi/2$ we get for the expansion of g_{33} and g_{55}

$$g_{33} = -a\frac{C_5}{(\xi - \frac{\pi}{2})^2} + O((\xi - \frac{\pi}{2})^0)\,, \qquad g_{55} = a\frac{C_6}{4(\xi - \frac{\pi}{2})^2} + O((\xi - \frac{\pi}{2})^0)\,, \tag{10.63}$$

which forces $C_5 = 0$ and $C_6 = 0$ for regularity. This, however, implies with the conditions (10.60) and (10.62) that regularity demands the vanishing of all integration constants. Let us stress that we can satisfy the regularity for a non-trivial solution on both sides, $\xi \to 0$ and $\xi \to \pi/2$, independently but not at the same time.

We thus have to report that (at least in this setup) there is no first order deformation around the constant warp factor/dilaton solution of section 10.2 which allows to turn on a non-constant warp factor/dilaton for the sourceless case. However, the observation that this only fails due to special boundary conditions strongly suggest that one could resolve this problem by the introduction of localized source terms of the form

$$\mathrm{d}F_2 = -j = K_1\delta(\xi - \xi_0)\mathrm{d}\xi \wedge e^{12} + K_2\delta(\xi - \xi_0)\mathrm{d}\xi \wedge e^{34} + K_3\delta(\xi - \xi_0)\mathrm{d}\xi \wedge (e^{13} + e^{24})\,. \tag{10.64}$$

Note that this are partially localized source terms at some point $\xi = \xi_0$ but still smeared along the other directions. Due to the delta distributions in the source term, the differential equations (10.57) and (10.58) would be completed with expressions involving delta distributions on the right hand side, and we thus would expect for the solutions of the differential equations to contain integration constants with different values on both sides of the source [6]. This offers the possibility to solve the boundary conditions for different integration constants for $\xi = 0$ and $\xi = \pi/2$, providing potentially a regular solution. To work out the explicit form of these integration constants and check whether this indeed resolves the problem of regularity would be very interesting, as it may be seen as a step towards the inclusion of localized sources.

[6] The difference of the integration constants on both sides of the source depends on the location ξ_0 of the source and the chosen constants K_i, $i = 1, 2, 3$, in (10.64).

Chapter 11

Effective type IIA action on coset spaces

In section 3.2, we discussed the procedure to derive the four-dimensional low-energy effective theory for a given compactification manifold. We now apply this procedure to derive the superpotential and the Kähler potential for compactifications on all the coset spaces which allow for a left-invariant strict SU(3)-structure. These coset models are given in table 9.1. As explained in section 3.3, we choose the fluxes as general as possible to cover the whole moduli space. For the first five models in table 9.1, we already know that there is a bubble that contains at least one supersymmetric AdS_4 solution.

There will be bubbles in the moduli space that do not contain any supersymmetric AdS_4 solution, whereas other bubbles contain one or more. We show for two models how to identify the number of supersymmetric AdS_4 solutions for a particular choice of bubble parameters. Note that in the full string theory the bubble parameters are quantized.

We will study the mass spectrum of the moduli fields around these supersymmetric AdS_4 solutions. In section 2.2.2 we discussed the problem of the separation of scales for an $\mathcal{N}=1$ AdS_4 solution, even before the uplifting. We have seen that requiring the manifold to be nearly Calabi-Yau (i.e., vanishing $\mathcal{W}_1^-$) and the possibility to choose μ so that it is close to its bound is one way to obtain a separation of scales between the light masses and the Kaluza-Klein scale. However, as we will see in the following, for the $\mathcal{N}=1$ AdS_4 solutions on the coset models this is not possible such that we can not prove the separation of scales for these solutions. In any case, as already mentioned, the position one can take is that this kind of question should be asked only after the uplifting.

Since the decoupling of the light Kaluza-Klein modes turns out to be difficult, we can not be sure that there are no other light Kaluza-Klein modes joining the light moduli fields based on the left-invariant expansion ansatz. However, a truncation to the set of left-invariant forms is believed to provide a consistent truncation [115, 60].

Indeed, in [116] the authors established the consistency of the *left-invariant* truncation ansatz by means of explicit examples based on the coset models $\frac{\mathrm{G}_2}{\mathrm{SU}(3)}$, $\frac{\mathrm{Sp}(2)}{\mathrm{S}(\mathrm{U}(2)\times\mathrm{U}(1))}$ and $\frac{\mathrm{SU}(3)}{\mathrm{U}(1)\times\mathrm{U}(1)}$ for the sourceless case. It seems very plausible that their argument also applies for the other coset models we study in this thesis in the presence of smeared *left-invariant* source terms. We thus have confidence that solutions to the effective four-dimensional theories we derive in this chapter lift to consistent solutions of the ten-dimensional equations of motion and that there is no coupling between the preserved left-invariant modes and the truncated non-invariant modes. This also implies that the mass spectrum we compute for the left-invariant modes is not influenced by the potentially light non-invariant Kaluza-Klein modes.

A necessary condition for a strict SU(3)-structure is the compatibility condition $J \wedge \Omega = 0$ (see eq. (2.6)). This condition is automatically satisfied if there are no G-invariant five-forms. On the other hand, if there are such five-forms, the compatibility is not automatically satisfied, and the condition fixes the parameters for J and Ω in the $\mathcal{N} = 1$ solution (see for instance the parameters a, b and c in the solution (10.12)). If we now turn on fluctuations around such a vacuum solution, fluctuations are possible that violate the compatibility condition. One approach to still satisfy the compatibility condition is to impose some constraints on the fluctuations. However, a more natural approach is to impose from the beginning an orientifold projection that projects out the one- and five-forms. With this procedure, we again automatically satisfy the compatibility condition for all the fluctuations.

Let us stress that consistency requires that the source term which follows from the Bianchi identities is then consistent with the orientifold involutions we imposed. In this chapter, we will follow the second approach and impose an orientifold projection when the models allow for G-invariant one- and five-forms. We will make the simplification that the orientifold planes are perpendicular to the coordinate frame [1], except for $\frac{\mathrm{SU}(3)\times\mathrm{U}(1)}{\mathrm{SU}(2)}$, where we will demonstrate a procedure how to find more general orientifold planes and for $\mathrm{SU}(2)\times\mathrm{SU}(2)$, which does not allow for perpendicular orientifold planes.

11.1 Effective type IIA action on $\frac{\mathbf{G}_2}{\mathbf{SU(3)}}$

With the given set of G-invariant forms (9.3), we choose the expansion forms as follows

$$\begin{aligned} Y^{(2-)}: &\qquad (e^{12} - e^{34} + e^{56})\,, \\ Y^{(3+)}: &\qquad (e^{145} - e^{246} - e^{235} - e^{136})\,, \end{aligned} \tag{11.1}$$

and the standard volume $V_s = -\int e^{123456}$. We expand according to eq. (3.37)

$$\begin{aligned} J_c &= J - i\delta B = t^1 Y^{(2-)}\,, \\ \Omega_c &= e^{\Phi}\mathrm{Im}\,\Omega + i\delta C_3 = z^0 Y^{(3+)}\,, \end{aligned} \tag{11.2}$$

[1] To be precise, here we mean orientifold involutions which act as $e^i \to \pm e^i$ on the left-invariant one-forms.

where we denote by $t^1 = k^1 - ib^1$ the complex modulus in the Kähler sector and by $z^0 = u^0 + ic^0$ the complex modulus in the complex structure/dilaton sector. Note that since there is only one even three-form, this coset space has no complex structure and the real part of z^0 encodes the dilaton. There are no G-invariant one- or five-forms in this model, hence the compatibility condition (2.6a) is automatically satisfied for the basis (11.1) and thus for all choices of the moduli t^1 and z^0. The metric is easily evaluated via the Hitchin procedure explained in appendix C and reads

$$g = \mathrm{diag}(k^1, k^1, k^1, k^1, k^1, k^1)\,, \tag{11.3}$$

such that $k^1 > 0$ ensures metric positivity. The Betti numbers of $\frac{\mathrm{G}_2}{\mathrm{SU}(3)}$ are given in eq. (9.4). As was explained in section 3.3 we choose accordingly the background fluxes to be

$$\begin{aligned} \hat{H} &= 0\,, \\ \hat{F}_0 &= m\,, \\ \hat{F}_2 &= nY^{(2-)}\,, \\ \hat{F}_4 &= 0\,, \\ \hat{F}_6 &= f' e^{123456}\,, \end{aligned} \tag{11.4}$$

The quantized parameters m, n and f' specify the bubble of moduli space. Remember that is is not possible to reach another bubble by finite fluctuations of the moduli fields.

With this data, the superpotential, whose derivation is explained in section 3.2, reads

$$\mathcal{W}_{\mathrm{E}} = -\frac{ie^{-i\theta}}{4\kappa_{10}^2} V_s \left(f' + 3n(t^1)^2 - 4\sqrt{3}t^1 z^0 - im(t^1)^3 \right)\,, \tag{11.5}$$

and for the Kähler potential, we derive from eq. (3.35)

$$\mathcal{K} = -\ln\left((t^1 + \bar{t}^1)^3\right) - \ln\left(4(z^0 + \bar{z}^0)^4\right) + 3\ln(8\kappa_{10}^2 M_P^2 V_s^{-1})\,. \tag{11.6}$$

11.1.1 Mass spectrum around the supersymmetric vacuum

We have seen in section 10.1 that this coset space admits a supersymmetric $\mathcal{N} = 1$ AdS$_4$ vacuum. An application of the effective theory developed in this chapter is to compute the mass spectrum around the vacuum. By means of the explicit mass spectrum we can for instance identify the number of tachyonic masses and the number of massless moduli. However, since the vacuum solution is in AdS$_4$ space, it is not enough to find a tachyonic field for an instability to be present. Tachyonic fields whose negative mass-squared are above the Breitenlohner-Freedman bound (3.11) do not generate an instability [61]. However, after an uplift procedure these tachyonic modes become eventually unstable, and one has to reconsider the stability of the solution.

To derive the masses of the scalar fields around the supersymmetric vacuum, it is convenient to choose the background fluxes, which we plug in the expression for the superpotential, to be the fluxes of the solution (10.4). With this choice we automatically are in the bubble of moduli space containing the supersymmetric solution. For the calculation we choose this time expansion forms adapted to the solution,

$$\begin{aligned} Y^{(2-)}: &\qquad a(e^{12} - e^{34} + e^{56})\,; \\ Y^{(3+)}: &\qquad a^{3/2}(-e^{235} - e^{246} + e^{145} - e^{136})\,, \end{aligned} \tag{11.7}$$

and the standard volume $V_s = -\int a^3\, e^{123456}$. It is further convenient to take out the background dilaton, $e^{-\hat{\Phi}}$, from the definition of z^i in eq. (3.37b), i.e., we choose the expansion as follows

$$e^{-\Phi}\mathrm{Im}\,\Omega + i\delta C_3 = z^i e^{-\hat{\Phi}} Y_i^{(3+)}\,. \tag{11.8}$$

The supersymmetric solution then corresponds to the values of the moduli fields $t^1 = 1$ and $z^0 = 1$.

With these assumptions and the background fields in eq. (10.4) we get the following superpotential

$$\mathcal{W}_{\mathrm{E}} = \frac{ie^{-i\theta}e^{-\hat{\Phi}}}{4\kappa_{10}^2} V_s a^{-1/2} \left(-\frac{3\sqrt{3}}{2} + \frac{8\tilde{m}i}{5} z^0 - \frac{9\tilde{m}i}{5} t^1 + 4\sqrt{3} z^0 t^1 - \frac{\sqrt{3}}{2}(t^1)^2 + i\tilde{m}(t^1)^3 \right)\,, \tag{11.9}$$

whereas the Kähler potential is given by

$$\mathcal{K} = -\ln\left((t^1 + \bar{t}^1)^3\right) - \ln\left(4(z^0 + \bar{z}^0)^4\right) + 3\ln(8\kappa_{10}^2 M_P^2 V_s^{-1} e^{4\hat{\Phi}/3})\,. \tag{11.10}$$

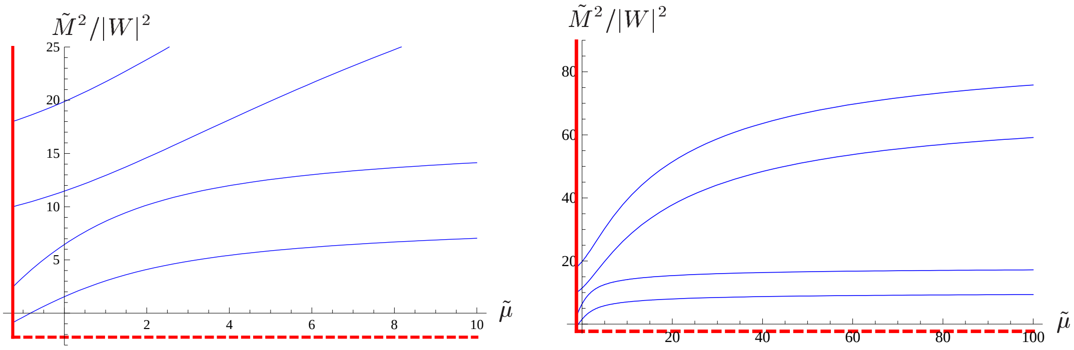
Indeed, one easily verifies that the F-terms $D_i\mathcal{W}_{\mathrm{E}} \equiv \partial_i\mathcal{W}_{\mathrm{E}} + (\partial_i\mathcal{K})\mathcal{W}_{\mathrm{E}}$, vanish for the values of the moduli fields $k^1 = 1$, $b^1 = 0$, $u^0 = 1$ and $c^0 = 0$.

By means of eq. (3.25), we now easily calculate the effective potential V and the mass matrix according to eq. (8.5). The resulting mass spectrum is plotted in figure 11.1. We plot $\tilde{M}^2/|W|^2$ such that the overall scale a drops out and the only parameter is the reduced orientifold tension $\tilde{\mu}$. The dashed and solid red line represent the Breitenlohner-Freedman bound (3.11) and the bound (2.34) for $\tilde{\mu}$, respectively. We see that all four moduli masses are above the Breitenlohner-Freedman bound as is expected. Moreover, all masses are positive for $\tilde{\mu} > -0.82$.

In section 2.2.2 we have seen that $|\mathcal{W}_1^-|L_{\mathrm{int}} \ll 1$ is one way to obtain a separation of scales between the light masses and the Kaluza-Klein masses even before the uplifting. However, as can be seen from eq. (10.2), this is impossible to achieve for this coset.

11.1.2 Number of supersymmetric solutions

We explained in section 3.3 that for different choices of the flux parameters m, n and f' in eq. (11.4) we are in different bubbles of the moduli space that are not connected by fluctuations of the moduli fields.



(a) Behaviour for small $\tilde{\mu}$

(b) Behaviour for large $\tilde{\mu}$

Figure 11.1: Mass spectrum of $\frac{\mathrm{G}_2}{\mathrm{SU}(3)}$.

An interesting question is whether each of these different bubbles of the moduli space characterized through the bubble parameters m, n and f' has a supersymmetric $\mathcal{N}=1$ AdS$_4$ solution and if, how many different solutions there are. To answer this question we may proceed backwards: Given a supersymmetric vacuum characterised through the supersymmetric solution parameters a, μ and e^{Φ}, we derive the corresponding bubble parameters m, n, f' in function of a, $\tilde{\mu} = a\mu$ and e^{Φ}. Inverting these equations one obtains the values for the bubble parameters that contain supersymmetric solutions.

From the supersymmetric solution for $\frac{\mathrm{G}_2}{\mathrm{SU}(3)}$ in eq. (10.2), the Bianchi identities and the fluctuations of the fluxes in eq. (3.44) we arrive at the following equations

$$
\begin{aligned}
m &= s_1\, e^{-\Phi} a^{-1/2} \sqrt{\frac{5}{4} + \tilde{\mu}}\,, \\
n &= -\frac{1}{\sqrt{3}} s_2\, a^{1/2} e^{-\Phi} \frac{2}{5} \tilde{\mu}\,, \\
f' &= -s_2\, e^{-\Phi} a^{5/2} \frac{4(20+\tilde{\mu})(25+2\tilde{\mu})}{375\sqrt{3}}\,,
\end{aligned}
\tag{11.11}
$$

where $s_1 = \pm 1$ and $s_2 = \pm 1$ are two signs further specifying the supersymmetric solution. Note that in the special case without source, $\tilde{\mu} = 0$, we find $n = 0$, and we can always find a supersymmetric vacuum by solving

$$
\begin{aligned}
m &= s_1\, e^{-\Phi} a^{-1/2} \frac{\sqrt{5}}{2}\,, \\
f' &= -s_2\, e^{-\Phi} a^{5/2} \frac{16}{3\sqrt{3}}\,,
\end{aligned}
\tag{11.12}
$$

for e^{Φ} and a. This results in

$$
a = \left(\frac{3\sqrt{15}}{32} \left| \frac{f'}{m} \right| \right)^{1/3}, \quad e^{\Phi} = \frac{\sqrt{5}}{2\,|m|} \left(\frac{3\sqrt{15}}{32} \left| \frac{f'}{m} \right| \right)^{-1/6}, \tag{11.13}
$$

such that, for arbitrary choice of the bubble parameters m and f', we find a supersymmetric solution characterized by a and e^{Φ}.

For $\tilde{\mu} \neq 0$, we can eliminate e^{Φ} and a by calculating

$$h = \frac{f'm^2}{\mu'^3} = \frac{(5+4\tilde{\mu})(20+\tilde{\mu})(25+2\tilde{\mu})}{8\sqrt{3}\tilde{\mu}^3}\,, \tag{11.14}$$

which can be rewritten as

$$(8-8h)\,\tilde{\mu}^3 + 270\tilde{\mu}^2 + 2325\tilde{\mu} + 2500 = 0\,. \tag{11.15}$$

For the values $h < 0$ and $h > 1$, this equation has exactly one solution satisfying further the bound coming from the first equation in (11.11): $5/4 + \tilde{\mu} > 0$ (note that we assumed $m \neq 0$). We conclude that there is no supersymmetric solution for the choice of bubble parameters satisfying $0 \leq \frac{f'm^2}{n^3} \leq 1$. Otherwise there is exactly one supersymmetric solution.

11.2 Effective type IIA action on $\frac{\mathbf{Sp(2)}}{\mathbf{S(U(2)\times U(1))}}$

From the given set of G-invariant forms (9.6), we define the expansion forms as follows

$$\begin{aligned} Y_i^{(2-)}: &\qquad (e^{12}+e^{34}), -e^{56}\,, \\ Y^{(3+)}: &\qquad (e^{235}+e^{246}+e^{145}-e^{136})\,, \end{aligned} \tag{11.16}$$

and the standard volume $V_s = \int e^{123456}$. According to eq. (3.37), we expand the SU(3)-structure as follows

$$\begin{aligned} J_c &= J - i\delta B = t^1(e^{12}+e^{34}) - t^2 e^{56}\,, \\ \Omega_c &= e^{\Phi}\mathrm{Im}\Omega + i\delta C_3 = z^0(e^{235}+e^{246}+e^{145}-e^{136})\,, \end{aligned} \tag{11.17}$$

which yields the metric

$$g = \mathrm{diag}(k^1, k^1, k^1, k^1, k^2, k^2)\,, \tag{11.18}$$

such that $k_i > 0$, $i = 1, 2$, ensures metric positivity. According to the Betti numbers (9.7), there is a closed two-form and we thus have only one non-closed two-form for $\hat{F}_2$. Since $\hat{F}_4 \in H^4(\mathcal{M}, \mathbb{R})$, we choose as background fluxes the following

$$\begin{aligned} \hat{H} &= 0\,, \\ \hat{F}_0 &= m\,, \\ \hat{F}_2 &= ne^{56}\,, \\ \hat{F}_4 &= \omega e^{1234}\,, \\ \hat{F}_6 &= -f'e^{123456}\,. \end{aligned} \tag{11.19}$$

The superpotential reads

$$\mathcal{W}_{\mathrm{E}} = -\frac{ie^{-i\theta}}{4\kappa_{10}^2} V_s \left(f' - i\omega t^2 + n(t^1)^2 + im(t^1)^2 t^2 - 2z^0(t^2 + 2t^1) \right) , \tag{11.20}$$

whereas the Kähler potential is given by

$$\mathcal{K} = -\ln\left((t^1 + \bar{t}^1)^2 (t^2 + \bar{t}^2) \right) - \ln\left(4(z^0 + \bar{z}^0)^4 \right) + 3\ln(8\kappa_{10}^2 M_P^2 V_s^{-1}) . \tag{11.21}$$

11.2.1 Mass spectrum around the supersymmetric vacuum

We choose the expansion forms suitable for the solution in section 10.2 as follows:

$$\begin{aligned} Y^{(2-)}: &\qquad a(e^{12} + e^{34}), -ae^{56} ; \\ Y^{(3+)}: &\qquad a^{3/2}(e^{235} + e^{246} + e^{145} - e^{136}) , \end{aligned} \tag{11.22}$$

and the standard volume $V_s = -\int a^3\, e^{123456}$. We find the following superpotential (where we use the redefinition of eq. (11.8))

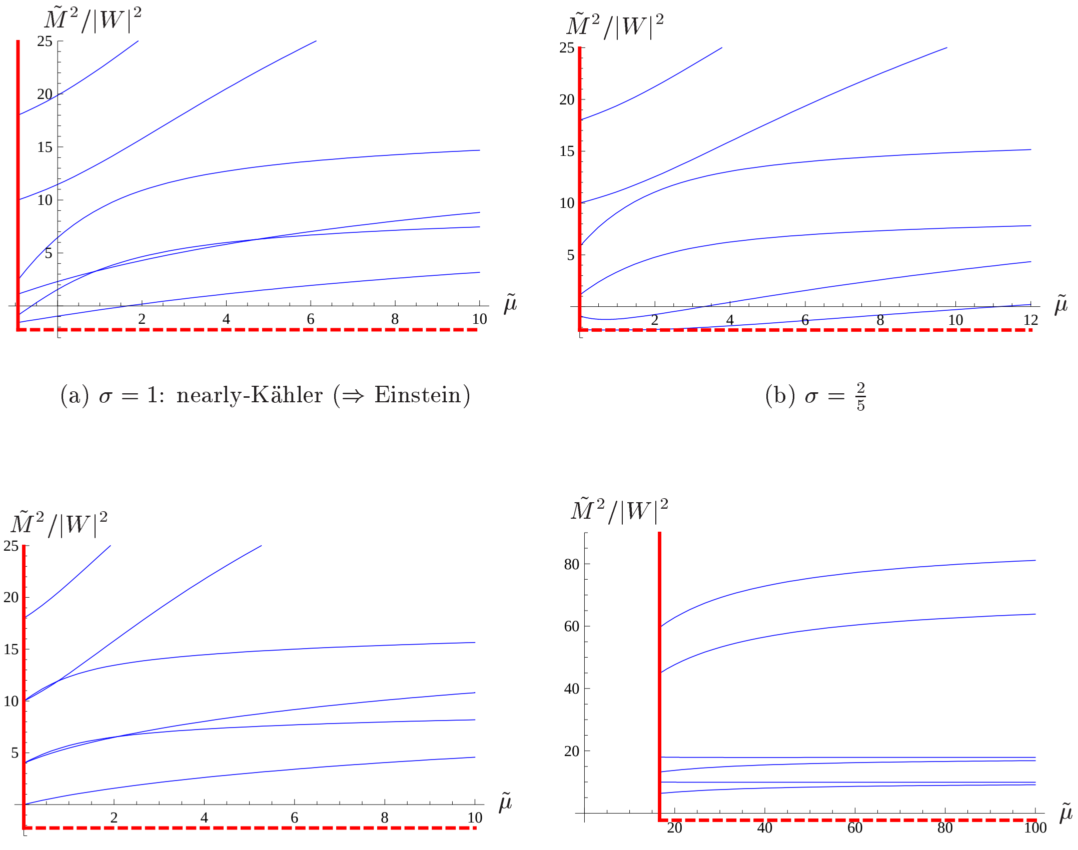
$$\begin{aligned} \mathcal{W}_{\mathrm{E}} = \frac{ie^{-i\theta}e^{-\hat{\Phi}}}{4\kappa_{10}^2} V_s a^{-1/2} \Bigg(& -\tilde{f}\sigma + \frac{8\tilde{m}i}{5}\sigma^{1/2} z^0 - \frac{3\tilde{m}i}{5}(2\sigma t^1 + t^2) - 2(2t^1 + t^2) z^0 \\ & + i\tilde{m}(t^1)^2 t^2 + \sigma^{1/2}\left(\frac{3}{2} - \frac{5}{4}\sigma\right)(t^1)^2 - \left(\sigma^{-1/2} - \frac{3}{2}\sigma^{1/2}\right) t^1 t^2 \Bigg) , \end{aligned} \tag{11.23}$$

and Kähler potential

$$\mathcal{K} = -\ln\left((t^1 + \bar{t}^1)^2 (t^2 + \bar{t}^2) \right) - \ln\left(4(z^0 + \bar{z}^0)^4 \right) + 3\ln(8\kappa_{10}^2 M_P^2 V_s^{-1} e^{4\hat{\Phi}/3}) . \tag{11.24}$$

This time the solution has next to the overall scale a two free parameters: the "shape" $\sigma = c/a$ and the orientifold tension $\tilde{\mu}$. In figure 11.2 we display plots for the mass spectrum for several values of σ: $\sigma = 1$ is the nearly-Kähler point, while for $\sigma = 2/5$ and $\sigma = 2$ the lower bound for $\tilde{\mu}$ from (2.37) is exactly zero. These were extreme points in [51] since outside the interval $[2/5, 2]$ the lower bound is above zero and solutions without orientifolds are no longer possible. Moreover, for $\tilde{\mu} = 0$ also $m = 0$, and these solutions can be lifted to M-theory. We also display a plot for large σ, here $\sigma = 13$. We see that the lower bound for $\tilde{\mu}$ is indeed positive so that there must be net orientifold charge. Again we see that in all cases all masses are above the Breitenlohner-Freedman bound and by choosing $\tilde{\mu}$ large enough, they are all positive.

Again we would like to have $|\mathcal{W}_1^-| L_{\mathrm{int}} \ll 1$ in order to decouple the Kaluza-Klein modes. From eq. (10.7) we see that this is not possible since this would imply putting $\sigma \to -2$, which should be positive. We thus can not prove the decoupling of the Kaluza-Klein modes for this model.



(a) $\sigma = 1$: nearly-Kähler ($\Rightarrow$ Einstein)

(b) $\sigma = \frac{2}{5}$

(c) $\sigma = 2$: Einstein

(d) $\sigma = 13$

Figure 11.2: Mass spectrum of $\frac{\mathrm{Sp}(2)}{\mathrm{S}(\mathrm{U}(2)\times\mathrm{U}(1))}$.

11.2.2 Number of supersymmetric solutions

With the same procedure we proposed in section 11.1.2 we can identify for this model the number of supersymmetric AdS$_4$ solutions for each choice of bubble parameters m, n, ω and f'. Starting from a supersymmetric solution specified by four parameters a, $\sigma = c/a$, e^{Φ} and $\tilde{\mu} = a\mu$ we find for this model

$$\begin{aligned}
m &= s_1 e^{-\Phi} a^{-1/2} \sqrt{\frac{5}{16\sigma}\left(-4 - 5\sigma^2 + 12\sigma\right) + \tilde{\mu}}\,, \\
n &= -s_2 \frac{4}{5} e^{-\Phi} a^{1/2} \tilde{\mu}\sqrt{\sigma}\,, \\
w &= -s_1 e^{-\Phi} \frac{4a^{3/2}(\sigma-1)(\sigma(10+3\tilde{\mu})-5)}{5\sqrt{60\sigma^2 + 16\tilde{\mu}\sigma^2 - 20\sigma - 25\sigma^3}}\,, \\
f' &= -s_2 e^{-\Phi} \frac{8a^{5/2}\sigma^{1/2}\left(150 + 40\tilde{\mu} - 425\sigma - 6\tilde{\mu}\sigma(35+4\tilde{\mu}) + 15\sigma^2(10+\tilde{\mu})\right)}{25\left(60\sigma + 16\tilde{\mu}\sigma - 20 - 25\sigma^2\right)}\,.
\end{aligned} \tag{11.25}$$

We can solve these equations for $\tilde{\mu}$ and σ by calculating

$$\begin{aligned}
\frac{4mw}{n^2} &= -\frac{5(\sigma-1)(\sigma(10+3\tilde{\mu})-5)}{4\tilde{\mu}^2\sigma^2} \equiv h_1\,, \\
-\frac{8f'm^2}{n^3} &= \frac{-50(15+4\tilde{\mu}) + 5\sigma(425 + 6\tilde{\mu}(35+4\tilde{\mu})) - 75\sigma^2(10+\tilde{\mu})}{16\tilde{\mu}^3\sigma^2} \equiv h_2\,.
\end{aligned} \tag{11.26}$$

with the solution for $\tilde{\mu}$

$$\begin{aligned}
0 = {} & (-256h_1h_2 + 256h_1^2 + 64h_2^2)\tilde{\mu}^6 \\
& + (-690h_2 + 1920h_1^2 - 690h_1h_2 + 1920h_1)\tilde{\mu}^5 \\
& + (-5900h_2 + 19000h_1 + 3600 + 3600h_1^2)\tilde{\mu}^4 \\
& + (44250 + 53250h_1 - 4500h_2)\tilde{\mu}^3 + (169375 + 34375h_1)\tilde{\mu}^2 \\
& + 206250\tilde{\mu} + 78125\,,
\end{aligned} \tag{11.27}$$

and the solution for σ

$$\sigma = \frac{5\left(750 + \tilde{\mu}\left[775 + 8\tilde{\mu}(15 - 2h_2\tilde{\mu} + h_1(15+4\tilde{\mu}))\right]\right)}{10000 + \tilde{\mu}\left\{13500 + \tilde{\mu}\left[4125 - 24\tilde{\mu}(2h_2(5+\tilde{\mu}) - 15) + 4h_1(6\tilde{\mu}(35+4\tilde{\mu}) + 425)\right]\right\}}\,. \tag{11.28}$$

In figure 11.3 we show the values for $h_1 = \frac{4mw}{n^2}$ and $h_2 = -\frac{8f'm^2}{n^3}$ for which eq. (11.27) has one or two solutions that are real and respect the bounds $\frac{5}{16\sigma}\left(-4 - 5\sigma^2 + 12\sigma\right) + \tilde{\mu} > 0$ and $\sigma > 0$. Hence, there are bubbles with zero, one or two supersymmetric solutions.

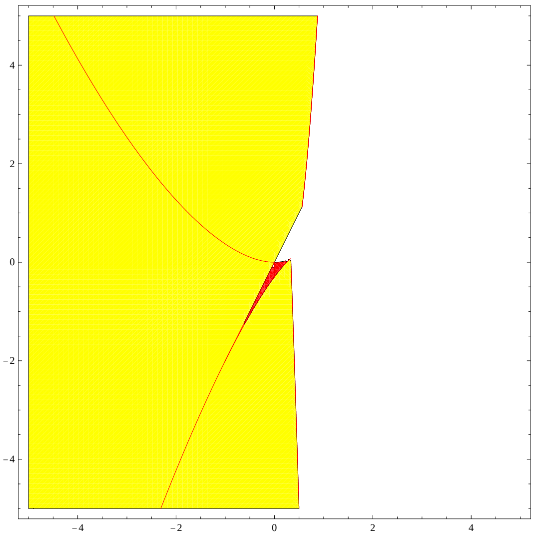


Figure 11.3: Regions in bubble parameter space in which there are one/two supersymmetric solutions (red/yellow) for $\frac{\mathrm{Sp}(2)}{\mathrm{S}(\mathrm{U}(2)\times\mathrm{U}(1))}$. On the x-axis (y-axis) is the value $h_1 = \frac{4mw}{n^2}$ ($h_2 = -\frac{8f'm^2}{n^3}$).

11.3 Effective type IIA action on $\frac{\mathbf{SU(3)}}{\mathbf{U(1)\times U(1)}}$

From the given set of G-invariant forms (9.9), we define the expansion forms as follows

$$\begin{aligned} Y_i^{(2-)}: &\qquad -e^{12}, e^{34}, -e^{56}\,, \\ Y^{(3+)}: &\qquad (e^{235}+e^{136}+e^{246}-e^{145})\,, \end{aligned} \tag{11.29}$$

and the standard volume $V_s = \int e^{123456}$. For this choice the metric reads

$$g = \mathrm{diag}(k^1, k^1, k^2, k^2, k^3, k^3)\,, \tag{11.30}$$

such that $k^i > 0$ ensures metric positivity. According to the Betti numbers (9.10), we choose a simple non-closed two-form $\hat{F}_2$ and $\hat{F}_4 \in H^4(\mathcal{M}, \mathbb{R})$ as background fluxes as follows

$$\begin{aligned} \hat{H} &= 0\,, \\ \hat{F}_0 &= m\,, \\ \hat{F}_2 &= ne^{12}\,, \\ \hat{F}_4 &= \omega_1 e^{1234} + \omega_2 e^{1256}\,, \\ \hat{F}_6 &= -f' e^{123456}\,. \end{aligned} \tag{11.31}$$

The superpotential reads

$$\mathcal{W}_{\mathrm{E}} = -\frac{ie^{-i\theta}}{4\kappa_{10}^2} V_s \left(f' - i(\omega_1 t^3 - \omega_2 t^2) - nt^2t^3 - imt^1t^2t^3 - 2z^0(t^1+t^2+t^3)\right)\,, \tag{11.32}$$

whereas the Kähler potential is given by

$$\mathcal{K} = -\ln\left((t^1+\bar{t}^1)(t^2+\bar{t}^2)(t^3+\bar{t}^3)\right) - \ln\left(4(z^0+\bar{z}^0)^4\right) + 3\ln(8\kappa_{10}^2 M_P^2 V_s^{-1})\,. \quad (11.33)$$

11.3.1 Mass spectrum around the supersymmetric vacuum

In this case we choose the expansion forms in (3.37) as follows:

$$\begin{aligned} Y^{(2-)}: &\qquad -ae^{12}, ae^{34}, -ae^{56}\,; \\ Y^{(3+)}: &\qquad a^{3/2}(e^{235}+e^{246}+e^{136}-e^{145})\,, \end{aligned} \quad (11.34)$$

and the standard volume $V_s = \int a^3\, e^{123456}$.

Using the expression (3.34) for the superpotential in the SU(3)-structure case and the expansion given in (3.37), we derive the superpotential (again using the redefinition of z as in eq. (11.8))

$$\begin{aligned} \mathcal{W}_{\mathrm{E}} = &-\frac{ie^{-i\theta}e^{-\hat{\Phi}}}{4\kappa_{10}^2} V_s a^{-1/2}\Bigg(\tilde{f}\rho\sigma - \frac{8\tilde{m}i}{5}\sqrt{\rho\sigma}z + \frac{3\tilde{m}i}{5}(\rho\sigma t^1+\sigma t^2+\rho t^3) \\ &+\frac{1}{4\sqrt{\rho\sigma}}\Big((3\sigma+3\rho\sigma-5\sigma^2)t^1t^2+(3\rho-5\rho^2+3\rho\sigma)t^1t^3+(-5+3\rho+3\sigma)t^2t^3\Big) \\ &-2z(t^1+t^2+t^3)-i\tilde{m}t^1t^2t^3\Bigg)\,. \end{aligned} \quad (11.35)$$

The Kähler potential is

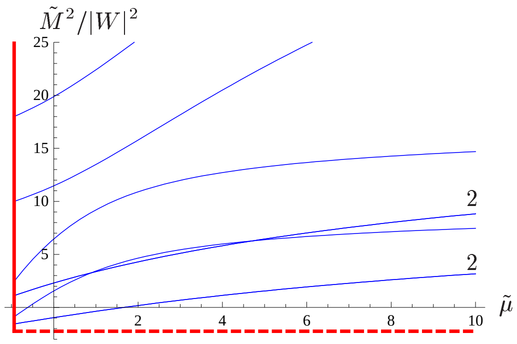
$$\mathcal{K} = -\ln\left(\prod_{i=1}^{3}(t^i+\bar{t}^i)\right) - \ln\left(4(z+\bar{z})^4\right) + 3\ln(8\kappa_{10}^2 M_P^2 V_s^{-1} e^{4\hat{\Phi}/3})\,. \quad (11.36)$$

The model has this time two shape parameters: $\rho = b/a$ and $\sigma = c/a$. We display the mass spectrum for a number of selected values of these parameters in figure 11.4. There is a symmetry under permuting (a, b, c) which translates into a symmetry under $\rho \leftrightarrow \sigma$ and $(\rho, \sigma, \tilde{\mu}) \leftrightarrow (\rho/\sigma, 1/\sigma, \sigma\tilde{\mu})$. Applying these symmetries leads to identical mass spectra. Moreover, the mass spectra for $\rho = 1$ are apart from two more eigenvalues identical to the mass spectra of $\frac{\mathrm{Sp(2)}}{\mathrm{S(U(2)\times U(1))}}$. We also display an example with $\sigma, \rho \neq 1$.

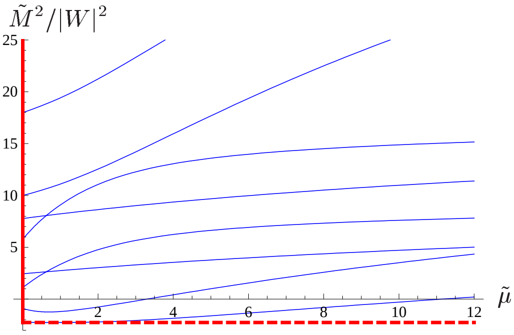
For this model, we have to choose $\rho + \sigma = -1$ in order to approach the nearly Calabi-Yau limit to decouple the Kaluza- Klein modes, which is again not possible.

11.4 Effective type IIA action on $\frac{\mathbf{SU(3)\times U(1)}}{\mathbf{SU(2)}}$

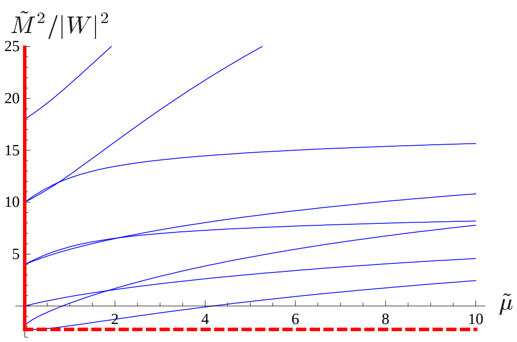
Since this coset space contains G-invariant one-forms, e^5 and e^6, one has to be careful satisfying the compatibility conditions for an SU(3)-structure given in eq. (2.6a).



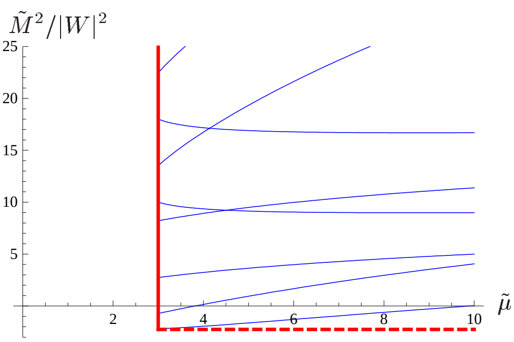
(a) $\rho = \sigma = 1$: nearly-Kähler. Lines indicated with 2 have multiplicity 2.



(b) $\rho = 1$ and $\sigma = \frac{2}{5}$.



(c) $\rho = 1$ and $\sigma = 2$.



(d) $\rho = \frac{5}{2}$ and $\sigma = \frac{1}{2}$.

Figure 11.4: Mass spectrum of $\frac{\mathrm{SU}(3)}{\mathrm{U}(1)\times\mathrm{U}(1)}$.

One way to guarantee the compatibility conditions for the fluctuations is to impose an orientifold projection that removes the left-invariant one- and five-forms. To find appropriate orientifold planes, let us therefore start with the most general, non-closed two-form as an ansatz for $\hat{F}_2$,

$$\hat{F}_2 = a^1(e^{13} - e^{24}) + a^2(e^{14} + e^{23}) + a^3 e^{56}\,, \tag{11.37}$$

where we assume non-vanishing coefficients a^i, $i = 1, 2, 3$. With this choice of $\hat{F}_2$ we get via the Bianchi identity (B.9a) the source term (note that since $b_3 = 0$ we choose $\hat{H} = 0$ such that there is no contribution to the Bianchi from $\hat{H}$)

$$j^6 = \sqrt{3}\left(-a^1(e^{145} + e^{235}) + a^2(e^{135} - e^{245}) + \frac{a^3}{2}(e^{126} + e^{346})\right)\,, \tag{11.38}$$

that can be written by a coordinate transformation consistent with the structure constants similar to (10.16) [2]

$$\begin{aligned} &e^{1'} = e^1\,, \quad e^{2'} = e^2\,, \quad e^{5'} = e^5\,, \quad e^{6'} = e^6\,, \\ &e^{3'} = \frac{1}{\sqrt{(a^1)^2 + (a^2)^2}}\left(-a^2 e^3 + a^1 e^4\right)\,, \quad e^{4'} = \frac{1}{\sqrt{(a^1)^2 + (a^2)^2}}\left(-a^1 e^3 - a^2 e^4\right)\,, \end{aligned} \tag{11.39}$$

as a sum of four decomposable terms to which we can associate four orientifold involutions (see also the discussion in appendix D),

$$j^6 = \sqrt{3}\left(\sqrt{(a^1)^2 + (a^2)^2}(e^{2'4'5'} - e^{1'3'5'}) + \frac{a^3}{2}(e^{1'2'6'} + e^{3'4'6'})\right)\,. \tag{11.40}$$

Under these orientifold involutions there are no one- and five-forms surviving, and we easy obtain the set of invariant two- and three-forms. By transforming back to the original coordinates, we get the following set of left-invariant (odd/even) two- and three-forms

$$\begin{aligned} \mathrm{Y}_i^{(2-)}: &\qquad \left[-(e^{13} - e^{24}) - \frac{a^2}{a^1}(e^{14} + e^{23})\right]\,, \; e^{56}\,, \\ \mathrm{Y}_i^{(3+)}: &\qquad \left[(e^{146} + e^{236}) - \frac{a^2}{a^1}(e^{136} + e^{246})\right]\,, \; e^{125} + e^{345}\,, \end{aligned} \tag{11.41}$$

where the quantity $\frac{a^2}{a^1} \equiv \lambda$ is actually related to the choice of the orientifolds. We now proceed as usual: with this choice of expansion forms the metric is positive for $k^i > 0$, $i = 1, 2$, and $u^1 u^2 < 0$ and is then given by

$$g = \mathrm{diag}\sqrt{1+\lambda^2}\left(k^1, k^1, k^1, k^1, \frac{k^2}{1+\lambda^2}\left|\frac{u^2}{u^1}\right|, k^2\left|\frac{u^1}{u^2}\right|\right)\,. \tag{11.42}$$

[2] Without loss of generality we assumed here $a^1 > 0$.

From the Betti numbers (9.15) and the set of G-invariant forms (9.14) we thus get

$$\begin{aligned}
\hat{H} &= 0\,, \\
\hat{F}_0 &= m\,, \\
\hat{F}_2 &= n^1\left[(e^{13}-e^{24})+\lambda(e^{14}+e^{23})\right]+n^2e^{56}\,, \\
\hat{F}_4 &= 0\,, \\
\hat{F}_6 &= -f'(1+\lambda^2)e^{123456}\,.
\end{aligned} \tag{11.43}$$

Putting everything together we arrive at the superpotential

$$\mathcal{W}_{\mathrm{E}} = -\frac{ie^{-i\theta}}{4\kappa_{10}^2}V_s\left(f'-2n_1t^1t^2+n_2(t^1)^2-im(t^1)^2t^2+2\sqrt{3}t^1z^1-\frac{\sqrt{3}}{1+\lambda^2}t^2z^2\right)\,, \tag{11.44}$$

where we defined a standard volume as $V_s=\int(1+\lambda^2)e^{123456}$. The Kähler potential is given by

$$\mathcal{K} = -\ln\left((t^1+\bar{t}^1)^2(t^2+\bar{t}^2)\right)-\ln\left(\frac{4}{1+\lambda^2}(z^1+\bar{z}^1)^2(z^2+\bar{z}^2)^2\right)+3\ln(8\kappa_{10}^2M_P^2V_s^{-1})\,. \tag{11.45}$$

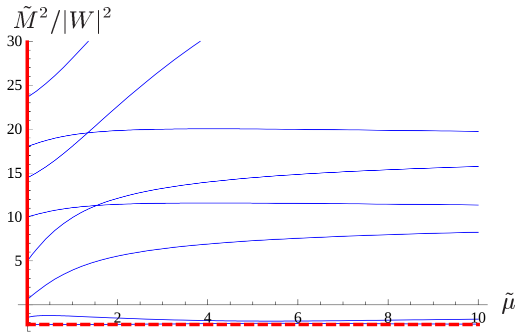
11.4.1 Mass spectrum around the supersymmetric vacuum

We choose the expansion forms suitable for the supersymmetric solution of section 10.4 as follows:

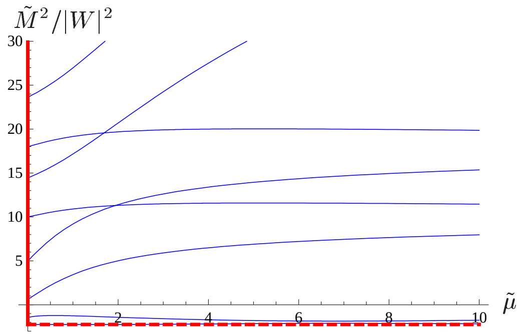
$$\begin{aligned}
Y^{(2-)}:&\qquad -a[(e^{13}-e^{24})-\rho(e^{14}+e^{23})],\ ae^{56}\,; \\
Y^{(3+)}:&\qquad a^{3/2}[(e^{13}-e^{24})+\rho^{-1}(e^{14}+e^{23})]\wedge e^6,\ a^{3/2}(e^{125}+e^{345})\,,
\end{aligned} \tag{11.46}$$

and the standard volume $V_s=\int a^3(1+\rho^2)e^{123456}$. The superpotential and Kähler potential read (using the redefinition (11.8)):

$$\begin{aligned}
\mathcal{W}_{\mathrm{E}} = &-\frac{ie^{-i\theta}e^{-\hat{\Phi}}}{4\kappa_{10}^2}V_sa^{-1/2}\left(\tilde{f}\sigma+\frac{3i\tilde{m}}{5}\sigma(2t^1+\frac{1}{\sigma}t^2)\right.\\
&+\sqrt{\frac{3}{2}}(1+\rho^2)^{-\frac{1}{4}}\left(-t^1t^2+\frac{\sigma}{2}(t^1)^2\right)-i\tilde{m}(t^1)^2t^2\\
&\left.-\frac{4\sqrt{2}i\tilde{m}}{5\rho}(1+\rho^2)^{\frac{1}{4}}z^1+\frac{2\sqrt{2}i\tilde{m}}{5}\sigma(1+\rho^2)^{-\frac{3}{4}}z^2+\frac{2\sqrt{3}}{\rho}z^1t^1-\sqrt{3}(1+\rho^2)^{-1}t^2z^2\right)\,,
\end{aligned} \tag{11.47}$$



(a) $\rho = \sigma = 1$.



(b) $\rho = \frac{1}{2}$ and $\sigma = 2$.

Figure 11.5: Mass spectrum of $\frac{\mathrm{SU}(3)\times\mathrm{U}(1)}{\mathrm{SU}(2)}$.

and

$$\begin{aligned}\mathcal{K} = &- \ln\left((t^1+\bar{t}^1)^2(t^2+\bar{t}^2)\right) - \ln\left(4\frac{1}{\rho^2(1+\rho^2)}(z^1+\bar{z}^1)^2(z^2+\bar{z}^2)^2\right) \\ &+ 3\ln(8\kappa_{10}^2 M_P^2 V_s^{-1} e^{4\hat{\Phi}/3})\,.\end{aligned} \tag{11.48}$$

This model has two shape parameters $\rho = b/a$ and $\sigma = c/a$, and a symmetry under $(\rho, \sigma, \tilde{\mu}) \leftrightarrow (1/\rho, \sigma/\rho, \rho\tilde{\mu})$. In figure 11.5, we show the mass spectrum for some values of the parameters. The mass spectrum at $\mu = 0$ turns out to be independent of the parameters ρ, σ. There always seem to be two negative $\tilde{M}^2$ eigenvalues. Note that there is no choice of parameters for this solution to obtain a NCY-limit, which was our proposal to decouple the Kaluza-Klein modes. This can be seen from eq. (10.13).

11.5 Effective type IIA action on SU(2) × SU(2)

Since SU(2) × SU(2) is a trivial coset space, all the left-invariant forms e^i are G-invariant. As we suggested in the introduction of this section, in order to satisfy the condition (2.6a) automatically, we must eliminate the one- and five-forms. We do so by introducing at least three mutually supersymmetric orientifolds, compatible with the structure constants. This model does not allow for O6-planes that are perpendicular to the coordinate frame. However, in section 10.5 and appendix D we explained how to perform a suitable basis transformation in order to identify the orientifold involutions such that the fields and structure constants have the right transformation properties. The result of that analysis are the following expansion forms (see also eq. (D.19))

$$
\begin{aligned}
Y_1^{(2-)} =& e^{14}, \qquad Y_2^{(2-)} = e^{25}, \qquad Y_3^{(2-)} = e^{36}, \\
Y^{(3-)1} =& \frac{1}{4}\left(e^{156} - e^{234} - e^{246} + e^{135} + e^{345} - e^{126} + e^{123} - e^{456}\right), \\
Y^{(3-)2} =& \frac{1}{4}\left(e^{156} - e^{234} + e^{246} - e^{135} - e^{345} + e^{126} + e^{123} - e^{456}\right), \\
Y^{(3-)3} =& \frac{1}{4}\left(e^{156} - e^{234} + e^{246} - e^{135} + e^{345} - e^{126} - e^{123} + e^{456}\right), \\
Y^{(3-)4} =& \frac{1}{4}\left(-e^{156} + e^{234} + e^{246} - e^{135} + e^{345} - e^{126} + e^{123} - e^{456}\right), \\
Y_1^{(3+)} =& \frac{1}{2}\left(e^{156} + e^{234} - e^{246} - e^{135} + e^{345} + e^{126} + e^{123} + e^{456}\right), \\
Y_2^{(3+)} =& \frac{1}{2}\left(e^{156} + e^{234} + e^{246} + e^{135} - e^{345} - e^{126} + e^{123} + e^{456}\right), \\
Y_3^{(3+)} =& \frac{1}{2}\left(e^{156} + e^{234} + e^{246} + e^{135} + e^{345} + e^{126} - e^{123} - e^{456}\right), \\
Y_4^{(3+)} =& \frac{1}{2}\left(-e^{156} - e^{234} + e^{246} + e^{135} + e^{345} + e^{126} + e^{123} + e^{456}\right).
\end{aligned}
\tag{11.49}
$$

To simplify notation, it is convenient to define a matrix r_{iI} as in eq. (5.65) and we find with (11.49) the following matrix

$$
r = \begin{pmatrix} 1 & 1 & 1 & -1 \\ 1 & -1 & -1 & -1 \\ 1 & -1 & 1 & 1 \end{pmatrix}. \tag{11.50}
$$

For SU(2) $\times$ SU(2), we calculated the third Betti numbers in (9.17) to be $b_3 = 2$. One of the two three-forms in $H^3(\mathcal{M}, \mathbb{R})$ is odd and we thus make the most general ansatz for the background fields as follows

$$
\begin{aligned}
\hat{H} &= p\left(Y_1^{(3-)} + Y_2^{(3-)} - Y_3^{(3-)} + Y_4^{(3-)}\right), \\
\hat{F}_0 &= m, \\
\hat{F}_2 &= m^i Y_i^{(2-)}, \\
\hat{F}_4 &= 0, \\
\hat{F}_6 &= 0.
\end{aligned}
\tag{11.51}
$$

Plugging these background fluxes in the expression for the superpotential, we find

$$
\mathcal{W}_{\mathrm{E}} = -\frac{ie^{-i\theta}}{4\kappa_{10}^2} V_s\left(m^1 t^2 t^3 + m^2 t^1 t^3 + m^3 t^1 t^2 - imt^1 t^2 t^3 - ip(z^1 + z^2 - z^3 + z^4) + r_{iI} t^i z^I\right), \tag{11.52}
$$

and the Kähler potential

$$\mathcal{K} = -\ln \prod_{i=1}^{3} \left(t^i + \bar{t}^i\right) - \ln 16 \prod_{I=1}^{4} \left(z^I + \bar{z}^I\right) + 3 \ln \left(8\kappa_{10}^2 M_P^2 V_s^{-1}\right) , \tag{11.53}$$

where $V_s = -\int e^{123456}$. Note that for $p \neq 0$ the superpotential depends on all the moduli so there are no flat directions in this model.

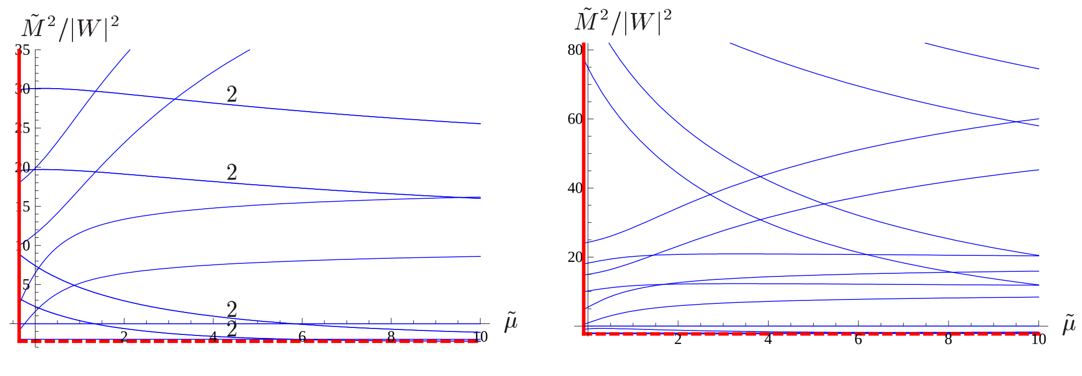
11.5.1 Mass spectrum around the supersymmetric vacuum

For the analysis of the mass spectrum around the supersymmetric solution, we choose the following suitable basis for the expansion forms

$$\begin{aligned}
&Y_1^{(2-)} = ae^{14} , \qquad Y_2^{(2-)} = be^{25} , \qquad Y_3^{(2-)} = ce^{36} , \\
&Y_1^{(3+)} = \frac{-h}{4c_1(a+b+c)}(e^{123} + e^{456} + e^{126} + e^{345} + e^{315} + e^{264} + e^{156} + e^{234}) , \\
&Y_2^{(3+)} \frac{h}{4c_1(-a+b+c)}(e^{123} + e^{456} - e^{126} - e^{345} - e^{315} - e^{264} + e^{156} + e^{234}) , \\
&Y_3^{(3+)} \frac{-h}{4c_1(a-b+c)}(-e^{123} - e^{456} + e^{126} + e^{345} - e^{315} - e^{264} + e^{156} + e^{234}) , \\
&Y_4^{(3+)} \frac{h}{4c_1(a+b-c)}(e^{123} + e^{456} + e^{126} + e^{345} - e^{315} - e^{264} - e^{156} - e^{234}) ,
\end{aligned} \tag{11.54}$$

and the standard volume $V_s = -\int abc\, e^{1\dots 6}$. One finds with eq. (10.22) the superpotential (with the redefinition (11.8)):

$$\begin{aligned}
\mathcal{W}_{\mathrm{E}} = \frac{ie^{-i\theta}e^{-\hat{\Phi}}}{4\kappa_{10}^2} V_s a^{-1/2} \Bigg\{ &\frac{3}{2}\tilde{c}_1 + i\tilde{m} \left(t^1t^2t^3 - \frac{3}{5}(t^1 + t^2 + t^3) - \frac{2}{5}(z^1 + z^2 + z^3 + z^4) \right) \\
&+ \frac{3}{2}\tilde{c}_1(t^1t^2 + t^2t^3 + t^1t^3) \\
&+ \frac{\tilde{c}_1}{\tilde{h}^2} \Big\{ 4\left[t^2t^3(1-\rho^2-\sigma^2) + t^1t^3\rho^2(-1+\rho^2-\sigma^2) + t^1t^2\sigma^2(-1-\rho^2+\sigma^2)\right] \\
&+ \left[t^1(-1+\rho^2+\sigma^2) + t^2\rho^2(1-\rho^2+\sigma^2) + t^3\sigma^2(1+\rho^2-\sigma^2)\right](z^1+z^2+z^3+z^4) \\
&+ \rho\sigma\left[-2t^1 + t^2(1+\rho^2-\sigma^2) + t^3(1-\rho^2+\sigma^2)\right](z^1+z^2-z^3-z^4) \\
&+ \sigma\left[t^1(1+\rho^2-\sigma^2) - 2\rho^2t^2 + t^3(-1+\rho^2+\sigma^2)\right](z^1-z^2+z^3-z^4) \\
&+ \rho\left[t^1(1-\rho^2+\sigma^2) + t^2(-1+\rho^2+\sigma^2) - 2\sigma^2t^3\right](z^1-z^2-z^3+z^4) \Big\} \Bigg\} ,
\end{aligned} \tag{11.55}$$



(a) $\rho = \sigma = 1$: nearly-Kähler. Lines indicated with 2 have multiplicity 2.

(b) $\sigma = 1$ and $\rho = \frac{2}{5}$.

Figure 11.6: Mass spectrum of SU(2)×SU(2).

and the Kähler potential

$$\mathcal{K} = -\ln\left(\prod_{i=1}^{3}(t^i + \bar{t}^i)\right) - \ln\left(4\prod_{i=1}^{4}\left(z^i + \bar{z}^i\right)\right) + 3\ln(8\kappa_{10}^2 M_P^2 V_s^{-1} e^{4\hat{\Phi}/3})\,. \tag{11.56}$$

There are again two shape parameters $\rho = b/a$ and $\sigma = c/a$ and the symmetries $\rho \leftrightarrow \sigma$, $(\rho, \sigma, \tilde{\mu}) \leftrightarrow (\rho/\sigma, 1/\sigma, \sigma\tilde{\mu})$. In figure 11.6 we display the mass spectrum for some values of the parameters. This time there will always be one unstabilized massless axion [3] ($\tilde{M}^2$=0) and a corresponding tachyonic complex structure modulus with $\tilde{M}^2/|W|^2 = -2$.

In the limit $\mathcal{W}_1^- \to 0$, $\mathcal{W}_2^-$ blows up just as the lower bound for $\tilde{\mu}$. Hence, we cannot satisfy (2.52) for negative a and the decoupling of the Kaluza-Klein modes is not guaranteed.

11.6 Effective type IIA action on $\frac{\mathbf{SU(2)}^2}{\mathbf{U(1)}} \times \mathbf{U(1)}$

This coset space has no supersymmetric AdS$_4$ solution. Nevertheless, one can define an SU(3)-structure on it. In order to eliminate the one- and five-forms, we introduce a set of suitable orientifolds. The possible orientifolds that are perpendicular to the coordinate frame and compatible with the structure constants are along [4]

$$123\,, \qquad 345, \qquad 256\,, \qquad 146\,, \qquad 246\,, \qquad 156\,. \tag{11.57}$$

[3]One may wonder why there is a flat axionic direction around the supersymmetric solution whereas we claimed that for $p \neq 0$ there are no flat directions arising with the superpotential (11.52). The reason is that the bubble containing the supersymmetric solution has bubble parameter $p = 0$ (see eq. (11.51)), since the background flux for the supersymmetric solution $\hat{H} \propto \mathrm{Re}\,\Omega \propto \mathrm{d}J$ is always exact and thus pure fluctuation.

[4]To be precise, e.g. 123 means for the orientifold involution $e^1 \to e^1$, $e^2 \to e^2$, $e^3 \to e^3$, $e^4 \to -e^4$, $e^5 \to -e^5$, $e^6 \to -e^6$.

In order to remove one- and five-forms, it turns out that we have to introduce at least two orientifolds, in particular one of $\{123, 345\}$ and one of $\{256, 146, 246, 156\}$. It does not matter for the analysis which particular choice is made, but for definiteness let us choose the following set

	1	2	3	4	5	6
O6			$\otimes$	$\otimes$	$\otimes$	
O6		$\otimes$			$\otimes$	$\otimes$
O6	$\otimes$			$\otimes$		$\otimes$
O6	$\otimes$	$\otimes$	$\otimes$			

From the set of G-invariant forms given in eq. (9.20) the following forms survive the orientifold projection

$$\begin{aligned} \text{odd 2-forms:} \quad & (e^{15}+e^{24})\,, \quad e^{36}\,, \\ \text{even 3-forms:} \quad & e^{123}\,, \quad (e^{256}-e^{146})\,, \quad e^{345}\,, \\ \text{odd 3-forms:} \quad & e^{126}\,, \quad (e^{235}-e^{134})\,, \quad e^{456}\,, \end{aligned} \tag{11.58}$$

which we then plug in eq. (3.37). There is always a change of basis such that we can assume $k^i > 0$, $i = 1, 2$. The conditions for metric positivity then become

$$u^1 u^2 > 0\,, \qquad u^1 u^3 > 0\,. \tag{11.59}$$

With the reduced set of forms (11.58) the Betti-numbers are $b_2^- = 0$ and $b_3^- = 1$. The most general background fluxes are thus chosen to be

$$\begin{aligned} \hat{H} &= p\left(e^{126} - e^{456}\right)\,, \\ \hat{F}_0 &= m\,, \\ \hat{F}_2 &= n_1 e^{36} + n_2(e^{15}+e^{24})\,, \\ \hat{F}_4 &= 0\,, \\ \hat{F}_6 &= 0\,, \end{aligned} \tag{11.60}$$

where we used the closed part of δC_3 to put $\hat{F}_6$ to zero as explained in section 3.3.

Note that one easily verifies that this choice of background fluxes reproduces with the Bianchi identity $\mathrm{d}\hat{F}_2 + m\hat{H} = -j^6$ exactly the expected source terms from our choice of the orientifold involutions. We find for the superpotential

$$\mathcal{W}_{\mathrm{E}} = -\frac{ie^{-i\theta}}{4\kappa_{10}^2} V_s\left(n_1(t^2)^2 + 2n_2 t^1 t^2 - imt^1(t^2)^2 + ip(z^1 - z^3) - t^1(z^1+z^3) - 2t^2 z^2\right)\,, \tag{11.61}$$

and the Kähler potential

$$\mathcal{K} = -\ln\left((t^1+\bar{t}^1)(t^2+\bar{t}^2)^2\right) - \ln\left(4(z^1+\bar{z}^1)(z^2+\bar{z}^2)^2(z^3+\bar{z}^3)\right) + 3\ln\left(8\kappa_{10}^2 M_P^2 V_s^{-1}\right) , \tag{11.62}$$

Let us mention that we also can consider the choice $\hat{H} = 0$ in eq. (11.60), which then implies that we have to choose $\hat{F}_6 = -f' e^{123456}$. For this choice the superpotential reads

$$\mathcal{W}_{\mathrm{E}} = -\frac{ie^{-i\theta}}{4\kappa_{10}^2} V_s \left(f' + n_1(t^2)^2 + 2n_2 t^1 t^2 - imt^1(t^2)^2 - t^1(z^1+z^3) - 2t^2 z^2\right) , \tag{11.63}$$

whereas the Kähler potential is not changed. Note that with this choice of the background fluxes, we have an axionic flat direction in the model, since the combination $(z^1 - z^3)$ drops out.

11.7 Effective type IIA action on SU(2) × U(1)^3

Again, for this trivial coset space all the left-invariant forms are G-invariant. There are ten possible orientifold planes perpendicular to the coordinate frame and compatible with the structure constants. It turns out that in order to remove the one- and five-forms we have to choose at least three mutually supersymmetric orientifolds and that it does not matter for the analysis which ones we choose. For definiteness, let us take the following choice

	1	2	3	4	5	6
O6	$\otimes$	$\otimes$	$\otimes$			
O6			$\otimes$		$\otimes$	$\otimes$
O6	$\otimes$			$\otimes$	$\otimes$	
O6		$\otimes$		$\otimes$		$\otimes$

With these orientifolds, we get the following expansion forms to be used in eq. (3.37)

$$\begin{aligned} &\text{odd 2-forms:} \quad e^{16}, \quad e^{25}, \quad e^{34}, \\ &\text{even 3-forms:} \quad e^{123}, \quad e^{356}, \quad -e^{246}, \quad e^{145}. \end{aligned} \tag{11.64}$$

Again there is always a change of basis such that we can assume $k^i > 0$, $i = 1, 2$. The positivity of the metric demands that

$$u^1 u^2 > 0 , \quad u^1 u^3 > 0 , \quad u^1 u^4 > 0 . \tag{11.65}$$

The Betti-numbers are this time $b_2^- = 0$ and $b_3^- = 1$ such that the most general background fluxes are

$$\begin{aligned}
\hat{H} &= pe^{456}\,, \\
\hat{F}_0 &= m\,, \\
\hat{F}_2 &= n_1 e^{16} + n_2 e^{25} + n_3 e^{34}\,, \\
\hat{F}_4 &= 0\,, \\
\hat{F}_6 &= 0\,.
\end{aligned} \tag{11.66}$$

Again one easily shows that the Bianchi identity reproduces the expected source term coming from our choice of orientifold involutions. The superpotential for this model reads

$$\mathcal{W}_{\mathrm{E}} = -\frac{ie^{-i\theta}}{4\kappa_{10}^2} V_s \left(n_1 t^2 t^3 + n_2 t^1 t^3 + n_3 t^1 t^2 - imt^1 t^2 t^3 - ipz^1 - t^1 z^4 - t^2 z^3 - t^3 z^2 \right)\,, \tag{11.67}$$

whereas the Kähler potential is

$$\mathcal{K} = -\ln\left(\prod_{i=1}^{3}(t^i + \bar{t}^i)\right) - \ln\left(4\prod_{i=1}^{4}(z^i + \bar{z}^i)\right) + 3\ln\left(8\kappa_{10}^2 M_P^2 V_s^{-1}\right)\,. \tag{11.68}$$

Again, one could choose $\hat{H} = 0$ and instead $\hat{F}_6 = -f' e^{123456}$ in eq. (11.66). The superpotential for this choice reads

$$\mathcal{W}_{\mathrm{E}} = -\frac{ie^{-i\theta}}{4\kappa_{10}^2} V_s \left(f' + n_1 t^2 t^3 + n_2 t^1 t^3 + n_3 t^1 t^2 - imt^1 t^2 t^3 - t^1 z^4 - t^2 z^3 - t^3 z^2 \right)\,, \tag{11.69}$$

whereas the Kähler potential does not change. Note that we obtain again a flat direction by turning off $\hat{H}$ since the superpotential (11.69) does not depend on z^1.

Chapter 12

Coset models with static SU(2)-structure

Within the class of coset geometries we can also try to find suitable coset spaces for compactifications with more general G-structures than strict SU(3)-structure. Let us focus in the following on compactifications with static SU(2)-structure. As we have seen in chapter 6, for the nilmanifolds there exists a type IIB AdS_4 $\mathcal{N} = 1$ solution with static SU(2)-structure which turned out to be related via a T-duality to both, the torus solution and the Iwasawa solution (at least for some values of the parameters). This motivates to look in type IIB for possible compactifications with static SU(2)-structure on the coset spaces. Indeed, in [34] it was mentioned that there is a static SU(2) type IIB $\mathcal{N} = 1$ compactification to AdS_4 on $\frac{\mathrm{SU(3)}\times\mathrm{U(1)}}{\mathrm{SU(2)}}$ that is T-dual to the strict SU(3) type IIA solution on the same coset (the solution of section 10.4) and a further static SU(2) type IIB $\mathcal{N} = 1$ AdS_4 solution on $\frac{\mathrm{SU(2)}^2}{\mathrm{U(1)}} \times \mathrm{U(1)}$ which is T-dual to the SU(3) type IIA solution on $\mathrm{SU(2)} \times \mathrm{SU(2)}$ of section 10.5 (see also [117]).

Here we do not only want to study type IIB $\mathcal{N} = 1$ compactifications to AdS_4, but follow our approach and compute the effective four-dimensional theory for all coset models that allow for a static SU(2)-structure. We thus derive the set of G-invariant forms for these models and expand the static SU(2)-structure quantities given in eq. (3.42) in the appropriate forms. The SU(2)-structure conditions (2.16) impose non-trivial conditions on these fluctuations. An elegant way to solve these compatibility conditions for the fluctuations is to introduce (smeared) O5/O7 orientifolds. In eq. (3.40) the transformation properties of the SU(2)-structure quantities are given such that we can expand these quantities in the G-invariant forms transforming correspondingly. It turns out that the compatibility conditions are then automatically satisfied for all the fluctuations. Note that this is similar to the approach we followed for the strict SU(3)-structure compactifications of chapter 11, where we removed one- and five-forms by choosing appropriate O6 orientifold involutions.

In the following we will study the six-dimensional coset spaces G/H of table 4.1 that have structure group SU(2). A necessary condition on H is that $H \subseteq \mathrm{SU(2)}$ [34], which restricts the possible coset candidates to the last four entries in table 4.1. We immediately can exclude the coset model $\frac{\mathrm{SU}(2)^3}{\mathrm{SU}(2)}$ as can be seen as follows [1]. When SU(2) is embedded diagonally in $\mathrm{SU}(2)^3$, the coset space admits no G-invariant one-forms (which are needed for a static SU(2)-structure), and if SU(2) is embedded in the last two SU(2) factors, the given set of G-invariant one- and two-forms is

$$\begin{aligned} \text{one-forms}: &\qquad e^1, e^2, e^3\,, \\ \text{two-forms}: &\qquad e^{12}, e^{13}, e^{23}\,, \end{aligned} \tag{12.1}$$

with which we can not satisfy condition (2.16a).

In the following we will study the remaining possible coset spaces, that are $\frac{\mathrm{SU}(3)\times\mathrm{U}(1)}{\mathrm{SU}(2)}$, $\frac{\mathrm{SU}(2)^2}{\mathrm{U}(1)}\times\mathrm{U}(1)$, $\mathrm{SU}(2)\times\mathrm{SU}(2)$ and $\mathrm{SU}(2)\times\mathrm{U}(1)^3$. We will restrict ourselves to O5/O7 orientifold planes that are perpendicular to the coordinate frame [2].

12.1 Effective type IIB action on $\frac{\mathbf{SU(3)\times U(1)}}{\mathbf{SU(2)}}$

We first derive all possible O5/O7 orientifold planes that are perpendicular to the coordinate frame and compatible with the structure constants (i.e. the structure constant tensor (4.29) is even under the orientifold involutions), and we obtain the following list of possible orientifold involutions

$$\begin{aligned} \text{O5:} &\quad 13, 14, 23, 24, 56\,, \\ \text{O7:} &\quad 1256, 3456\,. \end{aligned} \tag{12.2}$$

Choosing the O5 orientifold along 56, we would end up with even G-invariant one-forms e^5 and e^6 under this O5 orientifold, which is not appropriate to expand the SU(2)-structure quantity V in eq. (3.42), since the one-forms have to be odd under O5-orientifolds, see eq. (3.40). We thus exclude the orientifold along 56. Which compatible combination of the remaining O5-planes we choose does not matter for the following analysis, so let us choose for definiteness the following orientifold planes:

	1	2	3	4	5	6
O5	$\otimes$			$\otimes$		
O5		$\otimes$	$\otimes$			
O7	$\otimes$	$\otimes$			$\otimes$	$\otimes$
O7			$\otimes$	$\otimes$	$\otimes$	$\otimes$

From the set of G-invariant forms for this model (9.14), we obtain the following basis of forms transforming as indicated under the O5/O7 orientifold planes:

[1] For the details on the structure constants of this coset space we refer the reader to [34].

[2] Contrary to O6-planes, $\mathrm{SU}(2)\times\mathrm{SU}(2)$ allows for perpendicular O5/O7-planes.

type under O5/O7	basis	name
odd/even 1-form	e^5, e^6	$Y_i^{(1-+)}$
even/odd 2-form	$e^{14}+e^{23}$	$Y^{(2+-)}$
odd/odd 2-form	$e^{13}-e^{24}$	$Y^{(2--)}$
odd/even 4-form	$e^{1256}+e^{3456}$	$Y^{(4-+)}$

Putting these expansion forms in (3.42) we end up with four complex moduli fields $\tau = x + iy$, $t^1 = k^1 - ib^1$, $z^1 = u^1 + ic^1$ and $w^1 = v^1 + ih^1$. The SU(2)-structure quantities are expanded as follows

$$\begin{aligned} \omega_2 &= k^1(e^{13}-e^{24})\,, \\ e^{-\Phi}\mathrm{Im}\,\Omega_2 &= u^1(e^{14}+e^{23})\,, \\ -ie^{-2\Phi}2V\wedge\bar{V}\wedge\mathrm{Re}\,\Omega_2 &= v^1(e^{1256}+e^{3456})\,, \\ 2V &= C(ie^5-\tau e^6)\,, \end{aligned} \tag{12.3}$$

and one can easily check that the SU(2)-structure compatibility conditions (2.16) are automatically satisfied for all the fluctuations. Necessary conditions for metric positivity are $x > 0$, $k^1 > 0$ and $u^1 v^1 < 0$.

Note also that there are no vector fields arising in the spectrum. For instance, for the metric or the B-field, we would get a gauge field from the metric for every even/even one-form and a gauge field from the B-field for every odd/odd one-form under O5/O7. However, these one-forms do not appear after the orientifold projection. Similarly, one easily shows that there are no gauge fields arising from the RR-sector. The same applies for the other models in this section.

Next we come to the choice of background fluxes. As explained in section 3.3 we choose for this model the following background fluxes

$$\begin{aligned} \hat{H} &= 0\,, \\ \hat{F}_1 &= m_1 e^5 + m_2 e^6\,, \\ \hat{F}_3 &= f_3(e^{136}-e^{246})\,, \\ \hat{F}_5 &= f_5 e^{12345}\,, \end{aligned} \tag{12.4}$$

where $\hat{F}_1$ is the most general one-form which is odd/even under the O5/O7 orientifolds, $\hat{H} \in H^{3--}(M,\mathbb{R})$ (this fixes for this model δB in eq. (3.46) completely), $\hat{F}_3$, which is even/odd under O5/O7, is chosen up to exact forms and $\hat{F}_5 \in H^{5-+}(M,\mathbb{R})$.

Note that there is also a non-closed G-invariant three-form that is odd/odd under the O5/O7 planes, $e^{146}+e^{236}$. This means that whenever we turn on this $\hat{H}$ flux we automatically have NS5-branes, $\mathrm{d}\hat{H} = j_{\mathrm{NS5}} \neq 0$. However, in the following we will put this contribution to zero since we do not know if the expression we have given for the superpotential in eq. (3.38) takes the contribution from the NS5-brane properly into account.

The Bianchi identities for $\hat{F}_1$ and $\hat{F}_3$ read

$$\begin{aligned} \mathrm{d}\hat{F}_1 &= -j_{\mathrm{O7}} = -\frac{\sqrt{3}}{2} m_1 (e^{12} + e^{34}) \,, \\ \mathrm{d}\hat{F}_3 + \hat{F}_1 \wedge \hat{H} &= -j_{\mathrm{O5}} = \sqrt{3} f_3 (e^{1456} + e^{2346}) \,, \end{aligned} \tag{12.5}$$

as it is expected from our choice of the orientifolds. Plugging these expansions and the choice for the background fluxes in the expression for the superpotential (3.38), we arrive at

$$\mathcal{W}_{\mathrm{E}} = -\frac{iC}{4\kappa_{10}^2} V_s \left(f_5 \tau + 2 f_3 t^1 - m_1 (t^1)^2 \tau - i m_2 (t^1)^2 + 2\sqrt{3} t^1 z^1 \tau - \sqrt{3} w^1 \right) \,, \tag{12.6}$$

where we defined a standard volume $V_s = \int e^{123456}$. For the Kähler potential we obtain from (3.39)

$$\begin{aligned} \mathcal{K} = &- \ln \left((\tau + \bar{\tau})(t^1 + \bar{t}^1)^2 \right) - \ln \left(4 (z^1 + \bar{z}^1)^2 (w^1 + \bar{w}^1)^2 \right) \\ &+ 3 \ln (8 \kappa_{10}^2 M_P^2 V_s^{-1}) - \ln |C|^2 \,. \end{aligned} \tag{12.7}$$

We can eliminate the complex scalar C appearing in the superpotential and the Kähler potential by performing a Kähler transformation (3.31).

Let us now perform a T-duality on this solution along the direction 6. Following [102], T-duality acts on the RR-fields by adding/dropping the index we T-dualize on. From the choice of background fluxes in eq. (12.4) we infer that m_2 turns into the bubble parameter for F_0 (i.e. the Romans mass) on the type IIA side, m_1 and f_3 turn accordingly into bubble parameters for F_2 and f_5 into the parameter for F_6. Indeed, using T-duality on the level of the superpotential (see also eq. (8.8))

$$\mathcal{W}_{\mathrm{E,IIA}} \to \tau \mathcal{W}_{\mathrm{E,IIB}} \left(\tau \to \frac{1}{\tau} \right) \,, \tag{12.8}$$

we arrive at the T-dual type IIA superpotential

$$\mathcal{W}_{\mathrm{E,IIA}} = -\frac{i}{4\kappa_{10}^2} V_s \left(f_5 + 2 f_3 \tau t^1 - m_1 (t^1)^2 - i m_2 \tau (t^1)^2 + 2\sqrt{3} t^1 z^1 - \sqrt{3} \tau w^1 \right) \,. \tag{12.9}$$

With the identification $\tau \to t^2$, $f_5 \to f'$, $f_3 \to -n_1$, $m_1 \to -n_2$, $m_2 \to m$, $w^1 \to z^2$ and the choice $\lambda = 0$ (which reflects our simple choice of perpendicular orientifolds in this analysis) this is exactly the superpotential for the same coset on type IIA with strict SU(3)-structure that we obtained in eq. (11.44) with the expected scaling of the RR-fluxes with the moduli fields. This is also expected by looking at the structure constants of this model (see eq. (9.12)): since there is no structure constant with lower or upper index 6, we can T-dualize along 6 without changing the structure constants and we end up (for trivial $\hat{H}$) with the same model in type IIA. Thus, these two compactifications are related by T-duality, as was already suggested in [34] on the level of AdS$_4$ $\mathcal{N} = 1$ vacua.

12.2 Effective type IIB action on $\frac{\mathbf{SU(2)}^2}{\mathbf{U(1)}} \times \mathbf{U(1)}$

For this model we choose the following O5/O7 planes which are perpendicular to the coordinate frame and compatible with the structure constants (the choice is again unique up to relabeling of the coordinates):

	1	2	3	4	5	6
O5	$\otimes$				$\otimes$	
O5		$\otimes$		$\otimes$		
O7	$\otimes$	$\otimes$	$\otimes$			$\otimes$
O7			$\otimes$	$\otimes$	$\otimes$	$\otimes$

and we obtain the following basis of G-invariant forms transforming as indicated under the O5/O7 orientifold planes:

type under O5/O7	basis	name
odd/even 1-form	e^3, e^6	$Y_i^{(1-+)}$
even/odd 2-form	$e^{15} + e^{24}$	$Y^{(2+-)}$
odd/odd 2-form	$e^{25} - e^{14}$	$Y^{(2--)}$
odd/even 4-form	e^{1236} , e^{3456}	$Y_i^{(4-+)}$

This expansion basis gives rise to five complex moduli fields expanded as in eq. (3.42) with $\tau = x + iy$, $t^1 = k^1 - ib^1$, $z^1 = u^1 + ic^1$, $w^1 = v^1 + ih^1$ and $w^2 = v^2 + ih^2$. One can easily check that the SU(2)-structure compatibility conditions (2.16) are satisfied for all the fluctuations. Necessary conditions for metric positivity are $x > 0$, $k^1 > 0$, $v^1 v^2 > 0$ and $u^1 v^1 > 0$.

For this model, we choose according to the discussion in section 3.3 the background fluxes as follows

$$
\begin{aligned}
\hat{H} &= 0\,, \\
\hat{F}_1 &= m_1 e^3 + m_2 e^6\,, \\
\hat{F}_3 &= f_3(e^{256} - e^{146})\,, \\
\hat{F}_5 &= f_5 e^{12345}\,.
\end{aligned}
\tag{12.10}
$$

Let us mention that there is again room for an NS5-brane source since there is a non-closed G-invariant three-form, $e^{156}+e^{246}$, transforming exactly as the H-flux under the O5/O7 planes. However, we will put to zero this contribution.

The Bianchi identities for $\hat{F}_1$ and $\hat{F}_3$ read

$$
\begin{aligned}
\mathrm{d}\hat{F}_1 &= -j_{O7} = -m_1(e^{12} + e^{45}) \\
\mathrm{d}\hat{F}_3 + \hat{F}_1 \wedge \hat{H} &= -j_{O5} = f_3(e^{1356} + e^{2346})
\end{aligned}
\tag{12.11}
$$

as it is expected. For the superpotential, we derive from eq. (3.38) the expression

$$\mathcal{W}_{\mathrm{E}} = -\frac{iC}{4\kappa_{10}^2} V_s \left(f_5\tau + 2f_3t^1 - m_1(t^1)^2\tau - im_2(t^1)^2 - (w^1 + w^2) - 2t^1z^1\tau \right) , \quad (12.12)$$

where the standard volume is $V_s = \int e^{123456}$. For the Kähler potential we obtain from eq. (3.39)

$$\begin{aligned} \mathcal{K} = &- \ln\left((\tau + \bar{\tau})(t^1 + \bar{t}^1)^2\right) - \ln\left(4(z^1 + \bar{z}^1)^2(w^1 + \bar{w}^1)(w^2 + \bar{w}^2)\right) \\ &+ 3\ln(8\kappa_{10}^2 M_P^2 V_s^{-1}) - \ln|C|^2 . \end{aligned} \quad (12.13)$$

We can eliminate the complex scalar C by performing a Kähler transformation (3.31).

Let us now perform a T-duality along the 6 direction. The same considerations as in the previous section leads, using the T-duality rule (12.8), to the type IIA superpotential

$$\mathcal{W}_{\mathrm{E,IIA}} = -\frac{i}{4\kappa_{10}^2} V_s \left(f_5 + 2f_3\tau t^1 - m_1(t^1)^2 - im_2\tau(t^1)^2 - \tau(w^1 + w^2) - 2t^1z^1 \right) . \quad (12.14)$$

Under the identification $\tau \to t^1$, $t^1 \to t^2$, $w^1 \to z^1$, $w^2 \to z^3$, $z^1 \to z^2$ for the moduli fields and $f_5 \to f'$, $f_3 \to n_2$, $m_1 \to -n_1$ and $m_2 \to m$ for the bubble parameters, we obtain the same superpotential as we obtained for the same coset with strict SU(3)-structure in eq. (11.63). Hence, these two compactifications on the same coset are again related by a T-duality (which is also expected since the structure constants have no lower or upper 6 index, see eq. (9.19)). Note that we have an axionic flat direction in this model.

12.3 Effective type IIB action on SU(2) × SU(2)

For this model we choose the following perpendicular O5/O7-planes (again unique up to relabeling of the coordinates)

	1	2	3	4	5	6
O5	⊗			⊗		
O5		⊗			⊗	
O7	⊗	⊗	⊗			⊗
O7			⊗	⊗	⊗	⊗

such that we obtain the following basis of left-invariant forms transforming as indicated under the O5/O7 orientifolds

type under O5/O7	basis	name
odd/even 1-form	e^3, e^6	$Y_i^{(1-+)}$
even/odd 2-form	e^{14}, e^{25}	$Y_i^{(2+-)}$
odd/odd 2-form	e^{15}, e^{24}	$Y_i^{(2--)}$
odd/even 4-form	e^{1236}, e^{3456}	$Y^{(4-+)}$

We thus have 7 complex moduli fields which we expand as indicated in eq. (3.42). Again, one easily can verify that the compatibility conditions for the SU(2)-structure (2.16) are satisfied for all the fluctuations. Necessary conditions for metric positivity are $x > 0$, $k^1 > 0$, $k^2 > 0$ and $v^1 v^2 > 0$, $u^1 u^2 < 0$ and $u^2 v^2 < 0$.

The background fluxes are chosen according to our discussion in section 3.3 as follows

$$\begin{aligned} \hat{H} &= 0\,, \\ \hat{F}_1 &= m_1 e^3 + m_2 e^6\,, \\ \hat{F}_3 &= f_1(e^{135} + e^{246}) + f_2(e^{156} + e^{234})\,, \\ \hat{F}_5 &= 0\,. \end{aligned} \tag{12.15}$$

Note that there again exist non-closed invariant three-forms, $(e^{134} - e^{256})$ and $(e^{146} - e^{235})$, which transform the same way as $\hat{H}$ does under the orientifold involutions. Hence, we could have NS5-branes by turning on these fluxes. However, we will again put to zero these fluxes since the superpotential (3.38) may not be correct in the presence of NS5-branes.

The superpotential reads for this choice

$$\begin{aligned} \mathcal{W}_{\mathrm{E}} = -\frac{iC}{4\kappa_{10}^2} V_s \big(& f_1(t^1 + it^2\tau) + f_2(t^2 + it^1\tau) - m_1 t^1 t^2 \tau - i m_2 t^1 t^2 \\ & + it^1 z^2 - it^2 z^1 - t^1 z^1 \tau + t^2 z^2 \tau - i w^1 \tau - w^2 \big)\,, \end{aligned} \tag{12.16}$$

where we defined $V_s = \int e^{123456}$. The Kähler potential reads

$$\begin{aligned} \mathcal{K} = & -\ln\big((\tau + \bar{\tau})(t^1 + \bar{t}^1)(t^2 + \bar{t}^2)\big) - \ln\big(-4(z^1 + \bar{z}^1)(z^2 + \bar{z}^2)(w^1 + \bar{w}^1)(w^2 + \bar{w}^2)\big) \\ & + 3\ln(8\kappa_{10}^2 M_P^2 V_s^{-1}) - \ln|C|^2\,. \end{aligned} \tag{12.17}$$

Again, we can eliminate the complex scalar C by performing a Kähler transformation (3.31).

This superpotential is not T-dual to a type IIA model with *geometric fluxes* only. This can for example be seen by performing a T-duality along the 6 direction as in the previous sections, ending up with a type IIA superpotential

$$\begin{aligned} \mathcal{W}_{\mathrm{E,IIA}} = -\frac{i}{4\kappa_{10}^2} V_s \big(& i(f_1 t^2 + f_2 t^1) + f_1 t^1 \tau + f_2 t^2 \tau - m_1 t^1 t^2 - i m_2 t^1 t^2 \tau \\ & - t^1 z^1 + t^2 z^2 - i w^1 - w^2 \tau + i t^1 z^2 \tau - i t^2 z^1 \tau \big)\,, \end{aligned} \tag{12.18}$$

where the terms in the first line come from F_4, F_2 and F_0 fluxes, respectively. The first four terms in the second line come from geometric fluxes but the last two terms are non-geometric Q-fluxes (note the combination of two Kähler moduli and one complex structure moduli in those terms). Such non-geometric fluxes will also arise for a T-duality along any other direction than 6, since the structure constants have all six directions as lower and upper indices. Thus, a T-duality, which acts on geometric fluxes by raising/lowering the index we T-dualize on [118] (for a review on non-geometrical backgrounds see, e.g., [119])

$$H_{ijk} \xrightarrow{\mathrm{T}_i} f^i_{jk} \xrightarrow{\mathrm{T}_j} Q^{ij}{}_k\,, \tag{12.19}$$

results in a type IIA background with non-geometric fluxes Q. Hence, this is in fact a new model we did not study so far on the type IIA side.

12.4 Effective type IIB action on $\mathbf{SU(2)\times U(1)^3}$

The analysis of this model is quite similar to the analysis of the model $\mathrm{SU}(2)\times\mathrm{SU}(2)$, as one only turns off the structure constant $f^4{}_{56}=0$. Therefore, one can choose without loss of generality the same O5/O7-planes as in section 12.3 and the same expansion forms. The only difference is in the choice of background fluxes, since the cohomology changes, and we choose

$$\begin{aligned}
\hat{H} &= 0\,,\\
\hat{F}_1 &= m_1 e^3 + m_2 e^6\,,\\
\hat{F}_3 &= f_1 e^{156} + f_2 e^{246}\,,\\
\hat{F}_5 &= f_5 e^{12345}\,,
\end{aligned} \tag{12.20}$$

such that the superpotential reads

$$\mathcal{W}_{\mathrm{E}} = -\frac{iC}{4\kappa_{10}^2} V_s \left(f_5\tau + f_1 t^2 + f_2 t^1 - m_1 t^1 t^2 \tau - i m_2 t^1 t^2 - w^2 - t^1 z^1 \tau + t^2 z^2 \tau\right)\,, \tag{12.21}$$

and the Kähler potential is as in eq. (12.17). Note that there is an axionic flat direction since the superpotential does not depend on w^1.

Again, it is not difficult to find the identifications to show that this model is T-dual (along the direction 6) to the model on the same coset in type IIA with strict SU(3)-structure, see eq. (11.69).

Let us briefly summarize the result of this chapter. By turning on O5/O7 orientifold planes in order to satisfy the compatibility conditions (2.16), we computed the type IIB effective theory for all coset models that allow for a static SU(2)-structure (these are the last four entries in table 4.1). However, we noticed that for all except one of these models there is a T-duality relating the model to a type IIA strict SU(3)-structure compactification that we already analyzed in chapter 11. One model, however, is T-dual to a type IIA strict SU(3)-structure compactification with *non-geometric* fluxes and may thus be interesting for the phenomenological applications we study in the next chapter.

Chapter 13

On the cosmology of the coset models

We discussed in chapter 5 that an epoch of cosmic inflation in the early universe is the dominant lore to explain the fascinating data of recent astronomical observations, for instance the flatness and homogeneity of our universe. The inflationary phase took place even before the phase of the radiation dominated universe, as the universe had temperatures of at least 10 billions degrees. At extremely high energies quantum effects of gravity are expected to become important. String theory is believed to be a promising candidate to describe this physics appropriately - and as such should be able to realize inflation. We have seen that inflation can be driven by a scalar field, and the moduli fields of string theory provide us with natural candidates for an inflaton. Sufficient conditions to realize inflation within string theory are the so-called slow-roll conditions on the potential of the moduli fields. We reviewed these conditions in section 5.1.2.

Another important cosmological observation is that at present the universe is in a state of accelerated expansion. We thus want to look for a string theory vacuum with small positive cosmological constant, i.e. a de Sitter solution.

Of course we are now interested in the question, whether the models we consider in this thesis, for which we explicitly constructed the four-dimensional effective potential, are interesting candidates for inflation scenarios or have de Sitter solutions with small positive cosmological constant. As we mentioned already in the introduction, this would render these type IIA models extremely interesting, since type IIA orientifolds with intersecting D6-branes offer good prospects for deriving standard model-like sectors from strings.

However, the main problem to realize inflation or de Sitter vacua in the classical regime in type IIA is that there exist quite strong no-go theorems against slow-roll inflation and de Sitter vacua. These no-go theorems were discussed in section 5.2 and 5.3 and focus in particular on the role played by the curvature of the internal manifold. Let us briefly summarize the necessary conditions to *avoid* these no-go theorems:

$$V_f > 0 \,, \quad \text{or equivalently} \,, \quad R < 0 \,, \tag{13.1a}$$

$$m \neq 0 \,, \tag{13.1b}$$

$$DU \equiv -\sigma \frac{\partial U}{\partial \sigma} = -k^a \frac{\partial U}{\partial k^a} < 0 \,, \tag{13.1c}$$

where V_f is the contribution of the geometric fluxes to the scalar potential, R is the scalar curvature of the internal manifold (the expression for R is given in eq. (4.28)), m is the Roman mass and the expression for σ and the function U are given in eqs. (5.53) and (5.66), respectively. We further mentioned in section 5.2 that to avoid a runaway in τ-direction we need $V_{\mathrm{O6/D6}} < 0$.

We are always free to turn on a non-vanishing Romans mass m, such that condition (13.1b) is easy to satisfy. From the definition of U in eq. (5.66) and eq. (5.45), we get the relation between the scalar curvature and U

$$R \propto -\frac{U}{\mathrm{Vol}} \,, \tag{13.2}$$

such that the first condition in (13.1a) translates into the requirement that U is positive. It suffices therefore to derive for all coset models the function U. If U is negative for all values in the moduli space, the no-go theorem of section 5.2 applies, implying the bound on the slow-roll parameter $\epsilon \geq 27/13$, thus ruling out slow-roll inflation and de Sitter vacua. If it turns out that U can be positive for some region in the moduli space, we check the third condition (13.1c). If it turns out that $DU \geq 0$ the no-go theorem of section 5.3 applies and slow-roll inflation and de Sitter vacua are excluded for the corresponding model, since $\epsilon \geq 2$.

In the following we derive for each coset model of chapter 11 the scalar curvature R with eq. (4.28) and the metric g_{ij} induced by J and Ω. From eq. (5.66) we then calculate the function U.

In chapter 12 we identified type IIB static SU(2)-structure compactifications on the coset models. However, we showed for all but one model that there is a T-duality relating these models to type IIA strict SU(3)-structure compactifications which we already analyzed in chapter 11. Hence, for these models nothing new is expected. However, there is one type IIB model with a T-dual on type IIA involving non-geometric fluxes. Thus, the no-go theorems do not apply and the model could be interesting for inflation or de Sitter vacua.

13.1 Type IIA coset compactifications with a no-go theorem

In this section, we go through the list of coset models that admit a strict SU(3)-structure (see chapter 11). Unfortunately, as we will see in the following, we have to exclude all but one of the coset models for slow-roll inflation as well as de Sitter vacua, since we can apply one of the no-go theorems of chapter 5.

$\frac{\mathbf{G_2}}{\mathbf{SU(3)}}$

For this model we find for the function U (see eq. (5.66)):

$$U \propto -(k^1)^2 \,, \tag{13.3}$$

which is manifestly negative. This implies that V_f itself is manifestly negative so that the no-go theorem of [91], which we reviewed in section 5.2, already rules out this coset model [49].

$\frac{\mathbf{Sp(2)}}{\mathbf{S(U(2)\times U(1))}}$

For this coset model we calculate for the function U the following

$$U \propto (k^2)^2 - 4(k^1)^2 - 12k^1k^2 \,, \tag{13.4}$$

which is not negative on the whole moduli space (as one can see by choosing k^2 small and k^1 large). The no-go theorem (5.48) is thus not applicable and we therefore perform a more careful analysis using the refined no-go theorem of section 5.3. The only non-vanishing intersection number is κ_{112} and permutations thereof, so that k^2 plays the role of k^0, and we have

$$DU = -k^1\partial_{k^1}U \propto 8(k^1)^2 + 12k^1k^2 > 0 \,, \tag{13.5}$$

so that with $k^i > 0$ (because of metric positivity) the inequality (5.68) is strictly satisfied and this model is ruled out.

$\frac{\mathbf{SU(3)}}{\mathbf{U(1)\times U(1)}}$

For this coset space, we obtain

$$U \propto (k^1)^2 + (k^2)^2 + (k^3)^2 - 6k^1k^2 - 6k^2k^3 - 6k^1k^3 \,, \tag{13.6}$$

which can be positive for some values of k^a. The non-vanishing intersection numbers are of the type κ_{123} so that we can choose any one of the three k's as k^0. We will choose k^0 to be the biggest and assume without loss of generality that this is k^1, i.e. that $k^1 \geq k^2, k^3$. We then find that

$$DU = (-k^2\partial_{k^2} - k^3\partial_{k^3})U \propto (6k^1 - 2k^2)k^2 + (6k^1 - 2k^3)k^3 + 12k^2k^3 > 0, \tag{13.7}$$

so that with $k^i > 0$ (because of metric positivity) this coset space is also ruled out by the no-go theorem (5.68).

$\frac{\mathbf{SU(3)\times U(1)}}{\mathbf{SU(2)}}$

For this model, the function U depends on an extra constant λ related to the choice of orientifolds, see section 11.4. The function U turns out to be

$$U \propto (k^2)^2(u^2)^2 - 8k^1k^2|u^1u^2|(1+\lambda^2)\,, \tag{13.8}$$

and the non-vanishing intersection numbers are of the form κ_{112}. Thus k^2 plays the role of k^0, and we find that

$$DU = -k^1\partial_{k^1}U \propto 8k^1k^2|u^1u^2|(1+\lambda^2) > 0, \tag{13.9}$$

so that with $k^i > 0$ (because of metric positivity) this case is also ruled out.

$\frac{\mathbf{SU(2)}^2}{\mathbf{U(1)}} \times \mathbf{U(1)}$

The function U becomes for this coset model

$$U \propto \frac{-4k^1k^2u^2(u^1+u^3) + (k^2)^2\left[(u^1)^2+(u^3)^2\right]}{2\sqrt{u^1u^3}|u^2|}\,. \tag{13.10}$$

which can be positive for certain values of the Kähler moduli. The non-vanishing intersection number is κ_{112} so that k^2 plays the role of k^0, and we get for (5.68):

$$DU = -k^1\partial_{k^1}U \propto \frac{2k^1k^2u^2(u^1+u^3)}{\sqrt{u^1u^3}|u^2|} > 0\,, \tag{13.11}$$

which is positive using the conditions (11.59). Hence, this case is ruled out as well.

$\mathbf{SU(2)\times U(1)}^3$

For the quantity U we get this time

$$U \propto \frac{(k^1u^4)^2 + (k^2u^3)^2 + (k^3u^2)^2 - 2k^1u^4k^2u^3 - 2k^1u^4k^3u^2 - 2k^2u^3k^3u^2}{2\sqrt{u^1u^2u^3u^4}}\,, \tag{13.12}$$

which can be positive. The non-vanishing intersection number is κ_{123} so that each k^i can play the role of k^0. Without loss of generality we can assume $k^1u^4 \geq k^2u^3 > 0$, $k^1u^4 \geq k^3u^2 > 0$ and choose k^0 to be k^1. Thus we then find

$$DU = (-k^2\partial_{k^2} - k^3\partial_{k^3})U \propto \frac{-(k^2u^3-k^3u^2)^2 + k^1u^4(k^2u^3+k^3u^2)}{\sqrt{u^1u^2u^3u^4}} > 0\,, \tag{13.13}$$

so that we can also rule out this model.

SU(2) × SU(2)

Thus far, we have found that $\epsilon \geq 2$ for all other cases. For the remaining coset space SU(2) × SU(2), one finds

$$\begin{aligned} U \propto & \sum_{i=1}^{3}(k^i)^2 \left(\sum_{I=1}^{4}(u^I)^2\right) - 4k^2k^3(|u^1u^2| + |u^3u^4|) \\ & - 4k^1k^2(|u^1u^4| + |u^2u^3|) - 4k^1k^3 \left(|u^1u^3| + |u^2u^4|\right) , \end{aligned} \tag{13.14}$$

and the non-vanishing intersection numbers are of the form κ_{123} so that we could choose any one of the k's as k^0. However, it is not possible to apply the no-go theorem. This can be easily seen if we take for example $u^1 \gg u^2, u^3, u^4$. Then we have schematically $U \propto \vec{k}^2(u^1)^2$ and $DU \propto -k^a k^a (u^1)^2 < 0$. In [92] further no-go theorems have been derived but none of those apply to this case either. We therefore study this coset space in more detail in section 13.2.

To summarize, by means of the classical no-go theorems of chapter 5, we could rule out all but one coset model to allow for inflation or de Sitter vacua. To be precise, the lower bound on $\epsilon \geq 2$ implies that there are, for $V > 0$, directions in the field space that are too steep to realize inflation or a de Sitter minimum. Further, as we discussed in section 5.3.1, for the models in this section, the following additional ingredients cannot be added: NS5-, D4- and D8-branes, since there are no corresponding currents with the appropriate properties under all orientifold involutions. Also, an F-term uplift along the lines of O'KKLT [100, 101] does not work.

Note that we can not be sure that there are no other light Kaluza-Klein modes joining the light fields based on the left-invariant expansion ansatz, since a separation of scales turned out to be difficult. However, as we already mentioned, a truncation to the set of left-invariant forms is believed to provide a consistent truncation [115, 60, 116] and that there is no coupling between the set of preserved left-invariant fields and the truncated non-invariant fields. Hence, even if light fields from the Kaluza-Klein spectrum would eventually join the truncated effective theory, inflation and de Sitter vacua would still be excluded by the no-go theorems, since there are already in the truncated theory directions, that are too steep to allow for inflation and de Sitter vacua.

13.2 Numerical analysis for the SU(2) × SU(2) compactification on type IIA and on type IIB

In section 11.5 we derived the type IIA strict SU(3)-structure superpotential and Kähler potential for a compactification on this coset space. By means of eq. (3.25) it is straightforward to calculate the scalar potential and the slow-roll parameter ϵ as in eq. (5.31). However, the expression for ϵ is quite complicated so that we cannot minimize it analytically. On the other hand, we can minimize it numerically and it turns out that one indeed finds solutions with numerically vanishing ϵ (and we can conclude that in this case there is no undiscovered no-go theorem against small ϵ). For instance, such a solution is given by

$$\begin{aligned}
&m^1 = m^2 = m^3 = L\,, \qquad m = 2\,L^{-1}\,, \qquad p = 3\,L^2\,, \\
&k^1 = k^2 = k^3 \approx .8974\,L^2\,, \qquad b^1 = b^2 = b^3 \approx -.8167\,L^2\,, \\
&u^1 \approx 2.496\,L^3, \qquad u^2 = -u^3 = u^4 \approx -.05667\,L^3\,, \\
&c^1 \approx -2.574\,L^3\,, \qquad c^2 = -c^3 = c^4 \approx .3935\,L^3\,,
\end{aligned} \tag{13.15}$$

where L is an arbitrary length. While we can use L to scale up our solution with respect to the string length l_s, we stress that this does not correspond to a massless modulus, as it also changes the fluxes.

To obtain a trustworthy supergravity solution we would have to make sure that the internal space is large compared to the string length and that the string coupling is small (for which we could use our freedom in L). Furthermore, in the full string theory the fluxes have to be properly quantized. Although it is unlikely that this would prevent small ϵ, we will not try to find such a solution, because all the solutions with vanishing ϵ we found have a more serious problem, namely that $\eta \lesssim -2.4$. The eigenvalues of the mass matrix turn out to be generically all positive except for one, with the one tachyonic direction being a mixture of all the light fields, in particular the axions. This means that we have a saddle point rather than a de Sitter minimum. A similar instability was found in related models in [92].

In [120], a no-go theorem preventing de Sitter vacua and slow-roll inflation of general four-dimensional supergravity theories was derived by studying the eigenvalues of the mass matrix. Allowing for an arbitrary tuning of the superpotential it was shown that for certain Kähler potentials the Goldstino mass is always negative. For the examples we found, this mass is always positive so that the no-go theorem of [120] does not apply. This means that allowing for an arbitrary superpotential it should be possible to remove the tachyonic direction. In our case, however, the superpotential is of course not arbitrary.

Since the no-go theorems against slow-roll inflation do not apply and we have found solutions with vanishing ϵ, we checked whether our solutions allow for small η in the vicinity of the de Sitter extrema. Unfortunately, this is not the case. In fact, we found that η does not change much in the vicinity of our solutions where ϵ is still small. However, let us stress that our numerical search is possibly not exhaustive and we cannot completely rule out the existence of de Sitter vacua or inflating regions for this case.

On the other hand, on the same coset a type IIB static SU(2)-structure compactification is possible. In section 12.3 we derived the explicit superpotential and Kähler potential for this compactification. Further we showed that the type IIB compactification is not T-dual to a type IIA compactification with *geometric fluxes* only. Hence, it is not possible to apply the no-go theorems of section 5.2 and 5.3, and the model may still be interesting for phenomenological applications.

Again it is straightforward to derive the scalar potential from eq. (3.25) and the slow-roll parameter ϵ. Although we cannot analytically minimize ϵ, we will again do it numerically. However, this time the numerical analysis seems to give a lower bound for epsilon: $\epsilon \gtrsim 9/7$. This numerical analysis strongly suggest the existence of a so far undiscovered no-go theorem for type IIB compactifications (or, from the T-dual type IIA perspective, also in the presence of non-geometric fluxes) and it would be very interesting to further explore this possibility.

Chapter 14

Conclusions

In this thesis we analyzed a large number of type IIA strict SU(3)-structure compactifications with fluxes and O6/D6-sources, as well as type IIB static SU(2)-structure compactifications with fluxes and O5/O7-sources. Restricting to structures and fluxes that are constant in the basis of left-invariant one-forms, these models are tractable enough to allow for an explicit derivation of the four-dimensional low-energy effective theory.

The six-dimensional compact manifolds we studied in this thesis are nilmanifolds based on nilpotent Lie-algebras, and, on the other hand, coset spaces based on semi-simple and U(1)-groups, which admit a left-invariant strict SU(3)- or static SU(2)-structure. In particular, from the set of 34 distinct nilmanifolds we identified two nilmanifolds, the torus and the Iwasawa manifold, that allow for an AdS_4, $\mathcal{N}=1$ type IIA strict SU(3)-structure solution and one nilmanifold allowing for an AdS_4, $\mathcal{N}=1$ type IIB static SU(2)-structure solution. From the set of all the possible six-dimensional coset spaces given in table 4.1, we identified seven coset spaces suitable for strict SU(3)-structure compactifications, four of which also allow for a static SU(2)-structure compactification. For all these models, we calculated the four-dimensional low-energy effective theory using $\mathcal{N}=1$ supergravity techniques. In order to write down the most general four-dimensional effective action, we also studied how to classify the different disconnected "bubbles" in moduli space.

Some of the coset spaces allow for four-dimensional (massive) type IIA $\mathcal{N}=1$ AdS_4 solutions. For these coset models and the three nilmanifold models, we calculated the mass spectrum of the moduli fields around the supersymmetric solution. For the nilmanifold examples we have found that there are always three unstabilized moduli corresponding to axions in the RR-sector. On the other hand, the $\mathcal{N}=1$ solutions on the coset models, except for $\text{SU}(2)\times\text{SU}(2)$, have all moduli stabilized. For the torus and the Iwasawa solution, we also performed an explicit Kaluza-Klein reduction, which led to the same result as the analysis with supergravity techniques, supporting the validity of the effective supergravity approach also in the presence of geometric fluxes. Furthermore, we have demonstrated that this superpotential and Kähler potential lead to sensible results in type IIB string theory with static SU(2)-structure.

The necessary and sufficient conditions for $\mathcal{N} = 1$ compactifications of type IIA supergravity to AdS$_4$ with the strict SU(3)-structure ansatz force, for non-vanishing Romans mass, the warp factor and the dilaton to be constant. On the other hand, provided that we set the Romans mass to zero, nothing prevents the warp factor and the dilaton to be non-constant. We analyzed the necessary and sufficient conditions for an AdS$_4$ $\mathcal{N} = 1$ compactification of this type in section 2.3. However, to find explicit solutions of this type turns out to be difficult. One reason is that one has to leave the convenient notion of left-invariant forms that drops the explicit coordinate dependence. In addition, as we inferred from the analysis in section 10.6, where we turned on a small non-constant deformation for the warp factor, a non-constant warp factor seems in general to require the presence of localized sources.

Two of the coset models of table 4.1 do admit a strict SU(3)-structure, but no type IIA $\mathcal{N} = 1$ AdS$_4$ vacuum. Choosing for simplicity the O-planes such that the one- and five-forms are projected out and restricting to O-planes that are perpendicular to the coordinate frame, we could compute the four-dimensional low-energy effective action. In the same spirit, including appropriate O5/O7-planes, we computed the effective action for the four type IIB static SU(2)-structure compactifications on coset spaces. However, for three of these type IIB models we found a T-duality relating them to type IIA models with strict SU(3)-structure that we already studied. On the other hand, one model is new, since it is T-dual to a type IIA model with non-geometric fluxes.

Once the effective potential is known, one can study many interesting questions. For instance, we discussed for some models how to identify the bubbles in moduli space that contain one or more $\mathcal{N} = 1$ AdS$_4$ solutions. Ultimately, we would like to uplift the AdS$_4$ solutions to a de Sitter space-time with a small, positive cosmological constant. This might be accomplished by incorporating a suitable additional uplifting term in the potential along the lines of, e.g, [23]. Although a negative mass squared for a light field in AdS$_4$ does not necessarily signal an instability, after the uplift all fields should have positive mass squared. Unless the uplifting potential can change the sign of the squared masses, it is thus desirable that they are all positive even before the uplifting. We found that this can be arranged for the coset models $\frac{\mathrm{G}_2}{\mathrm{SU}(3)}$, $\frac{\mathrm{Sp}(2)}{\mathrm{S}(\mathrm{U}(2)\times\mathrm{U}(1))}$ and $\frac{\mathrm{SU}(3)}{\mathrm{U}(1)\times\mathrm{U}(1)}$ for suitable values of the orientifold charge.

An alternative approach towards obtaining meta-stable de Sitter vacua could also be to search for non-trivial de Sitter minima in the original flux potential away from the AdS$_4$ vacuum. This approach is also appropriate for the models without an $\mathcal{N} = 1$ AdS$_4$ solution. However, there exist strong no-go theorems against slow-roll inflation and de Sitter minima in type IIA string theory at tree level. We discussed the necessary conditions to circumvent these no-go theorems. For instance, the dilaton-volume dependence in type IIA SU(3)-structure compactifications forbids de Sitter vacua or slow-roll inflation unless the compact space has negative scalar curvature induced by the geometric fluxes (or other more complex ingredients are introduced). Regions in moduli space with negative scalar curvature are indeed possible for most of the coset models we studied. To study these models further we adapted a refined no-go theorem [92] and identified a geometrical criterion that allows one to separate interesting SU(3)-structure compactifications from non-realistic ones.

As a matter of fact, after this analysis, only two of the coset models are not directly ruled out by any known no-go theorem and remain interesting candidates to realize slow-roll inflation or stable de Sitter minima (without the inclusion of other ingredients). These are the type IIA strict SU(3)- and type IIB static SU(2)-structure compactifications on the model $\mathrm{SU}(2) \times \mathrm{SU}(2)$. For the former compactification, a numerical analysis indeed reveals critical points (corresponding to numerically vanishing ϵ) with positive energy density, but only at the price of a tachyonic direction, corresponding to a large negative eta-parameter, $\eta \lesssim -2.4$. Interestingly, this tachyonic direction does not correspond to the one used in the different types of no-go theorems of [120]. As our numerical search is possibly not exhaustive, we cannot completely rule out the existence of de Sitter vacua or inflating regions for this case. One may try to rule out this case by means of another no-go theorem, perhaps by using methods similar in spirit to [120], although a direct application of their results to this case does not seem possible.

On the other hand, the numerical analysis for the type IIB static SU(2)-structure compactification reveals a lower bound on the first slow-roll parameter, $\epsilon \gtrsim 9/7$, which strongly suggest the existence of a so far undiscovered no-go theorem for type IIB compactifications (or, from the T-dual type IIA perspective, also in the presence of non-geometric fluxes). To extend our study in this direction would be very interesting. Following [98, 99] or [121, 122, 123], one could also try to incorporate additional structures such as NS5-branes or quantum corrections of various types. In section 5.3.1, however, we found that at least for our type IIA models, the following additional ingredients cannot be added or do not work: NS5-, D4- and D8-branes as well as an F-term uplift along the lines of O'KKLT [100, 101]. Perhaps also methods similar to the ones in [124] for non-supersymmetric Minkowski or AdS_4 vacua might be useful for the direct ten-dimensional construction of de Sitter compactifications.

Part IV

Appendix and Bibliography

Appendix A

Conventions

We define an l-form as

$$A = \frac{1}{l!} A_{\mu_1 \dots \mu_l} \mathrm{d}x^{\mu_1} \wedge \cdots \wedge \mathrm{d}x^{\mu_l} , \tag{A.1}$$

and the exterior product of a p-form A and a q-form B as

$$(A \wedge B)_{\mu_1 \dots \mu_{p+q}} = \frac{(p+q)!}{p!q!} A_{[\mu_1 \dots \mu_p} B_{\mu_{p+1} \dots \mu_{p+q}]} , \tag{A.2}$$

where the antisymmetrization is understood with factors,

$$A_{[\mu_1 \dots \mu_l]} = \frac{1}{l!} (A_{\mu_1 \dots \mu_l} + \text{antisymmetric permutations}) . \tag{A.3}$$

The exterior derivative is $\mathrm{d} = \mathrm{d}x^\mu \partial_\mu$ and given by

$$\mathrm{d}A_{\mu_1 \dots \mu_{l+1}} = (l+1) \partial_{[\mu_1} A_{\mu_2 \dots \mu_{l+1}]} . \tag{A.4}$$

The contraction of an l-form A with with a vector $v = v^i \frac{\partial}{\partial x^i}$ is defined by

$$\iota_v A = \frac{1}{(l-1)!} v^j A_{j \mu_2 \dots \mu_l} \mathrm{d}x^{\mu_2} \wedge \cdots \wedge \mathrm{d}x^{\mu_l} . \tag{A.5}$$

The operator α acts on forms by reversing the order of their indices, i.e.,

$$\alpha(A) = \frac{1}{l!} A_{\mu_1 \dots \mu_l} \mathrm{d}x^{\mu_l} \wedge \cdots \wedge \mathrm{d}x^{\mu_1} . \tag{A.6}$$

Note that this results for an l-form in

$$\alpha(A) = (-1)^{\frac{l(l-1)}{2}} A . \tag{A.7}$$

The Hodge dual tensor of an l-form A and a given metric g is given by

$$(\star A)_{\mu_1 \dots \mu_{D-l}} = \frac{1}{l!} \sqrt{|g|} \epsilon_{\mu_1 \dots \mu_{D-l} \nu_1 \dots \nu_l} g^{\nu_1 \rho_1} \cdots g^{\nu_l \rho_l} A_{\rho_1 \dots \rho_l} , \tag{A.8}$$

where ϵ is the totally antisymmetric tensor such that $\epsilon_{01\ldots D} = 1$. With this definition we obtain

$$\star \star A = (-1)^{l(D-l)} \mathrm{sign}(g) A \,. \tag{A.9}$$

It follows for the kinetic terms for the RR-fields

$$-\frac{1}{2} \int \mathrm{d}^D x \sqrt{|g|} \frac{1}{n!} F_{\mu_1 \ldots \mu_n} F^{\mu_1 \ldots \mu_n} = -\frac{(-1)^{n(D-n)}}{2} \int F_n \wedge \star F_n \,. \tag{A.10}$$

The volume form is defined as

$$\star 1 = \mathrm{vol} \,. \tag{A.11}$$

We often use the $10 \to 4+6$ split which induces forms of the type $\alpha_p \wedge \beta_q$, where α_p is an external p-form and β_q an internal q-form. We can use

$$\star_{10} (\alpha_p \wedge \beta_q) = (-1)^{pq} \star_4 \alpha_p \wedge \star_6 \beta_q \,, \tag{A.12}$$

which implies the useful relations

$$\begin{aligned} \star_{10} \beta = \mathrm{vol}_4 \wedge \star_6 \beta \,, \qquad \star_{10} (\mathrm{vol}_4 \wedge \beta) = - \star_6 \beta \,, \\ \star_{10} \mathrm{vol}_4 = -\mathrm{vol}_6 \,, \qquad \star_{10} \mathrm{vol}_6 = \mathrm{vol}_4 \,. \end{aligned} \tag{A.13}$$

We define an inner product on forms as follows

$$(\alpha, \beta) = (-1)^{l(D-l)} \int \alpha \wedge \star \beta \,, \tag{A.14}$$

where l is the dimension of both α and β. Further we define the adjoint $\mathrm{d}^\dagger$ of the exterior derivative as follows

$$(\mathrm{d}\alpha, \beta) = (\alpha, \mathrm{d}^\dagger \beta) \,. \tag{A.15}$$

We find using eq. (A.14)

$$\mathrm{d}^\dagger = \left\{ \begin{array}{ccc} \mathrm{sign}(g) \star \mathrm{d} \star & \text{for} \quad D & \text{even} \\ (-1)^{l+1} \mathrm{sign}(g) \star \mathrm{d} \star & \text{for} \quad D & \text{odd} \end{array} \right. . \tag{A.16}$$

The Laplacian is defined as follows

$$\Delta = \mathrm{d}^\dagger \mathrm{d} + \mathrm{d}\mathrm{d}^\dagger \,. \tag{A.17}$$

For the contraction of a (poly-)form with gamma matrices we introduce the following notation

$$\underline{A} = \not{A} = \sum_l \frac{1}{l!} A_{\mu_1 \ldots \mu_l} \gamma^{\mu_1 \ldots \mu_l} \,, \tag{A.18}$$

where we use the underline if the slash makes the expression unreadable.

Appendix B

Type IIA supergravity

The bosonic content of type II supergravity consists of a metric g, a dilaton Φ, an NSNS three-form H and RR-fields F_n. In the democratic formalism of [44], where the number of RR-fields is doubled, n runs over $0, 2, 4, 6, 8, 10$ in IIA and over $1, 3, 5, 7, 9$ in type IIB. We write n to denote the dimension of the RR-fields; for example $(-1)^n$ stands for $+1$ in type IIA and -1 in type IIB. After deriving the equations of motion from the action, the redundant RR-fields are to be removed by hand by means of the duality condition:

$$F_n = (-1)^{\frac{(n-1)(n-2)}{2}} e^{\frac{n-5}{2}\Phi} \star_{10} F_{(10-n)} \;, \tag{B.1}$$

given here in the Einstein frame. We will often collectively denote the RR-fields, and the corresponding potentials, with polyforms $F = \sum_n F_n$ and $C = \sum_n C_{(n-1)}$, so that: $F = \mathrm{d}_H C$.

In the Einstein frame, the bosonic part of the bulk action reads:

$$S_{\text{bulk}} = \frac{1}{2\kappa_{10}^2} \int \mathrm{d}^{10}x \sqrt{-g} \left[R - \frac{1}{2}(\partial\Phi)^2 - \frac{1}{2} e^{-\Phi} H^2 - \frac{1}{4} \sum_n e^{\frac{5-n}{2}\Phi} F_n^2 \right] , \tag{B.2}$$

where for an l-form A we define

$$A^2 = A \cdot A = \frac{1}{l!} A_{M_1 \ldots M_l} A_{N_1 \ldots N_l} g^{M_1 N_1} \cdots g^{M_l N_l} \,. \tag{B.3}$$

Since (B.1) needs to be imposed by hand this is strictly-speaking only a pseudoaction. Note that the doubling of the RR-fields leads to factors of $1/4$ in their kinetic terms.

The contribution from the calibrated (supersymmetric) sources can be written as:

$$S_{\text{source}} = \int \langle C, j \rangle - \sum_n e^{\frac{n}{4}\Phi} \int \langle \Psi_n, j \rangle \,, \tag{B.4}$$

with

$$\Psi_n = e^A \mathrm{d}t \wedge \frac{e^{-\Phi}}{(n-1)! \hat{\epsilon}_1{}^T \epsilon_1} \hat{\epsilon}_1{}^T \gamma_{M_1 \ldots M_{n-1}} \hat{\epsilon}_2 \, \mathrm{d}X^{M_1} \wedge \ldots \wedge \mathrm{d}X^{M_{n-1}} \,, \tag{B.5}$$

with $\hat{\epsilon}_{1,2}$ nine-dimensional internal supersymmetry generators. For space-filling sources in compactifications to AdS_4 this becomes [125]

$$\Psi_n = \mathrm{vol}_4 \wedge e^{4A-\Phi}\mathrm{Im}\,\Psi_{1\mathrm{E}}\big|_{n-4}\,, \tag{B.6}$$

with $\Psi_{1\mathrm{E}}$ the pure spinor Ψ_1 in the Einstein frame.

The dilaton equation of motion and the Einstein equation read

$$0 = \nabla^2\Phi + \frac{1}{2}e^{-\Phi}H^2 - \frac{1}{8}\sum_n (5-n)e^{\frac{5-n}{2}\Phi}F_n^2 + \frac{\kappa_{10}^2}{2}\sum_n (n-4)e^{\frac{n}{4}\Phi}\star\langle\Psi_n, j\rangle\,, \tag{B.7a}$$

$$\begin{aligned} 0 = {} & R_{MN} + g_{MN}\left(\frac{1}{8}e^{-\Phi}H^2 + \frac{1}{32}\sum_n (n-1)e^{\frac{5-n}{2}\Phi}F_n^2\right) \\ & - \frac{1}{2}\partial_M\Phi\partial_N\Phi - \frac{1}{2}e^{-\Phi}H_M\cdot H_N - \frac{1}{4}\sum_n e^{\frac{5-n}{2}\Phi}F_{n\,M}\cdot F_{n\,N} \\ & - 2\kappa_{10}^2\sum_n e^{\frac{n}{4}\Phi}\star\langle\left(-\frac{1}{16}n g_{MN} + \frac{1}{2}g_{P(M}dx^P\otimes\iota_{N)}\right)\Psi_n, j\rangle\,, \end{aligned} \tag{B.7b}$$

where we defined for an l-form A

$$A_M\cdot A_N = \frac{1}{(l-1)!}A_{MM_2\ldots M_l}A_{NN_2\ldots N_l}g^{M_2N_2}\cdots g^{M_lN_l}\,. \tag{B.8}$$

The Bianchi identities and the equations of motion for the RR-fields, including the contribution from the 'Chern-Simons' terms of the sources, take the form

$$0 = \mathrm{d}F + H\wedge F + 2\kappa_{10}^2\,j\,, \tag{B.9a}$$

$$0 = \mathrm{d}\left(e^{\frac{5-n}{2}\Phi}\star F_n\right) - e^{\frac{3-n}{2}\Phi}H\wedge\star F_{(n+2)} - 2\kappa_{10}^2\,\alpha(j)\,. \tag{B.9b}$$

Finally, for the equation of motion for H we have:

$$0 = \mathrm{d}(e^{-\Phi}\star H) - \frac{1}{2}\sum_n e^{\frac{5-n}{2}\Phi}\star F_n\wedge F_{(n-2)} + 2\kappa_{10}^2\sum_n e^{\frac{n}{4}\Phi}\Psi_n\wedge\alpha(j)\Big|_8\,. \tag{B.10}$$

In the above equations we can redefine j in order to absorb the factor of $2\kappa_{10}^2$,

$$(2\kappa_{10}^2)j \to j\,, \tag{B.11}$$

which we do in this thesis.

Appendix C

Basics of generalized geometry

In this appendix, we summarize the most important concepts of generalized geometry that will be of importance for this thesis. This treatment of generalized geometry is not complete, and we refer the interested reader to the literature. Very valuable lecture notes can be found in [35] and a brief introduction in [38]. A complete treatment of generalized geometry is presented in [126, 127].

C.1 Generalized complex structures and pure spinors

Generalized geometry is a generalization of ordinary geometry. In fact, it is a unification and generalization of the language of complex and symplectic geometry, which seems to be natural to describe supersymmetric compactifications of supergravity with fluxes. As we will see, the language of generalized geometry allows one to rewrite the equations for a supersymmetric solution in a very concise form making the analysis more tractable.

The main idea is to replace the ordinary tangent bundle TM of a d-dimensional manifold M by a sum of the tangent bundle and the cotangent bundle $TM \oplus T^\star M$, which we denote in the following as the *generalized tangent bundle*. A generalized vector $\mathbb{X}$ living on this generalized tangent bundle is the sum of an ordinary vector $X \in \Gamma(TM)$ and a one-form $\xi \in \Gamma(T^\star M)$, such that $\mathbb{X} = X + \xi$. On the generalized tangent bundle, there is a canonical metric $\mathcal{L}$, defined for $\mathbb{X} = X + \xi$ and $\mathbb{Y} = Y + \eta$ as

$$\mathcal{L}(\mathbb{X}, \mathbb{Y}) = \xi(Y) + \eta(X)\,. \tag{C.1}$$

This metric is maximally indefinite, i.e., it has signature (d, d), and thus it already reduces the structure group to $\mathrm{O}(d, d)$. In analogy to ordinary geometry we define a *generalized almost complex structure* as a map

$$\mathcal{J} : TM \oplus T^\star M \to TM \oplus T^\star M\,, \tag{C.2}$$

that squares to minus one, $\mathcal{J}^2 = -\mathbb{1}_{2d}$, and is hermitian with respect to the canonical metric $\mathcal{L}$

$$\mathcal{L}(\mathcal{J}\mathbb{X}, \mathcal{J}\mathbb{Y}) = \mathcal{L}(\mathbb{X}, \mathbb{Y})\,. \tag{C.3}$$

A generalized almost complex structure further reduces the structure group from $\mathrm{O}(d,d)$ to $\mathrm{U}(d/2,d/2)$ [1].

As an example, we can construct from an ordinary complex structure $\mathcal{I}$, $\mathcal{I}^2 = -\mathbb{1}$, or an ordinary symplectic structure J, the following generalized complex structures, respectively,

$$\mathcal{J}_1 = \begin{pmatrix} -\mathcal{I} & 0 \\ 0 & \mathcal{I}^T \end{pmatrix}, \quad \mathcal{J}_2 = \begin{pmatrix} 0 & J^{-1} \\ -J & 0 \end{pmatrix}, \tag{C.4}$$

This demonstrates that both essential parts of an SU(3)-structure, namely the complex structure and the symplectic structure, are described in the language of generalized geometry in a completely uniform way.

In generalized geometry, pure spinors are described by polyforms $\Psi \in \Lambda^\bullet T^\star M$. Indeed, a section $\mathbb{X} = (X, \xi)$ of the generalized tangent bundle acts on such a polyform Ψ in a natural way as follows

$$\mathbb{X} \cdot \Psi = \iota_X \Psi + \xi \wedge \Psi\,, \tag{C.5}$$

and it is easy to show that

$$\{\mathbb{X}, \mathbb{Y}\} \cdot \Psi = \mathcal{L}(\mathbb{X}, \mathbb{Y}) \Psi\,. \tag{C.6}$$

This is nothing else than the spin representation of $\mathrm{Spin}(d,d)$ and therefore polyforms Ψ can be thought of as spinors for $\mathrm{Spin}(d,d)$. The generalized gamma-matrices on $\mathrm{Spin}(d,d)$ are vectors X (acting by contraction, ι_X) and one-forms ξ (acting by $\xi\wedge$). We choose in the following a basis for the generalized gamma-matrices as follows

$$\begin{aligned} \Gamma_\Sigma &= \iota_m \quad \text{for} \quad m = \Sigma = 1, \dots, d\,, \\ \Gamma_\Sigma &= e^m \wedge \quad \text{for} \quad m + d = \Sigma = d+1, \dots, 2d\,. \end{aligned} \tag{C.7}$$

We can further decompose the set of polyforms Ψ into the spaces of even and odd forms: positive or negative parity spinors correspond to polyforms with all dimensions even or odd, respectively, which we denote by Ψ_+ and Ψ_-.

From the action (C.5) we can define the annihilator space L_Ψ of a spinor as follows

$$L_\Psi = \{\mathbb{X} \in TM \oplus T^\star M : \mathbb{X} \cdot \Psi = 0\}\,, \tag{C.8}$$

which is isotropic [2], since $\mathcal{L}(\mathbb{X}, \mathbb{Y})\Psi = (\mathbb{X}\cdot\mathbb{Y} + \mathbb{Y}\cdot\mathbb{X})\cdot\Psi = 0$ for all $\mathbb{X}, \mathbb{Y} \in L_\Psi$. If L_Ψ is maximally isotropic, i.e., if its rank is d, Ψ is a pure spinor. This somewhat mathematical concept of pure spinors works actually for the more familiar spinors

[1] Let us mention that the concept of integrability for a generalized almost complex structure has a natural generalization from ordinary geometry by replacing the Lie bracket with the Courant bracket. We refer the reader to the literature for the proper definition of an integrable generalized complex structure (see, e.g., [35]). In the following we drop the "almost".

[2] A subbundle L is isotropic if $\mathcal{L}(\mathbb{X}, \mathbb{Y}) = 0$ for all $\mathbb{X}, \mathbb{Y} \in L$.

of Spin(d), d even, in exactly the same way, i.e. a spinor is pure if the number of independent gamma-matrices which annihilate the spinor is $d/2$. As a matter of fact, in $d \leq 6$ every Weyl spinor is pure.

In ordinary geometry there is a one-to-one correspondence between a complex structure and a Weyl spinor (see for instance eq. (2.5)). An analogous property holds between a generalized complex structure and a pure spinor, where the latter are described by polyforms. Let us first define a fundamental two-form as follows

$$\mathcal{J}_{\Pi\Sigma} = \langle \mathrm{Im}\,\Psi, \Gamma_{\Pi\Sigma} \mathrm{Im}\,\Psi \rangle \,, \tag{C.9}$$

where $\Pi, \Sigma = 1, \dots, 2d$ and the generalized gamma matrices Γ_Σ are given in eq. (C.7). We can use the canonical metric $\mathcal{L}$ to raise one index forming $\mathcal{J}^{\Pi}{}_{\Sigma}$ which generally defines a generalized complex structure [3]. The Mukai pairing $\langle \cdot, \cdot \rangle$ in eq. (C.9) is given by

$$\langle \Psi_1, \Psi_2 \rangle = \Psi_1 \wedge \alpha(\Psi_2)|_{\mathrm{top}} \,, \tag{C.10}$$

where the operator α acts by inverting the order of indices on forms (see eq. (A.6)) and "top" indicates that we project on the top-form part, i.e., the part that is proportional to the volume form. The Mukai pairing has the following useful property:

$$\langle e^B \Psi_1, e^B \Psi_2 \rangle = \langle \Psi_1, \Psi_2 \rangle \,, \tag{C.11}$$

for an arbitrary two-form B.

Gualtieri established in [127] that every generalized complex structure is associated to a pure spinor that can be written as

$$\Psi = \Omega_k \wedge e^{i\omega + B} \,, \tag{C.12}$$

where ω, B are real two-forms and Ω_k a complex decomposable k-form, i.e. it can (locally) be written as the wedge product of one-forms, such that $\langle \Psi, \bar{\Psi} \rangle \neq 0$. k is called the type of the pure spinor.

The construction of a pure spinor of the form (C.12) is not straightforward. In particular, for a pure spinor of type $k > 1$ the condition that it is decomposable is quite cumbersome. As showed by Hitchin [128, 126] and reviewed in [27] the complex pure spinor can be constructed as a function of a real spinor. This Hitchin construction also guarantees that a pure spinor of type $k > 1$ is decomposable.

The procedure works as follows. Let us assume that we are given a real form χ which we want to consider as the imaginary part of the pure spinor Ψ to be constructed, $\chi = \mathrm{Im}\,\Psi$. Using the correspondence (C.9), we define the associated generalized complex structure $\mathcal{J}$. The problem is the proper normalization such that $\mathcal{J}^{\Pi}{}_{\Lambda}\mathcal{J}^{\Lambda}{}_{\Sigma} = -\delta^{\Pi}{}_{\Sigma}$,

[3] Note that the correspondence actually involves the imaginary part of the pure spinor. As we will discuss in the following, the imaginary part of the pure spinor completely determines the pure spinor. This will also affect the proper normalization of $\mathcal{J}$.

since the normalization of χ is not fixed due to $\langle\chi,\chi\rangle = 0$. Hitchin proposed to define a quartic function of $\chi = \mathrm{Im}\,\Psi$ given by

$$H(\chi) = \sqrt{-\frac{1}{12}\mathcal{J}^{\Pi}{}_{\Sigma}\mathcal{J}^{\Sigma}{}_{\Pi}}\,. \tag{C.13}$$

This function is called the Hitchin functional. The proper normalization for $\mathcal{J}$ is then the condition that $H(\chi) = 1$. A necessary condition on the real form χ to define a generalized complex structure via (C.9) is that $\mathcal{J}^{\Pi}{}_{\Sigma}\mathcal{J}^{\Sigma}{}_{\Pi} < 0$. Real forms satisfying this condition are called *stable* real forms, and these are sufficient to define a SU$(d/2, d/2)$-structure.

Via the Hitchin procedure we can construct the real part $\hat{\chi}$ that correspond to χ such that the complex decomposable pure spinor is given by $\Psi = \hat{\chi} + i\chi$ as follows

$$\hat{\chi} = -\frac{1}{6H(\chi)}\mathcal{J}^{\Pi\Sigma}\Gamma_{\Pi\Sigma}\chi\,. \tag{C.14}$$

and the Hitchin functional can be rewritten using (C.9) as [27]

$$H(\chi) = \frac{1}{2}\langle\hat{\chi},\chi\rangle = \frac{i}{4}\langle\Psi,\bar{\Psi}\rangle\,. \tag{C.15}$$

We use this expression to evaluate the Kähler potential given in eq. (3.30) and eq. (3.39), where we need to evaluate $\int\langle t,\bar{t}\rangle$ with $t = e^{-\Phi}\Psi_1$.

C.2 SU$(d/2)\times$SU$(d/2)$-structures from pure spinor pairs

As we have seen, the existence of a generalized complex structure reduces the structure group of $TM \oplus T^{\star}M$ from O(d,d) to U$(d/2,d/2)$. If it is possible to define two generalized complex structures, $\mathcal{J}_1$ and $\mathcal{J}_2$, that commute, $[\mathcal{J}_1,\mathcal{J}_2] = 0$, and such that the generalized metric $\mathcal{G} = -\mathcal{L}\mathcal{J}_1\mathcal{J}_2$ is positive definite, the structure group is further reduced to its maximal compact subgroup, U$(d/2)\times$U$(d/2)$.

An U$(d/2)\times$U$(d/2)$-structure, or equivalently two compatible generalized complex structures $(\mathcal{J}_1, \mathcal{J}_2)$, provide automatically a generalized metric (g, B), where g is an ordinary metric and B a two-form on TM. This works as follows. Let us define the product

$$G = -\mathcal{J}_1\mathcal{J}_2\,. \tag{C.16}$$

Since $\mathcal{J}_1, \mathcal{J}_2$ commute and square to -1, G squares to 1. Taking into account the hermiticity of $\mathcal{J}_1$ and $\mathcal{J}_2$ (see eq. (C.3)) it follows $G^T\mathcal{L} = \mathcal{L}G$ and it turns out that the most general form of G is given by

$$G = -\mathcal{J}_1\mathcal{J}_2 = \begin{pmatrix} -g^{-1}B & g^{-1} \\ g - Bg^{-1}B & Bg^{-1} \end{pmatrix}, \tag{C.17}$$

from which we easily read off the metric g on TM that is positive definite since we required that $\mathcal{L}G$ is positive definite.

Two such generalized complex structures defining a U($d/2$)×U($d/2$)-structure are said to be compatible. The condition that $\mathcal{J}_1$ and $\mathcal{J}_2$ commute is equivalent to the structure conditions (these conditions are sometimes called compatibility conditions) for a given structure. It can be shown that in terms of the associated pure spinors we can reexpress the compatibility condition as

$$\langle \Psi_1, \mathbb{X} \cdot \Psi_2 \rangle = \langle \Psi_1, \mathbb{X} \cdot \bar{\Psi}_2 \rangle = 0 \quad \forall \mathbb{X} \in TM \oplus T^\star M\,. \tag{C.18}$$

Applied for the special cases of strict SU(3)-structure and static SU(2)-structure this is equivalent to eq. (2.6a) and eq. (2.16), respectively.

A U($d/2$)×U($d/2$)-structure defines two compatible pure spinors only up to an overall scalar function. We can further reduce the structure group to SU($d/2$)×SU($d/2$)-structure by removing the ambiguity of rescaling the pure spinors requiring globally defined pure spinors such that $\langle \Psi_1, \bar{\Psi}_1 \rangle \neq 0$ and $\langle \Psi_2, \bar{\Psi}_2 \rangle \neq 0$. We can then normalize the pure spinors as follows

$$\langle \Psi_1, \bar{\Psi}_1 \rangle = \langle \Psi_2, \bar{\Psi}_2 \rangle \neq 0\,. \tag{C.19}$$

The SU($d/2$)×SU($d/2$)-structure is actually associated to two spinors $\eta^{(1)}$ and $\eta^{(2)}$ of Spin(d) defined on M. Given two Spin(d) spinors $\eta^{(1)}$ and $\eta^{(2)}$ that are in general independent (and define two in general independent SU($d/2$)-structures) we can construct two compatible pure spinors $\Psi_\pm$ via the familiar Clifford map as follows

$$\not{\Psi}_+ = \frac{8}{|a||b|}\eta_+^{(1)} \otimes \eta_+^{(2)\dagger}\,, \qquad \not{\Psi}_- = \frac{8}{|a||b|}\eta_+^{(1)} \otimes \eta_-^{(2)\dagger}\,, \tag{C.20}$$

where the Clifford map is given by the isomorphism

$$\Psi \leftrightarrow \not{\Psi} = \sum_l \frac{1}{l!}\Psi_{i_1 \ldots i_l}\gamma^{i_1 \ldots i_l}\,. \tag{C.21}$$

We can use the following useful Fierz identity

$$M = \frac{1}{8}\sum_l \frac{1}{l!}\mathrm{Tr}(\gamma_{i_1 \ldots i_l} M)\gamma^{i_l \ldots i_1}\,, \tag{C.22}$$

to derive

$$\Psi_{\pm\, i_1 \ldots i_l} = \frac{1}{|a||b|}\eta_\pm^{(2)\dagger}\gamma_{i_l \ldots i_1}\eta_+^{(1)}\,. \tag{C.23}$$

Let us now consider six-dimesional space. Following the conventions of [38], we can define the most general relation between two spinors as follows

$$\begin{aligned} \eta_+^{(1)} &= a\eta_+\,, \\ \eta_+^{(2)} &= b(k_{\|}\eta_+ + k_\perp V^i \gamma_i \eta_-)\,, \end{aligned} \tag{C.24}$$

where $2|V|^2 = |k_{||}|^2 + |k_\perp|^2 = 1$ and $|a| = |b|$. With these definitions and the definitions of ω_2 and Ω_2 in eq. (2.15) we can express the most general pure spinors from eq. (C.23) as follows

$$\begin{aligned}\Psi_+ &= e^{-i\theta} e^{2V\wedge V^*} (k_{||} e^{i\omega_2} - k_\perp \Omega_2)\,,\\ \Psi_- &= -2V \wedge (k_{||}\Omega_2 + k_\perp e^{i\omega_2})\,,\end{aligned} \tag{C.25}$$

Using the terminology of [38, 39] we may consider the following interesting cases:

- strict SU(3)-structure: $k_{||} = 1$, $k_\perp = 0$. The spinors $\eta^{(1)}$ and $\eta^{(2)}$ are parallel everywhere. The types of the pure spinors (Ψ_+, Ψ_-) are (0,3);
- static SU(2)-structure: $k_{||} = 0$, $k_\perp = 1$. The spinors $\eta^{(1)}$ and $\eta^{(2)}$ are orthogonal everywhere. The types of the pure spinors (Ψ_+, Ψ_-) are (2,1);
- intermediate SU(2)-structure: $k_{||} \neq 0$, $k_\perp \neq 0$. The spinors $\eta^{(1)}$ and $\eta^{(2)}$ are at a fixed angle, but neither a zero angle nor a right angle. The types of the pure spinors (Ψ_+, Ψ_-) are (0,1);
- dynamic SU(3)×SU(3)-structure: $k_{||} \neq 0$, $k_\perp \neq 0$. The angle between $\eta^{(1)}$ and $\eta^{(2)}$ varies, possibly becoming a zero angle, type (0,3), or a right angle, type (2,1), at a special locus.

In this thesis we will only consider strict SU(3)-structure and static SU(2)-structure compactifications, so let us in the following look at these cases in more detail.

C.3 Strict SU(3)-structure and static SU(2)-structure

Let us first consider the case of strict SU(3)-structure and specialize the expressions obtained so far in terms of generalized geometry to this case. The two spinors $\eta^{(1)}$ and $\eta^{(2)}$ are proportional

$$\eta_+^{(2)} = (b/a)\eta_+^{(1)}\,, \tag{C.26}$$

with $|\eta_+^{(1)}|^2 = |a|^2, |\eta_+^{(2)}|^2 = |b|^2$. In the following, we will assume $|a| = |b|$ such that $b/a = e^{i\theta}$ is just a phase. We will see in the following that this condition is implied by the orientifold projection [43]. From eq. (C.23) (or from eq. (C.25) with $k_{||} = 1$, $k_\perp = 0$) we get the pure spinors for the strict SU(3)-structure as follows

$$\Psi_- = -\Omega\,, \qquad \Psi_+ = e^{-i\theta} e^{iJ}\,, \tag{C.27}$$

where J and Ω are defined in eq. (2.5).

The derivation of the metric also simplifies for a strict SU(3)-structure: from eq. (C.4) and the generalized metric (C.17) we immediately conclude (for $B = 0$)

$$g_{mn} = \mathcal{I}_m{}^l J_{ln}\,, \tag{C.28}$$

where we can construct the complex structure $\mathcal{I}$ from $\mathrm{Im}\,\Omega$ as follows

$$\tilde{\mathcal{I}}^l{}_k = \varepsilon^{lm_1\ldots m_5}(\mathrm{Im}\,\Omega)_{km_1m_2}(\mathrm{Im}\,\Omega)_{m_3m_4m_5}\,, \tag{C.29}$$

This follows from eq. (C.9) and $\mathcal{J}_1$ in eq. (C.4). We then properly normalize it with the Hitchin functional, which for strict SU(3)-structure simplifies to the expression in the denominator of the following equation,

$$\mathcal{I} = \frac{\tilde{\mathcal{I}}}{\sqrt{-\mathrm{tr}\,\frac{1}{6}\tilde{\mathcal{I}}^2}}\,, \tag{C.30}$$

so that $\mathcal{I}^2 = -\mathbb{1}$.

It is a simple exercise to show that the compatibility condition (2.6a) and the normalization condition (2.6b) for J and Ω follow from eq. (C.18) and eq. (C.19) for the pure spinors (C.27).

The decomposition of the intrinsic torsion in terms of the five torsion classes is given in eq. (2.10). Note that by definition $\mathcal{W}_2$ is primitive, which means

$$\mathcal{W}_2 \wedge J \wedge J = 0\,. \tag{C.31}$$

One interesting property of a primitive (1,1)-form is

$$\star(\mathcal{W}_2 \wedge J) = -\mathcal{W}_2\,, \tag{C.32}$$

which can be shown using $J^{mn}\mathcal{W}_{2mn} = 0$ (which follows from the primitivity) and $J_m{}^n J_p{}^q \mathcal{W}_{nq} = \mathcal{W}_{mp}$ (which follows from the fact that $\mathcal{W}_2$ is of type (1,1)).

Let us calculate the part of $\mathrm{d}\mathcal{W}_2^-$ proportional to $\mathrm{Re}\,\Omega$:

$$\mathrm{d}\mathcal{W}_2^- = \alpha\,\mathrm{Re}\,\Omega + (2,1) + (1,2)\ , \tag{C.33}$$

for some α. Taking the exterior derivative of $\Omega \wedge \mathcal{W}_2^- = 0$ and using eq. (C.33) as well as eqs. (2.6b) and (2.10), we arrive at

$$\mathcal{W}_2^- \wedge \mathcal{W}_2^- \wedge J = \frac{2i}{3}\alpha J^3\ . \tag{C.34}$$

We can now use eq. (C.32) to show

$$\mathcal{W}_2^- \wedge \mathcal{W}_2^- \wedge J = \frac{1}{2}|\mathcal{W}_2^-|^2 \mathrm{vol}_6\,, \tag{C.35}$$

from which we obtain $\alpha = -i|\mathcal{W}_2|^2/8$, in accordance with (2.33).

For the static SU(2)-structure case we have two everywhere orthogonal spinors $\eta_+^{(1)}$ and $\eta_+^{(2)}$ and we can define a vector V as in section 2.1.2,

$$\eta_+^{(1)} = a\eta_+\,, \tag{C.36a}$$

$$\eta_+^{(2)} = bV^i\gamma_i\eta_- \,, \tag{C.36b}$$

where $|\eta_+^{(1)}|^2 = |a|^2$, $|\eta_+^{(2)}|^2 = |b|^2$ and $|a| = |b|$. Only the relative phase θ in $b/a = e^{i\theta}$ is physical. With these definitions we obtain from eq. (C.23) (or from eq. (C.25) with $k_{||} = 0$, $k_\perp = 1$) the pure spinors as follows

$$\Psi_+ = -e^{-i\theta} e^{2V\wedge V^*}\Omega_2 \,, \tag{C.37a}$$

$$\Psi_- = -2\,V \wedge e^{i\omega_2} \,, \tag{C.37b}$$

where Ω_2 and ω_2 are defined in eq. (2.18). In the following it will be convenient to absorb the phase $e^{-i\theta}$ in Ω_2. This time it is not a completely trivial exercise to show the compatibility conditions (2.16) from condition (C.18) and the pure spinors (C.37). However, in [39] it is shown that the conditions (C.18) are indeed vanishing provided one imposes the conditions (2.16).

To calculate the induced metric for a static SU(2)-structure we compute, with eq. (C.9), the corresponding generalized complex structures $\mathcal{J}_{1,2}$ for the pure spinors (C.37), and from eq. (C.17) we can read off the metric g.

Orientifolds

Following [43], we can identify the action of a supersymmetric orientifold on the pure spinors $\Psi_\pm$. An orientifold projection consists of modding out the theory by an operator $\mathcal{O} = \Omega_p\sigma$ for O5/O9- and O6-orientifold projections and $\mathcal{O} = \Omega_p(-1)^{F_L}\sigma$ for O3/O7- and O4/O8-orientifold projections [4]. Here, Ω_p is a reflection on the world-sheet exchanging the left-movers with the right-movers, σ is an internal involution ($\sigma^2 = 1$) which acts only on the internal manifold and leaves the external space-time untouched and $(-1)^{F_L}$, where F_L is the fermion number of the left-movers, is used in some cases to ensure that $\mathcal{O}^2 = 1$. Under a supersymmetric orientifold projection, the total ten-dimensional supersymmetry parameter $\epsilon_1^L + \epsilon_2^R$ has to be invariant. Since the world-sheet reflection Ω_p exchanges left- and right-movers, we end up with the action of the involution on the ten-dimensional supersymmetry generators

$$\text{O5/O9, O6}: \qquad \sigma^*\epsilon_1 = \epsilon_2 \,, \qquad \sigma^*\epsilon_2 = \epsilon_1 \,, \tag{C.38a}$$

$$\text{O3/O7, O4/O8}: \qquad \sigma^*\epsilon_1 = -\epsilon_2 \,, \qquad \sigma^*\epsilon_2 = \epsilon_1 \,. \tag{C.38b}$$

If we now plug the $\mathcal{N} = 2$ ansatz (2.25) into these equations we immediately see (since σ only acts on the internal spinors) that the two external supersymmetry generators ζ^1 and ζ^2 can not be chosen independent and should be proportional. Since we can absorb the proportionality factors in the definition of the internal spinors, we will put $\zeta^1 = \zeta^2 = \zeta$, and we end up with an $\mathcal{N} = 1$ theory with the ansatz (2.3) [5]. Further

[4] We take here the conventions of [43].

[5] An ansatz for $\mathcal{N} > 1$ is then only possible if there are more invariant internal spinors.

reducing eq. (C.38) to the internal spinors $\eta_\pm^{(i)}$ with the ansatz (2.3), we find for the cases we are interested in:

$$\text{O5}: \qquad \sigma^*\eta_\pm^{(1)} = \eta_\pm^{(2)}\,, \qquad \sigma^*\eta_\pm^{(2)} = \eta_\pm^{(1)}\,, \tag{C.39a}$$

$$\text{O6}: \qquad \sigma^*\eta_\pm^{(1)} = \eta_\mp^{(2)}\,, \qquad \sigma^*\eta_\pm^{(2)} = \eta_\mp^{(1)}\,, \tag{C.39b}$$

$$\text{O7}: \qquad \sigma^*\eta_\pm^{(1)} = -\eta_\pm^{(2)}\,, \qquad \sigma^*\eta_\pm^{(2)} = \eta_\pm^{(1)}\,, \tag{C.39c}$$

and, since we define $|\eta_+^{(1)}|^2 = |a|^2$, $|\eta_+^{(2)}|^2 = |b|^2$, it follows from $\sigma^2 = 1$ that $|a| = |b|$. Plugging eq. (C.39) into the definition of the pure spinors (C.20), we get [43] (see also [56, 38])

$$\text{O5}: \qquad \sigma^*\Psi_+ = \alpha(\bar\Psi_+)\,, \qquad \sigma^*\Psi_- = -\alpha(\Psi_-)\,, \tag{C.40a}$$

$$\text{O6}: \qquad \sigma^*\Psi_+ = \alpha(\Psi_+)\,, \qquad \sigma^*\Psi_- = \alpha(\bar\Psi_-)\,, \tag{C.40b}$$

$$\text{O7}: \qquad \sigma^*\Psi_+ = -\alpha(\bar\Psi_+)\,, \qquad \sigma^*\Psi_- = \alpha(\Psi_-)\,, \tag{C.40c}$$

Applying this to the explicit pure spinors for a strict SU(3)-structure (C.27) and a static SU(2)-structure (C.37) we arrive at eq. (2.40b) and eq. (3.40), respectively.

C.4 Supersymmetry conditions in generalized geometry language

Generalized geometry allows one to rewrite the $\mathcal{N} = 1$ supersymmetry conditions (2.20) with the ansatz for the spinors (2.3) in a very concise form. In order to obtain similar equations in type IIA and type IIB, we define

$$\Psi_1 = \Psi_\mp\,, \qquad \Psi_2 = \Psi_\pm\,, \tag{C.41}$$

with upper/lower sign for IIA/IIB. We collect all the RR-fields of the democratic formalism into one polyform and make the following compactification ansatz

$$F = \hat F + \mathrm{vol}_4 \wedge \tilde F\,, \tag{C.42}$$

with vol_4 the four-dimensional (AdS_4) volume form [6].

With these definitions the supersymmetry conditions (in string frame) take the following form in both type IIA and type IIB [38]

$$\mathrm{d}_H\left(e^{4A-\Phi}\mathrm{Im}\,\Psi_1\right) = 3e^{3A-\Phi}\mathrm{Im}\,(W^*\Psi_2) + e^{4A}\tilde F\,, \tag{C.43a}$$

$$\mathrm{d}_H\left[e^{3A-\Phi}\mathrm{Re}\,(W^*\Psi_2)\right] = 2|W|^2 e^{2A-\Phi}\mathrm{Re}\,\Psi_1\,, \tag{C.43b}$$

[6] In this thesis we will drop the hat on the purely internal part of the RR-flux F and hope that it is clear from the context whether we mean the full F or only the internal part. Instead, we use the hat to denote background values of the fields.

$$\mathrm{d}_H \left[e^{3A-\Phi} \mathrm{Im}(W^* \Psi_2) \right] = 0 \,, \tag{C.43c}$$

where we used $|a|^2 = |b|^2 \propto e^A$. Here W is defined in terms of the AdS Killing spinors

$$\nabla_\mu \zeta_- = \pm \frac{1}{2} W \gamma_\mu \zeta_+ \,, \tag{C.44}$$

for IIA/IIB. These equations should be supplemented with the Bianchi identities for the RR-fluxes (B.9a) where the (localized or smeared) sources j have to be calibrated

$$\langle \mathrm{Re}\,\Psi_1, j \rangle = 0 \,, \tag{C.45a}$$

$$\langle \Psi_2, \mathbb{X} \cdot j \rangle = 0 \,, \quad \forall \mathbb{X} \in \Gamma(T_M \oplus T_M^\star) \,. \tag{C.45b}$$

Analogously to the strict SU(3)-case, an easy way to solve these calibration conditions is to choose

$$j = -k \,\mathrm{Re}\,\Psi_1 \,, \tag{C.46}$$

for some function k.

An advantage of this formulation is that we only need to know how the exterior derivative d acts on the left-invariant forms in which we expand the pure spinors. For the nilmanifolds and the coset spaces we consider in this thesis, the action of the exterior derivative d is given by the Maurer-Cartan equation (4.4) and the structure constants.

Inserting the pure spinors for a strict SU(3)-structure (C.27) in the equations (C.43) for an $\mathcal{N} = 1$ supersymmetric solution and considering the type IIA case (where $\Psi_1 = \Psi_-$ and $\Psi_2 = \Psi_+$), we arrive at eqs. (2.27) and (2.30) (these equations were first derived in [31] using the language of SU(3)-structures). However, these solutions assume constant warp factor e^A and dilaton Φ, which is required for non-vanishing Romans mass. As we showed in section 2.3, choosing the Romans mass to be zero, we can derive a solution with non-constant warp factor and dilaton.

On the other hand, for the type IIB case (for which we exchange the role of Ψ_+ and Ψ_-) there is no AdS_4 solution possible, as already noted in [129]. The reason is that for this case the left-hand-side of eq. (C.43b) is a four-form, which would put the zero- and two-form part of $\Psi_+ = e^{-i\theta} e^{iJ}$ to zero, making (C.19) impossible to be satisfied. A way out is to put $W = 0$ implying the vanishing of the AdS_4 curvature. We conclude that there are no $\mathcal{N} = 1$ AdS_4 vacua for type IIB and strict SU(3)-structure.

On the other hand, plugging the ansatz (2.19) for a static SU(2)-structure in the supersymmetry conditions (C.43), one finds the necessary equations for the SU(2)-structure quantities V, ω_2 and Ω_2. However, these equations are quite complicated and it turns out that it is less complicated to try to solve these equations directly in terms of pure spinors.

Similar to the argument that excludes $\mathcal{N} = 1$ AdS_4 vacua for type IIB and strict SU(3)-structure, we conclude from (C.43b) and the ansatz (C.37) that there are no $\mathcal{N} = 1$ AdS_4 vacua for type IIA and static SU(2)-structure, as was already noted in [42]. Indeed, the left-hand-side of eq. (C.43b) is a three- and five-form which implies on the right-hand-side that $V = 0$. This makes it impossible to satisfy eq. (C.19). Once again, putting $W = 0$ resolves the problem. We summarize these results in table 2.2.

Appendix D

Smeared sources and orientifold involutions

In this appendix we propose a procedure to identify the orientifold involutions associated to a given source term j representing the Poincaré dual of smeared orientifolds. As we will see, the Hitchin functional defined in appendix C provides a useful criterion to classify the possible source terms j.

Orientifold involutions from decomposable forms

Let us first give an example for a *localized* orientifold in flat space. If we have an orientifold along the directions $\Sigma = (x^1, x^2, x^3)$ then the corresponding source is

$$j = T_{\mathrm{O}p}\, j_\Sigma = -T_{\mathrm{O}p}\, \delta(x^4, x^5, x^6)\, \mathrm{d}x^4 \wedge \mathrm{d}x^5 \wedge \mathrm{d}x^6 \,, \tag{D.1}$$

where $T_{\mathrm{O}p} < 0$ for an orientifold and j is the Poincaré dual of Σ satisfying

$$\int_\Sigma \phi = \int_{\mathcal{M}} \langle \phi, j_\Sigma \rangle = -\int_{\mathcal{M}} \phi \wedge j_\Sigma \,, \tag{D.2}$$

for an arbitrary form ϕ [1]. In this case the orientifold involution is of course

$$O6: \qquad x^4 \to -x^4 \,, \quad x^5 \to -x^5 \,, \quad x^6 \to -x^6 \,. \tag{D.3}$$

Suppose we now introduce many orientifolds and completely smear them in the directions (x^4, x^5, x^6) obtaining

$$j = -T_{\mathrm{Op}}\, c\, \mathrm{d}x^4 \wedge \mathrm{d}x^5 \wedge \mathrm{d}x^6 \,, \tag{D.4}$$

where c is a constant representing the orientifold density. We have now lost information about the exact location but we would still like to associate the orientifold involution

$$O6: \qquad \mathrm{d}x^4 \to -\mathrm{d}x^4 \,, \quad \mathrm{d}x^5 \to -\mathrm{d}x^5 \,, \quad \mathrm{d}x^6 \to -\mathrm{d}x^6 \,. \tag{D.5}$$

[1] The definition with the Mukai pairing is the one appropriate for generalizing to D-branes with world-volume gauge flux as explained in [130]. Here it will just give an extra minus sign.

An important observation is that $\mathrm{d}x^4 \wedge \mathrm{d}x^5 \wedge \mathrm{d}x^6$ is not just any form, it is a *decomposable* form, i.e. it can be written as a wedge product of three one-forms. These one-forms span the annihilator space of T_Σ, the tangent space of Σ. So if we are given a smeared orientifold current j we should write it as a sum of decomposable forms and then associate to each term an orientifold involution as above.

It is not straightforward to decide whether a given form is decomposable or not and how we could write j as a sum of decomposable forms in a unique way. Let us first give a mathematical definition of a decomposable form. Let $\mathbb{V}$ be a d-dimensional vector space and $\mathbb{V}^\star$ its dual [2]. A (real/complex) p-form $j \in \Lambda^p\mathbb{V}^\star$ is called *simple* or *decomposable* if it can be written as a wedge product of p one-forms [3].

In [131] a criterion for a simple form is given as follows. Be

$$j^\perp = \{X \in \mathbb{V} : \iota_X j = 0\} \subset \mathbb{V}, \tag{D.6}$$

and

$$W = \mathrm{Ann}(j^\perp) \subset \mathbb{V}^\star \,. \tag{D.7}$$

The form j is simple if and only if $\dim W = p$. Using this the following alternative criterion is shown:

Theorem: A p-form $j \in \Lambda^p\mathbb{V}^\star$ is simple if and only if for every $(p-1)$-polyvector $\xi \in \Lambda^{p-1}\mathbb{V}$,

$$\iota_\xi j \wedge j = 0 \ , \tag{D.8}$$

where $\iota_\xi j$ is the one-form contraction of j with ξ.

For us of importance is the special case of three-forms in six dimensions. For this case there is another useful theorem due to Hitchin [128].

Theorem: Consider a real three-form $j \in \Lambda^3\mathbb{V}^\star$ and calculate its Hitchin functional $H(j)$ defined in (C.13). Then

- $H(j) > 0$ if and only if $j = j_1 + j_2$ where j_1, j_2 are unique (up to ordering) real decomposable three-forms and $j_1 \wedge j_2 \neq 0$,
- $H(j) < 0$ if and only if $j = \alpha + \bar{\alpha}$ where α is a unique (up to complex conjugation) complex decomposable three-form and $\alpha \wedge \bar{\alpha} \neq 0$.

Now we have two base-independent characterizations of j: the Hitchin functional $H(j)$ and $\dim W$. Using these two characterizations we can classify the possible j and decompose it in simple terms:

[2] For nilmanifolds and coset spaces that we consider in this thesis we always have a basis of globally defined left-invariant one-forms.

[3] Note that a (real/complex) form of fixed dimension is a pure spinor if and only if it is simple. In fact, we could regard the notion of pure spinor as a generalization of the notion of decomposable forms to polyforms.

- if $H(j) > 0$ it follows immediately that j is a sum of exactly two real simple terms,
- if $H(j) < 0$ then j is a sum of exactly two (conjugate) complex simple terms and thus of exactly four real simple terms,
- if $H(j) = 0$ we have three cases. Either (D.8) is satisfied (equivalently $\dim W = 3$) and j is simple, either $\dim W = 5$ and then j will be a sum of two simple terms j_1 and j_2 such that $j_1 \wedge j_2 = 0$, or $\dim W = 6$ and j will be a sum of three simple terms. All this is easy to prove by looking at possible types of sums of two and three simple terms.

An important remark is in order: while the Hitchin theorem states that for $H(j) \neq 0$ the two real/complex forms in the decomposition of j are unique (up to ordering/complex conjugation), the choice of one-forms out of which these forms are made is *not* unique. In the case of $H(j) < 0$ it is the freedom of choosing a basis of complex one-forms belonging to a complex structure, which is SL(3,$\mathbb{C}$). As a consequence the choice of the four real forms in which j is decomposed is *not* unique. Indeed, suppose we choose one basis of complex one-forms and associated x and y coordinates: $e^{z^i} = e^{x^i} - ie^{y^i}$. Then j can be written as the sum of the following four terms:

$$j = \mathrm{Re}\,(e^{z^1 z^2 z^3}) = e^{x^1 x^2 x^3} - e^{x^1 y^2 y^3} - e^{y^1 x^2 y^3} - e^{y^1 y^2 x^3}\,, \tag{D.9}$$

which leads to the following orientifold involutions:

$$\begin{aligned}
O6: &\quad e^{x^1} \to -e^{x^1}\,, \quad e^{x^2} \to -e^{x^2}\,, \quad e^{x^3} \to -e^{x^3}\,, \\
O6: &\quad e^{x^1} \to -e^{x^1}\,, \quad e^{y^2} \to -e^{y^2}\,, \quad e^{y^3} \to -e^{y^3}\,, \\
O6: &\quad e^{y^1} \to -e^{y^1}\,, \quad e^{x^2} \to -e^{x^2}\,, \quad e^{y^3} \to -e^{y^3}\,, \\
O6: &\quad e^{y^1} \to -e^{y^1}\,, \quad e^{y^2} \to -e^{y^2}\,, \quad e^{x^3} \to -e^{x^3}\,.
\end{aligned} \tag{D.10}$$

If we perform a SL(3,$\mathbb{C}$) transformation, j takes exactly the same form, but now in the *new* basis. So alternatively we could have chosen four orientifold involutions taking the same form as the old ones, but now in the *new* basis, which is rotated. This means that our choice of orientifold involutions is not unique. We must then further choose them such that the structure constant tensor of the group or coset is even, and $\mathrm{Re}\,\Omega$ and J are odd.

In the case of $H(j) > 0$ the argument does not apply because the remaining freedom GL(3,$\mathbb{R}$)×GL(3,$\mathbb{R}$) leaves the two terms of the decomposition *separately* invariant and the choice of orientifold involutions is unique.

Application to SU(2)×SU(2)

Let us now apply the above procedure to the model of section 10.5. Calculating the Hitchin functional $H(j^6)$ of (10.23) we find that it is negative so that it contains four orientifold involutions. We must now fix the freedom of choosing them such that $\mathrm{Re}\,\Omega$ and J are odd, and the structure constant tensor f is even. Some reflection should make clear that if $\mathrm{Re}\,\Omega$ is to be odd it should be a sum of the same four terms as j^6, but with different coefficients. In fact, we could reverse the procedure and choose a complex basis e^{z^i} in which Ω and J take their standard form:

$$\Omega = e^{z^1 z^2 z^3} \,, \qquad J = -\frac{i}{2} \sum_i e^{z^i \bar{z}^i} \,. \tag{D.11}$$

Then $\mathrm{Re}\,\Omega$ and J are automatically odd under the associated orientifold involutions (D.10). However, this should of course also be the orientifold involutions that follow from j^6. This will be the case if and only if j^6 has the same terms as $\mathrm{Re}\,\Omega$ (but with different coefficients) or equivalently j^6 should take the form

$$j^6 = \mathrm{Re}\left(c^0 e^{z^1 z^2 z^3} + c^{11} e^{\bar{z}^1 z^2 z^3} + c^{22} e^{z^1 \bar{z}^2 z^3} + c^{33} e^{z^1 z^2 \bar{z}^3}\right) , \tag{D.12}$$

with all coefficients c real. To accomplish this we still have the freedom to make a base transformation such that Ω and J invariant, i.e. an SU(3)-transformation. A priori, j^6 is an arbitrary three-form which transforms under SU(3) as

$$20 = 1 + \bar{1} + 3 + \bar{3} + 6 + \bar{6} \,. \tag{D.13}$$

However, we know that j^6 has to satisfy the calibration conditions (2.36), which remove the $3 + \bar{3}$ representation and only leave the form proportional to $\mathrm{Re}\,\Omega$ out of $1 + \bar{1}$. Here the 6 is the $(3 \times 3)_S$ i.e. the symmetric product of two fundamental representations of SU(3). It follows that the most general j^6 satisfying the calibration conditions looks like

$$\begin{aligned} j^6 &= c_0 \mathrm{Re}\,\Omega + \mathrm{Re}\left[c^{ki} g_{(k|\bar{j}} \mathrm{d}\bar{z}^{\bar{j}} \wedge \iota_{z^{i})} \Omega\right] \\ &= c_0 \mathrm{Re}\,\Omega + \mathrm{Re}\Big[c^{11} e^{\bar{z}^1 z^2 z^3} + c^{22} e^{z^1 \bar{z}^2 z^3} + c^{33} e^{z^1 z^2 \bar{z}^3} \\ &\quad + c^{12}\left(e^{\bar{z}^2 z^2 z^3} + e^{z^1 \bar{z}^1 z^3}\right) + c^{13}\left(e^{\bar{z}^3 z^2 z^3} + e^{z^1 z^2 \bar{z}^1}\right) + c^{23}\left(e^{z^1 \bar{z}^3 z^3} + e^{z^1 z^2 \bar{z}^2}\right)\Big] , \end{aligned} \tag{D.14}$$

with c_0 real and the entries of the coefficient matrix

$$C = \begin{pmatrix} c^{11} & c^{12} & c^{13} \\ c^{21} & c^{22} & c^{23} \\ c^{31} & c^{32} & c^{33} \end{pmatrix} , \tag{D.15}$$

complex. Now we have to find an SU(3)-transformation to put j^6 in the form (D.12). c_0 does not transform but is luckily already of the right form, while the coefficient matrix transforms as

$$C \to U C U^T \,. \tag{D.16}$$

From (D.12) we see that we want to transform C to a diagonal real matrix. In fact, since the above transformation cannot change the determinant this is only possible if

$$\det C \in \mathbb{R}\,. \tag{D.17}$$

This is a condition we have to add to the calibration conditions. For the j^6 of (10.23) one can check that it is indeed satisfied and it is possible to find the complex coordinates with the required properties. Also, under the associated orientifold involution the structure constant tensor f is even as required. The complex coordinates are given in (10.24). Defining the associated complex one-forms $e^{z^i} = e^{x^i} - ie^{y^i}$ we arrive at the transformation

$$\begin{aligned}
e^{x^1} &= \frac{-a^2+(b-c)^2+h}{2c_1\sqrt{2bc(2bc-h)}}(e^1+e^4)\,, & e^{y^1} &= \frac{a^2-(b+c)^2+h}{2c_1\sqrt{2bc(2bc-h)}}(e^1-e^4)\,,\\
e^{x^2} &= \frac{-b^2+(a-c)^2+h}{2c_1\sqrt{2ac(2ac-h)}}(e^2+e^5)\,, & e^{y^2} &= \frac{b^2-(a+c)^2+h}{2c_1\sqrt{2ac(2ac-h)}}(e^2-e^5)\,,\\
e^{x^3} &= \frac{-c^2+(a+b)^2-h}{2c_1\sqrt{2ab(2ab-h)}}(e^3-e^6)\,, & e^{y^3} &= \frac{-c^2-(a-b)^2+h}{2c_1\sqrt{2ab(2ab-h)}}(e^3+e^6)\,,
\end{aligned} \tag{D.18}$$

and the orientifold involutions given in eq. (D.10). The odd two-forms and even three-forms under the involutions are then given by

$$\begin{aligned}
Y_i^{(2-)}:&\qquad e^{x^1y^1}, e^{x^2y^2}, e^{x^3y^3}\,,\\
Y_i^{(3+)}:&\qquad e^{x^1x^2y^3}, e^{x^1y^2x^3}, e^{y^1x^2x^3}, -e^{y^1y^2y^3}\,.
\end{aligned} \tag{D.19}$$

With the transformation (D.18) we obtain the invariant forms in the old basis e^i which we display in eq. (11.54).

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