TYPE IIA FLUX COMPACTIFICATIONS: VACUA, EFFECTIVE THEORIES AND COSMOLOGICAL CHALLENGES

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Contents

Chapter 1

Introduction

1.1 The motivation for string theory

The standard model of particle physics and Einstein's theory of general relativity constitute the fundament of modern theoretical physics, and they explain almost every experimental data from particle and astrophysics [1, 2]. Despite this impressive success there are several theoretical drawbacks, which make us believe that there exists a more fundamental theory underlying both.

First of all, the standard model of particle physics (SM) contains a scalar field, the Higgs boson, which is needed to generate the masses of the SM particles by the mechanism of spontaneous symmetry breaking. Even though it has not been observed so far, it would come as a great surprise if it will not be discovered in the upcoming experiments at the Large Hadron Collider (LHC) at CERN. But even if one assumes its existence it is well known that the Higgs boson suffers from the so-called *hierarchy problem*. It states that scalar fields should get masses of the order Λ^2 if the SM is valid up to an energy scale Λ. So if Λ is much larger than the electro-weak scale the bare value of the Higgs mass has to be fine tuned in such a way that the quantum corrections cancel up to some 100 GeV, which seems quite unsatisfactory. A natural solution to the hierarchy problem would be to take Λ to be of the order of the electro-weak scale and to replace the SM above that scale by a theory, which somehow does not give rise to quadratic corrections in its own cut-off Λ'.

But there are also more fundamental questions that do not find an answer within the SM. As a consistent quantum field theory (QFT) the SM appears to be highly arbitrary in the sense that there exists no mechanism, which chooses the observed particle spectrum, the gauge group or even four-dimensional space-time. Furthermore there are roughly 20 free parameters, whose values have to be determined by experiment.

Another shortcoming of the SM is related to the most important problem of general relativity (GR). In the same way the SM neglects any gravitational effects in its usual formulation in flat Minkowski space, GR appears as a classical theory, neglecting any quantum effects. Thus, even though modern theoretical physics is build upon both theories, they seem to ignore the existence of the respective other. This issue begs for an explanation within a unified theory of GR and QFT.

During the last decades several ideas were proposed to solve the above mentioned problems with different success. Supersymmetric QFTs (see [3] for an introduction and further references) for example have exactly the properties needed to avoid the hierarchy problem. The symmetry between bosons and fermions leads to a cancellation of quadratic divergences such that the quantum corrections to the Higgs mass depend only logarithmically on Λ', which could be as large as the Planck mass without leading to a fine tuning problem.

An attempt to reduce the arbitrariness of the SM is given by the so-called *grand unification theories* (GUTs). The idea here is the embedding of the SM gauge group $SU(3) \times SU(2) \times U(1)$ into a simple gauge group such as $SU(5)$, $SO(10)$ or E_6. In this scenario there is only one gauge group factor at some high energy scale, which then reduces to the SM gauge group by some generalized Higgs mechanism. It turns out that only in a supersymmetric extension of the SM the gauge couplings can consistently be unified. One may view this as another motivation for supersymmetry.

In order to achieve a unification of GR and the SM, the famous idea of Kaluza and Klein [4] was to assume more than four space-time dimensions. In order to make contact with observation the extra dimensions should be small enough to escape detection by todays accelerators. The isometry group of the internal space gives rise to gauge fields in four dimensions even if the higher dimensional theory only involves gravity. To make this more precise let us consider a five-dimensional metric g_{mn} with a circle as internal space. Regarded as four-dimensional field, it contains the four-dimensional metric $g_{\mu\nu}$, a vector field $g_{\mu 5}$ and a scalar g_{55}. The vector turns out to obey the Maxwell equations in a curved background. In this way one has a unified a four-dimensional theory of gravitation and electromagnetism into a five-dimensional theory of pure gravity. The value of the gauge coupling is related to the radius of the internal circle and thus gets a deeper geometrical origin. But already in this simple toy model there is a problem that persists to much more advanced realizations of the Kaluza-Klein (KK) idea. The radius R is related to the scalar field corresponding to the g_{55}-component of the metric, and the problem is that it turns out to be massless. Hence, nothing fixes the value of the gauge coupling, i.e. the radius R. Uncharged massless scalar fields are called *moduli* and the problem of generating masses for such fields goes under the name of *moduli stabilization* which plays an important role in this thesis.

To overcome the classical nature of GR, the most obvious idea would be to just quantize it as one does with ordinary classical field theories. But it turns out that this quantization leads to ultraviolet divergences which appear to be non-renormalizable (see however [5]).

Most of the different approaches to extend or unify the SM and GR merge naturally in string theory (see [6] for an introduction). The basic point of string theory is to replace point particles by strings, i.e. one-dimensional objects. Upon quantization the string spectrum, i.e. the vibrational modes of the string, contains particles as they occur in the SM and a spin two particle, the graviton, which turns string theory into a viable candidate for the unification of the SM with GR. But even better, roughly speaking, the extended nature of the strings smears out the location of interactions in a way that removes the ultraviolet divergences encountered in the conventional QFT approach towards quantum gravity. Although this is a great achievement, string theory has not fully solved the problem of quantizing gravity since it considers strings in a given background space-time. The gravitons in the string spectrum describe small fluctuations around this vacuum and string theory thus provides only a consistent perturbation theory of fluctuations around a given background.

Historically the motivation for the first formulation of string theory was rather different. In the late 1960s, the bosonic string gave the theoretical background to derive the *Veneziano amplitude*, which was proposed as an amplitude for meson scattering before the advent of quantum chromodynamics (QCD). After improved experimental data ruled out the Veneziano amplitude as a hadronic amplitude, string theory was reinterpreted as a unified theory of gravity and all other fundamental forces in 1974 [7] by studying the spectrum of the quantized theory. The presence of a tachyonic field and the lack of any fermionic fields in the bosonic string theory led to the formulation of supersymmetric string theories, called superstring theories. Thus supersymmetry appears in string theory at a much more fundamental level than just as an extension as it does for the SM.

It turns out that a QFT of one-dimensional objects is only consistent in a ten-dimensional space-time and this immediately brings the KK idea back into the game. Six of the dimensions have to be compactified in order to obtain our four-dimensional world. Another consequence of consistency is, that there are only three possible superstring theories, the type I and the type IIA/IIB string theories. Furthermore there are two so-called heterotic string theories, which are the result of a hybrid construction, combining type II and bosonic strings. The type II theories seemed to lead to $\mathcal{N} = 2$ supersymmetry in four dimensions and too small gauge groups which made them phenomenologically unattractive. During the so-called *first superstring revolution* in the mid 1980s, triggered by [8], compactifications of the other superstring theories, however, gave rise to quasi-realistic particle spectra and gauge groups large enough to contain the SM gauge group, naturally employing the idea of grand unification.

But some features of superstring theory remained unclear. Similar to the arbitrariness of the SM as a QFT, there were now different superstring theories and no mechanism to prefer one over the other. Furthermore string theory was only defined as a perturbative expansion, which could only be used directly at weak coupling. In the early 1990s the situation could be improved by the discovery of the so-called D-branes [9], which implicitly were always present in string theories as boundary conditions of open strings but now could be identified with solitonic objects arising in the effective ten-dimensional supergravity theories of type II string theory. This made it also possible to construct quasi-realistic compactifications in the type II string theories since D-branes can lead to larger gauge groups and supersymmetry breaking. Maybe even more important they triggered the so-called *second superstring revolution* in the mid 1990s in which it became clear that all the different superstring theories are related to each other. The central idea, called duality, is that the strong coupling limit of one theory is equivalent to the weak coupling limit of another theory. The complete picture is, that all the string theories are different limits of one unifying theory called *M-theory*, whose low energy effective theory is eleven-dimensional supergravity, the unique supersymmetric theory in the highest possible dimension. In this way string theory, or now M-theory, appears to be a unique theory.

However, this high degree of uniqueness is spoiled by the the requirement of choosing a background around which to expand the KK reduction, leaving many possibilities for the resulting four-dimensional theory. And even if one finds the background which gives exactly the SM spectrum and gauge group, one still has to explain why nature chooses this one. A related problem is that even for a fixed background, as we already saw, the KK reduction leads to the problem of massless scalar fields which in turn leaves physical quantities such as gauge couplings undetermined and renders the vacuum of the theory degenerate. Furthermore massless scalar fields may lead to an unobserved fifth force. So, all in all, progress in phenomenology has been much more limited than had been hoped in the mid 1980s. The origin of the structure of the SM is not better understood now than it was then. Advances in this area have been mostly internal and a decisive low-energy test of string theory does not seem possible, since in any terrestrial experiment, unless the string scale is extremely low, all new signatures such as supersymmetry or extra dimensions find an explanation within string theory but they do not prove string theory.

This implies that astrophysical observations might become more and more important in order to find any experimental signature of string theory. But for that one first has to know how string theory predicts cosmological observables. This is a relatively new area of research, called string cosmology, and it has a strong relation to the already mentioned problem of moduli stabilization as we will see in this thesis. Recent advances in observational cosmology have brought us closer to a fundamental understanding of the origin of structure in the universe. Observations of variations in the cosmic microwave background (CMB) temperature and of the spatial distribution of galaxies in the sky have yielded a consistent picture in which gravitational instability drives primordial fluctuations to condense into large-scale structures, such as our own galaxy. Moreover, quantum field theory and GR provide an elegant microphysical mechanism, *inflation*, for generating these primordial perturbations during an early period of accelerated expansion. The resulting paradigm of a universe undergoing inflation [10, 11] at early times, and dominated by cold dark matter and dark energy at late times, has sometimes been referred to as a standard model for cosmology. So, if string theory wants to be the theory of everything it has to explain all these cosmological observations. But in fact there exists a mutual relevance of string theory and cosmology, because if one evolves the expansion of the universe back in time using the equations of GR and the SM, one hits a regime in which both descriptions break down and physics beyond the SM and GR is required. In particular one would need a consistent description of quantized gravity, whose best developed candidate seems to be string theory. This immediately leads to the question whether one can implement the mechanism of inflation in string compactifications. As we will review later, the best developed models of inflation are based on a scalar field, the inflaton, moving in a non-trivial potential. This immediately suggests that one of the moduli present in string compactifications might play the role of the inflaton provided one finds a way to generate a potential for it. To find explicit examples of inflation in string theory is technically quite challenging because one needs detailed knowledge of the four-dimensional effective theory resulting from string theory for a given ten-dimensional background. We will make this more precise in the next chapter motivating also the topics of this thesis, but first we will briefly sketch in the next section how string theory is actually formulated.

Finally, let us also mention that, despite the slow phenomenological progress, string theory has led to many profound results such as mirror symmetry [12, 13], an exact microscopic calculation of the Bekenstein-Hawking black hole entropy [14] and the AdS/CFT correspondence [15], some with deep connections to apparently unrelated fields.

1.2 The formulation of string theory

In this section we will establish the basic concepts to formulate string theory in a way that is adapted to the topics of this thesis. For a broad introduction into string theory see e.g. [6].

Let us consider a string moving in a D-dimensional Minkowski space-time M_D with coordinates X^M. It can be described by the embedding of the string world-sheet, i.e. the two-dimensional surface swept out by the string as it propagates in time, into space-time. This is a map from a two-dimensional surface Σ into M_D, $X^M(\sigma^1, \sigma^2) : \Sigma \to M_D$, where σ^a are the coordinates on Σ. In analogy to the point particle, the action determining the classical equations of motion for the string is taken to be proportional to the area of the world-sheet. This is known as the *Nambu-Goto action* which is classically equivalent to the *Polyakov action*

$$S_P = \frac{1}{4\pi\alpha'} \int_\Sigma \mathrm{d}^2\sigma \sqrt{h} h^{\alpha\beta} \partial_\alpha X^M \partial_\beta X^N G_{MN} \ , \tag{1.1}$$

where G_{MN} is the ten-dimensional space-time metric and $h_{\alpha\beta}$ is the two-dimensional world-sheet metric. This action is usually taken as the starting point for defining the quantum theory. The symmetries of the Polyakov action are D-dimensional Poincaré invariance, invariance under diffeomorphisms of the world-sheet and two-dimensional Weyl-invariance. Weyl invariance plays a crucial role in string theory, because it is generally anomalous under quantization. In order to obtain a unitary theory one has to demand Weyl-invariance, which in turn imposes severe constraints on the theory.

We do not want to go into the details of the quantization of this theory and just focus on the results. The spectrum of the quantum theory consists of the vibrational modes of the string. It turns out that it contains a tachyon and no fermions. To remove the tachyon and to get space-time fermions one introduces fermionic degrees of freedom on the world-sheet. Demanding a vanishing Weyl anomaly then constrains the dimension of the space-time uniquely to be $D = 10$, which we will assume from now on. One ends up with space-time fermions but the tachyon is still present. It is possible to remove the tachyon by a suitable truncation of the spectrum known as the *GSO projection*. The remaining spectrum consists of a set of massless states and an infinite tower of massive excitations whose masses are quantized in units of the string scale $\alpha'^{-1/2}$. As one usually assumes this to be of the order of the Planck mass, these states are extremely heavy.

Actually there are several possibilities to introduce world-sheet fermions and to perform the GSO projection. Together with further consistency conditions one ends up with only five consistent string theories in $D = 10$ Minkowski space-time listed in table 1.1.

Type	Massless bosonic spectrum	Gauge group G	N
IIA	$g_{MN}, B_{MN}, \Phi, A_M, A_{MNP}$	U(1)	2
IIB	$g_{MN}, B_{MN}, \Phi, A, A_{MN}, A_{MNPQ}$	-	2
Heterotic $E_8 \times E_8$	$g_{MN}, B_{MN}, \Phi, A^a_M$	$E_8 \times E_8$	1
Heterotic SO(32)	$g_{MN}, B_{MN}, \Phi, A^a_M$	$SO(32)$	1
Type I	$g_{MN}, \Phi, A^a_M, A_{MN}$	SO(32)	1

Table 1.1: The five consistent string theories in $D = 10$

Every theory contains a graviton g_{MN} and a scalar field Φ called the dilaton whose vacuum expectation value sets the value of the string coupling g_s. Furthermore all string theories except the type I are based on closed strings and their spectrum includes an antisymmetric tensor gauge field B_{MN} which is called the *NS B-field*. Besides this 'universal' part of the spectrum each string theory has its individual massless bosonic excitations, consisting of non-abelian gauge fields A^a_M, $a = 1, \dots, \mathrm{dim}G$, or antisymmetric p-form gauge fields $A_{M_1 \dots M_p}$, the so called *RR p-forms*. Strings do not carry any charge of the RR p-form fields. However it was one of the big discoveries within string theory that it actually contains objects which do carry a charge of the RR fields. They are called Dp-branes where p denotes the number of their spatial dimensions.

So far we only discussed strings in flat backgrounds. If the space-time metric is curved, then the Weyl-invariance of the classical action is still manifest. But at the quantum level it becomes non-trivial and imposes restrictions on the space-time metric. The metric can be interpreted as the couplings of the two-dimensional field theory. One can define a modified beta function β, which measures the violation of Weyl invariance. In order to preserve Weyl invariance this beta function must vanish. It can be computed perturbatively, order by order in α'. In a target space with characteristic radius L_{int} the effective dimensionless expansion parameter is $\sqrt{\alpha'}L_{int}^{-1}$. Terms with more than two derivatives in the β-function are of higher order in the $\sqrt{\alpha'}L_{int}^{-1}$ expansion. Thus if $\sqrt{\alpha'}L_{int}^{-1} \ll 1$ perturbation theory in the two dimensional theory is valid and it is possible to truncate the equations of motion at the two derivative level. This is known as the regime of *low energy effective theory.* Furthermore in this limit it is allowed to neglect the heavy string modes and consider only the massless spectrum. The leading term of the β-function for the metric is given by

$$\beta^G_{MN} = \alpha' R_{MN} \; . \tag{1.2}$$

Thus, the space-time background has to be Ricci-flat, i.e. it satisfies the vacuum Einstein equation. The condition imposed on the background field by Weyl invariance on the world-sheet is its space-time equation of motion. This relation between world-sheet and space-time properties holds for other background fields as well and can be used as an efficient method to construct effective actions whose equations of motion just reproduce the β-functions.

The equations of motion for the massless space-time fields can also be derived in an alternative way. One calculates their n-point functions and the effective space-time action is determined by demanding that its classical scattering amplitudes should reproduce these n-point functions. From this effective action one derives the equations of motion.

For both ways it turns out that the leading terms in an α'-expansion, the low energy effective theories, describe ten-dimensional supergravities, either type I supergravity in case of heterotic and type I string theory or type IIA/IIB supergravity in case of IIA/IIB string theory. For example the ten-dimensional type II supergravity action describing to lowest order in α' the low energy effective theory of the massless states of type II string theory is in string frame given by

$$S = \frac{1}{2\kappa_{10}^2} \int \mathrm{d}^{10}x \sqrt{-g}\, e^{-2\Phi} \left[R + 4(\partial\Phi)^2 - \frac{1}{2}H^2 - \frac{1}{4}e^{-2\Phi} \sum_n F_n^2 \right] \; . \tag{1.3}$$

In appendix A we collect further definitions and conventions. To make contact with observation, one would like to consider such a theory on a background of the form $X_4 \times M_6$, where X_4 could in a first step be any maximally symmetric four-dimensional space, i.e. Minkowski, de Sitter or anti-de Sitter, and M_6 is some compact six-dimensional manifold.

Chapter 2

The topics of this thesis

This thesis studies compactifications of type IIA string theory on a background of the form $\mathrm{AdS}_4 \times M_6$ where M_6 is a six-dimensional compact manifold with SU(3)-structure. In this chapter we want to introduce the basic concepts and give some motivation for the study of such compactifications. We will first review briefly the preceding developments without explaining all the details before we will more carefully introduce the topics of this thesis in separate sections.

As already mentioned in the introduction, before the discovery of D-branes, compactifications of the heterotic string seemed to be the phenomenologically most promising scenarios because they allowed for large enough gauge groups to incorporate the SM gauge group. In such compactifications, one would like to obtain an $\mathcal{N} = 1$ supersymmetric theory in four dimensions. The reason for that is twofold. First, from the phenomenological side, e.g. the hierarchy problem, one expects supersymmetry to be broken at a much lower scale than the string scale. Another and maybe even stronger motivation comes from the theoretical side. It is pretty hard to find non-trivial solutions to the ten-dimensional equations of motion, which are second order. The first order supersymmetry conditions, on the other hand, are much easier to solve, and they often extend to solutions of the full equations of motion. It turned out that in order to preserve $\mathcal{N} = 1$ supersymmetry in four dimensions the internal space has to be a so-called *Calabi-Yau manifold* which we will introduce later.

After the discovery of D-branes, the focus shifted to the type II string theories because now it was also possible in these theories to construct large enough gauge groups to incorporate the SM. However, the compactification of type II string theories on the well studied class of Calabi-Yau manifolds leads to $\mathcal{N} = 2$ supersymmetric vacua in four dimensions which seems phenomenologically unattractive since, e.g., such theories do not allow for fermions with chiral gauge interactions. Moreover, as a consequence of the Gauss law, the RR charge carried by the D-branes has to be cancelled by some objects carrying opposite RR charge. In principle this could be achieved by anti-D-branes but since they break supersymmetry explicitly one would loose its nice phenomenological properties as well as its computational control. As it turns out, type II string theories include objects which do carry opposite D-brane charge (and tension) and at the same time allow for a controlled way of breaking supersymmetry. These are the so-called *orientifold-planes* (O-planes). O-planes arise in type II string theories by modding out world-sheet parity plus a geometric symmetry σ of $X_4 \times M_6$. The O-planes are given by the fixpoint-set of this symmetry. On the level of the full string theory this implies that non-orientable string world-sheets are allowed. Focusing on the effective action, O-planes break part or all of the supersymmetry of the low-energy theory by truncating the field content of the $\mathcal{N} = 2$ supersymmetric theory to $\mathcal{N} = 1$ or $\mathcal{N} = 0$.

But even after the inclusion of O-planes another problem is still present in compactifications on Calabi-Yau manifolds, namely the moduli problem already mentioned in the introduction. Massless scalar fields corresponding to deformations of the internal space are in conflict with experiment and physical quantities such as gauge couplings remain arbitrary. This problem could be addressed in so-called *flux compactifications*. The inclusion of fluxes, i.e. non-vanishing background values for the different field-strengths present in ten dimensions, allows one to generate a potential for the scalar fields. As we will see, fluxes arise quite naturally by demanding $\mathcal{N} = 1$ supersymmetry for the *vacuum* of type II compactifications. However, in this thesis we are interested in $\mathcal{N} = 1$ effective theories, i.e. *fluctuations* around a given vacuum, for which we will still need O-planes to truncate the spectrum. In general, these are also needed for charge cancellation since the fluxes contribute to the integrated Bianchi identities with the same sign as the D-branes do.

What makes the inclusion of fluxes delicate is that they backreact in general on the geometry in such a way that they deform it away from the well-known classes of Calabi-Yau manifolds, as we will explain later. Since in type IIB compactifications, based on the work of [16], examples have been constructed where this deformation is rather mild and the resulting geometry is still conformal to a Calabi-Yau, the main focus in type II compactifications was on the type IIB side. In the following years it was shown that the moduli problem could indeed in principle be solved in such compactifications. In [17] the dilaton and complex structure moduli, i.e. deformations that roughly correspond to the shape of the internal manifold, could be stabilized by fluxes, whereas the stabilization of the Kähler moduli, corresponding to size deformations, require the inclusion of quantum effects along the lines of [18]. However, a supersymmetric vacuum is only possible for a non-positive cosmological constant and one always has to find some mechanism that breaks supersymmetry in such a way that the resulting vacuum has positive cosmological constant in agreement with observation. Several proposals have been made for such an uplift (see e.g. [19], [20]) which then fueled a broad study of the phenomenology of such compactifications concerning the SM as well as cosmology.

On the type IIA side, the deformation by the fluxes away from the Calabi-Yau case is in general much more severe and this made it difficult for some time to obtain explicit examples of type IIA flux compactifications. However, the improved mathematical understanding of, at least, a certain class of such non-Calabi-Yau manifolds in recent years [21] made it possible to study such compactifications in more detail. There are several reasons which make such compactifications an attractive area of research:

- First, as opposed to the type IIB side, in compactifications with a four-dimensional AdS_4 space-time it is in principle possible to stabilize all moduli already at tree level in a controlled supergravity regime without the use of any quantum effects. It is then an interesting question whether these compactifications can be of phenomenological interest, e.g. after the inclusion of an additional uplifting potential so as to construct meta-stable dS minima. But even without an explicit uplift potential, one can investigate whether the potential already has meta-stable dS vacua away from the supersymmetric AdS minimum. Related to that is the question of implementing some inflationary scenario in such compactifications.

- Second, type IIA orientifolds with intersecting D6-branes (see e.g. [22, 23] for reviews and many more references) offer good prospects for deriving the Standard Model from strings, as was recently demonstrated in [24]. So, if cosmological aspects can likewise be modelled, one may study questions such as, e.g., reheating much more explicitly.

- Third, vacua of type IIA string theory with AdS_4 space-time are also interesting in the context of the AdS/CFT duality, which we will introduce later. Explicit examples have been constructed recently where the AdS part is given by type IIA string theory in a background of the form $\mathrm{AdS}_4 \times M_6$, where M_6 is given by $\mathbb{CP}^3$. These examples involve vacua with $\mathcal{N} = 1$ supersymmetry as well as non-supersymmetric vacua.

In this thesis we will mainly focus on the first point which can be divided into three steps. First of all one has to find an $\mathcal{N} = 1$ supersymmetric vacuum of the ten-dimensional type IIA supergravity on a background of the form $\mathrm{AdS}_4 \times M_6$. Once a solution is found the second step would be to study small fluctuations around that vacuum and to write down a four-dimensional effective theory for the light fluctuations. In particular, one would like to check whether all the moduli have been stabilized by the fluxes. In a third step the phenomenology of the obtained vacuum could be studied. Here one would like to know whether it is possible to obtain all the features of the SM like spectrum, gauge group and so on. However, as already indicated, in this thesis we will concentrate on another phenomenologically important question, namely on how to implement inflation or to find de Sitter vacua in such compactifications. For that we will focus on the scalar fields in the four-dimensional effective theory. We will study these questions in detail for different explicit internal spaces M_6.

However, in the last chapter we will also construct non-supersymmetric vacua for some of the examples studied in the preceding chapters. These non-supersymmetric AdS_4 vacua may serve as a starting point for more realistic models in the same way as the supersymmetric ones, although they are much more difficult to obtain. Since $\mathbb{CP}^3$, mentioned in the third point above, is one of our examples, the results of that chapter are also interesting in the context of the AdS/CFT correspondence. A natural question, e.g., could be how the dual field theory construction of those vacua looks like.

In the following sections we are going to introduce the different topics of this thesis in more detail. In **section 2.1** we review the conditions the ten-dimensional background has to satisfy in order to get an $\mathcal{N} = 1$ supersymmetric vacuum in four dimensions and which role fluxes play in this construction. We will specialize this in **chapter 3** to the case of type IIA supergravity with an AdS_4 space-time and a manifold with SU(3)-structure as internal space. We will present all known explicit examples of internal manifolds that satisfy those conditions.

In **section 2.2** we dwell on the so called moduli problem which arises in string compactifications and how fluxes may solve it by generating a potential for the scalar fields. This will be the topic of **chapter 4** and **chapter 5**, where we will study the low energy theory of the examples found earlier. These chapters summarize [25].

In **section 2.3** we introduce the basics of inflation that are needed in this thesis. Furthermore we comment on the attempts to realize inflation in four-dimensional effective low energy theories that have their origin in string theory. We outline the current problems in type IIA compactifications and how they might be circumvented. This will be the subject of **chapter 6** which is based on [26].

In **section 2.4** we will recall why non-supersymmetric vacua are interesting from a phenomenological point of view. Furthermore, we will very briefly give a rough picture of the AdS/CFT correspondence with special emphasis on the $\mathrm{AdS}_4/\mathrm{CFT}_3$ case. We do this because the non-supersymmetric vacua that we construct in **chapter 7** might be of interest in that context. The results of this chapter will appear in [27].

We give a more detailed outline of this thesis in **section 2.5**.

2.1 Type II supersymmetric backgrounds with flux

We want to review the conditions that allow for a four-dimensional $\mathcal{N} = 1$ supersymmetric vacuum of type II supergravity given in the first reference of [28]. In order to find a vacuum of the ten-dimensional type II effective supergravity theory, one has to solve the equations of motion for the fields, which are given by the graviton, the dilaton, the NS B-field and the RR p-form fields as can be seen from (A.2). As we will explain later in more detail, it turns out that supersymmetry simplifies these equations in such a way that it is enough to verify supersymmetry as well as the Bianchi identities for the form fields. The Einstein equation, the dilaton equation of motion and the equations of motion for the form fields are then automatically satisfied. Here we will only consider the supersymmetry conditions and postpone the discussion of the Bianchi identities to chapter 3.

In order to get a four-dimensional (4d) $\mathcal{N}=1$ supersymmetric theory, one makes an ansatz for the ten-dimensional (10d) background to be of the form $M_{10}=M_4\times M_6$, where M_6 is some six-dimensional (6d) compact space. If one further demands 4d maximal space-time symmetry (i.e. Minkowski, anti-de Sitter (AdS) or de Sitter (dS) space-time) the most general 10d metric is given by

$$\mathrm{d}s^2 = e^{2A(y)} g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + g_{mn}\mathrm{d}y^\mu \mathrm{d}y^\nu \ , \tag{2.1}$$

with $\mu = 1, \ldots, 3$, $m = 1, \ldots, 6$. A is a function of the internal coordinates and it is called *warp factor*. For maximal symmetry in four dimensions the vacuum expectation value of the fermionic fields has to vanish, which means the background is purely bosonic. Thus, for any fermion χ, one should have, in a supersymmetric vacuum, $< Q_\epsilon\chi >=< \delta_\epsilon\chi >= 0$, where Q is the preserved supersymmetry generator and ϵ is the corresponding supersymmetry parameter. In type II theories, the fermionic fields are two gravitinos ψ^A_M , $A = 1, 2$ and two dilatinos λ^A. The bosonic part of the supersymmetry transformation for the fermions is given in string frame by

$$\begin{aligned}
\delta\psi^1_M &= \left(\nabla_M + \frac{1}{4}\underline{H_M}\right)\epsilon^1 + \frac{e^\Phi}{16}\sum_n \underline{F_{(n)}}\Gamma_M\Gamma_{(10)}\epsilon^2 \ ,\\
\delta\psi^2_M &= \left(\nabla_M - \frac{1}{4}\underline{H_M}\right)\epsilon^2 - \frac{e^\Phi}{16}\sum_n \sigma(\underline{F_{(n)}})\Gamma_M\Gamma_{(10)}\epsilon^1 \ ,\\
\delta\lambda^1 &= \left(\underline{\partial}\Phi + \frac{1}{2}\underline{H}\right)\epsilon^1 + \frac{e^\Phi}{16}\sum_n \Gamma^M\underline{F_{(n)}}\Gamma_M\Gamma_{(10)}\epsilon^2 \ ,\\
\delta\lambda^2 &= \left(\underline{\partial}\Phi - \frac{1}{2}\underline{H}\right)\epsilon^2 - \frac{e^\Phi}{16}\sum_n \Gamma^M\sigma(\underline{F_{(n)}})\Gamma_M\Gamma_{(10)}\epsilon^1 \ .
\end{aligned} \tag{2.2}$$

In these equations $M = 0, ..., 10$, ψ_M stands for the column vector $\psi_M = \binom{\psi^1_M}{\psi^2_M}$ containing the two Majorana-Weyl spinors of the same chirality in type IIB, and of opposite chirality in IIA, and similarly for λ and ϵ. An underline means a contraction with gamma matrices in the form $F_n = \frac{1}{n!}F_{P_1\ldots P_N}\Gamma^{P_1\ldots P_N}$, and $H_M \equiv \frac{1}{2}H_{MNP}\Gamma^{NP}$. The NS and RR field strengths are defined as in in (A.2). We are using the democratic formulation of Ref. [29] for the RR fields, as explained in appendix A. However, the details are not so important here.

First we want to analyze the implications of this equation for the internal geometry in the absence of flux, i.e. in the absence of any background values for the field strengths H and F_n. To this end one needs to split the two supersymmetry spinors of type II supergravity into 4d and 6d spinors. As explained later, we will use only one internal Weyl spinor to do this decomposition, which then reads for IIA

$$\begin{aligned}\epsilon^1 &= \zeta^1_+ \otimes \eta_+ + \zeta^1_- \otimes \eta_- \ , \\ \epsilon^2 &= \zeta^2_+ \otimes \eta_+ + \zeta^2_- \otimes \eta_- \ . \end{aligned} \tag{2.3}$$

Inserting the decomposition (2.3) into the internal part of the gravitino variation given in (2.2) gives the condition

$$\nabla_m \eta_\pm = 0 \, . \tag{2.4}$$

The internal manifold should therefore have a globally defined spinor which is covariantly constant with respect to the Levi-Civita connection. This is a very strong requirement from the topological and differential geometrical point of view. A 6d manifold that has a globally well defined non-vanishing spinor has structure group SU(3) and vice versa. The structure group of a manifold is the group of transformations required to patch the orthonormal frame bundle. If this spinor is in addition covariantly constant the manifold is said to have holonomy group SU(3), or a subgroup thereof. A 6d manifold with SU(3) holonomy is called a Calabi-Yau manifold. It admits one covariantly constant spinor. To have more than one, the holonomy group should be smaller than SU(3) which results in a larger number of supersymmetries preserved. In this thesis we will only consider manifolds with one globally defined spinor, although when turning on fluxes it does not have to be covariantly constant anymore, as one can anticipate by looking at (2.2). This explains the use of only one internal spinor in (2.3). All in all, we see that for one covariantly constant internal spinor equation (2.3) tells us that there are two 4d supersymmetry parameters, ζ^1 and ζ^2 leading to $\mathcal{N} = 2$ supersymmetry in four dimensions.

Turning on fluxes has two effects in (2.2). First, we see that the two supersymmetry parameters ϵ^1 and ϵ^2 are not independent anymore and this typically leads to $\mathcal{N} = 1$ supersymmetry instead of $\mathcal{N} = 2$. Second, the spinors do not have to be covariantly constant anymore with respect to the Levi-Civita connection[1], or in other words the differential constraint can be relaxed. In this thesis, we will keep for the 6d internal manifold the (minimal) topological assumption of SU(3)-structure, but we will drop the assumption of SU(3) holonomy. On a manifold with SU(3)-structure, the spinor representation in six dimensions, the **4** of SO(6), can be further decomposed into representations of SU(3) as $\mathbf{4} \to \mathbf{3} + \mathbf{1}$. We see a SU(3) singlet in the decomposition, which means that there is a spinor that depends trivially on the tangent bundle of the manifold and is therefore well-defined and non-vanishing. It turns out that there are also singlets in the decomposition of 2-forms and 3-forms. Thus, we also have a non-vanishing globally well defined real 2-form and a complex 3-form. They are called J and Ω. One does not find any invariant five-forms, which

[1] On manifolds with SU(3)-structure one can always define a connection with respect to which the spinor is covariantly constant.

means $J \wedge \Omega = 0$. J and Ω can be expressed in terms of the internal spinor, and they determine a metric as we demonstrate in appendix B[2].

For a Calaby-Yau space it turns out that J and Ω are both closed. One can parameterize the deviation of a 6d manifold with SU(3)-structure from the Calabi-Yau case by five torsion classes $W_1, \ldots, W_5$ which appear in the exterior derivative of J and Ω as follows

$$\begin{aligned} \mathrm{d}J &= \frac{3}{2}\mathrm{Im}(\mathcal{W}_1\Omega^*) + \mathcal{W}_4 \wedge J + \mathcal{W}_3 \,, \\ \mathrm{d}\Omega &= \mathcal{W}_1 J \wedge J + \mathcal{W}_2 \wedge J + \mathcal{W}_5^* \wedge \Omega \,, \end{aligned} \tag{2.5}$$

where $\mathcal{W}_1$ is a scalar, $\mathcal{W}_2$ is a primitive (1,1)-form, $\mathcal{W}_3$ is a real primitive $(1,2)+(2,1)$-form, $\mathcal{W}_4$ is a real one-form and $\mathcal{W}_5$ a complex (1,0)-form. This deviation from the Calabi-Yau case, i.e. the non-vanishing torsion classes, is sometimes called *geometric flux*. Geometric flux is not a terribly well-defined concept and for us the internal manifold will have geometric flux if the Ricci scalar R is non-zero. This is consistent with the above description since Calabi-Yau manifolds are Ricci flat.

Let us now come to the differential condition in the presence of fluxes. As already mentioned fluxes relate the two spinors ϵ^1 and ϵ^2 and in particular the two external spinors ζ^1 and ζ^2 to each other. Demanding maximal 4d symmetry only allows a trivial relation between ζ^1 and ζ^2, namely they should be proportional. The complex constant of proportionality can actually be a function of the internal space, which can be included in the definition of the 6d spinors. We will therefore write

$$\begin{aligned} \epsilon^1 &= \zeta^1_+ \otimes a\eta_+ + \zeta^1_- \otimes \bar{a}\eta_- \,, \\ \epsilon^2 &= \zeta^2_+ \otimes b\eta_+ + \zeta^2_- \otimes \bar{b}\eta_- \,. \end{aligned} \tag{2.6}$$

$\mathcal{N} = 1$ supersymmetry links a and b, and how they are related tells us how the $\mathcal{N} = 1$ vacuum sits in the underlying $\mathcal{N} = 2$ effective 4d effective theory.

When (2.3) is inserted in the supersymmetry variations (2.2), the 4d piece can be factored out, and one is left with equations involving only the 6d parts of the spinors. In this way, one obtains relations between the non-vanishing fluxes and the internal geometry, described by the spinors. Since the SU(3)-structure (J, Ω) can be constructed out of the internal spinors this leads to a relation between the non-vanishing fluxes and the torsion classes introduced in (2.5). We will postpone the result of this calculation for the special case of type IIA AdS_4 compactifications to **chapter 3**, where we will also have to impose the Bianchi identities for the form fields. Furthermore, we will have to clarify, how to deal with sources such as D-branes and O-planes in those equations. We present all known solutions on internal manifolds for

[2]In appendix B we will use the language of generalized geometry, which in fact constitutes a generalization of the SU(3)-structure case to the case with two different internal spinors. However, since it allows for a very elegant formulation of the supersymmetry conditions, we will use this language in that appendix and specialize it to the SU(3)-structure case.

which one can explicitly find a vacuum of the 10d theory in the special case of type IIA AdS_4 compactifications. These manifolds are so-called *nilmanifolds* and *coset spaces* introduced in appendix C. The key feature of such manifolds is that they allow for *left-invariant* (globally defined) one-forms and that the exterior derivative of those one-forms, when expanded in two-forms, only has constant coefficients. As we will see, this makes it possible to perform explicit calculations for those manifolds.

2.2 Flux compactifications and the moduli problem

In this section, we want to sketch the problem of moduli stabilization that plagued string compactifications for a long time and how it can be resolved by fluxes. In the last section we saw that fluxes arise in the breaking of the $\mathcal{N} = 2$ supersymmetry of the vacuum down to $\mathcal{N} = 1$. Another, but related, nice feature of the inclusion of background fluxes is the possibility of generating masses for the 4d scalar fields which in fluxless backgrounds would stay massless. This is also the key advance in implementing inflation in string theory, and it goes under the name of moduli stabilization (see [28] for the current status and more references). Let us see how this works.

To obtain the 4d effective theory for a given background, one should perform a KK reduction of the 10d type II supergravity on a compact internal manifold, and keep only some finite set of light fields. Take for example a scalar $\Phi(x, y)$ fulfilling the 10d Laplace equation of motion $\Delta_{10}\Phi = 0$ in the 10d space of the form (2.1). The KK reduction consists of considering small fluctuations of the 10d fields around a given vacuum leading to the equation $\Delta_{10}(\Phi(x, y) + \delta\Phi(x, y)) = 0$. The 10d Laplace operator splits as $\Delta_{10} = \Delta_4 + \Delta_6$ and we may apply the fact that Δ_6 on a compact space has a discrete spectrum. The fluctuations $\delta\Phi(x, y)$ are then expanded into eigenfunctions of the internal Laplace operator Δ_6. The coefficients arising in this expansion are fields depending only on the external coordinates. From a 4d point of view, the term $\Delta_6\delta\Phi$ thus appears as a mass term. One ends up with an infinite tower of massive states with masses quantized in terms of $1/R$, where R is the radius of the internal manifold. Choosing the internal manifold to be small enough the massive KK states become heavy and can be integrated out. However, this way of decoupling the KK tower only works in the simplest examples and we will have to come back to this issue. As we will see, the O-planes present in our constructions might help here. The resulting effective theory encodes the dynamics of the 4d fields associated with the massless KK modes satisfying

$$\Delta_6\Phi(x, y) = 0 \ . \tag{2.7}$$

This procedure can be generalized to all fields present in 10d supergravity theories including the metric. The ansatz (2.1) specifies the 10d background metric and a gravity theory is given by fluctuations around this background. In the external directions these correspond to the 4d graviton and the effective action reduces to the standard Einstein-Hilbert term for the metric in 4d. In the compact directions the fluctuations of the metric such as changes of the size and shape of the internal manifold correspond to massless scalar fields in the 4d effective theory. Since for manifolds with SU(3)-structure the metric is completely determined by the real two-form J and the complex three-form Ω, one can divide the scalar fields corresponding to metric deformations into *Kähler moduli*, corresponding to deformations of J, and *complex structure moduli*, corresponding to deformations of Ω. In order to write the resulting 4d theory in a manifest supersymmetric form, one has to complexify these real scalar fields with the scalar fields descending from the reduction of the 10d p-form potentials. Since in Calabi-Yau compactifications without fluxes there is no potential for the scalar fields, they are not driven to any particular value which is problematic for different reasons. First of all massless scalar fields typically (though not always) lead to modifications to the gravitational force law, which are not observed. Furthermore the parameters such as, e.g., the gauge kinetic function depend on these scalars and thus physics depends on their value. In this way one finds a parameterized family of physically distinct vacua, the moduli space, connected by simply varying massless fields. This is in contrast to the well known Goldstone bosons arising in the process of symmetry breaking, where the physics of any constant configuration of this field is the same. A first idea to solve the problem of massless scalar fields appearing at some early stage of the analysis would be to incorporate higher order corrections to the potential at some later stage. Indeed, in non-supersymmetric theories there is no reason the effective potential should not depend on all of the fields. But for supersymmetric QFTs there exist quite powerful non-renormalization theorems, such that moduli spaces often persist to all orders in perturbation theory or even beyond. However, in the end we will have to break supersymmetry and so they might get masses of the order of the supersymmetry breaking scale. But in the case of low scale supersymmetry breaking, which seems phenomenologically desirable, this will be a very small mass leading to the so-called Polonyi problem [30], wherein the light moduli fields carry too much energy in the early universe, leading to overclosure.

Therefore one needs to find a mechanism in string theory which induces a potential leading to larger masses for the moduli. This mechanism is given by background fluxes. To see this qualitatively, take as an example a tensor field B_2. If its field strength $H_3 = dB_2$ admits a background flux $H_3^{flux} = \langle \mathrm{d}B_2^{flux} \rangle$, the kinetic term of B_2 yields a contribution

$$\int_{M_{10}} H_3^{flux} \wedge \star H_3^{flux} \ , \tag{2.8}$$

which via the Hodge-$\star$ couples to the metric and its deformations. In this way a non-trivial potential for the size and shape deformations of the internal manifold is induced.

The light modes of the effective theory all appear as form-field zero modes of the Laplace operator on the given manifold. For Calabi-Yau manifolds such harmonic forms are in one-to-one correspondence with non-trivial elements of the cohomology groups of the Calabi-Yau, which means that they are closed. The interactions of the low energy Lagrangian are given by the KK reduction of the ten-dimensional Lagrangian. This low energy theory is found to be a 4d $\mathcal{N} = 2$ supergravity coupled to vector- and hypermultiplets.

One way to deal with background fluxes in string compactifications is the so called *Calabi-Yau with fluxes approximation.* If the typical energy scale of the fluxes is much lower than the KK scale, one can assume that the spectrum is the same as in the fluxless case, except that some of the massless modes acquire a mass due to fluxes. This allows one still to use the powerful Calaby-Yau machinery to extract the 4d effective theory, or in other words, one still uses the basis of harmonic forms on the Calabi-Yau in which one expands the 10d fields.

But, as already explained in the last section, the fluxes backreact through the supersymmetry variations (2.2) on the geometry deforming it away from the well-understood class of Calabi-Yau manifolds to the more general case of manifolds with SU(3)-structure or even beyond that. By looking at (2.5) we see that in general one now has to use non-closed forms in the KK reduction. Unfortunately, it is still unclear how to construct a suitable basis of expansion forms for this case *in general.* A detailed discussion of the general constraints on such a basis appeared in [31] (see also [32, 33] for related work). However, as already mentioned in the last section, on the manifolds studied in this thesis, namely nilmanifolds and coset spaces (see appendix C), a natural set of expansion forms, namely left-invariant forms, exists. These forms are not necessarily closed anymore, which somehow reflects the fact that we are going beyond ordinary Calabi-Yau manifolds. This makes it possible to construct the effective action for these examples explicitly.

Interestingly for supersymmetric theories there exists an alternative, although less direct, approach to derive the low energy effective action, which we will call *effective supergravity.* The scalar potential of any 4d $\mathcal{N} = 1$ supersymmetric theory is completely specified by a Kähler potential $\mathcal{K}$ and a holomorphic superpotential $\mathcal{W}$. For theories descending from string compactifications there exist general expressions for these quantities in terms of the internal geometry and the fluxes [34, 33, 35, 36]. For more work see also [37, 38, 39]. Using these expressions, one only has to plug in the values of the background fluxes, the expansion of the geometric quantities J and Ω that define the SU(3)-structure and the expansion of the form field potentials to obtain the whole scalar potential.

In this thesis we will make use of both the effective supergravity approach as well as the KK reduction. The computation of the scalar masses of the 4d low energy effective action resulting from a KK reduction of the nilmanifold examples will be the topic of **chapter 4**. The result will serve as a check on the potential obtained by the effective supergravity approach used in **chapter 5**. Having established consistency of both we will stick to the latter and compute the scalar potential for the coset space examples. We are then able to check whether it is indeed possible to stabilize all the moduli at tree level. Furthermore the knowledge of the full potential opens up the possibility to look for cosmological applications.

2.3 Inflation in string theory

In this section we want to introduce the concept of inflation and how it may be realized in string compactifications. By far the most important property of inflation is that it can generate irregularities in the universe, which may lead to the formation of structure. The general properties of the spectrum of inflationary inhomogeneities were predicted long ago ([40]) and are in beautiful agreement with recent observations by WMAP ([41]). However, the historical motivation for inflation was rather different. It has originally been formulated to solve the so called *flatness*-, *horizon*- and *defect problem*. The first problem concerns the spatial flatness of the present-day universe, which is suggested by observations of the temperature fluctuations in the CMB. The second problem asks why the initial universe is so very homogeneous. In particular, the temperature fluctuations of the CMB only arise at the level of 1 part in 10^5, and the question is why this temperature should be so incredibly uniform across the sky. A third problem, called the *defect problem*[3], can arise if one extrapolates the Big Bang back to times much earlier than the epoch of Big Bang Nucleosynthesis. It predicts a much larger abundance of magnetic monopoles than observed.

As an illustration we will just sketch the first problem and how inflation may solve it. The most general space-time metric consistent with homogeneity and isotropy of our three-dimensional space is given by the Friedmann-Robertson-Walker (FRW) metric

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta + \sin^2\theta d\phi^2)\right] , \tag{2.9}$$

where k can take the values $1, 0 - 1$ and $a(t)$ is the time-dependent scale factor of three-dimensional space. If one now assumes the perfect fluid form for the energy-momentum tensor of cosmological matter and applies the Einstein equation to the FRW metric one resulting equation is the *Friedman equation*

$$\Omega - 1 = \frac{k}{H^2 a^2} , \tag{2.10}$$

where Ω is the total energy density of the universe and the *Hubble parameter* H is defined by

$$H \equiv \frac{\dot{a}}{a} , \tag{2.11}$$

[3]Sometimes also known as the *monopole problem*.

where an overdot denotes a derivative with respect to time. We know observationally that at at present time Ω is not hugely different from unity. On the other hand aH is a decreasing function of time during radiation or matter domination so that the right hand side of (2.10) increases. This means that at much earlier times, e.g. at the time of nucleosynthesis, Ω must be yet closer to 1. The flatness problem states that such finely tuned initial conditions seem extremely unlikely.

The fundamental idea of inflation is that the universe undergoes a period of accelerated expansion, defined as a period when $\ddot{a} > 0$, at early times. The effect of this acceleration is to quickly expand a small region of space to a huge size, diminishing spatial curvature in this process, making the universe extremely close to flat. By further examining the Einstein equation applied to the FRW metric and a perfect fluid energy-momentum tensor, one can show that in order to get $\ddot{a} > 0$ one needs a material with the unusual property of a negative pressure. Such material may be given by scalar fields. In the last section, we saw how fluxes helped us to obtain masses, i.e. a potential, for the scalar fields of string compactifications. Here we learn that scalar fields might also provide a mechanism to realize inflation in the low energy theory. As we will demonstrate this is only possible if there exists a non-vanishing potential for the scalar fields. So, the non-vanishing scalar potential induced by the inclusion of background fluxes does not only allow for a solution to the moduli problem but it also provides a way to realize inflation in string theory. Let us see how scalars fields can realize inflation.

For simplicity we will specialize to the homogeneous case, in which all quantities depend only on cosmological time and set $k = 1$. The equation of motion for a scalar field is given by

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \frac{dV}{d\phi} = 0 \ , \tag{2.12}$$

which can be thought of the usual equation of motion for a scalar field in Minkowski space, but with a friction term due to the expansion of the universe. The Friedmann equation with the scalar field as the only energy source is given by

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3M_P^2}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right] \ . \tag{2.13}$$

If $\dot{\phi}^2 \ll V(\phi)$ we get from this equation

$$a(t) \propto e^{\sqrt{V(\phi)}} \ , \tag{2.14}$$

so that the resulting expansion is certainly accelerating. In a loose sense the negligence of the kinetic energy is equivalent to the field slowly rolling down its potential which we will now make more precise.

Technically, the *slow-roll approximation* for inflation involves neglecting the $\ddot{\phi}$ term in (2.12) and the kinetic energy of ϕ compared to the potential energy in (2.10).

The scalar field equation of motion (2.12) and the Friedmann equation (2.13) then become

$$H^2 \simeq \frac{V(\phi)}{3M_P^2} \;, \qquad 3H\dot{\phi} \simeq -V'(\phi) \;, \tag{2.15}$$

where a prime denotes a derivative with respect to ϕ. These conditions will hold if the two *slow-roll conditions* are satisfied. They are given by

$$\epsilon \ll 1 \quad \text{and} \quad |\eta| \ll 1 \,, \tag{2.16}$$

where the *slow-roll parameters* are defined as

$$\epsilon \equiv \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2 \quad \text{and} \quad \eta \equiv M_P^2 \frac{V''}{V} \;. \tag{2.17}$$

It is easy to see that the slow-roll conditions yield inflation. If one differentiates the definition of the Hubble-parameter with respect to time, one gets

$$\frac{\ddot{a}}{aH^2} = \frac{\dot{H}}{H^2} + 1 \;. \tag{2.18}$$

This should be larger than one to get inflation which means

$$\frac{\dot{H}}{H^2} > -1 \;. \tag{2.19}$$

But in slow-roll one has

$$\frac{\dot{H}}{H^2} \simeq \epsilon \;, \tag{2.20}$$

which will be small. Smallness of the η-parameter helps to ensure that inflation will last long enough.

As already mentioned in the introduction one may also hope to test string theory by cosmology. However, a direct test seems difficult because any signal that arises in string theory can also arise in a suitable low-energy effective QFT, as it is the case for any earth based experiment. But if one is extremely lucky, some high-energy phenomenon does not decouple at low energies. An example is given by cosmic strings and their detection would certainly be one of the greatest discoveries ever made. A more conservative approach would be to check for signals, which are generic in string-derived effective Lagrangians, but are highly unnatural from a conventional field-theory viewpoint. For example in many string based inflationary models the primordial tensor signal is very small. Hence, an observation would eliminate the majority of presently known models of inflation implemented in string theory.

Let us briefly sketch how inflationary models in string theory have been constructed so far. For the current status of inflation in string theory see [42]. Some earlier developments in string cosmology relied on the hope that whatever mechanism eventually stabilizes the moduli it would not have important side effects for models of inflation which resulted in the two step strategy of first fixing all the moduli and the adding some additional ingredient to realize inflation. Over the last years it turned out that this hope is often violated so that the problem of moduli stabilization and inflation in string theory are ultimatively linked together in a wide class of models.

The most prominent and detailed examples of inflationary models in string theory were obtained in type IIB flux compactifications with orientifolds and D3/D7-branes. As already mentioned, in these models the backreaction of the fluxes on the geometry is rather mild, and the internal manifold turns out to be still conformal to a Calabi-Yau manifold. This allows one to still use the whole machinery of Calabi-Yau compactifications and makes it possible to obtain the 4d effective potential for the scalar fields. However, in these models the fluxes turn out to stabilize only the dilaton and the complex structure moduli [17], while the Kähler moduli stabilization requires the use of quantum effects, e.g. along the lines of KKLT [18]. In addition one still needs a mechanism to uplift the resulting AdS_4 minimum to a dS vacuum. In [18] this is done by the inclusion of an $\overline{\mathrm{D3}}$-brane, which breaks supersymmetry explicitly. The role of the inflaton is played by the open string modulus corresponding to the separation of a D3/$\overline{\mathrm{D3}}$. Another uplift mechanism is given in [20] where one switches on some flux on a D7-brane, breaking supersymmetry only spontaneously. The inflaton is this time given by the separation of the D3-brane from a D7-brane. There also exist models in which the inflaton is played by some closed string moduli, e.g. in the large volume compactifications of [43].

In contrast to type IIB string theory, comparatively little is known about inflation in type IIA string theory. In [44] an example was given in which all moduli were stabilized. This example only made use of 3-form NSNS-flux, RR-fluxes, D6-branes and O6-planes. In addition to these ingredients [45, 46, 47] also included geometric fluxes. The advantage of such models is their explicitness and the possibility to stabilize the moduli at tree level in a well-controlled regime (corresponding to large volume and small string coupling) with power law parametric control (instead of logarithmic as in type IIB constructions along the lines of [18]). Possible cosmological applications were subsequently explored in a number of papers, with surprisingly little success. In [48], for instance, a simple F-term uplift to a meta-stable de Sitter vacuum based on an effective O'Raifeartaigh sector was found to be impossible. Using similar arguments, the authors of [49, 50] could also formulate a no-go theorem against slow-roll inflation and de Sitter vacua for general type IIA models with only 3-form NSNS-flux, RR-fluxes, D6-branes and O6-planes. As additional ingredients that can circumvent this no-go theorem, the authors of [50] identified geometric fluxes, NS5-branes and/or the more exotic non-geometric fluxes.[4]

Since the explicit examples of string compactifications, given in this thesis, contain geometric fluxes, i.e. they deviate from the Calabi-Yau case, they circumvent the

[4]Recent progress obtaining inflation with these ingredients appeared in [51].

above mentioned no-go theorem and thus might allow for dS vacua or inflation. We will deal with this question in **chapter 6**.

2.4 Non-supersymmetric vacua

Most string compactifications to four space-time dimensions built so far preserve at least $\mathcal{N} = 1$ supersymmetry. The main reason to focus on supersymmetric string vacua is two-fold. First, supersymmetric vacua are relatively easy to construct. The underlying supersymmetry equations are first-order differential equations, whose solutions are known in several instances. Second, from the phenomenological point of view, supersymmetric vacua are a good starting point, since a promising scenario is to assume that space-time supersymmetry is broken at the TeV scale, much below a string scale or a compactification scale not far from the Planck mass.

On the other hand we know that supersymmetry is eventually broken in nature. Hence, a stringy realization of our observed world should involve, in some sense, a non-supersymmetric string vacuum. It is a challenging task to find such non-supersymmetric vacua directly because one has to solve the full string equations of motion. Even in the supergravity approximation, this implies solving generically cumbersome second order differential equations whose solutions are complicated and to a large extent unknown. In practice, however, one may still hope to break supersymmetry in a controlled way, by modifying a certain supersymmetric background. One may then try to add some additional structure to uplift these vacua to dS in the same way as one does for the supersymmetric vacua.

Another strong motivation for the study of AdS_4 vacua, independent of the amount of preserved supersymmetry, is related to the AdS/CFT correspondence [52]. We only want to give a rough picture of where the results obtained in this thesis might find an application in that correspondence. We already mentioned in the introduction that there exist some remarkable dualities relating the different string theories or M-theory to each other. However, with the AdS/CFT correspondence an entirely new class of dualities has been conjectured. It relates conventional (non-gravitational) quantum field theories to string theories and M-theory. The AdS/CFT correspondences are dualities in the usual sense: when one description is weakly coupled, the dual description is strongly coupled. Thus, assuming that the conjecture is correct, it allows the use of weak-coupling perturbative methods in one theory to learn non-trivial facts about the strongly coupled dual theory.

The basic idea of the AdS/CFT duality and its generalizations is that string theory or M-theory in the near-horizon geometry of a collection of coincident D-branes or M-branes is equivalent to the low-energy world-volume theory of the corresponding branes. To make this more precise consider for example type IIB string theory. Its low energy effective action is given by the type IIB supergravity theory given in (1.3). Dp-branes arise as solitonic solutions to the equations of motion resulting from this action. Because it is the case that is best understood, let us take as an example D3-branes. They fill the four space-time dimensions and have six transverse directions. The resulting metric describes asymptotically a flat Minkowski space, but taking the near-horizon limit leads to a space of the form $\mathrm{AdS}_4 \times S^5$. The correspondence now states that type IIB string theory on this near-horizon space is dual to the D3-brane world-volume theory, which is given by $\mathcal{N} = 4$ super Yang-Mills theory. The string theory background corresponds to the ground state of the gauge theory, and excitations and interactions in one description correspond to excitations and interactions in the dual description. In this specific case, for example, one might hope to get insight into the strong coupling limit of a 4d gauge theory such as QCD by studying the weakly coupled string theory. Of course, realistic models of QCD should be able to explain confinement and chiral symmetry-breaking, properties which are not present in $\mathcal{N} = 4$ super Yang-Mills theories due to the large amount of unbroken supersymmetry. However, there is a variety of ways to break these symmetries so as to get richer models.

During the last years another example attracted more and more attention, namely that of M2-branes arising as solitonic objects in eleven-dimensional supergravity, the low-energy theory of M-theory. A non-perturbative understanding of M-theory is of great interest from the theoretical side since M-theory is believed to be the unifying theory of all string theories. The near-horizon geometry is given by $\mathrm{AdS}_4 \times S^7$ and only very recently there was progress in the understanding of the world-volume theory of coincident M2-branes [53, 54]. Again one can hope to learn something about the field theory side from the gravity side. Three dimensional conformal field theories could for example describe interesting conformal fix points in condensed matter systems. But also the other direction seems now interesting. The $\mathrm{AdS}_4/\mathrm{CFT}_3$ correspondence opens up the possibility to study some portion of the landscape of 4d backgrounds of string theory with negative cosmological constant.

In [54] a three-dimensional Chern-Simons-matter theory with gauge group $U(N)_k \times U(N)_{-k}$, where k denotes the level of the Chern-Simons theory, were constructed which explicitly realized $\mathcal{N} = 6$ superconformal symmetry. It was argued that this theory at level k describes the low energy limit of N M2-branes probing a $\mathbb{C}^4/\mathbb{Z}_k$ singularity. At large N this theory is then dual to M-theory on $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$. This description is weakly curved for $N \gg k^5$, while for larger values of k a circle in the M-theory description becomes small, and the more appropriate descriptions is in terms of type IIA string theory on $\mathrm{AdS}_4 \times \mathbb{CP}^3$. These gravity duals are old solutions [55, 56] that, of course, also have $\mathcal{N} = 6$ supersymmetry and only involve fluxes for F_2 and F_6, so in particular no flux for F_0.

In [57] it was realized that by allowing for different levels k_1 and k_2 for each $U(N)$ factor in the gauge groups of the Chern-Simons theory it was possible to relate the difference of both to the F_0 flux: $F_0 = k_1 - k_2$, leading to a field theory interpretation of the F_0 flux on the gravity side. And in fact in [58, 59] solutions of type IIA string theory with non-vanishing F_0 have been constructed on a space whose topology is $\mathbb{CP}^3$. These solutions have only $\mathcal{N} = 1$ supersymmetry but they happen to have a parameter space that, although discretized by flux quantization, gets arbitrarily close to the $\mathcal{N} = 6$ solutions of [55, 56]. Exploiting this fact, Chern-Simons theories have been constructed in [57] which are, in a sense, small deformations of the original $\mathcal{N} = 6$ Chern-Simons theory. Four different ways of deforming this theory have been identified, leading to $\mathcal{N} = 0, 1, 2, 3$ supersymmetric Chern-Simons theories. The gravity duals of the $\mathcal{N} = 2, 3$ cases have been constructed in [60] but they will not play any role in this thesis. The gravity duals for the $\mathcal{N} = 0, 1$ cases have been identified already in [57]. As anticipated, the $\mathcal{N} = 1$ case corresponds to the solution of [58, 59], whereas the $\mathcal{N} = 0$ solution was constructed in [57, 61]. It is here were the results of this thesis might find their application. Among others we will consider compactifications of type IIA string theory on a space that is topologically equivalent to $\mathbb{CP}^3$. In chapter 3 we will reproduce the solution found in [58, 59]. In **chapter 7** we will then try to find non-supersymmetric vacua ($\mathcal{N} = 0$) for this particular case. We will find the $N = 0$ solution of [57] as well as some other known non-supersymmetric solutions given in [62, 63] and [64]. But we will also find new non-supersymmetric solutions not discussed in the literature before.

2.5 Outline of this thesis

After the general introduction into string theory in **chapter 1** and the somewhat more detailed crash-course on flux compactifications and their relation to inflation in **chapter 2**, we now make things concrete for the case of type IIA string theory.

In **chapter 3** we solve the equations of motion for the ten-dimensional fields for the case in which the 10d background space takes the form (2.1) with the external part being 4d AdS space-time and the internal manifold has SU(3)-structure. To do so we will have to solve the supersymmetry variations (2.2) and to impose the Bianchi identities for the form fields. Furthermore, we will comment on the introduction of sources such as D-branes and O-planes in our equations. The result will be a set of conditions which have to be satisfied by the internal manifold in order to allow for a supersymmetric vacuum of type IIA supergravity. Finally we will have to make sure that our construction is self-consistent, i.e. that we are in a parameter regime in which the supergravity description is valid. We will present solutions on a class of manifolds, namely nilmanifolds and coset spaces introduced in appendix C, which are tractable enough to find such vacua explicitly. This consists of two steps. First, we will have to make sure that a given manifold admits an SU(3)-structure at all, and, second, this manifold has to meet the derived conditions for a supersymmetric vacuum. We will see that this leaves only a few examples. This chapter is mostly based on [59] while some results appeared in [25]. Based on this chapter, we will pursue three directions in this thesis, which all can be studied independently.

First of all, having found such explicit vacuum solutions, we will perform in **chapter 4** for the nilmanifolds the KK reduction of the 10d fluctuations around the vacuum and compute the masses for the 4d scalar fields. In **chapter 5** we will first use the effective supergravity approach to compute the scalar potential for the nilmanifolds and compare the resulting masses of the two approaches. The consistency with the KK reduction will provide a non-trivial check on the effective supergravity approach. Having confirmed its applicability we will use it to compute the scalar potential for the coset spaces. This chapter is entirely based on [25].

Secondly, we want to study the question of implementing inflation in the obtained low energy effective theories. This amounts to analyze the scalar potentials and their applicability for slow-roll inflation. In the first part of **chapter 6** we are able to prove in most cases the impossibility of implementing inflation. For that we only use the geometry of the internal manifold which makes this part independent from the preceding chapters. In a second part we will study the only case for which we were not able to exclude inflation and in this case we need the potential computed before. This chapter is based on [26].

Finally, in **chapter 7** we will construct non-supersymmetric vacua for some specific cosets of the preceding chapters. These examples play a prominent role in the AdS_4/CFT_3 correspondence, and our results should be of interest in that context. The results of this chapter will appear in [27].

We will summarize and conclude in **chapter 8**. Definitions and conventions, theoretical background material and computational details are delegated to the appendices. In **appendix A** we derive the equations of motion for type II supergravity. In **appendix B** we briefly review generalized geometry which allows for a very elegant formulation of the $\mathcal{N} = 1$ supersymmetry conditions for type II theories. In **appendix C** we introduce the manifolds that we study in this thesis. Finally, in **appendix D** we comment on a computational subtlety that we will encounter later.

Chapter 3

Supersymmetric type IIA AdS_4 compactifications

In this chapter we review the conditions that lead to a supersymmetric $\mathcal{N}=1$ vacuum of type IIA supergravity, i.e. a solution of the equations of motion, with an AdS_4 space-time and an SU(3)-structure manifold as internal space. Let us mention that up to now all the known explicit ten-dimensional examples of $\mathcal{N}=1$ supersymmetric compactifications to AdS_4 fall within the class of type IIA SU(3)-structure compactifications and T-duals thereof. By analyzing integrability conditions, it was proved in [65, 66] that, in the context of type II supergravity, a background that is supersymmetric and whose fluxes satisfy Bianchi identities and the equations of motion is a solution to the full equations of motion (whenever there are no mixed external-internal components of the Einstein tensor, which will be our case). We also discuss how to obtain a controlled parameter regime in which the string coupling is small and supergravity is valid such that these vacua of supergravity lift to true vacua of string theory. Finally, we give the list of all known manifolds for which it is possible to find explicit solutions. These manifolds are nilmanifolds and coset spaces whose properties we review in appendix C. For additional background material and a summary of our conventions the reader is referred to appendices A and B.

3.1 Conditions for a supersymmetric vacuum

As sketched in the last chapter for an $\mathcal{N}=1$ ansatz, the supersymmetry variations (2.2) of the fermionic fields relate the internal geometry to the fluxes. By direct inspection of the these variations, the most general form of $\mathcal{N}=1$ compactifications of IIA supergravity to AdS_4 with SU(3)-structure was given in [66]. There exists a framework for IIA/IIB supergravities, called generalized geometry, which allows for a very elegant and compact description of the supersymmetry conditions for both theories leading to the same result. Since we do not really need and use this framework in this thesis, we will only mention it at some places an refer to appendix B for more details. We review the derivation of the results of [66] using generalized geometry in appendix B.1 and just state here the result. It turns out that the vacua must have constant warp factor and constant dilaton[1], Φ. Setting the warp factor to one, the solutions of [66] are given by:

$$H = \frac{2m}{5} e^{\Phi} \mathrm{Re}\,\Omega\,, \tag{3.1a}$$

$$F_2 = \frac{f}{9} J + F_2'\,, \tag{3.1b}$$

$$F_4 = f \mathrm{vol}_4 + \frac{3m}{10} J \wedge J\,, \tag{3.1c}$$

$$W e^{i\theta} = -\frac{1}{5} e^{\Phi} m + \frac{i}{3} e^{\Phi} f\,, \tag{3.1d}$$

where H is the NSNS three-form, and F_n denote the RR n-forms. Furthermore, (J, Ω) is the SU(3)-structure (defining a metric, see appendix B.2 for definitions and further details) of the internal six-manifold , i.e. J is a real two-form, and Ω is a decomposable complex three form such that:

$$\Omega \wedge J = 0\,, \tag{3.2a}$$

$$\Omega \wedge \Omega^* = \frac{4i}{3} J^3 \neq 0\,. \tag{3.2b}$$

f, m are constants parameterizing the solution: f is the Freund-Rubin parameter, while m is the mass of Romans' supergravity [67] – which can be identified with F_0 in the 'democratic' formulation [29]. $e^{i\theta}$ is the constant of proportionality between the internal supersymmetry generators: $\eta_+^{(2)} = e^{i\theta}\eta_+^{(1)}$. This reflects the fact that we are dealing with an SU(3)-structure which arises as a special case of the more general SU(3)×SU(3)-structure as explained in appendix B. The constant W is defined by the following relation for the AdS$_4$ Killing spinors, $\zeta_\pm$,

$$\nabla_\mu \zeta_- = \frac{1}{2} W \gamma_\mu \zeta_+\,, \tag{3.3}$$

so that the radius of AdS$_4$ is given by $|W|^{-1}$. The two-form F_2' is the primitive part of F_2 (i.e. it is in the **8** of SU(3)).

Furthermore, for the above solutions most of the torsion classes have to vanish

$$\mathcal{W}_1^+ = \mathcal{W}_2^+ = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0\,, \tag{3.4}$$

where the plus sign denotes the real part. The only non-zero torsion classes of the internal manifold are

$$\mathcal{W}_1^- = -\frac{4i}{9} e^{\Phi} f\,, \qquad \mathcal{W}_2^- = -ie^{\Phi} F_2'\,, \tag{3.5}$$

[1] For the case of vanishing Romans mass non-constant warp factor and dilaton are possible. We will not discuss this in this thesis.

where we have defined $\mathcal{W}_{1,2}^- = i\mathrm{Im}\,\mathcal{W}_{1,2}$. Thus (2.5) reads (see also (B.19))

$$\mathrm{d}J = -\frac{3}{2} i\,\mathcal{W}_1^- \mathrm{Re}\,\Omega\,, \tag{3.6a}$$

$$\mathrm{d}\Omega = \mathcal{W}_1^- J \wedge J + \mathcal{W}_2^- \wedge J\,. \tag{3.6b}$$

The only extra condition that follows from the Bianchi identities and equations of motion of the form fields is given by:

$$\mathrm{d}F_2' = (\frac{2}{27} f^2 - \frac{2}{5} m^2) e^{\Phi} \mathrm{Re}\,\Omega - j^6\,, \tag{3.7}$$

where we allow for a non-vanishing source-term, j^6, for D6-branes/O6-planes on the right-hand side. A somewhat delicate feature of our models is that the sources have to be smeared. The reason for this is that the supersymmetry conditions of [66] (for constant Romans mass) force the warp factor to be constant. Considering the back-reaction of a localized orientifold, on the other hand, one would expect a non-constant warp factor, at least close to the orientifold source. A possible way around this contradiction is that taking into account α'-corrections might allow for a non-constant warp factor (see also [68] for an alternative discussion). A helpful interpretation of the smearing of a localized source, whose Poincaré dual is given, roughly-speaking, by a delta-function, is that it corresponds to Fourier-expanding the delta-function and discarding all but the zero mode. In this thesis, we will adopt the pragmatic point of view that the smeared orientifolds are an unavoidable feature of our models that is consistent with a Kaluza-Klein reduction in the approximation where only the lowest modes are kept.

As already mentioned in the introduction the inclusion of sources is motivated by several reasons. First, we will find examples, which do not allow for an $\mathcal{N}=1$ vacuum without sources. Second, as we will see in the next chapter, in which we compute the effective theories of these vacua, they might provide a mechanism to decouple the KK tower. Finally, we are interested in 4d, $\mathcal{N}=1$ supersymmetric low energy effective theories, for which O-planes are necessary. The question of how to associate orientifold involutions to a smeared source turns out to be somewhat subtle. We will make the natural assumption that the different orientifolds correspond to the decomposable (simple) terms in the orientifold current. The rationale and details behind this are explained in appendix B.3. The general properties of supersymmetric sources and their consequences for the integrability of the supersymmetry equations were recently discussed in [69] within the framework of generalized geometry. It was shown in this reference that, under certain mild assumptions, supersymmetry guarantees that the appropriately source-modified Einstein equation and dilaton equation of motion are automatically satisfied if the source-modified Bianchi identities are satisfied. For this to work the source must be supersymmetric, which means it must be generalized calibrated as in [70].

But for the moment let us imagine the case $j^6 = 0$. For a given geometry to correspond to a vacuum without orientifold sources, we find from plugging (3.7) into (3.5) and using (B.22) together with the result below (B.24) that the following bound on $(\mathcal{W}_1^-, \mathcal{W}_2^-)$ has to be satisfied

$$\frac{16}{5} e^{2\Phi} m^2 = 3|\mathcal{W}_1^-|^2 - |\mathcal{W}_2^-|^2 \geq 0 \; , \tag{3.8}$$

where we have defined $|\Theta|^2 := \Theta^*_{mn}\Theta^{mn}$, for any two-form Θ.

Still assuming $j^6 = 0$ we get from (3.5) and (3.7)

$$\mathrm{d}\mathcal{W}_2^- \propto \mathrm{Re}\Omega \; . \tag{3.9}$$

So in the absence of sources the necessary and sufficient conditions for $\mathcal{N} = 1$ compactification of type IIA supergravity to four-dimensional anti-de Sitter space on manifolds with SU(3)-structure are the conditions (3.4), (3.8) and (3.9) on the torsion classes of the internal six-dimensional manifold. The fluxes are then given by (3.5) and (3.1). Defining τ as the intrinsic torsion these conditions are summarized in table 3.1.

$\tau \in \mathcal{W}_1^- \oplus \mathcal{W}_2^-$
$3\vert\mathcal{W}_1^-\vert^2 \geq \vert\mathcal{W}_2^-\vert^2$
$d\mathcal{W}_2^- \propto \mathrm{Re}\Omega$

Table 3.1: Necessary and sufficient conditions on the internal six-dimensional SU(3)-structure manifold for $\mathcal{N} = 1$ compactification to four-dimensional anti-de Sitter space, in the absence of sources.

However, the second constraint (3.8) can be relaxed by allowing for an orientifold source, $j^6 \neq 0$. As a particular example, let us consider:

$$j^6 = -\frac{2}{5} e^{-\Phi} \mu \mathrm{Re}\Omega \; , \tag{3.10}$$

where μ is a *discrete*, real parameter of dimension (mass)^2, so that $-\mu$ is proportional to the orientifold/D6-brane charge (μ is positive for net orientifold charge and negative for net D6-brane charge). In this thesis we will make the assumption that we can tune this parameter by adding orientifolds or D-branes. For D-branes this should not be a problem since they are physical objects whose number we may vary. For orientifolds, however, this seems problematic since they arise as fixpoint loci of a geometric symmetry. In a true string compactification their charge is a fixed number. In our supergravity approximation we will consider them as charged objects in the same way as the D-branes and it remains an open question, which values for the charge are possible from string theory. The addition of the source term in (3.10) was first considered in [71]. Eq. (3.10) above guarantees that the calibration conditions, which for D6-branes/O6-planes read

$$j^6 \wedge \mathrm{Re}\Omega = 0\,, \qquad j^6 \wedge J = 0\,, \tag{3.11}$$

are satisfied and thus the source wraps supersymmetric cycles. The bound (3.8) changes to

$$e^{2\Phi} m^2 = \mu + \frac{5}{16}\left(3|\mathcal{W}_1^-|^2 - |\mathcal{W}_2^-|^2\right) \geq 0\,. \tag{3.12}$$

Since μ is arbitrary, the above equation can always be satisfied, and therefore no longer imposes any constraint on the torsion classes of the manifold. For this form of the source-term, the third condition in table 3.1, (3.9), still applies.

Furthermore it is also possible to relax this condition by the inclusion of more general supersymmetric orientifold six-plane sources that do *not* satisfy eq. (3.10). Requiring this source to satisfy the calibration conditions (3.11), we find that it is now of the following form:

$$j^6 = -\frac{2}{5} e^{-\Phi} \mu \mathrm{Re}\Omega + w_3\ , \tag{3.13}$$

with w_3 a primitive (2,1)+(1,2)-form. From the Bianchi identity (3.7) we find

$$w_3 = -ie^{-\Phi} \mathrm{d}\mathcal{W}_2^- \Big|_{(2,1)+(1,2)}\,, \tag{3.14}$$

and (3.12) still unchanged.

In appendix B.3 we will explain how to associate orientifold involutions to a smeared source. Under each orientifold involution the dilaton, metric and fluxes must transform as follows [69]:

$$\begin{aligned} &\textbf{Even}: \quad \sigma^* e^{\Phi} = e^{\Phi}\,, \quad \sigma^* F_0 = F_0\,, \quad \sigma^* F_4 = F_4\,, \\ &\textbf{Odd}: \quad \sigma^* H = -H\,, \quad \sigma^* F_2 = -F_2\,, \end{aligned} \tag{3.15a}$$

whereas the SU(3)-structure transforms as

$$\begin{aligned} &\textbf{Even}: \quad \sigma^* \mathrm{Im}\Omega = \mathrm{Im}\Omega\,, \\ &\textbf{Odd}: \quad \sigma^* \mathrm{Re}\Omega = -\mathrm{Re}\Omega\,, \quad \sigma^* J = -J\,. \end{aligned} \tag{3.15b}$$

So if one allows for sources of the type described above the only non-trivial condition for an $\mathcal{N} = 1$ vacuum of type IIA supergravity on a given manifold with SU(3)-structure is the first one in table 3.1, which is (3.4). The fluxes then follow from (3.5) and (3.1). The Bianchi identity (3.7) tells us if we need sources and whether they are of the form (3.10) or even (3.13). The source parameter μ is bounded from below by (3.12).

3.2 Hierarchy of scales

To promote a given supergravity vacuum to a trustworthy approximation of a string theory vacuum we need to show that we can consistently take the string coupling constant to be small ($g_s = e^{\Phi} \ll 1$), so that string loops can be safely ignored, and that the volume of the internal manifold is large in string units ($L_{int}/l \gg 1$, where L_{int} is the characteristic length of the internal manifold), so that α'-corrections can be neglected. This can be seen by essentially employing the following scaling argument:

In the full quantum theory, all fluxes have to be quantized according to

$$\frac{1}{l^{p-1}} \int_{\mathcal{C}_p} F_p = n_p \ , \tag{3.16}$$

where $l := 2\pi\sqrt{\alpha'}$, $\mathcal{C}_p$ is a cycle in the internal manifold, and $n_p \in \mathbb{Z}$. By combining the first equation in (3.1) with (3.6a) we see that the NSNS three-form turns out to be exact in our models, hence its integral over any internal three-cycle vanishes; it therefore suffices to impose (3.16) for the RR fluxes. The issue of quantization is studied in more detail in [58]. Let $f_p/(g_s L_{int})$ be the norm of the flux density F_p, for some numbers f_p depending on the internal geometry (but not on the overall scale L_{int}). The quantization conditions (3.16) imply:

$$g_s = (f_0^3 f_4)^{\frac{1}{4}} (n_0^3 n_4)^{-\frac{1}{4}} \ ; \qquad \frac{L_{int}}{l} = \left(\frac{f_0}{f_4}\right)^{\frac{1}{4}} \left(\frac{n_4}{n_0}\right)^{\frac{1}{4}} \ ; \tag{3.17}$$

together with

$$\frac{n_2}{\sqrt{n_0 n_4}} = \frac{f_2}{\sqrt{f_0 f_4}} \ ; \qquad \frac{n_0 n_6}{n_2 n_4} = \frac{f_0 f_6}{f_2 f_4} \ . \tag{3.18}$$

It can then be easily verified that, given a solution $\{n_p\}$ to the quantization conditions (3.16), there are several different possible scalings $n_p \to N^{\lambda_p} n_p$, for $N, \lambda_p \in \mathbb{N}$, which leave the f_p's invariant and at the same time ensure that g_s is parametrically small while L_{int}/l is parametrically large (with large parameter N). This schematic argument can be made precise, by taking into account the specifics of the geometry of each internal manifold, as in [58]. Despite the fact that we are allowing for large flux quanta, it can be shown that higher-order flux corrections can also be neglected. Indeed it is not difficult to see that the parameter $|g_s F_p|^2$, which controls the size of these corrections, scales with a negative power of the large parameter N.

3.3 Solutions on nilmanifolds

In the next two sections we want to use the manifolds introduced in appendix C to construct explicit examples of the type of compactifications reviewed in section 3.1.

By trying to solve the condition for a supersymmetric vacuum, one would like to find manifolds on which one can explicitly compute the exterior derivatives appearing in (3.6). Examples for such manifolds are given by nilmanifolds and coset spaces with the restriction to left-invariant forms, as explained in appendix C. Since one obtains a global description of these manifolds it becomes quite easy to explicitly solve the supersymmetry conditions (3.1). We review the results of [25] and [59] where the solutions for nilmanifolds and coset spaces have been presented, respectively.

As follows from the discussion of section 3.1, it suffices to look for all possible six-dimensional nilmanifolds whose only non-zero torsion classes are $\mathcal{W}_{1,2}^-$. A systematic scan yields exactly two possibilities in type IIA, namely the six-torus and the nilmanifold 4.7 of Table 4 of [72] (also known as the Iwasawa manifold), which (for some values of the parameters) turn out to be related by T-duality along two directions[2].

Let us note that condition (3.8) turns out to be too stringent to be satisfied for any nilmanifold whose only non-zero torsion classes are $\mathcal{W}_{1,2}^-$. This implies that without orientifolds there are no solutions on nilmanifolds. To obtain a solution the most general ansatz for (J, Ω) would involve all 15 two-forms and 20 three-forms. It turns out that some components of J and Ω are related by coordinate transformations, which have to be compatible with the structure constants. This allows one to reduce the number of forms appearing in Ω, and it is always possible to bring J into the form $J = ae^1 \wedge e^2 + be^3 \wedge e^4 + ce^5 \wedge e^6$.

With this ansatz we impose the SU(3)-structure conditions (3.2) (or (B.17)) and we have to demand that the resulting metric (B.28) implicitly defined by (J, Ω) is positive definite. Next we impose the conditions (3.4) on the torsion classes. When there is a solution, we can read of the fluxes by using (3.5) in (3.1). Finally, we read of the form of the source term from (3.7), where (3.12) puts a lower bound on the source parameter μ. One can then check that the resulting orientifold projection is consistent with the resulting background. In this way one obtains the following two solutions.

3.3.1 The T^6 solution

Our first IIA solution is obtained by taking the internal manifold to be a six-dimensional torus. Let us define a left-invariant basis $\{e^i\}$ such that:

$$\mathrm{d}e^i = 0, \qquad i = 1, \dots, 6 \ . \tag{3.19}$$

On the torus we can just choose $e^i = \mathrm{d}y^i$, where y^i are the internal coordinates. The SU(3)-structure is given by

$$\begin{aligned} J &= e^{12} + e^{34} + e^{56} \, , \\ \Omega &= (ie^1 + e^2) \wedge (ie^3 + e^4) \wedge (ie^5 + e^6) \ , \end{aligned} \tag{3.20}$$

[2]We also found a type IIB solution with static SU(2)-structure on the nilmanifold 5.1, which forms the intermediate step after one T-duality.

It readily follows that all torsion classes vanish

$$\mathcal{W}_1^- = 0\,, \qquad \mathcal{W}_2^- = 0\,. \tag{3.21}$$

Note, however, that there are non-vanishing H and F_4 fields given by (3.1)

$$\begin{aligned} H &= \frac{2}{5} e^{\Phi} m \left(e^{246} - e^{136} - e^{145} - e^{235}\right) , \\ F_4 &= \frac{3}{5} m \left(e^{1234} + e^{1256} + e^{3456}\right) . \end{aligned} \tag{3.22}$$

From (3.7) we find that there is an orientifold source of the type (3.10) with $\mu = e^{2\Phi} m^2$, which corresponds to smeared orientifolds along $(1,3,5)$, $(2,4,5)$, $(2,3,6)$ and $(1,4,6)$. The corresponding orientifold involutions are

$$\begin{aligned} O6: &\quad e^2 \to -e^2\,, \quad e^4 \to -e^4\,, \quad e^6 \to -e^6\,, \\ O6: &\quad e^1 \to -e^1\,, \quad e^3 \to -e^3\,, \quad e^6 \to -e^6\,, \\ O6: &\quad e^1 \to -e^1\,, \quad e^4 \to -e^4\,, \quad e^5 \to -e^5\,, \\ O6: &\quad e^2 \to -e^2\,, \quad e^3 \to -e^3\,, \quad e^5 \to -e^5\,. \end{aligned} \tag{3.23}$$

3.3.2 The Iwasawa solution

The second IIA solution is obtained by taking the internal manifold to be the Iwasawa manifold. The left-invariant basis is defined by:

$$\begin{aligned} \mathrm{d}e^a &= 0, \quad a = 1, \ldots, 4\,, \\ \mathrm{d}e^5 &= e^{13} - e^{24}\,, \\ \mathrm{d}e^6 &= e^{14} + e^{23}\,, \end{aligned} \tag{3.24}$$

and is usually denoted by $(0,0,0,0,13-24,14+23)$. Up to basis transformations there is a unique SU(3)-structure satisfying the supersymmetry conditions of section 3.1:

$$\begin{aligned} J &= e^{12} + e^{34} + \beta^2 e^{65}\,, \\ \Omega &= \beta\,(ie^5 - e^6) \wedge (ie^1 + e^2) \wedge (ie^3 + e^4)\,, \end{aligned} \tag{3.25}$$

In the left-invariant basis, the metric is given by $g = \mathrm{diag}(1,1,1,1,\beta^2,\beta^2)$, and the non-vanishing torsion classes are given by

$$\begin{aligned} \mathcal{W}_1^- &= -\frac{2i}{3}\beta\,, \\ \mathcal{W}_2^- &= -\frac{4i}{3}\beta\,\left(e^{12} + e^{34} + 2\,\beta^2\, e^{56}\right) . \end{aligned} \tag{3.26}$$

By using (3.5) the fluxes follow from (3.1). Furthermore we compute from (3.26)

$$|\mathcal{W}_1^-|^2 = \frac{4}{9}\beta^2\,, \qquad |\mathcal{W}_2^-|^2 = \frac{64}{3}\beta^2\,. \tag{3.27}$$

We therefore find from (3.12) a non-zero net orientifold six-plane charge

$$\mu \geq \frac{25}{4}\beta^2\ . \tag{3.28}$$

Finally one can verify that $\mathrm{d}\mathcal{W}_2^-$ is proportional to $\mathrm{Re}\,\Omega$:

$$\mathrm{d}\mathcal{W}_2^- = -\frac{8i}{3}\beta^2\mathrm{Re}\,\Omega\ , \tag{3.29}$$

which means we have a source of the form (3.10), and the orientifold involution is the same as in (3.23).

The solution (3.25) has one continuous parameter, β, corresponding essentially to the first torsion class $\mathcal{W}_1^-$. An additional second parameter can be introduced by noting that the defining SU(3)-structure equations (B.17) are invariant under the rescaling

$$J \to \gamma^2 J; \qquad \Omega \to \gamma^3\Omega\ . \tag{3.30}$$

The additional scalar γ is related to the volume modulus via $\mathrm{vol}_6 = -\gamma^6\beta^2 e^{1\ldots6}$, as can be seen from eq. (B.18).

For the case $m = 0$, for which the bound (3.28) is saturated, the above example can also be obtained by performing two T-dualities on the torus solution of section 3.3.1, as can be checked explicitly by using the T-duality rules of [73]. We find then that $\beta = \frac{2}{5}m_T e^{\Phi}$ where m_T is the mass parameter of the dual torus solution.

3.4 Solutions on coset spaces

We will now present the IIA solutions of the type described in section 3.1 where the internal manifold is a coset, $\mathcal{M}_6 = G/H$, equipped with a left-invariant SU(3)-structure, introduced in appendix C. They can be found in [59], which also incorporates solutions that were already known [55, 74, 75, 76, 58, 77, 78] into the single unifying framework of left-invariant SU(3)-structures on coset spaces. In [58] an alternative description in terms of twistor bundles is used for the cosets of sections 3.4.2 and 3.4.3. Although this description does not allow to describe the complete parameter space on the coset $\frac{\mathrm{SU(3)}\times\mathrm{U(1)}}{\mathrm{SU(2)}}$, it is more accurate for the nearly Calabi-Yau limit in which, as we will see, the shape parameters take negative values and the coset description is not valid anymore.

We will proceed in the same way as for the nilmanifolds, although for most of the cosets we do not need to gauge away some of the possible forms appearing in the ansatz for (J, Ω), because the set of leftinvariant forms is very restricted right from the start. We will see this in the examples.

So we start by imposing the SU(3)-structure conditions (3.2) (or (B.17)) for the most general ansatz for (J, Ω). The resulting metric (B.28), implicitly defined by (J, Ω), has to be positive definite. Next we impose (3.4). In case of a solution, the fluxes are given by (3.1) where we have to use (3.5). The source term follows from (3.7), where (3.12) puts a lower bound on the source parameter μ. Again we have to show that the resulting background is consistent with the orientifold projection. This means in particular that the structure constant tensor following from (C.20) has to be even under the orientifold involution in order to ensure that the exterior derivative is even.

For the coset spaces, we will find solutions that admit $\mu \geq 0$, i.e. solutions with zero orientifold or even with net D6-brane charge. However, we will always assume that there are orientifolds present in our construction, whose charge may then be balanced by an appropriate number of D6-branes. In this way we will always end up with an $\mathcal{N} = 1$ theory. We obtain the following five solutions.

3.4.1 The $\frac{\mathbf{G_2}}{\mathbf{SU(3)}}$ solution

The G_2 structure constants can be written as:

$$\begin{aligned}
& f^1{}_{63} = f^1{}_{45} = f^2{}_{53} = f^2{}_{64} = \frac{1}{\sqrt{3}}\,, \\
& f^7{}_{36} = f^7{}_{45} = f^8{}_{53} = f^8{}_{46} = f^9{}_{56} = f^9{}_{34} = f^{10}{}_{16} = f^{10}{}_{52} \\
& = f^{11}{}_{51} = f^{11}{}_{62} = f^{12}{}_{41} = f^{12}{}_{32} = f^{13}{}_{31} = f^{13}{}_{24} = \frac{1}{2}\,, \\
& f^{14}{}_{43} = f^{14}{}_{56} = \frac{1}{2\sqrt{3}}\,, \qquad f^{14}{}_{21} = \frac{1}{\sqrt{3}}\,, \\
& f^{i+6}{}_{j+6,k+6} = f_{\mathrm{GM}ijk}\,,
\end{aligned} \tag{3.31}$$

where $f_{\mathrm{GM}ijk}$ are the Gell-Mann structure constants.

The G-invariant two-forms and three-forms are spanned by

$$\{e^{12} - e^{34} + e^{56}\}\,, \tag{3.32}$$

$$\{\rho = e^{245} + e^{135} + e^{146} - e^{236}, \hat{\rho} = -e^{235} - e^{246} + e^{145} - e^{136}\}\,, \tag{3.33}$$

respectively[3], and there are no invariant one-forms.

[3] $\hat{\rho}$ can be found by lowering one index of the purely $\mathcal{K}_i$-part of the structure constant tensor with the Cartan-Killing metric, and ρ is its Hodge dual, so they are both left-invariant. Moreover, since the structure constant tensor should be even under all orientifold involutions and the Hodge dual is odd, we find that $\hat{\rho}$ is even and ρ odd. We can immediately conclude that they should be proportional to $\mathrm{Im}\,\Omega$ and $\mathrm{Re}\,\Omega$ respectively. Of course a priori there could have been more left-invariant three-forms.

The most general solution is then given by

$$\begin{aligned} J &= a(e^{12} - e^{34} + e^{56})\,, \\ \Omega &= d\left[(e^{245} + e^{146} + e^{135} - e^{236}) + i(e^{145} - e^{246} - e^{235} - e^{136})\right]\,, \end{aligned} \tag{3.34}$$

with

$$\begin{aligned} d^2 &= a^3\,, \qquad \text{normalization of } \Omega\,, \\ a &> 0\,, \qquad \text{metric positivity}\,, \end{aligned} \tag{3.35}$$

such that a, the overall scale, is the only free parameter. For the non-vanishing torsion classes (3.5) we find

$$\mathcal{W}_1^- = -i\frac{2a}{\sqrt{3}d}\,, \qquad \mathcal{W}_2^- = 0\,. \tag{3.36}$$

Thus, the only possibility for this coset is the nearly-Kähler geometry. It will be convenient to isolate the scale a and introduce the *reduced flux parameters*

$$\tilde{m} \equiv a^{1/2} e^{\Phi} m\,, \qquad \tilde{f} \equiv a^{1/2} e^{\Phi} f\,, \qquad \tilde{\mu} \equiv a\mu\,, \tag{3.37}$$

in terms of which the background fluxes in (3.1) take the form:

$$\begin{aligned} H &= \frac{2\tilde{m}}{5} a(e^{245} + e^{135} + e^{146} - e^{236})\,, \\ e^{\Phi} F_2 &= \frac{a^{1/2}}{2\sqrt{3}}\left(e^{12} - e^{34} + e^{56}\right)\,, \\ e^{\Phi} F_4 &= a^{-1/2}\tilde{f}\mathrm{vol}_4 - \frac{3}{5}\tilde{m}a^{3/2}\left(e^{1234} - e^{1256} + e^{3456}\right)\,. \end{aligned} \tag{3.38}$$

Furthermore, we compute for the source term (3.7)

$$e^{\Phi} j^6 = -\frac{2}{5} a^{1/2}\tilde{\mu}(e^{245} + e^{135} + e^{146} - e^{236})\,, \tag{3.39}$$

which shows that j is of the form (3.10), as was already clear from (3.36) or the fact that we only have one odd three-form. The bound (3.12) gives

$$\tilde{m}^2 - \tilde{\mu} = \frac{5a^3}{4d^2} \tag{3.40}$$

As mentioned before, $\tilde{\mu} > 0$ ($\Leftrightarrow \mu > 0$) corresponds to net orientifold charge. Solutions with $\mu \leq 0$ — i.e. with net D-brane charge — are possible, but in that case we still assume that smeared orientifolds are present, which then should be compensated by introducing enough smeared D-branes. It can be easily read off from j^6 that the orientifolds are along the directions $(1,3,6), (2,4,6), (2,3,5)$ and $(1,4,5)$, leading to four orientifold involutions. One can check that all fields and the SU(3)-structure transform as in (3.15) under *each* of the orientifold involutions. Also, the structure constant tensor is even.

3.4.2 The $\frac{\mathbf{Sp(2)}}{\mathbf{S(U(2)\times U(1))}}$ solution

The structure constants are totally antisymmetric. The non-zero ones are given by:

$$\begin{aligned} f^5{}_{41} = f^5{}_{32} = f^6{}_{13} = f^6{}_{42} = \frac{1}{2}\,, \qquad\qquad f^7{}_{56} = f^{10}{}_{89} = -1\,, \\ f^7{}_{21} = f^7{}_{43} = f^8{}_{14} = f^8{}_{32} = f^9{}_{13} = f^9{}_{24} = f^{10}{}_{34} = f^{10}{}_{21} = \frac{1}{2}\,, \end{aligned} \tag{3.41}$$

corresponding to the nonmaximal embedding. The G-invariant two-forms and three-forms are spanned by

$$\{e^{12} + e^{34}, e^{56}\}\,, \tag{3.42}$$

$$\{\rho = e^{245} - e^{135} - e^{146} - e^{236}, \hat{\rho} = e^{235} + e^{246} + e^{145} - e^{136}\}\,, \tag{3.43}$$

respectively, and there are no invariant one-forms. Again the source (if present) must be proportional to $\mathrm{Re}\Omega$. The most general solution is then given by

$$\begin{aligned} J &= a(e^{12} + e^{34}) - ce^{56}\,, \\ \Omega &= d\left[(e^{245} - e^{236} - e^{146} - e^{135}) + i(e^{246} + e^{235} + e^{145} - e^{136})\right]\,, \end{aligned} \tag{3.44}$$

with

$$\begin{aligned} &a > 0\,, \quad c > 0, && \text{metric positivity}\,, \\ &d^2 = a^2 c\,, && \text{normalization of } \Omega\,, \end{aligned} \tag{3.45}$$

such that a and c are the free parameters. For the non-vanishing torsion classes (3.5) we find

$$\begin{aligned} \mathcal{W}_1^- &= i\frac{2a+c}{3d}\,, \\ \mathcal{W}_2^- &= -\frac{2i}{3d}\left[a(a-c)(e^{12}+e^{34}) + 2c(a-c)e^{56}\right]\,, \\ |\mathcal{W}_2^-|^2 &= \frac{16}{3a^2c}(a-c)^2\,. \end{aligned} \tag{3.46}$$

The nearly-Kähler limit corresponds to setting $a = c$. The two parameters correspond to the overall scale a and a parameter $\sigma \equiv c/a$ that measures the deviation from the nearly-Kähler limit, and we can make contact with the results of [58] as in [59].

For the background fluxes and source we find in terms of the reduced flux parameters (3.37):

$$
\begin{aligned}
H &= \frac{2\tilde{m}}{5} a\sigma^{1/2}(e^{245} - e^{135} - e^{146} - e^{236})\,, \\
e^{\Phi}F_2 &= \frac{a^{1/2}}{4}\sigma^{-1/2}\left[(2-3\sigma)(e^{12}+e^{34}) + (6\sigma - 5\sigma^2)e^{56}\right]\,, \\
e^{\Phi}F_4 &= a^{-1/2}\tilde{f}\mathrm{vol}_4 + \frac{3}{5}a^{3/2}\tilde{m}\left(e^{1234} - \sigma e^{1256} - \sigma e^{3456}\right)\,.
\end{aligned}
\tag{3.47}
$$

Furthermore, we compute for the source term (3.7)

$$
e^{\Phi} j^6 = -\frac{2}{5} a^{1/2}\tilde{\mu}\sigma^{1/2}(e^{245} - e^{135} - e^{146} - e^{236})\,, \tag{3.48}
$$

which shows that j is again of the form (3.10). The bound (3.12) gives

$$
\tilde{m}^2 - \tilde{\mu} = \frac{5}{16ac}\left(-4a^2 - 5c^2 + 12ac\right)\,. \tag{3.49}
$$

We introduce the same orientifold involutions as in section 3.4.1 and check that all fields and the structure constants transform appropriately.

3.4.3 The $\frac{\mathbf{SU(3)}}{\mathbf{U(1)\times U(1)}}$ solution

We choose a basis such that the structure constants of SU(3) are given by

$$
\begin{aligned}
&f^1{}_{54} = f^1{}_{36} = f^2{}_{46} = f^2{}_{35} = f^3{}_{47} = f^5{}_{76} = \frac{1}{2}\,, \\
&f^1{}_{27} = 1\,, \quad f^3{}_{48} = f^5{}_{68} = \frac{\sqrt{3}}{2}\,, \qquad \text{and all cyclic}\,.
\end{aligned}
\tag{3.50}
$$

The G-invariant two-forms and three-forms are spanned by

$$
\{e^{12}, e^{34}, e^{56}\}\,, \tag{3.51}
$$

$$
\{\rho = e^{245} + e^{135} + e^{146} - e^{236}, \quad \hat{\rho} = e^{235} + e^{136} + e^{246} - e^{145}\}\,, \tag{3.52}
$$

respectively, and there are no invariant one-forms. The source (if present) must again be proportional to $\mathrm{Re}\,\Omega$.

The most general solution is then given by

$$
\begin{aligned}
J &= -ae^{12} + be^{34} - ce^{56}\,, \\
\Omega &= d\left[(e^{245} + e^{135} + e^{146} - e^{236}) + i(e^{235} + e^{136} + e^{246} - e^{145})\right]\,,
\end{aligned}
\tag{3.53}
$$

$$\begin{aligned} &a>0,\ b>0,\ c>0\,, && \text{metric positivity}\,,\\ &d^2=abc, && \text{normalization of }\Omega\,, \end{aligned} \tag{3.54}$$

with a, b and c three free parameters.

For the non-vanishing torsion classes (3.5) we find

$$\begin{aligned} \mathcal{W}_1^- &= -i\frac{a+b+c}{3d}\,,\\ \mathcal{W}_2^- &= -\frac{2i}{3d}\left[a(2a-b-c)e^{12}+b(a-2b+c)e^{34}+c(-a-b+2c)e^{56}\right]\,,\\ |\mathcal{W}_2^-|^2 &= \frac{16}{3abc}\left(a^2+b^2+c^2-(ab+ac+bc)\right)\,. \end{aligned} \tag{3.55}$$

Putting $a=b$ we end up with a model that is very similar to the one of section 3.4.2, while further putting $a=b=c$ corresponds to the nearly-Kähler limit. Next to the overall scale a, we have this time two shape parameters $\rho\equiv b/a$ and $\sigma\equiv c/a$. For a comparison with the results of [58] see [59]. Introducing again the reduced flux parameters (3.37) we find for the fluxes and source

$$\begin{aligned} H &= \frac{2\tilde{m}}{5}a(\rho\sigma)^{1/2}(e^{245}+e^{135}+e^{146}-e^{236})\,,\\ e^{\Phi}F_2 &= \frac{a^{1/2}}{4}(\rho\sigma)^{-1/2}\left[(5-3\rho-3\sigma)e^{12}+(3\rho-5\rho^2+3\rho\sigma)e^{34}+(-3\sigma-3\rho\sigma+5\sigma^2)e^{56}\right]\,,\\ e^{\Phi}F_4 &= a^{-1/2}\tilde{f}\mathrm{vol}_4-\frac{3}{5}a^{3/2}\tilde{m}\left(\rho e^{1234}-\sigma e^{1256}+\rho\sigma e^{3456}\right)\,. \end{aligned} \tag{3.56}$$

Furthermore, we compute for the source term (3.7)

$$e^{\Phi}j^6 = -\frac{2}{5}a^{1/2}\tilde{\mu}(\rho\sigma)^{1/2}(e^{135}+e^{146}+e^{245}-e^{236})\,, \tag{3.57}$$

which verifies that j is again of the form (3.10). The bound (3.12) gives

$$\tilde{m}^2-\tilde{\mu} = \frac{5}{16abc}\left[-5(a^2+b^2+c^2)+6(ab+ac+bc)\right]\,, \tag{3.58}$$

while the orientifold involutions are still as in section 3.4.1.

3.4.4 The SU(2)×SU(2) solution

The structure constants in this case are

$$f^1{}_{23}=f^4{}_{56}=1\,,\qquad \text{and cyclic}\,. \tag{3.59}$$

This time, the coset structure does not eliminate any forms so one might think, that we would have to introduce some orientifolds before we can proceed. In particular this time we have all the six one-forms available. As we will see the resulting orientifold will project them all out. What makes the analysis tractable again is the fact that it was shown in [79] that there is always a change of basis preserving the form of the structure constants which brings J to the form

$$J = ae^{14} + be^{25} + ce^{36} \,. \tag{3.60}$$

With this result the most general solution to eqs. (3.4), (3.5),(3.7), (3.12) and (3.13) is then given by

$$\begin{aligned} J &= ae^{14} + be^{25} + ce^{36} \,, \\ \Omega &= d\Big\{ a(e^{234} - e^{156}) + b(e^{246} - e^{135}) + c(e^{126} - e^{345}) \\ &\quad - \frac{i}{h}\Big[-2\,abc(e^{123} + e^{456}) + a(b^2 + c^2 - a^2)(e^{234} + e^{156}) + b(a^2 + c^2 - b^2)(e^{153} + e^{426}) \\ &\quad + c(a^2 + b^2 - c^2)(e^{345} + e^{126})\Big]\Big\} \,, \end{aligned} \tag{3.61}$$

$$\begin{aligned} \text{with} \quad & h \equiv \sqrt{2\,a^2b^2 + 2\,b^2c^2 + 2\,a^2c^2 - a^4 - b^4 - c^4} \,, \\ \text{and thus} \quad & 0 < 2\,a^2b^2 + 2\,b^2c^2 + 2\,a^2c^2 - a^4 - b^4 - c^4 \,. \end{aligned}$$

Again a, b and c are free parameters with

$$\begin{aligned} abc &> 0 \,, && \text{metric positivity} \,, \\ d^2 &= \frac{2abc}{h} \,, && \text{normalization of } \Omega \,. \end{aligned} \tag{3.62}$$

For the non-vanishing torsion classes (3.5) we find

$$\begin{aligned} \mathcal{W}_1^- &= -\frac{2i}{3d} \,, \\ \mathcal{W}_2^- &= -\frac{2i}{3h}\sqrt{\frac{2abc}{h}}\Bigg[\frac{(b^2 - c^2)^2 + a^2(-2a^2 + b^2 + c^2)}{bc}e^{14} + \frac{(c^2 - a^2)^2 + b^2(-2b^2 + c^2 + a^2)}{ac}e^{25} \\ &\quad + \frac{(a^2 - b^2)^2 + c^2(-2c^2 + a^2 + b^2)}{ab}e^{36}\Bigg] \,. \end{aligned} \tag{3.63}$$

By a suitable change of basis we can always arrange for $a > 0, b > 0$ and $c > 0$, which we will assume from now on. In terms of the reduced flux parameters (3.37), to which we add

$$\tilde{h} = a^{-2}h\,, \quad \tilde{d} = a^{-1/2}d\,, \tag{3.64}$$

we find for the fluxes

$$\begin{aligned}
H &= -\frac{2\tilde{m}}{5}\tilde{d}\left[(e^{156}-e^{234}) + \rho(e^{135}-e^{246}) + \sigma(e^{345}-e^{126})\right]\,,\\
F_2 &= -\frac{a^{1/2}}{2\tilde{d}\tilde{h}^2}\Big\{\left[3(\rho^4+\sigma^4) - 5 + 2(\rho^2+\sigma^2) - 6\rho^2\sigma^2\right]e^{14}\\
&\qquad + \rho\left[3(1+\sigma^4) - 5\rho^4 + 2\rho^2(1+\sigma^2) - 6\sigma^2\right]e^{25}\\
&\qquad + \sigma\left[3(1+\rho^4) - 5\sigma^4 + 2\sigma^2(1+\rho^2) - 6\rho^2\right]e^{36}\Big\}\,,\\
F_4 &= a^{-1/2}\tilde{f}\mathrm{vol}_4 - a^{3/2}\frac{3\tilde{m}}{5}(\rho e^{1245} + \sigma e^{1346} + \rho\sigma e^{2356})\,.
\end{aligned} \tag{3.65}$$

This time we compute for the source (3.7)

$$\begin{aligned}
e^{\Phi}j &= -i\mathrm{d}\mathcal{W}_2^- + \left(\frac{2}{27}f^2 - \frac{2}{5}m^2\right)e^{2\Phi}\mathrm{Re}\Omega\,,\\
&= j_1(e^{234}-e^{156}) + j_2(e^{246}-e^{135}) + j_3(e^{126}-e^{345})\,.
\end{aligned} \tag{3.66}$$

with j_1, j_2 and j_3 some complicated factors depending on a, b and c whose exact form does not matter for the moment. It contains the same terms as $\mathrm{Re}\Omega$ but with different coefficients. In fact, one can check that j^6 is *not* proportional to $\mathrm{Re}\Omega$ unless $|a| = |b| = |c|$, which reduces the solution to a nearly-Kähler geometry. This time it is not immediately obvious how to choose the orientifold projection. Choosing them naively along the six terms leads to the fields and structure constants having the wrong transformation properties. In appendix B.3 we outline how to find the orientifold involutions associated to a smeared source in general and then apply the procedure to the case at hand. In order to present the resulting involutions, it is convenient to define complex one-forms as follows

$$\begin{aligned}
e^{z^1} &= \pm\frac{e^{\frac{i3\pi}{4}}d}{2\sqrt{bc(2bc-h)}}\left\{[2bc-h+i(a^2-b^2-c^2)]e^1 + [a^2-b^2-c^2+i(2bc-h)]e^4\right\}\,,\\
e^{z^2} &= \pm\frac{e^{\frac{i3\pi}{4}}d}{2\sqrt{ac(2ac-h)}}\left\{[2ac-h+i(b^2-a^2-c^2)]e^2 + [b^2-a^2-c^2+i(2ac-h)]e^5\right\}\,,\\
e^{z^3} &= \pm\frac{e^{\frac{i\pi}{4}}d}{2\sqrt{ab(2ab-h)}}\left\{[2ab-h+i(c^2-a^2-b^2)]e^3 + [c^2-a^2-b^2+i(2ab-h)]e^6\right\}\,,
\end{aligned} \tag{3.67}$$

where the signs must be chosen such that $\Omega = e^{z^1 z^2 z^2}$. Defining further the associated x and y one-forms $e^{z^i} = e^{x^i} - ie^{y^i}$, the orientifold involutions are given as in (B.38).

3.4.5 The $\frac{\mathbf{SU(3)\times U(1)}}{\mathbf{SU(2)}}$ solution

We construct the algebra by taking

$$\begin{aligned} &E_i = G_{i+3}, \quad i = 1,\dots,5; \quad E_6 = M; \\ &E_7 = G_1; \quad E_8 = G_2; \quad E_9 = G_3 \ , \end{aligned} \tag{3.68}$$

where the G_i's are the Gell-Mann matrices generating su(3), M generates a u(1), and the su(2) subalgebra is generated by E_7, E_8 and E_9. It follows that the SU(2) subgroup is embedded entirely inside the SU(3), so that the total space is given by $\frac{\mathrm{SU(3)}}{\mathrm{SU(2)}} \times \mathrm{U}(1) \simeq S^5 \times S^1$. The structure constants are

$$\begin{aligned} &f^7{}_{89} = 1, \quad f^7{}_{14} = f^7{}_{32} = f^8{}_{13} = f^8{}_{24} = f^9{}_{12} = f^9{}_{43} = 1/2 \ , \\ &f^5{}_{12} = f^5{}_{34} = \frac{\sqrt{3}}{2}, \quad \text{all cyclic} \ . \end{aligned} \tag{3.69}$$

Invariant one-forms are generated by $\{e^5, e^6\}$, and, like in the last example, the resulting orientifold will project them out. The invariant two- and three-forms are given by

$$\{e^{12} + e^{34}, e^{13} - e^{24}, e^{14} + e^{23}, e^{56}\} \ , \tag{3.70}$$

$$\{e^{145} + e^{235}, e^{135} - e^{245}, e^{126} + e^{346}, e^{146} + e^{236}, e^{136} - e^{246}, e^{125} + e^{345}\} \ . \tag{3.71}$$

The most general solution is then given by

$$\begin{aligned} J &= -a(e^{13} - e^{24}) + b(e^{14} + e^{23}) + ce^{56} \ , \\ \Omega &= d\Big\{ \left[2a(e^{145} + e^{235}) + 2b(e^{135} - e^{245}) + c(e^{126} + e^{346})\right] \\ &- \frac{i}{\sqrt{a^2+b^2}} \left[ac(e^{146} + e^{236}) + bc(e^{136} - e^{246}) - 2(a^2+b^2)(e^{125} + e^{345})\right] \Big\} , \end{aligned} \tag{3.72}$$

with

$$\begin{aligned} &c > 0 \, , \quad a^2 + b^2 \neq 0 \, , \qquad \text{metric positivity} \, , \\ &d^2 = \frac{1}{2}\sqrt{a^2+b^2}, \qquad \text{normalization of } \Omega \, , \end{aligned} \tag{3.73}$$

and a, b and c three free parameters. For the non-vanishing torsion classes (3.5) we find

$$\begin{aligned} \mathcal{W}_1^- &= -\frac{i}{\sqrt{3}d} \, , \\ \mathcal{W}_2^- &= -\frac{i\,d}{2\sqrt{3}\sqrt{a^2+b^2}} \left[-a(e^{13} - e^{24}) + b(e^{14} + e^{23}) - 2ce^{56}\right] \ . \end{aligned} \tag{3.74}$$

By a suitable change of basis we can always arrange for $a > 0$ and $b > 0$, which we will assume from now on. Note that $d\mathcal{W}_2^-$ is not proportional to $\mathrm{Re}\,\Omega$, hence the source is not of the form (3.10). Interestingly, if we take the part of the source along $\mathrm{Re}\,\Omega$ to be zero, i.e. $j^6 \wedge \mathrm{Im}\,\Omega = 0$, we find from the last equation in (3.73) that $m = 0$. This would amount to a combination of smeared D6-branes and O6-planes such that the total tension is zero. Allowing for negative total tension (more orientifolds), we could have $m > 0$. For an arbitrary m we find the background

$$\begin{aligned} H &= \frac{\sqrt{3}\tilde{m}\,\tilde{d}}{5} a\left[2(e^{145}+e^{235}) + 2\rho(e^{135}-e^{245}) + \sigma(e^{126}+e^{346})\right] , \\ e^{\Phi}F_2 &= -\frac{a^{1/2}}{2\tilde{d}}\left[(e^{13}-e^{24}) - \rho(e^{14}+e^{23}) + \sigma e^{56}\right] , \\ e^{\Phi}F_4 &= a^{-1/2}\tilde{f}\mathrm{vol}_4 + \frac{3}{5}a^{3/2}\tilde{m}\left[(1+\rho^2)e^{1234} - \sigma(e^{1356}-e^{2456}) + \rho\sigma(e^{1456}+e^{2356})\right] , \end{aligned} \tag{3.75}$$

where we defined $\rho = b/a$ and $\sigma = c/a$ and used again (3.37). From (3.7) we compute for the source

$$\begin{aligned} e^{\Phi}j^{O6} = &\frac{\sqrt{3}\tilde{d}}{10}a^{1/2}\left(\frac{5}{\tilde{d}^2} - 4\tilde{m}^2\right)\left[e^{145}+e^{235}+\rho(e^{135}-e^{245})\right] \\ &- \frac{\sqrt{3}\tilde{d}}{20}a^{1/2}\sigma\left(\frac{5}{\tilde{d}^2} + 4\tilde{m}^2\right)\left(e^{126}+e^{346}\right) . \end{aligned} \tag{3.76}$$

One can check that for the background the source satisfies the calibration conditions (3.11). If we make the following coordinate transformation

$$e^{1'} = e^1 \,,\; e^{2'} = e^2 \,,\; e^{3'} = e^3 + \rho^{-1}e^4 \,,\; e^{4'} = e^3 - \rho e^4 \,,\; e^{5'} = e^5 \,,\; e^{6'} = e^6 \,, \tag{3.77}$$

we see clearly that j is a sum of four decomposable terms

$$\begin{aligned} e^{\Phi}j^6 = &-\frac{\sqrt{3}}{10}\tilde{d}a^{1/2}\left(\frac{5}{\tilde{d}^2} - 4\tilde{m}^2\right)(e^{1'3'5'} - e^{2'4'5'})\sqrt{1+\rho^2} \\ &+ \frac{\sqrt{3}\tilde{d}}{20}a^{1/2}\sigma\left(\frac{5}{\tilde{d}^2} + 4\tilde{m}^2\right)\left(e^{1'2'6'} + e^{3'4'6'}\right) , \end{aligned} \tag{3.78}$$

to which we can associate four orientifold involutions.

Chapter 4

Low energy physics I: The Kaluza-Klein reduction

In this chapter we want to use the direct KK reduction to compute the mass matrices for the two nilmanifold examples, i.e. the torus and the Iwasawa manifold[1], described in the last chapter. The comparison to the result of the effective supergravity approach, described in the next chapter, will then serve as a non-trivial check on the latter in the case of non Calabi-Yau manifolds. In the first section we will review the general KK procedure for the case of an AdS_4 space time. We will also show how to express the fluctuations of the RR field strengths in terms of fluctuations of their potentials. In the subsequent section we comment on the problem of decoupling the KK tower. Finally we will apply the KK reduction to the two nilmanifolds of section 3.3 and compute the mass matrices for the light fluctuations[2]. This chapter is based on [25].

4.1 Kaluza-Klein reduction

We are interested in performing a Kaluza-Klein reduction on each of the $\mathrm{AdS}_4 \times \mathcal{M}_6$ solutions described in sections 3.3.1 and 3.3.2. Let x and y be 4d space-time and internal-manifold coordinates, respectively. Moreover, let $\hat{\Phi}(x,y)$ be a 'vacuum', i.e. a particular solution of the equations of motion of ten-dimensional supergravity. The Kaluza-Klein reduction (see [80] for a review) consists in expanding all ten-dimensional fields $\Phi(x,y)$ in 'small' fluctuations around the vacuum:

$$\Phi(x,y) = \hat{\Phi}(x,y) + \delta\Phi(x,y) \ , \tag{4.1}$$

[1] More precisely, we will do this for the case $m = 0$.

[2] As a general remark, we will not consider blow-up modes associated to the fixed points of the orientifold involutions. Ideally, we would like to argue that the blow-up modes will be stabilized by flux through the blown-up cycle at a size much smaller than the size of the internal manifold. Unfortunately, such an analysis is beyond the scope of this thesis. It may be possible, however, to argue for the stabilization of the blow-up modes using a local analysis of the singularities as in [44].

keeping only terms up to linear order in $\delta\Phi(x,y)$ in the equations of motion (corresponding to at most quadratic terms in the Lagrangian). From now on the hats indicate background quantities, and the δ's denote fluctuations. The fluctuations are Fourier-expanded in the internal space:

$$\delta\Phi(x,y) = \sum_n \phi_n(x)\omega_n(y) \ , \tag{4.2}$$

where $\phi_n(x)$ are four-dimensional space-time fields, and the $\omega_n(y)$'s form a basis of eigenforms of the Laplacian operator $\Delta = \mathrm{d}\mathrm{d}^\dagger + \mathrm{d}^\dagger\mathrm{d}$ in the six-dimensional space $\mathcal{M}$ (the internal part of the vacuum solution).

In the following, we will truncate all the higher Kaluza-Klein modes in the harmonic expansion (4.2) and keep only those $\omega_n(y)$'s in (4.2) that are left-invariant on $\mathcal{M}_6$. The resulting modes are not in general harmonic, but can be combined into eigenvectors of the Laplacian whose eigenvalues are of order of the geometric fluxes. One has to make sure that such a truncation is consistent. We want to argue in the next section that indeed we can tune our parameters in such a way that the higher KK modes (the KK tower) decouples.

Plugging the ansatz (4.1)-(4.2) into the ten-dimensional equations of motion and keeping at most linear-order terms in the fluctuations, one can read off the masses of the space-time fields, i.e. the 'spectrum'. In the present case, this is accomplished by comparing with the equations of motion for non-interacting fields propagating in AdS_4. Let M and Λ be the mass of the field and the cosmological constant of the AdS space, respectively, such that

$$\text{Scalar}: \qquad \Delta\phi + \left(M^2 + \frac{2}{3}\Lambda\right)\phi = 0 \ , \tag{4.3a}$$

$$\text{Vector}: \qquad \Delta\phi_\mu + \nabla_\mu\nabla^\nu\phi_\nu + M^2\phi_\mu = 0 \ , \tag{4.3b}$$

$$\text{Metric}: \qquad \Delta_L h_{\mu\nu} + 2\nabla_{(\mu}\nabla^\rho h_{\nu)\rho} - \nabla_{(\mu}\nabla_{\nu)}h^\rho{}_\rho + (M^2 - 2\Lambda)h_{\mu\nu} = 0 \ , \tag{4.3c}$$

where Δ_L is the Lichnerowicz operator defined by:

$$\Delta_L h_{\mu\nu} = -\nabla^2 h_{\mu\nu} - 2R_{\mu\rho\nu\sigma}h^{\rho\sigma} + 2R_{(\mu}{}^\rho h_{\nu)\rho} \ . \tag{4.4}$$

With the above definitions, the Breitenlohner-Freedman bound [81] is simply

$$M^2 \geq 0 \ , \tag{4.5}$$

for the metric and the vectors. For the scalars, however, a negative mass-squared is allowed:

$$M^2 \geq \frac{\Lambda}{12} = -\frac{|W|^2}{4} \ , \tag{4.6}$$

where W was defined in eq. (3.3). Actually, we will present the results for the mass spectrum of the scalars in terms of

$$\tilde{M}^2 = M^2 + \frac{2}{3}\Lambda\,, \tag{4.7}$$

for which the Breitenlohner-Freedman bound reads

$$\tilde{M}^2 \geq -\frac{9|W|^2}{4}\,. \tag{4.8}$$

We will take $\tilde{M} = 0$ as the definition of an unstabilized modulus since from (4.3a) we see that then, if it were not for the boundary conditions of AdS_4, a constant shift of ϕ would be a solution to the equations of motion. Therefore, a constant shift of ϕ leads to a new vacuum solution.

We want to apply this strategy to the nilmanifold vacua of section 3.3. The backgrounds for these two vacua are given in section 3.3.1 and 3.3.2, and by definition they are solutions to the equations of motion of type IIA supergravity, which are given by (A.7a), (A.7b), (A.9b) as well as (A.10), and to the Bianchi identities (A.9a).

It is possible to express the fluctuations of the RR field strengths δF in terms of the fluctuations of the potentials δC in such a way that the Bianchi identity $d_H F = -j$ is automatically satisfied. This analysis is complicated by the presence of a source. We assume that the source does not fluctuate since it is associated to smeared orientifolds. For the Bianchi identities of the background and the fluctuation we find then, respectively,

$$(\mathrm{d} + \hat{H})\hat{F} = -j\,, \tag{4.9a}$$

$$(\mathrm{d} + \hat{H} + \delta H)(\hat{F} + \delta F) = -j\,. \tag{4.9b}$$

The integrability equations read

$$(\mathrm{d} + \hat{H})j = 0\,, \tag{4.10a}$$

$$(\mathrm{d} + \hat{H} + \delta H)j = 0\,, \tag{4.10b}$$

from which follows

$$\delta H \wedge j = 0\,. \tag{4.11}$$

This implies also

$$(\mathrm{d} + \hat{H})(e^{\delta B} \wedge j) = 0\,, \tag{4.12}$$

so that, subtracting (4.9a), we can define (locally)

$$-(e^{\delta B} - 1) \wedge j = (\mathrm{d} + \hat{H})\delta\omega\,. \tag{4.13}$$

Now, for orientifold sources the left hand side of this equation always vanishes. This follows because the pull-back of δB to the orientifold, $\delta B|_\Sigma$, must be zero, which implies using (B.30):

$$\delta B \wedge j = 0\,, \tag{4.14}$$

and the same for all powers of δB. Then, we can also choose $\delta\omega = 0$.

The difference between (4.9a) and (4.9b) gives the Bianchi identity for the fluctuations

$$\left(\mathrm{d} + \hat{H} + \delta H\right)\delta F + \delta H \wedge \hat{F} = 0\,, \tag{4.15}$$

which can be rewritten as

$$\left(\mathrm{d} + \hat{H}\right)\left(e^{\delta B}\delta F\right) + \delta H \wedge e^{\delta B}\hat{F} = 0\,. \tag{4.16}$$

One can easily show that (with $\delta F_0 = 0$) this Bianchi identity can be satisfied by introducing potentials δC and putting

$$e^{\delta B}\delta F = (\mathrm{d} + \hat{H})\delta C - (e^{\delta B} - 1)\hat{F} + \delta\omega\,. \tag{4.17}$$

where we can set $\delta F_0 = \delta\omega = 0$ so that we obtain

$$e^{\delta B}\delta F = (\mathrm{d} + \hat{H})\delta C - (e^{\delta B} - 1)\hat{F}\,. \tag{4.18}$$

Expanding this expression we find for the IIA-fluctuations

$$\begin{aligned}
&\delta F_0 = 0\,,\\
&\delta F_2 = \mathrm{d}\delta C_1 - m\delta B\,,\\
&\delta F_4 = \mathrm{d}\delta C_3 + \hat{H}\wedge\delta C_1 - \delta B\wedge(\hat{F}_2 + \delta F_2) - \frac{1}{2}m(\delta B)^2\,,\\
&\delta F_6 = \mathrm{d}\delta C_5 + \hat{H}\wedge\delta C_3 - \delta B\wedge(\hat{F}_4 + \delta F_4) - \frac{1}{2}(\delta B)^2\wedge(\hat{F}_2 + \delta F_2) - \frac{1}{3!}m(\delta B)^3\,.
\end{aligned} \tag{4.19}$$

For the NSNS flux we can just write

$$H = \hat{H} + \delta H = \hat{H} + \mathrm{d}\delta B\,. \tag{4.20}$$

For the Kaluza-Klein reduction of the equations of motion we will only need the terms linear in the fluctuations while for an analysis of finite fluctuations of the action one would need higher orders too. Furthermore, in the Kaluza-Klein reduction we will only need fluctuations of the physical fields $\delta F_2, \delta F_4$ since the higher-form fluxes are removed from the equations of motion using (A.1), while in the superpotential approach, which is formulated in the democratic formalism, we should work with the internal part of δF_6 instead of the external part of δF_4 as we will explain later.

4.2 Decoupling the Kaluza-Klein tower

Consistency requires that the Kaluza-Klein tower can be decoupled. This means we have to make sure that the higher Kaluza-Klein fields are really much heavier than the ones that we kept in our analysis such that we can neglect them in an effective low-energy theory. Since the Compton wavelength of the lightest excitations above the Breitenlohner-Freedman bound in four dimensions is of the order of the AdS_4 radius, we need to show that the Compton wavelength of the Kaluza-Klein excitations (which is proportional to L_{int}) satisfies:

$$|\Lambda_{\mathrm{AdS}}|L_{int}^2 \ll 1 \ , \tag{4.21}$$

where Λ_{AdS} is the four-dimensional cosmological constant. In models without orientifolds this is impossible to achieve, since the characteristic length of the internal manifold turns out to be of the same order as the radius of AdS_4. This is the problem of separation of scales which, for example, plagues the compactifications of eleven-dimensional supergravity on the seven-sphere. Ultimately, we would like to uplift our models to a de Sitter space with a small, positive cosmological constant, and the position could be taken that the question of the mass spectra should be re-addressed only after this uplifting. However, let us now study whether it is possible to tune the orientifold source such that there is a hierarchy between the two scales even before the uplifting and (4.21) is obeyed.

Taking into account $|\Lambda_{\mathrm{AdS}}| \sim |W|^2$ and using (3.1d), we find that to decouple the Kaluza-Klein scale we must impose

$$|W|^2 L_{int}^2 = \frac{1}{25}(g_s)^2 m^2 L_{int}^2 + \frac{1}{9}(g_s)^2 f^2 L_{int}^2 \ll 1 \,, \tag{4.22}$$

which means that each of the two terms on the right-hand side of the equal sign must be separately much smaller than one. Tuning the orientifold charge we can accomplish $e^{2\Phi}m^2L_{int}^2 \ll 1$. Indeed, we just need to show that we can choose μ so that it is close to its bound (3.12):

$$\mu L_{int}^2 + \frac{5}{16}\left(3|\mathcal{W}_1^-|^2 - |\mathcal{W}_2^-|^2\right) L_{int}^2 \ll 1 \ . \tag{4.23}$$

In our conventions the discrete parameter μ, which is proportional to the net number of orientifold planes $n_{\mathcal{O}6}$, is given by (up to numerical factors of order one): $\mu \sim g_s n_{\mathcal{O}6} l L_{int}^{-3}$. Taking into account that the torsion classes are given by (again up to numerical factors of order one): $|\mathcal{W}_i^-|^2 \sim L_{int}^{-2}$, we can rewrite the above equation schematically as follows:

$$n_{\mathcal{O}6} g_s \left(\frac{l}{L_{int}}\right) + a \ll 1 \ , \tag{4.24}$$

where a is a number of order one. Since $g_s\left(\frac{l}{L_{int}}\right) \ll 1$, we can then satisfy this bound by choosing some large integer $n_{\mathcal{O}6}$. Note that in the examples where we study this limit, a turns out to be negative so that we can accomplish this with positive $n_{\mathcal{O}6}$, which corresponds to net orientifold charge (as opposed to net D-brane charge).

However, we must also make sure that the second square in (4.22) is small, which means that $f g_s L_{int} \propto |\mathcal{W}_1^-| L_{int}$ is small. Manifolds for which $\mathcal{W}_1^-$ vanishes (and only $\mathcal{W}_2^-$ is possibly non-zero) are called 'nearly Calabi-Yau' (NCY) see e.g. [82]; hence for the bound (4.21) to be satisfied, the internal manifold must admit an SU(3)-structure which is sufficiently close to the NCY limit.

Once a solution for $n_{\mathcal{O}6}$ is obtained in this way, we have to make sure that it is consistent with the conditions for a small string coupling and large volume found in section 3.2. It turns out that we do not have any problems with that because we are free to rescale $n_{\mathcal{O}6} \to N^q n_{\mathcal{O}6}$ leaving (4.24) invariant, provided we take: $q = (\lambda_0 + \lambda_4)/2 \in \mathbb{N}$. For example, the reader can verify that the rescaling $\{n_0 \to N^4 n_0, n_2 \to N^6 n_2, n_4 \to N^8 n_4, n_6 \to N^{10} n_6, n_{\mathcal{O}6} \to N^6 n_{\mathcal{O}6}\}$ leave eq. (4.24) and all the $f_p' s$ in eq. (3.17) invariant, so that:

$$g_s \sim N^{-5}\ , \qquad \frac{L_{int}}{l} \sim N\ , \qquad |\Lambda_{\text{AdS}}| L_{int}^2 = \text{fixed} \ll 1\ , \tag{4.25}$$

where we can take N large.

We were only able to identify this way as a possibility to decouple the KK tower, although there might exist another. Unfortunately, as we will see in due course, for some models we will have some problems to decouple the KK tower in the way presented here. However, we believe that the conclusions for these models are not affected by this problem. Indeed it was shown in [83] that the $\mathcal{N} = 2$ theory obtained from a reduction of type IIA string theory based on left-invariant forms on the three coset spaces $\frac{\mathrm{G}_2}{\mathrm{SU}(3)}$, $\frac{\mathrm{Sp}(2)}{\mathrm{S}(\mathrm{U}(2)\times\mathrm{U}(1))}$ and $\frac{\mathrm{SU}(3)}{\mathrm{U}(1)\times\mathrm{U}(1)}$ without any sources is a consistent truncation, i.e. solutions of the 4d equations of motion lift to solutions of the 10d equations of motion. It seems plausible that the inclusion of smeared left-invariant sources does not alter this conclusion and that it also holds for reductions based on left-invariant forms on other spaces. Based on the arguments of [83] and references therein one expects that the fields constituting a consistent truncation do not couple to other fields. This then means that our results will not be altered by the inclusion of more fields. We will come back to this point at the end of the next two chapters.

4.3 The nilmanifolds

With the preparations of the last section we are now ready to explicitly perform the KK reduction of our type IIA supergravity vacua of section 3.3.1 and 3.3.2.

For the Kaluza-Klein reduction on T^6, we will expand the fluctuations of the various fields in the following basis:

$$\delta B(x,y) = b^{i,\vec{n}}(x)\mathcal{Y}^{(2)}_{i,\vec{n}}(y) + b_1^{i,\vec{n}}(x)\mathcal{Y}^{(1)}_{i,\vec{n}}(y) + b_2^{\vec{n}}(x)\mathcal{Y}^{(0)}_{\vec{n}}(y)\,, \tag{4.26a}$$

$$\delta\phi(x,y) = \delta\phi^{\vec{n}}(x)\mathcal{Y}^{(0)}_{\vec{n}}(y)\,, \tag{4.26b}$$

$$\delta C^{(1)}(x,y) = c^{(1)i,\vec{n}}(x)\mathcal{Y}^{(1)}_{i,\vec{n}}(y) + c_1^{(1)\vec{n}}(x)\mathcal{Y}^{(0)}_{\vec{n}}(y)\,, \tag{4.26c}$$

$$\delta C^{(3)}(x,y) = c^{(3)i,\vec{n}}(x)\mathcal{Y}^{(3)}_{i,\vec{n}}(y) + c_1^{(3)i,\vec{n}}(x)\mathcal{Y}^{(2)}_{i,\vec{n}}(y) + c_2^{(3)i,\vec{n}}(x)\mathcal{Y}^{(1)}_{i,\vec{n}}(y) + c_3^{(3)\vec{n}}(x)\mathcal{Y}^{(0)}_{\vec{n}}(y)\,, \tag{4.26d}$$

$$\delta g(x,y) = h^{i,\vec{n}}(x)\mathcal{X}^{(2)}_{i,\vec{n}}(y) + h_1^{i,\vec{n}}(x)\mathcal{Y}^{(1)}_{i,\vec{n}}(y) + h_2^{\vec{n}}(x)\mathcal{Y}^{(0)}_{\vec{n}}(y)\,. \tag{4.26e}$$

The functions $\mathcal{Y}^{(l)}_{i,\vec{n}}(y)$ are the l-eigenforms of the Laplacian operator and are given by

$$\mathcal{Y}^{(l)}_{i,\vec{n}}(y) = Y_i^{(l)} e^{i\vec{p}\cdot\vec{y}}\,, \qquad \vec{p} = \frac{\vec{n}}{R}\,, \quad \vec{n} \in \mathbb{Z}^6\,. \tag{4.27}$$

For the torus the $Y_i^{(l)}$ form a basis of harmonic l-forms. $\mathcal{X}^{(2)}$ are symmetric two-tensors

$$\mathcal{X}^{(2)}_{i,\vec{n}}(y) = X_i^{(2)} e^{i\vec{p}\cdot\vec{y}}\,, \qquad \vec{p} = \frac{\vec{n}}{R}\,, \quad \vec{n} \in \mathbb{Z}^6\,, \tag{4.28}$$

Since we will restrict our analysis to the zero modes ($\vec{p}=0$), we only keep $\mathcal{Y}^{(l)}_{i,\vec{n}=0}(y) = Y_i^{(l)}$ and $\mathcal{X}^{(2)}_{i,\vec{n}=0}(y) = X_i^{(2)}$ in the expansions above and derivatives only act on the external fields. A basis for the harmonic l-forms $Y_i^{(l)}$ is simply given by all exterior products of the form $\mathrm{d}y^{m_1} \wedge \cdots \wedge \mathrm{d}y^{m_l} = e^{m_1 \dots m_l}$, $1 \le l \le 6$. Hence:

$$b_l = \begin{pmatrix} 6 \\ l \end{pmatrix}\,, \tag{4.29}$$

where b_l denotes the real dimension of the lth cohomology group of T^6.

For the Iwasawa manifold, we will use for the expansion forms $Y_i^{(l)}$ left-invariant forms, which will not necessarily be all harmonic. When exterior derivatives act on these forms terms will be generated of the order of the geometric fluxes.

In both cases we must then impose the orientifold projection which means that suitable expansion forms must be even or odd under *all* the orientifold involutions. For both, the torus and the Iwasawa, this involution is given by (3.23) which leads to the following forms

type	basis	name
odd 2-form	$e^{12},\ e^{34},\ e^{56}$	$Y_i^{(2-)}$
even 3-form	$e^{135},\ e^{146},\ e^{236},\ e^{245}$	$Y_i^{(3+)}$
odd 3-form	$e^{136},\ e^{145},\ e^{235},\ e^{246}$	$Y_i^{(3-)}$
even 4-form	$e^{1234},\ e^{1256},\ e^{3456}$	$Y_i^{(4+)}$
even symmetric 2-tensor	$e^1 \otimes e^1, e^2 \otimes e^2, \dots, e^6 \otimes e^6$	$X_i^{(2)}$

Under the orientifold projection, we find from (3.15) that Φ, g, F_0, C_3 are even, while B, C_1 are odd. This simplifies the expansion (4.26) considerably

$$\delta B(x,y) = b^i(x) Y_i^{(2-)}\,, \tag{4.30a}$$

$$\delta\Phi(x,y) = \Phi(x)\,, \tag{4.30b}$$

$$\delta C^{(3)}(x,y) = c^{(3)i}(x) Y_i^{(3+)} + c_3^{(3)}(x)\,, \tag{4.30c}$$

$$\delta g(x,y) = h^i(x) X_i^{(2)} + h_2(x)\,. \tag{4.30d}$$

Note in particular that the orientifold projection removes all four-dimensional gauge fields, which in fact holds for all type IIA models for which the orientifolds project out all one-forms and even two-forms. So far the discussion for the torus and the Iwasawa went parallel. Now we have to use the backgrounds of sections 3.3.1 and 3.3.2 to get the respective fluctuations of the field strengths given in (4.19). For the torus we find

$$\delta F_2 = -m\delta B\,, \tag{4.31a}$$

$$\delta F_4 = \mathrm{d}\delta C_3\,. \tag{4.31b}$$

while for the Iwasawa we get

$$\delta F_2 = 0\,, \tag{4.32a}$$

$$\delta F_4 = \mathrm{d}\delta C_3 - \delta B \wedge \hat{F}_2\,. \tag{4.32b}$$

So we first have to compute the variation of all the equations of motion (A.7a), (A.7b), (A.9b) and (A.10) to first order. Remember that we should use (A.1) to remove the redundant RR-fields so that the only RR-fluctuations are the ones above. For the torus we have to plug in the background of section 3.3.1 plus the fluctuations (4.30) and (4.31a), while for the Iwasawa we will have to use the background of section 3.3.2 and the fluctuations (4.32a). We will discuss the two cases separately in the next two subsections.

4.3.1 Kaluza-Klein reduction of the torus

Since we are only considering the internal zero modes we use that for the torus derivatives only act on the external fields. It turns out that the RR-fields together with H do not mix with the metric and the dilaton, so we can discuss their equations of motion separately.

RR and NS B-filed sector

Applying the steps described above we get from the equation of motion for H (A.10) the following equation, which has (external, internal) index structure $(0,2)$:

$$0 = \Delta(b^i Y_i^{(2-)}) - \star(\hat{F}_4 \wedge \mathrm{d}c_3^{(3)}) - m \star (\star\hat{F}_4 \wedge b^i Y_i^{(2-)}) + m^2 b^i Y_i^{(2-)} . \qquad (4.33)$$

From the variation of the equation of motion of F_4 (A.9b) we get a $(0,3)$-equation and a $(1,6)$-equation

$$0 = \Delta(c^{(3)i} Y_i^{(3+)}) - \star(\hat{H} \wedge \mathrm{d}c_3^{(3)}) , \qquad (4.34a)$$

$$0 = \mathrm{d} \star \mathrm{d}c_3^{(3)} + \mathrm{d}b^i \wedge Y_i^{(2-)} \wedge \hat{F}_4 + \hat{H} \wedge \mathrm{d}c^{(3)i} \wedge Y_i^{(3+)} , \qquad (4.34b)$$

and from F_2 a $(4,5)$- and $(3,6)$-equation

$$0 = \hat{H} \wedge \star \left[h^i X_i^{(2)} \cdot \hat{F}_4 \right] , \qquad (4.35a)$$

$$0 = \hat{H} \wedge \star(\mathrm{d}c^{(3)i} \wedge Y_i^{(3+)}) , \qquad (4.35b)$$

where the dot is defined in (A.3). Furthermore, we used in the upper equation the variation of the $\star$ given by

$$(\delta\star)F_l = \left(\frac{1}{2} g^{MN} \delta g_{MN} \right) \star F_l - \star[\delta g \cdot F_l] , \qquad (4.36)$$

where we defined

$$[\delta g \cdot F_l]_{M_1 \ldots M_l} \equiv l \cdot \delta g_{[M_1|A} g^{AB} F_{B|M_2 \ldots M_l]} . \qquad (4.37)$$

The equations (4.35) are automatically satisfied using the orientifold projection. Indeed, the right-hand sides should have contained an even internal five-form respectively six-form under all orientifold involutions, which do not exist, so they must vanish.

Next, we integrate (4.34b) and put the integration constant to zero because it would correspond to changing the background value of f. The result can the be used to eliminate $\mathrm{d}c_3^{(3)}$ in (4.33) and (4.34a). This procedure corresponds to dualizing $c_3^{(3)}$ as explained in [37, 34]. For more details see appendix D.

To proceed, we make a choice of expansion basis for the even three-forms

$$Y_0^{(3+)} = \mathrm{Im}\,\Omega , \qquad (4.38a)$$

$$Y_i^{(3+)} , \quad i = 1,2,3 : \quad 3 \text{ real (2,1)+(1,2) forms} , \qquad (4.38b)$$

and the odd two-forms

$$Y_0^{(2-)} = J , \qquad (4.39a)$$

$$Y_i^{(2-)} , \quad i = 1,2 : \quad 2 \text{ primitive real 2-forms} , \qquad (4.39b)$$

where a primitive two-form is defined in (B.20).

Using these in (4.33) and (4.34a) gives the equations of motion for the 4d scalar fields. Diagonalizing the mass matrix we obtain the following result for the eigenvalues $\tilde{M}^2 = M^2 + 2/3\Lambda$:

mass eigenmode	mass (in units $m^2/25$)
$b^i, \quad i = 1, 2$	10
$c^i, \quad i = 1, 2, 3$	0
$b^0 - 4c^{(3)0}$	10
$3b^0 + c^{(3)0}$	88

Dilaton and metric sector

With the same procedure as above, we get from the dilaton equation of motion (A.7a)

$$0 = (\Delta + \frac{67m^2}{25})\delta\Phi + \frac{7m^2}{25}\sum_{i=1}^{6} h^i\,, \tag{4.40}$$

and from the internal part of the Einstein equation (A.7b)

$$0 = \Delta h^i + \frac{8m^2}{25}h^i + \frac{7m^2}{50}g_{ii}\delta\Phi + \frac{m^2}{50}g_{ii}\sum_{j=1}^{6} h^j + \frac{2m^2}{5}g_{ii}h^{i-(-1)^i}\,. \tag{4.41}$$

The result of diagonalizing the mass matrix is

mass eigenmode	mass (in units $m^2/25$)
$-h^1 - h^2 + h^3 + h^4$	18
$-h^1 - h^2 + h^5 + h^6$	18
$-3\,\delta\Phi + 7\sum h_i$	18
$7\,\delta\Phi + \sum h_i$	70
$-h^1 + h^2$	-2
$-h^3 + h^4$	-2
$-h^5 + h^6$	-2

The external part of the Einstein equation on the other hand becomes

$$\frac{1}{2}\Delta_L h_{\mu\nu} + \nabla_{(\mu}\nabla^\rho h_{\nu)\rho} - \frac{1}{2}\nabla_{(\mu}\nabla_{\nu)}h^P{}_P + \frac{3}{25}m^2 h_{\mu\nu} - \frac{3}{20}m^2 g_{\mu\nu}\sum h_i - \frac{21}{100}m^2 g_{\mu\nu}\delta\Phi = 0\,. \tag{4.42}$$

At this point we have to take into account that so far we worked in the *ten*-dimensional Einstein frame. As we will show in (5.16) the conversion to the *four*-dimensional Einstein frame is given by

$$g_{\mathrm{E}\mu\nu} = c\,\sqrt{g_6}\,g_{\mu\nu}\,, \tag{4.43}$$

where the constant factor $c = M_P^{-2}\kappa_{10}^{-2}V_s$ does not matter here, so that

$$c^{-1}h_{\mathrm{E}\mu\nu} = \sqrt{g_6}\,h_{\mu\nu} + \frac{1}{2}\sqrt{g_6}\,g_{\mu\nu}\sum_i h_i\,. \tag{4.44}$$

Plugging this into (4.42) and using (4.41), we find for $h_{\mathrm{E}\mu\nu}$ exactly equation (4.3c) with $M^2 = 0$ so that $h_{\mathrm{E}\mu\nu}$ indeed describes a *massless* graviton.

4.3.2 Kaluza-Klein reduction of the Iwasawa

Again it turns out that the equations of motion for the RR-fields and the H field do not mix with the Einstein equation and the equation of motion for the dilaton, so we can discuss them separately.

RR and NS B-filed sector

Expanding the equation of motion for H (A.10) around the Iwasawa solution, we obtain

$$\begin{aligned} 0 =& \Delta b^i\, Y_i^{(2-)} + b^i\left(\star_6 \mathrm{d}\star_6 \mathrm{d}Y_i^{(2-)}\right) - c^{(3)i}\star_6\left(\star_6 \mathrm{d}Y_i^{3+}\wedge\hat{F}_2\right) \\ &+ b^i\star_6\left[\star_6\left(Y_i^{(2-)}\wedge\hat{F}_2\right)\wedge\hat{F}_2\right] + f c^{(3)i}\star_6\mathrm{d}Y_i^{3+} - b^i f\star_6\left(Y_i^{(2-)}\wedge\hat{F}_2\right)\,, \end{aligned} \tag{4.45}$$

while the equation of motion for F_4 (A.9b) splits in $(1,6)$ and $(4,3)$ index structures

$$0 = \mathrm{d}\star_4\mathrm{d}c_3^{(3)} + \frac{1}{2}f\mathrm{d}\left(\delta g^\mu{}_\mu - \delta g^m{}_m - \delta\Phi\right)\,, \tag{4.46a}$$

$$0 = \Delta c^{(3)i}\,Y_i^{(3+)} + c^{(3)i}\left(\star_6\mathrm{d}\star_6\mathrm{d}Y_i^{(3+)}\right) + f b^i\star_6\mathrm{d}Y_i^{(2-)} - b^i\star_6\mathrm{d}\star_6\left(Y_i^{(2-)}\wedge\hat{F}_2\right)\,. \tag{4.46b}$$

In a similar way as in the torus case, we integrate (4.46a), put the integration constant to zero and plug the result for $\mathrm{d}c_3^{(3)}$ in the other equations.

As expansion forms we take the same three-forms as in eq. (4.38), while for the two-forms we take this time

$$Y_0^{(2-)} = \beta^2 e^{56}\,, \tag{4.47a}$$

$$Y_1^{(2-)} = e^{12} + e^{34}\,, \tag{4.47b}$$

$$Y_2^{(2-)} = e^{12} - e^{34}\,. \tag{4.47c}$$

Note that this time $Y_0^{(3+)}$ and $Y_0^{(2-)}$ are not closed. Introducing m_T such that $\beta = \frac{2}{5}e^{\Phi}m_T$ (this is of course the Romans mass of the T-dual torus solution), we get the following masses:

mass eigenmode	mass (in units $m_T^2/25$)
$c^i, \quad i = 1,2,3$	0
$b^0 + b^1$	10
b^2	10
$8c^{(3)0} + 5b^0 + 3b^1$	10
$c^{(3)0} - b^0 + 2b^1$	88

Due to T-duality the mass eigenvalues are the same as for the torus solution.

Dilaton and metric sector

The equation for the variation of the dilaton equation (A.7a) reads

$$0 = (\Delta + \frac{27m_T^2}{25})\delta\phi - \frac{9m_T^2}{25}\sum_{i=5}^{6} h^i + \frac{3m_T^2}{25}\sum_{i=1}^{4} h^i \,. \tag{4.48}$$

For the Einstein equation (A.7b) we find for $i = 5, 6$:

$$0 = \Delta h^i + \frac{49m_T^2}{50}h^i + \frac{53m_T^2}{50}h^{i-(-1)^i} - \frac{11m_T^2}{50}\sum_{j=1}^{4} h^j - \frac{33m_T^2}{50}\delta\phi \,, \tag{4.49}$$

and for $i = 1, 2, 3, 4$:

$$0 = \Delta h^i + \frac{8m_T^2}{25}h^i + \frac{2m_T^2}{5}h^{i-(-1)^i} - \frac{3m_T^2}{10}\sum_{j=5}^{6} h^j + \frac{m_T^2}{10}\sum_{j=1}^{4} h^j + \frac{3m_T^2}{10}\delta\phi \,. \tag{4.50}$$

Here we used that

$$\delta R_{mn} = \frac{1}{2}\Delta_L \delta g_{mn} + \nabla_{(m}\nabla^P \delta g_{n)P} - \frac{1}{2}\nabla_m \nabla_n \delta g^Q{}_Q \,, \tag{4.51}$$

where Δ_L is the Lichnerowicz operator defined in (4.4) and all covariant derivatives and contractions are with respect to the background metric. In (4.51) the last two terms are vanishing.

Diagonalizing the mass matrix we find the following eigenmodes:

mass eigenmode	mass (in units $m_T^2/25$)
$-h^1 - h^2 + h^3 + h^4$	18
$11(h^1 + h^2) + 5(h^5 + h^6)$	18
$5\delta\Phi - 3(h^1 + h^2)$	18
$3\delta\Phi - 3(h^5 + h^6) + (h^1 + h^2 + h^3 + h^4)$	70
$-h^1 + h^2$	-2
$-h^3 + h^4$	-2
$-h^5 + h^6$	-2

Once again, we find the same masses as in the torus example.

4.3.3 Summary

The direct computation of the Kaluza-Klein reduction on the six-torus solution of section 3.3.1 and the Iwasawa solution of section 3.3.2 yields in both cases exactly the same mass spectrum. This is of course the expected result, since the two solutions are related by T-duality. We obtain the following mass eigenvalues $\tilde{M}^2/|W|^2$ for the scalar fields:[3]

Complex structure	$-2,\ -2,\ -2$
Kähler & dilaton	70, 18, 18, 18
Three axions of δC_3	0, 0, 0
δB & one more axion	88, 10, 10, 10

We see that all three axions correspond to massless moduli. This is a feature that is also discussed in [47]. It is argued there that, when one introduces D6-branes, these axions can provide Stückelberg masses to some of the U(1) gauge fields on the D-brane. We further notice that some masses are tachyonic, which is allowed because they are still above the Breitenlohner-Freedman bound (4.8). Scalars that are in the same supermultiplet, such as the complex structure moduli and the three corresponding axions, the dilaton and the remaining axion, the Kähler moduli and the B-field moduli have different masses. This is in fact a subtlety of the supersymmetry algebra of AdS_4 that no longer allows a definition for the mass as an invariant Casimir operator.

We can decouple the tower of Kaluza-Klein masses (see the discussion below (4.21)) when we take $m^2(e^{2\Phi}L^2_{int}) \ll 1$ for the torus or $\beta L_{int} \ll 1$ for the Iwasawa.

[3] The calculations in section 4.3.1 were made in the *ten-dimensional* Einstein frame, while the effective supergravity approach followed in later sections will lead to a result in the *four-dimensional* Einstein frame. By dividing out with $|W|^2$ we avoid conversion problems, since $\tilde{M}^2$ and $|W|^2$ transform in the same way under change of frame.

Chapter 5

Low energy physics II: Effective supergravity

In this chapter we compute the scalar potential of the 4d low energy effective theory for all the examples of section 3.3 and 3.4. As already mentioned in section 2.2, the easiest and most popular approach to do this is to use the more or less indirect techniques of $\mathcal{N}=1$ supergravity, where the scalar potential is entirely determined in terms of a Kähler potential and a superpotential[1]. For the Calabi Yau case, their general form is given in [84, 34], whereas a generalization to SU(3)-structure manifolds or even beyond is given in [33, 35, 36]. We will use this approach to compute the whole scalar potential for all our explicit $\mathcal{N}=1$ AdS_4 vacua. From this potential we will compute the scalar masses and check the stability of those vacua. For the nilmanifolds we will reproduce the results of the last chapter where we used the direct KK reduction on these backgrounds. Having confirmed that both techniques yield the same results we will then continue with the effective supergravity approach and study the Iwasawa solution with $m \neq 0$ as well as the coset models in the next section. This chapter is based on [25].

5.1 Effective supergravity

The superpotential and Kähler potential of the effective $\mathcal{N}=1$ supergravity have been derived in various ways in [33, 35, 36] (based on earlier work of [84, 34]). Here we summarize the main formulæ which will be used in the following. More details on the derivation can be found in appendix B.4.

The part of the effective four-dimensional action containing the graviton and the scalars reads:

$$S = \int \mathrm{d}^4x \sqrt{-g_4} \left(\frac{M_P^2}{2} R - M_P^2 \mathcal{K}_{i\bar{\jmath}} \partial_\mu \phi^i \partial^\mu \bar{\phi}^{\bar{\jmath}} - V(\phi, \bar{\phi}) \right) , \qquad (5.1)$$

[1] We do not consider any D-terms in this thesis

where M_P is the four-dimensional Planck mass. The scalar potential is given in terms of the superpotential via:[2]

$$V(\phi,\bar{\phi}) = M_P^{-2} e^{\mathcal{K}} \left(\mathcal{K}^{i\bar{j}} D_i \mathcal{W}_{\text{E}} D_{\bar{j}} \mathcal{W}_{\text{E}}^* - 3|\mathcal{W}_{\text{E}}|^2 \right) , \tag{5.2}$$

where the superpotential in the Einstein frame $\mathcal{W}_{\text{E}}$ reads (see equation (B.65))

$$\mathcal{W}_{\text{E}} = \frac{-ie^{-i\theta}}{4\kappa_{10}^2} \int_M \langle e^{i(J-i\delta B)}, \hat{F} - i\mathrm{d}_{\hat{H}} \left(e^{\delta B} e^{-\Phi} \mathrm{Im}\,\Omega + i\delta C_3 \right) \rangle \,, \tag{5.3}$$

and $\langle \cdot, \cdot \rangle$ indicates the Mukai pairing (B.4). The Kähler potential is given by (see equation (B.66))

$$\mathcal{K} = \mathcal{K}_k + \mathcal{K}_c + 3 \ln(8\kappa_{10}^2 M_P^2) \,, \tag{5.4}$$

where $\mathcal{K}_k$ and $\mathcal{K}_c$ are the parts containing, respectively, the Kähler and complex structure/dilaton moduli. They are given by

$$\mathcal{K}_k = -\ln \int_M \frac{4}{3} J^3 \,, \tag{5.5a}$$

$$\mathcal{K}_c = -2\ln \int_M 2\, e^{-\Phi} \mathrm{Im}\,\Omega \wedge e^{-\Phi} \mathrm{Re}\,\Omega \,, \tag{5.5b}$$

where $e^{-\Phi}\mathrm{Re}\,\Omega$ should be seen as a function of $e^{-\Phi}\mathrm{Im}\,\Omega$ (see appendix B).

On the fluctuations we must impose the orientifold projections (3.15). It turns out that for all our examples:

$$\delta B \wedge \mathrm{Im}\,\Omega = 0 \,, \tag{5.6}$$

since there are no odd five-forms. By expanding in a suitable basis of even and odd expansion forms (which have to be identified separately for each case), we find that the fluctuations organize naturally in complex scalars

$$J_c = J - i\delta B = (k^i - ib^i) Y_i^{(2-)} = t^i Y_i^{(2-)} \,, \tag{5.7a}$$

$$e^{-\Phi} \mathrm{Im}\,\Omega + i\delta C_3 = (u^i + ic^i) e^{-\hat{\Phi}} Y_i^{(3+)} = z^i e^{-\hat{\Phi}} Y_i^{(3+)} \,, \tag{5.7b}$$

where we took out the background $e^{-\hat{\Phi}}$ from the definition of z^i for further convenience. We have defined the geometrical scalars k^i and u^i slightly differently from the axionic scalars b^i and c^i in the sense that the geometrical scalars contain the background whereas the axionic scalars are pure fluctuation. In other words the supersymmetric vacuum we started with corresponds to the values $k^i = u^i = 1$ and

[2] In [38] the scalar potential was for general type II SU(3)×SU(3) compactifications directly derived from dimensional reduction of the action.

$b^i = c^i = 0$. To compute now the potential we only have to use the expansion (5.7) and plug it together with the background values of the fields given in section 3.3 and 3.4 into the superpotential (5.3) and the Kähler potential (5.4), which we then have to use in (5.2) to obtain the full potential. From there we compute the mass matrix and check the stability of our solution.

5.2 The nilmanifolds

Now we want to use the effective supergravity approach described in the last section to compute the potential of the nilmanifold solutions of section 3.3.

5.2.1 The torus potential

For convenience we choose a slightly different expansion basis as in section 4.3.1:

$$\begin{aligned} Y^{(2-)}: &\qquad e^{12}, e^{34}, e^{56}\,; \\ Y^{(3+)}: &\qquad -e^{135}, e^{146}, e^{236}, e^{245}\,. \end{aligned} \tag{5.8}$$

Using this basis in (5.7) and plugging the result together with the background of section 3.3.1 into (5.3), we obtain the superpotential

$$\mathcal{W}_{\text{E,Torus}} = \frac{e^{-i\theta}}{4\kappa_{10}^2} V_s m \left[-t^1 t^2 t^3 + \frac{3}{5}(t^1 + t^2 + t^3) - \frac{2}{5}(z^1 + z^2 + z^3 + z^4) \right] , \tag{5.9}$$

where V_s is a standard volume $V_s = \int e^{1\ldots 6}$, which does not depend on the moduli. By the same procedure we get from (5.4) the Kähler potential

$$\mathcal{K} = \mathcal{K}_k + \mathcal{K}_c + 3\ln(8\kappa_{10}^2 M_P^2 V_s^{-1} e^{4\hat{\Phi}/3})\,, \tag{5.10a}$$

where

$$\mathcal{K}_k = -\ln\left(\prod_{i=1}^{3}(t^i + \bar{t}^i)\right) \tag{5.10b}$$

is the Kähler potential in the Kähler-moduli sector, and

$$\mathcal{K}_c = -\ln\left(4\prod_{i=1}^{4}\left(z^i + \bar{z}^i\right)\right) \tag{5.10c}$$

is the Kähler potential in the complex structure moduli sector.

Using the expressions for the superpotential and the Kähler potential, it is straightforward to calculate the masses for the scalar fields from the quadratic terms in the potential (5.2). Before we comment on the results, let us first do the same calculation for the Iwasawa manifold.

5.2.2 The Iwasawa potential

We choose the following expansion basis:

$$
\begin{aligned}
Y^{(2-)}: &\qquad \beta^2 e^{65}, e^{12}, e^{34}\,; \\
Y^{(3+)}: &\qquad -\beta e^{135}, -\beta e^{146}, -\beta e^{236}, \beta e^{245}\,.
\end{aligned}
\tag{5.11}
$$

This implies that $\mathrm{d}Y_i^{(3+)} = -\beta e^{1234}$ for all $i = 1, \dots, 4$. Using this basis in (5.7) and plugging the result together with the background of section 3.3.2 into (5.3), we obtain the superpotential

$$
\mathcal{W}_{\mathrm{E,Iwasawa}} = \frac{-ie^{-i\theta}}{4\kappa_{10}^2} m_T V_s \left[\frac{3}{5} - \frac{2}{5}t^1(z^1 + z^2 + z^3 + z^4) + \frac{3}{5}(t^1t^2 + t^1t^3) - t^2t^3\right]\,,
\tag{5.12}
$$

where $V_s = \int -\beta^2 e^{1\dots6}$ is again a standard volume, and $m_T \equiv \frac{5}{2}e^{-\hat{\Phi}}\beta$ the Romans mass of the T-dual torus solution. We note here the following relation

$$
\mathcal{W}_{\mathrm{E,Iwasawa}} = -it^1 \mathcal{W}_{\mathrm{E,Torus}}(t^1 \to \frac{1}{t^1})\,,
\tag{5.13}
$$

which follows from T-duality. Repeating this procedure, we get from (5.4) the same Kähler potential (5.10) as for the torus.

Again the masses for the scalar fields follow from the quadratic terms in the potential (5.2), where we have to use the above results for the Kähler potential and superpotential.

5.2.3 Summary

From the four-dimensional Einstein-frame action (B.54) we compute the equation of motion for the scalar fields

$$
\Delta\phi^k + M_P^{-2}(\hat{\mathcal{K}}^{-1}\hat{M})^k{}_i\phi^i = 0\,,
\tag{5.14}
$$

where $\hat{M}_{ij} = \frac{1}{2}\frac{\partial^2 V}{\partial\phi^i\partial\phi^j}|_{\mathrm{background}}$ is the mass matrix and $\hat{\mathcal{K}}_{ij}$ is the Kähler metric in real coordinates in the background. Therefore, to compare the results for the masses in the analysis with the superpotential and the Kähler potential with the results from the Kaluza-Klein reduction, we need to diagonalize the matrix $M_P^{-2}\hat{\mathcal{K}}^{-1}\hat{M}$. We also have to take into account that the results from the Kaluza-Klein reduction were in the *ten-dimensional* Einstein frame, while here we get the result in the *four-dimensional* Einstein frame:

$$
\begin{aligned}
g_s &= e^{\frac{\Phi}{2}} g_{\mathrm{E}10}\,, \\
g_s &= M_P^2 \mathcal{N}^{-1} g_{\mathrm{E}4}\,,
\end{aligned}
\tag{5.15}
$$

where $\mathcal{N}$ is defined below (B.46), and thus

$$g_{\mathrm{E}10} = M_P^2 e^{-\Phi/2} \mathcal{N}^{-1} g_{\mathrm{E}4} = M_P^2 \kappa_{10}^2 e^{-2A} \mathrm{Vol}_{\mathrm{E}}^{-1} \, g_{\mathrm{E}4} \,, \tag{5.16}$$

where in the last expression we assumed A and Φ constant over the internal space. The conversion for the mass is

$$m_{\mathrm{E}}^2 = \kappa_{10}^2 M_P^2 e^{-2A} \mathrm{Vol}_{\mathrm{E}}^{-1} \, m_{\mathrm{E}10}^2 \,. \tag{5.17}$$

Upon noting that in the Kaluza-Klein analysis we set the background values for the warp factor and the dilaton equal to zero and Vol $= V_s$, we find for the torus and the Iwasawa exactly the same result as we did in section 4.3.1 and 4.3.2 by performing a direct KK reduction. This provides a consistency check on the ability of the superpotential/Kähler potential approach to handle geometric fluxes. After this non-trivial test we believe in the correctness of the effective supergravity approach and compute in the next section the potentials for the coset spaces.

But before we will do so, let us briefly comment on the Iwasawa solution for the case $m \neq 0$. Turning on m, one gets extra terms in the superpotential that look exactly like the torus superpotential, so we find:

$$\mathcal{W}_{\mathrm{E,Iwasawa},m\neq 0} = \mathcal{W}_{\mathrm{E,Iwasawa}}(m_T) + \mathcal{W}_{\mathrm{E,Torus}}(m) \,, \tag{5.18}$$

where $\mathcal{W}_{\mathrm{E,Torus}}(m)$ is the superpotential of the torus obtained and $\mathcal{W}_{\mathrm{E,Iwasawa}}(m_T)$ is the superpotential for the Iwasawa manifold that one obtains by T-dualizing the torus solution. The mass spectrum is the same upon replacing $m_T^2 \to m^2 + m_T^2$. Also, this time it is possible to decouple the Kaluza-Klein tower: in the limit $(m^2 + m_T^2)(e^{2\Phi} L_{int}^2) \ll 1$.

This ends the use of nilmanifolds in this thesis. We have mainly used them to justify the use of the easier effective supergravity approach to compute the scalar potential for the coset spaces in the next section. From a phenomenological point of view they do not seem very promising, because, as we saw, three axions correspond to massless moduli, which one would have to stabilize before turning to phenomenology. This problem might be solved by the Stückelberg mechanism to generate masses for some of the U(1) gauge fields living on the D6-branes, as it is argued in [47]. But as we will see later in the cosmological applications, our torus potential falls under a class of potentials whose suitability for slow roll inflation is ruled out by a no go theorem formulated in [50]. The same is then true for the Iwasawa manifold because of T-duality. So let us instead turn to the more promising coset spaces, since there we will find examples in the next section, which do not have any massless scalar fields. Furthermore, in the next chapter we will also see how they evade the no go theorem of [50].

5.3 The coset spaces

In this section, we compute the scalar potential for the coset space vacua of section 3.4. We will do this by using the effective supergravity approach of section 5.1. We will proceed for each coset in the same way as we did for the nilmanifolds. First, we will have to choose an expansion basis which to use in (5.7). To compute the Kähler potential and superpotential, we then plug the result together with the respective background from section 3.4 into (5.3) and (5.4). The potential is given by (5.2) from where we obtain the mass matrix.

5.3.1 The $\frac{\mathrm{G}_2}{\mathrm{SU(3)}}$ potential

We choose the expansion forms in (5.7) as follows:

$$\begin{aligned} Y^{(2-)}: \quad & a(e^{12} - e^{34} + e^{56})\,; \\ Y^{(3+)}: \quad & a^{3/2}(-e^{235} - e^{246} + e^{145} - e^{136})\,, \end{aligned} \tag{5.19}$$

and the standard volume $V_s = -\int a^3\, e^{123456}$.

The superpotential reads:

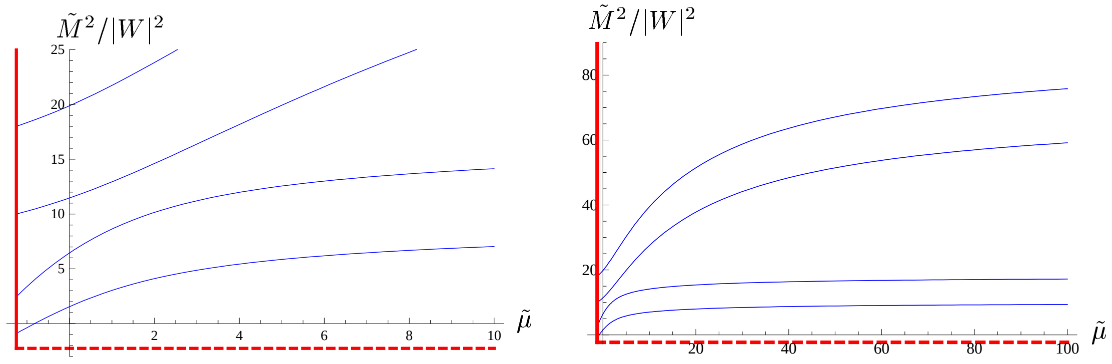
$$\mathcal{W}_{\mathrm{E}} = \frac{ie^{-i\theta}e^{-\hat{\Phi}}}{4\kappa_{10}^2} V_s a^{-1/2} \left(-\frac{3\sqrt{3}}{2} + \frac{8\tilde{m}i}{5} z^0 - \frac{9\tilde{m}i}{5} t^1 + 4\sqrt{3} z^0 t^1 - \frac{\sqrt{3}}{2}(t^1)^2 + i\tilde{m}(t^1)^3 \right)\,, \tag{5.20}$$

whereas the Kähler potential is

$$\mathcal{K} = -\ln\left((t^1 + \bar{t}^1)^3\right) - \ln\left(4(z^0 + \bar{z}^0)^4\right) + 3\ln(8\kappa_{10}^2 M_P^2 V_s^{-1} e^{4\hat{\Phi}/3})\,. \tag{5.21}$$

If we plot $\tilde{M}^2/|W|^2$, the overall scale a drops out, and the only parameter is the reduced orientifold tension $\tilde{\mu}$: see Figure 5.1, where the dashed and solid red line represent the Breitenlohner-Freedman bound (4.8) and the bound (3.8) for $\tilde{\mu}$, respectively. We see that all four moduli masses are above the Breitenlohner-Freedman bound. Moreover, all masses are positive for $\tilde{\mu} > -0.82$. For $\tilde{\mu} \to \infty$ the masses asymptote to $\tilde{M}^2/|W|^2 = (10, 18, 70, 88)$, which are the same as for the torus in section 4.3.1 (except there are no complex structure moduli and corresponding axions). In fact, this asymptotic behavior is universal for all models we studied. Indeed, for $\tilde{\mu} \to \infty$ we find from (3.12) that $m \to \infty$ regardless of the details $\mathcal{W}_1^-, \mathcal{W}_2^-$ of the model and exactly those terms in the superpotential become relevant that also appear in the superpotential of the torus..

In section 3.2, we have seen that $|\mathcal{W}_1^-|L_{int} \ll 1$ is one way to obtain a separation of scales between the light masses and the Kaluza-Klein masses even before the uplifting. However, as can be seen from eq. (3.38), this is impossible to achieve for this coset.



(a) Behavior for small $\tilde{\mu}$

(b) Behavior for large $\tilde{\mu}$

Figure 5.1: Mass spectrum of $\frac{\mathrm{G}_2}{\mathrm{SU}(3)}$.

5.3.2 The $\frac{\mathbf{Sp(2)}}{\mathbf{S(U(2)\times U(1))}}$ potential

We choose the expansion forms in (5.7) as follows:

$$\begin{aligned} Y^{(2-)}: \quad & a(e^{12}+e^{34}), -ae^{56}\,; \\ Y^{(3+)}: \quad & a^{3/2}(e^{235}+e^{246}+e^{145}-e^{136})\,, \end{aligned} \tag{5.22}$$

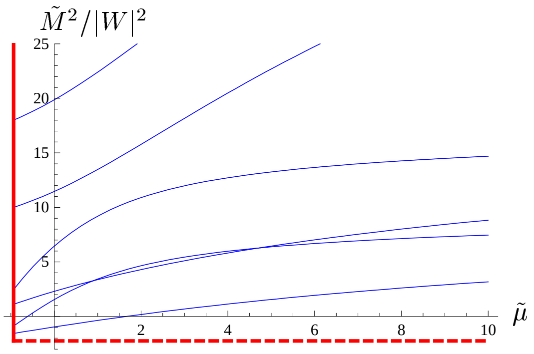
and the standard volume $V_s = -\int a^3\, e^{123456}$. We find the following superpotential

$$\begin{aligned} \mathcal{W}_{\rm E} = \frac{ie^{-i\theta}e^{-\hat{\Phi}}}{4\kappa_{10}^2} V_s a^{-1/2} \Big(& -\tilde{f}\sigma + \frac{8\tilde{m}i}{5}\sigma^{1/2}z^0 - \frac{3\tilde{m}i}{5}(2\sigma t^1+t^2) - 2(2t^1+t^2)z^0 \\ & + i\tilde{m}(t^1)^2t^2 + \sigma^{1/2}\left(\frac{3}{2}-\frac{5}{4}\sigma\right)(t^1)^2 - \left(\sigma^{-1/2}-\frac{3}{2}\sigma^{1/2}\right)t^1t^2 \Big)\,, \end{aligned} \tag{5.23}$$

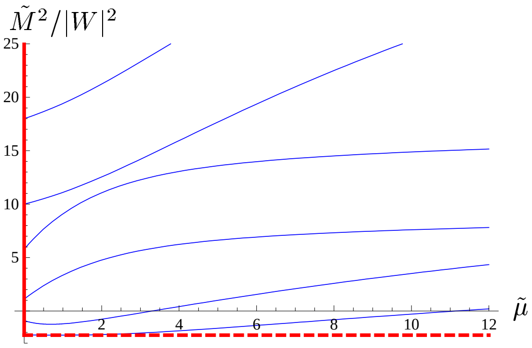
and Kähler potential

$$\mathcal{K} = -\ln\left((t^1+\bar{t}^1)^2(t^2+\bar{t}^2)\right) - \ln\left(4(z^0+\bar{z}^0)^4\right) + 3\ln(8\kappa_{10}^2 M_P^2 V_s^{-1} e^{4\hat{\Phi}/3})\,. \tag{5.24}$$

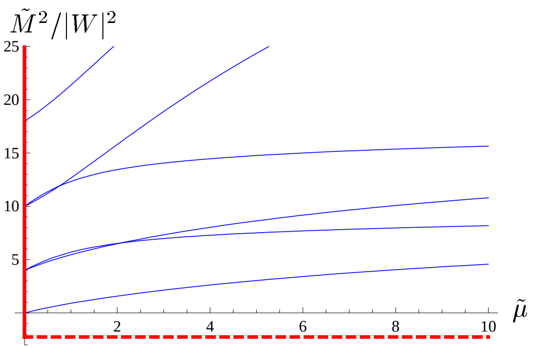
This time the solution has next to the overall scale a two free parameters: the “shape” $\sigma = c/a$ and the orientifold tension $\tilde{\mu}$. In Figure 5.2 we display plots for several values of σ: $\sigma = 1$ is the nearly-Kähler point while for $\sigma = 2/5$ and $\sigma = 2$ the lower bound for $\tilde{\mu}$ from (3.12) is exactly zero. These were extreme points in [58], since outside the interval $[2/5, 2]$ the lower bound is above zero and solutions without orientifolds are no longer possible. Moreover, for $\tilde{\mu} = 0$ also $m = 0$, and these solutions can be lifted to M-theory. We also display a plot for large σ, here $\sigma = 13$. We see that the lower bound for $\tilde{\mu}$ is indeed positive so that there must be net orientifold charge. The



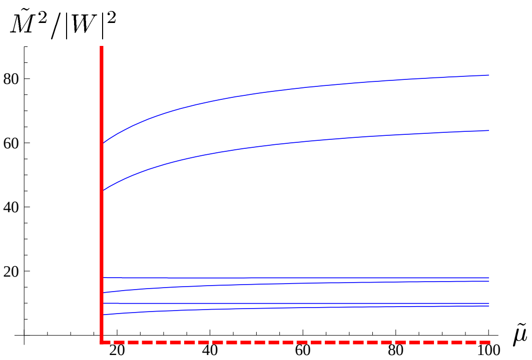
(a) $\sigma = 1$: nearly-Kähler ($\Rightarrow$ Einstein)



(b) $\sigma = \frac{2}{5}$



(c) $\sigma = 2$: Einstein

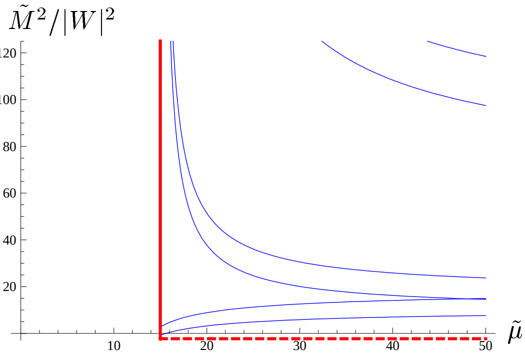


(d) $\sigma = 13$

Figure 5.2: Mass spectrum of $\frac{\mathrm{Sp}(2)}{\mathrm{S}(\mathrm{U}(2)\times\mathrm{U}(1))}$.

behavior is however already like the universal behavior for $\tilde{\mu} \to \infty$. Again we see that in all cases all masses are above the Breitenlohner-Freedman bound and by choosing $\tilde{\mu}$ large enough they are all positive.

Again we would like to get $|\mathcal{W}_1^-| L_{int} \ll 1$ to decouple the Kaluza-Klein modes. From eq. (3.46) we see that this can be formally obtained by putting $\sigma \to -2$, i.e. we need to analytically continue to negative values for σ. From [82] we learn that $\sigma < 0$ is indeed possible, but the model cannot be described as a left-invariant SU(3)-structure on the coset $\frac{\mathrm{Sp}(2)}{\mathrm{S}(\mathrm{U}(2)\times\mathrm{U}(1))}$ anymore. Rather it is a twistor bundle on a four-dimensional hyperbolic space. The precise agreement between the results of [58] (which is based on [82]) and [59] (wherever they overlap) suggests that the analytic continuation is possible. Strictly speaking, however, one should check that also the *mass spectrum* can be analytically continued to negative values for σ. Although this seems plausible, verifying it directly would require using entirely different technology, and lies beyond



(a) $\sigma = -2$

Figure 5.3: Mass spectrum of the continuation of $\frac{\mathrm{Sp}(2)}{\mathrm{S}(\mathrm{U}(2)\times\mathrm{U}(1))}$ to negative σ.

the scope of this thesis. In deriving the plot of Figure 5.3 for $\sigma = -2$, we have assumed that such analytic continuation of the mass spectrum is possible. We see that two mass eigenvalues stay light, while the others blow up if $\mathcal{W}_1^- \to 0$ and join the Kaluza-Klein masses. In this limit the light modes have $\tilde{M}^2/|W|^2 = (-38/49, 130/49)$.

5.3.3 The $\frac{\mathbf{SU(3)}}{\mathbf{U(1)\times U(1)}}$ potential

In this case we choose the expansion forms in (5.7) as follows:

$$\begin{aligned} Y^{(2-)}: &\qquad -ae^{12}, ae^{34}, -ae^{56}\,; \\ Y^{(3+)}: &\qquad a^{3/2}(e^{235} + e^{246} + e^{136} - e^{145})\,, \end{aligned} \tag{5.25}$$

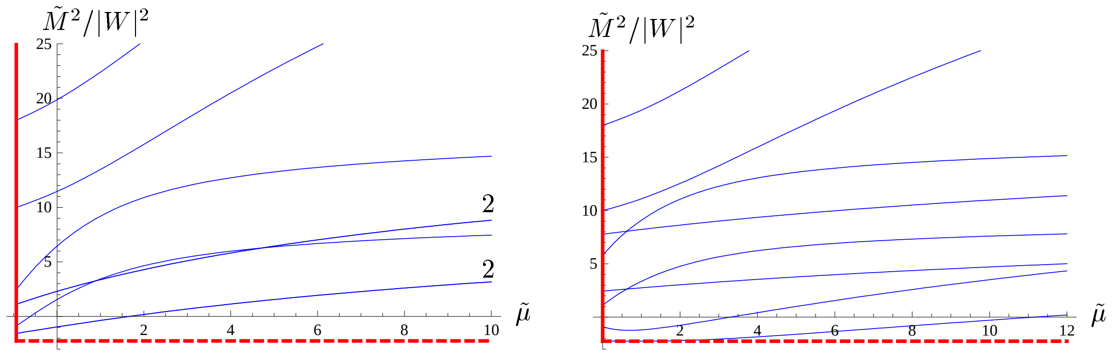
and the standard volume $V_s = \int a^3\, e^{123456}$.

Using the expression (B.65) for the superpotential and the expansion given in (5.7), we derive the superpotential

$$\begin{aligned} \mathcal{W}_{\mathrm{E}} = &-\frac{ie^{-i\theta}e^{-\hat{\Phi}}}{4\kappa_{10}^2} V_s a^{-1/2} \Bigg(\tilde{f}\rho\sigma - \frac{8\tilde{m}i}{5}(\rho\sigma)^{1/2} z^0 + \frac{3\tilde{m}i}{5}(\rho\sigma t^1 + \sigma t^2 + \rho t^3) \\ &+ \frac{1}{4}(\rho\sigma)^{-1/2}\Big((3\sigma + 3\rho\sigma - 5\sigma^2)t^1t^2 + (3\rho - 5\rho^2 + 3\rho\sigma)t^1t^3 + (-5 + 3\rho + 3\sigma)t^2t^3\Big) \\ &- 2z^0(t^1 + t^2 + t^3) - i\tilde{m}t^1t^2t^3 \Bigg)\,. \end{aligned} \tag{5.26}$$

The Kähler potential (5.4) becomes

$$\mathcal{K} = -\ln\left(\prod_{i=1}^{3}(t^i + \bar{t}^i)\right) - \ln\left(4(z^0 + \bar{z}^0)^4\right) + 3\ln(8\kappa_{10}^2 M_P^2 V_s^{-1} e^{4\hat{\Phi}/3})\,. \tag{5.27}$$



(a) $\rho = \sigma = 1$: nearly-Kähler. Lines indicated with 2 have multiplicity 2.

(b) $\rho = 1$ and $\sigma = \frac{2}{5}$.

Figure 5.4: Mass spectrum of $\frac{\mathrm{SU}(3)}{\mathrm{U}(1)\times\mathrm{U}(1)}$.

The model has this time two shape parameters: $\rho = b/a$ and $\sigma = c/a$. We display the mass spectrum for a number of selected values of these parameters in Figure 5.4. There is a symmetry under permuting (a, b, c) which translates into a symmetry under $\rho \leftrightarrow \sigma$ and $(\rho, \sigma, \tilde{\mu}) \leftrightarrow (\rho/\sigma, 1/\sigma, \sigma\tilde{\mu})$. Applying these symmetries leads to identical mass spectra. Moreover, the mass spectra for $\rho = 1$ are, apart from two more eigenvalues, identical to the mass spectra of $\frac{\mathrm{Sp}(2)}{\mathrm{S}(\mathrm{U}(2)\times\mathrm{U}(1))}$. We also display an example with $\sigma, \rho \neq 1$.

In the plots of Figure 5.5 we have analytically continued to $\rho < 0, \sigma < 0$ in order to approach the NCY limit, which we obtain for $\rho + \sigma = -1$. Again, two eigenvalues stay light with $\tilde{M}^2/|W|^2 = (-38/49, 130/49)$ in the limit while the other eigenvalues blow up to the Kaluza-Klein scale.

5.3.4 The SU(2)×SU(2) potential

The expansion forms are given by

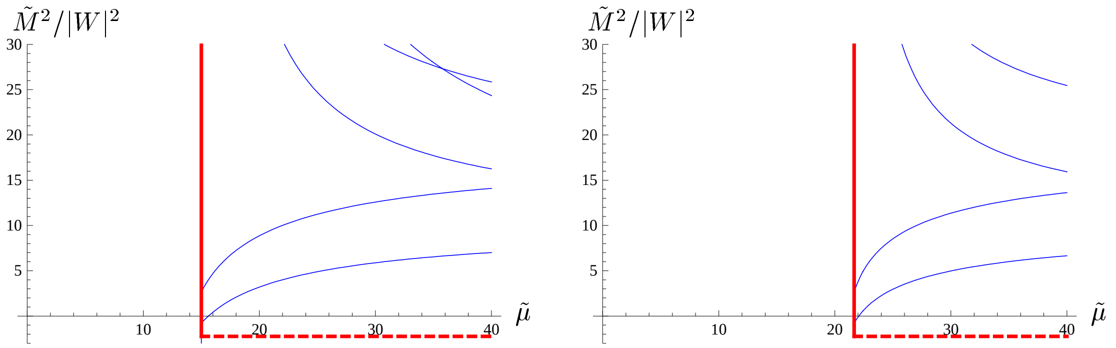
$$Y_1^{2-} = a e^{14}\,, \qquad Y_2^{2-} = b e^{25}\,, \qquad Y_3^{2-} = c e^{36}\,, \tag{5.28}$$

$$Y_1^{3+} = e^{x^1x^2y^3} = \frac{-h}{4c_1(a+b+c)}(e^{123} + e^{456} + e^{126} + e^{345} + e^{315} + e^{264} + e^{156} + e^{234})\,,$$

$$Y_2^{3+} = e^{x^1y^2x^3} = \frac{h}{4c_1(-a+b+c)}(e^{123} + e^{456} - e^{126} - e^{345} - e^{315} - e^{264} + e^{156} + e^{234})\,,$$

$$Y_3^{3+} = e^{y^1x^2x^3} = \frac{-h}{4c_1(a-b+c)}(-e^{123} - e^{456} + e^{126} + e^{345} - e^{315} - e^{264} + e^{156} + e^{234})\,,$$

$$Y_4^{3+} = -e^{y^1y^2y^3} = \frac{h}{4c_1(a+b-c)}(e^{123} + e^{456} + e^{126} + e^{345} - e^{315} - e^{264} - e^{156} - e^{234})\,,$$



(a) $\rho = \sigma = -\frac{1}{2}$. (b) $\rho = -\frac{3}{4}$ and $\sigma = -\frac{1}{4}$.

Figure 5.5: Mass spectrum of $\frac{\mathrm{SU}(3)}{\mathrm{U}(1)\times\mathrm{U}(1)}$ for negative σ and ρ.

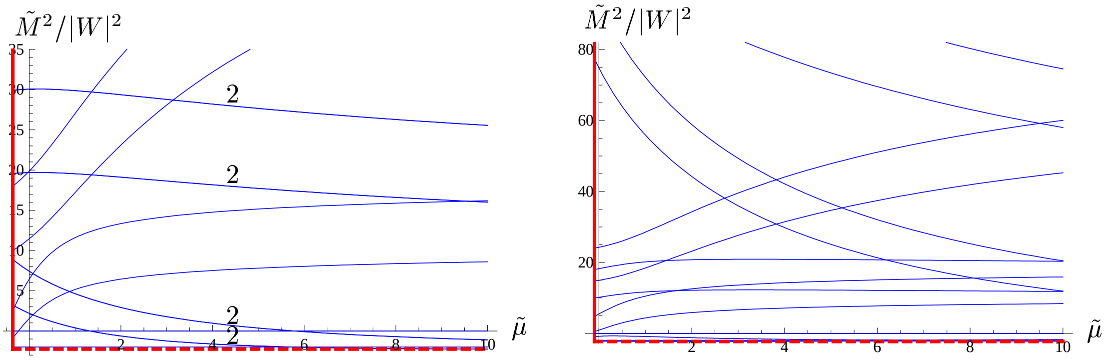
and the standard volume $V_s = -\int_M abc\, e^{1\ldots 6}$. One finds for the superpotential:

$$\begin{aligned}\mathcal{W} = \frac{ie^{-i\theta}e^{-\hat{\Phi}}}{4\kappa_{10}^2}V_s a^{-1/2}\Bigg\{ &\frac{3}{2}\tilde{c}_1 + i\tilde{m}\left(t^1t^2t^3 - \frac{3}{5}(t^1+t^2+t^3) - \frac{2}{5}(z^1+z^2+z^3+z^4)\right) \\ &+ \frac{3}{2}\tilde{c}_1(t^1t^2+t^2t^3+t^1t^3) \\ &+ \frac{\tilde{c}_1}{\tilde{h}^2}\Big\{4\left[t^2t^3(1-\rho^2-\sigma^2)+t^1t^3\rho^2(-1+\rho^2-\sigma^2)+t^1t^2\sigma^2(-1-\rho^2+\sigma^2)\right] \\ &+ \left[t^1(-1+\rho^2+\sigma^2)+t^2\rho^2(1-\rho^2+\sigma^2)+t^3\sigma^2(1+\rho^2-\sigma^2)\right](z^1+z^2+z^3+z^4) \\ &+ \rho\sigma\left[-2t^1+t^2(1+\rho^2-\sigma^2)+t^3(1-\rho^2+\sigma^2)\right](z^1+z^2-z^3-z^4) \\ &+ \sigma\left[t^1(1+\rho^2-\sigma^2)-2\rho^2t^2+t^3(-1+\rho^2+\sigma^2)\right](z^1-z^2+z^3-z^4) \\ &+ \rho\left[t^1(1-\rho^2+\sigma^2)+t^2(-1+\rho^2+\sigma^2)-2\sigma^2t^3\right](z^1-z^2-z^3+z^4)\Big\}\Bigg\}\,. \end{aligned} \tag{5.29}$$

The Kähler potential reads:

$$\mathcal{K} = -\ln\left(\prod_{i=1}^{3}(t^i+\bar{t}^i)\right) - \ln\left(4\prod_{i=1}^{4}\left(z^i+\bar{z}^i\right)\right) + 3\ln(8\kappa_{10}^2M_P^2V_s^{-1}e^{4\hat{\Phi}/3})\,. \tag{5.30}$$

There are again two shape parameters $\rho = b/a$ and $\sigma = c/a$, and the same symmetries $\rho \leftrightarrow \sigma$, $(\rho,\sigma,\tilde{\mu}) \leftrightarrow (\rho/\sigma, 1/\sigma, \sigma\tilde{\mu})$ as in the previous model. In Figure 5.6, we display the mass spectrum for some values of the parameters. This time there will always be one unstabilized massless axion ($\tilde{M}^2{=}0$) and a corresponding tachyonic complex structure modulus with $\tilde{M}^2/|W|^2 = -2$.



(a) $\rho = \sigma = 1$: nearly-Kähler. Lines indicated with 2 have multiplicity 2.

(b) $\sigma = 1$ and $\rho = \frac{2}{5}$.

Figure 5.6: Mass spectrum of SU(2)×SU(2).

In the limit $\mathcal{W}_1^- \to 0$, $\mathcal{W}_2^-$ blows up just as the lower bound for $\tilde{\mu}$. So in principle we could decouple the Kaluza-Klein modes this way, however it is quite difficult to study this singular limit.

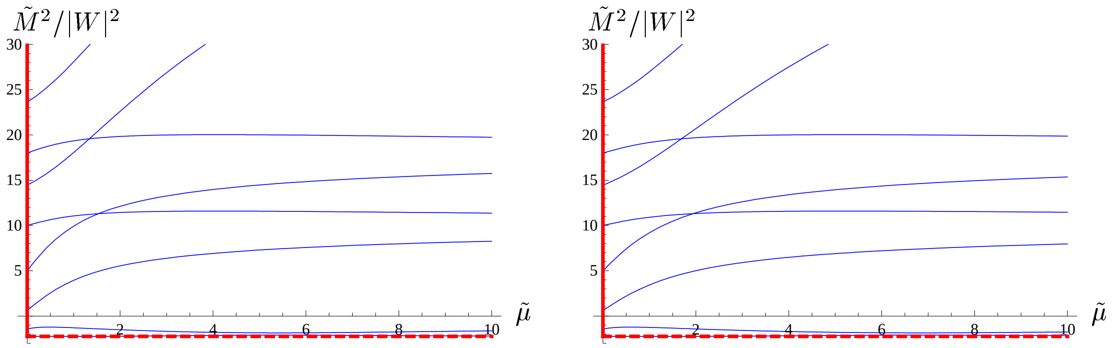
5.3.5 The $\frac{\mathbf{SU(3)\times U(1)}}{\mathbf{SU(2)}}$ potential

We display the general results here and refer the reader for the special case $5c_1^2 - 4e^{2\Phi}m^2 = 0$ to [25]. We choose the expansion forms in (5.7) as follows:

$$\begin{aligned} Y^{(2-)}: &\qquad -a[(e^{13} - e^{24}) - \rho(e^{14} + e^{23})],\ ae^{56}\,; \\ Y^{(3+)}: &\qquad a^{3/2}[(e^{13} - e^{24}) + \rho^{-1}(e^{14} + e^{23})] \wedge e^6,\ a^{3/2}(e^{125} + e^{345})\,, \end{aligned} \tag{5.31}$$

and the standard volume $V_s = \int a^3(1+\rho^2)e^{123456}$. The superpotential and Kähler potential read:

$$\begin{aligned} \mathcal{W}_{\rm E} = -\frac{ie^{-i\theta}e^{-\hat{\Phi}}}{4\kappa_{10}^2} V_s a^{-1/2} \Bigg(& \tilde{f}\sigma + \frac{3i\tilde{m}}{5}\sigma(2t^1 + \frac{1}{\sigma}t^2) \\ & + \sqrt{\frac{3}{2}}(1+\rho^2)^{-\frac{1}{4}}\left(-t^1t^2 + \frac{\sigma}{2}(t^1)^2\right) - i\tilde{m}(t^1)^2t^2 \\ & - \frac{4\sqrt{2}i\tilde{m}}{5\rho}(1+\rho^2)^{\frac{1}{4}}z^1 + \frac{2\sqrt{2}i\tilde{m}}{5}\sigma(1+\rho^2)^{-\frac{3}{4}}z^2 + \frac{2\sqrt{3}}{\rho}z^1t^1 - \sqrt{3}(1+\rho^2)^{-1}t^2z^2 \Bigg)\,, \end{aligned} \tag{5.32}$$



(a) $\rho = \sigma = 1$.　　(b) $\rho = \frac{1}{2}$ and $\sigma = 2$.

Figure 5.7: Mass spectrum of $\frac{\mathrm{SU}(3)\times\mathrm{U}(1)}{\mathrm{SU}(2)}$.

and

$$\begin{aligned}\mathcal{K} = &- \ln\left((t^1+\bar{t}^1)^2(t^2+\bar{t}^2)\right) - \ln\left(4\frac{1}{\rho^2(1+\rho^2)}(z^1+\bar{z}^1)^2(z^2+\bar{z}^2)^2\right)\\ &+ 3\ln(8\kappa_{10}^2 M_P^2 V_s^{-1} e^{4\hat{\Phi}/3})\,.\end{aligned} \tag{5.33}$$

This model has two shape parameters $\rho = b/a$ and $\sigma = c/a$, and a symmetry under $(\rho, \sigma, \tilde{\mu}) \leftrightarrow (1/\rho, \sigma/\rho, \rho\tilde{\mu})$. In Figure 5.7, we show the mass spectrum for some values of the parameters. The mass spectrum at $\mu = 0$ turns out to be independent of the parameters ρ, σ. There always seem to be two negative $\tilde{M}^2$ eigenvalues.

5.3.6 Summary

In this section we derived the scalar potential for type IIA SU(3)-structure compactifications on nilmanifolds and cosets, which are tractable enough to allow for an explicit calculation. In particular, we calculated the mass spectrum of the light scalar modes, using $\mathcal{N} = 1$ supergravity techniques. In the coset models, except for SU(2)×SU(2), all moduli are stabilized.

It would be interesting to study the uplifting of these models to de Sitter spacetimes. This might be accomplished by incorporating a suitable additional uplifting term in the potential along the lines of, e.g, [18]. Although a negative mass squared for a light field in AdS does not necessarily signal an instability, after the uplift all fields should have positive mass squared. Unless the uplifting potential can change the sign of the squared masses, it is thus desirable that they are all positive even before the uplifting. We find that this can be arranged in the coset models $\frac{\mathrm{G}_2}{\mathrm{SU}(3)}$, $\frac{\mathrm{Sp}(2)}{\mathrm{S}(\mathrm{U}(2)\times\mathrm{U}(1))}$ and $\frac{\mathrm{SU}(3)}{\mathrm{U}(1)\times\mathrm{U}(1)}$ for suitable values of the orientifold charge.

An alternative approach towards obtaining meta-stable de Sitter vacua could also be to search for non-trivial de Sitter minima in the original flux potential away from the AdS vacuum. In such a case, one would have to re-investigate the spectrum of the light fields and the issue of the Kaluza-Klein decoupling. We will come to this question in the next chapter.

We discussed the Kaluza-Klein decoupling for the original AdS vacua and found that it requires going to the nearly-Calabi Yau limit, which seems to be somewhat hard to do. Indeed, we found that for $\frac{\mathrm{Sp}(2)}{\mathrm{S}(\mathrm{U}(2)\times\mathrm{U}(1))}$ and $\frac{\mathrm{SU}(3)}{\mathrm{U}(1)\times\mathrm{U}(1)}$ one has to make a continuation to negative values of the "shape" parameters. Strictly speaking, this can no longer be described as a left-invariant SU(3)-structure on a coset anymore, but it can still be described in terms of a twistor bundle over a four-dimensional hyperbolic space. However, as explained in section 4.2, even if we are not able to decouple the KK tower our results should not be altered by the inclusion of other fields because the latter should not couple to fields constituting a consistent truncation.

Chapter 6

Cosmology

In this chapter we want to study whether the scalar potentials obtained in the last chapter might be suitable for some phenomenological application. The only thing that we know so far is that they posses a supersymmetric AdS_4 minimum. To make contact with observation, one possibility would be to try to modify the whole construction in a way that breaks supersymmetry and results in a 4d de Sitter minimum. But this is not what we want to do here. Instead we want to investigate whether the potentials computed in the last chapter allow for dS minima somewhere away from the original supersymmetric minimum. But as we will explain, we can answer this question by asking an even more general question, namely, whether there are regions somewhere in the potential that allow for slow-roll inflation.

The main problem of implementing inflation in type IIA compactifications is that there exist already quite strong no-go theorems against dS vacua and slow-roll inflation: extending the earlier work [49], the authors of [50] prove a no-go theorem against small ϵ in type IIA compactifications on Calabi-Yau manifolds with standard RR and NSNS-fluxes, D6-branes and O6-planes at large volume and with small string coupling. This no-go theorem uses the particular functional dependence of the corresponding scalar potentials on the volume modulus ρ and the 4d dilaton τ. Using only this (ρ, τ)-dependence, they could derive a no-go theorem in the absence of metric fluxes that puts a lower bound on the first slow-roll parameter,

$$\epsilon \equiv \frac{g^{ij}\partial_i V \partial_j V}{2V^2} \geq \frac{27}{13}, \quad \text{whenever } V > 0, \tag{6.1}$$

where g^{ij} denotes the inverse of g_{ij} appearing in front of the kinetic energy terms, and the indices i, j run over all moduli fields. This then not only excludes slow-roll inflation but also dS vacua (corresponding to $\epsilon = 0$).

As was already emphasized in [50], however, the inclusion of other ingredients such as NS5-branes, geometric fluxes and/or non-geometric fluxes evade the assumptions that underly this no-go theorem. In [85], a combination of geometric fluxes, KK5-branes and more ingredients was indeed argued to allow for dS minima. These ingredients were used in [51] to construct large field inflationary models with very interesting experimental predictions. As already mentioned in **??** the above no-go theorem directly rules out the torus example. Since the Iwasawa example is T-dual to the torus this manifold is also ruled out even though it has geometric flux.

In the recent work [86], F_0 flux (i.e. non-vanishing Romans mass) and geometric flux were identified as "minimal" additional ingredients in order to circumvent the no-go theorem of [50]. We want to discuss the question to what extent the type IIA $\mathcal{N}=1$ AdS_4 vacua with SU(3)-structure can be used for inflation or dS vacua. In particular, the coset models with SU(3)-structure could be candidates for circumventing the no-go theorem of [50], as they all have geometric fluxes and allow for non-vanishing Romans mass. Specifically, we investigate whether the scalar potentials in the closed string moduli sector can be flat enough in order to allow for inflation by one of the closed string moduli. For this to be the case the parameter ϵ must be small enough in some region of the positive scalar potential for the closed string moduli. In addition, this analysis is also relevant for open string inflation in these IIA vacua, since in this case we have to find closed string minima of the scalar potential, i.e. $\epsilon = 0$ somewhere in the closed string moduli space. Having a point with $\epsilon = 0$ would also be a necessary condition for a dS vacuum somewhere in moduli space.

In the next section we will first review the no-go theorems of [50] and [86] to see how our coset models evade them. After that we introduce yet another no-go theorem, first formulated in [87], which also includes geometric fluxes. We will then apply a slight modification of this no-go theorem to rule out all but one coset models to allow for dS minima or inflation. We will study the remaining coset and some further generalizations in the following sections.

6.1 A no-go theorem without geometric fluxes

We start by reviewing previously derived no-go theorems [50] (see also [85, 86]) that exclude slow-roll inflation and dS vacua in the simplest compactifications of massive type IIA supergravity, focusing in particular on the role played by the curvature of the internal space. In [50] the authors studied the dependence of this scalar potential on the volume modulus and the four-dimensional dilaton defined by

$$\rho \equiv (\mathrm{Vol})^{1/3}\,, \qquad\qquad \tau \equiv e^{-\phi}\sqrt{\mathrm{Vol}}\,. \tag{6.2}$$

The formulation of the no-go theorem then consists of two steps. First, they derive a general expression for the slow roll parameter ϵ, valid for any $\mathcal{N}=1$ supergravity theory. It is the sum of a positive term plus the gradient in the (ρ,τ)-plane. The second step consists of finding a lower bound for the gradient in the (ρ,τ)-plane, forbidding ϵ to become arbitrarily small.

Because we use similar arguments in the next sections let us review their construction here and start with the first step. A basic ingredient in the formulation of any 4d $\mathcal{N} = 1$ supergravity theory is the Kähler potential (see (5.4))

$$\mathcal{K} = \mathcal{K}_k + \mathcal{K}_c + 3\ln(8\kappa_{10}^2 M_P^2)\,, \tag{6.3}$$

where $\mathcal{K}_k$ and $\mathcal{K}_c$ are the parts containing, respectively, the Kähler and complex structure/dilaton moduli.

Let us now focus on the Kähler moduli, whose Kähler potential is given by (see (5.5a))

$$\mathcal{K}_k = -\ln \int_M \frac{4}{3} J^3 = -\ln \int_M 8\ \mathrm{dvol}_6\,, \tag{6.4a}$$

where we have used $J^3 = 6\ \mathrm{dvol}_6$ in the second equality. Since $\int_M \mathrm{dvol}_6 = \mathrm{Vol}$ we can use this to relate the volume modulus ρ defined in (6.2) to the Kähler moduli k^i appearing in the expansion of J. Namely, using the usual expansion of J given by

$$J = k^i Y_i^{(2-)}\,, \qquad\qquad (i = 1, \dots, h^{2-}) \tag{6.5}$$

and defining the triple intersection numbers κ_{ijk} as

$$\kappa_{ijk} = \int_M Y_i^{(2-)} \wedge Y_j^{(2-)} \wedge Y_k^{(2-)}\,, \tag{6.6}$$

we get from (6.4a)

$$\kappa_{ijk} k^i k^j k^k = 6\rho^3\,, \tag{6.7}$$

So we can relate ρ to the k^i if we write

$$k^i = \rho\gamma^i\,, \tag{6.8}$$

and impose the constraint

$$\kappa_{ijk}\gamma^i\gamma^j\gamma^k = 6\,. \tag{6.9}$$

Now we obtain an important piece of information by looking at the kinetic energy for the Kähler and complex structure moduli t^i and z^i given by

$$T = T_k + T_c = -\mathcal{K}_{i\bar{j}}\partial_\mu t^i \partial^\mu t^{\bar{j}} - \mathcal{K}_{i\bar{j}}\partial_\mu z^i \partial^\mu z^{\bar{j}}\,. \tag{6.10}$$

Let us focus again on the Kähler sector. Turning to real coordinates $t^i = k^i - ib^i$ we get

$$T_k = -\frac{1}{4}\frac{\partial^2 \mathcal{K}_k}{\partial k^i \partial k^j}\left(\partial_\mu k^i \partial^\mu k^j + \partial_\mu b^i \partial^\mu b^j\right). \tag{6.11}$$

Plugging in (6.8) and using $\partial_\mu(\kappa_{ijk}\gamma^i\gamma^j\gamma^k) = 0$, we obtain:

$$T_k = -\left[\frac{3(\partial_\mu\rho)^2}{4\rho^2} - \frac{1}{4}\kappa_{ijk}\gamma^k\partial_\mu\gamma^i\partial^\mu\gamma^j + \frac{\kappa_{ikl}\gamma^k\gamma^l\kappa_{jmn}\gamma^m\gamma^n - 4\kappa_{ijk}\gamma^k}{16\,\rho^2}\partial_\mu b^i\partial^\mu b^j\right] \quad (6.12)$$

We see that ρ does not have canonical kinetic energy, but we can define a $\hat{\rho}$, which does:

$$\hat{\rho} \equiv \sqrt{\frac{3}{2}}\ln\rho\,. \quad (6.13)$$

By switching from ρ to $\hat{\rho}$, we can read off the kinetic energy for $\hat{\rho}$. The remaining kinetic energy terms for γ^i and b^i are block diagonal (there are no cross terms involving $\partial_\mu\rho\,\partial^\mu\gamma^a$ etc), and this has an important consequence: We know that in the physical region the total kinetic energy must be positive, so *each* of the above 3 terms must be positive. Hence,

$$T_k = -(\partial_\mu\hat{\rho})^2/2 + \text{positive}. \quad (6.14)$$

For the complex structure/dilaton sector, the procedure is similar, although more subtle. Without going through the details here we, just give the result. Again, one pulls out the τ dependence by $u^i = \tau\tilde{u}^i$, where the $\tilde{u}^i$ are no longer independent anymore. One then has to define a canonically normalized field

$$\hat{\tau} \equiv \sqrt{2}\ln\tau \quad (6.15)$$

to obtain for the kinetic energy

$$T_c = -(\partial\hat{\tau})^2/2 + \text{positive}. \quad (6.16)$$

The kinetic energy is again block diagonal. In fact we know this *must* be true from the 10-dimensional point of view; the dilaton modulus is inherited directly from 10 dimensions, and so cannot possibly give rise to mixed kinetic terms with the complex structure moduli in four dimensions.

So all in all we know that the metric appearing in (6.1) is block diagonal in $\hat{\rho}, \hat{\tau}$ and the remaining moduli, which allows us to write

$$\epsilon = \frac{M_p^2}{2V^2}\left(V_{\hat{\rho}}V_{\hat{\rho}} + V_{\hat{\tau}}V_{\hat{\tau}} + \text{positive terms}\right)\,. \quad (6.17)$$

Thus we get for ϵ the following estimate derived in [50] :

$$\epsilon \geq \frac{M_p^2}{2}\left[\left(\frac{\partial\ln V}{\partial\hat{\rho}}\right)^2 + \left(\frac{\partial\ln V}{\partial\hat{\tau}}\right)^2\right]\,. \quad (6.18)$$

Writing this in terms of ρ and τ we get

$$\epsilon \geq \frac{M_p^2}{V^2}\left[\frac{1}{3}\left(\rho\frac{\partial V}{\partial \rho}\right)^2 + \frac{1}{4}\left(\tau\frac{\partial V}{\partial \tau}\right)^2\right] . \tag{6.19}$$

This can be written as

$$\epsilon \geq \frac{M_p^2}{39V^2}\left(-\rho\frac{\partial V}{\partial \rho} - 3\tau\frac{\partial V}{\partial \tau}\right)^2 + \frac{M_p^2}{13V^2}\left(2\rho\frac{\partial V}{\partial \rho} - \frac{1}{2}\tau\frac{\partial V}{\partial \tau}\right)^2 , \tag{6.20}$$

from which we get

$$\epsilon \geq \frac{M_p^2}{39V^2}\left(-\rho\frac{\partial V}{\partial \rho} - 3\tau\frac{\partial V}{\partial \tau}\right)^2 . \tag{6.21}$$

It is now surprisingly simple to derive a lower bound for the right hand side of (6.21). Classically, the four-dimensional scalar potentials of such compactifications may receive contributions from the NSNS H_3-flux, geometric fluxes, O6/D6-branes and the RR-fluxes F_p, $p = 0, 2, 4, 6$ leading to, respectively, the following terms:

$$V = V_3 + V_f + V_{O6/D6} + V_0 + V_2 + V_4 + V_6, \tag{6.22}$$

where $V_3, V_0, V_2, V_4, V_6 \geq 0$, and V_f and $V_{O6/D6}$ can a priori have either sign. V_f follows from the reduction of the Einstein Hilbert term in (A.2), and it is explicitly given by

$$V_f = -\frac{1}{2}M_P^4\kappa_{10}^2 e^{2\Phi}\mathrm{Vol}^{-1}R = -\frac{1}{2}M_P^4\kappa_{10}^2\tau^{-2}R , \tag{6.23}$$

where R is the Ricci scalar of the internal manifold. By looking at (A.2) and (A.4), it is not difficult to obtain the general scaling behavior of these terms with respect to ρ and τ,

$$V_3 \propto \rho^{-3}\tau^{-2}, \qquad V_p \propto \rho^{3-p}\tau^{-4}, \qquad V_{O6/D6} \propto \tau^{-3}, \qquad V_f \propto \rho^{-1}\tau^{-2} . \tag{6.24}$$

These scalings can also be found by analyzing the potential (5.2) arising in the effective supergravity approach. Using these scalings we get from (6.22)

$$-\rho\frac{\partial V}{\partial \rho} - 3\tau\frac{\partial V}{\partial \tau} = 9V + \sum_{p=2,4,6} pV_p - 2V_f. \tag{6.25}$$

Hence, whenever the contribution from the metric fluxes V_f is zero or negative this gives

$$-\rho\frac{\partial V}{\partial \rho} - 3\tau\frac{\partial V}{\partial \tau} \geq 9V . \tag{6.26}$$

Assuming a regime where $V > 0$, which is necessary for inflation, we can plug this into (6.21) to get

$$\epsilon \geq \frac{27}{13}, \tag{6.27}$$

as it has been derived in [50]. This directly rules out the torus example of section 5.2 as well as the T-dual Iwasawa example.

Avoiding this no-go theorem without introducing any new ingredients would thus require $V_f > 0$. Since $V_f \propto -R$, where R denotes the internal scalar curvature, this is equivalent to demanding that the internal space has negative curvature. Since all terms in V scale with a negative power of τ we see from (6.22) and (6.24) that we then also need $V_{O6/D6} < 0$ to avoid a runaway, which reflects the old result of [88].

Following a related argument in [86], one can identify another combination of derivatives with respect to ρ and τ that sets a bound for ϵ:

$$-3\rho\frac{\partial V}{\partial \rho} - 3\tau\frac{\partial V}{\partial \tau} = 9V + 6V_3 - 6V_0 + 6V_4 + 12V_6 \geq 9V - 6V_0. \tag{6.28}$$

In the case of vanishing mass parameter, we have $V_0 = 0$, and (6.28) implies $\epsilon \geq \frac{9}{7}$. Therefore, we need to have $V_f > 0$, $V_{O6/D6} < 0$ and $V_0 \neq 0$ in order to avoid the above no-go theorems. Note that the only real restriction here is that we have to have a compact space with negative curvature since in our examples we are always free to turn on F_0-flux and to do an orientifold projection. By computing the Ricci scalar (C.35) from the structure constants of the cosets and the metric, which depends on the geometric moduli, we will see that some of the cosets admit a negative curvature in a certain regime of the moduli space and are thus not affected by the no-go theorem of [50].

6.2 A modified no-go theorem for SU(3)-structure

Unfortunately, in [87] yet another no-go theorem has been derived, this time also applying to certain classes of compactifications with negative scalar curvature. We will review it in this section.

The coset examples of SU(3)-structure manifolds have special intersection numbers that allow a split of the index i of the Kähler moduli into $\{0, a\}$, $a = 1, \dots, (h^{2-} - 1)$, such that the only non-vanishing intersection numbers are

$$\kappa_{0ab} \equiv X_{ab}\,. \tag{6.29}$$

We now introduce a variable similar to ρ in (6.8) by defining

$$k^a = \sigma\chi^a\,, \tag{6.30}$$

where σ is the overall scale of $(h^{2-}-1)$ Kähler moduli, and the χ^a are constrained by $X_{ab}\chi^a\chi^b = 2$. The volume can now simply be written as $\mathrm{Vol} = k^0\sigma^2$. Now one does the same kind of computations as we did in the last chapter. Instead of (6.12) we get this time for the kinetic energy

$$T_k = -\left[\frac{(\partial_\mu\sigma)^2}{2\sigma^2} - \left(\frac{\partial_\mu k^0}{2k^0}\right)^2 + \frac{1}{4}X_{ab}\partial_\mu\chi^a\partial^\mu\chi^b - \frac{1}{4}K_{ab}\partial_\mu b^i\partial^\mu b^j\right], \tag{6.31}$$

where we used $\partial_\mu(X_{ab}\chi^a\chi^b) = 0$. This time the canonically normalized field is given by

$$\hat{\sigma} \equiv \ln\sigma\,, \tag{6.32}$$

which gives

$$T_k = -(\partial_\mu\hat{\sigma})^2/2 + \text{positive}. \tag{6.33}$$

The kinetic energy for $\hat{\tau}$ is still the same as in (6.16) so that we get the same bound as in (6.18), but now for σ instead of ρ:

$$\epsilon \geq \frac{1}{2}\left[\left(\frac{\partial \ln V}{\partial\hat{\sigma}}\right)^2 + \left(\frac{\partial \ln V}{\partial\hat{\tau}}\right)^2\right]. \tag{6.34}$$

Writing this in terms of σ and τ we get

$$\epsilon \geq \frac{1}{2V^2}\left[\left(\sigma\frac{\partial V}{\partial\sigma}\right)^2 + \frac{1}{2}\left(\tau\frac{\partial V}{\partial\tau}\right)^2\right]. \tag{6.35}$$

This can be written as

$$\epsilon \geq \frac{1}{18V^2}\left(\sigma\frac{\partial V}{\partial\sigma} + 2\tau\frac{\partial V}{\partial\tau}\right)^2 + \frac{1}{36V^2}\left(4\sigma\frac{\partial V}{\partial\sigma} - \tau\frac{\partial V}{\partial\tau}\right)^2. \tag{6.36}$$

from which we get

$$\epsilon \geq \frac{1}{18V^2}\left(-\sigma\frac{\partial V}{\partial\sigma} - 2\tau\frac{\partial V}{\partial\tau}\right)^2. \tag{6.37}$$

Again it is possible to derive a lower bound for the right hand side of (6.37). Without the geometric fluxes the scalings of the potentials in (6.22) become for the special intersection numbers (6.29)

$$V_3 \propto \frac{(k^0\sigma^2)^{-3}}{\tau^2}, \qquad V_0 \propto \frac{(k^0\sigma^2)^3}{\tau^4}, \qquad V_6 \propto \frac{(k^0\sigma^2)^{-3}}{\tau^4}, \qquad V_{O6/D6} \propto \frac{1}{\tau^3}, \tag{6.38}$$

$$V_2 \propto \frac{(\sigma^4 + (k^0)^2\sigma^2)}{(k^0\sigma^2)^1\tau^4}, \qquad V_4 \propto \frac{(\sigma^2 + (k^0)^2)}{(k^0\sigma^2)^1\tau^4}, \tag{6.39}$$

as it has been explicitly derived in [25].

Defining

$$DV \equiv (-\sigma\partial_\sigma - 2\tau\partial_\tau)\, V\,, \tag{6.40}$$

we obtain from (6.38)

$$\begin{aligned} DV_3 =& 6V_3\,, \\ DV_{O6} =& 6V_{O6}\,, \\ DV_0 =& 6V_0\,, \\ DV_2 =& 6V_2 + \text{positive term}\,, \\ DV_4 =& 8V_4 + \text{positive term}\,, \\ DV_6 =& 10V_6\,. \end{aligned} \tag{6.41}$$

In [87] it was shown that if one defines a matrix r_{iI} by

$$\mathrm{d}Y_i^{2-} = r_{iI} Y^{(3-)I}\,, \tag{6.42}$$

describing the geometric flux of J which is expanded in odd two-forms, the extra condition $r_{aI} = 0$ or $r_{0I} = 0$ leads to $DV_f = 6V_f$. Plugging this and (6.41) into (6.37) one would get

$$\epsilon \geq 2\,, \qquad \text{whenever} \;\; V > 0\,. \tag{6.43}$$

However, in the coset examples that we want to discuss, one always has $r_{aI} \neq 0$, and $r_{0I} \neq 0$ so the no-go theorem of [87] is not directly applicable. But one still can explicitly check for each case separately whether $DV_f \geq 6V_f$ is satisfied or not. In order to do so, it is convenient to write

$$V_f = \frac{1}{2\tau^2 \mathrm{Vol}} U\,, \tag{6.44}$$

so that

$$DV_f = 6V_f + \frac{1}{2\tau^2 \mathrm{Vol}} DU = 6V_f + \frac{1}{2\tau^2 \mathrm{Vol}} (-\sigma\partial_\sigma) U, \tag{6.45}$$

and the no-go theorem applies if we can show that

$$\boxed{-\sigma\partial_\sigma U = -k^a \partial_{k^a} U \geq 0\,.} \tag{6.46}$$

Furthermore, if the inequality (6.46) is strictly valid, Minkowski vacua are ruled out as well. This can be seen as follows. Using (6.41) and (6.45), we obtain

$$DV = 6V + 2V_4 + 4V_6 + \frac{1}{2\tau^2 \mathrm{Vol}}(-\sigma\partial_\sigma)U + \text{positive terms}\,, \tag{6.47}$$

so that for a vacuum, $DV = 0$, we find with (6.46)

$$V = -\frac{1}{6}\left(2V_4 + 4V_6 + \frac{1}{2\tau^2 \mathrm{Vol}}(-\sigma\partial_\sigma)U + \text{positive terms}\right) \leq 0\,. \tag{6.48}$$

So, if the inequality (6.46) holds strictly, also (6.48) holds strictly as well, and Minkowski vacua are ruled out.

Indeed, one can check that the coset models discussed in this thesis do not allow for *supersymmetric* Minkowski vacua with left-invariant SU(3)-structure. Strangely enough, this includes the case SU(2)×SU(2) for which eq. (6.46) can be violated. This model may still allow for a non-supersymmetric Minkowski vacuum. In the next section we will explicitly compute (6.46) for each coset.

6.3 Cosmology of cosets

In the previous section, we described a no-go theorem that rules out dS vacua and slow-roll inflation for type IIA compactifications on certain types of SU(3)-structure manifolds, namely those for which one coordinate in the triple intersection number κ_{ijk} can be separated as in eq. (6.29), and the geometric fluxes induce the relation (6.46). While these seem to be quite strong assumptions, it turns out that almost all the cosets do fall into this category, as we will show in this section. For that we will evaluate (6.46) for each coset explicitly. By looking at (6.44) and (6.23), we see that we first have to compute the Ricci scalar for each coset. It is given in (C.35) in terms of the structure constants and the metric.

6.3.1 The $\frac{\mathbf{G_2}}{\mathbf{SU(3)}}$ no-go

For this case, one finds for the function U of (6.44):

$$U \propto -(k^1)^2\,, \tag{6.49}$$

which is manifestly negative. This implies that V_f itself is manifestly negative so that the no-go theorem of [50], reviewed in section 6.1, already rules out this case. All other coset models allow for values of the moduli such that $V_f > 0$ and therefore require a more careful analysis using the refined no-go theorem of section 6.2.

6.3.2 The $\frac{\mathbf{Sp(2)}}{\mathbf{S(U(2)\times U(1))}}$ no-go

For this case, one has

$$U \propto (k^2)^2 - 4(k^1)^2 - 12k^1k^2 \,, \tag{6.50}$$

and the only non-vanishing intersection number is κ_{112} and permutations thereof, so that k^2 plays the role of k^0, and we have

$$DU = -k^1\partial_{k^1}U \propto 8(k^1)^2 + 12k^1k^2 > 0 \,, \tag{6.51}$$

so that with $k^i > 0$ (because of metric positivity) the inequality (6.46) is strictly satisfied and this model is ruled out.

6.3.3 The $\frac{\mathbf{SU(3)}}{\mathbf{U(1)\times U(1)}}$ no-go

For this coset space, we have

$$U \propto (k^1)^2 + (k^2)^2 + (k^3)^2 - 6k^1k^2 - 6k^2k^3 - 6k^1k^3 \,, \tag{6.52}$$

and the non-vanishing intersection numbers are of the type κ_{123} so that we can choose any one of the three k's as k^0. We will choose k^0 to be the biggest and assume without loss of generality that this is k^1, i.e. that $k^1 \geq k^2, k^3$. We then find that

$$DU = (-k^2\partial_{k^2} - k^3\partial_{k^3})U \propto (6k^1 - 2k^2)k^2 + (6k^1 - 2k^3)k^3 + 12k^2k^3 > 0, \tag{6.53}$$

so that with $k^i > 0$ (because of metric positivity) this coset space is also ruled out by the no-go theorem (6.46).

6.3.4 The $\frac{\mathbf{SU(3)\times U(1)}}{\mathbf{SU(2)}}$ no-go

For this model, the function U depends on an extra positive constant λ related to the choice of orientifolds. The function U turns out to be

$$U \propto (k^2)^2(u^2)^2\lambda - 8k^1k^2|u^1u^2|(1+\lambda^2) \,, \tag{6.54}$$

and the non-vanishing intersection numbers are of the form κ_{112}. Thus k^2 plays the role of k^0, and we find that

$$DU = -k^1\partial_{k^1}U \propto 8k^1k^2|u^1u^2|(1+\lambda^2) > 0, \tag{6.55}$$

so that with $k^i > 0$ (because of metric positivity) this case is also ruled out.

6.3.5 No SU(2) × SU(2) no-go

Thus far, we have found that $\epsilon \geq 2$ for all other cases. For the remaining coset space SU(2)×SU(2), one finds

$$\begin{aligned} U \propto & \sum_{i=1}^{3} (k^i)^2 \left(\sum_{I=1}^{4} (u^I)^2 \right) - 4k^2k^3(|u^1u^2| + |u^3u^4|) \\ & - 4k^1k^2(|u^1u^4| + |u^2u^3|) - 4k^1k^3 \left(|u^1u^3| + |u^2u^4| \right) , \end{aligned} \tag{6.56}$$

and the non-vanishing intersection numbers are of the form κ_{123} so that we could choose any one of the k's as k^0. However, it is not possible to apply the no-go theorem. This can be easily seen if we take for example $u^1 \gg u^2, u^3, u^4$. Then we have schematically $U \propto \vec{k}^2(u^1)^2$ and $DU \propto -k^a k^a (u^1)^2 < 0$. In [87] further no-go theorems have been derived but none of those apply to this case either. We will study this case in more detail in the next chapter.

6.4 The SU(2)×SU(2) coset

In the last section we have seen that the known no-go theorems cannot be used to rule out small ϵ for compactification on SU(2)×SU(2) even though in a numerical analysis we did not find small ϵ.

We will argue in this chapter that from a 4d effective supergravity perspective there are, in a sense we will have to specify, different inequivalent values for the fluxes possible, which lead to inequivalent superpotentials. The superpotentials we found for the cosets in chapter 5 by plugging in the supersymmetric background values for the fields given in chapter 3 are just one possibility. They are characterized by the fact that they allow by construction for a supersymmetric vacuum. In the next section we will make precise what we mean by inequivalent superpotentials. It turns out that there are values for the fluxes leading to superpotentials which do not allow for a supersymmetric minimum in the potential. Exactly for such a non-supersymmetric superpotential we will find that for SU(2)×SU(2) it is possible to get $\epsilon \approx 0$ and there are dS extrema. In principle one could to do such a classification of inequivalent potentials for all the coset spaces in order to study the full moduli space. Note, however, that for all the cosets in which we were able to prove a no-go theorem against inflation, this conclusion is not altered, because we only used the geometrical information, namely the geometrical flux potential for each coset, in this proof[1]. In order to find small ϵ this leaves as the only possibility out of the cosets studied so far the SU(2)×SU(2) model, although numerically we did not find small ϵ. From the viewpoint of this chapter, we should make this more precise by saying that there is no

[1]We are only considering the case of O6-planes. Allowing for other O-planes could change this conclusion

small ϵ in "bubbles", i.e. inequivalent choices of the background fluxes, which allow for a supersymmetric AdS_4 minimum. Indeed we will find configurations with $\epsilon \approx 0$ and $V > 0$ in bubbles that do not allow for supersymmetric AdS vacua.

Taking this 4d point of view it is also possible to study potentials resulting from cosets which do not allow for a supersymmetric vacuum at all. By still restricting for simplicity to cosets which allow for an SU(3)-structure, we will see that there are two more candidates in table C.1. In analyzing the whole moduli space there is one complication, namely the choice of compatible orientifolds. In our supersymmetric analysis they were obtained as a result of the solutions to the Bianchi identities of the fluxes, which were in turn fixed in terms of the geometry by the supersymmetry equations. This is no longer the case for the non-supersymmetric cases, and we will stick in our analysis to the case of O6-planes.

6.4.1 Classifying inequivalent potentials

In this section, we want to classify the different inequivalent superpotentials and the resulting potentials. In what follows, we will call a given set of flux parameters a "bubble". In a given bubble the potential is fixed, and one can reach different points of it by fluctuations of the fields. A natural idea would be to call two bubbles inequivalent when it is not possible to go from one bubble to the other by finite fluctuations of the moduli fields. From the 4d effective supergravity point of view one would then have to classify all inequivalent bubbles and study the potential for each bubble in order to analyze the full moduli space. In this way, we will find bubbles, which do not possess a supersymmetric AdS_4 vacuum and are thus not covered by our analysis so far. We follow here the standard approach of classifying the different bubbles by flux quanta, which is however complicated by the presence of Romans mass, H-field and O6-plane source. Classifying the different bubbles in terms of fluxes amounts to finding configurations that solve the Bianchi identities

$$\mathrm{d}H = 0\,, \tag{6.57a}$$

$$\mathrm{d}F_0 = 0\,, \tag{6.57b}$$

$$\mathrm{d}F_2 + mH = -j_3\,, \tag{6.57c}$$

$$\mathrm{d}F_4 + H \wedge F_2 = 0\,, \tag{6.57d}$$

while two configurations are considered equivalent if they are related by a fluctuation of the moduli fields, which after imposing the orientifold projection (and assuming it removes one-forms) is given by (4.19)

$$\delta H = \mathrm{d}\delta B\,, \tag{6.58a}$$

$$\delta F_0 = 0\,, \tag{6.58b}$$

$$\delta F_2 = -m\delta B\,, \tag{6.58c}$$

$$\delta F_4 = \mathrm{d}\delta C_3 - \delta B \wedge (F_2 + \delta F_2) - \frac{1}{2} m (\delta B)^2 \,, \tag{6.58d}$$

$$\delta F_6 = H \wedge \delta C_3 - \delta B \wedge (F_4 + \delta F_4) - \frac{1}{2} (\delta B)^2 \wedge (F_2 + \delta F_2) - \frac{1}{3!} m (\delta B)^3 \,. \tag{6.58e}$$

In other words, we want to find representatives of the cohomology of the Bianchi identities (6.57) modulo pure fluctuations of the potentials (6.58).

From eqs. (6.58b) we get immediately that F_0 is constant. Using the non-closed part of δB in (6.58a), we can remove the exact part of H and set $H \in H^3(M, \mathbb{R})$ in (6.57a). To analyze (6.57c) and (6.58c), we take the point of view that we choose the flux F_2, which then determines the source j_3. From here on, one has to discuss the case $F_0 \neq 0$ and $F_0 = 0$ separately.

If $F_0 \neq 0$, the closed part of F_2 can be set to zero by choosing the closed part of δB in (6.58c). Thus F_2 is the most general non-closed two form. Moving on to F_4, we find that in eq. (6.57d) $H \wedge F_2 = 0$, since we assumed there were no even five-forms under all the orientifold involutions. Moreover, with the fluctuations δC_3, we can remove the exact part of F_4 so that $F_4 \in H^4(M, \mathbb{R})$. This however, leaves the closed part of δC_3 undetermined, which, if we have chosen H non-trivial, can be used to put[2] $F_6 = 0$. Otherwise we should allow for $F_6 = f \mathrm{dvol}$.

If $F_0 = 0$, there is no δF_2 and F_2 is just the most general two form. Again with δC_3 we can remove the exact part of F_4 so that $F_4 \in H^4(M, \mathbb{R})$, which we can further simplify by using the freedom of choosing the closed part of δB. And also the closed part of δC_3 can, if we have chosen H non-trivial, be used again to put $F_6 = 0$. Otherwise we should allow for $F_6 = f \mathrm{dvol}$.

To illustrate the procedure, we can study the $\frac{\mathrm{G}_2}{\mathrm{SU}(3)}$ coset of section 3.4.1. For the case $F_0 \neq 0$, we obtain the following most general form of the fluxes

$$\begin{aligned} &\hat{F}_0 = m \,, \qquad \hat{F}_2 = \alpha (e^{12} - e^{34} + e^{56}) \,, \\ &\hat{F}_4 = 0 \,, \qquad \hat{F}_6 = f \mathrm{dvol} \,, \qquad \hat{H} = 0 \,, \end{aligned} \tag{6.59}$$

where m, f and α are free parameters. If we use the expansion

$$J_c = t^1 (e^{12} - e^{34} + e^{56}) \,, \tag{6.60}$$

$$\mathrm{Im}\,\Omega_c = z^1 (-e^{235} - e^{246} + e^{145} - e^{136}) \tag{6.61}$$

in (5.3) and (5.4), we obtain the same Kähler potential as in (5.21), and the superpotential is given by

$$\mathcal{W} = f + i m (t^1)^2 + 4\sqrt{3} t^1 z^1 - 3\alpha (t^1)^2 \,, \tag{6.62}$$

which already looks a bit nicer than (5.20). Now we can compute the potential as usual with (5.2).

[2]If there is non-trivial H there is always a δC_3 to put $F_0 = 0$.

6.4.2 Small ϵ for $\mathbf{SU(2)\times SU(2)}$

Now we come to the study of the SU(2)×SU(2) coset space for flux parameters which do not allow for a supersymmetric vacuum. In order to eliminate the one- and five-forms, we must introduce at least three mutually supersymmetric orientifolds, compatible with the structure constants. We can then always perform a basis transformation so that the odd two-forms and odd/even three-forms are the same as in section 5.3.4 and read explicitly

$$
\begin{aligned}
Y_1^{(2-)} =& e^{14}, \qquad Y_2^{(2-)} = e^{25}, \qquad Y_3^{(2-)} = e^{36}, \\
Y^{(3-)1} =& \frac{1}{4}\left(e^{156} - e^{234} - e^{246} + e^{135} + e^{345} - e^{126} + e^{123} - e^{456}\right), \\
Y^{(3-)2} =& \frac{1}{4}\left(e^{156} - e^{234} + e^{246} - e^{135} - e^{345} + e^{126} + e^{123} - e^{456}\right), \\
Y^{(3-)3} =& \frac{1}{4}\left(e^{156} - e^{234} + e^{246} - e^{135} + e^{345} - e^{126} - e^{123} + e^{456}\right), \\
Y^{(3-)4} =& \frac{1}{4}\left(-e^{156} + e^{234} + e^{246} - e^{135} + e^{345} - e^{126} + e^{123} - e^{456}\right), \\
Y_1^{(3+)} =& \frac{1}{2}\left(e^{156} + e^{234} - e^{246} - e^{135} + e^{345} + e^{126} + e^{123} + e^{456}\right), \\
Y_2^{(3+)} =& \frac{1}{2}\left(e^{156} + e^{234} + e^{246} + e^{135} - e^{345} - e^{126} + e^{123} + e^{456}\right), \\
Y_3^{(3+)} =& \frac{1}{2}\left(e^{156} + e^{234} + e^{246} + e^{135} + e^{345} + e^{126} - e^{123} - e^{456}\right), \\
Y_4^{(3+)} =& \frac{1}{2}\left(-e^{156} - e^{234} + e^{246} + e^{135} + e^{345} + e^{126} + e^{123} + e^{456}\right),
\end{aligned}
\tag{6.63}
$$

where the e^α $(\alpha = 1, \ldots, 6)$ are a basis of left-invariant 1-forms. The e^α satisfy

$$
\mathrm{d}e^\alpha = -\frac{1}{2} f^\alpha{}_{\beta\gamma} e^\beta \wedge e^\gamma \,, \tag{6.64}
$$

where the structure constants for SU(2) × SU(2) are $f^1{}_{23} = f^4{}_{56} = 1$, cyclic. From this we find

$$
\mathrm{d}Y_i^{(2-)} = r_{iI} Y^{(3-)I}, \quad \text{with} \quad r = \begin{pmatrix} 1 & 1 & 1 & -1 \\ 1 & -1 & -1 & -1 \\ 1 & -1 & 1 & 1 \end{pmatrix} . \tag{6.65}
$$

In terms of the above expansion forms, we can again define the complex moduli as in (5.7). The positivity of the metric demands

$$
u^1 u^2 < 0 \,, \quad u^3 u^4 < 0 \,, \quad u^1 u^4 < 0 \,. \tag{6.66}
$$

Next we turn to the choice of background fluxes as explained in section 6.4.

We already now from chapter 6.1 that we need a non-vanishing F_0 to get a small ϵ. Furthermore our numerical studies did not give small ϵ for the case of vanishing H flux, which one could choose in the potential of section 5.3.4. For the case where H is non-trivial in cohomology, $p \neq 0$ (see below), the most general form of the background fluxes is

$$F_0 = m, \tag{6.67a}$$

$$F_2 = m^i Y_i^{(2-)}, \tag{6.67b}$$

$$F_4 = 0, \tag{6.67c}$$

$$F_6 = 0, \tag{6.67d}$$

$$H = p\left(Y_1^{(3-)} + Y_2^{(3-)} - Y_3^{(3-)} + Y_4^{(3-)}\right). \tag{6.67e}$$

Plugging in these background values for the fluxes together with the expansion (5.7) in terms of the basis (6.63), we find for the superpotential (5.3)

$$\mathcal{W} = V_s(4\kappa_{10}^2)^{-1}\left(m^1t^2t^3 + m^2t^1t^3 + m^3t^1t^2 - imt^1t^2t^3 - ip(z^1 + z^2 - z^3 + z^4) + r_{iI}t^iz^I\right), \tag{6.68}$$

and the Kähler potential (5.4) reads

$$\mathcal{K} = -\ln\prod_{i=1}^{3}\left(t^i + \bar{t}^i\right) - \ln\prod_{I=1}^{4}\left(z^I + \bar{z}^I\right) + 3\ln\left(V_s^{-1}\kappa_{10}^2M_P^2\right) + \ln 32\,, \tag{6.69}$$

where $V_s = -\int_M e^{123456}$. Note that the superpotential depends on all the moduli so there are no flat directions in this model.

It is straightforward to calculate the scalar potential (5.2) and the slow-roll parameter ϵ (6.1) from the Kähler and superpotential. Although we cannot analytically minimize ϵ, we checked numerically that there is a solution with numerically vanishing ϵ, which means that in this case there is no lower bound for ϵ. To obtain a trustworthy supergravity solution, we would have to make sure that the internal space is large compared to the string length and that the string coupling is small. Furthermore, in the full string theory, the fluxes have to be properly quantized. Although we do not think that this would prevent small ϵ, we did not try to find such a solution because all the solutions with vanishing ϵ we found have a more serious problem, namely that $\eta \lesssim -2.4$. The eigenvalues of the mass matrix turn out to be generically all positive except for one, with the one tachyonic direction being a mixture of all the light fields, in particular the axions. This means that we have a saddle point rather than a dS minimum. A similar instability was found in related models in [87].

In [89], a no-go theorem preventing dS vacua and slow-roll inflation was derived by studying the eigenvalues of the mass matrix. Allowing for an arbitrary tuning of the superpotential, it was shown that for certain Kähler potentials the 'sGoldstino' mass is always negative. For the examples we found, this mass is always positive so that the no-go theorem of [89] does not apply. According to [89] this means that allowing for an arbitrary superpotential it should be possible to remove the tachyonic direction. In our case, however, the superpotential is of course not arbitrary.

Since the no-go theorems against slow-roll inflation do not apply and we have found solutions with vanishing ϵ, we checked whether our solutions allow for small η in the vicinity of the dS extrema. Unfortunately, this is not the case. In fact, we found that η does not change much in the vicinity of our solutions where ϵ is still small.

It would be very interesting to study the SU(2) $\times$ SU(2) model further to check whether one can prove that there is always at least one tachyonic direction or whether it allows for metastable dS vacua after all. Understanding this tachyonic direction better should also allow to decide whether there are points in the moduli space that allow for slow-roll inflation in this model.

6.5 SU(3)-structure cosets without supersymmetric vacuum

In this section, we study the only two coset spaces of the list given in table C.1 that do allow for an SU(3)-structure but not for a supersymmetric AdS_4 vacuum. To keep the analysis tractable we will restrict to *perpendicular* O6-planes, which are aligned along or perpendicular to the one-forms $e^1 \dots e^6$, although we already saw with SU(2)$\times$SU(2) an example where the O-planes are not perpendicular (3.66). As it turns out, it is again possible to apply the no-go theorem of section 6.2 to these cases, which only needs the potential part of the geometric fluxes. Thus, there is no need to compute the full potential.

6.5.1 $\frac{\mathbf{SU(2)}^2}{\mathbf{U(1)}} \times \mathbf{U(1)}$

It was shown in [59] that if the U(1) factor in the denominator does not sit completely in the $\mathrm{SU(2)}^2$, the resulting coset is equivalent to SU(2)$\times$SU(2), so we exclude this possibility here, as the above notation already suggests. The internal manifold is then in fact equivalent to $T^{1,1}\times$U(1). We choose the structure constants as follows (this is $a = 1$, $b = 0$ compared to [59])

$$\begin{aligned} f^1{}_{23} = f^7{}_{45} = 1, \quad \text{cyclic,} \\ f^3{}_{45} = f^2{}_{17} = f^1{}_{72} = 1. \end{aligned} \tag{6.70}$$

The possible orientifolds that are perpendicular to the coordinate frame and compatible with these structure constants are along

$$e^{123}\,, \qquad e^{345}, \qquad e^{256}\,, \qquad e^{146}\,, \qquad e^{246}\,, \qquad e^{156}\,. \tag{6.71}$$

In order to remove one-forms and five-forms, it turns out that we have to introduce two orientifolds, in particular one of $\{123, 345\}$ and one of $\{256, 146, 246, 156\}$. It does not matter for the analysis which particular choice is made, but for definiteness let us choose 345 and 256. We arrive then at the following expansion forms

$$\begin{aligned} &\text{odd 2-forms:} \quad (e^{15}+e^{24})\,, \quad e^{36}\,, \\ &\text{even 3-forms:} \quad e^{123}\,, \quad (e^{256}-e^{146})\,, \quad e^{345} \end{aligned} \tag{6.72}$$

for (5.7).

There is always a change of basis such that we can assume $k^i > 0$. The conditions for metric positivity then become

$$u^1u^2 > 0\,, \qquad u^1u^3 > 0\,. \tag{6.73}$$

U becomes

$$U \propto \frac{-4k^1k^2u^2(u^1+u^3) + (k^2)^2\left[(u^1)^2+(u^3)^2\right]}{2\sqrt{u^1u^3}|u^2|}\,. \tag{6.74}$$

The non-vanishing intersection number is κ_{112} so that k^2 plays the role of k^0, and we get for (6.46):

$$DU = -k^1\partial_{k^1}U \propto \frac{2k^1k^2u^2(u^1+u^3)}{\sqrt{u^1u^3}|u^2|} > 0\,, \tag{6.75}$$

which is positive using the conditions (6.73). Hence, this case is ruled out as well.

6.5.2 SU(2)×U(1)^3

In this case there are ten possible orientifold planes perpendicular to the coordinate frame and compatible with the structure constants. It turns out that in order to remove the one- and five-forms, we have to choose at least three mutually supersymmetric orientifolds and that it does not matter for the analysis which ones we choose. For definiteness, let us take

$$e^{123}\,, \qquad e^{356}, \qquad e^{246}\,. \tag{6.76}$$

With these orientifolds, we get the following expansion forms to be used in (5.7)

$$\begin{aligned} &\text{odd 2-forms:} \quad e^{16}\,, \quad e^{25}\,, \quad e^{34}\,, \\ &\text{even 3-forms:} \quad e^{123}\,, \quad e^{356}\,, \quad e^{264}\,, \quad e^{145}\,. \end{aligned} \tag{6.77}$$

Again there is always a change of basis such that we can assume $k^i > 0$. The positivity of the metric demands that

$$u^1 u^2 > 0\,, \quad u^1 u^3 > 0\,, \quad u^1 u^4 > 0\,. \tag{6.78}$$

For the quantity U as defined in (6.44) we get

$$U \propto \frac{(k^1u^4)^2 + (k^2u^3)^2 + (k^3u^2)^2 - 2k^1u^4k^2u^3 - 2k^1u^4k^3u^2 - 2k^2u^3k^3u^2}{2\sqrt{u^1u^2u^3u^4}}\,. \tag{6.79}$$

The non-vanishing intersection number is κ_{123} so that each k^i can play the role of k^0. Without loss of generality we can assume $k^1u^4 \geq k^2u^3 > 0$, $k^1u^4 \geq k^3u^2 > 0$ and choose k^0 to be k^1. Thus we then find

$$DU = (-k^2\partial_{k^2} - k^3\partial_{k^3})U \propto \frac{-(k^2u^3 - k^3u^2)^2 + k^1u^4(k^2u^3 + k^3u^2)}{\sqrt{u^1u^2u^3u^4}} > 0\,, \tag{6.80}$$

so that we can also rule out this model.

6.6 A comment on extra ingredients

Some ingredients that are not taken into account in the original no-go theorem of [50], see section 6.1, nor in the no-go theorems of [87], see section 6.2, are KK-monopoles, NS5-branes, D4-branes and D8-branes. Some of these ingredients were used in constructing simple dS-vacua in [85]. KK-monopoles would drastically change the topology and geometry of the internal manifold so that their introduction makes it difficult to obtain a clear ten-dimensional picture, hence we will not discuss this possibility further. NS5-branes, D4-branes and D8-branes would contribute through their respective currents j_{NS5}, j_{D4} and j_{D8} as follows to the Bianchi identities

$$\begin{aligned} \mathrm{d}H &= -j_{\mathrm{NS5}}\,, \\ \mathrm{d}F_4 + H \wedge F_2 &= -j_{\mathrm{D4}}\,, \\ \mathrm{d}F_0 &= -j_{\mathrm{D8}}\,. \end{aligned} \tag{6.81}$$

Since H and F_2 should be odd, and F_0 and F_4 even under all the orientifold involutions, we find that j_{NS5} is an odd four-form, j_{D4} an even five-form and j_{D8} an even one-form. In the approximation of left-invariant SU(3)-structure to be used in the next section, one should also impose these brane-currents to be left-invariant (making the branes itself smeared branes). For the concrete models studied in this thesis there are no such currents j_{NS5}, j_{D4} or j_{D8} with the appropriate properties under all orientifold involutions, implying that NS5-branes, D4- and D8-branes cannot be used in these models.

Let us briefly mention that an F-term uplifting along the lines of O'KKLT [48, 90] by combining the coset models with the quantum corrected O'Raifeartaigh model will not be a promising possibility either. The O'Raifeartaigh model is given by $\mathcal{W}_{\rm O} = -\mu^2 S$ and $\mathcal{K}_{\rm O} = S\bar{S} - \frac{(S\bar{S})^2}{\Lambda^2}$. The model has a dS minimum for $S = 0$ where $V_{\rm O} \approx \mu^4$. We combine the two models as follows (the subscript IIA refers to the previously discussed flux and brane contributions)

$$\mathcal{W} = \mathcal{W}_{\rm IIA} + \mathcal{W}_{\rm O}\,, \qquad \mathcal{K} = \mathcal{K}_{\rm IIA} + \mathcal{K}_{\rm O}\,. \tag{6.82}$$

In lowest order in S the total potential is then given by

$$V \approx V_{\rm IIA} + e^{\mathcal{K}_{\rm IIA}} V_{\rm O} + \ldots\,. \tag{6.83}$$

Note that we can then include the contribution of $V_{\rm up} = e^{\mathcal{K}_{\rm IIA}} V_{\rm O}$ in the no-go theorems, because the uplift potential $V_{\rm up}$ scales like F_6,

$$V_{\rm up} = \frac{A_{\rm up}}{\tau^4\,{\rm Vol}}\,. \tag{6.84}$$

Since we assume a positive uplift potential, $V_{\rm up} > 0$, the fact that $V_{\rm up}$ scales like F_6 tells us that adding this uplift potential does not help in circumventing the no-go theorems of section 6.1 or section 6.2.

6.7 Summary

The main result of this chapter is that we can apply, for all but one coset space, a refined no-go theorem of [87] that does *not* just use the volume modulus and the dilaton, but also some of the other Kähler moduli.[3] These would not have been ruled out by the no-go theorem of [50] (except for the example of positive curvature in 6.3.1) which already ruled out the nilmanifolds. Just as in [50], it is the epsilon parameter, i.e., first derivatives of the potential that cannot be made small. Our results in particular show that it is important to make sure that the potential has a critical point (or small first derivative) in *all* directions in moduli space. Moreover, the refined no-go theorem, just as the one of [50], is of a different nature than the no-go theorems developed in [89], which assume a vanishing (or small) first derivative and then show that, under certain conditions, the eta parameter defined in (2.17) cannot be made small enough.

The coset model we do not rule out by a no-go theorem corresponds to the group manifold SU(2)×SU(2)even though we could not find small ϵ by numerical analysis for the form of the superpotential given in section 5.3.4. However, generalizing the allowed fluxes as in section 6.4.1, we were indeed able to find critical points (corresponding to numerically vanishing ϵ) with positive energy density, but only at the price

[3] Problems with field directions orthogonal to the $(\rho,\,\tau)$-plane were also discussed in [86], where attempts were made to construct dS vacua on manifolds that are products of certain three-manifolds.

of a tachyonic direction, corresponding to a large negative eta-parameter, $\eta \lesssim -2.4$. Interestingly, this tachyonic direction does not correspond to the one used in the different types of no-go theorems of [89]. As our numerical search was not exhaustive, however, we cannot completely rule out the existence of dS vacua or inflating regions for this case. Since this case also does not allow for a supersymmetric Minkowski vacuum as mentioned below (6.48), our discussion covers all SU(3)-structure compactifications on semi-simple and U(1) cosets that have a supersymmetric vacuum.

Furthermore, we also studied the remaining two coset spaces of table C.1 that do admit an SU(3)-structure but no supersymmetric AdS vacuum. Choosing for simplicity the O-planes such that one-forms are projected out and restricting to O-planes perpendicular to the coordinate frame, we could again use the refined no-go theorem of section 6.2 to rule out dS vacua and slow-roll inflation for both of these cases as well. At the end we briefly excluded some of the most important extra ingredients that one can think of to modify the models in such a way as to allow for small ϵ.

Again we believe that our results are valid even if we are not able to decouple the KK tower for the same reasons as the ones given in section 5.3.6.

Chapter 7

Non-supersymmetric vacua

In this chapter, we want to study non-supersymmetric vacua on the three cosets spaces $\frac{\mathrm{G}_2}{\mathrm{SU}(3)}$, $\frac{\mathrm{Sp}(2)}{\mathrm{S}(\mathrm{U}(2)\times\mathrm{U}(1))}$ and $\frac{\mathrm{SU}(3)}{\mathrm{U}(1)\times\mathrm{U}(1)}$ whose supersymmetric vacua we have analyzed in the preceding chapters. In particular, we will be interested in the coset space $\frac{\mathrm{Sp}(2)}{\mathrm{S}(\mathrm{U}(2)\times\mathrm{U}(1))}$ which is topologically equivalent to $\mathbb{CP}^3$. The latter has played an important role in the recently conjectured $\mathrm{AdS}_4/\mathrm{CFT}_3$ correspondence [54], as already explained in section 2.4. To study this correspondence further, the non-supersymmetric vacua on this space are as important as the supersymmetric ones. We will not consider any sources in this chapter. The supersymmetric vacuum for $\mathbb{CP}^3$ was first constructed in [58] and, allowing for sources, in [59]. As we will review, a non-supersymmetric vacuum was constructed in [57, 64]. Moreover, there exist already some general mechanisms [62, 63] to produce non-supersymmetric vacua starting from a supersymmetric one. But with our ansatz, which is somehow trimmed to the explicit coset examples, we will find non-supersymmetric vacua that have not appeared in the literature so far.

As can be seen in chapter 3.4 the $\frac{\mathrm{G}_2}{\mathrm{SU}(3)}$ and the $\frac{\mathrm{Sp}(2)}{\mathrm{S}(\mathrm{U}(2)\times\mathrm{U}(1))}$ coset spaces are, in some sense, special cases of the $\frac{\mathrm{SU}(3)}{\mathrm{U}(1)\times\mathrm{U}(1)}$ coset and our analysis will be presented in a form which is adapted to the latter, but can then be easily specialized to the other two coset spaces. In the next section we will present our strategy to find non-supersymmetric vacua, namely to solve the equations of motion, before we will analyze the resulting equations for each of the three cosets separately, starting with $\frac{\mathrm{G}_2}{\mathrm{SU}(3)}$ which is the simplest. Of course, as already mentioned in section 2.4, for phenomenological applications these non-supersymmetric vacua are also of interest and one should study them in the same way as we studied the supersymmetric ones. In particular, one would have to check the stability of those vacua since, as opposed to supersymmetric vacua, they may have tachyonic directions. So strictly speaking, we will construct non-supersymmetric extrema and it remains to be checked whether they are true vacua of the theory.

7.1 Generalizing the supersymmetric solution

To construct non-supersymmetric vacua our strategy is to start from the supersymmetric solutions given in section 3.4. We will then keep the geometry, namely the SU(3)-structure (J, Ω) and the torsion classes $\mathcal{W}_{1,2}^-$ given in sections 3.4.1, 3.4.2 and 3.4.3, of the supersymmetric vacua unchanged, but write down the most general ansatz for the fluxes on these coset spaces. As we saw in section 3.4 for all three cosets there is always only one closed left-invariant three-form to expand H in and there are at most three linear independent two-forms leading to the following general ansatz for the fluxes:

$$\begin{aligned}
F_0 &= c_0 \\
F_2 &= c_4 J + i c_5 \mathcal{W}_2^- + c_8 P \\
F_4 &= c_1 J \wedge J + i c_6 J \wedge \mathcal{W}_2^- + c_7 J \wedge P \\
F_6 &= c_2 \mathrm{dvol}_6 \\
H &= c_3 \mathrm{Re}\Omega \,,
\end{aligned} \tag{7.1}$$

where the c_i are real parameters and the dilaton and warp factor have been put to zero. We have converted the external part of F_4 into an internal part of F_6 and expressed everything in terms of the torsion classes. For $\frac{\mathrm{SU}(3)}{\mathrm{U}(1)\times\mathrm{U}(1)}$ there are three linear independent two forms. Two of them are given by J and $\mathcal{W}_2^-$. One then finds a third linear independent closed primitive $(1,1)$-form P with the following properties:

$$\begin{aligned}
P \wedge \Omega = 0 \,, \qquad & \mathcal{W}_2^- \cdot P = 0 \\
P \wedge J \wedge \mathcal{W}_2^- = 0 \,, \qquad & \star P = -P \wedge J
\end{aligned} \tag{7.2}$$

Furthermore, by the same arguments as for $\mathcal{W}_2^-$ in (B.24) one can show the following relation:

$$P \wedge P \wedge J = -\frac{|P|^2}{2} \mathrm{vol}_6 \tag{7.3}$$

and we can normalize P such that

$$|P|^2 = |\mathcal{W}_2^-|^2 \,. \tag{7.4}$$

This susy solution is recovered by setting

$$\begin{aligned}
& c_5 = 1 \,, \quad c_6 = 0 \,, \quad c_7 = 0 \,, \quad c_8 = 0 \,, \\
& c_0 = m \,, \quad c_1 = \frac{3}{10} m \,, \quad c_2 = -\frac{9}{4} i \mathcal{W}_1^- \,, \quad c_3 = \frac{2}{5} m \,, \quad c_4 = \frac{i}{4} \mathcal{W}_1^- \,.
\end{aligned} \tag{7.5}$$

and using the relation

$$m^2 = \frac{15}{16}|\mathcal{W}_1^-|^2 - \frac{5}{16}|\mathcal{W}_2^-|^2\,. \tag{7.6}$$

The susy solution solves the full equations of motion and Bianchi identities of type IIA supergravity without the need of any sources given in appendix A. Now we want to use our ansatz (7.1) in those equations of motion and study the solutions to them. Without sources and vanishing dilaton they read

$$0 = \frac{1}{2}H^2 - \frac{1}{8}\sum_n (5-n)F_n^2\,, \tag{7.7}$$

$$0 = R_{MN} + g_{MN}\left(\frac{1}{8}H^2 + \frac{1}{32}\sum_n (n-1)F_n^2\right) - \frac{1}{2}H_M \cdot H_N - \frac{1}{4}\sum_n F_{n\,M}\cdot F_{n\,N}\,, \tag{7.8}$$

$$0 = \mathrm{d}\,(\star F_n) - H \wedge \star F_{(n+2)}\,, \tag{7.9}$$

$$0 = \mathrm{d}F + H \wedge F\,, \tag{7.10}$$

$$0 = \mathrm{d}\star H - \frac{1}{2}\sum_n \star F_n \wedge F_{(n-2)}\,. \tag{7.11}$$

The equation of motion for F_2 and the Bianchi for F_4 are trivially satisfied for our ansatz (7.1). From the Bianchi identity of F_2, the equation of motion for F_4 and the dilaton equation of motion we obtain

$$0 = 8c_3c_0 - 12i\mathcal{W}_1^- c_4 + c_5|\mathcal{W}_2^-|^2\,, \tag{7.12}$$

$$0 = 8c_3c_2 + 24ic_1\mathcal{W}_1^- + c_6|\mathcal{W}_2^-|^2\,, \tag{7.13}$$

$$0 = 16c_3^2 - 10c_0^2 - 18c_4^2 - 24c_1^2 + 2c_2^2 - (c_6^2 + 3c_5^2 + c_7^2 + 3c_8^2)|\mathcal{W}_2^-|^2\,. \tag{7.14}$$

The equation of motion for the H field (7.11) gives

$$\begin{aligned} 0 =& \Big(- ic_3\mathcal{W}_1^- - \frac{1}{2}c_4c_0 - 2c_1c_4 - c_2c_1\Big) J \wedge J \\ &+ \Big(- c_3 + c_5c_0 - 2c_1c_5 + c_4c_6 - c_2c_6\Big) i\mathcal{W}_2^- \wedge J \\ &+ \Big(c_8c_0 - 2c_1c_8 + c_7c_4 - c_2c_7\Big) J \wedge P + \Big(ic_5\mathcal{W}_2^- + c_8P\Big) \wedge \Big(ic_6\mathcal{W}_2^- + c_7P\Big) \end{aligned} \tag{7.15}$$

Since we know that there are at most three independent four-forms this leads to three independent equations, which we obtain by wedging this equation with J, $\mathcal{W}_2^-$ and P, respectively. The result can be simplified by using the primitivity of $\mathcal{W}_2^-$ and P

as well as (7.3). Furthermore, we define

$$\begin{aligned} X_1 \mathrm{vol}_6 \equiv W \wedge W \wedge W &= P \wedge P \wedge W \,, \\ X_2 \mathrm{vol}_6 \equiv W \wedge W \wedge P &= P \wedge P \wedge P \,, \end{aligned} \tag{7.16}$$

where the last equality follows from the properties (7.2) and (7.4) of P. With the above relations the three resulting equations that one obtains by wedging (7.15) with J, $\mathcal{W}_2^-$ and P, respectively, are given by

$$0 = 12\left(c_2 c_1 + 2c_1 c_4 + i c_3 \mathcal{W}_1^-\right) + 6c_4 c_0 + (c_5 c_6 + c_8 c_7)|\mathcal{W}_2^-|^2 \,, \tag{7.17}$$

$$0 = |\mathcal{W}_2^-|^2 \left(c_0 c_5 - c_3 - 2c_1 c_5 + c_4 c_6 - c_2 c_6\right) + 2i(c_5 c_6 - c_7 c_8) X_1 + 2(c_5 c_7 + c_6 c_8) X_2 \,,$$

$$0 = \left(c_0 c_8 - c_2 c_7 + c_7 c_4 - 2c_1 c_8\right)|\mathcal{W}_2^-|^2 + 2(c_5 c_6 - c_7 c_8) X_2 - 2i(c_5 c_7 + c_6 c_8) X_1 \,.$$

Since AdS_4 is Einstein, the whole information of the external part of the 10d Einstein equation (7.8) is in it's trace which is given by

$$8R_4 = 2c_0^2 - 16c_3^2 - 6c_4^2 - 72c_1^2 - 10c_2^2 - (c_5^2 + 3c_6^2 + c_8^2 + 3c_7^2)|\mathcal{W}_2^-|^2 \,. \tag{7.18}$$

We split the internal part of (7.8) into the trace and the traceless part defined by

$$R_{0mn} \equiv R_{mn} - \frac{1}{6} g_{mn} R \,. \tag{7.19}$$

The Ricci scalar for manifolds with SU(3)-structure is given by [91]

$$R = \frac{1}{4}(30|\mathcal{W}_1^-|^2 - |\mathcal{W}_2^-|^2) \,. \tag{7.20}$$

Using this formula we obtain from the trace of (7.8)

$$0 = 120|\mathcal{W}_1^-|^2 - 48c_3^2 - 6c_0^2 - 30c_4^2 - 18c_2^2 - 168c_1^2 - |\mathcal{W}_2^-|^2(5c_5^2 + 7c_6^2 + 5c_8^2 + 7c_7^2 + 4) \,, \tag{7.21}$$

where the trace of the external part drops out because of (7.18). For the traceless part we obtain

$$R_{0mn} = \frac{1}{2} F_{2m} \cdot F_{2n} + \frac{1}{2} F_{4m} \cdot F_{4n} - \frac{1}{6} g_{mn}\left(F_2^2 + 2F_4^2\right) \,. \tag{7.22}$$

Plugging in our ansatz (7.1) and subtracting the supersymmetric solution we get

$$\begin{aligned} 0 = {} & (c_5^2 - c_6^2 - 1)\left(\mathcal{W}_{2m}^{-\ x} \mathcal{W}_{2xn}^- - \frac{1}{6} g_{mn} |\mathcal{W}_2^-|^2\right) \\ & - (c_8^2 - c_7^2)\left(P_m^{\ x} P_{xn} + \frac{1}{6} g_{mn} |\mathcal{W}_2^-|^2\right) - i(c_5 c_8 - c_6 c_7) \mathcal{W}_{2(m}^{-\ x} P_{xn)} \\ & - J_{(m}^{\ x} \mathcal{W}_{2xn)}^- \left(2ic_1 c_6 + ic_4 c_5 + \frac{1}{4}\mathcal{W}_1^-\right) - J_{(m}^{\ x} P_{xn)} \left(c_8 c_4 + 2c_1 c_7\right) \,, \end{aligned} \tag{7.23}$$

where the symmetrization is with weight one and only affects uncontracted indices. As for the equation of motion for H there are again three independent parts of the Einstein equation. One is given by the trace part, which is obtained by the contraction with the inverse metric $g^{mn} = -J^{mx}J_x{}^n$. The other two parts are obtained by contracting (7.23) with $J^{(mx}\mathcal{W}^{-}_{2x}{}^{n)}$ and $J^{(mx}P_x{}^{n)}$. It is possible to express the resulting traces in terms of X_1, X_2 and $|\mathcal{W}^{-}_{2xn}|^2$. This results in

$$\begin{aligned}
0 &= (c_5^2 - c_6^2 + c_7^2 - c_8^2 - 1)X_1 - 2i(c_5c_8 - c_6c_7)X_2 + 2(2ic_1c_6 + ic_4c_5 + \frac{1}{4}\mathcal{W}_1^-)|\mathcal{W}_2^-|^2\,, \\
0 &= (c_5^2 - c_6^2 + c_7^2 - c_8^2 - 1)X_2 - 2i(c_5c_8 - c_6c_7)X_1 - 2(c_8c_4 + 2c_1c_7)|\mathcal{W}_2^-|^2\,.
\end{aligned} \tag{7.24}$$

So, in order to find a vacuum of type IIA supergravity for fluxes of the form (7.1) one has to solve the nine equations (7.13), (7.17), (7.21) and (7.24). in terms of the nine variables c_i. The equation (7.18) then determines the external scalar curvature. For a given solution to the above equations one can always produce three more by the following sign changes which each leave those equations invariant:

- keep c_3 and change all other signs;
- change c_3 together with c_0, c_1, c_6, c_7 and keep the rest. (7.25)

Thus, solutions to these equations will always come in quadruples. We will try to solve these equations for our three coset models in the next sections.

7.2 Non-supersymmetric vacua on $\frac{\mathrm{G}_2}{\mathrm{SU(3)}}$

For the coset space $\frac{\mathrm{G}_2}{\mathrm{SU(3)}}$ there is only one two-form, which is given by J. Thus there is no room for a second torsion class $\mathcal{W}_2^-$ or an additional two-form P and we have $c_5 = c_6 = c_7 = c_8 = 0$. This simplifies the equations to a huge extent. Plugging in the result for $\mathcal{W}_1^-$ from section 3.4.1, given by

$$\mathcal{W}_1^- = -\frac{2i}{\sqrt{3a}}\,, \tag{7.26}$$

and defining $C_i \equiv \sqrt{a}c_i$ $(i = 1 \ldots 4)$, we obtain from (7.13), (7.17) and (7.21)

$$\begin{aligned}
0 &= C_3C_0 - \sqrt{3}C_4\,, \\
0 &= 2\sqrt{3}C_1 + C_3C_2\,, \\
0 &= 16C_3^2 - 10C_0^2 - 18C_4^2 - 24C_1^2 + 2C_2^2\,, \\
0 &= 4C_3 + \sqrt{3}(C_4C_0 + 4C_1C_4 + 2C_2C_1)\,, \\
0 &= 160 - (48C_3^2 + 6C_0^2 + 30C_4^2 + 168C_1^2 + 48C_2^2)\,.
\end{aligned} \tag{7.27}$$

The equation (7.18) for the external curvature becomes

$$8cR_4 = 2C_0^2 - 16C_3^2 - 6C_4^2 - 72C_1^2 - 10C_2^2 \,. \tag{7.28}$$

Up to the sign changings (7.25) there are only three solutions to (7.27), which are given by

$$\begin{aligned}
&C_0 = \sqrt{\frac{5}{3}}\,, \quad C_1 = 0\,, \quad C_2 = \frac{5}{\sqrt{3}}\,, \quad C_3 = 0\,, \quad C_4 = 0\,, \\
&C_0 = 1\,, \quad C_1 = -\frac{1}{2}\,, \quad C_2 = \sqrt{3}\,, \quad C_3 = 1\,, \quad C_4 = \frac{1}{\sqrt{3}}\,, \\
&C_0 = -\frac{\sqrt{5}}{2}\,, \quad C_1 = -\frac{3}{4\sqrt{5}}\,, \quad C_2 = -\frac{3\sqrt{3}}{2}\,, \quad C_3 = -\frac{1}{\sqrt{5}}\,, \quad C_4 = \frac{1}{2\sqrt{3}}\,,
\end{aligned} \tag{7.29}$$

The last solution corresponds to the supersymmetric solution of section 3.4.1, whereas the other two solutions as well as the sign changings (7.25) of all the above solutions give rise to non-supersymmetric solutions. The above solutions are all of the type already found in [61], where non-supersymmetric vacua for Nearly-Kähler manifolds ($\mathcal{W}_2^- = 0$) have been constructed.

7.3 Non-supersymmetric vacua on $\frac{\mathrm{Sp}(2)}{\mathrm{S}(\mathrm{U}(2)\times\mathrm{U}(1))}$

For the coset space $\frac{\mathrm{Sp}(2)}{\mathrm{S}(\mathrm{U}(2)\times\mathrm{U}(1))}$ there are two linear independent two-forms, which we choose to be J and $\mathcal{W}_2^-$. Thus there is no room for the two-form P and we have $c_7 = c_8 = 0$. This still simplifies the equations considerably. The explicit values for X_1 and $|\mathcal{W}_2^-|^2$ follow from the solution in section 3.4.2 and are given by

$$\mathcal{W}_1^- = \frac{i}{3}\frac{2+\sigma}{\sqrt{c}}\,, \qquad |\mathcal{W}_2^-|^2 = \frac{16(1-\sigma)^2}{3c}\,, \qquad X_1 = -\frac{32i(1-\sigma)^3}{9c^{3/2}}\,, \tag{7.30}$$

where we have used the shape parameter σ, defined in section 3.45, which measures the deviation from the nearly-Kähler limit.

Defining $C_i \equiv \sqrt{c}c_i$ for $i = 1\ldots 4$, we get from (7.13), (7.17), (7.21) and (7.24):

$$\begin{aligned}
0 &= 6C_3C_0 + 3(2+\sigma)C_4 + 4c_5(1-\sigma)^2\,, \\
0 &= 3C_1(2+\sigma) - 2C_6(1-\sigma)^2 - 3C_3C_2\,, \\
0 &= 48C_3^2 - 30C_0^2 - 54C_4^2 - 72C_1^2 + 6C_2^2 - 16(1-\sigma)^2(3c_5^2 + c_6^2)\,, \\
0 &= 6C_3(2+\sigma) - 9C_4C_0 - 36C_1C_4 - 18C_2C_1 - 8c_5c_6(1-\sigma)^2\,, \\
0 &= 3(1-\sigma)^2(c_5C_0 - C_3 - 2C_1C_5 + C_4c_6 - C_2c_6) + 4c_5c_6(1-\sigma)^3\,, \\
0 &= 20(2+\sigma)^2 - 8(5c_5^2 + 7c_6^2 + 4)(1-\sigma)^2 - 9C_0^2 - 45C_4^2 - 252C_1^2 - 27C_2^2 - 72C_3^2\,, \\
0 &= (c_5^2 - c_6^2 - 1)(1-\sigma)^3 - (1-\sigma^2)(24C_1c_6 + 12C_4c_5 + (2+\sigma))\,,
\end{aligned} \tag{7.31}$$

The equation (7.18) for the external curvature becomes

$$8cR_4 = 2C_0^2 - 16C_3^2 - 6C_4^2 - 72C_1^2 - 10C_2^2 - (c_5^2 + 3c_6^2)\frac{16(1-\sigma)^2}{3}\,. \tag{7.32}$$

7.3.1 Reproducing known results

In the last section we saw that our ansatz did not lead to any new results for the cosets space $\frac{\mathrm{G}_2}{\mathrm{SU(3)}}$. This space is a Nearly-Kähler manifold, i.e. $\mathcal{W}_2^- = 0$, and its non-supersymmetric vacua all fall into the class described in [61]. The cosets space $\frac{\mathrm{Sp(2)}}{\mathrm{S(U(2)\times U(1))}}$, however, is in general not a Nearly-Kähler manifold. The shape of this coset is parameterized by σ and for special values of this parameter there exist already some results in the literature. Here, we want to reproduce these results before we will study the new vacua on this space.

Non-supersymmetric vacua on Nearly-Kähler manifolds

In [61] non-supersymmetric vacua for Nearly-Kähler manifolds were constructed. The coset $\frac{\mathrm{Sp(2)}}{\mathrm{S(U(2)\times U(1))}}$ becomes Nearly-Kähler only for the special value $\sigma = 1$. c_5 and c_6 are not determined in this case, because $\mathcal{W}_2^- = 0$. However, we could still modify our fluxes by a second two-form different from $\mathcal{W}_2^-$. This was different for the $\frac{\mathrm{G}_2}{\mathrm{SU(3)}}$ coset because in that case there is no other two-form than J. This kind of deformation needs a separate treatment. We get the following solutions to (7.31):

$$\begin{aligned}
&C_0 = \frac{\sqrt{3}}{2}\,, \quad C_1 = -\frac{\sqrt{3}}{4}\,, \quad C_2 = \frac{3}{2}\,, \quad C_3 = -\frac{\sqrt{3}}{2}\,, \quad C_4 = \frac{1}{2}\,,\\
&C_0 = \frac{\sqrt{5}}{2}\,, \quad C_1 = 0\,, \quad C_2 = \frac{5}{2}\,, \quad C_3 = 0\,, \quad C_4 = 0\,,\\
&C_0 = \frac{\sqrt{15}}{4}\,, \quad C_1 = \frac{3}{8}\sqrt{\frac{3}{5}}\,, \quad C_2 = \frac{9}{4}\,, \quad C_3 = \sqrt{\frac{3}{20}}\,, \quad C_4 = -\frac{1}{4}\,.
\end{aligned} \tag{7.33}$$

This was expected, since for $\sigma = 1$ the coset space $\frac{\mathrm{Sp(2)}}{\mathrm{S(U(2)\times U(1))}}$ looks like $\frac{\mathrm{G}_2}{\mathrm{SU(3)}}$ and the above solutions correspond to the ones found in (7.29) which already appeared in [61].

Non-supersymmetric vacua from M-theory

In [62, 63, 80] non-supersymmetric solutions in M-theory are discussed. Reducing these solutions to type IIA string theory implies solutions where one starts from a supersymmetric solution with only F_2 and F_6 non-vanishing (in particular $F_0 = 0$ which forces us to put $\sigma = 2$ or $\sigma = 2/5$ to reproduce their results) and obtains a non-supersymmetric solution with the same F_2, a modified F_6 and non-vanishing H as well as F_4. For $\sigma = 2$ these vacua are given by the following solutions to (7.31):

$$\begin{aligned} \text{Susy}: \quad & C_0 = 0,\, C_1 = 0,\, C_2 = 3,\, C_3 = 0,\, C_4 = -\frac{1}{3},\, c_5 = 1,\, c_6 = 0\,, \\ \text{Non-Susy}: \quad & C_0 = 0,\, C_1 = \frac{1}{2},\, C_2 = -2,\, C_3 = -1,\, C_4 = -\frac{1}{3},\, c_5 = 1,\, c_6 = 0\,, \end{aligned} \tag{7.34}$$

while for $\sigma = 2/5$ they are given by

$$\begin{aligned} \text{Susy}: \quad & C_0 = 0,\, C_1 = 0,\, C_2 = \frac{9}{5},\, C_3 = 0,\, C_4 = -\frac{1}{5},\, c_5 = 1,\, c_6 = 0\,, \\ \text{Non-Susy}: \quad & C_0 = 0,\, C_1 = \frac{3}{10},\, C_2 = -\frac{6}{5},\, C_3 = -\frac{3}{5},\, C_4 = -\frac{1}{5},\, c_5 = 1,\, c_6 = 0\,. \end{aligned} \tag{7.35}$$

We also get solutions corresponding to the sign changings (7.25) of the above solutions. We see exactly the expected behavior. F_2, specified by C_4 and c_5, stays the same while F_6 (C_2) gets modified. This is somehow compensated by turning on H (C_3) and F_4 (C_2).

Non-supersymmetric vacua on Einstein manifolds

In [64] and [57] solutions on Einstein manifolds are discussed where one starts from a supersymmetric solution with $c_0 = m = 0$ and $H = 0$ and gets a non-supersymmetric solution with $c_0 \neq 0$ keeping $H = 0$. Our coset becomes an Einstein manifold only for the special value $\sigma = 2$. Their ansatz for the fluxes is given by

$$H = 0\,, \quad F_0 = \alpha\,, \quad F_2 = \beta\tilde{J}\,, \quad F_4 = \frac{1}{2}\gamma\tilde{J}^2\,, \quad F_6 = \frac{1}{6}\delta\tilde{J}^3\,, \tag{7.36}$$

where $\tilde{J}$ is the Kähler form, i.e. it is closed. For the special case we are discussing here it is given by

$$\tilde{J} = \frac{1}{3}J - i\sqrt{c}\mathcal{W}_2^- \tag{7.37}$$

.

Since in our ansatz (7.1) for the fluxes the J is not the Kählerform this fixes our parameters c_5 and c_6 in terms of c_4 and c_1:

$$c_5 = -3\sqrt{c}c_4\,, \qquad c_6 = 6\sqrt{c}c_1\,. \tag{7.38}$$

Putting $c_3 = 0$ our ansatz (7.1) then reads

$$H = 0\,, \quad F_0 = c_0\,, \quad F_2 = 3c_4\tilde{J}\,, \quad F_4 = -3c_1\tilde{J}^2\,, \quad F_6 = -\frac{1}{6}c_2\tilde{J}^3\,, \tag{7.39}$$

which gives the following relation between our parameters c_i and the parameters (7.36) appearing in [64] and [57]:

$$c_0 = \alpha\,, \quad , c_2 = -\delta\,, \quad 3c_4 = \beta\,, \quad -6c_1 = \gamma\,. \tag{7.40}$$

Plugging these values into (7.31) only the third, fourth and sixth equation are non-trivial and read

$$\begin{aligned} 0 &= \alpha\beta + 2\gamma\beta + \gamma\delta\,, \\ 0 &= 15\alpha^2 + 27\beta^2 + 9\gamma^2 - 3\delta^2\,, \\ 16R &= 6\alpha^2 + 30\beta^2 + 42\gamma^2 + 18\delta^2\,. \end{aligned} \tag{7.41}$$

The other equations are trivially satisfied due to the closure of $\tilde{J}$. Furthermore from the external Einstein equation (7.18) we get

$$-12R_4 = -3\alpha^2 + 9\beta^2 + 27\gamma^2 + 15\delta^2\,. \tag{7.42}$$

These equations are equivalent to the equations (3.11), (3.12), (3.14) and (3.15) of [64] and we obtain exactly their solutions.

7.3.2 New non-supersymmetric vacua on $\frac{\mathbf{Sp(2)}}{\mathbf{S(U(2)\times U(1))}}$

Here we will give a preliminary analysis of the solutions to the equations (7.31) for all values of σ. To get a qualitative picture we plot in figure 7.1 the possible solutions for C_3, parameterizing H, against σ. The plots for the other variables C_i look very similar. The plot is symmetric under $C_3 \to -C_3$ due to the sign changings (7.25). Red points indicate the already known non-supersymmetric solutions from M-theory for the special values $\sigma = 2/5$ and $\sigma = 2$ as well as the supersymmetric solution, discussed in 7.3.1. We see that both solutions can be varied continuously between $\sigma = 2/5$ and $\sigma = 2$. Interestingly the non-supersymmetric solution also exists for a certain range beyond $\sigma = 2$. Green dots indicate the known solutions for Nearly-Kähler manifolds ($\sigma = 1$) also discussed in 7.3.1. We will have to leave a further study of the new non-supersymmetric vacua for future work [27]. A first step would be to analyze the stability of those vacua, i.e. to check whether they exhibit any tachyonic directions below the Breitenlohner-Friedman bound.

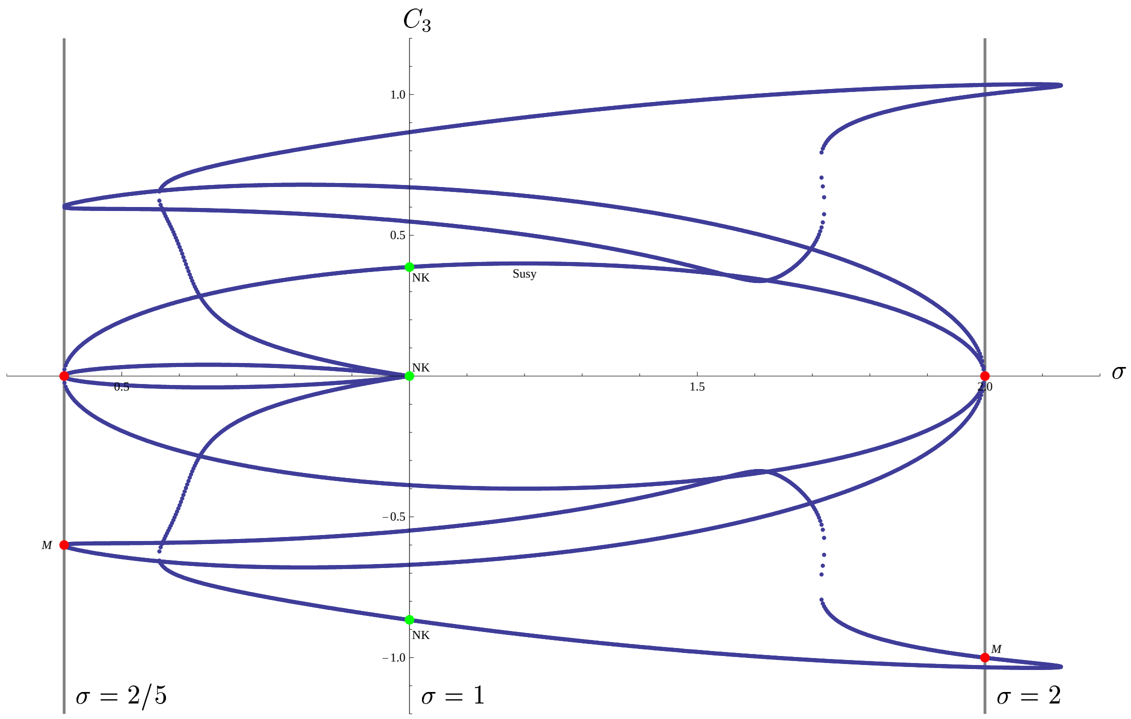


Figure 7.1: Solutions for C_3 for all possible values of σ

7.4 Non-supersymmetric vacua on $\frac{\mathbf{SU(3)}}{\mathbf{U(1)\times U(1)}}$

This is the most complicated case. We will express the equations in terms of the parameters given in section 3.4.3. We compute for the form P

$$P = \frac{2\sqrt{a}}{\sqrt{3\sigma\rho}}\left[(\rho-\sigma)e^{12} + \rho(1-\sigma)e^{34} + \sigma(1-\rho)e^{56}\right] , \tag{7.43}$$

where we have used the shape parameters σ and ρ, defined in section 3.4.3. The explicit values for X_1, X_2 and $|\mathcal{W}_2^-|^2$ follow from the solution in section 3.4.3 and are given by

$$\mathcal{W}_1^- = -\frac{i}{3}\frac{1+\rho+\sigma}{\sqrt{a\rho\sigma}} , \tag{7.44}$$

$$|\mathcal{W}_2^-|^2 = \frac{16}{3a\sigma\rho}(1+\rho^2+\sigma^2-\rho-\sigma\rho-\sigma) , \tag{7.45}$$

$$X_1 = -\frac{16i}{9(a\sigma\rho)^{3/2}}(2\sigma-1-\rho)(2-\rho-\sigma)(2\rho-1-\sigma) , \tag{7.46}$$

$$X_2 = \frac{16}{\sqrt{3}(a\sigma\rho)^{3/2}}(1-\rho)(1-\sigma)(\rho-\sigma) . \tag{7.47}$$

Plugging this into (7.13), (7.17), (7.21) and (7.24) one obtains the equations for this coset. However, we do not study this case any further here but leave this for future work. In principle, one would have to study the variations away from $\rho = 1$, since for that special value this coset looks like the $\frac{\mathrm{Sp}(2)}{\mathrm{S}(\mathrm{U}(2)\times\mathrm{U}(1))}$ model and we expect the same results.

Chapter 8

Conclusions

In this thesis, we studied a number of type IIA SU(3)-structure compactifications with O6-planes on nilmanifolds and cosets, which are tractable enough to allow for an explicit derivation of the low energy effective theory. In particular, in **chapter 5** we calculated the mass spectrum of the light scalar modes, using $\mathcal{N} = 1$ supergravity techniques. For the torus and the Iwasawa solution, we have also performed an explicit Kaluza-Klein reduction in **chapter 4**, which led to the same result, supporting the validity of the effective supergravity approach, with superpotential (5.3) and Kähler potential (5.4), also in the presence of geometric fluxes. For the nilmanifold examples we have found that there are always three unstabilized moduli corresponding to axions in the RR sector. On the other hand, in the coset models, except for SU(2)×SU(2), all moduli are stabilized.

We discussed the Kaluza-Klein decoupling in section 4.2 for the supersymmetric AdS vacua and found that it requires going to the Nearly-Calabi Yau limit. For our nilmanifolds, this can be arranged by tuning the parameters, while for our coset models it is somewhat harder. Indeed, we found that for $\frac{\mathrm{Sp}(2)}{\mathrm{S}(\mathrm{U}(2)\times\mathrm{U}(1))}$ and $\frac{\mathrm{SU}(3)}{\mathrm{U}(1)\times\mathrm{U}(1)}$ one has to make a continuation to negative values of the "shape" parameters. Strictly speaking, this can no longer be described as a left-invariant SU(3)-structure on a coset anymore, but it can still be described in terms of a twistor bundle over a four-dimensional hyperbolic space. It would be interesting to study these models in more detail, as there are more examples of this type. Another class of vacua may be obtained by quotienting out the internal manifold by a discrete group Γ, where Γ is a subgroup of SU(3). This possibility may be of interest for model-building. The results of chapter 4 and 5 all appeared in [25].

It would be interesting to study the uplifting of these models to de Sitter space-times. This might be accomplished by incorporating a suitable additional uplifting term in the potential along the lines of, e.g, [18]. Although a negative mass squared for a light field in AdS does not necessarily signal an instability, after the uplift all fields should have positive mass squared. Unless the uplifting potential can change the sign of the squared masses, it is thus desirable that they are all positive even before the uplifting. We find that this can be arranged in the coset models $\frac{\mathrm{G}_2}{\mathrm{SU}(3)}$, $\frac{\mathrm{Sp}(2)}{\mathrm{S}(\mathrm{U}(2)\times\mathrm{U}(1))}$ and $\frac{\mathrm{SU}(3)}{\mathrm{U}(1)\times\mathrm{U}(1)}$ for suitable values of the orientifold charge.

However, in **chapter 6**, we focused on an alternative approach towards obtaining meta-stable de Sitter vacua, namely we searched for non-trivial de Sitter minima in the original flux potential away from the AdS vacuum. This was motivated by the fact that the coset spaces allow for a negative scalar curvature circumventing recently proven no-go theorems for manifolds without curvature [50][1]. Using the 4D effective action worked out in chapter 5, we could rule out dS (as well as Minkowski) vacua and slow-roll inflation elsewhere in moduli space for four of the coset spaces by using a refined no-go theorem that probes the scalar potential also along a Kähler modulus different from the overall volume modulus (see also [87]). Just as the no-go theorem of [50], this no-go theorem works by establishing a certain lower bound on the first derivatives of the potential, and hence the epsilon parameter, for $V \geq 0$. It is thus different in spirit from the no-go theorems given in [89], which assume a small first derivative and consider consequences for the second derivatives, i.e. the eta parameter.

The only coset space that allows for supersymmetric vacua and that is not directly ruled out by any known no-go theorem is then the group manifold SU(2)×SU(2). For this case, we were indeed able to find critical points (corresponding to numerically vanishing ϵ) with positive energy density, but only at the price of a tachyonic direction, corresponding to a large negative eta-parameter, $\eta \lesssim -2.4$. Interestingly, this tachyonic direction does not correspond to the one used in the different types of no-go theorems of [89]. As our numerical search was not exhaustive, however, we cannot completely rule out the existence of dS vacua or inflating regions for this case. Since this case also does not allow for a supersymmetric Minkowski vacuum as mentioned at the end of section 6.2, our discussion covers all SU(3)-structure compactifications on semi-simple and U(1) cosets that have a supersymmetric vacuum.

Furthermore, we also studied the remaining two coset spaces of table C.1 that do admit an SU(3)-structure but no supersymmetric AdS vacuum. Choosing for simplicity the O-planes such that one-forms are projected out and restricting to O-planes perpendicular to the coordinate frame, we could again use the refined no-go theorem of section 6.2 to rule out dS vacua and slow-roll inflation for both of these cases as well. The results of chapter 6 are published in [26].

Our results show that a negative scalar curvature and a non-vanishing F_0 is in general not enough to ensure dS vacua or inflation (as also noted in [86]), and we give a geometric criterion that allows one to separate interesting SU(3)-structure compactifications from non-realistic ones.

Finally, in **chapter 7**, we focused on a family of three coset spaces and constructed non-supersymmetric vacua on them. For the $\frac{\mathrm{G}_2}{\mathrm{SU}(3)}$ coset we reproduced already known results and did not find any new vacua. For the $\frac{\mathrm{Sp}(2)}{\mathrm{S}(\mathrm{U}(2)\times\mathrm{U}(1))}$ model, however, we found

[1] Since the Iwasawa manifold is T-dual to the torus dS vacua and slow-roll inflation are ruled out already by [50].

new non-supersymmetric vacua that did not appear in the literature so far. This case is of special interest since it is topologically equivalent to $\mathbb{CP}_3$ which played a prominent role in the recently conjectured $\mathrm{AdS}_4/\mathrm{CFT}_3$ correspondence. We did not analyze the coset $\frac{\mathrm{SU}(3)}{\mathrm{U}(1)\times\mathrm{U}(1)}$, although we were able to write down a set of equations that one has to solve in order to find the vacua of this space. The results of chapter 7 as well as their further analysis will appear in [27].

The next step for these non-supersymmetric vacua would be to check whether they exhibit any tachyonic directions below the Breitenlohner-Friedman bound. If there are no such tachyons, there are basically two directions for further research. First, it would be interesting to study the phenomenology of those vacua in a similar way as we did for the supersymmetric vacua in this thesis. Second, regarding the AdS/CFT correspondence, it would be very interesting to identify on the dual field theory side the mechanism, that we used in this thesis to construct these vacua.

Our analysis of the low energy theory of string compactifications in chapter 4, 5 and 6 could be extended in several directions. For one thing, it would be extremely interesting to find explicit SU(3)-structure manifolds that do not fall under the class of coset spaces we have discussed here and to investigate their usefulness for cosmological applications along the lines of this thesis. The most obvious class of manifolds to study systematically would be the nil- and solvmanifolds. Another interesting direction might be the study of compactifications on manifolds with $\mathcal{N} = 1$ spinor ansätze more general than the SU(3)-structure case [92]. Concerning the SU(2)$\times$SU(2) model discussed in section 6.4, one might try to either find a working dS minimum, or rule it out based on another no-go theorem, perhaps by using methods similar in spirit to [89], although a direct application of their results to this case does not seem possible. Following [85, 51] or [93, 94], one could also try to incorporate additional structures such as NS5-branes or quantum corrections of various types. In section 6.6, however, we found that at least for our models, the following additional ingredients cannot be added or do not work: NS5-, D4- and D8-branes as well as an F-term uplift along the lines of O'KKLT [90, 48]. Perhaps also methods similar to the ones in [61] for non-supersymmetric Minkowski or AdS vacua might be useful for the direct 10D construction of dS compactifications. There is certainly a lot to improve about our understanding of cosmologically realistic compactifications of the type IIA string!

Appendix A

Type II supergravity

The bosonic content of type II supergravity consists of a metric g, a dilaton Φ, an NSNS 3-form H and RR-fields F_n. In the democratic formalism of [29], where the number of RR-fields is doubled, n runs over $0, 2, 4, 6, 8, 10$ in IIA and over $1, 3, 5, 7, 9$ in type IIB. We write n to denote the dimension of the RR-fields; for example $(-1)^n$ stands for $+1$ in type IIA and -1 in type IIB. After deriving the equations of motion from the action, the redundant RR-fields are to be removed by hand by means of the duality condition:

$$F_n = (-1)^{\frac{(n-1)(n-2)}{2}} e^{\frac{n-5}{2}\Phi} \star_{10} F_{(10-n)} \ , \tag{A.1}$$

given here in the Einstein frame. We will often collectively denote the RR-fields, and the corresponding potentials, with polyforms $F = \sum_n F_n$ and $C = \sum_n C_{(n-1)}$, so that $F = \mathrm{d}_H C$.

The conformal transformation $g_{sMN} = e^{\frac{\Phi}{2}} g_{EMN}$ brings the string frame action (1.3) to the Einstein frame action

$$S_{\text{bulk}} = \frac{1}{2\kappa_{10}^2} \int \mathrm{d}^{10}x\sqrt{-g} \left[R - \frac{1}{2}(\partial\Phi)^2 - \frac{1}{2}e^{-\Phi}H^2 - \frac{1}{4}\sum_n e^{\frac{5-n}{2}\Phi} F_n^2 \right] , \tag{A.2}$$

where for an l-form A we define

$$A^2 = A \cdot A = \frac{1}{l!} A_{M_1 \ldots M_l} A_{N_1 \ldots N_l} g^{M_1 N_1} \cdots g^{M_l N_l} \ . \tag{A.3}$$

Since (A.1) needs to be imposed by hand this is strictly-speaking only a pseudoaction. Note that the doubling of the RR-fields leads to factors of $1/4$ in their kinetic terms.

The contribution from the calibrated (supersymmetric) sources can be written as:

$$S_{\text{source}} = \int \langle C, j \rangle - \sum_n e^{\frac{n}{4}\Phi} \int \langle \Psi_n, j \rangle \, , \tag{A.4}$$

with

$$\Psi_n = e^A \mathrm{d}t \wedge \frac{e^{-\Phi}}{(n-1)!\hat{\epsilon}_1{}^T \epsilon_1} \hat{\epsilon}_1{}^T \gamma_{M_1 \dots M_{n-1}} \hat{\epsilon}_2 \, \mathrm{d}X^{M_1} \wedge \dots \wedge \mathrm{d}X^{M_{n-1}} \,, \tag{A.5}$$

with $\hat{\epsilon}_{1,2}$ nine-dimensional internal supersymmetry parameters. For space-filling sources in compactifications to AdS_4 this becomes [95]

$$\Psi_n = \mathrm{vol}_4 \wedge e^{4A-\Phi} \mathrm{Im}\, \Psi_{1\mathrm{E}} \big|_{n-4} \,, \tag{A.6}$$

with $\Psi_{1\mathrm{E}}$ the pure spinor Ψ_1 in the Einstein frame.

The dilaton equation of motion and the Einstein equation read

$$0 = \nabla^2 \Phi + \frac{1}{2} e^{-\Phi} H^2 - \frac{1}{8} \sum_n (5-n) e^{\frac{5-n}{2}\Phi} F_n^2 + \frac{\kappa_{10}^2}{2} \sum_n (n-4) e^{\frac{n}{4}\Phi} \star \langle \Psi_n, j \rangle \,, \tag{A.7a}$$

$$\begin{aligned} 0 = R_{MN} &+ g_{MN} \left(\frac{1}{8} e^{-\Phi} H^2 + \frac{1}{32} \sum_n (n-1) e^{\frac{5-n}{2}\Phi} F_n^2 \right) \\ &- \frac{1}{2} \partial_M \Phi \partial_N \Phi - \frac{1}{2} e^{-\Phi} H_M \cdot H_N - \frac{1}{4} \sum_n e^{\frac{5-n}{2}\Phi} F_{n\,M} \cdot F_{n\,N} \\ &- 2\kappa_{10}^2 \sum_n e^{\frac{n}{4}\Phi} \star \langle \left(-\frac{1}{16} n g_{MN} + \frac{1}{2} g_{P(M} dx^P \otimes \iota_{N)} \right) \Psi_n, j \rangle \,, \end{aligned} \tag{A.7b}$$

where we defined for an l-form A

$$A_M \cdot A_N = \frac{1}{(l-1)!} A_{M M_2 \dots M_l} A_{N N_2 \dots N_l} g^{M_2 N_2} \cdots g^{M_l N_l} \,. \tag{A.8}$$

The Bianchi identities and the equations of motion for the RR-fields, including the contribution from the ‘Chern-Simons’ terms of the sources, take the form

$$0 = \mathrm{d}F + H \wedge F + 2\kappa_{10}^2 \, j \,, \tag{A.9a}$$

$$0 = \mathrm{d}\left(e^{\frac{5-n}{2}\Phi} \star F_n \right) - e^{\frac{3-n}{2}\Phi} H \wedge \star F_{(n+2)} - 2\kappa_{10}^2 \, \alpha(j) \,. \tag{A.9b}$$

Finally, for the equation of motion for H we have:

$$0 = \mathrm{d}(e^{-\Phi} \star H) - \frac{1}{2} \sum_n e^{\frac{5-n}{2}\Phi} \star F_n \wedge F_{(n-2)} + 2\kappa_{10}^2 \sum_n e^{\frac{n}{4}\Phi} \Psi_n \wedge \alpha(j) \Big|_8 \,. \tag{A.10}$$

In the above equations we can redefine j in order to absorb the factor of $2\kappa_{10}^2$,

$$(2\kappa_{10}^2) j \to j \,, \tag{A.11}$$

which we do in this thesis.

The equations of motion resulting from $S_{\mathrm{bulk}} + S_{\mathrm{source}}$ were given in this form (in the string frame) in [69], where it was shown that, under certain mild assumptions, imposing the supersymmetry equations together with the Bianchi identities for the forms, is enough to guarantee that the dilaton and Einstein equations are also satisfied.

Appendix B

Generalized geometry

In this thesis we have assumed the following $\mathcal{N} = 1$ compactification ansatz for the ten-dimensional supersymmetry parameters [92]

$$\begin{aligned} \epsilon_1 &= \zeta_+ \otimes \eta_+^{(1)} \ + \ \zeta_- \otimes \eta_-^{(1)} \ , \\ \epsilon_2 &= \zeta_+ \otimes \eta_\mp^{(2)} \ + \ \zeta_- \otimes \eta_\pm^{(2)} \ , \end{aligned} \tag{B.1}$$

for IIA/IIB, where $\zeta_\pm$ are four-dimensional and $\eta_\pm^{(1,2)}$ six-dimensional Weyl spinors. The Majorana conditions for $\epsilon_{1,2}$ imply the four- and six-dimensional reality conditions $(\zeta_+)^* = \zeta_-$ and $(\eta_+^{(1,2)})^* = \eta_-^{(1,2)}$. This reduces the structure of the *generalized* tangent bundle to SU(3)×SU(3) [96]. The structure group of the tangent bundle itself, on the other hand, is a subgroup of SU(3), since there is at least one invariant internal spinor. The precise form of this subgroup depends on the relation between $\eta^{(1)}$ and $\eta^{(2)}$. Combining the terminology of [92] and [97], the following classification can be made:

- strict SU(3)-structure: $\eta^{(1)}$ and $\eta^{(2)}$ are parallel everywhere;
- static SU(2)-structure: $\eta^{(1)}$ and $\eta^{(2)}$ are orthogonal everywhere;
- intermediate SU(2)-structure: $\eta^{(1)}$ and $\eta^{(2)}$ at a fixed angle, but neither a zero angle nor a right angle;
- dynamic SU(3)×SU(3)-structure: the angle between $\eta^{(1)}$ and $\eta^{(2)}$ varies, possibly becoming a zero angle or a right angle at a special locus.

Since for static and intermediate SU(2)-structure there are two independent internal spinors, the structure of the tangent bundle reduces to SU(2), while for dynamic SU(3)×SU(3)-structure no extra constraints beyond SU(3) are imposed on the topology of the tangent bundle, since the two internal spinors $\eta^{(1)}$ and $\eta^{(2)}$ might not be everywhere independent.

In [36] it was realized that, in type IIB supergravity, strict SU(3) compactifications to $\mathcal{N} = 1$ AdS$_4$ are impossible[1]. Conversely it was shown in [65] that type IIA static SU(2) compactifications to AdS$_4$ are impossible. This was extended in [25] to intermediate SU(2)-structure AdS$_4$ vacua with *left-invariant* pure spinors for *both* type IIA and type IIB. The way out of this no-go theorem is that in type IIA we must allow $e^{2A-\Phi}\eta_+^{(2)\dagger}\eta_+^{(1)}$ to vary along the internal manifold, while in type IIB we need a genuine dynamic SU(3)×SU(3)-structure that changes type to static SU(2) on a non-zero locus. So the most interesting but also the most complicated case, the dynamic SU(3)×SU(3)-structure is still possible, but we will not consider that case here. Note that in [69, 97] examples of constant intermediate SU(2)-structure on *Minkowski* compactifications were provided. In this thesis, we focus on strict SU(3) $\mathcal{N} = 1$ AdS$_4$vacua in type IIA. In the first section of this appendix, we will review the formulation of the supersymmetry conditions for type II supergravity using the language of generalized geometry, specializing in the end to the SU(3)-structure case. Then we will recall the basic definitions of an SU(3)-structure independent of its formulation in terms of generalized geometry. Furthermore, we will clarify the role of the O-planes present in our constructions before we finally review the formulation of the 4d scalar potential in the language of generalized geometry.

B.1 $\mathcal{N} = 1$ AdS$_4$ susy equations

In the generalized geometry formalism the supersymmetry generators $\eta^{(1)}$ and $\eta^{(2)}$ from (B.1) are collected into two spinor bilinears, which using the Clifford map, can be associated with two polyforms of definite degree

$$\underline{\Psi}_+ = \frac{8}{|a||b|}\eta_+^{(1)} \otimes \eta_+^{(2)\dagger}\,, \qquad \underline{\Psi}_- = \frac{8}{|a||b|}\eta_+^{(1)} \otimes \eta_-^{(2)\dagger}\,. \tag{B.2}$$

It can be shown that these are associated to pure spinors of $SO(6,6)$ and that they satisfy the normalization

$$\langle \Psi_+, \Psi_+^* \rangle = \langle \Psi_-, \Psi_-^* \rangle \neq 0\,, \tag{B.3}$$

with the Mukai pairing $\langle \cdot, \cdot \rangle$ given by

$$\langle \phi_1, \phi_2 \rangle = \phi_1 \wedge \alpha(\phi_2)|_{\rm top}\,. \tag{B.4}$$

The operator α acts by inverting the order of indices on forms. The Mukai pairing has the following useful property:

$$\langle e^b\phi_1, e^b\phi_2 \rangle = \langle \phi_1, \phi_2 \rangle\,, \tag{B.5}$$

[1] That is at a pure classical level. Taking non-perturbative corrections into account the authors of [18] indeed constructed an AdS$_4$ vacuum with SU(3)-structure. See also [36] for a discussion.

for an arbitrary two-form b. Since there are two compatible invariant pure spinors the structure of the generalized tangent bundle is reduced to $SU(3) \times SU(3)$. In order to obtain similar equations in IIA and IIB, one redefines

$$\Psi_1 = \Psi_\mp \,, \qquad \Psi_2 = \Psi_\pm \,, \tag{B.6}$$

with upper/lower sign for IIA/IIB. We collect all the RR-fields of the democratic formalism into one polyform and make the following compactification ansatz

$$F = \hat{F} + \mathrm{vol}_4 \wedge \tilde{F} \,, \tag{B.7}$$

with vol_4 the four-dimensional (AdS$_4$) volume form. In fact, in this thesis we will drop the hat and hope that it is clear from the context whether we mean the full F or only the internal part.

With these definitions the supersymmetry conditions (in string frame) take the following concise form in both IIB and IIA [92]

$$\mathrm{d}_H \left(e^{4A-\Phi}\mathrm{Im}\,\Psi_1\right) = 3e^{3A-\Phi}\mathrm{Im}\,(W^*\Psi_2) + e^{4A}\tilde{F} \,, \tag{B.8a}$$

$$\mathrm{d}_H \left[e^{3A-\Phi}\mathrm{Re}\,(W^*\Psi_2)\right] = 2|W|^2 e^{2A-\Phi}\mathrm{Re}\,\Psi_1 \,, \tag{B.8b}$$

$$\mathrm{d}_H \left[e^{3A-\Phi}\mathrm{Im}\,(W^*\Psi_2)\right] = 0 \,, \tag{B.8c}$$

where we used $|a|^2 = |b|^2 \propto e^A$. From the above, the equations of motion for F follow as integrability conditions, as well as the following equation:

$$\mathrm{d}_H \left(e^{2A-\Phi}\mathrm{Re}\,\Psi_1\right) = 0 \,. \tag{B.9}$$

Here W is defined in terms of the AdS Killing spinors

$$\nabla_\mu \zeta_- = \pm\frac{1}{2}W\gamma_\mu\zeta_+ \,, \tag{B.10}$$

for IIA/IIB.

These equations should be supplemented with the Bianchi identities for the RR-fluxes (A.9a) where the (localized or smeared) sources j have to be calibrated

$$\langle \mathrm{Re}\,\Psi_1, j\rangle = 0 \,, \tag{B.11a}$$

$$\langle \Psi_2, \mathbb{X}\cdot j\rangle = 0 \,, \quad \forall \mathbb{X} \in \Gamma(T_M \oplus T^\star_M) \,. \tag{B.11b}$$

An easy way to solve these calibration conditions is to choose

$$j = -k\,\mathrm{Re}\,\Psi_1 \,, \tag{B.12}$$

for some function k, which is positive for net D-brane charge and negative for net orientifold charge. Applying an exterior derivative on (B.8a), taking (B.8b), (A.9a), (B.7) into account, it can be shown that

$$\pm\mathrm{d}_H \left\{\alpha\left[\star\mathrm{d}_H\left(e^{3A-\Phi}\mathrm{Im}\,\Psi_1\right)\right]\right\} = -e^{4A}j - 6|W|^2 e^{A-\Phi}\mathrm{Re}\,\Psi_1 \,, \tag{B.13}$$

for IIA/IIB.

When the internal supersymmetry generators of (B.1) are proportional,

$$\eta_+^{(2)} = (b/a)\eta_+^{(1)} , \tag{B.14}$$

with $|\eta^{(1)}|^2 = |a|^2, |\eta^{(2)}|^2 = |b|^2$, they define an SU(3)-structure whose properties we will review in the next section. First let us define a normalized spinor η_+ such that $\eta_+^{(1)} = a\eta_+$ and $\eta_+^{(2)} = b\eta_+$ and moreover we choose the phase of η such that $a = b^*$. Note that in compactifications to AdS_4 the supersymmetry imposes $|a|^2 = |b|^2$ such that $b/a = e^{i\theta}$ is just a phase. Now we can define J and Ω as follows

$$J_{mn} = i\eta_+^\dagger \gamma_{mn}\eta_+ , \qquad \Omega_{mnp} = \eta_-^\dagger \gamma_{mnp}\eta_+ . \tag{B.15}$$

Plugging in (B.14) into (B.2) and using the above definition we get

$$\Psi_- = -\Omega , \qquad \Psi_+ = e^{-i\theta} e^{iJ} . \tag{B.16}$$

By using (B.6) for IIA we can insert this into (B.8) and arrive at (3.1) as well as (3.4) and(3.5).

B.2 SU(3)-structure

A real non-degenerate two-form J and a complex decomposable three-form Ω completely specify an SU(3)-structure on the six-dimensional manifold $\mathcal{M}$ iff:

$$\Omega \wedge J = 0 , \tag{B.17a}$$

$$\Omega \wedge \Omega^* = \frac{4i}{3} J^3 \neq 0 , \tag{B.17b}$$

and the associated metric (B.28) is positive definite. Up to a choice of orientation, the volume normalization can be taken such that

$$\frac{1}{6}J^3 = -\frac{i}{8}\Omega \wedge \Omega^* = \mathrm{vol}_6 . \tag{B.18}$$

The intrinsic torsion of $\mathcal{M}$ decomposes into five modules (torsion classes) $\mathcal{W}_1, \ldots, \mathcal{W}_5$. These also appear in the SU(3) decomposition of the exterior derivative of J, Ω. Intuitively, this is because the intrinsic torsion parameterizes the failure of the manifold to be of special holonomy, which can also be thought of as the deviation from closure of J, Ω. More specifically we have:

$$\begin{aligned} \mathrm{d}J &= \frac{3}{2}\mathrm{Im}\,(\mathcal{W}_1\Omega^*) + \mathcal{W}_4 \wedge J + \mathcal{W}_3 , \\ \mathrm{d}\Omega &= \mathcal{W}_1 J \wedge J + \mathcal{W}_2 \wedge J + \mathcal{W}_5^* \wedge \Omega , \end{aligned} \tag{B.19}$$

where $\mathcal{W}_1$ is a scalar, $\mathcal{W}_2$ is a primitive (1,1)-form, $\mathcal{W}_3$ is a real primitive $(1,2)+(2,1)$-form, $\mathcal{W}_4$ is a real one-form and $\mathcal{W}_5$ a complex (1,0)-form. For the vacua of interest to us only the classes $\mathcal{W}_1$, $\mathcal{W}_2$ are non-vanishing and they are purely imaginary, which we will indicate with a minus superscript. Indeed, we can readily see that eq. (3.6a) follows from eq. (B.19) above, upon setting $\mathcal{W}_{3,4,5}$ to zero and imposing $\mathcal{W}_{1,2} = \mathcal{W}^-_{1,2} = i\mathrm{Im}\,\mathcal{W}^-_{1,2}$.

Note that by definition $\mathcal{W}_2$ is primitive, which means

$$\mathcal{W}_2 \wedge J \wedge J = 0\,. \tag{B.20}$$

One interesting property of a primitive (1,1)-form is

$$\star\,(\mathcal{W}_2 \wedge J) = -\mathcal{W}_2\,, \tag{B.21}$$

which can be shown using $J^{mn}\mathcal{W}_{2mn} = 0$ (which follows from the primitivity) and $J_m{}^n J_p{}^q \mathcal{W}_{nq} = \mathcal{W}_{mp}$ (which follows from the fact that $\mathcal{W}_2$ is of type (1,1)).

Let us now calculate the part of $\mathrm{d}\mathcal{W}_2^-$ proportional to $\mathrm{Re}\,\Omega$:

$$\mathrm{d}\mathcal{W}_2^- = \alpha\,\mathrm{Re}\,\Omega + (2,1) + (1,2)\ , \tag{B.22}$$

for some α. Taking the exterior derivative of $\Omega \wedge \mathcal{W}_2^- = 0$ and using (B.22) as well as the eqs. (B.17b), (2.5), we arrive at:

$$\mathcal{W}_2^- \wedge \mathcal{W}_2^- \wedge J = \frac{2i}{3}\alpha J^3\ . \tag{B.23}$$

We can now use (B.21) to show

$$\mathcal{W}_2^- \wedge \mathcal{W}_2^- \wedge J = \frac{1}{2}|\mathcal{W}_2^-|^2 \mathrm{vol}_6\,, \tag{B.24}$$

from which we obtain $\alpha = -i|\mathcal{W}_2|^2/8$.

From the SU(3)-structure (B.17b), we can read off the metric as follows [98]. From $\mathrm{Re}\,\Omega$ alone we can construct an almost complex structure. First we define

$$\tilde{\mathcal{I}}^l{}_k = -\varepsilon^{lm_1\ldots m_5}(\mathrm{Re}\,\Omega)_{km_1m_2}(\mathrm{Re}\,\Omega)_{m_3m_4m_5}\,, \tag{B.25}$$

where $\varepsilon^{m_1\ldots m_6} = \pm 1$ is the totally antisymmetric symbol in six dimensions, and then properly normalize it

$$\mathcal{I} = \frac{\tilde{\mathcal{I}}}{\sqrt{-\mathrm{tr}\,\frac{1}{6}\tilde{\mathcal{I}}^2}}\,, \tag{B.26}$$

so that $\mathcal{I}^2 = -\mathbb{1}$. Note that

$$H(\mathrm{Re}\,\Omega) = \mathrm{tr}\,\frac{1}{6}\tilde{\mathcal{I}}^2 \tag{B.27}$$

is called the Hitchin functional. The metric can then be constructed from $\mathcal{I}$ and J via:

$$g_{mn} = \mathcal{I}_m{}^l J_{ln}\ . \tag{B.28}$$

B.3 How to dress smeared sources with orientifold involutions

Suppose we are given a form j representing the Poincaré dual of smeared orientifolds. How do we decide what the orientifold involutions should be? Let us first give an example for a *localized* orientifold in flat space. If we have an orientifold along the directions $\Sigma = (x^1, x^2, x^3)$ then the corresponding source is

$$j = T_{\mathrm{O}p}\, j_\Sigma = -T_{\mathrm{O}p}\, \delta(x^4, x^5, x^6)\, \mathrm{d}x^4 \wedge \mathrm{d}x^5 \wedge \mathrm{d}x^6 \,, \tag{B.29}$$

where $T_{\mathrm{O}p} < 0$ for an orientifold and j is the Poincaré dual of Σ satisfying

$$\int_\Sigma \phi = \int_{\mathcal{M}} \langle \phi, j_\Sigma \rangle = -\int_{\mathcal{M}} \phi \wedge j_\Sigma \,, \tag{B.30}$$

for an arbitrary form ϕ [2]. In this case the orientifold involution is of course

$$O6: \qquad x^4 \to -x^4 \,, \quad x^5 \to -x^5 \,, \quad x^6 \to -x^6 \,. \tag{B.31}$$

Suppose we now introduce many orientifolds and completely smear them in the directions (x^1, x^2, x^3) obtaining

$$j = -T_{\mathrm{Op}}\, c\, \mathrm{d}x^4 \wedge \mathrm{d}x^5 \wedge \mathrm{d}x^6 \,, \tag{B.32}$$

where c is a constant representing the orientifold density. We have now lost information about the exact location but we would still like to associate the orientifold involution

$$O6: \qquad \mathrm{d}x^4 \to -\mathrm{d}x^4 \,, \quad \mathrm{d}x^5 \to -\mathrm{d}x^5 \,, \quad \mathrm{d}x^6 \to -\mathrm{d}x^6 \,. \tag{B.33}$$

An important observation is that $\mathrm{d}x^4 \wedge \mathrm{d}x^5 \wedge \mathrm{d}x^6$ is not just any form, it is a *decomposable* form, i.e. it can be written as a wedge product of three one-forms. These one-forms span the annihilator space of T_Σ, the tangent space of Σ. So if we are given a smeared orientifold current j we should write it as a sum of decomposable forms and then associate to each term an orientifold involution as above.

Let us now study more formally how we could write j as a sum of decomposable forms and whether the decomposition is unique. First, let us introduce a basis of forms $e^i \in \mathbb{V}^\star$ that span (locally) $T_{\mathcal{M}}$. Indeed, for the case of group manifolds we have such a basis, which is even defined globally. For the cosets left-invariant forms in this basis are also globally defined.

Now, let $\mathbb{V}$ be a d-dimensional vector space and $\mathbb{V}^\star$ its dual. A (real/complex) p-form $j \in \Lambda^p \mathbb{V}^\star$ is called *simple* or *decomposable* if it can be written as a wedge

[2] The definition with the Mukai pairing is the one appropriate for generalizing to D-branes with world-volume gauge flux as explained in [99]. Here it will just give an extra minus sign

product of p one-forms.[3] What we are interested in is that there is a one-to-one correspondence between $(d-p)$-planes (our orientifold planes) and decomposable p-forms (up to a proportionality factor). This isomorphism is called the *Plücker map*. A discussion of the criteria for having a simple form can be found in e.g. [100] pp. 209-211. We will use here the criterion based on

$$j^{\perp} = \{X \in \mathbb{V} : \iota_X j = 0\} \subset \mathbb{V}, \tag{B.34}$$

and

$$W = \mathrm{Ann}(j^{\perp}) \subset \mathbb{V}^{\star} . \tag{B.35}$$

In [100] it is shown that j is simple if and only if $\dim W = p$. Using this the following alternative criterion is shown:

Theorem: A p-form $j \in \Lambda^p \mathbb{V}^{\star}$ is simple if and only if for every $(p-1)$-polyvector $\xi \in \Lambda^{p-1}\mathbb{V}$,

$$\iota_\xi j \wedge j = 0 \ , \tag{B.36}$$

where $\iota_\xi j$ is the one-form contraction of j with ξ.

Now for the special case of three-forms in six dimensions there is another useful theorem due to Hitchin [98].

Theorem: Consider a real three-form $j \in \Lambda^3 \mathbb{V}^{\star}$ and calculate its Hitchin functional $H(j)$ defined in (B.27). Then

- $H(j) > 0$ if and only if $j = j_1 + j_2$ where j_1, j_2 are unique (up to ordering) real decomposable three-forms and $j_1 \wedge j_2 \neq 0$;
- $H(j) < 0$ if and only if $j = \alpha + \bar{\alpha}$ where α is a unique (up to complex conjugation) complex decomposable three-form and $\alpha \wedge \bar{\alpha} \neq 0$.

Now we have two base-independent characterizations of j: the Hitchin functional $H(j)$ and $\dim W$. Using these two characterizations the possible j's and their decomposition in simple terms are classified in [25]. Here we will focus on the case $H(j) < 0$ which is always the case for the examples in this thesis. From the above it follows that if $H(j) < 0$ then j is a sum of exactly two (conjugate) complex simple terms and thus of exactly four real simple terms.

An important remark is in order: while the Hitchin theorem states that the two complex forms in the decomposition of j are unique (up to complex conjugation), the choice of one-forms out of which these forms are made is *not* unique. One still has the freedom of choosing a basis of complex one-forms belonging to a complex structure,

[3] Note that a (real/complex) form of fixed dimension is a pure spinor if and only if it is simple. In fact, we could regard the notion of pure spinor as a generalization of the notion of decomposable forms to polyforms.

which is SL(3,$\mathbb{C}$). As a consequence the choice of the four real forms in which j is decomposed is *not* unique. Indeed, suppose we choose one basis of complex one-forms and associated x and y coordinates: $e^{z^i} = e^{x^i} - ie^{y^i}$. Then j can be written as the sum of the following four terms:

$$j = \mathrm{Re}(e^{z^1 z^2 z^3}) = e^{x^1 x^2 x^3} - e^{x^1 y^2 y^3} - e^{y^1 x^2 y^3} - e^{y^1 y^2 x^3} \,, \tag{B.37}$$

which leads to the following orientifold involutions:

$$\begin{aligned}
O6: &\quad e^{x^1} \to -e^{x^1}\,, \quad e^{x^2} \to -e^{x^2}\,, \quad e^{x^3} \to -e^{x^3}\,, \\
O6: &\quad e^{x^1} \to -e^{x^1}\,, \quad e^{y^2} \to -e^{y^2}\,, \quad e^{y^3} \to -e^{y^3}\,, \\
O6: &\quad e^{y^1} \to -e^{y^1}\,, \quad e^{x^2} \to -e^{x^2}\,, \quad e^{y^3} \to -e^{y^3}\,, \\
O6: &\quad e^{y^1} \to -e^{y^1}\,, \quad e^{y^2} \to -e^{y^2}\,, \quad e^{x^3} \to -e^{x^3}\,.
\end{aligned} \tag{B.38}$$

If we perform a SL(3,$\mathbb{C}$) transformation, j takes exactly the same form, but now in the *new* basis. So alternatively we could have chosen four orientifold involutions taking the same form as the old ones, but now in the *new* basis, which is rotated. This means that our choice of orientifold involutions is not unique. We must then further choose them such that the structure constant tensor of the group or coset is even, and $\mathrm{Re}\,\Omega$ and J are odd.

Application to SU(2)×SU(2)

Let us now apply the above procedure to the model of section 3.4.4. Calculating the Hitchin functional $H(j^6)$ of (3.66) we find that it is negative so that it contains four orientifold involutions. We must now fix the freedom of choosing them such that $\mathrm{Re}\,\Omega$ and J are odd, and the structure constant tensor f is even. Some reflection should make clear that if $\mathrm{Re}\,\Omega$ is to be odd, it should be a sum of the same four terms as j^6, but with different coefficients. In fact, we could reverse the procedure and choose a complex basis e^{z^i} in which Ω and J take their standard form:

$$\Omega = e^{z^1 z^2 z^3}\,, \qquad J = -\frac{i}{2}\sum_i e^{z^i \bar{z}^i}\,. \tag{B.39}$$

Then $\mathrm{Re}\,\Omega$ and J are automatically odd under the associated orientifold involutions (B.38). However, this should of course also be the orientifold involutions that follow from j^6. This will be the case if and only if j^6 has the same terms as $\mathrm{Re}\,\Omega$ (but with different coefficients). One can show that this is the case if j^6 is of the form

$$j^6 = \mathrm{Re}\left(c^0 e^{z^1 z^2 z^3} + c^{11} e^{\bar{z}^1 z^2 z^3} + c^{22} e^{z^1 \bar{z}^2 z^3} + c^{33} e^{z^1 z^2 \bar{z}^3}\right)\,, \tag{B.40}$$

with all coefficients c real. To bring j to this form we still have the freedom to make a base transformation such that Ω and J invariant, i.e. an SU(3)-transformation. A priori, j^6 is an arbitrary three-form which transforms under SU(3) as

$$20 = 1 + \bar{1} + 3 + \bar{3} + 6 + \bar{6}\,. \tag{B.41}$$

However, we know that j^6 has to satisfy the calibration conditions (3.11), which remove the $3 + \bar{3}$ representation and only leave the form proportional to $\mathrm{Re}\,\Omega$ out of $1 + \bar{1}$. Here the 6 is the $(3 \times 3)_S$ i.e. the symmetric product of two fundamental representations of SU(3). It follows that the most general j^6 satisfying the calibration conditions looks like

$$\begin{aligned} j^6 &= c_0 \mathrm{Re}\,\Omega + \mathrm{Re}\left[c^{ki} g_{(k|\bar{j}} \mathrm{d}\bar{z}^{\bar{j}} \wedge \iota_{z^{i})}\Omega\right] \\ &= c_0 \mathrm{Re}\,\Omega + \mathrm{Re}\Big[c^{11} e^{\bar{z}^1 z^2 z^3} + c^{22} e^{z^1 \bar{z}^2 z^3} + c^{33} e^{z^1 z^2 \bar{z}^3} \\ &+ c^{12}\left(e^{\bar{z}^2 z^2 z^3} + e^{z^1 \bar{z}^1 z^3}\right) + c^{13}\left(e^{\bar{z}^3 z^2 z^3} + e^{z^1 z^2 \bar{z}^1}\right) + c^{23}\left(e^{z^1 \bar{z}^3 z^3} + e^{z^1 z^2 \bar{z}^2}\right)\Big]\,, \end{aligned} \tag{B.42}$$

with c_0 real and the entries of the coefficient matrix

$$C = \begin{pmatrix} c^{11} & c^{12} & c^{13} \\ c^{21} & c^{22} & c^{23} \\ c^{31} & c^{32} & c^{33} \end{pmatrix}\,, \tag{B.43}$$

complex. Now we have to find an SU(3)-transformation to put j^6 in the form (B.40). c_0 does not transform but is luckily already of the right form, while the coefficient matrix transforms as

$$C \to UCU^T\,. \tag{B.44}$$

From (B.40) we see that we want to transform C to a diagonal real matrix. In fact, since the above transformation cannot change the determinant this is only possible if

$$\det C \in \mathbb{R}\,. \tag{B.45}$$

This is a condition we have to add to the calibration conditions. For the j^6 of (3.66), one can check that it is indeed satisfied and it is possible to find the complex coordinates with the required properties. Also, under the associated orientifold involution the structure constant tensor f is even as required. Note that alternatively, as we actually did in (3.67), we can also construct a complex basis associated to Ω such that f is even. This then automatically implies that j is odd and that it is a sum of the same four terms as $\mathrm{Re}\,\Omega$.

B.4 Effective supergravity

The superpotential for SU(3)× SU(3)-structure was derived in various ways in [33, 35, 36] (based on [84, 34]). Here we will follow the approach of [36], which calculated the superpotential and the (conformal) Kähler potential in the superconformal formalism of [101].

The bosonic part of the effective four-dimensional superconformal action takes the following form

$$S = \int \mathrm{d}^4x\sqrt{-g_4}\left(\frac{1}{2}\mathcal{N}R + 3\mathcal{N}_{I\bar{J}}\, g^{\mu\nu}D_\mu X^I D_\nu X^{*\bar{J}} + \frac{1}{3}\mathcal{W}_I\left(\mathcal{N}^{-1}\right)^{I\bar{J}}\mathcal{W}^*_{\bar{J}} + \cdots\right), \tag{B.46}$$

where the vector multiplet sector, including D-terms, has been omitted. Here the X^I are the $n+1$ scalars and $D_\mu X^I = \partial_\mu X^I - \frac{1}{3} i A_\mu X^I$, where A_μ is the gauge field associated to the U(1)-transformations, generated by α (see (B.49)), in the complex Weyl transformation. From dimensional reduction of the ten-dimensional supergravity action the conformal Kähler potential $\mathcal{N}$ and the superpotential $\mathcal{W}$ were found and read (here we reinstate dimensionful coupling constants)

$$\mathcal{N} = \frac{1}{\kappa_{10}^2}\int_M \mathrm{d}^6y\sqrt{\det h}\, e^{2A-2\Phi} = \frac{1}{8\kappa_{10}^2}\left(i\int_M e^{-4A}\langle \mathcal{Z},\bar{\mathcal{Z}}\rangle\right)^{1/3}\left(i\int_M e^{2A}\langle t,\bar{t}\rangle\right)^{2/3}, \tag{B.47a}$$

$$\mathcal{W} = \frac{1}{4\kappa_{10}^2}\int_M \langle \mathcal{Z}, F + i\,\mathrm{d}_H(\mathrm{Re}\mathcal{T})\rangle\ . \tag{B.47b}$$

Here $\mathcal{Z}$, $\mathrm{Re}\mathcal{T}$ and t are defined through

$$\mathcal{Z} = -ie^{3A-\Phi}\Psi_2\,, \tag{B.48a}$$

$$t = e^{-\Phi}\Psi_1\,, \tag{B.48b}$$

$$\mathrm{Re}\mathcal{T} = \mathrm{Im}\, t = e^{-\Phi}\mathrm{Im}\,\Psi_1\,. \tag{B.48c}$$

The dimensionally reduced action is naturally invariant under the following complex Weyl symmetry

$$A \to A + \sigma\,,\quad g \to e^{-2\sigma}g\,,\quad \mathcal{Z} \to e^{3\sigma+i\alpha}\mathcal{Z}\,,\quad \mathcal{N} \to e^{2\sigma}\mathcal{N}\,. \tag{B.49}$$

Since the scalars X^I transform as

$$X^I \to e^{\sigma+\frac{i}{3}\alpha}X^I\,, \tag{B.50}$$

we find that $\mathcal{Z}$ must be homogeneous of degree 3 in the X^I. To go to the usual Einstein frame, we must gauge-fix the Weyl symmetry. We first explicitly isolate the unphysical degree of freedom, which is called the conformon, as follows

$$X^I = Yx^I(\phi^i)\,,\qquad \mathcal{Z} = Y^3\mathcal{Z}(\phi^i)\qquad \mathcal{N} = |Y|^2e^{-\mathcal{K}/3},\qquad \mathcal{W} = Y^3M_P^{-3}\mathcal{W}_{\mathrm{E}}(\phi^i)\,, \tag{B.51}$$

where Y is the conformon, ϕ^i are the n scalar degrees of freedom in the Einstein frame and M_P the four-dimensional Planck mass. $\mathcal{K}$ and $\mathcal{W}_{\mathrm{E}}$ will turn out to be the Kähler potential and the Einstein-frame superpotential after gauge-fixing. Indeed, in the new coordinates the action (B.46) becomes

$$\begin{aligned} S = \int \mathrm{d}^4x\sqrt{-g_4}\,\Big[&\frac{1}{2}|Y|^2 e^{-\mathcal{K}/3} R - |Y|^2 e^{-\mathcal{K}/3}\,\mathcal{K}_{i\bar{\jmath}}\, g^{\mu\nu}\,\partial_\mu\phi^i\partial_\nu\bar{\phi}^{\bar{\jmath}} + \cdots \\ &-M_P^{-6}|Y|^4 e^{\mathcal{K}/3}\left(\mathcal{K}^{i\bar{\jmath}} D_i\mathcal{W}_{\mathrm{E}} D_{\bar{\jmath}}\mathcal{W}^*_{\mathrm{E}} - 3|\mathcal{W}_{\mathrm{E}}|^2\right) + \cdots\Big]\,, \end{aligned} \tag{B.52}$$

where for the kinetic term of the scalars we omitted pieces that will vanish after the gauge-fixing.

We then impose the following gauge

$$\mathcal{N} = |Y|^2 e^{-\mathcal{K}/3} = M_P^2\,, \tag{B.53}$$

which obviously gives us the usual Einstein-frame action

$$S = \int \mathrm{d}^4x\sqrt{-g_4}\left(\frac{M_P^2}{2}R - M_P^2\mathcal{K}_{i\bar{\jmath}}\partial_\mu\phi^i\partial^\mu\bar{\phi}^{\bar{\jmath}} - V(\phi,\bar{\phi})\right)\,, \tag{B.54}$$

and also leads to the standard expression for the potential

$$V(\phi,\bar{\phi}) = M_P^{-2} e^{\mathcal{K}}\left(\mathcal{K}^{i\bar{\jmath}} D_i\mathcal{W}_{\mathrm{E}} D_{\bar{\jmath}}\mathcal{W}^*_{\mathrm{E}} - 3|\mathcal{W}_{\mathrm{E}}|^2\right)\,. \tag{B.55}$$

The U(1)-symmetry must also be gauged, but for more details on this we refer to [101].

The Kähler potential reads

$$\mathcal{K} = -\ln i\int_M e^{-4A}\langle\mathcal{Z},\bar{\mathcal{Z}}\rangle - 2\ln i\int_M e^{2A}\langle t,\bar{t}\rangle + 3\ln(8\kappa_{10}^2|Y|^2)\,. \tag{B.56}$$

Note that in [102] it is shown that Imt is a function of Ret so that t can be seen as (non-holomorphically) dependent on $\mathcal{T}$. To take this relation properly into account we use the fact that the Kähler potential for the t-sector may be written as

$$\mathcal{K}_t = -2\ln 4\int_M e^{2A} H(\mathrm{Im}t)\,, \tag{B.57}$$

where $H(\mathrm{Im}t)$ is the Hitchin functional [98, 102, 33]. For stable pure spinors of $SO(6,6)$ it is defined as follows

$$H(\mathrm{Im}t) = \sqrt{-\frac{1}{12}\mathcal{J}^{\Pi}{}_{\Sigma}\mathcal{J}^{\Sigma}{}_{\Pi}}\,. \tag{B.58}$$

where $\mathcal{J}_{\Pi\Sigma} = \langle \mathrm{Im}t, \Gamma_{\Pi\Sigma}\mathrm{Im}t\rangle$ is a generalized complex structure and $\Pi,\Sigma = 1,\ldots,12$. The generalized $SO(6,6)$ gamma matrices Γ^{Σ} act on forms as

$$\Gamma_\Sigma = \iota_m \quad \text{for} \quad m=\Sigma=1,\ldots,6 \quad \text{and} \quad \Gamma_\Sigma = e^m\wedge \quad \text{for} \quad m+6=\Sigma=7,\ldots,12\,. \tag{B.59}$$

In the case of SU(3)-structure $\mathrm{Im}\,t = -\mathrm{Im}\,\Omega$, and the Hitchin functional reduces to (B.27).

Note that if we make an expansion of the warp factor A in harmonic modes

$$A = A^0 + \sum_{\vec{n}\neq 0} A^{\vec{n}} \mathcal{Y}^{(0)}_{\vec{n}}(y) \, = A^0 + \tilde{A}, \tag{B.60}$$

the Weyl transformation (B.49) only acts on A^0 since σ is constant in the internal coordinates (while of course it can depend on the four-dimensional coordinates). Suppose A and Φ are constant over the internal space (so $\tilde{A}$=0). A good choice of Y in (B.51) would be

$$Y = e^{A-\Phi/3} M_P \,, \tag{B.61}$$

where the M_P is introduced for convenience as it allows $\mathcal{K}$ to be dimensionless upon imposing the Einstein gauge (B.53). With this choice we find for the superpotential and the Kähler potential

$$\mathcal{K} = -\ln i \int_M \langle \Psi_2, \bar{\Psi}_2 \rangle - 2 \ln i \int_M \langle t, \bar{t} \rangle + 3 \ln(8\kappa_{10}^2 M_P^2) \,, \tag{B.62a}$$

$$\mathcal{W}_{\mathrm{E}} = \frac{-i}{4\kappa_{10}^2} \int_M \langle \Psi_2, F + i \, \mathrm{d}_H (\mathrm{Re}\,\mathcal{T}) \rangle \;. \tag{B.62b}$$

Note that another choice $Y' = fY$ would amount to a Kähler transformation

$$\mathcal{W}'_{\mathrm{E}} = f^{-3} \mathcal{W}_{\mathrm{E}} \,, \qquad \mathcal{K}' = \mathcal{K} + 3 \ln f + 3 \ln f^* \,. \tag{B.63}$$

Using the expansion in background and fluctuations of (4.18) and (4.20) we can rewrite the superpotential as

$$\mathcal{W}_{\mathrm{E}} = \frac{-i}{4\kappa_{10}^2} \int_M \langle \Psi_2 e^{\delta B}, \hat{F} + i \, \mathrm{d}_{\hat{H}} (e^{\delta B} \mathrm{Re}\,\mathcal{T} - i\delta C) \rangle \;, \tag{B.64}$$

where we used property (B.5). This shows how the fields organize in complex multiplets $\Psi_2 e^{\delta B}$ and $\mathrm{Re}\,\mathcal{T} - i\delta C$, which will be clearer in concrete examples.

Specializing to the SU(3) case with pure spinors (B.16) and the identification (B.6) for type IIA, the superpotential takes the form

$$\mathcal{W}_{\mathrm{E}} = \frac{-i e^{-i\theta}}{4\kappa_{10}^2} \int_M \langle e^{i(J - i\delta B)}, \hat{F} - i \mathrm{d}_{\hat{H}} \left(e^{\delta B} e^{-\Phi} \mathrm{Im}\,\Omega + i \delta C_3 \right) \rangle \,, \tag{B.65}$$

and the Kähler potential is given by

$$\mathcal{K} = -\ln \int_M \frac{4}{3} J^3 - 2 \ln \int_M 2\, e^{-\Phi} \mathrm{Im}\,\Omega \wedge e^{-\Phi} \mathrm{Re}\,\Omega + 3 \ln(8\kappa_{10}^2 M_P^2) \,, \tag{B.66}$$

where $e^{-\Phi}\mathrm{Re}\,\Omega$ should be seen as a function of $e^{-\Phi}\mathrm{Im}\,\Omega$.

Appendix C

Ten-dimensional geometries

In this appendix we introduce the ten-dimensional geometries that we want to use as the internal 6d compact manifolds with SU(3)-structure. These are so-called nilmanifolds and cosets spaces and they are totally characterized by the structure constants of the associated Lie algebra. We do not want to go into the details here but just want to collect the results appearing in the literature that we will need in this thesis. The key feature of such manifolds is that they allow for *left-invariant* (globally defined) one-forms and that the exterior derivative of those one-forms, when expanded in two-forms, only has constant coefficients. For later use we will also compute the scalar curvature of such spaces. Furthermore we need to make sure that we can make the non-compact examples compact by moding out a discrete symmetry. We will start with reviewing group-manifolds before we discuss nilmanifolds and coset spaces. Good reviews are given in [103] while an introduction into the topic can be found in [104].

C.1 Group-manifolds

A Lie group G is a manifold and group at the same time. Let y^m, $m = 1, \ldots, \dim(G)$, be local coordinates on G and let $L(y)$ be an element of G. The left action is defined as a map from G to G:

$$gL(y) = L(y') \,, \qquad (g \in G) \tag{C.1}$$

It induces a map between the tangent spaces at different points. Vector fields invariant under this map are called *left-invariant* and they define the Lie algebra $\mathcal{G}$ of G.

Since any left-invariant vector field is uniquely determined by its value at e, the identity element of G, $\mathcal{G}$ can be identified with $T_e(G)$. If we denote the basis of $T_e(G)$ as T_A with $A = 1 \ldots \dim(G)$ one has

$$[T_A, T_B] = f^C{}_{AB} T_C \,, \tag{C.2}$$

where the ${f^C}_{AB}$ are constants since the left hand side is left-invariant.

The left-invariant one-forms e^A are defined through the Lie-algebra valued one-form

$$E(y) \equiv L^{-1}(y)\mathrm{d}L(y) = e^A(y)T_A \ , \tag{C.3}$$

which we expanded in generators of G. This one-form is left-invariant and by definition it obeys the so called *Maurer-Cartan equations*

$$\mathrm{d}E = -E \wedge E \, . \tag{C.4}$$

Plugging in (C.3) and using (C.2), one gets

$$\mathrm{d}e^A = -\frac{1}{2}{f^A}_{BC}e^B \wedge e^C \, . \tag{C.5}$$

The Jacobi-identity for the structure constants ensures that taking another exterior derivative gives zero. If the Lie group G is non-compact one needs to make sure that one can make it compact by moding out a discrete subgroup Γ yielding $M = G/\Gamma$. We come to that point in the next sections.

So we see that for a Lie group the exterior derivative of the globally defined one-forms involves the structure constants of the Lie algebra. One can also show the other direction. A manifold M with $\dim(M)$ globally defined linear independent one-forms is called *parallelizable*. One can then of course always expand $\mathrm{d}e^i$ in the two-form basis $e^i \wedge e^j$, but not necessarily with constant coefficients. If they are constant, the manifold is called *homogeneous*. Imposing further $\mathrm{d}^2 e^i = 0$ forces the constant coefficients to satisfy the Jacobi identities, thus we can associate a Lie group G to them. If it is non-compact this means $M = G/\Gamma$ since we want M to be compact.

One possible metric on group manifolds is the so called *Cartan-Killing metric* defined by

$$\kappa_{AB} = {f^Y}_{AX}{f^X}_{BY} \, , \tag{C.6}$$

which has the property

$${f^C}_{A[B}g_{D]C} = 0 \, . \tag{C.7}$$

The Levi-Civita connection one-form ω^A_B of a metric g is uniquely determined by the two equations

$$0 = \mathrm{d}g_{AB} - {\omega^C}_A g_{CB} - {\omega^C}_B g_{AC} \, , \tag{C.8}$$

$$0 = \mathrm{d}e^A + {\omega^A}_B \wedge e^B \, . \tag{C.9}$$

For a left-invariant metric the second equation becomes

$$\omega_{AB} \equiv g_{AC}{\omega^C}_B = -\omega_{BA} \tag{C.10}$$

Using (C.2) in (C.8), one can show that the solution of (C.8) and (C.10) is given by

$$\omega^A{}_B = g^{AC}\left(\frac{1}{2} f^E{}_{CB} g_{ED} + f^E{}_{D[B} g_{C]E}\right) e^D \,. \tag{C.11}$$

Now it is straight forward to compute the curvature two-form

$$R^A{}_B = \frac{1}{2} R^A{}_{BCD} e^C \wedge e^D \equiv \mathrm{d}\omega^A{}_B + \omega^A{}_C \wedge \omega^C{}_B \,. \tag{C.12}$$

Using (C.2) and contracting indices we find for the Ricci scalar

$$R = -\frac{1}{2} g^{AB} f^C{}_{DA} f^D{}_{CB} - \frac{1}{4} g_{AB} g^{CD} g^{EF} f^A{}_{CE} f^B{}_{DF} \,, \tag{C.13}$$

where the first term is the contraction of the killing metric.

In the next two sections we will introduce the explicit Lie algebras that we want to study. Levi's theorem tells us that any Lie-algebra A can be written as the semi-direct sum of a *solvable* and a *semisimple* Lie algebra. We will look at examples which fall into the two extreme classes, namely either A is solvable or A is semisimple. Solvable Lie algebras are defined by a recursive series. If we set $A^0 = A$ and define the series $A^s \equiv [A^{s-1}, A^{s-1}]$, then A is called solvable if this series becomes zero after a finite number of steps. A particular subclass of solvable Lie algebras is given by *nilpotent* Lie algebras. They are defined in a similar way by demanding that the series $A^s \equiv [A^{s-1}, A]$ becomes zero after a finite number of steps. A special property of nilpotent algebras is that the Killing form (Killing metric) is identically zero. As explained in [72] they admit a generalized complex structure, which makes them good candidates to look for type II supergravity solutions. For Semisimple Lie algebras on the other hand, the Killing form is non-degenerate. There already exist some examples of type IIA solutions in the literature [58, 76], which gives hope that there might be more.

C.2 Nilmanifolds

Let us start with the nilpotent algebras. For these manifolds the construction of the leftinvariant one-forms and the action of the exterior derivative works exactly as in the last section. The question that arises is whether one can make them compact. If yes, the associated manifold $M = G/\Gamma$ is called a *nilmanifold*. Let's take as an example the Heisenberg algebra, which is nilpotent. The only non-vanishing structure constant is $f^3{}_{12}$ leading to

$$\mathrm{d}e^1 = 0\,; \qquad \mathrm{d}e^2 = 0\,; \qquad \mathrm{d}e^3 = N e^1 \wedge e^2 \,. \tag{C.14}$$

A compact notation for that is $(0, 0, N12)$. Let us choose a gauge where

$$e^1 = \mathrm{d}x^1\,; \qquad e^2 = \mathrm{d}x^2\,; \qquad \mathrm{d}e^3 = \mathrm{d}x^3 + N x^1 e^2 \,. \tag{C.15}$$

We can compactify this by making the identification $(x^1, x^2, x^3) \simeq (x^1, x^2 + a, x^3) \simeq (x^1, x^2, x^3 + b)$ with a,b integer but we can not do the same for x^1 because e^3 would not be single-valued. For that we need to *twist* the identification by $(x^1, x^2, x^3) \simeq (x^1 + c, x^2, x^3 - Ncx^2)$. The resulting nilmanifold G/Γ is an S^1 fibration over T^2, which is topologically distinct from T^3. More loosely, nilmanifolds are often called *twisted tori* and the structure constants are referred to as *metric fluxes*. A general nilmanifold is always an iteration of torus fibrations.

It is possible to perform a systematic scan for solutions on nilmanifolds because the nilpotent Lie algebras up to dimension seven have been classified and six is the highest dimension where there are finitely many. There are 34 isomorphism classes of simply-connected 6d nilpotent Lie groups. A list of them can be found in [72]. The classification, however, does not take into account whether it is possible to produce a compact manifold by modding out a discrete subgroup Γ. We only want to make sure that one Γ exists but do not care about whether there are more. By looking at (C.14), we see that already in three dimensions there are infinitely many nilmanifolds. However, they are all isomorphic via a rescaling of e^3. The information lost in this rescaling is which subgroup is being modded out. This choice does not matter for us because we only work with left-invariant forms, which have to have constant coefficients. It turns out that the necessary condition is $f^A{}_{BA} = 0$. This condition becomes sufficient for structure constants that are rational in some basis. It is easy to see that this condition is necessary. If $f^A{}_{BA} \neq 0$, the top form $\mathrm{dvol} \equiv e^1 \wedge \ldots \wedge e^6$ would be exact, but a compact manifold needs a top-form non-trivial in cohomology. Indeed, if $\alpha \equiv \epsilon_{A^1 \ldots A^n} \alpha^{A^1} e^{A^2} \wedge \ldots \wedge e^{A^n}$ with α^{A^1} constant, one has $\mathrm{d}\alpha = (f^A{}_{BA}\alpha^A)\mathrm{d}vol$ showing that the volume would be exact. This argument leaves open the possibility that $\mathrm{dvol} = fe^1 \wedge \ldots \wedge e^6$ with some function f. This would not be left-invariant and in general it is not clear that computing the cohomology using left-invariant forms gives the same as using all forms. However, it turns out that this is true for nilmanifolds after taking the quotient. This shows that $f^A{}_{BA} = 0$ is a necessary condition and a nice feature of nilmanifolds is that it is automatically satisfied.

The Ricci scalar (C.13) simplifies for nilmanifolds due to the vanishing of the Killing-form to

$$R = -\frac{1}{4} g_{AB} g^{CD} g^{EF} f^A{}_{CE} f^B{}_{DF} \,, \tag{C.16}$$

which is never positive.

C.3 Coset spaces

Let us now discuss the semisimple Lie algebras. To do so we will have to generalize the above definitions to the so called *coset spaces* $M = G/H$, where H is a subgroup of g which we divide out. We will only consider compact Lie groups.

Let y^m, $m = 1, \ldots, \dim(G) - \dim(H)$, be local coordinates on G/H and let $L(y)$ be a coset representative. The left action of G on G/H is now defined as:

$$gL(y) = L(y')h\,, \qquad (g \in G, h \in H)\,, \tag{C.17}$$

because by acting with g from the left on a coset representative $L(y)$, we will in general get an element belonging to a different coset whose representative we call $L(y')$. To bring $L(y')$ to that element we need an extra h transformation. It induces a map between the tangent spaces at different points. Vector fields invariant under this map are called left-invariant and they define the Lie algebra of G/H.

Let $\mathcal{H}_a$ be a basis of generators of the algebra $\mathcal{H}$, and let $\mathcal{K}_i$ be a basis of the complement $\mathcal{K}$ of $\mathcal{H}$ inside $\mathcal{G}$, i.e. $a = 1, \ldots, \dim(H)$ and $i = 1, \ldots, \dim(G) - \dim(H)$. We define the structure constants as follows:

$$\begin{aligned} [\mathcal{H}_a, \mathcal{H}_b] &= f^c{}_{ab}\mathcal{H}_c\,, \\ [\mathcal{H}_a, \mathcal{K}_i] &= f^j{}_{ai}\mathcal{K}_j\,, \\ [\mathcal{K}_i, \mathcal{K}_j] &= f^k{}_{ij}\mathcal{K}_k + f^a{}_{ij}\mathcal{H}_a\,, \end{aligned} \tag{C.18}$$

where we have used that for compact H one can always find a basis of generators $\{\mathcal{K}_i\}$ such that the structure constants $f^b{}_{ai}$ vanish [103]. In other words: $[\mathcal{H}, \mathcal{K}] \subset \mathcal{K}$, and in this case the coset G/H is called *reductive*.

Let y^m, $m = 1, \ldots, \dim(G) - \dim(H)$, be local coordinates on G/H and let $L(y)$ be a coset representative. The decomposition of the Lie-algebra valued one-form E is not left-invariant anymore, and it can be decomposed as

$$E(y) \equiv L^{-1}(y)dL(y) = e^i(y)\mathcal{K}_i + \omega^a(y)\mathcal{H}_a\,. \tag{C.19}$$

It still solves the Maurer Cartan equation (C.4) and by plugging its expansion into it and using (C.18), one arrives at

$$\mathrm{d}e^i = -\frac{1}{2}f^i{}_{jk}e^j \wedge e^k - f^i{}_{aj}\omega^a \wedge e^j\,, \tag{C.20}$$

$$\mathrm{d}\omega^a = -\frac{1}{2}f^a{}_{ij}e^i \wedge e^j - \frac{1}{2}f^a{}_{bc}\omega^b \wedge \omega^c\,. \tag{C.21}$$

Furthermore plugging (C.19) into (C.17) yields

$$e^i(y')\mathcal{K}_i + \omega^a(y')\mathcal{H}_a = e^i(y)h\mathcal{K}_ih^{-1} + \omega^a(y)h\mathcal{H}_ah^{-1} + h\mathrm{d}h^{-1}\,. \tag{C.22}$$

Since G/H is compact we know that $h\mathcal{K}h^{-1} \subset \mathcal{K}$ and we can define

$$D_j{}^i(h^{-1})\mathcal{K}_i \equiv h\mathcal{K}_Ih^{-1}\,. \tag{C.23}$$

This gives the transformation rule for the coframe e^i on G/H:

$$e^i(y') = e^j(y)D_j{}^i(h^{-1})\,. \tag{C.24}$$

We are interested in expanding in forms that are left-invariant under the action of G on G/H. Any covariant form B on G/H can be written as

$$B = \frac{1}{n!} B_{i^1 \dots i^n} e^{i^1} \wedge \dots \wedge e^{i^n} \tag{C.25}$$

and by using (C.24) left invariance of B then amounts to

$$B_{i^1 \dots i^n} = B_{j^1 \dots j^n} D_{i^1}{}^{j^1}(h) \dots D_{i^n}{}^{j^n}(h) \tag{C.26}$$

due to the action of H and

$$B_{i^1 \dots i^n} = \text{constant} \tag{C.27}$$

due to homogeneity. The infinitesimal version of (C.26) is

$$f^j{}_{a[i_1} B_{i_2 \dots i_p]j} = 0 \,, \tag{C.28}$$

where we have used the definition (C.23) and (C.18). If one now takes the exterior derivative $\mathrm{d}B$ this equation ensures that the part coming from the second term in (C.20) drops out and we get again a left-invariant form. Actually, one can reverse this procedure to obtain all the left-invariant forms on a coset space. One just computes for all possible forms the exterior derivative using (C.20) and keeps only those for which the second term drops out. This gives all left-invariant forms.

Similarly, a metric $g = g_{ij} e^i \otimes e^j$ is left-invariant if and only if its components g_{ij} are constants and

$$f^k{}_{a(i} g_{j)k} = 0 \,. \tag{C.29}$$

Again we compute the Levi-Civita connection one-form ω^i_j from

$$0 = \mathrm{d}g_{ij} - \omega^k{}_i g_{kj} - \omega^k{}_j g_{ik} \,, \tag{C.30}$$

$$0 = \mathrm{d}e^i + \omega^i{}_j \wedge e^j \,. \tag{C.31}$$

Choosing e^i to be the coframe given in (C.19) the second equation becomes for a left-invariant metric

$$\omega_{ij} \equiv g_{ik} \omega^k{}_j = -\omega_{ji} \tag{C.32}$$

Using (C.20) in (C.30) this time the solution of (C.30) and (C.32) is given by

$$\omega^i{}_j = f^i{}_{aj} \omega^a + g^{im} \left(\frac{1}{2} f^l{}_{mj} g_{lk} + f^l{}_{k[j} g_{m]l} \right) e^k \,, \tag{C.33}$$

which now has an extra term compared to (C.11). The curvature two-form is

$$R^i{}_j = \frac{1}{2} R^i{}_{jkl} e^k \wedge e^l \equiv \mathrm{d}\omega^i{}_j + \omega^i{}_k \wedge \omega^k{}_j \,, \tag{C.34}$$

and using (C.20) and contracting indices we find for the Ricci scalar:

$$R = -g^{ij} f^k{}_{ai} f^a{}_{kj} - \frac{1}{2} g^{ij} f^k{}_{li} f^l{}_{kj} - \frac{1}{4} g_{ij} g^{kl} g^{mn} f^i{}_{km} f^j{}_{ln} \,, \tag{C.35}$$

which also has an extra term compared to (C.13)

As was explained in [59], in order for a coset space G/H to allow for an SU(3)-structure, the group H should be contained in SU(3). The list of such six-dimensional cosets and the corresponding structure constants were given in and are summarized in table C.1. Out of these only five lead to $\mathcal{N} = 1$ AdS_4 solutions [59], as we have indicated in the table. We also indicated whether the coset admits an SU(3)-structure at all, which would be the first requirement.

G	H	SU(3)-structure	$\mathcal{N} = 1$ AdS_4
G_2	SU(3)	$\surd$	$\surd$
$\mathrm{SU}(3)\times\mathrm{SU}(2)^2$	SU(3)		
Sp(2)	S(U(2)×U(1))	$\surd$	$\surd$
$\mathrm{SU}(3)\times\mathrm{U}(1)^2$	S(U(2)×U(1))		
$\mathrm{SU}(2)^3\times\mathrm{U}(1)$	S(U(2)×U(1))		
SU(3)	U(1)×U(1)	$\surd$	$\surd$
$\mathrm{SU}(2)^2\times\mathrm{U}(1)^2$	U(1)×U(1)		
SU(3)×U(1)	SU(2)	$\surd$	$\surd$
$\mathrm{SU}(2)^3$	SU(2)		
$\mathrm{SU}(2)^2\times\mathrm{U}(1)$	U(1)	$\surd$	
$\mathrm{SU}(2)^2$	1	$\surd$	$\surd$
$\mathrm{SU}(2)\times\mathrm{U}(1)^3$	1	$\surd$	

Table C.1: All six-dimensional manifolds of the type $M = G/H$, where H is a subgroup of SU(3) and G and H are both products of semisimple and U(1)-groups. To be precise this list should be completed with the cosets obtained by replacing any number of SU(2) factors in G by $\mathrm{U}(1)^3$.

Appendix D

A note on integrating out $\mathrm{d}c_3^{(3)}$

Both in the torus and in the Iwasawa analysis we integrated out $dc_3^{(3)}$. In general one gets from the part of the equation of motion of F_4 with $(1,6)$ index structure

$$
\begin{aligned}
e^{\frac{1}{2}\Phi} \star_4 \mathrm{d}c_3^{(3)} \wedge \mathrm{vol}_6 = &+ \frac{1}{2} e^{\frac{1}{2}\Phi} f \left(\delta g^{\mu}{}_{\mu} - \delta g^{m}{}_{m} - \delta\Phi\right) \wedge \mathrm{vol}_6 \\
&+ c^{(3)i} \hat{H} \wedge Y_i^{(3+)} - b^i \wedge Y_i^{(2-)} \wedge \hat{F}_4 + \delta f \,,
\end{aligned} \tag{D.1}
$$

where the integration constant δf corresponds to a variation of the background flux f, which we put to zero.

This describes the external part of F_4, which equivalently can be described by the internal part of F_6. Indeed, from varying

$$
F_6 = e^{\frac{1}{2}\Phi} \star F_4 \,, \tag{D.2}
$$

which we got from (A.1), follows

$$
\delta F_{6,\mathrm{int}} = \frac{1}{2} e^{\frac{1}{2}\Phi} f \left(\delta g^{\mu}{}_{\mu} - \delta g^{m}{}_{m} - \delta\Phi\right) \wedge \mathrm{vol}_6 + e^{\frac{1}{2}\Phi} \star \mathrm{d}c_3^{(3)} \,, \tag{D.3}
$$

so that plugging in (D.1) we find

$$
\delta F_{6,\mathrm{int}} = c^{(3)i} \hat{H} \wedge Y_i^{(3+)} - b^i \wedge Y_i^{(2-)} \wedge \hat{F}_4 \,. \tag{D.4}
$$

This corresponds to the part of δF_6 in (4.19) that is first order in the fluctuations. We conclude that instead of introducing $\mathrm{d}c_3^{(3)}$, the external part of F_4, we might as well have worked with the internal part of F_6. That is exactly what we will do in the superpotential analysis.

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